

Flexible Multibody Dynamics Systems with Python and C++

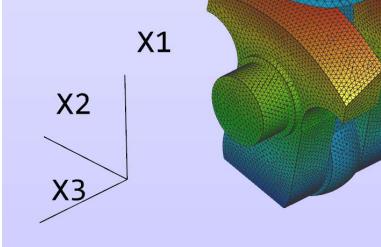
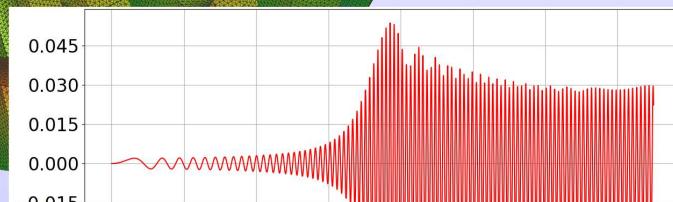
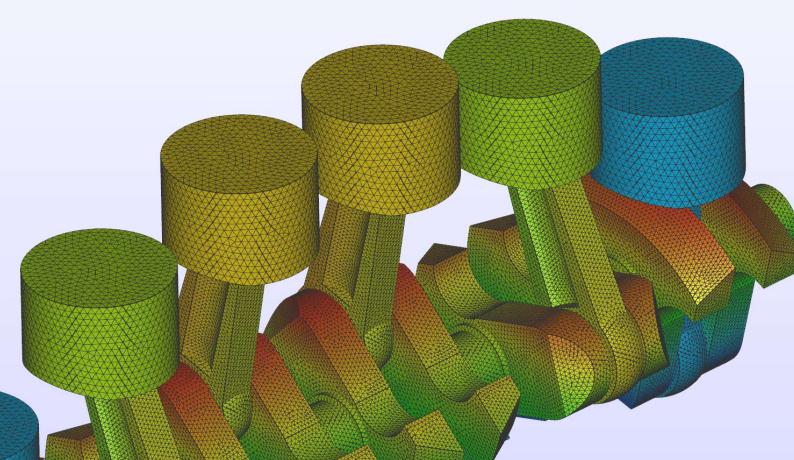
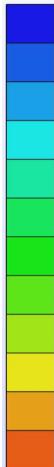
EXUDYN

USER DOCUMENTATION

EXUDYN

Solver finished successfully
time = 0.0355

Displacement(0)
min=-0.0904633,max=0.0758431



```
#import conrod CMS
cmsConrod = ObjectFFRFReducedOrderInterface(femConrod)
cmsConrodList.append(cmsConrod)
objFFRFconrod = cmsConrod.AddObjectFFRFReducedOrderWithUserFunctions(exu, mbs,
    positionRef=pRef,
    eulerParametersRef=eulerParametersInit,
    initialVelocity=[0,0,0],
    initialAngularVelocity=[0,0,0*fRotorStart*2*pi],
```

(mesh and FEM-model generated with NETGEN and NGsolve – 647058 total coordinates)

EXUDYN version = 1.0.201
CHECK section [Section 12](#) for changes from previous versions!!!

University of Innsbruck, Department of Mechatronics, May 3, 2021,

Johannes Gerstmayr

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Chapter 1

Getting Started

The documentation for EXUDYN is split into this introductory section, including a quick start up, code structure and important hints, as well as a couple of sections containing references to the available Python interfaces to interact with EXUDYN and finally some information on theory (e.g., 'Solver').

EXUDYN is hosted on [GitHub](#) [13]:

- web: <https://github.com/jgerstmayr/EXUDYN>

For any comments, requests, issues, bug reports, send an email to:

- email: reply.exudyn@gmail.com

Thanks for your contribution!

1.1 Getting started

This section will show:

1. What is EXUDYN?
2. Who is developing EXUDYN?
3. How to install EXUDYN
4. How to link EXUDYN and Python
5. Goals of EXUDYN
6. Run a simple example in Spyder
7. FAQ – Frequently asked questions

1.1.1 What is EXUDYN?

EXUDYN– (fLEXible mUltibody DYNamics – EXtend yoUr DYNamics)

EXUDYN is a C++ based Python library for efficient simulation of flexible multibody dynamics systems. It is the follow up code of the previously developed multibody code HOTINT, which Johannes Gerstmayr started during his PhD-thesis. The open source code HOTINT reached limits of

further (efficient) development and it seemed impossible to continue from this code as it is outdated regarding programming techniques and the numerical formulation.

EXUDYN is designed to easily set up complex multibody models, consisting of rigid and flexible bodies with joints, loads and other components. It shall enable automatized model setup and parameter variations, which are often necessary for system design but also for analysis of technical problems. The broad usability of python allows to couple a multibody simulation with environments such as optimization, statistics, data analysis, machine learning and others.

The multibody formulation is mainly based on redundant coordinates. This means that computational objects (rigid bodies, flexible bodies, ...) are added as independent bodies to the system. Hereafter, connectors (e.g., springs or constraints) are used to interconnect the bodies. The connectors are using Markers on the bodies as interfaces, in order to transfer forces and displacements. For details on the interaction of nodes, objects, markers and loads see [Section 2.2](#).

1.1.2 Developers of EXUDYN and thanks

EXUDYN is currently (5-2021) developed at the University of Innsbruck. In the first phase most of the core code is written by Johannes Gerstmayr, implementing ideas that followed out of the project HOTINT. 15 years of development led to a lot of lessons learned and after 20 years, a code must be re-designed.

Some specific earlier code parts for the coupling between C++ and Python have been written by Stefan Holzinger. Stefan also helped to set up the previous upload to GitLab and to test parallelization features. For the interoperability between C++ and Python, we extensively use **Pybind11**[24], originally written by Jakob Wenzel, see <https://github.com/pybind/pybind11>. Without **Pybind11** we couldn't have made this project.

Important discussions with researchers from the community were important for the design and development of EXUDYN, where we like to mention Joachim Schöberl from TU-Vienna who boosted the design of the code with great concepts.

The cooperation and funding within the EU H2020-MSCA-ITN project 'Joint Training on Numerical Modelling of Highly Flexible Structures for Industrial Applications' contributes to the development of the code.

The following people have contributed to the examples:

- Stefan Holzinger, Michael Pieber, Joachim Schöberl, Manuel Schieferle, Martin Knapp, Lukas March, Dominik Sponring, David Wibmer, Andreas Zwölfer

– thanks a lot! –

1.2 Installation instructions

1.2.1 How to install EXUDYN?

In order to run EXUDYN, you need an appropriate Python installation. We currently (2021-02) recommend to use

- **Anaconda, 64bit, Python 3.7.7¹**
- **Spyder 4.1.3** (with Python 3.7.7, 64bit), which is included in the Anaconda installation²

Many alternative options exist:

- In case that you have an older CPU, which does not support AVX2, use: Anaconda, 32bit, Python 3.6.5)³
- Users report successful use of EXUDYN with **Visual Studio Code** and **Jupyter** has been tested with some examples; both environments should work with default settings
- Anaconda 2020-11 with **Python 3.8** and Spyder 4.1.5: not much experience for now (2021-04), but TestSuite runs without errors for EXUDYN version 1.0.182.
- Alternative option with more stable Spyder (as compared to Spyder 4.1.3): Anaconda, 64bit, Python 3.6.5)⁴

If you plan to extend the C++ code, we recommend to use VS2017⁵ to compile your code, which offers Python 3.7 compatibility. Once again, remember that Python versions and the version of the EXUDYN module must be identical (e.g., Python 3.6 32 bit **both** in the EXUDYN module and in Spyder).

1.2.2 Install with Windows MSI installer

The simplest way on Windows 10 (and maybe also Windows 7), which works well **if you installed only one python version** and if you installed Anaconda with the option '**Register Anaconda as my default Python 3.x**' or similar, then you can use the provided .msi installers in the main/dist directory:

- For the 64bits python 3.6 version, double click on (version may differ):
`exudyn-1.0.8.win-amd64-py3.6.msi`
- Follow the instructions of the installer

¹Anaconda3 64bit with Python3.7.7 can be downloaded via the repository archive
<https://repo.anaconda.com/archive/> choosing `Anaconda3-2020.02-Windows-x86_64.exe`

²It is important that Spyder, Python and EXUDYN are **either** 32bit **or** 64bit and are compiled up to the same minor version, i.e., 3.7.x. There will be a strange .DLL error, if you mix up 32/64bit. It is possible to install both, Anaconda 32bit and Anaconda 64bit – then you should follow the recommendations of paths as suggested by Anaconda installer.

³Anaconda 32bit with Python3.6 can be downloaded via the repository archive <https://repo.anaconda.com/archive/> choosing `Anaconda3-5.2.0-Windows-x86.exe`.

⁴Anaconda 64bit with Python3.6 can be downloaded via the repository archive <https://repo.anaconda.com/archive/> choosing `Anaconda3-5.2.0-Windows-x86_64.exe` for 64bit.

⁵previously, VS2019 was recommended: However, VS2019 has problems with the library 'Eigen' and therefore leads to erroneous results with the sparse solver. VS2017 can also be configured with Python 3.7 now.

- If python / Anaconda is not found by the installer, provide the 'python directory' as the installation directory of Anaconda3, which usually is installed in:
C:\ProgramData\Anaconda3

1.2.3 Install from Wheel (UBUNTU and Windows)

The **standard way to install** the python package EXUDYN is to use the so-called 'wheels' (file ending .whl) provided at the directory wheels in the EXUDYN repository.

For UBUNTU18.04 (which by default uses Python 3.6) this may read (version number 1.0.20 may be different):

- Python 3.6, 64bit: pip3 install dist\exudyn-1.0.20-cp36-cp36-linux_x86_64.whl

For UBUNTU20.04 (which by default uses Python 3.8) this may read (version number 1.0.20 may be different):

- Python 3.8, 64bit: pip3 install dist\exudyn-1.0.20-cp38-cp38-linux_x86_64.whl

NOTE that your installation may have environments with different python versions, so install that EXUDYN version appropriately! If the wheel installation does not work on UBUNTU, it is highly recommended to build EXUDYN for your specific system as given in [Section 1.2.6](#).

Windows:

First, open an Anaconda prompt:

- EITHER calling: START->Anaconda->... OR go to anaconda/Scripts folder and call activate.bat
- You can check your python version then, by running `python6`, the output reads like:

```
Python 3.6.5 |Anaconda, Inc.| (default, Mar 29 2018, 13:32:41) [MSC v.1900 64 bit
(AMD64)] on win32
...

```

- → type `exit()` to close python

Go to the folder Exudyn_git/main (where `setup.py` lies) and choose the wheel in subdirectory `main/dist` according to your system (windows/UBUNTU), python version (3.6 or 3.7) and 32 or 64 bits.

For Windows the installation commands may read (version number 1.0.20 may be different):

- Python 3.6, 32bit: pip install dist\exudyn-1.0.20-cp36-cp36m-win32.whl
- Python 3.6, 64bit: pip install dist\exudyn-1.0.20-cp36-cp36m-win_amd64.whl
- Python 3.7, 64bit: pip install dist\exudyn-1.0.20-cp37-cp37m-win_amd64.whl

⁶python3 under UBUNTU 18.04

1.2.4 Work without installation and editing sys.path

The **uncommon and old way** (→ not recommended for EXUDYN versions $\geq 1.0.0$) is to use Python's sys module to link to your exudyn (previously WorkingRelease) directory, for example:

```
import sys
sys.path.append('C:/DATA/cpp/EXUDYN_git/bin/EXUDYN32bitsPython36')
```

The folder EXUDYN32bitsPython36 needs to be adapted to the location of the according EXUDYN package.

1.2.5 Build and install EXUDYN under Windows 10?

Note that there are a couple of pre-requisites, depending on your system and installed libraries. For Windows 10, the following steps proved to work:

- install your Anaconda distribution including Spyder
- close all Python programs (e.g. Spyder, Jupyter, ...)
- run an Anaconda prompt (may need to be run as administrator)
- if you cannot run Anaconda prompt directly, do:
 - open windows shell (cmd.exe) as administrator (START → search for cmd.exe → right click on app → 'run as administrator' if necessary)
 - go to your Scripts folder inside the Anaconda folder (e.g. C:\ProgramData\Anaconda\Scripts)
 - run 'activate.bat'
- go to 'main' of your cloned github folder of exudyn
- run: `python setup.py install`
- read the output; if there are errors, try to solve them by installing appropriate modules

You can also create your own wheels, doing the above steps to activate the according python version and then calling (requires installation of Microsoft Visual Studio; recommended: VS2017):

```
python setup.py bdist_wheel
```

This will add a wheel in the dist folder.

1.2.6 Build and install EXUDYN under UBUNTU?

Having a new UBUNTU 18.04 standard installation (e.g. using a VM virtual box environment), the following steps need to be done (python 3.6 is already installed on UBUNTU18.04, otherwise use `sudo apt install python3`)⁷:

First update ...

```
sudo apt-get update
```

⁷see also the youtube video: https://www.youtube.com/playlist?list=PLZduTa9mdcmOh5KVUqatD9GzVg_jtl6fx

Install necessary python libraries and pip3; `matplotlib` and `scipy` are not required for installation but used in EXUDYN examples:

```
sudo dpkg --configure -a  
sudo apt install python3-pip  
pip3 install numpy  
pip3 install matplotlib  
pip3 install scipy
```

Install `pybind11` (needed for running the `setup.py` file derived from the `pybind11` example):

```
pip3 install pybind11
```

If graphics is used (`#define USE(GLFW_GRAPHICS` in `BasicDefinitions.h`), you must install the according GLFW and OpenGL libs:

```
sudo apt-get install freeglut3 freeglut3-dev  
sudo apt-get install mesa-common-dev  
sudo apt-get install libglfw3 libglfw3-dev  
sudo apt-get install libx11-dev xorg-dev libglew1.5 libglew1.5-dev libglu1-mesa libglu1-mesa-dev libgl1-mesa-glx libgl1-mesa-dev
```

With all of these libs, you can run the `setup.py` installer (go to `Exudyn_git/main` folder), which takes some minutes for compilation (the `--user` option is used to install in local user folder):

```
sudo python3 setup.py install --user
```

Congratulation! **Now, run a test example** (will also open an OpenGL window if successful):

```
python3 pythonDev/Examples/rigid3Dexample.py
```

You can also create a UBUNTU wheel which can be easily installed on the same machine (x64), same operating system (UBUNTU18.04) and with same python version (e.g., 3.6):

```
sudo pip3 install wheel  
sudo python3 setup.py bdist_wheel
```

KNOWN issues for linux builds:

- Using **WSL2** (Windows subsystem for linux), there occur some conflicts during build because of incompatible windows and linux file systems and builds will not be copied to the `dist` folder; workaround: go to explorer, right click on 'build' directory and set all rights for authenticated user to 'full access'
- **compiler (gcc,g++) conflicts:** It seems that EXUDYN works well on UBUNTU18.04 with the original Python 3.6.9 and gcc-7.5.0 version as well as with UBUNTU20.04 with Python 3.8.5 and gcc-9.3.0. Upgrading gcc on a linux system with Python 3.6 to, e.g., gcc-8.2 showed us a linker error when loading the EXUDYN module in python – there are some common restriction using gcc versions different from those with which the Python version has been built. Starting `python` or `python3` on your linux machine shows you the `gcc` version it had been build with. Check your current `gcc` version with: `gcc -version`

1.2.7 Uninstall EXUDYN

To uninstall exudyn under Windows, run (may require admin rights):

```
pip uninstall exudyn
```

To uninstall under UBUNTU, run:

```
sudo pip3 uninstall exudyn
```

If you upgrade to a newer version, uninstall is usually not necessary!

1.2.8 How to install EXUDYN and using the C++ code (advanced)?

EXUDYN is still under intensive development of core modules. There are several ways to using the code, but you **cannot** install EXUDYN as compared to other executable programs and apps.

In order to make full usage of the C++ code and extending it, you can use:

- Windows / Microsoft Visual Studio 2017 and above:
 - get the files from git
 - put them into a local directory (recommended: C:/DATA/cpp/EXUDYN_git)
 - start main_sln.sln with Visual Studio
 - compile the code and run main/pythonDev/pytest.py example code
 - adapt pytest.py for your applications
 - extend the C++ source code
 - link it to your own code
 - NOTE: on Linux systems, you mostly need to replace '/' with '\'
- Linux, etc.: not fully supported yet; however, all external libraries are Linux-compatible and thus should run with minimum adaptation efforts.

1.3 Further notes

1.3.1 Goals of EXUDYN

After the first development phase (2019-2020), it shall

- be a small multibody library, which can be easily linked to other projects,
- allow to efficiently simulate small scale systems (compute 100000s time steps per second for systems with $n_{DOF} < 10$),
- allow to efficiently simulate medium scaled systems for problems with $n_{DOF} < 1\,000\,000$,
- safe and widely accessible module for Python,
- allow to add user defined objects in C++,
- allow to add user defined solvers in Python.

Future goals are:

- extend tests,
- add more multi-threaded parallel computing techniques (first trials implemented, improvements planned: Q3 2021),
- add vectorization,
- add specific and advanced connectors/constraints (3D revolute joint and prismatic joint instead of generic joint, extended wheels, contact, control connector)
- more interfaces for robotics,
- add 3D beams,
- extend floating frame of reference formulation with modal reduction

For specific open issues, see `trackerlog.html`.

1.4 Run a simple example in Spyder

After performing the steps of the previous section, this section shows a simplistic model which helps you to check if EXUDYN runs on your computer.

In order to start, run the python interpreter Spyder. For the following example, either

- open Spyder and copy the example provided in Listing 1.1 into a new file, or
- open `myFirstExample.py` from your `EXUDYN32bitsPython36`⁸ directory

Hereafter, press the play button or F5 in Spyder.

If successful, the IPython Console of Spyder will print something like:

```
runfile('C:/DATA/cpp/EXUDYN_git/main/bin/EXUDYN32bitsPython36/myFirstExample.py',
       wdir='C:/DATA/cpp/EXUDYN_git/main/bin/EXUDYN32bitsPython36')
+++++
EXUDYN V1.0.1 solver: implicit second order time integration
STEP100, t = 1 sec, timeToGo = 0 sec, Nit/step = 1
solver finished after 0.0007824 seconds.
```

If you check your current directory (where `myFirstExample.py` lies), you will find a new file `coordinatesSolution.txt`, which contains the results of your computation (with default values for time integration). The beginning and end of the file should look like:

```
#Exudyn generalized alpha solver solution file
#simulation started=2019-11-14,20:35:12
#columns contain: time, ODE2 displacements, ODE2 velocities, ODE2 accelerations, AE
#coordinates, ODE2 velocities
#number of system coordinates [nODE2, nODE1, nAlgebraic, nData] = [2,0,0,0]
#number of written coordinates [nODE2, nVel2, nAcc2, nODE1, nVel1, nAlgebraic, nData] =
#total columns exported (excl. time) = 6
#number of time steps (planned) = 100
#
```

⁸or any other directory according to your python version

Listing 1.1: My first example

```

import exudyn as exu           #EXUDYN package including C++ core part
from exudyn.itemInterface import * #conversion of data to exudyn dictionaries

SC = exu.SystemContainer()      #container of systems
mbs = SC.AddSystem()          #add a new system to work with

nMP = mbs.AddNode(NodePoint2D(referenceCoordinates=[0,0]))
mbs.AddObject(ObjectMassPoint2D(physicsMass=10, nodeNumber=nMP ))
mMP = mbs.AddMarker(MarkerNodePosition(nodeNumber = nMP))
mbs.AddLoad(Force(markerNumber = mMP, loadVector=[0.001,0,0]))

mbs.Assemble()                #assemble system and solve
simulationSettings = exu.SimulationSettings()
simulationSettings.timeIntegration.verboseMode=1 #provide some output
exu.SolveDynamic(mbs, simulationSettings)

```

```

0,0,0,0,0,0.0001,0
0.02,2e-08,0,2e-06,0,0.0001,0
0.03,4.5e-08,0,3e-06,0,0.0001,0
0.04,8e-08,0,4e-06,0,0.0001,0
0.05,1.25e-07,0,5e-06,0,0.0001,0

...
0.96,4.608e-05,0,9.6e-05,0,0.0001,0
0.97,4.7045e-05,0,9.7e-05,0,0.0001,0
0.98,4.802e-05,0,9.8e-05,0,0.0001,0
0.99,4.9005e-05,0,9.9e-05,0,0.0001,0
1,5e-05,0,0.0001,0,0.0001,0
#simulation finished=2019-11-14,20:35:12
#Solver Info: errorOccurred=0,converged=1,solutionDiverged=0,total time steps=100,total
Newton iterations=100,total Newton jacobians=100

```

Within this file, the first column shows the simulation time and the following columns provide solution of coordinates, their derivatives and Lagrange multipliers on system level. As expected, the x -coordinate of the point mass has constant acceleration $a = f/m = 0.001/10 = 0.0001$, the velocity grows up to 0.0001 after 1 second and the point mass moves 0.00005 along the x -axis.

1.5 Trouble shooting and FAQ

1.5.1 Trouble shooting

Python import errors:

- Sometimes the EXUDYN module cannot be loaded into Python. Typical **error messages if Python versions are not compatible** are:

```
Traceback (most recent call last):
```

```
  File "<ipython-input-14-df2a108166a6>", line 1, in <module>
    import exudynCPP
```

```
ImportError: Module use of python36.dll conflicts with this version of Python.
```

Typical error messages if 32/64 bits versions are mixed:

```
Traceback (most recent call last):
```

```
  File "<ipython-input-2-df2a108166a6>", line 1, in <module>
    import exudynCPP
```

```
ImportError: DLL load failed: \%1 is not a valid Win32 application.
```

There are several reasons and workarounds:

- You mixed up 32 and 64 bits (see below) version
- You are using an exudyn version for Python $x_1.y_1$ (e.g., 3.6.z₁) different from the Python $x_2.y_2$ version in your Anaconda (e.g., 3.7.z₂); note that $x_1 = x_2$ and $y_1 = y_2$ must be obeyed while z₁ and z₂ may be different
- **ModuleNotFoundError: No module named 'exudynCPP':**

A known reason is that your CPU⁹ does not support AVX2, while exudyn is compiled with the AVX2 option;

→ **workaround** to solve the AVX problem: use the Python 3.6 32bits version, which is compiled without AVX2; you can also compile for your specific Python version without AVX if you adjust the `setup.py` file in the main folder.

The **ModuleNotFoundError** may also happen if something went wrong during installation (paths, problems with Anaconda, ..) → very often a new installation of Anaconda and EXUDYN helps.

⁹modern Intel Core-i3, Core-i5 and Core-i7 processors as well as AMD processors, especially Zen and Zen-2 architectures should have no problems with AVX2; however, low-cost Celeron, Pentium and older AMD processors **do not support AVX2**, e.g., Intel Celeron G3900, Intel core 2 quad q6600, Intel Pentium Gold G5400T; check the system settings of your computer to find out the processor type; typical CPU manufacturer pages or Wikipedia provide information on this

Typical Python errors:

- Python **syntax errors** with missing braces

```
File "C:\DATA\cpp\EXUDYN_git\main\pythonDev\Examples\springDamperTutorial.py", line 42
    nGround=mbs.AddNode(NodePointGround(referenceCoordinates = [0,0,0]))
    ^
SyntaxError: invalid syntax
```

→ such an error points to the line of your code (line 42), but in fact the error may have been caused in previous code, such as in this case there was a missing brace in the line 40, which caused the error:

```
38 n1=mbs.AddNode(Point(referenceCoordinates = [L,0,0],
39                     initialCoordinates = [u0,0,0],
40                     initialVelocities= [v0,0,0])
41 #ground node
42 nGround=mbs.AddNode(NodePointGround(referenceCoordinates = [0,0,0]))
```

Typical solver errors:

- Typical **solver error due to redundant constraints or missing inertia terms**, could read as follows:

```
=====
SYSTEM ERROR [file 'C:\ProgramData\Anaconda3_64b37\lib\site-packages\exudyn\solver.py',
, line 207]:
CSolverBase::Newton: System Jacobian seems to be singular / not invertible!
time/load step #1, time = 0.0002
causing system equation number (coordinate number) = 42
```

→ this solver error shows that equation 42 is not solvable. The according coordinate is shown later in such an error message:

```
...
The causing system equation 42 belongs to a algebraic variable (Lagrange multiplier)
Potential object number(s) causing linear solver to fail: [7]
object 7, name='object7', type=JointGeneric
```

→ object 7 seems to be the reason, possibly there are too much (joint) constraints applied to your system, check this object.

→ show typical REASONS and SOLUTIONS, by using `showHints=True` in `exu.SolveDynamic(...)` or `exu.SolveStatic(...)`

→ You can also **highlight** object 7 by using the following code in the iPython console:

```
exu.StartRenderer()
HighlightItem(SC,mbs,7)
```

which draws the according object in red and others gray/transparent (but sometimes objects may be hidden inside other objects!). See the command's description for further options, e.g., to highlight nodes.

- Typical solver error if Newton does not converge:

```
+++++
EXUDYN V1.0.200 solver: implicit second order time integration
Newton (time/load step #1): convergence failed after 25 iterations; relative error =
  0.079958, time = 2
Newton (time/load step #1): convergence failed after 25 iterations; relative error =
  0.0707764, time = 1
Newton (time/load step #1): convergence failed after 25 iterations; relative error =
  0.0185745, time = 0.5
Newton (time/load step #2): convergence failed after 25 iterations; relative error =
  0.332953, time = 0.5
Newton (time/load step #2): convergence failed after 25 iterations; relative error =
  0.0783815, time = 0.375
Newton (time/load step #2): convergence failed after 25 iterations; relative error =
  0.0879718, time = 0.3125
Newton (time/load step #2): convergence failed after 25 iterations; relative error =
  2.84704e-06, time = 0.28125
Newton (time/load step #3): convergence failed after 25 iterations; relative error =
  1.9894e-07, time = 0.28125
STEP348, t = 20 sec, timeToGo = 0 sec, Nit/step = 7.00575
solver finished after 0.258349 seconds.
```

- this solver error is caused, because the nonlinear system cannot be solved using Newton's method.
- The static or dynamic solver by default tries to reduce step size to overcome this problem, but may fail finally (at minimum step size).
- possible reasons are: too large time steps (reduce step size by using more steps/second), inappropriate initial conditions, or inappropriate joints or constraints (remove joints to see if they are the reason), usually within a singular configuration. Sometimes a system may be just unsolvable in the way you set it up.

- Typical solver error if (e.g., syntax) **error in user function** (output may be very long, **read always message on top!**):

```
=====
SYSTEM ERROR [file 'C:\ProgramData\Anaconda3_64b37\lib\site-packages\exudyn\solver.py',
, line 214]:
Error in python USER FUNCTION 'LoadCoordinate::loadVectorUserFunction' (referred line
number my be wrong!):
NameError: name 'sin' is not defined

At:
  C:\DATA\cpp\DocumentationAndInformation\tests\springDamperUserFunctionTest.py(48):
    Sweep
  C:\DATA\cpp\DocumentationAndInformation\tests\springDamperUserFunctionTest.py(54):
    userLoad
  C:\ProgramData\Anaconda3_64b37\lib\site-packages\exudyn\solver.py(214): SolveDynamic
```

```

C:\DATA\cpp\DocumentationAndInformation\tests\springDamperUserFunctionTest.py(106):
    <module>
C:\ProgramData\Anaconda3_64b37\lib\site-packages\spyder_kernels\customize\
    spydercustomize.py(377): exec_code
C:\ProgramData\Anaconda3_64b37\lib\site-packages\spyder_kernels\customize\
    spydercustomize.py(476): runfile
<ipython-input-14-323569bebfb4>(1): <module>
C:\ProgramData\Anaconda3_64b37\lib\site-packages\IPython\core\interactiveshell.py
    (3331): run_code
...
...
; check your python code!
=====

```

Solver stopped! use showHints=True to show helpful information

→ this indicates an error in the user function `LoadCoordinate::loadVectorUserFunction`, because `sin` function has not been defined (must be imported, e.g., from `math`). It indicates that the error occurred in line 48 in `springDamperUserFunctionTest.py` within function `Sweep`, which has been called from function `userLoad`, etc.

1.5.2 FAQ

Some frequently asked questions:

1. When importing EXUDYN in python (windows) I get an error → see trouble shooting instructions above!
2. I do not understand the python errors – how can I find the reason of the error or crash?
 - Read trouble shooting section above!
 - First, you should read all error messages and warnings: from the very first to the last message. Very often, there is a definite line number which shows the error. Note, that if you are executing a string (or module) as a python code, the line numbers refer to the local line number inside the script or module.
 - If everything fails, try to execute only part of the code to find out where the first error occurs. By omitting parts of the code, you should find the according source of the error.
 - If you think, it is a bug: send an email with a representative code snippet, version, etc. to reply.exudyn@gmail.com
3. Spyder console hangs up, does not show error messages, ...:
 - very often a new start of Spyder helps; most times, it is sufficient to restart the kernel or to just press the 'x' in your IPython console, which closes the current session and restarts the kernel (this is much faster than restarting Spyder);
 - restarting the IPython console also brings back all error messages

List of Frequently asked questions:

1. Where do I find the '.exe' file?

- EXUDYN is only available via the python interface as a module 'exudyn', the C++ code being inside of `exudynCPP.pyd`, which is located in the `exudyn` folder where you installed the package. This means that you need to **run python** (best: Spyder) and import the EXUDYN module.
2. I get the error message 'check potential mixing of different (object, node, marker, ...) indices', what does it mean?
- probably you used wrong item indices, see beginning of [Section 4](#).
 - E.g., an object number `oNum = mbs.AddObject(...)` is used at a place where a `NodeIndex` is expected: `mbs.AddObject(MassPoint(nodeNumber=oNum, ...))`
 - Usually, this is an ERROR in your code, it does not make sense to mix up these indices!
 - In the exceptional case, that you want to convert numbers, see beginning of [Section 4](#).
3. Why does type auto completion does not work for `mbs` (Main system)?
- UPDATE 2020-06-01: with Spyder 4, using Python 3.7, type auto completion works much better, but may find too many completions.
 - most python environments (e.g., with Spyder 3) only have information up to the first sub-structure, e.g., `SC=exu.SystemContainer()` provides full access to `SC` in the type completion, but `mbs=SC.AddSystem()` is at the second sub-structure of the module and is not accessible.
WORKAROUND: type `mbs=MainSystem()` before the `mbs=SC.AddSystem()` command and the interpreter will know what type `mbs` is. This also works for settings, e.g., simulation settings 'Newton'.
4. How to add graphics?
- Graphics (lines, text, 3D triangular / STL mesh) can be added to all `BodyGraphicsData` items in objects. Graphics objects which are fixed with the background can be attached to a `ObjectGround` object. Moving objects must be attached to the `BodyGraphicsData` of a moving body. Other moving bodies can be realized, e.g., by adding a `ObjectGround` and changing its reference with time. Furthermore, `ObjectGround` allows to add fully user defined graphics.
5. What is the difference between `MarkerBodyPosition` and `MarkerBodyRigid`?
- Position markers (and nodes) do not have information on the orientation (rotation). For that reason, there is a difference between position based and rigid-body based markers. In case of a rigid body attached to ground with a `SpringDamper`, you can use both, `MarkerBodyPosition` or `MarkerBodyRigid`, markers. For a prismatic joint, you will need a `MarkerBodyRigid`.
6. I get an error in `exu.SolveDynamic(mbs, ...)` OR in `exu.SolveStatic(mbs, ...)` but no further information – how can I solve it?
- Typical time integration errors may look like:
- ```
File "C:/DATA/cpp/EXUDYN_git/main/pythonDev/...<file name>", line XXX, in <module>
solver.SolveSystem(...)

SystemError: <built-in method SolveSystem of PyCapsule object at 0x0CC63590> returned a result
with an error set
```

- The prechecks, which are performed to enable a crash-free simulation are insufficient for your model.
- As a first try, restart the IPython console in order to get all error messages, which may be blocked due to a previous run of EXUDYN.
- Very likely, you are using python user functions inside EXUDYN: They lead to an internal python error, which is not catched by EXUDYN. However, you can just check all your user functions, if they will run without EXUDYN. E.g., a load user function ULoad(mbs, t, load), which tries to access load[4] will fail internally.
- Use the print(...) command in python at many places to find a possible error in user functions (e.g., put `print("Start user function XYZ")` at the beginning of every user function).
- It is also possible, that you are using inconsistent data, which leads to the crash. In that case, you should try to change your model: omit parts and find out which part is causing your error
- see also *I do not understand the python errors – how can I find the cause?*

## 7. Why can't I get the focus of the simulation window on startup (render window hidden)?

- Starting EXUDYN out of Spyder might not bring the simulation window to front, because of specific settings in Spyder(version 3.2.8), e.g., Tools→Preferences→Editor→Advanced settings: uncheck 'Maintain focus in the Editor after running cells or selections'; Alternatively, set `SC.visualizationSettings.window.alwaysOnTop=True` **before** starting the renderer with `exu.StartRenderer()`



# Chapter 2

## Overview on EXUDYN

### 2.1 Module structure

This section will show:

- Overview of modules
- Conventions: dimension of nodes, objects and vectors
- Coordinates: reference coordinates and displacements
- Nodes, Objects, Markers and Loads

For an introduction to the solvers, see [Section 11](#).

#### 2.1.1 Overview of modules

Currently, the module structure is simple:

- Python parts:
  - `itemInterface`: contains the interface, which transfers python classes (e.g., of a `NodePoint`) to dictionaries that can be understood by the C++ module
  - `exudynUtilities`: contains helper classes in Python, which allows simpler working with EXUDYN
- C++ parts, see Figs. [2.1](#) and [2.2](#):
  - `exudyn`<sup>1</sup>: on this level, there are just very few functions: `SystemContainer()`, `StartRenderer()`, `StopRenderer()`
  - `SystemContainer`: contains the systems (most important), solvers (static, dynamics, ...), visualization settings

---

<sup>1</sup>For versions < 1.0.0: there is a second module, called `exudynFast`, which deactivates all range-, index- or memory allocation checks at the gain of higher speed (probably 30 percent in regular cases and up to 100 percent in the 64 bit version). This module is included by `import exudynFast as exu` and can be used same as `exudyn`. To check the version, just type `exu.__doc__` and you will see a note on 'exudynFast' in the `exudynFast` module.

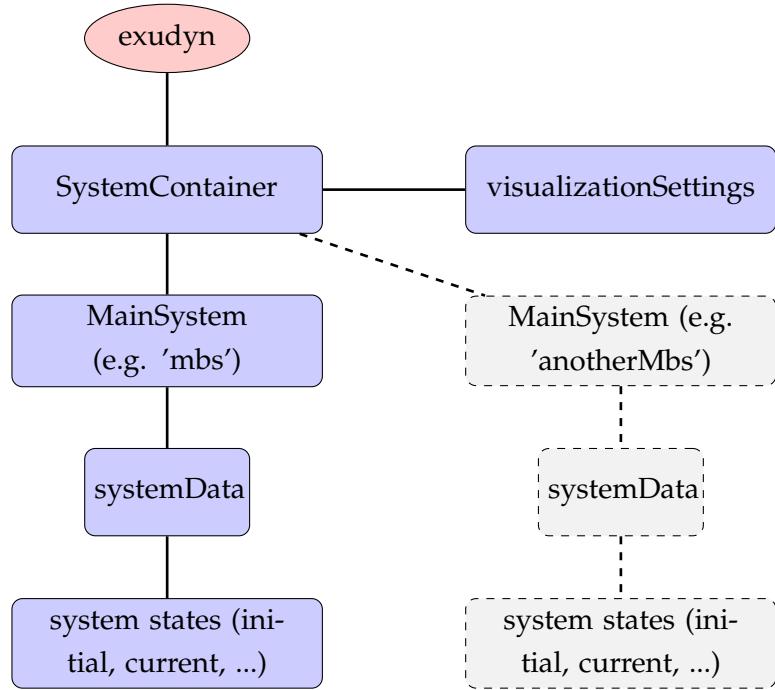


Figure 2.1: Overview of exudyn module.

- `mbs`: system created with `mbs = SC.AddSystem()`, this structure contains everything that defines a solvable multibody system; a large set of nodes, objects, markers, loads can be added to the system, see [Section 7](#);
- `mbs.systemData`: contains the initial, current, visualization, ... states of the system and holds the items, see Fig. [2.2](#)

### 2.1.2 Conventions: items, indices, coordinates

In this documentation, we will use the term **item** to identify nodes, objects, markers and loads:

$$\text{item} \in \{\text{node, object, marker, load}\} \quad (2.1)$$

#### Indices: arrays and vector starting with 0:

As known from Python, all **indices** of arrays, vectors, etc. are starting with 0. This means that the first component of the vector `v=[1, 2, 3]` is accessed with `v[0]` in Python (and also in the C++ part of EXUDYN). The range is usually defined as `range(0, 3)`, in which '3' marks the index after the last valid component of an array or vector.

#### Dimensionality of objects and vectors:

As a convention, quantities in EXUDYN are 3D, such as nodes, objects, markers, loads, measured quantities, etc. For that reason, we denote planar nodes, objects, etc. with the suffix '2D', but 3D objects do not get this suffix.

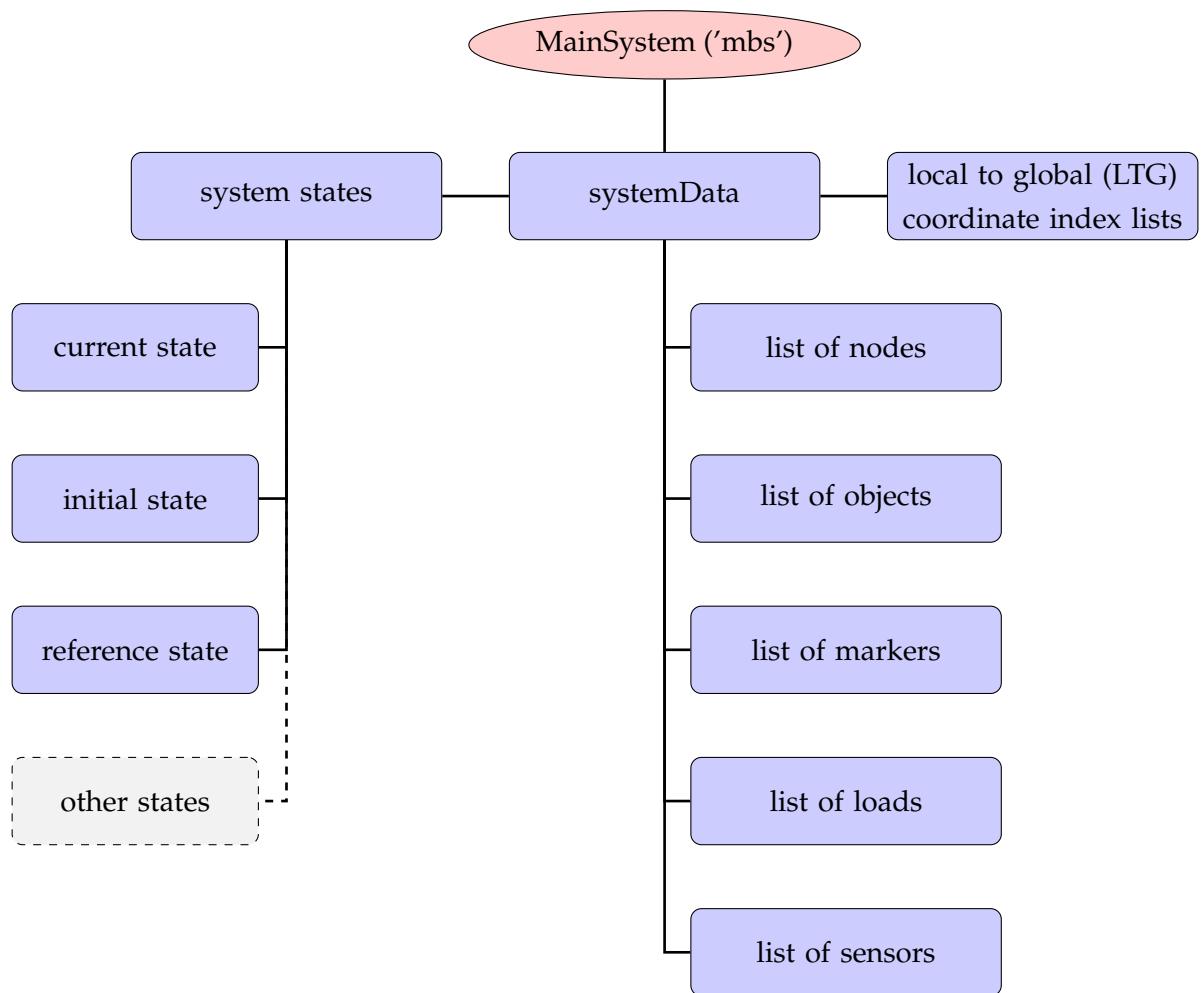


Figure 2.2: Overview of systemData, which connects items and states. Note that access to items is provided via functions in `system`.

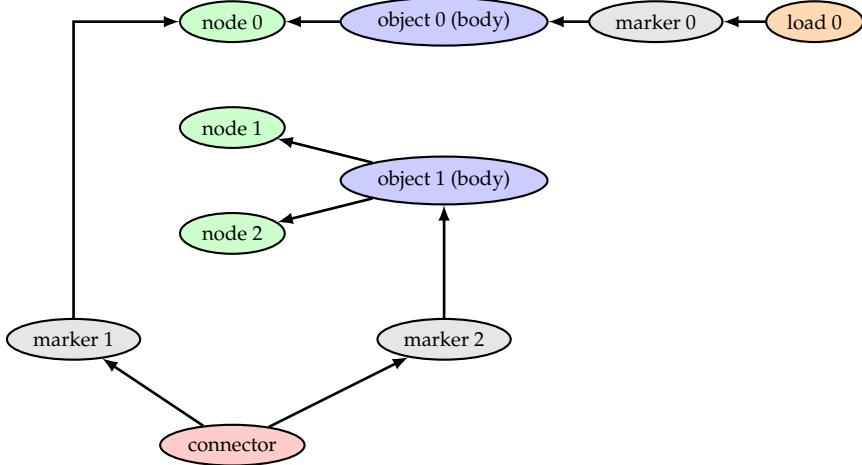


Figure 2.3: Typical interaction of items in a multibody system. Note that both, bodies and connectors/constraints are (computational) objects. The arrows indicate, that, e.g., object 1 has node 1 and node 2 (indices) and that marker 0 is attached to object 0, while load 0 uses marker 0 to apply the load. Sensors could additionally be attached to certain items.

Output and input to objects, markers, loads, etc. is usually given by 3D vectors (or matrices), such as (local) position, force, torque, rotation, etc. However, initial and reference values for nodes depend on their dimensionality. As an example, consider a `NodePoint2D`:

- `referenceCoordinates` is a 2D vector (but could be any dimension in general nodes)
- measuring the current position of `NodePoint2D` gives a 3D vector
- when attaching a `MarkerNodePosition` and a `LoadForceVector`, the force will be still a 3D vector

Furthermore, the local position in 2D objects is provided by a 3D vector. Usually, the dimensionality is given in the reference manual. User errors in the dimensionality will be usually detected either by the python interface (i.e., at the time the item is created) or by the system-preprocessor

## 2.2 Items: Nodes, Objects, Loads, Markers, Sensors, ...

In this section, the most important part of EXUDYN are provided. An overview of the interaction of the items is given in Fig. 2.3

### 2.2.1 Nodes

Nodes provide the coordinates (and the degrees of freedom) to the system. They have no mass, stiffness or whatsoever assigned. Without nodes, the system has no unknown coordinates. Adding a node provides (for the system unknown) coordinates. In addition we also need equations for every nodal coordinate – otherwise the system cannot be computed (NOTE: this is currently not checked by the preprocessor).

## 2.2.2 Objects

Objects are ‘computational objects’ and they provide equations to your system. Objects additionally often provide derivatives and have measurable quantities (e.g. displacement) and they provide access, which can be used to apply, e.g., forces.

Objects can be a:

- general object (e.g. a controller, user defined object, ...; no example yet)
- body: has a mass or mass distribution; markers can be placed on bodies; loads can be applied; constraints can be attached via markers; bodies can be:
  - ground object: has no nodes
  - simple body: has one node (e.g. mass point, rigid body)
  - finite element and more complicated body (e.g. FFRF-object): has more than one node
- connector: uses markers to connect nodes and/or bodies; adds additional terms to system equations either based on stiffness/damping or with constraints (and Lagrange multipliers). Possible connectors:
  - algebraic constraint (e.g. constrain two coordinates:  $q_1 = q_2$ )
  - classical joint
  - spring-damper or penalty constraint

## 2.2.3 Markers

Markers are interfaces between objects/nodes and constraints/loads. A constraint (which is also an object) or load cannot act directly on a node or object without a marker. As a benefit, the constraint or load does not need to know whether it is applied, e.g., to a node or to a local position of a body.

Typical situations are:

- Node – Marker – Load
- Node – Marker – Constraint (object)
- Body(object) – Marker – Load
- Body1 – Marker1 – Joint(object) – Marker2 – Body2

## 2.2.4 Loads

Loads are used to apply forces and torques to the system. The load values are static values. However, you can use Python functionality to modify loads either by linearly increasing them during static computation or by using the ‘mbs.SetPreStepUserFunction(...’ structure in order to modify loads in every integration step depending on time or on measured quantities (thus, creating a controller).

## 2.2.5 Sensors

Sensors are only used to measure output variables (values) in order to simpler generate the requested output quantities. They have a very weak influence on the system, because they are only evaluated after certain solver steps as requested by the user.

## 2.2.6 Reference coordinates and displacements

Nodes usually have separated reference and initial quantities. Here, `referenceCoordinates` are the coordinates for which the system is defined upon creation. Reference coordinates are needed, e.g., for definition of joints and for the reference configuration of finite elements. In many cases it marks the undeformed configuration (e.g., with finite elements), but not, e.g., for `ObjectConnectorSpringDamper`, which has its own reference length.

Initial displacement (or rotation) values are provided separately, in order to start a system from a configuration different from the reference configuration. As an example, the initial configuration of a `NodePoint` is given by `referenceCoordinates + initialCoordinates`, while the initial state of a dynamic system additionally needs `initialVelocities`.

## 2.3 Exudyn Basics

This section will show:

- Interaction with the EXUDYN module
- Simulation settings
- Visualization settings
- Generating output and results
- Graphics pipeline
- Generating animations

### 2.3.1 Interaction with the EXUDYN module

It is important that the EXUDYN module is basically a state machine, where you create items on the C++ side using the Python interface. This helps you to easily set up models using many other Python modules (`numpy`, `sympy`, `matplotlib`, ...) while the computation will be performed in the end on the C++ side in a very efficient manner.

#### Where do objects live?

Whenever a system container is created with `SC = exu.SystemContainer()`, the structure `SC` lives in C++ and will be modified via the python interface. Usually, the system container will hold at least one system, usually called `mbs`. Commands such as `mbs.AddNode(...)` add objects to the system `mbs`. The system will be prepared for simulation by `mbs.Assemble()` and can be solved (e.g., using `exu.SolveDynamic(...)`) and evaluated hereafter using the results files. Using `mbs.Reset()` will clear the system and allows to set up a new system. Items can be modified (`ModifyObject(...)`) after first initialization, even during simulation.

### 2.3.2 Simulation settings

The simulation settings consists of a couple of substructures, e.g., for `solutionSettings`, `staticSolver`, `timeIntegration` as well as a couple of general options – for details see Sections [8.1.1 – 8.1.10](#).

Simulation settings are needed for every solver. They contain solver-specific parameters (e.g., the way how load steps are applied), information on how solution files are written, and very specific control parameters, e.g., for the Newton solver.

The simulation settings structure is created with

```
simulationSettings = exu.SimulationSettings()
```

Hereafter, values of the structure can be modified, e.g.,

```
#10 seconds of simulation time:
simulationSettings.timeIntegration.endTime = 10

#1000 steps for time integration:
simulationSettings.timeIntegration.numberOfSteps = 1000

#assigns a new tolerance for Newton's method:
simulationSettings.timeIntegration.newton.relativeTolerance = 1e-9

#write some output while the solver is active (SLOWER):
simulationSettings.timeIntegration.verboseMode = 2

#write solution every 0.1 seconds:
simulationSettings.solutionSettings.solutionWritePeriod = 0.1

#use sparse matrix storage and solver (package Eigen):
simulationSettings.linearSolverType = exu.LinearSolverType.EigenSparse
```

### 2.3.3 Visualization settings

Visualization settings are used for user interaction with the model. E.g., the nodes, markers, loads, etc., can be visualized for every model. There are default values, e.g., for the size of nodes, which may be inappropriate for your model. Therefore, you can adjust those parameters. In some cases, huge models require simpler graphics representation, in order not to slow down performance – e.g., the number of faces to represent a cylinder should be small if there are 10000s of cylinders drawn. Even computation performance can be slowed down, if visualization takes lots of CPU power. However, visualization is performed in a separate thread, which usually does not influence the computation exhaustively. Details on visualization settings and its substructures are provided in Sections 8.2.1 – 8.2.14.

The visualization settings structure can be accessed in the system container SC (access per reference, no copying!), accessing every value or structure directly, e.g.,

```
SC.visualizationSettings.nodes.defaultSize = 0.001 #draw nodes very small

#change openGL parameters; current values can be obtained from SC.GetRenderState()
#change zoom factor:
SC.visualizationSettings.opengl.initialZoom = 0.2
#set the center point of the scene (can be attached to moving object):
SC.visualizationSettings.opengl.initialCenterPoint = [0.192, -0.0039, -0.075]
```

```

#turn off auto-fit:
SC.visualizationSettings.general.autoFitScene = False

#change smoothness of a cylinder:
SC.visualizationSettings.general.cylinderTiling = 100

#make round objects flat:
SC.visualizationSettings.opengl.shadeModelSmooth = False

#turn on coloured plot, using y-component of displacements:
SC.visualizationSettings.contour.outputVariable = exu.OutputVariableType.
 Displacement
SC.visualizationSettings.contour.outputVariableComponent = 1 #0=x, 1=y, 2=z

```

### 2.3.3.1 Storing the model view

There is a simple way to store the current view (zoom, centerpoint, orientation, etc.) by using `SC.GetRenderState()` and `SC.SetRenderState()`. A simple way is to reload the stored render state (model view) after simulating your model once at the end of the simulation<sup>2</sup>:

```

import exudyn as exu
SC=exu.SystemContainer()
SC.visualizationSettings.general.autoFitScene = False #prevent from autozoom
exu.StartRenderer()
if 'renderState' in exu.sys:
 SC.SetRenderState(exu.sys['renderState'])
#####
#do simulation here and adjust model view settings with mouse
#####

#store model view for next run:
StopRenderer() #stores render state in exu.sys['renderState']

```

Alternatively, you can obtain the current model view from the console after a simulation, e.g.,

```

In[1] : SC.GetRenderState()
Out[1]:
{'centerPoint': [1.0, 0.0, 0.0],
 'maxSceneSize': 2.0,
 'zoom': 1.0,
 'currentWindowSize': [1024, 768],

```

<sup>2</sup>note that `visualizationSettings.general.autoFitScene` should be set False if you want to use the stored zoom factor

```
'modelRotation': [[0.34202015, 0. , 0.9396926],
 [-0.60402274, 0.76604444, 0.21984631],
 [-0.7198463 , -0.6427876 , 0.26200265]]})}
```

which contains the last state of the renderer. Now copy the output and set this with `SC.SetRenderState` in your Python code to have a fixed model view in every simulation (`SC.SetRenderState` AFTER `exu.StartRenderer()`):

```
SC.visualizationSettings.general.autoFitScene = False #prevent from autozoom
exu.StartRenderer()
renderState={'centerPoint': [1.0, 0.0, 0.0],
 'maxSceneSize': 2.0,
 'zoom': 1.0,
 'currentWindowSize': [1024, 768],
 'modelRotation': [[0.34202015, 0. , 0.9396926],
 [-0.60402274, 0.76604444, 0.21984631],
 [-0.7198463 , -0.6427876 , 0.26200265]])}
SC.SetRenderState(renderState)
#.... further code for simulation here
```

---

### 2.3.4 Generating output and results

The solvers provide a number of options in `solutionSettings` to generate a solution file. As a default, exporting solution to the solution file is activated with a writing period of 0.01 seconds.

Typical output settings are:

```
#create a new simulationSettings structure:
simulationSettings = exu.SimulationSettings()

#activate writing to solution file:
simulationSettings.solutionSettings.writeSolutionToFile = True
#write results every 1ms:
simulationSettings.solutionSettings.solutionWritePeriod = 0.001

#assign new filename to solution file
simulationSettings.solutionSettings.coordinatesSolutionFileName= "myOutput.txt"

#do not export certain coordinates:
simulationSettings.solutionSettings.exportDataCoordinates = False
```

### 2.3.5 Graphics pipeline

There are basically two loops during simulation, which feed the graphics pipeline. The solver runs a loop:

- compute new step
- finish computation step; results are in current state
- copy current state to visualization state (thread safe)
- signal graphics pipeline that new visualization data is available

The openGL graphics thread (=separate thread) runs the following loop:

- render openGL scene with a given graphicsData structure (containing lines, faces, text, ...)
- go idle for some milliseconds
- check if openGL rendering needs an update (e.g. due to user interaction)
  - if update is needed, the visualization of all items is updated – stored in a graphicsData structure
- check if new visualization data is available and the time since last update is larger than a prescribed value, the graphicsData structure is updated with the new visualization state

### 2.3.6 Graphics user Python functions

There are some user functions in order to customize drawing:

- You can assign graphicsData to the visualization to most bodies, such as rigid bodies in order to change the shape. Graphics can also be imported from STL files (`GraphicsDataFromSTLfileTxt`).
- Some objects, e.g., `ObjectGenericODE2` or `ObjectRigidBody`, provide customized a function `graphicsDataUserFunction`. This user function just returns a list of `GraphicsData`, see [Section 9.3](#). With this function you can change the shape of the body in every step of the computation.
- Specifically, the `graphicsDataUserFunction` in `ObjectGround` can be used to draw any moving background in the scene.

Note that all kinds of `graphicsUserPythonFunctions` need to be called from the main (=computation) process as Python functions may not be called from separate threads (GIL). Therefore, the computation thread is interrupted to execute the `graphicsDataUserFunction` between two time steps, such that the graphics Python user function can be executed. There is a timeout variable for this interruption of the computation with a warning if scenes get too complicated.

### 2.3.7 Color and RGBA

Many functions and objects include color information. In order to allow transparency, all colors contain a list of 4 RGBA values, all values being in the range [0..1]:

- red (R) channel
- green (G) channel

- blue (B) channel
- alpha (A) value, representing transparency (A=0: fully transparent, A=1: solid)

E.g., red color with no transparency is obtained by the color=[1,0,0,1]. Color predefinitions are found in `exudynGraphicsDataUtilities.py`, e.g., `color4red` or `color4steelblue` as well a list of 10 colors `color4list`, which is convenient to be used in a loop creating objects.

### 2.3.8 Generating animations

In many dynamics simulations, it is very helpful to create animations in order to better understand the motion of bodies. Specifically, the animation can be used to visualize the model much slower or faster than the model is computed.

Animations are created based on a series of images (frames, snapshots) taken during simulation. It is important, that the current view is used to record these images – this means that the view should not be changed during the recording of images. To turn on recording of images during solving, set the following flag to a positive value

- `simulationSettings.solutionSettings.recordImagesInterval = 0.01`

which means, that after every 0.01 seconds of simulation time, an image of the current view is taken and stored in the directory and filename (without filename ending) specified by

- `SC.visualizationSettings.exportImages.saveImageFileName = "myFolder/frame"`

By default, a consecutive numbering is generated for the image, e.g., 'frame0000.tga, frame0001.tga,...'. Note that '.tga' files contain raw image data and therefore can become very large.

To create animation files, an external tool FFmpeg is used to efficiently convert a series of images into an animation. In windows, simple DOS batch files can do the job to convert frames given in the local directory to animations, e.g.:

---

```

echo off
REM 2019-12-23, Johannes Gerstmayr
REM helper file for EXUDYN to convert all frame0000.tga, frame0001.tga, ... files to a
REM video
REM for higher quality use crf option (standard: -crf 23, range: 0-51, lower crf value
REM means higher quality)

IF EXIST animation.mp4 (
 echo "animation.mp4 already exists! rename the file"
) ELSE (
 "C:\Program Files (x86)\FFMPEG\bin\ffmpeg.exe" -r 25 -start_number 0 -i frame%%05d.
 tga -c:v libx264 -vf "fps=25,format=yuv420p" animation.mp4
)

```

---

After the video has been created, you should delete the single images:

---

```
REM 2019-12-23, Johannes Gerstmayr
REM helper file for EXUDYN
REM delete all .tga images of current directory

del *.tga
```

---

## 2.4 C++ Code

This section covers some information on the C++ code. For more information see the Open source code and use doxygen.

Exudyn was developed for the efficient simulation of flexible multi-body systems. Exudyn was designed for rapid implementation and testing of new formulations and algorithms in multibody systems, whereby these algorithms can be easily implemented in efficient C++ code. The code is applied to industry-related research projects and applications.

### 2.4.1 Focus of the C++ code

**Four principles:**

1. developer-friendly
2. error minimization
3. efficiency
4. user-friendliness

The focus is therefore on:

- A developer-friendly basic structure regarding the C++ class library and the possibility to add new components.
- The basic libraries are slim, but extensively tested; only the necessary components are available
- Complete unit tests are added to new program parts during development; for more complex processes, tests are available in Python
- In order to implement the sometimes difficult formulations and algorithms without errors, error avoidance is always prioritized.
- To generate efficient code, classes for parallelization (vectorization and multithreading) are provided. We live the principle that parallelization takes place on multi-core processors with a central main memory, and thus an increase in efficiency through parallelization is only possible with small systems, as long as the program runs largely in the cache of the processor cores. Vectorization is tailored to SIMD commands as they have Intel processors, but could also be extended to GPGPUs in the future.
- The user interface (Python) provides a 1:1 image of the system and the processes running in it, which can be controlled with the extensive possibilities of Python.

## 2.4.2 C++ Code structure

The functionality of the code is based on systems (MainSystem/CSystem) representing the multibody system or similar physical systems to be simulated. Parts of the core structure of Exudyn are:

- CSystem / MainSystem: a multibody system which consists of nodes, objects, markers, loads, etc.
- SystemContainer: holds a set of systems; connects to visualization (container)
- node: used to hold coordinates (unknowns)
- (computational) object: leads to equations, using nodes
- marker: defines a consistent interface to objects (bodies) and nodes; write access ('AccessFunction')
  - provides jacobian and read access ('OutputVariable')
- load: acts on an object or node via a marker
- computational objects: efficient objects for computation = bodies, connectors, connectors, loads, nodes, ...
- visualization objects: interface between computational objects and 3D graphics
- main (manager) objects: do all tasks (e.g. interface to visualization objects, GUI, python, ...) which are not needed during computation
- static solver, kinematic solver, time integration
- python interface via pybind11; items are accessed with a dictionary interface; system structures and settings read/written by direct access to the structure (e.g. SimulationSettings, VisualizationSettings)
- interfaces to linear solvers; future: optimizer, eigenvalue solver, ... (mostly external or in python)

## 2.4.3 C++ Code: Modules

The following internal modules are used, which are represented by directories in `main/src`:

- Autogenerated: item (nodes, objects, markers and loads) classes split into main (management, python connection), visualization and computation
- Graphics: a general data structure for 2D and 3D graphical objects and a tiny OpenGL visualization; linkage to GLFW
- Linalg: Linear algebra with vectors and matrices; separate classes for small vectors (`SlimVector`), large vectors (`Vector` and `ResizableVector`), vectors without copying data (`LinkedDataVector`), and vectors with constant size (`ConstVector`)
- Main: mainly contains `SystemContainer`, `System` and `ObjectFactory`
- Objects: contains the implementation part of the autogenerated items
- Pymodules: manually created libraries for linkage to python via pybind; remaining linking to python is located in autogenerated folder
- pythonGenerator: contains python files for automatic generation of C++ interfaces and python interfaces of items;
- Solver: contains all solvers for solving a CSystem

- System: contains core item files (e.g., MainNode, CNode, MainObject, CObject, ...)
- Tests: files for testing of internal linalg (vector/matrix), data structure libraries (array, etc.) and functions
- Utilities: array structures for administrative/managing tasks (indices of objects ... bodies, forces, connectors, ...); basic classes with templates and definitions

The following main external libraries are linked to Exudyn:

- LEST: for testing of internal functions (e.g. linalg)
- GLFW: 3D graphics with openGL; cross-platform capabilities
- Eigen: linear algebra for large matrices, linear solvers, sparse matrices and link to special solvers
- pybind11: linking of C++ to python

#### 2.4.4 Code style and conventions

This section provides general coding rules and conventions, partly applicable to the C++ and python parts of the code. Many rules follow common conventions (e.g., google code style, but not always – see notation):

- write simple code (no complicated structures or uncommon coding)
- write readable code (e.g., variables and functions with names that represent the content or functionality; AVOID abbreviations)
- put a header in every file, according to Doxygen format
- put a comment to every (global) function, member function, data member, template parameter
- ALWAYS USE curly brackets for single statements in 'if', 'for', etc.; example: if (*i*<*n*) {*i* += 1;}
- use Doxygen-style comments (use '/\*!' Qt style and '@ date' with '@' instead of 'for commands')
- use Doxygen (with preceeding '@') 'test' for tests, 'todo' for todos and 'bug' for bugs
- USE 4-spaces-tab
- use C++11 standards when appropriate, but not exhaustively
- ONE class ONE file rule (except for some collectors of single implementation functions)
- add complete unit test to every function (every file has link to LEST library)
- avoid large classes (>30 member functions; > 15 data members)
- split up god classes (>60 member functions)
- mark changed code with your name and date
- REPLACE tabs by spaces: Extras->Options->C/C++->Tabstopps: tab stopp size = 4 (=standard) + KEEP SPACES=YES

#### 2.4.5 Notation conventions

The following notation conventions are applied (**no exceptions!**):

- use lowerCamelCase for names of variables (including class member variables), consts, c-define variables, ...; EXCEPTION: for algorithms following formulas, e.g.,  $f = M * q_t t + K * q$ , GBar, ...
- use UpperCamelCase for functions, classes, structs, ...
- Special cases for CamelCase: write 'ODEsystem', BUT: 'ODE1Equations'
- '[...]Init' ... in arguments, for initialization of variables; e.g. 'valueInit' for initialization of member variable 'value'
- use American English throughout: Visualization, etc.
- for (abbreviations) in capital letters, e.g. ODE, use a lower case letter afterwards:
- do not use consecutive capitalized words, e.g. DO NOT WRITE 'ODEAE'
- for functions use `ODEComputeCoords()`, for variables avoid 'ODE' at beginning: use `nODE` or write `odeCoords`
- do not use '\_' within variable or function names; exception: derivatives
- use name which exactly describes the function/variable: 'numberOfItems' instead of 'size' or 'l'
- examples for variable names: `secondOrderSize`, `massMatrix`, `mThetaTheta`
- examples for function/class names: `SecondOrderSize`, `EvaluateMassMatrix`, `Position(const Vector3D& localPosition)`
- use the Get/Set...() convention if data is retrieved from a class (Get) or something is set in a class (Set); Use `const T& Get() / T& Get` if direct access to variables is needed; Use Get/Set for pybind11
- example Get/Set: `Real* GetDataPointer()`, `Vector::SetAll(Real)`, `GetTransposed()`, `SetRotationalParam`, `SetColor(...)`, ...
- use 'Real' instead of double or float: for compatibility, also for AVX with SP/DP
- use 'Index' for array/vector size and index instead of `size_t` or `int`
- item: object, node, marker, load: anything handled within the computational/visualization systems

#### 2.4.6 No-abbreviations-rule

The code uses a **minimum set of abbreviations**; however, the following abbreviation rules are used throughout: In general: DO NOT ABBREVIATE function, class or variable names: `GetDataPointer()` instead of `GetPtr()`; exception: `cnt`, `i`, `j`, `k`, `x` or `v` in cases where it is really clear (5-line member functions).

Exceptions to the NO-ABBREVIATIONS-RULE:

- ODE ... ordinary differential equations;
- ODE2 ... marks parts related to second order differential equations (SOS2, EvalF2 in HOTINT)
- ODE1 ... marks parts related to first order differential equations (ES, EvalF in HOTINT)
- AE ... algebraic equations (IS, EvalG in HOTINT); write 'AEcoordinates' for 'algebraicEquation-sCoordinates'
- 'C[...]' ... Computational, e.g. for `ComputationalNode` ==> use 'CNode'
- min, max ... minimum and maximum

- write time derivatives with underscore: `_t`, `_tt`; example: `Position_t`, `Position_tt`, ...
- write space-wise derivatives ith underscore: `_x`, `_xx`, `_y`, ...
- if a scalar, write coordinate derivative with underscore: `_q`, `_v` (derivative w.r.t. velocity coordinates)
- for components, elements or entries of vectors, arrays, matrices: use 'item' throughout
- '`[...].Init`' ... in arguments, for initialization of variables; e.g. '`valueInit`' for initialization of member variable '`value`'

## 2.5 Changes

For continuous tracking of changes, see [Section 12](#).

# Chapter 3

## Tutorial

This section will show:

- A basic tutorial for a 1D mass and spring-damper with initial displacements, shortest possible model with practically no special settings
- A more advanced 2D rigid-body model (*coming soon*)
- Links to examples section

The python source code of this section can be found in the file:

```
main/pythonDev/Examples/springDamperTutorial.py
```

A large number of examples, some of them quite advanced, can be found in:

```
main/pythonDev/Examples
main/pythonDev/TestModels
```

This tutorial will set up a mass point and a spring damper, dynamically compute the solution and evaluate the reference solution.

We import the exudyn library and the interface for all nodes, objects, markers, loads and sensors:

```
import exudyn as exu
from exudyn.itemInterface import *
import numpy as np #for postprocessing
```

Next, we need a SystemContainer, which contains all computable systems and add a new system. Per default, you always should name your system 'mbs' (multibody system), in order to copy/paste code parts from other examples, tutorials and other projects:

```
SC = exu.SystemContainer()
mbs = SC.AddSystem()
```

In order to check, which version you are using, you can printout the current EXUDYN version. This version is in line with the issue tracker and marks the number of open/closed issues added to EXUDYN:

```
print('EXUDYN version=' + exu.__version__)
```

Using the powerful Python language, we can define some variables for our problem, which will also be used for the analytical solution:

```
L=0.5 #reference position of mass
mass = 1.6 #mass in kg
spring = 4000 #stiffness of spring-damper in N/m
damper = 8 #damping constant in N/(m/s)
f =80 #force on mass
```

For the simple spring-mass-damper system, we need initial displacements and velocities:

```
u0=-0.08 #initial displacement
v0=1 #initial velocity
x0=f/spring #static displacement
print('resonance frequency = '+str(np.sqrt(spring/mass)))
print('static displacement = '+str(x0))
```

We first need to add nodes, which provide the coordinates (and the degrees of freedom) to the system. The following line adds a 3D node for 3D mass point<sup>1</sup>:

```
n1=mbs.AddNode(Point(referenceCoordinates = [L,0,0],
 initialCoordinates = [u0,0,0],
 initialVelocities = [v0,0,0]))
```

Here, `Point` (=NodePoint) is a Python class, which takes a number of arguments defined in the reference manual. The arguments here are `referenceCoordinates`, which are the coordinates for which the system is defined. The initial configuration is given by `referenceCoordinates + initialCoordinates`, while the initial state additionally gets `initialVelocities`. The command `mbs.AddNode(...)` returns a `NodeIndex` `n1`, which basically contains an integer, which can only be used as node number. This node number will be used later on to use the node in the object or in the marker.

While `Point` adds 3 unknown coordinates to the system, which need to be solved, we also can add ground nodes, which can be used similar to nodes, but they do not have unknown coordinates – and therefore also have no initial displacements or velocities. The advantage of ground nodes (and ground bodies) is that no constraints are needed to fix these nodes. Such a ground node is added via:

```
nGround=mbs.AddNode(NodePointGround(referenceCoordinates = [0,0,0]))
```

In the next step, we add an object<sup>2</sup>, which provides equations for coordinates. The `MassPoint` needs at least a mass (kg) and a node number to which the mass point is attached. Additionally, graphical objects could be attached:

```
massPoint = mbs.AddObject(MassPoint(physicsMass = mass, nodeNumber = n1))
```

<sup>1</sup>Note: `Point` is an abbreviation for `NodePoint`, defined in `itemInterface.py`.

<sup>2</sup>For the moment, we just need to know that objects either depend on one or more nodes, which are usually bodies and finite elements, or they can be connectors, which connect (the coordinates of) objects via markers, see Section 2.1.

In order to apply constraints and loads, we need markers. These markers are used as local positions (and frames), where we can attach a constraint lateron. In this example, we work on the coordinate level, both for forces as well as for constraints. Markers are attached to the according ground and regular node number, additionally using a coordinate number (0 ... first coordinate):

```
groundMarker=mbs.AddMarker(MarkerNodeCoordinate(nodeNumber= nGround ,
 coordinate = 0))

#marker for springDamper for first (x-)coordinate:
nodeMarker = mbs.AddMarker(MarkerNodeCoordinate(nodeNumber= n1 ,
 coordinate = 0))
```

This means that loads can be applied to the first coordinate of node n1 via marker with number nodeMarker, which is in fact of type `MarkerIndex`.

Now we add a spring-damper to the markers with numbers `groundMarker` and the `nodeMarker`, providing stiffness and damping parameters:

```
nC = mbs.AddObject(CoordinateSpringDamper(markerNumbers = [groundMarker , nodeMarker] ,
 stiffness = spring ,
 damping = damper))
```

A load is added to marker `nodeMarker`, with a scalar load with value f:

```
nLoad = mbs.AddLoad(LoadCoordinate(markerNumber = nodeMarker ,
 load = f))
```

Finally, a sensor is added to the coordinate constraint object with number `nC`, requesting the `outputVariableType` `Force`:

```
mbs.AddSensor(SensorObject(objectNumber=nC, fileName='groundForce.txt',
 outputVariableType=exu.OutputVariableType.Force))
```

Note that sensors can be attached, e.g., to nodes, bodies, objects (constraints) or loads. As our system is fully set, we can print the overall information and assemble the system to make it ready for simulation:

```
print(mbs)
mbs.Assemble()
```

We will use time integration and therefore define a number of steps (fixed step size; must be provided) and the total time span for the simulation:

```
steps = 1000 #number of steps to show solution
tEnd = 1 #end time of simulation
```

All settings for simulation, see according reference section, can be provided in a structure given from `exu.SimulationSettings()`. Note that this structure will contain all default values, and only non-default values need to be provided:

```
simulationSettings = exu.SimulationSettings()
simulationSettings.solutionSettings.solutionWritePeriod = 5e-3 #output interval
general
```

```

simulationSettings.solutionSettings.sensorsWritePeriod = 5e-3 #output interval of
 sensors
simulationSettings.timeIntegration.numberOfSteps = steps
simulationSettings.timeIntegration.endTime = tEnd

```

We are using a generalized alpha solver, where numerical damping is needed for index 3 constraints. As we have only spring-dampers, we can set the spectral radius to 1, meaning no numerical damping:

```
simulationSettings.timeIntegration.generalizedAlpha.spectralRadius = 1
```

In order to visualize the results online, a renderer can be started. As our computation will be very fast, it is a good idea to wait for the user to press SPACE, before starting the simulation (uncomment second line):

```

exu.StartRenderer() #start graphics visualization
#mbs.WaitForUserToContinue() #wait for pressing SPACE bar to continue

```

As the simulation is still very fast, we will not see the motion of our node. Using e.g. `steps=10000000` in the lines above allows you online visualize the resulting oscillations.

Finally, we start the solver, by telling which system to be solved, solver type and the simulation settings:

```
exu.SolveDynamic(mbs, simulationSettings)
```

After simulation, our renderer needs to be stopped (otherwise it would stay in background and prohibit further simulations). Sometimes you would like to wait until closing the render window, using `WaitForRenderEngineStopFlag()`:

```
#SC.WaitForRenderEngineStopFlag()#wait for pressing 'Q' to quit
exu.StopRenderer() #safely close rendering window!
```

There are several ways to evaluate results, see the reference pages. In the following we take the final value of node n1 and read its 3D position vector:

```

#evaluate final (=current) output values
u = mbs.GetNodeOutput(n1, exu.OutputVariableType.Position)
print('displacement=', u)

```

The following code generates a reference (exact) solution for our example:

```

import matplotlib.pyplot as plt
import matplotlib.ticker as ticker

omega0 = np.sqrt(spring/mass) #eigen frequency of undamped system
dRel = damper/(2*np.sqrt(spring*mass)) #dimensionless damping
omega = omega0*np.sqrt(1-dRel**2) #eigen freq of damped system
C1 = u0-x0 #static solution needs to be considered!
C2 = (v0+omega0*dRel*C1) / omega #C1, C2 are coeffs for solution

refSol = np.zeros((steps+1,2))

```

```

for i in range(0,steps+1):
 t = tEnd*i/steps
 refSol[i,0] = t
 refSol[i,1] = np.exp(-omega0*dRel*t)*(C1*np.cos(omega*t)+C2*np.sin(omega*t))+x0

plt.plot(refSol[:,0], refSol[:,1], 'r-', label='displacement (m); exact solution')

```

Now we can load our results from the default solution file `coordinatesSolution.txt`, which is in the same directory as your python tutorial file. For convenient reading the file containing commented lines, we use a numpy feature and finally plot the displacement of coordinate 0 or our mass point<sup>3</sup>:

```

data = np.loadtxt('coordinatesSolution.txt', comments='#', delimiter=',')
plt.plot(data[:,0], data[:,1], 'b-', label='displacement (m); numerical solution')

```

The sensor result can be loaded in the same way. The sensor output format contains time in the first column and sensor values in the remaining columns. The number of columns depends on the sensor and the output quantity (scalar, vector, ...):

```

data = np.loadtxt('groundForce.txt', comments='#', delimiter=',')
plt.plot(data[:,0], data[:,1]*1e-3, 'g-', label='force (kN)')

```

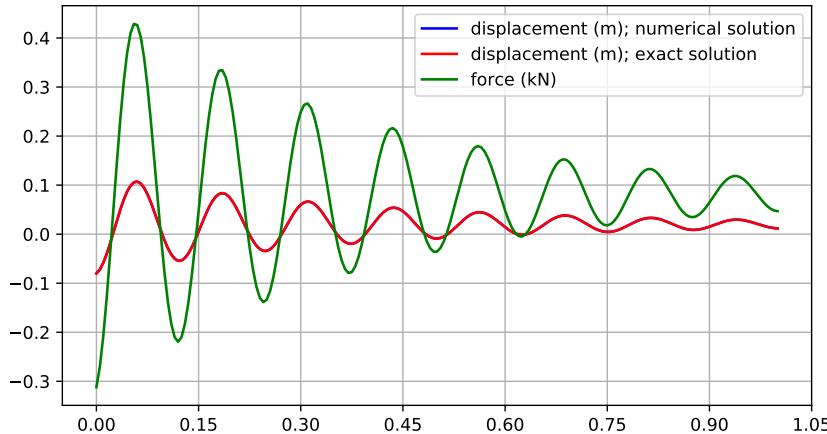
In order to get a nice plot within Spyder, the following options can be used<sup>4</sup>:

```

ax=plt.gca() # get current axes
ax.grid(True, 'major', 'both')
ax.xaxis.set_major_locator(ticker.MaxNLocator(10))
ax.yaxis.set_major_locator(ticker.MaxNLocator(10))
plt.legend() #show labels as legend
plt.tight_layout()
plt.show()

```

The matplotlib output should look like this:



<sup>3</sup>data[:,0] contains the simulation time, data[:,1] contains displacement of (global) coordinate 0, data[:,2] contains displacement of (global) coordinate 1, ...)

<sup>4</sup>note, in some environments you need finally the command plt.show()

Further examples can be found in your copy of exodyn:

`main/pythonDev/Examples`

`main/pythonDev/TestModels`

## Chapter 4

# Python-C++ command interface

This section lists the basic interface functions which can be used to set up a EXUDYN model in Python.

To import the module, just include the EXUDYN module in Python (for compatibility with examples and other users, we recommend to use the 'exu' abbreviation throughout)<sup>1</sup>:

```
import exudyn as exu
```

In addition, you may work with a convenient interface for your items, therefore also always include the line

```
from exudyn.itemInterface import *
```

Everything you work with is provided by the class `SystemContainer`, except for some very basic system functionality (which is inside the EXUDYN module).

You can create a new `SystemContainer`, which is a class that is initialized by assigning a system container to a variable, usually denoted as 'SC':

```
SC = exu.SystemContainer()
```

Note that creating a second `exu.SystemContainer()` will be independent of SC and therefore usually makes no sense.

Furthermore, there are a couple of commands available directly in the EXUDYN module, given in the following subsections. Regarding the **(basic) module access**, functions are related to the '`exudyn = exu`' module, see these examples:

```
import exudyn as exu
from exudyn.itemInterface import *
SC = exu.SystemContainer()
exu.InfoStat() #prints some statistics if available
exu.Go() #creates a systemcontainer and main system
nInvalid = exu.InvalidIndex() #the invalid index, depends on architecture and version
```

---

<sup>1</sup>note that there is a second module, called `exudynFast`, which does deactivates all range-, index- or memory allocation checks at the gain of higher speed (probably 30 percent in regular cases). To check which version you have, just type `exu.__doc__` and you will see a note on '`exudynFast`' in the `exudynFast` module.

Understanding the usage of functions for python object 'SystemContainer' provided by EXUDYN, the following examples might help:

```
import exudyn as exu
from exudyn.itemInterface import *
SC = exu.SystemContainer()
mbs = SC.AddSystem()
nSys = SC.NumberOfSystems()
print(nSys)
SC.Reset()
```

If you run a parameter variation (check Examples/parameterVariationExample.py), you may delete the created MainSystem mbs and the SystemContainer SC before creating new instances in order to avoid memory growth.

Many functions will work with node numbers ('NodeIndex'), object numbers ('ObjectIndex'), marker numbers ('MarkerIndex') and others. These numbers are special objects, which have been introduced in order to avoid mixing up, e.g., node and object numbers. For example, the command mbs.AddNode(...) returns a NodeIndex. For these indices, the following rules apply:

- mbs.Add[Node|Object|...](...) returns a specific NodeIndex, ObjectIndex, ...
- You can create any item index, e.g., using ni = NodeIndex(42) or oi = ObjectIndex(42)
- You can convert any item index, e.g., NodeIndex ni into an integer number using int(ni) or no.GetIndex()
- Still, you can use integers as initialization for item numbers, e.g.,
 

```
mbs.AddObject(MassPoint(nodeNumber=13, ...))
```

 However, it must be a pure integer type.
- You can also print item indices, e.g., **print**(ni) as it converts to string by default
- If you are unsure about the type of an index, use ni.GetTypeString() to show the index type

## 4.1 EXUDYN

These are the access functions to the EXUDYN module.

| function/structure name               | description                                                                                                                                                                                                                                                  |
|---------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| GetVersionString()                    | Get EXUDYN module version as string                                                                                                                                                                                                                          |
| RequireVersion(requiredVersionString) | Checks if the installed version is according to the required version. Major, micro and minor version must agree the required level. Example: RequireVersion("1.0.31")                                                                                        |
| StartRenderer(verbose = false)        | Start OpenGL rendering engine (in separate thread); use verbose=True to output information during OpenGL window creation; some of the information will only be seen in windows command (powershell) windows or linux shell, but not inside iPython or Spyder |

|                                                                                                                                                                                        |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| StopRenderer()                                                                                                                                                                         | Stop OpenGL rendering engine                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |
| SolveStatic(mbs, simulationSettings = exudyn.SimulationSettings(), updateInitialValues = False, storeSolver = True)                                                                    | Static solver function, mapped from module <code>solver</code> ; for details on the python interface see <a href="#">Section 5.15</a> ; for background on solvers, see <a href="#">Section 11</a>                                                                                                                                                                                                                                                                                                                                                              |
| SolveDynamic(mbs, simulationSettings = exudyn.SimulationSettings(), solverType = exudyn.DynamicSolverType.GeneralizedAlpha, updateInitialValues = False, storeSolver = True)           | Dynamic solver function, mapped from module <code>solver</code> ; for details on the python interface see <a href="#">Section 5.15</a> ; for background on solvers, see <a href="#">Section 11</a>                                                                                                                                                                                                                                                                                                                                                             |
| ComputeODE2Eigenvalues(mbs, simulationSettings = exudyn.SimulationSettings(), useSparseSolver = False, numberOfEigenvalues = -1, setInitialValues = True, convert2Frequencies = False) | Simple interface to scipy eigenvalue solver for eigenvalue analysis of the second order differential equations part in mbs, mapped from module <code>solver</code> ; for details on the python interface see <a href="#">Section 5.15</a>                                                                                                                                                                                                                                                                                                                      |
| SetOutputPrecision(numberOfDigits)                                                                                                                                                     | Set the precision (integer) for floating point numbers written to console (reset when simulation is started!)                                                                                                                                                                                                                                                                                                                                                                                                                                                  |
| SetLinalgOutputFormatPython(flagPythonFormat)                                                                                                                                          | true: use python format for output of vectors and matrices; false: use matlab format                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |
| SetWriteToConsole(flag)                                                                                                                                                                | set flag to write (true) or not write to console; default = true                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |
| SetWriteToFile(filename, flagWriteToFile = true, flagAppend = false)                                                                                                                   | set flag to write (true) or not write to console; default value of flagWriteToFile = false; flagAppend appends output to file, if set true; in order to finalize the file, write exu.SetWriteToFile("", False) to close the output file<br><b>EXAMPLE:</b><br><code>exu.SetWriteToConsole(False) #no output to console<br/>exu.SetWriteToFile('filename='testOutput.log', flagWriteToFile=True, flagAppend=False)<br/>exu.Print('print this to file')<br/>exu.SetWriteToFile("", False) #terminate writing to file which closes the file</code>                |
| SetPrintDelayMilliSeconds(delayMilliSeconds)                                                                                                                                           | add some delay (in milliSeconds) to printing to console, in order to let Spyder process the output; default = 0                                                                                                                                                                                                                                                                                                                                                                                                                                                |
| Print()                                                                                                                                                                                | this allows printing via exudyn with similar syntax as in python print(args) except for keyword arguments: print('test=42'); allows to redirect all output to file given by SetWriteToFile(...); does not output in case that SetWriteToConsole is set to false                                                                                                                                                                                                                                                                                                |
| InfoStat(writeOutput = true)                                                                                                                                                           | Retrieve list of global information on memory allocation and other counts as list:[array_new_counts, array_delete_counts, vector_new_counts, vector_delete_counts, matrix_new_counts, matrix_delete_counts, linkedDataVectorCast_counts]; May be extended in future; if writeOutput==True, it additionally prints the statistics; counts for new vectors and matrices should not depend on numberOfSteps, except for some objects such as ObjectGenericODE2 and for (sensor) output to files; Not available if code is compiled with __FAST_EXUDYN_LINALG flag |
| Go()                                                                                                                                                                                   | Creates a SystemContainer SC and a main system mbs                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |

|                |                                                                                                                                                                                                                                                                                                                 |
|----------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| InvalidIndex() | This function provides the invalid index, which may depend on the kind of 32-bit, 64-bit signed or unsigned integer; e.g. node index or item index in list; currently, the InvalidIndex() gives -1, but it may be changed in future versions, therefore you should use this function                            |
| variables      | this dictionary may be used by the user to store exudyn-wide data in order to avoid global python variables; usage: exu.variables["myvar"] = 42                                                                                                                                                                 |
| sys            | this dictionary is used and reserved by the system, e.g. for testsuite, graphics or system function to store module-wide data in order to avoid global python variables; the variable exu.sys['renderState'] contains the last render state after exu.StopRenderer() and can be used for subsequent simulations |

## 4.2 SystemContainer

The SystemContainer is the top level of structures in EXUDYN. The container holds all systems, solvers and all other data structures for computation. Currently, only one container shall be used. In future, multiple containers might be usable at the same time.

Example:

```
import exudyn as exu
SC = exu.SystemContainer()
mbs = SC.AddSystem()
```

| function/structure name                                          | description                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |
|------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| AddSystem()                                                      | add a new computational system                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |
| visualizationSettings                                            | this structure is read/writeable and contains visualization settings, which are immediately applied to the rendering window.<br><br>EXAMPLE:<br>SC = exu.SystemContainer()<br>SC.visualizationSettings.autoFitScene=False                                                                                                                                                                                                                                                                                                                                        |
| TimeIntegrationSolve(mainSystem, solverName, simulationSettings) | DEPRECATED, use exu.SolveDynamic(...) instead, see efSectionsec:solver:SolveDynamic! Call time integration solver for given system with solverName ('RungeKutta1'...explicit solver, 'GeneralizedAlpha'...implicit solver); use simulationSettings to individually configure the solver<br><br>EXAMPLE:<br>simSettings = exu.SimulationSettings()<br>simSettings.timeIntegration.numberOfSteps = 1000<br>simSettings.timeIntegration.endTime = 2<br>simSettings.timeIntegration.verboseMode = 1<br>SC.TimeIntegrationSolve(mbs, 'GeneralizedAlpha', simSettings) |

|                                                          |                                                                                                                                                                                                                                                                                                                                                                                                                |
|----------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| StaticSolve(mainSystem, simulationSettings)              | DEPRECATED, use exu.SolveStatic(...) instead<br>Call solver to compute a static solution of the system, considering acceleration and velocity coordinates to be zero (initial velocities may be considered by certain objects)<br><b>EXAMPLE:</b><br><code>simSettings = exu.SimulationSettings()<br/>simSettings.staticSolver.newton.relativeTolerance<br/>= 1e-6<br/>SC.StaticSolve(mbs, simSettings)</code> |
| GetRenderState()                                         | Get dictionary with current render state (openGL zoom, modelview, etc.)<br><b>EXAMPLE:</b><br><code>SC = exu.SystemContainer()<br/>renderState = SC.GetRenderState()<br/>print(renderState['zoom'])</code>                                                                                                                                                                                                     |
| SetRenderState(renderState)                              | Set current render state (openGL zoom, modelview, etc.) with given dictionary; usually, this dictionary has been obtained with GetRenderState<br><b>EXAMPLE:</b><br><code>SC = exu.SystemContainer()<br/>SC.SetRenderState(renderState)</code>                                                                                                                                                                 |
| WaitForRenderEngineStopFlag()                            | Wait for user to stop render engine (Press 'Q' or Escape-key)                                                                                                                                                                                                                                                                                                                                                  |
| RenderEngineZoomAll()                                    | Send zoom all signal, which will perform zoom all at next redraw request                                                                                                                                                                                                                                                                                                                                       |
| RedrawAndSaveImage()                                     | Redraw openGL scene and save image (command waits until process is finished)                                                                                                                                                                                                                                                                                                                                   |
| GetCurrentMouseCoordinates(useOpenGLcoordinates = False) | Get current mouse coordinates as list [x, y]; x and y being floats, as returned by GLFW, measured from top left corner of window; use GetCurrentMouseCoordinates(useOpenGLcoordinates=True) to obtain OpenGLcoordinates of projected plane                                                                                                                                                                     |
| Reset()                                                  | delete all systems and reset SystemContainer (including graphics)                                                                                                                                                                                                                                                                                                                                              |
| NumberOfSystems()                                        | obtain number of systems available in system container                                                                                                                                                                                                                                                                                                                                                         |
| GetSystem(systemNumber)                                  | obtain systems with index from system container                                                                                                                                                                                                                                                                                                                                                                |

## 4.3 MainSystem

This is the structure which defines a (multibody) system. In C++, there is a MainSystem (links to python) and a System (computational part). For that reason, the name is MainSystem on the python side, but it is often just called 'system'. It can be created, visualized and computed. Use the following functions for system manipulation.

Usage:

```

import exodyn as exu
SC = exu.SystemContainer()
mbs = SC.AddSystem()

```

| function/structure name               | description                                                                                                                                                                                                                                                                                                                                                                                         |
|---------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Assemble()                            | assemble items (nodes, bodies, markers, loads, ...);<br>Calls CheckSystemIntegrity(...), AssembleCoordinates(), AssembleLTGLists(), and AssembleInitializeSystemCoordinates()                                                                                                                                                                                                                       |
| AssembleCoordinates()                 | assemble coordinates: assign computational coordinates to nodes and constraints (algebraic variables)                                                                                                                                                                                                                                                                                               |
| AssembleLTGLists()                    | build local-to-global (ltg) coordinate lists for objects (used to build global ODE2RHS, MassMatrix, etc. vectors and matrices) and store special object lists (body, connector, constraint, ...)                                                                                                                                                                                                    |
| AssembleInitializeSystemCoordinates() | initialize all system-wide coordinates based on initial values given in nodes                                                                                                                                                                                                                                                                                                                       |
| Reset()                               | reset all lists of items (nodes, bodies, markers, loads, ...) and temporary vectors; deallocate memory                                                                                                                                                                                                                                                                                              |
| WaitForUserToContinue()               | interrupt further computation until user input -> 'pause' function                                                                                                                                                                                                                                                                                                                                  |
| SendRedrawSignal()                    | this function is used to send a signal to the renderer that the scene shall be redrawn because the visualization state has been updated                                                                                                                                                                                                                                                             |
| GetRenderEngineStopFlag()             | get the current stop simulation flag; true=user wants to stop simulation                                                                                                                                                                                                                                                                                                                            |
| SetRenderEngineStopFlag()             | set the current stop simulation flag; set to false, in order to continue a previously user-interrupted simulation                                                                                                                                                                                                                                                                                   |
| SetPreStepUserFunction()              | Sets a user function PreStepUserFunction(mbs, t) executed at beginning of every computation step; in normal case return True; return False to stop simulation after current step<br><b>EXAMPLE:</b><br><pre> def PreStepUserFunction(mbs, t):     print(mbs.systemData.NumberOfNodes())     if(t&gt;1):         return False     return True mbs.SetPreStepUserFunction(PreStepUserFunction) </pre> |

|                             |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |
|-----------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| SetPostNewtonUserFunction() | Sets a user function PostNewtonUserFunction(mbs, t) executed after successful Newton iteration in implicit or static solvers and after step update of explicit solvers, but BEFORE PostNewton functions are called by the solver; function returns list [discontinuousError, recommendedStepSize], containing a error of the PostNewtonStep, which is compared to [solver].discontinuous.iterationTolerance. The recommendedStepSize shall be negative, if no recommendation is given, 0 in order to enforce minimum step size or a specific value to which the current step size will be reduced and the step will be repeated; use this function, e.g., to reduce step size after impact or change of data variables<br><br><b>EXAMPLE:</b><br><pre>def PostNewtonUserFunction(mbs, t):     if(t&gt;1):         return [0, 1e-6]     return [0,0] mbs.SetPostNewtonUserFunction(PostNewtonUserFunction)</pre> |
| __repr__()                  | return the representation of the system, which can be, e.g., printed<br><br><b>EXAMPLE:</b><br><pre>print(mbs)</pre>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |
| systemIsConsistent          | this flag is used by solvers to decide, whether the system is in a solvable state; this flag is set to false as long as Assemble() has not been called; any modification to the system, such as Add...(), Modify...(), etc. will set the flag to false again; this flag can be modified (set to true), if a change of e.g. an object (change of stiffness) or load (change of force) keeps the system consistent, but would normally lead to systemIsConsistent=False                                                                                                                                                                                                                                                                                                                                                                                                                                           |
| interactiveMode             | set this flag to true in order to invoke a Assemble() command in every system modification, e.g. AddNode, AddObject, ModifyNode, ...; this helps that the system can be visualized in interactive mode.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |
| variables                   | this dictionary may be used by the user to store model-specific data, in order to avoid global python variables in complex models; mbs.variables["myvar"] = 42                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |
| sys                         | this dictionary is used by exudyn python libraries, e.g., solvers, to avoid global python variables                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |
| solverSignalJacobianUpdate  | this flag is used by solvers to decide, whether the jacobian should be updated; at beginning of simulation and after jacobian computation, this flag is set automatically to False; use this flag to indicate system changes, e.g. during time integration                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |
| systemData                  | Access to SystemData structure; enables access to number of nodes, objects, ... and to (current, initial, reference, ...) state variables (ODE2, AE, Data,...)                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |

### 4.3.1 MainSystem: Node

This section provides functions for adding, reading and modifying nodes. Nodes are used to define coordinates (unknowns to the static system and degrees of freedom if constraints are not present). Nodes can provide various types of coordinates for second/first order differential equations (ODE2/ODE1), algebraic equations (AE) and for data (history) variables – which are not providing unknowns in the nonlinear solver but will be solved in an additional nonlinear iteration for e.g. contact, friction or plasticity.

| function/structure name                                                            | description                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |
|------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| AddNode(pyObject)                                                                  | <p>add a node with nodeDefinition from Python node class; returns (global) node index (type NodeIndex) of newly added node; use int(nodeIndex) to convert to int, if needed (but not recommended in order not to mix up index types of nodes, objects, markers, ...)</p> <p><b>EXAMPLE:</b></p> <pre>item = Rigid2D( referenceCoordinates= [1,0.5,0], initialVelocities= [10,0,0]) mbs.AddNode(item) nodeDict = {'nodeType': 'Point', 'referenceCoordinates': [1.0, 0.0, 0.0], 'initialCoordinates': [0.0, 2.0, 0.0], 'name': 'example node'} mbs.AddNode(nodeDict)</pre> |
| GetNodeNumber(nodeName)                                                            | <p>get node's number by name (string)</p> <p><b>EXAMPLE:</b></p> <pre>n = mbs.GetNodeNumber('example node')</pre>                                                                                                                                                                                                                                                                                                                                                                                                                                                         |
| GetNode(nodeNumber)                                                                | <p>get node's dictionary by node number (type NodeIndex)</p> <p><b>EXAMPLE:</b></p> <pre>nodeDict = mbs.GetNode(0)</pre>                                                                                                                                                                                                                                                                                                                                                                                                                                                  |
| ModifyNode(nodeNumber, nodeDict)                                                   | <p>modify node's dictionary by node number (type NodeIndex)</p> <p><b>EXAMPLE:</b></p> <pre>mbs.ModifyNode(nodeNumber, nodeDict)</pre>                                                                                                                                                                                                                                                                                                                                                                                                                                    |
| GetNodeDefaults(typeName)                                                          | <p>get node's default values for a certain nodeType as (dictionary)</p> <p><b>EXAMPLE:</b></p> <pre>nodeType = 'Point' nodeDict = mbs.GetNodeDefaults(nodeType)</pre>                                                                                                                                                                                                                                                                                                                                                                                                     |
| GetNodeOutput(nodeNumber, variableType, configuration = ConfigurationType.Current) | <p>get the output of the node specified with the OutputVariableType; default configuration = 'current'; output may be scalar or array (e.g. displacement vector)</p> <p><b>EXAMPLE:</b></p> <pre>mbs.GetNodeOutput(nodeNumber=0, variableType='exu.OutputVariable.Displacement')</pre>                                                                                                                                                                                                                                                                                    |

|                                                    |                                                                                                                                                                              |
|----------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| GetNodeODE2Index(nodeNumber)                       | get index in the global ODE2 coordinate vector for the first node coordinate of the specified node<br><br><b>EXAMPLE:</b><br><code>mbs.GetNodeODE2Index(nodeNumber=0)</code> |
| GetNodeParameter(nodeNumber, parameterName)        | get nodes's parameter from node number (type NodeIndex) and parameterName; parameter names can be found for the specific items in the reference manual                       |
| SetNodeParameter(nodeNumber, parameterName, value) | set parameter 'parameterName' of node with node number (type NodeIndex) to value; parameter names can be found for the specific items in the reference manual                |

#### 4.3.2 MainSystem: Object

This section provides functions for adding, reading and modifying objects, which can be bodies (mass point, rigid body, finite element, ...), connectors (spring-damper or joint) or general objects. Objects provided terms to the residual of equations resulting from every coordinate given by the nodes. Single-noded objects (e.g. mass point) provides exactly residual terms for its nodal coordinates. Connectors constrain or penalize two markers, which can be, e.g., position, rigid or coordinate markers. Thus, the dependence of objects is either on the coordinates of the marker-objects/nodes or on nodes which the objects possess themselves.

| function/structure name                | description                                                                                                                                                                                                                                                                                                                                                                                                                        |
|----------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| AddObject(pyObject)                    | add an object with objectDefinition from Python object class; returns (global) object number (type ObjectIndex) of newly added object<br><br><b>EXAMPLE:</b><br><code>item = MassPoint(name='heavy object',<br/>nodeNumber=0, physicsMass=100)<br/>mbs.AddObject(item)<br/>objectDict = {'objectType': 'MassPoint',<br/>'physicsMass': 10,<br/>'nodeNumber': 0,<br/>'name': 'example object'}<br/>mbs.AddObject(objectDict)</code> |
| GetObjectNumber(objectName)            | get object's number by name (string)<br><br><b>EXAMPLE:</b><br><code>n = mbs.GetObjectNumber('heavy object')</code>                                                                                                                                                                                                                                                                                                                |
| GetObject(objectNumber)                | get object's dictionary by object number (type ObjectIndex)<br><br><b>EXAMPLE:</b><br><code>objectDict = mbs.GetObject(0)</code>                                                                                                                                                                                                                                                                                                   |
| ModifyObject(objectNumber, objectDict) | modify object's dictionary by object number (type ObjectIndex)<br><br><b>EXAMPLE:</b><br><code>mbs.ModifyObject(objectNumber, objectDict)</code>                                                                                                                                                                                                                                                                                   |

|                                                                                                                    |                                                                                                                                                                                                                                                                                                                                                                                                                                                               |
|--------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| GetObjectDefaults(typeName)                                                                                        | get object's default values for a certain objectType as (dictionary)<br><b>EXAMPLE:</b><br><code>objectType = 'MassPoint'<br/>objectDict = mbs.GetObjectDefaults(objectType)</code>                                                                                                                                                                                                                                                                           |
| GetObjectOutput(objectNumber, variableType)                                                                        | get object's current output variable from object number (type ObjectIndex) and OutputVariableType; can only be computed for exu.ConfigurationType.Current configuration!                                                                                                                                                                                                                                                                                      |
| GetObjectOutputBody(objectNumber, variableType, localPosition, configuration = ConfigurationType.Current)          | get body's output variable from object number (type ObjectIndex) and OutputVariableType<br><b>EXAMPLE:</b><br><code>u = mbs.GetObjectOutputBody(objectNumber = 1,<br/>variableType = exu.OutputVariableType.Position,<br/>localPosition=[1,0,0], configuration =<br/>exu.ConfigurationType.Initial)</code>                                                                                                                                                    |
| GetObjectOutputSuperElement(objectNumber, variableType, meshNodeNumber, configuration = ConfigurationType.Current) | get output variable from mesh node number of object with type SuperElement (GenericODE2, FFRF, FFRReduced - CMS) with specific OutputVariableType; the meshNodeNumber is the object's local node number, not the global node number!<br><b>EXAMPLE:</b><br><code>u = mbs.GetObjectOutputSuperElement(objectNumber<br/>= 1, variableType = exu.OutputVariableType.Position,<br/>meshNodeNumber = 12, configuration =<br/>exu.ConfigurationType.Initial)</code> |
| GetObjectParameter(objectNumber, parameterName)                                                                    | get objects's parameter from object number (type ObjectIndex) and parameterName; parameter names can be found for the specific items in the reference manual                                                                                                                                                                                                                                                                                                  |
| SetObjectParameter(objectNumber, parameterName, value)                                                             | set parameter 'parameterName' of object with object number (type ObjectIndex) to value; parameter names can be found for the specific items in the reference manual                                                                                                                                                                                                                                                                                           |

#### 4.3.3 MainSystem: Marker

This section provides functions for adding, reading and modifying markers. Markers define how to measure primal kinematical quantities on objects or nodes (e.g., position, orientation or coordinates themselves), and how to act on the quantities which are dual to the kinematical quantities (e.g., force, torque and generalized forces). Markers provide unique interfaces for loads, sensors and constraints in order to address these quantities independently of the structure of the object or node (e.g., rigid or flexible body).

| function/structure name | description |
|-------------------------|-------------|
|-------------------------|-------------|

|                                                        |                                                                                                                                                                                                                                                                                                                                                                        |
|--------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| AddMarker(pyObject)                                    | <p>add a marker with markerDefinition from Python marker class; returns (global) marker number (type MarkerIndex) of newly added marker</p> <p><b>EXAMPLE:</b></p> <pre>item = MarkerNodePosition(name='my marker',nodeNumber=1) mbs.AddMarker(item) markerDict = {'markerType': 'NodePosition', 'nodeNumber': 0, 'name': 'position0'} mbs.AddMarker(markerDict)</pre> |
| GetMarkerNumber(markerName)                            | <p>get marker's number by name (string)</p> <p><b>EXAMPLE:</b></p> <pre>n = mbs.GetMarkerNumber('my marker')</pre>                                                                                                                                                                                                                                                     |
| GetMarker(markerNumber)                                | <p>get marker's dictionary by index</p> <p><b>EXAMPLE:</b></p> <pre>markerDict = mbs.GetMarker(0)</pre>                                                                                                                                                                                                                                                                |
| ModifyMarker(markerNumber, markerDict)                 | <p>modify marker's dictionary by index</p> <p><b>EXAMPLE:</b></p> <pre>mbs.ModifyMarker(markerNumber, markerDict)</pre>                                                                                                                                                                                                                                                |
| GetMarkerDefaults(typeName)                            | <p>get marker's default values for a certain markerType as (dictionary)</p> <p><b>EXAMPLE:</b></p> <pre>markerType = 'NodePosition' markerDict = mbs.GetMarkerDefaults(markerType)</pre>                                                                                                                                                                               |
| GetMarkerParameter(markerNumber, parameterName)        | <p>get markers's parameter from markerNumber and parameterName; parameter names can be found for the specific items in the reference manual</p>                                                                                                                                                                                                                        |
| SetMarkerParameter(markerNumber, parameterName, value) | <p>set parameter 'parameterName' of marker with markerNumber to value; parameter names can be found for the specific items in the reference manual</p>                                                                                                                                                                                                                 |

#### 4.3.4 MainSystem: Load

This section provides functions for adding, reading and modifying operating loads. Loads are used to act on the quantities which are dual to the primal kinematic quantities, such as displacement and rotation. Loads represent, e.g., forces, torques or generalized forces.

| function/structure name | description |
|-------------------------|-------------|
|-------------------------|-------------|

|                                                    |                                                                                                                                                                                                                                                                                                                                                                                                                     |
|----------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| AddLoad(pyObject)                                  | <p>add a load with loadDefinition from Python load class; returns (global) load number (type LoadIndex) of newly added load</p> <p><b>EXAMPLE:</b></p> <pre>item = mbs.AddLoad(LoadForceVector(loadVector=[1,0,0], markerNumber=0, name='heavy load')) mbs.AddLoad(item) loadDict = {'loadType': 'ForceVector', 'markerNumber': 0, 'loadVector': [1.0, 0.0, 0.0], 'name': 'heavy load'} mbs.AddLoad(loadDict)</pre> |
| GetLoadNumber(loadName)                            | <p>get load's number by name (string)</p> <p><b>EXAMPLE:</b></p> <pre>n = mbs.GetLoadNumber('heavy load')</pre>                                                                                                                                                                                                                                                                                                     |
| GetLoad(loadNumber)                                | <p>get load's dictionary by index</p> <p><b>EXAMPLE:</b></p> <pre>loadDict = mbs.GetLoad(0)</pre>                                                                                                                                                                                                                                                                                                                   |
| ModifyLoad(loadNumber, loadDict)                   | <p>modify load's dictionary by index</p> <p><b>EXAMPLE:</b></p> <pre>mbs.ModifyLoad(loadNumber, loadDict)</pre>                                                                                                                                                                                                                                                                                                     |
| GetLoadDefaults(typeName)                          | <p>get load's default values for a certain loadType as (dictionary)</p> <p><b>EXAMPLE:</b></p> <pre>loadType = 'ForceVector' loadDict = mbs.GetLoadDefaults(loadType)</pre>                                                                                                                                                                                                                                         |
| GetLoadValues(loadNumber)                          | <p>Get current load values, specifically if user-defined loads are used; can be scalar or vector-valued return value</p>                                                                                                                                                                                                                                                                                            |
| GetLoadParameter(loadNumber, parameterName)        | <p>get loads's parameter from loadNumber and parameterName; parameter names can be found for the specific items in the reference manual</p>                                                                                                                                                                                                                                                                         |
| SetLoadParameter(loadNumber, parameterName, value) | <p>set parameter 'parameterName' of load with loadNumber to value; parameter names can be found for the specific items in the reference manual</p>                                                                                                                                                                                                                                                                  |

#### 4.3.5 MainSystem: Sensor

This section provides functions for adding, reading and modifying operating sensors. Sensors are used to measure information in nodes, objects, markers, and loads for output in a file.

| function/structure name | description |
|-------------------------|-------------|
|-------------------------|-------------|

|                                                                          |                                                                                                                                                                                                                                                                                                                                                                                                                                        |
|--------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| AddSensor(pyObject)                                                      | <p>add a sensor with sensor definition from Python sensor class; returns (global) sensor number (type SensorIndex) of newly added sensor</p> <p><b>EXAMPLE:</b></p> <pre>item = mbs.AddSensor(SensorNode(sensorType= exu.SensorType.Node, nodeNumber=0, name='test sensor')) mbs.AddSensor(item) sensorDict = {'sensorType': 'Node', 'nodeNumber': 0, 'fileName': 'sensor.txt', 'name': 'test sensor'} mbs.AddSensor(sensorDict)</pre> |
| GetSensorNumber(sensorName)                                              | <p>get sensor's number by name (string)</p> <p><b>EXAMPLE:</b></p> <pre>n = mbs.GetSensorNumber('test sensor')</pre>                                                                                                                                                                                                                                                                                                                   |
| GetSensor(sensorNumber)                                                  | <p>get sensor's dictionary by index</p> <p><b>EXAMPLE:</b></p> <pre>sensorDict = mbs.GetSensor(0)</pre>                                                                                                                                                                                                                                                                                                                                |
| ModifySensor(sensorNumber, sensorDict)                                   | <p>modify sensor's dictionary by index</p> <p><b>EXAMPLE:</b></p> <pre>mbs.ModifySensor(sensorNumber, sensorDict)</pre>                                                                                                                                                                                                                                                                                                                |
| GetSensorDefaults(typeName)                                              | <p>get sensor's default values for a certain sensorType as (dictionary)</p> <p><b>EXAMPLE:</b></p> <pre>sensorType = 'Node' sensorDict = mbs.GetSensorDefaults(sensorType)</pre>                                                                                                                                                                                                                                                       |
| GetSensorValues(sensorNumber, configuration = ConfigurationType.Current) | <p>get sensors's values for configuration; can be a scalar or vector-valued return value!</p>                                                                                                                                                                                                                                                                                                                                          |
| GetSensorParameter(sensorNumber, parameterName)                          | <p>get sensors's parameter from sensorNumber and parameterName; parameter names can be found for the specific items in the reference manual</p>                                                                                                                                                                                                                                                                                        |
| SetSensorParameter(sensorNumber, parameterName, value)                   | <p>set parameter 'parameterName' of sensor with sensorNumber to value; parameter names can be found for the specific items in the reference manual</p>                                                                                                                                                                                                                                                                                 |

## 4.4 SystemData

This is the data structure of a system which contains Objects (bodies/constraints/...), Nodes, Markers and Loads. The SystemData structure allows advanced access to this data, which HAS TO BE USED WITH CARE, as unexpected results and system crash might happen.

Usage:

```
#obtain current ODE2 system vector (e.g. after static simulation finished):
u = mbs.systemData.GetODE2Coordinates()
#set initial ODE2 vector for next simulation:
mbs.systemData.SetODE2Coordinates(coordinates=u, configurationType=exu.ConfigurationType.Initial)
```

| function/structure name                                              | description                                                                                                                                                                                         |
|----------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| NumberOfLoads()                                                      | return number of loads in system<br><b>EXAMPLE:</b><br><code>print(mbs.systemData.NumberOfLoads())</code>                                                                                           |
| NumberOfMarkers()                                                    | return number of markers in system<br><b>EXAMPLE:</b><br><code>print(mbs.systemData.NumberOfMarkers())</code>                                                                                       |
| NumberOfNodes()                                                      | return number of nodes in system<br><b>EXAMPLE:</b><br><code>print(mbs.systemData.NumberOfNodes())</code>                                                                                           |
| NumberOfObjects()                                                    | return number of objects in system<br><b>EXAMPLE:</b><br><code>print(mbs.systemData.NumberOfObjects())</code>                                                                                       |
| NumberOfSensors()                                                    | return number of sensors in system<br><b>EXAMPLE:</b><br><code>print(mbs.systemData.NumberOfSensors())</code>                                                                                       |
| ODE2Size(configurationType<br>exu.ConfigurationType.Current)         | = get size of ODE2 coordinate vector for given configuration<br>(only works correctly after mbs.Assemble() )<br><b>EXAMPLE:</b><br><code>print('ODE2 size=',mbs.systemData.ODE2Size())</code>       |
| ODE1Size(configurationType<br>exu.ConfigurationType.Current)         | = get size of ODE1 coordinate vector for given configuration<br>(only works correctly after mbs.Assemble() )<br><b>EXAMPLE:</b><br><code>print('ODE1 size=',mbs.systemData.ODE1Size())</code>       |
| AESize(configurationType<br>exu.ConfigurationType.Current)           | = get size of AE coordinate vector for given configuration<br>(only works correctly after mbs.Assemble() )<br><b>EXAMPLE:</b><br><code>print('AE size=',mbs.systemData.AEsize())</code>             |
| DataSize(configurationType<br>exu.ConfigurationType.Current)         | = get size of Data coordinate vector for given configuration<br>(only works correctly after mbs.Assemble() )<br><b>EXAMPLE:</b><br><code>print('Data size=',mbs.systemData.DataSize())</code>       |
| SystemSize(configurationType<br>exu.ConfigurationType.Current)       | = get size of System coordinate vector for given configuration<br>(only works correctly after mbs.Assemble() )<br><b>EXAMPLE:</b><br><code>print('System size=',mbs.systemData.SystemSize())</code> |
| GetTime(configurationType<br>exu.ConfigurationType.Current)          | = get configuration dependent time.<br><b>EXAMPLE:</b><br><code>mbs.systemData.GetTime(exu.ConfigurationType.Initial)</code>                                                                        |
| SetTime(newTime, configurationType<br>exu.ConfigurationType.Current) | = set configuration dependent time; use this access with care, e.g. in user-defined solvers.<br><b>EXAMPLE:</b><br><code>mbs.systemData.SetTime(10.,<br/>exu.ConfigurationType.Initial)</code>      |
| Info()                                                               | print detailed system information for every item; for short information use print(mbs)<br><b>EXAMPLE:</b><br><code>mbs.systemData.Info()</code>                                                     |

#### 4.4.1 SystemData: Access coordinates

This section provides access functions to global coordinate vectors. Assigning invalid values or using wrong vector size might lead to system crash and unexpected results.

| function/structure name                                                            | = | description                                                                                                                                                                                                                                   |
|------------------------------------------------------------------------------------|---|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| GetODE2Coordinates(configuration<br>exu.ConfigurationType.Current)                 | = | get ODE2 system coordinates (displacements) for given configuration (default: exu.Configuration.Current)<br><b>EXAMPLE:</b><br><code>uCurrent = mbs.systemData.GetODE2Coordinates()</code>                                                    |
| SetODE2Coordinates(coordinates, configuration<br>exu.ConfigurationType.Current)    | = | set ODE2 system coordinates (displacements) for given configuration (default: exu.Configuration.Current); invalid vector size may lead to system crash!<br><b>EXAMPLE:</b><br><code>mbs.systemData.SetODE2Coordinates(uCurrent)</code>        |
| GetODE2Coordinates_t(configuration<br>exu.ConfigurationType.Current)               | = | get ODE2 system coordinates (velocities) for given configuration (default: exu.Configuration.Current)<br><b>EXAMPLE:</b><br><code>vCurrent = mbs.systemData.GetODE2Coordinates_t()</code>                                                     |
| SetODE2Coordinates_t(coordinates, configuration<br>exu.ConfigurationType.Current)  | = | set ODE2 system coordinates (velocities) for given configuration (default: exu.Configuration.Current); invalid vector size may lead to system crash!<br><b>EXAMPLE:</b><br><code>mbs.systemData.SetODE2Coordinates_t(vCurrent)</code>         |
| GetODE2Coordinates_tt(configuration<br>exu.ConfigurationType.Current)              | = | get ODE2 system coordinates (accelerations) for given configuration (default: exu.Configuration.Current)<br><b>EXAMPLE:</b><br><code>vCurrent = mbs.systemData.GetODE2Coordinates_tt()</code>                                                 |
| SetODE2Coordinates_tt(coordinates, configuration<br>exu.ConfigurationType.Current) | = | set ODE2 system coordinates (accelerations) for given configuration (default: exu.Configuration.Current); invalid vector size may lead to system crash!<br><b>EXAMPLE:</b><br><code>mbs.systemData.SetODE2Coordinates_tt(aCurrent)</code>     |
| GetODE1Coordinates(configuration<br>exu.ConfigurationType.Current)                 | = | get ODE1 system coordinates (displacements) for given configuration (default: exu.Configuration.Current)<br><b>EXAMPLE:</b><br><code>qCurrent = mbs.systemData.GetODE1Coordinates()</code>                                                    |
| SetODE1Coordinates(coordinates, configuration<br>exu.ConfigurationType.Current)    | = | set ODE1 system coordinates (displacements) for given configuration (default: exu.Configuration.Current); invalid vector size may lead to system crash!<br><b>EXAMPLE:</b><br><code>mbs.systemData.SetODE1Coordinates(qCurrent)</code>        |
| GetAECoordinates(configuration<br>exu.ConfigurationType.Current)                   | = | get algebraic equations (AE) system coordinates for given configuration (default: exu.Configuration.Current)<br><b>EXAMPLE:</b><br><code>lambdaCurrent = mbs.systemData.GetAECoordinates()</code>                                             |
| SetAECoordinates(coordinates, configuration<br>exu.ConfigurationType.Current)      | = | set algebraic equations (AE) system coordinates for given configuration (default: exu.Configuration.Current); invalid vector size may lead to system crash!<br><b>EXAMPLE:</b><br><code>mbs.systemData.SetAECoordinates(lambdaCurrent)</code> |

|                                                                    |               |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |
|--------------------------------------------------------------------|---------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| GetDataCoordinates(configuration<br>exu.ConfigurationType.Current) | =             | get system data coordinates for given configuration<br>(default: exu.Configuration.Current)<br><b>EXAMPLE:</b><br><code>mbs.systemData.GetDataCoordinates()</code>                                                                                                                                                                                                                                                                                                                                                               |
| SetDataCoordinates(coordinates,<br>exu.ConfigurationType.Current)  | configuration | = set system data coordinates for given configuration (default: exu.Configuration.Current); invalid vector size may lead to system crash!<br><b>EXAMPLE:</b><br><code>mbs.systemData.SetDataCoordinates(dataCurrent)</code>                                                                                                                                                                                                                                                                                                      |
| GetSystemState(configuration<br>exu.ConfigurationType.Current)     | =             | get system state for given configuration (default: exu.Configuration.Current); state vectors do not include the non-state derivatives ODE1_t and ODE2_tt and the time; function is copying data - not highly efficient; format of pyList: [ODE2Coords, ODE2Coords_t, ODE1Coords, AEcoords, dataCoords]<br><b>EXAMPLE:</b><br><code>sysStateList = mbs.systemData.GetSystemState()</code>                                                                                                                                         |
| SetSystemState(systemStateList,<br>exu.ConfigurationType.Current)  | configuration | = set system data coordinates for given configuration (default: exu.Configuration.Current); invalid list of vectors / vector size may lead to system crash; write access to state vectors (but not the non-state derivatives ODE1_t and ODE2_tt and the time); function is copying data - not highly efficient; format of pyList: [ODE2Coords, ODE2Coords_t, ODE1Coords, AEcoords, dataCoords]<br><b>EXAMPLE:</b><br><code>mbs.systemData.SetSystemState(sysStateList,<br/>configuration = exu.ConfigurationType.Initial)</code> |

#### 4.4.2 SystemData: Get object local-to-global (LTG) coordinate mappings

This section provides access functions the LTG-lists for every object (body, constraint, ...) in the system.

| function/structure name        | description                                                                                                                                                                                                                             |
|--------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| GetObjectLTGODE2(objectNumber) | get local-to-global coordinate mapping (list of global coordinate indices) for ODE2 coordinates; only available after Assemble()<br><b>EXAMPLE:</b><br><code>ltgObject4 = mbs.systemData.GetObjectLTGODE2(4)</code>                     |
| GetObjectLTGODE1(objectNumber) | get local-to-global coordinate mapping (list of global coordinate indices) for ODE1 coordinates; only available after Assemble()<br><b>EXAMPLE:</b><br><code>ltgObject4 = mbs.systemData.GetObjectLTGODE1(4)</code>                     |
| GetObjectLTGAE(objectNumber)   | get local-to-global coordinate mapping (list of global coordinate indices) for algebraic equations (AE) coordinates; only available after Assemble()<br><b>EXAMPLE:</b><br><code>ltgObject4 = mbs.systemData.GetObjectLTGODE2(4)</code> |

|                                |                                                                                                                                                                                                                     |
|--------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| GetObjectLTGData(objectNumber) | get local-to-global coordinate mapping (list of global coordinate indices) for data coordinates; only available after Assemble()<br><b>EXAMPLE:</b><br><code>ltgObject4 = mbs.systemData.GetObjectLTGData(4)</code> |
|--------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

## 4.5 Type definitions

This section defines a couple of structures, which are used to select, e.g., a configuration type or a variable type. In the background, these types are integer numbers, but for safety, the types should be used as type variables.

Conversion to integer is possible:

```
x = int(exu.OutputVariableType.Displacement)
```

and also conversion from integer:

```
varType = exu.OutputVariableType(8)
```

### 4.5.1 OutputVariableType

This section shows the OutputVariableType structure, which is used for selecting output values, e.g. for GetObjectOutput(...) or for selecting variables for contour plot.

Available output variables and the interpretation of the output variable can be found at the object definitions. The OutputVariableType does not provide information about the size of the output variable, which can be either scalar or a list (vector). For vector output quantities, the contour plot option offers an additional parameter for selection of the component of the OutputVariableType. The components are usually out of {0,1,2}, representing {x,y,z} components (e.g., of displacements, velocities, ...), or {0,1,2,3,4,5} representing {xx,yy,zz,yz,xz,xy} components (e.g., of strain or stress). In order to compute a norm, chose component=-1, which will result in the quadratic norm for other vectors and to a norm specified for stresses (if no norm is defined for an outputVariable, it does not compute anything)

| function/structure name | description                                                                                       |
|-------------------------|---------------------------------------------------------------------------------------------------|
| _None                   | no value; used, e.g., to select no output variable in contour plot                                |
| Distance                | e.g., measure distance in spring damper connector                                                 |
| Position                | measure 3D position, e.g., of node or body                                                        |
| Displacement            | measure displacement; usually difference between current position and reference position          |
| DisplacementLocal       | measure local displacement, e.g. in local joint coordinates                                       |
| Velocity                | measure (translational) velocity of node or object                                                |
| VelocityLocal           | measure local (translational) velocity, e.g. in local body or joint coordinates                   |
| Acceleration            | measure (translational) acceleration of node or object                                            |
| RotationMatrix          | measure rotation matrix of rigid body node or object                                              |
| Rotation                | measure, e.g., scalar rotation of 2D body, Euler angles of a 3D object or rotation within a joint |
| AngularVelocity         | measure angular velocity of node or object                                                        |

|                      |                                                                                                                                                                                   |
|----------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| AngularVelocityLocal | measure local (body-fixed) angular velocity of node or object                                                                                                                     |
| AngularAcceleration  | measure angular acceleration of node or object                                                                                                                                    |
| Coordinates          | measure the coordinates of a node or object; coordinates usually just contain displacements, but not the position values                                                          |
| Coordinates_t        | measure the time derivative of coordinates (= velocity coordinates) of a node or object                                                                                           |
| Coordinates_tt       | measure the second time derivative of coordinates (= acceleration coordinates) of a node or object                                                                                |
| SlidingCoordinate    | measure sliding coordinate in sliding joint                                                                                                                                       |
| Director1            | measure a director (e.g. of a rigid body frame), or a slope vector in local 1 or x-direction                                                                                      |
| Director2            | measure a director (e.g. of a rigid body frame), or a slope vector in local 2 or y-direction                                                                                      |
| Director3            | measure a director (e.g. of a rigid body frame), or a slope vector in local 3 or z-direction                                                                                      |
| Force                | measure global force, e.g., in joint or beam (resultant force), or generalized forces; see description of according object                                                        |
| ForceLocal           | measure local force, e.g., in joint or beam (resultant force)                                                                                                                     |
| Torque               | measure torque, e.g., in joint or beam (resultant couple/-moment)                                                                                                                 |
| TorqueLocal          | measure local torque, e.g., in joint or beam (resultant couple/moment)                                                                                                            |
| StrainLocal          | measure local strain, e.g., axial strain in cross section frame of beam or Green-Lagrange strain                                                                                  |
| StressLocal          | measure local stress, e.g., axial stress in cross section frame of beam or Second Piola-Kirchoff stress; choosing component==1 will result in the computation of the Mises stress |
| CurvatureLocal       | measure local curvature; may be scalar or vectorial: twist and curvature of beam in cross section frame                                                                           |
| ConstraintEquation   | evaluates constraint equation (=current deviation or drift of constraint equation)                                                                                                |

#### 4.5.2 ConfigurationType

This section shows the ConfigurationType structure, which is used for selecting a configuration for reading or writing information to the module. Specifically, the ConfigurationType.Current configuration is usually used at the end of a solution process, to obtain result values, or the ConfigurationType.Initial is used to set initial values for a solution process.

| function/structure name | description                                                                                     |
|-------------------------|-------------------------------------------------------------------------------------------------|
| _None                   | no configuration; usually not valid, but may be used, e.g., if no configurationType is required |

|               |                                                                                                                                                          |
|---------------|----------------------------------------------------------------------------------------------------------------------------------------------------------|
| Initial       | initial configuration prior to static or dynamic solver; is computed during mbs.Assemble() or AssembleInitializeSystemCoordinates()                      |
| Current       | current configuration during and at the end of the computation of a step (static or dynamic)                                                             |
| Reference     | configuration used to define deformable bodies (reference configuration for finite elements) or joints (configuration for which some joints are defined) |
| StartOfStep   | during computation, this refers to the solution at the start of the step = end of last step, to which the solver falls back if convergence fails         |
| Visualization | this is a state completely de-coupled from computation, used for visualization                                                                           |
| EndOfEnumList | this marks the end of the list, usually not important to the user                                                                                        |

#### 4.5.3 ItemType

This section shows the ItemType structure, which is used for defining types of indices, e.g., in render window and will be also used in item dictionaries in future.

| function/structure name | description                     |
|-------------------------|---------------------------------|
| _None                   | item has no type                |
| Node                    | item or index is of type Node   |
| Object                  | item or index is of type Object |
| Marker                  | item or index is of type Marker |
| Load                    | item or index is of type Load   |
| Sensor                  | item or index is of type Sensor |

#### 4.5.4 DynamicSolverType

This section shows the DynamicSolverType structure, which is used for selecting dynamic solvers for simulation.

| function/structure name | description                                                                                                                                  |
|-------------------------|----------------------------------------------------------------------------------------------------------------------------------------------|
| GeneralizedAlpha        | an implicit solver for index 3 problems; allows to set variables also for Newmark and trapezoidal implicit index 2 solvers                   |
| TrapezoidalIndex2       | an implicit solver for index 3 problems with index2 reduction; uses generalized alpha solver with settings for Newmark with index2 reduction |
| ExplicitEuler           | an explicit 1st order solver (generally not compatible with constraints)                                                                     |
| ExplicitMidpoint        | an explicit 2nd order solver (generally not compatible with constraints)                                                                     |

|        |                                                                                                                                                                                                                                 |
|--------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| RK33   | an explicit 3 stage 3rd order Runge-Kutta method, aka "Heun"; (generally not compatible with constraints)                                                                                                                       |
| RK44   | an explicit 4 stage 4th order Runge-Kutta method, aka "classical Runge Kutta" (generally not compatible with constraints), compatible with Lie group integration and elimination of CoordinateConstraints                       |
| RK67   | an explicit 7 stage 6th order Runge-Kutta method, see 'On Runge-Kutta Processes of High Order', J. C. Butcher, J. Austr Math Soc 4, (1964); can be used for very accurate (reference) solutions, but without step size control! |
| ODE23  | an explicit Runge Kutta method with automatic step size selection with 3rd order of accuracy and 2nd order error estimation, see Bogacki and Shampine, 1989; also known as ODE23 in MATLAB                                      |
| DOPRI5 | an explicit Runge Kutta method with automatic step size selection with 5th order of accuracy and 4th order error estimation, see Dormand and Prince, 'A Family of Embedded Runge-Kutta Formulae.', J. Comp. Appl. Math. 6, 1980 |
| DVERK6 | [NOT IMPLEMENTED YET] an explicit Runge Kutta solver of 6th order with 5th order error estimation; includes adaptive step selection                                                                                             |

#### 4.5.5 KeyCode

This section shows the KeyCode structure, which is used for special key codes in keyPressUserFunction.

| function/structure name | description        |
|-------------------------|--------------------|
| SPACE                   | space key          |
| ENTER                   | enter (return) key |
| TAB                     |                    |
| BACKSPACE               |                    |
| RIGHT                   | cursor right       |
| LEFT                    | cursor left        |
| DOWN                    | cursor down        |
| UP                      | cursor up          |
| F1                      | function key F1    |
| F2                      | function key F2    |
| F3                      | function key F3    |
| F4                      | function key F4    |
| F5                      | function key F5    |
| F6                      | function key F6    |
| F7                      | function key F7    |
| F8                      | function key F8    |
| F9                      | function key F9    |
| F10                     | function key F10   |

#### 4.5.6 LinearSolverType

This section shows the LinearSolverType structure, which is used for selecting linear solver types, which are dense or sparse solvers.

| function/structure name | description                                                                                                                                                        |
|-------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| _None                   | no value; used, e.g., if no solver is selected                                                                                                                     |
| EXUDense                | use dense matrices and according solvers for densely populated matrices (usually the CPU time grows cubically with the number of unknowns)                         |
| EigenSparse             | use sparse matrices and according solvers; additional overhead for very small systems; specifically, memory allocation is performed during a factorization process |



# Chapter 5

## Python utility functions

This chapter describes in every subsection the functions and classes of the utility modules. These modules help to create multibody systems with the EXUDYN core module. Functions are implemented in Python and can be easily changed, extended and also verified by the user. **Check the source code** by entering these functions in Spyder and pressing **CTRL + left mouse button**. These Python functions are much slower than the functions available in the C++ core. Some matrix computations with larger matrices implemented in numpy and scipy, however, are parallelized and therefore very efficient.

Note that in usually functions accept lists and numpy arrays. If not, an error will occur, which is easily tracked. Furthermore, angles are generally provided in radian ( $2\pi$  equals  $360^\circ$ ) and no units are used for distances, but it is recommended to use SI units (m, kg, s) throughout.

Functions have been implemented, if not otherwise mentioned, by Johannes Gerstmayr.

### 5.1 Utility: ResultsMonitor

The results monitor is a special tool, which allows to monitor results during simulation. This is intended, e.g., to show results for long-term simulations or to visualize results for teaching. The tool can visualize time dependent data (e.g., from sensors or solution files) or data from optimization. The tool automatically detects the type of file and visualizes all given columns (default) or selected columns of the file.

