

# Supplemental Material: Valley Crossing Diagnostics

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# Chapter 1

## Introduction

This is the supplemental material for experiments with diagnostics and integrated valleys.

### 1.1 About our supplemental material

This supplemental material is hosted on GitHub using GitHub pages. The source code and configuration files used to generate this supplemental material can be found in this GitHub repository. We compiled our data analyses and supplemental documentation into this nifty web-accessible book using bookdown.

Our supplemental material includes the following paper figures and statistics:

- Exploitation rate results (Section 2)
- Ordered exploitation results (Section 3)
- Contradictory objectives results (Section 4)
- Multi-path exploration results (Section 5)

### 1.2 Contributing authors

- Jose Guadalupe Hernandez
- Alexander Lalejini
- Charles Ofria

### 1.3 Computer Setup

These analyses were conducted in the following computing environment:

```
print(version)
```

```
##  
## platform      _  
## arch          x86_64-pc-linux-gnu  
## os            linux-gnu  
## system        x86_64, linux-gnu  
## status  
## major         4  
## minor         3.1  
## year          2023  
## month         06  
## day           16
```

```
## svn rev      84548
## language     R
## version.string R version 4.3.1 (2023-06-16)
## nickname     Beagle Scouts
```

## 1.4 Experimental setup

Setting up required variables variables.

```
# libraries we are using
library(ggplot2)
library(cowplot)
library(dplyr)

##
## Attaching package: 'dplyr'

## The following objects are masked from 'package:stats':
##
##   filter, lag

## The following objects are masked from 'package:base':
##
##   intersect, setdiff, setequal, union

library(PupillometryR)

## Loading required package: rlang

# data diractory for gh-pages
DATA_DIR = '/opt/ECJ-2023-Suite-Of-Diagnostic-Metrics-For-Characterizing-Selection-Schemes/DATA/MVC_DIA

# data diractory for local testing
# DATA_DIR = '~/Desktop/Repositories/ECJ-2023-Suite-Of-Diagnostic-Metrics-For-Characterizing-Selection-

# graph variables
SHAPE = c(5,3,1,2,6,0,4,20,1)
cb_palette <- c('#332288', '#88CCEE', '#EE7733', '#EE3377', '#117733', '#882255', '#44AA99', '#CCBB44', '#000000')
TSIZE = 26
p_theme <- theme(
  plot.title = element_text( face = "bold", size = 20, hjust=0.5),
  panel.border = element_blank(),
  panel.grid.minor = element_blank(),
  legend.title=element_text(size=18, hjust = 0.5),
  legend.text=element_text(size=10),
  axis.title = element_text(size=18),
  axis.text = element_text(size=16),
  legend.position="bottom",
  panel.background = element_rect(fill = "#f1f2f5",
                                   colour = "white",
                                   linewidth = 0.5, linetype = "solid")
)

# default variables
DIMENSIONALITY = 100
GENERATIONS = 50000
```

```
# selection scheme related stuff
ACRO = c('tru','tor','lex','gfs','pfs','nds','nov','ran')
NAMES = c('Truncation (tru)','Tournament (tor)','Lexicase (lex)', 'Genotypic Fitness Sharing (gfs)', 'Ph
```





## Chapter 2

# Exploitation rate results

Here we present the results for **best performances** found by each selection scheme on the exploitation rate diagnostic with valley crossing integrated. 50 replicates are conducted for each scheme explored.

## 2.1 Data setup

```
DIR = paste(DATA_DIR, 'EXPLOITATION_RATE/', sep = "", collapse = NULL)
over_time_df <- read.csv(paste(DIR, 'over-time.csv', sep = "", collapse = NULL), header = TRUE, stringsAsFactors = FALSE)
over_time_df$scheme <- factor(over_time_df$scheme, levels = NAMES)

best_df <- read.csv(paste(DIR, 'best.csv', sep = "", collapse = NULL), header = TRUE, stringsAsFactors = FALSE)
best_df$acro <- factor(best_df$acro, levels = ACRO)
```

## 2.2 Performance over time

Best performance in a population over time. Data points on the graph is the average performance across 50 replicates every 2000 generations. Shading comes from the best and worse performance across 50 replicates.

```
lines = over_time_df %>%
  group_by(scheme, gen) %>%
  dplyr::summarise(
    min = min(pop_fit_max) / DIMENSIONALITY,
    mean = mean(pop_fit_max) / DIMENSIONALITY,
    max = max(pop_fit_max) / DIMENSIONALITY
  )

## `summarise()` has grouped output by 'scheme'. You can override using the
## `.groups` argument.

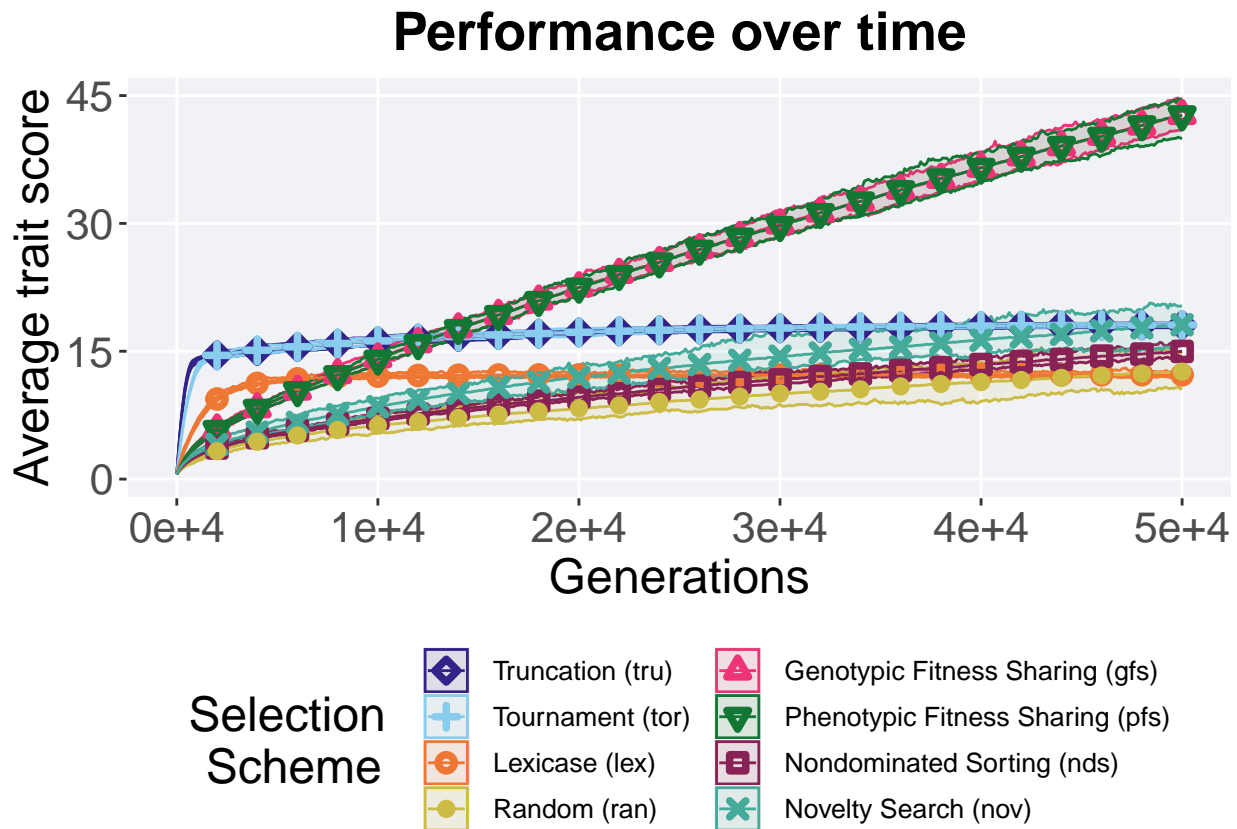
lines$scheme <- factor(lines$scheme, levels = c('Truncation (tru)', 'Tournament (tor)', 'Lexicase (lex)', 'Random (ran)'))

over_time_plot = ggplot(lines, aes(x=gen, y=mean, group = scheme, fill = scheme, color = scheme, shape = scheme)) +
  geom_ribbon(aes(ymin = min, ymax = max), alpha = 0.1) +
  geom_line(size = 0.5) +
  geom_point(data = filter(lines, gen %>% 2000 == 0 & gen != 0), size = 1.5, stroke = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name="Average trait score",
    limits=c(0, 45),
```

```

breaks=seq(0,45, 15)
) +
scale_x_continuous(
  name="Generations",
  limits=c(0, 50000),
  breaks=c(0, 10000, 20000, 30000, 40000, 50000),
  labels=c("0e+4", "1e+4", "2e+4", "3e+4", "4e+4", "5e+4")
) +
scale_shape_manual(values=c(5,3,1,20,2,6,0,4))+
scale_colour_manual(values = c('#332288','#88CCEE','#EE7733','#CCBB44','#EE3377','#117733','#882255','#332288'),
scale_fill_manual(values = c('#332288','#88CCEE','#EE7733','#CCBB44','#EE3377','#117733','#882255','#332288'),
ggtitle('Performance over time')+
p_theme +
guides(
  shape=guide_legend(ncol=2, title.position = "left", title = 'Selection \nScheme'),
  color=guide_legend(ncol=2, title.position = "left", title = 'Selection \nScheme'),
  fill=guide_legend(ncol=2, title.position = "left", title = 'Selection \nScheme')
)
over_time_plot

```



### 2.3 Best performance throughout

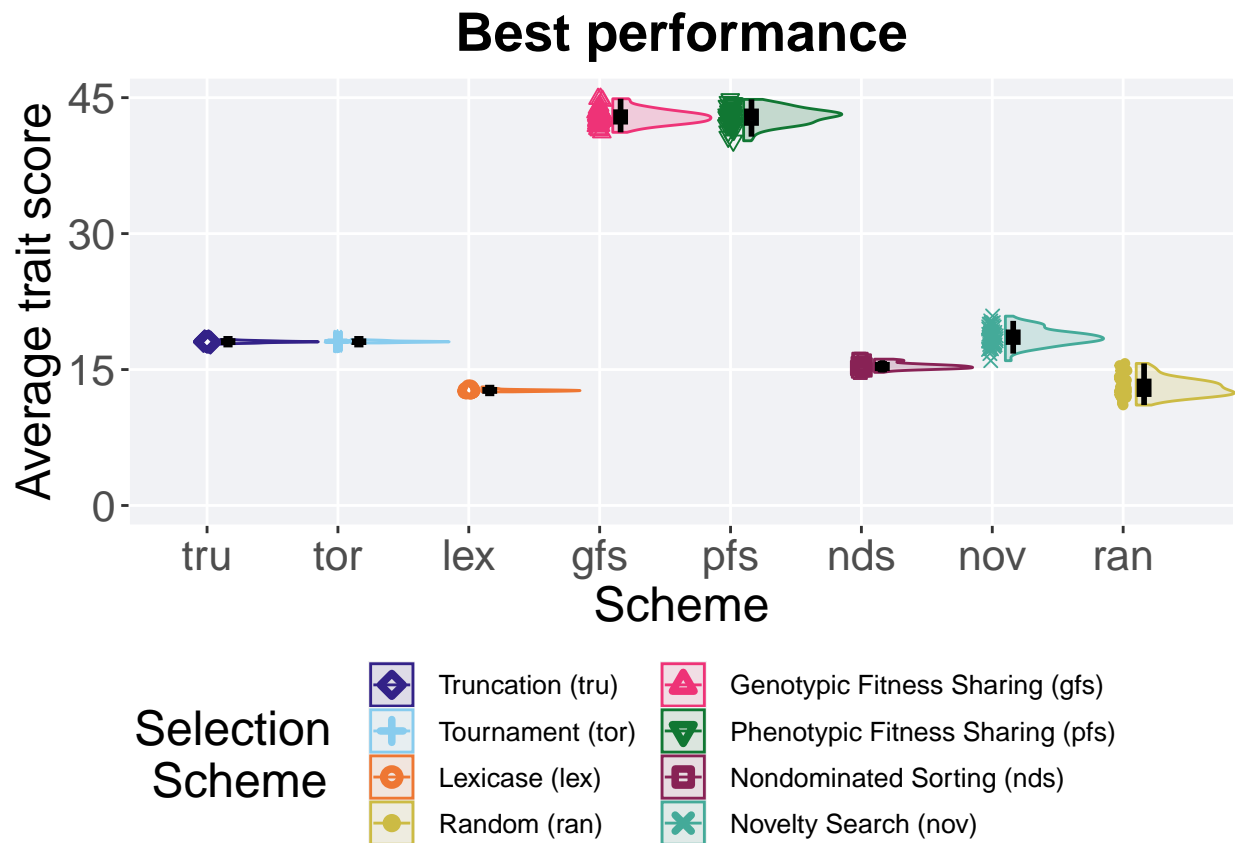
Best performance reached throughout 50,000 generations in a population.

```

plot = filter(best_df, var == 'pop_fit_max') %>%
  ggplot(., aes(x = acro, y = val / DIMENSIONALITY, color = acro, fill = acro, shape = acro)) +
  geom_flat_violin(position = position_nudge(x = .1, y = 0), scale = 'width', alpha = 0.2, width = 1.5) +
  geom_boxplot(color = 'black', width = .07, outlier.shape = NA, alpha = 0.0, size = 1.0, position = position_nudge(x = .1, y = 0)) +
  geom_point(position = position_jitter(width = 0.03, height = 0.02), size = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name="Average trait score",
    limits=c(0, 45),
    breaks=seq(0,45, 15)
  ) +
  scale_x_discrete(
    name="Scheme"
  ) +
  scale_shape_manual(values=SHAPE) +
  scale_colour_manual(values = cb_palette, ) +
  scale_fill_manual(values = cb_palette) +
  ggtitle('Best performance') +
  p_theme

plot_grid(
  plot +
    theme(legend.position="none"),
  legend,
  nrow=2,
  rel_heights = c(3,1)
)

