An Analysis of Quantity Caps with Multi-Product Sellers

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Abstract

I investigate the impacts of limiting the maximum quantity of one product in a two-goods market with adverse selection when the goods are either unrelated or related. The main goal is to explore the effects on consumer rents and consumption of both products. Following moderate and severe caps, a standard nonlinear pricing model predicts an increase in surplus for buyers with low preference for the capped good and high valuation for the unregulated product. This outcome holds with both independent and related goods. In general, the impacts on other types of buyers and on consumption are mixed and contingent on the model's parameters. Changes in screening are also interesting. Severe enough caps have opposite effects in the seller's segmentation scheme for unrelated and related goods. With independent products, the seller moves from full separation to bunching some types. With related components, the seller moves to separate buyers who have been previously served together. These results are applicable to the food retail industry and others, where limits on quantities are a regulatory alternative and the impact on consumer surplus is an important consideration.

Keywords: nonlinear pricing; quantity cap; adverse selection; multiple products

JEL classification: D82, I18, L51

1 Introduction

I present an analysis of the effects of quantity caps (caps) in a two-product market with four types of privately-informed buyers. Caps are policies restricting the default maximum quantities at which goods can be offered. I rely on a standard nonlinear pricing model to investigate the outcomes following a cap with both independent and related (e.g. complement in consumption) items. I aim to learn i) whether the intervention affects the consumption of both the target and the unregulated goods, ii) the effect on consumer surplus defined as the gross utility from consumption net of the price paid, and iii) the effect on the seller's segmentation strategy. This paper highlights how changes in consumer surplus and allocation following a cap are ultimately contingent on the way the seller adapts her screening scheme to accommodate the policy.

The baseline model is in the spirit of Armstrong and Rochet (1999). It features two products, pure bundling, and negatively correlated discrete buyer types. I refer to the goods as A and B. Roughly speaking, with independent items, the seller offers "small-small", "medium-large", "large-medium", and "large-large" A-B combos. This results in full separation of buyers and weakly increasing rents across types. Assuming that good A is subject to a cap, limiting any option of the good to be strictly lower than the "medium" unregulated portion, results in lower quantities of A consumed by all buyer types, and reductions in the quantity of good B for some buyers. The seller moves from serving a specific option for each type to offer three alternatives, one of which serves two buyers. Regarding consumer surplus, surprisingly, the buyer with low preference for A and high valuation for B is better off.

With related goods and no intervention, the seller offers large, medium, and small packages with equal proportions of A and B, bunching buyer types with mixed preferences (i.e. those with high valuation for one component and low for the other). Moderate caps on A result in a reduction of consumption of A for most buyers except for the lowest type. Naturally, a severe cap reduces the consumption of good A for all buyers. The impacts on consumption of B are mostly contingent on the values of the parameters. Following restrictive enough limits, the seller changes her screening scheme from bunching buyers with mixed preferences to offer tailored alternatives for each type. Interestingly, just as with independent goods, the buyer with high valuation for the unregulated good benefits from the cap.

The model incorporates three stylized observations. First, preferences are private information to the buyers and are taken as exogenous by the seller. It is fair to assume that in most markets taste can be considered exogenous and that sellers design incentive-compatible menus before any transaction occurs. Second, the seller offers more than one product. This reflects what is observed in the field, where most retailers are either multi-product vendors or offer one product which itself is composed of more than one component. Lastly, the seller decides the quantities and prices that characterize each package on the menu. In other words, she does not adopt a passive pricing scheme. Following a restriction in quantities, there is no reason to assume that the seller will not try to endogenously modify the menu to accommodate the intervention in ways that would impact how gains from trade are divided.

The multidimensional character of the model is an important feature. It highlights how, following a cap, surplus distribution and allocation in a market with adverse selection can change solely due to the multidimensional nature of the incentive constraints faced by the seller, even when mixed bundling strategies are absent and there are no complementary interactions between the products, although I relax the later assumption to test the sensitivity of the outcomes to complementarity in consumption.

The results I delineate are informative for industries where second-degree price discrimination is prevalent, quantity caps are a regulatory alternative available to policymakers, and the effect of the intervention on consumer welfare is a primary consideration. Quantity limits as presented in this document are analogous to several policies in the field, including food portion cap rules, limits on ingredients per serving, credence labeling, and even rules of origin in international trade treaties. I briefly present three examples for which the present analysis can be informative to varying degrees.

First, consider the so-called "New York City soda ban" originally proposed to have taken effect in 2013. The advanced plan intended to prohibit food vendors regulated by the city of New York from selling sugar-sweetened beverages in containers exceeding 16 ounces (Kansagra, 2012).[1] Ultimately, the measure was struck down in court. Nonetheless, the proposal sparked vibrant public discussions around its potential ills and virtues. Most debates departed from the premise that a cap would unavoidably hurt consumers.[2] The implication was that caps necessarily lower consumer welfare. This assumption is already shaping public policy. Mississippi's Bill 2687 (2013) interdicts against future restrictions of food sales based upon the product's nutrition information or upon its bundling with other items. This case motivated interesting work among food economists (e.g. Bourquard and Wu (2019)) and I aim to contribute to this body of knowledge. I argue that because sellers implement sophisticated pricing and bundling policies, even if a regulation modifies consumption of the target product, it does not necessarily follow that consumers are worse-off. Moreover, the effects

[1]As a reference, the "small", "medium", and "large" cup sizes typically found in popular American fast-food restaurants contain around 16, 21, and 32 ounces.

[2]Notes in the media where this premise is present to some degree abound. These are two examples Grynbaum (2012); and Grynbaum and Connelly (2012).

on the marketing of non-targeted products often sold in tandem with the regulated good (e.g. fries or burgers in the case of SSBs) remain largely unknown. My work can illuminate some of the consequences of such measures even though the model does not allow for mixed bundling. The insights would be more directly applicable in settings where a majority of the products are sold in combos and "value meals", such as in quick-service restaurants.

In light of studies linking larger portion sizes to increased consumption, foods judged to have deleterious impacts on human health are natural targets of potential caps (Young and Nestle (2002), Ledikwe et al. (2005), Rolls et al. (2006), and Flood et al. (2006)). If we consider foods to be bundles of ingredients, potential regulations limiting the amount of a specific ingredient to be contained in one serving can be analyzed with help of the model in this paper.[3] One example of such a regulation on ingredients use is the "New York City trans fat ban". After a phasing out stage spanning from 2006 to 2008, restaurants holding a New York City Health Department permit were not allowed to store, use or serve foods containing partially hydrogenated vegetable oils, shortenings, or margarines with 0.5 grams or more trans fat per serving. This regulation can be thought of as a cap on the maximum amount of trans fat per serving. The bi-dimensional model in this paper can inform the reader about the effects of rules such as this if we consider trans fats as the regulated good, and group all ingredients other than trans fat as a single composite unregulated product.

Lastly, consider credence labeling regarding country of origin and/or the organic status of the main ingredients. Typically, a producer must satisfy rules regarding the proportion

[3]Continuing with the SSBs example, one can imagine a regulation aiming to limit the maximum amount of sugar to be contained in a single container. One could consider a soda manufacturer deciding sugar-water (call them products A and B) combinations. In this example, the reader can interpret A as "sugar", and B as "all other ingredients". The "package" or bundle is a cup of soda with a particular sugar-water ratio, and the cap would regulate the concentration of sugar within portions.

of ingredients according to a government-defined standard.[4] This is akin to quantity cap on ingredients that do not follow the requirements. The work in this paper can illuminate the effects of such mandates if the sellers desire to exhibit the label in their products and were to strictly follow the requirements.

Even though the model presented in the paper is highly stylized, I abstract away from institutional particularities to examine outcomes from a basic screening problem. I am certain this decision will prove beneficial. Answering a question whose primitives are highly contingent on specific details of a rather narrow context, although important, does little to further our understanding of a general phenomenon. I am convinced that comprehending the way in which price-discriminating sellers react to a limit in quantity (broadly defined) in a fairly general environment will strengthen our understanding of the basic mechanisms behind the potential outcomes of quantity limits in the food retail industry and beyond.

2 Relevant literature

The design of incentive-compatible menus by sellers aiming to segment demand is a well-documented phenomenon. The single-product case has received more attention than the multidimensional scenario (See for example Myerson (1979), Maskin and Riley (1984), Mussa and Rosen (1978), Armstrong and Vickers (1993), and Sonderegger (2011)). Perhaps because multidimensional nonlinear pricing is notorious for being a source of research problems easy to state but difficult to solve analytically or otherwise, the literature on the topic

[4]Take the Canadian example. Only food products with 95% or more content satisfying their organic standards may carry the Canadian Organic Label. Likewise, a food producer may claim "Product of Canada" for her goods when, at most, 2% of the ingredients are non-Canadian.

has remained relatively small. However, the body of knowledge on multi-product screening continues to grow and remains a promising field of study. Knowledge in this area has experienced an impressive expansion from its relatively modest beginnings studying highly stylized instances, and the necessary conditions for screening and bundling to be profitable (Adams and Yellen (1976), McAfee et al. (1989), Armstrong (1999)) to the finding of general results (e.g. Carroll (2017)), albeit in most cases theoretical results remain highly sensitive to assumptions regarding the choice of key parameters.[5]

The progress in this field has provided researchers with a valuable toolkit to study the effects of regulating price-discriminating firms. Besanko et al. (1988) explore the effects of three regulatory measures intending to fix the quantity distortion characteristic of nonlinear pricing (price-discriminating firms distort quantity downward across the type space.): minimum quality standards, maximum price regulation, and rate of return regulation. Besanko and co-authors derive conditions under which the rate of return regulation lowers quantity for the high-types; they also demonstrate that maximum price interventions lower quantity for the high-types, although minimum quality standards do not modify the quantity consumed by the buyers with high valuation for the goods. Corts (1995) analytically studies the effect of imposing a price-cap on the lower level of quantity offered by a multi-product monopolist. Corts relies on a multidimensional version of the Spence-Mirrlees single crossing condition to analyze the multidimensional problem with a one-dimensional screening model. He finds mixed results regarding prices paid by different buyer types. Amrstong et al. (1995) consider two forms of regulations: a cap on the seller's average revenue, and a constraint that forces

[5] For discussions about of the literature on design of multi-product pricing and bundling and its evolution, I direct the reader to Armstrong (1996), and Armstrong (2016)

the seller to keep offering the option to buy a component at the uniform price. Armstrong and co-authors show that the average revenue constraint is preferred by the seller.

The study of quantity caps in multi-product markets is conspicuously absent from the literature. The case of quantity limits in a single-product market is studied by Bourquard and Wu (2019) who take the 2012/2013 New York City soda portion cap rule proposal as a motivating example. This article analytically studies the impacts of cap rules with single-product sellers trading with privately-informed heterogeneous buyers. The authors report that a portion cap reduces consumption without affecting consumer surplus. The reason is that as the cap limits quantity, the seller adjusts prices accordingly so as to leave consumer rents unaffected. My intention is to investigate the degree to which this finding holds in a more general case. Additionally, I aim to observe the indirect effects on screening and on the consumption of the good not directly targeted by the cap.

3 Model with independent goods

The model is largely based on Armstrong and Rochet (1999). The seller (she) is a monopolist offering goods A and B in contracts $\{q^A,\ q^B,\ p\}$, where p is the price charged for a package containing q^A and q^B units of the respective products. The ij-type buyer (he) has private preferences i for good A, and j for B. For each item, he can either have a high (H) or a low (L) preference. There are four buyer types: HH, HL, LH, and LL. The ij-type buyer is characterized by the vector of taste parameters $(\theta_i^A,\ \theta_j^B)$ for $i,\ j =$ H, L. I assume $\theta_H > \theta_L$. If the ij-type buyer pays price p_{ij} for a package containing quantities q_{ij}^A and q_{ij}^B, he earns the following in surplus:[6]

$$R_{ij} = \theta_i^A u(q_{ij}^A) + \theta_j^B u(q_{ij}^B) - p_{ij} \tag{1}$$

The subscripts i and j under R, q^A, q^B, and p indicate the type of consumer. For now, I assume away interactions between the components. I present the model with independent goods first to highlight the interactions between the multidimensional incentive constraints and the seller's pricing behavior across policy scenarios. This assumption provides a neutral background where changes across treatments can confidently be attributed to the impact of quantity restrictions on pricing behavior without the confounding effects of complementarity. Thus, I will show that a cap changes allocation and consumer surplus *even in the absence of complementarity*. I relax this assumption later.

I do not include health benefits in the consumer welfare analysis that would relevant in the case of cap rules in the food retailing industry. This modeling decision incorporates the assumption that health benefits from cap rules are null. I keep this assumption for three main reasons. First, it would be difficult to argue a precise measure of consumer welfare improvement due to the cap. Attempting to do so, would require me to adopt arbitrary assumptions regarding how precisely reduced consumption translates into consumer welfare gains. Second, the set of assumptions could be strategically chosen to generate any desired outcome. Lastly, by ignoring welfare effects from reduced consumption, I am likely underestimating the gains for buyers. The most salient result of the paper is that a subset of consumers is benefited by the cap *even without considering additional health benefits from*

[6] I use the terms "information rents", "rents", "consumer surplus", and "consumer earnings" interchangeably

lowered consumption of the presumed unhealthy food.

