Regulating Sugar-Sweetened Beverages: Portion-Size-Restrictions versus Taxes

Jose G. Nuno, Steven Y. Wu & Joseph V. Balagtas *

January 20, 2020

Abstract

We use a stylized nonlinear pricing model to evaluate the potential economic effects of two policy interventions designed to reduce sugar-sweetened beverage (SSB) consumption: taxes and portion size restrictions. We find that a tax would lead to decreases in retail serving sizes, consumer surplus, and retailer expected profit. Portion size restrictions would have similar effects, with the exception that consumer surplus would be unaffected unless the restriction is extreme. This surprising result suggests that portion size restrictions might be the preferred policy instrument in political environments in which consumer welfare is of primary concern.

JEL Codes:D04, D18, D82, H20, I18

KEYWORDS: beverage restrictions, beverage taxes, soda bans, nonlinear pricing, obesity, sugar consumption

*Jose Nuno (jnunoled@purdue.edu), Steven Wu (corresponding author, Agricultural Economics, 784B Krannert Building, 403 W. State Street, West Lafayette, IN 47907, (765)494-4299, sywu@purdue.edu) and Joseph Balagtas (balagtas@purdue.edu), are respectively, Assistant Professor at the University of Guelph, Associate Professor at Purdue University, and Associate Professor at Purdue University. This paper benefited greatly from comments from Stephen Hamilton and participants at the invited paper sessions of the AAEA meetings in Boston; as well as from Sven Anders and participants at the CAES & WAEA joint meetings in Victoria, Canada. Financial support from USDA-NIFA grant award no. 2014-67023-21852 is gratefully acknowledged.

Increased public awareness of the negative impacts of obesity has driven interest in public polices aimed at addressing obesity and its associated costs. Sugar consumption has been linked to increased risk of obesity and there is a growing, if still fluid, body of evidence that added sugars in sugar-sweetened beverages (SSBs) are uniquely harmful (Malik et al. (2006); Malik et al. (2010); Stanhope (2016)).[1] SSBs are low in satiety, have minimal nutritional value, and comprise a large portion of the added sugars in the American diet (Bray et al., 2004). Thus, much of the public policy debate surrounding obesity has focused on reducing consumption of SSBs. For example, Lustig et al. (2012) advocate for restrictions on sales of sugary foods using a public health argument that draws a parallel between sugar and tobacco and alcohol, and the World Health Organization (WHO) calls for taxes on SSBs as means to reduce sugar consumption to levels recommended by the institution's guidelines (World Health Organization, 2016).[2] However, there is also political opposition to SSB regulations and opponents tend to focus on how consumer welfare from the consumption of SSBs will be negatively impacted (Bourquard and Wu, 2019). Hence, the implementation of SSB regulations often hinge on the perceived tradeoff between health benefits versus the loss in consumer welfare due to decreased consumption choices.

In this paper, we analyze SSB taxes and portion size restrictions with a particular focus on how they impact consumer surplus. We also examine their impact on serving sizes and producer welfare. We focus on taxes and size-restrictions because (i) taxes are the

[1] Most definitions of sugar-sweetened beverages (SSB) are based on the criteria of "added sugars" and include, energy, sport, and regular soft drinks, among others (Jones et al., 2017). The category of "added sugars" does not include naturally-occurring sugars included in the broader group of "free sugars". The former are added into the beverage mix during the preparation process, while the later includes added sugars plus sugars naturally present in the beverage itself or in its ingredients (USDA, 2017).

[2] The WHO recommends an intake of free-sugars equivalent to less than 10 percent of total energy intake (World Health Organization, 2015).

focus of most of the scholarly work on economic policies targeting SSBs; (ii) portion caps are a relatively straightforward alternative that nonetheless incite polemic discussion when proposed; and (iii) there is a dearth of economic analysis, both theoretical or empirical, on the relative impact of size restrictions versus taxes, making it difficult to weigh the benefits and costs of these policies.

SSB taxes have been passed in Berkeley, CA, Philadelphia, PA, Cook Co., IL, Mexico, and elsewhere (Anwar (2016), Aubrey (2016b), Aubrey (2016a)). While SSB taxes forge ahead, there is only limited evidence that they effectively reduce consumption, and virtually no evidence of their broader economic effects. To date, research has focused on measuring the short-term effects of soda taxes on prices and consumption from recent policy experiments in the U.S. and abroad. These studies suggest that taxes have caused higher prices and reduced consumption (Falbe et al. (2015); Falbe et al. (2016); Grogger (2017); Silver et al. (2017); Cochero et al. (2017)). There are remaining questions on the long-term effects of such taxes on prices, consumption, and consumer welfare (Allcott et al., 2019).

Portion size restrictions have been less prevalent in practice. In 2013 Mayor Bloomberg of New York City proposed a rule prohibiting the sale of SSBs in cups exceeding 16 ounces, but the rule was later struck down in court (New York Statewide Coalition of Hispanic Chambers of Commerce v. New York City Department of Health and Mental Hygiene, 2014). Nonetheless, analyzing this policy still has some relevance since the state of Mississippi passed a bill in 2013 that has been called the "anti-Bloomberg bill" (Yan, 2013). The bill was passed ostensibly to protect consumer welfare from government interference of personal consumption choices. Moreover, size caps are still viable policy instruments given their simplicity. Additionally, some states are passing laws that prevent some of their local governments from enacting SSB taxes (O'Connor and Sanger-Katz, 2018) so that it is possible that size-restrictions might become practical alternatives to soda taxes, particularly in single-sitting environments. A McKinsey Global Institute report considers portion control schemes as the most cost-effective method for abatement of obesity (Dobbs et al., 2014).

One drawback of portion size restrictions is that, in some circumstances, they would be hard to enforce. For example, a size-restriction would be ineffective if consumers can buy SSBs to be stored and consumed over-time so that small containers can be offset by the purchase of more units. However, size restrictions are likely to be effective in single-sitting environments such as restaurants, fast food chains, and other venues where fountain drinks are sold for immediate consumption. Thus, our comparison of size-restrictions versus taxes is within the context of regulatory interventions in single-sitting environments where it is relatively easy to introduce accompanying rules to ensure compliance such as limits on number of units purchased or restrictions on the number of refills.[3]

In contrast to much of the extant literature, we use a nonlinear pricing framework that allows sellers (retailers) to endogenously adjust their strategic pricing schemes in response to exogenous policy interventions. Such responses could include not only price pass-through but also discrete changes in the nonlinear pricing structure typically used by soft-drink retailers. The economic effects of interventions in markets where sellers design their menus strategically are not easy to discern directly. Moreover, Bourquard and Wu (2019) suggest that SSB policy analysis using a nonlinear pricing framework may lead to different welfare conclusions than analysis conducted under a standard supply-demand framework with a

[3]The focus on single-sitting environments is not unique to our study. As mentioned in Bourquard and Wu (2019), government agencies are increasingly making rules that differ based on whether a product is meant to be consumed in one sitting or multiple sittings.

passive price taking seller.

Nonlinear pricing is consistent with stylized observations of SSB retailing. Typically, retailers in single-sitting environments offer consumers a menu of cup sizes/prices from which the consumer chooses. The canonical justification for nonlinear pricing is that the seller cannot observe willingness-to-pay (WTP) of consumers at the individual level. Since WTP is private information to the consumer, a menu is offered to incentivize each consumer to "reveal" her WTP through her choice.

Our analysis suggests that taxes and size-restrictions have very different consequences for consumer welfare. In particular, moderate size-restrictions can reduce consumption *without reducing consumer surplus*, whereas taxes unambiguously reduce consumer welfare. Intuitively, in order to segment the market after a portion restriction is imposed, the retailer needs to reduce the price to high-WTP consumers to ensure that they obtain the same information rent as they did prior to the cap. However, a tax puts upward pressure on prices so the retailer has to instead reduce the serving size to low-WTP consumers to achieve segmentation.[4] The reduction in sizes without porportional price reductions means consumers are worse off. Our finding that portion restrictions are more consumer welfare friendly than taxes is particularly important in debates where consumer advocates are concerned about the regressiveness of soda taxes.[5] Both taxes and size-restrictions create distortions, but in the latter case, all distortions are borne by sellers.

[4]Recall from textbook nonlinear pricing that low-WTP consumers have their serving sizes distorted downward to reduce information rents to high-WTP consumers. Once a portion cap is imposed, which makes serving size smaller to high types, the seller must either further reduce serving size to low types or reduce prices to high types to maintain separation. However, because serving sizes for low types were already distorted downward, it becomes increasingly distortionary to reduce it further. Thus, reducing the price to high types is a less costly way of maintaining menu separation but taxes makes this more difficult.

[5]Because consumption of sugary drinks in relation to overall diet is higher among low-income consumers, soda taxes are deemed to be potentially regressive (Ogden et al. (2012))

Prior Relevant Literature

Our work complements the growing economic and public health literature analyzing the consequences of either restraining portions or taxing consumption of beverages with added sugars. These studies typically look at either consumers' reaction to the policy or responses of retail prices, taking the seller's decisions as given. We emphasize the role of the seller, since the seller cannot be expected to remain passive after an intervention. This would in turn, influence the serving sizes and prices facing consumers. By using a nonlinear pricing framework, our paper provides a more complete explanation of the mechanisms behind changes in the variables of interest.

Bourquard and Wu (2019) (BW) is close in spirit to our analysis. BW analyze size restrictions with a similar nonlinear pricing framework. Their model includes a price-discriminating seller facing a buyer with with unknown (to the seller) willingness-to-pay for SSBs. BW conclude that portion caps can reduce serving sizes without decreasing consumer welfare. We extend their study by also examining the impact of SSB taxes.

Wilson et al. (2013) (WSF) conduct a behavioral study to determine how restrictions in cup sizes might affect SSB consumption. In a non-incentive-compatible experiment, human subjects are asked to declare hypothetical purchases. Two menus are offered, one where the largest cup had a capacity of 32oz, and a second menu where the largest cup is replaced for 16oz cups. Their key finding is that subjects presented with the restricted menu end up buying more soda. While interesting, this result seem counter-intuitive as it suggests that sellers could increase sales by offering only small servings. In practice most food retailers offer differentiated price-quantity options in their menus. John et al. (2017) (JDR) conducted a similar study, but included incentive compatible options and a budget constraint. JDR find that a restricted menu does reduce consumption of soda.

Bonnet and Réquillart (2013) (BR) highlight the importance of taking strategic pricing of sellers into account when evaluating food policy. Using French representative consumer panel data, BR used structural econometric models and policy simulations to evaluate the incidence levels of ad valorem and per-unit taxes on soft drinks. They find that strategic firms react differently to distinct tax regimes; per-unit taxes are overshifted to final prices while ad valorem taxes are undershifted. BR conclude that an incorrect assumption of passive pricing would lead to an underestimating (overestimating) the effects on consumption of an ad valorem (specific) tax. While BR focused on strategic relationships between manufacturers and retailers, we are interested on isolating the effects of taxes due to strategic interactions between the retailers and final consumers.

A large number of empirical studies have estimated the impact of soda-taxes on consumption. For example, using data from a nationally representative survey, Fletcher et al. (2010) studied SSB taxation using variation of tax rates across states and time in order to identify the effects on consumption by adolescents and children; they found a negative impact on consumption of SSB, but the effect was offset due to substitution of other calorie dense beverages. Falbe et al. (2016) use self-reported data to learn the past-tax impact on SSB consumption in Berkeley, California. They find a significant decline in self-declared levels of consumption. Grogger (2017) studied the case of the Mexican per-unit tax on SSB. He found evidence of over-shifting on prices of beverages with added sugars, and less evidence of an increase in prices of other calorie dense drinks. These studies concentrate on buyers' reaction to the policy. Open questions remain regarding the seller's reaction to the policy.

