(Mis-)Matchmaker

Job Market Paper

Click here for the latest version

Jana Gieselmann*

November 10, 2024

Abstract

As platforms collect more user data, they can tailor algorithms to better match users. At the same time, on matching platforms, users pay to be matched by the platform, while the platform makes money as long as it does not match them. This paper analyzes the matching rule of a profit-maximizing monopoly platform when the incentives between users and the platform are misaligned. Contrary to the intuition that more data about users might improve matching efficiency and speed, I show that more data allows the platform to design a matching rule that increases search time and distorts matching and sorting outcomes in the market. I demonstrate that frequently studied matching rules, such as random matching and positive assortative matching, can be suboptimal for the platform. Instead, the platform strategically lowers match quality to increase search time and thus profits, leading to unnecessary delays and potentially inefficient matches. Finally, I provide two explanations for why platforms adopt business models with misaligned incentives: targeted advertising and the presence of overconfident users.

JEL Classification: D83; D47; D42.
Keywords: Online Dating; Matching; Intermediary; Search Frictions; Two-Sided Market.

*I am grateful for the help of my supervisor Paul Heidhues. I also thank Mira Frick, Marina Halac, Johannes Hörner, Reinhold Kesler, Mats Köster, Ferdinand Pieroth, and David Zeimentz, as well as participants of the EEA-ESEM Conference 2024, EARIE 2024, and seminar audiences at DICE and Yale for their participation and feedback.
Contact: Düsseldorf Institute for Competition Economics, Heinrich-Heine University, Germany, gieselmann@dice.hhu.de

1. INTRODUCTION

The emergence of digital matchmakers has revolutionized the way people meet and interact. By reducing search frictions, these platforms have the potential to more efficiently match users. With the help of algorithms based on detailed user data, they promise to facilitate the search for suitable partners in many areas of life. In fact, online dating has become the most common way to meet potential partners in recent years, and for more than a decade, job searches have been conducted predominantly through such online platforms (Rosenfeld et al., 2019; Kircher, 2022). This paper investigates the impact of a platform with detailed user data on the resulting speed and assortativity of matching in the society. It highlights a novel source of mismatching: profit-driven, purposeful mismatching of platforms.

To do so, I study the matching rule of a profit-maximizing platform on which users search for a suitable match. To capture the two most prominent business models, I assume that the platform commits to either an amount of advertising or a payment per period in which the user is active.[1] In either case, spending their time searching is costly for users. To attract and keep users' attention, the platform offers users a recommended match in each period. First, I show that the most prominent search protocols used to study centralized or decentralized matching markets — the positive assortative matching rule (PAM) and a random matching rule — are strictly suboptimal. Instead, the platform uses its knowledge about users to strategically lower the quality of recommended matches. This induces agents to search longer and thereby increases the payments the platform can collect. Besides prolonging search, the resulting matching outcomes can be drastically different from the socially optimal outcome — positive assortative matching — and induce a substantial welfare loss.

Why do platforms then rely on business models that induce misaligned incentives? I provide two plausible explanations. First, when, as in many online markets, users are reluctant to make monetary payments but are willing to consume ads,[2] offering an ad-based model can be more profitable. Second, when users have arguably well-documented misperceptions such as as being overconfident regarding their desirability,[3] they underestimate their expected search duration and hence payments to the platforms for existing pay-per-month schemes.

After discussing the related literature in Section 2, Section 3 presents the model. A

[1]See Appendix C for evidence on the business model of dating and job search apps.

[2]Advertising-based models play a key role in online markets, including both fully ad-supported and "freemium" business models. Freemium refers to business models, where users can use a basic service for free in exchange for consuming ads, but need to pay a fee to use the premium service (without ads). Freemium has become the most popular pricing strategy for many apps (see ACM (2019) or `https://www.statista.com/chart/1733/app-monetization-strategies/`).

[3]Overconfidence has been widely documented in the experimental literature, see for example Burks et al. (2013) and Dubra (2015). Especially overconfidence with respect to one's own attractiveness is common (Greitemeyer, 2020). Psychologists argue that such overconfidence determines how individuals look and compete for potential partners (Murphy et al., 2015). In labor markets, Spinnewijn (2015) and Mueller et al. (2021) find that the unemployed overestimate how quickly they will find a job. Moreover, beliefs are not revised (sufficiently) downward after remaining unemployed. Both findings suggest that job seekers are persistently overconfident about their desirability to firms.

monopoly platform organizes a two-sided matching market in which users search for a partner on the opposite side. The platform commits to a matching rule that determines the probability that two users — each characterized by a vertical type — will meet. Additionally, the platform commits to a per-period cost that it collects from active users, which are either an amount of advertising or a search fee per period. After active users have paid the per-period cost, they receive a recommendation from the platform. Upon meeting, users simultaneously decide whether to accept or reject the proposed match. After rejecting, a user can continue to search. The analysis focuses on steady states; in these the inflow of new agents must equal the outflow under the platform's matching rule.

Section 4 starts by characterizing the users' search behavior. Then, fixing search costs, the platform's problem is to choose matching probabilities conditional on each users' type subject to participation constraints regarding the users' decision to join the platform, incentive constraints on the users acceptance decisions, feasibility constraints on the matching mechanism as well as steady-state constraints. This original problem is highly non-linear. Instead of analyzing the original problem, I make use of an auxiliary problem. This auxiliary problem is a linear programming problem in which the platform chooses masses of recommended matches and matched pairs accepting each other using the facts that: (i) the objective function is linear in steady-state masses; (ii) the steady-state constraints are linear in the mass of matched pairs and steady state masses; and (iii) the feasibility, participation and incentive constraints are linear in recommended matches and steady-state masses by using appropriate transformations. The profit-maximizing solution to this auxiliary problem is then transformed back to the solution of the original problem. Given the profit-maximizing matching rule, the platform chooses its advertising level or search fee. In the most general setting for any given finite set of users' types, I prove that an optimal solution to the platform's profit-maximization problem exists. Based on the reformulation, I show that the widely analyzed matching rules are suboptimal. Random matching is suboptimal, when at least two types on each side of the market participate. Moreover, whenever both market sides are fully symmetric I show that the positive assortative matching rule — where each user meets a user of their own type — can be suboptimal.

Considering the special case with two types on each side of the market and symmetric inflows, Section 4.2 illustrates the main insight of the model — the platform's incentive to recommend and foster mismatches. To induce users to search, the platform frequently recommends mismatches to users, i.e., a high type meets a low relatively more often than a high type. The socially efficient matching outcome in which users sort positively is only implemented by the platform if significantly more low than high types enter the market. Otherwise, the platform induces a weakly, or even non-assortative, matching outcome.

The platform's matching thus creates two intertwined inefficiencies: it distorts matching outcomes by inducing mismatches that deviate from the socially optimal outcome, and it increases users' search time, leading to higher search costs than necessary. Both inefficiencies have implications for real-world markets such as dating and labor markets. In particular, in labor markets, the amount of mismatch has a significant impact on productivity and long-term unemployment (Şahin et al., 2014; McGowan and Andrews, 2015). Moreover, prolonged search duration, i.e., time spent unemployed or in a mismatched job, has high economic and social costs (e.g., unemployment insurance). In marriage markets, sorting has been found to have important implications for income inequality and household decisions (Lee, 2016). In addition, the quality of the relationship or marriage is a determinant of overall well-being and health (Robles et al., 2014; Sharabi and Dorrance-Hall, 2024). In the special case with two types, I find that the socially efficient matching outcome can induce the longest search time of agents, while the search time of agents decreases when the platform implements a weakly assortative or non-assortative outcome.

Finally, Section 5 turns to the question of why platforms rely on business models in which the incentives between the platform and the users are misaligned. For example, a simple potential business model for platforms would be to collect an upfront participation fee from each type and provide them with the socially optimal match. In principle, this business model extracts the entire surplus from users. Under the realistic assumption that users are reluctant to pay upfront but are willing to consume ads, however, I show that an ad-based model can outperform the former business model if targeted advertising is sufficiently efficient. Alternatively, if users are overconfident about their desirability, this belief leads users to underestimate their search time. Therefore, under the pay-as-you-search business model they spend a higher amount ex post than anticipated ex ante. This, in turn, favors the prevailing business model.

Section 6 discusses robustness of my findings. Section 7 concludes and highlights that the tension arising from the misalignment of incentives becomes more important as the platform collects more data and develops more predictive algorithms.

2. RELATED LITERATURE

This article contributes to two central strands of literature, which I detail below. In contrast to the literature, I consider the profit-maximizing incentives of a matchmaker when agents are vertically differentiated and characterize the matching rule and resulting matching outcome.

Matching and Search Theory The vast literature on search-and-matching models, see for instance Burdett and Coles (1999), Eeckhout (1999), Bloch and Ryder (2000), and Smith (2006), provides insights into the functioning of decentralized markets in which agents meet at "random".[4] These matching models with heterogeneous agents build the foundation to investigate sorting and mismatch in markets such as labor and marriage markets when search frictions are present. In line with these models, agents in my model have vertical preferences that result in a unique stable matching. I follow Lauermann and Nöldeke (2014) and suppose that types are finite. The model at hand crucially departs from the literature on decentralized matching, which assumes that agents meet according to a random matching technology, by explicitly accounting for the design of the matching

[4] The aforementioned literature assumes that agents have non-transferable utility. Search-and-matching models with transferable utility have been analyzed, for example, by Becker (1973, 1974) and Shimer and Smith (2000). For an overview of the literature on search-and-matching models see Chade et al. (2017).

rule. With increasing access to user data about preferences and machine-learning tools, matching platforms can design their own recommendation and matching algorithms to maximize profits. While many platforms do not disclose the specifics of their matching algorithms, it is evident that their algorithms are far more sophisticated than random matching.[5]

The question of how to design the matching rule is related to the literature on centralized matching as pioneered by Gale and Shapley (1962), Roth (1982), and Roth and Sotomayor (1992), which studies match quality and implementation of efficient matching rules in two-sided markets.[6] The principal considers properties such as stability, strategy-proofness and Pareto efficiency of the matching rule. In contrast, I characterize the profit-maximizing solution for different given business models.

Search problems are widely studied not only on an individual level but researchers also rely on these to better understand job search and its implications on the functioning of the economy. Early articles include Pissarides (1985), Mortensen and Pissarides (1994), and Mortensen and Pissarides (1999), which focus on wage bargaining and unemployment dynamics and on-the-job search when agents are ex-ante homogeneous. Dolado et al. (2009) introduces heterogeneous types of workers and firms into job search models, which are also crucial in my model. A recent treatment on how job search has changed in the digital era is provided by Kircher (2022).

Finally, my paper is related to papers investigating biased beliefs of agents in matching and search markets. Closely related in a dating market, Antler and Bachi (2022) show that agents' coarse reasoning leads to overoptimism about their prospects in the market and induces them to search inefficiently long. In labor markets, Spinnewijn (2015) and Mueller et al. (2021) document that job seekers often hold overoptimistic beliefs and thereby underestimate their time to find a job. I contribute to this literature by showing how current platform business models exploit overconfident types.

Platform Markets Central to the literature that studies platform and (online) two-sided markets is the presence of network effects and how these shape the incentives and price setting of a platform that enables the interaction between two groups (Caillaud and Jullien, 2003; Rochet and Tirole, 2003, 2006). As a result, in most models agents are assumed to care only about the number of matches instead of match quality.

With the emergence of digital matchmakers, the literature extended to analyzing (customized) matching on platforms with a focus on the interaction between pricing and matching efficiency (Damiano and Li, 2007; Damiano and Hao, 2008), price discrimination (Gomes and Pavan, 2016, 2024), and auctions (Johnson, 2013; Fershtman and Pavan, 2022), all abstracting from search frictions and dynamics. In my model, the platform designs the matching rule in its online market place, but in contrast to the aforementioned

[5]Dating platforms such as Tinder or bumble provide a general description of their algorithm, see for example `https://www.help.tinder.com/hc/en-us/articles/7606685697037-Powering-Tinder-The-Method-Behind-Our-Matching`, whereas the dating platform "Hinge" claims to use the Gale-Shapley algorithm designed to find stable matchings.

[6]The literature on matching in two-sided markets can be divided into centralized and decentralized matching (see Echenique et al. (2023) for a recent overview).

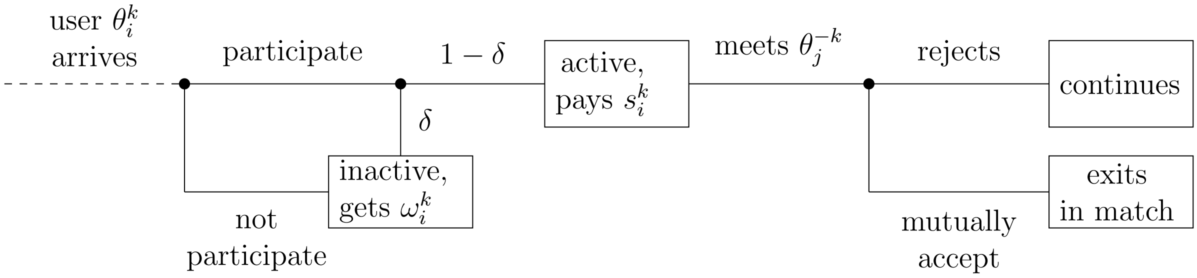

Figure 1: Within-Period Timing

articles, the platform has an incentive to not implement the efficient and full surplus extracting matching rule.

Within the analysis of digital matchmakers, Halaburda et al. (2018) and Antler et al. (2024a,b) also focus on applications to dating platforms. Most closely related is Antler et al. (2024b) who study a matchmaker's incentives in a model with horizontally differentiated types, which determine the fit of agents. The platform charges a single "upfront" fee in the second period after agents have joined and received their first match for free. The author draw a similar conclusion: the platform has an incentive to invest into a technology that increases the speed of search but not into a technology that improves the quality of matches. The main difference lies in modeling the matching technology. The authors restrict attention to a truncated random matching technology under which agents meet at random above a threshold and do not meet if their fit is below the threshold; in contrast, I solve for the optimal matching rule.

Within the platform literature models on platforms intermediating consumer search Hagiu and Jullien (2011, 2014), Eliaz and Spiegler (2011b, 2016), and Nocke and Rey (2024) are closely related. Hagiu and Jullien (2011) provide a rationale for intermediaries to divert search of their consumers away from preferred stores. Although the insight is closely related to the mismatching incentive in my model, the (one-sided) market in Hagiu and Jullien (2011) does not include the strategic component on the other side as stores would never reject a consumer willing to buy. Hence, there is no analogue to my finding that the platform prolongs search of lower types by recommending them to higher types knowing that they will reject those lower types. Additionally, there is no equivalent to overconfident users in their model. Finally, my model of a two-sided matching market offers insights into the allocative inefficiency and the length of search for labor and dating markets intermediated by matching platforms.

3. MODEL

A monopolist platform organizes a matching market in which a continuum of agents from two sides, $k = A, B$, search for a partner from the opposite side. The market operates in discrete time with an infinite horizon. I focus on steady state analysis. In slight abuse of notation, I therefore suppress time indices whenever it does not lead to confusion.

Agents Agents of each side are characterized by a type $\theta_i^k \in \Theta^k$, with $\Theta^k = \{\theta_1^k, \theta_2^k, ..., \theta_N^k\}$ finite. At the beginning of the period, agent θ_i^k decides whether to participate in the market or exit and get outside option ω_i^k. An agent, who participates in the market, becomes inactive with an exogenous probability δ, and exits the search process as well. The platform charges an active agent of type θ_i^k search cost s_i^k. Then, each active agent receives a single recommendation from the platform. After receiving a recommendation, two agents who meet, observe each other's type and simultaneously decide whether to accept or reject the other agent. The following payoffs realize based on their actions in the current period: (i) mutual acceptance yields a match utility of $u(\theta_i^k, \theta_j^{-k}) = \theta_i^k \theta_j^{-k}$, and (ii) (one-sided) rejection yields a utility of zero in the current period. Upon rejection, an agent can continue to search in the next period. The timing within each period is summarized in Figure 1.

Agents are assumed to use time- and history-independent strategies. A pair of functions $\sigma_k : \Theta^k \times \Theta^{-k} \to [0,1]$ and $\sigma_{-k} : \Theta^k \times \Theta^{-k} \to [0,1]$ and function $\eta_i^k : (\theta_i^k, \omega_i^k) \to [0,1]$ describe a strategy profile, where $0 \leq \sigma_k(\theta_i^k, \theta_j^{-k}) \leq 1$ is the probability that an agent of type θ_i^k on side k accepts a match with type θ_j^{-k} on the other side and $0 \leq \eta_i^k \leq 1$ is the probability that an agent of type θ_i^k participates. In other words, without loss of generality, I focus on strategies in which identical agents who are active on the same side of the market and have the same type use the same acceptance and participation strategy. Then,

$$\alpha(\theta_i^k, \theta_j^{-k}) = \sigma_k(\theta_i^k, \theta_j^{-k}) \cdot \sigma_{-k}(\theta_i^k, \theta_j^{-k})$$

denotes the probability of a mutual acceptance by type θ_i^k and θ_j^{-k}.

Matching A matching mechanism $\mathcal{M} := \{\phi^k(\cdot)\}_{k=A,B}$ consists of a (potentially stochastic) feasible matching rules $\phi^k(\cdot)$. Let $\hat{\Theta}^k$ be the set of participating types from side $k = A, B$. For any $\theta_i^k \in \Theta^k \setminus \hat{\Theta}^k$, $\phi^k(\theta_i^k) = \emptyset$. For $\theta_i^k \in \hat{\Theta}^k$, $\phi^k(\theta_i^k) \in \Delta(\hat{\Theta}^{-k} \cup \omega_i^k)$, which is a probability measure over $\hat{\Theta}^{-k} \cup \omega_i^k$. Intuitively, this describes the probability of meeting the various types of the opposing side as well as the outside option. Denote the steady state mass of agents of type θ_i^k on side k by $f(\theta_i^k)$. Matching rule $\mathcal{M}$ induces a distribution of matches pairs M

$$\left(\begin{pmatrix} f(\theta_1^k) \\ \vdots \\ f(\theta_N^k) \end{pmatrix}, \begin{pmatrix} f(\theta_1^{-k}) \\ \vdots \\ f(\theta_N^{-k}) \end{pmatrix} \right) \mapsto \begin{pmatrix} \Phi(\theta_1^k, \theta_1^{-k}) & \cdots & \Phi(\theta_1^k, \theta_N^{-k}) \\ \vdots & & \vdots \\ \Phi(\theta_N^k, \theta_1^{-k}) & \cdots & \Phi(\theta_N^k, \theta_N^{-k}) \end{pmatrix} \equiv M.$$

An entry of matrix M is the mass of agents that are recommended to each other under matching rule $\mathcal{M}$ and is given by

$$\Phi(\theta_i^k, \theta_j^{-k}) = f(\theta_i^k)\phi(\theta_j^{-k}|\theta_i^k) = f(\theta_j^{-k})\phi(\theta_i^k|\theta_j^{-k}),$$

where the masses are symmetric, i.e. the mass of agents of type θ_i^k on side k being matched to agents of type θ_j^{-k} on side $-k$ is equal to the mass of agents of type θ_j^{-k} on side $-k$ being matched to type θ_i^k on side k: $\Phi(\theta_i^k, \theta_j^{-k}) = \Phi(\theta_j^{-k}, \theta_i^k)$. Under matching rule $\mathcal{M}$, the mass of agents of type θ_i^k that are unmatched, i.e. do not receive a recommendation in a given period, is

$$\Phi(\theta_i^k, \omega_i^k) = f(\theta_i^k) - \sum_{\theta_j^{-k} \in \Theta^{-k}} \Phi(\theta_i^k, \theta_j^{-k}).$$

To capture the idea that the platform can only generate revenue by keeping users' attention and, hence, wants to match as many agents as possible, I impose the following assumption.

Assumption 1. *Let $\hat{k}$ be the short side of the market. For each agent on side $\hat{k}$, $\phi(\omega(\cdot)|\theta_i^k) = 0$.*

Under Assumption 1, feasibility of the matching rule can be expressed in terms of the masses of matched pairs.

$$\sum_{\theta_j^{-k} \in \Theta^{-k}} \Phi(\theta_i^k, \theta_j^{-k}) + \mathbf{1}_{k=\hat{k}} \Phi(\theta_i^k, \omega_i^k) = f(\theta_i^k), \forall i = 1, ..., N, k = A, B. \tag{1}$$

Timing and Population Dynamics At the beginning of a period t, agents who did not find a match in the last period arrive and a (time-invariant) inflow of new agents of type θ_i^k given by the mass $\{\beta_i^k\}_i^{k=A,B}$ enters the platform. Agents decide whether to participate on the platforms. Those who decide to participate become inactive with probability δ, while active agents are matched according to matching rule $\mathcal{M}$ resulting in matrix M_t. Based on their recommended match, agents make their acceptance decision resulting in mutual acceptance probabilities $\{\alpha_t(\theta_i^k, \theta_j^{-k})\}_{ij}$. At the end of the period, agents exit in pairs that mutually accepted each other. The total outflow of agents is then given by pairs that exit together in a match, agents that become inactive with probability δ and agents that decided not to participate.

Platform The platform commits to a matching mechanism $\mathcal{M} := \{\phi^k(\cdot)\}_k$. To capture the two most prominent business models, I suppose that the platform either commits to an extent of advertising or a given payment per period. Formally, this choice induces the type-dependent search cost s_i^k while generating revenue per search of type θ_i^k of $\nu(s_i^k)$. In case of payments, $\nu(s_i^k)$ is the identity function. In case of advertisements, $\nu(s_i^k)$ is an increasing and strictly concave function of the search costs, which for example captures the intuition that the agents' disutility of advertising is convex in the number of ads shown while the platform's profit is constant per ad. Let $s_i^k \in [0, \overline{u}]$, where $\overline{u}$ is the maximum match utility that the highest type can achieve on the platform. The platform discounts future profits according to ρ and thus maximizes

$$\Pi = \sum_{k=A,B} \sum_{\theta_i^k \in \Theta^k} \frac{(1-\delta)\nu(s_i^k)}{1-\rho} f(\theta_i^k).$$

Equilibrium Concept The model focuses on a steady state analysis in which the market is balanced: that is, the inflow of agents is equal to the outflow of agents under matching rule $\mathcal{M}$. Formally:

Definition 1. (Steady State) For given matching rule $\mathcal{M}$, a *steady state* is a tuple $(f(\theta_i^k), \alpha(\theta_i^k, \theta_j^{-k}))_{ij}^k$ that satisfies

$$\beta_i^k = f(\theta_i^k)\left[\eta_i^k + (1-\eta_i^k)\left(\delta + (1-\delta)\sum_{\theta_j^{-k}\in\Theta^{-k}} \alpha(\theta_i^k, \theta_j^{-k})\phi(\theta_j^{-k}|\theta_i^k)\right)\right], \qquad (2)$$

for all $\theta_i^k \in \Theta^k, k = A, B$. The left-hand side describes the inflow of agents of type θ_i^k, where the right-hand side is the outflow. The outflow is the mass of type θ_i^k agents times the probability that agents do not participate or the probability of becoming inactive and exiting in a match if agents participate.

A steady state is an equilibrium if

Definition 2. (Equilibrium) For a given choice of type-dependent search costs and matching rule $\mathcal{M}$, a steady state is an equilibrium whenever the profile of stationary strategies σ of agents satisfies:

1. Agents correctly anticipate $\mathcal{M}$ and other agents' strategies.
2. Agents accept a match if and only if the match yields a higher payoff than continuing to search.

The first part of the definition corresponds to the usual Nash assumption of correctly anticipating other players' strategies. The second part captures that agents maximize expected utility implicitly ruling out the case that a valuable pair is rejected because everyone is certain that their partner rejects.[7]

3.1 DISCUSSION OF ASSUMPTIONS

Search Costs Agents incur additive search cost s_i^k in each period, which are designed by the platform. They either represent the nuisance costs from advertising as, for example, in Anderson and Coate (2005), which are positively related to the advertising intensity, or the search fee that the platform charges periodically. Search frictions are modeled by introducing the exogenous exit probability δ. Following a literal interpretation, δ is the probability with which agents become inactive, i.e. the probability that an agent finds a job or a partner offline through other means. More generally, δ can be thought as modeling the force that leads agents to discount the future, which makes delayed matching more costly.

[7]This allows the current match partner to tremble with small probability. Alternatively, acceptance decisions could be made sequential in which case agents would have to accept a valuable match.

Business Model The platform is assumed to be a monopolist in the matching market. Following evidence from the dating market, the most popular dating platforms have a common owner. For simplicity, I assume that the dominant owner only offers one platform in my model.[8] More generally, we often observe platforms with large market power in two-sided markets, where joining a new platform is worthwhile only if others join. My monopoly setup is a simple setting capturing such market power.

