HW1_220B

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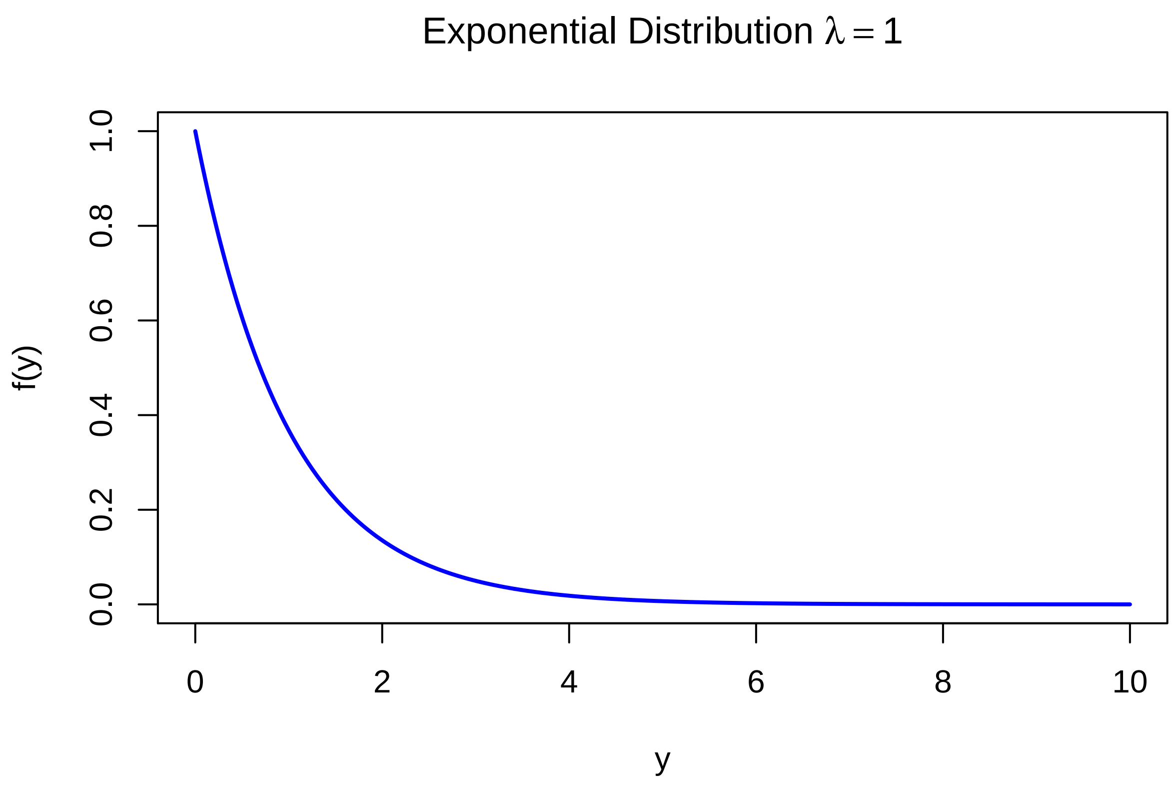
a)

```
n <- 1000
y <- seq(0,10,length=n)
fy <- dexp(y,rate=1)
plot(y,fy,type = "l",lwd=2, col=c("blue"), ylab='f(y)',
     xlab = 'y', main=expression(paste('Exponential Distribution ',lambda==1)))
```



The area underneath the curve is equal to 1 since it represents the probabilities of a random variable falling within a range of values, which when integrated over sums to 1.

b)

