Faking Network Size

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Abstract

Users have an incentive to join large platforms, and hence platforms want to convince users that they have a large user base. I consider a monopolist platform's incentive and ability to signal its user base when there is uncertainty regarding the amount of potential users. The platform can signal using either fake accounts or prices. When — as in some real-world examples — users are naive regarding the platform's ability to use fake accounts, small platforms use them to profit from the artificial increase in demand. If users are sophisticated, larger platforms use fake profiles to differentiate themselves from smaller ones, and — in contrast to the case of naive consumers — platforms would benefit from a ban on such business practices.

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1 Introduction

Joining a new platform often involves uncertainty for an individual user who cannot be sure of its overall popularity. With uncertainty about a platform's network size, users need to form beliefs about the participation decisions of others. Based on the data platforms collect and the in-depth knowledge of their business model, platforms naturally have an informational advantage relative to individual users. Moreover, they have an incentive to let consumers believe that joining will enable them to reap large network effects. Given these incentives, this paper investigates a platform's ability to use "fake profiles" to its advantage both when users are sophisticated about and when they are unaware of the platform's ability to do so.

The use of fake profiles is a common phenomenon on digital platforms. Prominent cases are those of Dating platforms. For example, the Federal Trade Commission (FTC) sued the Match Group for using fake profiles to persuade users to upgrade to a paid subscription. Other dating platforms use company-created fake profiles to interact with users on the platform, giving them the impression of a real contact with users often being unaware about this practice.[1] Further examples include cryptocurrency exchange platforms, which are under investigation by the SEC for engaging in trading financial assets themselves to artificially inflate the trading volume (so-called wash trading). Recent studies show that about 70% of unregulated trades are subject to wash trading (Cong et al., 2023). The economic costs to users and platforms are substantial. If fake profiles induce users to hold incorrect belief about the platform, they may make inefficient participation decisions. Furthermore, creating fake profiles is costly to the platform without generating additional value.

Formally, I investigate how a monopoly platform uses multiple signals to convince users of its network size. In particular, users can learn from the price they observe, a (cheap-talk) message, and the network size. Users are uncertain about the distribution of stand-alone values provided by the platform, while the platform has private information about this fundamental. Given the information asymmetry, suboptimal membership fees and fake profiles set by the platform are both costly signals about the fundamental. Fake profiles can increase the perceived network size but do not generate network effects ex post.

Users observe the membership fee first and then decide whether to join the platform. Thereafter, users who joined observe the perceived network size and decide whether to exit prior to paying the membership fee (following a "free-trial period"). In contrast to most of the signaling literature, I analyze a game with multiple signaling instruments, continuous signals and a continuum of receivers, where receivers (users) care not only about the sender's (platform's) action, but also about the action of other receivers (users).

Absent fake profiles, the platform's only signaling instrument about its size is the price. If users cannot observe demand and pay the price upfront, prices must be distorted upwards above full information prices to credibly signal a high fundamental. Only in

[1] See `https://www.ftc.gov/news-events/press-releases/2019/09/ftc-sues-owner-online-dating-service-matchcom-using-fake-love`, last visited 01.09.2020); `https://www.verbraucherzentrale.de/wissen/digitale-welt/onlinedienste/onlinedating-auf-diesen-portalen-flirten-fakeprofile-21848`, last visited 01.09.2020 or `https://www.faz.net/aktuell/wirtschaft/unternehmen/straftaten-schiessen-wegen-datingplattform-in-die-hoehe-18792428.html`.

higher states can a platform set inefficiently high prices to optimally separate from lower states. Including the possibility of creating fake profiles, users' understanding thereof is crucial when evaluating the market outcome. Before users pay the price, they observe the perceived network size without being able to distinguish between real and fake profiles. Sophisticated users, however, are fully aware of the platform's practice, whereas naive users are unaware of the possible use of fake profiles or believe that fake profiles are forbidden and hence not used.

Sophisticated users anticipate the platform's incentives correctly and hence, discount the perceived network size by the expected amount of fake profiles. In that case, both fake profiles and high prices are costly in that they reduce profits taken demand as given, and hence are substitutes for signaling a high fundamental. Abstracting from existence issues, the platform can always fully differentiate itself from those with less users through costly signaling based either on inefficiently high prices or the use of costly fake profiles. I identify parameter conditions such that the latter separating equilibrium exists, whereas the former always exists. Given its existence, in the unique separating equilibrium the platform with the lowest fundamental sets its full information price and all other platforms need to create fake profiles and distort their prices. Otherwise, the unique equilibrium has the same properties as the equilibrium absent fake profiles.

In contrast, if users dogmatically believe that every profile is real, i.e. they never considered the possibility of creating fake profiles, the platform uses the cheap-talk message to communicate the expected network size upfront. Users blindly believe this message upfront and the corresponding network size later on. The platform can exploit this misperception by using fake profiles to signal an unrealistic high network size, and thus value, to the users. In equilibrium, the platform always prefers to lie and deceive users by pretending that their network size is larger than it actually is. With a bounded state space, the platform with the highest fundamental, however, cannot induce unrealistic high user beliefs. Hence, depending on the costs of fake profiles, platforms below a threshold lie by creating fake profiles and set a higher price, and those above induce the highest possible belief about the state. This results in pooling on the observable instruments, but differentiation on the unobservable instrument.

The results imply that a platform would like to commit to refrain from using fake profiles with sophisticated users. Fake profiles are a wasteful investment for the platform used for separation. Rather than observing that platforms commit on not using fake profiles, it can be observed that platforms actually hide the use of fake profiles in their terms and conditions. This, however, likely indicates that users are mainly naive in these markets, which renders fake profiles profitable. With sophisticated users the platform would benefit from a regulation that provides commitment for the platform's claim that the observed network size is the true network size. This would result in full information prices being incentive-compatible as lying by platforms with a low fundamental is detectable and punished by exiting and non-paying users. If platforms are not able to credibly commit, however, they can profit from fake profiles with sophisticated users: signaling via fake profiles can lower the overall signaling costs when compared to signaling via distortionary prices only.

Methodologically, I apply an adjusted version of the D1 criterion developed by Banks and Sobel (1987) to refine the set of Perfect Bayesian Equilibria. It is well known that in certain classes of games, the D1 criterion selects a unique equilibrium outcome, which is separating, whenever there is a single receiver (or multiple receivers whose decisions are strategically independent). I extend this result to strategically interdependent receivers by imposing a restriction on the coordination problem of users' entry decision off the equilibrium path.

The remainder of the paper proceeds as follows. The related literature is discussed in Section 1.1. Section 2 describes the model and discusses potential applications. Section 3 analyzes the model when users are sophisticated, whereas Section 4 provides the analysis for naive users. Section 5 discusses common cases of fake profiles and Section 6 concludes. All omitted proofs are in Appendix B.

1.1 Related Literature

This paper is the first to introduce signaling into a model of platform adoption. As such it is related to models of platforms when there is incomplete information. Technically, it is related to the literature on signaling with multiple instruments. Since I allow for users who have incorrect beliefs, it is also related to papers on misleading consumers. I discuss these related papers below.

Platform Markets This paper belongs to the relatively sparse literature incorporating issues of incomplete information and asymmetric information (Halaburda et al., 2018a; Jullien and Pavan, 2019; Ke and Zhu, 2021; Kang and Muir, 2022) on platforms. Most models in the literature on platforms and two-sided markets assume complete information (Caillaud and Jullien, 2003; Rochet and Tirole, 2003, 2006; Armstrong, 2006; Halaburda et al., 2018b; Gal-Or, 2020). To the best of my knowledge, no paper has investigated asymmetric information between the platform and its users where the platform holds private information. The closest paper with respect to modelling the incomplete information is Jullien and Pavan (2019) who consider a platform market in which both users and platforms face uncertainty about participation decisions due to dispersion of information about their preferences. Especially given the growing importance of big data, I consider the more realistic case in which the platform has (superior) private information regarding its desirability to potential users. This, however, implies that the platform's choices act as a signal. Contrary to Jullien and Pavan (2019), I consider a platform with only one market side, or a platform on which both sides of the market are identical.

Signaling and Advertising As the monopoly platform has private information in my model, it is closely related to the literature on signaling (Kreps and Sobel, 1994), where signalling games with a continuum of states are studied by Mailath (1987) and in particular Mailath and von Thadden (2013). Fake profiles have not been studied in this context. Papers such as Kihlstrom and Riordan (1984), Milgrom and Roberts (1986), and Bagwell and Ramey (1988) that study the use of costly advertisement in combination with prices are conceptually closely related although they do not incorporate network effects. Fake profiles resemble persuasive advertisement, which is assumed to shift the willingness to pay of users (see Bagwell (2007) for an overview on advertisement). In a signaling model, Rhodes and Wilson (2018) analyzed false advertising used by firms to overstate the value of their products. False advertisement is only costly whenever it is punishable by a third-party. Buyers, nevertheless, may be affected by false advertisement in equilibrium.

A key difference is how the amount of fake profiles is determined in my model, in which not only the cost function but also equilibrium prices and demand determine the amount fake profiles. Furthermore, the fact that fake profiles might not be observable to users influence their equilibrium amount.

The paper adds to the literature of signaling by identifying a novel channel — network effects — that makes signaling via price or fake profiles credible. Main channels in the literature on signaling are: 1) repeated purchases, 2) cost differences between qualities, and 3) information differences between users. Although learning by users bears similarities to repeated purchases, price signaling in my model even works absent learning. Due to the presence of network effects an increase in the price has two effects. First, the price has a direct effect on users' utility lowering their willingness to participate on the platform. It follows that additionally, the price also has an indirect effect on users' utility through the reduced participation decision of others, which further reduces their willingness to participate. Without network effects, users would not care about the state as their participation decision would be independent of those of other users.

Due to equilibrium multiplicity in signaling games, a wide range of papers focuses on appropriate equilibrium refinements (Cho and Kreps, 1987; Banks and Sobel, 1987; Cho and Sobel, 1990). As users exert positive externalities on each other, in my model the most prominent refinements in the literature fail to select a unique equilibrium. Therefore, I adopt a version of the D1 criterion for a continuum of states as in (Ramey, 1996) and impose a further (weak) restriction on the receivers' strategies off-path: that they are rationalizable (Bernheim, 1984; Pearce, 1984) given a common belief.

Consumer Naïveté The model investigates the effects of different types of user sophistication on the market outcome when users face fake profiles thereby adding to the literature on consumer naïveté. See Heidhues and Kőszegi (2018) for a survey on the growing literature on how consumer naïveté affects market outcomes. While recent policy papers suggest (Crémer et al., 2019; Scott Morton et al., 2019; Fletcher et al., 2021), behavioral effects are particularly important in digital settings, academic research on this topic is scarce. My paper is among the first formal models to introduce consumer naïveté in platform markets among Johnen and Somogyi (2021) who analyze and compare the sellers' and the platforms' incentive to hide parts of the price from naive consumers. They find that a platform has strong incentives to shroud additional fees if it increases perceived consumer surplus. Conceptually, my paper is closely related to work on consumer naivete in cheap talk models (Ottaviani and Squintani, 2006; Kartik et al., 2007; Chen, 2011) that analyze the impact of naive or credulous consumers who blindly believe the sender's message. In contrast to these papers, creating fake profiles is costly giving rise to signaling issues.

Manipulating Consumer Expectations More broadly, the paper is connected to the literature that studies the manipulation of consumer expectations, especially in network markets. Early contributions focus on the expectations of early adopters of a network good. More recently, the emergence of fake reviews for products on platforms is studied. Evidence of fake reviews on for example Amazon.com is provided by He et al. (2022) or Expedia.com and TripAdvisor.com by Mayzlin et al. (2014). Theoretic treatments of fake reviews can be found in Glazer et al. (2021) and Yasui (2020), where the most closely related paper is Knapp (2022). The author analyzes a cheap-talk game in which a reviewer of a good may create a truthful or fake review but abstracts from the platform setting with network effects. Similar to my paper, consumers differ in their understanding of the possibility of fake reviews (naive or sophisticated).

2 Model

I analyze a sender-receiver model with two types of players: a platform (sender) and a group of potential users (receivers) of mass one. The platform has private information about a fundamental $\theta \in [\underline{\theta}, \overline{\theta}] \subset \mathbb{R}$ that determines the users' distribution of stand-alone values r on the platform $F(r|\theta) \equiv F_\theta(r)$. The common prior about the fundamental is $\mu_0(\theta)$, a continuous probability distribution, and has full support on Θ.

The platform and users engage in the following four-period game. First, nature draws the fundamental $\theta \in \Theta$. The platform observes θ and sets a price p and message m, where the message space is restricted to the type space. Additionally, the platform may invest in fake profiles, where the amount of fake profiles is given by ξ. Then, upon observing the platform's message and price (p, m), as well as their own stand-alone value r_i, users decide whether or not to enter. Users who joined observe the perceived number of users and decide whether to exit. The perceived number will depend on the actual mass of users and fake profiles, in a way detailed below. Lastly, the platform collects fees.

Users: Payoff Users have a common outside option normalized to zero. They vary, however, in the utility they receive from joining the platform — their stand-alone value r_i. User i obtains utility

$$v_i = r_i + \beta n - p,$$

where the distribution of stand-alone values r, F_θ, is continuous with full and strictly positive support. Users benefit from positive network effects, β, and from the mass of users that stay on the platform, n, but pay price p.

Users: Actions and Beliefs Upon having learned about the true fundamental, the platform sets a price, sends a message, and determines the number of fake profiles. First, after observing a price-message pair, users update about the fundamental and form a belief $\mu(\theta|p, m)$ and, then after learning their individual stand-alone value, form a belief $\mu_1(\theta|p, m, r_i)$. Second, after joining the platform, users observe the perceived mass of users, which is a function of the mass of users who have joined and the mass of fake profiles. The corresponding belief is denoted by $\mu_2(\theta|r, p, m, \mathbb{I})$, where $\mathbb{I}$ denotes the information structure. Depending on the users' ability to learn about or observe fake profiles, they update about the fundamental based on the information structure $\mathbb{I} = \{\emptyset, \{[0, 1], \mathbb{R}_0^+\}, [0, 1] + \mathbb{R}_0^+\}$. Users may either not observe the network size at all, $\mathbb{I} = \emptyset$, observe the true network size and fake profiles separately, $\mathbb{I} = \{[0, 1], \mathbb{R}_0^+\}$, or observe the sum of both, $\mathbb{I} = [0, 1] + \mathbb{R}_0^+$, which may include fake profiles. Among these users, sophisticated users are aware of the possibility of fake profiles, while naive users blindly believe the message sent and take the network size at face value for values below or equal to one.[2]

User i's entry strategy in the first period is a mapping $\sigma_i^1 : \mathbb{R} \times M \to [0,1]$ from prices and messages to entry. For given price p and belief $\mu_1(\theta|\cdot)$, a user enters if their expected utility from entering is higher than their outside option. The aggregate entry decision of users depends on the distribution of r in society. Following entry, users update their beliefs to $\mu_2(\theta|\cdot)$ and decide whether to exit the platform; formally, their exit strategy in period two — given that the user entered in period one — is given by $\sigma_i^2 : \mathbb{R} \times M \times \mathbb{I} \to [0,1]$.

Platform: Payoff and Actions The platform is a monopolist that chooses a price-message pair (p, m) with $p \in \mathbb{R}_+$ and $m \in M = \Theta$ and a number of fake profiles $\xi \in \mathbb{R}_0^+$. The platform's strategy maps the state space into prices, messages, and fake profiles $\sigma^P : \Theta \to \mathbb{R} \times \Theta \times \mathbb{R}_0^+$. The platform maximizes its profit with respect to prices and fake profiles

$$\max_{p,\xi} (p - c)n(\theta, \mu, p) - \gamma\xi,$$

where $n(\theta, \mu, p)$ is the mass of users that stay on the platform given the true fundamental, their belief about it, and price p. Let c denote the marginal cost of the platform to serve one user and γ the marginal cost of creating a fake profile.

Equilibrium Concept The equilibrium concept is a Perfect Bayesian Equilibrium (PBE) if all users are sophisticated. Strategies are optimal given beliefs at every information set. Beliefs of sophisticated users are updated via Bayes' rule whenever possible. At each information node, users optimize given their beliefs (sequentially rationality).