```



### 2.3.1 Stats

Summary statistics for the best performance.

```
performance = filter(best_df, var == 'pop_fit_max')
performance$acro = factor(performance$acro, levels = c('gfs','pfs','nov','tru','tor','nds','ran','lex'))
performance %>%
  group_by(acro) %>%
  dplyr::summarise(
    count = n(),
    na_cnt = sum(is.na(val)),
    min = min(val / DIMENSIONALITY, na.rm = TRUE),
    median = median(val / DIMENSIONALITY, na.rm = TRUE),
    mean = mean(val / DIMENSIONALITY, na.rm = TRUE),
    max = max(val / DIMENSIONALITY, na.rm = TRUE),
    IQR = IQR(val / DIMENSIONALITY, na.rm = TRUE)
  )
```

```
## # A tibble: 8 x 8
##   acro count na_cnt   min median   mean   max   IQR
##   <fct> <int>  <int> <dbl>  <dbl> <dbl> <dbl> <dbl>
## 1 gfs     50      0  41.2   42.9  42.9  44.9  1.05
## 2 pfs     50      0  40.2   43.1  42.9  44.8  1.28
## 3 nov     50      0  16.0   18.6  18.6  20.9  1.07
## 4 tru     50      0  17.8   18.1  18.1  18.3  0.123
## 5 tor     50      0  17.9   18.1  18.1  18.3  0.0974
## 6 nds     50      0  14.7   15.3  15.4  16.1  0.378
## 7 ran     50      0  11.1   13.0  13.1  15.7  1.39
## 8 lex     50      0  12.5   12.7  12.7  13.1  0.128
```

Kruskal-Wallis test illustrates evidence of statistical differences.

```
kruskal.test(val ~ acro, data = performance)
```

```
##
##   Kruskal-Wallis rank sum test
##
## data:  val by acro
## Kruskal-Wallis chi-squared = 366.61, df = 7, p-value < 2.2e-16
```

Results for post-hoc Wilcoxon rank-sum test with a Bonferroni correction.

```
pairwise.wilcox.test(x = performance$val, g = performance$acro, p.adjust.method = "bonferroni",
  paired = FALSE, conf.int = FALSE, alternative = 'l')
```

```
##
##   Pairwise comparisons using Wilcoxon rank sum test with continuity correction
##
## data:  performance$val and performance$acro
##
##      gfs      pfs      nov      tru      tor      nds      ran
## pfs 1.0000   -        -        -        -        -        -
## nov < 2e-16 < 2e-16 -        -        -        -        -
## tru < 2e-16 < 2e-16 0.0014 -        -        -        -
## tor < 2e-16 < 2e-16 0.0026 1.0000 -        -        -
## nds < 2e-16 < 2e-16 < 2e-16 < 2e-16 < 2e-16 -        -
## ran < 2e-16 < 2e-16 < 2e-16 < 2e-16 < 2e-16 1.2e-14 -
## lex < 2e-16 < 2e-16 < 2e-16 < 2e-16 < 2e-16 < 2e-16 1.0000
```

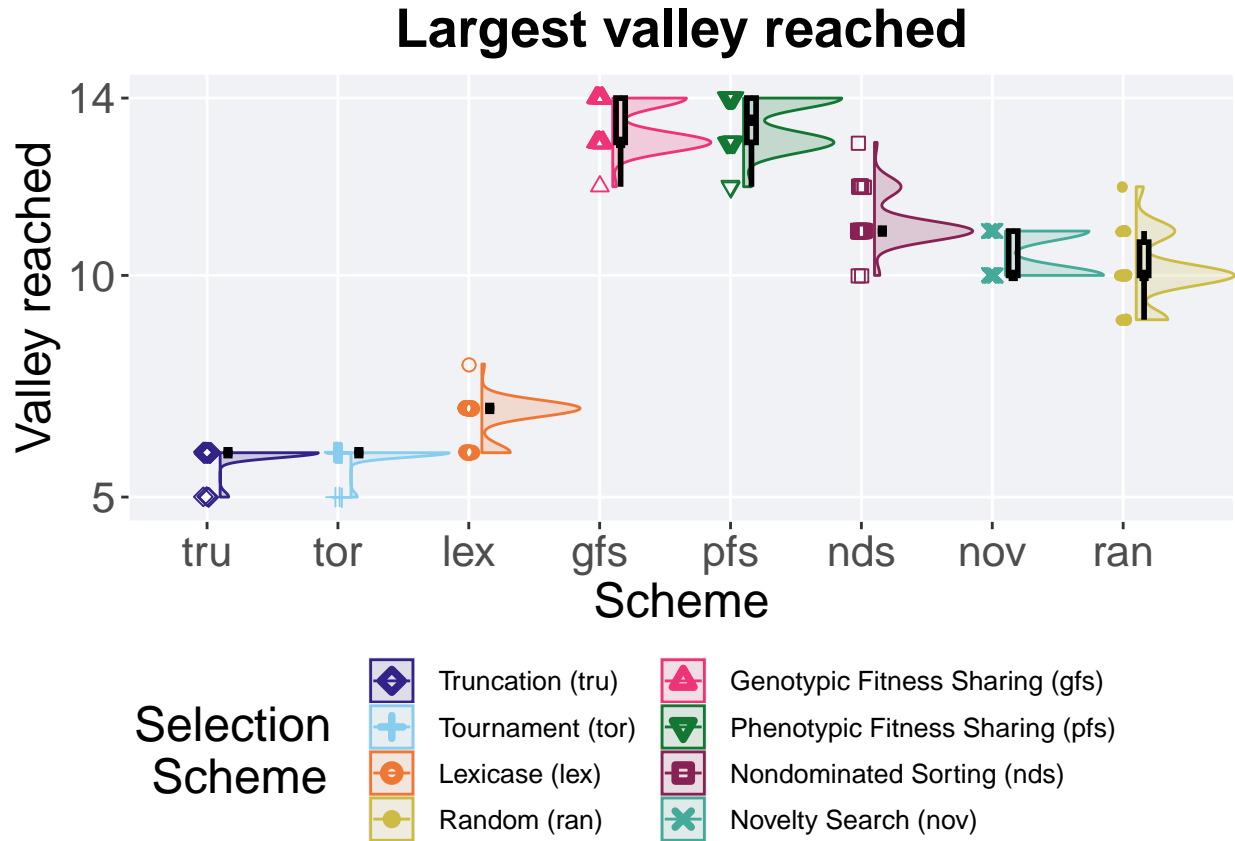
```
##
## P value adjustment method: bonferroni
```

## 2.4 Largest valley reached throughout

The largest valley reached in a single trait throughout an entire evolutionary run. To collect this value, we look through all the best-performing solutions each generation and find the largest valley reached.

```
plot = filter(best_df, var == 'ele_big_peak') %>%
  ggplot(., aes(x = acro, y = val, color = acro, fill = acro, shape = acro)) +
  geom_flat_violin(position = position_nudge(x = .1, y = 0), scale = 'width', alpha = 0.2, width = 1.5)
  geom_boxplot(color = 'black', width = .07, outlier.shape = NA, alpha = 0.0, size = 1.0, position = position_nudge(x = .1, y = 0))
  geom_point(position = position_jitter(width = 0.03, height = 0.02), size = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name="Valley reached",
    limits=c(4.9,14.1),
    breaks=c(5,10,14)
  ) +
  scale_x_discrete(
    name="Scheme"
  ) +
  scale_shape_manual(values=SHAPE) +
  scale_colour_manual(values = cb_palette, ) +
  scale_fill_manual(values = cb_palette) +
  ggtitle('Largest valley reached') +
  p_theme

plot_grid(
  plot +
    theme(legend.position="none"),
  legend,
  nrow=2,
  rel_heights = c(3,1)
)
```



#### 2.4.1 Stats

Summary statistics for the largest valley crossed.

```
valleys = filter(best_df, var == 'ele_big_peak')
valleys$acro = factor(valleys$acro, levels = c('gfs', 'pfs', 'nds', 'nov', 'ran', 'lex', 'tru', 'tor'))
valleys %>%
  group_by(acro) %>%
  dplyr::summarise(
    count = n(),
    na_cnt = sum(is.na(val)),
    min = min(val, na.rm = TRUE),
    median = median(val, na.rm = TRUE),
    mean = mean(val, na.rm = TRUE),
    max = max(val, na.rm = TRUE),
    IQR = IQR(val, na.rm = TRUE)
  )
```

```
## # A tibble: 8 x 8
##   acro  count na_cnt  min median  mean  max  IQR
##   <fct> <int>  <int> <dbl> <dbl> <dbl> <dbl> <dbl>
## 1 gfs     50     0    12    13  13.4    14    1
## 2 pfs     50     0    12   13.5  13.5    14    1
## 3 nds     50     0    10    11  11.2    13    0
## 4 nov     50     0    10    10  10.5    11    1
## 5 ran     50     0     9    10  10.1    12  0.75
## 6 lex     50     0     6     7   6.8     8    0
```

```
## 7 tru      50      0      5      6      5.92      6 0
## 8 tor      50      0      5      6      5.94      6 0
```

Kruskal-Wallis test illustrates evidence of statistical differences.

```
kruskal.test(val ~ acro, data = valleys)
```

```
##
## Kruskal-Wallis rank sum test
##
## data:  val by acro
## Kruskal-Wallis chi-squared = 377.23, df = 7, p-value < 2.2e-16
```

Results for post-hoc Wilcoxon rank-sum test with a Bonferroni correction.

```
pairwise.wilcox.test(x = valleys$val, g = valleys$acro, p.adjust.method = "bonferroni",
                     paired = FALSE, conf.int = FALSE, alternative = 'l')
```

```
##
## Pairwise comparisons using Wilcoxon rank sum test with continuity correction
##
## data:  valleys$val and valleys$acro
##
##      gfs      pfs      nds      nov      ran      lex      tru
## pfs 1.000      -      -      -      -      -      -
## nds < 2e-16 < 2e-16 -      -      -      -      -
## nov < 2e-16 < 2e-16 3.9e-08 -      -      -      -
## ran < 2e-16 < 2e-16 2.1e-10 0.099 -      -      -
## lex < 2e-16 < 2e-16 < 2e-16 < 2e-16 < 2e-16 -      -
## tru < 2e-16 < 2e-16 < 2e-16 < 2e-16 < 2e-16 4.6e-14 -
## tor < 2e-16 < 2e-16 < 2e-16 < 2e-16 < 2e-16 4.3e-14 1.000
##
## P value adjustment method: bonferroni
```





## Ordered exploitation results

### 3.1 Data setup

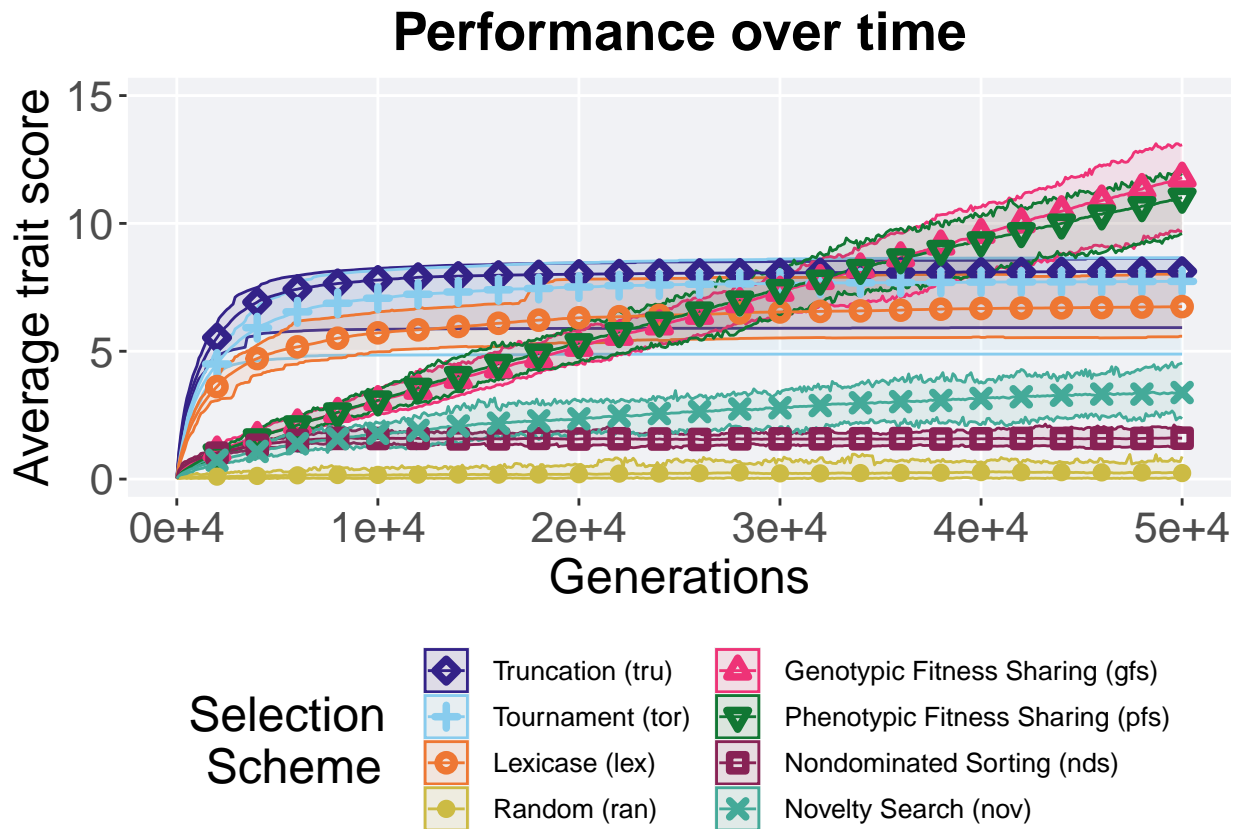
### 3.2 Performance over time

17

```

breaks=c(0,5,10,15)
) +
scale_x_continuous(
  name="Generations",
  limits=c(0, 50000),
  breaks=c(0, 10000, 20000, 30000, 40000, 50000),
  labels=c("0e+4", "1e+4", "2e+4", "3e+4", "4e+4", "5e+4")
) +
scale_shape_manual(values=c(5,3,1,20,2,6,0,4))+
scale_colour_manual(values = c('#332288','#88CCEE','#EE7733','#CCBB44','#EE3377','#117733','#882255','#332288'),
scale_fill_manual(values = c('#332288','#88CCEE','#EE7733','#CCBB44','#EE3377','#117733','#882255','#332288'),
ggtitle('Performance over time')+
p_theme +
guides(
  shape=guide_legend(ncol=2, title.position = "left", title = 'Selection \nScheme'),
  color=guide_legend(ncol=2, title.position = "left", title = 'Selection \nScheme'),
  fill=guide_legend(ncol=2, title.position = "left", title = 'Selection \nScheme')
)
over_time_plot