For running the results monitor, start a terminal (linux) or an Anaconda prompt (windows). Either you copy the `resultsLoader.py`, located in the `main/pythonDev/exudyn` subfolder of the repository, to your desired/current directory or you call it from a relative path from your current directory. Usage is described by typing `python resultsMonitor.py -h`, as given in the following listing:

---

```
usage for resultsLoader:
 python resultsLoader.py file.txt
options:
 -xcols i,j,...: comma-separated columns (NO SPACES!) to be plotted on x-axis
 -ycols i,j,...: comma-separated columns (NO SPACES!) to be plotted on y-axis
 -logx: use log scale for x-axis
 -logy: use log scale for y-axis
 -sizex float: float = x-size of one subplot in inches (default=5)
 -sizey float: float = y-size of one subplot in inches (default=5)
 -update float: float = update period in seconds
 -color char: char = line color code according to pyplot, default=b (blue)
 -style char: char = line symbol according to pyplot, default="-"
example: (to be called from windows Anaconda prompt or in linux terminal in the directory
 where file.txt lies)
```

```
python resultsMonitor.py file.txt -logy -xcols 0,1 -ycols 2,3 -update 0.2
```

---

## 5.2 Module: basicUtilities

Basic utility functions and constants, not depending on numpy or other python modules.

Author: Johannes Gerstmayr

Date: 2020-03-10 (created)

Notes: Additional constants are defined:

pi = 3.1415926535897932

sqrt2 = 2\*\*0.5

g=9.81

eye2D (2x2 diagonal matrix)

eye3D (3x3 diagonal matrix)

Two variables 'gaussIntegrationPoints' and 'gaussIntegrationWeights' define integration points and weights for function GaussIntegrate(...)

```
def ClearWorkspace ()
```

– **function description:**

clear all workspace variables except for system variables with '\_' at beginning,  
'func' or 'module' in name

– **notes:** It is recommended to call ClearWorkspace() at the very beginning of your models

– **example:**

```
#do this at the very beginning!
from exudyn.utilities import ClearWorkspace
ClearWorkspace() #clear old SC and mbs variables
#now import modules
import exudyn as exu
from exudyn.itemInterface import *
SC = exu.SystemContainer()
mbs = SC.AddSystem()
```

---

```
def DiagonalMatrix (rowsColumns, value= 1)
```

– **function description:** create a diagonal or identity matrix; used for interface.py, avoiding the need for numpy

– **input:**

*rowsColumns*: provides the number of rows and columns

*value*: initialization value for diagonal terms

- 
- **output:** list of lists representing a matrix

---

```
def NormL2 (vector)
```

- **function description:** compute L2 norm for vectors without switching to numpy or math module
  - **input:** vector as list or in numpy format
  - **output:** L2-norm of vector
- 

```
def VSum (vector)
```

- **function description:** compute sum of all values of vector
  - **input:** vector as list or in numpy format
  - **output:** sum of all components of vector
- 

```
def VAdd (v0, v1)
```

- **function description:** add two vectors instead using numpy
  - **input:** vectors v0 and v1 as list or in numpy format
  - **output:** component-wise sum of v0 and v1
- 

```
def VSub (v0, v1)
```

- **function description:** subtract two vectors instead using numpy: result = v0-v1
  - **input:** vectors v0 and v1 as list or in numpy format
  - **output:** component-wise difference of v0 and v1
- 

```
def VMult (v0, v1)
```

- **function description:** scalar multiplication of two vectors instead using numpy: result = v0'\*v1
- **input:** vectors v0 and v1 as list or in numpy format

- **output:** sum of all component wise products:  $c0[0]*v1[0] + v0[1]*v1[0] + \dots$
- 

```
def ScalarMult (scalar, v)
```

- **function description:** multiplication vectors with scalar: result = s\*v
  - **input:** value *scalar* and vector *v* as list or in numpy format
  - **output:** scalar multiplication of all components of *v*: [scalar\*v[0], scalar\*v[1], ...]
- 

```
def Normalize (v)
```

- **function description:** take a 3D vector and return a normalized 3D vector (L2Norm=1)
  - **input:** vector *v* as list or in numpy format
  - **output:** vector *v* multiplied with scalar such that L2-norm of vector is 1
- 

```
def Vec2Tilde (v)
```

- **function description:** apply tilde operator (skew) to 3D-vector and return skew matrix
- **input:** 3D vector *v* as list or in numpy format

– **output:** matrix as list of lists containing the skew-symmetric matrix computed from *v*: 
$$\begin{bmatrix} 0 & -v[2] & v[1] \\ v[2] & 0 & -v[0] \\ -v[1] & v[0] & 0 \end{bmatrix}$$

---

```
def Tilde2Vec (m)
```

- **function description:** take skew symmetric matrix and return vector (inverse of Skew(...))
  - **input:** list of lists containing a skew-symmetric matrix (3x3)
  - **output:** list containing the vector *v* (inverse function of Vec2Tilde(...))
- 

```
def GaussIntegrate (functionOfX, integrationOrder, a, b)
```

- **function description:** compute numerical integration of *functionOfX* in interval [a,b] using Gaussian integration

- **input:**

*functionOfX*: scalar, vector or matrix-valued function with scalar argument (X or other variable)

*integrationOrder*: odd number in {1,3,5,7,9}; currently maximum order is 9

*a*: integration range start

*b*: integration range end

- **output:** (scalar or vectorized) integral value

## 5.3 Module: FEM

Support functions and helper classes for import of meshes, finite element models (ABAQUS, ANSYS, NETGEN) and for generation of FFRF (floating frame of reference) objects.

Author: Johannes Gerstmayr, Stefan Holzinger (Abaqus and Ansys import utilities)

Date: 2020-03-10 (created)

Notes: internal CSR matrix storage format contains 3 float numbers per row: [row, column, value], can be converted to scipy csr sparse matrices with function CSRtoScipySparseCSR(...)

def **CompressedRowSparseToDenseMatrix** (*sparseData*)

- **function description:** convert zero-based sparse matrix data to dense numpy matrix

- **input:** *sparseData*: format (per row): [row, column, value] ==> converted into dense format

- **output:** a dense matrix as np.array

---

def **MapSparseMatrixIndices** (*matrix, sorting*)

- **function description:** resort a sparse matrix (internal CSR format) with given sorting for rows and columns; changes matrix directly! used for ANSYS matrix import

---

def **VectorDiadicUnitMatrix3D** (*v*)

- **function description:** compute diadic product of vector v and a 3D unit matrix = diadic(v,I<sub>3x3</sub>); used for ObjectFFRF and CMS implementation

---

def **CyclicCompareReversed** (*list1, list2*)

- **function description:** compare cyclic two lists, reverse second list; return True, if any cyclic shifted lists are same, False otherwise
- 

```
def AddEntryToCompressedRowSparseArray (sparseData, row, column, value)
```

- **function description:**
    - add entry to compressedRowSparse matrix, avoiding duplicates
    - value is either added to existing entry (avoid duplicates) or a new entry is appended
- 

```
def CSRtoRowsAndColumns (sparseMatrixCSR)
```

- **function description:** compute rows and columns of a compressed sparse matrix and return as tuple: (rows,columns)
- 

```
def CSRtoScipySparseCSR (sparseMatrixCSR)
```

- **function description:** convert internal compressed CSR to scipy.sparse csr matrix
- 

```
def ScipySparseCSRtoCSR (scipyCSR)
```

- **function description:** convert scipy.sparse csr matrix to internal compressed CSR
- 

```
def ResortIndicesOfCSRmatrix (mXXYYZZ, numberOfRows)
```

- **function description:**
    - resort indices of given CSR matrix in XXXYYYZZZ format to XYZXYZXYZ format; numberOfRows must be equal to columns
    - needed for import from NGsolve
-

```
def ConvertHexToTrigs (nodeNumbers)
```

- **function description:**

convert list of Hex8/C3D8 element with 8 nodes in nodeNumbers into triangle-List  
also works for Hex20 elements, but does only take the corner nodes!

---

```
def ConvertDenseToCompressedRowMatrix (denseMatrix)
```

- **function description:** convert numpy.array dense matrix to (internal) compressed row format

---

```
def ReadMatrixFromAnsysMMF (fileName, verbose= False)
```

- **function description:**

This function reads either the mass or stiffness matrix from an Ansys Matrix Market Format (MMF). The corresponding matrix can either be exported as dense matrix or sparse matrix.

- **input:** fileName of MMF file

- **output:** internal compressed row sparse matrix (as (nrows x 3) numpy array)

- **author:** Stefan Holzinger

- **notes:**

A MMF file can be created in Ansys by placing the following APDL code inside *the solution tree in Ansys Workbench*:

!!!!!!!!!!!!!!

! APDL code that exports sparse stiffness and mass matrix in MMF format. If

! the dense matrix is needed, replace \*SMAT with \*DMAT in the following

! APDL code.

! Export the stiffness matrix in MMF format

\*SMAT,MatKD,D,IMPORT,FULL,file.full,STIFF

\*EXPORT,MatKD,MMF,fileNameStiffnessMatrix,,

! Export the mass matrix in MMF format

\*SMAT,MatMD,D,IMPORT,FULL,file.full,MASS

\*EXPORT,MatMD,MMF,fileNameMassMatrix,,

!!!!!!!!!!!!!!

In case a lumped mass matrix is needed, place the following APDL Code inside *the Modal Analysis Tree*:

```
!!!!!!!!!!!!!!
! APDL code to force Ansys to use a lumped mass formulation (if available for
! used elements)
LUMPM, ON, , 0
!!!!!!!!!!!!!!
```

---

```
def ReadMatrixDOFmappingVectorFromAnsysTxt (fileName)
```

– **function description:**

read sorting vector for ANSYS mass and stiffness matrices and return sorting vector as np.array  
the file contains sorting for nodes and applies this sorting to the DOF (assuming 3 DOF per node!)  
the resulting sorted vector is already converted to 0-based indices

---

```
def ReadNodalCoordinatesFromAnsysTxt (fileName, verbose= False)
```

– **function description:** This function reads the nodal coordinates exported from Ansys.

– **input:** *fileName* (file name ending must be .txt!)

– **output:** nodal coordinates as numpy array

– **author:** Stefan Holzinger

– **notes:**

The nodal coordinates can be exported from Ansys by creating a named selection of the body whos mesh should to exported by choosing its geometry. Next, create a second named selction by using a worksheet. Add the named selection that was created first into the worksheet of the second named selection.

Inside the working sheet, choose 'convert' and convert the first created named selection to 'mesh node' (Netzknoten in german) and click on generate to create the second named selection. Next, right click on the second named selection tha was created and choose 'export' and save the nodal coordinates as .txt file.

---

```
def ReadElementsFromAnsysTxt (fileName, verbose= False)
```

– **function description:** This function reads the nodal coordinates exported from Ansys.

– **input:** fileName (file name ending must be .txt!)

– **output:** element connectivity as numpy array

– **author:** Stefan Holzinger

– **notes:**

The elements can be exported from Ansys by creating a named selection of the body whos mesh should be exported by choosing its geometry. Next, create a second named selection by using a worksheet. Add the named selection that was created first into the worksheet of the second named selection.

Inside the worksheet, choose 'convert' and convert the first created named selection to 'mesh element' (Netzelement in german) and click on generate to create the second named selection. Next, right click on the second named selection that was created and choose 'export' and save the elements as .txt file.

### 5.3.1 CLASS MaterialBaseClass (in module FEM)

**class description:** material base class, e.g., for FiniteElement

### 5.3.2 CLASS KirchhoffMaterial(MaterialBaseClass) (in module FEM)

**class description:** class for representation of Kirchhoff (linear elastic, 3D and 2D) material use planeStress=False for plane strain

### 5.3.3 CLASS FiniteElement (in module FEM)

**class description:** finite element base class for lateron implementations of other finite elements

### 5.3.4 CLASS Tet4(FiniteElement) (in module FEM)

**class description:** simplistic 4-noded tetrahedral interface to compute strain/stress at nodal points

### 5.3.5 CLASS ObjectFFRFinterface (in module FEM)

**class description:** compute terms necessary for ObjectFFRF class used internally in FEMinterface to compute ObjectFFRF object this class holds all data for ObjectFFRF user functions

`def __init__(self, femInterface)`

– **classFunction:**

initialize ObjectFFRFinterface with FEMinterface class  
initializes the ObjectFFRFinterface with nodes, modes, surface description and systemmatrices from FEMinterface  
data is then transferred to mbs object with classFunction AddObjectFFRF(...)

---

```
def AddObjectFFRF (self, exu, mbs, positionRef= [0,0,0], eulerParametersRef= [1,0,0,0], initialVelocity= [0,0,0],
initialAngularVelocity= [0,0,0], gravity= [0,0,0], constrainRigidBodyMotion= True, massProportionalDamping= 0,
stiffnessProportionalDamping= 0, color= [0.1,0.9,0.1,1.])
```

- **classFunction:** add according nodes, objects and constraints for FFRF object to MainSystem mbs; only implemented for Euler parameters
- **input:**
  - exu:* the exudyn module
  - mbs:* a MainSystem object
  - positionRef:* reference position of created ObjectFFRF (set in rigid body node underlying to ObjectFFRF)
  - eulerParametersRef:* reference euler parameters of created ObjectFFRF (set in rigid body node underlying to ObjectFFRF)
  - initialVelocity:* initial velocity of created ObjectFFRF (set in rigid body node underlying to ObjectFFRF)
  - initialAngularVelocity:* initial angular velocity of created ObjectFFRF (set in rigid body node underlying to ObjectFFRF)
  - gravity:* set [0,0,0] if no gravity shall be applied, or to the gravity vector otherwise
  - constrainRigidBodyMotion:* set True in order to add constraint (Tisserand frame) in order to suppress rigid motion of mesh nodes
  - color:* provided as list of 4 RGBA values
- add object to mbs as well as according nodes

---

```
def UForce (self, exu, mbs, t, q, q_t)
```

- **classFunction:** optional forceUserFunction for ObjectFFRF (per default, this user function is ignored)

---

```
def UFmassGenericODE2 (self, exu, mbs, t, q, q_t)
```

- **classFunction:** optional massMatrixUserFunction for ObjectFFRF (per default, this user function is ignored)

### 5.3.6 CLASS ObjectFFRFreducedOrderInterface (in module FEM)

**class description:** compute terms necessary for ObjectFFRFreducedOrder class used internally in FEMinterface to compute ObjectFFRFreducedOrder dictionary this class holds all data for ObjectFFRFreducedOrder user functions

```
def __init__(self, femInterface, rigidBodyNodeType= 'NodeType.RotationEulerParameters', roundMassMatrix= 1e-13, roundStiffnessMatrix= 1e-13)
```

– **classFunction:**

initialize ObjectFFRFreducedOrderInterface with FEMinterface class

initializes the ObjectFFRFreducedOrderInterface with nodes, modes, surface description and reduced system matrices from FEMinterface

data is then transferred to mbs object with classFunction AddObjectFFRFreducedOrderWithUserFunctions(...)

– **input:**

*femInterface*: must provide nodes, surfaceTriangles, modeBasis, massMatrix, stiffness

*roundMassMatrix*: use this value to set entries of reduced mass matrix to zero which are below the threshold

*roundStiffnessMatrix*: use this value to set entries of reduced stiffness matrix to zero which are below the threshold

---

```
def AddObjectFFRFreducedOrderWithUserFunctions (self, exu, mbs, positionRef= [0,0,0], initialVelocity= [0,0,0], rotationMatrixRef= [], initialAngularVelocity= [0,0,0], gravity= [0,0,0], UFforce= 0, UFmassMatrix= 0, massProportionalDamping= 0, stiffnessProportionalDamping= 0, color= [0.1,0.9,0.1,1.], eulerParametersRef= [])
```

– **classFunction:** add according nodes, objects and constraints for ObjectFFRFreducedOrder object to Main-System mbs; use this function with userfunctions=0 in order to use internal C++ functionality, which is approx. 10x faster; implementation of userfunctions also available for rotation vector (Lie group formulation), which needs further testing

– **input:**

*exu*: the exudyn module

*mbs*: a MainSystem object

*positionRef*: reference position of created ObjectFFRFreducedOrder (set in rigid body node underlying to ObjectFFRFreducedOrder)

*initialVelocity*: initial velocity of created ObjectFFRFreducedOrder (set in rigid body node underlying to ObjectFFRFreducedOrder)

*rotationMatrixRef*: reference rotation of created ObjectFFRFreducedOrder (set in rigid body node underlying to ObjectFFRFreducedOrder); if [], it becomes the unit matrix

*initialAngularVelocity*: initial angular velocity of created ObjectFFRFreducedOrder (set in rigid body node underlying to ObjectFFRFreducedOrder)

*eulerParametersRef*: DEPRECATED, use rotationParametersRef or rotationMatrixRef in future: reference euler parameters of created ObjectFFRFreducedOrder (set in rigid body node underlying to ObjectFFRFreducedOrder)

*gravity*: ONLY available if user functions are applied; otherwise use LoadMassProportional and add to ObjectFFRFreducedOrder; set [0,0,0] if no gravity shall be applied, or to the gravity vector otherwise

*UFforce*: (OPTIONAL, computation is slower) provide a user function, which computes the quadratic velocity vector and applied forces; usually this function reads like:

```
def UFforceFFRFreducedOrder(mbs, t, qReduced, qReduced_t):
 return cms.UFforceFFRFreducedOrder(exu, mbs, t, qReduced, qReduced_t)
```

*UFmassMatrix*: (OPTIONAL, computation is slower) provide a user function, which computes the quadratic velocity vector and applied forces; usually this function reads like:

```
def UFmassFFRFreducedOrder(mbs, t, qReduced, qReduced_t):
 return cms.UFmassFFRFreducedOrder(exu, mbs, t, qReduced, qReduced_t)
```

*massProportionalDamping*: Rayleigh damping factor for mass proportional damping, added to floating frame/modal coordinates only

*stiffnessProportionalDamping*: Rayleigh damping factor for stiffness proportional damping, added to floating frame/modal coordinates only

*color*: provided as list of 4 RGBA values

---

```
def UFmassFFRFreducedOrder(self, exu, mbs, t, qReduced, qReduced_t)
```

- **classFunction**: CMS mass matrix user function; qReduced and qReduced\_t contain the coordinates of the rigid body node and the modal coordinates in one vector!
- 

```
def UFforceFFRFreducedOrder(self, exu, mbs, t, qReduced, qReduced_t)
```

- **classFunction**: CMS force matrix user function; qReduced and qReduced\_t contain the coordinates of the rigid body node and the modal coordinates in one vector!
- 

```
def AddObjectFFRFreducedOrder(self, mbs, positionRef= [0,0,0], initialVelocity= [0,0,0], rotationMatrixRef= [],
initialAngularVelocity= [0,0,0], massProportionalDamping= 0, stiffnessProportionalDamping= 0, color=
[0.1,0.9,0.1,1.])
```

- **classFunction**: add according nodes, objects and constraints for ObjectFFRFreducedOrder object to Main-System mbs; use this function in order to use internal C++ functionality, which is approx. 10x faster than AddObjectFFRFreducedOrderWithUserFunctions(...)

- **input:**

*exu*: the exudyn module

*mbs*: a MainSystem object

*positionRef*: reference position of created ObjectFFRFreducedOrder (set in rigid body node underlying to ObjectFFRFreducedOrder)

*initialVelocity*: initial velocity of created ObjectFFRFreducedOrder (set in rigid body node underlying to ObjectFFRFreducedOrder)

*rotationMatrixRef*: reference rotation of created ObjectFFRFreducedOrder (set in rigid body node underlying to ObjectFFRFreducedOrder); if [], it becomes the unit matrix

*initialAngularVelocity*: initial angular velocity of created ObjectFFRFreducedOrder (set in rigid body node underlying to ObjectFFRFreducedOrder)

*massProportionalDamping*: Rayleigh damping factor for mass proportional damping, added to floating frame/modal coordinates only

*stiffnessProportionalDamping*: Rayleigh damping factor for stiffness proportional damping, added to floating frame/modal coordinates only

*color*: provided as list of 4 RGBA values

### 5.3.7 CLASS HCBstaticModeSelection(Enum) (in module FEM)

**class description:** helper calss for function ComputeHurtyCraigBamptonModes, declaring some computation options. It offers the following options:

- *allBoundaryNodes*: compute a single static mode for every boundary coordinate
- *RBE2*: static modes only for rigid body motion at boundary nodes
- *RBE3*: [yet NOT AVAILABLE] averaged rigid body interfaces; for future implementations
- *noStaticModes*: do not compute static modes, only eigen modes (not recommended; usually only for tests)

### 5.3.8 CLASS FEMinterface (in module FEM)

**class description:** general interface to different FEM / mesh imports and export to EXUDYN functions use this class to import meshes from different meshing or FEM programs (NETGEN/NGsolve, ABAQUS, ANSYS, ..) and store it in a unique format do mesh operations, compute eigenmodes and reduced basis, etc. load/store the data efficiently with LoadFromFile(...), SaveToFile(...) if import functions are slow export to EXUDYN objects

```
def __init__(self)
```

- **classFunction**: initailize all data of the FEMinterface by, e.g., `fem = FEMinterface()`

- **example**:

```
***** this is not an example, just a description for internal variables ****
#default values for member variables stored internally in FEMinterface fem and
#typical structure:
fem.nodes = {} # {'Position':[[x0,y0,z0],...], 'RigidBodyRxyz':[[[x0,y0,z0],...], ...], ...} #dictionary of different node
lists
```

```

fem.elements = [] # [{ 'Name' : 'identifier', 'Tet4' : [[n0,n1,n2,n3
],...], 'Hex8' : [[n0,...,n7],...], ...} ,...] #there may be several element
sets

fem.massMatrix = np.zeros((0,0)) # np.array([[r0,c0,value0],[r1,c1,value1],
...]) #currently only in SparseCSR format
allowed!

fem.stiffnessMatrix=np.zeros((0,0)) # np.array([[r0,c0,value0],[r1,c1,value1],
...]) #currently only in SparseCSR format
allowed!

fem.surface = [] # [{ 'Name' : 'identifier', 'Trigs' : [[n0,n1,n2],...],
'Quads' : [[n0,...,n3],...], ...}] #surface with faces

fem.nodeSets = [] # [{ 'Name' : 'identifier', 'NodeNumbers' : [n_0,...,
n_ns], 'NodeWeights' : [w_0,...,w_ns]},...] #for boundary conditions, etc.

fem.elementSets = [] # [{ 'Name' : 'identifier', 'ElementNumbers' : [n_0
,...,n_ns]},...] #for different volumes, etc.

fem.modeBasis = {} # { 'matrix' : [[Psi_00,Psi_01, ..., Psi_0m],...,[
Psi_n0,Psi_n1, ..., Psi_nm]], 'type' : 'NormalModes' } #'NormalModes' are
eigenmodes, 'HCBmodes' are Craig-Bampton modes including static modes

fem.eigenValues = [] # [ev0, ev1, ...]

eigenvalues according to eigenvectors in mode basis

fem.postProcessingModes = {} # { 'matrix' : <matrix containing stress components (
xx,yy,zz,yz,xz,xy) in each column, rows are for every mesh node>, '
outputVariableType' : exodyn.OutputVariableType.StressLocal}

```

---

### `def SaveToFile (self, fileName)`

- **classFunction:** save all data (nodes, elements, ...) to a data filename; this function is much faster than the text-based import functions
  - **input:** use filename without ending ==> ".npy" will be added
- 

### `def LoadFromFile (self, fileName)`

- **classFunction:**  
load all data (nodes, elements, ...) from a data filename previously stored with SaveToFile(...).  
this function is much faster than the text-based import functions
  - **input:** use filename without ending ==> ".npy" will be added
- 

### `def ImportFromAbaqusInputFile (self, fileName, typeName= 'Part', name= 'Part-1', verbose= False)`

– **classFunction:**

import nodes and elements from Abaqus input file and create surface elements  
node numbers in elements are converted from 1-based indices to python's 0-based indices  
only works for Hex8, Hex20, Tet4 and Tet10 (C3D4, C3D8, C3D10, C3D20) elements  
return node numbers as numpy array

---

```
def ReadMassMatrixFromAbaqus (self, fileName, type= 'SparseRowColumnValue')
```

– **classFunction:**

*read mass matrix from compressed row text format (exported from Abaqus); in order to export system matrices,  
write the following lines in your Abaqus input file:*

```
*STEP
*MATRIX GENERATE, STIFFNESS, MASS
*MATRIX OUTPUT, STIFFNESS, MASS, FORMAT=COORDINATE
*End Step
```

---

```
def ReadStiffnessMatrixFromAbaqus (self, fileName, type= 'SparseRowColumnValue')
```

– **classFunction:** read stiffness matrix from compressed row text format (exported from Abaqus)

---

```
def ImportMeshFromNGsolve (self, mesh, density, youngsModulus, poissonsRatio, verbose= False)
```

– **classFunction:** import mesh from NETGEN/NGsolve and setup mechanical problem

– **input:**

*mesh:* a previously created ngs.mesh (NGsolve mesh, see examples)  
*youngsModulus:* Young's modulus used for mechanical model  
*poissonsRatio:* Poisson's ratio used for mechanical model  
*density:* density used for mechanical model  
*verbose:* set True to print out some status information

– **notes:**

The interface to NETGEN/NGsolve has been created together with Joachim Schöberl, main developer of NETGEN/NGsolve; Thank's a lot!

*download NGsolve at:* <https://ngsolve.org/>

NGsolve needs Python 3.7 (64bit) ==> use according EXUDYN version!

note that node/element indices in the NGsolve mesh are 1-based and need to be converted to 0-base!

---

```
def GetMassMatrix (self, sparse= True)
```

- **classFunction:** get sparse mass matrix in according format
- 

```
def GetStiffnessMatrix (self, sparse= True)
```

- **classFunction:** get sparse stiffness matrix in according format
- 

```
def NumberOfNodes (self)
```

- **classFunction:** get total number of nodes
- 

```
def GetNodePositionsAsArray (self)
```

- **classFunction:** get node points as array; only possible, if there exists only one type of Position nodes
- 

```
def NumberOfCoordinates (self)
```

- **classFunction:** get number of total nodal coordinates
- 

```
def GetNodeAtPoint (self, point, tolerance= 1e-5, raiseException= True)
```

- **classFunction:**

get node number for node at given point, e.g.  $p=[0.1,0.5,-0.2]$ , using a tolerance (+/-) if coordinates are available only with reduced accuracy

if not found, it returns an invalid index

---

```
def GetNodesInPlane (self, point, normal, tolerance= 1e-5)
```

- **classFunction:**

get node numbers in plane defined by point p and (normalized) normal vector n using a tolerance for the distance to the plane

if not found, it returns an empty list

---

```
def GetNodesInCube (self, pMin, pMax)
```

- **classFunction:**

get node numbers in cube, given by pMin and pMax, containing the minimum and maximum x, y, and z coordinates

if not found, it returns an empty list

---

```
def GetNodesOnLine (self, p1, p2, tolerance= 1e-5)
```

- **classFunction:** get node numbers lying on line defined by points p1 and p2 and tolerance, which is accepted for points slightly outside the surface

---

```
def GetNodesOnCylinder (self, p1, p2, radius, tolerance= 1e-5)
```

- **classFunction:**

get node numbers lying on cylinder surface; cylinder defined by cylinder axes (points p1 and p2), cylinder radius and tolerance, which is accepted for points slightly outside the surface

if not found, it returns an empty list

---

```
def GetNodesOnCircle (self, point, normal, r, tolerance= 1e-5)
```

- **classFunction:**

get node numbers lying on a circle, by point p, (normalized) normal vector n (which is the axis of the circle) and radius r

using a tolerance for the distance to the plane

if not found, it returns an empty list

---

```
def GetSurfaceTriangles (self)
```

- **classFunction:** return surface trigs as node number list (for drawing in EXUDYN)
- 

```
def VolumeToSurfaceElements (self, verbose= False)
```

- **classFunction:**

generate surface elements from volume elements

stores the surface in self.surface

only works for one element list and one type ('Hex8') of elements

---

```
def GetGyroscopicMatrix (self, rotationAxis= 2, sparse= True)
```

- **classFunction:** get gyroscopic matrix in according format; rotationAxis=[0,1,2] = [x,y,z]
- 

```
def ScaleMassMatrix (self, factor)
```

- **classFunction:** scale (=multiply) mass matrix with factor
- 

```
def ScaleStiffnessMatrix (self, factor)
```

- **classFunction:** scale (=multiply) stiffness matrix with factor
- 

```
def AddElasticSupportAtNode (self, nodeNumber, springStiffness= [1e8,1e8,1e8])
```

- **classFunction:**

modify stiffness matrix to add elastic support (joint, etc.) to a node; nodeNumber zero based (as everywhere in the code...)

springStiffness must have length according to the node size

---

`def AddNodeMass (self, nodeNumber, addedMass)`

- **classFunction:** modify mass matrix by adding a mass to a certain node, modifying directly the mass matrix
- 

`def ComputeEigenmodes (self, nModes, excludeRigidBodyModes= 0, useSparseSolver= True)`

- **classFunction:** compute nModes smallest eigenvalues and eigenmodes from mass and stiffnessMatrix; store mode vectors in modeBasis, but exclude a number of 'excludeRigidBodyModes' rigid body modes from modeBasis; uses scipy for solution of generalized eigenvalue problem

- **input:**

*nModes*: prescribe the number of modes to be computed; total computed modes are (nModes+excludeRigidBodyMode) but only nModes with smallest absolute eigenvalues are considered and stored

*excludeRigidBodyModes*: if rigid body modes are expected (in case of free-free modes), then this number specifies the number of eigenmodes to be excluded in the stored basis (usually 6 modes in 3D)

*useSparseSolver*: for larger systems, the sparse solver needs to be used, which iteratively solves the problem and uses a random number generator (internally in ARPACK); therefore, results are not fully repeatable!!!

- **output:** eigenmodes are stored internally in FEMinterface as 'modeBasis' and eigenvalues as 'eigenValues'
- 

`def ComputeEigenModesWithBoundaryNodes (self, boundaryNodes, nEigenModes, useSparseSolver= True)`

- **classFunction:** compute eigenmodes, using a set of boundary nodes that are all fixed; very similar to ComputeEigenmodes, but with additional definition of (fixed) boundary nodes.

- **input:**

*boundaryNodes*: a list of boundary node indices, referring to 'Position' type nodes in FEMinterface; all coordinates of these nodes are fixed for the computation of the modes

*nEigenModes*: prescribe the number of modes to be computed; only nEigenModes with smallest abs(eigenvalues) are considered and stored

*useSparseSolver*: [yet NOT IMPLEMENTED] for larger systems, the sparse solver needs to be used, which iteratively solves the problem and uses a random number generator (internally in ARPACK); therefore, results are not fully repeatable!!!

- **output:** eigenmodes are stored internally in FEMinterface as 'modeBasis' and eigenvalues as 'eigenValues'

---

```
def ComputeHurtyCraigBamptonModes (self, boundaryNodesList, nEigenModes, useSparseSolver= True,
computationMode= HCBstaticModeSelection.RBE2)
```

- **classFunction:** compute static and eigen modes based on Hurty-Craig-Bampton, for details see theory part [Section 6.2](#). Note that this function may need significant time, depending on your hardware, but 50.000 nodes will require approx. 1-2 minutes and more nodes typically raise time more than linearly.
  - **input:**
    - boundaryNodesList:* [nodeList0, nodeList1, ...] a list of node lists, each of them representing a set of 'Position' nodes for which a rigid body interface (displacement/rotation and force/torque) is created; NOTE THAT boundary nodes may not overlap between the different node lists (no duplicated node indices!)
    - nEigenModes:* number of eigen modes in addition to static modes; eigen modes are computed for the case where all rigid body motions at boundaries are fixed (on average); only smallest nEigenModes absolute eigenvalues are considered
    - useSparseSolver:* for more than approx. 500 nodes, it is recommended to use the sparse solver
    - computationMode:* see class HCBstaticModeSelection for available modes; select RBE2 as standard, which is both efficient and accurate and which uses rigid-body-interfaces (6 independent modes) per boundary
  - **output:** stores computed modes in self.modeBasis and abs(eigenvalues) in self.eigenValues
- 

```
def GetEigenFrequenciesHz (self)
```

- **classFunction:** return list of eigenvalues in Hz of previously computed eigenmodes
- 

```
def ComputePostProcessingModes (self, material= 0, outputVariableType= 'OutputVariableType.StressLocal')
```

- **classFunction:** compute special stress or strain modes in order to enable visualization of stresses and strains in ObjectFFRFreducedOrder
- **input:**
  - material:* specify material properties for computation of stresses, using a material class, e.g. material = KirchhoffMaterial(Emodulus, nu, rho); not needed for strains
  - outputVariableType:* specify either exudyn.OutputVariableType.StressLocal or exudyn.OutputVariableType.StrainLocal as the desired output variables
- **output:** post processing modes are stored in FEMinterface in local variable postProcessingModes as a dictionary, where 'matrix' represents the modes and 'outputVariableType' stores the type of mode as a OutputVariableType

---

```
def ComputeCampbellDiagram (self, terminalFrequency, nEigenfrequencies= 10, frequencySteps= 25,
rotationAxis= 2, plotDiagram= False, verbose= False)
```

– **classFunction:**

compute Campbell diagram for given mechanical system

create a first order system  $Ax + Bx = 0$  with  $x = [q, qd]'$  and compute eigenvalues

takes mass M, stiffness K and gyroscopic matrix G from FEMinterface

currently only uses dense matrices, so it is limited to approx. 5000 unknowns!

– **input:**

*terminalFrequency*: frequency in Hz, up to which the campbell diagram is computed

*nEigenfrequencies*: gives the number of computed eigenfrequencies(modes), in addition to the rigid body mode 0

*frequencySteps*: gives the number of increments (gives *frequencySteps*+1 total points in campbell dia-  
gram)

*rotationAxis*:[0,1,2] = [x,y,z] provides rotation axis

*plotDiagram*: if True, plots diagram for *nEigenfrequencies* before terminating

*verbose*: if True, shows progress of computation

– **output:**

[listFrequencies, campbellFrequencies]

*listFrequencies*: list of computed frequencies

*campbellFrequencies*: array of campbell frequencies per eigenfrequency of system

---

```
def CheckConsistency (self)
```

– **classFunction:** perform some consistency checks

---

```
def ReadMassMatrixFromAnsys (self, fileName, dofMappingVectorFile, sparse= True, verbose= False)
```

– **classFunction:** read mass matrix from CSV format (exported from Ansys)

---

```
def ReadStiffnessMatrixFromAnsys (self, fileName, dofMappingVectorFile, sparse= True, verbose= False)
```

- 
- **classFunction:** read stiffness matrix from CSV format (exported from Ansys)

---

```
def ReadNodalCoordinatesFromAnsys (self, fileName, verbose= False)
```

- **classFunction:** read nodal coordinates (exported from Ansys as .txt-File)
- 

```
def ReadElementsFromAnsys (self, fileName, verbose= False)
```

- **classFunction:** read elements (exported from Ansys as .txt-File)

## 5.4 Module: graphicsDataUtilities

Utility functions for visualization, which provides functions for basic shapes like cube, cylinder, sphere, solid of revolution. Functions generate dictionaries which contain line, text or triangle primitives for drawing in Exudyn using OpenGL.

Author: Johannes Gerstmayr

Date: 2020-07-26 (created)

Notes: Some useful colors are defined, using RGBA (Red, Green, Blue and Alpha = opacity) channels in the range [0,1], e.g., red = [1,0,0,1].

Available colors are: color4red, color4green, color4blue, color4cyan, color4magenta, color4yellow, color4lightred, color4lightgreen, color4steelblue, color4grey, color4darkgrey, color4lightgrey, color4white

Additionally, a list of colors 'color4list' is available, which is intended to be used, e.g., for creating n bodies with different colors

```
def SwitchTripletOrder (vector)
```

- **function description:** helper function to switch order of three items in a list; mostly used for reverting normals in triangles
  - **input:** 3D vector as list or as np.array
  - **output:** interchanged 2nd and 3rd component of list
- 

```
def MergeGraphicsDataTriangleList (g1, g2)
```

- **function description:** merge 2 different graphics data with triangle lists
- **input:** graphicsData dictionaries g1 and g2 obtained from GraphicsData functions

- 
- **output:** one graphicsData dictionary with single triangle lists and compatible points and normals, to be used in visualization of EXUDYN objects

---

```
def GraphicsDataRectangle (xMin, yMin, xMax, yMax, color= [0.,0.,0.,1.])
```

- **function description:** generate graphics data for 2D rectangle
  - **input:** minimal and maximal cartesian coordinates in (x/y) plane; color provided as list of 4 RGBA values
  - **output:** graphicsData dictionary, to be used in visualization of EXUDYN objects
- 

---

```
def GraphicsDataOrthoCubeLines (xMin, yMin, zMin, xMax, yMax, zMax, color= [0.,0.,0.,1.])
```

- **function description:** generate graphics data for orthogonal cube drawn with lines
  - **input:** minimal and maximal cartesian coordinates for orthogonal cube; color provided as list of 4 RGBA values
  - **output:** graphicsData dictionary, to be used in visualization of EXUDYN objects
- 

---

```
def GraphicsDataOrthoCube (xMin, yMin, zMin, xMax, yMax, zMax, color= [0.,0.,0.,1.])
```

- **function description:** generate graphics data for orthogonal 3D cube with min and max dimensions
  - **input:** minimal and maximal cartesian coordinates for orthogonal cube; color provided as list of 4 RGBA values
  - **output:** graphicsData dictionary, to be used in visualization of EXUDYN objects
- 

---

```
def GraphicsDataOrthoCubePoint (centerPoint, size, color= [0.,0.,0.,1.])
```

- **function description:** generate graphics data for orthogonal 3D cube with center point and size
  - **input:** center point and size of cube (as 3D list or np.array); color provided as list of 4 RGBA values
  - **output:** graphicsData dictionary, to be used in visualization of EXUDYN objects
- 

---

```
def GraphicsDataCube (pList, color= [0.,0.,0.,1.], faces= [1,1,1,1,1,1])
```

- **function description:** generate graphics data for general cube with endpoints, according to given vertex definition
  - **input:**
    - pList:* is a list of points  $[[x_0, y_0, z_0], [x_1, y_1, z_1], \dots]$
    - color:* provided as list of 4 RGBA values
    - faces:* includes the list of six binary values (0/1), denoting active faces (value=1); set index to zero to hide face
  - **output:** graphicsData dictionary, to be used in visualization of EXUDYN objects
- 

```
def GraphicsDataSphere (point, radius, color= [0.,0.,0.,1.], nTiles= 8)
```

- **function description:** generate graphics data for a sphere with point p and radius
  - **input:**
    - point:* center of sphere (3D list or np.array)
    - radius:* positive value
    - color:* provided as list of 4 RGBA values
    - nTiles:* used to determine resolution of sphere  $\geq 3$ ; use larger values for finer resolution
  - **output:** graphicsData dictionary, to be used in visualization of EXUDYN objects
- 

```
def GraphicsDataCylinder (pAxis, vAxis, radius, color= [0.,0.,0.,1.], nTiles= 16, angleRange= [0,2*np.pi], lastFace= True, cutPlain= True, **kwargs)
```

- **function description:** generate graphics data for a cylinder with given axis, radius and color; nTiles gives the number of tiles (minimum=3)
- **input:**
  - pAxis:* axis point of one face of cylinder (3D list or np.array)
  - vAxis:* vector representing the cylinder's axis (3D list or np.array)
  - radius:* positive value representing radius of cylinder
  - color:* provided as list of 4 RGBA values
  - nTiles:* used to determine resolution of cylinder  $\geq 3$ ; use larger values for finer resolution
  - angleRange:* given in rad, to draw only part of cylinder (halfcylinder, etc.); for full range use  $[0..2 * \pi]$
  - lastFace:* if *angleRange*  $\neq [0,2*\pi]$ , then the faces of the open cylinder are shown with *lastFace = True*
  - cutPlain:* only used for *angleRange*  $\neq [0,2*\pi]$ ; if True, a plane is cut through the part of the cylinder; if False, the cylinder becomes a cake shape ...
  - alternatingColor:* if given, optionally another color in order to see rotation of solid; only works, if *angleRange=[0,2\*pi]*

- **output:** graphicsData dictionary, to be used in visualization of EXUDYN objects
- 

```
def GraphicsDataRigidLink (p0, p1, axis0= [0,0,0], axis1= [0,0,0], radius= [0.1,0.1], thickness= 0.05, width= [0.05,0.05], color= [0.,0.,0.,1.], nTiles= 16)
```

- **function description:** generate graphics data for a planar Link between the two joint positions, having two axes
  - **input:**
    - p0:* joint0 center position
    - p1:* joint1 center position
    - axis0:* direction of rotation axis at p0, if drawn as a cylinder; [0,0,0] otherwise
    - axis1:* direction of rotation axis of p1, if drawn as a cylinder; [0,0,0] otherwise
    - radius:* list of two radii [radius0, radius1], being the two radii of the joints drawn by a cylinder or sphere
    - width:* list of two widths [width0, width1], being the two widths of the joints drawn by a cylinder; ignored for sphere
    - thickness:* the thickness of the link (shaft) between the two joint positions; thickness in z-direction or diameter (cylinder)
    - color:* provided as list of 4 RGBA values
    - nTiles:* used to determine resolution of cylinder >=3; use larger values for finer resolution
  - **output:** graphicsData dictionary, to be used in visualization of EXUDYN objects
- 

```
def GraphicsDataFromSTLfileTxt (fileName, color= [0.,0.,0.,1.], verbose= False)
```

- **function description:** generate graphics data from STL file (text format!) and use color for visualization
  - **input:**
    - fileName:* string containing directory and filename of STL-file (in text / SCII format) to load
    - color:* provided as list of 4 RGBA values
    - verbose:* if True, useful information is provided during reading
  - **output:** interchanged 2nd and 3rd component of list
- 

```
def GraphicsDataSolidOfRevolution (pAxis, vAxis, contour, color= [0.,0.,0.,1.], nTiles= 16, smoothContour= False, **kwargs)
```

- **function description:** generate graphics data for a solid of revolution with given 3D point and axis, 2D point list for contour, (optional)2D normals and color;

– **input:**

*pAxis*: axis point of one face of solid of revolution (3D list or np.array)  
*vAxis*: vector representing the solid of revolution's axis (3D list or np.array)  
*contour*: a list of 2D-points, specifying the contour (*x*=axis, *y*=radius), e.g.: [[0,0],[0,0.1],[1,0.1]]  
*color*: provided as list of 4 RGBA values  
*nTiles*: used to determine resolution of solid; use larger values for finer resolution  
*smoothContour*: if True, the contour is made smooth by auto-computing normals to the contour  
*alternatingColor*: add a second color, which enables to see the rotation of the solid

– **output:** graphicsData dictionary, to be used in visualization of EXUDYN objects

– **example:**

```
#simple contour, using list of 2D points:
contour=[[0,0.2],[0.3,0.2],[0.5,0.3],[0.7,0.4],[1,0.4],[1,0.0]]
rev1 = GraphicsDataSolidOfRevolution(pAxis=[0,0.5,0], vAxis=[1,0,0],
 contour=contour, color=color4red,
 alternatingColor=color4grey)

#draw torus:
contour=[]
r = 0.2 #small radius of torus
R = 0.5 #big radius of torus
for i in range(nc+3): #+3 in order to remove boundary effects
 contour+=[[r*cos(i/nc*pi*2),R+r*sin(i/nc*pi*2)]]
#use smoothContour to make torus looking smooth
rev2 = GraphicsDataSolidOfRevolution(pAxis=[0,0.5,0], vAxis=[1,0,0],
 contour=contour, color=color4red,
 nTiles = 32*2, smoothContour=True)
```

---

def GraphicsDataArrow (*pAxis*, *vAxis*, *radius*, *color*=[0.,0.,0.,1.], *headFactor*=2, *headStretch*=4, *nTiles*=12)

– **function description:** generate graphics data for an arrow with given origin, axis, shaft radius, optional size factors for head and color; *nTiles* gives the number of tiles (minimum=3)

– **input:**

*pAxis*: axis point of the origin (base) of the arrow (3D list or np.array)  
*vAxis*: vector representing the vector pointing from the origin to the tip (head) of the error (3D list or np.array)  
*radius*: positive value representing radius of shaft cylinder  
*headFactor*: positive value representing the ratio between head's radius and the shaft radius  
*headStretch*: positive value representing the ratio between the head's radius and the head's length  
*color*: provided as list of 4 RGBA values  
*nTiles*: used to determine resolution of arrow (of revolution object) >=3; use larger values for finer resolution

- **output:** graphicsData dictionary, to be used in visualization of EXUDYN objects
- 

```
def GraphicsDataBasis (origin= [0,0,0], length= 1, colors= [color4red, color4green, color4blue], headFactor= 2, headStretch= 4, nTiles= 12, **kwargs)
```

- **function description:** generate graphics data for three arrows representing an orthogonal basis with point of origin, shaft radius, optional size factors for head and colors; nTiles gives the number of tiles (minimum=3)
  - **input:**
    - origin*: point of the origin of the base (3D list or np.array)
    - length*: positive value representing lengths of arrows for basis
    - colors*: provided as list of 3 colors (list of 4 RGBA values)
    - headFactor*: positive value representing the ratio between head's radius and the shaft radius
    - headStretch*: positive value representing the ratio between the head's radius and the head's length
    - nTiles*: used to determine resolution of arrows of basis (of revolution object) >=3; use larger values for finer resolution
    - radius*: positive value representing radius of arrows; default: radius = 0.01\*length
  - **output:** graphicsData dictionary, to be used in visualization of EXUDYN objects
- 

```
def GraphicsDataQuad (pList, color= [0.,0.,0.,1.], **kwargs)
```

- **function description:**
  - generate graphics data for simple quad with option for checkerboard pattern;
  - points are arranged counter-clock-wise*, e.g.: p0=[0,0,0], p1=[1,0,0], p2=[1,1,0], p3=[0,1,0]
- **input:**
  - pList*: list of 4 quad points [[x0,y0,z0],[x1,y1,z1],...]
  - color*: provided as list of 4 RGBA values
  - alternatingColor*: second color; if defined, a checkerboard pattern (default: 10x10) is drawn with color and alternatingColor
  - nTiles*: number of tiles for checkerboard pattern (default: 10)
  - nTilesY*: if defined, use number of tiles in y-direction different from x-direction (=nTiles)
- **output:** graphicsData dictionary, to be used in visualization of EXUDYN objects

#### – **example:**

```
plane = GraphicsDataQuad([[-8, 0, -8],[8, 0, -8],[8, 0, 8],[-8, 0, 8]],
 color4darkgrey, nTiles=8,
 alternatingColor=color4lightgrey)
oGround=mbs.AddObject(ObjectGround(referencePosition=[0,0,0],
 visualization=VObjectGround(graphicsData=[plane])))
```

---

```
def GraphicsDataCheckerBoard (point= [0,0,0], normal= [0,0,1], size= 1, color= color4lightgrey,
alternatingColor= color4lightgrey2, nTiles= 10, **kwargs)
```

– **function description:**

function to generate checkerboard background;

*points are arranged counter-clock-wise, e.g.:*

– **input:**

*point:* midpoint of pattern provided as list or np.array

*normal:* normal to plane provided as list or np.array

*size:* dimension of first side length of quad

*size2:* dimension of second side length of quad

*color:* provided as list of 4 RGBA values

*alternatingColor:* second color; if defined, a checkerboard pattern (default: 10x10) is drawn with color  
and alternatingColor

*nTiles:* number of tiles for checkerboard pattern in first direction

*nTiles2:* number of tiles for checkerboard pattern in second direction; default: nTiles

– **output:** graphicsData dictionary, to be used in visualization of EXUDYN objects

– **example:**

```
plane = GraphicsDataQuad([[-8, 0, -8],[8, 0, -8],[8, 0, 8],[-8, 0, 8]],
 color4darkgrey, nTiles=8,
 alternatingColor=color4lightgrey)
oGround=mbs .AddObject(ObjectGround(referencePosition=[0,0,0],
 visualization=VObjectGround(graphicsData=[plane])))
```

---

```
def ComputeTriangularMesh (vertices, segments)
```

– **function description:**

helper function to compute triangular mesh from list of vertices (=points) and segments;

computes triangular meshes for non-convex case. In order to make it efficient, it first computes

neighbors and then defines triangles at segments to be inside/outside. Finally neighboring

relations are used to define all triangles inside/outside

finally only returns triangles that are inside the segments

– **input:**

*vertices:* list of pairs of coordinates of vertices in mesh [x,y]

*segments:* list of segments, which are pairs of node numbers [i,j], defining the boundary of the mesh;

the ordering of the nodes is such that left triangle = inside, right triangle = outside, compare example with segment [V1,V2]:

inside  
V1 V2  
O——O  
outside

– **output:** triangulation structure of Delaunay(...), see `scipy.spatial.Delaunaystructure`, containing all simplices (=triangles)

– **example:**

```
points = np.array([[0, 0], [0, 2], [2, 2], [2, 1], [1, 1], [0, 1], [1, 0]])
segments = [len(points)-1,0]
for i in range(len(points)-1):
 segments += [i,i+1]
tri = ComputeTriangularMesh(points, segments)
print(tri.simplices)
```

---

def [GraphicsDataSolidExtrusion](#) (*vertices, segments, height, rot= np.diag([1,1,1]), pOff= [0,0,0], color= [0,0,0,1]*)

– **function description:**

create graphicsData for solid extrusion based on 2D points and segments;

additional transformations are possible to translate and rotate

– **input:**

*vertices*: list of pairs of coordinates of vertices in mesh [x,y], see `ComputeTriangularMesh(...)`

*segments*: list of segments, which are pairs of node numbers [i,j], defining the boundary of the mesh; the ordering of the nodes is such that left triangle = inside, right triangle = outside; see `ComputeTriangularMesh(...)`

*height*: height of extruded object

*rot*: rotation matrix, which the extruded object point coordinates are multiplied with before adding offset

*pOff*: 3D offset vector added to extruded coordinates; the z-coordinate of the extrusion object obtains 0 for the base plane, z=height for the top plane

– **output:** graphicsData dictionary, to be used in visualization of EXUDYN objects

## 5.5 Module: GUI

Helper functions and classes for graphical interaction with Exudyn

Author: Johannes Gerstmayer

Date: 2020-01-25

Notes: This is an internal library, which is only used inside Exudyn for modifying settings.

```
def EditDictionaryWithTypeInfo (dictionaryData, exu= None, dictionaryName= 'edit')
```

– **function description:** edit dictionaryData and return modified (new) dictionary

– **input:**

*dictionaryData*: dictionary obtained from SC.visualizationSettings.GetDictionaryWithTypeInfo()

*exu*: exudyn module

*dictionaryName*: name displayed in dialog

– **output:** returns modified dictionary, which can be used, e.g., for SC.visualizationSettings.SetDictionary(...)

## 5.6 Module: interactive

Utilities for interactive simulation and results monitoring

Author: Johannes Gerstmayr

Date: 2021-01-17 (created)

```
def AnimateModes (systemContainer, mainSystem, nodeNumber, period= 0.04, stepsPerPeriod= 30, showTime= True, renderWindowText= '')
```

– **function description:** animate modes of ObjectFFRFreducedOrder and other objects (changes periodically one nodal coordinate); for creating snapshots, press 'Static' and 'Record animation' and press 'Run' to save one figure in the image subfolder; for creating animations for one mode, use the same procedure but use 'One Cycle'

– **input:**

*systemContainer*: system container (usually SC) of your model, containing visualization settings

*mainSystem*: system (usually mbs) containing your model

*nodeNumber*: node number of which the coordinates shall be animated. In case of ObjectFFRFreducedOrder, this is the generic node, e.g., 'nGenericODE2' in the dictionary returned by the function AddObjectFFRFreducedOrderWithUserFunctions(...)

*period*: delay for animation of every frame; the default of 0.04 results in approximately 25 frames per second

*stepsPerPeriod*: number of steps into which the animation of one cycle of the mode is split into

*showTime*: show a virtual time running from 0 to  $2\pi$  during one mode cycle

*renderWindowText*: additional text written into renderwindow before 'Mode X' (use \n to add line breaks)

– **output:** opens interactive dialog with further settings

– **notes:**

Uses class InteractiveDialog in the background, which can be used to adjust animation creation.

Press 'Run' to start animation; Chose 'Mode shape', according component for contour plot; to record one cycle for animation, choose 'One cycle', run once to get the according range in the contour plot, press 'Record animation' and press 'Run', now images can be found in subfolder 'images' (for further info on animation creation see [Section 2.3.8](#)); now deactivate 'Record animation' by pressing 'Off' and chose another mode

### 5.6.1 CLASS InteractiveDialog (in module interactive)

**class description:** create an interactive dialog, which allows to interact with simulations the dialog has a 'Run' button, which initiates the simulation and a 'Stop' button which stops/pauses simulation; 'Quit' closes the simulation model for examples, see `simulateInteractively.py` and `massSpringFrictionInteractive.py` use `__init__` method to setup this class with certain buttons, edit boxes and sliders

#### - example:

```
#the following example is only demonstrating the structure of dialogItems and
#plots
#dialogItems structure:
#general items:
'type' can be out of:
'label' (simple text),
'button' (button with callback function),
'radio' (a radio button with several alternative options),
'slider' (with an adjustable range to choose a value)
'grid': (row, col, colspan) specifies the row, column and (optionally) the
span of columns the item is placed at;
exception in 'radio', where grid is a list of (row, col) for every
choice
'options': text options, where 'L' means flush left, 'R' means flush right
#suboptions of 'label':
'text': a text to be drawn
#suboptions of 'button':
'text': a text to be drawn on button
'callFunction': function which is called on button-press
#suboptions of 'radio':
'textValueList': [('text1',0),('text2',1)] a list of texts with
according values
'value': default value (choice) of radio buttons
'variable': according variable in mbs.variables, which is set to
current radio button value
#suboptions of 'slider':
'range': (min, max) a tuple containing minimum and maximum value
of slider
'value': default value of slider
'steps': number of steps in slider
'variable': according variable in mbs.variables, which is set to
current slider value
#example:
dialogItems = [{ 'type':'label', 'text':'Nonlinear oscillation simulator', 'grid':
:(0,0,2), 'options':['L'] },
{ 'type':'button', 'text':'test button', 'callFunction':ButtonCall,
'grid':(1,0,2)},
{ 'type':'radio', 'textValueList':[('linear',0),('nonlinear',1)], 'value':0, 'variable':'mode', 'grid': [(2,0),(2,1)]},
{ 'type':'label', 'text':'excitation frequency (Hz):', 'grid':(5,0)}
```

```

 },
 {'type':'slider', 'range':(3*f1/800, 3*f1), 'value':omegaInit/(2*pi), 'steps':800, 'variable':'frequency', 'grid':(5,1)},
 {'type':'label', 'text':'damping:', 'grid':(6,0)},
 {'type':'slider', 'range': (0, 40), 'value':damper, 'steps':800, 'variable':'damping', 'grid':(6,1)},
 {'type':'label', 'text':'stiffness:', 'grid':(7,0)},
 {'type':'slider', 'range':(0, 10000), 'value':spring, 'steps':800, 'variable':'stiffness', 'grid':(7,1)}]

#plots structure:
plots={'nPoints':500, #number of stored points in subplots (higher means slower drawing)
 'subplots':(2,1), #(rows, columns) arrangement of subplots
 #sensors defines per subplot (sensor, coordinate), xlabel and ylabel; if coordinate=0, time is used:
 'sensors':[[[(sensPos,0),(sensPos,1),'time','mass position'],
 [(sensFreq,0),(sensFreq,1),'time','excitation frequency']],
 'limitsX':[(0,2),(-5,5)], #x-range per subplot; if not provided, autoscale is applied
 'limitsY':[(-5,5),(0,10)], #y-range per subplot; if not provided, autoscale is applied
 'fontSize':16, #custom font size for figure
 'sizeInches':(12,12)} #specific x and y size of figure in inches (using 100 dpi)

def __init__(self, mbs, simulationSettings, simulationFunction, dialogItems, plots=[], period=0.04, realtimeFactor=1, userStartSimulation=False, title="", showTime=False, fontSize=12, doTimeIntegration=True)

```

– **classFunction:** initialize an InteractiveDialog

– **input:**

*mbs:* a multibody system to be simulated

*simulationSettings:* exudyn.SimulationSettings() according to user settings

*simulationFunction:* a function which is called before a simulation for the short period is started (e.g., assign special values, etc.)

*dialogItems:* a list of dictionaries, which describe the contents of the interactive items, where every dict has the structure 'type':[label, entry, button, slider, check] ... according to tkinter widgets, 'callFunction': a function to be called, if item is changed/button pressed, 'grid': (row,col) of item to be placed, 'rowSpan': number of rows to be used, 'columnSpan': number of columns to be used; for special item options see notes

*plots:* list of dictionaries to specify a sensor to be plotted live, see example

*period:* a simulation time span in seconds which is simulated with the simulationFunction in every iteration

*realtimeFactor:* if 1, the simulation is nearly performed in realtime (except for computation time); if > 1, it runs faster than realtime, if < 1, than it is slower

*userStartFunction:* a function F(flag) which is called every time after Run/Stop is pressed. The argument flag = False if button "Run" has been pressed, flag = True, if "Stop" has been pressed

*title*: title text for interactive dialog  
*showTime*: shows current time in dialog  
*fontSize*: adjust font size for all dialog items  
*doTimeIntegration*: performs internal time integration with given parameters

- **notes:** detailed description of dialogItems and plots list/dictionary is given in commented the example below
- 

`def OnQuit (self, event= None)`

- **classFunction:** function called when pressing escape or closing dialog
- 

`def StartSimulation (self, event= None)`

- **classFunction:** function called on button 'Run'
- 

`def ProcessWidgetStates (self)`

- **classFunction:** assign current values of radio buttons and sliders to mbs.variables
- 

`def ContinuousRunFunction (self, event= None)`

- **classFunction:** function which is repeatedly called when button 'Run' is pressed
- 

`def InitializePlots (self)`

- **classFunction:** initialize figure and subplots for plots structure
- 

`def UpdatePlots (self)`

- **classFunction**: update all subplots with current sensor values
- 

```
def InitializeSolver (self)
```

- **classFunction**: function to initialize solver for repeated calls
- 

```
def FinalizeSolver (self)
```

- **classFunction**: stop solver (finalize correctly)
- 

```
def RunSimulationPeriod (self)
```

- **classFunction**: function which performs short simulation for given period

## 5.7 Module: lieGroupBasics

Lie group methods and formulas for Lie group integration.

Author: Stefan Holzinger

Date: 2020-09-11

References:

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[Sonneville2017] Sonneville, Bruels, O., Bauchau, O.A.: Interpolation schemes for geometrically exact beams: A motion approach. International Journal for Numerical Methods in Engineering 112, 1129-1153 (2017)

[Terze2016] Terze, Z., Mueller, A., Zlatar, D.: Singularity-free time integration of rotational quaternions using non-redundant ordinary differential equations. Multibody System Dynamics 38(3), 201-225 (2016)

[Henderson1977] Henderson, D.: Euler angles, quaternions, and transformation matrices for space shuttle analysis. Tech. rep., NASA (1977)

[Holzinger2020] Holzinger, S., Gerstmayr, J.: Explicit time integration of multibody systems modelled with three rotation parameters. In: Proceedings of the ASME 2020 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference IDETC/CIE2020, August 17-19. Virtual, Online (2020)

[Holzinger2021] Holzinger, S., Gerstmayr, J.: Time integration of rigid bodies modelled with three rotation parameters, Multibody System Dynamics 2021

[Simo1988] Simo, J.C., Vu-Quoc, L.: On the dynamics in space of rods undergoing large motions—A geometrically exact approach. Computer methods in applied mechanics and engineering 66, 125-161 (1988)

[Mueller2017] Mueller, A.: Coordinate Mappings for Rigid Body Motions. Journal of Computational and Nonlinear Dynamics 12(2), 10 (2017)

def Sinc ( $x$ )

- **function description:** compute the cardinal sine function in radians
  - **input:** scalar float or int value
  - **output:** float value in radians
- 

def Cot ( $x$ )

- **function description:** compute the cotangent function  $\cot(x)=1/\tan(x)$  in radians
  - **input:** scalar float or int value
  - **output:** float value in radians
- 

def R3xSO3Matrix2RotationMatrix ( $G$ )

- **function description:** computes 3x3 rotation matrix from 7x7 R3xSO(3) matrix, see [6]
  - **input:**  $G$ : 7x7 matrix as np.array
  - **output:** 3x3 rotation matrix as np.array
- 

def R3xSO3Matrix2Translation ( $G$ )

- **function description:** computes translation part of R3xSO(3) matrix, see [6]
- **input:**  $G$ : 7x7 matrix as np.array
- **output:** 3D vector as np.array containg translational part of R3xSO(3)

---

```
def R3xSO3Matrix (x, R)
```

- **function description:** builds  $7 \times 7$  matrix as element of the Lie group  $\text{R3xSO}(3)$ , see [6]
  - **input:**
    - $x$ : 3D vector as np.array representing the translation part corresponding to R3
    - $R$ :  $3 \times 3$  rotation matrix as np.array
  - **output:**  $7 \times 7$  matrix as np.array
- 

```
def ExpSO3 (Omega)
```

- **function description:** compute the matrix exponential map on the Lie group  $\text{SO}(3)$ , see [26]
  - **input:** 3D rotation vector as np.array
  - **output:**  $3 \times 3$  matrix as np.array
- 

```
def ExpS3 (Omega)
```

- **function description:** compute the quaternion exponential map on the Lie group  $\text{S}(3)$ , see [34, 26]
  - **input:** 3D rotation vector as np.array
  - **output:**
    - 4D vector as np.array containing four Euler parameters
    - entry zero of output represent the scalar part of Euler parameters
- 

```
def LogSO3 (R)
```

- **function description:** compute the matrix logarithmic map on the Lie group  $\text{SO}(3)$ , see [33, 32]
  - **input:**  $3 \times 3$  rotation matrix as np.array
  - **output:**  $3 \times 3$  skew symmetric matrix as np.array
- 

```
def TExpSO3 (Omega)
```

- **function description:** compute the tangent operator corresponding to ExpSO3, see [6]
  - **input:** 3D rotation vector as np.array
  - **output:** 3x3 matrix as np.array
- 

```
def TExpSO3Inv (Omega)
```

- **function description:**
    - compute the inverse of the tangent operator TExpSO3, see [33]
    - this function was improved, see coordinateMaps.pdf by Stefan Holzinger
  - **input:** 3D rotation vector as np.array
  - **output:** 3x3 matrix as np.array
- 

```
def ExpSE3 (x)
```

- **function description:** compute the matrix exponential map on the Lie group SE(3), see [6]
  - **input:** 6D incremental motion vector as np.array
  - **output:** 4x4 homogeneous transformation matrix as np.array
- 

```
def LogSE3 (H)
```

- **function description:** compute the matrix logarithm on the Lie group SE(3), see [33]
  - **input:** 4x4 homogeneous transformation matrix as np.array
  - **output:** 4x4 skew symmetric matrix as np.array
- 

```
def TExpSE3 (x)
```

- **function description:** compute the tangent operator corresponding to ExpSE3, see [6]
  - **input:** 6D incremental motion vector as np.array
  - **output:** 6x6 matrix as np.array
-

```
def TExpSE3Inv (x)
```

- **function description:** compute the inverse of tangent operator TExpSE3, see [33]
  - **input:** 6D incremental motion vector as np.array
  - **output:** 6x6 matrix as np.array
- 

```
def ExpR3xSO3 (x)
```

- **function description:** compute the matrix exponential map on the Lie group R3xSO(3), see [6]
  - **input:** 6D incremental motion vector as np.array
  - **output:** 7x7 matrix as np.array
- 

```
def TExpR3xSO3 (x)
```

- **function description:** compute the tangent operator corresponding to ExpR3xSO3, see [6]
  - **input:** 6D incremental motion vector as np.array
  - **output:** 6x6 matrix as np.array
- 

```
def TExpR3xSO3Inv (x)
```

- **function description:** compute the inverse of tangent operator TExpR3xSO3
  - **input:** 6D incremental motion vector as np.array
  - **output:** 6x6 matrix as np.array
- 

```
def CompositionRuleDirectProductR3AndS3 ($q_0, incrementalMotionVector$)
```

- **function description:** compute composition operation for pairs in the Lie group R3xS3
- **input:**
  - $q_0$ : 7D vector as np.array containing position coordinates and Euler parameters
  - $incrementalMotionVector$ : 6D incremental motion vector as np.array
- **output:** 7D vector as np.array containing composed position coordinates and composed Euler parameters

---

```
def CompositionRuleSemiDirectProductR3AndS3 (q0, incrementalMotionVector)
```

- **function description:** compute composition operation for pairs in the Lie group R3 semiTimes S3 (corresponds to SE(3))
  - **input:**
    - q0*: 7D vector as np.array containing position coordinates and Euler parameters
    - incrementalMotionVector*: 6D incremental motion vector as np.array
  - **output:** 7D vector as np.array containing composed position coordinates and composed Euler parameters
- 

```
def CompositionRuleDirectProductR3AndR3RotVec (q0, incrementalMotionVector)
```

- **function description:**
    - compute composition operation for pairs in the group obtained from the direct product of R3 and R3,  
see [16]
    - the rotation vector is used as rotation parametrizations
    - this composition operation can be used in formulations which represent the translational velocities in  
the global (inertial) frame
  - **input:**
    - q0*: 6D vector as np.array containing position coordinates and rotation vector
    - incrementalMotionVector*: 6D incremental motion vector as np.array
  - **output:** 7D vector as np.array containing composed position coordinates and composed rotation vector
- 

```
def CompositionRuleSemiDirectProductR3AndR3RotVec (q0, incrementalMotionVector)
```

- **function description:**
  - compute composition operation for pairs in the group obtained from the direct product of R3 and R3.
  - the rotation vector is used as rotation parametrizations
  - this composition operation can be used in formulations which represent the translational velocities in  
the local (body-attached) frame
- **input:**
  - q0*: 6D vector as np.array containing position coordinates and rotation vector
  - incrementalMotionVector*: 6D incremental motion vector as np.array
- **output:** 6D vector as np.array containing composed position coordinates and composed rotation vector

---

```
def CompositionRuleDirectProductR3AndR3RotXYZAngles (q0, incrementalMotionVector)
```

– **function description:**

compute composition operation for pairs in the group obtained from the direct product of R3 and R3.  
Cardan-Tait/Bryan (CTB) angles are used as rotation parametrizations  
this composition operation can be used in formulations which represent the translational velocities in the global (inertial) frame

– **input:**

*q0*: 6D vector as np.array containing position coordinates and Cardan-Tait/Bryan angles  
*incrementalMotionVector*: 6D incremental motion vector as np.array

– **output:** 6D vector as np.array containing composed position coordinates and composed Cardan-Tait/Bryan angles

---

```
def CompositionRuleSemiDirectProductR3AndR3RotXYZAngles (q0, incrementalMotionVector)
```

– **function description:**

compute composition operation for pairs in the group obtained from the direct product of R3 and R3.  
Cardan-Tait/Bryan (CTB) angles are used as rotation parametrizations  
this composition operation can be used in formulations which represent the translational velocities in the local (body-attached) frame

– **input:**

*q0*: 6D vector as np.array containing position coordinates and Cardan-Tait/Bryan angles  
*incrementalMotionVector*: 6D incremental motion vector as np.array

– **output:** 6D vector as np.array containing composed position coordinates and composed Cardan-Tait/Bryan angles

---

```
def CompositionRuleForEulerParameters (q, p)
```

– **function description:**

compute composition operation for Euler parameters (unit quaternions)  
this composition operation is quaternion multiplication, see [34]

– **input:**

*q*: 4D vector as np.array containing Euler parameters  
*p*: 4D vector as np.array containing Euler parameters

- **output:** 4D vector as np.array containing composed (multiplied) Euler parameters
- 

```
def CompositionRuleForRotationVectors (v0, Omega)
```

- **function description:** compute composition operation for rotation vectors v0 and Omega, see [23]
  - **input:**
    - v0*: 3D rotation vector as np.array
    - Omega*: 3D (incremental) rotation vector as np.array
  - **output:** 3D vector as np.array containing composed rotation vector v
- 

```
def CompositionRuleRotXYZAnglesRotationVector (alpha0, Omega)
```

- **function description:** compute composition operation for RotXYZ angles, see [23]
- **input:**
  - alpha0*: 3D vector as np.array containing RotXYZ angles
  - Omega*: 3D vector as np.array containing the (incremental) rotation vector
- **output:** 3D vector as np.array containing composed RotXYZ angles

## 5.8 Module: physics

The physics library includes helper functions and data related to physics models and parameters; for rigid body inertia, see `rigidBodyUtilities`

Date: 2021-01-20

```
def StribeckFunction (vel, muDynamic, muStaticOffset, muViscous= 0, expVel= 1e-3, regVel= 1e-3)
```

- **function description:**

describes regularized Stribeck function with optial viscous part for given velocity,

$$f(v) = \begin{cases} (\mu_d + \mu_{s_{off}})v, & \text{if } |v| \leq v_{reg} \\ \text{Sign}(v) \left( \mu_d + \mu_{s_{off}} e^{-(|v|-v_{reg})/v_{exp}} + \mu_v (|v| - v_{reg}) \right), & \text{else} \end{cases}$$
- **input:**
  - vel*: input velocity *v*
  - muDynamic*: dynamic friction coefficient  $\mu_d$
  - muStaticOffset*:  $\mu_{s_{off}}$ , offset to dynamic friction, which gives  $\mu_{StaticFriction} = \mu_{Dynamic} + \mu_{StaticOffset}$
  - muViscous*:  $\mu_v$ , viscous part, acting proportional to velocity except for *regVel*

*regVel*:  $v_{reg}$ , small regularization velocity in which the friction is linear around zero velocity (e.g., to get Newton converged)

*expVel*:  $v_{exp}$ , velocity (relative to *regVel*, at which the *muStaticOffset* decreases exponentially, at  $vel=expVel$ , the factor to *muStaticOffset* is  $\exp(-1) = 36.8\%$ )

- **output**: returns velocity dependent friction coefficient (if *muDynamic* and *muStaticOffset* are friction coefficients) or friction force (if *muDynamic* and *muStaticOffset* are on force level)
- 

```
def RegularizedFrictionStep (x, x0, h0, x1, h1)
```

- **function description**: helper function for *RegularizedFriction*(...)
- 

```
def RegularizedFriction (vel, muDynamic, muStaticOffset, velStatic, velDynamic, muViscous= 0)
```

- **function description**: describes regularized friction function, with increased static friction, dynamic friction and optional viscous part

- **input**:

*vel*: input velocity

*muDynamic*: dynamic friction coefficient

*muStaticOffset*: offset to dynamic friction, which gives  $\mu_{StaticFriction} = \mu_{Dynamic} + \mu_{StaticOffset}$

*muViscous*: viscous part, acting proportional to velocity for velocities larger than *velDynamic*; extension to mentioned references

*velStatic*: small regularization velocity at which exactly the staticFriction is reached; for smaller velocities, the friction is smooth and zero-crossing (unphysical!) (e.g., to get Newton converged)

*velDynamic*: velocity at which *muDynamic* is reached for first time

- **output**: returns velocity dependent friction coefficient (if *muDynamic* and *muStaticOffset* are friction coefficients) or friction force (if *muDynamic* and *muStaticOffset* are on force level)

- **notes**: see references: Flores et al. [11], Qian et al. [29]

## 5.9 Module: plot

Plot utility functions based on matplotlib, including plotting of sensors and FFT.

Author: Johannes Gerstmayr

Date: 2020-09-16 (created)

Notes: For a list of plot colors useful for matplotlib, see also *utilities.PlotLineCode*(...)

```
def ParseOutputFileHeader (lines)
```

- **function description:** parse header of output file (solution file, sensor file, genetic optimization output, ...) given in file.readlines() format
  - **output:** return dictionary with 'type'=['sensor','solution','geneticOptimization','parameterVariation'], 'variableType',
- 

```
def PlotSensor (mbs, sensorNumbers, components= 0, **kwargs)
```

- **function description:** helper for matplotlib in order to easily visualize sensor output
- **input:**
  - mbs*: must be a valid MainSystem (*mbs*)
  - sensorNumbers*: consists of one or a list of sensor numbers (type SensorIndex) as returned by the *mbs* function AddSensor(...)
  - components*: consists of one or a list of components according to the component of the sensor to be plotted;
  - \*kwargs*: additional options, e.g.:
    - xLabel* -> string for text at x-axis (otherwise time is used)
    - yLabel* -> string for text at y-axis (otherwise outputvalues are used)
    - fontSize* -> default = 16, which is a little bit larger than default (12)
- **output:** plots the sensor data
- **example:**

```
s0=mbs.AddSensor(SensorNode(nodeNumber=0))
s1=mbs.AddSensor(SensorNode(nodeNumber=1))
Plot(mbs, s0, 0)
Plot(mbs, sensorNumbers=[s0,s1], components=[0,2], xlabel='time in seconds')
```

---

```
def PlotFFT (frequency, data, xLabel= 'frequency', yLabel= 'magnitude', label= "", freqStart= 0, freqEnd= -1, logScaleX= True, logScaleY= True, majorGrid= True, minorGrid= True)
```

- **function description:** plot fft spectrum of signal
- **input:**
  - frequency*: frequency vector (Hz, if time is in SECONDS)
  - data*: magnitude or phase as returned by ComputeFFT() in exudyn.signal
  - xLabel*: label for x-axis, default=frequency
  - yLabel*: label for y-axis, default=magnitude
  - label*: either empty string ("") or name used in legend
  - freqStart*: starting range for frequency
  - freqEnd*: end of range for frequency; if freqEnd== -1 (default), the total range is plotted

- logScaleX*: use log scale for x-axis
- logScaleY*: use log scale for y-axis
- majorGrid*: if True, plot major grid with solid line
- minorGrid*: if True, plot minor grid with dotted line
- **output**: creates plot and returns plot (plt) handle

## 5.10 Module: processing

The processing module supports multiple execution of EXUDYN models. It includes parameter variation and (genetic) optimization functionality.

Author: Johannes Gerstmayr

Date: 2020-11-17

Notes: Parallel processing, which requires multiprocessing library, can lead to considerable speedup (measured speedup factor > 50 on 80 core machine). The progress bar during multiprocessing requires the library tqdm.

```
def ProcessParameterList (parameterFunction, parameterList, addComputationIndex, useMultiProcessing, **kwargs)
```

- **function description**: processes parameterFunction for given parameters in parameterList, see ParameterVariation
- **input**:
  - parameterFunction*: function, which takes the form parameterFunction(parameterDict) and which returns any values that can be stored in a list (e.g., a floating point number)
  - parameterList*: list of parameter sets (as dictionaries) which are fed into the parameter variation, e.g., [‘mass’: 10, ‘mass’:20, ...]
  - addComputationIndex*: if True, key ‘computationIndex’ is added to every parameterDict in the call to parameterFunction(), which allows to generate independent output files for every parameter, etc.
  - useMultiProcessing*: if True, the multiprocessing lib is used for parallelized computation; WARNING: be aware that the function does not check if your function runs independently; DO NOT use GRAPHICS and DO NOT write to same output files, etc.!
  - numberOfThreads*: default: same as number of cpus (threads); used for multiprocessing lib;
  - resultsFile*: if provided, output is immediately written to resultsFile during processing
- **output**: returns values containing the results according to parameterList
- **notes**: options are passed from Parametervariation

```
def ParameterVariation (parameterFunction, parameters, useLogSpace= False, debugMode= False, addComputationIndex= False, useMultiProcessing= False, showProgress= True, **kwargs)
```

- **function description**:

calls successively the function parameterFunction(parameterDict) with variation of parameters in given range; parameterDict is a dictionary, containing the current values of parameters, e.g., `parameterDict=[‘mass’:13, ‘stiffness’:12000]` to be computed and returns a value or a list of values which is then stored for each parameter

– **input:**

`parameterFunction`: function, which takes the form `parameterFunction(parameterDict)` and which returns any values that can be stored in a list (e.g., a floating point number)

`parameters`: given as a dictionary, consist of name and tuple of (begin, end, `numberOfValues`) same as in `np.linspace(...)`, e.g. `‘mass’:(10,50,10)`, for a mass varied from 10 to 50, using 10 steps OR a list of values `[v0, v1, v2, ...]`, e.g. `‘mass’:[10,15,25,50]`

`useLogSpace`: (optional) if True, the parameters are varied at a logarithmic scale, e.g., `[1, 10, 100]` instead linear `[1, 50.5, 100]`

`debugMode`: if True, additional print out is done

`addComputationIndex`: if True, key ‘computationIndex’ is added to every parameterDict in the call to `parameterFunction()`, which allows to generate independent output files for every parameter, etc.

`useMultiProcessing`: if True, the multiprocessing lib is used for parallelized computation; WARNING: be aware that the function does not check if your function runs independently; DO NOT use GRAPHICS and DO NOT write to same output files, etc.!

`showProgress`: if True, shows for every iteration the progress bar (requires `tqdm` library)

`resultsFile`: if provided, output is immediately written to `resultsFile` during processing

`numberOfThreads`: default: same as number of cpus (threads); used for multiprocessing lib;

– **output:**

`returns [parameterList, values]`, containing, e.g., `parameterList=‘mass’:[1,1,2,2,3,3,3], ‘stiffness’:[4,5,6,4,5,6]` and the result values of the parameter variation accoring to the `parameterList`, `values=[7,8,9 ,3,4,5, 6,7,8]` (depends on solution of problem ..., can also contain tuples, etc.)

– **example:**

```
ParameterVariation(parameters={‘mass’: (1,10,10), ‘stiffness’: (1000,10000,10)},
 parameterFunction=Test, useMultiProcessing=True)
```

---

```
def GeneticOptimization(objectiveFunction, parameters, populationSize= 100, numberOfGenerations= 10,
elitistRatio= 0.1, crossoverProbability= 0.25, crossoverAmount= 0.5, rangeReductionFactor= 0.7, distanceFactor= 0.1,
debugMode= False, addComputationIndex= False, useMultiProcessing= False, showProgress= True, **kwargs)
```

– **function description:** compute minimum of given `objectiveFunction`

– **input:**

`objectiveFunction`: function, which takes the form `parameterFunction(parameterDict)` and which returns a value or list (or numpy array) which reflects the size of the objective to be minimized

`parameters`: given as a dictionary, consist of name and tuple containing the search range for this parameter (begin, end), e.g. `‘mass’:(10,50)`

`populationSize`: individuals in every generation

*initialPopulationSize*: number of random initial individuals; default: population size  
*numberOfGenerations*: number of generations; NOTE: it is required that elitistRatio\*populationSize >= 1  
*elitistRatio*: the number of surviving individuals in every generation is equal to the previous population times the elitistRatio  
*crossoverProbability*: if > 0: children are generated from two (randomly selected) parents by gene-crossover; if 0, no crossover is used  
*crossoverAmount*: if crossoverProbability > 0, then this amount is the probability of genes to cross; 0.1: small amount of genes cross, 0.5: 50  
*rangeReductionFactor*: reduction of mutation range (boundary) relative to range of last generation; helps algorithm to converge to more accurate values  
*distanceFactor*: children only survive at a certain relative distance of the current range; must be small enough (< 0.5) to allow individuals to survive; ignored if distanceFactor=0; as a rule of thumb, the distanceFactor should be zero in case that there is only one significant minimum, but if there are many local minima, the distanceFactor should be used to search at several different local minima  
*randomizerInitialization*: initialize randomizer at beginning of optimization in order to get reproducible results, provide any integer in the range between 0 and  $2^{**32} - 1$  (default: no initialization)  
*debugMode*: if True, additional print out is done  
*addComputationIndex*: if True, key 'computationIndex' is added to every parameterDict in the call to parameterFunction(), which allows to generate independent output files for every parameter, etc.  
*useMultiProcessing*: if True, the multiprocessing lib is used for parallelized computation; WARNING: be aware that the function does not check if your function runs independently; DO NOT use GRAPHICS and DO NOT write to same output files, etc.  
*showProgress*: if True, shows for every iteration the progress bar (requires tqdm library)  
*numberOfThreads*: default: same as number of cpus (threads); used for multiprocessing lib;  
*resultsFile*: if provided, the results are stored columnwise into the given file and written after every generation; use resultsMonitor.py to track results in realtime  
*numberOfChildren*: (DEPRECATED, UNUSED) number childrens of surviving population  
*survivingIndividuals*: (DEPRECATED) number of surviving individuals after children are born

- **output:**

returns [optimumParameter, optimumValue, parameterList, valueList], containing the optimum parameter set 'optimumParameter', optimum value 'optimumValue', the whole list of parameters parameterList with according objective values 'valueList'  
 values=[7,8,9 ,3,4,5, 6,7,8] (depends on solution of problem ..., can also contain tuples, etc.)

- **notes:** This function is still under development and shows an experimental state!

- **example:**

```
GeneticOptimization(objectiveFunction = f0pt, parameters={'mass':(1,10), 'stiffness':(1000,10000)})
```

---

```
def PlotOptimizationResults2D (parameterList, valueList, xLogScale= False, yLogScale= False)
```

- **function description:** visualize results of optimization for every parameter (2D plots)
- **input:**
  - parameterList*: taken from output parameterList of `GeneticOptimization`, containing a dictionary with lists of parameters
  - valueList*: taken from output valueList of `GeneticOptimization`; containing a list of floats that result from the objective function
  - xLogScale*: use log scale for x-axis
  - yLogScale*: use log scale for y-axis
- **output:** return [figList, axList] containing the corresponding handles; creates a figure for every parameter in parameterList

## 5.11 Module: rigidBodyUtilities

Advanced utility/mathematical functions for reference frames, rigid body kinematics and dynamics. Useful Euler parameter and Tait-Bryan angle conversion functions are included. A class for rigid body inertia creating and transformation is available.

Author: Johannes Gerstmayr, Stefan Holzinger (rotation vector and Tait-Bryan angles)

Date: 2020-03-10 (created)

---

def **ComputeOrthonormalBasis** (*vector0*)

- **function description:** compute orthogonal basis vectors (normal1, normal2) for given vector0 (non-unique solution!); if vector0 == [0,0,0], then any normal basis is returned
- 

def **GramSchmidt** (*vector0, vector1*)

- **function description:** compute Gram-Schmidt projection of given 3D vector 1 on vector 0 and return normalized triad (vector0, vector1, vector0 x vector1)
- 

def **Skew** (*vector*)

- **function description:** compute skew symmetric 3x3-matrix from 3x1- or 1x3-vector
- 

def **Skew2Vec** (*skew*)

- 
- **function description:** convert skew symmetric matrix m to vector

---

```
def ComputeSkewMatrix (v)
```

- **function description:** compute ( $3 \times 3^*n$ ) skew matrix from ( $3^*n$ ) vector
- 

---

```
def EulerParameters2G (eulerParameters)
```

- **function description:** convert Euler parameters (ep) to G-matrix ( $=\partial \omega / \partial \mathbf{p}_t$ )
  - **input:** vector of 4 eulerParameters as list or np.array
  - **output:** 3x4 matrix G as np.array
- 

---

```
def EulerParameters2GLocal (eulerParameters)
```

- **function description:** convert Euler parameters (ep) to local G-matrix ( $=\partial {}^b \omega / \partial \mathbf{p}_t$ )
  - **input:** vector of 4 eulerParameters as list or np.array
  - **output:** 3x4 matrix G as np.array
- 

---

```
def EulerParameters2RotationMatrix (eulerParameters)
```

- **function description:** compute rotation matrix from eulerParameters
  - **input:** vector of 4 eulerParameters as list or np.array
  - **output:** 3x3 rotation matrix as np.array
- 

---

```
def RotationMatrix2EulerParameters (rotationMatrix)
```

- **function description:** compute Euler parameters from given rotation matrix
- **input:** 3x3 rotation matrix as list of lists or as np.array
- **output:** vector of 4 eulerParameters as np.array

---

```
def AngularVelocity2EulerParameters_t (angularVelocity, eulerParameters)
```

- **function description:**

compute time derivative of Euler parameters from (global) angular velocity vector

note that for Euler parameters  $\mathbf{p}$ , we have  $\boldsymbol{\omega} = \mathbf{G}\dot{\mathbf{p}} \implies \mathbf{G}^T\boldsymbol{\omega} = \mathbf{G}^T \cdot \mathbf{G} \cdot \dot{\mathbf{p}} \implies \mathbf{G}^T\mathbf{G} = 4(\mathbf{I}_{4 \times 4} - \mathbf{p} \cdot \mathbf{p}^T)\dot{\mathbf{p}} = 4(\mathbf{I}_{4 \times 4})\dot{\mathbf{p}}$

- **input:**

*angularVelocity*: 3D vector of angular velocity in global frame, as lists or as np.array

*eulerParameters*: vector of 4 eulerParameters as np.array or list

- **output:** vector of time derivatives of 4 eulerParameters as np.array

---

```
def RotationVector2RotationMatrix (rotationVector)
```

- **function description:** rotation matrix from rotation vector, see appendix B in [31]

- **input:** 3D rotation vector as list or np.array

- **output:** 3x3 rotation matrix as np.array

---

```
def RotationMatrix2RotationVector (rotationMatrix)
```

- **function description:** compute rotation vector from rotation matrix

- **input:** 3x3 rotation matrix as list of lists or as np.array

- **output:** vector of 3 components of rotation vector as np.array

---

```
def ComputeRotationAxisFromRotationVector (rotationVector)
```

- **function description:** compute rotation axis from given rotation vector

- **input:** 3D rotation vector as np.array

- **output:** 3D vector as np.array representing the rotation axis

---

```
def RotationVector2G (rotationVector)
```

- **function description:** convert rotation vector (parameters) ( $\mathbf{v}$ ) to G-matrix ( $=\partial \boldsymbol{\omega} / \partial \mathbf{v}$ )
  - **input:** vector of rotation vector (len=3) as list or np.array
  - **output:** 3x3 matrix G as np.array
- 

```
def RotationVector2GLocal (eulerParameters)
```

- **function description:** convert rotation vector (parameters) ( $\mathbf{v}$ ) to local G-matrix ( $=\partial^b \boldsymbol{\omega} / \partial \mathbf{v}_t$ )
  - **input:** vector of rotation vector (len=3) as list or np.array
  - **output:** 3x3 matrix G as np.array
- 

```
def RotXYZ2RotationMatrix (rot)
```

- **function description:** compute rotation matrix from consecutive xyz rotations (Tait-Bryan angles);  $\mathbf{A} = \mathbf{A}_x * \mathbf{A}_y * \mathbf{A}_z$ ; rot=[rotX, rotY, rotZ]
  - **input:** 3D vector of Tait-Bryan rotation parameters [X,Y,Z] in radiant
  - **output:** 3x3 rotation matrix as np.array
- 

```
def RotationMatrix2RotXYZ (rotationMatrix)
```

- **function description:** convert rotation matrix to xyz Euler angles (Tait-Bryan angles);  $\mathbf{A} = \mathbf{A}_x * \mathbf{A}_y * \mathbf{A}_z$
  - **input:** 3x3 rotation matrix as list of lists or np.array
  - **output:** vector of Tait-Bryan rotation parameters [X,Y,Z] (in radiant) as np.array
- 

```
def RotXYZ2G (rot)
```

- **function description:** compute (global-frame) G-matrix for xyz Euler angles (Tait-Bryan angles) ( ${}^0\mathbf{G} = \partial {}^0\boldsymbol{\omega} / \partial \dot{\theta}$ )
  - **input:** 3D vector of Tait-Bryan rotation parameters [X,Y,Z] in radiant
  - **output:** 3x3 matrix G as np.array
-

```
def RotXYZ2G_t (rot, rot_t)
```

- **function description:** compute time derivative of (global-frame) G-matrix for xyz Euler angles (Tait-Bryan angles) ( ${}^0\mathbf{G} = \partial {}^0\boldsymbol{\omega} / \partial \dot{\theta}$ )
  - **input:**
    - rot*: 3D vector of Tait-Bryan rotation parameters [X,Y,Z] in radiant
    - rot\_t*: 3D vector of time derivative of Tait-Bryan rotation parameters [X,Y,Z] in radiant/s
  - **output:** 3x3 matrix  $\mathbf{G}_t$  as np.array
- 

```
def RotXYZ2GLocal (rot)
```

- **function description:** compute local (body-fixed) G-matrix for xyz Euler angles (Tait-Bryan angles) ( ${}^b\mathbf{G} = \partial {}^b\boldsymbol{\omega} / \partial \theta_t$ )
  - **input:** 3D vector of Tait-Bryan rotation parameters [X,Y,Z] in radiant
  - **output:** 3x3 matrix  $\mathbf{G}_{\text{Local}}$  as np.array
- 

```
def RotXYZ2GLocal_t (rot, rot_t)
```

- **function description:** compute time derivative of (body-fixed) G-matrix for xyz Euler angles (Tait-Bryan angles) ( ${}^b\mathbf{G} = \partial {}^b\boldsymbol{\omega} / \partial \theta_t$ )
  - **input:**
    - rot*: 3D vector of Tait-Bryan rotation parameters [X,Y,Z] in radiant
    - rot\_t*: 3D vector of time derivative of Tait-Bryan rotation parameters [X,Y,Z] in radiant/s
  - **output:** 3x3 matrix  $\mathbf{G}_{\text{Local}}_t$  as np.array
- 

```
def AngularVelocity2RotXYZ_t (angularVelocity, rotation)
```

- **function description:** compute time derivatives of angles RotXYZ from (global) angular velocity vector and given rotation
- **input:**
  - angularVelocity*: global angular velocity vector as list or np.array
  - rotation*: 3D vector of Tait-Bryan rotation parameters [X,Y,Z] in radiant
- **output:** time derivative of vector of Tait-Bryan rotation parameters [X,Y,Z] (in radiant) as np.array

---

```
def RotXYZ2EulerParameters (alpha)
```

- **function description:** compute four Euler parameters from given RotXYZ angles, see [22]
  - **input:** alpha: 3D vector as np.array containing RotXYZ angles
  - **output:**
    - 4D vector as np.array containing four Euler parameters
    - entry zero of output represent the scalar part of Euler parameters
- 

```
def RotationMatrixX (angleRad)
```

- **function description:** compute rotation matrix w.r.t. X-axis (first axis)
  - **input:** angle around X-axis in radiant
  - **output:** 3x3 rotation matrix as np.array
- 

```
def RotationMatrixY (angleRad)
```

- **function description:** compute rotation matrix w.r.t. Y-axis (second axis)
  - **input:** angle around Y-axis in radiant
  - **output:** 3x3 rotation matrix as np.array
- 

```
def RotationMatrixZ (angleRad)
```

- **function description:** compute rotation matrix w.r.t. Z-axis (third axis)
  - **input:** angle around Z-axis in radiant
  - **output:** 3x3 rotation matrix as np.array
- 

```
def HomogeneousTransformation (A, r)
```

- **function description:** compute homogeneous transformation matrix from rotation matrix A and translation vector r

---

```
def HTtranslate (r)
```

- **function description:** homogeneous transformation for translation with vector r
- 

```
def HT0 ()
```

- **function description:** identity homogeneous transformation:
- 

```
def HTrotateX (angle)
```

- **function description:** homogeneous transformation for rotation around axis X (first axis)
- 

```
def HTrotateY (angle)
```

- **function description:** homogeneous transformation for rotation around axis X (first axis)
- 

```
def HTrotateZ (angle)
```

- **function description:** homogeneous transformation for rotation around axis X (first axis)
- 

```
def HT2translation (T)
```

- **function description:** return translation part of homogeneous transformation
- 

```
def HT2rotationMatrix (T)
```

- **function description:** return rotation matrix of homogeneous transformation

---

```
def InverseHT (T)
```

- **function description:** return inverse homogeneous transformation such that  $\text{inv}(T)^*T = \text{np.eye}(4)$
- 

```
def GetRigidBodyNode (nodeType, position= [0,0,0], velocity= [0,0,0], rotationMatrix= [], rotationParameters= [], angularVelocity= [0,0,0])
```

- **function description:** get node item interface according to nodeType, using initialization with position, velocity, angularVelocity and rotationMatrix
  - **input:**
    - position:* reference position as list or numpy array with 3 components
    - velocity:* initial translational velocity as list or numpy array with 3 components
    - rotationMatrix:* 3x3 list or numpy matrix to define reference rotation; use EITHER  $\text{rotationMatrix}=[[],[],[]]$  (while  $\text{rotationParameters}=[]$ ) or  $\text{rotationParameters}=[...]$  (while  $\text{rotationMatrix}=[]$ )
    - rotationParameters:* reference rotation parameters; use EITHER  $\text{rotationMatrix}=[[],[],[]]$  (while  $\text{rotationParameters}=[]$ ) or  $\text{rotationParameters}=[...]$  (while  $\text{rotationMatrix}=[]$ )
    - angularVelocity:* initial angular velocity as list or numpy array with 3 components
  - **output:** returns list containing node number and body number: [nodeNumber, bodyNumber]
- 

```
def AddRigidBody (mainSys, inertia, nodeType, position= [0,0,0], velocity= [0,0,0], rotationMatrix= [], rotationParameters= [], angularVelocity= [0,0,0], gravity= [0,0,0], graphicsDataList= [])
```

- **function description:** adds a node (with `str(exu.NodeType. ...)`) and body for a given rigid body; all quantities (esp. velocity and angular velocity) are given in global coordinates!
- **input:**
  - position:* reference position as list or numpy array with 3 components
  - velocity:* initial translational velocity as list or numpy array with 3 components
  - rotationMatrix:* 3x3 list or numpy matrix to define reference rotation; use EITHER  $\text{rotationMatrix}=[[],[],[]]$  (while  $\text{rotationParameters}=[]$ ) or  $\text{rotationParameters}=[...]$  (while  $\text{rotationMatrix}=[]$ )
  - rotationParameters:* reference rotation parameters; use EITHER  $\text{rotationMatrix}=[[],[],[]]$  (while  $\text{rotationParameters}=[]$ ) or  $\text{rotationParameters}=[...]$  (while  $\text{rotationMatrix}=[]$ )
  - angularVelocity:* initial angular velocity as list or numpy array with 3 components
  - gravity:* if provided as list or numpy array with 3 components, it adds gravity force to the body at the COM, i.e.,  $fAdd = m^*gravity$
  - graphicsDataList:* list of graphicsData objects to define appearance of body
- **output:** returns list containing node number and body number: [nodeNumber, bodyNumber]

### 5.11.1 CLASS RigidBodyInertia (in module rigidBodyUtilities)

**class description:** helper class for rigid body inertia (see also derived classes Inertia...). Provides a structure to define mass, inertia and center of mass (com) of a rigid body. The inertia tensor and center of mass must correspond when initializing the body!

– **notes:** It is recommended to start with com=[0,0,0] and then to use Translated(...) and Rotated(...) to get transformed inertia parameters.

– **example:**

```
i0 = RigidBodyInertia(10,np.diag([1,2,3]))
i1 = i0.Rotated(RotationMatrixX(np.pi/2))
i2 = i1.Translated([1,0,0])
```

```
def __init__(self, mass= 0, inertiaTensor= np.zeros([3,3]), com= np.zeros(3))
```

– **classFunction:** initialize RigidBodyInertia with scalar mass, 3x3 inertiaTensor and center of mass com

---

```
def __add__(self, otherBodyInertia)
```

– **classFunction:**

add (+) operator allows adding another inertia information with SAME local coordinate system  
only inertias with same center of rotation can be added!

– **example:**

```
J = InertiaSphere(2,0.1) + InertiaRodX(1,2)
```

---

```
def Translated(self, vec)
```

– **classFunction:** returns a RigidBodyInertia with center of mass com shifted by vec; → transforms the returned inertiaTensor to the new center of rotation

---

```
def Rotated(self, rot)
```

– **classFunction:** returns a RigidBodyInertia rotated by 3x3 rotation matrix rot, such that for a given J, the new inertia tensor reads Jnew = rot\*J\*rot.T  
– **notes:** only allowed if COM=0 !

---

```
def GetInertia6D(self)
```

– **classFunction:** get vector with 6 inertia components (Jxx, Jyy, Jzz, Jyz, Jxz, Jxy) as needed in ObjectRigid-  
Body

### 5.11.2 CLASS InertiaCuboid(RigidBodyInertia) (in module rigidBodyUtilities)

**class description:** create RigidBodyInertia with moment of inertia and mass of a cuboid with density and side lengths sideLengths along local axes 1, 2, 3; inertia w.r.t. center of mass, com=[0,0,0]

– **example:**

```
InertiaCuboid(density=1000, sideLengths=[1, 0.1, 0.1])
```

### 5.11.3 CLASS InertiaRodX(RigidBodyInertia) (in module rigidBodyUtilities)

**class description:** create RigidBodyInertia with moment of inertia and mass of a rod with mass m and length L in local 1-direction (x-direction); inertia w.r.t. center of mass, com=[0,0,0]

### 5.11.4 CLASS InertiaMassPoint(RigidBodyInertia) (in module rigidBodyUtilities)

**class description:** create RigidBodyInertia with moment of inertia and mass of mass point with 'mass'; inertia w.r.t. center of mass, com=[0,0,0]

### 5.11.5 CLASS InertiaSphere(RigidBodyInertia) (in module rigidBodyUtilities)

**class description:** create RigidBodyInertia with moment of inertia and mass of sphere with mass and radius; inertia w.r.t. center of mass, com=[0,0,0]

### 5.11.6 CLASS InertiaHollowSphere(RigidBodyInertia) (in module rigidBodyUtilities)

**class description:** create RigidBodyInertia with moment of inertia and mass of hollow sphere with mass (concentrated at circumference) and radius; inertia w.r.t. center of mass, com=0

### 5.11.7 CLASS InertiaCylinder(RigidBodyInertia) (in module rigidBodyUtilities)

**class description:** create RigidBodyInertia with moment of inertia and mass of cylinder with density, length and outerRadius; axis defines the orientation of the cylinder axis (0=x-axis, 1=y-axis, 2=z-axis); for hollow cylinder use innerRadius != 0; inertia w.r.t. center of mass, com=[0,0,0]

## 5.12 Module: robotics

A library which includes support functions for robotics. The library is built on standard Denavit-Hartenberg Parameters and Homogeneous Transformations (HT) to describe transformations and coordinate systems

Author: Johannes Gerstmayr

Date: 2020-04-14

Example: see ComputeJointHT for the definition of the 'robot' dictionary.

```
def DH2HT (DHparameters)
```

- **function description:** compute homogeneous transformation HT from standard DH-parameters
- 

```
def ComputeJointHT (robot, configuration)
```

- **function description:** compute list of homogeneous transformations HT from base to every joint for given configuration

- **example:**

```
link0={'stdDH':[0,0,0,np.pi/2],
 'mass':20, #not needed!
 'inertia':np.diag([1e-8,0.35,1e-8]), #w.r.t. COM!
 'COM':[0,0,0]}
link1={'stdDH':[0,0,0.4318,0],
 'mass':17.4,
 'inertia':np.diag([0.13,0.524,0.539]), #w.r.t. COM!
 'COM':[-0.3638, 0.006, 0.2275]}
robot={'links':[link0, link1],
 'jointType':[1,1], #1=revolute, 0=prismatic
 'base':{'HT':HT0()},
 'tool':{'HT':HTtranslate([0,0,0.1])},
 'gravity':[0,0,9.81],
 'referenceConfiguration':[0]*2 #reference configuration for bodies; at
 which the robot is built
 }
HTlist = ComputeJointHT(robot, [np.pi/8]*2)
```

---

```
def ComputeCOMHT (robot, HT)
```

- **function description:** compute list of homogeneous transformations HT from base to every COM using HT list from ComputeJointHT
- 

```
def ComputeStaticTorques (robot, HT)
```

- **function description:** compute list joint torques for serial robot under gravity (gravity and mass as given in robot)
-

```
def Jacobian (robot, HT, toolPosition= [], mode= 'all')
```

- **function description:** compute jacobian for translation and rotation at toolPosition using joint HT
- 

```
def SerialRobot2MBS (mbs, robot, jointLoadUserFunctionList, baseMarker, *args, **kwargs)
```

- **function description:**

add items to existing mbs from the robot structure, a baseMarker (can be ground object or body) and the user function list for the joints; there are options that can be passed as args / kwargs, which can contains options as described below. For details, see the python file and `serialRobotTest.py` in `TestModels`

- **input:**

*mbs*: the multibody system, which will be extended

*robot*: the robot model as dictionary, described in function `ComputeJointHT`

*jointLoadUserFunctionList*: a list of user functions for actuation of joints according to a `LoadTorqueVector` `userFunction`, see `serialRobotTest.py` as an example; can be empty list

*baseMarker*: a rigid body marker, at which the robot will be placed (usually ground); note that the local coordinate system of the base must be in accordance with the DH-parameters, i.e., the z-axis must be the first rotation axis. For correction of the base coordinate system, use `rotationMarkerBase`

*rotationMarkerBase*: used in Generic joint between first joint and base; note, that for moving base, the static compensation does not work (base rotation must be updated)

*showCOM*: a list of 3 floats [a,b,c], indicates to show center of mass as rectangular block with size [a,b,c]

*bodyAlpha*: a float value in range [0..1], adds transparency to links if value < 1

*toolGraphicsSize*: list of 3 floats [sx,sy,sz], giving the size of the tool for graphics representation; set sx=0 to disable tool drawing or do not provide this optional variable

*drawLinkSize*: draw parameters for links as list of 3 floats [r,w,0], r=radius of joint, w=radius of link, set r=0 to disable link drawing

*rotationMarkerBase*: add a numpy 3x3 matrix for rotation of the base, in order that the robot can be attached to any rotated base marker; the `rotationMarkerBase` is according to the definition in `GenericJoint`

- **output:** the function returns a dictionary containing information on nodes, bodies, joints, markers, torques, for every joint

## 5.13 Module: roboticsSpecial

Library for additional support functions for robotics; The library is built on standard Denavit-Hartenberg Parameters and Homogeneous Transformations (HT) to describe transformations and coordinate systems

Author: Martin Sereinig

Date: 2020-12-08

```
def VelocityManipulability (robot, HT, mode)
```

- **function description:** compute velocity manipulability measure for given pose (homogenous transformation)
  - **input:**
    - robot*: robot structure
    - HT*: actual pose as hoogenous transformaton matrix
    - mode*: rotational or translational part of the movement
  - **output:** velocity manipulability measure as scalar value, defined as  $\sqrt{\det(JJ^T)}$
  - **notes:** compute velocity dependent manipulability definded by Yoshikawa, see [37]
- 

```
def ForceManipulability (robot, HT, mode)
```

- **function description:** compute force manipulability measure for given pose (homogenous transformation)
  - **input:**
    - robot*: robot structure
    - HT*: actual pose as hoogenous transformaton matrix
    - mode*: rotational or translational part of the movement
  - **output:** force manipulability measure as scalar value, defined as  $\sqrt{(\det(JJ^T))^{-1}}$
  - **notes:** compute force dependent manipulability definded by Yoshikawa, see [37]
- 

```
def StiffnessManipulability (robot, JointStiffness, HT, mode)
```

- **function description:** compute cartesian stiffness measure for given pose (homogenous transformation)
- **input:**
  - robot*: robot structure
  - JointStiffness*: joint stiffness matrix
  - HT*: actual pose as hoogenous transformaton matrix
  - mode*: rotational or translational part of the movement
- **output:**
  - stiffness manipulability measure as scalar value, defined as minimum Eigenvaluae of the Cartesian stiffness matrix
  - Cartesian stiffness matrix
- **notes:**

- **status:** this function is **currently under development** and under testing!
- 

```
def JointJacobian (robot, HT)
```

- **function description:** compute joint jacobian for each frame for given pose (homogenous transformation)
  - **input:**
    - robot:* robot structure
    - HT:* actual pose as hoogenous transformaton matrix
  - **output:** Link(body)-Jacobi matrix  $JJ_i = [{}^i J_{Ri}, {}^i J_{Ti}]$  for each link  $i$ , seperated in rotational ( $J_R$ ) and translational ( $J_T$ ) part of Jacobian matrix located in the  $i^{th}$  coordiante system, see [36]
  - **notes:** runs over number of HTs given in HT (may be less than number of links), caclulations in link coordinate system located at the end of each link regarding Standard Denavit-Hartenberg parameters, see [9]
- 

```
def MassMatrix (robot, HT, jointJacobian)
```

- **function description:** compute mass matrix from jointJacobian
  - **input:**
    - robot:* robot structure
    - HT:* actual pose as hoogenous transformaton matrix
    - jointJacobian:* provide list of jacobians as provided by function JointJacobian(...)
  - **output:** MM: Mass matrix
  - **notes:**
    - Mass Matrix calculation calculated in joint coordinates regarding (std) DH parameter:*
    - \*\* Dynamic equations in minimal coordinates as described in Mehrkoerpersysteme by Woernle, [36], p206, eq6.90.
    - \*\* Caclulations in link coordinate system at the end of each link
- 

```
def DynamicManipulability (robot, HT, MassMatrix, Tmax, mode)
```

- **function description:** compute dynamic manipulability measure for given pose (homogenous transformation)
- **input:**
  - robot:* robot structure

*HT*: actual pose as homogeneous transformaton matrix

*Tmax*: maximum joint torques

*mode*: rotational or translational part of the movement

*MassMatrix*: Mass (inertia) Maxtrix provided by the function MassMatrix

– **output:**

dynamic manipulability measure as scalar value, defined as minimum Eigenvalue of the dynamic manipulability matrix *N*

dynamic manipulability matrix

– **notes:** acceleration dependent manipulability definded by Chiacchio, see [7], eq.32. The eigenvectors and eigenvalues of *N* ([eigenvec eigenval]=eig(*N*))gives the direction and value of minimal and maximal accaleration )

– **status:** this function is **currently under development** and under testing!

## 5.14 Module: signal

The signal library supports processing of signals for import (e.g. measurement data) and for filtering result data.

Date: 2020-12-10

Notes: This module is still under construction and should be used with care!

def **FilterSensorOutput** (*signal*, *filterWindow*= 5, *polyOrder*= 3, *derivative*= 0, *centralDifferentiate*= True)

– **function description:** filter output of sensors (using numpy savgol filter) as well as numerical differentiation to compute derivative of signal

– **input:**

*signal*: numpy array (2D array with column-wise storage of signals, as exported by EXUDYN position, displacement, etc. sensors); first column = time, other columns = signals to operate on; note that it is assumed, that time devided in almost constant steps!

*derivative*: 0=no derivative, 1=first derivative, 2=second derivative, etc. (>2 only possible with filter)

*polyOrder*: order of polynomial for interpolation filtering

*filterWindow*: if zero: produces unfiltered derivative; if positive, must be ODD integer 1,3,5,... and > polyOrder; filterWindow determines the length of the filter window (e.g., to get rid of noise)

*centralDifferentiate*: if True, it uses a central differentiation for first order, unfiltered derivatives; leads to less phase shift of signal!

– **output:** numpy array containing same columns, but with filtered signal and according derivatives

---

def **FilterSignal** (*signal*, *samplingRate*= -1, *filterWindow*= 5, *polyOrder*= 3, *derivative*= 0, *centralDifferentiate*= True)

- **function description:** filter 1D signal (using numpy savgol filter) as well as numerical differentiation to compute derivative of signal
  - **input:**
    - signal:* 1D numpy array
    - samplingRate:* (time increment) of signal values, needed for derivatives
    - derivative:* 0=no derivative, 1=first derivative, 2=second derivative, etc. (>2 only possible with filter)
    - polyOrder:* order of polynomial for interpolation filtering
    - filterWindow:* if zero: produces unfiltered derivative; if positive, must be ODD integer 1,3,5,... and > polyOrder; filterWindow determines the length of the filter window (e.g., to get rid of noise)
    - centralDifferentiate:* if True, it uses a central differentiation for first order, unfiltered derivatives; leads to less phase shift of signal!
  - **output:** numpy array containing same columns, but with filtered signal and according derivatives
- 

```
def ComputeFFT (time, data)
```

- **function description:** computes fast-fourier-transform (FFT) resulting in frequency, magnitude and phase of signal data using numpy.fft of numpy
- **input:**
  - time ... time vector in SECONDS in numpy format, having constant sampling rate (not checked!)
  - data ... data vector in numpy format
- **output:**
  - frequency ... frequency vector (Hz, if time is in SECONDS)
  - magnitude ... magnitude vector
  - phase ... phase vector (in radiant)
- **author:** Stefan Holzinger
- **date:** 02.04.2020

## 5.15 Module: solver

The solver module provides interfaces to static, dynamic and eigenvalue solvers. Most of the solvers are implemented inside the C++ core.

Author: Johannes Gerstmayr

Date: 2020-12-02

Notes: Solver functions are included directly in exudyn and can be used with exu.SolveStatic(...)

```
def SolverErrorMessage (solver, mbs, isStatic= False, showCausingObjects= True, showCausingNodes= True,
showHints= True)
```

- **function description:** helper function for unique error and helper messages
- 

```
def SolveStatic (mbs, simulationSettings= exudyn.SimulationSettings(), updateInitialValues= False, storeSolver= True, showHints= False, showCausingItems= True,)
```

- **function description:** solves the static mbs problem using simulationSettings; check theDoc.pdf for Main-SolverStatic for further details of the static solver

- **input:**

*mbs*: the MainSystem containing the assembled system; note that mbs may be changed upon several runs of this function

*simulationSettings*: specific simulation settings used for computation of jacobian (e.g., sparse mode in static solver enables sparse computation)

*updateInitialValues*: if True, the results are written to initial values, such at a consecutive simulation uses the results of this simulation as the initial values of the next simulation

*storeSolver*: if True, the staticSolver object is stored in the mbs.sys dictionary as mbs.sys['staticSolver']

- **output:** returns True, if successful, False if fails; if storeSolver = True, mbs.sys contains staticSolver, which allows to investigate solver problems (check theDoc.pdf section [Section 8.3](#) and the items described in [Section 8.3.6](#))

- **example:**

```
import exudyn as exu
from exudyn.itemInterface import *
SC = exu.SystemContainer()
mbs = SC.AddSystem()
#create simple system:
ground = mbs.AddObject(ObjectGround())
mbs.AddNode(NodePoint())
body = mbs.AddObject(MassPoint(physicsMass=1, nodeNumber=0))
m0 = mbs.AddMarker(MarkerBodyPosition(bodyNumber=ground))
m1 = mbs.AddMarker(MarkerBodyPosition(bodyNumber=body))
mbs.AddObject(CartesianSpringDamper(markerNumbers=[m0,m1], stiffness
=[100,100,100]))
mbs.AddLoad(LoadForceVector(markerNumber=m1, loadVector=[10,10,10]))
mbs.Assemble()
sims = exu.SimulationSettings()
sims.timeIntegration.endTime = 10
success = exu.SolveStatic(mbs, sims, storeSolver = True)
print("success =", success)
print("iterations = ", mbs.sys['staticSolver'].it)
print("pos=", mbs.GetObjectOutputBody(body, localPosition=[0,0,0],
variableType=exu.OutputVariableType.Position))
```

---

```
def SolveDynamic (mbs, simulationSettings= exodyn.SimulationSettings(), solverType= exodyn.DynamicSolverType.GeneralizedAlpha, updateInitialValues= False, storeSolver= True, showHints= False, showCausingItems= True,)
```

- **function description:** solves the dynamic mbs problem using simulationSettings and solver type; check theDoc.pdf for MainSolverImplicitSecondOrder for further details of the dynamic solver

- **input:**

*mbs*: the MainSystem containing the assembled system; note that mbs may be changed upon several runs of this function

*simulationSettings*: specific simulation settings

*solverType*: use exodyn.DynamicSolverType to set specific solver (default=generalized alpha)

*updateInitialValues*: if True, the results are written to initial values, such at a consecutive simulation uses the results of this simulation as the initial values of the next simulation

*storeSolver*: if True, the staticSolver object is stored in the mbs.sys dictionary as mbs.sys['staticSolver']

*showHints*: show additional hints, if solver fails

*showCausingItems*: if linear solver fails, this option helps to identify objects, etc. which are related to a singularity in the linearized system matrix

- **output:** returns True, if successful, False if fails; if storeSolver = True, mbs.sys contains staticSolver, which allows to investigate solver problems (check theDoc.pdf section [Section 8.3](#) and the items described in [Section 8.3.6](#))

- **example:**

```
import exodyn as exu
from exodyn.itemInterface import *
SC = exu.SystemContainer()
mbs = SC.AddSystem()
#create simple system:
ground = mbs.AddObject(ObjectGround())
mbs.AddNode(NodePoint())
body = mbs.AddObject(MassPoint(physicsMass=1, nodeNumber=0))
m0 = mbs.AddMarker(MarkerBodyPosition(bodyNumber=ground))
m1 = mbs.AddMarker(MarkerBodyPosition(bodyNumber=body))
mbs.AddObject(CartesianSpringDamper(markerNumbers=[m0,m1], stiffness
 =[100,100,100]))
mbs.AddLoad(LoadForceVector(markerNumber=m1, loadVector=[10,10,10]))
mbs.Assemble()
sims = exu.SimulationSettings()
sims.timeIntegration.endTime = 10
success = exu.SolveDynamic(mbs, sims, storeSolver = True)
print("success =", success)
print("iterations = ", mbs.sys['dynamicSolver'].it)
print("pos=", mbs.GetObjectOutputBody(body, localPosition=[0,0,0],
 variableType=exu.OutputVariableType.Position))
```

```
def ComputeODE2Eigenvalues (mbs, simulationSettings= exudyn.SimulationSettings(), useSparseSolver= False, numberOfEigenvalues= 0, setInitialValues= True, convert2Frequencies= False)
```

- **function description:** compute eigenvalues for unconstrained ODE2 part of mbs, not considering the effects of algebraic constraints; the computation is done for the initial values of the mbs, independently of previous computations. If you would like to use the current state for the eigenvalue computation, you need to copy the current state to the initial state (using GetSystemState,SetSystemState, see [Section 4.4.1](#)).

- **input:**

*mbs*: the MainSystem containing the assembled system

*simulationSettings*: specific simulation settings used for computation of jacobian (e.g., sparse mode in static solver enables sparse computation)

*useSparseSolver*: if False (only for small systems), all eigenvalues are computed in dense mode (slow for large systems!); if True, only the *numberOfEigenvalues* are computed (*numberOfEigenvalues* must be set!); Currently, the matrices are exported only in DENSE MODE from mbs! NOTE that the sparsesolver accuracy is much less than the dense solver

*numberOfEigenvalues*: number of eigenvalues and eivenvectors to be computed; if *numberOfEigenvalues*==0, all eigenvalues will be computed (may be impossible for larger problems!)

*convert2Frequencies*: if True, the eigen values are converted into frequencies (Hz) and the output is [eigenFrequencies, eigenVectors]

- **output:** [eigenValues, eigenVectors]; eigenValues being a numpy array of eigen values ( $\omega_i^2$ , being the squared eigen frequencies in ( $\omega_i$  in rad/s!)), eigenVectors a numpy array containing the eigenvectors in every column

- **example:**

```
#take any example from the Examples or TestModels folder, e.g., '
 cartesianSpringDamper.py' and run it
#then execute the following commands in the console (or add it to the file):
[values, vectors] = exu.ComputeODE2Eigenvalues(mbs)
print("eigenvalues=", values)
#=>values contains the eigenvalues of the ODE2 part of the system in the
 current configuration
```

---

```
def CheckSolverInfoStatistics (solverName, infoStat, numberOfEvaluations)
```

- **function description:**

helper function for solvers to check e.g. if high number of memory allocations happened during simulation

This can happen, if large amount of sensors are attached and output is written in every time step

- **input:** stat=exudyn.InfoStat() from previous step, *numberOfEvaluations* is a counter which is proportional to number of RHS evaluations in method

## 5.16 Module: utilities

Basic support functions for simpler creation of Exudyn models. Advanced functions for loading and animating solutions and for drawing a graph of the mbs system. This library requires numpy (as well as time and copy)

Author: Johannes Gerstmayr

Date: 2019-07-26 (created)

```
def PlotLineCode (index)
```

- **function description:** helper functions for matplotlib, returns a list of 28 line codes to be used in plot, e.g. 'r-' for red solid line
  - **input:** index in range(0:28)
  - **output:** a color and line style code for matplotlib plot
- 

```
def FillInSubMatrix (subMatrix, destinationMatrix, destRow, destColumn)
```

- **function description:** fill submatrix into given destinationMatrix; all matrices must be numpy arrays
  - **input:**
    - subMatrix*: input matrix, which is filled into destinationMatrix
    - destinationMatrix*: the subMatrix is entered here
    - destRow*: row destination of subMatrix
    - destColumn*: column destination of subMatrix
  - **output:** destinationMatrix is changed after function call
  - **notes:** may be erased in future!
- 

```
def SweepSin (t, t1, f0, f1)
```

- **function description:** compute sin sweep at given time t
  - **input:**
    - t*: evaluate of sweep at time t
    - t1*: end time of sweep frequency range
    - f0*: start of frequency interval [f0,f1] in Hz
    - f1*: end of frequency interval [f0,f1] in Hz
  - **output:** evaluation of sin sweep (in range -1..+1)
-

```
def SweepCos (t, t1, f0, f1)
```

- **function description:** compute cos sweep at given time t
  - **input:**
    - t*: evaluate of sweep at time t
    - t1*: end time of sweep frequency range
    - f0*: start of frequency interval [f0,f1] in Hz
    - f1*: end of frequency interval [f0,f1] in Hz
  - **output:** evaluation of cos sweep (in range -1..+1)
- 

```
def FrequencySweep (t, t1, f0, f1)
```

- **function description:** frequency according to given sweep functions SweepSin, SweepCos
  - **input:**
    - t*: evaluate of frequency at time t
    - t1*: end time of sweep frequency range
    - f0*: start of frequency interval [f0,f1] in Hz
    - f1*: end of frequency interval [f0,f1] in Hz
  - **output:** frequency in Hz
- 

```
def SmoothStep (x, x0, x1, value0, value1)
```

- **function description:** step function with smooth transition from value0 to value1; transition is computed with cos function
  - **input:**
    - x*: argument at which function is evaluated
    - x0*: start of step ( $f(x) = \text{value0}$ )
    - x1*: end of step ( $f(x) = \text{value1}$ )
    - value0*: value before smooth step
    - value1*: value at end of smooth step
  - **output:** returns  $f(x)$
- 

```
def IndexFromValue (data, value, tolerance= 1e-7)
```

- **function description:** get index from value in given data vector (numpy array); usually used to get specific index of time vector
  - **input:**
    - data:* containing (almost) equidistant values of time
    - value:* e.g., time to be found in data
    - tolerance:* tolerance, which is accepted (default: tolerance=1e-7)
  - **output:** index
- 

```
def RoundMatrix (matrix, threshold= 1e-14)
```

- **function description:** set all entries in matrix to zero which are smaller than given threshold; operates directly on matrix
  - **input:** matrix as np.array, threshold as positive value
  - **output:** changes matrix
- 

```
def ComputeSkewMatrix (v)
```

- **function description:** compute ( $3 \times 3^*n$ ) skew matrix from ( $3^*n$ ) vector; used for ObjectFFRF and CMS implementation
  - **input:** a vector v in np.array format, containing  $3^*n$  components
  - **output:** ( $3 \times 3^*n$ ) skew matrix in np.array format
- 

```
def CheckInputVector (vector, length= -1)
```

- **function description:** check if input is list or array with according length; if length== -1, the length is not checked; raises Exception if the check fails
  - **input:**
    - vector:* a vector in np.array or list format
    - length:* desired length of vector; if length=-1, it is ignored
  - **output:** None
- 

```
def CheckInputIndexArray (indexArray, length= -1)
```

- **function description:** check if input is list or array with according length and positive indices; if length==1, the length is not checked; raises Exception if the check fails
  - **input:**
    - indexArray*: a vector in np.array or list format
    - length*: desired length of vector; if length=-1, it is ignored
  - **output:** None
- 

```
def FindObjectIndex (i, globalVariables)
```

- **function description:** simple function to find object index i within the local or global scope of variables
- **input:** i, the integer object number and globalVariables=globals()
- **example:**

```
FindObjectIndex(2, locals()) #usually sufficient
FindObjectIndex(2, globals()) #wider search
```

---

```
def FindNodeIndex (i, globalVariables)
```

- **function description:** simple function to find node index i within the local or global scope of variables
- **input:** i, the integer node number and globalVariables=globals()
- **example:**

```
FindObjectIndex(2, locals()) #usually sufficient
FindObjectIndex(2, globals()) #wider search
```

---

```
def ShowOnlyObjects (mbs, objectNumbers= [])
```

- **function description:** function to hide all objects in mbs except for those listed in objectNumbers
  - **input:**
    - mbs*: mbs containing object
    - objectNumbers*: integer object number or list of object numbers to be shown; if empty list [], then all objects are shown
    - showOthers*: if True, then all other objects are shown again
  - **output:** changes all colors in mbs, which is NOT reversible
- 

```
def HighlightItem (SC, mbs, itemNumber, itemType= exudyn.ItemType.Object, showNumbers= True)
```

- **function description:** to highlight a certain item with number itemNumber; set itemNumber to -1 to show again all objects
- 

```
def LoadSolutionFile (fileName)
```

- **function description:** read coordinates solution file (exported during static or dynamic simulation with option exu.SimulationSettings().solutionSettings.coordinatesSolutionFileName='...') into dictionary:
  - **input:** fileName: string containing directory and filename of stored coordinatesSolutionFile
  - **output:** dictionary with 'data': the matrix of stored solution vectors, 'columnsExported': a list with binary values showing the exported columns [nODE2, nVel2, nAcc2, nODE1, nVel1, nAlgebraic, nData], 'nColumns': the number of data columns and 'nRows': the number of data rows
- 

```
def SetSolutionState (exu, mbs, solution, row, configuration)
```

- **function description:** load selected row of solution dictionary (previously loaded with LoadSolutionFile) into specific state
- 

```
def SetVisualizationState (exu, mbs, solution, row)
```

- **function description:** load selected row of solution dictionary into visualization state and redraw
  - **input:**
    - exu: the exudyn library
    - mbs: the system, where the state is applied to
    - solution: solution dictionary previously loaded with LoadSolutionFile
    - row: the according row of the solution file which is visualized
  - **output:** renders the scene in mbs and changes the visualization state in mbs
- 

```
def AnimateSolution (exu, SC, mbs, solution, rowIncrement= 1, timeout= 0.04, createImages= False, runLoop= False)
```

- **function description:** consecutively load the rows of a solution file and visualize the result
- **input:**
  - exu: the exudyn library

*SC*: the system container, where the mbs lives in  
*mbs*: the system used for animation  
*solution*: solution dictionary previously loaded with LoadSolutionFile; will be played from first to last row  
*rowIncrement*: can be set larger than 1 in order to skip solution frames: e.g. *rowIncrement*=10 visualizes every 10th row (frame)  
*timeout*: in seconds is used between frames in order to limit the speed of animation; e.g. use *timeout*=0.04 to achieve approximately 25 frames per second  
*createImages*: creates consecutively images from the animation, which can be converted into an animation  
*runLoop*: if True, the animation is played in a loop until 'q' is pressed in render window

- **output:** renders the scene in mbs and changes the visualization state in mbs continuously

---

```
def DrawSystemGraph (mbs, showLoads= True, showSensors= True, useItemNames= False, useItemTypes= False)
```

- **function description:** helper function which draws system graph of a MainSystem (*mbs*); several options let adjust the appearance of the graph
- **input:**
  - showLoads*: toggle appearance of loads in mbs
  - showSensors*: toggle appearance of sensors in mbs
  - useItemNames*: if True, object names are shown instead of basic object types (Node, Load, ...)
  - useItemTypes*: if True, object type names (ObjectMassPoint, ...) are shown instead of basic object types (Node, Load, ...)
- **output:** nothing

---

```
def GenerateStraightLineANCFCable2D (mbs, positionOfNode0, positionOfNode1, numberOfElements,

cableTemplate, massProportionalLoad= [0,0,0], fixedConstraintsNode0= [0,0,0,0], fixedConstraintsNode1= [0,0,0,0],

vALE= 0, ConstrainAleCoordinate= True)
```

- **function description:** generate cable elements along straight line with certain discretization
- **input:**
  - mbs*: the system where ANCF cables are added
  - positionOfNode0*: 3D position (list or np.array) for starting point of line
  - positionOfNode1*: 3D position (list or np.array) for end point of line
  - numberOfElements*: for discretization of line
  - cableTemplate*: a ObjectANCFCable2D object, containing the desired cable properties; cable length and node numbers are set automatically

*massProportionalLoad*: a 3D list or np.array, containing the gravity vector or zero  
*fixedConstraintsNode0*: a list of 4 binary values, indicating the coordinate constraints on the first node (x,y-position and x,y-slope)  
*fixedConstraintsNode1*: a list of 4 binary values, indicating the coordinate constraints on the last node (x,y-position and x,y-slope)  
*vALE*: used for ObjectALEANCFCable2D objects

– **output**: returns a list [cableNodeList, cableObjectList, loadList, cableNodePositionList, cableCoordinateConstraintList]

– **example**:

see Examples/ANCF\_cantilever\_test.py

---

```
def GenerateSlidingJoint (mbs, cableObjectList, markerBodyPositionOfSlidingBody,
localMarkerIndexOfStartCable= 0, slidingCoordinateStartPosition= 0)
```

– **function description**: generate a sliding joint from a list of cables, marker to a sliding body, etc.

---

```
def GenerateAleSlidingJoint (mbs, cableObjectList, markerBodyPositionOfSlidingBody, AleNode,
localMarkerIndexOfStartCable= 0, AleSlidingOffset= 0, activeConnector= True, penaltyStiffness= 0)
```

– **function description**: generate an ALE sliding joint from a list of cables, marker to a sliding body, etc.

# Chapter 6

## Theory and formulations

This section includes some material on notation, formulations and general theoretical backgrounds for EXUDYN. Note that the formulation of nodes, objects, markers, etc. is presented right at the subsection of each object, see [Section 7](#). Furthermore, overview on the system assembly and system equations of motion is given in [Section 10](#), solvers are described in [Section 11](#).

