(Mis-)Matchmaker

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Abstract

As platforms collect more user data, they can tailor algorithms to better match users. At the same time, on matching platforms, users pay to be matched by the platform, while the platform makes money as long as it does not match them. This paper analyzes the matching rule of a profit-maximizing monopoly platform when the incentives between users and the platform are misaligned. Contrary to the intuition that more data about users might improve matching efficiency and speed, I show that more data allows the platform to design a matching rule that increases search time and distorts matching and sorting outcomes in the market. I demonstrate that frequently studied matching rules, such as random matching and positive assortative matching, can be suboptimal for the platform. Instead, the platform strategically lowers match quality to increase search time and thus profits, leading to unnecessary delays and potentially inefficient matches. Finally, I provide two explanations for why platforms adopt business models with misaligned incentives: targeted advertising and the presence of overconfident users.

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1. INTRODUCTION

The emergence of digital matchmakers has revolutionized the way people meet and interact. By reducing search frictions, these platforms have the potential to more efficiently match users. With the help of algorithms based on detailed user data, they promise to facilitate the search for suitable partners in many areas of life. In fact, online dating has become the most common way to meet potential partners in recent years, and for more than a decade, job searches have been conducted predominantly through such online platforms (Rosenfeld et al., 2019; Kircher, 2022). This paper investigates the impact of a platform with detailed user data on the resulting speed and assortativity of matching in the society. It highlights a novel source of mismatching: profit-driven, purposeful mismatching of platforms.

To do so, I study the matching rule of a profit-maximizing platform on which users search for a suitable match. To capture the two most prominent business models, I assume that the platform commits to either an amount of advertising or a payment per period in which the user is active.[1] In either case, spending their time searching is costly for users. To attract and keep users' attention, the platform offers users a recommended match in each period. First, I show that the most prominent search protocols used to study centralized or decentralized matching markets — the positive assortative matching rule (PAM) and a random matching rule — are strictly suboptimal. Instead, the platform uses its knowledge about users to strategically lower the quality of recommended matches. This induces agents to search longer and thereby increases the payments the platform can collect. Besides prolonging search, the resulting matching outcomes can be drastically different from the socially optimal outcome — positive assortative matching — and induce a substantial welfare loss.

Why do platforms then rely on business models that induce misaligned incentives? I provide two plausible explanations. First, when, as in many online markets, users are reluctant to make monetary payments but are willing to consume ads,[2] offering an ad-based model can be more profitable. Second, when users have arguably well-documented misperceptions such as being overconfident regarding their desirability,[3] they underestimate their expected search duration and hence payments to the platforms for existing pay-per-month schemes.

After discussing the related literature in Section 2, Section 3 presents the model. A

[1]See Appendix C for evidence on the business model of dating and job search apps.

[2]Advertising-based models play a key role in online markets, including both fully ad-supported and "freemium" business models. Freemium refers to business models, where users can use a basic service for free in exchange for consuming ads, but need to pay a fee to use the premium service (without ads). Freemium has become the most popular pricing strategy for many apps (see ACM (2019) or `https://www.statista.com/chart/1733/app-monetization-strategies/`).

[3]Overconfidence has been widely documented in the experimental literature, see for example Burks et al. (2013) and Dubra (2015). Especially overconfidence with respect to one's own attractiveness is common (Greitemeyer, 2020). Psychologists argue that such overconfidence determines how individuals look and compete for potential partners (Murphy et al., 2015). In labor markets, Spinnewijn (2015) and Mueller et al. (2021) find that the unemployed overestimate how quickly they will find a job. Moreover, beliefs are not revised (sufficiently) downward after remaining unemployed. Both findings suggest that job seekers are persistently overconfident about their desirability to firms.

monopoly platform organizes a two-sided matching market in which users search for a partner on the opposite side. The platform commits to a matching rule that determines the probability that two users — each characterized by a vertical type — will meet. Additionally, the platform commits to a per-period cost that it collects from active users, which are either an amount of advertising or a search fee per period. After active users have paid the per-period cost, they receive a recommendation from the platform. Upon meeting, users simultaneously decide whether to accept or reject the proposed match. After rejecting, a user can continue to search. The analysis focuses on steady states; in these the inflow of new agents must equal the outflow under the platform's matching rule.