```



### 3.3 Best performance throughout

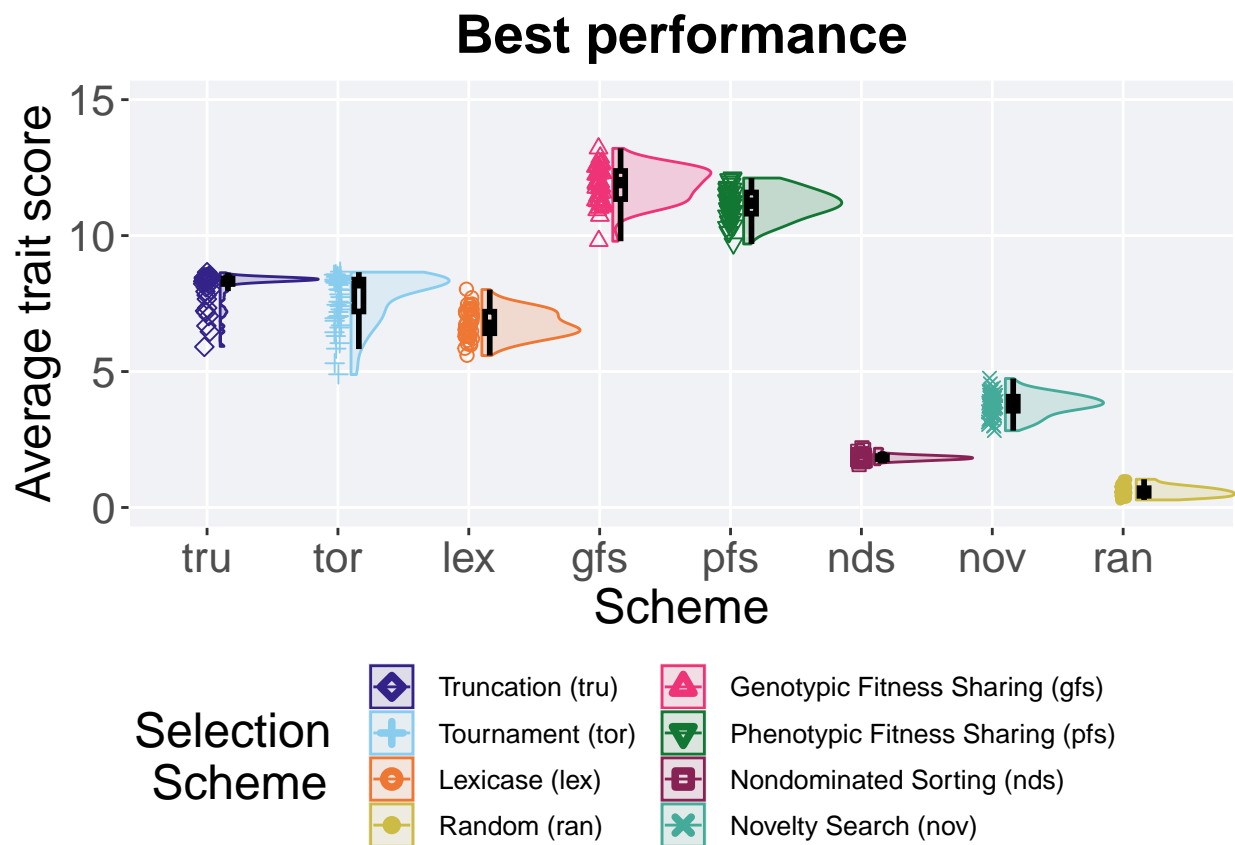
Best performance reached throughout 50,000 generations in a population.

```

plot = filter(best_df, var == 'pop_fit_max') %>%
  ggplot(., aes(x = acro, y = val / DIMENSIONALITY, color = acro, fill = acro, shape = acro)) +
  geom_flat_violin(position = position_nudge(x = .1, y = 0), scale = 'width', alpha = 0.2, width = 1.5) +
  geom_boxplot(color = 'black', width = .07, outlier.shape = NA, alpha = 0.0, size = 1.0, position = position_nudge(x = .1, y = 0)) +
  geom_point(position = position_jitter(width = 0.03, height = 0.02), size = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name="Average trait score",
    limits=c(0, 15),
    breaks=c(0,5,10,15)
  ) +
  scale_x_discrete(
    name="Scheme"
  ) +
  scale_shape_manual(values=SHAPE) +
  scale_colour_manual(values = cb_palette, ) +
  scale_fill_manual(values = cb_palette) +
  ggtitle('Best performance') +
  p_theme

plot_grid(
  plot +
    theme(legend.position="none"),
  legend,
  nrow=2,
  rel_heights = c(3,1)
)

```



### 3.3.1 Stats

Summary statistics for the best performance.

```
performance = filter(best_df, var == 'pop_fit_max')
performance$acro = factor(performance$acro, levels = c('gfs','pfs','tru','tor','lex','nov','nds','ran'))
performance %>%
  group_by(acro) %>%
  dplyr::summarise(
    count = n(),
    na_cnt = sum(is.na(val)),
    min = min(val / DIMENSIONALITY, na.rm = TRUE),
    median = median(val / DIMENSIONALITY, na.rm = TRUE),
    mean = mean(val / DIMENSIONALITY, na.rm = TRUE),
    max = max(val / DIMENSIONALITY, na.rm = TRUE),
    IQR = IQR(val / DIMENSIONALITY, na.rm = TRUE)
  )
```

```
## # A tibble: 8 x 8
##   acro count na_cnt   min median   mean   max   IQR
##   <fct> <int> <int> <dbl> <dbl> <dbl> <dbl> <dbl>
## 1 gfs     50      0  9.79  11.9  11.9  13.2  1.03
## 2 pfs     50      0  9.69  11.2  11.2  12.1  0.779
## 3 tru     50      0  5.92   8.36   8.13   8.63  0.203
## 4 tor     50      0  4.89   8.26   7.73   8.65  1.17
## 5 lex     50      0  5.59   6.70   6.76   8.02  0.792
## 6 nov     50      0  2.82   3.82   3.79   4.74  0.515
## 7 nds     50      0  1.57   1.83   1.84   2.18  0.116
## 8 ran     50      0  0.279  0.568  0.587   1.04  0.280
```

Kruskal-Wallis test illustrates evidence of statistical differences.

```
kruskal.test(val ~ acro, data = performance)
```

```
##
##   Kruskal-Wallis rank sum test
##
## data:  val by acro
## Kruskal-Wallis chi-squared = 379.83, df = 7, p-value < 2.2e-16
```

Results for post-hoc Wilcoxon rank-sum test with a Bonferroni correction.

```
pairwise.wilcox.test(x = performance$val, g = performance$acro, p.adjust.method = "bonferroni",
  paired = FALSE, conf.int = FALSE, alternative = 'l')
```

```
##
##   Pairwise comparisons using Wilcoxon rank sum test with continuity correction
##
## data:  performance$val and performance$acro
##
##      gfs      pfs      tru      tor      lex      nov      nds
## pfs 5.0e-06 -          -          -          -          -          -
## tru < 2e-16 < 2e-16 -          -          -          -          -
## tor < 2e-16 < 2e-16 0.33      -          -          -          -
## lex < 2e-16 < 2e-16 7.8e-13 2.8e-07 -          -          -
## nov < 2e-16 < 2e-16 < 2e-16 < 2e-16 < 2e-16 -          -
## nds < 2e-16 < 2e-16 < 2e-16 < 2e-16 < 2e-16 < 2e-16 -
## ran < 2e-16 < 2e-16 < 2e-16 < 2e-16 < 2e-16 < 2e-16 < 2e-16
```

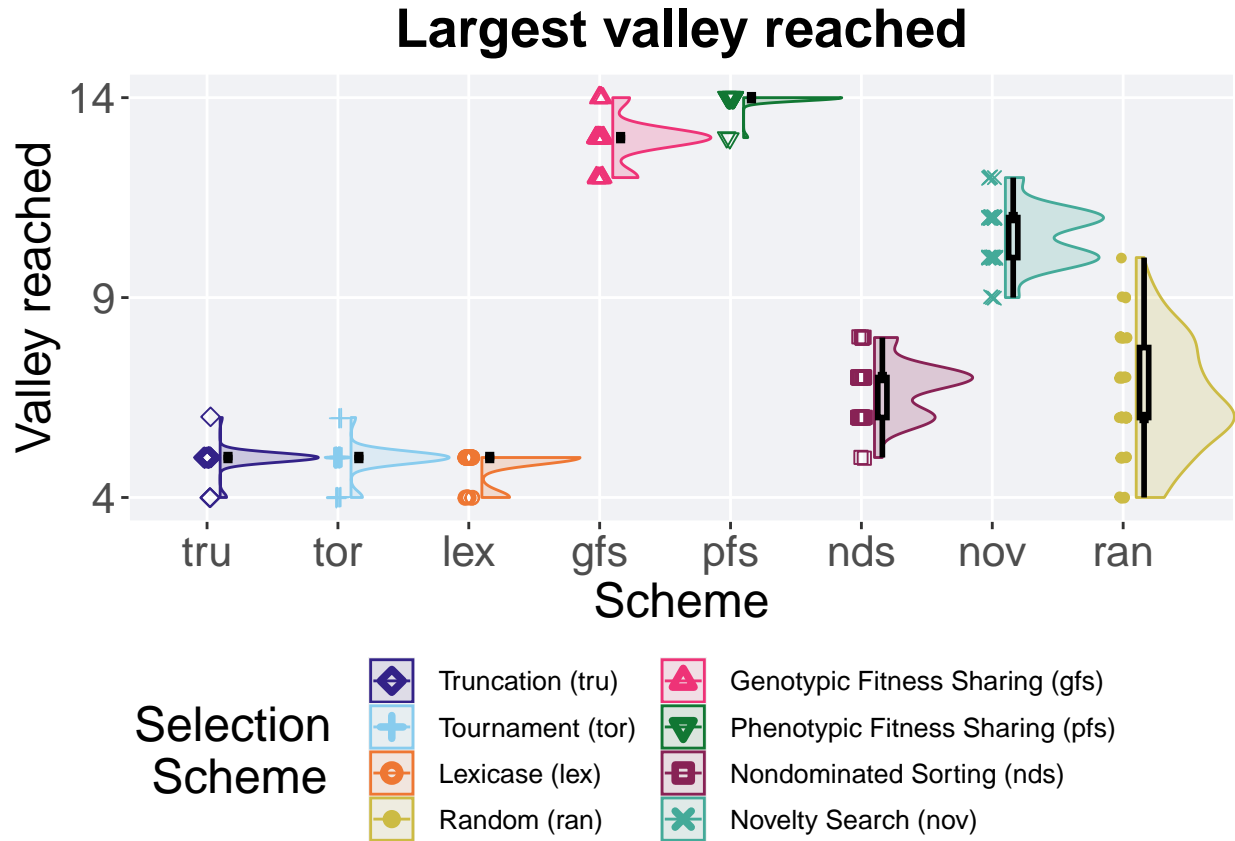
```
##
## P value adjustment method: bonferroni
```

## 3.4 Largest valley reached throughout

The largest valley reached in a single trait throughout an entire evolutionary run. To collect this value, we look through all the best-performing solutions each generation and find the largest valley reached.

```
plot = filter(best_df, var == 'ele_big_peak') %>%
  ggplot(., aes(x = acro, y = val, color = acro, fill = acro, shape = acro)) +
  geom_flat_violin(position = position_nudge(x = .1, y = 0), scale = 'width', alpha = 0.2, width = 1.5)
  geom_boxplot(color = 'black', width = .07, outlier.shape = NA, alpha = 0.0, size = 1.0, position = position_nudge(x = .1, y = 0))
  geom_point(position = position_jitter(width = 0.03, height = 0.02), size = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name="Valley reached",
    limits=c(3.9,14.1),
    breaks=c(4,9,14)
  ) +
  scale_x_discrete(
    name="Scheme"
  ) +
  scale_shape_manual(values=SHAPE) +
  scale_colour_manual(values = cb_palette, ) +
  scale_fill_manual(values = cb_palette) +
  ggtitle('Largest valley reached') +
  p_theme

plot_grid(
  plot +
    theme(legend.position="none"),
  legend,
  nrow=2,
  rel_heights = c(3,1)
)
```



### 3.4.1 Stats

Summary statistics for the largest valley crossed.

```
valleys = filter(best_df, var == 'ele_big_peak')
valleys$acro = factor(valleys$acro, levels = c('pfs', 'gfs', 'nov', 'nds', 'ran', 'tru', 'tor', 'lex'))
valleys %>%
  group_by(acro) %>%
  dplyr::summarise(
    count = n(),
    na_cnt = sum(is.na(val)),
    min = min(val, na.rm = TRUE),
    median = median(val, na.rm = TRUE),
    mean = mean(val, na.rm = TRUE),
    max = max(val, na.rm = TRUE),
    IQR = IQR(val, na.rm = TRUE)
  )
```

```
## # A tibble: 8 x 8
##   acro  count na_cnt  min median  mean  max  IQR
##   <fct> <int>  <int> <dbl> <dbl> <dbl> <dbl> <dbl>
## 1 pfs     50     0    13     14  14.0    14     0
## 2 gfs     50     0    12     13  12.9    14     0
## 3 nov     50     0     9     11  10.5    12     1
## 4 nds     50     0     5     7    6.72     8     1
## 5 ran     50     0     4     6    6.48    10    1.75
## 6 tru     50     0     4     5    4.96     6     0
```

```
## 7 tor      50      0      4      5 4.94      6 0
## 8 lex      50      0      4      5 4.78      5 0
```

Kruskal-Wallis test illustrates evidence of statistical differences.

```
kruskal.test(val ~ acro, data = valleys)
```

```
##
## Kruskal-Wallis rank sum test
##
## data:  val by acro
## Kruskal-Wallis chi-squared = 364.41, df = 7, p-value < 2.2e-16
```

Results for post-hoc Wilcoxon rank-sum test with a Bonferroni correction.

```
pairwise.wilcox.test(x = valleys$val, g = valleys$acro, p.adjust.method = "bonferroni",
                     paired = FALSE, conf.int = FALSE, alternative = 'l')
```

```
##
## Pairwise comparisons using Wilcoxon rank sum test with continuity correction
##
## data:  valleys$val and valleys$acro
##
##      pfs      gfs      nov      nds      ran      tru      tor
## gfs 3.3e-15 -          -          -          -          -          -
## nov < 2e-16 < 2e-16 -          -          -          -          -
## nds < 2e-16 < 2e-16 < 2e-16 -          -          -          -
## ran < 2e-16 < 2e-16 < 2e-16 1.00      -          -          -
## tru < 2e-16 < 2e-16 < 2e-16 < 2e-16 1.6e-09 -          -
## tor < 2e-16 < 2e-16 < 2e-16 < 2e-16 2.8e-09 1.00      -
## lex < 2e-16 < 2e-16 < 2e-16 < 2e-16 2.9e-10 0.21 0.72
##
## P value adjustment method: bonferroni
```





## Chapter 4

# Contradictory objectives results

Here we present the results for **activation gene coverage** and **satisfactory trait coverage** found by each selection scheme on the contradictory objectives diagnostic with valley crossing integrated. 50 replicates are conducted for each scheme explored.

