

Supplemental Material: Selection Scheme Parameter Sweep MVC Diagnostics

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Chapter 1

Introduction

This is the supplemental material for selection scheme parameter sweep experiments with diagnostics combined with valleys.

1.1 About our supplemental material

This supplemental material is hosted on GitHub using GitHub pages. The source code and configuration files used to generate this supplemental material can be found in this GitHub repository. We compiled our data analyses and supplemental documentation into this nifty web-accessible book using bookdown.

This supplemental material includes the following selection schemes:

- Truncation (Section 2)
- Tournament (Section 3)
- Genotypic fitness sharing (Section 4)
- Phenotypic fitness sharing (Section 5)
- Nondominated sorting (Section 6)
- Novelty search (Section 7)

1.2 Contributing authors

- Jose Guadalupe Hernandez
- Alexander Lalejini
- Charles Ofria

1.3 Computer Setup

These analyses were conducted in the following computing environment:

```
print(version)

##           _ 
## platform      x86_64-pc-linux-gnu
## arch        x86_64
## os          linux-gnu
## system      x86_64, linux-gnu
## status        
## major        4
## minor       3.1
```

```
## year           2023
## month          06
## day            16
## svn rev       84548
## language        R
## version.string R version 4.3.1 (2023-06-16)
## nickname      Beagle Scouts
```

1.4 Experimental setup

Setting up required variables variables.

```
# libraries we are using
library(ggplot2)
library(cowplot)
library(dplyr)

## 
## Attaching package: 'dplyr'

## The following objects are masked from 'package:stats':
## 
##     filter, lag

## The following objects are masked from 'package:base':
## 
##     intersect, setdiff, setequal, union
library(PupillometryR)

## Loading required package: rlang

# data directory for gh-pages
DATA_DIR = '/opt/ECJ-2023-Suite-Of-Diagnostic-Metrics-For-Characterizing-Selection-Schemes/DATA/PARAM_S

# data directory for local testing
# DATA_DIR = '~/Desktop/Repositories/ECJ-2023-Suite-Of-Diagnostic-Metrics-For-Characterizing-Selection-S

# graph variables
SHAPE = c(5,3,1,2,6,0,4,20,8)
cb_palette <- c('#332288', '#88CCEE', '#EE7733', '#EE3377', '#117733', '#882255', '#44AA99', '#CCBB44', '#000000')
TSIZE = 26
p_theme <- theme(
  text = element_text(size = 18),
  plot.title = element_text(face = "bold", size = 20, hjust=0.5),
  panel.border = element_blank(),
  panel.grid.minor = element_blank(),
  legend.title=element_text(size=12),
  legend.text=element_text(size=12),
  axis.title = element_text(size=18),
  axis.text = element_text(size=18),
  legend.position="bottom",
  panel.background = element_rect(fill = "#f1f2f5",
                                    colour = "white",
                                    linewidth = 0.5, linetype = "solid"))
)
```

```
# default variables
DIMENSIONALITY = 100
GENERATIONS = 50000

# selection scheme params exploring
TR_LIST = c('1','2','4','8','16','32','64','128','256')
TS_LIST = c('2','4','8','16','32','64','128','256','512')
FS_LIST = c('0','0.1','0.3','0.6','1.2','2.5','5')
ND_LIST = c('0','0.1','0.3','0.6','1.2','2.5','5')
NS_LIST = c('1','2','4','8','15','30')
```


Chapter 2

Truncation selection

Results for the truncation selection parameter sweep on the diagnostics with valleys.

2.1 Data setup

```
over_time_df <- read.csv(paste(DATA_DIR, 'OVER-TIME-MVC/tru.csv', sep = "", collapse = NULL), header = TRUE)
over_time_df$T <- factor(over_time_df$T, levels = TR_LIST)

best_df <- read.csv(paste(DATA_DIR, 'BEST-MVC/tru.csv', sep = "", collapse = NULL), header = TRUE, stringsAsFactors = FALSE)
best_df$T <- factor(best_df$T, levels = TR_LIST)
```

2.2 Exploitation rate results

Here we present the results for **best performances** found by each selection scheme parameter on the exploitation rate diagnostic with valleys. 50 replicates are conducted for each scheme explored.

2.2.1 Performance over time

Best performance in a population over time. Data points on the graph is the average performance across 50 replicates every 2000 generations. Shading comes from the best and worse performance across 50 replicates.

```
lines = filter(over_time_df, acro == 'exp') %>%
  group_by(T, gen) %>%
  dplyr::summarise(
    min = min(pop_fit_max) / DIMENSIONALITY,
    mean = mean(pop_fit_max) / DIMENSIONALITY,
    max = max(pop_fit_max) / DIMENSIONALITY
  )

## `summarise()` has grouped output by 'T'. You can override using the `.`groups` argument.

over_time_plot = ggplot(lines, aes(x=gen, y=mean, group = T, fill = T, color = T, shape = T)) +
  geom_ribbon(aes(ymin = min, ymax = max), alpha = 0.1) +
  geom_line(size = 0.5) +
  geom_point(data = filter(lines, gen %% 2000 == 0 & gen != 0), size = 1.5, stroke = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name="Average trait score",
```

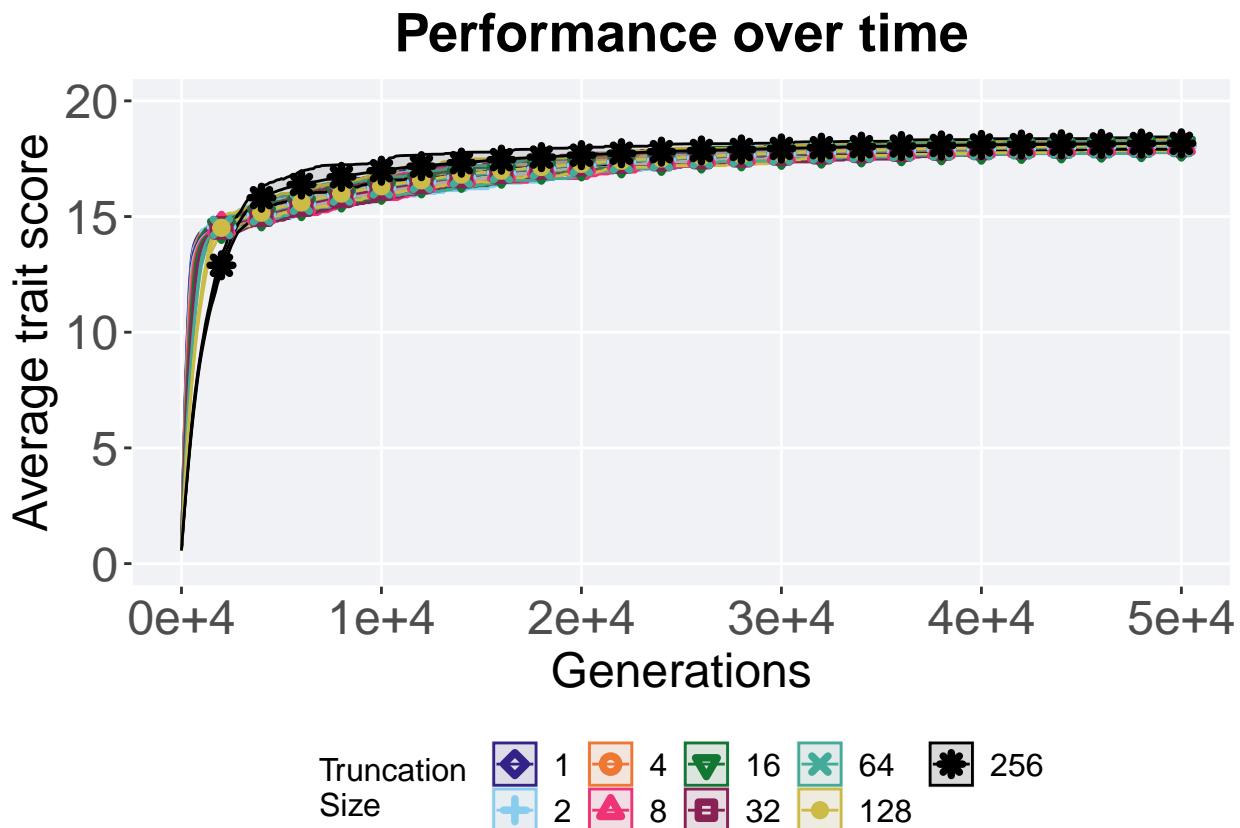
```

    limits=c(0, 20)
) +
scale_x_continuous(
  name="Generations",
  limits=c(0, 50000),
  breaks=c(0, 10000, 20000, 30000, 40000, 50000),
  labels=c("0e+4", "1e+4", "2e+4", "3e+4", "4e+4", "5e+4")

) +
scale_shape_manual(values=SHAPE) +
scale_colour_manual(values = cb_palette) +
scale_fill_manual(values = cb_palette) +
ggtitle('Performance over time') +
p_theme +
guides(
  shape=guide_legend(nrow=2, title.position = "left", title = 'Truncation \nSize'),
  color=guide_legend(nrow=2, title.position = "left", title = 'Truncation \nSize'),
  fill=guide_legend(nrow=2, title.position = "left", title = 'Truncation \nSize')
)

over_time_plot

```



2.2.2 Best performance throughout

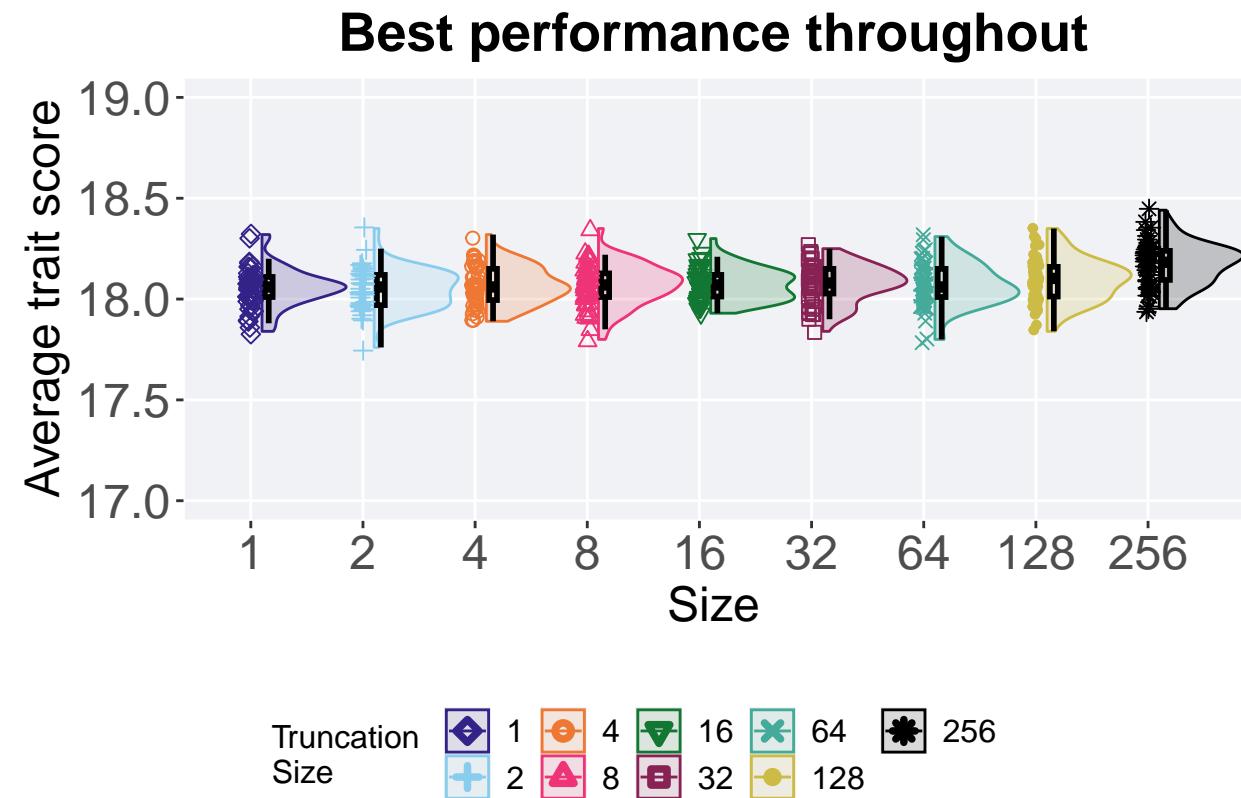
Best performance reached throughout 50,000 generations in a population.

```

plot = filter(best_df, acro == 'exp' & var == 'pop_fit_max') %>%
  ggplot(., aes(x = T, y = val / DIMENSIONALITY, color = T, fill = T, shape = T)) +
  geom_flat_violin(position = position_nudge(x = .1, y = 0), scale = 'width', alpha = 0.2, width = 1.5) +
  geom_boxplot(color = 'black', width = .07, outlier.shape = NA, alpha = 0.0, size = 1.0, position = position_nudge(x = -.1, y = 0), width = 1.5) +
  geom_point(position = position_jitter(width = 0.03, height = 0.02), size = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name = "Average trait score",
    limits = c(17, 19)
  ) +
  scale_x_discrete(
    name = "Size"
  ) +
  scale_shape_manual(values = SHAPE) +
  scale_color_manual(values = cb_palette, ) +
  scale_fill_manual(values = cb_palette) +
  ggtitle('Best performance throughout') +
  p_theme

plot_grid(
  plot +
    theme(legend.position = "none"),
  legend,
  nrow = 2,
  rel_heights = c(3, 1)
)

```



2.2.2.1 Stats

Summary statistics for the best performance.

```
performance = filter(best_df, acro == 'exp' & var == 'pop_fit_max')
performance$T = factor(performance$T, levels = TR_LIST)
performance %>%
  group_by(T) %>%
  dplyr::summarise(
    count = n(),
    na_cnt = sum(is.na(val)),
    min = min(val / DIMENSIONALITY, na.rm = TRUE),
    median = median(val / DIMENSIONALITY, na.rm = TRUE),
    mean = mean(val / DIMENSIONALITY, na.rm = TRUE),
    max = max(val / DIMENSIONALITY, na.rm = TRUE),
    IQR = IQR(val / DIMENSIONALITY, na.rm = TRUE)
  )

## # A tibble: 9 x 8
##   T      count  na_cnt   min  median   mean   max     IQR
##   <fct> <int> <int> <dbl> <dbl> <dbl> <dbl> <dbl>
## 1 1       50     0  17.8   18.1   18.1   18.3 0.0999
## 2 2       50     0  17.8   18.1   18.1   18.3 0.150
## 3 4       50     0  17.9   18.1   18.1   18.3 0.155
## 4 8       50     0  17.8   18.1   18.1   18.3 0.117
## 5 16      50     0  17.9   18.1   18.1   18.3 0.107
## 6 32      50     0  17.8   18.1   18.1   18.2 0.122
## 7 64      50     0  17.8   18.1   18.1   18.3 0.140
## 8 128     50     0  17.8   18.1   18.1   18.3 0.147
## 9 256     50     0  18.0   18.2   18.2   18.4 0.144
```

Kruskal–Wallis test illustrates evidence of statistical differences.

```
kruskal.test(val ~ T, data = performance)
```

```
##
##  Kruskal-Wallis rank sum test
##
## data: val by T
## Kruskal-Wallis chi-squared = 40.639, df = 8, p-value = 2.435e-06
```

Results for post-hoc Wilcoxon rank-sum test with a Bonferroni correction.

```
pairwise.wilcox.test(x = performance$val, g = performance$T, p.adjust.method = "bonferroni",
                      paired = FALSE, conf.int = FALSE, alternative = 't')

##
##  Pairwise comparisons using Wilcoxon rank sum test with continuity correction
##
## data: performance$val and performance$T
##
##      1      2      4      8      16     32     64     128
## 2  1.00000 -      -      -      -      -      -
## 4  1.00000 1.00000 -      -      -      -      -
## 8  1.00000 1.00000 1.00000 -      -      -      -
## 16 1.00000 1.00000 1.00000 1.00000 -      -      -
## 32 1.00000 1.00000 1.00000 1.00000 1.00000 -      -
## 64 1.00000 1.00000 1.00000 1.00000 1.00000 1.00000 -
```

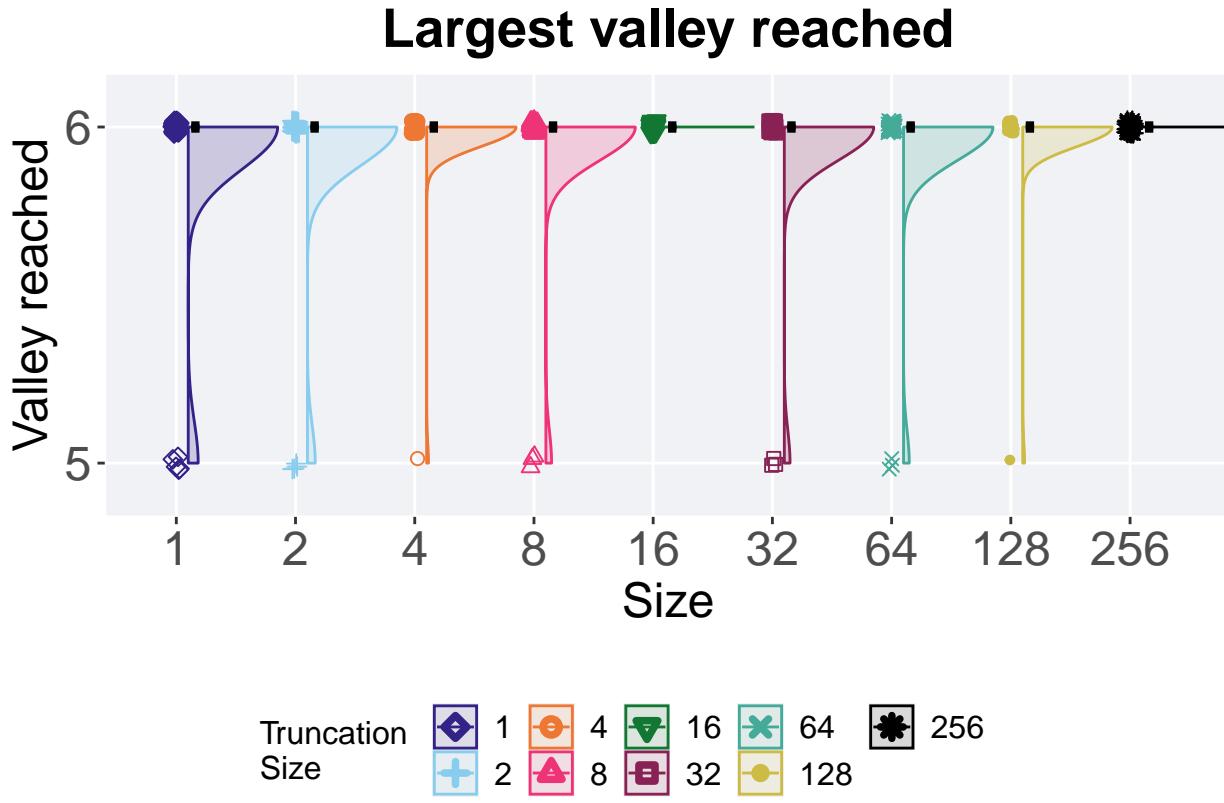
```
## 128 1.00000 1.00000 1.00000 1.00000 1.00000 1.00000 1.00000 -
## 256 1.7e-05 1.6e-05 0.00018 0.00043 0.00023 0.00104 0.00114 0.03366
##
## P value adjustment method: bonferroni
```

2.2.3 Largest valley reached throughout

Largest valley reached in a single trait by the best performing solution throughout an entire evolutionary run.

```
plot = filter(best_df, acro == 'exp' & var == 'ele_big_peak') %>%
  ggplot(., aes(x = T, y = val, color = T, fill = T, shape = T)) +
  geom_flat_violin(position = position_nudge(x = .1, y = 0), scale = 'width', alpha = 0.2, width = 1.5) +
  geom_boxplot(color = 'black', width = .07, outlier.shape = NA, alpha = 0.0, size = 1.0, position = position_nudge(x = -.1, y = 0)) +
  geom_point(position = position_jitter(width = 0.03, height = 0.02), size = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name="Valley reached",
    limits=c(4.9,6.1),
    breaks = c(5,6)
  ) +
  scale_x_discrete(
    name="Size"
  )+
  scale_shape_manual(values=SHAPE)+
  scale_colour_manual(values = cb_palette, ) +
  scale_fill_manual(values = cb_palette) +
  ggtitle('Largest valley reached')+
  p_theme

plot_grid(
  plot +
    theme(legend.position="none"),
  legend,
  nrow=2,
  rel_heights = c(3,1)
)
```



2.2.3.1 Stats

Summary statistics for the largest valley crossed.

```
valleys = filter(best_df, acro == 'exp' & var == 'ele_big_peak')
valleys$T = factor(valleys$T, levels = TR_LIST)
valleys %>%
  group_by(T) %>%
  dplyr::summarise(
    count = n(),
    na_cnt = sum(is.na(val)),
    min = min(val, na.rm = TRUE),
    median = median(val, na.rm = TRUE),
    mean = mean(val, na.rm = TRUE),
    max = max(val, na.rm = TRUE),
    IQR = IQR(val, na.rm = TRUE)
  )
```

```
## # A tibble: 9 x 8
##   T      count na_cnt   min median   mean   max   IQR
##   <fct> <int> <int> <dbl> <dbl> <dbl> <dbl> <dbl>
## 1 1       50     0     5     6    5.9     6     0
## 2 2       50     0     5     6    5.92    6     0
## 3 4       50     0     5     6    5.98    6     0
## 4 8       50     0     5     6    5.94    6     0
## 5 16      50     0     6     6     6     6     0
## 6 32      50     0     5     6    5.94    6     0
```

```
## 7 64      50     0     5     6  5.94     6     0
## 8 128     50     0     5     6  5.98     6     0
## 9 256     50     0     6     6   6     6     0
```

Kruskal-Wallis test illustrates evidence of **no statistical differences**.

```
kruskal.test(val ~ T, data = valleys)

##
## Kruskal-Wallis rank sum test
##
## data: val by T
## Kruskal-Wallis chi-squared = 12.008, df = 8, p-value = 0.1508
```

2.3 Ordered exploitation results

Here we present the results for **best performances** found by each selection scheme parameter on the ordered exploitation diagnostic with valleys. 50 replicates are conducted for each scheme explored.

2.3.1 Performance over time

Best performance in a population over time. Data points on the graph is the average performance across 50 replicates every 2000 generations. Shading comes from the best and worse performance across 50 replicates.

```
lines = filter(over_time_df, acro == 'ord') %>%
  group_by(T, gen) %>%
  dplyr::summarise(
    min = min(pop_fit_max) / DIMENSIONALITY,
    mean = mean(pop_fit_max) / DIMENSIONALITY,
    max = max(pop_fit_max) / DIMENSIONALITY
  )

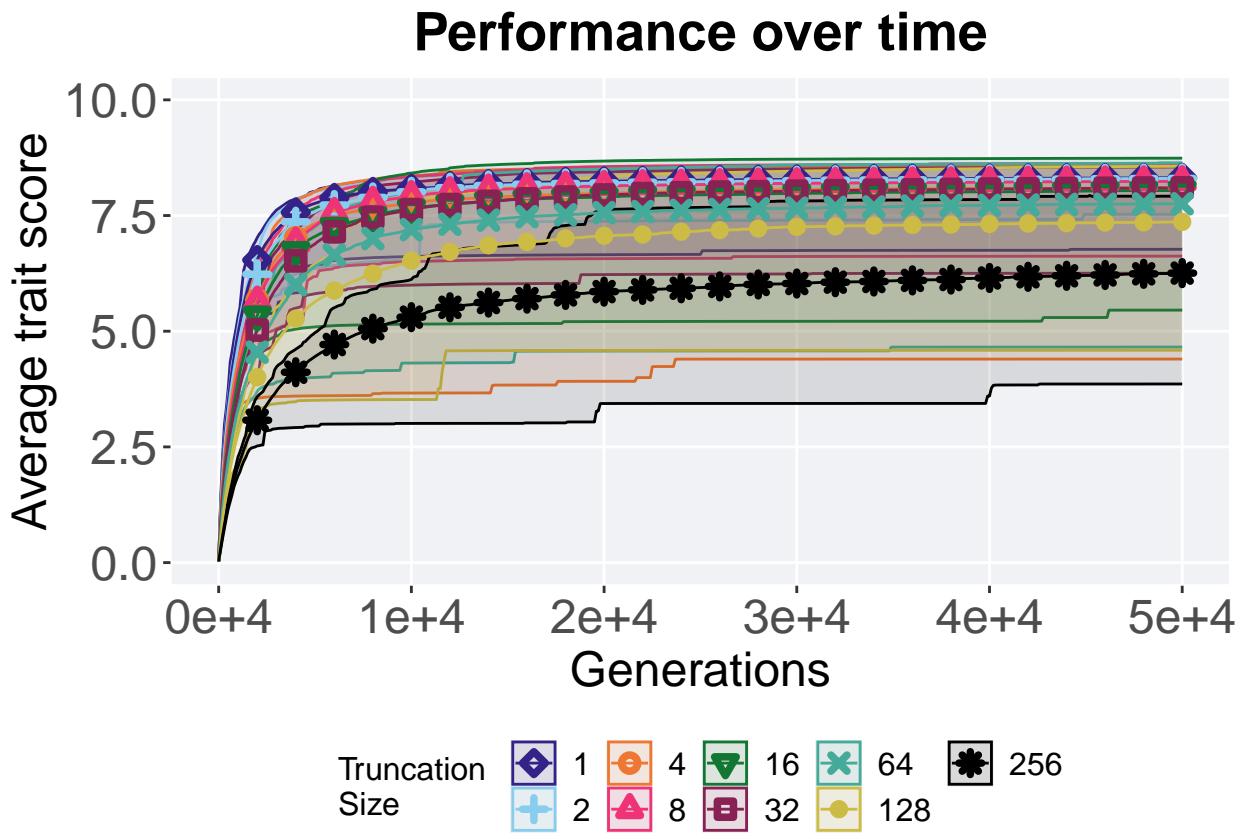
## `summarise()` has grouped output by 'T'. You can override using the `.`groups` argument.

over_time_plot = ggplot(lines, aes(x=gen, y=mean, group = T, fill = T, color = T, shape = T)) +
  geom_ribbon(aes(ymin = min, ymax = max), alpha = 0.1) +
  geom_line(size = 0.5) +
  geom_point(data = filter(lines, gen %% 2000 == 0 & gen != 0), size = 1.5, stroke = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name="Average trait score",
    limits=c(0, 10)
  ) +
  scale_x_continuous(
    name="Generations",
    limits=c(0, 50000),
    breaks=c(0, 10000, 20000, 30000, 40000, 50000),
    labels=c("0e+4", "1e+4", "2e+4", "3e+4", "4e+4", "5e+4")
  ) +
  scale_shape_manual(values=SHAPE) +
  scale_colour_manual(values = cb_palette) +
  scale_fill_manual(values = cb_palette) +
  ggtitle('Performance over time') +
  p_theme +
  guides(
```

```

    shape=guide_legend(nrow=2, title.position = "left", title = 'Truncation \nSize'),
    color=guide_legend(nrow=2, title.position = "left", title = 'Truncation \nSize'),
    fill=guide_legend(nrow=2, title.position = "left", title = 'Truncation \nSize')
)
over_time_plot

```



2.3.2 Best performance throughout

Best performance reached throughout 50,000 generations in a population.

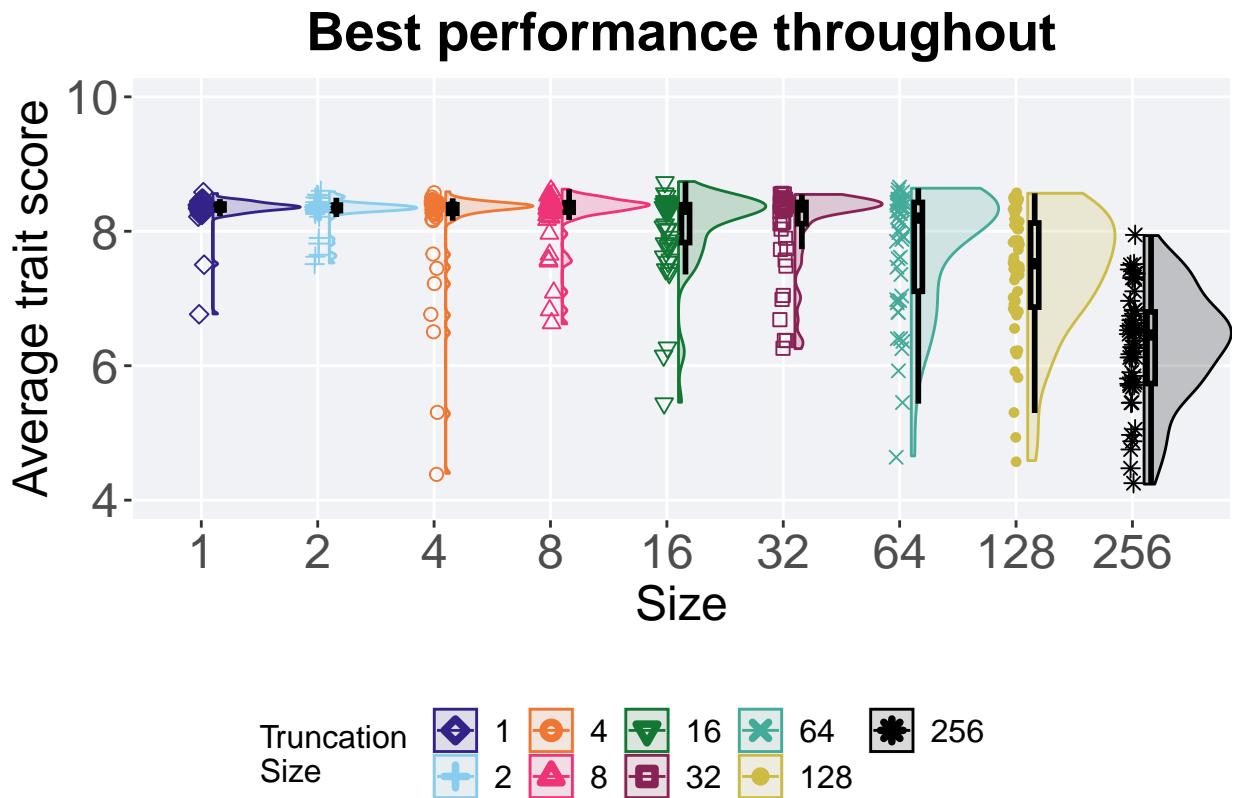
```

plot = filter(best_df, acro == 'ord' & var == 'pop_fit_max') %>%
  ggplot(., aes(x = T, y = val / DIMENSIONALITY, color = T, fill = T, shape = T)) +
  geom_flat_violin(position = position_nudge(x = .1, y = 0), scale = 'width', alpha = 0.2, width = 1.5) +
  geom_boxplot(color = 'black', width = .07, outlier.shape = NA, alpha = 0.0, size = 1.0, position = position_nudge(x = -.1, y = 0)) +
  geom_point(position = position_jitter(width = 0.03, height = 0.02), size = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name="Average trait score",
    limits=c(4, 10)
  ) +
  scale_x_discrete(
    name="Size"
  )+
  scale_shape_manual(values=SHAPE)+
  scale_colour_manual(values = cb_palette, ) +
  scale_fill_manual(values = cb_palette) +

```

```
ggtitle('Best performance throughout')+
  p_theme

plot_grid(
  plot +
    theme(legend.position="none"),
  legend,
  nrow=2,
  rel_heights = c(3,1)
)
```



2.3.2.1 Stats

Summary statistics for the best performance.

```
performance = filter(best_df, acro == 'ord' & var == 'pop_fit_max')
performance$T = factor(performance$T, levels = TR_LIST)
performance %>%
  group_by(T) %>%
  dplyr::summarise(
  count = n(),
  na_cnt = sum(is.na(val)),
  min = min(val / DIMENSIONALITY, na.rm = TRUE),
  median = median(val / DIMENSIONALITY, na.rm = TRUE),
  mean = mean(val / DIMENSIONALITY, na.rm = TRUE),
  max = max(val / DIMENSIONALITY, na.rm = TRUE),
```

```
IQR = IQR(val / DIMENSIONALITY, na.rm = TRUE)
)

## # A tibble: 9 x 8
##   T      count na_cnt   min median   mean   max     IQR
##   <fct> <int> <int> <dbl> <dbl> <dbl> <dbl> <dbl>
## 1 1       50     0  6.77  8.36  8.32  8.56 0.0867
## 2 2       50     0  7.53  8.35  8.30  8.59 0.0808
## 3 4       50     0  4.40  8.36  8.10  8.59 0.128
## 4 8       50     0  6.62  8.37  8.24  8.63 0.142
## 5 16      50     0  5.46  8.33  8.05  8.74 0.563
## 6 32      50     0  6.25  8.36  8.10  8.55 0.316
## 7 64      50     0  4.66  8.20  7.75  8.64 1.32
## 8 128     50     0  4.59  7.52  7.36  8.57 1.25
## 9 256     50     0  3.86  6.42  6.26  7.94 1.06
```

Kruskal–Wallis test illustrates evidence of statistical differences.

```
kruskal.test(val ~ T, data = performance)
```

```
##
## Kruskal-Wallis rank sum test
##
## data: val by T
## Kruskal-Wallis chi-squared = 148.77, df = 8, p-value < 2.2e-16
```

Results for post-hoc Wilcoxon rank-sum test with a Bonferroni correction.

```
pairwise.wilcox.test(x = performance$val, g = performance$T, p.adjust.method = "bonferroni",
                      paired = FALSE, conf.int = FALSE, alternative = 't')
```

```
##
## Pairwise comparisons using Wilcoxon rank sum test with continuity correction
##
## data: performance$val and performance$T
##
##    1     2     4     8     16    32    64   128
## 2 1.00000 -     -     -     -     -     -     -
## 4 1.00000 1.00000 -     -     -     -     -     -
## 8 1.00000 1.00000 1.00000 -     -     -     -     -
## 16 1.00000 1.00000 1.00000 1.00000 -     -     -     -
## 32 1.00000 1.00000 1.00000 1.00000 1.00000 -     -     -
## 64 0.91020 1.00000 1.00000 0.95152 1.00000 1.00000 -     -
## 128 6.0e-08 8.7e-08 3.7e-05 1.3e-06 0.00104 0.00013 0.34001 -
## 256 6.6e-16 3.4e-16 8.6e-13 2.9e-15 2.9e-13 2.9e-13 2.9e-09 1.7e-06
##
## P value adjustment method: bonferroni
```

2.3.3 Largest valley reached throughout

Largest valley reached in a single trait by the best performing solution throughout an entire evolutionary run.

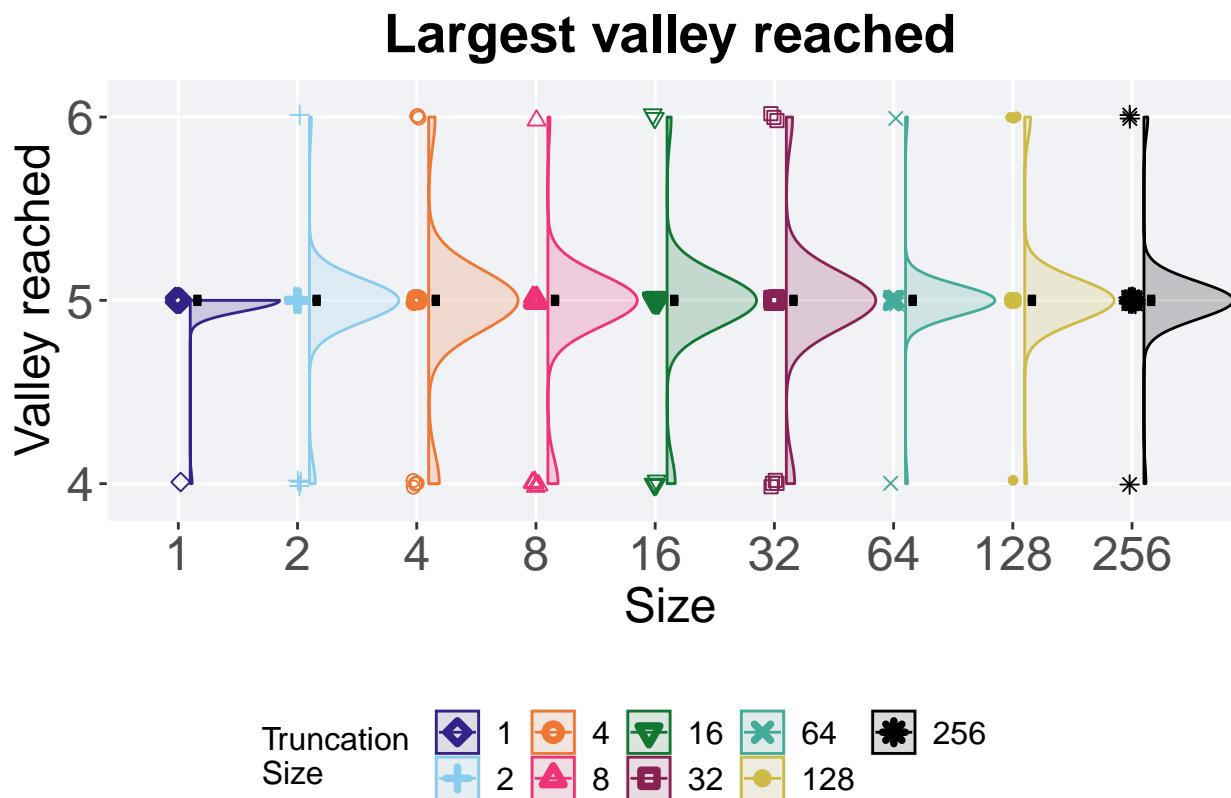
```
plot = filter(best_df, acro == 'ord' & var == 'ele_big_peak') %>%
  ggplot(., aes(x = T, y = val, color = T, fill = T, shape = T)) +
  geom_flat_violin(position = position_nudge(x = .1, y = 0), scale = 'width', alpha = 0.2, width = 1.5) +
  geom_boxplot(color = 'black', width = .07, outlier.shape = NA, alpha = 0.0, size = 1.0, position = pos) +
  geom_point(position = position_jitter(width = 0.03, height = 0.02), size = 2.0, alpha = 1.0) +
```

```

scale_y_continuous(
  name="Valley reached",
  limits=c(3.9,6.1),
  breaks = c(4,5,6)
) +
scale_x_discrete(
  name="Size"
) +
scale_shape_manual(values=SHAPE) +
scale_colour_manual(values = cb_palette, ) +
scale_fill_manual(values = cb_palette) +
ggtitle('Largest valley reached') +
p_theme

plot_grid(
  plot +
  theme(legend.position="none"),
  legend,
  nrow=2,
  rel_heights = c(3,1)
)
)

```



2.3.3.1 Stats

Summary statistics for the largest valley crossed.

```

valleys = filter(best_df, acro == 'ord' & var == 'ele_big_peak')
valleys$T = factor(valleys$T, levels = TR_LIST)
valleys %>%
  group_by(T) %>%
  dplyr::summarise(
    count = n(),
    na_cnt = sum(is.na(val)),
    min = min(val, na.rm = TRUE),
    median = median(val, na.rm = TRUE),
    mean = mean(val, na.rm = TRUE),
    max = max(val, na.rm = TRUE),
    IQR = IQR(val, na.rm = TRUE)
  )

## # A tibble: 9 x 8
##   T      count na_cnt   min median   mean   max   IQR
##   <fct> <int> <int> <dbl> <dbl> <dbl> <dbl> <dbl>
## 1 1       50     0     4     5    4.98     5     0
## 2 2       50     0     4     5    4.96     6     0
## 3 4       50     0     4     5    4.96     6     0
## 4 8       50     0     4     5    4.92     6     0
## 5 16      50     0     4     5    4.96     6     0
## 6 32      50     0     4     5    4.98     6     0
## 7 64      50     0     4     5    5.0       6     0
## 8 128     50     0     4     5    5.04     6     0
## 9 256     50     0     4     5    5.02     6     0

```

Kruskal–Wallis test illustrates evidence of **no statistical differences**.

```

kruskal.test(val ~ T, data = valleys)

##
##  Kruskal-Wallis rank sum test
##
## data: val by T
## Kruskal-Wallis chi-squared = 5.7698, df = 8, p-value = 0.673

```

2.4 Contradictory objectives results

Here we present the results for **activation gene coverage** and **satisfactory trait coverage** found by each selection scheme parameter on the contradictory objectives diagnostic with valleys. 50 replicates are conducted for each scheme parameters explored.

2.4.1 Activation gene coverage over time

Activation gene coverage in a population over time. Data points on the graph is the average activation gene coverage across 50 replicates every 2000 generations. Shading comes from the best and worse coverage across 50 replicates.

```

lines = filter(over_time_df, acro == 'con') %>%
  group_by(T, gen) %>%
  dplyr::summarise(
    min = min(uni_str_pos),
    mean = mean(uni_str_pos),
    max = max(uni_str_pos)
  )

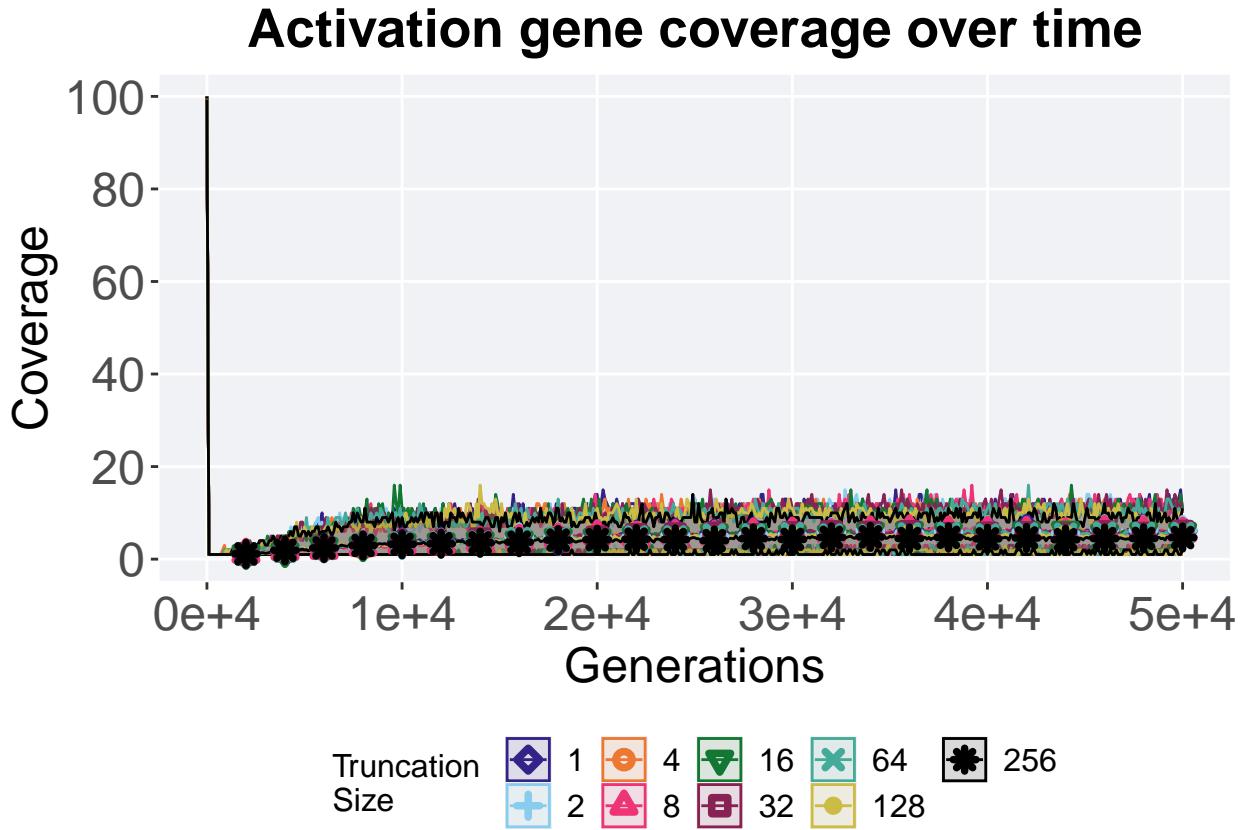
```

```

    max = max(uni_str_pos)
)

## `summarise()` has grouped output by 'T'. You can override using the `groups` argument.
ggplot(lines, aes(x=gen, y=mean, group = T, fill = T, color = T, shape = T)) +
  geom_ribbon(aes(ymin = min, ymax = max), alpha = 0.1) +
  geom_line(size = 0.5) +
  geom_point(data = filter(lines, gen %% 2000 == 0 & gen != 0), size = 1.5, stroke = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name="Coverage",
    limits=c(0, 100),
    breaks=seq(0,100, 20),
    labels=c("0", "20", "40", "60", "80", "100")
  ) +
  scale_x_continuous(
    name="Generations",
    limits=c(0, 50000),
    breaks=c(0, 10000, 20000, 30000, 40000, 50000),
    labels=c("0e+4", "1e+4", "2e+4", "3e+4", "4e+4", "5e+4")
  ) +
  scale_shape_manual(values=SHAPE) +
  scale_colour_manual(values = cb_palette) +
  scale_fill_manual(values = cb_palette) +
  ggtitle('Activation gene coverage over time') +
  p_theme +
  guides(
    shape=guide_legend(nrow=2, title.position = "left", title = 'Truncation \nSize'),
    color=guide_legend(nrow=2, title.position = "left", title = 'Truncation \nSize'),
    fill=guide_legend(nrow=2, title.position = "left", title = 'Truncation \nSize')
  )

```



2.4.2 Final activation gene coverage

Activation gene coverage found in the final population at 50,000 generations.

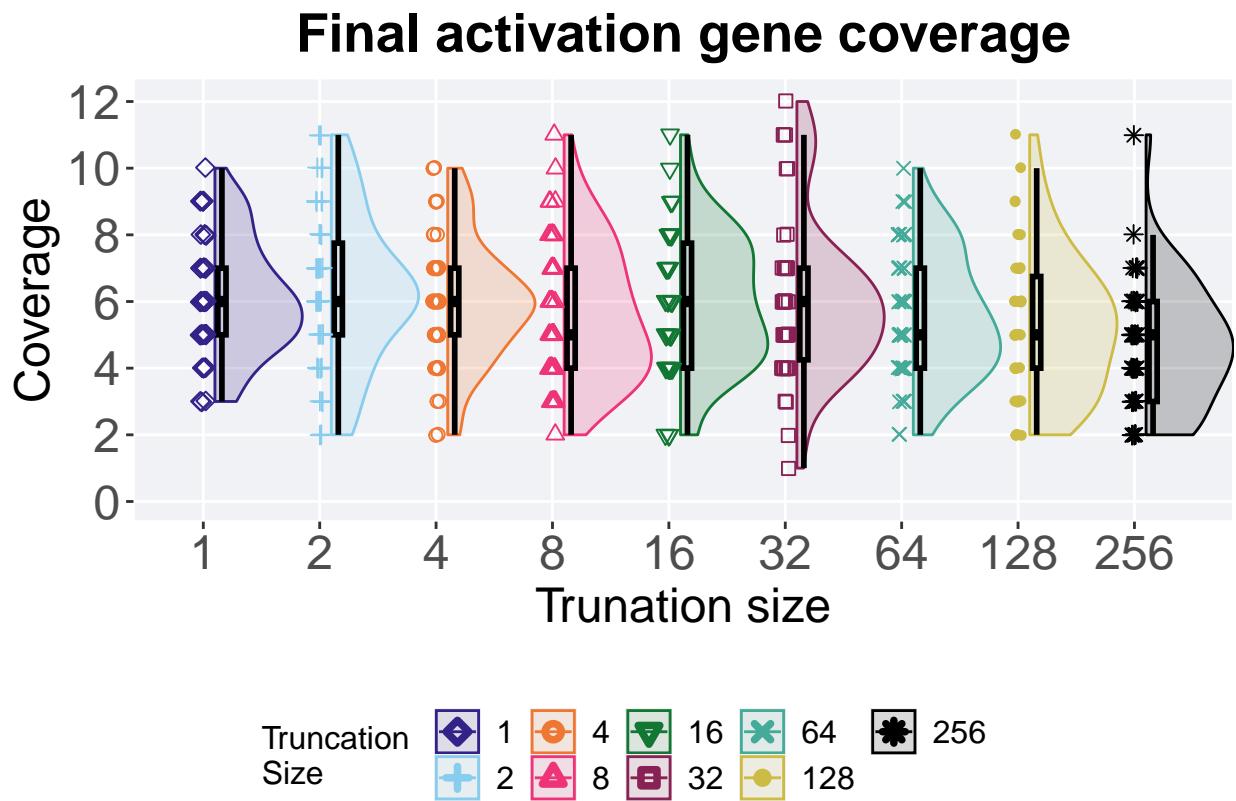
```
plot = filter(over_time_df, gen == 50000 & acro == 'con') %>%
  ggplot(., aes(x = T, y = uni_str_pos, color = T, fill = T, shape = T)) +
  geom_flat_violin(position = position_nudge(x = .1, y = 0), scale = 'width', alpha = 0.2, width = 1.5)
  geom_boxplot(color = 'black', width = .07, outlier.shape = NA, alpha = 0.0, size = 1.0, position = position_nudge(x = -.1, y = 0))
  geom_point(position = position_jitter(width = 0.03, height = 0.02), size = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name="Coverage",
    limits=c(0, 12.1),
    breaks=seq(0,12,2)
  ) +
  scale_x_discrete(
    name="Truncation size"
  )+
  scale_shape_manual(values=SHAPE)+
  scale_colour_manual(values = cb_palette, ) +
  scale_fill_manual(values = cb_palette) +
  ggtitle('Final activation gene coverage')+
  p_theme

plot_grid(
  plot +
    theme(legend.position="none"),
  nrow=1, ncol=1)
```

```

  legend,
  nrow=2,
  rel_heights = c(3,1)
)

```



2.4.2.1 Stats

Summary statistics for the generation a satisfactory solution is found.

```

act_coverage = filter(over_time_df, gen == 50000 & acro == 'con')
act_coverage$T = factor(act_coverage$T, levels = TR_LIST)
act_coverage %>%
  group_by(T) %>%
  dplyr::summarise(
    count = n(),
    na_cnt = sum(is.na(uni_str_pos)),
    min = min(uni_str_pos, na.rm = TRUE),
    median = median(uni_str_pos, na.rm = TRUE),
    mean = mean(uni_str_pos, na.rm = TRUE),
    max = max(uni_str_pos, na.rm = TRUE),
    IQR = IQR(uni_str_pos, na.rm = TRUE)
  )

## # A tibble: 9 x 8
##   T      count  na_cnt   min  median   mean   max   IQR
##   <fct> <int>   <int> <dbl> <dbl> <dbl> <int> <dbl>
## 1 1       50      0     3     6    6.1    10     2

```

```
## 2 2      50    0    2     6   6.28    11  2.75
## 3 4      50    0    2     6   5.98    10  2
## 4 8      50    0    2     5   5.68    11  3
## 5 16     50    0    2     6   5.94    11  3.75
## 6 32     50    0    1     6   5.96    12  2.75
## 7 64     50    0    2     5   5.54    10  3
## 8 128    50    0    2     5   5.28    11  2.75
## 9 256    50    0    2     5   4.66    11  3
```

Kruskal–Wallis test illustrates evidence of statistical differences.

```
kruskal.test(uni_str_pos ~ T, data = act_coverage)
```

```
##
## Kruskal-Wallis rank sum test
##
## data: uni_str_pos by T
## Kruskal-Wallis chi-squared = 24.29, df = 8, p-value = 0.002049
```

Results for post-hoc Wilcoxon rank-sum test with a Bonferroni correction.

```
pairwise.wilcox.test(x = act_coverage$uni_str_pos, g = act_coverage$T, p.adjust.method = "bonferroni",
                      paired = FALSE, conf.int = FALSE, alternative = 't')
```

```
##
## Pairwise comparisons using Wilcoxon rank sum test with continuity correction
##
## data: act_coverage$uni_str_pos and act_coverage$T
##
##      1     2     4     8    16    32    64   128
## 2 1.0000 -    -    -    -    -    -    -
## 4 1.0000 1.0000 -    -    -    -    -    -
## 8 1.0000 1.0000 1.0000 -    -    -    -    -
## 16 1.0000 1.0000 1.0000 1.0000 -    -    -    -
## 32 1.0000 1.0000 1.0000 1.0000 1.0000 -    -    -
## 64 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 -    -
## 128 1.0000 0.8123 1.0000 1.0000 1.0000 1.0000 1.0000 -
## 256 0.0053 0.0079 0.0194 0.6738 0.0578 0.1290 0.8827 1.0000
##
## P value adjustment method: bonferroni
```

2.4.3 Satisfactory trait coverage over time

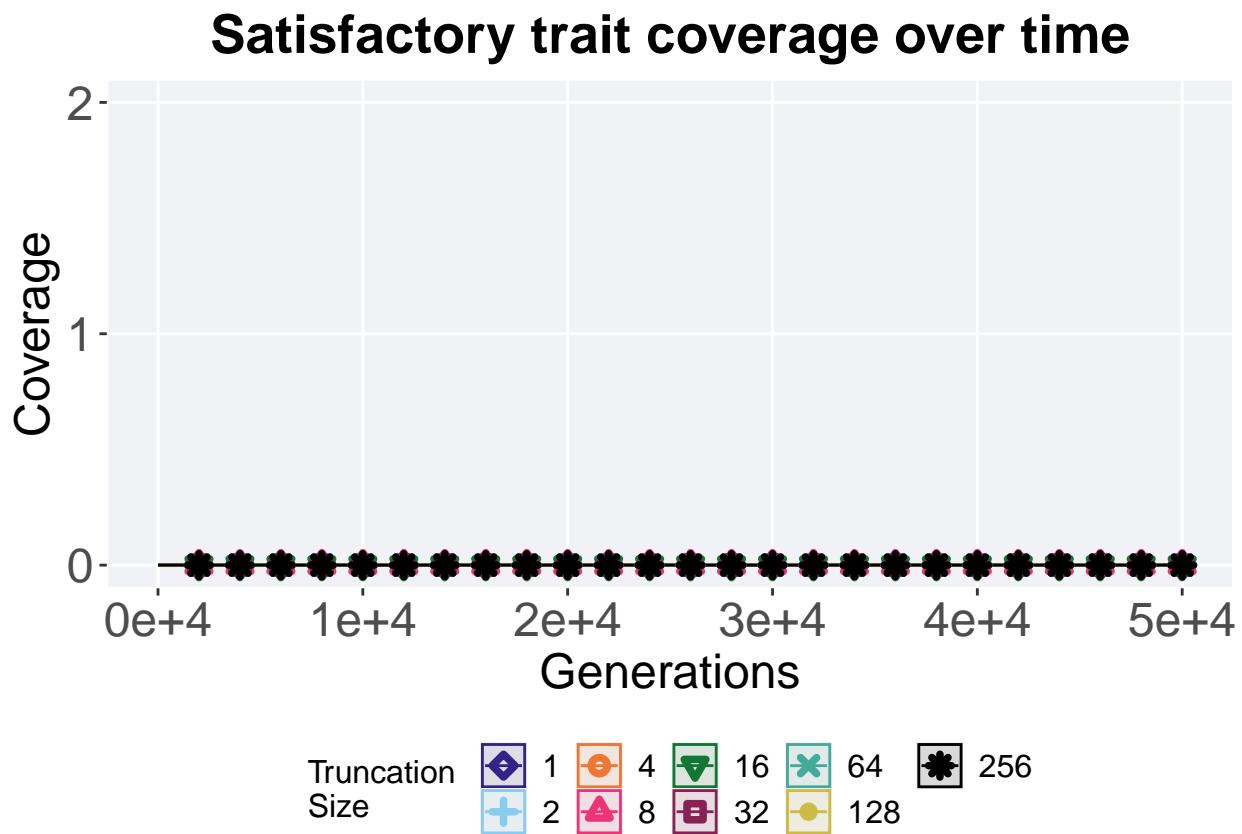
Satisfactory trait coverage in a population over time. Data points on the graph is the average activation gene coverage across 50 replicates every 2000 generations. Shading comes from the best and worse coverage across 50 replicates.

```
lines = filter(over_time_df, acro == 'con') %>%
  group_by(T, gen) %>%
  dplyr::summarise(
    min = min(pop_uni_obj),
    mean = mean(pop_uni_obj),
    max = max(pop_uni_obj)
  )

## `summarise()` has grouped output by 'T'. You can override using the `groups` argument.
```

```
ggplot(lines, aes(x=gen, y=mean, group = T, fill = T, color = T, shape = T)) +
  geom_ribbon(aes(ymin = min, ymax = max), alpha = 0.1) +
  geom_line(size = 0.5) +
  geom_point(data = filter(lines, gen %% 2000 == 0 & gen != 0), size = 1.5, stroke = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name="Coverage",
    limits=c(0, 2),
    breaks=c(0,1,2)
  ) +
  scale_x_continuous(
    name="Generations",
    limits=c(0, 50000),
    breaks=c(0, 10000, 20000, 30000, 40000, 50000),
    labels=c("0e+4", "1e+4", "2e+4", "3e+4", "4e+4", "5e+4")
  ) +
  scale_shape_manual(values=SHAPE) +
  scale_colour_manual(values = cb_palette) +
  scale_fill_manual(values = cb_palette) +
  ggtitle('Satisfactory trait coverage over time') +
  p_theme +
  guides(
    shape=guide_legend(nrow=2, title.position = "left", title = 'Truncation \nSize'),
    color=guide_legend(nrow=2, title.position = "left", title = 'Truncation \nSize'),
    fill=guide_legend(nrow=2, title.position = "left", title = 'Truncation \nSize')
  )

```



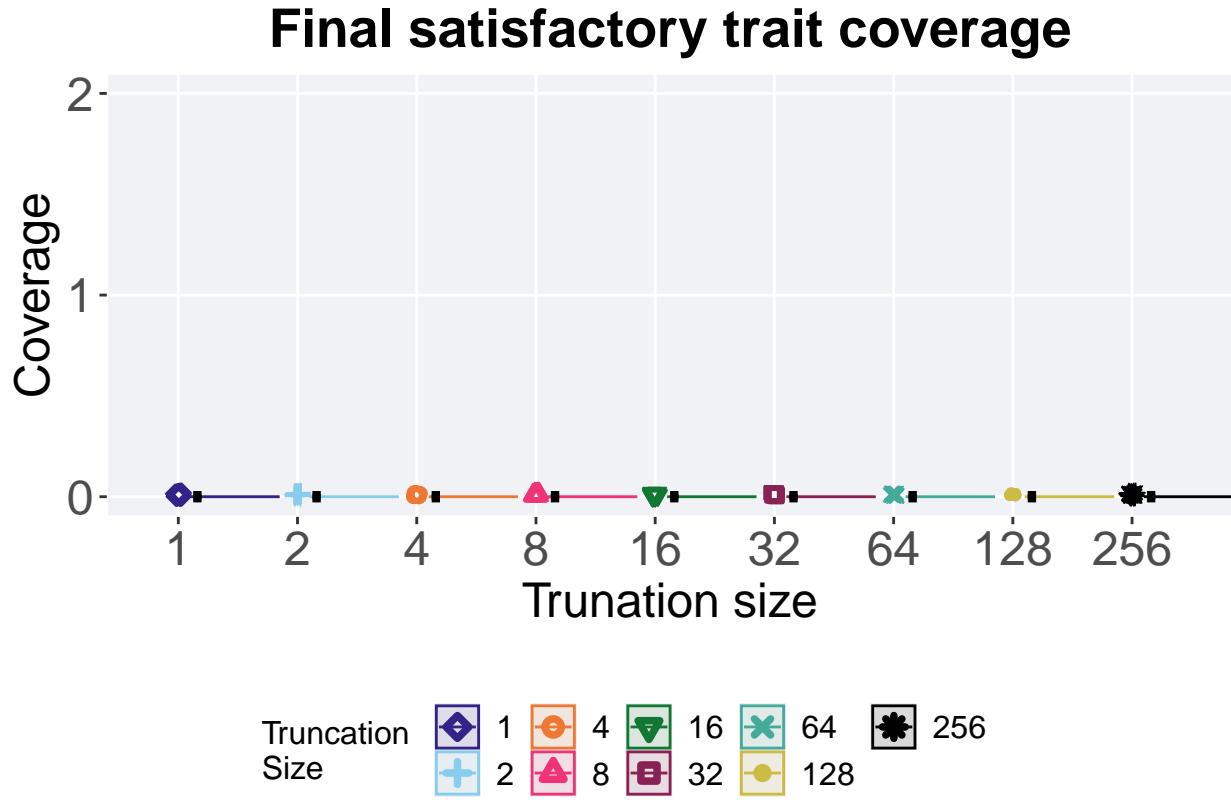
2.4.4 Final satisfactory trait coverage

Satisfactory trait coverage found in the final population at 50,000 generations.

```
plot = filter(over_time_df, gen == 50000 & acro == 'con') %>%
  ggplot(., aes(x = T, y = pop_uni_obj, color = T, fill = T, shape = T)) +
  geom_flat_violin(position = position_nudge(x = .1, y = 0), scale = 'width', alpha = 0.2, width = 1.5) +
  geom_boxplot(color = 'black', width = .07, outlier.shape = NA, alpha = 0.0, size = 1.0, position = position_nudge(x = .1, y = 0)) +
  geom_point(position = position_jitter(width = 0.03, height = 0.02), size = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name = "Coverage",
    limits = c(0, 2),
    breaks = c(0, 1, 2)
  ) +
  scale_x_discrete(
    name = "Trunation size"
  ) +
  scale_shape_manual(values = SHAPE) +
  scale_colour_manual(values = cb_palette, ) +
  scale_fill_manual(values = cb_palette) +
  ggtitle('Final satisfactory trait coverage') +
  p_theme

plot_grid(
  plot +
    theme(legend.position = "none"),
  legend,
  nrow = 2,
  rel_heights = c(3, 1)
)

## Warning: Removed 250 rows containing missing values (`geom_point()`).
```



2.4.4.1 Stats

Summary statistics for the generation a satisfactory solution is found.

```
sat_coverage = filter(over_time_df, gen == 50000 & acro == 'con')
sat_coverage$T = factor(sat_coverage$T, levels = TR_LIST)
sat_coverage %>%
  group_by(T) %>%
  dplyr::summarise(
    count = n(),
    na_cnt = sum(is.na(pop_uni_obj)),
    min = min(pop_uni_obj, na.rm = TRUE),
    median = median(pop_uni_obj, na.rm = TRUE),
    mean = mean(pop_uni_obj, na.rm = TRUE),
    max = max(pop_uni_obj, na.rm = TRUE),
    IQR = IQR(pop_uni_obj, na.rm = TRUE)
  )
```

```
## # A tibble: 9 x 8
##   T      count na_cnt  min median  mean   max   IQR
##   <fct> <int> <int> <dbl> <dbl> <dbl> <dbl>
## 1 1       50     0     0     0     0     0     0
## 2 2       50     0     0     0     0     0     0
## 3 4       50     0     0     0     0     0     0
## 4 8       50     0     0     0     0     0     0
## 5 16      50     0     0     0     0     0     0
## 6 32      50     0     0     0     0     0     0
```

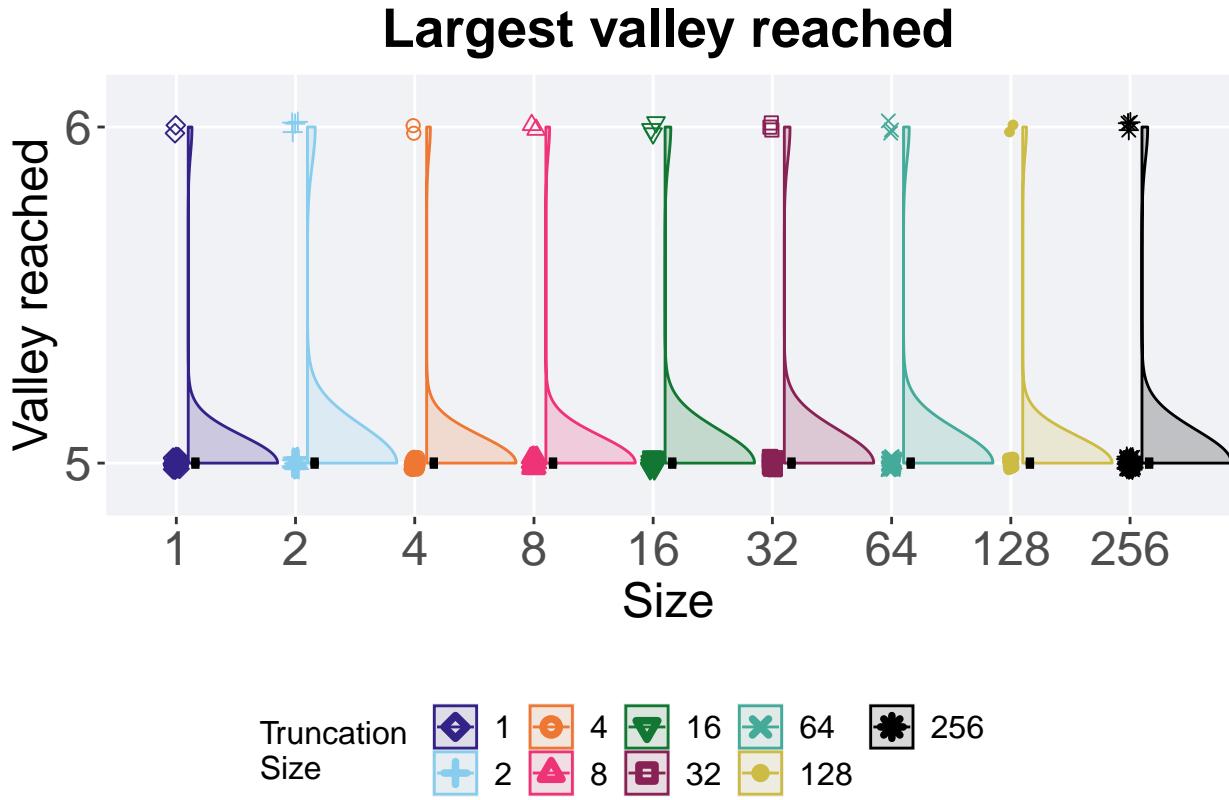
```
## 7 64      50    0    0    0    0    0    0
## 8 128     50    0    0    0    0    0    0
## 9 256     50    0    0    0    0    0    0
```

2.4.5 Largest valley reached throughout

Largest valley reached in a single trait by the best performing solution throughout an entire evolutionary run.

```
plot = filter(best_df, acro == 'con' & var == 'ele_big_peak') %>%
  ggplot(., aes(x = T, y = val, color = T, fill = T, shape = T)) +
  geom_flat_violin(position = position_nudge(x = .1, y = 0), scale = 'width', alpha = 0.2, width = 1.5)
  geom_boxplot(color = 'black', width = .07, outlier.shape = NA, alpha = 0.0, size = 1.0, position = position_nudge(x = -.1, y = 0))
  geom_point(position = position_jitter(width = 0.03, height = 0.02), size = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name="Valley reached",
    limits=c(4.9,6.1),
    breaks = c(5,6)
  ) +
  scale_x_discrete(
    name="Size"
  )+
  scale_shape_manual(values=SHAPE)+
  scale_colour_manual(values = cb_palette, ) +
  scale_fill_manual(values = cb_palette) +
  ggtitle('Largest valley reached')+
  p_theme

plot_grid(
  plot +
    theme(legend.position="none"),
  legend,
  nrow=2,
  rel_heights = c(3,1)
)
```



2.4.5.1 Stats

Summary statistics for the largest valley crossed.

```
valleys = filter(best_df, acro == 'con' & var == 'ele_big_peak')
valleys$T = factor(valleys$T, levels = TR_LIST)
valleys %>%
  group_by(T) %>%
  dplyr::summarise(
    count = n(),
    na_cnt = sum(is.na(val)),
    min = min(val, na.rm = TRUE),
    median = median(val, na.rm = TRUE),
    mean = mean(val, na.rm = TRUE),
    max = max(val, na.rm = TRUE),
    IQR = IQR(val, na.rm = TRUE)
  )
```

```
## # A tibble: 9 x 8
##   T      count na_cnt  min median  mean   max   IQR
##   <fct> <int> <int> <dbl> <dbl> <dbl> <dbl> <dbl>
## 1 1       50     0     5     5  5.04     6     0
## 2 2       50     0     5     5  5.08     6     0
## 3 4       50     0     5     5  5.04     6     0
## 4 8       50     0     5     5  5.04     6     0
## 5 16      50     0     5     5  5.06     6     0
## 6 32      50     0     5     5  5.06     6     0
```

```
## 7 64      50     0     5     5  5.06     6     0
## 8 128     50     0     5     5  5.04     6     0
## 9 256     50     0     5     5  5.06     6     0
```

Kruskal–Wallis test illustrates evidence of **no statistical differences**.

```
kruskal.test(val ~ T, data = valleys)

##
## Kruskal-Wallis rank sum test
##
## data: val by T
## Kruskal-Wallis chi-squared = 1.581, df = 8, p-value = 0.9913
```

2.5 Multi-path exploration results

Here we present the results for best performances and activation gene coverage found by each selection scheme parameter on the multi-path exploration diagnostic with valleys. 50 replicates are conducted for each scheme parameter explored.