### 6.1 Notation

#### 6.1.1 Common types in item descriptions

There are certain types, which are heavily used in the description of items:

- `float` ... a single-precision floating point number (note: in Python, 'float' is used also for double precision numbers; in EXUDYN, internally floats are single precision numbers especially for graphics objects and OpenGL)
- `Real` ... a double-precision floating point number (note: in Python this is also of type 'float')
- `UReal` ... same as `Real`, but may not be negative
- `PReal` ... same as `Real`, but must be positive, non-zero (e.g., step size may never be zero)
- `Index` ... deprecated, represents unsined integer, `UInt`
- `Int` ... a (signed) integer number, which converts to 'int' in Python, 'int' in C++
- `UInt` ... an unsigned integer number, which converts to 'int' in Python
- `PInt` ... an positive integer number ( $> 0$ ), which converts to 'int' in Python
- `NodeIndex`, `MarkerIndex`, ... ... a special (non-negative) integer type to represent indices of nodes, markers, ...; specifically, an unintentional conversion from one index type to the other is not possible (e.g., to convert `NodeIndex` to `MarkerIndex`)
- `String` ... a string
- `ArrayIndex` ... a list of integer numbers (either list or in some cases numpy arrays may be allowed)
- `ArrayNodeIndex` ... a list of node indices
- `Bool` ... a boolean parameter: either `True` or `False` ('bool' in Python)
- `VObjectMassPoint`, `VObjectRigidBody`, `VObjectGround`, etc. ... represents the visualization object of the underlying object; 'V' is put in front of object name
- `BodyGraphicsData` ... see [Section 9.3](#)
- `Vector2D` ... a list or numpy array of 2 real numbers
- `Vector3D` ... a list or numpy array of 3 real numbers

- `Vector'X'D` ... a list or numpy array of 'X' real numbers
- `Float4` ... a list of 4 float numbers
- `Vector` ... a list or numpy array of real numbers (length given by according object)
- `NumpyVector` ... a 1D numpy array with real numbers (size given by according object); similar as `Vector`, but not accepting list
- `Matrix3D` ... a list of lists or numpy array with  $3 \times 3$  real numbers
- `NumpyMatrix` ... a 2D numpy array (matrix) with real numbers (size given by according object)
- `NumpyMatrixI` ... a 2D numpy array (matrix) with integer numbers (size given by according object)

### 6.1.2 States and coordinate attributes

The following subscripts are used to define configurations of a quantity, e.g., for a vector of displacement coordinates  $\mathbf{q}$ :

- $\mathbf{q}_{\text{config}}$  ...  $\mathbf{q}$  in any configuration
- $\mathbf{q}_{\text{ref}}$  ...  $\mathbf{q}$  in reference configuration, e.g., reference coordinates:  $\mathbf{c}_{\text{ref}}$
- $\mathbf{q}_{\text{ini}}$  ...  $\mathbf{q}$  in initial configuration, e.g., initial displacements:  $\mathbf{u}_{\text{ini}}$
- $\mathbf{q}_{\text{cur}}$  ...  $\mathbf{q}$  in current configuration
- $\mathbf{q}_{\text{vis}}$  ...  $\mathbf{q}$  in visualization configuration
- $\mathbf{q}_{\text{start of step}}$  ...  $\mathbf{q}$  in start of step configuration

As written in the introduction, the coordinates are attributed to certain types of equations and therefore, the following attributes are used (usually as subscript, e.g.,  $\mathbf{q}_{\text{ODE2}}$ ):

- ODE2 ... second order differential equations coordinates
- ODE1 ... first order differential equations coordinates; CURRENTLY NOT AVAILABLE
- AE ... algebraic equations coordinates
- Data ... data coordinates (history variables)

Time is usually defined as 'time' or  $t$ . The cross product or vector product ' $\times$ ' is often replaced by the skew symmetric matrix using the tilde ' $\sim$ ' symbol,

$$\mathbf{a} \times \mathbf{b} = \tilde{\mathbf{a}} \mathbf{b} = -\tilde{\mathbf{b}} \mathbf{a} . \quad (6.1)$$

For the length of a vector we often use the abbreviation

$$\|\mathbf{a}\| = \sqrt{\mathbf{a}^T \mathbf{a}} . \quad (6.2)$$

### 6.1.3 Symbols and abbreviations in equations

The following table contains the common notation:

| python name (or description)    | symbol                                | description                                                                                                                                                                                                             |
|---------------------------------|---------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| displacement coordinates (ODE2) | $\mathbf{q} = [q_0, \dots, q_n]^T$    | vector of $n$ displacement based coordinates in any configuration; used for second order differential equations                                                                                                         |
| rotation coordinates (ODE2)     | $\psi = [\psi_0, \dots, \psi_\eta]^T$ | vector of $\eta$ rotation based coordinates in any configuration; these coordinates are added to reference rotation parameters to provide the current rotation parameters; used for second order differential equations |

| coordinates (ODE1)                         | $\mathbf{y} = [y_0, \dots, y_n]^T$                                                                                                   | vector of $n$ coordinates for first order ordinary differential equations (ODE1) in any configuration                                             |
|--------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------|
| algebraic coordinates                      | $\mathbf{z} = [z_0, \dots, z_m]^T$                                                                                                   | vector of $m$ algebraic coordinates if not Lagrange multipliers in any configuration                                                              |
| Lagrange multipliers                       | $\boldsymbol{\lambda} = [\lambda_0, \dots, \lambda_m]^T$                                                                             | vector of $m$ Lagrange multipliers (=algebraic coordinates) in any configuration                                                                  |
| data coordinates                           | $\mathbf{x} = [x_0, \dots, x_l]^T$                                                                                                   | vector of $l$ data coordinates in any configuration                                                                                               |
| python name: OutputVariable                | symbol                                                                                                                               | description                                                                                                                                       |
| Coordinate                                 | $\mathbf{c} = [c_0, \dots, c_n]^T$                                                                                                   | coordinate vector with $n$ generalized coordinates $c_i$ in any configuration; the letter $c$ is used both for ODE1 and ODE2 coordinates          |
| Coordinate_t                               | $\dot{\mathbf{c}} = [c_0, \dots, c_n]^T$                                                                                             | time derivative of coordinate vector                                                                                                              |
| Displacement                               | ${}^0\mathbf{u} = [u_0, u_1, u_2]^T$                                                                                                 | global displacement vector with 3 displacement coordinates $u_i$ in any configuration; in 1D or 2D objects, some of there coordinates may be zero |
| Rotation                                   | $\boldsymbol{\theta} = [\theta_0, \dots, \theta_n]^T$                                                                                | vector of <b>rotation parameters</b> (e.g., Euler parameters, Tait Bryan angles, ...) with $n$ coordinates $\theta_i$ in any configuration        |
| RotationMatrix                             | ${}^{0b}\mathbf{A} = \begin{bmatrix} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{bmatrix}$ | a 3D rotation matrix, which transforms local (e.g., body $b$ ) to global coordinates (0): ${}^0\mathbf{x} = {}^{0b}\mathbf{A} {}^b\mathbf{x}$     |
| Position                                   | ${}^0\mathbf{p} = [p_0, p_1, p_2]^T$                                                                                                 | global position vector with 3 position coordinates $p_i$ in any configuration                                                                     |
| Velocity                                   | ${}^0\mathbf{v} = {}^0\dot{\mathbf{u}} = [v_0, v_1, v_2]^T$                                                                          | global velocity vector with 3 displacement coordinates $v_i$ in any configuration                                                                 |
| AngularVelocity                            | ${}^0\boldsymbol{\omega} = [\omega_0, \dots, \omega_2]^T$                                                                            | global angular velocity vector with 3 coordinates $\omega_i$ in any configuration                                                                 |
| Acceleration                               | ${}^0\mathbf{a} = {}^0\ddot{\mathbf{u}} = [a_0, a_1, a_2]^T$                                                                         | global acceleration vector with 3 displacement coordinates $a_i$ in any configuration                                                             |
| AngularAcceleration                        | ${}^0\boldsymbol{\alpha} = {}^0\dot{\boldsymbol{\omega}} = [\alpha_0, \dots, \alpha_2]^T$                                            | global angular acceleration vector with 3 coordinates $\alpha_i$ in any configuration                                                             |
| VelocityLocal                              | ${}^b\mathbf{v} = [v_0, v_1, v_2]^T$                                                                                                 | local (body-fixed) velocity vector with 3 displacement coordinates $v_i$ in any configuration                                                     |
| AngularVelocityLocal                       | ${}^b\boldsymbol{\omega} = [\omega_0, \dots, \omega_2]^T$                                                                            | local (body-fixed) angular velocity vector with 3 coordinates $\omega_i$ in any configuration                                                     |
| Force                                      | ${}^0\mathbf{f} = [f_0, \dots, f_2]^T$                                                                                               | vector of 3 force components in global coordinates                                                                                                |
| Torque                                     | ${}^0\boldsymbol{\tau} = [\tau_0, \dots, \tau_2]^T$                                                                                  | vector of 3 torque components in global coordinates                                                                                               |
| python name: input to nodes, markers, etc. | symbol                                                                                                                               | description                                                                                                                                       |
| referenceCoordinates                       | $\mathbf{c}_{\text{ref}} = [c_0, \dots, c_n]_{\text{ref}}^T = [c_{\text{Ref},0}, \dots, c_{\text{Ref},n}]_{\text{ref}}^T$            | $n$ coordinates of reference configuration<br>(can usually be set at initialization of nodes)                                                     |

|                    |                                                     |                                                                                                                                              |
|--------------------|-----------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------|
| initialCoordinates | $\mathbf{c}_{\text{ini}}$                           | initial coordinates with generalized or mixed displacement/rotation quantities (can usually be set at initialization of nodes)               |
| localPosition      | ${}^b\mathbf{p} = [{}^b p_0, {}^b p_1, {}^b p_2]^T$ | local (body-fixed) position vector with 3 position coordinates $p_i$ in any configuration; used for local position of markers, sensors, etc. |

### 6.1.4 Reference and current coordinates

An important fact on the coordinates is upon the splitting of quantities (e.g. position, rotation parameters, etc.) into reference and current (initial/visualization/...) coordinates. For the current position of a point node we have, e.g.,

$$\mathbf{p}_{\text{cur}} = \mathbf{p}_{\text{ref}} + \mathbf{u}_{\text{cur}} \quad (6.3)$$

The same holds, e.g., for rotation parameters,

$$\theta_{\text{cur}} = \theta_{\text{ref}} + \psi_{\text{cur}} \quad (6.4)$$

### 6.1.5 Coordinate Systems

The left indices provide information about the coordinate system, e.g.,

$${}^0\mathbf{u} \quad (6.5)$$

is the displacement vector in the global (inertial) coordinate system 0, while

$${}^{m1}\mathbf{u} \quad (6.6)$$

represents the displacement vector in marker 1 ( $m1$ ) coordinates. Typical coordinate systems:

- ${}^0\mathbf{u}$  ... global coordinates
- ${}^b\mathbf{u}$  ... body-fixed, local coordinates
- ${}^{m0}\mathbf{u}$  ... local coordinates of (the body or node of) marker  $m0$
- ${}^{m1}\mathbf{u}$  ... local coordinates of (the body or node of) marker  $m1$

To transform the local coordinates  ${}^{m0}\mathbf{u}$  of marker 0 into global coordinates  ${}^0\mathbf{x}$ , we use

$${}^0\mathbf{u} = {}^{0,m0}\mathbf{A} {}^{m0}\mathbf{u} \quad (6.7)$$

in which  ${}^{0,m0}\mathbf{A}$  is the transformation matrix of (the body or node of) the underlying marker 0.

## 6.2 Model order reduction and component mode synthesis

This section describes the process how to create general flexible multibody system models using the floating frame of reference formulation with model order reduction. The according object `ObjectFFRFreducedOrder` is described in [Section 7.2.11](#).

### 6.2.1 Eigenmodes

This section will describe the computation of eigenmodes using FEMinterface.

The FEMinterface in the module FEM has various functionality to import finite element meshes from finite element software. We create a FEMinterface by means of

```
fem = FEMinterface()
```

which allows us to use the variable `fem` from now.

Meshes can be imported from NETGEN/NGsolve ([Section 5.3.8](#)), Abaqus (see [Section 5.3.8](#) and other sections related to ABAQUS), ANSYS (see [Section 5.3.8](#) and other sections related to ANSYS). The import procedure, which can also be done manually, needs to include `massMatrix M` and `stiffnessMatrix K` from any finite element model. Note that many functions are based on the requirement that nodes are 3D displacement-based nodes, without rotation or other coordinates.

For any functionality with `ObjectFFRFreducedOrder` and for the computation of Hurty-Craig-Bampton modes as described in the next section, `nodes` are required. Finally, `elements` need to be included for visualization, and a surface needs to be reconstructed from the element connectivity, which is available for tetrahedral and hexahedral elements for most import functions.

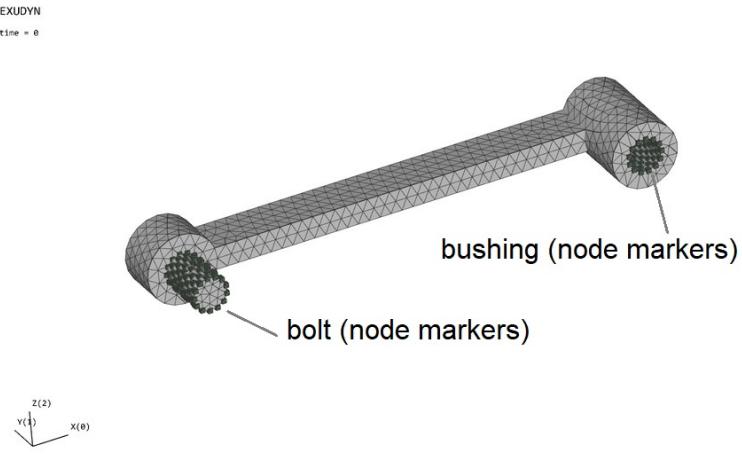


Figure 6.1: Test model and mesh for hinge created with Netgen (linear tetrahedral elements).

As an example, we consider a part denoted as 'hinge' in the following, see Fig. 6.1. The test example can be found in `Examples/NGsolveCMSTutorial.py` with lots of additional features.

After import of mass and stiffness matrix, eigenmodes and eigenfrequencies can be computed using `fem.ComputeEigenFrequencies(...)`, which computes the quantities `fem.modeBasis` and `fem.eigenValues`. The eigenvalues in Hz can be retrieved also with the function `fem.GetEigenFrequenciesHz()`. The function `fem.ComputeEigenFrequencies(...)` is available for dense and sparse matrices, and uses `scipy.linalg` to compute eigenvalues of the linear, undamped mechanical system

$$\mathbf{M}\ddot{\mathbf{q}}(t) + \mathbf{K}\dot{\mathbf{q}}(t) = \mathbf{f}(t) . \quad (6.8)$$

Here, the total number of coordinates of the system is  $n$ , thus having the vector of system coordinates  $\mathbf{q} \in \mathbb{R}^n$ , vector of applied forces  $\mathbf{f} \in \mathbb{R}^n$ , mass matrix  $\mathbf{M} \in \mathbb{R}^{n \times n}$  and stiffness matrix  $\mathbf{K} \in \mathbb{R}^{n \times n}$ . If we are interested in free vibrations of the system, without any boundary conditions or interconnections to other bodies, Eq. (6.8) can be converted to a generalized eigenvalue problem. Using the approach  $\dot{\mathbf{q}}(t) = \mathbf{v}e^{\beta\omega t}$  in Eq. (6.8), and thus  $\ddot{\mathbf{q}}(t) = -\omega^2\mathbf{q}(t)$ , we obtain

$$[(-\omega^2\mathbf{M} + \mathbf{K})\mathbf{v}]e^{i\omega t} = 0. \quad (6.9)$$

Assuming that Eq. (6.9) is valid for all times, the **generalized eigenvalue problem** follows that

$$(-\omega^2\mathbf{M} + \mathbf{K})\mathbf{v} = 0, \quad (6.10)$$

which can be rewritten as

$$\det(-\omega^2\mathbf{M} + \mathbf{K}) = 0, \quad (6.11)$$

and which defines the eigenvalues  $\omega_i^2$  of the linear system, where  $i \in \{0, \dots, n-1\}$ . Note that in this case, the eigenvalues are the squared eigenfrequencies (in rad/s). We can use eigenvalue algorithms to compute the eigenvalues  $\omega_i^2$  and according eigenvectors  $\mathbf{v}_i$  from Python. The function `fem.ComputeEigenmodes(...)` uses `eigh(...)` from `scipy.linalg` in the dense matrix mode, and in the sparse mode `eigsh(...)` from `scipy.sparse.linalg`, the latter being restricted to pure symmetric matrices. Using special shift-inverted techniques in `eigsh(...)`, it performs much better than standard settings. However, you may tune your specific eigenvalue problem by modifying the solver procedure (just copy that function and adjust to your needs). As an output, we obtain the smallest `nModes` eigenvectors (=eigenmodes)<sup>1</sup> of the system. Here, we will also use synonymously the terms ‘eigenmodes’ and ‘normal modes’, which result from an eigenvalue/eigenvector computation using certain (or even no) boundary conditions.

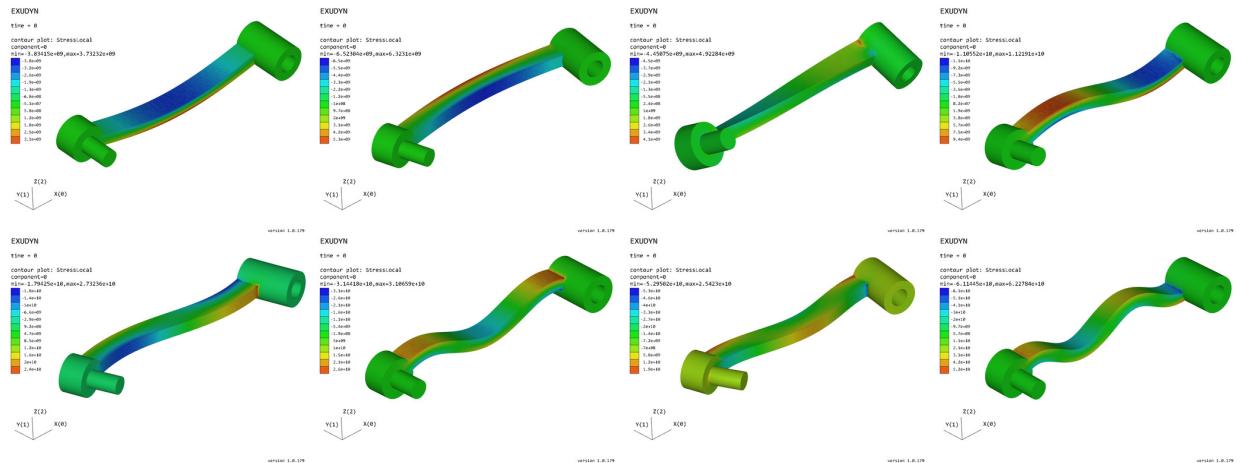


Figure 6.2: Lowest 8 free-free modes for hinge finite element model, contour plot for  $xx$ -stress component.

Clearly, if there are no supports included in the stiffness matrix, the resulting eigenmodes will contain 6 rigid body modes and we will also call this case for the computation of eigenmodes the ‘free-free’ case, in analogy to a simply supported beam. This rigid body modes, which are usually not needed (=unwanted) in the succeeding computation, can be excluded with an according option in

```
fem.ComputeEigenFrequencies(excludeRigidBodyModes = ...)
```

<sup>1</sup>Eigenvectors are the result of the eigenvalue algorithm, such as the QR algorithm. The mechanical interpretation of eigenvectors are eigenmodes, that can be visualized as shown in the figures of this section.

For our test example, 8 eigenmodes are shown in Fig. 6.2, where the 6 rigid body modes have been excluded (so in total, 14 eigenvectors were computed). The 8 eigenfrequencies for the chosen coarse mesh with mesh size  $h = 0.01$  and 1216 nodes result as

$$f_{0.7} = [671.59, 707.17, 1298.50, 1929.97, 1971.76, 3141.47, 3595.34, 4317.51] \text{Hz} \quad (6.12)$$

Note, that a computation with a finer mesh, using mesh size  $h = 0.002$  and 100224 nodes, leads to significantly different eigenfrequencies, starting with  $f_0 = 371.50 \text{ Hz}$ . This shows that quadratic finite elements would be more appropriate for this case.

After the computation of modes, it is always a good idea to visualize and/or animate these modes. We can do this, using the function `AnimateModes(...)` available in `exudyn.interactive`, which allows us to inspect and animate modes and to create animations for these modes, see the mentioned example.

Clearly, the free-free modes in Fig. 6.2 are not well suited for the modeling of the deformations within the hinge, if the bolt and the bushing shall be fixed to ground or to another part. Therefore, we can use modes based on ideas of Hurty [2] and Craig-Bampton [3], as shown in the following.

### 6.2.2 Hurty-Craig-Bampton modes

This section will describe the computation of static and eigen (normal) modes using FEMinterface. The theory is based on Hurty [2] and Craig-Bampton [3], but often only attributed to Craig-Bampton. Furthermore, boundaries are also called interfaces<sup>2</sup>, as they either represent surface sections of our finite element model which are connected to the ground or they represent interfaces to joints and are connected to other bodies.

The computation of so-called static and normal modes follows a simple concept based on finite element mass and stiffness matrices. The final goal of the computation of modes is to approximate the solution  $\mathbf{q} \in \mathbb{R}^n$  by means of a reduction basis  $\Psi \in \mathbb{R}^{n \times m}$  and a reduced set of coordinates  $\mathbf{p} \in \mathbb{R}^m$ , for which we assume  $m \ll n$ .

In order to include boundary/interface effects, we separate our nodes and the nodal coordinates into

- a) boundary nodes  $\mathbf{q}_b \in \mathbb{R}^{n_b}$  and
- b) internal or inner nodes  $\mathbf{q}_i \in \mathbb{R}^{n_i}$ .

We assume that internal nodes are not exposed to boundary/interface conditions or to forces.

Therefore, we may rewrite Eq. (6.8) as follows

$$\begin{bmatrix} \mathbf{M}_{bb} & \mathbf{M}_{bi} \\ \mathbf{M}_{ib} & \mathbf{M}_{ii} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{q}}_b \\ \ddot{\mathbf{q}}_i \end{bmatrix} + \begin{bmatrix} \mathbf{K}_{bb} & \mathbf{K}_{bi} \\ \mathbf{K}_{ib} & \mathbf{K}_{ii} \end{bmatrix} \begin{bmatrix} \mathbf{q}_b \\ \mathbf{q}_i \end{bmatrix} = \begin{bmatrix} \mathbf{f}_b \\ \mathbf{0} \end{bmatrix} \quad (6.13)$$

or, equivalently,

$$\mathbf{M}_{bb}\ddot{\mathbf{q}}_b + \mathbf{M}_{bi}\ddot{\mathbf{q}}_i + \mathbf{K}_{bb}\mathbf{q}_b + \mathbf{K}_{bi}\mathbf{q}_i = \mathbf{f}_b \quad (6.14)$$

$$\mathbf{M}_{ib}\ddot{\mathbf{q}}_b + \mathbf{M}_{ii}\ddot{\mathbf{q}}_i + \mathbf{K}_{ib}\mathbf{q}_b + \mathbf{K}_{ii}\mathbf{q}_i = \mathbf{0}. \quad (6.15)$$

A pure static condensation follows from Eq. (6.15) with the assumption that inertia terms are neglected, leading to the static result for internal nodes,

$$\mathbf{q}_{i,\text{stat}} = -\mathbf{K}_{ii}^{-1}\mathbf{K}_{ib}\mathbf{q}_b. \quad (6.16)$$

A pure static condensation, also denoted as Guyan-Irons method, keeps boundary coordinates but removes all internal modes, using the approximation

$$\begin{bmatrix} \mathbf{q}_b \\ \mathbf{q}_i \end{bmatrix} \approx \begin{bmatrix} \mathbf{I} \\ -\mathbf{K}_{ii}^{-1}\mathbf{K}_{ib} \end{bmatrix} \mathbf{q}_b = \Psi^{GI} \mathbf{q}_b, \quad (6.17)$$

---

<sup>2</sup>Here, and in the description of various Python functions, we will use boundary and interface often synonymously, as flexible bodies can be either connected to ground in the sense of a classical ‘support-type’ boundary condition, or they can represent the boundary of the flexible body as an interface to joints (via markers).

which leads to no approximations ('exact') results for the static case, but poor performance in highly dynamic problems.

Significant improvement result from the Hurty-Craig-Bampton method, which adds eigenmodes of the internal coordinates (internal nodes). We assume that  $\Psi_{ii}$  is the matrix of eigenvectors as a solution to the eigenvalue problem

$$(-\omega^2 \mathbf{M}_{ii} + \mathbf{K}_{ii}) \mathbf{v} = \mathbf{0}, \quad (6.18)$$

Hereafter, we will only keep the lowest (or other appropriate)  $m$  eigenmodes in a reduced eigenmode matrix,

$$\Psi_{ii}^{(red)} = [\Psi_{ii,0}, \dots, \Psi_{ii,m-1}] \quad (6.19)$$

Combining these 'fixed-fixed' eigenvectors with the Guyan-Irons reduction (6.17), we obtain the Hurty-Craig-Bampton modes as

$$\begin{bmatrix} \mathbf{q}_b \\ \mathbf{q}_i \end{bmatrix} \approx \begin{bmatrix} \mathbf{I} \\ -\mathbf{K}_{ii}^{-1} \mathbf{K}_{ib} \end{bmatrix} \mathbf{q}_b + \begin{bmatrix} \mathbf{0} \\ \Psi_{r,i} \end{bmatrix} \mathbf{p}_r, \quad (6.20)$$

or in matrix form

$$\begin{bmatrix} \mathbf{q}_b \\ \mathbf{q}_i \end{bmatrix} \approx \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ -\mathbf{K}_{ii}^{-1} \mathbf{K}_{ib} & \Psi_{r,i} \end{bmatrix} \begin{bmatrix} \mathbf{q}_b \\ \mathbf{p}_r \end{bmatrix} = \Psi^{HCB} \mathbf{p}^{HCB}. \quad (6.21)$$

The disadvantage of Eq. (6.21) is evident by the fact that there may be a large number of boundary/interface nodes, leading to a huge number of static modes (100s or 1000s) and thus making the model reduction inefficient. Therefore, we can switch to other interfaces, as described in the following.

### 6.2.2.1 Definition of RBE2 interfaces

A powerful extension, which is available in many finite element as well as flexible multibody codes, is the definition of special boundary/interface conditions, based on pure rigid body motion. The so-called RBE2 boundaries are defined such that they are firmly connected to a rigid frame, thus the boundary or interface can only undergo rigid body motion. The advantage of this procedure is that, in comparison to Eq. (6.21), the number of boundary/interface modes is given by 6 *rigid body* modes, which allow simple integration into standard joints of multibody systems, e.g., the `GenericJoint`.

### 6.2.2.2 Computation of Hurty-Craig-Bampton modes with RBE2 interfaces

In the following section, we show the procedure for the computation of static modes for the RBE2 rigid-body interfaces.

First, we use the index  $j$  here as a node index, having the clear correspondence to the coordinate index  $i$ , that node  $j$  has coordinates  $[3 \cdot j, 3 \cdot j + 1, 3 \cdot j + 2]$ . Furthermore, nodes are split into boundary and internal nodes, which then leads to according internal and boundary coordinates. We shall note that this sorting is never done in the finite element model or matrices, but just some indexing (referencing) lists are generated and used throughout, using valuable features of `numpy.linalg` and `scipy.sparse`.

For a certain boundary node set  $B = [j_0, j_1, j_2, \dots] \in \mathbb{N}^{n_b}$  with certain  $n_b$  node indices  $j_0, \dots$ , we define one boundary set. The following transformations need to be performed for every set of boundary node lists. We also assume that weighting of all boundary nodes is equal, which may not be appropriate in all cases.

If we assume that there may only occur rigid body translation and rotation for the whole boundary node set, which is according to the idea of so-called RBE2 boundary conditions, it follows that the translation of all boundary nodes is given by

$$\mathbf{T}_t = [\mathbf{I} \ \mathbf{I} \ \dots \ \mathbf{I}]^T \in \mathbb{R}^{3n_b \times 3} \quad (6.22)$$

with  $Im \in \mathbb{R}^{3 \times 3}$  identity matrices. The nodal translation coordinates on boundary  $B$  are denoted as  $\mathbf{q}_{B,t} \in \mathbb{R}^3$ . The translation of the boundary/interface is mapped to the boundary coordinates as follows (assuming only one boundary  $B$ ),

$$\mathbf{q}_{b,t} = \mathbf{T}_t \mathbf{q}_{B,t} \quad (6.23)$$

The nodal rotation coordinates on boundary  $B$  are denoted as  $\mathbf{q}_{B,r} \in \mathbb{R}^3$ . The rotation of the boundary/interface is mapped to the boundary coordinates as follows (assuming only one boundary  $B$ ),

$$\mathbf{q}_{b,r} = \mathbf{T}_r \mathbf{q}_{B,r} \quad (6.24)$$

The computation of matrix  $\mathbf{T}_r$  is more involved. It is based on nodal (reference) position vectors  $\mathbf{r}_j^{(0)}$ ,  $j \in B$ , the midpoint of all boundary nodes,

$$\mathbf{r}^{(m)} = \frac{1}{n_b} \sum_{j=0}^{n_b-1} \mathbf{r}_j^{(0)} \quad (6.25)$$

and the position relative to the midpoint, denoted as

$$\mathbf{r}_j = \mathbf{r}_j^{(0)} - \mathbf{r}^{(m)} . \quad (6.26)$$

The transformation for rotation follows from

$$\mathbf{T}_r = [\tilde{\boldsymbol{\Omega}}_x \mathbf{r}_{j_0} \ \tilde{\boldsymbol{\Omega}}_y \mathbf{r}_{j_0} \ \tilde{\boldsymbol{\Omega}}_z \mathbf{r}_{j_0} \ \tilde{\boldsymbol{\Omega}}_x \mathbf{r}_{j_1} \ \tilde{\boldsymbol{\Omega}}_y \mathbf{r}_{j_1} \ \tilde{\boldsymbol{\Omega}}_z \mathbf{r}_{j_1} \dots]^T \in \mathbb{R}^{3n_b \times 3} \quad (6.27)$$

with the special tensors, representing rotation about (x,y,z)-axes,

$$\tilde{\boldsymbol{\Omega}}_x = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}, \quad \tilde{\boldsymbol{\Omega}}_y = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}, \quad \tilde{\boldsymbol{\Omega}}_z = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} . \quad (6.28)$$

The total nodal coordinates at the boundary, representing translations and rotations, follow as

$$\mathbf{q}_B = \begin{bmatrix} \mathbf{q}_{B,t} \\ \mathbf{q}_{B,r} \end{bmatrix} , \quad (6.29)$$

and the transformation matrix for the translation and rotation simply reads

$$\mathbf{T} = [\mathbf{T}_t \ \mathbf{T}_r] \in \mathbb{R}^{3n_b \times 6} , \quad (6.30)$$

which provides the total mapping of boundary rigid body motion

$$\mathbf{q}_b = \mathbf{T} \mathbf{q}_B , \quad (6.31)$$

which is the sum of translation and rotation.

As an example, having the boundary nodes sorted for two boundary node set  $B_0$  and  $B_1$ , we obtain the following transformation for the Hurty-Craig-Bampton method with only 6 modes per boundary node set,

$$\begin{bmatrix} \mathbf{q}_b \\ \mathbf{q}_i \end{bmatrix} \approx \begin{bmatrix} \mathbf{T}_0 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{T}_1 & \mathbf{0} \\ -\mathbf{K}_{ii}^{-1} \mathbf{K}_{ib} \begin{bmatrix} \mathbf{T}_0 \\ \mathbf{0} \end{bmatrix} & -\mathbf{K}_{ii}^{-1} \mathbf{K}_{ib} \begin{bmatrix} \mathbf{0} \\ \mathbf{T}_1 \end{bmatrix} & \mathbf{\Psi}_{r,i} \end{bmatrix} \begin{bmatrix} \mathbf{q}_{B_0} \\ \mathbf{q}_{B_1} \\ \mathbf{p}_r \end{bmatrix} . \quad (6.32)$$

with the new boundary node vector  $\mathbf{q}_b = [\mathbf{q}_{B_0}^T \ \mathbf{q}_{B_1}^T]^T$ .

**Notes:**

- The inverse  $\mathbf{K}_{ii}^{-1}$  is not computed, but this matrix is LU-factorized using sparse techniques.
- The factorization only needs to be applied to six vectors for every relevant boundary node set.
- One set of boundary nodes can be omitted from the final static modes in Eq. (6.32), because keeping all boundary modes, would introduce six rigid body motions to our mode basis, what is usually not wanted nor needed.

Using again the examples given in Fig. 6.1, we now obtain a set of modified modes using the function `fem.ComputeHurtyCraigBamptonModes(...)`. Fig. 6.3 shows the first 6 rigid body modes. Note that these modes would be automatically removed in the function `fem.ComputeHurtyCraigBamptonModes(...)`. Fig. 6.4 shows the second set of 6 rigid body modes. Finally, 8 eigenmodes have been computed for the fixed-fixed case (where all boundary/interfaces nodes are fixed), see Fig. 6.5. The eigenfrequencies for this case now are significantly higher than in the free-free case, reading

$$f_{0.7} = [1277.35, 1469.86, 3336.91, 3584.28, \dots] \quad (6.33)$$

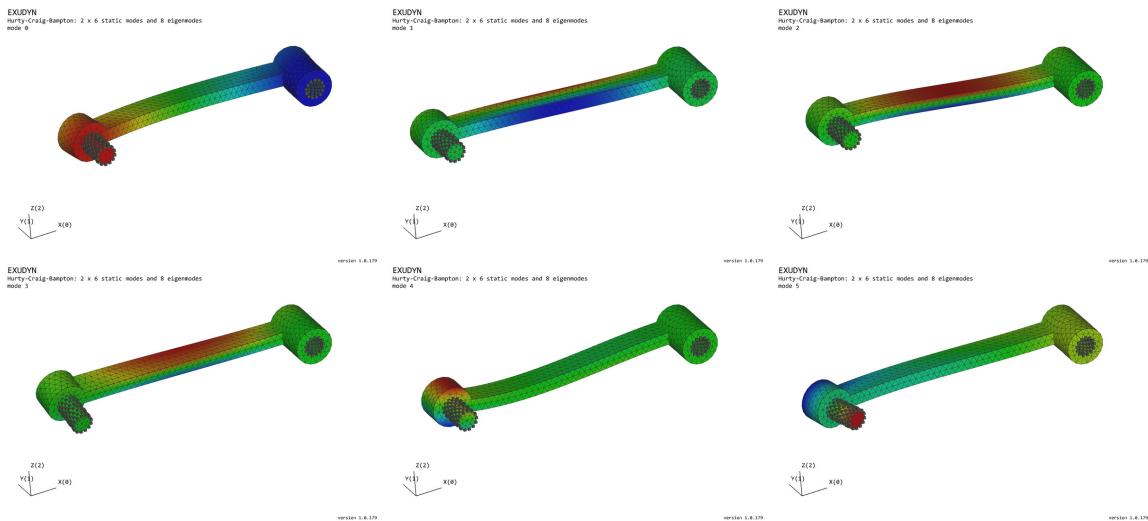


Figure 6.3: Static modes for bolt rigid body interface, using Hurty-Craig-Bampton method; top three images show  $(x,y,z)$ -translation modes, bottom three images show  $(x,y,z)$ -rotation modes; contour color represents norm of displacements.

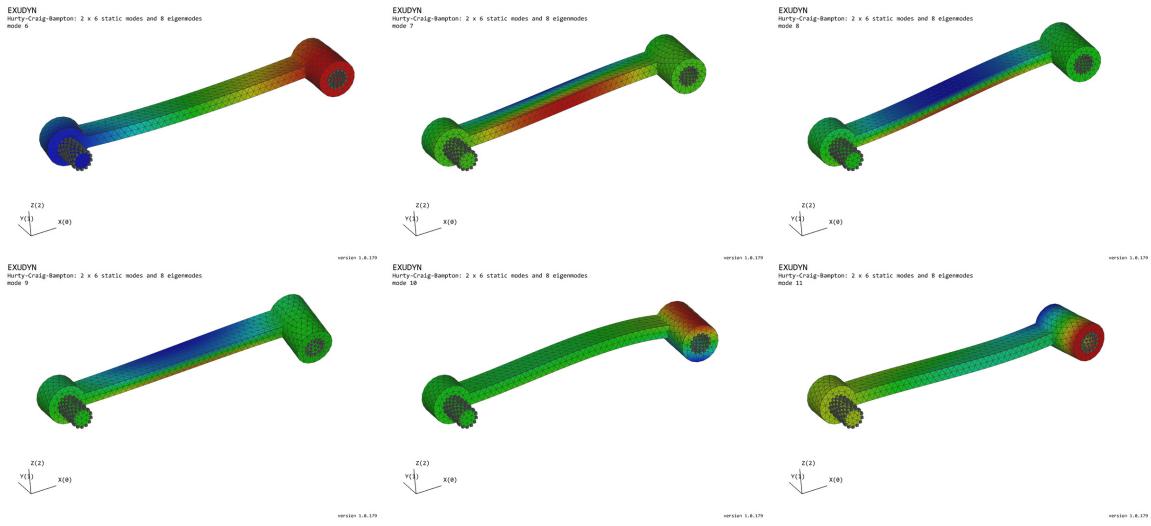


Figure 6.4: Static modes for bushing rigid body interface, using Hurty-Craig-Bampton method; top three images show  $(x,y,z)$ -translation modes, bottom three images show  $(x,y,z)$ -rotation modes; contour color represents norm of displacements.

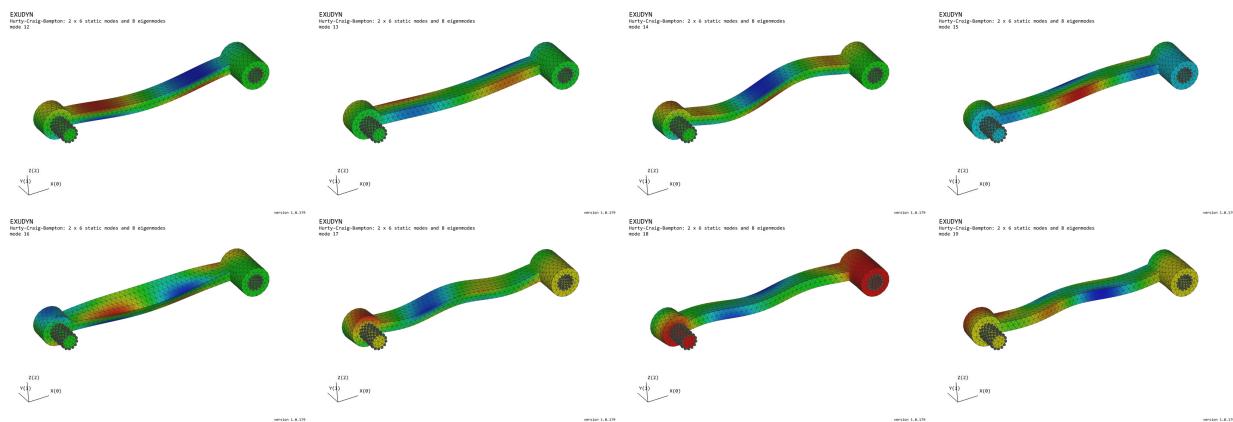


Figure 6.5: Eigenmodes for fixed-fixed case, resulting from Hurty-Craig-Bampton method; contour color represents norm of displacements.



## **Chapter 7**

# **Objects, nodes, markers, loads and sensors reference manual**

This chapter includes the reference manual for all objects (bodies/constraints), nodes, markers, loads and sensors (=items).

## 7.1 Nodes

### 7.1.1 NodePoint

A 3D point node for point masses or solid finite elements which has 3 displacement degrees of freedom for second order differential equations (ODE2).

**Additional information for NodePoint:**

- The Node has the following types = Position
- **Short name for Python** = Point
- **Short name for Python (visualization object)** = VPoint

The item **NodePoint** with type = 'Point' has the following parameters:

| Name                 | type       | size | default value | description                                                                                                            |
|----------------------|------------|------|---------------|------------------------------------------------------------------------------------------------------------------------|
| name                 | String     |      | "             | node's unique name                                                                                                     |
| referenceCoordinates | Vector3D   | 3    | [0.,0.,0.]    | reference coordinates of node, e.g. ref. coordinates for finite elements; global position of node without displacement |
| initialCoordinates   | Vector3D   | 3    | [0.,0.,0.]    | initial displacement coordinate                                                                                        |
| initialVelocities    | Vector3D   | 3    | [0.,0.,0.]    | initial velocity coordinate                                                                                            |
| visualization        | VNodePoint |      |               | parameters for visualization of item                                                                                   |

The item VNodePoint has the following parameters:

| Name     | type   | size | default value     | description                                                                                                         |
|----------|--------|------|-------------------|---------------------------------------------------------------------------------------------------------------------|
| show     | Bool   |      | True              | set true, if item is shown in visualization and false if it is not shown                                            |
| drawSize | float  |      | -1.               | drawing size (diameter, dimensions of underlying cube, etc.) for item; size == -1.f means that default size is used |
| color    | Float4 | 4    | [-1.,-1.,-1.,-1.] | Default RGBA color for nodes; 4th value is alpha-transparency; R=-1.f means, that default color is used             |

#### 7.1.1.1 DESCRIPTION of NodePoint:

**Information on input parameters:**

| input parameter      | symbol                                                                                                                | description see tables above |
|----------------------|-----------------------------------------------------------------------------------------------------------------------|------------------------------|
| referenceCoordinates | $\mathbf{q}_{\text{ref}} = [q_0, q_1, q_2]_{\text{ref}}^T = \mathbf{p}_{\text{ref}} = [r_0, r_1, r_2]^T$              |                              |
| initialCoordinates   | $\mathbf{q}_{\text{ini}} = [q_0, q_1, q_2]_{\text{ini}}^T = \mathbf{u}_{\text{ini}} = [u_0, u_1, u_2]_{\text{ini}}^T$ |                              |
| initialVelocities    | $\dot{\mathbf{q}}_{\text{ini}} = \mathbf{v}_{\text{ini}} = [\dot{q}_0, \dot{q}_1, \dot{q}_2]_{\text{ini}}^T$          |                              |

The following output variables are available as `OutputVariableType` in sensors, `Get...Output()` and other functions:

| output variable | symbol                                                                                                                    | description                                                      |
|-----------------|---------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------|
| Position        | $\mathbf{p}_{\text{config}} = [p_0, p_1, p_2]^T_{\text{config}} = \mathbf{u}_{\text{config}} + \mathbf{p}_{\text{ref}}$   | global 3D position vector of node; $\mathbf{u}_{\text{ref}} = 0$ |
| Displacement    | $\mathbf{u}_{\text{config}} = [q_0, q_1, q_2]^T_{\text{config}}$                                                          | global 3D displacement vector of node                            |
| Velocity        | $\mathbf{v}_{\text{config}} = [\dot{q}_0, \dot{q}_1, \dot{q}_2]^T_{\text{config}}$                                        | global 3D velocity vector of node                                |
| Acceleration    | $\mathbf{a}_{\text{config}} = \ddot{\mathbf{q}}_{\text{config}} = [\ddot{q}_0, \ddot{q}_1, \ddot{q}_2]^T_{\text{config}}$ | global 3D acceleration vector of node                            |
| Coordinates     | $\mathbf{c}_{\text{config}} = \mathbf{u}_{\text{config}} = [q_0, q_1, q_2]^T_{\text{config}}$                             | coordinate vector of node                                        |
| Coordinates_t   | $\dot{\mathbf{c}}_{\text{config}} = \mathbf{v}_{\text{config}} = [\dot{q}_0, \dot{q}_1, \dot{q}_2]^T_{\text{config}}$     | velocity coordinates vector of node                              |
| Coordinates_tt  | $\ddot{\mathbf{c}}_{\text{config}} = \mathbf{a}_{\text{config}} = [\ddot{q}_0, \ddot{q}_1, \ddot{q}_2]^T_{\text{config}}$ | acceleration coordinates vector of node                          |

**Detailed information:** The node provides  $n_c = 3$  displacement coordinates. Equations of motion need to be provided by an according object (e.g., `MassPoint`, finite elements, ...). Usually, the nodal coordinates are provided in the global frame. However, the coordinate system is defined by the object (e.g. `MassPoint` uses global coordinates, but floating frame of reference objects use local frames). Note that for this very simple node, coordinates are identical to the nodal displacements, same for time derivatives. This is not the case, e.g. for nodes with orientation.

Example for `NodePoint`: see `ObjectMassPoint`, [Section 7.2.2](#)

For further examples on `NodePoint` see Examples:

- [`interactiveTutorial.py`](#)
- [`Spring\_with\_constraints.py`](#)
- [`ALEANCF\_pipe.py`](#)
- [`ANCF\_cantilever\_test\_dyn.py`](#)
- [`ANCF\_contact\_circle.py`](#)
- [`ANCF\_contact\_circle2.py`](#)
- [`ANCF\_moving\_rigidbody.py`](#)
- [`ANCF\_slidingJoint2D.py`](#)
- [`ANCF\_switchingSlidingJoint2D.py`](#)
- [`ANCF\_tests2.py`](#)
- ...

For further examples on `NodePoint` see TestModels:

- [`objectGenericODE2Test.py`](#)
- [`sensorUserFunctionTest.py`](#)
- [`ANCFcontactCircleTest.py`](#)
- [`ANCFcontactFrictionTest.py`](#)
- [`computeODE2EigenvaluesTest.py`](#)
- [`explicitLieGroupIntegratorPythonTest.py`](#)
- [`explicitLieGroupIntegratorTest.py`](#)
- [`explicitLieGroupMBSTest.py`](#)
- [`fourBarMechanismTest.py`](#)
- [`genericODE2test.py`](#)
- ...

### 7.1.2 NodePoint2D

A 2D point node for point masses or solid finite elements which has 2 displacement degrees of freedom for second order differential equations.

**Additional information for NodePoint2D:**

- The Node has the following types = Position2D, Position
- **Short name for Python** = **Point2D**
- **Short name for Python (visualization object)** = **VPoint2D**

The item **NodePoint2D** with type = 'Point2D' has the following parameters:

| Name                 | type         | size | default value | description                                                                                                               |
|----------------------|--------------|------|---------------|---------------------------------------------------------------------------------------------------------------------------|
| name                 | String       |      | "             | node's unique name                                                                                                        |
| referenceCoordinates | Vector2D     | 2    | [0.,0.]       | reference coordinates of node ==> e.g. ref. coordinates for finite elements; global position of node without displacement |
| initialCoordinates   | Vector2D     | 2    | [0.,0.]       | initial displacement coordinate                                                                                           |
| initialVelocities    | Vector2D     | 2    | [0.,0.]       | initial velocity coordinate                                                                                               |
| visualization        | VNodePoint2D |      |               | parameters for visualization of item                                                                                      |

The item VNodePoint2D has the following parameters:

| Name     | type   | size | default value     | description                                                                                                         |
|----------|--------|------|-------------------|---------------------------------------------------------------------------------------------------------------------|
| show     | Bool   |      | True              | set true, if item is shown in visualization and false if it is not shown                                            |
| drawSize | float  |      | -1.               | drawing size (diameter, dimensions of underlying cube, etc.) for item; size == -1.f means that default size is used |
| color    | Float4 | 4    | [-1.,-1.,-1.,-1.] | Default RGBA color for nodes; 4th value is alpha-transparency; R=-1.f means, that default color is used             |

#### 7.1.2.1 DESCRIPTION of NodePoint2D:

**Information on input parameters:**

| input parameter      | symbol                                                                                            | description see tables above |
|----------------------|---------------------------------------------------------------------------------------------------|------------------------------|
| referenceCoordinates | $\mathbf{q}_{\text{ref}} = [q_0, q_1]_{\text{ref}}^T = \mathbf{p}_{\text{ref}} = [r_0, r_1]^T$    |                              |
| initialCoordinates   | $\mathbf{q}_{\text{ini}} = [q_0, q_1]_{\text{ini}}^T = [u_0, u_1]_{\text{ini}}^T$                 |                              |
| initialVelocities    | $\dot{\mathbf{q}}_{\text{ini}} = \mathbf{v}_{\text{ini}} = [\dot{q}_0, \dot{q}_1]_{\text{ini}}^T$ |                              |

**The following output variables are available as OutputVariableType in sensors, Get...Output() and other functions:**

| output variable | symbol                                                                                                                | description                                                      |
|-----------------|-----------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------|
| Position        | $\mathbf{p}_{\text{config}} = [p_0, p_1, 0]_{\text{config}}^T = \mathbf{u}_{\text{config}} + \mathbf{p}_{\text{ref}}$ | global 3D position vector of node; $\mathbf{u}_{\text{ref}} = 0$ |

|                |                                                                                                               |                                         |
|----------------|---------------------------------------------------------------------------------------------------------------|-----------------------------------------|
| Displacement   | $\mathbf{u}_{\text{config}} = [q_0, q_1, 0]^T_{\text{config}}$                                                | global 3D displacement vector of node   |
| Velocity       | $\mathbf{v}_{\text{config}} = [\dot{q}_0, \dot{q}_1, 0]^T_{\text{config}}$                                    | global 3D velocity vector of node       |
| Acceleration   | $\mathbf{a}_{\text{config}} = [\ddot{q}_0, \ddot{q}_1, 0]^T_{\text{config}}$                                  | global 3D acceleration vector of node   |
| Coordinates    | $\mathbf{c}_{\text{config}} = [q_0, q_1]^T_{\text{config}}$                                                   | coordinate vector of node               |
| Coordinates_t  | $\dot{\mathbf{c}}_{\text{config}} = [\dot{q}_0, \dot{q}_1]^T_{\text{config}}$                                 | velocity coordinates vector of node     |
| Coordinates_tt | $\ddot{\mathbf{c}}_{\text{config}} = \mathbf{a}_{\text{config}} = [\ddot{q}_0, \ddot{q}_1]^T_{\text{config}}$ | acceleration coordinates vector of node |

**Detailed information:** The node provides  $n_c = 2$  displacement coordinates. Equations of motion need to be provided by an according object (e.g., MassPoint2D). Coordinates are identical to the nodal displacements, except for the third coordinate  $u_2$ , which is zero, because  $q_2$  does not exist.

Note that for this very simple node, coordinates are identical to the nodal displacements, same for time derivatives. This is not the case, e.g. for nodes with orientation.

**Example** for NodePoint2D: see ObjectMassPoint2D, [Section 7.2.3](#)

For further examples on NodePoint2D see Examples:

- [myFirstExample.py](#)
- [pendulum2Dconstraint.py](#)
- [sliderCrank3DwithANCFbeltDrive2.py](#)
- [SpringDamperMassUserFunction.py](#)
- [ALEANCF\\_pipe.py](#)
- [ANCF\\_cantilever\\_test\\_dyn.py](#)
- [ANCF\\_contact\\_circle.py](#)
- [ANCF\\_contact\\_circle2.py](#)
- [ANCF\\_moving\\_rigidbody.py](#)
- [ANCF\\_slidingJoint2D.py](#)
- ...

For further examples on NodePoint2D see TestModels:

- [sparseMatrixSpringDamperTest.py](#)
- [ANCFcontactCircleTest.py](#)
- [ANCFcontactFrictionTest.py](#)
- [computeODE2EigenvaluesTest.py](#)
- [manualExplicitIntegrator.py](#)
- [modelUnitTests.py](#)
- [sliderCrankFloatingTest.py](#)

### 7.1.3 NodeRigidBodyEP

A 3D rigid body node based on Euler parameters for rigid bodies or beams; the node has 3 displacement coordinates (displacements of center of mass - COM: ux,uy,uz) and four rotation coordinates (Euler parameters = quaternions).

**Additional information for NodeRigidBodyEP:**

- The Node has the following types = Position, Orientation, RigidBody, RotationEulerParameters
- **Short name for Python** = **RigidEP**
- **Short name for Python** (visualization object) = **VRigidEP**

The item **NodeRigidBodyEP** with type = 'RigidBodyEP' has the following parameters:

| Name                 | type             | size | default value        | description                                                                                                                                                                                     |
|----------------------|------------------|------|----------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| name                 | String           |      | "                    | node's unique name                                                                                                                                                                              |
| referenceCoordinates | Vector7D         | 7    | [0.,0.,0., 0.,0.,0.] | reference coordinates (3 position coordinates and 4 Euler parameters) of node ==> e.g. ref. coordinates for finite elements or reference position of rigid body (e.g. for definition of joints) |
| initialCoordinates   | Vector7D         | 7    | [0.,0.,0., 0.,0.,0.] | initial displacement coordinates and 4 Euler parameters relative to reference coordinates                                                                                                       |
| initialVelocities    | Vector7D         | 7    | [0.,0.,0., 0.,0.,0.] | initial velocity coordinates: time derivatives of initial displacements and Euler parameters                                                                                                    |
| visualization        | VNodeRigidBodyEP |      |                      | parameters for visualization of item                                                                                                                                                            |

The item **VNodeRigidBodyEP** has the following parameters:

| Name     | type   | size | default value     | description                                                                                                         |
|----------|--------|------|-------------------|---------------------------------------------------------------------------------------------------------------------|
| show     | Bool   |      | True              | set true, if item is shown in visualization and false if it is not shown                                            |
| drawSize | float  |      | -1.               | drawing size (diameter, dimensions of underlying cube, etc.) for item; size == -1.f means that default size is used |
| color    | Float4 | 4    | [-1.,-1.,-1.,-1.] | Default RGBA color for nodes; 4th value is alpha-transparency; R=-1.f means, that default color is used             |

---

#### 7.1.3.1 DESCRIPTION of NodeRigidBodyEP:

**Information on input parameters:**

| input parameter      | symbol                                                                                                                                                       | description see tables above |
|----------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------|
| referenceCoordinates | $\mathbf{q}_{\text{ref}} = [q_0, q_1, q_2, \psi_0, \psi_1, \psi_2, \psi_3]_{\text{ref}}^T = [\mathbf{p}_{\text{ref}}^T, \boldsymbol{\psi}_{\text{ref}}^T]^T$ |                              |

|                    |                                                                                                                                                                                                                          |  |
|--------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|
| initialCoordinates | $\mathbf{q}_{\text{ini}} = [q_0, q_1, q_2, \psi_0, \psi_1, \psi_2, \psi_3]_{\text{ini}}^T = [\mathbf{u}_{\text{ini}}^T, \boldsymbol{\psi}_{\text{ini}}^T]^T$                                                             |  |
| initialVelocities  | $\dot{\mathbf{q}}_{\text{ini}} = [\dot{q}_0, \dot{q}_1, \dot{q}_2, \dot{\psi}_0, \dot{\psi}_1, \dot{\psi}_2, \dot{\psi}_3]_{\text{ini}}^T = [\dot{\mathbf{u}}_{\text{ini}}^T, \dot{\boldsymbol{\psi}}_{\text{ini}}^T]^T$ |  |

The following output variables are available as `OutputVariableType` in sensors, `Get...Output()` and other functions:

| output variable      | symbol                                                                                                                                                   | description                                                                                                                                                                                                                  |
|----------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Position             | ${}^0\mathbf{p}_{\text{config}} = {}^0[p_0, p_1, p_2]_{\text{config}}^T = {}^0\mathbf{u}_{\text{config}} + {}^0\mathbf{p}_{\text{ref}}$                  | global 3D position vector of node; $\mathbf{u}_{\text{ref}} = 0$                                                                                                                                                             |
| Displacement         | ${}^0\mathbf{u}_{\text{config}} = [q_0, q_1, q_2]_{\text{config}}^T$                                                                                     | global 3D displacement vector of node                                                                                                                                                                                        |
| Velocity             | ${}^0\mathbf{v}_{\text{config}} = [\dot{q}_0, \dot{q}_1, \dot{q}_2]_{\text{config}}^T$                                                                   | global 3D velocity vector of node                                                                                                                                                                                            |
| Acceleration         | ${}^0\mathbf{a}_{\text{config}} = [\ddot{q}_0, \ddot{q}_1, \ddot{q}_2]_{\text{config}}^T$                                                                | global 3D acceleration vector of node                                                                                                                                                                                        |
| Coordinates          | $\mathbf{c}_{\text{config}} = [q_0, q_1, q_2, \psi_0, \psi_1, \psi_2, \psi_3]_{\text{config}}^T$                                                         | coordinate vector of node, having 3 displacement coordinates and 4 Euler parameters                                                                                                                                          |
| Coordinates_t        | $\dot{\mathbf{c}}_{\text{config}} = [\dot{q}_0, \dot{q}_1, \dot{q}_2, \dot{\psi}_0, \dot{\psi}_1, \dot{\psi}_2, \dot{\psi}_3]_{\text{config}}^T$         | velocity coordinates vector of node                                                                                                                                                                                          |
| Coordinates_tt       | $\ddot{\mathbf{c}}_{\text{config}} = [\ddot{q}_0, \ddot{q}_1, \ddot{q}_2, \ddot{\psi}_0, \ddot{\psi}_1, \ddot{\psi}_2, \ddot{\psi}_3]_{\text{config}}^T$ | acceleration coordinates vector of node                                                                                                                                                                                      |
| RotationMatrix       | $[A_{00}, A_{01}, A_{02}, A_{10}, \dots, A_{21}, A_{22}]_{\text{config}}^T$                                                                              | vector with 9 components of the rotation matrix ${}^{0b}\mathbf{A}_{\text{config}}$ in row-major format, in any configuration; the rotation matrix transforms local ( $b$ ) to global ( $0$ ) coordinates                    |
| Rotation             | $[\varphi_0, \varphi_1, \varphi_2]_{\text{config}}^T$                                                                                                    | vector with 3 components of the Euler angles in xyz-sequence ( ${}^{0b}\mathbf{A}_{\text{config}} =: \mathbf{A}_0(\varphi_0) \cdot \mathbf{A}_1(\varphi_1) \cdot \mathbf{A}_2(\varphi_2)$ ), recomputed from rotation matrix |
| AngularVelocity      | ${}^0\boldsymbol{\omega}_{\text{config}} = {}^0[\omega_0, \omega_1, \omega_2]_{\text{config}}^T$                                                         | global 3D angular velocity vector of node                                                                                                                                                                                    |
| AngularVelocityLocal | ${}^b\boldsymbol{\omega}_{\text{config}} = {}^b[\omega_0, \omega_1, \omega_2]_{\text{config}}^T$                                                         | local (body-fixed) 3D angular velocity vector of node                                                                                                                                                                        |
| AngularAcceleration  | ${}^0\boldsymbol{\alpha}_{\text{config}} = {}^0[\alpha_0, \alpha_1, \alpha_2]_{\text{config}}^T$                                                         | global 3D angular acceleration vector of node                                                                                                                                                                                |

**Detailed information:** All coordinates  $\mathbf{c}_{\text{config}}$  lead to second order differential equations, but there is one additional constraint equation for the quaternions. The additional constraint equation, which needs to be provided by the object, reads

$$1 - \sum_{i=0}^3 \theta_i^2 = 0. \quad (7.1)$$

The rotation matrix  ${}^{0b}\mathbf{A}_{\text{config}}$  transforms local (body-fixed) 3D positions  ${}^b\mathbf{p} = {}^b[p_0, p_1, p_2]^T$  to global 3D positions,

$${}^0\mathbf{p}_{\text{config}} = {}^{0b}\mathbf{A}_{\text{config}} {}^b\mathbf{p} \quad (7.2)$$

Note that the Euler parameters  $\theta_{\text{cur}}$  are computed as sum of current coordinates plus reference coordinates,

$$\theta_{\text{cur}} = \boldsymbol{\psi}_{\text{cur}} + \boldsymbol{\psi}_{\text{ref}}. \quad (7.3)$$

The rotation matrix is defined as function of the rotation parameters  $\theta = [\theta_0, \theta_1, \theta_2, \theta_3]^T$

$${}^{0b}\mathbf{A} = \begin{bmatrix} -2\theta_3^2 - 2\theta_2^2 + 1 & -2\theta_3\theta_0 + 2\theta_2\theta_1 & 2*\theta_3\theta_1 + 2*\theta_2\theta_0 \\ 2\theta_3\theta_0 + 2\theta_2\theta_1 & -2\theta_3^2 - 2\theta_1^2 + 1 & 2\theta_3\theta_2 - 2\theta_1\theta_0 \\ -2\theta_2\theta_0 + 2\theta_3\theta_1 & 2\theta_3\theta_2 + 2\theta_1\theta_0 & -2\theta_2^2 - 2\theta_1^2 + 1 \end{bmatrix} \quad (7.4)$$

The derivatives of the angular velocity vectors w.r.t. the rotation velocity coordinates  $\dot{\theta} = [\dot{\theta}_0, \dot{\theta}_1, \dot{\theta}_2, \dot{\theta}_3]^T$  lead to the  $\mathbf{G}$  matrices, as used in the equations of motion for rigid bodies,

$${}^0\boldsymbol{\omega} = {}^0\mathbf{G} \dot{\boldsymbol{\theta}}, \quad (7.5)$$

$${}^b\boldsymbol{\omega} = {}^b\mathbf{G} \dot{\boldsymbol{\theta}}. \quad (7.6)$$

For creating a `NodeRigidBodyEP`, there is a `rigidBodyUtilities` function `AddRigidBody`, see [Section 5.11](#), which simplifies the setup of a rigid body significantly!

For further examples on `NodeRigidBodyEP` see Examples:

- [`rigid3Dexample.py`](#)
- [`rigidBodyIMUtest.py`](#)
- [`rigidRotor3DbasicBehaviour.py`](#)
- [`rigidRotor3DFWBW.py`](#)
- [`rigidRotor3Dnutation.py`](#)
- [`rigidRotor3Drunup.py`](#)
- [`mouseInteractionExample.py`](#)
- [`rigidBodyTutorial.py`](#)
- [`rigidBodyTutorial2.py`](#)
- [`sliderCrank3DwithANCFbeltDrive2.py`](#)
- ...

For further examples on `NodeRigidBodyEP` see TestModels:

- [`explicitLieGroupIntegratorPythonTest.py`](#)
- [`explicitLieGroupIntegratorTest.py`](#)
- [`explicitLieGroupMBSTest.py`](#)
- [`genericJointUserFunctionTest.py`](#)
- [`heavyTop.py`](#)
- [`objectFFRFTest.py`](#)
- [`sphericalJointTest.py`](#)
- [`carRollingDiscTest.py`](#)
- [`driveTrainTest.py`](#)
- [`mecanumWheelRollingDiscTest.py`](#)
- ...

### 7.1.4 NodeRigidBodyRxyz

A 3D rigid body node based on Euler / Tait-Bryan angles for rigid bodies or beams; all coordinates lead to second order differential equations; NOTE that this node has a singularity if the second rotation parameter reaches  $\psi_1 = (2k - 1)\pi/2$ , with  $k \in \mathbb{N}$  or  $-k \in \mathbb{N}$ .

**Additional information for NodeRigidBodyRxyz:**

- The Node has the following types = Position, Orientation, RigidBody, RotationRxyz
- **Short name for Python** = **RigidRxyz**
- **Short name for Python (visualization object)** = **VRigidRxyz**

The item **NodeRigidBodyRxyz** with type = 'RigidBodyRxyz' has the following parameters:

| Name                 | type               | size | default value        | description                                                                                                                                                                         |
|----------------------|--------------------|------|----------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| name                 | String             |      | "                    | node's unique name                                                                                                                                                                  |
| referenceCoordinates | Vector6D           | 6    | [0.,0.,0., 0.,0.,0.] | reference coordinates (3 position and 3 xyz Euler angles) of node ==> e.g. ref. coordinates for finite elements or reference position of rigid body (e.g. for definition of joints) |
| initialCoordinates   | Vector6D           | 6    | [0.,0.,0., 0.,0.,0.] | initial displacement coordinates: ux,uy,uz and 3 Euler angles (xyz) relative to reference coordinates                                                                               |
| initialVelocities    | Vector6D           | 6    | [0.,0.,0., 0.,0.,0.] | initial velocity coordinate: time derivatives of ux,uy,uz and of 3 Euler angles (xyz)                                                                                               |
| visualization        | VNodeRigidBodyRxyz |      |                      | parameters for visualization of item                                                                                                                                                |

The item **VNodeRigidBodyRxyz** has the following parameters:

| Name     | type   | size | default value     | description                                                                                                         |
|----------|--------|------|-------------------|---------------------------------------------------------------------------------------------------------------------|
| show     | Bool   |      | True              | set true, if item is shown in visualization and false if it is not shown                                            |
| drawSize | float  |      | -1.               | drawing size (diameter, dimensions of underlying cube, etc.) for item; size == -1.f means that default size is used |
| color    | Float4 | 4    | [-1.,-1.,-1.,-1.] | Default RGBA color for nodes; 4th value is alpha-transparency; R=-1.f means, that default color is used             |

---

#### 7.1.4.1 DESCRIPTION of NodeRigidBodyRxyz:

**Information on input parameters:**

| input parameter      | symbol                                                                                                                                               | description see tables above |
|----------------------|------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------|
| referenceCoordinates | $\mathbf{q}_{\text{ref}} = [q_0, q_1, q_2, \psi_0, \psi_1, \psi_2]_{\text{ref}}^T = [\mathbf{p}_{\text{ref}}^T, \boldsymbol{\psi}_{\text{ref}}^T]^T$ |                              |

|                    |                                                                                                                                                                                                            |  |
|--------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|
| initialCoordinates | $\mathbf{q}_{\text{ini}} = [q_0, q_1, q_2, \psi_0, \psi_1, \psi_2]_{\text{ini}}^T = [\mathbf{u}_{\text{ini}}^T, \boldsymbol{\psi}_{\text{ini}}^T]^T$                                                       |  |
| initialVelocities  | $\dot{\mathbf{q}}_{\text{ini}} = [\dot{q}_0, \dot{q}_1, \dot{q}_2, \dot{\psi}_0, \dot{\psi}_1, \dot{\psi}_2]_{\text{ini}}^T = [\dot{\mathbf{u}}_{\text{ini}}^T, \dot{\boldsymbol{\psi}}_{\text{ini}}^T]^T$ |  |

The following output variables are available as **OutputVariableType** in sensors, **Get...Output()** and other functions:

| output variable      | symbol                                                                                                                                       | description                                                                                                                                                                                                                               |
|----------------------|----------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Position             | ${}^0\mathbf{p}_{\text{config}} = {}^0[p_0, p_1, p_2]_{\text{config}}^T = {}^0\mathbf{u}_{\text{config}} + {}^0\mathbf{p}_{\text{ref}}$      | global 3D position vector of node; $\mathbf{u}_{\text{ref}} = 0$                                                                                                                                                                          |
| Displacement         | ${}^0\mathbf{u}_{\text{config}} = [q_0, q_1, q_2]_{\text{config}}^T$                                                                         | global 3D displacement vector of node                                                                                                                                                                                                     |
| Velocity             | ${}^0\mathbf{v}_{\text{config}} = [\dot{q}_0, \dot{q}_1, \dot{q}_2]_{\text{config}}^T$                                                       | global 3D velocity vector of node                                                                                                                                                                                                         |
| Acceleration         | ${}^0\mathbf{a}_{\text{config}} = [\ddot{q}_0, \ddot{q}_1, \ddot{q}_2]_{\text{config}}^T$                                                    | global 3D acceleration vector of node                                                                                                                                                                                                     |
| Coordinates          | $\mathbf{c}_{\text{config}} = [q_0, q_1, q_2, \psi_0, \psi_1, \psi_2]_{\text{config}}^T$                                                     | coordinate vector of node, having 3 displacement coordinates and 3 Euler angles                                                                                                                                                           |
| Coordinates_t        | $\dot{\mathbf{c}}_{\text{config}} = [\dot{q}_0, \dot{q}_1, \dot{q}_2, \dot{\psi}_0, \dot{\psi}_1, \dot{\psi}_2]_{\text{config}}^T$           | velocity coordinates vector of node                                                                                                                                                                                                       |
| Coordinates_tt       | $\ddot{\mathbf{c}}_{\text{config}} = [\ddot{q}_0, \ddot{q}_1, \ddot{q}_2, \ddot{\psi}_0, \ddot{\psi}_1, \ddot{\psi}_2]_{\text{config}}^T$    | acceleration coordinates vector of node                                                                                                                                                                                                   |
| RotationMatrix       | $[A_{00}, A_{01}, A_{02}, A_{10}, \dots, A_{21}, A_{22}]_{\text{config}}^T$                                                                  | vector with 9 components of the rotation matrix ${}^{0b}\mathbf{A}_{\text{config}}$ in row-major format, in any configuration; the rotation matrix transforms local ( $b$ ) to global ( $0$ ) coordinates                                 |
| Rotation             | $[\varphi_0, \varphi_1, \varphi_2]_{\text{config}}^T = [\psi_0, \psi_1, \psi_2]_{\text{ref}}^T + [\psi_0, \psi_1, \psi_2]_{\text{config}}^T$ | vector with 3 components of the Euler / Tait-Bryan angles in xyz-sequence ( ${}^{0b}\mathbf{A}_{\text{config}} := \mathbf{A}_0(\varphi_0) \cdot \mathbf{A}_1(\varphi_1) \cdot \mathbf{A}_2(\varphi_2)$ ), recomputed from rotation matrix |
| AngularVelocity      | ${}^0\boldsymbol{\omega}_{\text{config}} = {}^0[\omega_0, \omega_1, \omega_2]_{\text{config}}^T$                                             | global 3D angular velocity vector of node                                                                                                                                                                                                 |
| AngularVelocityLocal | ${}^b\boldsymbol{\omega}_{\text{config}} = {}^b[\omega_0, \omega_1, \omega_2]_{\text{config}}^T$                                             | local (body-fixed) 3D angular velocity vector of node                                                                                                                                                                                     |
| AngularAcceleration  | ${}^0\boldsymbol{\alpha}_{\text{config}} = {}^0[\alpha_0, \alpha_1, \alpha_2]_{\text{config}}^T$                                             | global 3D angular acceleration vector of node                                                                                                                                                                                             |

**Detailed information:** The node has 3 displacement coordinates  $[q_0, q_1, q_2]^T$  and 3 rotation coordinates  $[\psi_0, \psi_1, \psi_2]^T$  for consecutive rotations around the 0, 1 and 2-axis ( $x$ ,  $y$  and  $z$ ). All coordinates  $\mathbf{c}_{\text{config}}$  lead to second order differential equations. The rotation matrix  ${}^{0b}\mathbf{A}_{\text{config}}$  transforms local (body-fixed) 3D positions  ${}^b\mathbf{p} = {}^b[p_0, p_1, p_2]^T$  to global 3D positions,

$${}^0\mathbf{p}_{\text{config}} = {}^{0b}\mathbf{A}_{\text{config}} {}^b\mathbf{p} \quad (7.7)$$

Note that the Euler angles  $\boldsymbol{\theta}_{\text{cur}}$  are computed as sum of current coordinates plus reference coordinates,

$$\boldsymbol{\theta}_{\text{cur}} = \boldsymbol{\psi}_{\text{cur}} + \boldsymbol{\psi}_{\text{ref}}. \quad (7.8)$$

The rotation matrix is defined as function of the rotation parameters  $\boldsymbol{\theta} = [\theta_0, \theta_1, \theta_2]^T$

$${}^{0b}\mathbf{A} = \mathbf{A}_0(\theta_0)\mathbf{A}_1(\theta_1)\mathbf{A}_2(\theta_2) \quad (7.9)$$

The derivatives of the angular velocity vectors w.r.t. the rotation velocity coordinates  $\dot{\boldsymbol{\theta}} = [\dot{\theta}_0, \dot{\theta}_1, \dot{\theta}_2]^T$  lead to the  $\mathbf{G}$  matrices, as used in the equations of motion for rigid bodies,

$${}^0\boldsymbol{\omega} = {}^0\mathbf{G} \dot{\boldsymbol{\theta}}, \quad (7.10)$$

$${}^b\boldsymbol{\omega} = {}^b\mathbf{G} \dot{\boldsymbol{\theta}}. \quad (7.11)$$

For creating a `NodeRigidBodyRxyz`, there is a `rigidBodyUtilities` function `AddRigidBody`, see [Section 5.11](#), which simplifies the setup of a rigid body significantly!

---

For further examples on `NodeRigidBodyRxyz` see Examples:

- [`performanceMultiThreadingNG.py`](#)

For further examples on `NodeRigidBodyRxyz` see TestModels:

- [`explicitLieGroupIntegratorPythonTest.py`](#)
- [`explicitLieGroupIntegratorTest.py`](#)
- [`explicitLieGroupMBSTest.py`](#)
- [`heavyTop.py`](#)
- [`connectorRigidBodySpringDamperTest.py`](#)

### 7.1.5 NodeRigidBodyRotVecLG

A 3D rigid body node based on rotation vector and Lie group methods for rigid bodies or beams; the node has 3 displacement coordinates and three rotation coordinates.

**Additional information for NodeRigidBodyRotVecLG:**

- The Node has the following types = Position, Orientation, RigidBody, RotationRotationVector, RotationLieGroup
- Short name for Python = **RigidRotVecLG**
- Short name for Python (visualization object) = **VRigidRotVecLG**

The item **NodeRigidBodyRotVecLG** with type = 'RigidBodyRotVecLG' has the following parameters:

| Name                 | type                   | size | default value       | description                                                                                                                                                                           |
|----------------------|------------------------|------|---------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| name                 | String                 |      | "                   | node's unique name                                                                                                                                                                    |
| referenceCoordinates | Vector6D               | 3    | [0.,0.,0.,0.,0.,0.] | reference coordinates (position and rotation vector $\nu$ ) of node ==> e.g. ref. coordinates for finite elements or reference position of rigid body (e.g. for definition of joints) |
| initialCoordinates   | Vector6D               | 3    | [0.,0.,0.,0.,0.,0.] | initial displacement coordinates $\mathbf{u}$ and rotation vector $\nu$ relative to reference coordinates                                                                             |
| initialVelocities    | Vector6D               | 3    | [0.,0.,0.,0.,0.,0.] | initial velocity coordinate: time derivatives of displacement and angular velocity vector                                                                                             |
| visualization        | VNodeRigidBodyRotVecLG |      |                     | parameters for visualization of item                                                                                                                                                  |

The item **VNodeRigidBodyRotVecLG** has the following parameters:

| Name     | type   | size | default value     | description                                                                                                         |
|----------|--------|------|-------------------|---------------------------------------------------------------------------------------------------------------------|
| show     | Bool   |      | True              | set true, if item is shown in visualization and false if it is not shown                                            |
| drawSize | float  |      | -1.               | drawing size (diameter, dimensions of underlying cube, etc.) for item; size == -1.f means that default size is used |
| color    | Float4 | 4    | [-1.,-1.,-1.,-1.] | Default RGBA color for nodes; 4th value is alpha-transparency; R=-1.f means, that default color is used             |

#### 7.1.5.1 DESCRIPTION of NodeRigidBodyRotVecLG:

**Information on input parameters:**

| input parameter      | symbol                                                                                                                                       | description see tables above |
|----------------------|----------------------------------------------------------------------------------------------------------------------------------------------|------------------------------|
| referenceCoordinates | $\mathbf{q}_{\text{ref}} = [q_0, q_1, q_2, \nu_0, \nu_1, \nu_2]_{\text{ref}}^T = [\mathbf{p}_{\text{ref}}^T, \mathbf{\nu}_{\text{ref}}^T]^T$ |                              |
| initialCoordinates   | $\mathbf{q}_{\text{ini}} = [q_0, q_1, q_2, \nu_0, \nu_1, \nu_2]_{\text{ini}}^T = [\mathbf{u}_{\text{ini}}^T, \mathbf{\nu}_{\text{ini}}^T]^T$ |                              |

|                   |                                                                                                                                                                                            |  |
|-------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|
| initialVelocities | $\dot{\mathbf{q}}_{\text{ini}} = [\dot{q}_0, \dot{q}_1, \dot{q}_2, \dot{v}_0, \dot{v}_1, \dot{v}_2]_{\text{ini}}^T = [\dot{\mathbf{u}}_{\text{ini}}^T, \dot{\mathbf{v}}_{\text{ini}}^T]^T$ |  |
|-------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|

The following output variables are available as `OutputVariableType` in `sensors`, `Get...Output()` and other functions:

| output variable      | symbol                                                                                                                                  | description                                                                                                                                                                                                                              |
|----------------------|-----------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Position             | ${}^0\mathbf{p}_{\text{config}} = {}^0[p_0, p_1, p_2]_{\text{config}}^T = {}^0\mathbf{u}_{\text{config}} + {}^0\mathbf{p}_{\text{ref}}$ | global 3D position vector of node; $\mathbf{u}_{\text{ref}} = 0$                                                                                                                                                                         |
| Displacement         | ${}^0\mathbf{u}_{\text{config}} = [q_0, q_1, q_2]_{\text{config}}^T$                                                                    | global 3D displacement vector of node                                                                                                                                                                                                    |
| Velocity             | ${}^0\mathbf{v}_{\text{config}} = [\dot{q}_0, \dot{q}_1, \dot{q}_2]_{\text{config}}^T$                                                  | global 3D velocity vector of node                                                                                                                                                                                                        |
| Coordinates          | $\mathbf{c}_{\text{config}} = [q_0, q_1, q_2, v_0, v_1, v_2]_{\text{config}}^T$                                                         | coordinate vector of node, having 3 displacement coordinates and 3 Euler angles                                                                                                                                                          |
| Coordinates_t        | $\dot{\mathbf{c}}_{\text{config}} = [\dot{q}_0, \dot{q}_1, \dot{q}_2, \dot{v}_0, \dot{v}_1, \dot{v}_2]_{\text{config}}^T$               | velocity coordinates vector of node                                                                                                                                                                                                      |
| RotationMatrix       | $[A_{00}, A_{01}, A_{02}, A_{10}, \dots, A_{21}, A_{22}]_{\text{config}}^T$                                                             | vector with 9 components of the rotation matrix ${}^{0b}\mathbf{A}_{\text{config}}$ in row-major format, in any configuration; the rotation matrix transforms local ( $b$ ) to global ( $0$ ) coordinates                                |
| Rotation             | $[\varphi_0, \varphi_1, \varphi_2]_{\text{config}}^T$                                                                                   | vector with 3 components of the Euler / Tait-Bryan angles in xyz-sequence ( ${}^{0b}\mathbf{A}_{\text{config}} = \mathbf{A}_0(\varphi_0) \cdot \mathbf{A}_1(\varphi_1) \cdot \mathbf{A}_2(\varphi_2)$ ), recomputed from rotation matrix |
| AngularVelocity      | ${}^0\boldsymbol{\omega}_{\text{config}} = {}^0[\omega_0, \omega_1, \omega_2]_{\text{config}}^T$                                        | global 3D angular velocity vector of node                                                                                                                                                                                                |
| AngularVelocityLocal | ${}^b\boldsymbol{\omega}_{\text{config}} = {}^b[\omega_0, \omega_1, \omega_2]_{\text{config}}^T$                                        | local (body-fixed) 3D angular velocity vector of node                                                                                                                                                                                    |

**Detailed information:** For a detailed description on the rigid body dynamics formulation using this node, see Holzinger and Gerstmayr [16].

The node has 3 displacement coordinates  $[q_0, q_1, q_2]^T$  and three rotation coordinates, which is the rotation vector

$$\boldsymbol{\nu} = \varphi \mathbf{n} = \boldsymbol{\nu}_{\text{config}} + \boldsymbol{\nu}_{\text{ref}}, \quad (7.12)$$

with the rotation angle  $\varphi$  and the rotation axis  $\mathbf{n}$ . All coordinates  $\mathbf{c}_{\text{config}}$  lead to second order differential equations, however the rotation vector cannot be used as a conventional parameterization. It must be computed within a nonlinear update, using appropriate Lie group methods.

The rotation matrix  ${}^{0b}\mathbf{A}_{\text{config}}$  transforms local (body-fixed) 3D positions  ${}^b\mathbf{p} = {}^b[p_0, p_1, p_2]^T$  to global 3D positions,

$${}^0\mathbf{p}_{\text{config}} = {}^{0b}\mathbf{A}_{\text{config}} {}^b\mathbf{p} \quad (7.13)$$

A Lie group integrator must be used with this node, which is why the is used, the rotation parameter velocities are identical to the local angular velocity  ${}^b\boldsymbol{\omega}$  and thus the matrix  ${}^b\mathbf{G}$  becomes the identity matrix.

For creating a `NodeRigidBodyRotVecLG`, there is a `rigidBodyUtilities` function `AddRigidBody`, see [Section 5.11](#), which simplifies the setup of a rigid body significantly!

For further examples on `NodeRigidBodyRotVecLG` see `TestModels`:

- [`explicitLieGroupIntegratorPythonTest.py`](#)
- [`explicitLieGroupIntegratorTest.py`](#)
- [`explicitLieGroupMBSTest.py`](#)

### 7.1.6 NodeRigidBody2D

A 2D rigid body node for rigid bodies or beams; the node has 2 displacement degrees of freedom and one rotation coordinate (rotation around z-axis: uphi). All coordinates are ODE2, used for second order differential equations.

**Additional information for NodeRigidBody2D:**

- The Node has the following types = Position2D, Orientation2D, Position, Orientation, RigidBody
- **Short name for Python** = **Rigid2D**
- **Short name for Python** (visualization object) = **VRigid2D**

The item **NodeRigidBody2D** with type = 'RigidBody2D' has the following parameters:

| Name                 | type             | size | default value | description                                                                                                                                          |
|----------------------|------------------|------|---------------|------------------------------------------------------------------------------------------------------------------------------------------------------|
| name                 | String           |      | "             | node's unique name                                                                                                                                   |
| referenceCoordinates | Vector3D         | 3    | [0.,0.,0.]    | reference coordinates (x-pos,y-pos and rotation) of node ==> e.g. ref. coordinates for finite elements; global position of node without displacement |
| initialCoordinates   | Vector3D         | 3    | [0.,0.,0.]    | initial displacement coordinates and angle (relative to reference coordinates)                                                                       |
| initialVelocities    | Vector3D         | 3    | [0.,0.,0.]    | initial velocity coordinates                                                                                                                         |
| visualization        | VNodeRigidBody2D |      |               | parameters for visualization of item                                                                                                                 |

The item VNodeRigidBody2D has the following parameters:

| Name     | type   | size | default value     | description                                                                                                         |
|----------|--------|------|-------------------|---------------------------------------------------------------------------------------------------------------------|
| show     | Bool   |      | True              | set true, if item is shown in visualization and false if it is not shown                                            |
| drawSize | float  |      | -1.               | drawing size (diameter, dimensions of underlying cube, etc.) for item; size == -1.f means that default size is used |
| color    | Float4 | 4    | [-1.,-1.,-1.,-1.] | Default RGBA color for nodes; 4th value is alpha-transparency; R=-1.f means, that default color is used             |

#### 7.1.6.1 DESCRIPTION of NodeRigidBody2D:

**Information on input parameters:**

| input parameter      | symbol                                                                                                                      | description see tables above |
|----------------------|-----------------------------------------------------------------------------------------------------------------------------|------------------------------|
| referenceCoordinates | $\mathbf{q}_{\text{ref}} = [q_0, q_1, \psi_0]_{\text{ref}}^T$                                                               |                              |
| initialCoordinates   | $\mathbf{q}_{\text{ini}} = [q_0, q_1, \psi_0]_{\text{ini}}^T$                                                               |                              |
| initialVelocities    | $\dot{\mathbf{q}}_{\text{ini}} = [\dot{q}_0, \dot{q}_1, \dot{\psi}_0]_{\text{ini}}^T = [v_0, v_1, \omega_2]_{\text{ini}}^T$ |                              |

The following output variables are available as OutputVariableType in sensors, Get...Output() and other functions:

| output variable      | symbol                                                                                                                                | description                                                                                                                                                                                             |
|----------------------|---------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Position             | ${}^0\mathbf{p}_{\text{config}} = {}^0[p_0, p_1, 0]^T_{\text{config}} = {}^0\mathbf{u}_{\text{config}} + {}^0\mathbf{p}_{\text{ref}}$ | global 3D position vector of node; $\mathbf{u}_{\text{ref}} = 0$                                                                                                                                        |
| Displacement         | ${}^0\mathbf{u}_{\text{config}} = [q_0, q_1, 0]^T_{\text{config}}$                                                                    | global 3D displacement vector of node                                                                                                                                                                   |
| Velocity             | ${}^0\mathbf{v}_{\text{config}} = [\dot{q}_0, \dot{q}_1, 0]^T_{\text{config}}$                                                        | global 3D velocity vector of node                                                                                                                                                                       |
| Acceleration         | ${}^0\mathbf{a}_{\text{config}} = [\ddot{q}_0, \ddot{q}_1, 0]^T_{\text{config}}$                                                      | global 3D acceleration vector of node                                                                                                                                                                   |
| AngularVelocity      | ${}^0\boldsymbol{\omega}_{\text{config}} = [0, 0, \dot{\psi}_0]^T_{\text{config}}$                                                    | global 3D angular velocity vector of node                                                                                                                                                               |
| Coordinates          | $\mathbf{c}_{\text{config}} = [q_0, q_1, \psi_0]^T_{\text{config}}$                                                                   | coordinate vector of node, having 2 displacement coordinates and 1 angle                                                                                                                                |
| Coordinates_t        | $\dot{\mathbf{c}}_{\text{config}} = [\dot{q}_0, \dot{q}_1, \dot{\psi}_0]^T_{\text{config}}$                                           | velocity coordinates vector of node                                                                                                                                                                     |
| Coordinates_tt       | $\ddot{\mathbf{c}}_{\text{config}} = [\ddot{q}_0, \ddot{q}_1, \ddot{\psi}_0]^T_{\text{config}}$                                       | acceleration coordinates vector of node                                                                                                                                                                 |
| RotationMatrix       | $[A_{00}, A_{01}, A_{02}, A_{10}, \dots, A_{21}, A_{22}]^T_{\text{config}}$                                                           | vector with 9 components of the rotation matrix ${}^0b\mathbf{A}_{\text{config}}$ in row-major format, in any configuration; the rotation matrix transforms local ( $b$ ) to global ( $0$ ) coordinates |
| Rotation             | $[\theta_0]^T_{\text{config}} = [\psi_0]^T_{\text{ref}} + [\psi_0]^T_{\text{config}}$                                                 | vector with 1 angle around out of plane axis                                                                                                                                                            |
| AngularVelocityLocal | ${}^b\boldsymbol{\omega}_{\text{config}} = [0, 0, \dot{\psi}_0]^T_{\text{config}}$                                                    | local (body-fixed) 3D angular velocity vector of node                                                                                                                                                   |
| AngularAcceleration  | ${}^0\boldsymbol{\alpha}_{\text{config}} = [0, 0, \ddot{\psi}_0]^T_{\text{config}}$                                                   | global 3D angular acceleration vector of node                                                                                                                                                           |

**Detailed information:** The node provides 2 displacement coordinates (displacement of center of mass, COM,  $(q_0, q_1)$ ) and 1 rotation parameter ( $\theta_0$ ). According equations need to be provided by an according object (e.g., RigidBody2D). Using the rotation parameter  $\theta_{0\text{config}} = \psi_{0\text{ref}} + \psi_{0\text{config}}$ , the rotation matrix is defined as

$${}^0b\mathbf{A}_{\text{config}} = \begin{bmatrix} \cos(\theta_0) & -\sin(\theta_0) & 0 \\ \sin(\theta_0) & \cos(\theta_0) & 0 \\ 0 & 0 & 1 \end{bmatrix}_{\text{config}} \quad (7.14)$$

**Example** for NodeRigidBody2D: see ObjectRigidBody2D

For further examples on NodeRigidBody2D see Examples:

- [sliderCrank3DwithANCFbeltDrive2.py](#)
- [ANCF\\_moving\\_rigidbody.py](#)
- [ANCF\\_slidingJoint2D.py](#)
- [ANCF\\_switchingSlidingJoint2D.py](#)
- [geneticOptimizationSliderCrank.py](#)
- [lavalRotor2Dtest.py](#)
- [rigid\\_pendulum.py](#)
- [SliderCrank.py](#)
- [sliderCrank3DwithANCFbeltDrive.py](#)
- [slidercrankWithMassSpring.py](#)
- ...

For further examples on NodeRigidBody2D see TestModels:

- [scissorPrismaticRevolute2D.py](#)

- [ACNFslidingAndALEjointTest.py](#)
- [ANCFcontactFrictionTest.py](#)
- [ANCFmovingRigidBodyTest.py](#)
- [compareFullModifiedNewton.py](#)
- [geometricallyExactBeam2Dtest.py](#)
- [modelUnitTests.py](#)
- [PARTS\\_ATEs\\_moving.py](#)
- [pendulumFriction.py](#)
- [sliderCrankFloatingTest.py](#)
- ...

### 7.1.7 Node1D

A node with one ODE2 coordinate for one dimensional (1D) problems; use e.g. for scalar dynamic equations (Mass1D) and mass-spring-damper mechanisms, representing either translational or rotational degrees of freedom: in most cases, Node1D is equivalent to NodeGenericODE2 using one coordinate, however, it offers a transformation to 3D translational or rotational motion and allows to couple this node to 2D or 3D bodies.

#### Additional information for Node1D:

- The Node has the following types = GenericODE2

The item **Node1D** with type = '1D' has the following parameters:

| Name                 | type    | size | default value | description                                      |
|----------------------|---------|------|---------------|--------------------------------------------------|
| name                 | String  |      | "             | node's unique name                               |
| referenceCoordinates | Vector  |      | [0.]          | reference coordinate of node (in vector form)    |
| initialCoordinates   | Vector  |      | [0.]          | initial displacement coordinate (in vector form) |
| initialVelocities    | Vector  |      | [0.]          | initial velocity coordinate (in vector form)     |
| visualization        | VNode1D |      |               | parameters for visualization of item             |

The item VNode1D has the following parameters:

| Name | type | size | default value | description                                                                                                                                                                                                                                     |
|------|------|------|---------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| show | Bool |      | False         | set true, if item is shown in visualization and false if it is not shown; The node1D is represented as reference position and displacement along the global x-axis, which must not agree with the representation in the object using the Node1D |

#### 7.1.7.1 DESCRIPTION of Node1D:

##### Information on input parameters:

| input parameter      | symbol                       | description see tables above |
|----------------------|------------------------------|------------------------------|
| referenceCoordinates | $[q_0]_{\text{ref}}^T$       |                              |
| initialCoordinates   | $[q_0]_{\text{ini}}^T$       |                              |
| initialVelocities    | $[\dot{q}_0]_{\text{ini}}^T$ |                              |

The following output variables are available as OutputVariableType in sensors, Get...Output() and other functions:

| output variable | symbol                                                             | description                                       |
|-----------------|--------------------------------------------------------------------|---------------------------------------------------|
| Coordinates     | $\mathbf{q}_{\text{config}} = [q_0]_{\text{config}}^T$             | ODE2 coordinate of node (in vector form)          |
| Coordinates_t   | $\dot{\mathbf{q}}_{\text{config}} = [\dot{q}_0]_{\text{config}}^T$ | ODE2 velocity coordinate of node (in vector form) |

|                |                                                                     |                                                       |
|----------------|---------------------------------------------------------------------|-------------------------------------------------------|
| Coordinates_tt | $\ddot{\mathbf{q}}_{\text{config}} = [\dot{q}_0]^T_{\text{config}}$ | ODE2 acceleration coordinate of node (in vector form) |
|----------------|---------------------------------------------------------------------|-------------------------------------------------------|

**Detailed information:** The current position/rotation coordinate of the 1D node is computed from

$$p_0 = q_{0\text{ref}} + q_{0\text{cur}} \quad (7.15)$$

The coordinate leads to one second order differential equation. The graphical representation and the (internal) position of the node is

$$p_{\text{config}} = \begin{bmatrix} p_{0\text{config}} \\ 0 \\ 0 \end{bmatrix} \quad (7.16)$$

The (internal) velocity vector is  $[p_{0\text{config}}, 0, 0]^T$ .

For further examples on Node1D see Examples:

- [multiprocessingTest.py](#)
- [sliderCrankCMSacme.py](#)

For further examples on Node1D see TestModels:

- [driveTrainTest.py](#)

### 7.1.8 NodePoint2DSlope1

A 2D point/slope vector node for planar Bernoulli-Euler ANCF (absolute nodal coordinate formulation) beam elements; the node has 4 displacement degrees of freedom (2 for displacement of point node and 2 for the slope vector 'slopex'); all coordinates lead to second order differential equations; the slope vector defines the directional derivative w.r.t the local axial (x) coordinate, denoted as ()'; in straight configuration aligned at the global x-axis, the slope vector reads  $\mathbf{r}' = [r'_x \ r'_y]^T = [1 \ 0]^T$ .

#### Additional information for NodePoint2DSlope1:

- The Node has the following types = Position2D, Orientation2D, Point2DSlope1, Position, Orientation
- Short name for Python = **Point2DS1**
- Short name for Python (visualization object) = **VPoint2DS1**

The item **NodePoint2DSlope1** with type = 'Point2DSlope1' has the following parameters:

| Name                 | type               | size | default value | description                                                                                                   |
|----------------------|--------------------|------|---------------|---------------------------------------------------------------------------------------------------------------|
| name                 | String             |      | "             | node's unique name                                                                                            |
| referenceCoordinates | Vector4D           | 4    | [0.,0.,1.,0.] | reference coordinates (x-pos,y-pos; x-slopex, y-slopex) of node; global position of node without displacement |
| initialCoordinates   | Vector4D           | 4    | [0.,0.,0.,0.] | initial displacement coordinates: ux, uy and x/y 'displacements' of slopex                                    |
| initialVelocities    | Vector4D           | 4    | [0.,0.,0.,0.] | initial velocity coordinates                                                                                  |
| visualization        | VNodePoint2DSlope1 |      |               | parameters for visualization of item                                                                          |

The item VNodePoint2DSlope1 has the following parameters:

| Name     | type   | size | default value     | description                                                                                                         |
|----------|--------|------|-------------------|---------------------------------------------------------------------------------------------------------------------|
| show     | Bool   |      | True              | set true, if item is shown in visualization and false if it is not shown                                            |
| drawSize | float  |      | -1.               | drawing size (diameter, dimensions of underlying cube, etc.) for item; size == -1.f means that default size is used |
| color    | Float4 | 4    | [-1.,-1.,-1.,-1.] | Default RGBA color for nodes; 4th value is alpha-transparency; R=-1.f means, that default color is used             |

---

#### 7.1.8.1 DESCRIPTION of NodePoint2DSlope1:

The following output variables are available as **OutputVariableType** in sensors, **Get...Output()** and other functions:

| output variable | symbol | description                                                          |
|-----------------|--------|----------------------------------------------------------------------|
| Position        |        | global 3D position vector of node (=displacement+reference position) |
| Displacement    |        | global 3D displacement vector of node                                |

|                |  |                                                                                                                     |
|----------------|--|---------------------------------------------------------------------------------------------------------------------|
| Velocity       |  | global 3D velocity vector of node                                                                                   |
| Coordinates    |  | coordinates vector of node (2 displacement coordinates + 2 slope vector coordinates)                                |
| Coordinates_t  |  | velocity coordinates vector of node (derivative of the 2 displacement coordinates + 2 slope vector coordinates)     |
| Coordinates_tt |  | acceleration coordinates vector of node (derivative of the 2 displacement coordinates + 2 slope vector coordinates) |

---

For further examples on NodePoint2DSlope1 see Examples:

- [sliderCrank3DwithANCFbeltDrive2.py](#)
- [ALEANCF\\_pipe.py](#)
- [ANCF\\_cantilever\\_test\\_dyn.py](#)
- [ANCF\\_contact\\_circle.py](#)
- [ANCF\\_contact\\_circle2.py](#)
- [ANCF\\_moving\\_rigidbody.py](#)
- [ANCF\\_slidingJoint2D.py](#)
- [ANCF\\_switchingSlidingJoint2D.py](#)
- [ANCF\\_tests2.py](#)
- [ANCF\\_test\\_halfcircle.py](#)
- ...

For further examples on NodePoint2DSlope1 see TestModels:

- [ANCFcontactCircleTest.py](#)
- [ANCFcontactFrictionTest.py](#)
- [computeODE2EigenvaluesTest.py](#)
- [manualExplicitIntegrator.py](#)
- [modelUnitTests.py](#)

### 7.1.9 NodeGenericODE2

A node containing a number of ODE2 variables; use e.g. for scalar dynamic equations (Mass1D) or for the ALECable element. Note that referenceCoordinates and all initialCoordinates(\_t) must be initialized, because no default values exist.

#### Additional information for NodeGenericODE2:

- The Node has the following types = GenericODE2

The item **NodeGenericODE2** with type = 'GenericODE2' has the following parameters:

| Name                    | type             | size | default value | description                                                                            |
|-------------------------|------------------|------|---------------|----------------------------------------------------------------------------------------|
| name                    | String           |      | ''            | node's unique name                                                                     |
| referenceCoordinates    | Vector           |      | []            | generic reference coordinates of node; must be consistent with numberOfODE2Coordinates |
| initialCoordinates      | Vector           |      | []            | initial displacement coordinates; must be consistent with numberOfODE2Coordinates      |
| initialCoordinates_t    | Vector           |      | []            | initial velocity coordinates; must be consistent with numberOfODE2Coordinates          |
| numberOfODE2Coordinates | PInt             |      | 0             | number of generic ODE2 coordinates                                                     |
| visualization           | VNodeGenericODE2 |      |               | parameters for visualization of item                                                   |

The item VNodeGenericODE2 has the following parameters:

| Name | type | size | default value | description                                                              |
|------|------|------|---------------|--------------------------------------------------------------------------|
| show | Bool |      | False         | set true, if item is shown in visualization and false if it is not shown |

#### 7.1.9.1 DESCRIPTION of NodeGenericODE2:

##### Information on input parameters:

| input parameter         | symbol                                                                            | description see tables above |
|-------------------------|-----------------------------------------------------------------------------------|------------------------------|
| referenceCoordinates    | $\mathbf{q}_{\text{ref}} = [q_0, \dots, q_{nc}]^T_{\text{ref}}$                   |                              |
| initialCoordinates      | $\mathbf{q}_{\text{ini}} = [q_0, \dots, q_{nc}]^T_{\text{ini}}$                   |                              |
| initialCoordinates_t    | $\dot{\mathbf{q}}_{\text{ini}} = [\dot{q}_0, \dots, \dot{q}_{nc}]^T_{\text{ini}}$ |                              |
| numberOfODE2Coordinates | $n_c$                                                                             |                              |

The following output variables are available as OutputVariableType in sensors, Get...Output() and other functions:

| output variable | symbol                                                                                     | description                             |
|-----------------|--------------------------------------------------------------------------------------------|-----------------------------------------|
| Coordinates     | $\mathbf{q}_{\text{config}} = [q_0, \dots, q_{nc}]^T_{\text{config}}$                      | coordinates vector of node              |
| Coordinates_t   | $\dot{\mathbf{q}}_{\text{config}} = [\dot{q}_0, \dots, \dot{q}_{nc}]^T_{\text{config}}$    | velocity coordinates vector of node     |
| Coordinates_tt  | $\ddot{\mathbf{q}}_{\text{config}} = [\ddot{q}_0, \dots, \ddot{q}_{nc}]^T_{\text{config}}$ | acceleration coordinates vector of node |

---

For further examples on NodeGenericODE2 see Examples:

- [ALEANCF\\_pipe.py](#)
- [ANCFALEtest.py](#)
- [ANCF\\_moving\\_rigidbody.py](#)
- [simulateInteractively.py](#)

For further examples on NodeGenericODE2 see TestModels:

- [ACNFslidingAndALEjointTest.py](#)
- [ANCFmovingRigidBodyTest.py](#)
- [solverExplicitODE1ODE2test.py](#)

### 7.1.10 NodeGenericODE1

A node containing a number of ODE1 variables; use e.g. linear state space systems. Note that referenceCoordinates and initialCoordinates must be initialized, because no default values exist.

The item **NodeGenericODE1** with type = 'GenericODE1' has the following parameters:

| Name                    | type             | size | default value | description                                                                            |
|-------------------------|------------------|------|---------------|----------------------------------------------------------------------------------------|
| name                    | String           |      | "             | node's unique name                                                                     |
| referenceCoordinates    | Vector           |      | []            | generic reference coordinates of node; must be consistent with numberOfODE1Coordinates |
| initialCoordinates      | Vector           |      | []            | initial displacement coordinates; must be consistent with numberOfODE1Coordinates      |
| numberOfODE1Coordinates | PInt             |      | 0             | number of generic ODE1 coordinates                                                     |
| visualization           | VNodeGenericODE1 |      |               | parameters for visualization of item                                                   |

The item VNodeGenericODE1 has the following parameters:

| Name | type | size | default value | description                                                              |
|------|------|------|---------------|--------------------------------------------------------------------------|
| show | Bool |      | False         | set true, if item is shown in visualization and false if it is not shown |

#### 7.1.10.1 DESCRIPTION of NodeGenericODE1:

**Information on input parameters:**

| input parameter         | symbol                                   | description see tables above |
|-------------------------|------------------------------------------|------------------------------|
| referenceCoordinates    | $y_{ref} = [y_0, \dots, y_{nc}]^T_{ref}$ |                              |
| initialCoordinates      | $y_{ini} = [y_0, \dots, y_{nc}]^T_{ini}$ |                              |
| numberOfODE1Coordinates | $n_c$                                    |                              |

**The following output variables are available as OutputVariableType in sensors, Get...Output() and other functions:**

| output variable | symbol                                                           | description                              |
|-----------------|------------------------------------------------------------------|------------------------------------------|
| Coordinates     | $y_{config} = [y_0, \dots, y_{nc}]^T_{config}$                   | ODE1 coordinates vector of node          |
| Coordinates_t   | $\dot{y}_{config} = [\dot{y}_0, \dots, \dot{y}_{nc}]^T_{config}$ | ODE1 velocity coordinates vector of node |

For further examples on NodeGenericODE1 see TestModels:

- [solverExplicitODE1ODE2test.py](#)

### 7.1.11 NodeGenericData

A node containing a number of data (history) variables; use e.g. for contact (active set), friction or plasticity (history variable).

**Additional information for NodeGenericData:**

- The Node has the following types = GenericData

The item **NodeGenericData** with type = 'GenericData' has the following parameters:

| Name                    | type             | size | default value | description                                            |
|-------------------------|------------------|------|---------------|--------------------------------------------------------|
| name                    | String           |      | "             | node's unique name                                     |
| initialCoordinates      | Vector           |      | []            | initial data coordinates                               |
| numberOfDataCoordinates | UInt             |      | 0             | number of generic data coordinates (history variables) |
| visualization           | VNodeGenericData |      |               | parameters for visualization of item                   |

The item **VNodeGenericData** has the following parameters:

| Name | type | size | default value | description                                                              |
|------|------|------|---------------|--------------------------------------------------------------------------|
| show | Bool |      | False         | set true, if item is shown in visualization and false if it is not shown |

#### 7.1.11.1 DESCRIPTION of NodeGenericData:

**Information on input parameters:**

| input parameter         | symbol                                                           | description see tables above |
|-------------------------|------------------------------------------------------------------|------------------------------|
| initialCoordinates      | $\mathbf{x}_{\text{ini}} = [x_0, \dots, x_{n_c}]_{\text{ini}}^T$ |                              |
| numberOfDataCoordinates | $n_c$                                                            |                              |

**The following output variables are available as OutputVariableType in sensors, Get...Output() and other functions:**

| output variable | symbol                                                                 | description                                         |
|-----------------|------------------------------------------------------------------------|-----------------------------------------------------|
| Coordinates     | $\mathbf{x}_{\text{config}} = [x_0, \dots, x_{n_c}]_{\text{config}}^T$ | data coordinates (history variables) vector of node |

For further examples on NodeGenericData see Examples:

- [ANCF\\_contact\\_circle.py](#)
- [ANCF\\_contact\\_circle2.py](#)
- [ANCF\\_moving\\_rigidbody.py](#)
- [ANCF\\_slidingJoint2D.py](#)
- [ANCF\\_switchingSlidingJoint2D.py](#)
- [sliderCrank3DwithANCFbeltDrive.py](#)

- [sliderCrank3DwithANCFbeltDrive2.py](#)

For further examples on NodeGenericData see TestModels:

- [ACNFslidingAndALEjointTest.py](#)
- [ANCFcontactCircleTest.py](#)
- [ANCFcontactFrictionTest.py](#)
- [ANCFmovingRigidBodyTest.py](#)
- [carRollingDiscTest.py](#)
- [mecanumWheelRollingDiscTest.py](#)
- [modelUnitTests.py](#)
- [postNewtonStepContactTest.py](#)
- [rollingCoinPenaltyTest.py](#)

### 7.1.12 NodePointGround

A 3D point node fixed to ground. The node can be used as NodePoint, but it does not generate coordinates. Applied or reaction forces do not have any effect. This node can be used for 'blind' or 'dummy' ODE2 and ODE1 coordinates to which CoordinateSpringDamper or CoordinateConstraint objects are attached to.

**Additional information for NodePointGround:**

- The Node has the following types = Ground, Position2D, Position, GenericODE2
- **Short name for Python = PointGround**
- **Short name for Python (visualization object) = VPointGround**

The item **NodePointGround** with type = 'PointGround' has the following parameters:

| Name                 | type             | size | default value | description                                                                                                               |
|----------------------|------------------|------|---------------|---------------------------------------------------------------------------------------------------------------------------|
| name                 | String           |      | "             | node's unique name                                                                                                        |
| referenceCoordinates | Vector3D         | 3    | [0.,0.,0.]    | reference coordinates of node ==> e.g. ref. coordinates for finite elements; global position of node without displacement |
| visualization        | VNodePointGround |      |               | parameters for visualization of item                                                                                      |

The item **VNodePointGround** has the following parameters:

| Name     | type   | size | default value     | description                                                                                                         |
|----------|--------|------|-------------------|---------------------------------------------------------------------------------------------------------------------|
| show     | Bool   |      | True              | set true, if item is shown in visualization and false if it is not shown                                            |
| drawSize | float  |      | -1.               | drawing size (diameter, dimensions of underlying cube, etc.) for item; size == -1.f means that default size is used |
| color    | Float4 | 4    | [-1.,-1.,-1.,-1.] | Default RGBA color for nodes; 4th value is alpha-transparency; R=-1.f means, that default color is used             |

#### 7.1.12.1 DESCRIPTION of NodePointGround:

**Information on input parameters:**

| input parameter      | symbol                                                                                                   | description see tables above |
|----------------------|----------------------------------------------------------------------------------------------------------|------------------------------|
| referenceCoordinates | $\mathbf{q}_{\text{ref}} = [q_0, q_1, q_2]^T_{\text{ref}} = \mathbf{p}_{\text{ref}} = [r_0, r_1, r_2]^T$ |                              |

**The following output variables are available as OutputVariableType in sensors, Get...Output() and other functions:**

| output variable | symbol                                                                                     | description                                             |
|-----------------|--------------------------------------------------------------------------------------------|---------------------------------------------------------|
| Position        | $\mathbf{p}_{\text{config}} = [p_0, p_1, p_2]^T_{\text{config}} = \mathbf{p}_{\text{ref}}$ | global 3D position vector of node (=reference position) |
| Displacement    | $\mathbf{u}_{\text{config}} = [0, 0, 0]^T_{\text{config}}$                                 | zero 3D vector                                          |

|               |                                                            |                       |
|---------------|------------------------------------------------------------|-----------------------|
| Velocity      | $\mathbf{v}_{\text{config}} = [0, 0, 0]^T_{\text{config}}$ | zero 3D vector        |
| Coordinates   | $\mathbf{c}_{\text{config}} = []$                          | vector of length zero |
| Coordinates_t | $\dot{\mathbf{c}}_{\text{config}} = []$                    | vector of length zero |

---

For further examples on NodePointGround see Examples:

- [ALEANCF\\_pipe.py](#)
- [ANCFALEtest.py](#)
- [ANCF\\_cantilever\\_test.py](#)
- [ANCF\\_cantilever\\_test\\_dyn.py](#)
- [ANCF\\_contact\\_circle.py](#)
- [ANCF\\_contact\\_circle2.py](#)
- [ANCF\\_moving\\_rigidbody.py](#)
- [ANCF\\_slidingJoint2D.py](#)
- [ANCF\\_switchingSlidingJoint2D.py](#)
- [ANCF\\_tests2.py](#)
- ...

For further examples on NodePointGround see TestModels:

- [ACNFslidingAndALEjointTest.py](#)
- [ANCFcontactCircleTest.py](#)
- [ANCFcontactFrictionTest.py](#)
- [ANCFmovingRigidBodyTest.py](#)
- [computeODE2EigenvaluesTest.py](#)
- [connectorRigidBodySpringDamperTest.py](#)
- [geneticOptimizationTest.py](#)
- [geometricallyExactBeam2Dtest.py](#)
- [manualExplicitIntegrator.py](#)
- [modelUnitTests.py](#)
- ...

## 7.2 Objects

### 7.2.1 ObjectGround

A ground object behaving like a rigid body, but having no degrees of freedom; used to attach body-connectors without an action. For examples see spring dampers and joints.

#### Additional information for ObjectGround:

- The Object has the following types = Ground, Body

The item **ObjectGround** with type = 'Ground' has the following parameters:

| Name              | type          | size | default value | description                                                                                                    |
|-------------------|---------------|------|---------------|----------------------------------------------------------------------------------------------------------------|
| name              | String        |      | "             | objects's unique name                                                                                          |
| referencePosition | Vector3D      | 3    | [0.,0.,0.]    | reference position for ground object; local position is added on top of reference position for a ground object |
| visualization     | VObjectGround |      |               | parameters for visualization of item                                                                           |

The item **VObjectGround** has the following parameters:

| Name                     | type                   | size | default value     | description                                                                                                                                |
|--------------------------|------------------------|------|-------------------|--------------------------------------------------------------------------------------------------------------------------------------------|
| show                     | Bool                   |      | True              | set true, if item is shown in visualization and false if it is not shown                                                                   |
| graphicsDataUserFunction | PyFunctionGraphicsData | 0    |                   | A python function which returns a body-GraphicsData object, which is a list of graphics data in a dictionary computed by the user function |
| color                    | Float4                 |      | [-1.,-1.,-1.,-1.] | RGB node color; if R== -1, use default color                                                                                               |
| graphicsData             | BodyGraphicsData       |      |                   | Structure contains data for body visualization; data is defined in special list / dictionary structure                                     |

#### 7.2.1.1 DESCRIPTION of ObjectGround:

The following output variables are available as **OutputVariableType** in sensors, **Get...Output()** and other functions:

| output variable | symbol | description                                                     |
|-----------------|--------|-----------------------------------------------------------------|
| Position        |        | global position vector of rotated and translated local position |
| Displacement    |        | global displacement vector of local position                    |
| Velocity        |        | global velocity vector of local position                        |
| AngularVelocity |        | angular velocity of body                                        |
| RotationMatrix  |        | rotation matrix in vector form (stored in row-major order)      |

### 7.2.1.2 Equations

ObjectGround has no equations, as it only provides a static object, at which joints and connectors can be attached. The object cannot move and forces or torques do not have an effect.

---

#### Userfunction: `graphicsDataUserFunction(mbs, itemNumber)`

A user function, which is called by the visualization thread in order to draw user-defined objects. The function can be used to generate any BodyGraphicsData, see Section 9.3. Use `graphicsDataUtilities` functions, see Section 5.4, to create more complicated objects. Note that `graphicsDataUserFunction` needs to copy lots of data and is therefore inefficient and only designed to enable simpler tests, but not large scale problems.

| arguments / return        | type or size     | description                                                                                    |
|---------------------------|------------------|------------------------------------------------------------------------------------------------|
| <code>mbs</code>          | MainSystem       | provides reference to mbs, which can be used in user function to access all data of the object |
| <code>itemNumber</code>   | Index            | integer number of the object in mbs, allowing easy access                                      |
| <code>return value</code> | BodyGraphicsData | list of GraphicsData dictionaries, see Section 9.3                                             |

---

#### User function example:

```
import exudyn as exu
from math import sin, cos, pi
from exudyn.itemInterface import *
from exudyn.graphicsDataUtilities import *
SC = exu.SystemContainer()
mbs = SC.AddSystem()
#create simple system:
mbs.AddNode(NodePoint())
body = mbs.AddObject(MassPoint(physicsMass=1, nodeNumber=0))

#user function for moving graphics:
def UFgraphics(mbs, objectNum):
 t = mbs.systemData.GetTime(exu.ConfigurationType.Visualization) #get time if needed
 #draw moving sphere on ground
 graphics1=GraphicsDataSphere(point=[sin(t*2*pi), cos(t*2*pi), 0],
 radius=0.1, color=color4red, nTiles=32)
 return [graphics1]

#add object with graphics user function
ground = mbs.AddObject(ObjectGround(visualization=VObjectGround(
 graphicsDataUserFunction=UFgraphics)))
mbs.Assemble()
sim=exu.SimulationSettings()
```

---

```

sims.timeIntegration.numberOfSteps = 10000000 #many steps to see graphics
exu.StartRenderer() #perform zoom all (press 'a' several times) after startup to see
 the sphere
exu.SolveDynamic(mbs, sims)
exu.StopRenderer()

```

---

For further examples on ObjectGround see Examples:

- [ALEANCF\\_pipe.py](#)
- [ANCF\\_cantilever\\_test.py](#)
- [ANCF\\_cantilever\\_test\\_dyn.py](#)
- [ANCF\\_contact\\_circle.py](#)
- [ANCF\\_contact\\_circle2.py](#)
- [ANCF\\_moving\\_rigidbody.py](#)
- [ANCF\\_slidingJoint2D.py](#)
- [ANCF\\_switchingSlidingJoint2D.py](#)
- [ANCF\\_tests2.py](#)
- [ANCF\\_test\\_halfcircle.py](#)
- ...

For further examples on ObjectGround see TestModels:

- [ACNFslidingAndALEjointTest.py](#)
- [ANCFcontactCircleTest.py](#)
- [ANCFcontactFrictionTest.py](#)
- [ANCFmovingRigidBodyTest.py](#)
- [carRollingDiscTest.py](#)
- [compareFullModifiedNewton.py](#)
- [connectorRigidBodySpringDamperTest.py](#)
- [driveTrainTest.py](#)
- [explicitLieGroupIntegratorPythonTest.py](#)
- [explicitLieGroupIntegratorTest.py](#)
- ...

## 7.2.2 ObjectMassPoint

A 3D mass point which is attached to a position-based node, usually NodePoint.

**Additional information for ObjectMassPoint:**

- The Object has the following types = Body, SingleNoded
- Requested node type = Position
- Short name for Python = **MassPoint**
- Short name for Python (visualization object) = **VMassPoint**

The item **ObjectMassPoint** with type = 'MassPoint' has the following parameters:

| Name          | type             | size | default value | description                                 |
|---------------|------------------|------|---------------|---------------------------------------------|
| name          | String           |      | "             | objects's unique name                       |
| physicsMass   | UReal            |      | 0.            | mass [SI:kg] of mass point                  |
| nodeNumber    | NodeIndex        |      | MAXINT        | node number (type NodeIndex) for mass point |
| visualization | VObjectMassPoint |      |               | parameters for visualization of item        |

The item VObjectMassPoint has the following parameters:

| Name         | type             | size | default value | description                                                                                            |
|--------------|------------------|------|---------------|--------------------------------------------------------------------------------------------------------|
| show         | Bool             |      | True          | set true, if item is shown in visualization and false if it is not shown                               |
| graphicsData | BodyGraphicsData |      |               | Structure contains data for body visualization; data is defined in special list / dictionary structure |

### 7.2.2.1 DESCRIPTION of ObjectMassPoint:

**Information on input parameters:**

| input parameter | symbol | description see tables above |
|-----------------|--------|------------------------------|
| physicsMass     | $m$    |                              |
| nodeNumber      | $n_0$  |                              |

**The following output variables are available as OutputVariableType in sensors, Get...Output() and other functions:**

| output variable | symbol                                                                                                                                             | description                                                                                                    |
|-----------------|----------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------|
| Position        | ${}^0\mathbf{p}_{\text{config}}({}^b\mathbf{p}) = {}^0\mathbf{u}_{\text{config}} + {}^0\mathbf{p}_{\text{ref}} + {}^{0b}\mathbf{A} {}^b\mathbf{p}$ | global position vector of translated local position; local (body) coordinate system = global coordinate system |
| Displacement    | ${}^0\mathbf{u}_{\text{config}} = [q_0, q_1, q_2]^T_{\text{config}}$                                                                               | global displacement vector of mass point                                                                       |
| Velocity        | ${}^0\mathbf{v}_{\text{config}} = {}^0\dot{\mathbf{u}}_{\text{config}} = [\dot{q}_0, \dot{q}_1, \dot{q}_2]^T_{\text{config}}$                      | global velocity vector of mass point                                                                           |
| Acceleration    | ${}^0\mathbf{a}_{\text{config}} = {}^0\ddot{\mathbf{u}}_{\text{config}} = [\ddot{q}_0, \ddot{q}_1, \ddot{q}_2]^T_{\text{config}}$                  | global acceleration vector of mass point                                                                       |

### 7.2.2.2 Definition of quantities

| intermediate variables | symbol                                                                                                                       | description                                                                                                                                                         |
|------------------------|------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| node position          | ${}^0\mathbf{p}_{\text{config}} = {}^0\mathbf{p}(n_0)_{\text{config}}$                                                       | position of mass point which is provided by node $n_0$ in any configuration                                                                                         |
| node displacement      | ${}^0\mathbf{u}_{\text{config}} = [q_0, q_1, q_2]_{\text{config}}^T = {}^0\mathbf{u}(n_0)_{\text{config}}$                   | displacement of mass point which is provided by node $n_0$ in any configuration                                                                                     |
| node velocity          | ${}^0\mathbf{v}_{\text{config}} = [\dot{q}_0, \dot{q}_1, \dot{q}_2]_{\text{config}}^T = {}^0\mathbf{v}(n_0)_{\text{config}}$ | velocity of mass point which is provided by node $n_0$ in any configuration                                                                                         |
| transformation matrix  | ${}^{0b}\mathbf{A} = \mathbf{I}^{3 \times 3}$                                                                                | transformation of local body ( $b$ ) coordinates to global (0) coordinates; this is the constant unit matrix, because local = global coordinates for the mass point |
| residual forces        | ${}^0\mathbf{f} = [f_0, f_1, f_2]^T$                                                                                         | residual of all forces on mass point                                                                                                                                |
| applied forces         | ${}^0\mathbf{f}_a = [f_0, f_1, f_2]^T$                                                                                       | applied forces (loads, connectors, joint reaction forces, ...)                                                                                                      |

### 7.2.2.3 Equations of motion

$$\begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{bmatrix} \begin{bmatrix} \ddot{q}_0 \\ \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} = \begin{bmatrix} f_0 \\ f_1 \\ f_2 \end{bmatrix}. \quad (7.17)$$

For example, a LoadCoordinate on coordinate 1 of the node would add a term in  $f_1$  on the RHS.

Position-based markers can measure position  $\mathbf{p}_{\text{config}}$ . The **position jacobian**

$$\mathbf{J}_{\text{pos}} = \partial \mathbf{p}_{\text{cur}} / \partial \mathbf{c}_{\text{cur}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (7.18)$$

transforms the action of global applied forces  ${}^0\mathbf{f}_a$  of position-based markers on the coordinates  $\mathbf{c}$

$$\mathbf{Q} = \mathbf{J}_{\text{pos}} {}^0\mathbf{f}_a. \quad (7.19)$$

### 7.2.2.4 MINI EXAMPLE for ObjectMassPoint

```

node = mbs.AddNode(NodePoint(referenceCoordinates = [1,1,0],
 initialCoordinates=[0.5,0,0],
 initialVelocities=[0.5,0,0]))
mbs.AddObject(MassPoint(nodeNumber = node, physicsMass=1))

#assemble and solve system for default parameters
mbs.Assemble()
exu.SolveDynamic(mbs)

#check result

```

```
exodynTestGlobals.testResult = mbs.GetNodeOutput(node, exu.OutputVariableType.
Position)[0]
#final x-coordinate of position shall be 2
```

---

For further examples on ObjectMassPoint see Examples:

- [interactiveTutorial.py](#)
- [ANCF\\_slidingJoint2D.py](#)
- [cartesianSpringDamper.py](#)
- [coordinateSpringDamper.py](#)
- [geneticOptimizationExample.py](#)
- [geneticOptimizationSliderCrank.py](#)
- [massSpringFrictionInteractive.py](#)
- [parameterVariationExample.py](#)
- [pendulum2Dconstraint.py](#)
- [simulateInteractively.py](#)
- ...

For further examples on ObjectMassPoint see TestModels:

- [fourBarMechanismTest.py](#)
- [genericODE2test.py](#)
- [geneticOptimizationTest.py](#)
- [modelUnitTests.py](#)
- [postNewtonStepContactTest.py](#)
- [sensorUserFunctionTest.py](#)
- [sliderCrankFloatingTest.py](#)
- [sparseMatrixSpringDamperTest.py](#)
- [springDamperUserFunctionTest.py](#)

### 7.2.3 ObjectMassPoint2D

A 2D mass point which is attached to a position-based 2D node.

#### Additional information for ObjectMassPoint2D:

- The Object has the following types = Body, SingleNoded
- Requested node type = Position2D + Position
- Short name for Python = **MassPoint2D**
- Short name for Python (visualization object) = **VMassPoint2D**

The item **ObjectMassPoint2D** with type = 'MassPoint2D' has the following parameters:

| Name          | type               | size | default value | description                                 |
|---------------|--------------------|------|---------------|---------------------------------------------|
| name          | String             |      | "             | objects's unique name                       |
| physicsMass   | UReal              |      | 0.            | mass [SI:kg] of mass point                  |
| nodeNumber    | NodeIndex          |      | MAXINT        | node number (type NodeIndex) for mass point |
| visualization | VObjectMassPoint2D |      |               | parameters for visualization of item        |

The item **VObjectMassPoint2D** has the following parameters:

| Name         | type             | size | default value | description                                                                                            |
|--------------|------------------|------|---------------|--------------------------------------------------------------------------------------------------------|
| show         | Bool             |      | True          | set true, if item is shown in visualization and false if it is not shown                               |
| graphicsData | BodyGraphicsData |      |               | Structure contains data for body visualization; data is defined in special list / dictionary structure |

#### 7.2.3.1 DESCRIPTION of ObjectMassPoint2D:

##### Information on input parameters:

| input parameter | symbol | description see tables above |
|-----------------|--------|------------------------------|
| physicsMass     | $m$    |                              |
| nodeNumber      | $n_0$  |                              |

The following output variables are available as OutputVariableType in sensors, Get...Output() and other functions:

| output variable | symbol                                                                                                                                             | description                                                                                                    |
|-----------------|----------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------|
| Position        | ${}^0\mathbf{p}_{\text{config}}({}^b\mathbf{p}) = {}^0\mathbf{u}_{\text{config}} + {}^0\mathbf{p}_{\text{ref}} + {}^{0b}\mathbf{A} {}^b\mathbf{p}$ | global position vector of translated local position; local (body) coordinate system = global coordinate system |
| Displacement    | ${}^0\mathbf{u}_{\text{config}} = [q_0, q_1, 0]^T_{\text{config}}$                                                                                 | global displacement vector of mass point                                                                       |
| Velocity        | ${}^0\mathbf{v}_{\text{config}} = {}^0\dot{\mathbf{u}}_{\text{config}} = [\dot{q}_0, \dot{q}_1, 0]^T_{\text{config}}$                              | global velocity vector of mass point                                                                           |
| Acceleration    | ${}^0\mathbf{a}_{\text{config}} = {}^0\ddot{\mathbf{u}}_{\text{config}} = [\ddot{q}_0, \ddot{q}_1, 0]^T_{\text{config}}$                           | global acceleration vector of mass point                                                                       |

### 7.2.3.2 Definition of quantities

| intermediate variables | symbol                                                                                                               | description                                                                                                                                                             |
|------------------------|----------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| node position          | ${}^0\mathbf{p}_{\text{config}} = {}^0\mathbf{p}(n_0)_{\text{config}}$                                               | position of mass point which is provided by node $n_0$ in any configuration                                                                                             |
| node displacement      | ${}^0\mathbf{u}_{\text{config}} = [q_0, q_1, 0]^T_{\text{config}} = {}^0\mathbf{u}(n_0)_{\text{config}}$             | displacement of mass point which is provided by node $n_0$ in any configuration                                                                                         |
| node velocity          | ${}^0\mathbf{v}_{\text{config}} = [\dot{q}_0, \dot{q}_1, 0]^T_{\text{config}} = {}^0\mathbf{v}(n_0)_{\text{config}}$ | velocity of mass point which is provided by node $n_0$ in any configuration                                                                                             |
| transformation matrix  | ${}^0\mathbf{A} = \mathbf{I}^{3 \times 3}$                                                                           | transformation of local body ( $b$ ) coordinates to global ( $0$ ) coordinates; this is the constant unit matrix, because local = global coordinates for the mass point |
| residual forces        | ${}^0\mathbf{f} = [f_0, f_1]^T$                                                                                      | residual of all forces on mass point                                                                                                                                    |
| applied forces         | ${}^0\mathbf{f}_a = [f_0, f_1, f_2]^T$                                                                               | applied forces (loads, connectors, joint reaction forces, ...)                                                                                                          |

### 7.2.3.3 Equations of motion

$$\begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} \begin{bmatrix} \ddot{q}_0 \\ \ddot{q}_1 \end{bmatrix} = \begin{bmatrix} f_0 \\ f_1 \end{bmatrix}. \quad (7.20)$$

For example, a LoadCoordinate on coordinate 1 of the node would add a term in  $f_1$  on the RHS.

Position-based markers can measure position  $\mathbf{p}_{\text{config}}$ . The **position jacobian**

$$\mathbf{J}_{\text{pos}} = \partial \mathbf{p}_{\text{cur}} / \partial \mathbf{c}_{\text{cur}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad (7.21)$$

transforms the action of global applied forces  ${}^0\mathbf{f}_a$  of position-based markers on the coordinates  $\mathbf{c}$

$$\mathbf{Q} = \mathbf{J}_{\text{pos}} {}^0\mathbf{f}_a. \quad (7.22)$$

### 7.2.3.4 MINI EXAMPLE for ObjectMassPoint2D

```

node = mbs.AddNode(NodePoint2D(referenceCoordinates = [1,1],
 initialCoordinates=[0.5,0],
 initialVelocities=[0.5,0]))
mbs.AddObject(MassPoint2D(nodeNumber = node, physicsMass=1))

#assemble and solve system for default parameters
mbs.Assemble()
exu.SolveDynamic(mbs)

#check result
exudynTestGlobals.testResult = mbs.GetNodeOutput(node, exu.OutputVariableType.
 Position)[0]
#final x-coordinate of position shall be 2

```

---

For further examples on ObjectMassPoint2D see Examples:

- [myFirstExample.py](#)
- [ANCF\\_slidingJoint2D.py](#)
- [geneticOptimizationSliderCrank.py](#)
- [pendulum2Dconstraint.py](#)
- [SliderCrank.py](#)
- [sliderCrank3DwithANCFbeltDrive2.py](#)
- [slidercrankWithMassSpring.py](#)
- [SpringDamperMassUserFunction.py](#)
- [switchingConstraintsPendulum.py](#)

For further examples on ObjectMassPoint2D see TestModels:

- [modelUnitTests.py](#)
- [sliderCrankFloatingTest.py](#)
- [sparseMatrixSpringDamperTest.py](#)

## 7.2.4 ObjectMass1D

A 1D (translational) mass which is attached to Node1D. Note, that the mass does not need to have the interpretation as a translational mass.

