AJAE appendix for “An Analysis of Portion Cap Rules with a Multi-Product Seller”

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March 2021

Note: The material contained herein is supplementary to the article names in the title and published in the *American Journal of Agricultural Economics*.

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Proofs

The solution to the relaxed problem is the solution to the fully constrained program. I show this is the case for IC-Structure Γ. The proofs for the other IC Structures are very similar. For the purposes of this proof, I assume symmetry, that is: $\theta_i^A = \theta_i^L = \theta_i$ for $i =\ H, L$. Proofs for asymmetric cases are similar.

The seller maximizes expected profit subject to the following restrictions:

$$R_{LL} = \bar{u}$$

$$R_{LH} = q_{LL}'^B \Delta + \bar{u}$$

$$R_{HL} = q_{LL}'^A \Delta + \bar{u}$$

$$R_{HH} = \Delta[q_{LL}'^A + q_{LL}'^B] + \Delta[q_{LH}'^A - q_{LL}'^A] + \Delta[q_{HL}'^B - q_{LL}'^B] + \bar{u}$$

$$q_{HH}^A \geq q_{LH}^A,\ q_{HL}^A \geq q_{LL}^A,\ q_{HH}^B \geq q_{HL}^B, q_{LH}^B \geq q_{LL}^B$$

Where: $\Delta \equiv \theta_H - \theta_L$

- Goods A and B are served in a "small" portions. $q_{LL}^* \equiv q_{LL}^{A*} = q_{LL}^{B*}$.
- There is a "medium" portion. $q_{LH}^* \equiv q_{LH}^{A*} = q_{HL}^{B*}$.
- There are "large" portions. $q_{HL}^* \equiv q_{HL}^{A*} = q_{LH}^{B*}$ and $q_{HH*} \equiv q_{HH}^{A*} = q_{HH}^{A*}$.
- The quantities consumed by the LL, LH, HL and HH, respectively are: (q_{LL}^*, q_{LL}^*), (q_{LH}^*, q_{HL}^*), (q_{HL}^*, q_{LH}^*), and (q_{HH}^*, q_{HH}^*).

Consumer Surplus:

$$R^*_{LL} = \bar{u}$$

$$R^*_{LH} = \Delta q'^*_{LL} + \bar{u}$$

$$R^*_{HL} = \Delta q'^*_{LL} + \bar{u}$$

$$R^*_{HH} = 2\Delta q'^*_{LH} + \bar{u}$$

***Proposition** A1. I closely follow the proofs in Armstrong and Rochet (1999). Maximizing 2 subject to 4 (in main text) gives the solution to the seller's fully constrained problem.*

Proof of proposition A1:

Together, $R_{LL} = 0$, the monotonicity constraints, and the four downward binding constraints imply the satisfaction of the omitted IC constraints:

$$R_{LL} > R_{LH} + q'_{LH}(\theta_L - \theta_H)$$

$$R_{LL} > R_{HL} + q'_{HL}(\theta_L - \theta_H)$$

$$R_{LL} > R_{HH} + 2[q'_{HH}(\theta_L - \theta_H)]$$

From the corresponding first order conditions, it is straightforward to conclude that $q_{HL} > q_{LH}$, thus:

$$R_{LH} > R_{HL} + q'_{HL}(\theta_L - \theta_H) + q'_{LH}(\theta_H - \theta_L)$$

$$R_{HL} > R_{LH} + q'_{LH}(\theta_H - \theta_L) + q'_{HL}(\theta_L - \theta_H)$$

Lastly, the single crossing condition implies:

$$R_{LH} > R_{HH} + q'_{HH}(\theta_H - \theta_L)$$

$$R_{HL} > R_{HH} + q'_{HH}(\theta_L - \theta_H)$$

Proof of proposition 1

There are three parts to show:

First, a mild cap does not change the set of relevant IC constraints. A mild cap is defined as that where the limit is set strictly below the maximum unregulated portion of good A, and at or above the unregulated second largest portion of A. Effectively, the only quantity affected is q^A_{HH}. A new IC constraint would be added to the set to consider if the cap causes it to start binding with equality. The only downward IC restrictions affected are the following, all of which contain q^A_{HH} in the left hand side of the inequality:

$$R_{HH} \geq R_{LL} + (\theta^A_H - \theta^A_L)q'^A_{LL} + (\theta^B_H - \theta^B_L)q'^B_{LL} + \bar{u}$$

$$R_{HH} \geq R_{LH} + (\theta^A_H - \theta^A_L)q'^A_{LH} + (\theta^B_H - \theta^B_H)q'^B_{LH} + \bar{u}$$

$$R_{HH} \geq R_{HL} + (\theta^A_H - \theta^A_H)q'^A_{HL} + (\theta^B_H - \theta^B_L)q'^B_{HL} + \bar{u}$$

These are the same IC already included in the original regulation-free IC-structures. The reduction in q^A_{HH} is small enough that none of them change from potentially binding to always binding.