2.5.1 Activation gene coverage over time

Activation gene coverage in a population over time. Data points on the graph is the average activation gene coverage across 50 replicates every 2000 generations. Shading comes from the best and worse coverage across 50 replicates.

```
lines = filter(over_time_df, acro == 'mpe') %>%
  group_by(T, gen) %>%
  dplyr::summarise(
    min = min(uni_str_pos),
    mean = mean(uni_str_pos),
    max = max(uni_str_pos)
  )

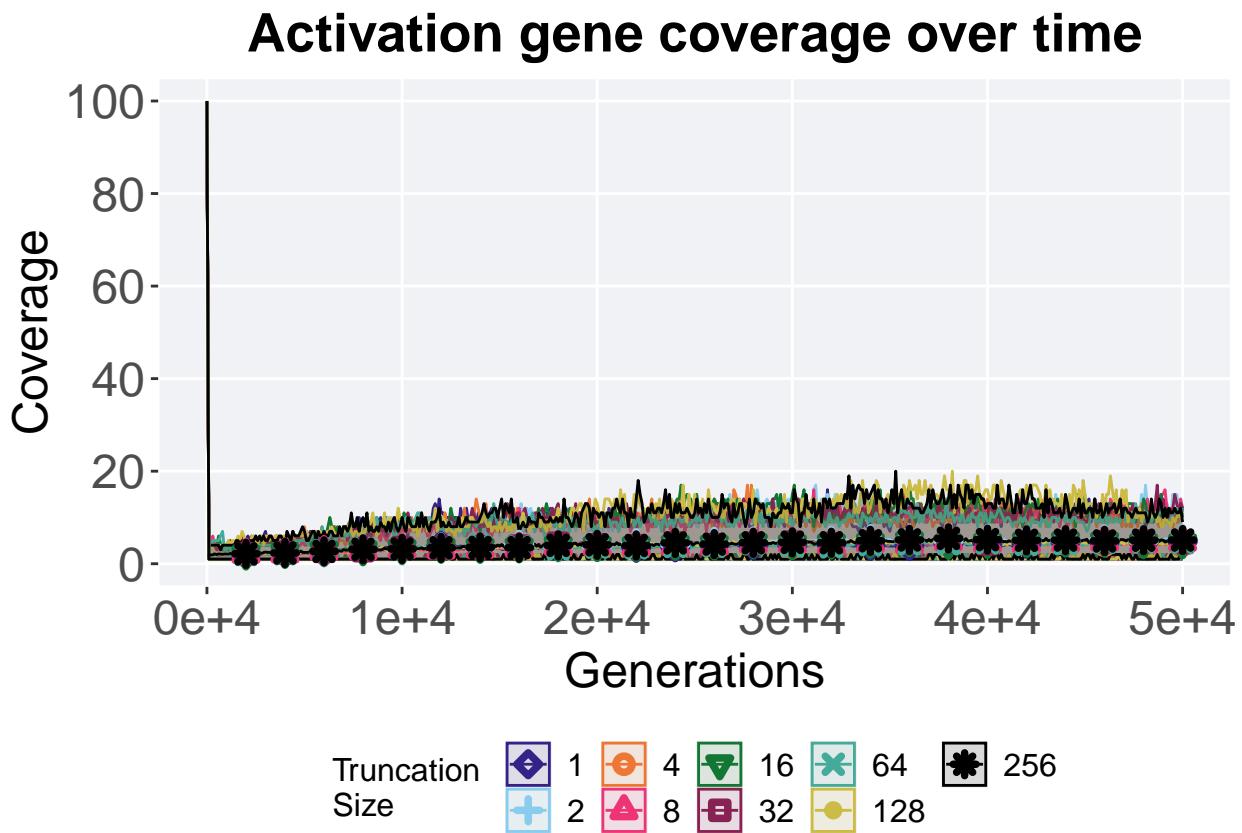
## `summarise()` has grouped output by 'T'. You can override using the `.groups` argument.

ggplot(lines, aes(x=gen, y=mean, group = T, fill = T, color = T, shape = T)) +
  geom_ribbon(aes(ymin = min, ymax = max), alpha = 0.1) +
  geom_line(size = 0.5) +
  geom_point(data = filter(lines, gen %% 2000 == 0 & gen != 0), size = 1.5, stroke = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name="Coverage",
    limits=c(0, 100),
    breaks=seq(0,100, 20),
    labels=c("0", "20", "40", "60", "80", "100")
  ) +
  scale_x_continuous(
    name="Generations",
    limits=c(0, 50000),
    breaks=c(0, 10000, 20000, 30000, 40000, 50000),
    labels=c("0e+4", "1e+4", "2e+4", "3e+4", "4e+4", "5e+4")
  ) +
  scale_shape_manual(values=SHAPE) +
  scale_colour_manual(values = cb_palette) +
```

```

scale_fill_manual(values = cb_palette) +
ggtitle('Activation gene coverage over time')+
p_theme +
guides(
  shape=guide_legend(nrow=2, title.position = "left", title = 'Truncation \nSize'),
  color=guide_legend(nrow=2, title.position = "left", title = 'Truncation \nSize'),
  fill=guide_legend(nrow=2, title.position = "left", title = 'Truncation \nSize')
)

```



2.5.2 Final activation gene coverage

Activation gene coverage found in the final population at 50,000 generations.

```

plot = filter(over_time_df, gen == 50000 & acro == 'mpe') %>%
  ggplot(., aes(x = T, y = uni_str_pos, color = T, fill = T, shape = T)) +
  geom_flat_violin(position = position_nudge(x = .1, y = 0), scale = 'width', alpha = 0.2, width = 1.5) +
  geom_boxplot(color = 'black', width = .07, outlier.shape = NA, alpha = 0.0, size = 1.0, position = position_nudge(x = -.1, y = 0)) +
  geom_point(position = position_jitter(width = 0.03, height = 0.02), size = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name="Coverage",
    limits=c(0, 12.1),
    breaks=seq(0,12,2)
  ) +
  scale_x_discrete(
    name="Truncation size"
  )

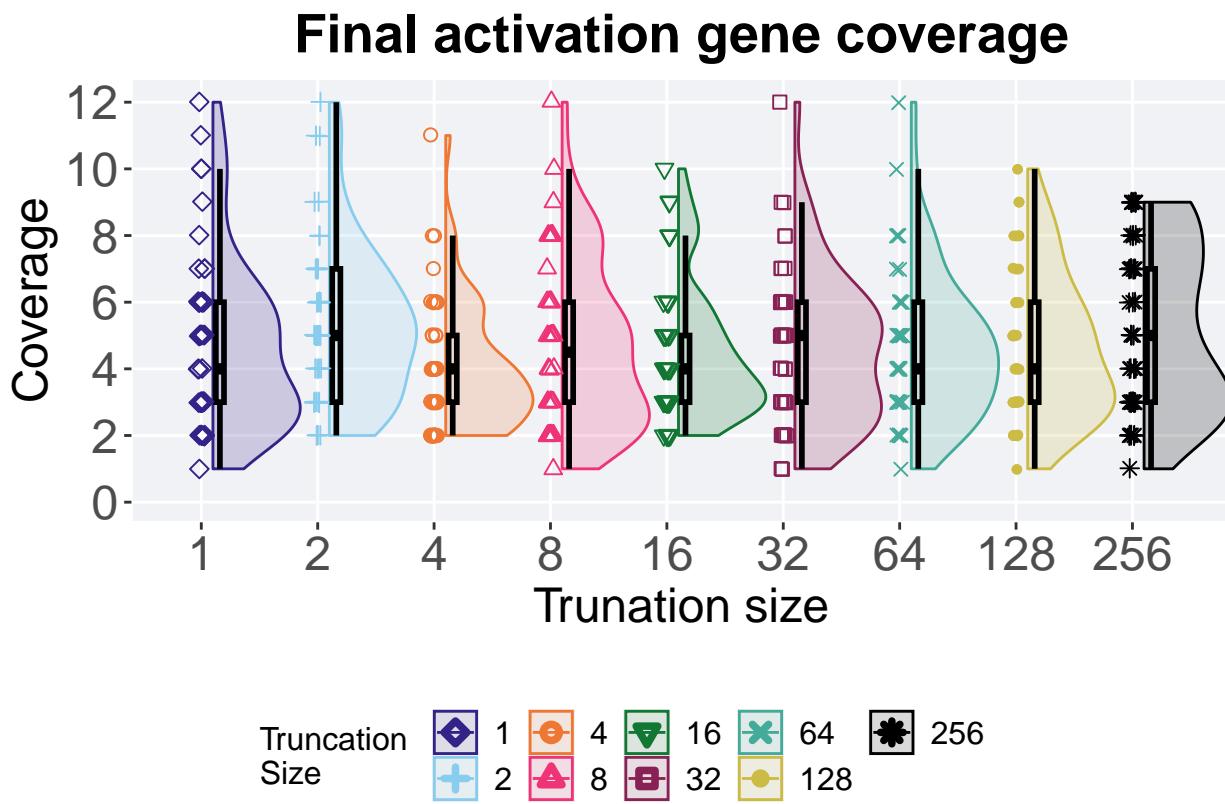
```

```

scale_shape_manual(values=SHAPE) +
scale_colour_manual(values = cb_palette, ) +
scale_fill_manual(values = cb_palette) +
ggtitle('Final activation gene coverage') +
p_theme

plot_grid(
  plot +
  theme(legend.position="none"),
  legend,
  nrow=2,
  rel_heights = c(3,1)
)

```



2.5.2.1 Stats

Summary statistics for the generation a satisfactory solution is found.

```

act_coverage = filter(over_time_df, gen == 50000 & acro == 'mpe')
act_coverage$T = factor(act_coverage$T, levels = TR_LIST)
act_coverage %>%
  group_by(T) %>%
  dplyr::summarise(
    count = n(),
    na_cnt = sum(is.na(uni_str_pos)),
    min = min(uni_str_pos, na.rm = TRUE),
    max = max(uni_str_pos),
    mean = mean(uni_str_pos),
    median = median(uni_str_pos),
    sd = sd(uni_str_pos),
    IQR = IQR(uni_str_pos)
  )

```

```

median = median(uni_str_pos, na.rm = TRUE),
mean = mean(uni_str_pos, na.rm = TRUE),
max = max(uni_str_pos, na.rm = TRUE),
IQR = IQR(uni_str_pos, na.rm = TRUE)
)

## # A tibble: 9 x 8
##   T      count na_cnt  min median  mean  max  IQR
##   <fct> <int> <int> <int> <dbl> <dbl> <int> <dbl>
## 1 1       50     0     1     4    4.68    12     3
## 2 2       50     0     2     5    5.34    12     4
## 3 4       50     0     2     4    4.04    11     2
## 4 8       50     0     1     4    4.5     4.7    12     3
## 5 16      50     0     2     4    4.46    10     2
## 6 32      50     0     1     5    4.58    12     3
## 7 64      50     0     1     4    4.64    12     3
## 8 128     50     0     1     4    4.66    10     3
## 9 256     50     0     1     5    5.12     9     4

```

Kruskal–Wallis test illustrates evidence of **no statistical differences**.

```
kruskal.test(uni_str_pos ~ T, data = act_coverage)
```

```

##
## Kruskal-Wallis rank sum test
##
## data: uni_str_pos by T
## Kruskal-Wallis chi-squared = 10.094, df = 8, p-value = 0.2585

```

2.5.3 Performance over time

Best performance in a population over time. Data points on the graph is the average performance across 50 replicates every 2000 generations. Shading comes from the best and worse performance across 50 replicates.

```

lines = filter(over_time_df, acro == 'mpe') %>%
  group_by(T, gen) %>%
  dplyr::summarise(
    min = min(pop_fit_max) / DIMENSIONALITY,
    mean = mean(pop_fit_max) / DIMENSIONALITY,
    max = max(pop_fit_max) / DIMENSIONALITY
  )

## `summarise()` has grouped output by 'T'. You can override using the `.groups` argument.

over_time_plot = ggplot(lines, aes(x=gen, y=mean, group = T, fill = T, color = T, shape = T)) +
  geom_ribbon(aes(ymin = min, ymax = max), alpha = 0.1) +
  geom_line(size = 0.5) +
  geom_point(data = filter(lines, gen %% 2000 == 0 & gen != 0), size = 1.5, stroke = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name="Average trait score",
    limits=c(0, 10)
  ) +
  scale_x_continuous(
    name="Generations",
    limits=c(0, 50000),

```

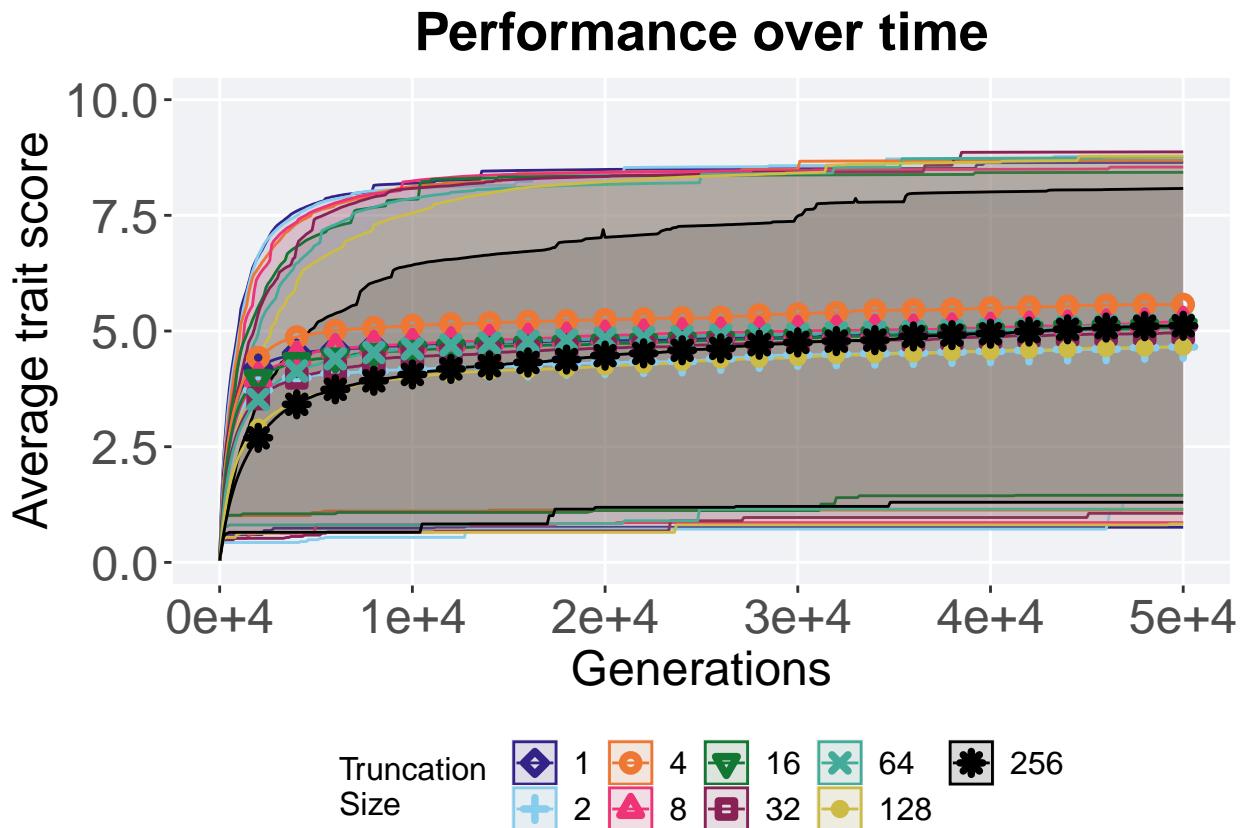
```

breaks=c(0, 10000, 20000, 30000, 40000, 50000),
labels=c("0e+4", "1e+4", "2e+4", "3e+4", "4e+4", "5e+4")

) +
scale_shape_manual(values=SHAPE)+
scale_colour_manual(values = cb_palette) +
scale_fill_manual(values = cb_palette) +
ggtitle('Performance over time')+
p_theme +
guides(
  shape=guide_legend(nrow=2, title.position = "left", title = 'Truncation \nSize'),
  color=guide_legend(nrow=2, title.position = "left", title = 'Truncation \nSize'),
  fill=guide_legend(nrow=2, title.position = "left", title = 'Truncation \nSize')
)

over_time_plot

```



2.5.4 Best performance throughout

Best performance reached throughout 50,000 generations in a population.

```

plot = filter(best_df, acro == 'mpe' & var == 'pop_fit_max') %>%
  ggplot(., aes(x = T, y = val / DIMENSIONALITY, color = T, fill = T, shape = T)) +
  geom_flat_violin(position = position_nudge(x = .1, y = 0), scale = 'width', alpha = 0.2, width = 1.5)
  geom_boxplot(color = 'black', width = .07, outlier.shape = NA, alpha = 0.0, size = 1.0, position = pos)
  geom_point(position = position_jitter(width = 0.03, height = 0.02), size = 2.0, alpha = 1.0) +

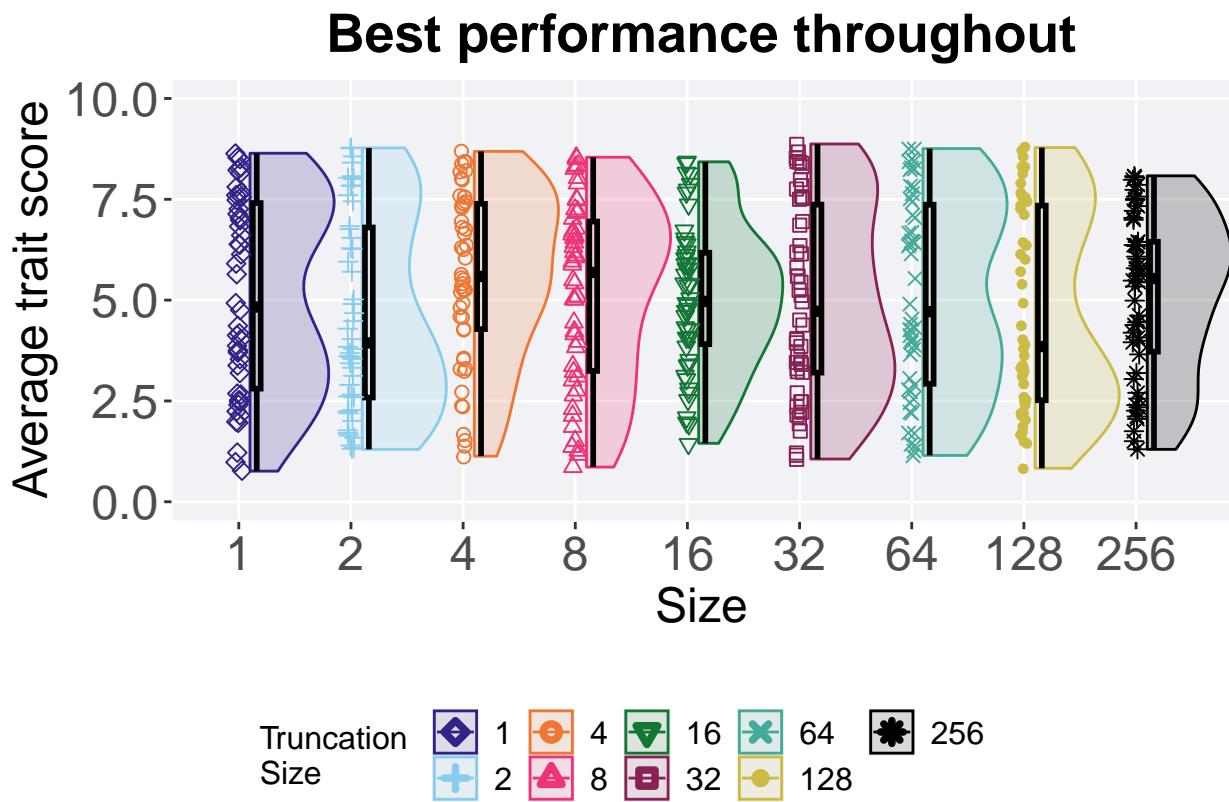
```

```

scale_y_continuous(
  name="Average trait score",
  limits=c(0, 10)
) +
scale_x_discrete(
  name="Size"
) +
scale_shape_manual(values=SHAPE) +
scale_colour_manual(values = cb_palette, ) +
scale_fill_manual(values = cb_palette) +
ggtitle('Best performance throughout') +
p_theme

plot_grid(
  plot +
  theme(legend.position="none"),
  legend,
  nrow=2,
  rel_heights = c(3,1)
)
)

```



2.5.4.1 Stats

Summary statistics for the best performance.

```

performance = filter(best_df, acro == 'mpe' & var == 'pop_fit_max')
performance$T = factor(performance$T, levels = TR_LIST)
performance %>%
  group_by(T) %>%
  dplyr::summarise(
    count = n(),
    na_cnt = sum(is.na(val)),
    min = min(val / DIMENSIONALITY, na.rm = TRUE),
    median = median(val / DIMENSIONALITY, na.rm = TRUE),
    mean = mean(val / DIMENSIONALITY, na.rm = TRUE),
    max = max(val / DIMENSIONALITY, na.rm = TRUE),
    IQR = IQR(val / DIMENSIONALITY, na.rm = TRUE)
  )

## # A tibble: 9 x 8
##   T     count na_cnt   min   median   mean   max   IQR
##   <fct> <int> <int> <dbl> <dbl> <dbl> <dbl> <dbl>
## 1 1      50     0  0.760  4.83  5.09  8.64  4.60
## 2 2      50     0  1.30   3.94  4.66  8.77  4.22
## 3 4      50     0  1.13   5.59  5.57  8.69  3.11
## 4 8      50     0  0.860  5.69  5.20  8.54  3.70
## 5 16     50     0  1.45   4.97  5.08  8.43  2.27
## 6 32     50     0  1.06   4.72  4.96  8.87  4.17
## 7 64     50     0  1.15   4.71  5.13  8.76  4.44
## 8 128    50     0  0.830  3.85  4.66  8.78  4.83
## 9 256    50     0  1.3    5.54  5.12  8.08  2.72

```

Kruskal–Wallis test illustrates evidence of **no statistical differences**.

```
kruskal.test(val ~ T, data = performance)
```

```

##
##  Kruskal-Wallis rank sum test
##
## data: val by T
## Kruskal-Wallis chi-squared = 5.6719, df = 8, p-value = 0.6839

```

2.5.5 Largest valley reached throughout

Largest valley reached in a single trait by the best performing solution throughout an entire evolutionary run.

```

plot = filter(best_df, acro == 'mpe' & var == 'ele_big_peak') %>%
  ggplot(., aes(x = T, y = val, color = T, fill = T, shape = T)) +
  geom_flat_violin(position = position_nudge(x = .1, y = 0), scale = 'width', alpha = 0.2, width = 1.5)
  geom_boxplot(color = 'black', width = .07, outlier.shape = NA, alpha = 0.0, size = 1.0, position = position_nudge(x = -.1, y = 0))
  geom_point(position = position_jitter(width = 0.03, height = 0.02), size = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name="Valley reached",
    limits=c(3.9,6.1),
    breaks = c(4,5,6)
  ) +
  scale_x_discrete(
    name="Size"
  )+
  scale_shape_manual(values=SHAPE)+
  scale_colour_manual(values = cb_palette, )

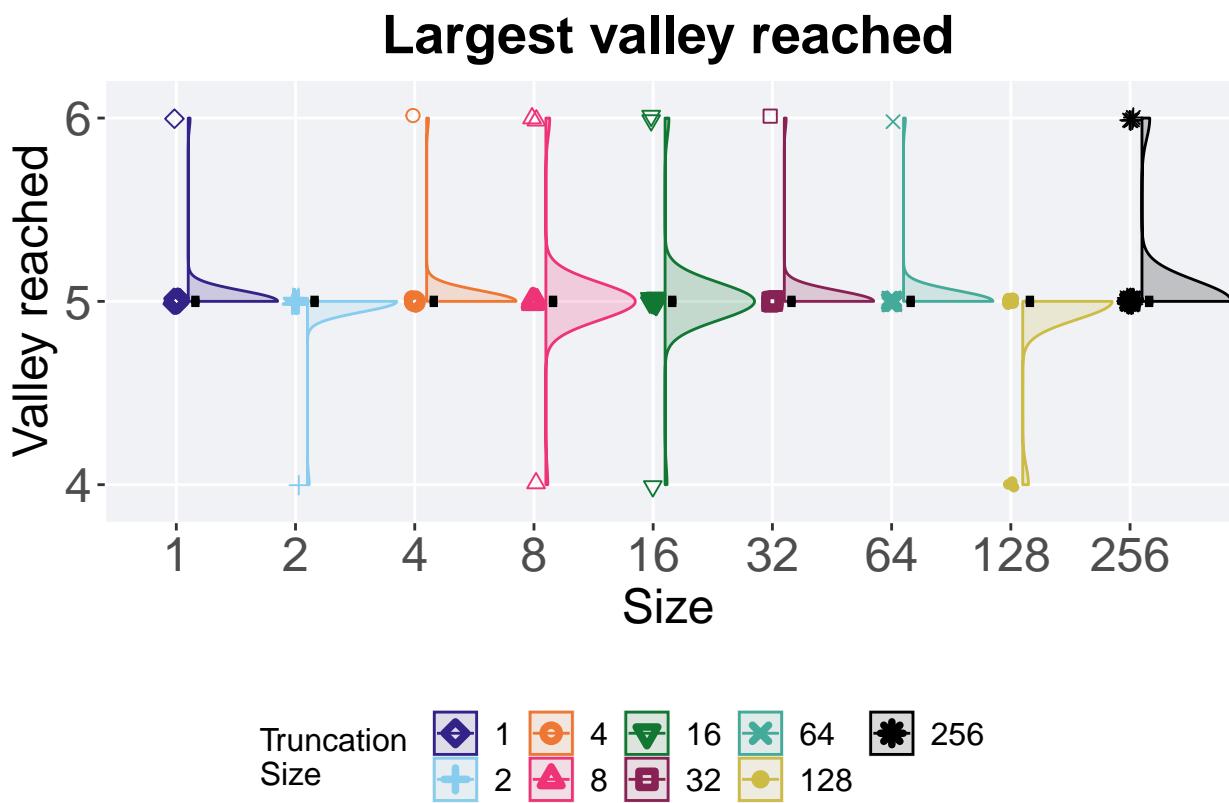
```

```

scale_fill_manual(values = cb_palette) +
ggtitle('Largest valley reached')+
p_theme

plot_grid(
  plot +
    theme(legend.position="none"),
  legend,
  nrow=2,
  rel_heights = c(3,1)
)

```



2.5.5.1 Stats

Summary statistics for the largest valley crossed.

```

valleys = filter(best_df, acro == 'mpe' & var == 'ele_big_peak')
valleys$T = factor(valleys$T, levels = TR_LIST)
valleys %>%
  group_by(T) %>%
  dplyr::summarise(
    count = n(),
    na_cnt = sum(is.na(val)),
    min = min(val, na.rm = TRUE),
    median = median(val, na.rm = TRUE),
    mean = mean(val, na.rm = TRUE),
    sd = sd(val, na.rm = TRUE)
  )

```

```

    max = max(val, na.rm = TRUE),
    IQR = IQR(val, na.rm = TRUE)
)

## # A tibble: 9 x 8
##   T      count na_cnt   min median   mean   max   IQR
##   <fct> <int> <int> <dbl> <dbl> <dbl> <dbl> <dbl>
## 1 1       50     0     5     5  5.02     6     0
## 2 2       50     0     4     5  4.98     5     0
## 3 4       50     0     5     5  5.02     6     0
## 4 8       50     0     4     5  5.02     6     0
## 5 16      50     0     4     5  5.02     6     0
## 6 32      50     0     5     5  5.02     6     0
## 7 64      50     0     5     5  5.02     6     0
## 8 128     50     0     4     5  4.94     5     0
## 9 256     50     0     5     5  5.08     6     0

```

Kruskal–Wallis test illustrates evidence of **no statistical differences**.

```

kruskal.test(val ~ T, data = valleys)

##
##  Kruskal-Wallis rank sum test
##
## data: val by T
## Kruskal-Wallis chi-squared = 13.997, df = 8, p-value = 0.08184

```

Chapter 3

Tournament selection

Results for the tournament selection parameter sweep on the diagnostics with valleys.

3.1 Data setup

```
over_time_df <- read.csv(paste(DATA_DIR, 'OVER-TIME-MVC/tor.csv', sep = "", collapse = NULL), header = TRUE)
over_time_df$T <- factor(over_time_df$T, levels = TS_LIST)

best_df <- read.csv(paste(DATA_DIR, 'BEST-MVC/tor.csv', sep = "", collapse = NULL), header = TRUE, stringsAsFactors = FALSE)
best_df$T <- factor(best_df$T, levels = TS_LIST)
```

3.2 Exploitation rate results

Here we present the results for **best performances** found by each selection scheme parameter on the exploitation rate diagnostic with valleys. 50 replicates are conducted for each scheme explored.

3.2.1 Performance over time

Best performance in a population over time. Data points on the graph is the average performance across 50 replicates every 2000 generations. Shading comes from the best and worse performance across 50 replicates.

```
lines = filter(over_time_df, acro == 'exp') %>%
  group_by(T, gen) %>%
  dplyr::summarise(
    min = min(pop_fit_max) / DIMENSIONALITY,
    mean = mean(pop_fit_max) / DIMENSIONALITY,
    max = max(pop_fit_max) / DIMENSIONALITY
  )

## `summarise()` has grouped output by 'T'. You can override using the `.`groups` argument.

over_time_plot = ggplot(lines, aes(x=gen, y=mean, group = T, fill = T, color = T, shape = T)) +
  geom_ribbon(aes(ymin = min, ymax = max), alpha = 0.1) +
  geom_line(size = 0.5) +
  geom_point(data = filter(lines, gen %% 2000 == 0 & gen != 0), size = 1.5, stroke = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name="Average trait score",
```

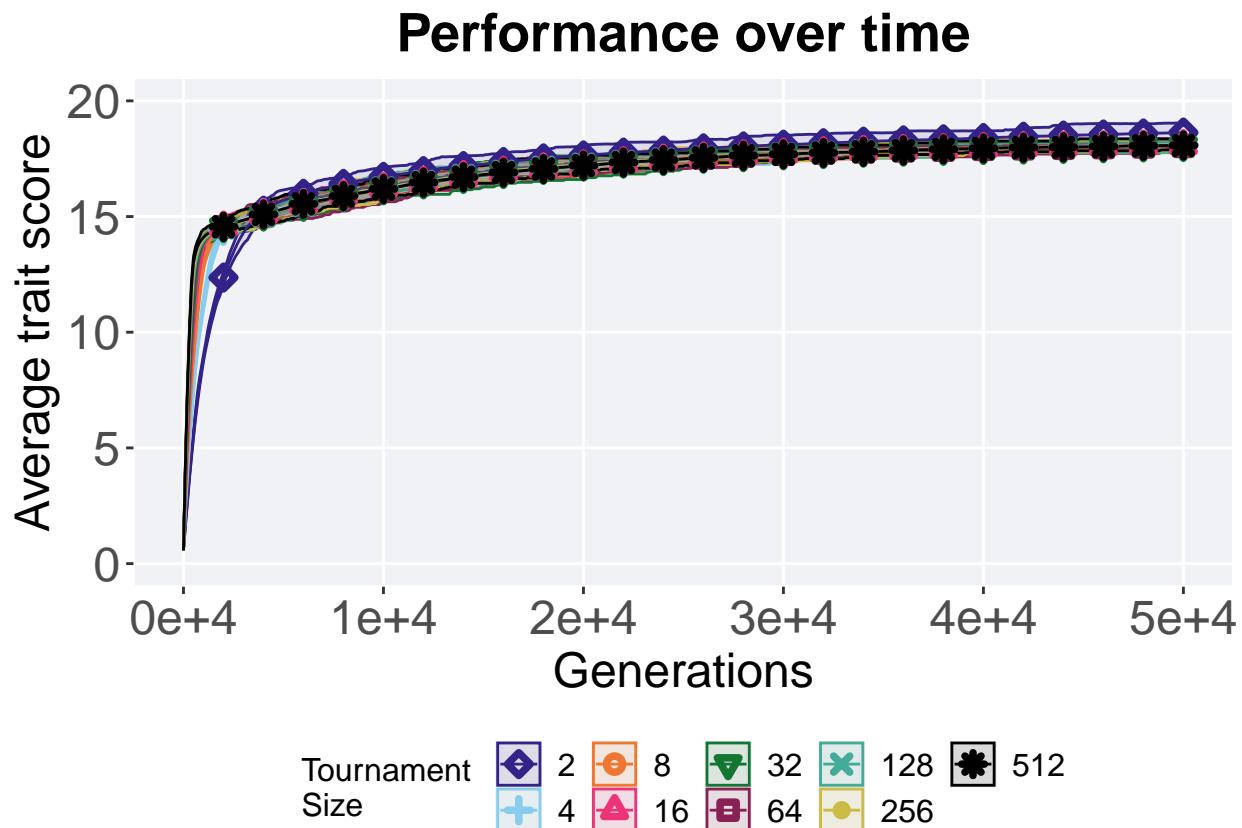
```

    limits = c(0,20)
) +
scale_x_continuous(
  name="Generations",
  limits=c(0, 50000),
  breaks=c(0, 10000, 20000, 30000, 40000, 50000),
  labels=c("0e+4", "1e+4", "2e+4", "3e+4", "4e+4", "5e+4")

) +
scale_shape_manual(values=SHAPE) +
scale_colour_manual(values = cb_palette) +
scale_fill_manual(values = cb_palette) +
ggtitle('Performance over time') +
p_theme +
guides(
  shape=guide_legend(nrow=2, title.position = "left", title = 'Tournament \nSize'),
  color=guide_legend(nrow=2, title.position = "left", title = 'Tournament \nSize'),
  fill=guide_legend(nrow=2, title.position = "left", title = 'Tournament \nSize')
)

over_time_plot

```



3.2.2 Best performance throughout

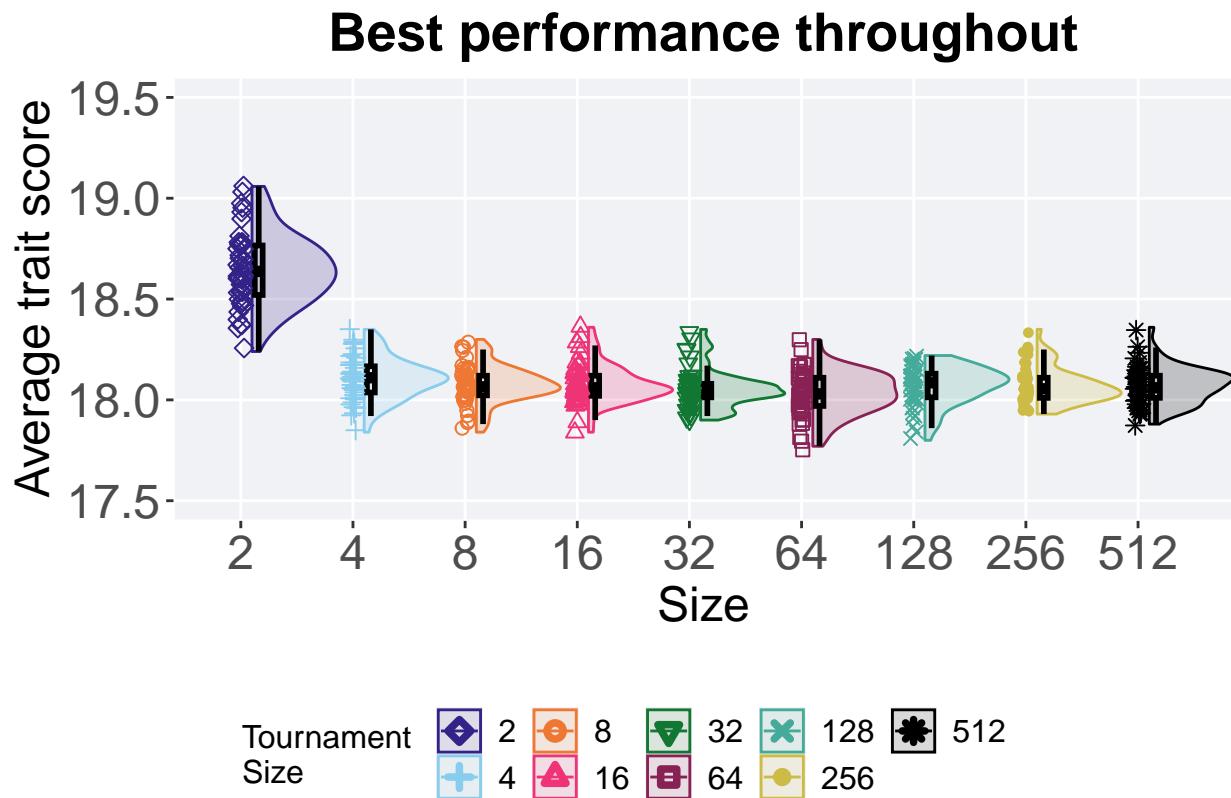
Best performance reached throughout 50,000 generations in a population.

```

plot = filter(best_df, acro == 'exp' & var == 'pop_fit_max') %>%
  ggplot(., aes(x = T, y = val / DIMENSIONALITY, color = T, fill = T, shape = T)) +
  geom_flat_violin(position = position_nudge(x = .1, y = 0), scale = 'width', alpha = 0.2, width = 1.5) +
  geom_boxplot(color = 'black', width = .07, outlier.shape = NA, alpha = 0.0, size = 1.0, position = position_nudge(x = -.1, y = 0), width = 1.5) +
  geom_point(position = position_jitter(width = 0.03, height = 0.02), size = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name = "Average trait score",
    limits = c(17.5, 19.5)
  ) +
  scale_x_discrete(
    name = "Size"
  ) +
  scale_shape_manual(values = SHAPE) +
  scale_color_manual(values = cb_palette, ) +
  scale_fill_manual(values = cb_palette) +
  ggtitle('Best performance throughout') +
  p_theme

plot_grid(
  plot +
    theme(legend.position = "none"),
  legend,
  nrow = 2,
  rel_heights = c(3, 1)
)

```



3.2.2.1 Stats

Summary statistics for the best performance.

```
performance = filter(best_df, acro == 'exp' & var == 'pop_fit_max')
performance$T = factor(performance$T, levels = TS_LIST)
performance %>%
  group_by(T) %>%
  dplyr::summarise(
    count = n(),
    na_cnt = sum(is.na(val)),
    min = min(val / DIMENSIONALITY, na.rm = TRUE),
    median = median(val / DIMENSIONALITY, na.rm = TRUE),
    mean = mean(val / DIMENSIONALITY, na.rm = TRUE),
    max = max(val / DIMENSIONALITY, na.rm = TRUE),
    IQR = IQR(val / DIMENSIONALITY, na.rm = TRUE)
  )

## # A tibble: 9 x 8
##   T      count  na_cnt   min  median   mean   max     IQR
##   <fct> <int> <int> <dbl> <dbl> <dbl> <dbl> <dbl>
## 1 2        50     0  18.2   18.6   18.6  19.1  0.246
## 2 4        50     0  17.8   18.1   18.1  18.3  0.130
## 3 8        50     0  17.8   18.1   18.1  18.3  0.0975
## 4 16       50     0  17.8   18.1   18.1  18.4  0.100
## 5 32       50     0  17.9   18.0   18.1  18.3  0.0675
## 6 64       50     0  17.8   18.0   18.0  18.3  0.140
## 7 128      50     0  17.8   18.1   18.1  18.2  0.117
## 8 256      50     0  17.9   18.1   18.1  18.3  0.0999
## 9 512      50     0  17.9   18.1   18.1  18.4  0.110
```

Kruskal–Wallis test illustrates evidence of statistical differences.

```
kruskal.test(val ~ T, data = performance)
```

```
##
## Kruskal-Wallis rank sum test
##
## data: val by T
## Kruskal-Wallis chi-squared = 144.89, df = 8, p-value < 2.2e-16
```

Results for post-hoc Wilcoxon rank-sum test with a Bonferroni correction.

```
pairwise.wilcox.test(x = performance$val, g = performance$T, p.adjust.method = "bonferroni",
                      paired = FALSE, conf.int = FALSE, alternative = 't')

##
## Pairwise comparisons using Wilcoxon rank sum test with continuity correction
##
## data: performance$val and performance$T
##
##      2      4      8     16     32     64    128    256
## 4 3.9e-16 -      -      -      -      -      -      -
## 8 3.2e-16 1.000 -      -      -      -      -      -
## 16 3.6e-16 1.000 1.000 -      -      -      -      -
## 32 3.4e-16 0.188 1.000 1.000 -      -      -      -
## 64 2.8e-16 0.058 1.000 1.000 1.000 -      -      -
## 128 2.5e-16 1.000 1.000 1.000 1.000 1.000 -      -
```

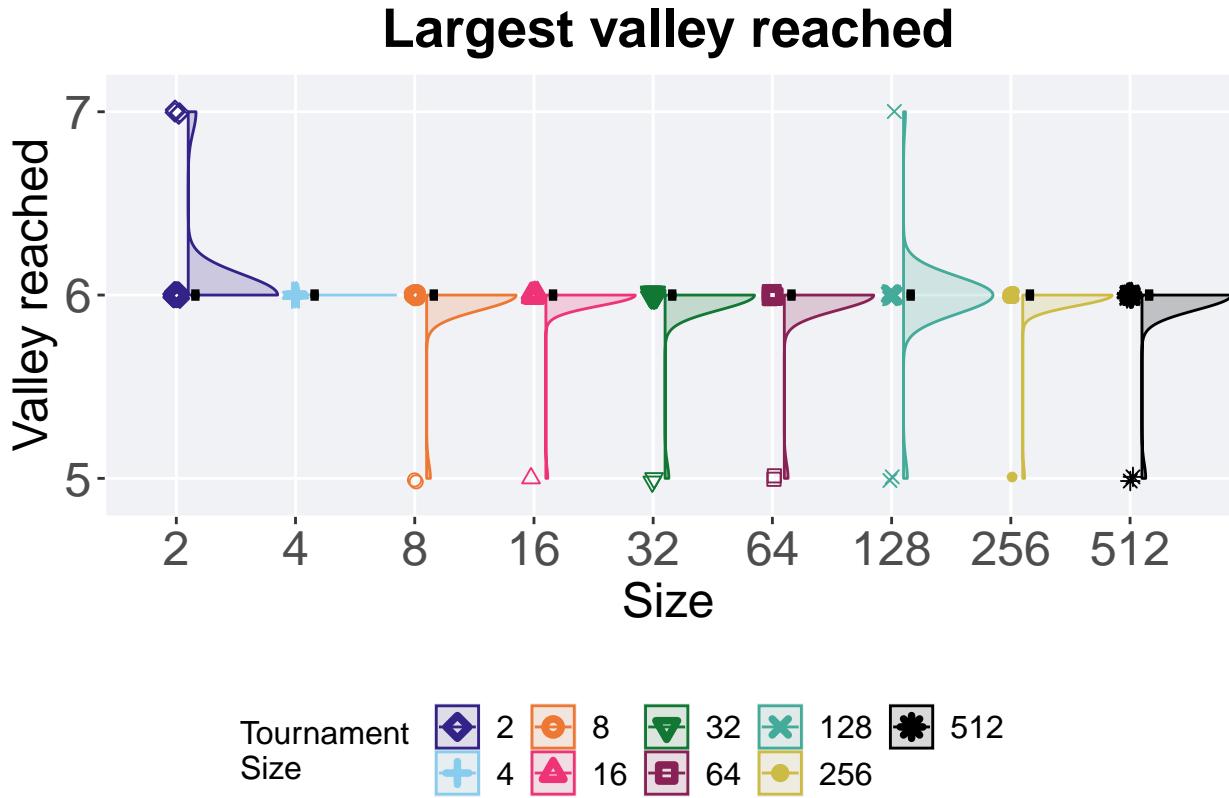
```
## 256 3.2e-16 1.000 1.000 1.000 1.000 1.000 1.000 -
## 512 3.2e-16 1.000 1.000 1.000 1.000 1.000 1.000
##
## P value adjustment method: bonferroni
```

3.2.3 Largest valley reached throughout

Largest valley reached in a single trait by the best performing solution throughout an entire evolutionary run.

```
plot = filter(best_df, acro == 'exp' & var == 'ele_big_peak') %>%
  ggplot(., aes(x = T, y = val, color = T, fill = T, shape = T)) +
  geom_flat_violin(position = position_nudge(x = .1, y = 0), scale = 'width', alpha = 0.2, width = 1.5)
  geom_boxplot(color = 'black', width = .07, outlier.shape = NA, alpha = 0.0, size = 1.0, position = position_nudge(x = -.1, y = 0))
  geom_point(position = position_jitter(width = 0.03, height = 0.02), size = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name="Valley reached",
    limits=c(4.9,7.1),
    breaks = c(5,6,7)
  ) +
  scale_x_discrete(
    name="Size"
  )+
  scale_shape_manual(values=SHAPE)+
  scale_colour_manual(values = cb_palette, ) +
  scale_fill_manual(values = cb_palette) +
  ggtitle('Largest valley reached')+
  p_theme

plot_grid(
  plot +
    theme(legend.position="none"),
  legend,
  nrow=2,
  rel_heights = c(3,1)
)
```



3.2.3.1 Stats

Summary statistics for the largest valley crossed.

```
valleys = filter(best_df, acro == 'exp' & var == 'ele_big_peak')
valleys$T = factor(valleys$T, levels = TS_LIST)
valleys %>%
  group_by(T) %>%
  dplyr::summarise(
    count = n(),
    na_cnt = sum(is.na(val)),
    min = min(val, na.rm = TRUE),
    median = median(val, na.rm = TRUE),
    mean = mean(val, na.rm = TRUE),
    max = max(val, na.rm = TRUE),
    IQR = IQR(val, na.rm = TRUE)
  )
```

```
## # A tibble: 9 x 8
##   T      count na_cnt  min median  mean   max   IQR
##   <fct> <int> <int> <dbl> <dbl> <dbl> <dbl> <dbl>
## 1 2       50     0     6     6  6.08     7     0
## 2 4       50     0     6     6     6     6     0
## 3 8       50     0     5     6  5.96     6     0
## 4 16      50     0     5     6  5.98     6     0
## 5 32      50     0     5     6  5.96     6     0
## 6 64      50     0     5     6  5.96     6     0
```

```
## 7 128      50      0      5      6  5.98      7      0
## 8 256      50      0      5      6  5.98      6      0
## 9 512      50      0      5      6  5.96      6      0
```

Kruskal–Wallis test illustrates evidence of statistical differences.

```
kruskal.test(val ~ T, data = valleys)

##
##  Kruskal-Wallis rank sum test
##
## data: val by T
## Kruskal-Wallis chi-squared = 15.545, df = 8, p-value = 0.04938
```

Results for post-hoc Wilcoxon rank-sum test with a Bonferroni correction.

```
pairwise.wilcox.test(x = valleys$val, g = valleys$T, p.adjust.method = "bonferroni",
                      paired = FALSE, conf.int = FALSE, alternative = 't')
```

```
##
##  Pairwise comparisons using Wilcoxon rank sum test with continuity correction
##
## data: valleys$val and valleys$T
##
##    2     4     8    16    32    64   128   256
## 4  1.00  -    -    -    -    -    -    -
## 8  0.53  1.00  -    -    -    -    -    -
## 16 0.91  1.00  1.00  -    -    -    -    -
## 32 0.53  1.00  1.00  1.00  -    -    -    -
## 64  0.53  1.00  1.00  1.00  1.00  -    -    -
## 128 1.00  1.00  1.00  1.00  1.00  1.00  -    -
## 256 0.91  1.00  1.00  1.00  1.00  1.00  1.00  -
## 512 0.53  1.00  1.00  1.00  1.00  1.00  1.00  1.00
##
## P value adjustment method: bonferroni
```

3.3 Ordered exploitation results

Here we present the results for **best performances** found by each selection scheme parameter on the ordered exploitation diagnostic with valleys. 50 replicates are conducted for each scheme explored.

3.3.1 Performance over time

Best performance in a population over time. Data points on the graph is the average performance across 50 replicates every 2000 generations. Shading comes from the best and worse performance across 50 replicates.

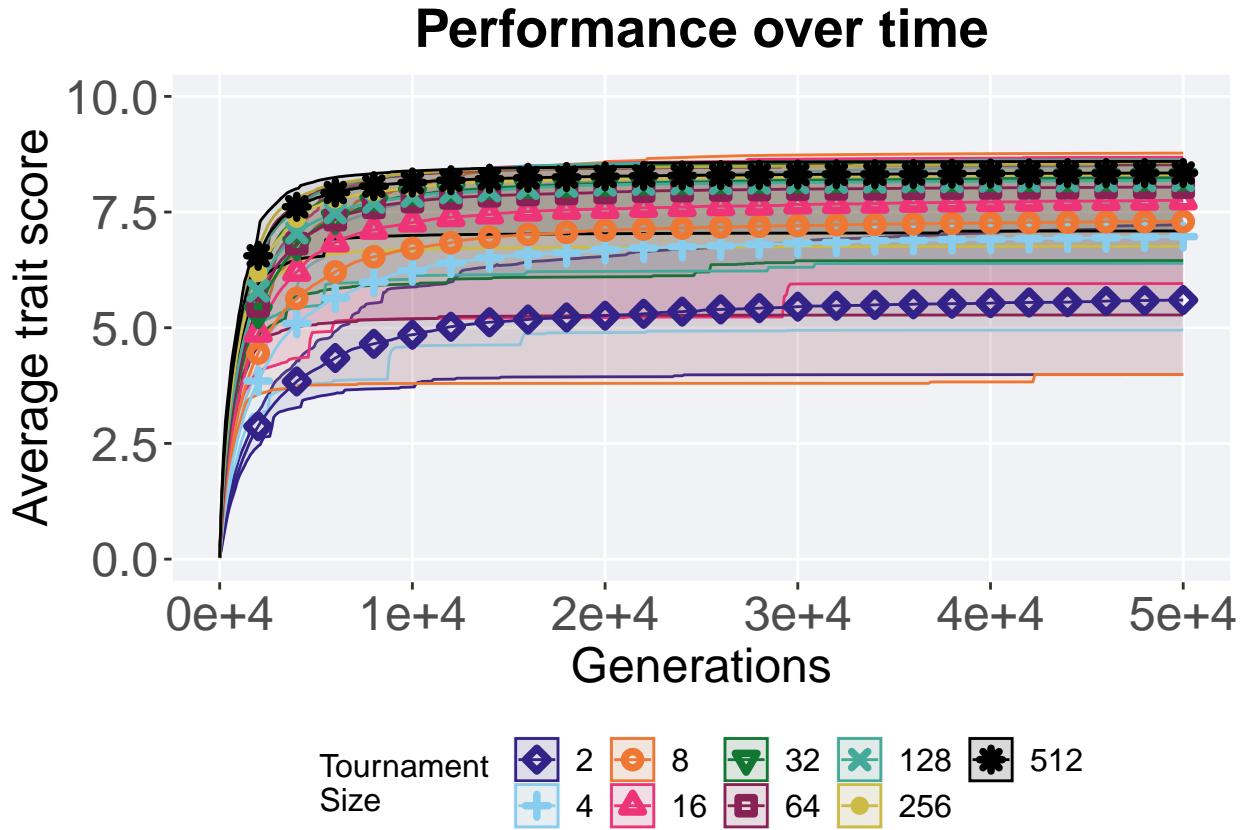
```
lines = filter(over_time_df, acro == 'ord') %>%
  group_by(T, gen) %>%
  dplyr::summarise(
    min = min(pop_fit_max) / DIMENSIONALITY,
    mean = mean(pop_fit_max) / DIMENSIONALITY,
    max = max(pop_fit_max) / DIMENSIONALITY
  )

## `summarise()` has grouped output by 'T'. You can override using the `groups` argument.
```

```
over_time_plot = ggplot(lines, aes(x=gen, y=mean, group = T, fill = T, color = T, shape = T)) +
  geom_ribbon(aes(ymin = min, ymax = max), alpha = 0.1) +
  geom_line(size = 0.5) +
  geom_point(data = filter(lines, gen %% 2000 == 0 & gen != 0), size = 1.5, stroke = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name="Average trait score",
    limits=c(0, 10)
  ) +
  scale_x_continuous(
    name="Generations",
    limits=c(0, 50000),
    breaks=c(0, 10000, 20000, 30000, 40000, 50000),
    labels=c("0e+4", "1e+4", "2e+4", "3e+4", "4e+4", "5e+4")

  ) +
  scale_shape_manual(values=SHAPE) +
  scale_colour_manual(values = cb_palette) +
  scale_fill_manual(values = cb_palette) +
  ggtitle('Performance over time') +
  p_theme +
  guides(
    shape=guide_legend(nrow=2, title.position = "left", title = 'Tournament \nSize'),
    color=guide_legend(nrow=2, title.position = "left", title = 'Tournament \nSize'),
    fill=guide_legend(nrow=2, title.position = "left", title = 'Tournament \nSize')
  )

over_time_plot
```



3.3.2 Best performance throughout

Best performance reached throughout 50,000 generations in a population.

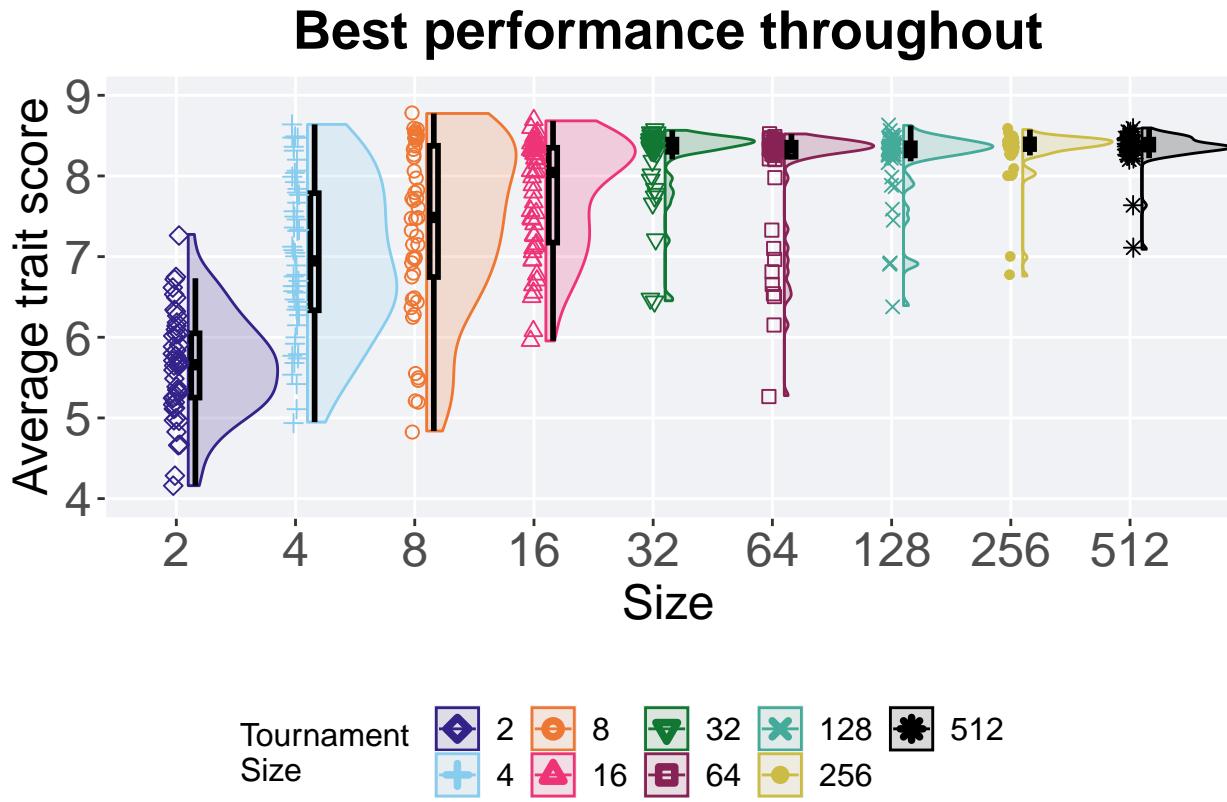
```
plot = filter(best_df, acro == 'ord' & var == 'pop_fit_max') %>%
  ggplot(., aes(x = T, y = val / DIMENSIONALITY, color = T, fill = T, shape = T)) +
  geom_flat_violin(position = position_nudge(x = .1, y = 0), scale = 'width', alpha = 0.2, width = 1.5) +
  geom_boxplot(color = 'black', width = .07, outlier.shape = NA, alpha = 0.0, size = 1.0, position = position_nudge(x = -.1, y = 0)) +
  geom_point(position = position_jitter(width = 0.03, height = 0.02), size = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name="Average trait score",
    limits=c(4, 9)
  ) +
  scale_x_discrete(
    name="Size"
  ) +
  scale_shape_manual(values=SHAPE) +
  scale_colour_manual(values = cb_palette, ) +
  scale_fill_manual(values = cb_palette) +
  ggtitle('Best performance throughout') +
  p_theme

plot_grid(
  plot +
    theme(legend.position="none"),
  legend,
```

```

nrow=2,
rel_heights = c(3,1)
)

```



3.3.2.1 Stats

Summary statistics for the best performance.

```

performance = filter(best_df, acro == 'ord' & var == 'pop_fit_max')
performance$T = factor(performance$T, levels = TS_LIST)
performance %>%
  group_by(T) %>%
  dplyr::summarise(
    count = n(),
    na_cnt = sum(is.na(val)),
    min = min(val / DIMENSIONALITY, na.rm = TRUE),
    median = median(val / DIMENSIONALITY, na.rm = TRUE),
    mean = mean(val / DIMENSIONALITY, na.rm = TRUE),
    max = max(val / DIMENSIONALITY, na.rm = TRUE),
    IQR = IQR(val / DIMENSIONALITY, na.rm = TRUE)
  )

## # A tibble: 9 x 8
##   T      count na_cnt   min  median   mean   max    IQR
##   <fct> <int>   <int> <dbl>   <dbl>   <dbl> <dbl> <dbl>
## 1 2       50      0  4.00    5.65    5.61  7.28  0.840
## 2 4       50      0  4.95    6.95    6.97  8.64  1.45

```

```
## 3 8      50     0 3.99  7.48  7.29  8.77 1.82
## 4 16     50     0 5.95  8.04  7.75  8.68 1.18
## 5 32     50     0 6.45  8.39  8.24  8.57 0.151
## 6 64     50     0 5.28  8.35  8.04  8.52 0.175
## 7 128    50     0 6.39  8.35  8.18  8.63 0.150
## 8 256    50     0 6.76  8.39  8.31  8.58 0.120
## 9 512    50     0 7.09  8.37  8.34  8.60 0.124
```

Kruskal–Wallis test illustrates evidence of statistical differences.

```
kruskal.test(val ~ T, data = performance)
```

```
##
## Kruskal-Wallis rank sum test
##
## data: val by T
## Kruskal-Wallis chi-squared = 198.69, df = 8, p-value < 2.2e-16
```

Results for post-hoc Wilcoxon rank-sum test with a Bonferroni correction.

```
pairwise.wilcox.test(x = performance$val, g = performance$T, p.adjust.method = "bonferroni",
                      paired = FALSE, conf.int = FALSE, alternative = 't')
```

```
##
## Pairwise comparisons using Wilcoxon rank sum test with continuity correction
##
## data: performance$val and performance$T
##
##   2     4     8     16    32    64   128   256
## 4 7.6e-09 -
## 8 2.7e-09 1.00000 -
## 16 5.1e-15 0.00201 1.00000 -
## 32 5.5e-16 4.4e-09 0.00320 0.00108 -
## 64 9.7e-15 2.2e-06 0.22098 0.34689 0.99449 -
## 128 4.4e-16 2.1e-08 0.01684 0.05029 1.00000 1.00000 -
## 256 2.9e-16 2.9e-10 0.00114 0.00011 1.00000 0.32009 1.00000 -
## 512 2.7e-16 9.6e-11 0.00037 8.8e-05 1.00000 1.00000 1.00000 1.00000
##
## P value adjustment method: bonferroni
```

3.3.3 Largest valley reached throughout

Largest valley reached in a single trait by the best performing solution throughout an entire evolutionary run.

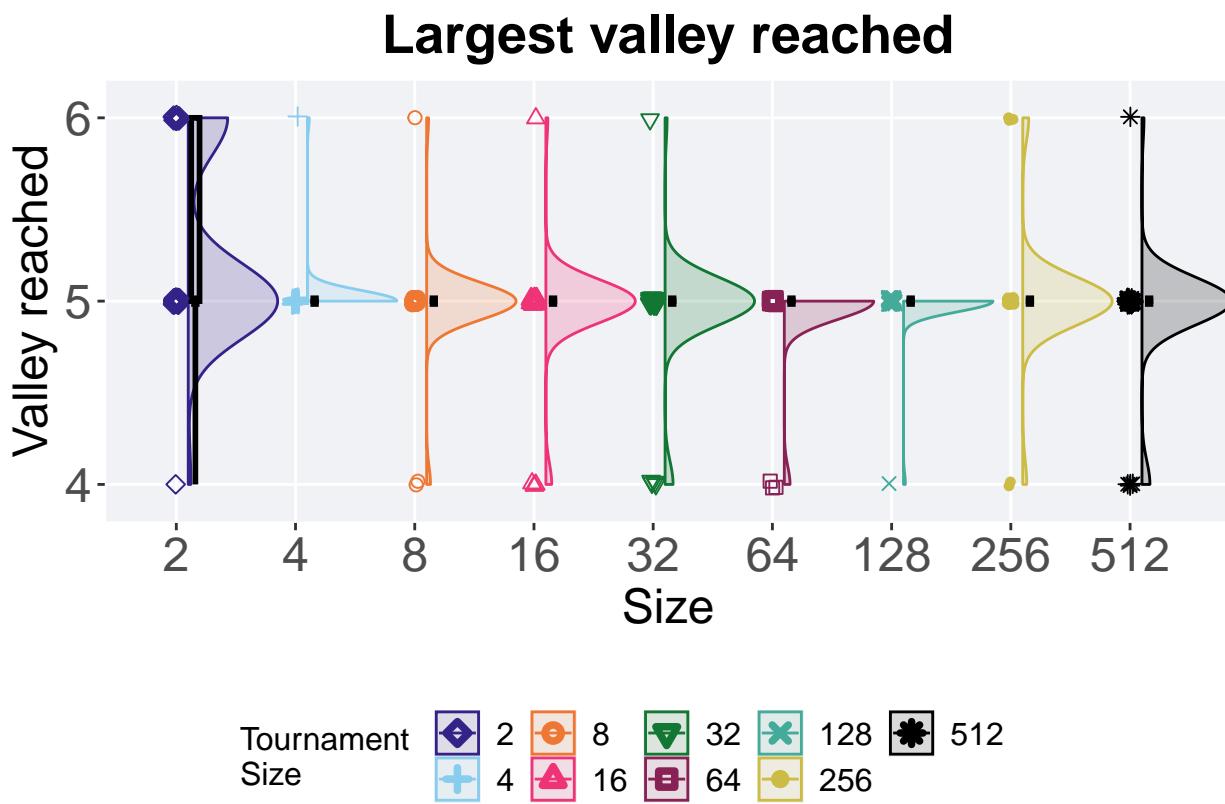
```
plot = filter(best_df, acro == 'ord' & var == 'ele_big_peak') %>%
  ggplot(., aes(x = T, y = val, color = T, fill = T, shape = T)) +
  geom_flat_violin(position = position_nudge(x = .1, y = 0), scale = 'width', alpha = 0.2, width = 1.5) +
  geom_boxplot(color = 'black', width = .07, outlier.shape = NA, alpha = 0.0, size = 1.0, position = pos) +
  geom_point(position = position_jitter(width = 0.03, height = 0.02), size = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name="Valley reached",
    limits=c(3.9,6.1),
    breaks = c(4,5,6)
  ) +
  scale_x_discrete(
    name="Size"
  )+
```

```

scale_shape_manual(values=SHAPE) +
scale_colour_manual(values = cb_palette, ) +
scale_fill_manual(values = cb_palette) +
ggtitle('Largest valley reached') +
p_theme

plot_grid(
  plot +
  theme(legend.position="none"),
  legend,
  nrow=2,
  rel_heights = c(3,1)
)

```



3.3.3.1 Stats

Summary statistics for the largest valley crossed.

```

valleys = filter(best_df, acro == 'ord' & var == 'ele_big_peak')
valleys$T = factor(valleys$T, levels = TS_LIST)
valleys %>%
  group_by(T) %>%
  dplyr::summarise(
    count = n(),
    na_cnt = sum(is.na(val)),
    min = min(val, na.rm = TRUE),
    max = max(val, na.rm = TRUE),
    mean = mean(val, na.rm = TRUE),
    median = median(val, na.rm = TRUE),
    sd = sd(val, na.rm = TRUE),
    iqr = IQR(val, na.rm = TRUE),
    range = range(val, na.rm = TRUE)
  )

```

```

median = median(val, na.rm = TRUE),
mean = mean(val, na.rm = TRUE),
max = max(val, na.rm = TRUE),
IQR = IQR(val, na.rm = TRUE)
)

## # A tibble: 9 x 8
##   T      count na_cnt  min median  mean  max  IQR
##   <fct> <int> <int> <dbl> <dbl> <dbl> <dbl> <dbl>
## 1 2       50     0     4     5    5.28     6     1
## 2 4       50     0     5     5    5.02     6     0
## 3 8       50     0     4     5    4.98     6     0
## 4 16      50     0     4     5    4.96     6     0
## 5 32      50     0     4     5    4.94     6     0
## 6 64      50     0     4     5    4.94     5     0
## 7 128     50     0     4     5    4.98     5     0
## 8 256     50     0     4     5    5.02     6     0
## 9 512     50     0     4     5    4.94     6     0

```

Kruskal–Wallis test illustrates evidence of statistical differences.

```
kruskal.test(val ~ T, data = valleys)
```

```

##
## Kruskal-Wallis rank sum test
##
## data: val by T
## Kruskal-Wallis chi-squared = 48.274, df = 8, p-value = 8.755e-08

```

Results for post-hoc Wilcoxon rank-sum test with a Bonferroni correction.

```

pairwise.wilcox.test(x = valleys$val, g = valleys$T, p.adjust.method = "bonferroni",
                      paired = FALSE, conf.int = FALSE, alternative = 't')

##
## Pairwise comparisons using Wilcoxon rank sum test with continuity correction
##
## data: valleys$val and valleys$T
##
##      2      4      8     16     32     64    128    256
## 4 0.0209 -     -     -     -     -     -     -
## 8 0.0098 1.0000 -     -     -     -     -     -
## 16 0.0066 1.0000 1.0000 -     -     -     -     -
## 32 0.0044 1.0000 1.0000 1.0000 -     -     -     -
## 64 0.0017 1.0000 1.0000 1.0000 1.0000 -     -     -
## 128 0.0037 1.0000 1.0000 1.0000 1.0000 1.0000 -     -
## 256 0.0885 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 -
## 512 0.0044 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000
##
## P value adjustment method: bonferroni

```

3.4 Contradictory objectives results

Here we present the results for **activation gene coverage** and **satisfactory trait coverage** found by each selection scheme parameter on the contradictory objectives diagnostic with valleys. 50 replicates are conducted for each scheme parameters explored.

3.4.1 Activation gene coverage over time

Activation gene coverage in a population over time. Data points on the graph is the average activation gene coverage across 50 replicates every 2000 generations. Shading comes from the best and worse coverage across 50 replicates.

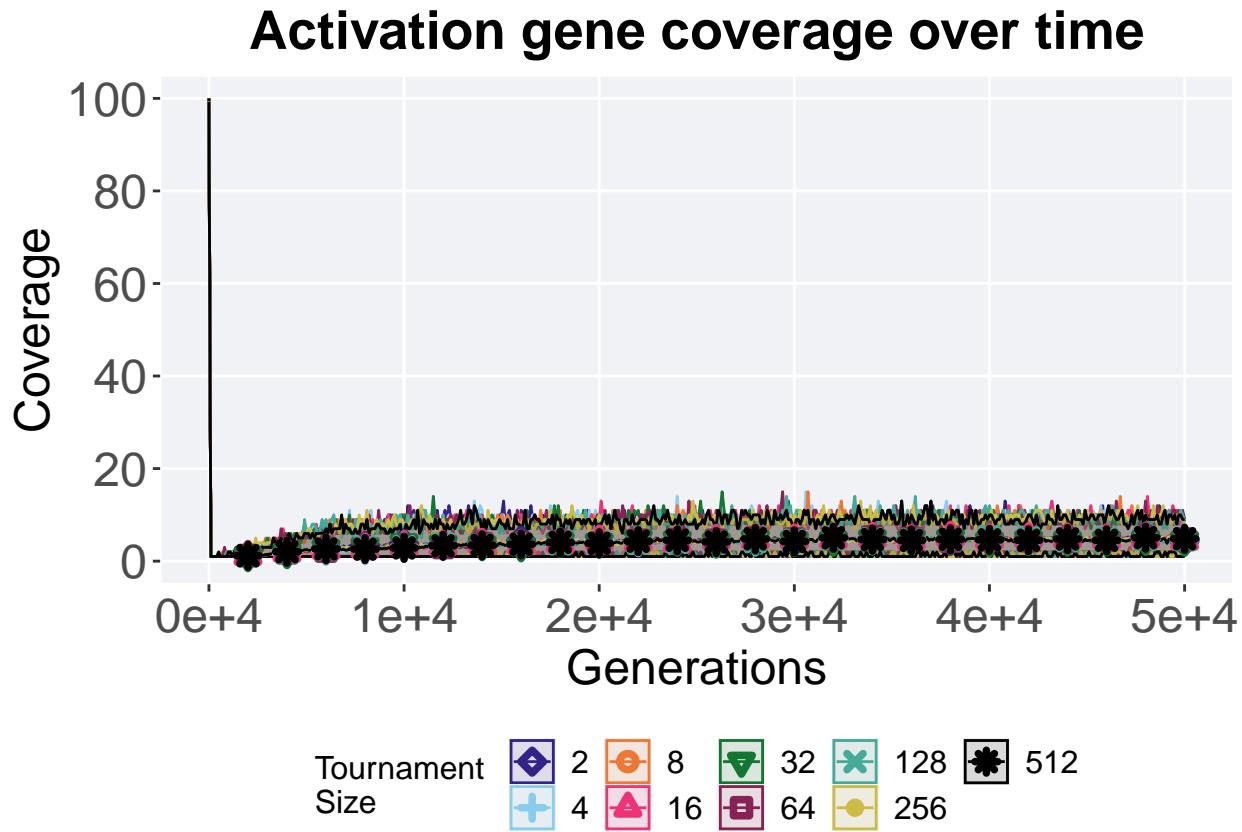
```

lines = filter(over_time_df, acro == 'con') %>%
  group_by(T, gen) %>%
  dplyr::summarise(
    min = min(uni_str_pos),
    mean = mean(uni_str_pos),
    max = max(uni_str_pos)
  )

## `summarise()` has grouped output by 'T'. You can override using the `.`groups` argument.

ggplot(lines, aes(x=gen, y=mean, group = T, fill = T, color = T, shape = T)) +
  geom_ribbon(aes(ymin = min, ymax = max), alpha = 0.1) +
  geom_line(size = 0.5) +
  geom_point(data = filter(lines, gen %% 2000 == 0 & gen != 0), size = 1.5, stroke = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name="Coverage",
    limits=c(0, 100),
    breaks=seq(0,100, 20),
    labels=c("0", "20", "40", "60", "80", "100")
  ) +
  scale_x_continuous(
    name="Generations",
    limits=c(0, 50000),
    breaks=c(0, 10000, 20000, 30000, 40000, 50000),
    labels=c("0e+4", "1e+4", "2e+4", "3e+4", "4e+4", "5e+4")
  ) +
  scale_shape_manual(values=SHAPE) +
  scale_colour_manual(values = cb_palette) +
  scale_fill_manual(values = cb_palette) +
  ggtitle('Activation gene coverage over time') +
  p_theme +
  guides(
    shape=guide_legend(nrow=2, title.position = "left", title = 'Tournament \nSize'),
    color=guide_legend(nrow=2, title.position = "left", title = 'Tournament \nSize'),
    fill=guide_legend(nrow=2, title.position = "left", title = 'Tournament \nSize')
  )

```



3.4.2 Final activation gene coverage

Activation gene coverage found in the final population at 50,000 generations.