**Additional information for ObjectMass1D:**

- The Object has the following types = Body, SingleNoded
- Requested node type = GenericODE2
- **Short name for Python** = **Mass1D**
- **Short name for Python (visualization object)** = **VMass1D**

The item **ObjectMass1D** with type = 'Mass1D' has the following parameters:

| Name              | type          | size | default value               | description                                                                                  |
|-------------------|---------------|------|-----------------------------|----------------------------------------------------------------------------------------------|
| name              | String        |      | "                           | objects's unique name                                                                        |
| physicsMass       | UReal         |      | 0.                          | mass [SI:kg] of mass                                                                         |
| nodeNumber        | NodeIndex     |      | MAXINT                      | node number (type NodeIndex) for Node1D                                                      |
| referencePosition | Vector3D      | 3    | [0.,0.,0.]                  | a reference position, used to transform the 1D coordinate to a position                      |
| referenceRotation | Matrix3D      |      | [[1,0,0], [0,1,0], [0,0,1]] | the constant body rotation matrix, which transforms body-fixed (b) to global (0) coordinates |
| visualization     | VObjectMass1D |      |                             | parameters for visualization of item                                                         |

The item VObjectMass1D has the following parameters:

| Name         | type             | size | default value | description                                                                                            |
|--------------|------------------|------|---------------|--------------------------------------------------------------------------------------------------------|
| show         | Bool             |      | True          | set true, if item is shown in visualization and false if it is not shown                               |
| graphicsData | BodyGraphicsData |      |               | Structure contains data for body visualization; data is defined in special list / dictionary structure |

---

### 7.2.4.1 DESCRIPTION of ObjectMass1D:

**Information on input parameters:**

| input parameter   | symbol                                            | description see tables above |
|-------------------|---------------------------------------------------|------------------------------|
| physicsMass       | $m$                                               |                              |
| nodeNumber        | $n_0$                                             |                              |
| referencePosition | ${}^0\mathbf{p}_0$                                |                              |
| referenceRotation | ${}^{0b}\mathbf{A}_0 \in \mathbb{R}^{3 \times 3}$ |                              |

**The following output variables are available as OutputVariableType in sensors, Get...Output() and other functions:**

| output variable      | symbol                                    | description                                                                                                                                                                     |
|----------------------|-------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Position             | ${}^0\mathbf{p}_{\text{config}}$          | global position vector; for interpretation see intermediate variables                                                                                                           |
| Displacement         | ${}^0\mathbf{u}_{\text{config}}$          | global displacement vector; for interpretation see intermediate variables                                                                                                       |
| Velocity             | ${}^0\mathbf{v}_{\text{config}}$          | global velocity vector; for interpretation see intermediate variables                                                                                                           |
| RotationMatrix       | ${}^{0b}\mathbf{A}$                       | vector with 9 components of the rotation matrix (row-major format)                                                                                                              |
| Rotation             |                                           | vector with 3 components of the Euler angles in xyz-sequence ( $\mathbf{R} = \mathbf{R}_x * \mathbf{R}_y * \mathbf{R}_z$ ), recomputed from rotation matrix ${}^{0b}\mathbf{A}$ |
| AngularVelocity      | ${}^0\boldsymbol{\omega}_{\text{config}}$ | angular velocity of body                                                                                                                                                        |
| AngularVelocityLocal | ${}^b\boldsymbol{\omega}_{\text{config}}$ | local (body-fixed) 3D velocity vector of node                                                                                                                                   |

#### 7.2.4.2 Definition of quantities

| intermediate variables  | symbol                                                                                                                                | description                                                                |
|-------------------------|---------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------|
| position coordinate     | $p_{0\text{config}} = c_{0\text{config}} + c_{0\text{ref}}$                                                                           | position coordinate of node (nodal coordinate $c_0$ ) in any configuration |
| displacement coordinate | $u_{0\text{config}} = c_{0\text{config}}$                                                                                             | displacement coordinate of mass node in any configuration                  |
| velocity coordinate     | $u_{0\text{config}}$                                                                                                                  | velocity coordinate of mass node in any configuration                      |
| Position                | ${}^0\mathbf{p}_{\text{config}} = {}^0\mathbf{p}_0 + {}^{0b}\mathbf{A}_0 \begin{bmatrix} p_0 \\ 0 \\ 0 \end{bmatrix}_{\text{config}}$ | (translational) position of mass object in any configuration               |
| Displacement            | ${}^0\mathbf{u}_{\text{config}} = {}^{0b}\mathbf{A}_0 \begin{bmatrix} q_0 \\ 0 \\ 0 \end{bmatrix}_{\text{config}}$                    | (translational) displacement of mass object in any configuration           |
| Velocity                | ${}^0\mathbf{v}_{\text{config}} = {}^{0b}\mathbf{A}_0 \begin{bmatrix} \dot{q}_0 \\ 0 \\ 0 \end{bmatrix}_{\text{config}}$              | (translational) velocity of mass object in any configuration               |
| residual force          | $f$                                                                                                                                   | residual of all forces on mass object                                      |
| applied force           | ${}^0\mathbf{f}_a = [f_0, f_1, f_2]^T$                                                                                                | 3D applied force (loads, connectors, joint reaction forces, ...)           |
| applied torque          | ${}^0\boldsymbol{\tau}_a = [\tau_0, \tau_1, \tau_2]^T$                                                                                | 3D applied torque (loads, connectors, joint reaction forces, ...)          |

A rigid body marker (e.g., MarkerBodyRigid) may be attached to this object and forces/torques can be applied. However, torques will have no effect and forces will only have effect in 'direction' of the coordinate.

#### 7.2.4.3 Equations of motion

$$m \cdot \ddot{q}_0 = f. \quad (7.23)$$

Note that  $f$  is computed from all connectors and loads upon the object. E.g., a 3D force vector  ${}^0\mathbf{f}_a$  is transformed to  $f$  as

$$f = {}^b[1, 0, 0] {}^{b0}\mathbf{A}_0 {}^0\mathbf{f}_a \quad (7.24)$$

Thus, the **position jacobian** reads

$$\mathbf{J}_{pos} = \partial \mathbf{p}_{cur} / \partial q_{0,cur} = {}^b[1, 0, 0] {}^{b0}\mathbf{A}_0 \quad (7.25)$$

---

#### 7.2.4.4 MINI EXAMPLE for ObjectMass1D

```
node = mbs.AddNode(Node1D(referenceCoordinates = [1],
 initialCoordinates=[0.5],
 initialVelocities=[0.5]))
mass = mbs.AddObject(Mass1D(nodeNumber = node, physicsMass=1))

#assemble and solve system for default parameters
mbs.Assemble()
exu.SolveDynamic(mbs)

#check result, get current mass position at local position [0,0,0]
exodynTestGlobals.testResult = mbs.GetObjectOutputBody(mass, exu.OutputVariableType.
 Position, [0,0,0])[0]
#final x-coordinate of position shall be 2
```

---

For further examples on ObjectMass1D see Examples:

- [multiprocessingTest.py](#)
- [sliderCrankCMSacme.py](#)

For further examples on ObjectMass1D see TestModels:

- [driveTrainTest.py](#)

## 7.2.5 ObjectRotationalMass1D

A 1D rotational inertia (mass) which is attached to Node1D.

### Additional information for ObjectRotationalMass1D:

- The Object has the following types = Body, SingleNoded
- Requested node type = GenericODE2
- Short name for Python = **Rotor1D**
- Short name for Python (visualization object) = **VRotor1D**

The item **ObjectRotationalMass1D** with type = 'RotationalMass1D' has the following parameters:

| Name              | type                    | size | default value               | description                                                                                  |
|-------------------|-------------------------|------|-----------------------------|----------------------------------------------------------------------------------------------|
| name              | String                  |      | "                           | objects's unique name                                                                        |
| physicsInertia    | UReal                   |      | 0.                          | inertia components [SI:kgm <sup>2</sup> ] of rotor / rotational mass                         |
| nodeNumber        | NodeIndex               |      | MAXINT                      | node number (type NodeIndex) of Node1D, providing rotation coordinate $\psi_0 = c_0$         |
| referencePosition | Vector3D                | 3    | [0.,0.,0.]                  | a constant reference position, used to assign joint constraints accordingly and for drawing  |
| referenceRotation | Matrix3D                |      | [[1,0,0], [0,1,0], [0,0,1]] | an intermediate rotation matrix, which transforms the 1D coordinate into 3D, see description |
| visualization     | VObjectRotationalMass1D |      |                             | parameters for visualization of item                                                         |

The item VObjectRotationalMass1D has the following parameters:

| Name         | type             | size | default value | description                                                                                            |
|--------------|------------------|------|---------------|--------------------------------------------------------------------------------------------------------|
| show         | Bool             |      | True          | set true, if item is shown in visualization and false if it is not shown                               |
| graphicsData | BodyGraphicsData |      |               | Structure contains data for body visualization; data is defined in special list / dictionary structure |

### 7.2.5.1 DESCRIPTION of ObjectRotationalMass1D:

#### Information on input parameters:

| input parameter   | symbol                                            | description see tables above |
|-------------------|---------------------------------------------------|------------------------------|
| physicsInertia    | $J$                                               |                              |
| nodeNumber        | $n_0$                                             |                              |
| referencePosition | ${}^0\mathbf{p}_0$                                |                              |
| referenceRotation | ${}^{0i}\mathbf{A}_0 \in \mathbb{R}^{3 \times 3}$ |                              |

The following output variables are available as OutputVariableType in sensors, Get...Output() and other functions:

| output variable      | symbol                                    | description                                                                                                                           |
|----------------------|-------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------|
| Position             | ${}^0\mathbf{p}_{\text{config}}$          | global position vector; for interpretation see intermediate variables                                                                 |
| Displacement         | ${}^0\mathbf{u}_{\text{config}}$          | global displacement vector; for interpretation see intermediate variables                                                             |
| Velocity             | ${}^0\mathbf{v}_{\text{config}}$          | global velocity vector; for interpretation see intermediate variables                                                                 |
| RotationMatrix       | ${}^{0b}\mathbf{A}$                       | vector with 9 components of the rotation matrix (row-major format)                                                                    |
| Rotation             |                                           | vector with 3 components of the Euler angles in xyz-sequence ( $R=R_x*R_y*R_z$ ), recomputed from rotation matrix ${}^{0b}\mathbf{A}$ |
| AngularVelocity      | ${}^0\boldsymbol{\omega}_{\text{config}}$ | angular velocity of body                                                                                                              |
| AngularVelocityLocal | ${}^b\boldsymbol{\omega}_{\text{config}}$ | local (body-fixed) 3D velocity vector of node                                                                                         |

### 7.2.5.2 Definition of quantities

| intermediate variables  | symbol                                                                                                                                                           | description                                                                                              |
|-------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------|
| position coordinate     | $\theta_0_{\text{config}} = c_0_{\text{config}} + c_0_{\text{ref}}$                                                                                              | total rotation coordinate of node (e.g., Node1D) in any configuration (nodal coordinate $c_0$ )          |
| displacement coordinate | $\psi_0_{\text{config}} = c_0_{\text{config}}$                                                                                                                   | change of rotation coordinate of mass node (e.g., Node1D) in any configuration (nodal coordinate $c_0$ ) |
| velocity coordinate     | $\dot{\psi}_0_{\text{config}}$                                                                                                                                   | rotation velocity coordinate of mass node (e.g., Node1D) in any configuration                            |
| Position                | ${}^0\mathbf{p}_{\text{config}} = {}^0\mathbf{p}_0$                                                                                                              | constant (translational) position of mass object in any configuration                                    |
| Displacement            | ${}^0\mathbf{u}_{\text{config}} = [0, 0, 0]^T$                                                                                                                   | (translational) displacement of mass object in any configuration                                         |
| Velocity                | ${}^0\mathbf{v}_{\text{config}} = [0, 0, 0]^T$                                                                                                                   | (translational) velocity of mass object in any configuration                                             |
| AngularVelocity         | ${}^0\boldsymbol{\omega}_{\text{config}} = {}^{0i}\mathbf{A}_0 \begin{bmatrix} 0 \\ 0 \\ \dot{\psi}_0 \end{bmatrix}^T$                                           |                                                                                                          |
| AngularVelocityLocal    | ${}^b\boldsymbol{\omega}_{\text{config}} = \begin{bmatrix} 0 \\ 0 \\ \dot{\psi}_0 \end{bmatrix}^T$                                                               |                                                                                                          |
| RotationMatrix          | ${}^{0b}\mathbf{A} = {}^{0i}\mathbf{A}_0 \begin{bmatrix} \cos(\theta_0) & -\sin(\theta_0) & 0 \\ \sin(\theta_0) & \cos(\theta_0) & 0 \\ 0 & 0 & 1 \end{bmatrix}$ | transformation of local body ( $b$ ) coordinates to global ( $0$ ) coordinates                           |
| residual force          | $\tau$                                                                                                                                                           | residual of all forces on mass object                                                                    |
| applied force           | ${}^0\mathbf{f}_a = [f_0, f_1, f_2]^T$                                                                                                                           | 3D applied force (loads, connectors, joint reaction forces, ...)                                         |
| applied torque          | ${}^0\boldsymbol{\tau}_a = [\tau_0, \tau_1, \tau_2]^T$                                                                                                           | 3D applied torque (loads, connectors, joint reaction forces, ...)                                        |

A rigid body marker (e.g., MarkerBodyRigid) may be attached to this object and forces/torques can be applied. However, forces will have no effect and torques will only have effect in 'direction' of the coordinate.

### 7.2.5.3 Equations of motion

$$J \cdot \ddot{\psi}_0 = \tau. \quad (7.26)$$

Note that  $\tau$  is computed from all connectors and loads upon the object. E.g., a 3D torque vector  ${}^0\boldsymbol{\tau}_a$  is transformed to  $\tau$  as

$$\tau = {}^b[0, 0, 1] {}^{b0}\mathbf{A}_0 {}^0\boldsymbol{\tau}_a \quad (7.27)$$

Thus, the **rotation jacobian** reads

$$\mathbf{J}_{rot} = \partial \boldsymbol{\omega}_{cur} / \partial \dot{q}_{0,cur} = {}^b[0, 0, 1] {}^{b0}\mathbf{A}_0 \quad (7.28)$$


---

### 7.2.5.4 MINI EXAMPLE for ObjectRotationalMass1D

```

node = mbs.AddNode(Node1D(referenceCoordinates = [1], #\psi_0ref
 initialCoordinates=[0.5], #\psi_0ini
 initialVelocities=[0.5])) #\psi_t0ini
rotor = mbs.AddObject(Rotor1D(nodeNumber = node, physicsInertia=1))

#assemble and solve system for default parameters
mbs.Assemble()
exu.SolveDynamic(mbs)

#check result, get current rotor z-rotation at local position [0,0,0]
exodynTestGlobals.testResult = mbs.GetObjectOutputBody(rotor, exu.OutputVariableType.
 Rotation, [0,0,0])
#final z-angle of rotor shall be 2

```

---

For further examples on ObjectRotationalMass1D see TestModels:

- [driveTrainTest.py](#)

## 7.2.6 ObjectRigidBody

A 3D rigid body which is attached to a 3D rigid body node. The rotation parametrization of the rigid body follows the rotation parametrization of the node. Use Euler parameters in the general case (no singularities) in combination with implicit solvers (GeneralizedAlpha or TrapezoidalIndex2), Tait-Bryan angles for special cases, e.g., rotors where no singularities occur if you rotate about x or z axis, or use Lie-group formulation with rotation vector together with explicit solvers. REMARK: Use the class `RigidBodyInertia`, see [Section 5.11.1](#) and `AddRigidBody(...)`, see [Section 5.11](#), of `exudyn.rigidBodyUtilities` to handle inertia, COM and mass.

**Additional information for ObjectRigidBody:**

- The Object has the following types = `Body`, `SingleNoded`
- Requested node type = `Position + Orientation + RigidBody`
- **Short name for Python = `RigidBody`**
- **Short name for Python (visualization object) = `VRigidBody`**

The item `ObjectRigidBody` with type = 'RigidBody' has the following parameters:

| Name                | type             | size | default value        | description                                                                                                                                                                                                                                                                                                                                                                         |
|---------------------|------------------|------|----------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| name                | String           |      | "                    | objects's unique name                                                                                                                                                                                                                                                                                                                                                               |
| physicsMass         | UReal            |      | 0.                   | mass [SI:kg] of rigid body                                                                                                                                                                                                                                                                                                                                                          |
| physicsInertia      | Vector6D         |      | [0.,0.,0., 0.,0.,0.] | inertia components [SI:kgm <sup>2</sup> ]: [ $J_{xx}, J_{yy}, J_{zz}, J_{yz}, J_{xz}, J_{xy}$ ] in body-fixed coordinate system and w.r.t. to the reference point of the body, NOT necessarily w.r.t. to center of mass; use the class <code>RigidBodyInertia</code> and <code>AddRigidBody(...)</code> of <code>exudynRigidBodyUtilities.py</code> to handle inertia, COM and mass |
| physicsCenterOfMass | Vector3D         | 3    | [0.,0.,0.]           | local position of center of mass (COM) relative to the body's reference point; if the vector of the COM is [0,0,0], the computation will not consider additional terms for the COM and it is faster                                                                                                                                                                                 |
| nodeNumber          | NodeIndex        |      | MAXINT               | node number (type <code>NodeIndex</code> ) for rigid body node                                                                                                                                                                                                                                                                                                                      |
| visualization       | VObjectRigidBody |      |                      | parameters for visualization of item                                                                                                                                                                                                                                                                                                                                                |

The item `VObjectRigidBody` has the following parameters:

| Name                     | type                   | size | default value | description                                                                                                                                                                                                                                                                     |
|--------------------------|------------------------|------|---------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| show                     | Bool                   |      | True          | set true, if item is shown in visualization and false if it is not shown                                                                                                                                                                                                        |
| graphicsDataUserFunction | PyFunctionGraphicsData | 0    |               | A python function which returns a <code>bodyGraphicsData</code> object, which is a list of graphics data in a dictionary computed by the user function; the graphics elements need to be defined in the local body coordinates and are transformed by mbs to global coordinates |

|              |                  |  |                                                                                                        |
|--------------|------------------|--|--------------------------------------------------------------------------------------------------------|
| graphicsData | BodyGraphicsData |  | Structure contains data for body visualization; data is defined in special list / dictionary structure |
|--------------|------------------|--|--------------------------------------------------------------------------------------------------------|

---

### 7.2.6.1 DESCRIPTION of ObjectRigidBody:

Information on input parameters:

| input parameter     | symbol                 | description see tables above |
|---------------------|------------------------|------------------------------|
| physicsMass         | $m$                    |                              |
| physicsInertia      | ${}^b\mathbf{j}_6$     |                              |
| physicsCenterOfMass | ${}^b\mathbf{r}_{COM}$ |                              |
| nodeNumber          | $n0$                   |                              |

The following output variables are available as OutputVariableType in sensors, Get...Output() and other functions:

| output variable      | symbol                                                                                                                                                                                                                                     | description                                                                                                |
|----------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------|
| Position             | ${}^0\mathbf{p}_{config}({}^b\mathbf{p}) = {}^0\mathbf{u}_{config} + {}^0\mathbf{p}_{ref} + {}^{0b}\mathbf{A} {}^b\mathbf{p}$                                                                                                              | global position vector of body-fixed point given by local position vector                                  |
| Displacement         | ${}^0\mathbf{u}_{config} + {}^{0b}\mathbf{A} {}^b\mathbf{p}$                                                                                                                                                                               | global displacement vector of body-fixed point given by local position vector                              |
| Velocity             | ${}^0\mathbf{v}_{config}({}^b\mathbf{p}) = {}^0\dot{\mathbf{u}}_{config} + {}^{0b}\mathbf{A}({}^b\boldsymbol{\omega} \times {}^b\mathbf{p}_{config})$                                                                                      | global velocity vector of body-fixed point given by local position vector                                  |
| RotationMatrix       |                                                                                                                                                                                                                                            | vector with 9 components of the rotation matrix (row-major format)                                         |
| Rotation             |                                                                                                                                                                                                                                            | vector with 3 components of the Euler angles in xyz-sequence (R=Rx*Ry*Rz), recomputed from rotation matrix |
| AngularVelocity      | ${}^0\boldsymbol{\omega}_{config}$                                                                                                                                                                                                         | angular velocity of body                                                                                   |
| AngularVelocityLocal | ${}^b\boldsymbol{\omega}_{config}$                                                                                                                                                                                                         | local (body-fixed) 3D velocity vector of node                                                              |
| Acceleration         | ${}^0\mathbf{a}_{config}({}^b\mathbf{p}) = {}^0\ddot{\mathbf{u}} + {}^0\boldsymbol{\alpha} \times ({}^{0b}\mathbf{A} {}^b\mathbf{p}) + {}^0\boldsymbol{\omega} \times ({}^0\boldsymbol{\omega} \times ({}^{0b}\mathbf{A} {}^b\mathbf{p}))$ | global acceleration vector of body-fixed point given by local position vector                              |
| AngularAcceleration  | ${}^0\boldsymbol{\alpha}_{config}$                                                                                                                                                                                                         | angular acceleration vector of body                                                                        |

### 7.2.6.2 Definition of quantities

| intermediate variables | symbol                                                                                                                            | description                                                                                                                               |
|------------------------|-----------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------|
| inertia tensor         | ${}^b\mathbf{J} = \begin{bmatrix} J_{xx} & J_{xy} & J_{xz} \\ J_{xy} & J_{yy} & J_{yz} \\ J_{xz} & J_{yz} & J_{zz} \end{bmatrix}$ | symmetric inertia tensor, based on components of ${}^b\mathbf{j}_6$ , in body-fixed (local) coordinates and w.r.t. body's reference point |
| reference coordinates  | $\mathbf{q}_{ref} = [\mathbf{p}_{ref}^T \ \boldsymbol{\psi}_{ref}^T]^T$                                                           | defines reference configuration, DIFFERENT meaning from body's reference position!                                                        |

|                                     |                                                                                                                                                                                         |                                                                                                                                                                                         |
|-------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| (relative) current coordinates      | $\mathbf{q}_{\text{cur}} = [\mathbf{p}_{\text{cur}}^T, \boldsymbol{\psi}_{\text{cur}}^T]^T$                                                                                             | unknowns in solver; <b>relative</b> to the reference coordinates; current coordinates at initial configuration = initial coordinates $\mathbf{q}_{\text{ini}}$                          |
| current velocity coordinates        | $\dot{\mathbf{q}}_{\text{cur}} = [\mathbf{v}_{\text{cur}}^T, \dot{\boldsymbol{\psi}}_{\text{cur}}^T]^T = [\dot{\mathbf{p}}_{\text{cur}}^T, \dot{\boldsymbol{\theta}}_{\text{cur}}^T]^T$ | current velocity coordinates                                                                                                                                                            |
| body's reference position           | ${}^0\mathbf{p}_{\text{config}} = {}^0\mathbf{p}(n_0)_{\text{config}}$                                                                                                                  | position of <b>body's reference point</b> provided by node $n_0$ in any configuration; if ${}^b\mathbf{r}_{\text{COM}} = [0, 0, 0]^T$ , this position becomes equal to the COM position |
| current body's reference position   | $\mathbf{p}_{\text{cur}} = \mathbf{p}_{\text{cur}} + \mathbf{p}_{\text{ref}}$                                                                                                           | position of <b>body's reference point</b> , including reference configuration                                                                                                           |
| body's reference point displacement | ${}^0\mathbf{u}_{\text{config}} = [q_0, q_1, q_2]_{\text{config}}^T = {}^0\mathbf{u}(n_0)_{\text{config}}$                                                                              | displacement of <b>body's reference point</b> which is provided by node $n_0$ in any configuration                                                                                      |
| body's reference point velocity     | ${}^0\mathbf{v}_{\text{config}} = [\dot{q}_0, \dot{q}_1, \dot{q}_2]_{\text{config}}^T = {}^0\mathbf{v}(n_0)_{\text{config}}$                                                            | velocity of <b>body's reference point</b> which is provided by node $n_0$ in any configuration                                                                                          |
| body's reference point acceleration | ${}^0\mathbf{a}_{\text{config}} = [\ddot{q}_0, \ddot{q}_1, \ddot{q}_2]_{\text{config}}^T$                                                                                               | acceleration of <b>body's reference point</b> which is provided by node $n_0$ in any configuration                                                                                      |
| rotation coordinates                | $\boldsymbol{\theta}_{\text{config}} = \boldsymbol{\psi}(n_0)_{\text{ref}} + \boldsymbol{\psi}(n_0)_{\text{config}}$                                                                    | (total) rotation parameters of body as provided by node $n_0$ in any configuration                                                                                                      |
| rotation parameters                 | $\boldsymbol{\theta}_{\text{config}} = \boldsymbol{\psi}(n_0)_{\text{ref}} + \boldsymbol{\psi}(n_0)_{\text{config}}$                                                                    | (total) rotation parameters of body as provided by node $n_0$ in any configuration                                                                                                      |
| body rotation matrix                | ${}^{0b}\mathbf{A}_{\text{config}} = {}^{0b}\mathbf{A}(n_0)_{\text{config}}$                                                                                                            | rotation matrix which transforms local to global coordinates as given by node                                                                                                           |
| local position                      | ${}^b\mathbf{p} = [{}^b p_0, {}^b p_1, {}^b p_2]^T$                                                                                                                                     | local position as used by markers or sensors                                                                                                                                            |
| angular velocity                    | ${}^0\boldsymbol{\omega}_{\text{config}} = {}^0[\omega_0(n_0), \omega_1(n_0), \omega_2(n_0)]_{\text{config}}^T$                                                                         | global angular velocity of body as provided by node $n_0$ in any configuration                                                                                                          |
| local angular velocity              | ${}^b\boldsymbol{\omega}_{\text{config}}$                                                                                                                                               | local angular velocity of body as provided by node $n_0$ in any configuration                                                                                                           |
| body angular acceleration           | ${}^0\boldsymbol{\alpha}_{\text{config}} = {}^0\dot{\boldsymbol{\omega}}_{\text{config}}$                                                                                               | angular acceleratoin of body as provided by node $n_0$ in any configuration                                                                                                             |
| applied forces                      | ${}^0\mathbf{f}_a = [f_0, f_1, f_2]^T$                                                                                                                                                  | calculated from loads, connectors, ...                                                                                                                                                  |
| applied torques                     | ${}^0\boldsymbol{\tau}_a = [\tau_0, \tau_1, \tau_2]^T$                                                                                                                                  | calculated from loads, connectors, ...                                                                                                                                                  |
| constraint reaction forces          | ${}^0\mathbf{f}_\lambda = [f_{\lambda 0}, f_{\lambda 1}, f_{\lambda 2}]^T$                                                                                                              | calculated from joints or constraint)                                                                                                                                                   |
| constraint reaction torques         | ${}^0\boldsymbol{\tau}_\lambda = [\tau_{\lambda 0}, \tau_{\lambda 1}, \tau_{\lambda 2}]^T$                                                                                              | calculated from joints or constraints                                                                                                                                                   |

### 7.2.6.3 Rotation parametrization

The equations of motion of the rigid body build upon a specific parameterization of the rigid body coordinates. Rigid body coordinates are defined by the underlying node given by nodeNumber  $n_0$ . Appropriate nodes are

- **NodeRigidBodyEP** (Euler parameters)
- **NodeRigidBodyRxyz** (Euler angles / Tait Bryan angles)
- **NodeRigidBodyRotVecLG** (Rotation vector with Lie group integration option)

Note that all operations for rotation parameters, such as the computation of the rotation matrix, must be performed with the rotation parameters  $\boldsymbol{\theta}$ , see table above, which are the sum of reference and current coordinates.

The angular velocity in body-fixed coordinates is related to the rotation parameters by means of a matrix  ${}^b\mathbf{G}_{rp}$ ,

$${}^b\boldsymbol{\omega} = {}^b\mathbf{G}_{rp} \dot{\boldsymbol{\theta}} = {}^b\mathbf{G}_{rp} \dot{\boldsymbol{\psi}}, \quad (7.29)$$

and is specific for any rotation parametrization  $rp$ . The angular velocity in global coordinates is related to the rotation parameters by means of a matrix  ${}^0\mathbf{G}_{rp}$ ,

$${}^0\boldsymbol{\omega} = {}^0\mathbf{G}_{rp} \dot{\boldsymbol{\theta}}. \quad (7.30)$$

The local angular accelerations follow as

$${}^b\boldsymbol{\alpha} = {}^b\dot{\boldsymbol{\omega}} = {}^b\mathbf{G}_{rp} \ddot{\boldsymbol{\theta}} + {}^b\dot{\mathbf{G}}_{rp} \dot{\boldsymbol{\theta}}, \quad (7.31)$$

remember that derivatives for angular velocities can also be done in the local frame. In case of Euler parameters and the Lie-group rotation vector we find that  ${}^b\dot{\mathbf{G}}_{rp} \dot{\boldsymbol{\theta}} = \mathbf{0}$ .

#### 7.2.6.4 Equations of motion for COM

The equations of motion for a rigid body, the so-called Newton-Euler equations, can be written for the special case of the reference point = COM (center of mass) and split for translations and rotations, using a coordinate-free notation,

$$\begin{bmatrix} m\mathbf{I}_3 & \mathbf{0} \\ \mathbf{0} & \mathbf{J} \end{bmatrix} \begin{bmatrix} \mathbf{a}_{COM} \\ \boldsymbol{\alpha} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ -\tilde{\omega}\mathbf{J}\boldsymbol{\omega} \end{bmatrix} + \begin{bmatrix} \mathbf{f}_a \\ \boldsymbol{\tau}_a \end{bmatrix} + \begin{bmatrix} \mathbf{f}_\lambda \\ \boldsymbol{\tau}_\lambda \end{bmatrix} \quad (7.32)$$

with the  $3 \times 3$  unit matrix  $\mathbf{I}_3$  and forces  $\mathbf{f}$  resp. torques  $\boldsymbol{\tau}$  as described in the table above. A change of the reference point, using the vector  $\mathbf{r}_{COM}$  from the body's reference point  $\mathbf{p}$  to the COM position, is simple by replacing COM accelerations using the common relation known from Euler

$$\mathbf{a}_{COM} = \mathbf{a} + \tilde{\alpha}\mathbf{r}_{COM} + \tilde{\omega}\tilde{\omega}\mathbf{r}_{COM}, \quad (7.33)$$

which is inserted into the first line of Eq. (7.32). Additionally, the second line of Eq. (7.32) (second Euler equation related to rate of angular momentum) is rewritten for an arbitrary reference point,  $\mathbf{r}_{COM}$  denoting the vector to COM, using the well known relation

$$m\tilde{\mathbf{r}}_{COM}\boldsymbol{\alpha} + \mathbf{J}\boldsymbol{\alpha} + \tilde{\omega}\mathbf{J}\boldsymbol{\omega} = \boldsymbol{\tau}_a + \boldsymbol{\tau}_\lambda \quad (7.34)$$

#### 7.2.6.5 Equations of motion for arbitrary reference point

This immediately leads to the equations of motion for the rigid body with respect to an arbitrary reference point ( $\neq$  COM), see e.g. [36](page 258ff.), which have the general coordinate-free form

$$\begin{bmatrix} m\mathbf{I}_3 & -m\tilde{\mathbf{r}}_{COM} \\ m\tilde{\mathbf{r}}_{COM} & \mathbf{J} \end{bmatrix} \begin{bmatrix} \mathbf{a} \\ \boldsymbol{\alpha} \end{bmatrix} = \begin{bmatrix} -m\tilde{\omega}\tilde{\omega}\mathbf{r}_{COM} \\ -\tilde{\omega}\mathbf{J}\boldsymbol{\omega} \end{bmatrix} + \begin{bmatrix} \mathbf{f}_a \\ \boldsymbol{\tau}_a \end{bmatrix} + \begin{bmatrix} \mathbf{f}_\lambda \\ \boldsymbol{\tau}_\lambda \end{bmatrix}, \quad (7.35)$$

in which  $\mathbf{J}$  is the inertia tensor w.r.t. the chosen reference point (which has local coordinates  ${}^b[0, 0, 0]^T$ ). Eq. (7.35) can be written in the global frame (0),

$$\begin{bmatrix} m\mathbf{I}_3 & -m{}^0\tilde{\mathbf{r}}_{COM} \\ m{}^0\tilde{\mathbf{r}}_{COM} & {}^0\mathbf{J} \end{bmatrix} \begin{bmatrix} {}^0\mathbf{a} \\ {}^0\boldsymbol{\alpha} \end{bmatrix} = \begin{bmatrix} -m{}^0\tilde{\omega}{}^0\tilde{\omega}{}^0\mathbf{r}_{COM} \\ -{}^0\tilde{\omega}{}^0\mathbf{J}{}^0\boldsymbol{\omega} \end{bmatrix} + \begin{bmatrix} {}^0\mathbf{f}_a \\ {}^0\boldsymbol{\tau}_a \end{bmatrix} + \begin{bmatrix} {}^0\mathbf{f}_\lambda \\ {}^0\boldsymbol{\tau}_\lambda \end{bmatrix}. \quad (7.36)$$

Expressing the translational part (first line) of Eq. (7.36) in the global frame (0), using local coordinates (b) for quantities that are constant in the body-fixed frame,  ${}^b\mathbf{J}$  and  ${}^b\mathbf{r}_{COM}$ , thus expressing also the angular velocity  ${}^b\boldsymbol{\omega}$  in the body-fixed frame, applying Eq. (7.29) and Eq. (7.31), and using the relations

$${}^0\tilde{\omega}{}^0\tilde{\omega}{}^0\mathbf{r}_{COM} = {}^{0b}\mathbf{A}{}^b\tilde{\omega}{}^b\tilde{\omega}{}^b\mathbf{r}_{COM} = -{}^{0b}\mathbf{A}{}^b\tilde{\omega}{}^b\tilde{\mathbf{r}}_{COM}{}^b\boldsymbol{\omega} = -{}^{0b}\mathbf{A}{}^b\tilde{\omega}{}^b\tilde{\mathbf{r}}_{COM}{}^b\mathbf{G}_{rp} \dot{\boldsymbol{\theta}}, \quad (7.37)$$

$$-m{}^0\tilde{\mathbf{r}}_{COM}{}^0\tilde{\boldsymbol{\alpha}} = -m{}^{0b}\mathbf{A}{}^b\tilde{\mathbf{r}}_{COM}{}^b\tilde{\boldsymbol{\alpha}} = -m{}^{0b}\mathbf{A}{}^b\tilde{\mathbf{r}}_{COM}({}^b\mathbf{G}_{rp} \ddot{\boldsymbol{\theta}} + {}^b\dot{\mathbf{G}}_{rp} \dot{\boldsymbol{\theta}}), \quad (7.38)$$

we obtain

$$\begin{aligned} & \begin{bmatrix} m\mathbf{I}_3 & -m^{0b}\mathbf{A}^b\tilde{\mathbf{r}}_{COM}^b\mathbf{G}_{rp} \\ m^b\mathbf{G}_{rp}^T\tilde{\mathbf{r}}_{COM}^{0b}\mathbf{A}^T & {}^b\mathbf{G}_{rp}^T{}^b\mathbf{J}^b\mathbf{G}_{rp} \end{bmatrix} \begin{bmatrix} {}^0\mathbf{a} \\ \dot{\boldsymbol{\theta}} \end{bmatrix} \\ & = \begin{bmatrix} m^{0b}\mathbf{A}^b\tilde{\boldsymbol{\omega}}^b\tilde{\mathbf{r}}_{COM}^b\boldsymbol{\omega} + m^{0b}\mathbf{A}^b\tilde{\mathbf{r}}_{COM}^b\dot{\mathbf{G}}_{rp}^b\dot{\boldsymbol{\theta}} \\ {}^b\mathbf{G}_{rp}^T\tilde{\boldsymbol{\omega}}^b\mathbf{J}^b\boldsymbol{\omega} - {}^b\mathbf{G}_{rp}^T{}^b\mathbf{J}^b\dot{\mathbf{G}}_{rp}^b\dot{\boldsymbol{\theta}} \end{bmatrix} + \begin{bmatrix} {}^0\mathbf{f}_a \\ {}^0\mathbf{G}_{rp}^T{}^0\boldsymbol{\tau}_a \end{bmatrix} + \begin{bmatrix} {}^0\mathbf{f}_{\lambda} \\ \mathbf{f}_{\theta,\lambda} \end{bmatrix} \end{aligned} \quad (7.39)$$

with constraint reaction forces  $\mathbf{f}_{\theta,\lambda}$  for the rotation parameters. Note that the last line has been pre-multiplied with  ${}^b\mathbf{G}_{rp}^T$  (in order to make the mass matrix symmetric) and that  ${}^b\dot{\mathbf{G}}_{rp}^b\dot{\boldsymbol{\theta}} = \mathbf{0}$  in case of Euler parameters and the Lie-group rotation vector.

### 7.2.6.6 Euler parameters

In case of Euler parameters, a constraint equation is automatically added, reading for the index 3 case

$$g_\theta(\boldsymbol{\theta}) = \theta_0^2 + \theta_1^2 + \theta_2^2 + \theta_3^2 - 1 = 0 \quad (7.40)$$

and for the index 2 case

$$\dot{g}_\theta(\boldsymbol{\theta}) = 2\theta_0\dot{\theta}_0 + 2\theta_1\dot{\theta}_1 + 2\theta_2\dot{\theta}_2 + 2\theta_3\dot{\theta}_3 = 0 \quad (7.41)$$

Given a Lagrange parameter (algebraic variable)  $\lambda_\theta$  related to the Euler parameter constraint (7.40), the constraint reaction forces in Eq. (7.39) then read

$$\mathbf{f}_{\theta,\lambda} = \frac{\partial g_\theta}{\partial \boldsymbol{\theta}^T} \lambda_\theta = [2\theta_0, 2\theta_1, 2\theta_2, 2\theta_3]^T \quad (7.42)$$


---

### Userfunction: `graphicsDataUserFunction(mbs, itemNumber)`

A user function, which is called by the visualization thread in order to draw user-defined objects. The function can be used to generate any BodyGraphicsData, see Section 9.3. Use `graphicsDataUtilities` functions, see Section 5.4, to create more complicated objects. Note that `graphicsDataUserFunction` needs to copy lots of data and is therefore inefficient and only designed to enable simpler tests, but not large scale problems.

For an example for `graphicsDataUserFunction` see ObjectGround, [Section 7.2.1](#).

| arguments / return        | type or size     | description                                                                                    |
|---------------------------|------------------|------------------------------------------------------------------------------------------------|
| <code>mbs</code>          | MainSystem       | provides reference to mbs, which can be used in user function to access all data of the object |
| <code>itemNumber</code>   | Index            | integer number of the object in mbs, allowing easy access                                      |
| <code>return value</code> | BodyGraphicsData | list of GraphicsData dictionaries, see Section 9.3                                             |

For creating a `ObjectRigidBody`, there is a `rigidBodyUtilities` function `AddRigidBody`, see [Section 5.11](#), which simplifies the setup of a rigid body significantly!

---

For further examples on `ObjectRigidBody` see Examples:

- [`rigid3Dexample.py`](#)
- [`rigidBodyIMUtest.py`](#)
- [`mouseInteractionExample.py`](#)

- [`rigidBodyTutorial.py`](#)
- [`rigidBodyTutorial2.py`](#)
- [`sliderCrank3DwithANCFbeltDrive2.py`](#)
- [`stiffFlyballGovernor2.py`](#)
- [`ANCF\_moving\_rigidbody.py`](#)
- [`ANCF\_slidingJoint2D.py`](#)
- [`ANCF\_switchingSlidingJoint2D.py`](#)
- ...

For further examples on ObjectRigidBody see TestModels:

- [`explicitLieGroupIntegratorPythonTest.py`](#)
- [`explicitLieGroupIntegratorTest.py`](#)
- [`explicitLieGroupMBSTest.py`](#)
- [`genericJointUserFunctionTest.py`](#)
- [`heavyTop.py`](#)
- [`objectFFRFTest.py`](#)
- [`sphericalJointTest.py`](#)
- [`carRollingDiscTest.py`](#)
- [`driveTrainTest.py`](#)
- [`mecanumWheelRollingDiscTest.py`](#)
- ...

## 7.2.7 ObjectRigidBody2D

A 2D rigid body which is attached to a rigid body 2D node. The body obtains coordinates, position, velocity, etc. from the underlying 2D node

**Additional information for ObjectRigidBody2D:**

- The Object has the following types = Body, SingleNoded
- Requested node type = Position2D + Orientation2D + Position + Orientation
- **Short name for Python** = **RigidBody2D**
- **Short name for Python (visualization object)** = **VRigidBody2D**

The item **ObjectRigidBody2D** with type = 'RigidBody2D' has the following parameters:

| Name           | type               | size | default value | description                                                        |
|----------------|--------------------|------|---------------|--------------------------------------------------------------------|
| name           | String             |      | ''            | objects's unique name                                              |
| physicsMass    | UReal              |      | 0.            | mass [SI:kg] of rigid body                                         |
| physicsInertia | UReal              |      | 0.            | inertia [SI:kgm <sup>2</sup> ] of rigid body w.r.t. center of mass |
| nodeNumber     | NodeIndex          |      | MAXINT        | node number (type NodeIndex) for 2D rigid body node                |
| visualization  | VObjectRigidBody2D |      |               | parameters for visualization of item                               |

The item VObjectRigidBody2D has the following parameters:

| Name                     | type                   | size | default value | description                                                                                                                                                                                                                                                         |
|--------------------------|------------------------|------|---------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| show                     | Bool                   |      | True          | set true, if item is shown in visualization and false if it is not shown                                                                                                                                                                                            |
| graphicsDataUserFunction | PyFunctionGraphicsData |      | 0             | A python function which returns a body-GraphicsData object, which is a list of graphics data in a dictionary computed by the user function; the graphics elements need to be defined in the local body coordinates and are transformed by mbs to global coordinates |
| graphicsData             | BodyGraphicsData       |      |               | Structure contains data for body visualization; data is defined in special list / dictionary structure                                                                                                                                                              |

---

### 7.2.7.1 DESCRIPTION of ObjectRigidBody2D:

**Information on input parameters:**

| input parameter | symbol | description see tables above |
|-----------------|--------|------------------------------|
| physicsMass     | $m$    |                              |
| physicsInertia  | $J$    |                              |
| nodeNumber      | $n_0$  |                              |

The following output variables are available as `OutputVariableType` in sensors, `Get...Output()` and other functions:

| output variable     | symbol                                                                                                                                                                                                                                            | description                                                                   |
|---------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------|
| Position            | ${}^0\mathbf{p}_{\text{config}}({}^b\mathbf{p}) = {}^0\mathbf{u}_{\text{config}} + {}^0\mathbf{p}_{\text{ref}} + {}^{0b}\mathbf{A} {}^b\mathbf{p}$                                                                                                | global position vector of body-fixed point given by local position vector     |
| Displacement        | ${}^0\mathbf{u}_{\text{config}} + {}^{0b}\mathbf{A} {}^b\mathbf{p}$                                                                                                                                                                               | global displacement vector of body-fixed point given by local position vector |
| Velocity            | ${}^0\mathbf{v}_{\text{config}}({}^b\mathbf{p}) = {}^0\dot{\mathbf{u}}_{\text{config}} + {}^{0b}\mathbf{A}({}^b\omega \times {}^b\mathbf{p}_{\text{config}})$                                                                                     | global velocity vector of body-fixed point given by local position vector     |
| Rotation            | $\theta_{0\text{config}}$                                                                                                                                                                                                                         | scalar rotation angle of body                                                 |
| AngularVelocity     | ${}^0\boldsymbol{\omega}_{\text{config}}$                                                                                                                                                                                                         | angular velocity vector of body                                               |
| RotationMatrix      | $\text{vec}({}^{0b}\mathbf{A}) = [A_{00}, A_{01}, A_{02}, A_{10}, \dots, A_{21}, A_{22}]_{\text{config}}^T$                                                                                                                                       | rotation matrix in vector form (stored in row-major order)                    |
| Acceleration        | ${}^0\mathbf{a}_{\text{config}}({}^b\mathbf{p}) = {}^0\ddot{\mathbf{u}} + {}^0\boldsymbol{\alpha} \times ({}^{0b}\mathbf{A} {}^b\mathbf{p}) + {}^0\boldsymbol{\omega} \times ({}^0\boldsymbol{\omega} \times ({}^{0b}\mathbf{A} {}^b\mathbf{p}))$ | global acceleration vector of body-fixed point given by local position vector |
| AngularAcceleration | ${}^0\boldsymbol{\alpha}_{\text{config}}$                                                                                                                                                                                                         | angular acceleration vector of body                                           |

### 7.2.7.2 Definition of quantities

| intermediate variables    | symbol                                                                                                                 | description                                                                           |
|---------------------------|------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------|
| COM position              | ${}^0\mathbf{p}_{\text{config}} = {}^0\mathbf{p}(n_0)_{\text{config}}$                                                 | position of center of mass (COM) which is provided by node $n_0$ in any configuration |
| COM displacement          | ${}^0\mathbf{u}_{\text{config}} = [q_0, q_1, 0]_{\text{config}}^T = {}^0\mathbf{u}(n_0)_{\text{config}}$               | displacement of center of mass which is provided by node $n_0$ in any configuration   |
| COM velocity              | ${}^0\mathbf{v}_{\text{config}} = [\dot{q}_0, \dot{q}_1, 0]_{\text{config}}^T = {}^0\mathbf{v}(n_0)_{\text{config}}$   | velocity of center of mass which is provided by node $n_0$ in any configuration       |
| body rotation             | ${}^0\theta_{0\text{config}} = \theta_0(n_0)_{\text{config}} = \psi_0(n_0)_{\text{ref}} + \psi_0(n_0)_{\text{config}}$ | rotation of body as provided by node $n_0$ in any configuration                       |
| body rotation matrix      | ${}^{0b}\mathbf{A}_{\text{config}} = {}^{0b}\mathbf{A}(n_0)_{\text{config}}$                                           | rotation matrix which transforms local to global coordinates as given by node         |
| local position            | ${}^b\mathbf{p} = [{}^b p_0, {}^b p_1, 0]^T$                                                                           | local position as used by markers or sensors                                          |
| body angular velocity     | ${}^0\boldsymbol{\omega}_{\text{config}} = {}^0[\omega_0(n_0), 0, 0]_{\text{config}}^T$                                | rotation of body as provided by node $n_0$ in any configuration                       |
| (generalized) coordinates | $\mathbf{c}_{\text{config}} = [q_0, q_1, \psi_0]^T$                                                                    | generalized coordinates of body (= coordinates of node)                               |
| generalized forces        | ${}^0\mathbf{f} = [f_0, f_1, \tau_2]^T$                                                                                | generalized forces applied to body                                                    |
| applied forces            | ${}^0\mathbf{f}_a = [f_0, f_1, 0]^T$                                                                                   | applied forces (loads, connectors, joint reaction forces, ...)                        |
| applied torques           | ${}^0\boldsymbol{\tau}_a = [0, 0, \tau_2]^T$                                                                           | applied torques (loads, connectors, joint reaction forces, ...)                       |

### 7.2.7.3 Equations of motion

$$\begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & J \end{bmatrix} \begin{bmatrix} \ddot{q}_0 \\ \ddot{q}_1 \\ \ddot{\psi}_0 \end{bmatrix} = \begin{bmatrix} f_0 \\ f_1 \\ \tau_2 \end{bmatrix} = \mathbf{f}. \quad (7.43)$$

For example, a LoadCoordinate on coordinate 2 of the node would add a torque  $\tau_2$  on the RHS.

Position-based markers can measure position  $\mathbf{p}_{\text{config}}$  depending on the local position  ${}^b\mathbf{p}$ . The **position jacobian** depends on the local position  ${}^b\mathbf{p}$  and is defined as,

$$\mathbf{J}_{\text{pos}} = \partial \mathbf{p}_{\text{cur}} / \partial \mathbf{c}_{\text{cur}} = \begin{bmatrix} 1 & 0 & -\sin(\theta) {}^b p_0 & -\cos(\theta) {}^b p_1 \\ 0 & 1 & \cos(\theta) {}^b p_0 & -\sin(\theta) {}^b p_1 \\ 0 & 0 & 0 \end{bmatrix} \quad (7.44)$$

which transforms the action of global forces  ${}^0\mathbf{f}$  of position-based markers on the coordinates  $\mathbf{c}$ ,

$$\mathbf{Q} = \mathbf{J}_{\text{pos}} {}^0\mathbf{f}_a \quad (7.45)$$

The **rotation jacobian**

$$\mathbf{J}_{\text{rot}} = \partial \mathbf{p}_{\text{cur}} / \partial \mathbf{c}_{\text{cur}} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (7.46)$$

transforms the action of global torques  ${}^0\boldsymbol{\tau}$  of orientation-based markers on the coordinates  $\mathbf{c}$ ,

$$\mathbf{Q} = \mathbf{J}_{\text{rot}} {}^0\boldsymbol{\tau}_a \quad (7.47)$$


---

### Userfunction: `graphicsDataUserFunction(mbs, itemNumber)`

A user function, which is called by the visualization thread in order to draw user-defined objects. The function can be used to generate any `BodyGraphicsData`, see Section 9.3. Use `graphicsDataUtilities` functions, see Section 5.4, to create more complicated objects. Note that `graphicsDataUserFunction` needs to copy lots of data and is therefore inefficient and only designed to enable simpler tests, but not large scale problems.

For an example for `graphicsDataUserFunction` see `ObjectGround`, [Section 7.2.1](#).

| arguments / return        | type or size                  | description                                                                                                  |
|---------------------------|-------------------------------|--------------------------------------------------------------------------------------------------------------|
| <code>mbs</code>          | <code>MainSystem</code>       | provides reference to <code>mbs</code> , which can be used in user function to access all data of the object |
| <code>itemNumber</code>   | <code>int</code>              | integer number of the object in <code>mbs</code> , allowing easy access                                      |
| <code>return value</code> | <code>BodyGraphicsData</code> | list of <code>GraphicsData</code> dictionaries, see Section 9.3                                              |

---

#### 7.2.7.4 MINI EXAMPLE for `ObjectRigidBody2D`

```

node = mbs.AddNode(NodeRigidBody2D(referenceCoordinates = [1,1,0.25*np.pi],
 initialCoordinates=[0.5,0,0],
 initialVelocities=[0.5,0,0.75*np.pi]))
mbs.AddObject(RigidBody2D(nodeNumber = node, physicsMass=1, physicsInertia=2))

#assemble and solve system for default parameters
mbs.Assemble()
exu.SolveDynamic(mbs)

```

```

#check result
exudynTestGlobals.testResult = mbs.GetNodeOutput(node, exu.OutputVariableType.
 Position)[0]
exudynTestGlobals.testResult+= mbs.GetNodeOutput(node, exu.OutputVariableType.
 Coordinates)[2]
#final x-coordinate of position shall be 2, angle theta shall be np.pi

```

---

For further examples on ObjectRigidBody2D see Examples:

- [sliderCrank3DwithANCFbeltDrive2.py](#)
- [ANCF\\_moving\\_rigidbody.py](#)
- [ANCF\\_slidingJoint2D.py](#)
- [ANCF\\_switchingSlidingJoint2D.py](#)
- [geneticOptimizationSliderCrank.py](#)
- [lavalRotor2Dtest.py](#)
- [rigid\\_pendulum.py](#)
- [SliderCrank.py](#)
- [sliderCrank3DwithANCFbeltDrive.py](#)
- [slidercrankWithMassSpring.py](#)
- ...

For further examples on ObjectRigidBody2D see TestModels:

- [ANCFslidingAndALEjointTest.py](#)
- [ANCFcontactFrictionTest.py](#)
- [ANCFmovingRigidBodyTest.py](#)
- [compareFullModifiedNewton.py](#)
- [modelUnitTests.py](#)
- [PARTS\\_ATEs\\_moving.py](#)
- [pendulumFriction.py](#)
- [scissorPrismaticRevolute2D.py](#)
- [sliderCrankFloatingTest.py](#)

## 7.2.8 ObjectGenericODE2

A system of  $n$  second order ordinary differential equations (ODE2), having a mass matrix, damping/gyroscopic matrix, stiffness matrix and generalized forces. It can combine generic nodes, or node points. User functions can be used to compute mass matrix and generalized forces depending on given coordinates. NOTE that all matrices, vectors, etc. must have the same dimensions  $n$  or  $(n \times n)$ , or they must be empty ( $0 \times 0$ ), except for the mass matrix which always needs to have dimensions  $(n \times n)$ .

### Additional information for ObjectGenericODE2:

- The Object has the following types = Body, MultiNoded, SuperElement
- Requested node type: read detailed information of item

The item **ObjectGenericODE2** with type = 'GenericODE2' has the following parameters:

| Name                   | type                             | size | default value | description                                                                                                                                                                   |
|------------------------|----------------------------------|------|---------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| name                   | String                           |      | "             | objects's unique name                                                                                                                                                         |
| nodeNumbers            | ArrayNodeIndex                   |      | []            | node numbers which provide the coordinates for the object (consecutively as provided in this list)                                                                            |
| massMatrix             | NumpyMatrix                      |      | Matrix[]      | mass matrix of object in python numpy format                                                                                                                                  |
| stiffnessMatrix        | NumpyMatrix                      |      | Matrix[]      | stiffness matrix of object in python numpy format                                                                                                                             |
| dampingMatrix          | NumpyMatrix                      |      | Matrix[]      | damping matrix of object in python numpy format                                                                                                                               |
| forceVector            | NumpyVector                      |      | []            | generalized force vector added to RHS                                                                                                                                         |
| forceUserFunction      | PyFunctionVectorMbsScalar2Vector |      | 0             | A python user function which computes the generalized user force vector for the ODE2 equations; see description below                                                         |
| massMatrixUserFunction | PyFunctionMatrixMbsScalar2Vector |      | 0             | A python user function which computes the mass matrix instead of the constant mass matrix; see description below                                                              |
| coordinateIndexPerNode | ArrayIndex                       |      | []            | this list contains the local coordinate index for every node, which is needed, e.g., for markers; the list is generated automatically every time parameters have been changed |
| tempCoordinates        | NumpyVector                      |      | []            | temporary vector containing coordinates                                                                                                                                       |
| tempCoordinates_t      | NumpyVector                      |      | []            | temporary vector containing velocity coordinates                                                                                                                              |
| tempCoordinates_tt     | NumpyVector                      |      | []            | temporary vector containing acceleration coordinates                                                                                                                          |
| visualization          | VObjectGenericODE2               |      |               | parameters for visualization of item                                                                                                                                          |

The item VObjectGenericODE2 has the following parameters:

| Name | type | size | default value | description                                                              |
|------|------|------|---------------|--------------------------------------------------------------------------|
| show | Bool |      | True          | set true, if item is shown in visualization and false if it is not shown |

|                          |                        |   |                   |                                                                                                                                                                                                                                                                                                                                                                                |
|--------------------------|------------------------|---|-------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| color                    | Float4                 | 4 | [-1.,-1.,-1.,-1.] | RGBA color for object; 4th value is alpha-transparency; R=-1.f means, that default color is used                                                                                                                                                                                                                                                                               |
| triangleMesh             | NumpyMatrixI           |   | MatrixI[]         | a matrix, containing node number triples in every row, referring to the node numbers of the GenericODE2 object; the mesh uses the nodes to visualize the underlying object; contour plot colors are still computed in the local frame!                                                                                                                                         |
| showNodes                | Bool                   |   | False             | set true, nodes are drawn uniquely via the mesh, eventually using the floating reference frame, even in the visualization of the node is show=False; node numbers are shown with indicator 'NF'                                                                                                                                                                                |
| graphicsDataUserFunction | PyFunctionGraphicsData | 0 |                   | A python function which returns a body-GraphicsData object, which is a list of graphics data in a dictionary computed by the user function; the graphics data is drawn in global coordinates; it can be used to implement user element visualization, e.g., beam elements or simple mechanical systems; note that this user function may significantly slow down visualization |

### 7.2.8.1 DESCRIPTION of ObjectGenericODE2:

Information on input parameters:

| input parameter        | symbol                                          | description see tables above |
|------------------------|-------------------------------------------------|------------------------------|
| nodeNumbers            | $\mathbf{n}_n = [n_0, \dots, n_n]^T$            |                              |
| massMatrix             | $\mathbf{M} \in \mathbb{R}^{n \times n}$        |                              |
| stiffnessMatrix        | $\mathbf{K} \in \mathbb{R}^{n \times n}$        |                              |
| dampingMatrix          | $\mathbf{D} \in \mathbb{R}^{n \times n}$        |                              |
| forceVector            | $\mathbf{f} \in \mathbb{R}^n$                   |                              |
| forceUserFunction      | $\mathbf{f}_{user} \in \mathbb{R}^n$            |                              |
| massMatrixUserFunction | $\mathbf{M}_{user} \in \mathbb{R}^{n \times n}$ |                              |
| tempCoordinates        | $\mathbf{c}_{temp} \in \mathbb{R}^n$            |                              |
| tempCoordinates_t      | $\dot{\mathbf{c}}_{temp} \in \mathbb{R}^n$      |                              |
| tempCoordinates_tt     | $\ddot{\mathbf{c}}_{temp} \in \mathbb{R}^n$     |                              |

The following output variables are available as OutputVariableType in sensors, Get...Output() and other functions:

| output variable | symbol | description                       |
|-----------------|--------|-----------------------------------|
| Coordinates     |        | all ODE2 coordinates              |
| Coordinates_t   |        | all ODE2 velocity coordinates     |
| Coordinates_tt  |        | all ODE2 acceleration coordinates |

|       |  |                                                                                                                         |
|-------|--|-------------------------------------------------------------------------------------------------------------------------|
| Force |  | generalized forces for all coordinates (residual of all forces except mass*acceleration; corresponds to ComputeODE2LHS) |
|-------|--|-------------------------------------------------------------------------------------------------------------------------|

### 7.2.8.2 Additional output variables for superelement node access

Functions like `GetObjectOutputSuperElement(...)`, see [Section 4.3.2](#), or `SensorSuperElement`, see [Section 4.3.5](#), directly access special output variables (`OutputVariableType`) of the mesh nodes of the superelement. Additionally, the contour drawing of the object can make use the `OutputVariableType` of the meshnodes.

For this object, all nodes of `ObjectGenericODE2` map their `OutputVariableType` to the meshnode → see at the according node for the list of `OutputVariableType`.

### 7.2.8.3 Equations of motion

An object with node numbers  $[n_0, \dots, n_n]$  and according numbers of nodal coordinates  $[n_{c_0}, \dots, n_{c_n}]$ , the total number of equations (=coordinates) of the object is

$$n = \sum_i n_{c_i}, \quad (7.48)$$

which is used throughout the description of this object.

### 7.2.8.4 Equations of motion

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{D}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{f} + \mathbf{f}_{user}(mbs, t, \mathbf{q}, \dot{\mathbf{q}}) \quad (7.49)$$

Note that the user function  $\mathbf{f}_{user}(mbs, t, \mathbf{q}, \dot{\mathbf{q}})$  may be empty ( $=0$ ).

In case that a user mass matrix is specified, Eq. (7.49) is replaced with

$$\mathbf{M}_{user}(mbs, t, \mathbf{q}, \dot{\mathbf{q}})\ddot{\mathbf{q}} + \mathbf{D}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{f} + \mathbf{f}_{user}(mbs, t, \mathbf{q}, \dot{\mathbf{q}}) \quad (7.50)$$

CoordinateLoads are added for the respective ODE2 coordinate on the RHS of the latter equation.

---

### Userfunction: `forceUserFunction(mbs, t, q, q_t)`

A user function, which computes a force vector depending on current time and states of object. Can be used to create any kind of mechanical system by using the object states.

| arguments / return  | type or size              | description                                                                                                  |
|---------------------|---------------------------|--------------------------------------------------------------------------------------------------------------|
| <code>mbs</code>    | MainSystem                | provides MainSystem <code>mbs</code> to which object belongs                                                 |
| <code>t</code>      | Real                      | current time in <code>mbs</code>                                                                             |
| <code>q</code>      | Vector $\in \mathbb{R}^n$ | object coordinates (e.g., nodal displacement coordinates) in current configuration, without reference values |
| <code>q_t</code>    | Vector $\in \mathbb{R}^n$ | object velocity coordinates (time derivative of <code>q</code> ) in current configuration                    |
| <b>return value</b> | Vector $\in \mathbb{R}^n$ | returns force vector for object                                                                              |

---

### Userfunction: massMatrixUserFunction(mbs, t, q, q\_t)

A user function, which computes a mass matrix depending on current time and states of object. Can be used to create any kind of mechanical system by using the object states.

| arguments / return | type or size                              | description                                                                                                  |
|--------------------|-------------------------------------------|--------------------------------------------------------------------------------------------------------------|
| mbs                | MainSystem                                | provides MainSystem mbs to which object belongs to                                                           |
| t                  | Real                                      | current time in mbs                                                                                          |
| q                  | Vector $\in \mathbb{R}^n$                 | object coordinates (e.g., nodal displacement coordinates) in current configuration, without reference values |
| q_t                | Vector $\in \mathbb{R}^n$                 | object velocity coordinates (time derivative of q) in current configuration                                  |
| return value       | NumpyMatrix $\in \mathbb{R}^{n \times n}$ | returns mass matrix for object                                                                               |

---

### Userfunction: graphicsDataUserFunction(mbs, itemNumber)

A user function, which is called by the visualization thread in order to draw user-defined objects. The function can be used to generate any BodyGraphicsData, see Section 9.3. Use graphicsDataUtilities functions, see Section 5.4, to create more complicated objects. Note that graphicsDataUserFunction needs to copy lots of data and is therefore inefficient and only designed to enable simpler tests, but not large scale problems.

For an example for graphicsDataUserFunction see ObjectGround, [Section 7.2.1](#).

| arguments / return | type or size     | description                                                                                    |
|--------------------|------------------|------------------------------------------------------------------------------------------------|
| mbs                | MainSystem       | provides reference to mbs, which can be used in user function to access all data of the object |
| itemNumber         | Index            | integer number of the object in mbs, allowing easy access                                      |
| return value       | BodyGraphicsData | list of GraphicsData dictionaries, see Section 9.3                                             |

---

### User function example:

```
#user function, using variables M, K, ... from mini example, replacing
ObjectGenericODE2(...)

KD = numpy.diag([200,100])
#nonlinear force example
def UFforce(mbs, t, q, q_t):
 return np.dot(KD, q_t*q) #add nonlinear function for q_t and q, q_t*q gives
 vector

#non-constant mass matrix:
def UFmass(mbs, t, q, q_t):
 return return (q[0]+1)*M #uses mass matrix from mini example
```

```

#non-constant mass matrix:
def UFgraphics(mbs, objectNum):
 t = mbs.systemData.GetTime(exu.ConfigurationType.Visualization) #get time if
 needed
 p = mbs.GetObjectOutputSuperElement(objectNumber=objectNum, variableType = exu.
 OutputVariableType.Position,
 meshNodeNumber = 0, #get first node's
 position
 configuration = exu.ConfigurationType.
 Visualization)
 graphics1=GraphicsDataSphere(point=p, radius=0.1, color=color4red)
 graphics2 = {'type':'Line', 'data': list(p)+[0,0,0], 'color':color4blue}
 return [graphics1, graphics2]

#now add object instead of object in mini-example:
oGenericODE2 = mbs.AddObject(ObjectGenericODE2(nodeNumbers=[nMass0,nMass1],
 massMatrix=M, stiffnessMatrix=K, dampingMatrix=D,
 forceUserFunction=UFForce, massMatrixUserFunction=UFmass,
 visualization=VObjectGenericODE2(graphicsDataUserFunction=
 UFgraphics))

```

---

### 7.2.8.5 MINI EXAMPLE for ObjectGenericODE2

```

#set up a mechanical system with two nodes; it has the structure: |~~M0~~M1
nMass0 = mbs.AddNode(NodePoint(referenceCoordinates=[0,0,0]))
nMass1 = mbs.AddNode(NodePoint(referenceCoordinates=[1,0,0]))

mass = 0.5 * np.eye(3) #mass of nodes
stif = 5000 * np.eye(3) #stiffness of nodes
damp = 50 * np.eye(3) #damping of nodes
Z = 0. * np.eye(3) #matrix with zeros
#build mass, stiffness and damping matrices (:
M = np.block([[mass, 0.*np.eye(3)],
 [0.*np.eye(3), mass]])
K = np.block([[2*stif, -stif],
 [-stif, stiff]])
D = np.block([[2*damp, -damp],
 [-damp, damp]])

oGenericODE2 = mbs.AddObject(ObjectGenericODE2(nodeNumbers=[nMass0,nMass1],
 massMatrix=M,
 stiffnessMatrix=K,
 dampingMatrix=D))

mNode1 = mbs.AddMarker(MarkerNodePosition(nodeNumber=nMass1))

```

```

mbs.AddLoad(Force(markerNumber = mNode1, loadVector = [10, 0, 0])) #static solution
=10*(1/5000+1/5000)=0.0004

#assemble and solve system for default parameters
mbs.Assemble()

exu.SolveDynamic(mbs, solverType = exudyn.DynamicSolverType.TrapezoidalIndex2)

#check result at default integration time
exudynTestGlobals.testResult = mbs.GetNodeOutput(nMass1, exu.OutputVariableType.
Position)[0]

```

---

For further examples on ObjectGenericODE2 see Examples:

- [simulateInteractively.py](#)

For further examples on ObjectGenericODE2 see TestModels:

- [genericODE2test.py](#)
- [objectFFRFTest.py](#)
- [objectGenericODE2Test.py](#)
- [solverExplicitODE1ODE2test.py](#)

### 7.2.9 ObjectGenericODE1

A system of  $n$  second order ordinary differential equations (ODE1), having a system matrix, a rhs vector, but mostly it will use a user function to describe special ODE1 systems. It is based on NodeGenericODE1 nodes. NOTE that all matrices, vectors, etc. must have the same dimensions  $n$  or  $(n \times n)$ , or they must be empty ( $0 \times 0$ ), using [] in python.

**Additional information for ObjectGenericODE1:**

- The Object has the following types = MultiNoded
- Requested node type: read detailed information of item

The item **ObjectGenericODE1** with type = 'GenericODE1' has the following parameters:

| Name                   | type                            | size | default value | description                                                                                                                                                                   |
|------------------------|---------------------------------|------|---------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| name                   | String                          |      | "             | objects's unique name                                                                                                                                                         |
| nodeNumbers            | ArrayNodeIndex                  |      | []            | node numbers which provide the coordinates for the object (consecutively as provided in this list)                                                                            |
| systemMatrix           | NumpyMatrix                     |      | Matrix[]      | system matrix (state space matrix) of first order ODE                                                                                                                         |
| rhsVector              | NumpyVector                     |      | []            | a constant rhs vector (e.g., for constant input)                                                                                                                              |
| rhsUserFunction        | PyFunctionVectorMbsScalarVector |      | 0             | A python user function which computes the right-hand-side (rhs) of the first order ODE; see description below                                                                 |
| coordinateIndexPerNode | ArrayIndex                      |      | []            | this list contains the local coordinate index for every node, which is needed, e.g., for markers; the list is generated automatically every time parameters have been changed |
| tempCoordinates        | NumpyVector                     |      | []            | temporary vector containing coordinates                                                                                                                                       |
| tempCoordinates_t      | NumpyVector                     |      | []            | temporary vector containing velocity coordinates                                                                                                                              |
| visualization          | VObjectGenericODE1              |      |               | parameters for visualization of item                                                                                                                                          |

The item VObjectGenericODE1 has the following parameters:

| Name | type | size | default value | description                                                              |
|------|------|------|---------------|--------------------------------------------------------------------------|
| show | Bool |      | True          | set true, if item is shown in visualization and false if it is not shown |

#### 7.2.9.1 DESCRIPTION of ObjectGenericODE1:

**Information on input parameters:**

| input parameter | symbol                               | description see tables above |
|-----------------|--------------------------------------|------------------------------|
| nodeNumbers     | $\mathbf{n}_n = [n_0, \dots, n_n]^T$ |                              |

|                   |                                            |  |
|-------------------|--------------------------------------------|--|
| systemMatrix      | $\mathbf{A} \in \mathbb{R}^{n \times n}$   |  |
| rhsVector         | $\mathbf{f} \in \mathbb{R}^n$              |  |
| rhsUserFunction   | $\mathbf{f}_{user} \in \mathbb{R}^n$       |  |
| tempCoordinates   | $\mathbf{c}_{temp} \in \mathbb{R}^n$       |  |
| tempCoordinates_t | $\dot{\mathbf{c}}_{temp} \in \mathbb{R}^n$ |  |

The following output variables are available as `OutputVariableType` in `sensors`, `Get...Output()` and other functions:

| output variable | symbol | description                   |
|-----------------|--------|-------------------------------|
| Coordinates     |        | all ODE1 coordinates          |
| Coordinates_t   |        | all ODE1 velocity coordinates |

### 7.2.9.2 Equations of motion

An object with node numbers  $[n_0, \dots, n_n]$  and according numbers of nodal coordinates  $[n_{c_0}, \dots, n_{c_n}]$ , the total number of equations (=coordinates) of the object is

$$n = \sum_i n_{c_i}, \quad (7.51)$$

which is used throughout the description of this object.

### 7.2.9.3 Equations of motion

$$\dot{\mathbf{q}} = \mathbf{f} + \mathbf{f}_{user}(mbs, t, \mathbf{q}) \quad (7.52)$$

Note that the user function  $\mathbf{f}_{user}(mbs, t, \mathbf{q})$  may be empty ( $=0$ ). CoordinateLoads are added for the respective ODE1 coordinate on the RHS of the latter equation.

---

#### Userfunction: `rhsUserFunction(mbs, t, q)`

A user function, which computes a RHS vector depending on current time and states of the object. Can be used to create any kind of first order system, especially state space equations (inputs are added via CoordinateLoads to every node).

| arguments / return | type or size              | description                                                                                                  |
|--------------------|---------------------------|--------------------------------------------------------------------------------------------------------------|
| mbs                | MainSystem                | provides MainSystem mbs to which object belongs                                                              |
| t                  | Real                      | current time in mbs                                                                                          |
| q                  | Vector $\in \mathbb{R}^n$ | object coordinates (composed from ODE1 nodal coordinates) in current configuration, without reference values |
| return value       | Vector $\in \mathbb{R}^n$ | returns force vector for object                                                                              |

---

#### User function example:

```
A = numpy.diag([200, 100])
#simple linear user function returning A*q
def UFrhs(mbs, t, q, q_t):
```

```

 return np.dot(A, q) + np.array([0,2])

nODE1 = mbs.AddNode(NodeGenericODE1(referenceCoordinates=[0,0],
 initialCoordinates=[1,0]))

#now add object instead of object in mini-example:
oGenericODE1 = mbs.AddObject(ObjectGenericODE1(nodeNumbers=[nODE1],
 rhsUserFunction=UFrhs))

```

---

#### 7.2.9.4 MINI EXAMPLE for ObjectGenericODE1

```

#set up a 2-DOF system
nODE1 = mbs.AddNode(NodeGenericODE1(referenceCoordinates=[0,0],
 initialCoordinates=[1,0],
 numberODE1Coordinates=2))

#build system matrix and force vector
#undamped mechanical system with m=1, K=100, f=1
A = np.array([[0,1],
 [-100,0]])
b = np.array([0,1])

oGenericODE1 = mbs.AddObject(ObjectGenericODE1(nodeNumbers=[nODE1],
 systemMatrix=A,
 rhsVector=b))

#assemble and solve system for default parameters
mbs.Assemble()

sims=exu.SimulationSettings()
solverType = exu.DynamicSolverType.RK44
exu.SolveDynamic(mbs, solverType=solverType, simulationSettings=sims)

#check result at default integration time
exudynTestGlobals.testResult = mbs.GetNodeOutput(nODE1, exu.OutputVariableType.
 Coordinates)[0]

```

---

For further examples on ObjectGenericODE1 see TestModels:

- [solverExplicitODE1ODE2test.py](#)

## 7.2.10 ObjectFFRF

This object is used to represent equations modelled by the floating frame of reference formulation (FFRF). It contains a RigidBodyNode (always node 0) and a list of other nodes representing the finite element nodes used in the FFRF. Note that temporary matrices and vectors are subject of change in future.

### Additional information for ObjectFFRF:

- The Object has the following types = Body, MultiNoded, SuperElement
- Requested node type: read detailed information of item

The item **ObjectFFRF** with type = 'FFRF' has the following parameters:

| Name              | type              | size             | default value     | description                                                                                                                                                                                                                                                                                                           |
|-------------------|-------------------|------------------|-------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| name              | String            |                  | ''                | objects's unique name                                                                                                                                                                                                                                                                                                 |
| nodeNumbers       | ArrayNodeIndex    |                  | []                | node numbers which provide the coordinates for the object (consecutively as provided in this list); the ( $n_{nf} + 1$ ) nodes represent the nodes of the FE mesh (except for node 0); the global nodal position needs to be reconstructed from the rigid-body motion of the reference frame                          |
| massMatrixFF      | PyMatrixContainer |                  | PyMatrixContainer | [] body-fixed and ONLY flexible coordinates part of mass matrix of object given in python numpy format (sparse (CSR) or dense, converted to sparse matrix); internally data is stored in triplet format                                                                                                               |
| stiffnessMatrixFF | PyMatrixContainer |                  | PyMatrixContainer | [] body-fixed and ONLY flexible coordinates part of stiffness matrix of object in python numpy format (sparse (CSR) or dense, converted to sparse matrix); internally data is stored in triplet format                                                                                                                |
| dampingMatrixFF   | PyMatrixContainer |                  | PyMatrixContainer | [] body-fixed and ONLY flexible coordinates part of damping matrix of object in python numpy format (sparse (CSR) or dense, converted to sparse matrix); internally data is stored in triplet format                                                                                                                  |
| forceVector       | NumpyVector       |                  | []                | generalized, force vector added to RHS; the rigid body part $f_r$ is directly applied to rigid body coordinates while the flexible part $f_f$ is transformed from global to local coordinates; note that this force vector only allows to add gravity forces for bodies with COM at the origin of the reference frame |
| forceUserFunction | PyFunctionVector  | MbsScalar2Vector | 0                 | A python user function which computes the generalized user force vector for the ODE2 equations; note the different coordinate systems for rigid body and flexible part; The function takes the time, coordinates q (without reference values) and coordinate velocities $q\_t$ ; see description below                |

|                        |                  |                             |                                                                                                                                                                                                                                   |
|------------------------|------------------|-----------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| massMatrixUserFunction | PyFunctionMatrix | MbsScalar2Vector<br>0       | A python user function which computes the TOTAL mass matrix (including reference node) and adds the local constant mass matrix; note the different coordinate systems as described in the FFRF mass matrix; see description below |
| computeFFRFterms       | Bool             | True                        | flag decides whether the standard FFRF terms are computed; use this flag for user-defined definition of FFRF terms in mass matrix and quadratic velocity vector                                                                   |
| coordinateIndexPerNode | ArrayIndex       | []                          | this list contains the local coordinate index for every node, which is needed, e.g., for markers; the list is generated automatically every time parameters have been changed                                                     |
| objectIsInitialized    | Bool             | False                       | ALWAYS set to False! flag used to correctly initialize all FFRF matrices; as soon as this flag is False, internal (constant) FFRF matrices are recomputed during Assemble()                                                       |
| physicsMass            | UReal            | 0.                          | total mass [SI:kg] of FFRF object, auto-computed from mass matrix ${}^b\mathbf{M}$                                                                                                                                                |
| physicsInertia         | Matrix3D         | [[1,0,0], [0,1,0], [0,0,1]] | inertia tensor [SI:kgm <sup>2</sup> ] of rigid body w.r.t. to the reference point of the body, auto-computed from the mass matrix ${}^b\mathbf{M}$                                                                                |
| physicsCenterOfMass    | Vector3D         | 3 [0.,0.,0.]                | local position of center of mass (COM); auto-computed from mass matrix ${}^b\mathbf{M}$                                                                                                                                           |
| PHItTM                 | NumpyMatrix      | Matrix[]                    | projector matrix; may be removed in future                                                                                                                                                                                        |
| referencePositions     | NumpyVector      | []                          | vector containing the reference positions of all flexible nodes                                                                                                                                                                   |
| tempVector             | NumpyVector      | []                          | temporary vector                                                                                                                                                                                                                  |
| tempCoordinates        | NumpyVector      | []                          | temporary vector containing coordinates                                                                                                                                                                                           |
| tempCoordinates_t      | NumpyVector      | []                          | temporary vector containing velocity coordinates                                                                                                                                                                                  |
| tempRefPosSkew         | NumpyMatrix      | Matrix[]                    | temporary matrix with skew symmetric local (deformed) node positions                                                                                                                                                              |
| tempVelSkew            | NumpyMatrix      | Matrix[]                    | temporary matrix with skew symmetric local node velocities                                                                                                                                                                        |
| visualization          | VObjectFFRF      |                             | parameters for visualization of item                                                                                                                                                                                              |

The item VObjectFFRF has the following parameters:

| Name | type | size | default value | description                                                                                                                                                                                                                         |
|------|------|------|---------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| show | Bool |      | True          | set true, if item is shown in visualization and false if it is not shown; use visualizationSettings.bodies.deformationScaleFactor to draw scaled (local) deformations; the reference frame node is shown with additional letters RF |

|              |              |   |                   |                                                                                                                                                                                                                                        |
|--------------|--------------|---|-------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| color        | Float4       | 4 | [-1.,-1.,-1.,-1.] | RGBA color for object; 4th value is alpha-transparency; R=-1.f means, that default color is used                                                                                                                                       |
| triangleMesh | NumpyMatrixI |   | MatrixI[]         | a matrix, containing node number triples in every row, referring to the node numbers of the GenericODE2 object; the mesh uses the nodes to visualize the underlying object; contour plot colors are still computed in the local frame! |
| showNodes    | Bool         |   | False             | set true, nodes are drawn uniquely via the mesh, eventually using the floating reference frame, even in the visualization of the node if show=False; node numbers are shown with indicator 'NF'                                        |

---

### 7.2.10.1 DESCRIPTION of ObjectFFRF:

Information on input parameters:

| input parameter        | symbol                                                                                          | description see tables above |
|------------------------|-------------------------------------------------------------------------------------------------|------------------------------|
| nodeNumbers            | $\mathbf{n}_f = [n_0, \dots, n_{n_f}]^T$                                                        |                              |
| massMatrixFF           | ${}^b\mathbf{M} \in \mathbb{R}^{n_f \times n_f}$                                                |                              |
| stiffnessMatrixFF      | ${}^b\mathbf{K} \in \mathbb{R}^{n_f \times n_f}$                                                |                              |
| dampingMatrixFF        | ${}^b\mathbf{D} \in \mathbb{R}^{n_f \times n_f}$                                                |                              |
| forceVector            | ${}^0\mathbf{f} = [{}^0\mathbf{f}_r, {}^0\mathbf{f}_f]^T \in \mathbb{R}^{n_c}$                  |                              |
| forceUserFunction      | $\mathbf{f}_{user} = [{}^0\mathbf{f}_{r,user}, {}^b\mathbf{f}_{f,user}]^T \in \mathbb{R}^{n_c}$ |                              |
| massMatrixUserFunction | $\mathbf{M}_{user} \in \mathbb{R}^{n_c \times n_c}$                                             |                              |
| physicsMass            | $m$                                                                                             |                              |
| physicsInertia         | $J_r \in \mathbb{R}^{3 \times 3}$                                                               |                              |
| physicsCenterOfMass    | ${}^b\mathbf{p}_{COM}$                                                                          |                              |
| PHITM                  | $\Phi_t^T \in \mathbb{R}^{n_f \times 3}$                                                        |                              |
| referencePositions     | $\mathbf{x}_{ref} \in \mathbb{R}^{n_f}$                                                         |                              |
| tempVector             | $\mathbf{v}_{temp} \in \mathbb{R}^{n_f}$                                                        |                              |
| tempCoordinates        | $\mathbf{c}_{temp} \in \mathbb{R}^{n_f}$                                                        |                              |
| tempCoordinates_t      | $\dot{\mathbf{c}}_{temp} \in \mathbb{R}^{n_f}$                                                  |                              |
| tempRefPosSkew         | $\ddot{\mathbf{p}}_t \in \mathbb{R}^{n_f \times 3}$                                             |                              |
| tempVelSkew            | $\dot{\mathbf{c}}_f \in \mathbb{R}^{n_f \times 3}$                                              |                              |

The following output variables are available as OutputVariableType in sensors, Get...Output() and other functions:

| output variable | symbol | description                                                                                                             |
|-----------------|--------|-------------------------------------------------------------------------------------------------------------------------|
| Coordinates     |        | all ODE2 coordinates                                                                                                    |
| Coordinates_t   |        | all ODE2 velocity coordinates                                                                                           |
| Coordinates_tt  |        | all ODE2 acceleration coordinates                                                                                       |
| Force           |        | generalized forces for all coordinates (residual of all forces except mass*acceleration; corresponds to ComputeODE2LHS) |

### 7.2.10.2 Additional output variables for superelement node access

Functions like `GetObjectOutputSuperelement(...)`, see [Section 4.3.2](#), or `SensorSuperelement`, see [Section 4.3.5](#), directly access special output variables (`OutputVariableType`) of the mesh nodes  $n_i$  of the superelement. Additionally, the contour drawing of the object can make use the `OutputVariableType` of the meshnodes.

### 7.2.10.3 Super element output variables

| super element output variables | symbol                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | description                                                                                                                                                                                             |
|--------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Position                       | ${}^0\mathbf{p}_{\text{config}}(n_i) = {}^0\mathbf{p}_{t,\text{config}} + {}^{0b}\mathbf{A}_{\text{config}} {}^b\mathbf{r}_{\text{config}}(n_i)$                                                                                                                                                                                                                                                                                                                                                                             | global position of mesh node $n_i$ including rigid body motion and flexible deformation                                                                                                                 |
| Displacement                   | ${}^0\mathbf{c}_{\text{config}}(n_i) = {}^0\mathbf{p}_{\text{config}}(n_i) - {}^0\mathbf{p}_{\text{ref}}(n_i)$                                                                                                                                                                                                                                                                                                                                                                                                               | global displacement of mesh node $n_i$ including rigid body motion and flexible deformation                                                                                                             |
| Velocity                       | ${}^0\mathbf{v}_{\text{config}}(n_i) = {}^0\dot{\mathbf{p}}_{t,\text{config}} + {}^{0b}\mathbf{A}_{\text{config}} {}^b\dot{\mathbf{q}}_{\text{f config}}(n_i) + {}^b\boldsymbol{\omega}_{\text{config}} \times {}^b\mathbf{r}_{\text{config}}(n_i)$                                                                                                                                                                                                                                                                          | global velocity of mesh node $n_i$ including rigid body motion and flexible deformation                                                                                                                 |
| Acceleration                   | ${}^0\mathbf{a}_{\text{config}}(n_i) = {}^0\ddot{\mathbf{p}}_{t,\text{config}} + {}^{0b}\mathbf{A}_{\text{config}} {}^b\ddot{\mathbf{q}}_{\text{f config}}(n_i) + 2 {}^0\boldsymbol{\omega}_{\text{config}} \times {}^{0b}\mathbf{A}_{\text{config}} {}^b\dot{\mathbf{q}}_{\text{f config}}(n_i) + {}^0\boldsymbol{\alpha}_{\text{config}} \times {}^0\mathbf{r}_{\text{config}}(n_i) + {}^0\boldsymbol{\omega}_{\text{config}} \times ({}^0\boldsymbol{\omega}_{\text{config}} \times {}^0\mathbf{r}_{\text{config}}(n_i))$ | global acceleration of mesh node $n_i$ including rigid body motion and flexible deformation; note that ${}^0\mathbf{r}_{\text{config}}(n_i) = {}^{0b}\mathbf{A} {}^b\mathbf{r}_{\text{config}}(n_i)$    |
| DisplacementLocal              | ${}^b\mathbf{d}_{\text{config}}(n_i) = {}^b\mathbf{r}_{\text{config}}(n_i) - {}^b\mathbf{x}_{\text{ref}}(n_i)$                                                                                                                                                                                                                                                                                                                                                                                                               | local displacement of mesh node $n_i$ , representing the flexible deformation within the body frame; note that ${}^0\mathbf{u}_{\text{config}} \neq {}^{0b}\mathbf{A} {}^b\mathbf{d}_{\text{config}}$ ! |
| VelocityLocal                  | ${}^b\dot{\mathbf{q}}_{\text{f config}}(n_i)$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | local velocity of mesh node $n_i$ , representing the rate of flexible deformation within the body frame                                                                                                 |

### 7.2.10.4 Definition of quantities

| intermediate variables                   | symbol                                                                                                                                 | description                                                                                                                          |
|------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------|
| object coordinates                       | $\mathbf{q} = [\mathbf{q}_t^T, \mathbf{q}_r^T, \mathbf{q}_f^T]^T$                                                                      | object coordinates                                                                                                                   |
| rigid body coordinates                   | $\mathbf{q}_{\text{rigid}} = [\mathbf{q}_t^T, \mathbf{q}_r^T]^T = [q_0, q_1, q_2, \psi_0, \psi_1, \psi_2, \psi_3]^T$                   | rigid body coordinates in case of Euler parameters                                                                                   |
| reference frame (rigid body) position    | ${}^0\mathbf{p}_{t,\text{config}} = {}^0\mathbf{q}_{t,\text{config}} + {}^0\mathbf{q}_{t,\text{ref}}$                                  | global position of underlying rigid body node $n_0$ which defines the reference frame origin                                         |
| reference frame (rigid body) orientation | ${}^{0b}\mathbf{A}(\boldsymbol{\theta})_{\text{config}}$                                                                               | transformation matrix for transformation of local (reference frame) to global coordinates, given by underlying rigid body node $n_0$ |
| local nodal position                     | ${}^b\mathbf{r}^{(i)} = {}^b\mathbf{x}_{\text{ref}}^{(i)} + {}^b\mathbf{q}_f^{(i)}$                                                    | vector of body-fixed (local) position of node $(i)$ , including flexible part                                                        |
| local nodal positions                    | ${}^b\mathbf{r} = {}^b\mathbf{x}_{\text{ref}} + {}^b\mathbf{q}_f$                                                                      | vector of all body-fixed (local) nodal positions including flexible part                                                             |
| rotation coordinates                     | $\boldsymbol{\theta}_{\text{cur}} = [\psi_0, \psi_1, \psi_2, \psi_3]_{\text{ref}}^T + [\psi_0, \psi_1, \psi_2, \psi_3]_{\text{cur}}^T$ | rigid body coordinates in case of Euler parameters                                                                                   |
| flexible coordinates                     | ${}^b\mathbf{q}_f$                                                                                                                     | flexible, body-fixed coordinates                                                                                                     |

|                                        |                                                                               |                                                                                                                  |
|----------------------------------------|-------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------|
| transformation of flexible coordinates | ${}^0b\mathbf{A}_{bd} = \text{diag}({}^0b\mathbf{A}, \dots, {}^0b\mathbf{A})$ | block diagonal transformation matrix, which transforms all flexible coordinates from local to global coordinates |
|----------------------------------------|-------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------|

The derivations follow Zwölfer and Gerstmayr [39] with only small modifications in the notation.

### 7.2.10.5 Nodal coordinates

Consider an object with  $n = 1 + n_{\text{nf}}$  nodes,  $n_{\text{nf}}$  being the number of 'flexible' nodes and one additional node is the rigid body node for the reference frame. The list of node numbers is  $[n_0, \dots, n_{n_{\text{nf}}}]$  and the according numbers of nodal coordinates are  $[n_{c_0}, \dots, n_{c_n}]$ , where  $n_0$  denotes the rigid body node. This gives  $n_c$  total nodal coordinates,

$$n_c = \sum_{i=0}^{n_{\text{nf}}} n_{c_i} , \quad (7.53)$$

whereof the number of flexible coordinates is

$$n_f = 3 \cdot n_{\text{nf}} . \quad (7.54)$$

The total number of equations (=coordinates) of the object is  $n_c$ . The first node  $n_0$  represents the rigid body motion of the underlying reference frame with  $n_{c_r} = n_{c_0}$  coordinates<sup>1</sup>.

### 7.2.10.6 Kinematics

We assume a finite element mesh with The kinematics of the floating frame of reference formulation (FFRF) is based on a splitting of translational ( $\mathbf{c}_t \in \mathbb{R}^{n_t}$ ), rotational ( $\mathbf{c}_r \in \mathbb{R}^{n_r}$ ) and flexible ( $\mathbf{c}_f \in \mathbb{R}^{n_f}$ ) nodal displacements,

$${}^0\mathbf{c} = {}^0\mathbf{c}_t + {}^0\mathbf{c}_r + {}^0\mathbf{c}_f . \quad (7.55)$$

which are written in global coordinates in Eq. (7.55) but will be transformed to other coordinates later on.

In the present formulation of ObjectFFRF, we use the following set of object coordinates (unknowns)

$$\mathbf{q} = \begin{bmatrix} {}^0\mathbf{q}_t^T & \boldsymbol{\theta}^T & {}^b\mathbf{q}_f^T \end{bmatrix}^T \in \mathbb{R}^{n_c} \quad (7.56)$$

with  ${}^0\mathbf{q}_t \in \mathbb{R}^3$ ,  $\boldsymbol{\theta} \in \mathbb{R}^4$  and  ${}^b\mathbf{q}_f \in \mathbb{R}^{n_f}$ . Note that parts of the coordinates  $\mathbf{q}$  can be already interpreted in specific coordinate systems, which is therefore added.

With the relations

$$\boldsymbol{\Phi}_t = [\mathbf{I}, \dots, \mathbf{I}]^T \in \mathbb{R}^{n_t \times 3} , \quad (7.57)$$

$${}^0\mathbf{c}_t = \boldsymbol{\Phi}_t {}^0\mathbf{q}_t , \quad (7.58)$$

$${}^0\mathbf{c}_r = ({}^0b\mathbf{A}_{bd} - \mathbf{I}_{bd}) {}^b\mathbf{x}_{\text{ref}} , \quad (7.59)$$

$${}^0\mathbf{c}_f = {}^0b\mathbf{A}_{bd} {}^b\mathbf{q}_f , \text{and} \quad (7.60)$$

$$\mathbf{I}_{bd} = \text{diag}(\mathbf{I}, \dots, \mathbf{I}) \in \mathbb{R}^{n_t \times n_t} , \quad (7.61)$$

we obtain the total relation of (global) nodal displacements to the object coordinates

$${}^0\mathbf{c} = \boldsymbol{\Phi}_t {}^0\mathbf{q}_t + ({}^0b\mathbf{A}_{bd} - \mathbf{I}_{bd}) {}^b\mathbf{x}_{\text{ref}} + {}^0b\mathbf{A}_{bd} {}^b\mathbf{q}_f . \quad (7.62)$$

---

<sup>1</sup>e.g.,  $n_{c_r} = 6$  coordinates for Euler angles and  $n_{c_r} = 7$  coordinates in case of Euler parameters; currently only the Euler parameter case is implemented.

On velocity level, we have

$${}^0\dot{\mathbf{c}} = \mathbf{L}\dot{\mathbf{q}}, \quad (7.63)$$

with the matrix  $\mathbf{L} \in \mathbb{R}^{n_f \times n_c}$

$$\mathbf{L} = \begin{bmatrix} \Phi_t, & {}^{0b}\mathbf{A}_{bd} {}^b\tilde{\mathbf{r}} {}^b\mathbf{G}, & {}^{0b}\mathbf{A}_{bd} \end{bmatrix} \quad (7.64)$$

with the rotation parameters specific matrix  ${}^b\mathbf{G}$ , implicitly defined in the rigid body node by the relation  ${}^b\omega = {}^b\mathbf{G} \theta$  and the body-fixed nodal position vector (for node  $i$ )

$${}^b\mathbf{r} = {}^b\mathbf{x}_{\text{ref}} + {}^b\mathbf{q}_f, \quad {}^b\mathbf{r}^{(i)} = {}^b\mathbf{x}_{\text{ref}}^{(i)} + {}^b\mathbf{q}_{f,i}^{(i)} \quad (7.65)$$

and the special tilde matrix for vectors  $\mathbf{r} \in \mathbb{R}^{3n_{nf}}$ ,

$${}^b\tilde{\mathbf{r}} = \begin{bmatrix} {}^b\tilde{\mathbf{r}}^{(i)} \\ \vdots \\ {}^b\tilde{\mathbf{r}}^{(i)} \end{bmatrix} \in \mathbb{R}^{3n_{nf} \times 3}. \quad (7.66)$$

with the tilde operator for a  $\mathbf{r}^{(i)} \in \mathbb{R}^3$  defined in the common notations section.

### 7.2.10.7 Equations of motion

We use the Lagrange equations extended for constraint  $\mathbf{g}$ ,

$$\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{\mathbf{q}}^T} \right) - \frac{\partial T}{\partial \mathbf{q}^T} + \frac{\partial V}{\partial \mathbf{q}^T} + \frac{\partial \lambda^T \mathbf{g}}{\partial \mathbf{q}^T} = \frac{\partial W}{\partial \mathbf{q}^T} \quad (7.67)$$

with the quantities

$$T({}^0\dot{\mathbf{c}}(\mathbf{q}, \dot{\mathbf{q}})) = \frac{1}{2} {}^0\dot{\mathbf{c}}^T {}^0\mathbf{M} {}^0\dot{\mathbf{c}} = \frac{1}{2} {}^0\dot{\mathbf{c}}^T {}^{0b}\mathbf{A}_{bd} {}^b\mathbf{M} {}^{0b}\mathbf{A}_{bd} {}^T {}^0\dot{\mathbf{c}} = \frac{1}{2} {}^0\dot{\mathbf{c}}^T {}^b\mathbf{M} {}^0\dot{\mathbf{c}} \quad (7.68)$$

$$V({}^0\mathbf{q}_f) = \frac{1}{2} {}^b\mathbf{q}_f^T {}^b\mathbf{K} {}^b\mathbf{q}_f \quad (7.69)$$

$$\delta W({}^0\mathbf{c}(\mathbf{q}), t) = {}^b\delta\mathbf{c}^T \mathbf{f} \quad (7.70)$$

$$\mathbf{g}(\mathbf{q}, t) = \mathbf{0} \quad (7.71)$$

$$(7.72)$$

Note that  ${}^b\mathbf{M}$  and  ${}^b\mathbf{K}$  are the conventional finite element mass and stiffness matrices defined in the body frame.

Elementary differentiation rules of the Lagrange equations lead to

$$\mathbf{L}^T \mathbf{M} \mathbf{L} \ddot{\mathbf{q}} + \mathbf{L}^T \mathbf{M} \dot{\mathbf{L}} \dot{\mathbf{q}} + \hat{\mathbf{K}} \mathbf{q} + \frac{\partial \mathbf{g}}{\partial \mathbf{q}^T} \lambda = \mathbf{L}^T \mathbf{f} \quad (7.73)$$

with  $\mathbf{M} = {}^b\mathbf{M}$  and  $\hat{\mathbf{K}}$  becoming obvious in Eq. (7.74). Note that Eq. (7.73) is given in global coordinates for the translational part, in terms of rotation parameters for the rotation part and in body-fixed coordinates for the flexible part of the equations.

In case that `computeFFRFterms` = True, the equations 7.73 can be transformed into the equations of motion,

$$\left( \mathbf{M}_{user}(mbs, t, \mathbf{q}, \dot{\mathbf{q}}) + \begin{bmatrix} \mathbf{M}_{tt} & \mathbf{M}_{tr} & \mathbf{M}_{tf} \\ \mathbf{M}_{rr} & \mathbf{M}_{rf} & {}^b\mathbf{M} \\ \text{sym.} & & {}^b\mathbf{M} \end{bmatrix} \right) \ddot{\mathbf{q}} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & {}^b\mathbf{D} \end{bmatrix} \dot{\mathbf{q}} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & {}^b\mathbf{K} \end{bmatrix} \mathbf{q} = \mathbf{f}_v(\mathbf{q}, \dot{\mathbf{q}}) + \begin{bmatrix} \mathbf{f}_r \\ {}^{0b}\mathbf{A}_{bd}^T \mathbf{f}_f \end{bmatrix} + \mathbf{f}_{user}(mbs, t, \mathbf{q}, \dot{\mathbf{q}}) \quad (7.74)$$

The mass terms are given as

$$\mathbf{M}_{tt} = \Phi_t^T {}^b\mathbf{M} \Phi_t, \quad (7.75)$$

$$\mathbf{M}_{tr} = -{}^{0b}\mathbf{A} \Phi_t^T {}^b\mathbf{M} {}^b\tilde{\mathbf{r}} {}^b\mathbf{G}, \quad (7.76)$$

$$\mathbf{M}_{tf} = {}^{0b}\mathbf{A} \Phi_t^T {}^b\mathbf{M}, \quad (7.77)$$

$$\mathbf{M}_{rr} = {}^b\mathbf{G}^T {}^b\tilde{\mathbf{r}}^T {}^b\mathbf{M} {}^b\tilde{\mathbf{r}} {}^b\mathbf{G}, \quad (7.78)$$

$$\mathbf{M}_{rf} = -{}^b\mathbf{G}^T {}^b\tilde{\mathbf{r}}^T {}^b\mathbf{M}. \quad (7.79)$$

In case that `computeFFRFterms = False`, the mass terms  $\mathbf{M}_{tt}, \mathbf{M}_{tr}, \mathbf{M}_{tf}, \mathbf{M}_{rr}, \mathbf{M}_{rf}, {}^b\mathbf{M}$  in Eq. (7.74) are set to zero (and not computed) and the quadratic velocity vector  $\mathbf{f}_v = \mathbf{0}$ . Note that the user functions  $\mathbf{f}_{user}(mbs, t, \mathbf{q}, \dot{\mathbf{q}})$  and  $\mathbf{M}_{user}(mbs, t, \mathbf{q}, \dot{\mathbf{q}})$  may be empty (=0). The detailed equations of motion for this element can be found in [38].

The quadratic velocity vector follows as

$$\mathbf{f}_v(\mathbf{q}, \dot{\mathbf{q}}) = \begin{bmatrix} -{}^{0b}\mathbf{A} \Phi_t^T {}^b\mathbf{M} \left( {}^b\tilde{\omega}_{bd} {}^b\tilde{\omega}_{bd} {}^b\mathbf{r} + 2 {}^b\tilde{\omega}_{bd} {}^b\dot{\mathbf{q}}_f - {}^b\tilde{\mathbf{r}} {}^b\dot{\mathbf{G}} \dot{\theta} \right) \\ {}^b\mathbf{G}^T {}^b\tilde{\mathbf{r}}^T {}^b\mathbf{M} \left( {}^b\tilde{\omega}_{bd} {}^b\tilde{\omega}_{bd} {}^b\mathbf{r} + 2 {}^b\tilde{\omega}_{bd} {}^b\dot{\mathbf{q}}_f - {}^b\tilde{\mathbf{r}} {}^b\dot{\mathbf{G}} \dot{\theta} \right) \\ -{}^b\mathbf{M} \left( {}^b\tilde{\omega}_{bd} {}^b\tilde{\omega}_{bd} {}^b\mathbf{r} + 2 {}^b\tilde{\omega}_{bd} {}^b\dot{\mathbf{q}}_f - {}^b\tilde{\mathbf{r}} {}^b\dot{\mathbf{G}} \dot{\theta} \right) \end{bmatrix} \quad (7.80)$$

with the special matrix

$${}^b\tilde{\omega}_{bd} = \text{diag}\left({}^b\tilde{\omega}_{bd}, \dots, {}^b\tilde{\omega}_{bd}\right) \in \mathbb{R}^{n_t \times n_t} \quad (7.81)$$

CoordinateLoads are added for each ODE2 coordinate on the RHS of the latter equation.

If the rigid body node is using Euler parameters  $\theta = [\theta_0, \theta_1, \theta_2, \theta_3]^T$ , an **additional constraint** (constraint nr. 0) is added automatically for the Euler parameter norm, reading

$$1 - \sum_{i=0}^3 \theta_i^2 = 0. \quad (7.82)$$

In order to suppress the rigid body motion of the mesh nodes, you should apply a `ObjectConnectorCoordinateVector` object with the following constraint equations which impose constraints of a so-called Tisserand frame, giving 3 constraints for the position of the center of mass

$$\Phi_t^T {}^b\mathbf{M} \mathbf{q}_f = 0 \quad (7.83)$$

and 3 constraints for the rotation,

$$\tilde{\mathbf{x}}_f^T {}^b\mathbf{M} \mathbf{q}_f = 0 \quad (7.84)$$

### Userfunction: `forceUserFunction(mbs, t, q, q_t)`

A user function, which computes a force vector depending on current time and states of object. Can be used to create any kind of mechanical system by using the object states.

| arguments / return  | type or size                  | description                                                                                                                         |
|---------------------|-------------------------------|-------------------------------------------------------------------------------------------------------------------------------------|
| <code>mbs</code>    | MainSystem                    | provides MainSystem <code>mbs</code> to which object belongs                                                                        |
| <code>t</code>      | Real                          | current time in <code>mbs</code>                                                                                                    |
| <code>q</code>      | Vector $\in \mathbb{R}_c^n$   | object coordinates (nodal displacement coordinates of rigid body and mesh nodes) in current configuration, without reference values |
| <code>q_t</code>    | Vector $\in \mathbb{R}_c^n$   | object velocity coordinates (time derivative of <code>q</code> ) in current configuration                                           |
| <b>return value</b> | Vector $\in \mathbb{R}^{n_c}$ | returns force vector for object                                                                                                     |

---

**Userfunction: `massMatrixUserFunction(mbs, t, q, q_t)`**

A user function, which computes a mass matrix depending on current time and states of object. Can be used to create any kind of mechanical system by using the object states.

| arguments / return  | type or size                                  | description                                                                                                                         |
|---------------------|-----------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------|
| <code>mbs</code>    | MainSystem                                    | provides MainSystem mbs to which object belongs                                                                                     |
| <code>t</code>      | Real                                          | current time in mbs                                                                                                                 |
| <code>q</code>      | Vector $\in \mathbb{R}_c^n$                   | object coordinates (nodal displacement coordinates of rigid body and mesh nodes) in current configuration, without reference values |
| <code>q_t</code>    | Vector $\in \mathbb{R}_c^n$                   | object velocity coordinates (time derivative of <code>q</code> ) in current configuration                                           |
| <b>return value</b> | NumpyMatrix $\in \mathbb{R}^{n_c \times n_c}$ | returns mass matrix for object                                                                                                      |

---

For further examples on ObjectFFRF see Examples:

- [sliderCrankCMSacme.py](#)

For further examples on ObjectFFRF see TestModels:

- [objectFFRFTest.py](#)
- [objectFFRFTest2.py](#)

### 7.2.11 ObjectFFRFreducedOrder

This object is used to represent modally reduced flexible bodies using the floating frame of reference formulation (FFRF) and the component mode synthesis (CMS). It can be used to model real-life mechanical systems imported from finite element codes or python tools such as NETGEN/NGsolve, see the [FEMinterface](#) in [Section 5.3.8](#). It contains a RigidBodyNode (always node 0) and a NodeGenericODE2 representing the modal coordinates. Currently, equations must be defined within user functions, which are available in the FEM module, see class `ObjectFFRFreducedOrderInterface`, especially the user functions `UFmassFFRFreducedOrder` and `UFforceFFRFreducedOrder`, [Section 5.3.6](#).

**Additional information for ObjectFFRFreducedOrder:**

- The Object has the following types = Body, MultiNoded, SuperElement
- Requested node type: read detailed information of item
- **Short name for Python = CMSobject**
- **Short name for Python (visualization object) = VCMSobject**

The item **ObjectFFRFreducedOrder** with type = 'FFRFreducedOrder' has the following parameters:

| Name                   | type                             | size              | default value | description                                                                                                                                                                                                                |
|------------------------|----------------------------------|-------------------|---------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| name                   | String                           |                   | "             | objects's unique name                                                                                                                                                                                                      |
| nodeNumbers            | ArrayNodeIndex                   | []                |               | node numbers of rigid body node and NodeGenericODE2 for modal coordinates; the global nodal position needs to be reconstructed from the rigid-body motion of the reference frame, the modal coordinates and the mode basis |
| massMatrixReduced      | PyMatrixContainer                | PyMatrixContainer | []            | body-fixed and ONLY flexible coordinates part of reduced mass matrix; provided as MatrixContainer(sparse/dense matrix)                                                                                                     |
| stiffnessMatrixReduced | PyMatrixContainer                | PyMatrixContainer | []            | body-fixed and ONLY flexible coordinates part of reduced stiffness matrix; provided as MatrixContainer(sparse/dense matrix)                                                                                                |
| dampingMatrixReduced   | PyMatrixContainer                | PyMatrixContainer | []            | body-fixed and ONLY flexible coordinates part of reduced damping matrix; provided as MatrixContainer(sparse/dense matrix)                                                                                                  |
| forceUserFunction      | PyFunctionVectorMbsScalar2Vector | 0                 |               | A python user function which computes the generalized user force vector for the ODE2 equations; see description below                                                                                                      |
| massMatrixUserFunction | PyFunctionMatrixMbsScalar2Vector | 0                 |               | A python user function which computes the TOTAL mass matrix (including reference node) and adds the local constant mass matrix; see description below                                                                      |
| computeFFRFterms       | Bool                             | True              |               | flag decides whether the standard FFRF/CMS terms are computed; use this flag for user-defined definition of FFRF terms in mass matrix and quadratic velocity vector                                                        |
| modeBasis              | NumpyMatrix                      | Matrix[]          |               | mode basis, which transforms reduced coordinates to (full) nodal coordinates, written as a single vector $[u_{x,n_0}, u_{y,n_0}, u_{z,n_0}, \dots, u_{x,n_n}, u_{y,n_n}, u_{z,n_n}]^T$                                     |

|                             |                         |                             |                                                                                                                                                                                               |
|-----------------------------|-------------------------|-----------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| outputVariableModeBasis     | NumpyMatrix             | Matrix[]                    | mode basis, which transforms reduced coordinates to output variables per mode and per node; $s_{OV}$ is the size of the output variable, e.g., 6 for stress modes ( $S_{xx}, \dots, S_{xy}$ ) |
| outputVariableTypeModeBasis | OutputVariableType      | OutputVariableType          | the <code>None</code> must be the output variable type of the <code>outputVariableModeBasis</code> , e.g. <code>exu.OutputVariableType.Stress</code>                                          |
| referencePositions          | NumpyVector             | []                          | vector containing the reference positions of all flexible nodes, needed for graphics                                                                                                          |
| objectIsInitialized         | Bool                    | False                       | ALWAYS set to False! flag used to correctly initialize all FFRF matrices; as soon as this flag is False, some internal (constant) FFRF matrices are recomputed during <code>Assemble()</code> |
| physicsMass                 | UReal                   | 0.                          | total mass [SI:kg] of FFRReducedOrder object                                                                                                                                                  |
| physicsInertia              | Matrix3D                | [[1,0,0], [0,1,0], [0,0,1]] | inertia tensor [SI:kgm <sup>2</sup> ] of rigid body w.r.t. to the reference point of the body                                                                                                 |
| physicsCenterOfMass         | Vector3D                | 3 [0.,0.,0.]                | local position of center of mass (COM)                                                                                                                                                        |
| mPsiTildePsi                | NumpyMatrix             | Matrix[]                    | special FFRReducedOrder matrix, computed in ObjectFFRRReducedOrderInterface                                                                                                                   |
| mPsiTildePsiTilde           | NumpyMatrix             | Matrix[]                    | special FFRReducedOrder matrix, computed in ObjectFFRRReducedOrderInterface                                                                                                                   |
| mPhitTPsi                   | NumpyMatrix             | Matrix[]                    | special FFRReducedOrder matrix, computed in ObjectFFRRReducedOrderInterface                                                                                                                   |
| mPhitTPsiTilde              | NumpyMatrix             | Matrix[]                    | special FFRReducedOrder matrix, computed in ObjectFFRRReducedOrderInterface                                                                                                                   |
| mXRefTildePsi               | NumpyMatrix             | Matrix[]                    | special FFRReducedOrder matrix, computed in ObjectFFRRReducedOrderInterface                                                                                                                   |
| mXRefTildePsiTilde          | NumpyMatrix             | Matrix[]                    | special FFRReducedOrder matrix, computed in ObjectFFRRReducedOrderInterface                                                                                                                   |
| physicsCenterOfMassTilde    | Matrix3D                | [[0,0,0], [0,0,0], [0,0,0]] | tilde matrix from local position of COM; autocomputed during initialization                                                                                                                   |
| tempUserFunctionForce       | NumpyVector             | []                          | temporary vector for UF force                                                                                                                                                                 |
| visualization               | VObjectFFRRReducedOrder |                             | parameters for visualization of item                                                                                                                                                          |

The item `VObjectFFRRReducedOrder` has the following parameters:

| Name | type | size | default value | description |
|------|------|------|---------------|-------------|
|------|------|------|---------------|-------------|

|              |              |   |                   |                                                                                                                                                                                                                                        |
|--------------|--------------|---|-------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| show         | Bool         |   | True              | set true, if item is shown in visualization and false if it is not shown; use visualizationSettings.bodies.deformationScaleFactor to draw scaled (local) deformations; the reference frame node is shown with additional letters RF    |
| color        | Float4       | 4 | [-1.,-1.,-1.,-1.] | RGBA color for object; 4th value is alpha-transparency; R=-1.f means, that default color is used                                                                                                                                       |
| triangleMesh | NumpyMatrixI |   | MatrixI[]         | a matrix, containing node number triples in every row, referring to the node numbers of the GenericODE2 object; the mesh uses the nodes to visualize the underlying object; contour plot colors are still computed in the local frame! |
| showNodes    | Bool         |   | False             | set true, nodes are drawn uniquely via the mesh, eventually using the floating reference frame, even in the visualization of the node is show=False; node numbers are shown with indicator 'NF'                                        |

### 7.2.11.1 DESCRIPTION of ObjectFFRFreducedOrder:

Information on input parameters:

| input parameter          | symbol                                                                             | description see tables above |
|--------------------------|------------------------------------------------------------------------------------|------------------------------|
| nodeNumbers              | $\mathbf{n} = [n_0, n_1]^T$                                                        |                              |
| massMatrixReduced        | $\mathbf{M}_{\text{red}} \in \mathbb{R}^{n_m \times n_m}$                          |                              |
| stiffnessMatrixReduced   | $\mathbf{K}_{\text{red}} \in \mathbb{R}^{n_m \times n_m}$                          |                              |
| dampingMatrixReduced     | $\mathbf{D}_{\text{red}} \in \mathbb{R}^{n_m \times n_m}$                          |                              |
| forceUserFunction        | $\mathbf{f}_{\text{user}} \in \mathbb{R}^{n_{\text{ODE2}}}$                        |                              |
| massMatrixUserFunction   | $\mathbf{M}_{\text{user}} \in \mathbb{R}^{n_{\text{ODE2}} \times n_{\text{ODE2}}}$ |                              |
| modeBasis                | ${}^b\boldsymbol{\Psi} \in \mathbb{R}^{n_t \times n_m}$                            |                              |
| outputVariableModeBasis  | ${}^b\boldsymbol{\Psi}_{OV} \in \mathbb{R}^{n_t \times (n_m \cdot s_{OV})}$        |                              |
| referencePositions       | ${}^b\mathbf{x}_{\text{ref}} \in \mathbb{R}^{n_t}$                                 |                              |
| physicsMass              | $m$                                                                                |                              |
| physicsInertia           | $\mathbf{J}_r \in \mathbb{R}^{3 \times 3}$                                         |                              |
| physicsCenterOfMass      | ${}^b\mathbf{p}_{\text{COM}}$                                                      |                              |
| physicsCenterOfMassTilde | ${}^b\tilde{\mathbf{p}}_{\text{COM}}$                                              |                              |
| tempUserFunctionForce    | $\mathbf{f}_{\text{temp}} \in \mathbb{R}^{n_{\text{ODE2}}}$                        |                              |

The following output variables are available as OutputVariableType in sensors, Get...Output() and other functions:

| output variable | symbol | description          |
|-----------------|--------|----------------------|
| Coordinates     |        | all ODE2 coordinates |

|               |  |                                                                                                                                                                                                                                                       |
|---------------|--|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Coordinates_t |  | all ODE2 velocity coordinates                                                                                                                                                                                                                         |
| Force         |  | generalized forces for all coordinates (residual of all forces except mass*acceleration; corresponds to ComputeODE2LHS)                                                                                                                               |
| StressLocal   |  | allows to compute linearized, corotational nodal stresses (in mesh nodes, in body frame) based on modal stress values provided in outputVariableModeBasis; the flag outputVariableTypeModeBasis must be set in this case to exu.Outputvariable.Stress |
| StrainLocal   |  | allows to compute linearized, corotational nodal strains (in mesh nodes, in body frame) based on modal strain values provided in outputVariableModeBasis; the flag outputVariableTypeModeBasis must be set in this case to exu.Outputvariable.Strain  |

### 7.2.11.2 Super element output variables

Functions like `GetObjectOutputSuperElement(...)`, see [Section 4.3.2](#), or `SensorSuperElement`, see [Section 4.3.5](#), directly access special output variables (`OutputVariableType`) of the mesh nodes of the superelement. Additionally, the contour drawing of the object can make use the `OutputVariableType` of the meshnodes.

| super element output variables     | symbol                                                                                                                                                                                                                                                                            | description                                                                                                                                                                                                                                                                                               |
|------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| DisplacementLocal (mesh node $i$ ) | ${}^b\mathbf{u}_f^{(i)} = ({}^b\Psi \mathbf{p}_{(\text{red})})_{3,i\dots 3,i+2} = \begin{bmatrix} {}^b\mathbf{q}_{f,i,3} \\ {}^b\mathbf{q}_{f,i,3+1} \\ {}^b\mathbf{q}_{f,i,3+2} \end{bmatrix}$                                                                                   | local nodal mesh displacement in reference (body) frame, measuring only flexible part of displacement                                                                                                                                                                                                     |
| VelocityLocal (mesh node $(i)$ )   | ${}^b\dot{\mathbf{u}}_f^{(i)} = ({}^b\Psi \dot{\mathbf{p}}_{(\text{red})})_{3,i\dots 3,i+2}$                                                                                                                                                                                      | local nodal mesh velocity in reference (body) frame, only for flexible part of displacement                                                                                                                                                                                                               |
| Displacement (mesh node $(i)$ )    | ${}^0\mathbf{u}_{\text{config}}^{(i)} = {}^0\mathbf{q}_{t,\text{config}} + {}^{0b}\mathbf{A}_{\text{config}} {}^b\mathbf{P}_{f,\text{config}}^{(i)} - ({}^0\mathbf{q}_{t,\text{ref}} + {}^{0b}\mathbf{A}_{\text{ref}} {}^b\mathbf{x}_{\text{ref}}^{(i)})$                         | nodal mesh displacement in global coordinates                                                                                                                                                                                                                                                             |
| Position (mesh node $(i)$ )        | ${}^0\mathbf{p}^{(i)} = {}^0\mathbf{p}_t + {}^{0b}\mathbf{A} {}^b\mathbf{p}_f^{(i)}$                                                                                                                                                                                              | nodal mesh position in global coordinates                                                                                                                                                                                                                                                                 |
| Velocity (mesh node $(i)$ )        | ${}^0\dot{\mathbf{u}}^{(i)} = {}^0\dot{\mathbf{q}}_t + {}^{0b}\mathbf{A} ({}^b\dot{\mathbf{u}}_f^{(i)} + {}^b\tilde{\omega} {}^b\mathbf{p}_f^{(i)})$                                                                                                                              | nodal mesh velocity in global coordinates                                                                                                                                                                                                                                                                 |
| Acceleration (mesh node $(i)$ )    | ${}^0\mathbf{a}^{(i)} = {}^0\ddot{\mathbf{q}}_t + {}^{0b}\mathbf{A} {}^b\ddot{\mathbf{u}}_f^{(i)} + 2 {}^0\omega \times {}^{0b}\mathbf{A} {}^b\dot{\mathbf{u}}_f^{(i)} + {}^0\alpha \times {}^0\mathbf{p}_f^{(i)} + {}^0\omega \times ({}^0\omega \times {}^0\mathbf{p}_f^{(i)})$ | global acceleration of mesh node $n_i$ including rigid body motion and flexible deformation; note that ${}^0\mathbf{x}(n_i) = {}^{0b}\mathbf{A} {}^b\mathbf{x}(n_i)$                                                                                                                                      |
| Stress (mesh node $(i)$ )          | ${}^b\sigma^{(i)} = {}^b\Psi_{OV} \bar{\boldsymbol{\zeta}}_{3,i\dots 3,i+5}$                                                                                                                                                                                                      | linearized stress components of mesh node $(i)$ in reference frame; $\boldsymbol{\sigma} = [\sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \sigma_{yz}, \sigma_{xz}, \sigma_{xy}]^T$ ; ONLY available, if ${}^b\Psi_{OV}$ is provided and <code>outputVariableTypeModeBasis==exu.OutputVariableType.Stress</code> |

|                            |                                                                                             |                                                                                                                                                                                                                                                                                                |
|----------------------------|---------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Strain (mesh node ( $i$ )) | ${}^b\boldsymbol{\varepsilon}^{(i)} = ({}^b\Psi_{OV}\zeta)_{3 \cdot i \dots 3 \cdot i + 5}$ | linearized stress components of mesh node ( $i$ ) in reference frame; $\sigma = [\sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \sigma_{yz}, \sigma_{xz}, \sigma_{xy}]^T$ ; ONLY available, if ${}^b\Psi_{OV}$ is provided and <code>outputVariableTypeModeBasis==exu.OutputVariableType.Strain</code> |
|----------------------------|---------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

| intermediate variables                     | symbol                                                                                                                                                                                                                                                                                                  | description                                                                                                                                                                                                                                                                                                    |
|--------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| reference frame                            | $b$                                                                                                                                                                                                                                                                                                     | the body-fixed / local frame is always denoted by $b$                                                                                                                                                                                                                                                          |
| number of rigid body coordinates           | $n_{rigid}$                                                                                                                                                                                                                                                                                             | number of rigid body node coordinates: 6 in case of Euler angles (not fully available for ObjectFFRFreducedOrder) and 7 in case of Euler parameters                                                                                                                                                            |
| number of flexible / mesh coordinates      | $n_f = 3 \cdot n_n$                                                                                                                                                                                                                                                                                     | with number of nodes $n_n$ ; relevant for visualization                                                                                                                                                                                                                                                        |
| number of modal coordinates                | $n_m \ll n_f$                                                                                                                                                                                                                                                                                           | the number of reduced or modal coordinates, computed from number of columns given in <code>modeBasis</code>                                                                                                                                                                                                    |
| total number object coordinates            | $n_{ODE2} = n_m + n_{rigid}$                                                                                                                                                                                                                                                                            |                                                                                                                                                                                                                                                                                                                |
| local vector of flexible coordinates       | ${}^b\mathbf{q}_f = {}^b\Psi\zeta$                                                                                                                                                                                                                                                                      | represents mesh displacements; vector of alternating $x, y$ , and $z$ coordinates of local (in body frame) mesh displacements reconstructed from modal coordinates $\zeta$ ; only evaluated for selected node points (e.g., sensors) during computation; corresponds to same vector in <code>ObjectFFRF</code> |
| local nodal positions                      | ${}^b\mathbf{p}_f = {}^b\mathbf{q}_f + {}^b\mathbf{x}_{ref}$                                                                                                                                                                                                                                            | vector of all body-fixed nodal positions including flexible part; only evaluated for selected node points during computation                                                                                                                                                                                   |
| local position of node ( $i$ )             | ${}^b\mathbf{p}_f^{(i)} = {}^b\mathbf{u}_f^{(i)} + {}^b\mathbf{x}_{ref}^{(i)} = \begin{bmatrix} {}^b\mathbf{q}_{f,i,3} \\ {}^b\mathbf{q}_{f,i,3+1} \\ {}^b\mathbf{q}_{f,i,3+2} \\ \vdots \\ {}^b\mathbf{x}_{ref,i,3} \\ {}^b\mathbf{x}_{ref,i,3+1} \\ {}^b\mathbf{x}_{ref,i,3+2} \end{bmatrix} + \dots$ | body-fixed, deformed nodal mesh position (including flexible part)                                                                                                                                                                                                                                             |
| vector of modal coordinates                | $\zeta = [\zeta_0, \dots, \zeta_{n_m-1}]^T$                                                                                                                                                                                                                                                             | vector of modal or reduced coordinates; these coordinates can either represent amplitudes of eigenmodes, static modes or general modes, depending on your mode basis                                                                                                                                           |
| coordinate vector                          | $\mathbf{q} = [{}^0\mathbf{q}_t, \psi, \zeta]$                                                                                                                                                                                                                                                          | vector of object coordinates; $\mathbf{q}_t$ and $\psi$ are the translation and rotation part of displacements of the reference frame, provided by the rigid body node (node number 0)                                                                                                                         |
| flexible coordinates transformation matrix | ${}^{0b}\mathbf{A}_{bd} = \text{diag}([{}^{0b}\mathbf{A}, \dots, {}^{0b}\mathbf{A}])$                                                                                                                                                                                                                   | block diagonal transformation matrix, which transforms all flexible coordinates from local to global coordinates                                                                                                                                                                                               |
| reference frame origin                     | ${}^0\mathbf{p}_t = {}^0\mathbf{q}_t + {}^0\mathbf{q}_{t,ref}$                                                                                                                                                                                                                                          | reference frame position (origin)                                                                                                                                                                                                                                                                              |

|                             |                                                                                                     |                                                                                                                                      |
|-----------------------------|-----------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------|
| reference frame rotation    | $\theta_{\text{config}} = \theta_{\text{config}} + \theta_{\text{ref}}$                             | reference frame rotation parameters in any configuration except reference                                                            |
| reference frame orientation | ${}^0{}^b \mathbf{A}_{\text{config}} = {}^0{}^b \mathbf{A}_{\text{config}}(\theta_{\text{config}})$ | transformation matrix for transformation of local (reference frame) to global coordinates, given by underlying rigid body node $n_0$ |

### 7.2.11.3 Modal reduction and reduced inertia matrices

The formulation is based on the EOM of ObjectFFRF, also regarding parts of notation and some input parameters, [Section 7.2.10](#), and can be found in Zwölfer and Gerstmayr [39] with only small modifications in the notation. The notation of kinematics quantities follows the floating frame of reference idea with quantities given in the tables above and sketched in Fig. ??.

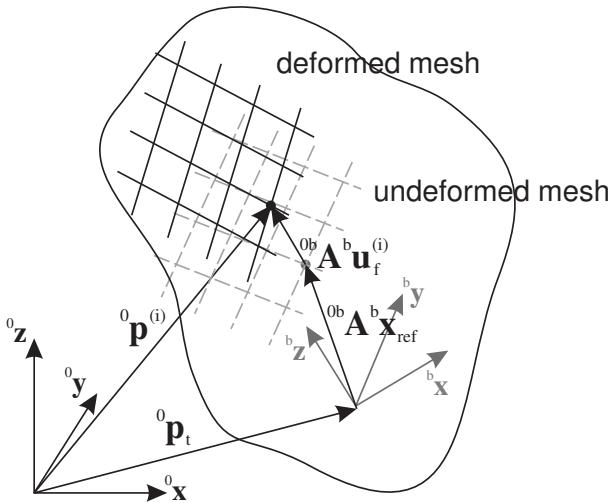


Figure 7.1: Floating frame of reference with exemplary position of a mesh node  $i$ .

The reduced order FFRF formulation is based on an approximation of flexible coordinates  ${}^b \mathbf{q}_f$  by means of a reduction or mode basis  ${}^b \Psi$  (`modeBasis`) and the modal coordinates  $\zeta$ ,

$${}^b \mathbf{q}_f \approx {}^b \Psi \zeta \quad (7.85)$$

The mode basis  ${}^b \Psi$  contains so-called mode shape vectors in its columns, which may be computed from eigen analysis, static computation or more advanced techniques, see the helper functions in module `exudyn.FEM`, within the class `FEMInterface`. To compute eigen modes, use `FEMInterface.ComputeEigenmodes(...)` or `FEMInterface.ComputeHurtyCraigBamptonModes(...)`. For details on model order reduction and component mode synthesis, see [Section 6.2](#). In many applications,  $n_m$  can typically have a size between 10 and 50, depending on the desired accuracy of the model.

The ObjectFFRF coordinates and Eqs. (7.74)<sup>2</sup> can be reduced by the matrix  $\mathbf{H} \in \mathbb{R}^{(n_f+n_{\text{rigid}}) \times n_{\text{ODE2}}}$ ,

$$\mathbf{q}_{\text{FFRF}} = \begin{bmatrix} \mathbf{q}_t \\ \boldsymbol{\theta} \\ {}^b \mathbf{q}_f \end{bmatrix} = \begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_r & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & {}^b \Psi \end{bmatrix} \begin{bmatrix} \mathbf{q}_t \\ \boldsymbol{\theta} \\ \zeta \end{bmatrix} = \mathbf{H} \mathbf{q} \quad (7.86)$$

<sup>2</sup>this is not done for user functions and `forceVector`

with the  $4 \times 4$  identity matrix  $\mathbf{I}_r$  in case of Euler parameters and the reduced coordinates  $\mathbf{q}$ .

The reduced equations follow from the reduction of system matrices in Eqs. (7.74),

$$\mathbf{K}_{\text{red}} = {}^b\Psi^T {}^b\mathbf{K} {}^b\Psi, \quad (7.87)$$

$$\mathbf{M}_{\text{red}} = {}^b\Psi^T {}^b\mathbf{M} {}^b\Psi, \quad (7.88)$$

$$(7.89)$$

the computation of rigid body inertia

$${}^b\Theta_u = {}^b\tilde{x}_{\text{ref}}^T {}^b\mathbf{M} {}^b\tilde{x}_{\text{ref}} \quad (7.90)$$

$$(7.91)$$

the center of mass (and according tilde matrix), using  $\Phi_t$  from Eq. (7.57),

$${}^b\chi_u = \frac{1}{m} \Phi_t^T {}^b\mathbf{M} {}^b\mathbf{x}_{\text{ref}} \quad (7.92)$$

$${}^b\tilde{\chi}_u = \frac{1}{m} \Phi_t^T {}^b\mathbf{M} {}^b\tilde{\mathbf{x}}_{\text{ref}} \quad (7.93)$$

$$(7.94)$$

and seven inertia-like matrices [39],

$$\mathbf{M}_{AB} = \mathbf{A}^T {}^b\mathbf{M} \mathbf{B}, \quad \text{using } \mathbf{AB} \in [\Psi\Psi, \tilde{\Psi}\Psi, \tilde{\Psi}\tilde{\Psi}, \Phi_t\Psi, \Phi_t\tilde{\Psi}, \tilde{x}_{\text{ref}}\Psi, \tilde{x}_{\text{ref}}\tilde{\Psi}] \quad (7.95)$$

Note that the special tilde operator for vectors  $\mathbf{r} \in \mathbb{R}^{n_f}$  of Eq. (7.66) is frequently used.