Second, $LH \to LL$ and $HL \to LL$ remain unchanged and binding in all caps because these IC restrictions do not involve q^A_{HH}.

Lastly, with moderate and severe caps, $LH \to HL$ binds with equality and substitutes $HH \to LL$, $HH \to LH$, and $HH \to HL$. The analysis below concerns to the moderate cap.

To proof this part, first recall that the general form of the IC constraints is the following:

$$R_{ij} \geq R_{kl} + \bar{u} + q_{kl}^{\prime A}(\theta_i^A - \theta_k^A) + q_{kl}^{\prime B}(\theta_j^B - \theta_l^B) \ \forall \ ij \text{ and } kl; i \neq k \text{ and } j \neq l$$

With a moderate cap the specific forms of the all IC program in the complete program are the following:

$$
\begin{aligned}
LL \rightarrow LH: &\quad R_{LL} \geq R_{LH} - \Delta^B q_{LH}^{\prime B} \\
LL \rightarrow HL: &\quad R_{LL} \geq R_{HL} - \Delta^A \bar{q}' \\
LL \rightarrow HH: &\quad R_{LL} \geq R_{HH} - \Delta^A \bar{q}' - \Delta^B q_{HH}^{\prime B} \\
LH \rightarrow LL: &\quad R_{LH} \geq R_{LL} + \Delta^B q_{LL}^{\prime B} \\
LH \rightarrow HL: &\quad R_{LH} \geq R_{HL} - \Delta^A \bar{q}' + \Delta^B q_{HL}^{\prime B} \\
LH \rightarrow HH: &\quad R_{LH} \geq R_{HH} - \Delta^A \bar{q}' \\
HL \rightarrow LL: &\quad R_{HL} \geq R_{LL} + \Delta^A q_{LL}^{\prime A} \\
HL \rightarrow LH: &\quad R_{HL} \geq R_{LH} + \Delta^A \bar{q}' - \Delta^B q_{LH}^{\prime B} \\
HL \rightarrow HH: &\quad R_{HL} \geq R_{HH} - \Delta^B q_{HH}^{\prime B} \\
HH \rightarrow LL: &\quad R_{HH} \geq R_{LL} + \Delta^A q_{LL}^{\prime A} + \Delta^B q_{LL}^{\prime B} \\
HH \rightarrow LH: &\quad R_{HH} \geq R_{LH} + \Delta^A \bar{q}' \\
HH \rightarrow HL: &\quad R_{HH} \geq R_{HL} + \Delta^B q_{HL}^{\prime B}
\end{aligned}
$$

Because profit maximization necessitates the satisfaction of the participation constraint for the lowest type, the rents earned by the LL-type continue to be $R_{LL} = 0$.

The cap could have changed the set of binding constraints. Regarding the medium types HL and LH, there are three possible candidates for the form of their surplus functions following the cap:

$$\text{Candidate 1: } \begin{cases} R_{LH} = \Delta^B q'^B_{LL} + \bar{u} \\ R_{HL} = \Delta^A q'^A_{LL} + \bar{u} \end{cases}$$

This first candidate set implies that $LH \to LL$ and $HL \to LL$ are the only binding constraints regarding the medium types (HL and LH types) and the LL-type buyer.

$$\text{Candidate 2: } \begin{cases} R_{LH} = \Delta^B q'^B_{LL} + \bar{u} \\ R_{HL} = \Delta^A q'^A_{LL} + \Delta^B q'^B_{LL} + \Delta^A \bar{q'} - \Delta^B q'^B_{LH} + \bar{u} \end{cases}$$

This second candidate set implies that $LH \to LL$, $HL \to LL$, and $HL \to LH$ are binding constraints regarding the medium types (HL and LH types) and the LL-type buyer.

$$\text{Candidate 3: } \begin{cases} R_{LH} = \Delta^B q'^B_{LL} + \Delta^A q'^A_{LL} + \Delta^B q'^B_{HL} - \Delta^A \bar{q'} + \bar{u} \\ R_{HL} = \Delta^A q'^A_{LL} + \bar{u} \end{cases}$$

This last candidate set implies that $LH \to LL$, $HL \to LL$, and $LH \to HL$ are binding constraints regarding the medium types (HL and LH types) and the LL-type buyer.

Candidates 1 and 2 would result in the violation of IC $HH \to LL$. Candidate 2 does not violate the set of IC constraints. Thus, as a result of the cap, the IC constraints $LH \to LL$, $HL \to LL$, and $LH \to HL$ bind with equality.