I assume the utility function $u(\cdot)$ to be continuous, also $u(0) = 0$, $u'(q) > 0$ and $u''(q) < 0$. The seller and the buyers have reservation values of zero. I assume both goods to have the same differentiable, increasing and convex cost function $c(\cdot)$. Also, $\theta_H u'(0) > c'(0)$ and $\lim_{q\to\infty}\theta_H u'(q) < c'(q)$ for both products, so that trade is possible at least with the HH-type, and total quantity supplied is finite. $\sum_{ij}\beta_{ij} = 1$, so β_{ij} represents the probability that a given buyer is of an ij-type. The seller's expected profit is $\mathbb{E}[\pi] = \sum_{ij}\beta_{ij}\Big[p_{ij} - c(q^A_{ij}) - c(q^B_{ij})\Big]$. It is useful to represent expected profit in terms of total and consumer surpluses:

$$\mathbb{E}[\pi] = \underbrace{\sum_{ij}\beta_{ij}[\theta^A_i u(q^A_{ij}) + \theta^B_j u(q^B_{ij}) - c(q^A_{ij}) - c(q^B_{ij})]}_{\text{Expected total surplus}} - \underbrace{\sum_{ij}\beta_{ij}[\theta^A_i u(q^A_{ij}) + \theta^B_j u(q^B_{ij}) - p_{ij}]}_{\text{Expected consumer surplus}} \quad (2)$$

The seller considers one set of participation constraints (PC), and another of incentive-compatibility restrictions (IC). The satisfaction of PC implies that all types are at least indifferent between buying and opting-out from the trade. Their general form is PC: $R_{ij} \geq 0 \;\forall\; ij$.

The IC restrictions are self-selection conditions providing incentives for the ij-type not to purchase a package originally intended to serve the kl-type buyer ($i \neq k$, and $j \neq l$). In other words, at the optimum, quantities and prices are such that the ij-type buyer is weakly better-off by choosing contract $\{q^A_{ij}, q^B_{ij}, p_{ij}\}$ over an alternative $\{q^A_{kl}, q^B_{kl}, p_{kl}\}$. The seller designs a menu of options such that the ij-type receives a positive surplus in the form of a temptation payoff known as information rents. These are exactly equal to the extraordinary benefit the ij-type would have gained had he chosen the alternative intended for the kl-type from a menu with linear prices. The IC constraints take the following form:

$$\text{IC: } R_{ij} \geq R_{kl} + \underbrace{u(q^A_{kl})(\theta^A_i - \theta^A_k) + u(q^B_{kl})(\theta^B_j - \theta^B_l)}_{\text{Rent gained by the } ij\text{-type from posing as a } kl\text{-type}} \quad \forall\ ij \text{ and } kl; i \neq k \text{ and } j \neq l \tag{3}$$

The complete optimization program includes 8 PC and 12 IC restrictions. The seller's goal is to design options $\{q^A_{ij}, q^B_{ij}, p_{ij}\}$ to maximize expected profit (2) subject to PC and IC constraints. The resulting pricing mechanism is incentive-compatible if it satisfies monotonicity conditions: $q^A_{HH} \geq q^A_{LH}$, $q^A_{HL} \geq q^A_{LL}$, $q^B_{HH} \geq q^B_{HL}$, and $q^B_{LH} \geq q^B_{LL}$. In words, these mean that the quantity of either good is weakly increasing with the corresponding valuation.

In a "relaxed" version of the problem, only a subset of the constraints is included. In this version of the program, the seller ignores the possibility of lower types misrepresenting their preferences and only the "downward" incentive restrictions are incorporated.[7] Similarly, as long as the PC restriction for the LL-type is satisfied, the rest of PCs are also met. In the appendix, I show that the solution to the simplified problem is the solution to the fully constrained program. The relevant IC constraints are graphically depicted in panel (a) of figure 1, where solid lines denote equations binding with equality, and dashed lines indicate restrictions that may or may not bind with strict equality depending on the problem's parameters, especially the distribution of types and taste dispersion ($\Delta \equiv \theta_H - \theta_L$).

[Figure 1 about here.]

[7] In other words, the seller does not consider the possibility of the HL-type choosing the packages intended for either the LH-type or the LL-types and, similarly, she does not have to worry about the LH-type buyer choosing the option designed to serve the HL and/or the HH-type.

3.1 Incentive Compatibility Structures and Bundling

Critically, the information rents R_{ij} received by each buyer type depend on the combination of active incentive constraints. Each set of binding IC restrictions correspond to a particular *Incentive Compatibility structure* (IC-structure, or ICS). With independent goods and no cap, there are four possible IC structures (call them ICS A, B, C, and D). In each of them the rents earned by the LL, LH, and HL types are the following:

$$\begin{aligned} R_{HL} &= \Delta^A u(q_{LL}^A) \\ R_{LH} &= \Delta^B u(q_{LL}^B) \\ R_{LL} &= 0 \end{aligned} \tag{4}$$

Where $\Delta^g \equiv \theta_H^g - \theta_L^g$ for $g = A, B$. Regarding the specific form of the HH-type's IC constraint, this varies across ICS in the following manner:

$$\begin{aligned} &\text{ICS A: } R_{HH} = \Delta^A u(q_{LH}^A) + \Delta^B u(q_{HL}^B) \\ &\text{ICS B: } R_{HH} = \Delta^A u(q_{LH}^A) + \Delta^B u(q_{LL}^B) \\ &\text{ICS C: } R_{HH} = \Delta^A u(q_{LL}^A) + \Delta^B u(q_{HL}^B) \\ &\text{ICS D: } R_{HH} = \Delta^A u(q_{LL}^A) + \Delta^B u(q_{LL}^B) \end{aligned} \tag{5}$$

Although each ICS implies solutions with different point predictions regarding allocation and consumer surplus, some patterns are consistent. Without regulation, consumption weakly increases with type; bundling is observed in the form of larger sizes of product i when the preference for good j rises (more on this later); and consumer rents weakly increase with preferences, with the LL-type earning no rents, the HH-type gaining the largest surplus, and the medium HL and LH-types earning rents in between. Panel A of figure 2 is a helpful illustration of the resulting allocation.[8] Naturally, the effects of a quantity limit are contingent on the IC structure characterizing the regulation-free market. Before continuing with an explanation about what this implies for the consequences of limiting the quantity of one of the goods, I dedicate a few paragraphs to the phenomenon of commodity bundling.

In this document, the seller practices pure bundling: both products are offered together as components of a single package. The products are said to be bundled if the variance in price across different packages is not entirely explained by differences in marginal cost of production. Particularly, if at the solution the quantity of product i increases with the preference for good j, i.e. when $q^A_{LL} < q^A_{LH}$, and/or $q^A_{HL} < q^A_{HH}$, and/or $q^B_{LL} < q^B_{HL}$, and/or $q^B_{LH} < q^B_{HH}$. Roughly speaking, each product is offered in "large", "medium", and "small" options.

The assumption of pure bundling comes with advantages. It reduces the complexity of the multidimensional pricing model and facilitates interpretation. Easiness of interpretation is a scarce virtue among multidimensional screening models, and every opportunity to facilitate the understanding of outcomes is highly valuable. Additionally, the assumption can help to illuminate the effects of caps on pricing practices in products we typically do not think of as bundles. For example, a single food product can be thought of as either the bundle of different ingredients or a composite of ingredients from different origins; likewise, an organic food product is a bundle of organic and non-organic ingredients. Each component of these bundles can be subject to a quantity cap.

[8] I omit scale labels along the vertical axis of both figures because the specific values of these variables depend on the chosen parameters. For some parameter combinations, the LL-type is excluded from participation, and rents for the LL, LH, and HL types are null. These examples better represent the symmetric case ($\theta^A_H \equiv \theta^B_H$, $\theta^A_L \equiv \theta^B_L$, and $\beta_{HL} = \beta_{LH}$). The essence of the result is the same for non-symmetric cases.

The decision to bundle is highly sensitive to the distribution of buyer types. A given distribution of types is conducive to bundling when the correlation of preferences, $\rho = \beta_{HH}\beta_{LL} - \beta_{HL}\beta_{LH}$, is weak enough (Armstrong and Rochet (1999), McAfee et al. (1989), and Adams and Yellen (1976)). Bundling is profitable as long as $\rho < \frac{\beta_{HL}\beta_{LH}}{\beta_{LL}}$. I present results for the case when $\rho < 0 < \frac{\beta^2}{\beta_{LL}}$; this is when the incentive to bundle is the strongest. I take this decision for two reasons. First, I am interested in learning the consequences of adopting caps in markets where bundling is rampant, thus I select the conditions that provide sellers with the strongest incentive to bundle. Second, the pricing strategy followed by a multi-product vendor that does not bundle is akin to single-product nonlinear pricing applied separately to each good, and this case has already been studied (Bourquard and Wu, 2019).

3.2 Quantity cap with independent goods

Without loss of generality, suppose that the quantity limit applies to good A. With the enacted regulation, bundles cannot have quantities of A larger than $\bar{q}$. The seller's objective is to maximize expected profit subject to participation and incentive-compatibility constraints, plus the following quantity cap (QC) restriction:

$$\text{QC:} \;\; q_{ij}^{A} \leq \bar{q} \;\text{ for } i,\, j = L,\, H \tag{6}$$

I analyze three levels of severity at which the cap can be set. The limit can be 1) mild if the limit is set below the largest quantity available without regulation but above the quantity of the second-largest unregulated alternative; 2) moderate if the cap is set below the second-largest unregulated option of good A, but also over the quantity contained in the smallest regulation-free alternative, or 3) severe if the limit on quantity is set at a level lower than the small alternative without the cap. I present the first order conditions characterizing the solution for each level of severity in the online appendix.

It is important to pay attention to how the caps distort the set of binding IC constraints in the expected profit maximization program. These are shown in panel (c) of figure 1.

Proposition *1. With independent goods, a mild cap does not modify the set of binding constraints regardless of the original regulation-free IC-structure. With moderate and severe caps, the set of IC constraints to consider in the seller's problem is:* $\{LH \to LL;\ LH \to HL;\ HL \to LL\}$*, each of them binding with equality.*

Proofs for this and later propositions are in the appendix. It is important to notice that a mild cap does not modify the set of binding constraints, and the seller offers as much of A as she is allowed for the HH and HL types. The moderate restriction lies below the size offered to the LH type but above the option serving the lowest type in the regulation-free baseline. The moderate and severe limits modify the set of binding IC constraints. Under moderate and severe regulations, the size of product A shrinks so much that the seller no longer separates the HH and LH types. Therefore, the seller defaults to offer a single option to serve both of these buyers. This "bunching" of types occurs regardless of the original regulation-free ICS. Panel (c) in figure 1 shows the set of active incentive constraints under moderate and severe restrictions. As the quantity of product A becomes smaller due to more restrictive caps, the seller has to be mindful of the possibility of the LH-type misrepresenting himself as an HL-type and increase the temptation payoff accordingly. With a moderate or severe cap, the incentive constraint preventing unfaithful representation of the LH-type buyer as an HL-type is binding. This can be confirmed using equation 3. This modification renders the downward incentive constraints involving the HH-type redundant.

The effects of the caps on the quantities of both products are summarized in Table 1, and listed in the proposition below. The proofs result from simple direct comparisons between the corresponding first order conditions, thus I omit them. The FOCs are shown in the online appendix. There are two salient effects on the quantities of the regulated good. First, a mild cap affects exclusively the largest options. Second, a moderate restriction not only directly reduces the large and medium options, as intended, but also indirectly impacts the portion of A in the small alternative.

***Proposition** 2.* *With independent goods and regardless of the original ICS, a mild cap affects only the target product and the portion served to the HH-type diminishes. The effects on quantities consumed associated with moderate and severe caps are the following:*

- *Moderate and severe caps reduce the quantity of A consumed by all buyer types.*
- *When the regulation-base IC structures are either ICS A or ICS C, types HL and LL consume less of product B. For the reminding ICS, only the HL-type buyer reduces his consumption of B.*

Because so far interactions between the products are absent, it is surprising to find that the model suggests changes in the quantity of the *unregulated* good B. Moderate and severe caps affect the unregulated product. When passing from regulation-free ICS A or ICS C to a moderate or severe cap, not only the corresponding quantities of good A are reduced, but also q^B_{HL} and q^B_{LL} are affected. When passing from regulation-free ICS B or D to a moderate or severe cap, q^B_{HL} drops. To facilitate the interpretation of the results, panel (b) in figure 2 shows the allocation resulting from a moderate cap for the symmetric case.

The effect on consumer rents is also interesting. As I posit in the propositions below, only moderate and severe caps modify consumer rents. Let wide tildes $(\widetilde{\cdot})$ denote solutions under a moderate restriction, and wide hats $(\widehat{\cdot})$ denote solutions under severe caps.

***Proposition** 3.* *When products are unrelated, only moderate and severe caps modify consumer surplus.*

***Proposition** 4.* *When products are unrelated, moderate and severe caps have the following effect on the rents earned by the LL, LH, and HL types:*

- R_{LL} *is unaffected.*
- R_{HL} *is negatively affected by both moderate and severe cap.*
- R_{LH} *increases following either a moderate or a severe cap.*

Additionally, the LH and HH types are bunched together and served the same alternative.

The LH-type buyer is benefited by moderate and severe caps on A. As shown by the equations in 4, the LH-type earns $\Delta^B u(q^B_{LL})$ without regulation (and with a mild cap) regardless of the original ICS. With moderate and severe caps, the specific functional form of the LH-type rents change in the following manner:

$$\begin{aligned} &\textbf{Moderate cap: } \widetilde{R_{LH}} = \Delta^A[u(\widetilde{q^A_{LL}}) - u(\bar{q})] + \Delta^B u(\widetilde{q^B_{LL}}) + \Delta^B u(\widetilde{q^B_{HL}}) \\ &\textbf{Severe cap: } \widehat{R_{LH}} = \Delta^B u(\widehat{q^B_{LL}}) + \Delta^B u(\widehat{q^B_{HL}}) \end{aligned} \tag{7}$$

Notice that $\widehat{R_{LH}} > \widetilde{R_{LH}}$ because $\Delta^A[u(\widetilde{q^A_{LL}}) - u(\bar{q})] < 0$. In general, I can expect R_{LH} to increase.