While we examine the impact of regulation on serving sizes rather than consumption explicitly, the literature on portion sizes suggests that food/beverage intake tends to track the portion size, which is known as the "portion size effect" (Rolls et al., 2004; Almiron-Roig et al., 2015; English et al., 2014). There is evidence that increases in portion sizes have coincided with the rise in obesity (Young and Nestle, 2002).

Model Setup Without Regulations in Effect

We begin by establishing a benchmark for the retailer's pricing behavior in the absence of regulation. This allows us to make subsequent comparisons with respect to the impact of regulation on serving sizes, expected profit and consumer welfare. With regard to consumer welfare, we primarily focus on consumer welfare from the *consumption* of SSBs and do not account for potential health benefits from reduced sugar consumption. We do this for three reasons. First, much of the opposition against SSB regulations focus on how such regulations might hurt consumers via reduced choice and consumption. Second, incorporating health benefits in a model is fraught with arbitrary assumptions and it would be easy for us to generate nearly any conclusion by strategically choosing our assumptions. Our approach allows us to focus on the claim that regulation would reduce consumer welfare from consumption. Third, omitting health benefits makes our results robust to substitution effects in that, even if consumers shift to other unhealthy beverages, we do not run the danger of over-estimating consumer benefits.[6]

[6] Indeed, one way to interpret our findings is how consumers might be impacted even if the regulations yield little to no net health benefits.

Our model is a fairly standard nonlinear pricing model where the seller (the principal) offers a menu of different price-size combinations of a product to a privately informed buyer (the agent). Let q represent serving size (e.g. number of ounces contained in the cup).[7] We assume that the seller's cost is $c(q) = cq$ where $c'(q) = c > 0$ is a positive constant. The buyer's per-serving profit is $t(q) - cq$, where $t(q)$ is the price of a serving of size q. Note that if p is the average per-unit (e.g. per-ounce) price, then $t = p \cdot q$.

The seller cannot observe a buyer's preferences (or WTP). However, she does know the distribution of two types of buyers in the population. High type (H-types) customers have a high WTP for the SSB and prefer to consume more at a given price than low type (L-types).[8] There is a proportion $\beta \in [0, 1]$ of L-types in the population. Correspondingly, there is a proportion $(1 - \beta)$ of H-types. Letting $t_i = t(q_i)$ to conserve on notation, the i-type buyer's surplus is $U_i = \theta_i v(q_i) - t_i$, for $i = H, L$. We assume that $v(0) = 0$, $v'(q) > 0$ and $v''(q) < 0$ $\forall\ q \geq 0$. The taste parameter θ_i is such that $\theta_H > \theta_L$, so that the H-type has higher WTP and the Spence-Mirrlees single crossing condition is satisfied.

The seller designs an optimal pricing strategy that maximizes her expected profit subject

[7]Throughout the paper, we use the words "cup", "serving" and "package" interchangeably.

[8]Adding more than two-types to our model would only complicate the analysis, and reduce clarity and intuition without altering the general conclusions. Continuous type models would over-model the way SSBs are typically sold which is only in a few sizes. Even adding one additional type would not alter the main qualitative conclusions as pointed out by Bourquard and Wu (2019)

to incentive-compatibility (IC) and individual-rationality (IR) constraints:

$$\begin{aligned}
&\underset{(t_L,q_L),(t_H,q_H)}{\text{maximize}} \quad E[\pi] = (\beta)[t_L - cq_L] + (1-\beta)[t_H - cq_H] \quad s.t. \\
&\text{ICH} : \theta_H v(q_H) - t_H \geq \theta_H v(q_L) - t_L \\
&\text{ICL} : \theta_L v(q_L) - t_L \geq \theta_L v(q_H) - t_H \\
&\text{IRH} : \theta_H v(q_H) - t_H \geq \bar{v_H} \\
&\text{IRL} : \theta_L v(q_L) - t_L \geq \bar{v_L} \\
&\qquad q_L \geq 0 \qquad q_H \geq 0
\end{aligned} \tag{1}$$

Where $\bar{v_i}$ is the i-type's reservation utility. Without loss of generality, we let $\bar{v_i} = 0$. It is well known that IRH and ICL will not bind at the optimum and can be omitted from the optimization program. At the optimum, ICH and IRL bind with equality, which gives us the pricing rules:

$$t_L = \theta_L v(q_L) \tag{2}$$

$$t_H = \theta_H v(q_H) - (\theta_H - \theta_L)v(q_L) \tag{3}$$

Substituting (2) and (3) into the objective function yields:

$$\max_{q_L,q_H} \quad E[\pi] = \beta[\theta_L v(q_L) - cq_L] + (1-\beta)[\theta_H v(q_H) - (\theta_H - \theta_L)v(q_L) - cq_H] \tag{4}$$

$$q_L \geq 0 \qquad q_H \geq 0$$

The first order Kuhn-Tucker (KT) conditions of this problem are:

$$\text{FOC}[q_H] : \frac{\partial E[\pi]}{\partial q_H} = (1-\beta)[\theta_H v'(q_H) - c] \leq 0,\ q_H \geq 0 \ \text{ and } \ \frac{\partial E[\pi]}{\partial q_H} \cdot q_H = 0 \tag{5}$$

$$\text{FOC}[q_L] : \frac{\partial E[\pi]}{\partial q_L} = \beta[\theta_L v'(q_L) - c] + (1-\beta)[-(\theta_H - \theta_L)v'(q_L)] \leq 0,\ q_L \geq 0 \ \text{ and } \ \frac{\partial E[\pi]}{\partial q_L} \cdot q_L = 0 \tag{6}$$

We assume that the above K-T conditions can encompass one of three schemes. The retailer can i) adopt a "separating" strategy: offer two differentiated price-size combinations intended to serve each type of buyer, ii) implement an "exclusive" scheme: concentrate on serving H-type buyers exclusively or iii) apply a "pooling" or one-size-fits-all strategy: attempt to cover the entire consumer base with a single price-size serving. For the unregulated case, we assume that the separating strategy is the default case which is consistent with the stylized observation that SSB retailers typically offer multiple sizes to customers. To conserve space, we do not cover the unregulated exclusive or pooling strategies in the main article but do provide brief a treatment in Appendices A and B. Nonetheless, we will explore the possibility that a tax or size-restriction can cause the retailer to endogenously switch from a separating to an exclusive or pooling scheme in subsequent sections of the main paper.

Case IA - Unregulated Benchmark

Our default assumption is that, in the absence of regulation, the retailer uses a separating strategy by offering a menu of two distinct price-size options with $q_H > 0$ and $q_L > 0$. The

K-T conditions (5) and (6) bind with strict equality so that:

$$\theta_H v'(q_H) = c \tag{7}$$

$$\theta_L v'(q_L) = c + \left(\frac{1-\beta}{\beta}\right)(\theta_H - \theta_L) v'(q_L) \tag{8}$$

Note from (7) that there is no distortion at the top in that the large cup contains H-type's first-best level of quantity. On the other hand, the small cup contains a quantity level that is lower than the L-type's first best. Incentive compatibility requires that the H-type receives an information rent to choose the larger cup.[9] This information rent can be reduced by making the small cup less attractive to the H-type by making it smaller. The term $\left(\frac{1-\beta}{\beta}\right)(\theta_H - \theta_L) v'(q_L)$ that is attached to marginal cost of serving L-types determines the optimal downward distortion of the small cup.[10]

The optimal serving prices are obtained from the pricing rules 2 and 3 to obtain $t_H^{ia} = \theta_H v(q_H^{ia}) - (\theta_H - \theta_L) v(q_L^{ia})$ and $t_L^{ia} = \theta_L v(q_L^{ia})$. The retailer's expected profit is:

$$\pi^{ia} = (\beta)[\theta_L v(q_L^{ia}) - c q_L^{ia}] + (1-\beta)[\theta_H v(q_H^{ia}) - (\theta_H - \theta_L) v(q_L^{ia}) - c q_H^{ia}] \tag{9}$$

These results are summarized in proposition 1.

Proposition 1. *Suppose that the retailer adopts a separating strategy in the absence of regulation. Then:*

[9] The information rent is the excess rent that the H-type must receive from choosing the large serving. This ensures incentive compatibility because the H-type earns at least as much utility from choosing the package meant for her rather than the small serving that is meant for the L-type.

[10] It is not optimal to decrease the size of the small cup beyond this point because it becomes increasingly distortionary. Thus, the seller must weigh the gains from information rent savings against the profit loss from serving L-types in a distortionary way

1. *$\theta_H v'(q_H^{ia}) = c$ so that H-types are offered an efficient serving size.*

2. *$\theta_L v'(q_L^{ia}) = \frac{c}{\left[1-\left(\frac{1-\beta}{\beta}\right)\left(\frac{\theta_H-\theta_L}{\theta_L}\right)\right]} > c$ so that L-types are offered a serving size that is distorted downward relative to first best.*

3. *$t_H^{ia} = \theta_H v(q_H^{ia}) - (\theta_H - \theta_L)v(q_L^{ia})$ so that the price of the H-type serving is discounted by an information rent.*

4. *$t_L^{ia} = \theta_L v(q_L^{ia})$ so that the price of the L-type serving extracts all of the surplus from L-type buyers.*

5. *The seller's value optimized profit is expressed in equation (9).*

6. *The H-type consumer's value function is $U_H^{ia} = (\theta_H - \theta_L)v(q_L^{ia})$.*

7. *The L-type consumer's value function is $U_L^{ia} = 0$.*

Put simply, the retailer will offer two differentiated price-size options to consumers. The L-type option is inefficiently small and priced so that the L-type is indifferent between buying and not buying. The H-type serving is larger and of efficient size for H-type consumers. Moreover, H-types receive a quantity discount that leaves them with some excess surplus above reservation level.

Incorporating Taxation into the Model

We expect a SSB tax to have two major effects. First, it could directly impact cup sizes and prices, and second, it may indirectly cause the retailer to alter her pricing strategy (e.g. separating, exclusive, or pooling). We start by analyzing the direct effects of the tax on sizes and prices *holding the seller's pricing strategy constant.* Once, we obtain maximized profit for each pricing strategy, then we can determine which strategy the retailer will adopt in response to a tax.

Let us define a tax regime (τ_s, τ_v) as any mixture of specific $(\tau_s \geq 0)$ and *ad valorem* $(\tau_v \in [0, 1))$ taxes, such that both of them are not zero at the same time. To avoid divisions by zero, we exclude combinations where $\tau_v = 1$. Note that $(\tau_s, \tau_v) = (0, 0)$ represents the event of no taxation. Specific taxes modify the objective function in a way akin to a change in the principal's cost function. Ad valorem taxes alter the objective function in two ways: by modifying the cost function, and scaling down expected profit. The seller's problem is:

$$\max_{q_L, q_H} \; E[\pi] = (1-\tau_v)\Big\{(\beta)\big[\theta_L v(q_L) - \Psi_L\big] + (1-\beta)\big[\theta_H v(q_H) - (\theta_H - \theta_L)v(q_L) - \Psi_H\big]\Big\} \quad (10)$$

where $\Psi_i \equiv (\tau_s q_i + c q_i) \div (1-\tau_v)$ is the effective cost function. Let $\psi \equiv \frac{d\Psi_i}{dq_i} = (\tau_s + c) \div (1-\tau_v)$ denote effective marginal cost. First order K-T conditions are:

$$\text{FOC}[q_H] : \frac{\partial E[\pi]}{\partial q_H} = (1-\tau_v)(1-\beta)[\theta_H v'(q_H) - \psi] \leq 0, \; q_H \geq 0 \;\text{ and }\; \frac{\partial E[\pi]}{\partial q_H} \cdot q_H = 0 \quad (11)$$

$$\text{FOC}[q_L] : \frac{\partial E[\pi]}{\partial q_L} = (1-\tau_v)\big\{\beta(\theta_L v'(q_L) - \psi) + (1-\beta)[-(\theta_H - \theta_L)v'(q_L)]\big\} \leq 0, \quad (12)$$

$$q_L \geq 0 \;\text{ and }\; \frac{\partial E[\pi]}{\partial q_L} \cdot q_L = 0$$

Below, we describe how taxes affect outcomes, retailer profit, and consumer welfare within each major pricing strategy.