Notice that $HH \to LH$ and $LH \to HH$ are equivalent under the cap. This implies the bunching together of buyers HH and LH. The IC restrictions $HH \to LH$ and $HH \to HL$

are satisfied as long as $LH \to LL$, $HL \to LL$, and $LH \to HL$ bind with equality. In other words, if the downward IC restrictions involving the LH-type are satisfied, the HH-type will not purchase an option intended to serve either the HL or the LL types.

In other words, as the quantity of product A becomes smaller due to more and more restrictive cap rules, the incentive constraint $LH \to HL$ becomes relevant.

The analysis for the severe cap is analogous.

Proof of proposition 3

To proof this proposition, it is sufficient to notice that a mild cap does not distort any downward incentive compatibility constraint. A mild cap only reduces q^A_{HH}, and this variable is absent in all IC restrictions shown in equations 4 and 5.

Proof of proposition 4

This proof has two parts. First, I will show that a moderate cap (where the regulation is intended to reduce the portion below the second largest alternative) impacts q^A_{HH}, q^A_{HL}, and q^A_{LH}. This facilitates showing the welfare effects. The second part addresses each of the three claims within proposition 4 separately.

First part: Without intervention, the second largest portion is q^A_{LH}. This can be verified as follows. By examining the corresponding first order conditions, it can be deduced that $q^A_{HH} = q^A_{HL}$ in all IC structures. Thus, the candidates for "second largest" portion are either q^A_{LH} and q^A_{LL}. We can find ot which one is largest by comparing the corresponding FOCs.

In IC structures Ψ and Ω, it is straightforward to find out that $q^A_{LH} > q^A_{LL}$.

Moving to IC structures Γ and Υ, bundling requires $\beta_{HH}\beta_{LL} - \beta_{LH}\beta_{HL} < 0$, this implies $\frac{\beta_{HH}}{\beta_{LH}} < \frac{\beta_{HL}}{\beta_{LL}}$. Bearing this in mind, comparing the corresponding FOCs implies $q^A_{LH} > q^A_{LL}$.

Second part: There are three claims within proposition 4.

First, R_{LL} is unaffected. This is straightforward to corroborate, because the participation constraint of this buyer type always binds with equality. In other words, this buyer was already receiving no rents before the cap. Because the LL-type's outside option is zero, no restriction can push R_{LL} below this value.

Second, R_{HL} is affected by the both moderate and severe caps. This is straightforward to corroborate as $R_{HL} = \Delta^B q'^A_{LL}$ and q^A_{LL} is reduced either indirectly or directly by the moderate and severe caps.

Third, R_{LH} increases following either a moderate or severe cap. Before the cap, $R_{LH} = \Delta^B q'^B_{LL}$. As shown in the proof of proposition 1, the rents earned by the LH-type with caps are the following:

$$\text{With moderate cap: } R_{LH} = \Delta^B u(\widetilde{q^B_{LL}}) + \Delta^A \widetilde{q'^A_{LL}} + \Delta^B \widetilde{q'^B_{HL}} - \Delta^A \bar{q}' + \bar{u}$$

$$\text{With severe cap: } R_{LH} = \Delta^B \widehat{q'^B_{LL}} + \Delta^B \widehat{q'^B_{HL}} + \bar{u}$$

Both of them are strictly larger than the base unregulated R_{LH}.

Impact of moderate and severe caps on R_{HH}

***Proposition** A2.* *Moderate and severe caps reduce R_{HH}, the rents earned by the HH-type are negatively affected as long as the following inequalities hold:*

$$\textbf{\textit{ICS}}\ \Gamma\textbf{\textit{:}}\quad \begin{cases} \textit{Moderate: } \Delta^A q'^A_{LH} + \Delta^B q'^B_{HL} > \Delta^A \widetilde{q'^A_{LL}} + \Delta^B \widetilde{q'^B_{LL}} + \Delta^B \widetilde{q'^B_{HL}} + \bar{u} \\[2ex] \textit{Severe: } \Delta^A q'^A_{LH} + \Delta^B q'^B_{HL} > \Delta^A \bar{q}' + \Delta^B \widehat{q'^B_{LL}} + \Delta^B \widehat{q'^B_{HL}} + \bar{u} \end{cases}$$

$$\textbf{\textit{ICS}}\ \Upsilon\textbf{\textit{:}}\quad \begin{cases} \textit{Moderate: } \Delta^A q'^A_{LH} + \Delta^B q'^B_{LL} > \Delta^A \widetilde{q'^A_{LL}} + \Delta^B \widetilde{q'^B_{LL}} + \Delta^B \widetilde{q'^B_{HL}} + \bar{u} \\[2ex] \textit{Severe: } \Delta^A q'^A_{LH} + \Delta^B q'^B_{LL} > \Delta^A \bar{q}' + \Delta^B \widehat{q'^B_{LL}} + \Delta^B \widehat{q'^B_{HL}} + \bar{u} \end{cases}$$