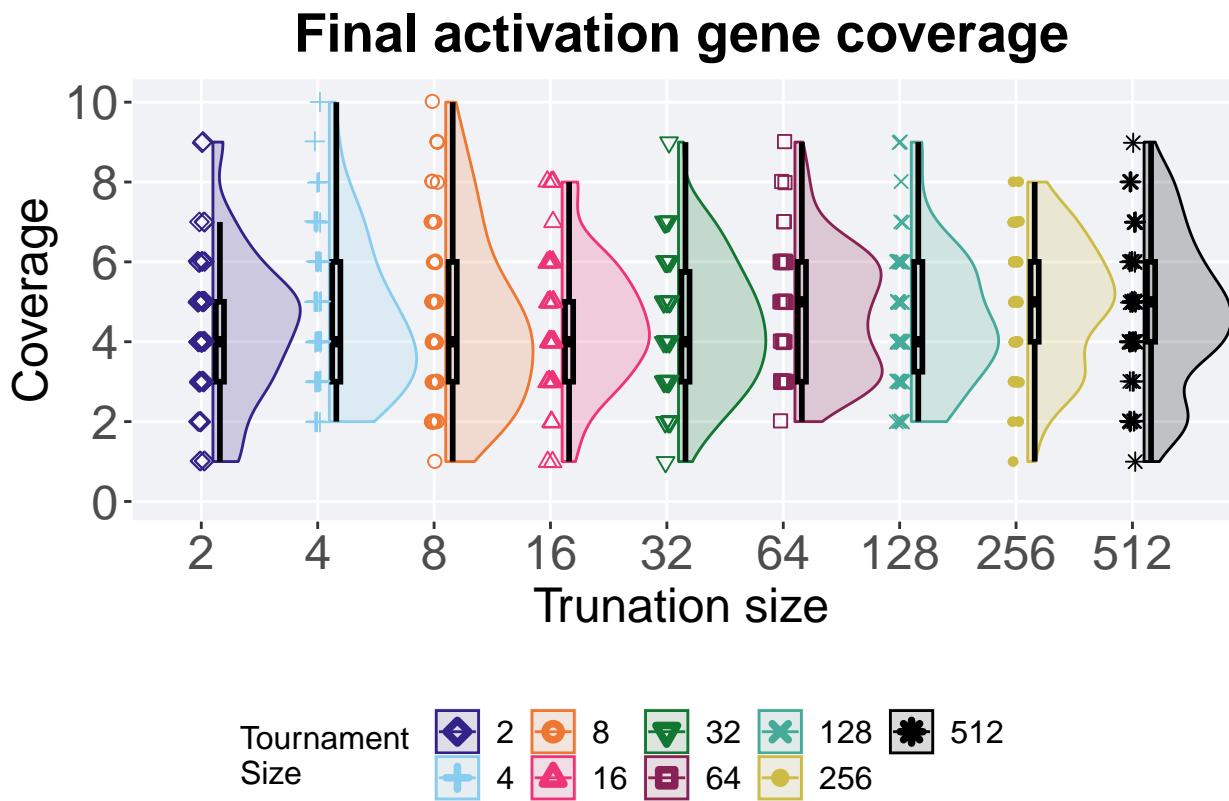
```
plot = filter(over_time_df, gen == 50000 & acro == 'con') %>%
  ggplot(., aes(x = T, y = uni_str_pos, color = T, fill = T, shape = T)) +
  geom_flat_violin(position = position_nudge(x = .1, y = 0), scale = 'width', alpha = 0.2, width = 1.5) +
  geom_boxplot(color = 'black', width = .07, outlier.shape = NA, alpha = 0.0, size = 1.0, position = position_nudge(x = -.1, y = 0), width = .07) +
  geom_point(position = position_jitter(width = 0.03, height = 0.02), size = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name="Coverage",
    limits=c(0, 10.1),
    breaks=seq(0,10,2)
  ) +
  scale_x_discrete(
    name="Trunation size"
  )+
  scale_shape_manual(values=SHAPE)+
  scale_colour_manual(values = cb_palette, ) +
  scale_fill_manual(values = cb_palette) +
  ggtitle('Final activation gene coverage')+
  p_theme

plot_grid(
  plot +
    theme(legend.position="none"),
  nrow=1, ncol=1)
```

```

  legend,
  nrow=2,
  rel_heights = c(3,1)
)

```



3.4.2.1 Stats

Summary statistics for the generation a satisfactory solution is found.

```

act_coverage = filter(over_time_df, gen == 50000 & acro == 'con')
act_coverage$T = factor(act_coverage$T, levels = TS_LIST)
act_coverage %>%
  group_by(T) %>%
  dplyr::summarise(
    count = n(),
    na_cnt = sum(is.na(uni_str_pos)),
    min = min(uni_str_pos, na.rm = TRUE),
    median = median(uni_str_pos, na.rm = TRUE),
    mean = mean(uni_str_pos, na.rm = TRUE),
    max = max(uni_str_pos, na.rm = TRUE),
    IQR = IQR(uni_str_pos, na.rm = TRUE)
  )

## # A tibble: 9 x 8
##   T      count  na_cnt   min  median   mean   max   IQR
##   <fct> <int>   <int>   <dbl>   <dbl>   <int>   <dbl>
## 1 2       50       0     1     4.32    4.32    9     2
## 2 4       50       0     1     4.32    4.32    9     2
## 3 8       50       0     1     4.32    4.32    9     2
## 4 16      50       0     1     4.32    4.32    9     2
## 5 32      50       0     1     4.32    4.32    9     2
## 6 64      50       0     1     4.32    4.32    9     2
## 7 128     50       0     1     4.32    4.32    9     2
## 8 256     50       0     1     4.32    4.32    9     2
## 9 512     50       0     1     4.32    4.32    9     2

```

```
## 2 4      50    0    2      4  4.6     10   3
## 3 8      50    0    1      4  4.54    10   3
## 4 16     50    0    1      4  4.42     8   2
## 5 32     50    0    1      4  4.42     9  2.75
## 6 64     50    0    2      5  4.76     9   3
## 7 128    50    0    2      4  4.6     9  2.75
## 8 256    50    0    1      5  4.92     8   2
## 9 512    50    0    1      5  4.82     9   2
```

Kruskal–Wallis test illustrates evidence of **no statistical differences**.

```
kruskal.test(uni_str_pos ~ T, data = act_coverage)
```

```
##
## Kruskal-Wallis rank sum test
##
## data: uni_str_pos by T
## Kruskal-Wallis chi-squared = 6.0815, df = 8, p-value = 0.6381
```

3.4.3 Satisfactory trait coverage over time

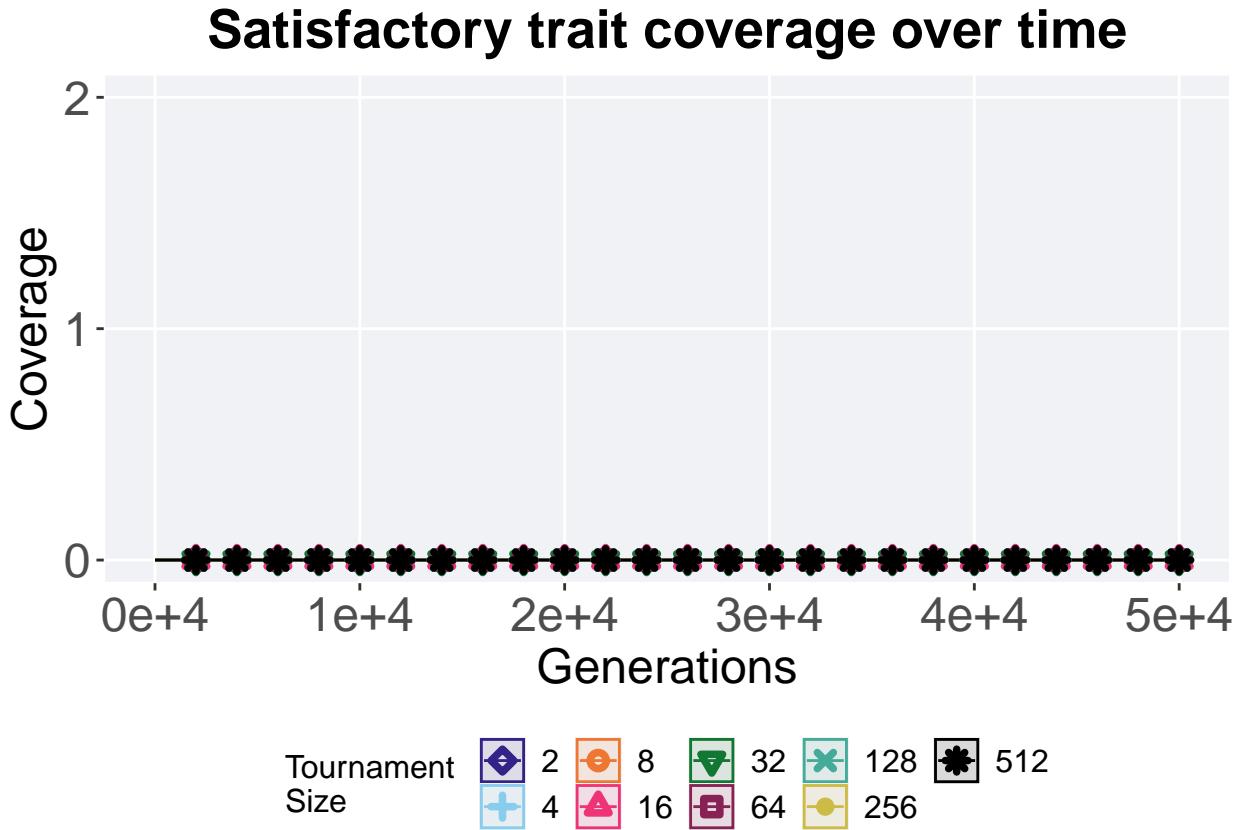
Satisfactory trait coverage in a population over time. Data points on the graph is the average activation gene coverage across 50 replicates every 2000 generations. Shading comes from the best and worse coverage across 50 replicates.

```
lines = filter(over_time_df, acro == 'con') %>%
  group_by(T, gen) %>%
  dplyr::summarise(
    min = min(pop_uni_obj),
    mean = mean(pop_uni_obj),
    max = max(pop_uni_obj)
  )

## `summarise()` has grouped output by 'T'. You can override using the `.groups` argument.

ggplot(lines, aes(x=gen, y=mean, group = T, fill = T, color = T, shape = T)) +
  geom_ribbon(aes(ymin = min, ymax = max), alpha = 0.1) +
  geom_line(size = 0.5) +
  geom_point(data = filter(lines, gen %% 2000 == 0 & gen != 0), size = 1.5, stroke = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name="Coverage",
    limits=c(0, 2),
    breaks=c(0,1,2)
  ) +
  scale_x_continuous(
    name="Generations",
    limits=c(0, 50000),
    breaks=c(0, 10000, 20000, 30000, 40000, 50000),
    labels=c("0e+4", "1e+4", "2e+4", "3e+4", "4e+4", "5e+4")
  ) +
  scale_shape_manual(values=SHAPE) +
  scale_colour_manual(values = cb_palette) +
  scale_fill_manual(values = cb_palette) +
  ggtitle('Satisfactory trait coverage over time') +
  p_theme +
```

```
guides(
  shape=guide_legend(nrow=2, title.position = "left", title = 'Tournament \nSize'),
  color=guide_legend(nrow=2, title.position = "left", title = 'Tournament \nSize'),
  fill=guide_legend(nrow=2, title.position = "left", title = 'Tournament \nSize')
)
```



3.4.4 Final satisfactory trait coverage

Satisfactory trait coverage found in the final population at 50,000 generations.

```
plot = filter(over_time_df, gen == 50000 & acro == 'con') %>%
  ggplot(., aes(x = T, y = pop_uni_obj, color = T, fill = T, shape = T)) +
  geom_flat_violin(position = position_nudge(x = .1, y = 0), scale = 'width', alpha = 0.2, width = 1.5) +
  geom_boxplot(color = 'black', width = .07, outlier.shape = NA, alpha = 0.0, size = 1.0, position = position_nudge(x = -.1, y = 0)) +
  geom_point(position = position_jitter(width = 0.03, height = 0.02), size = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name = "Coverage",
    limits = c(0, 2),
    breaks = c(0, 1, 2)
  ) +
  scale_x_discrete(
    name = "Trunction size"
  ) +
  scale_shape_manual(values = SHAPE) +
  scale_color_manual(values = cb_palette, ) +
  scale_fill_manual(values = cb_palette) +
```

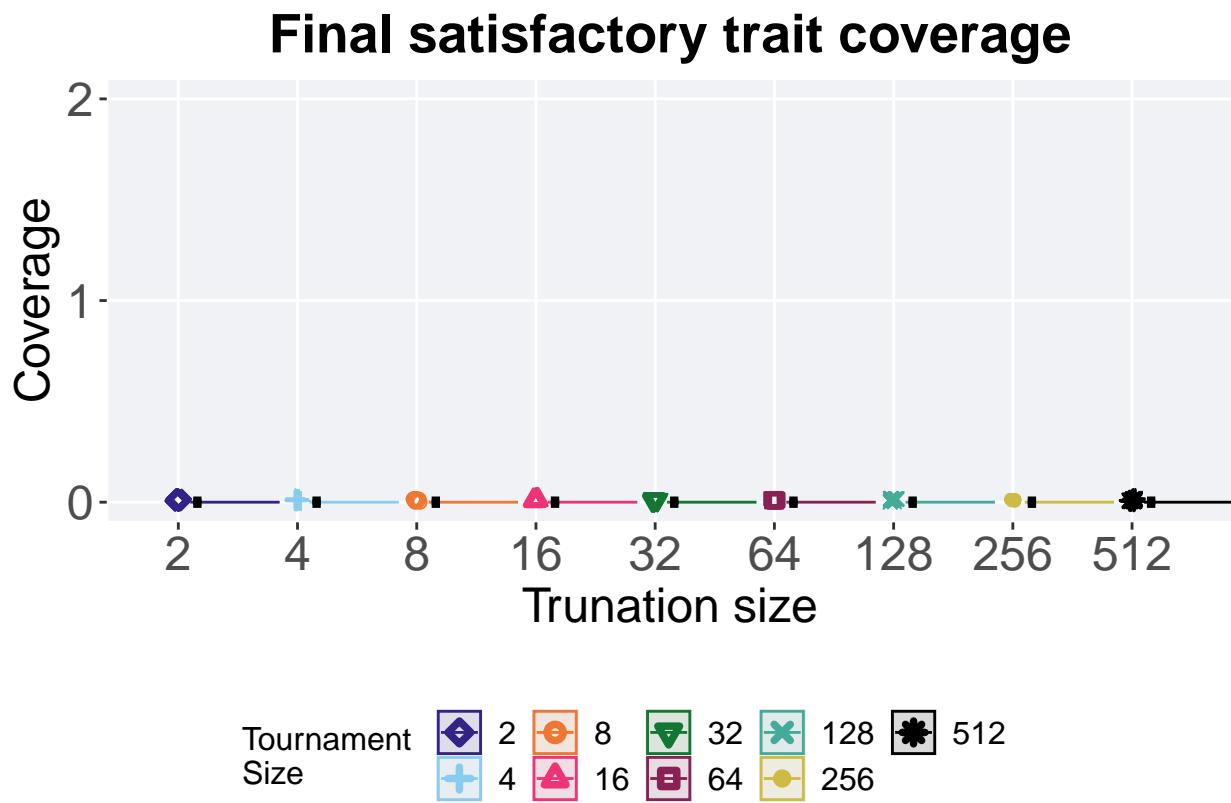
```

ggtitle('Final satisfactory trait coverage')+
  p_theme

plot_grid(
  plot +
    theme(legend.position="none"),
  legend,
  nrow=2,
  rel_heights = c(3,1)
)

## Warning: Removed 217 rows containing missing values (`geom_point()`).

```



3.4.4.1 Stats

Summary statistics for the generation a satisfactory solution is found.

```

sat_coverage = filter(over_time_df, gen == 50000 & acro == 'con')
sat_coverage$T = factor(sat_coverage$T, levels = TS_LIST)
sat_coverage %>%
  group_by(T) %>%
  dplyr::summarise(
    count = n(),
    na_cnt = sum(is.na(pop_uni_obj)),
    min = min(pop_uni_obj, na.rm = TRUE),
    median = median(pop_uni_obj, na.rm = TRUE),
    mean = mean(pop_uni_obj, na.rm = TRUE),

```

```

    max = max(pop_uni_obj, na.rm = TRUE),
    IQR = IQR(pop_uni_obj, na.rm = TRUE)
)

## # A tibble: 9 x 8
##   T      count na_cnt  min median  mean  max  IQR
##   <fct> <int>  <int> <dbl> <dbl> <dbl> <int> <dbl>
## 1 2       50     0     0     0     0     0     0     0
## 2 4       50     0     0     0     0     0     0     0
## 3 8       50     0     0     0     0     0     0     0
## 4 16      50     0     0     0     0     0     0     0
## 5 32      50     0     0     0     0     0     0     0
## 6 64      50     0     0     0     0     0     0     0
## 7 128     50     0     0     0     0     0     0     0
## 8 256     50     0     0     0     0     0     0     0
## 9 512     50     0     0     0     0     0     0     0

```

3.4.5 Largest valley reached throughout

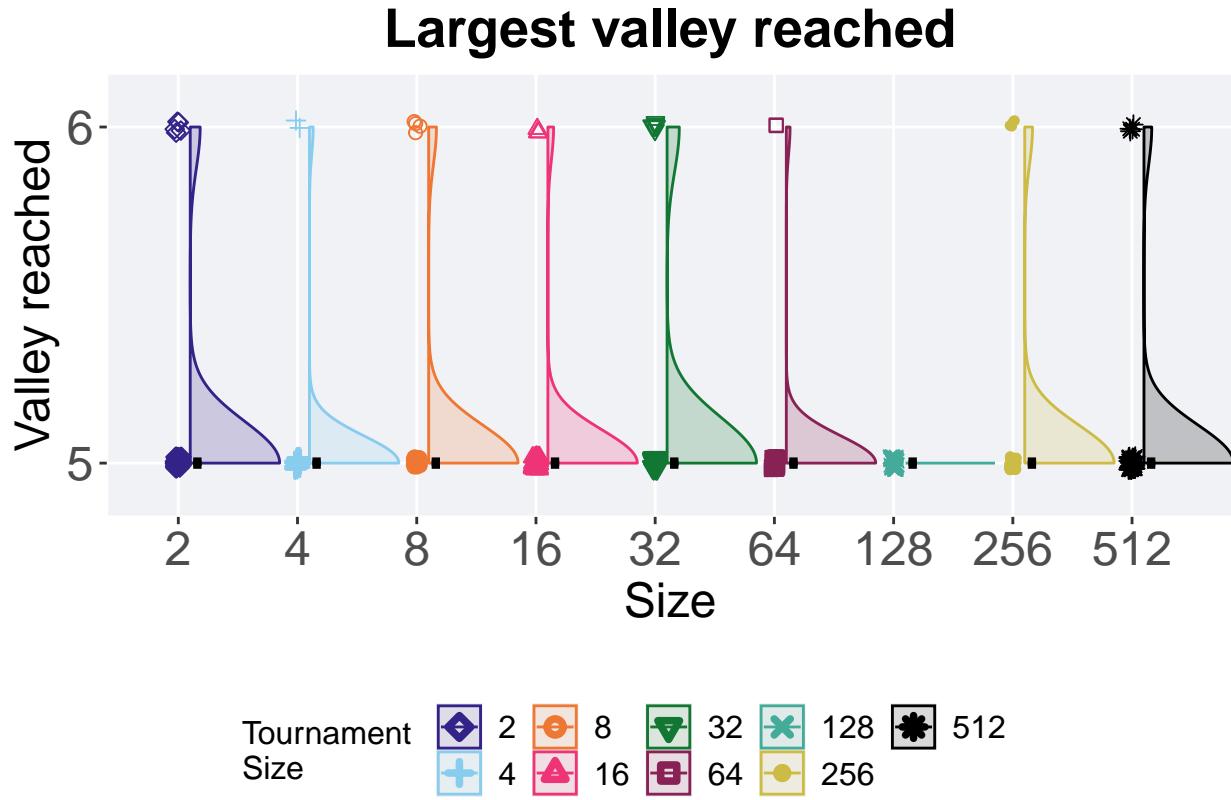
Largest valley reached in a single trait by the best performing solution throughout an entire evolutionary run.

```

plot = filter(best_df, acro == 'con' & var == 'ele_big_peak') %>%
  ggplot(., aes(x = T, y = val, color = T, fill = T, shape = T)) +
  geom_flat_violin(position = position_nudge(x = .1, y = 0), scale = 'width', alpha = 0.2, width = 1.5)
  geom_boxplot(color = 'black', width = .07, outlier.shape = NA, alpha = 0.0, size = 1.0, position = position_nudge(x = -.1, y = 0))
  geom_point(position = position_jitter(width = 0.03, height = 0.02), size = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name="Valley reached",
    limits=c(4.9,6.1),
    breaks = c(5,6)
  ) +
  scale_x_discrete(
    name="Size"
  )+
  scale_shape_manual(values=SHAPE)+
  scale_colour_manual(values = cb_palette, ) +
  scale_fill_manual(values = cb_palette) +
  ggtitle('Largest valley reached')+
  p_theme

plot_grid(
  plot +
    theme(legend.position="none"),
  legend,
  nrow=2,
  rel_heights = c(3,1)
)

```



3.4.5.1 Stats

Summary statistics for the largest valley crossed.

```

valleys = filter(best_df, acro == 'con' & var == 'ele_big_peak')
valleys$T = factor(valleys$T, levels = TS_LIST)
valleys %>%
  group_by(T) %>%
  dplyr::summarise(
    count = n(),
    na_cnt = sum(is.na(val)),
    min = min(val, na.rm = TRUE),
    median = median(val, na.rm = TRUE),
    mean = mean(val, na.rm = TRUE),
    max = max(val, na.rm = TRUE),
    IQR = IQR(val, na.rm = TRUE)
  )

## # A tibble: 9 x 8
##   T      count na_cnt   min median   mean   max   IQR
##   <fct> <int>   <int> <dbl>  <dbl>  <dbl> <dbl> <dbl>
## 1 2       50     0     5     5  5.1     6     0
## 2 4       50     0     5     5  5.04    6     0
## 3 8       50     0     5     5  5.08    6     0
## 4 16      50     0     5     5  5.06    6     0
## 5 32      50     0     5     5  5.12    6     0
## 6 64      50     0     5     5  5.04    6     0

```

```
## 7 128      50     0     5     5 5       5     0
## 8 256      50     0     5     5 5.08     6     0
## 9 512      50     0     5     5 5.08     6     0
```

Kruskal–Wallis test illustrates evidence of **no statistical differences**.

```
kruskal.test(val ~ T, data = valleys)

##
## Kruskal-Wallis rank sum test
##
## data: val by T
## Kruskal-Wallis chi-squared = 8.3386, df = 8, p-value = 0.4011
```

3.5 Multi-path exploration results

Here we present the results for best performances and activation gene coverage found by each selection scheme parameter on the multi-path exploration diagnostic with valleys. 50 replicates are conducted for each scheme parameter explored.

3.5.1 Activation gene coverage over time

Activation gene coverage in a population over time. Data points on the graph is the average activation gene coverage across 50 replicates every 2000 generations. Shading comes from the best and worse coverage across 50 replicates.

```
lines = filter(over_time_df, acro == 'mpe') %>%
  group_by(T, gen) %>%
  dplyr::summarise(
    min = min(uni_str_pos),
    mean = mean(uni_str_pos),
    max = max(uni_str_pos)
  )

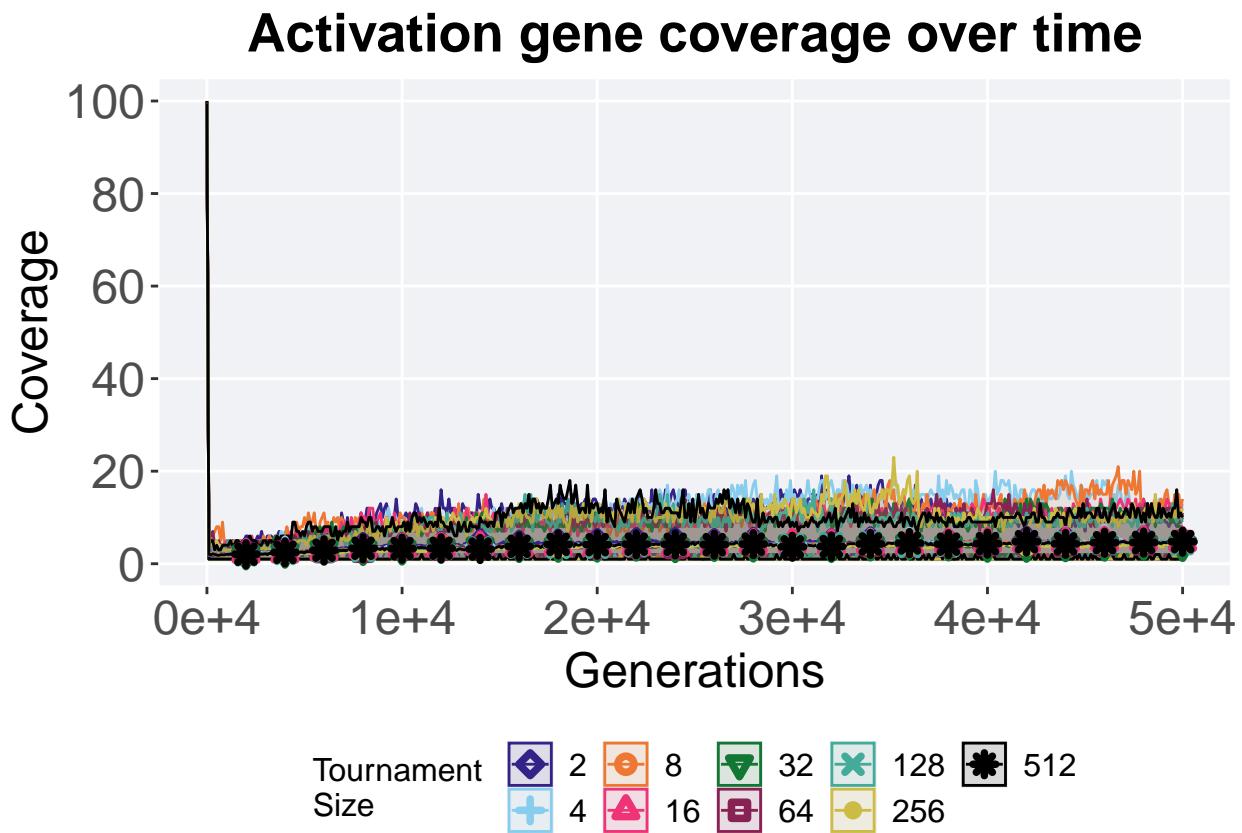
## `summarise()` has grouped output by 'T'. You can override using the `.groups` argument.

ggplot(lines, aes(x=gen, y=mean, group = T, fill = T, color = T, shape = T)) +
  geom_ribbon(aes(ymin = min, ymax = max), alpha = 0.1) +
  geom_line(size = 0.5) +
  geom_point(data = filter(lines, gen %% 2000 == 0 & gen != 0), size = 1.5, stroke = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name="Coverage",
    limits=c(0, 100),
    breaks=seq(0,100, 20),
    labels=c("0", "20", "40", "60", "80", "100")
  ) +
  scale_x_continuous(
    name="Generations",
    limits=c(0, 50000),
    breaks=c(0, 10000, 20000, 30000, 40000, 50000),
    labels=c("0e+4", "1e+4", "2e+4", "3e+4", "4e+4", "5e+4")
  ) +
  scale_shape_manual(values=SHAPE) +
  scale_colour_manual(values = cb_palette) +
```

```

scale_fill_manual(values = cb_palette) +
ggtitle('Activation gene coverage over time')+
p_theme +
guides(
  shape=guide_legend(nrow=2, title.position = "left", title = 'Tournament \nSize'),
  color=guide_legend(nrow=2, title.position = "left", title = 'Tournament \nSize'),
  fill=guide_legend(nrow=2, title.position = "left", title = 'Tournament \nSize')
)

```



3.5.2 Final activation gene coverage

Activation gene coverage found in the final population at 50,000 generations.

```

plot = filter(over_time_df, gen == 50000 & acro == 'mpe') %>%
  ggplot(., aes(x = T, y = uni_str_pos, color = T, fill = T, shape = T)) +
  geom_flat_violin(position = position_nudge(x = .1, y = 0), scale = 'width', alpha = 0.2, width = 1.5) +
  geom_boxplot(color = 'black', width = .07, outlier.shape = NA, alpha = 0.0, size = 1.0, position = position_nudge(x = -.1, y = 0)) +
  geom_point(position = position_jitter(width = 0.03, height = 0.02), size = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name="Coverage",
    limits=c(0, 14.1),
    breaks=seq(0,14,2)
  ) +
  scale_x_discrete(
    name="Trunation size"
  )

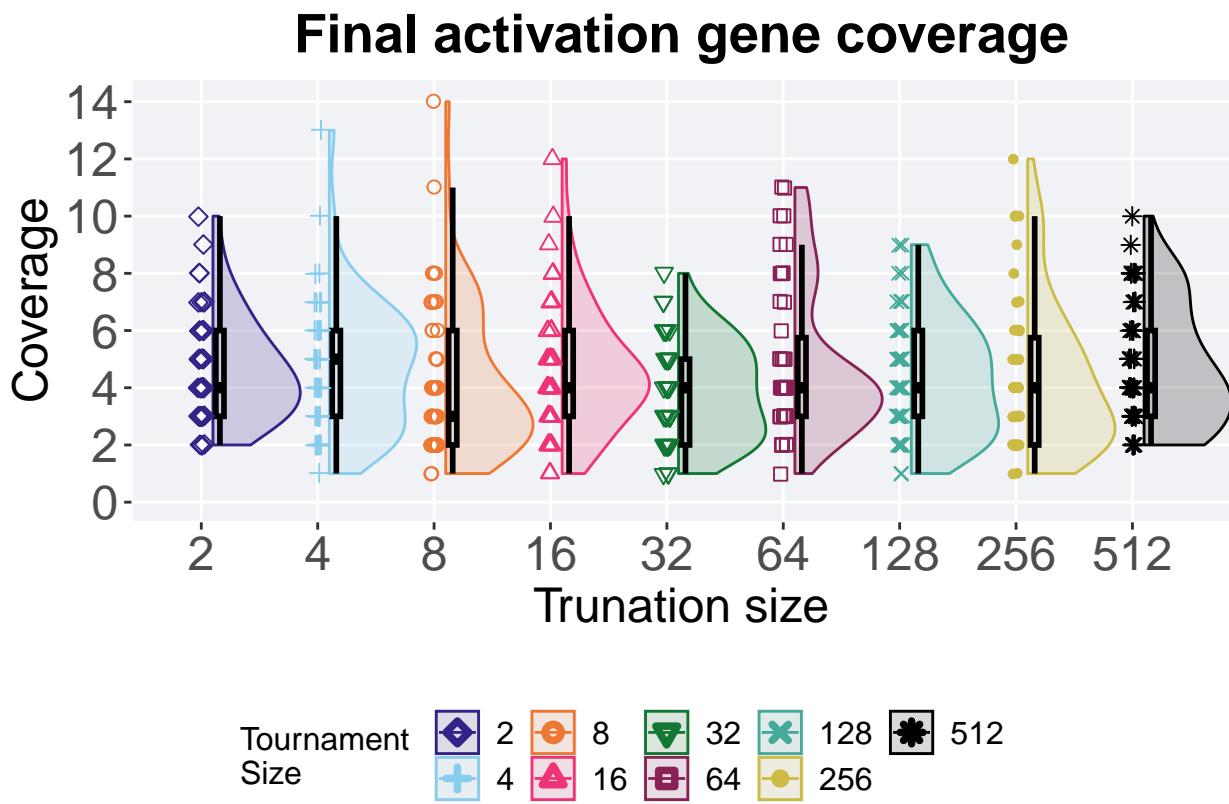
```

```

scale_shape_manual(values=SHAPE) +
scale_colour_manual(values = cb_palette, ) +
scale_fill_manual(values = cb_palette) +
ggtitle('Final activation gene coverage') +
p_theme

plot_grid(
  plot +
  theme(legend.position="none"),
  legend,
  nrow=2,
  rel_heights = c(3,1)
)

```



3.5.2.1 Stats

Summary statistics for the generation a satisfactory solution is found.

```

act_coverage = filter(over_time_df, gen == 50000 & acro == 'mpe')
act_coverage$T = factor(act_coverage$T, levels = TS_LIST)
act_coverage %>%
  group_by(T) %>%
  dplyr::summarise(
    count = n(),
    na_cnt = sum(is.na(uni_str_pos)),
    min = min(uni_str_pos, na.rm = TRUE),
    max = max(uni_str_pos),
    mean = mean(uni_str_pos),
    median = median(uni_str_pos),
    sd = sd(uni_str_pos),
    iqr = IQR(uni_str_pos)
  )

```

```

median = median(uni_str_pos, na.rm = TRUE),
mean = mean(uni_str_pos, na.rm = TRUE),
max = max(uni_str_pos, na.rm = TRUE),
IQR = IQR(uni_str_pos, na.rm = TRUE)
)

## # A tibble: 9 x 8
##   T      count na_cnt  min median  mean  max  IQR
##   <fct> <int> <int> <dbl> <dbl> <int> <dbl>
## 1 2       50     0     2     4    4.66    10    3
## 2 4       50     0     1     5    4.76    13    3
## 3 8       50     0     1     3    4.26    14    4
## 4 16      50     0     1     4    4.66    12    3
## 5 32      50     0     1     4    3.86     8    3
## 6 64      50     0     1     4    4.78    11   2.75
## 7 128     50     0     1     4    4.36     9    3
## 8 256     50     0     1     4    4.34    12   3.75
## 9 512     50     0     2     4    4.76    10    3

```

Kruskal–Wallis test illustrates evidence of **no statistical differences**.

```
kruskal.test(uni_str_pos ~ T, data = act_coverage)
```

```

##
## Kruskal-Wallis rank sum test
##
## data: uni_str_pos by T
## Kruskal-Wallis chi-squared = 9.3181, df = 8, p-value = 0.3162

```

3.5.3 Performance over time

Best performance in a population over time. Data points on the graph is the average performance across 50 replicates every 2000 generations. Shading comes from the best and worse performance across 50 replicates.

```

lines = filter(over_time_df, acro == 'mpe') %>%
  group_by(T, gen) %>%
  dplyr::summarise(
    min = min(pop_fit_max) / DIMENSIONALITY,
    mean = mean(pop_fit_max) / DIMENSIONALITY,
    max = max(pop_fit_max) / DIMENSIONALITY
  )

## `summarise()` has grouped output by 'T'. You can override using the `.groups` argument.

over_time_plot = ggplot(lines, aes(x=gen, y=mean, group = T, fill = T, color = T, shape = T)) +
  geom_ribbon(aes(ymin = min, ymax = max), alpha = 0.1) +
  geom_line(size = 0.5) +
  geom_point(data = filter(lines, gen %% 2000 == 0 & gen != 0), size = 1.5, stroke = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name="Average trait score",
    limits=c(0, 10)
  ) +
  scale_x_continuous(
    name="Generations",
    limits=c(0, 50000),

```

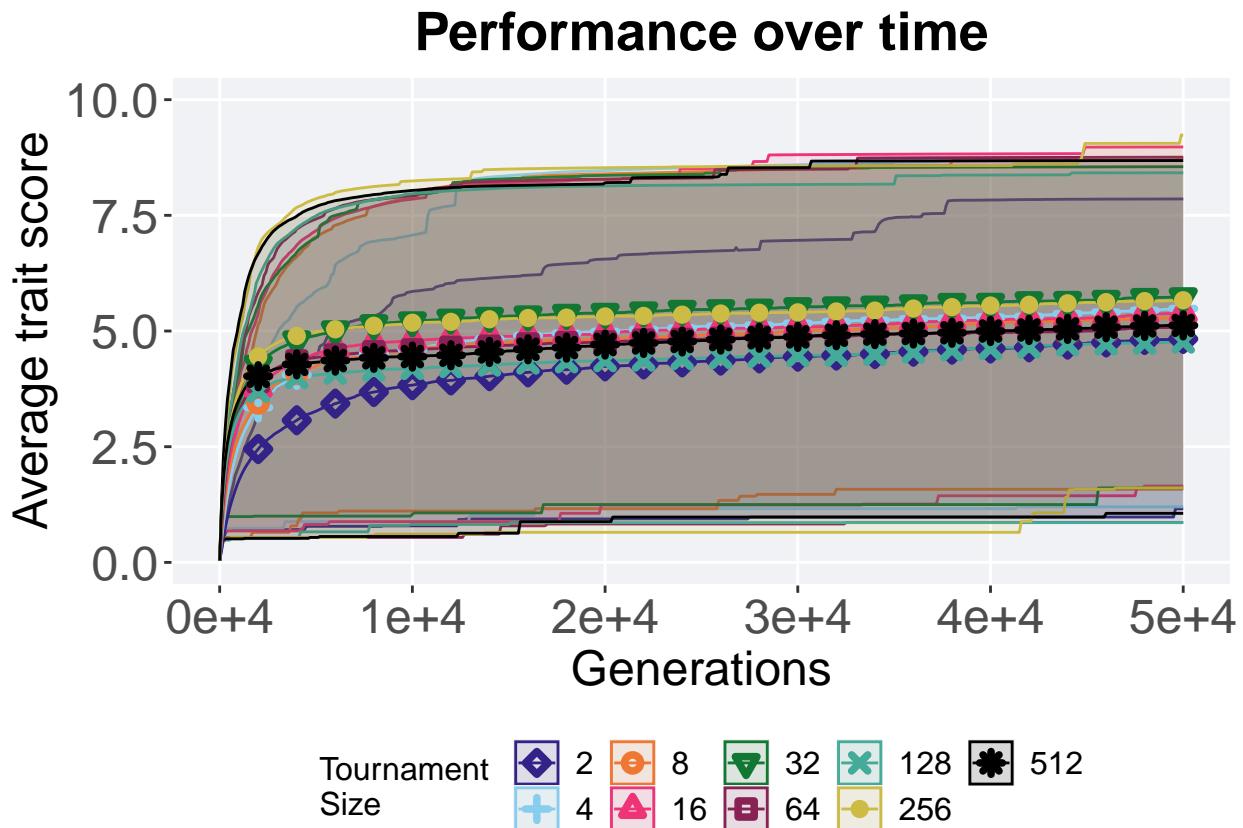
```

breaks=c(0, 10000, 20000, 30000, 40000, 50000),
labels=c("0e+4", "1e+4", "2e+4", "3e+4", "4e+4", "5e+4")

) +
scale_shape_manual(values=SHAPE)+
scale_colour_manual(values = cb_palette) +
scale_fill_manual(values = cb_palette) +
ggtitle('Performance over time')+
p_theme +
guides(
  shape=guide_legend(nrow=2, title.position = "left", title = 'Tournament \nSize'),
  color=guide_legend(nrow=2, title.position = "left", title = 'Tournament \nSize'),
  fill=guide_legend(nrow=2, title.position = "left", title = 'Tournament \nSize')
)

over_time_plot

```



3.5.4 Best performance throughout

Best performance reached throughout 50,000 generations in a population.

```

plot = filter(best_df, acro == 'mpe' & var == 'pop_fit_max') %>%
  ggplot(., aes(x = T, y = val / DIMENSIONALITY, color = T, fill = T, shape = T)) +
  geom_flat_violin(position = position_nudge(x = .1, y = 0), scale = 'width', alpha = 0.2, width = 1.5)
  geom_boxplot(color = 'black', width = .07, outlier.shape = NA, alpha = 0.0, size = 1.0, position = pos)
  geom_point(position = position_jitter(width = 0.03, height = 0.02), size = 2.0, alpha = 1.0) +

```

```

scale_y_continuous(
  name="Average trait score",
  limits=c(0, 10)
) +
scale_x_discrete(
  name="Size"
) +
scale_shape_manual(values=SHAPE) +
scale_colour_manual(values = cb_palette, ) +
scale_fill_manual(values = cb_palette) +
ggtitle('Best performance throughout') +
p_theme

plot_grid(
  plot +
    theme(legend.position="none"),
  legend,
  nrow=2,
  rel_heights = c(3,1)
)
)

```



3.5.4.1 Stats

Summary statistics for the best performance.

```

performance = filter(best_df, acro == 'mpe' & var == 'pop_fit_max')
performance$T = factor(performance$T, levels = TS_LIST)
performance %>%
  group_by(T) %>%
  dplyr::summarise(
    count = n(),
    na_cnt = sum(is.na(val)),
    min = min(val / DIMENSIONALITY, na.rm = TRUE),
    median = median(val / DIMENSIONALITY, na.rm = TRUE),
    mean = mean(val / DIMENSIONALITY, na.rm = TRUE),
    max = max(val / DIMENSIONALITY, na.rm = TRUE),
    IQR = IQR(val / DIMENSIONALITY, na.rm = TRUE)
  )

## # A tibble: 9 x 8
##   T     count na_cnt   min   median   mean   max   IQR
##   <fct> <int> <int> <dbl> <dbl> <dbl> <dbl> <dbl>
## 1 2       50     0 1.16   5.57   4.85   7.96   3.61
## 2 4       50     0 1.20   5.56   5.49   8.73   2.83
## 3 8       50     0 1.58   5.07   5.29   8.75   4.24
## 4 16      50     0 1.65   5.32   5.39   8.98   3.52
## 5 32      50     0 1.61   5.84   5.73   8.55   3.28
## 6 64      50     0 0.860  5.15   5.09   8.76   3.21
## 7 128     50     0 0.860  4.65   4.76   8.42   3.72
## 8 256     50     0 1.61   5.98   5.67   9.24   4.41
## 9 512     50     0 1.06   5.57   5.12   8.68   3.30

```

Kruskal–Wallis test illustrates evidence of **no statistical differences**.

```
kruskal.test(val ~ T, data = performance)
```

```

##
## Kruskal-Wallis rank sum test
##
## data: val by T
## Kruskal-Wallis chi-squared = 9.8288, df = 8, p-value = 0.2772

```

3.5.5 Largest valley reached throughout

Largest valley reached in a single trait by the best performing solution throughout an entire evolutionary run.

```

plot = filter(best_df, acro == 'mpe' & var == 'ele_big_peak') %>%
  ggplot(., aes(x = T, y = val, color = T, fill = T, shape = T)) +
  geom_flat_violin(position = position_nudge(x = .1, y = 0), scale = 'width', alpha = 0.2, width = 1.5)
  geom_boxplot(color = 'black', width = .07, outlier.shape = NA, alpha = 0.0, size = 1.0, position = position_dodge(.7))
  geom_point(position = position_jitter(width = 0.03, height = 0.02), size = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name="Valley reached",
    limits=c(3.9,6.1),
    breaks = c(4,5,6)
  ) +
  scale_x_discrete(
    name="Size"
  ) +
  scale_shape_manual(values=SHAPE) +
  scale_colour_manual(values = cb_palette, )

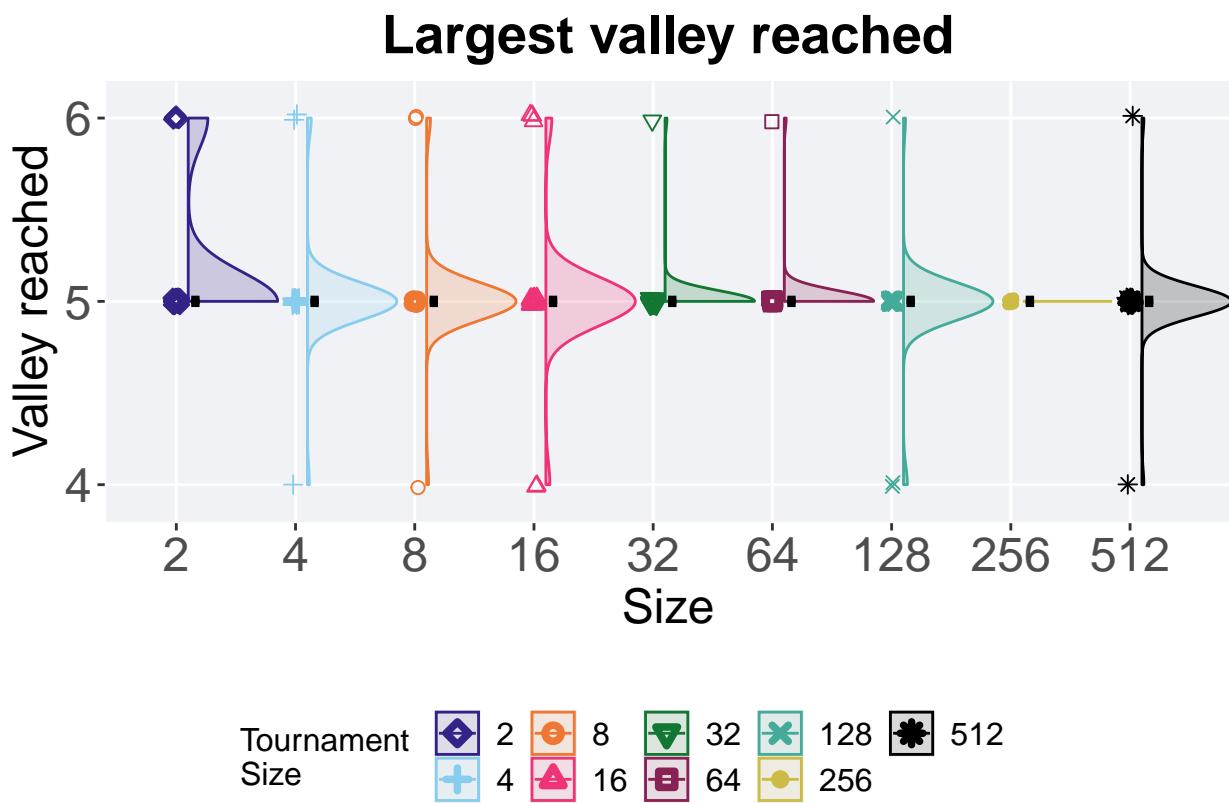
```

```

scale_fill_manual(values = cb_palette) +
ggtitle('Largest valley reached')+
p_theme

plot_grid(
  plot +
    theme(legend.position="none"),
  legend,
  nrow=2,
  rel_heights = c(3,1)
)

```



3.5.5.1 Stats

Summary statistics for the largest valley crossed.

```

valleys = filter(best_df, acro == 'mpe' & var == 'ele_big_peak')
valleys$T = factor(valleys$T, levels = TS_LIST)
valleys %>%
  group_by(T) %>%
  dplyr::summarise(
    count = n(),
    na_cnt = sum(is.na(val)),
    min = min(val, na.rm = TRUE),
    median = median(val, na.rm = TRUE),
    mean = mean(val, na.rm = TRUE),
    )

```

```

  max = max(val, na.rm = TRUE),
  IQR = IQR(val, na.rm = TRUE)
)

## # A tibble: 9 x 8
##   T      count na_cnt  min median  mean    max    IQR
##   <fct> <int> <int> <dbl> <dbl> <dbl> <dbl> <dbl>
## 1 2       50     0     5     5  5.18     6     0
## 2 4       50     0     4     5  5.02     6     0
## 3 8       50     0     4     5  5.02     6     0
## 4 16      50     0     4     5  5.02     6     0
## 5 32      50     0     5     5  5.02     6     0
## 6 64      50     0     5     5  5.02     6     0
## 7 128     50     0     4     5  4.98     6     0
## 8 256     50     0     5     5  5      5     0
## 9 512     50     0     4     5  5      6     0

```

Kruskal–Wallis test illustrates evidence of **no statistical differences**.

```
kruskal.test(val ~ T, data = valleys)
```

```

##
## Kruskal-Wallis rank sum test
##
## data: val by T
## Kruskal-Wallis chi-squared = 23.215, df = 8, p-value = 0.003099

```

Results for post-hoc Wilcoxon rank-sum test with a Bonferroni correction.

```
pairwise.wilcox.test(x = valleys$val, g = valleys$T, p.adjust.method = "bonferroni",
                      paired = FALSE, conf.int = FALSE, alternative = 't')
```

```

##
## Pairwise comparisons using Wilcoxon rank sum test with continuity correction
##
## data: valleys$val and valleys$T
##
##      2      4      8     16     32     64    128    256
## 4  0.612 -      -      -      -      -      -      -
## 8  0.612 1.000 -      -      -      -      -      -
## 16 1.000 1.000 1.000 -      -      -      -      -
## 32 0.293 1.000 1.000 1.000 -      -      -      -
## 64 0.293 1.000 1.000 1.000 1.000 -      -      -
## 128 0.117 1.000 1.000 1.000 1.000 1.000 -      -
## 256 0.065 1.000 1.000 1.000 1.000 1.000 1.000 -
## 512 0.186 1.000 1.000 1.000 1.000 1.000 1.000 1.000
##
## P value adjustment method: bonferroni
```

Chapter 4

Genotypic fitness sharing

Results for the genotypic fitness sharing parameter sweep on the diagnostics with valleys.

4.1 Data setup

```
over_time_df <- read.csv(paste(DATA_DIR, 'OVER-TIME-MVC/gfs.csv', sep = "", collapse = NULL), header = TRUE)
over_time_df$Sigma <- factor(over_time_df$Sigma, levels = FS_LIST)

best_df <- read.csv(paste(DATA_DIR, 'BEST-MVC/gfs.csv', sep = "", collapse = NULL), header = TRUE, stringsAsFactors = FALSE)
best_df$Sigma <- factor(best_df$Sigma, levels = FS_LIST)
```

4.2 Exploitation rate results

Here we present the results for **best performances** found by each selection scheme parameter on the exploitation rate diagnostic with valleys. 50 replicates are conducted for each scheme explored.

4.2.1 Performance over time

Best performance in a population over time. Data points on the graph is the average performance across 50 replicates every 2000 generations. Shading comes from the best and worse performance across 50 replicates.

```
lines = filter(over_time_df, acro == 'exp') %>%
  group_by(Sigma, gen) %>%
  dplyr::summarise(
    min = min(pop_fit_max) / DIMENSIONALITY,
    mean = mean(pop_fit_max) / DIMENSIONALITY,
    max = max(pop_fit_max) / DIMENSIONALITY
  )

## `summarise()` has grouped output by 'Sigma'. You can override using the
## `.groups` argument.

over_time_plot = ggplot(lines, aes(x=gen, y=mean, group = Sigma, fill = Sigma, color = Sigma, shape = Sigma))
  geom_ribbon(aes(ymin = min, ymax = max), alpha = 0.1) +
  geom_line(size = 0.5) +
  geom_point(data = filter(lines, gen %% 2000 == 0 & gen != 0), size = 1.5, stroke = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name="Average trait score",
```

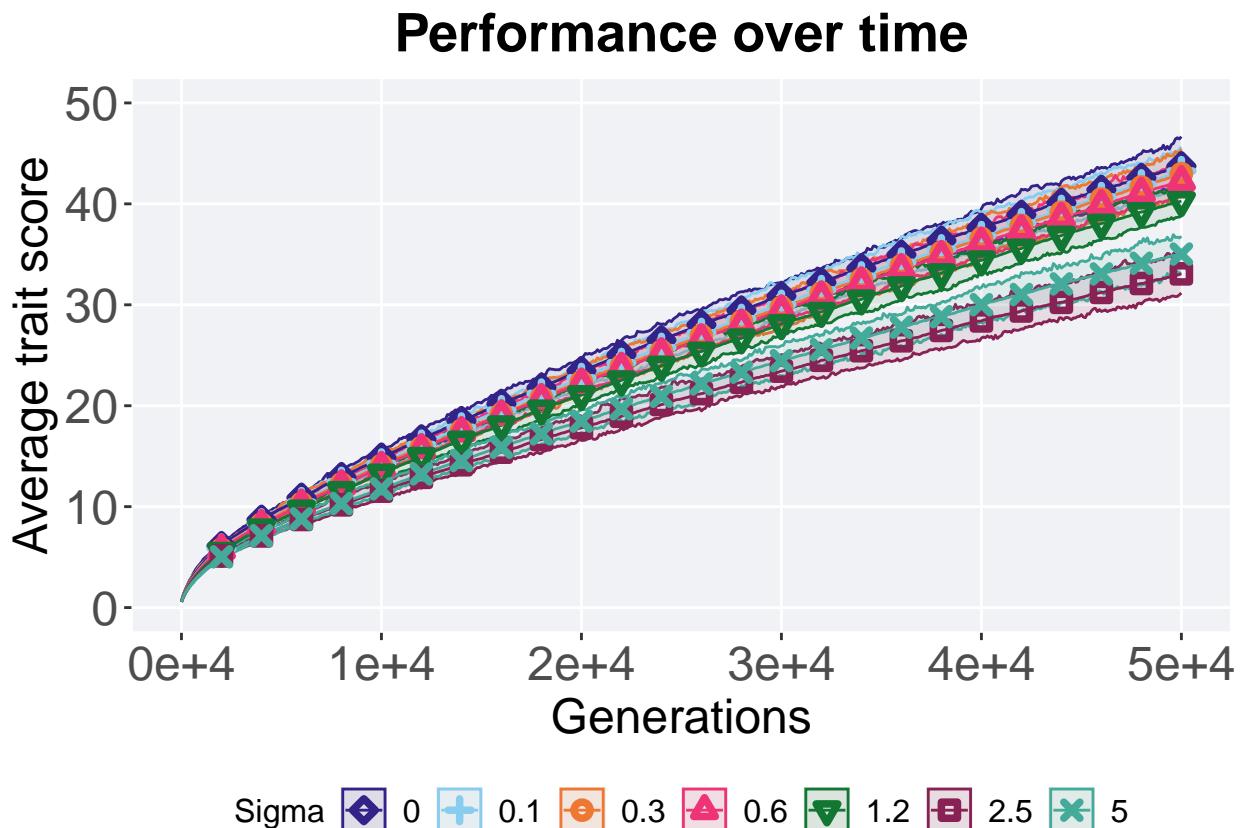
```

    limits = c(0,50)
) +
scale_x_continuous(
  name="Generations",
  limits=c(0, 50000),
  breaks=c(0, 10000, 20000, 30000, 40000, 50000),
  labels=c("0e+4", "1e+4", "2e+4", "3e+4", "4e+4", "5e+4")

) +
scale_shape_manual(values=SHAPE)+
scale_colour_manual(values = cb_palette) +
scale_fill_manual(values = cb_palette) +
ggtitle('Performance over time')+
p_theme +
guides(
  shape=guide_legend(nrow=1, title.position = "left", title = 'Sigma'),
  color=guide_legend(nrow=1, title.position = "left", title = 'Sigma'),
  fill=guide_legend(nrow=1, title.position = "left", title = 'Sigma')
)

over_time_plot

```



4.2.2 Best performance throughout

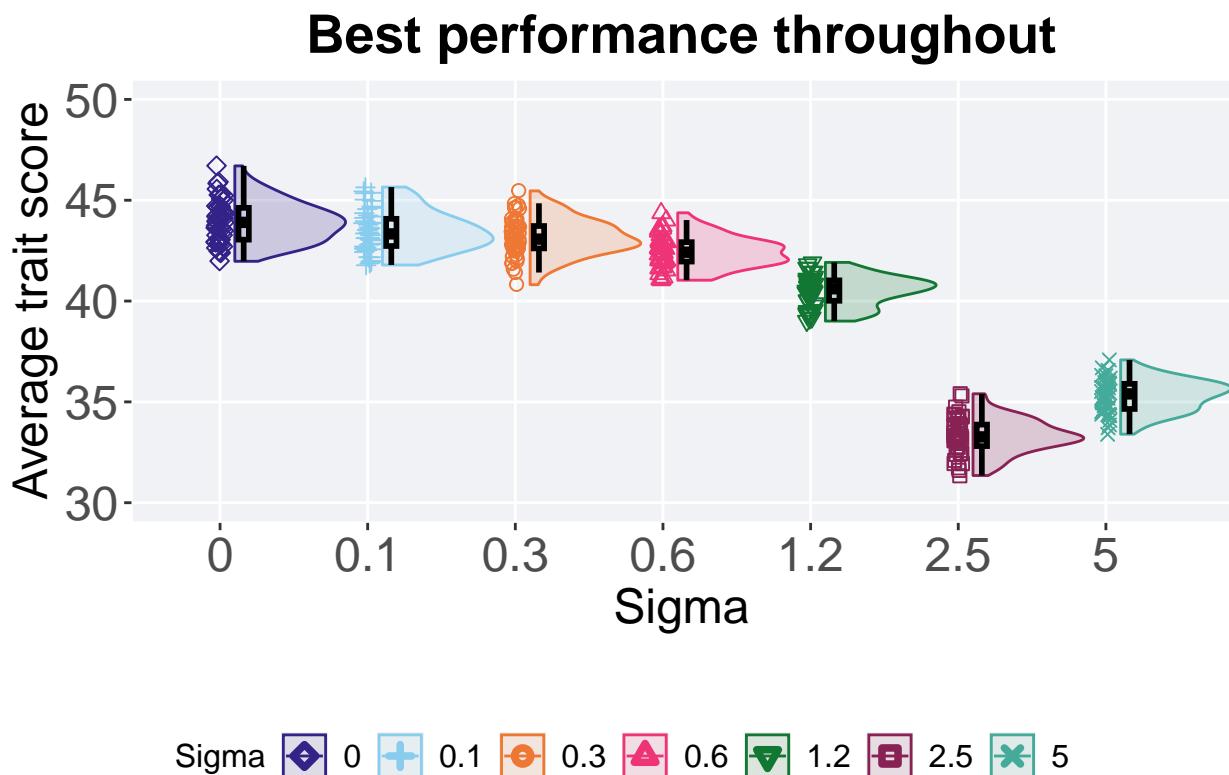
Best performance reached throughout 50,000 generations in a population.

```

plot = filter(best_df, acro == 'exp' & var == 'pop_fit_max') %>%
  ggplot(., aes(x = Sigma, y = val / DIMENSIONALITY, color = Sigma, fill = Sigma, shape = Sigma)) +
  geom_flat_violin(position = position_nudge(x = .1, y = 0), scale = 'width', alpha = 0.2, width = 1.5) +
  geom_boxplot(color = 'black', width = .07, outlier.shape = NA, alpha = 0.0, size = 1.0, position = position_nudge(x = -.1, y = 0)) +
  geom_point(position = position_jitter(width = 0.03, height = 0.02), size = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name = "Average trait score",
    limits = c(30, 50)
  ) +
  scale_x_discrete(
    name = "Sigma"
  ) +
  scale_shape_manual(values = SHAPE) +
  scale_color_manual(values = cb_palette, ) +
  scale_fill_manual(values = cb_palette) +
  ggtitle('Best performance throughout') +
  p_theme

plot_grid(
  plot +
    theme(legend.position = "none"),
  legend,
  nrow = 2,
  rel_heights = c(3, 1)
)

```



4.2.2.1 Stats

Summary statistics for the best performance.

```
performance = filter(best_df, acro == 'exp' & var == 'pop_fit_max')
performance$Sigma = factor(performance$Sigma, levels = FS_LIST)
performance %>%
  group_by(Sigma) %>%
  dplyr::summarise(
    count = n(),
    na_cnt = sum(is.na(val)),
    min = min(val / DIMENSIONALITY, na.rm = TRUE),
    median = median(val / DIMENSIONALITY, na.rm = TRUE),
    mean = mean(val / DIMENSIONALITY, na.rm = TRUE),
    max = max(val / DIMENSIONALITY, na.rm = TRUE),
    IQR = IQR(val / DIMENSIONALITY, na.rm = TRUE)
  )

## # A tibble: 7 x 8
##   Sigma count na_cnt   min median   mean   max   IQR
##   <fct> <int> <int> <dbl> <dbl> <dbl> <dbl>
## 1 0      50     0  42.0  43.9  43.9  46.7  1.58
## 2 0.1    50     0  41.8  43.3  43.5  45.7  1.35
## 3 0.3    50     0  40.8  43.1  43.2  45.5  1.11
## 4 0.6    50     0  41.0  42.4  42.4  44.4  0.963
## 5 1.2    50     0  39.0  40.6  40.5  41.9  1.01
## 6 2.5    50     0  31.3  33.3  33.3  35.4  1.07
## 7 5      50     0  33.4  35.4  35.3  37.1  1.25
```

Kruskal–Wallis test illustrates evidence of statistical differences.

```
kruskal.test(val ~ Sigma, data = performance)
```

```
##
## Kruskal-Wallis rank sum test
##
## data: val by Sigma
## Kruskal-Wallis chi-squared = 294.25, df = 6, p-value < 2.2e-16
```

Results for post-hoc Wilcoxon rank-sum test with a Bonferroni correction.

```
pairwise.wilcox.test(x = performance$val, g = performance$Sigma, p.adjust.method = "bonferroni",
                      paired = FALSE, conf.int = FALSE, alternative = 't')
```

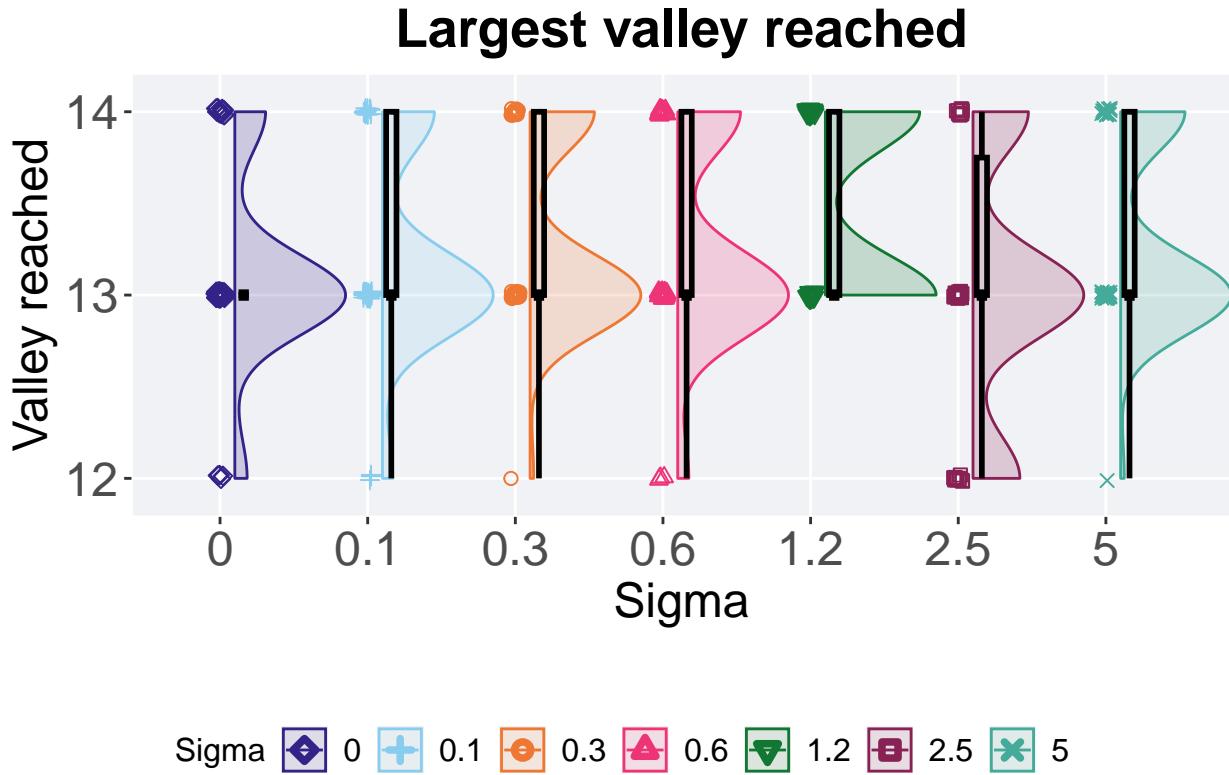
```
##
## Pairwise comparisons using Wilcoxon rank sum test with continuity correction
##
## data: performance$val and performance$Sigma
##
##          0       0.1      0.3      0.6      1.2      2.5
## 0.1 0.6669  -       -       -       -       -
## 0.3 0.0125  1.0000  -       -       -       -
## 0.6 1.5e-09 1.5e-05 0.0024  -       -       -
## 1.2 < 2e-16 < 2e-16 1.1e-15 6.0e-15  -       -
## 2.5 < 2e-16 < 2e-16 < 2e-16 < 2e-16 < 2e-16  -
## 5    < 2e-16 < 2e-16 < 2e-16 < 2e-16 < 2e-16 1.7e-13
##
## P value adjustment method: bonferroni
```

4.2.3 Largest valley reached throughout

Largest valley reached in a single trait by the best performing solution throughout an entire evolutionary run.

```
plot = filter(best_df, acro == 'exp' & var == 'ele_big_peak') %>%
  ggplot(., aes(x = Sigma, y = val, color = Sigma, fill = Sigma, shape = Sigma)) +
  geom_flat_violin(position = position_nudge(x = .1, y = 0), scale = 'width', alpha = 0.2, width = 1.5) +
  geom_boxplot(color = 'black', width = .07, outlier.shape = NA, alpha = 0.0, size = 1.0, position = position_nudge(x = -.1, y = 0)) +
  geom_point(position = position_jitter(width = 0.03, height = 0.02), size = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name = "Valley reached",
    limits = c(11.9, 14.1),
    breaks = c(11, 12, 13, 14)
  ) +
  scale_x_discrete(
    name = "Sigma"
  ) +
  scale_shape_manual(values = SHAPE) +
  scale_colour_manual(values = cb_palette, ) +
  scale_fill_manual(values = cb_palette) +
  ggtitle('Largest valley reached') +
  p_theme

plot_grid(
  plot +
    theme(legend.position = "none"),
  legend,
  nrow = 2,
  rel_heights = c(3, 1)
)
```



4.2.3.1 Stats

Summary statistics for the largest valley crossed.

```

valleys = filter(best_df, acro == 'exp' & var == 'ele_big_peak')
valleys$Sigma = factor(valleys$Sigma, levels = FS_LIST)
valleys %>%
  group_by(Sigma) %>%
  dplyr::summarise(
    count = n(),
    na_cnt = sum(is.na(val)),
    min = min(val, na.rm = TRUE),
    median = median(val, na.rm = TRUE),
    mean = mean(val, na.rm = TRUE),
    max = max(val, na.rm = TRUE),
    IQR = IQR(val, na.rm = TRUE)
  )

## # A tibble: 7 x 8
##   Sigma count na_cnt  min  median  mean   max    IQR
##   <fct> <int>   <int> <dbl>   <dbl> <dbl> <dbl> <dbl>
## 1 0        50      0    12     13.1  13.1    14     0
## 2 0.1      50      0    12     13.2  13.2    14     1
## 3 0.3      50      0    12     13.3  13.3    14     1
## 4 0.6      50      0    12     13.3  13.3    14     1
## 5 1.2      50      0    13     13.5  13.5    14     1
## 6 2.5      50      0    12     13.0  13.0    14    0.75

```

```
## 7 5      50      0    12     13   13.3    14   1
```

Kruskal–Wallis test illustrates evidence of statistical differences.

```
kruskal.test(val ~ Sigma, data = valleys)
```

```
##
## Kruskal-Wallis rank sum test
##
## data: val by Sigma
## Kruskal-Wallis chi-squared = 16.366, df = 6, p-value = 0.01192
```

Results for post-hoc Wilcoxon rank-sum test with a Bonferroni correction.

```
pairwise.wilcox.test(x = valleys$val, g = valleys$Sigma, p.adjust.method = "bonferroni",
                      paired = FALSE, conf.int = FALSE, alternative = 't')
```

```
##
## Pairwise comparisons using Wilcoxon rank sum test with continuity correction
##
## data: valleys$val and valleys$Sigma
##
##      0     0.1    0.3    0.6    1.2    2.5
## 0.1 1.000 -     -     -     -     -
## 0.3 0.869 1.000 -     -     -     -
## 0.6 1.000 1.000 1.000 -     -     -
## 1.2 0.042 1.000 1.000 1.000 -     -
## 2.5 1.000 1.000 0.589 1.000 0.046 -
## 5   0.869 1.000 1.000 1.000 1.000 0.589
##
## P value adjustment method: bonferroni
```

4.3 Ordered exploitation results

Here we present the results for **best performances** found by each selection scheme parameter on the ordered exploitation diagnostic with valleys. 50 replicates are conducted for each scheme explored.

4.3.1 Performance over time

Best performance in a population over time. Data points on the graph is the average performance across 50 replicates every 2000 generations. Shading comes from the best and worse performance across 50 replicates.

```
lines = filter(over_time_df, acro == 'ord') %>%
  group_by(Sigma, gen) %>%
  dplyr::summarise(
    min = min(pop_fit_max) / DIMENSIONALITY,
    mean = mean(pop_fit_max) / DIMENSIONALITY,
    max = max(pop_fit_max) / DIMENSIONALITY
  )

## `summarise()` has grouped output by 'Sigma'. You can override using the
## `.groups` argument.

over_time_plot = ggplot(lines, aes(x=gen, y=mean, group = Sigma, fill = Sigma, color = Sigma, shape = S
  geom_ribbon(aes(ymin = min, ymax = max), alpha = 0.1) +
  geom_line(size = 0.5) +
  geom_point(data = filter(lines, gen %% 2000 == 0 & gen != 0), size = 1.5, stroke = 2.0, alpha = 1.0) -
```

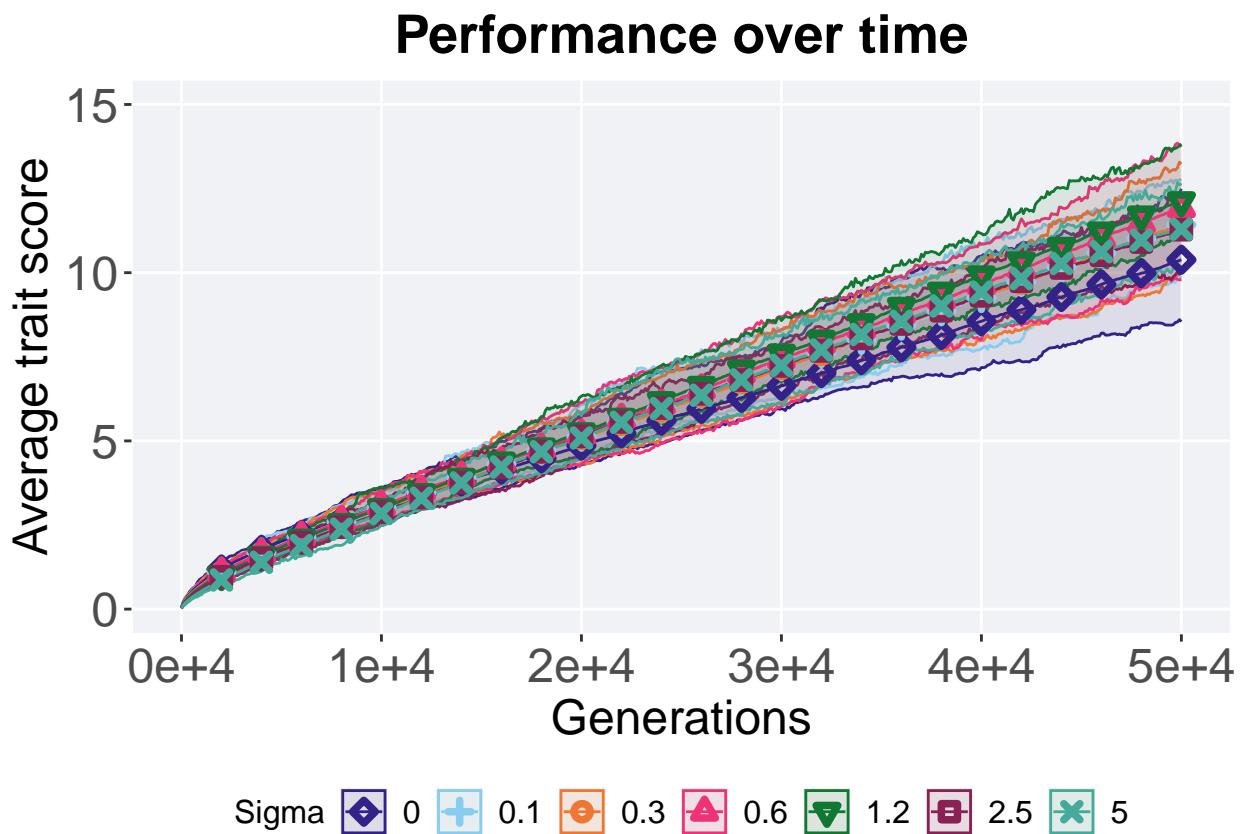
```

scale_y_continuous(
  name="Average trait score",
  limits = c(0,15)
) +
scale_x_continuous(
  name="Generations",
  limits=c(0, 50000),
  breaks=c(0, 10000, 20000, 30000, 40000, 50000),
  labels=c("0e+4", "1e+4", "2e+4", "3e+4", "4e+4", "5e+4")

) +
scale_shape_manual(values=SHAPE) +
scale_colour_manual(values = cb_palette) +
scale_fill_manual(values = cb_palette) +
ggtitle('Performance over time') +
p_theme +
guides(
  shape=guide_legend(nrow=1, title.position = "left", title = 'Sigma'),
  color=guide_legend(nrow=1, title.position = "left", title = 'Sigma'),
  fill=guide_legend(nrow=1, title.position = "left", title = 'Sigma')
)

over_time_plot

```

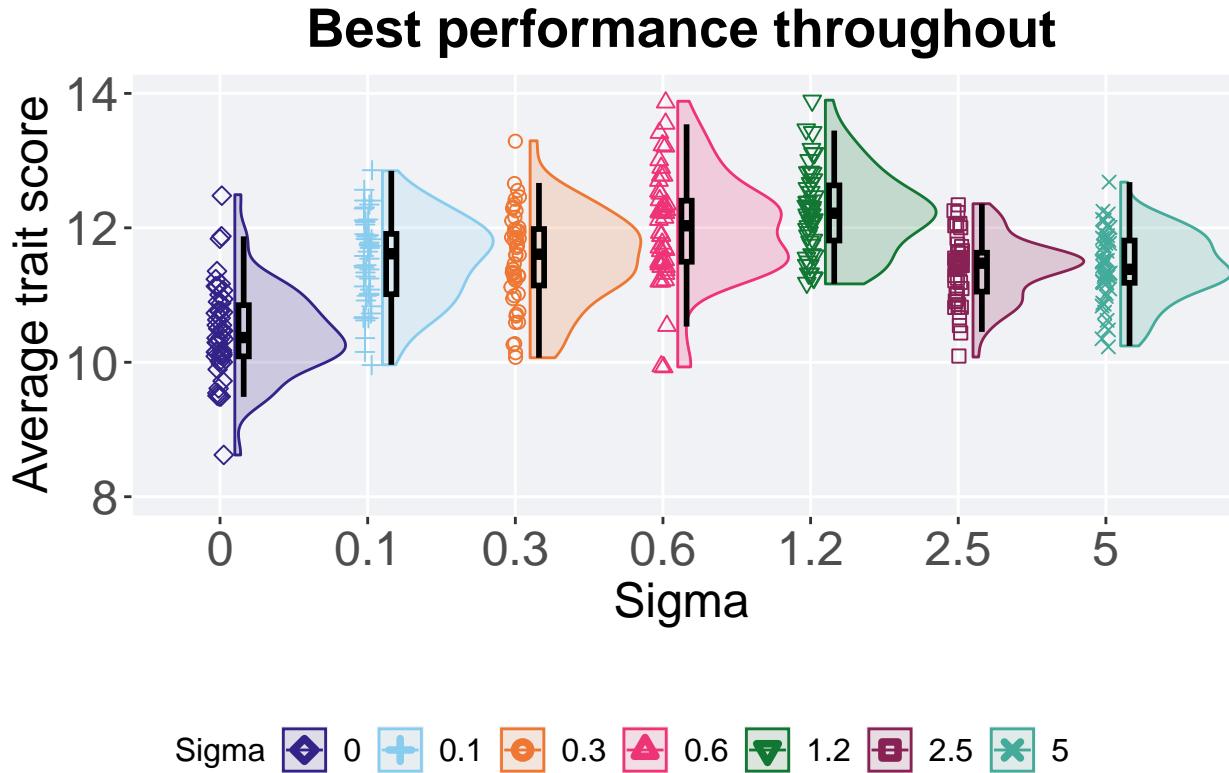


4.3.2 Best performance throughout

Best performance reached throughout 50,000 generations in a population.

```
plot = filter(best_df, acro == 'ord' & var == 'pop_fit_max') %>%
  ggplot(., aes(x = Sigma, y = val / DIMENSIONALITY, color = Sigma, fill = Sigma, shape = Sigma)) +
  geom_flat_violin(position = position_nudge(x = .1, y = 0), scale = 'width', alpha = 0.2, width = 1.5) +
  geom_boxplot(color = 'black', width = .07, outlier.shape = NA, alpha = 0.0, size = 1.0, position = position_nudge(x = -.1, y = 0)) +
  geom_point(position = position_jitter(width = 0.03, height = 0.02), size = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name = "Average trait score",
    limits = c(8, 14)
  ) +
  scale_x_discrete(
    name = "Sigma"
  ) +
  scale_shape_manual(values = SHAPE) +
  scale_colour_manual(values = cb_palette, ) +
  scale_fill_manual(values = cb_palette) +
  ggtitle('Best performance throughout') +
  p_theme

plot_grid(
  plot +
    theme(legend.position = "none"),
  legend,
  nrow = 2,
  rel_heights = c(3, 1)
)
```



4.3.2.1 Stats

Summary statistics for the best performance.

```
performance = filter(best_df, acro == 'ord' & var == 'pop_fit_max')
performance$Sigma = factor(performance$Sigma, levels = FS_LIST)
performance %>%
  group_by(Sigma) %>%
  dplyr::summarise(
```

```
  count = n(),
  na_cnt = sum(is.na(val)),
  min = min(val / DIMENSIONALITY, na.rm = TRUE),
  median = median(val / DIMENSIONALITY, na.rm = TRUE),
  mean = mean(val / DIMENSIONALITY, na.rm = TRUE),
  max = max(val / DIMENSIONALITY, na.rm = TRUE),
  IQR = IQR(val / DIMENSIONALITY, na.rm = TRUE)
)
```

```
## # A tibble: 7 x 8
##   Sigma count na_cnt  min  median  mean  max    IQR
##   <fct> <int>   <int> <dbl>   <dbl> <dbl> <dbl> <dbl>
## 1 0        50      0  8.62    10.4  10.5  12.5  0.761
## 2 0.1      50      0  9.96    11.6  11.5  12.8  0.898
## 3 0.3      50      0 10.1     11.6  11.5  13.3  0.840
## 4 0.6      50      0  9.93    12.0  12.0  13.9  0.908
## 5 1.2      50      0 11.2     12.2  12.2  13.9  0.821
## 6 2.5      50      0 10.1     11.5  11.4  12.4  0.579
```

```
## 7 5      50      0 10.2    11.4 11.4 12.7 0.629
```

Kruskal–Wallis test illustrates evidence of statistical differences.

```
kruskal.test(val ~ Sigma, data = performance)
```

```
##
## Kruskal-Wallis rank sum test
##
## data: val by Sigma
## Kruskal-Wallis chi-squared = 128.96, df = 6, p-value < 2.2e-16
```

Results for post-hoc Wilcoxon rank-sum test with a Bonferroni correction.

```
pairwise.wilcox.test(x = performance$val, g = performance$Sigma, p.adjust.method = "bonferroni",
                      paired = FALSE, conf.int = FALSE, alternative = 't')
```

```
##
## Pairwise comparisons using Wilcoxon rank sum test with continuity correction
##
## data: performance$val and performance$Sigma
##
##          0       0.1      0.3      0.6      1.2      2.5
## 0.1 5.3e-09 -       -       -       -       -
## 0.3 6.1e-09 1.00000 -       -       -       -
## 0.6 4.5e-12 0.04909 0.06661 -       -       -
## 1.2 8.0e-15 3.3e-05 0.00010 1.00000 -       -
## 2.5 1.9e-09 1.00000 1.00000 0.00035 2.6e-08 -
## 5   2.0e-09 1.00000 1.00000 0.00056 8.8e-08 1.00000
##
## P value adjustment method: bonferroni
```

4.3.3 Largest valley reached throughout

Largest valley reached in a single trait by the best performing solution throughout an entire evolutionary run.