Section 4 starts by characterizing the users' search behavior. Then, fixing search costs, the platform's problem is to choose matching probabilities conditional on each users' type subject to participation constraints regarding the users' decision to join the platform, incentive constraints on the users acceptance decisions, feasibility constraints on the matching mechanism as well as steady-state constraints. This original problem is highly non-linear. Instead of analyzing the original problem, I make use of an auxiliary problem. This auxiliary problem is a linear programming problem in which the platform chooses masses of recommended matches and matched pairs accepting each other using the facts that: (i) the objective function is linear in steady-state masses, and (ii) the constraints are linear in the mass of recommended and matched pairs by using appropriate transformations. The profit-maximizing solution to this auxiliary problem is then transformed back to the solution of the original problem. Given the profit-maximizing matching rule, the platform chooses its advertising level or search fee. In the most general setting for any given finite set of users' types, I prove that an optimal solution to the platform's profit-maximization problem exists using the auxiliary problem. Based on the reformulation, I show that the widely analyzed matching rules are suboptimal. Random matching is suboptimal, when at least two types on each side of the market participate. Moreover, whenever both market sides are fully symmetric I show that the positive assortative matching rule — where each user meets a user of their own type — can be suboptimal.

Considering the special case with two types on each side of the market and symmetric inflows, Section 4.2 illustrates the main insight of the model — the platform's incentive to recommend and foster mismatches. To induce users to search, the platform frequently recommends mismatches to users, i.e., a high type meets a low relatively more often than a high type. The socially efficient matching outcome in which users sort positively is only implemented by the platform if significantly more low than high types enter the market. Otherwise, the platform induces a weakly, or even non-assortative, matching outcome.

The platform's matching thus creates two intertwined inefficiencies: it distorts matching outcomes by inducing mismatches that deviate from the socially optimal outcome, and it increases users' search time, leading to higher search costs than necessary. Both inefficiencies have implications for real-world markets such as dating and labor markets. In particular, in labor markets, the extent of mismatch has a significant impact on productivity and long-term unemployment (Şahin et al., 2014; McGowan and Andrews, 2015). Moreover, prolonged search duration, i.e., time spent unemployed or in a mismatched job, has high economic and social costs (e.g., unemployment insurance). In marriage markets, sorting has been found to have important implications for income inequality and household decisions (Lee, 2016). In addition, the quality of the relationship or marriage is a determinant of overall well-being and health (Robles et al., 2014; Sharabi and Dorrance-Hall, 2024). In the special case with two types, I find that the socially efficient matching outcome can induce the longest search time of agents, while the search time of agents decreases when the platform implements a weakly assortative or non-assortative outcome.

Finally, Section 5 turns to the question of why platforms rely on business models in which the incentives between the platform and the users are misaligned. For example, a simple potential business model for platforms would be to collect high personalized search fees from each type and provide them with the socially optimal match in the first period. In principle, this business model extracts the entire surplus from users. Under the realistic assumption that users are reluctant to pay upfront but are willing to consume ads, however, I show that an ad-based model can outperform the former business model if targeted advertising is sufficiently efficient. Alternatively, if users are overconfident about their desirability, this belief leads users to underestimate their search time when incentivized to search. Therefore, under the pay-as-you-search business model they spend a higher amount ex post than anticipated ex ante. This, in turn, favors the prevailing business model.

Section 6 concludes and highlights that the tension arising from the misalignment of incentives becomes more important as the platform collects more data and develops more predictive algorithms.

2. RELATED LITERATURE

This article contributes to two central strands of literature, which I detail below. In contrast to the literature, I consider the profit-maximizing incentives of a matchmaker when agents are vertically differentiated and characterize the matching rule and resulting matching outcome.

Matching and Search Theory The vast literature on search-and-matching models, see for instance Burdett and Coles (1999), Eeckhout (1999), Bloch and Ryder (2000), and Smith (2006), provides insights into the functioning of decentralized markets in which agents meet at "random".[4] These matching models with heterogeneous agents build the foundation to investigate sorting and mismatch in markets such as labor and marriage markets when search frictions are present. In line with these models, agents in my model have vertical preferences that result in a unique stable matching. I follow Lauermann and Nöldeke (2014) and suppose that types are finite. The model at hand crucially departs from the literature on decentralized matching, which assumes that agents meet according to a random matching technology, by explicitly accounting for the design of the matching

[4]The aforementioned literature assumes that agents have non-transferable utility. Search-and-matching models with transferable utility have been analyzed, for example, by Becker (1973, 1974) and Shimer and Smith (2000). For an overview of the literature on search-and-matching models see Chade et al. (2017).

rule. With increasing access to user data about preferences and machine-learning tools, matching platforms can design their own recommendation and matching algorithms to maximize profits. While many platforms do not disclose the specifics of their matching algorithms, it is evident that their algorithms are far more sophisticated than random matching.[5]

The question of how to design the matching rule is related to the literature on centralized matching as pioneered by Gale and Shapley (1962), Roth (1982), and Roth and Sotomayor (1992), which studies match quality and implementation of efficient matching rules in two-sided markets.[6] The principal considers properties such as stability, strategy-proofness and Pareto efficiency of the matching rule. In contrast, I characterize the profit-maximizing solution for different given business models.

