Recitation 05

2.6 #23 $\lim_{\theta\to 0} \frac{\sec\theta - 1}{\theta} = \lim_{\theta\to 0} \frac{\frac{1}{\cos\theta} - 1}{\theta} =$

$\lim_{\theta\to 0} \frac{\left(\frac{1-\cos\theta}{\cos\theta}\right)}{\theta} = \lim_{\theta\to 0} \frac{1-\cos\theta}{\cos\theta} \cdot \frac{1}{\theta}$

$\lim_{\theta\to 0} \frac{1-\cos\theta}{\theta\cos\theta} = \lim_{\theta\to 0} \frac{1-\cos\theta}{\theta} \cdot \frac{1}{\cos\theta} =$

$\lim_{\theta\to 0} \left(\frac{1-\cos\theta}{\theta}\right) \cdot \lim_{\theta\to 0}\left(\frac{1}{\cos\theta}\right) = 0 \cdot 1 = \boxed{0}$

2.6 #27

$L = \lim_{x\to 0} \frac{\sin 14x}{x}$ let $\theta = 14x \to x = \theta/14$

Notice $\theta = 14x \to 0$ as $x \to 0$

So $L = \lim_{\theta\to 0} \frac{\sin\theta}{\theta/14} = \lim_{\theta\to 0} 14 \frac{\sin\theta}{\theta} = 14 \cdot 1 = \boxed{14}$

2.6 #9

$\lim_{x\to 1} (x-1) \sin \frac{\pi}{x-1}$; use squeeze theorum

$-1 \le ? \le (x-1) \sin\frac{\pi}{x-1} \le ? \le 1$

$\boxed{-|x| \le x \le |x|}$

$\left|(x-1) \sin \frac{\pi}{x-1}\right| \le |x-1| \left|\sin\frac{\pi}{x-1}\right|$

$-|x-1| \le (x-1) \sin\frac{\pi}{x-1} \le |x-1| \cdot 1$

$-|x-1| \le (x-1) \sin\frac{\pi}{x-1} \le |x-1|$

So $\lim_{x\to 1} (x-1) \sin\frac{\pi}{x-1} = 0$ by squeeze thm.

2.6#7 $\lim_{x\to 0} x^2 \cos\frac{1}{x}$

$-1 \le \cos\frac{1}{x} \le 1 \Rightarrow \underbrace{-x^2}_{\to 0} \le x^2\cos\frac{1}{x} \le \underbrace{x^2}_{\to 0}$

So $\lim_{x\to 0} x^2\cos\frac{1}{x} = 0$ by squeeze thm.

2.6#11

$\lim_{t\to 0} (2^t-1)\cos\frac{1}{t}$

$|(2^t-1)\cos\frac{1}{t}| \le |2^t-1|$

$-|2^t-1| \le (2^t-1)\cos\frac{1}{t} \le |2^t-1|$

$\lim_{t\to 0} -|2^t-1| = \lim_{t\to 0}|2^t+1| = 0$, so $\lim_{t\to 0}(2^t-1)\cos\frac{1}{t} = 0$

by squeeze thm

2.6#12 $\lim_{x\to 0^+} \sqrt{x}\, e^{\cos(\pi/x)} =$

$-1 \le \underbrace{\cos\frac{\pi}{x} \le 1 \Rightarrow}$ $e^{-1} \le e^{\cos(\pi/x)} \le e^1 \Rightarrow$

true because e^x is an increasing function

$\sqrt{x}e^{-1} \le \sqrt{x}\cos^{(\pi/x)} \le \sqrt{x}\,e$

$\lim_{x\to 0^+} e^{-1}\sqrt{x} = e^{-1}\lim_{x\to 0^+}\sqrt{x} = \frac{0}{e} = \boxed{0}$

$\lim_{x\to 0^+} e\sqrt{x} = e\lim_{x\to 0^+}\sqrt{x} = 0\cdot e = \boxed{0}$

$\lim_{x\to 0^+} e^{-1}\sqrt{x} = \lim_{x\to 0^+} e\sqrt{x} = 0$ so $\lim_{x\to 0^+}\sqrt{x}e^{\cos(\pi/x)} = 0$ by

squeeze thm.

2.6 #39 $\lim_{\theta \to 0} \frac{\sin(-3\theta)}{(4\theta)} = \lim_{\theta \to 0} \frac{\sin(-3\theta)}{-3\theta} \cdot \frac{4\theta}{\sin(4\theta)} \cdot \frac{-3\theta}{4\theta} =$

$-\frac{3}{4}\left(\lim_{\theta \to 0} \frac{\sin(-3\theta)}{-3\theta} \cdot \frac{4\theta}{\sin(4\theta)}\right) = -\frac{3}{4} \cdot 1 \cdot 1 = \boxed{-\frac{3}{4}}$

2.6 #47 $\lim_{\theta \to 0} \frac{\cos 2\theta - \cos\theta}{\theta} = \lim_{\theta \to 0} \frac{\cos^2\theta - \sin^2\theta - \cos\theta}{\theta} =$

$\lim_{\theta \to 0} \frac{\cos\theta(\cos\theta - 1) - \sin^2\theta}{\theta} = \theta$