$$\textbf{\textit{ICS}}\ \Psi\textbf{\textit{:}}\quad \begin{cases} \textit{Moderate: } \Delta^A q'^A_{LL} + \Delta^B q'^B_{HL} > \Delta^A \widetilde{q'^A_{LL}} + \Delta^B \widetilde{q'^B_{LL}} + \Delta^B \widetilde{q'^B_{HL}} + \bar{u} \\[2ex] \textit{Severe: } \Delta^A q'^A_{LL} + \Delta^B q'^B_{HL} > \Delta^A \bar{q}' + \Delta^B \widehat{q'^B_{LL}} + \Delta^B \widehat{q'^B_{HL}} + \bar{u} \end{cases}$$

$$\textbf{\textit{ICS}}\ \Omega\textbf{\textit{:}}\quad \begin{cases} \textit{Moderate: } \Delta^A q'^A_{LL} + \Delta^B q'^B_{LL} > \Delta^A \widetilde{q'^A_{LL}} + \Delta^B \widetilde{q'^B_{LL}} + \Delta^B \widetilde{q'^B_{HL}} + \bar{u} \\[2ex] \textit{Severe: } \Delta^A q'^A_{LL} + \Delta^B q'^B_{LL} > \Delta^A \bar{q}' + \Delta^B \widehat{q'^B_{LL}} + \Delta^B \widehat{q'^B_{HL}} + \bar{u} \end{cases}$$

The proof is a simple comparison between the rents earned by the HH-type before and after the cap, so I omit it.

First Order Conditions

IC-Structure Γ baseline:

$$FOC[q^A_{HH}] : \theta^A_H \frac{\partial q'^A_{HH}}{\partial q^A_{HH}} = \frac{\partial c(q^A_{HH})}{\partial q^A_{HH}} \qquad FOC[q^B_{HH}] : \theta^B_H \frac{\partial q'^B_{HH}}{\partial q^B_{HH}} = \frac{\partial c(q^B_{HH})}{\partial q^B_{HH}}$$

$$FOC[q^A_{HL}] : \theta^A_H \frac{\partial q'^A_{HL}}{\partial q^A_{HL}} = \frac{\partial c(q^A_{HL})}{\partial q^A_{HL}} \qquad FOC[q^B_{HL}] : \theta^B_L \frac{\partial q'^B_{HL}}{\partial q^B_{HL}} = \frac{\frac{\partial c(q^B_{HL})}{\partial q^B_{HL}}}{\left(1 - \frac{\beta_{HH}}{\beta_{HL}} \frac{\Delta^B}{\theta^B_L}\right)}$$

$$FOC[q^A_{LH}] : \theta^A_L \frac{\partial q'^A_{LH}}{\partial q^A_{LH}} = \frac{\frac{\partial c(q^A_{LH})}{\partial q^A_{LH}}}{\left(1 - \frac{\beta_{HH}}{\beta_{LH}} \frac{\Delta^A}{\theta^A_L}\right)} \qquad FOC[q^B_{LH}] : \theta^B_H \frac{\partial q'^B_{LH}}{\partial q^B_{LH}} = \frac{\partial c(q^B_{LH})}{\partial q^B_{LH}}$$

$$FOC[q^A_{LL}] : \theta^A_L \frac{\partial q'^A_{LL}}{\partial q^A_{LL}} = \frac{\frac{\partial c(q^A_{LL})}{\partial q^A_{LL}}}{\left(1 - \frac{\beta_{HL}}{\beta_{LL}} \frac{\Delta^A}{\theta^A_L}\right)} \qquad FOC[q^B_{LL}] : \theta^B_L \frac{\partial q'^B_{LL}}{\partial q^B_{LH}} = \frac{\frac{\partial c(q^B_{LL})}{\partial q^B_{LL}}}{\left(1 - \frac{\beta_{LH}}{\beta_{LL}} \frac{\Delta^B}{\theta^B_L}\right)}$$

