MULTI-PRODUCT NONLINEAR PRICING WITH QUANTITY CAPS

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Abstract

I investigate the impacts of limiting the maximum quantity of one product in a two-goods market with adverse selection when the goods are either unrelated or related. An experiment with independent goods complements the study. The main goal is to explore the effects on consumer rents and consumption of both items. Following moderate and severe caps, a standard nonlinear pricing model predicts an increase in surplus for buyers with low preference for the capped good and high valuation for the unregulated product; a reduction in rents earned by the highest type, and no effect on the rents of the rest. These outcomes hold with both independent and related goods. In general, the impact on quantities is mixed and contingent on the model's parameters. Under regulation and with independent goods, the seller bunches some types and serves them with the same option. With related components, the seller moves to separate buyers who have been previously served together. The experimental data largely corroborates the hypotheses. These results are applicable to industries where limits on quantities are a regulatory alternative and impacts on consumer surplus is an important consideration.

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1 Introduction

I present an analysis on the effects of quantity caps (caps) in a two-product market with four types of privately-informed buyers. Caps are policies restricting the default maximum quantities at which products can be offered. I rely on a standard nonlinear pricing model to investigate the outcomes following a cap with both independent and related (e.g. complement in consumption) products. I supplement the theoretical analysis with an experiment testing the model's predictions with independent goods. I aim to learn i) whether the intervention reduces consumption of the target good, ii) the impact on purchased quantity of the unregulated product, and iii) the effect on consumer surplus defined as the gross utility from consumption net of the price paid. This paper highlights how changes in consumer surplus and allocation following a cap are ultimately contingent on the way the seller adapts her screening scheme to accommodate the policy.

In the model, I refer to the goods in the market as A and B. With independent goods, the seller offers "small-small", "medium-large", "large-medium", and "large-large" A-B. This results in weakly increasing rents across buyer types. Assuming that good A is subject to a cap, limiting any option of A to be strictly lower than the "medium" unregulated portion, results in lower quantities of A consumed by all buyer types, and reductions in the quantity of good B for some buyer types. The seller moves from serving a specific option for each type, to offer three alternatives one of which covers two buyers at once. Regarding consumer surplus, the highest type is negatively impacted, the buyer with low preference for A and high valuation for B is better off, while the surplus earned by the rest of consumers remains unchanged.

With related goods and no intervention, the seller offers large, medium and small packages with equal proportions of A and B, bunching buyer types with mixed preferences (those with high valuation for one component and low for the other). Moderate caps on A result in a reduction of consumption of A for most buyers except for the lowest type. Naturally, a severe cap reduces consumption of good A for all buyers. The impacts on B with a severe cap are contingent on the values of the parameters. The seller changes her screening scheme from bunching buyers with mixed preferences to offer tailored alternatives for each buyer type. Interestingly, the effects on the distribution of consumer rents is the same as with independent goods: one buyer type is benefited by the limit, one loses and the rest remain unaffected.

In the experiment, I incorporate three stylized observations. First, buyers have private information regarding preferences and these are taken as exogenous by the seller. It is fair to assume that in most markets taste can be considered exogenous and that sellers design incentive-compatible menus before any transaction occurs. Second, the seller offers more than one product. This reflects what is observed in the field, where most retailers are either multi-product vendors or offer one product which itself is composed by more than one component. Lastly, the seller decides the quantities and prices that characterize each package in the menu. In other words, she does not adopt a passive pricing scheme. Following a restriction in quantities, there is no reason to assume that the seller will not try to endogenously modify the menu to accommodate the intervention in ways that will impact how seller and buyers divide gains from trade. In the experiment, I allow for flexible contract design. Instead of fixing the number of contracts a given seller can offer thereby limiting their tasks to merely specifying quantities and prices, my subjects taking the role of sellers are allowed to choose the number of bundles they want to offer, their mix of quantities, and their prices. This is consistent with how sellers are assumed to behave in standard screening models. I chose to test the predictions with independent goods to better highlight how surplus distribution and allocation in a market with adverse selection can change due to the multidimensional nature of the incentive constraints faced by the seller, even without consumption interactions between the products.

The results I delineate in this paper are informative for industries where second degree price discrimination is prevalent, where quantity caps are a regulatory alternative available to policy makers, and where the effect of the intervention on consumer welfare is a primary consideration. Quantity limits as presented in this document are analogous to a number of policies in the field, these include food portion cap rules, limits on ingredients per serving, credence labeling, and rules of origin in international trade treaties. I briefly discuss three examples.

First, consider the so-called "New York City soda ban" originally proposed to have taken effect on 2013. The advanced plan intended to prohibit food vendors regulated by the city of New York from selling sugar-sweetened beverages (SSBs) in containers exceeding 16 ounces (Kansagra, 2012).[1] Ultimately, the measure was struck down in court. Nonetheless, the proposal sparked a vibrant public discussion around its potential ills and virtues. Most debates departed from the premise that a cap would unavoidably hurt consumers.[2] The implication is that caps necessarily lower consumer welfare. This assumption is already shaping public policy. Mississippi's Bill 2687 (2013) interdicts against future restrictions of

[1]As a reference, the "small", "medium", and "large" cup sizes typically found in popular American fast-food restaurants contain around 16, 21, and 32 ounces.

[2]Notes in the media where this premise is present to some degree abound. These are two examples Grynbaum (2012); and Grynbaum and Connelly (2012).

food sales based upon the product's nutrition information or upon its bundling with other items. I argue that because sellers implement sophisticated pricing and bundling policies, even if a regulation modifies consumption of the target product, it does not necessarily follow that consumers are worse-off. Moreover, the effects on the marketing of non-targeted products often sold in tandem with the regulated good (e.g. fries or burgers in the case of SSBs) remain largely unknown. My work can illuminate some of the consequences of such measures even though the model does not allow for mixed bundling. The insights would be applicable in settings where a majority of the products are sold in combos and "value meals", such as in quick service restaurants.

In light of studies linking larger portion sizes to increased consumption, foods (other than SSBs) judged to have deleterious impacts on human health are natural targets of potential caps (Young and Nestle (2002), Ledikwe et al. (2005), Rolls et al. (2006), and Flood et al. (2006)). If we consider foods to be bundles of ingredients, potential regulations limiting the amount of a specific ingredient to be contained in one serving could be analyzed with help of insights from this paper.[3] One example of such a regulation on ingredients usage is the "New York City trans fat ban". After a phasing out stage spanning from 2006 to 2008, restaurants holding a New York City Health Department permit were not allowed to store, use or serve foods containing partially hydrogenated vegetable oils, shortenings, or margarines with 0.5 grams or more trans fat per serving. This regulation can be thought of as a cap on the maximum amount of trans fat per serving. The bi-dimensional model

[3]Continuing with the SSBs example, one can imagine a regulation aiming to limit the maximum amount of sugar to be contained in a single container. One could consider a soda manufacturer deciding sugar-water (call them products A and B) combinations. In this example, the reader can interpret A as "sugar", and B as "all other ingredients". The "package" or bundle is a cup of soda with a particular sugar-water ratio, and the cap would regulate the concentration of sugar within portions.

in this paper can inform us about the effects of rules such as this if we consider trans fats as the regulated good, and group all ingredients other than trans fat as single composite unregulated product.

The final example is that of food labeling referring to, for instance, country of origin and organic status of the main ingredients. Typically, a producer must satisfy rules regarding the proportion of ingredients satisfying a government-defined standard.[4] This is akin to quantity cap on ingredients that do not follow the requirements. The work in this paper can illuminate the effects of such mandates if the sellers desire to exhibit the label in their products and were to strictly follow the requirements.

2 Relevant literature

The design of incentive-compatible menus by sellers aiming to segment demand is a well documented phenomenon. The single-product case has received more attention than the multidimensional scenario, (See for example Myerson (1979), Maskin and Riley (1984), Mussa and Rosen (1978), Armstrong and Vickers (1993), and Sonderegger (2011)). Perhaps because multidimensional nonlinear pricing is notorious for being a source of research problems easy to state but difficult to solve analytically or otherwise, the literature on the topic has remained relatively small. However, the body of knowledge on multi-product screening continuous to grow and remains a promising field of study. Knowledge in this area has experienced an impressive expansion from its relatively modest beginnings studying highly

[4]Take the Canadian example. Only food products with 95% or more content satisfying their organic standards may carry the Canadian Organic Label. Likewise, a food producer may claim "Product of Canada" for her goods when, at most, 2% of the ingredients are non-Canadian.

stylized instances, and the necessary conditions for screening and bundling to be profitable (Adams and Yellen (1976), McAfee et al. (1989), Armstrong (1999)) to the finding of general results (e.g. Carroll (2017)), albeit in most cases theoretical results remain highly sensitive to assumptions regarding the choice of key parameters. [5].

The progress in this field has provided researchers with a valuable toolkit to study the effects of regulating price-discriminating firms. In a theoretical study, Besanko et al. (1988) explore the effects of three regulatory measures intending to fix the quantity distortion characteristic of nonlinear pricing:[6] minimum quality standards, maximum price regulation, and rate of return regulation. Besanko and co-authors derive conditions under which the rate of return regulation lowers quantity for the high-types; they also demonstrate that maximum price interventions lower quantity for the high-types, although minimum quality standards do not modify the quantity consumed by the buyers with high valuation for the goods. Corts (1995) analytically studies the effect of imposing a price-cap on the lower level of quantity offered by a multi-product monopolist. Corts relies on a multidimensional version of the Spence-Mirrlees single crossing condition to analyze the multidimensional problem with a one-dimensional screening model. He finds mixed results regarding prices paid by different buyer types. In a numerical example where the multi-item single-crossing assumption is relaxed, Corts show how socially sub-optimal un-bundling may arise as consequence of the intervention. Amrstong et al. (1995) consider two forms of regulations: a cap on the seller's average revenue, and a constraint that forces the seller to keep offering the option to buy a component at the uniform price. Armstrong and co-authors show that the average revenue

[5] For discussions about of the literature on design of multi-product pricing and bundling and its evolution, I direct the reader to Armstrong (1996), and Armstrong (2016)

[6] Price-discriminating firms distort quantity downward along the type space.

constraint is preferred by the seller.

The study of quantity caps in multi-product markets is conspicuously absent from the literature. The case of quantity limits in a single-product market is studied by Bourquard and Wu (2019). This article analytically studies the impacts of cap rules with single-product sellers trading with privately-informed heterogeneous buyers. The authors report that a portion cap reduces consumption without affecting consumer surplus. The reason is that as the cap limits quantity, the seller adjusts prices accordingly so as to leave consumer rents unaffected. My intention is to investigate the degree to which this finding holds in a more general case, and additionally, to observe the indirect effects on buyer bunching and on consumption of goods not directly targeted by the cap.

I contribute to the growing body of experimental literature addressing questions in multi-commodity settings. Caliskan et al. (2007) and Hinloopen et al. (2014) are largely concerned with evaluating outcomes from the leverage theory of product bundling. In this article, I am concerned with learning about pricing strategies of a regulated multi-product monopolist with presence in a single market, thus my research speaks to a different, although closely related, literature: multi-dimensional screening. One experimental article evaluating screening theory is Hoppe and Schmitz (2015), which tests the canonical adverse selection model wherein a seller makes a contract to try to separate a privately informed buyer who has preferences over a low and a high quality item.

More directly related to the topic of regulating food vendors, Wilson et al. (2013) conduct a behavioral study. They aim to determine how a limit on sugary drink portions might affect consumption patterns. The authors put to the consideration of human subjects a hypothetical menu of options, and the subjects were asked to choose how much food they would like to consume. The authors contrast consumption choices made under two types of menus: a baseline menu where the vendor offers soda cups without any regulation, and an active group where the seller replaces large cups (say of 32oz) with smaller containers (say of 16oz). Their main finding is that buyers decide to purchase more soda with the regulated menu featuring the cap rule. This study is useful because it provides an insight regarding potential framing effects that could alter subjects' purchase decisions. My study complements this work in two dimensions. First, I concentrate on the seller's side of the story. A complete explanation of the consequences of an intervention ought to include analyses of reactions from buyers and sellers. Second, my experiment ties monetary rewards to subjects' performance. I reward subjects for taking actions that would make the hypothetical market player they are playing for better off.

3 Theory

The model is largely based on Armstrong and Rochet (1999), although simplified to facilitate experimental implementation and retain ease of interpretation of the analytical results. This section is divided in four subsections. The first two present the baseline and regulated scenarios when the products are independent (i.e. neither complements or substitutes). The next two subsections correspond to the baseline and restricted cases with complementarity. The characterization of the optimal solutions can be found in the appendix. In each subsection, I present succinct discussions of the optimal solutions.

3.1 Baseline with independent goods

The seller (she) is a monopolist offering goods A and B in contracts $\{q^A,\ q^B,\ p\}$, where p is the price charged for a package containing q^A and q^B units of the respective products A and B. The ij-type buyer (he) has private preferences i for good A, and j for B. For each item, he can either have a high (H) or a low (L) preference. There are four buyer types denoted HH, HL, LH, and LL. The ij-type buyer is characterized by the vector of taste parameters $(\theta_i^A,\ \theta_j^B)$ for $i,\ j =$ H, L. I assume $\theta_H > \theta_L$. If the ij-type buyer pays price p_{ij} for a package containing quantities q_{ij}^A and q_{ij}^B, he earns the following in surplus:

$$R_{ij} = \theta_i^A u(q_{ij}^A) + \theta_j^B u(q_{ij}^B) - p_{ij} \tag{1}$$

The subscripts i and j under R, q^A, q^B, and p indicate the type of consumer. At the moment, I assume away interactions between the components. I present the model with independent goods first to highlight the interactions between the multidimensional incentive constraints and the seller's pricing behavior across policy scenarios. This assumption provides a neutral background where changes across treatments can confidently be attributed to the impact of quantity restrictions on pricing behavior without the confounding effects of complementarity. Thus, I will show that a cap changes allocation and consumer surplus *even in the absence of complementarity*. I relax this assumption later.

I assume the utility function $u(\cdot)$ to be continuous, also $u(0) = 0$, $u'(q) > 0$ and $u''(q) < 0$. The seller and the buyers have reservation values of zero. I assume both goods to have the same differentiable, increasing and convex cost function $c(\cdot)$. Also, $\theta_H u'(q) > c'(q)$ and $\lim_{q\to\infty}\theta_H u'(q) < c'(q)$ for both products, so that trade is possible at least with the HH-type, and total quantity supplied is finite. $\sum_{ij} \beta_{ij} = 1$, so β_{ij} represents the probability that a given buyer is of an ij-type. The seller's expected profit is $\mathbb{E}[\pi] = \sum_{ij} \beta_{ij} \Big[p_{ij} - c(q^A_{ij}) - c(q^B_{ij})\Big]$. It is useful to represent expected profit in terms of total and consumer surpluses:

$$\mathbb{E}[\pi] = \underbrace{\sum_{ij} \beta_{ij}[\theta^A_i u(q^A_{ij}) + \theta^B_j u(q^B_{ij}) - c(q^A_{ij}) - c(q^B_{ij})]}_{\text{Expected total surplus}} - \underbrace{\sum_{ij} \beta_{ij}[\theta^A_i u(q^A_{ij}) + \theta^B_j u(q^B_{ij}) - p_{ij}]}_{\text{Expected consumer surplus}} \quad (2)$$

The seller considers one set of participation constraints (PC), and another of incentive-compatibility restrictions (IC). Satisfaction of PC implies that all types are at least indifferent between buying and opting out from trade. Their general form is PC: $R_{ij} \geq 0 \; \forall \; ij$.