#### 7.2.11.4 Equations of motion

Equations of motion, in case that `computeFFRFterms = True`:

$$\left( \mathbf{M}_{user}(mbs, t, \mathbf{q}, \dot{\mathbf{q}}) + \begin{bmatrix} \mathbf{M}_{tt} & \mathbf{M}_{tr} & \mathbf{M}_{tf} \\ & \mathbf{M}_{rr} & \mathbf{M}_{rf} \\ \text{sym.} & & \mathbf{M}_{ff} \end{bmatrix} \right) \ddot{\mathbf{q}} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \mathbf{D}_{ff} \end{bmatrix} \dot{\mathbf{q}} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \mathbf{K}_{ff} \end{bmatrix} \mathbf{q} = \mathbf{f}_v(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{f}_{user}(mbs, t, \mathbf{q}, \dot{\mathbf{q}}) \quad (7.96)$$

<sup>3</sup> Note that in case of Euler parameters for the parameterization of rotations for the reference frame, the Euler parameter constraint equation is added automatically by this object. The single terms of the mass matrix are defined as [39]

$$\mathbf{M}_{tt} = m\mathbf{I} \quad (7.98)$$

$$\mathbf{M}_{tr} = -{}^{0b}\mathbf{A} \left[ m {}^b\tilde{\chi}_u + \mathbf{M}_{\Phi_t\tilde{\Psi}} (\zeta \otimes \mathbf{I}) \right] {}^b\mathbf{G} \quad (7.99)$$

$$\mathbf{M}_{tf} = {}^{0b}\mathbf{A} \mathbf{M}_{\Phi_t\Psi} \quad (7.100)$$

$$\mathbf{M}_{rr} = {}^b\mathbf{G}^T \left[ {}^b\Theta_u + \mathbf{M}_{\tilde{x}_{\text{ref}}\tilde{\Psi}} (\zeta \otimes \mathbf{I}) + (\zeta \otimes \mathbf{I})^T \mathbf{M}_{\tilde{x}_{\text{ref}}\tilde{\Psi}}^T + (\zeta \otimes \mathbf{I})^T \mathbf{M}_{\tilde{\Psi}\tilde{\Psi}} (\zeta \otimes \mathbf{I}) \right] {}^b\mathbf{G} \quad (7.101)$$

$$\mathbf{M}_{rf} = -{}^b\mathbf{G}^T \left[ \mathbf{M}_{\tilde{x}_{\text{ref}}\Psi} + (\zeta \otimes \mathbf{I})^T \mathbf{M}_{\tilde{\Psi}\Psi} \right] \quad (7.102)$$

$$\mathbf{M}_{ff} = \mathbf{M}_{\Psi\Psi} \quad (7.103)$$

---

<sup>3</sup>NOTE that currently the internal (C++) computed terms are zero,

$$\begin{bmatrix} \mathbf{M}_{tt} & \mathbf{M}_{tr} & \mathbf{M}_{tf} \\ & \mathbf{M}_{rr} & \mathbf{M}_{rf} \\ \text{sym.} & & \mathbf{M}_{ff} \end{bmatrix} = \mathbf{0} \quad \text{and} \quad \mathbf{f}_v(\mathbf{q}, \dot{\mathbf{q}}) = \mathbf{0}, \quad (7.97)$$

but they are implemented in predefined user functions, see `FEM.py`, [Section 5.3.6](#). In near future, these terms will be implemented in C++ and replace the user functions.

with the Kronecker product<sup>4</sup>,

$$\zeta \otimes \mathbf{I} = \begin{bmatrix} \zeta_0 \mathbf{I} \\ \vdots \\ \zeta_{m-1} \mathbf{I} \end{bmatrix} \quad (7.104)$$

The quadratic velocity vector  $\mathbf{f}_v(\mathbf{q}, \dot{\mathbf{q}}) = [\mathbf{f}_{vt}^T, \mathbf{f}_{vr}^T, \mathbf{f}_{vf}^T]^T$  reads

$$\begin{aligned} \mathbf{f}_{vt} &= {}^{0b}\mathbf{A} {}^b\tilde{\omega} [m {}^b\tilde{\chi}_u + \mathbf{M}_{\Phi_l\bar{\Psi}}(\zeta \otimes \mathbf{I})] {}^b\omega + 2 {}^{0b}\mathbf{A} \mathbf{M}_{\Phi_l\bar{\Psi}}(\zeta \otimes \mathbf{I}) {}^b\omega \\ &\quad + {}^{0b}\mathbf{A} [m {}^b\tilde{\chi}_u + \mathbf{M}_{\Phi_l\bar{\Psi}}(\zeta \otimes \mathbf{I})] {}^b\dot{\mathbf{G}} \dot{\theta}, \end{aligned} \quad (7.105)$$

$$\begin{aligned} \mathbf{f}_{vr} &= -{}^b\mathbf{G}^T {}^b\tilde{\omega} \left[ {}^b\Theta_u + \mathbf{M}_{\tilde{x}_{ref}\bar{\Psi}}(\zeta \otimes \mathbf{I}) + (\zeta \otimes \mathbf{I})^T \mathbf{M}_{\tilde{x}_{ref}\bar{\Psi}}^T + (\zeta \otimes \mathbf{I})^T \mathbf{M}_{\bar{\Psi}\bar{\Psi}}(\zeta \otimes \mathbf{I}) \right] {}^b\omega \\ &\quad - 2 {}^b\mathbf{G}^T \left[ \mathbf{M}_{\tilde{x}_{ref}\bar{\Psi}}(\zeta \otimes \mathbf{I}) + (\zeta \otimes \mathbf{I})^T \mathbf{M}_{\bar{\Psi}\bar{\Psi}}(\zeta \otimes \mathbf{I}) \right] {}^b\omega \\ &\quad - {}^b\mathbf{G}^T \left[ {}^b\Theta_u + \mathbf{M}_{\tilde{x}_{ref}\bar{\Psi}}(\zeta \otimes \mathbf{I}) + (\zeta \otimes \mathbf{I})^T \mathbf{M}_{\tilde{x}_{ref}\bar{\Psi}}^T + (\zeta \otimes \mathbf{I})^T \mathbf{M}_{\bar{\Psi}\bar{\Psi}}(\zeta \otimes \mathbf{I}) \right] {}^b\dot{\mathbf{G}} \dot{\theta}, \end{aligned} \quad (7.106)$$

$$\begin{aligned} \mathbf{f}_{vf} &= (\mathbf{I}_\zeta \otimes {}^b\omega)^T \left[ \mathbf{M}_{\tilde{x}_{ref}\bar{\Psi}}^T + \mathbf{M}_{\bar{\Psi}\bar{\Psi}}(\zeta \otimes \mathbf{I}) \right] {}^b\omega + 2 \mathbf{M}_{\bar{\Psi}\bar{\Psi}}^T(\zeta \otimes \mathbf{I}) {}^b\omega \\ &\quad + \left[ \mathbf{M}_{\tilde{x}_{ref}\bar{\Psi}}^T + \mathbf{M}_{\bar{\Psi}\bar{\Psi}}^T(\zeta \otimes \mathbf{I}) \right] {}^b\dot{\mathbf{G}} \dot{\theta}. \end{aligned} \quad (7.107)$$

Note that terms including  ${}^b\dot{\mathbf{G}} \dot{\theta}$  vanish in case of Euler parameters or in case that  ${}^b\dot{\mathbf{G}} = \mathbf{0}$ , and we use another Kronecker product with the unit matrix  $\mathbf{I}_\zeta \in \mathbb{R}^{n_m \times n_m}$ ,

$$\mathbf{I}_\zeta \otimes {}^b\omega = \begin{bmatrix} {}^b\omega & & \\ & \ddots & \\ & & {}^b\omega \end{bmatrix} \in \mathbb{R}^{3n_m \times n_m} \quad (7.108)$$

In case that `computeFFRFterms = False`, the mass terms  $\mathbf{M}_{tt} \dots \mathbf{M}_{ff}$  are zero (not computed) and the quadratic velocity vector  $\mathbf{f}_Q = \mathbf{0}$ . Note that the user functions  $\mathbf{f}_{user}(mbs, t, \mathbf{q}, \dot{\mathbf{q}})$  and  $\mathbf{M}_{user}(mbs, t, \mathbf{q}, \dot{\mathbf{q}})$  may be empty (=0). The detailed equations of motion for this element can be found in [39].

### 7.2.11.5 Position Jacobian

For joints and loads, the position jacobian of a node is needed in order to compute forces applied to averaged displacements and rotations at nodes. Recall that the modal coordinates  $\zeta$  are transformed to node coordinates by means of the mode basis  ${}^b\Psi$ ,

$${}^b\mathbf{q}_f = {}^b\Psi \zeta. \quad (7.109)$$

The local displacements  ${}^b\mathbf{u}_f^{(i)}$  of a specific node  $i$  can be reconstructed in this way by means of

$${}^b\mathbf{u}_f^{(i)} = \begin{bmatrix} {}^b\mathbf{q}_{f,i,3} \\ {}^b\mathbf{q}_{f,i,3+1} \\ {}^b\mathbf{q}_{f,i,3+2} \end{bmatrix}, \quad (7.110)$$

and the global position of a node, see tables above, reads

$${}^0\mathbf{p}^{(i)} = {}^0\mathbf{p}_t + {}^{0b}\mathbf{A} \left( {}^b\mathbf{u}_f^{(i)} + {}^b\mathbf{x}_{ref}^{(i)} \right) \quad (7.111)$$

Thus, the jacobian of the global position reads

$${}^0\mathbf{J}_{pos}^{(i)} = \frac{\partial {}^0\mathbf{p}^{(i)}}{\partial [\mathbf{q}_t, \theta, \zeta]} = \left[ \mathbf{I}, -{}^{0b}\mathbf{A} \left( {}^b\tilde{\mathbf{u}}_f^{(i)} + {}^b\tilde{\mathbf{x}}_{ref}^{(i)} \right) {}^b\mathbf{G}, {}^{0b}\mathbf{A} \begin{bmatrix} {}^b\Psi_{r=3i}^T \\ {}^b\Psi_{r=3i+1}^T \\ {}^b\Psi_{r=3i+2}^T \end{bmatrix} \right], \quad (7.112)$$

<sup>4</sup>In python numpy module this is computed by `numpy.kron(zeta, Im)`.

in which  ${}^b\Psi_{r=...}$  represents the row  $r$  of the mode basis (matrix)  ${}^b\Psi$ , and the matrix

$$\begin{bmatrix} {}^b\Psi_{r=3i}^T \\ {}^b\Psi_{r=3i+1}^T \\ {}^b\Psi_{r=3i+2}^T \end{bmatrix} \in \mathbb{R}^{3 \times n_m} \quad (7.113)$$

Furthermore, the jacobian of the local position reads

$${}^bJ_{\text{pos}}^{(i)} = \frac{\partial {}^bP_f^{(i)}}{\partial [\mathbf{q}_t, \boldsymbol{\theta}, \zeta]} = \left[ \mathbf{0}, \mathbf{0}, \begin{bmatrix} {}^b\Psi_{r=3i}^T \\ {}^b\Psi_{r=3i+1}^T \\ {}^b\Psi_{r=3i+2}^T \end{bmatrix} \right], \quad (7.114)$$

which is used in `MarkerSuperElementRigid`.

#### 7.2.11.6 Joints and Loads

Use special `MarkerSuperElementPosition` to apply forces, SpringDampers or spherical joints. This marker can be attached to a single node of the underlying mesh or to a set of nodes, which is then averaged, see the according marker description.

Use special `MarkerSuperElementRigid` to apply torques or special joints (e.g., `JointGeneric`). This marker must be attached to a set of nodes which can represent rigid body motion. The rigid body motion is then averaged for all of these nodes, see the according marker description.

For application of mass proportional loads (gravity), you can use conventional `MarkerBodyMass`. However, **do not use** `MarkerBodyPosition` or `MarkerBodyRigid` for `ObjectFFRFreducedOrder`, unless wanted, because it only attaches to the floating frame. This means, that a force to a `MarkerBodyPosition` would only be applied to the (rigid) floating frame, but not onto the deformable body and results depend strongly on the choice of the reference frame (or the underlying mode shapes).

`CoordinateLoads` are added for each ODE2 coordinate on the RHS of the equations of motion.

---

#### Userfunction: `forceUserFunction(mbs, t, q, q_t)`

A user function, which computes a force vector depending on current time and states of object. Can be used to create any kind of mechanical system by using the object states.

| arguments / return  | type or size                            | description                                                                                                                           |
|---------------------|-----------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------|
| <code>mbs</code>    | MainSystem                              | provides MainSystem <code>mbs</code> to which object belongs                                                                          |
| <code>t</code>      | Real                                    | current time in <code>mbs</code>                                                                                                      |
| <code>q</code>      | Vector $\in \mathbb{R}_{\text{ODE2}}^n$ | FFRF object coordinates (rigid body coordinates and reduced coordinates in a list) in current configuration, without reference values |
| <code>q_t</code>    | Vector $\in \mathbb{R}_{\text{ODE2}}^n$ | object velocity coordinates (time derivatives of <code>q</code> ) in current configuration                                            |
| <b>return value</b> | Vector $\in \mathbb{R}_{\text{ODE2}}^n$ | returns force vector for object                                                                                                       |

---

#### Userfunction: `massMatrixUserFunction(mbs, t, q, q_t)`

A user function, which computes a mass matrix depending on current time and states of object. Can be used to create any kind of mechanical system by using the object states.

| arguments / return  | type or size                                            | description                                                                                                                           |
|---------------------|---------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------|
| mbs                 | MainSystem                                              | provides MainSystem mbs to which object belongs                                                                                       |
| t                   | Real                                                    | current time in mbs                                                                                                                   |
| q                   | Vector $\in \mathbb{R}_{ODE2}^n$                        | FFRF object coordinates (rigid body coordinates and reduced coordinates in a list) in current configuration, without reference values |
| q_t                 | Vector $\in \mathbb{R}_{ODE2}^n$                        | object velocity coordinates (time derivatives of q) in current configuration                                                          |
| <b>return value</b> | NumpyMatrix $\in \mathbb{R}^{n_{ODE2} \times n_{ODE2}}$ | returns mass matrix for object                                                                                                        |

For further examples on ObjectFFRFreducedOrder see Examples:

- [NGsolvePistonEngine.py](#)
- [objectFFRFreducedOrderNetgen.py](#)
- [sliderCrankCMSacme.py](#)

For further examples on ObjectFFRFreducedOrder see TestModels:

- [NGsolveCrankShaftTest.py](#)
- [objectFFRFreducedOrderAccelerations.py](#)
- [objectFFRFreducedOrderShowModes.py](#)
- [objectFFRFreducedOrderStressModesTest.py](#)
- [superElementRigidJointTest.py](#)

## 7.2.12 ObjectANCF Cable2D

A 2D cable finite element using 2 nodes of type NodePoint2DSlope1. The beam with length  $L=\text{physicsLength}$  uses a localPosition $\in [0, L]$ .

**Additional information for ObjectANCF Cable2D:**

- Requested node type = Position2D + Orientation2D + Point2DSlope1 + Position + Orientation
- Short name for Python = **Cable2D**
- Short name for Python (visualization object) = **VCable2D**

The item **ObjectANCF Cable2D** with type = 'ANCF Cable2D' has the following parameters:

| Name                        | type                | size | default value    | description                                                                                                                                                                                                                           |
|-----------------------------|---------------------|------|------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| name                        | String              |      | "                | object's unique name                                                                                                                                                                                                                  |
| physicsLength               | UReal               |      | 0.               | [SI:m] reference length of beam; such that the total volume (e.g. for volume load) gives $\rho AL$ ; must be positive                                                                                                                 |
| physicsMassPerLength        | UReal               |      | 0.               | [SI:kg/m] mass per length of beam                                                                                                                                                                                                     |
| physicsBendingStiffness     | UReal               |      | 0.               | [SI:Nm <sup>2</sup> ] bending stiffness of beam; the bending moment is $m = EI(\kappa - \kappa_0)$ , in which $\kappa$ is the material measure of curvature                                                                           |
| physicsAxialStiffness       | UReal               |      | 0.               | [SI:N] axial stiffness of beam; the axial force is $f_{ax} = EA(\varepsilon - \varepsilon_0)$ , in which $\varepsilon =  \dot{\mathbf{r}}'  - 1$ is the axial strain                                                                  |
| physicsBendingDamping       | UReal               |      | 0.               | [SI:Nm <sup>2</sup> /s] bending damping of beam ; the additional virtual work due to damping is $\delta W_\kappa = \int_0^L \dot{\kappa} \delta \kappa dx$                                                                            |
| physicsAxialDamping         | UReal               |      | 0.               | [SI:N/s] axial stiffness of beam; the additional virtual work due to damping is $\delta W_\varepsilon = \int_0^L \dot{\varepsilon} \delta \varepsilon dx$                                                                             |
| physicsReferenceAxialStrain | Real                |      | 0.               | [SI:1] reference axial strain of beam (pre-deformation) of beam; without external loading the beam will statically keep the reference axial strain value                                                                              |
| physicsReferenceCurvature   | Real                |      | 0.               | [SI:1/m] reference curvature of beam (pre-deformation) of beam; without external loading the beam will statically keep the reference curvature value                                                                                  |
| nodeNumbers                 | NodeIndex2          |      | [MAXINT, MAXINT] | two node numbers ANCF cable element                                                                                                                                                                                                   |
| useReducedOrderIntegration  | Bool                |      | False            | false: use Gauss order 9 integration for virtual work of axial forces, order 5 for virtual work of bending moments; true: use Gauss order 7 integration for virtual work of axial forces, order 3 for virtual work of bending moments |
| visualization               | VObjectANCF Cable2D |      |                  | parameters for visualization of item                                                                                                                                                                                                  |

The item **VObjectANCF Cable2D** has the following parameters:

| <b>Name</b> | <b>type</b> | <b>size</b> | <b>default value</b> | <b>description</b>                                                       |
|-------------|-------------|-------------|----------------------|--------------------------------------------------------------------------|
| show        | Bool        |             | True                 | set true, if item is shown in visualization and false if it is not shown |
| drawHeight  | float       |             | 0.                   | if beam is drawn with rectangular shape, this is the drawing height      |
| color       | Float4      |             | [-1.,-1.,-1.,-1.]    | RGB color of the object; if R==1, use default color                      |

---

### 7.2.12.1 DESCRIPTION of ObjectANCF Cable2D:

Information on input parameters:

| <b>input parameter</b>      | <b>symbol</b>   | <b>description see tables above</b> |
|-----------------------------|-----------------|-------------------------------------|
| physicsLength               | $L$             |                                     |
| physicsMassPerLength        | $\rho A$        |                                     |
| physicsBendingStiffness     | $EI$            |                                     |
| physicsAxialStiffness       | $EA$            |                                     |
| physicsBendingDamping       | $d_e$           |                                     |
| physicsAxialDamping         | $d_K$           |                                     |
| physicsReferenceAxialStrain | $\varepsilon_0$ |                                     |
| physicsReferenceCurvature   | $\kappa_0$      |                                     |

The following output variables are available as OutputVariableType in sensors, Get...Output() and other functions:

| <b>output variable</b> | <b>symbol</b> | <b>description</b>                                                          |
|------------------------|---------------|-----------------------------------------------------------------------------|
| Position               |               | global position vector of local axis (1) and cross section (2) position     |
| Displacement           |               | global displacement vector of local axis (1) and cross section (2) position |
| Velocity               |               | global velocity vector of local axis (1) and cross section (2) position     |
| Director1              |               | (axial) slope vector of local axis position                                 |
| StrainLocal            |               | axial strain (scalar)                                                       |
| CurvatureLocal         |               | axial strain (scalar)                                                       |
| ForceLocal             |               | (local) section normal force (scalar)                                       |
| TorqueLocal            |               | (local) bending moment (scalar)                                             |

A 2D cable finite element using 2 nodes of type NodePoint2DSlope1. The Bernoulli-Euler beam is capable of large deformation as it employs the material measure of curvature for the bending. The following section summarizes the equations of motion. For further details, see Gerstmayr and Irschik [17].

### 7.2.12.2 Kinematics and interpolation

ANCF elements follow the original concept proposed by Shabana [30]. The present 2D element is based on the interpolation used by Bezeri and Shabana [5], but the formulation (especially of the elastic forces) is according to Gerstmayr and Irschik [17].

The current position of an arbitrary element at local axial position  $x \in [0, L]$ , where  $L$  is the beam length, reads

$$\mathbf{r} = \mathbf{r}(x, t), \quad (7.115)$$

The derivative of the position w.r.t. the axial reference coordinate is denoted as slope vector,

$$\mathbf{r}' = \frac{\partial \mathbf{r}(x, t)}{\partial x} \quad (7.116)$$

The interpolation is based on cubic (spline) interpolation of position, displacements and velocities. The generalized coordinates  $\mathbf{q} \in \mathbb{R}^8$  of the beam element is defined by

$$\mathbf{q} = [\mathbf{r}_0^T \ \mathbf{r}_0'^T \ \mathbf{r}_1^T \ \mathbf{r}_1'^T]^T. \quad (7.117)$$

in which  $\mathbf{r}_0$  is the position of node 0 and  $\mathbf{r}_1$  is the position of node 1,  $\mathbf{r}_0'$  the slope at node 0 and  $\mathbf{r}_1'$  the slope at node 1. Position and slope are interpolated with shape functions.

The position and slope along the beam are interpolated by means of

$$\mathbf{r} = \mathbf{S}\mathbf{q} \quad \text{and} \quad \mathbf{r}' = \mathbf{S}'\mathbf{q}. \quad (7.118)$$

in which  $\mathbf{S}$  is the shape function matrix,

$$\mathbf{S}(x) = [S_1(x)\mathbf{I} \ S_2(x)\mathbf{I} \ S_3(x)\mathbf{I} \ S_4(x)\mathbf{I}]. \quad (7.119)$$

with identity matrix  $\mathbf{I} \in \mathbb{R}^{2 \times 2}$  the shape functions

$$\begin{aligned} S_1(x) &= 1 - 3\frac{x^2}{L^2} + 2\frac{x^3}{L^3}, & S_2(x) &= x - 2\frac{x^2}{L} + \frac{x^3}{L^2} \\ S_3(x) &= 3\frac{x^2}{L^2} - 2\frac{x^3}{L^3}, & S_4(x) &= -\frac{x^2}{L} + \frac{x^3}{L^2} \end{aligned} \quad (7.120)$$

Velocity simply follows as

$$\frac{\partial \mathbf{r}}{\partial t} = \dot{\mathbf{r}} = \mathbf{S}\dot{\mathbf{q}}. \quad (7.121)$$

### 7.2.12.3 Elastic forces

The elastic forces  $\mathbf{Q}_e$  are implicitly defined by the relation to the virtual work of elastic forces,  $\delta W_e$ , of applied forces,  $\delta W_a$  and of viscous forces,  $\delta W_v$ ,

$$\mathbf{Q}_e^T \delta \mathbf{q} = \delta W_e + \delta W_a + \delta W_v. \quad (7.122)$$

The virtual work of elastic forces reads

$$\delta W_e = \int_0^L (N\delta\varepsilon + M\delta K) dx, \quad (7.123)$$

in which the axial strain is defined as [17]

$$\varepsilon = \|\mathbf{r}'\| - 1. \quad (7.124)$$

and the material measure of curvature (bending strain) is given as

$$K = \mathbf{e}_3^T \frac{\mathbf{r}' \times \mathbf{r}''}{\|\mathbf{r}'\|^2}. \quad (7.125)$$

in which  $\mathbf{e}_3$  is the unit vector which is perpendicular to the plane of the planar beam element.

By derivation, we obtain the variation of axial strain

$$\delta\varepsilon = \frac{\partial\varepsilon}{\partial q_i} \delta q_i = \frac{1}{\|\mathbf{r}'\|} \mathbf{r}'^T \mathbf{S}'_i \delta q_i. \quad (7.126)$$

and the variation of  $K$

$$\begin{aligned}\delta K &= \frac{\partial}{\partial q_i} \left( \frac{(\mathbf{r}'^T \times \mathbf{r}'')^T \mathbf{e}_3}{\|\mathbf{r}'\|^2} \right) \delta q_i \\ &= \frac{1}{\|\mathbf{r}'\|^4} \left[ \|\mathbf{r}'\|^2 (\mathbf{S}'_i \times \mathbf{r}'' + \mathbf{r}' \times \mathbf{S}'_i) - 2(\mathbf{r}' \times \mathbf{r}'') (\mathbf{r}'^T \mathbf{S}'_i) \right]^T \mathbf{e}_3 \delta q_i\end{aligned}\quad (7.127)$$

The normal force (axial force)  $N$  in the beam reads

$$N = EA(\varepsilon - \varepsilon_0). \quad (7.128)$$

in which  $\varepsilon_0$  includes the (pre-)stretch of the beam, e.g., due to temperature or plastic deformation. The bending moment  $M$  in the beam reads

$$M = EI(K - K_0). \quad (7.129)$$

in which  $K_0$  includes the (pre-)curvature of the undeformed beam. Using the latter definitions, the elastic forces follow from Eq. (7.122).

The virtual work of viscous damping forces, assuming viscous effects proportional to axial stretching and bending, is defined as

$$\delta W_v = \int_0^L (d_\varepsilon \dot{\varepsilon} \delta \varepsilon + d_K \dot{K} \delta K) dx. \quad (7.130)$$

with material coefficients  $d_\varepsilon$  and  $d_K$ . The time derivatives of axial strain  $\dot{\varepsilon}_p$  follows by elementary differentiation

$$\dot{\varepsilon} = \frac{\partial}{\partial t} (\|\mathbf{r}'\| - 1) = \frac{1}{\|\mathbf{r}'\|} \mathbf{r}'^T \mathbf{S}' \dot{\mathbf{q}} \quad (7.131)$$

as well as the derivative of the curvature,

$$\begin{aligned}\dot{K} &= \frac{\partial}{\partial t} \left( \mathbf{e}_3^T \frac{\mathbf{r}' \times \mathbf{r}''}{\|\mathbf{r}'\|^2} \right) \\ &= \frac{\mathbf{e}_3^T}{(\mathbf{r}'^T \mathbf{r}')^2} \left( (\mathbf{r}'^T \mathbf{r}') \frac{\partial (\mathbf{r}' \times \mathbf{r}'')^T}{\partial t} - (\mathbf{r}' \times \mathbf{r}'')^T \frac{\partial (\mathbf{r}'^T \mathbf{r}')}{\partial t} \right) \\ &= \frac{\mathbf{e}_3^T}{(\mathbf{r}'^T \mathbf{r}')^2} \left( (\mathbf{r}'^T \mathbf{r}') ((\mathbf{S}' \dot{\mathbf{q}}) \times \mathbf{r}'' + (\mathbf{S}'' \dot{\mathbf{q}}) \times \mathbf{r}') - (\mathbf{r}' \times \mathbf{r}'') (2\mathbf{r}'^T (\mathbf{S}' \dot{\mathbf{q}})) \right).\end{aligned}\quad (7.132)$$

The virtual work of applied forces reads

$$\delta W_a = \sum_i \mathbf{f}_i^T \delta \mathbf{r}_i(x_f) + \int_0^L \mathbf{b}^T \delta \mathbf{r}(x) dx, \quad (7.133)$$

in which  $\mathbf{f}_i$  are forces applied to a certain position  $x_f$  at the beam centerline. The second term contains a load per length  $\mathbf{b}$ , which in case of gravity vector  $\mathbf{g}$  reads

$$\mathbf{b} = \rho \mathbf{g}. \quad (7.134)$$

Note that the variation of  $\mathbf{r}$  simply follows as

$$\delta \mathbf{r} = \mathbf{S} \delta \mathbf{q} \quad (7.135)$$

For further examples on ObjectANCFBeam2D see Examples:

- [sliderCrank3DwithANCFbeltDrive2.py](#)
- [ALEANCF\\_pipe.py](#)

- [ANCF\\_cantilever\\_test.py](#)
- [ANCF\\_cantilever\\_test\\_dyn.py](#)
- [ANCF\\_contact\\_circle.py](#)
- [ANCF\\_contact\\_circle2.py](#)
- [ANCF\\_moving\\_rigidbody.py](#)
- [ANCF\\_slidingJoint2D.py](#)
- [ANCF\\_switchingSlidingJoint2D.py](#)
- [ANCF\\_tests2.py](#)
- ...

For further examples on ObjectANFCable2D see TestModels:

- [ANCFcontactCircleTest.py](#)
- [ANCFcontactFrictionTest.py](#)
- [computeODE2EigenvaluesTest.py](#)
- [manualExplicitIntegrator.py](#)
- [modelUnitTests.py](#)

### 7.2.13 ObjectALEANCFCable2D

A 2D cable finite element using 2 nodes of type NodePoint2DSlope1 and a axially moving coordinate of type NodeGenericODE2. The beam with length  $L=\text{physicsLength}$  uses a localPosition $\in [0, L]$ .

#### Additional information for ObjectALEANCFCable2D:

- Requested node type: read detailed information of item
- **Short name for Python = ALECFable2D**
- **Short name for Python (visualization object) = VALECFable2D**

The item **ObjectALEANCFCable2D** with type = 'ALEANCFCable2D' has the following parameters:

| Name                        | type   | size | default value | description                                                                                                                                                                                                                                                                                                                                                                            |
|-----------------------------|--------|------|---------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| name                        | String |      | "             | objects's unique name                                                                                                                                                                                                                                                                                                                                                                  |
| physicsLength               | UReal  |      | 0.            | [SI:m] reference length of beam; such that the total volume (e.g. for volume load) gives $\rho AL$ ; must be positive                                                                                                                                                                                                                                                                  |
| physicsMassPerLength        | UReal  |      | 0.            | [SI:kg/m] total mass per length of beam (including axially moving parts / fluid)                                                                                                                                                                                                                                                                                                       |
| physicsMovingMassFactor     | UReal  |      | 1.            | this factor denotes the amount of $\rho A$ which is moving; physicsMovingMassFactor=1 means, that all mass is moving; physicsMovingMassFactor=0 means, that no mass is moving; factor can be used to simulate e.g. pipe conveying fluid, in which $\rho A$ is the mass of the pipe+fluid, while $\text{physicsMovingMassFactor} \cdot \rho A$ is the mass per unit length of the fluid |
| physicsBendingStiffness     | UReal  |      | 0.            | [SI:Nm <sup>2</sup> ] bending stiffness of beam; the bending moment is $m = EI(\kappa - \kappa_0)$ , in which $\kappa$ is the material measure of curvature                                                                                                                                                                                                                            |
| physicsAxialStiffness       | UReal  |      | 0.            | [SI:N] axial stiffness of beam; the axial force is $f_{ax} = EA(\varepsilon - \varepsilon_0)$ , in which $\varepsilon =  \dot{\mathbf{r}}'  - 1$ is the axial strain                                                                                                                                                                                                                   |
| physicsBendingDamping       | UReal  |      | 0.            | [SI:Nm <sup>2</sup> /s] bending damping of beam ; the additional virtual work due to damping is $\delta W_\kappa = \int_0^L \dot{\kappa} \delta \kappa dx$                                                                                                                                                                                                                             |
| physicsAxialDamping         | UReal  |      | 0.            | [SI:N/s] axial stiffness of beam; the additional virtual work due to damping is $\delta W_\varepsilon = \int_0^L \dot{\varepsilon} \delta \varepsilon dx$                                                                                                                                                                                                                              |
| physicsReferenceAxialStrain | Real   |      | 0.            | [SI:1] reference axial strain of beam (pre-deformation) of beam; without external loading the beam will statically keep the reference axial strain value                                                                                                                                                                                                                               |
| physicsReferenceCurvature   | Real   |      | 0.            | [SI:1/m] reference curvature of beam (pre-deformation) of beam; without external loading the beam will statically keep the reference curvature value                                                                                                                                                                                                                                   |

|                            |                       |                          |                                                                                                                                                                                                                                       |
|----------------------------|-----------------------|--------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| physicsUseCouplingTerms    | Bool                  | True                     | true: correct case, where all coupling terms due to moving mass are respected; false: only include constant mass for ALE node coordinate, but deactivate other coupling terms (behaves like ANCFCable2D then)                         |
| nodeNumbers                | NodeIndex3            | [MAXINT, MAXINT, MAXINT] | two node numbers ANCF cable element, third node=ALE GenericODE2 node                                                                                                                                                                  |
| useReducedOrderIntegration | Bool                  | False                    | false: use Gauss order 9 integration for virtual work of axial forces, order 5 for virtual work of bending moments; true: use Gauss order 7 integration for virtual work of axial forces, order 3 for virtual work of bending moments |
| visualization              | VObjectALEANCFCable2D |                          | parameters for visualization of item                                                                                                                                                                                                  |

The item VObjectALEANCFCable2D has the following parameters:

| Name       | type   | size | default value     | description                                                              |
|------------|--------|------|-------------------|--------------------------------------------------------------------------|
| show       | Bool   |      | True              | set true, if item is shown in visualization and false if it is not shown |
| drawHeight | float  |      | 0.                | if beam is drawn with rectangular shape, this is the drawing height      |
| color      | Float4 |      | [-1.,-1.,-1.,-1.] | RGBA color of the object; if R==1, use default color                     |

### 7.2.13.1 DESCRIPTION of ObjectALEANCFCable2D:

**Information on input parameters:**

| input parameter             | symbol          | description see tables above |
|-----------------------------|-----------------|------------------------------|
| physicsLength               | $L$             |                              |
| physicsMassPerLength        | $\rho A$        |                              |
| physicsBendingStiffness     | $EI$            |                              |
| physicsAxialStiffness       | $EA$            |                              |
| physicsBendingDamping       | $d_e$           |                              |
| physicsAxialDamping         | $d_K$           |                              |
| physicsReferenceAxialStrain | $\varepsilon_0$ |                              |
| physicsReferenceCurvature   | $\kappa_0$      |                              |

**The following output variables are available as OutputVariableType in sensors, Get...Output() and other functions:**

| output variable | symbol | description                                                                 |
|-----------------|--------|-----------------------------------------------------------------------------|
| Position        |        | global position vector of local axis (1) and cross section (2) position     |
| Displacement    |        | global displacement vector of local axis (1) and cross section (2) position |

|                |  |                                                                         |
|----------------|--|-------------------------------------------------------------------------|
| Velocity       |  | global velocity vector of local axis (1) and cross section (2) position |
| Director1      |  | (axial) slope vector of local axis position                             |
| StrainLocal    |  | axial strain (scalar)                                                   |
| CurvatureLocal |  | axial strain (scalar)                                                   |
| ForceLocal     |  | (local) section normal force (scalar)                                   |
| TorqueLocal    |  | (local) bending moment (scalar)                                         |

A 2D cable finite element using 2 nodes of type NodePoint2DSlope1 and an axially moving coordinate of type NodeGenericODE2. The element has 8+1 coordinates and uses cubic polynomials for position interpolation. In addition to ANCF Cable2D the element adds an Eulerian axial velocity by the GenericODE2 coordinate. The parameter `physicsMovingMassFactor` allows to control the amount of mass, which moves with the Eulerian velocity (e.g., the fluid), and which is not moving (the pipe). A factor of `physicsMovingMassFactor=1` gives an axially moving beam.

The Bernoulli-Euler beam is capable of large deformation as it employs the material measure of curvature for the bending. Note that damping (`physicsBendingDamping`, `physicsAxialDamping`) only acts on the non-moving part of the beam, as it is the case for the pipe.

Note that most functions act on the underlying cable finite element, which is not co-moving axially. E.g., if you apply constraints to the nodal coordinates, the cable can be fixed, while still the axial component is freely moving. If you apply a LoadForce using a MarkerPosition, the force is acting on the beam finite element, but not on the axially moving coordinate. In contrast to the latter, the ObjectJointALEMoving2D and the MarkerBodyMass are acting on the moving coordinate as well.

A detailed paper on this element is yet under submission, but a similar formulation can be found in [28] and the underlying beam element is identical to ObjectANCF Cable2D.

---

For further examples on ObjectALEANCF Cable2D see Examples:

- [ALEANCF\\_pipe.py](#)
- [ANCFALEtest.py](#)
- [ANCF\\_moving\\_rigidbody.py](#)

## 7.2.14 ObjectBeamGeometricallyExact2D

A 2D geometrically exact beam finite element, currently using 2 nodes of type NodeRigidBody2D. The beam with length  $L=\text{physicsLength}$  uses a localPosition $\in [-L/2, L/2]$ .

**Additional information for ObjectBeamGeometricallyExact2D:**

- The Object has the following types = Body, MultiNoded
- Requested node type = Position2D + Orientation2D + Position + Orientation
- **Short name for Python** = **Beam2D**
- **Short name for Python (visualization object)** = **VBeam2D**

The item **ObjectBeamGeometricallyExact2D** with type = 'BeamGeometricallyExact2D' has the following parameters:

| Name                       | type                            | size | default value     | description                                                                                                                                                 |
|----------------------------|---------------------------------|------|-------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------|
| name                       | String                          |      | "                 | objects's unique name                                                                                                                                       |
| nodeNumbers                | NodeIndex2                      |      | [MAXINT, MAX-INT] | two node numbers for beam element                                                                                                                           |
| physicsLength              | UReal                           |      | 0.                | [SI:m] reference length of beam; such that the total volume (e.g. for volume load) gives $\rho AL$ ; must be positive                                       |
| physicsMassPerLength       | UReal                           |      | 0.                | [SI:kg/m] mass per length of beam                                                                                                                           |
| physicsCrossSectionInertia | UReal                           |      | 0.                | [SI:kg m] cross section mass moment of inertia; inertia acting against rotation of cross section                                                            |
| physicsBendingStiffness    | UReal                           |      | 0.                | [SI:Nm <sup>2</sup> ] bending stiffness of beam; the bending moment is $m = EI(\kappa - \kappa_0)$ , in which $\kappa$ is the material measure of curvature |
| physicsAxialStiffness      | UReal                           |      | 0.                | [SI:N] axial stiffness of beam; the axial force is $f_{ax} = EA(\varepsilon - \varepsilon_0)$ , in which $\varepsilon$ is the axial strain                  |
| physicsShearStiffness      | UReal                           |      | 0.                | [SI:N] effective shear stiffness of beam, including stiffness correction                                                                                    |
| visualization              | VObjectBeamGeometricallyExact2D |      |                   | parameters for visualization of item                                                                                                                        |

The item VObjectBeamGeometricallyExact2D has the following parameters:

| Name       | type   | size | default value     | description                                                              |
|------------|--------|------|-------------------|--------------------------------------------------------------------------|
| show       | Bool   |      | True              | set true, if item is shown in visualization and false if it is not shown |
| drawHeight | float  |      | 0.                | if beam is drawn with rectangular shape, this is the drawing height      |
| color      | Float4 |      | [-1.,-1.,-1.,-1.] | RGBA color of the object; if R==1, use default color                     |

### 7.2.14.1 DESCRIPTION of ObjectBeamGeometricallyExact2D:

Information on input parameters:

| input parameter            | symbol   | description see tables above |
|----------------------------|----------|------------------------------|
| physicsLength              | $L$      |                              |
| physicsMassPerLength       | $\rho A$ |                              |
| physicsCrossSectionInertia | $\rho J$ |                              |
| physicsBendingStiffness    | $EI$     |                              |
| physicsAxialStiffness      | $EA$     |                              |
| physicsShearStiffness      | $GA$     |                              |

The following output variables are available as OutputVariableType in sensors, Get...Output() and other functions:

| output variable | symbol | description                                                                   |
|-----------------|--------|-------------------------------------------------------------------------------|
| Position        |        | global position vector of local axis (1) and cross section (2) position       |
| Displacement    |        | global displacement vector of local axis (1) and cross section (2) position   |
| Velocity        |        | global velocity vector of local axis (1) and cross section (2) position       |
| Rotation        |        | 3D Tait-Bryan rotation components, containing rotation around z-axis only     |
| StrainLocal     |        | 6 strain components, containing only axial ( $xx$ ) and shear strain ( $xy$ ) |
| CurvatureLocal  |        | 3D vector of curvature, containing only curvature w.r.t. z-axis               |

See paper of Simo and Vu-Quoc (1986). Detailed description coming later.

For further examples on ObjectBeamGeometricallyExact2D see TestModels:

- [geometricallyExactBeam2Dtest.py](#)

## 7.2.15 ObjectConnectorSpringDamper

An simple spring-damper element with additional force; connects to position-based markers.

### Additional information for ObjectConnectorSpringDamper:

- The Object has the following types = Connector
- Requested marker type = Position
- Short name for Python = **SpringDamper**
- Short name for Python (visualization object) = **VSpringDamper**

The item **ObjectConnectorSpringDamper** with type = 'ConnectorSpringDamper' has the following parameters:

| Name                    | type                         | size | default value       | description                                                                                                                                                                                             |
|-------------------------|------------------------------|------|---------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| name                    | String                       |      | "                   | connector's unique name                                                                                                                                                                                 |
| markerNumbers           | ArrayMarkerIndex             |      | [ MAXINT, MAX-INT ] | list of markers used in connector                                                                                                                                                                       |
| referenceLength         | PReal                        |      | 0.                  | reference length [SI:m] of spring                                                                                                                                                                       |
| stiffness               | UReal                        |      | 0.                  | stiffness [SI:N/m] of spring; acts against (length-initialLength)                                                                                                                                       |
| damping                 | UReal                        |      | 0.                  | damping [SI:N/(m s)] of damper; acts against d/dt(length)                                                                                                                                               |
| force                   | Real                         |      | 0.                  | added constant force [SI:N] of spring; scalar force; f=1 is equivalent to reducing initialLength by 1/stiffness; f > 0: tension; f < 0: compression; can be used to model actuator force                |
| activeConnector         | Bool                         |      | True                | flag, which determines, if the connector is active; used to deactivate (temporarily) a connector or constraint                                                                                          |
| springForceUserFunction | PyFunctionMbsScalar6         |      | 0                   | A python function which defines the spring force with parameters; the python function will only be evaluated, if activeConnector is true, otherwise the SpringDamper is inactive; see description below |
| visualization           | VObjectConnectorSpringDamper |      |                     | parameters for visualization of item                                                                                                                                                                    |

The item VObjectConnectorSpringDamper has the following parameters:

| Name     | type   | size | default value     | description                                                                               |
|----------|--------|------|-------------------|-------------------------------------------------------------------------------------------|
| show     | Bool   |      | True              | set true, if item is shown in visualization and false if it is not shown                  |
| drawSize | float  |      | -1.               | drawing size = diameter of spring; size == -1.f means that default connector size is used |
| color    | Float4 |      | [-1.,-1.,-1.,-1.] | RGBA connector color; if R== -1, use default color                                        |

### 7.2.15.1 DESCRIPTION of ObjectConnectorSpringDamper:

**Information on input parameters:**

| input parameter | symbol       | description see tables above |
|-----------------|--------------|------------------------------|
| markerNumbers   | $[m0, m1]^T$ |                              |
| referenceLength | $L_0$        |                              |
| stiffness       | $k$          |                              |
| damping         | $d$          |                              |
| force           | $f_a$        |                              |

The following output variables are available as **OutputVariableType** in sensors, **Get...Output()** and other functions:

| output variable | symbol | description                               |
|-----------------|--------|-------------------------------------------|
| Distance        |        | distance between both points              |
| Displacement    |        | relative displacement between both points |
| Velocity        |        | relative velocity between both points     |
| Force           |        | spring-damper force                       |

### 7.2.15.2 Definition of quantities

| intermediate variables | symbol                | description                                            |
|------------------------|-----------------------|--------------------------------------------------------|
| marker m0 position     | ${}^0\mathbf{p}_{m0}$ | current global position which is provided by marker m0 |
| marker m1 position     | ${}^0\mathbf{p}_{m1}$ |                                                        |
| marker m0 velocity     | ${}^0\mathbf{v}_{m0}$ | current global velocity which is provided by marker m0 |
| marker m1 velocity     | ${}^0\mathbf{v}_{m1}$ |                                                        |

| output variables | symbol                  | formula                                     |
|------------------|-------------------------|---------------------------------------------|
| Distance         | $L$                     | $ \Delta {}^0\mathbf{p} $                   |
| Displacement     | $\Delta {}^0\mathbf{p}$ | ${}^0\mathbf{p}_{m1} - {}^0\mathbf{p}_{m0}$ |
| Velocity         | $\Delta {}^0\mathbf{v}$ | ${}^0\mathbf{v}_{m1} - {}^0\mathbf{v}_{m0}$ |
| Force            | $\mathbf{f}$            | see below                                   |

### 7.2.15.3 Connector forces

The unit vector in force direction reads (raises SysError if  $L = 0$ ),

$$\mathbf{v}_f = \frac{1}{L} \Delta {}^0\mathbf{p} \quad (7.136)$$

If `activeConnector = True`, the scalar spring force is computed as

$$f_{SD} = k \cdot (L - L_0) + d \cdot \Delta {}^0\mathbf{v}^T \mathbf{v}_f + f_a \quad (7.137)$$

If the `springForceUserFunction` UF is defined,  $\mathbf{f}$  instead becomes ( $t$  is current time)

$$f_{SD} = \text{UF}(mbs, t, L - L_0, \Delta {}^0\mathbf{v}^T \mathbf{v}_f, k, d, f_a) \quad (7.138)$$

if `activeConnector = False`,  $f_{SD}$  is set to zero.: The vector of the spring force applied at both markers finally reads

$$\mathbf{f} = f_{SD} \mathbf{v}_f \quad (7.139)$$

## Userfunction: springForceUserFunction(mbs, t, deltaL, deltaL\_t, stiffness, damping, force)

A user function, which computes the spring force depending on time, object variables (deltaL, deltaL\_t) and object parameters (stiffness, damping, force). The object variables are provided to the function using the current values of the SpringDamper object

| arguments / return  | type or size | description                                            |
|---------------------|--------------|--------------------------------------------------------|
| mbs                 | MainSystem   | provides MainSystem mbs to which object belongs        |
| t                   | Real         | current time in mbs                                    |
| deltaL              | Real         | $L - L_0$ , spring elongation                          |
| deltaL_t            | Real         | $\Delta^0 \mathbf{v}^T \mathbf{v}_f$ , spring velocity |
| stiffness           | Real         | copied from object                                     |
| damping             | Real         | copied from object                                     |
| force               | Real         | copied from object; constant force                     |
| <b>return value</b> | Real         | scalar value of computed spring force                  |

---

### User function example:

```
#define nonlinear force
def UFforce(mbs, t, u, v, k, d, F0):
 return k*u + d*v + F0
#markerNumbers taken from mini example
mbs.AddObject(ObjectConnectorSpringDamper(markerNumbers=[m0,m1],
 referenceLength = 1,
 stiffness = 100, damping = 1,
 springForceUserFunction = UFforce))
```

---

#### 7.2.15.4 MINI EXAMPLE for ObjectConnectorSpringDamper

```
node = mbs.AddNode(NodePoint(referenceCoordinates = [1.05,0,0]))
oMassPoint = mbs.AddObject(MassPoint(nodeNumber = node, physicsMass=1))

m0 = mbs.AddMarker(MarkerBodyPosition(bodyNumber=oGround, localPosition=[0,0,0]))
m1 = mbs.AddMarker(MarkerBodyPosition(bodyNumber=oMassPoint, localPosition=[0,0,0]))

mbs.AddObject(ObjectConnectorSpringDamper(markerNumbers=[m0,m1],
 referenceLength = 1, #shorter than initial
 distance
 stiffness = 100,
 damping = 1))

#assemble and solve system for default parameters
mbs.Assemble()
```

```
exu.SolveDynamic(mbs)

#check result at default integration time
exdynTestGlobals.testResult = mbs.GetNodeOutput(node, exu.OutputVariableType.
Position)[0]
```

---

For further examples on ObjectConnectorSpringDamper see Examples:

- [SpringDamperMassUserFunction.py](#)
- [ANCF\\_contact\\_circle.py](#)
- [ANCF\\_contact\\_circle2.py](#)
- [ANCF\\_slidingJoint2D.py](#)
- [Spring\\_with\\_constraints.py](#)
- [stiffFlyballGovernor2.py](#)

For further examples on ObjectConnectorSpringDamper see TestModels:

- [ANCFcontactCircleTest.py](#)
- [modelUnitTests.py](#)
- [PARTS\\_ATEs\\_moving.py](#)
- [stiffFlyballGovernor.py](#)

### 7.2.16 ObjectConnectorCartesianSpringDamper

An 3D spring-damper element acting accordingly in three (global) directions (x,y,z) which connects to position-based markers.

**Additional information for ObjectConnectorCartesianSpringDamper:**

- The Object has the following types = Connector
- Requested marker type = Position
- Short name for Python = **CartesianSpringDamper**
- Short name for Python (visualization object) = **VCartesianSpringDamper**

The item **ObjectConnectorCartesianSpringDamper** with type = 'ConnectorCartesianSpringDamper' has the following parameters:

| Name                    | type                                  | size | default value       | description                                                                                                                        |
|-------------------------|---------------------------------------|------|---------------------|------------------------------------------------------------------------------------------------------------------------------------|
| name                    | String                                |      | "                   | connector's unique name                                                                                                            |
| markerNumbers           | ArrayMarkerIndex                      |      | [ MAXINT, MAX-INT ] | list of markers used in connector                                                                                                  |
| stiffness               | Vector3D                              |      | [0.,0.,0.]          | stiffness [SI:N/m] of springs; act against relative displacements in 0, 1, and 2-direction                                         |
| damping                 | Vector3D                              |      | [0.,0.,0.]          | damping [SI:N/(m s)] of dampers; act against relative velocities in 0, 1, and 2-direction                                          |
| offset                  | Vector3D                              |      | [0.,0.,0.]          | offset between two springs                                                                                                         |
| springForceUserFunction | PyFunctionVector3DmbScalar5Vector3D   | 0    |                     | A python function which computes the 3D force vector between the two marker points, if activeConnector=True; see description below |
| activeConnector         | Bool                                  |      | True                | flag, which determines, if the connector is active; used to deactivate (temporarily) a connector or constraint                     |
| visualization           | VObjectConnectorCartesianSpringDamper |      |                     | parameters for visualization of item                                                                                               |

The item VObjectConnectorCartesianSpringDamper has the following parameters:

| Name     | type   | size | default value     | description                                                                               |
|----------|--------|------|-------------------|-------------------------------------------------------------------------------------------|
| show     | Bool   |      | True              | set true, if item is shown in visualization and false if it is not shown                  |
| drawSize | float  |      | -1.               | drawing size = diameter of spring; size == -1.f means that default connector size is used |
| color    | Float4 |      | [-1.,-1.,-1.,-1.] | RGBA connector color; if R==1, use default color                                          |

---

#### 7.2.16.1 DESCRIPTION of ObjectConnectorCartesianSpringDamper:

**Information on input parameters:**

| input parameter | symbol       | description see tables above |
|-----------------|--------------|------------------------------|
| markerNumbers   | $[m0, m1]^T$ |                              |
| stiffness       | $k$          |                              |
| damping         | $d$          |                              |
| offset          | $v_{off}$    |                              |

The following output variables are available as **OutputVariableType** in sensors, **Get...Output()** and other functions:

| output variable | symbol                                                           | description                                           |
|-----------------|------------------------------------------------------------------|-------------------------------------------------------|
| Displacement    | $\Delta^0\mathbf{p} = {}^0\mathbf{p}_{m1} - {}^0\mathbf{p}_{m0}$ | relative displacement in global coordinates           |
| Distance        | $L =  \Delta^0\mathbf{p} $                                       | scalar distance between both marker points            |
| Velocity        | $\Delta^0\mathbf{v} = {}^0\mathbf{v}_{m1} - {}^0\mathbf{v}_{m0}$ | relative translational velocity in global coordinates |
| Force           | $\mathbf{f}_{SD}$                                                | joint force in global coordinates, see equations      |

### 7.2.16.2 Definition of quantities

| intermediate variables | symbol                | description                                            |
|------------------------|-----------------------|--------------------------------------------------------|
| marker m0 position     | ${}^0\mathbf{p}_{m0}$ | current global position which is provided by marker m0 |
| marker m1 position     | ${}^0\mathbf{p}_{m1}$ |                                                        |
| marker m0 velocity     | ${}^0\mathbf{v}_{m0}$ | current global velocity which is provided by marker m0 |
| marker m1 velocity     | ${}^0\mathbf{v}_{m1}$ |                                                        |

### 7.2.16.3 Connector forces

Displacement between marker m0 to marker m1 positions,

$$\Delta^0\mathbf{p} = {}^0\mathbf{p}_{m1} - {}^0\mathbf{p}_{m0} \quad (7.140)$$

and relative velocity,

$$\Delta^0\mathbf{v} = {}^0\mathbf{v}_{m1} - {}^0\mathbf{v}_{m0} \quad (7.141)$$

If **activeConnector** = True, the spring force vector is computed as

$$\mathbf{f}_{SD} = (\mathbf{k} \cdot (\Delta^0\mathbf{p} - \mathbf{v}_{off}) + \mathbf{d}\Delta^0\mathbf{v}) \quad (7.142)$$

If the **springForceUserFunction** UF is defined,  $\mathbf{f}_{SD}$  instead becomes ( $t$  is current time)

$$\mathbf{f}_{SD} = \text{UF}(mbs, t, \Delta^0\mathbf{p}, \Delta^0\mathbf{v}, \mathbf{k}, \mathbf{d}, \mathbf{v}_{off}) \quad (7.143)$$

if **activeConnector** = False,  $\mathbf{f}_{SD}$  is set to zero.:

**Userfunction:** **springForceUserFunction(mbs, t, displacement, velocity, stiffness, damping, offset)**

A user function, which computes the 3D spring force vector depending on time, object variables (deltaL, deltaL\_t) and object parameters (stiffness, damping, force). The object variables are provided to the function using the current values of the SpringDamper object

| arguments / return  | type or size | description                                                 |
|---------------------|--------------|-------------------------------------------------------------|
| mbs                 | MainSystem   | provides MainSystem mbs in which underlying item is defined |
| t                   | Real         | current time in mbs                                         |
| displacement        | Vector3D     | $\Delta^0 p$                                                |
| velocity            | Vector3D     | $\Delta^0 v$                                                |
| stiffness           | Vector3D     | copied from object                                          |
| damping             | Vector3D     | copied from object                                          |
| offset              | Vector3D     | copied from object                                          |
| <b>return value</b> | Vector3D     | list or numpy array of computed spring force                |

---

### User function example:

```
#define simple force for spring-damper:
def UFForce(mbs, t, u, v, k, d, offset):
 return [u[0]*k[0],u[1]*k[1],u[2]*k[2]]

#markerNumbers and parameters taken from mini example
mbs.AddObject(CartesianSpringDamper(markerNumbers = [mGround, mMass],
 stiffness = [k,k,k],
 damping = [0,k*0.05,0], offset = [0,0,0],
 springForceUserFunction = UFForce))
```

---

#### 7.2.16.4 MINI EXAMPLE for ObjectConnectorCartesianSpringDamper

```
#example with mass at [1,1,0], 5kg under load 5N in -y direction
k=5000
nMass = mbs.AddNode(NodePoint(referenceCoordinates=[1,1,0]))
oMass = mbs.AddObject(MassPoint(physicsMass = 5, nodeNumber = nMass))

mMass = mbs.AddMarker(MarkerNodePosition(nodeNumber=nMass))
mGround = mbs.AddMarker(MarkerBodyPosition(bodyNumber=oGround, localPosition =
[1,1,0]))
mbs.AddObject(CartesianSpringDamper(markerNumbers = [mGround, mMass],
 stiffness = [k,k,k],
 damping = [0,k*0.05,0], offset = [0,0,0]))
mbs.AddLoad(Force(markerNumber = mMass, loadVector = [0, -5, 0])) #static solution
=-5/5000=-0.001m

#assemble and solve system for default parameters
mbs.Assemble()
exu.SolveDynamic(mbs)
```

```
#check result at default integration time
exudynTestGlobals.testResult = mbs.GetNodeOutput(nMass, exu.OutputVariableType.
 Displacement)[1]
```

---

For further examples on ObjectConnectorCartesianSpringDamper see Examples:

- [mouseInteractionExample.py](#)
- [rigid3Dexample.py](#)
- [sliderCrank3DwithANCFbeltDrive2.py](#)
- [ANCF\\_contact\\_circle.py](#)
- [ANCF\\_contact\\_circle2.py](#)
- [ANCF\\_slidingJoint2D.py](#)
- [cartesianSpringDamper.py](#)
- [flexibleRotor3Dtest.py](#)
- [lavalRotor2Dtest.py](#)
- [NGsolvePistonEngine.py](#)
- ...

For further examples on ObjectConnectorCartesianSpringDamper see TestModels:

- [scissorPrismaticRevolute2D.py](#)
- [sphericalJointTest.py](#)
- [ANCFcontactCircleTest.py](#)
- [carRollingDiscTest.py](#)
- [driveTrainTest.py](#)
- [explicitLieGroupMBSTest.py](#)
- [genericODE2test.py](#)
- [mecanumWheelRollingDiscTest.py](#)
- [modelUnitTests.py](#)
- [objectFFRFreducedOrderAccelerations.py](#)
- ...

### 7.2.17 ObjectConnectorRigidBodySpringDamper

An 3D spring-damper element acting on relative displacements and relative rotations of two rigid body (position+orientation) markers; connects to (position+orientation)-based markers and represents a penalty-based rigid joint (or prismatic, revolute, etc.)

**Additional information for ObjectConnectorRigidBodySpringDamper:**

- The Object has the following types = Connector
- Requested marker type = Position + Orientation
- Requested node type = GenericData
- **Short name for Python = RigidBodySpringDamper**
- **Short name for Python (visualization object) = VRigidBodySpringDamper**

The item **ObjectConnectorRigidBodySpringDamper** with type = 'ConnectorRigidBodySpringDamper' has the following parameters:

| Name            | type             | size | default value               | description                                                                                                                                                                                                                                                                                                                                                                                               |
|-----------------|------------------|------|-----------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| name            | String           |      | "                           | connector's unique name                                                                                                                                                                                                                                                                                                                                                                                   |
| markerNumbers   | ArrayMarkerIndex |      | [ MAXINT, MAX-INT ]         | list of markers used in connector                                                                                                                                                                                                                                                                                                                                                                         |
| nodeNumber      | NodeIndex        |      | MAXINT                      | node number of a NodeGenericData (size depends on application) for dataCoordinates for user functions (e.g., implementing contact/friction user function)                                                                                                                                                                                                                                                 |
| stiffness       | Matrix6D         |      | np.zeros([6,6])             | stiffness [SI:N/m or Nm/rad] of translational, torsional and coupled springs; act against relative displacements in x, y, and z-direction as well as the relative angles (calculated as Euler angles); in the simplest case, the first 3 diagonal values correspond to the local stiffness in x,y,z direction and the last 3 diagonal values correspond to the rotational stiffness around x,y and z axis |
| damping         | Matrix6D         |      | np.zeros([6,6])             | damping [SI:N/(m/s) or Nm/(rad/s)] of translational, torsional and coupled dampers; very similar to stiffness, however, the rotational velocity is computed from the angular velocity vector                                                                                                                                                                                                              |
| rotationMarker0 | Matrix3D         |      | [[1,0,0], [0,1,0], [0,0,1]] | local rotation matrix for marker 0; stiffness, damping, etc. components are measured in local coordinates relative to rotationMarker0                                                                                                                                                                                                                                                                     |
| rotationMarker1 | Matrix3D         |      | [[1,0,0], [0,1,0], [0,0,1]] | local rotation matrix for marker 1; stiffness, damping, etc. components are measured in local coordinates relative to rotationMarker1                                                                                                                                                                                                                                                                     |
| offset          | Vector6D         |      | [0.,0.,0.,0.,0.,0.]         | translational and rotational offset considered in the spring force calculation                                                                                                                                                                                                                                                                                                                            |
| activeConnector | Bool             |      | True                        | flag, which determines, if the connector is active; used to deactivate (temporarily) a connector or constraint                                                                                                                                                                                                                                                                                            |

|                               |                                                                      |   |                                                                                                                                                     |
|-------------------------------|----------------------------------------------------------------------|---|-----------------------------------------------------------------------------------------------------------------------------------------------------|
| springForceTorqueUserFunction | PyFunctionVector6Dmbs4Vector3D2Matrix6D2Matrix3DVector6DVector6D     | 0 | which computes the 6D force-torque vector (3D force + 3D torque) between the two rigid body markers, if activeConnector=True; see description below |
| postNewtonStepUserFunction    | PyFunctionVectorMbs4VectorVector3D2Matrix6D2Matrix3DVector6DVector6D | 0 | which computes the error of the PostNewtonStep; see description below                                                                               |
| visualization                 | VObjectConnectorRigidBodySpringDamper                                |   | parameters for visualization of item                                                                                                                |

The item VObjectConnectorRigidBodySpringDamper has the following parameters:

| Name     | type   | size | default value     | description                                                                               |
|----------|--------|------|-------------------|-------------------------------------------------------------------------------------------|
| show     | Bool   |      | True              | set true, if item is shown in visualization and false if it is not shown                  |
| drawSize | float  |      | -1.               | drawing size = diameter of spring; size == -1.f means that default connector size is used |
| color    | Float4 |      | [-1.,-1.,-1.,-1.] | RGBA connector color; if R==1, use default color                                          |

### 7.2.17.1 DESCRIPTION of ObjectConnectorRigidBodySpringDamper:

#### Information on input parameters:

| input parameter | symbol | description see tables above |
|-----------------|--------|------------------------------|
| nodeNumber      | $n_d$  |                              |

The following output variables are available as OutputVariableType in sensors, Get...Output() and other functions:

| output variable      | symbol                                                          | description                                                                                                                                                                                                      |
|----------------------|-----------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| DisplacementLocal    | ${}^{j0}\Delta\mathbf{p}$                                       | relative displacement in local joint0 coordinates                                                                                                                                                                |
| VelocityLocal        | ${}^{j0}\Delta\mathbf{v}$                                       | relative translational velocity in local joint0 coordinates                                                                                                                                                      |
| Rotation             | ${}^{j0}\boldsymbol{\theta} = [\theta_0, \theta_1, \theta_2]^T$ | relative rotation parameters (Tait Bryan Rxyz); these are the angles used for calculation of joint torques (e.g. if cX is the diagonal rotational stiffness, the moment for axis X reads $mX=cX*\phi_iX$ , etc.) |
| AngularVelocityLocal | ${}^{j0}\Delta\boldsymbol{\omega}$                              | relative angular velocity in local joint0 coordinates                                                                                                                                                            |
| ForceLocal           | ${}^{j0}\mathbf{f}$                                             | joint force in local joint0 coordinates                                                                                                                                                                          |
| TorqueLocal          | ${}^{j0}\mathbf{m}$                                             | joint torque in local joint0 coordinates                                                                                                                                                                         |

#### 7.2.17.2 Definition of quantities

| input parameter  | symbol                                     | description                                                             |
|------------------|--------------------------------------------|-------------------------------------------------------------------------|
| stiffness        | $\mathbf{k} \in \mathbb{R}^{6 \times 6}$   | stiffness in $J0$ coordinates                                           |
| damping          | $\mathbf{d} \in \mathbb{R}^{6 \times 6}$   | damping in $J0$ coordinates                                             |
| offset           | ${}^{J0}\mathbf{v}_{off} \in \mathbb{R}^6$ | offset in $J0$ coordinates                                              |
| rotationMarker0  | ${}^{m0,J0}\mathbf{A}$                     | rotation matrix which transforms from joint 0 into marker 0 coordinates |
| rotationMarker1  | ${}^{m1,J1}\mathbf{A}$                     | rotation matrix which transforms from joint 1 into marker 1 coordinates |
| markerNumbers[0] | $m0$                                       | global marker number $m0$                                               |
| markerNumbers[1] | $m1$                                       | global marker number $m1$                                               |

| intermediate variables  | symbol                             | description                                                                                                                                      |
|-------------------------|------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------|
| marker $m0$ position    | ${}^0\mathbf{p}_{m0}$              | current global position which is provided by marker $m0$                                                                                         |
| marker $m0$ orientation | ${}^{0,m0}\mathbf{A}$              | current rotation matrix provided by marker $m0$                                                                                                  |
| marker $m1$ position    | ${}^0\mathbf{p}_{m1}$              | accordingly                                                                                                                                      |
| marker $m1$ orientation | ${}^{0,m1}\mathbf{A}$              | current rotation matrix provided by marker $m1$                                                                                                  |
| marker $m0$ velocity    | ${}^0\mathbf{v}_{m0}$              | current global velocity which is provided by marker $m0$                                                                                         |
| marker $m1$ velocity    | ${}^0\mathbf{v}_{m1}$              | accordingly                                                                                                                                      |
| marker $m0$ velocity    | ${}^b\boldsymbol{\omega}_{m0}$     | current local angular velocity vector provided by marker $m0$                                                                                    |
| marker $m1$ velocity    | ${}^b\boldsymbol{\omega}_{m1}$     | current local angular velocity vector provided by marker $m1$                                                                                    |
| Displacement            | ${}^0\Delta\mathbf{p}$             | ${}^0\mathbf{p}_{m1} - {}^0\mathbf{p}_{m0}$                                                                                                      |
| Velocity                | ${}^0\Delta\mathbf{v}$             | ${}^0\mathbf{v}_{m1} - {}^0\mathbf{v}_{m0}$                                                                                                      |
| DisplacementLocal       | ${}^{J0}\Delta\mathbf{p}$          | $({}^{0,m0}\mathbf{A} {}^{m0,J0}\mathbf{A})^T {}^0\Delta\mathbf{p}$                                                                              |
| VelocityLocal           | ${}^{J0}\Delta\mathbf{v}$          | $({}^{0,m0}\mathbf{A} {}^{m0,J0}\mathbf{A})^T {}^0\Delta\mathbf{v}$                                                                              |
| AngularVelocityLocal    | ${}^{J0}\Delta\boldsymbol{\omega}$ | $({}^{0,m0}\mathbf{A} {}^{m0,J0}\mathbf{A})^T ({}^{0,m1}\mathbf{A} {}^{m1}\boldsymbol{\omega} - {}^{0,m0}\mathbf{A} {}^{m0}\boldsymbol{\omega})$ |

### 7.2.17.3 Connector forces

If `activeConnector = True`, the vector spring force is computed as

$$\begin{bmatrix} {}^{J0}\mathbf{f}_{SD} \\ {}^{J0}\mathbf{m}_{SD} \end{bmatrix} = \mathbf{k} \left( \begin{bmatrix} {}^{J0}\Delta\mathbf{p} \\ {}^{J0}\boldsymbol{\theta} \end{bmatrix} - {}^{J0}\mathbf{v}_{off} \right) + \mathbf{d} \begin{bmatrix} {}^{J0}\Delta\mathbf{v} \\ {}^{J0}\Delta\boldsymbol{\omega} \end{bmatrix} \quad (7.144)$$

For the application of joint forces to markers,  $[{}^{J0}\mathbf{f}_{SD}, {}^{J0}\mathbf{m}_{SD}]^T$  is transformed into global coordinates. if `activeConnector = False`,  ${}^{J0}\mathbf{f}_{SD}$  and  ${}^{J0}\mathbf{m}_{SD}$  are set to zero.:.

If the `springForceTorqueUserFunction` UF is defined and `activeConnector = True`,  $\mathbf{f}_{SD}$  instead becomes ( $t$  is current time)

$$\mathbf{f}_{SD} = \text{UF}(mbs, t, {}^{J0}\Delta\mathbf{p}, {}^{J0}\boldsymbol{\theta}, {}^{J0}\Delta\mathbf{v}, {}^{J0}\Delta\boldsymbol{\omega}, stiffness, damping, rotationMarker0, rotationMarker1, offset) \quad (7.145)$$

**Userfunction: `springForceTorqueUserFunction(mbs, t, displacement, rotation, velocity, angularVelocity, stiffness, damping, rotJ0, rotJ1, offset)`**

A user function, which computes the 6D spring-damper force-torque vector depending on mbs, time, local quantities (displacement, rotation, velocity, angularVelocity, stiffness), which are evaluated at current time, which are relative quantities between both markers and which are defined in joint J0 coordinates. As relative rotations are defined by Tait-Bryan rotation parameters, it is recommended to use this connector for small relative rotations only (except for rotations about one axis). Furthermore, the user function contains object parameters (stiffness, damping, rotationMarker0/1, offset).

Detailed description of the arguments and local quantities:

| arguments / return  | type or size | description                                                 |
|---------------------|--------------|-------------------------------------------------------------|
| mbs                 | MainSystem   | provides MainSystem mbs in which underlying item is defined |
| t                   | Real         | current time in mbs                                         |
| displacement        | Vector3D     | $J^0 \Delta p$                                              |
| rotation            | Vector3D     | $J^0 \theta$                                                |
| velocity            | Vector3D     | $J^0 \Delta v$                                              |
| angularVelocity     | Vector3D     | $J^0 \Delta \omega$                                         |
| stiffness           | Vector6D     | copied from object                                          |
| damping             | Vector6D     | copied from object                                          |
| rotJ0               | Matrix3D     | rotationMarker0 copied from object                          |
| rotJ1               | Matrix3D     | rotationMarker1 copied from object                          |
| offset              | Vector6D     | copied from object                                          |
| <b>return value</b> | Vector6D     | list or numpy array of computed spring force-torque         |

**Userfunction: `postNewtonStepUserFunction(mbs, t, dataCoordinates, displacement, rotation, velocity, angularVelocity, stiffness, damping, rotJ0, rotJ1, offset)`**

A user function which computes the error of the PostNewtonStep  $\varepsilon_{PN}$ , a recommended for stepsize reduction  $t_{recom}$  (use values  $> 0$  to recommend step size or values  $< 0$  else; 0 gives minimum step size) and the updated dataCoordinates  $\mathbf{d}^k$  of NodeGenericData  $n_d$ . Except from dataCoordinates, the arguments are the same as in `springForceTorqueUserFunction`. The `postNewtonStepUserFunction` should be used together with the dataCoordinates in order to implement a active set or switching strategy for discontinuous events, such as in contact, friction, plasticity, fracture or similar.

Detailed description of the arguments and local quantities:

| arguments / return  | type or size | description                                                                               |
|---------------------|--------------|-------------------------------------------------------------------------------------------|
| mbs                 | MainSystem   | provides MainSystem mbs in which underlying item is defined                               |
| t                   | Real         | current time in mbs                                                                       |
| dataCoordinates     | Vector       | $\mathbf{d}^{k-1} = [d_0^{k-1}, d_1^{k-1}, \dots]$ for previous post Newton step $k - 1$  |
| ...                 | ...          | other arguments see <code>springForceTorqueUserFunction</code>                            |
| <b>return value</b> | Vector       | $[\varepsilon_{PN}, t_{recom}, d_0^k, d_1^k, \dots]$ where $k$ indicates the current step |

### User function example:

```
#define simple force for spring-damper:
def UFForce(mbs, t, displacement, rotation, velocity, angularVelocity,
 stiffness, damping, rotJ0, rotJ1, offset):
 k = stiffness #passed as list
 u = displacement
 return [u[0]*k[0][0], u[1]*k[1][1], u[2]*k[2][2], 0,0,0]

#markerNumbers and parameters taken from mini example
mbs.AddObject(RigidBodySpringDamper(markerNumbers = [mGround, mBody],
 stiffness = np.diag([k,k,k, 0,0,0]),
 damping = np.diag([0,k*0.01,0, 0,0,0]),
 offset = [0,0,0, 0,0,0],
 springForceTorqueUserFunction = UFForce))
```

---

#### 7.2.17.4 MINI EXAMPLE for ObjectConnectorRigidBodySpringDamper

```
#example with rigid body at [0,0,0], 1kg under initial velocity
k=500
nBody = mbs.AddNode(RigidRxyz(initialVelocities=[0,1e3,0, 0,0,0]))
oBody = mbs.AddObject(RigidBody(physicsMass=1, physicsInertia=[1,1,1,0,0,0],
 nodeNumber=nBody))

mBody = mbs.AddMarker(MarkerNodeRigid(nodeNumber=nBody))
mGround = mbs.AddMarker(MarkerBodyRigid(bodyNumber=oGround,
 localPosition = [0,0,0]))
mbs.AddObject(RigidBodySpringDamper(markerNumbers = [mGround, mBody],
 stiffness = np.diag([k,k,k, 0,0,0]),
 damping = np.diag([0,k*0.01,0, 0,0,0]),
 offset = [0,0,0, 0,0,0]))

#assemble and solve system for default parameters
mbs.Assemble()
exu.SolveDynamic(mbs, exu.SimulationSettings())

#check result at default integration time
exudynTestGlobals.testResult = mbs.GetNodeOutput(nBody, exu.OutputVariableType.
 Displacement)[1]
```

---

For further examples on ObjectConnectorRigidBodySpringDamper see Examples:

- [stiffFlyballGovernor2.py](#)

For further examples on ObjectConnectorRigidBodySpringDamper see TestModels:

- [connectorRigidBodySpringDamperTest.py](#)

- stiffFlyballGovernor.py
- superElementRigidJointTest.py

## 7.2.18 ObjectConnectorCoordinateSpringDamper

A 1D (scalar) spring-damper element acting on single ODE2 coordinates; connects to coordinate-based markers; NOTE that the coordinate markers only measure the coordinate (=displacement), but the reference position is not included as compared to position-based markers!; the spring-damper can also act on rotational coordinates.

**Additional information for ObjectConnectorCoordinateSpringDamper:**

- The Object has the following types = Connector
- Requested marker type = Coordinate
- **Short name for Python** = **CoordinateSpringDamper**
- **Short name for Python (visualization object)** = **VCoordinateSpringDamper**

The item **ObjectConnectorCoordinateSpringDamper** with type = 'ConnectorCoordinateSpringDamper' has the following parameters:

| Name                        | type                                   | size | default value       | description                                                                                                                                     |
|-----------------------------|----------------------------------------|------|---------------------|-------------------------------------------------------------------------------------------------------------------------------------------------|
| name                        | String                                 |      | "                   | connector's unique name                                                                                                                         |
| markerNumbers               | ArrayMarkerIndex                       |      | [ MAXINT, MAX-INT ] | list of markers used in connector                                                                                                               |
| stiffness                   | Real                                   |      | 0.                  | stiffness [SI:N/m] of spring; acts against relative value of coordinates                                                                        |
| damping                     | Real                                   |      | 0.                  | damping [SI:N/(m s)] of damper; acts against relative velocity of coordinates                                                                   |
| offset                      | Real                                   |      | 0.                  | offset between two coordinates (reference length of springs), see equation                                                                      |
| dryFriction                 | Real                                   |      | 0.                  | dry friction force [SI:N] against relative velocity; assuming a normal force $f_N$ , the friction force can be interpreted as $f_\mu = \mu f_N$ |
| dryFrictionProportionalZone | Real                                   |      | 0.                  | limit velocity [m/s] up to which the friction is proportional to velocity (for regularization / avoid numerical oscillations)                   |
| activeConnector             | Bool                                   |      | True                | flag, which determines, if the connector is active; used to deactivate (temporarily) a connector or constraint                                  |
| springForceUserFunction     | PyFunctionMbsScalar8                   |      | 0                   | A python function which defines the spring force with 8 parameters, see equations section / see description below                               |
| visualization               | VObjectConnectorCoordinateSpringDamper |      |                     | parameters for visualization of item                                                                                                            |

The item **VObjectConnectorCoordinateSpringDamper** has the following parameters:

| Name     | type   | size | default value     | description                                                                               |
|----------|--------|------|-------------------|-------------------------------------------------------------------------------------------|
| show     | Bool   |      | True              | set true, if item is shown in visualization and false if it is not shown                  |
| drawSize | float  |      | -1.               | drawing size = diameter of spring; size == -1.f means that default connector size is used |
| color    | Float4 |      | [-1.,-1.,-1.,-1.] | RGBA connector color; if R==1, use default color                                          |

---

### 7.2.18.1 DESCRIPTION of ObjectConnectorCoordinateSpringDamper:

**Information on input parameters:**

| input parameter             | symbol           | description see tables above |
|-----------------------------|------------------|------------------------------|
| stiffness                   | $k$              |                              |
| damping                     | $d$              |                              |
| offset                      | $l_{\text{off}}$ |                              |
| dryFriction                 | $f_\mu$          |                              |
| dryFrictionProportionalZone | $v_\mu$          |                              |

The following output variables are available as OutputVariableType in sensors, Get...Output() and other functions:

| output variable | symbol     | description                                        |
|-----------------|------------|----------------------------------------------------|
| Displacement    | $\Delta q$ | relative scalar displacement of marker coordinates |
| Velocity        | $\Delta v$ | difference of scalar marker velocity coordinates   |
| Force           | $f_{SD}$   | scalar spring force                                |

### 7.2.18.2 Definition of quantities

| intermediate variables        | symbol   | description                                                                                            |
|-------------------------------|----------|--------------------------------------------------------------------------------------------------------|
| marker m0 coordinate          | $q_{m0}$ | current displacement coordinate which is provided by marker m0; does NOT include reference coordinate! |
| marker m1 coordinate          | $q_{m1}$ |                                                                                                        |
| marker m0 velocity coordinate | $v_{m0}$ | current velocity coordinate which is provided by marker m0                                             |
| marker m1 velocity coordinate | $v_{m1}$ |                                                                                                        |

### 7.2.18.3 Connector forces

Displacement between marker m0 to marker m1 coordinates (does NOT include reference coordinates),

$$\Delta q = q_{m1} - q_{m0} \quad (7.146)$$

and relative velocity,

$$\Delta v = v_{m1} - v_{m0} \quad (7.147)$$

If  $f_\mu > 0$ , the friction force is computed as

$$f_{\text{friction}} = \begin{cases} \text{Sgn}(\Delta v) \cdot f_\mu & \text{if } |\Delta v| \geq v_\mu \\ \frac{\Delta v}{v_\mu} f_\mu & \text{if } |\Delta v| < v_\mu \end{cases} \quad (7.148)$$

If activeConnector = True, the scalar spring force vector is computed as

$$f_{SD} = k(\Delta q - l_{\text{off}}) + d \cdot \Delta v + f_{\text{friction}} \quad (7.149)$$

If the `springForceUserFunction` UF is defined,  $f_{SD}$  instead becomes ( $t$  is current time)

$$f_{SD} = \text{UF}(mbs, t, \Delta q, \Delta v, k, d, l_{off}, f_\mu, v_\mu) \quad (7.150)$$

if `activeConnector = False`,  $f_{SD}$  is set to zero.:

---

### Userfunction: `springForceUserFunction(mbs, t, displacement, velocity, stiffness, damping, offset, dryFriction, dryFrictionProportionalZone)`

A user function, which computes the scalar spring force depending on time, object variables (displacement, velocity) and object parameters . The object variables are passed to the function using the current values of the `CoordinateSpringDamper` object.

| arguments / return                       | type or size | description                                                              |
|------------------------------------------|--------------|--------------------------------------------------------------------------|
| <code>mbs</code>                         | MainSystem   | provides MainSystem <code>mbs</code> in which underlying item is defined |
| <code>t</code>                           | Real         | current time in <code>mbs</code>                                         |
| <code>displacement</code>                | Real         | $\Delta q$                                                               |
| <code>velocity</code>                    | Real         | $\Delta v$                                                               |
| <code>stiffness</code>                   | Real         | copied from object                                                       |
| <code>damping</code>                     | Real         | copied from object                                                       |
| <code>offset</code>                      | Real         | copied from object                                                       |
| <code>dryFriction</code>                 | Real         | copied from object                                                       |
| <code>dryFrictionProportionalZone</code> | Real         | copied from object                                                       |
| <b>return value</b>                      | Real         | scalar value of computed force                                           |

---

### User function example:

```
#see also mini example!
def UFForce(mbs, t, u, v, k, d, offset, dryFriction, dryFrictionProportionalZone):
 return k*(u-offset) + d*v
```

---

#### 7.2.18.4 MINI EXAMPLE for ObjectConnectorCoordinateSpringDamper

```
def springForce(mbs, t, u, v, k, d, offset, dryFriction, dryFrictionProportionalZone)
:
 return 0.1*k*u+k*u**3+v*d

nMass=mbs.AddNode(Point(referenceCoordinates = [2,0,0]))
massPoint = mbs.AddObject(MassPoint(physicsMass = 5, nodeNumber = nMass))

groundMarker=mbs.AddMarker(MarkerNodeCoordinate(nodeNumber= nGround, coordinate = 0))
nodeMarker =mbs.AddMarker(MarkerNodeCoordinate(nodeNumber= nMass, coordinate = 0))

#Spring-Damper between two marker coordinates
mbs.AddObject(CoordinateSpringDamper(markerNumbers = [groundMarker, nodeMarker],
```

```

 stiffness = 5000, damping = 80,
 springForceUserFunction = springForce))
loadCoord = mbs.AddLoad(LoadCoordinate(markerNumber = nodeMarker, load = 1)) #static
 linear solution:0.002

#assemble and solve system for default parameters
mbs.Assemble()
exu.SolveDynamic(mbs)

#check result at default integration time
exudynTestGlobals.testResult = mbs.GetNodeOutput(nMass, exu.OutputVariableType.
 Displacement)[0]

```

---

For further examples on ObjectConnectorCoordinateSpringDamper see Examples:

- [slidercrankWithMassSpring.py](#)
- [ANCFALEtest.py](#)
- [ANCF\\_switchingSlidingJoint2D.py](#)
- [coordinateSpringDamper.py](#)
- [geneticOptimizationExample.py](#)
- [geneticOptimizationSliderCrank.py](#)
- [massSpringFrictionInteractive.py](#)
- [mouseInteractionExample.py](#)
- [parameterVariationExample.py](#)
- [simulateInteractively.py](#)
- ...

For further examples on ObjectConnectorCoordinateSpringDamper see TestModels:

- [scissorPrismaticRevolute2D.py](#)
- [ANCFslidingAndALEjointTest.py](#)
- [ANCFcontactFrictionTest.py](#)
- [geneticOptimizationTest.py](#)
- [modelUnitTests.py](#)
- [objectGenericODE2Test.py](#)
- [postNewtonStepContactTest.py](#)
- [sliderCrankFloatingTest.py](#)
- [springDamperUserFunctionTest.py](#)

## 7.2.19 ObjectConnectorDistance

Connector which enforces constant or prescribed distance between two bodies/nodes.

**Additional information for ObjectConnectorDistance:**

- The Object has the following types = Connector, Constraint
- Requested marker type = Position
- Short name for Python = **DistanceConstraint**
- Short name for Python (visualization object) = **VDistanceConstraint**

The item **ObjectConnectorDistance** with type = 'ConnectorDistance' has the following parameters:

| Name            | type                     | size | default value       | description                                                                                                    |
|-----------------|--------------------------|------|---------------------|----------------------------------------------------------------------------------------------------------------|
| name            | String                   |      | "                   | constraints's unique name                                                                                      |
| markerNumbers   | ArrayMarkerIndex         |      | [ MAXINT, MAX-INT ] | list of markers used in connector                                                                              |
| distance        | UReal                    |      | 0.                  | prescribed distance [SI:m] of the used markers                                                                 |
| activeConnector | Bool                     |      | True                | flag, which determines, if the connector is active; used to deactivate (temporarily) a connector or constraint |
| visualization   | VObjectConnectorDistance |      |                     | parameters for visualization of item                                                                           |

The item **VObjectConnectorDistance** has the following parameters:

| Name     | type   | size | default value     | description                                                                      |
|----------|--------|------|-------------------|----------------------------------------------------------------------------------|
| show     | Bool   |      | True              | set true, if item is shown in visualization and false if it is not shown         |
| drawSize | float  |      | -1.               | drawing size = link size; size == -1.f means that default connector size is used |
| color    | Float4 |      | [-1.,-1.,-1.,-1.] | RGBA connector color; if R== -1, use default color                               |

### 7.2.19.1 DESCRIPTION of ObjectConnectorDistance:

**Information on input parameters:**

| input parameter | symbol         | description see tables above |
|-----------------|----------------|------------------------------|
| markerNumbers   | $[m_0, m_1]^T$ |                              |
| distance        | $d_0$          |                              |

The following output variables are available as **OutputVariableType** in sensors, **Get...Output()** and other functions:

| output variable | symbol         | description                                 |
|-----------------|----------------|---------------------------------------------|
| Displacement    | ${}^0\Delta p$ | relative displacement in global coordinates |

|          |                          |                                                                             |
|----------|--------------------------|-----------------------------------------------------------------------------|
| Velocity | ${}^0\Delta\mathbf{v}$   | relative translational velocity in global coordinates                       |
| Distance | $ {}^0\Delta\mathbf{p} $ | distance between markers (should stay constant; shows constraint deviation) |
| Force    | $\lambda_0$              | joint force (=scalar Lagrange multiplier)                                   |

### 7.2.19.2 Definition of quantities

| intermediate variables | symbol                 | description                                            |
|------------------------|------------------------|--------------------------------------------------------|
| marker m0 position     | ${}^0\mathbf{p}_{m0}$  | current global position which is provided by marker m0 |
| marker m1 position     | ${}^0\mathbf{p}_{m1}$  | accordingly                                            |
| marker m0 velocity     | ${}^0\mathbf{v}_{m0}$  | current global velocity which is provided by marker m0 |
| marker m1 velocity     | ${}^0\mathbf{v}_{m1}$  | accordingly                                            |
| relative displacement  | ${}^0\Delta\mathbf{p}$ | ${}^0\mathbf{p}_{m1} - {}^0\mathbf{p}_{m0}$            |
| relative velocity      | ${}^0\Delta\mathbf{v}$ | ${}^0\mathbf{v}_{m1} - {}^0\mathbf{v}_{m0}$            |
| algebraicVariable      | $\lambda_0$            | Lagrange multiplier = force in constraint              |

### 7.2.19.3 Connector forces constraint equations

If `activeConnector = True`, the index 3 algebraic equation reads

$$|{}^0\Delta\mathbf{p}| - d_0 = 0 \quad (7.151)$$

The index 2 (velocity level) algebraic equation reads

$$\left( \frac{{}^0\Delta\mathbf{p}}{|{}^0\Delta\mathbf{p}|} \right)^T \Delta\mathbf{v} = 0 \quad (7.152)$$

if `activeConnector = False`, the algebraic equation reads

$$\lambda_0 = 0 \quad (7.153)$$

### 7.2.19.4 MINI EXAMPLE for ObjectConnectorDistance

```
#example with 1m pendulum, 50kg under gravity
nMass = mbs.AddNode(NodePoint2D(referenceCoordinates=[1,0]))
oMass = mbs.AddObject(MassPoint2D(physicsMass = 50, nodeNumber = nMass))

nMass = mbs.AddMarker(MarkerNodePosition(nodeNumber=nMass))
mGround = mbs.AddMarker(MarkerBodyPosition(bodyNumber=oGround, localPosition =
[0,0,0]))
oDistance = mbs.AddObject(DistanceConstraint(markerNumbers = [mGround, mMass],
distance = 1))
```

```

mbs.AddLoad(Force(markerNumber = mMass, loadVector = [0, -50*9.81, 0]))

#assemble and solve system for default parameters
mbs.Assemble()

sims=exu.SimulationSettings()
sims.timeIntegration.generalizedAlpha.spectralRadius=0.7
exu.SolveDynamic(mbs, sims)

#check result at default integration time
exudynTestGlobals.testResult = mbs.GetNodeOutput(nMass, exu.OutputVariableType.
Position)[0]

```

---

For further examples on ObjectConnectorDistance see Examples:

- [pendulum2Dconstraint.py](#)

For further examples on ObjectConnectorDistance see TestModels:

- [fourBarMechanismTest.py](#)
- [modelUnitTests.py](#)
- [PARTS\\_ATEs\\_moving.py](#)

## 7.2.20 ObjectConnectorCoordinate

A coordinate constraint which constrains two (scalar) coordinates of Marker[Node|Body]Coordinates attached to nodes or bodies. The constraint acts directly on coordinates, but does not include reference values, e.g., of nodal values. This constraint is computationally efficient and should be used to constrain nodal coordinates.

**Additional information for ObjectConnectorCoordinate:**

- The Object has the following types = Connector, Constraint
- Requested marker type = Coordinate
- **Short name for Python** = **CoordinateConstraint**
- **Short name for Python (visualization object)** = **VCoordinateConstraint**

The item **ObjectConnectorCoordinate** with type = 'ConnectorCoordinate' has the following parameters:

| Name                 | type                       | size | default value       | description                                                                                                                                                                                            |
|----------------------|----------------------------|------|---------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| name                 | String                     |      | "                   | constraints's unique name                                                                                                                                                                              |
| markerNumbers        | ArrayMarkerIndex           |      | [ MAXINT, MAX-INT ] | list of markers used in connector                                                                                                                                                                      |
| offset               | Real                       |      | 0.                  | An offset between the two values                                                                                                                                                                       |
| factorValue1         | Real                       |      | 1.                  | An additional factor multiplied with value1 used in algebraic equation                                                                                                                                 |
| velocityLevel        | Bool                       |      | False               | If true: connector constrains velocities (only works for ODE2 coordinates!); offset is used between velocities; in this case, the offsetUserFunction_t is considered and offsetUserFunction is ignored |
| offsetUserFunction   | PyFunctionMbsScalar2       |      | 0                   | A python function which defines the time-dependent offset; see description below                                                                                                                       |
| offsetUserFunction_t | PyFunctionMbsScalar2       |      | 0                   | time derivative of offsetUserFunction; needed for velocity level constraints; see description below                                                                                                    |
| activeConnector      | Bool                       |      | True                | flag, which determines, if the connector is active; used to deactivate (temporarily) a connector or constraint                                                                                         |
| visualization        | VObjectConnectorCoordinate |      |                     | parameters for visualization of item                                                                                                                                                                   |

The item **VObjectConnectorCoordinate** has the following parameters:

| Name     | type   | size | default value     | description                                                                      |
|----------|--------|------|-------------------|----------------------------------------------------------------------------------|
| show     | Bool   |      | True              | set true, if item is shown in visualization and false if it is not shown         |
| drawSize | float  |      | -1.               | drawing size = link size; size == -1.f means that default connector size is used |
| color    | Float4 |      | [-1.,-1.,-1.,-1.] | RGBA connector color; if R==1, use default color                                 |

### 7.2.20.1 DESCRIPTION of ObjectConnectorCoordinate:

**Information on input parameters:**

| input parameter      | symbol                                | description see tables above |
|----------------------|---------------------------------------|------------------------------|
| markerNumbers        | $[m0, m1]^T$                          |                              |
| offset               | $l_{\text{off}}$                      |                              |
| factorValue1         | $k_{m1}$                              |                              |
| offsetUserFunction   | $\text{UF}(mbs, t, l_{\text{off}})$   |                              |
| offsetUserFunction_t | $\text{UF}_t(mbs, t, l_{\text{off}})$ |                              |

The following output variables are available as **OutputVariableType** in sensors, **Get...Output()** and other functions:

| output variable    | symbol       | description                                                                    |
|--------------------|--------------|--------------------------------------------------------------------------------|
| Displacement       | $\Delta q$   | relative scalar displacement of marker coordinates, not including factorValue1 |
| Velocity           | $\Delta v$   | difference of scalar marker velocity coordinates, not including factorValue1   |
| ConstraintEquation | $\mathbf{c}$ | (residuum of) constraint equation                                              |
| Force              | $\lambda_0$  | scalar constraint force (Lagrange multiplier)                                  |

### 7.2.20.2 Definition of quantities

| intermediate variables             | symbol                       | description                                                                                            |
|------------------------------------|------------------------------|--------------------------------------------------------------------------------------------------------|
| marker m0 coordinate               | $q_{m0}$                     | current displacement coordinate which is provided by marker m0; does NOT include reference coordinate! |
| marker m1 coordinate               | $q_{m1}$                     |                                                                                                        |
| marker m0 velocity coordinate      | $v_{m0}$                     | current velocity coordinate which is provided by marker m0                                             |
| marker m1 velocity coordinate      | $v_{m1}$                     |                                                                                                        |
| difference of coordinates          | $\Delta q = q_{m1} - q_{m0}$ | Displacement between marker m0 to marker m1 coordinates (does NOT include reference coordinates)       |
| difference of velocity coordinates | $\Delta v = v_{m1} - v_{m0}$ |                                                                                                        |

### 7.2.20.3 Connector constraint equations

If **activeConnector** = True, the index 3 algebraic equation reads

$$\mathbf{c}(q_{m0}, q_{m1}) = k_{m1} \cdot q_{m1} - q_{m0} - l_{\text{off}} = 0 \quad (7.154)$$

If the offsetUserFunction UF is defined, **c** instead becomes (*t* is current time)

$$\mathbf{c}(q_{m0}, q_{m1}) = k_{m1} \cdot q_{m1} - q_{m0} - \text{UF}(mbs, t, l_{\text{off}}) = 0 \quad (7.155)$$

The **activeConnector** = True, index 2 (velocity level) algebraic equation reads

$$\dot{\mathbf{c}}(\dot{q}_{m0}, \dot{q}_{m1}) = k_{m1} \cdot \dot{q}_{m1} - \dot{q}_{m0} - d = 0 \quad (7.156)$$

The factor  $d$  in velocity level equations is zero, except if parameters.velocityLevel = True, then  $d = l_{\text{off}}$ . If velocity level constraints are active and the velocity level offsetUserFunction\_t  $\text{UF}_t$  is defined,  $\mathbf{c}$  instead becomes ( $t$  is current time)

$$\dot{\mathbf{c}}(\dot{q}_{m0}, \dot{q}_{m1}) = k_{m1} \cdot \dot{q}_{m1} - \dot{q}_{m0} - \text{UF}_t(mbs, t, l_{\text{off}}) = 0 \quad (7.157)$$

Note that the index 2 equations are used, if the solver uses index 2 formulation OR if the flag parameters.velocityLevel = True (or both). The user functions include dependency on time  $t$ , but this time dependency is not respected in the computation of initial accelerations. Therefore, it is recommended that UF and  $\text{UF}_t$  does not include initial accelerations.

If `activeConnector = False`, the (index 1) algebraic equation reads for ALL cases:

$$\mathbf{c}(\lambda_0) = \lambda_0 = 0 \quad (7.158)$$


---

### Userfunction: `offsetUserFunction(mbs, t, 10ffset)`

A user function, which computes scalar offset for the coordinate constraint, e.g., in order to move a node on a prescribed trajectory. It is NECESSARY to use sufficiently smooth functions, having **initial offsets** consistent with **initial configuration** of bodies, either zero or compatible initial offset-velocity, and no initial accelerations. The `offsetUserFunction` is **ONLY used** in case of static computation or index3 (generalizedAlpha) time integration. In order to be on the safe side, provide both `offsetUserFunction` and `offsetUserFunction_t`.

The user function gets time and the offset parameter as an input and returns the computed offset:

| arguments / return   | type or size | description                                                 |
|----------------------|--------------|-------------------------------------------------------------|
| <code>mbs</code>     | MainSystem   | provides MainSystem mbs in which underlying item is defined |
| <code>t</code>       | Real         | current time in mbs                                         |
| <code>10ffset</code> | Real         | $l_{\text{off}}$                                            |
| <b>return value</b>  | Real         | computed offset for given time                              |

---

### Userfunction: `offsetUserFunction_t(mbs, t, 10ffset)`

A user function, which computes scalar offset velocity for the coordinate constraint. It is NECESSARY to use sufficiently smooth functions, having **initial offset velocities** consistent with **initial velocities** of bodies. The `offsetUserFunction_t` is used instead of `offsetUserFunction` in case of `velocityLevel = True`, or for index2 time integration and needed for computation of initial accelerations in second order implicit time integrators.

The user function gets time and the offset parameter as an input and returns the computed offset velocity:

| arguments / return   | type or size | description                                                 |
|----------------------|--------------|-------------------------------------------------------------|
| <code>mbs</code>     | MainSystem   | provides MainSystem mbs in which underlying item is defined |
| <code>t</code>       | Real         | current time in mbs                                         |
| <code>10ffset</code> | Real         | $l_{\text{off}}$                                            |
| <b>return value</b>  | Real         | computed offset velocity for given time                     |

---

### User function example:

```

#see also mini example!
from math import sin, cos, pi
def UFoffset(mbs, t, l0ffset):
 return 0.5*l0ffset*(1-cos(0.5*pi*t))

def UFoffset_t(mbs, t, l0ffset): #time derivative of UFoffset
 return 0.5*l0ffset*0.5*pi*sin(0.5*pi*t)

nMass=mbs.AddNode(Point(referenceCoordinates = [2,0,0]))
massPoint = mbs.AddObject(MassPoint(physicsMass = 5, nodeNumber = nMass))

groundMarker=mbs.AddMarker(MarkerNodeCoordinate(nodeNumber= nGround, coordinate = 0))
nodeMarker =mbs.AddMarker(MarkerNodeCoordinate(nodeNumber= nMass, coordinate = 0))

#Spring-Damper between two marker coordinates
mbs.AddObject(CoordinateConstraint(markerNumbers = [groundMarker, nodeMarker],
 offset = 0.1,
 offsetUserFunction = UFoffset,
 offsetUserFunction_t = UFoffset_t))

```

---

#### 7.2.20.4 MINI EXAMPLE for ObjectConnectorCoordinate

```

def OffsetUF(mbs, t, l0ffset): #gives 0.05 at t=1
 return 0.5*(1-np.cos(2*3.141592653589793*0.25*t))*l0ffset

nMass=mbs.AddNode(Point(referenceCoordinates = [2,0,0]))
massPoint = mbs.AddObject(MassPoint(physicsMass = 5, nodeNumber = nMass))

groundMarker=mbs.AddMarker(MarkerNodeCoordinate(nodeNumber= nGround, coordinate = 0))
nodeMarker =mbs.AddMarker(MarkerNodeCoordinate(nodeNumber= nMass, coordinate = 0))

#Spring-Damper between two marker coordinates
mbs.AddObject(CoordinateConstraint(markerNumbers = [groundMarker, nodeMarker],
 offset = 0.1, offsetUserFunction = OffsetUF))

#assemble and solve system for default parameters
mbs.Assemble()
exu.SolveDynamic(mbs)

#check result at default integration time
exodynTestGlobals.testResult = mbs.GetNodeOutput(nMass, exu.OutputVariableType.
 Displacement)[0]

```

For further examples on ObjectConnectorCoordinate see Examples:

- [sliderCrank3DwithANCFbeltDrive2.py](#)

- [ALEANCF\\_pipe.py](#)
- [ANCFALEtest.py](#)
- [ANCF\\_cantilever\\_test\\_dyn.py](#)
- [ANCF\\_contact\\_circle.py](#)
- [ANCF\\_contact\\_circle2.py](#)
- [ANCF\\_moving\\_rigidbody.py](#)
- [ANCF\\_slidingJoint2D.py](#)
- [ANCF\\_switchingSlidingJoint2D.py](#)
- [ANCF\\_tests2.py](#)
- ...

For further examples on ObjectConnectorCoordinate see TestModels:

- [ACNFslidingAndALEjointTest.py](#)
- [ANCFcontactCircleTest.py](#)
- [ANCFcontactFrictionTest.py](#)
- [ANCFmovingRigidBodyTest.py](#)
- [driveTrainTest.py](#)
- [explicitLieGroupIntegratorPythonTest.py](#)
- [explicitLieGroupIntegratorTest.py](#)
- [explicitLieGroupMBSTest.py](#)
- [fourBarMechanismTest.py](#)
- [geometricallyExactBeam2Dtest.py](#)
- ...

### 7.2.21 ObjectConnectorCoordinateVector

A constraint which constrains the coordinate vectors of two markers Marker[Node|Object|Body]Coordinates attached to nodes or bodies. The marker uses the objects LTG-lists to build the according coordinate mappings.

**Additional information for ObjectConnectorCoordinateVector:**

- The Object has the following types = Connector, Constraint
- Requested marker type = Coordinate
- Short name for Python = **CoordinateVectorConstraint**
- Short name for Python (visualization object) = **VCoordinateVectorConstraint**

The item **ObjectConnectorCoordinateVector** with type = 'ConnectorCoordinateVector' has the following parameters:

| Name            | type                             | size | default value       | description                                                                                                                                                                                            |
|-----------------|----------------------------------|------|---------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| name            | String                           |      | "                   | constraints's unique name                                                                                                                                                                              |
| markerNumbers   | ArrayMarkerIndex                 |      | [ MAXINT, MAX-INT ] | list of markers used in connector                                                                                                                                                                      |
| scalingMarker0  | NumpyMatrix                      |      | Matrix[]            | linear scaling matrix for coordinate vector of marker 0; matrix provided in python numpy format                                                                                                        |
| scalingMarker1  | NumpyMatrix                      |      | Matrix[]            | linear scaling matrix for coordinate vector of marker 1; matrix provided in python numpy format                                                                                                        |
| offset          | NumpyVector                      |      | []                  | offset added to constraint equation; only active, if no userFunction is defined                                                                                                                        |
| velocityLevel   | Bool                             |      | False               | If true: connector constrains velocities (only works for ODE2 coordinates!); offset is used between velocities; in this case, the offsetUserFunction_t is considered and offsetUserFunction is ignored |
| activeConnector | Bool                             |      | True                | flag, which determines, if the connector is active; used to deactivate (temporarily) a connector or constraint                                                                                         |
| visualization   | VObjectConnectorCoordinateVector |      |                     | parameters for visualization of item                                                                                                                                                                   |

The item **VObjectConnectorCoordinateVector** has the following parameters:

| Name  | type   | size | default value     | description                                                              |
|-------|--------|------|-------------------|--------------------------------------------------------------------------|
| show  | Bool   |      | True              | set true, if item is shown in visualization and false if it is not shown |
| color | Float4 |      | [-1.,-1.,-1.,-1.] | RGBA connector color; if R==1, use default color                         |

### 7.2.21.1 DESCRIPTION of ObjectConnectorCoordinateVector:

**Information on input parameters:**

| input parameter | symbol                                                      | description see tables above |
|-----------------|-------------------------------------------------------------|------------------------------|
| markerNumbers   | $[m0, m1]^T$                                                |                              |
| scalingMarker0  | $\mathbf{X}_{m0} \in \mathbb{R}^{n_{ae} \times n_{q_{m0}}}$ |                              |
| scalingMarker1  | $\mathbf{X}_{m1} \in \mathbb{R}^{n_{ae} \times n_{q_{m1}}}$ |                              |
| offset          | $\mathbf{v}_{off} \in \mathbb{R}^{n_{ae}}$                  |                              |

The following output variables are available as OutputVariableType in sensors, Get...Output() and other functions:

| output variable    | symbol             | description                                                                                             |
|--------------------|--------------------|---------------------------------------------------------------------------------------------------------|
| Displacement       | $\Delta\mathbf{q}$ | relative scalar displacement of marker coordinates, not including scaling matrices                      |
| Velocity           | $\Delta\mathbf{v}$ | difference of scalar marker velocity coordinates, not including scaling matrices                        |
| ConstraintEquation | $\mathbf{c}$       | (residuum of) constraint equations                                                                      |
| Force              | $\lambda$          | constraint force vector (vector of Lagrange multipliers), resulting from action of constraint equations |

### 7.2.21.2 Definition of quantities

| intermediate variables               | symbol                                                             | description                                                                                                                       |
|--------------------------------------|--------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------|
| marker m0 coordinate vector          | $\mathbf{q}_{m0} \in \mathbb{R}^{n_{q_{m0}}}$                      | coordinate vector provided by marker $m0$ ; depending on the marker, the coordinates may or may not include reference coordinates |
| marker m1 coordinate vector          | $\mathbf{q}_{m1} \in \mathbb{R}^{n_{q_{m1}}}$                      | coordinate vector provided by marker $m1$ ; depending on the marker, the coordinates may or may not include reference coordinates |
| marker m0 velocity coordinate vector | $\dot{\mathbf{q}}_{m0} \in \mathbb{R}^{n_{q_{m0}}}$                | velocity coordinate vector provided by marker $m0$                                                                                |
| marker m1 velocity coordinate vector | $\dot{\mathbf{q}}_{m1} \in \mathbb{R}^{n_{q_{m1}}}$                | velocity coordinate vector provided by marker $m1$                                                                                |
| number of algebraic equations        | $n_{ae}$                                                           | number of algebraic equations must be same as number of rows in $\mathbf{X}_{m0}$ and $\mathbf{X}_{m1}$                           |
| difference of coordinates            | $\Delta\mathbf{q} = \mathbf{q}_{m1} - \mathbf{q}_{m0}$             | Displacement between marker $m0$ to marker $m1$ coordinates                                                                       |
| difference of velocity coordinates   | $\Delta\mathbf{v} = \dot{\mathbf{q}}_{m1} - \dot{\mathbf{q}}_{m0}$ |                                                                                                                                   |

### 7.2.21.3 Remarks

The number of algebraic equations depends on the number of rows in  $\mathbf{X}_{m0}$ , which must be same as the number of rows in  $\mathbf{X}_{m1}$ . The number of columns in  $\mathbf{X}_{m0}$  must agree with the length of the coordinate vector  $\mathbf{q}_{m0}$  and the number of columns in  $\mathbf{X}_{m1}$  must agree with the length of the coordinate vector  $\mathbf{q}_{m1}$ . If one marker  $k$  is a ground marker (node/object), the length of  $\mathbf{q}_{m,k}$  is zero and also the according matrix  $\mathbf{X}_{m,k}$  has zero size and will not be considered in the computation of the constraint equations.

If the number of rows of  $\mathbf{X}_{m0}$  plus the number of rows of  $\mathbf{X}_{m1}$  is larger than the total number of coordinates ( $\mathbf{q}_{m0}$  and  $\mathbf{q}_{m1}$ ), the algebraic equations are underdetermined and probably not solvable.

#### 7.2.21.4 Connector constraint equations

If `activeConnector = True`, the index 3 algebraic equations

$$\mathbf{c}(\mathbf{q}_{m0}, \mathbf{q}_{m1}) = \mathbf{X}_{m1} \cdot \mathbf{q}_{m1} - \mathbf{X}_{m0} \mathbf{q}_{m0} - \mathbf{v}_{\text{off}} = 0 \quad (7.159)$$

If the `offsetUserFunction` UF is defined,  $\mathbf{c}$  instead becomes ( $t$  is current time)

$$\mathbf{c}(\mathbf{q}_{m0}, \mathbf{q}_{m1}) = \mathbf{X}_{m1} \cdot \mathbf{q}_{m1} - \mathbf{X}_{m0} \mathbf{q}_{m0} - \text{UF}(mbs, t, \mathbf{v}_{\text{off}}) = 0 \quad (7.160)$$

The `activeConnector = True`, index 2 (velocity level) algebraic equation reads

$$\dot{\mathbf{c}}(\dot{\mathbf{q}}_{m0}, \dot{\mathbf{q}}_{m1}) = \mathbf{X}_{m1} \cdot \dot{\mathbf{q}}_{m1} - \mathbf{X}_{m0} \dot{\mathbf{q}}_{m0} - \mathbf{d}_{\text{off}} = 0 \quad (7.161)$$

The vector  $d\mathbf{v}$  in velocity level equations is zero, except if `parameters.velocityLevel = True`, then  $\mathbf{d} = \mathbf{v}_{\text{off}}$ .

If velocity level constraints are active and the velocity level `offsetUserFunction_t`  $\text{UF}_t$  is defined,  $\dot{\mathbf{c}}$  instead becomes ( $t$  is current time)

$$\dot{\mathbf{c}}(\dot{\mathbf{q}}_{m0}, \dot{\mathbf{q}}_{m1}) = \mathbf{X}_{m1} \cdot \dot{\mathbf{q}}_{m1} - \mathbf{X}_{m0} \dot{\mathbf{q}}_{m0} - \text{UF}_t(mbs, t, \mathbf{v}_{\text{off}}) = 0 \quad (7.162)$$

Note that the index 2 equations are used, if the solver uses index 2 formulation OR if the flag `parameters.velocityLevel = True` (or both). The user functions include dependency on time  $t$ , but this time dependency is not respected in the computation of initial accelerations. Therefore, it is recommended that UF and  $\text{UF}_t$  does not include initial accelerations.

If `activeConnector = False`, the (index 1) algebraic equation reads for ALL cases:

$$\mathbf{c}(\lambda) = \lambda = 0 \quad (7.163)$$

## 7.2.22 ObjectConnectorRollingDiscPenalty

A (flexible) connector representing a rolling rigid disc (marker 1) on a flat surface (marker 0, ground body, not moving) in global  $x$ - $y$  plane. The connector is based on a penalty formulation and adds friction and slipping. The constraints works for discs as long as the disc axis and the plane normal vector are not parallel. Parameters may need to be adjusted for better convergence (e.g., dryFrictionProportionalZone). The formulation is still under development and needs further testing. Note that the rolling body must have the reference point at the center of the disc.

**Additional information for ObjectConnectorRollingDiscPenalty:**

- The Object has the following types = Connector
- Requested marker type = Position + Orientation
- Requested node type = GenericData
- **Short name for Python = RollingDiscPenalty**
- **Short name for Python (visualization object) = VRollingDiscPenalty**

The item **ObjectConnectorRollingDiscPenalty** with type = 'ConnectorRollingDiscPenalty' has the following parameters:

| Name                        | type             | size | default value       | description                                                                                                                                                                                                                                                                                                                                         |
|-----------------------------|------------------|------|---------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| name                        | String           |      | "                   | constraints's unique name                                                                                                                                                                                                                                                                                                                           |
| markerNumbers               | ArrayMarkerIndex | 2    | [ MAXINT, MAX-INT ] | list of markers used in connector; $m_0$ represents the ground, which can undergo translations but not rotations, and $m_1$ represents the rolling body, which has its reference point (=local position [0,0,0]) at the disc center point                                                                                                           |
| nodeNumber                  | NodeIndex        |      | MAXINT              | node number of a NodeGenericData (size=3) for 3 dataCoordinates                                                                                                                                                                                                                                                                                     |
| dryFrictionAngle            | Real             |      | 0.                  | angle [SI:1 (rad)] which defines a rotation of the local tangential coordinates dry friction; this allows to model Mecanum wheels with specified roll angle                                                                                                                                                                                         |
| contactStiffness            | Real             |      | 0.                  | normal contact stiffness [SI:N/m]                                                                                                                                                                                                                                                                                                                   |
| contactDamping              | Real             |      | 0.                  | normal contact damping [SI:N/(m s)]                                                                                                                                                                                                                                                                                                                 |
| dryFriction                 | Vector2D         |      | [0,0]               | dry friction coefficients [SI:1] in local marker 1 joint $J_1$ coordinates; if $\alpha_t == 0$ , lateral direction $l = x$ and forward direction $f = y$ ; assuming a normal force $f_n$ , the local friction force can be computed as<br>$\begin{bmatrix} f_{t,x} \\ f_{t,y} \end{bmatrix} = \begin{bmatrix} \mu_x f_n \\ \mu_y f_n \end{bmatrix}$ |
| dryFrictionProportionalZone | Real             |      | 0.                  | limit velocity [m/s] up to which the friction is proportional to velocity (for regularization / avoid numerical oscillations)                                                                                                                                                                                                                       |
| rollingFrictionViscous      | Real             |      | 0.                  | rolling friction [SI:1], which acts against the velocity of the trail on ground and leads to a force proportional to the contact normal force; currently, only implemented for disc axis parallel to ground!                                                                                                                                        |

|                 |                                    |         |                                                                                                                                       |
|-----------------|------------------------------------|---------|---------------------------------------------------------------------------------------------------------------------------------------|
| activeConnector | Bool                               | True    | flag, which determines, if the connector is active; used to deactivate (temporarily) a connector or constraint                        |
| discRadius      | Real                               | 0       | defines the disc radius                                                                                                               |
| planeNormal     | Vector3D                           | [0,0,1] | normal to the contact / rolling plane (ground); Currently, this is not co-rotating with the ground body, but will do so in the future |
| visualization   | VObjectConnectorRollingDiscPenalty |         | parameters for visualization of item                                                                                                  |

The item VObjectConnectorRollingDiscPenalty has the following parameters:

| Name      | type   | size | default value     | description                                                              |
|-----------|--------|------|-------------------|--------------------------------------------------------------------------|
| show      | Bool   |      | True              | set true, if item is shown in visualization and false if it is not shown |
| discWidth | float  |      | 0.1               | width of disc for drawing                                                |
| color     | Float4 |      | [-1.,-1.,-1.,-1.] | RGBA connector color; if R==1, use default color                         |

### 7.2.22.1 DESCRIPTION of ObjectConnectorRollingDiscPenalty:

#### Information on input parameters:

| input parameter             | symbol                                              | description see tables above |
|-----------------------------|-----------------------------------------------------|------------------------------|
| markerNumbers               | $[m_0, m_1]^T$                                      |                              |
| nodeNumber                  | $n_d$                                               |                              |
| dryFrictionAngle            | $\alpha_t$                                          |                              |
| contactStiffness            | $k_c$                                               |                              |
| contactDamping              | $d_c$                                               |                              |
| dryFriction                 | $[\mu_x, \mu_y]^T$                                  |                              |
| dryFrictionProportionalZone | $v_\mu$                                             |                              |
| rollingFrictionViscous      | $\mu_r$                                             |                              |
| planeNormal                 | ${}^0\mathbf{v}_{PN}$ , $ {}^0\mathbf{v}_{PN}  = 1$ |                              |

The following output variables are available as OutputVariableType in sensors, Get...Output() and other functions:

| output variable | symbol             | description                                                                                                                                |
|-----------------|--------------------|--------------------------------------------------------------------------------------------------------------------------------------------|
| Position        | ${}^0\mathbf{p}_G$ | current global position of contact point between rolling disc and ground                                                                   |
| VelocityLocal   | ${}^D\mathbf{v}_G$ | current velocity of the trail (contact) point in disc coordinates; this is the velocity with which the contact moves over the ground plane |

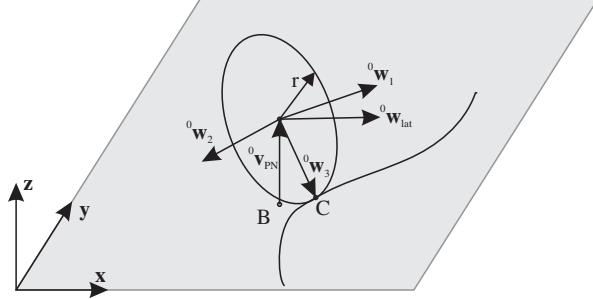
|            |                     |                                                                                                                                                             |
|------------|---------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------|
| ForceLocal | ${}^J_1 \mathbf{f}$ | disc-ground force in special marker 1 joint coordinates, $f_0$ being the lateral force, $f_1$ being the longitudinal force and $f_2$ being the normal force |
|------------|---------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------|

### 7.2.22.2 Definition of quantities

| intermediate variables     | symbol                                                                                                         | description                                                                                                                                                                                                               |
|----------------------------|----------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| marker m0 position         | ${}^0 \mathbf{p}_{m0}$                                                                                         | current global position which is provided by marker m0, any ground reference position; currently unused                                                                                                                   |
| marker m0 orientation      | ${}^{0,m0} \mathbf{A}$                                                                                         | current rotation matrix provided by marker m0; currently unused                                                                                                                                                           |
| marker m1 position         | ${}^0 \mathbf{p}_{m1}$                                                                                         | center of disc                                                                                                                                                                                                            |
| marker m1 orientation      | ${}^{0,m1} \mathbf{A}$                                                                                         | current rotation matrix provided by marker m1                                                                                                                                                                             |
| data coordinates           | $\mathbf{x} = [x_0, x_1, x_2]^T$                                                                               | data coordinates for $[x_0, x_1]$ : hold the sliding velocity in lateral and longitudinal direction of last discontinuous iteration; $x_2$ : represents gap of last discontinuous iteration (in contact normal direction) |
| marker m1 velocity         | ${}^0 \mathbf{v}_{m1}$                                                                                         | accordingly                                                                                                                                                                                                               |
| marker m1 angular velocity | ${}^0 \boldsymbol{\omega}_{m1}$                                                                                | current angular velocity vector provided by marker m1                                                                                                                                                                     |
| ground normal vector       | ${}^0 \mathbf{v}_{PN}$                                                                                         | normalized normal vector to the (moving, but not rotating) ground, by default [0,0,1]                                                                                                                                     |
| ground position B          | ${}^0 \mathbf{p}_B$                                                                                            | disc center point projected on ground (normal projection)                                                                                                                                                                 |
| ground position C          | ${}^0 \mathbf{p}_C$                                                                                            | contact point of disc with ground                                                                                                                                                                                         |
| ground velocity C          | ${}^0 \mathbf{v}_C$                                                                                            | velocity of disc at ground contact point (must be zero at end of iteration)                                                                                                                                               |
| wheel axis vector          | ${}^0 \mathbf{w}_1 = {}^{0,m1} \mathbf{A} \cdot [1, 0, 0]^T$                                                   | normalized disc axis vector, currently $[1, 0, 0]^T$ in local coordinates                                                                                                                                                 |
| longitudinal vector        | ${}^0 \mathbf{w}_2$                                                                                            | vector in longitudinal (motion) direction                                                                                                                                                                                 |
| lateral vector             | ${}^0 \mathbf{w}_l = {}^0 \mathbf{v}_{PN} \times {}^0 \mathbf{w}_2 = [-\mathbf{w}_{2,y}, \mathbf{w}_{2,x}, 0]$ | vector in lateral direction, lies in ground plane                                                                                                                                                                         |
| contact point vector       | ${}^0 \mathbf{w}_3$                                                                                            | normalized vector from disc center point in direction of contact point C                                                                                                                                                  |
| connector forces           | ${}^J_1 \mathbf{f} = [f_{t,x}, f_{t,y}, f_n]^T$                                                                | joint force vector at contact point in joint 1 coordinates: x=lateral direction, y=longitudinal direction, z=plane normal (contact normal)                                                                                |

### 7.2.22.3 Geometric relations

The main geometrical setup is shown in the following figure:



First, the contact point  ${}^0\mathbf{p}_C$  must be computed. With the helper vector,

$${}^0\mathbf{x} = {}^0\mathbf{w}_1 \times {}^0\mathbf{v}_{PN} \quad (7.164)$$

we create a disc coordinate system  $({}^0\mathbf{w}_1, {}^0\mathbf{w}_2, {}^0\mathbf{w}_3)$ , representing the longitudinal direction,

$${}^0\mathbf{w}_2 = \frac{1}{|{}^0\mathbf{x}|} {}^0\mathbf{x} \quad (7.165)$$

and the vector to the contact point,

$${}^0\mathbf{w}_3 = {}^0\mathbf{w}_1 \times {}^0\mathbf{w}_2 \quad (7.166)$$

The contact point can be computed from

$${}^0\mathbf{p}_C = {}^0\mathbf{p}_{m1} + r \cdot {}^0\mathbf{w}_3 \quad (7.167)$$

The velocity of the contact point at the disc is computed from,

$${}^0\mathbf{v}_C = {}^0\mathbf{v}_{m1} + {}^0\omega_{m1} \times (r \cdot {}^0\mathbf{w}_3) \quad (7.168)$$

A second coordinate system is defined by  $({}^0\mathbf{w}_{lat}, {}^0\mathbf{w}_2, {}^0\mathbf{v}_{PN})$ , using

$${}^0\mathbf{w}_{lat} = {}^0\mathbf{v}_{PN} \times {}^0\mathbf{w}_2 \quad (7.169)$$

Note that **in the case that** the rolling axis  ${}^0\mathbf{w}_1$  lies in the rolling plane, we obtain the special case  ${}^0\mathbf{w}_{lat} = {}^0\mathbf{w}_1$  and  ${}^0\mathbf{w}_1 = -{}^0\mathbf{v}_{PN}$ .

#### 7.2.22.4 Computation of normal and tangential forces

The connector forces at the contact point C are computed as follows. The normal contact force reads

$$f_n = (k_c \cdot {}^0\mathbf{p}_C + d_c \cdot {}^0\mathbf{v}_C)^T {}^0\mathbf{v}_{PN} \quad (7.170)$$

The inplane velocity in joint coordinates,

$${}^{J1}\mathbf{v}_t = [{}^0\mathbf{v}_C^T \ {}^0\mathbf{w}_{lat}, \ {}^0\mathbf{v}_C^T \ {}^0\mathbf{w}_2]^T, \quad (7.171)$$

is used for the computation of tangential forces,

$${}^{J1}\mathbf{f}_t = [f_{t,x}, f_{t,y}]^T = {}^{J1}\boldsymbol{\mu} \cdot (\phi(|\mathbf{v}_t|, v_\mu) \cdot f_n \cdot {}^{J1}\mathbf{e}_t), \quad (7.172)$$

with the regularization function, see Geradin and Cardona [12] (Sec. 7.9.3):

$$\phi(v, v_\mu) = \begin{cases} \left(2 - \frac{v}{v_\mu}\right) \frac{v}{v_\mu} & \text{if } v \leq v_\mu \\ 1 & \text{if } v > v_\mu \end{cases} \quad (7.173)$$

The direction of tangential slip is given as

$${}^J_1 \mathbf{e}_t = \begin{cases} \frac{{}^J_1 \mathbf{v}_t}{|\mathbf{v}_t|} & \text{if } |\mathbf{v}_t| > 0 \\ \begin{bmatrix} 0 \\ 0 \end{bmatrix} & \text{else} \end{cases} \quad (7.174)$$

The friction coefficient matrix  ${}^J_1 \boldsymbol{\mu}$  is given in joint coordinates and computed from

$${}^J_1 \boldsymbol{\mu} = \begin{bmatrix} \mu_x & 0 \\ 0 & \mu_y \end{bmatrix} \quad (7.175)$$

where for isotropic behaviour of surface and wheel, it will give a diagonal matrix with the friction coefficient in the diagonal. In case that the dry friction angle  $\alpha_t$  is not zero, the  $\boldsymbol{\mu}$  changes to

$${}^J_1 \boldsymbol{\mu} = \begin{bmatrix} \cos(\alpha_t) & \sin(\alpha_t) \\ -\sin(\alpha_t) & \cos(\alpha_t) \end{bmatrix} \begin{bmatrix} \mu_x & 0 \\ 0 & \mu_y \end{bmatrix} \begin{bmatrix} \cos(\alpha_t) & -\sin(\alpha_t) \\ \sin(\alpha_t) & \cos(\alpha_t) \end{bmatrix} \quad (7.176)$$

#### 7.2.22.5 Connector forces

Finally, the connector forces read in joint coordinates

$${}^J_1 \mathbf{f} = \begin{bmatrix} f_{t,x} \\ f_{t,y} \\ f_n \end{bmatrix} \quad (7.177)$$

and in global coordinates, they are computed from

$${}^0 \mathbf{f} = f_{t,x} {}^0 \mathbf{w}_1 + f_{t,y} {}^0 \mathbf{w}_2 + f_n {}^0 \mathbf{v}_{PN} \quad (7.178)$$

The moment caused by the contact forces are given as

$${}^0 \mathbf{f} = (\mathbf{r} \cdot {}^0 \mathbf{w}_3) \times {}^0 \mathbf{f} \quad (7.179)$$

Note that if `activeConnector = False`, we replace Eq. (7.177) with

$${}^J_1 \mathbf{f} = \mathbf{0} \quad (7.180)$$

For further examples on ObjectConnectorRollingDiscPenalty see TestModels:

- [`carRollingDiscTest.py`](#)
- [`mecanumWheelRollingDiscTest.py`](#)
- [`rollingCoinPenaltyTest.py`](#)

### 7.2.23 ObjectContactCoordinate

A penalty-based contact condition for one coordinate; the contact gap  $g$  is defined as  $g = marker.value[1] - marker.value[0] - offset$ ; the contact force  $f_c$  is zero for  $gap > 0$  and otherwise computed from  $f_c = g * contactStiffness + \dot{g} * contactDamping$ ; during Newton iterations, the contact force is activated only, if  $dataCoordinate[0] \leq 0$ ;  $dataCoordinate$  is set equal to  $gap$  in nonlinear iterations, but not modified in Newton iterations.

**Additional information for ObjectContactCoordinate:**

- The Object has the following types = Connector
- Requested marker type = Coordinate
- Requested node type = GenericData

The item **ObjectContactCoordinate** with type = 'ContactCoordinate' has the following parameters:

| Name             | type                     | size | default value       | description                                                                                                                                |
|------------------|--------------------------|------|---------------------|--------------------------------------------------------------------------------------------------------------------------------------------|
| name             | String                   |      | "                   | connector's unique name                                                                                                                    |
| markerNumbers    | ArrayMarkerIndex         |      | [ MAXINT, MAX-INT ] | markers define contact gap                                                                                                                 |
| nodeNumber       | NodeIndex                |      | MAXINT              | node number of a NodeGenericData for 1 dataCoordinate (used for active set strategy ==> holds the gap of the last discontinuous iteration) |
| contactStiffness | UReal                    |      | 0.                  | contact (penalty) stiffness [SI:N/m]; acts only upon penetration                                                                           |
| contactDamping   | UReal                    |      | 0.                  | contact damping [SI:N/(m s)]; acts only upon penetration                                                                                   |
| offset           | Real                     |      | 0.                  | offset [SI:m] of contact                                                                                                                   |
| activeConnector  | Bool                     |      | True                | flag, which determines, if the connector is active; used to deactivate (temporarily) a connector or constraint                             |
| visualization    | VObjectContactCoordinate |      |                     | parameters for visualization of item                                                                                                       |

The item **VObjectContactCoordinate** has the following parameters:

| Name     | type   | size | default value     | description                                                                               |
|----------|--------|------|-------------------|-------------------------------------------------------------------------------------------|
| show     | Bool   |      | True              | set true, if item is shown in visualization and false if it is not shown                  |
| drawSize | float  |      | -1.               | drawing size = diameter of spring; size == -1.f means that default connector size is used |
| color    | Float4 |      | [-1.,-1.,-1.,-1.] | RGBA connector color; if R== -1, use default color                                        |

### **7.2.23.1 DESCRIPTION of ObjectContactCoordinate:**

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For further examples on ObjectContactCoordinate see Examples:

- [ANCF\\_contact\\_circle.py](#)
- [ANCF\\_contact\\_circle2.py](#)

For further examples on ObjectContactCoordinate see TestModels:

- [ANCFcontactCircleTest.py](#)

## 7.2.24 ObjectContactCircleCable2D

A very specialized penalty-based contact condition between a 2D circle (=marker0, any Position-marker) on a body and an ANCFCable2DShape (=marker1, Marker: BodyCable2DShape), in xy-plane; a node NodeGenericData is required with the number of coordinates according to the number of contact segments; the contact gap  $g$  is integrated (piecewise linear) along the cable and circle; the contact force  $f_c$  is zero for  $gap > 0$  and otherwise computed from  $f_c = g * contactStiffness + \dot{g} * contactDamping$ ; during Newton iterations, the contact force is activated only, if  $dataCoordinate[0] \leq 0$ ; dataCoordinate is set equal to gap in nonlinear iterations, but not modified in Newton iterations.

**Additional information for ObjectContactCircleCable2D:**

- The Object has the following types = Connector
- Requested marker type = \_None
- Requested node type = GenericData

The item **ObjectContactCircleCable2D** with type = 'ContactCircleCable2D' has the following parameters:

| Name                    | type                        | size | default value       | description                                                                                                                                                                                    |
|-------------------------|-----------------------------|------|---------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| name                    | String                      |      | "                   | connector's unique name                                                                                                                                                                        |
| markerNumbers           | ArrayMarkerIndex            |      | [ MAXINT, MAX-INT ] | markers define contact gap                                                                                                                                                                     |
| nodeNumber              | NodeIndex                   |      | MAXINT              | node number of a NodeGenericData for nSegments dataCoordinates (used for active set strategy ==> hold the gap of the last discontinuous iteration and the friction state)                      |
| numberOfContactSegments | Index                       |      | 3                   | number of linear contact segments to determine contact; each segment is a line and is associated to a data (history) variable; must be same as in according marker                             |
| contactStiffness        | UReal                       |      | 0.                  | contact (penalty) stiffness [SI:N/m/(contact segment)]; the stiffness is per contact segment; specific contact forces (per length) $f_N$ act in contact normal direction only upon penetration |
| contactDamping          | UReal                       |      | 0.                  | contact damping [SI:N/(m s)/(contact segment)]; the damping is per contact segment; acts in contact normal direction only upon penetration                                                     |
| circleRadius            | UReal                       |      | 0.                  | radius [SI:m] of contact circle                                                                                                                                                                |
| offset                  | Real                        |      | 0.                  | offset [SI:m] of contact, e.g. to include thickness of cable element                                                                                                                           |
| activeConnector         | Bool                        |      | True                | flag, which determines, if the connector is active; used to deactivate (temporarily) a connector or constraint                                                                                 |
| visualization           | VObjectContactCircleCable2D |      |                     | parameters for visualization of item                                                                                                                                                           |

The item VObjectContactCircleCable2D has the following parameters:

| <b>Name</b> | <b>type</b> | <b>size</b> | <b>default value</b> | <b>description</b>                                                                        |
|-------------|-------------|-------------|----------------------|-------------------------------------------------------------------------------------------|
| show        | Bool        |             | True                 | set true, if item is shown in visualization and false if it is not shown                  |
| drawSize    | float       |             | -1.                  | drawing size = diameter of spring; size == -1.f means that default connector size is used |
| color       | Float4      |             | [-1.,-1.,-1.,-1.]    | RGBA connector color; if R== -1, use default color                                        |

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#### 7.2.24.1 DESCRIPTION of ObjectContactCircleCable2D:

#### 7.2.24.2 Connector equations

Geometry and equations are very similar to ObjectContactFrictionCircleCable2D, while friction is not used and no torque is transferred to the circle object.