Recall that without regulation, the HH-type's rents can take different values depending on the original IC structure. Under moderate and severe caps, this is no longer the case. With moderate and severe caps, the specific form of R_{HH} is the same rents regardless of the original "regulation-free" ICS. The rents earned by the HH-type are negatively affected as long as the regulation-free rents from the corresponding ICS in 5 are strictly less than the following rents under regulation:

$$\begin{aligned} &\text{Moderate cap} : \widetilde{R_{HH}} = \Delta^A u(\widetilde{q^A_{LL}}) + \Delta^B u(\widetilde{q^B_{LL}}) + \Delta^B u(\widetilde{q^B_{HL}}) \\ &\text{Severe cap} : \widehat{R_{HH}} = \Delta^A u(\bar{q}) + \Delta^B u(\widehat{q^B_{LL}}) + \Delta^B u(\widehat{q^B_{HL}}) \end{aligned} \tag{8}$$

***Proposition** 5.* *When products are unrelated, moderate and severe caps reduce R_{HH}, the* *rents earned by the HH-type are negatively affected as long as the following inequalities hold:*

$$\textbf{\textit{ICS A:}} \begin{cases} \textit{Moderate: } \Delta^A u(q^A_{LH}) + \Delta^B u(q^B_{HL}) > \Delta^A u(\widetilde{q^A_{LL}}) + \Delta^B u(\widetilde{q^B_{LL}}) + \Delta^B u(\widetilde{q^B_{HL}}) \\ \textit{Severe: } \Delta^A u(q^A_{LH}) + \Delta^B u(q^B_{HL}) > \Delta^A u(\bar{q}) + \Delta^B u(\widehat{q^B_{LL}}) + \Delta^B u(\widehat{q^B_{HL}}) \end{cases}$$

$$\textbf{\textit{ICS B:}} \begin{cases} \textit{Moderate: } \Delta^A u(q^A_{LH}) + \Delta^B u(q^B_{LL}) > \Delta^A u(\widetilde{q^A_{LL}}) + \Delta^B u(\widetilde{q^B_{LL}}) + \Delta^B u(\widetilde{q^B_{HL}}) \\ \textit{Severe: } \Delta^A u(q^A_{LH}) + \Delta^B u(q^B_{LL}) > \Delta^A u(\bar{q}) + \Delta^B u(\widehat{q^B_{LL}}) + \Delta^B u(\widehat{q^B_{HL}}) \end{cases}$$

$$\textbf{\textit{ICS C:}} \begin{cases} \textit{Moderate: } \Delta^A u(q^A_{LL}) + \Delta^B u(q^B_{HL}) > \Delta^A u(\widetilde{q^A_{LL}}) + \Delta^B u(\widetilde{q^B_{LL}}) + \Delta^B u(\widetilde{q^B_{HL}}) \\ \textit{Severe: } \Delta^A u(q^A_{LL}) + \Delta^B u(q^B_{HL}) > \Delta^A u(\bar{q}) + \Delta^B u(\widehat{q^B_{LL}}) + \Delta^B u(\widehat{q^B_{HL}}) \end{cases}$$

$$\textbf{\textit{ICS D:}} \begin{cases} \textit{Moderate: } \Delta^A u(q^A_{LL}) + \Delta^B u(q^B_{LL}) > \Delta^A u(\widetilde{q^A_{LL}}) + \Delta^B u(\widetilde{q^B_{LL}}) + \Delta^B u(\widetilde{q^B_{HL}}) \\ \textit{Severe: } \Delta^A u(q^A_{LL}) + \Delta^B u(q^B_{LL}) > \Delta^A u(\bar{q}) + \Delta^B u(\widehat{q^B_{LL}}) + \Delta^B u(\widehat{q^B_{HL}}) \end{cases}$$

The proof is a simple comparison between the rents earned by the HH-type before and after the cap, so I omit it. In sum, a severe enough cap on good A will directly reduce the quantity of A consumed by the HH and HL types; indirectly (directly) diminish the size of A served to the LL-type when the cap is moderate (severe); indirectly decrease consumption of B by the HL type regardless of the original ICS, and indirectly diminish the consumption of B by the LL-type for ICS A and C. Likewise, the model predicts a positive effect on the information rents of the LH-type.

A brief discussion on the mechanism driving the model's outcomes may be beneficial. Once the cap is implemented, the seller's desire to price-discriminate continues. The restriction merely reduces her choice space. To accommodate the policy while continuing to segment the demand, the seller has to modify all of the endogenous variables to her disposal, including quantities of product B and the segmentation strategy (i.e. whether to bunch or separate buyer types).

According to the model, the HL-type buyer is offered less of product B when the cap is moderate or severe. The LL buyer might receive less of B depending on the original ICS. I first discuss the adjustments made to the small package. In essence, these are driven by the LL-type's participation constraint and the need to provide positive rents to the LH-type to purchase his own package. Without regulation, information rents for the LH-type are driven by a larger quantity of A compared to the level received by the LL-type buyer. With a moderate cap on A, the LH-type (as well as the HL, and HH types) consumes less of the regulated good A. However, the profit-maximizing seller still needs to provide positive information rents to the LH-type in order to make sure that this buyer will not purchase the small combo designed to serve the LL buyer. Because there is a limit on A, the only way the seller can increase the difference in quantity of A offered to the LL and LH types is by decreasing the quantity of A served to the LL-type buyer. Thus, the LL ought to receive less product A. To maintain the satisfaction of the LL-type's participation constraint, the seller modifies the quantity of B served to this type.

I now turn to discuss the modifications in the package sold to the HL-buyer. These are explained by changes in the smallest package (served to the LL-type consumer), and the fact that the need to separate the HL from the LL-type remains, but the incentives need not be as strong under regulation. Due to the cap, the seller is unable to offer the first best quantity of A to the HL-type buyer. Indeed, the HL buyer purchases considerably less compared to the baseline. The seller still needs to provide incentives to the HL-type in the form of a larger portion of B compared to the LL package. Because the quantity of A contained in the smallest package (that serving the LL-type) is low and indeed smaller compared to the baseline unregulated case, the extra amount of B granted to the HL-type consumer to generate information rents need not be as large.

Regarding the impacts on buyers' surplus, the model predicts an *increase* in the surplus earned by the LH-type (R_{LH}). The intuition behind the increase in the LH-type's well-being is the following. In the unregulated baseline, the LH-type is purchasing a "medium" portion A for which he has a *low* preference. This buyer would prefer a price-discounted "small-large" A-B package; the closest option for them in the unregulated baseline is a price-discounted "medium-large" combo; the "small-small" alternative has too little of product B, whereas the "large-large" package is just too expensive for this buyer. A quantity limit on good A shapes the set of contracts such that the package designed by the seller to serve the buyers with low-high valuation, is closer to this buyers' ideal contract.

4 Model with related goods

In this subsection, I explore the more general case of related goods (products that are complement or substitute in consumption). The most important modification to the model is related to the taste parameters. For complements (superscript C), the following inequalities hold: ($\theta^C_{HH} > \theta^A_H + \theta^B_H$), ($\theta^C_{HL} > \theta^A_H + \theta^B_L$), ($\theta^C_{LH} > \theta^A_L + \theta^B_H$), and ($\theta^C_{LL} > \theta^A_L + \theta^B_L$). Additionally, $\theta^C_{HH} > \theta^C_{HL}$, $\theta^C_{HH} > \theta^C_{LH}$, $\theta^C_{HH} > \theta^C_{LL}$, $\theta^C_{HL} > \theta^C_{LL}$, and $\theta^C_{LH} > \theta^C_{LL}$. For substitutes (superscript S), the following holds: ($\theta^S_{HH} < \theta^A_H + \theta^B_H$), ($\theta^S_{HL} < \theta^A_H + \theta^B_L$), ($\theta^S_{LH} < \theta^A_L + \theta^B_H$), and ($\theta^S_{LL} < \theta^A_L + \theta^B_L$). Additionally, $\theta^S_{HH} > \theta^S_{HL}$, $\theta^S_{HH} > \theta^S_{LH}$, $\theta^S_{HH} > \theta^S_{LL}$, $\theta^S_{HL} > \theta^S_{LL}$, and $\theta^S_{LH} > \theta^S_{LL}$.

Because the analysis below is identical for complements and substitutes, I drop the superscript. Throughout the text, I use complements and related goods as interchangeable terms. The analysis and conclusions hold without change for substitute products. The seller's expected profit is the following:

$$\mathbb{E}[\pi] = \underbrace{\sum_{ij} \beta_{ij}[\theta_{ij}u(q_{ij}^A) + \theta_{ij}u(q_{ij}^B) - c(q_{ij}^A) - c(q_{ij}^B)]}_{\text{Expected total surplus}} - \underbrace{\sum_{ij} \beta_{ij}[\theta_{ij}u(q_{ij}^A) + \theta_{ij}u(q_{ij}^B) - p_{ij}]}_{\text{Expected consumer surplus}} \quad (9)$$

The general form of the PC constraints remains PC: $R_{ij} \geq 0 \; \forall \; ij$. The IC constraints take the following form:

$$\text{IC: } R_{ij} \geq R_{kl} + \underbrace{u(q_{kl}^A)(\theta_{ij} - \theta_{kl}) + u(q_{kl}^B)(\theta_{ij} - \theta_{kl})}_{\text{Rent gained by the } ij\text{-type from posing as a } kl\text{-type}} \quad \forall \; ij \text{ and } kl; i \neq k \text{ and } j \neq l \quad (10)$$

The following definitions will be useful:

$$\theta_{HH} - \theta_{LL} \equiv \Delta_1, \quad \theta_{HH} - \theta_{HL} \equiv \Delta_2, \quad \theta_{HH} - \theta_{LH} \equiv \Delta_3,$$

$$\theta_{HL} - \theta_{LL} \equiv \Delta_4, \quad \theta_{LH} - \theta_{LL} \equiv \Delta_5, \quad \theta_{LH} - \theta_{HL} \equiv \Delta_6,$$

$$\theta_{HL} - \theta_{LH} \equiv \Delta_7$$

Only the downward IC constraints are incorporated into the maximization problem, just as in the case with unrelated products. The set of relevant incentive constraints is illustrated in panel (b) of figure 1. As with independent goods, there are four possible IC structures with complement goods. I will also refer to these as ICS A, B, C, and D.

The set of first order conditions characterizing the solution to the seller's problem with related goods is shown in the appendix. Recall that in this model the goods are said to be bundled if the portion of item i increases with good j. With complement goods, the seller does not bundle the products. Instead, the seller offers three packages with either only "large", only "medium" or only "small" portions of both goods each. The HL and LH-types are served the same option. Panel (c) in figure 2 serves as an example of the effects in the symmetric case.

Moving on to consumer surplus, without a cap, the HL and LL buyer types receive the same rents independently of the original ICS:

$$\begin{aligned} &R_{HL} = \Delta_4[u(q^A_{LL}) + u(q^B_{LL})] \\ &R_{LL} = 0 \end{aligned} \tag{11}$$

The rents earned by the LH-type in the regulation-free baseline vary in the following way:

$$\begin{aligned} &\text{ICS A, B, and C: } R_{LH} = \Delta_5[u(q^A_{LL}) + u(q^B_{LL})] \\ &\text{ICS D: } R_{LH} = (\Delta_5 + \Delta_4)[u(q^A_{LL}) + u(q^B_{LL})] + \Delta_6[u(q^A_{HL}) + u(q^B_{HL})] \end{aligned} \tag{12}$$

The information rents received by the HH-type depends on the IC structure as follows:

ICS A: $R_{HH} = (\Delta_4 + \Delta_5)[u(q^A_{LL}) + u(q^B_{LL})] + \Delta_3[u(q^A_{LH}) + u(q^B_{LH})] + \Delta_2[u(q^A_{HL}) + u(q^B_{HL})]$

ICS B: $R_{HH} = (\Delta_4 + \Delta_5)[u(q^A_{LL}) + u(q^B_{LL})] + \Delta_3[u(q^A_{LH}) + u(q^B_{LH})] + (\Delta_6 + \Delta_2)[u(q^A_{HL}) + u(q^B_{HL})]$

ICS C: $R_{HH} = (\Delta_1 + \Delta_4 + \Delta_5)[u(q^A_{LL}) + u(q^B_{LL})] + \Delta_3[u(q^A_{LH}) + u(q^B_{LH})] + \Delta_1[u(q^A_{HL}) + u(q^B_{HL})]$

ICS D: $R_{HH} = (\Delta_1 + \Delta_4 + \Delta_5)[u(q^A_{LL}) + u(q^B_{LL})] + \Delta_3[u(q^A_{LH}) + u(q^B_{LH})] + (\Delta_6 + \Delta_2)[u(q^A_{HL}) + u(q^B_{HL})]$ (13)

Regarding consumer surplus; the HH-type receives the most information rents; the LL-type the less, and between these types, the HL and LH buyers receive the same level of surplus.

4.1 Quantity cap with related goods

I continue to study the same three levels of cap severity on product A introduced in the section where I look at the case with independent goods. Recall that the mild cap directly limits only the large portion of A, the moderate regulation directly affects both medium and large portions of A, while the severe limit directly affects all options of A. The seller's goal is to maximize expected profit 9 subject to the relevant IC and PC restrictions plus the quantity cap in 6.

