Credibility of Secretly Colluding Manufactures in Retail Contracting*

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Abstract

With two exclusive manufacturer-retailer pairs and private contracting, we show that potential strategic misinterpretations and misunderstandings by retailers are important for the feasibility of manufacturer collusion in vertically related markets. We model the retailers' (potentially incorrect) expectations about their competitors' wholesale price offers. If retailers believe collusion to be infeasible or do not foresee manufacturers' punishment strategies, it is impossible for manufacturers to collude. By contrast, if retailers anticipate the collusive strategy and condition their action on past offers, collusion becomes feasible. We introduce the property of opportunism-proofness that excludes profitable joint deviations by the collusive entity and discuss *adaptive* beliefs.

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1 Introduction

Manufacturers who secretly start an illegal cartel must increase their wholesale prices for initially often unsuspecting downstream retailers. These retailers may, however, refuse to accept higher prices for the fear of being outcompeted by fellow downstream firms that (continue to) receive better offers. There is evidence from cartel cases in which manufacturers have failed to achieve a retail price increase without explicit communication to their retailers. For example, in the German coffee cartel, the coffee roasters were only able to sustain higher wholesale prices after coordinating a retail price increase with their retailers (Holler and Rickert, 2022). Similarly, manufacturers coordinated prices with retailers in cartels involving Anheuser Busch (beer), Haribo (gummi bears), Ritter (chocolate), and Melitta (coffee). The same issue of convincing retailers to accept higher wholesale prices appears to be the underlying problem in a number of so-called hub-and-spoke cartels.[1] Although this obstacle to collusion has been widely discussed and documented in practice, it is hitherto unmodelled.

Conceptually, we argue that with strategic uncertainty of retailers, in the sense that, if retailers are unaware or uncertain about manufacturer collusion within a secret-contracting setting, the simple Nash-equilibrium logic is inadequate to study the initial formation of manufacturer collusion.[2] According to this logic, retailers know the manufacturers' strategies even off the equilibrium path, which is why retailers can perfectly predict whether manufacturers collude. We suggest that potential strategic misinterpretations and misunderstandings are important for the feasibility of collusion in vertically related markets.

In collusion models, the number of equilibria is usually infinite, creating scope for coordination problems. It is intuitive in the presence of a competitive market equilibrium that manufacturers attempt to replace with a collusive outcome. To formalize this, we express retailers' incorrect expectations by allowing for potentially incorrect beliefs off the equilibrium path. In contrast to a Nash equilibrium in which downstream retailers anticipate the specific collusive strategy, retailers may incorrectly believe that collusion is unlikely or even infeasible. This is important not only when starting to collude, but also for the problem of opportunism that even a monopolist faces. For instance, if retailers are convinced of collusion, manufacturers may also exploit such beliefs. The equilibrium concept of weak Perfect Bayesian Equilibrium provides sufficient freedom off the equilibrium path to model beliefs that are not only shaped by the strategies, but also by exogenous events, such as the experience of a firm in a different market. To capture the problem of the initial formation of collusion we characterize the collusive equilibrium and ask under which conditions it can be reached.

Formally, we analyze an infinitely repeated pricing game that features private contracting within each of two exclusive manufacturer-retailer pairs. We focus on manufacturer collusion

[1] See, for example, Harrington (2018) for a description of the cheese cartel in the UK.

[2] Strategic uncertainty and collusion have been analyzed by Blume and Heidhues (2008); however, they consider strategic uncertainty among cartel members only. Instead, to capture the many settings described below in which retailers did not suspect the cartel's existence, we focus on strategic mistakes or uncertainty by retailers.

and retailers who maximize short-term profits. The equilibrium concept of (weak) perfect Bayesian equilibria requires that retailers' beliefs are correct on the equilibrium path. Following a deviation, however, the market outcome depends on the retailers' responses. Because these responses are shaped by the retailers' beliefs, it is useful to contemplate which beliefs retailers hold and act upon. We study how retailers' beliefs may react dynamically to the actions observed in previous periods. In our model, *within-period* passive beliefs capture that retailers view a deviation by one manufacturer as independent from the other manufacturer's behavior. Passive beliefs are also prominently considered in the literature on private contracts in one-shot games (see, for example, Segal 1999; Rey and Vergé 2004).

In our analysis, we first consider beliefs that do not change in past behavior. For these beliefs, we find that manufacturers employing efficient non-linear tariffs and grim-trigger strategies are incapable of sustaining *any* price above the competitive price. Hence, such non-reactive beliefs are self-fulfilling, rendering self-sustaining collusion infeasible. Two forces are driving this result. First, a colluding manufacturer cannot commit that the other manufacturer will offer the same contract, such that its retailer may fear that the rival receives a better offer. Therefore, in contrast to a monopolist, it is not the lack of own commitment that drives the result, but the inability to credibly commit in the name of the other manufacturer. Second in line with the above cited evidence from various collusion cases, the retailer lacks the (equilibrium) knowledge that collusion occurs. This effect is novel and practically important, but conceptually difficult to capture.

We then turn to beliefs that adapt to observed past behavior of the manufacturers. First, we consider beliefs that (correctly) anticipate the manufacturers' trigger strategies. The retailers thus implicitly understand not only that manufacturers collude, but also how they collude. For such beliefs that capture perfect anticipation of the collusive behavior, collusion at industry-profit-maximizing prices is feasible and may be stable with respect to unilateral deviations, contrasting the result with non-reactive beliefs. Note that there are indeed cases in which upstream firms were apparently able to establish collusion without having to resort to communication with downstream firms.[3] At the same time, however, collusion may not be *opportunism-proof* because the manufacturers may have a joint incentive to lower prices, similarly to the monopolist in Hart et al. (1990). We show that the opportunism condition is more restrictive than the stability condition.

Opportunism-proofness relates to the concept of renegotiation-proofness introduced by Farrell and Maskin (1989) in the sense that opportunism-proofness is a necessary condition for renegotiation-proofness. We apply their concept to the coalition of colluding manufactur-

[3] A number of cartels have been detected in the automotive industry, where suppliers coordinated on high prices at the expense of carmakers and eventually final consumers. The cases involved rolling bearings (Frankfurter Allgemeine Zeitung, "Schaeffler has to pay a cartel fine of 370 million euros", 03/19/2014), safety belts, airbags, and steering wheels (Tagesschau, ""Millions in fines against auto suppliers"", 03/05/2019), as well as doors and electric window lifts (European Commission, "Antitrust: Commission fines car parts suppliers of € 18 million in cartel settlement", 09/29/2020) ; last accessed 11/27/2023. Recently, authorities in several European countries have started an investigation into the market for fragrances and fragrance ingredients involving the world's largest manufacturers. See, for example, Tagesschau, "DAX group Symrise under suspicion of collusion", 03/08/2023; last accessed 11/27/2023.

ers and define opportunism in this context as a joint deviation from the collusive agreement. This reflects that manufacturers may want to renegotiate the collusive wholesale price if a joint deviation is profitable for given actions of the retailers. Moreover, to ensure that manufacturers do not want to renegotiate the wholesale price during the punishment phase, we additionally consider a condition on *credible punishment.* If both conditions on opportunism-proofness and credible punishment are met, the collusive equilibrium is renegotiation-proof.

Collusion with trigger strategies usually is not renegotiation-proof (see, for example, Bernheim and Ray (1989) and Farrell and Maskin (1989)): firms have an incentive to jointly deviate in the punishment phases by renegotiating higher prices – undermining the credibility of the necessary threat to punish. Although a few solutions for certain settings are known, such as asymmetric punishment, the problem of renegotiation appears to be simply ignored in the applied literature.[4] We find that if the colluding firms sell via retailers using private contracts, renegotiation-proof equilibria with the usual symmetric punishment exist. If retailers fully anticipate the collusive strategy, they expect a punishment phase once a manufacturer deviates by making an out-of-equilibrium offer. This expectation of punishment can render punishment credible because retailers are not willing to accept the original collusive contracts during the punishment phase. Hence, if the stricter opportunism-proofness condition holds, with beliefs that anticipate a collusive strategy, collusion can become a renegotiation-proof equilibrium of the game for sufficiently patient manufacturers.

The case descriptions mentioned in the first paragraph reveal that manufacturers occasionally struggle to establish collusion because retailers do not accept the increased wholesale prices. This raises the question under which condition retailers' responses allow for the formation of collusion. We formally define the *formability* of collusion that requires the existence of a potential path from a non-collusive history to collusion. For example, with beliefs that anticipate collusion with grim-trigger strategies, retailers' responses make collusion impossible after observing a single period of competition. Hence, pure grim-trigger beliefs and also history-independent beliefs do not support the formation of collusion.

We introduce an adaptive belief whereby retailers expect collusion or competition depending on the past behavior of the manufacturers. These intuitive beliefs can describe settings in which players cannot perfectly anticipate the actions of other players in parts of the game but are able to learn and adapt to the equilibrium. We parameterize the adaption speed that specifies how fast a retailer who believes in competitive prices but receives collusive wholesale price offers switches to believing in collusive prices. The adaption speed may range from a single period to many periods. We find that collusion can be formable with these adaptive beliefs, but such adaptive beliefs perform poorly in the punishment phase in the sense that punishment is not renegotiation-proof. We find that a faster speed of adaption makes collusion easier to form but harder to sustain: collusion becomes less opportunism-proof.

[4] An exception is McCutcheon (1997). We differ from McCutcheon (1997) who builds on the model of costly renegotiation proposed by Blume (1994). By contrast, following Farrell and Maskin (1989) we assume that renegotiation is costless and establish that the collusive equilibrium can nevertheless be renegotiation-proof when considering renegotiation between the collusive players.

In summary, our key contributions are as follows. First, from an application point of view, we show that, in a vertically related structure, manufacturer collusion may be difficult to establish. We even find that, for certain retailer beliefs and supply contracts, collusion cannot be enforced at all. Also, we demonstrate that the opportunism problem, which is well established in the monopoly context, has rich implications for the theory of cartel stability and formation because it may occur during both collusive and punishment periods. Furthermore, starting to collude can be complicated because retailers may not be willing to accept the new collusive contract. Starting to collude is in various instances more difficult than sustaining collusion. Second, from a conceptual point of view, we go beyond the stability condition, which is the main focus of the analysis in many theories on collusion, and model other factors that may hinder collusion.

The remainder of the article is structured as follows. Section 2 relates our model to the relevant literature. We set up the model in Section 3 and subsequently analyze beliefs that are independent of the history of the game in Section 4.1. In Section 4.2, we study trigger beliefs and adaptive beliefs. We review symmetric beliefs in Section 5.1. We provide a competition policy discussion in Section 6, where we relate our theory to business practices which manufacturers may use to deal with the problem of formation and opportunism. These practices include communicating non-binding price increase announcements, resale price maintenance, vertical integration, communication between retailers, downsizing of packs, and buyback policies. Section 7 concludes.

2 Related Literature

Our paper contributes to four aspects that have been analyzed in the literature: (i) manufacturer collusion, (ii) a monopolist's opportunism problem, (iii) downstream retailers' types of beliefs, and (iv) cartel formation.

Manufacturer collusion. Nocke and White (2007) and Normann (2009) analyze tacit collusion among manufacturers in vertical relationships but, in contrast to us, focus on whether vertical integration makes tacit collusion easier to sustain. Both articles consider perfect Bertrand competition among manufactures and compare an industry with no integration to one in which one pair of firms is vertically integrated. In Nocke and White (2007), manufacturers compete in two-part tariffs. The authors show that it is easier for manufacturers to sustain collusion in a scenario with vertical integration. Normann (2009) shows that this finding carries over to a situation in which manufacturers set linear prices, even though double marginalization leads to different collusive and deviation profits. Piccolo and Miklós-Thal (2012) find similar results for the case in which retailers have full bargaining power. Under public contracts, Schinkel et al. (2008) show that, when manufacturers have full bargaining power but need to make sure that retailers do not sue for private damages, upstream collusion requires low wholesale prices and possibly negative franchise fees. Piccolo and Reisinger

(2011) analyze the impact of exclusive territories granted to retailers on the feasibility of manufacturer collusion. Under observable contracts, establishing exclusive territories has two opposing effects on collusive stability. Exclusive territories soften punishment but they also reduce deviation profits. The second effect is due to the fact that, when a manufacturer deviates, retailers of competing products adjust their prices, whereas retailers of the same product do not. Because the effect on deviation tends to dominate, exclusive territories tend to facilitate tacit collusion.

Our contribution to this literature is twofold: First, we study the opportunism problem that colluding manufacturers face with secret contracting. Second, we demonstrate the relevance of retailers' behavior under strategic uncertainty for establishing and maintaining collusion.[5]

Opportunism problem. We relate to the classic opportunism problem of a monopolist in a vertical structure with secret contracting and two-part tariffs (Hart et al., 1990; O'Brien and Shaffer, 1992; McAfee and Schwartz, 1994) as colluding manufacturers are similar to a monopolist. In such a scenario, the upstream firm that deals with multiple competing downstream firms through bilateral contracts may – as discussed further below – encounter the following problem: The upstream firm is interested in maintaining high prices and profits but it cannot commit to refraining from opportunistic moves. Indeed, the upstream firm has an incentive to increase bilateral surplus with one downstream firm, which is anticipated by the other downstream firm(s). The existence of this opportunism problem has been evoked as an explanation for vertical mergers and various vertical restraints as measures aimed at restoring the upstream firm's market power (O'Brien and Shaffer, 1992; McAfee and Schwartz, 1994; Rey and Vergé, 2004). The restraints include exclusive dealing, non-discrimination clauses, and industry-wide RPM. We also relate to Gaudin (2019) who shows that, in the framework of Hart et al. (1990), linear contracting can lead to higher retail prices absent collusion as well. Do and Miklos-Thal (2021) explore shortcomings of the seminal papers by considering a version of sequential (re)contracting between upstream and downstream firms. The opportunism problem that we study in detail has been neglected in the collusion literature so far and we establish that it has important non-trivial consequences. We complement the literature by introducing an opportunism-proofness condition that is related to concept of renegotiation proofness by Farrell and Maskin (1989).