### 4.1 Data setup

```
DIR = paste(DATA_DIR, 'CONTRADICTIONARY_OBJECTIVES/', sep = "", collapse = NULL)
over_time_df <- read.csv(paste(DIR, 'over-time.csv', sep = "", collapse = NULL), header = TRUE, stringsAsFactors = FALSE)
over_time_df$uni_str_pos = over_time_df$uni_str_pos + over_time_df$arc_acti_gene - over_time_df$overlap
over_time_df$scheme <- factor(over_time_df$scheme, levels = NAMES)
over_time_df$acro <- factor(over_time_df$acro, levels = ACRO)

best_df <- read.csv(paste(DIR, 'best.csv', sep = "", collapse = NULL), header = TRUE, stringsAsFactors = FALSE)
best_df$acro <- factor(best_df$acro, levels = ACRO)
```

### 4.2 Activation gene coverage over time

Activation gene coverage in a population over time. Data points on the graph is the average activation gene coverage across 50 replicates every 2000 generations. Shading comes from the best and worse coverage across 50 replicates.

```
lines = over_time_df %>%
  group_by(scheme, gen) %>%
  dplyr::summarise(
    min = min(uni_str_pos),
    mean = mean(uni_str_pos),
    max = max(uni_str_pos)
  )

## `summarise()` has grouped output by 'scheme'. You can override using the
## `.groups` argument.

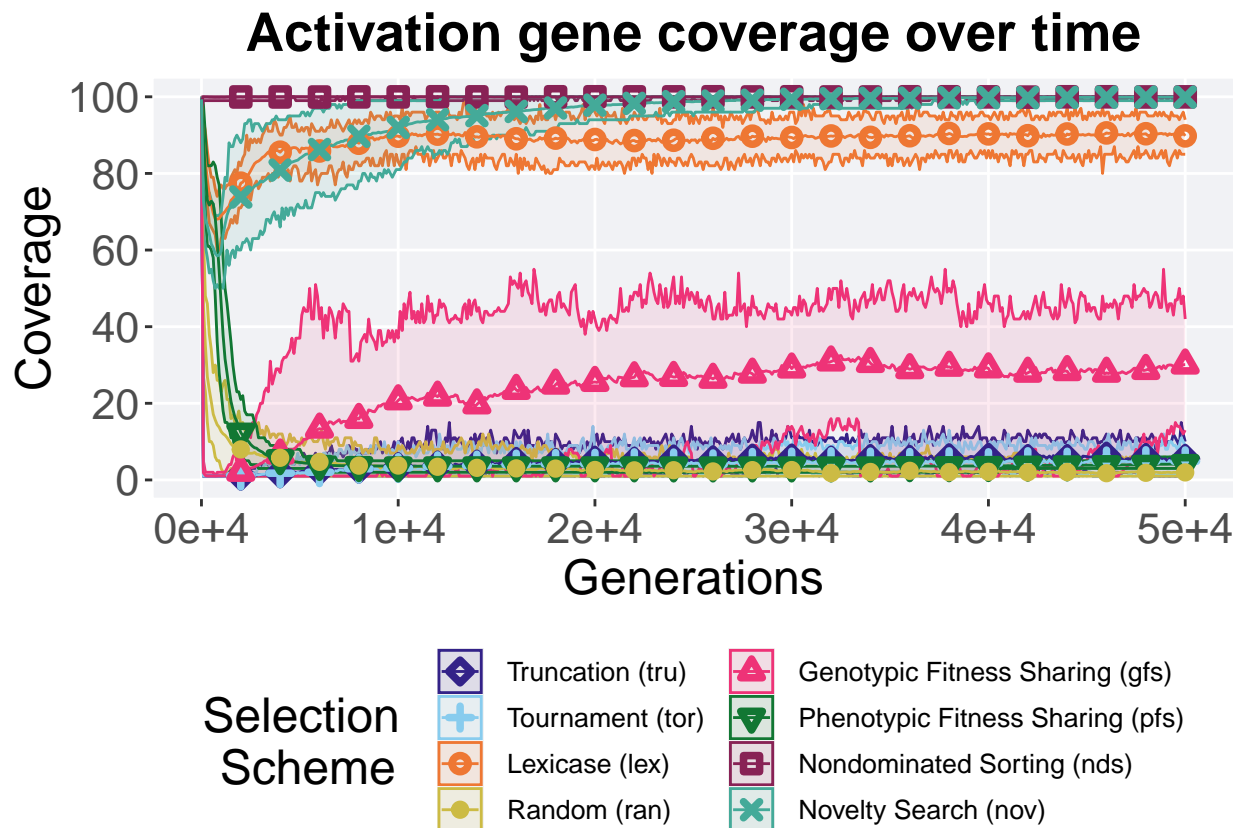
lines$scheme <- factor(lines$scheme, levels = c('Truncation (tru)', 'Tournament (tor)', 'Lexicase (lex)', 'Random (ran)'))

over_time_plot = ggplot(lines, aes(x=gen, y=mean, group = scheme, fill = scheme, color = scheme, shape = scheme)) +
  geom_ribbon(aes(ymin = min, ymax = max), alpha = 0.1) +
  geom_line(size = 0.5) +
```

```

geom_point(data = filter(lines, gen %% 2000 == 0 & gen != 0), size = 1.5, stroke = 2.0, alpha = 1.0) +
scale_y_continuous(
  name="Coverage",
  limits=c(0, 100.1),
  breaks=seq(0,100, 20),
  labels=c("0", "20", "40", "60", "80", "100")
) +
scale_x_continuous(
  name="Generations",
  limits=c(0, 50000),
  breaks=c(0, 10000, 20000, 30000, 40000, 50000),
  labels=c("0e+4", "1e+4", "2e+4", "3e+4", "4e+4", "5e+4")
) +
scale_shape_manual(values=c(5,3,1,20,2,6,0,4))+
scale_colour_manual(values = c('#332288', '#88CCEE', '#EE7733', '#CCBB44', '#EE3377', '#117733', '#882255',
scale_fill_manual(values = c('#332288', '#88CCEE', '#EE7733', '#CCBB44', '#EE3377', '#117733', '#882255', '#
ggtitle('Activation gene coverage over time')+
p_theme +
guides(
  shape=guide_legend(ncol=2, title.position = "left", title = 'Selection \nScheme'),
  color=guide_legend(ncol=2, title.position = "left", title = 'Selection \nScheme'),
  fill=guide_legend(ncol=2, title.position = "left", title = 'Selection \nScheme')
)
over_time_plot

```



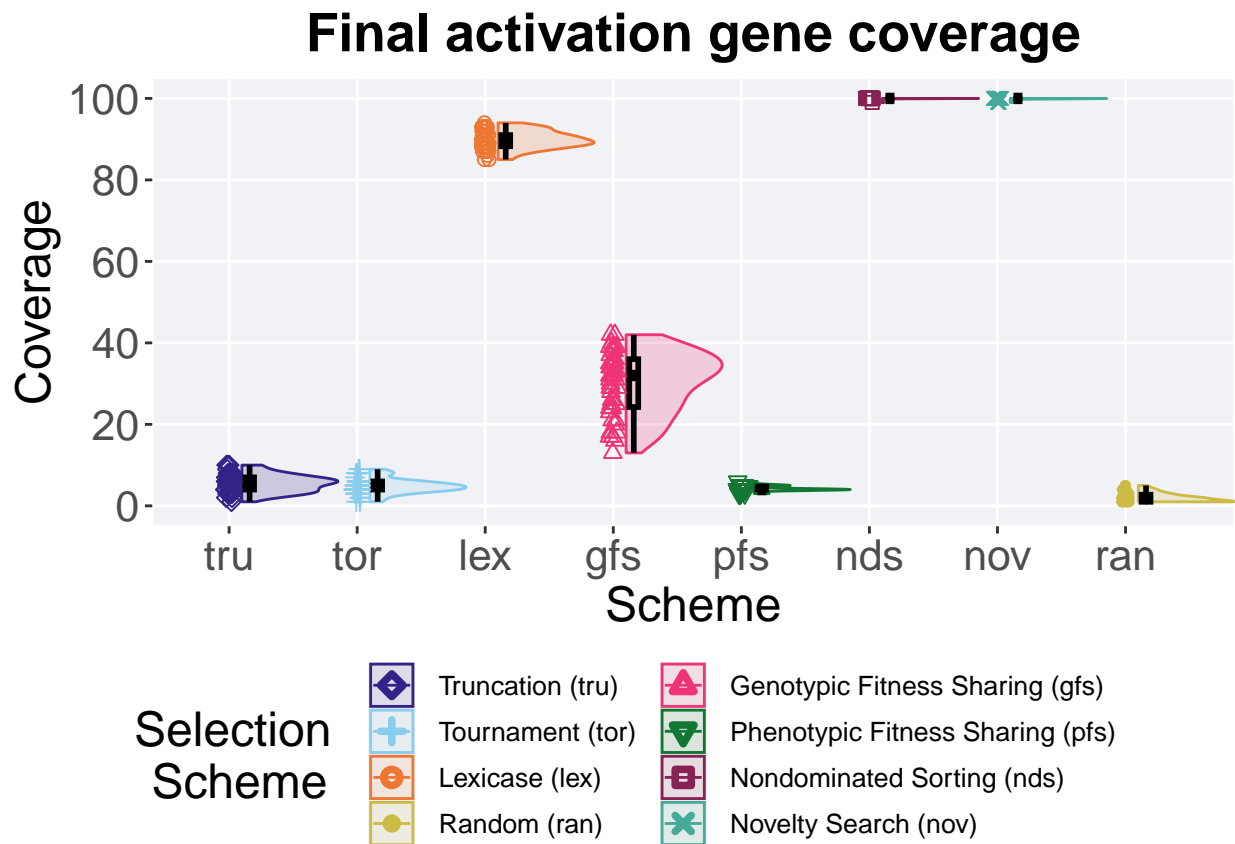
### 4.3 Final activation gene coverage

Activation gene coverage found in the final population at 50,000 generations.

```
plot = filter(over_time_df, gen == 50000) %>%
  ggplot(., aes(x = acro, y = uni_str_pos, color = acro, fill = acro, shape = acro)) +
  geom_flat_violin(position = position_nudge(x = .1, y = 0), scale = 'width', alpha = 0.2, width = 1.5) +
  geom_boxplot(color = 'black', width = .07, outlier.shape = NA, alpha = 0.0, size = 1.0, position = position_nudge(x = .1, y = 0)) +
  geom_point(position = position_jitter(width = 0.03, height = 0.02), size = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name="Coverage",
    limits=c(0, 100.1),
    breaks=seq(0,100, 20),
    labels=c("0", "20", "40", "60", "80", "100")
  ) +
  scale_x_discrete(
    name="Scheme"
  ) +
  scale_shape_manual(values=SHAPE) +
  scale_colour_manual(values = cb_palette, ) +
  scale_fill_manual(values = cb_palette) +
  ggtitle('Final activation gene coverage') +
  p_theme

plot_grid(
```

```
plot +
  theme(legend.position="none"),
  legend,
  nrow=2,
  rel_heights = c(3,1)
)
```



#### 4.3.1 Stats

Summary statistics for the coverage found in the final population.

```
act_coverage = filter(over_time_df, gen == 50000)
act_coverage$acro = factor(act_coverage$acro, levels = c('nds', 'nov', 'lex', 'gfs', 'tru', 'tor', 'pfs', 'ran'))
act_coverage %>%
  group_by(acro) %>%
  dplyr::summarise(
    count = n(),
    na_cnt = sum(is.na(uni_str_pos)),
    min = min(uni_str_pos, na.rm = TRUE),
    median = median(uni_str_pos, na.rm = TRUE),
    mean = mean(uni_str_pos, na.rm = TRUE),
    max = max(uni_str_pos, na.rm = TRUE),
    IQR = IQR(uni_str_pos, na.rm = TRUE)
  )
```

```
## # A tibble: 8 x 8
```

```
##   acro   count na_cnt   min median   mean   max   IQR
```

```
##   <fct> <int>  <int> <int>  <dbl>  <dbl> <int> <dbl>
## 1 nds      50      0   99    100 100.    100  0
## 2 nov      50      0   99    100 99.9    100  0
## 3 lex      50      0   85     90 89.8     94 2.75
## 4 gfs      50      0   13     32 30.1     42 11.8
## 5 tru      50      0    1     6  5.5     10  3
## 6 tor      50      0    1     5  4.86     9  2
## 7 pfs      50      0    3     4  4.18     6 0.75
## 8 ran      50      0    1     2  1.96     5 1.75
```

