1   **A Beta Way: A Tutorial For Using Beta Regression in Psychological Research**

2   Jason Geller[1], Robert Kubinec[2], Chelsea M. Parlett Pelleriti[3], and Matti Vuorre[4]

3   [1]Department of Psychology and Neuroscience, Boston College

4   [2]University of South Carolina

5   [3]Canva

6   [4]Tilburg University

Author Note

Jason Geller ⬤ https://orcid.org/0000-0002-7459-4505

Robert Kubinec ⬤ https://orcid.org/0000-0001-6655-4119

Chelsea M. Parlett Pelleriti ⬤ https://orcid.org/0000-0001-9301-1398

Matti Vuorre ⬤ https://orcid.org/0000-0001-5052-066X

Correspondence concerning this article should be addressed to Jason Geller, Department of Psychology and Neuroscience, Boston College, McGuinn 300z, Chestnut Hill, MA 2467, USA, Email: drjasongeller@gmail.com

Abstract

Rates, percentages, and proportions are common outcomes in psychology and the social sciences. These outcomes are often analyzed using models that assume normality, but this practice overlooks important features of the data, such as their natural bounds at 0 and 1. As a result, estimates can become distorted. In contrast, treating such outcomes as beta-distributed respects these limits and can yield more accurate estimates. Despite these advantages, the use of beta models in applied research remains limited. Our goal is to provide researchers with practical guidance for adopting beta regression models, illustrated with an example drawn from the psychological literature. We begin by introducing the beta distribution and beta regression, emphasizing key components and assumptions. Next, using data from a learning and memory study, we demonstrate how to fit a beta regression model in R with the Bayesian package {brms} and how to interpret results on the response scale. We also discuss model extensions, including zero-inflated, zero- and one-inflated, and ordered beta models. Basic familiarity with regression modeling and R is assumed. To promote wider adoption of these methods, we provide detailed code and materials at https://doi.org/10.5281/zenodo.15830595.

Keywords: beta regression, beta distribution, R tutorial, psychology, learning and memory

A Beta Way: A Tutorial For Using Beta Regression in Psychological Research

> ⚠️ Preprint
>
> This manuscript is currently **under review** and has not been peer-reviewed. Content is **subject to change**. Please feel free to provide feedback!

Introduction

Many outcomes in psychological research are naturally expressed as proportions or percentages. These include the proportion of correct responses on a test (e.g., Kornell and Bjork (2008)), the proportion of time a participant fixates on a stimulus in an eye-tracking task (e.g., James et al., 2025), or the proportion of respondents endorsing a particular belief (e.g., Costello et al., 2024). A central question for studies involving proportional data is how to analyze these outcomes in a statistically appropriate and interpretable way.

Researchers frequently default to linear models that assume Gaussian (normal) distributions, such as t-tests, ANOVAs, and linear regression. However, these models make strong assumptions: (1) residuals are normally distributed, (2) the outcome is unbounded (from $-\infty$ to ∞), and (3) variance is constant across the range of the data. These assumptions are rarely satisfied in practice (Sladekova & Field, 2024), and they are especially ill-suited for proportional outcomes, which are bounded between 0 and 1 and often exhibit heteroscedasticity—non-constant variance, particularly near the boundaries (Ferrari & Cribari-Neto, 2004; Paolino, 2001; Smithson & Verkuilen, 2006). Violating these assumptions can lead to biased estimates and spurious inferences, especially when many observations cluster near 0 or 1.

In some cases, a generalized linear model (GLM) can relax the assumption of normality. For example, binomial and Bernoulli models—often referred to as logistic regression when used with a logit link—are appropriate when the outcome is binary (0/1) or represents the number of successes out of a fixed number of trials. These models, however, rely on data that arise from discrete trial structures. They tend to perform poorly when the observed proportions are truly continuous or when the data show extra variability (overdispersion), particularly when many

24   values occur near the boundaries of the scale (0 and 1).

25   The challenges of analyzing proportional data are not new (see Bartlett, 1936).

26   Fortunately, several existing approaches address the limitations of commonly used models. One

27   such approach is beta regression, an extension of the generalized linear model that employs the

28   beta distribution (Ferrari & Cribari-Neto, 2004; Paolino, 2001). Beta regression offers a flexible

29   and robust solution for modeling proportional data directly by accounting for boundary effects and

30   over-dispersion, making it a valuable alternative to traditional binomial models. This approach is

31   particularly well-suited for psychological research because it can handle both the bounded nature

32   of proportional data and the non-constant variance often encountered in these datasets (Sladekova

33   & Field, 2024). In addition, the direct modeling of proportions allows comparability across tasks

34   and scales, and can be particularly valuable when only the proportional data is available, as is

35   often the case with secondary data that lack item-level structure or point values. While usage of

36   these models has faced obstacles due to theoretical and computational limitations, as we argue in

37   this paper, beta regression and its extensions now provide an accessible and more robust method

38   to traditional linear modeling.

39   While in this paper we will focus on proportional-responses that lie between 0 and 1–it is

40   important to note that our analysis applies to any bounded continuous scale. Any bounded scale

41   can be mapped to lie within 0 and 1 without resulting in a loss of information as the

42   transformation is linear.[1] Consequently, a scale that has natural end points of -1,234 and

43   +8,451–or any other end points on the real number line short of infinity—can be modeled using

44   the approaches we describe in this paper.

45   **A Beta Way Is Possible**

46   With the widespread availability of open-source software such as R (R Core Team, 2024)

47   and its extensive ecosystem of user-developed packages, advanced models like beta regression

48   have become increasingly accessible to applied researchers. Yet, their adoption in psychology

---

[1] Specifically, for any continuous bounded variable x, we can rescale this variable to lie within 0 and 1 by using the formula $x' = \frac{x - \min(x)}{\max(x) - \min(x)}$ where $0 \leq x' \leq 1$.

49 remains relatively limited. One contributing factor may be the lack of domain-specific examples

50 that demonstrate how these models address common challenges in psychological data. Although

51 recent years have seen a growing interest in beta regression, and a number of useful tutorials are

52 available (Bendixen & Purzycki, 2023; Coretta & Bürkner, 2025; Heiss, 2021; e.g., Smithson &

53 Verkuilen, 2006; Vuorre, 2019), these resources often either focus on basic implementation or

54 briefly mention extensions without detailing how they can be applied to psychologically relevant

55 research questions.

56 The present tutorial aims to help bridge this gap by providing a comprehensive, applied

57 introduction to beta regression and several of its extensions. In addition to the standard beta

58 model, we walk through zero-inflated, zero-one-inflated, and ordered beta regression. These

59 models are particularly useful for researchers working with proportion outcomes that include

60 boundary values (e.g., exact 0s or 1s) or responses with an inherent ordinal structure. Our goal is

61 to offer practical guidance that enables psychological researchers to implement, interpret, and

62 report these models in ways that directly support their empirical questions.

63 Beyond model specification, we place strong emphasis on interpreting results on the

64 response scale–that is, in terms of probabilities and proportions–rather than relying on often

65 difficult to interpret parameters. This focus makes the models more accessible and meaningful for

66 psychological applications, where effects are often easier to communicate when framed on the

67 original scale of the outcome (e.g., changes in recall accuracy or task performance). Throughout,

68 we provide reproducible code and annotated examples to help readers implement and interpret

69 these models in their own work.

70 We begin the tutorial with a non-technical overview of the beta distribution and its core

71 parameters. We then walk through the process of estimating beta regression models using the R

72 package {brms} (Bürkner, 2017), illustrating each step with applied examples. To guide

73 interpretation, we emphasize coefficients, predicted probabilities, and marginal effects calculated

74 using the {marginaleffects} package (Arel-Bundock et al., 2024). We also introduce several useful

75 extensions–zero-inflated (ZIB), zero-one-inflated (ZOIB), and ordered beta regression–that enable

researchers to model outcomes that include boundary values. Finally, all code and materials used

in this tutorial are fully reproducible and available via our GitHub repository:

https://github.com/jgeller112/beta_regression_tutorial and on Zenodo

(https://doi.org/10.5281/zenodo.15830595) [2].

Beta Distribution

Proportional data pose some challenges for standard modeling approaches: The data are

bounded between 0 and 1 and often exhibit non-constant variance (heteroscedasticity) (Ferrari &

Cribari-Neto, 2004; Paolino, 2001). Common distributions used within the generalized linear

model frameworks often fail to capture these properties adequately, which can necessitate

alternative modeling strategies.

While we do not have time to delve fully into its derivation, the beta distribution is useful

for modeling bounded continuous scales because it is the distribution for the probability of an

event. Given that a probability can take on any value from near 0 (the event will not occur with

certainty) to 1 (the event will occur with certainty), the beta distribution can likewise take on

virtually any value in that bounded interval. As a consequence, the beta distribution is the

maximum entropy distribution for *any* bounded continuous random variable, which means that the

beta distribution can represent the full range of possibilities of such a scale.[3] As a consequence, if

---

[2] In this article, we try to limit code where possible; however, the online version has all the code needed to reproduce

all analyses herein. Furthermore, to promote transparency and reproducibility, the tutorial was written in R version

4.5.1 (R Core Team (2024)) using Quarto (v.1.5.54), an open-source publishing system that allows for dynamic and

static documents. This allows figures, tables, and text to be programmatically included directly in the manuscript,

ensuring that all results are seamlessly integrated into the document. In addition, we use the `rix` (Rodrigues &

Baumann, 2025) R package which harnesses the power of the `nix` (Dolstra & contributors, 2006) ecosystem to to

help with computational reproducibility. Not only does this give us a snapshot of the packages used to create the

current manuscript, but it also takes a snapshot of system dependencies used at run-time. This way reproducers can

easily re-use the exact same environment by installing the `nix` package manager and using the included default.nix

file to set up the right environment. The README file in the GitHub repository contains detailed information on how

to set this up to reproduce the contents of the current manuscript, including a video.

[3] Technically, this maximum entropy condition is satisfied because the beta(1,1) distribution is uniform over its

93 we have a continuous scale with upper and lower bounds–and no other special conditions–the beta

94 distribution will in principle provide a very good approximation of the uncertainty of the scale.

95   Typically, the expected value (or mean) of the response variable is the central estimand

96 scholars want to estimate. A model should specify how this expected value depends on

97 explanatory variables through two main components: a linear predictor, which combines the

98 explanatory variables in a linear form ($a + b_1x_1 + b_2x_2$, etc.), and a link function, which connects

99 the expected value of the response variable to the linear predictor (e.g,

100 $E[Y] = g(a + b_1x_1 + b_2x_2)$). In addition, a random component specifies the distribution of the

101 response variable around its expected value (such as Poisson or binomial distributions, which

102 belong to the exponential family) (Nelder & Wedderburn, 1972). Together, these components

103 provide a flexible framework for modeling data with different distributional properties.

104   The beta distribution is continuous and restricted to values between 0 and 1 (exclusive). Its

105 two parameters–commonly called shape1 (α) and shape2 (β)–govern the distribution's location,

106 skewness, and spread. By adjusting these parameters, the distribution can take many functional

107 forms (e.g., it can be symmetric, skewed, U-shaped, or even approximately uniform; see Figure 1).

108   To illustrate, consider a test question worth seven points. Suppose a participant scores five

109 out of seven. The number of points received (5) can be treated as α, and the number of points

110 missed (2) as β. The resulting beta distribution would be skewed toward higher values, reflecting

111 a high performance (pink line in Figure 1; "beta(5, 2)"). Reversing these values would produce a

112 distribution skewed toward lower values, representing poorer performance (orange line in
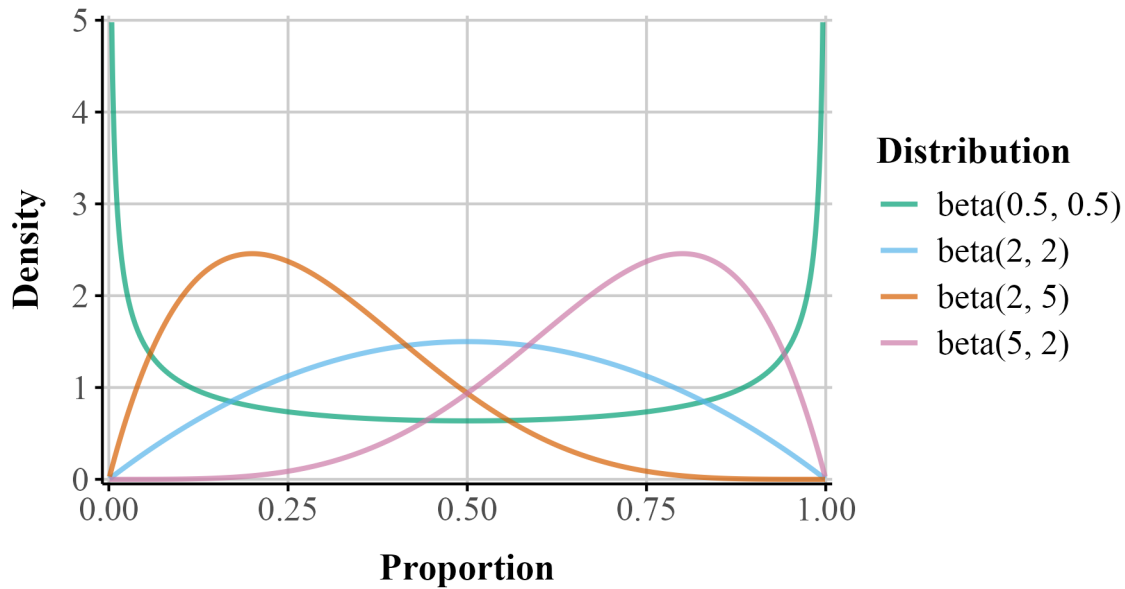
113 Figure 1; "beta(2, 5)").

I Can't Believe It's Not beta

115   While the standard parameterization of the beta distribution uses α and β, a

116 reparameterization to a mean (μ) and precision (ϕ) is more useful for regression models. The

117 mean represents the expected value of the distribution, while the dispersion, which is inversely

118 related to variance, reflects how concentrated the distribution is around the mean, with higher

———————

support. In addition, we assume that the scale has been re-scaled to the [0, 1] interval as we describe above.

Figure 1

beta distributions with different shape1 and shape2 parameters.

values indicating a narrower distribution and lower values indicating a wider one. The

connections between the beta distribution's parameters are shown in Equation 1. Importantly, the

variance depends on the average value of the response because uncertainty intervals need to adjust

for how close the value of the response is to the boundary.

$$
\begin{aligned}
\text{Shape 1:} \quad a &= \mu\phi & \text{Mean:} \quad \mu &= \frac{a}{a+b} \\
\text{Shape 2:} \quad b &= (1-\mu)\phi & \text{Precision:} \quad \phi &= a+b \\
& & \text{Variance:} \quad var &= \frac{\mu \cdot (1-\mu)}{1+\phi}
\end{aligned}
\tag{1}
$$

Thus, beta regression allows modeling both the mean and precision of the outcome

distribution. To ensure that μ stays between 0 and 1, we apply a link function, which allows linear

modeling of the mean on an unbounded scale. A common link-function choice is the logit, but

other functions such as the probit or complementary log-log are possible.