Taxed Case IIA: Separating Pricing Strategy

Under a separating contract, both types of consumers are offered options such that $q_H > 0$ and $q_L > 0$. Thus, the K-T conditions (11) and (12) hold with strict equality:

$$\theta_H v'(q_H) = \psi \tag{13}$$

$$\theta_L v'(q_L) = \psi + \left(\frac{1-\beta}{\beta}\right)(\theta_H - \theta_L)v'(q_L) \tag{14}$$

These conditions indicate that the *effective* marginal cost of supplying a large cup equals the H-type's marginal utility of consumption. Thus, while there is distortion relative to the untaxed case, there is "no distortion at the top" relative to the effective marginal cost. The seller continues to distort the size of the L-type package downwards even with respect to the effective marginal cost due to the presence of the term $\left(\frac{1-\beta}{\beta}\right)(\theta_H - \theta_L)v'(q_L)$. However, relative to the untaxed scenario, both types of consumers get less than their first-best optimal quantities because the effective marginal cost is altered by the tax so that $\psi > c$. L-types receive an even smaller quantity than the unregulated case I-A.

The price rules associated with this case are $t_L^{iia} = \theta_L v(q_L^{iia})$, and $t_H^{iia} = \theta_H v(q_H^{iia}) - (\theta_H - \theta_L)v(q_L^{iia})$. The seller's expected profit is:

$$\pi^{iia} = (1-\tau_v)\Big\{(\beta)[\theta_L v(q_L^{iia}) - \psi q_L^{iia}] + (1-\beta)\big\{[\theta_H v(q_H^{iia}) - (\theta_H - \theta_L)v(q_L^{iia})] - \psi q_H^{iia}\big\}\Big\} \tag{15}$$

In the next proposition, we compare the outcomes above with the results derived in our benchmark case IA, where there is no regulation.

Proposition 2. *Assume the government enforces a tax regime* (τ_s, τ_v) *with at least one type of tax strictly positive. Suppose that the retailer serves both type of buyers with a separating pricing strategy. Then:*

1. *The tax reduces H-type serving sizes so that* $q_H^{ia} > q_H^{iia}$.
2. *The L-type serving size is still distorted downward. Additionally,* $q_L^{ia} > q_L^{iia}$ *so that serving size is even smaller under the tax.*
3. *The L-type serving price is lower under the tax; i.e.* $t_L^{ia} > t_L^{iia}$.
4. *The H-type serving price is lower under the tax; i.e.* $t_H^{ia} > t_H^{iia}$.
5. *The retailer's value function is given by (15) so that profit is lower under the tax; i.e.* $E\pi^{ia} > E\pi^{iia}$.
6. *The H-type buyer's value function is* $U_H^{iia} = (\theta_H - \theta_L)v(q_L^{iia})$ *so that there is consumer welfare loss for H-types; i.e.* $U_H^{ia} > U_H^{iia}$.
7. *L-type buyer's value function remains* $U_L = 0$*; i.e.* $U_L^{ia} = U_L^{iia}$.

The proof is omitted since it is just a straightforward comparison. In sum and compared to the corresponding benchmark, both packages are smaller due to distortions caused by the tax. H-type consumers still receive information rents, although these are smaller so there is welfare loss. L-types are held at their reservation values, and the seller sees her expected profit unambiguously diminished. While it may seem counter-intuitive that serving prices are lower after a tax, keep in mind that we focus on *serving size* prices as opposed to per-unit prices (e.g. price per-ounce).[11]

[11] In the standard textbook problem, a tax causes the demand function to shift downward and lead to a

Taxed Case II-B: The Seller Serves only H-type consumers

We now consider the case where the seller serves only H-types ($q_H > 0$ and $q_L = 0$) because we ultimately want to know whether a tax can cause the retailer to switch from the default separating strategy to the H-type only strategy, and, if so, the economic consequences associated with such a shift.

Let the superscript *iib* denote variables under this strategy. This scheme implies that FOC[q_L] in (12) does not bind with equality. Using FOC[q_H] from (11), pricing rule (3), and our normalizing assumption $v(0) = 0$, we obtain:

$$\theta_H v'(q_H^{iib}) = \psi \tag{16}$$

$$t_H^{iib} = \theta_H v(q_H^{iib}) \tag{17}$$

The seller no longer needs to elicit truthful revelation of information and therefore does not grant information rents.[12] Expected profit is

$$\pi^{iib} = (1 - \tau_v)(1 - \beta)[\theta_H v(q_H^{iib}) - \psi q_H^{iib}] \tag{18}$$

Proposition 3. *Assume that the government enforces a tax regime (τ_s, τ_v) with at least one type of tax strictly positive. Suppose that the retailer decides to offer one single cup size designed to serve H-type buyers solely. Then:*

higher uniform price for all units. However, in the nonlinear pricing problem that we study, a serving size price is designed to extract as much consumer surplus as possible. As such, the implicit unit price may vary across different units.

[12]L-types drop out of the market because they would earn negative utility given the price.

1. *$\theta_H v'(q_H^{iib}) = \psi > c$. There is a tax induced reduction in q_H^{iib} below first best. Thus, $q_H^{iib} < q_H^{ia}$*

2. *L-type buyers are excluded and do not engage in trade.*

3. *The serving price is $t_H^{iib} = \theta_H v(q_H^{iib})$ which does not include an information rent.*

4. *The seller's value functions is expressed by equation (18).*

5. *Both buyer types are held at their reservation values; i.e. $U_H = U_L = 0$.*

The proof is just a straightforward comparison so we exclude it.

Taxed Case II-C: One-Size-Fits-All

Another case that must be considered is when the retailer chooses a single size and price to serve both types of buyers. The retailer's optimization problem can be written as follows:

$$\begin{aligned} &\max_{p,q} E[\pi] = (1-\tau_v)t_L - (\tau_s + c)q_L \\ &\text{subject to:} \\ &\text{IRL} : \theta_L v(q) - t_L \geq 0 \end{aligned} \tag{19}$$

Given that $\theta_H > \theta_L$, if the L-type's individual rationality constraint is satisfied, then the H-type's will be satisfied as well. The optimization conditions imply:

$$\theta_L v'(q_L^{iic}) = \psi \tag{20}$$

$$t_L^{iic} = \theta_L v(q_L^{iic}) \tag{21}$$

While 20 implies marginal benefits are equal to marginal cost, recall that ψ is the effective marginal cost post-tax. Thus, the L-type does not get his first-best level of consumption. The value functions for the seller and buyers are:

$$\pi^{iic} = (1-\tau_v)[\theta_L v(q_L^{iic}) - \psi \cdot q_L^{iic}] \tag{22}$$

$$\begin{aligned} U_L^{iic} &= 0 \\ U_H^{iic} &= (\theta_H - \theta_L)v(q_L^{iic}) \end{aligned} \tag{23}$$

We summarize the results in the next proposition.

Proposition 4. *Assume the government enforces a tax regime (τ_s, τ_v) with at least one type of tax strictly positive. If the retailer decides not to screen the market and offers a one-size-fits-all package designed to serve both types of buyers, then:*

1. *$\theta_L v'(q_L^{iic}) = \psi$ so that buyers are provided with a quantity, q_L^{iic}, that is smaller than the L-type first best.*
2. *The price per serving is $t_L^{iic} = \theta_L v(q_L^{iic})$.*
3. *The seller's value function is given by equation (22).*
4. *The L-type consumer value function is $U_L^{iic} = 0$.*
5. *The H-type consumer value function is $U_H^{iic} = (\theta_H - \theta_L)v(q_L^{iic}) > 0$.*

The proof is straightforward and therefore excluded. Note that H-type buyers still earn excess rents though this is not due to screening driven information rents.

How does taxation affect retailers' choice of scheme?

In the previous section, we examined how the tax might affect outcomes under each major discrete pricing strategy. Recall that in the absence of regulation, we assume that the retailer adopts the separating screening strategy which is akin to second degree pricing discrimination. The question is: will this change after a tax?

Answering this question is straightforward given that we now have the retailer's value function under each discrete strategy. First, we want to know whether the implementation of a tax might cause the retailer to switch from the default separating strategy to an H-exclusive strategy. To help answer this, we make the following claim.

Claim 1. *The separating strategy is more profitable than the H-exclusive strategy if and only if* $\beta \geq \underline{\beta}_E = \frac{[\theta_H - \theta_L]v(q_L^{iia})}{\theta_H v(q_L^{iia}) - c q_L^{iia}}$.

The above claim pins down a minimum threshold $\underline{\beta}_E$ that β must exceed in order for the separating strategy to be more profitable than a H-exclusive strategy. Claim 1 then leads naturally to a key result.

Proposition 5. *Suppose that a tax regime,* (τ_s, τ_v)*, comes into effect. Then,* $\underline{\beta}_E$ *increases, which reduces the range of* β *for which the separating strategy is more profitable than the H-exclusive strategy.*

Proof: The proof is in Appendix C.

Because a tax reduces the range of β for which the separating strategy is more profitable than the H-exclusive strategy, it increases the possibility that retailers might endogenously switch to the H-exclusive strategy. We will discuss the welfare implications of this switch in more detail in a subsequent section.

The next question is, how might a tax influence the retailer's propensity to switch from the status quo separating strategy to an one-siz-fits-all strategy?

Claim 2. *The separating strategy is more profitable than the one-size-fits-all pricing strategy if and only if* $\beta \geq \underline{\beta}_O = \frac{[\theta_H - \theta_L] v(q_L^{iia}) + \theta_L v(q_L^{iic}) - \psi q_L^{iic} - [\theta_H v(q_H^{iia}) - \psi q_H^{iia}]}{\theta_H v(q_L^{iia}) - \psi q_L^{iia} - [\theta_H v(q_H^{iia}) - \psi q_H^{iia}]}$.

The above claim pins down the minimum threshold, $\underline{\beta}_O$, that β must exceed for the separating strategy to remain more profitable than the one-size-fits-all strategy.

Proposition 6. *Suppose that a tax regime* (τ_s, τ_v)*, comes into effect. Then* $\underline{\beta}_O$ *decreases, which increases the range of* β *for which the separating strategy is more profitable than the one-size-fits-all strategy.*

In contrast to the previous proposition, Proposition 6 suggests that a tax actually increases the probability that retailers will not switch from a separating strategy to a one-size-fits-all strategy. The intuition is that a one-size-fits-all-strategy has to be priced reasonably low to ensure that L-type consumers participate. Thus, it is a relatively low profit margin strategy that relies on volume to make money. However, a rise in taxes raises marginal cost which puts pressure on the already low margins.

The key take-away is that if the retailer choses the separating strategy pre-tax, which is true by assumption, then the retailer will not switch to an one-size-fits-all strategy post-tax. Thus, we need not consider the one-size-fits-all strategy further when assessing the tax.

Policy Implications

First, we address whether the tax will have the intended effect of reducing SSB consumption. One way to address this question is to ask whether an implementation of the tax will reduce serving size. Rolls et al. (2004); Almiron-Roig et al. (2015); English et al. (2014) have noted that consumption of food and beverage products tends to track portion size. Second, we want to know how the tax affects consumer surplus because political opponents of SSB regulations often claim that these regulations hurt consumers and/or are regressive in that certain demographic groups are disporportinately affected (Grynbaum (2012); Nestle (2012)). Finally, we want to know how the tax affects sellers of SSB.

Effects on serving sizes (quantity)

An important question is whether a tax achieves the intended effect of reducing serving size. To answer this question, we compare the size of the packages offered in the pre-tax scenario I-A to the relevant post-tax packages. The relevant tax cases under consideration are cases IIA and IIB: the separating and H-exclusive strategies.