Kruskal-Wallis test illustrates evidence of statistical differences.

```
kruskal.test(uni_str_pos ~ acro, data = act_coverage)
```

```
##
## Kruskal-Wallis rank sum test
##
## data: uni_str_pos by acro
## Kruskal-Wallis chi-squared = 369.27, df = 7, p-value < 2.2e-16
```

Results for post-hoc Wilcoxon rank-sum test with a Bonferroni correction.

```
pairwise.wilcox.test(x = act_coverage$uni_str_pos, g = act_coverage$acro, p.adjust.method = "bonferroni",
                     paired = FALSE, conf.int = FALSE, alternative = 'l')
```

```
##
## Pairwise comparisons using Wilcoxon rank sum test with continuity correction
##
## data: act_coverage$uni_str_pos and act_coverage$acro
##
##      nds      nov      lex      gfs      tru      tor      pfs
## nov 1.0000 -          -          -          -          -          -
## lex < 2e-16 < 2e-16 -          -          -          -          -
## gfs < 2e-16 < 2e-16 < 2e-16 -          -          -          -
## tru < 2e-16 < 2e-16 < 2e-16 < 2e-16 -          -          -
## tor < 2e-16 < 2e-16 < 2e-16 < 2e-16 1.0000 -          -
## pfs < 2e-16 < 2e-16 < 2e-16 < 2e-16 0.0095 0.1827 -
## ran < 2e-16 < 2e-16 < 2e-16 < 2e-16 1.7e-12 1.3e-11 5.0e-13
##
## P value adjustment method: bonferroni
```

## 4.4 Satisfactory trait coverage over time

Satisfactory trait coverage in a population over time. Data points on the graph is the average activation gene coverage across 50 replicates every 2000 generations. Shading comes from the best and worse coverage across 50 replicates.

```
lines = over_time_df %>%
  group_by(scheme, gen) %>%
  dplyr::summarise(
    min = min(pop_uni_obj),
    mean = mean(pop_uni_obj),
    max = max(pop_uni_obj)
  )
```

```
## `summarise()` has grouped output by 'scheme'. You can override using the
## `.groups` argument.
```

```

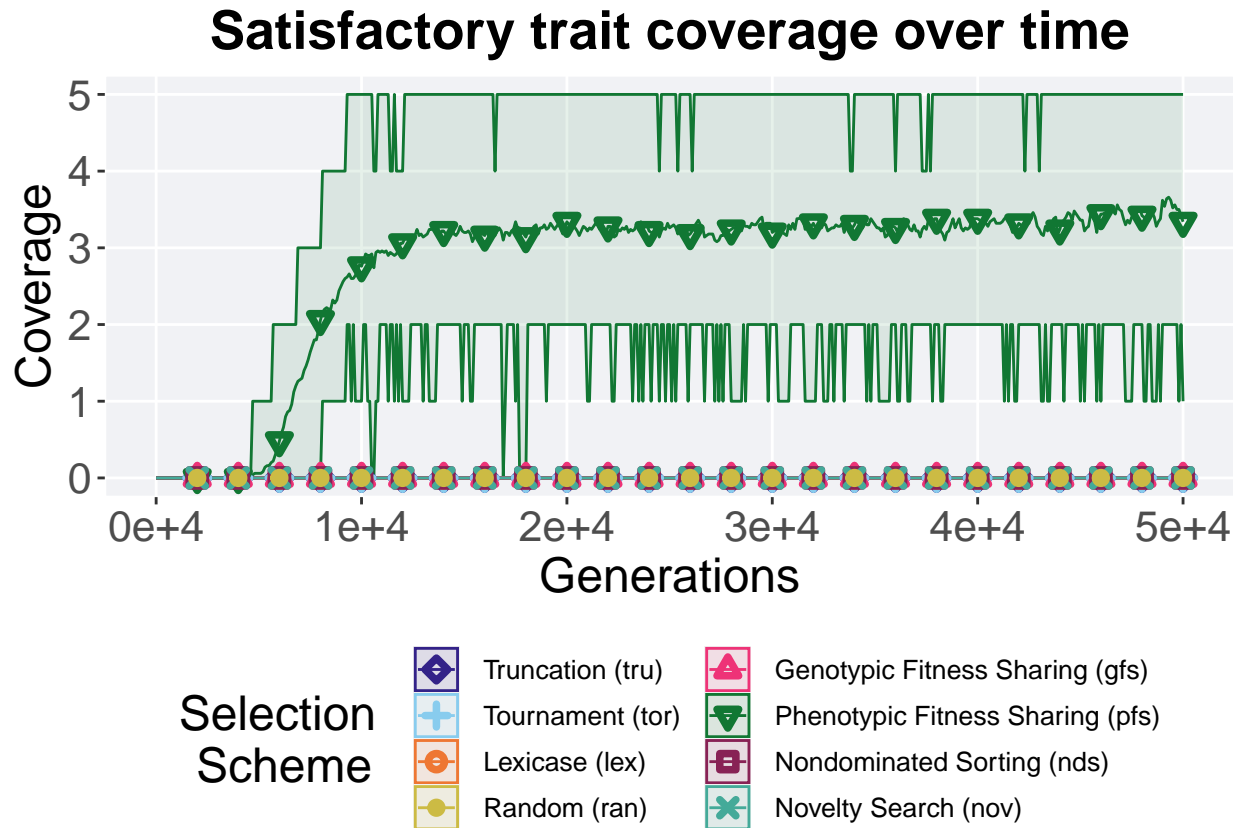
lines$scheme <- factor(lines$scheme, levels = c('Truncation (tru)', 'Tournament (tor)', 'Lexicase (lex)',

over_time_plot = ggplot(lines, aes(x=gen, y=mean, group = scheme, fill = scheme, color = scheme, shape =
  geom_ribbon(aes(ymin = min, ymax = max), alpha = 0.1) +
  geom_line(size = 0.5) +
  geom_point(data = filter(lines, gen %% 2000 == 0 & gen != 0), size = 1.5, stroke = 2.0, alpha = 1.0)
  scale_y_continuous(
    name="Coverage"
  ) +
  scale_x_continuous(
    name="Generations",
    limits=c(0, 50000),
    breaks=c(0, 10000, 20000, 30000, 40000, 50000),
    labels=c("0e+4", "1e+4", "2e+4", "3e+4", "4e+4", "5e+4")

  ) +
  scale_shape_manual(values=c(5,3,1,20,2,6,0,4))+
  scale_colour_manual(values = c('#332288', '#88CCEE', '#EE7733', '#CCBB44', '#EE3377', '#117733', '#882255',
  scale_fill_manual(values = c('#332288', '#88CCEE', '#EE7733', '#CCBB44', '#EE3377', '#117733', '#882255', '#
  ggtitle('Satisfactory trait coverage over time')+
  p_theme +
  guides(
    shape=guide_legend(ncol=2, title.position = "left", title = 'Selection \nScheme'),
    color=guide_legend(ncol=2, title.position = "left", title = 'Selection \nScheme'),
    fill=guide_legend(ncol=2, title.position = "left", title = 'Selection \nScheme')
  )

over_time_plot

```



## 4.5 Final satisfactory trait coverage

Satisfactory trait coverage found in the final population at 50,000 generations.

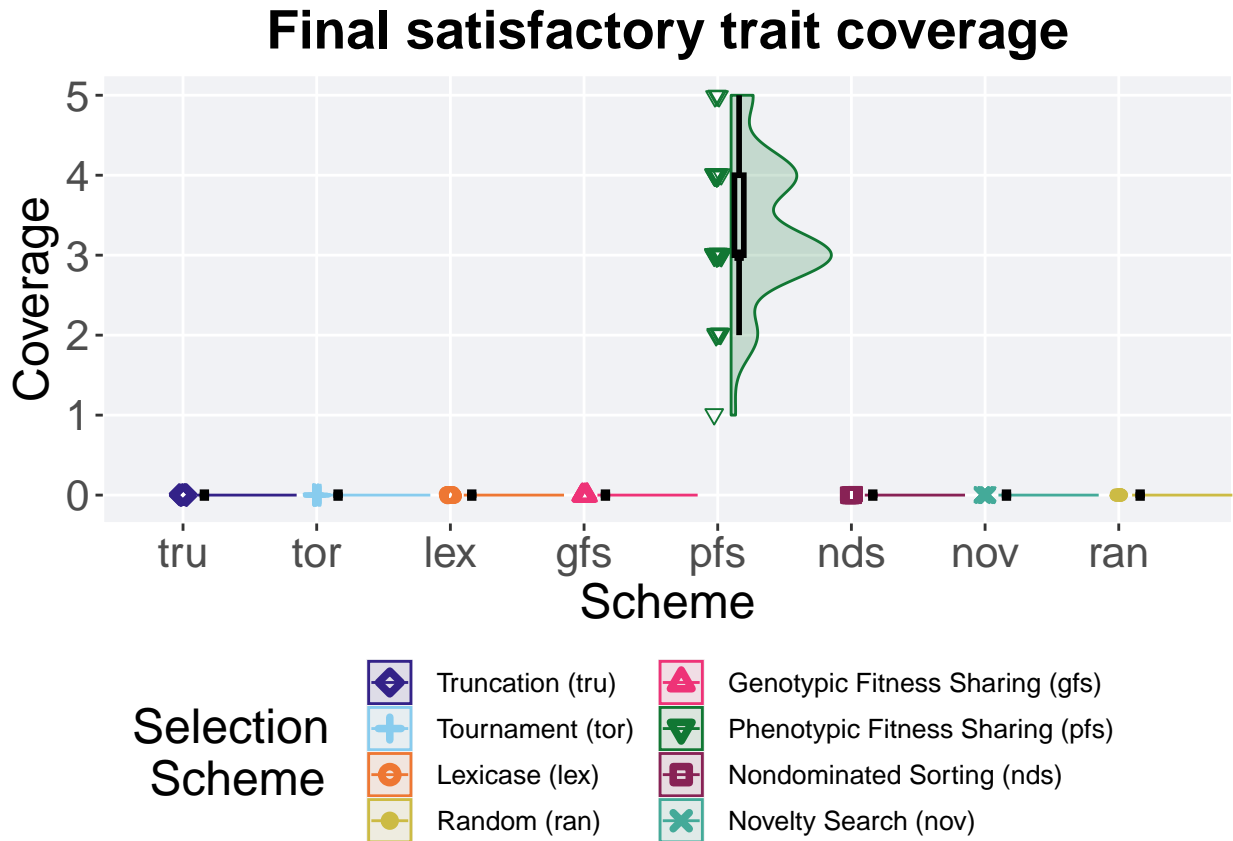
```
plot = filter(over_time_df, gen == 50000) %>%
  ggplot(., aes(x = acro, y = pop_uni_obj, color = acro, fill = acro, shape = acro)) +
  geom_flat_violin(position = position_nudge(x = .1, y = 0), scale = 'width', alpha = 0.2, width = 1.5) +
  geom_boxplot(color = 'black', width = .07, outlier.shape = NA, alpha = 0.0, size = 1.0, position = position_nudge(x = .1, y = 0)) +
  geom_point(position = position_jitter(width = 0.03, height = 0.02), size = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name="Coverage",
    limits=c(-0.1, 5)
  ) +
  scale_x_discrete(
    name="Scheme"
  ) +
  scale_shape_manual(values=SHAPE) +
  scale_colour_manual(values = cb_palette, ) +
  scale_fill_manual(values = cb_palette) +
  ggtitle('Final satisfactory trait coverage') +
  p_theme

plot_grid(
  plot +
  theme(legend.position="none"),
```

```

legend,
nrow=2,
rel_heights = c(3,1)
)

```



#### 4.5.1 Stats

Summary statistics for the coverage found in the final population.

```

act_coverage = filter(over_time_df, gen == 50000)
act_coverage$acro = factor(act_coverage$acro, levels = c('pfs', 'nds', 'lex', 'gfs', 'tor', 'tru', 'nov', 'ran'))
act_coverage %>%
  group_by(acro) %>%
  dplyr::summarise(
    count = n(),
    na_cnt = sum(is.na(pop_uni_obj)),
    min = min(pop_uni_obj, na.rm = TRUE),
    median = median(pop_uni_obj, na.rm = TRUE),
    mean = mean(pop_uni_obj, na.rm = TRUE),
    max = max(pop_uni_obj, na.rm = TRUE),
    IQR = IQR(pop_uni_obj, na.rm = TRUE)
  )

```

```

## # A tibble: 8 x 8
##   acro  count na_cnt  min median  mean  max  IQR
##   <fct> <int>  <int> <int>  <dbl> <dbl> <int> <dbl>
## 1 pfs     50      0     1     3  3.34     5     1

```



```
## 2 nds      50      0      0      0 0      0      0
## 3 lex      50      0      0      0 0      0      0
## 4 gfs      50      0      0      0 0      0      0
## 5 tor      50      0      0      0 0      0      0
## 6 tru      50      0      0      0 0      0      0
## 7 nov      50      0      0      0 0      0      0
## 8 ran      50      0      0      0 0      0      0
```

Kruskal–Wallis test illustrates evidence of statistical differences.

```
kruskal.test(pop_uni_obj ~ acro, data = act_coverage)
```

```
##
## Kruskal-Wallis rank sum test
##
## data: pop_uni_obj by acro
## Kruskal-Wallis chi-squared = 396.94, df = 7, p-value < 2.2e-16
```

Results for post-hoc Wilcoxon rank-sum test with a Bonferroni correction.

```
pairwise.wilcox.test(x = act_coverage$pop_uni_obj, g = act_coverage$acro, p.adjust.method = "bonferroni",
                     paired = FALSE, conf.int = FALSE, alternative = 'l')
```

```
##
## Pairwise comparisons using Wilcoxon rank sum test with continuity correction
##
## data: act_coverage$pop_uni_obj and act_coverage$acro
##
##      pfs      nds lex gfs tor tru nov
## nds <2e-16 -    -    -    -    -
## lex <2e-16 1    -    -    -    -
## gfs <2e-16 1    1    -    -    -
## tor <2e-16 1    1    1    -    -
## tru <2e-16 1    1    1    1    -
## nov <2e-16 1    1    1    1    1
## ran <2e-16 1    1    1    1    1
##
## P value adjustment method: bonferroni
```