The logit function, $\text{logit}(\mu) = \log\left(\frac{\mu}{1-\mu}\right)$ links the mean to log-odds which are unbounded,

making linear modeling possible. The logit here no longer carries the same literal *odds*

interpretation because there are no corresponding counts of "successes" and "failures." Instead, the logit transform here simply maps the mean of the distribution to the real line. The inverse of the logit, called the logistic function, maps the linear predictor η back to the original scale of the data $\left(\mu = \frac{1}{1+e^{-\eta}}\right)$. The coefficients describe how predictors shift the *average proportion* on the logit scale. Similarly, the strictly positive dispersion parameter is usually modeled through a log link function, ensuring it remains positive.

By accounting for the observations' natural limits and non-constant variance, the beta distribution is useful in psychology where outcomes like performance rates or response scales frequently exhibit these features.

Bayesian Approach to Beta Regression

Beta regression models can be estimated using either frequentist or Bayesian methods. In this paper, we adopt a Bayesian framework because it facilitates the estimation and interpretation of more complex models (Gelman et al., 2013; Johnson et al., 2022; McElreath, 2020). Additionally, the use of Bayesin statistics in psychology has been steadily growing (Pfadt et al., 2025). In principle, frequentist methods like maximum likelihood can be framed as Bayesian models with uninformative priors, and as a result, the modeling perspective we put forward in this paper can apply to either approach. Nonetheless, we note that in non-linear and hierarchical models, frequentist estimation may require additional adjustments such as bootstrapping to obtain proper uncertainty intervals, whereas Bayesian modeling handles these extensions more naturally via exploration of the full joint posterior distribution.[4]

There are several important differences between our Bayesian analysis and the frequentist

---

[4] A common concern is that Bayesian methods are slower than frequentist ones. While this is true in general, modern Bayesian computation engines are efficient, and in explanatory modeling our priority is to specify appropriate estimands rather than to optimize for computation speed. Moreover, we use the {brms} package (Bürkner, 2017), which provides a high-level interface to Stan (Team, 2023) and uses familiar R formula syntax (similar to `lm()`), making advanced Bayesian modeling accessible to researchers with standard regression backgrounds. The package also supports parallelization, which substantially reduces computation time for large datasets.

150   methods readers may be more familiar with—most notably, the absence of *t*- and *p*-values. To

151   estimate models, the {brms} package uses Stan's computational algorithms to draw random

152   samples from the posterior distribution, which represents uncertainty about the model parameters.

153   This posterior is conceptually analogous to a frequentist sampling distribution. By default,

154   Bayesian models run 4 chains with 2,000 iterations each.[5] The first 1,000 iterations per chain are

155   warmup and are discarded. The remaining 1,000 iterations per chain are retained as posterior

156   draws, yielding 4,000 total post-warmup draws across all chains. From these draws, we can

157   compute the posterior mean (analogous to a frequentist point estimate) and the 95% credible

158   interval (Cr.I.), which is often compared to a confidence interval.

159   In addition, an important part of Bayesian analyses is prior specification. Priors encode

160   our assumptions about plausible parameter values before observing the data and allow the model

161   to regularize estimates, especially when data are sparse or parameters are weakly identified. To

162   help bridge the conceptual gap for users more familiar with frequentist models, we begin with the

163   default priors (flat/non-informative, or weakly informative in some cases) provided by {brms}.

164   These priors are intentionally non-informative, and in many applications produce results that

165   closely align with frequentist estimates, while still offering the flexibility and interpretive

166   advantages of a Bayesian framework. We strongly urge readers to consider prior specification

167   strongly in all their work.

168   To ease readers into Bayesian data analysis we provide a metric known as the *probability*

169   *of direction* (pd), which reflects the probability that a parameter is positive or negative. When a

170   uniform prior is used (all values equally likely in the prior), pds of 95%, 97.5%, 99.5%, and

171   99.95% corresponds approximately to two-sided p-values of .10, .05, .01, and .001 (i.e., pd $\approx$ 1 $-$

172   p/2 for symmetric posteriors with weak/flat priors) (see Figure 2 for an illustrative comparison).

173   For directional hypotheses, the pd can be interpreted as roughly equivalent to one minus the

----

[5] The Hamiltonian Monte Carlo sampler employed by Stan, which we also use in this paper, can converge with

significantly fewer iterations, though rapid convergence depends on model complexity, which is why we use a more

conservative standard in this paper.

174 *p*-value (Marsman & Wagenmakers, 2016).

175   For reasons of space, we refer readers unfamiliar to Bayesian data analysis to several

176 existing books on the topic (Gelman et al., 2013; Kruschke, 2015; McElreath, 2020). In addition,

177 we assume readers are familiar with R, but those in need of a refresher should find Wickham et al.

178 (2023) useful.

<div align="center">**Beta Regression Tutorial**</div>
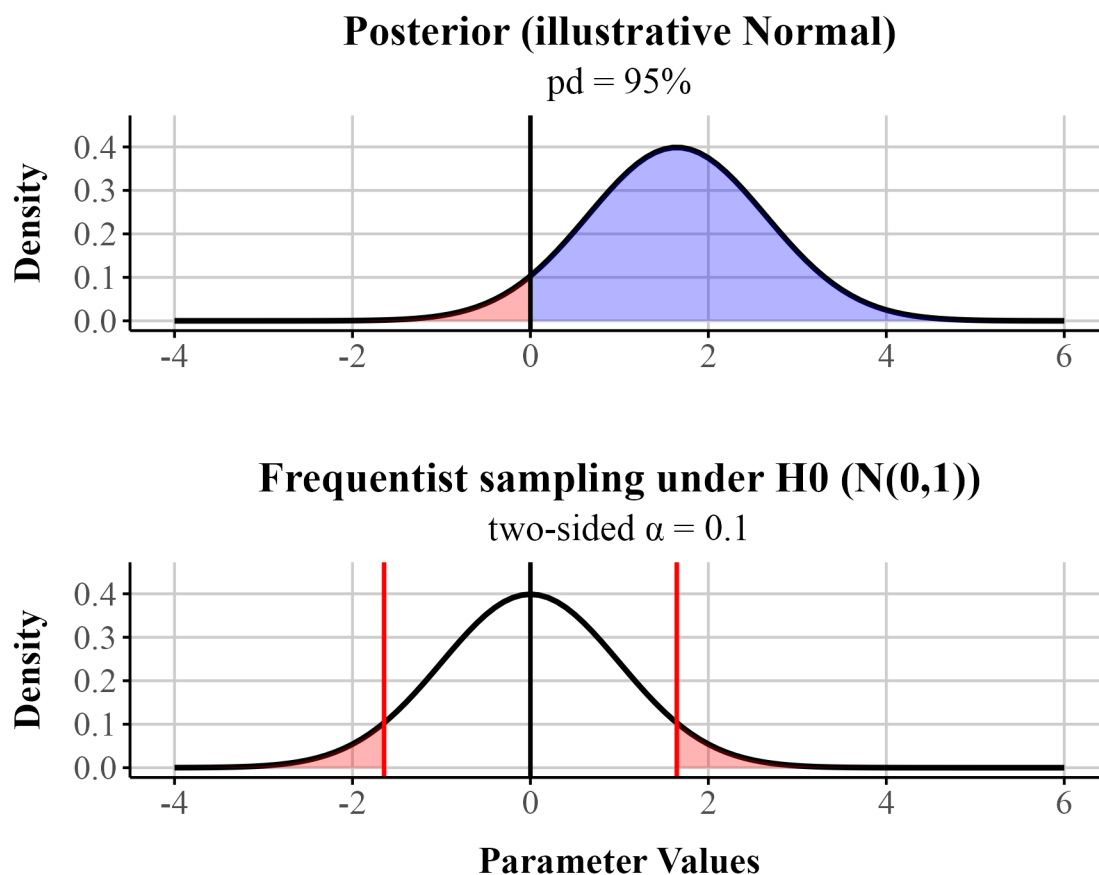
Example Data

181   Throughout this tutorial, we analyze data from a memory experiment examining whether

182 the fluency of an instructor's delivery affects recall performance (Wilford et al., 2020, Experiment

183 1A). Instructor fluency—marked by expressive gestures, dynamic vocal tone, and confident

184 pacing—has been shown to influence students' perceptions of learning, often leading learners to

185 rate fluent instructors more favorably (Carpenter et al., 2013). However, previous research

186 suggests that these impressions do not reliably translate into improved memory performance (e.g.,

187 Carpenter et al., 2013; Toftness et al., 2017; Witherby & Carpenter, 2022). In contrast, Wilford et

188 al. (2020) found that participants actually recalled more information after watching a fluent

189 instructor compared to a disfluent one. This surprising finding makes the dataset a compelling

190 case study for analyzing proportion data, as recall was scored out of 10 possible idea units per

191 video.

192   In Experiment 1A, ninety-six participants watched two short instructional videos, each

193 delivered either fluently or disfluently. Fluent videos featured instructors with smooth delivery

194 and natural pacing, while disfluent videos included hesitations, monotone speech, and awkward

195 pauses. After a distractor task, participants completed a free recall test, writing down as much

196 content as they could remember from each video within a three-minute window. Their recall was

197 then scored for the number of idea units correctly remembered.

198   Our primary outcome variable is the proportion of idea units recalled on the final test,

199 calculated by dividing the number of correct units by 10. We show a sample of these data in

200 Table 1. The dataset can be downloaded from GitHub (Listing 1). Because this is a bounded

Figure 2

A Bayesian posterior distribution (assuming a uniform prior) centered at a point estimate chosen

so that the probability of direction (pd) equals 0.95, and a frequentist sampling distribution (under

the null; centered at 0). In the Bayesian posterior distribution, the blue area represents the pd,

and the red area represents the remaining 1 − pd of the distribution. In the frequentist sampling

distribution, the red tail areas represent the rejection region at α = 0.10. In this example, the

posterior mean lies exactly at the $1 - \frac{\alpha}{2}$ quantile of the null sampling distribution. For symmetric

posteriors with flat priors, the pd is numerically equivalent to the one-sided p-value.

Listing 1 Data needed to run examples

```
# get data here from project folder

fluency_data <- read_csv(here::here("manuscript", "data", "fluency_data.csv"))
```

---

201   continuous variable (i.e., it ranges from 0 to 1), it violates the assumptions of typical linear

202   regression models that assume normally distributed residual errors. Despite this, it remains

203   common in psychological research to analyze proportion data using models that assume

204   normality. In what follows, we reproduce Wilford et al. (2020)'s analysis and then re-analyze the

205   data using beta regression and highlight how it can improve our inferences.

Table 1

Four observations from Wilford et al. (2020). Accuracy refers to the proportion of correctly

recalled idea units.

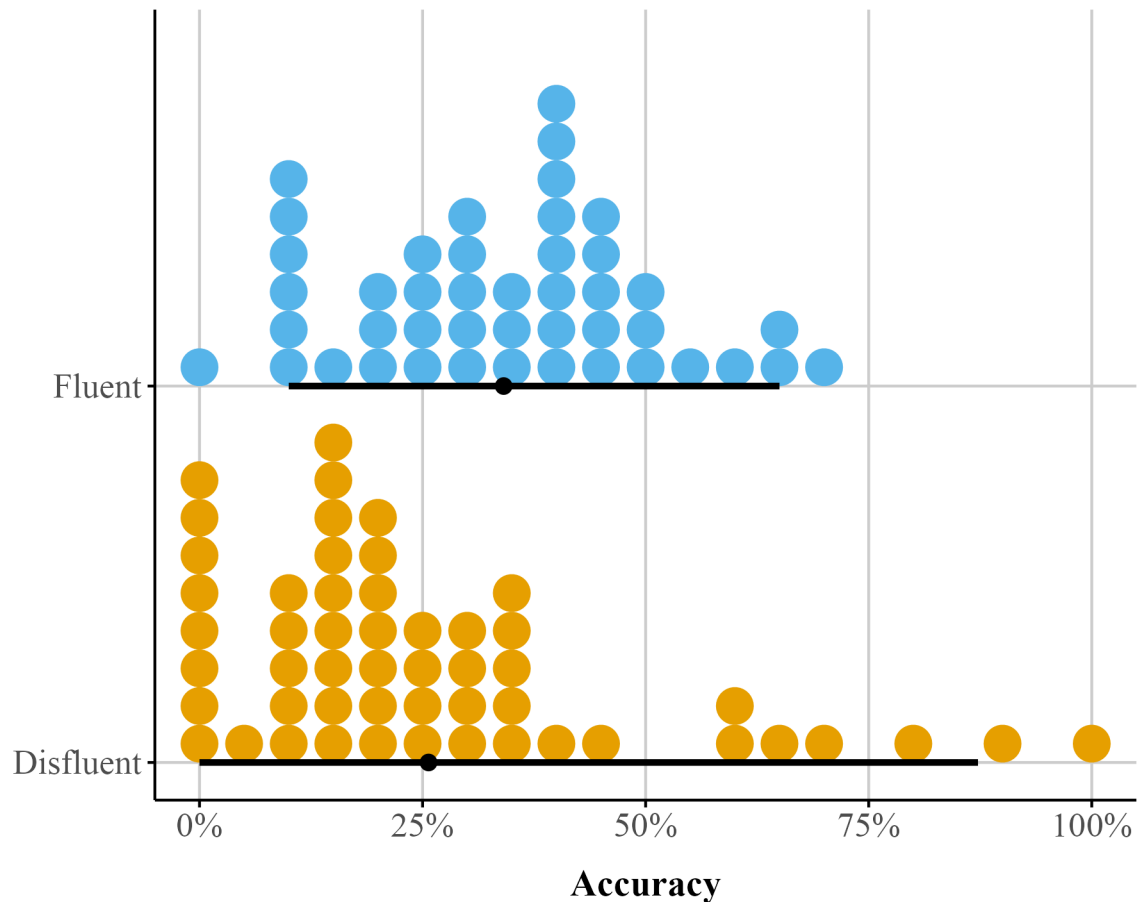| Participant	Fluency	Accuracy
64	Disfluent	0.10
30	Fluent	0.60
12	Fluent	0.10
37	Fluent	0.35

206   **Reanalysis of Wilford et al. Experiment 1A**

207   In their original analysis of Experiment 1A, Wilford et al. (2020) compared memory

208   performance between fluent and disfluent instructor conditions using a traditional

209   independent-samples t-test on mean accuracy for 96 participants. They found that participants

210   who watched the fluent instructor recalled significantly more idea units than those who viewed the

211   disfluent version (see Figure 3).

212   We first replicate this analysis in a regression framework using {brms}. We model final

213   test mean accuracy–the proportion of correctly recalled idea units across the videos–as the

Figure 3

Dot plot depicting accuracy distributions for the Fluent and Disfluent conditions. Each condition shows individual data points and summary statistics (mean and 95% percentile interval) to illustrate variability and central tendency.

214   dependent variable. Our predictor is instructor fluency, with two levels: Fluent and Disfluent. We

215   use treatment (dummy) coding, which is the default in R. This coding scheme sets the first level of

216   a factor (in alphabetical order) as the reference level. In this case, Disfluent is the reference, and

217   the coefficient for Fluent reflects the contrast between fluent and disfluent instructor conditions.

218   *Regression Analysis*

219   We first start by loading the {brms} (Bürkner, 2017) and {cmdstanr} (Gabry et al., 2024)

220   packages (Listing 2). We use the cmdstanr backend for Stan (Team, 2023) because it's faster than

the default used to run models (i.e., `rstan`),[6] though all of these models can also be fit with `brms`

defaults.