The model examines two prevalent business models: an advertisement-based approach and periodic search fees. Many platforms adopt the former— (targeted) advertising — by monetizing user attention through selling advertising slots to firms. In return for users' attention, the platform provides its matching service for free. In this setup, keeping user attention is crucial for the platform's revenue.[9] This is why I assume that the platforms earns no revenue when not capturing the user's attention through offering a potential match. Alternatively, platforms implement search fees, which they collect from active users. Examples include "pay-per-click" or "pay-per-contact" fees, though monthly subscription plans are also common. These fees are typically low, distinguishing them significantly from participation fees, which are far less common but used by some selective matching platforms.[10]

An advertising-based stream of revenues continues to be a prominent part of platform business models, especially with transaction costs. Platforms have transaction costs when setting up a payment system, while many users are reluctant to give their credit card data to platforms. Overall, privacy concerns, risk aversion and uncertainty when using new products (platforms) can play a role why users (initially) prefer to use the matching service for "free" while watching advertisement over signing up to a subscription plan or paying a participation fee. As a consequence, many platforms rely on these so-called "freemium" business models, which have become even more popular since the emergence of mobile applications (apps). Here, "freemium" describes business models where a basic service is available to users for free (with advertisement), whereas an upgraded service can be accessed through purchases.[11] Other platforms, however, rely only on advertising or fees. I return to the question of why platforms refrain from collecting a fixed fee for a certain promised match in Section 5.

No Agent is Unmatched The key assumption of the matching rule, Assumption 1, states that if possible each agent receives a recommended match in any period.[12] As many

[8] The dating market is highly concentrated with the Match Group Inc. owning many of the most popular dating platforms: Tinder, Hinge, PlentyofFish, Match, OkCupid etc. (see `https://www.bamsec.com/filing/89110323000114?cik=891103`), while other dating platforms are highly differentiated and for example, cater to specific religious groups. Recent experimental evidence from Dertwinkel-Kalt et al. (2024) suggest that even the most closest competitors, Tinder and bumble, are viewed to be almost independent instead of substitutes by consumers.

[9] Recent papers that study different aspects of attention on platforms are for example Prat and Valletti (2022), Chen (2022), and Srinivasan (2023).

[10] For an overview of the most common platforms and their fee structure see Appendix C.

[11] For empirical evidence see for example, Kummer and Schulte (2019) for studying privacy concerns in the mobile app market and Deng et al. (2023) for studying freemium pricing of mobile applications.

[12] In the literature on search-and-matching models time is often continuous, such that matching opportunities arrive at a constant rate. Similarly, Antler et al. (2024a,b) make the assumption that matches

online platforms take on a dual role as attention intermediaries and need to attract consumers' attention to sell to advertisers, providing a constant stream of potential matches aims at grabbing and keeping consumers' attention.[13]

To grab users' attention, the platform makes a recommendation any time the agent enters and is active on the platform. The recommendation of a potential match can be viewed as being part of a menu that the platform offers. Following the idea of Eliaz and Spiegler (2011a), the platform offers a menu that consists of an attention-grabbing component and its true value of the service. In reality, the attention grabbing component is supported by push notifications or emails, while the value from the platform's service is determined by the expected utility from getting a match. The modeling choice is further supported by a recent lawsuit against the MatchGroup Inc., owner of a majority of the most popular dating platforms.[14] In the complaint, the plaintiff accuses Match to monopolize users' attention and claim that "Push Notifications prey on users' fear of missing out on any potential matches with a strategic notification system designed to capture and retain attention at all times of the day".

4. ANALYSIS

To analyze the equilibrium, I need to characterize the agents' behavior and the platform's optimization problem. The agents' search process is characterized by a set of participation and incentive constraints that determine whether an agent is willing to incur the search costs as well as accepts or rejects a recommended match.

Agents' Search Process. Consider the strategy of agent θ_i^k being active in the matching market. Upon meeting an agent θ_j^{-k}, the agents decides whether to accept or reject the *recommended* match. Mutual acceptance results in a *match* and both agents leave the market as a pair. If at least one of the agents rejects the match, agent θ_i^k continues to search.

Due to the stationarity of the environment, the continuation value of agent θ_i^k, $V^C(\theta_i^k)$, is defined by the following recursive equation

$$\begin{aligned} V^C(\theta_i^k) =& \delta\omega_i^k + (1-\delta)\left[-s_i^k + \sum_j \alpha(\theta_i^k, \theta_j^{-k})\phi(\theta_j^{-k}|\theta_i^k)\theta_i^k\theta_j^{-k}\right. \\ &\left. + (1 - \sum_j \alpha(\theta_i^k, \theta_j^{-k})\phi(\theta_j^{-k}|\theta_i^k))V^C(\theta_i^k)\right]. \end{aligned}$$

The first term represents the case in which agent θ_i^k will become inactive with probability δ and gets its outside option ω_i^k. If the agent remains active with probability $1-\delta$, it

arrive at a constant rate even in the presence of a matchmaking platform.

[13]In a recent experiment, Aridor (Forthcoming) provides evidence that users allocate their attention across product categories and offline when facing restriction in their time spent on a specific platform. The results suggest that competition for attention spans across multiple markets.

[14]Oksayan v. MatchGroup Inc., N.D. Cal., No. 3:24-cv-00888, 2/14/24.

incurs the search cost s_i^k. The expected utility from leaving in a match is given by the utility from a match with type θ_j^{-k}, which is equal to the product of both types, and the probability of meeting and mutually accepting type θ_j^{-k}. With the counterprobability, the match was not mutually accepted and agent θ_i^k continues to search.

Solving for the continuation value yields

$$V^C(\theta_i^k) = \frac{\delta\omega_i^k + (1-\delta)\left(-s_i^k + \sum_j \alpha(\theta_i^k, \theta_j^{-k})\phi(\theta_j^{-k}|\theta_i^k)\theta_i^k\theta_j^{-k}\right)}{\delta + (1-\delta)\left(\sum_j \alpha(\theta_i^k, \theta_j^{-k})\phi(\theta_j^{-k}|\theta_i^k)\right)}. \tag{3}$$

The continuation value then characterizes the payoff of an agent who rejects a match and returns to the search process, whereas the match payoff $\theta_i^k\theta_j^{-k}$ characterizes the payoff of an agent who accepts a match with type θ_j^k (and is accepted by them). By Definition 2, if the match value $\theta_i^k\theta_j^{-k}$ is smaller (larger) than the continuation value $V^C(\theta_i^k)$, agent-θ_i^k will reject (accept) a recommended match with agent-θ_j^{-k}.

The optimal strategy of an agent who uses an time-and history-independent strategy is then a cutoff strategy:

$$\sigma_k(\theta_i^k, \theta_j^{-k}) = \begin{cases} 0 & \text{if } \theta_i^k\theta_j^{-k} < V^C(\theta_i^k) \\ 1 & \text{if } \theta_i^k\theta_j^{-k} > V^C(\theta_i^k) \end{cases}, \text{ for } k = A, B. \tag{4}$$

If the match value with a type $\hat{\theta}_j^{-k}$ is larger than the continuation value, agent θ_i^k will accept a recommended match with agent $\hat{\theta}_j^{-k}$ and all agents of types higher than $\hat{\theta}_j^{-k}$. The optimality of the cutoff strategy follows directly from the supermodularity of the match payoff.

An agent participates if the contiunation value is larger than the agent's outside option. Due to the stationarity and history-independence of strategies, if an agent decides to participate in the matching market, they will not exit during the search process and search until they exit in a match or become inactive with probability δ.

Remark. The strategy of an agent of type θ_i^k is increasing in its second argument $\sigma_k(\theta_i^k, \theta_N^{-k}) \geq \sigma_k(\theta_i^k, \theta_{N-1}^{-k}) \geq \cdots \geq \sigma_k(\theta_i^k, \theta_1^{-k})$, but may be neither in- nor decreasing in its first argument.

For random matching or positive assortative matching $\sigma_k(\theta_i^k, \theta_j^{-k})$ is decreasing in its first argument: $\sigma_k(\theta_N^k, \theta_j^{-k}) \leq \cdots \leq \sigma_k(\theta_1^k, \theta_j^{-k})$. A random matching rule yields the same meeting probabilities for all types. Due to the supermodularity of the payoff function, higher types will reject (weakly) higher types than lower types do. With positive assortative matching, the matching probabilities conditional on being a higher type first-order stochastically dominates the matching probabilities conditional on being a lower type. Hence, higher types will reject strictly higher types than lower types do. In contrast, a negative assortative matching rule, which recommends (almost exclusively) higher types to lower types, and vice versa, can cause lower types to reject lower types while higher types are willing to accept them. Indeed, I will explicitly provide an example of such an equilibrium in Section 4.2.

Given the agent's strategy in Equation 4, the acceptance probabilities satisfy

$$\alpha(\theta_i^k, \theta_j^{-k}) = \begin{cases} 0 & \text{if } \theta_i\theta_j < V^C(\theta_i^k) \text{ or } \theta_i\theta_j < V^C(\theta_j^{-k}) \\ 1 & \text{if } \theta_i\theta_j > V^C(\theta_i^k) \text{ and } \theta_i\theta_j > V^C(\theta_j^{-k}) \end{cases} . \tag{5}$$

Equation 5 establishes the relationship between acceptance probabilities and matching outcomes. Mutual acceptance requires that whenever two types of agents meet, both must find it optimal to stop searching.

4.1 N TYPES

Consider the case with N types of agents such that $\Theta^k = \{\theta_1^k, ..., \theta_N^k\}$ where $\theta_N^k > ... > \theta_1^k$. The following section provides general results on the existence of an equilibrium, optimal solution and their properties. Let s_i^k be exogenous. The first result states that any feasible matching rule, which must satisfy Equation 1, induces an equilibrium among agents, in which agents maximize their expected utility and correctly anticipate the strategy of other agents.[15]

Proposition 1. *For any exogenously given matching rule that is feasible, there exists an equilibrium.*

The statement implies that for a given matching rule, there exists a steady state in which the profile of strategies of the agents is consistent with utility maximization (i.e., Equation 4). Furthermore, as agents use cutoff strategies, the matching rule implements acceptance probabilities consistent with Equation 5. Lastly, Proposition 1 shows that the matching rule is feasible in the equilibrium that is implemented by the matching rule.

I prove Proposition 1 by induction. First, I show that if the type space only consists of one type on each side, $\Theta^k = \{\theta_1^k\}$, an equilibrium exists. Intuitively, if search costs are too high on at least one side of the market, there only exists an equilibrium in which agents do not participate. If search cost, however, are sufficiently low there exists a participation equilibrium in which agents on both sides mutually accept each other consistent with expected utility maximization. The steady state mass follows from Equation 2 given the probability that both types meet each other and leave. Second, if there exists an equilibrium for the case $\Theta^k = \{\theta_1^k, ..., \theta_N^k\}$, then I show that it must also hold for the case $\Theta^k = \{\theta_1^k, ..., \theta_{N+1}^k\}$, which concludes the proof.

Next, to determine the profit-maximizing matching rule $\mathcal{M}$, it is useful to define the matching outcome. Intuitively, the matching outcome is defined as the matrix that describes the distribution of pairs under matching rule $\mathcal{M}$ that exit in a match. Recall that matrix M describes the masses of recommended pairs under matching rule $\mathcal{M}$ and let A denote the matrix of agents' mutual acceptance probabilities

$$A \equiv \begin{pmatrix} \alpha(\theta_1^k, \theta_1^{-k}) & \cdots & \alpha(\theta_1^k, \theta_N^{-k}) \\ \vdots & & \vdots \\ \alpha(\theta_N^k, \theta_1^{-k}) & \cdots & \alpha(\theta_N^k, \theta_N^{-k}) \end{pmatrix}.$$

[15]Note that I can always construct an equilibrium in which no agent participates; however, I am looking for an equilibrium in which participation is maximized given that the platform finds it optimal to do so.

Formally, the matching outcome is defined as the componentwise multiplication (Hadamard product) of matrix A and M:

Definition 3. The matching outcome is defined by the matrix

$$A \odot M = \begin{bmatrix} \alpha(\theta_1^k, \theta_1^{-k})\Phi(\theta_1^k, \theta_1^{-k}) & \cdots & \alpha(\theta_1^k, \theta_N^{-k})\Phi(\theta_1^k, \theta_N^{-k}) \\ & \vdots & \\ \alpha(\theta_N^k, \theta_1^k)\Phi(\theta_N^k, \theta_1^{-k}) & \cdots & \alpha(\theta_N^k, \theta_N^{-k})\Phi(\theta_N^k, \theta_N^{-k}) \end{bmatrix} \equiv O(\cdot).$$

The matching outcome is said to be (i) assortative if $O(\cdot)$ has positive entries only along the main diagonal, (ii) weakly assortative if $O(\cdot)$ has positive entries along the main diagonal and to the right if and only if the entries below are also positive, and (iii) non-assortative otherwise.

Denote by $m(\theta_i^k, \theta_j^{-k}) = \alpha(\theta_i^k, \theta_j^{-k})\Phi(\theta_i^k, \theta_j^{-k})$ an entry of matrix $O(\mathcal{M})$. Each entry is the mass of pairs that are recommended to each other times their mutual acceptance probabilities. Therefore, $m(\theta_i^k, \theta_j^{-k})$ is the mass of *matched* pairs that exit the market together. For a given matching rule, an equilibrium induces at most one matching outcome since the mutual acceptance probabilities and steady state masses are pinned down in equilibrium.

To find the profit-maximizing matching rule and the associated matching outcome, I proceed in two steps. First, I fix the acceptance probabilities associated with a matching outcome and determine the optimal feasible matching rule that implements the chosen matching outcome. Second, supposing the optimal matching rule from step one is used to implement any chosen matching outcome, I choose the matching outcome that yields the highest platform profits.

In the following, I make use of the fact that the platform's profit-maximization problem can be transformed into a linear program. For given search cost s_i^k, recall that the platform's objective is to maximize

$$\max_{\mathcal{M}} \sum_{k=A,B} \sum_{\theta_i^k \in \Theta^k} \frac{(1-\delta)s_i^k}{1-\rho} f(\theta_i^k),$$

i.e., the platform maximizes the steady state mass of active agents with weight s_i^k. The maximization problem underlies a set of constraints. First, the matching rule must implement a steady state. By expanding the steady state condition in Equation 2, the condition is

$$\beta_i^k = f(\theta_i^k)\delta + (1-\delta) \sum_{\theta_j^{-k} \in \Theta^{-k}} \underbrace{\alpha(\theta_i^k, \theta_j^{-k})\Phi(\theta_j^{-k}|\theta_i^k)}_{=m(\theta_i^k, \theta_j^{-k})}. \tag{Steady State}$$

In the steady state, the inflow of agents of θ_i^k is equal to the mass of agents that become inactive in a period with probability δ and the mass of active agents that exit in matched pairs. In a steady state, the mass of agents of type θ_i^k can be restated in the following way

$$f(\theta_i^k) = \frac{\beta_i^k - (1-\delta)\sum_j m(\theta_i^k, \theta_j^{-k})}{\delta}, \tag{Steady-State Mass}$$

and therefore, depends positively on the inflow, β_i^k, and negatively on the mass of matched pairs that include type θ_i^k. Second, the matching rule determines whether agents participate in the market and whether agents search according to the platform's recommendations. For participating agents, it must hold that the agent prefers participating in the market to accepting the outside option

$$\omega_i^k \leq \frac{\delta\omega_i^k + (1-\delta)\left(-s_i^k + \sum_j \alpha(\theta_i^k, \theta_j^{-k})\phi(\theta_j^{-k}|\theta_i^k)\theta_i^k\theta_j^{-k}\right)}{\delta + (1-\delta)\left(\sum_j \alpha(\theta_i^k, \theta_j^{-k})\phi(\theta_j^{-k}|\theta_i^k)\right)} = V^C(\theta_i^k).$$

To induce agents to search, the platform can be understood as selecting a critical lowest type that is accepted with positive probability or just rejected by an agent of type θ_i^k. Following from the cutoff strategy, agent θ_i^k rejects all types below the critical lowest type. The incentive constraint for agent θ_i^k to follow the recommendation of the platform to (weakly) reject an agent θ_j^{-k} reads[16]

$$\theta_i^k\theta_j^{-k} \leq V^C(\theta_i^k).$$

By using the steady state condition, the participation and obedience constraints can be reformulated. Note that the denominator of the continuation value is equal to the probability that an agent exists, which is equal to $\beta_i^k/f(\theta_i^k)$ by Equation 2. Inserting into the continuation value and rearranging yields

$$\beta_i^k\omega_i^k \leq \delta f(\theta_i^k)\omega_i^k - (1-\delta)f(\theta_i^k)s_i^k + (1-\delta)\sum_j \underbrace{\alpha(\theta_i^k, \theta_j^{-k})\Phi(\theta_j^{-k}|\theta_i^k)}_{=m(\theta_i^k,\theta_j^{-k})}\theta_i^k\theta_j^{-k}, \tag{PC}$$

$$\beta_i^k\theta_i^k\theta_j^{-k} \leq \delta f(\theta_i^k)\omega_i^k - (1-\delta)f(\theta_i^k)s_i^k + (1-\delta)\sum_j \underbrace{\alpha(\theta_i^k, \theta_j^{-k})\Phi(\theta_j^{-k}|\theta_i^k)}_{=m(\theta_i^k,\theta_j^{-k})}\theta_i^k\theta_j^{-k}. \tag{IC}$$

Lastly, the platform's matching rule must satisfy the feasibility constraints. Without loss of generality, let side B be of smaller or same size as side A. Then on side A, the sum over the mass of each recommended pair that includes type θ_i^A must be equal to the steady state mass of θ_i^A. On side B, the sum over the mass of each recommended pair that includes type θ_i^B and the mass of agents of type θ_i^B that are unmatched must be equal to the steady state mass of type θ_i^B

$$\sum_{\theta_j^{-k}\in\Theta^{-k}} \Phi(\theta_i^k, \theta_j^{-k}) + \mathbf{1}_{k=B}\Phi(\theta_i^k, \omega_i^k) = f(\theta_i^k), k = A, B. \tag{Feasibility}$$

As stated, for given matrix A the above constraints and the objective function are all linear functions of the steady state masses, matched pairs, and recommended pairs. The steady-state mass in turn is also a linear functions of the mass of matched pairs. To complete the reformulation as linear program, one needs to be careful to ensure that

[16]In mechanism design, this is often referred to as an obedience constraints because there is not private information throughout the model.

incentive constraints hold with equality if the agent mixes in its acceptance probability, as well as use the appropriate direction of the inequality otherwise. Appendix A.1 formally does so, leading to:

Lemma 1. *The platform's problem can be restated as a linear programming problem in the mass of matched and recommended pairs:* $\{m(\theta_i^k, \theta_j^{-k})\}, \{\Phi(\theta_i^k, \theta_j^{-k})\}_{ij}$.

Given a solution of the linear program — the auxiliary problem — the optimal matching rule to the original problem results from

$$\phi(\theta_j^{-k}|\theta_i^k) = \frac{\Phi(\theta_i^k, \theta_j^{-k})}{f(\theta_i^k)}.$$

Let $\mathcal{F}$ be the set of equilibrium matching outcomes that can be implemented by the platform. That is, $\mathcal{F}$ is a subset of the set that contains all possible matching outcomes. An equilibrium matching outcome is the componentwise multiplication of matrix M — the mass of recommended pairs in equilibrium — and matrix A — the mutual acceptance probabilities in equilibrium. By Proposition 1, $\mathcal{F}$ is not empty.

Next, define the subset $\mathcal{F}^* \subset \mathcal{F}$ as follows. Denote the set of matrices A that can be implemented through $\mathcal{F}$ by $\mathcal{A}$. Now, construct a finite subset $\mathcal{A}^* \subset \mathcal{A}$ as follows: If $A' \in \mathcal{A}$ has only entries composed of $\alpha(\theta_i^k, \theta_j^{-k}) \in \{0, 1\}$, then $A' \in \mathcal{A}^*$. For any $A' \in \mathcal{A}$ with $\alpha(\theta_i^k, \theta_j^{-k}) \in (0, 1)$, add matrix A'' to $\mathcal{A}^*$ in which entry $\alpha(\theta_i^k, \theta_j^{-k})$ is replaced with the generic variable α_{ij}. By construction $\mathcal{A}^*$ is finite. Observe that any $\alpha_{ij} \in (0, 1)$ induces the same constraints in the linear program in Appendix A.1. The linear program, in turn, is solved over the mass of matched and recommended pairs and not acceptance probabilities. Solving this for all (finite) possible combinations of constraints gives a finite number of candidate solution. Using the profit maximizing solution, it is easy to select the optimal acceptance probabilities $\alpha_{ij} \in (0, 1)$ when the agent is indifferent by dividing the matched pairs through the recommended ones. Then, for each $A \in \mathcal{A}^*$, find the mass of recommended and matched pairs that implement the profit-maximizing matching outcome $\mathcal{F}^*(A)$ and let $\mathcal{F}^* = \bigcup_{A \in \mathcal{A}^*} \mathcal{F}^*(A)$.

Lemma 2. *The set* $\mathcal{F}^*$ *is non-empty and finite.*

Key to the proof is to show that the linear program for any given matrix $A \in \mathcal{A}$ is (i) not unbounded and (ii) not infeasible, i.e. the feasible region is non-empty. This implies that $\mathcal{F}^*$ is non-empty. Given that both (i) and (ii) are satisfied, an optimal solution to the linear program exists (Dantzig, 1963).[17]

Given that Lemma 2 established that $\mathcal{F}^*$ is non-empty and finite, the platform selects the matching rule $\mathcal{M}^*$ that implements the matching outcome among the set of matching outcomes in $\mathcal{F}^*$ that yields the highest profit.

[17] Existence follows from the fact that the constraint set is a convex polyhedron. Because the objective is linear and the constraint set is convex, any local extremum will be the global extremum. As the objective is linear, the extremum will be obtained at one of the extreme points of the constraint set, i.e., at the vertices of the polyhedron.

Theorem 1. *For given $s_i^k \in [0, \overline{u}]$ for all $\theta_i^k \in \Theta^k$, $k = A, B$, there exists an optimal solution with $\Pi^* \equiv \max_{\mathcal{F}^*} \Pi$. Let the platform choose $s_i^k \in [0, \overline{u}]$ for all $\theta_i^k \in \Theta^k$, $k = A, B$. There exists an optimal solution $\Pi^{*, s_i^k} \equiv \max_{\mathcal{F}^*, s_i^k} \Pi$*

By Lemma 2 the set $\mathcal{F}^*$ is finite and non-empty, such that $\max_{\mathcal{F}^*} \Pi$ is well-defined. Together with Proposition 1, this implies the existence of an optimal solution. The second part follows from the fact that the platform's profit is compact-valued and upper hemicontinuous in s_i^k.

To identify properties of the optimal solution, first consider two prominently studied matching rules. As discussed in Section 2, in decentralized matching-and-search markets agents are often assumed to meet according to a random matching technology. A natural question to consider is whether a platform that has access to extensive user data would commit to a random meeting technology as well. The answer, however, is that random matching is generically suboptimal for the platform.[18]

Proposition 2. *Random matching is generically suboptimal for $N > 1$.*

Under random matching, the conditional probability of meeting a type θ_i^k on side k is the same for all types $\theta_j^{-k} \in \Theta^{-k}$ on side $-k$ and corresponds to the proportion of type θ_i^k in the population. As shown in Appendix A.2, the probability of meeting a type θ_i^k is a function of the inflow, β_i^k, and the probability δ. In contrast, the optimal solution of the linear program is a function of s_i^k and internalizes changes in the search cost. Therefore, random matching is generically suboptimal, although it may coincide with the optimal solution for knife-edge $s_i^k, \theta_i^k, \delta$, and β_i^k. This result highlights that a platform, which has increasing access to user data, does not commit to a random matching technology.

Second, consider the positive assortative matching rule (PAM) under the assumption that both sides are symmetric with respect to the inflow of new agents: $\beta_i^A = \beta_i^B$, their type space $\Theta^k = \Theta$ and outside options. In this particular case, PAM is of special interest in the literature as it maximizes match productivity when the match utility is supermodular. Furthermore, the resulting matching outcome, i.e., the positive assortative matching outcome, is equivalent to the set of stable matchings (Roth and Sotomayor, 1992). That is, matches are individually rational, i.e., yield a utility greater than their outside option, and are pairwise stable, i.e., there exists no blocking pair of agent that would prefer to be matched to each other instead of the equilibrium matching. The next proposition shows under which circumstances PAM is not profit-maximizing.

Proposition 3. *If both market sides are fully symmetric, the positive assortative matching rule (PAM) is*

(i) profit-maximizing for all values of the exogenous parameters if and only if the platform can charge search costs $s_i \geq \min\{\theta_i(\theta_i - \theta_1), \theta_i^2 - \omega_i\}$, $\forall \theta_i \in \Theta \setminus \{\theta_1\}$ and $s_1 \geq \theta_1^2 - \omega_1$,

[18]Consider the following definition for *generically suboptimal.* The probability of the case in which random matching is optimal occurs with probability zero when the model parameters are randomly drawn from continuous intervals as defined in the proof.