Beliefs. The literature on secret contracting between manufacturers and retailers, which dates back to Hart et al. (1990), McAfee and Schwartz (1994) and Segal (1999), emphasizes the relevance of the retailers' beliefs. The literature mostly focuses on passive, symmetric,

[5] The literature also contains explanations of how resale price maintenance (RPM) can facilitate manufacturer collusion but abstracts from the relevance of retailer beliefs. Jullien and Rey (2007) study how (RPM) affects collusion when only the retailers observe local shocks on demand. They assume that colluding manufacturers reveal all wholesale prices to all retailers. Hunold and Muthers (2020) show that, absent any uncertainty and asymmetric information, RPM can still facilitate manufacturer collusion when there is retail bargaining power. They focus on subgame perfect Nash equilibria.

or wary beliefs in static settings. Whereas passive beliefs suppose that the agents treat unexpected offers as mistakes, symmetric beliefs could correspond to a rule of thumb, where agents conjecture that identical principals make identical offers (Pagnozzi and Piccolo, 2011). Wary beliefs – according to which a retailer anticipate that rivals get the offer that maximize the manufacturer's profits – are often used when passive beliefs are implausible or induce non-existence of equilibria (Rey and Vergé, 2004; Rey and Tirole, 2007; Miklós-Thal and Shaffer, 2016). Empirical evidence on passive and symmetric beliefs was tested by Zhang (2021) and Martin et al. (2001) in experiments. Moreover, Aoyagi et al. (2021) study beliefs in finitely and infinitely repeated games experimentally and find that the same history of play can lead to different beliefs, and the same belief can lead to different action choices. We contribute to this literature by providing an in-depth analysis of how retailers' beliefs affect manufacturer collusion in a dynamic setting with secret contracting. In our analysis, we show how the retailers' contract acceptance decisions change depending on their perception of the collusive strategy ranging from fully anticipating the collusive (trigger-) strategies to not expecting any collusion.

In repeated games, the notion of passive beliefs differs across papers as the exact notion from static settings cannot be applied. Within this literature, our paper is most closely related to Piccolo and Reisinger (2011), Gilo and Yehezkel (2020), and Liu and Thomes (2020). In an extension, Piccolo and Reisinger (2011) consider secret contacting to investigate strategic delegation in a model with colluding manufacturers. The authors consider a perfect Bayesian equilibrium with passive beliefs. Their use of passive beliefs requires that those are correct on and *off* the equilibrium path. Similar to their approach, Liu and Thomes (2020) use weak perfect Bayesian equilibrium and consider passive beliefs as well as symmetric beliefs. They do not define passive beliefs explicitly but employ them in the sense that out-of-equilibrium offers do not change the expectation about the other manufacturer (in deviation periods). In punishment periods, they assume that retailers understand that there is punishment going on, such that the outcome is the competitive outcome of the stage game. Thereby, they implicitly assume that the beliefs of retailers are correct in the punishment period, which is the case if retailers anticipate the complete strategy of the manufacturers. That is, their definition is equivalent to a definition of passive beliefs whereby retailer beliefs are consistent with the strategy profile of the manufacturers in any subgame.

This consistency does not follow from the definition of weak PBE, which is silent on beliefs in nodes that are not reached on the equilibrium path with positive probability. It presumes that retailers have a correct expectation even in subgames that they never observe, given the strategy of the manufacturers, and thus cannot have any experience with. The beliefs thus rely on information that retailers cannot obtain on the equilibrium path. In addition, in this game, a similar equilibrium path can be sustained by many different manufacturer strategies that imply different behaviors off the equilibrium path. Thus, it is also theoretically impossible for retailers to predict the off-equilibrium behavior with certainty. In this context, we consider different beliefs explicitly as they correspond to different understandings about how the market works.

Gilo and Yehezkel (2020) study secret contracts and vertical collusion involving retailers and a single manufacturer. Their approach mirrors ours in the sense that they analyze the role of the manufacturer in retailer collusion, whereas we look at the retailers' role in manufacturer collusion. The major difference is that we abstract from retailer collusion by assuming short-lived retailers, whereas repeated retailer interaction is at the heart of their analysis.Gilo and Yehezkel consider symmetric pure-strategy, perfect Bayesian equilibria. Whereas the asymmetry in information about the contracts is similar, the timing differs from ours because the retailers in their model are those who offer contracts. They use the concept of "rational beliefs," which they define as players anticipating the rational action of others in off-equilibrium situations. They argue that this implies that manufacturers and retailers both understand whether a period is collusive or not after a deviation.

Cartel formation. The literature on cartel membership formation has focused on a variety of different aspects. In this literature, the question of how to initiate cartels typically focuses, among other things, on contracts (Selten, 1973; d'Aspremont et al., 1983), on stochastic opportunities to form a cartel (Harrington and Chang, 2009), on the heterogeneity with regard to capacities and umbrella pricing (Bos and Harrington, 2010), on signals and beliefs of producers (Harrington and Zhao, 2012; Harrington, 2017), on quality differentiation (Bos et al., 2020), and the ability to overcome strategic uncertainty absent communication (Blume and Heidhues, 2008). Selten (1973) analyzes the case of quantity competition and assumes that cartels can be enforced via contracts, and that a cartel acts as a Stackelberg leader. He shows that a cartel is stable in the sense that outsiders do not want to be part of the cartel, and insiders cannot profitably leave the cartel as long as the number of cartel members is relatively small. d'Aspremont et al. (1983) obtain a similar result for the case of price leadership.

Harrington and Chang (2009) consider a set of heterogeneous industries in which stochastic opportunities to form a cartel arise to explain the birth and death of cartels and to inform antitrust authorities about the extent of cartels that have not been discovered. Bos and Harrington (2010) endogenize the composition of a cartel in an industry in which heterogeneous firms differ in their capacities. They show that non-all-inclusive cartels set umbrella prices, and that mergers involving moderate-sized firms may result in the most severe coordinated effects. Harrington and Zhao (2012) analyze whether different types of players (patient and impatient) manage to cooperate via grim-trigger strategies when players signal and coordinate through their actions. The authors show that there is always a positive chance of cooperation, but cooperation may fail altogether. Moreover, the longer cooperation does not occur, the less likely it is to occur in the next period. Harrington (2017) focuses on mutual beliefs to coordinate prices. In the context of price leadership, firms are assumed to commonly believe that price increases will be at least matched. The firms, however, lack any shared understanding about who will lead, when they will, and at what prices. Sufficient conditions are derived, which ensure that supra-competitive prices emerge, but price is bounded below the maximal equilibrium price.

In contrast to the literature on cartel membership formation, we address the question whether firms can transit to a collusive equilibrium once they have reached – possibly explicitly – a common understanding to collude in a currently non-collusive industry. We thereby focus on the process of establishing collusive outcomes that firms need to go through until they may reach a stable collusive equilibrium. At the core of our analysis is the response of retailers in the transition process.

3 Model

We study manufacturer collusion in an infinitely repeated stage game with time $t = 0, \ldots, \infty$. There are two symmetric manufacturers, M_A and M_B, that compete by selling horizontally differentiated products to their exclusive retailers R_A and R_B. Each manufacturer makes an exclusive and secret two-part tariff offer with a unit wholesale price w_i and a franchise fee F_i to its retailer, with $i \in \{A, B\}$.

3.1 Set-Up

Timing and information. In each period, the following stage game unfolds:

1. Each manufacturer makes a private contract offer to its retailer.
2. Each retailer decides whether to accept its offer. Post contract acceptance, the fixed fees are sunk.
3. The retailers simultaneously and non-cooperatively set their retail prices p_i, $i \in \{A, B\}$.

Because the manufacturers' contract offers are secret, a retailer cannot observe the contract offered to the rival.[6] When competing in the downstream market, the retailers are thus unable to observe each others' input prices.

At the end of each period, all actions are revealed to all players. All players thus know the complete history of the game at the end of a period. We focus on the secret contracting problem between manufacturers and retailers and avoid that manufacturer collusion is directly hampered by the long-term unobservability of manufacturers' actions to each other.

The supergame is a game of complete information but unobservable actions.The manufacturers are long-lived and discount next-period profits with the common discount factor $\delta > 0$. The retailers have a discount factor of zero, such that they do not take future profits into account. This assumption ensures that the retailers cannot collude. One can similarly assume that the retailers are short-lived but one should keep in mind that the retailers do see the history of the game.[7]

[6]Note that we do not consider a scenario in which (colluding) manufacturers try to get retailers on board, that is, there is no communication between manufacturers and retailers on the issue of collusion. This appears to be in line with the cartel cases mentioned in the Introduction.

[7]Related literature assumes that retailers are short-lived or only live for one period. See, for example,

Assumptions on costs, demand, and profits. All (fundamental) costs are zero. We assume that the outside option (opportunity cost) of each retailer is equal to zero. We consider general demand functions that fulfill the standard properties summarized in

Assumption 1. *Demand $D_i(p_i, p_{-i})$ (with $i \in \{A, B\}$)*

- *decreases in the own price p_i ($\partial D_i(p_i, p_{-i})/\partial p_i < 0$),*
- *increases in the other product's price p_{-i} ($\partial D_i(p_i, p_{-i})/\partial p_{-i} > 0$), and*
- *decreases when all prices increase ($\partial D_i(p_i, p_{-i})/\partial p_i + \partial D_i(p_i, p_{-i})/\partial p_{-i} < 0$).*

To ensure that there exists a unique and stable equilibrium in the downstream market, we assume that the Hessian matrix of $D_i(p_i, p_{-i})$ has a negative and dominant main diagonal. This results in well-behaved retail profits that are twice differentiable and concave. Note that this also implies that the retailers' reactions behave normally, such that $\partial p_i\left(w_{it}, p_{-i}\right)/\partial w_{it} > 0$ and, consequently, $\partial D_i\left(p_i, p_{-i}\right)/\partial p_i < 0$ hold.[8]

Our assumptions on retailer profits mostly carry over to manufacturer profits because manufacturers internalize retailer profits using two-part tariffs. In some cases, however, manufacturers' true actions and retailers' beliefs differ in such a way off the equilibrium path that the behavior of the manufacturers' profit is not identical to that of the retailers' profit. In those cases, we analogously assume that manufacturers' profits are well behaved, such that optimal behavior can be derived from the respective first-order conditions. We comment on these cases below.

Equilibrium concept. We consider (pure-strategy) weak perfect Bayesian equilibria (weak PBE) and focus on symmetric equilibria.[9] We use the formal definition of a weak perfect Bayesian equilibrium from Mas-Colell et al. (1995): A profile of strategies and a system of beliefs is a weak PBE in extensive form games if it has the following properties:

1. The strategy profile is sequentially rational for the given belief system (each player maximizes expected utility at each node).
2. The system of beliefs is derived from the strategy profile through Bayes' rule whenever possible.

This implies that the retailers' beliefs are consistent with equilibrium strategies on the equilibrium path. However, when observing an out-of-equilibrium offer, the second condition

Piccolo and Reisinger, 2011 and Jullien and Rey, 2007. Such an assumption rules out hub-and-spoke collusion as well as vertical collusion Gilo and Yehezkel (2020), where retailers are a part of the collusion.

[8]This can be shown by applying the implicit function theorem on the retailer's first-order condition for optimal pricing.

[9]The game defined includes two groups of players, manufacturers and retailers. Within each group, the players are symmetric. The equilibria we study are strongly symmetric in the sense that the (symmetric) manufacturers use a common continuation strategy on and off the equilibrium path (Athey et al., 2004; Jullien and Rey, 2007). The retailers play symmetric Nash equilibrium actions in the downstream market.

does not apply as this information set is reached with zero probability, meaning that Bayes' rule does not restrict these beliefs. Throughout the analysis we focus on the case that the manufacturers correctly anticipate the retailer beliefs.

We impose an additional condition of "no-signaling-what-you-don't-know" for most of the analysis.

Condition 1. The belief of retailer i about its rival retailer $-i$ at the beginning of period $t+1$ depends only on the history up to date t ($\mathcal{H}_t$), but not on the action of manufacturer i at date t ("no-signaling-what-you-don't-know").

This condition captures the idea that the deviation of a manufacturer should not signal (private) information that the manufacturer does not have. Hence, a retailer's belief should not change.[10] Condition 1 has an intuitive interpretation in our game: A deviation of one manufacturer should not change the retailer's belief about the other manufacturer's contract offer. The condition, then, is equivalent to within-period passive beliefs.

We use weak PBE in conjunction with condition 1 to be explicit about our equilibrium definition as there is a lack of clarity in the literature about the definition of perfect Bayesian equilibrium (PBE). According to the definition in Fudenberg and Tirole (1991a), our equilibrium would be considered a PBE. Other definitions of PBE, e.g., in González-Díaz and Meléndez-Jiménez (2014), require subgame perfection, which we do not want to generally impose. In our setting, subgame perfection would require that the retailers correctly anticipate the manufacturers' actions in information sets that are reached with zero probability. We explicitly want to allow for retailers' incorrect judgment in these nodes. For instance, this allows retailers to misjudge which of the different trigger strategies manufacturers employ if they have the same equilibrium path but different off-equilibrium punishment strategies.

Our game is reduced-form but reflects a game that could be richer, for instance, by including a cheap-talk stage. If retailers consider it possible that manufacturers may communicate, manufacturers' actions might be correlated when a deviation occurs. Given this interpretation, condition 1 does not naturally capture the spirit of sequential equilibrium in this "extended game."[11]

[10]Condition 1 is derived from the definition of the PBE that Fudenberg and Tirole (1991a,b) provide. This definition is suited for games of incomplete information with independent types. Because in our game of complete information the private information of the retailers is generated by manufacturers' actions, the concept can only be applied analogously. Similarly to Pagnozzi and Piccolo (2011), we adapt condition B(iii) ("no-signaling-what-you-don't-know") on p. 332 in Fudenberg and Tirole (1991a) to our game. Originally, the condition means that in a signaling game, the actions of other players with independent types have no effect on beliefs about a player's type if this player acts the same. In our setting, because retailers act simultaneously, one retailer cannot observe the other retailer's action such that the condition translates into: A retailer cannot infer anything about a rival's current pricing from the current action of its own manufacturer.

The condition is also in the spirit of sequential equilibrium, where (out-of equilibrium) beliefs are naturally passive within a period because deviations are the result of a random mistake ("tremble"). Note that we cannot apply sequential equilibrium because we have a continuous action space.

[11]For instance, the symmetric beliefs we consider in Section 5.1 violate condition 1 but capture the idea that retailers may anticipate communication between manufacturers.