Listing 2 Load the {brms} and {cmdstanr} packages

```r
library(brms)

library(cmdstanr)
```

Listing 3 Fitting a gaussian model with brm().

```r
bayes_reg_model <- brm(

  Accuracy ~ Fluency,

  data = fluency_data,

  family = gaussian(),

  file = here::here("manuscript", "models", "model_reg_bayes")

)
```

We fit the model using the `brm()` function from the {brms} package (Listing 3). Although

not shown here, we ran the models using four chains (the default), executed in parallel across four

cores. When the model is run in Listing 3, the model summary output will appear in the R

console. The output from `bayes_reg_model` shows each parameter's posterior summary: The

posterior distribution's mean and standard deviation (analogous to the frequentist standard error)

and its 95% credible interval, which indicate the 95% of the most credible parameter values. In

{brms}, the reported Cr.I is an equal-tailed interval, meaning that the probability mass excluded

from the interval is split equally between the lower and upper tails. Additionally, the output

indicates numerical estimates of the sampling algorithm's performance: Rhat should be close to

one, and the ESS (effective sample size) metrics should be as large as possible given the number

---

[6] In order to use the cmdstanr backend you will need to first install the package (see

https://mc-stan.org/cmdstanr/) and also run `cmdstanr::install_cmdstan()` if you have not done so

already.

of iterations specified (default is 4000). Generally, ESS >= 1000 is recommended (Bürkner,

2017). For the models we present in this paper, convergence is trivial with standard linear models,

though we note that these metrics are still important to pay attention to in case of model misfit.

Our main question of interest is: what is the causal effect of instructor fluency on final test

performance? In order to answer this question, we will have to look at the output summary

produced by Listing 3 (also see Table 8 under Bayesian LM). the `Intercept` refers to the

posterior mean accuracy in the disfluent condition, $M = 0.256$, as fluency was dummy-coded.

The fluency coefficient (`FluencyFluent`) reflects the mean posterior difference in recall accuracy

between the fluent and disfluent conditions: $b = 0.085$. The 95% Cr.I for this estimate spans from

0 to 0.167. These values are shown in the "95% Cr.I" columns of the output. These results closely

mirror the findings reported by Wilford et al. (2020) (Experiment 1A).

```
 Family: gaussian
  Links: mu = identity
Formula: Accuracy ~ Fluency
   Data: fluency_data (Number of observations: 96)


Regression Coefficients:
              Estimate Est.Error l-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
Intercept         0.26      0.03     0.20     0.31 1.00     4190     2492
FluencyFluent     0.08      0.04     0.00     0.17 1.00     4386     3152


Further Distributional Parameters:
      Estimate Est.Error l-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
sigma     0.21      0.02     0.18     0.24 1.00     4259     2944
```

The output also includes the effective sample size (ESS) and R (R-hat) values, both of

which fall within acceptable ranges, indicating good model convergence. Throughout the tutorial,

we focus primarily on posterior mean estimates and their 95% credible intervals. In addition, we

report the pd measure in the main summary table (Table 8), provided by the {bayestestR} package

(Makowski, Ben-Shachar, Chen, et al., 2019; Makowski, Ben-Shachar, & Lüdecke, 2019). This

measure offers an intuitive parallel to *p*-values, which many readers may find familiar. For

example, the fluency effect has a pd of .977, indicating a high probability that the effect is positive

rather than negative.

Importantly, pd does not indicate whether an effect is meaningfully different from a point

value—it only reflects the proportion of the posterior in one direction. To address questions of

practical significance, readers can consider the Region of Practical Equivalence (ROPE) with the

Cr.Is (Kruschke, 2015). Unlike a hypothesis test of a point null (e.g., exactly zero), the ROPE

defines a range of values that are deemed too small to be of substantive importance. As a rule of

thumb (see Kruschke, 2018), if more than 95% of the posterior distribution lies inside the ROPE,

the effect can be considered practically equivalent to that negligible range. If less than 5% lies

inside, the effect can be considered meaningfully different. Intermediate cases are typically

labeled undecided.

The `rope()` function in the {bayestestR} package computes the proportion of the posterior

within the ROPE to facilitate this evaluation. By default, from bayesian models fit via {brms}, the

package determines a ROPE based on the data (roughly reflecting "negligible" effects), but these

defaults should be used cautiously. The choice of ROPE should always be guided by theoretical

considerations, previous research, and the substantive context of the study. In Listing 4, we show

how to compute this using {bayestestR}. Running the function with default settings suggests that

less than 5% lies within the default ROPE (indicating the effect is larger than .02) (see Figure 4).
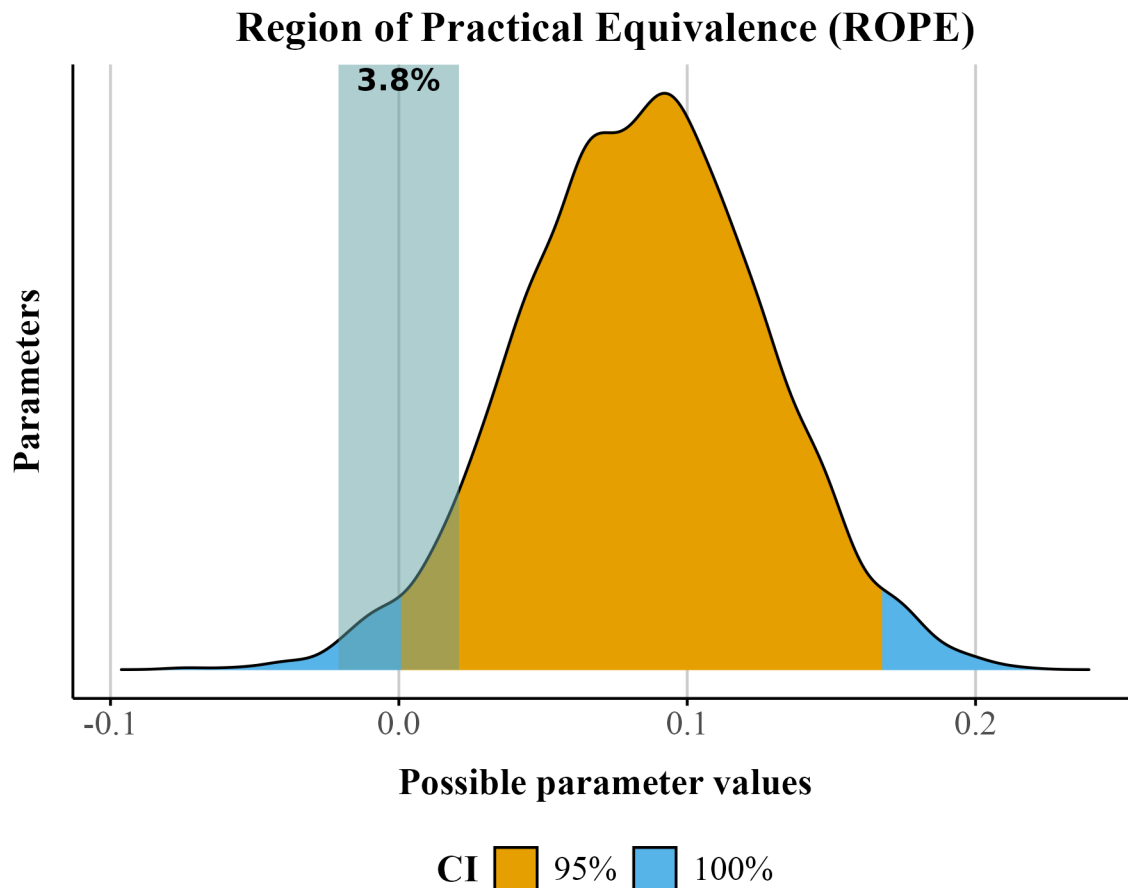

Listing 4 Getting ROPE from `bayes_reg_model` obect using `rope` function from {bayestestR}

```
brms_rope <- bayestestR::rope(bayes_reg_model, ci = .95, ci_method = "ETI")
```

Assumptions. Wilford et al. (2020) observed that instructor fluency impacts actual

learning, using a standard *t*-test on the mean accuracy. But recall this approach assumes normality

of residuals and homoskedasticity. These assumptions are unrealistic when the response values

Figure 4

Posterior distribution for the fluency effect showing the ROPE(shaded area) with 95% credible interval (orange) and 100% credible interval (blue). The percentage indicates the proportion of the posterior within the ROPE.
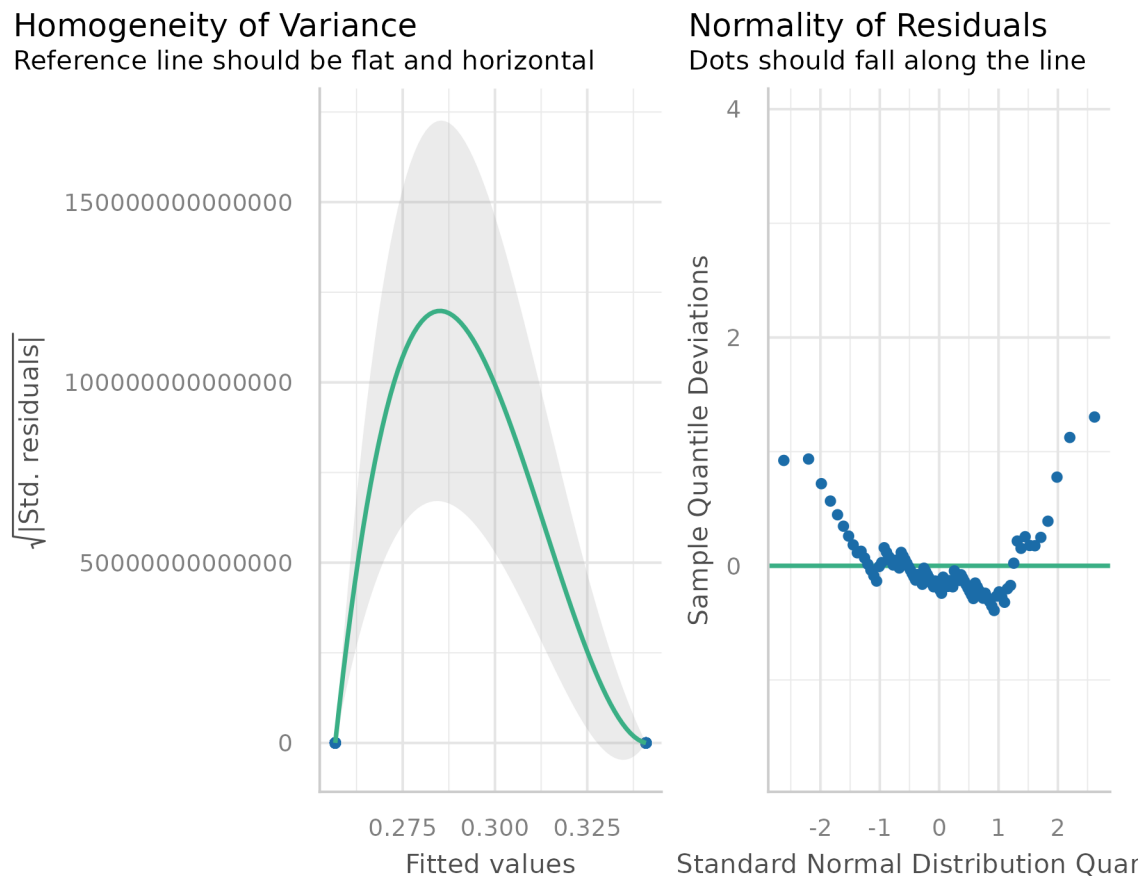
Region of Practical Equivalence (ROPE)

approach the scale boundaries (Sladekova & Field, 2024). Does the data we have meet those
assumptions? We can use the function `check_model()` from {easystats} (Lüdecke et al., 2022) to
check our assumptions easily. The code in Listing 5 produces Figure 5. We can see some issues
with our model. Specifically, there appears to be violations of constant variance across the values
of the scale (homoskedasticity). In plain terms, this type of model mis-specfication means that a
standard OLS model can predict non-sensical values outside the bounds of the scale.

Listing 5 Checking assumptions with the `check_model()` from {easystats} package .

```
check_model(bayes_reg_model, check = c("homogeneity", "qq"))
```

Figure 5

Two assumption checks for our OLS model: Normality (right) and Homoskedasticity (left)

We can also examine how well the data fits the model by performing a posterior predictive
check using the `pp_check()` function from {brms}. A posterior predictive check involves looking
at multiple model-predicted values and plotting them against the observed data. Ideally, the
predicted values (the light blue lines) should show reasonable resemblance with the observed data
(dark blue line). In our example (see Figure 12 (A)) the model-predicted density is slightly too
peaked and narrow compared to the data. In addition, some of the predicted accuracy values are

296   negative.

Distributional Regression - Beta

298   It is important to note that there are several justifiable approaches for addressing the

299   distributional issues observed in the data. For instance, one could analyze median accuracy

300   instead of the mean, use some type of robust estimator for heterogeneity, or apply non-parametric

301   methods to relax some of the model assumptions. Alternatively, we can address these issues

302   directly by fitting distributional models (Kneib et al., 2023; Kruschke, 2013). A key advantage of

303   distributional models is that they are not limited to modeling only the mean or median of the

304   outcome, but can also model parameters such as the variance (or other shape parameters) as

305   functions of predictors. This allows examining how instructor fluency may influence not only

306   average performance, but also the variability in performance across students. If we wanted to keep

307   our mean accuracy variable and continue to use a Guassian model, we could use a distributional

308   approach and model the effect of fluency on σ.

309   Given the outcome variable is proportional, another solution would be to run a beta

310   regression model. Again, we can create the beta regression model in {brms}. In {brms}, we

311   model each parameter independently. Recall from the introduction that in a beta model we model

312   two parameters–μ and ϕ. Again we do this by using the `bf()` function from {brms} (Listing 6).

313   We specify two formulas, one for μ and one for ϕ and store it in the `model_beta_bayes` object

314   below. In the below `bf()` call, we are modeling accuracy as a function of fluency only for the μ

315   parameter. For the ϕ parameter, we are only modeling the intercept value. This is saying

316   dispersion does not change as a function of fluency.

317   To run our beta regression model, we need to exclude 0s and 1s in our data set. If we try to

318   run a model with our data `data_fluency` we get an error: `Error: Family 'beta' requires`

319   `response greater than 0`. This is because the beta distribution only supports observations in

320   the 0 to 1 interval *excluding exact 0s and 1s*. We need make sure there are no 0s and 1s in our

321   dataset.

322   The dataset contains nine 0s and one 1. One approach is to nudge our 0s towards .01 and

323 our 1s to .99, or apply a special formula (Smithson & Verkuilen, 2006) so values fall within the [0,

324 1] interval. We implore readers not to engage in this practice. Kubinec (2022) showed that this

325 practice can result in serious distortion of the outcome as the sample size   grows larger, resulting

326 in ever smaller values that are "nudged". Because the beta distribution is a non-linear model of

327 the outcome, values that are very close to the boundary, such as 0.00001 or 0.99999, will be

328 highly influential outliers. To run this beta model we will remove the 0s and 1s, and later in this

329 article we will show how to jointly model these scale end points with the rest of the data. The

330 model from Listing 6 uses a transformed `data_fluency` object (called `data_beta`) where 0s and

331 1s are removed. When we run this code we should not get an error.

332   ***Model Parameters.*** In Table 8, under the beta regression column, the coefficient with `b_`

333 represents how fluency of instructor influences the μ parameter estimates (which is the mean of

334 the distribution here). These coefficients are linear on the logit-scale, but not on the raw accuracy

335 scale. The intercept term (`b_Intercept`) represents the log odds of the mean on accuracy for the

336 fluent instructor. Log odds that are negative indicate that it is more likely a "success" (like getting

337 the correct answer) will not happen than that it will happen. Similarly, regression coefficients in

338 log odds forms that are negative indicate that an increase in that predictor leads to a decrease in

339 the predicted probability of a "success".