***Proposition** 6.* *Moderate and severe caps change the information rents earned by the HH and LH types to the following:*

- *Moderate cap:*

  $\widetilde{R_{HH}} = (\Delta_4 + \Delta_1)[u(\widetilde{q^A_{LL}}) + u(\widetilde{q^B_{LL}})] + \Delta_2[u(\bar{q}) + u(\widetilde{q^B_{HL}})]$

  $\widetilde{R_{LH}} = (\Delta_5 + \Delta_4)[u(\widetilde{q^A_{LL}}) + u(\widetilde{q^B_{LL}})] + \Delta_6[u(\bar{q}) + u(\widetilde{q^B_{HL}})]$

- *Severe cap:*

  $\widehat{R_{HH}} = (\Delta_4 + \Delta_1)[u(\bar{q}) + u(\widehat{q^B_{LL}})] + \Delta_2[u(\bar{q}) + u(\widehat{q^B_{HL}})]$

  $\widehat{R_{LH}} = (\Delta_5 + \Delta_4)[u(\bar{q}) + u(\widehat{q^B_{LL}})] + \Delta_6[u(\bar{q}) + u(\widehat{q^B_{HL}})]$

Direct comparison with the corresponding information rents equations yield the following results. Following moderate and severe caps and compared to the rents earned without regulation:

- R_{HH} *is diminished.*

- *The effect on* R_{HL} *is ambiguous and depends on the model's specific parameter values.*

- R_{LH} *unambiguously increases for regulation free IC-structures A, B, and C. For IC-structure D,* R_{LH} *increases as long as:*

  Moderate:

  $(\Delta_5+\Delta_4)[u(q^A_{LL})-u(\widetilde{q^A_{LL}})+u(q^B_{LL})-u(\widetilde{q^B_{LL}})]+\Delta_6[u(q^A_{LL})-u(\bar{q})+u(q^B_{HL})-u(\widetilde{q^B_{HL}})] < 0$

  Severe:

  $(\Delta_5+\Delta_4)[u(\bar{q})-u(\widehat{q^A_{LL}})+u(q^B_{LL})-u(\widehat{q^B_{LL}})]+\Delta_6[u(q^A_{LL})-u(\bar{q})+u(q^B_{HL})-u(\widehat{q^B_{HL}})] < 0$

- R_{LL} *remains unaffected.*

The information rents granted to the LH-type are larger under regulation-free ICS A, B and C. For ICS D, the effect on LH is likely to be positive because $u(q^A_{LL}) - u(\widetilde{q^A_{LL}}) < 0$; $u(q^B_{LL}) - u(\widetilde{q^B_{LL}}) < 0$, and $u(q^B_{HL}) - u(\widetilde{q^B_{HL}}) < 0$; likewise $u(q^B_{LL}) - u(\widehat{q^B_{LL}}) < 0$, and $u(q^B_{HL}) - u(\widehat{q^B_{HL}}) < 0$.

***Proposition** 7.*

A mild cap only affects q^A_{HH}. *Following a moderate cap:*

- *q^A_{HH}, q^A_{HL}, and q^A_{LH} are directly affected.*

- *q^A_{LL} increases for original IC-Structure D, and the effect is contingent to the parametrization for the rest of IC-structures.*

- *q_{HH} does not change.*

- *q^B_{HL} unambiguously decreases for ICS A, unambiguously increases for ICS D, and depends on specific parameter values for the rest of IC-structures.*

- *q^B_{LH} unambiguously increases for IC-structures A, B, and C. It decreases for ICS D.*

- *q^B_{LL} increases for the original ICS D, and the effect depends on the specific parameter values for the rest of IC-structures.*

These changes are straightforwardly corroborated by a simple comparison between the corresponding first order conditions. Table 2 summarizes the effect of the cap for each severity level comparing the resulting quantities to the quantities allocated to each type under no regulation. The first order conditions characterizing the solutions are in the appendix. Just as in the case with independent goods, the mild cap does not modify the quantities of the products beyond the largest alternatives.

Moderate and severe caps have more nuanced effects on quantities. I first discuss the effects on product A. By design, a moderate cap directly reduces the large and medium portions of good A. The moderate cap also indirectly affects the quantity of A contained in the small package. When under no regulation, the market is characterized by IC structure A, B, or C a moderate cap can either reduce or increase the quantity of A contained in the smallest package, depending on the value of the model's parameters as shown in the relevant footnote in table 2. For ICS D, where R_{HH} takes the form of last equation in 13, a moderate restriction unambiguously increases the portion of A contained in the small option.

Moving to the impacts on the unregulated good B, with regulation-free ICS A, the quantity of good B served to the HL type unambiguously decreases; with ICS D, this type is served a larger portion of B; while with IC structures B and C the effect is ambiguous and might decrease contingent on particular parameter values. The quantity of product B consumed by buyer type LH increases regardless of the original ICS. While the portion served to buyer type LL unambiguously increases for ICS D, and may decrease for the rest of IC structures depending on parameter values.

For an interpretation of the results observed with related goods, consider seller of SSBs deciding sugar-“water” combinations (possible A-B products, where “water” is a composite good that includes ingredients such as flavoring). Suppose that the context is such that we observe allocations resembling panels (c) and (d) in figure 2. The “package” is a bottle of soda with a particular sugar-water ratio. In this context, the baseline outcomes can be interpreted as follows. Without regulation, the seller decides to produce bottles of soda in three different presentations: small, medium, and large servings all with a one-to-one sugar-water ratio. If the government enacts a limit on the maximum amount grams of sugar contained in a single serving, the seller would accommodate the policy by offering the following four choices. First, a “light” large alternative with low sugar-water ratio serving the HH-type (who, after all, also highly values the ingredients other than sugar contained in the beverage); second, a relatively small option with a concentrated formula with a high sugar-water ratio designed for the HL-type’s sweet taste; third, a smaller “light” alternative serving the health-conscious LH-type; and lastly, a mini serving of the “traditional” formula targeting the LL-type buyer.

5 Conclusion

I present a theoretical analysis of the effects of quantity caps on consumer rents, consumption, and segmentation scheme when sellers offer two products in a pure bundle to serve privately-informed buyers. With independent goods, moderate and severe caps do reduce consumption of the target good by all buyer types. Also, depending on the original IC structure, the consumption of the non-target good B can decrease as well. With related goods, the effects on quantities are mixed. Moderate and severe caps cause the seller of independent goods to bunch together some buyer types who previously were offered tailored alternatives, while the seller of related goods moves from bunching types to finer market segmentation.

Quantity caps can increase consumer well-being for a subset of buyers. The benefited consumer has low valuation for the regulated product but high preference for the unregulated good. Absent a quantity limit, the seller has an incentive to engage in commodity bundling and offer information rents in the form of a relatively larger quantity of the product he values lowly. The cap reduces the extent to which bundling can be leveraged as a sorting device. The offered alternative under a cap is closer to the LH-type's first-best option.

The analysis I present in this paper is based on a partial equilibrium model with no market failures. This limits the degree to which the conclusions can be applied to normative questions about whether government intervention is granted based on social welfare. As one of the first studies on the effects of quantity caps in multi-product markets, this paper serves as a starting point for assessing conclusions that seem plausible and therefore influence policy design without a prior careful analysis; for example, the intuitive idea that caps necessarily hurt consumers. This type of analysis is as important as normative studies because often the success of a particular policy hinges on the ability of proponents to stress benefits over costs.

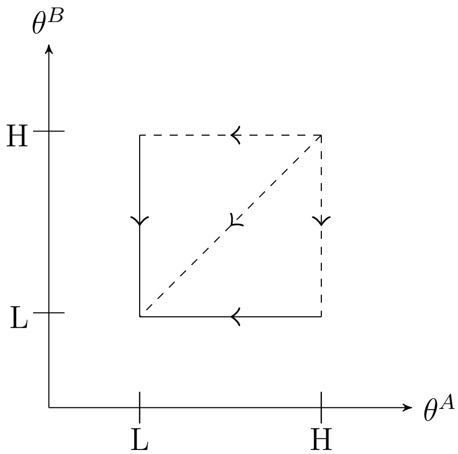
Other researchers can expand the analysis in this paper in multiple forms. For example, by including behavioral or general equilibrium effects. Additional work can produce formal comparisons between the impacts of quantity limits and other popular measures such as excise taxes. Additionally, both theoretical and empirical work on the effects of caps when the seller can practice mixed bundling are natural extensions.

Tables and figures

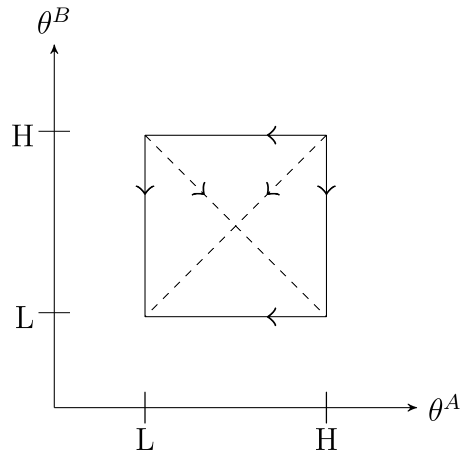
Table 1: Theoretical change in quantities: Independent goods

	q^A_{HH}	q^B_{HH}	q^A_{HL}	q^B_{HL}	q^A_{LH}	q^B_{LH}	q^A_{LL}	q^B_{LL}
IC-Structure A								
Mild	$\downarrow$	$=$	$\downarrow$	$=$	$=$	$=$	$=$	$=$
Moderate	$\downarrow$	$=$	$\downarrow$	$\downarrow$	$\downarrow$	$=$	$\downarrow$	$\downarrow$
Severe	$\downarrow$	$=$	$\downarrow$	$\downarrow$	$\downarrow$	$=$	$\downarrow$	$\downarrow$
IC-Structure B								
Mild	$\downarrow$	$=$	$\downarrow$	$=$	$=$	$=$	$=$	$=$
Moderate	$\downarrow$	$=$	$\downarrow$	$\downarrow$	$\downarrow$	$=$	$\downarrow$	$=$
Severe	$\downarrow$	$=$	$\downarrow$	$\downarrow$	$\downarrow$	$=$	$\downarrow$	$=$
IC-Structure C								
Mild	$\downarrow$	$=$	$\downarrow$	$=$	$=$	$=$	$=$	$=$
Moderate	$\downarrow$	$=$	$\downarrow$	$\downarrow$	$\downarrow$	$=$	$\downarrow$	$\downarrow$
Severe	$\downarrow$	$=$	$\downarrow$	$\downarrow$	$\downarrow$	$=$	$\downarrow$	$\downarrow$
IC-Structure D								
Mild	$\downarrow$	$=$	$\downarrow$	$=$	$=$	$=$	$=$	$=$
Moderate	$\downarrow$	$=$	$\downarrow$	$\downarrow$	$\downarrow$	$=$	$\downarrow$	$=$
Severe	$\downarrow$	$=$	$\downarrow$	$\downarrow$	$\downarrow$	$=$	$\downarrow$	$=$

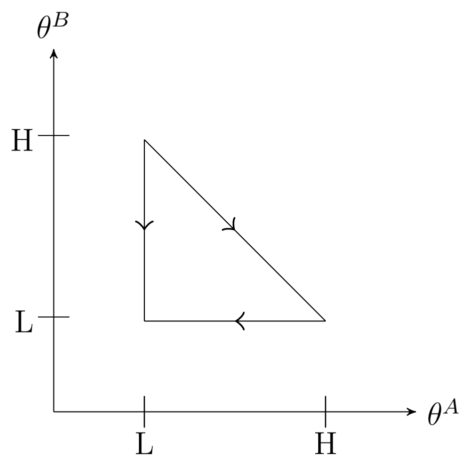
In each case, the comparison is against the baseline scenario.



(a) Baseline - Independent goods



(b) Baseline - Complements



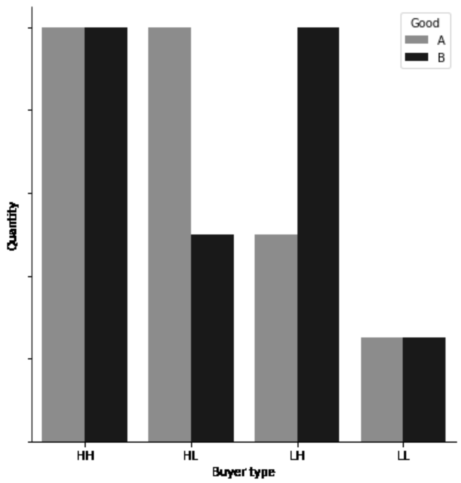
(c) Cap - Complements

Figure 1: Set of IC constraints

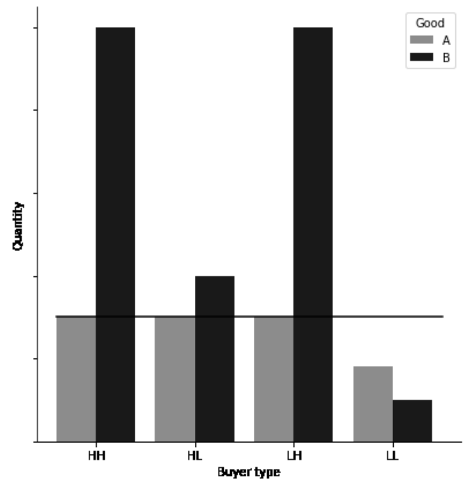
Table 2: Theoretical change in quantities: Complement and substitute goods

	q^A_{HH}	q^B_{HH}	q^A_{HL}	q^B_{HL}	q^A_{LH}	q^B_{LH}	q^A_{LL}	q^B_{LL}
IC-Structure A								
Mild	$\downarrow$	$=$	$=$	$=$	$=$	$=$	$=$	$=$
Moderate	$\downarrow$	$=$	$\downarrow$	$\downarrow$	$\downarrow$	$\uparrow$	$\downarrow^1$	$\downarrow^1$
Severe	$\downarrow$	$=$	$\downarrow$	$\downarrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\downarrow^1$
IC-Structure B								
Mild	$\downarrow$	$=$	$=$	$=$	$=$	$=$	$=$	$=$
Moderate	$\downarrow$	$=$	$\downarrow$	$\downarrow^2$	$\downarrow$	$\uparrow$	$\downarrow^3$	$\downarrow^1$
Severe	$\downarrow$	$=$	$\downarrow$	$\downarrow^2$	$\downarrow$	$\uparrow$	$\downarrow$	$\downarrow^1$
IC-Structure C								
Mild	$\downarrow$	$=$	$=$	$=$	$=$	$=$	$=$	$=$
Moderate	$\downarrow$	$=$	$\downarrow$	$\downarrow^4$	$\downarrow$	$\uparrow$	$\downarrow^5$	$\downarrow^5$
Severe	$\downarrow$	$=$	$\downarrow$	$\downarrow^4$	$\downarrow$	$\uparrow$	$\downarrow$	$\downarrow^5$
IC-Structure D								
Mild	$\downarrow$	$=$	$=$	$=$	$=$	$=$	$=$	$=$
Moderate	$\downarrow$	$=$	$\downarrow$	$\uparrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\uparrow$
Severe	$\downarrow$	$=$	$\downarrow$	$\uparrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\uparrow$

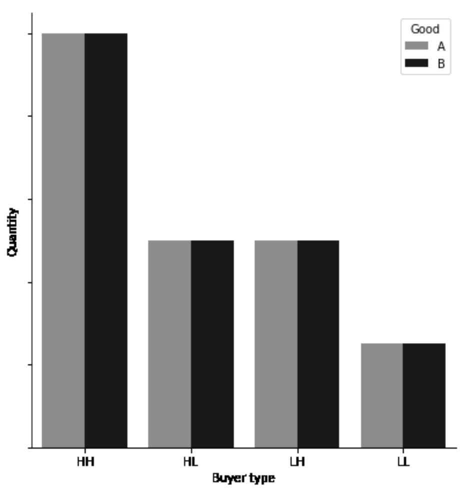
In each case, the comparison is against the baseline scenario. For arrows with superscript, the effect holds if the following inequalities hold:
$^1\,\frac{\beta_{HH}}{\beta_{LH}} < \frac{\Delta_4}{\Delta_5-\Delta_1}$. $^2\beta_{HH} < \beta_{LH}$. $^3\Delta_5 < \Delta_1 + \Delta_4$. $^4\,\frac{\beta_{HH}}{\beta_{LH}} < \frac{\Delta_6}{\Delta_1-\Delta_2}$. $^5\,\frac{\beta_{HH}}{\beta_{LH}} < \frac{\Delta_4}{\Delta_5}$.