Each retailer forms a belief about the contract offer made to the rival and about how the rival reacts to its contract offer when accepting the contract in the second stage. Following Rey and Vergé (2004), we focus on retailers that form beliefs about the resulting retail price of the rival, which is the payoff-relevant information that retailers are lacking.[12] In our setting there is a unique mapping from the belief a retailer has about its rival's wholesale price to the expected retail price of the rival.[13]

3.2 Formability and Renegotiation-Proofness

We focus on the phenomenon of collusion among manufacturers and aim to address additional issues beyond the standard stability and incentive compatibility problem. In particular, we shed light on the possibility for manufacturers to form collusion, that is, the transition from a competitive to a collusive market outcome. Moreover, we highlight the robustness of collusion to the opportunity of manufacturers to renegotiate, both during the collusive phase and after a deviation. Both issues are theoretically and empirically appealing because collusion in real-world markets has various important aspects besides the stability problem. To characterize whether the collusive equilibrium allows for renegotiation or cartel formation, we define conditions that can be checked in a similar fashion to the traditional stability condition.

Definitions. First, we characterize how manufacturers can overcome the coordination problem by defining under which conditions collusion is formable in the sense that collusion can be started and maintained. For example, suppose that manufacturers have been competing for a large number of periods and want to start collusion with trigger strategies. Formability addresses the question whether the system of retailers' beliefs allows for a transition path to a collusive equilibrium. We define a weak PBE as formable if there is a transition path to a collusive equilibrium in the sense that, in this transition, each player maximizes its expected utility under the belief system of the weak PBE. Thus, formability is an additional property of an equilibrium that imposes a condition on the system of beliefs.

Definition 1. (Formable) A collusive weak PBE $\mathcal{C}$, consisting of a strategy profile and a system of beliefs, is formable if there exists a transition strategy profile, such that for the system of beliefs of $\mathcal{C}$ after an *arbitrary* history $\mathcal{H}_{t_0}$:

1. In the transition phase starting in period t_0 and lasting for x periods, with $x < \infty$, until $t_0 + x$ (exclusively), the transition strategy profile in the continuation game is sequentially rational for the system of beliefs of C;

2. The collusive phase starts in period $t_0 + x$. In the continuation game, the strategy profile of $\mathcal{C}$, adjusted with $t_0 + x$ as the first period, together with the system of beliefs of C, is a weak PBE.

[12] See Section 3 in Rey and Vergé (2004).

[13] For any history $\mathcal{H}_t$ and Assumption 1, retailer expect a certain level of wholesale prices that results in a unique expected price level at the rival of p^e_{it}. We prove this in Proposition 1.

Note that beliefs in the transition phase can be incorrect, such that the strategy profile and system of beliefs after the alternative history do not generally constitute a weak PBE.[14] The number of transition periods x can be long, such that our definition is not restrictive with regard to the speed of formation.

Next, we define how the concept of renegotiation-proofness relates to our model. Specifically, we rely on the notion of renegotiation-proofness by Farrell and Maskin (1989). We adapt the concept by restricting renegotiation to manufacturers. Renegotiation takes place implicitly in our model whenever *manufacturers* have a collective interest in revising their agreement. This assumption is in line with other theories on renegotiation-proofness (see, for example, Bernheim et al., 1987 and Bernheim and Ray, 1989).[15]

Definition 2. A collusive weak PBE $\mathcal{C}$, consisting of a strategy profile and a system of beliefs, is *weakly* renegotiation-proof if it features opportunism-proofness and renegotiation-proof punishment:

1. (Opportunism-Proofness) A collusive equilibrium strategy for given beliefs (a weak PBE) is said to be opportunism-proof if the manufacturers, on the equilibrium path, do not benefit from jointly changing their contract offers.

2. (Renegotiation-Proof Punishment) A wPBE has the property of renegotiation-proof punishment if there is no punishment period in which the manufacturers would benefit if they jointly changed their contract offers.

The first condition excludes the scope for opportunistic manufacturer behavior during the collusive phase. Because demand does not change over time, manufacturers have no incentive to renegotiate the collusive price on the equilibrium path, but a coordinated one-shot deviation (against their retailers) may be profitable. Without opportunism-proofness, the implicit "collusive agreement" between the manufacturers is not renegotiation-proof. Whereas opportunism-proofness considers renegotiation-proofness of the collusive strategy on the collusive path, the criterion of renegotiation-proof punishment does so for the punishment phase after deviations. Intuitively, the condition states whether manufacturers want to negotiate the terms of the price war that follows after a unilateral deviation. Together, opportunism-proofness and renegotiation-proof punishment ensure *weak* renegotiation-proofness of the equilibrium as defined in Farrell and Maskin (1989). *Strong* renegotiation-proofness requires that not only all the continuation game equilibria do not invite joint deviation, but also that there is no other renegotiation-proof strategy profile that Pareto-dominates the candidate equilibrium of the whole game.

[14]The definition, however, requires that the strategy profile of the continuation game starting with t_0 is perception perfect, i.e., sequentially rational given the beliefs.

[15]Our approach is line with the original idea presented by Farrell and Maskin (1989) who consider subgame perfect Nash equilibria (SPNE) in a collusive game and treat the buyers as non-strategic. Similarly, renegotiation-proofness is usually applied to the parties of a *relational contract* only, in our case the collusive agreement, e.g., Buehler and Gärtner (2013); Goldlücke and Kranz (2013). Alternatively, one can understand our approach as applying the exact concept of Farrell and Maskin (1989) to the reduced form SPNE taking retailers' strategies and beliefs from the wPBE as fixed.

Discussion. In a standard model of horizontal collusion, the usual stability condition is necessary for collusion and also sufficient if the manufacturers can coordinate on the collusive outcome through a "meeting of the minds." If, however, colluding upstream firms sell to competing downstream firms, stability is not sufficient. Retailers might not accept the new contract offers with collusive prices, especially if the contract offers to downstream rivals are secret. Our formation condition captures the feasibility of the manufacturers to implement a collusive agreement vis-a-vis the downstream firms. For example, in the coffee cartel, collusion among the coffee roasters turned out to be unsuccessful because their retailers did not accept the collusive wholesale prices (Holler and Rickert, 2022). Collusion only became successful after introducing resale price maintenance.

Furthermore, we consider renegotiation of the collusive strategy and the option to renegotiate after a deviation. The collusive strategy is often viewed as all or nothing and cannot be adjusted. We show that manufacturers might want to adjust their collusive strategy (against their retailers) in a stable market environment, which we refer to as opportunism. Additionally, we check whether manufacturers can and want to renegotiate in the punishment phase after a deviation. As stated by Levenstein and Suslow (2006), "one of the most clearly established stylized facts is that cartels form, endure for a period, appear to break down, and then re-form again". They report that cheating and disagreement in cartels happens quite often but cartels frequently have multiple episodes of cooperation. Hence, it might be that cartel members renegotiate during price wars to return to the collusive outcome. As referenced in Levenstein and Suslow (2006), empiricists are able to differentiate between bargaining price wars and price wars following a punishment strategy. We will show that collusion can be sustainable while not being renegotiation-proof and while being renegotiation-proof, explaining both kinds of price-wars.

4 Solution

4.1 History-Independent Beliefs

In this section, we consider retailer beliefs that are independent of the history of actions in the game. Such beliefs arise naturally when the manufacturers' equilibrium strategies are stationary. For example, the competitive equilibrium in which both manufacturers set the competitive wholesale prices in each period is consistent with retailers having a constant and thus history-independent belief about wholesale prices. With these history-independent beliefs, we show that collusion is infeasible.

Definition 3. (History-Independent Beliefs) The price expectation p^e_{-it} of retailer i in period t about the price of retailer $-i$ is independent of the history of the actions in the game up to period $t-1$ and independent of the offer (w_{it}, F_{it}) made by its supplier in period t.

The proposed belief refinement above defines the retailers' out-of-equilibrium beliefs about the retail price of their competitor to be independent of the history. As discussed before, the fact that beliefs are passive within a period follows from condition 1. The definition, however, does not impose a restriction on how the beliefs react to the past play of actions.

We first solve for the equilibrium of the game that results when both manufacturers maximize stage-game profits. In the last stage within each period, on the equilibrium path, each retailer has accepted the contract, but the rival's wholesale price remains secret. Each retailer i faces the own wholesale price w_{it} and holds a history-independent belief p^e_{-it} about the retail price of the rival. The retailers set the retail prices p_{it} simultaneously. Thus, the profit of retailer i is

$$\pi_{it}\left(p_{it}, p^e_{-it}\right) = \left(p_{it} - w_{it}\right) D_i\left(p_{it}, p^e_{-it}\right) - F_{it}.$$

Retailer i maximizes its profit with respect to p_{it}. The first-order condition is

$$D_i\left(p_{it}, p^e_{-it}\right) + \left(p_{it} - w_{it}\right)\frac{\partial D_i\left(p_{it}, p^e_{-it}\right)}{\partial p_{it}} = 0 \tag{1}$$

and defines retailer i's reaction function $p_i\left(w_{it}, p^e_{-it}\right)$. Anticipating the pricing outcome and resulting profits, each retailer decides whether to accept the wholesale tariff offered by its manufacturer in the second stage. Under Assumption 1, there exists a unique symmetric Nash equilibrium in the downstream market for offers (w_{it}, F_{it}) made by the manufacturers given retailers accept the offers.

In the first stage, the manufacturers offer their wholesale tariffs. Under manufacturer competition, each manufacturer i maximizes its profit, anticipating that its retailer sets a price of $p_i\left(w_{it}, p^e_{-it}\right)$:

$$\max_{w_{it}, F_{it}} w_{it} \cdot D_i\left(p_i\left(w_{it}, p^e_{-it}\right), p_{-i}(w_{-it}, p^e_{it})\right) + F_{it},$$

subject to the retailer's participation constraint

$$\left(p_i\left(w_{it}, p^e_{-it}\right) - w_{it}\right) \cdot D_i\left(p_i\left(w_{it}, p^e_{-it}\right), p^e_{-it}\right) - F_{it} \geq 0. \tag{2}$$

The profit is determined by two parts. The variable part consists of the units sold times the unit wholesale price, and the fixed part consists of an up-front payment from the retailer to the manufacturer. The maximum fixed payment cannot be larger than the revenue that the retailer earns. This participation constraint binds in equilibrium.

Consider the one-shot game. In the one-shot equilibrium, the belief of the retailer about the retail price of the rival coincides with the expectation of the manufacturer about the retail prices. This implies $p_{-i}(w_{-it}, p^e_{it}) = p^e_{-it}$.

The optimality condition of the manufacturer problem implies that the only solution has $w_{it} = 0$. Denote the competitive wholesale price by w^P and the resulting retail price by $p^P = p_i(0, p^P)$. We have the following result:

Lemma 1. *With history-independent beliefs, in the equilibrium with competing manufacturers, the wholesale prices are equal to zero ($w^P = 0$), resulting in a competitive retail price level of p^P.*

Proof. See Appendix. □

The intuition behind the result is that, with selling the product at marginal costs of zero, each manufacturer ensures that retailers maximize the joint profits of a manufacturer-retailer pair, which can then be extracted via the fixed fee.

This history-independent benchmark defines the competitive level of wholesale prices which is at $w_i = 0$, the corresponding fixed fees, and the resulting retail prices. The fixed fee is equal to the retailer's profit, $F_i = \pi_i(p^P, p^P)$, corresponding to a non-colluding industry. In the next steps, we analyze the equilibrium of the repeated game if retailers hold history-independent beliefs. To show that firms are unable to collude on a price different from the competitive wholesale price of zero, we take the whole dynamic game into account. Manufacturers may collude using any dynamic strategy (for example, grim-trigger strategies). Each strategy, however, is a mapping from the previous history to an action chosen in period t.

Proposition 1. *Suppose retailers have history-independent beliefs. Then, there exists a unique symmetric perfect Bayesian equilibrium in which the manufacturers set wholesale prices equal to zero and extract all profits of retailers via the fixed fee. Collusion cannot increase prices above the competitive level.*

Proof. See Appendix. □

This impossibility result can be interpreted as showing that the beliefs that support the competitive equilibrium cannot support a collusive equilibrium. Thus, for manufacturers in a competitive industry striving to start collusion, their joint will is not sufficient to increase prices above the competitive level. The strict impossibility result does not extend to linear tariffs but depends on efficient contracts (See Section 5.2).[16]

The intuition for the result is that the punishment strategy that underlies any collusive equilibrium becomes ineffective with these beliefs. Each manufacturer can ensure a profit larger than the collusive profit level that only depends on its own actions and the belief of its own retailer but is independent of the action of the other manufacturer. Thus independence of the other manufacturer's action eliminates any credible punishment. Punishment, however, is necessary for collusion and thus there is an incentive to deviate from any supra-competitive price level.

Next we consider beliefs that are consistent with collusion. They are history-dependent and meet condition 1, which implies that they are within-period passive.

[16] The classical opportunism problem can be mitigated under linear contract as shown by Gaudin (2019), where the monopolist achieves supra-competitive (marginal) wholesale prices. In contrast, with two-part tariffs, the monopolist sets the wholesale prices (w) equal to its marginal costs (McAfee and Schwartz, 1994).

4.2 Belief Dynamics

Consider beliefs that depend on the history of the game. We focus on the history of wholesale prices and disregard the history of retail prices, such that the retailers' beliefs cannot directly depend on the competing retailers' past actions. Otherwise, such beliefs could support retail collusion, which we want to abstract from.[17] Thus, the relevant history of the game in period t is $\mathcal{H}_t \equiv [(w_{A0}, w_{B0})\,; \ldots ; (w_{At-1}, w_{Bt-1})]$, that is, the set of the pairs of wholesale prices that the manufacturers have set in all periods up to period $t-1$. More formally, for each retailer i, the belief is a function $p^e_{-it}(\mathcal{H}_t)$.

4.2.1 Grim-Trigger Beliefs

We construct history-dependent beliefs that have a grim-trigger property. For this, consider that manufacturers play grim-trigger strategies as described below.

We focus on industry-profit-maximizing collusion, such that $w^C = w^M$. Any deviation by one manufacturer causes the other manufacturer to set the punitive price w^P forever. Because we are solving for perfect Bayesian equilibria, the punishment with w^P must be individually rational. This implies that $w^P = 0$ must hold in equilibrium, which follows from the logic presented in the proof of Proposition 1: Otherwise, each manufacturer would have an incentive to deviate from a $w^P \neq 0$ in a punishment period in which future beliefs and actions are fixed and unaffected by current actions. In punishment periods, each manufacturer must thus maximize short-term profits.