340   The other component we need to pay attention to is the dispersion or precision parameter

341 coefficients labeled as `phi` in Table 8. The dispersion (ϕ) parameter tells us how precise our

342 estimate is. Specifically, ϕ in beta regression tells us about the variability of the response variable

343 around its mean. Specifically, a higher dispersion parameter indicates a narrower distribution,

344 reflecting less variability. Conversely, a lower dispersion parameter suggests a wider distribution,

345 reflecting greater variability. The main difference between a dispersion parameter and the

346 variance is that the dispersion has a different interpretation depending on the value of the

347 outcome, as we show below. The best way to understand dispersion is to examine visual changes

348 in the distribution as the dispersion increases or decreases.

349   Understanding the dispersion parameter helps us gauge the precision of our predictions

Listing 6 Fitting a beta model without 0s and 1s in brm().

```r
# set up model formual

model_beta_bayes <- bf(

  Accuracy ~ Fluency, # fit mu model

  phi ~ 1 # fit phi model

)


# transform 0 to 0.1 and 1 to .99

data_beta <- fluency_data |>

  filter(

    Accuracy != 0,

    Accuracy != 1

  )


beta_brms <- brm(

  model_beta_bayes,

  data = data_beta,

  family = Beta(),

  file = here::here("manuscript","models", "model_beta_bayes_reg_01")

)
```

350 and the consistency of the response variable. In `beta_brms` we only modeled the dispersion of

351 the intercept. When ϕ is not specified, the intercept is modeled by default (see Table 8). It

352 represents the overall dispersion in the outcome across all conditions. Instead, we can model

353 different dispersions across levels of the `Fluency` factor. To do so, we add `Fluency` to the phi

354 model in `bf()`. We model the precision (`phi`) of the `Fluency` factor by using a ~ and adding

355 factors of interest to the right of it (Listing 7).

Listing 7 Fitting beta model with dispersion.

```r
model_beta_bayes_disp <- bf(

  Accuracy ~ Fluency, # Model of the mean

  phi ~ Fluency # Model of the precision

)


beta_brms_dis <- brm(

  model_beta_bayes_disp,

  data = data_beta,

  family = Beta(),

  file = here::here("manuscript","models", "model_beta_bayes_dis_run01")

)
```

Table 8 displays the model summary with the precision parameter labeled as

`phi_Fluency`. Since ϕ is modeled on the log scale, the coefficients represent changes in log-ϕ

rather than φ itself. To see the effect in the original units, we convert the values back by

exponentiating. Thus, the effect of the Fluent condition can be understood by comparing the

exponentiated predicted ϕ in the Fluent condition to that in the baseline condition.

The ϕ parameters are estimated on the log scale. The term $\beta^{(\phi)}_{\text{Intercept}}$ represents the

log-precision for the reference (disfluent) condition. The coefficient $\beta^{(\phi)}_{\text{FluencyFluent}}$ represents the

change in log-precision when moving from the disfluent to the fluent condition.

To obtain precision on the original scale, we exponentiate the linear predictor:

$$\phi_{\text{disfluent}} = \exp\left(\beta^{(\phi)}_{\text{Intercept}}\right), \qquad \phi_{\text{fluent}} = \exp\left(\beta^{(\phi)}_{\text{Intercept}} + \beta^{(\phi)}_{\text{Fluency}}\right).$$

The coefficient $\beta^{(\phi)}_{\text{Fluency}}$ therefore describes a *multiplicative* change in precision.

Specifically,

$$\frac{\phi_{\text{fluent}}}{\phi_{\text{disfluent}}} = \exp\left(\beta^{(\phi)}_{\text{Fluency}}\right).$$

367   Because the 95% credible interval for $\beta^{(\phi)}_{\text{Fluency}}$ does not include zero, we infer that there is

368 a credible difference in precision between the fluent and disfluent conditions.

369   It is important to note that these estimates are not the same as the marginal effects we

370 discussed earlier. Changes in dispersion affect the spread or variability of the response

371 distribution without necessarily altering its mean. This makes dispersion particularly relevant for

372 research questions that focus on features of the distribution beyond the average—such as how

373 concentrated responses are. For instance, high dispersion might indicate that individuals cluster at

374 the extremes (e.g., very high or very low ratings), suggesting clustering in the outcome.

375   A critical assumption of the linear model is homoscedasticity, which means constant

376 variance of the errors. With beta regression model we can include a dispersion parameter for

377 Fluency. Properly accounting for dispersion is crucial because it impacts the precision of our mean

378 estimates and, consequently, the substantive inferences we might make about the coefficients. The

379 inclusion of dispersion in the model increased the uncertainty of the μ coefficient (see Figure 6).

380 This highlights the potential utility of an approach like beta regression over a traditional approach

381 as beta regression can explicitly model dispersion and address issues of heteroscedasticity.

382   While it is advisable to model precision, if there is uncertainty about the best model, a

383 relatively agnostic approach would be to compare models, for example with leave one out (loo)

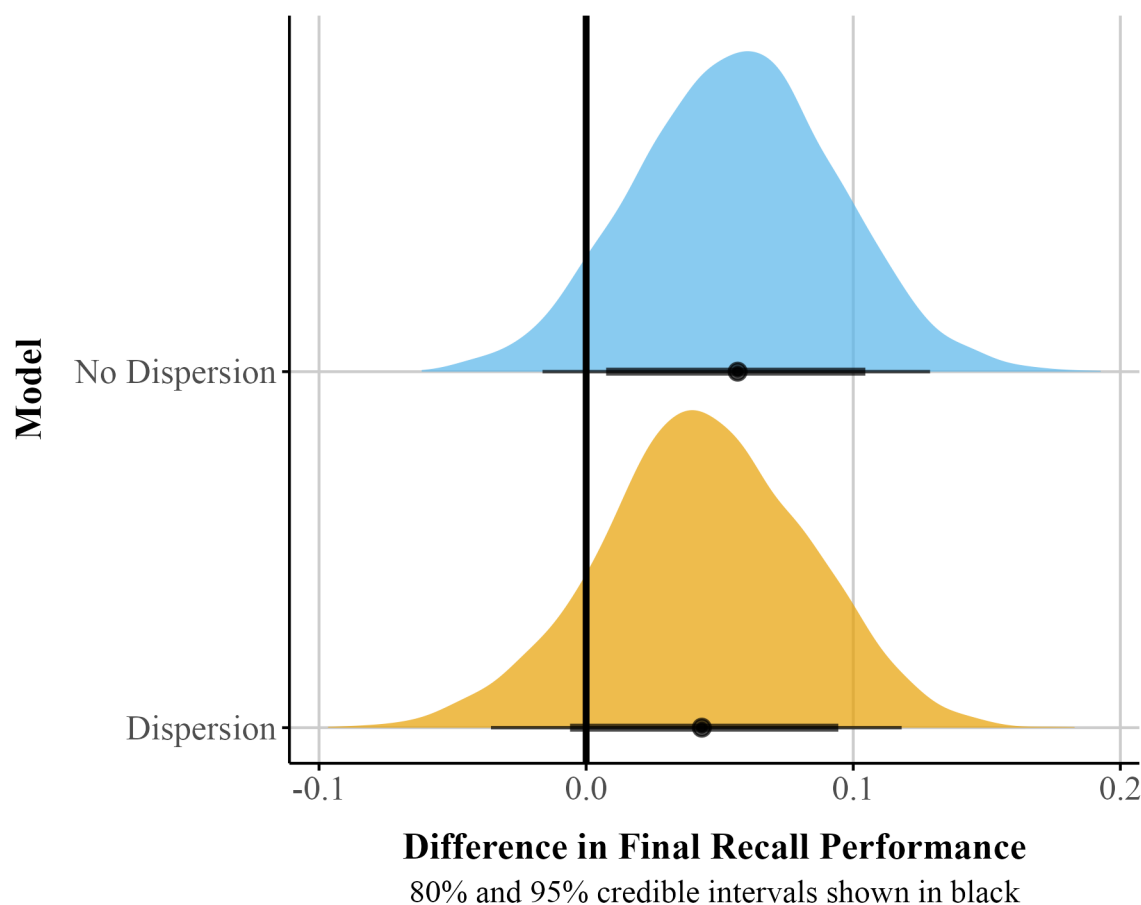384 cross validation, to examine if a dispersion parameter should be considered in our model.[7]

385 *Predicted Probabilities*

386   Parameter estimates can be difficult to interpret, and researchers can instead discuss effects

387 on the actual outcome scale (in this case the 0-1 scale). The logit link allows us to transform back

388 and forth between the scale of a linear model and the nonlinear scale of the outcome, which is

389 bounded by 0 and 1. By using the inverse of the logit, we can easily transform our linear

---

[7] The model fit statistic LOO-CV can be compared for any set of fitted brms models with the function `loo()`.

Figure 6

Comparison of posterior distributions for the risk difference in fluency: Simple model (no dispersion for Fluency) vs. complex model with dispersion

Difference in Final Recall Performance
80% and 95% credible intervals shown in black

coefficients to obtain average effects on the scale of the proportions or percentages, which is usually easier to interpret. In a simple case, we can do this manually, but when there are many factors in your model this can be quite complex.

In our example, we can use the `plogis()` function in base R to convert estimates from the logit scale to the probability scale. The intercept of our model is -0.918, which reflects the logit of the mean accuracy in the disfluent condition. If the estimated difference between the fluent and disfluent conditions is 0.24 on the logit scale, we first add this value to the intercept value (-0.918) to get the logit for the fluent condition: `-0.83 + 0.20 = -0.63`. We then use `plogis()` to convert both logit values to probabilities (Fluent = 35%, Disfluent = 30%).

399   With single coefficients this calculation is trivial, but in more complex models with

400   interactions, it can be quite cumbersome. To help us extract predictions from our model and

401   visualize them we will use a package called {marginaleffects} (Arel-Bundock et al., 2024) (see

402   Listing 8). To get the proportions for each of our categorical predictors on the μ parameter we can

403   use the function from the package called `avg_predictions()`. These are displayed in Table 2.

404   These probabilities match what we calculated above.

Listing 8 Load the {marginaleffects} package.

```r
library(marginaleffects)

options(marginaleffects_posterior_center = mean) # make sure returns mean
```

Listing 9 Predictions from the beta model for each level of Fluency (μ).

```r
avg_predictions(

  beta_brms_dis,

  # need to specify the levels of the categorical predictor

  variables = "Fluency"

)
```

Table 2

Predicted probabilities for fluency factor (μ).

Fluency	Mean	95% Cr.I
Disfluent	0.304	[0.249, 0.363]
Fluent	0.348	[0.301, 0.398]

405   For the `Fluency` factor, we can interpret the estimates under the `Mean` column as a

406   proportion or percentage. That is, participants who watched the fluent instructor scored on

407   average 35% on the final exam compared to 30% for those who watched the disfluent instructor.

408 We can also visualize these from {marginaleffects} using the `plot_predictions()` function

409 (see Listing 10).

Listing 10 Plot predicted probabilities using `plot_predictions()` from {marginaleffects}

```
beta_plot <- plot_predictions(beta_brms_dis, by = "Fluency")
```
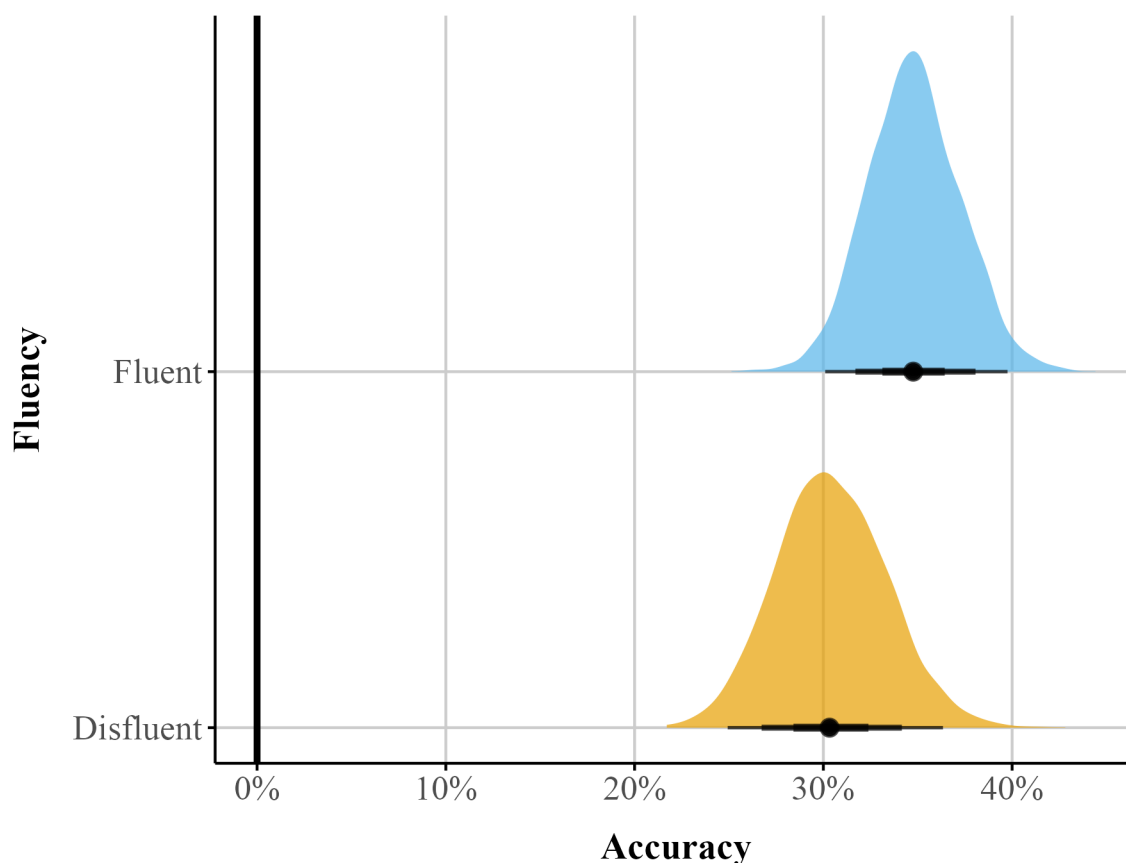
---

410   The `plot_predictions()` function will only display the point estimate with the 95%

411 credible interval. However, Bayesian estimation methods generate distributions for each

412 parameter. This approach allows visualizing full uncertainty estimates beyond points and

413 intervals. Using the {marginaleffects} package, we can obtain samples from the posterior

414 distribution with the `posterior_draws()` function (see Listing 11). We can then plot these

415 results to illustrate the range of plausible values for our estimates at different levels of uncertainty

416 (see Figure 7).

Listing 11 Extracting posterior draws from the beta regression model.

```
# Add a model identifier to each dataset

pred_draws_beta <- avg_predictions(beta_brms_dis, variables = "Fluency") |>
  posterior_draws()
```

---

417 *Marginal Effects*

418   Marginal effects offer an interpretable way to quantify how changes in a predictor

419 influence an outcome, while holding other factors constant in a specific manner. In recent years,

420 there has been a thrust to move away from reporting regression coefficients alone, focusing instead

421 on estimates that are easier to interpret and communicate—particularly in non-linear models

422 (McCabe et al., 2021; Rohrer & Arel-Bundock, 2025). Technically, marginal effects are computed

423 as partial derivatives for continuous variables or as finite differences for categorical (and

424 sometimes continuous) predictors, depending on the structure of the data and the research

425 question. Substantively, these procedures translate raw regression coefficients into quantities that

Figure 7

Predicted probablity posterior distributions by instructor fluency

Accuracy

50%, 80% and 95% credible intervals shown in black

reflect changes in the bounded outcome—for example, an $x\%$ change in the value of a proportion.