Search problems are widely studied not only on an individual level but researchers also rely on these to better understand job search and its implications on the functioning of the economy. Early articles include Pissarides (1985), Mortensen and Pissarides (1994), and Mortensen and Pissarides (1999), which focus on wage bargaining and unemployment dynamics and on-the-job search when agents are ex-ante homogeneous. Dolado et al. (2009) introduces heterogeneous types of workers and firms into job search models, which are also crucial in my model. A recent treatment on how job search has changed in the digital era is provided by Kircher (2022).

Finally, my paper is related to papers investigating biased beliefs of agents in matching and search markets. Closely related in a dating market, Antler and Bachi (2022) show that agents' coarse reasoning leads to overoptimism about their prospects in the market and induces them to search inefficiently long. In labor markets, Spinnewijn (2015) and Mueller et al. (2021) document that job seekers often hold overoptimistic beliefs and thereby underestimate their time to find a job. I contribute to this literature by showing how current platform business models exploit overconfident types.

Platform Markets Central to the literature that studies platform and (online) two-sided markets is the presence of network effects and how these shape the incentives and price setting of a platform that enables the interaction between two groups (Caillaud and Jullien, 2003; Rochet and Tirole, 2003, 2006). As a result, in most models agents are assumed to care only about the number of matches instead of match quality.

With the emergence of digital matchmakers, the literature extended to analyzing (customized) matching on platforms with a focus on the interaction between pricing and matching efficiency (Damiano and Li, 2007; Damiano and Hao, 2008), price discrimination (Gomes and Pavan, 2016, 2024), and auctions (Johnson, 2013; Fershtman and Pavan, 2022), all abstracting from search frictions and dynamics. In my model, the platform designs the matching rule in its online market place, but in contrast to the aforementioned

[5]Dating platforms such as Tinder or bumble provide a general description of their algorithm, see for example `https://www.help.tinder.com/hc/en-us/articles/7606685697037-Powering-Tinder-The-Method-Behind-Our-Matching`, whereas the dating platform "Hinge" claims to use the Gale-Shapley algorithm designed to find stable matchings.

[6]The literature on matching in two-sided markets can be divided into centralized and decentralized matching (see Echenique et al. (2023) for a recent overview).

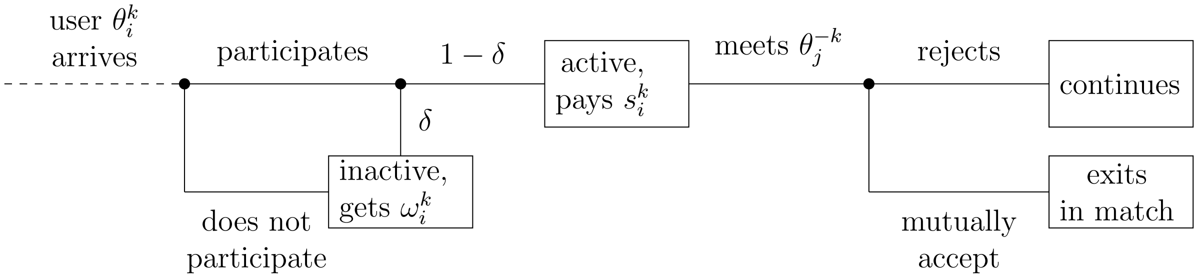

Figure 1: Within-Period Timing

articles, the platform has an incentive to not implement the efficient and full surplus extracting matching rule.

Within the analysis of digital matchmakers, Halaburda et al. (2018) and Antler et al. (2024a,b) also focus on applications to dating platforms. Most closely related is Antler et al. (2024b) who study a matchmaker's incentives in a model with horizontally differentiated types, which determine the fit of agents. The platform charges a single "upfront" fee in the second period after agents have joined and received their first match for free. The authors draw a similar conclusion: the platform has an incentive to invest into a technology that increases the speed of search but not into improving match quality. The main difference lies in modeling the matching technology. The authors restrict attention to a truncated random matching technology under which agents meet at random above a threshold and do not meet if their fit is below the threshold; in contrast, I solve for the optimal matching rule.