Collusive strategy. We consider collusion at the prices that maximize the integrated industry profit:

$$p^M = \max_{p_i} p_i \cdot D_i\left(p_i, p_{-i}\right) + p_{-i} \cdot D_{-i}\left(p_{-i}, p_i\right), \tag{3}$$

and define w^M implicitly through $p^M = p_i\left(w^M, w^M\right)$. Denote the maximal industry profit by

$$\Pi^M := p^M \cdot D_i\left(p^M, p^M\right) + p^M \cdot D_{-i}\left(p^M, p^M\right). \tag{4}$$

The optimal collusive wholesale price w^M and the respective belief about the retail price, p^M, are then determined by equation (3) as well. Thus, whenever the manufacturers collude at the industry-profit-maximizing wholesale prices w^M, each manufacturer earns a profit of

$$\Pi^C := \frac{\Pi^M}{2}.$$

The optimal collusive wholesale price maximizes the joint profit of the manufacturers given that the retailers' belief is identical to that price. The collusive profits are Pareto-optimal for

[17] Suppose that each retailer believes that the price of the competitor is the monopoly price unless they have observed a different price in the past in which case they would believe that the price is the competitive price. Such beliefs could support a collusive action profile even with otherwise history-independent strategies.

the manufacturers if there is no joint deviation that is more profitable (see Definition 1). If weak renegotiation-proofness is fulfilled, then the equilibrium is also strongly renegotiation-proof because the profit on the equilibrium path would be Pareto-efficient for the colluding manufacturers.

Histories and beliefs. If manufacturers play grim-trigger strategies, their actions are only conditional on two kinds of histories: the collusive history $\mathcal{H}^{\mathcal{C}}$, where both manufacturers have only played w^C, and the deviation histories $\mathcal{H}^{\mathcal{D}}$ (any history other than $\mathcal{H}^{\mathcal{C}}$). Define the grim-trigger strategy as follows: Manufacturers set the collusive wholesale price w^Cin the first period. Then, in the t^{th} period, if both manufacturers have set the collusive price in each of the $t-1$ previous periods (history $\mathcal{H}^{\mathcal{C}}$), they set the collusive wholesale price w^C; otherwise, after histories $\mathcal{H}^{\mathcal{D}}$, manufacturers set a punishment price $w^P \neq w^C$ forever. Grim-trigger beliefs match these strategies by assigning two different beliefs to these histories. Grim-trigger beliefs $p_i^e(\mathcal{H})$ are thus history-dependent and differentiate between the two histories $\mathcal{H}^{\mathcal{C}}$ and $\mathcal{H}^{\mathcal{D}}$. Grim-trigger beliefs can be interpreted as the beliefs resulting from a subgame-perfect PBE with retailers fully anticipating manufacturers' strategies in every subgame.

Definition 4. (Grim-trigger beliefs)

- In the first period, retailers believe that the rival sets a retail price of p^C.
- As long as both manufacturers play w^C, that is, the collusive history $\mathcal{H}^C$ prevails, each retailer believes that the other retailer sets p^C in the current period.
- Once one manufacturer has deviated, the history is $\mathcal{H}^{\mathcal{D}}$, and both retailers believe that the other retailer sets the competitive price p^P. This corresponds to a situation in which both retailers have common knowledge that the wholesale prices are $w^P = 0$ in the current period.

Given the passive nature of beliefs in a period (condition 1), the beliefs are not correct in deviation periods. Neither a retailer that is offered the equilibrium contract updates its belief, nor updates the retailer that receives a deviating offer.

Because a deviation does not occur on the equilibrium path, the beliefs are nevertheless correct on the equilibrium path. Off the equilibrium path, the retailers' beliefs anticipate that manufacturers play grim-trigger strategies in that they also punish a deviation by one manufacturer in period t in all future periods.

Equilibrium. To determine an equilibrium of the dynamic game, we must consider deviations from the collusive strategy. In equilibrium, each manufacturer realizes per-period profits of $\Pi^C = \Pi^M/2$. In a deviation period, both retailers believe that the wholesale price is w^C and anticipate that the other retailer sets p^C. This results in a belief $p_{it}^e := p_i\left(w_{it}, p^C\right)$. Suppose that manufacturer i maximizes its deviating profit in period t in view of history $\mathcal{H}^{\mathcal{C}}$.

When there is a deviation, that is, $w_{it} \neq w^C$ holds, grim-trigger beliefs imply that the level of w_{it} has no impact on future beliefs. The deviation profit is given by

$$\begin{aligned} & w_{it} \cdot D_i\left(p_i\left(w_{it}, p^C\right), p^C\right) + \left[p_i\left(w_{it}, p^C\right) - w_{it}\right] \cdot D_i\left(p_i\left(w_{it}, p^C\right), p^C\right) \\ & = p_i\left(w_{it}, p^C\right) \cdot D_i\left(p_i\left(w_{it}, p^C\right), p^C\right). \end{aligned} \tag{5}$$

Maximizing with respect to w_{it} yields the first-order condition

$$\begin{aligned} \frac{\partial p_i\left(w_{it}, p^C\right)}{\partial w_{it}} D_i\left(p_i\left(w_{it}, p^C\right), p^C\right) + \frac{\partial D_i\left(p_i\left(w_{it}, p^C\right), p^C\right)}{\partial p_i} \frac{\partial p_i\left(w_{it}, p^C\right)}{\partial w_{it}} p_i(w_{it}, p^C) = 0 \\ \iff \frac{\partial p_{it}\left(w_{it}, p^C\right)}{\partial w_{it}} \frac{\partial D_{it}\left(p_{it}\left(w_{it}, p^C\right), p^C\right)}{\partial p_{it}} w_{it} = 0. \end{aligned} \tag{6}$$

The last step follows from the first-order condition of the retailers in equation (1). Inserting the expression in the first line above yields the second line. Because the first two factors in equation (6) are non-zero, the manufacturer optimally deviates to $w^D = 0$. This results in

$$\Pi^D \coloneqq p_i(0, p^C) \cdot D_i\left(p_i(0, p^C), p^C\right).$$

After any deviation by a manufacturer, the beliefs revert to $p^e_{-i} = p^P$ forever, that is, the belief in the punishment period. This results in profits of

$$\Pi^P \coloneqq p^P \cdot D_i(p^P, p^P), \tag{7}$$

where $p^P = p_{it}(0, p^P)$ is the competitive price.

Collusion is sustainable when no manufacturer wants to deviate from the grim-trigger strategy. Using the one-shot deviation principle, the relevant incentive constraint for stability is

$$\frac{\Pi^C}{1-\delta} \geq \Pi^D + \frac{\delta \Pi^P}{1-\delta}. \tag{8}$$

The left-hand side contains the present value on the equilibrium path and the right-hand side the present value of a deviation. We can rewrite the incentive constraint for manufacturer i as follows:

$$\frac{p^C \cdot D_i(p^C, p^C)}{(1-\delta)} \geq p_i(0, p^C) \cdot D_i\left(p_i(0, p^C), p^C\right) + \frac{\delta}{1-\delta}\left(p^P \cdot D_i(p^P, p^P)\right). \tag{9}$$

This inequality is equivalent to the incentive constraint for standard horizontal collusion (when manufacturers and retailers are pairwise integrated), where the following order holds: $\Pi^D > \Pi^C = \Pi^M/2 > \Pi^P$.

Let us check whether the equilibrium is opportunism-proof (see Definition 1). Jointly, the manufacturers may have an incentive to reduce their wholesale prices for given beliefs. Suppose $\mathcal{H}_C$ is the history of the game, such that each retailer believes that its competitor sets p^C and anticipates to set $p_i\left(w_{it}, p^C\right)$. First, we show that jointly deviating manufacturers set a wholesale price of $w_{it} = w_{-it} < 0$ to maximize spot profits. To see this, let us inspect the profit per manufacturer in the case of a joint deviation:

$$\begin{aligned}\Pi^{JD} :=& \frac{1}{2}\max_{w_{it}, w_{-it}} w_{it} \cdot D_i\left(p_i(w_{it}, p^C), p_{-i}(w_{-it}, p^C)\right) + \left[p_i(w_{it}, p^C) - w_{it}\right] \cdot D_i\left(p_i(w_{it}, p^C), p^C\right) \\ &+ w_{-it} \cdot D_{-i}\left(p_{-i}(w_{-it}, p^C), p_i(w_{it}, p^C)\right) + \left[p_{-i}(w_{-it}, p^C) - w_{-it}\right] \cdot D_{-i}\left(p_{-i}(w_{-it}, p^C), p^C\right).\end{aligned}$$

We assume that this profit is quasi-concave, such that we can use first-order conditions.[18] We rewrite the first-order conditions in equation (1) by applying symmetry. This is possible because we assume that the manufacturer profits are well behaved in the sense that the optimal joint action of the manufacturers is symmetric. This yields

$$\left[\underbrace{D_i\left(p_i(w, p^C), p_{-i}(w, p^C)\right) - D_i\left(p_i(w, p^C), p^C\right)}_{<0}\right] + w\underbrace{\frac{\partial p_i}{\partial w_{it}}}_{>0}\left[\underbrace{\frac{\partial D_i}{\partial p_i} + \frac{\partial D_{-i}}{\partial p_i}}_{<0}\right] = 0, \forall i. \tag{10}$$

Equation (10) only holds for $w < 0$. Hence, manufacturers optimally deviate to $w < 0$ jointly. A manufacturer makes a higher profit in the case of a joint deviation than when deviating unilaterally: $\Pi^{JD} > \Pi^D$. To explain the last inequality, note that the manufacturers could replicate the profit of Π^D for each of them by setting $w = 0$. The manufacturers, however, optimally set a price w below zero because this yields strictly larger profits. Setting a price below zero has a negative externality on the rival retailer they do not internalize; it enables them to profitably exploit the incorrect beliefs. Following a deviation, the manufacturers make profits Π^P in future periods due to the grim-trigger beliefs. This results in the following opportunism-proofness condition

$$\frac{\Pi^C}{1-\delta} \geq \Pi^{JD} + \frac{\delta \Pi^P}{1-\delta}. \tag{11}$$

Comparing equation (11) with equation (8), the only difference is the deviation profit on the right-hand side. Because $\Pi^{JD} > \Pi^D$, the condition for opportunism-proofness is harder to satisfy than the stability condition above.

Proposition 2. *With grim-trigger beliefs, there exists an equilibrium in which manufacturers are able to sustain collusion on the industry-profit-maximizing wholesale price using grim trigger-strategies if the discount factor is high enough to satisfy equation* (9)*. This condition is equivalent to the incentive constraint when manufacturers and retailers are pairwise integrated, that is, under horizontal collusion. Under the stronger condition* (11)*, collusion is also opportunism-proof. The punishment is renegotiation-proof, but collusion is not formable.*

Proof. See Appendix. □

[18] Note that this holds for linear demand. In general, quasi-concavity depends on higher-order derivatives of demand at different loci of demand.

As derived above, the opportunism problem gives rise to the incentive condition (11) that is harder to satisfy (that is, only for more patient firms) than condition (8) for stable collusion. Hence, opportunism can make cartels less sustainable. Similar to the literature on the opportunism problem in single-shot games, this result depends on the beliefs. Due to their passive beliefs, the retailers do not react immediately to opportunism, which allows the manufacturers to 'trick' the retailers because the price that retailers expect will turn out to be incorrect if manufacturers jointly deviate. This joint deviation differs from a unilateral deviation of one manufacturer-retailer pair from a candidate equilibrium in which instead the belief of the deviating retailer is correct. In the latter case, it is optimal for the deviating manufacturer to set the wholesale price equal to the true costs. In the former case of a joint deviation, each retailer wrongly believes that the other retailer will buy at a high wholesale price and will thus sell at a high price, such that demand is high. It is, hence, profitable for the manufacturer to set a high fixed fee in return for a marginal wholesale price below costs because the retailer believes that it can sell a large quantity. A marginal wholesale price below costs, however, only becomes profitable for a manufacturer when the retailer has a wrong belief. It could thus signal the retailer that its belief is wrong because, if it were correct, the manufacturer's offer would be dominated by an offer with a wholesale price of $w = 0$. The stricter condition for opportunism-proofness is thus an implication of the passive nature of the beliefs. We later consider symmetric beliefs, where retailers' beliefs instantly react to any change in wholesale prices.

If marginal wholesale prices below marginal costs are impossible (for instance, due to competition law), the incentive constraint for opportunism and for stability are identical, implying that any stable cartel is also opportunism-proof. In any case, the opportunism problem is mitigated under grim-trigger beliefs compared to the one-shot game, or history-independent beliefs, because retailers with grim-trigger beliefs react to the deviation from the expected cartel wholesale prices by adapting their beliefs from the next period on to the level of competitive prices. This effectively punishes manufacturer opportunism, such that joint deviations are not profitable when manufacturers are sufficiently patient.

Interestingly, however, we find that under grim-trigger beliefs, collusion is *not* formable. Because these beliefs are unforgiving, they are beneficial in supporting the collusive equilibrium, but do not allow for new cycles of collusion, even after a long time.

The characterized equilibrium is also renegotiation-proof in the sense of (Farrell and Maskin, 1989). Because the opportunism-proofness condition (11) ensures that manufacturers have no incentives to deviate jointly from the collusive agreement, and credible punishment is fulfilled as well, the equilibrium is *weakly* renegotiation-proof. Because the equilibrium features collusion on the industry-profit-maximizing prices that are Pareto-optimal for the manufacturers, the equilibrium is also *strongly* renegotiation-proof.

Corollary 1. *The equilibrium with grim-trigger beliefs described in Proposition 2 is strongly renegotiation-proof if the opportunism-proofness condition* (11) *is satisfied.*

Note that this corollary extends naturally to general trigger-strategies and more general beliefs as employed in the next section. Recall that the concept of renegotiation-proofness is applied to the coalition of manufacturers; but the result may be more general if retailers are allowed to be part of the negotiation, as long as beliefs are not affected. Recall that we focus on collusion in which retailers do not take part. Fixing beliefs, however, manufacturers leave zero profits to retailers, making them effectively indifferent in renegotiation, such that a renegotiation-proof equilibrium would also be renegotiation-proof when including retailers.

For reference, consider a standard model of horizontal collusion absent a vertical dimension. In our model, this arises when each manufacturer-retailer pair is vertically integrated. Suppose that firms collude with grim-trigger strategies at the industry-profit-maximizing level and the usual stability condition is met. Such a collusion is formable because switching to grim-trigger strategies is an equilibrium of the continuation game independent of the history of actions in the game. Collusion is then also opportunism-proof because joint profits are maximized. In this industry structure, punishment is not renegotiation-proof because the firms would prefer to renegotiate to return to collusive prices. Hence, there are no renegotiation-proof equilibria in the case of standard horizontal collusion. By contrast, we will show that secret contracting can feature renegotiation-proof punishment and that opportunism-proofness can impose additional conditions on collusion.