(ii) suboptimal when the platform commits to advertisement if $\frac{\nu(s_i)}{s_i} < 1$ at $s_i = \theta_i^2 - \omega_i$ for at least one $\theta_i \in \Theta$.

When the platform commits to a (time-constant) deterministic matching rule such as PAM, agents will accept the recommended match in the first period. Therefore, all agents search for exactly one period, which results in a steady state population equal to the inflow for each type.

First, if the platform uses an advertisement-based business model, PAM is suboptimal if advertising is inefficient. If the return to showing advertising to the platform is greater than the cost to the agents, agents "pay with attention". To keep agents engaged and induce them to watch ads, however, the platform prolongs the search time of agents. Under PAM, agents search for one period only, hence, to be profit-maximizing the platform would need to charge the highest possible search cost to each agent. Under the condition in the proposition, however, the return to advertising to the platform is smaller than the cost to the agents implying that advertisement is inefficient at such high search costs. Due to the concavity of the return to advertising, the platform benefits from lowering the search costs and in turn by inducing the agents to search.

Second, PAM is indeed profit-maximizing when the platform charges type-dependent fees that extract the full surplus from agents, i.e., a fee equal to the match value from an assortative match minus the outside option. In this case, the "search fee" is effectively a participation fee as the fee is only paid once upfront for the first search. Consider, however, the case in which the platform cannot commit to high participation fees as there is an upper bound on the fee that the platform can charge $\overline{s}$.

Let $\overline{s}$ be such that s_i violates the condition in Proposition 3 for at least one type $\theta_i \in \Theta$ and for exposition, let this type be not the lowest type. Then, the platform earns $1 \cdot \overline{s} < \min\{\theta_i(\theta_i - \theta_1), \theta_i^2 - \omega_i\}$ from an agent of type θ_i. Then, PAM is not profit-maximizing, as the platform has an incentive to deviate to a matching rule under which type θ_i and the lowest type θ_1 meet with mass ε and additionally decrease the fee for the lowest type s_1. As $\overline{s}$ is such that whenever type θ_i would meet type θ_1 under PAM, θ_i would strictly reject θ_1, then θ_i (weakly) rejects θ_1 under the new matching rule. This implies that type θ_i searches longer than one period such that the platform earns more from type θ_i. If the platform does not decrease the search fee for the lowest type, the lowest type would not participate on the platform as it was just indifferent under PAM. Instead the platform decreases the search fee for the lowest type such that the lowest type is indifferent between participating or not under the new matching rule. Hence, the platform earns the same as before from the lowest types. Overall, this deviation increased the platforms profits.

For example, fees for in-app purchases in Apple's App store are capped at 999.99$, i.e., $\overline{s} = 999.99\$$. The estimated life time utility from a match and hence, potential willingness to pay for a partner could be well above 999.99$. Traditional matchmakers charge over ten times the amount.[19]

[19] See `https://www.nytimes.com/2024/02/13/business/dating-bounty-roy-zaslavskiy.html?unlocked_article_code=1.VU0.XqAb.q2iJT-p0bHz1&smid=nytcore-ios-share&referringSource=articleShare`

When does positive sorting occur, i.e. when is the matching outcome at least (weakly) positive assortative? To answer this question, consider the case with binary types as an illustration.

4.2 BINARY TYPES

Suppose there are only two types $\Theta = \{\theta_h, \theta_l\}$ in the market with $\theta_h > \theta_l$, which have an outside option of zero.[20] In the previous section, I showed that random matching is suboptimal for the platform, while PAM can also be suboptimal. Both matching rules implement matching outcomes that are (weakly) positively assortative. In order to answer when the matching outcome is positively assortative under a matching rule chosen by the platform, the analysis will characterize all possible matching outcomes. For simplicity, the main text is presented for the case where δ goes to zero, while the formal proofs hold for $\delta \geq 0$.

The section leads with the case where both types of agents face the same search cost designed by the platform $s_i = s$. A possible interpretation is that the two types of agents use a basic service of a (freemium) platform. In this case, the platform is assumed to decide on an amount of advertising to be shown to each agent using the basic service. Alternatively, in the case of payments, agents use one of the (discrete) tiers of the platform, where agents using the same tier pay the same amount, as for example on dating platforms. For job platforms, companies often pay the same price per click when they advertise in the same submarket. Finally, I discuss the implications of price discrimination.

The first result is Lemma 4, which characterizes the optimal matching rule that implements the mutual acceptance probabilities that are consistent with Equation 5. Lemma 4 and its proof can be found in Appendix B. To identify the optimal matching rule for the platform, suppose for now that s is exogenous and the platform maximizes the mass of active users.

Given the platform's matching rule, high type agents can either (i) accept only other high types, (ii) accept low types with positive probability, or (iii) accept low types with probability one. This results in five possible constellations of mutual acceptance probabilities and thus matching outcomes. In case (i), low types will always accept high types, resulting in a positive assortative matching outcome — only agents of the same type accept each other. Depending on the matching rule in cases (ii) and (iii), low types may accept low types, resulting in a weak assortative matching outcome — high and low types mutually accept the same types of agents. Alternatively, low types may reject low types, resulting in a non-assortative matching outcome — high types accept low types, but low types do not.

The lemma shows that for each of the five possible matching outcomes, there exists an optimal matching rule that implements the outcome for a range of parameters. The implementation of the matching outcomes depends crucially on feasibility. Given the total mass of agents that join, the ratio of new high to low type agents, $0 < \beta_h/\beta_l < \infty$,

[20] The following analysis qualitatively unaffected as long as the outside options are $\omega_l < \theta_l^2$ and $\omega_h < \theta_h\theta_l$. The platform's profit, however, is quantitatively affected as the platform can extract less rent from each agent.

determines which outcome can be implemented, as the ratio affects the steady state population of both types. The positive assortative matching outcome can be implemented over the whole range, whereas the weakly assortative and non-assortative outcomes cannot be implemented for all β_h/β_l. Then, given the existence of an optimal matching rule, which matching outcome maximizes the mass of active users on the platform?

First, consider the maximum rent that the platform can extract from agents through search. A high type agent is willing to search the longest for a match with another high type. In this case, the maximum rent the platform can extract from a high type agent is proportional to $\theta_h(\theta_h - \theta_l)$, which is the value of its own type times the *match premium*. The match premium is the gain from being in a final match with a high type instead of leaving with a low type. If the platform were to extract more rent, high types would start accepting low types as well, and thus not search at all. Conversely, if high types always reject low types and only search for high types, the maximum rent the platform can extract from low types is proportional to θ_l^2, since low types have an outside option of zero. The matching outcome is positive assortative.

Due to feasibility constraints, the platform is constrained by the ratio of high to low types when choosing the matching rule. The platform can extract the rent from both types — as described above — at

$$0 < \left(\frac{\beta_h}{\beta_l}\right)^* = \frac{\theta_l^2 - s}{\theta_h(\theta_h - \theta_l) - s} < 1. \tag{6}$$

At this "optimal" ratio, high types are just indifferent between accepting and rejecting low types, while low types are just indifferent between participating or not, which results in

$$\begin{aligned} \theta_h\theta_l &= \frac{-s + \phi(\theta_h|\theta_h)\theta_h^2}{\phi(\theta_h|\theta_h)} \Leftrightarrow \phi(\theta_h|\theta_h) = \frac{s}{\theta_h(\theta_h - \theta_l)}, \\ 0 &= \frac{-s + \phi(\theta_l|\theta_l)\theta_l^2}{\phi(\theta_l|\theta_l)} \Leftrightarrow \phi(\theta_l|\theta_l) = \frac{s}{\theta_l^2}. \end{aligned}$$

Due to feasibility constraints, the incentive and participation constraints cannot generally bind at the same time while implementing a positive assortative matching outcome. As the ratio increases, relatively more high type agents enter compared to low type agents. In this case, high types inevitably meet high types more often, so the platforms makes the participation constraint binding for low types. The platform must increase the probability of a high type meeting a high type such that high types are left with a rent greater than $\theta_h\theta_l$. As the ratio decreases, relatively few high type agents enter compared to low type agents. The platform makes the incentive constraint binding for high types, leaving a positive rent for low types by increasing the probability of a low type meeting a low type. In both cases, the platform potentially forgoes a significant amount of rent when moving away from the "optimal" ratio.

Second, consider the second-best matching outcome for the platform when it cannot implement the positive assortative matching outcome at the ratio in Equation 6. Suppose the ratio of high to low types is greater than in Equation 6. Then, the platform can commit to a matching rule in which high types randomize over accepting and rejecting low types, while low types remain indifferent between participating and their outside option. The platform can extract less rent from high type agents because the expected match utility for a high type agent is now a linear combination of the match utility with a high type and a low type. The platform, however, can extract more rent from low types as their expected match utility increases. The randomization probability of high types and the matching rule are substitutes for the platform, so the platform prefers randomization to a slack incentive constraint.

Third, as the ratio continues to increase, more high types enter the market. Since low types meet and mutually accept relatively many high types, the platform finds it profitable to implement a matching rule under which low types start rejecting low types. The rent extracted from low types is then proportional to $\theta_l(\theta_h - \theta_l)$, the value of their own type times the *match premium*. The next proposition summarizes the results.

Proposition 4. *For given $s \leq \overline{s}$, the platform implements a*

(a) positive assortative matching outcome if

$$\frac{\beta_h}{\beta_l} \leq \frac{\theta_l^2 - s}{\theta_h(\theta_h - \theta_l) - s}$$

,

(b) weakly assortative matching outcome if

$$\frac{\theta_l^2 - s}{\theta_h(\theta_h - \theta_l) - s} \leq \frac{\beta_h}{\beta_l} \leq \max\left\{\frac{\theta_h(\theta_h - \theta_l)}{\theta_h(\theta_h - \theta_l) - s}, \left(\frac{\beta_h}{\beta_l}\right)'\right\},$$

(c) non-assortative matching outcome if

$$\max\left\{\frac{\theta_h(\theta_h - \theta_l)}{\theta_h(\theta_h - \theta_l) - s}, \left(\frac{\beta_h}{\beta_l}\right)'\right\} \leq \frac{\beta_h}{\beta_l} \leq \frac{\theta_h^2 - \theta_l^2 - s}{\theta_h(\theta_h - \theta_l) - s},$$

(d) positive assortative matching outcome if

$$\frac{\theta_h^2 - \theta_l^2 - s}{\theta_h(\theta_h - \theta_l) - s} \leq \frac{\beta_h}{\beta_l}.$$

For a given inflow of high and low types, $\frac{\beta_h}{\beta_l}$, Proposition 4 presents the matching outcomes that the platform prefers to implement. The assortativity of the matching outcomes is non-monotonic in the ratio of high to low types. For example, the platform implements the positive assortative matching outcome in markets in which one type dominates. In contrast, the platform implements mismatch in relatively balanced markets. Similarly, the optimal matching rule can recommend more or less assortative matches depending on the ratio of high to low types. Overall, the positive assortative, and socially optimal, outcome is implemented over a smaller range of β_h/β_l if search cost, s, increases and the type difference, $\theta_h - \theta_l$, decreases.

Turning to the optimal matching rule, a first general insight is the following:

Corollary 1. *The platform strategically lowers the quality of recommended matches.*

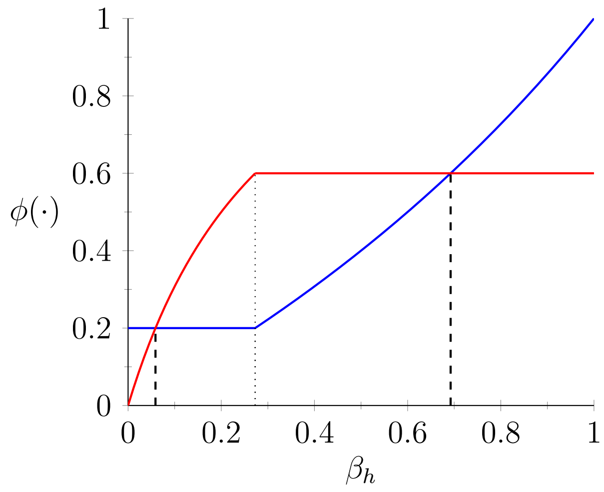

Figure 2: Recommended Mismatches

In other words, the platform recommends mismatches to agents if feasible. Therefore, if the platform can identify the agent's types perfectly high types meet high types less often than low types do for a large range of parameters.

Figure 2 shows the conditional probabilities of a high (and low) type meeting a high type when the platform implements positive assortative matching. For a wide range of parameters, the platform recommends mismatches, i.e. low types meet a high type more often than high types do. The result is subject to feasibility, so if the market is highly skewed towards one type, the matching must necessarily be more assortative.

Note that in all equilibria the conditional probability of a high type to meet a high type fulfills

$$\underbrace{s}_{\text{search cost}} \quad \cdot \underbrace{\phi(\theta_h|\theta_h)}_{\text{cond. meeting prob.}} \geq \underbrace{\theta_h(\theta_h - \theta_l)}_{\text{match premium}},$$

where the inequality is binding in all equilibria for $\frac{\beta_h}{\beta_l} \leq \frac{\theta_h^2 - \theta_l^2 - s}{\theta_h(\theta_h - \theta_l) - s}$. The conditional probability of a low type meeting a high type changes throughout the equilibria. The conditional probabilities for a high type do not generically coincide with random matching, but are designed to prolong the search time for the highest type.

Corollary 2. *For a given matching outcome, the optimal matching rule, $\mathcal{M}^*$, becomes more assortative if search cost, s, increase and the type difference, $\theta_h - \theta_l$, decreases.*

Intuitively, if the search cost increases, i.e., the platform charges a higher search fee or increases the advertising load, the matching rule must become more assortative as the platform extracts more rent per period, so that the search time must decrease. If the type difference increases, i.e. the types become less similar, the matching rule becomes less assortative as high types become more willing to search for a high type instead of accepting a now less valuable match with a low type.

Next, consider the welfare implications, measured as (i) the amount of mismatch and (ii) the inefficient length of search for agents. Let the welfare loss from match productivity be given by

$$\mathcal{W} = \sum_{i \in h,l} \sum_{j \in h,l} \alpha(\theta_i, \theta_j) \Phi(\theta_i, \theta_j)(\theta_i \theta_j - \theta_i^2).$$

The cost from search is s per agent per period, such that the resulting costs per agent during the expected usage time are given by the stopping time

$$\mathcal{T}(\theta_i) = s \cdot \frac{1}{\sum_{j=h,l} \alpha(\theta_i, \theta_j) \phi(\theta_j | \theta_i)}.$$

For example, consider the stopping time in the positive assortative matching outcome

$$\begin{aligned} \theta_h\text{-type:} \quad \frac{1}{\phi(\theta_h|\theta_h)} &= \frac{\theta_h(\theta_h - \theta_l)}{s}, \\ \theta_l\text{-type:} \quad \frac{s}{\phi(\theta_l|\theta_l)} &= \frac{\beta_h(\theta_h(\theta_h - \theta_l) + (\beta_l - \beta_h)s}{\beta_l s}, \end{aligned}$$

where low types search longest if $\frac{\beta_h}{\beta_l} = \left(\frac{\beta_h}{\beta_l}\right)^*$ with $\frac{s}{\phi(\theta_l|\theta_l)} = \frac{\theta_l^2}{s}$.

Proposition 5. *The welfare loss from match productivity is always larger under the non-assortative outcome than the weakly assortative matching outcome. In contrast, the welfare loss from search costs is highest under the positive assortative matching outcome if* $\frac{\beta_h}{\beta_l} = \left(\frac{\beta_h}{\beta_l}\right)^*$.

In unbalanced markets, the platform implements the positive assortative matching outcome, where final matches occur only within each group of types, i.e., types sort positively. The positive assortative matching outcome maximizes the overall match productivity. Therefore, the platform realizes the "efficient" matching outcome. In contrast, in more balanced markets, the platform prefers to induce mismatches.

The proof furthermore shows that the welfare loss from mismatch in the non-assortative case decreases with increasing β_h/β_l. In the weakly assortative matching outcome, the welfare loss is greater if search costs increase (decrease) as long as $\beta_l > \beta_h$ ($\beta_l < \beta_h$). Furthermore, the welfare loss is greater with increasing type difference. Differentiating between types, high types search the longest in the positive assortative matching outcome for low β_h/β_l, whereas low types search the longest in the weakly assortartive matching outcome.

4.3 ADVERTISING AND SEARCH FEE

Suppose the platform sells its users attention to advertisers. The platform decides over the advertising intensity that is related to the nuisance cost that users experience. Let $\nu(s)$ be the revenue per unit of nuisance cost to users. For the analysis, I make the following assumption

Assumption 2. *Advertisement revenue $\nu(s)$ is an increasing, concave function of the nuisance cost s with $\nu(0) = 0$. The semi-elasticity of advertisement $\frac{\nu(s)}{\nu'(s)}$ is an increasing, (strictly) convex function of the nuisance cost.*

The assumption excludes functions that are convex, i.e. under which the platform could prefer an advertising intensity that induces users to stay for one period only and therefore, substantially decreases the mass of active users. As advertisers might be willing to pay less for advertisement when a platform is smaller, i.e. their ads reach a smaller audience, $\nu(\cdot)$ might positively depend on the mass of active users on the platform. As a consequence, if with increasing nuisance demand decreases, the marginal rate of $\nu(\cdot)$ might decrease. Convexity implies that the semi-elasticity is sufficiently sensitive to changes in s for larger s.

The profit under advertisement is

$$\Pi = \frac{\nu(s)}{1-\rho}\left(\sum_k \sum_i f(\theta_i^k)(s)\right),$$

where the mass of agents of type θ_i^k is given by Lemma 4. The platform maximizes the advertising intensity through s and chooses $s = s^{A,*}$ such that

$$\frac{\nu(s)}{\nu'(s)} = -\frac{\sum_k \sum_i f(\theta_i^k)}{\frac{\partial \sum_k \sum_i f(\theta_i^k)}{\partial s}}. \tag{7}$$

Under the above condition the marginal cost of advertising, given by the semi-elasticity of demand on the right-hand side, and the marginal benefit of advertising, given by the semi-elasticity of advertisement, are equal.

The semi-price elasticity of advertisement changes depending on the distribution of high versus low types that join the platform. The optimal solution is either characterized by an interior solution s^A (given existence) that solves Equation 7 or a corner solution. To ensure that the profit-maximization problem is concave and has an interior solution, the function $\nu(s)$ must fulfill the following additional assumption.

Proposition 6. *The platform sets $s^{A,*} \in (0, \overline{s})$ if an interior solution of the optimal advertisement level exists. Otherwise, the platform selects $s^{A,*} = \overline{s}$. For the optimal $s^{A,*}$, the platform implements the matching outcomes in Proposition 4. If an interior solution exist, the advertising level is highest in the positive assortative matching outcome.*

Next, consider the case in which the platform sets a linear search fee, i.e. $\nu(s) = s$ is the identify function.

Proposition 7. *Let $s \in (0, \overline{s})$. The platform sets $s = \frac{\beta_l \theta_l^2 - \beta_h \theta_h(\theta_h - \theta_l)}{\beta_l - \beta_h}$ to implement the positive assortative matching outcome if*

$$0 \leq \frac{\beta_h}{\beta_l} \leq \frac{\theta_l^2}{\theta_h(\theta_h - \theta_l)}.$$

Otherwise, the platform either sets the smallest possible fee $s = \varepsilon > 0$, or the highest possible fee $s = \overline{s}$ to implement matching outcome (b) to (d) in Proposition 4.

The proposition characterizes the optimal solution for the platform if s only consists of the search fee that the agents pay on the platform. The choice of the search fee has the following implications for the matching outcomes.

In equilibrium (a), the platform chooses s to solve

$$\frac{\beta_h}{\beta_l} = \frac{\theta_l^2 - s}{\theta_h(\theta_h - \theta_l) - s},$$

In equilibrium (b), the platform prefers to choose search fee as small as possible, whereas it prefers to choose search fee as high as possible in (c). As a result, agent search for the longest in (b), but search for the smallest amount of time in (c). In equilibrium (d), the platform is indifferent between any $s \in (0, \overline{s}]$, but prefers to set low search fee to increase the range of the equilibrium. Lastly, the platform decreases the search fee again in (e).

Overall, the platform prefers to choose search fees that are relatively small and increase the search time instead of charging a high search fee and inducing less search.

Proposition 8. *Let the platform choose $s_h \in [0, \theta_h(\theta_h - \theta_l)]$ and $s_l \in [0, \theta_l^2]$. The platform can always implement the positive assortative matching outcome by choosing*

$$(s_h, s_l) : \frac{\beta_h}{\beta_l} = \frac{s_h}{s_l} \frac{(\theta_l^2 - s_l)}{(\theta_h(\theta_h - \theta_l) - s_h)}.$$

5. EXPLANATIONS

The optimal contract is a set of personalized participation fees if $\nu(s_i^k) = s_i^k$. The platform maximizes match productivity as in Appendix A.2. Considering the simplified model from Section 4.2, the platform commits to the positive assortative matching rule and personalized fees $(s_h = \theta_h^2, s_l = \theta_l^2)$ (see Proposition 3). The platform's profit is

$$\Pi^{PD} = \frac{2(1-\delta)}{(1-\rho)} (\beta_h \theta_h^2 + \beta_l \theta_l^2).$$

5.1 ADVERTISEMENT

Advertisement plays a key role in the digital economy. More specifically, in the light of the application to dating and job search platforms, a substantial share of these platforms rely on advertisement as a source of revenue, see Appendix C for an overview of dating and job search apps that show advertisement.

In the following example, I highlight that a (partly) advertising-based business model can outperform profits generated by personalized prices that extract the total consumer surplus from agents. Consider the following example.

Example 1. Let $\beta_h = \frac{3}{11}, \beta_l = \frac{8}{11}, \theta_h = 1, \theta_l = \frac{1}{2}$ and $\nu(s) = e^{-\frac{1}{10s}}$. The optimal intensity of advertising corresponds to $s = \frac{1}{10}$ A short calculation yields

$$\Pi^A = \frac{2}{1-\rho} \frac{10}{e} \left(\beta_h \theta_h (\theta_h - \theta_l) + \beta_l \theta_l^2 \right) = \frac{2}{1-\rho} 1.18 \tag{8}$$

$$\Pi^{PD} = \frac{2}{1-\rho}\left(\beta_h\theta_h^2 + \beta_l\theta_l^2\right) = \frac{2}{1-\rho}0.45$$

Given Proposition 4, the profit from advertisement can be bounded by

$$\frac{2(1-\delta)\nu(s^A)}{1-\rho}\frac{2\beta_l\theta_l^2 + (\beta_h - \beta_l)s^A}{s^A} \leq \Pi^A \leq \frac{2(1-\delta)\nu(s^A)}{1-\rho}\frac{\beta_h\theta_h(\theta_h - \theta_l) + \beta_l\theta_l^2}{s^A},$$

where advertising is more profitable in balanced markets (of high to low types). If the market is extremely unbalanced, advertising profits are low especially if many high types are in the market. Suppose the platform can implement the upper bound on the profit. If the revenue per disutility of advertisement exceeds the inefficiency in profits, then advertising can outperform the optimal contract

$$\frac{\nu(s)}{s} \geq \frac{\beta_h\theta_h^2 + \beta_l\theta_l^2}{\beta_h\theta_h(\theta_h - \theta_l) + \beta_l\theta_l^2},$$

where $\frac{\nu(s)}{s} \simeq \frac{10}{7}$ for the numbers in the example.

Proposition 9. *An advertisement-based business model can generate higher revenue than with personalized prices if advertisement revenue is sufficiently efficient compared to its nuisance.*

5.2 OVERCONFIDENCE

Up to this point, the model has assumed that agents behave rationally and have a correct expectation about their own type. In the following, I will introduce a fraction of overconfident agents, i.e., agents who perceive themselves to be of a higher type than they actually are. In the simplest example, an overconfident low type perceives itself as a high type.

Overconfidence is a widely documented bias in the psychology and behavioral economics literature (Burks et al., 2013; Dubra, 2015). Especially in dating markets overconfidence is thought to be prevalent for example, when it comes to a person's own attractiveness. Bruch and Newman (2018, 2019) analyze the structure of online dating markets in US cities and provide suggestive evidence for the fact that the majority of users contacts a partner who is more desirable than they are instead of contacting a partner who is as desirable than they are. In an experiment Egebark et al. (2021), document that both women and men prefer attractive over unattractive profiles regardless of their own attractiveness. More generally, Greitemeyer (2020) documents that more unattractive people are unaware of their (un-)attractiveness from a psychological perspective. In labor markets, Spinnewijn (2015) and Mueller et al. (2021) find that the unemployed overestimate how quickly they will find a job and are persistently overconfident about their desirability to firms. Lastly, Dargnies et al. (2019) document in an experiment that agents who are overconfident are less likely to accept earlier offers in a matching market.

Following this evidence, consider the following simple extension to the model in Section 4.2. There exists a symmetric share of λ overconfident users on each side of the market.