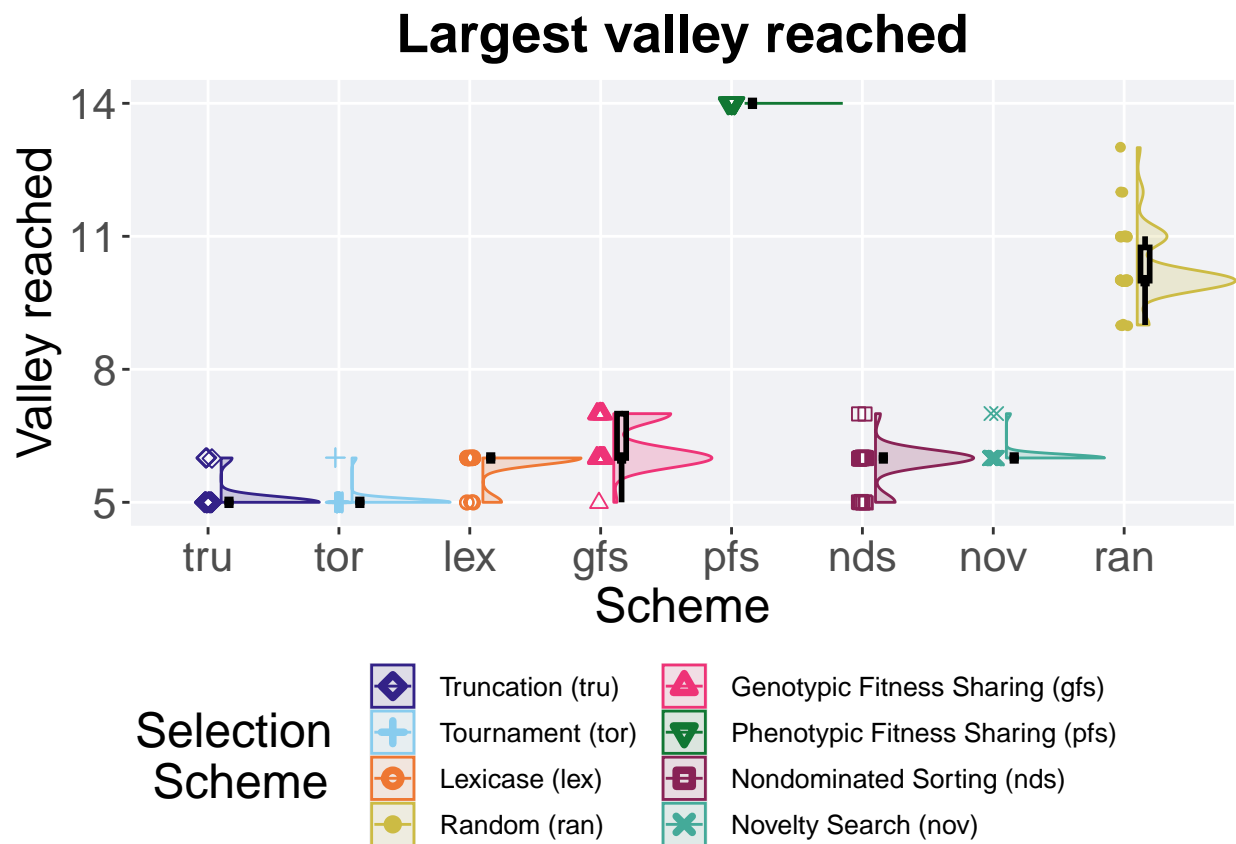
## 4.6 Largest valley reached throughout

The largest valley reached in a single trait throughout an entire evolutionary run. To collect this value, we look through all the best-performing solutions each generation and find the largest valley reached.

```
plot = filter(best_df, var == 'ele_big_peak') %>%
  ggplot(., aes(x = acro, y = val, color = acro, fill = acro, shape = acro)) +
  geom_flat_violin(position = position_nudge(x = .1, y = 0), scale = 'width', alpha = 0.2, width = 1.5) +
  geom_boxplot(color = 'black', width = .07, outlier.shape = NA, alpha = 0.0, size = 1.0, position = position_nudge(x = .1, y = 0)) +
  geom_point(position = position_jitter(width = 0.03, height = 0.02), size = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name = "Valley reached",
    limits = c(4.9, 14.1),
    breaks = c(5, 8, 11, 14)
  ) +
  scale_x_discrete(
    name = "Scheme"
```

```
)+
scale_shape_manual(values=SHAPE)+
scale_colour_manual(values = cb_palette, ) +
scale_fill_manual(values = cb_palette) +
ggtitle('Largest valley reached')+
p_theme

plot_grid(
  plot +
    theme(legend.position="none"),
  legend,
  nrow=2,
  rel_heights = c(3,1)
)
```



#### 4.6.1 Stats

Summary statistics for the largest valley crossed.

```
valleys = filter(best_df, var == 'ele_big_peak')
valleys$acro = factor(valleys$acro, levels = c('pfs', 'ran', 'gfs', 'nov', 'nds', 'lex', 'tru', 'tor'))
valleys %>%
  group_by(acro) %>%
  dplyr::summarise(
    count = n(),
    na_cnt = sum(is.na(val)),
```

```

min = min(val, na.rm = TRUE),
median = median(val, na.rm = TRUE),
mean = mean(val, na.rm = TRUE),
max = max(val, na.rm = TRUE),
IQR = IQR(val, na.rm = TRUE)
)

```

```

## # A tibble: 8 x 8
##   acro count na_cnt   min median  mean   max   IQR
##   <fct> <int> <int> <dbl> <dbl> <dbl> <dbl> <dbl>
## 1 pfs     50     0    14     14  14    14     0
## 2 ran     50     0     9     10 10.3    13  0.75
## 3 gfs     50     0     5      6  6.34     7  1
## 4 nov     50     0     6      6  6.04     7  0
## 5 nds     50     0     5      6  5.88     7  0
## 6 lex     50     0     5      6  5.84     6  0
## 7 tru     50     0     5      5  5.1      6  0
## 8 tor     50     0     5      5  5.04     6  0

```

Kruskal-Wallis test illustrates evidence of statistical differences.

```
kruskal.test(val ~ acro, data = valleys)
```

```

##
## Kruskal-Wallis rank sum test
##
## data:  val by acro
## Kruskal-Wallis chi-squared = 352.03, df = 7, p-value < 2.2e-16

```

Results for post-hoc Wilcoxon rank-sum test with a Bonferroni correction.

```

pairwise.wilcox.test(x = valleys$val, g = valleys$acro, p.adjust.method = "bonferroni",
                     paired = FALSE, conf.int = FALSE, alternative = 'l')

```

```

##
## Pairwise comparisons using Wilcoxon rank sum test with continuity correction
##
## data:  valleys$val and valleys$acro
##
##      pfs      ran      gfs      nov      nds      lex      tru
## ran < 2e-16 -          -          -          -          -          -
## gfs < 2e-16 < 2e-16 -          -          -          -          -
## nov < 2e-16 < 2e-16 0.00347 -          -          -          -
## nds < 2e-16 < 2e-16 0.00018 0.26915 -          -          -
## lex < 2e-16 < 2e-16 1.3e-05 0.01917 1.00000 -          -
## tru < 2e-16 < 2e-16 < 2e-16 < 2e-16 2.8e-12 2.3e-12 -
## tor < 2e-16 < 2e-16 < 2e-16 < 2e-16 2.2e-14 1.6e-14 1.00000
##
## P value adjustment method: bonferroni

```



## Chapter 5

# Multi-path exploration results

Here we present the results for **best performances** found by each selection scheme on the multi-path exploration diagnostic with valley crossing integrated. 50 replicates are conducted for each scheme explored.

### 5.1 Data setup

```
DIR = paste(DATA_DIR, 'MULTIPATH_EXPLORATION/', sep = "", collapse = NULL)
over_time_df <- read.csv(paste(DIR, 'over-time.csv', sep = "", collapse = NULL), header = TRUE, stringsAsFactors = FALSE)
over_time_df$uni_str_pos = over_time_df$uni_str_pos + over_time_df$arc_acti_gene - over_time_df$overlap
over_time_df$scheme <- factor(over_time_df$scheme, levels = NAMES)
over_time_df$acro <- factor(over_time_df$acro, levels = ACRO)

best_df <- read.csv(paste(DIR, 'best.csv', sep = "", collapse = NULL), header = TRUE, stringsAsFactors = FALSE)
best_df$acro <- factor(best_df$acro, levels = ACRO)
```

### 5.2 Activation gene coverage over time

Activation gene coverage in a population over time. Data points on the graph is the average activation gene coverage across 50 replicates every 2000 generations. Shading comes from the best and worse coverage across 50 replicates.

```
lines = over_time_df %>%
  group_by(scheme, gen) %>%
  dplyr::summarise(
    min = min(uni_str_pos),
    mean = mean(uni_str_pos),
    max = max(uni_str_pos)
  )

## `summarise()` has grouped output by 'scheme'. You can override using the
## `.groups` argument.

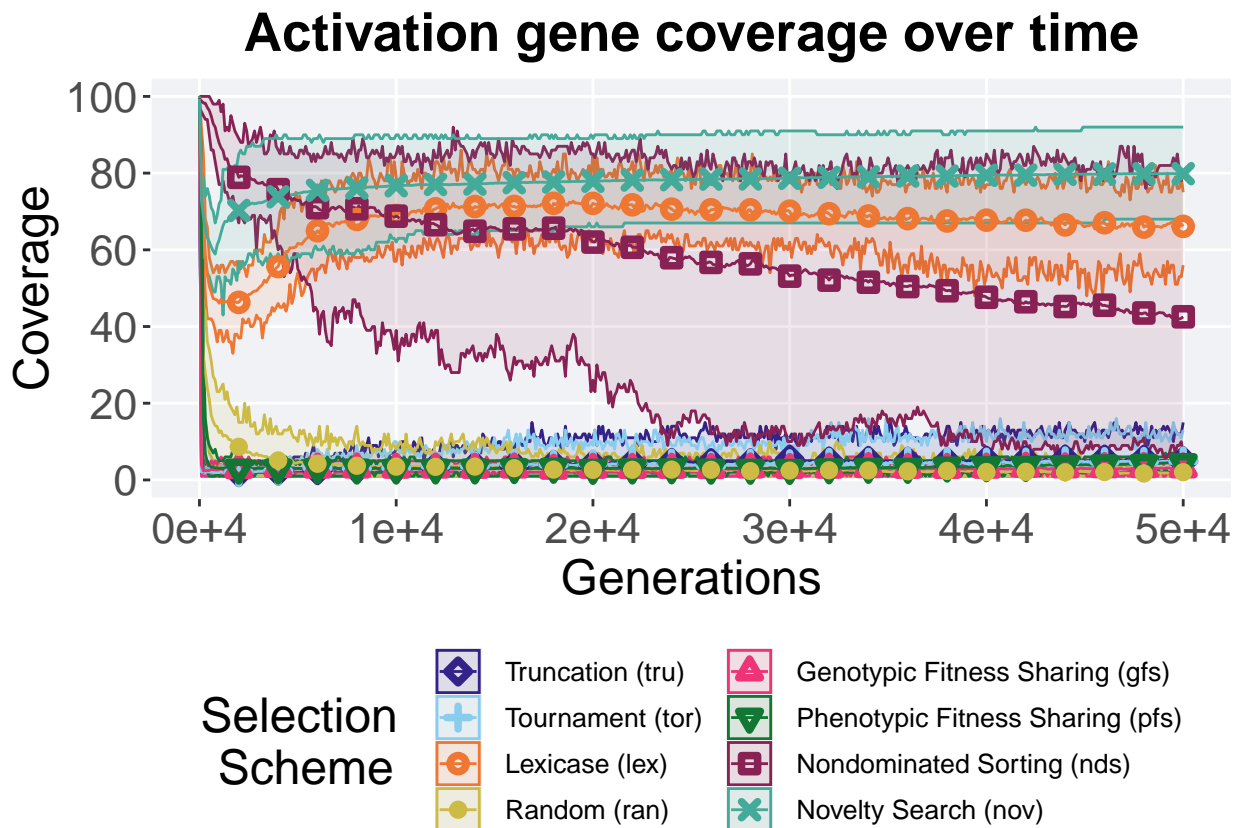
lines$scheme <- factor(lines$scheme, levels = c('Truncation (tru)', 'Tournament (tor)', 'Lexicase (lex)', 'Random (ran)'))

over_time_plot = ggplot(lines, aes(x=gen, y=mean, group = scheme, fill = scheme, color = scheme, shape = scheme)) +
  geom_ribbon(aes(ymin = min, ymax = max), alpha = 0.1) +
  geom_line(size = 0.5) +
  geom_point(data = filter(lines, gen %% 2000 == 0 & gen != 0), size = 1.5, stroke = 2.0, alpha = 1.0)
```

```

scale_y_continuous(
  name="Coverage",
  limits=c(0, 100),
  breaks=seq(0,100, 20),
  labels=c("0", "20", "40", "60", "80", "100")
) +
scale_x_continuous(
  name="Generations",
  limits=c(0, 50000),
  breaks=c(0, 10000, 20000, 30000, 40000, 50000),
  labels=c("0e+4", "1e+4", "2e+4", "3e+4", "4e+4", "5e+4")
) +
scale_shape_manual(values=c(5,3,1,20,2,6,0,4))+
scale_colour_manual(values = c('#332288','#88CCEE','#EE7733','#CCBB44','#EE3377','#117733','#882255','#332288'),
scale_fill_manual(values = c('#332288','#88CCEE','#EE7733','#CCBB44','#EE3377','#117733','#882255','#332288'),
ggtitle('Activation gene coverage over time')+
p_theme +
guides(
  shape=guide_legend(ncol=2, title.position = "left", title = 'Selection \nScheme'),
  color=guide_legend(ncol=2, title.position = "left", title = 'Selection \nScheme'),
  fill=guide_legend(ncol=2, title.position = "left", title = 'Selection \nScheme')
)
over_time_plot