The IC restrictions are self-selection conditions providing incentives for the ij-type not to purchase a package originally intended to serve a kl-type buyer ($i \neq k$, and $j \neq l$). In other words, at the optimum, quantities and prices are such that the ij-type buyer is weakly better-off by choosing contract $\{q^A_{ij}, q^B_{ij}, p_{ij}\}$ over an alternative $\{q^A_{kl}, q^B_{kl}, p_{kl}\}$. The seller designs a menu of options such that the ij-type receives positive surplus in the form of a temptation payoff known as information rents. These are exactly equal to the extraordinary benefit the ij-type would have gained had he chosen the alternative intended for the kl-type from a menu with linear prices. The IC constraints take the following form:

$$\text{IC: } R_{ij} \geq R_{kl} + \underbrace{u(q^A_{kl})(\theta^A_i - \theta^A_k) + u(q^B_{kl})(\theta^B_j - \theta^B_l)}_{\text{Rent gained by the } ij\text{-type from posing as a } kl\text{-type}} \quad \forall \; ij \text{ and } kl; i \neq k \text{ and } j \neq l \qquad (3)$$

The complete optimization program includes 8 PC and 12 IC restrictions. The seller's goal is to design options $\{q^A_{ij}, q^B_{ij}, p_{ij}\}$ to maximize expected profit (2) subject to PC and IC constraints. The resulting pricing mechanism is incentive-compatible if it satisfies monotonicity conditions: $q^A_{HH} \geq q^A_{LH}$, $q^A_{HL} \geq q^A_{LL}$, $q^B_{HH} \geq q^B_{HL}$, and $q^B_{LH} \geq q^B_{LL}$. These state that the quantity of either good is weakly increasing with the corresponding valuation.

In a "relaxed" version of the problem, only a subset of the constraints are included. The seller ignores the possibility of lower types misrepresenting their preferences. In this case, only the "downward" incentive restrictions are incorporated.[7] Similarly, as long as the PC restriction for the LL-type is satisfied, all types' own PCs are also met. In the appendix, I show that the solution to the simplified problem is the solution to the fully constrained program. The relevant IC constraints are graphically depicted in panel (a) of Figure 1, where solid lines denote equations binding with equality, and dashed lines indicate restrictions that may or may not bind with strict equality depending on the problem's parameters, especially the distribution of types and taste dispersion ($\Delta \equiv \theta_H - \theta_L$).

[Figure 1 about here.]

Critically, the information rents for each buyer type R_{ij} depend on the combination of active constraints. Each set of binding IC constraints correspond to a particular *Incentive Compatibility structure* (IC structure, or ICS). With independent goods, there are four IC structures (call them ICS A, B, C and D). In each of them the rents earned by the LL, LH, and HL types are the following:

[7] The seller does not consider the possibility of the HL-type choosing the packages intended for either the LH-type or the LL-types and, similarly, she does not have to worry about the LH-type buyer choosing the option designed to serve the HL and/or the HH-type.

$$
\begin{aligned}
&R_{HL} = \Delta^A u(q_{LL}^A) \\
&R_{LH} = \Delta^B u(q_{LL}^B) \\
&R_{LL} = 0
\end{aligned}
\tag{4}
$$

Where $\Delta^g \equiv \theta_H^g - \theta_L^g$ for $g = A,\ B$. Regarding the form of the HH-type's IC constraint R_{HH}, it is different in each ICS:

$$
\begin{aligned}
&\text{ICS A: } R_{HH} = \Delta^A u(q_{LH}^A) + \Delta^B u(q_{HL}^B) \\
&\text{ICS B: } R_{HH} = \Delta^A u(q_{LH}^A) + \Delta^B u(q_{LL}^B) \\
&\text{ICS C: } R_{HH} = \Delta^A u(q_{LL}^A) + \Delta^B u(q_{HL}^B) \\
&\text{ICS D: } R_{HH} = \Delta^A u(q_{LL}^A) + \Delta^B u(q_{LL}^B)
\end{aligned}
\tag{5}
$$

Although each ICS implies solutions with different point predictions regarding allocation and consumer surplus distribution, some patterns are consistent. Without regulation, consumption increases with type; bundling is observed in the form of larger sizes of product i when preference for good j rises (more on this later); and Consumer rents weakly increase with preferences, with the LL-type earning no rents, the HH-type gaining the largest surplus, and the medium HL and LH-types earning rents in between. Panel A of Figures 2 and 3 are helpful illustrations of the resulting allocation and surplus distribution. [8] Naturally, the effects of a quantity limit are contingent on the IC structure characterizing the regulation-free market. Before continuing with an explanation about what this implies for the consequences

[8] I omit scale labels along the vertical axis of both figures because the specific values of these variables depend on the chosen parameters. For some parameter combinations, the LL-type is excluded from participation, and rents for the LL, LH, and HL types are null. These examples better represent the symmetric case ($\theta_H^A \equiv \theta_H^B$, $\theta_L^A \equiv \theta_L^B$, and $\beta_{HL} = \beta_{LH}$). The essence of the result is the same for non-symmetric cases.

of limiting the quantity of one the goods, I dedicate a few paragraphs to the phenomenon of commodity bundling.

In this document, the seller practices pure bundling: both products are offered together as components of a single package. The products are said to be bundled if the variance in price across different packages is not entirely explained by differences in marginal cost of production. Particularly, if at the solution the quantity of product i increases with the preference for good j, the menu of options is said to feature bundling i.e. when $q^A_{LL} < q^A_{LH}$, and/or $q^A_{HL} < q^A_{HH}$, and/or $q^B_{LL} < q^B_{HL}$, and/or $q^B_{LH} < q^B_{HH}$. Roughly speaking, each product is offered in "large", "medium", and "small" options.

The assumption of pure bundling comes with advantages. First, it reduces the complexity of the multidimensional pricing model and facilitates interpretation. Easiness of interpretation is a scarce attribute in models of multidimensional screening, and every opportunity to facilitate the understanding of outcomes is highly valuable. Second, it simplifies experimental implementation and makes it easy for subjects to understand their task. Lastly, the assumption can help to illuminate the effects of caps on pricing practices in products we typically do not think of as bundles. For example, a single food product can be thought as the bundle of different ingredients, a vehicle can be thought as a bundle of components from different origins, and an organic food product is a bundle of organic and non-organic ingredients.

The decision to bundle is highly sensitive to the distribution of buyer types. A given distribution of types is conducive to bundling when the correlation of preferences, $\rho = \beta_{HH}\beta_{LL} - \beta_{HL}\beta_{LH}$, is weak enough (Armstrong and Rochet (1999), McAfee et al. (1989), and Adams and Yellen (1976)). Bundling is profitable as long as $\rho < \frac{\beta_{HL}\beta_{LH}}{\beta_{LL}}$. I present results for the case when $\rho < 0 < \frac{\beta^2}{\beta_{LL}}$; this is when the incentive to bundle is the strongest. I take this decision for two reasons. First, I am interested in learning the consequences of adopting caps in markets where bundling is rampant, thus I select the conditions that provide sellers with the strongest incentive to bundle. Second, the pricing strategy followed by a multi-product vendor that does not bundle is identical to single-product nonlinear pricing applied separately to each good, and this case has already been studied (Bourquard and Wu, 2019).

3.2 Quantity cap with independent goods

Without loss of generality, suppose that the quantity limit applies to good A. With the enacted regulation, bundles cannot have quantities of A larger than $\bar{q}$. The seller's objective is to maximize expected profit subject to participation and incentive-compatibility constraints, plus the following quantity cap (QC) restriction:

$$\text{QC:} \;\; q_{ij}^{A} \leq \bar{q} \;\text{ for } i,\, j = L,\, H \tag{6}$$

I analyze three levels of severity at which the cap can be set. The limit can be 1) mild if the limit is set below the "largest" quantity available without regulation but above the quantity of the "medium" unregulated alternative; 2) moderate if the cap is set below the "medium" unregulated option of good A, but also over the quantity contained in the "smallest" regulation-free alternative, or 3) severe if the limit on quantity is set at a level lower than the "small" alternative without the cap.

I present the First Order Conditions characterizing the solution for each level of severity in the appendix. It is important to notice that a mild cap does not modify the set of binding constraints, and the seller offers as much of A as she is allowed for the HH and HL types. The moderate restriction lies below the size offered to the LH type but above the option serving the lowest type in the regulation-free baseline. The moderate and severe limits modify the set of binding IC constraints. Under moderate and severe regulations, the size of product A shrinks so much that the seller is no longer able to separate the HH and LH types. She therefore defaults to offer a single option to serve both of them. This "bunching" of types occurs regardless of the original regulation-free ICS. Panel (c) in Figure 1 shows the set of active incentive constraints under moderate and severe restrictions. As the quantity of product A becomes smaller due to more restrictive caps, the seller has to be mindful of the possibility of the LH-type misrepresenting himself as an HL-type and increase the temptation payoff accordingly. With a moderate or severe cap, the incentive constraint preventing unfaithful representation of the LH-type buyer as an HL-type is binding. This can be confirmed using equation 3. This modification renders the downward incentive constraints involving the HH-type redundant.

The effects of the caps on the quantities of both products are shown in table 1. There are two salient effects on the quantities of the regulated good. First, a mild cap affects exclusively the largest options. Second, a moderate restriction not only directly reduces the large and medium options, as intended, but also indirectly impacts the small alternative.

Because so far interactions between the products are absent, it is surprising to find that the model suggests changes in the quantity of the *unregulated* good B. Moderate and severe caps affect the *unregulated* product. When passing from regulation-free ICS A or C to a moderate or severe cap, not only the corresponding quantities of good A are reduced, but also q^B_{HL} and q^B_{LL}. When passing from regulation-free ICS B or D to a moderate or severe cap, q^B_{HL} drops. To facilitate the interpretation of the results, panel (b) in figure 2 shows the allocation resulting from a moderate cap for the symmetric case.

The effect on consumer rents is also interesting. Only moderate and severe caps modify the buyers' rents. Only the HH and LH buyer types are affected. Let wide tildes $(\widetilde{\cdot})$ denote solutions under a moderate restrictions, and wide hats $(\widehat{\cdot})$ denote solutions under severe caps. Recall that without regulation, the HH-type's rents can take different values depending on the IC structure. Under moderate and severe caps, this is no longer the case. With moderate and severe caps, the HH-type buyer earns the same rents regardless of the original "regulation-free" ICS. The rents earned by the HH-type are negatively affected as long as the regulation free rents from the corresponding ICS in 5 are strictly less than the following rents under regulation:

$$\begin{aligned} &\text{Moderate cap}: \widetilde{R_{HH}} = \Delta^A u(\widetilde{q^A_{LL}}) + \Delta^B u(\widetilde{q^B_{LL}}) + \Delta^B u(\widetilde{q^B_{HL}}) \\ &\text{Severe cap}: \widehat{R_{HH}} = \Delta^A u(\bar{q}) + \Delta^B u(\widehat{q^B_{LL}}) + \Delta^B u(\widehat{q^B_{HL}}) \end{aligned} \tag{7}$$

The necessary conditions for R_{HH} to be lower under moderate and severe caps compared to the baseline are the following. Again, hats and tildes correspond to the regulated outcomes, the unmarked variables denote solutions under no intervention.

ICS A:

Moderate cap: $\Delta^A u(q^A_{LH}) + \Delta^B u(q^B_{HL}) > \Delta^A u(\widetilde{q^A_{LL}}) + \Delta^B u(\widetilde{q^B_{LL}}) + \Delta^B u(\widetilde{q^B_{HL}})$

Severe cap: $\Delta^A u(q^A_{LH}) + \Delta^B u(q^B_{HL}) > \Delta^A u(\bar{q}) + \Delta^B u(\widehat{q^B_{LL}}) + \Delta^B u(\widehat{q^B_{HL}})$

ICS B:

Moderate cap: $\Delta^A u(q^A_{LH}) + \Delta^B u(q^B_{LL}) > \Delta^A u(\widetilde{q^A_{LL}}) + \Delta^B u(\widetilde{q^B_{LL}}) + \Delta^B u(\widetilde{q^B_{HL}})$

Severe cap: $\Delta^A u(q^A_{LH}) + \Delta^B u(q^B_{LL}) > \Delta^A u(\bar{q}) + \Delta^B u(\widehat{q^B_{LL}}) + \Delta^B u(\widehat{q^B_{HL}})$

ICS C:

Moderate cap: $\Delta^A u(q^A_{LL}) + \Delta^B u(q^B_{HL}) > \Delta^A u(\widetilde{q^A_{LL}}) + \Delta^B u(\widetilde{q^B_{LL}}) + \Delta^B u(\widetilde{q^B_{HL}})$

Severe cap: $\Delta^A u(q^A_{LL}) + \Delta^B u(q^B_{HL}) > \Delta^A u(\bar{q}) + \Delta^B u(\widehat{q^B_{LL}}) + \Delta^B u(\widehat{q^B_{HL}})$

ICS D:

Moderate cap: $\Delta^A u(q^A_{LL}) + \Delta^B u(q^B_{LL}) > \Delta^A u(\widetilde{q^A_{LL}}) + \Delta^B u(\widetilde{q^B_{LL}}) + \Delta^B u(\widetilde{q^B_{HL}})$

Severe cap: $\Delta^A u(q^A_{LL}) + \Delta^B u(q^B_{LL}) > \Delta^A u(\bar{q}) + \Delta^B u(\widehat{q^B_{LL}}) + \Delta^B u(\widehat{q^B_{HL}})$

Because, in general, the quantity variables involved in the inequalities above tend be smaller under regulation than without a cap, the HH-type buyer tends to be harmed by the regulation.

The other buyer type impacted by moderate and severe caps is the LH-type. This buyer is benefited by the cap. As shown by the equations in 4, the LH-type earns $\Delta^B u(q^B_{LL})$ without regulation (and with a mild cap) regardless of the original ICS. With moderate and severe caps, the LH-type rents change in the following manner:

$$\begin{aligned} &\textbf{Moderate cap: } \widetilde{R_{LH}} = \Delta^A[u(\widetilde{q^A_{LL}}) - u(\bar{q})] + \Delta^B u(\widetilde{q^B_{LL}}) + \Delta^B u(\widetilde{q^B_{HL}}) \\ &\textbf{Severe cap: } \widehat{R_{LH}} = \Delta^B u(\widehat{q^B_{LL}}) + \Delta^B u(\widehat{q^B_{HL}}) \end{aligned} \tag{8}$$

Notice that $\widehat{R_{LH}} > \widetilde{R_{LH}}$ because $\Delta^A[u(\widetilde{q^A_{LL}}) - u(\bar{q})] < 0$. To better illustrate the effects of the caps on consumer rents, consider panel (b) in figure 3. In general, once a moderate (or severe) cap is adopted, we can expect R_{HH} to drop, and R_{LH} to increase. However, R_{HH} will remain slightly higher than R_{LH} even with a severe cap, as it can be corroborated by comparing the equations in 7 and 8.

