

A Beta Way: A Tutorial For Using Beta Regression in Psychological Research

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Abstract

Rates, percentages, and proportions are common outcomes in psychology and the social sciences. These outcomes are often analyzed using models that assume normality, but this practice overlooks important features of the data, such as their natural bounds at 0 and 1. As a result, estimates can become distorted. In contrast, treating such outcomes as beta-distributed respects these limits and can yield more accurate estimates. Despite these advantages, the use of beta models in applied research remains limited. Our goal is to provide researchers with practical guidance for adopting beta regression models, illustrated with an example drawn from the psychological literature. We begin by introducing the beta distribution and beta regression, emphasizing key components and assumptions. Next, using data from a learning and memory study, we demonstrate how to fit a beta regression model in R with the Bayesian package `{brms}` and how to interpret results on the response scale. We also discuss model extensions, including zero-inflated, zero- and one-inflated, and ordered beta models. Basic familiarity with regression modeling and R is assumed. To promote wider adoption of these methods, we provide detailed code and materials at <https://zenodo.org/records/16895241>.

Keywords: beta regression, beta distribution, R tutorial, psychology, learning and memory

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1

2

Introduction

3 Many outcomes in psychological research are naturally expressed as proportions or percentages.
4 These include the proportion of correct responses on a test (e.g., Kornell and Bjork (2008)), the proportion
5 of time a participant fixates on a stimulus in an eye-tracking task (e.g., James et al., 2025), or the proportion of
6 respondents endorsing a particular belief (e.g., Costello et al., 2024). A central question for studies involving
7 proportional data is how to analyze these outcomes in a statistically appropriate and interpretable way.

8 Researchers frequently default to linear models that assume Gaussian (normal) distributions, such
9 as t-tests, ANOVAs, and linear regression. However, these models make strong assumptions: (1) residuals
10 are normally distributed, (2) the outcome is unbounded (from $-\infty$ to ∞), and (3) variance is constant across
11 the range of the data. These assumptions are rarely satisfied in practice (Sladekova & Field, 2024), and they
12 are especially ill-suited for proportional outcomes, which are bounded between 0 and 1 and often exhibit
13 heteroscedasticity—non-constant variance, particularly near the boundaries (Ferrari & Cribari-Neto, 2004;
14 Paolino, 2001; Smithson & Verkuilen, 2006). Violating these assumptions can lead to biased estimates and
15 spurious inferences, especially when many observations cluster near 0 or 1.

16 In some cases, a generalized linear model (GLM) can relax the assumption of normality. For example,
17 binomial and Bernoulli models—often referred to as logistic regression when used with a logit link—are
18 appropriate when the outcome is binary (0/1) or represents the number of successes out of a fixed number
19 of trials. These models, however, rely on data that arise from discrete trial structures. They tend to perform
20 poorly when the observed proportions are truly continuous or when the data show extra variability (overdis-
21 pension), particularly when many values occur near the boundaries of the scale (0 and 1).

22 The challenges of analyzing proportional data are not new (see Bartlett, 1936). Fortunately, several
23 existing approaches address the limitations of commonly used models. One such approach is beta regression,
24 an extension of the generalized linear model that employs the beta distribution (Ferrari & Cribari-Neto, 2004;
25 Paolino, 2001). Beta regression offers a flexible and robust solution for modeling proportional data directly by
26 accounting for boundary effects and over-dispersion, making it a valuable alternative to traditional binomial
27 models. This approach is particularly well-suited for psychological research because it can handle both
28 the bounded nature of proportional data and the non-constant variance often encountered in these datasets
29 (Sladekova & Field, 2024). In addition, the direct modeling of proportions allows comparability across tasks
30 and scales, and can be particularly valuable when only the proportional data is available, as is often the case
31 with secondary data that lack item-level structure or point values. While usage of these models has faced
32 obstacles due to theoretical and computational limitations, as we argue in this paper, beta regression and its
33 extensions now provide an accessible and more robust method to traditional linear modeling.

34 While in this paper we will focus on proportional-responses that lie between 0 and 1—it is important
35 to note that our analysis applies to any bounded continuous scale. Any bounded scale can be mapped to lie
36 within 0 and 1 without resulting in a loss of information as the transformation is linear.¹ Consequently, a
37 scale that has natural end points of -1,234 and +8,451—or any other end points on the real number line short
38 of infinity—can be modeled using the approaches we describe in this paper.

¹Specifically, for any continuous bounded variable x , we can rescale this variable to lie within 0 and 1 by using the formula $x' = \frac{x - \min(x)}{\max(x) - \min(x)}$ where $0 \leq x' \leq 1$.

39 **A Beta Way Is Possible**

40 With the widespread availability of open-source software such as R (R Core Team, 2024) and its
41 extensive ecosystem of user-developed packages, advanced models like beta regression have become increasingly
42 accessible to applied researchers. Yet, their adoption in psychology remains relatively limited. One
43 contributing factor may be the lack of domain-specific examples that demonstrate how these models address
44 common challenges in psychological data. Although recent years have seen a growing interest in beta regression,
45 and a number of useful tutorials are available (Bendixen & Purzycki, 2023; Coretta & Bürkner, 2025;
46 Heiss, 2021; e.g., Smithson & Verkuilen, 2006; Vuorre, 2019), these resources often either focus on basic
47 implementation or briefly mention extensions without detailing how they can be applied to psychologically
48 relevant research questions.

49 The present tutorial aims to help bridge this gap by providing a comprehensive, applied introduction
50 to beta regression and several of its extensions. In addition to the standard beta model, we walk through zero-
51 inflated, zero-one-inflated, and ordered beta regression. These models are particularly useful for researchers
52 working with proportion outcomes that include boundary values (e.g., exact 0s or 1s) or responses with an
53 inherent ordinal structure. Our goal is to offer practical guidance that enables psychological researchers to
54 implement, interpret, and report these models in ways that directly support their empirical questions.

55 Beyond model specification, we place strong emphasis on interpreting results on the response scale—
56 that is, in terms of probabilities and proportions—rather than relying on often difficult to interpret parameters.
57 This focus makes the models more accessible and meaningful for psychological applications, where effects
58 are often easier to communicate when framed on the original scale of the outcome (e.g., changes in recall
59 accuracy or task performance). Throughout, we provide reproducible code and annotated examples to help
60 readers implement and interpret these models in their own work.

61 We begin the tutorial with a non-technical overview of the beta distribution and its core parameters.
62 We then walk through the process of estimating beta regression models using the R package `{brms}` (Bürkner,
63 2017), illustrating each step with applied examples. To guide interpretation, we emphasize coefficients,
64 predicted probabilities, and marginal effects calculated using the `{marginaleffects}` package (Arel-Bundock
65 et al., 2024). We also introduce several useful extensions—zero-inflated (ZIB), zero-one-inflated (ZOIB), and
66 ordered beta regression—that enable researchers to model outcomes that include boundary values. Finally,
67 all code and materials used in this tutorial are fully reproducible and available via our GitHub repository:
68 https://github.com/jgeller112/beta_regression_tutorial and on Zenodo <https://zenodo.org/records/16895241>
69 ².

70 **Beta Distribution**

71 Proportional data pose some challenges for standard modeling approaches: The data are bounded
72 between 0 and 1 and often exhibit non-constant variance (heteroscedasticity) (Ferrari & Cribari-Neto, 2004;
73 Paolino, 2001). Common distributions used within the generalized linear model frameworks often fail to
74 capture these properties adequately, which can necessitate alternative modeling strategies.

²In this article, we try to limit code where possible; however, the online version has all the code needed to reproduce all analyses herein. Furthermore, to promote transparency and reproducibility, the tutorial was written in R version 4.5.1 (R Core Team (2024)) using Quarto (v.1.5.54), an open-source publishing system that allows for dynamic and static documents. This allows figures, tables, and text to be programmatically included directly in the manuscript, ensuring that all results are seamlessly integrated into the document. In addition, we use the `rix` (Rodrigues & Baumann, 2025) R package which harnesses the power of the `nix` (Dolstra & contributors, 2006) ecosystem to help with computational reproducibility. Not only does this give us a snapshot of the packages used to create the current manuscript, but it also takes a snapshot of system dependencies used at run-time. This way reproducers can easily re-use the exact same environment by installing the `nix` package manager and using the included `default.nix` file to set up the right environment. The README file in the GitHub repository contains detailed information on how to set this up to reproduce the contents of the current manuscript, including a video.

75 While we do not have time to delve fully into its derivation, the beta distribution is useful for modeling bounded continuous scales because it is the distribution for the probability of an event. Given that a
 76 probability can take on any value from near 0 (the event will not occur with certainty) to 1 (the event will
 77 occur with certainty), the beta distribution can likewise take on virtually any value in that bounded interval.
 78 As a consequence, the beta distribution is the maximum entropy distribution for *any* bounded continuous
 79 random variable, which means that the beta distribution can represent the full range of possibilities of such a
 80 scale.³ As a consequence, if we have a continuous scale with upper and lower bounds—and no other special
 81 conditions—the beta distribution will in principle provide a very good approximation of the uncertainty of the
 82 scale.

83 Typically, the expected value (or mean) of the response variable is the central estimand scholars want
 84 to estimate. A model should specify how this expected value depends on explanatory variables through two
 85 main components: a linear predictor, which combines the explanatory variables in a linear form ($a + b_1x_1 +$
 86 b_2x_2 , etc.), and a link function, which connects the expected value of the response variable to the linear
 87 predictor (e.g., $E[Y] = g(a + b_1x_1 + b_2x_2)$). In addition, a random component specifies the distribution
 88 of the response variable around its expected value (such as Poisson or binomial distributions, which belong
 89 to the exponential family) (Nelder & Wedderburn, 1972). Together, these components provide a flexible
 90 framework for modeling data with different distributional properties.

91 The beta distribution is continuous and restricted to values between 0 and 1 (exclusive). Its two
 92 parameters—commonly called shape1 (α) and shape2 (β)—govern the distribution’s location, skewness, and
 93 spread. By adjusting these parameters, the distribution can take many functional forms (e.g., it can be sym-
 94 metric, skewed, U-shaped, or even approximately uniform; see Figure 1).

95 To illustrate, consider a test question worth seven points. Suppose a participant scores five out of
 96 seven. The number of points received (5) can be treated as α , and the number of points missed (2) as β . The
 97 resulting beta distribution would be skewed toward higher values, reflecting a high performance (yellow line
 98 in Figure 1; “beta(5, 2)”). Reversing these values would produce a distribution skewed toward lower values,
 99 representing poorer performance (green line in Figure 1; “beta(2, 5)”).

101 I Can’t Believe It’s Not beta

102 While the standard parameterization of the beta distribution uses α and β , a reparameterization to a
 103 mean (μ) and precision (ϕ) is more useful for regression models. The mean represents the expected value
 104 of the distribution, while the dispersion, which is inversely related to variance, reflects how concentrated
 105 the distribution is around the mean, with higher values indicating a narrower distribution and lower values
 106 indicating a wider one. The connections between the beta distribution’s parameters are shown in Equation 1.
 107 Importantly, the variance depends on the average value of the response because uncertainty intervals need to
 108 adjust for how close the value of the response is to the boundary.

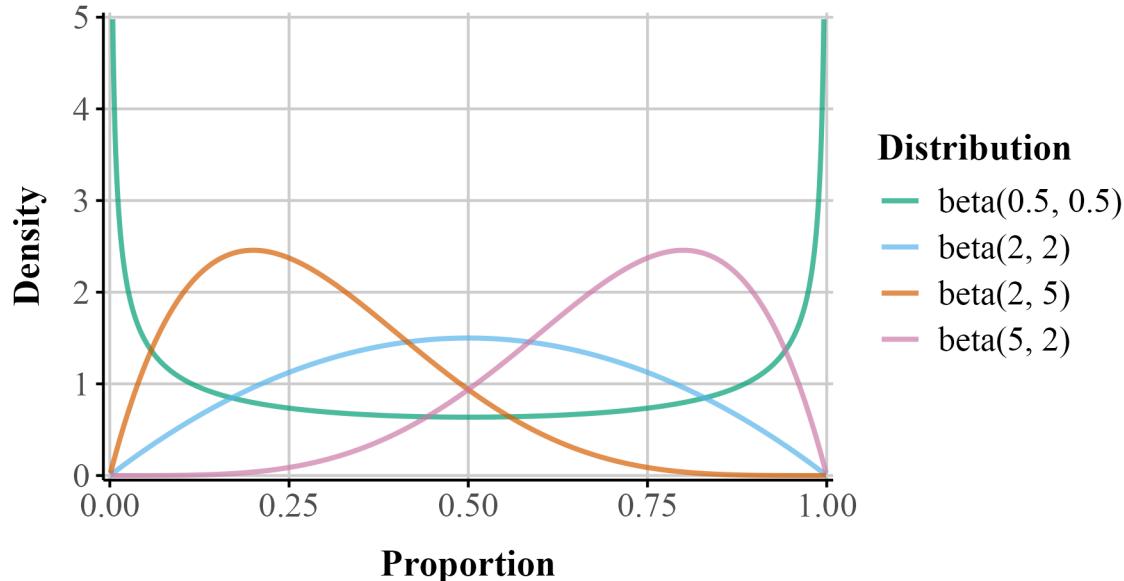
Shape 1: $a = \mu\phi$	Mean: $\mu = \frac{a}{a + b}$	(1)
Shape 2: $b = (1 - \mu)\phi$	Precision: $\phi = a + b$	
	Variance: $var = \frac{\mu \cdot (1 - \mu)}{1 + \phi}$	

109 Thus, beta regression allows modeling both the mean and precision of the outcome distribution. To
 110 ensure that μ stays between 0 and 1, we apply a link function, which allows linear modeling of the mean on
 111 an unbounded scale. A common link-function choice is the logit, but other functions such as the probit or
 112 complementary log-log are possible.

³Technically, this maximum entropy condition is satisfied because the beta(1,1) distribution is uniform over its support. In addition, we assume that the scale has been re-scaled to the [0, 1] interval as we describe above.