Within the platform literature models on platforms intermediating consumer search Hagiu and Jullien (2011, 2014), Eliaz and Spiegler (2011b, 2016), and Nocke and Rey (2024) are closely related. Hagiu and Jullien (2011) provide a rationale for intermediaries to divert search of their consumers away from preferred stores. Although the insight is closely related to the mismatching incentive in my model, the (one-sided) market in Hagiu and Jullien (2011) does not include the strategic component on the other side as stores would never reject a consumer willing to buy. Hence, there is no analogue to my finding that the platform prolongs search of lower types by recommending them to higher types knowing that they will reject those lower types. Additionally, there is no equivalent to overconfident users in their model. Finally, my model of a two-sided matching market offers insights into the allocative inefficiency and the length of search for labor and dating markets intermediated by matching platforms.

3. MODEL

A monopolist platform organizes a matching market in which a continuum of agents from two sides, $k = A, B$, search for a partner from the opposite side. The market operates in discrete time with an infinite horizon. I focus on steady state analysis. In slight abuse of notation, I therefore suppress time indices whenever it does not lead to confusion.

Agents Agents of each side are characterized by a type $\theta_i^k \in \Theta^k$, with $\Theta^k = \{\theta_1^k, \theta_2^k, ..., \theta_{N^k}^k\}$ finite. At the beginning of each period, an agent θ_i^k decides whether to enter the market or to exit and take outside option ω_i^k. An agent that participates in the market becomes inactive with an exogenous probability $\delta > 0$ and also leaves the search process. The platform charges an active agent of type θ_i^k a search cost s_i^k. Then, each active agent receives a single recommendation from the platform. After receiving a recommendation, two agents who meet observe each other's type and simultaneously decide whether to accept or reject the other agent. The following payoffs are realized based on their actions in the current period: (i) mutual acceptance yields a match utility of $u(\theta_i^k, \theta_j^{-k}) = \theta_i^k \theta_j^{-k}$, and (ii) (one-sided) rejection yields a utility of zero in the current period. After a rejection, an agent can continue searching in the next period. The timing within each period is summarized in Figure 1.

Agents are assumed to use time- and history-independent strategies. A pair of functions $\sigma_k : \Theta^k \times \Theta^{-k} \to [0,1]$ and $\sigma_{-k} : \Theta^k \times \Theta^{-k} \to [0,1]$ describe the acceptance strategies, where $0 \leq \sigma_k(\theta_i^k, \theta_j^{-k}) \leq 1$ is the probability that an agent of type θ_i^k on side k accepts a match with type θ_j^{-k} on the other side. The function $\eta_i^k : (\theta_i^k, \omega_i^k) \to [0,1]$ describes the participation strategy of an agent of type θ_i^k with outside option ω_i^k. In other words, without loss of generality, I focus on strategies where identical agents, active on the same side of the market and of the same type, use the same acceptance and participation strategy. Then,

$$\alpha(\theta_i^k, \theta_j^{-k}) = \sigma_k(\theta_i^k, \theta_j^{-k}) \cdot \sigma_{-k}(\theta_i^k, \theta_j^{-k})$$

denotes the probability of a mutual acceptance by type θ_i^k and θ_j^{-k}.

Matching A matching mechanism $\mathcal{M} := \{\phi^k(\cdot)\}_{k=A,B}$ consists of (potentially stochastic) matching rules $\phi^k(\cdot)$. Let $\hat{\Theta}^k$ be the set of participating types from side $k = A, B$. For $\theta_i^k \in \hat{\Theta}^k$, $\phi^k(\cdot|\theta_i^k) \in \Delta(\hat{\Theta}^{-k} \cup \omega_i^k)$, which is a probability measure over $\hat{\Theta}^{-k} \cup \omega_i^k$. Intuitively, this describes the probability of meeting the various types of the opposing side as well as the outside option. Any $\theta_i^k \in \Theta^k \setminus \hat{\Theta}^k$ who does not participate is assumed to be meet their outside option with probability one, $\phi(\omega_i^k|\theta_i^k) = 1$. Denote the mass of agents of type θ_i^k on side k by $f(\theta_i^k)$. Matching mechanism $\mathcal{M}$ induces a distribution of matched pairs M

$$\left(\begin{pmatrix} f(\theta_1^k) \\ \vdots \\ f(\theta_{N^k}^k) \end{pmatrix}, \begin{pmatrix} f(\theta_1^{-k}) \\ \vdots \\ f(\theta_{N^{-k}}^{-k}) \end{pmatrix} \right) \mapsto \begin{pmatrix} \Phi(\theta_1^k, \theta_1^{-k}) & \cdots & \Phi(\theta_1^k, \theta_{N^{-k}}^{-k}) \\ \vdots & & \vdots \\ \Phi(\theta_{N^k}^k, \theta_1^{-k}) & \cdots & \Phi(\theta_{N^k}^k, \theta_{N^{-k}}^{-k}) \end{pmatrix} \equiv M.$$