In sum, a severe enough cap on good A will directly reduce the quantity of A consumed by the HH and HL types; indirectly (directly) diminish the size of A served to the LL-type when the cap is moderate (severe); indirectly decrease consumption of B by the HL type regardless of the original ICS, and indirectly diminish the consumption of B by the LL-type for ICS A and C. Likewise, the model predicts a negative effect on the rents earned by the HH-type and a positive effect on the information rents of the LH-type.

A brief discussion on the mechanism driving the model's outcomes may proof beneficial. Once the cap is implemented, the seller's desire to price-discriminate continues. The restriction merely reduces her choice space. To accommodate the policy while continuing to segment the demand, the seller has to modify all of the endogenous variables to her disposal, including the quantities of product B.

According to the model, the HL-type buyer is offered less of product B when the cap is moderate or severe. The LL buyer might receives less of B depending on the original ICS.

I first discuss the adjustments made to the small package. In essence, these are driven by the LL-type's participation constraint and the need to provide positive rents to the LH-type to purchase his own package. Without regulation, information rents for the LH-type are driven by a larger quantity of A compared to the level received by the LL-type buyer. With a moderate cap on A, the LH-type (as well as the HL, and HH types) consumes less of the regulated good A. However, the profit-maximizing seller still needs to provide positive information rents to the LH-type in order to make sure that this buyer will not purchase the small combo designed to serve the LL buyer. Because there is a limit on A, the only way the seller can increase the difference in quantity of A offered to the LL and LH types is by decreasing the quantity of A served to the LL-type buyer. Thus, the LL ought to receive less product A. To maintain the satisfaction of the LL-type's participation constraint, the seller modifies the quantity of B served to this type.

I now turn to discuss the modifications in the package sold to the HL-buyer. These are explained by changes in the smallest package (served to the LL-type consumer), and the fact that the need to separate the HL from the LL-type remains, but the incentives need not be as strong under regulation. Due to the cap, the seller is unable to offer the first best quantity of A to the HL-type buyer. Indeed the HL buyer purchases considerably less compared to the baseline. The seller still needs to provide incentives to the HL-type in the form of a larger portion of B compared to the LL package. Because the quantity of A contained in the smallest package (that serving the LL-type) is low and indeed smaller compared to the baseline unregulated case, the extra amount of B granted to the HL-type consumer to generate information rents need not be as large.

Regarding the impacts on buyers' surplus, the model predicts a reduction in the rents granted to the HH-type (R_{HH}) and an *increase* in the surplus earned by the LH-type (R_{LH}). The reason behind the reduction of the HH-type's surplus is straightforward. The HH-type buyer is worse-off because he is receiving significantly less of a product he values highly and the reduction in price is not large enough to compensate for the diminished size of the package. The intuition behind the increase in the LH-type's well-being is the following. In the unregulated baseline, the LH-type is purchasing a "medium" portion A for which he has a *low* preference. This buyer would prefer a price-discounted "small-large" A-B package; the closest option for them in the unregulated baseline is a price-discounted "medium-large" combo; the "small-small" alternative has too little of product B, whereas the "large-large" package is just too expensive for this buyer. A quantity limit on good A shapes the set of contracts such that the package designed by the seller to serve the buyers with low-high valuation, is closer this buyers' ideal contract.

3.3 Baseline with related goods

In this subsection, I explore the more general case of related goods (complement and/or substitute). The most important modification is related to the taste parameters θ. For complements (superscript C), the following holds: ($\theta^C_{HH} > \theta^A_H + \theta^B_H$), ($\theta^C_{HL} > \theta^A_H + \theta^B_L$), ($\theta^C_{LH} > \theta^A_L + \theta^B_H$), and ($\theta^C_{LL} > \theta^A_L + \theta^B_L$). Additionally, $\theta^C_{HH} > \theta^C_{HL}$, $\theta^C_{HH} > \theta^C_{LH}$, $\theta^C_{HH} > \theta^C_{LL}$, $\theta^C_{HL} > \theta^C_{LL}$, and $\theta^C_{LH} > \theta^C_{LL}$. For substitutes (superscript S), the following holds: ($\theta^S_{HH} < \theta^A_H + \theta^B_H$), ($\theta^S_{HL} < \theta^A_H + \theta^B_L$), ($\theta^S_{LH} < \theta^A_L + \theta^B_H$), and ($\theta^S_{LL} < \theta^A_L + \theta^B_L$). Additionally, $\theta^S_{HH} > \theta^S_{HL}$, $\theta^S_{HH} > \theta^S_{LH}$, $\theta^S_{HH} > \theta^S_{LL}$, $\theta^S_{HL} > \theta^S_{LL}$, and $\theta^S_{LH} > \theta^S_{LL}$.

Because the analysis below looks identical for both complements and substitutes, I drop the superscript. Throughout the text, I use complements and related goods as interchangeable terms. The conclusions hold for substitute products, keeping in mind that the inequalities just described must hold. The seller's expected profit is the following:

$$\mathbb{E}[\pi] = \underbrace{\sum_{ij} \beta_{ij}[\theta_{ij}u(q^A_{ij}) + \theta_{ij}u(q^B_{ij}) - c(q^A_{ij}) - c(q^B_{ij})]}_{\text{Expected total surplus}} - \underbrace{\sum_{ij} \beta_{ij}[\theta_{ij}u(q^A_{ij}) + \theta_{ij}u(q^B_{ij}) - p_{ij}]}_{\text{Expected consumer surplus}} \tag{9}$$

The general form of the PC constraints remains PC: $R_{ij} \geq 0 \; \forall \; ij$. The IC constraints take the following form:

$$\text{IC: } R_{ij} \geq R_{kl} + \underbrace{u(q^A_{kl})(\theta_{ij} - \theta_{kl}) + u(q^B_{kl})(\theta_{ij} - \theta_{kl})}_{\text{Rent gained by the } ij\text{-type from posing as a } kl\text{-type}} \quad \forall \; ij \text{ and } kl; i \neq k \text{ and } j \neq l \tag{10}$$

The following definitions will be useful:

$$\theta_{HH} - \theta_{LL} \equiv \Delta_1, \quad \theta_{HH} - \theta_{HL} \equiv \Delta_2, \quad \theta_{HH} - \theta_{LH} \equiv \Delta_3,$$
$$\theta_{HL} - \theta_{LL} \equiv \Delta_4, \quad \theta_{LH} - \theta_{LL} \equiv \Delta_5, \quad \theta_{LH} - \theta_{HL} \equiv \Delta_6,$$
$$\theta_{HL} - \theta_{LH} \equiv \Delta_7$$

Only the downward IC constraints are incorporated to the maximization problem, just as in the case with unrelated products. The set of relevant incentive constraints is illustrated in panel (b) of figure 1. As with independent goods, there are four possible IC structures with complement goods. I will also refer to these as ICS A, B, C, and D.

The set of first order conditions characterizing the solution to the seller's problem with related goods is shown in the appendix. Recall that in this model the goods are said to be bundled if the portion of item i increases with good j. With complement goods, the seller does not bundle the products. Instead, the seller offers three packages with either only "large", only "medium" or only "small" portions of both goods each.

Moving on to consumer surplus, without cap, the HL and LL buyer types receive the same rents independently of the original ICS:

$$\begin{aligned} &R_{HL} = \Delta_4[u(q^A_{LL}) + u(q^B_{LL})] \\ &R_{LL} = 0 \end{aligned} \tag{11}$$

The rents earned by the LH-type in the regulation-free baseline vary in the following way:

$$\begin{aligned} &\text{ICS A, B, and C: } R_{LH} = \Delta_5[u(q^A_{LL}) + u(q^B_{LL})] \\ &\text{ICS D: } R_{LH} = (\Delta_5 + \Delta_4)[u(q^A_{LL}) + u(q^B_{LL})] + \Delta_6[u(q^A_{HL}) + u(q^B_{HL})] \end{aligned} \tag{12}$$

The information rents received by the HH-type depends on the IC structure as follows:

$$\begin{aligned} &\text{ICS A: } R_{HH} = (\Delta_4 + \Delta_5)[u(q^A_{LL}) + u(q^B_{LL})] + \Delta_3[u(q^A_{LH}) + u(q^B_{LH})] + \Delta_2[u(q^A_{HL}) + u(q^B_{HL})] \\ &\text{ICS B: } R_{HH} = (\Delta_4 + \Delta_5)[u(q^A_{LL}) + u(q^B_{LL})] + \Delta_3[u(q^A_{LH}) + u(q^B_{LH})] + (\Delta_6 + \Delta_2)[u(q^A_{HL}) + u(q^B_{HL})] \\ &\text{ICS C: } R_{HH} = (\Delta_1 + \Delta_4 + \Delta_5)[u(q^A_{LL}) + u(q^B_{LL})] + \Delta_3[u(q^A_{LH}) + u(q^B_{LH})] + \Delta_1[u(q^A_{HL}) + u(q^B_{HL})] \\ &\text{ICS D: } R_{HH} = (\Delta_1 + \Delta_4 + \Delta_5)[u(q^A_{LL}) + u(q^B_{LL})] + \Delta_3[u(q^A_{LH}) + u(q^B_{LH})] + (\Delta_6 + \Delta_2)[u(q^A_{HL}) + u(q^B_{HL})] \end{aligned} \tag{13}$$

Illustrations of the resulting allocation and surplus distributions are depicted in panels (c) and (a) of figures 2 and 3, correspondingly. In short, only three contracts are offered:

one with large quantities of both products, a second package with medium quantities of the two goods, and a third option with small sizes of both items. Regarding consumer surplus; the HH-type receives the most information rents; the LL-type the less, and between these types, the HL and LH buyers receive the same level of surplus.

3.4 Quantity cap with related goods

I continue to present analysis on the same three levels of cap severity on product A as I did with independent goods. Recall that the mild cap directly limits only the large portion of A, the moderate regulation directly affects both medium and large portions of A, while the severe limit directly affects all options of A. Thus, the seller's goal is to maximize expected profit 9 subject to the relevant IC and PC restrictions plus the quantity cap in 6.

Table 2 shows the effect of the cap for each severity level comparing the resulting quantities to the quantities allocated to each type under no regulation. The first order conditions characterizing the solutions are in the appendix. Just as in the case with independent goods, the mild cap does not modify the quantities of the products beyond the largest alternatives.

Moderate and severe caps have more nuanced effects on quantities. I first discuss the effects on product A. By design, a moderate cap directly reduces the large and medium portions of good A. The moderate cap also indirectly affects the quantity of A contained in the small package. When under no regulation, the market is characterized by IC structure A, B, or C a moderate cap can either reduce or increase the quantity of A contained in the smallest package, depending on the value of the model's parameters as shown in the relevant footnote in table 2. For ICS D, where R_{HH} takes the form of last equation in 13, a moderate restriction unambiguously increases the portion of A contained in the small option.

Moving to the impacts on the unregulated good B, with regulation-free ICS A, the quantity of good B served to the HL type unambiguously decreases; with ICS D, this type is served a larger portion of B; while with IC structures B and C the effect is ambiguous and might decrease contingent on particular parameter values. The quantity of product B consumed by buyer type LH increases regardless of the original ICS. While the portion served to buyer type LL unambiguously increases for ICS D, and may decrease for the rest of IC structures depending on parameter values. To better illustrate the outcomes, panel (d) in figure 2 shows a possible outcome from imposing a moderate cap on a market defined by ICS A.

The HL and LL buyer types do not see their information rents affected. The equations defining the information rents granted to the HH and LH types are modified under moderate and severe caps in the following way. Again, wide tildes and hats denote solutions under moderate and severe caps, respectively:

$$
\begin{aligned}
&\textbf{Mderate cap:}\\
&\widetilde{R_{HH}} = (\Delta_4 + \Delta_1)[u(\widetilde{q^A_{LL}}) + u(\widetilde{q^B_{LL}})] + \Delta_2[u(\bar{q}) + u(\widetilde{q^B_{HL}})]\\
&\widetilde{R_{LH}} = (\Delta_5 + \Delta_4)[u(\widetilde{q^A_{LL}}) + u(\widetilde{q^B_{LL}})] + \Delta_6[u(\bar{q}) + u(\widetilde{q^B_{HL}})]\\
&\textbf{Severe cap:}\\
&\widehat{R_{HH}} = (\Delta_4 + \Delta_1)[u(\bar{q}) + u(\widehat{q^B_{LL}})] + \Delta_2[u(\bar{q}) + u(\widehat{q^B_{HL}})]\\
&\widehat{R_{LH}} = (\Delta_5 + \Delta_4)[u(\bar{q}) + u(\widehat{q^B_{LL}})] + \Delta_6[u(\bar{q}) + u(\widehat{q^B_{HL}})]
\end{aligned}
\tag{14}
$$

Comparing the relevant equations from 12 and 13 I can infer the impact on consumer surplus. The HH-type sees his rents reduced by moderate and severe caps. The effect on the information rents granted to the LH-type are larger under regulation-free ICS A, B and C. For ICS D, the effect on LH is likely to be positive even though the corresponding equations in 12 and 13 are very similar. The reason is that even though $q^A_{HL} > \bar{q}$, the other quantities are larger under regulation, which is likely to more than compensate, rendering $\widetilde{R_{LH}} > R_{LH}$ and $\widehat{R_{LH}} > R_{LH}$. Panel (b) in figure 3 depicts a possible outcome from imposing a moderate cap on ICS A.

For an interpretation of the results observed with related goods, consider seller of SSBs deciding sugar-"water" combinations (possible A-B products, where "water" is a composite goods that includes ingredients such as flavoring). Suppose that the context is such that we observe allocations resembling panels (c) and (d) in figure 2. The "package" is a bottle of soda with a particular sugar-water ratio. In this context, the baseline outcomes can be interpreted as follows. Without regulation, the seller decides to produce bottles of soda in three different presentations: small, medium, and large servings all with a one-to-one sugar-water ratio. If the government enacts a limit on the maximum amount grams of sugar contained in a single serving, the seller would accommodate the policy by offering the following four choices. First, a "light" large alternative with low sugar-water ratio serving the HH-type (who, after all, also highly values the ingredients other than sugar contained in the beverage); second, a relatively small option with a concentrated formula with a high sugar-water ratio designed for the HL-type's sweet taste; third, a smaller "light" alternative serving the health-conscious LH-type; and lastly, a mini serving of the "traditional" formula targeting the LL-type buyer.

4 Experimental design

In this section, I present an experiment conducted to test the model's predictions regarding a moderate cap on a market with independent goods characterized, under no regulation, by IC structure A. Before discussing the experimental design, I list the set of hypotheses to evaluate.

Hypothesis *1. Consumption of A.* All buyer types reduce their consumption of the regulated product A.