Figure 1

beta distributions with different shape1 and shape2 parameters.



113 The logit function, $\text{logit}(\mu) = \log\left(\frac{\mu}{1-\mu}\right)$ links the mean to log-odds which are unbounded, making
 114 linear modeling possible. The logit here no longer carries the same literal *odds* interpretation because there
 115 are no corresponding counts of “successes” and “failures.” Instead, the logit transform here simply maps
 116 the mean of the distribution to the real line. The inverse of the logit, called the logistic function, maps the
 117 linear predictor η back to the original scale of the data $(\mu = \frac{1}{1+e^{-\eta}})$. The coefficients describe how predictors
 118 shift the *average proportion* on the logit scale. Similarly, the strictly positive dispersion parameter is usually
 119 modeled through a log link function, ensuring it remains positive.

120 By accounting for the observations’ natural limits and non-constant variance, the beta distribution
 121 is useful in psychology where outcomes like performance rates or response scales frequently exhibit these
 122 features.

123 Bayesian Approach to Beta Regression

124 Beta regression models can be estimated using either frequentist or Bayesian methods. In this paper,
 125 we adopt a Bayesian framework because it facilitates the estimation and interpretation of more complex
 126 models (Gelman et al., 2013; Johnson et al., 2022; McElreath, 2020). Additionally, the use of Bayesian
 127 statistics in psychology has been steadily growing (Pfadt et al., 2025). In principle, frequentist methods
 128 like maximum likelihood can be framed as Bayesian models with uninformative priors, and as a result, the
 129 modeling perspective we put forward in this paper can apply to either approach. Nonetheless, we note that
 130 in non-linear and hierarchical models, frequentist estimation may require additional adjustments such as
 131 bootstrapping to obtain proper uncertainty intervals, whereas Bayesian modeling handles these extensions
 132 more naturally via exploration of the full joint posterior distribution.⁴

⁴A common concern is that Bayesian methods are slower than frequentist ones. While this is true in general, modern Bayesian computation engines are efficient, and in explanatory modeling our priority is to specify appropriate estimands rather than to optimize for computation speed. Moreover, we use the `{brms}` package (Bürkner, 2017), which provides a high-level interface to Stan (Team, 2023) and uses familiar R formula syntax (similar to `lm()`), making advanced Bayesian modeling accessible to researchers with standard regression backgrounds. The package also supports parallelization, which substantially reduces computation time for large

133 There are several important differences between our Bayesian analysis and the frequentist methods
 134 readers may be more familiar with—most notably, the absence of t - and p -values. To estimate models, the
 135 `{brms}` package uses Stan’s computational algorithms to draw random samples from the posterior distribu-
 136 tion, which represents uncertainty about the model parameters. This posterior is conceptually analogous to
 137 a frequentist sampling distribution. By default, Bayesian models run 4 chains with 2,000 iterations each.⁵
 138 The first 1,000 iterations per chain are warmup and are discarded. The remaining 1,000 iterations per chain
 139 are retained as posterior draws, yielding 4,000 total post-warmup draws across all chains. From these draws,
 140 we can compute the posterior mean (analogous to a frequentist point estimate) and the 95% credible interval
 141 (Cr.I.), which is often compared to a confidence interval.

142 In addition, an important part of Bayesian analyses is prior specification. Priors encode our assump-
 143 tions about plausible parameter values before observing the data and allow the model to regularize estimates,
 144 especially when data are sparse or parameters are weakly identified. To help bridge the conceptual gap for
 145 users more familiar with frequentist models, we begin with the default priors (flat/non-informative, or weakly
 146 informative in some cases) provided by `{brms}`. These priors are intentionally non-informative, and in many
 147 applications produce results that closely align with frequentist estimates, while still offering the flexibility
 148 and interpretive advantages of a Bayesian framework. We strongly urge readers to consider prior specification
 149 strongly in all their work.

150 To ease readers into Bayesian data analysis we provide a metric known as the *probability of direction*
 151 (*pd*), which reflects the probability that a parameter is positive or negative. When a uniform prior is used
 152 (all values equally likely in the prior), *pds* of 95%, 97.5%, 99.5%, and 99.95% corresponds approximately
 153 to two-sided *p*-values of .10, .05, .01, and .001 (i.e., $pd \approx 1 - p/2$ for symmetric posteriors with weak/flat
 154 priors) (see Figure 2 for an illustrative comparison). For directional hypotheses, the *pd* can be interpreted as
 155 roughly equivalent to one minus the *p*-value (Marsman & Wagenmakers, 2016).

156 For reasons of space, we refer readers unfamiliar to Bayesian data analysis to several existing books
 157 on the topic (Gelman et al., 2013; Kruschke, 2015; McElreath, 2020). In addition, we assume readers are
 158 familiar with R, but those in need of a refresher should find Wickham et al. (2023) useful.

159 Beta Regression Tutorial

160 Example Data

161 Throughout this tutorial, we analyze data from a memory experiment examining whether the flu-
 162 ency of an instructor’s delivery affects recall performance (Wilford et al., 2020, Experiment 1A). Instructor
 163 fluency—marked by expressive gestures, dynamic vocal tone, and confident pacing—has been shown to
 164 influence students’ perceptions of learning, often leading learners to rate fluent instructors more favorably
 165 (Carpenter et al., 2013). However, previous research suggests that these impressions do not reliably translate
 166 into improved memory performance (e.g., Carpenter et al., 2013; Toftness et al., 2017; Witherby &
 167 Carpenter, 2022). In contrast, Wilford et al. (2020) found that participants actually recalled more information
 168 after watching a fluent instructor compared to a disfluent one. This surprising finding makes the dataset a
 169 compelling case study for analyzing proportion data, as recall was scored out of 10 possible idea units per
 170 video.

171 In Experiment 1A, ninety-six participants watched two short instructional videos, each delivered
 172 either fluently or disfluently. Fluent videos featured instructors with smooth delivery and natural pacing,
 173 while disfluent videos included hesitations, monotone speech, and awkward pauses. After a distractor task,
 174 participants completed a free recall test, writing down as much content as they could remember from each

datasets.

⁵The Hamiltonian Monte Carlo sampler employed by Stan, which we also use in this paper, can converge with significantly fewer iterations, though rapid convergence depends on model complexity, which is why we use a more conservative standard in this paper.

Figure 2

A Bayesian posterior distribution (assuming a uniform prior) centered at a point estimate chosen so that the probability of direction (pd) equals 0.95, and a frequentist sampling distribution (under the null; centered at 0). In the Bayesian posterior distribution, the blue area represents the pd , and the red area represents the remaining $1 - pd$ of the distribution. In the frequentist sampling distribution, the red tail areas represent the rejection region at $\alpha = 0.10$. In this example, the posterior mean lies exactly at the $1 - \frac{\alpha}{2}$ quantile of the null sampling distribution. For symmetric posteriors with flat priors, the pd is numerically equivalent to the one-sided p -value.

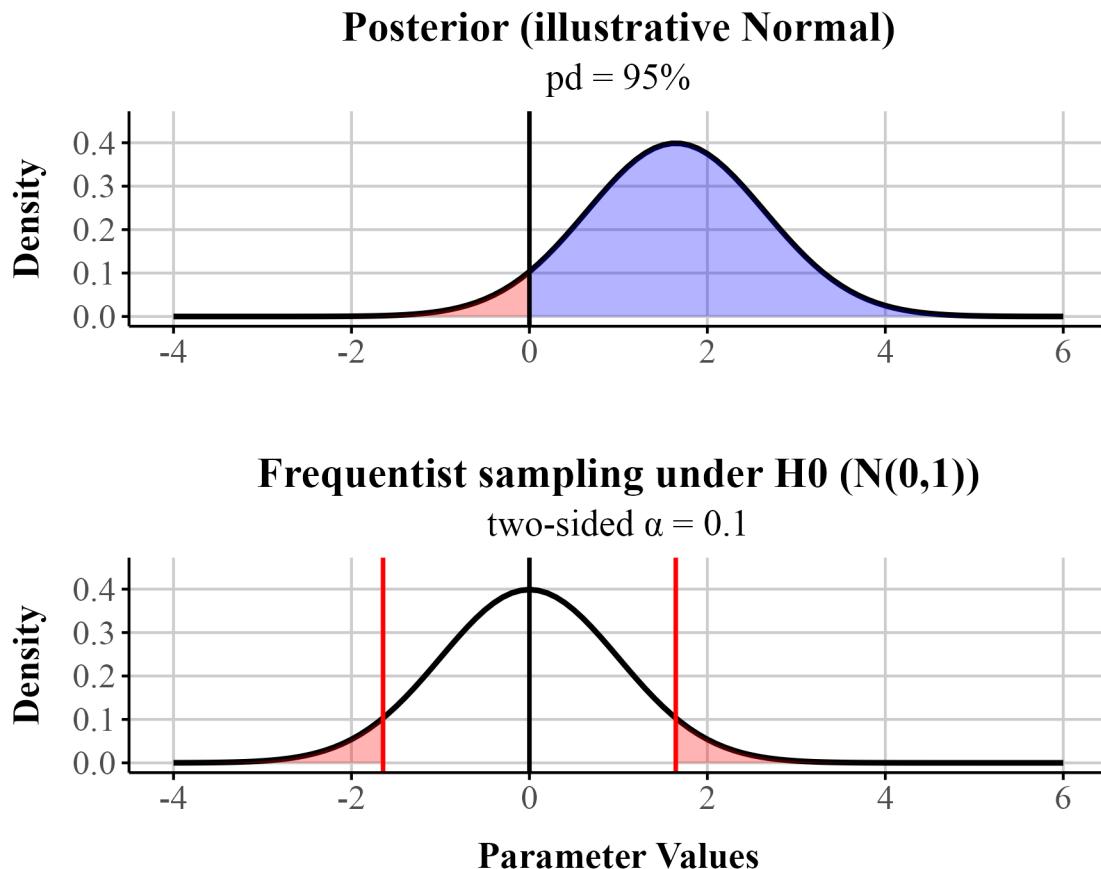


Table 1

Four observations from Wilford et al. (2020). Accuracy refers to the proportion of correctly recalled idea units.

Participant	Fluency	Accuracy
64	Disfluent	0.10
30	Fluent	0.60
12	Fluent	0.10
37	Fluent	0.35

175 video within a three-minute window. Their recall was then scored for the number of idea units correctly
 176 remembered.

Listing 1 Data needed to run examples

```
# get data here from project folder
fluency_data <- read_csv(here::here("manuscript", "data", "fluency_data.csv"))
```

177 Our primary outcome variable is the proportion of idea units recalled on the final test, calculated by
 178 dividing the number of correct units by 10. We show a sample of these data in Table 1. The dataset can be
 179 downloaded from GitHub (Listing 1). Because this is a bounded continuous variable (i.e., it ranges from 0 to
 180 1), it violates the assumptions of typical linear regression models that assume normally distributed residual
 181 errors. Despite this, it remains common in psychological research to analyze proportion data using models
 182 that assume normality. In what follows, we reproduce Wilford et al. (2020)'s analysis and then re-analyze
 183 the data using beta regression and highlight how it can improve our inferences.

184 Reanalysis of Wilford et al. Experiment 1A

185 In their original analysis of Experiment 1A, Wilford et al. (2020) compared memory performance
 186 between fluent and disfluent instructor conditions using a traditional independent-samples t-test on mean
 187 accuracy for 96 participants. They found that participants who watched the fluent instructor recalled signifi-
 188 cantly more idea units than those who viewed the disfluent version (see Figure 3).

189 We first replicate this analysis in a regression framework using {brms}. We model final test mean
 190 accuracy—the proportion of correctly recalled idea units across the videos—as the dependent variable. Our
 191 predictor is instructor fluency, with two levels: Fluent and Disfluent. We use treatment (dummy) coding,
 192 which is the default in R. This coding scheme sets the first level of a factor (in alphabetical order) as the
 193 reference level. In this case, Disfluent is the reference, and the coefficient for Fluent reflects the contrast
 194 between fluent and disfluent instructor conditions.

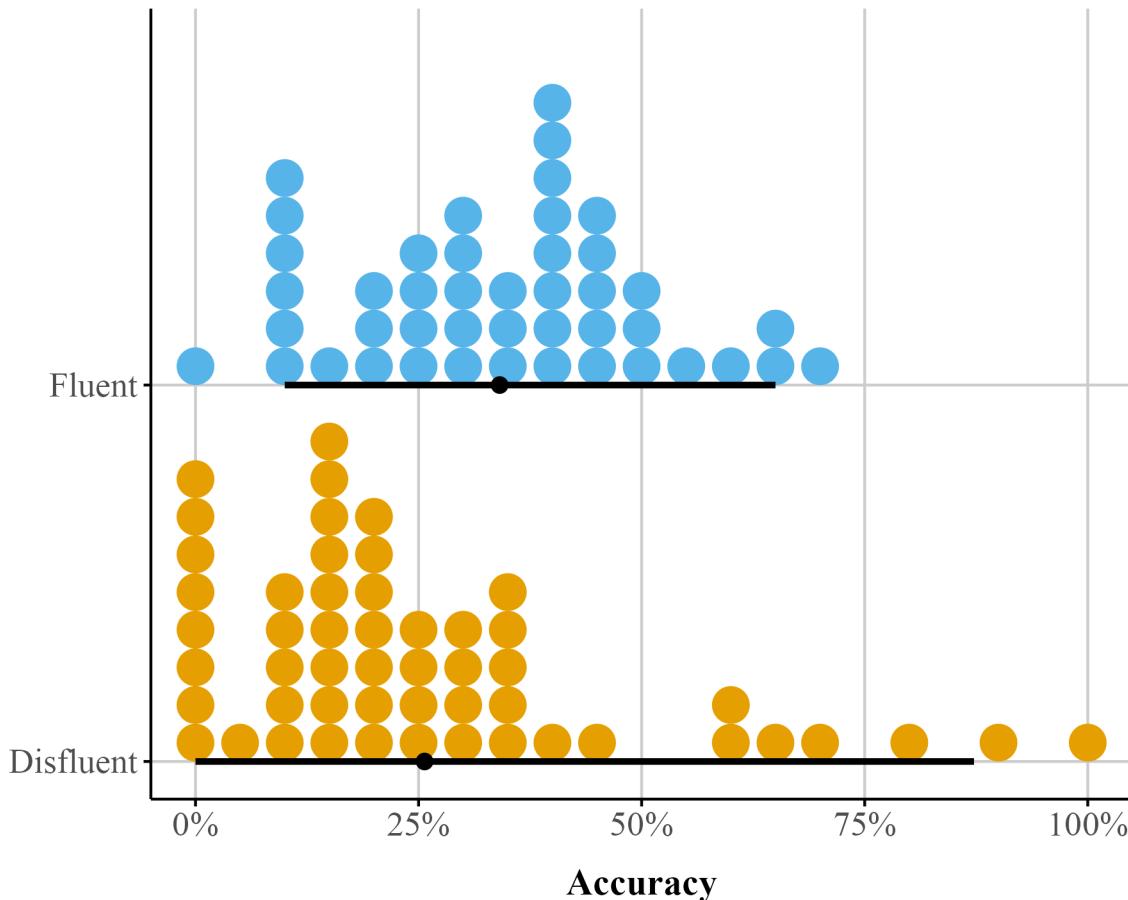
195 Regression Model

196 We first start by loading the {brms} (Bürkner, 2017) and {cmdstanr} (Gabry et al., 2024) packages
 197 (Listing 2). We use the cmdstanr backend for Stan (Team, 2023) because it's faster than the default used to
 198 run models (i.e., rstan),⁶ though all of these models can also be fit with brms defaults.