There are several types of marginal effects, and their computation can vary across software packages. For example, the popular {emmeans} package (Lenth, 2025) computes marginal effects by holding all predictors at their means (marginal effects at the mean; MEM). In this tutorial, we use the {marginaleffects} package (Arel-Bundock et al., 2024), which by default computes average marginal effects (AMEs). AMEs are based on counterfactual predictions: the dataset is conceptually replicated across all unique values of the predictor of interest, predictions are generated for each row under each counterfactual scenario, and the resulting differences are then averaged. This approach maintains a strong connection to the observed data—because predictions are made using each participant's actual values—while providing a clear and interpretable

436 summary of the effect of interest.

437   One practical use of AMEs is to estimate the average difference between two groups or

438 conditions which corresponds to the average treatment effect (ATE). Using the

439 `avg_comparisons()` function in the {marginaleffects}package (Listing 12), we can compute this

440 quantity directly. By default, the function returns the discrete difference between groups. When

441 we take the difference in proportions between two groups it is called the risk difference.

442 Depending on the audience and modeling goals, the function can also produce alternative effect

443 size metrics, such as odds ratios or risk ratios. This flexibility makes it a powerful approach for

444 summarizing and communicating regression results.

Listing 12 Calculating the difference between probabilities (μ) for fluency with `avg_comparisons()`

```r
# get risk difference by default
beta_avg_comp <- avg_comparisons(
  beta_brms_dis,
  variables = "Fluency", # counterfactural difference
  comparison = "difference"
)
```

Table 3

Fluency difference (μ)

Contrast	Mean	95% Cr.I	pd
Fluent - Disfluent	0.044	[-0.036, 0.118]	0.87

445   μ **Difference.** Table 3 presents the estimated difference for the `Fluency` factor (Mean

446 column). The difference between the fluent and disfluent conditions is 0.06, indicating that

447 participants who watched a fluent instructor scored, on average, 6% higher on the final recall test

448 than those who watched a disfluent instructor. However, the 95% credible interval includes 0

449 among the most credible values, suggesting we cannot rule out the possibility of a null or weakly

450 negative effect. The pd also indicates the effect is non-significant ($p > .05$). In the basic regression

451 model above, the ROPE value was extracted using the {bayestestR} package.

Listing 13 Extracting ROPE using {marginaleffects} for μ difference

```
# get risk difference by default

beta_avg_comp_rope <- avg_comparisons(

  beta_brms_dis,

  variables = "Fluency", # counterfactural difference

  comparison = "difference",

  equivalence=c(-0.02, .02)

)
```

---

452   ,The ROPE can also be obtained using the {marginaleffects} package by specifying the

453 `equivalence` bounds in `avg_comparisons()`. Running the code in Listing 13 returns a ROPE

454 probability of 23%, indicating that there is insufficient evidence to conclude that the effect

455 exceeds 2% or is practically equivalent.

456   ϕ **difference.** In addition, with {marginaleffects}, we can get the actual precision

457 difference between the two groups on ϕ using similar code to above by setting `dpar` to "phi"

458 (Listing 14).

459   **Effect Size.** In psychology, it is common to report effect size measures like Cohen's d

460 (Cohen, 1977). When working with proportions we can calculate something similar called

461 Cohen's h. Taking our proportions, we can use the below equation (Equation 2) to calculate

462 Cohen's h along with the 95% Cr.I around it. Using this metric we see the effect size is small

463 (0.107), 95% credible interval [-0.002, 0.361].

$$h = 2 \cdot \left(\arcsin\left(\sqrt{p_1}\right) - \arcsin\left(\sqrt{p_2}\right) \right) \tag{2}$$

Listing 14 Calculating ϕ difference with `avg_comparisons()`

```
# get risk difference by default


beta_avg_phi <- avg_comparisons(

  beta_brms_dis,

  variables = "Fluency",

  dpar = "phi",

  comparison = "difference"

)
```

Posterior Predictive Check

Figure 12 (B) shows the predictive check for our beta model. The model's predictions generally conform to the data as the predictions are now between constrained to the 0-1 interval. However, the model's predictive performance can be improved the bounds of the scale are explicitly specified.

Zero-Inflated beta (ZIB) Regression

A limitation of the beta regression model is that it can only accommodate values strictly between 0 and 1—a probability cannot take on values of 0 (the event will not occur with certainty) or 1 (the event will occur with certainty). In our dataset, we observed 9 rows where Accuracy equals zero. To fit a beta regression model, we removed these values, but we have left out potentially valuable information from our model–especially if the end points of the scale are distinctive in some way. In our case, these 0s may be structural—that is, they represent real, systematic instances where participants failed to answer correctly (rather than random noise or measurement error). For example, the fluency of the instructor might be a key factor in predicting these zero responses. We will discuss two approaches for jointly modeling these end points with the continuous data. First, we can use a zero-inflated beta (ZIB) model. This model still estimates the mean (μ) and precision (ϕ) of the beta distribution for values between 0 and 1, but it also

481 includes an additional parameter, α, which captures the probability of observing structural 0s.

482   The zero-inflated beta models a mixture of the data-generating process. The α parameter

483 uses a logistic regression to model whether the data is 0 or not. Substantively, this could be a

484 useful model when we think that 0s come from a process that is relatively distinct from the data

485 that is greater than 0. For example, if we had a dataset with proportion of looks or eye fixations to

486 certain areas on marketing materials, we might want a separate model for those that do not look at

487 certain areas on the screen because individuals who do not look might be substantively different

488 than those that look.

489   We can fit a ZIB model using `brms()` and use the {marginaleffects} package to make

490 inferences about our parameters of interest. Before we run a zero-inflated beta model, we will

491 need to transform our data again and remove the one 1 value in our data–we can keep our 0s.

492 Similar to our beta regression model we fit in `brms`, we will use the `bf()` function to fit several

493 models. We fit our μ and ϕ parameters as well as our zero-inflated parameter (α; here labeled as

494 `zi`). In `brms` we can use the zero_inflated_beta family (see Listing 15).

Posterior Predictive Check

496   The ZIB model does a bit better at capturing the structure of the data then the beta

497 regression model (see Figure 12). Specifically, the ZIB model more accurately captures the

498 increased density of values near the lower end of the scale (i.e., near zero), which the standard

499 beta model underestimates. The ZIB model's predictive distributions also align more closely with

500 the observed data across the entire range, particularly in the peak and tail regions. This improved

501 fit likely reflects the ZIB model's ability to explicitly model excess 0s (or near-zero values) via its

502 inflation component, allowing it to better account for features in the data that a standard beta

503 distribution cannot accommodate.

Predicted Probabilities and Marginal Effects

505   Table 8, under the zero-inflated beta regression column, provides a summary of the

506 posterior distribution for each parameter. As stated before, it is preferable to back-transform our

507 estimates to get probabilities. To get the predicted probabilities we can again use the

Listing 15 Fitting zib model with `brm()`

```r
# keep 0 but remove 1

data_beta_0 <- fluency_data |>

  filter(Accuracy != 1)


# set up model formual for zero-inflated beta in brm

zib_model <- bf(

  Accuracy ~ Fluency, # The mean of the 0-1 values, or mu

  phi ~ Fluency, # The precision of the 0-1 values, or phi

  zi ~ Fluency, # The zero-or-one-inflated part, or alpha

  family = zero_inflated_beta()

)


# fit zib model with brm

fit_zi <- brm(

  formula = zib_model,

  data = data_beta_0,

  file = here::here("manuscript","models", "bayes_zib_model0not1.rds")

)
```

508 `avg_predictions()` and `avg_comparisons()` functions from {marginaleffects} package

509 (Arel-Bundock, 2024) to get predicted probabilities and the probability difference between the

510 levels of each factor. We can model the parameters separately using the `dpar` argument setting to:

511 μ, ϕ, α. Here we look at the risk difference for Fluency under each parameter. If one were

512 interested in the average effect for the entire model, the `dpar` argument could be removed.

513 **Mu.** As shown in Table 4, there is little evidence for an effect of Fluency – the 95% Cr.I

514 includes zero, suggesting substantial uncertainty about the direction and magnitude of the

515 effect–that is, though most of the posterior density supports positive effects, nil and weakly

516 negative effects cannot be ruled out.

Table 4

Probablity fluency difference (μ)

Contrast	Mean	95% Cr.I	pd
Fluent - Disfluent	0.043	[-0.031, 0.118]	0.875

517   **Dispersion.** As shown in Table 5, the posterior estimates suggest a credible effect of

518 Fluency on dispersion (ϕ), with disfluent responses showing greater variability. The 95% Cr.I for

519 the fluency contrast does not include zero, indicating a high probability in differences in precision.
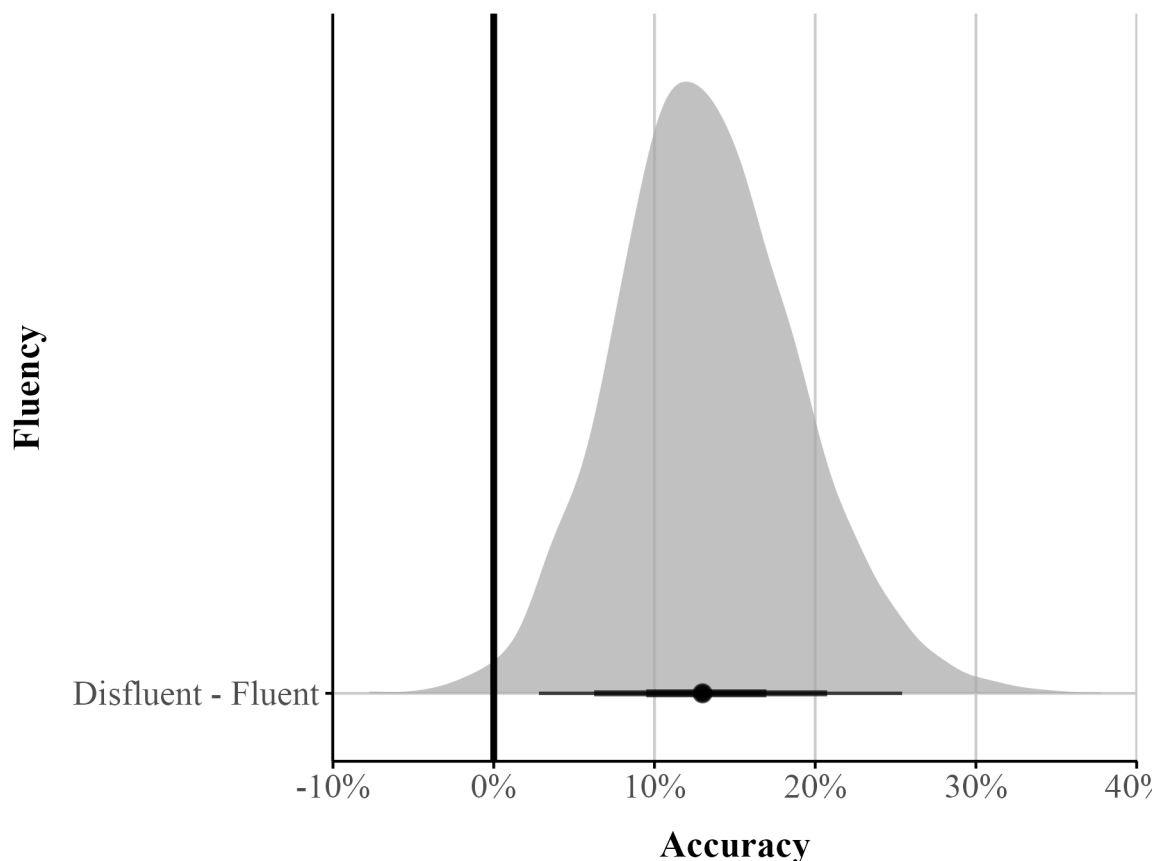
Table 5

Probablity fluency difference (φ)

Contrast	Mean	95% Cr.I	pd
Fluent - Disfluent	2.68	[-0.894, 6.45]	0.926

520 *Zero-Inflation*

521   We can use {marginaleffects} to estimate and plot the posterior difference between the

522 fluent and disfluent conditions (see Figure 8). In Figure 8, the posterior distribution for this

523 contrast lies mostly below zero, indicating that a fluent instructor is associated with a lower

524 probability of zero responses. The estimated reduction is approximately 13%. The 95% credible

525 interval does not include zero, which indicates that the data provide consistent evidence for a

526 reduction in zero responses under fluent instruction.

Figure 8

Visualization of the predicted difference for zero-inflated part of model

50%, 80% and 95% credible intervals shown in black

Zero-One-Inflated beta (ZOIB)

The ZIB model works well if there are 0s in your data, but not 1s.[8] In our previous examples we either got rid of both 0s and 1s (beta regression), or removed the 1s (ZIB). Sometimes it is theoretically useful to model both 0s and 1s as separate processes or to consider

---

[8] In cases where your data include exact 1s but no 0s, you can fit a one-inflated beta regression model in {brms} by setting the `coi` parameter to 1. This tells the model that all point masses occur at 1, rather than being split between 0 and 1. In other words, `coi = 1` assumes that any inflation in the data is due entirely to values at 1. In our data, we have exactly one value equal to 1[^6]. While probably not significant to alter our findings, we can model 1s with a special type of model called the zero-one-inflated beta (ZOIB) model (Liu & Kong, 2015) if we believe that both 0s and 1s are distinct outcomes.

these values as essentially similar parts of the continuous response, as we show later in the

ordered beta regression model. For example, this is important in visual analog scale data where

there might be a prevalence of responses at the bounds (Kong & Edwards, 2016), in JOL tasks

(Wilford et al., 2020), or in a free-list task where individuals provide open responses to some

question or topic which are then recoded to fall between 0-1 (Bendixen & Purzycki, 2023). Here

0s and 1s are meaningful; 0 means item was not listed and 1 means the item was listed first.

Similar to the beta and zero-inflated models discussed above, we can fit a

zero-and-one-inflated beta (ZOIB) model in {brms} using the `zero_one_inflated_beta` family.

This formulation simultaneously estimates the mean μ and precision ϕ of the Beta component, as

well as two inflation parameters: α , the probability that an observation is at either boundary (0 or

1), and γ , the conditional probability that, given an observation falls on a boundary, it takes the

value 1 rather than 0. In other words, α determines how often responses occur exactly at the

endpoints, and γ determines the balance between zeros and ones among those endpoint values.

This specification allows the model to capture both the continuous variation in the interior of the

(0, 1) interval and the presence of exact boundary values.