Hypothesis *2. Consumption of B.* i) The HH and LH-type buyers do not modify their consumption of the unregulated product B; ii) the HL and LL-type purchase less of B.

Hypothesis *3. Consumer rents.* i) the LH-type receives more consumer surplus; ii) the HH-type buyer earns a smaller consumer rent, and iii) the LL and HL-type's consumer surpluses remain unaffected.

In table 3, I show the parameters I use in the experiment. Other than to have been chosen to provide incentives to bundle, the selected distribution of buyer types is fairly generic, its properties are not particular and can be considered to be fairly representative of other probability-combinations with negative correlation. Because it is symmetric (the probability of the buyer being a LL-type is the same with the probability of being an HH-type, and similarly for the HL and LH types) and it can be expressed with probabilities with only one decimal, this distribution simplifies the experimental instructions.

[Table 3 about here]

In total, 82 subjects were randomly assigned to one of two experimental treatments:

Baseline or *Cap* named after their policy environment. There were three sessions per treatment with 12 to 14 subjects each. Payoff functions and the ranges of choice variables given to the subjects can be seen in table 4. Subjects were recruited via ORSEE (Greiner, 2015). The experimental interface was designed with oTree (Chen et al., 2016). No subject participated in more than one session.

[Table 4 about here]

In all sessions, subjects play "trading periods" in which the seller submits a menu of choices and the buyer makes consumption decisions. There are 6 training periods with no financial consequences to familiarize subjects with the interface and game structure. Following the training phase, each subject plays 11 paying effective periods. Every menu of choices submitted and the corresponding purchase decision constitute an observation in my database. Excluding training periods, the final database contains 902 observations, 440 from the *Baseline* group and 462 from the *Cap* treatment. All subjects are assigned to the role of a seller and did not interact with any other human subject in the room. A computer program mimics the choices of a rational consumer whose type was randomly and independently assigned before each trading period. Throughout the experiment, earnings are denominated in points. Final earnings are converted into cash at the exchange rate was 31 points per US Dollar following protocol I describe below. All sessions had the same structure: first, subjects answered a pre-experimental quiz; second, there were six "training" non-paying trading periods; then, eleven "effective" trading rounds were played; lastly, subjects answered a post-experimental survey.

The game in each trading period closely mirrors the screening problem I described in the previous sections. At the beginning of each trading round, the seller chooses to offer a number of packages from one to four, or not to offer any package at all. Next, the seller specifies quantities and prices. Thus, the seller is designing a menu consisting of up to four packages, each with three arguments: quantity of A, quantity of B, and price. Following the design of the menu, the offer is submitted to the computerized buyer for consideration. The buyer can purchase only one package per period. The buyer chooses the package that maximizes his payoff, but rejects the entire menu if all packages resulted in earnings lower than the reservation value of zero. If more than one packages results in the same non-negative earnings for the buyer, then the first of these packages (in the order they were submitted by the seller) is chosen. The seller and buyer payoffs in points are determined using the purchased package, if any. If no menu is submitted or if the buyer rejects the entire menu, both parties receive zero points. At the end of each trading period, the seller is shown the terms of the menu she offered, the choice made my the seller and her period earnings in points. They can take notes in a paper earning tracking sheet provided by the experimenter. Subjects also have access to a calculator during the menu-design phase of the trading periods. With this calculator, they can experiment with different quantities-price combinations and learn how these would translate into profit, cost of production, and consumer surplus per buyer type.

The sum of points earned in four out of the eleven effective trading periods determined the final experimental earnings for the subjects. These were randomly chosen via the following protocol. Labeled from 1 to 330, the experimenter had a list with all possible combinations of four periods. A computer application that randomly chooses numbers between 1 to 330, all equally likely is activated before the experiment started. The application was activated three time. The number that appeared the third time represented the label of the selected combination of paying periods. The selected paying combination was shown to each subject after they finished with all of their tasks. If the sum earnings of the four randomly selected periods was negative, the earnings of the subject was set to zero. This protocol was detailed in the experimental instructions.

5 Experimental results

I first offer an overview of the the data. If these follow general patterns consistent with the model predictions, I can be reasonably convinced that my experimental design appropriately captures the essence of the theory, and that subjects understood the instructions. The theory predicts that, without regulation, sellers engage in bundling when facing privately informed buyers where the distribution of types is negatively correlated. If I take all of the menus with one or more packages submitted during the baseline treatment, order the packages within a menu by the sum of their quantities (from the package containing more units of bot products to the option with less units of the goods), and average across menus, the result is figure 4, where panel (a) and (b) correspond to the control and the capped treatments, respectively. Remember that two goods are said to be bundled if the quantity of product j increases with preference for component i. This is graphically confirmed to occur in panel (a) of figure 4, assuming that the smaller, and second smaller packages target LL and LH types, and the largest and the second largest target HH and HL types, respectively. This is a crude approximation to the sellers' pricing scheme in the sense that it is not immediately obvious which of the two "medium" packages (the options between the smallest and the largest) would be consumed by either the HL or the LH type. Moreover, it ignores the possibility that some sellers engaging in bunching (serving more than one type with a single package), and exclusion. However, it is not one of my objectives to formally test the theory of multidimensional screening. Therefore, I consider the pattern of offered quantities shown in the figure to be sufficient evidence of sellers in both treatments behaving as expected.

[Figure 4 about here]

I now turn to the way in which the characteristics of the menus evolved across periods. Evidence of learning during the experiment would provide the reader with a high degree of confidence on the data because it would indicate that the subjects not only understood the instructions, but they took non-random decisions and increased their pricing accuracy as the experiment progressed. To elicit price discrimination, subjects were informed that they were going to be matched with a single buyer each trading round but the type of the buyer would change across periods according to a known vector of probabilities. From the submitted menus, I can infer which packages would each type of buyer would have purchased had he been presented with the submitted menu. These packages and their associated payoffs are the data I use to test hypotheses during the rest of this document. In table 5 I show average price and quantities of the packages purchased by each buyer type in both treatments. In both treatments, price and quantities are larger in later periods.

The evolution of both prices and quantities would be evidence of a greater degree of pricing sophistication if buyers' information rents are lower and seller's per-period payoffs are larger in the later periods of the experiment. Table 6 shows that this is generally the case. As the experiment progresses, subjects seem to learn to more precisely price their packages and extract more surplus from the buyers as a result.

[Tables 5 and 6 about here]

5.1 Main experimental results

For all menus of contracts that subjects submitted during the trading periods, I compute which package each type of buyer would have purchased; how much they would have paid; the seller's expected profit; the information rents for all buyers, and the associated experimental payoffs in points. I use these quantities in the estimations below.

I start by looking at the impacts on quantity purchased by type of buyer. In table 7, I show econometric estimates of the cap's impact on quantities purchased by each buyer type. I find significant reductions in consumption of A by all buyer types. I do not find statistically significant evidence of a change in consumption of product B by any of the consumer types. These are the main two findings regarding impacts on consumption.

***Main Result** 1. In accordance with hypothesis 1, compared to the unregulated baseline, all consumers reduce their consumption of product A.*

***Main Result** 2. In accordance with hypothesis 2, the cap rule does not impact the quantity of product B purchased by the HH and LH-type buyers. In opposition to hypothesis 2, the HL and LL-type buyers do not reduce their consumption of B.*

Under regulation, all buyer types reduced their consumption of product A. I do not find evidence of a significant change in purchases of product B by any type. I turn to the distributional impacts of the cap shown in table 8. For completeness, I show the effect of the regulation on expected profit. I expect the LL and HL types to reain unaffected, LH-type to be benefited by the cap and the HH-type to be worse off.

Main Result *3. The reduction in consumer surplus earned by the HH-type buyers is not statistically significant either. In alignment with hypothesis 3 the LH-type buyer earns a larger surplus. The HL and LL-type's surpluses remain unchanged.*

As predicted, the LH-type is better off after the cap. This buyer is no longer pressed to buy more of the product he has a low valuation for in order to get the large portion of the good he values the most. The cap moves the set of options closer to the ideal for this buyer's preferences. Contrary to the hypotheses, the HH-type buyer is not impacted by the cap. The main reason can be found in table 9. The HH-buyer is buying less of A which he values largely, however he is also paying less for the package he is purchasing, the reduction in price compensates for the reduction in consumption.

[Tables 7, 8 and 9 about here]

To put the empirical results in perspective, it is useful to compare them with the model's predictions. For the LL buyer, the model predicts a reduction in the portion of both goods. As shown in table 7, I do observe a statistically significant reduction in quantity of A purchased by the LL-type buyer (q^A_{LL}), which aligns with the hypothesis. On the other hand, I do not find a statistically significant change in the consumption of good B by this buyer type (q^B_{LL}). As expected, these changes in the mix of quantities consumed by the LL-type result in a null impact on his consumer rents (table 8). Because the price paid by this buyer type remained unaffected across treatments, as can be seen in table 9, the LL-type's rents are held constant.

Recall that the model predicts the HL-type would reduce his consumption of both products when the cap is enacted. The empirical estimates in table 7 suggest that this buyer does reduce his consumption of A (q^A_{HL}), but does not modify his consumption of B (q^B_{HL}). The model predicts a null impact on the information rents earned by this buyer and this what I find in the data (see table 8). Because the cap limits the consumption of the product for which this consumer has a high valuation, in order to keep his consumer surplus unchanged, the seller must decrease the price of the package she offers to him.

The notable result of more surplus earned by the LH-type buyer following a cap is corroborated by the experimental data. Looking at the estimates in tables 7 and 8, I conclude that the increase in surplus is explained by the reduction in the quantity of A acquired by this buyer (the product for which this buyer has a low valuation): the cap moves the choice set closer to this buyer's first best.

The model generated the following hypotheses regarding the HH-type consumer: lower quantity of A q^A_{HH}, no effect on consumed quantity of B q^B_{HH}, and therefore a lower consumer surplus R_{HH}. In the experimental data I find support for the predictions involving quantities (see table 7). Although the observed effect on the surplus earned by this buyer has the expected sign, it is not significant. This is because, during the experiment, this buyer paid lower prices (see table 9) and the reduction is large enough to keep his rents constant.

6 Conclusion

In this paper, I present a theoretical analysis on the effects of quantity caps on payoffs and consumption when sellers offer two products in a pure bundle. I supplement the theoretical study with an experiment designed to evaluate the model's prediction when the goods are unrelated.

According to the nonlinear pricing model, with independent goods, moderate and sever caps do reduce consumption of the target good by all buyer types. Also, depending on the original IC structure, consumption of the non-target good B can decrease as well. With related goods, the effects on quantities are mixed. However, the outcomes on rents are similar. Following the regulation, the LL and HL types are expected to remain unaffected; the HH-type is expected to lose; while the LH-type's rents increase following moderate and severe caps.

The experiment corroborate most of the model's predictions with unrelated goods. The data suggest that a moderate quantity cap would be successful at reducing consumption of the targeted product for all consumer types, with neither increased consumption of the unregulated component nor negative impacts on consumer rents. Indeed, one type of buyer is better-off as a result of the policy, namely the consumer with low valuation for the regulated product A and high preference for good B. The buyer with high-high valuations for the A-B goods are surprisingly not worse-off after the policy, this is because during the experimental sessions, this type of buyer paid lower prices for the packages he purchased, the reduction in per-package price is significant and would have left information rents for this buyer unmodified after the cap. The buyers who benefit have low valuation for the regulated product but high preference for the unregulated goods. Absent a quantity limit, the seller has an incentive to offer information rents in the form of a relatively larger quantity of the product this buyer values lowly. The cap reduces the extent to witch this strategy can be leveraged.

The assumption that quantity caps negatively impact consumer well-being is an important driver of public discourse surrounding food policy and at it is already shaping public policy. I show that these worries are not justified. A cap can increase consumer well-being for some buyers. The benefited buyers have low valuation for the regulated product but high preference for the unregulated goods. Absent a quantity limit, the seller has an incentive to engage in commodity bundling and offer to these buyers information rents in the form of a relatively larger quantity of the product he values lowly. The cap reduces the extent to witch bundling (in the form of larger sizes of the lowly-preferred product) can be leveraged as a sorting device .

Future work can produce formal comparisons between the impacts of quantity limits and other popular measures such as excise taxes. Additionally, both theoretical and experimental work on the effects of caps when the seller can practice mixed bundling are natural extensions.

Tables and figures

Table 1: Theoretical change in quantities: Independent goods

	q^A_{HH}	q^B_{HH}	q^A_{HL}	q^B_{HL}	q^A_{LH}	q^B_{LH}	q^A_{LL}	q^B_{LL}
IC-Structure A								
Mild	↓	=	↓	=	=	=	=	=
Moderate	↓	=	↓	↓	↓	=	↓	↓
Severe	↓	=	↓	↓	↓	=	↓	↓
IC-Structure B								
Mild	↓	=	↓	=	=	=	=	=
Moderate	↓	=	↓	↓	↓	=	↓	=
Severe	↓	=	↓	↓	↓	=	↓	=
IC-Structure C								
Mild	↓	=	↓	=	=	=	=	=
Moderate	↓	=	↓	↓	↓	=	↓	↓
Severe	↓	=	↓	↓	↓	=	↓	↓
IC-Structure D								
Mild	↓	=	↓	=	=	=	=	=
Moderate	↓	=	↓	↓	↓	=	↓	=
Severe	↓	=	↓	↓	↓	=	↓	=

In each case, the comparison is against the baseline scenario.

Table 2: Theoretical change in quantities: Complement and substitute goods

	q^A_{HH}	q^B_{HH}	q^A_{HL}	q^B_{HL}	q^A_{LH}	q^B_{LH}	q^A_{LL}	q^B_{LL}
IC-Structure A								
Mild	$\downarrow$	=	=	=	=	=	=	=
Moderate	$\downarrow$	=	$\downarrow$	$\downarrow$	$\downarrow$	$\uparrow$	$\downarrow^1$	$\downarrow^1$
Severe	$\downarrow$	=	$\downarrow$	$\downarrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\downarrow^1$
IC-Structure B								
Mild	$\downarrow$	=	=	=	=	=	=	=
Moderate	$\downarrow$	=	$\downarrow$	$\downarrow^2$	$\downarrow$	$\uparrow$	$\downarrow^3$	$\downarrow^1$
Severe	$\downarrow$	=	$\downarrow$	$\downarrow^2$	$\downarrow$	$\uparrow$	$\downarrow$	$\downarrow^1$
IC-Structure C								
Mild	$\downarrow$	=	=	=	=	=	=	=
Moderate	$\downarrow$	=	$\downarrow$	$\downarrow^4$	$\downarrow$	$\uparrow$	$\downarrow^5$	$\downarrow^5$
Severe	$\downarrow$	=	$\downarrow$	$\downarrow^4$	$\downarrow$	$\uparrow$	$\downarrow$	$\downarrow^5$
IC-Structure D								
Mild	$\downarrow$	=	=	=	=	=	=	=
Moderate	$\downarrow$	=	$\downarrow$	$\uparrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\uparrow$
Severe	$\downarrow$	=	$\downarrow$	$\uparrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\uparrow$

In each case, the comparison is against the baseline scenario. For arrows with superscript, the effect holds if the following inequalities hold:
[1] $\frac{\beta_{HH}}{\beta_{LH}} < \frac{\Delta_4}{\Delta_5-\Delta_1}$. [2] $\beta_{HH} < \beta_{LH}$. [3] $\Delta_5 < \Delta_1 + \Delta_4$. [4] $\frac{\beta_{HH}}{\beta_{LH}} < \frac{\Delta_6}{\Delta_1-\Delta_2}$. [5] $\frac{\beta_{HH}}{\beta_{LH}} < \frac{\Delta_4}{\Delta_5}$.