⁶In order to use the cmdstanr backend you will need to first install the package (see <https://mc-stan.org/cmdstanr/>) and also run cmdstanr::install_cmdstan() if you have not done so already.

Figure 3

Dot plot depicting accuracy distributions for the Fluent and Disfluent conditions. Each condition shows individual data points and summary statistics (mean and 95% percentile interval) to illustrate variability and central tendency.



Listing 2 Load the `brms` and `cmdstanr` packages

```
library(brms)
library(cmdstanr)
```

199 We fit the model using the `brm()` function from the `{brms}` package (Listing 3). Although not
 200 shown here, we ran the models using four chains (the default), executed in parallel across four cores. When
 201 the model is run in Listing 3, the model summary output will appear in the R console. The output from
 202 `bayes_reg_model` shows each parameter's posterior summary: The posterior distribution's mean and stan-
 203 dard deviation (analogous to the frequentist standard error) and its 95% credible interval, which indicate the
 204 95% of the most credible parameter values. In `{brms}`, the reported Cr.I is an equal-tailed interval, meaning
 205 that the probability mass excluded from the interval is split equally between the lower and upper tails. Ad-
 206 ditionally, the output indicates numerical estimates of the sampling algorithm's performance: Rhat should
 207 be close to one, and the ESS (effective sample size) metrics should be as large as possible given the number
 208 of iterations specified (default is 4000). Generally, ESS ≥ 1000 is recommended (Bürkner, 2017). For the

Listing 3 Fitting a gaussian model with brm().

```

bayes_reg_model <- brm(
  Accuracy ~ Fluency,
  data = fluency_data,
  family = gaussian(),
  file = here::here("manuscript", "models", "model_reg_bayes")
)

```

209 models we present in this paper, convergence is trivial with standard linear models, though we note that these
210 metrics are still important to pay attention to in case of model misfit.

211 Our main question of interest is: what is the causal effect of instructor fluency on final test performance?
212 In order to answer this question, we will have to look at the output summary produced by Listing 3
213 (also see Table 8 under Bayesian LM). the Intercept refers to the posterior mean accuracy in the disfluent
214 condition, $M = 0.256$, as fluency was dummy-coded. The fluency coefficient (FluencyFluent) reflects the
215 mean posterior difference in recall accuracy between the fluent and disfluent conditions: $b = 0.086$. The 95%
216 Cr.I for this estimate spans from 0.004 to 0.17. These values are shown in the “95% Cr.I” columns of the
217 output. These results closely mirror the findings reported by Wilford et al. (2020) (Experiment 1A).

```

218 Family: gaussian
219 Links: mu = identity; sigma = identity
220 Formula: Accuracy ~ Fluency
221 Data: fluency_data (Number of observations: 96)
222
223 Regression Coefficients:
224             Estimate Est.Error 1-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
225 Intercept       0.26      0.03    0.20    0.31 1.00     3852    2651
226 FluencyFluent   0.09      0.04    0.00    0.17 1.00     4148    2475
227
228 Further Distributional Parameters:
229             Estimate Est.Error 1-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
230 sigma        0.21      0.02    0.18    0.24 1.00     3974    2960

```

231 The output also includes the effective sample size (ESS) and R (R-hat) values, both of which fall
232 within acceptable ranges, indicating good model convergence. Throughout the tutorial, we focus primarily
233 on posterior mean estimates and their 95% credible intervals. In addition, we report the pd measure in the
234 main summary table (Table 8), provided by the {bayestestR} package (Makowski, Ben-Shachar, Chen, et
235 al., 2019; Makowski, Ben-Shachar, & Lüdecke, 2019). This measure offers an intuitive parallel to p -values,
236 which many readers may find familiar. For example, the fluency effect has a pd of .977, indicating a high
237 probability that the effect is positive rather than negative.

238 Importantly, pd does not indicate whether an effect is meaningfully different from a point value—it
239 only reflects the proportion of the posterior in one direction. To address questions of practical significance,
240 readers can consider the Region of Practical Equivalence (ROPE) with the Cr.Is (Kruschke, 2015). Unlike
241 a hypothesis test of a point null (e.g., exactly zero), the ROPE defines a range of values that are deemed
242 too small to be of substantive importance. As a rule of thumb (see Kruschke, 2018), if more than 95%
243 of the posterior distribution lies inside the ROPE, the effect can be considered practically equivalent to that

²⁴⁴ negligible range. If less than 5% lies inside, the effect can be considered meaningfully different. Intermediate
²⁴⁵ cases are typically labeled undecided.

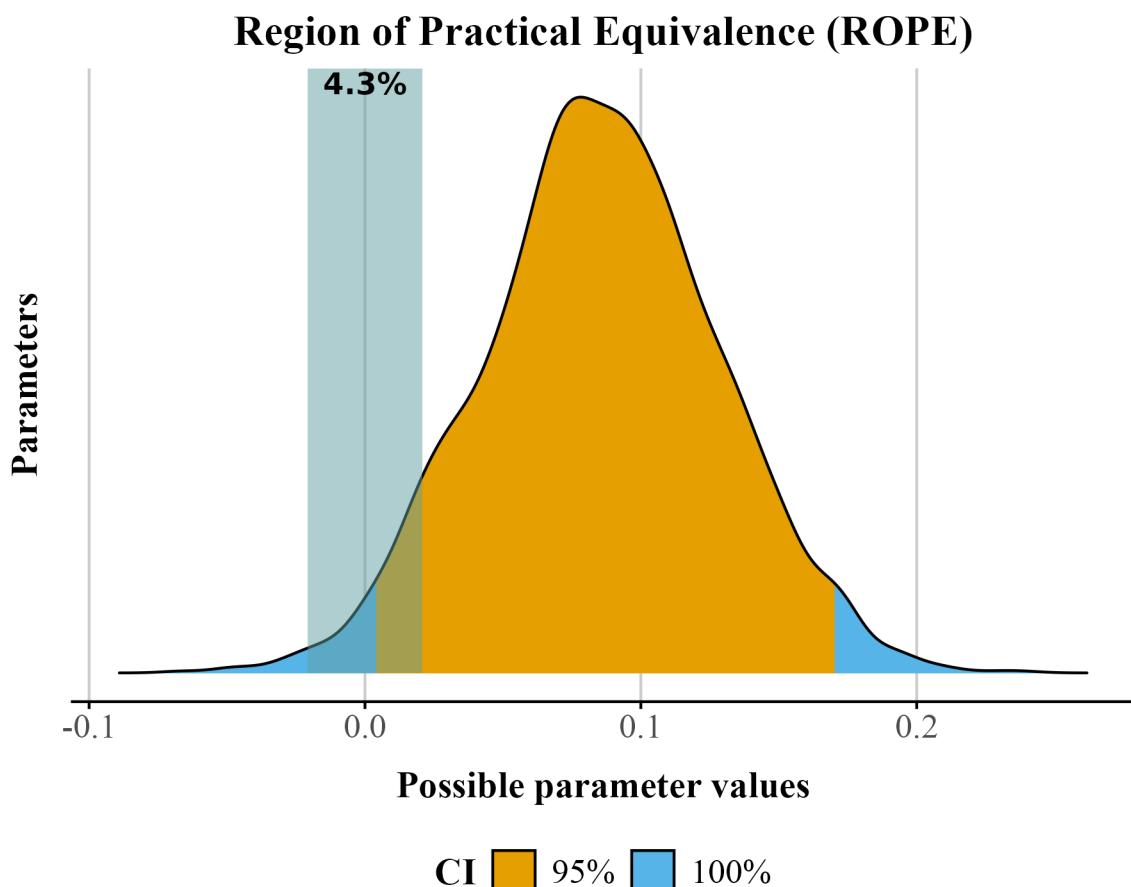
²⁴⁶ The `rope()` function in the `{bayestestR}` package computes the proportion of the posterior within
²⁴⁷ the ROPE to facilitate this evaluation. By default, from bayesian models fit via `{brms}` the package deter-
²⁴⁸mines a ROPE based on the data (roughly reflecting “negligible” effects), but these defaults should be used
²⁴⁹cautiously. The choice of ROPE should always be guided by theoretical considerations, previous research,
²⁵⁰and the substantive context of the study. In Listing 4, we show how to compute this using `{bayestestR}`.
²⁵¹Running the function with default settings suggests that less than 5% of the posterior distribution lies within
²⁵²the default ROPE (indicating the effect is larger than .02) (see Figure 4).

Listing 4 Getting ROPE from `bayes_reg_model` object using `rope` function from `{bayestestR}`

```
brms_rope <- bayestestR::rope(bayes_reg_model, ci = .95, ci_method = "ETI")
```

Figure 4

Posterior distribution for the fluency effect showing the ROPE(shaded area) with 95% credible interval (orange) and 100% credible interval (blue). The percentage indicates the proportion of the posterior within the ROPE.



²⁵³ Wilford et al. (2020) observed that instructor fluency impacts actual learning, using a standard *t*-test

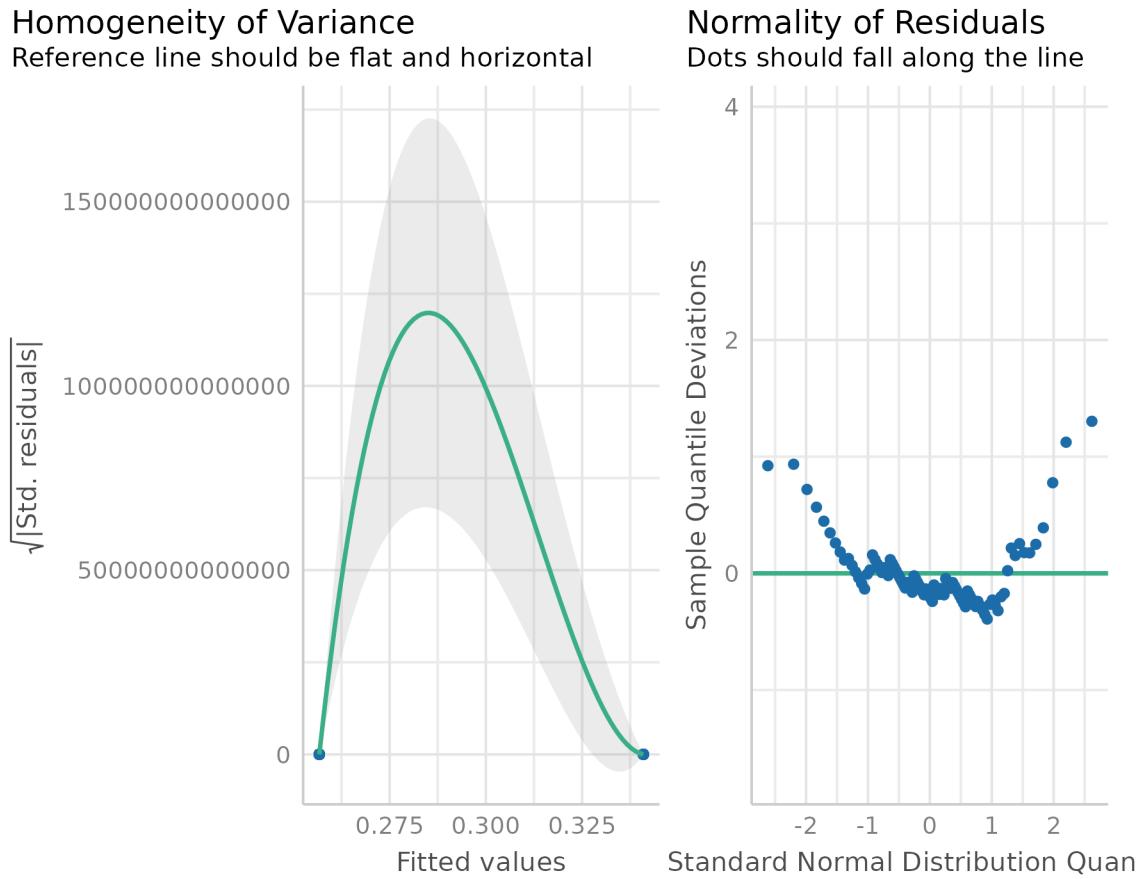
on the mean accuracy. But recall this approach assumes normality of residuals and homoskedasticity. These assumptions are unrealistic when the response values approach the scale boundaries (Sladekova & Field, 2024). Does the data we have meet those assumptions? We can use the function `check_model()` from `{easystats}` (Lüdecke et al., 2022) to check our assumptions easily. The code in Listing 5 produces Figure 5. We can see some issues with our model. Specifically, there appears to be violations of constant variance across the values of the scale (homoskedasticity). In plain terms, this type of model mis-specification means that a standard OLS model can predict non-sensical values outside the bounds of the scale.