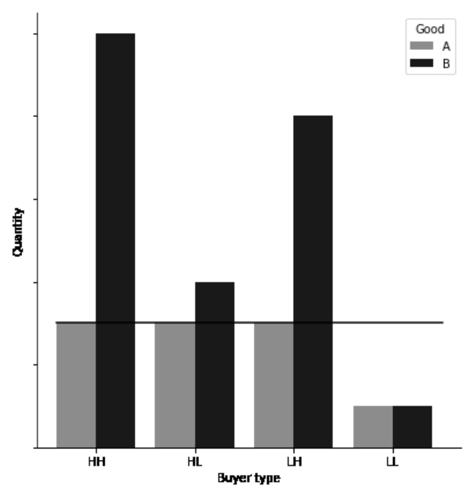
(a) Baseline - Independent goods



(b) Cap - Independent goods



(c) Baseline - Related goods



(d) Cap - Related goods

Figure 2: Allocation by Buyer Types (Theory)

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Appendix A: Proofs for online publication.

The solution to the relaxed problem is the solution to the fully constrained program.

I show this is the case for IC-Structure A. The proofs for the other IC Structures are very similar. For the purposes of this proof, I assume symmetry, that is: $\theta_i^A = \theta_i^L = \theta_i$ for $i =$ H, L. The seller maximizes expected profit subject to the following restrictions:

$$R_{LL} = 0$$

$$R_{LH} = u(q_{LL}^B)\Delta$$

$$R_{HL} = u(q_{LL}^A)\Delta$$

$$R_{HH} = \Delta[u(q_{LL}^A) + u(q_{LL}^B)] + \Delta[u(q_{LH}^A) - u(q_{LL}^A)] + \Delta[u(q_{HL}^B) - u(q_{LL}^B)]$$

$$q_{HH}^A \geq q_{LH}^A,\ q_{HL}^A \geq q_{LL}^A,\ q_{HH}^B \geq q_{HL}^B, q_{LH}^B \geq q_{LL}^B$$

Where: $\Delta \equiv \theta_H - \theta_L$

- Goods A and B are served in a "small" portions. $q_{LL}^* \equiv q_{LL}^{A*} = q_{LL}^{B*}$.
- There is a "medium" portion. $q_{LH}^* \equiv q_{LH}^{A*} = q_{HL}^{B*}$.
- There are "large" portions. $q_{HL}^* \equiv q_{HL}^{A*} = q_{LH}^{B*}$ and $q_{HH*} \equiv q_{HH}^{A*} = q_{HH}^{A*}$.
- The quantities consumed by the LL, LH, HL and HH, respectively are: (q_{LL}^*, q_{LL}^*), (q_{LH}^*, q_{HL}^*), (q_{HL}^*, q_{LH}^*), and (q_{HH}^*, q_{HH}^*).

Consumer Surplus:

$$R^*_{LL} = 0$$

$$R^*_{LH} = \Delta u(q^*_{LL})$$

$$R^*_{HL} = \Delta u(q^*_{LL})$$

$$R^*_{HH} = 2\Delta u(q^*_{LH})$$

***Proposition** 8. I closely follow the proofs in Armstrong and Rochet (1999). Maximizing 2 subject to 5 gives the solution to the seller's fully constrained problem.*

Proof of proposition 8:

Together, $R_{LL} = 0$, the monotonicity constraints, and the four downward binding constraints imply the satisfaction of the omitted IC constraints:

$$R_{LL} > R_{LH} + u(q_{LH})(\theta_L - \theta_H)$$

$$R_{LL} > R_{HL} + u(q_{HL})(\theta_L - \theta_H)$$

$$R_{LL} > R_{HH} + 2[u(q_{HH})(\theta_L - \theta_H)]$$

From the corresponding first order conditions, it is straightforward to conclude that $q_{HL} > q_{LH}$, thus:

$$R_{LH} > R_{HL} + u(q_{HL})(\theta_L - \theta_H) + u(q_{LH})(\theta_H - \theta_L)$$

$$R_{HL} > R_{LH} + u(q_{LH})(\theta_H - \theta_L) + u(q_{HL})(\theta_L - \theta_H)$$

Lastly, the single crossing condition implies:

$$R_{LH} > R_{HH} + u(q_{HH})(\theta_H - \theta_L)$$

$$R_{HL} > R_{HH} + u(q_{HH})(\theta_L - \theta_H)$$

Proof of proposition 1

There are three parts to show:

First, a mild cap does not change the set of relevant IC constraints. A mild cap is defined as that where the limit is set strictly below the maximum unregulated portion of good A, and at or above the unregulated second largest portion of A. Effectively, the only quantity affected is q^A_{HH}. A new IC constraint would be added to the set to consider if the cap causes it to start binding with equality. The only downward IC restrictions affected are the following, all of which contain q^A_{HH} in the left hand side of the inequality:

$$R_{HH} \geq R_{LL} + (\theta^A_H - \theta^A_L)u(q^A_{LL}) + (\theta^B_H - \theta^B_L)u(q^B_{LL})$$

$$R_{HH} \geq R_{LH} + (\theta^A_H - \theta^A_L)u(q^A_{LH}) + (\theta^B_H - \theta^B_H)u(q^B_{LH})$$

$$R_{HH} \geq R_{HL} + (\theta^A_H - \theta^A_H)u(q^A_{HL}) + (\theta^B_H - \theta^B_L)u(q^B_{HL})$$

These are the same IC already included in the original regulation-free IC-structures. The reduction in q^A_{HH} is small enough that none of them change from potentially binding to always binding.

Second, $LH \to LL$ and $HL \to LL$ remain unchanged and binding in all caps because these IC restrictions do not involve q^A_{HH}.

Lastly, with moderate and severe caps, $LH \to HL$ binds with equality and substitutes $HH \to LL$, $HH \to LH$, and $HH \to HL$. The analysis below concerns to the moderate cap.

To proof this part, first recall that the general form of the IC constraints is the following:

$$R_{ij} \geq R_{kl} + u(q_{kl}^A)(\theta_i^A - \theta_k^A) + u(q_{kl}^B)(\theta_j^B - \theta_l^B) \ \forall \ ij \text{ and } kl; i \neq k \text{ and } j \neq l$$

With a moderate cap the specific forms of the all IC program in the complete program are the following:

$$\begin{aligned}
LL \to LH: & \quad R_{LL} \geq R_{LH} - \Delta^B u(q_{LH}^B) \\
LL \to HL: & \quad R_{LL} \geq R_{HL} - \Delta^A u(\bar{q}) \\
LL \to HH: & \quad R_{LL} \geq R_{HH} - \Delta^A u(\bar{q}) - \Delta^B u(q_{HH}^B) \\
LH \to LL: & \quad R_{LH} \geq R_{LL} + \Delta^B u(q_{LL}^B) \\
LH \to HL: & \quad R_{LH} \geq R_{HL} - \Delta^A u(\bar{q}) + \Delta^B u(q_{HL}^B) \\
LH \to HH: & \quad R_{LH} \geq R_{HH} - \Delta^A u(\bar{q}) \\
HL \to LL: & \quad R_{HL} \geq R_{LL} + \Delta^A u(q_{LL}^A) \\
HL \to LH: & \quad R_{HL} \geq R_{LH} + \Delta^A u(\bar{q}) - \Delta^B u(q_{LH}^B) \\
HL \to HH: & \quad R_{HL} \geq R_{HH} - \Delta^B u(q_{HH}^B) \\
HH \to LL: & \quad R_{HH} \geq R_{LL} + \Delta^A u(q_{LL}^A) + \Delta^B u(q_{LL}^B) \\
HH \to LH: & \quad R_{HH} \geq R_{LH} + \Delta^A u(\bar{q}) \\
HH \to HL: & \quad R_{HH} \geq R_{HL} + \Delta^B u(q_{HL}^B)
\end{aligned}$$

Because profit maximization necessitates the satisfaction of the participation constraint for the lowest type, the rents earned by the LL-type continue to be $R_{LL} = 0$.

The cap could have changed the set of binding constraints. Regarding the medium types HL and LH, there are three possible candidates for the form of their information rents following the cap:

$$\text{Candidate 1: } \begin{cases} R_{LH} = \Delta^B u(q^B_{LL}) \\ \\ R_{HL} = \Delta^A u(q^A_{LL}) \end{cases}$$

This first candidate set implies that $LH \rightarrow LL$ and $HL \rightarrow LL$ are the only binding constraints regarding the medium types (HL and LH types) and the LL-type buyer.

$$\text{Candidate 2: } \begin{cases} R_{LH} = \Delta^B u(q^B_{LL}) \\ \\ R_{HL} = \Delta^A u(q^A_{LL}) + \Delta^B u(q^B_{LL}) + \Delta^A u(\bar{q}) - \Delta^B u(q^B_{LH}) \end{cases}$$

This second candidate set implies that $LH \rightarrow LL$, $HL \rightarrow LL$, and $HL \rightarrow LH$ are binding constraints regarding the medium types (HL and LH types) and the LL-type buyer.

$$\text{Candidate 3: } \begin{cases} R_{LH} = \Delta^B u(q^B_{LL}) + \Delta^A u(q^A_{LL}) + \Delta^B u(q^B_{HL}) - \Delta^A u(\bar{q}) \\ \\ R_{HL} = \Delta^A u(q^A_{LL}) \end{cases}$$

This last candidate set implies that $LH \rightarrow LL$, $HL \rightarrow LL$, and $LH \rightarrow HL$ are binding constraints regarding the medium types (HL and LH types) and the LL-type buyer.

Candidates 1 and 2 would result in the violation of IC $HH \rightarrow LL$. Candidate 2 does not violate the set of IC constraints. Thus, as a result of the cap, the IC constraints $LH \rightarrow LL$, $HL \rightarrow LL$, and $LH \rightarrow HL$ bind with equality.

Notice that $HH \rightarrow LH$ and $LH \rightarrow HH$ are equivalent under the cap. This implies the bunching together of buyers HH and LH. The IC restrictions $HH \to LH$ and $HH \to HL$ are satisfied as long as $LH \to LL$, $HL \to LL$, and $LH \to HL$ bind with equality. In other words, if the downward IC restrictions involving the LH-type are satisfied, the HH-type will not purchase an option intended to serve either the HL or the LL types.

In other words, as the quantity of product A becomes smaller due to more and more restrictive cap rules, the incentive constraint $LH \to HL$ becomes relevant.

The analysis for the severe cap is analogous.

Proof of proposition 3

To proof this proposition, it is sufficient to notice that a mild cap does not distort any downward incentive compatibility constraint. A mild cap only reduces q^A_{HH}, and this variable is absent in all IC restrictions shown in 4 and 5.

Proof of proposition 4

This proof has two parts. First, I will show that a moderate cap (where the regulation is intended to reduce the portion below the second largest alternative) impacts q^A_{HH}, q^A_{HL}, and q^A_{LH}. This facilitates showing the welfare effects. The second part addresses each of the three claims within proposition 4 separately.

First part: With unrelated goods and without intervention, the second largest portion is q^A_{LH}. This can be verified as follows. By examining the corresponding first order conditions, it can be deduced that $q^A_{HH} = q^A_{HL}$ in all IC structures. Thus, the candidates for "second largest" portion are either q^A_{LH} and q^A_{LL}. We can find ot which one is largest by comparing the corresponding FOCs.

In IC structures C and D, it is straightforward to find out that $q^A_{LH} > q^A_{LL}$.

Moving to IC structures A and B, bundling requires $\beta_{HH}\beta_{LL} - \beta_{LH}\beta_{HL} < 0$, this implies $\frac{\beta_{HH}}{\beta_{LH}} < \frac{\beta_{HL}}{\beta_{LL}}$. Bearing this in mind, comparing the corresponding FOCs implies $q^A_{LH} > q^A_{LL}$.

Second part: There are three claims within proposition 4.

First, R_{LL} is unaffected. This is straightforward to corroborate, because the participation constraint of this buyer type always binds with equality. In other words, this buyer was already receiving no rents before the cap. Because the LL-type's outside option is zero, no restriction can push R_{LL} below this value.

Second, R_{HL} is affected by the both moderate and severe caps. This is straightforward to corroborate as $R_{HL} = \Delta^B u(q^A_{LL})$ and q^A_{LL} is reduced either indirectly or directly by the moderate and severe caps.

Third, R_{LH} increases following either a moderate or severe cap. Before the cap, $R_{LH} = \Delta^B u(q^B_{LL})$. As shown in the proof of proposition 1, the rents earned by the LH-type with caps are the following:

$$\text{With moderate cap: } R_{LH} = \Delta^B u(\widetilde{q^B_{LL}}) + \Delta^A u(\widetilde{q^A_{LL}}) + \Delta^B u(\widetilde{q^B_{HL}}) - \Delta^A u(\bar{q})$$

$$\text{With severe cap: } R_{LH} = \Delta^B u(\widehat{q^B_{LL}}) + \Delta^B u(\widehat{q^B_{HL}})$$

Both of them are strictly larger than the base unregulated R_{LH}.