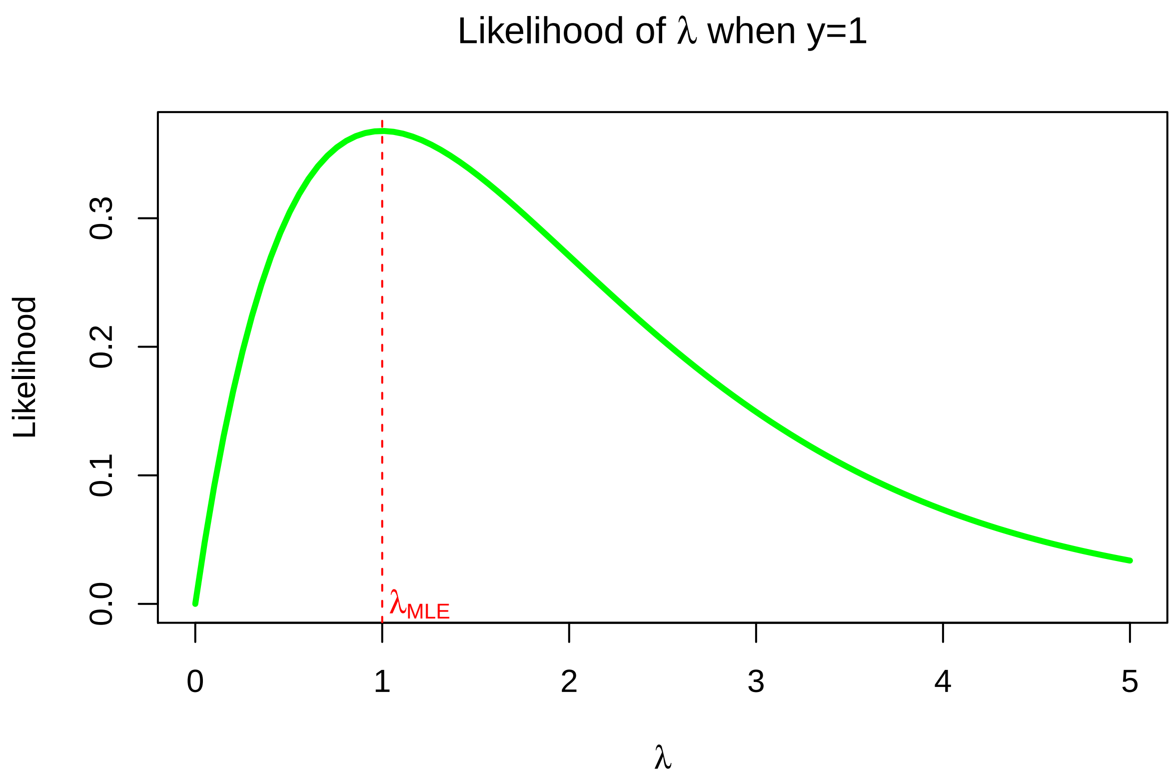
$P(Y > y) = e^{-\lambda x}$

c)

```r
#need to have the likelihood function of exponential distribution, then generate random vector with mea
exp_likeh <- function(lambda,y){
  ans <- (lambda)*exp(-lambda*y)
  return(ans)
}

y <- 1
lambdas <- seq(0,5,length.out = 100)
lamb.lh <- exp_likeh(lambdas,y)

plot(lambdas,lamb.lh, type='l',col='green',lwd=3,
     ylab='Likelihood',xlab=expression(lambda),
     main=expression(paste('Likelihood of ',lambda,' when y=1')))
abline(v=1,col='red',lty=2)
text(1.2,0,labels=expression(lambda['MLE']),col='red')
```



d)

Theoretical mean:

$$E[Y] = \int_0^\infty yf(y)dy \tag{1}$$
$$E[Y] = \int_0^\infty y\lambda e^{-\lambda y}dy \tag{2}$$
$$Let\ v' = \lambda e^{-\lambda y},\ u = y,\ v = -e^{-\lambda y},\ u' = 1,\ du = dy,\ then: \tag{3}$$
$$= uv - \int vdu \tag{4}$$
$$= [-ye^{-\lambda y}]_0^\infty - \int -e^{-\lambda y}dy \tag{5}$$
$$Let\ u = -\lambda y,\ du = -\lambda dy,\ then: \tag{6}$$
$$= (0-0) - \int e^u \frac{du}{\lambda} \tag{7}$$
$$= -[-\frac{e^{-\lambda y}}{\lambda}]_0^\infty \tag{8}$$
$$E[Y] = \frac{1}{\lambda} \tag{9}$$