Listing 5 Checking assumptions with the `check_model()` from `easystats` package .

```
check_model(bayes_reg_model, check = c("homogeneity", "qq"))
```

Figure 5

Two assumption checks for our OLS model: Normality (right) and Homoskedasticity (left)



We can also examine how well the data fits the model by performing a posterior predictive check using the `pp_check()` function from `{brms}`. A posterior predictive check involves looking at multiple model-predicted values and plotting them against the observed data. Ideally, the predicted values (the light blue lines) should show reasonable resemblance with the observed data (dark blue line). In our example (see Figure 12 (A)) the model-predicted density is slightly too peaked and narrow compared to the data. In

266 addition, some of the predicted accuracy values are negative.

267 **Distributional Regression - Beta Regression**

268 It is important to note that there are several justifiable approaches for addressing the distributional
 269 issues observed in the data. For instance, one could analyze median accuracy instead of the mean, use some
 270 type of robust estimator for heterogeneity, or apply non-parametric methods to relax some of the model
 271 assumptions. Alternatively, we can address these issues directly by fitting distributional models (Kneib et
 272 al., 2023; Kruschke, 2013). A key advantage of distributional models is that they are not limited to modeling
 273 only the mean or median of the outcome, but can also model parameters such as the variance (or other shape
 274 parameters) as functions of predictors. This allows examining how instructor fluency may influence not only
 275 average performance, but also the variability in performance across students. If we wanted to keep our mean
 276 accuracy variable and continue to use a Gaussian model, we could use a distributional approach and model
 277 the effect of fluency on σ .

278 Given the outcome variable is proportional, another solution would be to run a beta regression model.
 279 Again, we can create the beta regression model in {brms}. In {brms}, we model each parameter indepen-
 280 dently. Recall from the introduction that in a beta model we model two parameters— μ and ϕ . Again we do
 281 this by using the `bf()` function from {brms} (Listing 6). We specify two formulas, one for μ and one for ϕ
 282 and store it in the `model_beta_bayes` object below. In the below `bf()` call, we are modeling accuracy as a
 283 function of fluency only for the μ parameter. For the ϕ parameter, we are only modeling the intercept value.
 284 This is saying dispersion does not change as a function of fluency.

285 To run our beta regression model, we need to exclude 0s and 1s in our data set. If we try to run
 286 a model with our data `data_fluency` we get an error: `Error: Family 'beta' requires response`
 287 `greater than 0`. This is because the beta distribution only supports observations in the 0 to 1 interval
 288 `excluding exact 0s and 1s`. We need make sure there are no 0s and 1s in our dataset.

289 The dataset contains nine 0s and one 1. One approach is to nudge our 0s towards .01 and our 1s to
 290 .99, or apply a special formula (Smithson & Verkuilen, 2006) so values fall within the [0, 1] interval. We
 291 implore readers not to engage in this practice. Kubinec (2022) showed that this practice can result in serious
 292 distortion of the outcome as the sample size grows larger, resulting in ever smaller values that are “nudged”.
 293 Because the beta distribution is a non-linear model of the outcome, values that are very close to the boundary,
 294 such as 0.00001 or 0.99999, will be highly influential outliers. To run this beta model we will remove the 0s
 295 and 1s, and later in this article we will show how to jointly model these scale end points with the rest of the
 296 data. The model from Listing 6 uses a transformed `data_fluency` object (called `data_beta`) where 0s and
 297 1s are removed. When we run this code we should not get an error.

298 **Model Parameters.** In Table 8, under the beta regression column, the coefficient with `b_` repre-
 299 sents how fluency of instructor influences the μ parameter estimates (which is the mean of the distribution
 300 here). These coefficients are linear on the logit-scale, but not on the raw accuracy scale. The intercept term
 301 (`b_Intercept`) represents the log odds of the mean on accuracy for the fluent instructor. Log odds that are
 302 negative indicate that it is more likely a “success” (like getting the correct answer) will not happen than that
 303 it will happen. Similarly, regression coefficients in log odds forms that are negative indicate that an increase
 304 in that predictor leads to a decrease in the predicted probability of a “success”.

305 The other component we need to pay attention to is the dispersion or precision parameter coefficients
 306 labeled as `phi` in Table 8. The dispersion (ϕ) parameter tells us how precise our estimate is. Specifically,
 307 ϕ in beta regression tells us about the variability of the response variable around its mean. Specifically, a
 308 higher dispersion parameter indicates a narrower distribution, reflecting less variability. Conversely, a lower
 309 dispersion parameter suggests a wider distribution, reflecting greater variability. The main difference between
 310 a dispersion parameter and the variance is that the dispersion has a different interpretation depending on the
 311 value of the outcome, as we show below. The best way to understand dispersion is to examine visual changes

Listing 6 Fitting a beta model without 0s and 1s in brm().

```
# set up model formual
model_beta_bayes <- bf(
  Accuracy ~ Fluency, # fit mu model
  phi ~ 1 # fit phi model
)

# transform 0 to 0.1 and 1 to .99
data_beta <- fluency_data |>
  filter(
    Accuracy != 0,
    Accuracy != 1
  )

beta_brms <- brm(
  model_beta_bayes,
  data = data_beta,
  family = Beta(),
  file = here::here("manuscript", "models", "model_beta_bayes_reg_01")
)
```

312 in the distribution as the dispersion increases or decreases.

313 Understanding the dispersion parameter helps us gauge the precision of our predictions and the con-
 314 sistency of the response variable. In `beta_brms` we only modeled the dispersion of the intercept. When
 315 ϕ is not specified, the intercept is modeled by default (see Table 8). It represents the overall dispersion in
 316 the outcome across all conditions. Instead, we can model different dispersions across levels of the Fluency
 317 factor. To do so, we add `Fluency` to the `phi` model in `bf()`. We model the precision (`phi`) of the `Fluency`
 318 factor by using a `~` and adding factors of interest to the right of it (Listing 7).

Listing 7 Fitting beta model with dispersion in brm().

```
model_beta_bayes_disp <- bf(
  Accuracy ~ Fluency, # Model of the mean
  phi ~ Fluency # Model of the precision
)

beta_brms_dis <- brm(
  model_beta_bayes_disp,
  data = data_beta,
  family = Beta(),
  file = here::here("manuscript", "models", "model_beta_bayes_dis_run01")
)
```

319 Table 8 displays the model summary with the precision parameter labeled as `phi_Fluency`. Since ϕ
 320 is modeled on the log scale, the coefficients represent changes in $\log\phi$ rather than ϕ itself. To see the effect

321 in the original units, we convert the values back by exponentiating. Thus, the effect of the Fluent condition
 322 can be understood by comparing the exponentiated predicted ϕ in the Fluent condition to that in the baseline
 323 condition.

324 The ϕ parameters are estimated on the log scale. The term $\beta_{\text{Intercept}}^{(\phi)}$ represents the log-precision for
 325 the reference (disfluent) condition. The coefficient $\beta_{\text{FluencyFluent}}^{(\phi)}$ represents the change in log-precision when
 326 moving from the disfluent to the fluent condition.

327 To obtain precision on the original scale, we exponentiate the linear predictor:

$$\phi_{\text{disfluent}} = \exp(\beta_{\text{Intercept}}^{(\phi)}), \quad \phi_{\text{fluent}} = \exp(\beta_{\text{Intercept}}^{(\phi)} + \beta_{\text{Fluency}}^{(\phi)}).$$

328 The coefficient $\beta_{\text{Fluency}}^{(\phi)}$ therefore describes a *multiplicative* change in precision. Specifically,

$$\frac{\phi_{\text{fluent}}}{\phi_{\text{disfluent}}} = \exp(\beta_{\text{Fluency}}^{(\phi)}).$$

329 Because the 95% credible interval for $\beta_{\text{FluencyFluent}}^{(\phi)}$ does not include zero, we infer that there is a
 330 credible difference in precision between the fluent and disfluent conditions.

331 It is important to note that these estimates are not the same as the marginal effects we discussed
 332 earlier. Changes in dispersion affect the spread or variability of the response distribution without necessarily
 333 altering its mean. This makes dispersion particularly relevant for research questions that focus on features
 334 of the distribution beyond the average—such as how concentrated responses are. For instance, high dispersion
 335 might indicate that individuals cluster at the extremes (e.g., very high or very low ratings), suggesting
 336 clustering in the outcome.

337 A critical assumption of the linear model is homoscedasticity, which means constant variance of the
 338 errors. With beta regression model we can include a dispersion parameter for Fluency. Properly accounting
 339 for dispersion is crucial because it impacts the precision of our mean estimates and, consequently, the sub-
 340 stantive inferences we might make about the coefficients. The inclusion of dispersion in the model increased
 341 the uncertainty of the μ coefficient (see Figure 6). This highlights the potential utility of an approach like
 342 beta regression over a traditional approach as beta regression can explicitly model dispersion and address
 343 issues of heteroscedasticity.

344 While it is advisable to model precision, if there is uncertainty about the best model, a relatively
 345 agnostic approach would be to compare models, for example with leave one out (loo) cross validation, to
 346 examine if a dispersion parameter should be considered in our model.⁷

347 **Predicted Probabilities**

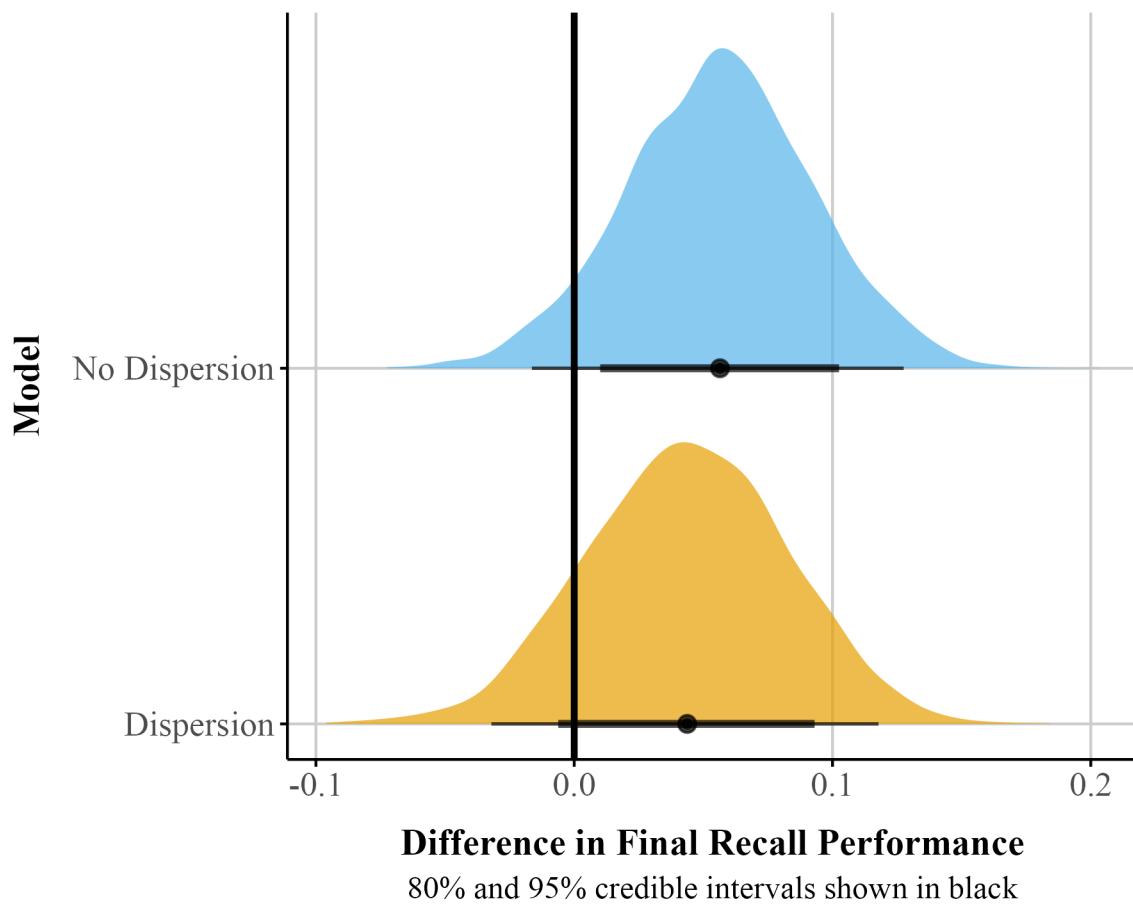
348 Parameter estimates can be difficult to interpret, and researchers can instead discuss effects on the
 349 actual outcome scale (in this case the 0-1 scale). The logit link allows us to transform back and forth between
 350 the scale of a linear model and the nonlinear scale of the outcome, which is bounded by 0 and 1. By using the
 351 inverse of the logit, we can easily transform our linear coefficients to obtain average effects on the scale of the
 352 proportions or percentages, which is usually easier to interpret. In a simple case, we can do this manually,
 353 but when there are many factors in your model this can be quite complex.

354 In our example, we can use the `plogis()` function in base R to convert estimates from the logit scale
 355 to the probability scale. The intercept of our model is -0.918, which reflects the logit of the mean accuracy
 356 in the disfluent condition. If the estimated difference between the fluent and disfluent conditions is 0.24 on
 357 the logit scale, we first add this value to the intercept value (-0.918) to get the logit for the fluent condition:
 358 $-0.83 + 0.20 = -0.63$. We then use `plogis()` to convert both logit values to probabilities (Fluent =
 359 35%, Disfluent = 30%).

⁷The model fit statistic LOO-CV can be compared for any set of fitted brms models with the function `loo()`.