If users are naive, I use a Perception-Perfect Equilibrium (PPE). Naive users form their beliefs through the following rule. In the first period, naive users blindly believe in the fundamental stated by the observed message, and thus are point beliefs. Following entry, users take the network size at face value (for a mass below or equal to one). If the observed network size confirms the expected network size given first period belief and price, the belief remains the same. If the observed network size, $\hat{n}$, does not match the expected network size naive users revise their belief. The naive users' new belief must satisfy the following condition

$$\mu_2^N \equiv \{\theta' \in \Theta | n(\theta', \mu_1^N, p) = \hat{n}\}.$$

Naive users maximize expected utility given their beliefs.

As a tie-breaking rule, I impose that users enter only if they expect to stay: Whenever a user is indifferent between not joining the platform or joining the platform but leaving in period 3, I assume that the user does not enter.

[2] As the true network size is at most equal to a mass of one when all users enter, naive users take the network size at face value as long as it does not exceed a value of one. For values above one, naive users are free to hold any belief about the state, where I restrict attention to naive users holding the most pessimistic belief. Imposing any other belief such as the most optimistic belief, however, does not affect the equilibrium as long as the message space is restricted to the type space and users understand that the type space is bounded.

Equilibrium Refinement For sophisticated users, off-path the set of equilibria is refined by adapting the D1 Criterion of Ramey (1996) to a signaling game in which the receivers strategically interact. Intuitively, under D1 users' out-of equilibrium beliefs put positive mass only to the types that are most likely to profit from a deviation from equilibrium. As users' participation decisions depend on the decision of other users and are thus not strategically independent, I impose a restriction on the coordination aspect of users' entry decision. For a given price, I suppose this induces a common receiver belief (as it does on the path of the play). Consumers take this common belief as given, and then resolve the coordination problem among themselves in the same way as they would if this belief was common knowledge. With common knowledge of the state, there is a unique rationalizable entry decision suggesting that the coordination problem should be resolved in exactly that way. The precise definition is given by Definition 4 in Appendix A. For naive users, I do not use an equilibrium refinement. Beliefs (on and off-path) are naively given by the simple rule specified above.

Assumptions To analyze the game, I impose regularity conditions on the family of distributions $F_\theta(r)$ and the strength of the network effect β.

Assumption 1. *The distributions $F_\theta(r)$ are*

1. *twice differentiable in r and θ with density $f_\theta(r)$,*
2. *where the corresponding densities $f_\theta(r)$ are single-peaked in r, and*
3. *the distributions have a (weakly) increasing hazard rate $\lambda(r;\theta)$ in r, and*
4. *common support.*

The above assumption on F_θ ensures that the optimization problem of the platform is well-behaved under complete information and that there exists a unique (monopoly) price. The assumption on common support and single-peakedness can be relaxed to allow for the family of uniform distributions as well.[3]

Assumption 2. *(MLRP) For $\theta > \theta'$, f_θ likelihood dominates $f_{\theta'}$: $\frac{f_\theta(r)}{f_{\theta'}(r)}$ is an increasing function.*

The monotone likelihood ratio property implies that F_θ first-order stochastically dominates $F_{\theta'}$ and hazard rate $\lambda(p,n;\theta)$ is strictly monotonically decreasing in θ. First-order stochastic dominance implies that higher states lead to higher demand. The latter yields that the price elasticity increases with increasing θ holding participation constant. Hence, a higher state induces higher monopoly prices c.p. Lastly, to exclude multiplicity of (continuation) equilibria network effects β cannot be too strong.

Assumption 3. *(Network effects) $\beta \in \{\beta \in \mathbb{R}_+ : 1/2 - \beta \max_{\theta,r} f_\theta(r) > 0,\ \theta \in \Theta\}$.*

Networks effects must be small enough to avoid multiplicity of continuation equilibria, and hence, guarantees uniqueness of continuation equilibria.

[3] Assuming non-common support has the following implication for the analysis. After observing the stand-alone value, users with a high stand-alone value form a belief that puts zero probability on states that are not possible. This does not affect the analysis of separating equilibria on-path, but plays a role for incentive-compatibility as a deviation to a lower state cannot be credible to those users. During most of the analysis, however, the relevant incentive compatibility is for a low type to mimic a high type. In pooling equilibria beliefs are dispersed and the analysis remains unchanged.

2.1 Preliminaries

Under the assumptions made for any price the platform has set, there exists a unique cutoff strategy for users, even if the information is incomplete. Each user has private information about their own reservation value. All users with reservation values above the cutoff participate in the platform, while users below the cutoff do not. The first lemma defines the cutoff.

Lemma 1. *(Unique cutoff) In any equilibrium in which users hold a common belief upon observing (p, m), users use a cutoff strategy. The unique cutoff is given by*

$$r_c = p - \beta \int_{\Theta} (1 - F_\theta(r_c))\mu(\theta|r_c, p)d\theta, \tag{1}$$

which results in $n(\theta, \mu, p) = 1 - F_\theta(r_c)$ agents.

This lemma implies that users also follow a unique cut-off strategy in an equilibrium in which some types pool, i.e. when there is incomplete separation. Users' beliefs are dispersed as although all users have a common prior, they draw inferences from their own r. As a result, after observing a price and their own reservation value, users hold different beliefs. To establish the lemma, however, it is sufficient to suppose that upon observing the price but not yet their standalone value, users hold a common belief. On path this must be fulfilled because all users rely on Bayes rule, whereas off-path the common belief assumption is imposed.

As a benchmark, the next lemma characterizes the full information benchmark which corresponds to the first-best solution in prices and user participation.

Lemma 2. *(First-best) Under full information, there exists a unique equilibrium. In this equilibrium, the platform's profit maximizing price $p^{FI}(\theta)$ satisfies*

$$p^{FI} - c = \frac{1 - F_\theta(r^{FI})}{f_\theta(r^{FI})}(1 - \beta f_\theta(r^{FI})), \tag{2}$$

where r^{FI} denotes the equilibrium cutoff given p^{FI}. The full information price is strictly monotonically increasing in θ if the density $f_\theta(r^{FI})$ is strictly decreasing in θ.

It follows that the mark-up is always positive and hence, the price is always above marginal cost. If $F_\theta(\cdot)$ is the exponential distribution with scale parameter θ, which satisfies the MLRP, then indeed $f^{1,\theta}(r^{FI}) < 0$ and thus, the full information price increases in θ. Lastly, Assumptions 1-3 imply that the platform's profit for $\mu = \hat{\theta}$ fulfills the (strict) single-crossing property.

Lemma 3. *The platform's profit function, $\pi(\theta, \hat{\theta}, p)$, satisfies the single-crossing property, namely*

$$\frac{\partial}{\partial \theta}\left(\frac{\dfrac{\partial \pi(\theta, \hat{\theta}, p)}{\partial p}}{\dfrac{\partial \pi(\theta, \hat{\theta}, p)}{\partial \hat{\theta}}}\right) > 0.$$

3 Price and Message as Signals

In this section, I will discuss a benchmark for analyzing the effectiveness of signaling on platforms in which the presence of fake profiles does not impact demand. The benchmark assumes that users enter the platform and decide whether to stay without being able to observe the network size. In other words, the timing is as if user pay for their membership before joining the platform. Users cannot learn from the network size before their purchase decision, making fake profiles irrelevant. Hence, the participation and purchasing decision happen simultaneously represented by the information structure $\mathbb{I} = \emptyset$.

3.1 Sophisticated Users

Sophisticated users are rational and fully understand the signaling game. For those users, the price is the only credible signal and the message is ignored. Hence, I will suppress the message in the section below. For ease of exposition, the main part will focus on the construction of separating equilibria. The platform uses a one-to-one strategy $\tau : \Theta \to \mathbb{R}$ that maps the state to its chosen price and therefore, users hold a common belief on the path of play. I will focus on differentiable separating strategies τ.

Definition 1. A separating equilibrium consist of the platform's strategy τ, users' strategy σ_i and beliefs, μ, such that:

1. For any $p \in \tau(\Theta), \mu(p) = \tau^{-1}(p)$,
2. For any $\theta \in \Theta.\tau(\theta) \in \arg\max_{p\in\mathbb{R}_+} \pi(\theta, \mu(p), p)$ (Incentive Compatibility).

The platform maximizes its profit with respect to the price given that users form their beliefs according to $\mu(\theta|p, m, r) = \tau^{-1}(p)$. With a slight abuse of notation $n(\theta, \tau^{-1}(p), p)$ denotes the network size based on the true state θ, the belief $\tau^{-1}(p)$ which is a Dirac measure, and the price. Therefore, when the platform increases its price, the effects on profit are two-fold. The first effect is the direct price effect on the mark-up and demand, whereas the second effect is the belief effect, i.e. a higher price potentially signals a higher state. The platform's pricing strategy is determined by

$$\{\tau(\theta)\} \equiv \underset{p\in\tau\in([\underline{\theta},\overline{\theta}])}{\arg\max} (p - c)n(\theta, \tau^{-1}(p), p).$$

Assumptions 1-3 ensure that the profit is differentiable. In any separating equilibrium, rational users learn about the true state from the separating strategy. Focusing on differentiable separating strategies the first-order condition can be used. The first-order condition given that in equilibrium beliefs are correct, i.e., $\tau^{-1}(p) = \theta$ yields

$$n(\theta, \theta, p) + (p - c)\frac{\partial n(\theta, \tau^{-1}(p), p)}{\partial p}\bigg|_{\tau^{-1}(p)=\theta} + (\tau^{-1}(p))'(p - c)\frac{\partial n(\theta, \tau^{-1}(p), p)}{\partial \tau^{-1}(p)}\bigg|_{\tau^{-1}(p)=\theta} = 0.$$

The separating strategy $\tau(\theta)$ is given by the differential equation

$$\tau'(\theta) = -\frac{(\tau - c)n_2(\theta, \theta, \tau)}{n(\theta, \theta, \tau) + (\tau - c)n_3(\theta, \theta, \tau)}, \tag{3}$$

where $n_2(\cdot)$ and $n_3(\cdot)$ denote the partial derivatives with respect to the second and third arguments, respectively. Observe that setting $p = p^{FI}(\theta)$ from Equation 2 sets the denominator equal zero. Hence, setting the complete information prices for all types is not a solution. Prices must be distorted. Sequential rationality implies setting the initial value condition to $\tau(\underline{\theta}) = p^{FI}(\underline{\theta})$, i.e., the lowest type cannot do better than setting their first-best price. Given the initial value condition, there exists a unique solution to the differential equation that minimizes the level of costly signaling.

Proposition 1. *Suppose $\mathbb{I} = \emptyset$. Then under the equilibrium refinement in Definition 4, there exists a differentiable separating equilibrium outcome in which the equilibrium price $p^{S,*}$ is given by Equation 3 with $\tau(\underline{\theta}) = p^{FI}(\underline{\theta})$.*

Given differentiability and the initial value condition, the (differentiable) separating equilibrium is unique (Mailath, 1987). In the separating equilibrium, the platform "burns money" to credibly communicate its type to its users taking the form of distorted prices. The price as signaling device is feasible as the marginal cost of a price increase depends on the demand curvature, which in turn is influenced by the platform's true state. As shown in Lemma 3, the platform is more willing to trade-off and increase in price against an increase in demand. This link between the true fundamental and price is established by the network effects that arise on the platform. Thus, the incentive-compatible separating strategy must be increasing in the state as signed in the next corollary.

Corollary 1. *The equilibrium price $p^{S,*}$ is increasing in θ and is always greater than the full information price.*

To sign the pricing distortion, it is useful to recall that the full information price in Lemma 2 might be either increasing or decreasing in the state, but is always greater than marginal cost. In contrast, the equilibrium price under price signaling is always increasing in the state. Together with Equation 3 and the fact that profits increase in $\hat{\theta}$, this implies that the price must be larger than the full information price (i.e. at $p^{S,*}$ the denominator of Equation 3 must be negative). Hence, signaling always takes the form of inflated prices.

Additionally, I show that there exist no equilibria in which types partially pool on prices by applying the equilibrium refinement.

Proposition 2. *There exists no equilibrium in which the platform in more than one state θ sets a price $p(\theta) = p'$ under the equilibrium refinement.*

Applying the adjusted D1 Criterion in Definition 4 rules out any equilibrium in which types partially pool on prices. The highest type in the pool always has an incentive to deviate. The single-crossing property in prices implies that there exists a small increase in both price and demand for which the highest type prefers to deviate, while lower types do not. Since lower types would not choose such a price, D1 beliefs assign positive probability only to higher types. Then, since D1 beliefs assign higher probability to higher types following an off-path deviation, the user response must increase accordingly. This ensures that the highest type finds the deviation profitable, thereby breaking the pooling equilibrium.

The results in this section provide a novel rationale for platforms that charge high prices, namely to signal their high quality. Many platforms offering "premium" services only charge high prices to demonstrate that they can attract users with higher stand-alone values through their services.[4]

4 Price, Message and Fake Profiles as Signals

In this section, I consider a setting where users observe the network size after joining but cannot distinguish fake from real profiles, modeled by the information structure $\mathbb{I} = [0,1] + \mathbb{R}_0^+$. Due to the timing of the game, signals are observed sequentially and beliefs are updated twice. I analyze the following two cases: First, sophisticated users who are aware of the possibility that the platform can create fake profiles but are unable to distinguish them ad hoc. Second, naive users who are unaware of the possibility of creating fake profiles, i.e., they simply believe that fake profiles are illegal or impossible to create.

4.1 Sophisticated Users

The analysis of sophisticated users, for whom fake profiles are unobservable but feasible, extends the analysis in Section 3.1 by introducing an additional signaling instrument. Throughout, I will continue to focus on differentiable separating equilibria as before.

Definition 2. A separating equilibrium consists of a platform's strategy that is a one-to-one mapping from the state to pairs of price and fake profiles $\rho : \Theta \to \mathbb{R}_+ \times \mathbb{R}_+,\ \theta \mapsto (p, \xi)$, users' strategy σ_i^1 and σ_i^2, and beliefs μ_1 and μ_2 such that:

1. For any $(p, \xi) \in \rho(\Theta), \mu_1(\cdot) = \rho^{-1}(p) = \mu_2(\cdot) = \rho^{-1}((p, \xi))$ (Belief Consistency).
2. For any $\theta, \theta' \in \Theta, \pi(\theta, \theta, \rho(\theta)) \geq \pi(\theta', \theta', \rho(\theta'))$ (Incentive Compatibility).

By construction, the platform faces a two-dimensional signaling problem as sophisticated users take both the price and the (expected) number of fake profiles as signal. The optimization problem can be formulated as the platform maximizing its profit given that users are able to infer the true state in the separating equilibrium:

$$\arg\max_{p,\xi} (p - c)n(\theta, \theta, p) - \gamma(\xi)$$

subject to incentive compatibility

$$\pi(\theta, \theta, \rho(\theta)) \geq \pi(\theta, \theta', \rho(\theta')), \forall \theta, \theta' \in \Theta,$$

given by

$$\begin{aligned} &(p(\theta) - c)n(\theta, \theta, p(\theta)) - \gamma(\xi(\theta)) \\ \geq\ &(p(\theta') - c)n(\theta, \theta', p(\theta')) - \gamma\left[\xi(\theta') + (n(\theta', \theta', p(\theta')) - n(\theta, \theta', p(\theta')))\right]. \end{aligned} \tag{IC}$$

[4] For example in the dating industry, platforms such as eHarmony.com or ElitePartner advertise their high quality services in comparison to other dating sites such as Match.com. ElitePartner, a dating site for academics, offers to create an account for free, but to take any action on the platform, users need to sign up for their membership which ranges between 70 Euro/month (6 months contract) to 35 Euro/month (24 month contract). To sign up on ElitePartner users need to certify their academic degrees.

A deviation in the equilibrium strategy $\rho(\theta)$ consists of a deviation in price $p(\theta)$ alongside a change in fake profiles $\xi(\theta)$. Although fake profiles bear similarities to the concept of advertising, note that the incentive constraints are different. As users only observe $n+\xi$, a deviation to price-fake profile pair (p', ξ') reveals information about the state. For a type θ to mimic a type θ', the platform instead needs to additionally adjust its fake profiles by the difference in demand when deviating to induce belief θ'.