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For further examples on ObjectContactCircleCable2D see Examples:

- [ANCF\\_contact\\_circle.py](#)
- [ANCF\\_contact\\_circle2.py](#)
- [ANCF\\_moving\\_rigidbody.py](#)
- [ANCF\\_slidingJoint2D.py](#)

For further examples on ObjectContactCircleCable2D see TestModels:

- [ACNFslidingAndALEjointTest.py](#)
- [ANCFcontactCircleTest.py](#)
- [ANCFmovingRigidBodyTest.py](#)

## 7.2.25 ObjectContactFrictionCircleCable2D

A very specialized penalty-based contact/friction condition between a 2D circle in the local x/y plane (=marker0, a Rigid-Body Marker) on a body and an ANCFCircle2DShape (=marker1, Marker: BodyCircle2DShape), in xy-plane; a node NodeGenericData is required with  $3 \times (\text{number of contact segments})$  – containing per segment: [contact gap, stick/slip (stick=1), last friction position]; Note that friction can be only considered in the dynamic case where velocities are available, while it is inactive in the static case.

**Additional information for ObjectContactFrictionCircleCable2D:**

- The Object has the following types = Connector
- Requested marker type = \_None
- Requested node type = GenericData

The item **ObjectContactFrictionCircleCable2D** with type = 'ContactFrictionCircleCable2D' has the following parameters:

| Name                    | type             | size | default value       | description                                                                                                                                                                                                                               |
|-------------------------|------------------|------|---------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| name                    | String           |      | "                   | connector's unique name                                                                                                                                                                                                                   |
| markerNumbers           | ArrayMarkerIndex |      | [ MAXINT, MAX-INT ] | markers define contact gap                                                                                                                                                                                                                |
| nodeNumber              | NodeIndex        |      | MAXINT              | node number of a NodeGenericData with $3 \times n_{cs}$ dataCoordinates (used for active set strategy → hold the gap of the last discontinuous iteration and the friction state)                                                          |
| numberOfContactSegments | PInt             |      | 3                   | number of linear contact segments to determine contact; each segment is a line and is associated to a data (history) variable; must be same as in according marker                                                                        |
| contactStiffness        | UReal            |      | 0.                  | contact (penalty) stiffness [SI:N/m/(contact segment)]; the stiffness is per contact segment; specific contact forces (per length) $f_n$ act in contact normal direction only upon penetration                                            |
| contactDamping          | UReal            |      | 0.                  | contact damping [SI:N/(m s)/(contact segment)]; the damping is per contact segment; acts in contact normal direction only upon penetration                                                                                                |
| frictionVelocityPenalty | UReal            |      | 0.                  | velocity dependent penalty coefficient for friction [SI:N/(m s)/(contact segment)]; the coefficient causes tangential (contact) forces against relative tangential velocities in the contact area                                         |
| frictionStiffness       | UReal            |      | 0.                  | CURRENTLY NOT IMPLEMENTED: displacement dependent penalty/stiffness coefficient for friction [SI:N/m/(contact segment)]; the coefficient causes tangential (contact) forces against relative tangential displacements in the contact area |
| frictionCoefficient     | UReal            |      | 0.                  | friction coefficient [SI: 1]; tangential specific friction forces (per length) $f_t$ must fulfill the condition $f_t \leq \mu f_n$                                                                                                        |

|                 |                                     |      |                                                                                                                |
|-----------------|-------------------------------------|------|----------------------------------------------------------------------------------------------------------------|
| circleRadius    | UReal                               | 0.   | radius [SI:m] of contact circle                                                                                |
| offset          | Real                                | 0.   | offset [SI:m] of contact, e.g. to include thickness of cable element                                           |
| activeConnector | Bool                                | True | flag, which determines, if the connector is active; used to deactivate (temporarily) a connector or constraint |
| visualization   | VObjectContactFrictionCircleCable2D |      | parameters for visualization of item                                                                           |

The item VObjectContactFrictionCircleCable2D has the following parameters:

| Name     | type   | size | default value     | description                                                                               |
|----------|--------|------|-------------------|-------------------------------------------------------------------------------------------|
| show     | Bool   |      | True              | set true, if item is shown in visualization and false if it is not shown                  |
| drawSize | float  |      | -1.               | drawing size = diameter of spring; size == -1.f means that default connector size is used |
| color    | Float4 |      | [-1.,-1.,-1.,-1.] | RGBA connector color; if R==1, use default color                                          |

### 7.2.25.1 DESCRIPTION of ObjectContactFrictionCircleCable2D:

#### Information on input parameters:

| input parameter         | symbol         | description see tables above |
|-------------------------|----------------|------------------------------|
| markerNumbers           | $[m_0, m_1]^T$ |                              |
| nodeNumber              | $n_g$          |                              |
| numberOfContactSegments | $n_{cs}$       |                              |
| contactStiffness        | $k_c$          |                              |
| contactDamping          | $d_c$          |                              |
| frictionVelocityPenalty | $\mu_v$        |                              |
| frictionStiffness       | $\mu_k$        |                              |
| frictionCoefficient     | $\mu$          |                              |
| circleRadius            | $r$            |                              |
| offset                  | $h_o$          |                              |

### 7.2.25.2 Definition of quantities

| intermediate variables | symbol                                       | description                                                    |
|------------------------|----------------------------------------------|----------------------------------------------------------------|
| marker m0 position     | ${}^0\mathbf{p}_{m0}$                        | represents current global position of the circle's centerpoint |
| marker m0 velocity     | ${}^0\mathbf{v}_{m0}$                        | current global velocity which is provided by marker m0         |
| marker m1              |                                              | represents the 2D ANCF cable                                   |
| data node              | $\mathbf{x} = [x_i, \dots, x_{3n_{cs}-1}]^T$ | coordinates of node with node number $n_{GD}$                  |

|                                                             |                                                |                                                                                                                                                                                          |
|-------------------------------------------------------------|------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| data coordinate with previous gap for segment $i$           | $x_i$ , with $i \in [0, n_{cs} - 1]$           | This data coordinate holds the gap of the last converged Newton iterations used for active set strategy. If this $x_i < 0$ , then contact is used in the current Newton iteration.       |
| data coordinate with previous tangent force for segment $i$ | $x_{n_{cs}+i}$ , with $i \in [0, n_{cs} - 1]$  | This data coordinate holds the tangent (friction) force of the last converged Newton iterations used for active set strategy. Currently not implemented.                                 |
| data coordinate with previous point of contact $i$          | $x_{2n_{cs}+i}$ , with $i \in [0, n_{cs} - 1]$ | This data coordinate holds relative point of contact of the last converged Newton iterations used for active set strategy for static computation of friction. Currently not implemented. |
| shortest distance to segment $s_i$                          | $\mathbf{d}_{g,i}$                             | shortest distance of center of circle to contact segment, considering the endpoint of the segment                                                                                        |

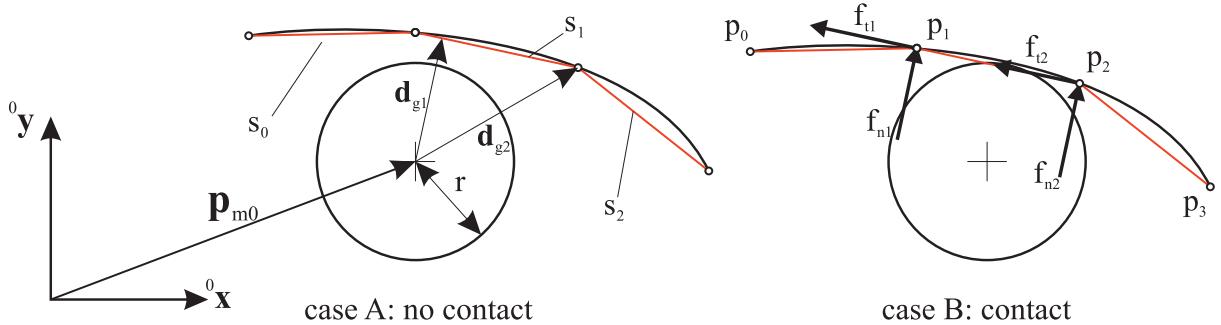


Figure 7.2: Sketch of cable, contact segments and circle; case A shows contact with  $|\mathbf{d}_{g1}| > r$ , while case B shows contact with  $|\mathbf{d}_{g1}| \leq r$ ; the shortest distance vector  $\mathbf{d}_{g1}$  is related to segment  $s_1$  (which is perpendicular to the segment line) and  $\mathbf{d}_{g2}$  is the shortest distance to the end point of segment  $s_2$ , not being perpendicular.

### 7.2.25.3 Connector forces

The cable is discretized by separating into segments  $s_i$ , located between points  $p_i$  and  $p_{i+1}$ . In order to compute the gap function for a line segment, the shortest distance of one line segment with points  $\mathbf{p}_i$ ,  $\mathbf{p}_{i+1}$ , the circle centerpoint given by the marker  $\mathbf{p}_{m0}$  are computed, all in the global coordinates system (0), including edge points of every segment.

With the intermediate quantities

$$\mathbf{v}_s = \mathbf{p}_{i+1} - \mathbf{p}_i, \quad \mathbf{v}_p = \mathbf{p}_{m0} - \mathbf{p}_i, \quad n = \mathbf{v}_s^T \mathbf{v}_p, \quad d = \mathbf{v}_s^T \mathbf{v}_s \quad (7.181)$$

and assuming that  $d \neq 0$  (otherwise the two segment points would be identical and the shortest distance would be  $d_g = |\mathbf{v}_p|$ ), we find the relative position  $\rho$  of the shortest (projected) point on the segment, which runs from 0 to 1 if lying on the segment, as

$$\rho = \frac{n}{d} \quad (7.182)$$

We distinguish 3 cases (see also Fig. 7.2 for cases 1 and 2):

1. If  $\rho \leq 0$ , the shortest distance would be the distance to point  $\mathbf{p}_p = \mathbf{p}_i$ , reading

$$d_g = |\mathbf{p}_{m0} - \mathbf{p}_i| \quad (\rho \leq 0) \quad (7.183)$$

2. If  $\rho \geq 1$ , the shortest distance would be the distance to point  $\mathbf{p}_p = \mathbf{p}_{i+1}$ , reading

$$d_g = |\mathbf{p}_{m0} - \mathbf{p}_{i+1}| \quad (\rho \geq 1) \quad (7.184)$$

3. Finally, if  $0 < \rho < 1$ , then the shortest distance has a projected point somewhere in between with the projected point

$$\mathbf{p}_p = \mathbf{p}_i + \rho \cdot \mathbf{v}_s \quad (7.185)$$

and the distance

$$d_g = |\mathbf{d}_g| = \sqrt{\mathbf{v}_p^T \mathbf{v}_p - (n^2)/d} \quad (7.186)$$

The contact gap for a specific point is in general defined as

$$g = d_g - r - h_0 . \quad (7.187)$$

in which  $d_g = |\mathbf{d}_g|$  is the shortest distance from the circle's center point to the cable.

The shortest distance vector for every segment results from the projected point  $\mathbf{p}_p$  of the above mentioned cases, see also Fig. 7.2, with the relation

$$\mathbf{d}_g = \mathbf{p}_{m0} - \mathbf{p}_p . \quad (7.188)$$

We define the contact normal vector  $\mathbf{n}$  and tangential vector  $\mathbf{t}$  as

$$\mathbf{n} = \frac{1}{|\mathbf{d}_g|} \mathbf{d}_g, \quad \mathbf{t} = [-n_1, n_0]^T \quad (7.189)$$

With that help, we can also define a gap velocity  $v_n$  ( $\neq \dot{g}$ ) using the formula

$$v_n = (\dot{\mathbf{p}}_p - \dot{\mathbf{p}}_{m0}) \mathbf{n} \quad (7.190)$$

In a similar, we may compute a tangential velocity, reading

$$v_t = (\dot{\mathbf{p}}_p - \dot{\mathbf{p}}_{m0}) \mathbf{t} \quad (7.191)$$

The contact force  $f_n$  is zero for  $g > 0$  and otherwise computed from

$$f_n = k_c \cdot g + d_c \cdot v_n \quad (7.192)$$

Friction forces are computed in the same manner, but only in the dynamic case, using the linear friction force

$$f_t^{(lin)} = \mu_v \cdot v_t , \quad (7.193)$$

which leads to the friction force due to limitation by the friction coefficient,

$$f_t = \begin{cases} f_t^{(lin)}, & \text{if } |f_t^{(lin)}| \leq \mu \cdot |f_c| \\ \mu \cdot |f_n| \cdot \text{Sign}(f_t^{(lin)}), & \text{else} \end{cases} \quad (7.194)$$

The contact force vector for one segment  $s_i$  then reads

$$\mathbf{f}_{s_i} = f_n \cdot \mathbf{n} + f_t \cdot \mathbf{t} \quad (7.195)$$

If **activeConnector = True**, contact forces  $\mathbf{f}_i$  with  $i \in [0, n_{cs}]$  are applied at the points  $p_i$ , and they are computed for every contact segments (i.e., two segments may contribute to contact forces of one point). For every contact computation, first all contact forces are set to zero. If a single segment  $s_i$  shows a gap  $g < 0$ , see Fig. 7.2(right), contact forces are summed up according to

$$\begin{aligned}\mathbf{f}_i &+ = (1 - \rho) \cdot \mathbf{f}_{s_i} \\ \mathbf{f}_{i+1} &+ = \rho \cdot \mathbf{f}_{s_i}\end{aligned}\quad (7.196)$$

These forces are then applied to the `ObjectANCF Cable2D` element as point loads via a position jacobian (using the according access function), for details see the C++ implementation.

The forces on the circle marker  $m0$  are computed as the total sum of all segment contact forces,

$$\mathbf{f}_{m0} = \sum_{s_i} \mathbf{f}_{s_i} \quad (7.197)$$

and additional torques on the circle's rotation simply follow from

$$\tau_{m0} = \sum_{s_i} r \cdot f_{t_{s_i}} . \quad (7.198)$$

During Newton iterations, the contact forces for segment  $s_i$  are considered only, if  $x_i \leq 0$ . The dataCoordinate  $x_i$  is not modified during Newton iterations, but computed during the DiscontinuousIteration, see Fig. 11.4 in the Solver description. Note that currently friction can be only considered in the dynamic case where velocities are available, while it is inactive in the static case.

If **activeConnector = False**, all contact and friction forces on the cable and the force and torque on the circle's marker are set to zero.

For further examples on `ObjectContactFrictionCircleCable2D` see Examples:

- [sliderCrank3DwithANCFbeltDrive.py](#)
- [sliderCrank3DwithANCFbeltDrive2.py](#)

For further examples on `ObjectContactFrictionCircleCable2D` see TestModels:

- [ACNFslidingAndALEjointTest.py](#)
- [ANCFcontactFrictionTest.py](#)
- [ANCFmovingRigidBodyTest.py](#)

## 7.2.26 ObjectJointGeneric

A generic joint in 3D; constrains components of the absolute position and rotations of two points given by PointMarkers or RigidMarkers; an additional local rotation can be used to define three rotation axes and/or sliding axes

**Additional information for ObjectJointGeneric:**

- The Object has the following types = Connector, Constraint
- Requested marker type = Position + Orientation
- **Short name for Python** = **GenericJoint**
- **Short name for Python (visualization object)** = **VGenericJoint**

The item **ObjectJointGeneric** with type = 'JointGeneric' has the following parameters:

| Name                         | type                                | size | default value               | description                                                                                                                                                                                            |
|------------------------------|-------------------------------------|------|-----------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| name                         | String                              |      | "                           | constraints's unique name                                                                                                                                                                              |
| markerNumbers                | ArrayMarkerIndex                    | 2    | [ MAXINT, MAX-INT ]         | list of markers used in connector                                                                                                                                                                      |
| constrainedAxes              | ArrayIndex                          | 6    | [1,1,1,1,1]                 | flag, which determines which translation (0,1,2) and rotation (3,4,5) axes are constrained; for $j_i$ , two values are possible: 0=free axis, 1=constrained axis                                       |
| rotationMarker0              | Matrix3D                            |      | [[1,0,0], [0,1,0], [0,0,1]] | local rotation matrix for marker $m_0$ ; translation and rotation axes for marker $m_0$ are defined in the local body coordinate system and additionally transformed by rotation-Marker0               |
| rotationMarker1              | Matrix3D                            |      | [[1,0,0], [0,1,0], [0,0,1]] | local rotation matrix for marker $m_1$ ; translation and rotation axes for marker $m_1$ are defined in the local body coordinate system and additionally transformed by rotation-Marker1               |
| activeConnector              | Bool                                |      | True                        | flag, which determines, if the connector is active; used to deactivate (temporarily) a connector or constraint                                                                                         |
| offsetUserFunctionParameters | Vector6D                            |      | [0.,0.,0.,0.,0.,0.]         | vector of 6 parameters for joint's offsetUserFunction                                                                                                                                                  |
| offsetUserFunction           | PyFunctionVector6DmbsScalarVector6D | 0    |                             | A python function which defines the time-dependent (fixed) offset of translation (indices 0,1,2) and rotation (indices 3,4,5) joint coordinates with parameters (mbs, t, offsetUserFunctionParameters) |
| offsetUserFunction_t         | PyFunctionVector6DmbsScalarVector6D | 0    |                             | (NOT IMPLEMENTED YET)time derivative of offsetUserFunction using the same parameters                                                                                                                   |
| visualization                | VObjectJointGeneric                 |      |                             | parameters for visualization of item                                                                                                                                                                   |

The item VObjectJointGeneric has the following parameters:

| <b>Name</b> | <b>type</b> | <b>size</b> | <b>default value</b> | <b>description</b>                                                       |
|-------------|-------------|-------------|----------------------|--------------------------------------------------------------------------|
| show        | Bool        |             | True                 | set true, if item is shown in visualization and false if it is not shown |
| axesRadius  | float       |             | 0.1                  | radius of joint axes to draw                                             |
| axesLength  | float       |             | 0.4                  | length of joint axes to draw                                             |
| color       | Float4      |             | [-1,-1,-1,-1.]       | RGBA connector color; if R==1, use default color                         |

---

### 7.2.26.1 DESCRIPTION of ObjectJointGeneric:

Information on input parameters:

| <b>input parameter</b>       | <b>symbol</b>                    | <b>description see tables above</b> |
|------------------------------|----------------------------------|-------------------------------------|
| markerNumbers                | $[m0, m1]^T$                     |                                     |
| constrainedAxes              | $\mathbf{j} = [j_0, \dots, j_5]$ |                                     |
| rotationMarker0              | ${}^{m0,J0}\mathbf{A}$           |                                     |
| rotationMarker1              | ${}^{m1,J1}\mathbf{A}$           |                                     |
| offsetUserFunctionParameters | $\mathbf{P}_{par}$               |                                     |
| offsetUserFunction           | $UF(mbs, t, \mathbf{p}_{par})$   |                                     |
| offsetUserFunction_t         | $UF_t(mbs, t, \mathbf{p}_{par})$ |                                     |

The following output variables are available as OutputVariableType in sensors, Get...Output() and other functions:

| <b>output variable</b> | <b>symbol</b>                                                   | <b>description</b>                                                                                                                                                                                              |
|------------------------|-----------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Position               | ${}^0\mathbf{p}_{m0}$                                           | current global position of position marker $m0$                                                                                                                                                                 |
| Velocity               | ${}^0\mathbf{v}_{m0}$                                           | current global velocity of position marker $m0$                                                                                                                                                                 |
| DisplacementLocal      | ${}^{J0}\Delta\mathbf{p}$                                       | relative displacement in local joint0 coordinates; uses local J0 coordinates even for spherical joint configuration                                                                                             |
| VelocityLocal          | ${}^{J0}\Delta\mathbf{v}$                                       | relative translational velocity in local joint0 coordinates                                                                                                                                                     |
| Rotation               | ${}^{J0}\boldsymbol{\theta} = [\theta_0, \theta_1, \theta_2]^T$ | relative rotation parameters (Tait Bryan Rxyz); if all axes are fixed, this output represents the rotational drift; for a revolute joint, it contains the rotation of this axis                                 |
| AngularVelocityLocal   | ${}^{J0}\Delta\boldsymbol{\omega}$                              | relative angular velocity in local joint0 coordinates; if all axes are fixed, this output represents the angular velocity constraint error; for a revolute joint, it contains the angular velocity of this axis |
| ForceLocal             | ${}^{J0}\mathbf{f}$                                             | joint force in local $J0$ coordinates                                                                                                                                                                           |
| TorqueLocal            | ${}^{J0}\mathbf{m}$                                             | joint torque in local $J0$ coordinates; depending on joint configuration, the result may not be the according torque vector                                                                                     |

### 7.2.26.2 Definition of quantities

| intermediate variables       | symbol                                                                        | description                                                                                                                                                                |
|------------------------------|-------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| marker m0 position           | ${}^0\mathbf{p}_{m0}$                                                         | current global position which is provided by marker m0                                                                                                                     |
| marker m0 orientation        | ${}^{0,m0}\mathbf{A}$                                                         | current rotation matrix provided by marker m0                                                                                                                              |
| joint J0 orientation         | ${}^{0,J0}\mathbf{A} = {}^{0,m0}\mathbf{A} \cdot {}^{m0,J0}\mathbf{A}$        | joint J0 rotation matrix                                                                                                                                                   |
| joint J0 orientation vectors | ${}^{0,J0}\mathbf{A} = [\mathbf{v}_{x0}, \mathbf{v}_{y0}, \mathbf{v}_{z0}]^T$ | orientation vectors used for definition of constraint equations                                                                                                            |
| marker m1 position           | ${}^0\mathbf{p}_{m1}$                                                         | accordingly                                                                                                                                                                |
| marker m1 orientation        | ${}^{0,m1}\mathbf{A}$                                                         | current rotation matrix provided by marker m1                                                                                                                              |
| joint J1 orientation         | ${}^{0,J1}\mathbf{A} = {}^{0,m1}\mathbf{A} \cdot {}^{m1,J1}\mathbf{A}$        | joint J1 rotation matrix                                                                                                                                                   |
| joint J1 orientation vectors | ${}^{0,J1}\mathbf{A} = [\mathbf{v}_{x1}, \mathbf{v}_{y1}, \mathbf{v}_{z1}]^T$ | orientation vectors used for definition of constraint equations                                                                                                            |
| marker m0 velocity           | ${}^0\mathbf{v}_{m0}$                                                         | current global velocity which is provided by marker m0                                                                                                                     |
| marker m1 velocity           | ${}^0\mathbf{v}_{m1}$                                                         | accordingly                                                                                                                                                                |
| marker m0 velocity           | ${}^b\omega_{m0}$                                                             | current local angular velocity vector provided by marker m0                                                                                                                |
| marker m1 velocity           | ${}^b\omega_{m1}$                                                             | current local angular velocity vector provided by marker m1                                                                                                                |
| Displacement                 | ${}^0\Delta\mathbf{p} = {}^0\mathbf{p}_{m1} - {}^0\mathbf{p}_{m0}$            | used, if all translational axes are constrained                                                                                                                            |
| Velocity                     | ${}^0\Delta\mathbf{v} = {}^0\mathbf{v}_{m1} - {}^0\mathbf{v}_{m0}$            | used, if all translational axes are constrained (velocity level)                                                                                                           |
| DisplacementLocal            | ${}^{J0}\Delta\mathbf{p}$                                                     | $({}^{0,m0}\mathbf{A} \cdot {}^{m0,J0}\mathbf{A})^T {}^0\Delta\mathbf{p}$                                                                                                  |
| VelocityLocal                | ${}^{J0}\Delta\mathbf{v}$                                                     | $({}^{0,m0}\mathbf{A} \cdot {}^{m0,J0}\mathbf{A})^T {}^0\Delta\mathbf{v}$ ... note that this is the global relative velocity projected into the local J0 coordinate system |
| AngularVelocityLocal         | ${}^{J0}\Delta\omega$                                                         | $({}^{0,m0}\mathbf{A} \cdot {}^{m0,J0}\mathbf{A})^T ({}^{0,m1}\mathbf{A} \cdot {}^{m1}\omega - {}^{0,m0}\mathbf{A} \cdot {}^{m0}\omega)$                                   |
| algebraic variables          | $\mathbf{z} = [\lambda_0, \dots, \lambda_5]^T$                                | vector of algebraic variables (Lagrange multipliers) according to the algebraic equations                                                                                  |

### 7.2.26.3 Connector constraint equations

Equations for translational part (`activeConnector = True`) :

If  $[j_0, \dots, j_2] = [1, 1, 1]^T$ , meaning that all translational coordinates are fixed, the translational index 3 constraints read ( $UF_{0,1,2}(mbs, t, \mathbf{p}_{par})$  is the translational part of the user function  $UF$ ),

$${}^0\mathbf{p}_{m1} - {}^0\mathbf{p}_{m0} - UF_{0,1,2}(mbs, t, \mathbf{p}_{par}) = \mathbf{0} \quad (7.199)$$

and the translational index 2 constraints read

$${}^0\mathbf{v}_{m1} - {}^0\mathbf{v}_{m0} - UF_{t;0,1,2}(mbs, t, \mathbf{p}_{par}) = \mathbf{0} \quad (7.200)$$

If  $[j_0, \dots, j_2] \neq [1, 1, 1]^T$ , meaning that at least one translational coordinate is free, the translational index 3 constraints read for every component  $k \in [0, 1, 2]$  of the vector  ${}^{J0}\Delta p$

$${}^{J0}\Delta p_k - UF_k(mbs, t, \mathbf{p}_{par}) = 0 \quad \text{if } j_k = 1 \quad \text{and} \quad (7.201)$$

$$\lambda_k = 0 \quad \text{if } j_k = 0 \quad (7.202)$$

$$(7.203)$$

and the translational index 2 constraints read for every component  $k \in [0, 1, 2]$  of the vector  ${}^{J0}\Delta v$

$${}^{J0}\Delta v_k - UF_{-t_k}(mbs, t, \mathbf{p}_{par}) = 0 \quad \text{if } j_k = 1 \quad \text{and} \quad (7.204)$$

$$\lambda_k = 0 \quad \text{if } j_k = 0 \quad (7.205)$$

$$(7.206)$$

### **Equations for rotational part (`activeConnector = True`) :**

The following equations are exemplarily for certain constrained rotation axes configurations, which shall represent all other possibilities. Equations are only given for the index 3 case; the index 2 case can be derived from these equations easily (see C++ code...). In case of user functions, the additional rotation matrix  ${}^{J0,J0U}\mathbf{A}$  ( $UF_{3,4,5}(mbs, t, \mathbf{p}_{par})$ ), in which the three components of  $UF_{3,4,5}$  are interpreted as Tait-Bryan angles that are added to the joint frame.

If **3 rotation axes are constrained**,  $[j_3, \dots, j_5] = [1, 1, 1]^T$ , the index 3 constraint equations read

$$\mathbf{v}_{z0}^T \mathbf{v}_{y1} = 0 \quad (7.207)$$

$$\mathbf{v}_{z0}^T \mathbf{v}_{x1} = 0 \quad (7.208)$$

$$\mathbf{v}_{x0}^T \mathbf{v}_{y1} = 0 \quad (7.209)$$

If **2 rotation axes are constrained**, e.g.,  $[j_3, \dots, j_5] = [0, 1, 1]^T$ , the index 3 constraint equations read

$$\lambda_3 = 0 \quad (7.210)$$

$$\mathbf{v}_{x0}^T \mathbf{v}_{y1} = 0 \quad (7.211)$$

$$\mathbf{v}_{x0}^T \mathbf{v}_{z1} = 0 \quad (7.212)$$

If **1 rotation axis is constrained**, e.g.,  $[j_3, \dots, j_5] = [1, 0, 0]^T$ , the index 3 constraint equations read

$$\mathbf{v}_{y0}^T \mathbf{v}_{z1} = 0 \quad (7.213)$$

$$\lambda_4 = 0 \quad (7.214)$$

$$\lambda_5 = 0 \quad (7.215)$$

if `activeConnector = False`,

$$\mathbf{z} = \mathbf{0} \quad (7.216)$$

### **Userfunction: `offsetUserFunction(mbs, t, offsetUserFunctionParameters)`**

A user function, which computes scalar offset for relative joint translation and joint rotation for the GenericJoint, e.g., in order to move or rotate a body on a prescribed trajectory. It is NECESSARY to use sufficiently smooth functions, having **initial offsets** consistent with **initial configuration** of bodies, either zero or compatible initial offset-velocity, and no initial accelerations. The `offsetUserFunction` is **ONLY used** in case of static computation or index3 (generalizedAlpha) time integration. In order to be on the safe side, provide both `offsetUserFunction` and `offsetUserFunction_t`.

The user function gets time and the `offsetUserFunctionParameters` as an input and returns the computed offset vector for all relative translational and rotational joint coordinates:

| arguments / return           | type or size | description                                                             |
|------------------------------|--------------|-------------------------------------------------------------------------|
| mbs                          | MainSystem   | provides MainSystem mbs in which underlying item is defined             |
| t                            | Real         | current time in mbs                                                     |
| offsetUserFunctionParameters | Real         | $p_{par}$ , set of parameters which can be freely used in user function |
| return value                 | Real         | computed offset vector for given time                                   |

---

### Userfunction: offsetUserFunction\_t(mbs, t, offsetUserFunctionParameters)

A user function, which computes an offset **velocity** vector for the GenericJoint. It is NECESSARY to use sufficiently smooth functions, having **initial offset velocities** consistent with **initial velocities** of bodies. The offsetUserFunction\_t is used instead of offsetUserFunction in case of velocityLevel = True, or for index2 time integration and needed for computation of initial accelerations in second order implicit time integrators.

The user function gets time and the offsetUserFunctionParameters as an input and returns the computed offset velocity vector for all relative translational and rotational joint coordinates:

| arguments / return           | type or size | description                                                             |
|------------------------------|--------------|-------------------------------------------------------------------------|
| mbs                          | MainSystem   | provides MainSystem mbs in which underlying item is defined             |
| t                            | Real         | current time in mbs                                                     |
| offsetUserFunctionParameters | Real         | $p_{par}$ , set of parameters which can be freely used in user function |
| return value                 | Real         | computed offset velocity vector for given time                          |

---

### User function example:

```
#simple example, computing only the translational offset for x-coordinate
from math import sin, cos, pi
def UOffset(mbs, t, offsetUserFunctionParameters):
 return [offsetUserFunctionParameters[0]*(1 - cos(t*10*2*pi)), 0,0,0,0]
```

For further examples on ObjectJointGeneric see Examples:

- [mouseInteractionExample.py](#)
- [NGsolveCMSTutorial.py](#)
- [NGsolveCraigBampton.py](#)
- [rigidBodyTutorial.py](#)
- [rigidBodyTutorial2.py](#)
- [sliderCrank3DwithANCFbeltDrive.py](#)
- [stiffFlyballGovernor2.py](#)

For further examples on ObjectJointGeneric see TestModels:

- [driveTrainTest.py](#)
- [carRollingDiscTest.py](#)

- [genericJointUserFunctionTest.py](#)
- [mecanumWheelRollingDiscTest.py](#)
- [objectFFRFreducedOrderAccelerations.py](#)
- [objectFFRFreducedOrderStressModesTest.py](#)
- [objectFFRFreducedOrderTest.py](#)
- [objectFFRFTTest.py](#)
- [rigidBodyCOMtest.py](#)
- [sliderCrank3Dbenchmark.py](#)
- ...

## 7.2.27 ObjectJointSpherical

A spherical joint, which constrains the relative translation between two position based markers.

**Additional information for ObjectJointSpherical:**

- The Object has the following types = Connector, Constraint
- Requested marker type = Position
- Short name for Python = **SphericalJoint**
- Short name for Python (visualization object) = **VsphericalJoint**

The item **ObjectJointSpherical** with type = 'JointSpherical' has the following parameters:

| Name            | type                  | size | default value       | description                                                                                                                                                              |
|-----------------|-----------------------|------|---------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| name            | String                |      | "                   | constraints's unique name                                                                                                                                                |
| markerNumbers   | ArrayMarkerIndex      | 2    | [ MAXINT, MAX-INT ] | list of markers used in connector; $m_1$ is the moving coin rigid body and $m_0$ is the marker for the ground body, which use the localPosition=[0,0,0] for this marker! |
| constrainedAxes | ArrayIndex            | 3    | [1,1,1]             | flag, which determines which translation (0,1,2) and rotation (3,4,5) axes are constrained; for $j_i$ , two values are possible: 0=free axis, 1=constrained axis         |
| activeConnector | Bool                  |      | True                | flag, which determines, if the connector is active; used to deactivate (temporarily) a connector or constraint                                                           |
| visualization   | VObjectJointSpherical |      |                     | parameters for visualization of item                                                                                                                                     |

The item **VObjectJointSpherical** has the following parameters:

| Name        | type   | size | default value     | description                                                              |
|-------------|--------|------|-------------------|--------------------------------------------------------------------------|
| show        | Bool   |      | True              | set true, if item is shown in visualization and false if it is not shown |
| jointRadius | float  |      | 0.1               | radius of joint to draw                                                  |
| color       | Float4 |      | [-1.,-1.,-1.,-1.] | RGBA connector color; if R==1, use default color                         |

---

### 7.2.27.1 DESCRIPTION of ObjectJointSpherical:

**Information on input parameters:**

| input parameter | symbol                  | description see tables above |
|-----------------|-------------------------|------------------------------|
| markerNumbers   | $[m_0, m_1]^T$          |                              |
| constrainedAxes | $j = [j_0, \dots, j_2]$ |                              |

**The following output variables are available as OutputVariableType in sensors, Get...Output() and other functions:**

| output variable | symbol                                                             | description                                                |
|-----------------|--------------------------------------------------------------------|------------------------------------------------------------|
| Position        | ${}^0\mathbf{p}_{m0}$                                              | current global position of position marker $m0$            |
| Velocity        | ${}^0\mathbf{v}_{m0}$                                              | current global velocity of position marker $m0$            |
| Displacement    | ${}^0\Delta\mathbf{p} = {}^0\mathbf{p}_{m1} - {}^0\mathbf{p}_{m0}$ | constraint drift or relative motion, if not all axes fixed |
| Force           | ${}^0\mathbf{f}$                                                   | joint force in global coordinates                          |

### 7.2.27.2 Definition of quantities

| intermediate variables | symbol                                                             | description                                                                               |
|------------------------|--------------------------------------------------------------------|-------------------------------------------------------------------------------------------|
| marker $m0$ position   | ${}^0\mathbf{p}_{m0}$                                              | current global position which is provided by marker $m0$                                  |
| marker $m1$ position   | ${}^0\mathbf{p}_{m1}$                                              | current global position which is provided by marker $m1$                                  |
| marker $m0$ velocity   | ${}^0\mathbf{v}_{m0}$                                              | current global velocity which is provided by marker $m0$                                  |
| marker $m1$ velocity   | ${}^0\mathbf{v}_{m1}$                                              | current global velocity which is provided by marker $m1$                                  |
| relative velocity      | ${}^0\Delta\mathbf{v} = {}^0\mathbf{v}_{m1} - {}^0\mathbf{v}_{m0}$ | constraint velocity error, or relative velocity if not all axes fixed                     |
| algebraic variables    | $\mathbf{z} = [\lambda_0, \dots, \lambda_2]^T$                     | vector of algebraic variables (Lagrange multipliers) according to the algebraic equations |

### 7.2.27.3 Connector constraint equations

**activeConnector = True:** If  $[j_0, \dots, j_2] = [1, 1, 1]^T$ , meaning that all translational coordinates are fixed, the translational index 3 constraints read

$${}^0\mathbf{p}_{m1} - {}^0\mathbf{p}_{m0} = \mathbf{0} \quad (7.217)$$

and the translational index 2 constraints read

$${}^0\mathbf{v}_{m1} - {}^0\mathbf{v}_{m0} = \mathbf{0} \quad (7.218)$$

If  $[j_0, \dots, j_2] \neq [1, 1, 1]^T$ , meaning that at least one translational coordinate is free, the translational index 3 constraints read for every component  $k \in [0, 1, 2]$  of the vector  ${}^0\Delta\mathbf{p}$

$${}^0\Delta p_k = 0 \quad \text{if } j_k = 1 \quad \text{and} \quad (7.219)$$

$$\lambda_k = 0 \quad \text{if } j_k = 0 \quad (7.220)$$

$$(7.221)$$

and the translational index 2 constraints read for every component  $k \in [0, 1, 2]$  of the vector  ${}^0\Delta\mathbf{v}$

$${}^0\Delta v_k = 0 \quad \text{if } j_k = 1 \quad \text{and} \quad (7.222)$$

$$\lambda_k = 0 \quad \text{if } j_k = 0 \quad (7.223)$$

$$(7.224)$$

```
activeConnector = False:
```

**z = 0**

(7.225)

---

For further examples on ObjectJointSpherical see Examples:

- [NGsolvePostProcessingStresses.py](#)
- [objectFFRFreducedOrderNetgen.py](#)
- [sliderCrank3DwithANCFbeltDrive.py](#)
- [sliderCrankCMSacme.py](#)

For further examples on ObjectJointSpherical see TestModels:

- [driveTrainTest.py](#)
- [genericJointUserFunctionTest.py](#)
- [objectFFRFreducedOrderAccelerations.py](#)
- [objectFFRFreducedOrderStressModesTest.py](#)
- [objectFFRFreducedOrderTest.py](#)
- [objectFFRFTTest.py](#)
- [objectFFRFTTest2.py](#)
- [sphericalJointTest.py](#)
- [superElementRigidJointTest.py](#)

## 7.2.28 ObjectJointRollingDisc

A joint representing a rolling rigid disc (marker 1) on a flat surface (marker 0, ground body) in global  $x$ - $y$  plane. The constraint is based on an idealized rolling formulation with no slip. The constraints works for discs as long as the disc axis and the plane normal vector are not parallel. It must be assured that the disc has contact to ground in the initial configuration (adjust z-position of body accordingly). The ground body can be a rigid body which is moving. In this case, the flat surface is assumed to be in the  $x$ - $y$ -plane at  $z = 0$ . Note that the rolling body must have the reference point at the center of the disc. NOTE: the case of a moving ground body needs to be tested further, check your results!

**Additional information for ObjectJointRollingDisc:**

- The Object has the following types = Connector, Constraint
- Requested marker type = Position + Orientation
- **Short name for Python** = **RollingDiscJoint**
- **Short name for Python (visualization object)** = **VRollingDiscJoint**

The item **ObjectJointRollingDisc** with type = 'JointRollingDisc' has the following parameters:

| Name            | type                    | size | default value       | description                                                                                                                                                                            |
|-----------------|-------------------------|------|---------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| name            | String                  |      | "                   | constraints's unique name                                                                                                                                                              |
| markerNumbers   | ArrayMarkerIndex        | 2    | [ MAXINT, MAX-INT ] | list of markers used in connector; $m_0$ represents the ground and $m_1$ represents the rolling body, which has its reference point (=local position [0,0,0]) at the disc center point |
| constrainedAxes | ArrayIndex              | 3    | [1,1,1]             | flag, which determines which constraints are active, in which $j_0, j_1$ represent the tangential motion and $j_2$ represents the normal (contact) direction                           |
| activeConnector | Bool                    |      | True                | flag, which determines, if the connector is active; used to deactivate (temporarily) a connector or constraint                                                                         |
| discRadius      | PReal                   |      | 0                   | defines the disc radius                                                                                                                                                                |
| planeNormal     | Vector3D                |      | [0,0,1]             | normal to the contact / rolling plane; cannot be changed at the moment                                                                                                                 |
| visualization   | VObjectJointRollingDisc |      |                     | parameters for visualization of item                                                                                                                                                   |

The item **VObjectJointRollingDisc** has the following parameters:

| Name      | type   | size | default value     | description                                                              |
|-----------|--------|------|-------------------|--------------------------------------------------------------------------|
| show      | Bool   |      | True              | set true, if item is shown in visualization and false if it is not shown |
| discWidth | float  |      | 0.1               | width of disc for drawing                                                |
| color     | Float4 |      | [-1.,-1.,-1.,-1.] | RGBA connector color; if R== -1, use default color                       |

### 7.2.28.1 DESCRIPTION of ObjectJointRollingDisc:

**Information on input parameters:**

| input parameter | symbol                  | description see tables above |
|-----------------|-------------------------|------------------------------|
| markerNumbers   | $[m0, m1]^T$            |                              |
| constrainedAxes | $j = [j_0, \dots, j_2]$ |                              |

The following output variables are available as **OutputVariableType** in sensors, Get...Output() and other functions:

| output variable | symbol                                                                                       | description                                                                                                                                                    |
|-----------------|----------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Position        | ${}^0\mathbf{p}_G$                                                                           | current global position of contact point between rolling disc and ground                                                                                       |
| VelocityLocal   | ${}^D\mathbf{v}_G$                                                                           | current velocity of the trail (contact) point in disc coordinates; this is the velocity with which the contact moves over the ground plane                     |
| ForceLocal      | ${}^D\mathbf{f} = [-\mathbf{z}^T \mathbf{w}_l, -\mathbf{z}^T \mathbf{w}_2, -\mathbf{z}_z]^T$ | contact forces acting on disc, in special disc coordinates, $f_x$ being the lateral force, $f_y$ being the longitudinal force and $f_z$ being the normal force |

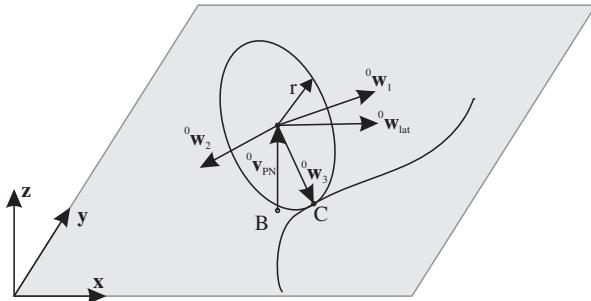
### 7.2.28.2 Definition of quantities

| intermediate variables     | symbol                         | description                                                                                                                                    |
|----------------------------|--------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------|
| marker m0 position         | ${}^0\mathbf{p}_{m0}$          | current global position of marker $m0$ body reference position (ground body reference position); needed only if body $m0$ is not a ground body |
| marker m0 orientation      | ${}^{0,m0}\mathbf{A}$          | current rotation matrix provided by marker $m0$ (assumed to be rigid body)                                                                     |
| marker m0 velocity         | ${}^0\mathbf{v}_{m0}$          | current global velocity which is provided by marker $m0$ (assumed to be rigid body)                                                            |
| marker m0 angular velocity | ${}^0\boldsymbol{\omega}_{m0}$ | current angular velocity vector provided by marker $m0$ (assumed to be rigid body)                                                             |
| marker m1 position         | ${}^0\mathbf{p}_{m1}$          | center of disc                                                                                                                                 |
| marker m1 orientation      | ${}^{0,m1}\mathbf{A}$          | current rotation matrix provided by marker $m1$                                                                                                |
| marker m1 velocity         | ${}^0\mathbf{v}_{m1}$          | accordingly                                                                                                                                    |
| marker m1 angular velocity | ${}^0\boldsymbol{\omega}_{m1}$ | current angular velocity vector provided by marker $m1$                                                                                        |
| ground normal vector       | ${}^0\mathbf{v}_{PN}$          | normalized normal vector to the ground plane, currently $[0,0,1]$                                                                              |
| ground position B          | ${}^0\mathbf{p}_B$             | disc center point projected on ground in plane normal (z-direction, $z = 0$ )                                                                  |
| ground position C          | ${}^0\mathbf{p}_C$             | contact point of disc with ground in global coordinates                                                                                        |
| ground velocity C          | ${}^0\mathbf{v}_{Cm1}$         | velocity of disc (marker 1) at ground contact point (must be zero if ground does not move)                                                     |

|                      |                                                                                                             |                                                                                                |
|----------------------|-------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------|
| ground velocity C    | ${}^0\mathbf{v}_{Cm2}$                                                                                      | velocity of ground (marker 0) at ground contact point (is always zero if ground does not move) |
| wheel axis vector    | ${}^0\mathbf{w}_1 = {}^{0,m1}\mathbf{A} \cdot [1, 0, 0]^T$                                                  | normalized disc axis vector, currently $[1, 0, 0]^T$ in local coordinates                      |
| longitudinal vector  | ${}^0\mathbf{w}_2$                                                                                          | vector in longitudinal (motion) direction                                                      |
| lateral vector       | ${}^0\mathbf{w}_l = {}^0\mathbf{v}_{PN} \times {}^0\mathbf{w}_2 = [-\mathbf{w}_{2,y}, \mathbf{w}_{2,x}, 0]$ | vector in lateral direction, lies in ground plane                                              |
| contact point vector | ${}^0\mathbf{w}_3$                                                                                          | normalized vector from disc center point in direction of contact point C                       |
| algebraic variables  | $\mathbf{z} = [\lambda_0, \lambda_1, \lambda_2]^T$                                                          | vector of algebraic variables (Lagrange multipliers) according to the algebraic equations      |

### 7.2.28.3 Geometric relations

The main geometrical setup is shown in the following figure:



First, the contact point  ${}^0\mathbf{p}_C$  must be computed. With the helper vector,

$${}^0\mathbf{x} = {}^0\mathbf{w}_1 \times {}^0\mathbf{v}_{PN} \quad (7.226)$$

we obtain a disc coordinate system, representing the longitudinal direction,

$${}^0\mathbf{w}_2 = \frac{1}{|{}^0\mathbf{x}|} {}^0\mathbf{x} \quad (7.227)$$

and the vector to the contact point,

$${}^0\mathbf{w}_3 = {}^0\mathbf{w}_1 \times {}^0\mathbf{w}_2 \quad (7.228)$$

The contact point C can be computed from

$${}^0\mathbf{p}_C = {}^0\mathbf{p}_{m1} + r \cdot {}^0\mathbf{w}_3 \quad (7.229)$$

The velocity of the contact point at the disc is computed from,

$${}^0\mathbf{v}_{Cm1} = {}^0\mathbf{v}_{m1} + {}^0\omega_{m1} \times (r \cdot {}^0\mathbf{w}_3) \quad (7.230)$$

If marker 0 body is (moving) rigid body instead of a ground body, the contact point C is reconstructed in body of marker 0,

$${}^{m0}\mathbf{p}_C = {}^{m0,0}\mathbf{A} ({}^0\mathbf{p}_C - {}^0\mathbf{p}_{m0}) \quad (7.231)$$

The velocity of the contact point at the marker 0 body reads

$${}^0\mathbf{v}_{Cm0} = {}^0\mathbf{v}_{m0} + {}^0\omega_{m0} \times ({}^{0,m0}\mathbf{A} {}^{m0}\mathbf{p}_C) \quad (7.232)$$

#### 7.2.28.4 Connector constraint equations

**activeConnector = True:**

The non-holonomic, index 2 constraints for the tangential and normal contact follow from (an index 3 formulation would be possible, but is not implemented yet because of mixing different jacobians)

$$\begin{bmatrix} {}^0\mathbf{v}_{Cm1,x} \\ {}^0\mathbf{v}_{Cm1,y} \\ {}^0\mathbf{v}_{Cm1,z} \end{bmatrix} - \begin{bmatrix} {}^0\mathbf{v}_{Cm0,x} \\ {}^0\mathbf{v}_{Cm0,y} \\ {}^0\mathbf{v}_{Cm0,z} \end{bmatrix} = \mathbf{0} \quad (7.233)$$

**activeConnector = False:**

$$\mathbf{z} = \mathbf{0} \quad (7.234)$$

---

For further examples on ObjectJointRollingDisc see TestModels:

- [rollingCoinTest.py](#)

## 7.2.29 ObjectJointRevolute2D

A revolute joint in 2D; constrains the absolute 2D position of two points given by PointMarkers or RigidMarkers

**Additional information for ObjectJointRevolute2D:**

- The Object has the following types = Connector, Constraint
- Requested marker type = Position
- **Short name for Python = RevoluteJoint2D**
- **Short name for Python (visualization object) = VRevoluteJoint2D**

The item **ObjectJointRevolute2D** with type = 'JointRevolute2D' has the following parameters:

| Name            | type                   | size | default value       | description                                                                                                    |
|-----------------|------------------------|------|---------------------|----------------------------------------------------------------------------------------------------------------|
| name            | String                 |      | "                   | constraints's unique name                                                                                      |
| markerNumbers   | ArrayMarkerIndex       |      | [ MAXINT, MAX-INT ] | list of markers used in connector                                                                              |
| activeConnector | Bool                   |      | True                | flag, which determines, if the connector is active; used to deactivate (temporarily) a connector or constraint |
| visualization   | VObjectJointRevolute2D |      |                     | parameters for visualization of item                                                                           |

The item **VObjectJointRevolute2D** has the following parameters:

| Name     | type   | size | default value     | description                                                                                     |
|----------|--------|------|-------------------|-------------------------------------------------------------------------------------------------|
| show     | Bool   |      | True              | set true, if item is shown in visualization and false if it is not shown                        |
| drawSize | float  |      | -1.               | drawing size = radius of revolute joint; size == -1.f means that default connector size is used |
| color    | Float4 |      | [-1.,-1.,-1.,-1.] | RGBA connector color; if R== -1, use default color                                              |

---

### 7.2.29.1 DESCRIPTION of ObjectJointRevolute2D:

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For further examples on ObjectJointRevolute2D see Examples:

- [sliderCrank3DwithANCFbeltDrive2.py](#)
- [ANCF\\_slidingJoint2D.py](#)
- [geneticOptimizationSliderCrank.py](#)
- [rigid\\_pendulum.py](#)
- [SliderCrank.py](#)
- [sliderCrank3DwithANCFbeltDrive.py](#)
- [slidercrankWithMassSpring.py](#)

- [switchingConstraintsPendulum.py](#)

For further examples on ObjectJointRevolute2D see TestModels:

- [compareFullModifiedNewton.py](#)
- [modelUnitTests.py](#)
- [PARTS\\_ATEs\\_moving.py](#)
- [pendulumFriction.py](#)
- [scissorPrismaticRevolute2D.py](#)
- [sliderCrankFloatingTest.py](#)

### 7.2.30 ObjectJointPrismatic2D

A prismatic joint in 2D; allows the relative motion of two bodies, using two RigidMarkers; the vector  $\mathbf{t}_0 = \text{axisMarker0}$  is given in local coordinates of the first marker's (body) frame and defines the prismatic axis; the vector  $\mathbf{n}_1 = \text{normalMarker1}$  is given in the second marker's (body) frame and is the normal vector to the prismatic axis; using the global position vector  $\mathbf{p}_0$  and rotation matrix  $\mathbf{A}_0$  of marker0 and the global position vector  $\mathbf{p}_1$  rotation matrix  $\mathbf{A}_1$  of marker1, the equations for the prismatic joint follow as

$$(\mathbf{p}_1 - \mathbf{p}_0)^T \cdot \mathbf{A}_1 \cdot \mathbf{n}_1 = 0 \quad (7.235)$$

$$(\mathbf{A}_0 \cdot \mathbf{t}_0)^T \cdot \mathbf{A}_1 \cdot \mathbf{n}_1 = 0 \quad (7.236)$$

The lagrange multipliers follow for these two equations  $[\lambda_0, \lambda_1]$ , in which  $\lambda_0$  is the transverse force and  $\lambda_1$  is the torque in the joint.

#### Additional information for ObjectJointPrismatic2D:

- The Object has the following types = Connector, Constraint
- Requested marker type = Position + Orientation
- **Short name for Python** = PrismaticJoint2D
- **Short name for Python (visualization object)** = VPrismaticJoint2D

The item **ObjectJointPrismatic2D** with type = 'JointPrismatic2D' has the following parameters:

| Name              | type                    | size | default value       | description                                                                                                                                                                       |
|-------------------|-------------------------|------|---------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| name              | String                  |      | "                   | constraints's unique name                                                                                                                                                         |
| markerNumbers     | ArrayMarkerIndex        |      | [ MAXINT, MAX-INT ] | list of markers used in connector                                                                                                                                                 |
| axisMarker0       | Vector3D                |      | [1.,0.,0.]          | direction of prismatic axis, given as a 3D vector in Marker0 frame                                                                                                                |
| normalMarker1     | Vector3D                |      | [0.,1.,0.]          | direction of normal to prismatic axis, given as a 3D vector in Marker1 frame                                                                                                      |
| constrainRotation | Bool                    |      | True                | flag, which determines, if the connector also constrains the relative rotation of the two objects; if set to false, the constraint will keep an algebraic equation set equal zero |
| activeConnector   | Bool                    |      | True                | flag, which determines, if the connector is active; used to deactivate (temporarily) a connector or constraint                                                                    |
| visualization     | VObjectJointPrismatic2D |      |                     | parameters for visualization of item                                                                                                                                              |

The item **VObjectJointPrismatic2D** has the following parameters:

| Name     | type   | size | default value     | description                                                                                     |
|----------|--------|------|-------------------|-------------------------------------------------------------------------------------------------|
| show     | Bool   |      | True              | set true, if item is shown in visualization and false if it is not shown                        |
| drawSize | float  |      | -1.               | drawing size = radius of revolute joint; size == -1.f means that default connector size is used |
| color    | Float4 |      | [-1.,-1.,-1.,-1.] | RGBA connector color; if R== -1, use default color                                              |

---

### 7.2.30.1 DESCRIPTION of ObjectJointPrismatic2D:

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For further examples on ObjectJointPrismatic2D see Examples:

- [sliderCrank3DwithANCFbeltDrive2.py](#)
- [geneticOptimizationSliderCrank.py](#)

For further examples on ObjectJointPrismatic2D see TestModels:

- [PARTS\\_ATEs\\_moving.py](#)
- [scissorPrismaticRevolute2D.py](#)
- [sliderCrankFloatingTest.py](#)

### 7.2.31 ObjectJointSliding2D

A specialized sliding joint (without rotation) in 2D between a Cable2D (marker1) and a position-based marker (marker0); the data coordinate x[0] provides the current index in slidingMarkerNumbers, and x[1] the local position in the cable element at the beginning of the timestep.

**Additional information for ObjectJointSliding2D:**

- The Object has the following types = Connector, Constraint
- Requested marker type = \_None
- Requested node type = GenericData
- **Short name for Python** = SlidingJoint2D
- **Short name for Python (visualization object)** = VSlidingJoint2D

The item **ObjectJointSliding2D** with type = 'JointSliding2D' has the following parameters:

| Name                 | type             | size | default value      | description                                                                                                                                                                                                                                                                                                                     |
|----------------------|------------------|------|--------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| name                 | String           |      | "                  | constraints's unique name                                                                                                                                                                                                                                                                                                       |
| markerNumbers        | ArrayMarkerIndex |      | [ MAXINT, MAXINT ] | marker m0: position-marker of mass point or rigid body; marker m1: updated marker to Cable2D element, where the sliding joint currently is attached to; must be initialized with an appropriate (global) marker number according to the starting position of the sliding object; this marker changes with time (PostNewtonStep) |
| slidingMarkerNumbers | ArrayMarkerIndex |      | []                 | these markers are used to update marker m1, if the sliding position exceeds the current cable's range; the markers must be sorted such that marker $m_{si}$ at $x=cable(i).length$ is equal to marker(i+1) at $x=0$ of cable(i+1)                                                                                               |
| slidingMarkerOffsets | Vector           |      | []                 | this list contains the offsets of every sliding object (given by slidingMarkerNumbers) w.r.t. to the initial position (0): marker m0: offset=0, marker m1: offset=Length(cable0), marker m2: offset=Length(cable0)+Length(cable1), ...                                                                                          |
| nodeNumber           | NodeIndex        |      | MAXINT             | node number of a NodeGenericData for 1 dataCoordinate showing the according marker number which is currently active and the start-of-step (global) sliding position                                                                                                                                                             |
| classicalFormulation | Bool             |      | True               | uses a formulation with 3 equations, including the force in sliding direction to be zero; forces in global coordinates, only index 3; alternatively: use local formulation, which only needs two equations and can be used with index 2 formulation                                                                             |
| activeConnector      | Bool             |      | True               | flag, which determines, if the connector is active; used to deactivate (temporarily) a connector or constraint                                                                                                                                                                                                                  |

|               |                       |                                      |
|---------------|-----------------------|--------------------------------------|
| visualization | VObjectJointSliding2D | parameters for visualization of item |
|---------------|-----------------------|--------------------------------------|

The item VObjectJointSliding2D has the following parameters:

| Name     | type   | size | default value     | description                                                                                     |
|----------|--------|------|-------------------|-------------------------------------------------------------------------------------------------|
| show     | Bool   |      | True              | set true, if item is shown in visualization and false if it is not shown                        |
| drawSize | float  |      | -1.               | drawing size = radius of revolute joint; size == -1.f means that default connector size is used |
| color    | Float4 |      | [-1.,-1.,-1.,-1.] | RGBA connector color; if R==1, use default color                                                |

### 7.2.31.1 DESCRIPTION of ObjectJointSliding2D:

Information on input parameters:

| input parameter      | symbol                      | description see tables above |
|----------------------|-----------------------------|------------------------------|
| markerNumbers        | $[m_0, m_1]^T$              |                              |
| slidingMarkerNumbers | $[m_{s0}, \dots, m_{sn}]^T$ |                              |
| slidingMarkerOffsets | $[d_{s0}, \dots, d_{sn}]$   |                              |
| nodeNumber           | $n_{GD}$                    |                              |

The following output variables are available as OutputVariableType in sensors, Get...Output() and other functions:

| output variable   | symbol | description                                                                                                                                 |
|-------------------|--------|---------------------------------------------------------------------------------------------------------------------------------------------|
| Position          |        | position vector of joint given by marker0                                                                                                   |
| Velocity          |        | velocity vector of joint given by marker0                                                                                                   |
| SlidingCoordinate |        | global sliding coordinate along all elements; the maximum sliding coordinate is equivalent to the reference lengths of all sliding elements |
| Force             |        | joint force vector (3D)                                                                                                                     |

### 7.2.31.2 Definition of quantities

| intermediate variables | symbol                                  | description                                                                                                                             |
|------------------------|-----------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------|
| data node              | $\mathbf{x} = [x_{data0}, x_{data1}]^T$ | coordinates of node with node number $n_{GD}$                                                                                           |
| data coordinate 0      | $x_{data0}$                             | the current index in slidingMarkerNumbers                                                                                               |
| data coordinate 1      | $x_{data1}$                             | the global sliding coordinate (ranging from 0 to the total length of all sliding elements) at start-of-step - beginning of the timestep |
| marker m0 position     | ${}^0\mathbf{p}_{m0}$                   | current global position which is provided by marker m0                                                                                  |

|                                |                                                                                     |                                                                                                                                                                                                                                         |
|--------------------------------|-------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| marker m0 velocity             | ${}^0\mathbf{v}_{m0}$                                                               | current global velocity which is provided by marker m0                                                                                                                                                                                  |
| cable coordinates              | $\mathbf{q}_{ANCF,m1}$                                                              | current coordinates of the ANCF cable element with the current marker $m1$ is referring to                                                                                                                                              |
| sliding position               | ${}^0\mathbf{r}_{ANCF} = \mathbf{S}(s_{el})\mathbf{q}_{ANCF,m1}$                    | current global position at the ANCF cable element, evaluated at local sliding position $s_{el}$                                                                                                                                         |
| sliding position slope         | ${}^0\mathbf{r}'_{ANCF} = \mathbf{S}'(s_{el})\mathbf{q}_{ANCF,m1} = [r'_0, r'_1]^T$ | current global slope vector of the ANCF cable element, evaluated at local sliding position $s_{el}$                                                                                                                                     |
| sliding velocity               | ${}^0\mathbf{v}_{ANCF} = \mathbf{S}(s_{el})\dot{\mathbf{q}}_{ANCF,m1}$              | current global velocity at the ANCF cable element, evaluated at local sliding position $s_{el}$ ( $s_{el}$ not differentiated!!!)                                                                                                       |
| sliding velocity slope         | ${}^0\mathbf{v}'_{ANCF} = \mathbf{S}'(s_{el})\dot{\mathbf{q}}_{ANCF,m1}$            | current global slope velocity vector of the ANCF cable element, evaluated at local sliding position $s_{el}$                                                                                                                            |
| sliding normal vector          | ${}^0\mathbf{n} = [-r'_1, r'_0]$                                                    | 2D normal vector computed from slope $\mathbf{r}' = {}^0\mathbf{r}'_{ANCF}$                                                                                                                                                             |
| sliding normal velocity vector | ${}^0\dot{\mathbf{n}} = [-\dot{r}'_1, \dot{r}'_0]$                                  | time derivative of 2D normal vector computed from slope velocity $\dot{\mathbf{r}'} = {}^0\dot{\mathbf{r}}'_{ANCF}$                                                                                                                     |
| algebraic coordinates          | $\mathbf{z} = [\lambda_0, \lambda_1, s]^T$                                          | algebraic coordinates composed of Lagrange multipliers $\lambda_0$ and $\lambda_1$ (in local cable coordinates: $\lambda_0$ is in axis direction) and the current sliding coordinate $s$ , which is local in the current cable element. |
| local sliding coordinate       | $s$                                                                                 | local incremental sliding coordinate $s$ : the (algebraic) sliding coordinate <b>relative to the start-of-step value</b> . Thus, $s$ only contains small local increments.                                                              |

| output variables  | symbol                | formula                                         |
|-------------------|-----------------------|-------------------------------------------------|
| Position          | ${}^0\mathbf{p}_{m0}$ | current global position of position marker $m0$ |
| Velocity          | ${}^0\mathbf{v}_{m0}$ | current global velocity of position marker $m0$ |
| SlidingCoordinate | $s_g = s + x_{data1}$ | current value of the global sliding coordinate  |
| Force             | $\mathbf{f}$          | see below                                       |

### 7.2.31.3 Geometric relations

Assume we have given the sliding coordinate  $s$  (e.g., as a guess of the Newton method or beginning of the time step). The element sliding coordinate (in the local coordinates of the current sliding element) is computed as

$$s_{el} = s + x_{data1} - d_{m1} = s_g - d_{m1}. \quad (7.237)$$

The vector (=difference; error) between the marker  $m0$  and the marker  $m1$  ( $=\mathbf{r}_{ANCF}$ ) positions reads

$${}^0\Delta\mathbf{p} = {}^0\mathbf{r}_{ANCF} - {}^0\mathbf{p}_{m0} \quad (7.238)$$

The vector (=difference; error) between the marker  $m0$  and the marker  $m1$  velocities reads

$${}^0\Delta\mathbf{v} = {}^0\dot{\mathbf{r}}_{ANCF} - {}^0\mathbf{v}_{m0} \quad (7.239)$$

#### 7.2.31.4 Connector constraint equations (classicalFormulation=True)

The 2D sliding joint is implemented having 3 equations, using the special algebraic coordinates  $\mathbf{z}$ . The algebraic equations read

$${}^0\Delta\mathbf{p} = \mathbf{0}, \quad \dots \text{two index 3 equations, ensuring the sliding body to stay at cable} \quad (7.240)$$

$$[\lambda_0, \lambda_1] \cdot {}^0\dot{\mathbf{r}}_{ANCF} = \mathbf{0}, \quad \dots \text{one index 1 equation, ensuring force in sliding direction} = 0 \quad (7.241)$$

$$(7.242)$$

No index 2 case exists, because no time derivative exists for  $s_{el}$ . The jacobian matrices for algebraic and ODE2 coordinates read

$$\mathbf{J}_{AE} = \begin{bmatrix} 0 & 0 & r'_0 \\ 0 & 0 & r'_1 \\ r'_0 & r'_1 & r''_0\lambda_0 + r''_1\lambda_1 \end{bmatrix} \quad (7.243)$$

$$\mathbf{J}_{ODE2} = \begin{bmatrix} -J_{pos,m0} & \mathbf{S}(s_{el}) \\ \mathbf{0}^T & [\lambda_0, \lambda_1] \cdot \mathbf{S}'(s_{el}) \end{bmatrix} \quad (7.244)$$

if `activeConnector = False`, the algebraic equations are changed to:

$$\lambda_0 = 0, \quad (7.245)$$

$$\lambda_1 = 0, \quad (7.246)$$

$$s = 0 \quad (7.247)$$

#### 7.2.31.5 Connector constraint equations (classicalFormulation=False)

The 2D sliding joint is implemented having 3 equations (first equation is dummy and could be eliminated), using the special algebraic coordinates  $\mathbf{z}$ . The algebraic equations read

$$\lambda_0 = 0, \quad \dots \text{this equation is not necessary, but can be used for switching to other mode} \quad (7.248)$$

$${}^0\Delta\mathbf{p}^T {}^0\mathbf{n} = 0, \quad \dots \text{equation ensures that sliding body stays at cable centerline; index3 equation} \quad (7.249)$$

$${}^0\Delta\mathbf{p}^T {}^0\dot{\mathbf{r}}_{ANCF} = 0. \quad \dots \text{resolves the sliding coordinate } s; \text{ index1 equation!} \quad (7.250)$$

In the index 2 case, the second equation reads

$${}^0\Delta\mathbf{v}^T {}^0\mathbf{n} + {}^0\Delta\mathbf{p}^T {}^0\dot{\mathbf{n}} = 0 \quad (7.251)$$

if `activeConnector = False`, the algebraic equations are changed to:

$$\lambda_0 = 0, \quad (7.252)$$

$$\lambda_1 = 0, \quad (7.253)$$

$$s = 0 \quad (7.254)$$

### 7.2.31.6 Post Newton Step

After the Newton solver has converged, a PostNewtonStep is performed for the element, which updates the marker  $m1$  index if necessary.

$$\begin{aligned} s_{el} < 0 &\rightarrow x_{data0} -= 1 \\ s_{el} > L &\rightarrow x_{data0} += 1 \end{aligned} \quad (7.255)$$

Furthermore, it is checked, if  $x_{data0}$  becomes smaller than zero, which raises a warning and keeps  $x_{data0} = 0$ . The same results if  $x_{data0} \geq sn$ , then  $x_{data0} = sn$ . Finally, the data coordinate is updated in order to provide the starting value for the next step,

$$x_{data1} += s. \quad (7.256)$$

---

For further examples on ObjectJointSliding2D see Examples:

- [ANCF\\_moving\\_rigidbody.py](#)
- [ANCF\\_slidingJoint2D.py](#)
- [ANCF\\_switchingSlidingJoint2D.py](#)

For further examples on ObjectJointSliding2D see TestModels:

- [modelUnitTests.py](#)

### 7.2.32 ObjectJointALEMoving2D

A specialized axially moving joint (without rotation) in 2D between a ALE Cable2D (marker1) and a position-based marker (marker0); ALE=Arbitrary Lagrangian Eulerian; the data coordinate  $x[0]$  provides the current index in slidingMarkerNumbers, and the ODE2 coordinate  $q[0]$  provides the (given) moving coordinate in the cable element.

**Additional information for ObjectJointALEMoving2D:**

- The Object has the following types = Connector, Constraint
- Requested marker type = \_None
- Requested node type: read detailed information of item
- **Short name for Python** = **ALEMovingJoint2D**
- **Short name for Python (visualization object)** = **VALEMovingJoint2D**

The item **ObjectJointALEMoving2D** with type = 'JointALEMoving2D' has the following parameters:

| Name                  | type                    | size | default value      | description                                                                                                                                                                                                                                                                                                                          |
|-----------------------|-------------------------|------|--------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| name                  | String                  |      | "                  | constraints's unique name                                                                                                                                                                                                                                                                                                            |
| markerNumbers         | ArrayMarkerIndex        |      | [ MAXINT, MAXINT ] | marker m0: position-marker of mass point or rigid body; marker m1: updated marker to ANCF Cable2D element, where the sliding joint currently is attached to; must be initialized with an appropriate (global) marker number according to the starting position of the sliding object; this marker changes with time (PostNewtonStep) |
| slidingMarkerNumbers  | ArrayMarkerIndex        |      | []                 | a list of sn (global) marker numbers which are used to update marker1                                                                                                                                                                                                                                                                |
| slidingMarkerOffsets  | Vector                  |      | []                 | this list contains the offsets of every sliding object (given by slidingMarkerNumbers) w.r.t. to the initial position (0): marker0: offset=0, marker1: offset=Length(cable0), marker2: offset=Length(cable0)+Length(cable1), ...                                                                                                     |
| slidingOffset         | Real                    |      | 0.                 | sliding offset list [SI:m]: a list of sn scalar offsets, which represent the (reference arc) length of all previous sliding cable elements                                                                                                                                                                                           |
| nodeNumbers           | ArrayNodeIndex          |      | [ MAXINT, MAXINT ] | node number of NodeGenericData (GD) with one data coordinate and of NodeGenericODE2 (ALE) with one ODE2 coordinate                                                                                                                                                                                                                   |
| usePenaltyFormulation | Bool                    |      | False              | flag, which determines, if the connector is formulated with penalty, but still using algebraic equations (IsPenaltyConnector() still false)                                                                                                                                                                                          |
| penaltyStiffness      | Real                    |      | 0.                 | penalty stiffness [SI:N/m] used if usePenaltyFormulation=True                                                                                                                                                                                                                                                                        |
| activeConnector       | Bool                    |      | True               | flag, which determines, if the connector is active; used to deactivate (temporarily) a connector or constraint                                                                                                                                                                                                                       |
| visualization         | VObjectJointALEMoving2D |      |                    | parameters for visualization of item                                                                                                                                                                                                                                                                                                 |

The item VObjectJointALEMoving2D has the following parameters:

| <b>Name</b> | <b>type</b> | <b>size</b> | <b>default value</b> | <b>description</b>                                                                              |
|-------------|-------------|-------------|----------------------|-------------------------------------------------------------------------------------------------|
| show        | Bool        |             | True                 | set true, if item is shown in visualization and false if it is not shown                        |
| drawSize    | float       |             | -1.                  | drawing size = radius of revolute joint; size == -1.f means that default connector size is used |
| color       | Float4      |             | [-1.,-1.,-1.,-1.]    | RGBA connector color; if R== -1, use default color                                              |

### 7.2.32.1 DESCRIPTION of ObjectJointALEMoving2D:

Information on input parameters:

| <b>input parameter</b> | <b>symbol</b>               | <b>description see tables above</b> |
|------------------------|-----------------------------|-------------------------------------|
| markerNumbers          | $[m_0, m_1]^T$              |                                     |
| slidingMarkerNumbers   | $[m_{s0}, \dots, m_{sn}]^T$ |                                     |
| slidingMarkerOffsets   | $[d_{s0}, \dots, d_{sn}]$   |                                     |
| slidingOffset          | $s_{off}$                   |                                     |
| nodeNumbers            | $[n_{GD}, n_{ALE}]$         |                                     |
| penaltyStiffness       | $k$                         |                                     |

The following output variables are available as OutputVariableType in sensors, Get...Output() and other functions:

| <b>output variable</b> | <b>symbol</b>             | <b>description</b>                                                                                                            |
|------------------------|---------------------------|-------------------------------------------------------------------------------------------------------------------------------|
| Position               | ${}^0\mathbf{p}_{m0}$     | current global position of position marker $m_0$                                                                              |
| Velocity               | ${}^0\mathbf{v}_{m0}$     | current global velocity of position marker $m_0$                                                                              |
| SlidingCoordinate      | $s_g = q_{ALE} + s_{off}$ | current value of the global sliding ALE coordinate, including offset; note that reference coordinate of $q_{ALE}$ is ignored! |
| Coordinates            | $[x_{data0}, q_{ALE}]^T$  | provides two values: [0] = current sliding marker index, [1] = ALE sliding coordinate                                         |
| Coordinates_t          | $[\dot{q}_{ALE}]^T$       | provides ALE sliding velocity                                                                                                 |
| Force                  | $\mathbf{f}$              | joint force vector (3D)                                                                                                       |

### 7.2.32.2 Definition of quantities

| <b>intermediate variables</b> | <b>symbol</b>                | <b>description</b>                                                                                               |
|-------------------------------|------------------------------|------------------------------------------------------------------------------------------------------------------|
| generic data node             | $\mathbf{x} = [x_{data0}]^T$ | coordinates of node with node number $n_{GD}$                                                                    |
| generic ODE2 node             | $\mathbf{q} = [q_0]^T$       | coordinates of node with node number $n_{ALE}$ , which is shared with all ALE-ANCF and ALE sliding joint objects |

|                        |                                                                                                               |                                                                                                                                                  |
|------------------------|---------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------|
| data coordinate        | $x_{data0}$                                                                                                   | the current index in slidingMarkerNumbers                                                                                                        |
| ALE coordinate         | $q_{ALE} = q_0$                                                                                               | current ALE coordinate (in fact this is the Eulerian coordinate in the ALE formulation); note that reference coordinate of $q_{ALE}$ is ignored! |
| marker m0 position     | ${}^0\mathbf{p}_{m0}$                                                                                         | current global position which is provided by marker m0                                                                                           |
| marker m0 velocity     | ${}^0\mathbf{v}_{m0}$                                                                                         | current global velocity which is provided by marker m0                                                                                           |
| cable coordinates      | $\mathbf{q}_{ANCF,m1}$                                                                                        | current coordinates of the ANCF cable element with the current marker $m1$ is referring to                                                       |
| sliding position       | ${}^0\mathbf{r}_{ANCF} = \mathbf{S}(s_{el})\mathbf{q}_{ANCF,m1}$                                              | current global position at the ANCF cable element, evaluated at local sliding position $s_{el}$                                                  |
| sliding position slope | ${}^0\mathbf{r}'_{ANCF} = \mathbf{S}'(s_{el})\mathbf{q}_{ANCF,m1}$                                            | current global slope vector of the ANCF cable element, evaluated at local sliding position $s_{el}$                                              |
| sliding velocity       | ${}^0\mathbf{v}_{ANCF} = \mathbf{S}(s_{el})\dot{\mathbf{q}}_{ANCF,m1} + \dot{q}_{ALE} {}^0\mathbf{r}'_{ANCF}$ | current global velocity at the ANCF cable element, evaluated at local sliding position $s_{el}$ , including convective term                      |
| sliding normal vector  | ${}^0\mathbf{n} = [-r'_1, r'_0]$                                                                              | 2D normal vector computed from slope $\mathbf{r}' = {}^0\mathbf{r}'_{ANCF}$                                                                      |
| algebraic variables    | $\mathbf{z} = [\lambda_0, \lambda_1]^T$                                                                       | algebraic variables (Lagrange multipliers) according to the algebraic equations                                                                  |

### 7.2.32.3 Geometric relations

The element sliding coordinate (in the local coordinates of the current sliding element) is computed from the ALE coordinate

$$s_{el} = q_{ALE} + s_{off} - d_{m1} = s_g - d_{m1}. \quad (7.257)$$

For the description of the according quantities, see the description above. The distance  $d_{m1}$  is obtained from the `slidingMarkerOffsets` list, using the current (local) index  $x_{data0}$ . The vector (=difference; error) between the marker  $m0$  and the marker  $m1$  ( $=\mathbf{r}_{ANCF}$ ) positions reads

$${}^0\Delta\mathbf{p} = {}^0\mathbf{r}_{ANCF} - {}^0\mathbf{p}_{m0} \quad (7.258)$$

Note that  ${}^0\mathbf{p}_{m0}$  represents the current position of the marker  $m0$ , which could represent the midpoint of a mass sliding along the beam. The position  ${}^0\mathbf{r}_{ANCF}$  is computed from the beam represented by marker  $m1$ , using the local beam coordinate  $x = s_{el}$ . The marker and the according beam finite element changes during movement using the list `slidingMarkerNumbers` and the index is updated in the PostNewtonStep. The vector (=difference; error) between the marker  $m0$  and the marker  $m1$  velocities reads

$${}^0\Delta\mathbf{v} = {}^0\mathbf{v}_{ANCF} - {}^0\mathbf{v}_{m0} \quad (7.259)$$

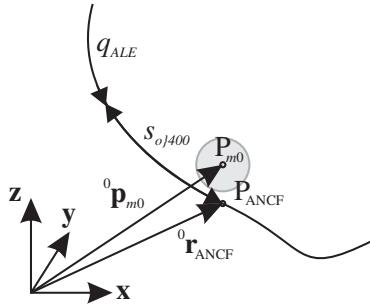


Figure 7.3: Geometrical relations for ALE sliding joint.

#### 7.2.32.4 Connector constraint equations

The 2D sliding joint is implemented having 2 equations, using the Lagrange multipliers  $\mathbf{z}$ . The algebraic (index 3) equations read

$${}^0 \Delta \mathbf{p} = 0 \quad (7.260)$$

Note that the Lagrange multipliers  $[\lambda_0, \lambda_1]^T$  are the global forces in the joint. In the index 2 case the algebraic equations read

$${}^0 \Delta \mathbf{v} = 0 \quad (7.261)$$

If `usePenalty = True`, the algebraic equations are changed to:

$${}^0 \Delta \mathbf{p} - \frac{1}{k} \mathbf{z} = 0. \quad (7.262)$$

If `activeConnector = False`, the algebraic equations are changed to:

$$\lambda_0 = 0, \quad (7.263)$$

$$\lambda_1 = 0. \quad (7.264)$$

#### 7.2.32.5 Post Newton Step

After the Newton solver has converged, a `PostNewtonStep` is performed for the element, which updates the marker  $m1$  index if necessary.

$$\begin{aligned} s_{el} < 0 &\rightarrow x_{data0} -= 1 \\ s_{el} > L &\rightarrow x_{data0} += 1 \end{aligned} \quad (7.265)$$

Furthermore, it is checked, if  $x_{data0}$  becomes smaller than zero, which raises a warning and keeps  $x_{data0} = 0$ . The same results if  $x_{data0} \geq sn$ , then  $x_{data0} = sn$ . Finally, the data coordinate is updated in order to provide the starting value for the next step,

$$x_{data1} += s. \quad (7.266)$$

For further examples on `ObjectJointALEMoving2D` see Examples:

- [`ANCF\_moving\_rigidbody.py`](#)

For further examples on `ObjectJointALEMoving2D` see TestModels:

- [`ANCFmovingRigidBodyTest.py`](#)

## 7.3 Markers

### 7.3.1 MarkerBodyMass

A marker attached to the body mass; use this marker to apply a body-load (e.g. gravitational force).

**Additional information for MarkerBodyMass:**

- The Marker has the following types = Object, Body, BodyMass

The item **MarkerBodyMass** with type = 'BodyMass' has the following parameters:

| Name          | type            | size | default value | description                                |
|---------------|-----------------|------|---------------|--------------------------------------------|
| name          | String          |      | "             | marker's unique name                       |
| bodyNumber    | ObjectIndex     |      | MAXINT        | body number to which marker is attached to |
| visualization | VMarkerBodyMass |      |               | parameters for visualization of item       |

The item VMarkerBodyMass has the following parameters:

| Name | type | size | default value | description                                                              |
|------|------|------|---------------|--------------------------------------------------------------------------|
| show | Bool |      | True          | set true, if item is shown in visualization and false if it is not shown |

---

#### 7.3.1.1 DESCRIPTION of MarkerBodyMass:

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For further examples on MarkerBodyMass see Examples:

- [ALEANCF\\_pipe.py](#)
- [ANCF\\_moving\\_rigidbody.py](#)
- [ANCF\\_slidingJoint2D.py](#)
- [ANCF\\_switchingSlidingJoint2D.py](#)
- [NGsolveCMSTutorial.py](#)
- [NGsolveCraigBampton.py](#)
- [NGsolvePostProcessingStresses.py](#)
- [rigid3Dexample.py](#)
- [rigidBodyIMUtest.py](#)

For further examples on MarkerBodyMass see TestModels:

- [genericJointUserFunctionTest.py](#)
- [modelUnitTests.py](#)
- [sphericalJointTest.py](#)

### 7.3.2 MarkerBodyPosition

A position body-marker attached to local position (x,y,z) of the body.

#### Additional information for MarkerBodyPosition:

- The Marker has the following types = Object, Body, Position

The item **MarkerBodyPosition** with type = 'BodyPosition' has the following parameters:

| Name          | type                | size | default value | description                                                                               |
|---------------|---------------------|------|---------------|-------------------------------------------------------------------------------------------|
| name          | String              |      | "             | marker's unique name                                                                      |
| bodyNumber    | ObjectIndex         |      | MAXINT        | body number to which marker is attached to                                                |
| localPosition | Vector3D            | 3    | [0,0,0.]      | local body position of marker; e.g. local (body-fixed) position where force is applied to |
| visualization | VMarkerBodyPosition |      |               | parameters for visualization of item                                                      |

The item VMarkerBodyPosition has the following parameters:

| Name | type | size | default value | description                                                              |
|------|------|------|---------------|--------------------------------------------------------------------------|
| show | Bool |      | True          | set true, if item is shown in visualization and false if it is not shown |

---

#### 7.3.2.1 DESCRIPTION of MarkerBodyPosition:

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For further examples on MarkerBodyPosition see Examples:

- [ANCF\\_contact\\_circle.py](#)
- [ANCF\\_contact\\_circle2.py](#)
- [ANCF\\_moving\\_rigidbody.py](#)
- [ANCF\\_slidingJoint2D.py](#)
- [ANCF\\_switchingSlidingJoint2D.py](#)
- [cartesianSpringDamper.py](#)
- [coordinateSpringDamper.py](#)
- [flexibleRotor3Dtest.py](#)
- [geneticOptimizationSliderCrank.py](#)
- [lavalRotor2Dtest.py](#)
- ...

For further examples on MarkerBodyPosition see TestModels:

- [ACNFslidingAndALEjointTest.py](#)
- [ANCFcontactCircleTest.py](#)

- [ANCFcontactFrictionTest.py](#)
- [ANCFmovingRigidBodyTest.py](#)
- [compareFullModifiedNewton.py](#)
- [explicitLieGroupIntegratorPythonTest.py](#)
- [explicitLieGroupIntegratorTest.py](#)
- [explicitLieGroupMBSTest.py](#)
- [genericODE2test.py](#)
- [heavyTop.py](#)
- ...

### 7.3.3 MarkerBodyRigid

A rigid-body (position+orientation) body-marker attached to local position (x,y,z) of the body.

#### Additional information for MarkerBodyRigid:

- The Marker has the following types = Object, Body, Position, Orientation

The item **MarkerBodyRigid** with type = 'BodyRigid' has the following parameters:

| Name          | type             | size | default value | description                                                                               |
|---------------|------------------|------|---------------|-------------------------------------------------------------------------------------------|
| name          | String           |      | "             | marker's unique name                                                                      |
| bodyNumber    | ObjectIndex      |      | MAXINT        | body number to which marker is attached to                                                |
| localPosition | Vector3D         | 3    | [0,0,0.]      | local body position of marker; e.g. local (body-fixed) position where force is applied to |
| visualization | VMarkerBodyRigid |      |               | parameters for visualization of item                                                      |

The item VMarkerBodyRigid has the following parameters:

| Name | type | size | default value | description                                                              |
|------|------|------|---------------|--------------------------------------------------------------------------|
| show | Bool |      | True          | set true, if item is shown in visualization and false if it is not shown |

---

#### 7.3.3.1 DESCRIPTION of MarkerBodyRigid:

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For further examples on MarkerBodyRigid see Examples:

- [ANCF\\_contact\\_circle.py](#)
- [ANCF\\_contact\\_circle2.py](#)
- [ANCF\\_slidingJoint2D.py](#)
- [ANCF\\_tests2.py](#)
- [ANCF\\_test\\_halfcircle.py](#)
- [flexibleRotor3Dtest.py](#)
- [geneticOptimizationSliderCrank.py](#)
- [mouseInteractionExample.py](#)
- [NGsolveCMSTutorial.py](#)
- [NGsolveCraigBampton.py](#)
- ...

For further examples on MarkerBodyRigid see TestModels:

- [ACNFslidingAndALEjointTest.py](#)
- [ANCFcontactCircleTest.py](#)

- [ANCFcontactFrictionTest.py](#)
- [ANCFmovingRigidBodyTest.py](#)
- [carRollingDiscTest.py](#)
- [connectorRigidBodySpringDamperTest.py](#)
- [driveTrainTest.py](#)
- [genericJointUserFunctionTest.py](#)
- [manualExplicitIntegrator.py](#)
- [mecanumWheelRollingDiscTest.py](#)
- ...

### 7.3.4 MarkerNodePosition

A node-Marker attached to a position-based node.

#### Additional information for MarkerNodePosition:

- The Marker has the following types = Node, Position

The item **MarkerNodePosition** with type = 'NodePosition' has the following parameters:

| Name          | type                | size | default value | description                                |
|---------------|---------------------|------|---------------|--------------------------------------------|
| name          | String              |      | ''            | marker's unique name                       |
| nodeNumber    | NodeIndex           |      | MAXINT        | node number to which marker is attached to |
| visualization | VMarkerNodePosition |      |               | parameters for visualization of item       |

The item VMarkerNodePosition has the following parameters:

| Name | type | size | default value | description                                                              |
|------|------|------|---------------|--------------------------------------------------------------------------|
| show | Bool |      | True          | set true, if item is shown in visualization and false if it is not shown |

---

#### 7.3.4.1 DESCRIPTION of MarkerNodePosition:

---

For further examples on MarkerNodePosition see Examples:

- [ALEANCF\\_pipe.py](#)
- [ANCF\\_cantilever\\_test.py](#)
- [ANCF\\_cantilever\\_test\\_dyn.py](#)
- [ANCF\\_contact\\_circle.py](#)
- [ANCF\\_contact\\_circle2.py](#)
- [ANCF\\_slidingJoint2D.py](#)
- [ANCF\\_tests2.py](#)
- [ANCF\\_test\\_halfcircle.py](#)
- [flexibleRotor3Dtest.py](#)
- [geneticOptimizationSliderCrank.py](#)
- ...

For further examples on MarkerNodePosition see TestModels:

- [ANCFcontactCircleTest.py](#)
- [ANCFcontactFrictionTest.py](#)
- [fourBarMechanismTest.py](#)
- [manualExplicitIntegrator.py](#)
- [modelUnitTests.py](#)

- objectFFRFTest.py
- pendulumFriction.py
- sliderCrankFloatingTest.py
- sphericalJointTest.py

### 7.3.5 MarkerNodeRigid

A rigid-body (position+orientation) node-marker attached to a rigid-body node.

#### Additional information for MarkerNodeRigid:

- The Marker has the following types = Node, Position, Orientation

The item **MarkerNodeRigid** with type = 'NodeRigid' has the following parameters:

| Name          | type             | size | default value | description                                |
|---------------|------------------|------|---------------|--------------------------------------------|
| name          | String           |      | ''            | marker's unique name                       |
| nodeNumber    | NodeIndex        |      | MAXINT        | node number to which marker is attached to |
| visualization | VMarkerNodeRigid |      |               | parameters for visualization of item       |

The item VMarkerNodeRigid has the following parameters:

| Name | type | size | default value | description                                                              |
|------|------|------|---------------|--------------------------------------------------------------------------|
| show | Bool |      | True          | set true, if item is shown in visualization and false if it is not shown |

---

#### 7.3.5.1 DESCRIPTION of MarkerNodeRigid:

---

For further examples on MarkerNodeRigid see Examples:

- [ANCF\\_contact\\_circle.py](#)
- [ANCF\\_contact\\_circle2.py](#)
- [ANCF\\_slidingJoint2D.py](#)
- [ANCF\\_tests2.py](#)
- [ANCF\\_test\\_halfcircle.py](#)
- [NGsolveCMSTutorial.py](#)
- [NGsolveCraigBampton.py](#)
- [NGsolvePostProcessingStresses.py](#)
- [objectFFRFreducedOrderNetgen.py](#)
- [sliderCrankCMSacme.py](#)
- ...

For further examples on MarkerNodeRigid see TestModels:

- [ANCFcontactCircleTest.py](#)
- [connectorRigidBodySpringDamperTest.py](#)
- [geometricallyExactBeam2Dtest.py](#)
- [manualExplicitIntegrator.py](#)
- [objectFFRFreducedOrderAccelerations.py](#)

- [objectFFRFreducedOrderStressModesTest.py](#)
- [objectFFRFreducedOrderTest.py](#)
- [objectFFRFTTest.py](#)
- [objectFFRFTTest2.py](#)
- [superElementRigidJointTest.py](#)
- ...

### 7.3.6 MarkerNodeCoordinate

A node-Marker attached to a ODE2 coordinate of a node; for other coordinates (ODE1,...) other markers need to be defined.

**Additional information for MarkerNodeCoordinate:**

- The Marker has the following types = Node, Coordinate

The item **MarkerNodeCoordinate** with type = 'NodeCoordinate' has the following parameters:

| Name          | type                  | size | default value | description                                       |
|---------------|-----------------------|------|---------------|---------------------------------------------------|
| name          | String                |      | "             | marker's unique name                              |
| nodeNumber    | NodeIndex             |      | MAXINT        | node number to which marker is attached to        |
| coordinate    | UInt                  |      | MAXINT        | coordinate of node to which marker is attached to |
| visualization | VMarkerNodeCoordinate |      |               | parameters for visualization of item              |

The item **VMarkerNodeCoordinate** has the following parameters:

| Name | type | size | default value | description                                                              |
|------|------|------|---------------|--------------------------------------------------------------------------|
| show | Bool |      | True          | set true, if item is shown in visualization and false if it is not shown |

---

#### 7.3.6.1 DESCRIPTION of MarkerNodeCoordinate:

---

For further examples on MarkerNodeCoordinate see Examples:

- [ALEANCF\\_pipe.py](#)
- [ANCFALEtest.py](#)
- [ANCF\\_cantilever\\_test.py](#)
- [ANCF\\_cantilever\\_test\\_dyn.py](#)
- [ANCF\\_contact\\_circle.py](#)
- [ANCF\\_contact\\_circle2.py](#)
- [ANCF\\_moving\\_rigidbody.py](#)
- [ANCF\\_slidingJoint2D.py](#)
- [ANCF\\_switchingSlidingJoint2D.py](#)
- [ANCF\\_tests2.py](#)
- ...

For further examples on MarkerNodeCoordinate see TestModels:

- [ACNFslidingAndALEjointTest.py](#)
- [ANCFcontactCircleTest.py](#)

- [ANCFcontactFrictionTest.py](#)
- [ANCFmovingRigidBodyTest.py](#)
- [computeODE2EigenvaluesTest.py](#)
- [driveTrainTest.py](#)
- [explicitLieGroupIntegratorPythonTest.py](#)
- [explicitLieGroupIntegratorTest.py](#)
- [explicitLieGroupMBSTest.py](#)
- [fourBarMechanismTest.py](#)
- ...

### 7.3.7 MarkerNodeCoordinate

A node-Marker attached to a ODE2 coordinate of a node; this marker allows to connect a coordinate-based constraint or connector to a nodal coordinate (also NodeGround); for ODE1 coordinates use MarkerNodeODE1Coordinate.

**Additional information for MarkerNodeCoordinate:**

- The Marker has the following types = Node, Coordinate

The item **MarkerNodeCoordinate** with type = 'NodeCoordinate' has the following parameters:

| Name          | type                  | size | default value | description                                       |
|---------------|-----------------------|------|---------------|---------------------------------------------------|
| name          | String                |      | ''            | marker's unique name                              |
| nodeNumber    | NodeIndex             |      | MAXINT        | node number to which marker is attached to        |
| coordinate    | UInt                  |      | MAXINT        | coordinate of node to which marker is attached to |
| visualization | VMarkerNodeCoordinate |      |               | parameters for visualization of item              |

The item **VMarkerNodeCoordinate** has the following parameters:

| Name | type | size | default value | description                                                              |
|------|------|------|---------------|--------------------------------------------------------------------------|
| show | Bool |      | True          | set true, if item is shown in visualization and false if it is not shown |

---

#### 7.3.7.1 DESCRIPTION of MarkerNodeCoordinate:

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For further examples on MarkerNodeCoordinate see Examples:

- [ALEANCF\\_pipe.py](#)
- [ANCFALTest.py](#)
- [ANCF\\_cantilever\\_test.py](#)
- [ANCF\\_cantilever\\_test\\_dyn.py](#)
- [ANCF\\_contact\\_circle.py](#)
- [ANCF\\_contact\\_circle2.py](#)
- [ANCF\\_moving\\_rigidbody.py](#)
- [ANCF\\_slidingJoint2D.py](#)
- [ANCF\\_switchingSlidingJoint2D.py](#)
- [ANCF\\_tests2.py](#)
- ...

For further examples on MarkerNodeCoordinate see TestModels:

- [ACNFslidingAndALEjointTest.py](#)

- [ANCFcontactCircleTest.py](#)
- [ANCFcontactFrictionTest.py](#)
- [ANCFmovingRigidBodyTest.py](#)
- [computeODE2EigenvaluesTest.py](#)
- [driveTrainTest.py](#)
- [explicitLieGroupIntegratorPythonTest.py](#)
- [explicitLieGroupIntegratorTest.py](#)
- [explicitLieGroupMBSTest.py](#)
- [fourBarMechanismTest.py](#)
- ...

### 7.3.8 MarkerNodeODE1Coordinate

A node-Marker attached to a ODE1 coordinate of a node.

#### Additional information for MarkerNodeODE1Coordinate:

- The Marker has the following types = Node, Coordinate

The item **MarkerNodeODE1Coordinate** with type = 'NodeODE1Coordinate' has the following parameters:

| Name          | type                      | size | default value | description                                       |
|---------------|---------------------------|------|---------------|---------------------------------------------------|
| name          | String                    |      | ''            | marker's unique name                              |
| nodeNumber    | NodeIndex                 |      | MAXINT        | node number to which marker is attached to        |
| coordinate    | UInt                      |      | MAXINT        | coordinate of node to which marker is attached to |
| visualization | VMarkerNodeODE1Coordinate |      |               | parameters for visualization of item              |

The item VMarkerNodeODE1Coordinate has the following parameters:

| Name | type | size | default value | description                                                                                       |
|------|------|------|---------------|---------------------------------------------------------------------------------------------------|
| show | Bool |      | False         | currently not available; set true, if item is shown in visualization and false if it is not shown |

### 7.3.9 MarkerNodeRotationCoordinate

A node-Marker attached to a node containing rotation; the Marker measures a rotation coordinate (Tait-Bryan angles) or angular velocities on the velocity level.

**Additional information for MarkerNodeRotationCoordinate:**

- The Marker has the following types = Node, Orientation, Coordinate

The item **MarkerNodeRotationCoordinate** with type = 'NodeRotationCoordinate' has the following parameters:

| Name               | type                          | size | default value | description                                |
|--------------------|-------------------------------|------|---------------|--------------------------------------------|
| name               | String                        |      | "             | marker's unique name                       |
| nodeNumber         | NodeIndex                     |      | MAXINT        | node number to which marker is attached to |
| rotationCoordinate | UInt                          |      | MAXINT        | rotation coordinate: 0=x, 1=y, 2=z         |
| visualization      | VMarkerNodeRotationCoordinate |      |               | parameters for visualization of item       |

The item VMarkerNodeRotationCoordinate has the following parameters:

| Name | type | size | default value | description                                                              |
|------|------|------|---------------|--------------------------------------------------------------------------|
| show | Bool |      | True          | set true, if item is shown in visualization and false if it is not shown |

---

#### 7.3.9.1 DESCRIPTION of MarkerNodeRotationCoordinate:

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For further examples on MarkerNodeRotationCoordinate see Examples:

- [rigidRotor3DbasicBehaviour.py](#)
- [sliderCrank3DwithANCFbeltDrive2.py](#)

For further examples on MarkerNodeRotationCoordinate see TestModels:

- [driveTrainTest.py](#)

### 7.3.10 MarkerSuperElementPosition

A position marker attached to a SuperElement, such as ObjectFFRF, ObjectGenericODE2 and ObjectFFRFReducedOrder (for which it is in its current implementation inefficient for large number of meshNodeNumbers). The marker acts on the mesh (interface) nodes, not on the underlying nodes of the object.

#### Additional information for MarkerSuperElementPosition:

- The Marker has the following types = Object, Body, Position

The item **MarkerSuperElementPosition** with type = 'SuperElementPosition' has the following parameters:

| Name             | type                        | size | default value | description                                                                                                                                                                                                                                                                                                                                            |
|------------------|-----------------------------|------|---------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| name             | String                      |      | ''            | marker's unique name                                                                                                                                                                                                                                                                                                                                   |
| bodyNumber       | ObjectIndex                 |      | MAXINT        | body number to which marker is attached to                                                                                                                                                                                                                                                                                                             |
| meshNodeNumbers  | ArrayIndex                  |      | []            | a list of $n_m$ mesh node numbers of superelement (=interface nodes) which are used to compute the body-fixed marker position; the related nodes must provide 3D position information, such as NodePoint, NodePoint2D, NodeRigidBody[.]; in order to retrieve the global node number, the generic body needs to convert local into global node numbers |
| weightingFactors | Vector                      |      | []            | a list of $n_m$ weighting factors per node to compute the final local position; the sum of these weights shall be 1, such that a summation of all nodal positions times weights gives the average position of the marker                                                                                                                               |
| visualization    | VMarkerSuperElementPosition |      |               | parameters for visualization of item                                                                                                                                                                                                                                                                                                                   |

The item VMarkerSuperElementPosition has the following parameters:

| Name            | type | size | default value | description                                                                   |
|-----------------|------|------|---------------|-------------------------------------------------------------------------------|
| show            | Bool |      | True          | set true, if item is shown in visualization and false if it is not shown      |
| showMarkerNodes | Bool |      | True          | set true, if all nodes are shown (similar to marker, but with less intensity) |

---

#### 7.3.10.1 DESCRIPTION of MarkerSuperElementPosition:

##### Information on input parameters:

| input parameter  | symbol                      | description see tables above |
|------------------|-----------------------------|------------------------------|
| bodyNumber       | $n_b$                       |                              |
| meshNodeNumbers  | $[k_0, \dots, k_{n_m-1}]^T$ |                              |
| weightingFactors | $[w_0, \dots, w_{n_m-1}]^T$ |                              |

## Definition of marker quantities:

| intermediate variables | symbol                                                 | description                                                                                                                   |
|------------------------|--------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------|
| number of mesh nodes   | $n_m$                                                  | size of meshNodeNumbers and weightingFactors which are marked; this must not be the number of mesh nodes in the marked object |
| mesh node number       | $i = k_i$                                              | abbreviation                                                                                                                  |
| mesh node points       | ${}^0\mathbf{p}_i$                                     | position of mesh node $k_i$ in object $n_b$                                                                                   |
| mesh node velocities   | ${}^0\mathbf{v}_i$                                     | velocity of mesh node $i$ in object $n_b$                                                                                     |
| marker position        | ${}^0\mathbf{p}_m = \sum_i w_i \cdot {}^0\mathbf{p}_i$ | current global position which is provided by marker                                                                           |
| marker velocity        | ${}^0\mathbf{v}_m = \sum_i w_i \cdot {}^0\mathbf{v}_i$ | current global velocity which is provided by marker                                                                           |

### 7.3.10.2 Marker quantities

The marker provides a 'position' jacobian, which is the derivative of the marker velocity w.r.t. the object velocity coordinates  $\dot{\mathbf{q}}_{n_b}$ ,

$$\mathbf{J}_{m,pos} = \frac{\partial {}^0\mathbf{v}_m}{\partial \dot{\mathbf{q}}_{n_b}} = \sum_i w_i \cdot \mathbf{J}_{i,pos} \quad (7.267)$$

in which  $\mathbf{J}_{i,pos}$  denotes the position jacobian of mesh node  $i$ ,

$$\mathbf{J}_{i,pos} = \frac{\partial {}^0\mathbf{v}_i}{\partial \dot{\mathbf{q}}_{n_b}} \quad (7.268)$$

The jacobian  $\mathbf{J}_{i,pos}$  usually contains mostly zeros for ObjectGenericODE2, because the jacobian only affects one single node. In ObjectFFRFreducedOrder, the jacobian may affect all reduced coordinates.

Note that  $\mathbf{J}_{m,pos}$  is actually computed by the ObjectSuperElement within the function GetAccessFunctionSuperElement.

### 7.3.10.3 MINI EXAMPLE for MarkerSuperElementPosition

```
#set up a mechanical system with two nodes; it has the structure: |~~M0~~M1
#==>further examples see objectGenericODE2Test.py, objectFFRFTest2.py, etc.
nMass0 = mbs.AddNode(NodePoint(referenceCoordinates=[0,0,0]))
nMass1 = mbs.AddNode(NodePoint(referenceCoordinates=[1,0,0]))
nGround = mbs.AddMarker(MarkerBodyPosition(bodyNumber=oGround, localPosition =
[1,0,0]))

mass = 0.5 * np.eye(3) #mass of nodes
stif = 5000 * np.eye(3) #stiffness of nodes
damp = 50 * np.eye(3) #damping of nodes
Z = 0. * np.eye(3) #matrix with zeros
#build mass, stiffness and damping matrices :
M = np.block([[mass, 0.*np.eye(3)],
 [0.*np.eye(3), mass]])
K = np.block([[2*stif, -stif],
```

```

 [-stif, stiff]])
D = np.block([[2*damp, -damp],
 [-damp, damp]]]

oGenericODE2 = mbs.AddObject(ObjectGenericODE2(nodeNumbers=[nMass0,nMass1],
 massMatrix=M,
 stiffnessMatrix=K,
 dampingMatrix=D))

#EXAMPLE for single node marker on super element body, mesh node 1; compare results
to ObjectGenericODE2 example!!!
mSuperElement = mbs.AddMarker(MarkerSuperElementPosition(bodyNumber=oGenericODE2,
 meshNodeNumbers=[1], weightingFactors=[1]))
mbs.AddLoad(Force(markerNumber = mSuperElement, loadVector = [10, 0, 0]))

#assemble and solve system for default parameters
mbs.Assemble()

exu.SolveDynamic(mbs, solverType = exudyn.DynamicSolverType.TrapezoidalIndex2)

#check result at default integration time
exudynTestGlobals.testResult = mbs.GetNodeOutput(nMass1, exu.OutputVariableType.
 Position)[0]

```

For further examples on MarkerSuperElementPosition see Examples:

- [NGsolvePistonEngine.py](#)
- [NGsolvePostProcessingStresses.py](#)
- [objectFFRFreducedOrderNetgen.py](#)
- [sliderCrankCMSacme.py](#)

For further examples on MarkerSuperElementPosition see TestModels:

- [objectFFRFreducedOrderAccelerations.py](#)
- [objectFFRFreducedOrderStressModesTest.py](#)
- [objectFFRFreducedOrderTest.py](#)
- [objectFFRFTTest.py](#)
- [objectFFRFTTest2.py](#)
- [objectGenericODE2Test.py](#)
- [superElementRigidJointTest.py](#)

### 7.3.11 MarkerSuperElementRigid

A position and orientation (rigid-body) marker attached to a SuperElement, such as ObjectFFRF, ObjectGenericODE2 and ObjectFFRFreducedOrder (for which it may be inefficient). The marker acts on the mesh nodes, not on the underlying nodes of the object. Note that in contrast to the MarkerSuperElementPosition, this marker needs a set of interface nodes which are not aligned at one line, such that these node points can represent a rigid body motion. Note that definitions of marker positions are slightly different from MarkerSuperElementPosition.

#### Additional information for MarkerSuperElementRigid:

- The Marker has the following types = Object, Body, Position, Orientation

The item **MarkerSuperElementRigid** with type = 'SuperElementRigid' has the following parameters:

| Name                   | type                     | size | default value | description                                                                                                                                                                                                                                                                                                                                                                             |
|------------------------|--------------------------|------|---------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| name                   | String                   |      | "             | marker's unique name                                                                                                                                                                                                                                                                                                                                                                    |
| bodyNumber             | ObjectIndex              |      | MAXINT        | body number to which marker is attached to                                                                                                                                                                                                                                                                                                                                              |
| offset                 | Vector3D                 | 3    | [0.,0.,0.]    | local marker SuperElement reference position offset used to correct the center point of the marker, which is computed from the weighted average of reference node positions (which may have some offset to the desired joint position). Note that this offset shall be small and larger offsets can cause instability in simulation models (better to have symmetric meshes at joints). |
| meshNodeNumbers        | ArrayIndex               |      | []            | a list of $n_m$ mesh node numbers of superelement (=interface nodes) which are used to compute the body-fixed marker position and orientation; the related nodes must provide 3D position information, such as NodePoint, NodePoint2D, NodeRigidBody[...]; in order to retrieve the global node number, the generic body needs to convert local into global node numbers                |
| weightingFactors       | Vector                   |      | []            | a list of $n_m$ weighting factors per node to compute the final local position and orientation; these factors could be based on surface integrals of the constrained mesh faces                                                                                                                                                                                                         |
| useAlternativeApproach | Bool                     |      | True          | this flag switches between two versions for the computation of the rotation and angular velocity of the marker; alternative approach uses skew symmetric matrix of reference position; follows the inertia concept                                                                                                                                                                      |
| visualization          | VMarkerSuperElementRigid |      |               | parameters for visualization of item                                                                                                                                                                                                                                                                                                                                                    |

The item VMarkerSuperElementRigid has the following parameters:

| <b>Name</b>     | <b>type</b> | <b>size</b> | <b>default value</b> | <b>description</b>                                                            |
|-----------------|-------------|-------------|----------------------|-------------------------------------------------------------------------------|
| show            | Bool        |             | True                 | set true, if item is shown in visualization and false if it is not shown      |
| showMarkerNodes | Bool        |             | True                 | set true, if all nodes are shown (similar to marker, but with less intensity) |

---

### 7.3.11.1 DESCRIPTION of MarkerSuperElementRigid:

Information on input parameters:

| <b>input parameter</b> | <b>symbol</b>               | <b>description see tables above</b> |
|------------------------|-----------------------------|-------------------------------------|
| bodyNumber             | $n_b$                       |                                     |
| offset                 | ${}^r\mathbf{o}_{ref}$      |                                     |
| meshNodeNumbers        | $[k_0, \dots, k_{n_m-1}]^T$ |                                     |
| weightingFactors       | $[w_0, \dots, w_{n_m-1}]^T$ |                                     |

Definition of marker quantities:

| <b>intermediate variables</b>                      | <b>symbol</b>                                                                          | <b>description</b>                                                                                                                                                                                                                               |
|----------------------------------------------------|----------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| number of mesh nodes                               | $n_m$                                                                                  | size of meshNodeNumbers and weightingFactors which are marked; this must not be the number of mesh nodes in the marked object                                                                                                                    |
| mesh node number                                   | $i = k_i$                                                                              | abbreviation, runs over all marker mesh nodes                                                                                                                                                                                                    |
| mesh node local displacement                       | ${}^r\mathbf{u}^{(i)}$                                                                 | current local (within reference frame $r$ ) displacement of mesh node $k_i$ in object $n_b$                                                                                                                                                      |
| mesh node local position                           | ${}^r\mathbf{p}^{(i)} = {}^r\mathbf{x}_{ref}^{(i)} + {}^r\mathbf{u}^{(i)}$             | current local (within reference frame $r$ , which is the body frame $b$ , e.g., in ObjectFFRFreducedOrder) position of mesh node $k_i$ in object $n_b$                                                                                           |
| mesh node local reference position                 | ${}^r\mathbf{x}_{ref}^{(i)}$                                                           | local (within reference frame $r$ ) reference position of mesh node $k_i$ in object $n_b$ , see e.g. ObjectFFRFreducedOrder                                                                                                                      |
| averaged local reference position                  | ${}^r\mathbf{x}_{ref}^{avg} = \sum_i w_i {}^r\mathbf{x}_{ref}^{(i)}$                   | midpoint reference position of marker; averaged local reference positions of all mesh nodes $k_i$ , using weighting for averaging; may not coincide with center point of your idealized joint surface (e.g., midpoint of cylinder), see Fig. 7.4 |
| marker centered mesh node local reference position | ${}^r\mathbf{p}_{ref}^{(i)} = {}^r\mathbf{x}_{ref}^{(i)} - {}^r\mathbf{x}_{ref}^{avg}$ | local reference position of mesh node $k_i$ relative to the center position of marker                                                                                                                                                            |
| mesh node local velocity                           | ${}^r\mathbf{v}^{(i)}$                                                                 | current local (within reference frame $r$ ) velocity of mesh node $k_i$ in object $n_b$                                                                                                                                                          |
| super element reference position                   | ${}^0\mathbf{p}_r$ ( $= {}^0\mathbf{p}_t$ in ObjectFFRFreducedOrder)                   | current position (origin) of super element's floating frame ( $r$ ), which is zero, if the object does not provide a reference frame (such as GenericODE2)                                                                                       |

|                                |                                                                                                                                                                                                                                          |                                                                                                                                                                                                                                                    |
|--------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| super element rotation matrix  | ${}^0r\mathbf{A}$                                                                                                                                                                                                                        | current rigid body transformation matrix of super element's floating frame (r), which is the identity matrix, if the object does not provide a reference frame (such as GenericODE2)                                                               |
| super element angular velocity | ${}^r\boldsymbol{\omega}_r$                                                                                                                                                                                                              | current local angular velocity of super element's floating frame (r), which is zero, if the object does not provide a reference frame (such as GenericODE2)                                                                                        |
| marker position                | ${}^0\mathbf{p}_m = {}^0\mathbf{p}_r + {}^0r\mathbf{A} \left( {}^r\mathbf{o}_{ref} + \sum_i w_i \cdot {}^r\mathbf{p}^{(i)} \right)$                                                                                                      | current global position which is provided by marker; note offset ${}^r\mathbf{o}_{ref}$ added, if used as a correction of marker mesh nodes                                                                                                        |
| marker velocity                | $\begin{aligned} {}^0\mathbf{v}_m &= {}^0\dot{\mathbf{p}}_r + \\ {}^0r\mathbf{A} \left( {}^r\tilde{\boldsymbol{\omega}}_r \sum_i (w_i \cdot {}^r\mathbf{p}^{(i)}) + \sum_i (w_i \cdot {}^r\dot{\mathbf{u}}^{(i)}) \right) \end{aligned}$ | current global velocity which is provided by marker                                                                                                                                                                                                |
| marker rotation matrix         | ${}^0r\mathbf{A}_m = {}^0r\mathbf{A} \cdot \exp({}^r\boldsymbol{\theta}_m)$                                                                                                                                                              | current rotation matrix, which transforms the local marker coordinates and adds the rigid body transformation of floating frames ${}^0r\mathbf{A}$ ; uses exponential map for SO3, assumes that $\boldsymbol{\theta}$ represents a rotation vector |
| marker local rotation          | ${}^r\boldsymbol{\theta}_m$                                                                                                                                                                                                              | current local linearized rotations (rotation vector); for the computation, see below for the standard and alternative approach                                                                                                                     |
| marker local angular velocity  | ${}^r\boldsymbol{\omega}_m$                                                                                                                                                                                                              | local angular velocity due to mesh node velocity only; for the computation, see below for the standard and alternative approach                                                                                                                    |
| marker global angular velocity | ${}^0\boldsymbol{\omega}_m = {}^0\boldsymbol{\omega}_r + {}^0r\mathbf{A} {}^r\boldsymbol{\omega}_m$                                                                                                                                      | current global angular velocity                                                                                                                                                                                                                    |

### 7.3.11.2 Marker background

The marker allows to realize a multi-point constraint (assuming that the marker is used in a joint constraint), connecting to averaged nodal displacements and rotations (also known as RBE3 in NASTRAN), see e.g. [21]. However, using Craig-Bampton RBE2 modes, will create RBE2 multi-point constraints for ObjectFFRFreducedOrder objects.

For more information on the various quantities and their coordinate systems, see table above and Fig. 7.4.

### 7.3.11.3 Marker quantities

The marker provides a 'position' jacobian, which is the derivative of the global marker velocity w.r.t. the object velocity coordinates  $\dot{\mathbf{q}}_{n_b}$ ,

$${}^0\mathbf{J}_{m, pos} = \frac{\partial {}^0\mathbf{v}_m}{\partial \dot{\mathbf{q}}_{n_b}} . \quad (7.269)$$

In case of ObjectGenericODE2, assuming pure displacement based nodes, the jacobian will consist of zeros and unit matrices  $\mathbf{I}$ ,

$${}^0\mathbf{J}_{m, pos}^{GenericODE2} = \frac{\partial {}^0\mathbf{v}_m}{\partial \dot{\mathbf{q}}_{n_b}} = [0, \dots, 0, \mathbf{I}, 0, \dots, 0, \mathbf{I}, 0, \dots, 0] , \quad (7.270)$$

in which the  $\mathbf{I}$  matrices are placed at the according indices of marker nodes.

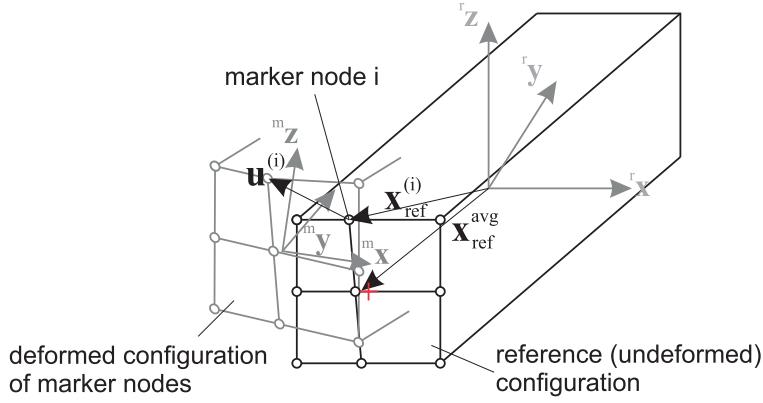


Figure 7.4: Sketch of marker nodes, exemplary node  $i$ , reference coordinates and marker coordinate system; note the difference of the center of the marker ‘surface’ (rectangle) marked with the red cross, and the averaged of the averaged local reference position.

In case of `ObjectFFRFreducedOrder`, this jacobian is computed as weighted sum of the position jacobians, see `ObjectFFRFreducedOrder`,

$${}^0\mathbf{J}_{m, pos}^{FFRFreduced} = \frac{\partial {}^0\mathbf{v}_m}{\partial \dot{\mathbf{q}}_{nb}} = \sum_i w_i {}^0\mathbf{J}_{pos}^{(i)} = \left[ \mathbf{I}, - {}^{0r}\mathbf{A} \left( \sum_i ({}^r\tilde{\mathbf{u}}_f^{(i)} + {}^r\tilde{\mathbf{x}}_{ref}^{(i)}) \right) {}^r\mathbf{G}, \sum_i w_i {}^{0r}\mathbf{A} \begin{bmatrix} {}^r\Psi_{r=3i}^T \\ {}^r\Psi_{r=3i+1}^T \\ {}^r\Psi_{r=3i+2}^T \end{bmatrix} \right]. \quad (7.271)$$

In `ObjectFFRFreducedOrder`, the jacobian usually affects all reduced coordinates.

#### 7.3.11.4 Standard approach for computation of rotation (`useAlternativeApproach = False`)

As compared to `MarkerSuperElementPosition`, `MarkerSuperElementRigid` also links the marker to the orientation of the set of nodes provided. For this reason, the check performed in `mbs.assemble()` will take care that the nodes are capable to describe rotations. The first approach, here called as a standard, follows the idea that displacements contribute to rotation are weighted by their quadratic distance, cf. [21], and gives the (small rotation) rotation vector

$${}^r\boldsymbol{\theta}_m = \frac{\sum_i w_i {}^r\mathbf{p}_{ref}^{(i)} \times {}^r\mathbf{u}^{(i)}}{\sum_i w_i |{}^r\mathbf{p}_{ref}^{(i)}|^2} \quad (7.272)$$

Note that  $\mathbf{p}_{ref}^{(i)}$  is not the reference position in the `ObjectFFRFreducedOrder` object, but it is relative to the midpoint reference position all marker nodes, given in  ${}^r\mathbf{x}_{ref}^{avg}$ . Accordingly, the marker local angular velocity can be calculated as

$${}^r\boldsymbol{\omega}_m = {}^r\dot{\boldsymbol{\theta}}_m = \frac{\sum_i w_i {}^r\tilde{\mathbf{p}}_{ref}^{(i)} {}^r\mathbf{v}_i}{\sum_i w_i |{}^r\mathbf{p}_{ref}^{(i)}|^2} \quad (7.273)$$

The marker also provides a ‘rotation’ jacobian, which is the derivative of the marker angular velocity  ${}^0\boldsymbol{\omega}_m$  w.r.t. the object velocity coordinates  $\dot{\mathbf{q}}_{nb}$ ,

$${}^0\mathbf{J}_{m, rot} = \frac{\partial {}^0\boldsymbol{\omega}_m}{\partial \dot{\mathbf{q}}_{nb}} = \frac{\partial {}^{0r}\mathbf{A} ({}^r\boldsymbol{\omega}_r + {}^r\boldsymbol{\omega}_m)}{\partial \dot{\mathbf{q}}_{nb}} = {}^{0r}\mathbf{A} \left( \frac{\partial {}^r\boldsymbol{\omega}_r}{\partial \dot{\mathbf{q}}_{nb}} + \frac{\sum_i w_i {}^r\tilde{\mathbf{p}}_{ref}^{(i)} {}^r\mathbf{J}_{pos}^{(i)}}{\sum_i w_i |{}^r\mathbf{p}_{ref}^{(i)}|^2} \right) \quad (7.274)$$

In case of `ObjectFFRFreducedOrder`, this jacobian is computed as

$${}^0\mathbf{J}_{m,rot}^{FFRFreduced} = \left[ \mathbf{0}, {}^{0r}\mathbf{A} {}^r\mathbf{G}_{local}, {}^{0r}\mathbf{A} \frac{\sum_i w_i {}^r\tilde{\mathbf{p}}_{ref}^{(i)} {}^r\mathbf{J}_{pos,f}^{(i)}}{\sum_i w_i |{}^r\tilde{\mathbf{p}}_{ref}^{(i)}|^2} \right] \quad (7.275)$$

in which you should know that

- we used  $\frac{\partial {}^r\boldsymbol{\omega}_r}{\partial \theta_r} = {}^r\mathbf{G}_{local}$ ,
- $\theta_r$  represent the rotation parameters for the rigid body node of `ObjectFFRFreducedOrder`,
- ${}^r\mathbf{J}_{pos,f}^{(i)}$  is the **local** jacobian, which only includes the flexible part of the local jacobian for a single mesh node,  ${}^r\mathbf{J}_{pos}^{(i)}$  (note the small  $r$  on the upper left), as defined in `ObjectFFRFreducedOrder`.

For further quantities also consult the according description in `ObjectFFRFreducedOrder`.

### 7.3.11.5 Alternative computation of rotation (`useAlternativeApproach = True`)

Note that this approach is **still under development** and needs further validation. However, tests show that this model is superior to the standard approach, as it improves the averaging of motion w.r.t. rotations at the marker nodes.

In the alternative approach, the weighting matrix  $\mathbf{W}$  has the interpretation of an inertia tensor built from nodes using weights equal to node masses. In such an interpretation, the 'local angular momentum' w.r.t. the marker (averaged) position can be computed as

$$\mathbf{W} {}^r\boldsymbol{\omega}_m = \sum_i w_i {}^r\tilde{\mathbf{p}}_{ref}^{(i)} ({}^r\mathbf{v}^{(i)} - {}^r\mathbf{v}^{\text{avg}}) = - \sum_i (w_i {}^r\tilde{\mathbf{p}}_{ref}^{(i)} {}^r\tilde{\mathbf{p}}_{ref}^{(i)}) {}^r\boldsymbol{\omega}_m \quad (7.276)$$

which implicitly defines the weighting matrix  $\mathbf{W}$ , which must be invertable (but it is only a  $3 \times 3$  matrix!),

$$\mathbf{W} = - \sum_i w_i {}^r\tilde{\mathbf{p}}_{ref}^{(i)} {}^r\tilde{\mathbf{p}}_{ref}^{(i)} \quad (7.277)$$

Furthermore, we need to introduce the averaged velocity of the marker averaged reference position, using  ${}^r\dot{\mathbf{u}}^{(i)} = {}^r\mathbf{v}^{(i)}$ , which is defined as

$${}^r\mathbf{v}^{\text{avg}} = \sum_i w_i {}^r\mathbf{v}^{(i)}, \quad (7.278)$$

similar to the averaged local reference position  ${}^r\mathbf{x}_{ref}^{\text{avg}}$  given in the table above, see also Fig. 7.4.

In the alternative approach, thus the marker local rotations read

$${}^r\boldsymbol{\theta}_{m,alt} = \mathbf{W}^{-1} \sum_i w_i {}^r\tilde{\mathbf{p}}_{ref}^{(i)} ({}^r\mathbf{u}^{(i)} - {}^r\mathbf{x}_{ref}^{\text{avg}}), \quad (7.279)$$

and the marker local angular velocity is defined as

$${}^r\boldsymbol{\omega}_{m,alt} = \mathbf{W}^{-1} \sum_i w_i {}^r\tilde{\mathbf{p}}_{ref}^{(i)} ({}^r\mathbf{v}^{(i)} - {}^r\mathbf{v}^{\text{avg}}). \quad (7.280)$$

Note that, the average velocity  ${}^r\mathbf{v}^{\text{avg}}$  would cancel out in a symmetric mesh, but would cause spurious angular velocities in unsymmetric (w.r.t. the axis of rotation) distribution of mesh nodes. This could even lead to spurious rotations or angular velocities in pure translatory motion.

In the alternative mode, the Jacobian for the rotation / angular velocity is defined as

$${}^0\mathbf{J}_{m,rot,alt} = \frac{\partial {}^0\boldsymbol{\omega}_m}{\partial \dot{\mathbf{q}}_{nb}} = \frac{\partial {}^{0r}\mathbf{A} ({}^r\boldsymbol{\omega}_r + {}^r\boldsymbol{\omega}_m)}{\partial \dot{\mathbf{q}}_{nb}} = {}^{0r}\mathbf{A} \left( \frac{\partial {}^r\boldsymbol{\omega}_r}{\partial \dot{\mathbf{q}}_{nb}} + \mathbf{W}^{-1} \sum_i w_i {}^r\tilde{\mathbf{p}}_{ref}^{(i)} {}^r\mathbf{J}_{pos}^{(i)} \right) \quad (7.281)$$

In case of `ObjectFFRFreducedOrder`, this jacobian is computed as

$${}^0\mathbf{J}_{m,rot,alt}^{FFRFreduced} = \left[ \mathbf{0}, {}^0r\mathbf{A} {}^r\mathbf{G}_{local}, {}^0r\mathbf{A} \mathbf{W}^{-1} \sum_i w_i {}^r\tilde{\mathbf{p}}_{ref}^{(i)} {}^r\mathbf{J}_{pos,f}^{(i)} \right] \quad (7.282)$$

see also the descriptions given after Eq. (7.275) in the ‘standard’ approach.

#### EXAMPLE for marker on body 4, mesh nodes 10,11,12,13:

```
MarkerSuperElementRigid(bodyNumber = 4, meshNodeNumber = [10, 11, 12, 13], weightingFactors = [0.25, 0.25, 0.25, 0.25], referencePosition=[0,0,0])
```

For detailed examples, see `TestModels`.

For further examples on `MarkerSuperElementRigid` see Examples:

- [`NGsolveCMSTutorial.py`](#)
- [`NGsolveCraigBampton.py`](#)

For further examples on `MarkerSuperElementRigid` see `TestModels`:

- [`superElementRigidJointTest.py`](#)

### 7.3.12 MarkerObjectODE2Coordinates

A Marker attached to all coordinates of an object (currently only body is possible), e.g. to apply special constraints or loads on all coordinates. The measured coordinates INCLUDE reference + current coordinates. This marker is under development and UNTESTED.

#### Additional information for MarkerObjectODE2Coordinates:

- The Marker has the following types = Object, Body, Coordinate

The item **MarkerObjectODE2Coordinates** with type = 'ObjectODE2Coordinates' has the following parameters:

| Name          | type                         | size | default value | description                                |
|---------------|------------------------------|------|---------------|--------------------------------------------|
| name          | String                       |      | "             | marker's unique name                       |
| objectNumber  | ObjectIndex                  |      | MAXINT        | body number to which marker is attached to |
| visualization | VMarkerObjectODE2Coordinates |      |               | parameters for visualization of item       |

The item VMarkerObjectODE2Coordinates has the following parameters:

| Name | type | size | default value | description                                                              |
|------|------|------|---------------|--------------------------------------------------------------------------|
| show | Bool |      | True          | set true, if item is shown in visualization and false if it is not shown |

### 7.3.13 MarkerBodyCable2DShape

A special Marker attached to a 2D ANCF beam finite element with cubic interpolation and 8 coordinates.

#### Additional information for MarkerBodyCable2DShape:

- The Marker has the following types = Object, Body, Coordinate

The item **MarkerBodyCable2DShape** with type = 'BodyCable2DShape' has the following parameters:

| Name             | type                    | size | default value | description                                                                                                                             |
|------------------|-------------------------|------|---------------|-----------------------------------------------------------------------------------------------------------------------------------------|
| name             | String                  |      | "             | marker's unique name                                                                                                                    |
| bodyNumber       | ObjectIndex             |      | MAXINT        | body number to which marker is attached to                                                                                              |
| numberOfSegments | PInt                    |      | 3             | number of segments; each segment is a line and is associated to a data (history) variable; must be same as in according contact element |
| visualization    | VMarkerBodyCable2DShape |      |               | parameters for visualization of item                                                                                                    |

The item **VMarkerBodyCable2DShape** has the following parameters:

| Name | type | size | default value | description                                                              |
|------|------|------|---------------|--------------------------------------------------------------------------|
| show | Bool |      | True          | set true, if item is shown in visualization and false if it is not shown |

---

#### 7.3.13.1 DESCRIPTION of MarkerBodyCable2DShape:

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For further examples on MarkerBodyCable2DShape see Examples:

- [ANCF\\_contact\\_circle.py](#)
- [ANCF\\_contact\\_circle2.py](#)
- [ANCF\\_moving\\_rigidbody.py](#)
- [ANCF\\_slidingJoint2D.py](#)
- [sliderCrank3DwithANCFbeltDrive.py](#)
- [sliderCrank3DwithANCFbeltDrive2.py](#)

For further examples on MarkerBodyCable2DShape see TestModels:

- [ACNFslidingAndALEjointTest.py](#)
- [ANCFcontactCircleTest.py](#)
- [ANCFcontactFrictionTest.py](#)
- [ANCFmovingRigidBodyTest.py](#)

### 7.3.14 MarkerBodyCable2DCoordinates

A special Marker attached to the coordinates of a 2D ANCF beam finite element with cubic interpolation.

**Additional information for MarkerBodyCable2DCoordinates:**

- The Marker has the following types = Object, Body, Coordinate

The item **MarkerBodyCable2DCoordinates** with type = 'BodyCable2DCoordinates' has the following parameters:

| Name          | type                          | size | default value | description                                |
|---------------|-------------------------------|------|---------------|--------------------------------------------|
| name          | String                        |      | "             | marker's unique name                       |
| bodyNumber    | ObjectIndex                   |      | MAXINT        | body number to which marker is attached to |
| visualization | VMarkerBodyCable2DCoordinates |      |               | parameters for visualization of item       |

The item VMarkerBodyCable2DCoordinates has the following parameters:

| Name | type | size | default value | description                                                              |
|------|------|------|---------------|--------------------------------------------------------------------------|
| show | Bool |      | True          | set true, if item is shown in visualization and false if it is not shown |

---

#### 7.3.14.1 DESCRIPTION of MarkerBodyCable2DCoordinates:

---

For further examples on MarkerBodyCable2DCoordinates see Examples:

- [ANCF\\_moving\\_rigidbody.py](#)
- [ANCF\\_slidingJoint2D.py](#)
- [ANCF\\_switchingSlidingJoint2D.py](#)

For further examples on MarkerBodyCable2DCoordinates see TestModels:

- [ANCFmovingRigidBodyTest.py](#)
- [modelUnitTests.py](#)

## 7.4 Loads

### 7.4.1 LoadForceVector

Load with (3D) force vector; attached to position-based marker.