Figure 6

Comparison of posterior distributions for the risk difference in fluency: Simple model (no dispersion for Fluency) vs. complex model with dispersion



With single coefficients this calculation is trivial, but in more complex models with interactions, it can be quite cumbersome. To help us extract predictions from our model and visualize them we will use a package called `{marginaleffects}` (Arel-Bundock et al., 2024) (see Listing 8). To get the proportions for each of our categorical predictors on the μ parameter we can use the function from the package called `predictions()`. These are displayed in Table 2. These probabilities match what we calculated above.

Listing 8 Load the `{marginaleffects}` package.

```
library(marginaleffects)
options(marginaleffects_posterior_center = mean) # make sure returns mean
```

For the Fluency factor, we can interpret Mean as proportions or percentages. That is, participants who watched the fluent instructor scored on average 35% on the final exam compared to 30% for those who watched the disfluent instructor. We can also visualize these from `{marginaleffects}` using the `plot_predictions()` function (see Listing 10).

The `plot_predictions()` function will only display the point estimate with the 95% credible inter-

Listing 9 Predictions from the beta model for each level of Fluency.

```
predictions(
  beta_brms,
  # need to specify the levels of the categorical predictor
  newdata = datagrid(Fluency = c("Disfluent", "Fluent"))
)
```

Table 2*Predicted probabilities for fluency factor.*

Fluency	Mean	95% Cr.I
Disfluent	0.297	[0.249, 0.349]
Fluent	0.354	[0.302, 0.407]

370 val. However, Bayesian estimation methods generate distributions for each parameter. This approach allows
 371 visualizing full uncertainty estimates beyond points and intervals. Using the `{marginaleffects}` package, we
 372 can obtain samples from the posterior distribution with the `posterior_draws()` function (see Listing 11).
 373 We can then plot these results to illustrate the range of plausible values for our estimates at different levels
 374 of uncertainty (see Figure 7).

375 **Marginal Effects**

376 Marginal effects offer an interpretable way to quantify how changes in a predictor influence an out-
 377 come, while holding other factors constant in a specific manner. In recent years, there has been a thrust to
 378 move away from reporting regression coefficients alone, focusing instead on estimates that are easier to in-
 379 terpret and communicate—particularly in non-linear models (McCabe et al., 2021; Rohrer & Arel-Bundock,
 380 2025). Technically, marginal effects are computed as partial derivatives for continuous variables or as finite
 381 differences for categorical (and sometimes continuous) predictors, depending on the structure of the data and
 382 the research question. Substantively, these procedures translate raw regression coefficients into quantities
 383 that reflect changes in the bounded outcome—for example, an $x\%$ change in the value of a proportion.

384 There are various types of marginal effects, and their calculation can vary across software packages.
 385 For example, the popular `{emmeans}` package (Lenth, 2025) computes marginal effects by holding all pre-
 386 dictors at their means (MEM). In this tutorial, we will use the `{marginaleffects}` package (Arel-Bundock et
 387 al., 2024), which focuses on average marginal effects (AMEs) by default. AMEs summarize effects by gen-
 388 erating predictions for each row of the original dataset and then averaging these predictions. This approach
 389 retains a strong connection to the original data while offering a straightforward summary of the effect of
 390 interest.

391 One practical use of AMEs is to estimate the average difference between two groups or conditions
 392 which corresponds to the average treatment effect (ATE). Using the `avg_comparisons()` function in the
 393 `{marginaleffects}` package (Listing 12), we can compute this quantity directly. By default, the function returns
 394 the discrete difference between groups. When we take the difference in proportions between two groups it
 395 is called the risk difference. Depending on the audience and modeling goals, the function can also produce
 396 alternative effect size metrics, such as odds ratios or risk ratios. This flexibility makes it a powerful approach
 397 for summarizing and communicating regression results.

398 Table 3 presents the estimated difference for the Fluency factor (Mean column). The difference

Listing 10 Plot predicted probabilities using `plot_predictions()` from `{marginaleffects}`

```
beta_plot <- plot_predictions(beta_brms, by = "Fluency")
```

Listing 11 Extracting posterior draws from the beta regression model.

```
# Add a model identifier to each dataset
pred_draws_beta <- avg_predictions(beta_brms, variables = "Fluency") |>
  posterior_draws()
```

399 between the fluent and disfluent conditions is 0.06, indicating that participants who watched a fluent instructor
 400 scored, on average, 6% higher on the final recall test than those who watched a disfluent instructor. However,
 401 the 95% credible interval includes 0 among the most credible values, suggesting we cannot rule out the
 402 possibility of a null or weakly negative effect.

403 We can also use `{marginaleffects}` to get the actual precision difference between the two groups on
 404 ϕ using similar code to above by setting `dpar` to "phi" {Listing 13}.

405 In psychology, it is common to report effect size measures like Cohen's d (Cohen, 1977). When
 406 working with proportions we can calculate something similar called Cohen's h . Taking our proportions, we
 407 can use the below equation (Equation 2) to calculate Cohen's h along with the 95% Cr.I around it. Using this
 408 metric we see the effect size is small (0.107), 95% credible interval [-0.002, 0.361].

$$h = 2 \cdot (\arcsin(\sqrt{p_1}) - \arcsin(\sqrt{p_2})) \quad (2)$$

409 **Posterior Predictive Check**

410 Figure 12 (B) shows the predictive check for our beta model. The model's predictions generally
 411 conform to the data as the predictions are now between constrained to the 0-1 interval. However, we can
 412 further improve the model's predictive performance if we take into account the bounds of the scale more
 413 explicitly.

414 **Zero-Inflated beta (ZIB) Regression**

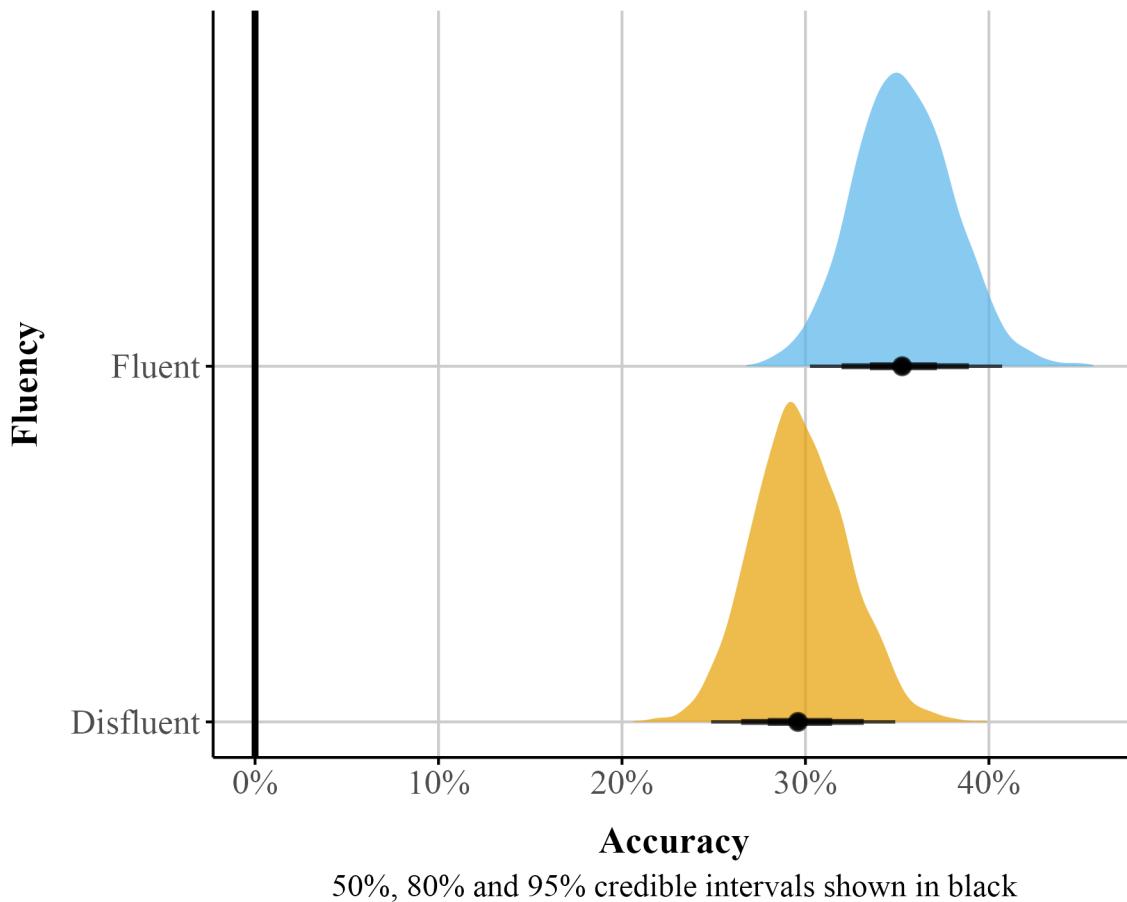
415 A limitation of the beta regression model is that it can only accommodate values strictly between
 416 0 and 1—a probability cannot take on values of 0 (the event will not occur with certainty) or 1 (the event
 417 will occur with certainty). In our dataset, we observed 9 rows where Accuracy equals zero. To fit a beta
 418 regression model, we removed these values, but we have left out potentially valuable information from our
 419 model—especially if the end points of the scale are distinctive in some way. In our case, these 0s may be
 420 structural—that is, they represent real, systematic instances where participants failed to answer correctly
 421 (rather than random noise or measurement error). For example, the fluency of the instructor might be a
 422 key factor in predicting these zero responses. We will discuss two approaches for jointly modeling these end
 423 points with the continuous data. First, we can use a zero-inflated beta (ZIB) model. This model still estimates

Table 3

Fluency difference

Figure 7

Predicted probability posterior distributions by fluency



Listing 12 Calculating the difference between probabilities with `avg_comparisons()`

```
# get risk difference by default
beta_avg_comp <- avg_comparisons(beta_brms, comparison = "difference")
```

424 the mean (μ) and precision (ϕ) of the beta distribution for values between 0 and 1, but it also includes an
 425 additional parameter, α , which captures the probability of observing structural 0s.

426 The zero-inflated beta models a mixture of the data-generating process. The α parameter uses a
 427 logistic regression to model whether the data is 0 or not. Substantively, this could be a useful model when we
 428 think that 0s come from a process that is relatively distinct from the data that is greater than 0. For example,
 429 if we had a dataset with proportion of looks or eye fixations to certain areas on marketing materials, we might
 430 want a separate model for those that do not look at certain areas on the screen because individuals who do
 431 not look might be substantively different than those that look.

432 We can fit a ZIB model using `brms()` and use the `{marginaleffects}` package to make inferences
 433 about our parameters of interest. Before we run a zero-inflated beta model, we will need to transform our
 434 data again and remove the one 1 value in our data—we can keep our 0s. Similar to our beta regression model
 435 we fit in `brms`, we will use the `bf()` function to fit several models. We fit our μ and ϕ parameters as well as

Listing 13 Calculating ϕ difference with avg_comparisons()

```
# get risk difference by default

beta_avg_phi <- avg_comparisons(
  beta_brms_dis,
  dpar = "phi",
  comparison = "difference"
)
```

436 our zero-inflated parameter (α ; here labeled as `zi`). In `brms` we can use the `zero_inflated_beta` family (see
437 Listing 14).

Listing 14 Fitting zib model with brm()

```
# keep 0 but remove 1
data_beta_0 <- fluency_data |>
  filter(Accuracy != 1)

# set up model formula for zero-inflated beta in brm
zib_model <- bf(
  Accuracy ~ Fluency, # The mean of the 0-1 values, or mu
  phi ~ Fluency, # The precision of the 0-1 values, or phi
  zi ~ Fluency, # The zero-or-one-inflated part, or alpha
  family = zero_inflated_beta()
)

# fit zib model with brm
fit_zi <- brm(
  formula = zib_model,
  data = data_beta_0,
  file = here::here("manuscript", "models", "bayes_zib_model0not1.rds")
)
```

438 **Posterior Predictive Check**

439 The ZIB model does a bit better at capturing the structure of the data than the beta regression model
440 (see Figure 12). Specifically, the ZIB model more accurately captures the increased density of values near
441 the lower end of the scale (i.e., near zero), which the standard beta model underestimates. The ZIB model's
442 predictive distributions also align more closely with the observed data across the entire range, particularly in
443 the peak and tail regions. This improved fit likely reflects the ZIB model's ability to explicitly model excess
444 0s (or near-zero values) via its inflation component, allowing it to better account for features in the data that
445 a standard beta distribution cannot accommodate.

Table 4*Probability fluency difference (μ)*

Contrast	Mean	95% Cr.I	pd
Fluent - Disfluent	0.044	[-0.029, 0.118]	0.881

Table 5*Probability fluency difference (ϕ)*

Contrast	Mean	95% Cr.I	pd
Fluent - Disfluent	2.72	[-0.744, 6.628]	0.934

446 Predicted Probabilities and Marginal Effects

447 Table 8, under the zero-inflated beta regression column, provides a summary of the posterior distribution
 448 for each parameter. As stated before, it is preferable to back-transform our estimates to get probabilities.
 449 To get the predicted probabilities we can again use the `avg_predictions()` and `avg_comparisons()`
 450 functions from `{marginaleffects}` package (Arel-Bundock, 2024) to get predicted probabilities and the prob-
 451 ability difference between the levels of each factor. We can model the parameters separately using the `dpar`
 452 argument setting to: μ , ϕ , α . Here we look at the risk difference for Fluency under each parameter. If one
 453 were interested in the average effect for the entire model, the `dpar` argument could be removed.

454 **Mu.** As shown in Table 4, there is little evidence for an effect of Fluency – the 95% Cr.I includes
 455 zero, suggesting substantial uncertainty about the direction and magnitude of the effect—that is, though most
 456 of the posterior density supports positive effects, nil and weakly negative effects cannot be ruled out.

457 **Dispersion.** As shown in Table 5, the posterior estimates suggest a credible effect of Fluency on
 458 dispersion (ϕ), with disfluent responses showing greater variability. The 95% Cr.I for the fluency contrast
 459 does not include zero, indicating a high probability in differences in precision.