Turning to analyzing the incentive constraint, note that IC must bind. Setting the IC slack would imply that the platform could decrease the difference between $\xi(\theta)$ and $\xi(\theta')$ and save costs. Rearranging yields

$$\gamma[\xi(\theta') - \xi(\theta)] = \\ (p(\theta') - c)n(\theta, \theta', p(\theta')) - (p(\theta) - c)n(\theta, \theta, p(\theta)) - \gamma\left[n(\theta', \theta', p(\theta')) - n(\theta, \theta', p(\theta'))\right]. \tag{IC*}$$

which pins down the fake profile strategy of type θ as a function of the price $p(\theta)$. Note that I restrict fake profiles to be non-negative throughout the analysis. Another possibility is to pin down the pricing strategy as a function of fake profiles. Hence, it is possible to construct a continuum of separating equilibria as separation can be achieved either via the price or fake profiles. Consider the following condition:

Condition 1.

$$(p - c)n_2(\theta, \theta, p) > \gamma n_1(\theta, \theta, p), \forall \theta \in \Theta.$$

Under Condition 1, the signaling benefit from an extra fake profile, the left-hand side of the inequality, outweighs the cost of an extra fake profile, the right-hand side of the inequality.

Proposition 3. *Suppose Condition 1 is fulfilled.*

(i) There always exists a separating equilibrium in which the platform sets zero fake profiles and prices are set to

$$\tau'(\theta) = -\frac{(\tau(\theta) - c)n_2(\theta, \theta, \tau(\theta))}{(\tau(\theta) - c)n_3(\theta, \theta, \tau(\theta)) + n(\theta, \theta, \tau(\theta))}, \tag{4}$$

with the initial value condition $p(\underline{\theta}) = p^{FI}(\underline{\theta})$.

(ii) There exists a separating equilibrium in which the platform sets a positive level of fake profiles given by

$$\gamma\xi = (p(\theta) - c)n(\theta, \theta, p(\theta)) - (p(\underline{\theta}) - c)n(\underline{\theta}, \underline{\theta}, p(\underline{\theta})) - \int_{\underline{\theta}}^{\theta} (p(t) - c - \gamma)n_1(t, t, p(t))dt, \tag{5}$$

and prices maximize equilibrium profits

$$p(\theta) - c - \gamma = -\frac{n_1(\theta, \theta, p)}{n_{13}(\theta, \theta, p)}, \text{ for } \theta > \underline{\theta}. \tag{6}$$

Suppose Condition 1 is violated. Then, there exists a separating equilibrium in which the platform sets zero fake profiles and full information prices in each state.

Suppose Condition 1 holds. Then, there always exists a separating equilibrium in which the platform uses only prices as a signal, as in Proposition 1. Since the equilibrium outcome in Proposition 1 satisfies the equilibrium refinement, so does the equilibrium outcome in Proposition 3. The proof of Proposition 1 shows that there is no price to which a type wants to deviate off-path. The same logic applies here. Additionally, independent of any belief, the platform does not want to deviate to positive fake profiles, as once users have entered—given any belief—no additional users can enter. Thus, fake profiles are costly but do not increase demand.

There can exist a second equilibrium under additional condition in which the platform creates a positive number of fake profiles. There are two main differences compared to the analysis of advertisement. First, in the IC fake profiles require that the additional term $\gamma\left[n(\theta,\theta',p(\theta')) - n(\theta',\theta',p(\theta'))\right]$ is present. This reduces, ceteris paribus, the slope of the fake profile function in equilibrium which must be created to ensure incentive-compatibility. Intuitively, when the platform with a low fundamental wants to mimic a platform with a high fundamental, the latter has an advantage of an already larger network size. Second, the price in a model with advertisement would be independent of the costs γ which appear as a mark-up on the right-hand side of Equation 4. Therefore, the platform can shift a part of the marginal costs of creating fake profiles to its users.

As in Section 3, I apply the equilibrium refinement to show that there exist no (partial) pooling equilibria.

Proposition 4. *There exists no equilibrium in which the platform sets a price p' and fake profiles $\xi(\theta)$ such that $n(\theta,\mu',p') + \xi(\theta) = n(\theta',\mu',p') + \xi(\theta')$ in more than one state.*

4.2 Naive Users

This section turns to the analysis of naive users. Users are assumed to be misspecified about fake profiles as they do not take the possibility into account that fake profiles can be created. Therefore, naive users take the network size on a platform at face value, i.e. suppose real profiles are equal to the profiles they see. As users cannot see the network size upfront, they observe the platform's message $m \in \Theta$ and take this as a literal statement about its network size. Given a price p, the platform's message $m \in [\underline{\theta}, \overline{\theta}]$ can be interpreted as sending a message about the feasible network size $m \in [n(\underline{\theta},\underline{\theta},p), n(\overline{\theta},\overline{\theta},p)]$.

The notion of naive users is motivated by wrong legal beliefs, as users may believe that fake profiles are simply illegal or impossible to create in practice.[5] A majority of users that is new to those platforms is surprised afterwards about the use of fake profiles. In other cases, users form beliefs about these practices in traditional markets, where the use of fake profiles or similar practices is forbidden, and take over their beliefs to online markets or digital platforms. Fake profiles are often legal or the creation of fake profiles is legal as long as firms disclose their use in the terms and conditions. In the terms and conditions, however, the phrases are well-hidden (see Section 5). Hence, if users are naive with respect to the possibility of fake profiles, they take any demand that they deem feasible at face value. This approach is combined with the notion of credulity of users. The platform's message about the network size is technically cheap-talk, but users blindly believe in the

[5]See Armstrong and Vickers (2012) who make a similar argument towards naivete with respect to hidden prices.

message as they take the potential network size at face value. In online markets, this message could be the announcement of membership statistics or advertisement about the network size. Ottaviani and Squintani (2006) and Kartik et al. (2007) define the notion of credulous users for cheap-talk games. In the model at hand, the credulity stems from the naivete about the network size.

The equilibrium analysis is greatly simplified due to the fact that users put probability one onto the state that the platform announces (which corresponds to a mass of users on the platform). Hence, all subsequent beliefs in this section are point-beliefs, i.e. the Dirac measure on θ'. To analyze the equilibrium of the game with naive users, I need to define the platform's strategy and equilibrium concept.[6]

Definition 3. The platform's strategy ν is a LSHP (low types separate and high types pool) strategy if, for any price p, there exists a $\tilde{\theta} \in [\underline{\theta}, \overline{\theta}]$ such that:

1. For all $\theta < \tilde{\theta}, \nu(\theta) \in \{m(\theta)|m \in \Theta \setminus \{\overline{\theta}\}\}$, with $\nu(\theta) \neq \nu(\theta') \forall \theta \neq \theta'$.
2. For all $\theta \geq \tilde{\theta}, \nu(\theta) = \overline{\theta}$.

Given a price p, a perceived separating equilibrium consists of

1. A LSHP strategy on messages $\nu(\theta)$.
2. User beliefs $\mu(m) = m$.
3. A fake profile strategy $\xi(m) = n(m, m, p) - n(\theta, m, p)$.

While the platform may separate in some states but pool in others, naive users, however, hold separating beliefs and thus form a point belief after observing the message and perceived network size. The equilibrium is therefore termed a *perceived* separating equilibrium. Since *full* separation is not feasible under low costs, as specified below, there exists a cutoff state: the platform pools on the highest message if the fundamental is above the cutoff and separates if it is below. Note that the fake profile strategy mirrors the difference in real demand and forms part of the equilibrium. This definition is not restrictive, as the platform chooses the fake profile strategy optimally as shown in the following lemma.

Lemma 4. *The platform optimally sets fake profiles equal to* $\xi(\mu^*(m)) = n(\mu^*(m), \mu^*(m), p) - n(\theta^*, \mu(m), p)$ *such that* m *and* ξ *induce the same belief* $\mu^*(m)$.

Users hold two relevant beliefs for the platform, once during their entry decision $\mu(\theta|p, m, r) = m$ and once during their exit decision $\mu(\theta|p, m, r, n)$. If the announced message (or announced demand) differs from the actual observed demand on the platform, these two beliefs are not the same. Again, with a slight abuse of notation $n(\theta, m, p)$ denotes the true network size based on the true state θ, the belief $\mu(\cdot) = m$ which is a Dirac measure, and the price. Conversely, $n(m, m, p)$ denotes the believed network size of naive users given belief $\mu(\cdot) = m$. The platform's maximization problem is

$$\underset{\{m \in \Theta, p \in \mathbb{R}_+\}}{\arg\max} \; (p - c)n(\theta, m, p) - \gamma\left(n(m, m, p) - n(\theta, m, p)\right)$$

[6]This definition is based on Kartik (2009) who defines a strategy about a message as a LSHP strategy in the context of a cheap-talk game.

The first-order conditions with respect to p and m result in

$$p - c = -\frac{n(\theta, m, p)}{n_3(\theta, m, p)} + \gamma\left(\frac{n_3(m, m, p)}{n_3(\theta, m, p)} - 1\right) \tag{7}$$

$$(p - c + \gamma)n_2(\theta, m, p) = \gamma\left(n_1(m, m, p) + n_2(m, m, p)\right). \tag{8}$$

The first equation determines the optimal price given a chosen message m. If the message is equal to the true state, the optimal price is equal to the full information price. The second equation determines the choice of the optimal message. For a given price, the left-hand side is the marginal benefit of fake profiles, i.e. the increase in users' beliefs, and the right-hand side are the marginal costs. The first-order conditions are only applicable for $m \geq \theta$. If $m < \theta$, the platform does not set any fake profiles as the number of fake profiles is bounded away from zero.

Next, I determine the cutoff, i.e., the state at which the platform first chooses the highest message. To solve for the cutoff type $\tilde{\theta} < \overline{\theta}$, I examine the indifference condition. Let the profit of the platform in the indifferent state $\tilde{\theta}$ be

$$\pi(\tilde{\theta}, m, \xi(\tilde{\theta}, m), p) \equiv \pi(\tilde{\theta}, m),$$

which solves $\overline{\theta} = \arg\max_{m \in \Theta} \pi(\tilde{\theta}, m)$:

$$\left(-\frac{n(\tilde{\theta}, \overline{\theta}, p)}{n_3(\tilde{\theta}, \overline{\theta}, p)} + \gamma\frac{n_3(\overline{\theta}, \overline{\theta}, p)}{n_3(\tilde{\theta}, \overline{\theta}, p)}\right) n_2(\tilde{\theta}, \overline{\theta}, p) = \gamma\left(n_1(\overline{\theta}, \overline{\theta}, p) + n_2(\overline{\theta}, \overline{\theta}, p)\right). \tag{9}$$

Rewriting the equation gives the next lemma.

Lemma 5. *The indifferent type $\tilde{\theta}$ is the solution to*

$$\beta(1 - F_{\tilde{\theta}}(r(\overline{\theta}, p)) = \gamma, \tag{10}$$

which has a unique solution and solves for $\tilde{\theta} \in \Theta$ if

$$\underline{\gamma} \equiv \beta(1 - F_{\underline{\theta}}(r(\overline{\theta}, p)) \leq \gamma \leq \beta(1 - F_{\overline{\theta}}(r(\overline{\theta}, p)) \equiv \overline{\gamma}$$

The equilibrium is characterized in the following proposition.

Proposition 5. *The equilibrium with naive users is characterized as follows:*

(i) If $\underline{\gamma} \leq \gamma \leq \overline{\gamma}$. The indifferent type solves Equation 10. Types $\theta < \tilde{\theta}$ separate with $\nu(\theta) > \theta$ and types $\theta \geq \tilde{\theta}$ pool on $\nu(\theta) = \overline{\theta}$.

(ii) If $\gamma < \underline{\gamma}$, all types pool on $\nu(\theta) = \overline{\theta}$.

(iii) If $\gamma > \overline{\gamma}$, each type chooses $\nu(\theta) = \theta$.

Given message m, users believe $\mu(m) = m$, equilibrium prices are given by Equation 7 and the number of fake profiles is $\xi(m) = n(m, m, p) - n(\theta, m, p)$.

Suppose that γ lies within the specified upper and lower bounds. Then, by Lemma 5, Equation 10 has a unique solution. All types above the indifferent type pool on the highest message, while the types below choose m according to

$$\beta(1 - F_\theta(r(m,p)) = \gamma.$$

Setting $m = \theta$, i.e., every type reveals its true type to the naïve users, does not satisfy the equation, as the marginal benefit from lying upwards exceeds the cost. Hence, every type uses an inflated message $m > \theta$, such that complete separation (from the platform's perspective) is not possible. Due to the bounded state space, the highest type runs out of claims to make, and therefore higher types pool on the highest possible message. All types except the highest set a higher price than under full information, and even the lowest type creates fake profiles. The number of fake profiles increases up to the indifferent type and decreases afterward; only the highest type creates no fake profiles. To exploit consumers' naivete, a bound is imposed on the feasible strategies. In contrast to the fake profile strategy in Section 4.1, which was not bounded, consumer naivete makes the restriction of the state space binding, thereby influencing the strategy space.

Corollary 2. *Suppose $\underline{\gamma} \leq \gamma \leq \overline{\gamma}$. Then $\tilde{\theta}$, is increasing in γ and decreasing in β.*

In other words, the larger the network effects, the lower the indifferent type, as the benefit from creating fake profiles increases. Conversely, if the marginal costs increase, the indifferent type rises.

Suppose that $\gamma \leq \underline{\gamma}$. Then the lowest type already finds it optimal to send the message $m = \overline{\theta}$, and thus so do all types above. Therefore, all types pool on the message and the perceived network size. This implies that the lowest type creates fake profiles, and moreover, the number of fake profiles is decreasing in type. That is, the lowest type creates the most, and the highest creates none. Lastly, if $\gamma \geq \overline{\gamma}$. Then all types find it optimal to send a message equal to their true state. They create no fake profiles and set their full information prices.

From a welfare perspective, participation on the platform is distorted if $\gamma \leq \overline{\gamma}$. The new indifferent user does not benefit from the use of fake profiles, as they pay an inflated price for a non-existent network size. Users, however, who would have entered in the game without fake profiles and full information might benefit indirectly from fake profiles if they value network effects via β sufficiently. Due to excessive entry, the real network size on the platform increases, which may offset the higher prices for some users.

4.3 Regulation

This section analyzes possible remedies for and regulation to deal with the use of fake profiles. More specifically, I consider a ban of fake profiles, labeling fake profiles on the platform, mandatory disclosure of fake profiles upfront. It is assumed that the regulation is publicly known and users are educated and informed about the policy. The analysis considers the first three policies first and will show that all three will lead to the same unique market outcome. Lastly, it will be shown that educating users about the use of fake profiles is insufficient to prevent the use of fake profiles.

Banning Fake Profiles How does a ban of fake profiles impact the market outcome? Suppose the ban of fake profiles is public and users are informed about the policy. Sophisticated users will deduce that whenever they join a platform, they will observe the real network size. Hence, after joining sophisticated users are in a subgame of complete information in which the state is known. The unique equilibrium is summarized in Proposition 6.

Labeling Fake Profiles Suppose through labeling fake profiles, users can perfectly identify fake profiles and determine the real network size. In this case, labels must be clear, obvious and understood by users. Again, in the last period sophisticated users face a subgame of complete information (see Proposition 6). If fake profiles cannot serve as a signal due to perfect identification, no fake profiles are used by the platform. This follows directly from the fact that fake profiles are costly, but do not yield a positive benefit to the platform at this stage. Naive users behave as above.

Mandatory Disclosure Lastly, consider that platform must mandatory disclose their use of fake profiles upfront (either stating their use or abstention). Under signaling with price and fake profiles, or price only, the profit of a platform is always lower than the full information profit which is first-best as signaling is costly. In the presence of mandatory disclosure, the platform can choose to refrain from fake profiles credibly, which induces sophisticated users to deduce, again, that they will observe the real network size on the platform. As will be shown in the following proposition, this will enable the platform to achieve its full information profit.

Analysis All policies result in perfect knowledge of users about the real network size on the platform after joining, such that fake profiles cannot influence their perception. Users are sophisticated and take the price as the only costly signal. Hence, the platform maximizes its full information profit

$$\max_p (p-c)n(\theta,\theta,p), \text{ subject to } n(\theta,\mu_2(\cdot)=\theta,p) \leq n(\theta,\mu_1(\cdot)=\hat{\theta},p).$$

As users' participation decisions are made after observing the first-period price, the constraint imposes an upper bound on demand in the last period. The optimal prices are given by the first-order condition

$$n(\theta,\theta,p) + (p-c)\frac{\partial n(\theta,\theta,p)}{\partial p} = 0 \tag{11}$$

resulting in $p = p^{FI}(\theta)$, the full information benchmark price given that $\theta \leq \hat{\theta}$.

