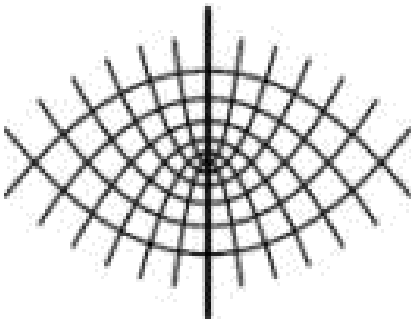
Joe Gibson
Vector Calculus
Weekly Problem 1
Coordinate Systems

Parabolic Coordinate System



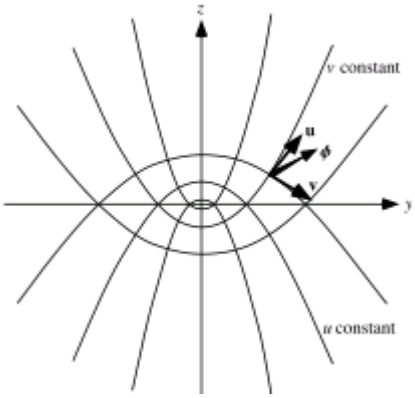
The parabolic coordinate system is a two dimension coordinate system where the two axes are defined by intersecting parabolas. For each parabola the focal point is located at what would be the origin in the Cartesian coordinate system. Within this coordinate system, sigma and tau define any point, where x and y in the Cartesian coordinate system would be defined:

$$x = \sigma\,\tau$$
$$y = \frac{1}{2}(\tau^2 + \sigma^2)$$

Sigma and tau represent along which parabolic curve the point rests, with sigma being the curves opening upwards and tau representing curves that open downwards.

Another annotation system uses u, v, and z, where:

$$x = uv\cos(\theta)$$
$$y = uv\sin(\theta)$$
$$z = \frac{1}{2}(u^2 - v^2)$$



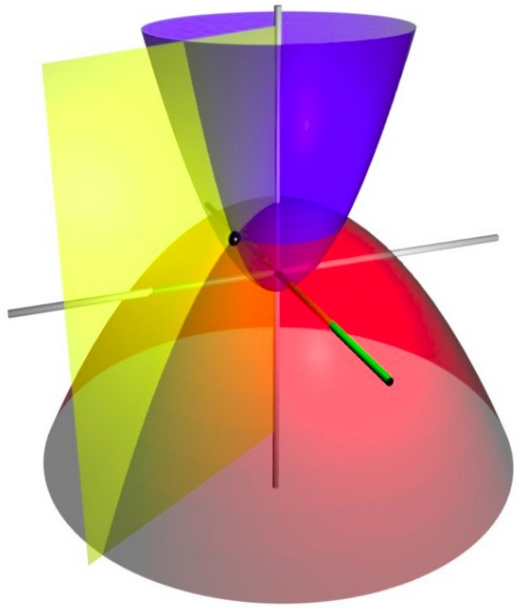
x is the x axis, y is the y axis, and z is the x axis after the point has been determined and the graph has been rotated. The difference in adding the z axis points at the major difference between the two systems, which is that in the first system, only points in the upper half of the grid can be defined because y is always positive with real points. Annotating the points on the parabolic coordinate system in this method is essentially a mapping of the three dimensional cylindrical parabolic coordinate system unto a two dimensional plane. Thinking about the grid this way would mean that the x axis is running perpendicular from this page, intersecting at the origin.

This coordinate system has specific uses, and is broadly used for anything that exhibits parabolic nature. For instance, the Stark effect, which is the change of spectral lines of molecules and atoms due to an external static electric field, exhibits a parabolic nature when the second-order effect is analyzed.

Cylindrical Parabolic Coordinate System

The cylindrical parabolic coordinate system is similar to the two dimensional version, where the grid system is rotated around the z axis in the previous figure, and the x axis is perpendicular to the page at the origin. The definition of points is the same as the second method of describing the parabolic coordinate system.

Uses for this system include potential theory, which is the study of harmonic functions. Many phenomena in nature are modeled using parabolic functions, such as the fundamental forces, which makes them easy to model in the cylindrical parabolic coordinate system.



Information and graphics taken from Wikipedia and Wolfram Alpha