To illustrate how α and γ shape the distribution, Figure 9 displays simulated data across a

range of parameter combinations. As α increases, more responses occur at the endpoints. As γ

increases, the proportion of those endpoint responses that are 1 increases relative to 0, producing

increasingly pronounced spikes at 1 as γ approaches 1. Together, these parameters give the ZOIB

model the flexibility to represent datasets with mixtures of continuous values and exact zeros and
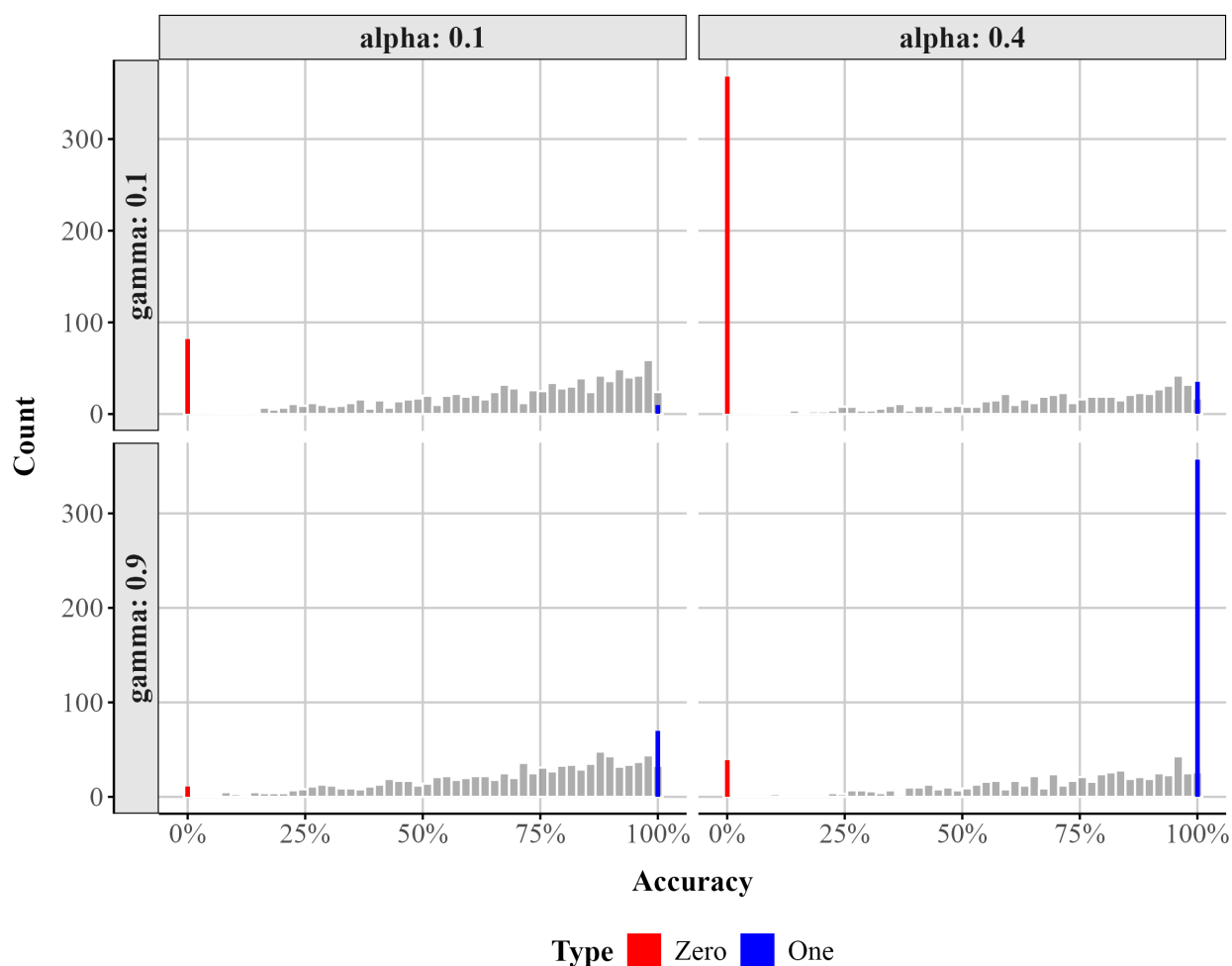
ones.

To fit a ZOIB model we use the `bf()` function. We model each parameter as a function of

Fluency. We then pass the `zoib_model` to our `brm()` function (see Listing 16). The summary of

the output is in Table 8 (under `ZOIB`).

Model Parameters

The output for the model is lengthy because we are estimating four distinct components,

each with their own independent responses and sub-models. All the coefficients are on the logit

Figure 9

Simulated data from a ZOIB model illustrating the effects of the zero-one inflation parameter (α) and the conditional one-inflation parameter (γ).

scale, except ϕ , which is on the log scale. Thankfully drawing inferences for all these different parameters, plotting their distributions, and estimating their average marginal effects looks exactly the same–all the `brms` and {marginaleffects} functions we used work the same.

Predictions and Marginal Effects

With {marginaleffects} we can choose marginalize over all the sub-models, averaged across the 0s, continuous responses, and 1s in the data, or we can model the parameters separately using the `dpar` argument like we did above setting it to: μ, ϕ, α, γ (see below). Using

Listing 16 Fitting a ZOIB model with `brm()`.

```r
# fit the zoib model

zoib_model <- bf(
  Accuracy ~ Fluency, # The mean of the 0-1 values, or mu
  phi ~ Fluency, # The precision of the 0-1 values, or phi
  zoi ~ Fluency, # The zero-or-one-inflated part, or alpha
  coi ~ Fluency, # The one-inflated part, conditional on the 1s, or gamma
  family = zero_one_inflated_beta()
)


fit_zoib <- brm(
  formula = zoib_model,
  data = fluency_data,
  file = here::here("manuscript", "models", "bayes_zoib_model")
)
```

`avg_predictions()` and not setting `dpar` we can get the predicted probabilities across all the sub-models. We can also plot the overall difference between fluency and disfluency for the whole model with `plot_predictions()`.

In addition, we show below how one can extract the predicted probabilities and marginal effects for γ (and a similar process for any other model component, `zoi`, etc.):

Listing 17 Extracting predicted probabilities and marginal effects for conditional-one parameter

```r
# get average predictions for coi param
coi_probs <- avg_predictions(fit_zoib, variables = "Fluency", dpar = "coi")
# get differene between the two conditions
coi_me <- avg_comparisons(fit_zoib, variables = c("Fluency"), dpar = "coi")
```

Ordered Beta Regression

Looking at the output from the ZOIB model (Table 8), we can see how running a model like this can become fairly complex as it is fitting distinct sub-models for each component of the scale. The ability to consider 0s and 1s as distinct processes from continuous values comes at a price in terms of complexity and interpretability. A simplified version of the zero-one-inflated beta (ZOIB) model, known as ordered beta regression (Kubinec, 2022; see also Makowski et al., 2025 for a reparameterized version called the *beta-Gate* model), has been recently proposed. The ordered beta regression model exploits the fact that, for most analyses, the continuous values (between 0-1) and the discrete outcomes (e.g., 0 or 1) are *ordered*. For example, as a covariate x increases or decreases, we should expect the bounded outcome y to increase or decrease monotonically as well from 0 to (0, 1) to 1. The ZOIB model does not impose this restriction; a covariate could increase and the response y could increase in its continuous values while *simultaneously* decreasing at *both* end points.[9] This complexity is not immediately obvious when fitting the ZOIB, nor is it a potential relationship that many scholars want to consider when examining how covariates influence a bounded scale.

To make the response ordered, the ordered beta regression model estimates a weighted combination of a standard beta regression model for continuous responses and a logit model for the discrete values of the response. By doing so, the amount of distinctiveness between the continuous responses and the discrete end points is a function of the data (and any informative priors) rather than strictly defined as fully distinct processes as in the ZOIB. For some datasets, the continuous and discrete responses will be fairly distinct, and in others less so.

The weights that average together the two parts of the outcome (i.e., discrete and continuous) are determined by cutpoints that are estimated in conjunction with the data in a similar manner to what is known as an ordered logit model. An in-depth explanation of ordinal regression is beyond the scope of this tutorial (Bürkner & Vuorre, 2019; but see Fullerton & Anderson, 2021). At a basic level, ordinal regression models are useful for outcome variables that

---

[9] For a more complete description of this issue, we refer the reader to Kubinec (2022).

are categorical in nature and have some inherent ordering (e.g., Likert scale items). To preserve

this ordering, ordinal models rely on the cumulative probability distribution. Within an ordinal

regression model it is assumed that there is a continuous but unobserved latent variable that

determines which of k ordinal responses will be selected. For example on a typical Likert scale

from 'Strongly Disagree' to 'Strongly Agree', you could assume that there is a continuous,

unobserved variable called 'Agreement'.

While we cannot measure Agreement directly, the ordinal response gives us some

indication about where participants are on the continuous Agreement scale. $k - 1$ cutoffs are then

estimated to indicate the point on the continuous Agreement scale at which your Agreement level

is high enough to push you into the next ordinal category (say Agree to Strongly Agree).

Coefficients in the model estimate how much different predictors change the estimated *continuous*

scale (here, Agreement). Since there's only one underlying process, there's only one set of

coefficients to work with (proportional odds assumption).

In an ordered beta regression, three ordered categories are modeled: (1) exactly zero, (2)

somewhere between zero and one, and (3) exactly one. In an ordered beta regression, (1) and (2)

are modeled with cumulative logits, where one cutpoint is the the boundary between Exactly 0

and Between 0 and 1 and the other cutpoint is the boundary between *Between 0 and 1* and *Exactly*

1. The continuous values in the middle, 0 to 1 (3), are modeled as a vanilla beta regression with

parameters reflecting the mean response on the logit scale as we have described previously.

Ultimately, employing cutpoints allows for a smooth transition between the bounds and the

continuous values, permitting both to be considered together rather than modeled separately as the

ZOIB requires.

The ordered beta regression model has shown to be more efficient and less biased than

some of the methods discussed (Kubinec, 2022) herein and has seen increasing use across the

biomedical and social sciences (Martin et al., 2024; Nouvian et al., 2023; Shrestha et al., 2024;

Smith et al., 2024; Wilkes et al., 2024) because it produces only a single set of coefficient

622 estimates in a similar manner to a standard beta regression or OLS.[10]

Fitting an Ordered Beta Regression

624 To fit an ordered beta regression in a Bayesian context we use the {ordbetareg} (Kubinec,

625 2023) package. {ordbetareg} is a front-end to the `brms` package that we described earlier; in

626 addition to the functions available in the package, most `brms` functions and plots, including the

627 diverse array of regression modeling options, will work with {ordbetareg} models. (We note that

628 the `ordbeta` model is also available as a maximum-likelihood variant in the R package

629 {glmmTMB}.) We first load the {ordbetareg} package (see Listing 18).

Listing 18 Load {ordbetareg}

```
library(ordbetareg)
```

630 The {ordbetareg} package uses `brms` on the front-end so all the arguments we used

631 previously apply here. Instead of the `brm()` function we use `ordbetareg()`. To fit a model where

632 dispersion does not vary as a function of fluency we can use the below code (see Listing 19).

Listing 19 Fitting ordered beta model with `ordbetareg()`

```
ord_fit_brms <- ordbetareg(
  Accuracy ~ Fluency,
  data = fluency_data,
  file = here::here("manuscript","models", "bayes_ordbeta_model")
)
```

633 However, if we want dispersion to vary as a function of fluency we can easily do that (see

634 Listing 20). Note the addition of the `phi_reg` argument in `m.phi`. This argument allows us to

---

[10] Please note that there are other models available that can model this continuous process like the beta-gate model
(Makowski et al., 2025) and the censored extended beta regression model (Kosmidis & Zeileis, 2025).

include a model that explicitly models the dispersion parameter. Because we are modeling ϕ as a function of fluency, we set the the argument to `both`.

Listing 20 Fitting ordered beta model with dispersion using `ordbetareg()`

```
ord_beta_phi <- bf(Accuracy ~ Fluency, phi ~ Fluency)


m.phi <- ordbetareg(
  ord_beta_phi,
  data = fluency_data,
  phi_reg = 'both',
  file = here::here("manuscript", "models", "bayes_ordbeta_phi_model")
)
```

Marginal Effects. Table 8 presents the posterior summary. We can use {marginaleffects} to calculate differences on the response scale that average over (or marginalize over) all our parameters.

Table 6

Marginal effect of fluency ordered beta model

Contrast	Mean	95% Cr.I	pd
Fluent - Disfluent	0.061	[-0.016, 0.139]	0.94

In Table 6 the credible interval is close enough to zero relative to its uncertainty that we can conclude there likely aren't differences between the conditions after taking dispersion and the 0s and 1s in our data into account.

Cutpoints. The model cutpoints are not reported by default in the summary output, but we can access them with the R package `posterior` (Bürkner et al., 2025) and the functions `as_draws` and `summary_draws`.

Table 7

Cutzero and cutone parameter summary

Parameter	Mean	95% Cr.I
cutzero	-2.97	[-3.57, -2.44]
cutone	1.85	[1.64, 2.07]

646   In Table 7, `cutzero` is the first cutpoint (the difference between 0 and continuous values)

647 and `cutone` is the second cutpoint (the difference between the continuous values and 1). These

648 cutpoints are on the logit scale and as such the numbers do not have a simple substantive meaning.

649 In general, as the cutpoints increase in absolute value (away from zero), then the discrete/boundary

650 observations are more distinct from the continuous values. This will happen if there is a clear gap

651 or bunching in the outcome around the bounds. This type of empirical feature of the distribution

652 may be useful to scholars if they want to study differences in how people perceive the ends of the

653 scale versus the middle. It is possible, though beyond the scope of this article, to model the

654 location of the cutpoints with hierarchical (non-linear) covariates in `brms`. In the most recent

655 version of `ordbeta`, it is possible to test the influence of different factors on these boundaries.
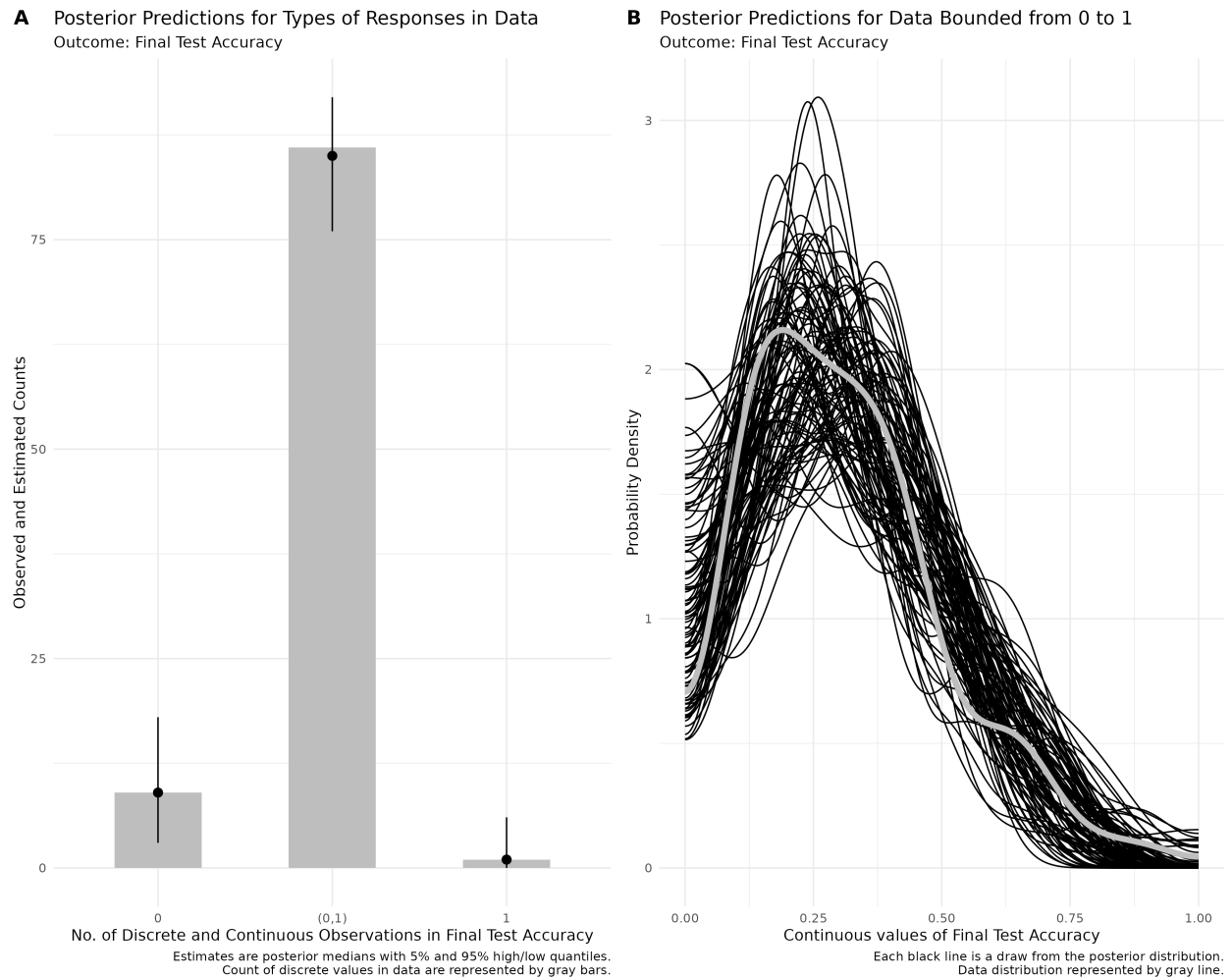
Model Fit

657   The best way to visualize model fit is to plot the full predictive distribution relative to the

658 original outcome. Because ordered beta regression is a mixed discrete/continuous model, a

659 separate plotting function, `pp_check_ordbetareg`, is included in the {ordbetareg} package that

660 accurately handles the unique features of this distribution. The default plot in `brms` will collapse

661 these two features of the outcome together, which will make the fit look worse than it actually is.