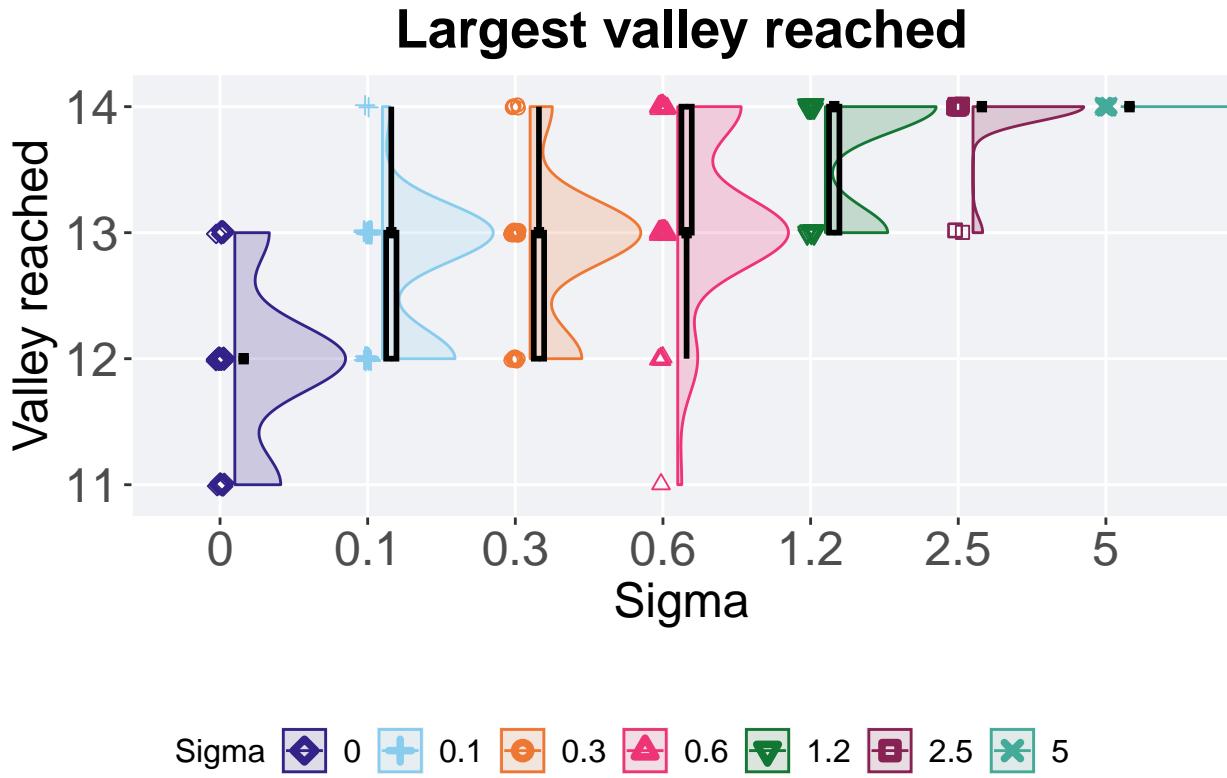
```
plot = filter(best_df, acro == 'ord' & var == 'ele_big_peak') %>%
  ggplot(., aes(x = Sigma, y = val, color = Sigma, fill = Sigma, shape = Sigma)) +
  geom_flat_violin(position = position_nudge(x = .1, y = 0), scale = 'width', alpha = 0.2, width = 1.5) +
  geom_boxplot(color = 'black', width = .07, outlier.shape = NA, alpha = 0.0, size = 1.0, position = position_nudge(x = -.1, y = 0)) +
  geom_point(position = position_jitter(width = 0.03, height = 0.02), size = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name = "Valley reached",
    limits = c(10.9, 14.1),
    breaks = c(11, 12, 13, 14)
  ) +
  scale_x_discrete(
    name = "Sigma"
  ) +
  scale_shape_manual(values = SHAPE) +
  scale_colour_manual(values = cb_palette, ) +
  scale_fill_manual(values = cb_palette) +
  ggtitle('Largest valley reached') +
  p_theme

plot_grid(
  plot +
```

```

    theme(legend.position="none"),
  legend,
  nrow=2,
  rel_heights = c(3,1)
)

```



4.3.3.1 Stats

Summary statistics for the largest valley crossed.

```

valleys = filter(best_df, acro == 'ord' & var == 'ele_big_peak')
valleys$Sigma = factor(valleys$Sigma, levels = FS_LIST)
valleys %>%
  group_by(Sigma) %>%
  dplyr::summarise(
    count = n(),
    na_cnt = sum(is.na(val)),
    min = min(val, na.rm = TRUE),
    median = median(val, na.rm = TRUE),
    mean = mean(val, na.rm = TRUE),
    max = max(val, na.rm = TRUE),
    IQR = IQR(val, na.rm = TRUE)
  )

## # A tibble: 7 x 8
##   Sigma count na_cnt  min  median  mean   max    IQR
##   <fct> <int>   <int> <dbl>   <dbl> <dbl> <dbl> <dbl>
## 1 0       10      10  11.0  12.0  12.0  13.0  1.0
## 2 0.1     10      10  11.8  12.8  12.8  13.8  1.0
## 3 0.3     10      10  12.0  13.0  13.0  14.0  1.0
## 4 0.6     10      10  11.8  13.0  13.0  14.2  1.0
## 5 1.2     10      10  12.8  13.8  13.8  14.0  1.0
## 6 2.5     10      10  13.0  14.0  14.0  14.0  1.0
## 7 5       10      10  14.0  14.0  14.0  14.0  1.0

```

```
## 1 0      50    0    11    12   11.9    13    0
## 2 0.1    50    0    12    13   12.7    14    1
## 3 0.3    50    0    12    13   12.8    14    1
## 4 0.6    50    0    11    13   13.2    14    1
## 5 1.2    50    0    13    14   13.6    14    1
## 6 2.5    50    0    13    14   13.9    14    0
## 7 5      50    0    14    14   14     14    0
```

Kruskal–Wallis test illustrates evidence of statistical differences.

```
kruskal.test(val ~ Sigma, data = valleys)
```

```
##
##  Kruskal-Wallis rank sum test
##
## data: val by Sigma
## Kruskal-Wallis chi-squared = 231.15, df = 6, p-value < 2.2e-16
```

Results for post-hoc Wilcoxon rank-sum test with a Bonferroni correction.

```
pairwise.wilcox.test(x = valleys$val, g = valleys$Sigma, p.adjust.method = "bonferroni",
                      paired = FALSE, conf.int = FALSE, alternative = 't')
```

```
##
##  Pairwise comparisons using Wilcoxon rank sum test with continuity correction
##
## data: valleys$val and valleys$Sigma
##
##          0       0.1      0.3      0.6      1.2      2.5
## 0.1 7.9e-06 -
## 0.3 1.1e-07 1.0000 -
## 0.6 6.4e-11 0.0013 0.1424 -
## 1.2 1.0e-15 1.2e-10 9.6e-08 0.0093 -
## 2.5 < 2e-16 3.3e-16 9.9e-14 1.6e-08 0.0165 -
## 5   < 2e-16 < 2e-16 3.9e-16 3.8e-11 6.8e-05 0.9092
##
## P value adjustment method: bonferroni
```

4.4 Contradictory objectives results

Here we present the results for **activation gene coverage** and **satisfactory trait coverage** found by each selection scheme parameter on the contradictory objectives diagnostic with valleys. 50 replicates are conducted for each scheme parameters explored.

4.4.1 Activation gene coverage over time

Activation gene coverage in a population over time. Data points on the graph is the average activation gene coverage across 50 replicates every 2000 generations. Shading comes from the best and worse coverage across 50 replicates.

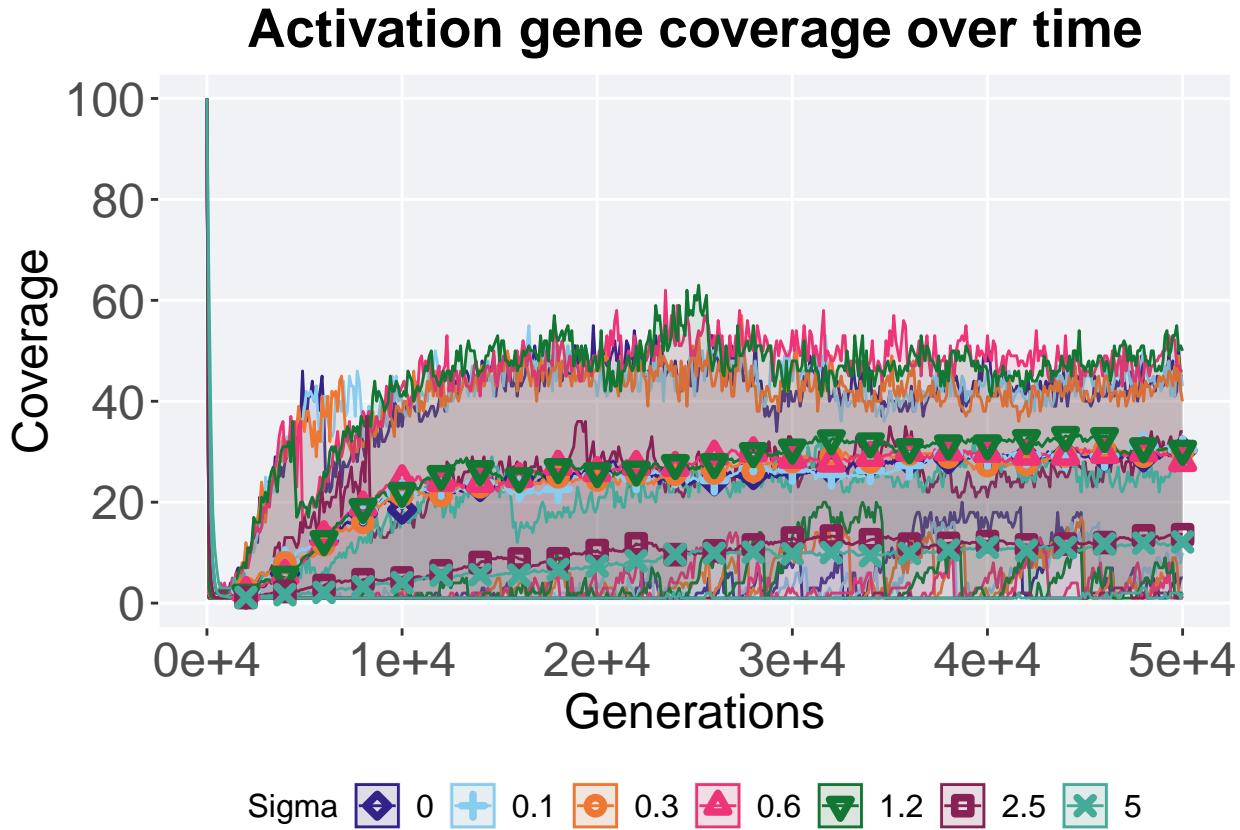
```
lines = filter(over_time_df, acro == 'con') %>%
  group_by(Sigma, gen) %>%
  dplyr::summarise(
    min = min(uni_str_pos),
    mean = mean(uni_str_pos),
    max = max(uni_str_pos)
```

```
)
```



```
## `summarise()` has grouped output by 'Sigma'. You can override using the
## ` `.groups` argument.

ggplot(lines, aes(x=gen, y=mean, group = Sigma, fill = Sigma, color = Sigma, shape = Sigma)) +
  geom_ribbon(aes(ymin = min, ymax = max), alpha = 0.1) +
  geom_line(size = 0.5) +
  geom_point(data = filter(lines, gen %% 2000 == 0 & gen != 0), size = 1.5, stroke = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name="Coverage",
    limits=c(0, 100),
    breaks=seq(0,100, 20),
    labels=c("0", "20", "40", "60", "80", "100")
  ) +
  scale_x_continuous(
    name="Generations",
    limits=c(0, 50000),
    breaks=c(0, 10000, 20000, 30000, 40000, 50000),
    labels=c("0e+4", "1e+4", "2e+4", "3e+4", "4e+4", "5e+4")
  ) +
  scale_shape_manual(values=SHAPE) +
  scale_colour_manual(values = cb_palette) +
  scale_fill_manual(values = cb_palette) +
  ggtitle('Activation gene coverage over time') +
  p_theme +
  guides(
    shape=guide_legend(nrow=1, title.position = "left", title = 'Sigma'),
    color=guide_legend(nrow=1, title.position = "left", title = 'Sigma'),
    fill=guide_legend(nrow=1, title.position = "left", title = 'Sigma')
  )
```



4.4.2 Final activation gene coverage

Activation gene coverage found in the final population at 50,000 generations.

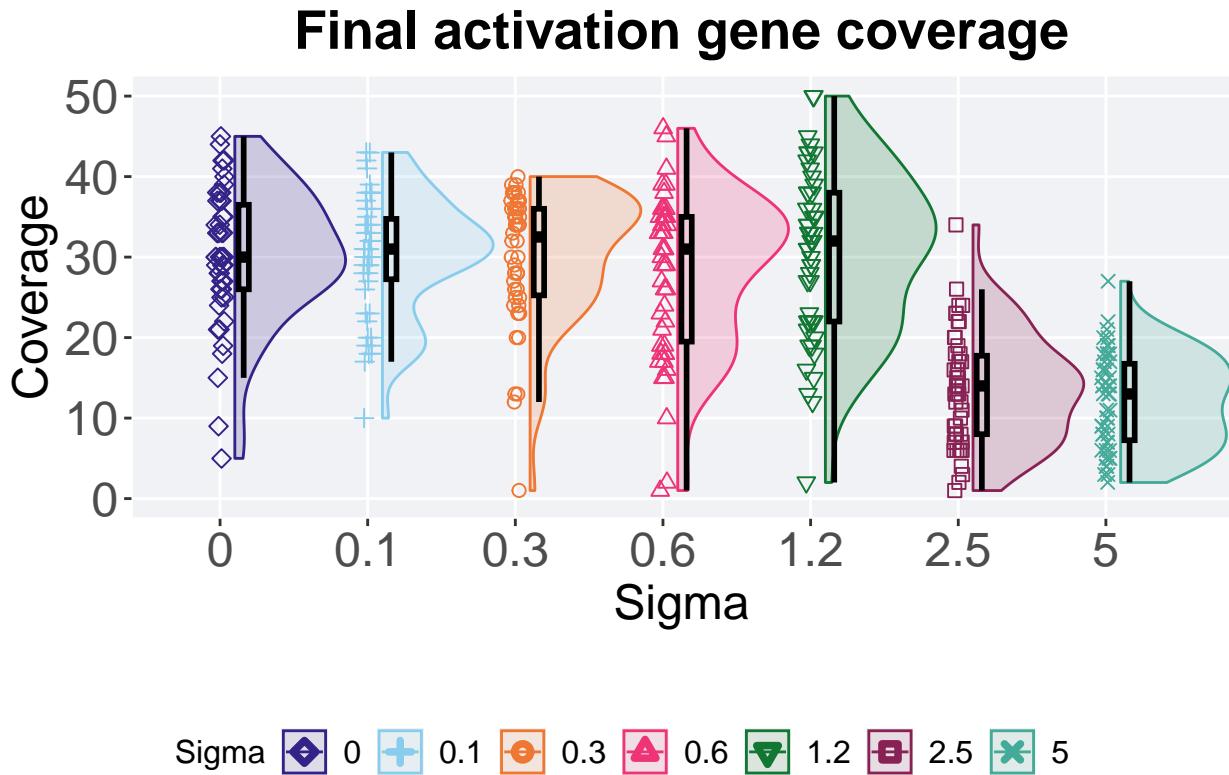
```
plot = filter(over_time_df, gen == 50000 & acro == 'con') %>%
  ggplot(., aes(x = Sigma, y = uni_str_pos, color = Sigma, fill = Sigma, shape = Sigma)) +
  geom_flat_violin(position = position_nudge(x = .1, y = 0), scale = 'width', alpha = 0.2, width = 1.5) +
  geom_boxplot(color = 'black', width = .07, outlier.shape = NA, alpha = 0.0, size = 1.0, position = position_nudge(x = -.1, y = 0)) +
  geom_point(position = position_jitter(width = 0.03, height = 0.02), size = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name="Coverage",
    limits=c(0, 50.1)
  ) +
  scale_x_discrete(
    name="Sigma"
  ) +
  scale_shape_manual(values=SHAPE) +
  scale_colour_manual(values = cb_palette,) +
  scale_fill_manual(values = cb_palette) +
  ggtitle('Final activation gene coverage') +
  p_theme

plot_grid(
  plot +
    theme(legend.position="none"),
  legend,
```

```

nrow=2,
rel_heights = c(3,1)
)

```



4.4.2.1 Stats

Summary statistics for the generation a satisfactory solution is found.

```

act_coverage = filter(over_time_df, gen == 50000 & acro == 'con')
act_coverage$Sigma = factor(act_coverage$Sigma, levels = FS_LIST)
act_coverage %>%
  group_by(Sigma) %>%
  dplyr::summarise(
    count = n(),
    na_cnt = sum(is.na(uni_str_pos)),
    min = min(uni_str_pos, na.rm = TRUE),
    median = median(uni_str_pos, na.rm = TRUE),
    mean = mean(uni_str_pos, na.rm = TRUE),
    max = max(uni_str_pos, na.rm = TRUE),
    IQR = IQR(uni_str_pos, na.rm = TRUE)
  )

## # A tibble: 7 x 8
##   Sigma count na_cnt  min median  mean  max    IQR
##   <fct> <int>   <int> <dbl> <dbl> <dbl> <dbl>
## 1 0         50      0     5    30   30.2    45 10.5
## 2 0.1       50      0    10    31   30.3    43  7.5

```

```
## 3 0.3      50     0     1   32.5   30      40 10.8
## 4 0.6      50     0     1    31    27.9    46 15.5
## 5 1.2      50     0     2    32    30.4    50 16
## 6 2.5      50     0     1    14    13.6    34 9.75
## 7 5        50     0     2    13    12.1    27 9.5
```

Kruskal-Wallis test illustrates evidence of statistical differences.

```
kruskal.test(uni_str_pos ~ Sigma, data = act_coverage)
```

```
##
## Kruskal-Wallis rank sum test
##
## data: uni_str_pos by Sigma
## Kruskal-Wallis chi-squared = 157.28, df = 6, p-value < 2.2e-16
```

Results for post-hoc Wilcoxon rank-sum test with a Bonferroni correction.

```
pairwise.wilcox.test(x = act_coverage$uni_str_pos, g = act_coverage$Sigma, p.adjust.method = "bonferroni",
                      paired = FALSE, conf.int = FALSE, alternative = 't')
```

```
##
## Pairwise comparisons using Wilcoxon rank sum test with continuity correction
##
## data: act_coverage$uni_str_pos and act_coverage$Sigma
##
##          0       0.1      0.3      0.6      1.2      2.5
## 0.1 1      -       -       -       -       -
## 0.3 1      1      -       -       -       -
## 0.6 1      1      1      -       -       -
## 1.2 1      1      1      1      -       -
## 2.5 2.7e-12 7.4e-13 6.8e-12 2.1e-09 9.8e-11 -
## 5    1.5e-13 2.0e-14 4.5e-13 1.1e-10 1.8e-12 1
##
## P value adjustment method: bonferroni
```

4.4.3 Satisfactory trait coverage over time

Satisfactory trait coverage in a population over time. Data points on the graph is the average activation gene coverage across 50 replicates every 2000 generations. Shading comes from the best and worse coverage across 50 replicates.

```
lines = filter(over_time_df, acro == 'con') %>%
  group_by(Sigma, gen) %>%
  dplyr::summarise(
    min = min(pop_uni_obj),
    mean = mean(pop_uni_obj),
    max = max(pop_uni_obj)
  )

## `summarise()` has grouped output by 'Sigma'. You can override using the
## `.groups` argument.

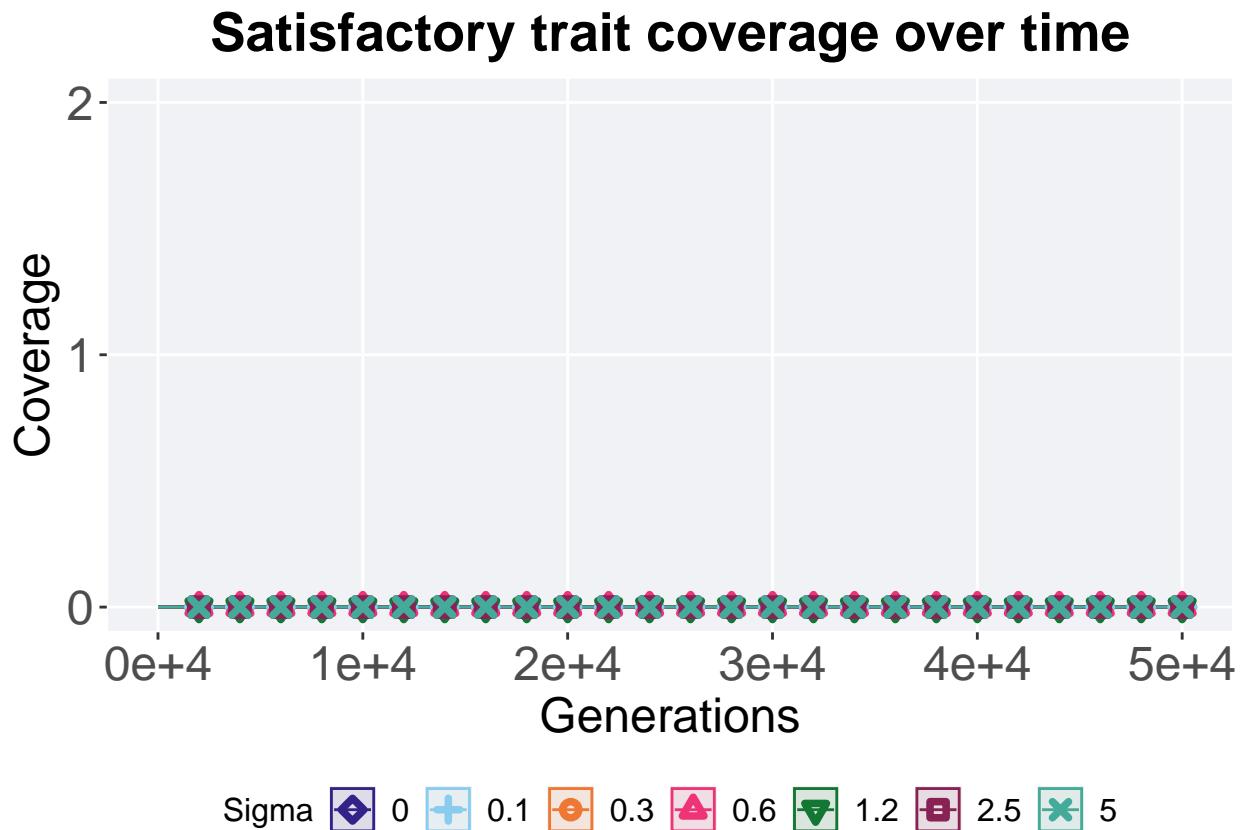
ggplot(lines, aes(x=gen, y=mean, group = Sigma, fill = Sigma, color = Sigma, shape = Sigma)) +
  geom_ribbon(aes(ymin = min, ymax = max), alpha = 0.1) +
  geom_line(size = 0.5) +
  geom_point(data = filter(lines, gen %% 2000 == 0 & gen != 0), size = 1.5, stroke = 2.0, alpha = 1.0) +
  scale_y_continuous()
```

```

name="Coverage",
limits=c(0, 2),
breaks=c(0,1,2)
) +
scale_x_continuous(
name="Generations",
limits=c(0, 50000),
breaks=c(0, 10000, 20000, 30000, 40000, 50000),
labels=c("0e+4", "1e+4", "2e+4", "3e+4", "4e+4", "5e+4")

) +
scale_shape_manual(values=SHAPE) +
scale_colour_manual(values = cb_palette) +
scale_fill_manual(values = cb_palette) +
ggtitle('Satisfactory trait coverage over time') +
p_theme +
guides(
shape=guide_legend(nrow=1, title.position = "left", title = 'Sigma'),
color=guide_legend(nrow=1, title.position = "left", title = 'Sigma'),
fill=guide_legend(nrow=1, title.position = "left", title = 'Sigma')
)

```



4.4.4 Final satisfactory trait coverage

Satisfactory trait coverage found in the final population at 50,000 generations.

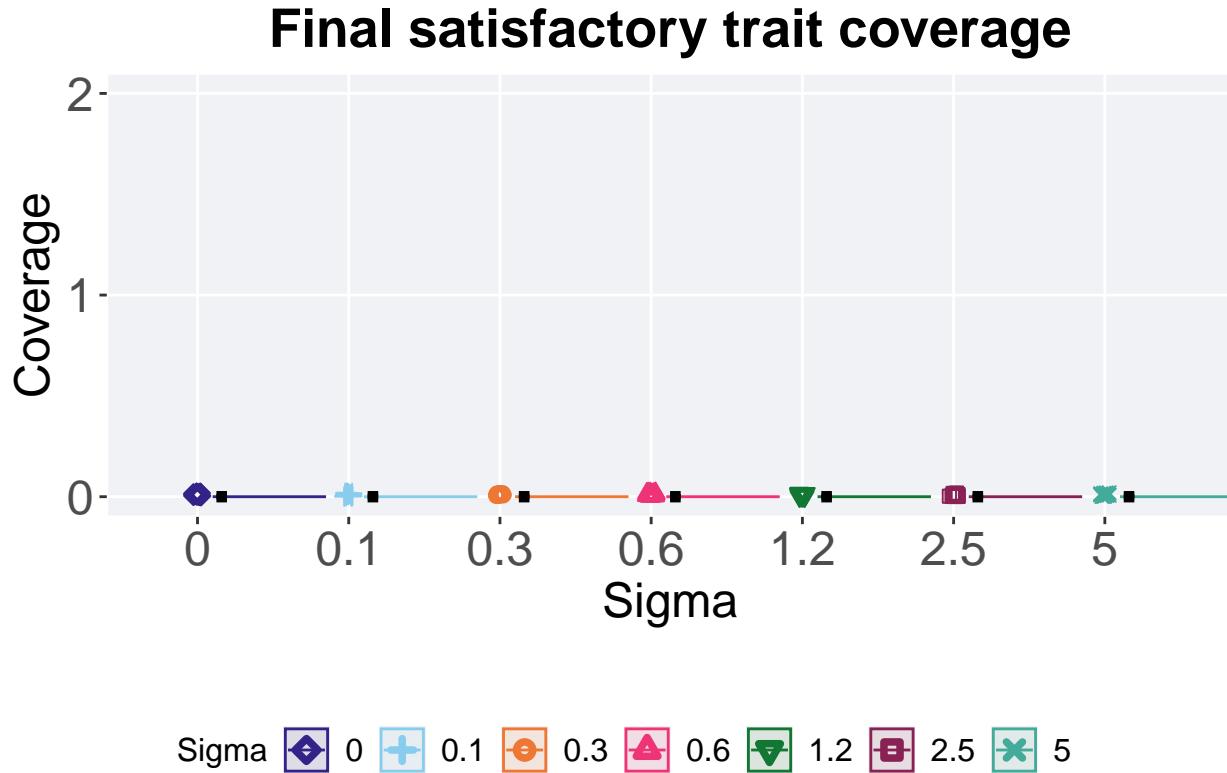
```

plot = filter(over_time_df, gen == 50000 & acro == 'con') %>%
  ggplot(., aes(x = Sigma, y = pop_uni_obj, color = Sigma, fill = Sigma, shape = Sigma)) +
  geom_flat_violin(position = position_nudge(x = .1, y = 0), scale = 'width', alpha = 0.2, width = 1.5)
  geom_boxplot(color = 'black', width = .07, outlier.shape = NA, alpha = 0.0, size = 1.0, position = position_nudge(x = .1, y = 0))
  geom_point(position = position_jitter(width = 0.03, height = 0.02), size = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name = "Coverage",
    limits = c(0, 2),
    breaks = c(0, 1, 2)
  ) +
  scale_x_discrete(
    name = "Sigma"
  ) +
  scale_shape_manual(values = SHAPE) +
  scale_color_manual(values = cb_palette) +
  scale_fill_manual(values = cb_palette) +
  ggtitle('Final satisfactory trait coverage') +
  p_theme

plot_grid(
  plot +
    theme(legend.position = "none"),
  legend,
  nrow = 2,
  rel_heights = c(3, 1)
)

```

Warning: Removed 192 rows containing missing values (`geom_point()`).



4.4.4.1 Stats

Summary statistics for the generation a satisfactory solution is found.

```

sat_coverage = filter(over_time_df, gen == 50000 & acro == 'con')
sat_coverage$Sigma = factor(sat_coverage$Sigma, levels = FS_LIST)
sat_coverage %>%
  group_by(Sigma) %>%
  dplyr::summarise(
    count = n(),
    na_cnt = sum(is.na(pop_uni_obj)),
    min = min(pop_uni_obj, na.rm = TRUE),
    median = median(pop_uni_obj, na.rm = TRUE),
    mean = mean(pop_uni_obj, na.rm = TRUE),
    max = max(pop_uni_obj, na.rm = TRUE),
    IQR = IQR(pop_uni_obj, na.rm = TRUE)
  )

## # A tibble: 7 x 8
##   Sigma count na_cnt  min median  mean  max  IQR
##   <fct> <int>   <int> <dbl> <dbl> <dbl> <dbl>
## 1 0        50      0     0     0     0     0     0
## 2 0.1      50      0     0     0     0     0     0
## 3 0.3      50      0     0     0     0     0     0
## 4 0.6      50      0     0     0     0     0     0
## 5 1.2      50      0     0     0     0     0     0
## 6 2.5      50      0     0     0     0     0     0

```

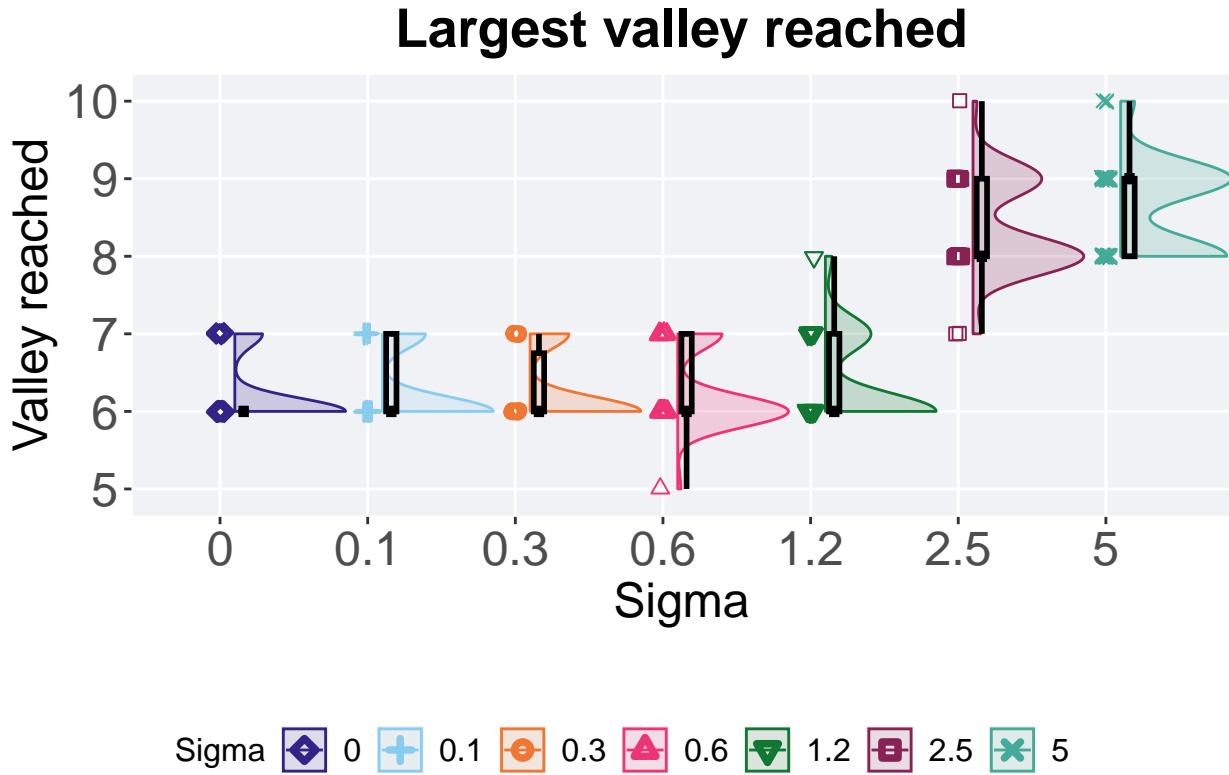
```
## 7 5      50      0      0      0      0      0      0
```

4.4.5 Largest valley reached throughout

Largest valley reached in a single trait by the best performing solution throughout an entire evolutionary run.

```
plot = filter(best_df, acro == 'con' & var == 'ele_big_peak') %>%
  ggplot(., aes(x = Sigma, y = val, color = Sigma, fill = Sigma, shape = Sigma)) +
  geom_flat_violin(position = position_nudge(x = .1, y = 0), scale = 'width', alpha = 0.2, width = 1.5) +
  geom_boxplot(color = 'black', width = .07, outlier.shape = NA, alpha = 0.0, size = 1.0, position = position_nudge(x = -.1, y = 0)) +
  geom_point(position = position_jitter(width = 0.03, height = 0.02), size = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name = "Valley reached",
    limits = c(4.9, 10.1)
  ) +
  scale_x_discrete(
    name = "Sigma"
  ) +
  scale_shape_manual(values = SHAPE) +
  scale_colour_manual(values = cb_palette, ) +
  scale_fill_manual(values = cb_palette) +
  ggtitle('Largest valley reached') +
  p_theme

plot_grid(
  plot +
    theme(legend.position = "none"),
  legend,
  nrow = 2,
  rel_heights = c(3, 1)
)
```



4.4.5.1 Stats

Summary statistics for the largest valley crossed.

```

valleys = filter(best_df, acro == 'con' & var == 'ele_big_peak')
valleys$Sigma = factor(valleys$Sigma, levels = FS_LIST)
valleys %>%
  group_by(Sigma) %>%
  dplyr::summarise(
    count = n(),
    na_cnt = sum(is.na(val)),
    min = min(val, na.rm = TRUE),
    median = median(val, na.rm = TRUE),
    mean = mean(val, na.rm = TRUE),
    max = max(val, na.rm = TRUE),
    IQR = IQR(val, na.rm = TRUE)
  )

## # A tibble: 7 x 8
##   Sigma count na_cnt  min median  mean  max    IQR
##   <fct> <int>   <int> <dbl>  <dbl>  <dbl> <dbl> <dbl>
## 1 0        50      0     6     6.2     7     0
## 2 0.1      50      0     6     6.28    7     1
## 3 0.3      50      0     6     6.26    7     0.75
## 4 0.6      50      0     5     6     6.26    7     1
## 5 1.2      50      0     6     6.36    8     1
## 6 2.5      50      0     7     8     8.36   10     1

```

```
## 7 5      50      0      8      9  8.6      10  1
```

Kruskal–Wallis test illustrates evidence of statistical differences.

```
kruskal.test(val ~ Sigma, data = valleys)
```

```
##
## Kruskal-Wallis rank sum test
##
## data: val by Sigma
## Kruskal-Wallis chi-squared = 248.66, df = 6, p-value < 2.2e-16
```

Results for post-hoc Wilcoxon rank-sum test with a Bonferroni correction.

```
pairwise.wilcox.test(x = valleys$val, g = valleys$Sigma, p.adjust.method = "bonferroni",
                      paired = FALSE, conf.int = FALSE, alternative = 't')
```

```
##
## Pairwise comparisons using Wilcoxon rank sum test with continuity correction
##
## data: valleys$val and valleys$Sigma
##
##      0     0.1    0.3    0.6    1.2    2.5
## 0.1 1     -     -     -     -     -
## 0.3 1     1     -     -     -     -
## 0.6 1     1     1     -     -     -
## 1.2 1     1     1     1     -     -
## 2.5 <2e-16 <2e-16 <2e-16 <2e-16 <2e-16 -
## 5    <2e-16 <2e-16 <2e-16 <2e-16 <2e-16 1
##
## P value adjustment method: bonferroni
```

4.5 Multi-path exploration results

Here we present the results for best performances and activation gene coverage found by each selection scheme parameter on the multi-path exploration diagnostic with valleys. 50 replicates are conducted for each scheme parameter explored.

4.5.1 Activation gene coverage over time

Activation gene coverage in a population over time. Data points on the graph is the average activation gene coverage across 50 replicates every 2000 generations. Shading comes from the best and worse coverage across 50 replicates.

```
lines = filter(over_time_df, acro == 'mpe') %>%
  group_by(Sigma, gen) %>%
  dplyr::summarise(
    min = min(uni_str_pos),
    mean = mean(uni_str_pos),
    max = max(uni_str_pos)
  )

## `summarise()` has grouped output by 'Sigma'. You can override using the
## `.groups` argument.

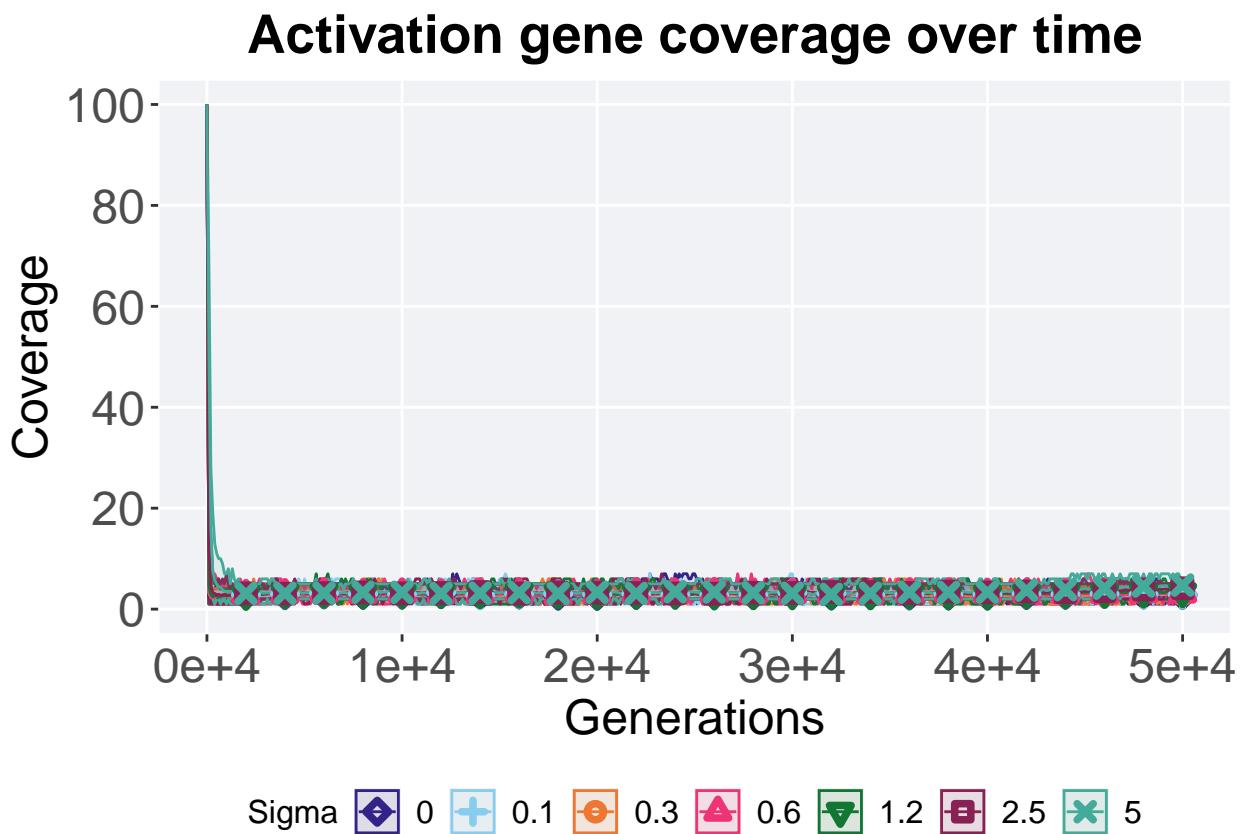
ggplot(lines, aes(x=gen, y=mean, group = Sigma, fill = Sigma, color = Sigma, shape = Sigma)) +
  geom_ribbon(aes(ymin = min, ymax = max), alpha = 0.1) +
```

```

geom_line(size = 0.5) +
geom_point(data = filter(lines, gen %% 2000 == 0 & gen != 0), size = 1.5, stroke = 2.0, alpha = 1.0) +
scale_y_continuous(
  name="Coverage",
  limits=c(0, 100),
  breaks=seq(0,100, 20),
  labels=c("0", "20", "40", "60", "80", "100")
) +
scale_x_continuous(
  name="Generations",
  limits=c(0, 50000),
  breaks=c(0, 10000, 20000, 30000, 40000, 50000),
  labels=c("0e+4", "1e+4", "2e+4", "3e+4", "4e+4", "5e+4")

) +
scale_shape_manual(values=SHAPE) +
scale_colour_manual(values = cb_palette) +
scale_fill_manual(values = cb_palette) +
ggtitle('Activation gene coverage over time') +
p_theme +
guides(
  shape=guide_legend(nrow=1, title.position = "left", title = 'Sigma'),
  color=guide_legend(nrow=1, title.position = "left", title = 'Sigma'),
  fill=guide_legend(nrow=1, title.position = "left", title = 'Sigma')
)

```

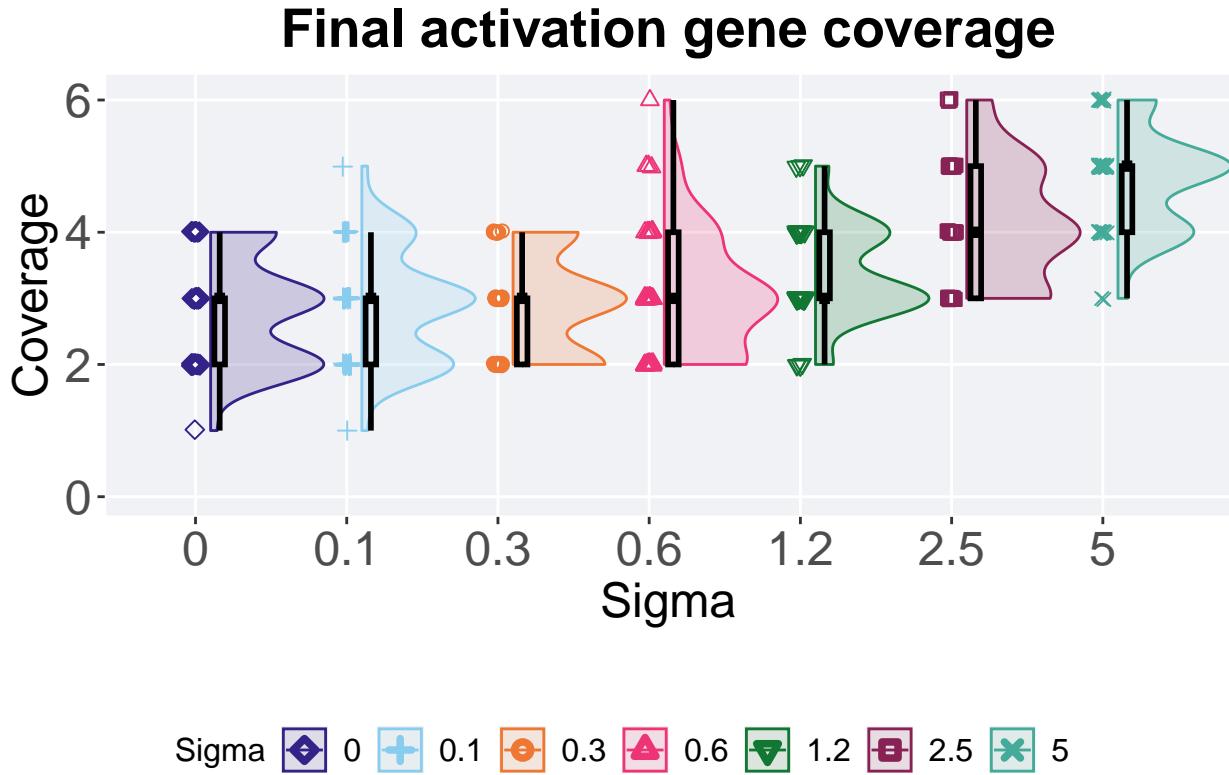


4.5.2 Final activation gene coverage

Activation gene coverage found in the final population at 50,000 generations.

```
plot = filter(over_time_df, gen == 50000 & acro == 'mpe') %>%
  ggplot(., aes(x = Sigma, y = uni_str_pos, color = Sigma, fill = Sigma, shape = Sigma)) +
  geom_flat_violin(position = position_nudge(x = .1, y = 0), scale = 'width', alpha = 0.2, width = 1.5) +
  geom_boxplot(color = 'black', width = .07, outlier.shape = NA, alpha = 0.0, size = 1.0, position = position_nudge(x = -.1, y = 0)) +
  geom_point(position = position_jitter(width = 0.03, height = 0.02), size = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name = "Coverage",
    limits = c(0, 6.1),
    breaks = seq(0, 6, 2)
  ) +
  scale_x_discrete(
    name = "Sigma"
  ) +
  scale_shape_manual(values = SHAPE) +
  scale_colour_manual(values = cb_palette, ) +
  scale_fill_manual(values = cb_palette) +
  ggtitle('Final activation gene coverage') +
  p_theme

plot_grid(
  plot +
    theme(legend.position = "none"),
  legend,
  nrow = 2,
  rel_heights = c(3, 1)
)
```



4.5.2.1 Stats

Summary statistics for the generation a satisfactory solution is found.

```
act_coverage = filter(over_time_df, gen == 50000 & acro == 'mpe')
act_coverage$Sigma = factor(act_coverage$Sigma, levels = FS_LIST)
act_coverage %>%
  group_by(Sigma) %>%
  dplyr::summarise(
    count = n(),
    na_cnt = sum(is.na(uni_str_pos)),
    min = min(uni_str_pos, na.rm = TRUE),
    median = median(uni_str_pos, na.rm = TRUE),
    mean = mean(uni_str_pos, na.rm = TRUE),
    max = max(uni_str_pos, na.rm = TRUE),
    IQR = IQR(uni_str_pos, na.rm = TRUE)
  )

## # A tibble: 7 x 8
##   Sigma count na_cnt  min median  mean   max   IQR
##   <fct> <int>   <int> <int>  <dbl> <dbl> <int> <dbl>
## 1 0        50      0     1     3    2.8     4     1
## 2 0.1      50      0     1     3    2.86    5     1
## 3 0.3      50      0     2     3    2.9     4     1
## 4 0.6      50      0     2     3    3.08    6     2
## 5 1.2      50      0     2     3    3.38    5     1
## 6 2.5      50      0     3     4    4.14    6     2
```

```
## 7 5      50      0      3      5  4.76      6      1
```

Kruskal–Wallis test illustrates evidence of statistical differences.

```
kruskal.test(uni_str_pos ~ Sigma, data = act_coverage)
```

```
##
## Kruskal-Wallis rank sum test
##
## data: uni_str_pos by Sigma
## Kruskal-Wallis chi-squared = 138.2, df = 6, p-value < 2.2e-16
```

Results for post-hoc Wilcoxon rank-sum test with a Bonferroni correction.

```
pairwise.wilcox.test(x = act_coverage$uni_str_pos, g = act_coverage$Sigma, p.adjust.method = "bonferroni",
                      paired = FALSE, conf.int = FALSE, alternative = 't')
```

```
##
## Pairwise comparisons using Wilcoxon rank sum test with continuity correction
##
## data: act_coverage$uni_str_pos and act_coverage$Sigma
##
##          0       0.1      0.3      0.6      1.2      2.5
## 0.1 1.0000  -       -       -       -       -
## 0.3 1.0000  1.0000  -       -       -       -
## 0.6 1.0000  1.0000  1.0000  -       -       -
## 1.2 0.0128  0.0347  0.0682  0.7788  -       -
## 2.5 1.7e-08 6.2e-08 7.8e-08 1.1e-05 0.0012  -
## 5   4.8e-14 1.6e-13 8.5e-14 4.3e-11 9.6e-11 0.0129
##
## P value adjustment method: bonferroni
```

4.5.3 Performance over time

Best performance in a population over time. Data points on the graph is the average performance across 50 replicates every 2000 generations. Shading comes from the best and worse performance across 50 replicates.

```
lines = filter(over_time_df, acro == 'mpe') %>%
  group_by(Sigma, gen) %>%
  dplyr::summarise(
    min = min(pop_fit_max) / DIMENSIONALITY,
    mean = mean(pop_fit_max) / DIMENSIONALITY,
    max = max(pop_fit_max) / DIMENSIONALITY
  )
```

```
## `summarise()` has grouped output by 'Sigma'. You can override using the
## `.groups` argument.
```

```
over_time_plot = ggplot(lines, aes(x=gen, y=mean, group = Sigma, fill = Sigma, color = Sigma, shape = S)
  geom_ribbon(aes(ymin = min, ymax = max), alpha = 0.1) +
  geom_line(size = 0.5) +
  geom_point(data = filter(lines, gen %% 2000 == 0 & gen != 0), size = 1.5, stroke = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name="Average trait score",
    limits=c(0, 15)
  ) +
  scale_x_continuous(
    name="Generations",
```

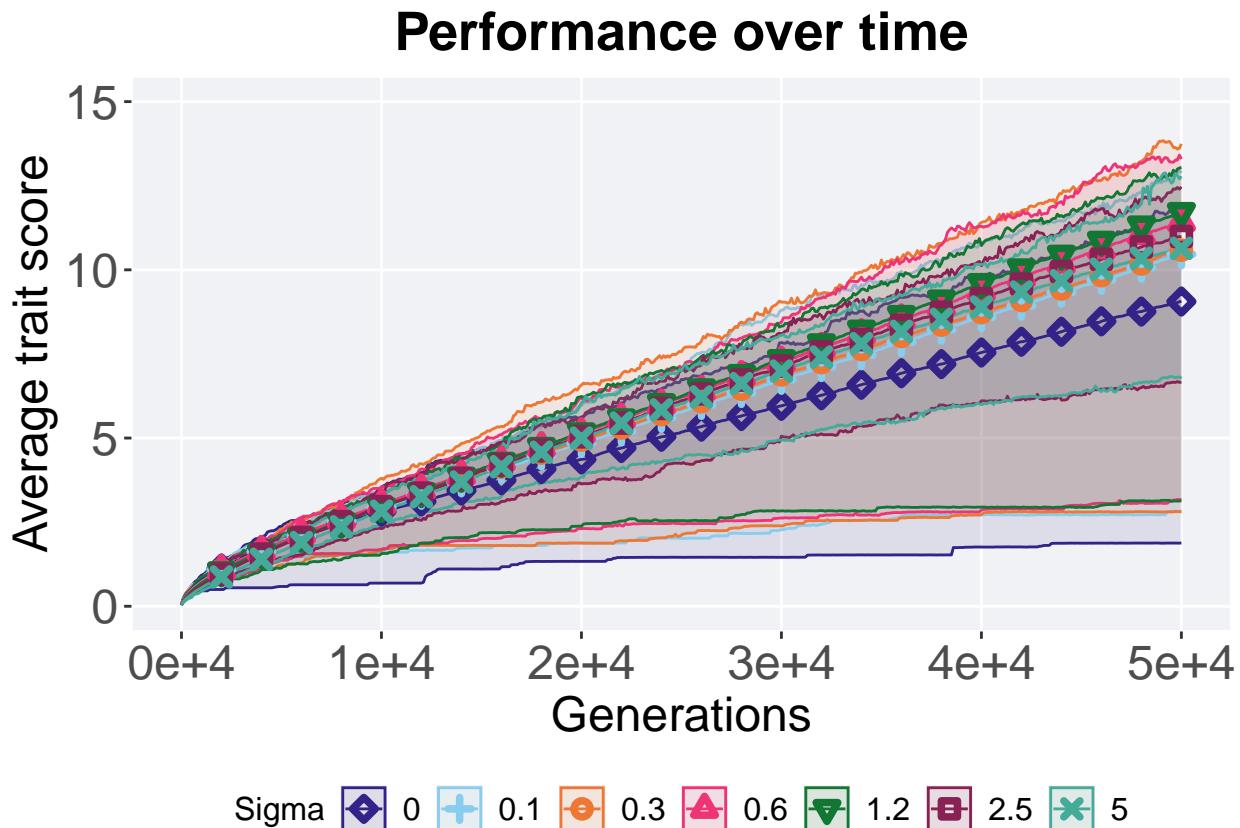
```

limits=c(0, 50000),
breaks=c(0, 10000, 20000, 30000, 40000, 50000),
labels=c("0e+4", "1e+4", "2e+4", "3e+4", "4e+4", "5e+4")

) +
scale_shape_manual(values=SHAPE) +
scale_colour_manual(values = cb_palette) +
scale_fill_manual(values = cb_palette) +
ggtitle('Performance over time') +
p_theme +
guides(
  shape=guide_legend(nrow=1, title.position = "left", title = 'Sigma'),
  color=guide_legend(nrow=1, title.position = "left", title = 'Sigma'),
  fill=guide_legend(nrow=1, title.position = "left", title = 'Sigma')
)

over_time_plot

```



4.5.4 Best performance throughout

Best performance reached throughout 50,000 generations in a population.

```

plot = filter(best_df, acro == 'mpe' & var == 'pop_fit_max') %>%
  ggplot(., aes(x = Sigma, y = val / DIMENSIONALITY, color = Sigma, fill = Sigma, shape = Sigma)) +
  geom_flat_violin(position = position_nudge(x = .1, y = 0), scale = 'width', alpha = 0.2, width = 1.5) +
  geom_boxplot(color = 'black', width = .07, outlier.shape = NA, alpha = 0.0, size = 1.0, position =

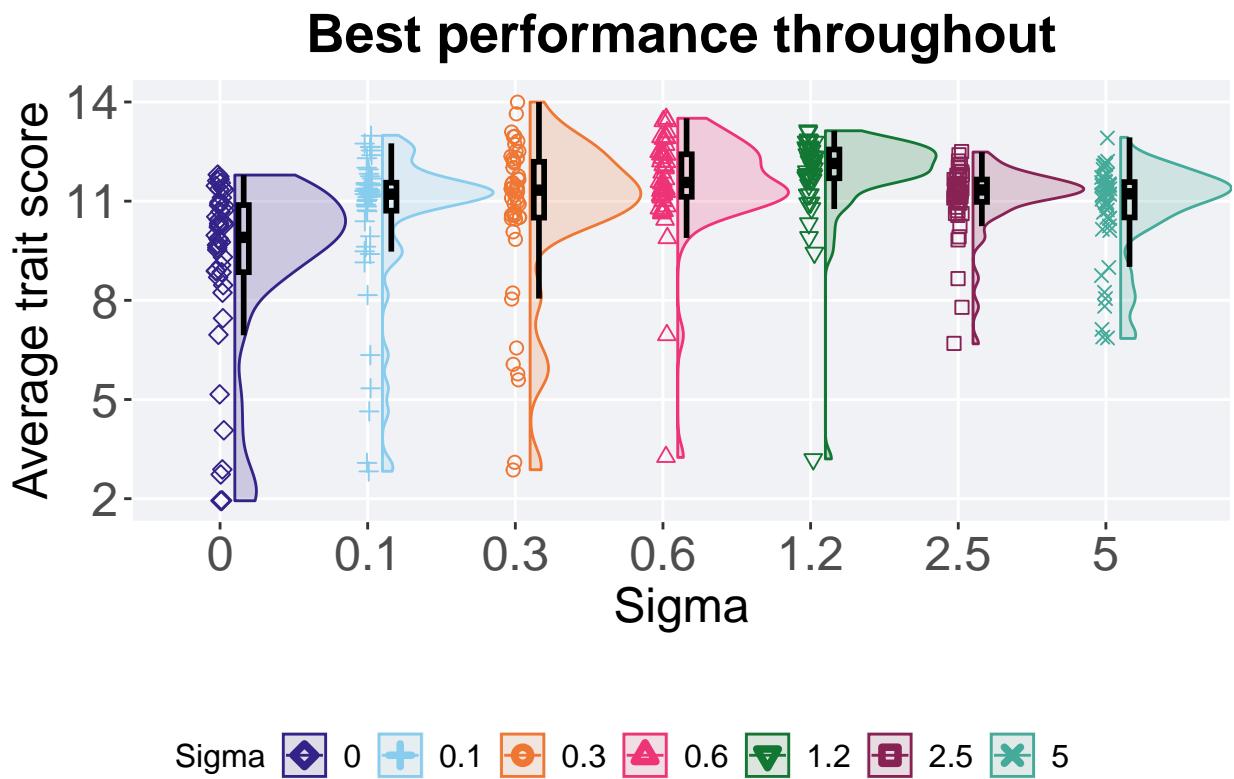
```

```

geom_point(position = position_jitter(width = 0.03, height = 0.02), size = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name="Average trait score",
    limits=c(1.9, 14.1),
    breaks=c(2,5,8,11,14)
  ) +
  scale_x_discrete(
    name="Sigma"
  ) +
  scale_shape_manual(values=SHAPE) +
  scale_colour_manual(values = cb_palette, ) +
  scale_fill_manual(values = cb_palette) +
  ggtitle('Best performance throughout') +
  p_theme

plot_grid(
  plot +
    theme(legend.position="none"),
  legend,
  nrow=2,
  rel_heights = c(3,1)
)
)

```



4.5.4.1 Stats

Summary statistics for the best performance.

```

performance = filter(best_df, acro == 'mpe' & var == 'pop_fit_max')
performance$Sigma = factor(performance$Sigma, levels = FS_LIST)
performance %>%
  group_by(Sigma) %>%
  dplyr::summarise(
    count = n(),
    na_cnt = sum(is.na(val)),
    min = min(val / DIMENSIONALITY, na.rm = TRUE),
    median = median(val / DIMENSIONALITY, na.rm = TRUE),
    mean = mean(val / DIMENSIONALITY, na.rm = TRUE),
    max = max(val / DIMENSIONALITY, na.rm = TRUE),
    IQR = IQR(val / DIMENSIONALITY, na.rm = TRUE)
  )

```

```

## # A tibble: 7 x 8
##   Sigma count na_cnt   min median   mean   max   IQR
##   <fct> <int> <int> <dbl> <dbl> <dbl> <dbl> <dbl>
## 1 0         50     0  1.94   9.90   9.12  11.8  2.03
## 2 0.1       50     0  2.83  11.2   10.5  13.0  0.846
## 3 0.3       50     0  2.88  11.3   10.7  14.0  1.69
## 4 0.6       50     0  3.25  11.6   11.5  13.5  1.29
## 5 1.2       50     0  3.20  12.1   11.8  13.1  0.870
## 6 2.5       50     0  6.68  11.3   11.1  12.5  0.677
## 7 5         50     0  6.85  11.2   10.8  12.9  1.07

```

Kruskal–Wallis test illustrates evidence of statistical differences.

```
kruskal.test(val ~ Sigma, data = performance)
```

```

##
## Kruskal-Wallis rank sum test
##
## data: val by Sigma
## Kruskal-Wallis chi-squared = 80.554, df = 6, p-value = 2.745e-15

```

Results for post-hoc Wilcoxon rank-sum test with a Bonferroni correction.

```
pairwise.wilcox.test(x = performance$val, g = performance$Sigma, p.adjust.method = "bonferroni",
                      paired = FALSE, conf.int = FALSE, alternative = 't')
```

```

##
## Pairwise comparisons using Wilcoxon rank sum test with continuity correction
##
## data: performance$val and performance$Sigma
##
##          0      0.1      0.3      0.6      1.2      2.5
## 0.1 0.00157 -      -      -      -      -
## 0.3 0.00067 1.00000 -      -      -      -
## 0.6 1.6e-08 0.16193 1.00000 -      -      -
## 1.2 6.8e-12 8.0e-05 0.01514 1.00000 -      -
## 2.5 2.3e-06 1.00000 1.00000 0.58011 8.3e-06 -
## 5    0.00075 1.00000 1.00000 0.08218 3.1e-06 1.00000
##
## P value adjustment method: bonferroni

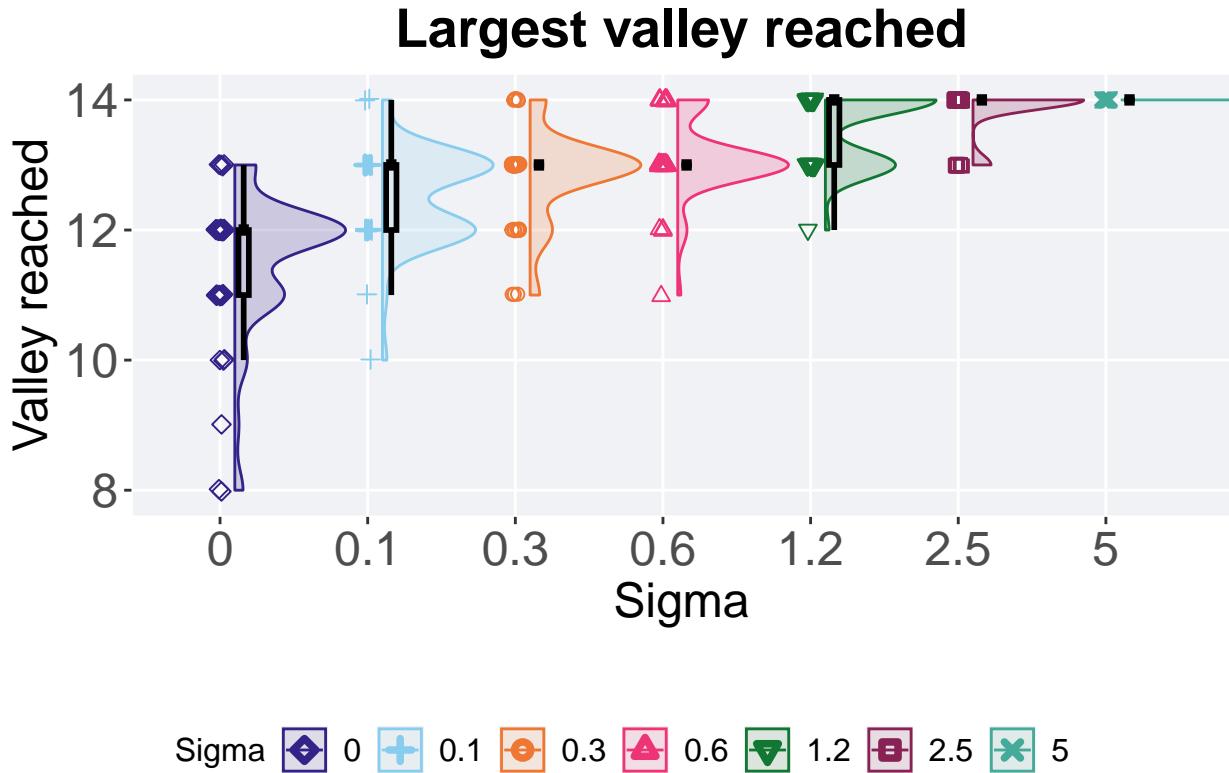
```

4.5.5 Largest valley reached throughout

Largest valley reached in a single trait by the best performing solution throughout an entire evolutionary run.

```
plot = filter(best_df, acro == 'mpe' & var == 'ele_big_peak') %>%
  ggplot(., aes(x = Sigma, y = val, color = Sigma, fill = Sigma, shape = Sigma)) +
  geom_flat_violin(position = position_nudge(x = .1, y = 0), scale = 'width', alpha = 0.2, width = 1.5) +
  geom_boxplot(color = 'black', width = .07, outlier.shape = NA, alpha = 0.0, size = 1.0, position = position_nudge(x = -.1, y = 0)) +
  geom_point(position = position_jitter(width = 0.03, height = 0.02), size = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name="Valley reached",
    limits=c(7.9,14.1)
  ) +
  scale_x_discrete(
    name="Sigma"
  ) +
  scale_shape_manual(values=SHAPE) +
  scale_colour_manual(values = cb_palette, ) +
  scale_fill_manual(values = cb_palette) +
  ggtitle('Largest valley reached') +
  p_theme

plot_grid(
  plot +
    theme(legend.position="none"),
  legend,
  nrow=2,
  rel_heights = c(3,1)
)
```



4.5.5.1 Stats

Summary statistics for the largest valley crossed.

```

valleys = filter(best_df, acro == 'mpe' & var == 'ele_big_peak')
valleys$Sigma = factor(valleys$Sigma, levels = FS_LIST)
valleys %>%
  group_by(Sigma) %>%
  dplyr::summarise(
    count = n(),
    na_cnt = sum(is.na(val)),
    min = min(val, na.rm = TRUE),
    median = median(val, na.rm = TRUE),
    mean = mean(val, na.rm = TRUE),
    max = max(val, na.rm = TRUE),
    IQR = IQR(val, na.rm = TRUE)
  )

## # A tibble: 7 x 8
##   Sigma count na_cnt  min median  mean  max  IQR
##   <fct> <int>   <int> <dbl>  <dbl>  <dbl> <dbl> <dbl>
## 1 0        50      0     8    12.  11.5    13     1
## 2 0.1      50      0    10    13.  12.5    14     1
## 3 0.3      50      0    11    13.  12.8    14     0
## 4 0.6      50      0    11    13.  13.1    14     0
## 5 1.2      50      0    12    14.  13.6    14     1
## 6 2.5      50      0    13    14.  13.9    14     0

```

```
## 7 5      50      0    14     14    14      0
```

Kruskal–Wallis test illustrates evidence of statistical differences.

```
kruskal.test(val ~ Sigma, data = valleys)
```

```
##  
## Kruskal-Wallis rank sum test  
##  
## data: val by Sigma  
## Kruskal-Wallis chi-squared = 238.55, df = 6, p-value < 2.2e-16
```

Results for post-hoc Wilcoxon rank-sum test with a Bonferroni correction.

```
pairwise.wilcox.test(x = valleys$val, g = valleys$Sigma, p.adjust.method = "bonferroni",  
                      paired = FALSE, conf.int = FALSE, alternative = 't')
```

```
##  
## Pairwise comparisons using Wilcoxon rank sum test with continuity correction  
##  
## data: valleys$val and valleys$Sigma  
##  
##          0       0.1      0.3      0.6      1.2      2.5  
## 0.1 2.2e-06 -       -       -       -  
## 0.3 4.4e-10 0.19650 -       -       -  
## 0.6 2.1e-13 0.00022 0.88310 -       -  
## 1.2 6.9e-16 4.6e-10 1.2e-06 0.00092 -  
## 2.5 < 2e-16 5.9e-15 1.2e-12 9.9e-10 0.07000 -  
## 5   < 2e-16 < 2e-16 < 2e-16 2.1e-14 1.4e-05 0.13628  
##  
## P value adjustment method: bonferroni
```


Chapter 5

Phenotypic fitness sharing

Results for the phenotypic fitness sharing parameter sweep on the diagnostics with valleys.

5.1 Data setup

```
over_time_df <- read.csv(paste(DATA_DIR, 'OVER-TIME-MVC/pfs.csv', sep = "", collapse = NULL), header = TRUE)
over_time_df$Sigma <- factor(over_time_df$Sigma, levels = FS_LIST)

best_df <- read.csv(paste(DATA_DIR, 'BEST-MVC/pfs.csv', sep = "", collapse = NULL), header = TRUE, stringsAsFactors = FALSE)
best_df$Sigma <- factor(best_df$Sigma, levels = FS_LIST)
```

5.2 Exploitation rate results

Here we present the results for **best performances** found by each selection scheme parameter on the exploitation rate diagnostic with valleys. 50 replicates are conducted for each scheme explored.