Proposition 7. *Suppose that a tax regime* (τ_s, τ_v) *is implemented. Then serving sizes for both types of consumers decline.*

To understand the intuition, previous propositions suggest that a tax will either cause the retailer to remain with the separating strategy or switch to the H-type strategy. In the former case, a tax reduces the serving size of both H and L-types. In the latter case, the post-tax H-exclusive strategy delivers the same serving size as the post-tax separating strategy, which is smaller than the pre-tax separating strategy. The L-type is no longer served. So in either case, serving sizes are reduced.

Effect of Taxation on Consumers' Surplus

The impact on consumer surplus is arguably one of the most important policy effects given that consumer welfare is often mentioned in political debates over SSB regulation; i.e. policy success may depend on how consumer surplus is impacted.

Proposition 8. *Suppose that a tax regime* (τ_s, τ_v) *is implemented. Then consumer surplus for H-types declines. Consumer surplus for L-types is unaffected.*

Intuitively, if a tax does not cause the retailer to switch away from a separating strategy, the tax still causes the L-type serving size to drop, which lowers the H-type information rent. Thus, H-type consumer welfare decreases from $U_H^{ia} = (\theta_H - \theta_L)v(q_L^{ia})$ to U_H^{iia} to $(\theta_H - \theta_L)v(q_L^{iia})$ where $q_L^{ia} > q_L^{iia}$. If instead, the tax causes the retailer to switch to a H-type exclusive strategy, then since the retailer need not worry about screening and paying an information rent, the retailer can just hold the H-type to his reservation utility. In either case, the H-type consumer surplus declines.

L-type consumer surplus is unaffected by the tax because the L-type earns reservation utility in all relevant scenarios. While technically, the L-type is no longer served under the H-type exclusive scheme, the L-type is consuming his next best beverage and thus still earns the reservation utility.

Effect of Taxation on Seller Surplus

Another political argument against SSB taxes is that they put pressure on businesses. According to White (2019), the beverage industry launched a campaign to influence the business community in Pennsylvania to pressure legislators to pass a state law that would prohibit Pennsylvania cities from passing local soda taxes. Our model predicts that indeed SSB taxes will reduce retailer profitability irrespective of whether retailers switch away from the separating strategy or not.

Proposition 9. *Suppose that a tax regime* (τ_s, τ_v) *is implemented. Then retailer surplus unambiguously declines.*

Intuitively, the tax causes the sizes of the servings under the separating strategy to be distorted downward relative to the optimal sizes that maximize the retailer's profits. Even if the tax induces the retailer to switch to the H-exclusive strategy, this only means the H-exclusive strategy is more profitable than the post-tax separating strategy, not the pre-tax separating strategy.

Comparing Taxes and Portion Size Restrictions

Bourquard and Wu (2016) analyze portion size restrictions with a similar nonlinear pricing framework. We purposely chose our model to be similar to theirs to enhance comparability of results. The only modification we make is to the form of the cost function to ensure that the model is consistent with the model we used in the previous sections to evaluate SSB taxes. No qualitative results will be affected by our modification.

The retailer maximizes expected profit subject to the usual incentive-compatibility and participation constraints, plus a restriction on the maximum size per serving (PS). In the economically interesting case, PS is binding, thus $q_H = \hat{q}$.

$$
\begin{aligned}
&\underset{(t_L,q_L),(t_H,q_H)}{\text{maximize}} \ (\beta)[t_L - cq_L] + (1-\beta)[t_H - cq_H] \text{ s.t.} \\
&\text{IC} : \theta_H v(q_H) - t_H \geq \theta_H v(q_L) - t_L \\
&\text{IR} : \theta_L v(q_L) - t_L \geq 0 \\
&\text{PS} : 0 \leq q_i \leq \hat{q}, \text{ for } i = \{H,\, L\}
\end{aligned}
\tag{24}
$$

To conserve space, we do not solve the above problem in the main body of the paper as the derivation follows Bourquard and Wu (2016) closely. We provide the details in Appendix D.

Table 1 organizes qualitative results under both policies so that the reader can quickly compare outcomes under the tax versus the size-restriction. We highlight several results starting with consumer surplus. Under the size restriction, consumer surplus is unaffected when the cap only limits the largest serving size whereas the tax causes a decrease in consumer surplus for for all non-zero tax rates.

Result 1: *A portion size restriction that only limits the H-type serving but not the L-type serving will have no impact on consumer surplus. Any non-zero tax will decrease H-type consumer surplus and weakly decrease L-type consumer surplus.*

Note that the key qualification is that the size restriction cannot limit the L-size serving or there would be a decrease in consumer surplus. For example, if prior to regulation, a H-type serving is 32 ounces and an L-type serving is 16 ounces, then any size restriction strictly above 16 ounces would have no impact on consumer surplus.

Result 2: *A size restriction that only caps the H-type serving will not affect the L-type serving. Any positive tax will induce decreases in serving sizes for both types.*

If society decides to reduce soda consumption to H-types, both policy instruments can induce a reduction. A non-zero tax will also reduce serving size to L-types whereas the portion size restriction would have to be explicitly designed to cap L-type serving size. Keep in mind that the general results are qualitative in nature.

Result 3: *Both the size restriction and the tax will reduce seller surplus.*

Both the tax and size restriction would face political challenges because they could potentially hurt businesses. Given that many retailers are small businesses, and larger retailers employ many people, this could pose a challenge to actually passing legislation. However, in political environments where there is concern about consumer welfare or where the impact of regulation is regressive, then the size restriction has a distinct advantage. Where taxes have an advantage is that they could generate government revenue which can be used to sell the policy to constituents by promising that the money would fund public programs.[13]

Parametric Example: Effects on Quantities and Weight Loss

Imagine two customers (H and L) who eat lunch at a quick service restaurant every day. Customer H drinks one 31 ounces cup of soda every day, while L consumes one 7.8 ounce cup. Suppose that the government wants to impose a maximum cup size of 17 ounces. The

[13] When arguing for a soda tax, the mayor of Philadelphia pledged to spend tax generated funds on public programs such as universal pre-kindergarten (Cohen (2016)).

retailer's problem takes the following parametric form:

$$
\begin{aligned}
&\underset{(t_L,q_L),(t_H,q_H)}{\text{maximize}}\ (\beta)[(1-\tau_v)t_L-\tau_s q_L-kq_L^m]+(1-\beta)[(1-\tau_v)t_H-\tau_s q_H-kq_H^m]\text{ s.t.}\\
&\text{IC}: \theta_H q_H^\delta - t_H \geq \theta_H q_L^\delta - t_L\\
&\text{IR}: \theta_L q_L^\delta - t_L \geq 0\\
&\text{PS}: 0 \leq q_i \leq \hat{q},\text{ for } i=\{H,\, L\}
\end{aligned}
\tag{25}
$$

Where τ_v represents an ad valorem tax, τ_s a per-unit tax. PS defines the portion size restriction and $\hat{q}$ is the size of the restriction (17 ounces in our example). In the unregulated case, $(\tau_s,\, \tau_v) = (0,\, 0)$, and the restriction PS is inactive. Under taxation, we have either τ_s, τ_v or both strictly positive, and PS is inactive. Under portion size restriction, $(\tau_s,\, \tau_v) = (0,\, 0)$ and PS is binding. Given a possible set of parameter values, Table 2 shows the optimal pricing schedules across policy environments.

If the government enforces a portion size restriction, consumption by H will drop from 31 to 17 and the small cup size will remain unaffected. A specific tax high enough to reach the same size reduction for the large cup will also affect the size of the small cup. Therefore, a tax would decrease both the H and L serving sizes.

The important points to note from our numeric example are the following:

- The portion cap restriction decreases only the H-type serving whereas the tax decreases both sizes.

- Consumer welfare is unaffected by the portion-cap whereas the tax reduces the H-type utility from 70.6 to 39.8.

- Both the portion-cap and tax reduce expected profit of the seller and the reduction is greater under the tax.

We might also want to know how these numeric results translate into weight loss. Assuming that neither customer substitutes soda for any other product, we follow Cutler et al. (2003), and Grogger (2017) and calculate an expected weight change resulting from each intervention using the Harris-Benedict formula (Harris and Benedict, 1918). The HB formula establishes a relationship between weight and basal metabolic rate (BMR), which is the daily number of calories required to maintain the human body at complete rest. This equation takes the following form:

$$\mathrm{BMR} = \alpha + \delta \mathrm{W} \tag{26}$$

where α is a function of age, sex and height; while δ depends on the person's sex. According to Roza and Shizgal (1984), $\delta = 13.397$ for men and $\delta = 9.247$ for women. We also need the following relationship:

$$I = \gamma(\mathrm{BMR}) \tag{27}$$

where I denotes a person's caloric needs. According to Douglas et al. (2007), γ equals 1.2 or 1.5 if the individual is sedentary or moderately active, respectively. From equation 27 we can estimate a steady-state change in weight $\Delta\mathrm{W}$ given the difference in caloric intake before and after implementing the regulation:

$$\Delta\mathrm{W} = \frac{\Delta\mathrm{I}}{\gamma \cdot \delta} \tag{28}$$

We assume that one ounce of soda contains 11.83 calories, and that our hypothetical consumers are moderately active ($\gamma = 1.5$).

In table 3, we simulate steady-state changes in pounds for each type (ΔW_H and ΔW_L) under different policy environments relative to the unregulated benchmark. The size restrictions causes the serving size to drop from 31 ounces to 17 ounces. The results highlight the potential for both taxes and size restrictions to limit portion sizes to desired quantity targets (17 in our example). The table also highlights that both policies impact H-type weight changes more than L-type weight changes. But the size restriction has the advantage of being both less distortionary and less regressive in terms of impacting consumer surplus.[14]

Conclusion

To our knowledge, our study is one of the first to theoretical compare SSB taxes and portion size restrictions within a nonlinear pricing framework. Nonlinear pricing is consistent with the way SSBs are sold in practice, particularly in single sitting environments, such as restaurants and/or convenience stores. Thus, our study can add nuanced economic content to the debate on how SSB regulations might impact consumers and sellers.

The most important findings from our paper are that both the tax and portion-size restriction can be effective at curbing portion sizes to heavy consumers (those with high willingness-to-pay) of SSBs, but the portion size restriction can do this without negatively impacting consumer surplus. Thus, political arguments that claim that portion size restric-

[14] The actual weight loss numbers are rather sizable and perhaps overestimates of what might happen in practice due to leakage and enforcement problems. However, the purpose of this example was largely pedagogical as the analytic model presented in the earlier sections can be fairly abstract.

tions hurt consumers are largely misplaced. However, both policies would decrease seller surplus so concerns that the restrictions might hurt small business or employment in the beverage sector have merit.

The preferred policy depends on the preferences of the regulator and the political environment. If there is political concern for consumer welfare, then size caps are superior to taxes, since they can curb consumption while leaving consumer surplus unaffected. If the success of the policy is also tied to tax revenue generation to fund social welfare programs, then taxes should be preferred. Moreover the tax mix should be crafted such that the negative impact on consumption is the lowest because the larger the reduction in consumption, the smaller the tax revenue (Bonnet and Réquillart, 2013). If the goal is to reduce consumption, not of SSB, but of added sugars, then more work is needed so as to discern which policy specifically targeting this ingredient reduces its intake in the most economical manner.

Ideally, a theoretical study such as ours should be combined with data to verify the predictions. However, because portion-caps have only been proposed but not passed, no data is current available. Another potential concern of our model is that it is based on a hypothetical monopolist retailer who engages in nonlinear pricing. However, as pointed out by Bourquard and Wu (2019), there is no scope for nonlinear pricing under perfect competition, which is inconsistent with what is observed in practice, and the introduction of oligopolistic competition only serves to increase consumer welfare. The fact that our monopoly model may underestimate consumer welfare suggests that the result that consumer welfare is unaffected under the portion cap restriction is robust. Moreover, while competition would reduce distortions under the tax, it would not eliminate them. Hence, our qualitative results are robust to the introduction of competition.

References

Allcott, H., B. Lockwood, and D. Taubinsky (2019). Should we tax sugar-sweetened beverages? an overview of theory and evidence. *Journal of Economic Perspectives 33*(3), 202–227.