Theoretical variance:

$$Var[Y] = E[Y^2] - (E[Y])^2 \tag{10}$$
$$E[Y^2] = \int_0^\infty y^2\lambda e^{-\lambda x}dy \tag{11}$$
$$Let\ u = y^2, v' = \lambda e^{-\lambda y}, then: \tag{12}$$
$$= -y^2e^{-\lambda x} - \int -e^{\lambda y}2ydy \tag{13}$$
$$Let\ u = 2y, v' = -e^{-\lambda x}, then: \tag{14}$$
$$= -x^2e^{-\lambda y} - \frac{2ye^{-\lambda y}}{\lambda} + \int 2\frac{e^{-\lambda y}}{\lambda} \tag{15}$$
$$= -x^2e^{-\lambda y} - \frac{2ye^{-\lambda y}}{\lambda} - \frac{2e^{-\lambda y}}{\lambda^2} \tag{16}$$
$$= [-e^{-\lambda y}(y^2 + \frac{2y}{\lambda} + \frac{2}{\lambda^2})]_0^\infty \tag{17}$$
$$\tag{18}$$
$$E[Y^2] = \frac{2}{\lambda^2} \tag{19}$$
$$Var[Y] = \frac{2}{\lambda^2} - (\frac{1}{\lambda})^2 \tag{20}$$
$$Var[Y] = \frac{1}{\lambda^2} \tag{21}$$

h)

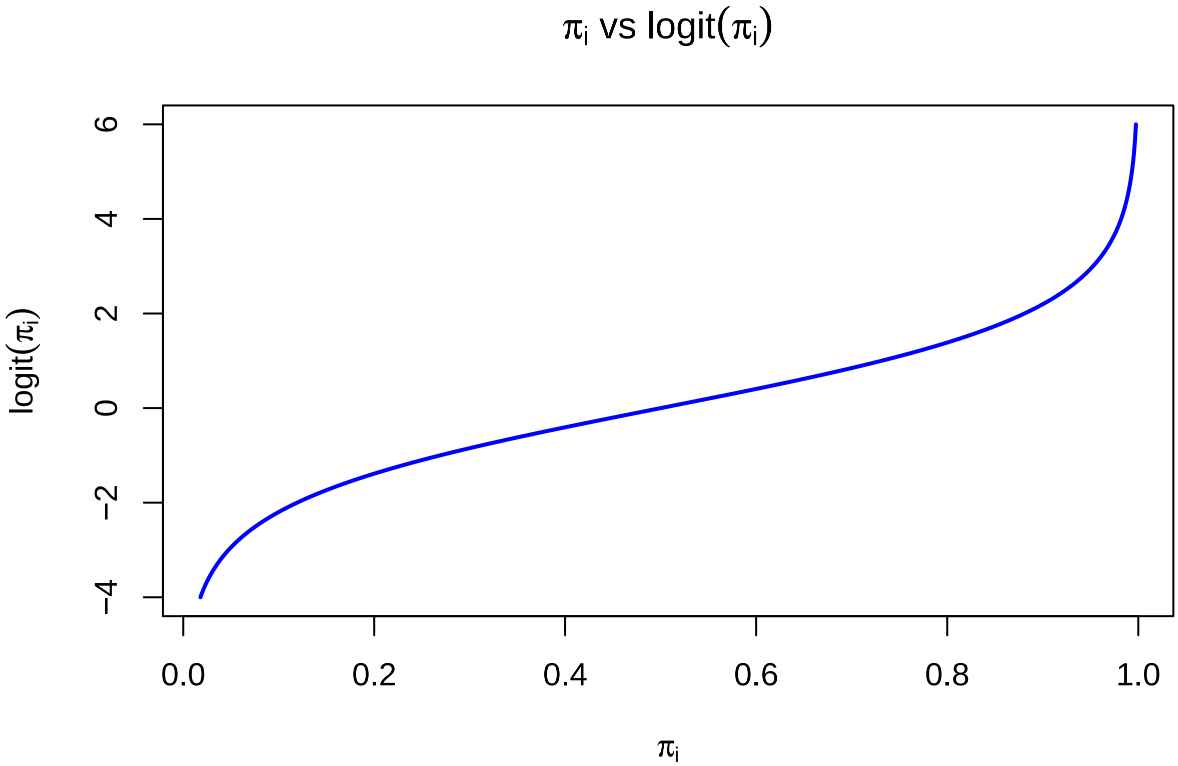
$$
\begin{aligned}
f(y|\lambda) &= \lambda exp(-\lambda y) && (22)\\
&= exp[ln(\lambda exp(-\lambda y))] && (23)\\
&= exp[ln(\lambda) - \lambda y] && (24)\\
&= exp[-\lambda y + ln(\lambda)] && (25)\\
& && (26)\\
\theta_i &= -\lambda && (27)\\
b(\theta_i) = -ln(\lambda) &= -ln(-\theta_i) && (28)\\
a(\phi) &= 1 && (29)\\
\phi &= 1 && (30)\\
c(y, \phi) &= 0 && (31)
\end{aligned}
$$