IC-Structure ϒ baseline:

$$FOC[q^A_{HH}] : \theta^A_H \frac{\partial q'^A_{HH}}{\partial q^A_{HH}} = \frac{\partial c(q^A_{HH})}{\partial q^A_{HH}} \qquad FOC[q^B_{HH}] : \theta^B_H \frac{\partial q'^B_{HH}}{\partial q^B_{HH}} = \frac{\partial c(q^B_{HH})}{\partial q^B_{HH}}$$

$$FOC[q^A_{HL}] : \theta^A_H \frac{\partial q'^A_{HL}}{\partial q^A_{HL}} = \frac{\partial c(q^A_{HL})}{\partial q^A_{HL}} \qquad FOC[q^B_{HL}] : \theta^B_L \frac{\partial q'^B_{HL}}{\partial q^B_{HL}} = \frac{\partial c(q^B_{HL})}{\partial q^B_{HL}}$$

$$FOC[q^A_{LH}] : \theta^A_L \frac{\partial q'^A_{LH}}{\partial q^A_{LH}} = \frac{\frac{\partial c(q^A_{LH})}{\partial q^A_{LH}}}{\left(1 - \frac{\beta_{HH}}{\beta_{LH}} \frac{\Delta^A}{\theta^A_L}\right)} \qquad FOC[q^B_{LH}] : \theta^B_H \frac{\partial q'^B_{LH}}{\partial q^B_{LH}} = \frac{\partial c(q^B_{LH})}{\partial q^B_{LH}}$$

$$FOC[q^A_{LL}] : \theta^A_L \frac{\partial q'^A_{LL}}{\partial q^A_{LL}} = \frac{\frac{\partial c(q^A_{LL})}{\partial q^A_{LL}}}{\left(1 - \frac{\beta_{HL}}{\beta_{LL}} \frac{\Delta^A}{\theta^A_L}\right)} \qquad FOC[q^B_{LL}] : \theta^B_L \frac{\partial q'^B_{LL}}{\partial q^B_{LL}} = \frac{\frac{\partial c(q^B_{LL})}{\partial q^B_{LL}}}{\left(1 - \frac{\beta_{LH}+\beta_{HH}}{\beta_{LL}} \frac{\Delta^B}{\theta^B_L}\right)}$$

IC-Structure Ψ baseline:

$$FOC[q^A_{HH}] : \theta^A_H \frac{\partial q'^A_{HH}}{\partial q^A_{HH}} = \frac{\partial c(q^A_{HH})}{\partial q^A_{HH}} \qquad FOC[q^B_{HH}] : \theta^B_H \frac{\partial q'^B_{HH}}{\partial q^B_{HH}} = \frac{\partial c(q^B_{HH})}{\partial q^B_{HH}}$$

$$FOC[q^A_{HL}] : \theta^A_H \frac{\partial q'^A_{HL}}{\partial q^A_{HL}} = \frac{\partial c(q^A_{HL})}{\partial q^A_{HL}} \qquad FOC[q^B_{HL}] : \theta^B_L \frac{\partial q'^B_{HL}}{\partial q^B_{HL}} = \frac{\frac{\partial c(q^B_{HL})}{\partial q^B_{HL}}}{\left(1 - \frac{\beta_{HH}}{\beta_{HL}} \frac{\Delta^B}{\theta^B_L}\right)}$$

$$FOC[q^A_{LH}] : \theta^A_L \frac{\partial q'^A_{LH}}{\partial q^A_{LH}} = \frac{\partial c(q^A_{LH})}{\partial q^A_{LH}} \qquad FOC[q^B_{LH}] : \theta^B_H \frac{\partial q'^B_{LH}}{\partial q^B_{LH}} = \frac{\partial c(q^B_{LH})}{\partial q^B_{LH}}$$

$$FOC[q^A_{LL}] : \theta^A_L \frac{\partial q'^A_{LL}}{\partial q^A_{LL}} = \frac{\frac{\partial c(q^A_{LL})}{\partial q^A_{LL}}}{\left(1 - \frac{\beta_{HL}+\beta_{HH}}{\beta_{LL}} \frac{\Delta^A}{\theta^A_L}\right)} \qquad FOC[q^B_{LL}] : \theta^B_L \frac{\partial q'^B_{LL}}{\partial q^B_{LL}} = \frac{\frac{\partial c(q^B_{LL})}{\partial q^B_{LL}}}{\left(1 - \frac{\beta_{LH}}{\beta_{LL}} \frac{\Delta^B}{\theta^B_L}\right)}$$

IC-Structure Ω baseline:

$$FOC[q^A_{HH}] : \theta^A_H \frac{\partial q'^A_{HH}}{\partial q^A_{HH}} = \frac{\partial c(q^A_{HH})}{\partial q^A_{HH}} \qquad FOC[q^B_{HH}] : \theta^B_H \frac{\partial q'^B_{HH}}{\partial q^B_{HH}} = \frac{\partial c(q^B_{HH})}{\partial q^B_{HH}}$$

$$FOC[q^A_{HL}] : \theta^A_H \frac{\partial q'^A_{HL}}{\partial q^A_{HL}} = \frac{\partial c(q^A_{HL})}{\partial q^B_{HL}} \qquad FOC[q^B_{HL}] : \theta^B_L \frac{\partial q'^B_{HL}}{\partial q^B_{HL}} = \frac{\partial c(q^B_{HL})}{\partial q^B_{HL}}$$