```

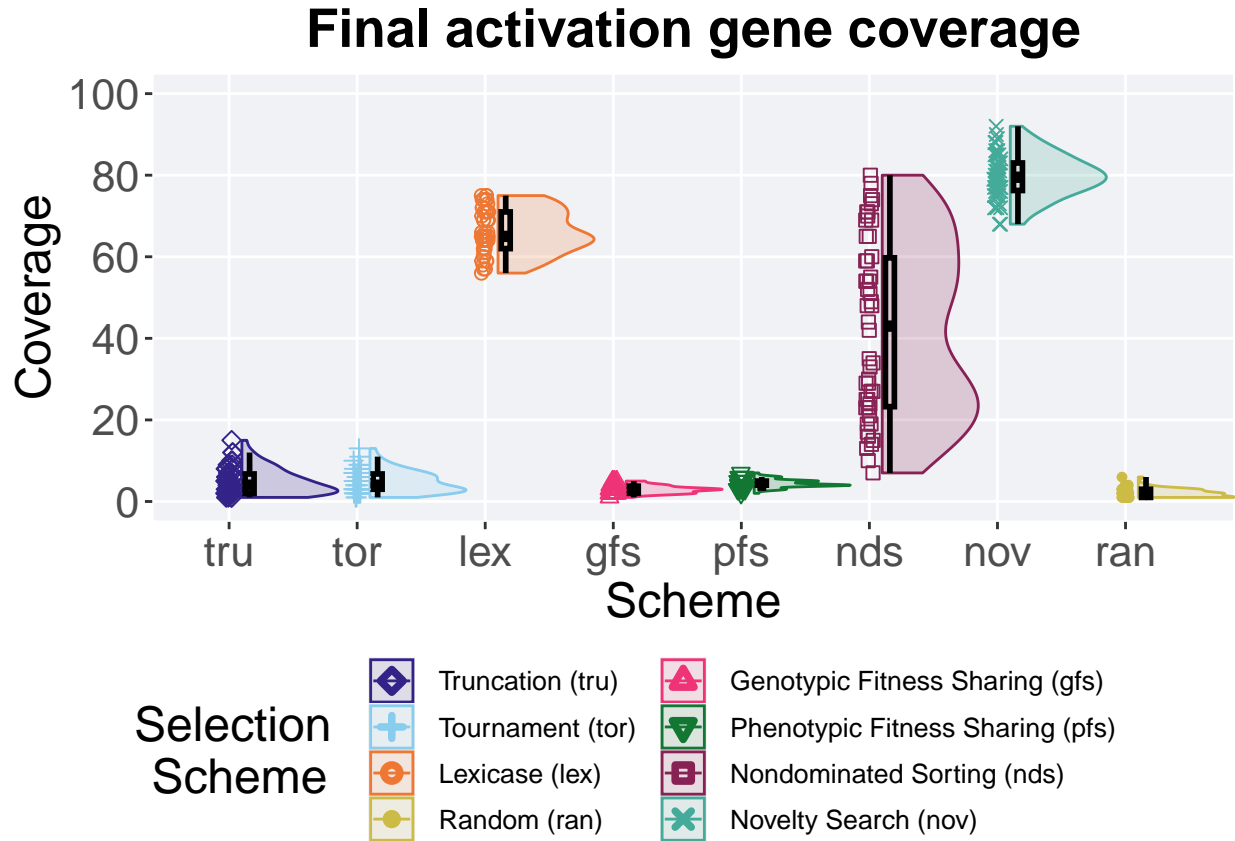


## 5.3 Final activation gene coverage

Activation gene coverage found in the final population at 50,000 generations.

```
plot = filter(over_time_df, gen == 50000) %>%
  ggplot(., aes(x = acro, y = uni_str_pos, color = acro, fill = acro, shape = acro)) +
  geom_flat_violin(position = position_nudge(x = .1, y = 0), scale = 'width', alpha = 0.2, width = 1.5)
  geom_boxplot(color = 'black', width = .07, outlier.shape = NA, alpha = 0.0, size = 1.0, position = position_nudge(x = .1, y = 0))
  geom_point(position = position_jitter(width = 0.03, height = 0.02), size = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name="Coverage",
    limits=c(0, 100),
    breaks=seq(0,100, 20),
    labels=c("0", "20", "40", "60", "80", "100")
  ) +
  scale_x_discrete(
    name="Scheme"
  ) +
  scale_shape_manual(values=SHAPE) +
  scale_colour_manual(values = cb_palette, ) +
  scale_fill_manual(values = cb_palette) +
  ggtitle('Final activation gene coverage') +
  p_theme

plot_grid(
  plot +
    theme(legend.position="none"),
  legend,
  nrow=2,
  rel_heights = c(3,1)
)
```



### 5.3.1 Stats

Summary statistics for the coverage found in the final population.

```
act_coverage = filter(over_time_df, gen == 50000)
act_coverage$acro = factor(act_coverage$acro, levels = c('nov', 'lex', 'nds', 'tor', 'tru', 'pfs', 'gfs', 'ran'))
act_coverage %>%
  group_by(acro) %>%
  dplyr::summarise(
    count = n(),
    na_cnt = sum(is.na(uni_str_pos)),
    min = min(uni_str_pos, na.rm = TRUE),
    median = median(uni_str_pos, na.rm = TRUE),
    mean = mean(uni_str_pos, na.rm = TRUE),
    max = max(uni_str_pos, na.rm = TRUE),
    IQR = IQR(uni_str_pos, na.rm = TRUE)
  )
```

```
## # A tibble: 8 x 8
##   acro  count na_cnt  min median  mean  max  IQR
##   <fct> <int>  <int> <int>  <dbl> <dbl> <int> <dbl>
## 1 nov      50      0    68   79.5  79.9   92  6.75
## 2 lex      50      0    56    65  66.1   75    9
## 3 nds      50      0     7    43  42.5   80 36.5
## 4 tor      50      0     1     4   4.84   13  3.75
## 5 tru      50      0     1     4   4.9    15  4.75
## 6 pfs      50      0     2     4   4.4     7    1
```



```
## 7 gfs      50      0      1      3      3      5 1.75
## 8 ran      50      0      1      2      2.02    6 2
```

Kruskal–Wallis test illustrates evidence of statistical differences.

```
kruskal.test(uni_str_pos ~ acro, data = act_coverage)
```

```
##
## Kruskal-Wallis rank sum test
##
## data: uni_str_pos by acro
## Kruskal-Wallis chi-squared = 324.89, df = 7, p-value < 2.2e-16
```

Results for post-hoc Wilcoxon rank-sum test with a Bonferroni correction.

```
pairwise.wilcox.test(x = act_coverage$uni_str_pos, g = act_coverage$acro, p.adjust.method = "bonferroni",
                     paired = FALSE, conf.int = FALSE, alternative = 'l')
```

```
##
## Pairwise comparisons using Wilcoxon rank sum test with continuity correction
##
## data: act_coverage$uni_str_pos and act_coverage$acro
##
##      nov      lex      nds      tor      tru      pfs      gfs
## lex 1.4e-14 -          -          -          -          -          -
## nds 8.1e-15 1.5e-06 -          -          -          -          -
## tor < 2e-16 < 2e-16 < 2e-16 -          -          -          -
## tru < 2e-16 < 2e-16 < 2e-16 2.7e-16 1.000    -          -
## pfs < 2e-16 < 2e-16 < 2e-16 1.000    1.000    -          -
## gfs < 2e-16 < 2e-16 < 2e-16 0.011    0.157    3.8e-08 -
## ran < 2e-16 < 2e-16 < 2e-16 3.7e-08 2.7e-06 1.3e-13 4.0e-05
##
## P value adjustment method: bonferroni
```

## 5.4 Performance over time

Best performance in a population over time. Data points on the graph is the average performance across 50 replicates every 2000 generations. Shading comes from the best and worse performance across 50 replicates.

```
lines = over_time_df %>%
  group_by(scheme, gen) %>%
  dplyr::summarise(
    min = min(pop_fit_max) / DIMENSIONALITY,
    mean = mean(pop_fit_max) / DIMENSIONALITY,
    max = max(pop_fit_max) / DIMENSIONALITY
  )

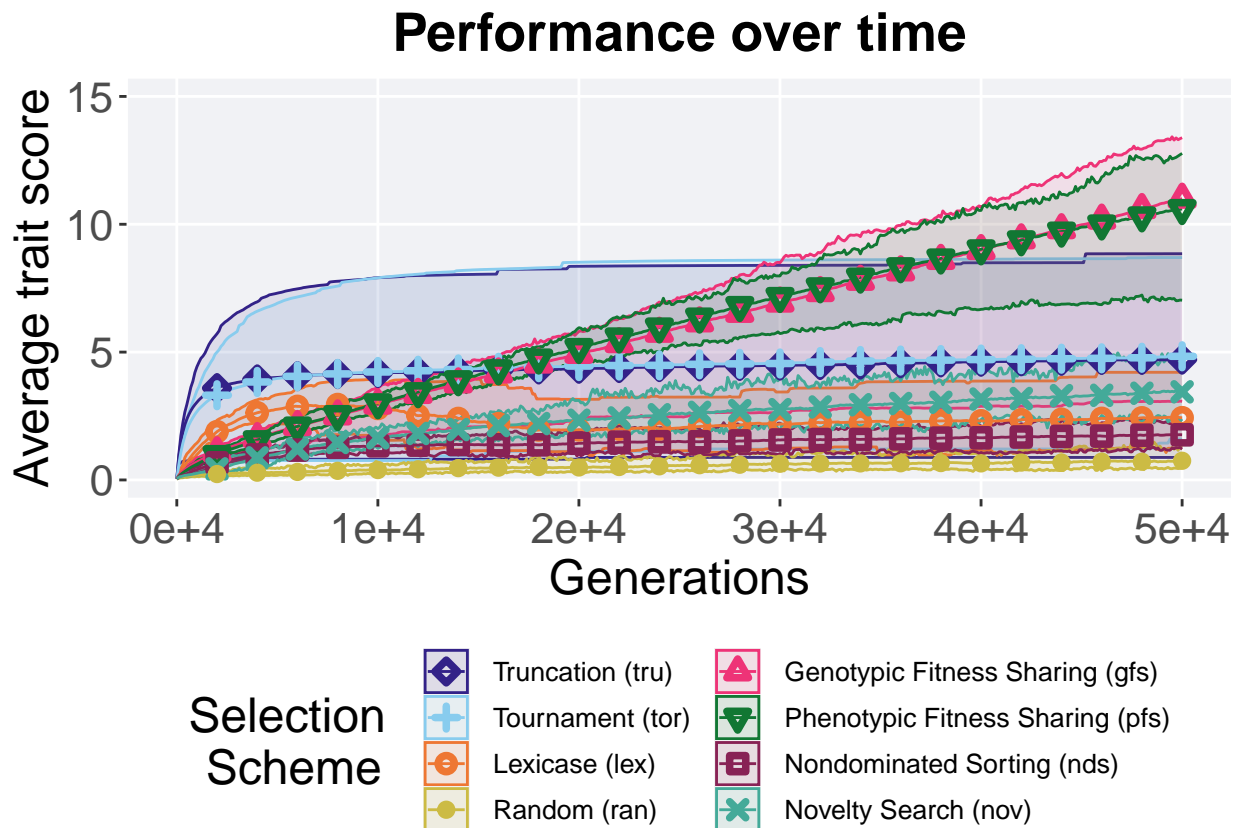
## `summarise()` has grouped output by 'scheme'. You can override using the
## `.groups` argument.

lines$scheme <- factor(lines$scheme, levels = c('Truncation (tru)', 'Tournament (tor)', 'Lexicase (lex)',
over_time_plot = ggplot(lines, aes(x=gen, y=mean, group = scheme, fill = scheme, color = scheme, shape =
  geom_ribbon(aes(ymin = min, ymax = max), alpha = 0.1) +
  geom_line(size = 0.5) +
  geom_point(data = filter(lines, gen % 2000 == 0 & gen != 0), size = 1.5, stroke = 2.0, alpha = 1.0) +
  scale_y_continuous(
```

```

name="Average trait score",
limits=c(0, 15),
breaks=c(0,5,10,15)
) +
scale_x_continuous(
  name="Generations",
  limits=c(0, 50000),
  breaks=c(0, 10000, 20000, 30000, 40000, 50000),
  labels=c("0e+4", "1e+4", "2e+4", "3e+4", "4e+4", "5e+4")
) +
scale_shape_manual(values=c(5,3,1,20,2,6,0,4))+
scale_colour_manual(values = c('#332288','#88CCEE','#EE7733','#CCBB44','#EE3377','#117733','#882255','#332288','#88CCEE','#EE7733','#CCBB44','#EE3377','#117733','#882255','#332288','#88CCEE','#EE7733','#CCBB44','#EE3377','#117733','#882255'))
scale_fill_manual(values = c('#332288','#88CCEE','#EE7733','#CCBB44','#EE3377','#117733','#882255','#332288','#88CCEE','#EE7733','#CCBB44','#EE3377','#117733','#882255','#332288','#88CCEE','#EE7733','#CCBB44','#EE3377','#117733','#882255'))
ggtitle('Performance over time')+
p_theme +
guides(
  shape=guide_legend(ncol=2, title.position = "left", title = 'Selection \nScheme'),
  color=guide_legend(ncol=2, title.position = "left", title = 'Selection \nScheme'),
  fill=guide_legend(ncol=2, title.position = "left", title = 'Selection \nScheme')
)
over_time_plot