Proof of proposition 6

The analysis follows closely what I show in the proof of proposition

$$
\begin{aligned}
LL \to LH: &\quad R_{LL} \geq R_{LH} - \Delta_5 u(\bar{q}) - \Delta_5 u(q^B_{LH}) \\
LL \to HL: &\quad R_{LL} \geq R_{HL} - \Delta_4 u(\bar{q}) - \Delta_4 u(q^B_{HL}) \\
LL \to HH: &\quad R_{LL} \geq R_{HH} - \Delta_1 u(\bar{q}) - \Delta_1 u(q^B_{HH}) \\
LH \to LL: &\quad R_{LH} \geq R_{LL} + \Delta_5 u(q^A_{LL}) + \Delta_5 u(q^B_{LL}) \\
LH \to HL: &\quad R_{LH} \geq R_{HL} + \Delta_6 u(\bar{q}) + \Delta_6 u(q^B_{HL}) \\
LH \to HH: &\quad R_{LH} \geq R_{HH} - \Delta_3 u(\bar{q}) - \Delta_3 u(q^B_{HH}) \\
HL \to LL: &\quad R_{HL} \geq R_{LL} + \Delta_4 u(q^A_{LL}) + \Delta_4 u(q^B_{LL}) \\
HL \to LH: &\quad R_{HL} \geq R_{LH} + \Delta_7 u(\bar{q}) + \Delta_7 u(q^B_{LH}) \\
HL \to HH: &\quad R_{HL} \geq R_{HH} - \Delta_2 u(\bar{q}) - \Delta_2 u(q^B_{HH}) \\
HH \to LL: &\quad R_{HH} \geq R_{LL} + \Delta_1 u(q^A_{LL}) + \Delta_1 u(q^B_{LL}) \\
HH \to LH: &\quad R_{HH} \geq R_{LH} + \Delta_3 u(\bar{q}) + \Delta_3 u(q^B_{LH}) \\
HH \to HL: &\quad R_{HH} \geq R_{HL} + \Delta_2 u(\bar{q}) + \Delta_2 u(q^B_{HL})
\end{aligned}
$$

From the possible candidate combinations of forms of R_{HL} and R_{LH}, the following do not violate any of the IC constraints listed above:

$$
\begin{aligned}
&\widetilde{R_{LH}} = (\Delta_5 + \Delta_4)[u(\widetilde{q^A_{LL}}) + u(\widetilde{q^B_{LL}})] + \Delta_6[u(\bar{q}) + u(\widetilde{q^B_{HL}})] \\
&\widetilde{R_{HL}} = \Delta_4[u(\widetilde{q^A_{LL}}) + u(\widetilde{q^B_{LL}})]
\end{aligned}
$$

Appendix B: First Order Conditions - Independent Goods

IC-Structure A baseline:

$$FOC[q^A_{HH}] : \theta^A_H u'(q^A_{HH}) = c'(q^A_{HH}) \qquad FOC[q^B_{HH}] : \theta^B_H u'(q^B_{HH}) = c'(q^B_{HH})$$

$$FOC[q^A_{HL}] : \theta^A_H u'(q^A_{HL}) = c'(q^A_{HL}) \qquad FOC[q^B_{HL}] : \theta^B_L u'(q^B_{HL}) = \frac{c'(q^B_{HL})}{\left(1 - \frac{\beta_{HH}}{\beta_{HL}} \frac{\Delta^B}{\theta^B_L}\right)}$$

$$FOC[q^A_{LH}] : \theta^A_L u'(q^A_{LH}) = \frac{c'(q^A_{LH})}{\left(1 - \frac{\beta_{HH}}{\beta_{LH}} \frac{\Delta^A}{\theta^A_L}\right)} \qquad FOC[q^B_{LH}] : \theta^B_H u'(q^B_{LH}) = c'(q^B_{LH})$$

$$FOC[q^A_{LL}] : \theta^A_L u'(q^A_{LL}) = \frac{c'(q^A_{LL})}{\left(1 - \frac{\beta_{HL}}{\beta_{LL}} \frac{\Delta^A}{\theta^A_L}\right)} \qquad FOC[q^B_{LL}] : \theta^B_L u'(q^B_{LL}) = \frac{c'(q^B_{LL})}{\left(1 - \frac{\beta_{LH}}{\beta_{LL}} \frac{\Delta^B}{\theta^B_L}\right)}$$

IC-Structure B baseline:

$$FOC[q^A_{HH}] : \theta^A_H u'(q^A_{HH}) = c'(q^A_{HH}) \qquad FOC[q^B_{HH}] : \theta^B_H u'(q^B_{HH}) = c'(q^B_{HH})$$

$$FOC[q^A_{HL}] : \theta^A_H u'(q^A_{HL}) = c'(q^A_{HL}) \qquad FOC[q^B_{HL}] : \theta^B_L u'(q^B_{HL}) = c'(q^B_{HL})$$

$$FOC[q^A_{LH}] : \theta^A_L u'(q^A_{LH}) = \frac{c'(q^A_{LH})}{\left(1 - \frac{\beta_{HH}}{\beta_{LH}} \frac{\Delta^A}{\theta^A_L}\right)} \qquad FOC[q^B_{LH}] : \theta^B_H u'(q^B_{LH}) = c'(q^B_{LH})$$

$$FOC[q^A_{LL}] : \theta^A_L u'(q^A_{LL}) = \frac{c'(q^A_{LL})}{\left(1 - \frac{\beta_{HL}}{\beta_{LL}} \frac{\Delta^A}{\theta^A_L}\right)} \qquad FOC[q^B_{LL}] : \theta^B_L u'(q^B_{LL}) = \frac{c'(q^B_{LL})}{\left(1 - \frac{\beta_{LH}+\beta_{HH}}{\beta_{LL}} \frac{\Delta^B}{\theta^B_L}\right)}$$

IC-Structure C baseline:

$$FOC[q^A_{HH}]: \theta^A_H u'(q^A_{HH}) = c'(q^A_{HH}) \qquad FOC[q^B_{HH}]: \theta^B_H u'(q^B_{HH}) = c'(q^B_{HH})$$

$$FOC[q^A_{HL}]: \theta^A_H u'(q^A_{HL}) = c'(q^A_{HL}) \qquad FOC[q^B_{HL}]: \theta^B_L u'(q^B_{HL}) = \frac{c'(q^B_{HL})}{\left(1 - \frac{\beta_{HH}}{\beta_{HL}} \frac{\Delta^B}{\theta^B_L}\right)}$$

$$FOC[q^A_{LH}]: \theta^A_L u'(q^A_{LH}) = c'(q^A_{LH}) \qquad FOC[q^B_{LH}]: \theta^B_H u'(q^B_{LH}) = c'(q^B_{LH})$$

$$FOC[q^A_{LL}]: \theta^A_L u'(q^A_{LL}) = \frac{c'(q^A_{LL})}{\left(1 - \frac{\beta_{HL}+\beta_{HH}}{\beta_{LL}} \frac{\Delta^A}{\theta^A_L}\right)} \qquad FOC[q^B_{LL}]: \theta^B_L u'(q^B_{LL}) = \frac{c'(q^B_{LL})}{\left(1 - \frac{\beta_{LH}}{\beta_{LL}} \frac{\Delta^B}{\theta^B_L}\right)}$$

IC-Structure D baseline:

$$FOC[q^A_{HH}]: \theta^A_H u'(q^A_{HH}) = c'(q^A_{HH}) \qquad FOC[q^B_{HH}]: \theta^B_H u'(q^B_{HH}) = c'(q^B_{HH})$$

$$FOC[q^A_{HL}]: \theta^A_H u'(q^A_{HL}) = c'(q^A_{HL}) \qquad FOC[q^B_{HL}]: \theta^B_L u'(q^B_{HL}) = c'(q^B_{HL})$$

$$FOC[q^A_{LH}]: \theta^A_L u'(q^A_{LH}) = c'(q^A_{LH}) \qquad FOC[q^B_{LH}]: \theta^B_H u'(q^B_{LH}) = c'(q^B_{LH})$$

$$FOC[q^A_{LL}]: \theta^A_L u'(q^A_{LL}) = \frac{c'(q^A_{LL})}{\left(1 - \frac{\beta_{HL}+\beta_{HH}}{\beta_{LL}} \frac{\Delta^A}{\theta^A_L}\right)} \qquad FOC[q^B_{LL}]: \theta^B_L u'(q^B_{LL}) = \frac{c'(q^B_{LL})}{\left(1 - \frac{\beta_{LH}+\beta_{HH}}{\beta_{LL}} \frac{\Delta^B}{\theta^B_L}\right)}$$

Mild IC-Structure A:

$$FOC[\bar{q}] : \theta_H^A u'(\bar{q}) = c'(\bar{q}) \qquad FOC[q_{HH}^B] : \theta_H^B u'(q_{HH}^B) = c'(q_{HH}^B)$$

$$FOC[q_{HL}^A] : \theta_H^A u'(q_{HL}^A) = c'(q_{HL}^A) \qquad FOC[q_{HL}^B] : \theta_L^B u'(q_{HL}^B) = \frac{c'(q_{HL}^B)}{\left(1 - \frac{\beta_{HH}}{\beta_{HL}} \frac{\Delta^B}{\theta_L^B}\right)}$$

$$FOC[q_{LH}^A] : \theta_L^A u'(q_{LH}^A) = \frac{c'(q_{LH}^A)}{\left(1 - \frac{\beta_{HH}}{\beta_{LH}} \frac{\Delta^A}{\theta_L^A}\right)} \qquad FOC[q_{LH}^B] : \theta_H^B u'(q_{LH}^B) = c'(q_{LH}^B)$$

$$FOC[q_{LL}^A] : \theta_L^A u'(q_{LL}^A) = \frac{c'(q_{LL}^A)}{\left(1 - \frac{\beta_{HL}}{\beta_{LL}} \frac{\Delta^A}{\theta_L^A}\right)} \qquad FOC[q_{LL}^B] : \theta_L^B u'(q_{LL}^B) = \frac{c'(q_{LL}^B)}{\left(1 - \frac{\beta_{LH}}{\beta_{LL}} \frac{\Delta^B}{\theta_L^B}\right)}$$

Mild cap IC-Structure B:

$$FOC[\bar{q}] : \theta_H^A u'(\bar{q}) = c'(\bar{q}) \qquad FOC[q_{HH}^B] : \theta_H^B u'(q_{HH}^B) = c'(q_{HH}^B)$$

$$FOC[q_{HL}^A] : \theta_H^A u'(q_{HL}^A) = c'(q_{HL}^A) \qquad FOC[q_{HL}^B] : \theta_L^B u'(q_{HL}^B) = c'(q_{HL}^B)$$

$$FOC[q_{LH}^A] : \theta_L^A u'(q_{LH}^A) = \frac{c'(q_{LH}^A)}{\left(1 - \frac{\beta_{HH}}{\beta_{LH}} \frac{\Delta^A}{\theta_L^A}\right)} \qquad FOC[q_{LH}^B] : \theta_H^B u'(q_{LH}^B) = c'(q_{LH}^B)$$

$$FOC[q_{LL}^A] : \theta_L^A u'(q_{LL}^A) = \frac{c'(q_{LL}^A)}{\left(1 - \frac{\beta_{HL}}{\beta_{LL}} \frac{\Delta^A}{\theta_L^A}\right)} \qquad FOC[q_{LL}^B] : \theta_L^B u'(q_{LL}^B) = \frac{c'(q_{LL}^B)}{\left(1 - \frac{\beta_{LH}+\beta_{HH}}{\beta_{LL}} \frac{\Delta^B}{\theta_L^B}\right)}$$

Mild cap IC-Structure C:

$$FOC[\bar{q}] : \theta_H^A u'(\bar{q}) = c'(\bar{q}) \qquad FOC[q_{HH}^B] : \theta_H^B u'(q_{HH}^B) = c'(q_{HH}^B)$$

$$FOC[q_{HL}^A] : \theta_H^A u'(q_{HL}^A) = c'(q_{HL}^A) \qquad FOC[q_{HL}^B] : \theta_L^B u'(q_{HL}^B) = \frac{c'(q_{HL}^B)}{\left(1 - \frac{\beta_{HH}}{\beta_{HL}} \frac{\Delta^B}{\theta_L^B}\right)}$$

$$FOC[q_{LH}^A] : \theta_L^A u'(q_{LH}^A) = c'(q_{LH}^A) \qquad FOC[q_{LH}^B] : \theta_H^B u'(q_{LH}^B) = c'(q_{LH}^B)$$

$$FOC[q_{LL}^A] : \theta_L^A u'(q_{LL}^A) = \frac{c'(q_{LL}^A)}{\left(1 - \frac{\beta_{HL}+\beta_{HH}}{\beta_{LL}} \frac{\Delta^A}{\theta_L^A}\right)} \qquad FOC[q_{LL}^B] : \theta_L^B u'(q_{LL}^B) = \frac{c'(q_{LL}^B)}{\left(1 - \frac{\beta_{LH}}{\beta_{LL}} \frac{\Delta^B}{\theta_L^B}\right)}$$

Mild IC-Structure D:

$$FOC[\bar{q}] : \theta_H^A u'(\bar{q}) = c'(\bar{q}) \qquad FOC[q_{HH}^B] : \theta_H^B u'(q_{HH}^B) = c'(q_{HH}^B)$$

$$FOC[q_{HL}^A] : \theta_H^A u'(q_{HL}^A) = c'(q_{HL}^A) \qquad FOC[q_{HL}^B] : \theta_L^B u'(q_{HL}^B) = c'(q_{HL}^B)$$

$$FOC[q_{LH}^A] : \theta_L^A u'(q_{LH}^A) = c'(q_{LH}^A) \qquad FOC[q_{LH}^B] : \theta_H^B u'(q_{LH}^B) = c'(q_{LH}^B)$$

$$FOC[q_{LL}^A] : \theta_L^A u'(q_{LL}^A) = \frac{c'(q_{LL}^A)}{\left(1 - \frac{\beta_{HL}+\beta_{HH}}{\beta_{LL}} \frac{\Delta^A}{\theta_L^A}\right)} \quad FOC[q_{LL}^B] : \theta_L^B u'(q_{LL}^B) = \frac{c'(q_{LL}^B)}{\left(1 - \frac{\beta_{LH}+\beta_{HH}}{\beta_{LL}} \frac{\Delta^B}{\theta_L^B}\right)}$$