4.2.2 Trigger Beliefs

In this section, we define trigger beliefs and derive the conditions for the sustainability and for other properties of collusion. First, consider that the manufacturers play trigger-strategies similar to the grim-trigger strategy above, but a deviation is forgiven after κ periods.

We define trigger-strategies as in Green and Porter (1984):

Formally, let $y = (w^C, w^C)$ be a profile of collusive wholesale prices and let $z = (w^P, w^P)$ be a punishment wholesale price profile. Let κ denote a time length measured in periods. Define period t to be collusive if

(a) $t = 0$, or

(b) $t - 1$ was collusive and $w_{it-1} = w^C$ for all i, or

(c) $t - \kappa$ was collusive and $w_{it-\kappa} \neq w^C$ for some i.

Define t to be reversionary otherwise. Manufacturer i sets

$$w_{it} = \begin{cases} y_i & \text{if } t \text{ is collusive,} \\ z_i & \text{if } t \text{ is reversionary.} \end{cases}$$

Assume that the collusive price equals the monopoly price ($w^C = w^M$), as defined in Subsection 4.2.1. Any deviation by one manufacturer causes the other manufacturer to carry out a punitive action of $w^P = 0$. Again, $w^P = 0$ must hold because a deviation during the reversionary periods is not punished as the future actions and beliefs are fixed. Hence, the punishment action must be the same as the short-term optimal action, that is $w^P = 0$ as we demonstrated before. This maximizes the manufacturer profits because it aligns the incentives of retailer and manufacturer. Similar to the previous section, trigger beliefs emerge when considering a wPBE in which manufacturers use trigger strategies instead of grim-trigger strategies.

Definition 5. (Trigger beliefs)

Choose a collusive price level p^C and a punishment price level p^P. Formally, the retailers' beliefs correspond to the manufacturer strategies for collusive and reversionary periods as defined above:

$$p^e_{-it} = \begin{cases} p^C & \text{if } t \text{ is collusive,} \\ p^P & \text{if } t \text{ is reversionary.} \end{cases}$$

When both manufacturers play trigger-strategies with actions w^C and w^P, the retailers' corresponding trigger-beliefs are correct on the collusive equilibrium path. They are also correct in the punishment phase.

Qualitatively, our insights for grim-trigger strategies carry over to more general trigger strategies. Trigger strategies with limited punishment imply stricter conditions for stability and opportunism-proofness compared to grim-trigger strategies. Nevertheless, they may be attractive for very relevant reasons that we do not model, including cost and demand shocks as well as other uncertainty that could result in unwarranted punishment, which – in case of grim-trigger strategies – would be very costly. We summarize our findings in the following proposition:

Proposition 3. *With trigger beliefs, there exists an equilibrium in which manufacturers are able to sustain collusion on the industry-profit-maximizing wholesale price (with trigger strategies) if the discount factor is high enough to satisfy condition* (20)*. This condition is equivalent to the incentive constraint when manufacturers and retailers are pairwise integrated. Furthermore, the condition requires more patience than under grim-trigger beliefs and strategies. Only under the stricter condition* (21)*, collusion is also opportunism-proof. Punishment is always renegotiation-proof, and collusion is never formable.*

Proof. See Appendix. □

The derived equilibrium with trigger beliefs is Pareto-efficient and features renegotiation-proof punishment. If, in addition, the manufacturers are patient enough for the condition of opportunism-proofness to hold, the equilibrium is strongly renegotiation-proof.

Corollary 2. *The equilibrium with trigger beliefs described in Proposition 3 is strongly renegotiation-proof if the opportunism-proofness condition* (21) *is satisfied.*

4.2.3 Adaptive Beliefs

After analyzing beliefs that anticipate collusion with (grim-)trigger strategies, we provide a simple example of a belief system where retailers are initially agnostic whether manufactures collude or compete. These beliefs try to capture that retailers learn heuristically about future wholesale price offers by adapting to past observed behavior. Imposing condition (1) implies that retailers do not revise their belief within a period, but they 'learn' afterwards from manufacturers' observed behavior.We use the so-called adaptive beliefs to analyze equilibria with (grim-)trigger strategies. We define the beliefs in a way that they only depend on the actions of the manufacturers in the last T periods and not on the full history of the game.

Definition 6. (Adaptive Beliefs)

Beliefs are passive within each period t, that is, w_{it} does not affect the belief p^e_{-it}. Beliefs are dynamic: p^e_{-it} can depend on the history of past wholesale prices. There are three relevant histories:

1. In period t, the manufacturers offer contracts that are identical to the ex ante beliefs in period t. Then, both retailers retain the same belief in period $t+1$.

2. In period t, both manufacturers play the same $w \in W^*$ that differs from the ex ante beliefs of the retailers in period t. The same holds for all previous periods up to $t-(T-1)$, with $T \in \{1, 2, ...\}$ being a parameter measuring the adaptation length in periods. In $t+1$, both retailers hold the new (passive) belief p^*. The set W^* contains wholesale prices that the retailers accept as possible equilibria (for example, the collusive wholesale price w^C).

3. In period t, at least one of the manufacturers does not play a price consistent with a retailer's ex ante belief, and it is not the case that both manufacturers play a price $w \in W^*$. In $t+1$, both retailers hold the new belief p^P, where p^P is the wholesale price of the perfect Bayesian Nash equilibrium of the stage game (Nash reversal).

The definition of adaptive beliefs differs from the definition of (grim-) trigger beliefs in that we allow retailers to take on any new belief $w \in W^*$ after observing w for sufficiently many periods (see point 2). The remaining requirements (point 1 and 3) are also present for (grim-) trigger beliefs.

In the following, we use the example of grim-trigger strategies to analyze the necessary incentive constraints implied by opportunism-proofness, renegotiation-proof punishment, and formability of collusion that arise from considering adaptive beliefs. We also solve for trigger strategies and present the results in the proposition, relegating the exposition to the proof.

Stability. The stability condition for collusion, once established, is

$$\frac{\Pi^C}{1-\delta} \geq \Pi^D + \frac{\delta \Pi^P}{1-\delta}. \tag{12}$$

In this case, retailers with adaptive beliefs already have the belief p^C and revert to the belief p^P after a deviation. Hence, with grim-trigger strategies, the condition is the same as the incentive condition (8). As a consequence, collusion is stable if manufacturers are sufficiently patient (δ is large enough). Again, the condition is identical to the stability condition under horizontal collusion.

Formability. Formability is fulfilled if there exists a transition path to the collusive wPBE, where – on the path – the manufacturers' actions are mutually best responses. Suppose that, starting from any history in period s, the manufacturers both start to play the collusive wholesale price w^C with the usual grim-trigger strategies that punish any deviation. If the manufacturers want to form collusion, the worst history in terms of our beliefs is that there was competition in $s-1$ (with profits Π^P). This implies that, for the next T periods, the retailers' beliefs are fixed at p^P, such that the manufacturers can extract lower transfers. Hence, in the transition periods, the manufacturer profits are lower than under competition, $\Pi^F < \Pi^P$. The reason is that the beliefs are identical in both cases, and whereas the manufacturers play their unique best response to the belief in periods of competition, which results in Π^P, they play a worse action with respect to stage-game profits as response to the same belief in transition periods, resulting in a profit of Π^F.

With grim-trigger strategies, manufacturers have an incentive to jointly start to collude in a competitive period, such that they eventually arrive at the collusive equilibrium path if

$$\frac{\Pi^P}{1-\delta} \leq \frac{1-\delta^T}{1-\delta}\Pi^F + \frac{\delta^T}{1-\delta}\Pi^C. \tag{13}$$

The left-hand side contains the present value of perpetual competition. The first term on the right-hand-side is the discounted profit of the T periods in which retailers' belief is p^P, while manufacturers actually set w^C; the second term is the discounted profit of perpetual collusion starting after T formation periods.

Because the manufacturer profits are lower during the formation period than under competition, each manufacturer may have an incentive to deviate to a lower wholesale price during formation. Deviating during the formation phase yields a period profit of $\Pi^{F,D}$ but triggers a punitive action forever. Consider the incentives to stick to w^C in the formation phase. No manufacturer wants to deviate unilaterally in the formation phase, which implies that actions in the transition are mutually best responses, if

$$\frac{1-\delta^T}{1-\delta}\Pi^F + \frac{\delta^T}{1-\delta}\Pi^C \geq \Pi^{F,D} + \frac{\delta\Pi^P}{1-\delta}. \tag{14}$$

Comparing the inequalities (13) and (14) shows that the latter is stricter if and only if $\Pi^{F,D} \geq \Pi^P$. This is always the case, as we demonstrate in the proof of Proposition 4. Hence, the deviation condition (14) is not only necessary but also sufficient for formability. Note that formability decreases in T, that is, it holds for a smaller set of discount factors because the left-hand side decreases in T. This holds because an increase in T, which only affects the left-hand side of condition (14), shifts the weight Π^C to the smaller term Π^F.

Renegotiation-proofness. Collusion is opportunism-proof if the manufacturers have no incentive to deviate jointly from the collusive price. Suppose the manufacturers jointly behave opportunistically in the present period. They can earn an opportunism profit of Π^{JD} by lowering w_i and increasing F_i for each retailer. As a result, the retailers believe in competition in the next period. Confronted with competitive beliefs, the reformation phase starts so that the manufacturers need to play w^C for T periods to convince retailers of collusive prices again. This yields the following condition of opportunism-proofness:

$$\frac{\Pi^C}{1-\delta} \geq \Pi^{JD} + \delta\left(\frac{\left(1-\delta^T\right)}{1-\delta}\Pi^F + \frac{\delta^T}{1-\delta}\Pi^C\right). \tag{15}$$

Collusion is formable and opportunism-proof if the manufacturers are sufficiently patient, that is, if condition (14) for formability and condition (15) for opportunism-proofness hold. Increasing the adaptation length T of the beliefs makes collusion less formable but more opportunism-proof, that is, relaxes condition (15), but tightens condition (14). Opportunsim-proofness is harder to satisfy than stability whenever collusion is formable. To see this, compare the right-hand side of condition (15) with the stability condition (12) and note that $\Pi^{JD} > \Pi^D$.

Punishment is not renegotiation-proof whenever collusion is formable. To see this, note that whenever the formability condition holds, and the manufacturers are supposed to punish, they are better off forming collusion again. This implies that formation dominates competitive pricing in the punishment phase and manufacturers prefer to enter the cooperative phase again. Note that grim-trigger strategies are a special case of trigger strategies with $\kappa \to \infty$. For trigger strategies, we generalize the result in the following proposition:

Proposition 4. *With adaptive beliefs and trigger strategies, the stability condition is the same as for vertically integrated collusion with the equivalent trigger strategies. Stability increases in the number of punishment periods κ. Collusion is formable and opportunism-proof if δ is sufficiently large. Increasing the adaptation periods T of the retailer beliefs makes collusion less formable but more opportunism-proof. Punishment is not renegotiation-proof for any $\kappa > 0$ whenever collusion is formable, but collusion may be sustained even without punishment, that is, for $\kappa = 0$.*

Proof. See Appendix. □

The take-away is that there exist adaptive beliefs that make collusion formable, sustainable, and opportunism-proof. Formability requires that beliefs can adapt to collusion after a 'history' of competition or punishment. The adaptivity can have the cost that punishment is not renegotiation-proof. This is different from trigger-beliefs where retailers do not accept collusive contracts after a deviation. Instead, with adaptive beliefs they do so, such that, whenever an adaptive belief features formability, punishment is not renegotiation-proof. An interesting observation is that, for adaptive beliefs, there is a trade-off between opportunism-proofness and formability: The longer beliefs take to adapt the harder it is to start collusion, whereas opportunistic behavior that counts on restarting collusion becomes less of a problem.

As a polar case, we find that the strategy to always collude, the degenerate trigger strategy with a punishment length of zero, can support a collusive equilibrium. If manufacturers play "always collusion", the incentive constraint for stability is

$$\frac{\Pi^C}{1-\delta} \geq \Pi^D + \delta\left(\frac{1-\delta^T}{1-\delta}\Pi^F + \frac{\delta^T}{1-\delta}\Pi^C\right). \tag{16}$$

Note that the stability is supported by the beliefs because deviation requires a new formation of collusion of length T. While this strategy is the least stable strategy, it is the only strategy that features formability and, in addition, features renegotiation-proof punishment.

Corollary 3. *For $\kappa > 0$, there exists a strongly renegotiation-proof equilibrium that is not formable if T and δ are sufficiently large. For $\kappa > 0$ and if the formability conditions* (13) *and* (14) *hold, there is no renegotiation-proof equilibrium with trigger strategies. For $\kappa = 0$, a strongly renegotiation-proof equilibrium exists if the opportunism-proofness condition* (15) *holds (which implies stability).*

Proof. See Appendix.

□

5 Extensions

5.1 Symmetric Beliefs

After considering passive beliefs, we turn to the analysis of the case that retailers have symmetric beliefs defined as follows:

Definition 7. (Symmetric Beliefs) The price expectation p^e_{-it} of retailer i in period t about the price of retailer $-i$ is $p_i\left(w_{it}, p(w_{it})\right)$.

In other words, when a retailer receives an unexpected offer deviating from the candidate equilibrium, the retailer revises its belief and believes that its rival has received the same offer by its manufacturer. Symmetric beliefs are history-independent because they only rely on the information contained in the current wholesale price offer. We assume that manufacturers play (grim-)trigger as in the previous sections. The case of symmetric beliefs is also analyzed in Liu and Thomes (2020) who consider the linear demand case and, hence, offer closed-form solutions for the critical discount factor. Symmetric beliefs may be particularly plausible if retailers expect that manufacturers coordinate their actions and expect that they do this in a symmetric way.

We focus again on industry-profit-maximizing collusion that naturally arises when manufacturers jointly maximize their profits given symmetric beliefs. Denote the price expectation of retailer i with symmetric beliefs by p^e_{it} as above. This allows the manufacturer to essentially choose the symmetric price level, such that the joint maximization problem of the manufacturers can be rewritten as

$$\Pi^C := \frac{1}{2}\max_p p \cdot D_i\left(p, p\right) + p \cdot D_{-i}\left(p, p\right) = \Pi^M.$$

Hence, the joint-profit-maximizing wholesale price of the manufacturers is equal to the industry-profit-maximizing price. Moreover, this implies that any joint deviation by the manufacturers will always be the industry-profit-maximizing price if retailers hold symmetric beliefs. In contrast to (grim-)trigger beliefs, where the optimal joint deviation of the manufacturers is to charge a wholesale price below zero, such a deviation is not optimal because only the collusive price maximizes the manufacturers' profits. Because the manufacturers collude at the Pareto-efficient level, collusion is opportunism-proof. Additionally, punishment is not renegotiation-proof because the manufacturers would prefer to revert back to collusion in every punishment period.