An entry of matrix M is the mass of agents that are recommended to each other under matching mechanism $\mathcal{M}$ and is given by

$$\Phi(\theta_i^k, \theta_j^{-k}) = f(\theta_i^k)\phi(\theta_j^{-k}|\theta_i^k) = f(\theta_j^{-k})\phi(\theta_i^k|\theta_j^{-k}),$$

where the masses are symmetric, i.e. the mass of agents of type θ_i^k on side k being matched to agents of type θ_j^{-k} on side $-k$ is equal to the mass of agents of type θ_j^{-k} on side $-k$ being matched to type θ_i^k on side k: $\Phi(\theta_i^k, \theta_j^{-k}) = \Phi(\theta_j^{-k}, \theta_i^k)$. Under matching mechanism $\mathcal{M}$, the mass of agents of type θ_i^k that are unmatched, i.e. do not receive a recommendation in a given period, is

$$\Phi(\theta_i^k, \omega_i^k) = f(\theta_i^k) - \sum_{\theta_j^{-k} \in \Theta^{-k}} \Phi(\theta_i^k, \theta_j^{-k}).$$

To capture the idea that the platform can only generate revenue by keeping users' attention and, hence, wants to match as many agents as possible, I impose the following assumption.

Assumption 1. *Let $\hat{k}$ be the short side of the market. For each agent on side $\hat{k}$, $\phi(\omega_i^k | \theta_i^{\hat{k}}) = 0$.*

Under Assumption 1, feasibility of the matching rule can be expressed in terms of the masses of matched pairs.

Definition 1. A matching mechanism $\mathcal{M}$ is feasible if

$$\sum_{\theta_j^{-k} \in \Theta^{-k}} \Phi(\theta_i^k, \theta_j^{-k}) + \mathbf{1}_{k=\hat{k}} \Phi(\theta_i^k, \omega_i^k) = \eta_i^k f(\theta_i^k), \forall \theta_i^k \in \Theta^k, k = A, B. \tag{1}$$

Timing and Population Dynamics At the beginning of a period t, agents who did not find a match in the last period arrive and a (time-invariant) inflow of new agents of type θ_i^k given by the mass $\{\beta_i^k\}_i^{k=A,B}$ enters the platform. Agents decide whether to participate on the platforms. Those who decide to participate become inactive with probability δ, while active agents are matched according to matching mechanism $\mathcal{M}$ resulting in matrix M_t. Based on their recommended match, agents make their acceptance decision resulting in mutual acceptance probabilities $\{\alpha_t(\theta_i^k, \theta_j^{-k})\}_{ij}$. At the end of the period, agents that mutually accepted each other exit in pairs. The total outflow of agents is then given by pairs that exit together in a match, agents that become inactive with probability δ and agents that decided not to participate.

Platform The platform commits to a matching mechanism $\mathcal{M} := \{\phi^k(\cdot)\}_k$. To capture the two most prominent business models, I suppose that the platform either commits to an extent of advertising or a given payment per period. Formally, this choice induces the type-dependent search cost s_i^k while generating revenue per search of type θ_i^k of $\nu(s_i^k)$. In case of payments, $\nu(s_i^k)$ is the identity function. In case of advertisements, $\nu(s_i^k)$ is an increasing and strictly concave function of the search costs, which for example captures the intuition that the agents' disutility of advertising is convex in the number of ads shown while the platform's profit is constant per ad. Let $s_i^k \in [0, \overline{u}]$, where $\overline{u}$ is the maximum match utility that the highest type can achieve on the platform. The platform discounts future profits according to ρ and thus maximizes

$$\Pi = \sum_{k=A,B} \sum_{\theta_i^k \in \Theta^k} \frac{(1-\delta)\eta_i^k}{1-\rho} \nu(s_i^k) f(\theta_i^k).$$

Equilibrium Concept The model focuses on a steady state analysis in which the market is balanced: that is, the inflow of agents is equal to the outflow of agents under matching mechanism $\mathcal{M}$. Formally:

Definition 2. (Steady State) For given matching mechanism $\mathcal{M}$, a *steady state* is a tuple $(f(\theta_i^k), \alpha(\theta_i^k, \theta_j^{-k}), \eta_i^k)_{ij}^k$ that satisfies

$$\beta_i^k = f(\theta_i^k) \left[(1-\eta_i^k) + \eta_i^k \left(\delta + (1-\delta) \sum_{\theta_j^{-k} \in \Theta^{-k}} \alpha(\theta_i^k, \theta_j^{-k}) \phi(\theta_j^{-k} | \theta_i^k) \right) \right], \tag{2}$$

for all $\theta_i^k \in \Theta^k, k = A, B$. The left-hand side describes the inflow of agents of type θ_i^k, where the right-hand side is the outflow. The outflow is the mass of type θ_i^k agents times the probability that agents do not participate plus the probability of becoming inactive or exiting in a match.