Table 3: Parameter values used in this study

Parameter	Value	Description
β_{HH}	0.1	Probability of the buyer being a HH-type
β	0.4	Probability of the buyer being a HL-type
β_{LL}	0.1	Probability of the buyer being a LL-type
θ_H	15	Taste parameter when preference is high
θ_L	10	Taste parameter when preference is low
$\theta_i u(q)$	$\theta_i\sqrt{q}$	Buyer's gross utility
$c(q)$	$q^2/500$	Seller's cost of producing q units of a given good
$\bar{q_A}$	75	Maximum-quantity cap on good A in the cap treatment

The probability of the buyer being an LH-type is also β.

Table 4: Experimental treatments

	Payoffs		Choice variables: ranges		
Treatment	Seller	ij-type buyer	Product A	Product B	Price
Baseline	$p - \frac{(q_A)^2+(q_B)^2}{500}$	$\theta_i\sqrt{q_A} + \theta_j\sqrt{q_B} - p$	$[0,\ldots,250]$	$[0,\ldots,250]$	$[0,\ldots,500]$
Cap	$p - \frac{(q_A)^2+(q_B)^2}{500}$	$\theta_i\sqrt{q_A} + \theta_j\sqrt{q_B} - p$	$[0,\ldots,75]$	$[0,\ldots,250]$	$[0,\ldots,500]$

Table 5: Average paid prices and purchased quantities per buyer type

	Baseline				Cap			
	LL	LH	HL	HH	LL	LH	HL	HH
All periods:								
Mean price	160.14	209.66	211.35	218.80	128.97	180.04	170.19	184.22
Mean q^A	93.27	98.32	117.15	115.87	48.71	48.57	56.59	55.32
Mean q^B	90.58	112.93	97.11	107.98	95.21	120.12	97.03	117.19
First 5 periods:								
Mean price	145.49	197.63	200.18	204.39	121.30	171.55	162.18	177.25
Mean q^A	84.82	92.07	112.38	108.53	37.50	45.76	54.05	52.97
Mean q^B	83.80	107.98	91.26	101.30	88.67	114.05	92.13	112.22
Last 6 periods:								
Mean price	173.66	219.69	220.75	230.57	136.26	187.14	176.79	190.03
Mean q^A	101.06	103.53	121.16	121.87	45.16	51.19	57.28	57.28
Mean q^B	96.83	117.05	102.03	113.45	101.43	125.19	101.07	121.33

Table 6: Average per-period earnings

	Number of observed packages (Baseline)				Number of observed packages (Cap)			
	0	1	2	3	0	1	2	3
All periods:								
#Obs/Total (Share)	4/440 (0.9)	251/440 (57.0)	170/440 (38.6)	15/440 (3.4)	2/462 (0.4)	300/462 (64.9)	121/462 (26.2)	39/462 (8.4)
Mean R_{LL}	0	10.96	8.51	8.01	0	15.69	9.85	5.10
Mean R_{LH}	0	40.25	33.81	36.51	0	52.18	36.68	34.75
Mean R_{HL}	0	41.48	34.04	36.60	0	35.29	27.93	25.47
Mean R_{HH}	0	90.44	79.59	74.73	0	87.13	72.34	62.26
Mean payoff seller	0	142.33	144.51	140.93	0	126.00	135.33	134.41
Mean $\mathbb{E}[\pi]$	0	107.15	110.39	117.62	0	95.93	102.04	117.38
First 5 periods:								
#Obs/Total (Share)	4/200 (2.0)	111/200 (55.5)	76/200 (38.0)	9/200 (4.5)	1/210 (0.5)	133/210 (63.3)	58/210 (27.6)	18/210 (8.6)
Mean R_{LL}	0	16.70	8.10	8.95	0	17.75	12.17	3.24
Mean R_{LH}	0	48.44	33.76	39.25	0	53.60	41.71	34.66
Mean R_{HL}	0	49.52	33.53	39.84	0	36.50	33.22	25.02
Mean R_{HH}	0	96.49	78.13	77.96	0	87.31	76.13	62.23
Mean payoff seller	0	136.68	142.38	134.61	0	117.70	127.62	144.21
Mean $\mathbb{E}[\pi]$	0	102.34	109.46	121.51	0	92.17	105.36	120.87
Last 6 periods:								
#Obs/Total (Share)	0/240 (0.0)	140/240 (58.3)	94/240 (39.1)	6/240 (2.5)	1/252 (0.4)	167/252 (66.3)	63/252 (25.0)	21/252 (8.3)
Mean R_{LL}	0	6.41	8.84	6.61	0	14.05	7.71	6.70
Mean R_{LH}	0	33.76	33.85	32.40	0	51.05	32.04	34.83
Mean R_{HL}	0	35.10	34.45	31.74	0	34.32	23.05	25.85
Mean R_{HH}	0	85.65	80.77	69.88	0	86.99	68.85	62.29
Mean payoff seller	0	146.81	146.23	150.41	0	132.61	142.43	126.01
Mean $\mathbb{E}[\pi]$	0	110.96	111.14	111.78	0	98.92	99.00	114.39

Table 7: Estimates: impact of the quantity cap on per-period quantities purchased per buyer type

	q^A_{HH}	q^B_{HH}	q^A_{HL}	q^B_{HL}	q^A_{LH}	q^B_{LH}	q^A_{LL}	q^B_{LL}
Cap dummy	-60.065***	9.504	-61.061***	0.609	-49.066***	7.587	-44.567***	9.480
	(8.635)	(11.694)	(9.803)	(9.616)	(8.761)	(12.153)	(5.779)	(9.374)
Period	1.593***	1.647***	1.269***	1.670***	1.512***	1.485***	1.940***	1.984***
	(0.341)	(0.494)	(0.228)	(0.355)	(0.464)	(0.427)	(0.444)	(0.656)
Constant	105.752***	97.726***	109.174***	86.587***	88.623***	103.507***	71.523***	70.542***
	(7.841)	(6.929)	(8.510)	(6.474)	(5.725)	(10.393)	(3.824)	(7.203)
Observations	896	896	872	872	890	890	467	467

* P < 0.10, ** P < 0.05, *** P < 0.01. Regressions estimated using multi-level random effects at the session and subject levels. Robust standard errors clustered at the session level in parentheses. Cap dummy takes a value of 1 if the observation belongs to the cap treatment, 0 otherwise.

Table 8: Estimates: impact of the quantity cap on per-period earnings

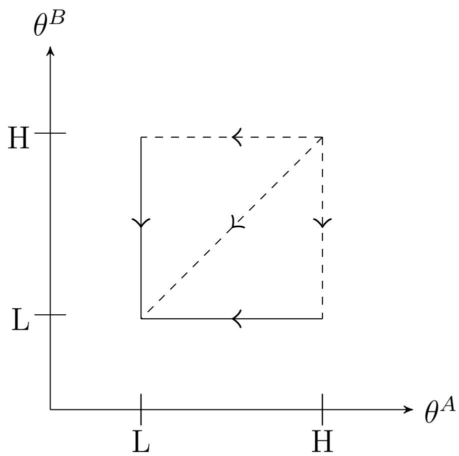
	Seller's earning		Buyers' earnings			
	$\mathbb{E}[\pi]$	Observed profit	R_{HH}	R_{HL}	R_{LH}	R_{LL}
Cap dummy	-8.975	-13.382	-4.109	-5.683	9.151**	3.388
	(8.765)	(12.179)	(3.903)	(4.946)	(4.227)	(3.548)
Period	0.992**	2.108***	-0.509	-0.966***	-1.071***	-0.796***
	(0.387)	(0.358)	(0.406)	(0.357)	(0.344)	(0.214)
Constant	101.952***	129.345***	87.950***	43.866***	43.703***	14.592***
	(8.437)	(10.601)	(5.338)	(6.214)	(5.534)	(4.275)
Observations	902	902	902	902	902	902

* P < 0.10, ** P < 0.05, *** P < 0.01. Regressions estimated using multi-level random effects at the session and subject levels. Robust standard errors clustered at the session level in parentheses. Cap dummy takes a value of 1 if the observation belongs to the cap treatment, 0 otherwise.

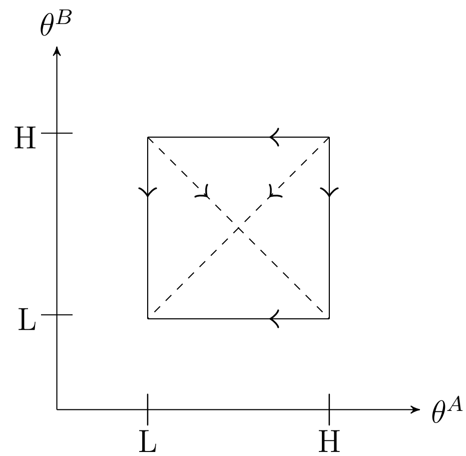
Table 9: Estimates: impact of the quantity cap on per-period prices

	p_{HH}	p_{HL}	p_{LH}	p_{LL}
Cap dummy	-34.163*	-41.644**	-28.745	-18.345
	(19.122)	(19.860)	(18.661)	(15.219)
Period	3.355***	3.200***	3.180***	3.596***
	(0.680)	(0.524)	(0.568)	(0.792)
Constant	198.185***	191.572***	189.616***	118.163***
	(16.693)	(16.872)	(16.421)	(13.467)
Observations	896	872	890	467

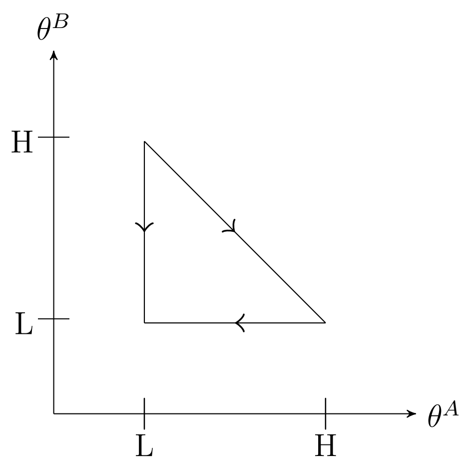
* $P < 0.10$, ** $P < 0.05$, *** $P < 0.01$. Regressions estimated using multi-level random effects at the session and subject levels. Robust standard errors clustered at the session level in parentheses. Cap dummy takes a value of 1 if the observation belongs to the cap treatment, 0 otherwise.



(a) Baseline - Independent goods

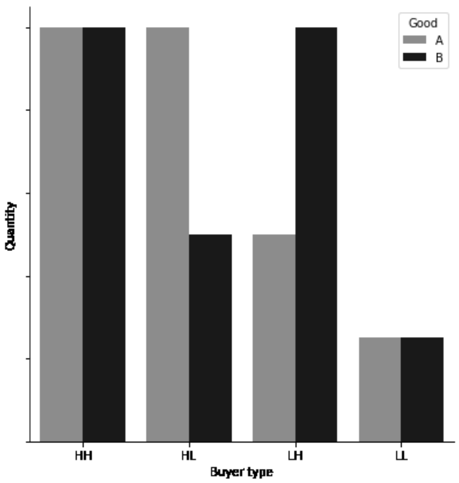


(b) Baseline - Complements

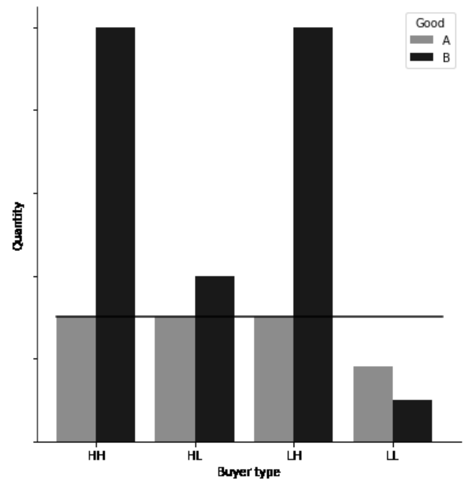


(c) Cap - Complements

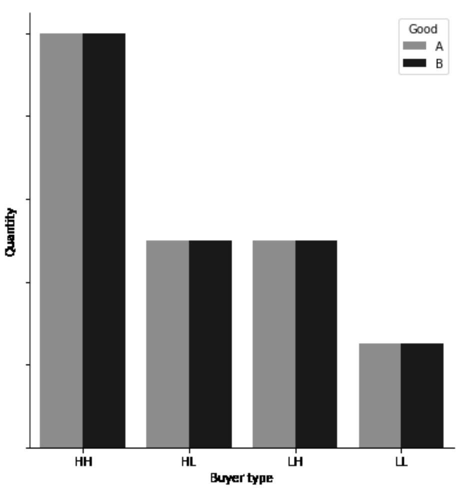
Figure 1: IC constraints in the relaxed problem