Collusion is formable because symmetric beliefs instantly adapt to the new wholesale price in every period. Manufacturers only need to agree on the collusive price and set it in any period. In a period in which the manufacturers set wholesale prices of w^C, the retailers' expectations are immediately equal to $p_i\left(w^C, p(w^C)\right) = p_i(w^C, p^C) = p^C$, which corresponds to the expectation of collusion. Forming collusion immediately leads to stable collusion as long as the stability condition is fulfilled.

From Pagnozzi and Piccolo (2011), we know that symmetric beliefs affect competition between vertically separated manufacturers. Competition is less fierce due to a so-called belief effect, which increases the competitive wholesale price above marginal costs. If punishment, however, relies on the competitive wholesale prices and profits, the stability of collusion is affected. Manufacturers must be more patient to satisfy the condition of stable collusion.

Symmetric beliefs violate the "no-signaling-what-you-don't-know" condition 1.[19] We thus look for a perfect Bayesian equilibrium in the context of symmetric beliefs:

Proposition 5. *With symmetric beliefs, there exists a collusive equilibrium with (grim-)trigger strategies if condition 26 holds. Collusion is also formable if the stability condition holds. Collusion is always opportunism-proof, but punishment is not renegotiation-proof. If condition 26 holds, both the competitive equilibrium and the collusive equilibrium are not renegotiation-proof.*

Proof. See Appendix. □

With symmetric beliefs there is no opportunism problem because lowering the wholesale prices negatively affects the belief such that retailers do not accept collusive contracts. Because the symmetric belief follows the manufacturers' actions, there is no long-term response of retailers and thus retailers do not discipline the manufacturers as, for example, the retailers' response does when they are holding trigger beliefs. This implies that formation, in the sense of needing to convince retailers, is not an issue with symmetric beliefs. As a downside for collusion, symmetric beliefs do not support the renegotiation-proofness of punishment because they make the punishment phase prone to renegotiation incentives. The reason is that the retailers always accept the collusive contracts, believing that wholesale prices are at

[19] Pagnozzi and Piccolo (2011) discuss this observation and also show that, with common cost shocks, symmetric beliefs can be consistent with a PBE.

the collusive level in the whole industry. During a punishment phase both manufacturers are able to renegotiate a joint increase of their wholesale prices again, which in turn will be accepted by each retailer. Therefore, manufacturers are able to return to the Pareto-dominant collusive equilibrium. This intuition also applies to the competitive equilibrium given that the manufacturers discount factor is sufficiently large to potentially support collusion.

5.2 Linear Wholesale Prices

For this extension we restrict the manufacturers' contracts to be linear. Each manufacturer offers a contract that only includes a wholesale price w_i.

Let us reconsider the case of history-independent beliefs. For two-part tariffs we have shown that there is no collusive equilibrium. In contrast, for linear tariffs we construct grim-trigger equilibria and show that for sufficient patience an equilibrium with collusive prices above the competitive price level always exists. Let us first define the competitive equilibrium. The one-shot wholesale prices with correct beliefs $p^N \equiv p^*(w^N, w^N)$ are defined by

$$w^N = \arg\max_{w} \; wD(p^*(w, p^N), p^*(w^N, p^N)).$$

This results in the competitive profits $\Pi^N = \max_w wD(p^*(w, p^N), p^N)$. Playing w^N in each period and retailers believing p^N is a wPBE of the repeated game.

Consider that manufacturers collude symmetrically at wholesale price of w^C using grim-trigger strategies. We denote the wholesale prices in punishment periods by w^P. A deviation manufacturer chooses a wholesale price denoted by w^D.

If there exists an equilibrium with a collusive price level of w^C, the beliefs have to be correct on the equilibrium path and equal $p^e = p^*(w^C, p^e)$. We construct such an equilibrium with time-constant beliefs to show the existence of a collusive equilibrium. If manufacturer A deviates in a period, the belief of the retailer of the non-deviating manufacturer B are still incorrectly at p^e. Similarly, in punishment periods, both retailers may hold incorrect beliefs, expecting the price to be at the collusive level, whereas the actual price level depends on w^P that is generally not equal to w^C. Different to the case of two-part tariffs, wrong retailers do not render collusion impossible with linear tariffs as the retailers mechanically accept different wholesale price levels, so that the effect of the wholesale prices on manufacturer profits are maintained.

Let us define the profits necessary to evaluate the stability of collusion:

$$\Pi^C \equiv w^C D(p^e, p^e),$$

$$\Pi^D = \max_{w} w \cdot D(p^*(w, p^e), p^e).$$

The punishment prices being mutual best responses yields

$$\begin{aligned} w^P &= \arg\max_w w \cdot D(p^*(w, p^e), p^*(w^P, p^e)), \\ \Pi^P &= w^P \cdot D(p^*(w^P, p^e), p^*(w^P, p^e)), \end{aligned}$$

where $p^*(w^P, p^e) < p^e$ holds for any $w^P < w^C$.

Assumption 2. *We assume that at the locus at $w^C = w^D = w^P = w^N$, the implicitly defined wholesale prices w^N ,w^D, and w^P are unique and $0 < \frac{\partial w^D(w^C)}{\partial w^C} < 1$ and $0 < \frac{\partial w^P(w^C)}{\partial w^C} < 1$ hold.*

This assumption holds globally for linear demand. The assumption implies that the wholesale prices have the usual ordering in a neighborhood around the competitive price level: If $w^C > w^N$, that is, if there is effective collusion, then $w^C > w^D > w^P > w^N$ holds (see proof of Proposition 6).

An equilibrium with grim-trigger strategies exists if there is a critical discount factor below one, such that the stability condition holds. This can be rewritten as

$$\Pi^D - \Pi^C < \Pi^C - \Pi^P. \tag{17}$$

Proposition 6. *Suppose that retailers hold history-independent beliefs and Assumption 2 holds. With linear wholesale prices, there is a collusive equilibrium with $w^C > w^N$ if manufacturers are sufficiently patient. This collusive equilibrium is also formable.*

Proof. See Appendix. □

While collusive equilibria with linear tariffs and prices just above the competitive price level exist if manufacturers are sufficiently patient, linear tariffs reduce the ability of manufactures to extract retail rents. Thus, overall, linear tariffs do not necessarily result in higher profits for manufacturers than two-part tariffs.

6 Policy Discussion

Our results provide new insights for various business practices from the perspective of competition policy.

Resale price maintenance (RPM). There is an ongoing policy debate about RPM that is typically considered anti-competitive in European competition policy and more benign in the US, at least since the Supreme Court's Leegin decision.[20] Our theory suggests that RPM can be pro-collusive by helping colluding manufacturers to convince retailers that competing products will have high prices. For this it is necessary that RPM is not just implemented as a secret contract clause but rather communicated in a way that competing retailers become

[20] See Leegin Creative Leather Products, Inc. v. PSKS, Inc., 551 U.S. 877 (2007).

aware of it. This may happen through the trade press, press releases, or other announcements – at least if RPM is legal. If it is illegal, recommended retail prices, which may for instance be printed on the products' price tags, or so-called minimum advertised price (MAP) restrictions can serve a similar purpose if the manufacturers incentivize the retailers to not deviate from them.[21]

Vertical integration. Vertical integration is often viewed as beneficial by solving various coordination issues within supply chains, such as that of double marginalization. A major concern, however, is market foreclosure (Rey and Tirole, 2007). Our theory suggests that vertical integration can facilitate collusion because retailers no longer need to be convinced of higher competing prices, which, as we show, may fail under various retailer beliefs. A potential downside of vertical integration is that, in certain cases, punishments for a deviation from collusion may not be renegotiation proof, although it can be for certain retailer beliefs under vertical separation (see Corollary 1).

Communication. Communication within supply chains is generally viewed to be beneficial to overcome coordination problems, whereas communication between competitors about sales prices and similar strategic variables is typically considered to be suspicious and may constitute a violation of the cartel prohibition.[22] Our theory suggests that communication among competing retailers about simultaneous new wholesale tariff offers of different manufacturers may help to turn competitive retailer beliefs into collusive beliefs, which facilitates manufacturer collusion. Hence, communication between retailers might not only facilitate collusion among them, but can also facilitate collusion in the upstream market.

Collusion through downsizing of packs. There are instances of coordinated downsizing of pack sizes by manufacturers, as observed in the chocolate case in Germany (Ritter). The collective reduction of the pack sizes while maintaining the old price of the larger pack may be used to reduce strategic uncertainty for retailers. The colluding manufacturers do not have to implement a price increase, which, depending on the retailer beliefs, may fail. Instead, it might be sufficient to show their retailers smaller packs of the competing brands to convince them that selling the smaller pack at the old price is competitive. If manufacturers present the retailers in a price negotiation with actually downsized packs of the own and a competing product, the claim may be more credible than the claim that other retailers face higher wholesale prices for competing products, especially if the adjustment of pack size is costly.

[21]See Asker and Bar-Isaac (2020) for a review of MAP restrictions. They also find that MAP facilitates collusion among manufacturers, where our model adds an additional channel, namely to overcome the opportunism problem towards retailers.

[22]See the "Guidelines on the applicability of Article 101 of the Treaty on the Functioning of the European Union to horizontal co-operation agreements", European Commission, Commission Communication 2023/C 259/01.

Buyback policies. Buyback policies can reduce the risk of retailers ending up with unsold units if their prices are not competitive. If retailers need to be convinced that the higher (collusive) wholesale prices are competitive to accept the manufacturer's contract, a buyback policy that insures the retailer in case its offer is not competitive can help.

7 Conclusion

Our approach of explicitly characterizing belief systems and discussing the equilibria consistent with such beliefs differs from the usual logic of Nash equilibrium, where players know other players' strategies fully. In particular, we consider that retailers do not fully anticipate the collusive strategy of manufacturers. While retailers' beliefs are correct on the equilibrium path, that is, retailers predict the correct price level in any collusive equilibrium, the existence of such an equilibrium depends also on retailers playing their role in the punishment phase which is necessary to support the equilibrium. We analyze three problems for the existence and plausibility of manufacturer collusion. First, the collusive equilibrium may fail to exist due to a lack of effective punishment. Second, the collusive equilibrium may lack credibility if the implied agreement is not renegotiation-proof, either on the collusive equilibrium path or during a punishment phase. In this context, we find that an opportunism problem, like the one a monopolist with secret contracts faces, is recreated by colluding firms.[23] Third, a collusive equilibrium may be implausible if formation, such as the transition from a competitive market to a collusive market, is not possible given the retailers' reactions.

When retailer beliefs do not anticipate manufacturer collusion, we demonstrate that collusion is infeasible with two-part tariffs. Such beliefs may arise in industries that have long-standing competitive conduct. Belief differences between industries may explain why some industries stay competitive, whereas other industries give birth to collusion repeatedly. Because these beliefs give rise to equilibria of the infinitely repeated game and, consequently, are correct on the equilibrium path, they are self-fulfilling and may never be challenged.

When retailer beliefs react to observed past actions, trigger-based manufacturer collusion becomes feasible and the punishment may even be renegotiation-proof, although this would not be the case under vertical integration of the industry. We show that opportunism can still be the most important challenge for the colluding manufacturers, more so than the usual unilateral deviation incentives.

Trigger beliefs and particularly grim-trigger beliefs can feature renegotiation-proof punishment and opportunism-proofness because they make it difficult for manufacturers to obtain high wholesale prices again after a breakdown. However, they do not allow for the formation of collusion. Formation requires that, to transition from a competitive to a collusive outcome, beliefs must adapt to the change in contract offers. The intuition here is that trigger beliefs cannot handle the transition from a non-collusive state to successful collusion because they

[23]This highlights that communication among manufacturers may be a source of misinterpretation for retailers. This contrasts the understanding that communication usually reduces strategic uncertainty, as discussed by Blume (1994) and Blume and Heidhues (2008).

would be too "pessimistic" about the future whenever they observe a non-collusive price in the history of the game.

We introduce new beliefs that are adaptive to the manufacturers' pricing over time. These retailer beliefs facilitate the formation of manufacturer collusion. As the beliefs do not mirror the collusive strategy, they do not support renegotiation-proof punishment; they may still support a credible collusive strategy because retailers 'punish' deviation by believing in competitive conduct after a deviation. Adaptive beliefs can also satisfy the conditions for stability and opportunism-proofness.

Because manufacturer cartels are ubiquitous, our results can help competition authorities to screen markets. Our model shows that, whenever supply contracts are not public or easily renegotiable, the ability to form and sustain collusion critically depends on retailers' beliefs about the supply conditions of other retailers. This may make it easier to sustain collusion in markets in which the retailers are used to manufacturer collusion. Our findings suggest that it should be in the interest of colluding manufacturers to manage and influence their retailers' beliefs about the conditions in the wholesale market. One conjecture is thus that the opportunism problem may be one of the causes behind the widespread use of resale price maintenance and hub-and-spoke arrangements when manufacturers collude. A more direct control of retail prices by manufacturers in form of resale-price maintenance may circumvent the problem of skeptical retailer beliefs. Similarly, coordinated downsizing of packages by manufacturers, as observed in the chocolate case in Germany, may be used to reduce strategic uncertainty for retailers.[24] Consequently, coordinated behavior and communication of manufacturers vis à vis their retailers may deserve more antitrust scrutiny because such coordination can be essential for making manufacturer cartels work.

Competition authorities should also be aware that announcements by industry associations to raise prices can help member firms to overcome the opportunism problem in the context of collusion. When such an announcement comes from an authoritative body, downstream firms can expect that price increases will affect the whole industry and that they will not lose out vis à vis their competitors when accepting higher input prices.

Another aspect that should be of interest for competition authorities is that (sudden) changes of (industry-wide) input prices – for example, due to inflation or an uncertain economic environment – can be used by manufacturers to increase their prices collusively to a larger extent than such cost changes would imply. These changes may lead to a situation in which downstream firms' expectations are less skeptical with regard to price increases, such that the transition to supra-competitive prices and profits is facilitated.[25]

[24]See Section 1.

[25]Recent developments in the retail food sector point in this direction (see, for example, Der Spiegel, "Is retail ripping off consumers?", 03/23/2023; last access 06/25/2023).