To see that $p^{FI}(\theta)$ is an incentive-compatible separating strategy, suppose that the platform in state θ sets a price $p^{FI}(\theta') < p^{FI}(\theta)$ for $\theta' < \theta$. This influences demand in two ways: first, a price decrease leads to more demand holding all else constant and second, a price decrease influences the believed state $\hat{\theta}$ and leads to a lower expected state. This in turn, decreases demand all else constant. Suppose first that demand overall increases and more user enter than in equilibrium. Then, in the last period users observe the realized demand given price and belief $(p^{FI}(\theta'), \theta')$ and the true state θ. As too many users entered given belief θ', users exit again such that

$$n(\theta, \theta, p^{FI}(\theta') = 1 - F_\theta(p^{FI}(\theta') - \beta n(\theta, \theta, p^{FI}(\theta'))).$$

As the price and realized demand $n(\theta, \theta, p^{FI}(\theta'))$ are not profit maximizing in state θ, the platform does not face a profitable deviation. A similar argument can be constructed if demand overall decreases. Then, following the deviating price $p^{FI}(\theta')$ too few users join the platform than optimal.

Proposition 6. *Suppose the government regulates platforms by either banning fake profiles, forcing them to label fake profiles, or mandatory disclosing fake profiles. If users are aware of these policies, there exists a unique equilibrium that is separating and first-best. In equilibrium, the platform sets the full information price and zero fake profiles are used.*

This result stresses the importance of observing the network size before paying the membership fee. Compared to Section 3.1 in which users paid the membership fee without observing the network size, the platform can increase its profit by offering a free-trial period before collecting the (one-time) membership fee.[7] This "free-trial" period is desirable from a platform's perspective as full information prices are incentive compatible. The platform does not incur a loss in profit due to wasteful signaling. In equilibrium, both users and the platform are better off compared to the Section 3.1. In comparison to the signaling literature, the learning-from-demand-stage resembles repeat purchases for regular products or a full warranty/money-back guarantee.

The effect of educating naive users about the possibility of fake profiles are ambiguous. Users and the platform might be both worse off. If naive users are educated and no other action on fake profiles are taken, the platform might still use fake profiles for signaling as in Theorem 3. Due to the consumer sophistication, the platform still needs to engage in costly signaling. Comparing the equilibrium outcome in Theorem 3 and 5, the platform makes losses when moving from the latter to the first. With naive users, the platform makes higher profits than under full information, whereas with sophisticated users the platform makes lower profit than under full information. The effect on users depends on the prices and network effects. Naive users benefit from an increase in the real network size compared to the equilibrium in Theorem 3. Additionally, prices might also increase compared to the latter equilibrium.

5 Discussion

Convincing Users to Upgrade into a Premium Subscription The Dating platform "Match.com" presumably utilized third-party fake profiles to persuade (male) users into a paid subscription.[8] Following the model, users are able to sign up to the platform for free. Initially, undecided users, who did not pay for the membership, received emails from potentially interested users. In the stylized version, users are assumed to view the

[7] The model abstracts from discounting between periods. Otherwise, the firm must be sufficiently patient or the free-trial period must be sufficiently short.

[8] `https://www.ftc.gov/news-events/press-releases/2019/09/ftc-sues-owner-online-dating-service-matchcom-using-fake-love`

total perceived network size $\tilde{n}$. To interact with the other users, they needed to upgrade their free trial. The platform's network size included fake profiles. The platform allegedly used the third-party fake profiles ξ to direct messages towards non-paying users which lead them to upgrade to a premium membership and pay p. Due to the platform's intentional use of fake profiles, i.e. identifying the third-party fake profiles, directing those to non-paying users but keeping those away from paying members, it is plausible to assume that the platform incurred small (effort) costs γ.

Manipulating the Network Size: Wash Trades One of the largest cryptocurrency exchange platforms ("Binance") is under investigation by the SEC for "manipulative trading that artificially inflated the platform's trading volume". They engaged in so-called Wash trading. More precisely, another associated company ("Sigma Chain") owned by the same entity ("Zhao") as the crypto exchange platform manipulated the platform's trading volume by selling and buying the same financial assets, therefore artificially inflating the platform's volume.[9] Furthermore, the U.S. based affiliate of "Binance" called "BAM Trading Services" is accused of misleading investors about non-existent trading controls on Binance.US. Wash trading is prohibited in offline (financial) markets, e.g. in the US by the Commodity Exchange Act.[10] For example, the Intercontinental Exchange (ICE) takes measures to prevent self-trade to comply with regulations.[11]

In this application, the platform is a cryptocurrency trading platform and its users are potential investors both buying and selling assets on the platform. Network effects take the form of caring for liquidity. A platform with a large network, i.e. a high trading volume, has more liquid assets and is more credible. Fake profiles are financial assets that are self-traded by the platform and hence inflate the network size.

Manipulating the Network Size: Dating Platforms Other dating platforms use company-created fake profiles; a list of several dating sites using this practice has been published by the Verbraucherzentrale Bayern (Center for Consumer Advise Bavaria) in Germany. These platforms employ paid workers to create profiles, and interact with users on the platform, giving them the impression of a real contact.[12] It is not commonly known that platforms themselves create fake users to possibly stimulate demand, although it is legal to do so as long as it is mentioned in the terms and conditions. There are companies that specialize in providing employees as chat moderators to these platforms.[13] These chat moderators set up fake profiles and engage in conversations with the users of the platform pretending to be a real profile.

[9] https://www.sec.gov/news/press-release/2023-101

[10] https://www.law.cornell.edu/uscode/text/7/chapter-1

[11] https://www.theice.com/publicdocs/futures/IFEU_Self_Trade_Prevention_FAQ.pdf

[12] See https://www.verbraucherzentrale.de/wissen/digitale-welt/onlinedienste/onlinedating-auf-diesen-portalen-flirten-fakeprofile-21848, last visited 01.09.2020.

[13] For example, Cloudworkers or Agentur da Chatdeife are companies that employ freelancers to work for and on one or more social-community platforms. See also https://www.spiegel.de/wirtschaft/service/singleboersen-ein-moderator-von-fake-profilen-spricht-ueber-seinen-job-a-1113937.html. and https://www.ndr.de/fernsehen/sendungen/panorama_die_reporter/Undercover-als-Chatschreiberin-Abzocke-Flirtportal,sendung1098906.html for an interview (in German) and https://www.marieclaire.fr/,dating-assistant,750821.asp for an article (in French).

Furthermore, the UK Consumer and Markets Authority (CMA) confirms in its report about the online dating industry that dating platforms may use ”pseudo profiles” or provider-generated profiles that could possibly mislead consumers. The CMA states that if these fake profiles are not disclosed as such, it may be in breach with the ”Consumer Protection from Unfair Trading Regulations”.[14] In another industry report issued by the Australian Competition and Consumer Commission (ACCC), the ACCC acknowledges that fake profiles generated by providers exist, but stress that this issue lies beyond the scope of their investigation mandate.[15] This shows that the use of fake profiles might be more common than initially expected and might not be restricted to the examples given above.

Evidence that chat bots might have been used by dating platforms exists for the dating site “Ashley Madison”. Ashley Madison was subject to a large data leak by hackers.[16] The dating site used “chat hostesses” before 2011 to engage men, which coincides with the notion of fake profiles in this context. After 2011, however, it is reported that they stopped employing “chat hostesses”. Instead, the dating platform allegedly used chat bots to deceive users to spend money on the platform. Although one might think that chats bots are easier to be identified, this might not have been the case. Users seem to have spent a reasonable amount of money on communicating with chat bots.

Lastly, there is evidence on dating platforms that use methods to create a similar effect as with platform-generated fake profiles. The CMA investigated the case of Venntro Media Group Ltd, a company that operates several dating sites. To inflate the network size on their dating sites, Venntro cross-registered their members on various sites and not only the site they originally signed up for.[17]

Launching Strategy for Start-Ups Upon launching a new platform, founders often generate artificial demand (or supply) to onboard producers (or consumers). This practice is documented in the business and management literature, e.g. Schirrmacher et al. (2017) or Reillier and Reillier (2017). Evans and Schmalensee (2016) describe the practice as “self-supply”. In case studies by Schirrmacher et al. (2017) some platforms self-supplied in the beginning at launch to influence participants' beliefs, whereas one platform simulated fake demand.

6 Conclusion

Especially on Dating platforms the use of fake profiles is heavily relied upon. Suggestive evidence from a data leak of the platform ”Ashley Madison” shows that fake profiles were used excessively. Most of the female users were in fact fake profiles. The data, however, included credit card transactions (mostly from men) indicating that many users spend a

[14]See `https://assets.publishing.service.gov.uk/media/5b114a8040f0b634abe911e7/compliance_statement.pdf`.

[15]See `https://www.accc.gov.au/system/files/927_ICPEN%20Dating%20Industry%20Report_D09.pdf`.

[16]See `https://financialpost.com/fp-tech-desk/inside-ashley-madison-calls-from-crying-spouses-fake-profiles-and-the-hack-that-changed-everything?__lsa=b245-a155`.

[17]See `https://www.gov.uk/government/news/online-dating-giant-vows-clearer-path-to-love`.

lot of money on the platform even though the chance of encountering a real women was surprisingly low.

Economic papers exploring the regulation of platform markets are scarce, although policy papers such as Fletcher et al. (2021) investigate common issues on platform that may need to be regulated. For fake profiles, one suggested policy is banning fake profiles. German cases suggest, however, that the disclosure cannot be hidden in terms and conditions. Similarly, one could consider mandatory disclosure policies. Voluntary and mandatory disclosure has been discussed by scholars such as Grossman (1981), Mathios (2000), or Fishman and Hagerty (2003).

In a classical model with rational users and voluntary disclosure all but the lowest type should disclose their type and state that they would not use fake profiles. In my model a platform would like to commit to refrain using fake profiles with sophisticated users as they are costly, which would indicate that if users are sophisticated voluntary disclosure on fake profiles should be observed in online markets. Instead, their actual use is mentioned in the terms and conditions, and consumer protection and competition authorities try to inform unknowing consumers about these. In contrast, there is no evidence of information campaigns or initiatives of firms committing not to use fake profiles. Such voluntary disclosure might fail as the presence of naive users eliminates the incentives to voluntary disclose the own type.

Combining suggestive evidence and the failure to observe voluntary disclosure in these markets suggests that users are mainly naive. This speaks in favor of consumer protection policies against practices that influence network effects such as a ban of fake profiles or mandatory disclosure.

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A Appendix: Definition of Equilibrium Refinement

Definition 4. Equilibrium Refinement for Sophisticated Users
Denote the equilibrium strategy profile by $\Sigma = ((p^*, m^*), r^*(p^*, m^*))$, where $r(\cdot)$ denotes the equilibrium cut-off mapping. The equilibrium profit of a platform of type θ is $\pi^*(\theta, \Sigma)$.

For a given price p, an arbitrary non-empty subset of sender type space $\tilde{\Theta} \subseteq \Theta$, and a non-empty subset of the other receivers action spaces $\tilde{\mathbf{Y}}_{-i}$ let

$$\mathrm{BR}_i(\tilde{\Theta}, \tilde{\mathbf{Y}}_{-i}) = \cup_{\rho_i \sim \Delta(\tilde{\Theta} \times \tilde{\mathbf{Y}}_{-i})} \arg\max_{y_i \in [0,1]} \mathbb{E}_{(\theta, \mathbf{y}_{-i}) \sim \rho_i}[u_i(\theta, y_i, \mathbf{y}_{-i}, p)] \forall i$$

be the set of user i's best responses to p for some belief ρ_i over sender type and the other receivers action pairs with support contained in $\tilde{\Theta} \times \tilde{\mathbf{Y}}_{-i}$. For an arbitrary non-empty subset of sender type space $\tilde{\Theta} \subseteq \Theta$ and $k \in \{0, 1, 2, ...\}$ let

$$Y_i^k(\tilde{\Theta}) = \mathrm{BR}_i(\tilde{\Theta}, \tilde{\mathbf{Y}}_{-i}^{k-1}(\tilde{\Theta})), \text{ and } Y_i^\infty(\tilde{\Theta}) = \cap_{k \in \{0,1,2,...\}} Y_i^k(\tilde{\Theta}) \forall i$$

be the set of rationalizable actions given $\tilde{\Theta}$ for receiver i. Denote by $\mathcal{R}^\infty(\tilde{\Theta}, p)$ the set of rationalizable receiver action profiles for given p and $\tilde{\Theta}$.

For a given out-of equilibrium price p and for each type θ, find all rationalizable action profiles $\alpha \in \mathcal{R}^\infty(\Theta, p)$ by users that would cause θ to deviate from equilibrium. For $\theta \in \Theta$, p, and equilibrium profile Σ,

$$D_\theta = \{\alpha \in \mathcal{R}^\infty(\Theta, p) : \pi^*(\theta, \Sigma) < \mathbb{E}_{r \sim \alpha} \pi(\theta, p, r)\},$$

is the set of receiver rationalizable actions for which type θ is strictly better-off deviating towards p, and

$$D_\theta^0 = \{\alpha \in \mathcal{R}^\infty(\Theta, p) : \pi^*(\theta, \Sigma) = \mathbb{E}_{r \sim \alpha} \pi(\theta, p, r)\},$$

is the set of receiver rationalizable actions for which type θ is indifferent between deviating towards p and setting equilibrium price p^*. If for some type θ there exists another type θ' such that

$$D_\theta \cup D_\theta^0 \subset D_{\theta'},$$

then (θ, p) may be pruned from the game. The set of types that cannot be deleted is denoted by $\Theta^*(p)$. A PBE violates D1 if there exists a type and action (θ, p) such that

$$\min_{\alpha \in \mathcal{R}^\infty(\Theta^*(p), p)} \pi(\theta, p, r) > \pi^*(\theta, \Sigma) \text{ for some } \theta \in \Theta^*(p). \tag{D1}$$

Discussion Ramey (1996) shows that under the following assumptions the unique D1 equilibrium is separating. The set of types is given by non-degenerate interval $[\underline{\theta}, \overline{\theta}]$, where prior beliefs are given by a continuous probability distribution $\mu(\theta)$ with full support. Signals are $p \in \mathbb{R}^k$ and the (single) receiver's response r is chosen from the real line. Payoff functions are continuously differentiable; the sender's payoff π increases in the receiver's response. The receiver's utility function u is strictly quasi-concave in its action r for each signal and type. The receiver's payoff is maximized in r by $r^*(\theta, p)$, which is strictly increasing in θ. Furthermore, $r^*(\theta, p)$ is uniformly bounded above and for

$k = 1, .., n : \lim_{p_k \to +/-\infty} \pi(\theta, r, p) = -\infty$. Enger's Incentive Montonicity Condition holds for the k signals (weaker condition than the multi-dimensional single-crossing property).

In the model at hand the assumptions on the receiver's payoff functions and actions are not fulfilled. A receiver's response given price p is binary and depends on the belief over the other receivers' actions. The sender, however, does not care about the action of a single receiver, but cares about the aggregate action taken by the receivers. The receivers' payoffs are instead quasi-concave (linear) in the aggregate response. Additionally, the aggregate response $n^*(\theta, p)$ is strictly increasing in θ due to the assumptions on F_θ.

B Appendix: Omitted Proofs

Recall that stand-alone values r are distributed according to the cumulative distribution function $F_\theta(r)$, with associated density $f_\theta(r)$. Denote the derivative of the density with respect to r by $f'_\theta(r)$. Let $F^{1,\theta}(r)$ and $f^{1,\theta}(r)$ represent the first derivatives of the distribution function and the density, respectively, with respect to θ.

Proof of Lemma 1 First, consider the case of complete information, in which the assumption that users hold a common belief upon observing (p, m) is trivially fulfilled. Users play sequentially rationalizable strategies, i.e. they play a best-response to a symmetric cutoff of other. To establish the unique cutoff $r_c \in [\underline{r}, \overline{r}] \subseteq [-\infty, \infty]$, I will iterate on the best-responses of users once from above starting at $r_0 = \overline{r}$ and once from below starting at $r_0 = \underline{r}$.

Step (i) Iteration starting from $r_0 = \overline{r}$.