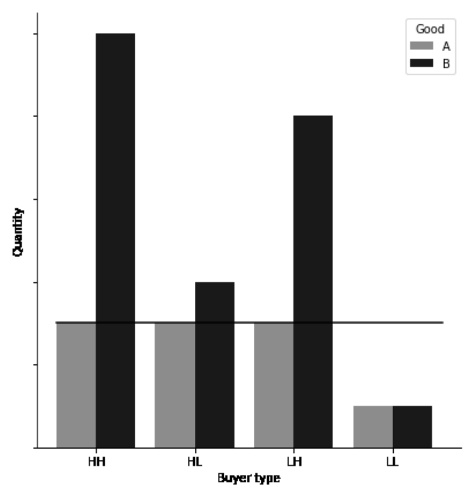
(a) Baseline - Independent goods



(b) Cap - Independent goods

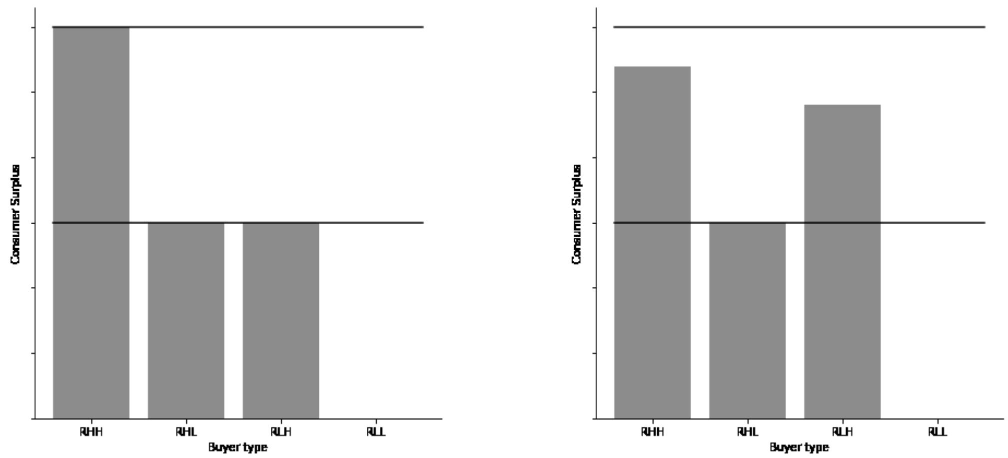


(c) Baseline - Related goods



(d) Cap - Related goods

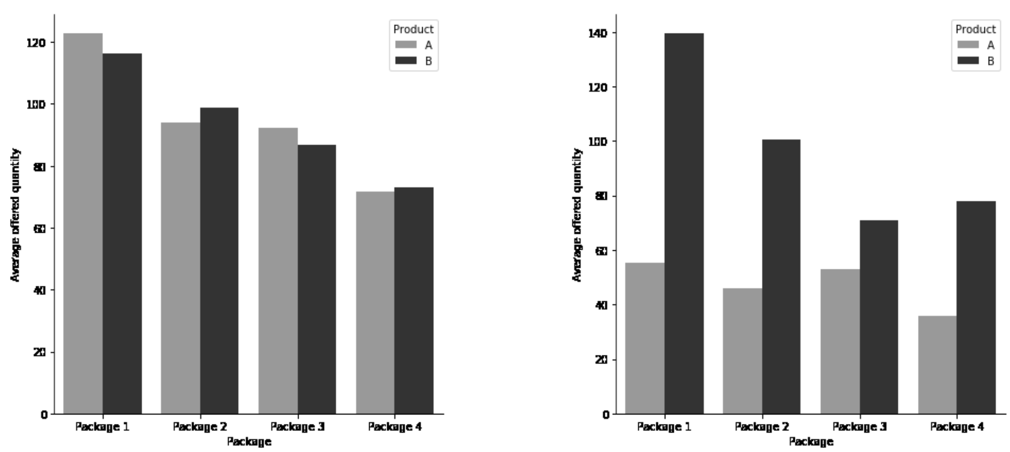
Figure 2: Allocation by Buyer Types (Theory)



(a) Baseline

(b) Quantity cap

Figure 3: Consumer surplus by types (Theory)



(a) Baseline

(b) Quantity cap

Figure 4: Packages by sum of offered quantities in the experiment

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Appendix A: Proofs

The solution to the relaxed problem is the solution to the fully constrained program. I show this is the case for IC-Structure A. The proofs for the other IC Structures are very similar. For the purposes of this proof, I assume symmetry, that is: $\theta_i^A = \theta_i^L = \theta_i$ for $i =$ H, L. The seller maximizes expected profit subject to the following restrictions:

$$R_{LL} = 0$$

$$R_{LH} = u(q_{LL}^B)\Delta$$

$$R_{HL} = u(q_{LL}^A)\Delta$$

$$R_{HH} = \Delta[u(q_{LL}^A) + u(q_{LL}^B)] + \Delta[u(q_{LH}^A) - u(q_{LL}^A)] + \Delta[u(q_{HL}^B) - u(q_{LL}^B)]$$

$$q_{HH}^A \geq q_{LH}^A,\ q_{HL}^A \geq q_{LL}^A,\ q_{HH}^B \geq q_{HL}^B, q_{LH}^B \geq q_{LL}^B$$

Where: $\Delta \equiv \theta_H - \theta_L$

The solution implies the following outcomes:

- Goods A and B are served in a "small" portions. $q_{LL}^* \equiv q_{LL}^{A*} = q_{LL}^{B*}$.
- There is a "medium" portion. $q_{LH}^* \equiv q_{LH}^{A*} = q_{HL}^{B*}$.
- There are "large" portions. $q_{HL}^* \equiv q_{HL}^{A*} = q_{LH}^{B*}$ and $q_{HH*} \equiv q_{HH}^{A*} = q_{HH}^{A*}$.
- The quantities consumed by the LL, LH, HL and HH, respectively are: (q_{LL}^*, q_{LL}^*), (q_{LH}^*, q_{HL}^*), (q_{HL}^*, q_{LH}^*), and (q_{HH}^*, q_{HH}^*).

Consumer Surplus:

$$R^*_{LL} = 0$$

$$R^*_{LH} = \Delta u(q^*_{LL})$$

$$R^*_{HL} = \Delta u(q^*_{LL})$$

$$R^*_{HH} = 2\Delta u(q^*_{LH})$$

***Proposition** 1. I closely follow the proofs in Armstrong and Rochet (1999). Maximizing 2 subject to 6 gives the solution to the seller's fully constrained problem.*

Proof. Proposition 1. Together, $R_{LL} = 0$, the monotonicity constraints, and the four downward binding constraints imply the satisfaction of the omitted IC constraints:

$$R_{LL} > R_{LH} + u(q_{LH})(\theta_L - \theta_H)$$

$$R_{LL} > R_{HL} + u(q_{HL})(\theta_L - \theta_H)$$

$$R_{LL} > R_{HH} + 2[u(q_{HH})(\theta_L - \theta_H)]$$

From the corresponfing first order conditions, it is straightforward to conclude that $q_{HL} > q_{LH}$, thus:

$$R_{LH} > R_{HL} + u(q_{HL})(\theta_L - \theta_H) + u(q_{LH})(\theta_H - \theta_L)$$

$$R_{HL} > R_{LH} + u(q_{LH})(\theta_H - \theta_L) + u(q_{HL})(\theta_L - \theta_H)$$

Lastly, the single crossing condition implies:

$$R_{LH} > R_{HH} + u(q_{HH})(\theta_H - \theta_L)$$

$$R_{HL} > R_{HH} + u(q_{HH})(\theta_L - \theta_H)$$

Appendix B: Characterization - Independent Goods

IC-Structure A baseline:

$$FOC[q^A_{HH}] : \theta^A_H u'(q^A_{HH}) = c'(q^A_{HH}) \qquad FOC[q^B_{HH}] : \theta^B_H u'(q^B_{HH}) = c'(q^B_{HH})$$

$$FOC[q^A_{HL}] : \theta^A_H u'(q^A_{HL}) = c'(q^A_{HL}) \qquad FOC[q^B_{HL}] : \theta^B_L u'(q^B_{HL}) = \frac{c'(q^B_{HL})}{\left(1 - \frac{\beta_{HH}}{\beta_{HL}} \frac{\Delta^B}{\theta^B_L}\right)}$$

$$FOC[q^A_{LH}] : \theta^A_L u'(q^A_{LH}) = \frac{c'(q^A_{LH})}{\left(1 - \frac{\beta_{HH}}{\beta_{LH}} \frac{\Delta^A}{\theta^A_L}\right)} \qquad FOC[q^B_{LH}] : \theta^B_H u'(q^B_{LH}) = c'(q^B_{LH})$$

$$FOC[q^A_{LL}] : \theta^A_L u'(q^A_{LL}) = \frac{c'(q^A_{LL})}{\left(1 - \frac{\beta_{HL}}{\beta_{LL}} \frac{\Delta^A}{\theta^A_L}\right)} \qquad FOC[q^B_{LL}] : \theta^B_L u'(q^B_{LL}) = \frac{c'(q^B_{LL})}{\left(1 - \frac{\beta_{LH}}{\beta_{LL}} \frac{\Delta^B}{\theta^B_L}\right)}$$

IC-Structure B baseline:

$$FOC[q^A_{HH}] : \theta^A_H u'(q^A_{HH}) = c'(q^A_{HH}) \qquad FOC[q^B_{HH}] : \theta^B_H u'(q^B_{HH}) = c'(q^B_{HH})$$

$$FOC[q^A_{HL}] : \theta^A_H u'(q^A_{HL}) = c'(q^A_{HL}) \qquad FOC[q^B_{HL}] : \theta^B_L u'(q^B_{HL}) = c'(q^B_{HL})$$

$$FOC[q^A_{LH}] : \theta^A_L u'(q^A_{LH}) = \frac{c'(q^A_{LH})}{\left(1 - \frac{\beta_{HH}}{\beta_{LH}} \frac{\Delta^A}{\theta^A_L}\right)} \qquad FOC[q^B_{LH}] : \theta^B_H u'(q^B_{LH}) = c'(q^B_{LH})$$

$$FOC[q^A_{LL}] : \theta^A_L u'(q^A_{LL}) = \frac{c'(q^A_{LL})}{\left(1 - \frac{\beta_{HL}}{\beta_{LL}} \frac{\Delta^A}{\theta^A_L}\right)} \qquad FOC[q^B_{LL}] : \theta^B_L u'(q^B_{LL}) = \frac{c'(q^B_{LL})}{\left(1 - \frac{\beta_{LH}+\beta_{HH}}{\beta_{LL}} \frac{\Delta^B}{\theta^B_L}\right)}$$

IC-Structure C baseline:

$$FOC[q^A_{HH}] : \theta^A_H u'(q^A_{HH}) = c'(q^A_{HH}) \qquad FOC[q^B_{HH}] : \theta^B_H u'(q^B_{HH}) = c'(q^B_{HH})$$

$$FOC[q^A_{HL}] : \theta^A_H u'(q^A_{HL}) = c'(q^A_{HL}) \qquad FOC[q^B_{HL}] : \theta^B_L u'(q^B_{HL}) = \frac{c'(q^B_{HL})}{\left(1 - \frac{\beta_{HH}}{\beta_{HL}} \frac{\Delta^B}{\theta^B_L}\right)}$$

$$FOC[q^A_{LH}] : \theta^A_L u'(q^A_{LH}) = c'(q^A_{LH}) \qquad FOC[q^B_{LH}] : \theta^B_H u'(q^B_{LH}) = c'(q^B_{LH})$$

$$FOC[q^A_{LL}] : \theta^A_L u'(q^A_{LL}) = \frac{c'(q^A_{LL})}{\left(1 - \frac{\beta_{HL}+\beta_{HH}}{\beta_{LL}} \frac{\Delta^A}{\theta^A_L}\right)} \qquad FOC[q^B_{LL}] : \theta^B_L u'(q^B_{LL}) = \frac{c'(q^B_{LL})}{\left(1 - \frac{\beta_{LH}}{\beta_{LL}} \frac{\Delta^B}{\theta^B_L}\right)}$$

IC-Structure D baseline:

$$FOC[q^A_{HH}] : \theta^A_H u'(q^A_{HH}) = c'(q^A_{HH}) \qquad FOC[q^B_{HH}] : \theta^B_H u'(q^B_{HH}) = c'(q^B_{HH})$$

$$FOC[q^A_{HL}] : \theta^A_H u'(q^A_{HL}) = c'(q^A_{HL}) \qquad FOC[q^B_{HL}] : \theta^B_L u'(q^B_{HL}) = c'(q^B_{HL})$$

$$FOC[q^A_{LH}] : \theta^A_L u'(q^A_{LH}) = c'(q^A_{LH}) \qquad FOC[q^B_{LH}] : \theta^B_H u'(q^B_{LH}) = c'(q^B_{LH})$$

$$FOC[q^A_{LL}] : \theta^A_L u'(q^A_{LL}) = \frac{c'(q^A_{LL})}{\left(1 - \frac{\beta_{HL}+\beta_{HH}}{\beta_{LL}} \frac{\Delta^A}{\theta^A_L}\right)} \qquad FOC[q^B_{LL}] : \theta^B_L u'(q^B_{LL}) = \frac{c'(q^B_{LL})}{\left(1 - \frac{\beta_{LH}+\beta_{HH}}{\beta_{LL}} \frac{\Delta^B}{\theta^B_L}\right)}$$

Mild IC-Structure A:

$$FOC[\bar{q}] : \theta_H^A u'(\bar{q}) = c'(\bar{q}) \qquad FOC[q_{HH}^B] : \theta_H^B u'(q_{HH}^B) = c'(q_{HH}^B)$$

$$FOC[q_{HL}^A] : \theta_H^A u'(q_{HL}^A) = c'(q_{HL}^A) \qquad FOC[q_{HL}^B] : \theta_L^B u'(q_{HL}^B) = \frac{c'(q_{HL}^B)}{\left(1 - \frac{\beta_{HH}}{\beta_{HL}} \frac{\Delta^B}{\theta_L^B}\right)}$$

$$FOC[q_{LH}^A] : \theta_L^A u'(q_{LH}^A) = \frac{c'(q_{LH}^A)}{\left(1 - \frac{\beta_{HH}}{\beta_{LH}} \frac{\Delta^A}{\theta_L^A}\right)} \qquad FOC[q_{LH}^B] : \theta_H^B u'(q_{LH}^B) = c'(q_{LH}^B)$$

$$FOC[q_{LL}^A] : \theta_L^A u'(q_{LL}^A) = \frac{c'(q_{LL}^A)}{\left(1 - \frac{\beta_{HL}}{\beta_{LL}} \frac{\Delta^A}{\theta_L^A}\right)} \qquad FOC[q_{LL}^B] : \theta_L^B u'(q_{LL}^B) = \frac{c'(q_{LL}^B)}{\left(1 - \frac{\beta_{LH}}{\beta_{LL}} \frac{\Delta^B}{\theta_L^B}\right)}$$

Mild cap IC-Structure B:

$$FOC[\bar{q}] : \theta_H^A u'(\bar{q}) = c'(\bar{q}) \qquad FOC[q_{HH}^B] : \theta_H^B u'(q_{HH}^B) = c'(q_{HH}^B)$$

$$FOC[q_{HL}^A] : \theta_H^A u'(q_{HL}^A) = c'(q_{HL}^A) \qquad FOC[q_{HL}^B] : \theta_L^B u'(q_{HL}^B) = c'(q_{HL}^B)$$

$$FOC[q_{LH}^A] : \theta_L^A u'(q_{LH}^A) = \frac{c'(q_{LH}^A)}{\left(1 - \frac{\beta_{HH}}{\beta_{LH}} \frac{\Delta^A}{\theta_L^A}\right)} \qquad FOC[q_{LH}^B] : \theta_H^B u'(q_{LH}^B) = c'(q_{LH}^B)$$

$$FOC[q_{LL}^A] : \theta_L^A u'(q_{LL}^A) = \frac{c'(q_{LL}^A)}{\left(1 - \frac{\beta_{HL}}{\beta_{LL}} \frac{\Delta^A}{\theta_L^A}\right)} \qquad FOC[q_{LL}^B] : \theta_L^B u'(q_{LL}^B) = \frac{c'(q_{LL}^B)}{\left(1 - \frac{\beta_{LH}+\beta_{HH}}{\beta_{LL}} \frac{\Delta^B}{\theta_L^B}\right)}$$