An overconfident users is of type θ_l, but persistently believes to be of type θ_h. Following Definition 2, I assume that an overconfident user correctly predicts agents' strategies given their perceived type, but mispredicts agents' actual acceptance behavior based on their true type. Furthermore, an overconfident user maximizes expected utility given their perceived type. As before, an overconfident users incurs search cost s and becomes inactive with probability δ.[21]

For simplicity, assume that the platform can perfectly identify overconfident users and classifies them as $\hat{\theta}_h$. In this case, the platform chooses the matching rule $\mathcal{M}$, which consists of $\phi(\cdot|\theta_i)$ for $\theta_i \in \{\theta_l, \theta_h, \hat{\theta}_h\}$. Consider the positive assortative matching outcome from Proposition 4, in which high types only accept high types, but low types accept all types. The incentive of participation constraint of high and low types are

$$\theta_h\theta_l \leq \frac{(1-\delta)(-s+\phi(\theta_h|\theta_h)\theta_h^2)}{\delta+(1-\delta)\phi(\theta_h|\theta_h)}, \tag{IC-θ_h}$$

$$0 \leq \frac{(1-\delta)(-s+\phi(\theta_l|\theta_l)\theta_l^2)}{\delta+(1-\delta)\phi(\theta_l|\theta_l)}. \tag{PC-θ_l}$$

An overconfident agent is characterized by a perceived incentive constraint, which determines their acceptance behavior, and their actual incentive constraint, which determines their search time. The perceived incentive constraint is

$$\hat{\theta}_h\theta_l \leq \frac{(1-\delta)(-s+\phi(\theta_h|\theta_h)\hat{\theta}_h\theta_h)}{\delta+(1-\delta)\phi(\theta_h|\theta_h)}, \tag{PIC-$\hat{\theta}_h$}$$

which coincides with the incentive constraint of high types, whereas their actual participation constraint is violated

$$\frac{-s}{\delta} < 0. \tag{PC-$\hat{\theta}_h$}$$

As overconfident users mispredict the true acceptance behavior of agents towards them, overconfident users end up rejecting low types and accepting high types. This leads them to search until they exogenously exit with probability δ as high types reject overconfident users in equilibrium.

Remark. Overconfident users search too intensively.

Following the incentive and participation constraints, the platform maximizes its profit subject to the feasibility and steady state constraints.

Proposition 10. *(Overconfidence) The platform makes larger profits with per-period payments than with personalized participation fees if $\lambda \geq \lambda^*$ for*

$$\lambda^* \equiv \frac{\beta_h\delta\theta_h\left(\theta_l^2(\theta_l-\delta\theta_h)+\delta\theta_h^3\right)}{\beta_l\left(\theta_l^2+\delta(\theta_h^2-\theta_l^2)\right)\left(\theta_l^2-\delta\theta_h^2\right)},$$

and charges a search fee of $s = \theta_l^2$.

[21]Note that δ can have an additional interpretation in the presence of overconfident users. If overconfident users do not find a match, δ can be interpreted with the probability that an overconfident agents looses due to growing dissatisfaction with the platform.

Anecdotes from Dating Apps, such as Tinder, provide evidence for the fact that less than 10% of users account for a disproportional amount of revenue.[22] On Tinder, an average user spends around 30$ in in-app purchases and subscriptions, whereas "heavy" users would spend 10 times the amount.

Consider the following example to illustrate that in markets with many low types, already a small percentage of overconfident users can be sufficient to achieve higher profits.

Example 2. Let $\beta_h = \frac{1}{4}, \beta_l = \frac{3}{4}, \theta_h = 1, \theta_l = \frac{1}{2}$. Then,

$$\lambda \geq 4.7\% \text{ if } \delta = \frac{1}{20},$$
$$\lambda \geq 13.7\% \text{ if } \delta = \frac{1}{10}.$$

For low values of δ, a relatively small percentage of overconfident users is necessary to substantially increase the platforms profit. Note that δ is directly related to the stopping time of overconfident users, i.e. in the first (second) case overconfident users search for 20 (10) period before they exit. More generally, consider the following comparative statics.

Corollary 3. *λ^* increases in δ, $\frac{\beta_h}{\beta_l}$ and $\theta_h - \theta_l$.*

Intuitively, the necessary share of overconfident users decreases if δ becomes small as overconfident users search for more periods. If the ratio $\frac{\beta_h}{\beta_l}$ increases, i.e. there are more high types than low types in the market, the platform needs to rely more on overconfident users. The reason is that given that the platform implements the positive assortative matching outcomes, the inefficiency between per-period payments and personalized upfront prices increases with more high types as the platform extracts too little surplus from high types. Similarly, if the type differences increase, a large amount of the platform's profit is driven by high types.

6. DISCUSSION

Exogenous Search Cost Consider the following version of the model. Agents incur additive search cost s_i^k in each period, which are the sum of exogenous search cost $\tilde{s}_i^k$ and endogenous search costs $\hat{s}_i^k$. The endogenous search costs are designed by the platform as before. Agents can differ in their exogenous search costs. For example, on dating platforms accepting or rejecting a recommendation requires attention for inspecting the corresponding profile. In labor markets, job seekers need to upload a CV and cover letter on the platform when applying for a job. In both examples, agents might face cost of attention when making their acceptance decisions. In the literature on search-and-matching markets, many articles assume that search frictions arise as time costs due to

[22] See https://uxdesign.cc/how-tinder-drives-over-1-6-billion-in-revenue-8006e718e761 and the referenced podcast therein, https://open.spotify.com/episode/1ZfL2Mq1n0NzyVKKerynvZ?si=UBlpCunARLW8jPfNNYK4dw.

discounting. Exceptions are Chade (2001) and Atakan (2006) who study explicit search costs.

As detailed above, my model can also accommodate explicit additive search costs. Whereas agents incur $\tilde{s}_i^k + \hat{s}_i^k$, the platform now optimizes over $\hat{s}_i^k$. This change does not affect the existence of an optimal solution, but it does affect the optimal matching rule, since positive search costs, $\tilde{s}_i^k > 0$, decrease the continuation value (c.p.).

Exogenous search costs can be a source of additional mismatch. For example, the result that the positive assortative matching outcome can always be implemented with price discrimination in search fees and binary types in Proposition 8 is no longer true with exogenous search costs. Instead, small search costs for one or both types imply that the platform cannot implement the positive assortative matching outcome for all ratios of high to low types. Therefore, the platform can prefer to implement a matching outcome in which non-assortative pairs accept each other as in Section 4.2.

Application to Other Platform Objectives The proof and tools in Section 4.1 can be analogously applied to other objectives of the platform, as long as the objective remains linear. For example, if the platform is concerned about its reputation the objective could be the weighted average of the demand and welfare from match productivity, i.e. the sum over the mass of matched pairs with match productivity of $u(\theta_i^k, \theta_j^k)$. For the weight $\mu \in [0,1]$, the objective takes the following form

$$\frac{1-\delta}{1-\rho}\left(\mu \sum_{k=A,B}\sum_{\theta_j^{-k}\in\Theta^{-k}} \nu(s_i^k) f(\theta_i^k) + (1-\mu)\sum_{k=A,B}\sum_{\theta_j^{-k}\in\Theta^{-k}} m(\theta_i^k, \theta_j^{-k})\theta_i^k\theta_j^{-k}\right),$$

where the first term maximizes demand given function $\nu(s_i^k)$ and the second term maximizes the welfare from match productivity. The extreme case $\mu = 1$ has been analyzed throughout the main part of the paper, while $\mu = 0$ is equivalent to maximizing the socially optimal matching as in Appendix A.2.

Hybrid Business Model The main model considers a platform that either commits to either an amount of advertising or search fees. The model, however, can incorporate hybrid business models such as the freemium business model. In the freemium business model, a portion of users use the platform for free but see advertisements, while the other portion of users pay search fees and do not see advertisements. To implement this business model, the platform segments the market into types and selects a subset of types for each service. The platform shows advertisements to those user types where $\nu(s_i^k)/s_i^k$ is greater than one at the optimal advertising level, i.e., the return of advertising per unit of disutility is greater than one. Otherwise, the platform will charge these users a search fee.

Labor Markets The model in Section 3 assumes that utility is non-transferable. In labor markets, however, workers and firms often negotiate about the wages. Therefore, in the literature utility is often modeled to be transferable. The results of the model are qualitatively unaffected when assuming that the utility from a match is $u(\theta_i^k, \theta_j^{-k}) = 2\theta_i^k \theta_j^{-k}$ for the pair and utility is transferable if agents bargain over the surplus via the Nash bargaining solution.

7. CONCLUSION

On matching platforms, the misalignment of incentives between users and the platform becomes more problematic as platforms collect more data and develop more predictive algorithms. This paper presents a model in which a platform has perfect information about its users' types and matches them to its advantage. In contrast, random matching corresponds to the case where the platform has no information about its users' types. The platform benefits from more information about its users' types: Random matching is strictly suboptimal.

Both sorting and search time have implications for real-world markets. The platform's algorithm can support the socially optimal matching. But even absent exogenous search costs and search frictions, the algorithm can also foster non-assortative matching outcomes in fully symmetric markets resulting in mismatch. Additionally, it increases users' search time by recommending unsuitable matches. While mismatch has a negative impact on productivity and long-term unemployment in labor markets (Şahin et al., 2014; McGowan and Andrews, 2015), assortative mating in marriage markets is a driver of household inequality (Pestel, 2017; Eika et al., 2019; Almar et al., 2023). Therefore, if policies aim to reduce mismatch — as in labor markets — policymakers should be concerned about matching platforms that employ the business models described above. Rather than relying on platforms to reduce search frictions, the platform's algorithm is a potential source of additional mismatch. In contrast, dating apps can make a positive contribution to reducing household inequality.

Empirical evidence on online matching and search platforms is mixed. For example, in dating markets Hitsch et al. (2010) show that matches are approximately efficient and stable. The authors, however, rely on data before the advent of large dating apps. In contrast, more recent evidence, such as Sharabi and Dorrance-Hall (2024), finds that people who meet online are less satisfied in their marriages. In labor markets, Kroft and Pope (2014) shows that Craigslist has no effect on the unemployment rate. Similarly, Gürtzgen et al. (2021) provide evidence that online searches do not affect employment stability or wage outcomes, but instead increase the proportion of unsuitable candidates in job applications.

REFERENCES

ACM, "Market Study into Mobile App Stores," Technical Report 2019.

Almar, Frederik, Benjamin Friedrich, Ana Reynoso, Bastian Schulz, and Rune Vejlin, "Marital Sorting and Inequality: How Educational Categorization Matters," 2023.

Anderson, Simon P and Stephen Coate, "Market Provision of Broadcasting: A Welfare Analysis," *The Review of Economic Studies*, 2005, *72* (4), 947–972.

Antler, Yair and Benjamin Bachi, "Searching Forever After," *American Economic Journal: Microeconomics*, 2022, *14* (3), 558–590.

_ , Daniel Bird, and Daniel Fershtman, *Search, Dating, and Segregation in Marriage*, Foerder Institute for Economic Research, Tel Aviv University, The Eitan . . . , 2024.

_ , _ , and _ , *Search, Matching, and Online Platforms*, Centre for Economic Policy Research, 2024.

Aridor, Guy, "Market Definition in the Attention Economy: An Experimental Approach," *American Economic Journal: Microeconomics*, Forthcoming.

Atakan, Alp E, "Assortative Matching with Explicit Search Costs," *Econometrica*, 2006, *74* (3), 667–680.

Becker, Gary S, "A Theory of Marriage: Part I," *Journal of Political Economy*, 1973, *81* (4), 813–846.

_ , "A Theory of Social Interactions," *Journal of Political Economy*, 1974, *82* (6), 1063–1093.

Bertsimas, Dimitris and John N Tsitsiklis, *Introduction to Linear Optimization*, Vol. 6, Athena Scientific Belmont, MA, 1997.

Bloch, Francis and Harl Ryder, "Two-Sided Search, Marriages, and Matchmakers," *International Economic Review*, 2000, *41* (1), 93–116.

Bruch, Elizabeth E and Mark EJ Newman, "Aspirational Pursuit of Mates in Online Dating Markets," *Science Advances*, 2018, *4* (8), eaap9815.

_ and MEJ Newman, "Structure of Online Dating Markets in US Cities," *Sociological Science*, 2019, *6*, 219–234.

Burdett, Kenneth and Melvyn G Coles, "Long-Term Partnership Formation: Marriage and Employment," *The Economic Journal*, 1999, *109* (456), 307–334.

Burks, Stephen V, Jeffrey P Carpenter, Lorenz Goette, and Aldo Rustichini, "Overconfidence and Social Signalling," *Review of Economic Studies*, 2013, *80* (3), 949–983.

Caillaud, Bernard and Bruno Jullien, "Chicken & Egg: Competition among Intermediation Service Providers," *RAND Journal of Economics*, 2003, pp. 309–328.

Chade, Hector, "Two-Sided Search and Perfect Segregation with Fixed Search Costs," *Mathematical Social Sciences*, 2001, *42* (1), 31–51.

_ , Jan Eeckhout, and Lones Smith, "Sorting Through Search and Matching Models in Economics," *Journal of Economic Literature*, 2017, *55* (2), 493–544.

Chen, Daniel, "The Market for Attention," *Available at SSRN 4024597*, 2022.

Damiano, Ettore and Hao Li, "Price Discrimination and Efficient Matching," *Eco-*

nomic Theory, 2007, *30* (2), 243–263.

_ **and Li Hao**, "Competing Matchmaking," *Journal of the European Economic Association*, 2008, *6* (4), 789–818.

Dantzig, George B., *Linear Programming and Extensions*, Princeton: Princeton University Press, 1963.

Dargnies, Marie-Pierre, Rustamdjan Hakimov, and Dorothea Kübler, "Self-Confidence and Unraveling in Matching Markets," *Management Science*, 2019, *65* (12), 5603–5618.

Deng, Yiting, Anja Lambrecht, and Yongdong Liu, "Spillover Effects and Freemium Strategy in the Mobile App Market," *Management Science*, 2023, *69* (9), 5018–5041.

Dertwinkel-Kalt, Markus, Vincent Eulenberg, and Christian Wey, "Defining What the Relevant Market is: A New Method for Consumer Research and Antitrust," *Available at SSRN*, 2024.

Dolado, Juan J, Marcel Jansen, and Juan F Jimeno, "On-the-Job Search in a Matching Model with Heterogeneous Jobs and Workers," *The Economic Journal*, 2009, *119* (534), 200–228.

Dubra, Jean-Pierre Benoît Juan, "Does the Better-Than-Average Effect Show That People Are Overconfident?: Two Experiments," *Journal of the European Economic Association*, 2015, *13* (2), 293–329.

Echenique, Federico, Nicole Immorlica, and Vijay V Vazirani, *Online and Matching-Based Market Design*, Cambridge University Press, 2023.

Eeckhout, Jan, "Bilateral search and Vertical Heterogeneity," *International Economic Review*, 1999, *40* (4), 869–887.

Egebark, Johan, Mathias Ekström, Erik Plug, and Mirjam Van Praag, "Brains or Beauty? Causal Evidence on the Returns to Education and Attractiveness in the Online Dating Market," *Journal of Public Economics*, 2021, *196*, 104372.

Eika, Lasse, Magne Mogstad, and Basit Zafar, "Educational Assortative Mating and Household Income Inequality," *Journal of Political Economy*, 2019, *127* (6), 2795–2835.

Eliaz, Kfir and Ran Spiegler, "On the Strategic Use of Attention Grabbers," *Theoretical Economics*, 2011, *6* (1), 127–155.

_ **and** _ , "A simple model of search engine pricing," *The Economic Journal*, 2011, *121* (556), F329–F339.

_ **and** _ , "Search Design and Broad Matching," *American Economic Review*, 2016, *106* (3), 563–586.

Fershtman, Daniel and Alessandro Pavan, "Matching Auctions," *The RAND Journal of Economics*, 2022, *53* (1), 32–62.

Gale, David and Lloyd S Shapley, "College Admissions and the Stability of Marriage," *The American Mathematical Monthly*, 1962, *69* (1), 9–15.

Gomes, Renato and Alessandro Pavan, "Many-to-Many Matching and Price Discrimination," *Theoretical Economics*, 2016, *11* (3), 1005–1052.

_ **and** _ , "Price Customization and Targeting in Matching Markets," *The RAND Jour-*

nal of Economics, 2024, *55* (2), 230–265.

Greitemeyer, Tobias, "Unattractive People are Unaware of Their (Un) Attractiveness," *Scandinavian Journal of Psychology*, 2020, *61* (4), 471–483.

Gürtzgen, Nicole, Benjamin Lochner, Laura Pohlan, and Gerard J van den Berg, "Does Online Search Improve the Match Quality of New Hires?," *Labour Economics*, 2021, *70*, 101981.

Hagiu, Andrei and Bruno Jullien, "Why Do Intermediaries Divert Search?," *The RAND Journal of Economics*, 2011, *42* (2), 337–362.

_ **and** _ , "Search Diversion and Platform Competition," *International Journal of Industrial Organization*, 2014, *33*, 48–60.

Halaburda, Hanna, Mikołaj Jan Piskorski, and Pınar Yıldırım, "Competing by Restricting Choice: The Case of Matching Platforms," *Management Science*, 2018, *64* (8), 3574–3594.

Hardy, Godfrey Harold, John Edensor Littlewood, and George Pólya, *Inequalities*, Cambridge University Press, 1952.

Hitsch, Günter J, Ali Hortaçsu, and Dan Ariely, "Matching and Sorting in Online Dating," *American Economic Review*, 2010, *100* (1), 130–163.

Johnson, Terence R, "Matching Through Position Auctions," *Journal of Economic Theory*, 2013, *148* (4), 1700–1713.

Kircher, Philipp, "Schumpeter Lecture 2022: Job Search in the 21St Century," *Journal of the European Economic Association*, 2022, *20* (6), 2317–2352.

Koopmans, Tjalling C and Martin Beckmann, "Assignment Problems and the Location of Economic Activities," *Econometrica: journal of the Econometric Society*, 1957, pp. 53–76.

Kroft, Kory and Devin G Pope, "Does Online Search Crowd out Traditional Search and Improve Matching Efficiency? Evidence from Craigslist," *Journal of Labor Economics*, 2014, *32* (2), 259–303.

Kummer, Michael and Patrick Schulte, "When Private Information Settles the Bill: Money and Privacy in Google's Market for Smartphone Applications," *Management Science*, 2019, *65* (8), 3470–3494.

Lauermann, Stephan and Georg Nöldeke, "Stable Marriages and Search Frictions," *Journal of Economic Theory*, 2014, *151*, 163–195.

Lee, Soohyung, "Effect of Online Dating on Assortative Mating: Evidence from South Korea," *Journal of Applied Econometrics*, 2016, *31* (6), 1120–1139.

McGowan, Müge Adalet and Dan Andrews, "Labour Market Mismatch and Labour Productivity: Evidence from PIAAC Data," 2015.

Mortensen, Dale T and Christopher A Pissarides, "Job Creation and Job Destruction in the Theory of Unemployment," *The Review of Economic Studies*, 1994, *61* (3), 397–415.

_ **and** _ , "New Developments in Models of Search in the Labor Market," *Handbook of Labor Economics*, 1999, *3*, 2567–2627.

Mueller, Andreas I, Johannes Spinnewijn, and Giorgio Topa, "Job Seekers' Perceptions and Employment Prospects: Heterogeneity, Duration Dependence, and Bias," *American Economic Review*, 2021, *111* (1), 324–363.

Murphy, Sean C, William von Hippel, Shelli L Dubbs, Michael J Angilletta Jr, Robbie S Wilson, Robert Trivers, and Fiona Kate Barlow, "The Role of Overconfidence in Romantic Desirability and Competition," *Personality and Social Psychology Bulletin*, 2015, *41* (8), 1036–1052.

Nocke, Volker and Patrick Rey, "Consumer Search, Steering, and Choice Overload," *Journal of Political Economy*, 2024, *132* (5), 000–000.

Pestel, Nico, "Marital Sorting, Inequality and the Role of Female Labour Supply: Evidence from East and West Germany," *Economica*, 2017, *84* (333), 104–127.

Pissarides, Christopher A, "Short-Run Equilibrium Dynamics of Unemployment, Vacancies, and Real Wages," *The American Economic Review*, 1985, *75* (4), 676–690.

Prat, Andrea and Tommaso Valletti, "Attention Oligopoly," *American Economic Journal: Microeconomics*, 2022, *14* (3), 530–557.

Robles, Theodore F, Richard B Slatcher, Joseph M Trombello, and Meghan M McGinn, "Marital Quality and Health: A Meta-Analytic Review.," *Psychological Bulletin*, 2014, *140* (1), 140.

Rochet, Jean-Charles and Jean Tirole, "Platform Competition in Two-Sided Markets," *Journal of the European Economic Association*, 2003, *1* (4), 990–1029.

_ **and** _ , "Two-Sided Markets: A Progress Report," *The RAND Journal of Economics*, 2006, *37* (3), 645–667.

Rosenfeld, Michael J, Reuben J Thomas, and Sonia Hausen, "Disintermediating your Friends: How Online Dating in the United States Displaces other Ways of Meeting," *Proceedings of the National Academy of Sciences*, 2019, *116* (36), 17753–17758.

Roth, Alvin E, "The Economics of Matching: Stability and Incentives," *Mathematics of Operations Research*, 1982, *7* (4), 617–628.

_ **and Marilda Sotomayor**, "Two-Sided Matching," *Handbook of Game Theory with Economic Applications*, 1992, *1*, 485–541.

Şahin, Ayşegül, Joseph Song, Giorgio Topa, and Giovanni L Violante, "Mismatch Unemployment," *American Economic Review*, 2014, *104* (11), 3529–3564.

Shapley, Lloyd S and Martin Shubik, "The Assignment Game I: The Core," *International Journal of game theory*, 1971, *1* (1), 111–130.

Sharabi, Liesel L and Elizabeth Dorrance-Hall, "The Online Dating Effect: Where a Couple Meets Predicts the Quality of their Marriage," *Computers in Human Behavior*, 2024, *150*, 107973.

Shimer, Robert and Lones Smith, "Assortative Matching and Search," *Econometrica*, 2000, *68* (2), 343–369.

Smith, Lones, "The Marriage Model with Search Frictions," *Journal of Political Economy*, 2006, *114* (6), 1124–1144.

Spinnewijn, Johannes, "Unemployed but Optimistic: Optimal Insurance Design with Biased Beliefs," *Journal of the European Economic Association*, 2015, *13* (1), 130–167.

Srinivasan, Karthik, "Paying Attention," Technical Report, Mimeo 2023.

Wets, Roger J-B, "On the Continuity of the Value of a Linear Program and of Related Polyhedral-Valued Multifunctions," *Mathematical Programming Essays in Honor of*

George B. Dantzig Part I, 1985, pp. 14–29.