Almiron-Roig, E., M. Tsiountsioura, H. Lewis, J. Wu, I. Solis-Trapala, and S. Jebb (2015). Large portion sizes increase bite size and eating rate in overweight women. *Physiology and Behavior 139*, 297–302.

Anwar, Y. (2016, August). Soda tax linked to drop in sugary beverage drinking in berkeley - berkeley news. http://news.berkeley.edu/2016/08/23/sodadrinking/. Accessed on 10/01/2016.

Aubrey, A. (2016a, November). Souring On Sweet? Voters In 4 Cities Pass Soda Tax Measures. *The Salt : NPR.* Retrieved from http://www.npr.org/sections/thesalt/2016/11/09/501472007/souring-on-sweet-voters-in-4-cities-pass-soda-tax-measures.

Aubrey, A. (2016b, June). Taxing sugar: 5 things to know about philly's soda tax : The salt : Npr. http://www.npr.org/sections/thesalt/2016/06/09/481390378/taxing-sugar-5-things-to-know-about-phillys-proposed-soda-tax. Accessed on 10/01/2016.

Bonnet, C. and V. Réquillart (2013). Tax incidence with strategic firms in the soft drink market. *Journal of Public Economics 106*, 77–88.

Bourquard, B. and S. Wu (2019). An analysis of beverage size restrictions. *American Journal of Agricultural Economics*, Forthcoming.

Bourquard, B. and S. Y. Wu (2016). An Economic Analysis of Beverage Size Restrictions. *Purdue University, mimeo*.

Bray, G. A., S. J. Nielsen, and B. M. Popkin (2004). Consumption of high-fructose corn syrup in beverages may play a role in the epidemic of obesity. *The American Journal of Clinical Nutrition 79*(4), 537–543.

Cochero, M. A., J. Rivera-Dommarco, B. M. Popkin, and S. W. Ng (2017). In mexico, evidence of sustained consumer response two years after implementing a sugar-sweetened beverage tax. *Health Affairs*, 10–1377.

Cohen, L. (2016, June). Philadelphia mayor rewrites playbook for soda-tax advocates. www.reuters.com/article/us-beverages-philadelphia-sodatax-mayor-idUSKCN0Z20BI. Accessed on 04/17/2017.

Cutler, D. M., E. L. Glaeser, and J. M. Shapiro (2003). Why have americans become more obese? *The Journal of Economic Perspectives 17*(3), 93–118.

Dobbs, R., C. Sawers, F. Thompson, J. Manyika, J. R. Woetzel, P. Child, S. McKenna, and A. Spatharou (2014). *Overcoming obesity: An initial economic analysis*. McKinsey Global Institute.

Douglas, C. C., J. C. Lawrence, N. C. Bush, R. A. Oster, B. A. Gower, and B. E. Darnell (2007). Ability of the harris-benedict formula to predict energy requirements differs with weight history and ethnicity. *Nutrition Research 27*(4), 194–199.

English, L., M. Lasschuijt, and K. Keller (2014). Mechanisms of the portion size effect. what is known and where do we go from here? *Appetite 88*, 39–49.

Falbe, J., N. Rojas, A. H. Grummon, and K. A. Madsen (2015). Higher retail prices of sugar-sweetened beverages 3 months after implementation of an excise tax in berkeley, california. *American Journal of Public Health 105*(11), 2194–2201.

Falbe, J., H. R. Thompson, C. M. Becker, N. Rojas, C. E. McCulloch, and K. A. Madsen (2016). Impact of the Berkeley excise tax on sugar-sweetened beverage consumption. *American Journal of Public Health 106*(10), 1865–1871.

Fletcher, J. M., D. E. Frisvold, and N. Tefft (2010). The effects of soft drink taxes on child and adolescent consumption and weight outcomes. *Journal of Public Economics 94*(11), 967–974.

Grogger, J. (2017). Soda taxes and the prices of sodas and other drinks: Evidence from Mexico. *American Journal of Agricultural Economics 99*(2), 481.

Grynbaum, M. (2012). Soda makers begin their push against new york ban, *The New York Times*, New York, NY, July 1. http://www.nytimes.com/2012/07/02/nyregion/in-fight-against-nyc-soda-ban-industry-focuses-on-personal-choice.html?pagewanted=all.

Harris, J. A. and F. G. Benedict (1918). A biometric study of human basal metabolism. *Proceedings of the National Academy of Sciences 4*(12), 370–373.

John, L. K., G. E. Donnelly, and C. A. Roberto (2017). Psychologically informed implementations of sugary-drink portion limits. *Psychological Science*, 0956797617692041.

Jones, A., J. L. Veerman, and D. Hammond (2017, March). The health and economic impact of a tax on sugary drinks in canada.

Lustig, R. H., L. A. Schmidt, and C. D. Brindis (2012). Public health: the toxic truth about sugar. *Nature 482*(7383), 27–29.

Malik, V. S., B. M. Popkin, G. A. Bray, J.-P. Després, W. C. Willett, and F. B. Hu (2010). Sugar-sweetened beverages and risk of metabolic syndrome and type 2 diabetes a meta-analysis. *Diabetes care 33*(11), 2477–2483.

Malik, V. S., M. B. Schulze, and F. B. Hu (2006). Intake of sugar-sweetened beverages and weight gain: a systematic review. *The American Journal of Clinical Nutrition 84*(2), 274–288.

Nestle, M. (2012). Why the 'consumer choice' argument is an automatic win for big soda, *The Atlantic*, Washington, D.C., July 2. http://http://www.theatlantic.com/health/archive/2012/07/why-the-consumer-choice-argument-is-an-automatic-win-for-big-soda/259293/.

New York Statewide Coalition of Hispanic Chambers of Commerce v. New York City Department of Health and Mental Hygiene (2014, N.Y.). 16 N.E. 3d 538.

O'Connor, A. and M. Sanger-Katz (2018). California, of all places, has banned soda taxes. how a new industry strategy is succeeding, *New York Times*, New York, NY, June 27. https://www.nytimes.com/2018/06/27/upshot/california-banning-soda-taxes-a-new-industry-strategy-is-stunning-some-lawmakers.html.

Ogden, C., B. Kit, M. Carroll, and S. Park (2012). Consumption of sugar drinks in the united states, 2005–2008. nchs data brief, no 71. hyattsville, md: National center for health statistics. 2011. *US Department of Agriculture, Agricultural Research Service, National Agricultural Library. National nutrient database for standard reference, release 25.*

Rolls, B., L. Roe, T. Kral, J. Meengs, and D. Wall (2004). Increasing the portion size of a packaged snack increases energy intake in men and women. *Appetite 42*(1), 63–69.

Roza, A. M. and H. M. Shizgal (1984). The harris benedict equation reevaluated: resting energy requirements and the body cell mass. *The American Journal of Clinical Nutrition 40*(1), 168–182.

Silver, L. D., S. W. Ng, S. Ryan-Ibarra, L. S. Taillie, M. Induni, D. R. Miles, J. M. Poti, and B. M. Popkin (2017). Changes in prices, sales, consumer spending, and beverage consumption one year after a tax on sugar-sweetened beverages in berkeley, california, us: A before-and-after study. *PLoS medicine 14*(4), e1002283.

Stanhope, K. L. (2016). Sugar consumption, metabolic disease and obesity: The state of the controversy. *Critical reviews in clinical laboratory sciences 53*(1), 52–67.

USDA (2017). What are added sugars? choose myplate. https://www.choosemyplate.gov/what-are-added-sugars. (Accessed on 04/15/2017).

White, J. (2019). Is big soda winning the soft drink wars?, *Politico: The Agenda*, Arlington County, VA, August 13, 2019. https://www.politico.com/agenda/story/2019/08/13/soda-tax-california-public-health-000940.

Wilson, B. M., S. Stolarz-Fantino, and E. Fantino (2013). Regulating the way to obesity: unintended consequences of limiting sugary drink sizes. *PloS one 8*(4), e61081.

World Health Organization (2015). Guideline: Sugars intake for adults and children.

World Health Organization (2016). Fiscal policies for diet and the prevention of noncommunicable diseases.

Yan, H. (2013). No Soda Ban Here: Mississippi passes 'Anti-Bloomberg' Bill, CNN, Atlanta, GA, March 21. http://www.cnn.com/2013/03/21/us/mississippi-anti-bloomberg-bill/.

Young, L. and M. Nestle (2002). The contribution of expanding portion sizes to the us obesity epidemic. *American Journal of Public Health 92*, 246–249.

Table 1: **Qualitative comparison of outcomes (relative to no regulation)**

	Size restriction	Severe size restriction[a]	Tax
Seller's optimal pricing strategy	Unchanged	Switch to single-price[b]	Increased liklihood of H-exclusive strategy
H-type serving (q_H)	Decrease	Decrease	Decrease
L-type serving (q_L)	Unchanged	Decrease	Decrease
H-type consumer surplus	Unchanged	Decrease	Decrease
L-type consumer surplus	Unchanged	Weakly decreasing	Weakly decreasing
Producer/seller surplus	Decrease	Decrease	Decrease

[a]A severe size restriction is one that includes a cap that falls below the optimal L-size serving size under the unregulated segmentation strategy. In other words, the restriction is so severe, it also caps the size of the small serving.

[b]The single-price strategy can be either H-exclusive or one-size-fits-all depending on the size of β.

Table 2: Optimal Pricing Schedules

	Equilibrium Values						
Policy	q_H	q_L	t_H	t_L	U_H	U_L	$E[\pi]$
No regulation	31.1	7.8	7784.7	2047.1	70.6	0.0	245.8
Portion cap: $\hat{q} = 17$	17.0	7.8	4355.8	2047.1	70.6	0.0	222.6
Tax: $(\tau_s, \tau_v) = (7.35, 0)$	17.0	4.3	4388.6	1154.0	39.8	0.0	138.3
$\theta_H = 300$; $\theta_L = 290$; $\beta = 0.5$; $\delta = 0.95$; $m = 1$; $k = 240$.							

Table 3: Steady-State Weight Changes

[Sex] Policy	$\Delta \mathrm{W}_H$	ΔU_H	$\Delta \mathrm{W}_L$	ΔU_L
[Male] Portion cap	-18.17	-	0	-
[Male] Tax	-18.7	$\downarrow$	-4.55	-
[Female] Portion cap	-11.94	-	0	-
[Female] Tax	-11.94	$\downarrow$	-6.59	-

All MATERIAL BELOW ARE APPENDICES FOR ONLINE PUBLICATION

Appendix A - Treatment of Unregulated "Exclusive" Scheme that Serves only H-types

In this scenario, $q_H > 0$ and $q_L = 0$. Let the superscript ib denote variables that maximize the retailer's benefit under this strategy and policy environment. Using FOC[q_H] from (5), and pricing rule (3) we obtain:

$$\theta_H v'(q_H^{ib}) = c \tag{29}$$

$$t_H^{ib} = \theta_H v(q_H^{ib}) \tag{30}$$

The per-serving price is higher than the default separating case ia because the seller no longer needs to elicit truthful revelation of private information and therefore does not need to grant information rents. Both consumer types are held at their reservation values. Expected profit is in equation (31).

$$\pi^{ib} = (1-\beta)[\theta_H v(q_H^{ib}) - cq_H^{ib}] \tag{31}$$

Proposition ib summarizes the key results.

Proposition ib *Suppose that the retailer decides to serve high willingness to pay consumers exclusively. Assume that the sale of the product is unregulated. Then:*

1. $\theta_H v'(q_H^{ib}) = c$ *So that H-types are offered a serving size equal to the H-type first best.*

2. $t_H^{ib} = \theta_H v(q_H^{ib})$ *So that the retailer extracts all of the surplus from H-type buyers.*

3. *The seller's value optimized profit is expressed in equation (31).*

4. *The H-type consumer's value function is* $U_H^{ib} = 0$.

5. *The L-type consumer's value function is* $U_L^{ib} = 0$.