**Additional information for LoadForceVector:**

- Requested marker type = Position
- **Short name for Python** = **Force**
- **Short name for Python (visualization object)** = **VForce**

The item **LoadForceVector** with type = 'ForceVector' has the following parameters:

| Name                   | type                                | size | default value | description                                                                                                                                     |
|------------------------|-------------------------------------|------|---------------|-------------------------------------------------------------------------------------------------------------------------------------------------|
| name                   | String                              |      | "             | load's unique name                                                                                                                              |
| markerNumber           | MarkerIndex                         |      | MAXINT        | marker's number to which load is applied                                                                                                        |
| loadVector             | Vector3D                            |      | [0.,0.,0.]    | vector-valued load [SI:N]                                                                                                                       |
| bodyFixed              | Bool                                |      | False         | if bodyFixed is true, the load is defined in body-fixed (local) coordinates, leading to a follower force; if false: global coordinates are used |
| loadVectorUserFunction | PyFunctionVector3DmbsScalarVector3D |      | 0             | A python function which defines the time-dependent load                                                                                         |
| visualization          | VLoadForceVector                    |      |               | parameters for visualization of item                                                                                                            |

The item **VLoadForceVector** has the following parameters:

| Name | type | size | default value | description                                                              |
|------|------|------|---------------|--------------------------------------------------------------------------|
| show | Bool |      | True          | set true, if item is shown in visualization and false if it is not shown |

#### 7.4.1.1 DESCRIPTION of LoadForceVector:

**Information on input parameters:**

| input parameter | symbol | description see tables above |
|-----------------|--------|------------------------------|
| loadVector      | f      |                              |

#### 7.4.1.2 Details

The load vector acts on a body or node via the local (`bodyFixed = True`) or global coordinates of a body or at a node. The marker transforms the (translational) force via the according jacobian matrix of the object (or node) to object (or node) coordinates.

### Userfunction: `loadVectorUserFunction(mbs, t, loadVector)`

A user function, which computes the force vector depending on time and object parameters, which is hereafter applied to object or node.

| arguments / return  | type or size | description                                                                                                                                       |
|---------------------|--------------|---------------------------------------------------------------------------------------------------------------------------------------------------|
| mbs                 | MainSystem   | provides MainSystem mbs to which load belongs                                                                                                     |
| t                   | Real         | current time in mbs                                                                                                                               |
| loadVector          | Vector3D     | f copied from object; WARNING: this parameter does not work in combination with static computation, as it is changed by the solver over step time |
| <b>return value</b> | Vector3D     | computed force vector                                                                                                                             |

---

### User function example:

```
from math import sin, cos, pi
def UFForce(mbs, t, loadVector):
 return [loadVector[0]*sin(t*10*2*pi),0,0]
```

---

For further examples on LoadForceVector see Examples:

- [interactiveTutorial.py](#)
- [SpringDamperMassUserFunction.py](#)
- [ANCF\\_cantilever\\_test.py](#)
- [ANCF\\_cantilever\\_test\\_dyn.py](#)
- [ANCF\\_contact\\_circle.py](#)
- [ANCF\\_contact\\_circle2.py](#)
- [ANCF\\_moving\\_rigidbody.py](#)
- [ANCF\\_slidingJoint2D.py](#)
- [ANCF\\_switchingSlidingJoint2D.py](#)
- [ANCF\\_tests2.py](#)
- ...

For further examples on LoadForceVector see TestModels:

- [ANCFslidingAndALEjointTest.py](#)
- [ANCFcontactCircleTest.py](#)
- [ANCFcontactFrictionTest.py](#)
- [ANCFmovingRigidBodyTest.py](#)
- [compareFullModifiedNewton.py](#)
- [explicitLieGroupIntegratorPythonTest.py](#)
- [explicitLieGroupIntegratorTest.py](#)
- [explicitLieGroupMBSTest.py](#)
- [fourBarMechanismTest.py](#)
- [genericODE2test.py](#)
- ...

## 7.4.2 LoadTorqueVector

Load with (3D) torque vector; attached to rigidbody-based marker.

**Additional information for LoadTorqueVector:**

- Requested marker type = Orientation
- Short name for Python = **Torque**
- Short name for Python (visualization object) = **VTorque**

The item **LoadTorqueVector** with type = 'TorqueVector' has the following parameters:

| Name                   | type                                | size | default value | description                                                                                                                                                                                                                                                                                                                                          |
|------------------------|-------------------------------------|------|---------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| name                   | String                              |      | "             | load's unique name                                                                                                                                                                                                                                                                                                                                   |
| markerNumber           | MarkerIndex                         |      | MAXINT        | marker's number to which load is applied                                                                                                                                                                                                                                                                                                             |
| loadVector             | Vector3D                            |      | [0.,0.,0.]    | vector-valued load [SI:N]                                                                                                                                                                                                                                                                                                                            |
| bodyFixed              | Bool                                |      | False         | if bodyFixed is true, the load is defined in body-fixed (local) coordinates, leading to a follower torque; if false: global coordinates are used                                                                                                                                                                                                     |
| loadVectorUserFunction | PyFunctionVector3DmbsScalarVector3D | 0    |               | A python function which defines the time-dependent load with parameters (Real t, Vector3D load); the load represents the current value of the load; WARNING: this factor does not work in combination with static computation (loadFactor); Example for python function: def f(mbs, t, loadVector): return [loadVector[0]*np.sin(t*10*2*3.1415),0,0] |
| visualization          | VLoadTorqueVector                   |      |               | parameters for visualization of item                                                                                                                                                                                                                                                                                                                 |

The item **VLoadTorqueVector** has the following parameters:

| Name | type | size | default value | description                                                              |
|------|------|------|---------------|--------------------------------------------------------------------------|
| show | Bool |      | True          | set true, if item is shown in visualization and false if it is not shown |

---

### 7.4.2.1 DESCRIPTION of LoadTorqueVector:

**Information on input parameters:**

| input parameter | symbol | description see tables above |
|-----------------|--------|------------------------------|
| loadVector      | $\tau$ |                              |

### 7.4.2.2 Details

The torque vector acts on a body or node via the local (`bodyFixed = True`) or global coordinates of a body or at a node. The marker transforms the torque via the according jacobian matrix of the object (or node) to object (or node) coordinates.

---

#### Userfunction: `loadVectorUserFunction(mbs, t, loadVector)`

A user function, which computes the torque vector depending on time and object parameters, which is hereafter applied to object or node.

| arguments / return        | type or size | description                                                                                                                                            |
|---------------------------|--------------|--------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>mbs</code>          | MainSystem   | provides MainSystem mbs to which load belongs                                                                                                          |
| <code>t</code>            | Real         | current time in mbs                                                                                                                                    |
| <code>loadVector</code>   | Vector3D     | $\tau$ copied from object; WARNING: this parameter does not work in combination with static computation, as it is changed by the solver over step time |
| <code>return value</code> | Vector3D     | computed torque vector                                                                                                                                 |

---

#### User function example:

```
from math import sin, cos, pi
def UForce(mbs, t, loadVector):
 return [loadVector[0]*sin(t*10*2*pi), 0, 0]
```

---

For further examples on LoadTorqueVector see Examples:

- [sliderCrank3DwithANCFbeltDrive2.py](#)
- [ANCF\\_contact\\_circle.py](#)
- [ANCF\\_contact\\_circle2.py](#)
- [ANCF\\_slidingJoint2D.py](#)
- [ANCF\\_tests2.py](#)
- [ANCF\\_test\\_halfcircle.py](#)
- [flexibleRotor3Dtest.py](#)
- [rigid3Dexample.py](#)
- [rigidBodyIMUtest.py](#)
- [rigidRotor3DbasicBehaviour.py](#)
- ...

For further examples on LoadTorqueVector see TestModels:

- [ANCFcontactCircleTest.py](#)
- [ANCFcontactFrictionTest.py](#)
- [carRollingDiscTest.py](#)
- [geometricallyExactBeam2Dtest.py](#)
- [manualExplicitIntegrator.py](#)

- mecanumWheelRollingDiscTest.py
- modelUnitTests.py
- objectFFRFreducedOrderAccelerations.py
- objectFFRFreducedOrderStressModesTest.py
- objectFFRFreducedOrderTest.py
- ...

### 7.4.3 LoadMassProportional

Load attached to MarkerBodyMass marker, applying a 3D vector load (e.g. the vector [0,-g,0] is used to apply gravitational loading of size g in negative y-direction).

**Additional information for LoadMassProportional:**

- Requested marker type = Body + BodyMass
- **Short name for Python = Gravity**
- **Short name for Python (visualization object) = VGravity**

The item **LoadMassProportional** with type = 'MassProportional' has the following parameters:

| Name                   | type                                | size | default value | description                                                                                                        |
|------------------------|-------------------------------------|------|---------------|--------------------------------------------------------------------------------------------------------------------|
| name                   | String                              |      | "             | load's unique name                                                                                                 |
| markerNumber           | MarkerIndex                         |      | MAXINT        | marker's number to which load is applied                                                                           |
| loadVector             | Vector3D                            |      | [0.,0.,0.]    | vector-valued load [SI:N/kg = m/s <sup>2</sup> ]; typically, this will be the gravity vector in global coordinates |
| loadVectorUserFunction | PyFunctionVector3DmbsScalarVector3D |      | 0             | A python function which defines the time-dependent loadVector.                                                     |
| visualization          | VLoadMassProportional               |      |               | parameters for visualization of item                                                                               |

The item **VLoadMassProportional** has the following parameters:

| Name | type | size | default value | description                                                              |
|------|------|------|---------------|--------------------------------------------------------------------------|
| show | Bool |      | True          | set true, if item is shown in visualization and false if it is not shown |

#### 7.4.3.1 DESCRIPTION of LoadMassProportional:

**Information on input parameters:**

| input parameter | symbol | description see tables above |
|-----------------|--------|------------------------------|
| loadVector      | b      |                              |

#### 7.4.3.2 Details

The load applies a (translational) and distributed load proportional to the distributed body's density. The marker of type MarkerBodyMass transforms the loadVector via an according jacobian matrix to object coordinates.

#### Userfunction: **loadVectorUserFunction(mbs, t, loadVector)**

A user function, which computes the mass proportional load vector depending on time and object parameters, which is hereafter applied to object or node.

| arguments / return  | type or size | description                                                                                                                                              |
|---------------------|--------------|----------------------------------------------------------------------------------------------------------------------------------------------------------|
| mbs                 | MainSystem   | provides MainSystem mbs to which load belongs                                                                                                            |
| t                   | Real         | current time in mbs                                                                                                                                      |
| loadVector          | Vector3D     | <b>b</b> copied from object; WARNING: this parameter does not work in combination with static computation, as it is changed by the solver over step time |
| <b>return value</b> | Vector3D     | computed load vector                                                                                                                                     |

Example of user function: functionality same as in LoadForceVector

---

#### 7.4.3.3 MINI EXAMPLE for LoadMassProportional

```

node = mbs.AddNode(NodePoint(referenceCoordinates = [1,0,0]))
body = mbs.AddObject(MassPoint(nodeNumber = node, physicsMass=2))
mMass = mbs.AddMarker(MarkerBodyMass(bodyNumber=body))
mbs.AddLoad(LoadMassProportional(markerNumber=mMass, loadVector=[0,0,-9.81]))

#assemble and solve system for default parameters
mbs.Assemble()
exu.SolveDynamic(mbs)

#check result
exodynTestGlobals.testResult = mbs.GetNodeOutput(node, exu.OutputVariableType.
 Position)[2]
#final z-coordinate of position shall be -g/2 due to constant acceleration with g
 =-9.81
#result independent of mass

```

---

For further examples on LoadMassProportional see Examples:

- [NGsolveCMSTutorial.py](#)
- [NGsolveCraigBampton.py](#)
- [NGsolvePostProcessingStresses.py](#)
- [ALEANCF\\_pipe.py](#)
- [ANCF\\_moving\\_rigidbody.py](#)
- [ANCF\\_slidingJoint2D.py](#)
- [ANCF\\_switchingSlidingJoint2D.py](#)
- [rigid3Dexample.py](#)
- [rigidBodyIMUtest.py](#)

For further examples on LoadMassProportional see TestModels:

- [genericJointUserFunctionTest.py](#)
- [modelUnitTests.py](#)
- [sphericalJointTest.py](#)

#### 7.4.4 LoadCoordinate

Load with scalar value, which is attached to a coordinate-based marker; the load can be used e.g. to apply a force to a single axis of a body, a nodal coordinate of a finite element or a torque to the rotatory DOF of a rigid body.

**Additional information for LoadCoordinate:**

- Requested marker type = Coordinate

The item **LoadCoordinate** with type = 'Coordinate' has the following parameters:

| Name             | type                 | size | default value | description                                                                    |
|------------------|----------------------|------|---------------|--------------------------------------------------------------------------------|
| name             | String               |      | "             | load's unique name                                                             |
| markerNumber     | MarkerIndex          |      | MAXINT        | marker's number to which load is applied                                       |
| load             | Real                 |      | 0.            | scalar load [SI:N]                                                             |
| loadUserFunction | PyFunctionMbsScalar2 |      | 0             | A python function which defines the time-dependent load; see description below |
| visualization    | VLoadCoordinate      |      |               | parameters for visualization of item                                           |

The item **VLoadCoordinate** has the following parameters:

| Name | type | size | default value | description                                                              |
|------|------|------|---------------|--------------------------------------------------------------------------|
| show | Bool |      | True          | set true, if item is shown in visualization and false if it is not shown |

##### 7.4.4.1 DESCRIPTION of LoadCoordinate:

**Information on input parameters:**

| input parameter | symbol | description see tables above |
|-----------------|--------|------------------------------|
| load            | $f$    |                              |

##### 7.4.4.2 Details

The scalar load is applied on a coordinate defined by a Marker of type 'Coordinate', e.g., **MarkerNodeCoordinate**. This can be used to create simple 1D problems, or to simply apply a translational force on a Node or even a torque on a rotation coordinate (but take care for its meaning).

##### Userfunction: **loadUserFunction(mbs, t, load)**

A user function, which computes the scalar load depending on time and the object's load parameter.

| arguments / return | type or size | description                                   |
|--------------------|--------------|-----------------------------------------------|
| mbs                | MainSystem   | provides MainSystem mbs to which load belongs |

|                     |      |                                                                                                                                                          |
|---------------------|------|----------------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>t</b>            | Real | current time in mbs                                                                                                                                      |
| load                | Real | <b>b</b> copied from object; WARNING: this parameter does not work in combination with static computation, as it is changed by the solver over step time |
| <b>return value</b> | Real | computed load                                                                                                                                            |

---

### User function example:

```

from math import sin, cos, pi
#this example uses the object's stored parameter load to compute a time-dependent
load
def UFload(mbs, t, load):
 return load*sin(10*(2*pi)*t)

n0=mbs.AddNode(Point())
nodeMarker = mbs.AddMarker(MarkerNodeCoordinate(nodeNumber=n0, coordinate=0))
mbs.AddLoad(LoadCoordinate(markerNumber = markerCoordinate,
 load = 10,
 loadUserFunction = UFload))

```

For further examples on LoadCoordinate see Examples:

- [coordinateSpringDamper.py](#)
- [geneticOptimizationExample.py](#)
- [geneticOptimizationSliderCrank.py](#)
- [lavalRotor2Dtest.py](#)
- [massSpringFrictionInteractive.py](#)
- [parameterVariationExample.py](#)
- [simulateInteractively.py](#)
- [slidercrankWithMassSpring.py](#)
- [springDamperTutorial.py](#)

For further examples on LoadCoordinate see TestModels:

- [ACNFslidingAndALEjointTest.py](#)
- [driveTrainTest.py](#)
- [geneticOptimizationTest.py](#)
- [modelUnitTests.py](#)
- [postNewtonStepContactTest.py](#)
- [sliderCrankFloatingTest.py](#)
- [springDamperUserFunctionTest.py](#)

## 7.5 Sensors

### 7.5.1 SensorNode

A sensor attached to a ODE2 or ODE1 node. The sensor measures OutputVariables and outputs values into a file, showing per line [time, sensorValue[0], sensorValue[1], ...]. Use SensorUserFunction to modify sensor results (e.g., transforming to other coordinates) and writing to file.

The item **SensorNode** with type = 'Node' has the following parameters:

| Name               | type               | size | default value                                     | description                                                                                                                                                                |
|--------------------|--------------------|------|---------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| name               | String             |      | "                                                 | sensor's unique name                                                                                                                                                       |
| nodeNumber         | NodeIndex          |      | MAXINT                                            | node number to which sensor is attached to                                                                                                                                 |
| writeToFile        | Bool               |      | True                                              | true: write sensor output to file                                                                                                                                          |
| fileName           | String             |      | "                                                 | directory and file name for sensor file output; default: empty string generates sensor + sensorNumber + outputVariableType; directory will be created if it does not exist |
| outputVariableType | OutputVariableType |      | OutputVariableType: OutputVariableType for sensor |                                                                                                                                                                            |
| visualization      | VSensorNode        |      |                                                   | parameters for visualization of item                                                                                                                                       |

The item VSensorNode has the following parameters:

| Name | type | size | default value | description                                                              |
|------|------|------|---------------|--------------------------------------------------------------------------|
| show | Bool |      | True          | set true, if item is shown in visualization and false if it is not shown |

---

#### 7.5.1.1 DESCRIPTION of SensorNode:

---

For further examples on SensorNode see Examples:

- [ANCFALEtest.py](#)
- [geneticOptimizationSliderCrank.py](#)
- [massSpringFrictionInteractive.py](#)
- [mouseInteractionExample.py](#)
- [simulateInteractively.py](#)
- [sliderCrank3DwithANCFbeltDrive.py](#)
- [sliderCrankCMSacme.py](#)
- [stiffFlyballGovernor2.py](#)

For further examples on SensorNode see TestModels:

- [driveTrainTest.py](#)
- [explicitLieGroupIntegratorPythonTest.py](#)

- [explicitLieGroupIntegratorTest.py](#)
- [explicitLieGroupMBSTest.py](#)
- [heavyTop.py](#)
- [objectFFRFreducedOrderAccelerations.py](#)
- [objectFFRFreducedOrderStressModesTest.py](#)
- [objectFFRFreducedOrderTest.py](#)
- [objectFFRFTTest.py](#)
- [objectFFRFTTest2.py](#)
- ...

## 7.5.2 SensorObject

A sensor attached to any object except bodies (connectors, constraint, spring-damper, etc). As a difference to other SensorBody, the connector sensor measures quantities without a local position. The sensor measures OutputVariable and outputs values into a file, showing per line [time, sensorValue[0], sensorValue[1], ...]. Use SensorUserFunction to modify sensor results (e.g., transforming to other coordinates) and writing to file.

The item **SensorObject** with type = 'Object' has the following parameters:

| Name               | type               | size | default value                                     | description                                                                                                                                                                |
|--------------------|--------------------|------|---------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| name               | String             |      | ''                                                | sensor's unique name                                                                                                                                                       |
| objectNumber       | ObjectIndex        |      | MAXINT                                            | object (e.g. connector) number to which sensor is attached to                                                                                                              |
| writeToFile        | Bool               |      | True                                              | true: write sensor output to file                                                                                                                                          |
| fileName           | String             |      | ''                                                | directory and file name for sensor file output; default: empty string generates sensor + sensorNumber + outputVariableType; directory will be created if it does not exist |
| outputVariableType | OutputVariableType |      | OutputVariableType: OutputVariableType for sensor |                                                                                                                                                                            |
| visualization      | VSensorObject      |      |                                                   | parameters for visualization of item                                                                                                                                       |

The item VSensorObject has the following parameters:

| Name | type | size | default value | description                                                                                                                                                                                                              |
|------|------|------|---------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| show | Bool |      | True          | set true, if item is shown in visualization and false if it is not shown; sensors can be shown at the position associated with the object - note that in some cases, there might be no such position (e.g. data object)! |

### 7.5.2.1 DESCRIPTION of SensorObject:

For further examples on SensorObject see Examples:

- [geneticOptimizationExample.py](#)
- [parameterVariationExample.py](#)
- [sliderCrank3DwithANCFbeltDrive.py](#)
- [sliderCrankCMSacme.py](#)
- [springDamperTutorial.py](#)

For further examples on SensorObject see TestModels:

- [carRollingDiscTest.py](#)
- [geneticOptimizationTest.py](#)

- [mecanumWheelRollingDiscTest.py](#)
- [objectFFRFTest.py](#)
- [rollingCoinPenaltyTest.py](#)
- [rollingCoinTest.py](#)
- [serialRobotTest.py](#)

### 7.5.3 SensorBody

A sensor attached to a body-object with local position. As a difference to other ObjectSensors, the body sensor has a local position at which the sensor is attached to. The sensor measures OutputVariableBody and outputs values into a file, showing per line [time, sensorValue[0], sensorValue[1], ...]. Use SensorUserFunction to modify sensor results (e.g., transforming to other coordinates) and writing to file.

The item **SensorBody** with type = 'Body' has the following parameters:

| Name               | type               | size | default value           | description                                                                                                                                                                |
|--------------------|--------------------|------|-------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| name               | String             |      | ''                      | sensor's unique name                                                                                                                                                       |
| bodyNumber         | ObjectIndex        |      | MAXINT                  | body (=object) number to which sensor is attached to                                                                                                                       |
| localPosition      | Vector3D           | 3    | [0.,0.,0.]              | local (body-fixed) body position of sensor                                                                                                                                 |
| writeToFile        | Bool               |      | True                    | true: write sensor output to file                                                                                                                                          |
| fileName           | String             |      | ''                      | directory and file name for sensor file output; default: empty string generates sensor + sensorNumber + outputVariableType; directory will be created if it does not exist |
| outputVariableType | OutputVariableType |      | OutputVariableType:None | OutputVariableType for sensor                                                                                                                                              |
| visualization      | VSensorBody        |      |                         | parameters for visualization of item                                                                                                                                       |

The item VSensorBody has the following parameters:

| Name | type | size | default value | description                                                              |
|------|------|------|---------------|--------------------------------------------------------------------------|
| show | Bool |      | True          | set true, if item is shown in visualization and false if it is not shown |

#### 7.5.3.1 DESCRIPTION of SensorBody:

For further examples on SensorBody see Examples:

- [rigidBodyIMUtest.py](#)
- [rigidBodyTutorial2.py](#)
- [rigidRotor3DBasicBehaviour.py](#)
- [rigidRotor3DFWBW.py](#)
- [rigidRotor3DRunup.py](#)
- [sliderCrank3DwithANCFbeltDrive.py](#)
- [sliderCrank3DwithANCFbeltDrive2.py](#)

For further examples on SensorBody see TestModels:

- [carRollingDiscTest.py](#)
- [driveTrainTest.py](#)

- [explicitLieGroupIntegratorPythonTest.py](#)
- [explicitLieGroupIntegratorTest.py](#)
- [explicitLieGroupMBSTest.py](#)
- [heavyTop.py](#)
- [mecanumWheelRollingDiscTest.py](#)
- [pendulumFriction.py](#)
- [rollingCoinPenaltyTest.py](#)
- [rollingCoinTest.py](#)
- ...

### 7.5.4 SensorSuperElement

A sensor attached to a SuperElement-object with mesh node number. As a difference to other ObjectSensors, the SuperElement sensor has a mesh node number at which the sensor is attached to. The sensor measures OutputVariableSuperElement and outputs values into a file, showing per line [time, sensorValue[0], sensorValue[1], ...]. Use SensorUserFunction to modify sensor results (e.g., transforming to other coordinates) and writing to file.

The item **SensorSuperElement** with type = 'SuperElement' has the following parameters:

| <b>Name</b>        | <b>type</b>         | <b>size</b> | <b>default value</b>  | <b>description</b>                                                                                                                                                                                                                         |
|--------------------|---------------------|-------------|-----------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| name               | String              |             | "                     | sensor's unique name                                                                                                                                                                                                                       |
| bodyNumber         | ObjectIndex         |             | MAXINT                | body (=object) number to which sensor is attached to                                                                                                                                                                                       |
| meshNodeNumber     | UInt                |             | MAXINT                | mesh node number, which is a local node number within in the object (starting with 0); the node number may represent a real Node in mbs, or may be virtual and reconstructed from the object coordinates such as in ObjectFFRFreducedOrder |
| writeToFile        | Bool                |             | True                  | true: write sensor output to file                                                                                                                                                                                                          |
| fileName           | String              |             | "                     | directory and file name for sensor file output; default: empty string generates sensor + sensorNumber + outputVariableType; directory will be created if it does not exist                                                                 |
| outputVariableType | OutputVariableType  |             | OutputVariableType:ON | OutputVariableType for sensor, based on the output variables available for the mesh nodes (see special section for super element output variables, e.g, in ObjectFFRFreducedOrder, <a href="#">Section 7.2.11.2</a> )                      |
| visualization      | VSensorSuperElement |             |                       | parameters for visualization of item                                                                                                                                                                                                       |

The item VSensorSuperElement has the following parameters:

| <b>Name</b> | <b>type</b> | <b>size</b> | <b>default value</b> | <b>description</b>                                                       |
|-------------|-------------|-------------|----------------------|--------------------------------------------------------------------------|
| show        | Bool        |             | True                 | set true, if item is shown in visualization and false if it is not shown |

---

#### 7.5.4.1 DESCRIPTION of SensorSuperElement:

---

For further examples on SensorSuperElement see Examples:

- [NGsolveCraigBampton.py](#)
- [NGsolvePostProcessingStresses.py](#)
- [objectFFRFreducedOrderNetgen.py](#)

- [sliderCrankCMSacme.py](#)

For further examples on SensorSuperElement see TestModels:

- [objectFFRFreducedOrderAccelerations.py](#)
- [objectFFRFreducedOrderStressModesTest.py](#)
- [objectFFRFreducedOrderTest.py](#)
- [objectFFRFTTest.py](#)
- [objectFFRFTTest2.py](#)
- [objectGenericODE2Test.py](#)
- [superElementRigidJointTest.py](#)

## 7.5.5 SensorMarker

A sensor attached to a marker. The sensor measures the selected marker values and outputs values into a file, showing per line [time, sensorValue[0], sensorValue[1], ...]. Depending on markers, it can measure Coordinates (MarkerNodeCoordinate), Position and Velocity (MarkerXXXPosition), Position, Velocity, Rotation and AngularVelocityLocal (MarkerXXXRigid). Note that marker values are only available for the current configuration. Use SensorUserFunction to modify sensor results (e.g., transforming to other coordinates) and writing to file

The item **SensorMarker** with type = 'Marker' has the following parameters:

| <b>Name</b>        | <b>type</b>        | <b>size</b> | <b>default value</b>    | <b>description</b>                                                                                                                                                         |
|--------------------|--------------------|-------------|-------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| name               | String             |             | "                       | sensor's unique name                                                                                                                                                       |
| markerNumber       | MarkerIndex        |             | MAXINT                  | marker number to which sensor is attached to                                                                                                                               |
| writeToFile        | Bool               |             | True                    | true: write sensor output to file                                                                                                                                          |
| fileName           | String             |             | "                       | directory and file name for sensor file output; default: empty string generates sensor + sensorNumber + outputVariableType; directory will be created if it does not exist |
| outputVariableType | OutputVariableType |             | OutputVariableType:None | NoneVariableType for sensor; output variables are only possible according to markertype, see general description of SensorMarker                                           |
| visualization      | VSensorMarker      |             |                         | parameters for visualization of item                                                                                                                                       |

The item VSensorMarker has the following parameters:

| <b>Name</b> | <b>type</b> | <b>size</b> | <b>default value</b> | <b>description</b>                                                       |
|-------------|-------------|-------------|----------------------|--------------------------------------------------------------------------|
| show        | Bool        |             | True                 | set true, if item is shown in visualization and false if it is not shown |

### 7.5.5.1 DESCRIPTION of SensorMarker:

For further examples on SensorMarker see Examples:

- [NGsolveCMSTutorial.py](#)

For further examples on SensorMarker see TestModels:

- [pendulumFriction.py](#)

## 7.5.6 SensorLoad

A sensor attached to a load. The sensor measures the load values and outputs values into a file, showing per line [time, sensorValue[0], sensorValue[1], ...]. Use SensorUserFunction to modify sensor results (e.g., transforming to other coordinates) and writing to file.

The item **SensorLoad** with type = 'Load' has the following parameters:

| Name          | type        | size | default value | description                                                                                                                                                                |
|---------------|-------------|------|---------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| name          | String      |      | "             | sensor's unique name                                                                                                                                                       |
| loadNumber    | LoadIndex   |      | MAXINT        | load number to which sensor is attached to                                                                                                                                 |
| writeToFile   | Bool        |      | True          | true: write sensor output to file                                                                                                                                          |
| fileName      | String      |      | "             | directory and file name for sensor file output; default: empty string generates sensor + sensorNumber + outputVariableType; directory will be created if it does not exist |
| visualization | VSensorLoad |      |               | parameters for visualization of item                                                                                                                                       |

The item VSensorLoad has the following parameters:

| Name | type | size | default value | description                                                                                                              |
|------|------|------|---------------|--------------------------------------------------------------------------------------------------------------------------|
| show | Bool |      | True          | set true, if item is shown in visualization and false if it is not shown; sensor visualization CURRENTLY NOT IMPLEMENTED |

### 7.5.6.1 DESCRIPTION of SensorLoad:

---

For further examples on SensorLoad see Examples:

- [simulateInteractively.py](#)
- [sliderCrank3DwithANCFbeltDrive2.py](#)

For further examples on SensorLoad see TestModels:

- [serialRobotTest.py](#)
- [springDamperUserFunctionTest.py](#)

### 7.5.7 SensorUserFunction

A sensor defined by a user function. The sensor is intended to collect sensor values of a list of given sensors and recombine the output into a new value for output or control purposes. It is also possible to use this sensor without any dependence on other sensors in order to generate output for, e.g., any quantities in mbs or solvers.

The item **SensorUserFunction** with type = 'UserFunction' has the following parameters:

| Name               | type                                      | size | default value | description                                                                                                                                                                |
|--------------------|-------------------------------------------|------|---------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| name               | String                                    |      | "             | sensor's unique name                                                                                                                                                       |
| sensorNumbers      | ArraySensorIndex                          |      | []            | optional list of $n$ sensor numbers for use in user function                                                                                                               |
| factors            | Vector                                    |      | []            | optional list of $m$ factors which can be used, e.g., for weighting sensor values                                                                                          |
| writeToFile        | Bool                                      |      | True          | true: write sensor output to file                                                                                                                                          |
| fileName           | String                                    |      | "             | directory and file name for sensor file output; default: empty string generates sensor + sensorNumber + outputVariableType; directory will be created if it does not exist |
| sensorUserFunction | PyFunctionVectorMbsScalarArrayIndexVector | 0    |               | A python function which defines the time-dependent user function, which usually evaluates one or several sensors and computes a new sensor value, see example              |
| visualization      | VSensorUserFunction                       |      |               | parameters for visualization of item                                                                                                                                       |

The item **VSensorUserFunction** has the following parameters:

| Name | type | size | default value | description                                                                                                              |
|------|------|------|---------------|--------------------------------------------------------------------------------------------------------------------------|
| show | Bool |      | True          | set true, if item is shown in visualization and false if it is not shown; sensor visualization CURRENTLY NOT IMPLEMENTED |

#### 7.5.7.1 DESCRIPTION of SensorUserFunction:

**Information on input parameters:**

| input parameter | symbol                               | description see tables above |
|-----------------|--------------------------------------|------------------------------|
| sensorNumbers   | $\mathbf{n}_s = [s_0, \dots, s_n]^T$ |                              |
| factors         | $\mathbf{f}_s = [f_0, \dots, f_m]^T$ |                              |

**Userfunction: sensorUserFunction(mbs, t, sensorNumbers, factors, configuration)**

A user function, which computes a sensor output from other sensor outputs (or from generic time dependent

functions). The configuration in general will be the `exudyn.ConfigurationType.Current`, but others could be used as well except for `SensorMarker`. The user function arguments are as follows:

| arguments / return         | type or size                          | description                                                                                                                                          |
|----------------------------|---------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>mbs</code>           | MainSystem                            | provides MainSystem <code>mbs</code> to which object belongs                                                                                         |
| <code>t</code>             | Real                                  | current time in <code>mbs</code>                                                                                                                     |
| <code>sensorNumbers</code> | Array $\in \mathbb{N}^n$              | list of sensor numbers                                                                                                                               |
| <code>factors</code>       | Vector $\in \mathbb{R}^n$             | list of factors that can be freely used for the user function                                                                                        |
| <code>configuration</code> | <code>exudyn.ConfigurationType</code> | usually the <code>exudyn.ConfigurationType.Current</code> , but could also be different in user defined functions.                                   |
| <b>return value</b>        | Vector $\in \mathbb{R}^{n_r}$         | returns list or numpy array of sensor output values; size $n_r$ is implicitly defined by the returned list and may not be changed during simulation. |

### User function example:

```

import exudyn as exu
from exudyn.itemInterface import *
from math import pi, atan2
SC = exu.SystemContainer()
mbs = SC.AddSystem()
node = mbs.AddNode(NodePoint(referenceCoordinates = [1,1,0],
 initialCoordinates=[0,0,0],
 initialVelocities=[0,-1,0]))
mbs.AddObject(MassPoint(nodeNumber = node, physicsMass=1))

sNode = mbs.AddSensor(SensorNode(nodeNumber=node, fileName='solution/sensorTest.txt',
 outputVariableType=exu.OutputVariableType.Position))

#user function for sensor, convert position into angle:
def UFsensor(mbs, t, sensorNumbers, factors, configuration):
 val = mbs.GetSensorValues(sensorNumbers[0]) #x,y,z
 phi = atan2(val[1],val[0]) #compute angle from x,y: atan2(y,x)
 return [factors[0]*phi] #return angle in degree

sUser = mbs.AddSensor(SensorUserFunction(sensorNumbers=[sNode], factors=[180/pi],
 fileName='solution/sensorTest2.txt',
 sensorUserFunction=UFsensor))

#assemble and solve system for default parameters
mbs.Assemble()
exu.SolveDynamic(mbs)

```

```
if False:
 from exudyn.plot import PlotSensor
 PlotSensor(mbs, [sNode, sNode, sUser], [0, 1, 0])
```

---

For further examples on SensorUserFunction see TestModels:

- [sensorUserFunctionTest.py](#)



# Chapter 8

# EXUDYN Settings and Solver Structures

This section includes the reference manual for settings which are available in the python interface, e.g. simulation settings, visualization settings, and structures for solvers. The data is auto-generated from the according interfaces in order to keep fully up-to-date with changes.

## 8.1 Simulation settings

This section includes hierarchical structures for simulation settings, e.g., time integration, static solver, Newton iteration and solution file export.

### 8.1.1 SolutionSettings

General settings for exporting the solution (results) of a simulation.

SolutionSettings has the following items:

| Name                | type/function return type | size | default value / function args | description                                                                                                                                                                                                                                                    |
|---------------------|---------------------------|------|-------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| writeSolutionToFile | bool                      |      | True                          | flag (true/false), which determines if (global) solution vector is written to file; standard quantities that are written are: solution is written as displacements and coordinatesODE1; for additional coordinates in the solution file, see the options below |
| appendToFile        | bool                      |      | False                         | flag (true/false); if true, solution and solver-information is appended to existing file (otherwise created)                                                                                                                                                   |
| writeFileHeader     | bool                      |      | True                          | flag (true/false); if true, file header is written (turn off, e.g. for multiple runs of time integration)                                                                                                                                                      |
| writeFileFooter     | bool                      |      | True                          | flag (true/false); if true, information at end of simulation is written: convergence, total solution time, statistics                                                                                                                                          |
| solutionWritePeriod | UReal                     |      | 0.01                          | time span (period), determines how often the solution is written during a simulation                                                                                                                                                                           |
| exportVelocities    | bool                      |      | True                          | add ODE2 velocities to solution file                                                                                                                                                                                                                           |
| exportAccelerations | bool                      |      | True                          | add ODE2 accelerations to solution file                                                                                                                                                                                                                        |

|                             |          |                           |                                                                                                                                                                                                                                                                                                                                                                                                                                 |
|-----------------------------|----------|---------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| exportODE1Velocities        | bool     | True                      | add coordinatesODE1_t to solution file                                                                                                                                                                                                                                                                                                                                                                                          |
| exportAlgebraicCoordinates  | bool     | True                      | add algebraicCoordinates (=Lagrange multipliers) to solution file                                                                                                                                                                                                                                                                                                                                                               |
| exportDataCoordinates       | bool     | True                      | add DataCoordinates to solution file                                                                                                                                                                                                                                                                                                                                                                                            |
| coordinatesSolutionFileName | FileName | 'coordinatesSolution.txt' | filename and (relative) path of solution file containing all coordinates versus time; directory will be created if it does not exist; character encoding of string is up to your filesystem, but for compatibility, it is recommended to use letters, numbers and '_' only                                                                                                                                                      |
| sensorsAppendToFile         | bool     | False                     | flag (true/false); if true, sensor output is appended to existing file (otherwise created)                                                                                                                                                                                                                                                                                                                                      |
| sensorsWriteFileHeader      | bool     | True                      | flag (true/false); if true, file header is written for sensor output (turn off, e.g. for multiple runs of time integration)                                                                                                                                                                                                                                                                                                     |
| sensorsWritePeriod          | UReal    | 0.01                      | time span (period), determines how often the sensor output is written during a simulation                                                                                                                                                                                                                                                                                                                                       |
| solverInformationFileName   | FileName | 'solverInformation.txt'   | filename and (relative) path of text file showing detailed information during solving; detail level according to yourSolver.verboseModeFile; if solutionSettings.appendToFile is true, the information is appended in every solution step; directory will be created if it does not exist; character encoding of string is up to your filesystem, but for compatibility, it is recommended to use letters, numbers and '_' only |
| solutionInformation         | String   | "                         | special information added to header of solution file (e.g. parameters and settings, modes, ...); character encoding may be UTF-8, restricted to characters in <a href="#">Section 9.4</a> , but for compatibility, it is recommended to use ASCII characters only (95 characters, see wiki)                                                                                                                                     |
| outputPrecision             | UInt     | 10                        | precision for floating point numbers written to solution and sensor files                                                                                                                                                                                                                                                                                                                                                       |
| recordImagesInterval        | Real     | -1.                       | record frames (images) during solving: amount of time to wait until next image (frame) is recorded; set recordImages = -1. if no images shall be recorded; set, e.g., recordImages = 0.01 to record an image every 10 milliseconds (requires that the time steps / load steps are sufficiently small!); for file names, etc., see VisualizationSettings.exportImages                                                            |

### 8.1.2 NumericalDifferentiationSettings

Settings for numerical differentiation of a function (needed for computation of numerical jacobian e.g. in implizit integration); HOTINT1: `relativeEpsilon * Maximum(minimumCoordinateSize, fabs(x(i)))`.

`NumericalDifferentiationSettings` has the following items:

| Name                             | type/function return type | size | default value / function args | description                                                                                                                                                                                                                                                                                                                                                                                                                                                                |
|----------------------------------|---------------------------|------|-------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| relativeEpsilon                  | UReal                     |      | 1e-7                          | relative differentiation parameter epsilon; the numerical differentiation parameter $\varepsilon$ follows from the formula ( $\varepsilon = \varepsilon_{\text{relative}} * \max(q_{\min},  q_i + [q_i^{\text{Ref}}] )$ ), with $\varepsilon_{\text{relative}} = \text{relativeEpsilon}$ , $q_{\min} = \text{minimumCoordinateSize}$ , $q_i$ is the current coordinate which is differentiated, and $q_{\text{Ref}}$ is the reference coordinate of the current coordinate |
| minimumCoordinateSize            | UReal                     |      | 1e-2                          | minimum size of coordinates in relative differentiation parameter                                                                                                                                                                                                                                                                                                                                                                                                          |
| doSystemWideDifferentiation      | bool                      |      | False                         | True: system wide differentiation (e.g. all ODE2 equations w.r.t. all ODE2 coordinates); False: only local (object) differentiation                                                                                                                                                                                                                                                                                                                                        |
| addReferenceCoordinatesToEpsilon | bool                      |      | False                         | True: for the size estimation of the differentiation parameter, the reference coordinate $q_i^{\text{Ref}}$ is added to ODE2 coordinates $\rightarrow$ see; False: only the current coordinate is used for size estimation of the differentiation parameter                                                                                                                                                                                                                |

### 8.1.3 DiscontinuousSettings

Settings for discontinuous iterations, as in contact, friction, plasticity and general switching phenomena.

`DiscontinuousSettings` has the following items:

| Name                | type/function return type | size | default value / function args | description                                                                                                                  |
|---------------------|---------------------------|------|-------------------------------|------------------------------------------------------------------------------------------------------------------------------|
| maxIterations       | UInt                      |      | 5                             | maximum number of discontinuous (post Newton) iterations                                                                     |
| ignoreMaxIterations | bool                      |      | True                          | continue solver if maximum number of discontinuous (post Newton) iterations is reached (ignore tolerance)                    |
| iterationTolerance  | UReal                     |      | 1                             | absolute tolerance for discontinuous (post Newton) iterations; the errors represent absolute residuals and can be quite high |

### 8.1.4 NewtonSettings

Settings for Newton method used in static or dynamic simulation.

NewtonSettings has the following items:

| Name                         | type/function return type        | size | default value / function args | description                                                                                                                                                                                                                                                 |
|------------------------------|----------------------------------|------|-------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| numericalDifferentiation     | NumericalDifferentiationSettings |      |                               | numerical differentiation parameters for numerical jacobian (e.g. Newton in static solver or implicit time integration)                                                                                                                                     |
| useNumericalDifferentiation  | bool                             |      | False                         | flag (true/false); false = perform direct computation of jacobian, true = use numerical differentiation for jacobian                                                                                                                                        |
| useNewtonSolver              | bool                             |      | True                          | flag (true/false); false = linear computation, true = use Newton solver for nonlinear solution                                                                                                                                                              |
| relativeTolerance            | UReal                            |      | 1e-8                          | relative tolerance of residual for Newton (general goal of Newton is to decrease the residual by this factor)                                                                                                                                               |
| absoluteTolerance            | UReal                            |      | 1e-10                         | absolute tolerance of residual for Newton (needed e.g. if residual is fulfilled right at beginning); condition: $\text{sqrt}(q^*q)/\text{numberOfCoordinates} \leq \text{absoluteTolerance}$                                                                |
| weightTolerancePerCoordinate | bool                             |      | False                         | flag (true/false); false = compute error as L2-Norm of residual; true = compute error as $(\text{L2-Norm of residual}) / (\text{sqrt}(\text{number of coordinates}))$ , which can help to use common tolerance independent of system size                   |
| newtonResidualMode           | UInt                             |      | 0                             | 0 ... use residual for computation of error (standard); 1 ... use ODE2 and ODE1 newton increment for error (set relTol and absTol to same values!) ==> may be advantageous if residual is zero, e.g., in kinematic analysis; TAKE CARE with this flag       |
| adaptInitialResidual         | bool                             |      | True                          | flag (true/false); false = standard; True: if initialResidual is very small (or zero), it may increase dramatically in first step; to achieve relativeTolerance, the initialResidual will be updated by a higher residual within the first Newton iteration |
| modifiedNewtonContractivity  | PReal                            |      | 0.5                           | maximum contractivity (=reduction of error in every Newton iteration) accepted by modified Newton; if contractivity is greater, a Jacobian update is computed                                                                                               |
| useModifiedNewton            | bool                             |      | False                         | True: compute Jacobian only at first step; no Jacobian updates per step; False: Jacobian computed in every step                                                                                                                                             |

|                                    |       |       |                                                                                                                                                                                                                                                                                                                                |
|------------------------------------|-------|-------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| modifiedNewtonJacUpdatePerStep     | bool  | False | True: compute Jacobian at every time step, but not in every iteration (except for bad convergence ==> switch to full Newton)                                                                                                                                                                                                   |
| maxIterations                      | UInt  | 25    | maximum number of iterations (including modified + restart Newton steps); after that iterations, the static/dynamic solver stops with error                                                                                                                                                                                    |
| maxModifiedNewtonIterations        | UInt  | 8     | maximum number of iterations for modified Newton (without Jacobian update); after that number of iterations, the modified Newton method gets a jacobian update and is further iterated                                                                                                                                         |
| maxModifiedNewtonRestartIterations | UInt  | 7     | maximum number of iterations for modified Newton after a Jacobian update; after that number of iterations, the full Newton method is started for this step                                                                                                                                                                     |
| maximumSolutionNorm                | UReal | 1e38  | this is the maximum allowed value for solutionU.L2NormSquared() which is the square of the square norm (value= $u_1^2+u_2^2+\dots$ ), and solutionV/A...; if the norm of solution vectors are larger, Newton method is stopped; the default value is chosen such that it would still work for single precision numbers (float) |

### 8.1.5 GeneralizedAlphaSettings

Settings for generalized-alpha, implicit trapezoidal or Newmark time integration methods.

GeneralizedAlphaSettings has the following items:

| Name                 | type/function return type | size | default value / function args | description                                                                                                    |
|----------------------|---------------------------|------|-------------------------------|----------------------------------------------------------------------------------------------------------------|
| newmarkBeta          | UReal                     |      | 0.25                          | value beta for Newmark method; default value beta = $\frac{1}{4}$ corresponds to (undamped) trapezoidal rule   |
| newmarkGamma         | UReal                     |      | 0.5                           | value gamma for Newmark method; default value gamma = $\frac{1}{2}$ corresponds to (undamped) trapezoidal rule |
| useIndex2Constraints | bool                      |      | False                         | set useIndex2Constraints = true in order to use index2 (velocity level constraints) formulation                |
| useNewmark           | bool                      |      | False                         | if true, use Newmark method with beta and gamma instead of generalized-Alpha                                   |

|                             |       |      |                                                                                                                                                                                                                                       |
|-----------------------------|-------|------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| spectralRadius              | UReal | 0.9  | spectral radius for Generalized-alpha solver; set this value to 1 for no damping or to $0 < \text{spectralRadius} < 1$ for damping of high-frequency dynamics; for position-level constraints (index 3), spectralRadius must be $< 1$ |
| computeInitialAccelerations | bool  | True | True: compute initial accelerations from system EOM in acceleration form; NOTE that initial accelerations that are following from user functions in constraints are not considered for now! False: use zero accelerations             |

### 8.1.6 ExplicitIntegrationSettings

Settings for generalized-alpha, implicit trapezoidal or Newmark time integration methods.

ExplicitIntegrationSettings has the following items:

| Name                   | type/function return type | size | default value / function args | description                                                                                                                                                                                                           |
|------------------------|---------------------------|------|-------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| eliminateConstraints   | bool                      |      | True                          | True: make explicit solver work for simple CoordinateConstraints, which are eliminated for ground constraints (e.g. fixed nodes in finite element models). False: incompatible constraints are ignored (BE CAREFUL)!  |
| useLieGroupIntegration | bool                      |      | True                          | True: use Lie group integration for rigid body nodes; must be turned on for Lie group nodes, but also improves integration of other rigid body nodes. Only available for RK44 integrator.                             |
| dynamicSolverType      | DynamicSolverType         |      | DynamicSolverType::DOPRI5     | selection of explicit solver type (DOPRI5, ExplicitEuler, ExplicitMidpoint, RK44, RK67, ...), for detailed description see DynamicSolverType, <a href="#">Section 4.5.4</a> , but only referring to explicit solvers. |

### 8.1.7 TimeIntegrationSettings

General parameters used in time integration; specific parameters are provided in the according solver settings, e.g. for generalizedAlpha.

TimeIntegrationSettings has the following items:

| Name                      | type/function return type | size | default value / function args | description                                                                                                                                                                                                                                                                   |
|---------------------------|---------------------------|------|-------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| newton                    | NewtonSettings            |      |                               | parameters for Newton method; used for implicit time integration methods only                                                                                                                                                                                                 |
| discontinuous             | DiscontinuousSettings     |      |                               | parameters for treatment of discontinuities                                                                                                                                                                                                                                   |
| startTime                 | UReal                     | 0    |                               | $t_{start}$ : start time of time integration (usually set to zero)                                                                                                                                                                                                            |
| endTime                   | UReal                     | 1    |                               | $t_{end}$ : end time of time integration                                                                                                                                                                                                                                      |
| numberOfSteps             | PInt                      | 100  |                               | $n_{steps}$ : number of steps in time integration; (maximum) stepSize $h$ is computed from $h = \frac{t_{end}-t_{start}}{n_{steps}}$ ; for automatic stepsize control, this stepSize is the maximum steps size, $h_{max} = h$                                                 |
| adaptiveStep              | bool                      | True |                               | True: the step size may be reduced if step fails; no automatic stepsize control                                                                                                                                                                                               |
| adaptiveStepRecoverySteps | UInt                      | 10   |                               | Number of steps needed after which steps will be increased after previous step reduction due to discontinuousIteration or Newton errors                                                                                                                                       |
| adaptiveStepIncrease      | UReal                     | 2    |                               | Multiplicative factor (MUST BE > 1) for step size to increase after previous step reduction due to discontinuousIteration or Newton errors                                                                                                                                    |
| adaptiveStepDecrease      | UReal                     | 0.5  |                               | Multiplicative factor (MUST BE: 0 < factor < 1) for step size to decrese due to discontinuousIteration or Newton errors                                                                                                                                                       |
| automaticStepSize         | bool                      | True |                               | True: for specific integrators with error control (e.g., DOPRI5), compute automatic step size based on error estimation; False: constant step size (step may be reduced if adaptiveStep=True); the maximum stepSize reads $h = h_{max} = \frac{t_{end}-t_{start}}{n_{steps}}$ |
| minimumStepSize           | PReal                     | 1e-8 |                               | $h_{min}$ : if automaticStepSize=True or adaptiveStep=True: lower limit of time step size, before integrator stops with adaptiveStep; lower limit of automaticStepSize control (continues but raises warning)                                                                 |
| initialStepSize           | UReal                     | 0    |                               | $h_{init}$ : if automaticStepSize=True, initial step size; if initialStepSize==0, max. stepSize, which is $(endTime-startTime)/numberOfSteps$ , is used as initial guess; a good choice of initialStepSize may help the solver to start up faster.                            |
| absoluteTolerance         | UReal                     | 1e-8 |                               | $a_{tol}$ : if automaticStepSize=True, absolute tolerance for the error control; must fulfill $a_{tol} > 0$ ; see <a href="#">Section 11.1</a>                                                                                                                                |
| relativeTolerance         | UReal                     | 1e-8 |                               | $r_{tol}$ : if automaticStepSize=True, relative tolerance for the error control; must fulfill $r_{tol} \geq 0$ ; see <a href="#">Section 11.1</a>                                                                                                                             |

|                     |                             |       |                                                                                                                                                                                                                                                                  |
|---------------------|-----------------------------|-------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| stepSizeSafety      | UReal                       | 0.90  | $r_{sfty}$ : if automaticStepSize=True, a safety factor added to estimated optimal step size, in order to prevent from many rejected steps, see <a href="#">Section 11.1</a> . Make this factor smaller if many steps are rejected.                              |
| stepSizeMaxIncrease | UReal                       | 2     | $f_{maxInc}$ : if automaticStepSize=True, maximum increase of step size per step, see <a href="#">Section 11.1</a> ; make this factor smaller (but > 1) if too many rejected steps                                                                               |
| preStepPyExecute    | String                      | "     | DEPRECATED, use mbs.SetPreStepUserFunction(...); Python code to be executed prior to every step and after last step, e.g. for postprocessing                                                                                                                     |
| simulateInRealtime  | bool                        | False | True: simulate in realtime; the solver waits for computation of the next step until the CPU time reached the simulation time; if the simulation is slower than realtime, it simply continues                                                                     |
| realtimeFactor      | PReal                       | 1     | if simulateInRealtime=True, this factor is used to make the simulation slower than realtime (factor < 1) or faster than realtime (factor > 1)                                                                                                                    |
| verboseMode         | UInt                        | 0     | 0 ... no output, 1 ... show short step information every 2 seconds (error), 2 ... show every step information, 3 ... show also solution vector, 4 ... show also mass matrix and jacobian (implicit methods), 5 ... show also Jacobian inverse (implicit methods) |
| verboseModeFile     | UInt                        | 0     | same behaviour as verboseMode, but outputs all solver information to file                                                                                                                                                                                        |
| stepInformation     | UInt                        | 2     | 0 ... only current step time, 1 ... show time to go, 2 ... show newton iterations (Nit) per step, 3 ... show discontinuous iterations (Dit) and newton jacobians (jac) per step                                                                                  |
| generalizedAlpha    | GeneralizedAlphaSettings    |       | parameters for generalized-alpha, implicit trapezoidal rule or Newmark (options only apply for these methods)                                                                                                                                                    |
| explicitIntegration | ExplicitIntegrationSettings |       | special parameters for explicit time integration                                                                                                                                                                                                                 |

### 8.1.8 StaticSolverSettings

Settings for static solver linear or nonlinear (Newton).

StaticSolverSettings has the following items:

| Name | type/function return type | size | default value / function args | description |
|------|---------------------------|------|-------------------------------|-------------|
|      |                           |      |                               |             |

|                           |                       |       |                                                                                                                                                                                                                                                                                                                                                                                                      |
|---------------------------|-----------------------|-------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| newton                    | NewtonSettings        |       | parameters for Newton method (e.g. in static solver or time integration)                                                                                                                                                                                                                                                                                                                             |
| discontinuous             | DiscontinuousSettings |       | parameters for treatment of discontinuities                                                                                                                                                                                                                                                                                                                                                          |
| numberOfLoadSteps         | PInt                  | 1     | number of load steps; if numberOfLoadSteps=1, no load steps are used and full forces are applied at once                                                                                                                                                                                                                                                                                             |
| loadStepDuration          | PReal                 | 1     | quasi-time for all load steps (added to current time in load steps)                                                                                                                                                                                                                                                                                                                                  |
| loadStepStart             | UReal                 | 0     | a quasi time, which can be used for the output (first column) as well as for time-dependent forces; quasi-time is increased in every step i by loadStepDuration/numberOfLoadSteps; loadStepTime = loadStepStart + i*loadStepDuration/numberOfLoadSteps, but loadStepStart untouched ==> increment by user                                                                                            |
| loadStepGeometric         | bool                  | False | if loadStepGeometric=false, the load steps are incremental (arithmetic series, e.g. 0.1,0.2,0.3,...); if true, the load steps are increased in a geometric series, e.g. for n = 8 numberOfLoadSteps and d = 1000 loadStepGeometricRange, it follows: $1000^{1/8}/1000 = 0.00237$ , $1000^{2/8}/1000 = 0.00562$ , $1000^{3/8}/1000 = 0.0133$ , ..., $1000^{7/8}/1000 = 0.422$ , $1000^{8/8}/1000 = 1$ |
| loadStepGeometricRange    | PReal                 | 1000  | if loadStepGeometric=true, the load steps are increased in a geometric series, see loadStepGeometric                                                                                                                                                                                                                                                                                                 |
| useLoadFactor             | bool                  | True  | True: compute a load factor $\in [0, 1]$ from static step time; all loads are scaled by the load factor; False: loads are always scaled with 1 – use this option if time dependent loads use a userFunction                                                                                                                                                                                          |
| stabilizerODE2term        | UReal                 | 0     | add mass-proportional stabilizer term in ODE2 part of jacobian for stabilization (scaled), e.g. of badly conditioned problems; the diagonal terms are scaled with $stabilizer = (1 - loadStepFactor^2)$ , and go to zero at the end of all load steps: $loadStepFactor = 1 \rightarrow stabilizer = 0$                                                                                               |
| adaptiveStep              | bool                  | True  | True: use step reduction if step fails; False: fixed step size                                                                                                                                                                                                                                                                                                                                       |
| adaptiveStepRecoverySteps | UInt                  | 4     | Number of steps needed after which steps will be increased after previous step reduction due to discontinuousIteration or Newton errors                                                                                                                                                                                                                                                              |

|                      |        |      |                                                                                                                                                                                                  |
|----------------------|--------|------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| adaptiveStepIncrease | UReal  | 2    | Multiplicative factor (MUST BE > 1) for step size to increase after previous step reduction due to discontinuousIteration or Newton errors                                                       |
| adaptiveStepDecrease | UReal  | 0.25 | Multiplicative factor (MUST BE: 0 < factor < 1) for step size to decrease due to discontinuousIteration or Newton errors                                                                         |
| minimumStepSize      | PReal  | 1e-8 | lower limit of step size, before nonlinear solver stops                                                                                                                                          |
| verboseMode          | UInt   | 1    | 0 ... no output, 1 ... show errors and load steps, 2 ... show short Newton step information (error), 3 ... show also solution vector, 4 ... show also jacobian, 5 ... show also Jacobian inverse |
| verboseModeFile      | UInt   | 0    | same behaviour as verboseMode, but outputs all solver information to file                                                                                                                        |
| stepInformation      | UInt   | 2    | 0 ... only current step time, 1 ... show time to go, 2 ... show newton iterations (Nit) per step, 3 ... show discontinuous iterations (Dit) and newton jacobians (jac) per step                  |
| preStepPyExecute     | String | "    | DEPRECATED, use mbs.SetPreStepUserFunction(...); Python code to be executed prior to every load step and after last step, e.g. for postprocessing                                                |

### 8.1.9 LinearSolverSettings

Settings for linear solver, both dense and sparse (Eigen).

LinearSolverSettings has the following items:

| Name                       | type/function return type | size | default value / function args | description                                                                                                                                                                                                                                                                                                                               |
|----------------------------|---------------------------|------|-------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| pivotThreshold             | PReal                     | 0    |                               | threshold for dense linear solver, can be used to detect close to singular solutions, setting this to, e.g., 1e-12; solver then reports on equations that are causing close to singularity                                                                                                                                                |
| ignoreRedundantConstraints | bool                      |      | False                         | [ONLY implemented for dense matrices]<br>False: standard way, fails if redundant equations or singular matrices occur; True: if redundant constraints appear, the solver tries to resolve them by setting according Lagrange multipliers to zero; in case of redundant constraints, this may help, but it may lead to erroneous behaviour |

|                        |      |       |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |
|------------------------|------|-------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| ignoreSingularJacobian | bool | False | [ONLY implemented for dense matrices]<br>False: standard way, fails if jacobian is singular; True: if singularities appear in jacobian (e.g. no equation attributed to a node, redundant equations, zero mass matrix, zero eigenvalue for static problem, etc.), the jacobian inverse is resolved such that according solution variables are set to zero; this may help, but it MAY LEAD TO ERRONEOUS BEHAVIOUR; for static problems, this may suppress static motion or resolve problems in case of instabilities, but should in general be considered with care! |
| showCausingItems       | bool | True  | False: no output, if solver fails; True: if redundant equations appear, they are resolved such that according solution variables are set to zero; in case of redundant constraints, this may help, but it may lead to erroneous behaviour; for static problems, this may suppress static motion or resolve problems in case of instabilities, but should in general be considered with care!                                                                                                                                                                       |

### 8.1.10 SimulationSettings

General Settings for simulation; according settings for solution and solvers are given in subitems of this structure.

SimulationSettings has the following items:

| Name                 | type/function return type | size | default value / function args | description                                                                                                                                                                                    |
|----------------------|---------------------------|------|-------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| timeIntegration      | TimeIntegrationSettings   |      |                               | time integration parameters                                                                                                                                                                    |
| solutionSettings     | SolutionSettings          |      |                               | settings for solution files                                                                                                                                                                    |
| staticSolver         | StaticSolverSettings      |      |                               | static solver parameters                                                                                                                                                                       |
| linearSolverSettings | LinearSolverSettings      |      |                               | linear solver parameters (used for dense and sparse solvers)                                                                                                                                   |
| linearSolverType     | LinearSolverType          |      | LinearSolverType::EXUdense    | selection of numerical linear solver:<br>exu.LinearSolverType.EXUdense<br>(dense matrix inverse),<br>exu.LinearSolverType.EigenSparse (sparse matrix LU-factorization), ... (enumeration type) |
| cleanUpMemory        | bool                      |      | False                         | True: solvers will free memory at exit (recommended for large systems); False: keep allocated memory for repeated computations to increase performance                                         |

|                        |      |       |                                                                                                                       |
|------------------------|------|-------|-----------------------------------------------------------------------------------------------------------------------|
| displayStatistics      | bool | False | display general computation information at end of time step (steps, iterations, function calls, step rejections, ...) |
| displayComputationTime | bool | False | display computation time statistics at end of solving                                                                 |
| pauseAfterEachStep     | bool | False | pause after every time step or static load step (user press SPACE)                                                    |
| outputPrecision        | UInt | 6     | precision for floating point numbers written to console; e.g. values written by solver                                |
| numberOfThreads        | PInt | 1     | number of threads used for parallel computation (1 == scalar processing); not yet implemented (status: Nov 2019)      |

## 8.2 Visualization settings

This section includes hierarchical structures for visualization settings, e.g., drawing of nodes, bodies, connectors, loads and markers and furthermore OpenGL, window and save image options.

### 8.2.1 VSettingsGeneral

General settings for visualization.

VSettingsGeneral has the following items:

| Name                         | type/function return type | size | default value / function args | description                                                                                                                                                                                                                                                                                                                                                                                         |
|------------------------------|---------------------------|------|-------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| graphicsUpdateInterval       | float                     |      | 0.1                           | interval of graphics update during simulation in seconds; 0.1 = 10 frames per second; low numbers might slow down computation speed                                                                                                                                                                                                                                                                 |
| autoFitScene                 | bool                      |      | True                          | automatically fit scene within first second after StartRenderer()                                                                                                                                                                                                                                                                                                                                   |
| textSize                     | float                     |      | 12.                           | general text size (font size) in pixels if not overwritten; if useWindowsMonitorScaleFactor=True, the the textSize is multiplied with the windows monitor scaling factor for larger texts on on high resolution monitors; for bitmap fonts, the maximum size of any font (standard/large/huge) is limited to 256 (which is not recommended, especially if you do not have a powerful graphics card) |
| textColor                    | Float4                    | 4    | [0.,0.,0.,1.0]                | general text color (default); used for system texts in render window                                                                                                                                                                                                                                                                                                                                |
| useWindowsMonitorScaleFactor | bool                      |      | True                          | the windows monitor scaling is used for increased visibility of texts on high resolution monitors; based on GLFW glfwGetWindowContentScale                                                                                                                                                                                                                                                          |

|                          |        |                     |                                                                                                                                                                        |
|--------------------------|--------|---------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| useBitmapText            | bool   | True                | if true, texts are displayed using pre-defined bitmaps for the text; may increase the complexity of your scene, e.g., if many (>10000) node numbers shown              |
| minSceneSize             | float  | 0.1                 | minimum scene size for initial scene size and for autoFitScene, to avoid division by zero; SET GREATER THAN ZERO                                                       |
| backgroundColor          | Float4 | 4 [1.0,1.0,1.0,1.0] | red, green, blue and alpha values for background color of render window (white=[1,1,1,1]; black = [0,0,0,1])                                                           |
| backgroundColorBottom    | Float4 | 4 [0.8,0.8,1.0,1.0] | red, green, blue and alpha values for bottom background color in case that useGradientBackground = True                                                                |
| useGradientBackground    | bool   | False               | true = use vertical gradient for background;                                                                                                                           |
| coordinateSystemSize     | float  | 5.                  | size of coordinate system relative to font size                                                                                                                        |
| drawCoordinateSystem     | bool   | True                | false = no coordinate system shown                                                                                                                                     |
| drawWorldBasis           | bool   | False               | true = draw world basis coordinate system at (0,0,0)                                                                                                                   |
| worldBasisSize           | float  | 1.0                 | size of world basis coordinate system                                                                                                                                  |
| showComputationInfo      | bool   | True                | true = show (hide) all computation information including EXUDYN and version                                                                                            |
| showSolutionInformation  | bool   | True                | true = show solution information (from simulationSettings.solution)                                                                                                    |
| showSolverInformation    | bool   | True                | true = solver name and further information shown in render window                                                                                                      |
| showSolverTime           | bool   | True                | true = solver current time shown in render window                                                                                                                      |
| renderWindowString       | String | "                   | string shown in render window (use this, e.g., for debugging, etc.; written below EXUDYN, similar to solutionInformation in SimulationSettings.solutionSettings)       |
| pointSize                | float  | 0.01                | global point size (absolute)                                                                                                                                           |
| circleTiling             | PInt   | 16                  | global number of segments for circles; if smaller than 2, 2 segments are used (flat)                                                                                   |
| cylinderTiling           | PInt   | 16                  | global number of segments for cylinders; if smaller than 2, 2 segments are used (flat)                                                                                 |
| sphereTiling             | PInt   | 6                   | global number of segments for spheres; if smaller than 2, 2 segments are used (flat)                                                                                   |
| axesTiling               | PInt   | 12                  | global number of segments for drawing axes cylinders and cones (reduce this number, e.g. to 4, if many axes are drawn)                                                 |
| threadSafeGraphicsUpdate | bool   | True                | true = updating of visualization is thread-safe, but slower for complicated models; deactivate this to speed up computation, but activate for generation of animations |

## 8.2.2 VSettingsContour

Settings for contour plots; use these options to visualize field data, such as displacements, stresses, strains, etc. for bodies, nodes and finite elements.

VSettingsContour has the following items:

| Name                    | type/function re-<br>turn type | size | default value / func-<br>tion args | description                                                                                                                                                                                                                                                                                                                                                                                      |
|-------------------------|--------------------------------|------|------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| outputVariableComponent | Int                            | 1    | 0                                  | select the component of the chosen output variable; e.g., for displacements, 3 components are available: 0 == x, 1 == y, 2 == z component; for stresses, 6 components are available, see OutputVariableType description; to draw the norm of a outputVariable, set component to -1; if a certain component is not available by certain objects or nodes, no value is drawn (using default color) |
| outputVariable          | OutputVariableType             |      | OutputVariableType:_None           | selected contour plot output variable type; select OutputVariableType._None to deactivate contour plotting.                                                                                                                                                                                                                                                                                      |
| minValue                | float                          | 1    | 0                                  | minimum value for contour plot; set manually, if automaticRange == False                                                                                                                                                                                                                                                                                                                         |
| maxValue                | float                          | 1    | 1                                  | maximum value for contour plot; set manually, if automaticRange == False                                                                                                                                                                                                                                                                                                                         |
| automaticRange          | bool                           |      | True                               | if true, the contour plot value range is chosen automatically to the maximum range                                                                                                                                                                                                                                                                                                               |
| reduceRange             | bool                           |      | True                               | if true, the contour plot value range is also reduced; better for static computation; in dynamic computation set this option to false, it can reduce visualization artifacts; you should also set minVal to max(float) and maxVal to min(float)                                                                                                                                                  |
| showColorBar            | bool                           |      | True                               | show the colour bar with minimum and maximum values for the contour plot                                                                                                                                                                                                                                                                                                                         |
| colorBarTiling          | PInt                           | 1    | 12                                 | number of tiles (segements) shown in the colorbar for the contour plot                                                                                                                                                                                                                                                                                                                           |

## 8.2.3 VSettingsNodes

Visualization settings for nodes.

VSettingsNodes has the following items:

| Name        | type/function re-<br>turn type | size | default value / func-<br>tion args | description                                      |
|-------------|--------------------------------|------|------------------------------------|--------------------------------------------------|
| show        | bool                           |      | True                               | flag to decide, whether the nodes are shown      |
| showNumbers | bool                           |      | False                              | flag to decide, whether the node number is shown |

|                  |        |                  |                                                                                                                                                        |
|------------------|--------|------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------|
| drawNodesAsPoint | bool   | True             | simplified/faster drawing of nodes; uses general->pointSize as drawing size; if drawNodesAsPoint==True, the basis of the node will be drawn with lines |
| showBasis        | bool   | False            | show basis (three axes) of coordinate system in 3D nodes                                                                                               |
| basisSize        | float  | 0.2              | size of basis for nodes                                                                                                                                |
| tiling           | PInt   | 4                | tiling for node if drawn as sphere; used to lower the amount of triangles to draw each node; if drawn as circle, this value is multiplied with 4       |
| defaultSize      | float  | -1.              | global node size; if -1.f, node size is relative to openGL.initialMaxSceneSize                                                                         |
| defaultColor     | Float4 | 4 [0.2,0.2,1,1.] | default cRGB color for nodes; 4th value is alpha-transparency                                                                                          |
| showNodalSlopes  | UInt   | False            | draw nodal slope vectors, e.g. in ANCF beam finite elements                                                                                            |

## 8.2.4 VSettingsBeams

Visualization settings for beam finite elements.

VSettingsBeams has the following items:

| Name        | type/function return type | size | default value / function args | description                                     |
|-------------|---------------------------|------|-------------------------------|-------------------------------------------------|
| axialTiling | PInt                      | 8    |                               | number of segments to discretise the beams axis |

## 8.2.5 VSettingsBodies

Visualization settings for bodies.

VSettingsBodies has the following items:

| Name         | type/function return type | size | default value / function args | description                                               |
|--------------|---------------------------|------|-------------------------------|-----------------------------------------------------------|
| show         | bool                      |      | True                          | flag to decide, whether the bodies are shown              |
| showNumbers  | bool                      |      | False                         | flag to decide, whether the body(=object) number is shown |
| defaultSize  | Float3                    | 3    | [1.,1.,1.]                    | global body size of xyz-cube                              |
| defaultColor | Float4                    | 4    | [0.3,0.3,1,1.]                | default cRGB color for bodies; 4th value is               |

|                        |                |   |                                                                                                                                    |
|------------------------|----------------|---|------------------------------------------------------------------------------------------------------------------------------------|
| deformationScaleFactor | float          | 1 | global deformation scale factor; also applies to nodes, if drawn; used for scaled drawing of (linear) finite elements, beams, etc. |
| beams                  | VSettingsBeams |   | visualization settings for beams (e.g. ANCFCable or other beam elements)                                                           |

## 8.2.6 VSettingsConnectors

Visualization settings for connectors.

VSettingsConnectors has the following items:

| Name                     | type/function re-<br>turn type | size | default value / func-<br>tion args | description                                                                     |
|--------------------------|--------------------------------|------|------------------------------------|---------------------------------------------------------------------------------|
| show                     | bool                           |      | True                               | flag to decide, whether the connectors are shown                                |
| showNumbers              | bool                           |      | False                              | flag to decide, whether the connector(=object) number is shown                  |
| defaultSize              | float                          |      | 0.1                                | global connector size; if -1.f, connector size is relative to maxSceneSize      |
| showJointAxes            | bool                           |      | False                              | flag to decide, whether contact joint axes of 3D joints are shown               |
| jointAxesLength          | float                          |      | 0.2                                | global joint axes length                                                        |
| jointAxesRadius          | float                          |      | 0.02                               | global joint axes radius                                                        |
| showContact              | bool                           |      | False                              | flag to decide, whether contact points, lines, etc. are shown                   |
| springNumberOfWindings   | PInt                           |      | 8                                  | number of windings for springs drawn as helical spring                          |
| contactPointsDefaultSize | float                          |      | 0.02                               | global contact points size; if -1.f, connector size is relative to maxSceneSize |
| defaultColor             | Float4                         | 4    | [0.2,0.2,1.,1.]                    | default cRGB color for connectors; 4th value is alpha-transparency              |

## 8.2.7 VSettingsMarkers

Visualization settings for markers.

VSettingsMarkers has the following items:

| Name           | type/function re-<br>turn type | size | default value / func-<br>tion args | description                                          |
|----------------|--------------------------------|------|------------------------------------|------------------------------------------------------|
| show           | bool                           |      | True                               | flag to decide, whether the markers are shown        |
| showNumbers    | bool                           |      | False                              | flag to decide, whether the marker numbers are shown |
| drawSimplified | bool                           |      | True                               | draw markers with simplified symbols                 |

|              |        |   |                  |                                                                      |
|--------------|--------|---|------------------|----------------------------------------------------------------------|
| defaultSize  | float  |   | -1.              | global marker size; if -1.f, marker size is relative to maxSceneSize |
| defaultColor | Float4 | 4 | [0.1,0.5,0.1,1.] | default cRGB color for markers; 4th value is alpha-transparency      |

## 8.2.8 VSettingsLoads

Visualization settings for loads.

VSettingsLoads has the following items:

| Name           | type/function return type | size | default value / function args | description                                                                                                           |
|----------------|---------------------------|------|-------------------------------|-----------------------------------------------------------------------------------------------------------------------|
| show           | bool                      |      | True                          | flag to decide, whether the loads are shown                                                                           |
| showNumbers    | bool                      |      | False                         | flag to decide, whether the load numbers are shown                                                                    |
| defaultSize    | float                     |      | 0.2                           | global load size; if -1.f, load size is relative to maxSceneSize                                                      |
| defaultRadius  | float                     |      | 0.005                         | global radius of load axis if drawn in 3D                                                                             |
| fixedLoadSize  | bool                      |      | True                          | if true, the load is drawn with a fixed vector length in direction of the load vector, independently of the load size |
| drawSimplified | bool                      |      | True                          | draw markers with simplified symbols                                                                                  |
| loadSizeFactor | float                     |      | 0.1                           | if fixedLoadSize=false, then this scaling factor is used to draw the load vector                                      |
| defaultColor   | Float4                    | 4    | [0.7,0.1,0.1,1.]              | default cRGB color for loads; 4th value is alpha-transparency                                                         |

## 8.2.9 VSettingsSensors

Visualization settings for sensors.

VSettingsSensors has the following items:

| Name           | type/function return type | size | default value / function args | description                                                          |
|----------------|---------------------------|------|-------------------------------|----------------------------------------------------------------------|
| show           | bool                      |      | True                          | flag to decide, whether the sensors are shown                        |
| showNumbers    | bool                      |      | False                         | flag to decide, whether the sensor numbers are shown                 |
| drawSimplified | bool                      |      | True                          | draw sensors with simplified symbols                                 |
| defaultSize    | float                     |      | -1.                           | global sensor size; if -1.f, sensor size is relative to maxSceneSize |

|              |        |   |                  |                                                                 |
|--------------|--------|---|------------------|-----------------------------------------------------------------|
| defaultColor | Float4 | 4 | [0.6,0.6,0.1,1.] | default cRGB color for sensors; 4th value is alpha-transparency |
|--------------|--------|---|------------------|-----------------------------------------------------------------|

## 8.2.10 VSettingsWindow

Window and interaction settings for visualization; handle changes with care, as they might lead to unexpected results or crashes.

VSettingsWindow has the following items:

| Name                        | type/function return type | size | default value / function args | description                                                                                                                                                                                                                                                                                                                                                                                                                                                              |
|-----------------------------|---------------------------|------|-------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| renderWindowSize            | Index2                    | 2    | [1024,768]                    | initial size of OpenGL render window in pixel                                                                                                                                                                                                                                                                                                                                                                                                                            |
| startupTimeout              | PInt                      |      | 2500                          | OpenGL render window startup timeout in ms (change might be necessary if CPU is very slow)                                                                                                                                                                                                                                                                                                                                                                               |
| alwaysOnTop                 | bool                      |      | False                         | True: OpenGL render window will be always on top of all other windows                                                                                                                                                                                                                                                                                                                                                                                                    |
| maximize                    | bool                      |      | False                         | True: OpenGL render window will be maximized at startup                                                                                                                                                                                                                                                                                                                                                                                                                  |
| showWindow                  | bool                      |      | True                          | True: OpenGL render window is shown on startup; False: window will be iconified at startup (e.g. if you are starting multiple computations automatically)                                                                                                                                                                                                                                                                                                                |
| keyPressUserFunction        | KeyPressUserFunction      | 0    |                               | add a Python function f(key, action, mods) here, which is called every time a key is pressed; Example:<br>def f(key, action, mods):<br>print('key=',key);<br>use chr(key) to convert key codes [32 ...96] to ascii; special key codes (>256) are provided in the exudyn.KeyCode enumeration type; key action needs to be checked (0=released, 1=pressed, 2=repeated); mods provide information (binary) for SHIFT (1), CTRL (2), ALT (4), Super keys (8), CAP-SLOCK (16) |
| showMouseCoordinates        | bool                      |      | False                         | True: show OpenGL coordinates and distance to last left mouse button pressed position; switched on/off with key 'F3'                                                                                                                                                                                                                                                                                                                                                     |
| ignoreKeys                  | bool                      |      | False                         | True: ignore keyboard input except escape and 'F2' keys; used for interactive mode, e.g., to perform kinematic analysis; This flag can be switched with key 'F2'                                                                                                                                                                                                                                                                                                         |
| ResetKeyPressUserFunction() | void                      |      |                               | set keyPressUserFunction to zero (no function); because this cannot be assigned to the variable itself                                                                                                                                                                                                                                                                                                                                                                   |

## 8.2.11 VSettingsOpenGL

OpenGL settings for 2D and 2D rendering. For further details, see the OpenGL functionality.

VSettingsOpenGL has the following items:

| <b>Name</b>          | <b>type/function re-<br/>turn type</b> | <b>size</b> | <b>default value / func-<br/>tion args</b>    | <b>description</b>                                                                                                                                                                                                                                                           |
|----------------------|----------------------------------------|-------------|-----------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| initialCenterPoint   | Float3                                 | 3           | [0.,0.,0.]                                    | centerpoint of scene (3D) at renderer startup; overwritten if autoFitScene = True                                                                                                                                                                                            |
| initialZoom          | float                                  |             | 1.                                            | initial zoom of scene; overwritten/ignored if autoFitScene = True                                                                                                                                                                                                            |
| initialMaxSceneSize  | float                                  |             | 1.                                            | initial maximum scene size (auto: diagonal of cube with maximum scene coordinates); used for 'zoom all' functionality and for visibility of objects; overwritten if autoFitScene = True                                                                                      |
| initialModelRotation | StdArray33F                            | 3x3         | [Matrix3DF[3,3,1.,0,0.,<br>0.,1.,0., 0,0,1.]] | initial model rotation matrix for OpenGL; in python use e.g.: initialModelRotation=[[1,0,0],[0,1,0],[0,0,1]]                                                                                                                                                                 |
| multiSampling        | PInt                                   | 1           | 1                                             | multi sampling turned off (<=1) or turned on to given values (2, 4, 8 or 16); increases the graphics buffers and might crash due to graphics card memory limitations; only works if supported by hardware; if it does not work, try to change 3D graphics hardware settings! |
| lineWidth            | float                                  | 1           | 1.                                            | width of lines used for representation of lines, circles, points, etc.                                                                                                                                                                                                       |
| lineSmooth           | bool                                   | 1           | True                                          | draw lines smooth                                                                                                                                                                                                                                                            |
| textLineWidth        | float                                  | 1           | 1.                                            | width of lines used for representation of text                                                                                                                                                                                                                               |
| textLineSmooth       | bool                                   | 1           | False                                         | draw lines for representation of text smooth                                                                                                                                                                                                                                 |
| showFaces            | bool                                   | 1           | True                                          | show faces of triangles, etc.; using the options showFaces=false and showFaceEdges=true gives are wire frame representation                                                                                                                                                  |
| facesTransparent     | bool                                   | 1           | False                                         | True: show faces transparent independent of transparency (A)-value in color of objects; allow to show otherwise hidden node/marker/object numbers                                                                                                                            |
| showFaceEdges        | bool                                   | 1           | False                                         | show edges of faces; using the options showFaces=false and showFaceEdges=true gives are wire frame representation                                                                                                                                                            |
| shadeModelSmooth     | bool                                   | 1           | True                                          | True: turn on smoothing for shaders, which uses vertex normals to smooth surfaces                                                                                                                                                                                            |

|                            |        |   |                  |                                                                                                                                                                                                                                                                                                 |
|----------------------------|--------|---|------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| materialAmbientAndDiffuse  | Float4 | 4 | [0.6,0.6,0.6,1.] | 4f ambient color of material                                                                                                                                                                                                                                                                    |
| materialShininess          | float  | 1 | 32.              | shininess of material                                                                                                                                                                                                                                                                           |
| materialSpecular           | Float4 | 4 | [0.6,0.6,0.6,1.] | 4f specular color of material                                                                                                                                                                                                                                                                   |
| enableLighting             | bool   | 1 | True             | generally enable lighting (otherwise, colors of objects are used); OpenGL: glEnable(GL_LIGHTING)                                                                                                                                                                                                |
| lightModelLocalViewer      | bool   | 1 | False            | select local viewer for light; maps to OpenGL glLightModel(GL_LIGHT_MODEL_LOCAL_VIEWER,...)                                                                                                                                                                                                     |
| lightModelTwoSide          | bool   | 1 | True             | enlighten also backside of object; maps to OpenGL glLightModel(GL_LIGHT_MODEL_TWO_SIDE,...)                                                                                                                                                                                                     |
| lightModelAmbient          | Float4 | 4 | [0.,0.,0.,1.]    | global ambient light; maps to OpenGL glLightModel(GL_LIGHT_MODEL_AMBIENT,[r,g,b,a])                                                                                                                                                                                                             |
| enableLight0               | bool   | 1 | True             | turn on/off light0                                                                                                                                                                                                                                                                              |
| light0position             | Float4 | 4 | [0.2,0.2,10.,0.] | 4f position vector of GL_LIGHT0; 4th value should be 0 for lights like sun, but 1 for directional lights (and for attenuation factor being calculated); see opengl manuals                                                                                                                      |
| light0ambient              | float  | 1 | 0.3              | ambient value of GL_LIGHT0                                                                                                                                                                                                                                                                      |
| light0diffuse              | float  | 1 | 0.6              | diffuse value of GL_LIGHT0                                                                                                                                                                                                                                                                      |
| light0specular             | float  | 1 | 0.5              | specular value of GL_LIGHT0                                                                                                                                                                                                                                                                     |
| light0constantAttenuation  | float  | 1 | 1.0              | constant attenuation coefficient of GL_LIGHT0, this is a constant factor that attenuates the light source; attenuation factor = $1/(k_c + k_l \cdot d + k_q \cdot d^2)$ ; $(k_c, k_l, k_q) = (1, 0, 0)$ means no attenuation; only used for lights, where last component of light position is 1 |
| light0linearAttenuation    | float  | 1 | 0.0              | linear attenuation coefficient of GL_LIGHT0, this is a linear factor for attenuation of the light source with distance                                                                                                                                                                          |
| light0quadraticAttenuation | float  | 1 | 0.0              | quadratic attenuation coefficient of GL_LIGHT0, this is a quadratic factor for attenuation of the light source with distance                                                                                                                                                                    |
| enableLight1               | bool   | 1 | True             | turn on/off light1                                                                                                                                                                                                                                                                              |
| light1position             | Float4 | 4 | [1.,1.,-10.,0.]  | 4f position vector of GL_LIGHT0; 4th value should be 0 for lights like sun, but 1 for directional lights (and for attenuation factor being calculated); see opengl manuals                                                                                                                      |
| light1ambient              | float  | 1 | 0.0              | ambient value of GL_LIGHT1                                                                                                                                                                                                                                                                      |
| light1diffuse              | float  | 1 | 0.5              | diffuse value of GL_LIGHT1                                                                                                                                                                                                                                                                      |

|                            |       |   |       |                                                                                                                                                                                                                              |
|----------------------------|-------|---|-------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| light1specular             | float | 1 | 0.6   | specular value of GL_LIGHT1                                                                                                                                                                                                  |
| light1constantAttenuation  | float | 1 | 1.0   | constant attenuation coefficient of GL_LIGHT1, this is a constant factor that attenuates the light source; attenuation factor = $1/(kx + kl*d + kq*d^2)$ ; only used for lights, where last component of light position is 1 |
| light1linearAttenuation    | float | 1 | 0.0   | linear attenuation coefficient of GL_LIGHT1, this is a linear factor for attenuation of the light source with distance                                                                                                       |
| light1quadraticAttenuation | float | 1 | 0.0   | quadratic attenuation coefficient of GL_LIGHT1, this is a quadratic factor for attenuation of the light source with distance                                                                                                 |
| drawFaceNormals            | bool  | 1 | False | draws triangle normals, e.g. at center of triangles; used for debugging of faces                                                                                                                                             |
| drawVertexNormals          | bool  | 1 | False | draws vertex normals; used for debugging                                                                                                                                                                                     |
| drawNormalsLength          | float | 1 | 0.1   | length of normals; used for debugging                                                                                                                                                                                        |

## 8.2.12 VSettingsExportImages

Functionality to export images to files (.tga format) which can be used to create animations; to activate image recording during the solution process, set SolutionSettings.recordImagesInterval accordingly.

VSettingsExportImages has the following items:

| Name                 | type/function return type | size | default value / function args | description                                                                                                                                                                             |
|----------------------|---------------------------|------|-------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| saveImageTimeOut     | PInt                      |      | 5000                          | timeout in milliseconds for saving a frame as image to disk; this is the amount of time waited for redrawing; increase for very complex scenes                                          |
| saveImageFileName    | FileName                  |      | 'images/frame'                | filename (without extension!) and (relative) path for image file(s) with consecutive numbering (e.g., frame0000.tga, frame0001.tga,...); directory will be created if it does not exist |
| saveImageFileCounter | UInt                      |      | 0                             | current value of the counter which is used to consecutively save frames (images) with consecutive numbers                                                                               |
| saveImageSingleFile  | bool                      |      | False                         | True: only save single files with given filename, not adding numbering; False: add numbering to files, see saveImageFileName                                                            |

### 8.2.13 VSettingsInteractive

Functionality to interact with render window; will include left and right mouse press actions and others in future.

VSettingsInteractive has the following items:

| Name                    | type/function re-<br>turn type | size | default value / func-<br>tion args | description                                                                                                        |
|-------------------------|--------------------------------|------|------------------------------------|--------------------------------------------------------------------------------------------------------------------|
| keypressRotationStep    | float                          |      | 5.                                 | rotation increment per keypress in degree (full rotation = 360 degree)                                             |
| mouseMoveRotationFactor | float                          |      | 1.                                 | rotation increment per 1 pixel mouse movement in degree                                                            |
| keypressTranslationStep | float                          |      | 0.1                                | translation increment per keypress relative to window size                                                         |
| zoomStepFactor          | float                          |      | 1.15                               | change of zoom per keypress (keypad +/-) or mouse wheel increment                                                  |
| highlightItemIndex      | Int                            |      | -1                                 | index of item that shall be highlighted (e.g., need to find item due to errors); if set -1, no item is highlighted |
| highlightItemType       | ItemType                       |      | ItemType::None                     | item type (Node, Object, ...) that shall be highlighted (e.g., need to find item due to errors)                    |
| highlightColor          | Float4                         | 4    | [0.8,0.05,0.05,0.75]               | cRGB color for highlighted item; 4th value is alpha-transparency                                                   |
| highlightOtherColor     | Float4                         | 4    | [0.5,0.5,0.5,0.4]                  | cRGB color for other items (which are not highlighted); 4th value is alpha-transparency                            |

### 8.2.14 VisualizationSettings

Settings for visualization.

VisualizationSettings has the following items:

| Name       | type/function re-<br>turn type | size | default value / func-<br>tion args | description                                   |
|------------|--------------------------------|------|------------------------------------|-----------------------------------------------|
| general    | VSettingsGeneral               |      |                                    | general visualization settings                |
| contour    | VSettingsContour               |      |                                    | contour plot visualization settings           |
| nodes      | VSettingsNodes                 |      |                                    | node visualization settings                   |
| bodies     | VSettingsBodies                |      |                                    | body visualization settings                   |
| connectors | VSettingsConnectors            |      |                                    | connector visualization settings              |
| markers    | VSettingsMarkers               |      |                                    | marker visualization settings                 |
| loads      | VSettingsLoads                 |      |                                    | load visualization settings                   |
| sensors    | VSettingsSensors               |      |                                    | sensor visualization settings                 |
| window     | VSettingsWindow                |      |                                    | visualization window and interaction settings |
| openGL     | VSettingsOpenGL                |      |                                    | OpenGL rendering settings                     |

|              |                       |  |                                                                               |
|--------------|-----------------------|--|-------------------------------------------------------------------------------|
| interactive  | VSettingsInteractive  |  | Settings for interaction with renderer                                        |
| exportImages | VSettingsExportImages |  | settings for exporting (saving) images to files in order to create animations |

## 8.3 Solver substructures

This section includes structures contained in the solver, which can be accessed via the python interface during solution or for building a customized solver in python.

### 8.3.1 CSolverTimer

Structure for timing in solver. Each Real variable is used to measure the CPU time which certain parts of the solver need. This structure is only active if the code is not compiled with the `_FAST_EXUDYN_LINALG` option and if `displayComputationTime` is set True. Timings will only be filled, if `useTimer` is True.

`CSolverTimer` has the following items:

| Name               | type/function return type | size | default value / function args | description                                                                |
|--------------------|---------------------------|------|-------------------------------|----------------------------------------------------------------------------|
| useTimer           | bool                      |      | True                          | flag to decide, whether the timer is used (true) or not                    |
| total              | Real                      |      | 0.                            | total time measured between start and end of computation (static/dynamics) |
| factorization      | Real                      |      | 0.                            | solve or inverse                                                           |
| newtonIncrement    | Real                      |      | 0.                            | $\text{Jac}^{-1} * \text{RHS}$ ; backsubstitution                          |
| integrationFormula | Real                      |      | 0.                            | time spent for evaluation of integration formulas                          |
| ODE2RHS            | Real                      |      | 0.                            | time for residual evaluation of ODE2 right-hand-side                       |
| ODE1RHS            | Real                      |      | 0.                            | time for residual evaluation of ODE1 right-hand-side                       |
| AERHS              | Real                      |      | 0.                            | time for residual evaluation of algebraic equations right-hand-side        |
| totalJacobian      | Real                      |      | 0.                            | time for all jacobian computations                                         |
| jacobianODE1       | Real                      |      | 0.                            | jacobian w.r.t. coordinates of ODE1 equations (not counted in sum)         |
| jacobianODE2       | Real                      |      | 0.                            | jacobian w.r.t. coordinates of ODE2 equations (not counted in sum)         |
| jacobianODE2_t     | Real                      |      | 0.                            | jacobian w.r.t. coordinates_t of ODE2 equations (not counted in sum)       |
| jacobianAE         | Real                      |      | 0.                            | jacobian of algebraic equations (not counted in sum)                       |
| massMatrix         | Real                      |      | 0.                            | mass matrix computation                                                    |
| reactionForces     | Real                      |      | 0.                            | $\text{CqT} * \text{lambda}$                                               |
| postNewton         | Real                      |      | 0.                            | post newton step                                                           |
| errorEstimator     | Real                      |      | 0.                            | for explicit solvers, additional evaluation                                |
| writeSolution      | Real                      |      | 0.                            | time for writing solution                                                  |

|                 |        |                |                                                                                                          |
|-----------------|--------|----------------|----------------------------------------------------------------------------------------------------------|
| overhead        | Real   | 0.             | overhead, such as initialization, copying and some matrix-vector multiplication                          |
| python          | Real   | 0.             | time spent for python functions                                                                          |
| visualization   | Real   | 0.             | time spent for visualization in computation thread                                                       |
| Reset(...)      | void   | useSolverTimer | reset solver timings to initial state by assigning default values; useSolverTimer sets the useTimer flag |
| Sum()           | Real   |                | compute sum of all timers (except for those counted multiple, e.g., jacobians)                           |
| StartTimer(...) | void   | value          | start timer function for a given variable; subtracts current CPU time from value                         |
| StopTimer(...)  | void   | value          | stop timer function for a given variable; adds current CPU time to value                                 |
| ToString()      | String |                | converts the current timings to a string                                                                 |

### 8.3.2 SolverLocalData

Solver local data structure for solution vectors, system matrices and temporary vectors and data structures. SolverLocalData has the following items:

| Name                         | type/function return type | size | default value / function args | description                                                                          |
|------------------------------|---------------------------|------|-------------------------------|--------------------------------------------------------------------------------------|
| nODE2                        | Index                     | 0    |                               | number of second order ordinary diff. eq. coordinates                                |
| nODE1                        | Index                     | 0    |                               | number of first order ordinary diff. eq. coordinates                                 |
| nAE                          | Index                     | 0    |                               | number of algebraic coordinates                                                      |
| nData                        | Index                     | 0    |                               | number of data coordinates                                                           |
| nSys                         | Index                     | 0    |                               | number of system (unknown) coordinates = nODE2+nODE1+nAE                             |
| startAE                      | Index                     | 0    |                               | start of algebraic coordinates, but set to zero if nAE==0                            |
| systemResidual               | ResizableVector           |      |                               | system residual vector (vectors will be linked to this vector!)                      |
| newtonSolution               | ResizableVector           |      |                               | Newton decrement (computed from residual and jacobian)                               |
| tempODE2                     | ResizableVector           |      |                               | temporary vector for ODE2 quantities; use in initial accelerations and during Newton |
| temp2ODE2                    | ResizableVector           |      |                               | second temporary vector for ODE2 quantities; use in static computation               |
| tempODE2F0                   | ResizableVector           |      |                               | temporary vector for ODE2 Jacobian                                                   |
| tempODE2F1                   | ResizableVector           |      |                               | temporary vector for ODE2 Jacobian                                                   |
| tempODE1F0                   | ResizableVector           |      |                               | temporary vector for ODE1 Jacobian                                                   |
| tempODE1F1                   | ResizableVector           |      |                               | temporary vector for ODE1 Jacobian                                                   |
| startOfStepStateAAlgorithmic | ResizableVector           |      |                               | additional term needed for generalized alpha (startOfStep state)                     |

|                          |                  |                  |                                                                                                     |
|--------------------------|------------------|------------------|-----------------------------------------------------------------------------------------------------|
| aAlgorithmic             | ResizableVector  |                  | additional term needed for generalized alpha (current state)                                        |
| CleanUpMemory()          | void             |                  | if desired, temporary data is cleaned up to safe memory                                             |
| SetLinearSolverType(...) | void             | linearSolverType | set linear solver type and matrix version: links system matrices to according dense/sparse versions |
| GetLinearSolverType()    | LinearSolverType |                  | return current linear solver type (dense/sparse)                                                    |

### 8.3.3 SolverIterationData

Solver internal structure for counters, steps, step size, time, etc.; solution vectors, residuals, etc. are SolverLocalData. The given default values are overwritten by the simulationSettings when initializing the solver. SolverIterationData has the following items:

| Name                | type/function return type | size | default value / function args | description                                                                                                                                                                                                                     |
|---------------------|---------------------------|------|-------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| maxStepSize         | Real                      |      | 0.                            | constant or maximum stepSize                                                                                                                                                                                                    |
| minStepSize         | Real                      |      | 0.                            | minimum stepSize for static/dynamic solver; only used, if automaticStepSize is activated                                                                                                                                        |
| initialStepSize     | Real                      |      | 1e-6                          | initial stepSize for dynamic solver; only used, if automaticStepSize is activated                                                                                                                                               |
| lastStepSize        | Real                      |      | 0.                            | stepSize suggested from last step or by initial step size; only used, if automaticStepSize is activated                                                                                                                         |
| currentStepSize     | Real                      |      | 0.                            | stepSize of current step                                                                                                                                                                                                        |
| recommendedStepSize | Real                      |      | -1.                           | recommended step size $h_{recom}$ after Post-Newton(...): $h_{recom} < 0$ : no recommendation, $h_{recom} == 0$ : use minimum step size, $h_{recom} > 0$ : use specific step size, if no smaller size requested by other reason |
| numberOfSteps       | Index                     |      | 0                             | number of time steps (if fixed size); $n$                                                                                                                                                                                       |
| currentStepIndex    | Index                     |      | 0                             | current step index; $i$                                                                                                                                                                                                         |
| adaptiveStep        | bool                      |      | True                          | True: the step size may be reduced if step fails; no automatic stepsize control                                                                                                                                                 |
| automaticStepSize   | bool                      |      | True                          | True: if timeIntegration.automaticStepSize == True AND chosen integrators supports automatic step size control (e.g., DOPRI5); False: constant step size used (step may be reduced if adaptiveStep=True)                        |
| currentTime         | Real                      |      | 0.                            | holds the current simulation time, copy of state.current.time; interval is [start-Time,tEnd]; in static solver, duration is loadStepDuration                                                                                    |
| startTime           | Real                      |      | 0.                            | time at beginning of time integration                                                                                                                                                                                           |

|                                |        |    |                                                                                                                                |
|--------------------------------|--------|----|--------------------------------------------------------------------------------------------------------------------------------|
| endTime                        | Real   | 0. | end time of static/dynamic solver                                                                                              |
| discontinuousIteration         | Index  | 0  | number of current discontinuous iteration                                                                                      |
| newtonSteps                    | Index  | 0  | number of current newton steps                                                                                                 |
| newtonStepsCount               | Index  | 0  | count total Newton steps                                                                                                       |
| newtonJacobiCount              | Index  | 0  | count total Newton jacobian computations                                                                                       |
| rejectedModifiedNewtonSteps    | Index  | 0  | count the number of rejected modified Newton steps (switch to full Newton)                                                     |
| discontinuousIterationsCount   | Index  | 0  | count total number of discontinuous iterations (min. 1 per step)                                                               |
| rejectedAutomaticStepSizeSteps | Index  | 0  | count the number of rejected steps in case of automatic step size control (rejected steps are repeated with smaller step size) |
| automaticStepSizeError         | Real   | 0  | estimated error (relative to atol + rtol*solution) of last step; must be $\leq 1$ for a step to be accepted                    |
| ToString()                     | String |    | convert iteration statistics to string; used for displayStatistics option                                                      |

### 8.3.4 SolverConvergenceData

Solver internal structure for convergence information: residua, iteration loop errors and error flags. For detailed behavior of these flags, visit the source code!

SolverConvergenceData has the following items:

| Name                             | type/function return type | size | default value / function args | description                                                                                                                               |
|----------------------------------|---------------------------|------|-------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------|
| stepReductionFailed              | bool                      |      | False                         | true, if iterations over time/static steps failed (finally, cannot be recovered)                                                          |
| discontinuousIterationSuccessful | bool                      |      | True                          | true, if last discontinuous iteration had success (failure may be recovered by adaptive step)                                             |
| linearSolverFailed               | bool                      |      | False                         | true, if linear solver failed to factorize                                                                                                |
| linearSolverCausingRow           | Index                     |      | -1                            | -1 if successful, 0 ... n-1, the system equation (=coordinate) index which may have caused the problem, at which the linear solver failed |
| newtonConverged                  | bool                      |      | False                         | true, if Newton has (finally) converged                                                                                                   |
| newtonSolutionDiverged           | bool                      |      | False                         | true, if Newton diverged (may be recovered)                                                                                               |
| jacobianUpdateRequested          | bool                      |      | True                          | true, if a jacobian update is requested in modified Newton (determined in previous step)                                                  |
| massMatrixNotInvertible          | bool                      |      | True                          | true, if mass matrix is not invertable during initialization or solution (explicit solver)                                                |
| discontinuousIterationError      | Real                      |      | 0.                            | error of discontinuous iterations (contact, friction, ...) outside of Newton iteration                                                    |
| residual                         | Real                      |      | 0.                            | current Newton residual                                                                                                                   |

|                       |      |    |                                                                            |
|-----------------------|------|----|----------------------------------------------------------------------------|
| lastResidual          | Real | 0. | last Newton residual to determine contractivity                            |
| contractivity         | Real | 0. | Newton contractivity = geometric decay of error in every step              |
| errorCoordinateFactor | Real | 1. | factor may include the number of system coordinates to reduce the residual |
| InitializeData()      | void |    | initialize SolverConvergenceData by assigning default values               |

### 8.3.5 SolverOutputData

Solver internal structure for output modes, output timers and counters.

SolverOutputData has the following items:

| Name                 | type/function re-<br>turn type | size | default value / func-<br>tion args | description                                                                                         |
|----------------------|--------------------------------|------|------------------------------------|-----------------------------------------------------------------------------------------------------|
| finishedSuccessfully | bool                           |      | False                              | flag is false until solver finished successfully (can be used as external trigger)                  |
| verboseMode          | Index                          |      | 0                                  | this is a copy of the solvers verboseMode used for console output                                   |
| verboseModeFile      | Index                          |      | 0                                  | this is a copy of the solvers verboseModeFile used for file                                         |
| stepInformation      | Index                          |      | 0                                  | this is a copy of the solvers stepInformation used for console output                               |
| writeToSolutionFile  | bool                           |      | False                              | if false, no solution file is generated and no file is written                                      |
| writeToSolverFile    | bool                           |      | False                              | if false, no solver output file is generated and no file is written                                 |
| sensorValuesTemp     | ResizableVector                |      |                                    | temporary vector for per sensor values (overwritten for every sensor; usually contains last sensor) |
| lastSolutionWritten  | Real                           |      | 0.                                 | simulation time when last solution has been written                                                 |
| lastSensorsWritten   | Real                           |      | 0.                                 | simulation time when last sensors have been written                                                 |
| lastImageRecorded    | Real                           |      | 0.                                 | simulation time when last image has been recorded                                                   |
| cpuStartTime         | Real                           |      | 0.                                 | CPU start time of computation (starts counting at computation of initial conditions)                |
| cpuLastTimePrinted   | Real                           |      | 0.                                 | CPU time when output has been printed last time                                                     |
| InitializeData()     | void                           |      |                                    | initialize SolverOutputData by assigning default values                                             |

### 8.3.6 MainSolverStatic

PyBind interface (trampoline) class for static solver. With this interface, the static solver and its substructures can be accessed via python. NOTE that except from SolveSystem(...), these functions are only intended for experienced users and they need to be handled with care, as unexpected crashes may happen if used inappropriate. Furthermore, the functions have a lot of overhead (performance much lower than internal solver) due to python interfaces, and should thus be used for small systems. To access the solver in python, write:

```
solver = MainSolverStatic()
```

and hereafter you can access all data and functions via 'solver'.

MainSolverStatic has the following items:

| Name                             | type/function return type | size | default value / function args            | description                                                                                                        |
|----------------------------------|---------------------------|------|------------------------------------------|--------------------------------------------------------------------------------------------------------------------|
| timer                            | CSolverTimer              |      |                                          | timer which measures the CPU time of solver sub functions                                                          |
| it                               | SolverIterationData       |      |                                          | all information about iterations (steps, discontinuous iteration, newton,...)                                      |
| conv                             | SolverConvergenceData     |      |                                          | all information about tolerances, errors and residuals                                                             |
| output                           | SolverOutputData          |      |                                          | output modes and timers for exporting solver information and solution                                              |
| newton                           | NewtonSettings            |      |                                          | copy of newton settings from timeint or staticSolver                                                               |
| loadStepGeometricFactor          | Real                      |      |                                          | multiplicative load step factor; this factor is computed from loadStepGeometric parameters in SolveSystem(...)     |
| CheckInitialized(...)            | bool                      |      | mainSystem                               | check if MainSolver and MainSystem are correctly initialized ==> otherwise raise SysError                          |
| ComputeLoadFactor(...)           | Real                      |      | simulationSettings                       | for static solver, this is a factor in interval [0,1]; MUST be overwritten                                         |
| GetSolverName()                  | std::string               |      |                                          | get solver name - needed for output file header and visualization window                                           |
| IsStaticSolver()                 | bool                      |      |                                          | return true, if static solver; needs to be overwritten in derived class                                            |
| GetSimulationEndTime(...)        | Real                      |      | simulationSettings                       | compute simulation end time (depends on static or time integration solver)                                         |
| ReduceStepSize(...)              | bool                      |      | mainSystem, simulationSettings, severity | reduce step size (1..normal, 2..severe problems); return true, if reduction was successful                         |
| IncreaseStepSize(...)            | void                      |      | mainSystem, simulationSettings           | increase step size if convergence is good                                                                          |
| InitializeSolver(...)            | bool                      |      | mainSystem, simulationSettings           | initialize solverSpecific,data,it,conv; set/compute initial conditions (solver-specific!); initialize output files |
| PreInitializeSolverSpecific(...) | void                      |      | mainSystem, simulationSettings           | pre-initialize for solver specific tasks; called at beginning of InitializeSolver, right after Solver data reset   |

|                                        |       |                                              |                                                                                                                                                                                     |
|----------------------------------------|-------|----------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| InitializeSolverOutput(...)            | void  | mainSystem, simulationSettings               | initialize output files; called from InitializeSolver()                                                                                                                             |
| InitializeSolverPreChecks(...)         | bool  | mainSystem, simulationSettings               | check if system is solvable; initialize dense/sparse computation modes                                                                                                              |
| InitializeSolverData(...)              | void  | mainSystem, simulationSettings               | initialize all data,it,conv; called from InitializeSolver()                                                                                                                         |
| InitializeSolverInitialConditions(...) | void  | mainSystem, simulationSettings               | set/compute initial conditions (solver-specific!); called from InitializeSolver()                                                                                                   |
| PostInitializeSolverSpecific(...)      | void  | mainSystem, simulationSettings               | post-initialize for solver specific tasks; called at the end of InitializeSolver                                                                                                    |
| SolveSystem(...)                       | bool  | mainSystem, simulationSettings               | solve System: InitializeSolver, SolveSteps, FinalizeSolver                                                                                                                          |
| FinalizeSolver(...)                    | void  | mainSystem, simulationSettings               | write concluding information (timer statistics, messages) and close files                                                                                                           |
| SolveSteps(...)                        | bool  | mainSystem, simulationSettings               | main solver part: calls multiple InitializeStep(...)/ DiscontinuousIteration(...)/ FinishStep(...); do step reduction if necessary; return true if success, false else              |
| UpdateCurrentTime(...)                 | void  | mainSystem, simulationSettings               | update currentTime (and load factor); MUST be overwritten in special solver class                                                                                                   |
| InitializeStep(...)                    | void  | mainSystem, simulationSettings               | initialize static step / time step; python-functions; do some outputs, checks, etc.                                                                                                 |
| FinishStep(...)                        | void  | mainSystem, simulationSettings               | finish static step / time step; write output of results to file                                                                                                                     |
| DiscontinuousIteration(...)            | bool  | mainSystem, simulationSettings               | perform discontinuousIteration for static step / time step; CALLS ComputeNewtonResidual                                                                                             |
| Newton(...)                            | bool  | mainSystem, simulationSettings               | perform Newton method for given solver method                                                                                                                                       |
| ComputeNewtonResidual(...)             | Real  | mainSystem, simulationSettings               | compute residual for Newton method (e.g. static or time step); store residual vector in systemResidual and return scalar residual (specific computation may depend on solver types) |
| ComputeNewtonUpdate(...)               | void  | mainSystem, simulationSettings, initial=true | compute update for currentState from newtonSolution (decrement from residual and jacobian); if initial, this is for the initial update with newtonSolution=0                        |
| ComputeNewtonJacobian(...)             | void  | mainSystem, simulationSettings               | compute jacobian for newton method of given solver method; store result in systemJacobian                                                                                           |
| WriteSolutionFileHeader(...)           | void  | mainSystem, simulationSettings               | write unique file header, depending on static/ dynamic simulation                                                                                                                   |
| WriteCoordinatesToFile(...)            | void  | mainSystem, simulationSettings               | write unique coordinates solution file                                                                                                                                              |
| IsVerboseCheck(...)                    | bool  | level                                        | return true, if file or console output is at or above the given level                                                                                                               |
| VerboseWrite(...)                      | void  | level, str                                   | write to console and/or file in case of level                                                                                                                                       |
| GetODE2size()                          | Index |                                              | number of ODE2 equations in solver                                                                                                                                                  |

|                                |             |                                                                               |                                                                                                                                                                                                                                                                                      |
|--------------------------------|-------------|-------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| GetODE1size()                  | Index       |                                                                               | number of ODE1 equations in solver (not yet implemented)                                                                                                                                                                                                                             |
| GetAEmode()                    | Index       |                                                                               | number of algebraic equations in solver                                                                                                                                                                                                                                              |
| GetDataSize()                  | Index       |                                                                               | number of data (history) variables in solver                                                                                                                                                                                                                                         |
| GetSystemJacobian()            | NumpyMatrix |                                                                               | get locally stored / last computed system jacobian of solver                                                                                                                                                                                                                         |
| GetSystemMassMatrix()          | NumpyMatrix |                                                                               | get locally stored / last computed mass matrix of solver                                                                                                                                                                                                                             |
| GetSystemResidual()            | NumpyVector |                                                                               | get locally stored / last computed system residual                                                                                                                                                                                                                                   |
| GetNewtonSolution()            | NumpyVector |                                                                               | get locally stored / last computed solution (=increment) of Newton                                                                                                                                                                                                                   |
| SetSystemJacobian(...)         | void        | systemJacobian                                                                | set locally stored system jacobian of solver; must have size nODE2+nODE1+nAE                                                                                                                                                                                                         |
| SetSystemMassMatrix(...)       | void        | systemMassMatrix                                                              | set locally stored mass matrix of solver; must have size nODE2+nODE1+nAE                                                                                                                                                                                                             |
| SetSystemResidual(...)         | void        | systemResidual                                                                | set locally stored system residual; must have size nODE2+nODE1+nAE                                                                                                                                                                                                                   |
| ComputeMassMatrix(...)         | void        | mainSystem, scalarFactor=1.                                                   | compute systemMassMatrix (multiplied with factor) in cSolver and return mass matrix                                                                                                                                                                                                  |
| ComputeJacobianODE2RHS(...)    | void        | mainSystem, scalarFactor=1.                                                   | set systemJacobian to zero and add jacobian (multiplied with factor) of ODE2RHS to systemJacobian in cSolver                                                                                                                                                                         |
| ComputeJacobianODE2RHS_t(...)  | void        | mainSystem, scalarFactor=1.                                                   | add jacobian of ODE2RHS_t (multiplied with factor) to systemJacobian in cSolver                                                                                                                                                                                                      |
| ComputeJacobianAE(...)         | void        | mainSystem, scalarFactor_ODE2=1., scalarFactor_ODE2_t=1., velocityLevel=false | add jacobian of algebraic equations (multiplied with factor) to systemJacobian in cSolver; the scalarFactors are scaling the derivatives w.r.t. ODE2 coordinates and w.r.t. ODE2_t (velocity) coordinates; if velocityLevel == true, the constraints are evaluated at velocity level |
| ComputeODE2RHS(...)            | void        | mainSystem                                                                    | compute the RHS of ODE2 equations in systemResidual in range(0,nODE2)                                                                                                                                                                                                                |
| ComputeAlgebraicEquations(...) | void        | mainSystem, velocityLevel=false                                               | compute the algebraic equations in systemResidual in range(nODE2+nODE1, nODE2+nODE1+nAE)                                                                                                                                                                                             |

### 8.3.7 MainSolverImplicitSecondOrder

PyBind interface (trampoline) class for dynamic implicit solver. Note that this solver includes the classical Newmark method (set useNewmark True; with option of index 2 reduction) as well as the generalized-alpha method. With the interface, the dynamic implicit solver and its substructures can be accessed via python. NOTE that except from SolveSystem(...), these functions are only intended for experienced users and they need to be handled with care, as unexpected crashes may happen if used inappropriate. Furthermore, the functions have a lot of overhead (still fast, but performance much lower than internal solver) due to python interfaces, and should thus be used for small systems. To access the solver in python, write

```
solver = MainSolverImplicitSecondOrder()
```

and hereafter you can access all data and functions via 'solver'. In this solver, user functions are possible to extend the solver at certain parts, while keeping the overall C++ performance. User functions, which are added with SetUserFunction...(...), have the arguments (MainSolver, MainSystem, simulationSettings), except for ComputeNewtonUpdate which adds the initial flag as an additional argument and ComputeNewtonResidual, which returns the scalar residual.

MainSolverImplicitSecondOrder has the following items:

| Name                               | type/function re-<br>turn type | size | default value / func-<br>tion args | description                                                                                                                                                                                                                                                         |
|------------------------------------|--------------------------------|------|------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| timer                              | CSolverTimer                   |      |                                    | timer which measures the CPU time of solver sub functions; note that solver structures can only be written indirectly, e.g., timer=dynamicSolver.timer; timer.useTimer = False; dynamicSolver.timer=timer; however, dynamicSolver.timer.useTimer cannot be written. |
| it                                 | SolverIterationData            |      |                                    | all information about iterations (steps, discontinuous iteration, newton,...)                                                                                                                                                                                       |
| conv                               | SolverConvergenceData          |      |                                    | all information about tolerances, errors and residuals                                                                                                                                                                                                              |
| output                             | SolverOutputData               |      |                                    | output modes and timers for exporting solver information and solution                                                                                                                                                                                               |
| newton                             | NewtonSettings                 |      |                                    | copy of newton settings from timeint or staticSolver                                                                                                                                                                                                                |
| newmarkBeta                        | Real                           |      |                                    | copy of parameter in timeIntegration.generalizedAlpha                                                                                                                                                                                                               |
| newmarkGamma                       | Real                           |      |                                    | copy of parameter in timeIntegration.generalizedAlpha                                                                                                                                                                                                               |
| alphaM                             | Real                           |      |                                    | copy of parameter in timeIntegration.generalizedAlpha                                                                                                                                                                                                               |
| alphaF                             | Real                           |      |                                    | copy of parameter in timeIntegration.generalizedAlpha                                                                                                                                                                                                               |
| spectralRadius                     | Real                           |      |                                    | copy of parameter in timeIntegration.generalizedAlpha                                                                                                                                                                                                               |
| factJacAlgorithmic                 | Real                           |      |                                    | locally computed parameter from generalizedAlpha parameters                                                                                                                                                                                                         |
| CheckInitialized(...)              | bool                           |      | mainSystem                         | check if MainSolver and MainSystem are correctly initialized ==> otherwise raise SysError                                                                                                                                                                           |
| ComputeLoadFactor(...)             | Real                           |      | simulationSettings                 | for static solver, this is a factor in interval [0,1]; MUST be overwritten                                                                                                                                                                                          |
| GetAAAlgorithmic()                 | NumpyVector                    |      |                                    | get locally stored / last computed algorithmic accelerations                                                                                                                                                                                                        |
| GetStartOfStepStateAAAlgorithmic() | NumpyVector                    |      |                                    | get locally stored / last computed algorithmic accelerations at start of step                                                                                                                                                                                       |
| SetUserFunctionUpdateCurrent       | Time(...)<br>void              |      | mainSystem, user-<br>Function      | set user function                                                                                                                                                                                                                                                   |
| SetUserFunctionInitializeStep(...) | void                           |      | mainSystem, user-<br>Function      | set user function                                                                                                                                                                                                                                                   |

|                                            |             |                                          |                                                                                                                                                                      |
|--------------------------------------------|-------------|------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| SetUserFunctionFinishStep(...)             | void        | mainSystem, userFunction                 | set user function                                                                                                                                                    |
| SetUserFunctionDiscontinuousIteration(...) | void        | mainSystem, userFunction                 | set user function                                                                                                                                                    |
| SetUserFunctionNewton(...)                 | void        | mainSystem, userFunction                 | set user function                                                                                                                                                    |
| SetUserFunctionComputeNewtonUpdate(...)    | void        | mainSystem, userFunction                 | set user function                                                                                                                                                    |
| SetUserFunctionComputeNewtonResidual(...)  | void        | mainSystem, userFunction                 | set user function                                                                                                                                                    |
| SetUserFunctionComputeNewtonJacobian(...)  | void        | mainSystem, userFunction                 | set user function                                                                                                                                                    |
| SetUserFunctionPostNewton(...)             | void        | mainSystem, userFunction                 | set user function                                                                                                                                                    |
| GetSolverName()                            | std::string |                                          | get solver name - needed for output file header and visualization window                                                                                             |
| IsStaticSolver()                           | bool        |                                          | return true, if static solver; needs to be overwritten in derived class                                                                                              |
| GetSimulationEndTime(...)                  | Real        | simulationSettings                       | compute simulation end time (depends on static or time integration solver)                                                                                           |
| ReduceStepSize(...)                        | bool        | mainSystem, simulationSettings, severity | reduce step size (1..normal, 2..severe problems); return true, if reduction was successful                                                                           |
| IncreaseStepSize(...)                      | void        | mainSystem, simulationSettings           | increase step size if convergence is good                                                                                                                            |
| InitializeSolver(...)                      | bool        | mainSystem, simulationSettings           | initialize solverSpecific,data,it,conv; set/compute initial conditions (solver-specific!); initialize output files                                                   |
| PreInitializeSolverSpecific(...)           | void        | mainSystem, simulationSettings           | pre-initialize for solver specific tasks; called at beginning of InitializeSolver, right after Solver data reset                                                     |
| InitializeSolverOutput(...)                | void        | mainSystem, simulationSettings           | initialize output files; called from InitializeSolver()                                                                                                              |
| InitializeSolverPreChecks(...)             | bool        | mainSystem, simulationSettings           | check if system is solvable; initialize dense/sparse computation modes                                                                                               |
| InitializeSolverData(...)                  | void        | mainSystem, simulationSettings           | initialize all data,it,conv; called from InitializeSolver()                                                                                                          |
| InitializeSolverInitialConditions(...)     | void        | mainSystem, simulationSettings           | set/compute initial conditions (solver-specific!); called from InitializeSolver()                                                                                    |
| PostInitializeSolverSpecific(...)          | void        | mainSystem, simulationSettings           | post-initialize for solver specific tasks; called at the end of InitializeSolver                                                                                     |
| SolveSystem(...)                           | bool        | mainSystem, simulationSettings           | solve System: InitializeSolver, SolveSteps, FinalizeSolver                                                                                                           |
| FinalizeSolver(...)                        | void        | mainSystem, simulationSettings           | write concluding information (timer statistics, messages) and close files                                                                                            |
| SolveSteps(...)                            | bool        | mainSystem, simulationSettings           | main solver part: calls multiple InitializeStep(...)/DiscontinuousIteration(...)/FinishStep(...); do step reduction if necessary; return true if success, false else |

|                              |             |                                              |                                                                                                                                                                                     |
|------------------------------|-------------|----------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| UpdateCurrentTime(...)       | void        | mainSystem, simulationSettings               | update currentTime (and load factor); MUST be overwritten in special solver class                                                                                                   |
| InitializeStep(...)          | void        | mainSystem, simulationSettings               | initialize static step / time step; python-functions; do some outputs, checks, etc.                                                                                                 |
| FinishStep(...)              | void        | mainSystem, simulationSettings               | finish static step / time step; write output of results to file                                                                                                                     |
| DiscontinuousIteration(...)  | bool        | mainSystem, simulationSettings               | perform discontinuousIteration for static step / time step; CALLS ComputeNewtonResidual                                                                                             |
| Newton(...)                  | bool        | mainSystem, simulationSettings               | perform Newton method for given solver method                                                                                                                                       |
| PostNewton(...)              | Real        | mainSystem, simulationSettings               | call PostNewton for all relevant objects (contact, friction, ... iterations); returns error for discontinuous iteration                                                             |
| ComputeNewtonResidual(...)   | Real        | mainSystem, simulationSettings               | compute residual for Newton method (e.g. static or time step); store residual vector in systemResidual and return scalar residual (specific computation may depend on solver types) |
| ComputeNewtonUpdate(...)     | void        | mainSystem, simulationSettings, initial=true | compute update for currentState from newtonSolution (decrement from residual and jacobian); if initial, this is for the initial update with newtonSolution=0                        |
| ComputeNewtonJacobian(...)   | void        | mainSystem, simulationSettings               | compute jacobian for newton method of given solver method; store result in systemJacobian                                                                                           |
| WriteSolutionFileHeader(...) | void        | mainSystem, simulationSettings               | write unique file header, depending on static/ dynamic simulation                                                                                                                   |
| WriteCoordinatesToFile(...)  | void        | mainSystem, simulationSettings               | write unique coordinates solution file                                                                                                                                              |
| IsVerboseCheck(...)          | bool        | level                                        | return true, if file or console output is at or above the given level                                                                                                               |
| VerboseWrite(...)            | void        | level, str                                   | write to console and/or file in case of level                                                                                                                                       |
| GetODE2size()                | Index       |                                              | number of ODE2 equations in solver                                                                                                                                                  |
| GetODE1size()                | Index       |                                              | number of ODE1 equations in solver (not yet implemented)                                                                                                                            |
| GetAEmode()                  | Index       |                                              | number of algebraic equations in solver                                                                                                                                             |
| GetDataSize()                | Index       |                                              | number of data (history) variables in solver                                                                                                                                        |
| GetSystemJacobian()          | NumpyMatrix |                                              | get locally stored / last computed system jacobian of solver                                                                                                                        |
| GetSystemMassMatrix()        | NumpyMatrix |                                              | get locally stored / last computed mass matrix of solver                                                                                                                            |
| GetSystemResidual()          | NumpyVector |                                              | get locally stored / last computed system residual                                                                                                                                  |
| GetNewtonSolution()          | NumpyVector |                                              | get locally stored / last computed solution (=increment) of Newton                                                                                                                  |
| SetSystemJacobian(...)       | void        | systemJacobian                               | set locally stored system jacobian of solver; must have size nODE2+nODE1+nAE                                                                                                        |

|                                |      |                                                                               |                                                                                                                                                                                                                                                                                      |
|--------------------------------|------|-------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| SetSystemMassMatrix(...)       | void | systemMassMatrix                                                              | set locally stored mass matrix of solver; must have size nODE2+nODE1+nAE                                                                                                                                                                                                             |
| SetSystemResidual(...)         | void | systemResidual                                                                | set locally stored system residual; must have size nODE2+nODE1+nAE                                                                                                                                                                                                                   |
| ComputeMassMatrix(...)         | void | mainSystem, scalarFactor=1.                                                   | compute systemMassMatrix (multiplied with factor) in cSolver and return mass matrix                                                                                                                                                                                                  |
| ComputeJacobianODE2RHS(...)    | void | mainSystem, scalarFactor=1.                                                   | set systemJacobian to zero and add jacobian (multiplied with factor) of ODE2RHS to systemJacobian in cSolver                                                                                                                                                                         |
| ComputeJacobianODE2RHS_t(...)  | void | mainSystem, scalarFactor=1.                                                   | add jacobian of ODE2RHS_t (multiplied with factor) to systemJacobian in cSolver                                                                                                                                                                                                      |
| ComputeJacobianAE(...)         | void | mainSystem, scalarFactor_ODE2=1., scalarFactor_ODE2_t=1., velocityLevel=false | add jacobian of algebraic equations (multiplied with factor) to systemJacobian in cSolver; the scalarFactors are scaling the derivatives w.r.t. ODE2 coordinates and w.r.t. ODE2_t (velocity) coordinates; if velocityLevel == true, the constraints are evaluated at velocity level |
| ComputeODE2RHS(...)            | void | mainSystem                                                                    | compute the RHS of ODE2 equations in systemResidual in range(0,nODE2)                                                                                                                                                                                                                |
| ComputeODE1RHS(...)            | void | mainSystem                                                                    | compute the RHS of ODE1 equations in systemResidual in range(0,nODE1)                                                                                                                                                                                                                |
| ComputeAlgebraicEquations(...) | void | mainSystem, velocityLevel=false                                               | compute the algebraic equations in systemResidual in range(nODE2+nODE1, nODE2+nODE1+nAE)                                                                                                                                                                                             |

### 8.3.8 MainSolverExplicit

PyBind interface (trampoline) class for dynamic explicit solver. Note that this solver includes the 1st order explicit Euler scheme and the 4th order Runge-Kutta scheme with 5th order error estimation (DOPRI5). With the interface, the solver and its substructures can be accessed via python. NOTE that except from SolveSystem(...), these functions are only intended for experienced users and they need to be handled with care, as unexpected crashes may happen if used inappropriate. Furthermore, the functions have a lot of overhead (still fast, but performance much lower than internal solver) due to python interfaces, and should thus be used for small systems. To access the solver in python, write

```
solver = MainSolverExplicit()
```

and hereafter you can access all data and functions via 'solver'. In this solver, no user functions are possible, but you can use SolverImplicitSecondOrder instead (turning off Newton gives explicit scheme ...).

MainSolverExplicit has the following items:

| Name  | type/function return type | size | default value / function args | description                                               |
|-------|---------------------------|------|-------------------------------|-----------------------------------------------------------|
| timer | CSolverTimer              |      |                               | timer which measures the CPU time of solver sub functions |

|                                        |                       |                                          |                                                                                                                                                                      |
|----------------------------------------|-----------------------|------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| it                                     | SolverIterationData   |                                          | all information about iterations (steps, discontinuous iteration, newton,...)                                                                                        |
| conv                                   | SolverConvergenceData |                                          | all information about tolerances, errors and residuals                                                                                                               |
| output                                 | SolverOutputData      |                                          | output modes and timers for exporting solver information and solution                                                                                                |
| ComputeLoadFactor(...)                 | Real                  | simulationSettings                       | for static solver, this is a factor in interval [0,1]; MUST be overwritten                                                                                           |
| GetNumberOfStages()                    | Index                 |                                          | return number of stages in current method                                                                                                                            |
| GetMethodOrder()                       | Index                 |                                          | return order of method (higher value in methods with automatic step size, e.g., DOPRI5=5)                                                                            |
| GetSolverName()                        | std::string           |                                          | get solver name - needed for output file header and visualization window                                                                                             |
| IsStaticSolver()                       | bool                  |                                          | return true, if static solver; needs to be overwritten in derived class                                                                                              |
| GetSimulationEndTime(...)              | Real                  | simulationSettings                       | compute simulation end time (depends on static or time integration solver)                                                                                           |
| ReduceStepSize(...)                    | bool                  | mainSystem, simulationSettings, severity | reduce step size (1..normal, 2..severe problems); return true, if reduction was successful                                                                           |
| IncreaseStepSize(...)                  | void                  | mainSystem, simulationSettings           | increase step size if convergence is good                                                                                                                            |
| InitializeSolver(...)                  | bool                  | mainSystem, simulationSettings           | initialize solverSpecific,data,it,conv; set/compute initial conditions (solver-specific!); initialize output files                                                   |
| PreInitializeSolverSpecific(...)       | void                  | mainSystem, simulationSettings           | pre-initialize for solver specific tasks; called at beginning of InitializeSolver, right after Solver data reset                                                     |
| InitializeSolverOutput(...)            | void                  | mainSystem, simulationSettings           | initialize output files; called from InitializeSolver()                                                                                                              |
| InitializeSolverPreChecks(...)         | bool                  | mainSystem, simulationSettings           | check if system is solvable; initialize dense/sparse computation modes                                                                                               |
| InitializeSolverData(...)              | void                  | mainSystem, simulationSettings           | initialize all data,it,conv; called from InitializeSolver()                                                                                                          |
| InitializeSolverInitialConditions(...) | void                  | mainSystem, simulationSettings           | set/compute initial conditions (solver-specific!); called from InitializeSolver()                                                                                    |
| PostInitializeSolverSpecific(...)      | void                  | mainSystem, simulationSettings           | post-initialize for solver specific tasks; called at the end of InitializeSolver                                                                                     |
| SolveSystem(...)                       | bool                  | mainSystem, simulationSettings           | solve System: InitializeSolver, SolveSteps, FinalizeSolver                                                                                                           |
| FinalizeSolver(...)                    | void                  | mainSystem, simulationSettings           | write concluding information (timer statistics, messages) and close files                                                                                            |
| SolveSteps(...)                        | bool                  | mainSystem, simulationSettings           | main solver part: calls multiple InitializeStep(...)/DiscontinuousIteration(...)/FinishStep(...); do step reduction if necessary; return true if success, false else |
| UpdateCurrentTime(...)                 | void                  | mainSystem, simulationSettings           | update currentTime (and load factor); MUST be overwritten in special solver class                                                                                    |

|                              |             |                                              |                                                                                                                                                                                     |
|------------------------------|-------------|----------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| InitializeStep(...)          | void        | mainSystem, simulationSettings               | initialize static step / time step; python-functions; do some outputs, checks, etc.                                                                                                 |
| FinishStep(...)              | void        | mainSystem, simulationSettings               | finish static step / time step; write output of results to file                                                                                                                     |
| DiscontinuousIteration(...)  | bool        | mainSystem, simulationSettings               | perform discontinuousIteration for static step / time step; CALLS ComputeNewtonResidual                                                                                             |
| Newton(...)                  | bool        | mainSystem, simulationSettings               | perform Newton method for given solver method                                                                                                                                       |
| ComputeNewtonResidual(...)   | Real        | mainSystem, simulationSettings               | compute residual for Newton method (e.g. static or time step); store residual vector in systemResidual and return scalar residual (specific computation may depend on solver types) |
| ComputeNewtonUpdate(...)     | void        | mainSystem, simulationSettings, initial=true | compute update for currentState from newtonSolution (decrement from residual and jacobian); if initial, this is for the initial update with newtonSolution=0                        |
| ComputeNewtonJacobian(...)   | void        | mainSystem, simulationSettings               | compute jacobian for newton method of given solver method; store result in systemJacobian                                                                                           |
| WriteSolutionFileHeader(...) | void        | mainSystem, simulationSettings               | write unique file header, depending on static/ dynamic simulation                                                                                                                   |
| WriteCoordinatesToFile(...)  | void        | mainSystem, simulationSettings               | write unique coordinates solution file                                                                                                                                              |
| IsVerboseCheck(...)          | bool        | level                                        | return true, if file or console output is at or above the given level                                                                                                               |
| VerboseWrite(...)            | void        | level, str                                   | write to console and/or file in case of level                                                                                                                                       |
| GetODE2size()                | Index       |                                              | number of ODE2 equations in solver                                                                                                                                                  |
| GetODE1size()                | Index       |                                              | number of ODE1 equations in solver (not yet implemented)                                                                                                                            |
| GetAEmode()                  | Index       |                                              | number of algebraic equations in solver                                                                                                                                             |
| GetDataSize()                | Index       |                                              | number of data (history) variables in solver                                                                                                                                        |
| GetSystemMassMatrix()        | NumpyMatrix |                                              | get locally stored / last computed mass matrix of solver                                                                                                                            |
| GetSystemResidual()          | NumpyVector |                                              | get locally stored / last computed system residual                                                                                                                                  |
| SetSystemMassMatrix(...)     | void        | systemMassMatrix                             | set locally stored mass matrix of solver; must have size nODE2+nODE1+nAE                                                                                                            |
| SetSystemResidual(...)       | void        | systemResidual                               | set locally stored system residual; must have size nODE2+nODE1+nAE                                                                                                                  |
| ComputeMassMatrix(...)       | void        | mainSystem, scalarFactor=1.                  | compute systemMassMatrix (multiplied with factor) in cSolver and return mass matrix                                                                                                 |
| ComputeODE2RHS(...)          | void        | mainSystem                                   | compute the RHS of ODE2 equations in systemResidual in range(0,nODE2)                                                                                                               |
| ComputeODE1RHS(...)          | void        | mainSystem                                   | compute the RHS of ODE1 equations in systemResidual in range(0,nODE1)                                                                                                               |

# Chapter 9

## 3D Graphics Visualization

The 3D graphics visualization window is kept simple, but useful to see the animated results of the multibody system. The graphics output is restricted to a 3D window (renderwindow) into which the renderer draws the visualization state of the `MainSystem.mbs`.