A. APPENDIX

A.1 LINEAR PROGRAMMING FORMULATION

The linear programming formulation of the platform's problem 1 is given in the following. For $\alpha(\theta_i^k, \theta_j^{-k}) \in \{0,1\}$, the platform's optimization problem can be represented by the following (mixed integer) linear program:

$$\max \sum_{k=A,B} \sum_{\theta_i^k \in \Theta^k} \frac{(1-\delta)s_i^k}{1-\rho} f(\theta_i^k), \tag{9}$$

subject to participation constraints

$$\beta_i^k \omega_i^k \leq f(\theta_i^k)(\delta\omega_i^k - (1-\delta)s_i^k) + (1-\delta)\sum_j m(\theta_i^k, \theta_j^{-k})\theta_i^k\theta_j^{-k}, \forall \theta_i^k \in \Theta^k,\ k = A, B, \tag{10}$$

incentive constraints

$$\begin{aligned} \beta_i^k\theta_i^k\theta_j^{-k} + \alpha(\theta_i^k, \theta_j^{-k})(-\beta_i^k\theta_i^k\theta_j^{-k}) \leq & f(\theta_i^k)(\delta\omega_i^k - (1-\delta)s_i^k) + (1-\delta)\sum_j m(\theta_i^k, \theta_j^{-k})\theta_i^k\theta_j^{-k} \\ & \leq \left(\frac{\beta_i^k}{\delta}\theta_i^k\theta_j^{-k} - \beta_i^k\theta_i^k\theta_j^{-k}\right)(1 - \alpha(\theta_i^k, \theta_j^{-k})) + \beta_i^k\theta_i^k\theta_j^{-k}, \\ & \forall \theta_i^k \in \Theta^k,\ k = A, B, \end{aligned} \tag{11}$$

feasibility and steady state constraints

$$\sum_{\theta_j^{-k} \in \Theta^{-k}} \Phi(\theta_i^k, \theta_j^{-k}) + \mathbf{1}_{k=B}\Phi(\theta_i^k, \omega_i^k) = f(\theta_i^k), \forall \theta_i^k \in \Theta^k,\ k = A, B, \tag{12}$$

$$f(\theta_i^k) = \frac{\beta_i^k - (1-\delta)\sum_j m(\theta_i^k, \theta_j^{-k})}{\delta}, \forall \theta_i^k \in \Theta^k,\ k = A, B, \tag{13}$$

and constraints on the matched and recommended pairs $\forall(\theta_i^k, \theta_j^{-k}) \in \Theta^k \times \Theta^{-k}$. First, the mass of recommended and matched pairs must be non-negative and the mass of matched pairs cannot be greater than the mass of recommended pairs

$$\Phi(\theta_i^k, \theta_j^{-k}) \geq 0, m(\theta_i^k, \theta_j^{-k}) \geq 0,, \tag{14}$$

$$m(\theta_i^k, \theta_j^{-k}) \leq \Phi(\theta_i^k, \theta_j^{-k}), \tag{15}$$

second, the mass of matched pairs must be smaller than the largest possible mass times the acceptance probability, but larger than the mass of recommended pairs minus the largest possible mass times the probability of a rejection

$$m(\theta_i^k, \theta_j^{-k}) \leq \frac{\min\{\beta_i^k, \beta_j^{-k}\}}{\delta}\alpha(\theta_i^k, \theta_j^{-k}), \tag{16}$$

$$m(\theta_i^k, \theta_j^{-k}) \geq \Phi(\theta_i^k, \theta_j^{-k}) - \frac{\min\{\beta_i^k, \beta_j^{-k}\}}{\delta}(1 - \alpha(\theta_i^k, \theta_j^{-k})). \tag{17}$$

This ensures that, for example if $\alpha(\theta_i^k, \theta_j^{-k}) = 0$, the mass of matched pairs cannot be greater than zero and it must be strictly smaller than the mass of recommended pairs. To accommodate for mixed acceptance probabilities of agents, consider an agent of type θ_m^k that is indifferent between accepting and rejecting a type θ_s^{-k}. Hence, θ_m^k could randomize over the acceptance probability towards type θ_s^{-k}: $\sigma_k(\theta_m^k, \theta_s^{-k}) \in (0,1)$. Conceptually, this imposes indifference or equality on some constraints rather than inequalities in the original formulation above. For any pair $(\theta_m^k, \theta_s^{-k}) \in \Theta^k \times \Theta^{-k}$ for which $\alpha(\theta_m^k, \theta_s^{-k}) \in (0,1)$, the adjusted incentive constraints are

$$\beta_m^k \theta_m^k \theta_s^{-k} = f(\theta_m^k)(\delta\omega_m^k - (1-\delta)s_m^k) + (1-\delta)\sum_j m(\theta_m^k, \theta_j^{-k})\theta_m^k \theta_j^{-k}, \text{ for } \theta_m^k, \tag{18}$$

$$\beta_s^{-k} \theta_m^k \theta_s^{-k} \geq f(\theta_s^{-k})(\delta\omega_s^{-k} - (1-\delta)s_s^{-k}) + (1-\delta)\sum_j m(\theta_m^k, \theta_j^{-k})\theta_m^k \theta_j^{-k}, \text{ for } \theta_s^k, \tag{19}$$

where θ_m^k is indifferent between accepting and rejecting θ_s^{-k} and θ_s^{-k} (weakly) accepts θ_m^k. The constraints on the mass of recommended and matched pairs are

$$m(\theta_m^k, \theta_s^{-k}) \leq \frac{\min\{\beta_m^k, \beta_s^{-k}\}}{\delta}, \text{ for } (\theta_m^k, \theta_s^{-k}), \tag{20}$$

$$m(\theta_m^k, \theta_j^{-k}) \leq \Phi(\theta_m^k, \theta_s^{-k}), \text{ for } (\theta_m^k, \theta_s^{-k}). \tag{21}$$

The linear program can be summarized in the subsequent lemma.

Lemma 3 (Linear Program)**.** *For any pair $(\theta_i^k, \theta_j^{-k}) \in \Theta^k \times \Theta^{-k}$ for which $\alpha(\theta_i^k, \theta_j^{-k}) \in \{0,1\}$, the linear program is given by Equation 9 through 14. For any pair $(\theta_m^k, \theta_s^{-k}) \in \Theta^k \times \Theta^{-k}$ for which $\alpha(\theta_s^k, \theta_m^{-k}) \in (0,1)$, replace Equation 11 for θ_m^k by Equation 18 and replace Equation 11 for θ_s^k by Equation 19 and replace Equations 16 to 17 for $(\theta_m^k, \theta_s^{-k})$ by Equations 20 to 21.*

Note on Standard Form of a Linear Program To abbreviate future arguments, I relate the linear program to the standard form of a linear program. The matrix notation is

$$\max xc^T,$$
$$s.t. Hx \leq b, x \geq 0.$$

where $x \in \mathbb{R}^n$ is the variable vector — the mass of recommended and matched pairs — consisting of n variables and $c \in \mathbb{R}^n$. The m inequalities are given by matrix $H \in \mathbb{R}^{m \times n}$. Equalities, such as the feasibility constraints, can be expressed as two opposite inequalities. Vector $b \in \mathbb{R}^m$ captures the right-hand side of the inequalities. $\mathcal{P} \equiv \{x \in \mathbb{R}^n | Hx \leq b\}$ is the feasible region given by the inequality constraints.

A.2 BENCHMARKS

This section analyzes two polar cases, in which the intermediary has full information about agent's types and is able to extract the full rent from the matching output or the intermediary has no information about agent's types and must match agents at random.

Socially-Optimal Matching The first benchmark constitutes the case in which the intermediary (or a social planner) provides the socially-optimal matching under the premise that agent's types can be identified perfectly. The intermediary or social planner maximizes the sum of total matching outputs. The matching output function is supermodular, i.e. types of both sides are complements. The socially-optimal matching is the solution to the linear program

$$\max_{M} \sum_{k=A,B} \sum_{\theta_j^{-k} \in \Theta^{-k}} \sum_{\theta_i^k \in \Theta^k} \theta_i^k \theta_j^{-k} m(\theta_i^k, \theta_j^{-k})$$

subject to feasibility

$$\sum_{\theta_j^{-k} \in \Theta^{-k}} m(\theta_i^k, \theta_j^{-k}) \leq \beta_i^k, \forall \theta_i^k \in \Theta^k,$$

$$\sum_{\theta_i^k \in \Theta^k} m(\theta_i^k, \theta_j^{-k}) \leq \beta_j^{-k}, \forall \theta_j^{-k} \in \Theta^{-k},$$

$$m(\theta_i^k, \theta_j^{-k}) \geq 0, \forall (\theta_i^k, \theta_j^{-k}) \in \Theta^k \times \Theta^{-k}.$$

The linear program follows the optimal assignment problem by Koopmans and Beckmann (1957) and Shapley and Shubik (1971). Both agents that form the match $(\theta_i^k, \theta_j^{-k})$ receive the output $\theta_i^k \cdot \theta_j^{-k}$.

Remark. If markets are fully symmetric, the socially optimal matching is $m(\theta_i^k, \theta_j^{-k}) = \beta_i^k$ if $\theta_i^k = \theta_j^{-k}$. The outcome is said to exhibit positive assortative matching.

If market sides are fully symmetric, $\beta_i^A = \beta_i^B$, the solution to the linear program is attained with $m(\theta_i^k, \theta_j^{-k}) \in \{0, \beta_i^k\}$, that is a pair is either matched with probability one or not matched. Although the linear program permits partial or fractional matching of agents, Dantzig (1963) showed that the maximum value of the objective is attained with probabilities in $\{0, 1\}$.

For symmetric populations of agents, optimality requires that no individual remains unmatched, such that the feasibility constraints must hold with equality. Otherwise, the social planner can increase welfare by assigning an unmatched agent to another unmatched agent as the value of their match is greater than zero. The objective is maximized if $m(\theta_i^k, \theta_j^{-k}) = \beta_i^k$ when $\theta_i^k = \theta_j^{-k}$ by applying the rearrangement inequality (Hardy et al., 1952).

Random Matching The second benchmark is a random matching market. For example, if an intermediary has no information (data) about agents' types, and thus cannot condition on any observables, the intermediary's matching rule incorporates random meetings between agents. A random matching market may also reflect offline meetings between agents that are not intermediated by any platform.

A random matching market is a tuple $(\hat{\Theta}^k, f(\theta_i^k))_{k=A,B}$ with parameters (s_i^k, δ).The analysis builds on the model of Lauermann and Nöldeke (2014).[23]

[23] In contrast to Lauermann and Nöldeke (2014), agents may face explicit search cost s_i^k in addition to δ. Furthermore, the speed of meetings and mass of meetings per unit of time is normalized to one.

Given that meetings are random, the fraction of meetings that involve type θ_i^k on side k and type θ_j^{-k} on side $-k$ is

$$\frac{f(\theta_i^k)f(\theta_j^{-k})}{\overline{f}^k \cdot \overline{f}^{-k}}$$

where the total mass of agents on side k is $\overline{f}^k = \sum_{\theta_i^k \in \Theta^k} f(\theta_i^k)$. The probability to meet type θ_j^{-k} on side $-k$ conditional on being an agent of type θ_i^k on side k is

$$\phi(\theta_j^{-k}) = \frac{f(\theta_j^{-k})}{\overline{f}^k \cdot \overline{f}^{-k}},$$

where the probability that type θ_i^k on side k exits the search process in a match with type θ_j^{-k} is

$$\mu(\theta_i^k, \theta_j^{-k}) = \frac{(1-\delta)\alpha(\theta_i^k, \theta_j^{-k})\phi(\theta_j^{-k})}{\delta + (1-\delta)\sum_{\theta_j^{-k}} \alpha(\theta_i^k, \theta_j^{-k})\phi(\theta_j^{-k})},$$

where $\mu(\theta_i^k, \omega_i^k) = 1 - \sum_{\theta_j^{-k}} \mu(\theta_i^k, \theta_j^{-k})$ is the probability that type θ_i^k remains unmatched.

Let $(f(\theta_i^k), \alpha(\theta_i^k, \theta_j^{-k})_{ij})_{k=A,B}$ be a steady state. Then M with entries given by

$$m(\theta_i^k, \theta_j^{-k}) = \frac{\alpha(\theta_i, \theta_j) f(\theta_i) g(\theta_j)}{\overline{f} \cdot \overline{g}}. \tag{22}$$

is the unique matching outcome induced by the steady state under random matching. Vice versa, if M is a steady state matching outcome then $f(\theta_i^k), \alpha(\theta_i^k, \theta_j^{-k})$ is given by

$$f(\theta_i^k) = \frac{\beta_i^k}{\delta}\mu(\theta_i^k, \omega_i^k), \tag{23}$$

$$\alpha(\theta_i, \theta_j) = m(\theta_i^k, \theta_j^{-k}) \frac{\overline{f}^k \cdot \overline{f}^{-k}}{f(\theta_i^k) f(\theta_j^{-k})}, \tag{24}$$

where $\alpha(\theta_i^k, \theta_j^{-k}) \leq 1$ for all $(\theta_i^k, \theta_j^{-k}) \in \hat{\Theta}^k \times \hat{\Theta}^{-k}$ and $m(\theta_i^k, \omega_i^k)$ is the probability of ending up with one's outside option. Matching M is an **equilibrium** matching if and only if

$$m(\theta_i^k, \theta_j^{-k}) = \begin{cases} 0 & \text{if } \theta_i^k \theta_j^{-k} < V^C(\theta_i^k) \text{ or } \theta_i^k, \theta_j^{-k} < V^C(\theta_j^{-k}) \\ \frac{f(\theta_i^k) f(\theta_j^{-k})}{\overline{f}^k \cdot \overline{f}^{-k}} & \text{if } \theta_i^k \theta_j^{-k} > V^C(\theta_i^k) \text{ and } \theta_i^k \theta_j^{-k} > V^C(\theta_j^{-k}) \end{cases}$$

holds for all $(\theta_i^k, \theta_j^{-k}) \in \hat{\Theta}^k \times \hat{\Theta}^{-k}$.

B. APPENDIX: PROOFS

B.1 N TYPES

Proof of Proposition 1 First, recall the definition of an equilibrium. For given type-dependent costs s_i^k and matching mechanism $\mathcal{M}$, an equilibrium is a steady state in which agents' acceptance probabilities maximize their expected utility and agents correctly anticipate each others strategies (by Definition 2).

Second, note that an equilibrium is defined for any matching mechanism $\mathcal{M}$, which consists of a *feasible* matching rule. Therefore, the feasible matching rule must satisfy the feasibility condition of Equation 1 in equilibrium.

Then, for any exogenously given matching rule that is feasible, an equilibrium exists if and only if the following conditions are met: (i) the steady state condition in Equation 2 must be met, (ii) agents' acceptance probabilities must maximize their expected utility and be consistent with Equation 5, and (iii) the matching rule must be feasible for all participating types $\theta_i^k \in \hat{\Theta}^k$ following Equation 1 and for all non-participating types $\theta_i^k \in \Theta^k \setminus \hat{\Theta}^k$ with $\phi(\theta_i^k) = \emptyset$.

The "if"- part follows directly from the definition of an equilibrium and feasible matching rule above. For the "only-if", note that for a given matching rule that is feasible, a steady state in which agents maximize their expected utility and the matching rule is feasible in implies the existence of an equilibrium.

Let β_i^k be the inflow of new agents of type θ_i^k on side k. Without loss of generality assume that $\sum_i \beta_i^A \geq \sum_i \beta_i^B$, i.e. market side B is equal or smaller than market side A. The proof proceeds as follows: for any exogenously given matching rule that is feasible, I construct an equilibrium according to condition (i) to (iii).

Base case: For any exogenously given matching rule that is feasible, there exists an equilibrium if $\Theta^k = \{\theta_1^k\}$ for $k = A, B$. Let $s_i^k > \theta_1^k \theta_1^{-k} - \omega_i^k$ for $k = A \vee B$, then for any exogenously given matching rule that is feasible, there exists only one equilibrium, in which no agent participates on the platform. The feasible matching rule is $\phi(\cdot) = \emptyset$ for any non-participating type. This is indeed an equilibrium as (i) it is a steady state as no agent enters the platform and hence, no agent leaves the platform. (ii) Agents maximize their expected utility as for $s_i^k > \theta_1^k \theta_1^{-k} - \omega_i^k$, the search cost exceeds the maximum match utility that an agent can obtain on the platform against its outside option, therefore violating the participation constraint of agents. If agents do not participate on one side of the market, there are no match opportunities for the other side of the market, such that no agent is willing to participate on the platform. (iii) The matching rule is feasible as no agent participates.

Let s_i^k be sufficiently small such that agents on both market sides participate. The matching rule is a tuple

$$(\phi(\theta_1^B|\theta_1^A), \phi(\theta_1^A|\theta_1^B), \phi(\omega_1^A|\theta_1^A), \phi(\omega_1^B|\theta_1^B)).$$

Let $\phi(\theta_1^A|\theta_1^B) \in [0, 1]$ be exogenously given. First, $\phi(\omega_1^B|\theta_1^B) = 1 - \phi(\theta_1^A|\theta_1^B)$, which follows from the fact that the matching probabilities conditional on a type θ_i^k must add up to one. Second, from feasibility, (iii), and the fact that the mass of matched pairs is symmetric, it must hold that

$$\phi(\theta_1^B|\theta_1^A)f(\theta_1^A) = \phi(\theta_1^A|\theta_1^B)f(\theta_1^B), \tag{25}$$

such that $\phi(\theta_1^B|\theta_1^A)$ is pinned down uniquely by the agents' acceptance behavior and resulting steady state mass. Note that whenever θ_1^A meets θ_1^B, it follows from (ii) expected utility maximization that both agents mutually accept each other as their continuation value is smaller than $\theta_1^A\theta_1^B$ (due to $s_i^k > 0$ and/or $\delta > 0$).

Consider the continuation value of type θ_1^B for exogenously given $\phi(\theta_1^A|\theta_1^B)$:

$$V^C(\theta_1^B) = \frac{\delta\omega_1^B + (1-\delta)(-s_1^B + \phi(\theta_1^A|\theta_1^B)\theta_1^A\theta_1^B)}{\delta + (1-\delta)\phi(\theta_1^A|\theta_1^B)},$$

which takes values within the compact interval $[\omega_1^B - \frac{(1-\delta)s_1^B}{\delta}, \delta\omega_1^B + (1-\delta)(-s_1^B + \theta_1^A\theta_1^B)]$. Note that type θ_1^B participates if $V^C(\theta_1^B) \geq \omega_1^B$. Otherwise, θ_1^B does not participate and the only equilibrium is the non-participation equilibrium from above. Given participation, the steady state mass of type θ_1^B is

$$f(\theta_1^B) = \frac{\beta_1^B}{\delta + (1-\delta)\phi(\theta_1^A|\theta_1^B)},$$

which follows from Equation 2 and analogously,

$$f(\theta_1^A) = \frac{\beta_1^A}{\delta + (1-\delta)\phi(\theta_1^B|\theta_1^A)},$$

given participation of type θ_1^B (otherwise, the only equilibrium is again the non-participation equilibrium). The steady state mass trivially satisfy (i) as they directly follow from rearranging Equation 2. Lastly, $\phi(\theta_1^B|\theta_1^A)$ then follows from Equation 25, and $\phi(\omega_1^B|\theta_1^B)$ from the fact $\phi(\omega_1^B|\theta_1^B) = 1 - \phi(\theta_1^B|\theta_1^A)$, which satisfies (iii).

Induction hypothesis: Let $\Theta^k = \{\theta_1^k, ..., \theta_N^k\}, k = A, B$. For any exogenously given matching rule that is feasible, there exists an equilibrium.

Induction step: Let $\Theta^k = \{\theta_1^k, ..., \theta_N^k, \theta_{N+1}^k\}, k = A, B$ and let s_i^k be sufficiently small to induce participation for all types, otherwise an equilibrium exists for the case with $\Theta^k = \{\theta_1^k, ..., \theta_N^k\}, k = A, B$ by the induction hypothesis.

The matching rule for type θ_i^k is a (discrete) probability distribution $\phi(\cdot|\theta_i^k) \in \Delta(\Theta^{-k} \cup \omega_i^k)$, which is defined by $N+2$ probabilities (one for each type and one for the outside option). For $N+1$ types, the matching rule is in total given by $(N+1)(N+2)$ conditional probabilities. Let the following conditional probabilities be exogenously given

$$\begin{aligned}
&\phi(\theta_{N+1}^A|\theta_{N+1}^B), ..., \phi(\theta_1^A|\theta_{N+1}^B),\\
&\phi(\theta_N^A|\theta_N^B), ..., \phi(\theta_1^A|\theta_N^B),\\
&\vdots\\
&\phi(\theta_1^A|\theta_1^B).
\end{aligned}$$

As in the base case, the remaining probabilities follow from the feasibility conditions

$$\sum_{\theta_j^{-k}\in\Theta^{-k}} \phi(\theta_j^{-k}|\theta_i^k) + \phi(\omega_i^k|\theta_i^k) = 1, \text{ for } k = A, B, \tag{26}$$

$$f(\theta_i^k)\phi(\theta_j^{-k}|\theta_i^k) = f(\theta_j^{-k})\phi(\theta_i^k|\theta_j^{-k}), \forall\theta_j^{-k} \in \Theta^{-k}, \theta_i^k \in \Theta^k. \tag{27}$$

First, given the exogenous conditional probabilities, I argue that the agents' acceptance probabilities and mutual acceptance probabilities, $\{\alpha(\theta_i^k, \theta_j^{-k}\}_{ij}$ follow from expected utility maximization by considering the following procedure. Start by considering the acceptance probabilities of the highest type θ_{N-1}^B on side B. Given participation, its continuation value takes values within a compact interval $[\omega_{N+1}^B, \delta\omega_{N+1}^B + (1-\delta)(-s_{N+1}^B + \theta_{N+1}^B\theta_{N+1}^A)]$. Given the exogenous conditional probabilities, the continuation value then determines the acceptance probabilities, $(\sigma_B(\theta_i^A, \theta_{N+1}^B))_i$, according to Equation 4. As all types θ_i^A accept a match with the highest type on side B, these are equivalent to the mutual acceptance probabilities, $(\alpha(\theta_i^A, \theta_{N+1}^B))_i$.

Second, given the exogenous conditional probabilities and type θ_{N+1}^B's mutual acceptance probabilities, the steady state mass follows from Equation 1

$$f(\theta_{N+1}^B) = \frac{\beta_{N+1}^B}{\delta + (1-\delta)\sum_i \alpha(\theta_i^A, \theta_{N+1}^B)\phi(\theta_i^A|\theta_{N+1}^B)}.$$

Next, determine the acceptance probabilities $(\sigma_A(\theta_i^A, \theta_N^B))_i$ of a type θ_i^A on side A towards type θ_N^B on side B. Suppose for now that type θ_i^A rejects type θ_N^B. Under this assumption and given $(\alpha(\theta_i^A, \theta_{N+1}^B))_i$, I can construct the continuation value of a type θ_i^A and check whether

$$\theta_i^A\theta_N^B \leq \frac{\delta\omega_i^k + (1-\delta)(-s_i^A + \alpha(\theta_i^A, \theta_{N+1}^B)\phi(\theta_{N+1}^B|\theta_i^A)\theta_i^A\theta_{N+1}^B}{\delta + (1-\delta)(\alpha(\theta_i^A, \theta_{N+1}^B)\phi(\theta_{N+1}^B|\theta_i^A)},$$

holds. Note that whenever $\alpha(\theta_i^A, \theta_{N+1}^B) = 0$, it is easy to see that type θ_i^A cannot reject type θ_N^B when maximizing utility.

Continue to type θ_N^B on side B. Given the acceptance probabilities on side A, $(\alpha(\theta_i^A, \theta_N^B))_i$ can be determined and the resulting steady state mass. Follow by determining the acceptance probabilities $(\sigma_A(\theta_i^A, \theta_{N-1}^B))_i$ of a type θ_i^A on side A towards type θ_{N-1}^B on side B and so on.

Lastly, by construction the matching rule is feasible as the remaining probabilities follow from Equation 27 given the mutual acceptance probabilities and the steady state masses.

Conclusion: Since both the base case and the general case have been proved as true, by mathematical induction the statement holds for every natural number N. □

Proof of Lemma 2 As defined in the Section 4.1, the set $\mathcal{F}^*$ is a subset of the set $\mathcal{F}$. I show that the set $\mathcal{F}^*$ is (i) finite and (ii) non-empty.

To define set $\mathcal{F}^*$, recall the following definitions from the text: (i) Define a subset $\mathcal{A}^* \subset \mathcal{A}$, where $\mathcal{A}$ are the mutual acceptance matrices that can be implemented through $\mathcal{F}$. If $A' \in \mathcal{A}$ has only entries composed of $\alpha(\theta_i^k, \theta_j^{-k}) \in \{0,1\}$, then $A' \in \mathcal{A}^*$. For any $A' \in \mathcal{A}$ with an entry $\alpha(\theta_i^k, \theta_j^{-k}) \in (0,1)$, add matrix A'' to $\mathcal{A}^*$ in which any entry with $\alpha(\theta_i^k, \theta_j^{-k}) \in (0,1)$ is replaced with a variable α_{ij}. (ii) For each $A \in \mathcal{A}^*$, find the matching rule that implements the profit-maximizing matching outcome $\mathcal{F}^*(A)$. Then, $\mathcal{F}^* = \mathcal{F}^*(A) \cup \mathcal{A}^*$.

(a) $\mathcal{F}^*$ is non-empty.

Recall from Proposition 1 that $\mathcal{F}$ is non-empty and thus, also $\mathcal{A}^*$ is non-empty. By Proposition 1, as long as at least one type participates in the market, for any matching rule there exists an equilibrium. This implies that there exists at least one matching outcome that can be implemented. Next, I will show that for any $A \in \mathcal{A}^*$, there exists an optimal matching rule to implement A such that $A \cup \mathcal{F}^*(A) \in \mathcal{F}^*$.

To do so, fix $A \in \mathcal{A}^*$, which yields a linear program as defined in Lemma 3 in Appendix A.1. To show that an optimal solution exists, I show that: (i) the objective of the linear program is bounded, i.e., the linear program is not unbounded, and (ii) the feasible region of the variable vector, $\mathcal{P}$, is non-empty. From both it follows that there exists an optimal solution by Dantzig (1963); Bertsimas and Tsitsiklis (1997).

(i) First, I show that the objective is bounded. For a maximization problem to be bounded there must exists a constant $C \in \mathbb{R}$ such that for all feasible $x \in \mathbb{R}^n$ $c^T x \leq C$ holds. The objective is bounded as

$$\sum_{k=A,B} \sum_{\theta_i^k \in \Theta^k} \frac{(1-\delta)s_i^k}{(1-\rho)} f(\theta_i^k) < \sum_{k=A,B} \sum_{\theta_i^k \in \Theta^k} \frac{(1-\delta)s_i^k}{(1-\rho)} \frac{\beta_i^k}{\delta} \equiv C. \tag{28}$$

(ii) Second, I show that the feasible region is non-empty. The feasible region is defined by the set $\mathcal{P} = \{x \in \mathbb{R}^n : Hx \leq b\}$. For any $A \in \mathcal{A}^*$, there exists a matching rule under which the constraints are not inconsistent by Proposition 1. This follows from the fact that $\mathcal{A}^* \subset \mathcal{A}$ and the definition of $\mathcal{A}$ implies that $A \in \mathcal{A}$ if and only if there exists an exogenous matching rule for which an equilibrium exists that implements A. Therefore, the feasible region is not empty.