5.2.1 Performance over time

Best performance in a population over time. Data points on the graph is the average performance across 50 replicates every 2000 generations. Shading comes from the best and worse performance across 50 replicates.

```
lines = filter(over_time_df, acro == 'exp') %>%
  group_by(Sigma, gen) %>%
  dplyr::summarise(
    min = min(pop_fit_max) / DIMENSIONALITY,
    mean = mean(pop_fit_max) / DIMENSIONALITY,
    max = max(pop_fit_max) / DIMENSIONALITY
  )

## `summarise()` has grouped output by 'Sigma'. You can override using the
## `.groups` argument.

over_time_plot = ggplot(lines, aes(x=gen, y=mean, group = Sigma, fill = Sigma, color = Sigma, shape = Sigma))
  geom_ribbon(aes(ymin = min, ymax = max), alpha = 0.1) +
  geom_line(size = 0.5) +
  geom_point(data = filter(lines, gen %% 2000 == 0 & gen != 0), size = 1.5, stroke = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name="Average trait score",
```

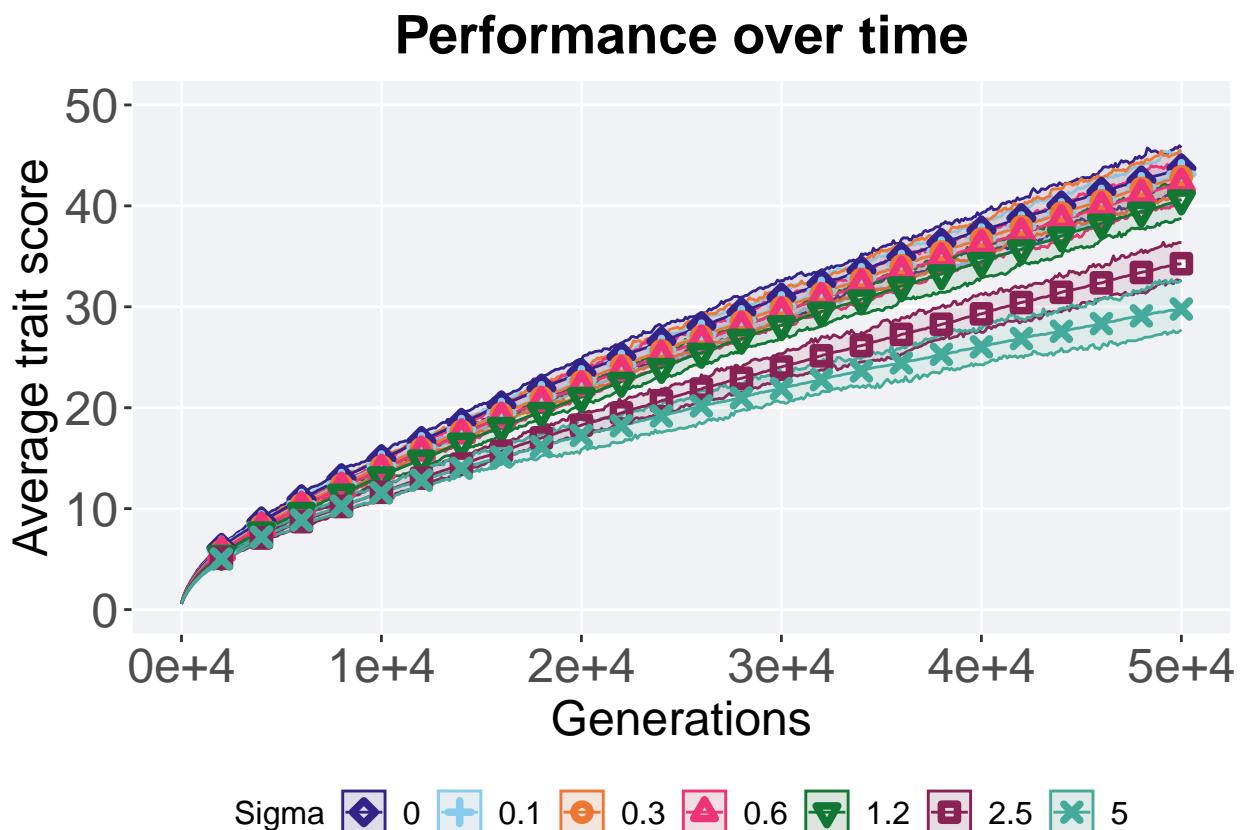
```

    limits = c(0,50)
) +
scale_x_continuous(
  name="Generations",
  limits=c(0, 50000),
  breaks=c(0, 10000, 20000, 30000, 40000, 50000),
  labels=c("0e+4", "1e+4", "2e+4", "3e+4", "4e+4", "5e+4")

) +
scale_shape_manual(values=SHAPE) +
scale_colour_manual(values = cb_palette) +
scale_fill_manual(values = cb_palette) +
ggtitle('Performance over time') +
p_theme +
guides(
  shape=guide_legend(nrow=1, title.position = "left", title = 'Sigma'),
  color=guide_legend(nrow=1, title.position = "left", title = 'Sigma'),
  fill=guide_legend(nrow=1, title.position = "left", title = 'Sigma')
)

over_time_plot

```



5.2.2 Best performance throughout

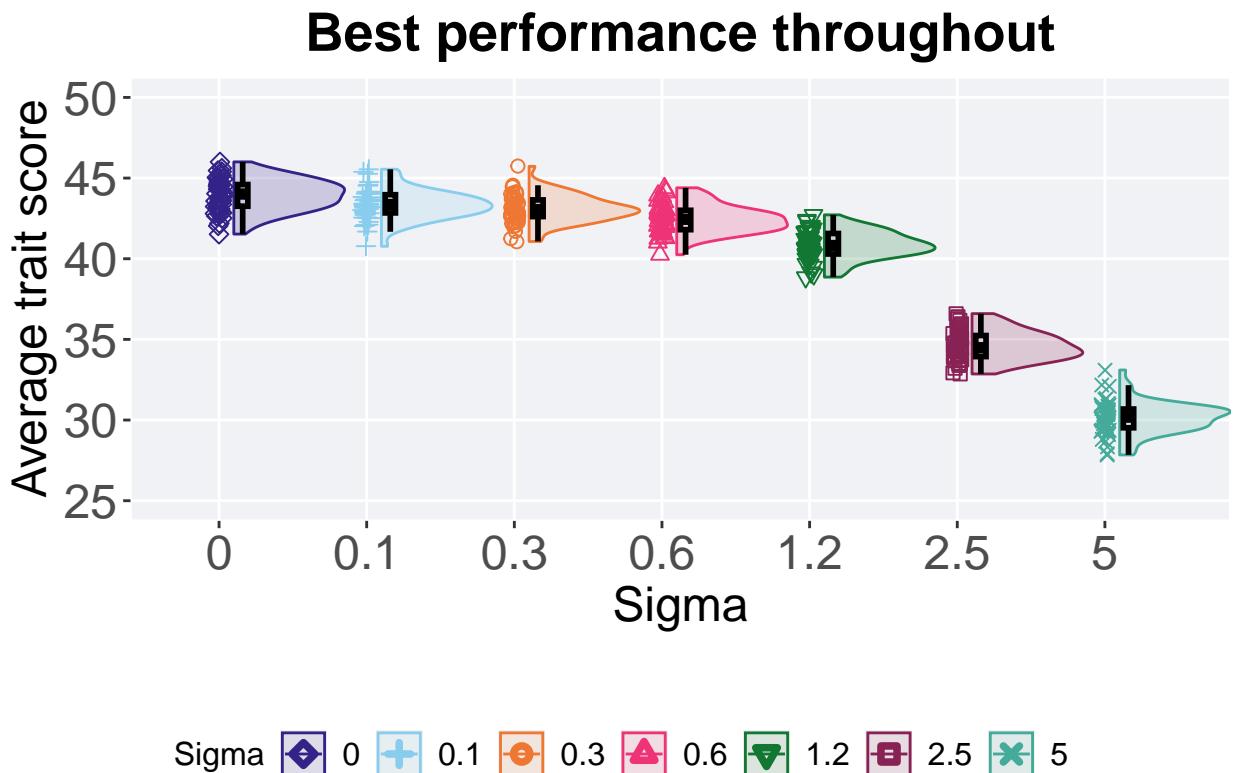
Best performance reached throughout 50,000 generations in a population.

```

plot = filter(best_df, acro == 'exp' & var == 'pop_fit_max') %>%
  ggplot(., aes(x = Sigma, y = val / DIMENSIONALITY, color = Sigma, fill = Sigma, shape = Sigma)) +
  geom_flat_violin(position = position_nudge(x = .1, y = 0), scale = 'width', alpha = 0.2, width = 1.5) +
  geom_boxplot(color = 'black', width = .07, outlier.shape = NA, alpha = 0.0, size = 1.0, position = position_nudge(x = -.1, y = 0)) +
  geom_point(position = position_jitter(width = 0.03, height = 0.02), size = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name = "Average trait score",
    limits = c(25, 50)
  ) +
  scale_x_discrete(
    name = "Sigma"
  ) +
  scale_shape_manual(values = SHAPE) +
  scale_color_manual(values = cb_palette,) +
  scale_fill_manual(values = cb_palette) +
  ggtitle('Best performance throughout') +
  p_theme

plot_grid(
  plot +
    theme(legend.position = "none"),
  legend,
  nrow = 2,
  rel_heights = c(3, 1)
)
)

```



5.2.2.1 Stats

Summary statistics for the best performance.

```
performance = filter(best_df, acro == 'exp' & var == 'pop_fit_max')
performance$Sigma = factor(performance$Sigma, levels = FS_LIST)
performance %>%
  group_by(Sigma) %>%
  dplyr::summarise(
    count = n(),
    na_cnt = sum(is.na(val)),
    min = min(val / DIMENSIONALITY, na.rm = TRUE),
    median = median(val / DIMENSIONALITY, na.rm = TRUE),
    mean = mean(val / DIMENSIONALITY, na.rm = TRUE),
    max = max(val / DIMENSIONALITY, na.rm = TRUE),
    IQR = IQR(val / DIMENSIONALITY, na.rm = TRUE)
  )

## # A tibble: 7 x 8
##   Sigma count na_cnt   min median   mean   max   IQR
##   <dbl> <int>   <dbl> <dbl> <dbl> <dbl> <dbl>
## 1 0      50     0  41.5  44.0  43.9  46.0  1.34
## 2 0.1    50     0  40.8  43.4  43.4  45.5  1.13
## 3 0.3    50     0  41.1  43.1  43.1  45.7  1.04
## 4 0.6    50     0  40.2  42.4  42.5  44.4  1.20
## 5 1.2    50     0  38.9  40.8  40.9  42.7  1.27
## 6 2.5    50     0  32.9  34.5  34.6  36.6  1.31
## 7 5      50     0  27.8  30.2  30.1  33.1  1.16
```

Kruskal–Wallis test illustrates evidence of statistical differences.

```
kruskal.test(val ~ Sigma, data = performance)
```

```
##
## Kruskal-Wallis rank sum test
##
## data: val by Sigma
## Kruskal-Wallis chi-squared = 290.52, df = 6, p-value < 2.2e-16
```

Results for post-hoc Wilcoxon rank-sum test with a Bonferroni correction.

```
pairwise.wilcox.test(x = performance$val, g = performance$Sigma, p.adjust.method = "bonferroni",
                      paired = FALSE, conf.int = FALSE, alternative = 't')

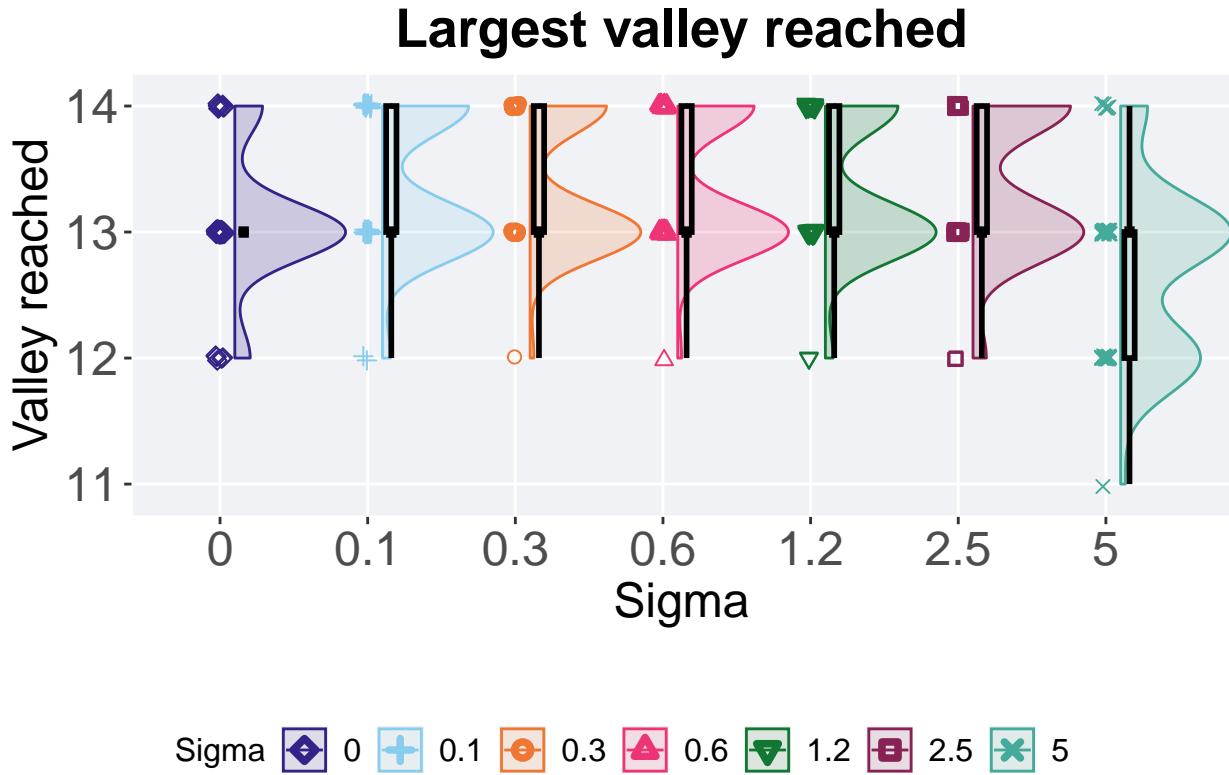
##
## Pairwise comparisons using Wilcoxon rank sum test with continuity correction
##
## data: performance$val and performance$Sigma
##
##          0       0.1      0.3      0.6      1.2      2.5
## 0.1 0.08040 -       -       -       -       -
## 0.3 0.00051 1.00000 -       -       -       -
## 0.6 4.4e-09 8.2e-05 0.00565 -       -       -
## 1.2 5.2e-16 3.8e-15 1.0e-14 5.6e-11 -       -
## 2.5 < 2e-16 < 2e-16 < 2e-16 < 2e-16 < 2e-16 -
## 5   < 2e-16 < 2e-16 < 2e-16 < 2e-16 < 2e-16 < 2e-16
##
## P value adjustment method: bonferroni
```

5.2.3 Largest valley reached throughout

Largest valley reached in a single trait by the best performing solution throughout an entire evolutionary run.

```
plot = filter(best_df, acro == 'exp' & var == 'ele_big_peak') %>%
  ggplot(., aes(x = Sigma, y = val, color = Sigma, fill = Sigma, shape = Sigma)) +
  geom_flat_violin(position = position_nudge(x = .1, y = 0), scale = 'width', alpha = 0.2, width = 1.5) +
  geom_boxplot(color = 'black', width = .07, outlier.shape = NA, alpha = 0.0, size = 1.0, position = position_nudge(x = -.1, y = 0)) +
  geom_point(position = position_jitter(width = 0.03, height = 0.02), size = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name = "Valley reached",
    limits = c(10.9, 14.1),
    breaks = c(10, 11, 12, 13, 14)
  ) +
  scale_x_discrete(
    name = "Sigma"
  ) +
  scale_shape_manual(values = SHAPE) +
  scale_colour_manual(values = cb_palette, ) +
  scale_fill_manual(values = cb_palette) +
  ggtitle('Largest valley reached') +
  p_theme

plot_grid(
  plot +
    theme(legend.position = "none"),
  legend,
  nrow = 2,
  rel_heights = c(3, 1)
)
```



5.2.3.1 Stats

Summary statistics for the largest valley crossed.

```

valleys = filter(best_df, acro == 'exp' & var == 'ele_big_peak')
valleys$Sigma = factor(valleys$Sigma, levels = FS_LIST)
valleys %>%
  group_by(Sigma) %>%
  dplyr::summarise(
    count = n(),
    na_cnt = sum(is.na(val)),
    min = min(val, na.rm = TRUE),
    median = median(val, na.rm = TRUE),
    mean = mean(val, na.rm = TRUE),
    max = max(val, na.rm = TRUE),
    IQR = IQR(val, na.rm = TRUE)
  )

## # A tibble: 7 x 8
##   Sigma count na_cnt  min  median  mean  max  IQR
##   <fct> <int>   <int> <dbl>   <dbl>   <dbl> <dbl>
## 1 0        50      0    12    13.1  13.1    14     0
## 2 0.1      50      0    12    13.4  13.4    14     1
## 3 0.3      50      0    12    13.4  13.4    14     1
## 4 0.6      50      0    12    13.4  13.4    14     1
## 5 1.2      50      0    12    13.3  13.3    14     1
## 6 2.5      50      0    12    13.4  13.4    14     1

```

```
## 7 5      50      0    11    13 12.7    14      1
```

Kruskal–Wallis test illustrates evidence of statistical differences.

```
kruskal.test(val ~ Sigma, data = valleys)
```

```
##
## Kruskal-Wallis rank sum test
##
## data: val by Sigma
## Kruskal-Wallis chi-squared = 43.54, df = 6, p-value = 9.117e-08
```

Results for post-hoc Wilcoxon rank-sum test with a Bonferroni correction.

```
pairwise.wilcox.test(x = valleys$val, g = valleys$Sigma, p.adjust.method = "bonferroni",
                      paired = FALSE, conf.int = FALSE, alternative = 't')
```

```
##
## Pairwise comparisons using Wilcoxon rank sum test with continuity correction
##
## data: valleys$val and valleys$Sigma
##
##      0     0.1    0.3    0.6    1.2    2.5
## 0.1 0.15268 -      -      -      -      -
## 0.3 0.14162 1.00000 -      -      -      -
## 0.6 0.14162 1.00000 1.00000 -      -      -
## 1.2 0.39290 1.00000 1.00000 1.00000 -      -
## 2.5 0.16234 1.00000 1.00000 1.00000 1.00000 -
## 5   0.09970 9.4e-05 6.3e-05 6.3e-05 0.00024 0.00014
##
## P value adjustment method: bonferroni
```

5.3 Ordered exploitation results

Here we present the results for **best performances** found by each selection scheme parameter on the ordered exploitation diagnostic with valleys. 50 replicates are conducted for each scheme explored.

5.3.1 Performance over time

Best performance in a population over time. Data points on the graph is the average performance across 50 replicates every 2000 generations. Shading comes from the best and worse performance across 50 replicates.

```
lines = filter(over_time_df, acro == 'ord') %>%
  group_by(Sigma, gen) %>%
  dplyr::summarise(
    min = min(pop_fit_max) / DIMENSIONALITY,
    mean = mean(pop_fit_max) / DIMENSIONALITY,
    max = max(pop_fit_max) / DIMENSIONALITY
  )

## `summarise()` has grouped output by 'Sigma'. You can override using the
## `.groups` argument.

over_time_plot = ggplot(lines, aes(x=gen, y=mean, group = Sigma, fill = Sigma, color = Sigma, shape = S
  geom_ribbon(aes(ymin = min, ymax = max), alpha = 0.1) +
  geom_line(size = 0.5) +
  geom_point(data = filter(lines, gen %% 2000 == 0 & gen != 0), size = 1.5, stroke = 2.0, alpha = 1.0) -
```

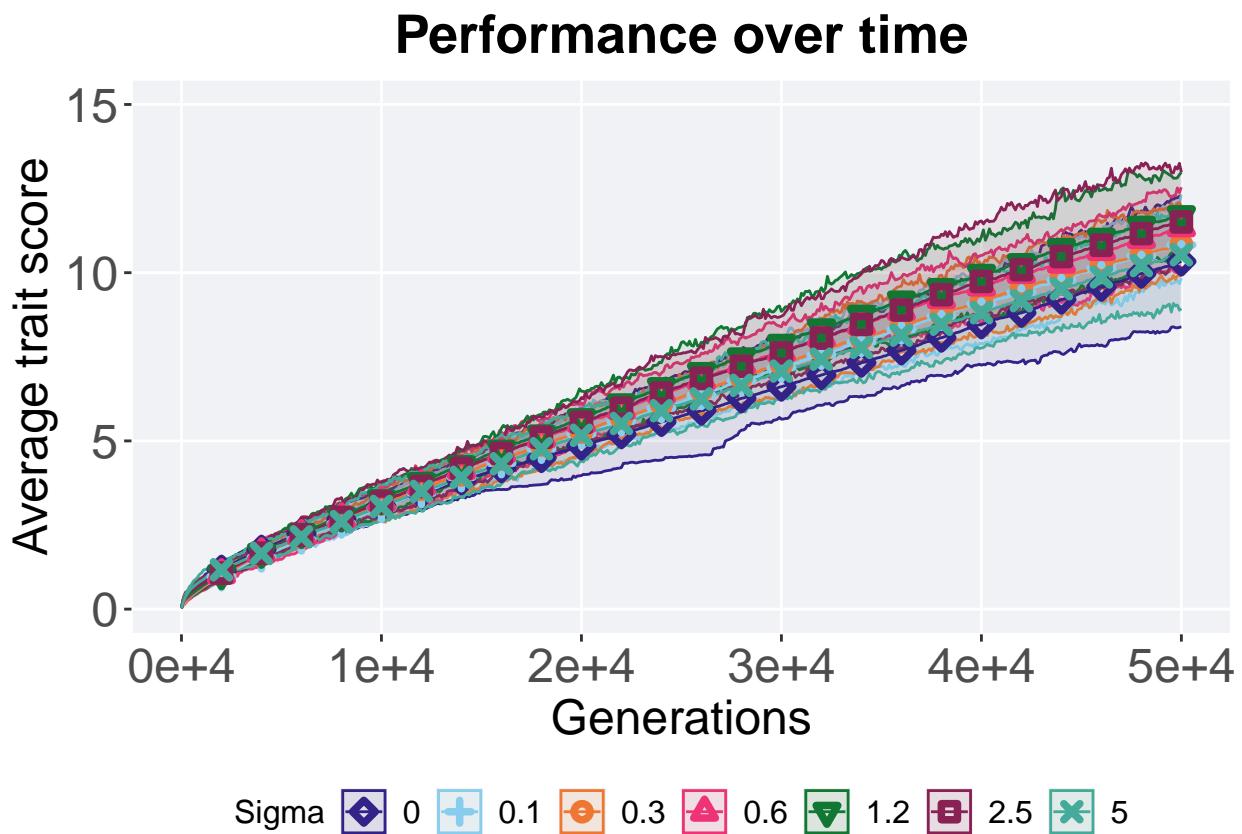
```

scale_y_continuous(
  name="Average trait score",
  limits = c(0,15)
) +
scale_x_continuous(
  name="Generations",
  limits=c(0, 50000),
  breaks=c(0, 10000, 20000, 30000, 40000, 50000),
  labels=c("0e+4", "1e+4", "2e+4", "3e+4", "4e+4", "5e+4")

) +
scale_shape_manual(values=SHAPE) +
scale_colour_manual(values = cb_palette) +
scale_fill_manual(values = cb_palette) +
ggtitle('Performance over time') +
p_theme +
guides(
  shape=guide_legend(nrow=1, title.position = "left", title = 'Sigma'),
  color=guide_legend(nrow=1, title.position = "left", title = 'Sigma'),
  fill=guide_legend(nrow=1, title.position = "left", title = 'Sigma')
)

over_time_plot

```

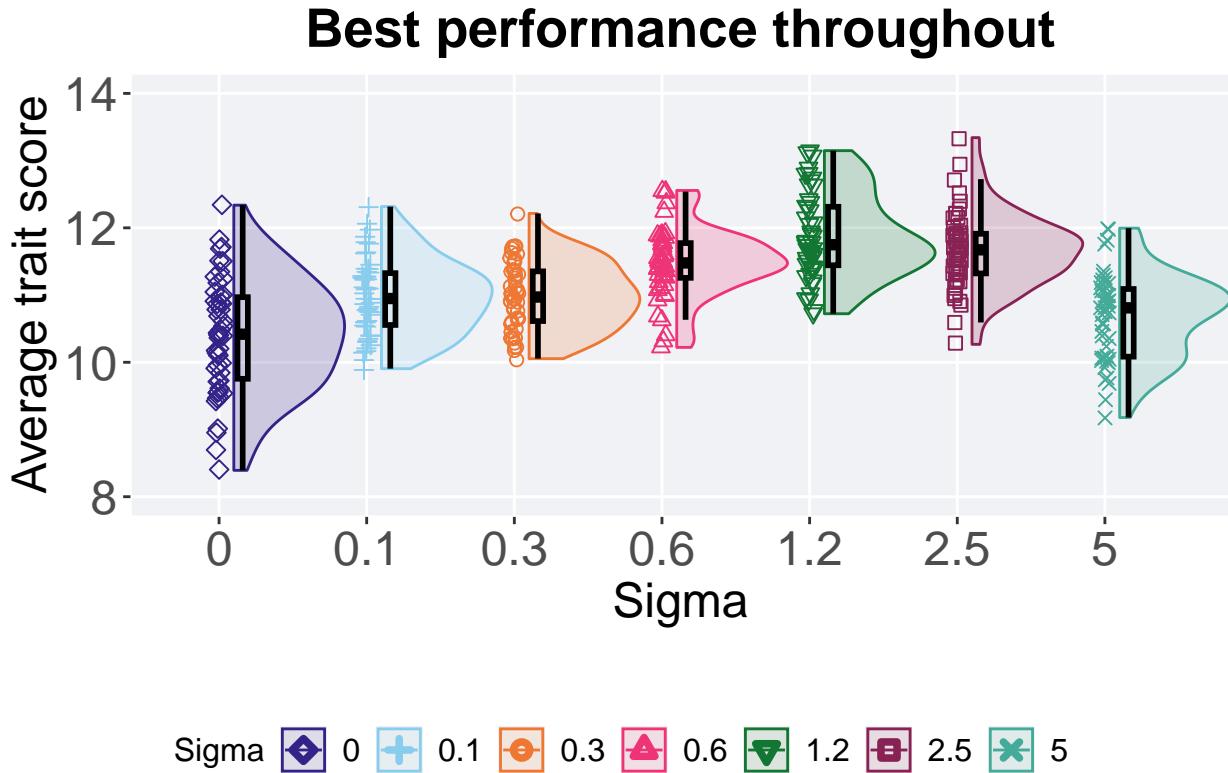


5.3.2 Best performance throughout

Best performance reached throughout 50,000 generations in a population.

```
plot = filter(best_df, acro == 'ord' & var == 'pop_fit_max') %>%
  ggplot(., aes(x = Sigma, y = val / DIMENSIONALITY, color = Sigma, fill = Sigma, shape = Sigma)) +
  geom_flat_violin(position = position_nudge(x = .1, y = 0), scale = 'width', alpha = 0.2, width = 1.5) +
  geom_boxplot(color = 'black', width = .07, outlier.shape = NA, alpha = 0.0, size = 1.0, position = position_nudge(x = -.1, y = 0)) +
  geom_point(position = position_jitter(width = 0.03, height = 0.02), size = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name = "Average trait score",
    limits = c(8, 14)
  ) +
  scale_x_discrete(
    name = "Sigma"
  ) +
  scale_shape_manual(values = SHAPE) +
  scale_colour_manual(values = cb_palette, ) +
  scale_fill_manual(values = cb_palette) +
  ggtitle('Best performance throughout') +
  p_theme

plot_grid(
  plot +
    theme(legend.position = "none"),
  legend,
  nrow = 2,
  rel_heights = c(3, 1)
)
```



5.3.2.1 Stats

Summary statistics for the best performance.

```
performance = filter(best_df, acro == 'ord' & var == 'pop_fit_max')
performance$Sigma = factor(performance$Sigma, levels = FS_LIST)
performance %>%
  group_by(Sigma) %>%
```

```
dplyr::summarise(
  count = n(),
  na_cnt = sum(is.na(val)),
  min = min(val / DIMENSIONALITY, na.rm = TRUE),
  median = median(val / DIMENSIONALITY, na.rm = TRUE),
  mean = mean(val / DIMENSIONALITY, na.rm = TRUE),
  max = max(val / DIMENSIONALITY, na.rm = TRUE),
  IQR = IQR(val / DIMENSIONALITY, na.rm = TRUE)
)
```

```
## # A tibble: 7 x 8
##   Sigma count na_cnt   min median   mean   max   IQR
##   <fct> <int>   <int>   <dbl>   <dbl>   <dbl>   <dbl>
## 1 0        50      0  8.39  10.4  10.4  12.3  1.21
## 2 0.1     50      0  9.90  10.9  11.0  12.3  0.771
## 3 0.3     50      0 10.1   11.0  11.0  12.2  0.739
## 4 0.6     50      0 10.2   11.5  11.5  12.6  0.517
## 5 1.2     50      0 10.7   11.7  11.9  13.1  0.870
## 6 2.5     50      0 10.3   11.7  11.7  13.3  0.590
```

```
## 7 5      50      0  9.18  10.8 10.7 12.0 1.01
```

Kruskal–Wallis test illustrates evidence of statistical differences.

```
kruskal.test(val ~ Sigma, data = performance)
```

```
##
## Kruskal-Wallis rank sum test
##
## data: val by Sigma
## Kruskal-Wallis chi-squared = 140.32, df = 6, p-value < 2.2e-16
```

Results for post-hoc Wilcoxon rank-sum test with a Bonferroni correction.

```
pairwise.wilcox.test(x = performance$val, g = performance$Sigma, p.adjust.method = "bonferroni",
                      paired = FALSE, conf.int = FALSE, alternative = 't')
```

```
##
## Pairwise comparisons using Wilcoxon rank sum test with continuity correction
##
## data: performance$val and performance$Sigma
##
##          0       0.1      0.3      0.6      1.2      2.5
## 0.1 0.01089 -      -      -      -      -
## 0.3 0.00397 1.00000 -      -      -      -
## 0.6 9.5e-09 0.00020 0.00017 -      -      -
## 1.2 9.3e-12 1.7e-08 6.6e-09 0.07200 -      -
## 2.5 1.6e-10 2.1e-06 1.0e-06 1.00000 1.00000 -
## 5   1.00000 0.71439 0.38977 5.5e-08 2.5e-11 6.5e-10
##
## P value adjustment method: bonferroni
```

5.3.3 Largest valley reached throughout

Largest valley reached in a single trait by the best performing solution throughout an entire evolutionary run.

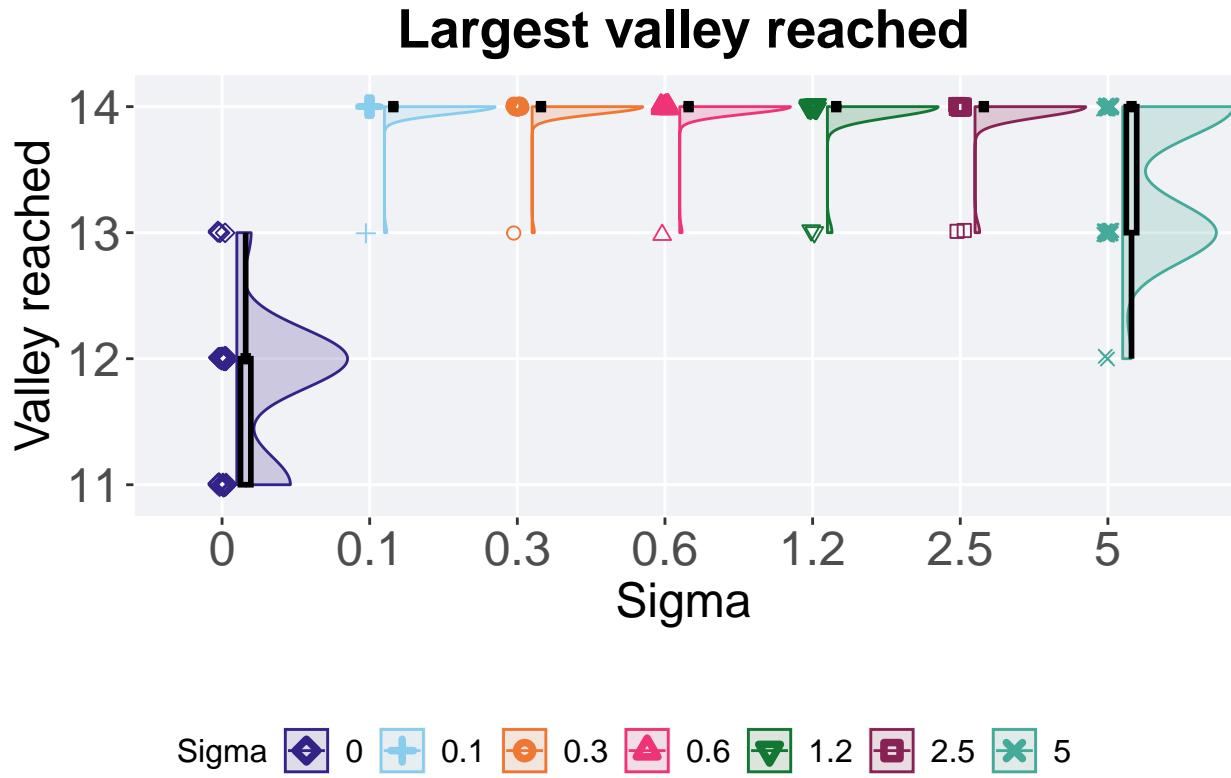
```
plot = filter(best_df, acro == 'ord' & var == 'ele_big_peak') %>%
  ggplot(., aes(x = Sigma, y = val, color = Sigma, fill = Sigma, shape = Sigma)) +
  geom_flat_violin(position = position_nudge(x = .1, y = 0), scale = 'width', alpha = 0.2, width = 1.5) +
  geom_boxplot(color = 'black', width = .07, outlier.shape = NA, alpha = 0.0, size = 1.0, position = position_dodge(.7)) +
  geom_point(position = position_jitter(width = 0.03, height = 0.02), size = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name = "Valley reached",
    limits = c(10.9, 14.1),
    breaks = c(11, 12, 13, 14)
  ) +
  scale_x_discrete(
    name = "Sigma"
  ) +
  scale_shape_manual(values = SHAPE) +
  scale_colour_manual(values = cb_palette, ) +
  scale_fill_manual(values = cb_palette) +
  ggtitle('Largest valley reached') +
  p_theme

plot_grid(
  plot +
```

```

  theme(legend.position="none"),
  legend,
  nrow=2,
  rel_heights = c(3,1)
)

```



5.3.3.1 Stats

Summary statistics for the largest valley crossed.

```

valleys = filter(best_df, acro == 'ord' & var == 'ele_big_peak')
valleys$Sigma = factor(valleys$Sigma, levels = FS_LIST)
valleys %>%
  group_by(Sigma) %>%
  dplyr::summarise(
    count = n(),
    na_cnt = sum(is.na(val)),
    min = min(val, na.rm = TRUE),
    median = median(val, na.rm = TRUE),
    mean = mean(val, na.rm = TRUE),
    max = max(val, na.rm = TRUE),
    IQR = IQR(val, na.rm = TRUE)
  )

## # A tibble: 7 x 8
##   Sigma count na_cnt   min median   mean   max   IQR
##   <fct> <int> <int> <dbl> <dbl> <dbl> <dbl> <dbl>
## 1 0       10     0 11.0 12.0 12.0 13.0 1.0
## 2 0.1     10     0 13.0 14.0 14.0 14.0 1.0
## 3 0.3     10     0 13.0 14.0 14.0 14.0 1.0
## 4 0.6     10     0 13.0 14.0 14.0 14.0 1.0
## 5 1.2     10     0 13.0 14.0 14.0 14.0 1.0
## 6 2.5     10     0 13.0 14.0 14.0 14.0 1.0
## 7 5       10     0 12.0 13.0 13.0 14.0 1.0

```

```
## 1 0      50    0    11    12   11.8    13    1
## 2 0.1    50    0    13    14   14.0    14    0
## 3 0.3    50    0    13    14   14.0    14    0
## 4 0.6    50    0    13    14   14.0    14    0
## 5 1.2    50    0    13    14   14.0    14    0
## 6 2.5    50    0    13    14   14.0    14    0
## 7 5      50    0    12    14   13.5    14    1
```

Kruskal–Wallis test illustrates evidence of statistical differences.

```
kruskal.test(val ~ Sigma, data = valleys)
```

```
##
##  Kruskal-Wallis rank sum test
##
## data: val by Sigma
## Kruskal-Wallis chi-squared = 265.8, df = 6, p-value < 2.2e-16
```

Results for post-hoc Wilcoxon rank-sum test with a Bonferroni correction.

```
pairwise.wilcox.test(x = valleys$val, g = valleys$Sigma, p.adjust.method = "bonferroni",
                      paired = FALSE, conf.int = FALSE, alternative = 't')
```

```
##
##  Pairwise comparisons using Wilcoxon rank sum test with continuity correction
##
## data: valleys$val and valleys$Sigma
##
##          0     0.1    0.3    0.6    1.2    2.5
## 0.1 < 2e-16 -    -    -    -    -
## 0.3 < 2e-16 1    -    -    -    -
## 0.6 < 2e-16 1    1    -    -    -
## 1.2 < 2e-16 1    1    1    -    -
## 2.5 < 2e-16 1    1    1    1    -
## 5    1.3e-15 2.8e-06 2.8e-06 2.8e-06 1.3e-05 1.3e-05
##
## P value adjustment method: bonferroni
```

5.4 Contradictory objectives results

Here we present the results for **activation gene coverage** and **satisfactory trait coverage** found by each selection scheme parameter on the contradictory objectives diagnostic with valleys. 50 replicates are conducted for each scheme parameters explored.

5.4.1 Activation gene coverage over time

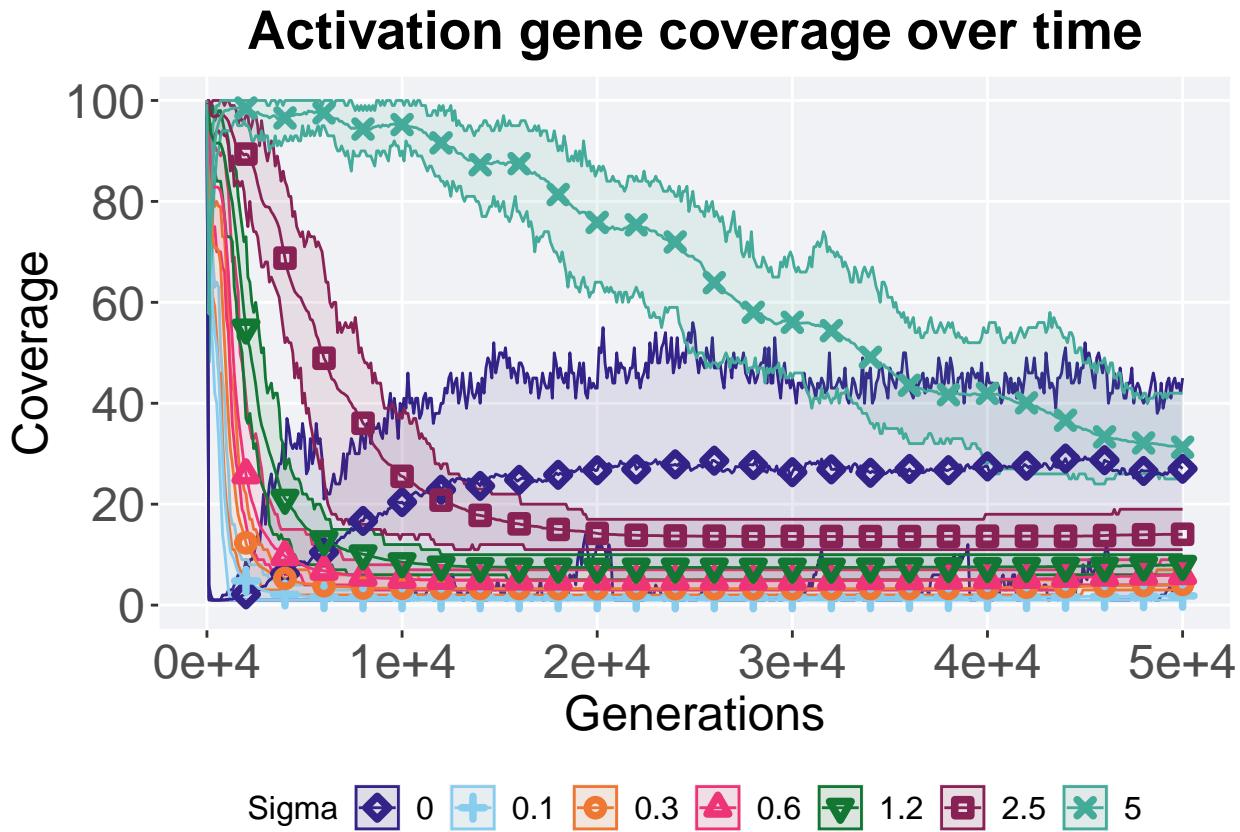
Activation gene coverage in a population over time. Data points on the graph is the average activation gene coverage across 50 replicates every 2000 generations. Shading comes from the best and worse coverage across 50 replicates.

```
lines = filter(over_time_df, acro == 'con') %>%
  group_by(Sigma, gen) %>%
  dplyr::summarise(
    min = min(uni_str_pos),
    mean = mean(uni_str_pos),
    max = max(uni_str_pos)
```

```
)
```

```
## `summarise()` has grouped output by 'Sigma'. You can override using the
## `.` argument.

ggplot(lines, aes(x=gen, y=mean, group = Sigma, fill = Sigma, color = Sigma, shape = Sigma)) +
  geom_ribbon(aes(ymin = min, ymax = max), alpha = 0.1) +
  geom_line(size = 0.5) +
  geom_point(data = filter(lines, gen %% 2000 == 0 & gen != 0), size = 1.5, stroke = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name="Coverage",
    limits=c(0, 100),
    breaks=seq(0,100, 20),
    labels=c("0", "20", "40", "60", "80", "100")
  ) +
  scale_x_continuous(
    name="Generations",
    limits=c(0, 50000),
    breaks=c(0, 10000, 20000, 30000, 40000, 50000),
    labels=c("0e+4", "1e+4", "2e+4", "3e+4", "4e+4", "5e+4")
  ) +
  scale_shape_manual(values=SHAPE) +
  scale_colour_manual(values = cb_palette) +
  scale_fill_manual(values = cb_palette) +
  ggtitle('Activation gene coverage over time') +
  p_theme +
  guides(
    shape=guide_legend(nrow=1, title.position = "left", title = 'Sigma'),
    color=guide_legend(nrow=1, title.position = "left", title = 'Sigma'),
    fill=guide_legend(nrow=1, title.position = "left", title = 'Sigma')
  )
```



5.4.2 Final activation gene coverage

Activation gene coverage found in the final population at 50,000 generations.

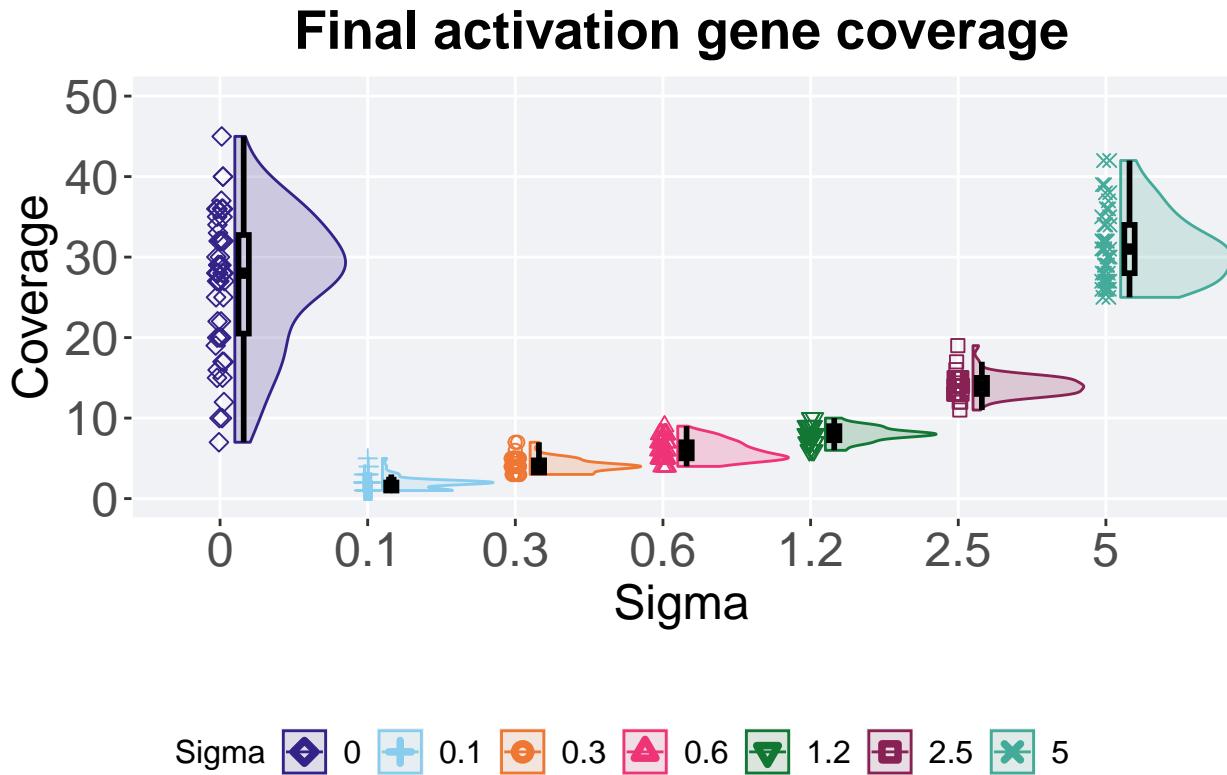
```
plot = filter(over_time_df, gen == 50000 & acro == 'con') %>%
  ggplot(., aes(x = Sigma, y = uni_str_pos, color = Sigma, fill = Sigma, shape = Sigma)) +
  geom_flat_violin(position = position_nudge(x = .1, y = 0), scale = 'width', alpha = 0.2, width = 1.5)
  geom_boxplot(color = 'black', width = .07, outlier.shape = NA, alpha = 0.0, size = 1.0, position = position_nudge(x = -.1, y = 0))
  geom_point(position = position_jitter(width = 0.03, height = 0.02), size = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name="Coverage",
    limits=c(0, 50.1)
  ) +
  scale_x_discrete(
    name="Sigma"
  ) +
  scale_shape_manual(values=SHAPE) +
  scale_colour_manual(values = cb_palette,) +
  scale_fill_manual(values = cb_palette) +
  ggtitle('Final activation gene coverage') +
  p_theme

plot_grid(
  plot +
    theme(legend.position="none"),
  legend,
```

```

nrow=2,
rel_heights = c(3,1)
)

```



5.4.2.1 Stats

Summary statistics for the generation a satisfactory solution is found.

```

act_coverage = filter(over_time_df, gen == 50000 & acro == 'con')
act_coverage$Sigma = factor(act_coverage$Sigma, levels = FS_LIST)
act_coverage %>%
  group_by(Sigma) %>%
  dplyr::summarise(
    count = n(),
    na_cnt = sum(is.na(uni_str_pos)),
    min = min(uni_str_pos, na.rm = TRUE),
    median = median(uni_str_pos, na.rm = TRUE),
    mean = mean(uni_str_pos, na.rm = TRUE),
    max = max(uni_str_pos, na.rm = TRUE),
    IQR = IQR(uni_str_pos, na.rm = TRUE)
  )

## # A tibble: 7 x 8
##   Sigma count na_cnt  min median  mean  max    IQR
##   <fct> <int>   <int> <dbl> <dbl> <dbl> <dbl>
## 1 0         50      0     7    28  27.0    45  12.2
## 2 0.1       50      0     1     2   1.84     5   1

```

```
## 3 0.3      50     0     3     4   4.1      7   1.5
## 4 0.6      50     0     4     6   5.92      9   2
## 5 1.2      50     0     6     8   8          10  1.75
## 6 2.5      50     0    11    14  14.0      19  2
## 7 5        50     0    25    31  31.3      42  6
```

Kruskal-Wallis test illustrates evidence of statistical differences.

```
kruskal.test(uni_str_pos ~ Sigma, data = act_coverage)
```

```
##
## Kruskal-Wallis rank sum test
##
## data: uni_str_pos by Sigma
## Kruskal-Wallis chi-squared = 325.94, df = 6, p-value < 2.2e-16
```

Results for post-hoc Wilcoxon rank-sum test with a Bonferroni correction.

```
pairwise.wilcox.test(x = act_coverage$uni_str_pos, g = act_coverage$Sigma, p.adjust.method = "bonferroni",
                      paired = FALSE, conf.int = FALSE, alternative = 't')
```

```
##
## Pairwise comparisons using Wilcoxon rank sum test with continuity correction
##
## data: act_coverage$uni_str_pos and act_coverage$Sigma
##
##          0       0.1      0.3      0.6      1.2      2.5
## 0.1 < 2e-16 -       -       -       -       -
## 0.3 < 2e-16 6.8e-15 -       -       -       -
## 0.6 < 2e-16 < 2e-16 4.9e-10 -       -       -
## 1.2 1.2e-15 < 2e-16 < 2e-16 2.6e-10 -       -
## 2.5 2.4e-11 < 2e-16 < 2e-16 < 2e-16 < 2e-16 -
## 5    0.43    < 2e-16 < 2e-16 < 2e-16 < 2e-16 < 2e-16
##
## P value adjustment method: bonferroni
```

5.4.3 Satisfactory trait coverage over time

Satisfactory trait coverage in a population over time. Data points on the graph is the average activation gene coverage across 50 replicates every 2000 generations. Shading comes from the best and worse coverage across 50 replicates.

```
lines = filter(over_time_df, acro == 'con') %>%
  group_by(Sigma, gen) %>%
  dplyr::summarise(
    min = min(pop_uni_obj),
    mean = mean(pop_uni_obj),
    max = max(pop_uni_obj)
  )

## `summarise()` has grouped output by 'Sigma'. You can override using the
## `.groups` argument.

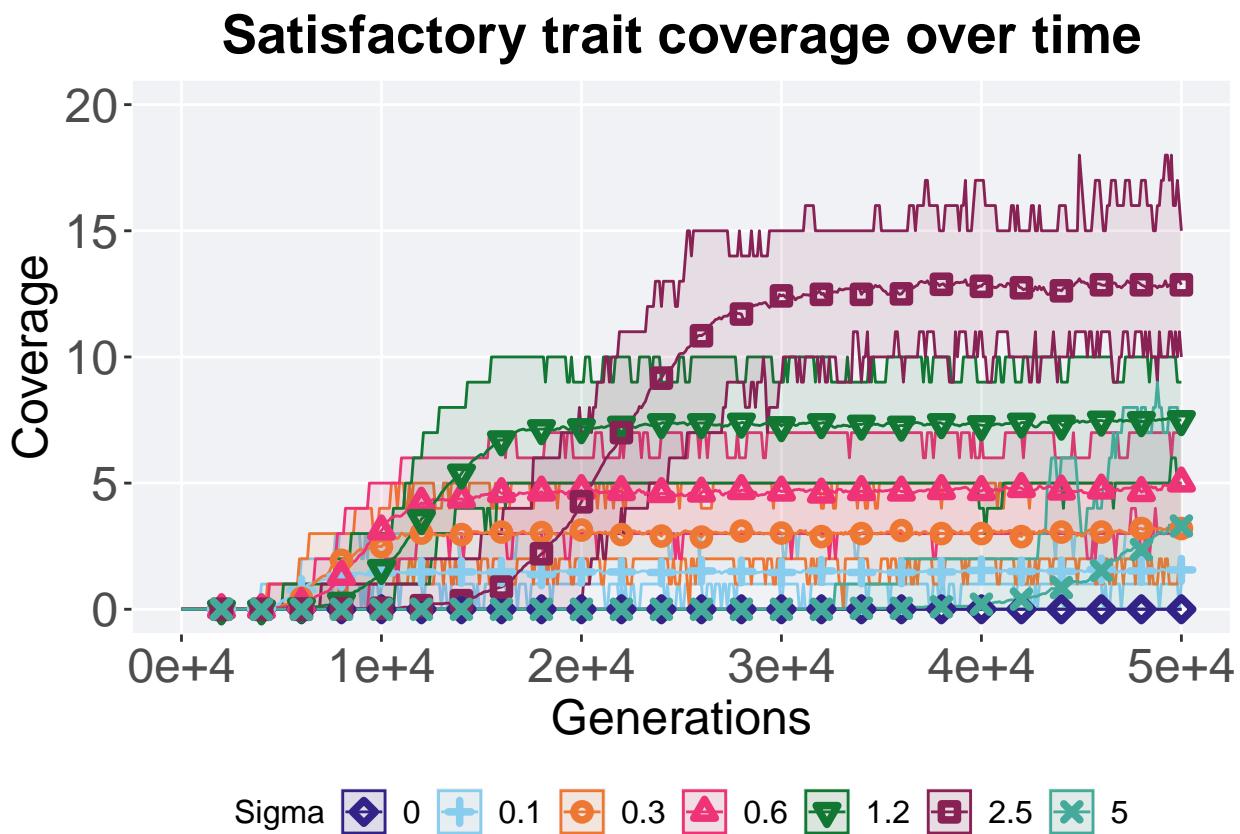
ggplot(lines, aes(x=gen, y=mean, group = Sigma, fill = Sigma, color = Sigma, shape = Sigma)) +
  geom_ribbon(aes(ymin = min, ymax = max), alpha = 0.1) +
  geom_line(size = 0.5) +
  geom_point(data = filter(lines, gen %% 2000 == 0 & gen != 0), size = 1.5, stroke = 2.0, alpha = 1.0) +
  scale_y_continuous()
```

```

    name="Coverage",
    limits=c(0, 20)
) +
scale_x_continuous(
  name="Generations",
  limits=c(0, 50000),
  breaks=c(0, 10000, 20000, 30000, 40000, 50000),
  labels=c("0e+4", "1e+4", "2e+4", "3e+4", "4e+4", "5e+4")

) +
scale_shape_manual(values=SHAPE) +
scale_colour_manual(values = cb_palette) +
scale_fill_manual(values = cb_palette) +
ggtitle('Satisfactory trait coverage over time') +
p_theme +
guides(
  shape=guide_legend(nrow=1, title.position = "left", title = 'Sigma'),
  color=guide_legend(nrow=1, title.position = "left", title = 'Sigma'),
  fill=guide_legend(nrow=1, title.position = "left", title = 'Sigma')
)

```



5.4.4 Final satisfactory trait coverage

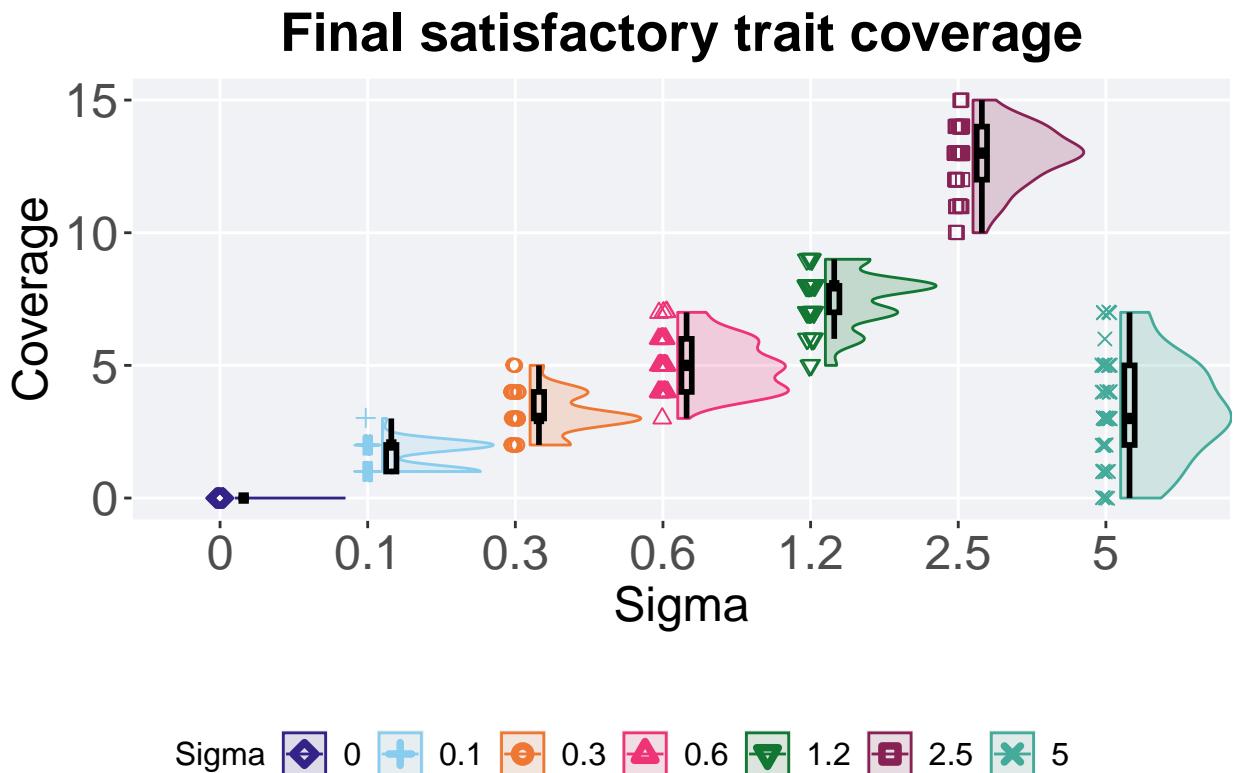
Satisfactory trait coverage found in the final population at 50,000 generations.

```

plot = filter(over_time_df, gen == 50000 & acro == 'con') %>%
  ggplot(., aes(x = Sigma, y = pop_uni_obj, color = Sigma, fill = Sigma, shape = Sigma)) +
  geom_flat_violin(position = position_nudge(x = .1, y = 0), scale = 'width', alpha = 0.2, width = 1.5) +
  geom_boxplot(color = 'black', width = .07, outlier.shape = NA, alpha = 0.0, size = 1.0, position = position_nudge(x = .1, y = 0)) +
  geom_point(position = position_jitter(width = 0.03, height = 0.02), size = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name = "Coverage",
    limits = c(-0.1, 15.1)
  ) +
  scale_x_discrete(
    name = "Sigma"
  ) +
  scale_shape_manual(values = SHAPE) +
  scale_color_manual(values = cb_palette, ) +
  scale_fill_manual(values = cb_palette) +
  ggtitle('Final satisfactory trait coverage') +
  p_theme

plot_grid(
  plot +
    theme(legend.position = "none"),
  legend,
  nrow = 2,
  rel_heights = c(3, 1)
)

```



5.4.4.1 Stats

Summary statistics for the generation a satisfactory solution is found.

```
sat_coverage = filter(over_time_df, gen == 50000 & acro == 'con')
sat_coverage$Sigma = factor(sat_coverage$Sigma, levels = FS_LIST)
sat_coverage %>%
  group_by(Sigma) %>%
  dplyr::summarise(
    count = n(),
    na_cnt = sum(is.na(pop_uni_obj)),
    min = min(pop_uni_obj, na.rm = TRUE),
    median = median(pop_uni_obj, na.rm = TRUE),
    mean = mean(pop_uni_obj, na.rm = TRUE),
    max = max(pop_uni_obj, na.rm = TRUE),
    IQR = IQR(pop_uni_obj, na.rm = TRUE)
  )

## # A tibble: 7 x 8
##   Sigma count na_cnt  min median  mean  max  IQR
##   <fct> <int> <int> <dbl> <dbl> <int> <dbl>
## 1 0         50     0     0     0     0     0     0
## 2 0.1       50     0     1     2     1.56    3     1
## 3 0.3       50     0     2     3     3.2     5     1
## 4 0.6       50     0     3     5     5.02    7     2
## 5 1.2       50     0     5     8     7.5     9     1
## 6 2.5       50     0    10    13    12.9    15     2
## 7 5         50     0     0     3     3.3     7     3
```

Kruskal–Wallis test illustrates evidence of statistical differences.

```
kruskal.test(pop_uni_obj ~ Sigma, data = sat_coverage)
```

```
##
##  Kruskal-Wallis rank sum test
##
## data: pop_uni_obj by Sigma
## Kruskal-Wallis chi-squared = 313.86, df = 6, p-value < 2.2e-16
```

Results for post-hoc Wilcoxon rank-sum test with a Bonferroni correction.

```
pairwise.wilcox.test(x = sat_coverage$pop_uni_obj, g = sat_coverage$Sigma, p.adjust.method = "bonferroni",
                      paired = FALSE, conf.int = FALSE, alternative = 't')

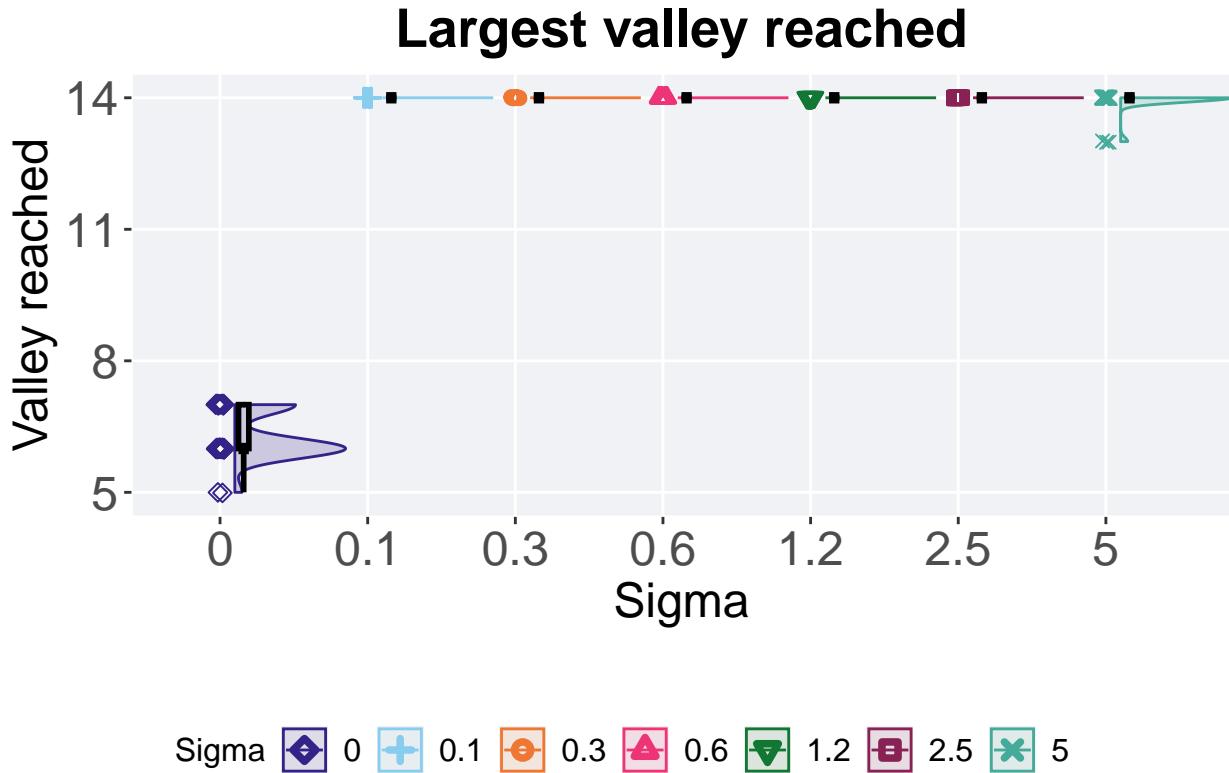
##
##  Pairwise comparisons using Wilcoxon rank sum test with continuity correction
##
## data: sat_coverage$pop_uni_obj and sat_coverage$Sigma
##
##          0      0.1     0.3     0.6     1.2     2.5
## 0.1 < 2e-16 -      -      -      -      -
## 0.3 < 2e-16 3.5e-14 -      -      -      -
## 0.6 < 2e-16 < 2e-16 4.5e-12 -      -      -
## 1.2 < 2e-16 < 2e-16 < 2e-16 2.1e-13 -      -
## 2.5 < 2e-16 < 2e-16 < 2e-16 < 2e-16 < 2e-16 -
## 5     < 2e-16 4.7e-06 1      2.3e-05 7.3e-15 < 2e-16
##
## P value adjustment method: bonferroni
```

5.4.5 Largest valley reached throughout

Largest valley reached in a single trait by the best performing solution throughout an entire evolutionary run.

```
plot = filter(best_df, acro == 'con' & var == 'ele_big_peak') %>%
  ggplot(., aes(x = Sigma, y = val, color = Sigma, fill = Sigma, shape = Sigma)) +
  geom_flat_violin(position = position_nudge(x = .1, y = 0), scale = 'width', alpha = 0.2, width = 1.5) +
  geom_boxplot(color = 'black', width = .07, outlier.shape = NA, alpha = 0.0, size = 1.0, position = position_nudge(x = -.1, y = 0)) +
  geom_point(position = position_jitter(width = 0.03, height = 0.02), size = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name = "Valley reached",
    limits = c(4.9, 14.1),
    breaks = c(5, 8, 11, 14)
  ) +
  scale_x_discrete(
    name = "Sigma"
  ) +
  scale_shape_manual(values = SHAPE) +
  scale_colour_manual(values = cb_palette, ) +
  scale_fill_manual(values = cb_palette) +
  ggtitle('Largest valley reached') +
  p_theme

plot_grid(
  plot +
    theme(legend.position = "none"),
  legend,
  nrow = 2,
  rel_heights = c(3, 1)
)
```



5.4.5.1 Stats

Summary statistics for the largest valley crossed.

```
valleys = filter(best_df, acro == 'con' & var == 'ele_big_peak')
valleys$Sigma = factor(valleys$Sigma, levels = FS_LIST)
valleys %>%
  group_by(Sigma) %>%
  dplyr::summarise(
    count = n(),
    na_cnt = sum(is.na(val)),
    min = min(val, na.rm = TRUE),
    median = median(val, na.rm = TRUE),
    mean = mean(val, na.rm = TRUE),
    max = max(val, na.rm = TRUE),
    IQR = IQR(val, na.rm = TRUE)
  )

## # A tibble: 7 x 8
##   Sigma count na_cnt   min median   mean   max   IQR
##   <fct> <int>   <int> <dbl>  <dbl>  <dbl>  <dbl> <dbl>
## 1 0        50      0     5     6    6.3     7     1
## 2 0.1      50      0    14    14    14     14     0
## 3 0.3      50      0    14    14    14     14     0
## 4 0.6      50      0    14    14    14     14     0
## 5 1.2      50      0    14    14    14     14     0
## 6 2.5      50      0    14    14    14     14     0
```

```
## 7 5      50      0    13     14   13.9     14      0
```

Kruskal–Wallis test illustrates evidence of statistical differences.

```
kruskal.test(val ~ Sigma, data = valleys)
```

```
##
## Kruskal-Wallis rank sum test
##
## data: val by Sigma
## Kruskal-Wallis chi-squared = 331.14, df = 6, p-value < 2.2e-16
```

Results for post-hoc Wilcoxon rank-sum test with a Bonferroni correction.

```
pairwise.wilcox.test(x = valleys$val, g = valleys$Sigma, p.adjust.method = "bonferroni",
                      paired = FALSE, conf.int = FALSE, alternative = 't')
```

```
##
## Pairwise comparisons using Wilcoxon rank sum test with continuity correction
##
## data: valleys$val and valleys$Sigma
##
##      0     0.1 0.3 0.6 1.2 2.5
## 0.1 <2e-16 - - - - -
## 0.3 <2e-16 - - - - -
## 0.6 <2e-16 - - - - -
## 1.2 <2e-16 - - - - -
## 2.5 <2e-16 - - - - -
## 5    <2e-16 0.9 0.9 0.9 0.9 0.9
##
## P value adjustment method: bonferroni
```

5.5 Multi-path exploration results

Here we present the results for best performances and activation gene coverage found by each selection scheme parameter on the multi-path exploration diagnostic with valleys. 50 replicates are conducted for each scheme parameter explored.

5.5.1 Activation gene coverage over time

Activation gene coverage in a population over time. Data points on the graph is the average activation gene coverage across 50 replicates every 2000 generations. Shading comes from the best and worse coverage across 50 replicates.