Mild cap IC-Structure C:

$$FOC[\bar{q}] : \theta_H^A u'(\bar{q}) = c'(\bar{q}) \qquad FOC[q_{HH}^B] : \theta_H^B u'(q_{HH}^B) = c'(q_{HH}^B)$$

$$FOC[q_{HL}^A] : \theta_H^A u'(q_{HL}^A) = c'(q_{HL}^A) \qquad FOC[q_{HL}^B] : \theta_L^B u'(q_{HL}^B) = \frac{c'(q_{HL}^B)}{\left(1 - \frac{\beta_{HH}}{\beta_{HL}} \frac{\Delta^B}{\theta_L^B}\right)}$$

$$FOC[q_{LH}^A] : \theta_L^A u'(q_{LH}^A) = c'(q_{LH}^A) \qquad FOC[q_{LH}^B] : \theta_H^B u'(q_{LH}^B) = c'(q_{LH}^B)$$

$$FOC[q_{LL}^A] : \theta_L^A u'(q_{LL}^A) = \frac{c'(q_{LL}^A)}{\left(1 - \frac{\beta_{HL}+\beta_{HH}}{\beta_{LL}} \frac{\Delta^A}{\theta_L^A}\right)} \qquad FOC[q_{LL}^B] : \theta_L^B u'(q_{LL}^B) = \frac{c'(q_{LL}^B)}{\left(1 - \frac{\beta_{LH}}{\beta_{LL}} \frac{\Delta^B}{\theta_L^B}\right)}$$

Mild IC-Structure D:

$$FOC[\bar{q}] : \theta_H^A u'(\bar{q}) = c'(\bar{q}) \qquad FOC[q_{HH}^B] : \theta_H^B u'(q_{HH}^B) = c'(q_{HH}^B)$$

$$FOC[q_{HL}^A] : \theta_H^A u'(q_{HL}^A) = c'(q_{HL}^A) \qquad FOC[q_{HL}^B] : \theta_L^B u'(q_{HL}^B) = c'(q_{HL}^B)$$

$$FOC[q_{LH}^A] : \theta_L^A u'(q_{LH}^A) = c'(q_{LH}^A) \qquad FOC[q_{LH}^B] : \theta_H^B u'(q_{LH}^B) = c'(q_{LH}^B)$$

$$FOC[q_{LL}^A] : \theta_L^A u'(q_{LL}^A) = \frac{c'(q_{LL}^A)}{\left(1 - \frac{\beta_{HL}+\beta_{HH}}{\beta_{LL}} \frac{\Delta^A}{\theta_L^A}\right)} \quad FOC[q_{LL}^B] : \theta_L^B u'(q_{LL}^B) = \frac{c'(q_{LL}^B)}{\left(1 - \frac{\beta_{LH}+\beta_{HH}}{\beta_{LL}} \frac{\Delta^B}{\theta_L^B}\right)}$$

Moderate cap:

$$FOC[\bar{q}] : \theta_H^A u'(q_{HH}^A) = \frac{\beta_{HH} + \beta_{HL} + \beta_{LH}}{\beta_{HH} + \beta_{HL} + \beta_{LH}\left(\frac{\theta_L^A}{\theta_H^A} + \frac{\Delta^A}{\theta_H^A}\right)} \cdot c'(q_{HH}^A)$$

$$FOC[q_{HH}^B] : \theta_H^B u'(q_{HH}^B) = c'(q_{HH}^B)$$

$$FOC[q_{HL}^B] : \theta_L^B u'(q_{HL}^B) = \frac{c'(q_{HL}^B)}{\left(1 - \frac{\beta_{LH}+\beta_{HH}}{\beta_{HL}} \frac{\Delta^B}{\theta_L^B}\right)}$$

$$FOC[q_{LH}^B] : \theta_H^B u'(q_{LH}^B) = c'(q_{LH}^B)$$

$$FOC[q_{LL}^A] : \theta_L^A u'(q_{LL}^A) = \frac{c'(q_{LL}^A)}{\left(1 - \frac{\beta_{LH}+\beta_{HL}+\beta_{HH}}{\beta_{LL}} \frac{\Delta^A}{\theta_L^A}\right)}$$

$$FOC[q_{LL}^B] : \theta_L^B u'(q_{LL}^B) = \frac{c'(q_{LL}^B)}{\left(1 - \frac{\beta_{LH}+\beta_{HH}}{\beta_{LL}} \frac{\Delta^B}{\theta_L^B}\right)}$$

Severe cap

$$FOC[\bar{q}] : \theta_H^A u'(\bar{q}) = \frac{\theta_H^A}{\theta_L^A} c'(\bar{q})$$

$$FOC[q_{HH}^B] : \theta_H^B u'(q_{HH}^B) = c'(q_{HH}^B)$$

$$FOC[q_{HL}^B] : \theta_L^B u'(q_{HL}^B) = \frac{c'(q_{HL}^B)}{\left(1 - \frac{\beta_{HH}+\beta_{LH}}{\beta_{HL}} \frac{\Delta^B}{\theta_L^B}\right)}$$

$$FOC[q_{LH}^B] : \theta_H^B u'(q_{LH}^B) = c'(q_{LH}^B)$$

$$FOC[q_{LL}^B] : \theta_L^B u'(q_{LL}^B) = \frac{c'(q_{LL}^B)}{\left(1 - \frac{\beta_{HH}+\beta_{LH}}{\beta_{LL}} \frac{\Delta^B}{\theta_L^B}\right)}$$

Appendix C: Characterization - Related (Complement and Substitute) Goods

For complements (superscript C), the following holds: ($\theta^C_{HH} > \theta^A_H + \theta^B_H$), ($\theta^C_{HL} > \theta^A_H + \theta^B_L$), ($\theta^C_{LH} > \theta^A_L + \theta^B_H$), *and* ($\theta^C_{LL} > \theta^A_L + \theta^B_L$). *Additionally,* $\theta^C_{HH} > \theta^C_{HL}$, $\theta^C_{HH} > \theta^C_{LH}$, $\theta^C_{HH} > \theta^C_{LL}$, $\theta^C_{HL} > \theta^C_{LL}$, *and* $\theta^C_{LH} > \theta^C_{LL}$.

For substitutes (superscript S), the following holds: ($\theta^S_{HH} < \theta^A_H + \theta^B_H$), ($\theta^S_{HL} < \theta^A_H + \theta^B_L$), ($\theta^S_{LH} < \theta^A_L + \theta^B_H$), *and* ($\theta^S_{LL} < \theta^A_L + \theta^B_L$). *Additionally,* $\theta^S_{HH} > \theta^S_{HL}$, $\theta^S_{HH} > \theta^S_{LH}$, $\theta^S_{HH} > \theta^S_{LL}$, $\theta^S_{HL} > \theta^S_{LL}$, *and* $\theta^S_{LH} > \theta^S_{LL}$.

Because the analysis below looks identical for both complements and substitutes, I drop the superscript. Consider the following definitions:

$$
\begin{aligned}
&\theta_{HH} - \theta_{LL} \equiv \Delta_1 \quad && \theta_{HH} - \theta_{HL} \equiv \Delta_2 \\
&\theta_{HH} - \theta_{LH} \equiv \Delta_3 \quad && \theta_{HL} - \theta_{LL} \equiv \Delta_4 \\
&\theta_{LH} - \theta_{LL} \equiv \Delta_5 \quad && \theta_{LH} - \theta_{HL} \equiv \Delta_6 \\
&\theta_{HL} - \theta_{LH} \equiv \Delta_7
\end{aligned}
$$

IC-Structure A (complements) baseline:

$$FOC[q_{HH}^{A}] : \theta_{HH}u'(q_{HH}^{A}) = c'(q_{HH}^{A})$$

$$FOC[q_{HH}^{B}] : \theta_{HH}u'(q_{HH}^{B}) = c'(q_{HH}^{B})$$

$$FOC[q_{HL}^{A}] : \theta_{HL}u'(q_{HL}^{A}) = \frac{c'(q_{HL}^{A})}{\left(1 - \frac{\beta_{HH}}{\beta_{HL}}\frac{\Delta_2}{\theta_{HL}}\right)}$$

$$FOC[q_{HL}^{B}] : \theta_{HL}u'(q_{HL}^{B}) = \frac{c'(q_{HL}^{B})}{\left(1 - \frac{\beta_{HH}}{\beta_{HL}}\frac{\Delta_2}{\theta_{HL}}\right)}$$

$$FOC[q_{LH}^{A}] : \theta_{LH}u'(q_{LH}^{A}) = \frac{c'(q_{LH}^{A})}{\left(1 - \frac{\beta_{HH}}{\beta_{LH}}\frac{\Delta_3}{\theta_{LH}}\right)}$$

$$FOC[q_{LH}^{B}] : \theta_{LH}u'(q_{LH}^{B}) = \frac{c'(q_{LH}^{B})}{\left(1 - \frac{\beta_{HH}}{\beta_{LH}}\frac{\Delta_3}{\theta_{LH}}\right)}$$

$$FOC[q_{LL}^{A}] : \theta_{LL}u'(q_{LL}^{A}) = \frac{c'(q_{LL}^{A})}{\left(1 - \frac{\beta_{LH}+\beta_{HH}}{\beta_{LL}}\frac{\Delta_5}{\theta_{LL}} - \frac{\beta_{HL}+\beta_{HH}}{\beta_{LL}}\frac{\Delta_4}{\theta_{LL}}\right)}$$

$$FOC[q_{LL}^{B}] : \theta_{LL}u'(q_{LL}^{B}) = \frac{c'(q_{LL}^{B})}{\left(1 - \frac{\beta_{LH}+\beta_{HH}}{\beta_{LL}}\frac{\Delta_5}{\theta_{LL}} - \frac{\beta_{HL}+\beta_{HH}}{\beta_{LL}}\frac{\Delta_4}{\theta_{LL}}\right)}$$

IC-Structure B (complements) baseline:

$$FOC[q^A_{HH}] : \theta_{HH} u'(q^A_{HH}) = c'(q^A_{HH})$$

$$FOC[q^B_{HH}] : \theta_{HH} u'(q^B_{HH}) = c'(q^B_{HH})$$

$$FOC[q^A_{HL}] : \theta_{HL} u'(q^A_{HL}) = \frac{c'(q^A_{HL})}{\left(1 - \frac{\beta_{HH}}{\beta_{HL}} \frac{\Delta_6+\Delta_2}{\theta_{HL}}\right)}$$

$$FOC[q^B_{HL}] : \theta_{HL} u'(q^B_{HL}) = \frac{c'(q^B_{HL})}{\left(1 - \frac{\beta_{HH}}{\beta_{HL}} \frac{\Delta_6+\Delta_2}{\theta_{HL}}\right)}$$

$$FOC[q^A_{LH}] : \theta_{LH} u'(q^A_{LH}) = \frac{c'(q^A_{LH})}{\left(1 - \frac{\beta_{HH}}{\beta_{LH}} \frac{\Delta_3}{\theta_{LH}}\right)}$$

$$FOC[q^B_{LH}] : \theta_{LH} u'(q^B_{LH}) = \frac{c'(q^B_{LH})}{\left(1 - \frac{\beta_{HH}}{\beta_{LH}} \frac{\Delta_3}{\theta_{LH}}\right)}$$

$$FOC[q^A_{LL}] : \theta_{LL} u'(q^A_{LL}) = \frac{c'(q^A_{LL})}{\left(1 - \frac{\beta_{LH}+\beta_{HH}}{\beta_{LL}} \frac{\Delta_5}{\theta_{LL}} - \frac{\beta_{HL}+\beta_{HH}}{\beta_{LL}} \frac{\Delta_4}{\theta_{LL}}\right)}$$

$$FOC[q^B_{LL}] : \theta_{LL} u'(q^B_{LL}) = \frac{c'(q^B_{LL})}{\left(1 - \frac{\beta_{LH}+\beta_{HH}}{\beta_{LL}} \frac{\Delta_5}{\theta_{LL}} - \frac{\beta_{HL}+\beta_{HH}}{\beta_{LL}} \frac{\Delta_4}{\theta_{LL}}\right)}$$

IC-Structure C (complements) baseline:

$$FOC[q_{HH}^A] : \theta_{HH} u'(q_{HH}^A) = c'(q_{HH}^A)$$

$$FOC[q_{HH}^B] : \theta_{HH} u'(q_{HH}^B) = c'(q_{HH}^B)$$

$$FOC[q_{HL}^A] : \theta_{HL} u'(q_{HL}^A) = \frac{c'(q_{HL}^A)}{\left(1 - \frac{\beta_{HH}}{\beta_{HL}} \frac{\Delta_1}{\theta_{HL}}\right)}$$

$$FOC[q_{HL}^B] : \theta_{HL} u'(q_{HL}^B) = \frac{c'(q_{HL}^B)}{\left(1 - \frac{\beta_{HH}}{\beta_{HL}} \frac{\Delta_1}{\theta_{HL}}\right)}$$

$$FOC[q_{LH}^A] : \theta_{LH} u'(q_{LH}^A) = \frac{c'(q_{LH}^A)}{\left(1 - \frac{\beta_{HH}}{\beta_{LH}} \frac{\Delta_3}{\theta_{LH}}\right)}$$

$$FOC[q_{LH}^B] : \theta_{LH} u'(q_{LH}^B) = \frac{c'(q_{LH}^B)}{\left(1 - \frac{\beta_{HH}}{\beta_{LH}} \frac{\Delta_3}{\theta_{LH}}\right)}$$

$$FOC[q_{LL}^A] : \theta_{LL} u'(q_{LL}^A) = \frac{c'(q_{LL}^A)}{\left(1 - \frac{\beta_{LH}+\beta_{HH}}{\beta_{LL}} \frac{\Delta_5}{\theta_{LL}} - \frac{\beta_{HL}+\beta_{HH}}{\beta_{LL}} \frac{\Delta_4}{\theta_{LL}} - \frac{\beta_{HH}}{\beta_{LL}} \frac{\Delta_1}{\theta_{LL}}\right)}$$

$$FOC[q_{LL}^B] : \theta_{LL} u'(q_{LL}^B) = \frac{c'(q_{LL}^B)}{\left(1 - \frac{\beta_{LH}+\beta_{HH}}{\beta_{LL}} \frac{\Delta_5}{\theta_{LL}} - \frac{\beta_{HL}+\beta_{HH}}{\beta_{LL}} \frac{\Delta_4}{\theta_{LL}} - \frac{\beta_{HH}}{\beta_{LL}} \frac{\Delta_1}{\theta_{LL}}\right)}$$