Moderate cap:

$$FOC[\bar{q}] : \theta_H^A u'(q_{HH}^A) = \frac{\beta_{HH} + \beta_{HL} + \beta_{LH}}{\beta_{HH} + \beta_{HL} + \beta_{LH}\left(\frac{\theta_L^A}{\theta_H^A} + \frac{\Delta^A}{\theta_H^A}\right)} \cdot c'(q_{HH}^A)$$

$$FOC[q_{HH}^B] : \theta_H^B u'(q_{HH}^B) = c'(q_{HH}^B)$$

$$FOC[q_{HL}^B] : \theta_L^B u'(q_{HL}^B) = \frac{c'(q_{HL}^B)}{\left(1 - \frac{\beta_{LH}+\beta_{HH}}{\beta_{HL}} \frac{\Delta^B}{\theta_L^B}\right)}$$

$$FOC[q_{LH}^B] : \theta_H^B u'(q_{LH}^B) = c'(q_{LH}^B)$$

$$FOC[q_{LL}^A] : \theta_L^A u'(q_{LL}^A) = \frac{c'(q_{LL}^A)}{\left(1 - \frac{\beta_{LH}+\beta_{HL}+\beta_{HH}}{\beta_{LL}} \frac{\Delta^A}{\theta_L^A}\right)}$$

$$FOC[q_{LL}^B] : \theta_L^B u'(q_{LL}^B) = \frac{c'(q_{LL}^B)}{\left(1 - \frac{\beta_{LH}+\beta_{HH}}{\beta_{LL}} \frac{\Delta^B}{\theta_L^B}\right)}$$

Severe cap

$$FOC[\bar{q}] : \theta_H^A u'(\bar{q}) = \frac{\theta_H^A}{\theta_L^A} c'(\bar{q})$$

$$FOC[q_{HH}^B] : \theta_H^B u'(q_{HH}^B) = c'(q_{HH}^B)$$

$$FOC[q_{HL}^B] : \theta_L^B u'(q_{HL}^B) = \frac{c'(q_{HL}^B)}{\left(1 - \frac{\beta_{HH}+\beta_{LH}}{\beta_{HL}} \frac{\Delta^B}{\theta_L^B}\right)}$$

$$FOC[q_{LH}^B] : \theta_H^B u'(q_{LH}^B) = c'(q_{LH}^B)$$

$$FOC[q_{LL}^B] : \theta_L^B u'(q_{LL}^B) = \frac{c'(q_{LL}^B)}{\left(1 - \frac{\beta_{HH}+\beta_{LH}}{\beta_{LL}} \frac{\Delta^B}{\theta_L^B}\right)}$$

Appendix C: First Order Conditions - Related (Complement and Substitute) Goods

For complements (superscript C), the following holds: $(\theta^C_{HH} > \theta^A_H + \theta^B_H)$, $(\theta^C_{HL} > \theta^A_H + \theta^B_L)$, $(\theta^C_{LH} > \theta^A_L + \theta^B_H)$, and $(\theta^C_{LL} > \theta^A_L + \theta^B_L)$. Additionally, $\theta^C_{HH} > \theta^C_{HL}$, $\theta^C_{HH} > \theta^C_{LH}$, $\theta^C_{HH} > \theta^C_{LL}$, $\theta^C_{HL} > \theta^C_{LL}$, and $\theta^C_{LH} > \theta^C_{LL}$.

For substitutes (superscript S), the following holds: $(\theta^S_{HH} < \theta^A_H + \theta^B_H)$, $(\theta^S_{HL} < \theta^A_H + \theta^B_L)$, $(\theta^S_{LH} < \theta^A_L + \theta^B_H)$, and $(\theta^S_{LL} < \theta^A_L + \theta^B_L)$. Additionally, $\theta^S_{HH} > \theta^S_{HL}$, $\theta^S_{HH} > \theta^S_{LH}$, $\theta^S_{HH} > \theta^S_{LL}$, $\theta^S_{HL} > \theta^S_{LL}$, and $\theta^S_{LH} > \theta^S_{LL}$.

Because the analysis below looks identical for both complements and substitutes, I drop the superscript. Consider the following definitions:

$$
\begin{aligned}
&\theta_{HH} - \theta_{LL} \equiv \Delta_1 \quad && \theta_{HH} - \theta_{HL} \equiv \Delta_2 \\
&\theta_{HH} - \theta_{LH} \equiv \Delta_3 \quad && \theta_{HL} - \theta_{LL} \equiv \Delta_4 \\
&\theta_{LH} - \theta_{LL} \equiv \Delta_5 \quad && \theta_{LH} - \theta_{HL} \equiv \Delta_6 \\
&\theta_{HL} - \theta_{LH} \equiv \Delta_7 &&
\end{aligned}
$$

IC-Structure A (complements) baseline:

$$FOC[q_{HH}^A] : \theta_{HH} u'(q_{HH}^A) = c'(q_{HH}^A)$$

$$FOC[q_{HH}^B] : \theta_{HH} u'(q_{HH}^B) = c'(q_{HH}^B)$$

$$FOC[q_{HL}^A] : \theta_{HL} u'(q_{HL}^A) = \frac{c'(q_{HL}^A)}{\left(1 - \frac{\beta_{HH}}{\beta_{HL}} \frac{\Delta_2}{\theta_{HL}}\right)}$$

$$FOC[q_{HL}^B] : \theta_{HL} u'(q_{HL}^B) = \frac{c'(q_{HL}^B)}{\left(1 - \frac{\beta_{HH}}{\beta_{HL}} \frac{\Delta_2}{\theta_{HL}}\right)}$$

$$FOC[q_{LH}^A] : \theta_{LH} u'(q_{LH}^A) = \frac{c'(q_{LH}^A)}{\left(1 - \frac{\beta_{HH}}{\beta_{LH}} \frac{\Delta_3}{\theta_{LH}}\right)}$$

$$FOC[q_{LH}^B] : \theta_{LH} u'(q_{LH}^B) = \frac{c'(q_{LH}^B)}{\left(1 - \frac{\beta_{HH}}{\beta_{LH}} \frac{\Delta_3}{\theta_{LH}}\right)}$$

$$FOC[q_{LL}^A] : \theta_{LL} u'(q_{LL}^A) = \frac{c'(q_{LL}^A)}{\left(1 - \frac{\beta_{LH}+\beta_{HH}}{\beta_{LL}} \frac{\Delta_5}{\theta_{LL}} - \frac{\beta_{HL}+\beta_{HH}}{\beta_{LL}} \frac{\Delta_4}{\theta_{LL}}\right)}$$

$$FOC[q_{LL}^B] : \theta_{LL} u'(q_{LL}^B) = \frac{c'(q_{LL}^B)}{\left(1 - \frac{\beta_{LH}+\beta_{HH}}{\beta_{LL}} \frac{\Delta_5}{\theta_{LL}} - \frac{\beta_{HL}+\beta_{HH}}{\beta_{LL}} \frac{\Delta_4}{\theta_{LL}}\right)}$$

IC-Structure B (complements) baseline:

$$FOC[q^A_{HH}] : \theta_{HH} u'(q^A_{HH}) = c'(q^A_{HH})$$

$$FOC[q^B_{HH}] : \theta_{HH} u'(q^B_{HH}) = c'(q^B_{HH})$$

$$FOC[q^A_{HL}] : \theta_{HL} u'(q^A_{HL}) = \frac{c'(q^A_{HL})}{\left(1 - \frac{\beta_{HH}}{\beta_{HL}} \frac{\Delta_6+\Delta_2}{\theta_{HL}}\right)}$$

$$FOC[q^B_{HL}] : \theta_{HL} u'(q^B_{HL}) = \frac{c'(q^B_{HL})}{\left(1 - \frac{\beta_{HH}}{\beta_{HL}} \frac{\Delta_6+\Delta_2}{\theta_{HL}}\right)}$$

$$FOC[q^A_{LH}] : \theta_{LH} u'(q^A_{LH}) = \frac{c'(q^A_{LH})}{\left(1 - \frac{\beta_{HH}}{\beta_{LH}} \frac{\Delta_3}{\theta_{LH}}\right)}$$

$$FOC[q^B_{LH}] : \theta_{LH} u'(q^B_{LH}) = \frac{c'(q^B_{LH})}{\left(1 - \frac{\beta_{HH}}{\beta_{LH}} \frac{\Delta_3}{\theta_{LH}}\right)}$$

$$FOC[q^A_{LL}] : \theta_{LL} u'(q^A_{LL}) = \frac{c'(q^A_{LL})}{\left(1 - \frac{\beta_{LH}+\beta_{HH}}{\beta_{LL}} \frac{\Delta_5}{\theta_{LL}} - \frac{\beta_{HL}+\beta_{HH}}{\beta_{LL}} \frac{\Delta_4}{\theta_{LL}}\right)}$$

$$FOC[q^B_{LL}] : \theta_{LL} u'(q^B_{LL}) = \frac{c'(q^B_{LL})}{\left(1 - \frac{\beta_{LH}+\beta_{HH}}{\beta_{LL}} \frac{\Delta_5}{\theta_{LL}} - \frac{\beta_{HL}+\beta_{HH}}{\beta_{LL}} \frac{\Delta_4}{\theta_{LL}}\right)}$$

IC-Structure C (complements) baseline:

$$FOC[q^A_{HH}] : \theta_{HH} u'(q^A_{HH}) = c'(q^A_{HH})$$

$$FOC[q^B_{HH}] : \theta_{HH} u'(q^B_{HH}) = c'(q^B_{HH})$$

$$FOC[q^A_{HL}] : \theta_{HL} u'(q^A_{HL}) = \frac{c'(q^A_{HL})}{\left(1 - \frac{\beta_{HH}}{\beta_{HL}} \frac{\Delta_1}{\theta_{HL}}\right)}$$

$$FOC[q^B_{HL}] : \theta_{HL} u'(q^B_{HL}) = \frac{c'(q^B_{HL})}{\left(1 - \frac{\beta_{HH}}{\beta_{HL}} \frac{\Delta_1}{\theta_{HL}}\right)}$$

$$FOC[q^A_{LH}] : \theta_{LH} u'(q^A_{LH}) = \frac{c'(q^A_{LH})}{\left(1 - \frac{\beta_{HH}}{\beta_{LH}} \frac{\Delta_3}{\theta_{LH}}\right)}$$

$$FOC[q^B_{LH}] : \theta_{LH} u'(q^B_{LH}) = \frac{c'(q^B_{LH})}{\left(1 - \frac{\beta_{HH}}{\beta_{LH}} \frac{\Delta_3}{\theta_{LH}}\right)}$$

$$FOC[q^A_{LL}] : \theta_{LL} u'(q^A_{LL}) = \frac{c'(q^A_{LL})}{\left(1 - \frac{\beta_{LH}+\beta_{HH}}{\beta_{LL}} \frac{\Delta_5}{\theta_{LL}} - \frac{\beta_{HL}+\beta_{HH}}{\beta_{LL}} \frac{\Delta_4}{\theta_{LL}} - \frac{\beta_{HH}}{\beta_{LL}} \frac{\Delta_1}{\theta_{LL}}\right)}$$

$$FOC[q^B_{LL}] : \theta_{LL} u'(q^B_{LL}) = \frac{c'(q^B_{LL})}{\left(1 - \frac{\beta_{LH}+\beta_{HH}}{\beta_{LL}} \frac{\Delta_5}{\theta_{LL}} - \frac{\beta_{HL}+\beta_{HH}}{\beta_{LL}} \frac{\Delta_4}{\theta_{LL}} - \frac{\beta_{HH}}{\beta_{LL}} \frac{\Delta_1}{\theta_{LL}}\right)}$$

IC-Structure D (complements) baseline:

$$FOC[q_{HH}^A] : \theta_{HH} u'(q_{HH}^A) = c'(q_{HH}^A)$$

$$FOC[q_{HH}^B] : \theta_{HH} u'(q_{HH}^B) = c'(q_{HH}^B)$$

$$FOC[q_{HL}^A] : \theta_{HL} u'(q_{HL}^A) = \frac{c'(q_{HL}^A)}{\left(1 - \frac{\beta_{LH}+\beta_{HH}}{\beta_{HL}} \frac{\Delta_6}{\theta_{HL}} - \frac{\beta_{HH}}{\beta_{HL}} \frac{\Delta_2}{\theta_{HL}}\right)}$$

$$FOC[q_{HL}^B] : \theta_{HL} u'(q_{HL}^B) = \frac{c'(q_{HL}^B)}{\left(1 - \frac{\beta_{LH}+\beta_{HH}}{\beta_{HL}} \frac{\Delta_6}{\theta_{HL}} - \frac{\beta_{HH}}{\beta_{HL}} \frac{\Delta_2}{\theta_{HL}}\right)}$$

$$FOC[q_{LH}^A] : \theta_{LH} u'(q_{LH}^A) = \frac{c'(q_{LH}^A)}{\left(1 - \frac{\beta_{HH}}{\beta_{LH}} \frac{\Delta_3}{\theta_{LH}}\right)}$$

$$FOC[q_{LH}^B] : \theta_{LH} u'(q_{LH}^B) = \frac{c'(q_{LH}^B)}{\left(1 - \frac{\beta_{HH}}{\beta_{LH}} \frac{\Delta_3}{\theta_{LH}}\right)}$$

$$FOC[q_{LL}^A] : \theta_{LL} u'(q_{LL}^A) = \frac{c'(q_{LL}^A)}{\left(1 - \frac{\beta_{LH}+\beta_{HH}}{\beta_{LL}} \frac{\Delta_5}{\theta_{LL}} - \frac{\beta_{HL}+\beta_{HH}}{\beta_{LL}} \frac{\Delta_4}{\theta_{LL}} - \frac{\beta_{HH}}{\beta_{LL}} \frac{\Delta_1}{\theta_{LL}}\right)}$$

$$FOC[q_{LL}^B] : \theta_{LL} u'(q_{LL}^B) = \frac{c'(q_{LL}^B)}{\left(1 - \frac{\beta_{LH}+\beta_{HH}}{\beta_{LL}} \frac{\Delta_5}{\theta_{LL}} - \frac{\beta_{HL}+\beta_{HH}}{\beta_{LL}} \frac{\Delta_4}{\theta_{LL}} - \frac{\beta_{HH}}{\beta_{LL}} \frac{\Delta_1}{\theta_{LL}}\right)}$$