Then, by strong duality (Dantzig, 1963), it follows that the linear program attains an optimal solution for any $A \in \mathcal{A}^*$: $\mathcal{F}^*(A)$ and $\mathcal{F}^*(A) \cup \mathcal{A}^*$ is non-empty.

(b) $\mathcal{F}^*$ is finite.

As $\mathcal{F}^* = \bigcup_{A \in \mathcal{A}^*} \mathcal{F}^*(A)$ and $\mathcal{A}^*$ is finite by construction, $\mathcal{F}^*$ is also finite. Recall that $\mathcal{A}^*$ is defined as a finite subset $\mathcal{A}^* \subset \mathcal{A}$: If $A' \in \mathcal{A}$ has only entries composed of $\alpha(\theta_i^k, \theta_j^{-k}) \in \{0,1\}$, then $A' \in \mathcal{A}^*$. For any $A' \in \mathcal{A}$ with $\alpha(\theta_i^k, \theta_j^{-k}) \in (0,1)$, add matrix A'' to $\mathcal{A}^*$ in which entry $\alpha(\theta_i^k, \theta_j^{-k})$ is replaced with the generic variable α_{ij}. By construction $\mathcal{A}^*$ is finite. First, there is a finite number of matrices with permutations of $\alpha(\theta_i^k, \theta_j^{-k}) \in \{0,1\}$ as the permutations of finite variables is finite. Second, observe that any value of $\alpha(\theta_i^k, \theta_j^{-k}) \in (0,1)$ for the same pair $(\theta_i^k, \theta_j^{-k}$ requires that the incentive constraints in Equation 11 in the linear program in Appendix A.1 are replaced by the incentive constraints in Equation 18 and 19. The linear program solves for an optimal mass of matched pairs, $m(\theta_i^k, \theta_j^{-k})$, and optimal mass of recommended pairs, $\Phi(\theta_i^k, \theta_j^{-k})$, which returns an optimal mutual acceptance probability of

$$\alpha(\theta_i^k, \theta_j^{-k}) = \frac{m(\theta_i^k, \theta_j^{-k})}{\Phi(\theta_i^k, \theta_j^{-k})}.$$

Therefore, it is without loss of generality to construct the set $\mathcal{A}^*$ to be finite. $\square$

Proof of Theorem 1 The first part of the theorem immediately follows from Lemma 2: *For any* $s_i^k \in \mathbb{R}_+^{|\Theta^k| \times |\Theta^{-k}|}$*, there exists an optimal solution.* By Lemma 2 the set $\mathcal{F}^*$ is finite and non-empty. Hence, $\max_{\mathcal{F}^*} \Pi$ is well-defined and a solution exists.

Next, I prove the second part of the theorem: *Let the platform choose* $s_i^k \in \mathbb{R}_+^{|\Theta^k| \times |\Theta^{-k}|}$*, there exists an optimal solution.* By Lemma 2, the set $\mathcal{F}^*$ is finite and non-empty for any $s_i^k \in \mathbb{R}_+^{|\Theta^k| \times |\Theta^{-k}|}$. For $s_i^k \geq \max_{\theta_j^{-k}}\{\theta_i^k \cdot \theta_j^{-k}\}$ for all $\theta_i^k \in \Theta^k$, however, the matching outcome is empty as no agent participates and the equilibrium profit is zero. Therefore, to make positive profits $s_i^k \leq \max_{\theta_j^{-k}}\{\theta_i^k \cdot \theta_j^{-k}\}\}$ for at least one $\theta_i^k \in \Theta^k$ such that the set of participating types $\hat{\Theta}^k$ is non-empty. For simplicity, denote $\max_{\theta_i^k} \max_{\theta_j^{-k}}\{\theta_i^k, \theta_j^{-k}\} = \overline{u}$ as the maximum utility that the highest type can attain on the platform.

Let $s_i^k \in [0, \overline{u}] \equiv \mathcal{S}$ and denote the vector $(s_i^k)_i^k \equiv \mathbf{s}$. Consider the correspondence from $\mathbf{s}$ to $\Pi(\mathbf{s}, \mathcal{F}^*(\mathbf{s}))$ by

$$\Psi(\mathbf{s}) : \mathcal{S}^{|\Theta^k| \times |\Theta^{-k}|} \rightrightarrows \mathcal{C},$$

which assigns to each point $\mathbf{s}$ of $\mathcal{S}^{|\Theta^k| \times |\Theta^{-k}|}$ a subset $\Psi(\mathbf{s})$ of $\mathcal{C} \subset \mathbb{R}_+$. Since $\mathcal{C}$ is a collection of finite profit levels for any given $\mathbf{s}$, the set is compact for any given $\mathbf{s}$. Hence, because $\mathbf{s}$ is chosen from a compact set, the correspondence is compact-valued. Therefore, an optimal solution exists if the correspondence is upper hemicontinuous. To show that $\Psi(\mathbf{s})$ is upper hemicontinuous, I first prove that the following: The steady-state mass is upper hemicontinuous in $\mathbf{s}$ for each linear program induced by a matrix $A_l \in \mathcal{A}^*$

The steady-state mass is a function of the mass of matched pairs, which is the solution to the linear program in Lemma 3 in Appendix A.1. As the steady-state mass is continuous in the mass of matched pairs and is only a function of $\mathbf{s}$ through the mass of matched pairs, I proceed to show that the mass of matched pairs, i.e. the solutions to the linear programs, are upper hemicontinuous. This in turn implies that the steady-state mass is upper hemicontinuous in $\mathbf{s}$.

First, note that when fixing one linear program induced by some $A_l \in \mathcal{A}^*$, the set of (primal) feasible solutions is upper hemicontinuous. Consider one linear program l for given $A_l \in \mathcal{A}^*$ following the notation in Appendix A.1:

$$\Pi(r)_l \equiv \sup_{x \in \mathbb{R}^n} \{cx | Hx \leq b, x \geq 0\},$$

where H and b are determined by A_l and where

$$
\begin{aligned}
r &\equiv (c; H_1; H_2; ...; H_m, b^T), \\
&= (c_1, ..., c_n; h_{11}, ..., h_{1n}; h_{m1}, ..., h_{mm}; b_q, ..., b_m).
\end{aligned}
$$

The set of primal feasible solutions of the linear program that defines Π is the (polyhedral-valued) multifunction (or set-valued function)

$$r \to P(r)_l \equiv \{x | Hx \leq b, x \geq 0\}.$$

As I have shown that the linear program induced by some $A \in \mathcal{A}^*$ has an optimal solution, the value of the linear program, $\Pi(r)_l$, is finite on set $\mathcal{J}_l \equiv \{r \in \mathbb{R}^{n+m \cdot n+m} | -\infty < \Pi_l(r) < \infty\}$.

As $\mathbf{s}$ continuously changes H, b and c, the solution $P(r)_l$ is upper hemicontinuous in $\mathbf{s}$ if it is upper hemicontinuous in r. Then taken from Wets (1985), $P(r)_l$ is upper hemicontinuous at r if

$$r = \lim_{v \to \infty} r_v, \;\; x_v \in P(r_v)_l, \;\text{ and } x = \lim_{v \to \infty} x_v,$$

implies that $x \in P(r)_l$. To prove, consider the following: Suppose that $\{r_v\}_v \in \mathcal{J}$ and $r = \lim_{n \to \infty} r_v$. Let $\{x_v\}_v$ be a sequence such that for all v, $x_v \in P(r)_l$: $H_v x_v \leq b_v$, and $\lim_{v \to \infty} = x$. Since

$$||H_v - H|| \to 0, \;\; ||x_v - x|| \to 0, \;\text{ and } ||b_v - b|| \to 0,$$

it follows that $Hx \leq b$ and $x \geq 0$ which yields $x \in P(r)_l$. This implies that $P(r)_l$ is in fact upper hemicontinuous in r.

Next, it follows that $\Pi(r)_l$ is also upper hemicontinuous on $\mathcal{J}_l$ as $\Pi(r)_l$ is the composition of a continuous single-valued function and a upper hemicontinuous set-valued functions. The product and sum of upper hemicontinuous (set-valued) functions preserve the property.

To summarize, $P(r)_l$ and $\Pi(r)_l$ are upper hemicontinuous on $\mathcal{J}_l$. It remains to show that the profit over all possible linear programs, $\Pi(r)$, is upper hemicontinuous on $\bigcup_{l \in L} \mathcal{J}_l$, where $|L|$ is the finite number of possible linear programs. Note that each linear program returns a finite set of feasible solutions and hence, a finite number of profit levels. Additionally, as the first part of the theorem notes, there exists at least one optimal matching outcome and thus, at least one optimal profit level for each $\mathbf{s}$.

Let $\Pi(r) = \bigcup_{l \in L} \Pi(r)_l$ be the finite union over the profit levels of each linear program. Show that $\Pi(r)$ is upper hemicontinuous in r on $\bigcup_{l \in L} \mathcal{J}_l$. If $|L| = 1$, this state is has been proven above. Let $|L| = 2$. By (yet another definition), $\Pi(r)_{l_1} \cup \Pi(r)_{l_2}$ is upper hemicontinuous at r_0, if for any open set $V \subseteq \mathcal{J}_{l_1} \cup \mathcal{J}_{l_2}$ with $\Pi(r_0) \subseteq V$, there exists an open neighborhood $U(r_0) \subseteq \bigcup_{l \in L} \mathcal{J}_l$ such that if $r \in U(r_0)$, then $\Pi(r) \subseteq V$.

As $\Pi(r)_l$ is finite on $\mathcal{J}_l$, $\Pi(r)_l$ is either finite or empty on $\bigcup_{l \in L} \mathcal{J}_l$ as the linear program is only feasible on $\mathcal{J}_l$ and is infeasible on $\bigcup_{l \in L} \mathcal{J}_l \setminus \mathcal{J}_l$ such that $P(r)_l =$.

Now, consider the following argument for $|L| = 2$: Since $\Pi(r) = \Pi(r_0)_{l_1} \cup \Pi(r_0)_{l_2} \subseteq V$, it follows that both $\Pi(r_0)_{l_1} \subseteq V$ and $\Pi(r_0)_{l_2} \subseteq V$. Because either both $\Pi(r_0)_{l_1}$ and $\Pi(r_0)_{l_2}$

are upper hemicontinuous, or at least one of the two is upper hemicontinuous (where the other is equal to the empty set), it holds that: There exists a neighborhood U_{l_1} of r_0 such that $\Pi(r_0)_{l_1} \subseteq V$ for all $r \in U_{l_1}$ (and for l_2 respectively). Let $U = U_{l_1} \cap U_{l_2}$. Then, for any $r \in U$, both $\Pi(r)_{l_1} \subseteq V$ and $\Pi(r)_{l_2} \subseteq V$ such that $\Pi(r)_{l_1} \cup \Pi(r)_{l_2} \subseteq V$. Therefore, $\Pi(r)$ is upper hemicontinuous. The argument can be extended to finite $|L|$ by proof via induction. Assume that the union of $\bigcup_{l \in L} \Pi(r)_l$ is upper hemicontinuous, then one needs to show that $\bigcup_{l \in L+1} \Pi(r)_l$ is upper hemicontinuous, which follows directly from the above argument. $\square$

Proof of Proposition 2 Define the parameters over the following sets: $\theta_i^k \in \Theta^k = [\underline{\theta}, \overline{\theta}] \subseteq \mathbb{R}_+$, $\beta_i^k \in [0,1]$, $\delta \in [0,1]$, $\omega_i^k \in \Omega = [\underline{\theta}, \overline{\theta}]$, and $s_i^k \in [0, \overline{u}]$. Then generically suboptimal implies that the probability of the case in which random matching is optimal occurs with probability zero when the parameters are randomly drawn from their respective compact intervals.

For given $A \in \mathcal{A}^*$, an optimal solution is a matching rule for which the objective function of the linear program in Appendix A.1 attains its maximum value. To verify optimality I rely on the geometry of linear programs. As defined in Appendix A.1, the feasible region $\mathcal{P} \equiv \{x \in \mathbb{R}^n | Hx \leq b\}$ is a convex polyhedron. From Bertsimas and Tsitsiklis (1997) the following are equivalent

(i) x is a vertex of $\mathcal{P}$.

(ii) x is an extreme point of $\mathcal{P}$.

(iii) x is a basic feasible solution of $\mathcal{P}$.

This implies, that if the vector that contains the recommended and matched pairs under random matching $x^{RM} \in \mathbf{R}^n$ for all given $A \in \mathcal{A}^*$ is not at a vertex of $\mathcal{P}$, then it is also not an optimal solution. Let $A_=$ be the matrix of constraints that are satisfied with equality under x. Then, x is a vertex if and only if $rank(A_=) = n$. In other words, there must be n linearly independent (in-)equalities that are binding (out of m (in-)equalities). For k feasibility constraints, which must be binding, optimality requires that $n - k \geq 1$ participation and incentive constraints must be binding.

Consider any given $A \in \mathcal{A}^*$ which belongs to the profit-maximizing matching outcome in $\mathcal{F}^*$. There are two cases: (i) $A \in \mathcal{A}^*$ is part of a non-assortative matching outcome, (ii) $A \in \mathcal{A}^*$ is part of a (weakly) assortative matching outcome, i.e., $\alpha(\theta_i^k, \theta_j^{-k})$ is weakly increasing in its second argument, as well as its first argument. In case (i), random matching can never implement the profit-maximizing outcome. As random matching always induces (weakly) positive assortative mutual acceptance probabilities as noted in Section 4. In case (ii) $A \in \mathcal{A}^*$ is (weakly) positive assortative, then random matching must induce $n-k$ binding participation and/or incentive constraints. Then for, $f(\theta_i^k)$ and $m(\theta_i^k, \theta_j^{-k})$ which under random matching are functions of β_i^k, δ and the probability of remaining unmatched $\mu(\theta_i^k, \omega_i^k)$, the participation and incentive constraints are generically non-binding

$$\beta_i^k \theta_i^k \theta_j^{-k} \leq f(\theta_i^k)(\delta\omega_i^k - (1-\delta)s_i^k) + (1-\delta)\sum_j m(\theta_i^k, \theta_j^{-k})\theta_i^k\theta_j^{-k},$$

$$\beta_i^k \omega_i^k \leq f(\theta_i^k)(\delta\omega_i^k - (1-\delta)s_i^k) + (1-\delta)\sum_j m(\theta_i^k, \theta_j^{-k})\theta_i^k\theta_j^{-k}.$$

Rearranging and using the steady state condition yields

$$\beta_i^k \left(\theta_i^k\theta_j^k - \omega_i^k + \frac{(1-\delta)}{\delta}s_i^k\right) \leq (1-\delta)\sum_j m(\theta_i^k, \theta_j^{-k})\left(\theta_i^k\theta_j^{-k} - \omega_i^k + \frac{(1-\delta)}{\delta}s_i^k\right),$$

$$\beta_i^k \frac{(1-\delta)}{\delta}s_i^k \leq (1-\delta)\sum_j m(\theta_i^k, \theta_j^{-k})\left(\theta_i^k\theta_j^{-k} - \omega_i^k + \frac{(1-\delta)}{\delta}s_i^k\right),$$

where under random matching $m(\theta_i^k, \theta_j^{-k}) = 0$ if $\alpha(\theta_i^k, \theta_j^{-k}) = 0$ and

$$m(\theta_i^k, \theta_j^{-k}) = \frac{\alpha(\theta_i^k, \theta_j^{-k})\beta_i^k\mu(\theta_i^k, \omega_i^k)\beta_j^{-k}\mu(\theta_j^{-k}, \omega_j^{-k})}{\left(\sum_{\theta_i^k}\beta_i^k\mu(\theta_i^k, \omega_i^k)\right)\cdot\left(\sum_{\theta_j^{-k}}\beta_j^{-k}\mu(\theta_j^{-k}, \omega_j^{-k})\right)}$$

if $\alpha(\theta_i^k, \theta_j^{-k}) \in (0,1]$ from Appendix A.2. Then for $A \in \mathcal{A}^*$, the probability that the mass of matched pairs under random matching induces a binding incentive or participation constraint is zero when $\beta_j^{-k}, s_i^k, \omega_i^k, \theta_i^k, \theta_j^{-k}$ and δ are drawn from continuous intervals. $\square$

Proof of Proposition 3 Suppose market sides are fully symmetric. Therefore, I drop the superscript k. The positive assortative matching rule is defined as $\phi(\theta_i|\theta_j) = 1$ if and only if $i = j$ and results in $f(\theta_i) = \beta_i$.

(i) Search Fee

(a) "if": PAM is optimal if $s_i = \theta_i^2$ for all $\theta_i \in \Theta$. To see this, note that as shown in Appendix A, PAM maximizes match productivity over all agents. By setting $s_i = \theta_i^2$, the platform extract the full surplus from agents. In this case, s_i is effectively a type-dependent participation fee.

(b) "only-if": Suppose for contradiction, PAM were optimal if $s_i < \theta_i(\theta_i - \max\{\theta_1, \omega_i\})$ for at least one $\theta_i \in \Theta \setminus \{\theta_1\}$. Then, the platform earns

$$\frac{2(1-\delta)}{1-\rho}\left(\sum_{\theta_j \in \Theta\setminus\{\theta_i\}}\beta_j\theta_j^2 + \beta_i s_i\right) < \frac{2(1-\delta)}{1-\rho}\left(\sum_{\theta_j \in \Theta\setminus\{\theta_i\}}\beta_j\theta_j^2 + \beta_i\theta_i(\theta_i - \min\{\theta_1, \omega_i\})\right)$$

Observe that for type θ_i, the incentive constraint towards the lowest type is slack. If type θ_i met type θ_1, it would strictly prefer to reject type θ_1.

$$\max\{\theta_i\theta_1, \omega_i\} < \delta\omega_i + (1-\delta)(\theta_i^2 - s_i) < \theta_i^2.$$

Consider a deviation to a matching rule in which types θ_1 and θ_i meet each other with mass $\epsilon > 0$ while keeping the matching rule for all other types fixed: $\Phi(\theta_i, \theta_1) = \epsilon$. Additionally, decrease s_1 from $s_1 = \theta_1^2 - \omega_1$ to s_1'. Then, there exists an $\epsilon > 0$ and s_1 that is feasible and incentive compatible which increases the platform's profit.

The feasibility constraints read

$$\frac{\beta_i}{\delta + (1-\delta)\phi(\theta_i|\theta_i)} = \frac{\beta_i \phi(\theta_i|\theta_i)}{\delta + (1-\delta)\phi(\theta_i|\theta_i)} + \epsilon,$$
$$\frac{\beta_1}{\delta + (1-\delta)\phi(\theta_1|\theta_1)} = \frac{\beta_1 \phi(\theta_1|\theta_1)}{\delta + (1-\delta)\phi(\theta_1|\theta_1)} + \epsilon,$$

which results in

$$\phi^D(\theta_i|\theta_i) = \frac{\beta_i - \epsilon\delta}{\beta_i + (1-\delta)\epsilon}, \ \phi^D(\theta_1|\theta_1) = \frac{\beta_1 - \epsilon\delta}{\beta_1 + (1-\delta)\epsilon}.$$

For $\epsilon > 0$, the assortative matching probabilities are strictly smaller than one. And let, $s_l = \frac{\beta_1 - \epsilon\delta}{\beta_1 + (1-\delta)\epsilon}(\theta_1^2 - \omega_1)$.

Next, to ensure that the acceptance probabilities are unaffected by the change of the matching rule, I examine the incentive and participation constraints. Given $s_i < \min\{\theta_i(\theta_i - \theta_1), \theta_i^2 - \omega_i)$ and PAM incentive constraint (or participation constraint) of θ_i is non-binding. As the deviation matching probabilities are continuous in ϵ, there exists a small movement in ϵ for which the incentive (or participation) constraint continues to be slack or is binding. As a result, the agent's search behavior of type θ_i does not change for a small enough change in $\epsilon > 0$. Therefore, type θ_i continues to reject θ_1. Additionally, the participation constraint of θ_1 is binding if $s_1 = \theta_1^2 - \omega_1$ under PAM. As θ_1 meets θ_i with positive probability, the participation constraint would be violated at $s_1 = \theta_1^2 - \omega_1$. By lowering s_1 as above, the participation constraint continues to be binding.

The platform's profit is

$$\Pi = \frac{2(1-\delta)}{1-\rho}\left(\frac{\beta_i s_i}{\delta + (1-\delta)\phi(\theta_i|\theta_i)} + \frac{\beta_1 s_1'}{\delta + (1-\delta)\phi(\theta_1|\theta_1)} + \sum_{j, j \neq i, 2} \beta_j s_j\right),$$

and strictly decreasing in $\phi(\theta_i|\theta_i), i = 1, 2$. Therefore, a deviation to $\phi^D(\cdot)$ is always profitable and generates a profit of

$$\Pi^D = \frac{2(1-\delta)}{1-\rho}\left((\beta_i + \epsilon)s_i + \beta_1\theta_1^2 + \sum_{j, i \neq i, 2} \beta_j s_j\right),$$

which is larger than the profit under PAM — a contradiction.

Suppose for contradiction, PAM were optimal if $s_1 < \theta_1^2 - \omega_1$ for θ_1 for all values of exogenous parameters: $\theta_i^k \in \Theta^k = [\underline{\theta}, \overline{\theta}] \subseteq \mathbb{R}_+$, $\beta_i^k \in [0, 1]$, $\delta \in [0, 1]$, and $\omega_i^k \in \Omega = [\underline{\theta}, \overline{\theta}]$. Then, consider the participation and incentive constraint of type θ_1 and a type θ_i. Then holding search fee and the matching rule for all other types constant, let the platform deviates to s_i and $s_1 < \theta_1^2 < \omega_i$ and matching rule for θ_i and θ_1 such that θ_i is indifferent between accepting and rejecting θ_1 and θ_1 is indifferent between participating or not

$$\frac{\delta\omega_i + (1-\delta)(-s_i + \phi(\theta_i|\theta_i)\theta_i^2)}{\delta + (1-\delta)\phi(\theta_i|\theta_i)} = \max\{\theta_i\theta_1, \omega_i\},$$
$$\frac{\delta\omega_1 + (1-\delta)(-s_i + \phi(\theta_1|\theta_1)\theta_1^2)}{\delta + (1-\delta)\phi(\theta_1|\theta_1)} = \omega_1,$$

where $\phi(\theta_i|\theta_i) = \frac{(1-\delta)s_i + \delta(\theta_i\theta_1 - \omega_i)}{(1-\delta)\theta_i(\theta_i - \theta_1)}$ and $\phi(\theta_1|\theta_1) = \frac{s_1}{\theta_1^2 - \omega_1}$ are smaller than one. The steady state mass is $f(\theta_j) = \frac{\beta_j}{\delta + (1-\delta)\phi(\theta_j|\theta_j)}$ for $\theta_j = \theta_i, \theta_1$. Then, feasibility requires that s_i and $s_1 < \theta_1^2 - \omega_1$ are such that $f(\theta_i)(1-\phi(\theta_i|\theta_i)) = f(\theta_1)(1-\phi(\theta_1|\theta_1)) = \Phi(\theta_i, \theta_1)$.

Then, there exists a ratio of $\frac{\beta_i}{\beta_1}$ such that the platform's deviation yields a higher profit than under PAM for non-generic parameter values — a contradiction. It holds that $s_1 < \theta_i - \omega_1$ such that $s_1 = \theta_i - \omega_i - \varepsilon$ for $\varepsilon > 0$. For $\delta \to 0$

$$\underbrace{\beta_i \min\{\theta_i(\theta_i - \theta_1), \theta_i^2 - \omega_i\} + \beta_1(\theta_1^2 - \omega_1)}_{=\Pi^D} \geq \underbrace{\beta_i(\theta_i^2 - \omega_i) + \beta_1(\theta_1^2 - \omega_1\varepsilon)}_{\Pi^{PAM}},$$

for $\beta_i\theta_i\theta_1 \leq \beta_i\varepsilon$ if $\omega_i < \theta_i\theta_1$. The inequality is strict if $\omega_i \geq \theta_i\theta_1$.