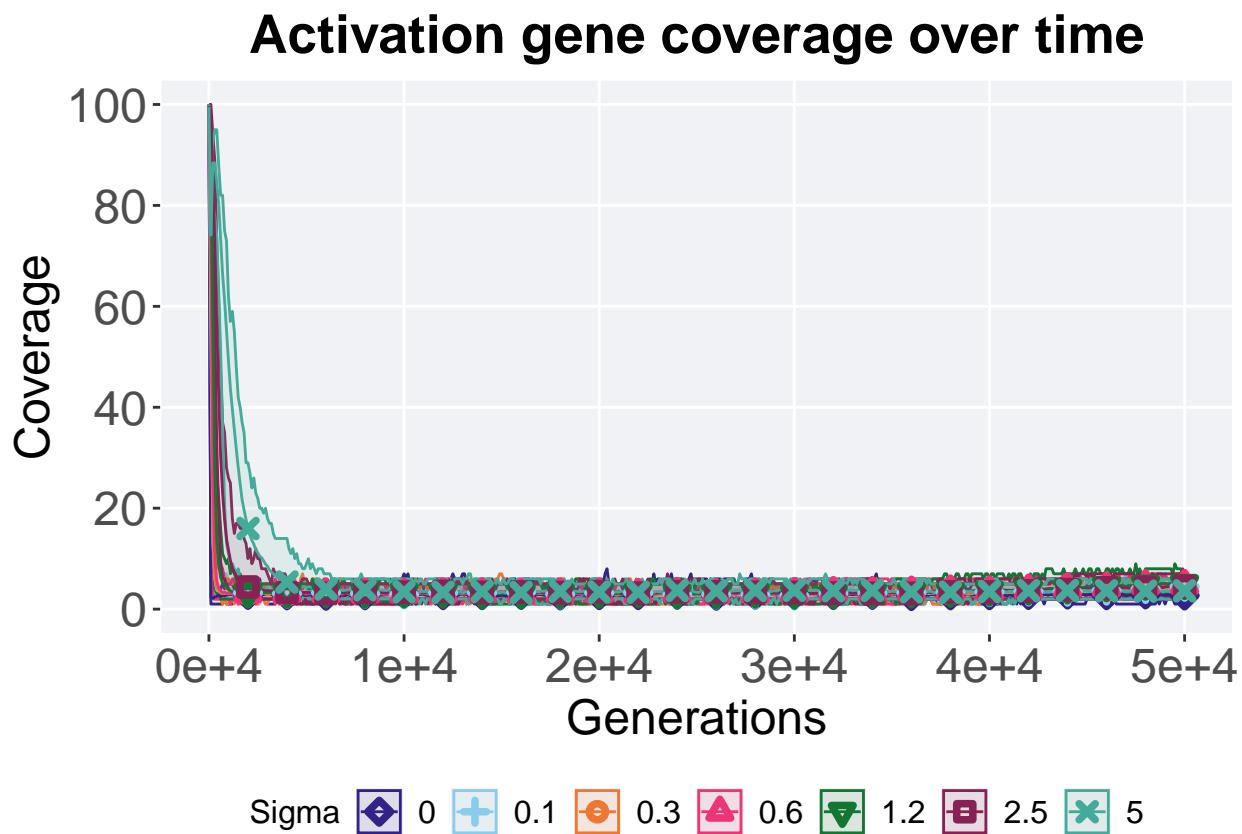
```
lines = filter(over_time_df, acro == 'mpe') %>%
  group_by(Sigma, gen) %>%
  dplyr::summarise(
    min = min(uni_str_pos),
    mean = mean(uni_str_pos),
    max = max(uni_str_pos)
  )

## `summarise()` has grouped output by 'Sigma'. You can override using the
## `.groups` argument.
ggplot(lines, aes(x=gen, y=mean, group = Sigma, fill = Sigma, color = Sigma, shape = Sigma)) +
  geom_ribbon(aes(ymin = min, ymax = max), alpha = 0.1) +
```

```

geom_line(size = 0.5) +
geom_point(data = filter(lines, gen %% 2000 == 0 & gen != 0), size = 1.5, stroke = 2.0, alpha = 1.0) +
scale_y_continuous(
  name="Coverage",
  limits=c(0, 100),
  breaks=seq(0,100, 20),
  labels=c("0", "20", "40", "60", "80", "100")
) +
scale_x_continuous(
  name="Generations",
  limits=c(0, 50000),
  breaks=c(0, 10000, 20000, 30000, 40000, 50000),
  labels=c("0e+4", "1e+4", "2e+4", "3e+4", "4e+4", "5e+4")
) +
scale_shape_manual(values=SHAPE) +
scale_colour_manual(values = cb_palette) +
scale_fill_manual(values = cb_palette) +
ggtitle('Activation gene coverage over time') +
p_theme +
guides(
  shape=guide_legend(nrow=1, title.position = "left", title = 'Sigma'),
  color=guide_legend(nrow=1, title.position = "left", title = 'Sigma'),
  fill=guide_legend(nrow=1, title.position = "left", title = 'Sigma')
)

```

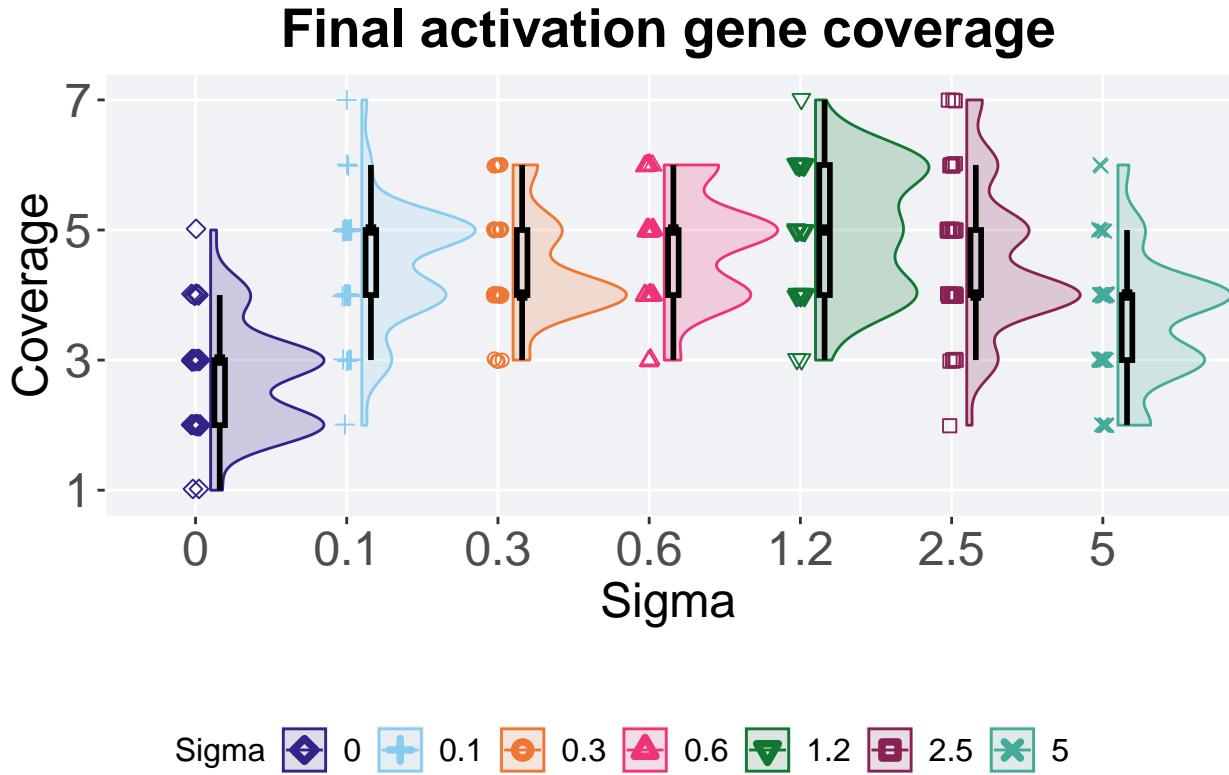


5.5.2 Final activation gene coverage

Activation gene coverage found in the final population at 50,000 generations.

```
plot = filter(over_time_df, gen == 50000 & acro == 'mpe') %>%
  ggplot(., aes(x = Sigma, y = uni_str_pos, color = Sigma, fill = Sigma, shape = Sigma)) +
  geom_flat_violin(position = position_nudge(x = .1, y = 0), scale = 'width', alpha = 0.2, width = 1.5) +
  geom_boxplot(color = 'black', width = .07, outlier.shape = NA, alpha = 0.0, size = 1.0, position = position_nudge(x = -.1, y = 0)) +
  geom_point(position = position_jitter(width = 0.03, height = 0.02), size = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name = "Coverage",
    limits = c(0.9, 7.1),
    breaks = c(1, 3, 5, 7)
  ) +
  scale_x_discrete(
    name = "Sigma"
  ) +
  scale_shape_manual(values = SHAPE) +
  scale_colour_manual(values = cb_palette, ) +
  scale_fill_manual(values = cb_palette) +
  ggtitle('Final activation gene coverage') +
  p_theme

plot_grid(
  plot +
    theme(legend.position = "none"),
  legend,
  nrow = 2,
  rel_heights = c(3, 1)
)
```



5.5.2.1 Stats

Summary statistics for the generation a satisfactory solution is found.

```
act_coverage = filter(over_time_df, gen == 50000 & acro == 'mpe')
act_coverage$Sigma = factor(act_coverage$Sigma, levels = FS_LIST)
act_coverage %>%
  group_by(Sigma) %>%
  dplyr::summarise(
    count = n(),
    na_cnt = sum(is.na(uni_str_pos)),
    min = min(uni_str_pos, na.rm = TRUE),
    median = median(uni_str_pos, na.rm = TRUE),
    mean = mean(uni_str_pos, na.rm = TRUE),
    max = max(uni_str_pos, na.rm = TRUE),
    IQR = IQR(uni_str_pos, na.rm = TRUE)
  )

## # A tibble: 7 x 8
##   Sigma count na_cnt  min median  mean  max  IQR
##   <fct> <int>   <int> <dbl> <dbl> <int> <dbl>
## 1 0        50      0     1     3    2.7     5     1
## 2 0.1      50      0     2     5    4.44    7     1
## 3 0.3      50      0     3     4    4.4     6     1
## 4 0.6      50      0     3     5    4.76    6     1
## 5 1.2      50      0     3     5     5     7     2
## 6 2.5      50      0     2     4    4.5     7     1
```

```
## 7 5      50      0      2      4   3.62      6      1
```

Kruskal–Wallis test illustrates evidence of statistical differences.

```
kruskal.test(uni_str_pos ~ Sigma, data = act_coverage)
```

```
##
## Kruskal-Wallis rank sum test
##
## data: uni_str_pos by Sigma
## Kruskal-Wallis chi-squared = 129.93, df = 6, p-value < 2.2e-16
```

Results for post-hoc Wilcoxon rank-sum test with a Bonferroni correction.

```
pairwise.wilcox.test(x = act_coverage$uni_str_pos, g = act_coverage$Sigma, p.adjust.method = "bonferroni",
                      paired = FALSE, conf.int = FALSE, alternative = 't')
```

```
##
## Pairwise comparisons using Wilcoxon rank sum test with continuity correction
##
## data: act_coverage$uni_str_pos and act_coverage$Sigma
##
##          0       0.1      0.3      0.6      1.2      2.5
## 0.1 1.3e-11 -       -       -       -       -
## 0.3 1.8e-12 1.00000 -       -       -       -
## 0.6 9.0e-14 1.00000 0.45482 -       -       -
## 1.2 3.0e-14 0.12465 0.03295 1.00000 -       -
## 2.5 3.1e-11 1.00000 1.00000 1.00000 0.29397 -
## 5    9.3e-05 0.00053 0.00084 1.2e-06 9.1e-08 0.00181
##
## P value adjustment method: bonferroni
```

5.5.3 Performance over time

Best performance in a population over time. Data points on the graph is the average performance across 50 replicates every 2000 generations. Shading comes from the best and worse performance across 50 replicates.

```
lines = filter(over_time_df, acro == 'mpe') %>%
  group_by(Sigma, gen) %>%
  dplyr::summarise(
    min = min(pop_fit_max) / DIMENSIONALITY,
    mean = mean(pop_fit_max) / DIMENSIONALITY,
    max = max(pop_fit_max) / DIMENSIONALITY
  )

## `summarise()` has grouped output by 'Sigma'. You can override using the
## `.groups` argument.

over_time_plot = ggplot(lines, aes(x=gen, y=mean, group = Sigma, fill = Sigma, color = Sigma, shape = S)
  geom_ribbon(aes(ymin = min, ymax = max), alpha = 0.1) +
  geom_line(size = 0.5) +
  geom_point(data = filter(lines, gen %% 2000 == 0 & gen != 0), size = 1.5, stroke = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name="Average trait score",
    limits=c(0, 15)
  ) +
  scale_x_continuous(
    name="Generations",
```

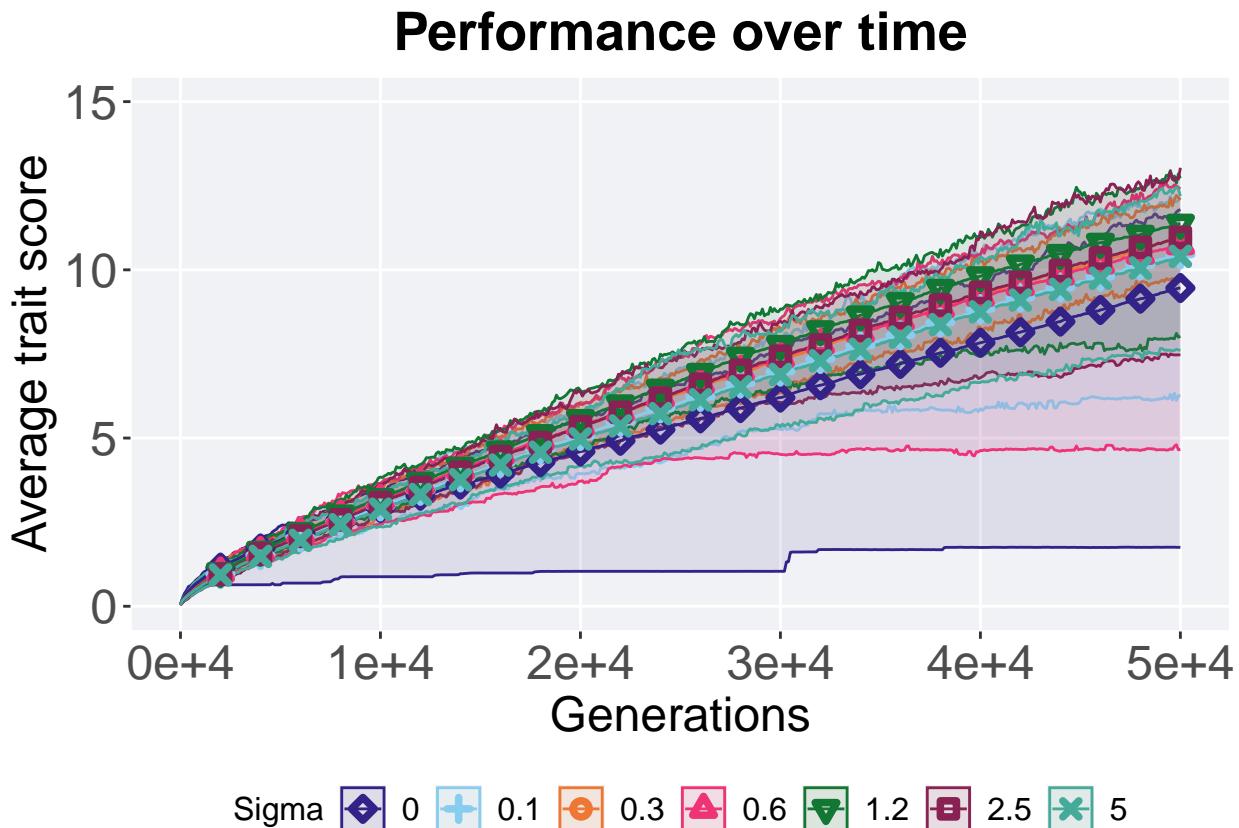
```

limits=c(0, 50000),
breaks=c(0, 10000, 20000, 30000, 40000, 50000),
labels=c("0e+4", "1e+4", "2e+4", "3e+4", "4e+4", "5e+4")

) +
scale_shape_manual(values=SHAPE) +
scale_colour_manual(values = cb_palette) +
scale_fill_manual(values = cb_palette) +
ggtitle('Performance over time') +
p_theme +
guides(
  shape=guide_legend(nrow=1, title.position = "left", title = 'Sigma'),
  color=guide_legend(nrow=1, title.position = "left", title = 'Sigma'),
  fill=guide_legend(nrow=1, title.position = "left", title = 'Sigma')
)

over_time_plot

```



5.5.4 Best performance throughout

Best performance reached throughout 50,000 generations in a population.

```

plot = filter(best_df, acro == 'mpe' & var == 'pop_fit_max') %>%
  ggplot(., aes(x = Sigma, y = val / DIMENSIONALITY, color = Sigma, fill = Sigma, shape = Sigma)) +
  geom_flat_violin(position = position_nudge(x = .1, y = 0), scale = 'width', alpha = 0.2, width = 1.5) +
  geom_boxplot(color = 'black', width = .07, outlier.shape = NA, alpha = 0.0, size = 1.0, position =

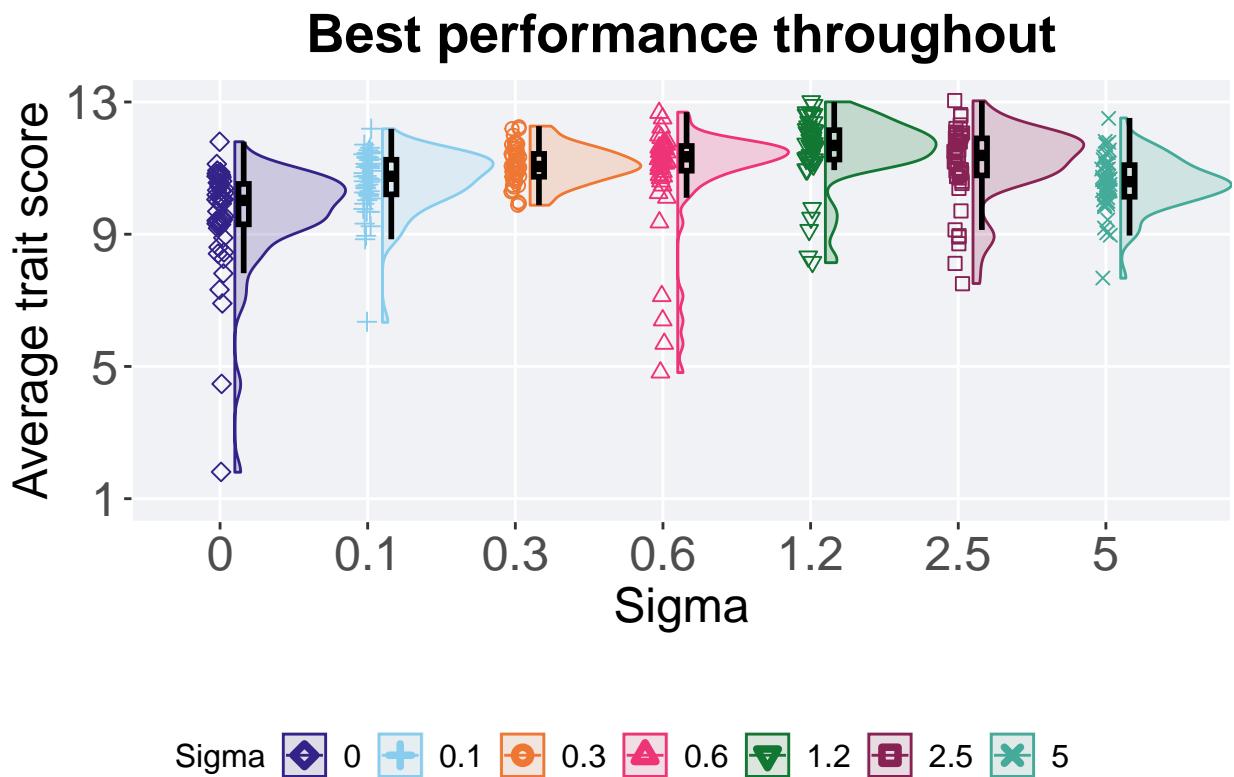
```

```

geom_point(position = position_jitter(width = 0.03, height = 0.02), size = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name="Average trait score",
    limits=c(0.9, 13.1),
    breaks=c(1,5,9,13)
  ) +
  scale_x_discrete(
    name="Sigma"
  ) +
  scale_shape_manual(values=SHAPE) +
  scale_colour_manual(values = cb_palette, ) +
  scale_fill_manual(values = cb_palette) +
  ggtitle('Best performance throughout') +
  p_theme

plot_grid(
  plot +
    theme(legend.position="none"),
  legend,
  nrow=2,
  rel_heights = c(3,1)
)
)

```



5.5.4.1 Stats

Summary statistics for the best performance.

```

performance = filter(best_df, acro == 'mpe' & var == 'pop_fit_max')
performance$Sigma = factor(performance$Sigma, levels = FS_LIST)
performance %>%
  group_by(Sigma) %>%
  dplyr::summarise(
    count = n(),
    na_cnt = sum(is.na(val)),
    min = min(val / DIMENSIONALITY, na.rm = TRUE),
    median = median(val / DIMENSIONALITY, na.rm = TRUE),
    mean = mean(val / DIMENSIONALITY, na.rm = TRUE),
    max = max(val / DIMENSIONALITY, na.rm = TRUE),
    IQR = IQR(val / DIMENSIONALITY, na.rm = TRUE)
  )
}

## # A tibble: 7 x 8
##   Sigma count na_cnt   min median   mean   max   IQR
##   <fct> <int> <int> <dbl> <dbl> <dbl> <dbl> <dbl>
## 1 0         50     0  1.80  10.0  9.53  11.8  1.22
## 2 0.1       50     0  6.33  10.8  10.6  12.2  1.05
## 3 0.3       50     0  9.88  11.0  11.1  12.3  0.690
## 4 0.6       50     0  4.81  11.3  10.9  12.7  0.747
## 5 1.2       50     0  8.14  11.7  11.5  13.0  0.889
## 6 2.5       50     0  7.51  11.4  11.1  13.0  1.12
## 7 5         50     0  7.67  10.6  10.5  12.5  0.956

```

Kruskal–Wallis test illustrates evidence of statistical differences.

```

kruskal.test(val ~ Sigma, data = performance)

##
##  Kruskal-Wallis rank sum test
##
## data: val by Sigma
## Kruskal-Wallis chi-squared = 108.01, df = 6, p-value < 2.2e-16

```

Results for post-hoc Wilcoxon rank-sum test with a Bonferroni correction.

```

pairwise.wilcox.test(x = performance$val, g = performance$Sigma, p.adjust.method = "bonferroni",
                      paired = FALSE, conf.int = FALSE, alternative = 't')

##
##  Pairwise comparisons using Wilcoxon rank sum test with continuity correction
##
## data: performance$val and performance$Sigma
##
##          0      0.1     0.3     0.6     1.2     2.5
## 0.1 0.00012 -      -      -      -      -
## 0.3 1.7e-10 0.19052 -      -      -      -
## 0.6 9.9e-09 0.00777 1.00000 -      -      -
## 1.2 1.9e-11 8.4e-07 0.00101 0.04375 -      -
## 2.5 3.6e-08 0.01477 1.00000 1.00000 0.45978 -
## 5    0.00262 1.00000 0.00346 0.00087 2.3e-07 0.00140
##
## P value adjustment method: bonferroni

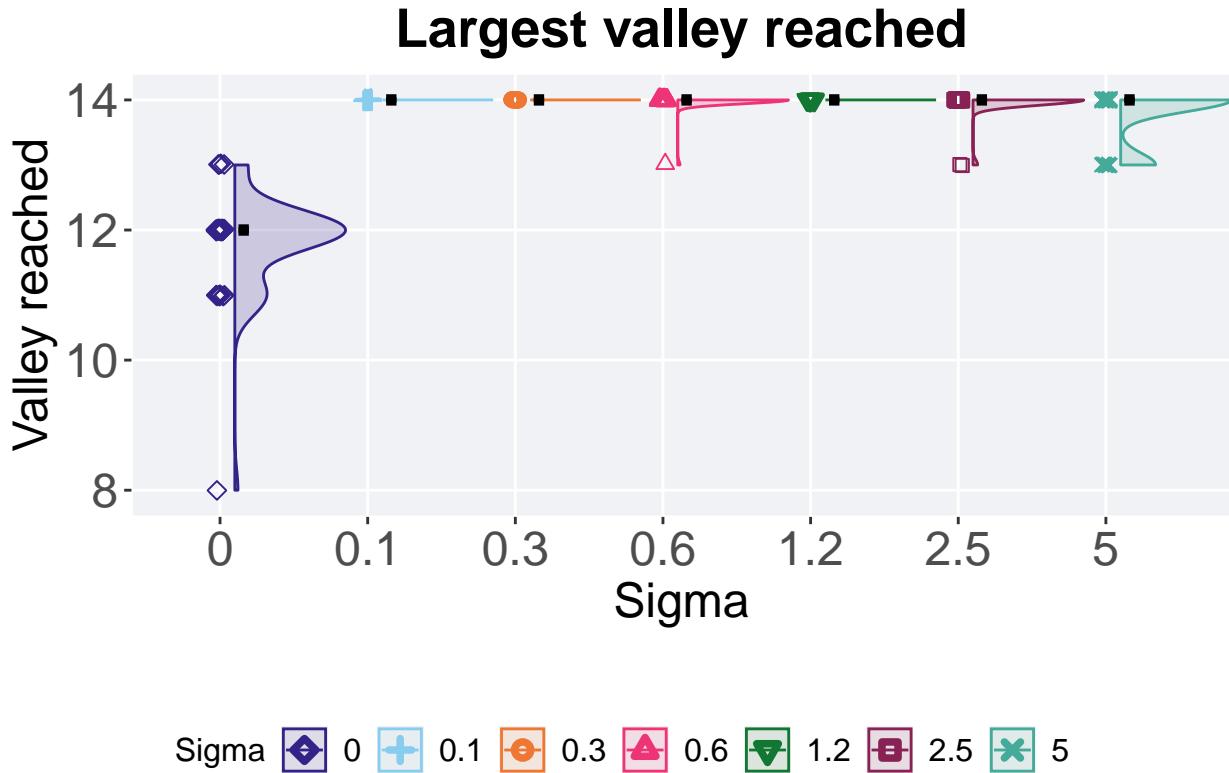
```

5.5.5 Largest valley reached throughout

Largest valley reached in a single trait by the best performing solution throughout an entire evolutionary run.

```
plot = filter(best_df, acro == 'mpe' & var == 'ele_big_peak') %>%
  ggplot(., aes(x = Sigma, y = val, color = Sigma, fill = Sigma, shape = Sigma)) +
  geom_flat_violin(position = position_nudge(x = .1, y = 0), scale = 'width', alpha = 0.2, width = 1.5) +
  geom_boxplot(color = 'black', width = .07, outlier.shape = NA, alpha = 0.0, size = 1.0, position = position_nudge(x = -.1, y = 0)) +
  geom_point(position = position_jitter(width = 0.03, height = 0.02), size = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name="Valley reached",
    limits=c(7.9,14.1)
  ) +
  scale_x_discrete(
    name="Sigma"
  ) +
  scale_shape_manual(values=SHAPE) +
  scale_colour_manual(values = cb_palette, ) +
  scale_fill_manual(values = cb_palette) +
  ggtitle('Largest valley reached') +
  p_theme

plot_grid(
  plot +
    theme(legend.position="none"),
  legend,
  nrow=2,
  rel_heights = c(3,1)
)
```



5.5.5.1 Stats

Summary statistics for the largest valley crossed.

```

valleys = filter(best_df, acro == 'mpe' & var == 'ele_big_peak')
valleys$Sigma = factor(valleys$Sigma, levels = FS_LIST)
valleys %>%
  group_by(Sigma) %>%
  dplyr::summarise(
    count = n(),
    na_cnt = sum(is.na(val)),
    min = min(val, na.rm = TRUE),
    median = median(val, na.rm = TRUE),
    mean = mean(val, na.rm = TRUE),
    max = max(val, na.rm = TRUE),
    IQR = IQR(val, na.rm = TRUE)
  )

## # A tibble: 7 x 8
##   Sigma count na_cnt   min median   mean   max   IQR
##   <fct> <int>   <int> <dbl>   <dbl>   <dbl> <dbl> <dbl>
## 1 0        50      0     8     12  11.8     13     0
## 2 0.1      50      0    14     14     14     14     0
## 3 0.3      50      0    14     14     14     14     0
## 4 0.6      50      0    13     14  14.0     14     0
## 5 1.2      50      0    14     14     14     14     0
## 6 2.5      50      0    13     14  14.0     14     0

```

```
## 7 5      50     0    13     14   13.8    14     0
```

Kruskal–Wallis test illustrates evidence of statistical differences.

```
kruskal.test(val ~ Sigma, data = valleys)
```

```
##  
## Kruskal-Wallis rank sum test  
##  
## data: val by Sigma  
## Kruskal-Wallis chi-squared = 288.11, df = 6, p-value < 2.2e-16
```

Results for post-hoc Wilcoxon rank-sum test with a Bonferroni correction.

```
pairwise.wilcox.test(x = valleys$val, g = valleys$Sigma, p.adjust.method = "bonferroni",  
                      paired = FALSE, conf.int = FALSE, alternative = 't')
```

```
##  
## Pairwise comparisons using Wilcoxon rank sum test with continuity correction  
##  
## data: valleys$val and valleys$Sigma  
##  
##      0     0.1    0.3    0.6    1.2    2.5  
## 0.1 <2e-16 -     -     -     -     -  
## 0.3 <2e-16 -     -     -     -     -  
## 0.6 <2e-16 1.0000 1.0000 -     -     -  
## 1.2 <2e-16 -     -     1.0000 -     -  
## 2.5 <2e-16 1.0000 1.0000 1.0000 1.0000 -  
## 5    <2e-16 0.0044 0.0044 0.0209 0.0044 0.0758  
##  
## P value adjustment method: bonferroni
```


Chapter 6

Nondominated sorting

Results for the nondominated sorting parameter sweep on the diagnostics with valleys.

6.1 Data setup

```
over_time_df <- read.csv(paste(DATA_DIR, 'OVER-TIME-MVC/nds.csv', sep = "", collapse = NULL), header = TRUE)
over_time_df$Sigma <- factor(over_time_df$Sigma, levels = ND_LIST)

best_df <- read.csv(paste(DATA_DIR, 'BEST-MVC/nds.csv', sep = "", collapse = NULL), header = TRUE, stringsAsFactors = FALSE)
best_df$Sigma <- factor(best_df$Sigma, levels = ND_LIST)
```

6.2 Exploitation rate results

Here we present the results for **best performances** found by each selection scheme parameter on the exploitation rate diagnostic with valleys. 50 replicates are conducted for each scheme explored.

6.2.1 Performance over time

Best performance in a population over time. Data points on the graph is the average performance across 50 replicates every 2000 generations. Shading comes from the best and worse performance across 50 replicates.

```
lines = filter(over_time_df, acro == 'exp') %>%
  group_by(Sigma, gen) %>%
  dplyr::summarise(
    min = min(pop_fit_max) / DIMENSIONALITY,
    mean = mean(pop_fit_max) / DIMENSIONALITY,
    max = max(pop_fit_max) / DIMENSIONALITY
  )

## `summarise()` has grouped output by 'Sigma'. You can override using the
## `.groups` argument.

over_time_plot = ggplot(lines, aes(x=gen, y=mean, group = Sigma, fill = Sigma, color = Sigma, shape = Sigma))
  geom_ribbon(aes(ymin = min, ymax = max), alpha = 0.1) +
  geom_line(size = 0.5) +
  geom_point(data = filter(lines, gen %% 2000 == 0 & gen != 0), size = 1.5, stroke = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name="Average trait score",
```

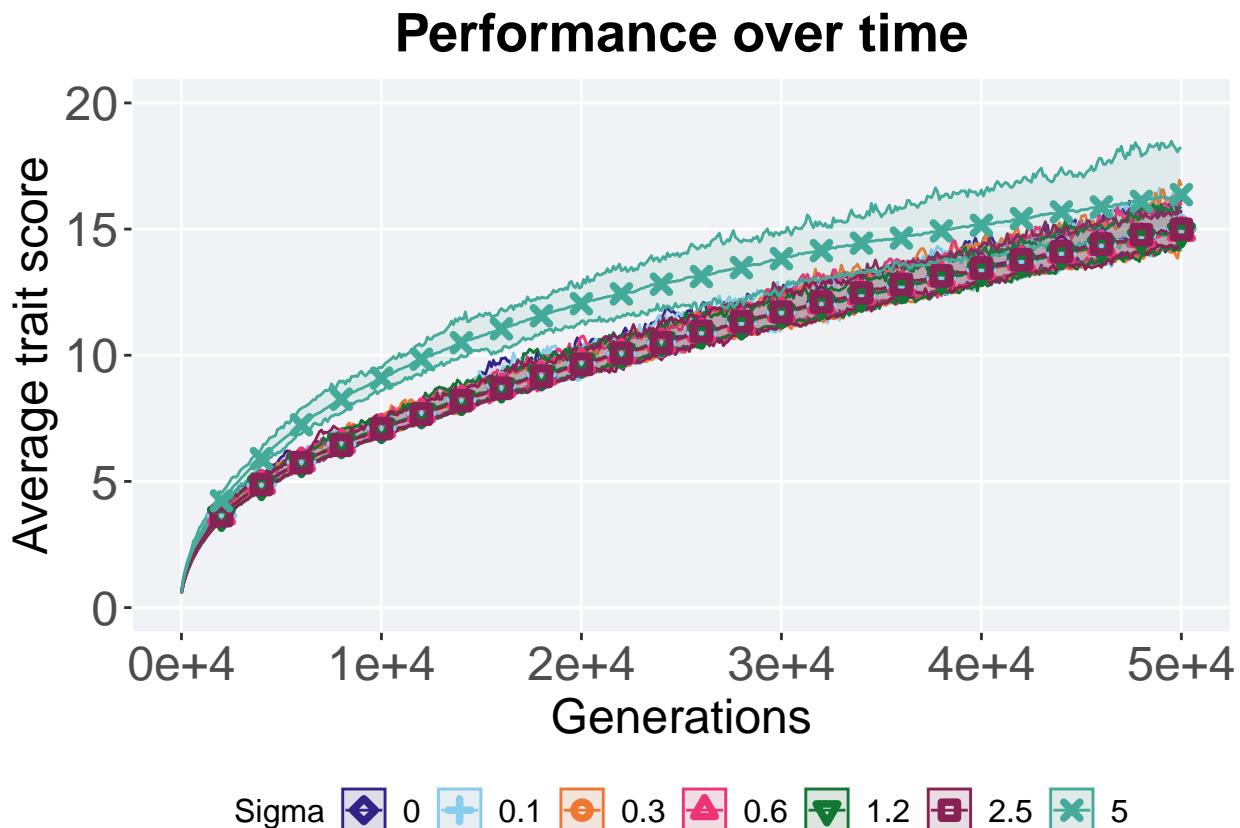
```

    limits = c(0,20)
) +
scale_x_continuous(
  name="Generations",
  limits=c(0, 50000),
  breaks=c(0, 10000, 20000, 30000, 40000, 50000),
  labels=c("0e+4", "1e+4", "2e+4", "3e+4", "4e+4", "5e+4")

) +
scale_shape_manual(values=SHAPE) +
scale_colour_manual(values = cb_palette) +
scale_fill_manual(values = cb_palette) +
ggtitle('Performance over time') +
p_theme +
guides(
  shape=guide_legend(nrow=1, title.position = "left", title = 'Sigma'),
  color=guide_legend(nrow=1, title.position = "left", title = 'Sigma'),
  fill=guide_legend(nrow=1, title.position = "left", title = 'Sigma')
)

over_time_plot

```



6.2.2 Best performance throughout

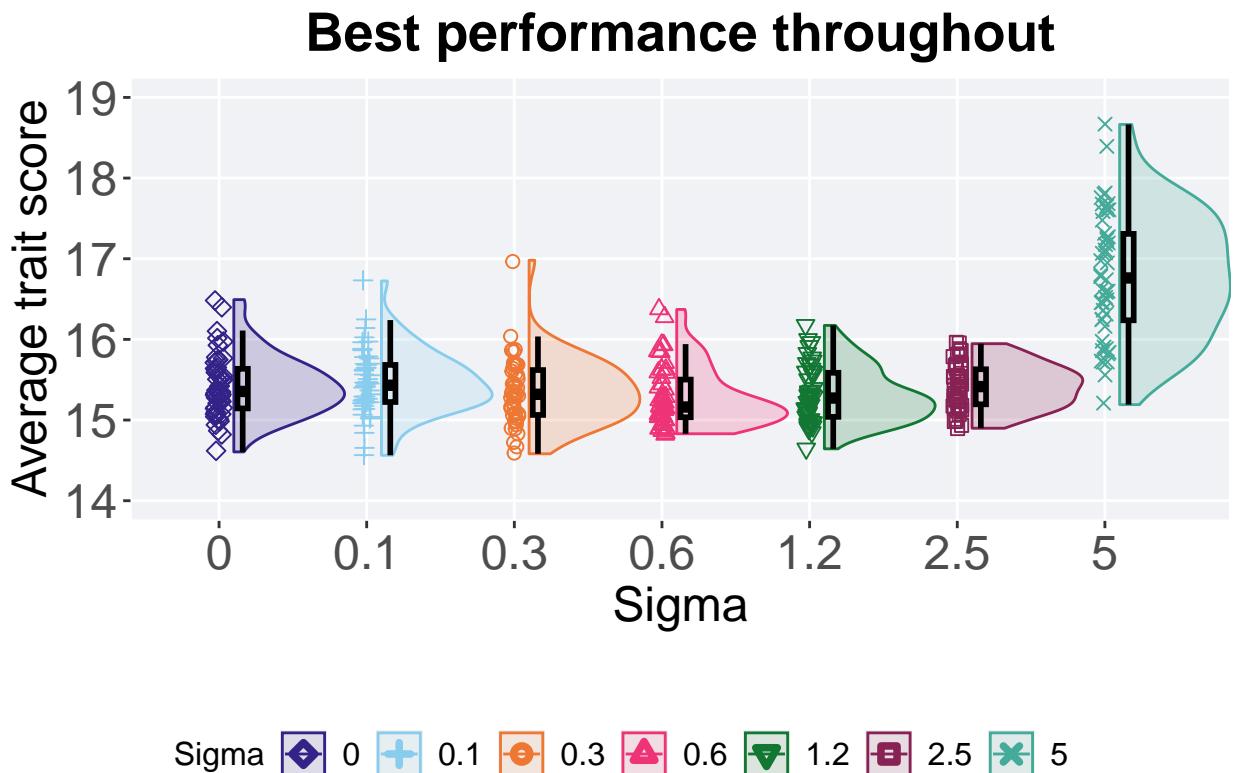
Best performance reached throughout 50,000 generations in a population.

```

plot = filter(best_df, acro == 'exp' & var == 'pop_fit_max') %>%
  ggplot(., aes(x = Sigma, y = val / DIMENSIONALITY, color = Sigma, fill = Sigma, shape = Sigma)) +
  geom_flat_violin(position = position_nudge(x = .1, y = 0), scale = 'width', alpha = 0.2, width = 1.5) +
  geom_boxplot(color = 'black', width = .07, outlier.shape = NA, alpha = 0.0, size = 1.0, position = position_nudge(x = -.1, y = 0)) +
  geom_point(position = position_jitter(width = 0.03, height = 0.02), size = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name = "Average trait score",
    limits = c(14, 19)
  ) +
  scale_x_discrete(
    name = "Sigma"
  ) +
  scale_shape_manual(values = SHAPE) +
  scale_color_manual(values = cb_palette,) +
  scale_fill_manual(values = cb_palette) +
  ggtitle('Best performance throughout') +
  p_theme

plot_grid(
  plot +
    theme(legend.position = "none"),
  legend,
  nrow = 2,
  rel_heights = c(3, 1)
)
)

```



6.2.2.1 Stats

Summary statistics for the best performance.

```
performance = filter(best_df, acro == 'exp' & var == 'pop_fit_max')
performance$Sigma = factor(performance$Sigma, levels = ND_LIST)
performance %>%
  group_by(Sigma) %>%
  dplyr::summarise(
    count = n(),
    na_cnt = sum(is.na(val)),
    min = min(val / DIMENSIONALITY, na.rm = TRUE),
    median = median(val / DIMENSIONALITY, na.rm = TRUE),
    mean = mean(val / DIMENSIONALITY, na.rm = TRUE),
    max = max(val / DIMENSIONALITY, na.rm = TRUE),
    IQR = IQR(val / DIMENSIONALITY, na.rm = TRUE)
  )

## # A tibble: 7 x 8
##   Sigma count na_cnt   min median   mean   max   IQR
##   <fct> <int> <int> <dbl> <dbl> <dbl> <dbl>
## 1 0         50     0 14.6  15.4  15.4  16.5 0.491
## 2 0.1       50     0 14.6  15.4  15.5  16.7 0.460
## 3 0.3       50     0 14.6  15.3  15.4  17.0 0.556
## 4 0.6       50     0 14.8  15.2  15.3  16.4 0.467
## 5 1.2       50     0 14.6  15.3  15.3  16.2 0.546
## 6 2.5       50     0 14.9  15.4  15.4  15.9 0.434
## 7 5         50     0 15.2  16.8  16.8  18.7 1.07
```

Kruskal–Wallis test illustrates evidence of statistical differences.

```
kruskal.test(val ~ Sigma, data = performance)
```

```
##
## Kruskal-Wallis rank sum test
##
## data: val by Sigma
## Kruskal-Wallis chi-squared = 116.31, df = 6, p-value < 2.2e-16
```

Results for post-hoc Wilcoxon rank-sum test with a Bonferroni correction.

```
pairwise.wilcox.test(x = performance$val, g = performance$Sigma, p.adjust.method = "bonferroni",
                      paired = FALSE, conf.int = FALSE, alternative = 't')

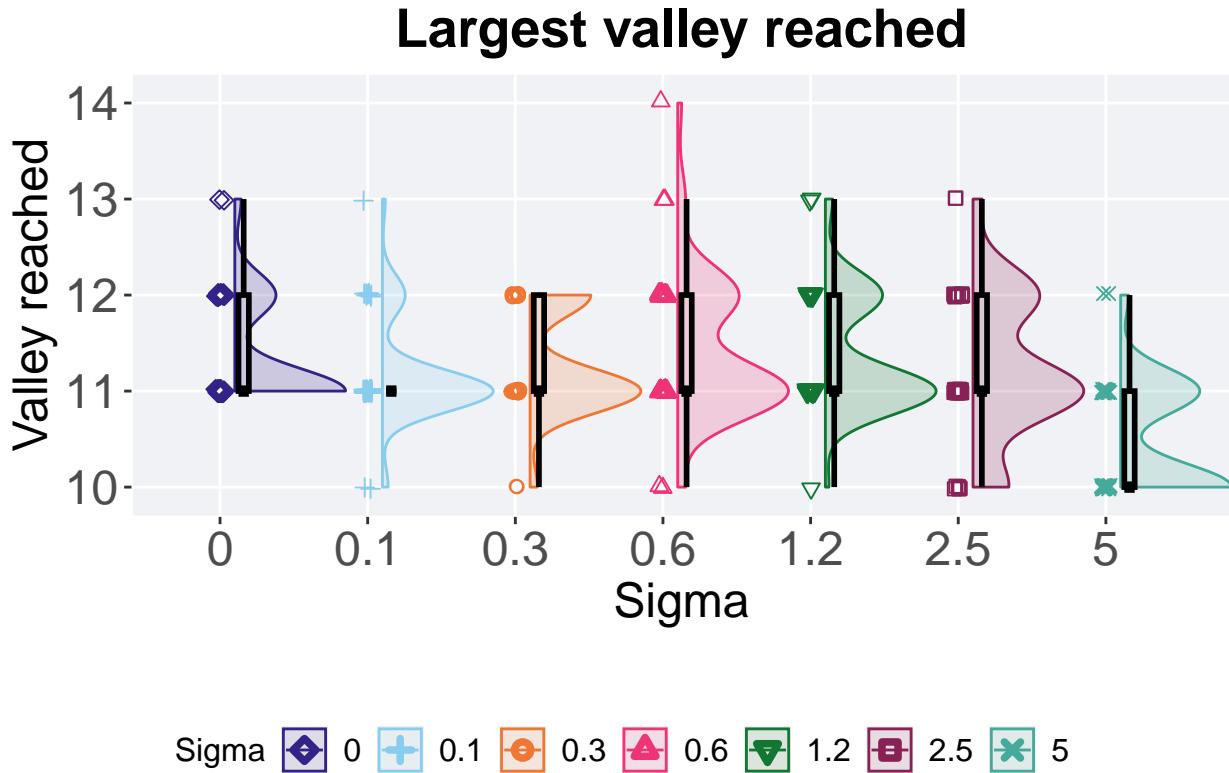
##
## Pairwise comparisons using Wilcoxon rank sum test with continuity correction
##
## data: performance$val and performance$Sigma
##
##          0      0.1     0.3     0.6     1.2     2.5
## 0.1 1.00   -      -      -      -      -
## 0.3 1.00   1.00   -      -      -      -
## 0.6 0.55   0.12   1.00   -      -      -
## 1.2 1.00   0.83   1.00   1.00   -      -
## 2.5 1.00   1.00   1.00   0.15   1.00   -
## 5   2.5e-13 7.3e-13 7.5e-14 3.1e-14 2.3e-14 2.9e-14
##
## P value adjustment method: bonferroni
```

6.2.3 Largest valley reached throughout

Largest valley reached in a single trait by the best performing solution throughout an entire evolutionary run.

```
plot = filter(best_df, acro == 'exp' & var == 'ele_big_peak') %>%
  ggplot(., aes(x = Sigma, y = val, color = Sigma, fill = Sigma, shape = Sigma)) +
  geom_flat_violin(position = position_nudge(x = .1, y = 0), scale = 'width', alpha = 0.2, width = 1.5) +
  geom_boxplot(color = 'black', width = .07, outlier.shape = NA, alpha = 0.0, size = 1.0, position = position_nudge(x = -.1, y = 0)) +
  geom_point(position = position_jitter(width = 0.03, height = 0.02), size = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name = "Valley reached",
    limits = c(9.9, 14.1),
    breaks = c(10, 11, 12, 13, 14)
  ) +
  scale_x_discrete(
    name = "Sigma"
  ) +
  scale_shape_manual(values = SHAPE) +
  scale_colour_manual(values = cb_palette, ) +
  scale_fill_manual(values = cb_palette) +
  ggtitle('Largest valley reached') +
  p_theme

plot_grid(
  plot +
    theme(legend.position = "none"),
  legend,
  nrow = 2,
  rel_heights = c(3, 1)
)
```



6.2.3.1 Stats

Summary statistics for the largest valley crossed.

```

valleys = filter(best_df, acro == 'exp' & var == 'ele_big_peak')
valleys$Sigma = factor(valleys$Sigma, levels = ND_LIST)
valleys %>%
  group_by(Sigma) %>%
  dplyr::summarise(
    count = n(),
    na_cnt = sum(is.na(val)),
    min = min(val, na.rm = TRUE),
    median = median(val, na.rm = TRUE),
    mean = mean(val, na.rm = TRUE),
    max = max(val, na.rm = TRUE),
    IQR = IQR(val, na.rm = TRUE)
  )

## # A tibble: 7 x 8
##   Sigma count na_cnt  min  median  mean  max  IQR
##   <fct> <int>   <int> <dbl>   <dbl> <dbl> <dbl> <dbl>
## 1 0        50      0    11     11  11.3    13     1
## 2 0.1      50      0    10     11  11.2    13     0
## 3 0.3      50      0    10     11  11.3    12     1
## 4 0.6      50      0    10     11  11.4    14     1
## 5 1.2      50      0    10     11  11.4    13     1
## 6 2.5      50      0    10     11  11.2    13     1

```

```
## 7 5      50      0     10     10.5    12      1
```

Kruskal–Wallis test illustrates evidence of statistical differences.

```
kruskal.test(val ~ Sigma, data = valleys)
```

```
##
## Kruskal-Wallis rank sum test
##
## data: val by Sigma
## Kruskal-Wallis chi-squared = 67.685, df = 6, p-value = 1.219e-12
```

Results for post-hoc Wilcoxon rank-sum test with a Bonferroni correction.

```
pairwise.wilcox.test(x = valleys$val, g = valleys$Sigma, p.adjust.method = "bonferroni",
                      paired = FALSE, conf.int = FALSE, alternative = 't')
```

```
##
## Pairwise comparisons using Wilcoxon rank sum test with continuity correction
##
## data: valleys$val and valleys$Sigma
##
##          0     0.1    0.3    0.6    1.2    2.5
## 0.1 1.00  -     -     -     -     -
## 0.3 1.00  1.00  -     -     -     -
## 0.6 1.00  0.84  1.00  -     -     -
## 1.2 1.00  0.89  1.00  1.00  -     -
## 2.5 1.00  1.00  1.00  1.00  1.00  -
## 5   8.1e-09 7.0e-07 5.5e-08 2.5e-08 1.0e-08 3.1e-05
##
## P value adjustment method: bonferroni
```

6.3 Ordered exploitation results

Here we present the results for **best performances** found by each selection scheme parameter on the ordered exploitation diagnostic with valleys. 50 replicates are conducted for each scheme explored.

6.3.1 Performance over time

Best performance in a population over time. Data points on the graph is the average performance across 50 replicates every 2000 generations. Shading comes from the best and worse performance across 50 replicates.

```
lines = filter(over_time_df, acro == 'ord') %>%
  group_by(Sigma, gen) %>%
  dplyr::summarise(
    min = min(pop_fit_max) / DIMENSIONALITY,
    mean = mean(pop_fit_max) / DIMENSIONALITY,
    max = max(pop_fit_max) / DIMENSIONALITY
  )

## `summarise()` has grouped output by 'Sigma'. You can override using the
## `.groups` argument.

over_time_plot = ggplot(lines, aes(x=gen, y=mean, group = Sigma, fill = Sigma, color = Sigma, shape = S
  geom_ribbon(aes(ymin = min, ymax = max), alpha = 0.1) +
  geom_line(size = 0.5) +
  geom_point(data = filter(lines, gen %% 2000 == 0 & gen != 0), size = 1.5, stroke = 2.0, alpha = 1.0) -
```

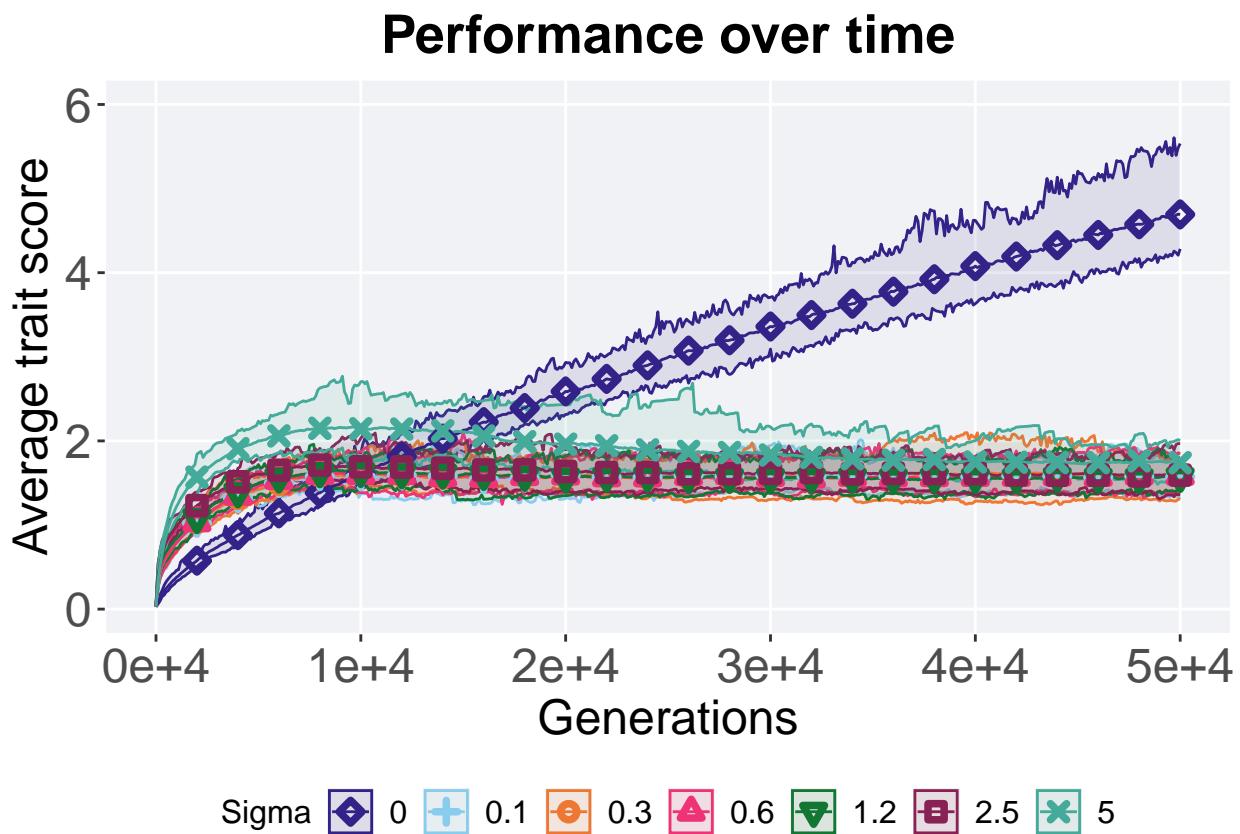
```

scale_y_continuous(
  name="Average trait score",
  limits = c(0,6)
) +
scale_x_continuous(
  name="Generations",
  limits=c(0, 50000),
  breaks=c(0, 10000, 20000, 30000, 40000, 50000),
  labels=c("0e+4", "1e+4", "2e+4", "3e+4", "4e+4", "5e+4")

) +
scale_shape_manual(values=SHAPE) +
scale_colour_manual(values = cb_palette) +
scale_fill_manual(values = cb_palette) +
ggtitle('Performance over time') +
p_theme +
guides(
  shape=guide_legend(nrow=1, title.position = "left", title = 'Sigma'),
  color=guide_legend(nrow=1, title.position = "left", title = 'Sigma'),
  fill=guide_legend(nrow=1, title.position = "left", title = 'Sigma')
)

over_time_plot

```

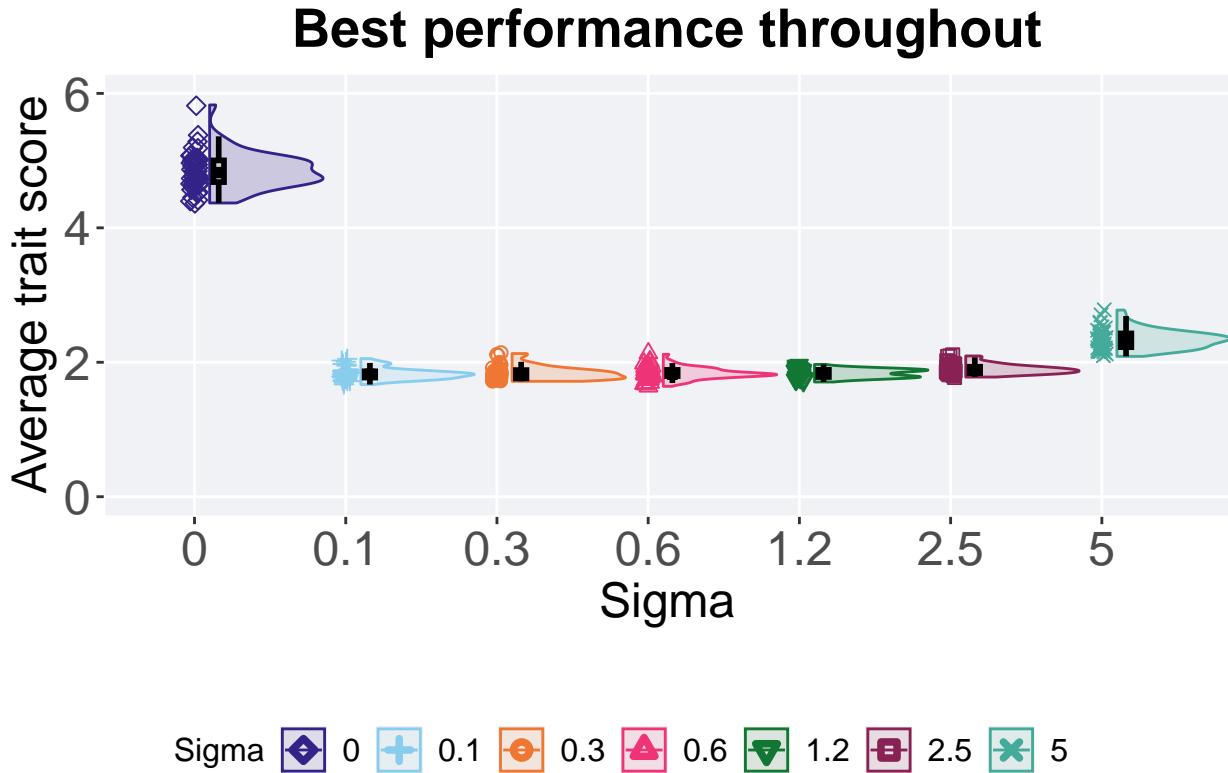


6.3.2 Best performance throughout

Best performance reached throughout 50,000 generations in a population.

```
plot = filter(best_df, acro == 'ord' & var == 'pop_fit_max') %>%
  ggplot(., aes(x = Sigma, y = val / DIMENSIONALITY, color = Sigma, fill = Sigma, shape = Sigma)) +
  geom_flat_violin(position = position_nudge(x = .1, y = 0), scale = 'width', alpha = 0.2, width = 1.5) +
  geom_boxplot(color = 'black', width = .07, outlier.shape = NA, alpha = 0.0, size = 1.0, position = position_nudge(x = -.1, y = 0)) +
  geom_point(position = position_jitter(width = 0.03, height = 0.02), size = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name = "Average trait score",
    limits = c(0, 6)
  ) +
  scale_x_discrete(
    name = "Sigma"
  ) +
  scale_shape_manual(values = SHAPE) +
  scale_colour_manual(values = cb_palette, ) +
  scale_fill_manual(values = cb_palette) +
  ggtitle('Best performance throughout') +
  p_theme

plot_grid(
  plot +
    theme(legend.position = "none"),
  legend,
  nrow = 2,
  rel_heights = c(3, 1)
)
```



6.3.2.1 Stats

Summary statistics for the best performance.

```
performance = filter(best_df, acro == 'ord' & var == 'pop_fit_max')
performance$Sigma = factor(performance$Sigma, levels = ND_LIST)
performance %>%
  group_by(Sigma) %>%
  dplyr::summarise(
```

```
  count = n(),
  na_cnt = sum(is.na(val)),
  min = min(val / DIMENSIONALITY, na.rm = TRUE),
  median = median(val / DIMENSIONALITY, na.rm = TRUE),
  mean = mean(val / DIMENSIONALITY, na.rm = TRUE),
  max = max(val / DIMENSIONALITY, na.rm = TRUE),
  IQR = IQR(val / DIMENSIONALITY, na.rm = TRUE)
)
```

```
## # A tibble: 7 x 8
##   Sigma count na_cnt   min median   mean   max     IQR
##   <fct> <int>   <int>   <dbl>   <dbl>   <dbl>   <dbl>
## 1 0        50      0  4.37    4.82    4.85   5.83  0.323
## 2 0.1      50      0  1.67    1.82    1.83   2.05  0.0929
## 3 0.3      50      0  1.72    1.83    1.84   2.13  0.125
## 4 0.6      50      0  1.64    1.82    1.84   2.12  0.0923
## 5 1.2      50      0  1.71    1.84    1.83   1.97  0.106
## 6 2.5      50      0  1.78    1.88    1.90   2.09  0.0952
```

```
## 7 5      50      0  2.09  2.34  2.34  2.78 0.205
```

Kruskal–Wallis test illustrates evidence of statistical differences.

```
kruskal.test(val ~ Sigma, data = performance)
```

```
##
## Kruskal-Wallis rank sum test
##
## data: val by Sigma
## Kruskal-Wallis chi-squared = 233.29, df = 6, p-value < 2.2e-16
```

Results for post-hoc Wilcoxon rank-sum test with a Bonferroni correction.

```
pairwise.wilcox.test(x = performance$val, g = performance$Sigma, p.adjust.method = "bonferroni",
                      paired = FALSE, conf.int = FALSE, alternative = 't')
```

```
##
## Pairwise comparisons using Wilcoxon rank sum test with continuity correction
##
## data: performance$val and performance$Sigma
##
##    0     0.1    0.3    0.6    1.2    2.5
## 0.1 < 2e-16 -     -     -     -     -
## 0.3 < 2e-16 1.00000 -     -     -     -
## 0.6 < 2e-16 1.00000 1.00000 -     -     -
## 1.2 < 2e-16 1.00000 1.00000 1.00000 -     -
## 2.5 < 2e-16 0.00025 0.00387 0.00169 0.00111 -
## 5    < 2e-16 < 2e-16 < 2e-16 < 2e-16 < 2e-16 < 2e-16
##
## P value adjustment method: bonferroni
```

6.3.3 Largest valley reached throughout

Largest valley reached in a single trait by the best performing solution throughout an entire evolutionary run.

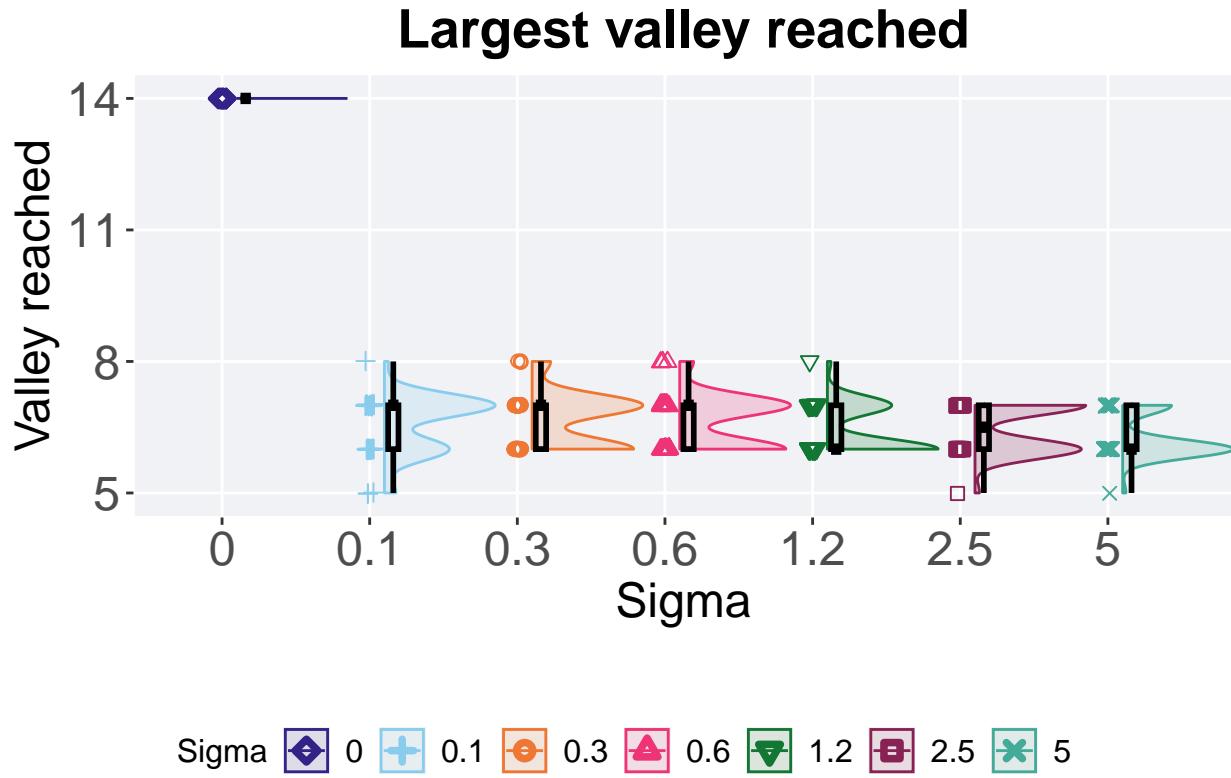
```
plot = filter(best_df, acro == 'ord' & var == 'ele_big_peak') %>%
  ggplot(., aes(x = Sigma, y = val, color = Sigma, fill = Sigma, shape = Sigma)) +
  geom_flat_violin(position = position_nudge(x = .1, y = 0), scale = 'width', alpha = 0.2, width = 1.5) +
  geom_boxplot(color = 'black', width = .07, outlier.shape = NA, alpha = 0.0, size = 1.0, position = position_dodge(.7)) +
  geom_point(position = position_jitter(width = 0.03, height = 0.02), size = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name = "Valley reached",
    limits = c(4.9, 14.1),
    breaks = c(5, 8, 11, 14)
  ) +
  scale_x_discrete(
    name = "Sigma"
  ) +
  scale_shape_manual(values = SHAPE) +
  scale_colour_manual(values = cb_palette, ) +
  scale_fill_manual(values = cb_palette) +
  ggtitle('Largest valley reached') +
  p_theme

plot_grid(
  plot +
```

```

  theme(legend.position="none"),
  legend,
  nrow=2,
  rel_heights = c(3,1)
)

```



6.3.3.1 Stats

Summary statistics for the largest valley crossed.

```

valleys = filter(best_df, acro == 'ord' & var == 'ele_big_peak')
valleys$Sigma = factor(valleys$Sigma, levels = ND_LIST)
valleys %>%
  group_by(Sigma) %>%
  dplyr::summarise(
    count = n(),
    na_cnt = sum(is.na(val)),
    min = min(val, na.rm = TRUE),
    median = median(val, na.rm = TRUE),
    mean = mean(val, na.rm = TRUE),
    max = max(val, na.rm = TRUE),
    IQR = IQR(val, na.rm = TRUE)
  )

## # A tibble: 7 x 8
##   Sigma count na_cnt   min median   mean   max   IQR
##   <fct> <int> <int> <dbl> <dbl> <dbl> <dbl> <dbl>
## 1 0       1     0 14.0 14.0 14.0 14.0 14.0
## 2 0.1     1     0 5.5 6.5 6.5 6.5 7.0
## 3 0.3     1     0 6.0 7.0 7.0 7.0 8.0
## 4 0.6     1     0 6.5 7.5 7.5 7.5 8.0
## 5 1.2     1     0 6.5 7.5 7.5 7.5 8.0
## 6 2.5     1     0 5.5 6.5 6.5 6.5 7.5
## 7 5       1     0 5.5 6.5 6.5 6.5 7.5

```

```
## 1 0      50    0    14   14   14    14    0
## 2 0.1    50    0    5    7   6.56    8    1
## 3 0.3    50    0    6    7   6.64    8    1
## 4 0.6    50    0    6    7   6.6    8    1
## 5 1.2    50    0    6    6   6.4    8    1
## 6 2.5    50    0    5   6.5  6.48    7    1
## 7 5      50    0    5    6   6.28    7    1
```

Kruskal–Wallis test illustrates evidence of statistical differences.

```
kruskal.test(val ~ Sigma, data = valleys)
```

```
##
##  Kruskal-Wallis rank sum test
##
## data: val by Sigma
## Kruskal-Wallis chi-squared = 158.07, df = 6, p-value < 2.2e-16
```

Results for post-hoc Wilcoxon rank-sum test with a Bonferroni correction.

```
pairwise.wilcox.test(x = valleys$val, g = valleys$Sigma, p.adjust.method = "bonferroni",
                      paired = FALSE, conf.int = FALSE, alternative = 't')
```

```
##
##  Pairwise comparisons using Wilcoxon rank sum test with continuity correction
##
## data: valleys$val and valleys$Sigma
##
##    0     0.1    0.3    0.6    1.2    2.5
## 0.1 <2e-16 -     -     -     -     -
## 0.3 <2e-16 1.000 -     -     -     -
## 0.6 <2e-16 1.000 1.000 -     -     -
## 1.2 <2e-16 1.000 1.000 1.000 -     -
## 2.5 <2e-16 1.000 1.000 1.000 1.000 -
## 5    <2e-16 0.171 0.083 0.172 1.000 1.000
##
## P value adjustment method: bonferroni
```

6.4 Contradictory objectives results

Here we present the results for **activation gene coverage** and **satisfactory trait coverage** found by each selection scheme parameter on the contradictory objectives diagnostic with valleys. 50 replicates are conducted for each scheme parameters explored.

6.4.1 Activation gene coverage over time

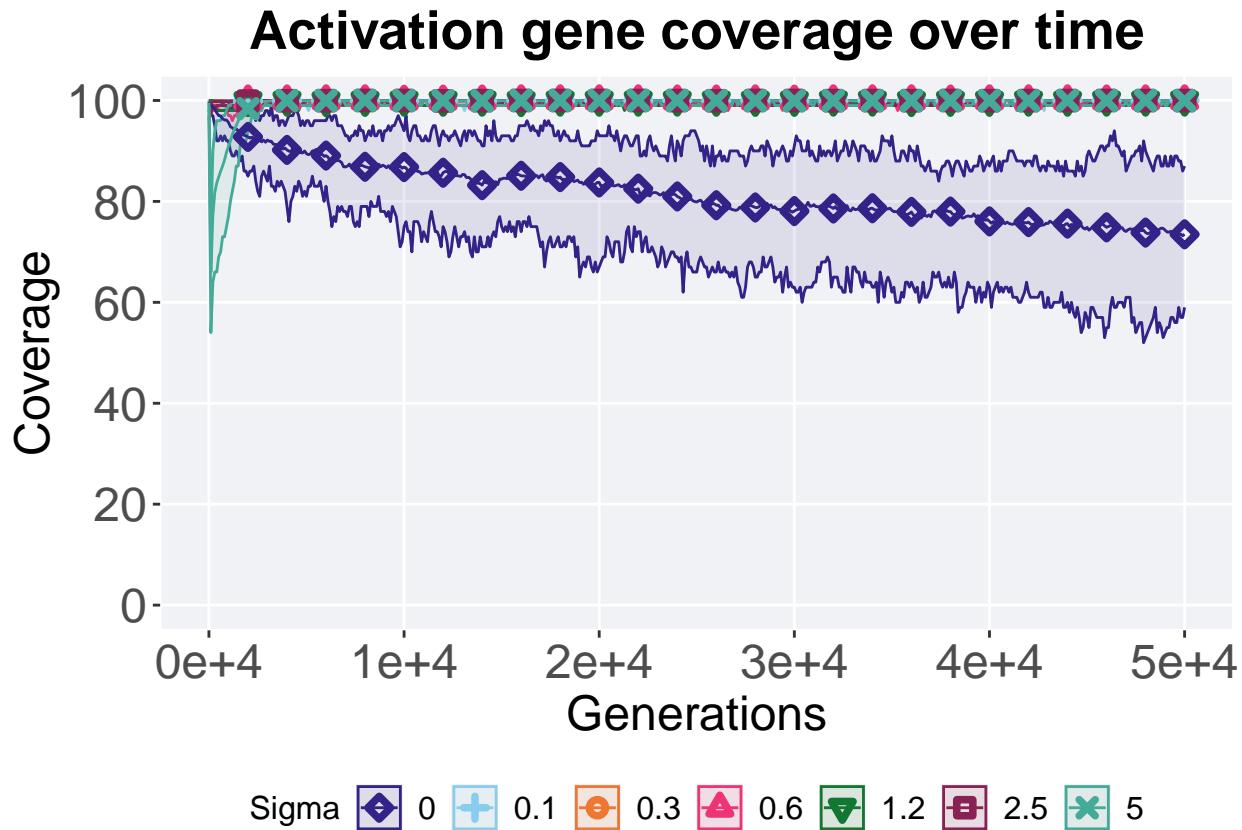
Activation gene coverage in a population over time. Data points on the graph is the average activation gene coverage across 50 replicates every 2000 generations. Shading comes from the best and worse coverage across 50 replicates.

```
lines = filter(over_time_df, acro == 'con') %>%
  group_by(Sigma, gen) %>%
  dplyr::summarise(
    min = min(uni_str_pos),
    mean = mean(uni_str_pos),
    max = max(uni_str_pos)
```

```
)
```

```
## `summarise()` has grouped output by 'Sigma'. You can override using the
## `.` argument.

ggplot(lines, aes(x=gen, y=mean, group = Sigma, fill = Sigma, color = Sigma, shape = Sigma)) +
  geom_ribbon(aes(ymin = min, ymax = max), alpha = 0.1) +
  geom_line(size = 0.5) +
  geom_point(data = filter(lines, gen %% 2000 == 0 & gen != 0), size = 1.5, stroke = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name="Coverage",
    limits=c(0, 100),
    breaks=seq(0,100, 20),
    labels=c("0", "20", "40", "60", "80", "100")
  ) +
  scale_x_continuous(
    name="Generations",
    limits=c(0, 50000),
    breaks=c(0, 10000, 20000, 30000, 40000, 50000),
    labels=c("0e+4", "1e+4", "2e+4", "3e+4", "4e+4", "5e+4")
  ) +
  scale_shape_manual(values=SHAPE) +
  scale_colour_manual(values = cb_palette) +
  scale_fill_manual(values = cb_palette) +
  ggtitle('Activation gene coverage over time') +
  p_theme +
  guides(
    shape=guide_legend(nrow=1, title.position = "left", title = 'Sigma'),
    color=guide_legend(nrow=1, title.position = "left", title = 'Sigma'),
    fill=guide_legend(nrow=1, title.position = "left", title = 'Sigma')
  )
```



6.4.2 Final activation gene coverage

Activation gene coverage found in the final population at 50,000 generations.