IC-Structure A (complements) mild cap:

$$FOC[\bar{q}] : \theta_{HH} u'(\bar{q}) = c'(\bar{q})$$

$$FOC[q^B_{HH}] : \theta_{HH} u'(q^B_{HH}) = c'(q^B_{HH})$$

$$FOC[q^A_{HL}] : \theta_{HL} u'(q^A_{HL}) = \frac{c'(q^A_{HL})}{\left(1 - \frac{\beta_{HH}}{\beta_{HL}} \frac{\Delta_2}{\theta_{HL}}\right)}$$

$$FOC[q^B_{HL}] : \theta_{HL} u'(q^B_{HL}) = \frac{c'(q^B_{HL})}{\left(1 - \frac{\beta_{HH}}{\beta_{HL}} \frac{\Delta_2}{\theta_{HL}}\right)}$$

$$FOC[q^A_{LH}] : \theta_{LH} u'(q^A_{LH}) = \frac{c'(q^A_{LH})}{\left(1 - \frac{\beta_{HH}}{\beta_{LH}} \frac{\Delta_3}{\theta_{LH}}\right)}$$

$$FOC[q^B_{LH}] : \theta_{LH} u'(q^B_{LH}) = \frac{c'(q^B_{LH})}{\left(1 - \frac{\beta_{HH}}{\beta_{LH}} \frac{\Delta_3}{\theta_{LH}}\right)}$$

$$FOC[q^A_{LL}] : \theta_{LL} u'(q^A_{LL}) = \frac{c'(q^A_{LL})}{\left(1 - \frac{\beta_{LH}+\beta_{HH}}{\beta_{LL}} \frac{\Delta_5}{\theta_{LL}} - \frac{\beta_{HL}+\beta_{HH}}{\beta_{LL}} \frac{\Delta_4}{\theta_{LL}}\right)}$$

$$FOC[q^B_{LL}] : \theta_{LL} u'(q^B_{LL}) = \frac{c'(q^B_{LL})}{\left(1 - \frac{\beta_{LH}+\beta_{HH}}{\beta_{LL}} \frac{\Delta_5}{\theta_{LL}} - \frac{\beta_{HL}+\beta_{HH}}{\beta_{LL}} \frac{\Delta_4}{\theta_{LL}}\right)}$$

IC-Structure B (complements) mild cap:

$$FOC[\bar{q}] : \theta_{HH} u'(\bar{q}) = c'(\bar{q})$$

$$FOC[q^B_{HH}] : \theta_{HH} u'(q^B_{HH}) = c'(q^B_{HH})$$

$$FOC[q^A_{HL}] : \theta_{HL} u'(q^A_{HL}) = \frac{c'(q^A_{HL})}{\left(1 - \frac{\beta_{HH}}{\beta_{HL}} \frac{\Delta_6 + \Delta_2}{\theta_{HL}}\right)}$$

$$FOC[q^B_{HL}] : \theta_{HL} u'(q^B_{HL}) = \frac{c'(q^B_{HL})}{\left(1 - \frac{\beta_{HH}}{\beta_{HL}} \frac{\Delta_6 + \Delta_2}{\theta_{HL}}\right)}$$

$$FOC[q^A_{LH}] : \theta_{LH} u'(q^A_{LH}) = \frac{c'(q^A_{LH})}{\left(1 - \frac{\beta_{HH}}{\beta_{LH}} \frac{\Delta_3}{\theta_{LH}}\right)}$$

$$FOC[q^B_{LH}] : \theta_{LH} u'(q^B_{LH}) = \frac{c'(q^B_{LH})}{\left(1 - \frac{\beta_{HH}}{\beta_{LH}} \frac{\Delta_3}{\theta_{LH}}\right)}$$

$$FOC[q^A_{LL}] : \theta_{LL} u'(q^A_{LL}) = \frac{c'(q^A_{LL})}{\left(1 - \frac{\beta_{LH} + \beta_{HH}}{\beta_{LL}} \frac{\Delta_5}{\theta_{LL}} - \frac{\beta_{HL} + \beta_{HH}}{\beta_{LL}} \frac{\Delta_4}{\theta_{LL}}\right)}$$

$$FOC[q^B_{LL}] : \theta_{LL} u'(q^B_{LL}) = \frac{c'(q^B_{LL})}{\left(1 - \frac{\beta_{LH} + \beta_{HH}}{\beta_{LL}} \frac{\Delta_5}{\theta_{LL}} - \frac{\beta_{HL} + \beta_{HH}}{\beta_{LL}} \frac{\Delta_4}{\theta_{LL}}\right)}$$

IC-Structure C (complements) mild cap:

$$FOC[\bar{q}] : \theta_{HH} u'(\bar{q}) = c'(\bar{q})$$

$$FOC[q^B_{HH}] : \theta_{HH} u'(q^B_{HH}) = c'(q^B_{HH})$$

$$FOC[q^A_{HL}] : \theta_{HL} u'(q^A_{HL}) = \frac{c'(q^A_{HL})}{\left(1 - \frac{\beta_{HH}}{\beta_{HL}} \frac{\Delta_1}{\theta_{HL}}\right)}$$

$$FOC[q^B_{HL}] : \theta_{HL} u'(q^B_{HL}) = \frac{c'(q^B_{HL})}{\left(1 - \frac{\beta_{HH}}{\beta_{HL}} \frac{\Delta_1}{\theta_{HL}}\right)}$$

$$FOC[q^A_{LH}] : \theta_{LH} u'(q^A_{LH}) = \frac{c'(q^A_{LH})}{\left(1 - \frac{\beta_{HH}}{\beta_{LH}} \frac{\Delta_3}{\theta_{LH}}\right)}$$

$$FOC[q^B_{LH}] : \theta_{LH} u'(q^B_{LH}) = \frac{c'(q^B_{LH})}{\left(1 - \frac{\beta_{HH}}{\beta_{LH}} \frac{\Delta_3}{\theta_{LH}}\right)}$$

$$FOC[q^A_{LL}] : \theta_{LL} u'(q^A_{LL}) = \frac{c'(q^A_{LL})}{\left(1 - \frac{\beta_{LH}+\beta_{HH}}{\beta_{LL}} \frac{\Delta_5}{\theta_{LL}} - \frac{\beta_{HL}+\beta_{HH}}{\beta_{LL}} \frac{\Delta_4}{\theta_{LL}} - \frac{\beta_{HH}}{\beta_{LL}} \frac{\Delta_1}{\theta_{LL}}\right)}$$

$$FOC[q^B_{LL}] : \theta_{LL} u'(q^B_{LL}) = \frac{c'(q^B_{LL})}{\left(1 - \frac{\beta_{LH}+\beta_{HH}}{\beta_{LL}} \frac{\Delta_5}{\theta_{LL}} - \frac{\beta_{HL}+\beta_{HH}}{\beta_{LL}} \frac{\Delta_4}{\theta_{LL}} - \frac{\beta_{HH}}{\beta_{LL}} \frac{\Delta_1}{\theta_{LL}}\right)}$$

IC-Structure D (complements) mild cap:

$$FOC[\bar{q}] : \theta_{HH} u'(\bar{q}) = c'(\bar{q})$$

$$FOC[q^B_{HH}] : \theta_{HH} u'(q^B_{HH}) = c'(q^B_{HH})$$

$$FOC[q^A_{HL}] : \theta_{HL} u'(q^A_{HL}) = \frac{c'(q^A_{HL})}{\left(1 - \frac{\beta_{LH}+\beta_{HH}}{\beta_{HL}} \frac{\Delta_6}{\theta_{HL}} - \frac{\beta_{HH}}{\beta_{HL}} \frac{\Delta_2}{\theta_{HL}}\right)}$$

$$FOC[q^B_{HL}] : \theta_{HL} u'(q^B_{HL}) = \frac{c'(q^B_{HL})}{\left(1 - \frac{\beta_{LH}+\beta_{HH}}{\beta_{HL}} \frac{\Delta_6}{\theta_{HL}} - \frac{\beta_{HH}}{\beta_{HL}} \frac{\Delta_2}{\theta_{HL}}\right)}$$

$$FOC[q^A_{LH}] : \theta_{LH} u'(q^A_{LH}) = \frac{c'(q^A_{LH})}{\left(1 - \frac{\beta_{HH}}{\beta_{LH}} \frac{\Delta_3}{\theta_{LH}}\right)}$$

$$FOC[q^B_{LH}] : \theta_{LH} u'(q^B_{LH}) = \frac{c'(q^B_{LH})}{\left(1 - \frac{\beta_{HH}}{\beta_{LH}} \frac{\Delta_3}{\theta_{LH}}\right)}$$

$$FOC[q^A_{LL}] : \theta_{LL} u'(q^A_{LL}) = \frac{c'(q^A_{LL})}{\left(1 - \frac{\beta_{LH}+\beta_{HH}}{\beta_{LL}} \frac{\Delta_5}{\theta_{LL}} - \frac{\beta_{HL}+\beta_{HH}}{\beta_{LL}} \frac{\Delta_4}{\theta_{LL}} - \frac{\beta_{HH}}{\beta_{LL}} \frac{\Delta_1}{\theta_{LL}}\right)}$$

$$FOC[q^B_{LL}] : \theta_{LL} u'(q^B_{LL}) = \frac{c'(q^B_{LL})}{\left(1 - \frac{\beta_{LH}+\beta_{HH}}{\beta_{LL}} \frac{\Delta_5}{\theta_{LL}} - \frac{\beta_{HL}+\beta_{HH}}{\beta_{LL}} \frac{\Delta_4}{\theta_{LL}} - \frac{\beta_{HH}}{\beta_{LL}} \frac{\Delta_1}{\theta_{LL}}\right)}$$

Moderate cap (complements):

$$FOC[\bar{q}] : \theta_{HH} u'(\bar{q}) = \frac{\theta_{HH}}{\theta_{HL}} c'(\bar{q})$$

$$FOC[q^B_{HH}] : \theta_{HH} u'(q^B_{HH}) = c'(q^B_{HH})$$

$$FOC[q^B_{HL}] : \theta_{HL} u'(q^B_{HL}) = \frac{c'(q^B_{HL})}{\left(1 - \frac{\beta_{LH}}{\beta_{HL}} \frac{\Delta_6}{\theta_{HL}} - \frac{\beta_{HH}}{\beta_{HL}} \frac{\Delta_2}{\theta_{HL}}\right)}$$

$$FOC[q^B_{LH}] : \theta_{LH} u'(q^B_{LH}) = c'(q^B_{LH})$$

$$FOC[q^A_{LL}] : \theta_{LL} u'(q^A_{LL}) = \frac{c'(q^A_{LL})}{\left(1 - \frac{\beta_{LH}}{\beta_{LL}} \frac{\Delta_5}{\theta_{LL}} - \frac{\beta_{LH}+\beta_{HL}+\beta_{HH}}{\beta_{LL}} \frac{\Delta_4}{\theta_{LL}} - \frac{\beta_{HH}}{\beta_{LL}} \frac{\Delta_1}{\theta_{LL}}\right)}$$

$$FOC[q^B_{LL}] : \theta_{LL} u'(q^B_{LL}) = \frac{c'(q^B_{LL})}{\left(1 - \frac{\beta_{LH}}{\beta_{LL}} \frac{\Delta_5}{\theta_{LL}} - \frac{\beta_{LH}+\beta_{HL}+\beta_{HH}}{\beta_{LL}} \frac{\Delta_4}{\theta_{LL}} - \frac{\beta_{HH}}{\beta_{LL}} \frac{\Delta_1}{\theta_{LL}}\right)}$$

Severe cap (complements):

$$FOC[\bar{q}] : \theta_{HH}u'(\bar{q}) = \frac{c'(\bar{q})}{\beta_{HH}\left(1 - \frac{\Delta_1+\Delta_2+\Delta_4}{\theta_{HH}}\right) + \beta_{LH}\left(\frac{\theta_{LH}-\Delta_5-\Delta_6-\Delta_4}{\theta_{HH}}\right) + (\beta_{HL} + \beta_{LL})\frac{\theta_{LL}}{\theta_{HH}}}$$

$$FOC[q^B_{HH}] : \theta_{HH}u'(q^B_{HH}) = c'(q^B_{HH})$$

$$FOC[q^B_{HL}] : \theta_{HL}u'(q^B_{HL}) = \frac{c'(q^B_{HH})}{\left(1 - \frac{\beta_{LH}}{\beta_{HL}}\frac{\Delta_6}{\theta_{HL}} - \frac{\beta_{HH}}{\beta_{HL}}\frac{\Delta_2}{\theta_{HL}}\right)}$$

$$FOC[q^B_{LL}] : \theta_{LL}u'(q^B_{LL}) = \frac{c'(q^B_{LL})}{\left(1 - \frac{\beta_{LH}}{\beta_{LL}}\frac{\Delta_5}{\theta_{LL}} - \frac{\beta_{LH}+\beta_{HL}+\beta_{HH}}{\beta_{LL}}\frac{\Delta_4}{\theta_{LL}} - \frac{\beta_{HH}}{\beta_{LL}}\frac{\Delta_1}{\theta_{LL}}\right)}$$