(ii) Advertisement

Suppose PAM were optimal, which implies that the platform's profit is equal to

$$\Pi^{PAM} = 2\sum_{\theta_i \in \Theta} \nu(s_i)\beta(\theta_i)$$

for some $s_i \in [0, \overline{u}]$. Note that $\nu(s_i)$ is strictly concave and hence, Π^{PAM} as β_i is constant in s_i. Under PAM,

$$\nu'(s_i^k)\beta_i^k > 0,$$

such that the platform chooses the highest feasible search costs $s_i = \theta_i - \omega_i$. First, if $\frac{\nu(s_i)}{s_i} < 1$ for $s_i = \theta_i - \omega_i$ for some $\theta_i \in \Theta$, the platform can increase its profit by constructing a deviation to any $s_i < \theta_i(\theta_i - \theta_1)$ as above in (b), which fulfills $\frac{\nu(s_i)}{s_i} \geq 1.\geq$

B.2 BINARY TYPES

Lemma 4 and its Proof

Lemma 4. *The optimal matching rule that implements*

(a) (Positive assortative) $(\alpha(\theta_h, \theta_h) = 1, \alpha(\theta_h, \theta_l) = 0, \alpha(\theta_l, \theta_l) = 1)$*:*

$$\begin{bmatrix} \frac{s}{\theta_h(\theta_h - \theta_l)} & 1 - \frac{s}{\theta_h(\theta_h-\theta_l)} \\ \frac{\beta_l s}{\beta_h\theta_h(\theta_h-\theta_l) + (\beta_l - \beta_h)s} & \frac{\beta_h(\theta_h(\theta_h-\theta_l)-s)}{\beta_h\theta_h(\theta_h-\theta_l)+(\beta_l-\beta_h)s} \end{bmatrix}, \ \textit{if}\ \frac{\beta_h}{\beta_l} \leq \frac{\theta_l^2 - s}{\theta_h(\theta_h - \theta_l) - s}, \tag{29}$$

or otherwise,

$$\begin{bmatrix} \frac{\beta_h s}{\beta_l \theta_l^2 + (\beta_h - \beta_l)s} & \frac{\beta_l(\theta_l^2 - s)}{\beta_l \theta_l^2 + (\beta_h - \beta_l)s} \\ \frac{s}{\theta_l^2} & 1 - \frac{s}{\theta_l^2} \end{bmatrix}, \text{ if } \frac{\beta_h}{\beta_l} \geq \frac{\theta_l^2 - s}{\theta_h(\theta_h - \theta_l) - s}, \tag{30}$$

where at equality both matrices coincide.

(b) (Non-assortative) $(\alpha(\theta_h, \theta_h) = 1, \alpha(\theta_h, \theta_l) = 1, \alpha(\theta_l, \theta_l) = 0)$*:*

$$\begin{bmatrix} \frac{\beta_h - \beta_l}{\beta_h} & \frac{\beta_l}{\beta_h} \\ 1 - \frac{s}{\theta_l(\theta_h - \theta_l)} & \frac{s}{\theta_l(\theta_h - \theta_l)} \end{bmatrix}, \text{ if } 1 \leq \frac{\beta_h}{\beta_l} \leq \frac{\theta_h(\theta_h - \theta_l)}{\theta_h(\theta_h - \theta_l) - s} \tag{31}$$

(c) (Weakly assortative) $(\alpha(\theta_h, \theta_h) = 1, \alpha(\theta_h, \theta_l) \in (0, 1), \alpha(\theta_l, \theta_l) = 1)$

$$\begin{bmatrix} \frac{s}{\theta_h(\theta_h - \theta_l)} & 1 - \frac{s}{\theta_h(\theta_h - \theta_l)} \\ \frac{(\beta_l(\theta_h^2 - s) - \beta_h(\theta_h(\theta_h - \theta_l) - s))s}{\theta_l(\theta_h - \theta_l)(\beta_h \theta_h(\theta_h - \theta_l) + \beta_l \theta_h \theta_l + (\beta_l - \beta_h)s)} & 1 - \frac{(\beta_l(\theta_h^2 - s) - \beta_h(\theta_h(\theta_h - \theta_l) - s))s}{\theta_l(\theta_h - \theta_l)(\beta_h \theta_h(\theta_h - \theta_l) + \beta_l \theta_h \theta_l + (\beta_l - \beta_h)s)} \end{bmatrix}, \tag{32}$$

$$\text{if } \frac{(\theta_l^2 - s)}{\theta_h(\theta_h - \theta_l) - s)} \leq \frac{\beta_h}{\beta_l} \leq \frac{(\theta_h^2 - s)}{\theta_h(\theta_h - \theta_l) - s)}.$$

(d) (Non-assortative) $(\alpha(\theta_h, \theta_h) = 1, \alpha(\theta_h, \theta_l) \in (0, 1), \alpha(\theta_l, \theta_l) = 0)$*:*

$$\begin{bmatrix} \frac{s}{\theta_h(\theta_h - \theta_l)} & 1 - \frac{s}{\theta_h(\theta_h - \theta_l)} \\ 1 - \frac{(\beta_h - \beta_l)(\theta_h(\theta_h - \theta_l) - s)}{(\theta_h - \theta_l)\theta_l \beta_l} & \frac{(\beta_h - \beta_l)(\theta_h(\theta_h - \theta_l) - s)}{(\theta_h - \theta_l)\theta_l \beta_l} \end{bmatrix}, \tag{33}$$

$$\text{if } \frac{\theta_h(\theta_h - \theta_l)}{\theta_h(\theta_h - \theta_l) - s} \leq \frac{\beta_h}{\beta_l} \leq \frac{\theta_h^2 - \theta_l^2 - s}{\theta_h(\theta_h - \theta_l) - s}$$

(e) The optimal matching rule that implements no search and weakly assortative matching outcome is and $\phi(\theta_h, \theta_h) \in [\max\{0, \frac{\beta_h \theta_l(\theta_h - \theta_l) - \beta_l s}{\beta_h \theta_l(\theta_h - \theta_l)}\}, \frac{s}{\theta_h(\theta_h, \theta_l)})$ *and* $\phi(\theta_l|\theta_l) = \max\{0, \frac{\beta_l - \beta_h(1 - \phi(\theta_h|\theta_h))}{\beta_h}\}$.

The proof proceeds as follows. Each tuple of mutual acceptance probabilities $(\alpha(\theta_h, \theta_h), \alpha(\theta_h, \theta_l), \alpha(\theta_l, \theta_l))$ induces a linear program for the platform. See Appendix A for a formal definition and derivation. The linear program in the binary case is given by

$$\max_{\Phi(\cdot)} \frac{2(1-\delta)s}{1-\rho} \left(f(\theta_h) + f(\theta_l) \right),$$

subject to feasibility

$$f(\theta_h) = \Phi(\theta_h, \theta_h) + \Phi(\theta_h, \theta_l),$$
$$f(\theta_l) = \Phi(\theta_l, \theta_l) + \Phi(\theta_h, \theta_l),$$

and the steady state conditions

$$\beta_h = f(\theta_h)\delta + (1-\delta)(\alpha(\theta_h, \theta_h)\Phi(\theta_h, \theta_h) + \alpha(\theta_h, \theta_l)\Phi(\theta_h, \theta_l)),$$
$$\beta_l = f(\theta_l)\delta + (1-\delta)(\alpha(\theta_l, \theta_l)\Phi(\theta_l, \theta_l) + \alpha(\theta_h, \theta_l)\Phi(\theta_h, \theta_l)).$$

Fixing the mutual acceptance probabilities, the optimization problem is linear in $\Phi(\cdot)$.

(a) Positive assortative matching outcome:

Consider $(\alpha(\theta_h,\theta_h)=1, \alpha(\theta_h,\theta_l)=0, \alpha(\theta_l,\theta_l)=1)$ that induce the following constraints. The incentive constraint for a high type to continue searching after meeting a low type must hold, as well as the participation constraint for a low type.

$$\theta_h\theta_l \leq \frac{(1-\delta)(-s+\phi(\theta_h|\theta_h)\theta_h^2)}{\delta+(1-\delta)\phi(\theta_h|\theta_h)} \equiv V^C(\theta_h,\phi), \tag{34}$$

$$0 \leq \frac{(1-\delta)(-s+\phi(\theta_l|\theta_l)\theta_l^2)}{\delta+(1-\delta)\phi(\theta_l|\theta_l)} \equiv V^C(\theta_l,\phi). \tag{35}$$

By using Equation 2, the constraints are also linear in $\Phi(\cdot)$

$$\beta_h(\delta\theta_h\theta_l+(1-\delta)s) \leq (1-\delta)\Phi(\theta_h|\theta_h)(\delta\theta_h^2+(1-\delta)s),$$
$$\beta_l(1-\delta)s \leq (1-\delta)\Phi(\theta_l|\theta_l)(\delta\theta_l^2+(1-\delta)s).$$

The feasibility and steady state constraints become

$$\frac{\beta_h-(1-\delta)\Phi(\theta_h,\theta_h)}{\delta} = \Phi(\theta_h,\theta_h)+\Phi(\theta_h,\theta_l),$$
$$\frac{\beta_l-(1-\delta)\Phi(\theta_l,\theta_l)}{\delta} = \Phi(\theta_l,\theta_l)+\Phi(\theta_h,\theta_l).$$

By strong duality, the linear program is either unbounded, infeasible, or has an optimal solution. As the linear program is not unbounded, nor infeasible, the linear program must have an optimal solution.

In the binary case, the optimal solution can easily be checked. As the platform maximizes the steady state mass, it chooses $\Phi(\theta_h,\theta_h)$ and $\Phi(\theta_l,\theta_l)$ to be as small as possible without violating the constraints. Here, $\Phi(\theta_h,\theta_h)$ and $\Phi(\theta_l,\theta_l)$ are minimal when Equation 34 and Equation 35 bind resulting in

$$\Phi^{(a)}(\theta_h,\theta_h) = \frac{\beta_h((1-\delta)s+\delta\theta_h\theta_l)}{(1-\delta)((1-\delta)s+\delta\theta_h^2)},$$
$$\Phi^{(a)}(\theta_l,\theta_l) = \frac{\beta_l s}{(1-\delta)s+\delta\theta_l^2}.$$

Both the incentive and participation constraint, however, can only bind at the same time whenever

$$\left(\frac{\beta_h}{\beta_l}\right)^{(a)} = \frac{(1-\delta)(\theta_l^2-s)((1-\delta)s+\delta\theta_h^2)}{(\theta_h(\theta_h-\theta_l)-(1-\delta)s-\delta\theta_h^2)((1-\delta)s+\delta\theta_l^2)},$$

due to the feasibility constraints.

The steady state mass can be calculated by inserting $\Phi^{(a)}(\theta_h,\theta_h)$ and $\Phi^{(a)}(\theta_l,\theta_l)$ into

$$f(\theta_h) = \frac{\beta_h-(1-\delta)\Phi(\theta_h,\theta_h)}{\delta},$$
$$f(\theta_l) = \frac{\beta_l-(1-\delta)\Phi(\theta_l,\theta_l)}{\delta}.$$

The optimal matching rule is then given by $\phi(\theta_i|\theta_i) = \frac{\Phi(\theta_i,\theta_i)}{f(\theta_i)}$ for $i = h, l$.

If $\frac{\beta_h}{\beta_l} > (\frac{\beta_h}{\beta_l})^{(a)}$, only the participation constraint can be binding such that $\Phi(\theta_l,\theta_l) = \Phi^{(a)}(\theta_l,\theta_l)$. Inserting $\Phi(\theta_l,\theta_l) = \Phi^{(a)}(\theta_l,\theta_l)$ into the feasibility constraint of the low types yields $\Phi(\theta_h,\theta_l)$, which in turn determines $\Phi(\theta_h,\theta_h)$ by inserting it into the feasibility constraint of the high type. If $\frac{\beta_h}{\beta_l} < (\frac{\beta_h}{\beta_l})^{(a)}$, only the incentive constraint of the high type can be binding such that $\Phi(\theta_h,\theta_h) = \Phi(\theta_h,\theta_h)^{(a)}$ and the steps above can be repeated respectively.

(b) Non-assortative outcome:

Consider $(\alpha(\theta_h,\theta_h) = 1, \alpha(\theta_h,\theta_l) = 1, \alpha(\theta_l,\theta_l) = 0)$, which induce the following constraints. The linear incentive constraints are

$$\beta_h((1-\delta)s + \delta\theta_h\theta_l) \geq (1-\delta)\Phi(\theta_h|\theta_h)((1-\delta)s + \delta\theta_h^2) + (1-\delta)\Phi(\theta_h,\theta_l)((1-\delta)s + \delta\theta_h\theta_l),$$
$$\beta_l((1-\delta)s + \delta\theta_l^2) \leq (1-\delta)\Phi(\theta_h|\theta_l)((1-\delta)s + \delta\theta_h\theta_l).$$

As high types accept both high and low types and search for only one period, the steady state mass of high types is equal to their inflow: $f(\theta_h) = \beta_h$. The platform's profit from high types is, therefore, independent of the matching rule for $\phi(\theta_h|\theta_h) \in \left[0, \frac{(1-\delta)s+\delta\theta_h\theta_l}{(1-\delta)\theta_h(\theta_h-\theta_l)}\right]$. To maximize steady state mass of low types, the platform minimizes $\Phi(\theta_h,\theta_l)$ such that

$$\Phi(\theta_h,\theta_l)^{(b)} = \frac{\beta_l((1-\delta)s + \delta\theta_l^2)}{(1-\delta)((1-\delta)s + \delta\theta_h\theta_l)},$$

where the incentive constraint of the low type binds. $\Phi(\theta_h,\theta_h)$ follows from the feasibility constraints, where $\Phi(\theta_h,\theta_h)$ and $\Phi(\theta_h,\theta_l)$ must be such that the incentive constraints of the high type is fulfilled, which is true if

$$1 < \frac{\beta_h}{\beta_l} \leq \frac{((1-\delta)s + \delta\theta_l^2)\,\theta_h(\theta_h-\theta_l)}{(\theta_h(\theta_h-\theta_l) - (1-\delta)s - \delta\theta_h^2)\,((1-\delta)s + \delta\theta_h\theta_l)}.$$

(c) Mixed-strategy: weakly assortative outcome:

Consider $(\alpha(\theta_h,\theta_h) = 1, \alpha(\theta_h,\theta_l) \in (0,1), \alpha(\theta_l,\theta_l) = 1)$. First, note that for the high types to accept low types with positive probability, they must be indifferent between searching and accepting low types:

$$\theta_h\theta_l = V^C(\theta_h,\phi),$$

which holds for $\phi(\theta_h|\theta_h) = \frac{(1-\delta)s+\delta\theta_h\theta_l}{(1-\delta)\theta_h(\theta_h-\theta_l)}$. For low types to participate, it must hold that

$$\beta_l(1-\delta)s \leq (1-\delta)\Phi(\theta_l,\theta_l)(\delta\theta_l^2 + (1-\delta)s) + (1-\delta)\alpha(\theta_h,\theta_l)\Phi(\theta_h,\theta_l)(\delta\theta_h\theta_l + (1-\delta)s).$$

From $\phi(\theta_h|\theta_h) = \frac{(1-\delta)s+\delta\theta_h\theta_l}{(1-\delta)\theta_h(\theta_h-\theta_l)}$ it follows

$$\Phi^{(c)}(\theta_h,\theta_h) = \phi(\theta_h|\theta_h)\,\underbrace{\frac{\beta_h}{\delta + (1-\delta)(\phi(\theta_h|\theta_h) + \alpha(\theta_h,\theta_l)(1-\phi(\theta_h|\theta_h)))}}_{=f(\theta_h)}$$

Then, $\Phi^{(c)}(\theta_h, \theta_l)$ follows from feasibility of the high type. Furthermore, $\Phi^{(c)}(\theta_l, \theta_l)$ follows from feasibility of the low type. All can be substituted into the incentive constraint of the low type. The low type is indifferent between participating and not participating if

$$\alpha^{WPAM} \equiv \left\{\alpha(\theta_h, \theta_l) : \beta_l s = \Phi^{(c)}(\theta_l, \theta_l)(\delta\theta_l^2 + (1-\delta)s) + \alpha(\theta_h, \theta_l)\Phi^{(c)}(\theta_h, \theta_l)(\delta\theta_h\theta_l + (1-\delta)s)\right\}.$$

For $\delta \to 0$, I get

$$\alpha^{WPAM} = \frac{s\left(\beta_h\theta_h(\theta_h - \theta_l) - \beta_l\theta_l^2 - (\beta_h - \beta_l)s\right)}{(\theta_h(\theta_h - \theta_l) - s)\left(\beta_h\theta_l(\theta_h - \theta_l) + \beta_l\theta_l^2 + (\beta_h - \beta_l)s\right)} \tag{36}$$

The mutual acceptance probability is then given by the above. For $\delta \to 0$, to ensure that both $\alpha^{WPAM} \in [0, 1]$ and $\phi(\theta_l|\theta_l) \in [0, 1]$, the conditions in the lemma must hold.

(d) Mixed-strategy: non-assortative outcome:

Consider $(\alpha(\theta_h, \theta_h) = 1, \alpha(\theta_h, \theta_l) \in (0, 1), \alpha(\theta_l, \theta_l) = 0)$. Again, $\phi(\theta_h|\theta_h) = \frac{(1-\delta)s+\delta\theta_h\theta_l}{(1-\delta)\theta_h(\theta_h-\theta_l)}$ must hold to ensure indifference of high types. Then, $\Phi^{(d)}(\theta_h, \theta_h) = \Phi^{(c)}(\theta_h, \theta_h)$ and $\Phi^{(d)}(\theta_h, \theta_l) = \Phi^{(c)}(\theta_h, \theta_l)$. Inserting into the incentive constraint of the low type, the low type rejects low types if

$$\beta_l((1-\delta)s + \delta\theta_l^2) \leq (1-\delta)\alpha(\theta_h, \theta_l)\Phi^{(d)}(\theta_h, \theta_l)((1-\delta)s + \delta\theta_h\theta_l),$$

which holds with equality, and $\delta \to 0$ for

$$\alpha^{NAM} = \frac{\beta_l s}{(\beta_h - \beta_l)(\theta_h(\theta_h - \theta_l) - s)} \tag{37}$$

The mutual acceptance probability is in $[0, 1]$ if the conditions in the lemma are fulfilled. Note that I do not need to consider the mixed strategy of low types as the solution is bang-bang. Profit either increase or decrease in $\alpha(\theta_l, \theta_l)$.

(e) Pure-strategy: weakly-assortative outcome:

Consider $(\alpha(\theta_h, \theta_h) = 1, \alpha(\theta_h, \theta_l) = 1, \alpha(\theta_l, \theta_l) = 1)$. Note that agents do not search if

$$\begin{aligned} \theta_h\theta_l &> V^C(\theta_h, \phi) > 0, \\ \theta_l^2 &> V^C(\theta_l, \phi) > 0 \end{aligned}$$

A continuum of matching rules exists that yield the same profit. The high type does not search if $\phi(\theta_h|\theta_h) = [0, \frac{(1-\delta)s+\delta\theta_h\theta_l}{(1-\delta)\theta_h(\theta_h-\theta_l)}]$. From the equivalence of masses, it follows

$$\phi(\theta_h|\theta_l) = \frac{\beta_h(1 - \phi(\theta_h|\theta_h))}{\beta_l}.$$

□

Proof of Proposition 4 To determine the platform's preferred outcome, consider the profits from the demand in matching outcome (a) to (e) in Lemma 4 for all values of s. As the lemma only considers situation in which both types participate, it should hold that $s \leq \theta_h\theta_l$ (or even $s \leq \theta_l^2$).

For the first part of the lemma, only the matching outcome in (a) (Equation 29) and (e) in Lemma 4 exist for $\frac{\beta_h}{\beta_l} < \frac{\theta_l^2 - s}{\theta_h(\theta_h-\theta_l)-s}$. As no agent searches in the matching outcome from (e), the profit is $\Pi^{(e)} = 2(\beta_h + \beta_l)$. The profit in (a) is $\Pi^{(a)} = 2\left(\frac{\beta_h\theta_h(\theta_h-\theta_l)}{s} + \frac{\beta_h\theta_h(\theta_h-\theta_l)+(\beta_l-\beta_h)s}{s}\right)$. For both outcomes to exist $s \leq \theta_l^2$, such that $\Pi^{(a)} > \Pi^{(e)}$ for all $s \leq \theta_l^2$. In general, $\Pi^{(e)}$ is dominated by any profit which induces some search for $s \leq \theta_l^2$.

For the second part of the lemma, only the matching outcome in (a) (Equation 30) and (c) exist for ratio just above the previous one: $\frac{\beta_h}{\beta_l} \geq \frac{\theta_l^2-s}{\theta_h(\theta_h-\theta_l)-s}$. By construction, the participation constraint is binding for the low type in both equilibria. In contrast, the incentive constraint of the high type is slack in equilibrium (a), whereas it is binding in (c). The profits are $\Pi^{(a)} = 2(\frac{\beta_l\theta_l^2+(\beta_h-\beta_l)s}{s} + \frac{\beta_l\theta_l^2}{s})$ and $\Pi^{(c)} = 2(\frac{\theta_h(\beta_h\theta_l(\theta_h-\theta_l)+\beta_l\theta_l^2+(\beta_h-\beta_l)s)}{\theta_h s+\theta_l s} + \frac{(\beta_h\theta_h^2-\beta_h\theta_h\theta_l+\beta_l\theta_h\theta_l-\beta_h s+\beta_l s)\theta_l}{s(\theta_h+\theta_l)})$, where the difference is strictly positive

$$\Pi^{(c)} - \Pi^{(a)} = \frac{4(\beta_h(\theta_h(\theta_h-\theta_l)-s) - \beta_l(\theta_l^2-s))}{s(\theta_h+\theta_l)},$$

as $\frac{\beta_h}{\beta_l} \geq \frac{\theta_l^2-s}{\theta_h(\theta_h-\theta_l)-s}$ holds. The equilibrium profit in (b) is $\Pi^{(b)} = 2(\beta_h + \frac{\beta_l\theta_l(\theta_h-\theta_l)}{s})$ is clearly dominated for $s < \theta_l^2$. Furthermore, $\Pi^{(b)} < \Pi^{(c)}$ for $\theta_l^2 \geq s \leq \theta_l(\theta_h - \theta_l)$ for $\frac{\beta_h}{\beta_l} \geq 1$.

For the third part of the lemma, it remains to compare when the profit in equilibrium (c) is larger or smaller then the profit from equilibrium (d). Only equilibrium (c) and (d) can exist for $\frac{\beta_h}{\beta_l} \geq \frac{\theta_h(\theta_h-\theta_l)}{\theta_h(\theta_h-\theta_l)-s}$. The profit in (d) is $\Pi^{(d)} = 2(\frac{(\beta_h s-\beta_l s)\theta_h(\theta_h-\theta_l)}{s^2} + \frac{\beta_l\theta_l(\theta_h-\theta_l)}{s})$. Then $\Pi^{(d)} \geq \Pi^{(c)}$ if $\frac{\theta_h^3-\theta_h^2\theta_l+\theta_h\theta_l^2+\theta_l^3-\theta_h s+\theta_l s}{\theta_h^3-2\theta_h^2\theta_l+\theta_h\theta_l^2-\theta_h s+\theta_l s} \leq \frac{\beta_h}{\beta_l}$.

For the fourth part of the lemma, note that only one equilibrium exists for $\frac{\beta_h}{\beta_l} \geq \frac{\theta_h^2-\theta_l^2-s}{\theta_h(\theta_h-\theta_l)-s}$ in which agents search (see Lemma 4). This yields higher profits than an equilibrium, in which agents do not search.

□

Proof of Proposition 5 The proof follows the matching outcomes in Proposition 4 from (a) to (d).

(a) The positive assortative matching outcome maximizes match productivity, but maximizes search time. At $\frac{\beta_h}{\beta_l} \leq \frac{\theta_l^2-s}{\theta_h(\theta_h-\theta_l)-s}$ agents search the longest. For $\frac{\beta_h}{\beta_l} \leq \frac{\theta_l^2-s}{\theta_h(\theta_h-\theta_l)-s}$, agents search for

$$\theta_h\text{-type: } \frac{1}{\phi(\theta_h|\theta_h)} = \frac{\theta_h(\theta_h-\theta_l)}{s},$$
$$\theta_l\text{-type: } \frac{s}{\phi(\theta_l|\theta_l)} = \frac{\beta_h(\theta_h(\theta_h-\theta_l)+(\beta_l-\beta_h)s}{\beta_l s},$$

High types search the longest in the positive assortative matching outcome compared to the other outcomes.

(b) The weakly assortative matching outcome has a mass of mismatched agents of

$$\frac{\beta_h(\theta_h(\theta_h-\theta_l)-s)-\beta_l(\theta_l^2-s)}{\theta_h^2-\theta_l^2},$$

which increases in s if $\beta_h<\beta_l$, and decreases in s if $\beta_h>\beta_l$. Furthermore, the mismatch increases if the type difference increases, i.e. types becomes less similar. The welfare loss from mismatch is

$$\begin{aligned}\mathcal{W}_P^{WPAM}&=\alpha(\theta_h,\theta_l)\Phi(\theta_h,\theta_l)\theta_l(\theta_h-\theta_l)-\alpha(\theta_h,\theta_l)\Phi(\theta_h,\theta_l)\theta_h(\theta_h-\theta_l),\\&=-\frac{\left(\beta_h\theta_h(\theta_h-\theta_l)-\beta_l\theta_l^2-\beta_h s+\beta_l s\right)(\theta_h-\theta_l)}{\theta_h+\theta_l},\end{aligned}$$

and agents search for

$$\begin{aligned}\theta_h\text{-type: }\quad&\frac{1}{\phi(\theta_h|\theta_h)+\alpha^{WPAM}(\theta_h,\theta_l)\phi(\theta_l|\theta_h)}=\frac{\theta_h\left(\beta_h\theta_h\theta_l-\beta_h\theta_l^2+\beta_l\theta_l^2+\beta_h s-\beta_l s\right)}{(\theta_h+\theta_l)\beta_h s},\\\theta_l\text{-type: }\quad&\frac{1}{\phi(\theta_l|\theta_l)+\alpha^{WPAM}(\theta_h,\theta_l)\phi(\theta_l|\theta_l)}=\frac{\theta_l\left(\beta_h\theta_h^2-\beta_h\theta_h\theta_l+\beta_l\theta_h\theta_l-\beta_h s+\beta_l s\right)}{(\theta_h+\theta_l)\beta_l s},\end{aligned}$$

Low types search the longest in the weakly positive assortative matching outcome compared to the other outcomes.