Consider the best response of an agent i given an arbitrary state θ, price and message pair (p, m) and the action profile of the other agents. The first iteration given the symmetric cutoff $r_0 = \overline{r}$ yields

$$BR_i^1(\{\theta\}, r_0 = \overline{r}) = \begin{cases} 1 & \text{if } r_i \geq p \\ 0 & \text{if } r_i < p. \end{cases}$$

In the first iteration, agents with a reservation value of $r_1 \equiv p$ or higher will always enter even if no one enters the platform (independent of their beliefs). Iterated elimination of not best responses yields a cutoff value of r_i in the $i + 1$'th iteration given by

$$r_{i+1} = p - \beta(1 - F_\theta(r_i))$$

This sequence is bounded by $r \in [\beta - p, p]$ and strictly decreasing by the assumptions made in Section 2. Hence, the sequence converges to its limit

$$r_c = \lim_{i\to\infty} r_{i+1} = \lim_{i\to\infty} p - \beta(1 - F_\theta(r_i)) = p - \beta(1 - \lim_{i\to\infty} F_\theta(r_i)) = p - \beta(1 - F_\theta(r_c)).$$

The last inequality follows from the fact that the probability function is assumed to be continuous. Then, the condition

$$\overline{r}_c = p - \beta(1 - F_\theta(\overline{r}_c))$$

Step (ii) Iteration starting from $r_0 = \underline{r}$.

The first iteration given the symmetric cutoff $r_0 = \underline{r}$ yields

$$BR_i^1(\{\theta\}, r_0 = \underline{r}) = \begin{cases} 1 & \text{if } r_i \geq p - \beta \\ 0 & \text{if } r_i < p - \beta. \end{cases}$$

In the first iteration, agents with a stand-alone value below $r_1 \equiv p - \beta$ will never enter even if all others join the platform (independent of their beliefs). The cutoff value of r_i in the $i+1$'th iteration is given by

$$r_{i+1} = p - \beta(1 - F_\theta(r_i)),$$

as the sequence is bounded by $r \in [\beta - p, p]$ and strictly increasing,it converges to its limit

$$\underline{r}_c = p - \beta(1 - F_\theta(\overline{r}_c)).$$

Step (iii) Show that $\overline{r}_c$ and $\underline{r}_c$ coincide.

Given Assumption 3, for any $p \in \mathbb{R}_+$ there exists one and only one solution to the equation

$$r + \beta(1 - F_\theta(r)) = p, \tag{12}$$

as r is increasing in r with slope one, whereas $\beta(1 - F_\theta(r))$ is decreasing in r with slope smaller than one. Hence, the left-hand side is strictly increasing in r. Then, $r_c \equiv \{r \in [\underline{r}, \overline{r}] : r + \beta(1 - F_\theta(r)) = p\}$ characterizes the unique cutoff which is the unique sequentially rationalize user profile given the action-pair (p, m) by the platform.

Second, consider the case of incomplete information under the assumption that users hold a common belief upon observing (p, m). Given the equilibrium definition of PBE, there are two relevant types of equilibria — separating and pooling equilibria. In both equilibria, the common belief assumption is fulfilled again. In a separating equilibrium, users hold point-beliefs after observing (p, m), whereas users' Bayesian update in a pooling equilibrium after observing (p, m) is equal to their common (full support) prior. The proof proceeds as follows. First, I will show that if users play cutoff strategies, there exists a unique cutoff. Second, I will show that users will play a cutoff strategy in any equilibrium.

Step (i) Unique cutoff.

Suppose there exist two cutoffs defined by

$$\underline{r}_c + \beta \int_\Theta (1 - F_\theta(\underline{r}_c)) d\mu(\theta | p, m, \underline{r}_c) = p, \text{where} \tag{13}$$

$$\mu(\theta | p, m, \underline{r}_c) = \frac{\mu(\theta | p.m) f_\theta(\underline{r}_c | \theta)}{\int_{\tilde{\theta} \in \Theta} \mu(\tilde{\theta} | p, m) f_\theta(\underline{r}_c | \tilde{\theta}) d\tilde{\theta}}, \tag{14}$$

and

$$\overline{r}_c + \beta \int_\Theta (1 - F_\theta(\overline{r}_c)) d\mu(\theta | p, m, \overline{r}_c) = p, \text{where} \tag{15}$$

$$\mu(\theta | p, m, \overline{r}_c) = \frac{\mu(\theta | p.m) f_\theta(\overline{r}_c | \theta)}{\int_{\tilde{\theta} \in \Theta} \mu(\tilde{\theta} | p, m) f_\theta(\overline{r}_c | \tilde{\theta}) d\tilde{\theta}}. \tag{16}$$

For the sake of contradiction, suppose that the cutoff differ, e.g. $\underline{r}_c < \overline{r}_c$. Denote $X(r_1, r_2) = \int_\Theta (1 - F_\theta(r_1)) d\mu(\theta | r_2)$.

Lemma 6. *$X(r_1, r_2)$ is strictly decreasing in r_1 and (weakly) increasing in r_2.*

The first part follows directly from the fact that $1 - F_\theta(r_1)$ is decreasing in r_1 for all θ and hence, $\int_\Theta (1 - F_\theta(r_1))d\mu(\theta|r_2)$ is decreasing in r_1 as well holding $\mu(\theta|r_2)$ fixed.

For the second part, note that for $r' > r''$, $\mu(\theta|p, m, r')$ has first-order stochastic dominance over $\mu(\theta|p, m, r'')$ due Assumption 2. Assumption 2 states that the family of densities $\{f_\theta(r) \equiv f(r|\theta)\}$ is assumed to have the monotone likelihood ratio property. By Milgrom (1981) (Proposition 2) a family of densities has the MLRP iff $r' > r''$ implies that r' is more favorable than r'' meaning that $\mu(\cdot|r')$ dominates $\mu(\cdot|r'')$.

Recall that $1 - F_\theta(r_1)$ is bounded by $[0, 1]$ and is strictly monotone increasing in θ by Assumption 1 and 2. Then, $\int_\Theta (1 - F_\theta(r_1))d\mu(\theta|r') \geq \int_\Theta (1 - F_\theta(r_1))d\mu(\theta|r'')$ holds as r' is more favorable than r'', where it holds with equality whenever $1 - F_\theta(r_1) \in \{0, 1\}$.

In equilibrium, both the left-hand side of Equation 13 and 15 must be equal to p. Therefore, $\underline{r}_c + \beta X (= \overline{r}_c + \beta \int_\Theta (1 - F_\theta(\overline{r}_c))d\mu(\theta|p, m, \overline{r}_c)$ must hold with $\underline{r}_c < \overline{r}_c$. Note that in equilibrium, both the left-hand side of Equation 13 and 15 must be equal to p. Therefore, $\underline{r}_c + \beta X (= \overline{r}_c + \beta \int_\Theta (1 - F_\theta(\overline{r}_c))d\mu(\theta|p, m, \overline{r}_c)$ must hold with $\underline{r}_c < \overline{r}_c$. Note that In equilibrium, both the left-hand side of Equation 13 and 15 must be equal to p. Therefore, $\underline{r}_c + \beta X(\underline{r}_c, \underline{r}_c) = \overline{r}_c + \beta X(\overline{r}_c, \overline{r}_c)$ needs hold with $\underline{r}_c < \overline{r}_c$; however,

$$\underline{r}_c + \beta X(\underline{r}_c, \underline{r}_c) < \overline{r}_c + \beta X(\overline{r}_c, \underline{r}_c) \leq \overline{r}_c + \beta X(\overline{r}_c, \overline{r}_c),$$

where the first inequality follows from Assumption 3 for any given θ and the second inequality follows from the lemma above. This contradicts the initial assumption, thus, $\underline{r}_c = \overline{r}_c$.

Step (ii) Users play cutoff strategies in any equilibrium.

Define $\tilde{r} \equiv \inf\{r_i : u(r_i, p) \geq 0\}$ to be the user with the lowest r_i of the set of users that have a non-negative utility from joining the platform. Similarly, let $\tilde{\tilde{r}} \equiv \sup\{r_i : u(r_i, p) \leq 0\}$ be the users with the largest r_i of the set of users that have a negative or zero utility from joining the platform.

Therefore, it needs to hold that $\tilde{\tilde{r}} + \beta X(\tilde{\tilde{r}}, \tilde{\tilde{r}}) \leq p \leq \tilde{r} + \beta X(\tilde{r}, \tilde{r})$ for $\tilde{r} > \tilde{\tilde{r}}$ by definition. Imposing that $X(r_1, r_2)$ *strictly* increases in r_2, I can use the previous argument from Step (i) to show that there agents play cutoff strategies. As long as $\tilde{r}, \tilde{\tilde{r}} \in (\underline{r}, \overline{r})$ and hence $X(r_1, r_2)$ *strictly* increases in r_2, the following holds

$$\tilde{\tilde{r}} + \beta X(\tilde{\tilde{r}}, \tilde{\tilde{r}}) < \tilde{\tilde{r}} + \beta X(\tilde{\tilde{r}}, \tilde{r}) < \tilde{r} + \beta X(\tilde{r}, \tilde{r})$$

Therefore, $\tilde{r} = \tilde{\tilde{r}}$ and users follow a cutoff strategy.

Then, the condition

$$r_c = p - \beta \int_\Theta (1 - F_\theta(r_c))d\mu(\theta|p, m, r_c), \tag{17}$$

characterizes the unique cutoff. Note that if $\mu(\theta|r_c) = \delta_{\tilde{\theta}}$, i.e. the belief concentrates on state θ with probability 1 (a.s.), this condition is the same as under complete information. Observe that

Remark. If belief $\mu(\theta|p, m, r)$ has first-order stochastic dominance over a belief $\mu'(\theta|p, m, r)$, $\int_\Theta (1 - F_\theta(r_c))\mu(\theta|r_c)d\theta$ increases and therefore, the cutoff r_c decreases.

The result follows directly from the Equation 17 and Lemma 6. □

Proof Lemma 2 Under complete information, equilibrium demand is determined by the unique solution to

$$\begin{aligned} n^* = Pr(r + \beta n^* - p \geq 0) = 1 - F_\theta(p - \beta n^*), \\ \Leftrightarrow n^* = 1 - F_\theta(r^*) \end{aligned}$$

given that Assumption 3. It is possible to rewrite this condition as $G(n^*; p) = 1 - F_\theta(p - \beta n^*) - n^*$. The implicit function theorem implies that a function g exists such that $n^* = g(p)$. Implicit differentiation yields

$$\begin{aligned} -f_\theta(p - \beta n^*) + \beta f_\theta(p - \beta n^*)\frac{\partial n}{\partial p} - \frac{\partial n}{\partial p} = 0 \\ \frac{\partial n}{\partial p} = \frac{-f_\theta(p - \beta n^*)}{1 - \beta f_\theta(p - \beta n^*)}. \end{aligned}$$

The platform faces the optimization problem $\max_p (p - c)n(p)$ and yields

$$1 - F_\theta(r^*) + (p - c)\frac{-f_\theta(r^*)}{1 - \beta f_\theta(r^*)} = 0,$$

which can be rewritten as

$$p - c = \underbrace{\frac{1 - F_\theta(r^*)}{f_\theta(r^*)}}_{\eta(\theta,p)} \underbrace{(1 - \beta f_\theta(r^*))}_{>0},$$

where $\eta(\theta, p)$ is the users price elasticity. Given Assumption 1 the hazard rate defined by $\lambda \equiv \frac{1}{\eta}$ is decreasing. Thus, the first-order condition solves for a unique price $p^*(\theta)$. The second-order condition is

$$-\left[2\frac{f_\theta(r^*)}{1 - \beta f_\theta(r^*)} + (p - c)\frac{f'_\theta(r^*)}{(1 - \beta f_\theta(r^*))^3}\right].$$

At p^* it holds that

$$\begin{aligned} &-\frac{1}{1 - \beta f_\theta(r^*)}\left[2f_\theta(r^*) + f'_\theta(r^*)\frac{1 - F_\theta(r^*)}{f_\theta(r^*)(1 - \beta f_\theta(r^*))}\right] \\ \Leftrightarrow &-\frac{1}{1 - \beta f_\theta(r^*)}\left[\frac{2(1 - \beta f_\theta(r^*))[f_\theta(r^*)]^2 + f'_\theta(r^*)(1 - F_\theta(r^*))}{f_\theta(r^*)}\right] < 0 \end{aligned}$$

Note that the term in rectangular brackets is positive if $2(1 - \beta f_\theta(r^*)) \geq 1$ which holds if $1/2 \geq \beta f_\theta(r^*)$. The denominator is always positive, however, the numerator must also be positive due to the assumption that the hazard rate is increasing in r given $2(1 - \beta f_\theta(r^*)) \geq 1$. To see this, take the first derivative of the hazard rate with respect to r

$$\lambda'(r) = \frac{[f_\theta(r^*)]^2 + f'_\theta(r^*)(1 - F_\theta(r^*))}{[1 - F_\theta(r^*)]^2} > 0, \text{ by Assumption 1.}$$

Lastly, to show that the equilibrium price can be increasing/decreasing or constant in state θ, note first that the hazard rate $\lambda(r,\theta)$ is strictly decreasing in θ by the MLRP property. It follows that

$$\frac{\partial\lambda(r,\theta)}{\partial\theta} = \frac{f^{1,\theta}(r)(1-F_\theta(r)) + f_\theta(r)F^{1,\theta}(r)}{(1-F_\theta)^2} < 0,$$

which allows to bound $f^{1,\theta}(r) < \frac{f_\theta(r)}{1-F_\theta(r)}(-F^{1,\theta}(r))$.

Taking the derivative of the first-order condition with respect to θ yields

$$\begin{aligned}\frac{\partial p}{\partial\theta} &= \frac{-\beta f^{1,\theta}(r)f_\theta(r)(1-F_\theta(r)) - (f^{1,\theta}(r)(1-F_\theta(r)) + f_\theta(r)F^{1,\theta}(r))(1-\beta f_\theta(r))}{f_\theta(r)^2}\\ &= \frac{-f^{1,\theta}(r)(1-F_\theta(r)) - f_\theta(r)F^{1,\theta}(r)(1-\beta f_\theta(r))}{f_\theta(r)^2}\\ &> 0\end{aligned}$$

if $f^{1,\theta}(r^{FI}) < 0$.

Proof of Lemma 3 To see that the single-crossing condition is fulfilled, denote profits by

$$\pi(\theta,\hat{\theta},p) = (p-c)(1-F_\theta(r_c)),$$

where

$$r_c = p - \beta(1-F_{\hat{\theta}}(r_c)).$$

Recall from Lemma 2 that the partial derivative of the cut-off with respect to p is

$$\frac{\partial r_c}{\partial p} = \frac{1}{1-\beta f_{\hat{\theta}}(r_c)} > 0.$$

The partial derivative of the cut-off with respect to $\hat{\theta}$ can be derived by totally differentiating the cut-off above:

$$\frac{\partial r_c}{\partial\hat{\theta}} = \frac{\beta F^{1,\hat{\theta}}}{1-\beta f_{\hat{\theta}}(r_c)} < 0.$$

The (strict) single-crossing property is satisfied if $\frac{\partial\pi(\theta,\hat{\theta},p)}{\partial p}/\frac{\partial\pi(\theta,\hat{\theta},p)}{\partial\hat{\theta}}$ is a strictly increasing function of θ. Taking the respective derivatives and rearranging, yields

$$\frac{\partial\pi(\theta,\hat{\theta},p)/\partial p}{\partial\pi(\theta,\hat{\theta},p)/\partial\hat{\theta}} = -\frac{(p-c)\left[\frac{-f_\theta(r)}{1-\beta f_{\hat{\theta}}(r_c)}\right] + 1 - F_\theta(r)}{(p-c)f_\theta(r)\left[\frac{-\beta F^{1,\hat{\theta}}(r)}{1-\beta f_{\hat{\theta}}(r_c)}\right]} \tag{18}$$

$$= \frac{1}{(-\beta F^{1,\hat{\theta}}(r))} + \frac{1-\beta f_{\hat{\theta}}(r_c)}{(p-c)(-\beta F^{1,\hat{\theta}}(r))}\frac{1-F_\theta(r)}{f_\theta(r)}, \tag{19}$$

where only the last term depends on θ. Recall that $1-F_\theta(r)/f_\theta(r)$ corresponds to the inverse hazard rate. Since the hazard rate is strictly decreasing in θ, the inverse hazard rate is strictly increasing in θ. Thus, the whole expression is strictly increasing in θ if $p > c$. □

Lemma 7. *Choose any price p', users' response r', and type θ'. For any type $\theta < \theta'$, there exists $h \in \mathbb{R}_+$ and λ, such that $0 < \lambda < \varepsilon$ implies*

$$\pi(\theta', p_\lambda, r_\lambda) > \pi(\theta', p', r') \tag{A1}$$

$$\pi(\tilde{\theta}, p_\lambda, r_\lambda) < \pi(\tilde{\theta}, p', r'), \ \forall \tilde{\theta} \leq \theta, \tag{A2}$$

where $(p_\lambda, r_\lambda) = (p' + h\lambda, r' - \lambda)$.