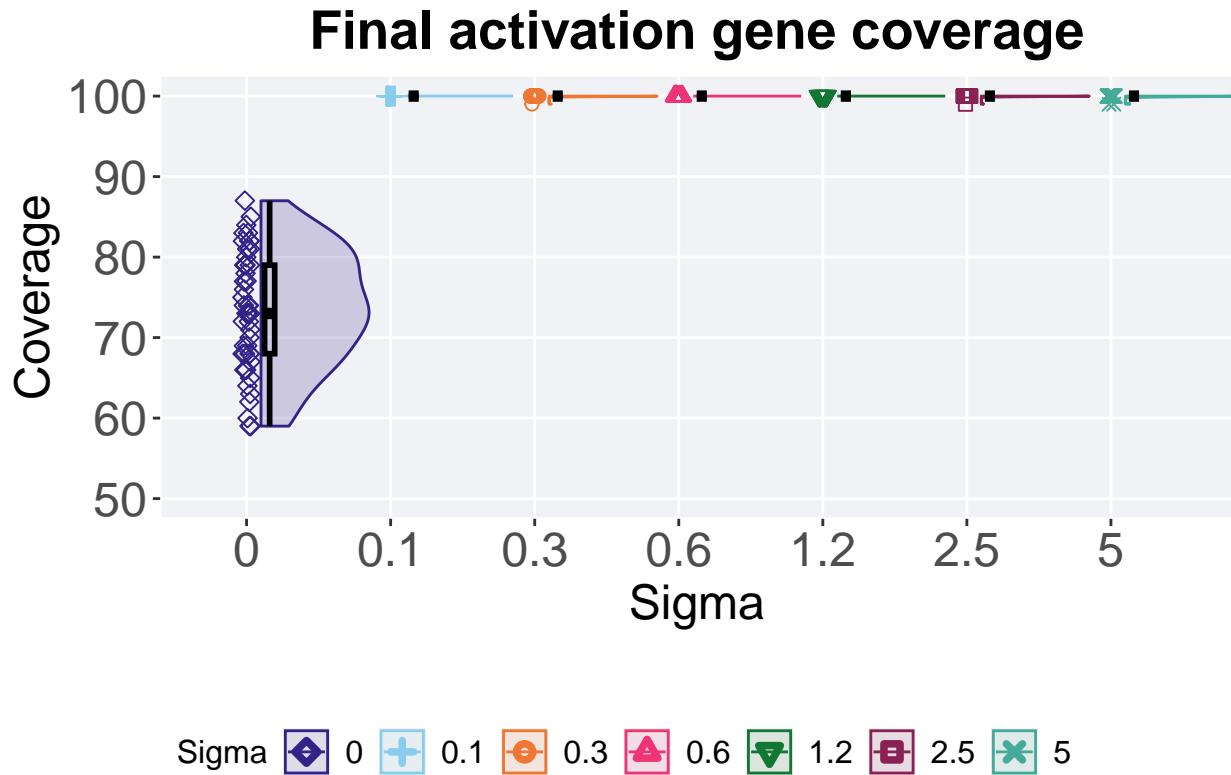
```
plot = filter(over_time_df, gen == 50000 & acro == 'con') %>%
  ggplot(., aes(x = Sigma, y = uni_str_pos, color = Sigma, fill = Sigma, shape = Sigma)) +
  geom_flat_violin(position = position_nudge(x = .1, y = 0), scale = 'width', alpha = 0.2, width = 1.5) +
  geom_boxplot(color = 'black', width = .07, outlier.shape = NA, alpha = 0.0, size = 1.0, position = position_nudge(x = -.1, y = 0)) +
  geom_point(position = position_jitter(width = 0.03, height = 0.02), size = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name = "Coverage",
    limits = c(50, 100.1)
  ) +
  scale_x_discrete(
    name = "Sigma"
  ) +
  scale_shape_manual(values = SHAPE) +
  scale_colour_manual(values = cb_palette,) +
  scale_fill_manual(values = cb_palette) +
  ggtitle('Final activation gene coverage') +
  p_theme

plot_grid(
  plot +
  theme(legend.position = "none"),
  legend,
```

```

  nrow=2,
  rel_heights = c(3,1)
)

```



6.4.2.1 Stats

Summary statistics for the generation a satisfactory solution is found.

```

act_coverage = filter(over_time_df, gen == 50000 & acro == 'con')
act_coverage$Sigma = factor(act_coverage$Sigma, levels = ND_LIST)
act_coverage %>%
  group_by(Sigma) %>%
  dplyr::summarise(
    count = n(),
    na_cnt = sum(is.na(uni_str_pos)),
    min = min(uni_str_pos, na.rm = TRUE),
    median = median(uni_str_pos, na.rm = TRUE),
    mean = mean(uni_str_pos, na.rm = TRUE),
    max = max(uni_str_pos, na.rm = TRUE),
    IQR = IQR(uni_str_pos, na.rm = TRUE)
  )

## # A tibble: 7 x 8
##   Sigma count na_cnt  min median  mean    max    IQR
##   <fct> <int> <int> <int> <dbl> <dbl> <int> <dbl>
## 1 0         50     0    59     73  73.5     87     11
## 2 0.1       50     0   100    100 100      100      0

```

```
## 3 0.3      50     0    99    100 100.    100     0
## 4 0.6      50     0   100    100 100     100     0
## 5 1.2      50     0   100    100 100     100     0
## 6 2.5      50     0    99    100 100.    100     0
## 7 5        50     0    99    100 100.    100     0
```

Kruskal-Wallis test illustrates evidence of statistical differences.

```
kruskal.test(uni_str_pos ~ Sigma, data = act_coverage)
```

```
##
## Kruskal-Wallis rank sum test
##
## data: uni_str_pos by Sigma
## Kruskal-Wallis chi-squared = 324.85, df = 6, p-value < 2.2e-16
```

Results for post-hoc Wilcoxon rank-sum test with a Bonferroni correction.

```
pairwise.wilcox.test(x = act_coverage$uni_str_pos, g = act_coverage$Sigma, p.adjust.method = "bonferroni",
                      paired = FALSE, conf.int = FALSE, alternative = 't')
```

```
##
## Pairwise comparisons using Wilcoxon rank sum test with continuity correction
##
## data: act_coverage$uni_str_pos and act_coverage$Sigma
##
##          0       0.1 0.3 0.6 1.2 2.5
## 0.1 <2e-16 -     -     -     -
## 0.3 <2e-16  1     -     -     -
## 0.6 <2e-16  -     1     -     -
## 1.2 <2e-16  -     1     -     -
## 2.5 <2e-16  1     1     1     -
## 5    <2e-16  1     1     1     1
##
## P value adjustment method: bonferroni
```

6.4.3 Satisfactory trait coverage over time

Satisfactory trait coverage in a population over time. Data points on the graph is the average activation gene coverage across 50 replicates every 2000 generations. Shading comes from the best and worse coverage across 50 replicates.

```
lines = filter(over_time_df, acro == 'con') %>%
  group_by(Sigma, gen) %>%
  dplyr::summarise(
    min = min(pop_uni_obj),
    mean = mean(pop_uni_obj),
    max = max(pop_uni_obj)
  )

## `summarise()` has grouped output by 'Sigma'. You can override using the
## `.groups` argument.

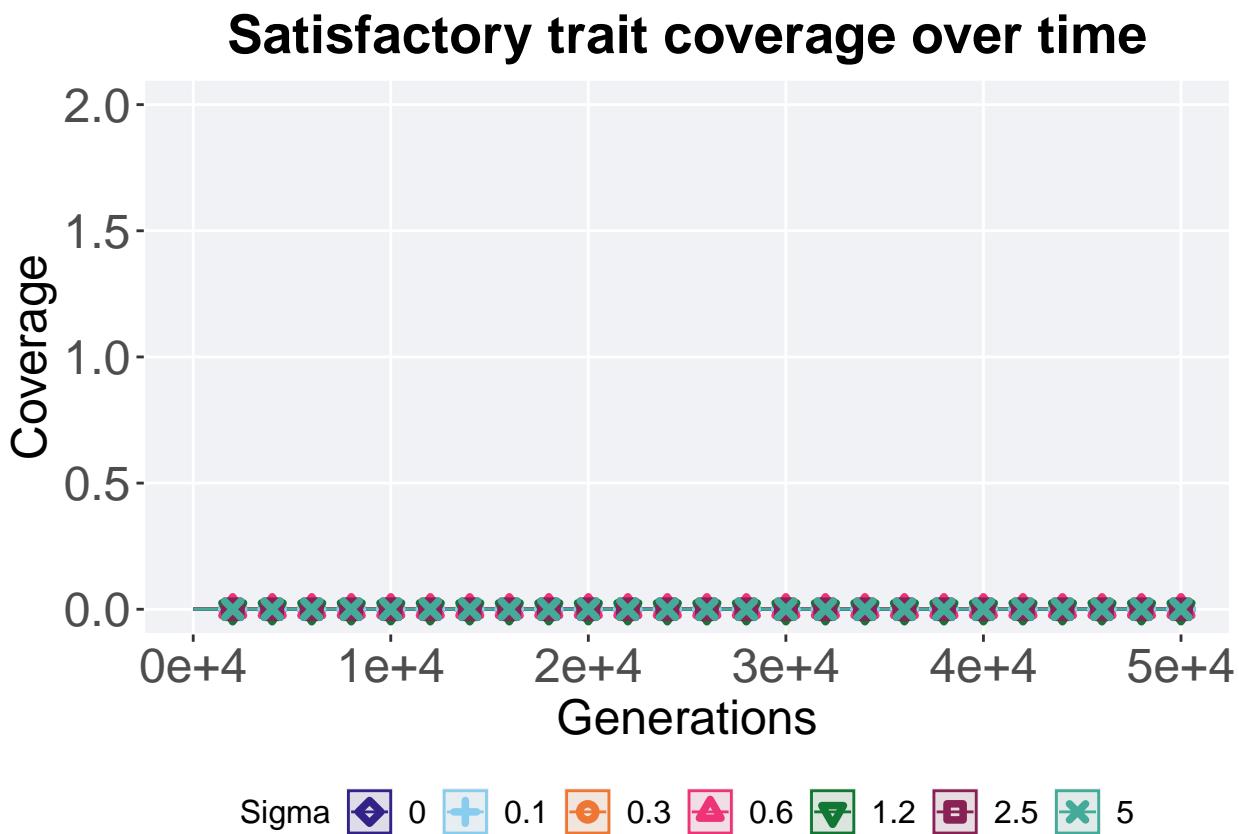
ggplot(lines, aes(x=gen, y=mean, group = Sigma, fill = Sigma, color = Sigma, shape = Sigma)) +
  geom_ribbon(aes(ymin = min, ymax = max), alpha = 0.1) +
  geom_line(size = 0.5) +
  geom_point(data = filter(lines, gen %% 2000 == 0 & gen != 0), size = 1.5, stroke = 2.0, alpha = 1.0) +
  scale_y_continuous()
```

```

    name="Coverage",
    limits=c(0, 2)
) +
scale_x_continuous(
  name="Generations",
  limits=c(0, 50000),
  breaks=c(0, 10000, 20000, 30000, 40000, 50000),
  labels=c("0e+4", "1e+4", "2e+4", "3e+4", "4e+4", "5e+4")

) +
scale_shape_manual(values=SHAPE) +
scale_colour_manual(values = cb_palette) +
scale_fill_manual(values = cb_palette) +
ggtitle('Satisfactory trait coverage over time') +
p_theme +
guides(
  shape=guide_legend(nrow=1, title.position = "left", title = 'Sigma'),
  color=guide_legend(nrow=1, title.position = "left", title = 'Sigma'),
  fill=guide_legend(nrow=1, title.position = "left", title = 'Sigma')
)

```



6.4.4 Final satisfactory trait coverage

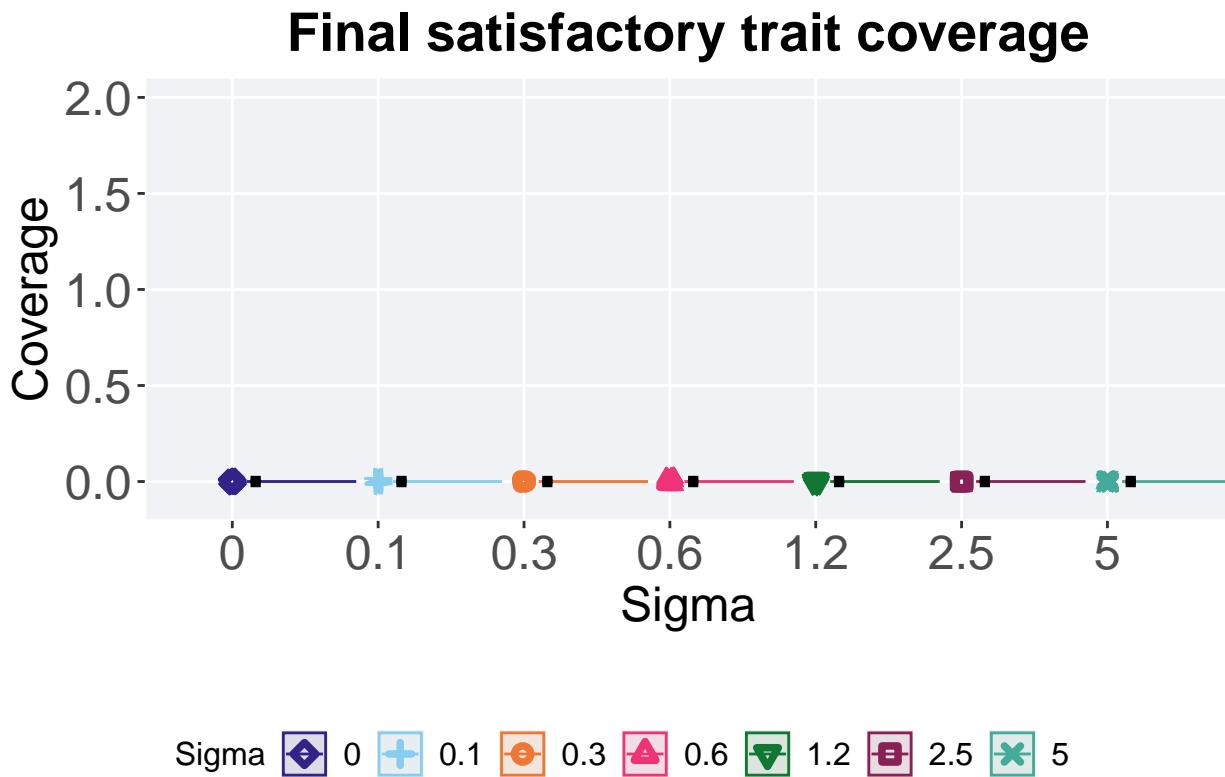
Satisfactory trait coverage found in the final population at 50,000 generations.

```

plot = filter(over_time_df, gen == 50000 & acro == 'con') %>%
  ggplot(., aes(x = Sigma, y = pop_uni_obj, color = Sigma, fill = Sigma, shape = Sigma)) +
  geom_flat_violin(position = position_nudge(x = .1, y = 0), scale = 'width', alpha = 0.2, width = 1.5)
  geom_boxplot(color = 'black', width = .07, outlier.shape = NA, alpha = 0.0, size = 1.0, position = position_nudge(x = .1, y = 0))
  geom_point(position = position_jitter(width = 0.03, height = 0.02), size = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name = "Coverage",
    limits = c(-0.1, 2)
  ) +
  scale_x_discrete(
    name = "Sigma"
  ) +
  scale_shape_manual(values = SHAPE) +
  scale_color_manual(values = cb_palette, ) +
  scale_fill_manual(values = cb_palette) +
  ggtitle('Final satisfactory trait coverage') +
  p_theme

plot_grid(
  plot +
    theme(legend.position = "none"),
  legend,
  nrow = 2,
  rel_heights = c(3, 1)
)

```



6.4.4.1 Stats

Summary statistics for the generation a satisfactory solution is found.

```
sat_coverage = filter(over_time_df, gen == 50000 & acro == 'con')
sat_coverage$Sigma = factor(sat_coverage$Sigma, levels = ND_LIST)
sat_coverage %>%
  group_by(Sigma) %>%
  dplyr::summarise(
    count = n(),
    na_cnt = sum(is.na(pop_uni_obj)),
    min = min(pop_uni_obj, na.rm = TRUE),
    median = median(pop_uni_obj, na.rm = TRUE),
    mean = mean(pop_uni_obj, na.rm = TRUE),
    max = max(pop_uni_obj, na.rm = TRUE),
    IQR = IQR(pop_uni_obj, na.rm = TRUE)
  )

## # A tibble: 7 x 8
##   Sigma count na_cnt  min median  mean  max  IQR
##   <fct> <int>   <int> <dbl> <dbl> <int> <dbl>
## 1 0         50      0     0     0     0     0     0
## 2 0.1       50      0     0     0     0     0     0
## 3 0.3       50      0     0     0     0     0     0
## 4 0.6       50      0     0     0     0     0     0
## 5 1.2       50      0     0     0     0     0     0
## 6 2.5       50      0     0     0     0     0     0
## 7 5         50      0     0     0     0     0     0
```

6.4.5 Largest valley reached throughout

Largest valley reached in a single trait by the best performing solution throughout an entire evolutionary run.

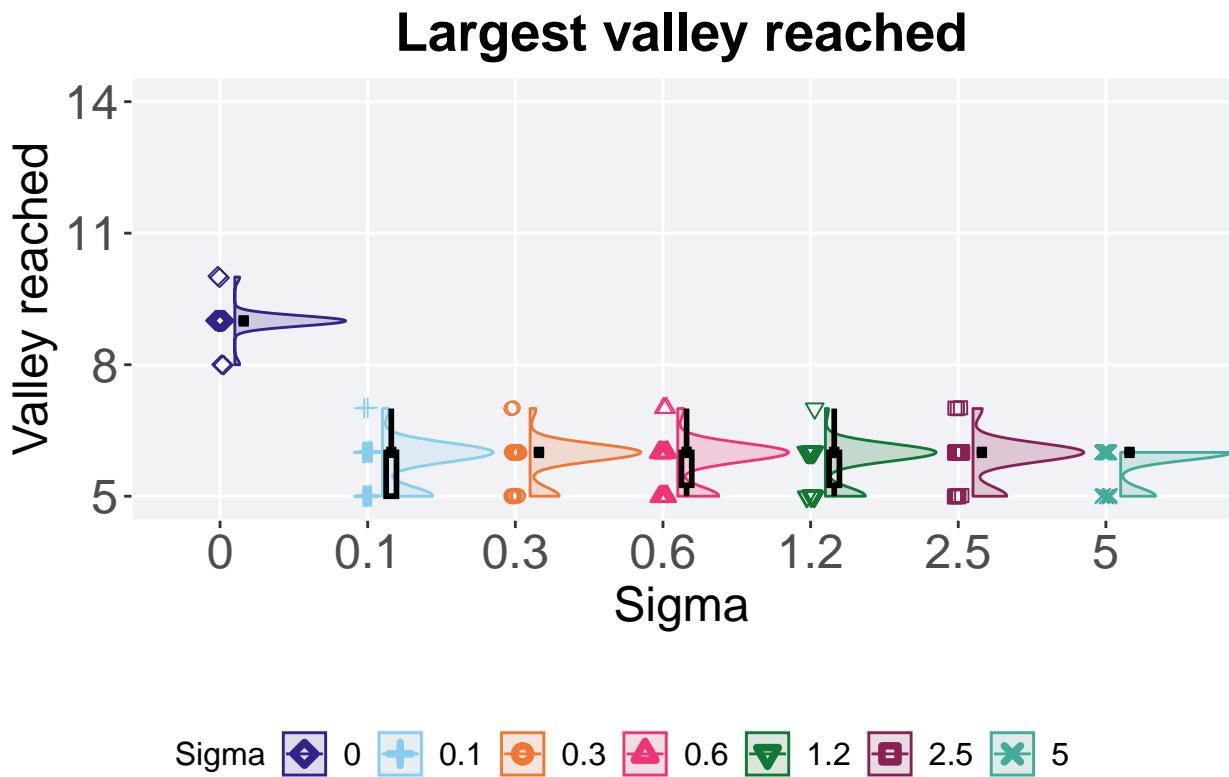
```
plot = filter(best_df, acro == 'con' & var == 'ele_big_peak') %>%
  ggplot(. , aes(x = Sigma, y = val, color = Sigma, fill = Sigma, shape = Sigma)) +
  geom_flat_violin(position = position_nudge(x = .1, y = 0), scale = 'width', alpha = 0.2, width = 1.5) +
  geom_boxplot(color = 'black', width = .07, outlier.shape = NA, alpha = 0.0, size = 1.0, position = position_nudge(x = -.1, y = 0)) +
  geom_point(position = position_jitter(width = 0.03, height = 0.02), size = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name="Valley reached",
    limits=c(4.9,14.1),
    breaks=c(5,8,11,14)
  ) +
  scale_x_discrete(
    name="Sigma"
  ) +
  scale_shape_manual(values=SHAPE) +
  scale_colour_manual(values = cb_palette, ) +
  scale_fill_manual(values = cb_palette) +
  ggtitle('Largest valley reached') +
  p_theme

plot_grid(
  plot +
    theme(legend.position="none"),
  legend,
```

```

nrow=2,
rel_heights = c(3,1)
)

```



6.4.5.1 Stats

Summary statistics for the largest valley crossed.

```

valleys = filter(best_df, acro == 'con' & var == 'ele_big_peak')
valleys$Sigma = factor(valleys$Sigma, levels = ND_LIST)
valleys %>%
  group_by(Sigma) %>%
  dplyr::summarise(
    count = n(),
    na_cnt = sum(is.na(val)),
    min = min(val, na.rm = TRUE),
    median = median(val, na.rm = TRUE),
    mean = mean(val, na.rm = TRUE),
    max = max(val, na.rm = TRUE),
    IQR = IQR(val, na.rm = TRUE)
  )

## # A tibble: 7 x 8
##   Sigma count na_cnt  min median  mean  max    IQR
##   <fct> <int>   <int> <dbl>  <dbl> <dbl> <dbl> <dbl>
## 1 0         50      0     8     9     9     10     0
## 2 0.1       50      0     5     6     5.74    7     1

```

```
## 3 0.3      50     0     5     6  5.84      7  0
## 4 0.6      50     0     5     6  5.78      7  0.75
## 5 1.2      50     0     5     6  5.76      7  0.75
## 6 2.5      50     0     5     6  5.84      7  0
## 7 5        50     0     5     6  5.76      6  0
```

Kruskal–Wallis test illustrates evidence of statistical differences.

```
kruskal.test(val ~ Sigma, data = valleys)

##
##  Kruskal-Wallis rank sum test
##
## data: val by Sigma
## Kruskal-Wallis chi-squared = 171.34, df = 6, p-value < 2.2e-16
```

Results for post-hoc Wilcoxon rank-sum test with a Bonferroni correction.

```
pairwise.wilcox.test(x = valleys$val, g = valleys$Sigma, p.adjust.method = "bonferroni",
                      paired = FALSE, conf.int = FALSE, alternative = 't')

##
##  Pairwise comparisons using Wilcoxon rank sum test with continuity correction
##
## data: valleys$val and valleys$Sigma
##
##      0      0.1 0.3 0.6 1.2 2.5
## 0.1 <2e-16 - - - - -
## 0.3 <2e-16 1 - - - - -
## 0.6 <2e-16 1 1 - - - -
## 1.2 <2e-16 1 1 1 - - -
## 2.5 <2e-16 1 1 1 1 - -
## 5   <2e-16 1 1 1 1 1
##
## P value adjustment method: bonferroni
```

6.5 Multi-path exploration results

Here we present the results for best performances and activation gene coverage found by each selection scheme parameter on the multi-path exploration diagnostic with valleys. 50 replicates are conducted for each scheme parameter explored.

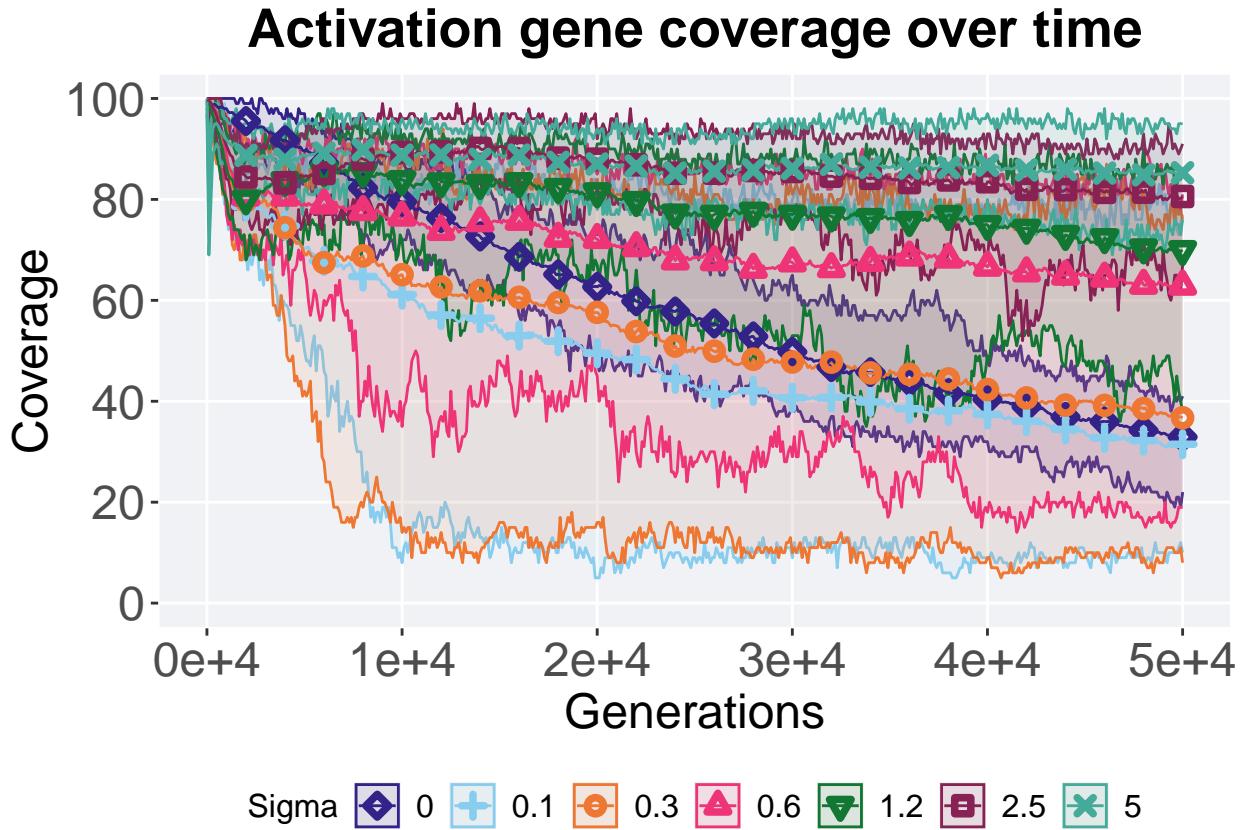
6.5.1 Activation gene coverage over time

Activation gene coverage in a population over time. Data points on the graph is the average activation gene coverage across 50 replicates every 2000 generations. Shading comes from the best and worse coverage across 50 replicates.

```
lines = filter(over_time_df, acro == 'mpe') %>%
  group_by(Sigma, gen) %>%
  dplyr::summarise(
    min = min(uni_str_pos),
    mean = mean(uni_str_pos),
    max = max(uni_str_pos)
  )

## `summarise()` has grouped output by 'Sigma'. You can override using the
```

```
## ` `.groups` argument.
ggplot(lines, aes(x=gen, y=mean, group = Sigma, fill = Sigma, color = Sigma, shape = Sigma)) +
  geom_ribbon(aes(ymin = min, ymax = max), alpha = 0.1) +
  geom_line(size = 0.5) +
  geom_point(data = filter(lines, gen %% 2000 == 0 & gen != 0), size = 1.5, stroke = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name="Coverage",
    limits=c(0, 100),
    breaks=seq(0,100, 20),
    labels=c("0", "20", "40", "60", "80", "100")
  ) +
  scale_x_continuous(
    name="Generations",
    limits=c(0, 50000),
    breaks=c(0, 10000, 20000, 30000, 40000, 50000),
    labels=c("0e+4", "1e+4", "2e+4", "3e+4", "4e+4", "5e+4")
  ) +
  scale_shape_manual(values=SHAPE) +
  scale_colour_manual(values = cb_palette) +
  scale_fill_manual(values = cb_palette) +
  ggtitle('Activation gene coverage over time') +
  p_theme +
  guides(
    shape=guide_legend(nrow=1, title.position = "left", title = 'Sigma'),
    color=guide_legend(nrow=1, title.position = "left", title = 'Sigma'),
    fill=guide_legend(nrow=1, title.position = "left", title = 'Sigma')
  )
```



6.5.2 Final activation gene coverage

Activation gene coverage found in the final population at 50,000 generations.

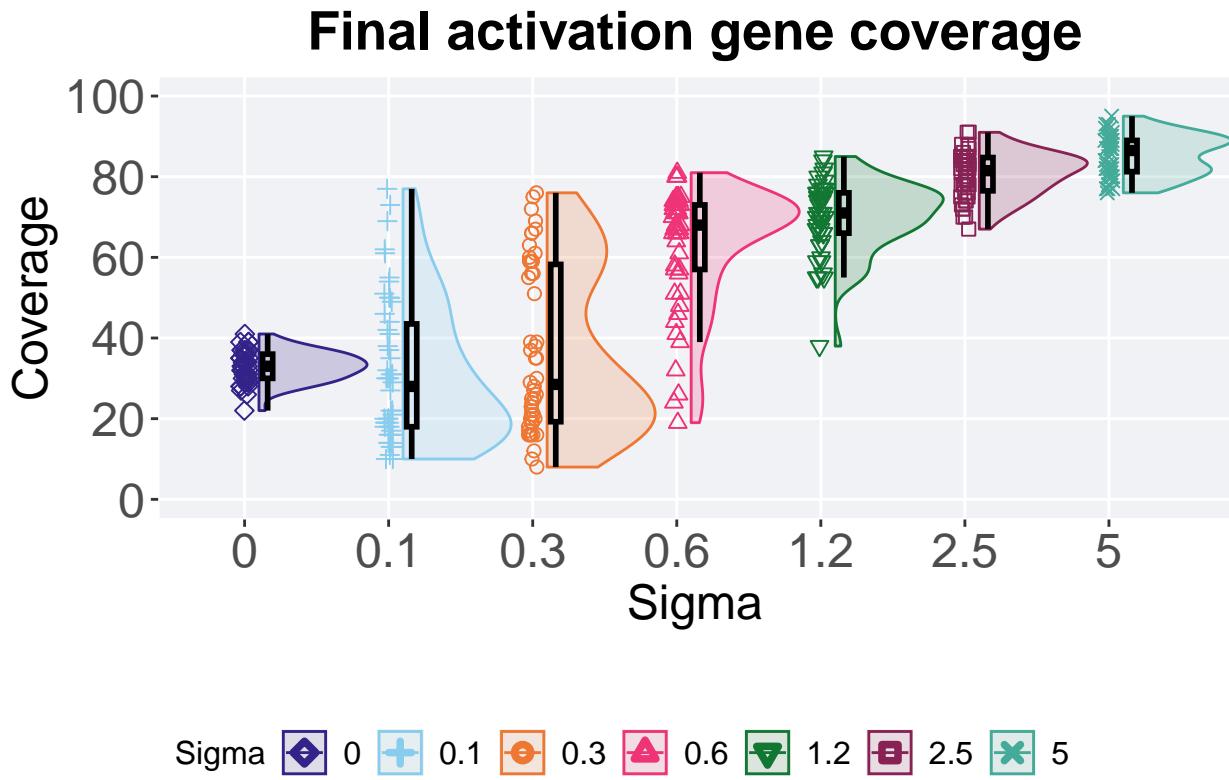
```
plot = filter(over_time_df, gen == 50000 & acro == 'mpe') %>%
  ggplot(., aes(x = Sigma, y = uni_str_pos, color = Sigma, fill = Sigma, shape = Sigma)) +
  geom_flat_violin(position = position_nudge(x = .1, y = 0), scale = 'width', alpha = 0.2, width = 1.5)
  geom_boxplot(color = 'black', width = .07, outlier.shape = NA, alpha = 0.0, size = 1.0, position = position_nudge(x = -.1, y = 0))
  geom_point(position = position_jitter(width = 0.03, height = 0.02), size = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name="Coverage",
    limits=c(0, 100),
    breaks=seq(0,100, 20),
    labels=c("0", "20", "40", "60", "80", "100")
  ) +
  scale_x_discrete(
    name="Sigma"
  ) +
  scale_shape_manual(values=SHAPE) +
  scale_colour_manual(values = cb_palette, ) +
  scale_fill_manual(values = cb_palette) +
  ggtitle('Final activation gene coverage') +
  p_theme

plot_grid(
  plot +
```

```

    theme(legend.position="none"),
  legend,
  nrow=2,
  rel_heights = c(3,1)
)

```



6.5.2.1 Stats

Summary statistics for the generation a satisfactory solution is found.

```

act_coverage = filter(over_time_df, gen == 50000 & acro == 'mpe')
act_coverage$Sigma = factor(act_coverage$Sigma, levels = ND_LIST)
act_coverage %>%
  group_by(Sigma) %>%
  dplyr::summarise(
    count = n(),
    na_cnt = sum(is.na(uni_str_pos)),
    min = min(uni_str_pos, na.rm = TRUE),
    median = median(uni_str_pos, na.rm = TRUE),
    mean = mean(uni_str_pos, na.rm = TRUE),
    max = max(uni_str_pos, na.rm = TRUE),
    IQR = IQR(uni_str_pos, na.rm = TRUE)
  )

## # A tibble: 7 x 8
##   Sigma count na_cnt  min  median  mean   max    IQR
##   <fct> <int> <int> <dbl> <dbl> <dbl> <int> <dbl>
## 1 0       7     1000  20.0  34.0  33.0  40.0  1000  10.0
## 2 0.1     7     1000  10.0  43.0  40.0  50.0  1000  10.0
## 3 0.3     7     1000  10.0  55.0  50.0  60.0  1000  10.0
## 4 0.6     7     1000  15.0  75.0  70.0  80.0  1000  10.0
## 5 1.2     7     1000  35.0  75.0  70.0  80.0  1000  10.0
## 6 2.5     7     1000  65.0  85.0  80.0  90.0  1000  10.0
## 7 5       7     1000  75.0  85.0  80.0  90.0  1000  10.0

```

```
## 1 0      50    0    22   33   32.9    41   6
## 2 0.1    50    0    10   28   31.5    77 25.5
## 3 0.3    50    0     8   28.5  36.7    76 39
## 4 0.6    50    0    19   68   62.9    81 16
## 5 1.2    50    0    38   71   70.0    85 10
## 6 2.5    50    0    67   81.5  80.6    91 8.25
## 7 5      50    0    76   86.5  85.3    95 7.75
```

Kruskal–Wallis test illustrates evidence of statistical differences.

```
kruskal.test(uni_str_pos ~ Sigma, data = act_coverage)
```

```
##
## Kruskal-Wallis rank sum test
##
## data: uni_str_pos by Sigma
## Kruskal-Wallis chi-squared = 268, df = 6, p-value < 2.2e-16
```

Results for post-hoc Wilcoxon rank-sum test with a Bonferroni correction.

```
pairwise.wilcox.test(x = act_coverage$uni_str_pos, g = act_coverage$Sigma, p.adjust.method = "bonferroni",
                      paired = FALSE, conf.int = FALSE, alternative = 't')
```

```
##
## Pairwise comparisons using Wilcoxon rank sum test with continuity correction
##
## data: act_coverage$uni_str_pos and act_coverage$Sigma
##
##          0       0.1      0.3      0.6      1.2      2.5
## 0.1 1.0000 -       -       -       -       -
## 0.3 1.0000 1.0000 -       -       -       -
## 0.6 3.5e-12 3.0e-10 1.0e-07 -       -       -
## 1.2 < 2e-16 1.1e-13 2.1e-11 0.3489 -       -
## 2.5 < 2e-16 4.6e-16 6.8e-16 3.7e-12 1.5e-08 -
## 5   < 2e-16 < 2e-16 < 2e-16 7.7e-16 5.1e-14 0.0033
##
## P value adjustment method: bonferroni
```

6.5.3 Performance over time

Best performance in a population over time. Data points on the graph is the average performance across 50 replicates every 2000 generations. Shading comes from the best and worse performance across 50 replicates.

```
lines = filter(over_time_df, acro == 'mpe') %>%
  group_by(Sigma, gen) %>%
  dplyr::summarise(
    min = min(pop_fit_max) / DIMENSIONALITY,
    mean = mean(pop_fit_max) / DIMENSIONALITY,
    max = max(pop_fit_max) / DIMENSIONALITY
  )

## `summarise()` has grouped output by 'Sigma'. You can override using the
## `.groups` argument.

over_time_plot = ggplot(lines, aes(x=gen, y=mean, group = Sigma, fill = Sigma, color = Sigma, shape = S
  geom_ribbon(aes(ymin = min, ymax = max), alpha = 0.1) +
  geom_line(size = 0.5) +
  geom_point(data = filter(lines, gen %% 2000 == 0 & gen != 0), size = 1.5, stroke = 2.0, alpha = 1.0) -
```

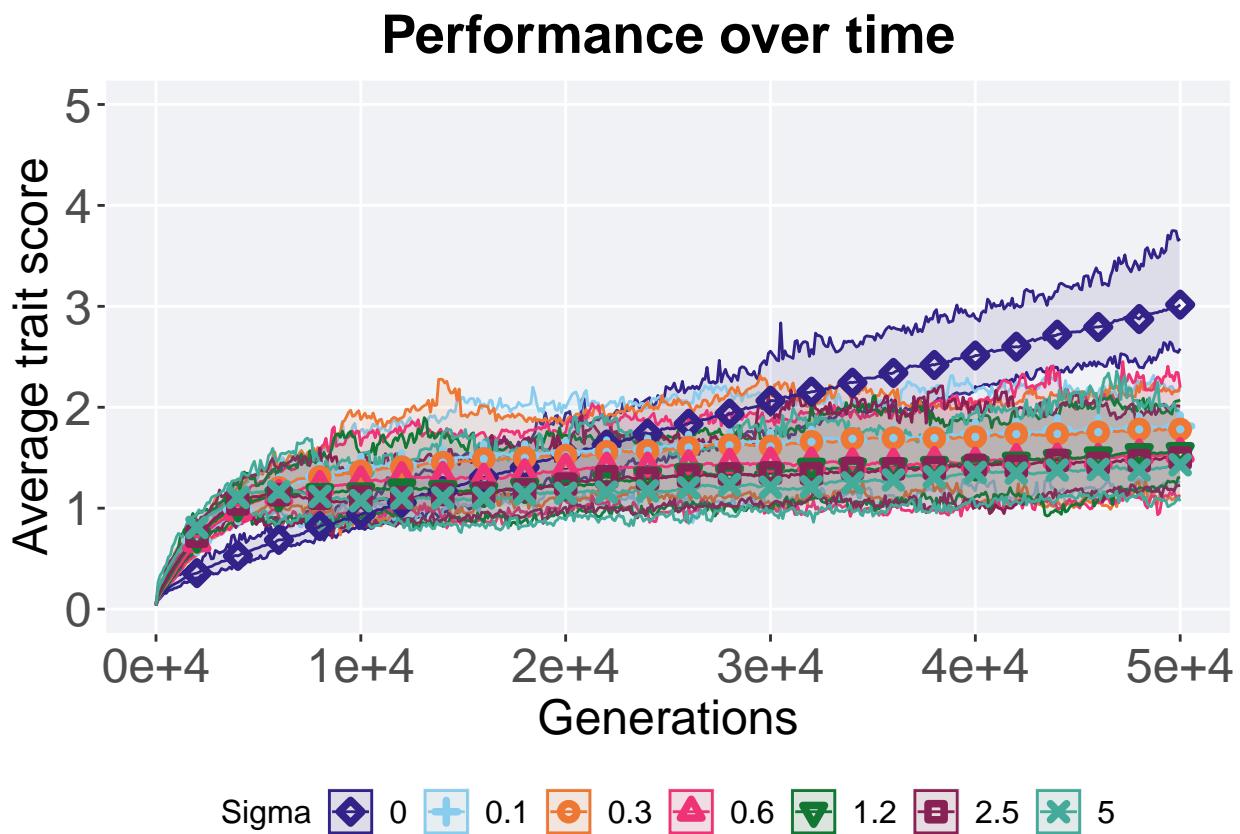
```

scale_y_continuous(
  name="Average trait score",
  limits=c(0, 5)
) +
scale_x_continuous(
  name="Generations",
  limits=c(0, 50000),
  breaks=c(0, 10000, 20000, 30000, 40000, 50000),
  labels=c("0e+4", "1e+4", "2e+4", "3e+4", "4e+4", "5e+4")

) +
scale_shape_manual(values=SHAPE) +
scale_colour_manual(values = cb_palette) +
scale_fill_manual(values = cb_palette) +
ggtitle('Performance over time') +
p_theme +
guides(
  shape=guide_legend(nrow=1, title.position = "left", title = 'Sigma'),
  color=guide_legend(nrow=1, title.position = "left", title = 'Sigma'),
  fill=guide_legend(nrow=1, title.position = "left", title = 'Sigma')
)

over_time_plot

```

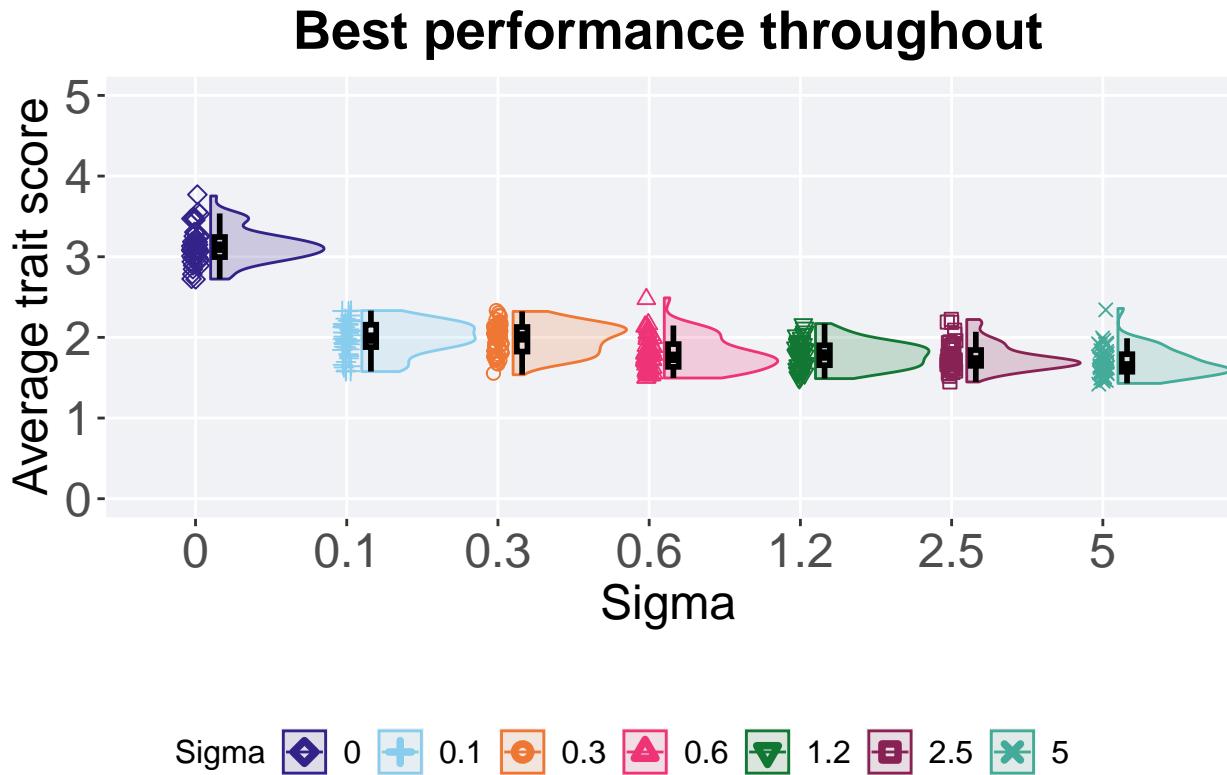


6.5.4 Best performance throughout

Best performance reached throughout 50,000 generations in a population.

```
plot = filter(best_df, acro == 'mpe' & var == 'pop_fit_max') %>%
  ggplot(., aes(x = Sigma, y = val / DIMENSIONALITY, color = Sigma, fill = Sigma, shape = Sigma)) +
  geom_flat_violin(position = position_nudge(x = .1, y = 0), scale = 'width', alpha = 0.2, width = 1.5) +
  geom_boxplot(color = 'black', width = .07, outlier.shape = NA, alpha = 0.0, size = 1.0, position = position_nudge(x = -.1, y = 0)) +
  geom_point(position = position_jitter(width = 0.03, height = 0.02), size = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name = "Average trait score",
    limits = c(0, 5)
  ) +
  scale_x_discrete(
    name = "Sigma"
  ) +
  scale_shape_manual(values = SHAPE) +
  scale_colour_manual(values = cb_palette,) +
  scale_fill_manual(values = cb_palette) +
  ggtitle('Best performance throughout') +
  p_theme

plot_grid(
  plot +
    theme(legend.position = "none"),
  legend,
  nrow = 2,
  rel_heights = c(3, 1)
)
```



6.5.4.1 Stats

Summary statistics for the best performance.

```
performance = filter(best_df, acro == 'mpe' & var == 'pop_fit_max')
performance$Sigma = factor(performance$Sigma, levels = ND_LIST)
performance %>%
  group_by(Sigma) %>%
  dplyr::summarise(
```

```
  count = n(),
  na_cnt = sum(is.na(val)),
  min = min(val / DIMENSIONALITY, na.rm = TRUE),
  median = median(val / DIMENSIONALITY, na.rm = TRUE),
  mean = mean(val / DIMENSIONALITY, na.rm = TRUE),
  max = max(val / DIMENSIONALITY, na.rm = TRUE),
  IQR = IQR(val / DIMENSIONALITY, na.rm = TRUE)
)
```

```
## # A tibble: 7 x 8
##   Sigma count na_cnt   min   median   mean   max   IQR
##   <fct> <int>   <int>   <dbl>   <dbl>   <dbl>   <dbl>
## 1 0         50       0  2.72    3.12    3.14    3.75  0.249
## 2 0.1       50       0  1.58    1.99    1.99    2.33  0.277
## 3 0.3       50       0  1.54    2.01    1.98    2.32  0.305
## 4 0.6       50       0  1.50    1.75    1.79    2.49  0.282
## 5 1.2       50       0  1.49    1.78    1.78    2.17  0.247
## 6 2.5       50       0  1.45    1.71    1.76    2.22  0.196
```

```
## 7 5      50      0 1.43 1.63 1.68 2.36 0.217
```

Kruskal–Wallis test illustrates evidence of statistical differences.

```
kruskal.test(val ~ Sigma, data = performance)
```

```
##
## Kruskal-Wallis rank sum test
##
## data: val by Sigma
## Kruskal-Wallis chi-squared = 186.67, df = 6, p-value < 2.2e-16
```

Results for post-hoc Wilcoxon rank-sum test with a Bonferroni correction.

```
pairwise.wilcox.test(x = performance$val, g = performance$Sigma, p.adjust.method = "bonferroni",
                      paired = FALSE, conf.int = FALSE, alternative = 't')
```

```
##
## Pairwise comparisons using Wilcoxon rank sum test with continuity correction
##
## data: performance$val and performance$Sigma
##
##          0       0.1      0.3      0.6      1.2      2.5
## 0.1 < 2e-16 -       -       -       -       -
## 0.3 < 2e-16 1.00000 -       -       -       -
## 0.6 < 2e-16 0.00026 0.00029 -       -       -
## 1.2 < 2e-16 3.2e-05 6.3e-05 1.00000 -       -
## 2.5 < 2e-16 9.6e-06 2.2e-06 1.00000 1.00000 -
## 5    < 2e-16 8.3e-09 5.8e-09 0.18672 0.05257 0.50321
##
## P value adjustment method: bonferroni
```

6.5.5 Largest valley reached throughout

Largest valley reached in a single trait by the best performing solution throughout an entire evolutionary run.

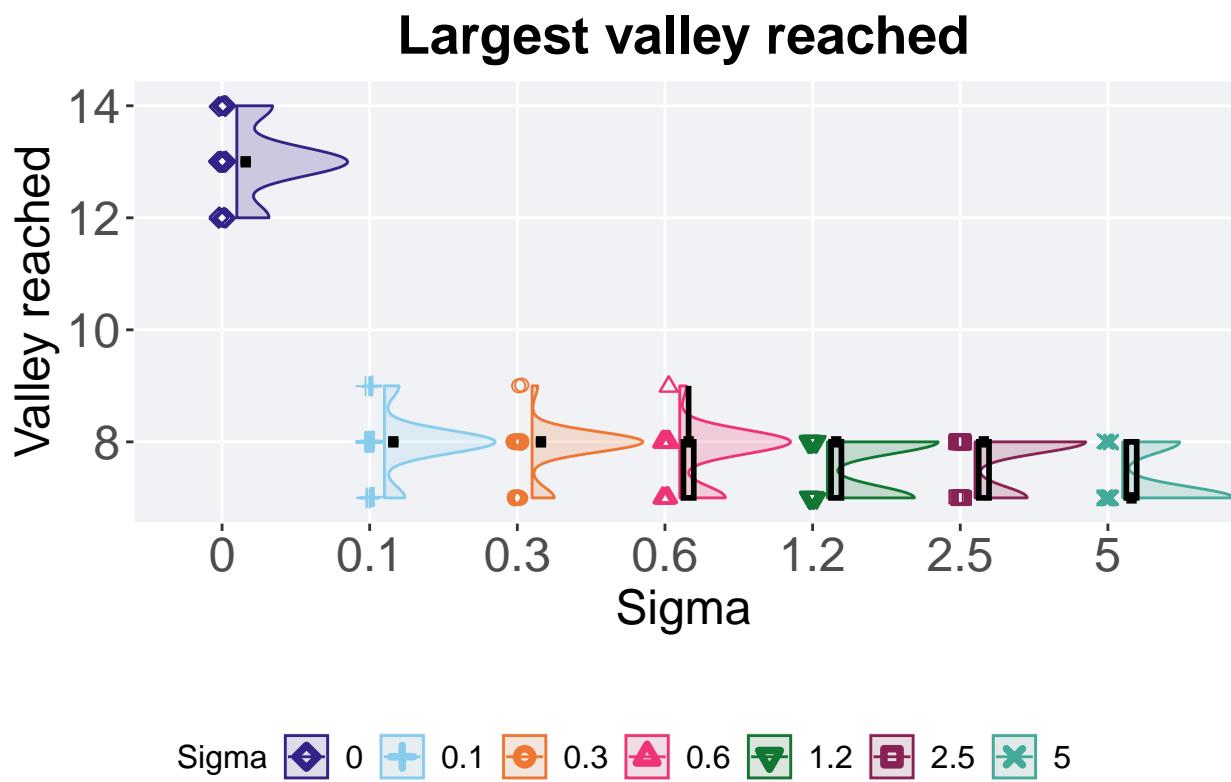
```
plot = filter(best_df, acro == 'mpe' & var == 'ele_big_peak') %>%
  ggplot(., aes(x = Sigma, y = val, color = Sigma, fill = Sigma, shape = Sigma)) +
  geom_flat_violin(position = position_nudge(x = .1, y = 0), scale = 'width', alpha = 0.2, width = 1.5) +
  geom_boxplot(color = 'black', width = .07, outlier.shape = NA, alpha = 0.0, size = 1.0, position = position_dodge(.7)) +
  geom_point(position = position_jitter(width = 0.03, height = 0.02), size = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name = "Valley reached",
    limits = c(6.9, 14.1)
  ) +
  scale_x_discrete(
    name = "Sigma"
  ) +
  scale_shape_manual(values = SHAPE) +
  scale_colour_manual(values = cb_palette,) +
  scale_fill_manual(values = cb_palette) +
  ggtitle('Largest valley reached') +
  p_theme

plot_grid(
  plot +
    theme(legend.position = "none"),
```

```

  legend,
  nrow=2,
  rel_heights = c(3,1)
)

```



6.5.5.1 Stats

Summary statistics for the largest valley crossed.

```

valleys = filter(best_df, acro == 'mpe' & var == 'ele_big_peak')
valleys$Sigma = factor(valleys$Sigma, levels = ND_LIST)
valleys %>%
  group_by(Sigma) %>%
  dplyr::summarise(
    count = n(),
    na_cnt = sum(is.na(val)),
    min = min(val, na.rm = TRUE),
    median = median(val, na.rm = TRUE),
    mean = mean(val, na.rm = TRUE),
    max = max(val, na.rm = TRUE),
    IQR = IQR(val, na.rm = TRUE)
  )

```

```

## # A tibble: 7 x 8
##   Sigma count na_cnt  min median  mean  max   IQR
##   <fct> <int>    <int> <dbl>  <dbl> <dbl> <dbl> <dbl>
## 1 0         50       0    12     13.0 13.0    14     0

```

```
## 2 0.1      50    0    7     8   7.96    9    0
## 3 0.3      50    0    7     8   7.88    9    0
## 4 0.6      50    0    7     8   7.76    9    1
## 5 1.2      50    0    7     8   7.56    8    1
## 6 2.5      50    0    7     8   7.68    8    1
## 7 5        50    0    7     7   7.34    8    1
```

Kruskal–Wallis test illustrates evidence of statistical differences.

```
kruskal.test(val ~ Sigma, data = valleys)
```

```
##
## Kruskal-Wallis rank sum test
##
## data: val by Sigma
## Kruskal-Wallis chi-squared = 187.01, df = 6, p-value < 2.2e-16
```

Results for post-hoc Wilcoxon rank-sum test with a Bonferroni correction.

```
pairwise.wilcox.test(x = valleys$val, g = valleys$Sigma, p.adjust.method = "bonferroni",
                      paired = FALSE, conf.int = FALSE, alternative = 't')
```

```
##
## Pairwise comparisons using Wilcoxon rank sum test with continuity correction
##
## data: valleys$val and valleys$Sigma
##
##          0       0.1      0.3      0.6      1.2      2.5
## 0.1 < 2e-16 -       -       -       -       -
## 0.3 < 2e-16 1.0000 -       -       -       -
## 0.6 < 2e-16 1.0000 1.0000 -       -       -
## 1.2 < 2e-16 0.0046 0.0282 1.0000 -       -
## 2.5 < 2e-16 0.1364 0.7287 1.0000 1.0000 -
## 5    < 2e-16 1.1e-06 6.4e-06 0.0022 0.5899 0.0152
##
## P value adjustment method: bonferroni
```

Chapter 7

Novelty search

Results for the novelty search parameter sweep on the diagnostics with valleys.

7.1 Data setup

```
over_time_df <- read.csv(paste(DATA_DIR, 'OVER-TIME-MVC/nov.csv', sep = "", collapse = NULL), header = TRUE)
over_time_df$K <- factor(over_time_df$K, levels = NS_LIST)
over_time_df$uni_str_pos = over_time_df$uni_str_pos + over_time_df$arc_acti_gene - over_time_df$overlap

best_df <- read.csv(paste(DATA_DIR, 'BEST-MVC/nov.csv', sep = "", collapse = NULL), header = TRUE, stringsAsFactors = FALSE)
best_df$K <- factor(best_df$K, levels = NS_LIST)
```

7.2 Exploitation rate results

Here we present the results for **best performances** found by each selection scheme parameter on the exploitation rate diagnostic with valleys. 50 replicates are conducted for each scheme explored.

7.2.1 Performance over time

Best performance in a population over time. Data points on the graph is the average performance across 50 replicates every 2000 generations. Shading comes from the best and worse performance across 50 replicates.

```
lines = filter(over_time_df, acro == 'exp') %>%
  group_by(K, gen) %>%
  dplyr::summarise(
    min = min(pop_fit_max) / DIMENSIONALITY,
    mean = mean(pop_fit_max) / DIMENSIONALITY,
    max = max(pop_fit_max) / DIMENSIONALITY
  )

## `summarise()` has grouped output by 'K'. You can override using the `.groups` argument.

over_time_plot = ggplot(lines, aes(x=gen, y=mean, group = K, fill = K, color = K, shape = K)) +
  geom_ribbon(aes(ymax = min, ymin = max), alpha = 0.1) +
  geom_line(size = 0.5) +
  geom_point(data = filter(lines, gen %% 2000 == 0 & gen != 0), size = 1.5, stroke = 2.0, alpha = 1.0) +
  scale_y_continuous()
```

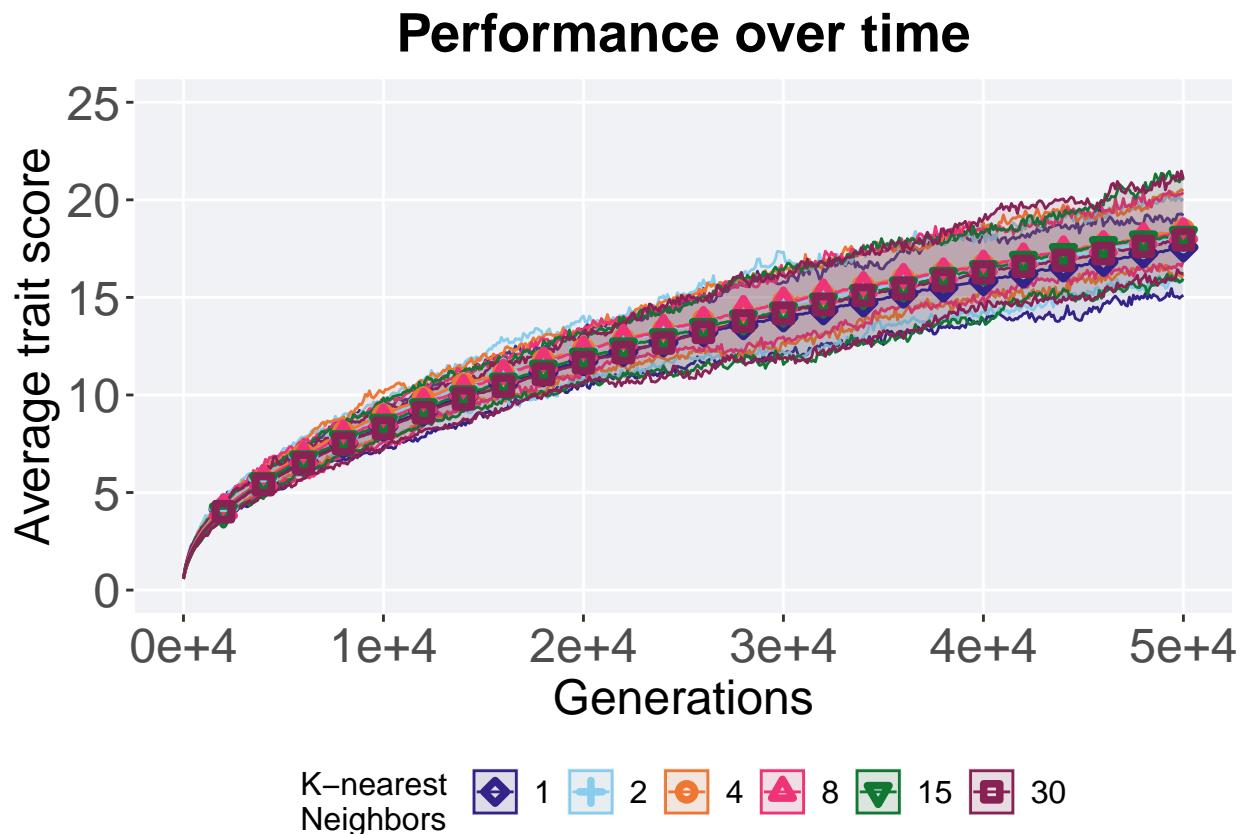
```

    name="Average trait score",
    limits = c(0,25)
) +
scale_x_continuous(
  name="Generations",
  limits=c(0, 50000),
  breaks=c(0, 10000, 20000, 30000, 40000, 50000),
  labels=c("0e+4", "1e+4", "2e+4", "3e+4", "4e+4", "5e+4")

) +
scale_shape_manual(values=SHAPE) +
scale_colour_manual(values = cb_palette) +
scale_fill_manual(values = cb_palette) +
ggtitle('Performance over time') +
p_theme +
guides(
  shape=guide_legend(nrow=1, title.position = "left", title = 'K-nearest \nNeighbors'),
  color=guide_legend(nrow=1, title.position = "left", title = 'K-nearest \nNeighbors'),
  fill=guide_legend(nrow=1, title.position = "left", title = 'K-nearest \nNeighbors')
)

over_time_plot

```



7.2.2 Best performance throughout

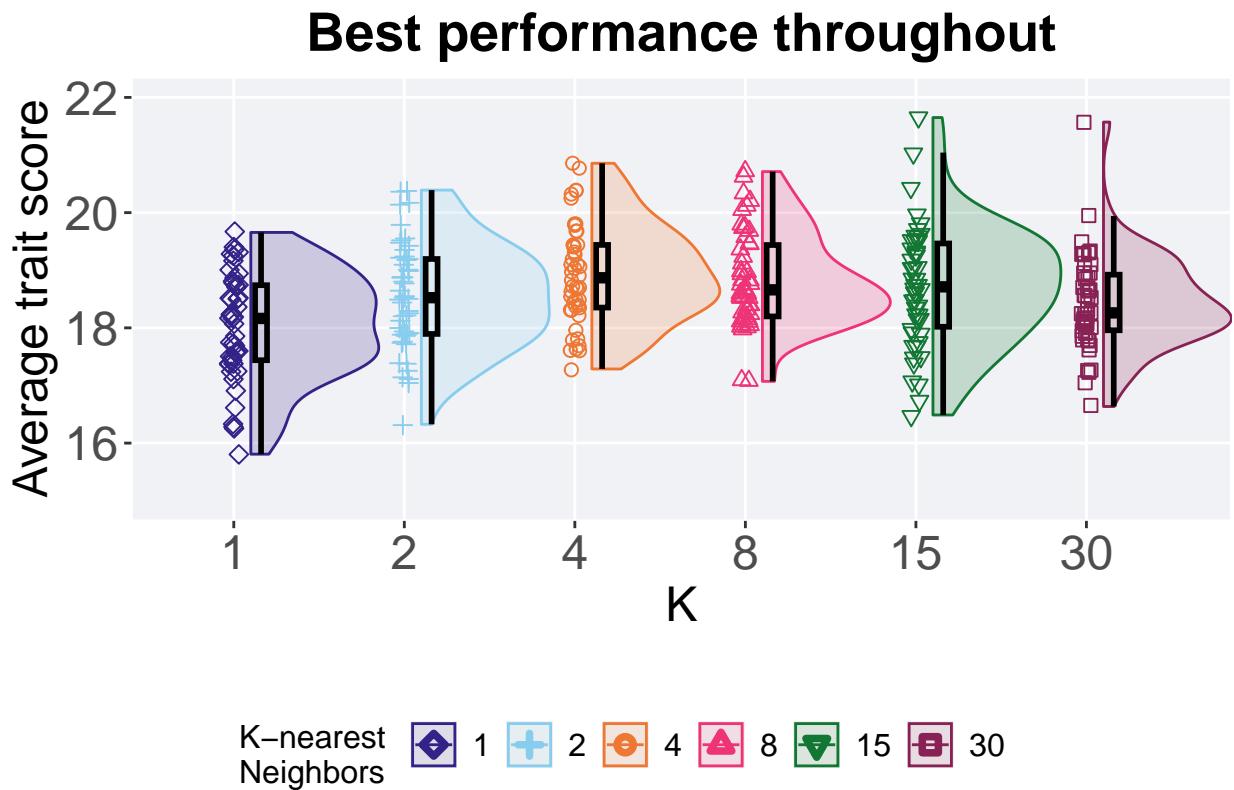
Best performance reached throughout 50,000 generations in a population.

```

plot = filter(best_df, acro == 'exp' & var == 'pop_fit_max') %>%
  ggplot(., aes(x = K, y = val / DIMENSIONALITY, color = K, fill = K, shape = K)) +
  geom_flat_violin(position = position_nudge(x = .1, y = 0), scale = 'width', alpha = 0.2, width = 1.5) +
  geom_boxplot(color = 'black', width = .07, outlier.shape = NA, alpha = 0.0, size = 1.0, position = position_nudge(x = -.1, y = 0)) +
  geom_point(position = position_jitter(width = 0.03, height = 0.02), size = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name = "Average trait score",
    limits = c(15, 22)
  ) +
  scale_x_discrete(
    name = "K"
  ) +
  scale_shape_manual(values = SHAPE) +
  scale_color_manual(values = cb_palette,) +
  scale_fill_manual(values = cb_palette) +
  ggtitle('Best performance throughout') +
  p_theme

plot_grid(
  plot +
    theme(legend.position = "none"),
  legend,
  nrow = 2,
  rel_heights = c(3, 1)
)

```



7.2.2.1 Stats

Summary statistics for the best performance.

```
performance = filter(best_df, acro == 'exp' & var == 'pop_fit_max')
performance$K = factor(performance$K, levels = NS_LIST)
performance %>%
  group_by(K) %>%
  dplyr::summarise(
    count = n(),
    na_cnt = sum(is.na(val)),
    min = min(val / DIMENSIONALITY, na.rm = TRUE),
    median = median(val / DIMENSIONALITY, na.rm = TRUE),
    mean = mean(val / DIMENSIONALITY, na.rm = TRUE),
    max = max(val / DIMENSIONALITY, na.rm = TRUE),
    IQR = IQR(val / DIMENSIONALITY, na.rm = TRUE)
  )

## # A tibble: 6 x 8
##   K     count na_cnt   min median   mean   max   IQR
##   <fct> <int>   <int> <dbl> <dbl> <dbl> <dbl> <dbl>
## 1 1       50      0  15.8   18.2   18.0  19.7  1.30
## 2 2       50      0  16.3   18.5   18.5  20.4  1.30
## 3 4       50      0  17.3   18.9   18.9  20.9  1.09
## 4 8       50      0  17.1   18.7   18.8  20.7  1.24
## 5 15      50      0  16.5   18.7   18.7  21.7  1.45
## 6 30      50      0  16.6   18.3   18.4  21.6  0.965
```

Kruskal–Wallis test illustrates evidence of statistical differences.

```
kruskal.test(val ~ K, data = performance)
```

```
##
## Kruskal-Wallis rank sum test
##
## data: val by K
## Kruskal-Wallis chi-squared = 27.662, df = 5, p-value = 4.237e-05
```

Results for post-hoc Wilcoxon rank-sum test with a Bonferroni correction.

```
pairwise.wilcox.test(x = performance$val, g = performance$K, p.adjust.method = "bonferroni",
                      paired = FALSE, conf.int = FALSE, alternative = 't')

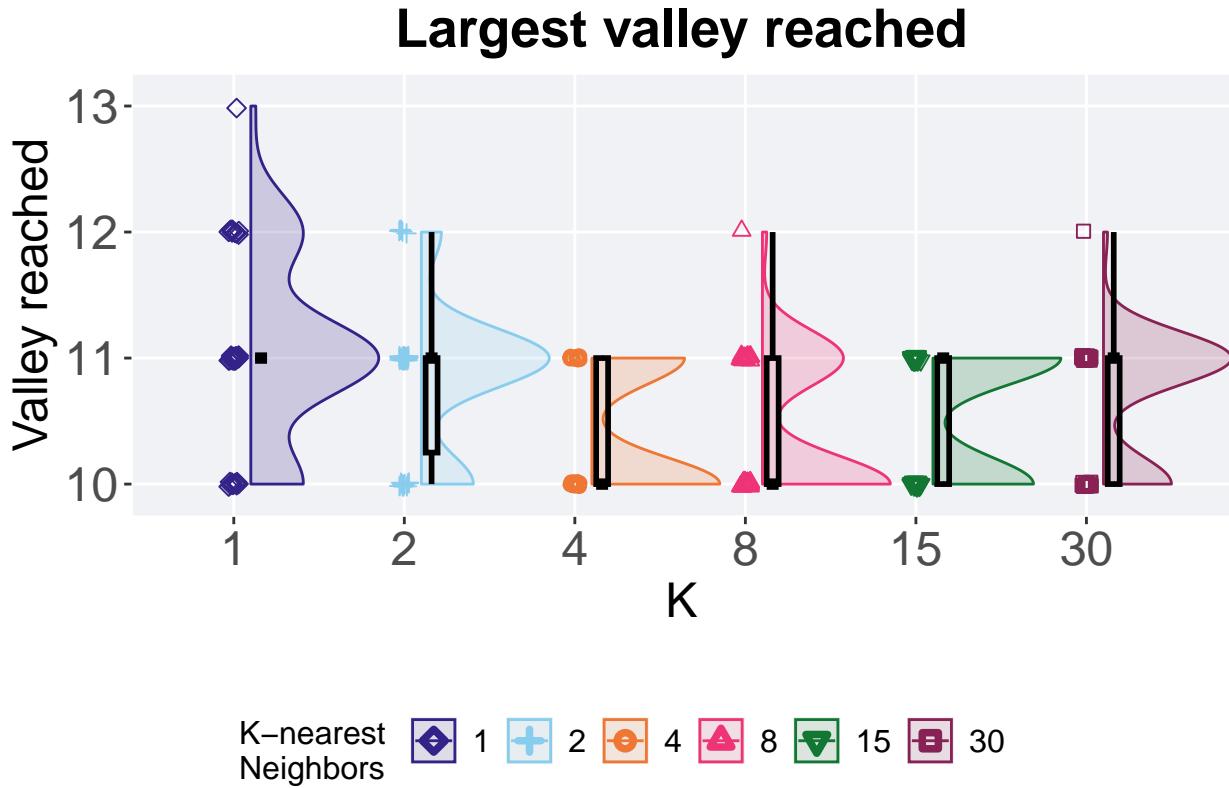
##
## Pairwise comparisons using Wilcoxon rank sum test with continuity correction
##
## data: performance$val and performance$K
##
##   1     2     4     8     15
## 2 0.19048 -     -     -     -
## 4 0.00014 0.65631 -     -     -
## 8 0.00204 1.00000 1.00000 -     -
## 15 0.02198 1.00000 1.00000 1.00000 -
## 30 0.91878 1.00000 0.03427 0.17619 1.00000
##
## P value adjustment method: bonferroni
```

7.2.3 Largest valley reached throughout

Largest valley reached in a single trait by the best performing solution throughout an entire evolutionary run.

```
plot = filter(best_df, acro == 'exp' & var == 'ele_big_peak') %>%
  ggplot(., aes(x = K, y = val, color = K, fill = K, shape = K)) +
  geom_flat_violin(position = position_nudge(x = .1, y = 0), scale = 'width', alpha = 0.2, width = 1.5) +
  geom_boxplot(color = 'black', width = .07, outlier.shape = NA, alpha = 0.0, size = 1.0, position = position_nudge(x = -.1, y = 0)) +
  geom_point(position = position_jitter(width = 0.03, height = 0.02), size = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name = "Valley reached",
    limits = c(9.9, 13.1),
    breaks = c(10, 11, 12, 13)
  ) +
  scale_x_discrete(
    name = "K"
  ) +
  scale_shape_manual(values = SHAPE) +
  scale_colour_manual(values = cb_palette, ) +
  scale_fill_manual(values = cb_palette) +
  ggtitle('Largest valley reached') +
  p_theme

plot_grid(
  plot +
    theme(legend.position = "none"),
  legend,
  nrow = 2,
  rel_heights = c(3, 1)
)
```



7.2.3.1 Stats

Summary statistics for the largest valley crossed.

```
valleys = filter(best_df, acro == 'exp' & var == 'ele_big_peak')
valleys$K = factor(valleys$K, levels = NS_LIST)
valleys %>%
  group_by(K) %>%
  dplyr::summarise(
    count = n(),
    na_cnt = sum(is.na(val)),
    min = min(val, na.rm = TRUE),
    median = median(val, na.rm = TRUE),
    mean = mean(val, na.rm = TRUE),
    max = max(val, na.rm = TRUE),
    IQR = IQR(val, na.rm = TRUE)
  )
```

```
## # A tibble: 6 x 8
##   K      count na_cnt   min median   mean   max   IQR
##   <fct> <int> <int> <dbl> <dbl> <dbl> <dbl> <dbl>
## 1 1       50     0    10    11  11.0    13    0
## 2 2       50     0    10    11  10.8    12  0.75
## 3 4       50     0    10    10  10.4    11    1
## 4 8       50     0    10    10  10.4    12    1
## 5 15      50     0    10    11  10.6    11    1
## 6 30      50     0    10    11  10.7    12    1
```

Kruskal–Wallis test illustrates evidence of statistical differences.

```
kruskal.test(val ~ K, data = valleys)

##
## Kruskal-Wallis rank sum test
##
## data: val by K
## Kruskal-Wallis chi-squared = 36.731, df = 5, p-value = 6.781e-07

Results for post-hoc Wilcoxon rank-sum test with a Bonferroni correction.

pairwise.wilcox.test(x = valleys$val, g = valleys$K, p.adjust.method = "bonferroni",
                      paired = FALSE, conf.int = FALSE, alternative = 't')

##
## Pairwise comparisons using Wilcoxon rank sum test with continuity correction
##
## data: valleys$val and valleys$K
##
##   1     2     4     8    15
## 2 1.00000 -    -    -    -
## 4 0.00019 0.00612 -    -    -
## 8 0.00020 0.00596 1.00000 -    -
## 15 0.00946 0.27532 1.00000 1.00000 -
## 30 0.14860 1.00000 0.20761 0.18174 1.00000
##
## P value adjustment method: bonferroni
```

7.3 Ordered exploitation results

Here we present the results for **best performances** found by each selection scheme parameter on the ordered exploitation diagnostic with valleys. 50 replicates are conducted for each scheme explored.