Appendix B - Treatment of Unregulated "Pooling" Scheme that Serves both Types with a One-size-fits-all Strategy

In this case, the seller pools the market; i.e. she stops customizing prize-size combinations and implements a one-size-fits-all scheme. This implies that only one cup with size $q_L > 0$ is offered by the seller. The serving size is denoted with the subscript L because it must, at minimum, satisfy the L-types individual rationality constraint. Moreover, because the retailer no longer attempts to segment the market, the retailer does not need to motivate revelation of private information from the H-type buyers. The retailer's optimization problem can be written as follows:

$$\begin{aligned} &\max_{t,q} E[\pi] = t_L - cq_L \\ &\text{subject to:} \\ &\text{PCL} : \theta_L v(q_L) - t_L \geq \bar{v}_L \end{aligned} \tag{32}$$

Since the Spence-Mirrlees single crossing condition is satisfied, if the package is individually rational for L-types, then it automatically satisfies individual rationality for H-types. Without loss of generality, we let $\bar{v}_L = 0$. Letting ic denote the superscript for this pooling case, profit maximization implies:

$$\theta_L v'(q_L^{ic}) = c \tag{33}$$

$$t_L^{ic} = \theta_L v(q_L^{ic}) \tag{34}$$

Thus, L-type consumers do get their first best quantity. The retailer's expected benefits are in (35) and the buyers' value functions in (36).

$$\pi^{iic} = \theta_L v(q_L^{ic}) - c q_L^{ic} \tag{35}$$

$$\begin{aligned} U_L^{ic} &= 0 \\ U_H^{ic} &= (\theta_H - \theta_L) v(q_L^{ic}) \end{aligned} \tag{36}$$

These results are summarized in proposition ic.

Proposition ic

Suppose that the retailer decides to implement a one-size-fits-all pricing scheme. In the absence of regulation, we have:

1. $\theta_L v'(q_L^{ic}) = c$ *So that L-types are offered a serving size equal to the L-type first-best quantity.*

2. $t_L^{ic} = \theta_H v(q_L^{ic})$ *So that the retailer extracts all of the surplus from L-type buyers.*

3. *The seller's value optimized profit is expressed in equation (35).*

4. *The H-type consumer's value function is* $U_H^{ic} = (\theta_H - \theta_L)v(q_L^{ic})$.

5. *The L-type consumer's value function is* $U_L^{ic} = 0$.

Appendix C

Proof of Claim 1

Proof. We will show that $\pi_{iia} \geq \pi_{iib}$ is equivalent to $\beta \geq \frac{[\theta_H - \theta_L]v(q_L^{iia})}{\theta_H v(q_L^{iia}) - \psi q_L^{iia}} = \underline{\beta}_E$.

Using the profit expressions under the taxed separating and H-exclusive strategies, note that $\pi_{iia} \geq \pi_{iib}$ can be expressed as $(1-\tau_v)\Big\{(\beta)[\theta_L v(q_L^{iia}) - \psi q_L^{iia}] + (1-\beta)\big\{[\theta_H v(q_H^{iia}) - (\theta_H - \theta_L)v(q_L^{iia})] - \psi q_H^{iia}\Big\} \geq (1-\tau_v)(1-\beta)[\theta_H v(q_H^{iib}) - \psi q_H^{iib}]$. Using the fact that the H-type first order conditions for taxed cases II-A and II-B are identical, we have $q_H^{iia} = q_L^{iib}$. This allows us to simplify the inequality to obtain $\beta[\theta_H v(q_L^{iia}) - \psi q_L^{iia}] \geq [\theta_H - \theta_L]v(q_L^{iia})$. Solving for β yields $\beta \geq \frac{[\theta_H - \theta_L]v(q_L^{iia})}{\theta_H v(q_L^{iia}) - \psi q_L^{iia}}$

□

Proof of Proposition 5

Proof. Recall that $\psi \equiv \frac{d\Psi_i}{dq_i} = (\tau_s + c) \div (1 - \tau_v)$. The no taxation case is nested within ψ when $\tau_s = \tau_v = 0$. When there is no tax, $\psi = c$ but if either τ_s or $\tau_v > 0$, then $\psi > c$. Since c is the lower bound of ψ, it follows that any positive tax implies an increase in ψ. One can easily see from $\beta_E = \frac{[\theta_H - \theta_L]v(q_L^{iia})}{\theta_H v(q_L^{iia}) - \psi q_L^{iia}}$ that an increase in ψ implies an increase in β_E. □

Proof of Claim 2

Proof. We will show that $\pi_{iia} \geq \pi_{iic}$ is equivalent to $\beta \geq \underline{\beta}_O = \frac{[\theta_H - \theta_L]v(q_L^{iia}) + \theta_L v(q_L^{iic}) - \psi q_L^{iic} - [\theta_H v(q_H^{iia}) - \psi q_H^{iia}]}{\theta_H v(q_L^{iia}) - \psi q_L^{iia} - [\theta_H v(q_H^{iia}) - \psi q_H^{iia}]}$.

.

Using the profit expressions under the taxed separating and one-size-fits-all strategies, note that $\pi_{iia} \geq \pi_{iic}$ can be expressed as $(1-\tau_v)\Big\{(\beta)[\theta_L v(q_L^{iia}) - \psi q_L^{iia}] + (1-\beta)\big\{[\theta_H v(q_H^{iia}) - (\theta_H - \theta_L)v(q_L^{iia})] - \psi q_H^{iia}\big\} \geq (1-\tau_v)[\theta_L v(q_L^{iic}) - \psi q_L^{iic}]$. Solving for β yields $\beta \geq \frac{[\theta_H - \theta_L]v(q_L^{iia}) + \theta_L v(q_L^{iic}) - \psi q_L^{iic} - [\theta_H v(q_H^{iia}) - \psi q_H^{iia}]}{\theta_H v(q_L^{iia}) - \psi q_L^{iia} - [\theta_H v(q_H^{iia}) - \psi q_H^{iia}]}$ $\underline{\beta}_O$.

$\square$

Proof of Proposition 6

Proof. Recall that $\psi \equiv \frac{d\Psi_i}{dq_i} = (\tau_s + c) \div (1 - \tau_v)$. The no taxation case is nested within ψ when $\tau_s = \tau_v = 0$. When there is no tax, $\psi = c$ but if either τ_s or $\tau_v > 0$, then $\psi > c$. Thus, taxation effectively increases marginal cost. By Claim 2, $\underline{\beta}_O = \frac{[\theta_H - \theta_L]v(q_L^{iia}) + \theta_L v(q_L^{iic}) - \psi q_L^{iic} - [\theta_H v(q_H^{iia}) - \psi q_H^{iia}]}{\theta_H v(q_L^{iia}) - \psi q_L^{iia} - [\theta_H v(q_H^{iia}) - \psi q_H^{iia}]}$. Applying the quotient rule, we have $\frac{\partial \underline{\beta}_O}{\partial \psi} = \frac{q_L^{iia} - q_L^{iic}}{\theta_H v(q_L^{iia}) - \psi q_L^{iia} - [\theta_H v(q_H^{iia}) - \psi q_H^{iia}]}$. Note that the denominator is squared and therefore must be positive. To sign the numerator, comparing the first order conditions 14 and 20, it is obvious that $q_L^{iic} > q_L^{iia}$. Hence, $\frac{\partial \underline{\beta}_O}{\partial \psi} < 0$. $\square$

Proof of Proposition 7

Proof. Suppose that a tax a tax regime (τ_s, τ_v) is implemented. Then we know from propositions 2 and 3 that $q_H^{ia} > q_H^{iia}$ and $q_H^{ia} > q_H^{iib}$. Therefore, regardless of whether the retailer continues with the separating strategy or switches to the H-exclusive strategy, the H-type serving will decline. Moreover, by proposition 6, we do not need to consider the one-size-fits-all strategy.

Also, by propsition 2, $q_L^{ia} > q_L^{iia}$ so the L-type serving size declines if the retailer continuous with the separating strategy post tax. If the retailer switches to the H-exclusive strategy, then by proposition 3, L-type consumers are excluded so that serving size trivially declines to zero. In either case, L-type serving size declines.

□

Proof of Proposition 8

Proof. Suppose that a tax regime (τ_s, τ_v) is implemented. Then we know from proposition 2 that $U_H^{ia} > U_H^{iia}$. Therefore, the H-type buyer's surplus declines.

Also, by propositions 1, 2, and 3, the L-type buyer is always held at his reservation utility. Thus, a tax does not affect the L-type's consumer surplus as his utility remains at the reservation both pre and post-tax.

□

Proof of Proposition 9

Proof. Suppose that a tax regime (τ_s, τ_v) is implemented. If the retailer continues to use the segmentation strategy post-tax, then by proposition 2, $\pi^{ia} > \pi^{iia}$.

If instead, the retailer switches to a the H-exclusive strategy, then the retailer's post-tax value function is $\pi^{iib} = (1 - \tau_v)(1 - \beta)[\theta_H v(q_H^{iib}) - \psi q_H^{iib}]$. Note that if the retailer had adopted a H-exclusive strategy pre-tax, then the retailer's value function would be $\pi^{ib} = (1 - \beta)[\theta_H v(q_H^{ib}) - \psi q_H^{ib}]$ where q_H^{ib} is the optimal H-type serving size in the absence of a tax. This would be determined by the first order condition $\theta_H v'(q_H^{ib}) = c$. However, note that Taxed Case II-B that the same condition for the post-tax H-exclusive strategy is

$\theta_H v'(q_H^{iib}) = \psi$. Because $\psi > c$, it follows that $q_H^{iib} < q_H^{ib}$ and therefore $\pi^{ib} > \pi^{iib}$. However, we know that, by assumption, the retailer adopts a separating strategy pre-tax so it must be the case that $\pi^{ia} > \pi^{ib}$. Hence, by transitivity, $\pi^{ia} > \pi^{iib}$. □

Appendix D

In this appendix, we will derive the qualitative results reported in Table 1 using a model that is very similar to Bourquard and Wu (2019). The key difference is that our model uses a cost function specification $c(q) = cq$ where c is a positive constant rather than a general cost function.

The seller maximizes expected profit subject to participation (PC) and incentive compatibility (IC) constraints:

$$\max_{q_L, q_H, t_L, t_H} \beta \left[t_L - cq_L\right] + (1-\beta)\left[t_H - cq_H\right] \quad s.t. \tag{37}$$

$$\theta_L v(q_L) - t_L \geq \overline{u} \quad (PC)$$

$$\theta_H v(q_H) - t_H \geq \theta_H v(q_L) - t_L \quad (IC)$$

$$0 \leq q_H \leq \hat{q} \qquad\qquad 0 \leq q_L \leq \hat{q}$$

where $\hat{q}$ is an exogenous portion-cap. Assuming that (IC) and (PC) are binding, they can be substituted into the objective function, which yields

$$\max_{q_L, q_H} \pi = \beta \left[\theta_L v(q_L) - \overline{u} - cq_L\right] + (1-\beta)\left[\theta_H v(q_H) - \overline{u} - (\theta_H - \theta_L) v(q_L) - cq_H\right] \tag{38}$$

$$0 \leq q_H \leq \hat{q} \qquad\qquad 0 \leq q_L \leq \hat{q}$$

Benchmark case: no regulation

The case of no regulation is nested in problem 38 via the assumption that the upper bound of the constraints $0 \leq q_H \leq \hat{q}$ and $0 \leq q_L \leq \hat{q}$ are not binding. The first order Kuhn-Tucker conditions become

$$\theta_H v'(q_H) = c \tag{39}$$

$$\theta_L v'(q_L) = c + \frac{(1-\beta)}{\beta} [\theta_H - \theta_L] v'(q_L) \tag{40}$$

These conditions imply that the seller chooses a first-best serving size for the H-type and a L-type serving size that is distorted downward relative to the L-type first-best.