Given the single-crossing property from Lemma 3, I show that there exists a small increase in price and increase in users' response (demand), i.e. a decrease in the cutoff r, that gives higher types scope to separate from lower types. Formally, the upcoming lemma provides a price-response pair for which a higher type would like to deviate whereas lower types do not. Note that both on-path and off-path user responses, i.e. the change in demand, are determined by the cutoff in Equation 1. Due to continuity, the cutoffs are $r \in [r(p, \delta_{\underline{\theta}}), r(p, \delta_{\overline{\theta}})]$ for given price p.

Proof of Lemma 7 The proof of this lemma follows Ramey (1996), but is simplified as there is only one signal p for which the one-dimensional single-crossing condition holds.

Take $\theta < \theta'$ and let $x \in \mathbb{R}$ be such that $x \geq MRS(\theta, p', r')$. Note that $x \neq MRS(\theta', p', r')$ and $\{MRS(\theta', p', r')\}$, $\{x\}$ are closed, convex sets as they are a singleton. Hence, it is possible to apply Minkowski's hyperplane separation theorem, which implies the existence of $h \in \mathbb{R}$, $h \neq 0$, such that

$$h \cdot MRS(\theta', p', r') < 1 < h \cdot x, \tag{20}$$

for some $h > 0$. Suppose $(p_\lambda, r_\lambda) = (p' + h\lambda, r' - \lambda)$ for $\lambda > 0$, i.e., a small increase in price and a small increase in demand (a small decrease in the cutoff). To determine whether $\pi(\tilde{\theta}, p_\lambda, r_\lambda) - \pi(\tilde{\theta}, p', r') < 0$, define

$$\zeta(\lambda, \tilde{\theta}) = \frac{-\pi(\tilde{\theta}, p_\lambda, r_\lambda) + \pi(\tilde{\theta}, p', r - \lambda)}{\pi(\tilde{\theta}, p', r - \lambda) - \pi(\tilde{\theta}, p', r')},$$

and then

$$\pi(\tilde{\theta}, p_\lambda, r_\lambda) - \pi(\tilde{\theta}, p', r') = \left[\pi(\tilde{\theta}, p', r - \lambda) - \pi(\tilde{\theta}, p', r')\right]\left[1 - \zeta(\lambda, \tilde{\theta})\right].$$

To determine the sign of $\zeta(\lambda, \tilde{\theta})$, observe that

$$\lim_{\lambda \to 0} \zeta(\lambda, \tilde{\theta}) = \lim_{\lambda \to 0} \frac{-\pi(\tilde{\theta}, p_\lambda, r_\lambda) + \pi(\tilde{\theta}, p', r - \lambda)}{\pi(\tilde{\theta}, p', r - \lambda) - \pi(\tilde{\theta}, p', r')} = -\frac{h\pi_p(\tilde{\theta}, p', r')}{\pi_r(\tilde{\theta}, p', r')} = h \cdot MRS(\tilde{\theta}, p', r'),$$

by l'Hospital rule and note that ${\partial \pi(\cdot)}/{\partial r} < 0$, so it is written as $-\pi_r(\cdot)$. Hence, it is possible to extend $\zeta(\cdot)$ continuously to $\lambda = 0$. Define

$$\zeta(\lambda, \tilde{\theta}) = \begin{cases} \frac{-\pi(\tilde{\theta}, p_\lambda, r_\lambda) + \pi(\tilde{\theta}, p', r - \lambda)}{\pi(\tilde{\theta}, p', r - \lambda) - \pi(\tilde{\theta}, p', r')} & \text{if } \lambda > 0 \\ -\frac{h\pi_p(\tilde{\theta}, p', r')}{\pi_r(\tilde{\theta}, p', r')} & \text{if } \lambda = 0 \end{cases}$$

In $\lambda \in \mathbb{R}_{>0}$ $\zeta(\lambda, \tilde{\theta})$ is differentiable as a composition of differentiable functions, however, the function is not differentiable in $\lambda = 0$ as $MRS(\tilde{\theta}, p', r') \neq 0$. For $\lambda > 0$ the function is strictly decreasing in λ

$$\frac{-h\pi_p(\tilde{\theta}, p', r')(\pi(\tilde{\theta}, p', r-\lambda) - \pi(\tilde{\theta}, p', r')) - (-\pi(\tilde{\theta}, p_\lambda, r_\lambda) + \pi(\tilde{\theta}, p', r-\lambda))(\pi_r(\tilde{\theta}, p', r'))}{(\pi(\tilde{\theta}, p', r-\lambda) - \pi(\tilde{\theta}, p', r'))^2} < 0$$

From Equation (20), it follows that $\zeta(\lambda = 0, \theta') < 1$ and $\zeta(\lambda, \theta') < 1$ as well, such that $\pi(\theta', p_\lambda, r_\lambda) - \pi(\theta', p', r') > 0$. Furthermore, $\zeta(\lambda = 0, \tilde{\theta}) > 1$ and hence $\zeta(\lambda, \tilde{\theta}) > 1$ for λ sufficiently small ($\lambda < \varepsilon$), such that $\pi(\tilde{\theta}, p_\lambda, r_\lambda) - \pi(\tilde{\theta}, p', r') < 0$, which needed to be shown. □

Proof of Proposition 1 By assumption $\tau(\theta)$ is a differentiable one-to-one strategy. Given that $\tau(\theta)$ is differentiable, it satisfies the differential equation in Equation 3 and hence, also the first-order condition implied by the incentive condition in Definition 1. Then, $\tau(\theta)$ satisfies the incentive condition if

$$\tau'(\theta)\pi_2(\theta, \hat{\theta}, p)\frac{d}{d\theta}\left\{\frac{\pi_3(\theta, \hat{\theta}, p)}{\pi_2(\theta, \hat{\theta}, p)}\right\} \geq 0, \tag{21}$$

which is proven in Theorem 6 of Mailath and von Thadden (2013). Note that the profit is monotonic in belief $\hat{\theta}$: $\pi_2(\theta, \hat{\theta}, p) = (p - c)n_2(\theta, \hat{\theta}, p) > 0$ for $p > c$. That is, the platform always has an incentive to manipulate beliefs in a way that users believe that the state is higher than it actually is. Taking the partial derivative, yields

$$\frac{\partial \pi(\theta, \hat{\theta}, p)}{\partial \hat{\theta}} = (p - c)f_\theta(r)\frac{-\beta F^{1,\hat{\theta}}(r)}{1 - \beta f_{\hat{\theta}}(r)} > 0.$$

Additionally, by Lemma 3 the last term is strictly positive if $p > c$. Thus, $\tau'(\theta)$ must be increasing for $p > c$ to fulfill Equation 21. Thus, for the platform to make positive profits, τ must be increasing in θ. From this, Corollary 1 follows.

Denote the equilibria inducing the equilibrium outcome in Proposition 1 by Γ_S, which only differ in their equilibrium messages. To show that the equilibrium outcome exists under the equilibrium refinement, suppose for contradiction that it fails to be one under the equilibrium refinement in Definition 4. Then there exists some price $p' \neq \{\tau(\theta)\}_{\theta\in\Theta}$ for which some type has a strict incentive to deviate when beliefs satisfy Definition 4. Define the cutoff r' under p' and set of types Θ' as follows

$$r' \equiv \{r|\pi(\theta, r, p') = \pi^{*,S}(\theta) \text{ for some } \theta\}$$
$$\Theta' \equiv \{\theta|\pi(\theta, r', p') = \pi^{*,S}(\theta)\},$$

where $\pi^{*,S}(\theta)$ are the equilibrium profits of type θ. Γ_S fails the equilibrium refinement if for any posterior satisfying $\mu(\theta|p') \subset \Theta'$ and any response $r(p')$, some type strictly prefers p' to the equilibrium action. That is, $r(p') < r'$ for some type to strictly prefer p' to the equilibrium action.

Then, fix $\theta' \in \Theta'$. By Lemma 7 for price p', cutoff r', and type θ', there exists a price-cutoff pair (p_λ, r_λ) such θ' can separate itself from lower types by choosing p_λ. Such separation ensures that no lower type would choose p_λ. By the definition of the equilibrium price, $\tau(\theta')$ is the least-cost signaling price, i.e. the smallest price for which type θ' can separate from lower types implying that $p_\lambda \geq \tau(\theta')$. Thus, type θ' has no incentive to deviate from the equilibrium price — a contradiction.

Proof of Proposition 2 I prove the following: In an equilibrium in which $p^*(\theta) = p'$ is set by more than one type, the highest type of the pool θ' can set price p_λ to break the equilibrium. For p_λ, there exists $r \in \mathcal{R}^\infty(\Theta^*(p_\lambda), p_\lambda)$ such that $\min_{r\in\mathcal{R}^\infty(\Theta^*(p_\lambda),p_\lambda)} \pi(\theta', r, p_\lambda) \leq \pi^*(\theta', \Sigma)$.

Consider an equilibrium candidate in which $p^*(\theta) = p'$ for more than one θ. Let $\theta' = sup\{\theta|p^*(\theta) = p'\}$ be the highest type in the pool and $r' = r(p')$ be the user response to observing price p' in equilibrium. Since $\{\theta|p^*(\theta) = p'\}$ is non-degenerate and $\mu_1(\theta|r, p')$ has full support on the closure of the set $\{\theta|p^*(\theta) = p'\}$, receivers place strictly positive probability on the set $cl\{\theta|p^*(\theta) = p'\} - \{\theta'\}$.

Given users use rationalizable strategies off-path, Lemma 1 provides a unique cutoff for given beliefs. Then, $n(\theta', p', r') < n(\theta', p', r^*)$, where the cutoff for the highest type in the pool is lower if users believe $\mu(\theta') = \delta_{\theta'}$ (r^*) than the cutoff r'

$$r' = p' - \beta \int_\Theta (1 - F_\theta(r_c))\mu_1(\theta|r', p)d\theta,$$
$$r^* = p' - \beta(1 - F_{\theta'}(r^*)),$$

such that $r^* < r'$ as the users place strictly positive probability on lower types other than θ'.

The rest of the proof follows Ramey (1996). Take a type θ sufficiently close to the highest type of the pool θ' that yields $r' > r(p', \delta_\theta)$. Given θ, there exist small moves upwards in price and receiver response (p_λ, r_λ) supplied by Lemma 7 that satisfies Equation A1 and Equation A2. By taking λ sufficiently small $r_\lambda > r(p_\lambda, \theta)$.

For p_λ, the user response is either such that Equation A1 and Equation A2 are satisfied, or if $\pi(\tilde{\theta}, p_\lambda, r) \geq \pi(\tilde{\theta}, p', r')$ for $\tilde{\theta}$ resulting in $r > r_\lambda$, then because of the single-crossing property $\pi(\theta', p_\lambda, r) > \pi(\theta', p', r')$ such that θ' has a stricter incentive to deviate. In both cases, $D_{\tilde{\theta}} \cup D^0_{\tilde{\theta}} \subset D_{\theta'}$ holds, i.e. for types $\tilde{\theta}$ there are less rationalizable strategy profiles for which it can improve. Then, D1 criterion requires the support of $\mu^*(\theta|p_\lambda)$ to be in $[\theta, \overline{\theta}]$, i.e. $\Theta^*(p_\lambda) = [\theta, \overline{\theta}]$ with $\theta' \in [\theta, \overline{\theta}]$. By Equation 1, it must be that $r(p_\lambda, \delta_\theta) > r(p_\lambda, \mu(\Theta^*(p_\lambda)))$, and Equation A1 implies that θ' has a profitable deviation breaking the equilibrium. □

Proof of Proposition 3 Suppose Condition 1 is satisfied. I will construct the two types of separating equilibria in Theorem 3 (i) and (ii).

(i) Price Signaling Suppose first that zero fake profiles are used in equilibrium, i.e. $\xi(\theta) = 0$. The incentive constraints for $\theta' > \theta$ read

$$(p(\theta) - c)n(\theta, \theta, p(\theta)) \geq (p(\theta') - c)n(\theta, \theta', p(\theta')) - \gamma\,[n(\theta', \theta', p(\theta')) - n(\theta, \theta', p(\theta'))]$$
$$(p(\theta') - c)n(\theta', \theta', p(\theta')) \geq (p(\theta) - c)n(\theta', \theta, p(\theta)).$$

Observe that the incentive constraints for upward and downward deviations are asymmetric. If type θ' deviates downward to mimic type θ, it does not need to create fake profiles to match the lower demand. Instead, it would have to create negative fake profiles to reduce demand—something that is not feasible. At the entry stage, users hold the belief $\mu_1(\cdot) = \delta_\theta$ after observing the price $p(\theta)$, which leads to an entering mass of $n(\theta', \theta, p(\theta))$. This mass is smaller than the number of users who would have entered under the true type θ', so the platform is constrained in demand after entry, even if the true type θ' is (partially) revealed afterward.

Now observe that the second incentive constraint must be binding, whereas the first incentive constraint is slack. If the first IC were binding, the second IC would not be satisfied, and higher types would prefer to set the prices of lower types. Given differentiability, the resulting differential equation is

$$p'(\theta) = -\frac{(p(\theta) - c)n_2(\theta, \theta, p(\theta))}{(p(\theta) - c)n_3(\theta, \theta, p(\theta)) + n(\theta, \theta, p(\theta))}$$

The separating strategy is the same as in Proposition 1 and is indeed separating as $p'(\theta) \neq 0$ and $n(\theta, \theta, p(\theta)) \neq n(\theta', \theta', p(\theta')), \forall \theta \neq \theta'$ which can easily be verified.

Given that the equilibrium pricing strategy satisfies the differential equation and thus local incentive compatibility, global incentive compatibility is ensured by the single-crossing condition in Equation 19. For deviations from a higher type to a lower type's price, the argument follows as in Proposition 1. Global incentive compatibility for deviations from a lower type to a higher type's price is also satisfied—and even more straightforwardly. To see this, consider the following. If a deviation from $p(\theta)$ to $p(\theta')$, as well as from $p(\theta')$ to $p(\theta'')$ for $\theta'' > \theta' > \theta$, is unprofitable (due to local incentive compatibility), then, by the single-crossing property, a deviation of θ from $p(\theta)$ to $p(\theta'')$ is also unprofitable. Additionally, when deviating from $p(\theta)$ to $p(\theta')$ to induce belief θ', type θ needs to create additional fake profiles. A deviation from $p(\theta)$ to $p(\theta'')$ requires even more fake profiles because

$$n(\theta'', \theta'', p(\theta'')) - n(\theta, \theta'', p(\theta'')) > n(\theta', \theta', p(\theta')) - n(\theta, \theta', p(\theta'))$$

making a global deviation even less profitable.