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Appendix: Proofs

Proof of Lemma 1. Plugging $p_{-i}(w_{-it}, p^e_{it}) = p^e_{-it}$ in the manufacturer problem 2. Simplifying yields

$$\Pi_{it}\left(w_{it}, p^e_{-it}\right) = p_i\left(w_{it}, p^e_{-it}\right) \cdot D_i\left(p_{it}\left(w_{it}, p^e_{-it}\right), p^e_{-it}\right).$$

Maximizing with respect to w_{it} gives the first-order condition

$$\frac{\partial p_i\left(w_{it}, p^e_{-it}\right)}{\partial w_{it}} D_i\left(p_i\left(w_{it}, p^e_{-it}\right), p^e_{-it}\right) + \frac{\partial D_i\left(p_i\left(w_{it}, p^e_{-it}\right), p^e_{-it}\right)}{\partial p_i} \frac{\partial p_i\left(w_{it}, p^e_{-it}\right)}{\partial w_{it}} p_i\left(w_{it}, p^e_{-it}\right) = 0$$
$$\iff \frac{\partial p_i\left(w_{it}, p^e_{-it}\right)}{\partial w_{it}} \frac{\partial D_i\left(p_i\left(w_{it}, p^e_{-it}\right), p^e_{-it}\right)}{\partial p_i} w_{it} = 0. \tag{18}$$

Because the first term on the left-hand side in the previous line is assumed to be strictly positive and the second term is assumed to be strictly negative (Assumption 1), the only solution to the first-order condition is $w_{it} = 0$.

Proof of Proposition 1.

Proof. Step (i): Let us construct this equilibrium by showing that it is uniquely optimal for each manufacturer to set $(w_i = 0, F_{it} = \pi^*_i(w_i = 0, p^e_{-it}))$ in each period independent of the strategy of the other manufacturer.

The participation constraint of the retailer is binding in equilibrium because otherwise the manufacturer could increase profits by raising the fixed fee without affecting p^e_{-it} by the assumption of passive beliefs. Hence, $F_{it} = \pi^*_i(w_i = 0, p^e_{-it}))$ holds.

On the equilibrium path, $p^e_{-it} = p^*_{-it}$, that is, retailers' beliefs are correct and thus identical to manufacturer's conjectures such that the in-period manufacturer profits can be simplified as in the stage game to:

$$\Pi_{it}\left(w_{it}, p^e_{-it}\right) = p_i\left(w_{it}, p^e_{-it}\right) \cdot D_i\left(p_{it}\left(w_{it}, p^e_{-it}\right), p^e_{-it}\right).$$

Note that this in-period profit is insulated from the actual actions of the other manufacturer, and w_{it} only affects the manufacturer profits through the price setting of the retailer. Because the manufacturers' profit in each period, on any equilibrium path, only depends on w_{it} and the belief p^e_{-it}, the discounted equilibrium profits of a manufacturer do not depend on the strategy of the other manufacturer either. Fixing p^e_{-it}, equation (18) implies that $w_i = 0$ is optimal (independent of w_{-it} in each period). Hence, there is an equilibrium path with each manufacturer setting $w_i = 0$ in every subgame and a matching time-constant belief by retailers, where the time-constant retail price follows because the retail price equilibrium is unique for $w_i = w_{-i} = 0$.

Step (ii) Next, we exclude other equilibrium paths in which $w_{it} \neq 0$ by contradiction: Suppose that there is an equilibrium path with $w_{it} \neq 0$ in some periods. It follows from equation 18 that each manufacturer can increase current period profits through setting $w_{it} = 0$ and $F_{it} = \pi(0, p^e_{-it})$, resulting in $\Pi_{it}\left(0, p^e_{-it}\right)$. In particular, in every period, manufacturer i can ensure at least a profit of $\Pi(0, p^e) = \max_p \pi(0, p^e) = \max_p pD(p, p^e)$, which is independent of the actual wholesale price of the other manufacturer. This profit is strictly larger than the profit in any candidate equilibrium with $w' \neq 0$. Formally, this is $\Pi(w', p^e) < \Pi(0, p^e) \Leftrightarrow w'D(p^e) + (p^e - w')\, D(p^e) = p^e D(p^e) < \max_p pD(p, p^e)$, where the strict inequality follows from the uniqueness of the maximizer and the strict monotonicity of p_i in w_i .

Since each manufacturer can ensure this deviation profit of $\Pi(0, p^e)$ in any period independently of the other manufacturers actions, a profitable deviation from any candidate equilibrium with $w' \neq 0$ always exists and this deviation is immune to punishment by the other manufacturer.

As we employ beliefs about p^e_{it} instead of beliefs about w^e_{it}, we establish that this is equivalent in our setting if retailers have common knowledge about the belief system and expect sequentially rational choices of their rivals. More precisely, for any history $\mathcal{H}_t$ and belief w^e_{it}, there exists a unique implied belief. Expecting a sequentially rational choice implies that the belief of retailer A, p^e_{Bt}, is the result of $p^e_{Bt} = \arg\max_{p_{Bt}} \pi_B(w^e_{Bt}, w^e_{At} = w^{2e}_{At})$, where w^{2e}_{At} is the higher-order belief of retailer A about retailer B's belief. The identity $w^e_{At} = w^{2e}_{At}$ follows from common knowledge about the belief system and condition 1. Due to condition 1 beliefs are within-period passive, such that the higher-order belief w^{2e}_{At} is independent of retailer A's private information w_{At}. Common knowledge about the belief system implies then that retailers hold correct conjectures about higher order beliefs. Therefore, given w^e_{-it}, retailer i correctly predicts w^{2e}_{it} and thus the unique $p^e_{-it} = \arg\max_{p_{-it}} \pi_{-i}(w^e_{-it}, w^{2e}_{it})$. This follows from Assumption 1, which implies that there is a unique solution to the maximization problem.

$\square$

Proof of Proposition 2.

Proof. We established in the main text that manufacturers are able to *sustain* collusion if equation (9) is fulfilled. To show that the incentive constraint is the same if manufacturers and retailers are pairwise integrated, it is sufficient to prove that the profits are the same: $\Pi^j_V = \Pi^j_I$ for $j = C, D, P$ (V: vertical, I: integrated). Because we define Π^C_V to be half of the integrated industry-maximizing profit (4), the profit from collusion in the integrated case Π^C_I is the same.

The deviation profit Π^D_V is given by the first line of equation (5) and simplifies further in the second line because the manufacturers' true actions and retailers' beliefs are aligned. The second line, however, is equal to the maximization problem of an integrated manufacturer that maximizes the profit with respect to the retail price. Lastly, the punishment profits are aligned as well, following the same argument as before. The punishment profits are given by equation (7). Manufacturers set $w^P = 0$, which results in the same retail prices and profits as in a vertically integrated industry.

To see that punishment is renegotiation-proof, note that with grim-trigger beliefs, any deviation leads to the belief p^P forever such that the current actions of the manufacturers have no effect on the belief of retailers. To establish this, we check for a joint action by manufacturers that would yield a Pareto improvement for manufactures. Note that it is the best response is, in each period, for each manufacturer, to set $w = 0$ individually. Next, we consider the profit maximization for manufacturers when they would optimize jointly during a punishment period:

$$\frac{1}{2} \max_{w_{it}, w_{-i,t}} w_{it} \cdot D_i \left(p_i(w_{it}, p^P), p_{-i}(w_{-it}, p^P) \right) + \left[p_i(w_{it}, p^P) - w_{it} \right] \cdot D_i \left(p_i(w_{it}, p^P), p^P \right)$$
$$+ w_{-it} \cdot D_{-i} \left(p_{-i}(w_{-it}, p^P), p_i(w_{it}, p^P) \right) + \left[p_{-i}(w_{-it}, p^P) - w_{-it} \right] \cdot D_{-i} \left(p_{-i}(w_{-it}, p^P), p^P \right).$$

We again assume that this profit is quasi-concave such that we can use first-order conditions. Using the retailers' first-order conditions and applying symmetry, using that manufacturer profits are well-behaved by assumption such that the optimum is symmetric, the first-order conditions can be rewritten as

$$\left[\underbrace{D_i \left(p_i(w, p^P), p_{-i}(w, p^P) \right) - D_i \left(p_i(w, p^P), p^P \right)}_{\geq 0} \right] + w \underbrace{\frac{\partial p_i}{\partial w_{it}}}_{>0} \left[\underbrace{\frac{\partial D_i}{\partial p_i} + \frac{\partial D_{-i}}{\partial p_i}}_{<0} \right] = 0 \ \forall i. \tag{19}$$

This holds for $w = 0$. Hence, playing $w = 0$ is the best manufacturers can do such that there is no continuation game that can be reached by manufacturers and that yields larger profits.

To see that collusion is *not formable*, consider any history with $w_i \neq w^C$ in $t = 0$. With this history, grim-trigger beliefs imply that the belief is p^P forever, which violates the second condition of the definition of formability that a collusive PBE can be obtained in some future period for any history. □

Proof of Proposition 3.

Proof. Consider the conditions that are needed to sustain collusion. After any deviation by a manufacturer, the beliefs revert to $p^e_{-i} = p^P$ for κ periods, which results in profits of $\Pi^P = p^P \cdot D_i(p^P, p^P)$. After κ periods, however, the retailers believe in collusion at a price of p^C again. Collusion is sustainable when no manufacturer wants to deviate from the trigger strategy given by the incentive condition:

$$\frac{\Pi^C}{1-\delta} \geq \Pi^D + \delta \left(\frac{1-\delta^\kappa}{1-\delta} \Pi^P + \frac{\delta^\kappa}{1-\delta} \Pi^C \right). \tag{20}$$

This condition is more difficult to fulfill than the incentive condition (9) for grim-trigger strategies and beliefs. The punishment in condition (20) is less harsh and ends after κ periods, such that the expression on the right-hand side is larger than in the condition with grim-trigger strategies. Recall that manufacturers make the same profits as pairwise integrated manufacturer-retailer pairs (Proposition 2). Note that the individual profits, Π^C, Π^D, and Π^P, are still identical to an integrated firm's profits. This implies that the stability condition (20) is the same for vertically separated and vertically integrated manufacturer-retailer pairs whenever they play trigger strategies of length κ.

To check whether the equilibrium is opportunism-proof, we must consider a revised version of condition (11) that applies to trigger beliefs. Because trigger beliefs are forgiving after κ periods, a joint deviation of both manufacturers is not "punished by the beliefs" forever.[26] Hence, the conditions can be written as

$$\frac{\Pi^C}{1-\delta} \geq \Pi^{JD} + \delta\left(\frac{1-\delta^\kappa}{1-\delta}\Pi^P + \frac{\delta^\kappa}{1-\delta}\Pi^C\right). \tag{21}$$

Again, the opportunism-proofness condition (21) resembles the stability condition (20), except that $\Pi^{JD} > \Pi^D$, which makes the condition harder to meet. If condition (21) holds, collusion is robust against joint deviations by the manufacturers, that is, opportunistic behavior.

The punishment is renegotiation-proof because, in a punishment phase, both retailers believe $w^P = 0$ to which the best response is $w = 0$ as well. Beliefs are constant in the wholesale price played in the punishment phase. Thus, the argument in Subsection 4.2.1 applies, such that focusing on short-term best responses is valid.

To see that collusion is not formable with trigger beliefs, consider an argument by contradiction. Formability requires that the PBE under consideration can arise after an arbitrary history of the game. Let us consider a specific history. Suppose that, for the first κ periods, the manufacturers do not collude. With this history, all the following periods starting with period $\kappa + 1$ are labeled as non-collusive. It follows that it is impossible for retailers to hold collusive beliefs at any future point of the game. Recall that periods are only labeled collusive after the first period if in period t, the previous period $t-1$ was collusive or period $t-\kappa$ was collusive. $\square$

Proof of Proposition 4.

Proof. Similarly to the incentive constraints above the proposition for grim-trigger strategies, we can also describe the incentive constraints when manufacturers play more general trigger strategies. The difference between these strategies is the length of the punishment phase κ. Because neither the formation condition (13) nor the opportunism-proofness condition (15) rely on punishment between the colluding firms, the conditions remain the same.

In the remainder of the proof, we characterize the stability condition and the stability condition of the formation phase for trigger strategies. If firms play trigger strategies with

[26] By contrast, the grim-trigger beliefs switch forever to the competitive price level in response to a deviation. This effectively punishes the manufacturers that face pessimistic retailers from then on.

punishment length κ, the incentive constraint for stability is

$$\frac{\Pi^C}{1-\delta} \geq \Pi^D + \delta\left(\frac{1-\delta^\kappa}{1-\delta}\right)\Pi^P + \delta^{\kappa+1}\left(\frac{1-\delta^T}{1-\delta}\right)\Pi^F + \delta^{\kappa+1}\frac{\delta^T}{1-\delta}\Pi^C.$$

As in all the above cases, the punishment must be $w^P = 0$ because there is no unilateral action in a punishment phase that has any effect on future beliefs. Because the future beliefs and actions of the rival manufacturer are fixed in any punishment period, each manufacturer must play the one-shot best response of $w^P = 0$.

The deviation of one manufacturer triggers a punishment of length $\kappa \in [0, \infty)$. After the punishment phase, collusion is resumed, but the adaptive retailer beliefs require a formation phase of length T. A deviation in the formation phase is not profitable if

$$\frac{1-\delta^T}{1-\delta}\Pi^F + \frac{\delta^T}{1-\delta}\Pi^C \geq \Pi^{F,D} + \delta\left(\frac{1-\delta^\kappa}{1-\delta}\right)\Pi^P + \delta^{\kappa+1}\left(\frac{1-\delta^T}{1-\delta}\right)\Pi^F + \delta^{\kappa+1}\frac{\delta^T}{1-\delta}\Pi^C, \quad (22)$$

which simplifies to

$$\delta^T\left(\Pi^C - \Pi^F\right) \geq \left(\frac{(1-\delta)}{1-\delta^{\kappa+1}}\right)\Pi^{F,D} - \Pi^F + \delta\left(\frac{1-\delta^\kappa}{1-\delta^{\kappa+1}}\right)\Pi^P. \quad (23)$$

Next, we show that condition (13) is always stricter than condition (22). We also demonstrate that this is the relevant condition for formation. To see this, we consider the polar cases that correspond to a strategy "always collude" for $\kappa = 0$ and the grim-trigger strategy as $\kappa \to \infty$. We already know from the analysis before the proposition that the postulated relation of the conditions holds for $\kappa \to \infty$.