$$FOC[q^A_{LH}] : \theta^A_L \frac{\partial q'^A_{LH}}{\partial q^A_{LH}} = \frac{\partial c(q^A_{LH})}{\partial q^A_{LH}} \qquad FOC[q^B_{LH}] : \theta^B_H \frac{\partial q'^B_{LH}}{\partial q^B_{LH}} = \frac{\partial c(q^B_{LH})}{\partial q^B_{LH}}$$

$$FOC[q^A_{LL}] : \theta^A_L \frac{\partial q'^A_{LL}}{\partial q^A_{LL}} = \frac{\frac{\partial c(q^A_{LL})}{\partial q^A_{LL}}}{\left(1 - \frac{\beta_{HL}+\beta_{HH}}{\beta_{LL}} \frac{\Delta^A}{\theta^A_L}\right)} \qquad FOC[q^B_{LL}] : \theta^B_L \frac{\partial q'^B_{LL}}{\partial q^B_{LL}} = \frac{\frac{\partial c(q^B_{LL})}{\partial q^B_{LL}}}{\left(1 - \frac{\beta_{LH}+\beta_{HH}}{\beta_{LL}} \frac{\Delta^B}{\theta^B_L}\right)}$$

Mild IC-Structure Γ:

$$FOC[\bar{q}] : \theta_H^A \frac{\partial \bar{q}'}{\partial \bar{q}} = \frac{\partial c(\bar{q})}{\partial \bar{q}}$$

$$FOC[q_{HH}^B] : \theta_H^B \frac{\partial q_{HH}'^B}{\partial q_{HH}^B} = \frac{\partial c(q_{HH}^B)}{\partial q_{HH}^B}$$

$$FOC[q_{HL}^A] : \theta_H^A \frac{\partial q_{HL}'^A}{\partial q_{HL}^A} = \frac{\partial c(q_{HL}^A)}{\partial q_{HL}^A}$$

$$FOC[q_{HL}^B] : \theta_L^B \frac{\partial q_{HL}'^B}{\partial q_{HL}^B} = \frac{\frac{\partial c(q_{HL}^B)}{\partial q_{HL}^B}}{\left(1 - \frac{\beta_{HH}}{\beta_{HL}} \frac{\Delta^B}{\theta_L^B}\right)}$$

$$FOC[q_{LH}^A] : \theta_L^A \frac{\partial q_{LH}'^A}{\partial q_{LH}^A} = \frac{\frac{\partial c(q_{LH}^A)}{\partial q_{LH}^A}}{\left(1 - \frac{\beta_{HH}}{\beta_{LH}} \frac{\Delta^A}{\theta_L^A}\right)}$$

$$FOC[q_{LH}^B] : \theta_H^B \frac{\partial q_{LH}'^B}{\partial q_{LH}^B} = \frac{\partial c(q_{LH}^B)}{\partial q_{LH}^B}$$

$$FOC[q_{LL}^A] : \theta_L^A \frac{\partial q_{LL}'^A}{\partial q_{LL}^A} = \frac{\frac{\partial c(q_{LL}^A)}{\partial q_{LL}^A}}{\left(1 - \frac{\beta_{HL}}{\beta_{LL}} \frac{\Delta^A}{\theta_L^A}\right)}$$

$$FOC[q_{LL}^B] : \theta_L^B \frac{\partial q_{LL}'^B}{\partial q_{LL}^B} = \frac{\frac{\partial c(q_{LL}^B)}{\partial q_{LL}^B}}{\left(1 - \frac{\beta_{LH}}{\beta_{LL}} \frac{\Delta^B}{\theta_L^B}\right)}$$

Mild cap IC-Structure Υ:

$$FOC[\bar{q}] : \theta_H^A \frac{\partial \bar{q}'}{\partial \bar{q}} = \frac{\partial c(\bar{q})}{\partial \bar{q}}$$

$$FOC[q_{HH}^B] : \theta_H^B \frac{\partial q_{HH}'^B}{\partial q_{HH}^B} = \frac{\partial c(q_{HH}^B)}{\partial q_{HH}^B}$$

$$FOC[q_{HL}^A] : \theta_H^A \frac{\partial q_{HL}'^A}{\partial q_{HL}^A} = \frac{\partial c(q_{HL}^A)}{\partial q_{HL}^A}$$

$$FOC[q_{HL}^B] : \theta_L^B \frac{\partial q_{HL}'^B}{\partial q_{HL}^B} = \frac{\partial c(q_{HL}^B)}{\partial q_{HL}^B}$$