IC-Structure D (complements) baseline:

$$FOC[q^A_{HH}] : \theta_{HH} u'(q^A_{HH}) = c'(q^A_{HH})$$

$$FOC[q^B_{HH}] : \theta_{HH} u'(q^B_{HH}) = c'(q^B_{HH})$$

$$FOC[q^A_{HL}] : \theta_{HL} u'(q^A_{HL}) = \frac{c'(q^A_{HL})}{\left(1 - \frac{\beta_{LH}+\beta_{HH}}{\beta_{HL}} \frac{\Delta_6}{\theta_{HL}} - \frac{\beta_{HH}}{\beta_{HL}} \frac{\Delta_2}{\theta_{HL}}\right)}$$

$$FOC[q^B_{HL}] : \theta_{HL} u'(q^B_{HL}) = \frac{c'(q^B_{HL})}{\left(1 - \frac{\beta_{LH}+\beta_{HH}}{\beta_{HL}} \frac{\Delta_6}{\theta_{HL}} - \frac{\beta_{HH}}{\beta_{HL}} \frac{\Delta_2}{\theta_{HL}}\right)}$$

$$FOC[q^A_{LH}] : \theta_{LH} u'(q^A_{LH}) = \frac{c'(q^A_{LH})}{\left(1 - \frac{\beta_{HH}}{\beta_{LH}} \frac{\Delta_3}{\theta_{LH}}\right)}$$

$$FOC[q^B_{LH}] : \theta_{LH} u'(q^B_{LH}) = \frac{c'(q^B_{LH})}{\left(1 - \frac{\beta_{HH}}{\beta_{LH}} \frac{\Delta_3}{\theta_{LH}}\right)}$$

$$FOC[q^A_{LL}] : \theta_{LL} u'(q^A_{LL}) = \frac{c'(q^A_{LL})}{\left(1 - \frac{\beta_{LH}+\beta_{HH}}{\beta_{LL}} \frac{\Delta_5}{\theta_{LL}} - \frac{\beta_{HL}+\beta_{HH}}{\beta_{LL}} \frac{\Delta_4}{\theta_{LL}} - \frac{\beta_{HH}}{\beta_{LL}} \frac{\Delta_1}{\theta_{LL}}\right)}$$

$$FOC[q^B_{LL}] : \theta_{LL} u'(q^B_{LL}) = \frac{c'(q^B_{LL})}{\left(1 - \frac{\beta_{LH}+\beta_{HH}}{\beta_{LL}} \frac{\Delta_5}{\theta_{LL}} - \frac{\beta_{HL}+\beta_{HH}}{\beta_{LL}} \frac{\Delta_4}{\theta_{LL}} - \frac{\beta_{HH}}{\beta_{LL}} \frac{\Delta_1}{\theta_{LL}}\right)}$$

IC-Structure A (complements) mild cap:

$$FOC[\bar{q}] : \theta_{HH}u'(\bar{q}) = c'(\bar{q})$$

$$FOC[q^B_{HH}] : \theta_{HH}u'(q^B_{HH}) = c'(q^B_{HH})$$

$$FOC[q^A_{HL}] : \theta_{HL}u'(q^A_{HL}) = \frac{c'(q^A_{HL})}{\left(1 - \frac{\beta_{HH}}{\beta_{HL}}\frac{\Delta_2}{\theta_{HL}}\right)}$$

$$FOC[q^B_{HL}] : \theta_{HL}u'(q^B_{HL}) = \frac{c'(q^B_{HL})}{\left(1 - \frac{\beta_{HH}}{\beta_{HL}}\frac{\Delta_2}{\theta_{HL}}\right)}$$

$$FOC[q^A_{LH}] : \theta_{LH}u'(q^A_{LH}) = \frac{c'(q^A_{LH})}{\left(1 - \frac{\beta_{HH}}{\beta_{LH}}\frac{\Delta_3}{\theta_{LH}}\right)}$$

$$FOC[q^B_{LH}] : \theta_{LH}u'(q^B_{LH}) = \frac{c'(q^B_{LH})}{\left(1 - \frac{\beta_{HH}}{\beta_{LH}}\frac{\Delta_3}{\theta_{LH}}\right)}$$

$$FOC[q^A_{LL}] : \theta_{LL}u'(q^A_{LL}) = \frac{c'(q^A_{LL})}{\left(1 - \frac{\beta_{LH}+\beta_{HH}}{\beta_{LL}}\frac{\Delta_5}{\theta_{LL}} - \frac{\beta_{HL}+\beta_{HH}}{\beta_{LL}}\frac{\Delta_4}{\theta_{LL}}\right)}$$

$$FOC[q^B_{LL}] : \theta_{LL}u'(q^B_{LL}) = \frac{c'(q^B_{LL})}{\left(1 - \frac{\beta_{LH}+\beta_{HH}}{\beta_{LL}}\frac{\Delta_5}{\theta_{LL}} - \frac{\beta_{HL}+\beta_{HH}}{\beta_{LL}}\frac{\Delta_4}{\theta_{LL}}\right)}$$

IC-Structure B (complements) mild cap:

$$FOC[\bar{q}] : \theta_{HH}u'(\bar{q}) = c'(\bar{q})$$

$$FOC[q^B_{HH}] : \theta_{HH}u'(q^B_{HH}) = c'(q^B_{HH})$$

$$FOC[q^A_{HL}] : \theta_{HL}u'(q^A_{HL}) = \frac{c'(q^A_{HL})}{\left(1 - \frac{\beta_{HH}}{\beta_{HL}}\frac{\Delta_6+\Delta_2}{\theta_{HL}}\right)}$$

$$FOC[q^B_{HL}] : \theta_{HL}u'(q^B_{HL}) = \frac{c'(q^B_{HL})}{\left(1 - \frac{\beta_{HH}}{\beta_{HL}}\frac{\Delta_6+\Delta_2}{\theta_{HL}}\right)}$$

$$FOC[q^A_{LH}] : \theta_{LH}u'(q^A_{LH}) = \frac{c'(q^A_{LH})}{\left(1 - \frac{\beta_{HH}}{\beta_{LH}}\frac{\Delta_3}{\theta_{LH}}\right)}$$

$$FOC[q^B_{LH}] : \theta_{LH}u'(q^B_{LH}) = \frac{c'(q^B_{LH})}{\left(1 - \frac{\beta_{HH}}{\beta_{LH}}\frac{\Delta_3}{\theta_{LH}}\right)}$$

$$FOC[q^A_{LL}] : \theta_{LL}u'(q^A_{LL}) = \frac{c'(q^A_{LL})}{\left(1 - \frac{\beta_{LH}+\beta_{HH}}{\beta_{LL}}\frac{\Delta_5}{\theta_{LL}} - \frac{\beta_{HL}+\beta_{HH}}{\beta_{LL}}\frac{\Delta_4}{\theta_{LL}}\right)}$$

$$FOC[q^B_{LL}] : \theta_{LL}u'(q^B_{LL}) = \frac{c'(q^B_{LL})}{\left(1 - \frac{\beta_{LH}+\beta_{HH}}{\beta_{LL}}\frac{\Delta_5}{\theta_{LL}} - \frac{\beta_{HL}+\beta_{HH}}{\beta_{LL}}\frac{\Delta_4}{\theta_{LL}}\right)}$$

IC-Structure C (complements) mild cap:

$$FOC[\bar{q}] : \theta_{HH} u'(\bar{q}) = c'(\bar{q})$$

$$FOC[q^B_{HH}] : \theta_{HH} u'(q^B_{HH}) = c'(q^B_{HH})$$

$$FOC[q^A_{HL}] : \theta_{HL} u'(q^A_{HL}) = \frac{c'(q^A_{HL})}{\left(1 - \frac{\beta_{HH}}{\beta_{HL}} \frac{\Delta_1}{\theta_{HL}}\right)}$$

$$FOC[q^B_{HL}] : \theta_{HL} u'(q^B_{HL}) = \frac{c'(q^B_{HL})}{\left(1 - \frac{\beta_{HH}}{\beta_{HL}} \frac{\Delta_1}{\theta_{HL}}\right)}$$

$$FOC[q^A_{LH}] : \theta_{LH} u'(q^A_{LH}) = \frac{c'(q^A_{LH})}{\left(1 - \frac{\beta_{HH}}{\beta_{LH}} \frac{\Delta_3}{\theta_{LH}}\right)}$$

$$FOC[q^B_{LH}] : \theta_{LH} u'(q^B_{LH}) = \frac{c'(q^B_{LH})}{\left(1 - \frac{\beta_{HH}}{\beta_{LH}} \frac{\Delta_3}{\theta_{LH}}\right)}$$

$$FOC[q^A_{LL}] : \theta_{LL} u'(q^A_{LL}) = \frac{c'(q^A_{LL})}{\left(1 - \frac{\beta_{LH}+\beta_{HH}}{\beta_{LL}} \frac{\Delta_5}{\theta_{LL}} - \frac{\beta_{HL}+\beta_{HH}}{\beta_{LL}} \frac{\Delta_4}{\theta_{LL}} - \frac{\beta_{HH}}{\beta_{LL}} \frac{\Delta_1}{\theta_{LL}}\right)}$$

$$FOC[q^B_{LL}] : \theta_{LL} u'(q^B_{LL}) = \frac{c'(q^B_{LL})}{\left(1 - \frac{\beta_{LH}+\beta_{HH}}{\beta_{LL}} \frac{\Delta_5}{\theta_{LL}} - \frac{\beta_{HL}+\beta_{HH}}{\beta_{LL}} \frac{\Delta_4}{\theta_{LL}} - \frac{\beta_{HH}}{\beta_{LL}} \frac{\Delta_1}{\theta_{LL}}\right)}$$

IC-Structure D (complements) mild cap:

$$FOC[\bar{q}] : \theta_{HH}u'(\bar{q}) = c'(\bar{q})$$

$$FOC[q^B_{HH}] : \theta_{HH}u'(q^B_{HH}) = c'(q^B_{HH})$$

$$FOC[q^A_{HL}] : \theta_{HL}u'(q^A_{HL}) = \frac{c'(q^A_{HL})}{\left(1 - \frac{\beta_{LH}+\beta_{HH}}{\beta_{HL}}\frac{\Delta_6}{\theta_{HL}} - \frac{\beta_{HH}}{\beta_{HL}}\frac{\Delta_2}{\theta_{HL}}\right)}$$

$$FOC[q^B_{HL}] : \theta_{HL}u'(q^B_{HL}) = \frac{c'(q^B_{HL})}{\left(1 - \frac{\beta_{LH}+\beta_{HH}}{\beta_{HL}}\frac{\Delta_6}{\theta_{HL}} - \frac{\beta_{HH}}{\beta_{HL}}\frac{\Delta_2}{\theta_{HL}}\right)}$$

$$FOC[q^A_{LH}] : \theta_{LH}u'(q^A_{LH}) = \frac{c'(q^A_{LH})}{\left(1 - \frac{\beta_{HH}}{\beta_{LH}}\frac{\Delta_3}{\theta_{LH}}\right)}$$

$$FOC[q^B_{LH}] : \theta_{LH}u'(q^B_{LH}) = \frac{c'(q^B_{LH})}{\left(1 - \frac{\beta_{HH}}{\beta_{LH}}\frac{\Delta_3}{\theta_{LH}}\right)}$$

$$FOC[q^A_{LL}] : \theta_{LL}u'(q^A_{LL}) = \frac{c'(q^A_{LL})}{\left(1 - \frac{\beta_{LH}+\beta_{HH}}{\beta_{LL}}\frac{\Delta_5}{\theta_{LL}} - \frac{\beta_{HL}+\beta_{HH}}{\beta_{LL}}\frac{\Delta_4}{\theta_{LL}} - \frac{\beta_{HH}}{\beta_{LL}}\frac{\Delta_1}{\theta_{LL}}\right)}$$

$$FOC[q^B_{LL}] : \theta_{LL}u'(q^B_{LL}) = \frac{c'(q^B_{LL})}{\left(1 - \frac{\beta_{LH}+\beta_{HH}}{\beta_{LL}}\frac{\Delta_5}{\theta_{LL}} - \frac{\beta_{HL}+\beta_{HH}}{\beta_{LL}}\frac{\Delta_4}{\theta_{LL}} - \frac{\beta_{HH}}{\beta_{LL}}\frac{\Delta_1}{\theta_{LL}}\right)}$$

Moderate cap (complements):

$$FOC[\bar{q}] : \theta_{HH}u'(\bar{q}) = \frac{\theta_{HH}}{\theta_{HL}}c'(\bar{q})$$

$$FOC[q^B_{HH}] : \theta_{HH}u'(q^B_{HH}) = c'(q^B_{HH})$$

$$FOC[q^B_{HL}] : \theta_{HL}u'(q^B_{HL}) = \frac{c'(q^B_{HL})}{\left(1 - \frac{\beta_{LH}}{\beta_{HL}}\frac{\Delta_6}{\theta_{HL}} - \frac{\beta_{HH}}{\beta_{HL}}\frac{\Delta_2}{\theta_{HL}}\right)}$$

$$FOC[q^B_{LH}] : \theta_{LH}u'(q^B_{LH}) = c'(q^B_{LH})$$

$$FOC[q^A_{LL}] : \theta_{LL}u'(q^A_{LL}) = \frac{c'(q^A_{LL})}{\left(1 - \frac{\beta_{LH}}{\beta_{LL}}\frac{\Delta_5}{\theta_{LL}} - \frac{\beta_{LH}+\beta_{HL}+\beta_{HH}}{\beta_{LL}}\frac{\Delta_4}{\theta_{LL}} - \frac{\beta_{HH}}{\beta_{LL}}\frac{\Delta_1}{\theta_{LL}}\right)}$$

$$FOC[q^B_{LL}] : \theta_{LL}u'(q^B_{LL}) = \frac{c'(q^B_{LL})}{\left(1 - \frac{\beta_{LH}}{\beta_{LL}}\frac{\Delta_5}{\theta_{LL}} - \frac{\beta_{LH}+\beta_{HL}+\beta_{HH}}{\beta_{LL}}\frac{\Delta_4}{\theta_{LL}} - \frac{\beta_{HH}}{\beta_{LL}}\frac{\Delta_1}{\theta_{LL}}\right)}$$

Severe cap (complements):

$$FOC[\bar{q}] : \theta_{HH} u'(\bar{q}) = \frac{c'(\bar{q})}{\beta_{HH}\Big(1 - \frac{\Delta_1 + \Delta_2 + \Delta_4}{\theta_{HH}}\Big) + \beta_{LH}\Big(\frac{\theta_{LH} - \Delta_5 - \Delta_6 - \Delta_4}{\theta_{HH}}\Big) + (\beta_{HL} + \beta_{LL})\frac{\theta_{LL}}{\theta_{HH}}}$$

$$FOC[q^B_{HH}] : \theta_{HH} u'(q^B_{HH}) = c'(q^B_{HH})$$

$$FOC[q^B_{HL}] : \theta_{HL} u'(q^B_{HL}) = \frac{c'(q^B_{HH})}{\Big(1 - \frac{\beta_{LH}}{\beta_{HL}}\frac{\Delta_6}{\theta_{HL}} - \frac{\beta_{HH}}{\beta_{HL}}\frac{\Delta_2}{\theta_{HL}}\Big)}$$

$$FOC[q^B_{LL}] : \theta_{LL} u'(q^B_{LL}) = \frac{c'(q^B_{LL})}{\Big(1 - \frac{\beta_{LH}}{\beta_{LL}}\frac{\Delta_5}{\theta_{LL}} - \frac{\beta_{LH} + \beta_{HL} + \beta_{HH}}{\beta_{LL}}\frac{\Delta_4}{\theta_{LL}} - \frac{\beta_{HH}}{\beta_{LL}}\frac{\Delta_1}{\theta_{LL}}\Big)}$$