662 The {ordbetareg} function returns a list with two plots, `discrete` and `continuous`, which can

663 either be printed and plotted or further modified as {ggplot2} objects (see Figure 10).

664   The discrete plot, which is a bar graph, shows that the posterior distribution accurately

665 captures the number of different types of responses (discrete or continuous) in the data. For the

Figure 10

Posterior predictive check for ordered beta regression model. A. Discrete posterior check. B.

Continuous posterior check.

A Posterior Predictions for Types of Responses in Data
Outcome: Final Test Accuracy

B Posterior Predictions for Data Bounded from 0 to 1
Outcome: Final Test Accuracy

continuous plot shown as a density plot with one line per posterior draw, the model does a very
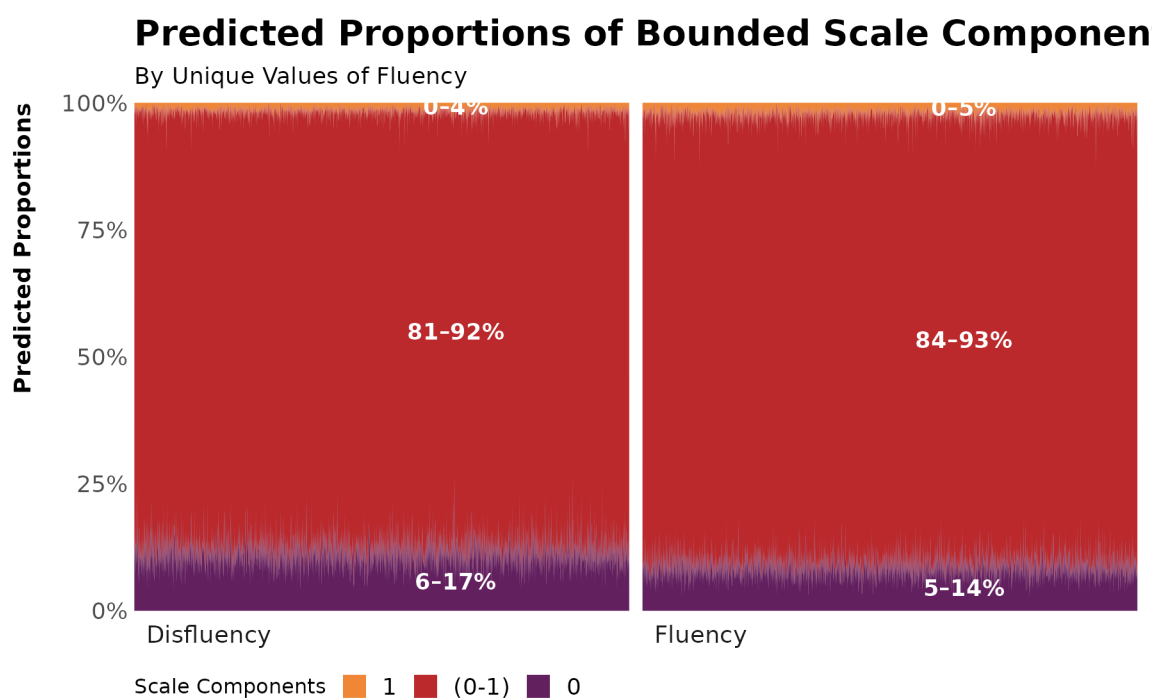
good job at capturing the distribution.

Overall, it is clear from the posterior distribution plot that the ordered beta model fits the

data well. To fully understand model fit, both of these plots need to be inspected as they are

conceptually distinct.

Model Visualization

{ordbetareg} provides a useful visualization function called `plot_heiss()` (Ye & Heiss,

2023) that can represent dispersion in the entire outcome as a function of discrete covariates. This

function produces a plot of predicted proportions across the range of our `Fluency` factor. In

Figure 11 we get predicted proportions for Fluency across the bounded scale. Looking at the

figure we can see there is much overlap between instructors in the middle portion (μ) . However,

we do see some small differences at the zero bounds.

Figure 11

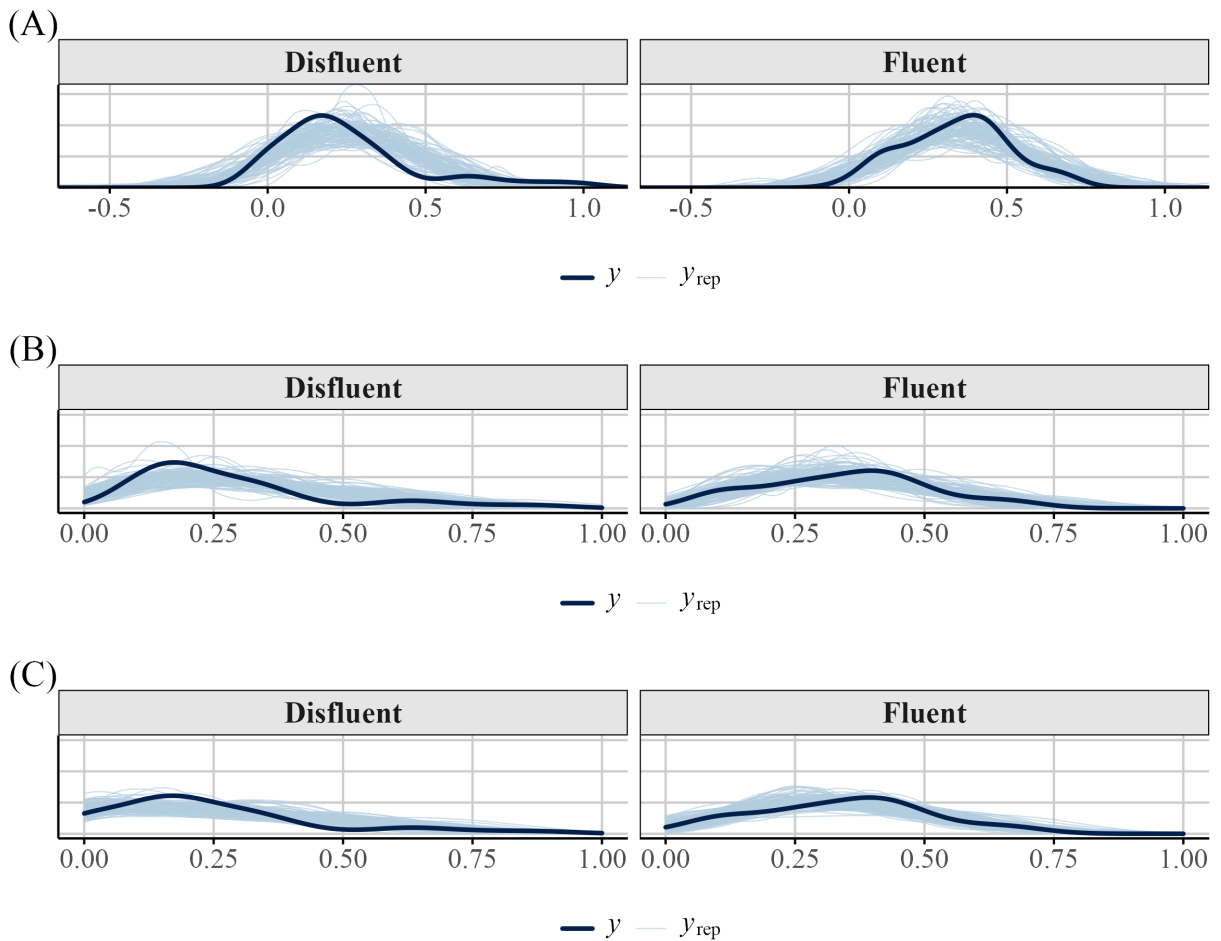*Heiss plot of predicted probabilities across the scale (0-100)*

Plot shows predicted proportions of the components of a bounded
scale, i.e. the predicted (expected) probability of the top value
of the scale, the intermediate continuous values, and the bottom
value of the scale. The predictions are subset for unique values of
a grouping factor. The predictions are shown for multiple posterior
draws to indicate uncertainty. Labels on components indicate posterior
quantiles for the probability of that component for each level of the
grouping variable.

Ordered Beta Scale

In the {ordbetareg} function there is a `true_bound` argument. In cases where your data is

not bounded between 0-1, this argument can be used to specify the bounds of the argument to fit

the ordered beta regression. For example, the response data might be bounded between 1 and 7. If

so, {ordbetareg} can model it within the [0,1] interval and {ordbetareg} will convert the model

predictions back to the true bounds after estimation.

Figure 12

The plots show 100 posterior predicted distributions with the label yrep (light blue), the

distribution of accuracy as function of fluency in dark blue for regular regression (A) , beta

regression (B), ZIB (C) and ZOIB (D).

Discussion

684

685   The use of beta regression in psychology, and the social sciences in general, is rare. With

686 this tutorial, we hope to turn the tides. Beta regression models are an attractive alternative to

687 models that impose unrealistic assumptions like normality, linearity, homoscedasticity, and

688 unbounded data. Beyond these models, there are a diverse array of different models that can be

689 used depending on your outcome of interest.

690   Throughout this tutorial our main aim was to help guide researchers in running analyses

691 with proportional or percentage outcomes using beta regression and some of its alternatives. In

692 the current example, we used real data from Wilford et al. (2020) and discussed how to fit these

693 models in R, interpret model parameters, extract predicted probabilities and marginal effects, and

694 visualize the results using the {marginaleffects} package.

695   Comparing our analysis with that of Wilford et al. (2020), we demonstrated that using a

696 traditional approach (e.g., *t*-test) to analyze mean accuracy data can lead to biased inferences.

697 Although we successfully reproduced one of their key findings, our use of beta regression and its

698 extensions revealed important nuances in the results. With a traditional beta regression

699 model–which accounts for both the mean and the precision (dispersion)–we observed similar

700 effects of instructor fluency on performance. However, the standard beta model does not

701 accommodate boundary values (i.e., 0s and 1s).

702   When we applied a ZIB model, which explicitly accounts for structural 0s, we found no

703 effect of fluency on the mean (μ) part of the model. Instead, the effect of fluency emerged in the

704 structural zero (inflated zero; α) component. This pattern was consistent when using a

705 zero-one-inflated beta (ZOIB) model. Furthermore, we fit an ordered beta regression model

706 (Kubinec, 2022), which appropriately models the full range of values, including 0s and 1s. Here,

707 we did not observe a reliable effect of fluency on the mean once we accounted for dispersion.

708   These analyses emphasize the importance of fitting a model that aligns with the nature of

709 the data. The simplest and recommended approach when dealing with data that contains 0s and/or

710 1s is to fit an ordered beta model, assuming the process is truly continuous. However, if you

believe the process is distinct in nature, a ZIB or ZOIB model might be a better choice. Ultimately, this decision should be guided by theory.

For instance, if we believe fluency influences the boundaries (0 and 1), we might want to model this process separately using a ZIB or ZOIB. With the current dataset, fluency might affect specific aspects of performance (such as the likelihood of complete failure) rather than general performance levels. This effect could be due to participant disengagement during the disfluent lecture. If students fail to pay attention because of features of disfluency, they may miss relevant information, leading to a floor effect at the test. Follwoing from this, disfluency would be expected to influence the boundary (O) and not the continuous part of the model. If this is the case, we would want to model this appropriately. However, if we believe fluency effects general performance levels (the continuous part), a model that takes in to account the entire process accounting for the 0s and 1s might be appropriate.

In the discussion section of Wilford et al. (2020), they were unable to offer a tenable explanation for performance differences based on instructor fluency. A model that accounts for the excess 0s in the dataset provides one testable explanation: watching a disfluent lecture may lead to lapses in attention, resulting in poorer performance in that group. These lapses, in turn, contribute to the observed differences in the fluent condition. This modeling approach opens a promising avenue for future research–one that would have remained inaccessible otherwise.

Not everyone will be eager to implement the techniques discussed herein. In such cases, the key question becomes: What is the least problematic approach to handling proportional data? One reasonable option is to fit multiple models tailored to the specific characteristics of your data. For example, if your data contain 0s, you might fit two models: a traditional linear model excluding the 0s, and a logistic model to account for the zero versus non-zero distinction. If your data contain both 0s and 1s, you could fit separate models for the 0s and 1s in addition to the OLS model. There are many defensible strategies to choose from depending on the context. However, we do not recommend transforming the values of your data (e.g., 0s to .01 and 1s to .99) or ignoring the properties of your data simply to fit traditional statistical models.

738   In this tutorial, we demonstrated how to analyze these models from a Bayesian perspective.

739 While we recognize that not everyone identifies as a Bayesian, implementing these models using a

740 Bayesian framework is relatively straightforward–it requires only a single package, lowering the

741 barrier to entry. For those who prefer frequentist analyses, several R packages are available. For

742 example, the {betareg} package (Cribari-Neto & Zeileis, 2010) {glmmTMB} (Brooks et al.,

743 2017) and {gamlss} (2005) are nice options. To this end, I have included supplemental materials

744 demonstrating how to use frequentist packages to analyze the data presented herein.

Conclusion

746   Overall, this tutorial emphasizes the importance of modeling the data you have. Although

747 the example provided is relatively simple (a one-factor model with two levels), we hope it

748 demonstrates that even with a basic dataset, there is much nuance in interpretation and inference.