```

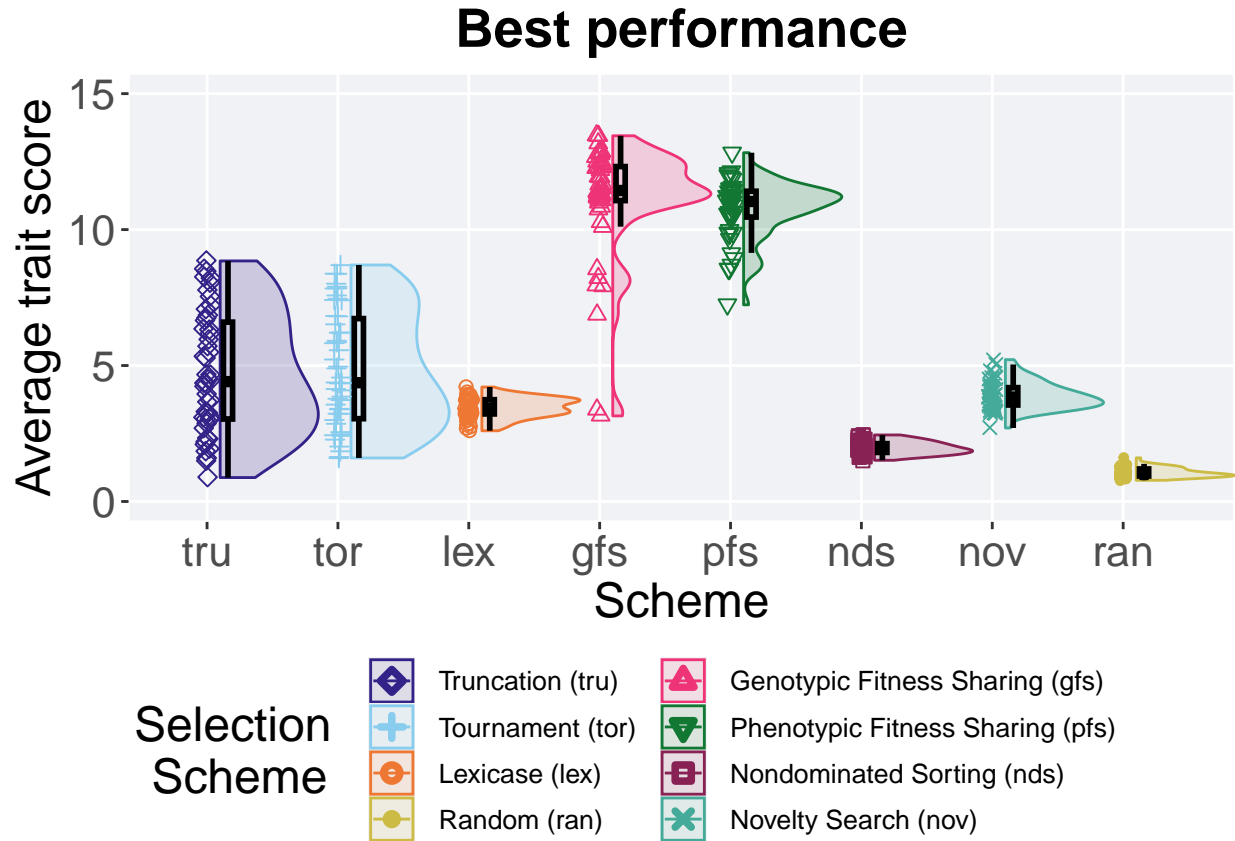


## 5.5 Best performance throughout

Best performance reached throughout 50,000 generations in a population.

```
plot = filter(best_df, var == 'pop_fit_max') %>%
  ggplot(., aes(x = acro, y = val / DIMENSIONALITY, color = acro, fill = acro, shape = acro)) +
  geom_flat_violin(position = position_nudge(x = .1, y = 0), scale = 'width', alpha = 0.2, width = 1.5) +
  geom_boxplot(color = 'black', width = .07, outlier.shape = NA, alpha = 0.0, size = 1.0, position = position_nudge(x = .1, y = 0)) +
  geom_point(position = position_jitter(width = 0.03, height = 0.02), size = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name="Average trait score",
    limits=c(0, 15),
    breaks=c(0,5,10,15)
  ) +
  scale_x_discrete(
    name="Scheme"
  ) +
  scale_shape_manual(values=SHAPE) +
  scale_colour_manual(values = cb_palette, ) +
  scale_fill_manual(values = cb_palette) +
  ggtitle('Best performance') +
  p_theme

plot_grid(
  plot +
    theme(legend.position="none"),
  legend,
  nrow=2,
  rel_heights = c(3,1)
)
```



### 5.5.1 Stats

Summary statistics for the best performance.

```
performance = filter(best_df, var == 'pop_fit_max')
performance$acro = factor(performance$acro, levels = c('gfs', 'pfs', 'tru', 'tor', 'nov', 'lex', 'nds', 'ran'))
performance %>%
  group_by(acro) %>%
  dplyr::summarise(
    count = n(),
    na_cnt = sum(is.na(val)),
    min = min(val / DIMENSIONALITY, na.rm = TRUE),
    median = median(val / DIMENSIONALITY, na.rm = TRUE),
    mean = mean(val / DIMENSIONALITY, na.rm = TRUE),
    max = max(val / DIMENSIONALITY, na.rm = TRUE),
    IQR = IQR(val / DIMENSIONALITY, na.rm = TRUE)
  )
```

```
## # A tibble: 8 x 8
##   acro count na_cnt min median mean max IQR
##   <fct> <int> <int> <dbl> <dbl> <dbl> <dbl> <dbl>
## 1 gfs     50     0 3.15  11.4  11.1  13.4  1.25
## 2 pfs     50     0 7.23  11.0  10.8  12.8  0.949
## 3 tru     50     0 0.880  4.41  4.71  8.85  3.56
## 4 tor     50     0 1.60  4.37  4.84  8.70  3.68
## 5 nov     50     0 2.70  3.84  3.89  5.22  0.639
## 6 lex     50     0 2.60  3.44  3.45  4.21  0.523
```

```
## 7 nds      50      0 1.52   1.93   1.97   2.45 0.322
## 8 ran      50      0 0.780  0.998  1.06   1.60 0.261
```

Kruskal–Wallis test illustrates evidence of statistical differences.

```
kruskal.test(val ~ acro, data = performance)
```

```
##
## Kruskal-Wallis rank sum test
##
## data:  val by acro
## Kruskal-Wallis chi-squared = 327.4, df = 7, p-value < 2.2e-16
```

Results for post-hoc Wilcoxon rank-sum test with a Bonferroni correction.

```
pairwise.wilcox.test(x = performance$val, g = performance$acro, p.adjust.method = "bonferroni",
                     paired = FALSE, conf.int = FALSE, alternative = 'l')
```

```
##
## Pairwise comparisons using Wilcoxon rank sum test with continuity correction
##
## data:  performance$val and performance$acro
##
##      gfs      pfs      tru      tor      nov      lex      nds
## pfs 0.03925 -      -      -      -      -      -      -
## tru 2.2e-14 < 2e-16 -      -      -      -      -      -
## tor 2.4e-14 < 2e-16 1.00000 -      -      -      -      -
## nov 2.1e-14 < 2e-16 1.00000 1.00000 -      -      -      -
## lex 5.3e-15 < 2e-16 0.23671 0.04294 0.00042 -      -      -
## nds < 2e-16 < 2e-16 5.5e-10 8.2e-13 < 2e-16 < 2e-16 < 2e-16 -
## ran < 2e-16 < 2e-16 1.4e-15 < 2e-16 < 2e-16 < 2e-16 < 2e-16 -
##
## P value adjustment method: bonferroni
```

## 5.6 Largest valley reached throughout

The largest valley reached in a single trait throughout an entire evolutionary run. To collect this value, we look through all the best-performing solutions each generation and find the largest valley reached.

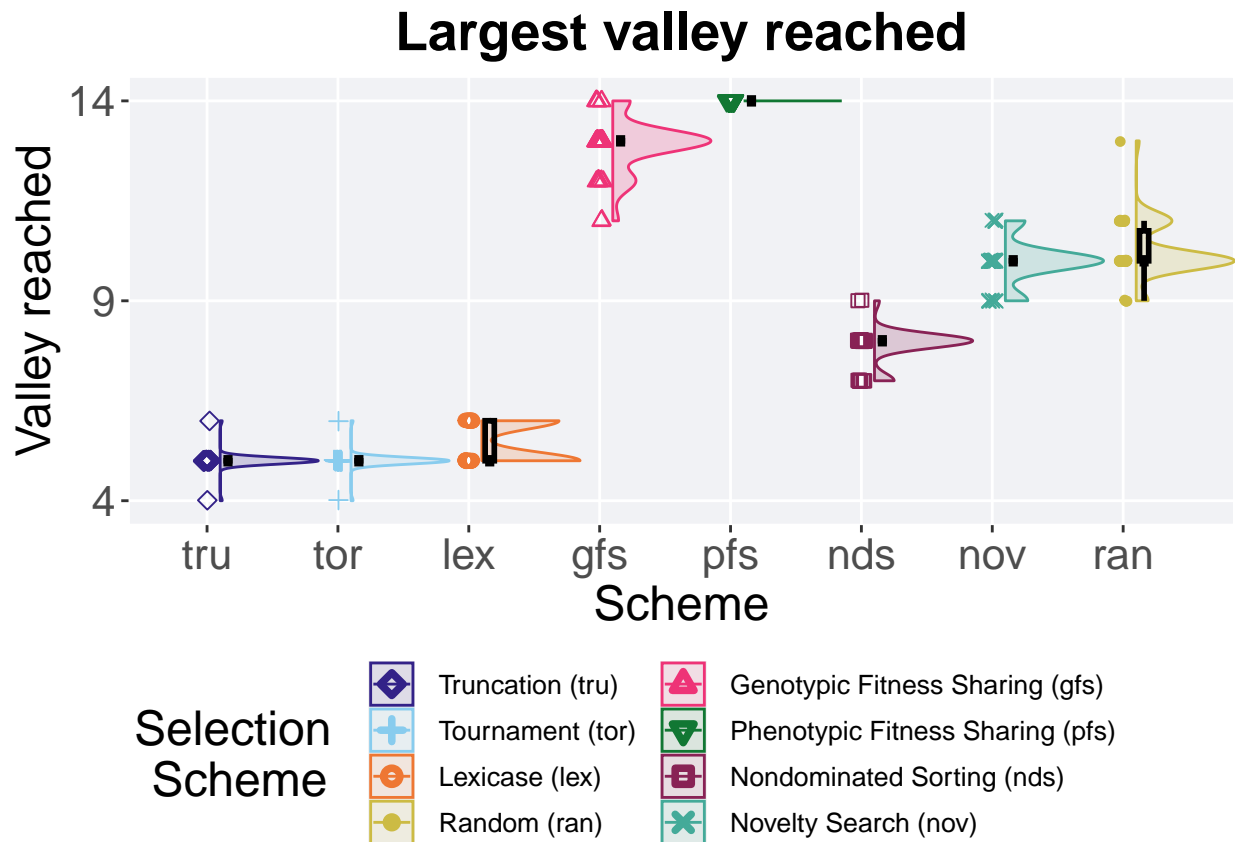
```
plot = filter(best_df, var == 'ele_big_peak') %>%
  ggplot(., aes(x = acro, y = val, color = acro, fill = acro, shape = acro)) +
  geom_flat_violin(position = position_nudge(x = .1, y = 0), scale = 'width', alpha = 0.2, width = 1.5) +
  geom_boxplot(color = 'black', width = .07, outlier.shape = NA, alpha = 0.0, size = 1.0, position = position_nudge(x = .1, y = 0)) +
  geom_point(position = position_jitter(width = 0.03, height = 0.02), size = 2.0, alpha = 1.0) +
  scale_y_continuous(
    name="Valley reached",
    limits=c(3.9,14.1),
    breaks=c(4,9,14)
  ) +
  scale_x_discrete(
    name="Scheme"
  ) +
  scale_shape_manual(values=SHAPE) +
  scale_colour_manual(values = cb_palette, ) +
  scale_fill_manual(values = cb_palette) +
  ggtitle('Largest valley reached') +
```

```

p_theme

plot_grid(
  plot +
    theme(legend.position="none"),
  legend,
  nrow=2,
  rel_heights = c(3,1)
)

```



#### 5.6.1 Stats

Summary statistics for the largest valley crossed.

```

valleys = filter(best_df, var == 'ele_big_peak')
valleys$acro = factor(valleys$acro, levels = c('pfs', 'gfs', 'ran', 'nov', 'nds', 'lex', 'tru', 'tor'))
valleys %>%
  group_by(acro) %>%
  dplyr::summarise(
    count = n(),
    na_cnt = sum(is.na(val)),
    min = min(val, na.rm = TRUE),
    median = median(val, na.rm = TRUE),
    mean = mean(val, na.rm = TRUE),
    max = max(val, na.rm = TRUE),
    IQR = IQR(val, na.rm = TRUE)
  )

```

```
)

## # A tibble: 8 x 8
##   acro count na_cnt   min median mean   max   IQR
##   <fct> <int>  <int> <dbl>  <dbl> <dbl> <dbl> <dbl>
## 1 pfs     50     0    14    14  14    14    0
## 2 gfs     50     0    11    13 12.9    14    0
## 3 ran     50     0     9    10 10.2    13  0.75
## 4 nov     50     0     9    10  9.98    11    0
## 5 nds     50     0     7     8  7.88     9    0
## 6 lex     50     0     5     5  5.44     6    1
## 7 tru     50     0     4     5   5       6    0
## 8 tor     50     0     4     5   5       6    0
```

Kruskal-Wallis test illustrates evidence of statistical differences.

```
kruskal.test(val ~ acro, data = valleys)
```

```
##
##  Kruskal-Wallis rank sum test
##
## data:  val by acro
## Kruskal-Wallis chi-squared = 385.68, df = 7, p-value < 2.2e-16
```

Results for post-hoc Wilcoxon rank-sum test with a Bonferroni correction.

```
pairwise.wilcox.test(x = valleys$val, g = valleys$acro, p.adjust.method = "bonferroni",
                     paired = FALSE, conf.int = FALSE, alternative = 'l')
```

```
##
##  Pairwise comparisons using Wilcoxon rank sum test with continuity correction
##
## data:  valleys$val and valleys$acro
##
##      pfs      gfs      ran      nov      nds      lex      tru
## gfs < 2e-16 -          -          -          -          -          -
## ran < 2e-16 < 2e-16 -          -          -          -          -
## nov < 2e-16 < 2e-16 1          -          -          -          -
## nds < 2e-16 < 2e-16 < 2e-16 < 2e-16 -          -          -
## lex < 2e-16 < 2e-16 < 2e-16 < 2e-16 < 2e-16 -          -
## tru < 2e-16 < 2e-16 < 2e-16 < 2e-16 < 2e-16 7.6e-06 -
## tor < 2e-16 < 2e-16 < 2e-16 < 2e-16 < 2e-16 7.6e-06 1
##
## P value adjustment method: bonferroni
```