For $\kappa = 0$, the incentive constraint for formation can be written as

$$\delta^T\left(\Pi^C - \Pi^F\right) \geq \Pi^P - \Pi^F.$$

A deviation from the collusive price in the formation phase results in a profit of $\Pi^{F,D}$. However, for $\kappa = 0$, no punishment is triggered. The stability condition for formation becomes

$$\frac{1-\delta^T}{1-\delta}\Pi^F + \frac{\delta^T}{1-\delta}\Pi^C \geq \Pi^{F,D} + \delta\left(\frac{1-\delta^T}{1-\delta}\frac{\Pi^F}{1-\delta} + \frac{\delta^T}{1-\delta}\Pi^C\right),$$

which simplifies to

$$\delta^T\left(\Pi^C - \Pi^F\right) \geq \Pi^{F,D} - \Pi^F. \quad (24)$$

Observe that a deviation in the formation phase is not profitable under a stricter condition than the incentive condition for formation if $\Pi^{F,D} \geq \Pi^P$. Additionally, a necessary condition is $\Pi^C \geq \Pi^{F,D}$ because the discount factor δ is in the range $(0, 1)$. Because $\delta < 1$, the left-hand side of condition (24) decreases in T, such that formability becomes harder to satisfy the larger T.

Thus, for both cases of $\kappa = 0$ and $\kappa \to \infty$, we demonstrated that sticking to the formation phase requires higher values of δ (the condition is more strict) if $\Pi^{F,D} \geq \Pi^P$. Let us now analyze intermediary values of κ.

Inequality (23) differs from the formation incentives. Recall that the formation condition (13) is

$$\delta^T \left(\Pi^C - \Pi^F\right) \geq \Pi^P - \Pi^F$$

and thus independent of κ. For a given value of δ, the weight of the term $\Pi^{F,D}$ is strictly monotonically decreasing in κ, whereas the weight of Π^P is strictly monotonically increasing in κ. Hence, the condition $\Pi^{F,D} \geq \Pi^P$ is sufficient for any $\kappa \in [0, \infty)$ to guarantee that condition (23) is tighter than condition (13). This implies that the stability condition for formation is the relevant condition for general trigger strategies.

Next, we show that

$$\Pi^{F,D} \geq \Pi^P$$

holds because the profit of i increases in the wholesale price of the competitor for given beliefs due to the fact that this increases the price of the competing retailer and increases demand for i. The profit under competition is

$$\Pi^P \left(w_i = 0, p^e_{-i} = p^P\right) = \underbrace{p_i \left(0, p^P\right)}_{p_i^P} \cdot D_i \left(p^P, p^P\right).$$

A firm that deviates from formation plays $w_i = 0$ because this maximizes the unilateral spot profit. The deviation profit $\Pi^{F,D}$ is similar to the punishment profit and only consists of the revenue $p_i(\cdot) \cdot D_i(\cdot)$. For $\Pi^{F,D}$, however, both the manufacturer and the retailer expect a higher wholesale price of w^C of the other manufacturer, yielding a higher price at the competing retailer. Because profits increase in the rival's price, $\Pi^{F,D} > \Pi^P$ holds.

Finally, observe that punishment is not renegotiation-proof whenever collusion is formable. To see this consider that the formability condition implies that is better to jointly deviate to a new formation of collusion in any period in which punishment profits are expected. This also implies that only in the case $\kappa = 0$, formability and renegotiation-proof punishment are both met because punishment is not part of the strategy. □

Proof of Corollary 3.

Proof. The equilibrium path is always Pareto-efficient by assumption, such that any weakly renegotiation-proof equilibrium is also strongly renegotiation-proof.

To find the critical value of T, such that collusion is not formable, but opportunism-proof for $\kappa > 0$, we must compare the relevant conditions. If collusion is not formable, punishment will be renegotiation-proof. The equilibrium is thus weakly renegotiation-proof for certain values of T. As shown in the proof of Proposition 4, the relevant condition for formation is the condition whereby a deviation from formation is not profitable:

$$\delta^T \left(\Pi^C - \Pi^F\right) \geq \left(\frac{1-\delta}{1-\delta^{\kappa+1}}\right) \Pi^{F,D} - \Pi^F + \delta \left(\frac{1-\delta^{\kappa}}{1-\delta^{\kappa+1}}\right) \Pi^P > 0.$$

Because $\delta < 1$, this condition is violated for sufficiently large T, such that δ^T and thus the left-hand side of the formation condition becomes arbitrarily small.

The condition for opportunism-proofness 15 can be written as

$$\delta^T \left(\Pi^C - \Pi^F\right) \leq \frac{1}{\delta} \left[\Pi^C - (1-\delta)\, \Pi^{JD} - \delta \Pi^F\right].$$

Because for $\delta < 1$, the left-hand side of the opportunism-proofness condition also becomes arbitrarily small as T increases. Hence, there exists a T, such that the opportunism condition holds, whenever the right-hand side is non-negative (which holds for sufficiently large δ), while the formation condition is violated. $\square$

Proof of Proposition 5.

Proof. We will show all results for general trigger strategies as defined in Subsection 4.2.2, which includes grim-trigger strategies as a subcase for $\kappa \to \infty$. Let us first consider the stability condition. As argued in the text before the proposition, manufacturers collude on the industry-maximizing level and earn profits of $\Pi^C = \Pi^M/2$. If the manufacturers make a one-sided deviation from the collusive agreement, they must consider that changing their wholesale price also influences the belief of their own retailer. A deviating manufacturer maximizes the following problem:

$$\begin{aligned} w^D = \arg\max_{w_i}\, & w_{it} \cdot D_i \left(p_i(w_{it}, p(w_{it})), p_{-i}(w^C, p(w^C))\right) \\ & + \left(p_i(w_{it}, p(w_{it})) - w_{it}\right) \cdot D_i \left(p_i(w_{it}, p(w_{it})), p(w_{it})\right)). \end{aligned}$$

This results in a profit Π^D for the deviating manufacturer. Note that $w^D > 0$. Punishment is assumed to be carried out on the competitive wholesale price level. The competitive benchmark corresponds to the case analyzed in Pagnozzi and Piccolo (2011). Under symmetric beliefs, manufactures solve

$$\max_{w_i} w_{it} \cdot D_i \left(p_i \left(w_{it}, p(w_{it})\right), p_{-i} \left(w_{-it}, p(w_{-it})\right)\right) + \left(p_i(w_{it}, p(w_{it})) - w_{it}\right) \cdot D_i \left(p_i \left(w_{it}, p(w_{it})\right), p(w_{it})\right).$$

This results in Π^P. By using the envelope theorem, the first-order condition simplifies to

$$\underbrace{w_i \underbrace{\frac{\partial D_i(\cdot)}{\partial p_i}}_{<0}}_{<0} + \underbrace{\left(p(w_i, p(w_i)) - w_i\right) \cdot \underbrace{\frac{\partial D_i(\cdot)}{\partial p_{-i}}}_{>0}}_{\text{belief effect:}>0} = 0. \tag{25}$$

Applying symmetry to equation (25) defines the equilibrium wholesale prices w^P. Note that the second term of (25) is positive at $w_i = 0$, which implies that $w^P > 0$. Under competition, with symmetric beliefs, prices are above the price level under competition with passive beliefs. The stability condition is given by

$$\frac{\Pi^C}{1-\delta} \geq \Pi^D + \delta \left(\frac{1-\delta^\kappa}{1-\delta} \Pi^P + \frac{\delta^\kappa}{1-\delta} \Pi^C \right). \tag{26}$$

To see why collusion is formable with symmetric beliefs, recall the definition of formability. Symmetric beliefs allow for forming collusion if there exists a strategy profile, such that for this belief, best responses are played in period $t \geq s$, and there exists a weak PBE that results in payoff V^C. Given any history before period s – the period in which collusion is about to be formed –, manufacturers play mutually best responses in the following periods when setting wholesale prices that result in the collusive price defined by equation (3). The joint profit maximization of both manufacturers and setting w^C is a weak PBE if the stability condition for collusion – equation (26) – is fulfilled. In this equilibrium, both manufacturers earn a payoff of $V^C = \Pi^C/(1-\delta)$. Hence, collusion is formable according to our definition.

As in Subsection 4.2.3, it should be considered whether there exist incentives to deviate from formation. Due to symmetric beliefs, collusion is in place directly after it is formed, such that the condition for deviating from formation is identical to deviating from collusion in equation (26).

To check whether collusion is opportunism-proof, we consider the joint maximization problem of the manufacturers. As shown in the text, this leads to the Pareto-efficient wholesale price level. That is, manufacturers always prefer to set w^C when jointly maximizing. Thus, there is no scope for opportunistic behavior because the beliefs directly react to any change in wholesale prices. Following the same argument, punishment is not renegotiation-proof because manufacturers would prefer to renegotiate and jointly revert to setting w^C.

The competitive equilibrium is not renegotiation proof whenever a collusive equilibrium with higher manufacturer profits exists. Hence, if condition 26 holds, there exists a Pareto-dominant equilibrium for manufacturers that can be reached by a joint deviation of the manufacturer. With symmetric beliefs retailers are immediately willing to accept the collusive contract expecting wholesale prices to increase in the whole industry. □

Proof of Proposition 6.

Proof. If $0 < \frac{\partial w^D(w^C)}{\partial w^C}|_{w^C = w^D} < 1$ holds, then increasing w^C implies $w^C > w^D$. Then, $w^C > w^N$ (by construction) implies that $w^D > w^N$. Using $w^C > w^D$, we show that

$w^D > w^P$ holds: Note that $w^D > w^P \Leftrightarrow p^e > p^*(w^P, p^e) \Leftrightarrow w^P < w^C$. The last inequality follows again from the argument that starting at $w^C = w^N$, increasing w^C implies by $\frac{\partial w^P(w^C)}{\partial w^C} < 1$ that $w^C > w^N$ implies $w^C > w^P$. Hence, $w^C > w^P$ implies that $w^D > w^P$must hold.

Rewrite condition (17) as

$$2\Pi^C - \Pi^P - \Pi^D > 0.$$

This can be rewritten to

$$\int_{w^p}^{w^c} wD(p^*(w,p^e),p^*(w,p^e))dw + \int_{w^D}^{w^c} wD(p^*(w,p^e),p^e)dw$$

$$= \int_{w^p}^{w^D} wD(p^*(w,p^e),p^*(w,p^e))dw + \int_{w^D}^{w^c} w\left[D(p^*(w,p^e),p^e) + D(p^*(w,p^e),p^e)\right]dw. \quad (27)$$

Note $2\Pi^C - \Pi^P - \Pi^D = 0$ holds at $w^C = w^N$. Thus if, at this locus, the derivative of (27) with respect to w^C is positive, then the stability condition holds for some $w^C > w^N$and some $\delta < 1$. Thus, there is a collusive equilibrium if, at $w^C = w^N$,

$$\partial\left(\int_{w^p}^{w^D} wD(p^*(w,p^e),p^*(w,p^e))dw + \int_{w^D}^{w^c} w\left[D(p^*(w,p^e),p^e) + D(p^*(w,p^e),p^e)\right]dw\right)/\partial w^C > 0.$$

Note that, at $w^C = w^N$, it holds that $w^D = w^P = w^N = w^C$. By Leibniz integral rule we have

$$\begin{aligned}
&\partial\left(\int_{w^p}^{w^D} wD(p^*(w,p^e),p^*(w,p^e))dw + \int_{w^D}^{w^c} w\left[D(p^*(w,p^e),p^e) + D(p^*(w,p^e),p^e)\right]dw\right)/\partial w^C \\
&= w^D D(p^*(w^D,p^e),p^*(w^D,p^e))w^{D\prime} - w^P D(p^*(w^P,p^e),p^*(w^P,p^e))w^{P\prime} \\
&+ \int_{w^p}^{w^D} \left[wD(p^*(w,p^e),p^*(w,p^e))\right]' dw + w^C\left[D(p^*(w^C,p^e),p^e) + D(p^*(w^C,p^e),p^e)\right] \\
&-w^D\left[D(p^*(w^D,p^e),p^e) + D(p^*(w^D,p^e),p^e)\right]w^{D\prime} + \int_{w^D}^{w^c}\left[w\left[D(p^*(w,p^e),p^e) + D(p^*(w,p^e),p^e)\right]\right]' dw
\end{aligned}$$

Evaluation the derivative at $w^D = w^P = w^N = w^C = w^*$ yields

$$\begin{aligned}
&w^D D(p^*(w^D,p^e),p^*(w^D,p^e))w^{D\prime} - w^P D(p^*(w^P,p^e),p^*(w^P,p^e))w^{P\prime} \\
&+ \underbrace{\int_{w^p}^{w^D} \left[wD(p^*(w,p^e),p^*(w,p^e))\right]' dw}_{=0} + w^C\left[D(p^*(w^C,p^e),p^e) + D(p^*(w^C,p^e),p^e)\right] \\
&-w^D\left[D(p^*(w^D,p^e),p^e) + D(p^*(w^D,p^e),p^e)\right]w^{D\prime} + \underbrace{\int_{w^D}^{w^c}\left[w\left[D(p^*(w,p^e),p^e) + D(p^*(w,p^e),p^e)\right]\right]' dw}_{=0}.
\end{aligned}$$

Define $D^* = D(p^*(w^*,p^e),p^*(w^*,p^e)) = D(p^*(w^*,p^e),p^e)$ with $p^*(w^*,p^e) = p^e$. The above expression can then be rewritten as

$$w^* D(p^*(w^*,p^e),p^*(w^*,p^e))(w^{D\prime} - w^{P\prime}) +$$

$$w^*\left[D(p^*(w^*,p^e),p^e) + D(p^*(w^*,p^e),p^e)\right](1 - w^{D\prime})$$

$$= w^* D^*(2 - w^{D\prime} - w^{P\prime}).$$

As $w^*D^* > 0$, this is larger than zero if and only if

$$w^{D\prime} + w^{P\prime} < 2,$$

where $w^{D\prime} = \frac{\partial w^D(w^C)}{\partial w^C}|_{w^D = w^C}$. This holds if $w^{D\prime} < 1$ and $w^{P\prime} < 1$, which holds by Assumption 2.

Formability requires that the wPBE can be reached from an arbitrary history through a sequentially rational transition. For the collusive wPBE, the system of beliefs in this case is simply that, starting in period t_0, the beliefs are constant in w^C for both retailers. Thus, if the stability condition for collusion holds, collusion is a wPBE of the continuation game starting in period t_0 (there is no transition required). $\square$