The PC and IC constraints can be used to generate the serving prices.

$$t_H = \theta_H v(q_H^*) - (\theta_H - \theta_L) v(\tilde{q}_L) - \overline{u} \tag{41}$$

$$t_L = \theta_L v(\tilde{q}_L) - \overline{u} \tag{42}$$

Finally, the serving prices and quantities can be substituted into the objective functions of the seller and consumers to obtain value functions, which allows for welfare analysis.

Proposition 10. *In the absence of a size-restriction regulation, the retailer's optimal non-linear pricing strategy yields the following benchmark results:*

1. *The H-type is offered a serving size that yields the first-best level of consumption, q_H^*, for a price, t_H, that provides an information rent driven quantity discount.*

2. *The L-type is offered a serving size, $\tilde{q}_L$, that is distorted downward relative to the L-type's first-best of q_L^*. The price, t_L, is set to extract the L-type's rents.*

3. *The retailer's value function (maximized expected profit) is:* $\Pi = (1-\beta)[\theta_H v(q_H^*) - \overline{u} - cq_H^* - [\theta_H - \theta_L]v(\tilde{q}_L)] + \beta[\theta_L v(\tilde{q}_L) - \overline{u} - c\tilde{q}_L]$

4. *The H-type's value function (welfare under the optimal nonlinear pricing scheme) is* $U_H = \overline{u} + [\theta_H - \theta_L]v(\tilde{q}_L)$ *(earns information rents).*

5. *The L-type's value function is* $U_L = \overline{u}$ *(earns no excess surplus).*

The proof is omitted as these are well-known nonlinear pricing results.

The impact of a size-restriction regulation

A beverage size restriction can have two potential effects. First, the sizes and prices of the beverages offered to the consumers might change. Second, it might cause retailers to switch to a different strategy. For example, if the retailer used the separating pricing strategy prior to the restriction, the restriction might induce the retailer to switch to an H-exclusive or one-size-fits-all strategy. Both possibilities must be accounted for to ensure accurate welfare analysis.

How are prices and quantities impacted by the size-restriction, holding the pricing strategy constant?

Consider the set of possible discrete pricing strategies:

- Case ib: Sell to both types of consumers with a menu of differentiated H-type and L-type price-size options.
- Case iib: Sell exclusively to H-types.
- Case iiib: Sell to all types using one-size-fits-all pricing.

Case ib: Sell to both types with a menu of H-type and L-type options.

Assuming that the size-restriction only caps the H-type serving so that the restriction has an upper corner solution, $0 \leq q_H \leq \hat{q}$, then the K-T conditions are

$$\theta_H v'(q_H) \geq c \quad \textit{where} \quad q_H = \hat{q} \tag{43}$$

$$\beta \left[\theta_L v'(q_L) - c\right] + (1-\beta)\left[-(\theta_H - \theta_L)v'(q_L)\right] \leq 0 \quad \textit{where} \quad q_L \geq 0 \quad \& \quad \frac{\partial \pi}{\partial q_L} q_L = 0 \tag{44}$$

These conditions imply that $q_H = \hat{q}$ and $\theta_L v'(q_L) = c + \frac{(1-\beta)}{\beta}\left[\theta_H - \theta_L\right] v'(q_L)$. But the latter is identical to 40 so that a beverage size-restriction would have no impact on q_L if the separating strategy is used post-regulation.

Furthermore, because $\tilde{q}_L$ is unchanged, and q_H^* decreases to $\hat{q}$, equations 41 and 42 suggest that t_H drops but t_L remains the same.

Lemma 1. *Suppose that there is a size-restriction $q_H \leq \hat{q}$ such that the retailer continues to use a separating pricing strategy where $0 < q_L < q_H = \hat{q}$. Then*

1. *The H-type's serving size declines to $q_H = \hat{q} < q_H^*$ and t_H drops from $t_H = \theta_H v(q_H^*) - (\theta_H - \theta_L)v(\tilde{q}_L) - \overline{u}$ to $\hat{t}_H = \theta_H v(\hat{q}) - (\theta_H - \theta_L)v(\tilde{q}_L) - \overline{u}$,*

2. *The L-type's serving size, $\tilde{q}_L$, and price, t_L, remain unchanged.*

3. *The retailer's profit declines to:* $\Pi_{ib} = \beta\left[\theta_L v(\tilde{q}_L) - c\tilde{q}_L - \overline{u}\right] +$

   $(1-\beta)\left[\theta_H v(\hat{q}) - c\hat{q} - (\theta_H - \theta_L)v(\tilde{q}_L) - \overline{u}\right]$

4. *The H-type's welfare (utility) remains unchanged at $U_{Hib} = \overline{u} + [\theta_H - \theta_L]v(\tilde{q}_L)$ (earns information rents).*

5. *The L-type's welfare remains unchanged at $U_{Lib} = \overline{u}$ (earns no excess surplus).*

Proof of Lemma 1

Proof. The proof for part (1) follows from the binding size-restriction, which yields K-T condition 43 and thus, $q_H = \hat{q}$. Also, t_H drops because q_H^* is replaced with the smaller $\hat{q}$ in the the optimal price function. Since the price function is a function of $v(q_H)$ and $v(q_H)$ is increasing in q_H $\forall$ $q_H < q_H^*$, it must be true that the new price $\hat{t}_H < t_H$ since $\hat{q} < q_H^*$.

Part (2) follows from the first order condition for q_L (44), which is unchanged from the unregulated case. Hence, the retailer will still offer the same $\tilde{q}_L$ as the unregulated case. Serving price t_L is unchanged because the L-type price is a function of only q_L (and not q_H).

The proofs for parts (3), (4), and (5) are easy to show by substituting the optimal prices and quantities into the objective functions of the retailers and consumers. □

Case iib: Sell to only high types with $q_L = 0$

Here, the seller only serves H-type consumers because it is too costly in terms of information rents to also serve L-types. Neither 43 nor 44 hold with strict equality so $q_H^* = \hat{q}$ and $\tilde{q}_L = 0$.

Because the size-restriction causes q_H^* to drop to $\hat{q}$, $t_H^* = \theta_H v(q_H^*) - \overline{u}$ (from case ii) drops to $\hat{t}_H = \theta_H v(\hat{q}) - \overline{u}$.

Lemma 2. *Suppose that there is a regulatory restriction of the form $q_H \leq \hat{q}$ and the retailer serves only H-type consumers. Then*

1. *The H-type's serving size declines to $q_H = \hat{q} < q_H^*$ and t_H drops from $t_H^* = \theta_H v(q_H^*) - \overline{u}$ to $\hat{t}_H = \theta_H v(\hat{q}) - \overline{u}$.*

2. *The retailer's profit declines to: $\Pi_{iib} = (1-\beta)[\theta_H v(\hat{q}) - c\hat{q} - \overline{u}]$*

3. *The H-type's consumer welfare is: $U_{Hiib} = \overline{u}$ (no excess rents).*

Proof of Lemma 2

Proof. Part (1) follows from the assumption of a binding restriction, $\hat{q}$, which yields K-T condition 43 so $q_H = \hat{q}$. The serving price, t_H, drops because q_H^* is replaced with the smaller $\hat{q}$ in the the optimal price function. Since the price function is a function of $v(q_H)$ and $v(q_H)$ is increasing in q_H $\forall$ $q_H < q_H^*$, it follow that $\hat{t}_H < t_H$ since $\hat{q} < q_H^*$.

The proofs for parts (2) and (3) follow from substituting the optimal prices and quantities into the objective functions for the retailer and consumers. □

Case iiib: Sell to both types with a one-sized fits all package

The optimal one-size-fits-all strategy under a size restriction is generated by solving:

$$\max_{t,q} [t - cq] \quad s.t. \tag{45}$$

$$\theta_L v(q) - t \geq \overline{u} \tag{46}$$

$$0 \leq q \leq \hat{q} \tag{47}$$

Because $\theta_L < \theta_H$, the H-type participation constraint is always satisfied so long as L-type constraint is satisfied. The binding participation constraint 46 can be substituted into the objective function to get:

$$\max_q] \left[\theta_L v(q) - cq - \overline{u}\right] \tag{48}$$

$$0 \leq q \leq \hat{q} \tag{49}$$

which yields the Kuhn-Tucker conditions:

$$\theta_L v'(q) \geq c \quad \& \quad q \leq \hat{q} \quad \& \quad \frac{\partial \pi}{\partial q}(\hat{q} - q) = 0 \tag{50}$$

Solving the K-T conditions yields the following proposition.

Lemma 3. *Suppose that there is a restriction of the form $q \leq \hat{q}$ and the retailer uses a one-size-fits-all strategy for both types of consumers. Then*

1. *The quantity offered to both types of consumers is $q = min\{q_L^*, \hat{q}\}$ where q_L^* is the first-best quantity for the L-type consumer.*
2. *The price is $t = \theta_L v(q) - \overline{u}$.*
3. *The retailer's profit is: $\Pi_{iiib} = \theta_L v(q) - cq - \overline{u}$.*
4. *The H-type's consumer welfare is: $U_{Hiiib} = \overline{u} + [\theta_H - \theta_L]v(\hat{q})$ (excess rents).*

5. *The L-type's consumer welfare is:* $U_{Liiib} = \overline{u}$ *(no excess rents).*

Proof of Lemma 3

Proof. Part (1) follows from K-T condition 50. That is, if the size-constraint is not binding so that $q < \hat{q}$, then the first order is $\theta_L v'(q) = c'(q)$ so the solution to 50 is clearly equal to the first best level of quantity for L-types, q_L^*. If the size constraint is binding, then, $\hat{q} \leq q_L^*$ in which case $q = \hat{q}$. Hence, $q = min\{q_L^*, \hat{q}\}$

Part (2) follows easily from the optimal q and the binding participation constraint.

Parts (3)-(5) follow from substituting the optimal q and t into the objective functions of the retailer, and consumers. □

How does the regulation affect retailers' choice of pricing strategy?

Now that we know how optimal prices and quantities respond to a size-restriction within each discrete pricing strategy, we can investigate which pricing strategy is optimal following the size-restriction. This involves comparing the value functions of the seller across the difference pricing strategies. The retailer will choose the strategy that yields the highest profit.

Following Bourquard and Wu (2019) (BB), it is also important to consider the magnitude of the size-restriction. BB consider three regions that define the severity of the size restriction:

- *Region 1*: $q_L^* \leq \hat{q} < q_H^*$
- *Region 2*: $\tilde{q}_L \leq \hat{q} < q_L^*$
- *Region 3*: $\hat{q} < \tilde{q}_L$.

Recall that q_H^* is the first-best level of consumption for the H-type, which is implemented by the retailer under the segmentation strategy in an unregulated market. The quantity $\tilde{q}_L$ is the profit maximizing L-type serving size under the unregulated segmentation strategy. Note that this is distorted downward relative to q_L^*, the first-best level for the L-type. It is also important to note that any size restriction that is more stringent than $\tilde{q}_L$ also caps the size of the L-type beverage. This would be a Region 3 size-restriction and we call this the "severe" size-restriction in Table 1.

Region 1: $q_L^* \leq \hat{q} < q_H^*$

To begin with, we show that a Region 1 restriction would not cause a seller to switch from the default, unregulated separating/segmentation strategy to a H-type exclusive strateg.

Lemma 4. *Suppose that, in an unregulated market, a retailer chooses the separating strategy that offers different price-size packages to H-types and L-types. A regulation of the form $q \leq \hat{q}$ cannot cause the retailer to switch to the exclusive H-types strategy.*

Proof of Lemma 4

Proof. The proof involves comparing Π_{ib} from Lemma 1 to Π_{iib} from Lemma 2. i.e. the retailer will not switch away from the segmentation strategy iff:

$$\Pi_{ib} = \beta\left[\theta_L v(\tilde{q}_L) - c\tilde{q}_L - \overline{u}\right] + (1-\beta)\left[\theta_H v(\hat{q}) - c\hat{q} - (\theta_H - \theta_L)v(\tilde{q}_L) - \overline{u}\right] \tag{51}$$

$$\geq (1-\beta)\left[\theta_H v(\hat{q}) - c\hat{q} - \overline{u}\right] = \Pi_{iib}$$

After some algebra, 51 reduces to

$$\beta\left[\theta_L v(\tilde{q}_L) - c\tilde{q}_L - \overline{u}\right] \geq (1-\beta)[\theta_H - \theta_L]v(\tilde{q}_L) \tag{52}$$

Note that 52 is completely independent of $\hat{q}$. Therefore, the regulation cannot reverse the inequality. Consequently, if the retailer chose the segmentation strategy prior to the regulation, it will continue to do so after the regulation. □

Can the restriction cause a retailer to switch from a segmentation strategy to a one-size-fits-all *single price* strategy that serves both types?