(c) The non-assortative matching outcome has a mass of mismatched agents of

$$\beta_l,$$

which decreases in $\frac{\beta_h}{\beta_l}$. The welfare loss from mismatch is

$$\begin{aligned}\mathcal{W}_P^{NAM}&=\alpha(\theta_h,\theta_l)\Phi(\theta_h,\theta_l)\theta_l(\theta_h-\theta_l)-\alpha(\theta_h,\theta_l)\Phi(\theta_h,\theta_l)\theta_h(\theta_h-\theta_l),\\&=-\beta_l(\theta_h-\theta_l)^2,\end{aligned}$$

and agents search for

$$\begin{aligned}\theta_h\text{-type: }\quad&\frac{1}{\phi(\theta_h|\theta_h)+\alpha^{WPAM}(\theta_h,\theta_l)\phi(\theta|\theta_h)}=\frac{\beta_h-\beta_l}{\beta_h}\frac{\theta_h(\theta_h-\theta_l)}{s},\\\theta_l\text{-type: }\quad&\frac{1}{\alpha^{WPAM}(\theta_h,\theta_l)\phi(\theta_h|\theta_l)}=\frac{\theta_l(\theta_h-\theta_l)}{s}.\end{aligned}$$

Then, $\mathcal{W}_P^{WPAM}\geq\mathcal{W}_P^{NAM}$ if

$$\frac{\beta_h}{\beta_l}\geq\frac{\theta_h^2-s}{\theta_h(\theta_h-\theta_l)-s}.$$

If, however, the above inequality holds, the platform cannot implement the positive assortative matching outcome (see Lemma 4).

(d) The positive assortative matching outcome does not induce any mismatch. Agents search for

$$\theta_h\text{-type: } \frac{1}{\phi(\theta_h|\theta_h)} = \frac{\beta_l\theta_l^2 + (\beta_h - \beta_l)s}{\beta_h s},$$
$$\theta_l\text{-type: } \frac{s}{\phi(\theta_l|\theta_l)} = \frac{\theta_l^2}{s}.$$

□

Proof of Proposition 6 The optimal level of advertising is determined by Equation 7, where demand is given by any of the four cases in Proposition 4.

(a) Positive assortative matching outcome for $\frac{\beta_h}{\beta_l} \leq \frac{\theta_l^2 - s}{\theta_h(\theta_h-\theta_l)-s}$.

For $\frac{\beta_h}{\beta_l} < \frac{\theta_l^2 - s}{\theta_h(\theta_h-\theta_l)-s}$, taking demand associated with the positive assortative matching outcome, Equation 7 becomes

$$\frac{\nu(s)}{\nu'(s)} = \frac{s(\beta_h(2\theta_h(\theta_h-\theta_l)-s)+\beta_l s)}{2\beta_h\theta_h(\theta_h-\theta_l)} = s + \frac{(\beta_l-\beta_h)s^2}{2\beta_h\theta_h(\theta_h-\theta_l)}.$$

The right-hand side is increasing and convex. For $\frac{\beta_h}{\beta_l} = \frac{\theta_l^2 - s}{\theta_h(\theta_h-\theta_l)-s}$, the right-hand side is equal to s. Demand is decreasing and concave in s as

$$\nu'(s) = -4\frac{\beta_h\theta_h(\theta_h-\theta_l)}{s^2},\ \nu''(s) = 8\frac{\beta_h\theta_h(\theta_h-\theta_l)}{s^3}.$$

(b) Weakly positive assortative matching outcome for $\frac{\beta_h}{\beta_l} > \frac{\theta_l^2 - s}{\theta_h(\theta_h-\theta_l)-s}$.

Taking demand associated with the weakly positive assortative matching outcome, Equation 7 becomes

$$\frac{\nu(s)}{\nu'(s)} = \frac{s(2\beta_h\theta_h^2\theta_l - 2\beta_h\theta_h\theta_l^2 + 2\beta_l\theta_h\theta_l^2 + \beta_h\theta_h s - \beta_h\theta_l s - \beta_l\theta_h s + \beta_l\theta_l s)}{2\theta_h\theta_l(\beta_h(\theta_h-\theta_l)+\beta_l\theta_l)},$$

with

$$\nu'(s) = -4\frac{\theta_h\theta_l\left(\beta_h\theta_h - \beta_h\theta_l + \beta_l\theta_l\right)}{(\theta_h+\theta_l)s^2},\ \nu''(s) = 8\frac{\theta_h\theta_l\left(\beta_h\theta_h - \beta_h\theta_l + \beta_l\theta_l\right)}{(\theta_h+\theta_l)s^3}.$$

(c) Negative assortative matching outcome for $\frac{\beta_h}{\beta_l} > \frac{\theta_h(\theta_h-\theta_l)}{\theta_h(\theta_h-\theta_l)-s}$.

Taking demand associated with the negative positive assortative matching outcome, Equation 7 becomes

$$\frac{\nu(s)}{\nu'(s)} = s,$$

with

$$\nu'(s) = -2\frac{(\theta_h-\theta_l)(\beta_h\theta_h - \beta_l\theta_h + \beta_l\theta_l)}{s^2},\ \nu''(s) = 4\frac{(\theta_h-\theta_l)(\beta_h\theta_h - \beta_l\theta_h + \beta_l\theta_l)}{s^3}.$$

(d) Positive assortative matching outcome for $\frac{\beta_h}{\beta_l} > \frac{\theta_h^2-\theta_l^2}{\theta_h(\theta_h-\theta_l)-s}$. Taking demand associated with the positive assortative matching outcome, Equation 7 becomes

$$\frac{\nu(s)}{\nu'(s)} = s + \frac{(\beta_h - \beta_l)s^2}{2\beta_l\theta_l^2},$$

with

$$\nu'(s) = -\frac{4\beta_l\theta_l^2}{s},\ \nu''(s) = \frac{8\beta_l\theta_l}{s^3}.$$

(e) Existence of an optimal solution

Demand is decreasing, but concave in s. Advertisement profits are concave for each segment if

$$\underbrace{\nu''(s)D(s)}_{<0} + \underbrace{2\nu'(s)D'(s)}_{<0} + \underbrace{\nu(s)D''(s)}_{>0} < 0,$$

or if

$$\nu(s)D''(s) < -\left(\nu''(s)D(s) + 2\nu'(s)D'(s)\right) \tag{38}$$

The following assumptions guarantees an optimal solution for each case (a)-(d).

Assumption 3. *Let* $\overline{s}$ *be such that for* $s = \overline{s}$*:* $f(\theta_l) = \beta_l$*. Then,*

$$\frac{\nu(\overline{s})}{\nu'(\overline{s})} > -\frac{\sum_k \sum_i f(\theta_i^k)}{\frac{\partial \sum_k \sum_i f(\theta_i^k)}{\partial s}}\Big|_{s=\overline{s}},$$

and

$$-\frac{\nu''(s)}{\nu'(s)} > \frac{-f''f + 2f'}{f'f}$$

Then, if Equation 7 for each cases solves for an s that is in the range of each equilibrium, an optimal solution exists. Note that due to the Assumption 2, both marginal benefit and cost of advertisement always intersect at least at the origin. If $\frac{\nu(s)}{\nu'(s)}$ is extremely convex, i.e. $\nu(s)$ is extremely concave, then the optimal solution tends close to zero.

If $\nu(s)$ is not sufficiently concave, then the optimal solution might be $s > \theta_l^2$. In that case, the optimal solution is a corner solution.

(f) Advertising intensity

The optimal advertising intensity is highest for the case in which the right-hand side has the steepest increase. Take case (c) as a benchmark. The increase in (a) is steeper. Case (b) is less steep than (c) if

$$\frac{s^2\left(\beta_h\theta_h - \beta_h\theta_l - \beta_l\theta_h + \beta_l\theta_l\right)}{2\theta_h\theta_l\left(\beta_h\theta_h - \beta_h\theta_l + \beta_l\theta_l\right)} < 0$$

which holds for $\beta_h < \beta_l$, and is more steep if $\beta_h > \beta_l$.

The optimal advertising intensity is thus lowest for (b) if $\beta_h < \beta_l$ and otherwise for (c).

Under optimal advertisement level $s^{A,*}$, the platform prefers to implement

(a) positive assortative matching for $\frac{\beta_h}{\beta_l} \leq \frac{\theta_l^2 - s^A}{\theta_h(\theta_h-\theta_l)-s^A}$,

(b) weakly assortative matching for $\frac{\theta_l^2 - s^A}{\theta_h(\theta_h-\theta_l)-s^A} \leq \frac{\beta_h}{\beta_l} \leq \max\{\frac{\theta_h(\theta_h-\theta_l)}{\theta_h(\theta_h-\theta_l)-s^A}, (\frac{\beta_h}{\beta_l})'\}$,

(c) non-assortative matching for $\max\{\frac{\theta_h(\theta_h-\theta_l)}{\theta_h(\theta_h-\theta_l)-s^A}, (\frac{\beta_h}{\beta_l})'\} \leq \frac{\beta_h}{\beta_l} \leq \frac{\theta_h^2-\theta_l^2-s^A}{\theta_h(\theta_h-\theta_l)-s^A}$,

(d) positive assortative matching for $\frac{\beta_h}{\beta_l} \geq \frac{\theta_h^2-\theta_l^2-s^A}{\theta_h(\theta_h-\theta_l)-s^A}$.

$\square$

Proof of Proposition 7 The search fees in each case are (a) $s = \frac{\beta_l\theta_l^2 - \beta_h\theta_h(\theta_h-\theta_l)}{\beta_l-\beta_h}$ and implements the positive assortative matching outcome if

$$0 \leq \frac{\beta_h}{\beta_l} \leq \frac{\theta_l^2}{\theta_h(\theta_h-\theta_l)},$$

(b) $s = \epsilon > 0$ and implements the weakly positive assortative matching outcome if

$$\frac{\theta_l^2}{\theta_h(\theta_h-\theta_l)} \leq \frac{\beta_h}{\beta_l} \leq 1,$$

(c) $s = \min\left\{\frac{\beta_h(\theta_h(\theta_h-\theta_l)+\beta_l\theta_h\theta_l)\theta_l}{\beta_h\theta_l+\beta_l\theta_h}, \frac{\beta_l\theta_l^2-\beta_h\theta_h(\theta_h-\theta_l)}{\beta_l-\beta_h}\right\}$ and implements the weakly positive assortative matching outcome if

$$1 \leq \frac{\beta_h}{\beta_l} \leq \frac{\beta_h}{\beta_l}'(s),$$

(d) $s = \epsilon > 0$ and implements the non-assortative matching outcome if

$$\frac{\beta_h}{\beta_l}'(s) \leq \frac{\beta_h}{\beta_l} \leq \frac{\theta_h^2-\theta_l^2}{\theta_h(\theta_h-\theta_l)},$$

(e) $s = \epsilon > 0$ and implements the positive assortative matching outcome if

$$\frac{\theta_h^2-\theta_l^2}{\theta_h(\theta_h-\theta_l)} \leq \frac{\beta_h}{\beta_l}$$

The optimal solution is the positive assortative matching outcome for $\frac{\beta_h}{\beta_l} \leq \frac{\theta_l^2}{\theta_h(\theta_h-\theta_l)}$. At $s = \frac{\beta_l\theta_l^2-\beta_h\theta_h(\theta_h-\theta_l)}{\beta_l-\beta_h}$, both the incentive constraint for the high type and the participation constraint for the low type are binding. The platform extracts the maximum rent from agents. The platform earns $\theta_h(\theta_h-\theta_l)$ from high types and θ_l^2 from low types. For $\frac{\theta_l^2}{\theta_h(\theta_h-\theta_l)} \leq \frac{\beta_h}{\beta_l}$, the platform can no longer extract the maximum rent.

From Lemma 4, it follows that for $\frac{\theta_l^2-s}{\theta_h(\theta_h-\theta_l-s)} \leq \frac{\beta_h}{\beta_l} \leq \frac{\theta_h^2-s}{\theta_h(\theta_h-\theta_l)-s}$, the platform wants to implement the weakly positive assortative outcome. To implement the outcome, s must be in

$$\frac{(\theta_l^2 - s)}{\theta_h(\theta_h - \theta_l) - s)} \leq \frac{\beta_h}{\beta_l} \leq \frac{(\theta_h^2 - s)}{\theta_h(\theta_h - \theta_l) - s)}.$$

To ensure that the mutual acceptance probability and matching probabilities are between zero and one.Rewriting the condition in terms of s, they become

$$\max\{0, \frac{\beta_h\theta_h(\theta_h - \theta_l) - \beta_l\theta_h^2}{\beta_h - \beta_l}\} \leq s \leq \min\{\frac{\beta_h\theta_h(\theta_h - \theta_l) - \beta_l\theta_l^2}{\beta_h - \beta_l}, \frac{\beta_h(\theta_h(\theta_h - \theta_l) + \beta_l\theta_h\theta_l)\theta_l}{\beta_h\theta_l + \beta_l\theta_h}\}$$

If $\frac{\theta_l^2}{\theta_h(\theta_h-\theta_l)} < \frac{\beta_h}{\beta_l} \leq 1$, it follows from the proof of Proposition 6, that the profit is decreasing in s. Therefore, the platform chooses $s \to 0$. If $\frac{\beta_h}{\beta_l} \geq 1$, the profit is increasing in s. The platform chooses $s^{WPAM} = \min\{\frac{\beta_h\theta_h(\theta_h-\theta_l)-\beta_l\theta_l^2}{\beta_h-\beta_l}, \frac{\beta_h(\theta_h(\theta_h-\theta_l)+\beta_l\theta_h\theta_l)\theta_l}{\beta_h\theta_l+\beta_l\theta_h}\}$.

For $\max\{\frac{\theta_h(\theta_h-\theta_l)}{\theta_h(\theta_h-\theta_l)-s^{WPAM}}, (\frac{\beta_h}{\beta_l})(s^{WPAM})'\} \leq \frac{\beta_h}{\beta_l} \leq \frac{\theta_h^2-\theta_l^2}{\theta_h(\theta_h-\theta_l)}$, the platform implements the non-assortative matching outcome. The platform is indifferent between any $s \in (0, \bar{s}]$, but prefers to increase the range of the equilibrium by setting $s \to 0$.

For $\frac{\theta_h^2-\theta_l^2}{\theta_h(\theta_h-\theta_l)} \leq \frac{\beta_h}{\beta_l}$, the platform implements the positive assortative matching outcome. The platform prefers to set $s = \min\{\theta_l^2, \frac{\beta_h\theta_h(\theta_h-\theta_l)-\beta_l\theta_l^2}{\beta_h-\beta_l}\}$.

$\square$

C. APPENDIX: TABLES

Application	App Price	Subscriptions	One-Time Purchases
Tinder	Free	Tinder Gold (1 Week): \$13.99 – 18.99 Tinder Gold (1 Month): \$14.99 – 24.99 Tinder Plus (1 Month): \$9.99	1 Boost: \$3.99 – 7.99 3 Super Likes: \$9.99 5 Super Likes: \$4.99
Bumble	Free	Bumble Premium (1 Year) \$129.99 – 169.99	5 Spotlights + Compliments \$24.99 – 29.99 15 Spotlights + Compliments \$44.99 – 59.99 30 Spotlights + Compliments \$79.99 – 99.99
Hinge	Free	Hinge+ Subscription (1 Week): \$16.99 Hinge Subscription (1 Month): \$29.99 – 34.99 Membership (1 Month): \$19.99 Hinge Subscription (1 Week): \$14.99 HingeX Subscription: \$24.99	Bundle of three Roses: \$9.99 Bundle of twelve Roses: \$29.99 Boost: \$9.99 – 19.99
Match	Free	Match (1 Month): \$19.99 – 42.99 Match (3 Months): \$74.99 Match (6 Months): \$129.99 Standard (1 Month): \$44.99 Basic (1 Months): \$44.99 Platinum (1 Week): \$29.99	1 Top Spot: \$2.99 Top Spot 10-Pack: \$19.99 Boost 1-Pack: \$5.99
Hily	Free	Hily Premium (1 Week): \$14.99 Profile boost (1 Week): \$5.99 – 9.99 Premium+ (1 Week): \$24.99 Hily Elixir (1 Week): \$19.99	1 Unblur: \$4.99 5 Unblur: \$12.99
Plenty of Fish	Free	Upgrade (1 Month): \$19.99 Upgrade (3 Months): \$38.99 Premium Membership (1 Month): \$29.99	1 Token: \$1.99 5 Tokens: \$8.99 10 Tokens: \$17.99
Badoo	Free	Badoo Premium (1 Week): \$5.99 – 8.99 Super Powers (1 Week): \$2.99 Super Powers (1 Months): \$11.99	Pack of 100 Credits: \$1.99 – 3.99
Coffee Meets Bagel	Free	Premium (1 Month): \$14.99 – 34.99 Premium (3 Months): \$74.99 Premium (6 Months): \$71.99 Platinum (1 Month): \$46.99 Platinum (3 Month): \$99.99	200 Coffee Beans: \$2.99 400 Coffee Beans: \$4.99 3000 Coffee Beans: \$24.99
Raya	Free	Membership (1 Month): \$24.99 Membership (6 Month): \$113.99 Raya+ Membership: \$49.99	30 Extra Likes: \$10.99 Skip the Wait: \$7.99 5 Skip the Waits: \$29.99 1 Direct Request: \$4.99 3 Direct Requests: \$12.99 12 Direct Requests: \$49.99
MeetMe	Free	MeetMe (1 Month): \$7.99 MeetMe (3 Months): \$17.99 MeetMe+ (1 Month): \$7.99	Pack of 200 Credits: \$1.99 Pack of 500 Credits: \$1.99 – 4.99 Pack of 1800 Credits: \$14.99 Pack of 14500 Credits: \$99.99 Pack of 3200 Credits: \$24.99

Table 1: A selection of dating apps in the US Apple Store

Application	App Price	Subscriptions	One-Time Purchases
Tinder	Free	Tinder Gold (1 Week): 13, 99 € Tinder Gold (1 Month): 8, 99 – 27, 49 € Tinder Platinum (1 Month): 32, 99 €	1 Boost: 7, 99 – 9, 99 € 3 Super-Likes: 11,99 € 5 Super-Likes: 5, 99 – 9, 99 €
Bumble	Free	Bumble Premium (1 Week): 14, 99 – 19, 99 € Bumble Boost (1 Week): 5, 99 – 6, 99 € Bumble Premium (1 Month): 34, 99 €	
Hinge	Free	Hinge+ Sub (1 Week): 14, 99 € Hinge+ Sub (1 Month): 24, 99 € HingeX Sub (1 Week): 24, 99 €	Bundle of twelve Roses: 24, 99 € Bundle of three Roses: 7, 99 € One Superboost: 14, 99 € One Boost: 7, 99 €
LOVOO	Free	Lovoo Premium (1 Month): 11, 99 – 24, 99 €	300 Credits: 5, 99 € 500 Credits: 4, 99 € 550 Credits: 7, 99 € 3000 Credits: 19, 99 € 5 Icebreaker: 5, 99 € Unbegrenzte Likes: 1, 19 €
Badoo	Free	Badoo Premium (1 Week): 5, 99 – 7, 99 € Badoo Premium (1 Month): 19, 99 €	100 Badoo Credits: 1, 99 – 4, 99 € 550 Badoo Credits: 12, 99 € Super Powers (1 Woche): 2, 99 € Super Powers (1 Monat): 8, 99 € Super Powers (1 Woche): 2, 99 €
Parship	Free	Premium lite (6 Month): 209, 99 – 229, 99 € Premium classic (1 Year): 224, 99 – 499, 99 € Premium Comfort: 249, 99 €	Parship Premium: 9,99 €
OkCupid	Free	OkCupid Premium (1 Month): 15, 99 – 32, 99 € OKCupid Premium (3 Month): 65, 99 €	1 Boost: 1, 99 – 7, 99 € 2 Superlikes: 7, 99 €
Raya	Free	Membership (1 Month): 18, 99 € Membership (6 Month): 83, 99 € Raya+ Membership (1 Month): 44, 99 €	Skip the Wait 7, 99 € 3 Direct Requests 12, 99 € 1 Direct Request 4, 99 € 30 Extra Likes 10, 99 € 5 Skip the Waits 29, 99 € 1 Travel Plan 9, 99 €
LoveScout24	Free	Lovescout24 (1 Month): 39, 99 € Mobile Plus (1 Month): 9, 99 € Mobile Plus (1 Week): 4, 99 € Lovescout24 (1 Week): 9, 99 € Lovescout24 (3 Month): 89, 99 €	1 Booster: 1, 99 € Wer sucht mich?: 1, 99 € Boost: 1, 99 € Dateroulette: 2, 99 € Favouriten-Funk: 1, 99 €
ElitePartner	Free	ElitePartner Premium Go: 3, 99 – 19, 99 € Premium plus (1 Year): 399, 99 € Premium basic (6 Months): 279, 99 € Premium comfort (2 years) : 599, 99 €	

Table 2: A selection of dating apps in the German Apple Store

App Name	Price	Contains Ads	Prices of In-App Purchases	Number of Installations
German Store				
happn	Free	Yes	0.59 – 274.99 €	100M+
Badoo	Free	Yes	0.39 – 244.99 €	100M+
Tinder	Free	Yes	0.29 – 324.99 €	100M+
SweetMeet	Free	Yes	0.59 – 219.99 €	50M+
Bumble	Free	No	0.29 – 314.99 €	50M+
BLOOM	Free	Yes	1.49 – 299.00 €	10M+
OkCupid	Free	Yes	0.71 – 194.99 €	10M+
Zoosk	Free	Yes	0.50 – 434.99 €	10M+
Mamba	Free	Yes	0.50 – 294.99 €	10M+
Boo	Free	Yes	0.46 – 218.85 €	10M+
US Store				
happn	Free	Yes	$0.49 – 224.99	100M+
Badoo	Free	Yes	$0.49 – 239.99	100M+
Tinder	Free	Yes	$0.49 – 299.99	100M+
SweetMeet	Free	Yes	$0.99 – 199.99	50M+
Bumble	Free	No	$0.49 – 259.99	50M+
BLOOM	Free	Yes	$1.99 – 349.00	10M+
OkCupid	Free	Yes	$0.99 – 179.99	10M+
Zoosk	Free	Yes	$0.49 – 399.99	10M+
Mamba	Free	Yes	$0.99 – 264.99	10M+
Boo	Free	Yes	$1.00 – 269.99	10M+

Table 3: Most Popular Dating Apps in the German and US Google Play Store

App Name	App Price	In-App Purchases	Price	Adds	In-App Purchases	No. of Downloads
US Apple Store			US Android Store			
LinkedIn	Free	Career (1 Month): $29, 99 – 39, 99 Business (1 Month): $69, 99	Free	Yes	$7.49 – 839.88	1B+
Indeed	Free	None	Free	Yes	none	100M+
Glassdoor	Free	None	Free	No	none	10M+
ZipRecruiter	Free	None	Free	No	none	10M+
Monster	Free	None	Free	No	none	5M+
German Apple Store			German Android Store			
LinkedIn	Free	Essentials (1 Month): 9, 99 € Career (1 Month): 29, 99 – 39, 99 € Business (1 Month): 69, 99 €	Free	Yes	7, 00 – 839, 88 €	1B+
Indeed	Free	None	Free	Yes	none	100M+
Glassdoor	Free	None	Free	Yes	none	10M+
Stepstone	Free	None	Free	Yes	none	10M+
Monster	Free	None	Free	Yes	none	5M+
Costs for Recruiters						
LinkedIn	The standard account is free. Premium accounts cost between 40 – 125 €/$ (See above)					
Indeed	There is an option for free listings. Costly adds are charged per click, with a minimum of 5 €/$ per day					
Glassdoor	No information					
Monster	Two Options: Monster+ Standard: Pay per Click and Monster+ Pro: 749€/$299 per month					
Stepstone	Multiple tiers: "Campus" 199 €,"Select": 329€, "Pro": 1399 €, Pro Plus: 1699 €, Pro Ultimate: 2399 €					
Zip Recruiter	Pricing depends on the number of job ads. Ads are charged per day and per add:"Standard": $16, "Premium": $24 Plans are charged per ad and per month: "Standard": $299, "Premium": $419, "Pro": $719					

Table 4: Job Platforms