(ii) Price and Fake Profile Signaling Suppose $\theta \in \hat{\Theta}$, where $\hat{\Theta} \subseteq [\underline{\theta}, \overline{\theta}]$. Fake profiles are given by

$$\gamma\xi = (p(\theta) - c)n(\theta, \theta, p(\theta)) - (p(\underline{\theta}) - c)n(\underline{\theta}, \underline{\theta}, p(\underline{\theta})) - \int_{\underline{\theta}}^{\theta} (p(t) - c - \gamma)n_1(t, t, p(t))dt,$$

and prices maximize equilibrium profits

$$\max_{p \in \mathbb{R}_+} (p(\underline{\theta}) - c)n(\underline{\theta}, \underline{\theta}, p(\underline{\theta})) + \int_{\underline{\theta}}^{\theta} (p(t) - c - \gamma)n_1(t, t, p(t))dt,$$

which results in

$$p^{**,S} \equiv p(\theta) = -\frac{n_1(\theta, \theta, p)}{n_{13}(\theta, \theta, p)} + c + \gamma. \tag{22}$$

The equilibrium exists under the following conditions: Let $\hat{\Theta}$ be such that $p^{**,S}(\theta) \leq p^{max}(\theta)$ holds for all $\theta \in \hat{\Theta}$, where $p^{max}(\theta)$ is given by the differential equation

$$p'(\theta) = -\frac{(p(\theta) - c)n_2(\theta, \theta, p(\theta)) - \gamma n_1(\theta, \theta, p(\theta))}{(p(\theta) - c)n_3(\theta, \theta, p(\theta)) + n(\theta, \theta, p(\theta))}.$$

For $p^{**,S}$ to maximize profits a necessary and sufficient condition is $f^{1,\theta}(r(p^{**,S})) > 0$. Lastly, γ must be sufficiently small

$$2\gamma \leq \left(-\frac{F^{1,\theta}(r)}{f^{1,\theta}(r)}(1-\beta f_\theta(r))\right)\frac{-F^{1,\theta}(r)}{-F^{1,\theta}(r')} - \left(-\frac{F^{1,\theta'}(r')}{f^{1,\theta'}(r')}(1-\beta f_{\theta'}(r'))\right). \tag{23}$$

I will construct the equilibrium outcome in the next steps. The incentive constraints for $\theta' > \theta$ are

$$(p(\theta)-c)n(\theta,\theta,p(\theta)) - \gamma\xi(\theta) \geq (p(\theta')-c)n(\theta,\theta',p(\theta')) - \gamma\left[\xi(\theta') + (n(\theta',\theta',p(\theta')) - n(\theta,\theta',p(\theta')))\right],$$

and

$$(p(\theta')-c)n(\theta',\theta',p(\theta')) - \gamma\xi(\theta') \geq (p(\theta)-c)n(\theta',\theta,p(\theta)) - \gamma\left[\xi(\theta) - (n(\theta',\theta,p(\theta)) - n(\theta,\theta,p(\theta)))\right],$$

if $\gamma\left[\xi(\theta) - (n(\theta',\theta,p(\theta)) - n(\theta,\theta,p(\theta)))\right] > 0$, or

$$(p(\theta')-c)n(\theta',\theta',p(\theta')) - \gamma\xi(\theta') \geq (p(\theta)-c)n(\theta',\theta,p(\theta)),$$

if $\gamma\left[\xi(\theta) - (n(\theta',\theta,p(\theta)) - n(\theta,\theta,p(\theta)))\right] \leq 0$. I will show that it suffices to impose the incentive constraint only for nearby types. For close θ to θ', the incentive constraints are

$$p'(\theta)[(p(\theta)-c)n_3(\theta,\theta,p(\theta)) + n(\theta,\theta,p(\theta))] + (p(\theta)-c)n_2(\theta,\theta,p(\theta)) - \gamma\xi'(\theta) \leq \gamma n_1(\theta,\theta,p(\theta)) \tag{24}$$

$$p'(\theta)[(p(\theta)-c)n_3(\theta,\theta,p(\theta)) + n(\theta,\theta,p(\theta))] + (p(\theta)-c)n_2(\theta,\theta,p(\theta)) - \gamma\xi'(\theta) \geq \gamma n_1(\theta,\theta,p(\theta)). \tag{25}$$

Setting the first IC to bind, implies that the second IC binds and vice versa. This results in a differential equation of the fake profile strategy as a function of prices:

$$\gamma\xi'(\theta) = p'(\theta)[(p(\theta)-c)n_3(\theta,\theta,p(\theta)) + n(\theta,\theta,p(\theta))] + (p(\theta)-c)n_2(\theta,\theta,p(\theta)) - \gamma n_1(\theta,\theta,p(\theta)).$$

Sequential rationality implies that once the lowest type, $\underline{\theta}$, is identified as such, it cannot do better than setting zero fake profiles. This implies the following initial value condition for the differential equation: $\xi(\underline{\theta}) = 0$. For a given price $p(\theta)$, the differential equation can be solved via the Fourier method, yielding a unique solution (up to a constant):

$$\gamma\xi(\theta) = (p(\theta)-c)n(\theta,\theta,p(\theta)) - (p(\underline{\theta})-c)n(\underline{\theta},\underline{\theta},p(\underline{\theta})) - \int_{\underline{\theta}}^{\theta}(p(t)-c-\gamma)n_1(t,t,p(t))dt.$$

Then, the equilibrium profit as a function of the price is given by

$$\begin{aligned}\Pi(\theta) &= (p(\theta)-c)n(\theta,\theta,p(\theta)) - \gamma\xi(\theta)\\ &= (p(\underline{\theta})-c)n(\underline{\theta},\underline{\theta},p(\underline{\theta})) + \int_{\underline{\theta}}^{\theta}(p(t)-c-\gamma)n_1(t,t,p(t))dt.\end{aligned}$$

and is maximized if prices are set to

$$\arg\max_{p(\theta)}(p(\underline{\theta})-c)n(\underline{\theta},\underline{\theta},p(\underline{\theta})) + \int_{\underline{\theta}}^{\theta}(p(t)-c-\gamma)n_1(t,t,p(t))dt\forall\theta\in\Theta.$$

Solving the maximization problem yields

$$p^{**,S} \equiv \begin{cases} p(\underline{\theta}) - c = \frac{1-F_{\underline{\theta}}(r)}{f_{\underline{\theta}}(r)}(1-\beta f_{\underline{\theta}})(r) & \theta = \underline{\theta} \\ p(\theta) - c - \gamma = -\frac{F^{1,\theta}(r)}{f^{1,\theta}(r)}(1-\beta f_{\theta}(r)) & \theta \in (\underline{\theta}, \overline{\theta}]. \end{cases}$$

The necessary and sufficient condition for $p^{**,S}$ to maximize the platform's profits are given in the following. For $\theta = \underline{\theta}$, the profit function is concave under the assumptions (see Lemma 2), hence the only profit-maximizing price is the first-best price. For $\theta \in (\underline{\theta}, \overline{\theta}]$, the profit function with respect to p is not necessarily single-peaked. The necessary and sufficient conditions are given by the first-order condition

$$(p - c - \gamma)n_{13}(\theta, \theta, p) + n_1(\theta, \theta, p) = 0,$$

and the second-order condition

$$(p - c - \gamma)n_{133}(\theta, \theta, p) + 2n_{13}(\theta, \theta, p).$$

For any price $p \in \mathbb{R}_+$, there exists a unique cutoff r by Lemma 1. If n_{13} changes sign (n_1 is strictly positive), there can be two price-response pairs solving the first-order condition. To do so, reformulate both conditions in terms of the underlying distribution

$$(p - c - \gamma)\frac{-f^{1,\theta}(r)}{1 - \beta f_\theta(r)} - F^{1,\theta}(r) = 0.$$

By Assumption 1, the density is single-peaked and hence $f^{1,\theta}(r)$ changes sign only once. If the density is right-skewed, $f^{1,\theta}(r)$ is strictly increasing in r. By the MLRP assumption, which implies that the hazard rate is monotonically decreasing in θ, it is possible to bound $f^{1,\theta}(r)$ by $f^{1,\theta}(r) < \frac{f_\theta(r)}{1-F_\theta(r)}(-F^{1,\theta}(r))$. Then, the first-order condition can exhibit (a) two solutions, or (b) only one solution.

(a) If the first-order solves for two solutions, one solution exists with $p - c - \gamma < 0$ if $f^{1,\theta} < 0$ and one solution exists with $p - c - \gamma > 0$ if $f^{1,\theta} > 0$. The second-order condition is

$$SOC := -\frac{F^{1,\theta}(r)}{f^{1,\theta}(r)} \frac{\partial f_\theta(r)/\partial r \partial\theta + f^{1,\theta}(r) f'(r)\beta / 1-\beta f_\theta(r)}{1 - \beta f_\theta(r)} + 2\frac{-f^{1,\theta}(r)}{1 - \beta f_\theta(r)}.$$

If $f^{1,\theta} > 0$, then $SOC < 0$, and if $f^{1,\theta} < 0$, then $SOC > 0$. This implies that the profit function is well-behaved, so that $p - c - \gamma > 0$ is both the local and global maximum. To see that the interior maximum is also the global maximum, note that profits are zero when $p = 0$ and negative as $p \to \infty$.

(b) As $f^{1,\theta}(r)$ is bounded from above, only one solution at $p' - c - \gamma < 0$ with $f^{1,\theta} < 0$ can exist. In that case, the first-order condition is always larger than zero for prices higher that p'. The solution p' is a local minimum and the profit-maximizing prices are a corner solution. As the profit-maximizing solution must still adhere to incentive-compatibility, prices are chosen as high as possible given incentive-compatibility. The profit-maximizing solution is given by (i).

If the density is left-skewed, $f^{1,\theta}(r)$ is strictly decreasing in θ, i.e. $f^{2,\theta}(r) < 0$. The first-order condition solves for one solution with $p - c - \gamma > 0$ and $f^{1,\theta} > 0$. The price is again profit-maximizing as the second-order condition is negative at the solution.

Lastly, to check global incentive compatibility note that prices fulfill the single-crossing condition by Lemma 3. The single-crossing condition for fake profiles is given by

$$\frac{\frac{\partial \pi(\theta,\hat{\theta},p,\xi)}{\partial \xi}}{\frac{\partial \pi(\theta,\hat{\theta},p,\xi)}{\partial \hat{\theta}}} = \frac{-\gamma}{(p-c)f_\theta(r)\frac{(-\beta F^{1,\hat{\theta}}(r))}{1-\beta f_{\hat{\theta}}(r)}},$$

which is strictly increasing in θ if $f^{1,\theta}(r) > 0$. Hence, as long as $p(\theta)-c-\gamma = -\frac{F^{1,\theta}(r)}{f^{1,\theta}(r)}(1-\beta f_\theta(r))$ is solved for a price-cutoff pair resulting in a positive mark-up ($p-c-\gamma > 0$) and a cutoff r such that $f^{1,\theta} > 0$, both signals meet the strict single-crossing condition.

Assume from now on $p - c - \gamma > 0$ and $f^{1,\theta} > 0$. As the fake profile strategy solves the differential equation, it thus, satisfies local incentive compatibility. To fulfill global incentive compatibility, for $\theta > \theta'$ (p, ξ) must satisfy

$$(p(\underline{\theta}) - c)n(\underline{\theta},\underline{\theta},p(\underline{\theta})) + \int_{\underline{\theta}}^{\theta} (p(t) - c - \gamma)n_1(t,t,p(t))dt \geq (p(\theta') - c)n(\theta,\theta',p(\theta')) \tag{26}$$

$$- [(p(\theta') - c)n(\theta',\theta',p(\theta')) - (p(\underline{\theta}) - c)n(\underline{\theta},\underline{\theta},p(\underline{\theta})) - \int_{\underline{\theta}}^{\theta'} (p(t) - c - \gamma)n_1(t,t,p(t))dt] \tag{27}$$

$$+ \gamma(n(\theta,\theta',p(\theta') - n(\theta',\theta',p(\theta')). \tag{28}$$

and for $\theta < \theta'$

$$(p(\underline{\theta}) - c)n(\underline{\theta},\underline{\theta},p(\underline{\theta})) + \int_{\underline{\theta}}^{\theta} (p(t) - c - \gamma)n_1(t,t,p(t))dt \geq (p(\theta') - c)n(\theta,\theta',p(\theta')) \tag{29}$$

$$- [(p(\theta') - c)n(\theta',\theta',p(\theta')) - (p(\underline{\theta}) - c)n(\underline{\theta},\underline{\theta},p(\underline{\theta})) - \int_{\underline{\theta}}^{\theta'} (p(t) - c - \gamma)n_1(t,t,p(t))dt] \tag{30}$$

$$- \gamma(n(\theta,\theta',p(\theta') - n(\theta',\theta',p(\theta')). \tag{31}$$

Observe that Equation 28 is harder to satisfy than 31 as the higher type must create less fake profiles to mimic the lower type. Hence, rewriting Equation 28 yields

$$\int_{\theta'}^{\theta} (p(t) - c - \gamma)n_1(t,t,p(t))dt \geq \int_{\theta'}^{\theta} (p(\theta') - c + \gamma)n_1(t,\theta',p(\theta'))dt.$$

A sufficient condition is

$$(\theta - \theta')\left[(p(\theta) - c - \gamma)n_1(\theta,\theta,p(\theta)) - (p(\theta') - c + \gamma)n_1(\theta,\theta',p(\theta'))\right] \geq 0,$$

such that the second term must be larger or equal to zero. Substituting the profit-maximizing prices $p^{**,S}$ into the condition and solving for γ yields

$$2\gamma \leq \left(-\frac{F^{1,\theta}(r)}{f^{1,\theta}(r)}(1-\beta f_\theta(r))\right)\frac{-F^{1,\theta}(r)}{-F^{1,\theta}(r')} - \left(-\frac{F^{1,\theta'}(r')}{f^{1,\theta'}(r')}(1-\beta f_{\theta'}(r'))\right). \tag{32}$$

Therefore, to fulfill global incentive compatibility, γ must be sufficiently small given by Equation 23.

Lastly, to guarantee that the non-negativity constraint on the fake profile strategy is not violated, the following condition must hold: For all $\theta \in \hat{\Theta}$,

$$p^{FP} \leq p^{max}(\theta), \tag{33}$$

where $p^{max}(\theta)$ is given by the differential equation $p'(\theta) = -\frac{(p(\theta)-c)n_2(\theta,\theta,p(\theta))-\gamma n_1(\theta,\theta,p(\theta))}{(p(\theta)-c)n_3(\theta,\theta,p(\theta))+n(\theta,\theta,p(\theta))}$. Note that $\gamma\xi'(\theta, p^{max}(\theta)) = 0$ and $\gamma\xi'(\theta, p^{FI}(\theta)) > 0$. Hence, the equilibrium with positive fake profiles exists for $\theta \in \hat{\Theta}$ for which the Inequality in Equation 33 holds.

Suppose Condition 1 is violated, then fake profiles are sufficiently costly for all types. As the cost of fake profiles are common knowledge, users anticipate that fake profiles cannot be used in equilibrium. Then, the equilibrium is the same as characterized in Proposition 6 (see proof of Proposition 6).

Proof of Proposition 4 The proof follows directly from the proof of Proposition 1 by considering the incentives for some type in the pool, $\theta' \in \Theta'$, to deviate from the pooled price p' in the first period. By Lemma 7, there continues to exist a price-cutoff pair (p_λ, r_λ) for type θ' that makes it possible to separate from lower types. This lemma is unaffected by the possibility of fake profiles as the strict single-crossing property holds for the price as a signal alone.[18] By the equilibrium refinement, users put strictly positive probability $\mu_1(\theta|p_\lambda)$ on a set Θ^* in support of strictly higher types than Θ'. Thus, in turn the cutoff decreases and the mass of entering users increases. As users cannot observe the number of fake profiles and do not know the true demand on the platform, the equilibrium refinement has no bite at this point and users do not revise their belief. Thus, $\mu_2(\theta|p_\lambda, n(\theta', \mu_1(\theta|p_\lambda), p_\lambda) + \xi)$ puts the same probability on set Θ^* and no user exits. Lastly, this implies that the profit of type θ' increases by the deviation, thereby breaking the pooling equilibrium.

Proof Lemma 4 Given the timing of the game, users "update" their beliefs twice. In the first period, users observe the message and hold beliefs $\mu_1(\theta|p, m, r) = m$. In the second period, users observe the perceived network size $\tilde{n} = n(\theta, \mu_1, p) + \xi$ and hold beliefs $\mu_2(\theta|p, m, r, \tilde{n})$. Note that $\pi_2(\cdot)$ is strictly increasing if the believed state increases.

Fix the price p and message m. After users' entry decisions, the true network size of users in equilibrium is bounded from above. Denote by $n(\theta, m, p)$ the number of users who enter given $\mu_1(\cdot) = m$, and by $n(\theta, \mu_2, p)$ the number of users on the platform given $\mu_2(\cdot)$. The number of users who remain on the platform are $\min\{n(\theta, m, p), n(\theta, \mu_2, p)\}$.

Suppose that $m \geq \theta$, where $m < \theta$ is irrelevant as profits increase in m; see above. Then to induce $\mu_2 \geq m$, the platform must create fake profiles such that:

$$\xi \geq n(m, m, p) - n(\theta, m, p). \tag{34}$$

Suppose the platform sets ξ such that $\mu_2(\cdot) > m$, i.e., the above inequality is strict. Then, the demand in equilibrium on the platform is $n(\theta, m, p) = min\{n(\theta, m, p), n(\theta, \mu_2, p)\}$. As demand is unaffected by the increase in fake profiles and profits decrease due to the cost of creating them, the platform does not set ξ such that $\mu_2(\cdot) > m$.