$$FOC[q_{LH}^A] : \theta_L^A \frac{\partial q_{LH}'^A}{\partial q_{LH}^A} = \frac{\frac{\partial c(q_{LH}^A)}{\partial q_{LH}^A}}{\left(1 - \frac{\beta_{HH}}{\beta_{LH}} \frac{\Delta^A}{\theta_L^A}\right)}$$

$$FOC[q_{LH}^B] : \theta_H^B \frac{\partial q_{LH}'^B}{\partial q_{LH}^B} = \frac{\partial c(q_{LH}^B)}{\partial q_{LH}^B}$$

$$FOC[q_{LL}^A] : \theta_L^A \frac{\partial q_{LL}'^A}{\partial q_{LL}^A} = \frac{\frac{\partial c(q_{LL}^A)}{\partial q_{LL}^A}}{\left(1 - \frac{\beta_{HL}}{\beta_{LL}} \frac{\Delta^A}{\theta_L^A}\right)}$$

$$FOC[q_{LL}^B] : \theta_L^B \frac{\partial q_{LL}'^B}{\partial q_{LL}^B} = \frac{\frac{\partial c(q_{LL}^B)}{\partial q_{LL}^B}}{\left(1 - \frac{\beta_{LH}+\beta_{HH}}{\beta_{LL}} \frac{\Delta^B}{\theta_L^B}\right)}$$

Mild cap IC-Structure Ψ:

$$FOC[\bar{q}] : \theta_H^A \frac{\partial \bar{q}'}{\partial \bar{q}} = \frac{\partial c(\bar{q})}{\partial \bar{q}}$$

$$FOC[q_{HH}^B] : \theta_H^B \frac{\partial q_{HH}'^B}{\partial q_{HH}^B} = \frac{\partial c(q_{HH}^B)}{\partial q_{HH}^B}$$

$$FOC[q_{HL}^A] : \theta_H^A \frac{\partial q_{HL}'^A}{\partial q_{HL}^A} = \frac{\partial c(q_{HL}^A)}{\partial q_{HL}^A}$$

$$FOC[q_{HL}^B] : \theta_L^B \frac{\partial q_{HL}'^B}{\partial q_{HL}^B} = \frac{\frac{\partial c(q_{HL}^B)}{\partial q_{HL}^B}}{\left(1 - \frac{\beta_{HH}}{\beta_{HL}} \frac{\Delta^B}{\theta_L^B}\right)}$$

$$FOC[q_{LH}^A] : \theta_L^A \frac{\partial q_{LH}'^A}{\partial q_{LH}^A} = \frac{\partial c(q_{LH}^A)}{\partial q_{LH}^A}$$

$$FOC[q_{LH}^B] : \theta_H^B \frac{\partial q_{LH}'^B}{\partial q_{LH}^B} = \frac{\partial c(q_{LH}^B)}{\partial q_{LH}^B}$$

$$FOC[q_{LL}^A] : \theta_L^A \frac{\partial q_{LL}'^A}{\partial q_{LL}^A} = \frac{\frac{\partial c(q_{LL}^A)}{\partial q_{LL}^A}}{\left(1 - \frac{\beta_{HL}+\beta_{HH}}{\beta_{LL}} \frac{\Delta^A}{\theta_L^A}\right)}$$

$$FOC[q_{LL}^B] : \theta_L^B \frac{\partial q_{LL}'^B}{\partial q_{LL}^B} = \frac{\frac{\partial c(q_{LL}^B)}{\partial q_{LL}^B}}{\left(1 - \frac{\beta_{LH}}{\beta_{LL}} \frac{\Delta^B}{\theta_L^B}\right)}$$

Mild IC-Structure Ω:

$$FOC[\bar{q}] : \theta_H^A \frac{\partial \bar{q'}}{\partial \bar{q}} = \frac{\partial c(\bar{q})}{\partial \bar{q}}$$

$$FOC[q_{HL}^A] : \theta_H^A \frac{\partial q'^A_{HL}}{\partial q_{HL}^A} = \frac{\partial c(q_{HL}^A)}{\partial q_{HL}^A}$$

$$FOC[q_{LH}^A] : \theta_L^A \frac{\partial q'^A_{LH}}{\partial q_{LH}^A} = \frac{\partial c(q_{LH}^A)}{\partial q_{LH}^A}$$

$$FOC[q_{LL}^A] : \theta_L^A \frac{\partial q'^A_{LL}}{\partial q_{LL}^A} = \frac{\frac{\partial c(q_{LL}^A)}{\partial q_{LL}^A}}{\left(1 - \frac{\beta_{HL}+\beta_{HH}}{\beta_{LL}} \frac{\Delta^A}{\theta_L^A}\right)}$$