460 Zero-Inflation

461 We can use `{marginaleffects}` to estimate and plot the posterior difference between the fluent and
 462 disfluent conditions (see Figure 8). In Figure 8, the posterior distribution for this contrast lies mostly below
 463 zero, indicating that a fluent instructor is associated with a lower probability of zero responses. The estimated
 464 reduction is approximately 13%. The 95% credible interval does not include zero, which indicates that the
 465 data provide consistent evidence for a reduction in zero responses under fluent instruction.

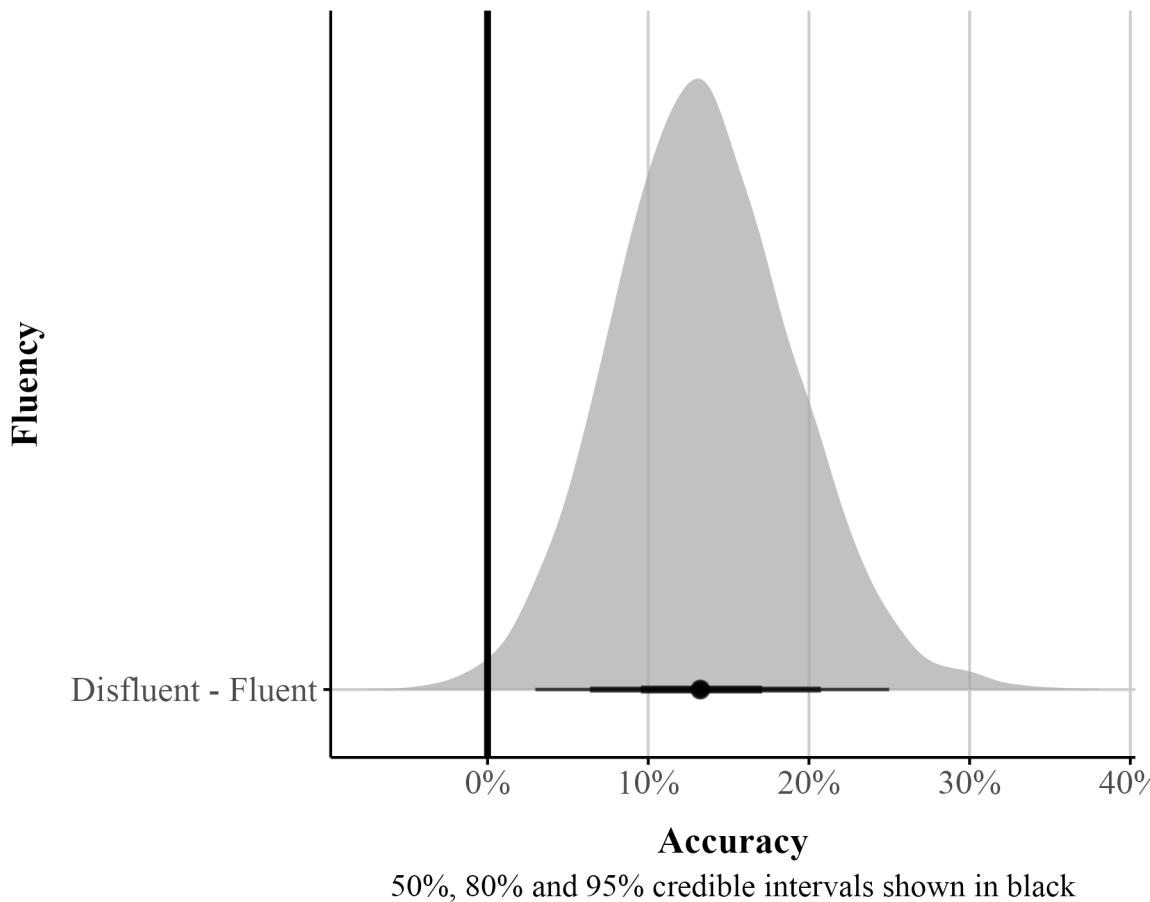
466 Zero-One-Inflated beta (ZOIB)

467 The ZIB model works well if there are 0s in your data, but not 1s.⁸ In our previous examples we
 468 either got rid of both 0s and 1s (beta regression), or removed the 1s (ZIB). Sometimes it is theoretically useful
 469 to model both 0s and 1s as separate processes or to consider these values as essentially similar parts of the
 470 continuous response, as we show later in the ordered beta regression model. For example, this is important
 471 in visual analog scale data where there might be a prevalence of responses at the bounds (Kong & Edwards,

⁸In cases where your data include exact 1s but no 0s, you can fit a one-inflated beta regression model in `{brms}` by setting the `coi` parameter to 1. This tells the model that all point masses occur at 1, rather than being split between 0 and 1. In other words, `coi = 1` assumes that any inflation in the data is due entirely to values at 1. In our data, we have exactly one value equal to 1^[^6]. While probably not significant to alter our findings, we can model 1s with a special type of model called the zero-one-inflated beta (ZOIB) model (Liu & Kong, 2015) if we believe that both 0s and 1s are distinct outcomes.

Figure 8

Visualization of the predicted difference for zero-inflated part of model



472 2016), in JOL tasks (Wilford et al., 2020), or in a free-list task where individuals provide open responses to
 473 some question or topic which are then recoded to fall between 0-1 (Bendixen & Purzycki, 2023). Here 0s
 474 and 1s are meaningful; 0 means item was not listed and 1 means the item was listed first.

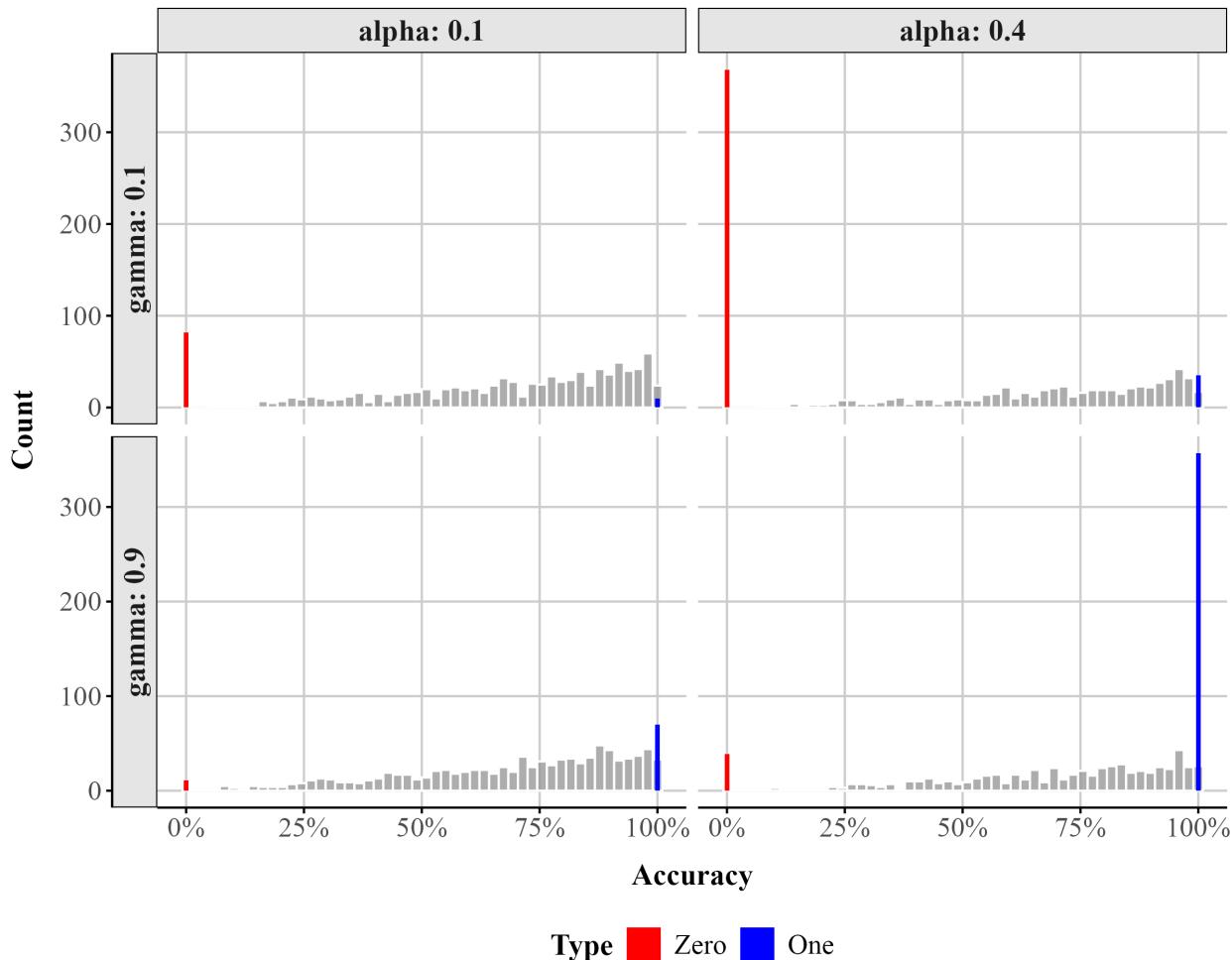
475 Similar to the beta and zero-inflated models discussed above, we can fit a zero-and-one-inflated beta
 476 (ZOIB) model in `{brms}` using the `zero_one_inflated_beta` family. This formulation simultaneously
 477 estimates the mean μ and precision ϕ of the Beta component, as well as two inflation parameters: α , the
 478 probability that an observation is at either boundary (0 or 1), and γ , the conditional probability that, given
 479 an observation falls on a boundary, it takes the value 1 rather than 0. In other words, α determines how often
 480 responses occur exactly at the endpoints, and γ determines the balance between zeros and ones among those
 481 endpoint values. This specification allows the model to capture both the continuous variation in the interior
 482 of the (0, 1) interval and the presence of exact boundary values.

483 To illustrate how α and γ shape the distribution, Figure 9 displays simulated data across a range
 484 of parameter combinations. As α increases, more responses occur at the endpoints. As γ increases, the
 485 proportion of those endpoint responses that are 1 increases relative to 0, producing increasingly pronounced
 486 spikes at 1 as γ approaches 1. Together, these parameters give the ZOIB model the flexibility to represent
 487 datasets with mixtures of continuous values and exact zeros and ones.

488 To fit a ZOIB model we use the `bf()` function. We model each parameter as a function of Fluency.
 489 We then pass the `zoib_model` to our `brm()` function (see Listing 15). The summary of the output is in

Figure 9

Simulated data from a ZOIB model illustrating the effects of the zero-one inflation parameter (α) and the conditional one-inflation parameter (γ).



490 Table 8 (under ZOIB).

491 **Model Parameters**

492 The output for the model is lengthy because we are estimating four distinct components, each with
 493 their own independent responses and sub-models. All the coefficients are on the logit scale, except ϕ , which is
 494 on the log scale. Thankfully drawing inferences for all these different parameters, plotting their distributions,
 495 and estimating their average marginal effects looks exactly the same—all the brms and {marginaleffects}
 496 functions we used work the same.

497 **Predictions and Marginal Effects**

498 With {marginaleffects} we can choose marginalize over all the sub-models, averaged across the 0s,
 499 continuous responses, and 1s in the data, or we can model the parameters separately using the dpar argument
 500 like we did above setting it to: μ , ϕ , α , γ (see below). Using avg_predictions() and not setting dpar we
 501 can get the predicted probabilities across all the sub-models. We can also plot the overall difference between

Listing 15 Fitting a ZOIB model with `brm()`.

```
# fit the zoib model
zoib_model <- bf(
  Accuracy ~ Fluency, # The mean of the 0-1 values, or mu
  phi ~ Fluency, # The precision of the 0-1 values, or phi
  zoi ~ Fluency, # The zero-or-one-inflated part, or alpha
  coi ~ Fluency, # The one-inflated part, conditional on the 1s, or gamma
  family = zero_one_inflated_beta()
)

fit_zoib <- brm(
  formula = zoib_model,
  data = fluency_data,
  file = here::here("manuscript", "models", "bayes_zoib_model")
)
```

502 fluency and disfluency for the whole model with `plot_predictions()`.

503 In addition, we show below how one can extract the predicted probabilities and marginal effects for
504 γ (and a similar process for any other model component, `zoi`, etc.):

Listing 16 Extracting predicted probabilities and marginal effects for conditional-one parameter

```
# get average predictions for coi param
coi_probs <- avg_predictions(fit_zoib, by = c("Fluency"), dpar = "coi")
# get difference between the two conditions
coi_me <- avg_comparisons(fit_zoib, variables = c("Fluency"), dpar = "coi")
```

505 **Ordered Beta Regression**

506 Looking at the output from the ZOIB model (Table 8), we can see how running a model like this
507 can become fairly complex as it is fitting distinct sub-models for each component of the scale. The ability
508 to consider 0s and 1s as distinct processes from continuous values comes at a price in terms of complexity
509 and interpretability. A simplified version of the zero-one-inflated beta (ZOIB) model, known as ordered
510 beta regression (Kubinec, 2022; see also Makowski et al., 2025 for a reparameterized version called the
511 *beta-Gate* model), has been recently proposed. The ordered beta regression model exploits the fact that,
512 for most analyses, the continuous values (between 0-1) and the discrete outcomes (e.g., 0 or 1) are *ordered*.
513 For example, as a covariate x increases or decreases, we should expect the bounded outcome y to increase
514 or decrease monotonically as well from 0 to (0, 1) to 1. The ZOIB model does not impose this restriction;
515 a covariate could increase and the response y could increase in its continuous values while *simultaneously*
516 decreasing at *both* end points.⁹ This complexity is not immediately obvious when fitting the ZOIB, nor is
517 it a potential relationship that many scholars want to consider when examining how covariates influence a
518 bounded scale.

⁹For a more complete description of this issue, we refer the reader to Kubinec (2022).