[18] Ramey (1996) notes that "With multiple signals, such separating movements remain possible as long as the MRS of any one signal is strictly decreasing in type at every point of the space of signals and responses" (p.511).

Suppose the platform sets ξ such that $\mu_2(\cdot) < m$, i.e, the inequality in Equation 34 is reversed. Then, demand in equilibrium on the platform is $n(\theta, \mu_2, p)$ and thus decreases. Since the platform has already chosen message m in the first period and found it optimal to do so, it will not set ξ such that $\mu_2(\cdot) < m$. This implies that the platform induces ξ such that $\mu_2(\cdot) = m$, i.e, the inequality in Equation 34 binds. $\square$

Proof of Lemma 5 First, I will show that the indifferent type is given as the unique solution to Equation 9. To get Equation 9, substitute the first-order condition in Equation 7 into the first-order condition in Equation 8. The indifferent type is given by the following equation evaluated at $m = \overline{\theta}$:

$$\left(-\frac{n(\theta,m,p)}{n_3(\theta,m,p)} + \gamma\frac{n_3(m,m,p)}{n_3(\theta,m,p)}\right) n_2(\theta,m,p) - \gamma\left(n_1(m,m,p) + n_2(m,m,p)\right) = 0,$$

which is equal to Equation 9 when evaluated at $m = \overline{\theta}$. Note that $r(m,p) \equiv \overline{r}$ is given by

$$\overline{r} = p - \beta(1 - F_m(\overline{r}))|_{m=\overline{\theta}},$$

and the respective derivatives are

$$\begin{aligned}
\frac{\partial \overline{r}}{\partial p} &= \left.\frac{1}{1-\beta f_m(\overline{r})}\right|_{m=\overline{\theta}}, \\
\frac{\partial \overline{r}}{\partial m} &= \left.\frac{(-\beta F^{1,m}(\overline{r}))}{1-\beta f_m(\overline{r})}\right|_{m=\overline{\theta}}.
\end{aligned}$$

Substituting the respective derivatives into Equation 9 yields

$$\begin{aligned}
&\left(\frac{1-F_\theta(\overline{r})}{f_\theta(\overline{r})}\left(1-\beta f_{\overline{\theta}}(\overline{r})\right) + \gamma\frac{f_{\overline{\theta}}(\overline{r})}{f_\theta(\overline{r})}\frac{1-\beta f_{\overline{\theta}}(\overline{r})}{1-\beta f_{\overline{\theta}}(\overline{r})}\right)\cdot\frac{f_\theta(\overline{r})\cdot(-\beta F^{1,\overline{\theta}}(\overline{r}))}{1-\beta f_{\overline{\theta}}(\overline{r})} \\
&= \gamma\left(-F^{1,\overline{\theta}}(\overline{r}) + f_{\overline{\theta}}(\overline{r})\cdot\frac{-\beta F^{1,\overline{\theta}}(\overline{r})}{1-\beta f_{\overline{\theta}}(\overline{r})}\right).
\end{aligned}$$

Simplifying and setting $\theta = \tilde{\theta}$ yields Equation 10

$$\beta(1 - F_\theta(\overline{r})) = \gamma, \tag{35}$$

for $\overline{r} = p - \beta(1 - F_{\overline{\theta}}(\overline{r}))$, where p is given by Equation 7. Then the indifferent type $\tilde{\theta} \in \Theta$ is the solution to Equation 10. Since the left-hand side is a constant and the right-hand side is strictly increasing in θ, the equation has a unique solution. Define

$$\begin{aligned}
\overline{\gamma} &\equiv \beta(1 - F_{\overline{\theta}}(\overline{r})), \\
\underline{\gamma} &\equiv \beta(1 - F_{\underline{\theta}}(\overline{r})).
\end{aligned}$$

The solution to Equation 10 is unique and solves for a $\tilde{\theta} \in \Theta$ if and only if $\underline{\gamma} \leq \gamma \leq \overline{\gamma}$. $\square$

Proof of Proposition 5 (i) Let $\underline{\gamma} \leq \gamma \leq \overline{\gamma}$. For $\theta < \tilde{\theta}$ the equilibrium strategy is given by Equation (8), which is separating. Note that the relevant separating strategy is defined with respect to m as users are unaware of $\xi(\theta)$ and ignore p as a signal. That is, $m \neq m'$ for $\theta \neq \theta'$. Fixing p, incentive compatibility is fulfilled as $\{p, m\} = \arg\max \pi(\theta, m, p)$, i.e., m is chosen to fulfill the first-order condition in Equation 8:

$$\beta[1 - F_\theta(r(m, p))] = \gamma.$$

Recall that users also believe in the message and the state conveyed by the network size off the equilibrium path. As the equilibrium outcome uniquely maximizes the platform's profit in each state, there exists no incentive to deviate from the equilibrium.

For $\theta > \tilde{\theta}$, types choose $m = \overline{\theta}$, resulting in pooling on the highest available message. These types have no incentive to deviate off-path. First, note that types cannot deviate upward in the message space: they would prefer to send a message above $\overline{\theta}$, but the type space is bounded above by $\overline{\theta}$, making such deviations infeasible. Second, consider off-path deviations in fake profiles for given $m = \overline{\theta}$. Since users also believe in m off-path, demand cannot exceed $n(\overline{\theta}, \overline{\theta}, p)$ after entry. Therefore, deviating in the fake profiles does not attract additional users and is only costly. Lastly, consider deviations in the price. Given that demand is $n(\overline{\theta}, \overline{\theta}, \underline{p})$, and that price p is optimally chosen by the platform conditional on the message $m = \overline{\theta}$, there is no profitable deviation in price either. Any deviation would reduce profits.

(ii) Let $\gamma < \underline{\theta}$, then for the lowest type it holds that

$$\beta(1 - F_{\underline{\theta}}(\overline{r})) > \gamma, \tag{36}$$

which implies that the lowest type sets $m(\underline{\theta}) = \overline{\theta}$. Since the left-hand side of the above equation is increasing in type, all types larger than the lowest type set $m(\underline{\theta}) = \overline{\theta}$ as well. There exists no incentive to deviate as prices are chosen optimally given message m.

(iii) Let $\gamma > \overline{\gamma}$, then for the highest type it holds that

$$\beta(1 - F_{\overline{\theta}}(\overline{r})) < \gamma, \tag{37}$$

which implies that the highest type sets $m(\overline{\theta}) = \overline{\theta}$. Since the left-hand side is increasing in type, all types smaller than the highest type set $m(\theta) = \theta$ as well. There exists no incentive to deviate as prices are equal to the full information prices. $\square$

Proof Proposition 6 The platform maximizes its profit

$$\max_{p,\xi} (p - c) n(\theta, \theta, p) - \gamma\xi,$$

subject to incentive compatibility

$$\pi(\theta, \theta, p(\theta)) \geq \pi(\theta, \theta', p(\theta')).$$

Additionally, the platform's equilibrium demand is bounded above by the demand of users that enter: $n(\theta, \mu_1, p) \geq n(\theta, \mu_2, p)$.

First, suppose that the government bans fake profiles, i.e., $\xi = 0$. I will characterize the equilibrium outcome and show that if (i) the platform needs to label fake profiles, or

(ii) must mandatorily disclose fake profiles, or (iii) Condition 1 is violated, this leads to the same equilibrium outcome.

Let $\xi = 0$. By Lemma 2, the unique solution to the first-order condition in Equation 11 is $p^{FI}(\theta)$. Suppose that the platform randomizes over messages in equilibrium.

Now, fix the equilibrium strategy $p^{FI}(\theta)$; this is separating as $\frac{\partial p^{FI}(\theta)}{\partial \theta} > 0$ if $f^{1,\theta}(r^{FI}) < 0$, and it is differentiable. To prove that this construction is a separating equilibrium, it must be shown that incentive compatibility is satisfied. As fake profiles are banned, the information structure is $\mathbb{I} = [0,1]$; that is, users observe the true network size. This implies that given separating beliefs $\mu_1(p^{FI}(\theta)) = \theta$ in the first period, after observing network size n' in the second period, users hold beliefs $\mu_2(\cdot)$ according to

$$\mu_2(\cdot) = \left\{\theta' \in \Theta | n(\theta', \theta, p^{FI}(\theta)) = n'\right\}.$$

The equation solves for a unique θ', as sophisticated users can predict the cutoff r_c from $(\mu_1 = \theta, p^{FI}(\theta)$. Due to the first-order stochastic dominance of $F_{\theta'}(r)$ with respect to the true θ', for a given $r = r_c$, there exists only one θ' that solves the equation.

The incentive compatibility constraints are thus

$$\pi(\theta, \theta, p^{FI}(\theta)) \geq \pi(\theta, \theta, p^{FI}(\theta')). \tag{38}$$

By Lemma 2, $p^{FI}(\theta)$ uniquely maximizes the profit of θ and thus, the inequality is always satisfied. Similarly, for any out-of-equilibrium beliefs satisfying the equilibrium refinement, there exists no profitable deviation for any type. Similarly to the logic above, for a given deviation p' and out-of-equilibrium beliefs μ', users can predict r' such that they perfectly know the true state θ in the second period. Therefore, deviating from the full information price is never profitable.

Now, consider cases (i) to (iii). In case (i), the information structure is $\mathbb{I} = \{[0,1], \mathbb{R}_+\}$, i.e. users can perfectly identify fake profiles. Setting $p^{FI}(\theta)$ as separating strategy implies that for any given amount of fake profiles, the incentive constraints are as in Equation 38. Since creating fake profiles is costly and the platform receives no benefit from doing so, it chooses $\xi = 0$.

In case (ii), the platform must mandatorily disclose its use of fake profiles. The equilibrium outcome with fake profiles is given by Theorem 3. The resulting profits, for given price and fake profile strategy $\rho(\tau) = (p, \xi)$, are

$$(p - c)n(\theta, \theta, p) - \gamma\xi,$$

for $p \neq p^{FI}(\theta)$ for $\theta \in \Theta \setminus \{\underline{\theta}\}$ and $\xi \geq 0$. This implies that profits are always smaller than under full information. Hence, disclosing $\xi = 0$ and setting full information prices dominates disclosing $\xi > 0$.

In case (iii), Condition 1 is violated. Then, there exists a separating equilibrium in which the platform sets the full information prices and zero fake profiles. Recall that the information structure is $\mathbb{I} = \{[0,1] + \mathbb{R}_+\}$. The separating equilibrium with full information prices and zero fake profiles is incentive compatible if

$$(p^{FI}(\theta) - c)n(\theta, \theta, p^{FI}(\theta)) \geq (p^{FI}(\theta') - c)n(\theta, \theta', p^{FI}(\theta')) - \gamma\xi', \tag{39}$$

where $\xi' = n(\theta, \theta', p^{FI}(\theta')) - n(\theta, \theta, p^{FI}(\theta))$. Since Condition 1 is violated, γ is such that type θ is not willing to marginally or discretely increase ξ, thereby satisfying Equation 39. If $\xi = 0$, the incentive constraint reduces to Equation 38, which is again satisfied. $\square$

C Appendix: Regularity Conditions

Denote the platform's profit function by $\pi(\theta, \hat{\theta}, p)$, where the first position denotes the true state, the second position the user's belief $\mu = \hat{\theta}$ and the last position the price. The platform's profit is twice differentiable: $\pi(\theta, \hat{\theta}, p)$ is C^2 on $\Theta^2 \times \mathbb{R}$. The platform's profit fulfills the following regularity conditions.

Condition 2. The first-best problem (under full information)

$$\max_p \pi(\theta, \theta, p)$$

has a unique solution, $p^{FI}(\theta)$, for which $\pi_{33}(\theta, \theta, p^{FI}(\theta)) < 0$, for all $\theta \in \Theta$.

By Lemma 2 for each $\theta \in \Theta$, there exists a unique profit-maximizing price $p^{FI}(\theta)$ for which $\pi_{33}(\theta, \theta, p^{FI}(\theta)) < 0$. Therefore, Condition 2 is fulfilled.

Condition 3. There exists an $\varepsilon > 0$ such that for all $(\theta, p) \in \Theta \times \mathbb{R}$, $\pi_{33}(\theta, \theta, p) \geq 0 \Rightarrow |\pi_3(\theta, \theta, p)| > \varepsilon$.

The condition requires that the first derivative of $\pi(\theta, \theta, p)$ with respect to p is bounded from below, whenever the second-order condition is non-negative. To verify, recall from the proof of Lemma 2 that $\pi_{33} \geq 0$ if

$$-(p-c) \geq 2\frac{f_\theta(r)}{f'_\theta(r)}(1-\beta f_\theta(r))^2.$$

Then, for $\varepsilon = \inf_r |2f_\theta(r)^2(1-\beta f_\theta(r)) + (1-F_\theta(r))f_\theta(r)'|$, $|\pi_3(\theta, \theta, p)| > \varepsilon$ since

$$\begin{aligned}
&|-(p-c)\frac{f_\theta(r)}{1-\beta f_\theta(r)} + 1 - F_\theta(r)| \geq |2\frac{f_\theta(r)^2}{f_\theta(r)'}(1-\beta f_\theta(r)) + 1 - F_\theta(r)| \\
&= |2f_\theta(r)^2(1-\beta f_\theta(r)) + (1-F_\theta(r))f_\theta(r)'| \\
&> \varepsilon.
\end{aligned}$$

By Assumption 1 and 3, $\varepsilon > 0$, such that Condition 3 is fulfilled.

Additionally, the profit is monotonic in belief $\hat{\theta}$. That is, the platform always has an incentive to manipulate beliefs in a way that users believe that the state is higher than it actually is.

Condition 4. (Belief Monotonicity) $\pi_2(\theta, \hat{\theta}, p) = (p-c)n_2(\theta, \hat{\theta}, p) > 0$ for $p > c$.

Taking the partial derivative proves the statement:

$$\frac{\partial \pi(\theta, \hat{\theta}, p)}{\partial \hat{\theta}} = (p-c)f_\theta(r)\frac{-\beta F^{1,\hat{\theta}}(r)}{1-\beta f_{\hat{\theta}}(r)} > 0.$$

Consider one-to-one strategy $\tau : \Theta \to \mathbb{R}$ and suppose τ satisfies incentive compatibility:

$$\begin{aligned}
\pi(\theta, \theta, \tau(\theta)) &\geq \pi(\theta, \theta', \tau(\theta')), \\
\pi(\theta', \theta, \tau(\theta)) &\leq \pi(\theta', \theta', \tau(\theta')).
\end{aligned}$$

Let

$$P = \{p \in \mathbb{R}_+ | \exists \theta, \pi(\theta, \theta, p) \geq \pi(\theta, \underline{\theta}, \tau(\underline{\theta}))\}.$$

Incentive compatibility implies that $\tau([\underline{\theta}, \overline{\theta}] \subset P$. P has compact closure, $\overline{P}$, by Condition 2 and 3. Then, observe that:

Lemma 8. $\pi(\theta, \theta', \tau(\theta'')) \to \pi(\theta, \theta', \tau(\theta'))$ *as* $\theta'' \to \theta'$.

Since $\pi(\cdot)$ is C^2 on $[\underline{\theta}, \overline{\theta}]^2 \times \overline{Y}$, a compact set, $\pi(\cdot)$ is uniformly continuous there. The proof directly follows from incentive compatibility and the continuity of $\pi(\cdot)$ (see Mailath (1987) Proposition 1 for the full argument). Next, if τ is continuous at θ', it is differentiable almost everywhere:

Lemma 9. *If* τ *is continuous at* θ' *and* $\tau(\theta') \neq p^{FI}(\theta')$, *then*

$$\lim_{\theta' \to \theta''} \frac{\tau(\theta'') - \tau(\theta')}{\theta'' - \theta'} = -\frac{\pi_2(\theta', \theta', \tau(\theta'))}{\pi_3(\theta', \theta', \tau(\theta'))}.$$

The proof can be found in Mailath (1987) Proposition 2.

Lemma 10. *Since the strict single-crossing property holds,* τ *must be strictly increasing in* θ.

First, note that τ is a one-to-one strategy and hence, must be monotonic. Consider the incentive constraints.