$$FOC[q_{HH}^B] : \theta_H^B \frac{\partial q'^B_{HH}}{\partial q_{HH}^B} = \frac{\partial c(q_{HH}^B)}{\partial q_{HH}^B}$$

$$FOC[q_{HL}^B] : \theta_L^B \frac{\partial q'^B_{HL}}{\partial q_{HL}^B} = \frac{\partial c(q_{HL}^B)}{\partial q_{HL}^B}$$

$$FOC[q_{LH}^B] : \theta_H^B \frac{\partial q'^B_{LH}}{\partial q_{LH}^B} = \frac{\partial c(q_{LH}^B)}{\partial q_{LH}^B}$$

$$FOC[q_{LL}^B] : \theta_L^B \frac{\partial q'^B_{LL}}{\partial q_{LL}^B} = \frac{\frac{\partial c(q_{LL}^B)}{\partial q_{LL}^B}}{\left(1 - \frac{\beta_{LH}+\beta_{HH}}{\beta_{LL}} \frac{\Delta^B}{\theta_L^B}\right)}$$

Moderate cap:

$$FOC[\bar{q}] : \theta_H^A \frac{\partial \bar{q'}}{\partial \bar{q}} = \frac{\beta_{HH}+\beta_{HL}+\beta_{LH}}{\beta_{HH}+\beta_{HL}+\beta_{LH}\left(\frac{\theta_L^A}{\theta_H^A}+\frac{\Delta^A}{\theta_H^A}\right)} \cdot \frac{\partial c(\bar{q})}{\partial \bar{q}}$$

$$FOC[q_{HH}^B] : \theta_H^B \frac{\partial q'^B_{HH}}{\partial q_{HH}^B} = \frac{\partial c(q_{HH}^B)}{\partial q_{HH}^B}$$

$$FOC[q_{HL}^B] : \theta_L^B \frac{\partial q'^B_{HL}}{\partial q_{HL}^B} = \frac{\frac{\partial c(q_{HL}^B)}{\partial q_{HL}^B}}{\left(1 - \frac{\beta_{LH}+\beta_{HH}}{\beta_{HL}} \frac{\Delta^B}{\theta_L^B}\right)}$$

$$FOC[q_{LH}^B] : \theta_H^B \frac{\partial q'^B_{LH}}{\partial q_{LH}^B} = \frac{\partial c(q_{LH}^B)}{\partial q_{LH}^B}$$

$$FOC[q_{LL}^A] : \theta_L^A \frac{\partial q'^A_{LL}}{\partial q_{LL}^A} = \frac{\frac{\partial c(q_{LL}^A)}{\partial q_{LL}^A}}{\left(1 - \frac{\beta_{LH}+\beta_{HL}+\beta_{HH}}{\beta_{LL}} \frac{\Delta^A}{\theta_L^A}\right)}$$

$$FOC[q_{LL}^B] : \theta_L^B \frac{\partial q'^B_{LL}}{\partial q_{LL}^B} = \frac{\frac{\partial c(q_{LL}^B)}{\partial q_{LL}^B}}{\left(1 - \frac{\beta_{LH}+\beta_{HH}}{\beta_{LL}} \frac{\Delta^B}{\theta_L^B}\right)}$$

Severe cap

$$FOC[\bar{q}] : \theta_H^A \frac{\partial \bar{q}'}{\partial \bar{q}} = \frac{\theta_H^A}{\theta_L^A} \frac{\partial c(\bar{q})}{\partial \bar{q}}$$

$$FOC[q_{HH}^B] : \theta_H^B \frac{\partial q'^B_{HH}}{\partial q_{HH}^B} = \frac{\partial c(q_{HH}^B)}{\partial q_{HH}^B}$$

$$FOC[q_{HL}^B] : \theta_L^B \frac{\partial q'^B_{HL}}{\partial q_{HL}^B} = \frac{\frac{\partial c(q_{HL}^B)}{\partial q_{HL}^B}}{\left(1 - \frac{\beta_{HH}+\beta_{LH}}{\beta_{HL}} \frac{\Delta^B}{\theta_L^B}\right)}$$

$$FOC[q_{LH}^B] : \theta_H^B \frac{\partial q'^B_{LH}}{\partial q_{LH}^B} = \frac{\partial c(q_{LH}^B)}{\partial q_{LH}^B}$$

$$FOC[q_{LL}^B] : \theta_L^B \frac{\partial q'^B_{LL}}{\partial q_{LL}^B} = \frac{\frac{\partial c(q_{LL}^B)}{\partial q_{LL}^B}}{\left(1 - \frac{\beta_{HH}+\beta_{LH}}{\beta_{LL}} \frac{\Delta^B}{\theta_L^B}\right)}$$