519 To make the response ordered, the ordered beta regression model estimates a weighted combination
 520 of a standard beta regression model for continuous responses and a logit model for the discrete values of
 521 the response. By doing so, the amount of distinctiveness between the continuous responses and the discrete
 522 end points is a function of the data (and any informative priors) rather than strictly defined as fully distinct
 523 processes as in the ZOIB. For some datasets, the continuous and discrete responses will be fairly distinct,
 524 and in others less so.

525 The weights that average together the two parts of the outcome (i.e., discrete and continuous) are
 526 determined by cutpoints that are estimated in conjunction with the data in a similar manner to what is known
 527 as an ordered logit model. An in-depth explanation of ordinal regression is beyond the scope of this tutorial
 528 (Bürkner & Vuorre, 2019; but see Fullerton & Anderson, 2021). At a basic level, ordinal regression models
 529 are useful for outcome variables that are categorical in nature and have some inherent ordering (e.g., Likert
 530 scale items). To preserve this ordering, ordinal models rely on the cumulative probability distribution.
 531 Within an ordinal regression model it is assumed that there is a continuous but unobserved latent variable
 532 that determines which of k ordinal responses will be selected. For example on a typical Likert scale from
 533 ‘Strongly Disagree’ to ‘Strongly Agree’, you could assume that there is a continuous, unobserved variable
 534 called ‘Agreement’.

535 While we cannot measure Agreement directly, the ordinal response gives us some indication about
 536 where participants are on the continuous Agreement scale. $k - 1$ cutoffs are then estimated to indicate the
 537 point on the continuous Agreement scale at which your Agreement level is high enough to push you into the
 538 next ordinal category (say Agree to Strongly Agree). Coefficients in the model estimate how much differ-
 539 ent predictors change the estimated *continuous* scale (here, Agreement). Since there’s only one underlying
 540 process, there’s only one set of coefficients to work with (proportional odds assumption).

541 In an ordered beta regression, three ordered categories are modeled: (1) exactly zero, (2) somewhere
 542 between zero and one, and (3) exactly one. In an ordered beta regression, (1) and (2) are modeled with
 543 cumulative logits, where one cutpoint is the boundary between Exactly 0 and Between 0 and 1 and the
 544 other cutpoint is the boundary between Between 0 and 1 and Exactly 1. The continuous values in the middle,
 545 0 to 1 (3), are modeled as a vanilla beta regression with parameters reflecting the mean response on the
 546 logit scale as we have described previously. Ultimately, employing cutpoints allows for a smooth transition
 547 between the bounds and the continuous values, permitting both to be considered together rather than modeled
 548 separately as the ZOIB requires.

549 The ordered beta regression model has shown to be more efficient and less biased than some of the
 550 methods discussed (Kubinec, 2022) herein and has seen increasing use across the biomedical and social
 551 sciences (Martin et al., 2024; Nouvian et al., 2023; Shrestha et al., 2024; Smith et al., 2024; Wilkes et al.,
 552 2024) because it produces only a single set of coefficient estimates in a similar manner to a standard beta
 553 regression or OLS.¹⁰

554 ***Fitting an Ordered Beta Regression***

555 To fit an ordered beta regression in a Bayesian context we use the `{ordbetareg}` (Kubinec, 2023) pack-
 556 age. `{ordbetareg}` is a front-end to the `brms` package that we described earlier; in addition to the functions
 557 available in the package, most `brms` functions and plots, including the diverse array of regression model-
 558 ing options, will work with `{ordbetareg}` models. (We note that the `ordbeta` model is also available as a
 559 maximum-likelihood variant in the R package `{glmmTMB}`.) We first load the `{ordbetareg}` package (see
 560 Listing 17).

561 The `{ordbetareg}` package uses `brms` on the front-end so all the arguments we used previously apply
 562 here. Instead of the `brm()` function we use `ordbetareg()`. To fit a model where dispersion does not vary

¹⁰Please note that there are other models available that can model this continuous process like the beta-gate model (Makowski et al., 2025) and the censored extended beta regression model (Kosmidis & Zeileis, 2025).

Listing 17 Load {ordbetareg}

```
library(ordbetareg)
```

Table 6*Marginal effect of fluency ordered beta model*

Contrast	Mean	95% Cr.I	pd
Fluent - Disfluent	0.06	[-0.02, 0.134]	0.934

563 as a function of fluency we can use the below code (see Listing 18).

Listing 18 Fitting ordered beta model with ordbetareg()

```
ord_fit_brms <- ordbetareg(
  Accuracy ~ Fluency,
  data = fluency_data,
  file = here::here("manuscript", "models", "bayes_ordbeta_model")
)
```

564 However, if we want dispersion to vary as a function of fluency we can easily do that (see Listing 19).
 565 Note the addition of the `phi_reg` argument in `m.phi`. This argument allows us to include a model that
 566 explicitly models the dispersion parameter. Because we are modeling ϕ as a function of fluency, we set the
 567 the argument to `both`.

Listing 19 Fitting ordered beta model with dispersion using ordbetareg()

```
ord_beta_phi <- bf(Accuracy ~ Fluency, phi ~ Fluency)

m.phi <- ordbetareg(
  ord_beta_phi,
  data = fluency_data,
  phi_reg = 'both',
  file = here::here("manuscript", "models", "bayes_ordbeta_phi_model")
)
```

568 **Marginal Effects.** Table 8 presents the posterior summary. We can use `{marginaleffects}` to calculate differences on the response scale that average over (or marginalize over) all our parameters.

569 In Table 6 the credible interval is close enough to zero relative to its uncertainty that we can conclude
 570 there likely aren't differences between the conditions after taking dispersion and the 0s and 1s in our data
 571 into account.

572 **Cutpoints.** The model cutpoints are not reported by default in the summary output, but we can
 573 access them with the R package `posterior` (Bürkner et al., 2025) and the functions `as_draws` and
 574 `summary_draws`.

Table 7

Cutzero and cutone parameter summary

Parameter	Mean	95% Cr.I
cutzero	-2.97	[-3.54, -2.41]
cutone	1.85	[1.64, 2.07]

576 In Table 7, `cutzero` is the first cutpoint (the difference between 0 and continuous values) and `cutone`
 577 is the second cutpoint (the difference between the continuous values and 1). These cutpoints are on the
 578 logit scale and as such the numbers do not have a simple substantive meaning. In general, as the cutpoints
 579 increase in absolute value (away from zero), then the discrete/boundary observations are more distinct from
 580 the continuous values. This will happen if there is a clear gap or bunching in the outcome around the bounds.
 581 This type of empirical feature of the distribution may be useful to scholars if they want to study differences
 582 in how people perceive the ends of the scale versus the middle. It is possible, though beyond the scope of
 583 this article, to model the location of the cutpoints with hierarchical (non-linear) covariates in `brms`. In the
 584 most recent version of `ordbeta`, it is possible to test the influence of different factors on these boundaries.

585 **Model Fit**

586 The best way to visualize model fit is to plot the full predictive distribution relative to the original
 587 outcome. Because ordered beta regression is a mixed discrete/continuous model, a separate plotting function,
 588 `pp_check_ordbetareg`, is included in the `{ordbetareg}` package that accurately handles the unique features
 589 of this distribution. The default plot in `brms` will collapse these two features of the outcome together, which
 590 will make the fit look worse than it actually is. The `{ordbetareg}` function returns a list with two plots,
 591 `discrete` and `continuous`, which can either be printed and plotted or further modified as `{ggplot2}` objects
 592 (see Figure 10).

593 The discrete plot, which is a bar graph, shows that the posterior distribution accurately captures the
 594 number of different types of responses (discrete or continuous) in the data. For the continuous plot shown as
 595 a density plot with one line per posterior draw, the model does a very good job at capturing the distribution.

596 Overall, it is clear from the posterior distribution plot that the ordered beta model fits the data well.
 597 To fully understand model fit, both of these plots need to be inspected as they are conceptually distinct.

598 **Model Visualization**

599 `{ordbetareg}` provides a useful visualization function called `plot_heiss()` (Ye & Heiss, 2023) that
 600 can represent dispersion in the entire outcome as a function of discrete covariates. This function produces a
 601 plot of predicted proportions across the range of our Fluency factor. In Figure 11 we get predicted propor-
 602 tions for Fluency across the bounded scale. Looking at the figure we can see there is much overlap between
 603 instructors in the middle portion (μ). However, we do see some small differences at the zero bounds.

604 **Ordered Beta Scale**

605 In the `{ordbetareg}` function there is a `true_bound` argument. In cases where your data is not
 606 bounded between 0-1, this argument can be used to specify the bounds of the argument to fit the ordered
 607 beta regression. For example, the response data might be bounded between 1 and 7. If so, `{ordbetareg}` can
 608 model it within the [0,1] interval and `{ordbetareg}` will convert the model predictions back to the true bounds
 609 after estimation.

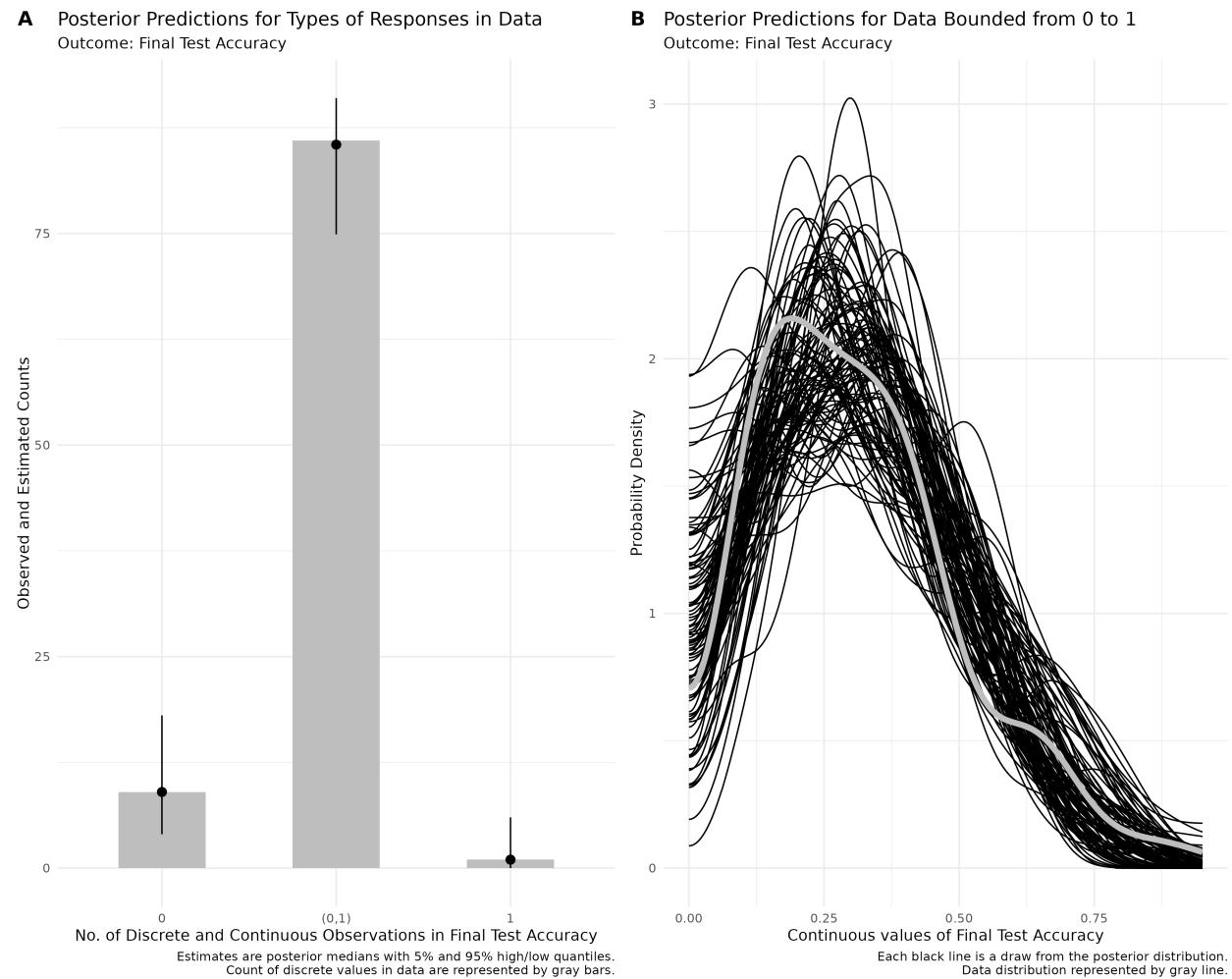
Table 8*Bayesian regression summaries for each model*

Parameter	Stat	Bayesian LM	Beta Regression	ZIB	ZOIB	Ordered Beta
b_Intercept	Mean	0.256	-0.830	-0.830	-0.831	-0.863
	Cr.I	[0.197, 0.314]	[-1.1, -0.548]	[-1.095, -0.569]	[-1.083, -0.576]	[-1.122, -0.594]
	pd	1.000***	1.000***	1.000***	1.000***	1.000***
b_Fluency	Mean	0.086	0.201	0.203	0.202	0.257
	Cr.I	[0.004, 0.17]	[-0.144, 0.543]	[-0.127, 0.541]	[-0.138, 0.528]	[-0.084, 0.585]
	pd	0.980*	0.870	0.881	0.889	0.934
sigma	Mean	0.208	-	-	-	-
	Cr.I	[0.181, 0.241]	-	-	-	-
	pd	1.000***	-	-	-	-
b_phi_Intercept	Mean	-	1.599	1.598	1.609	1.616
	Cr.I	-	[1.185, 1.986]	[1.191, 1.973]	[1.193, 1.989]	[1.216, 2.001]
	pd	-	1.000***	1.000***	1.000***	1.000***
b_phi_Fluency	Mean	-	0.432	0.429	0.420	0.403
	Cr.I	-	[-0.137, 1.005]	[-0.126, 1.002]	[-0.138, 0.983]	[-0.164, 0.974]
	pd	-	0.931	0.934	0.933	0.921
b_zi_Intercept	Mean	-	-	-1.663	-	-
	Cr.I	-	-	[-2.496, -0.978]	-	-
	pd	-	-	1.000***	-	-
b_zi_Fluency	Mean	-	-	-2.140	-	-
	Cr.I	-	-	[-4.73, -0.362]	-	-
	pd	-	-	0.994**	-	-
b_zoi_Intercept	Mean	-	-	-	-1.543	-
	Cr.I	-	-	-	[-2.312, -0.859]	-
	pd	-	-	-	1.000***	-
b_zoi_Fluency	Mean	-	-	-	-2.273	-
	Cr.I	-	-	-	[-4.587, -0.46]	-
	pd	-	-	-	0.996***	-
b_coi_Intercept	Mean	-	-	-	-2.043	-
	Cr.I	-	-	-	[-4.402, -0.312]	-
	pd	-	-	-	0.990**	-
b_coi_Fluency	Mean	-	-	-	0.211	-
	Cr.I	-	-	-	[-6.654, 5.48]	-
	pd	-	-	-	0.571	-

Note. Link functions: b_mean = logit; b_phi = log; b_zoi (zero-one inflation) = logit; b_coi (conditional one-inflation) = logit. Asterisks reflect approximate two-sided p-values derived from the posterior pd. pd \geq 0.975 ($p \leq .05$) = *; pd \geq 0.990 ($p \leq .01$) = **; pd \geq 0.998 ($p \leq .001$) = ***.