To answer this question, we need to determine whether $\Pi_{ib} \geq \Pi_{iiib}$ (profits from lemmas 1 and 3) will be reversed by the regulation. Writing out this inequality explicitly, we have:

$$\beta\left[\theta_L v(\tilde{q}_L) - c\tilde{q}_L - \overline{u}\right] + (1-\beta)\left[\theta_H v(\hat{q}) - (\theta_H - \theta_L)v(\tilde{q}_L) - c\hat{q} - \overline{u}\right] \geq \theta_L v(q_L^*) - cq_L^* - \overline{u} \tag{53}$$

If the inequality holds even after the regulation, then the retailer will not switch to the one-size-fits-all strategy.

Lemma 5. *The implementation of a size restriction in the range $q_L^* \leq \hat{q} < q_H^*$ will not cause a retailer to switch from a segmentation strategy to the one-size-fits all single price, single size pricing strategy identified in Case iiib.*

Proof of Lemma 5

Proof. By 43, the left-hand-side of 53 is monotonically increasing in $\hat{q}$ over the range $q_L^* \leq \hat{q} < q_H^*$. Also, the right-hand-side of 53 is independent of $\hat{q}$ since $\hat{q}$ does not constrain implementation of q_L^*. Hence, if 53 is satisfied even when $\hat{q} = q_L^*$, then it must continue to hold for all $\hat{q} \in [q_L^*, q_H^*]$ so that any restriction in this region cannot cause the retailer to switch from a segmentation strategy to a one-size-fits all strategy.

Suppose that $\hat{q} = q_L^*$, the lowest quality level in the range $[q_L^*, q_H^*]$. Then substituting into 53 yields:

$$\beta\left[\theta_L v(\tilde{q}_L) - c\tilde{q}_L - \overline{u}\right] + (1-\beta)\left[\theta_H v(q_L^*) - (\theta_H - \theta_L)v(\tilde{q}_L) - cq_L^* - \overline{u}\right] \geq \theta_L v(q_L^*) - cq_L^* - \overline{u} \quad (54)$$

The above can be simplified to:

$$\beta[cq_L^* - c\tilde{q}_L] + [v(q_L^*) - v(\tilde{q}_L)][\theta_H(1-\beta) - \theta_L] \geq 0 \quad (55)$$

Since the left-hand-side of 55 depends on β, we first make the following claim:

Claim 3. *The left-hand-side of 55 is decreasing in β.*

Proof. Taking the derivative with respect to β of the left-hand-side of 55 yields

$$-\{\theta_H[v(q_L^*) - v(\tilde{q}_L)] - [cq_L^* - c\tilde{q}_L]\} - \beta c'\tilde{q}_L \frac{d\tilde{q}_L}{d\beta} - v'(\tilde{q})\frac{d\tilde{q}_L}{d\beta} \quad (56)$$

Note that $-\{\theta_H[v(q_L^*) - v(\tilde{q}_L)] - [cq_L^* - c\tilde{q}_L]\}$ is non-positive given $\theta_H v'(q) > c$ for all $q < q_H^*$ and $q_L^* \geq \tilde{q}_L$. Thus, it remains to determine the sign of $\frac{d\tilde{q}_L}{d\beta}$. Assuming an interior solution for $\tilde{q}_L$, implicitly differentiating 44 yields

$$\frac{d\tilde{q}_L}{d\beta} = \frac{-[\theta_H - \theta_L]v'(\tilde{q}_L)}{\beta^2[\theta_L v''(\tilde{q}_L) - \frac{1-\beta}{\beta}[\theta_H - \theta_L]v''(\tilde{q}_L)]} \quad (57)$$

Since the denominator of 57 must be negative in order for the principal's objective function to be concave, it follows that 57 is positive so that $\tilde{q}_L$ is increasing in β. Since $\frac{d\tilde{q}_L}{d\beta} > 0$, it follows that 56 must be negative. Thus, the left-hand-side of 55 is decreasing in β. $\square$

Claim 3 suggests that the left-hand-side of 55 is minimized at $\beta = 1$ so if the left-hand-side remains non-negative for $\beta = 1$, then the profit from the segmentation strategy will be at least as great as the profit from the one-size-fits all strategy. Note that for $\beta = 1$, the first order condition 44 implies that $\tilde{q}_L = q_L^*$. At $\beta = 1$, the left-hand-side of 55 reduces to $cq_L^* - cq_L^* + [v(q_L^*) - v(q_L^*)][-\theta_L] = 0$. Consequently, over the range $q_L^* \leq \hat{q} < q_H^*$, a size-regulation cannot cause the expected profit from the segmentation strategy to fall below the profit from the one-size-fits all strategy. $\square$

The key point to remember is that any policy restriction in Region 1 will not cause the retailer to switch away from the segmentation strategy.

Region 2: $\tilde{q}_L \leq \hat{q} < q_L^*$

A Region 2 size-restriction is more stringent than a Region 1 restriction but is still not a "severe restriction" as defined in Table 1. This is because a Region 2 restriction would still not cap the L-type serving.

The first thing to note is that the retailer will not switch from a separation strategy to a one-size-fits-all strategy.

Lemma 6. *The implementation of a size-restriction in the range* $\tilde{q}_L \leq \hat{q} < q_L^*$ *will not cause a retailer to switch from a separating strategy to a one-size-fits-all strategy.*

Proof of Lemma 6

Proof. We must show that over the range $[\tilde{q}_L, q_L^*)$,

$$\beta\left[\theta_L v(\tilde{q}_L) - c\tilde{q}_L - \overline{u}\right] + (1-\beta)\left[\theta_H v(\hat{q}) - (\theta_H - \theta_L)v(\tilde{q}_L) - c\hat{q} - \overline{u}\right] \geq \theta_L v(\hat{q}) - c\hat{q} - \overline{u} \quad (58)$$

In the proof for Lemma 5, it was shown that the separating strategy profit is greater than the one-size-fits-all profit over the range $q_L^* \leq \hat{q} < q_H^*$. The key difference now is that the right-hand-side of 58 is decreasing in $\hat{q}$ over the interval $[\tilde{q}_L, q_L^*)$ whereas 53 is independent of $\hat{q}$ over the interval $[q_L^*, q_H^*)$. Therefore, the inequality becomes even more relaxed over the range $[\tilde{q}_L, q_L^*)$. Hence, if the inequality is satisfied over the range $[q_L^*, q_H^*)$, then it must be satisfied be satisfied over the range $[\tilde{q}_L, q_L^*)$. □

Region 3: $\hat{q} < \tilde{q}_L$

A Region 3 restriction is what is described as a "severe restriction" in Table 1. This is the case where the restriction is so stringent, that even the smaller L-type serving is capped.

The first-order conditions 43 and 44 would become:

$$\theta_H v'(q_H) > c'(q_H) \quad \textit{where} \quad q_H = \hat{q} \quad (59)$$

$$\beta\left[\theta_L v'(q_L) - c'(q_L)\right] + (1-\beta)\left[-(\theta_H - \theta_L)v'(q_L)\right] > 0 \quad \textit{where} \qquad q_L = \hat{q}(< \tilde{q}_L) \quad (60)$$

Thus, the optimal solutions are $q_H = q_L = \hat{q}$ so the separating strategy based on differentiated serving sizes is no longer optimal. However, the seller can still price to serve either both types or only H-types. A higher price will cause L-types to drop out but increase profit margin from H-types. A low price would induce both types to purchase but the profit margin would decrease. The optimal pricing strategy thus is determined by comparing seller profit in Lemma 2 to retailer profit in Lemma 3.

Lemma 7. *A size-restriction in the range $\hat{q} < \tilde{q}_L$ will induce a seller to adopt a one-size-fits-all strategy if $\frac{[\theta_H - \theta_L]v(\hat{q})}{[\theta_H v(\hat{q}) - c(\hat{q}) - \overline{u}]} \leq \beta$. On the other hand, if $\frac{[\theta_H - \theta_L]v(\hat{q})}{[\theta_H v(\hat{q}) - c(\hat{q}) - \overline{u}]} > \beta$, then the retailer will price only to serve H-types.*

The intuition is, if there is a higher probability of encountering L-types, then ignoring L-types would substantially decrease volume sold and result in an overall lower profit. On the other hand, if the probability of encountering an L-type is low, then the smaller loss in volume can be offset by the larger margin from serving H-types.

The Ultimate Impact of a Beverage Size Restriction on Serving Size and Welfare

The lemmas in the previous sections outline the optimal sizes and prices within each discrete pricing strategy and which strategies would ultimately be adopted under size-restrictions of various intensity. We can summarize the key insights as follows:

- Any size-restriction in the range $[\tilde{q}_L, q_H^*)$ will not cause the seller to switch from the default separating strategy.
- Any size-restriction $\hat{q} < \tilde{q}$ would cause the seller to switch to a single price strategy which serves either H-types only, or both types depending on model parameters.

These insights explain the qualitative results in the first two columns of row 1 Table 1. That is, a non-severe size restriction does not change the seller's optimal strategy, but a severe restriction causes the switch to a single price strategy.

The other lemmas provide the following insights:

- Under a non-severe size-restriction where the seller maintains the default separating strategy, the only thing that changes is that the H-type serving decreases due to the size-cap. However, L-type serving, and consumer surplus remain the same.

- Under a severe-size restriction that impedes on even the L-type serving, both serving sizes and consumer surplus decrease.

These insights justify the qualitative conclusions in the first two columns of Table 1 with the exception of the last row.

The last row, which claims that seller welfare will unambiguously decrease, is easy to show. By Proposition 10, the seller's unregulated expected profit is $\Pi = (1-\beta)[\theta_H v(q_H^*) - \overline{u} - c(q_H^*) - [\theta_H - \theta_L]v(\tilde{q}_L)] + \beta[\theta_L v(\tilde{q}_L) - \overline{u} - c(\tilde{q}_L)]$.

A Region 1 or 2 restriction does not cause the seller to switch away from the separating strategy. However, it does cause the serving size and price to decrease so by Lemma 1, the new profit is $\Pi_{ib} = (1-\beta)[\theta_H v(\hat{q}) - \overline{u} - c(\hat{q}) - [\theta_H - \theta_L]v(\tilde{q}_L)] + \beta[\theta_L v(\tilde{q}_L) - \overline{u} - c(\tilde{q}_L)]$. By first-order condition 43, Π_{ib} is increasing in $\hat{q}$ for any $\hat{q} < q_H^*$. Hence, it must be true that $\Pi_{ib} - \Pi < 0$ so there is seller welfare loss.

If the restriction is severe, and the retailer serves only H-types, then Lemma 2 tells us that $\Pi_{iib} = (1-\beta)[\theta_H v(\hat{q}) - c(\hat{q}) - \overline{u}]$. On the other hand, if the retailer serves both types with a one-size-fits-all strategy, then Lemma 3 tells us that $\Pi_{iiib} = \theta_L v(\hat{q}) - c(\hat{q}) - \overline{u}$. In either case, the first-order conditions 59 and 60, imply that profit is increasing in both q_H

and q_L in Region 3. Hence, $\Pi_{iib} - \Pi < 0$ and $\Pi_{iiib} - \Pi < 0$.

The above explains the qualitative results in the final row of Table 1.