A steady state is an equilibrium if the following is satisfied.

Definition 3. (Equilibrium) A steady state is an equilibrium if — given that agents anticipate other agents' strategies correctly — the profile of stationary strategies (σ, η) satisfies:

1. Agents accept a match if and only if the match yields a higher payoff than the expected utility from continuing to search.
2. Agents participate if and only if the expected utility from participating yields a higher payoff than their outside option.

Under the usual Nash assumption of correctly anticipating other players' strategies, the definition captures that agents maximize expected utility with respect to their acceptance strategy implicitly ruling out the case that a valuable pair is rejected because everyone is certain that their partner rejects.[7] The third part captures that agents maximize expected utility when deciding to participate on the platform.

3.1 DISCUSSION OF ASSUMPTIONS

Search Costs Agents incur additive search costs s_i^k in each period, which are designed by the platform. They either represent the nuisance costs from advertising as, for example, in Anderson and Coate (2005), which are positively related to the advertising intensity, or the search fee that the platform charges periodically. Search frictions are modeled by introducing the exogenous exit probability δ. Following a literal interpretation, δ is the probability with which agents become inactive, i.e. the probability that an agent finds a job or a partner offline through other means. More generally, δ can be thought of as modeling the force that leads agents to discount the future, which makes delayed matching more costly.

[7]This allows the current match partner to tremble with small probability. Alternatively, acceptance decisions could be made sequential in which case agents would have to accept a valuable match.

Business Model The platform is assumed to be a monopolist in the matching market. Following evidence from the dating market, the most popular dating platforms have a common owner. For simplicity, I assume that the dominant owner only offers one platform in my model.[8] More generally, we often observe platforms with large market power in two-sided markets, where joining a new platform is worthwhile only if others join. My monopoly setup is a simple setting capturing such market power.

The model examines two prevalent business models: an advertisement-based approach and periodic search fees. Many platforms adopt the former— (targeted) advertising — by monetizing user attention through selling advertising slots to firms. In return for users' attention, the platform provides its matching service for free. In this setup, keeping user attention is crucial for the platform's revenue.[9] This is why I assume that the platforms earns no revenue when not capturing the user's attention through offering a potential match. Alternatively, platforms implement search fees, which they collect from active users. Examples include "pay-per-click" or "pay-per-contact" fees, though monthly subscription plans are also common. These fees are typically low, distinguishing them significantly from participation fees, which are far less common but used by some selective matching platforms.[10]

An advertising-based stream of revenues continues to be a prominent part of platform business models, especially with transaction costs. Platforms have transaction costs when setting up a payment system, while many users are reluctant to give their credit card data to platforms. Overall, privacy concerns, risk aversion and uncertainty about new products (platforms) can play a role why users (initially) prefer to use the matching service for "free" while watching advertisement over signing up to a subscription plan or paying a participation fee. As a consequence, many platforms rely on these so-called "freemium" business models, which have become even more popular since the emergence of mobile applications (apps). Here, "freemium" describes business models where a basic service is available to users for free (with advertisement), whereas an upgraded service can be accessed through purchases.[11] Other platforms, however, rely only on advertising or fees. I return to the question of why platforms refrain from collecting a fixed fee for a certain promised match in Section 5.

No Agent is Unmatched The key assumption of the matching rule, Assumption 1, states that if possible each agent receives a recommended match in any period.[12] As many

[8] The dating market is highly concentrated with the Match Group Inc. owning many of the most popular dating platforms: Tinder, Hinge, PlentyofFish, Match, OkCupid etc. (see `https://www.bamsec.com/filing/89110323000114?cik=891103`), while other dating platforms are highly differentiated and for example, cater to specific religious groups. Recent experimental evidence from Dertwinkel-Kalt et al. (2024) suggest that even the closest competitors, Tinder and bumble, are viewed to be almost independent instead of substitutes by consumers.

[9] Recent papers that study different aspects of attention on platforms are for example Prat and Valletti (2022), Chen (2022), and Srinivasan (2023).

[10] For an overview of the most common platforms and their fee structure see Appendix C.

[11] For empirical evidence see for example, Kummer and Schulte (2019) for studying privacy concerns in the mobile app market and Deng et al. (2023) for studying freemium pricing of mobile applications.