6)

```
n = 1000
b0 <- 1
b1 <- 0.5
x <- seq(-10,10,length=n)
mu <- inv.logit(b0 + b1*x)
```

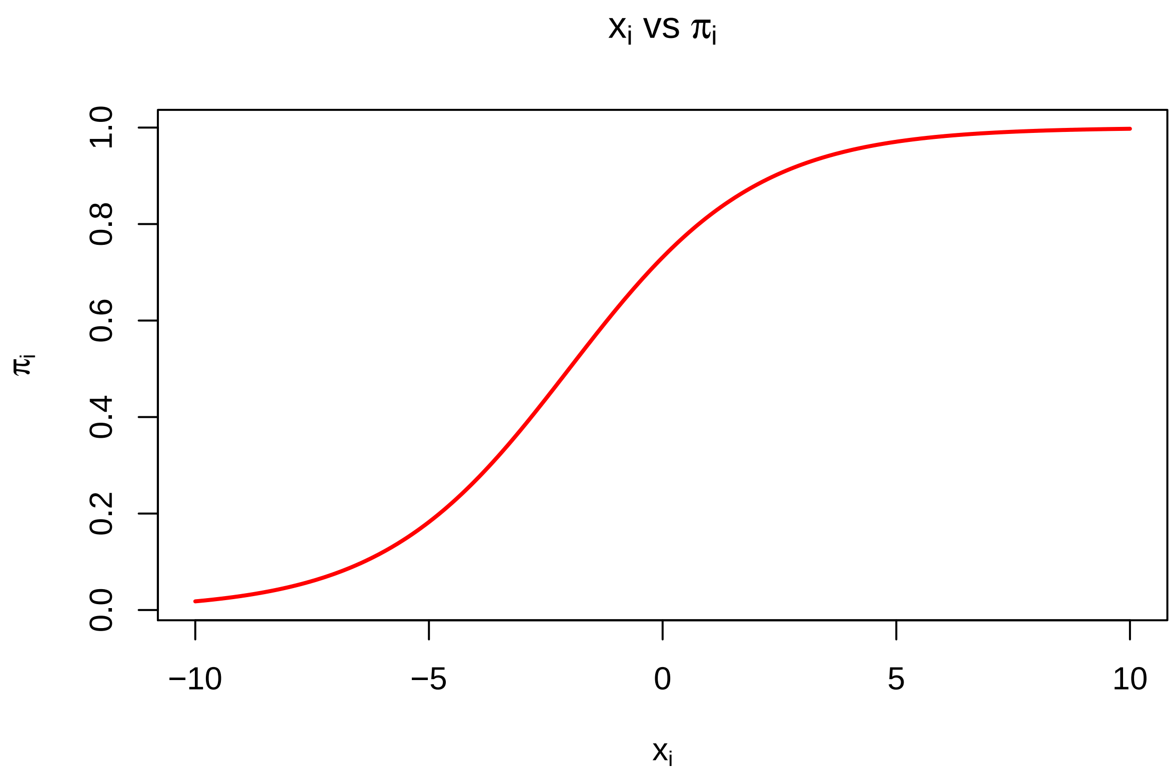
d)

```
plot(mu,logit(mu), xlab = expression(pi[i]), ylab = expression(logit(pi[i])),
     main = expression(paste(pi[i],' vs ',logit(pi[i]))),
     type = 'l', col='blue', lwd=2)
```



e)

```
plot(x,mu, xlab = expression(x[i]), ylab = expression(pi[i]),
     main = expression(paste(x[i],' vs ',pi[i])),
     type = 'l', col='red', lwd=2)
```



f)

```r
set.seed(4)
y <- rbinom(n,1,mu)
summary(glm1 <- glm(y ~ x, family="binomial"))
```

```
##
## Call:
## glm(formula = y ~ x, family = "binomial")
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -3.0960  -0.4025   0.1076   0.3948   2.7828
##
## Coefficients:
##             Estimate Std. Error z value Pr(>|z|)
## (Intercept)  1.06217    0.11898   8.928   <2e-16 ***
## x            0.51287    0.03146  16.300   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
```

```
##     Null deviance: 1344.38  on 999  degrees of freedom
## Residual deviance:  617.55  on 998  degrees of freedom
## AIC: 621.55
##
## Number of Fisher Scoring iterations: 6
```

$\beta_0 = 1, \beta_1 = 0.5$