749 Properly modeling your data can lead to deeper insights, far beyond what traditional measures

750 might offer. With the tools introduced in this tutorial, researchers now have the means to analyze

751 their data effectively, uncover patterns, make accurate predictions, and support their findings with

752 robust statistical evidence. By applying these modeling techniques, researchers can improve the

753 validity and reliability of their studies, ultimately leading to more informed decisions and

754 advancements in their respective fields.

<center>**References**</center>

756 Arel-Bundock, V. (2024). *Marginaleffects: Predictions, comparisons, slopes, marginal means,*

757   *and hypothesis tests*. https://CRAN.R-project.org/package=marginaleffects

758 Arel-Bundock, V., Greifer, N., & Heiss, A. (2024). How to interpret statistical models using

759   marginaleffects for R and Python. *Journal of Statistical Software*, *111*(9), 1–32.

760   https://doi.org/10.18637/jss.v111.i09

761 Bartlett, M. S. (1936). The square root transformation in analysis of variance. *Journal of the*

762   *Royal Statistical Society Series B: Statistical Methodology*, *3*(1), 68–78.

763   https://doi.org/10.2307/2983678

764 Bendixen, T., & Purzycki, B. G. (2023). Cognitive and cultural models in psychological science:

A tutorial on modeling free-list data as a dependent variable in bayesian regression.

Psychological Methods. https://doi.org/10.1037/met0000553

Brooks, M. E., Kristensen, K., van, K. J., Magnusson, A., Berg, C. W., Nielsen, A., Skaug, H. J.,

Maechler, M., & Bolker, B. M. (2017). *{glmmTMB} balances speed and flexibility among*

packages for zero-inflated generalized linear mixed modeling. 9.

https://doi.org/10.32614/RJ-2017-066

Bürkner, P.-C. (2017). *{Brms}: An {r} package for {bayesian} multilevel models using {stan}. 80.*

https://doi.org/10.18637/jss.v080.i01

Bürkner, P.-C., Gabry, J., Kay, M., & Vehtari, A. (2025). *posterior: Tools for working with*

posterior distributions. https://mc-stan.org/posterior/

Bürkner, P.-C., & Vuorre, M. (2019). Ordinal regression models in psychology: A tutorial.

Advances in Methods and Practices in Psychological Science, *2*(1), 77–101.

https://doi.org/10.1177/2515245918823199

Carpenter, S. K., Wilford, M. M., Kornell, N., & Mullaney, K. M. (2013). Appearances can be

deceiving: Instructor fluency increases perceptions of learning without increasing actual

learning. *Psychonomic Bulletin & Review*, *20*(6), 1350–1356.

https://doi.org/10.3758/s13423-013-0442-z

Cohen, J. (1977). *Statistical power analysis for the behavioral sciences, rev. ed.* Lawrence

Erlbaum Associates, Inc.

Coretta, S., & Bürkner, P.-C. (2025). *Bayesian beta regressions with brms in r: A tutorial for*

phoneticians. https://doi.org/10.31219/osf.io/f9rqg_v1.

Costello, T. H. et al. (2024). Durably reducing conspiracy beliefs through dialogues with AI.

Science, *385*, eadq1814. https://doi.org/10.1126/science.adq1814

Cribari-Neto, F., & Zeileis, A. (2010). *Beta regression in {r}. 34.*

https://doi.org/10.18637/jss.v034.i02

Dolstra, E., & contributors, T. N. (2006). *Nix* [Computer software]. https://nixos.org/

Ferrari, S., & Cribari-Neto, F. (2004). Beta regression for modelling rates and proportions.

Journal of Applied Statistics, *31*(7), 799–815. https://doi.org/10.1080/0266476042000214501

Fullerton, A. S., & Anderson, K. F. (2021). Ordered regression models: A tutorial. *Prevention Science*, *24*(3), 431–443. https://doi.org/10.1007/s11121-021-01302-y

Gabry, J., Češnovar, R., Johnson, A., & Bronder, S. (2024). *Cmdstanr: R interface to 'CmdStan'*. https://mc-stan.org/cmdstanr/

Gelman, A., Carlin, J. B., Stern, H. S., Dunson, D. B., Vehtari, A., & Rubin, D. B. (2013). *Bayesian data analysis* (Third). CRC. https://stat.columbia.edu/~gelman/book/

Heiss, A. (2021). *A guide to modeling proportions with bayesian beta and zero-inflated beta regression models*. http://dx.doi.org/10.59350/7p1a4-0tw75

James, A. N., Ryskin, R., Hartshorne, J. K., Backs, H., Bala, N., Barcenas-Meade, L., Bhattarai, S., Charles, T., Copoulos, G., Coss, C., Eisert, A., Furuhashi, E., Ginell, K., Guttman-McCabe, A., Harrison, E. (Chaz)., Hoban, L., Hwang, W. A., Iannetta, C., Koenig, K. M., … Leeuw, J. R. de. (2025). What paradigms can webcam eye-tracking be used for? Attempted replications of five cognitive science experiments. *Collabra: Psychology*, *11*(1). https://doi.org/10.1525/collabra.140755

Johnson, A., Ott, M., & Dogucu, M. (2022). *Bayes rules!: An introduction to applied bayesian modeling*. Routledge & CRC Press.

Kneib, T., Silbersdorff, A., & Säfken, B. (2023). Rage against the mean – a review of distributional regression approaches. *Econometrics and Statistics*, *26*, 99–123. https://doi.org/10.1016/j.ecosta.2021.07.006

Kong, E. J., & Edwards, J. (2016). Individual differences in categorical perception of speech: Cue weighting and executive function. *Journal of Phonetics*, *59*, 40–57. https://doi.org/10.1016/j.wocn.2016.08.006

Kornell, N., & Bjork, R. A. (2008). Learning concepts and categories. *Psychological Science*, *19*(6), 585–592. https://doi.org/10.1111/j.1467-9280.2008.02127.x

Kosmidis, I., & Zeileis, A. (2025). Extended-support beta regression for [0, 1] responses. *Journal of the Royal Statistical Society Series C: Applied Statistics*.

https://doi.org/10.1093/jrsssc/qlaf039

Kruschke, J. K. (2013). Bayesian estimation supersedes the t test. *Journal of Experimental Psychology: General*, *142*(2), 573–603. https://doi.org/10.1037/a0029146

Kruschke, J. K. (2015). *Doing bayesian data analysis: A tutorial with r, JAGS, and stan* (2nd ed.). Academic Press.

Kruschke, J. K. (2018). Rejecting or accepting parameter values in bayesian estimation. *Advances in Methods and Practices in Psychological Science*, *1*(2), 270–280. https://doi.org/10.1177/2515245918771304

Kubinec, R. (2022). Ordered beta regression: A parsimonious, well-fitting model for continuous data with lower and upper bounds. *Political Analysis*, *31*(4), 519–536. https://doi.org/10.1017/pan.2022.20

Kubinec, R. (2023). *Ordbetareg: Ordered beta regression models with 'brms'*. https://CRAN.R-project.org/package=ordbetareg

Lenth, R. V. (2025). *Emmeans: Estimated marginal means, aka least-squares means*. https://doi.org/10.32614/CRAN.package.emmeans

Liu, F., & Kong, Y. (2015). Zoib: An r package for bayesian inference for beta regression and zero/one inflated beta regression. *The R Journal*, *7*(2), 34. https://doi.org/10.32614/rj-2015-019

Lüdecke, D., Ben-Shachar, M. S., Patil, I., Wiernik, B. M., Bacher, E., Thériault, R., & Makowski, D. (2022). *Easystats: Framework for easy statistical modeling, visualization, and reporting*. https://easystats.github.io/easystats/

Makowski, D., Ben-Shachar, M. S., Chen, S. H. A., & Lüdecke, D. (2019). Indices of effect existence and significance in the bayesian framework. *Frontiers in Psychology*, *10*. https://doi.org/10.3389/fpsyg.2019.02767

Makowski, D., Ben-Shachar, M., & Lüdecke, D. (2019). bayestestR: Describing effects and their uncertainty, existence and significance within the bayesian framework. *Journal of Open Source Software*, *4*(40), 1541. https://doi.org/10.21105/joss.01541

Makowski, D., Neves, A., & Field, A. P. (2025). *Introducing the choice-confidence (CHOCO) model for bimodal data from subjective ratings: Application to the effect of attractiveness on reality beliefs about AI-generated faces*. https://doi.org/10.31234/osf.io/z68v3_v1

Marsman, M., & Wagenmakers, E.-J. (2016). Three Insights from a Bayesian Interpretation of the One-Sided P Value. *Educational and Psychological Measurement*. https://doi.org/10.1177/0013164416669201

Martin, K., Cornero, F. M., Clayton, N. S., Adam, O., Obin, N., & Dufour, V. (2024). Vocal complexity in a socially complex corvid: Gradation, diversity and lack of common call repertoire in male rooks. *Royal Society Open Science*, *11*(1), 231713. https://doi.org/10.1098/rsos.231713

McCabe, C. J., Halvorson, M. A., King, K. M., Cao, X., & Kim, D. S. (2021). Interpreting interaction effects in generalized linear models of nonlinear probabilities and counts. *Multivariate Behavioral Research*, *57*(2-3), 243–263. https://doi.org/10.1080/00273171.2020.1868966

McElreath, R. (2020). *Statistical rethinking: A bayesian course with examples in r and STAN* (2nd ed.). Chapman; Hall/CRC. https://doi.org/10.1201/9780429029608

Nelder, J. A., & Wedderburn, R. W. M. (1972). Generalized linear models. *Journal of the Royal Statistical Society. Series A (General)*, *135*(3), 370–384. https://doi.org/10.2307/2344614

Nouvian, M., Foster, J. J., & Weidenmüller, A. (2023). Glyphosate impairs aversive learning in bumblebees. *Science of The Total Environment*, *898*, 165527. https://www.sciencedirect.com/science/article/pii/S0048969723041505

Paolino, P. (2001). Maximum likelihood estimation of models with beta-distributed dependent variables. *Political Analysis*, *9*(4), 325–346. https://doi.org/10.1093/oxfordjournals.pan.a004873

Pfadt, J. M., Bartoš, F., Godmann, H. R., Waaijers, M., Groot, L., Heo, I., & Wagenmakers, E. …. (2025). A methodological metamorphosis: The rapid rise of bayesian inference and open science practices in psychology. *Preprint*. https://doi.org/10.31234/osf.io/ck3js_v1

R Core Team. (2024). *R: A language and environment for statistical computing*. R Foundation for

Statistical Computing. https://www.R-project.org/

R. A. Rigby, & D. M. Stasinopoulos. (2005). *Generalized additive models for location, scale and*

shape,(with discussion). *54*, 507–554.

Rodrigues, B., & Baumann, P. (2025). *Rix: Reproducible data science environments with 'nix'*.

https://docs.ropensci.org/rix/

Rohrer, J. M., & Arel-Bundock, V. (2025). *Models as prediction machines: How to convert*

confusing coefficients into clear quantities. https://doi.org/10.31234/osf.io/g4s2a_v1

Shrestha, S., Sigdel, K., Pokharel, M., & Columbus, S. (2024). Big five traits predict between-

and within-person variation in loneliness. *European Journal of Personality*,

08902070241239834. https://doi.org/10.1177/08902070241239834

Sladekova, M., & Field, A. P. (2024). *In search of unicorns: Assessing statistical assumptions in*

real psychology datasets. https://doi.org/10.31234/osf.io/4rznt

Smith, K. E., Panlilio, L. V., Feldman, J. D., Grundmann, O., Dunn, K. E., McCurdy, C. R.,

Garcia-Romeu, A., & Epstein, D. H. (2024). Ecological momentary assessment of

self-reported kratom use, effects, and motivations among US adults. *JAMA Network Open*,

7(1), e2353401. https://doi.org/10.1001/jamanetworkopen.2023.53401

Smithson, M., & Verkuilen, J. (2006). A better lemon squeezer? Maximum-likelihood regression

with beta-distributed dependent variables. *Psychological Methods*, *11*(1), 54–71.

https://doi.org/10.1037/1082-989x.11.1.54

Team, S. D. (2023). *Stan: A probabilistic programming language*. https://mc-stan.org

Toftness, A. R., Carpenter, S. K., Geller, J., Lauber, S., Johnson, M., & Armstrong, P. I. (2017).

Instructor fluency leads to higher confidence in learning, but not better learning.

Metacognition and Learning, *13*(1), 1–14. https://doi.org/10.1007/s11409-017-9175-0

Vuorre, M. (2019, February 18). *How to analyze visual analog (slider) scale data?* https://vuorre.

com/posts/2019-02-18-analyze-analog-scale-ratings-with-zero-one-inflated-beta-models

Wickham, H., Çetinkaya-Rundel, M., & Grolemund, G. (2023). *R for Data Science: Import, Tidy,*

Transform, Visualize, and Model Data. O'Reilly. https://r4ds.hadley.nz/

Wilford, M. M., Kurpad, N., Platt, M., & Weinstein-Jones, Y. (2020). Lecturer fluency can impact students' judgments of learning and actual learning performance. *Applied Cognitive Psychology*, *34*(6), 1444–1456. https://doi.org/10.1002/acp.3724

Wilkes, L. N., Barner, A. K., Keyes, A. A., Morton, D., Byrnes, J. E. K., & Dee, L. E. (2024). Quantifying co-extinctions and ecosystem service vulnerability in coastal ecosystems experiencing climate warming. *Global Change Biology*, *30*(7), e17422. https://doi.org/10.1111/gcb.17422

Witherby, A. E., & Carpenter, S. K. (2022). The impact of lecture fluency and technology fluency on students' online learning and evaluations of instructors. *Journal of Applied Research in Memory and Cognition*, *11*(4), 500–509. https://doi.org/10.1037/mac0000003

Ye, M., & Heiss, A. (2023). Enforcing Boundaries: China's Overseas NGO Law and Operational Constraints for Global Civil Society. *Working Paper*. https://stats.andrewheiss.com/compassionate-clam/manuscript/output/manuscript.html

Table 8

Bayesian regression summaries for each model

Parameter	Stat	Bayesian LM	Beta Regression	ZIB	ZOIB	Ordered Beta
b_Intercept	Mean	0.256	-0.831	-0.828	-0.826	-0.865
	Cr.I	[0.201, 0.313]	[-1.102, -0.561]	[-1.1, -0.564]	[-1.093, -0.556]	[-1.124, -0.606]
	pd	1.000***	1.000***	1.000***	1.000***	1.000***
b_Fluency	Mean	0.085	0.201	0.198	0.198	0.261
	Cr.I	[0, 0.167]	[-0.156, 0.552]	[-0.14, 0.543]	[-0.151, 0.54]	[-0.069, 0.597]
	pd	0.976*	0.871	0.875	0.870	0.940
sigma	Mean	0.208	-	-	-	-
	Cr.I	[0.182, 0.24]	-	-	-	-
	pd	1.000***	-	-	-	-
b_phi_Intercept	Mean	-	1.599	1.603	1.600	1.619
	Cr.I	-	[1.172, 1.986]	[1.183, 1.984]	[1.176, 1.985]	[1.204, 1.996]
	pd	-	1.000***	1.000***	1.000***	1.000***
b_phi_Fluency	Mean	-	0.429	0.424	0.434	0.396
	Cr.I	-	[-0.157, 0.996]	[-0.155, 0.993]	[-0.131, 1.021]	[-0.202, 0.966]
	pd	-	0.928	0.926	0.929	0.907
b_zi_Intercept	Mean	-	-	-1.670	-	-
	Cr.I	-	-	[-2.476, -0.953]	-	-