7.3.1 Performance over time

Best performance in a population over time. Data points on the graph is the average performance across 50 replicates every 2000 generations. Shading comes from the best and worse performance across 50 replicates.

```
lines = filter(over_time_df, acro == 'ord') %>%
  group_by(K, gen) %>%
  dplyr::summarise(
    min = min(pop_fit_max) / DIMENSIONALITY,
    mean = mean(pop_fit_max) / DIMENSIONALITY,
    max = max(pop_fit_max) / DIMENSIONALITY
  )

## `summarise()` has grouped output by 'K'. You can override using the `.`groups` argument.

over_time_plot = ggplot(lines, aes(x=gen, y=mean, group = K, fill = K, color = K, shape = K)) +
  geom_ribbon(aes(ymin = min, ymax = max), alpha = 0.1) +
  geom_line(size = 0.5) +
  geom_point(data = filter(lines, gen %% 2000 == 0 & gen != 0), size = 1.5, stroke = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name="Average trait score",
    limits = c(0,7)
```

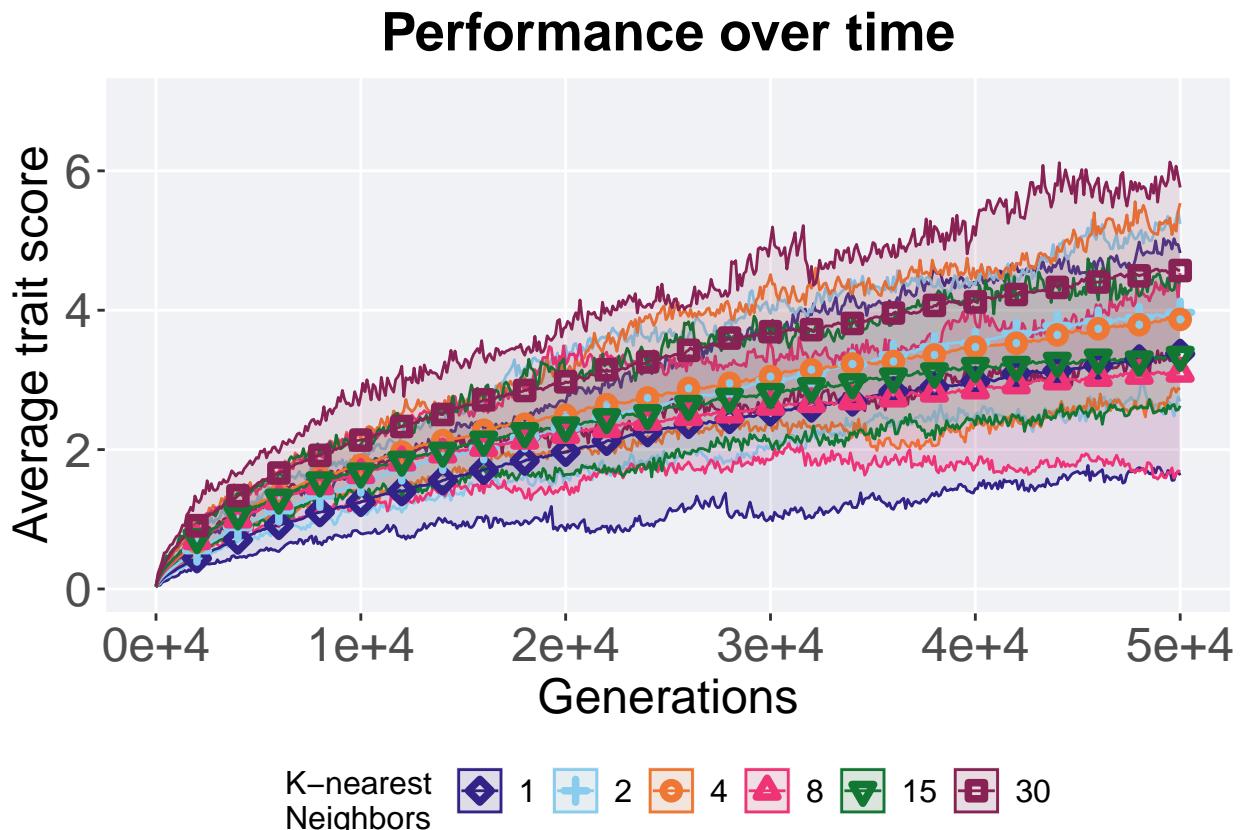
```

) +
scale_x_continuous(
  name="Generations",
  limits=c(0, 50000),
  breaks=c(0, 10000, 20000, 30000, 40000, 50000),
  labels=c("0e+4", "1e+4", "2e+4", "3e+4", "4e+4", "5e+4")

) +
scale_shape_manual(values=SHAPE) +
scale_colour_manual(values = cb_palette) +
scale_fill_manual(values = cb_palette) +
ggtitle('Performance over time') +
p_theme +
guides(
  shape=guide_legend(nrow=1, title.position = "left", title = 'K-nearest \nNeighbors'),
  color=guide_legend(nrow=1, title.position = "left", title = 'K-nearest \nNeighbors'),
  fill=guide_legend(nrow=1, title.position = "left", title = 'K-nearest \nNeighbors')
)

over_time_plot

```



7.3.2 Best performance throughout

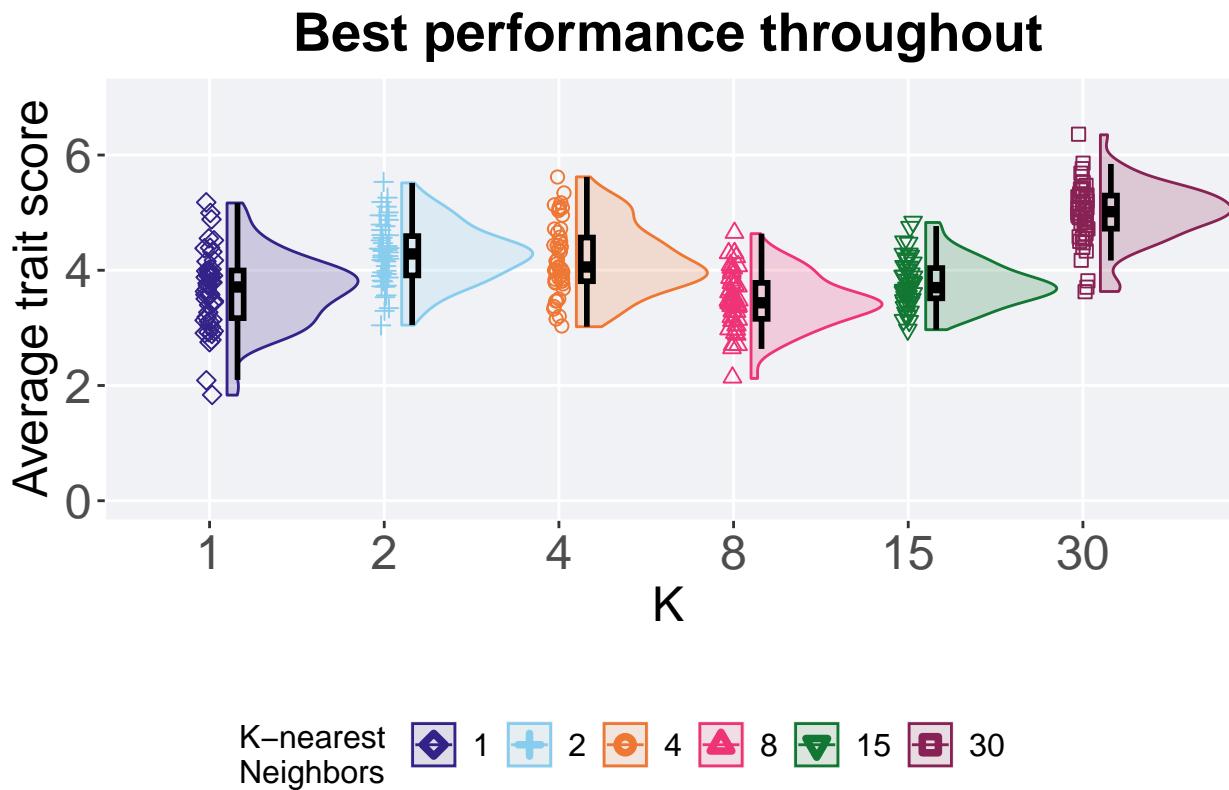
Best performance reached throughout 50,000 generations in a population.

```

plot = filter(best_df, acro == 'ord' & var == 'pop_fit_max') %>%
  ggplot(., aes(x = K, y = val / DIMENSIONALITY, color = K, fill = K, shape = K)) +
  geom_flat_violin(position = position_nudge(x = .1, y = 0), scale = 'width', alpha = 0.2, width = 1.5) +
  geom_boxplot(color = 'black', width = .07, outlier.shape = NA, alpha = 0.0, size = 1.0, position = position_nudge(x = -.1, y = 0)) +
  geom_point(position = position_jitter(width = 0.03, height = 0.02), size = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name = "Average trait score",
    limits = c(0, 7)
  ) +
  scale_x_discrete(
    name = "K"
  ) +
  scale_shape_manual(values = SHAPE) +
  scale_color_manual(values = cb_palette,) +
  scale_fill_manual(values = cb_palette) +
  ggtitle('Best performance throughout') +
  p_theme

plot_grid(
  plot +
    theme(legend.position = "none"),
  legend,
  nrow = 2,
  rel_heights = c(3, 1)
)

```



7.3.2.1 Stats

Summary statistics for the best performance.

```
performance = filter(best_df, acro == 'ord' & var == 'pop_fit_max')
performance$K = factor(performance$K, levels = NS_LIST)
performance %>%
  group_by(K) %>%
  dplyr::summarise(
    count = n(),
    na_cnt = sum(is.na(val)),
    min = min(val / DIMENSIONALITY, na.rm = TRUE),
    median = median(val / DIMENSIONALITY, na.rm = TRUE),
    mean = mean(val / DIMENSIONALITY, na.rm = TRUE),
    max = max(val / DIMENSIONALITY, na.rm = TRUE),
    IQR = IQR(val / DIMENSIONALITY, na.rm = TRUE)
  )

## # A tibble: 6 x 8
##   K     count na_cnt   min median   mean   max   IQR
##   <fct> <int>   <int> <dbl> <dbl> <dbl> <dbl> <dbl>
## 1 1       50      0  1.83   3.71   3.65   5.17  0.827
## 2 2       50      0  3.05   4.28   4.28   5.52  0.682
## 3 4       50      0  3.02   4.06   4.19   5.62  0.758
## 4 8       50      0  2.12   3.44   3.47   4.64  0.625
## 5 15      50      0  2.97   3.70   3.77   4.83  0.525
## 6 30      50      0  3.63   5.02   4.98   6.35  0.575
```

Kruskal–Wallis test illustrates evidence of statistical differences.

```
kruskal.test(val ~ K, data = performance)
```

```
##
## Kruskal-Wallis rank sum test
##
## data: val by K
## Kruskal-Wallis chi-squared = 134.98, df = 5, p-value < 2.2e-16
```

Results for post-hoc Wilcoxon rank-sum test with a Bonferroni correction.

```
pairwise.wilcox.test(x = performance$val, g = performance$K, p.adjust.method = "bonferroni",
                      paired = FALSE, conf.int = FALSE, alternative = 't')
```

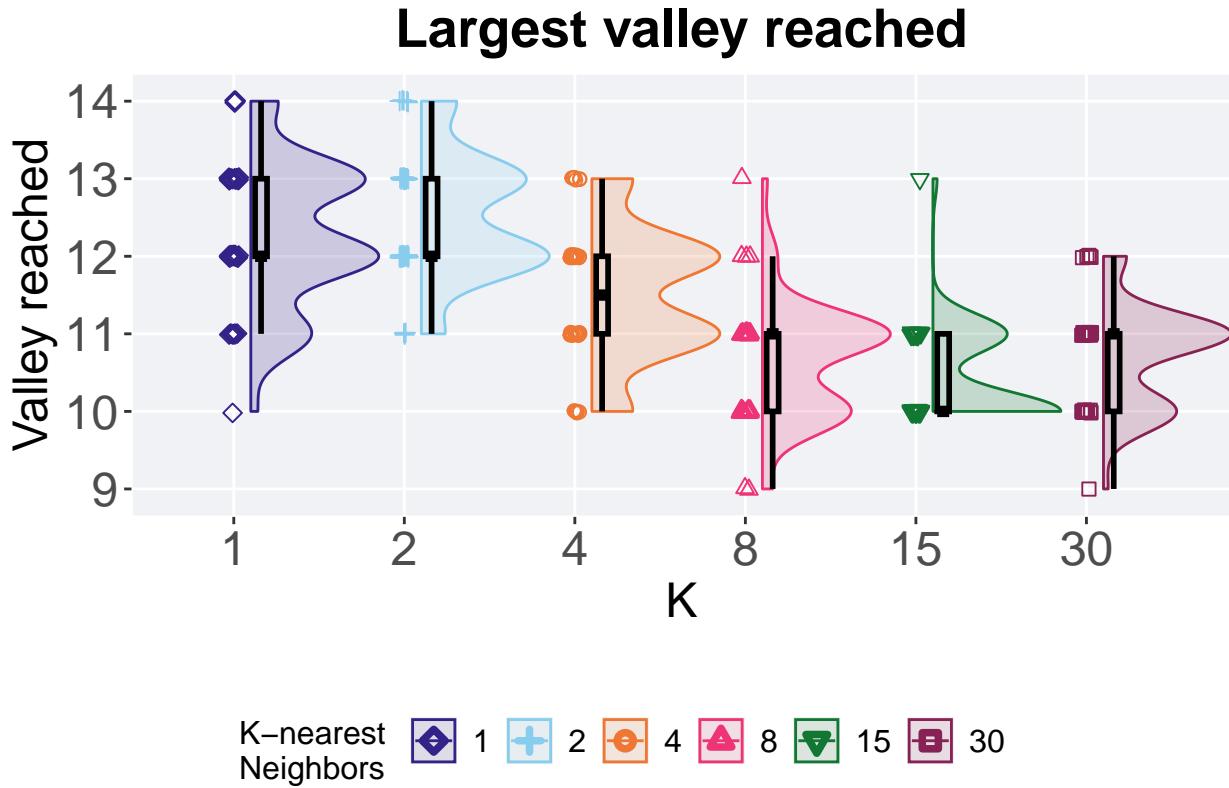
```
##
## Pairwise comparisons using Wilcoxon rank sum test with continuity correction
##
## data: performance$val and performance$K
##
##   1     2     4     8     15
## 2 2.7e-05 -     -     -     -
## 4 0.0016  1.0000 -     -     -
## 8 1.0000  1.8e-09 4.4e-07 -     -
## 15 1.0000  1.1e-05 0.0038  0.0195  -
## 30 6.8e-13 1.7e-07 3.2e-07 2.5e-15 8.8e-14
##
## P value adjustment method: bonferroni
```

7.3.3 Largest valley reached throughout

Largest valley reached in a single trait by the best performing solution throughout an entire evolutionary run.

```
plot = filter(best_df, acro == 'ord' & var == 'ele_big_peak') %>%
  ggplot(., aes(x = K, y = val, color = K, fill = K, shape = K)) +
  geom_flat_violin(position = position_nudge(x = .1, y = 0), scale = 'width', alpha = 0.2, width = 1.5) +
  geom_boxplot(color = 'black', width = .07, outlier.shape = NA, alpha = 0.0, size = 1.0, position = position_nudge(x = -.1, y = 0)) +
  geom_point(position = position_jitter(width = 0.03, height = 0.02), size = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name = "Valley reached",
    limits = c(8.9, 14.1),
    breaks = c(9, 10, 11, 12, 13, 14)
  ) +
  scale_x_discrete(
    name = "K"
  ) +
  scale_shape_manual(values = SHAPE) +
  scale_colour_manual(values = cb_palette, ) +
  scale_fill_manual(values = cb_palette) +
  ggtitle('Largest valley reached') +
  p_theme

plot_grid(
  plot +
    theme(legend.position = "none"),
  legend,
  nrow = 2,
  rel_heights = c(3, 1)
)
```



7.3.3.1 Stats

Summary statistics for the largest valley crossed.

```
valleys = filter(best_df, acro == 'ord' & var == 'ele_big_peak')
valleys$K = factor(valleys$K, levels = NS_LIST)
valleys %>%
  group_by(K) %>%
  dplyr::summarise(
    count = n(),
    na_cnt = sum(is.na(val)),
    min = min(val, na.rm = TRUE),
    median = median(val, na.rm = TRUE),
    mean = mean(val, na.rm = TRUE),
    max = max(val, na.rm = TRUE),
    IQR = IQR(val, na.rm = TRUE)
  )
```

```
## # A tibble: 6 x 8
##   K     count na_cnt  min median  mean   max   IQR
##   <fct> <int> <int> <dbl> <dbl> <dbl> <dbl> <dbl>
## 1 1      50     0    10    12    12.3    14     1
## 2 2      50     0    11    12    12.5    14     1
## 3 4      50     0    10    11.5   11.5    13     1
## 4 8      50     0     9    11    10.7    13     1
## 5 15     50     0    10    10    10.4    13     1
## 6 30     50     0     9    11    10.7    12     1
```

Kruskal–Wallis test illustrates evidence of statistical differences.

```
kruskal.test(val ~ K, data = valleys)

##
## Kruskal-Wallis rank sum test
##
## data: val by K
## Kruskal-Wallis chi-squared = 162.64, df = 5, p-value < 2.2e-16

Results for post-hoc Wilcoxon rank-sum test with a Bonferroni correction.

pairwise.wilcox.test(x = valleys$val, g = valleys$K, p.adjust.method = "bonferroni",
                      paired = FALSE, conf.int = FALSE, alternative = 't')

##
## Pairwise comparisons using Wilcoxon rank sum test with continuity correction
##
## data: valleys$val and valleys$K
##
##    1      2      4      8     15
## 2 1.00000 -     -     -     -
## 4 0.00118 3.1e-06 -     -     -
## 8 1.4e-11 3.4e-14 3.8e-05 -     -
## 15 7.6e-14 8.6e-16 2.0e-08 0.61124 -
## 30 2.0e-11 2.5e-14 0.00017 1.00000 0.07759
##
## P value adjustment method: bonferroni
```

7.4 Contradictory objectives results

Here we present the results for **activation gene coverage** and **satisfactory trait coverage** found by each selection scheme parameter on the contradictory objectives diagnostic with valleys. 50 replicates are conducted for each scheme parameters explored.

7.4.1 Activation gene coverage over time

Activation gene coverage in a population over time. Data points on the graph is the average activation gene coverage across 50 replicates every 2000 generations. Shading comes from the best and worse coverage across 50 replicates.

```
lines = filter(over_time_df, acro == 'con') %>%
  group_by(K, gen) %>%
  dplyr::summarise(
    min = min(uni_str_pos),
    mean = mean(uni_str_pos),
    max = max(uni_str_pos)
  )

## `summarise()` has grouped output by 'K'. You can override using the `.groups` argument.

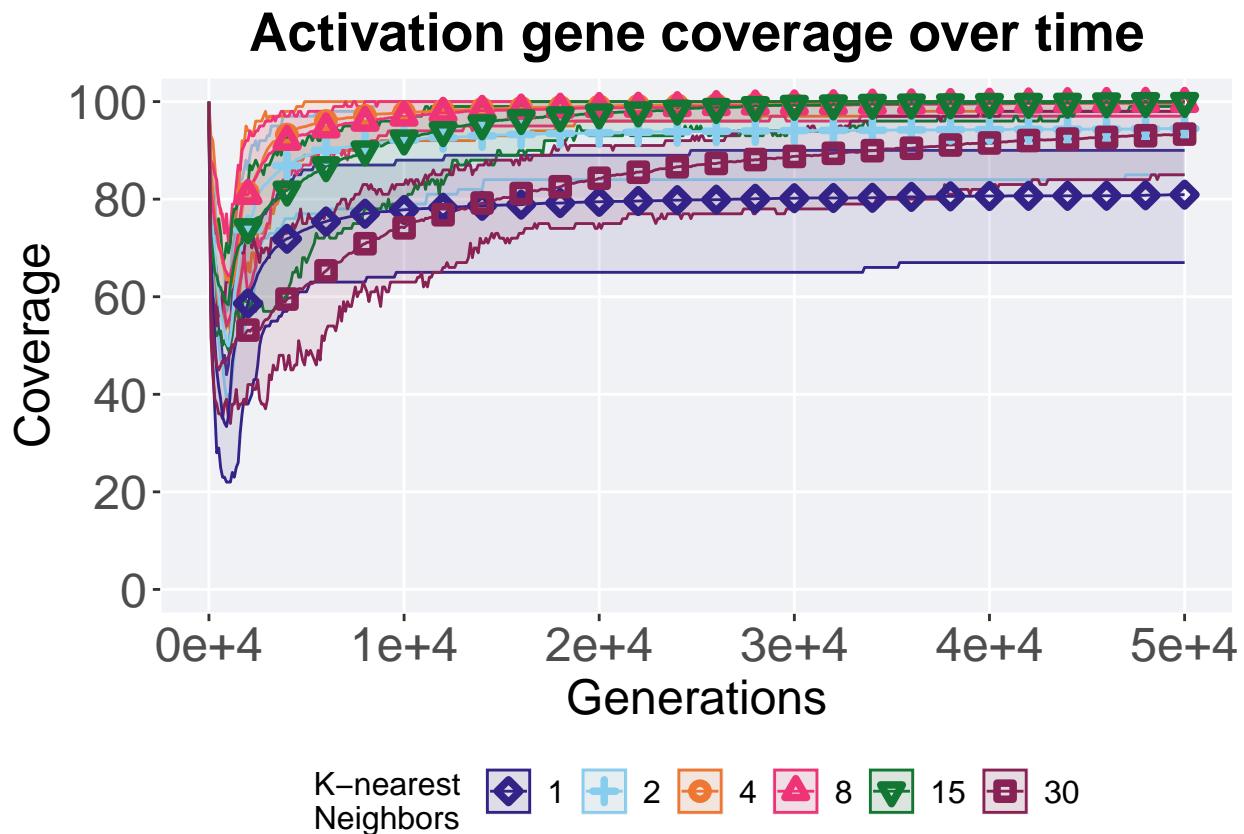
ggplot(lines, aes(x=gen, y=mean, group = K, fill = K, color = K, shape = K)) +
  geom_ribbon(aes(ymin = min, ymax = max), alpha = 0.1) +
  geom_line(size = 0.5) +
  geom_point(data = filter(lines, gen %% 2000 == 0 & gen != 0), size = 1.5, stroke = 2.0, alpha = 1.0) +
  scale_y_continuous()
```

```

name="Coverage",
limits=c(0, 100),
breaks=seq(0,100, 20),
labels=c("0", "20", "40", "60", "80", "100")
) +
scale_x_continuous(
  name="Generations",
  limits=c(0, 50000),
  breaks=c(0, 10000, 20000, 30000, 40000, 50000),
  labels=c("0e+4", "1e+4", "2e+4", "3e+4", "4e+4", "5e+4")

) +
scale_shape_manual(values=SHAPE) +
scale_colour_manual(values = cb_palette) +
scale_fill_manual(values = cb_palette) +
ggtitle('Activation gene coverage over time') +
p_theme +
guides(
  shape=guide_legend(nrow=1, title.position = "left", title = 'K-nearest \nNeighbors'),
  color=guide_legend(nrow=1, title.position = "left", title = 'K-nearest \nNeighbors'),
  fill=guide_legend(nrow=1, title.position = "left", title = 'K-nearest \nNeighbors')
)

```



7.4.2 Final activation gene coverage

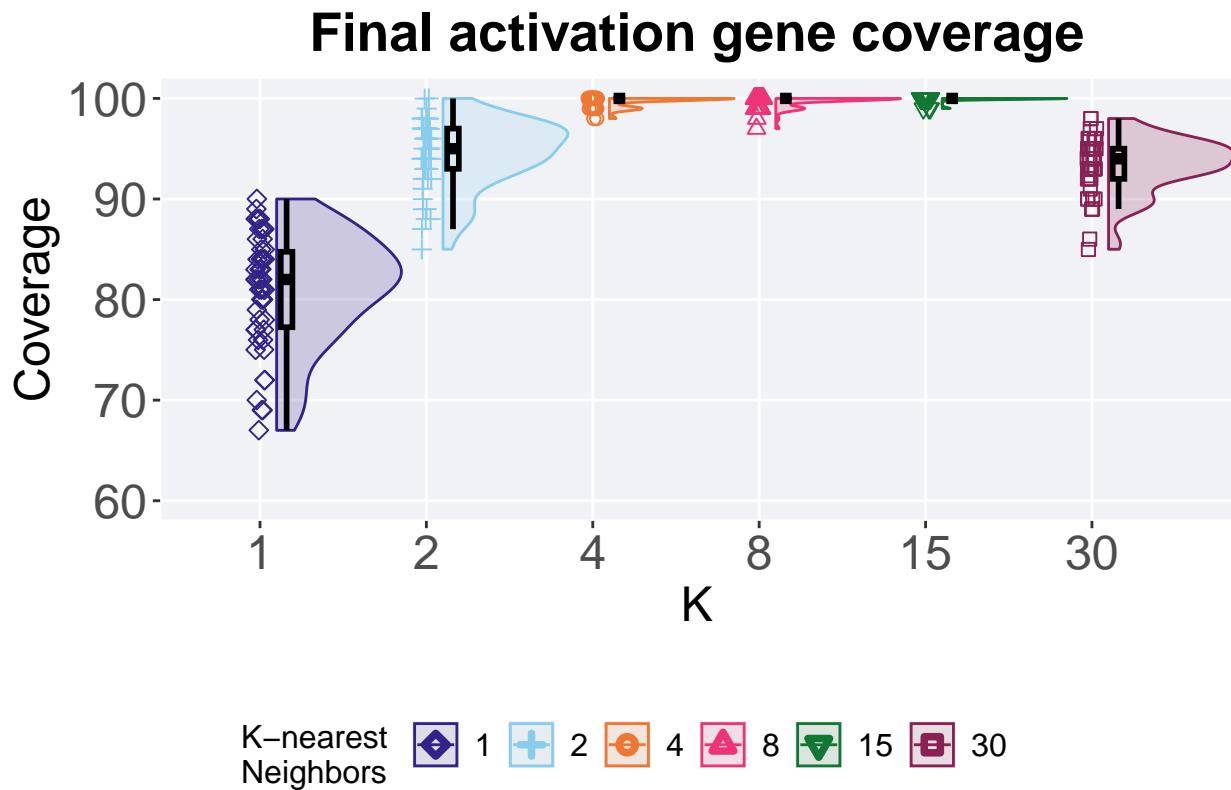
Activation gene coverage found in the final population at 50,000 generations.

```

plot = filter(over_time_df, gen == 50000 & acro == 'con') %>%
  ggplot(., aes(x = K, y = uni_str_pos, color = K, fill = K, shape = K)) +
  geom_flat_violin(position = position_nudge(x = .1, y = 0), scale = 'width', alpha = 0.2, width = 1.5) +
  geom_boxplot(color = 'black', width = .07, outlier.shape = NA, alpha = 0.0, size = 1.0, position = position_nudge(x = -.1, y = 0)) +
  geom_point(position = position_jitter(width = 0.03, height = 0.02), size = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name = "Coverage",
    limits = c(60, 100.1)
  ) +
  scale_x_discrete(
    name = "K"
  ) +
  scale_shape_manual(values = SHAPE) +
  scale_color_manual(values = cb_palette, ) +
  scale_fill_manual(values = cb_palette) +
  ggtitle('Final activation gene coverage') +
  p_theme

plot_grid(
  plot +
    theme(legend.position = "none"),
  legend,
  nrow = 2,
  rel_heights = c(3, 1)
)

```



7.4.2.1 Stats

Summary statistics for the generation a satisfactory solution is found.

```
act_coverage = filter(over_time_df, gen == 50000 & acro == 'con')
act_coverage$K = factor(act_coverage$K, levels = NS_LIST)
act_coverage %>%
  group_by(K) %>%
  dplyr::summarise(
    count = n(),
    na_cnt = sum(is.na(uni_str_pos)),
    min = min(uni_str_pos, na.rm = TRUE),
    median = median(uni_str_pos, na.rm = TRUE),
    mean = mean(uni_str_pos, na.rm = TRUE),
    max = max(uni_str_pos, na.rm = TRUE),
    IQR = IQR(uni_str_pos, na.rm = TRUE)
  )

## # A tibble: 6 x 8
##   K     count na_cnt   min median   mean   max   IQR
##   <fct> <int> <int> <int> <dbl> <dbl> <int> <dbl>
## 1 1      50     0    67     82  80.9    90    7.5
## 2 2      50     0    85     95  94.5   100     4
## 3 4      50     0    98    100  99.7   100     0
## 4 8      50     0    97    100  99.7   100     0
## 5 15     50     0    99    100  99.9   100     0
## 6 30     50     0    85    94  93.3    98     3
```

Kruskal–Wallis test illustrates evidence of statistical differences.

```
kruskal.test(uni_str_pos ~ K, data = act_coverage)
```

```
##
## Kruskal-Wallis rank sum test
##
## data: uni_str_pos by K
## Kruskal-Wallis chi-squared = 262.41, df = 5, p-value < 2.2e-16
```

Results for post-hoc Wilcoxon rank-sum test with a Bonferroni correction.

```
pairwise.wilcox.test(x = act_coverage$uni_str_pos, g = act_coverage$K, p.adjust.method = "bonferroni",
                      paired = FALSE, conf.int = FALSE, alternative = 't')
```

```
##
## Pairwise comparisons using Wilcoxon rank sum test with continuity correction
##
## data: act_coverage$uni_str_pos and act_coverage$K
##
##   1     2     4     8     15
## 2 5.6e-16 -     -     -     -
## 4 < 2e-16 7.3e-16 -     -     -
## 8 < 2e-16 8.5e-16 1.00   -     -
## 15 < 2e-16 < 2e-16 0.17   0.31   -
## 30 5.1e-16 0.42    < 2e-16 < 2e-16 < 2e-16
##
## P value adjustment method: bonferroni
```

7.4.3 Satisfactory trait coverage over time

Satisfactory trait coverage in a population over time. Data points on the graph is the average activation gene coverage across 50 replicates every 2000 generations. Shading comes from the best and worse coverage across 50 replicates.

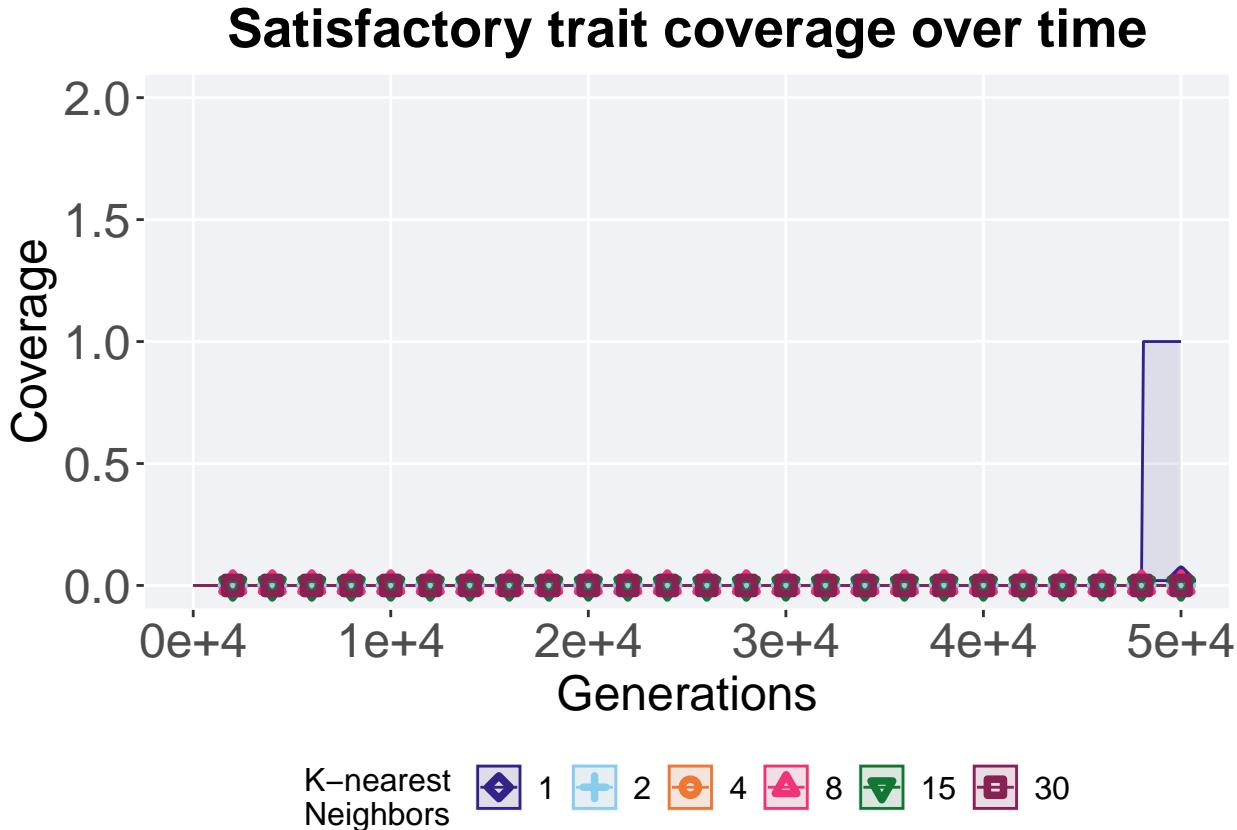
```

lines = filter(over_time_df, acro == 'con') %>%
  group_by(K, gen) %>%
  dplyr::summarise(
    min = min(pop_uni_obj),
    mean = mean(pop_uni_obj),
    max = max(pop_uni_obj)
  )

## `summarise()` has grouped output by 'K'. You can override using the `.`groups` argument.

ggplot(lines, aes(x=gen, y=mean, group = K, fill = K, color = K, shape = K)) +
  geom_ribbon(aes(ymin = min, ymax = max), alpha = 0.1) +
  geom_line(size = 0.5) +
  geom_point(data = filter(lines, gen %% 2000 == 0 & gen != 0), size = 1.5, stroke = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name="Coverage",
    limits=c(0, 2)
  ) +
  scale_x_continuous(
    name="Generations",
    limits=c(0, 50000),
    breaks=c(0, 10000, 20000, 30000, 40000, 50000),
    labels=c("0e+4", "1e+4", "2e+4", "3e+4", "4e+4", "5e+4")
  ) +
  scale_shape_manual(values=SHAPE) +
  scale_colour_manual(values = cb_palette) +
  scale_fill_manual(values = cb_palette) +
  ggtitle('Satisfactory trait coverage over time') +
  p_theme +
  guides(
    shape=guide_legend(nrow=1, title.position = "left", title = 'K-nearest \nNeighbors'),
    color=guide_legend(nrow=1, title.position = "left", title = 'K-nearest \nNeighbors'),
    fill=guide_legend(nrow=1, title.position = "left", title = 'K-nearest \nNeighbors')
  )

```



7.4.4 Final satisfactory trait coverage

Satisfactory trait coverage found in the final population at 50,000 generations.

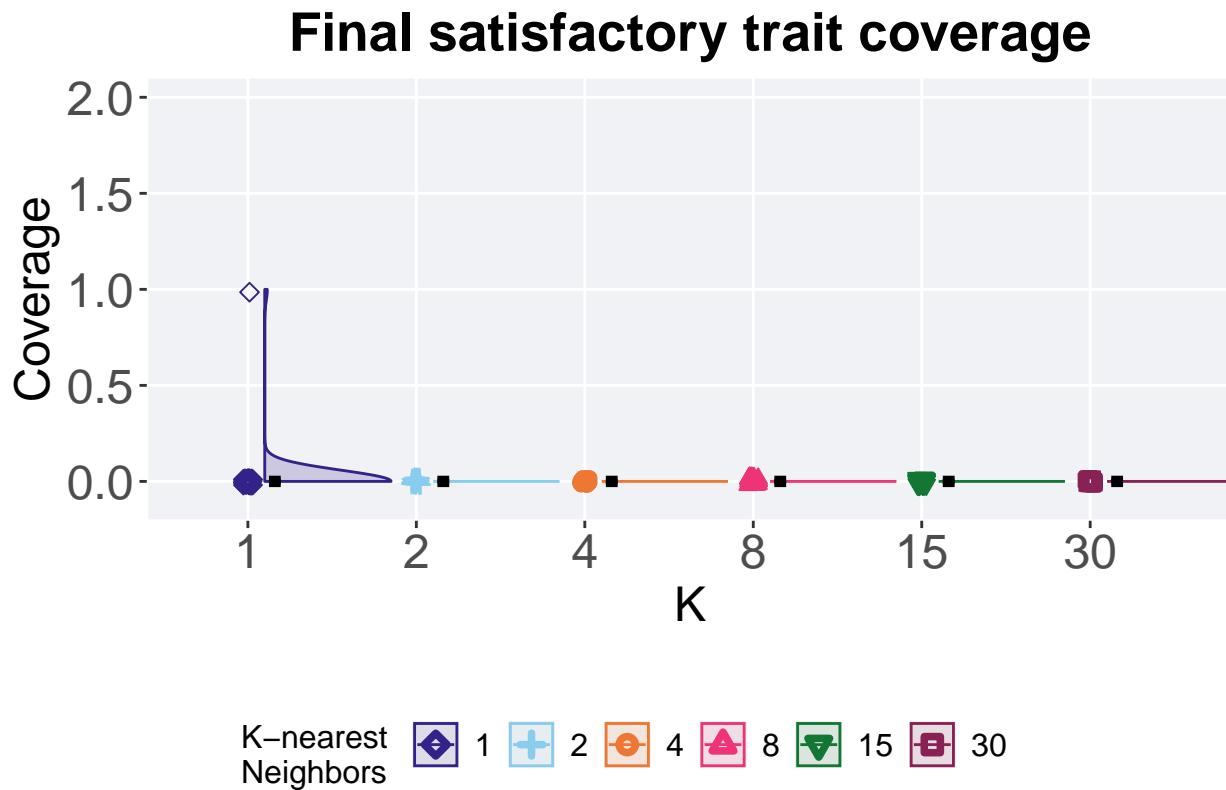
```
plot = filter(over_time_df, gen == 50000 & acro == 'con') %>%
  ggplot(., aes(x = K, y = pop_uni_obj, color = K, fill = K, shape = K)) +
  geom_flat_violin(position = position_nudge(x = .1, y = 0), scale = 'width', alpha = 0.2, width = 1.5) +
  geom_boxplot(color = 'black', width = .07, outlier.shape = NA, alpha = 0.0, size = 1.0, position = position_nudge(x = -.1, y = 0)) +
  geom_point(position = position_jitter(width = 0.03, height = 0.02), size = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name = "Coverage",
    limits = c(-0.1, 2)
  ) +
  scale_x_discrete(
    name = "K"
  ) +
  scale_shape_manual(values = SHAPE) +
  scale_colour_manual(values = cb_palette,) +
  scale_fill_manual(values = cb_palette) +
  ggtitle('Final satisfactory trait coverage') +
  p_theme

plot_grid(
  plot +
    theme(legend.position = "none"),
  legend,
```

```

nrow=2,
rel_heights = c(3,1)
)

```



7.4.4.1 Stats

Summary statistics for the generation a satisfactory solution is found.

```

sat_coverage = filter(over_time_df, gen == 50000 & acro == 'con')
sat_coverage$K = factor(sat_coverage$K, levels = NS_LIST)
sat_coverage %>%
  group_by(K) %>%
  dplyr::summarise(
    count = n(),
    na_cnt = sum(is.na(pop_uni_obj)),
    min = min(pop_uni_obj, na.rm = TRUE),
    median = median(pop_uni_obj, na.rm = TRUE),
    mean = mean(pop_uni_obj, na.rm = TRUE),
    max = max(pop_uni_obj, na.rm = TRUE),
    IQR = IQR(pop_uni_obj, na.rm = TRUE)
  )

## # A tibble: 6 x 8
##   K      count  na_cnt   min  median   mean   max    IQR
##   <fct> <int> <int> <dbl> <dbl> <dbl> <int> <dbl>
## 1 1        50     0     0     0    0.02     1     0
## 2 2        50     0     0     0    0       0     0

```

```
## 3 4      50    0    0    0  0    0    0
## 4 8      50    0    0    0  0    0    0
## 5 15     50    0    0    0  0    0    0
## 6 30     50    0    0    0  0    0    0
```

Kruskal–Wallis test illustrates evidence of **no statistical differences**.

```
kruskal.test(pop_uni_obj ~ K, data = sat_coverage)
```

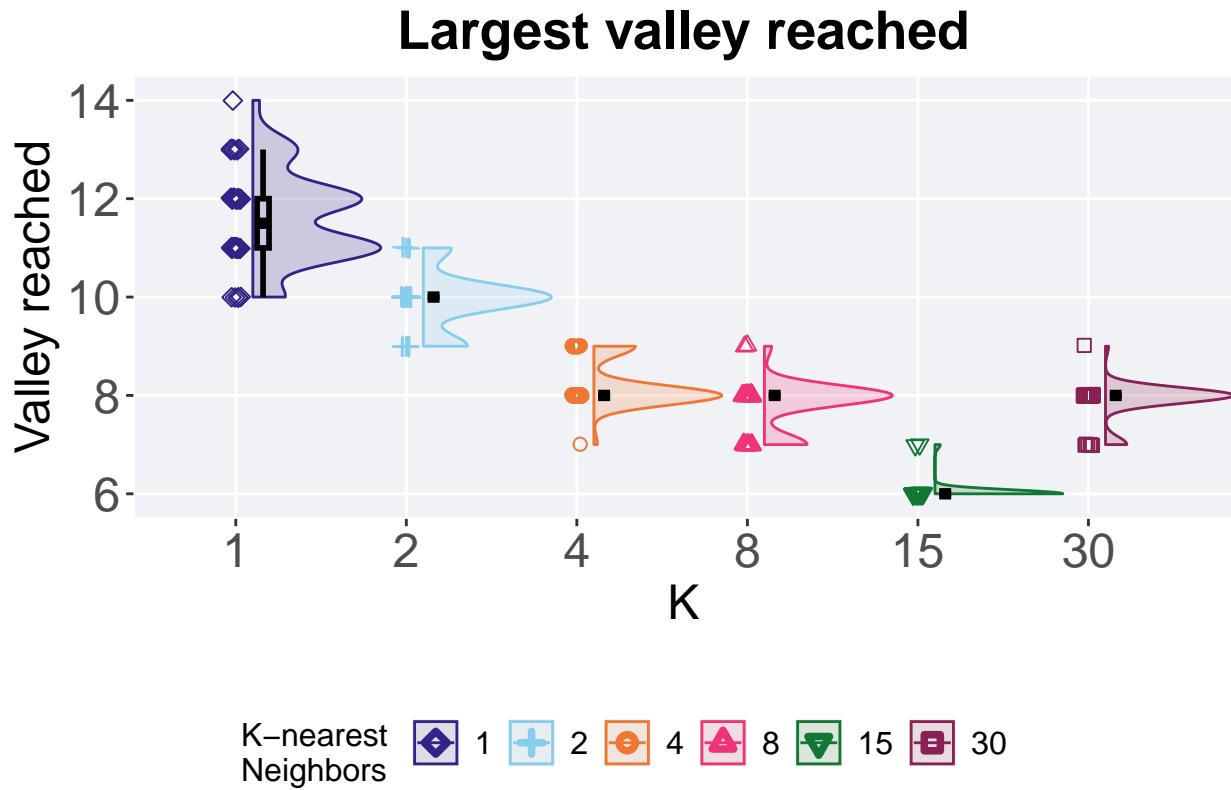
```
##
##  Kruskal-Wallis rank sum test
##
## data: pop_uni_obj by K
## Kruskal-Wallis chi-squared = 5, df = 5, p-value = 0.4159
```

7.4.5 Largest valley reached throughout

Largest valley reached in a single trait by the best performing solution throughout an entire evolutionary run.

```
plot = filter(best_df, acro == 'con' & var == 'ele_big_peak') %>%
  ggplot(., aes(x = K, y = val, color = K, fill = K, shape = K)) +
  geom_flat_violin(position = position_nudge(x = .1, y = 0), scale = 'width', alpha = 0.2, width = 1.5)
  geom_boxplot(color = 'black', width = .07, outlier.shape = NA, alpha = 0.0, size = 1.0, position = position_nudge(x = -.1, y = 0))
  geom_point(position = position_jitter(width = 0.03, height = 0.02), size = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name="Valley reached",
    limits=c(5.9,14.1),
    breaks=c(6,8,10,12,14)
  ) +
  scale_x_discrete(
    name="K"
  )+
  scale_shape_manual(values=SHAPE)+
  scale_colour_manual(values = cb_palette, ) +
  scale_fill_manual(values = cb_palette) +
  ggtitle('Largest valley reached')+
  p_theme

plot_grid(
  plot +
    theme(legend.position="none"),
  legend,
  nrow=2,
  rel_heights = c(3,1)
)
```



7.4.5.1 Stats

Summary statistics for the largest valley crossed.

```
valleys = filter(best_df, acro == 'con' & var == 'ele_big_peak')
valleys$K = factor(valleys$K, levels = NS_LIST)
valleys %>%
  group_by(K) %>%
  dplyr::summarise(
    count = n(),
    na_cnt = sum(is.na(val)),
    min = min(val, na.rm = TRUE),
    median = median(val, na.rm = TRUE),
    mean = mean(val, na.rm = TRUE),
    max = max(val, na.rm = TRUE),
    IQR = IQR(val, na.rm = TRUE)
  )
```

```
## # A tibble: 6 x 8
##   K      count na_cnt   min median   mean   max   IQR
##   <fct> <int>   <int> <dbl>  <dbl>  <dbl>  <dbl> <dbl>
## 1 1       50     0    10   11.5  11.6    14     1
## 2 2       50     0     9    10    9.92    11     0
## 3 4       50     0     7     8    8.22     9     0
## 4 8       50     0     7     8    7.8      9     0
## 5 15      50     0     6     6    6.04     7     0
## 6 30      50     0     7     8    7.88     9     0
```

Kruskal–Wallis test illustrates evidence of statistical differences.

```
kruskal.test(val ~ K, data = valleys)

##
## Kruskal-Wallis rank sum test
##
## data: val by K
## Kruskal-Wallis chi-squared = 273.17, df = 5, p-value < 2.2e-16

Results for post-hoc Wilcoxon rank-sum test with a Bonferroni correction.

pairwise.wilcox.test(x = valleys$val, g = valleys$K, p.adjust.method = "bonferroni",
                      paired = FALSE, conf.int = FALSE, alternative = 't')

##
## Pairwise comparisons using Wilcoxon rank sum test with continuity correction
##
## data: valleys$val and valleys$K
##
##    1      2      4      8     15
## 2 3.3e-13 -      -      -
## 4 < 2e-16 < 2e-16 -      -
## 8 < 2e-16 < 2e-16 0.00089 -
## 15 < 2e-16 < 2e-16 < 2e-16 -
## 30 < 2e-16 < 2e-16 0.00301 1.00000 < 2e-16
##
## P value adjustment method: bonferroni
```

7.5 Multi-path exploration results

Here we present the results for best performances and activation gene coverage found by each selection scheme parameter on the multi-path exploration diagnostic with valleys. 50 replicates are conducted for each scheme parameter explored.

7.5.1 Activation gene coverage over time

Activation gene coverage in a population over time. Data points on the graph is the average activation gene coverage across 50 replicates every 2000 generations. Shading comes from the best and worse coverage across 50 replicates.

```
lines = filter(over_time_df, acro == 'mpe') %>%
  group_by(K, gen) %>%
  dplyr::summarise(
    min = min(uni_str_pos),
    mean = mean(uni_str_pos),
    max = max(uni_str_pos)
  )

## `summarise()` has grouped output by 'K'. You can override using the `.groups` argument.

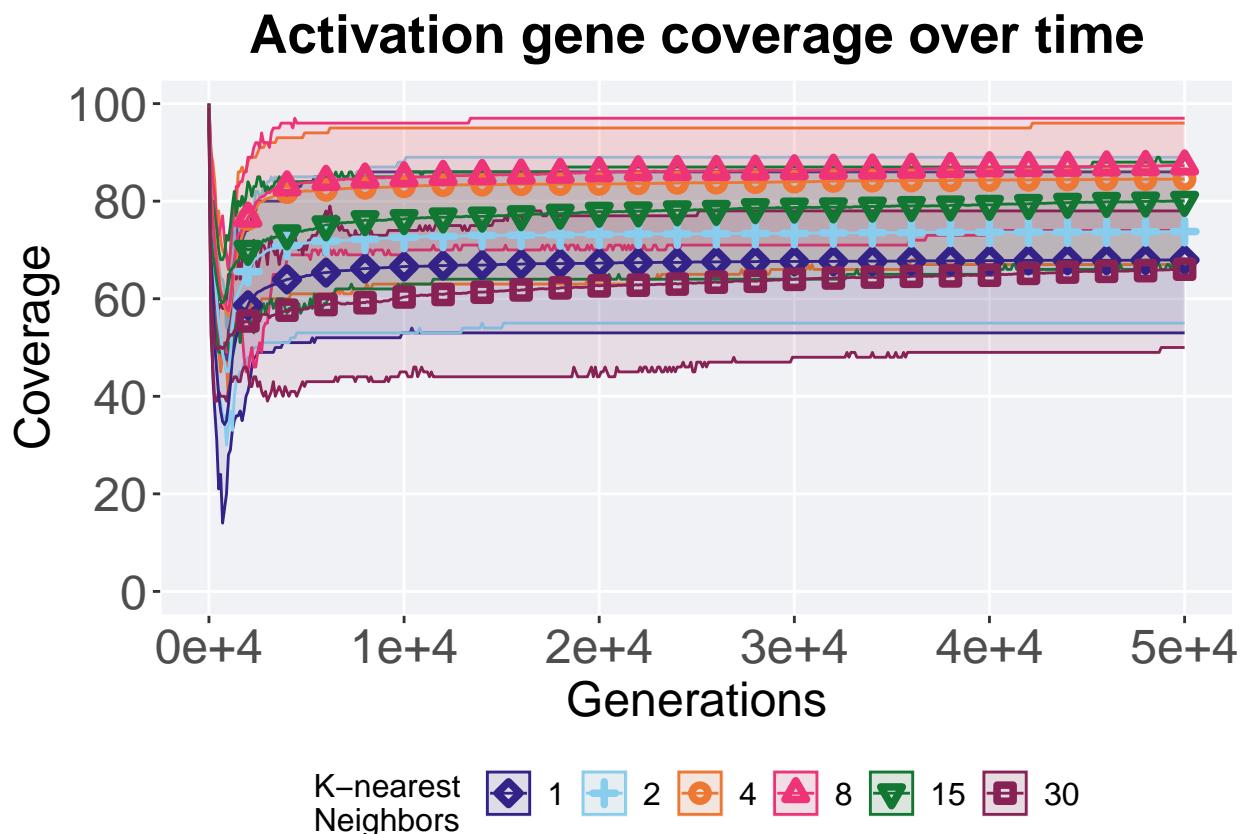
ggplot(lines, aes(x=gen, y=mean, group = K, fill = K, color = K, shape = K)) +
  geom_ribbon(aes(ymin = min, ymax = max), alpha = 0.1) +
  geom_line(size = 0.5) +
  geom_point(data = filter(lines, gen %% 2000 == 0 & gen != 0), size = 1.5, stroke = 2.0, alpha = 1.0) +
  scale_y_continuous()
```

```

name="Coverage",
limits=c(0, 100),
breaks=seq(0,100, 20),
labels=c("0", "20", "40", "60", "80", "100")
) +
scale_x_continuous(
  name="Generations",
  limits=c(0, 50000),
  breaks=c(0, 10000, 20000, 30000, 40000, 50000),
  labels=c("0e+4", "1e+4", "2e+4", "3e+4", "4e+4", "5e+4")

) +
scale_shape_manual(values=SHAPE) +
scale_colour_manual(values = cb_palette) +
scale_fill_manual(values = cb_palette) +
ggtitle('Activation gene coverage over time') +
p_theme +
guides(
  shape=guide_legend(nrow=1, title.position = "left", title = 'K-nearest \nNeighbors'),
  color=guide_legend(nrow=1, title.position = "left", title = 'K-nearest \nNeighbors'),
  fill=guide_legend(nrow=1, title.position = "left", title = 'K-nearest \nNeighbors')
)

```



7.5.2 Final activation gene coverage

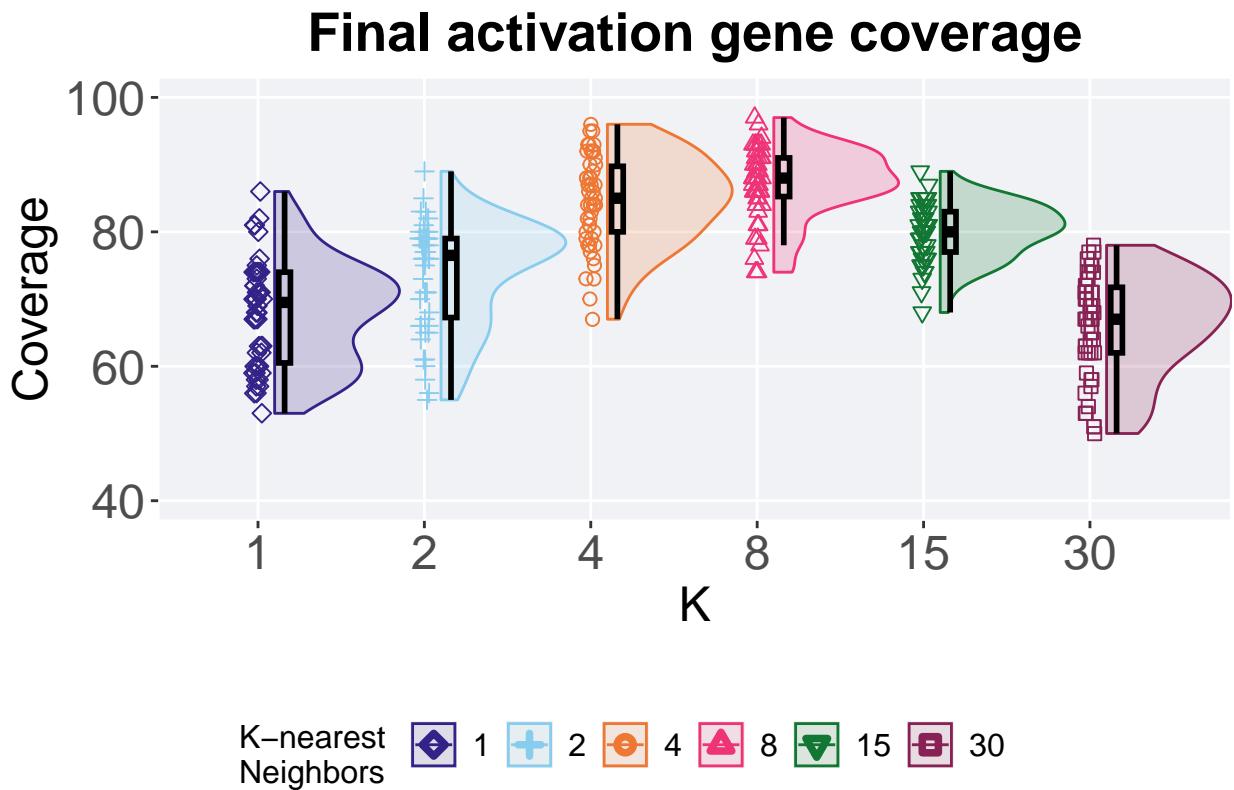
Activation gene coverage found in the final population at 50,000 generations.

```

plot = filter(over_time_df, gen == 50000 & acro == 'mpe') %>%
  ggplot(., aes(x = K, y = uni_str_pos, color = K, fill = K, shape = K)) +
  geom_flat_violin(position = position_nudge(x = .1, y = 0), scale = 'width', alpha = 0.2, width = 1.5) +
  geom_boxplot(color = 'black', width = .07, outlier.shape = NA, alpha = 0.0, size = 1.0, position = position_nudge(x = -.1, y = 0)) +
  geom_point(position = position_jitter(width = 0.03, height = 0.02), size = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name = "Coverage",
    limits = c(40, 100)
  ) +
  scale_x_discrete(
    name = "K"
  ) +
  scale_shape_manual(values = SHAPE) +
  scale_color_manual(values = cb_palette,) +
  scale_fill_manual(values = cb_palette) +
  ggtitle('Final activation gene coverage') +
  p_theme

plot_grid(
  plot +
    theme(legend.position = "none"),
  legend,
  nrow = 2,
  rel_heights = c(3, 1)
)

```



7.5.2.1 Stats

Summary statistics for the generation a satisfactory solution is found.

```
act_coverage = filter(over_time_df, gen == 50000 & acro == 'mpe')
act_coverage$K = factor(act_coverage$K, levels = NS_LIST)
act_coverage %>%
  group_by(K) %>%
  dplyr::summarise(
    count = n(),
    na_cnt = sum(is.na(uni_str_pos)),
    min = min(uni_str_pos, na.rm = TRUE),
    median = median(uni_str_pos, na.rm = TRUE),
    mean = mean(uni_str_pos, na.rm = TRUE),
    max = max(uni_str_pos, na.rm = TRUE),
    IQR = IQR(uni_str_pos, na.rm = TRUE)
  )

## # A tibble: 6 x 8
##   K     count na_cnt   min median   mean   max   IQR
##   <fct> <int> <int> <dbl> <dbl> <int> <dbl>
## 1 1      50     0    53    69.5  67.9    86  13.5
## 2 2      50     0    55    76.5  73.8    89  11.8
## 3 4      50     0    67    85     84.5   96  9.75
## 4 8      50     0    74    88     87.3   97  5.75
## 5 15     50     0    68    80     80.1   89  6
## 6 30     50     0    50    67    66.1    78  9.75
```

Kruskal–Wallis test illustrates evidence of statistical differences.

```
kruskal.test(uni_str_pos ~ K, data = act_coverage)
```

```
##
## Kruskal-Wallis rank sum test
##
## data: uni_str_pos by K
## Kruskal-Wallis chi-squared = 186.99, df = 5, p-value < 2.2e-16
```

Results for post-hoc Wilcoxon rank-sum test with a Bonferroni correction.

```
pairwise.wilcox.test(x = act_coverage$uni_str_pos, g = act_coverage$K, p.adjust.method = "bonferroni",
                      paired = FALSE, conf.int = FALSE, alternative = 't')
```

```
##
## Pairwise comparisons using Wilcoxon rank sum test with continuity correction
##
## data: act_coverage$uni_str_pos and act_coverage$K
##
##   1     2     4     8     15
## 2 0.00498 -     -     -     -
## 4 5.1e-13 2.1e-08 -     -     -
## 8 2.7e-15 2.0e-12 0.55005 -     -
## 15 1.8e-11 0.00017 0.00300 7.0e-09 -
## 30 1.00000 4.6e-05 8.7e-15 3.1e-16 2.5e-14
##
## P value adjustment method: bonferroni
```

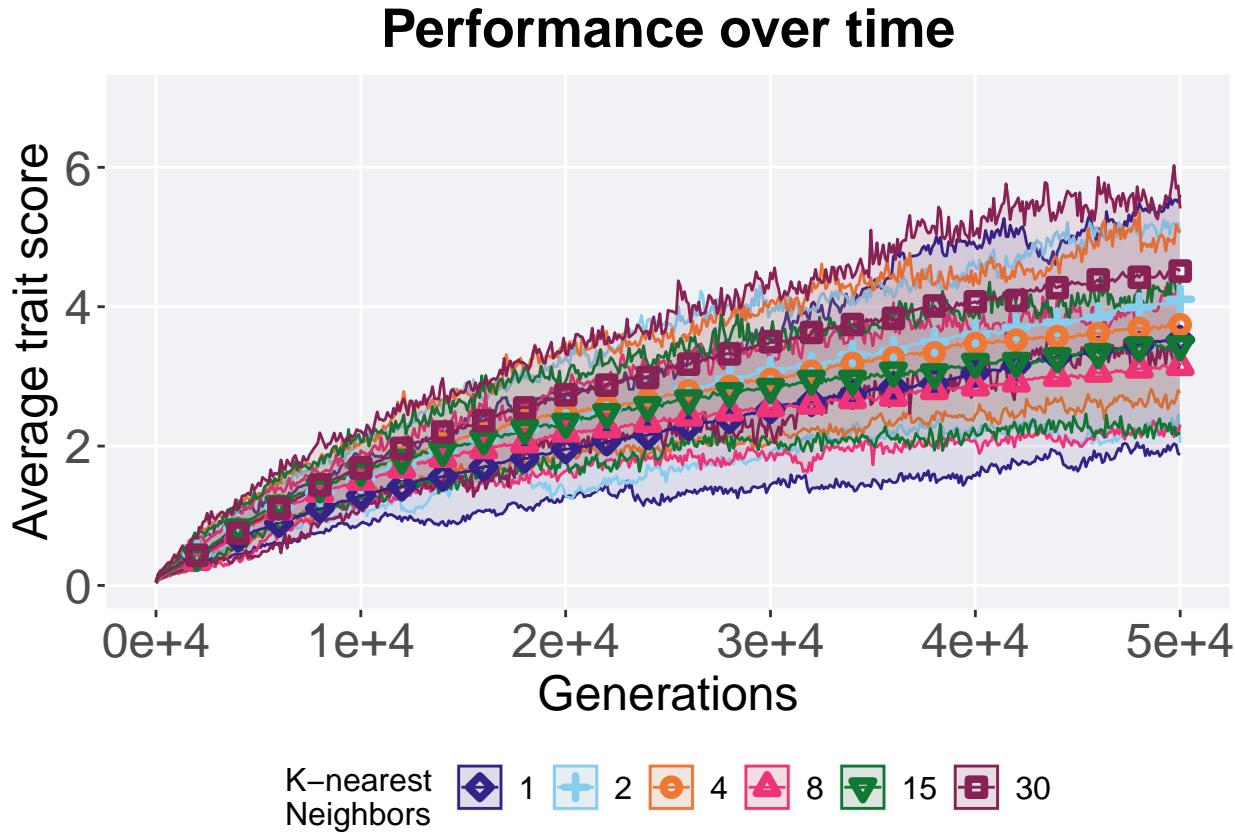
7.5.3 Performance over time

Best performance in a population over time. Data points on the graph is the average performance across 50 replicates every 2000 generations. Shading comes from the best and worse performance across 50 replicates.

```
lines = filter(over_time_df, acro == 'mpe') %>%
  group_by(K, gen) %>%
  dplyr::summarise(
    min = min(pop_fit_max) / DIMENSIONALITY,
    mean = mean(pop_fit_max) / DIMENSIONALITY,
    max = max(pop_fit_max) / DIMENSIONALITY
  )
```

```
## `summarise()` has grouped output by 'K'. You can override using the `.`groups` argument.
over_time_plot = ggplot(lines, aes(x=gen, y=mean, group = K, fill = K, color = K, shape = K)) +
  geom_ribbon(aes(ymin = min, ymax = max), alpha = 0.1) +
  geom_line(size = 0.5) +
  geom_point(data = filter(lines, gen %% 2000 == 0 & gen != 0), size = 1.5, stroke = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name="Average trait score",
    limits=c(0, 7)
  ) +
  scale_x_continuous(
    name="Generations",
    limits=c(0, 50000),
    breaks=c(0, 10000, 20000, 30000, 40000, 50000),
    labels=c("0e+4", "1e+4", "2e+4", "3e+4", "4e+4", "5e+4")
  ) +
  scale_shape_manual(values=SHAPE) +
  scale_colour_manual(values = cb_palette) +
  scale_fill_manual(values = cb_palette) +
  ggtitle('Performance over time') +
  p_theme +
  guides(
    shape=guide_legend(nrow=1, title.position = "left", title = 'K-nearest \nNeighbors'),
    color=guide_legend(nrow=1, title.position = "left", title = 'K-nearest \nNeighbors'),
    fill=guide_legend(nrow=1, title.position = "left", title = 'K-nearest \nNeighbors')
  )

over_time_plot
```



7.5.4 Best performance throughout

Best performance reached throughout 50,000 generations in a population.

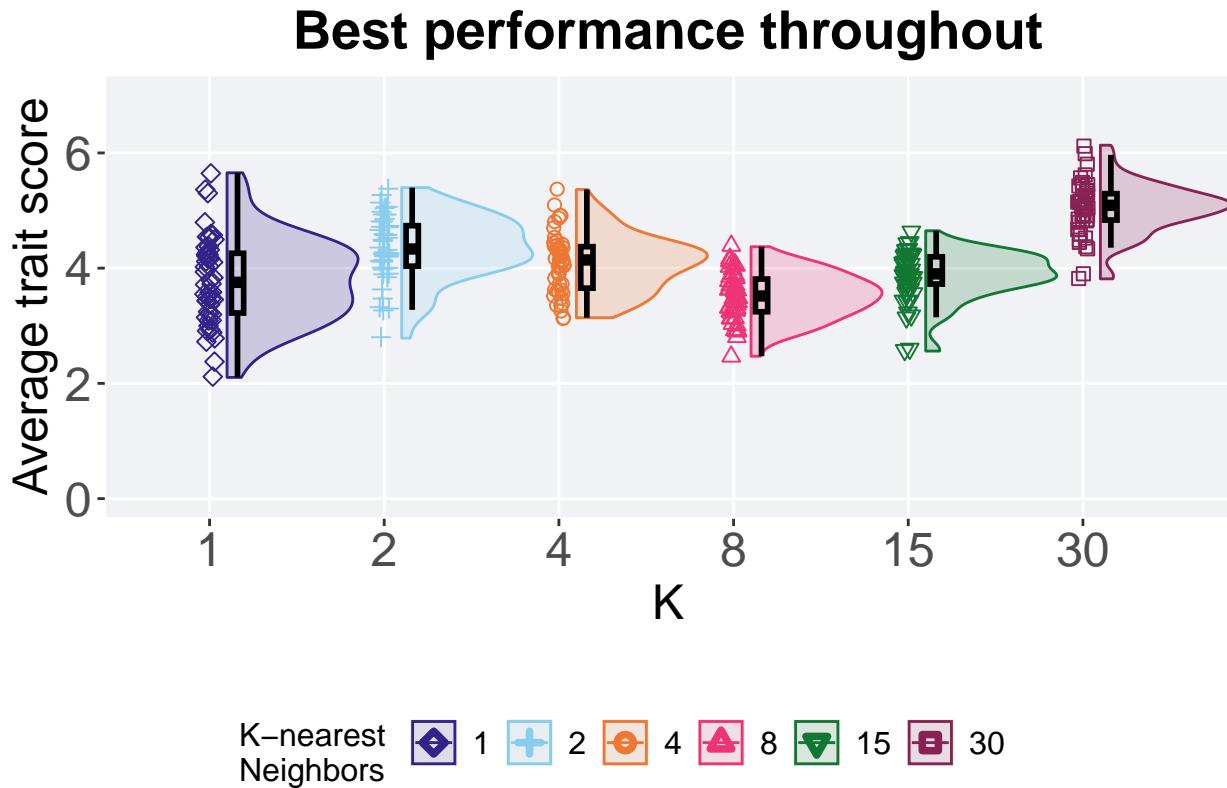
```
plot = filter(best_df, acro == 'mpe' & var == 'pop_fit_max') %>%
  ggplot(., aes(x = K, y = val / DIMENSIONALITY, color = K, fill = K, shape = K)) +
  geom_flat_violin(position = position_nudge(x = .1, y = 0), scale = 'width', alpha = 0.2, width = 1.5) +
  geom_boxplot(color = 'black', width = .07, outlier.shape = NA, alpha = 0.0, size = 1.0, position = position_nudge(x = -.1, y = 0)) +
  geom_point(position = position_jitter(width = 0.03, height = 0.02), size = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name = "Average trait score",
    limits = c(0, 7)
  ) +
  scale_x_discrete(
    name = "K"
  ) +
  scale_shape_manual(values = SHAPE) +
  scale_colour_manual(values = cb_palette,) +
  scale_fill_manual(values = cb_palette) +
  ggtitle('Best performance throughout') +
  p_theme

plot_grid(
  plot +
    theme(legend.position = "none"),
  legend,
```

```

nrow=2,
rel_heights = c(3,1)
)

```



7.5.4.1 Stats

Summary statistics for the best performance.

```

performance = filter(best_df, acro == 'mpe' & var == 'pop_fit_max')
performance$K = factor(performance$K, levels = NS_LIST)
performance %>%
  group_by(K) %>%
  dplyr::summarise(
    count = n(),
    na_cnt = sum(is.na(val)),
    min = min(val / DIMENSIONALITY, na.rm = TRUE),
    median = median(val / DIMENSIONALITY, na.rm = TRUE),
    mean = mean(val / DIMENSIONALITY, na.rm = TRUE),
    max = max(val / DIMENSIONALITY, na.rm = TRUE),
    IQR = IQR(val / DIMENSIONALITY, na.rm = TRUE)
  )

## # A tibble: 6 x 8
##   K     count  na_cnt   min  median   mean   max    IQR
##   <fct> <int>   <int> <dbl>   <dbl>   <dbl> <dbl>   <dbl>
## 1 1       50      0  2.10    3.75   3.79  5.65  1.04
## 2 2       50      0  2.79    4.33   4.36  5.40  0.704

```

```
## 3 4      50     0 3.14  4.14  4.09  5.37 0.720
## 4 8      50     0 2.47  3.52  3.51  4.37 0.568
## 5 15     50     0 2.56  3.90  3.87  4.65 0.473
## 6 30     50     0 3.81  5.09  5.04  6.13 0.458
```

Kruskal–Wallis test illustrates evidence of statistical differences.

```
kruskal.test(val ~ K, data = performance)
```

```
##
## Kruskal-Wallis rank sum test
##
## data: val by K
## Kruskal-Wallis chi-squared = 138.11, df = 5, p-value < 2.2e-16
```

Results for post-hoc Wilcoxon rank-sum test with a Bonferroni correction.

```
pairwise.wilcox.test(x = performance$val, g = performance$K, p.adjust.method = "bonferroni",
                      paired = FALSE, conf.int = FALSE, alternative = 't')
```

```
##
## Pairwise comparisons using Wilcoxon rank sum test with continuity correction
##
## data: performance$val and performance$K
##
##    1     2     4     8     15
## 2 0.00094 -
## 4 0.37925 0.15967 -
## 8 0.60413 3.0e-10 3.0e-06 -
## 15 1.00000 3.1e-05 0.62456 0.00030 -
## 30 6.3e-12 9.1e-08 9.0e-12 4.2e-16 5.4e-15
##
## P value adjustment method: bonferroni
```

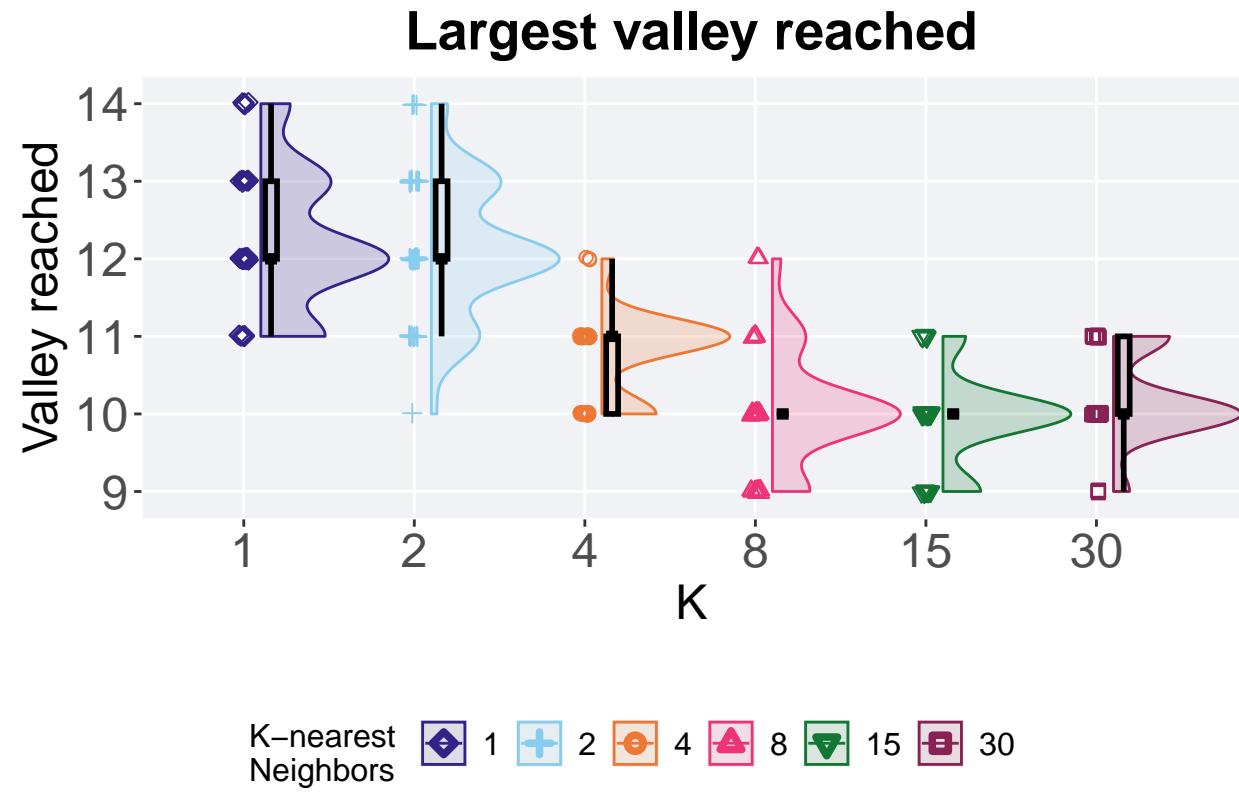
7.5.5 Largest valley reached throughout

Largest valley reached in a single trait by the best performing solution throughout an entire evolutionary run.

```
plot = filter(best_df, acro == 'mpe' & var == 'ele_big_peak') %>%
  ggplot(., aes(x = K, y = val, color = K, fill = K, shape = K)) +
  geom_flat_violin(position = position_nudge(x = .1, y = 0), scale = 'width', alpha = 0.2, width = 1.5) +
  geom_boxplot(color = 'black', width = .07, outlier.shape = NA, alpha = 0.0, size = 1.0, position = position_nudge(x = -.1, y = 0)) +
  geom_point(position = position_jitter(width = 0.03, height = 0.02), size = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name = "Valley reached",
    limits = c(8.9, 14.1)
  ) +
  scale_x_discrete(
    name = "K"
  ) +
  scale_shape_manual(values = SHAPE) +
  scale_colour_manual(values = cb_palette, ) +
  scale_fill_manual(values = cb_palette) +
  ggtitle('Largest valley reached') +
  p_theme

plot_grid()
```

```
plot +
  theme(legend.position="none"),
  legend,
  nrow=2,
  rel_heights = c(3,1)
)
```



7.5.5.1 Stats

Summary statistics for the largest valley crossed.

```
valleys = filter(best_df, acro == 'mpe' & var == 'ele_big_peak')
valleys$K = factor(valleys$K, levels = NS_LIST)
valleys %>%
  group_by(K) %>%
  dplyr::summarise(
  count = n(),
  na_cnt = sum(is.na(val)),
  min = min(val, na.rm = TRUE),
  median = median(val, na.rm = TRUE),
  mean = mean(val, na.rm = TRUE),
  max = max(val, na.rm = TRUE),
  IQR = IQR(val, na.rm = TRUE)
)

## # A tibble: 6 x 8
##   K      count na_cnt   min median   mean   max    IQR
##   <dbl>    <dbl>   <dbl> <dbl> <dbl> <dbl> <dbl> <dbl>
```

```
##   <fct> <int> <int> <dbl> <dbl> <dbl> <dbl> <dbl>
## 1 1      50     0    11    12 12.2    14     1
## 2 2      50     0    10    12 12.2    14     1
## 3 4      50     0    10    11 10.8    12     1
## 4 8      50     0     9    10 10.1    12     0
## 5 15     50     0     9    10  9.92   11     0
## 6 30     50     0     9    10 10.2    11     1
```

Kruskal–Wallis test illustrates evidence of statistical differences.

```
kruskal.test(val ~ K, data = valleys)
```

```
##
## Kruskal-Wallis rank sum test
##
## data: val by K
## Kruskal-Wallis chi-squared = 201.02, df = 5, p-value < 2.2e-16
```

Results for post-hoc Wilcoxon rank-sum test with a Bonferroni correction.

```
pairwise.wilcox.test(x = valleys$val, g = valleys$K, p.adjust.method = "bonferroni",
                      paired = FALSE, conf.int = FALSE, alternative = 't')
```

```
##
## Pairwise comparisons using Wilcoxon rank sum test with continuity correction
##
## data: valleys$val and valleys$K
##
##   1     2     4     8    15
## 2 1.00  -    -    -    -
## 4 4.2e-12 7.4e-12  -    -    -
## 8 2.9e-15 5.9e-15 2.9e-06  -    -
## 15 < 2e-16 < 2e-16 5.3e-09 1.00  -
## 30 1.1e-15 2.6e-15 5.9e-05 1.00  0.25
##
## P value adjustment method: bonferroni
```