[12] In the literature on search-and-matching models time is often continuous, such that matching opportunities arrive at a constant rate. Similarly, Antler et al. (2024a,b) make the assumption that matches

online platforms take on a dual role as attention intermediaries and need to attract consumers' attention to sell to advertisers, providing a constant stream of potential matches aims at grabbing and keeping consumers' attention.[13]

To grab users' attention, the platform makes a recommendation any time the agent enters and is active on the platform. The recommendation of a potential match can be viewed as being part of a menu that the platform offers. Following the idea of Eliaz and Spiegler (2011a), the platform offers a menu that consists of an attention-grabbing component and its true value of the service. In reality, the attention grabbing component is supported by push notifications or emails, while the value from the platform's service is determined by the expected utility from getting a match. The modeling choice is further supported by a recent lawsuit against the MatchGroup Inc., owner of a majority of the most popular dating platforms.[14] In the complaint, the plaintiff accuses Match to monopolize users' attention and claim that "Push Notifications prey on users' fear of missing out on any potential matches with a strategic notification system designed to capture and retain attention throughout the day".

4. ANALYSIS

To analyze the equilibrium, I need to characterize the agents' behavior and the platform's optimization problem. The agents' search process is characterized by a set of participation and incentive constraints that determine whether an agent is willing to incur the search costs as well as accepts or rejects a recommended match.

Agents' Search Process. Consider the strategy of agent θ_i^k being active in the matching market. Upon meeting an agent θ_j^{-k}, the agent decides whether to accept or reject the *recommended* match. Mutual acceptance results in a *match* and both agents leave the market as a pair. If at least one of the agents rejects the match, agent θ_i^k continues to search.

Due to the stationarity of the environment, the continuation value of agent θ_i^k, $V^C(\theta_i^k)$, is defined by the following recursive equation

$$\begin{aligned} V^C(\theta_i^k) =& \delta\omega_i^k + (1-\delta)\Bigg[-s_i^k + \sum_j \alpha(\theta_i^k, \theta_j^{-k})\phi(\theta_j^{-k}|\theta_i^k)\theta_i^k\theta_j^{-k} \\ & + (1 - \sum_j \alpha(\theta_i^k, \theta_j^{-k})\phi(\theta_j^{-k}|\theta_i^k))V^C(\theta_i^k)\Bigg]. \end{aligned}$$

The first term represents the case in which agent θ_i^k will become inactive with probability δ and gets its outside option ω_i^k. If the agent remains active with probability $1-\delta$, it

arrive at a constant rate even in the presence of a matchmaking platform.

[13]In a recent experiment, Aridor (Forthcoming) provides evidence that users allocate their attention across product categories and offline when facing restriction in their time spent on a specific platform. The results suggest that competition for attention spans across multiple markets.

[14]Oksayan v. MatchGroup Inc., N.D. Cal., No. 3:24-cv-00888, 2/14/24.

incurs the search cost s_i^k. The expected utility from leaving in a match is given by the utility from a match with type θ_j^{-k}, which is equal to the product of both types, and the probability of meeting and mutually accepting type θ_j^{-k}. With the counterprobability, the match was not mutually accepted and agent θ_i^k continues to search.

Solving for the continuation value yields

$$V^C(\theta_i^k) = \frac{\delta\omega_i^k + (1-\delta)\left(-s_i^k + \sum_j \alpha(\theta_i^k, \theta_j^{-k})\phi(\theta_j^{-k}|\theta_i^k)\theta_i^k\theta_j^{-k}\right)}{\delta + (1-\delta)\left(\sum_j \alpha(\theta_i^k, \theta_j^{-k})\phi(\theta_j^{-k}|\theta_i^k)\right)}. \tag{3}$$

The continuation value then characterizes the payoff of an agent who rejects a match and returns to the search process, whereas the match payoff $\theta_i^k\theta_j^{-k}$ characterizes the payoff of an agent who accepts a match with type θ_j^k (and is accepted by them). By Definition 3, if the match value $\theta_i^k\theta_j^{-k}$ is smaller (larger) than the continuation value $V^C(\theta_i^k)$, agent-θ_i^k will reject (accept) a recommended match with agent-θ_j^{-k}.