Figure 10

Posterior predictive check for ordered beta regression model. A. Discrete posterior check. B. Continuous posterior check.



610

Discussion

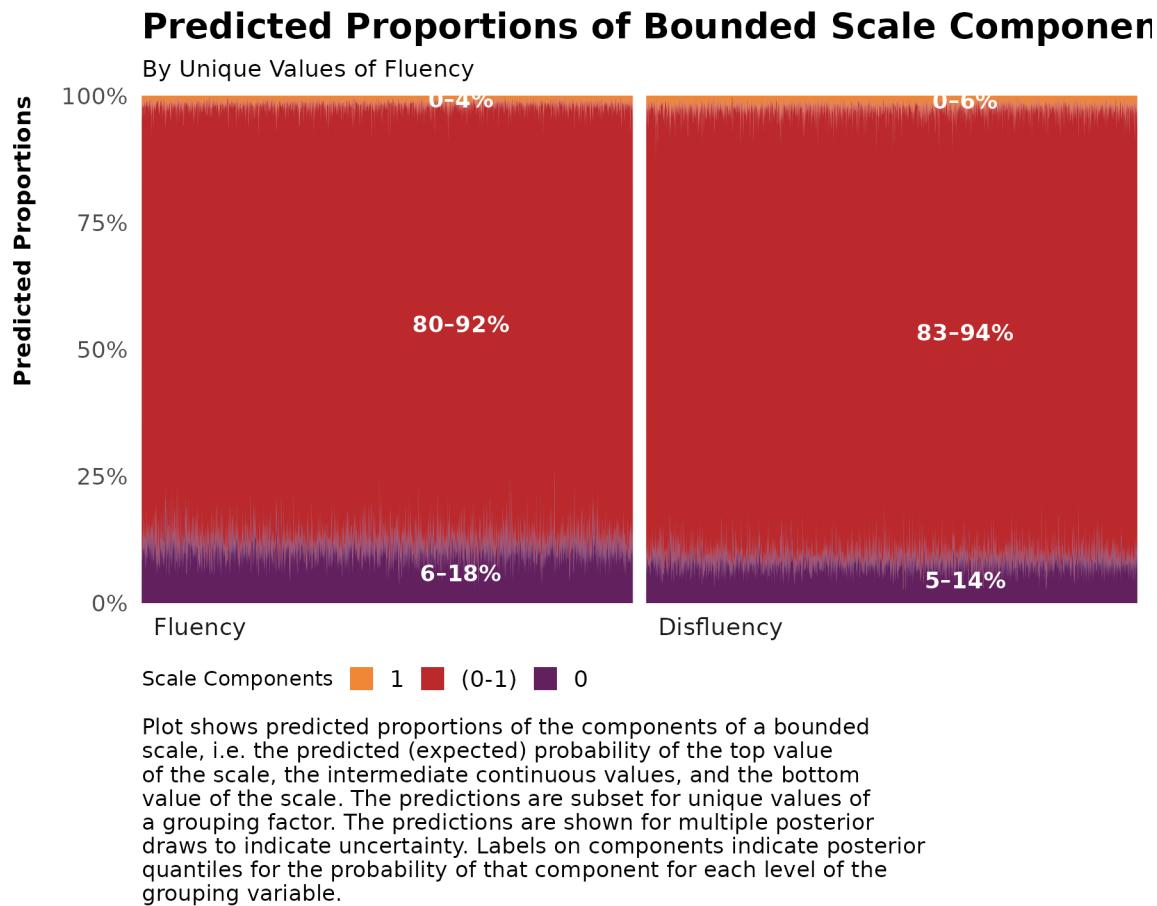
611 The use of beta regression in psychology, and the social sciences in general, is rare. With this tutorial,
 612 we hope to turn the tides. Beta regression models are an attractive alternative to models that impose un-
 613 realistic assumptions like normality, linearity, homoscedasticity, and unbounded data. Beyond these models,
 614 there are a diverse array of different models that can be used depending on your outcome of interest.

615 Throughout this tutorial our main aim was to help guide researchers in running analyses with pro-
 616 portional or percentage outcomes using beta regression and some of its alternatives. In the current example,
 617 we used real data from Wilford et al. (2020) and discussed how to fit these models in R, interpret model
 618 parameters, extract predicted probabilities and marginal effects, and visualize the results.

619 Comparing our analysis with that of Wilford et al. (2020), we demonstrated that using a traditional
 620 approache (e.g., t -test) to analyze mean accuracy data can lead to biased inferences. Although we successfully
 621 reproduced one of their key findings, our use of beta regression and its extensions revealed important nuances
 622 in the results. With a traditional beta regression model—which accounts for both the mean and the precision
 623 (dispersion)—we observed similar effects of instructor fluency on performance. However, the standard beta

Figure 11

Heiss plot of predicted probabilities across the scale (0-100)



624 model does not accommodate boundary values (i.e., 0s and 1s).

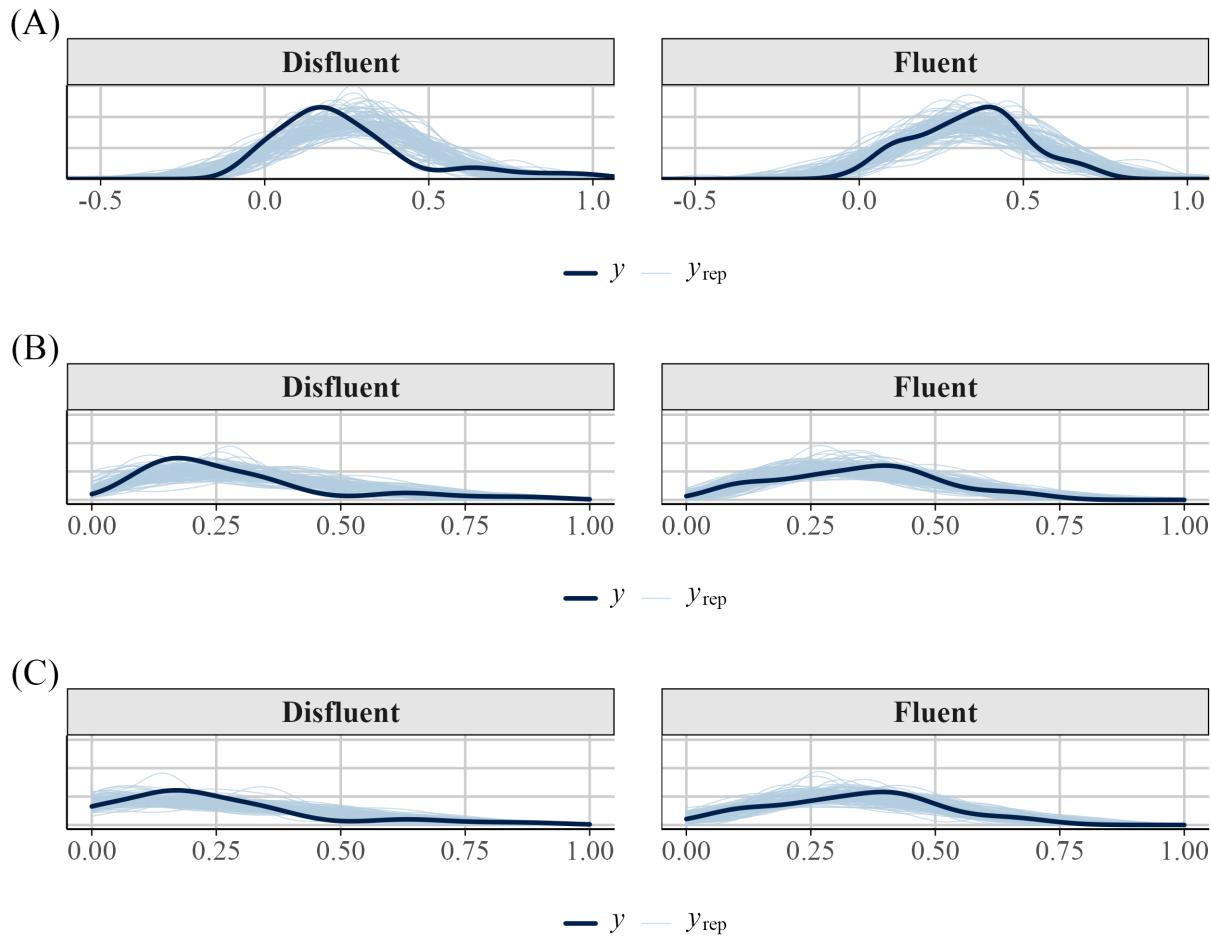
625 When we applied a ZIB model, which explicitly accounts for structural 0s, we found no effect of
 626 fluency on the mean (μ) part of the model. Instead, the effect of fluency emerged in the structural zero
 627 (inflated zero; α) component. This pattern was consistent when using a zero-one-inflated beta (ZOIB) model.
 628 Furthermore, we fit an ordered beta regression model (Kubinec, 2022), which appropriately models the full
 629 range of values, including 0s and 1s. Here, we did not observe a reliable effect of fluency on the mean once
 630 we accounted for dispersion.

631 These analyses emphasize the importance of fitting a model that aligns with the nature of the data.
 632 The simplest and recommended approach when dealing with data that contains 0s and/or 1s is to fit an ordered
 633 beta model, assuming the process is truly continuous. However, if you believe the process is distinct in nature,
 634 a ZIB or ZOIB model might be a better choice. Ultimately, this decision should be guided by theory.

635 For instance, if we believe fluency influences the boundaries (0 and 1), we might want to model
 636 this process separately using a ZIB or ZOIB. With the current dataset, fluency might affect specific aspects
 637 of performance (such as the likelihood of complete failure) rather than general performance levels. This
 638 effect could be due to participant disengagement during the disfluent lecture. If students fail to pay attention
 639 because of features of disfluency, they may miss relevant information, leading to a floor effect at the test.
 640 Following from this, disfluency would be expected to influence the boundary (0) and not the continuous part
 641 of the model. If this is the case, we would want to model this appropriately. However, if we believe fluency

Figure 12

The plots show 100 posterior predicted distributions with the label y_{rep} (light blue), the distribution of accuracy as function of fluency in dark blue for regular regression (A), beta regression (B), and ZIB (C) models



642 effects general performance levels (the continuous part), a model that takes in to account the entire process
 643 accounting for the 0s and 1s might be appropriate.

644 In the discussion section of Wilford et al. (2020), they were unable to offer a tenable explanation for
 645 performance differences based on instructor fluency. A model that accounts for the excess 0s in the dataset
 646 provides one testable explanation: watching a disfluent lecture may lead to lapses in attention, resulting
 647 in poorer performance in that group. These lapses, in turn, contribute to the observed differences in the
 648 fluent condition. This modeling approach opens a promising avenue for future research—one that would have
 649 remained inaccessible otherwise.

650 Not everyone will be eager to implement the techniques discussed herein. In such cases, the key
 651 question becomes: What is the least problematic approach to handling proportional data? One reasonable
 652 option is to fit multiple models tailored to the specific characteristics of your data. For example, if your data
 653 contain 0s, you might fit two models: a traditional linear model excluding the 0s, and a logistic model to
 654 account for the zero versus non-zero distinction. If your data contain both 0s and 1s, you could fit separate
 655 models for the 0s and 1s in addition to the OLS model. There are many defensible strategies to choose from
 656 depending on the context. However, we do not recommend transforming the values of your data (e.g., 0s to

657 .01 and 1s to .99) or ignoring the properties of your data simply to fit traditional statistical models.

658 In this tutorial, we demonstrated how to analyze these models from a Bayesian perspective. While we
659 recognize that not everyone identifies as a Bayesian, implementing these models using a Bayesian framework
660 is relatively straightforward—it requires only a single package, lowering the barrier to entry. For those who
661 prefer frequentist analyses, several R packages are available. For example, the `{betareg}` package (Cribari-
662 Neto & Zeileis, 2010) `{glmmTMB}` (Brooks et al., 2017) and `{gamlss}` (2005) are nice options. To this end,
663 I have included supplemental materials demonstrating how to use frequentist packages to analyze the data
664 presented herein.

665 Conclusion

666 Overall, this tutorial emphasizes the importance of modeling the data you have. Although the ex-
667 ample provided is relatively simple (a one-factor model with two levels), we hope it demonstrates that even
668 with a basic dataset, there is much nuance in interpretation and inference. Properly modeling your data
669 can lead to deeper insights, far beyond what traditional measures might offer. With the tools introduced in
670 this tutorial, researchers now have the means to analyze their data effectively, uncover patterns, make ac-
671 curate predictions, and support their findings with robust statistical evidence. By applying these modeling
672 techniques, researchers can improve the validity and reliability of their studies, ultimately leading to more
673 informed decisions and advancements in their respective fields.

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