### 9.1 Mouse input

The following table includes the mouse functions.

| Button                    | action       | remarks                                                                               |
|---------------------------|--------------|---------------------------------------------------------------------------------------|
| <b>left mouse button</b>  | move model   | keep left mouse button pressed to move the model in the current x/y plane             |
| <b>right mouse button</b> | rotate model | keep right mouse button pressed to rotate model around current current $X_1/X_2$ axes |
| <b>mouse wheel</b>        | zoom         | use mouse wheel to zoom (on touch screens ‘pinch-to-zoom’ might work as well)         |

Note that current mouse coordinates can be obtained via `SystemContainer.GetCurrentMouseCoordinates()`.

### 9.2 Keyboard input

The following table includes the keyboard shortcuts available in the window.

| Key(s)                               | action                     | remarks                                                                                                                   |
|--------------------------------------|----------------------------|---------------------------------------------------------------------------------------------------------------------------|
| <b>1,2,3,4 or 5</b>                  | visualization update speed | the entered digit controls the visualization update, which can be changed from 1=1 update per 20ms to 5=1 update per 100s |
| <b>‘.’ or KEYPAD +</b>               | zoom in                    | zoom one step into scene (additionally press CTRL to perform small zoom step)                                             |
| <b>‘,’ or KEYPAD -</b>               | zoom out                   | zoom one step out of scene (additionally press CTRL to perform small zoom step)                                           |
| <b>CTRL+1</b><br><b>SHIFT+CTRL+1</b> | <b>or</b><br>change view   | set view in 1/2-plane (+SHIFT: view from opposite side)                                                                   |

|                             |                             |             |                                                                                                                                                                                                                       |
|-----------------------------|-----------------------------|-------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>CTRL+2</b>               | <b>or</b>                   | change view | set view in 1/3-plane (+SHIFT: view from opposite side)                                                                                                                                                               |
| <b>SHIFT+CTRL+2</b>         |                             |             |                                                                                                                                                                                                                       |
| <b>CTRL+3</b>               | <b>or</b>                   | change view | set view in 2/3-plane (+SHIFT: view from opposite side)                                                                                                                                                               |
| <b>SHIFT+CTRL+3</b>         |                             |             |                                                                                                                                                                                                                       |
| <b>CTRL+4</b>               | <b>or</b>                   | change view | set view in 2/1-plane (+SHIFT: view from opposite side)                                                                                                                                                               |
| <b>SHIFT+CTRL+4</b>         |                             |             |                                                                                                                                                                                                                       |
| <b>CTRL+5</b>               | <b>or</b>                   | change view | set view in 3/1-plane (+SHIFT: view from opposite side)                                                                                                                                                               |
| <b>SHIFT+CTRL+5</b>         |                             |             |                                                                                                                                                                                                                       |
| <b>CTRL+6</b>               | <b>or</b>                   | change view | set view in 3/2-plane (+SHIFT: view from opposite side)                                                                                                                                                               |
| <b>SHIFT+CTRL+6</b>         |                             |             |                                                                                                                                                                                                                       |
| <b>A</b>                    | zoom all                    |             | set zoom such that the whole scene is visible                                                                                                                                                                         |
| <b>CURSOR UP, DOWN, ...</b> | move scene                  |             | use cursor keys to move the scene up, down, left, and right (use CTRL for small movements)                                                                                                                            |
| <b>KEYPAD 2/8,4/6,1/9</b>   | rotate scene                |             | about 1, 2 and 3-axis (use CTRL for small rotations)                                                                                                                                                                  |
| <b>C</b>                    | show/hide connectors        |             | pressing this key switches the visibility of connectors                                                                                                                                                               |
| <b>CTRL+C</b>               | show/hide connector numbers |             | pressing this key switches the visibility of connector numbers                                                                                                                                                        |
| <b>B</b>                    | show/hide bodies            |             | pressing this key switches the visibility of bodies                                                                                                                                                                   |
| <b>CTRL+B</b>               | show/hide body numbers      |             | pressing this key switches the visibility of body numbers                                                                                                                                                             |
| <b>L</b>                    | show/hide loads             |             | pressing this key switches the visibility of loads                                                                                                                                                                    |
| <b>CTRL+L</b>               | show/hide load numbers      |             | pressing this key switches the visibility of load numbers                                                                                                                                                             |
| <b>M</b>                    | show/hide markers           |             | pressing this key switches the visibility of markers                                                                                                                                                                  |
| <b>CTRL+M</b>               | show/hide marker numbers    |             | pressing this key switches the visibility of marker numbers                                                                                                                                                           |
| <b>N</b>                    | show/hide nodes             |             | pressing this key switches the visibility of nodes                                                                                                                                                                    |
| <b>CTRL+N</b>               | show/hide node numbers      |             | pressing this key switches the visibility of node numbers                                                                                                                                                             |
| <b>S</b>                    | show/hide sensors           |             | pressing this key switches the visibility of sensors                                                                                                                                                                  |
| <b>CTRL+S</b>               | show/hide sensor numbers    |             | pressing this key switches the visibility of sensor numbers                                                                                                                                                           |
| <b>T</b>                    | faces transparent           |             | add transparency to all faces (bodies, finite elements) in order to make node/marker/object numbers visible                                                                                                           |
| <b>Q</b>                    | stop simulation             |             | simulation is stopped and cannot be recovered                                                                                                                                                                         |
| <b>X</b>                    | execute command             |             | open dialog to enter a python command (in global python scope); dialog may appear behind the visualization window! User errors may lead to crash – be careful! Examples: 'print(mbs)', 'x=5', 'mbs.GetObject(0)',etc. |
| <b>V</b>                    | visualization settings      |             | open dialog to modify visualization settings; dialog may appear behind the visualization window!                                                                                                                      |
| <b>ESCAPE</b>               | close render window         |             | stops the simulation and closes the render window                                                                                                                                                                     |
| <b>SPACE</b>                | continue simulation         |             | if simulation is paused, it can be continued by pressing space; use SHIFT+SPACE to continuously activate 'continue simulation'                                                                                        |

## 9.3 GraphicsData

All graphics objects are defined by a `GraphicsData` structure. Note that currently the visualization is based on a very simple and ancient OpenGL implementation, as there is currently no simple platform independent alternative. However, most of the operations may be speed up as data is stored internally and may be transformed by very efficient OpenGL commands in the future. However, note that the number of triangles to

represent the object should be kept low ( $< 100000$ ) in order to obtain a reasonably fast response of the renderer.

Many objects include a `GraphicsData` dictionary structure for definition of attached visualization of the object. Typically, you can use primitives (cube, sphere, ...) or STL-data to define the objects appearance. `GraphicsData` dictionaries can be created with functions provided in the utilities module `exudyn.graphicsDataUtilities.py`, see [Section 5.4](#). `BodyGraphicsData` contains a list of `GraphicsData` items, i.e. `bodyGraphicsData = [graphicsItem1, graphicsItem2, ...]`. Every single `graphicsItem` may be defined as one of the following structures using a specific 'type':

| Name                          | type   | default value | description                                                                                                                                                                                                                                                  |
|-------------------------------|--------|---------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>type = 'Line':</b>         |        |               | <i>draws a polygonal line between all specified points</i>                                                                                                                                                                                                   |
| color                         | list   | [0,0,0,1]     | list of 4 floats to define RGB-color and transparency                                                                                                                                                                                                        |
| <b>type = 'Circle':</b>       |        |               | <i>draws a circle with center point, normal (defines plane of circle) and radius</i>                                                                                                                                                                         |
| color                         | list   | [0,0,0,1]     | list of 4 floats to define RGB-color and transparency                                                                                                                                                                                                        |
| radius                        | float  | mandatory     | radius                                                                                                                                                                                                                                                       |
| position                      | list   | mandatory     | list of float triples of x,y,z coordinates of center point of the circle                                                                                                                                                                                     |
| normal                        | list   | [0,0,1]       | list of float triples of x,y,z coordinates of normal to the plane of the circle; the default value gives a circle in the $(x, y)$ -plane                                                                                                                     |
| <b>type = 'Text':</b>         |        |               | <i>places the given text at position</i>                                                                                                                                                                                                                     |
| color                         | list   | [0,0,0,1]     | list of 4 floats to define RGB-color and transparency                                                                                                                                                                                                        |
| text                          | string | mandatory     | text to be displayed, using UTF-8 encoding (see <a href="#">Section 9.4</a> )                                                                                                                                                                                |
| position                      | list   | mandatory     | list of float triples of [x,y,z] coordinates of the left upper position of the text; e.g. <code>position=[20,10,0]</code>                                                                                                                                    |
| <b>type = 'TriangleList':</b> |        |               | <i>draws a flat triangle mesh for given points and connectivity</i>                                                                                                                                                                                          |
| points                        | list   | mandatory     | list $[x_0,y_0,z_0, x_1,y_1,z_1, \dots]$ containing $n \times 3$ floats (grouped $x_0,y_0,z_0, x_1,y_1,z_1, \dots$ ) to define x,y,z coordinates of points, $n$ being the number of points (=vertices)                                                       |
| colors                        | list   | empty         | list $[R_0,G_0,B_0,A_0, R_1,G_2,B_1,A_1, \dots]$ containing $n \times 4$ floats to define RGB-color and transparency A, where $n$ must be according to number of points; if field 'colors' does not exist, default colors will be used                       |
| normals                       | list   | empty         | list $[n_0x,n_0y,n_0z, \dots]$ containing $n \times 3$ floats to define normal direction of triangles per point, where $n$ must be according to number of points; if field 'normals' does not exist, default normals [0,0,0] will be used                    |
| triangles                     | list   | mandatory     | list $[T_0point0, T_0point1, T_0point2, \dots]$ containing $n_{trig} \times 3$ floats to define point indices of each vertex of the triangles (=connectivity); point indices start with index 0; the maximum index must be $\leq$ <code>points.size()</code> |

Examples of `GraphicsData` can be found in the Python examples and in `exudynUtilities.py`.

## 9.4 Character encoding: UTF-8

Character encoding is a major issue in computer systems, as different languages need a huge amount of different characters, see the amusing blog of Joel Spolsky:

[The Absolute Minimum Every Software Developer Absolutely, Positively Must Know About Unicode ...](#)

More about encoding can be found in [Wikipedia:UTF-8](#). UTF-8 encoding tables can be found within the wikipedia article and a comparison with the first 256 characters of unicode is provided at [UTF-8 char table](#).

For short, EXUDYN uses UTF-8 character encoding in texts / strings drawn in OpenGL renderer window. However, the set of available UTF-8 characters in EXUDYN is restricted to a very small set of characters (as compared to available characters in UTF-8), see the following table of available characters (using hex codes, e.g. `0x20 = 32`):

| unicode (hex code) | UTF-8 (hex code) | character                       |
|--------------------|------------------|---------------------------------|
| 20                 | 20               | ' '                             |
| 21                 | 21               | ' !'                            |
| ...                | ...              | ...                             |
| 30                 | 30               | ' 0'                            |
| ...                | ...              | ...                             |
| 39                 | 39               | ' 9'                            |
| 40                 | 40               | ' @'                            |
| 41                 | 41               | ' A'                            |
| ...                | ...              | ...                             |
| 5A                 | 5A               | ' Z'                            |
| ...                | ...              | ...                             |
| 7E                 | 7E               | ' ~'                            |
| 7F                 | 7F               | control, not shown              |
| ...                | ...              | ...                             |
| A0                 | C2 A0            | no-break space                  |
| A1                 | C2 A1            | inverted exclamation mark: ' ¡' |
| ...                | ...              | ...                             |
| BF                 | C2 BF            | inverted ' i'                   |
| C0                 | C3 80            | A with grave                    |
| ...                | ...              | ...                             |
| FF                 | C3 BF            | y with diaeresis: ' ÿ'          |

special characters (selected):

|  |             |                                            |
|--|-------------|--------------------------------------------|
|  | E2 89 88    | ≈                                          |
|  | E2 88 82    | ∂                                          |
|  | E2 88 AB    | ƒ                                          |
|  | E2 88 9A    | √                                          |
|  | CE B1       | α                                          |
|  | CE B2       | β                                          |
|  | ...         | (complete list of greek letters see below) |
|  | F0 9F 99 82 | smiley                                     |
|  | F0 9F 98 92 | frowney                                    |
|  | E2 88 9e    | infinity: ∞                                |

Greek characters include:  $\alpha, \beta, \gamma, \delta, \varepsilon, \zeta, \eta, \theta, \kappa, \lambda, \nu, \xi, \pi, \rho, \sigma, \varphi, \Delta, \Pi, \Sigma, \Omega$ . Note, that unicode character codes

are shown only for completeness, but they **cannot be encoded by EXUDYN!**



# Chapter 10

## Notation and System of equations of motion

The general idea of the code is to have objects, which provide equations (ODE2, ODE1, AE). The solver then assembles these equations and solves the static or dynamic problem. The system structure and solver are similar but much more advanced and modular as earlier solvers by the main developer [18, 14, 15].

### 10.1 LHS-RHS naming conventions in EXUDYN

Functions and variables contain the abbreviations LHS (*left-hand-side*) and RHS (*right-hand-side*), sometimes lower-case, in order to distinguish if terms are computed at the LHS or RHS.

The objects have the following LHS–RHS conventions:

- the acceleration term, e.g.,  $m \cdot \ddot{q}$  is always positive on the LHS
- objects, connectors, etc., use LHS conventions for most terms: mass, stiffness matrix, elastic forces, damping, etc., are computed at LHS of the object equation
- object forces are written at the RHS of the object equation
- in case of constraint or connector equations, there is no LHS or RHS, as there is no acceleration term. Therefore, the computation function evaluates the term as given in the description of the object, adding it to the LHS.

Object equations may read, e.g., for one coordinate  $q$ , mass  $m$ , damping coefficient  $d$ , stiffness  $k$  and applied force  $f$ ,

$$\underbrace{m \cdot \ddot{q} + d \cdot \dot{q} + k \cdot q}_{LHS} = \underbrace{f}_{RHS} \quad (10.1)$$

In this case, the C++ function `ComputeODE2LHS(const Vector& ode2Lhs)` will compute the term  $d \cdot \dot{q} + k \cdot q$  with positive sign. Note that the acceleration term  $m \cdot \ddot{q}$  is computed separately, as it is computed from mass matrix and acceleration.

However, system quantities (e.g. within the solver) are always written on RHS<sup>1</sup>:

$$\underbrace{M_{sys} \cdot \ddot{q}_{sys}}_{LHS} = \underbrace{f_{sys}}_{RHS} . \quad (10.2)$$

<sup>1</sup>except for the acceleration  $\times$  mass matrix and constraint reaction forces, see Eq. (10.13)

In the case of the object equation

$$m \cdot \ddot{q} + d \cdot \dot{q} + k \cdot q = f, \quad (10.3)$$

the RHS term becomes  $f_{sys} = -(d \cdot \dot{q} + k \cdot q) + f$  and it is computed by the C++ function `ComputeSystemODE2RHS`. This means, that during computation, terms which appear at the LHS of the object are transferred to the RHS of the system equation. This enables a simpler setup of equations for the solver.

## 10.2 System assembly

Assembling equations of motion is done within the C++ class `CSystem`, see the file `CSystem.cpp`. The general idea is to assemble, i.e. to sum up, (parts of) residuals attributed by different objects. The summation process is based on coordinate indices to which the single equations belong to. Let's assume that we have two simple `ObjectMass1D` objects, with object indices  $o0$  and  $o1$  and having mass  $m_0$  and  $m_1$ . They are connected to nodes of type `Node1D`  $n0$  and  $n1$ , with global coordinate indices  $c0$  and  $c1$ . The partial object residuals, which are fully independent equations, read

$$m_0 \cdot \ddot{q}_{c0} = RHS_{c0}, \quad (10.4)$$

$$m_1 \cdot \ddot{q}_{c1} = RHS_{c1}, \quad (10.5)$$

where  $RHS_{c0}$  and  $RHS_{c1}$  the right-hand-side of the respective equations/coordinates. They represent forces, e.g., from `LoadCoordinate` items (which directly are applied to coordinates of nodes), say  $f_{c0}$  and  $f_{c1}$ , that are in case also summed up on the right hand side. Let us for now assume that

$$RHS_{c0} = f_{c0} \quad \text{and} \quad RHS_{c1} = f_{c1}. \quad (10.6)$$

Now we add another `ObjectMass1D` object with object index  $o2$ , having mass  $m_2$ , but letting the object *again* use node  $n0$  with coordinate  $c0$ . In this case, the total object residuals read

$$(m_0 + m_2) \cdot \ddot{q}_{c0} = RHS_{c0}, \quad (10.7)$$

$$m_1 \cdot \ddot{q}_{c1} = RHS_{c1}. \quad (10.8)$$

It is clear, that now the mass in the first equation is increased due to the fact that two objects contribute to the same coordinate. The same would happen, if several loads are applied to the same coordinate.

Finally, if we add a `CoordinateSpringDamper`, assuming a spring  $k$  between coordinates  $c0$  and  $c1$ , the RHS of equations related to  $c0$  and  $c1$  is now augmented to

$$RHS_{c0} = f_{c0} + k \cdot (q_{c1} - q_{c0}), \quad (10.9)$$

$$RHS_{c1} = f_{c1} + k \cdot (q_{c0} - q_{c1}). \quad (10.10)$$

The system of equation would therefore read

$$(m_0 + m_2) \cdot \ddot{q}_{c0} = f_{c0} + k \cdot (q_{c1} - q_{c0}), \quad (10.11)$$

$$m_1 \cdot \ddot{q}_{c1} = f_{c1} + k \cdot (q_{c0} - q_{c1}). \quad (10.12)$$

It should be noted, that all (components of) residuals ('equations') are summed up for the according coordinates, and also all contributions to the mass matrix. Only constraint equations, which are related to Lagrange parameters always get their 'own' Lagrange multipliers, which are automatically assigned by the system and therefore independent for every constraint. Therefore, it is not possible right now to attach two `ObjectRigidBody` to the same node, as it would create two constraint equations. This will be resolved within issue #0564.

## 10.3 Nomenclature for system equations of motion and solvers

Using the basic notation for coordinates in [Section 6.1](#), we use the following quantities and symbols for equations of motion and solvers:

| quantity                     | symbol                                                                                                      | description                                                                                                                                                                              |
|------------------------------|-------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| number of ODE2 coordinates   | $n$                                                                                                         | ODE2 = 2 <sup>nd</sup> order differential equations                                                                                                                                      |
| number of ODE1 coordinates   | $n_y$                                                                                                       | ODE1 = 1 <sup>st</sup> order differential equations                                                                                                                                      |
| number of AE coordinates     | $m$                                                                                                         | AE = algebraic equations                                                                                                                                                                 |
| number of system coordinates | $n_s$                                                                                                       | system equations                                                                                                                                                                         |
| ODE2 coordinates             | $\mathbf{q} = [q_0, \dots, q_{n_q}]^T$                                                                      | ODE2, displacement-based coordinates (could also be rotation or deformation coordinates)                                                                                                 |
| ODE2 velocities              | $\mathbf{v} = \dot{\mathbf{q}} = [\dot{q}_0, \dots, \dot{q}_{n_q}]^T$                                       | ODE2 velocity coordinates                                                                                                                                                                |
| ODE2 accelerations           | $\ddot{\mathbf{q}} = [\ddot{q}_0, \dots, \ddot{q}_{n_q}]^T$                                                 | ODE2 acceleration coordinates                                                                                                                                                            |
| ODE1 coordinates             | $\mathbf{y} = [y_0, \dots, y_{n_y}]^T$                                                                      | vector of $n_y$ coordinates for first order ordinary differential equations (ODE1)                                                                                                       |
| ODE1 velocities              | $\dot{\mathbf{y}} = [\dot{y}_0, \dots, \dot{y}_{n_y}]^T$                                                    | vector of $n$ velocities for first order ordinary differential equations (ODE1)                                                                                                          |
| ODE2 Lagrange multipliers    | $\lambda = [\lambda_0, \dots, \lambda_m]^T$                                                                 | vector of $m$ Lagrange multipliers (=algebraic coordinates), representing the linear factors (often forces or torques) to fulfill the algebraic equations; for ODE1 and ODE2 coordinates |
| data coordinates             | $\mathbf{x} = [x_0, \dots, x_l]^T$                                                                          | vector of $l$ data coordinates in any configuration                                                                                                                                      |
| RHS ODE2                     | $\mathbf{f}_q \in \mathbb{R}^{n_q}$                                                                         | right-hand-side of ODE2 equations; (all terms except mass matrix $\times$ acceleration and joint reaction forces)                                                                        |
| RHS ODE1                     | $\mathbf{f}_y \in \mathbb{R}^{n_y}$                                                                         | right-hand-side of ODE1 equations                                                                                                                                                        |
| AE                           | $\mathbf{g} \in \mathbb{R}^m$                                                                               | algebraic equations                                                                                                                                                                      |
| mass matrix                  | $\mathbf{M} \in \mathbb{R}^{n_q \times n_q}$                                                                | mass matrix, only for ODE2 equations                                                                                                                                                     |
| (tangent) stiffness matrix   | $\mathbf{K} \in \mathbb{R}^{n_q \times n_q}$                                                                | includes all derivatives of $\mathbf{f}_q$ w.r.t. $\mathbf{q}$                                                                                                                           |
| damping/gyroscopic matrix    | $\mathbf{D} \in \mathbb{R}^{n_q \times n_q}$                                                                | includes all derivatives of $\mathbf{f}_q$ w.r.t. $\mathbf{v}$                                                                                                                           |
| step size                    | $h$                                                                                                         | current step size in time integration method                                                                                                                                             |
| residual                     | $\mathbf{r}_q \in \mathbb{R}^{n_q}, \mathbf{r}_y \in \mathbb{R}^{n_y}, \mathbf{r}_\lambda \in \mathbb{R}^m$ | residuals for each type of coordinates within static/time integration – depends on method                                                                                                |
| system residual              | $\mathbf{r} \in \mathbb{R}^{n_s}$                                                                           | system residual – depends on method                                                                                                                                                      |
| system coordinates           | $\xi$                                                                                                       | system coordinates and unknowns for solver; definition depends on solver                                                                                                                 |
| Jacobian                     | $\mathbf{J} \in \mathbb{R}^{n_s \times n_s}$                                                                | system Jacobian – depends on method                                                                                                                                                      |

### 10.3.1 System equations of motion

The system equations of motion in EXUDYN are represented as

$$\mathbf{M}\ddot{\mathbf{q}} + \frac{\partial \mathbf{g}}{\partial \mathbf{q}^T} \lambda = \mathbf{f}_q(\mathbf{q}, \dot{\mathbf{q}}, t) \quad (10.13)$$

$$\dot{\mathbf{y}} + \frac{\partial \mathbf{g}}{\partial \mathbf{y}^T} \lambda = \mathbf{f}_y(\mathbf{y}, t) \quad (10.14)$$

$$\mathbf{g}(\mathbf{q}, \dot{\mathbf{q}}, \mathbf{y}, \lambda, t) = 0 \quad (10.15)$$

Note that the term  $\frac{\partial \mathbf{g}}{\partial \mathbf{y}} \lambda_y$  is not yet implemented, such that algebraic equations may not yet depend on 1<sup>st</sup> order differential equations coordinates.

It is important to note, that for linear mechanical the term  $\mathbf{f}_q$  becomes

$$\mathbf{f}_q^{lin} = \mathbf{f}_a - \mathbf{K}\mathbf{q} - \mathbf{D}\dot{\mathbf{q}} \quad (10.16)$$

in which  $\mathbf{f}^a$  represents applied forces and stiffness matrix  $\mathbf{K}$  and damping matrix  $\mathbf{D}$  become part of the system Jacobian for time integration.

# Chapter 11

## Solvers

The description of solvers in this section follows the nomenclature given in [Chapter 10](#). Both in the static as well as in the dynamic case, the solvers run in a loop to solve a nonlinear system of (differential and/or algebraic) equations over a given time or load interval. Explicit solvers only perform a factorization of the mass matrix, but the Newton loop, see Fig. 11.5, is replaced by an explicit computation of the time step according to a given Runge-Kutta tableau.

In case of an implicit time integration, Fig. 11.1 shows the basic loops for the solution process. The inner loops are shown in Fig. 11.3 and Fig. 11.4. The static solver behaves very similar, while no velocities or accelerations need to be solved and time is replaced by load steps.

Settings for the solver substructures, like timer, output, iterations, etc. are described in Sections 8.3.1 – 8.3.5. The description of interfaces for solvers starts in [Section 8.3.6](#).

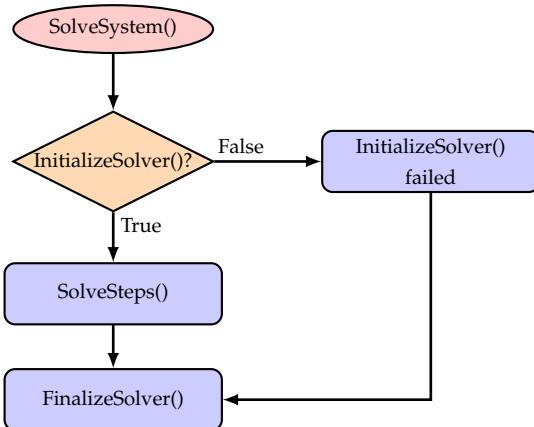


Figure 11.1: Basic solver flow chart for `SolveSystem()`. This flow chart is the same for static solver and for time integration.

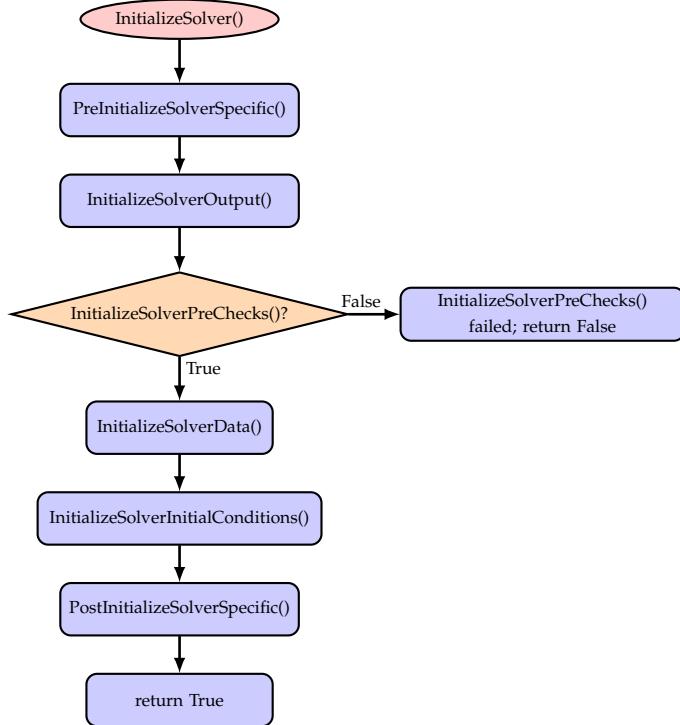


Figure 11.2: Basic solver flow chart for function `InitializeSolver()`.

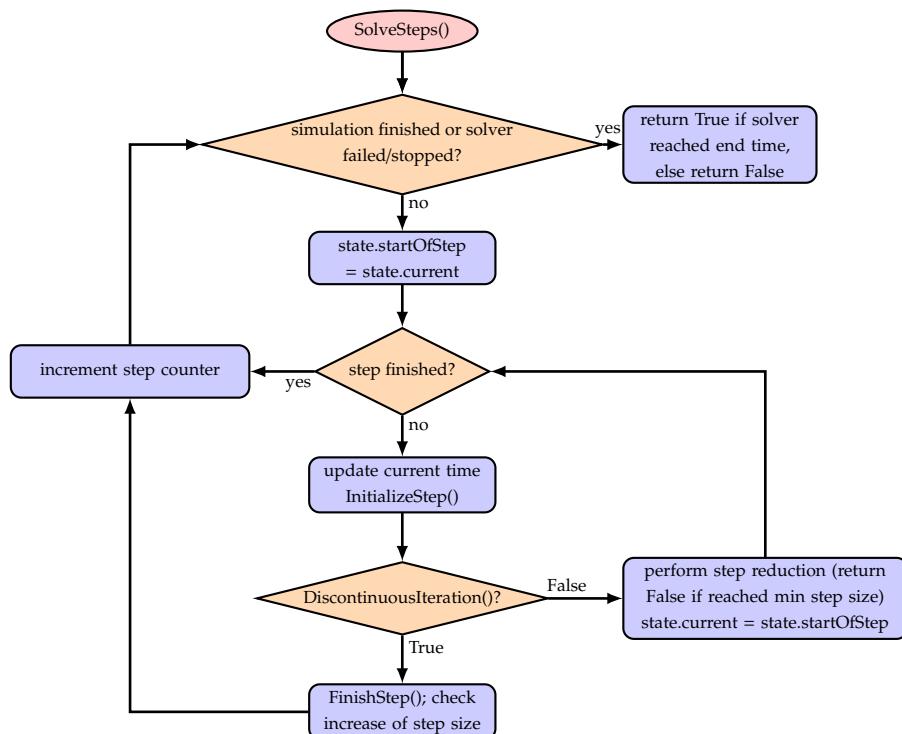


Figure 11.3: Solver flow chart for `SolveSteps()`, which is the inner loop of the solver.

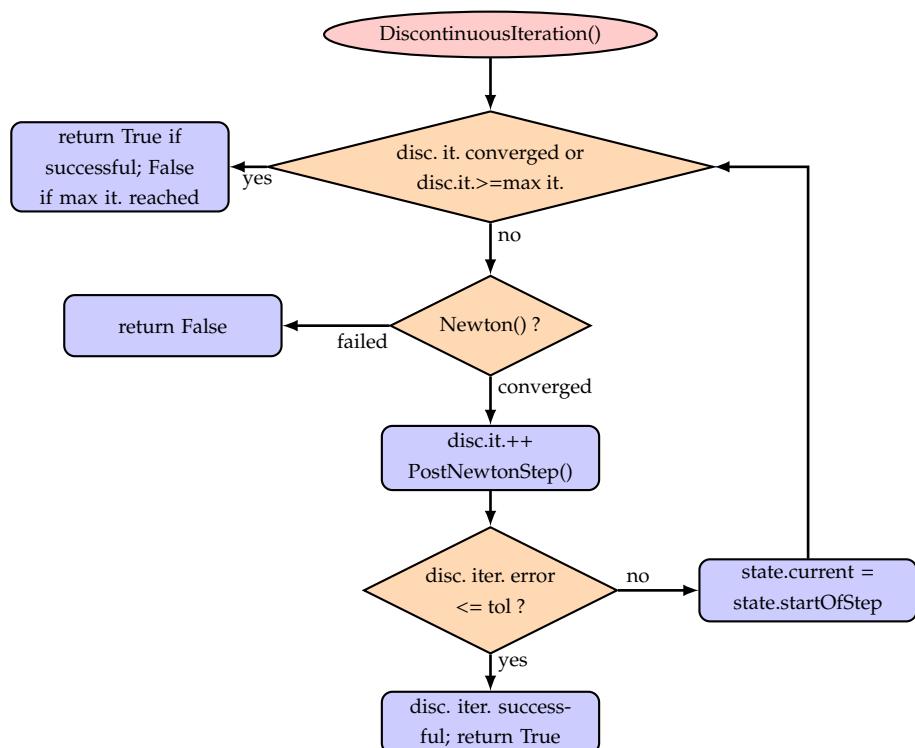


Figure 11.4: Solver flow chart for `DiscontinuousIteration()`, which is run for every solved step inside the static/dynamic solvers. If the `DiscontinuousIteration()` returns `False`, `SolveSteps()` will try to reduce the step size.

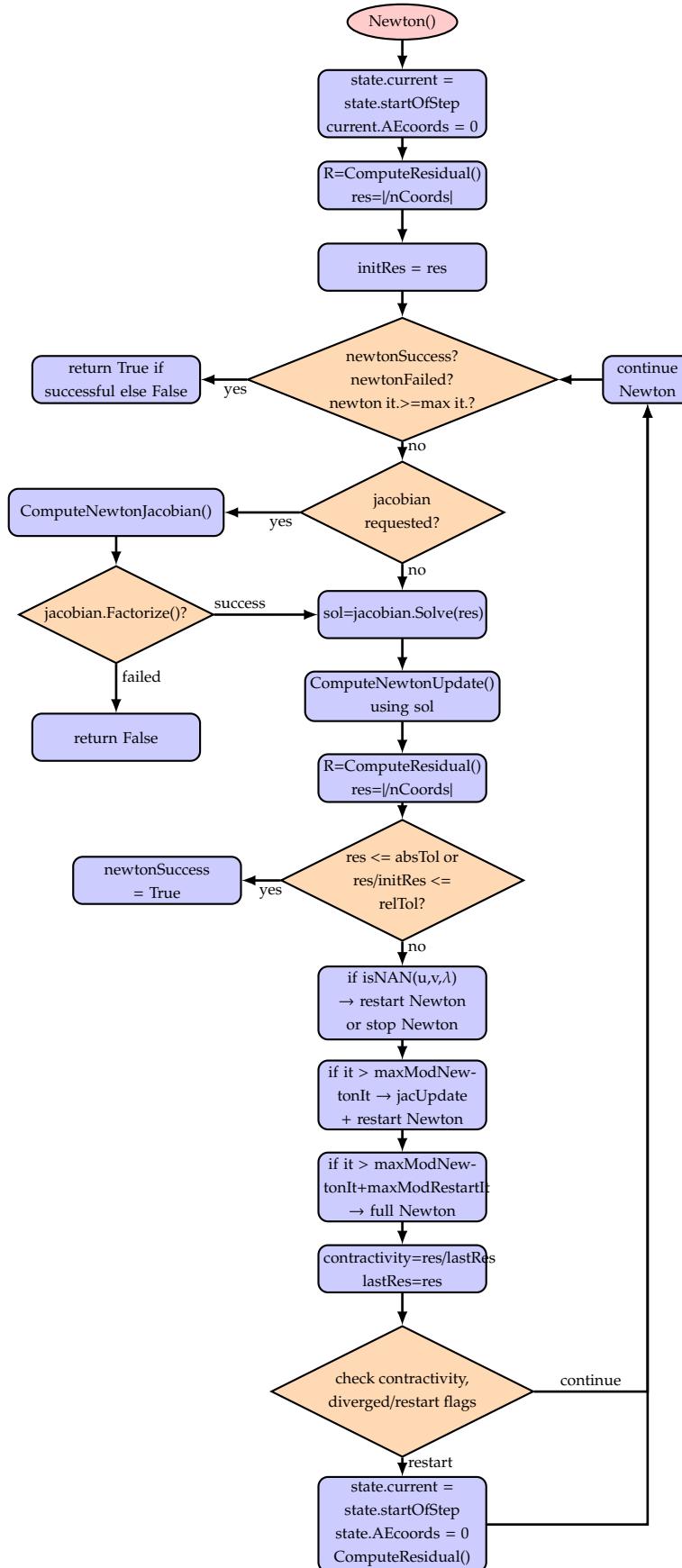


Figure 11.5: Solver flow chart for Newton(), which is run inside the DiscontinuousIteration(). The shown case is valid for newtonResidualMode = 0.

## 11.1 Explicit solvers

Explicit solvers are in general only applicable for systems without constraints (i.e., no joints!). However, some solvers accept simple `CoordinateConstraint`, e.g., fixing coordinates to the ground. Nevertheless, for constraint-free systems, e.g., with penalty constraints, can be solved for very high order and with great efficiency. A list of explicit solvers is available, see [Section 4.5.4](#), for an overview of all implicit and explicit solvers.

The solution vector  $\xi$  (denoted as  $y$  in the literature [20]), which is defined as

$$\xi = [\mathbf{q}^T \ \dot{\mathbf{q}}^T \ \mathbf{y}^T]^T \quad (11.1)$$

and which includes ODE2 coordinates and velocities and ODE1 coordinates. All coordinates are computed without reference values.

The ODE1 and ODE2 equations of Eq. (11.2), with  $\lambda = 0$ , are written in explicit form and converted to first order equations,

$$\begin{aligned} \dot{\mathbf{q}} &= \mathbf{v} \\ \ddot{\mathbf{v}} &= \mathbf{M}^{-1} \mathbf{f}_q(\mathbf{q}, \mathbf{v}, t) \end{aligned} \quad (11.2)$$

$$\dot{\mathbf{y}} = \mathbf{f}_y(\mathbf{y}, t) \quad (11.3)$$

The system first order differential equations for explicit solvers thus read

$$\dot{\xi} = \mathbf{f}_e(\xi, t) \quad (11.4)$$

### 11.1.1 Explicit Runge-Kutta method

Explicit time integration methods seek the solution  $\xi_{t+h}$  at time  $t + h$  for given initial value  $\xi_t$  (at the beginning of one step  $t$  or at the beginning of the simulation,  $t = 0$ ),

$$\xi_{t+h} = \xi_t + \Delta\xi. \quad (11.5)$$

For any given Runge-Kutta method, the integration of one step with step size  $h$  is performed by an approximation

$$\Delta\xi = \int_t^{t+h} \mathbf{f}_e(\tau, \xi(\tau)) d\tau \approx h [b_1 \mathbf{f}_e(t, \xi(t)) + b_2 \mathbf{f}_e(t + c_2 h, \xi(t + c_2 h)) + \dots + b_s \mathbf{f}_e(t + \xi_s h, u(t + \xi_s h))] \quad (11.6)$$

in which  $t + c_i h$  is the time for stage  $i$  and  $b_i$  the according weight given in the integration formula. Stages are within one step (therefor called one-step-methods), where  $c_i = 0$  represents the beginning of the step and  $c_i = 1$  the end. Note that  $c_1 = 0$  for explicit integration formulas.

The unknown solution vectors  $\xi$  at the stages are abbreviated by

$$\mathbf{g}_i \approx \xi(t + c_i h) \quad (11.7)$$

and computed by explicit integration (quadrature) formulas of lower order ( $g_i$  not to be mixed up with algebraic equations!),

$$\begin{aligned} \mathbf{g}_1 &= \xi_t \\ \mathbf{g}_2 &= \xi_t + h a_{21} \mathbf{f}_e(t, \mathbf{g}_1) \\ \mathbf{g}_3 &= \xi_t + h [a_{31} \mathbf{f}_e(t, \mathbf{g}_1) + a_{32} \mathbf{f}_e(t + c_2 h, \mathbf{g}_2)] \\ &\vdots \\ \mathbf{g}_s &= \xi_t + h [a_{s1} \mathbf{f}_e(t, \mathbf{g}_1) + a_{s2} \mathbf{f}_e(t + c_2 h, \mathbf{g}_2) + \dots + a_{s,s-1} \mathbf{f}_e(t + c_{s-1} h, \mathbf{g}_{s-1})] \end{aligned} \quad (11.8)$$

Explicit Euler method, number of stages  $s = 1$ , order  $p = 1$ :

|   |  |
|---|--|
| 0 |  |
| 1 |  |

Explicit midpoint rule, number of stages  $s = 2$ , order  $p = 2$ :

|        |  |
|--------|--|
| 0      |  |
| 1/2    |  |
|        |  |
| 0    1 |  |

Classical explicit Runge-Kutta method (RK44), number of stages  $s = 4$ , order  $p = 4$ :

|                          |  |
|--------------------------|--|
| 0                        |  |
| 1/2                      |  |
| 1/2                      |  |
| 1                        |  |
|                          |  |
| 1/6    1/3    1/3    1/6 |  |

Table 11.1: Several examples of tableaus for the implemented Runge-Kutta methods.

After all vectors  $\mathbf{g}_i$  have been consecutively evaluated, the step is updated by Eq. (11.6).

Conventional explicit Runge-Kutta solvers, such as `ExplicitMidPoint`, `RK44` or `RK67` are based on fixed step size and users must control the error by choosing an appropriate global step size. The tableaus for some lower order methods are given in Table 11.1, using the structure

|                |  |
|----------------|--|
| $\mathbf{c}$   |  |
| $\mathbf{b}^T$ |  |

with

|             |  |
|-------------|--|
| 0           |  |
| $c_2$       |  |
| ⋮           |  |
| $c_s$       |  |
|             |  |
| 0           |  |
| $a_{21}$    |  |
| ⋮           |  |
| $a_{s1}$    |  |
| $\cdots$    |  |
| $a_{s,s-1}$ |  |
| 0           |  |
| $b_1$       |  |
| $\cdots$    |  |
| $b_{s-1}$   |  |
| $b_s$       |  |

with  $\mathbf{c} = [c_0 = 0, c_1, \dots, c_s]$ ,  $\mathbf{b} = [b_0, b_1, \dots, b_s]$ , and  $\mathbf{A}$  having only entries in the lower left triangle. For number of stages  $s > 4$ , the maximum order of explicit methods is lower than the number of stages, such as for `RK67`, which is order 6 but 7 stages.

### 11.1.2 Automatic step size control

Advanced solvers, such as `ODE23` and `DOPRI5`, include automatic step size control<sup>1</sup>.

<sup>1</sup>activated with `timeIntegration.automaticStepSize = True` in `simulationSettings`

We estimate the error of a time step with current step size  $h$  by using an embedded Runge-Kutta formula, which includes two approximations 11.6 of order  $p$  and  $\hat{p} = p - 1$ , which is obtained by using two different integration formulas with common coefficients  $c_i$ , but two sets of weights  $b_i$  and  $\hat{b}_i$ , leading to two approximations  $\xi$  and  $\hat{\xi}$ . These so-called embedded Runge-Kutta formulas are widely used, for details see Hairer et al. [20].

The according apporximations  $\xi$  and  $\hat{\xi}$  are used to estimate an error

$$e_j = |\xi_j - \hat{\xi}_j| \quad (11.9)$$

for every component  $j$  of the solution vector  $\xi$ . A scaling is used for every component of the solution vector, evaluating at the beginning (0) and end (1) of the time step:

$$s_j = a_{tol} + r_{tol} \cdot \max(|\xi_{0j}|, |\xi_{1j}|) \quad (11.10)$$

Then the relative, scaled, scalar error for the step, which needs to fulfill  $err \leq 1$ , is computed as

$$err = \sqrt{\frac{1}{n} \sum_{j=1}^n \left( \frac{\xi_{1j} - \hat{\xi}_{1j}}{s_j} \right)^2} \quad (11.11)$$

The optimal step size then reads

$$h_{opt} = h \cdot \left( \frac{1}{err} \right)^{(1/(q+1))} \quad (11.12)$$

Currently we use the suggested step size as

$$h_{new} = \min(h_{max}, \min(h \cdot f_{maxInc}, \max(h_{min}, f_{sfty} \cdot h_{opt}))) \quad (11.13)$$

With the maximum step size  $h_{max} = \frac{t_{start} - t_{end}}{n_{steps}}$  and the minimum step size  $h_{min}$ , given in the `timeIntegration simulationSettings`. The factor  $f_{maxInc}$  limits the increase of the current step size  $h$ , the factor  $f_{sfty}$  is a safety factor for limiting the chosen step size relative to the optimal one in order to avoid frequent step rejections. If  $h_{new} \leq h$ , the current step is accepted, otherwise the step is recomputed with  $h_{new}$ . For more details, see Hairer et al. [20].

### 11.1.3 Stability limit

Note that there are hard limitations for every explicit integration method regarding the step size. Especially for stiff systems (basically with high stiffness parameters and small masses, but also with restrictions to damping), the **step size  $h$  has an upper limit**:  $h < h_{lim}$ . Above that limit the method is inherently unstable, which needs to be considered both for constant and automatic step size selection.

### 11.1.4 Explicit Lie group integrators

All explicit solvers including the automatic step size solvers (DOPRI5, ODE23) have been equiped with Lie group integration functionality – details will be published soon and some tests are made, but handle this with care.

Basically, the integration formulas, see [Section 11.1.1](#) are extended for special rotation parameters. Lie group integration is currently only available for `NodeRigidBodyRotVecLG` used in `ObjectRigidBody` (3D rigid body). `FFRFreducedOrder` will be extended to such nodes in the near future. To get Lie group integrators running with rigid body models, all 3D node types need to be set to `NodeRigidBodyRotVecLG` and set `explicitIntegration.useLieGroupIntegration == True`.

### 11.1.5 Constraints with explicit solvers

Explicit solvers generally do not solve for algebraic constraints, except for very simple CoordinateConstraint. All connectors having the additional type=Constraint, see the according object in [Section 7.2.15ff.](#), are in general not solvable by explicit solvers. Currently, only CoordinateConstraint with one coordinate fixed to ground can be accounted for, if `explicitIntegration.eliminateConstraints == True`. However, this offers the great flexibility to compute finite elements (imported meshes or ANCF beams) to be (partially) fixed to ground. A CoordinateConstraint that fixes a coordinate with index  $j$  to ground leads to the simple algebraic ODE2 equation

$$g_j(\mathbf{q}) = 0 \Leftrightarrow q_j = 0 \quad (11.14)$$

which can be solved by the implemented explicit solvers by just setting  $q_j = 0$  previously to every computation and  $\dot{q}_j = 0$  after every RHS evaluation.

NOTE that, if `explicitIntegration.eliminateConstraints == False`, constraints are ignored by the explicit solver (and all algebraic variables are set to zero). This may be wanted (e.g. to investigate the free motion of bodies), but in general leads to wrong and meaningless solution.

## 11.2 Implicit (trapezoidal rule-based, Newmark, generalized- $\alpha$ ) solver

This solver represents a class of solvers, which are – in the undamped case – based on the implicit trapezoidal rule (in the view of Runge-Kutta methods). The interpolation of the quantities for one step includes the start and the end value of the time step, thus being called trapezoidal integration rule. In some special cases in Newmark's method [27], the interpolation might only depend on the start value or the end value.

For now, all implemented solvers can be viewed as a generalization of Newmark's method, but there are called differently in the solver interfaces

- **Implicit trapezoidal rule** (= Newmark with  $\beta = \frac{1}{4}$  and  $\gamma = \frac{1}{2}$ )
- **Newmark's method** [27]
- **Generalized- $\alpha$  method** (= generalized Newmark method with additional parameters), see Chung and Hulbert [8] for the original method and Arnold and Brüls [1] for the application to multibody system dynamics.

### 11.2.1 Newmark and generalized- $\alpha$ method

Newmark's method has two parameters  $\beta$  and  $\gamma$ . The main ideas are

- Interpolate the displacements and the velocities linearly using the accelerations  $\ddot{\mathbf{q}}$  of the beginning of the time step (subindex '0') and the end of the time step (subindex 'T'). The 2<sup>nd</sup> order differential equations displacements and velocities and for 1<sup>st</sup> order differential equations coordinates are given by (definition of  $\mathbf{a}$  will become clear later):

$$\begin{aligned} \mathbf{q}_T &= \mathbf{q}_0 + h\dot{\mathbf{q}}_0 + h^2(\frac{1}{2} - \beta)\mathbf{a}_0 + h^2\beta\mathbf{a}_T \\ \dot{\mathbf{q}}_T &= \dot{\mathbf{q}}_0 + h(1 - \gamma)\mathbf{a}_0 + h\gamma\mathbf{a}_T \\ \mathbf{y}_T &= \mathbf{y}_0 + h(1 - \gamma_y)\mathbf{v}_y^0 + h\gamma_y\mathbf{v}_y^T \end{aligned} \quad (11.15)$$

- Solve the system equations at the end of the time step ( $T$ ) for the unknown accelerations as well as for 1<sup>st</sup> order differential equations and algebraic equations coordinates.

Remarks:

- The system of equations may be solved for accelerations  $\ddot{\mathbf{q}}$ , but also for displacements  $\mathbf{q}$  or even velocities as unknowns while the remaining quantities are reconstructed from Eq. (11.15). In case of displacements as unknowns, a scaling of the Jacobian is necessary, see later.
- For consistency reasons, one may set  $\gamma_y = \gamma$ , but currently we use  $\gamma_T = \frac{1}{2}$ , leading to no numerical damping for ODE1 variables  $\mathbf{y}$ .
- In the extension to the so-called generalized- $\alpha$  method [8], algorithmic accelerations  $\mathbf{a}$  are used in Eq. (11.15). Algorithmic accelerations are no longer equivalent to the time derivatives of displacements,  $\mathbf{a} \neq \ddot{\mathbf{q}}$ ; thus, both sets of variables are used independently. In case of Newmark or the implicit trapezoidal rule just use  $\mathbf{a} = \ddot{\mathbf{q}}$ .
- For generalized- $\alpha$ , the algorithmic accelerations  $\mathbf{a}$  are computed from the recurrence relation

$$(1 - \alpha_m)\mathbf{a}_T + \alpha_m\mathbf{a}_0 = (1 - \alpha_f)\ddot{\mathbf{u}}_T + \alpha_f\ddot{\mathbf{u}}_0 \quad (11.16)$$

which can be resolved for the unknown  $\mathbf{a}_T$ ,

$$\mathbf{a}_T = \frac{(1 - \alpha_f)\ddot{\mathbf{u}}_T + \alpha_f\ddot{\mathbf{u}}_0 - \alpha_m\mathbf{a}_0}{(1 - \alpha_m)} \quad (11.17)$$

For the first step, one can simply use  $\mathbf{a}_0 = \ddot{\mathbf{q}}_0$ .

### 11.2.2 Parameter selection for generalized- $\alpha$

Compared to alternative implicit integration methods (including the Newmark method), the generalized- $\alpha$  integrator's parameters break down to one single parameter  $\rho_\infty$ , which allows to chose numerical damping in a practical way.

Based on a simple single DOF mass-spring-damper model [4], having the eigen frequency  $\omega = 2\pi f$  with frequency  $f$  and period  $T = 1/f$ , the spectral radius  $\rho$  for the integrator defines the amount of damping for a given step size  $h$  related to  $T$ , thus using the dimensionless step size  $\bar{h} = h/T$ .

In Fig. 11.6 the spectral radius is shown versus  $\bar{h}$  for various spectral radii at infinity  $\rho_\infty$ . Here,  $\rho_\infty$  specifies the numerical damping of very time step for large step sizes (or very high frequencies). An amount of  $\rho_\infty = 0.9$  means that high frequency parts of the system ( $(\bar{h} \gg 1)$ ; high compared to the step rate) are damped to 90% in every step, reducing an initial value 1 to  $2.66e - 5$  after 100 steps, which is already much larger than usual physical damping in many cases.

Furthermore, low frequency parts of the system ( $\bar{h} \ll 1$ ) receive almost no numerical damping, see again Fig. 11.6. Exemplarily, consider  $\rho$  a low frequency situation with different  $\rho_\infty$ :

- $\rho(\bar{h} = 0.01, \rho_\infty = 0.9) = 1 - 1.13 \cdot 10^{-9}$
- $\rho(\bar{h} = 0.01, \rho_\infty = 0.6) = 1 - 1.22 \cdot 10^{-7}$

which shows that numerical damping is very low for moderately small step sizes (100 steps for one oscillation).

Obviously,  $\rho_\infty$  does not have a large influence for very high or low frequencies in the system as long as it is  $\neq 1$  and we could even use  $\rho_\infty = 0$ . Regarding differential algebraic equations (DAEs),  $\rho_\infty < 1$  allows to integrate index 3 DAEs. Typically a value of  $\rho_\infty = 0.7$  leads to a stable integration, but values depend on the structure of the multibody system.

Once having chosen  $\rho_\infty$ , all other parameters follow automatically [8], regarding the  $\alpha$ s

$$\alpha_m = \frac{2\rho_\infty - 1}{\rho_\infty + 1}, \quad \alpha_f = \frac{\rho_\infty}{\rho_\infty + 1} \quad (11.18)$$

and Newmarks's parameters,

$$\gamma = \frac{1}{2} - \alpha_m + \alpha_f, \quad \beta = \frac{1}{4}(1 - \alpha_m + \alpha_f)^2 \quad (11.19)$$

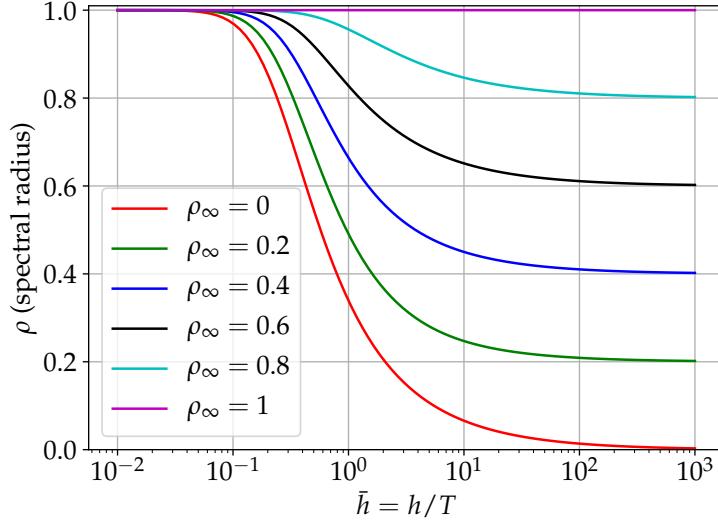


Figure 11.6: Spectral radius for generalized- $\alpha$  method depending on dimensionless step size  $\bar{h} = h/T$ , in which  $T$  is the period of an equivalent single DOF mass-spring-damper system.

### 11.2.3 Newton iteration

Thus, the residuals at the end of the time step ( $T$ ) read (put all terms to LHS):

$$\mathbf{r}_q^{G\alpha} = \mathbf{M}\ddot{\mathbf{q}}_T + \frac{\partial \mathbf{g}}{\partial \mathbf{q}^T} \lambda_T - \mathbf{f}_q(\mathbf{q}_T, \dot{\mathbf{q}}_T, t) = 0 \quad (11.20)$$

$$\mathbf{r}_y^{G\alpha} = \dot{\mathbf{y}}_T + \frac{\partial \mathbf{g}}{\partial \mathbf{y}^T} \lambda_T - \mathbf{f}_y(\mathbf{y}_T, t) = 0 \quad (11.21)$$

$$\mathbf{r}_\lambda^{G\alpha} = \mathbf{g}(\mathbf{q}_T, \dot{\mathbf{q}}_T, \mathbf{y}_T, \lambda_T, t) = 0 \quad (11.22)$$

We consider two options for 2<sup>nd</sup> order differential equations: (A) solve for unknown accelerations  $\ddot{\mathbf{q}}_T$ , or (B) for unknown displacements  $\mathbf{q}_T$ .

#### 11.2.3.1 (A) Solve for unknown accelerations

The unknowns for the Newton method then are

$$\xi_{k+1}^{G\alpha} = \begin{bmatrix} \ddot{\mathbf{q}}_T \\ \mathbf{y}_T \\ \lambda_T \end{bmatrix} \quad (11.23)$$

and at the beginning of the step, we have

$$\xi_k^{G\alpha} = \begin{bmatrix} \ddot{\mathbf{q}}_0 \\ \mathbf{y}_0 \\ \lambda_0 \end{bmatrix} \quad (11.24)$$

For the Newton method, we need to compute an update for the unknowns of Eq. (11.23), using the previous residual  $\mathbf{r}_k$  and the inverse of the Jacobian  $\mathbf{J}_k$  of Newton iteration  $k$ ,

$$\xi_{k+1}^{G\alpha} = \xi_k^{G\alpha} - \mathbf{J}^{-1}(\xi_k^{G\alpha}) \cdot \mathbf{r}^{G\alpha}(\xi_k^{G\alpha}) \quad (11.25)$$

The Jacobian has the following  $3 \times 3$  structure,

$$\mathbf{J} = \begin{bmatrix} \mathbf{J}_{qq} & \mathbf{J}_{qy} & \mathbf{J}_{q\lambda} \\ \mathbf{J}_{yq} & \mathbf{J}_{yy} & \mathbf{J}_{y\lambda} \\ \mathbf{J}_{\lambda q} & \mathbf{J}_{\lambda y} & \mathbf{J}_{\lambda\lambda} \end{bmatrix} = \begin{bmatrix} \mathbf{J}_{qq} & \mathbf{0} & \mathbf{J}_{q\lambda} \\ \mathbf{0} & \mathbf{J}_{yy} & \mathbf{J}_{y\lambda} \\ \mathbf{J}_{\lambda q} & \mathbf{J}_{\lambda y} & \mathbf{J}_{\lambda\lambda} \end{bmatrix} \quad (11.26)$$

in which we consider  $\mathbf{J}_{yq}$  and  $\mathbf{J}_{qy}$  to vanish in the current implementations, which means that coupling of ODE1 and ODE2 coordinates is only possible due to algebraic equations.

The remaining terms in the Jacobian read

$$\begin{aligned} \mathbf{J}_{qq} &= \frac{\partial \mathbf{r}_q^{G\alpha}}{\partial \ddot{\mathbf{q}}} = \frac{\partial \mathbf{r}_q^{G\alpha}}{\partial \mathbf{q}} \frac{\partial \mathbf{q}}{\partial \ddot{\mathbf{q}}} + \frac{\partial \mathbf{r}_q^{G\alpha}}{\partial \dot{\mathbf{q}}} \frac{\partial \dot{\mathbf{q}}}{\partial \ddot{\mathbf{q}}} = h^2 \beta \mathbf{K} + h\gamma \mathbf{D} \\ \mathbf{J}_{q\lambda} &= \frac{\partial \mathbf{r}_q^{G\alpha}}{\partial \lambda} = \frac{\partial \mathbf{g}}{\partial \mathbf{q}} \quad (\text{or } \frac{\partial \mathbf{g}}{\partial \dot{\mathbf{q}}} \text{ for constraints at velocity level}) \\ \mathbf{J}_{yy} &= \frac{\partial \mathbf{r}_y^{G\alpha}}{\partial \mathbf{y}} \\ \mathbf{J}_{\lambda q} &= \frac{\partial \mathbf{r}_\lambda^{G\alpha}}{\partial \ddot{\mathbf{q}}} = \frac{\partial \mathbf{g}}{\partial \ddot{\mathbf{q}}} = \frac{\partial \mathbf{g}}{\partial \mathbf{q}} \frac{\partial \mathbf{q}}{\partial \ddot{\mathbf{q}}} + \frac{\partial \mathbf{g}}{\partial \dot{\mathbf{q}}} \frac{\partial \dot{\mathbf{q}}}{\partial \ddot{\mathbf{q}}} = h^2 \beta \frac{\partial \mathbf{g}}{\partial \mathbf{q}} + h\gamma \frac{\partial \mathbf{g}}{\partial \dot{\mathbf{q}}} \\ \mathbf{J}_{\lambda y} &= \frac{\partial \mathbf{r}_\lambda^{G\alpha}}{\partial \mathbf{y}} \\ \mathbf{J}_{\lambda\lambda} &= \frac{\partial \mathbf{r}_\lambda^{G\alpha}}{\partial \lambda} = \frac{\partial \mathbf{g}}{\partial \lambda} \end{aligned} \quad (11.27)$$

Once an update  $\mathbf{q}_{k+1}^{\text{Newton}}$  has been computed, the interpolation formulas (11.15) need to be evaluated before the next residual and Jacobian can be computed.

### 11.2.3.2 (B) Solve for unknown displacements

This approach is similar to the previous approach and follows exactly the algorithm given by Arnold and Brüls [1], however, extended for ODE1 variables, which are integrated by the (undamped) trapezoidal rule. Documentation will be added lateron.

### 11.2.4 Initial accelerations

For the solvers based on the implicit trapezoidal rule, initial accelerations are necessary in order to significantly increase the accuracy of the first time step. For this reason, the constraints  $\mathbf{g}(\mathbf{q}_0, \dot{\mathbf{q}}_0, \mathbf{y}_0, \lambda_0, t) = 0$  in Eq. (10.13) are differentiated w.r.t. time,

$$\dot{\mathbf{g}}(\mathbf{q}_0, \dot{\mathbf{q}}_0, \mathbf{y}_0, \lambda_0, t) = \frac{\partial \mathbf{g}}{\partial \mathbf{q}} \dot{\mathbf{q}}_0 + \frac{\partial \mathbf{g}}{\partial \dot{\mathbf{q}}} \ddot{\mathbf{q}}_0 + \frac{\partial \mathbf{g}}{\partial \mathbf{y}} \dot{\mathbf{y}}_0 + \frac{\partial \mathbf{g}}{\partial \lambda} \dot{\lambda} + \frac{\partial \mathbf{g}}{\partial t} = 0. \quad (11.28)$$

Currently, we assume  $\frac{\partial \mathbf{g}}{\partial \lambda} = 0$  for all further derivations on initial accelerations. For velocity level constraints, Eq. (11.28) is used to extract initial accelerations  $\ddot{\mathbf{q}}_0$ ,

$$\frac{\partial \mathbf{g}}{\partial \dot{\mathbf{q}}} \dot{\mathbf{q}}_0 = - \frac{\partial \mathbf{g}}{\partial \mathbf{q}} \dot{\mathbf{q}}_0 - \frac{\partial \mathbf{g}}{\partial \mathbf{y}} \dot{\mathbf{y}}_0 - \frac{\partial \mathbf{g}}{\partial t}. \quad (11.29)$$

Finally, the equations for the computation of the initial accelerations read for velocity level constraints, note that  $\mathbf{y}_{init}$  are the nodal initial values for  $\mathbf{y}$ ,

$$\begin{bmatrix} \mathbf{M} & \mathbf{0} & \frac{\partial \mathbf{g}}{\partial \dot{\mathbf{q}}} \\ \mathbf{0} & \mathbf{I} & \mathbf{0} \\ \frac{\partial \mathbf{g}}{\partial \dot{\mathbf{q}}} & \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{q}}_0 \\ \mathbf{y}_0 \\ \lambda_0 \end{bmatrix} = \begin{bmatrix} \mathbf{f}_q(\mathbf{q}_T, \dot{\mathbf{q}}_T, t) \\ \mathbf{y}_{init} \\ -\frac{\partial \mathbf{g}}{\partial \mathbf{q}} \dot{\mathbf{q}}_0 - \frac{\partial \mathbf{g}}{\partial \mathbf{y}} \dot{\mathbf{y}}_0 - \frac{\partial \mathbf{g}}{\partial t} \end{bmatrix}, \quad (11.30)$$

The term  $\frac{\partial \mathbf{g}}{\partial t}$  can only occur in case of user functions and therefore currently not implemented, and the ODE1 term  $\frac{\partial \mathbf{g}}{\partial y} = 0$  is not used yet in constraints.

For position level constraints, we assume  $\frac{\partial \mathbf{g}}{\partial \dot{\mathbf{q}}} = 0$  and  $\frac{\partial \mathbf{g}}{\partial y} = 0$  in Eq. (11.28) and perform a second derivation w.r.t. time,

$$\ddot{\mathbf{g}}(\mathbf{q}_0, \dot{\mathbf{q}}_0, \mathbf{y}_0, \lambda_0, t) = \frac{\partial^2 \mathbf{g}}{\partial \mathbf{q}^2} \dot{\mathbf{q}}_0^2 + 2 \frac{\partial^2 \mathbf{g}}{\partial \mathbf{q} \partial t} \dot{\mathbf{q}}_0 + \frac{\partial \mathbf{g}}{\partial \mathbf{q}} \ddot{\mathbf{q}}_0 + \frac{\partial^2 \mathbf{g}}{\partial t^2} = 0. \quad (11.31)$$

For position level constraints, Eq. (11.31) is used to extract initial accelerations  $\ddot{\mathbf{q}}_0$ ,

$$\frac{\partial \mathbf{g}}{\partial \mathbf{q}} \ddot{\mathbf{q}}_0 = -2 \frac{\partial^2 \mathbf{g}}{\partial \mathbf{q} \partial t} \dot{\mathbf{q}}_0 - \frac{\partial^2 \mathbf{g}}{\partial \mathbf{q}^2} \dot{\mathbf{q}}_0^2 - \frac{\partial^2 \mathbf{g}}{\partial t^2}. \quad (11.32)$$

Finally, the equations for the computation of the initial accelerations for position level constraints read

$$\begin{bmatrix} \mathbf{M} & \mathbf{0} & \frac{\partial \mathbf{g}}{\partial \mathbf{q}^T} \\ \mathbf{0} & \mathbf{I} & \mathbf{0} \\ \frac{\partial \mathbf{g}}{\partial \mathbf{q}} & \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{q}}_0 \\ \mathbf{y}_0 \\ \lambda_0 \end{bmatrix} = \begin{bmatrix} \mathbf{f}_q(\mathbf{q}_T, \dot{\mathbf{q}}_T, t) \\ \mathbf{y}_{init} \\ -2 \frac{\partial^2 \mathbf{g}}{\partial \mathbf{q} \partial t} \dot{\mathbf{q}}_0 - \frac{\partial^2 \mathbf{g}}{\partial \mathbf{q}^2} \dot{\mathbf{q}}_0^2 - \frac{\partial^2 \mathbf{g}}{\partial t^2} \end{bmatrix}, \quad (11.33)$$

The linear system of equations 11.30 or 11.33 is solved prior to an implicit time integration if

```
simulationSettings.timeIntegration.generalizedAlpha.computeInitialAccelerations = True
```

which is the default value.

## 11.3 Optimization and parameter variation

The real benefit of powerful multi-body simulation emerges only if combined with modern but also simple analysis and evaluation methods. Therefore, EXUDYN has been integrated into the Python language, which offers a virtually unlimited number of methods of post-processing, evaluation and optimization. In this section, two methods that are directly integrated into EXUDYN are revisited.

### 11.3.1 Parameter Variation

Parameter variation is one of the simplest tools to evaluate the dependency of the solution of a problem on certain parameters. This usually requires the computation of an objective (goal, result) value for a single computation (e.g., some error norm, maximum vibration amplitude, maximum stress, maximum deflection, etc.) for every computation. Furthermore, it needs to be run for a set of parameters, e.g., using a `for` loop. While this could be done manually in EXUDYN, it is recommended to use built-in functions, which simplify evaluation and postprocessing and directly enable parallelization. The according function `ParameterVariation(...)`, see [Section 5.10](#), performs a set of multi-dimensional parameter variations using a dictionary that describes the variation of parameters. See also `parameterVariationExample.py` in the `Examples` folder for a simple example showing a 2D parameter variation. The function `ParameterVariation(...)` requires the `multiprocessing` Python module which enables simple multi-threaded parallelism and has been tested for up to 80 cores on the LEO4 supercomputer at the University of Innsbruck, achieving a speedup of 50 as compared to a serial computation.

### 11.3.2 Genetic Optimization

In engineering, we often need to find a set of unknown, independent parameters  $\mathbf{x} \in \mathbb{R}^n$ ,  $\mathbf{x}$  being denoted as design variables and  $\mathbb{R}^n$  as design space. Sometimes, the design space is further subjected to constraints  $\mathbf{g}(\mathbf{x}) = 0$  as well as inequalities  $\mathbf{h}(\mathbf{x}) \leq 0$ , which are not considered here. For simple solutions for constrained optimization problems using penalty methods, see the introductory literature [25].

Optimization problems are written in general in the form

$$\min_{\mathbf{x}} f(\mathbf{x}), \quad \mathbf{x} \in \mathbb{R}^n, \quad (11.34)$$

where  $f(\mathbf{x})$  denotes the *objective function* (=fitness function). If we would like to maximize a function  $\tilde{f}(\mathbf{x})$ , simply set  $f(\mathbf{x}) = -\tilde{f}(\mathbf{x})$ .

In engineering, the optimization problem could seek model parameters, e.g., the geometric dimensions and inertia parameters of a slider crank mechanism, in order to achieve smallest possible forces at the supports. Another example is the identification of unknown physical parameters, such as stiffness, damping or friction. This can be achieved by comparing measurement and simulation data (e.g., accelerations measured at relevant parts of a machine). Lets assume that  $\epsilon(t)$  is an error computed in every time step of a computation, then we can set the objective (=fitness) function, e.g., as

$$f(\mathbf{x}) = \frac{1}{T} \sqrt{\int_{t=0}^T \epsilon(t)^2 dt} \quad (11.35)$$

as the integral over the error  $\epsilon$  between measurement and simulation data. In general, a parameter variation would be sufficient to compute sufficient computations for all combinations within the design space, however, a 3D design space with 100 variations into every direction (e.g., varying the unknown damping coefficient between 1 and 100, etc.) would already require 1000.000 computations, which in an ideal case of 1 second/computation leads to almost 2 weeks of computation time.

As an alternative stochastic methods can be used to compute only the objective function for a smaller set of randomly generated design variables, which usually show regions with better parameters (lower  $f$ ) in scatter plots.

**Genetic algorithms**[19, 35] can significantly reduce the necessary amount of objective function evaluations in order to perform the optimization. Genetic identification algorithms have been already successfully applied to multibody system dynamics[10].

The general structure of a (canonical) genetic algorithm is depicted in Fig. 11.7. For details, see the cited

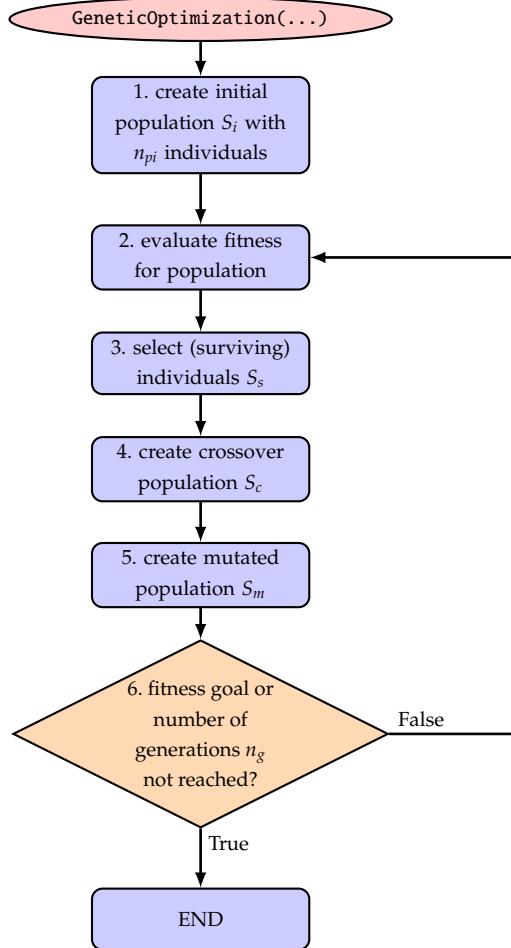


Figure 11.7: Basic solver flow chart genetic algorithm / optimization.

literature. Here, we focus on the implementation of the function `GeneticOptimization(...)`, see [Section 5.10](#). The initial population (step 1) is created with `initialPopulationSize` individuals with uniformly distributed random design variables  $[x_0, \dots, x_{n_{pi}-1}]^2$  in the search space, which is given in the dictionary `parameters`. Herafter (steps 2-6), we iteratively process a population for a certain `numberOfGenerations` generations.

In step 3, the surviving individuals  $S_s$  with best fitness (smallest value from evaluation of `objectiveFunction`) are selected and considered further in the optimization. If the `distanceFactor` is used, the surviving individuals must be located within a certain distance (measured relative to the range of the search space) to all other surviving individuals. This option guarantees the search within several local minima, while a conventional search often converges to one single minimum. Crossover (step 4) is performed using a crossover of all available parameters of two randomly selected parents when generating children from the surviving individuals.

<sup>2</sup> $\mathbf{x}_i = [x_{i0}, x_{i1}, \dots]$  being a set of genes, with single genes  $x_{i0}, x_{i1}, \dots$

The crossover of genes is performed only for a part of the new population, defined by `crossoverAmount`.

Finally, in step 5, we apply mutation to all genes, which extends the search to the surrounding design space of the individuals created by crossover. The mutation could be performed by means of certain distribution functions in order to focus on the currently best search regions. However, in the current implementation of `GeneticOptimization(...)` we simply use a uniform random variable to distribute the genes over a certain percentage of the design space, which is reduced in every generation defined by the `rangeReductionFactor`  $r_r$ . This allows us to restrict further search to a smaller subregion of the design space and in general allows a reduction of search space by means of  $r_r^{n_g}$ . In the ideal case, using sufficiently large population sizes and being lucky with the found random values, a range reduction factor  $r_r = 0.7$  reduces the search space by a factor of 100 after every 13 generations, allowing to obtain 4 digits of accuracy for design variables after 26 generations for suitable optimization problems.

It should be noted that still this optimization method is based on random values and thus may fail occasionally for any problem case. In order to get reproducible results, set `randomizerInitialization` to any integer value (simply: 0) in order to get identical results for repeated runs. Setting the latter variable guarantees that the Python (numpy) randomizer creates the same series of random values for initial population, mutation, etc.



# Chapter 12

## Issues and bugs

This section contains resolved issues per release and known bugs. Use this information to understand changes compared to previous versions. The mentioning of author is omitted if it was Johannes Gerstmayer (JG). BUG numbers refer to the according issue numbers. For major changes, also see [Section 2.5](#). For details, see the `trackerlog.html` file.

General information on current version:

- EXUDYN version = 1.0.201,
- last change = 2021-05-03
- Number of issues = 662,
- Number of resolved issues = 569 (201 in current version),

### 12.1 Resolved issues and resolved bugs

The following list contains the issues which have been **RESOLVED** in the according version:

- **Version 1.0.201:** resolved Issue 0659: **Troubleshooting** (docu)
  - description: Add Trouble shooting section, treating common Python and solver errors to theDoc.pdf
  - date resolved: **2021-05-03 10:18**, date raised: 2021-05-03
- **Version 1.0.200:** resolved Issue 0650: **Highlight item#** (extension)
  - description: Highlight item# for object/node/etc.; add to visualization options (itemType, item#, colorHighlightItem, colorOtherItems), draw all other items in gray
  - date resolved: **2021-05-03 01:18**, date raised: 2021-05-01
- **Version 1.0.199:** resolved Issue 0649: **add ItemType** (extension)
  - description: Add enum ItemType: Node, Object, ...
  - date resolved: **2021-05-03 01:18**, date raised: 2021-05-01
- **Version 1.0.198:** resolved Issue 0651: **add itemID to graphics objects** (extension)
  - description: Add itemID (nodes, objects, markers, loads, sensors, in that order) to graphics objects (for right-mouse-press)
  - date resolved: **2021-05-02 21:26**, date raised: 2021-05-01
- **Version 1.0.197:** resolved Issue 0658: **add VisualizationSettings() interactive** (change)
  - description: move visualization settings window functions keypressRotationStep, mouseMoveRotationFactor, keypressTranslationStep, zoomStepFactor to new substructure "interactive"
  - **notes: adapt your models if you used these options!**
  - date resolved: **2021-05-01 23:36**, date raised: 2021-05-01

- **Version 1.0.196:** resolved Issue 0654: **coordinates sizes** (extension)
  - description: add function ODE2Size(...), ODE1Size(...), SystemSize(...) to mbs.systemData to retrieve number of ODE2,ODE1,AE and Data coordinates for certain configurationType; only works after mbs.Assemble()
  - date resolved: **2021-05-01 23:23**, date raised: 2021-05-01
- **Version 1.0.195:** resolved Issue 0656: **mbs.systemData** (change)
  - description: removed GetcurrentTime() and SetVisualizationTime(...) which have been marked as deprecated already
  - **notes:** **Use GetTime(...) and SetTime(...) in mbs.systemData instead**
  - date resolved: **2021-05-01 23:08**, date raised: 2021-05-01
- **Version 1.0.194:** resolved Issue 0644: **solver messages** (extension)
  - description: add solver message if not converged with helpful hints (especially if invert fails or newton fails)
  - date resolved: **2021-05-01 02:31**, date raised: 2021-04-30
- **Version 1.0.193:** resolved Issue 0349: **add causing row** (extension)
  - description: output causing row/column (=coordinate) which leads to singular matrix; do this for Matrix.Invert as well as for SparseLU .info code; matrix class creates string with error message!
  - date resolved: **2021-05-01 02:30**, date raised: 2020-03-02
- **Version 1.0.192:** resolved Issue 0646: **jacobian singular** (extension)
  - description: resolve singularities in general jacobian: resolves coordinates which are still free for static problems, but is marked as unsafe
  - date resolved: **2021-05-01 02:29**, date raised: 2021-04-30
- **Version 1.0.191:** resolved Issue 0645: **redundant constraints** (extension)
  - description: resolve redundant constraints: add flag to ignore singular constraint jacobians
  - date resolved: **2021-05-01 02:29**, date raised: 2021-04-30
- **Version 1.0.190:** resolved Issue 0333: **node numbers with type** (extension)
  - description: extend node/object/... numbers as python class with type information to check if item numbers are mixed illegally
  - **notes:** **done already earlier, but still marked as unresolved**
  - date resolved: **2021-04-30 17:04**, date raised: 2020-02-06
- **Version 1.0.189:** resolved Issue 0641: **ObjectContactFrictionCircleCable2D** (docu)
  - description: add description and figure for theory and computation
  - date resolved: **2021-04-29 08:40**, date raised: 2021-04-29
- **Version 1.0.188:** **resolved BUG 0423: fix MarkerSuperElementRigidBody**
  - description: fix velocity level for MarkerSuperElementRigidBody (check constraint equations)
  - **notes:** **fixed several errors and test examples work now on velocity level, but further checks are necessary**
  - date resolved: **2021-04-27 18:24**, date raised: 2020-06-09
- **Version 1.0.187:** resolved Issue 0635: **AnimateModes** (check)
  - description: check, why animate modes has threading-conflicts; use std::cout to find issues
  - **notes:** **resolved threading conflicts, but visualization state set inbetween graphics update, which needs to resolve #634**
  - date resolved: **2021-04-27 18:20**, date raised: 2021-04-25
- **Version 1.0.186:** resolved Issue 0640: **MarkerSuperElementRigid** (extension)
  - description: remove referencePosition and add offset instead (to correct errors of midpoint due to small mesh-unsymmetries)
  - **notes:** **CHANGED interface: MarkerSuperElementRigid does not have a referencePosition anymore, but adds a parameter offset**
  - date resolved: **2021-04-27 18:18**, date raised: 2021-04-26

- **Version 1.0.185:** resolved **BUG 0639:** ObjectFFRFreducedOrder
  - description: incorrect AccessFunction AccessFunctionType::AngularVelocity\_qt, missing correct reference point = midpoint for computation of rotation
  - date resolved: **2021-04-26 21:47**, date raised: 2021-04-26
- **Version 1.0.184:** resolved Issue 0638: **MarkerSuperElementRigid** (extension)
  - description: use consistent reference point = midpoint for computation of rotation and use exponential Map for rotation matrix
  - date resolved: **2021-04-26 21:47**, date raised: 2021-04-26
- **Version 1.0.183:** resolved Issue 0636: **Python3.8** (extension)
  - description: add python 3.8 compilation tests; resolve issues with `__index__` method needed fore NodeIndex, MarkerIndex, etc.
  - date resolved: **2021-04-26 16:25**, date raised: 2021-04-26
- **Version 1.0.182:** resolved **BUG 0629:** mesh visualization
  - description: visualization artifacts in larger FE meshes due to multithreading
  - notes: **added flag threadSafeGraphicsUpdate to avoid thread conflicts between graphics and computation, which is by default set True and MAY SLOW DOWN your computation speed if True**
  - date resolved: **2021-04-25 22:28**, date raised: 2021-04-22
- **Version 1.0.181:** resolved Issue 0632: **CMS theory** (docu)
  - description: add theory section for Hurty-Craig-Bampton modes and eigenmode computation
  - date resolved: **2021-04-23 19:03**, date raised: 2021-04-23
- **Version 1.0.180:** resolved **BUG 0628:** `FEMinterface.GetNodesOnCylinder`
  - description: returns erroneous indices
  - notes: **corrected indexing and add warnings for illegal node types**
  - date resolved: **2021-04-22 22:59**, date raised: 2021-04-22
- **Version 1.0.179:** resolved Issue 0616: **Craig-Bampton** (extension)
  - description: add static modes (Hurty-Craig-Bampton) to computation of modes in CMSinterface
  - notes: **implemented in FEMinterface.ComputeHurtyCraigBamptonModes(...)**
  - date resolved: **2021-04-21 11:09**, date raised: 2021-03-21
- **Version 1.0.178:** resolved Issue 0624: **norm in contour plots** (extension)
  - description: show norm (of vectors or stresses) in contour plot, using special outputVariable component=-1
  - date resolved: **2021-04-09 18:18**, date raised: 2021-04-09
- **Version 1.0.177:** resolved Issue 0623: **postProcessingModes** (extension)
  - description: add function to compute stress or strain modes for postprocessing, working for linear tetrahedrons (Tet4); see Examples/NGsolvePostProcessingStresses.py
  - date resolved: **2021-04-09 15:46**, date raised: 2021-04-09
- **Version 1.0.176:** resolved Issue 0424: **show modes** (extension)
  - description: add feature to visualize eigenmodes, e.g. using ObjectFFRFreducedOrder and set one initialCoordinate nonzero
  - date resolved: **2021-04-07 13:48**, date raised: 2020-06-12
- **Version 1.0.175:** resolved Issue 0619: **Eigenmode visualizer** (extension)
  - description: visualize eigenmodes with interactive tools with new function `AnimateModes(...)` to show eigenmodes of system or ObjectFFRFreducedOrder (see [Section 5.6](#))
  - date resolved: **2021-04-07 13:47**, date raised: 2021-03-30
- **Version 1.0.174:** resolved Issue 0622: **InteractiveDialog** (extension)
  - description: improved functionality of InteractiveDialog in `interactive.py`, specially for animating modes
  - date resolved: **2021-04-07 12:20**, date raised: 2021-04-07

- **Version 1.0.173:** resolved **BUG 0621:** `mbs.GetNodeODE2Index`
  - description: Fails for NodeRigidBodyEP, because mix of AE and ODE2 variables
  - **notes:** added correct type check in `MainSystem::PyGetNodeODE2Index`
  - date resolved: **2021-04-07 09:12**, date raised: 2021-04-07
- **Version 1.0.172:** resolved Issue 0403: **CMS C++** (extension)
  - description: add ObjectFFRFreducedOrder (CMS) equations in C++ and clean up code
  - date resolved: **2021-03-30 16:57**, date raised: 2020-05-21
- **Version 1.0.171:** resolved Issue 0470: **geometrically exact beam2D** (extension)
  - description: add to CPP
  - date resolved: **2021-03-25 18:07**, date raised: 2020-11-21
- **Version 1.0.170:** resolved Issue 0563: **ODE1Coordinate** (extension)
  - description: add MarkerODE1Coordinate and extend LoadCoordinate for ODE1
  - date resolved: **2021-03-25 07:43**, date raised: 2021-01-27
- **Version 1.0.169:** resolved Issue 0615: **MarkerNodeCoordinate** (extension)
  - description: add check in CSystem for valid coordinate numbers in MarkerNodeCoordinate
  - date resolved: **2021-03-22 14:18**, date raised: 2021-03-21
- **Version 1.0.168:** resolved Issue 0611: **adaptiveStep** (extension)
  - description: add adaptiveStepIncrease, Decrease and RecoverySteps options to control behavior in case of discontinuous problems
  - date resolved: **2021-03-21 00:21**, date raised: 2021-03-21
- **Version 1.0.167:** resolved Issue 0603: **loadFactor** (change)
  - description: exclude load factor for loads with user functions in static computations
  - date resolved: **2021-03-20 23:24**, date raised: 2021-03-18
- **Version 1.0.166:** resolved Issue 0610: **startOfStep** (extension)
  - description: add access function for nodal coordinates at startOfStep configuration, used in `mbs.GetNodeOutput(configuration = exu.ConfigurationType.startOfStep)`
  - date resolved: **2021-03-20 23:23**, date raised: 2021-03-20
- **Version 1.0.165:** resolved Issue 0609: **SolveDynamic, SolveStatic** (change)
  - description: store dynamicSolver and staticSolver in mbs.sys dictionary immediately after creation, which allows to use these structures in user functions during static or dynamic solution
  - date resolved: **2021-03-20 23:23**, date raised: 2021-03-20
- **Version 1.0.164:** resolved Issue 0607: **test recommendedStepSize** (test)
  - description: test recommendedStepSize and PostNewtonUserFunction with simple elastic contact example
  - date resolved: **2021-03-20 23:23**, date raised: 2021-03-20
- **Version 1.0.163:** resolved Issue 0605: **UIndex, UReal** (extension)
  - description: change all relevant unsigned quantities to UIndex and UReal, as well as Vectors of UReal and Arrays of UIndex
  - date resolved: **2021-03-20 23:23**, date raised: 2021-03-19
- **Version 1.0.162:** resolved Issue 0604: **UIndex check** (extension)
  - description: add automatic check in item interface to check for correctness of UIndex and UReal quantities
  - date resolved: **2021-03-20 23:23**, date raised: 2021-03-19
- **Version 1.0.161:** resolved Issue 0337: **local quantities beam** (change)
  - description: change beam output of Force, Torque, Curvature, Stress and Strain to ForceLocal, TorqueLocal, CurvatureLocal, StressLocal, and StrainLocal

- notes: **WARNING: you need to adapt force, torque and stress, strain and curvature output variables accordingly as they may have changed specifically for ANCF beams; adapt all your model files regarding Force, Torque, etc.**
  - date resolved: **2021-03-20 23:22**, date raised: 2020-02-18
- **Version 1.0.160:** resolved Issue 0139: **Index** (change)
  - description: change Index to (signed) int and use UIIndex in python interface for unsigned parameters
  - notes: **ATTENTION: this change affects many routines. All TestSuite examples passed the change but there may still be open problems due to this major change.**
  - date resolved: **2021-03-20 23:21**, date raised: 2019-05-20
- **Version 1.0.159:** resolved Issue 0606: **resolve errors 32bit testsuite** (test)
  - description: add extra tolerances for 32bit
  - date resolved: **2021-03-20 23:19**, date raised: 2021-03-20
- **Version 1.0.158:** **resolved BUG 0575: new genAlpha solver**
  - description: new generalized alpha/implicit trapezoidal solver does not call solver user functions; Solution: derive CSolverImplicitSecondOrderTimeIntNew from CSolverBase and add user functions on top
  - notes: **solved by removing old solver structure; new solver fully supports user functions now**
  - date resolved: **2021-03-18 21:34**, date raised: 2021-02-08
- **Version 1.0.157:** resolved Issue 0602: **PostNewton step size recommendation** (extension)
  - description: add step recommendation as outcome of PostNewton function to improve contact and friction accuracy
  - date resolved: **2021-03-18 21:33**, date raised: 2021-03-18
- **Version 1.0.156:** resolved Issue 0601: **mbs.postNewtonUserFunction** (extension)
  - description: add function PostNewton(...) to be called after step update (Newton or explicit step)
  - date resolved: **2021-03-18 21:33**, date raised: 2021-03-18
- **Version 1.0.155:** resolved Issue 0600: **ImplicitSecondOrderSolver** (cleanup)
  - description: remove old solver
  - date resolved: **2021-03-18 21:33**, date raised: 2021-03-18
- **Version 1.0.154:** resolved Issue 0598: **rigidBodyUtilities** (extension)
  - description: add G matrices for Rxyz (Tait-Bryan angles) and also time derivatives of G
  - date resolved: **2021-03-18 17:05**, date raised: 2021-03-18
- **Version 1.0.153:** resolved Issue 0594: **CMS rotations** (extension)
  - description: test and extend CMS / ObjectFFRFreducedOrder object for other rotation parameterizations (Tait-Bryan and rotation vector/Lie group) such that they work with explicit codes
  - date resolved: **2021-03-18 17:04**, date raised: 2021-02-24
- **Version 1.0.152:** resolved Issue 0597: **ObjectRigidBody** (description)
  - description: fix description of equations of motion (missing m) and add steps in derivation
  - date resolved: **2021-03-18 08:22**, date raised: 2021-03-18
- **Version 1.0.151:** resolved Issue 0283: **cylinder with hole** (new feature)
  - description: add TriangleList for cylinder with hole
  - notes: **not implemented, because it can be easily created with GraphicsDataSolidOfRevolution**
  - date resolved: **2021-03-16 16:59**, date raised: 2019-12-07
- **Version 1.0.150:** resolved Issue 0396: **description** (description)
  - description: add latex description for ObjectFFRF
  - date resolved: **2021-03-16 16:57**, date raised: 2020-05-16
- **Version 1.0.149:** resolved Issue 0394: **description** (description)
  - description: add latex description for ObjectSuperElement

- date resolved: **2021-03-16 16:57**, date raised: 2020-05-16
- **Version 1.0.148:** resolved Issue 0595: **ObjectFFRFreducedOrder** (extension)
  - description: add general nodeType to AddObjectFFRFreducedOrderWithUserFunctions
  - date resolved: **2021-03-16 16:56**, date raised: 2021-03-14
- **Version 1.0.147:** resolved Issue 0461: **ObjectRigidBody** (check)
  - description: check description of output variables
  - date resolved: **2021-03-16 16:55**, date raised: 2020-11-12
- **Version 1.0.146:** resolved Issue 0596: **GetRigidBodyNode** (extension)
  - description: add function GetRigidBodyNode into rigidBodyUtilities, which returns a node item for an according node type, e.g., Euler parameters or rotation vector
  - date resolved: **2021-03-14 14:43**, date raised: 2021-03-14
- **Version 1.0.145:** resolved Issue 0583: **FFRF docu** (docu)
  - description: add documentation for FFRF and FFRReducedOrder (CMS) to documentation
  - date resolved: **2021-03-01 12:15**, date raised: 2021-02-16
- **Version 1.0.144:** resolved Issue 0455: **FEM help** (docu)
  - description: add comments to ObjectFFRF (Tisserand frame!) and reduced that there are convenient helper functions in FEM, etc. for creating objects
  - date resolved: **2021-02-24 21:21**, date raised: 2020-10-13
- **Version 1.0.143:** resolved Issue 0593: **Add MacOS support** (change)
  - description: make minor adjustments for MacOS to run setup.py
  - date resolved: **2021-02-22 13:10**, date raised: 2021-02-22
- **Version 1.0.142:** **resolved BUG 0592: StartRenderer**
  - description: flag verbose=True not working
  - date resolved: **2021-02-22 10:08**, date raised: 2021-02-22
- **Version 1.0.141:** resolved Issue 0590: **SensorMarker visualization** (extension)
  - description: add visualization for SensorMarker according to marker position
  - date resolved: **2021-02-19 10:21**, date raised: 2021-02-19
- **Version 1.0.140:** resolved Issue 0400: **add SensorMarker** (extension)
  - description: add sensor for markers, restricting to position/velocity and rotation/angular velocity
  - date resolved: **2021-02-18 19:15**, date raised: 2020-05-20
- **Version 1.0.139:** resolved Issue 0585: **UserSensor** (extension)
  - description: add user sensor, which enables the user to add any kind of sensor, specifically to combine several sensor outputs into one single sensor
  - date resolved: **2021-02-18 19:14**, date raised: 2021-02-17
- **Version 1.0.138:** resolved Issue 0589: **solver updateInitialValues** (change)
  - description: update also initial coordinates in order to avoid jumps in accelerations when continuing simulation
  - date resolved: **2021-02-18 18:15**, date raised: 2021-02-18
- **Version 1.0.137:** resolved Issue 0588: **Solver file header** (change)
  - description: move writing of solution file and sensor files headers from InitializeSolverPreChecks(...) to InitializeSolverInitialConditions(...) to avoid sensor evaluation for initial configuration
  - date resolved: **2021-02-18 16:23**, date raised: 2021-02-18
- **Version 1.0.136:** resolved Issue 0587: **ALEANCF Cable2D** (fix)
  - description: change mass proportional load to include force in direction of ALE coordinate
  - date resolved: **2021-02-17 18:19**, date raised: 2021-02-17
- **Version 1.0.135:** resolved Issue 0586: **GeneticOptimization** (fix)

- description: add special warnings and adaptations in order to catch cases where elitistRatio\*populationSize < 1 and if distanceFactor >= 1
  - date resolved: **2021-02-17 18:17**, date raised: 2021-02-17
- **Version 1.0.134:** resolved Issue 0584: **parameter variation** (extension)
  - description: add possibility to prescribe set of parameters using list, e.g., 'mass':[1,2,4,8] instead of tuple which describes the range: 'mass':(0,6,4)
  - date resolved: **2021-02-16 09:43**, date raised: 2021-02-16
- **Version 1.0.133:** resolved Issue 0582: **optimization** (docu)
  - description: add parameter variation and genetic optimization to documentation of solvers
  - date resolved: **2021-02-16 08:21**, date raised: 2021-02-16
- **Version 1.0.132:** **resolved BUG 0581:** **SetODE2Coordinates\_tt**
  - description: SetODE2Coordinates\_tt writes to velocities instead of accelerations
  - date resolved: **2021-02-14 11:12**, date raised: 2021-02-14
- **Version 1.0.131:** resolved Issue 0580: **ParameterVariation** (extension)
  - description: processing.ParameterVariation(...): write to resultsFile to show progress in resultsMonitor.py
  - date resolved: **2021-02-11 17:49**, date raised: 2021-02-11
- **Version 1.0.130:** resolved Issue 0576: **add ClearWorkspace** (extension)
  - description: add ClearWorkspace() to basicUtilities which allows simple and save cleanup of globals() in python environment; recommended to be called at beginning of complex models
  - date resolved: **2021-02-10 12:35**, date raised: 2021-02-08
- **Version 1.0.129:** resolved Issue 0569: **contour text** (fix)
  - description: add space to computation info text before contour plot text
  - date resolved: **2021-02-10 12:35**, date raised: 2021-02-05
- **Version 1.0.128:** resolved Issue 0568: **Renderer axes** (fix)
  - description: use X(0), Y(1) and Z(2) for axes description to be compliant with Python indexing starting with 0 as well as contour components
  - date resolved: **2021-02-10 12:35**, date raised: 2021-02-05
- **Version 1.0.127:** resolved Issue 0579: **ClearWorkspace** (extension)
  - description: add ClearWorkspacefunction to exdyn.basicUtilities, which allows to reset global variables in ipython; see example in function description
  - date resolved: **2021-02-10 12:13**, date raised: 2021-02-10
- **Version 1.0.126:** resolved Issue 0578: **SmoothStep** (extension)
  - description: add SmoothStep function to exdyn.utilities, which produces a smooth step function using cosine
  - date resolved: **2021-02-10 12:13**, date raised: 2021-02-10
- **Version 1.0.125:** resolved Issue 0572: **void** (fix)
  - description: redundant with issue 568
  - date resolved: **2021-02-10 12:05**, date raised: 2021-02-05
- **Version 1.0.124:** resolved Issue 0571: **void** (fix)
  - description: redundant with issue 568
  - date resolved: **2021-02-10 12:05**, date raised: 2021-02-05
- **Version 1.0.123:** resolved Issue 0570: **void** (fix)
  - description: redundant with issue 568
  - date resolved: **2021-02-10 12:05**, date raised: 2021-02-05
- **Version 1.0.122:** **resolved BUG 0577:** **PostNewtonStep**
  - description: perform PostNewtonStep and PostDiscontinuousIterationStep only for active objects

- date resolved: **2021-02-09 14:00**, date raised: 2021-02-09
- **Version 1.0.121:** resolved Issue 0355: **generalized alpha** (extension)
  - description: implement version of Brüls and Arnold for generalized alpha solver
  - **notes:** **WARNING: switched to new solver based on displacement increments (Arnold/Bruls,2007), which leads to DIFFERENT (but improved) RESULTS than previous dynamic implicit integrator; new implicit solver now works with ODE1 variables**
  - date resolved: **2021-02-08 01:56**, date raised: 2020-03-08
- **Version 1.0.120:** resolved Issue 0573: **merge solver documentation** (docu)
  - description: merge docu on solver in EXUDYN overview and in solver section
  - date resolved: **2021-02-07 17:33**, date raised: 2021-02-07
- **Version 1.0.119:** resolved Issue 0567: **solvers description** (docu)
  - description: extend description for equations of motion, explicit and implicit solvers
  - date resolved: **2021-02-04 01:14**, date raised: 2021-02-03
- **Version 1.0.118:** resolved Issue 0560: **Impl integrator ODE1** (extension)
  - description: add ODE1 coordinates to implicit integrator
  - date resolved: **2021-02-02 15:16**, date raised: 2021-01-26
- **Version 1.0.117:** resolved Issue 0508: **implicit Lie group integrator** (extension)
  - description: implement implicit index2/index3 Lie group integrator as python function
  - **notes:** **cancelled, because will be directly done in C++**
  - date resolved: **2021-02-02 15:15**, date raised: 2020-12-17
- **Version 1.0.116:** resolved Issue 0566: **memory alloc cnt** (check)
  - description: add control to check whether large amount of memory allocations happen during time integration+test suite
  - date resolved: **2021-02-01 01:38**, date raised: 2021-01-29
- **Version 1.0.115:** resolved Issue 0541: **objectODE1/2, constraint lists** (extension)
  - description: add lists of ODE1 and ODE2 objects, constraints, etc. in cSystemData in order to speed up processing
  - date resolved: **2021-02-01 01:38**, date raised: 2021-01-13
- **Version 1.0.114:** resolved Issue 0557: **RK with constraints** (extension)
  - description: add CoordinateConstraints to explicit Runge-Kutta solvers
  - **notes:** **only ground constraints included for now**
  - date resolved: **2021-01-27 17:38**, date raised: 2021-01-26
- **Version 1.0.113:** resolved Issue 0558: **Lie group tests** (test)
  - description: add Lie group integrator simple tests
  - date resolved: **2021-01-27 12:00**, date raised: 2021-01-26
- **Version 1.0.112:** resolved Issue 0550: **GraphicsDataArrow** (extension)
  - description: add arrow to graphicsDataUtilities
  - **notes:** **also added GraphicsDataBasis(...) for drawing 3 orthogonal basis vectors, GraphicsDataCheckerBoard(...) for simple drawing of checker board background and MergeGraphicsDataTriangleList(...) for merging graphicsData triangle lists**
  - date resolved: **2021-01-27 00:10**, date raised: 2021-01-17
- **Version 1.0.111:** resolved Issue 0495: **add ODE1 coordinates** (extension)
  - description: extend system (Jacobian, etc.) for ODE1 coordinates
  - date resolved: **2021-01-26 13:21**, date raised: 2020-12-09
- **Version 1.0.110:** resolved Issue 0556: **explicit RK tests** (test)
  - description: add tests for explicit Runge Kutta integrators to TestModels
  - date resolved: **2021-01-26 13:17**, date raised: 2021-01-25

- **Version 1.0.109:** resolved Issue 0555: **explicit Lie group integrator** (extension)
  - description: add existing Lie group integrator in C++
  - date resolved: **2021-01-26 13:17**, date raised: 2021-01-25
- **Version 1.0.108:** resolved Issue 0554: **explicit integrator** (extension)
  - description: add explicit integrator with automatic step size control (DOPRI5, ODE23); checkout [Section 11.1](#) for description of explicit solvers and [Section 4.5.4](#) for available solver types
  - date resolved: **2021-01-25 00:54**, date raised: 2021-01-24
- **Version 1.0.107:** resolved Issue 0513: **add RK4 integrator** (extension)
  - description: put existing python RK4 integrator into CPP
  - date resolved: **2021-01-25 00:54**, date raised: 2020-12-19
- **Version 1.0.106:** resolved Issue 0533: **ObjectGenericODE1** (extension)
  - description: add object ObjectGenericODE1 for generic first order ODEs
  - date resolved: **2021-01-21 17:27**, date raised: 2021-01-04
- **Version 1.0.105:** resolved Issue 0553: **create physics submodule** (extension)
  - description: create exudyn.physics and add friction functions
  - date resolved: **2021-01-20 10:25**, date raised: 2021-01-20
- **Version 1.0.104: resolved BUG 0552: DrawSystemGraph**
  - description: does not work with RigidBodySpringDamper due to invalid GenericNodeData number
  - date resolved: **2021-01-19 14:43**, date raised: 2021-01-19
- **Version 1.0.103:** resolved Issue 0551: **InteractiveDialog** (extension)
  - description: add interactive tkinter dialog and new submodule exudyn.interactive to interact with models
  - date resolved: **2021-01-19 00:26**, date raised: 2021-01-19
- **Version 1.0.102:** resolved Issue 0549: **show solver name and time** (extension)
  - description: add options to show/hide solver name and current time in render window
  - date resolved: **2021-01-17 17:42**, date raised: 2021-01-17
- **Version 1.0.101: resolved BUG 0545: mbs.WaitForUserToContinue()**
  - description: call to WaitForUserToContinue() does not always wait for keypress. Check StartRender() function and flag settings
  - date resolved: **2021-01-17 16:55**, date raised: 2021-01-15
- **Version 1.0.100: resolved BUG 0548: SolveDynamic/SolveStatic**
  - description: option updateInitialValues not working
  - notes: corrected SetSystemState call
  - date resolved: **2021-01-17 00:08**, date raised: 2021-01-17
- **Version 1.0.99:** resolved Issue 0542: **GeneticOptimization** (extension)
  - description: store values continuously to file, add automatic loader and animate optimized values
  - notes: added resultsMonitor.py to exudyn module
  - date resolved: **2021-01-15 15:18**, date raised: 2021-01-13
- **Version 1.0.98:** resolved Issue 0547: **realtimeSimulation** (extension)
  - description: add factor for timeIntegration.simulateInRealtime
  - date resolved: **2021-01-15 15:16**, date raised: 2021-01-15
- **Version 1.0.97:** resolved Issue 0546: **add \_\_version\_\_ version to module** (extension)
  - description: enable exudyn.\_\_version\_\_ as commonly used in other modules
  - date resolved: **2021-01-15 15:05**, date raised: 2021-01-15
- **Version 1.0.96:** resolved Issue 0544: **geneticOptimization** (extension)
  - description: add optional argument resultsFile to specify a file for output of results data

- date resolved: **2021-01-14 23:17**, date raised: 2021-01-14
- **Version 1.0.95:** resolved Issue 0543: **add results monitor** (extension)
  - description: add resultsMonitor.py to be called from command line for doing continuous visualization of sensors and geneticOptimization output
  - date resolved: **2021-01-14 23:08**, date raised: 2021-01-14
- **Version 1.0.94:** resolved Issue 0532: **NodeGenericODE1** (extension)
  - description: add node NodeGenericODE1 for arbitrary number of ODE1 coordinates
  - date resolved: **2021-01-13 20:12**, date raised: 2021-01-04
- **Version 1.0.93:** resolved Issue 0531: **solidExtrusion** (extension)
  - description: add graphicsData for solid extrusion (prismatic) body; based on 2D point and segment list for flat boundaries
  - date resolved: **2021-01-10 20:43**, date raised: 2021-01-04
- **Version 1.0.92:** resolved Issue 0539: **RigidBodySpringDamper** (extension)
  - description: add postNewtonStepUserFunction and dataCoordinates
  - date resolved: **2021-01-08 14:34**, date raised: 2021-01-08
- **Version 1.0.91:** **resolved BUG 0537: Render window**
  - description: double calling of Render(...) function could happen from RunLoop/Render and glfwSetWindowRefreshCallback (set in InitCreateWindow(...)); check if semaphore would remove visualization problems
  - **notes:** **added semaphore but FEM visualization anomalies are still there**
  - date resolved: **2021-01-07 11:23**, date raised: 2021-01-06
- **Version 1.0.90:** resolved Issue 0385: **add solver eigenvalues example** (extension)
  - description: add Examples/solverFunctionsTestEigenvalues to test suite
  - **notes:** **added ComputeODE2EigenvaluesTest.py using new functionality exudyn.solver.ComputeODE2Eigenvalues(...)**
  - date resolved: **2021-01-07 11:08**, date raised: 2020-05-06
- **Version 1.0.89:** resolved Issue 0494: **add all tests** (extension)
  - description: add all TestModel/\*.py to testsuite and also examples before making changes to solver
  - date resolved: **2021-01-07 11:04**, date raised: 2020-12-09
- **Version 1.0.88:** resolved Issue 0515: **user function connector** (extension)
  - description: add forceUserFunction for ObjectConnectorRigidBodySpringDamper to enable User connector
  - date resolved: **2021-01-07 11:03**, date raised: 2020-12-19
- **Version 1.0.87:** resolved Issue 0506: **utilities docu** (extension)
  - description: complete documentation for all exudyn python utilities and add unique headers for documentation
  - date resolved: **2021-01-06 22:57**, date raised: 2020-12-16
- **Version 1.0.86:** resolved Issue 0530: **solidOfRevolution** (extension)
  - description: add graphicsData for solid of revolution
  - date resolved: **2021-01-06 00:31**, date raised: 2021-01-04
- **Version 1.0.85:** resolved Issue 0536: **GraphicsDataPlane** (extension)
  - description: add graphicsData for simple rectangular plane with option for checkerboard pattern
  - date resolved: **2021-01-05 22:57**, date raised: 2021-01-05
- **Version 1.0.84:** resolved Issue 0535: **alternating color for cylinder** (extension)
  - description: add alternatingColor argument in GraphicsDataCylinder for visualization of rotation of cylindrical bodies
  - date resolved: **2021-01-05 21:46**, date raised: 2021-01-05
- **Version 1.0.83:** **resolved Issue 0529: add MainSystem to userFunctions** (change)

- description: add MainSystem "mbs" to all user functions as first argument (WARNING: this changes ALL user functions!!!)
  - date resolved: **2021-01-05 14:31**, date raised: 2021-01-04
- **Version 1.0.82:** resolved Issue 0447: **test examples** (check)
  - description: test all examples with new index types
  - date resolved: **2021-01-05 14:31**, date raised: 2020-09-09
- **Version 1.0.81:** **resolved BUG 0534: PlotSensor**
  - description: PlotSensor crashes for Load sensors because no outputVariableType exists
  - **notes: added special treatment for load sensors**
  - date resolved: **2021-01-04 20:11**, date raised: 2021-01-04
- **Version 1.0.80:** resolved Issue 0527: **faces transparent** (extension)
  - description: add general transparency flag for faces in visualizationSettings.opengl, switchable with button "T"; allows to make node/marker/object numbers visible
  - date resolved: **2021-01-03 21:53**, date raised: 2021-01-03
- **Version 1.0.79:** resolved Issue 0509: **ComputeODE2Eigenvalues** (test)
  - description: add example in TestModels
  - date resolved: **2021-01-03 10:44**, date raised: 2020-12-18
- **Version 1.0.78:** resolved Issue 0528: **textured fonts** (extension)
  - description: use TEXTURED based bitmap fonts based stored in glLists, allowing better interpolation, scalability (currently up to font size 64 without quality drop) and much higher performance
  - date resolved: **2021-01-03 10:29**, date raised: 2021-01-03
- **Version 1.0.77:** **resolved BUG 0526: solver.ComputeODE2Eigenvalues**
  - description: dense mode returned unsorted eigenvalues==>add sorting
  - date resolved: **2021-01-03 10:21**, date raised: 2021-01-03
- **Version 1.0.76:** resolved Issue 0524: **interpret UTF8** (change)
  - description: add conversion from UTF8 to unicode to interpret most central European characters + some important characters correctly (see [Section 9.3](#))
  - date resolved: **2021-01-02 20:13**, date raised: 2020-12-29
- **Version 1.0.75:** resolved Issue 0525: **opengl write UTF8** (extension)
  - description: use UTF8 encoding in opengl text output
  - date resolved: **2020-12-29 21:00**, date raised: 2020-12-29
- **Version 1.0.74:** resolved Issue 0523: **show version** (extension)
  - description: show current version info in openGL window; can be switched off with showComputationInfo=False
  - date resolved: **2020-12-27 01:33**, date raised: 2020-12-27
- **Version 1.0.73:** resolved Issue 0522: **openGl issues** (fix)
  - description: fix positioning problems of coordinate system and contour colorbar
  - **notes: now using pixel coordinates for info texts and different font sizes**
  - date resolved: **2020-12-27 01:31**, date raised: 2020-12-27
- **Version 1.0.72:** resolved Issue 0521: **useWindowsMonitorScaleFactor** (extension)
  - description: add new option useWindowsMonitorScaleFactor in visualizationSettings.general to include monitor scaling factor for font sizes
  - date resolved: **2020-12-27 01:25**, date raised: 2020-12-27
- **Version 1.0.71:** resolved Issue 0520: **useBitmapText** (extension)
  - description: add new option useBitmapText in visualizationSettings.general to activate bitmap fonts (deprecated; now using textured fonts)
  - date resolved: **2020-12-27 01:25**, date raised: 2020-12-27

- **Version 1.0.70:** resolved Issue 0516: **add bitmap font** (extension)
  - description: add font using OpenGL bitmaps to improve visibility of texts (deprecated, now using textured fonts)
  - date resolved: **2020-12-27 01:23**, date raised: 2020-12-21
- **Version 1.0.69:** resolved Issue 0519: **correct coordinateSystemSize** (change)
  - description: set visualizationSettings.general.coordinateSystemSize relative to fontSize which scales better with larger screens
  - date resolved: **2020-12-24 01:25**, date raised: 2020-12-24
- **Version 1.0.68:** resolved Issue 0518: **windows monitor scaling** (extension)
  - description: include windows monitor (screen) scaling into drawing of texts to increase visibility on high dpi screens
  - **notes: added flag in visualizationSettings: general.useWindowsMonitorScaleFactor**
  - date resolved: **2020-12-24 00:22**, date raised: 2020-12-24
- **Version 1.0.67:** resolved Issue 0511: **GeneticOptimization** (test)
  - description: add example in TestModels
  - date resolved: **2020-12-19 23:31**, date raised: 2020-12-19
- **Version 1.0.66:** resolved Issue 0510: **ParameterVariation** (test)
  - description: add example in TestModels
  - date resolved: **2020-12-19 23:31**, date raised: 2020-12-19
- **Version 1.0.65:** resolved Issue 0502: **rigidbodyinertia** (docu)
  - description: add description for rigidBodyUtilities class RigidBodyInertia
  - date resolved: **2020-12-19 23:28**, date raised: 2020-12-14
- **Version 1.0.64:** **resolved BUG 0512: testsuite**
  - description: EXUDYN build date referred shows wrong path
  - **notes: refer now to installed module**
  - date resolved: **2020-12-19 00:44**, date raised: 2020-12-19
- **Version 1.0.63:** resolved Issue 0507: **changes** (extension)
  - description: incorporate resolved issues and bugs into theDoc.pdf
  - date resolved: **2020-12-17**, date raised: 2020-12-16
- **Version 1.0.62:** resolved Issue 0505: **rigidBodyUtilities** (extension)
  - description: add description for RigidBodyInertia class
  - date resolved: **2020-12-17**, date raised: 2020-12-16
- **Version 1.0.61:** resolved Issue 0501: **geneticOptimization add crossover** (extension)
  - description: added crossover and improved parameters for GeneticOptimization
  - date resolved: **2020-12-14**, date raised: 2020-12-14
- **Version 1.0.60:** resolved Issue 0497: **genetic algorithm** (check)
  - description: check if stochsearch or genetic algorithm has simpler interface
  - date resolved: **2020-12-14**, date raised: 2020-12-10
- **Version 1.0.59:** resolved Issue 0500: **FilterSignal** (extension)
  - description: put in signal module, make it working for 1D signals as well
  - date resolved: **2020-12-11**, date raised: 2020-12-10
- **Version 1.0.58:** resolved Issue 0492: **FEMinterface GetNodesInOrthoCube** (extension)
  - description: add function which returns all nodes lying in cube aligned with global coordinate system, using [pMin, pMax], with tolerance
  - date resolved: **2020-12-11**, date raised: 2020-12-08
- **Version 1.0.57:** resolved Issue 0491: **FEMinterface GetNodesOnCylinder** (extension)

- description: add function which returns all nodes lying on specific cylinder, with tolerance
  - date resolved: **2020-12-11**, date raised: 2020-12-08
- **Version 1.0.56: resolved BUG 0499: key V gives error**
  - description: keypress v for visualization dialog gives error
  - date resolved: **2020-12-10**, date raised: 2020-12-10
- **Version 1.0.55: resolved BUG 0498: SensorObject position**
  - description: wrong position shown in sensor
  - date resolved: **2020-12-10**, date raised: 2020-12-10
- **Version 1.0.54: resolved Issue 0484: test DEAP (test)**
  - description: test genetic optimization with DEAP
  - **notes: too many parameters and too involved to simply include**
  - date resolved: **2020-12-10**, date raised: 2020-12-04
- **Version 1.0.53: resolved BUG 0493: CheckForValidFunction**
  - description: modify / add this check to setParameters; additional if for setting this to 0
  - date resolved: **2020-12-09**, date raised: 2020-12-09
- **Version 1.0.52: resolved BUG 0490: keypress crash**
  - description: find out causes for crash in keyPress user function; find way to deactivate the user function (set it to 0)
  - date resolved: **2020-12-09**, date raised: 2020-12-07
- **Version 1.0.51: resolved Issue 0389: MainSystem includes (cleanup)**
  - description: put SystemIntegrity item checks into separate file, to reduce includig MainSystem into every .cpp item file
  - date resolved: **2020-12-09**, date raised: 2020-05-13
- **Version 1.0.50: resolved Issue 0357: solver flag prolong solution (extension)**
  - description: add flag for solvers that current state at end of computation is set as initial state for next solving
  - **notes: added into new python interface of solver**
  - date resolved: **2020-12-09**, date raised: 2020-03-11
- **Version 1.0.49: resolved Issue 0489: add gradient background (extension)**
  - description: add according visualization.general option
  - date resolved: **2020-12-06**, date raised: 2020-12-06
- **Version 1.0.48: resolved BUG 0488: problem with coordinate sys**
  - description: fix problems with drawing of coordinate system: text moves strangely and axes dissappear after rotation
  - date resolved: **2020-12-06**, date raised: 2020-12-06
- **Version 1.0.47: resolved Issue 0487: draw world basis (extension)**
  - description: add option to draw coordinate system at origin (world basis)
  - date resolved: **2020-12-06**, date raised: 2020-12-06
- **Version 1.0.46: resolved Issue 0486: realtime (extension)**
  - description: add flag to time integration to simulate in realtime
  - date resolved: **2020-12-05**, date raised: 2020-12-05
- **Version 1.0.45: resolved Issue 0485: mouse coordinates (extension)**
  - description: store mouse coordinates in renderState
  - date resolved: **2020-12-05**, date raised: 2020-12-05
- **Version 1.0.44: resolved Issue 0467: mouse coordinates (extension)**
  - description: show mouse coordinates in render window (without transformation)

- date resolved: **2020-12-05**, date raised: 2020-11-19
- **Version 1.0.43:** resolved Issue 0325: **key callback** (extension)
  - description: add key callback function into graphics module to enable interactive settings, etc.; transfer latin letters, SHIFT, CTRL, ALT, 0-9,A-Z,,SPACE as ASCII code
  - date resolved: **2020-12-05**, date raised: 2020-01-26
- **Version 1.0.42:** resolved Issue 0460: **test accelerations** (test)
  - description: test GetODE2Coordinates\_tt, nodal accelerations and rigidbody2D/3D accelerations
  - date resolved: **2020-12-04**, date raised: 2020-11-12
- **Version 1.0.41:** resolved Issue 0482: **store model view** (extension)
  - description: store renderState in exudyn.sys dictionary after exu.StopRenderer() for subsequent simulations
  - date resolved: **2020-12-03**, date raised: 2020-12-03
- **Version 1.0.40:** resolved Issue 0478: **link examples** (docu)
  - description: automatically add links to examples in thedoc
  - date resolved: **2020-12-03**, date raised: 2020-12-02
- **Version 1.0.39:** resolved Issue 0477: **links in theDoc** (extension)
  - description: add links between user functions, add labels to item sections
  - date resolved: **2020-12-03**, date raised: 2020-12-02
- **Version 1.0.38:** resolved Issue 0463: **accelerations** (extension)
  - description: add accelerations Outputvariable to Super elements
  - date resolved: **2020-12-03**, date raised: 2020-11-18
- **Version 1.0.37:** resolved Issue 0481: **eigenvalue solver** (extension)
  - description: add eigenvalue computation interface for mbs in python
  - date resolved: **2020-12-02**, date raised: 2020-12-02
- **Version 1.0.36:** resolved Issue 0480: **python solver** (extension)
  - description: add solver interfaces in python for MainSolverStatic and MainSolverImplicitSecondOrder, helping to retrieve solver data and to make solvers accessible for users
  - date resolved: **2020-12-02**, date raised: 2020-12-02
- **Version 1.0.35:** resolved Issue 0479: **solver return** (extension)
  - description: add return value to solvers and copy solver structures to mbs.sys variables after finishing
  - **notes:** added python interfaces and kept old cpp solvers
  - date resolved: **2020-12-02**, date raised: 2020-12-02
- **Version 1.0.34:** resolved Issue 0469: **userfunctions** (extension)
  - description: put user function generation in objectdefinition, with separate U userfunction flag - this will automatically document the user function parameters (AND return values); this improves documentation and adds a unique interface in C++ using exception handling as well as GIL handling
  - date resolved: **2020-12-02**, date raised: 2020-11-20
- **Version 1.0.33:** resolved Issue 0458: **graphicsDataUserFunction** (docu)
  - description: add example to docu in ObjectGround and GenericODE2 and add more accurate docu to ALL python user functions
  - date resolved: **2020-12-02**, date raised: 2020-11-10
- **Version 1.0.32:** resolved Issue 0428: **queue user functions** (extension)
  - description: implement drawing user functions as global function similar to PyProcessQueue, in order to avoid messing up the CSys and visualization modules
  - date resolved: **2020-12-02**, date raised: 2020-06-26
- **Version 1.0.31:** resolved Issue 0476: **add RequireVersion** (extension)

- description: functionality to allow to add a simple check to see if the installed version meets the requirements
  - date resolved: **2020-11-30**, date raised: 2020-11-30
- **Version 1.0.30:** resolved Issue 0475: **rolling disc ext** (extension)
  - description: add force on ground and moving ground for ObjectJointRollingDisc
  - **notes: needs to be tested further**
  - date resolved: **2020-11-29**, date raised: 2020-11-26
- **Version 1.0.29:** resolved Issue 0474: **auto compilation** (check)
  - description: check automatic compilation; check version in wheels; check linux wheels
  - **notes: linux wheels can not be built with admin rights**
  - date resolved: **2020-11-29**, date raised: 2020-11-25
- **Version 1.0.28:** resolved Issue 0473: **no glfw option** (extension)
  - description: add simple option in setup.py to deactivate glfw both in setup.py as well as in C++ part
  - date resolved: **2020-11-29**, date raised: 2020-11-25
- **Version 1.0.27:** resolved Issue 0457: **GetVersionString** (extension)
  - description: put into docu with pybindings
  - date resolved: **2020-11-29**, date raised: 2020-11-07
- **Version 1.0.26:** resolved Issue 0472: **examples in utilities** (extension)
  - description: activate lstlisting for examples
  - date resolved: **2020-11-25**, date raised: 2020-11-25
- **Version 1.0.25:** resolved Issue 0468: **test WSL2** (test)
  - description: test compilation on WSL2 - Windows subsystem for Linux
  - **notes: WSL2 now used to automatically create linux wheels**
  - date resolved: **2020-11-21**, date raised: 2020-11-19
- **Version 1.0.24: resolved BUG 0465: SC.GetSystem(..)**
  - description: raises RuntimeError: should return reference instead of copy
  - date resolved: **2020-11-21**, date raised: 2020-11-18
- **Version 1.0.23:** resolved Issue 0446: **NodeIndex in arrays** (check)
  - description: use additional functionality to enable index type checks also in arrays, e.g., ArrayIndex of node numbers
  - **notes: not needed for now**
  - date resolved: **2020-11-21**, date raised: 2020-09-09
- **Version 1.0.22:** resolved Issue 0383: **pybind11 submodule** (extension)
  - description: used for advanced functions, not necessarily included in exudyn or make other module
  - **notes: not needed for now**
  - date resolved: **2020-11-21**, date raised: 2020-05-06
- **Version 1.0.21:** resolved Issue 0191: **Newton lambda** (check)
  - description: Check whether Newton can be implemented as lambda-function
  - **notes: not needed for now**
  - date resolved: **2020-11-21**, date raised: 2019-06-17
- **Version 1.0.20:** resolved Issue 0466: **main/bin** (change)
  - description: remove main/bin from github and from Tools folder
  - date resolved: **2020-11-19**, date raised: 2020-11-19
- **Version 1.0.19:** resolved Issue 0464: **processing module** (extension)
  - description: create processing module for parameter variation and optimization using multiprocessing library
  - date resolved: **2020-11-18**, date raised: 2020-11-18
- **Version 1.0.18:** resolved Issue 0462: **AVX Celeron problems** (docu)

- description: add info to documentation - FAQ AND common problems and installation instructions that CPUs without AVX support only work with 32bit version
  - date resolved: **2020-11-18**, date raised: 2020-11-16
- **Version 1.0.17:** resolved Issue 0459: **lie group utilities** (extension)
  - description: add documented lie group utilities to exudyn (python) lib
  - date resolved: **2020-11-11**, date raised: 2020-11-11 (resolved by: Stefan Holzinger)
- **Version 1.0.16:** resolved Issue 0454: **add item graph** (extension)
  - description: add graph containing nodes, objects, etc.
  - date resolved: **2020-10-08**, date raised: 2020-10-08
- **Version 1.0.15:** resolved Issue 0453: **systemdata.NumberOfSensors** (extension)
  - description: add access function for systemdata.NumberOfSensors()
  - date resolved: **2020-10-08**, date raised: 2020-10-08
- **Version 1.0.14:** resolved Issue 0449: **MT ngsolve** (extension)
  - description: add NGsolve multithreading library (task manager)
  - **notes: first tests made**
  - date resolved: **2020-09-16**, date raised: 2020-09-15
- **Version 1.0.13:** resolved Issue 0330: **correct ODE2RHS** (change)
  - description: correct ODE2RHS to ODE2Terms in objects because it is left-hand-side
  - **notes: changed object computation function from RHS to LHS, as it always computed the LHS (the system.cpp function ComputeODE2RHS then puts it to RHS)**
  - date resolved: **2020-09-10**, date raised: 2020-02-03
- **Version 1.0.12:** resolved Issue 0435: **check runtimeError** (check)
  - description: check if exception runtimeerror works for all catch cases (test in windows?)
  - date resolved: **2020-09-09**, date raised: 2020-07-21
- **Version 1.0.11:** resolved Issue 0445: **remove GetItemByName()** (change)
  - description: remove GetNodeByName, GetObjectByName, etc. from C++ interface; already disabled in python interface before
  - date resolved: **2020-09-08**, date raised: 2020-09-08
- **Version 1.0.10:** resolved Issue 0288: **Item::CallFunction** (change)
  - description: Disable Item::CallFunction functionality from EXUDYN; either outputvariables can be used, or some functions are automatically created including the documentation
  - **notes: already removed from python interface earlier**
  - date resolved: **2020-09-08**, date raised: 2019-12-10
- **Version 1.0.9:** resolved Issue 0443: **SensorObject** (warning)
  - description: add error message, if sensorobject is used for a body (and check if SensorBody excepts object other than body)
  - **notes: added test for SensorObject if attached to body**
  - date resolved: **2020-09-04**, date raised: 2020-09-03
- **Version 1.0.8: resolved BUG 0442: difference MSC and setuptools**
  - description: compilation with MSC and setuptools gives different results
  - **notes: problem with VS2019 compilation of Eigen; resolved by removing VS2019 installation**
  - date resolved: **2020-08-25**, date raised: 2020-08-24
- **Version 1.0.7:** resolved Issue 0431: **auto create dirs** (extension)
  - description: automatically create dictionaries if they do not exist
  - date resolved: **2020-08-25**, date raised: 2020-07-01
- **Version 1.0.6:** resolved Issue 0439: **setuptools** (extension)

- description: use setuptools for installation
  - date resolved: **2020-08-17**, date raised: 2020-08-13
- **Version 1.0.5:** resolved Issue 0381: **test pybind11\_2020** (test)
  - description: downloaded in Download folder
  - date resolved: **2020-08-17**, date raised: 2020-05-06
- **Version 1.0.4:** resolved Issue 0378: **setup tools** (extension)
  - description: use setup tools to install EXUDYN on local user accounts; use installed python version to decide which version to install
  - date resolved: **2020-08-17**, date raised: 2020-05-04
- **Version 1.0.3:** resolved Issue 0441: **remove WorkingRelease path** (change)
  - description: do not include WorkingRelease to sys.path any more, but require installation of modules
  - date resolved: **2020-08-14**, date raised: 2020-08-14
- **Version 1.0.2:** resolved Issue 0440: **exudyn package** (extension)
  - description: make a package with sub .py files in exudyn package - requires renaming of C++ module
  - date resolved: **2020-08-14**, date raised: 2020-08-13
- **Version 1.0.1:** resolved Issue 0438: **UBUNTU** (extension)
  - description: adapt setup.py and implementation for gcc and UBUNTU
  - date resolved: **2020-08-13**, date raised: 2020-08-13
- **Version 1.0.0:** **resolved BUG 0434: CheckSystemIntegrity**
  - description: gives wrong node, marker, etc. numbers for some checks
  - date resolved: **2020-07-20**, date raised: 2020-07-20

## 12.2 Known open bugs

- open **BUG 0631: ObjectFFReducedOrder**
  - description: freefree eigenmodes and Hurty-Craig-Bampton modes do not converge to same results
  - date raised: 2021-04-23
- open **BUG 0456: ObjectFFRF bug with GenericJoint**
  - description: raises error: CSolverBase::SolveSteps CObjectSuperElement::GetAccessFunctionSuperElement: AngularVelocity\_qt not implemented; cannot compute jacobian for orientation
  - date raised: 2020-10-13
- open **BUG 0448: ObjectGenericODE2 bug**
  - description: ObjectGenericODE2 crashes without message when initialized with invalid node numbers
  - date raised: 2020-09-09



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# Chapter 13

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