The optimal strategy of an agent who uses an time-and history-independent strategy satisfies:

$$\sigma_k(\theta_i^k, \theta_j^{-k}) = \begin{cases} 0 & \text{if } \theta_i^k\theta_j^{-k} < V^C(\theta_i^k) \\ r \in [0,1] & \text{if } \theta_i^k\theta_j^{-k} = V^C(\theta_i^k) \\ 1 & \text{if } \theta_i^k\theta_j^{-k} > V^C(\theta_i^k) \end{cases}, \text{ for } k = A, B. \tag{4}$$

If the match value with a type $\hat{\theta}_j^{-k}$ is larger than the continuation value, agent θ_i^k will accept a recommended match with agent $\hat{\theta}_j^{-k}$ and all agents of types higher than $\hat{\theta}_j^{-k}$. The optimality of this strategy follows directly from the supermodularity of the match payoff.

An agent participates if the continuation value is larger than the agent's outside option. Due to the stationarity and history-independence of strategies, if an agent decides to participate in the matching market, they will not exit during the search process and search until they exit in a match or become inactive with probability δ.

Remark. The strategy of an agent of type θ_i^k is increasing in its second argument $\sigma_k(\theta_i^k, \theta_{N^{-k}}^{-k}) \geq \sigma_k(\theta_i^k, \theta_{N-1}^{-k}) \geq \cdots \geq \sigma_k(\theta_i^k, \theta_1^{-k})$, but may be neither in- nor decreasing in its first argument.

The fact that the agent's strategy is increasing in its second argument follows directly from Equation 4. If the agent's outside options are weakly increasing in type, for matching rules such as random or positive assortative matching rules $\sigma_k(\theta_i^k, \theta_j^{-k})$ is additionally decreasing in its first argument: $\sigma_k(\theta_{N^k}^k, \theta_j^{-k}) \leq \cdots \leq \sigma_k(\theta_1^k, \theta_j^{-k})$. A random matching rule yields the same meeting probabilities for all types. Due to the supermodularity of the payoff function, higher types will reject (weakly) higher types than lower types do. With positive assortative matching, the matching probabilities conditional on being a higher type first-order stochastically dominates the matching probabilities conditional on being a lower type. Hence, higher types will reject strictly higher types than lower types do. In contrast, a negative assortative matching rule, which recommends (almost exclusively) higher types to lower types, and vice versa, can cause lower types to reject lower types while higher types are willing to accept them. Indeed, I will explicitly provide an example of such an equilibrium in Section 4.2.

Given the agent's strategy in Equation 4, the acceptance probabilities satisfy

$$\alpha(\theta_i^k, \theta_j^{-k}) = \begin{cases} 0 & \text{if } \theta_i\theta_j < V^C(\theta_i^k) \text{ or } \theta_i\theta_j < V^C(\theta_j^{-k}) \\ 1 & \text{if } \theta_i\theta_j > V^C(\theta_i^k) \text{ and } \theta_i\theta_j > V^C(\theta_j^{-k}) \end{cases} . \tag{5}$$

Equation 5 establishes the relationship between acceptance probabilities and matching outcomes. Mutual acceptance requires that whenever two types of agents meet, both must find it optimal to stop searching.

4.1 MULTIPLE TYPES

Consider the case with N^k types of agents such that $\Theta^k = \{\theta_1^k, ..., \theta_{N^k}^k\}$ on side $k = A, B$, where $\theta_{N^k}^k > ... > \theta_1^k$. The following section provides general results on the existence of an equilibrium, optimal solution and their properties. Let s_i^k be exogenous.

Lemma 1. *For a given feasible matching mechanism a steady-state equilibrium exists if and only if Equation 2 and 5 are satisfied.*

Suppose for a feasible matching mechanism, an equilibrium exists. Then, it must give rise to (i) a steady state and (ii) optimal strategies of agents, i.e. satisfy Definition 2 and Definition 3. Hence, by (i) Equation 2 (balance condition) must hold, and (ii) implies Equation 5 (optimal mutual acceptance) must hold. Conversely, if Equation 2 is violated the steady state (balance) condition fails and if Equation 5 is violated at least some agent behaves suboptimal. Thus, a feasible matching rule gives rise to an equilibrium if and only if Equation 2 and 5 hold.

Lemma 2. *There exists a feasible matching rule that gives rise to an equilibrium.*

In the most simple case consider the matching rule $\phi(\omega_i^k|\theta_i^k) = 1$ for all types $\theta_i^k \in \Theta^k$ on side $k = A, B$. Given that agents are matched with their outside option, no agent is willing to incur search costs. With no agent participating in the steady state, the matching rule is feasible and gives rise to a steady state equilibrium.

Next, to determine the profit-maximizing matching rule $\mathcal{M}$, it is useful to define the matching outcome. Intuitively, the matching outcome is defined as the matrix that describes the distribution of pairs under matching rule $\mathcal{M}$ that exit in a match. Recall that matrix M describes the masses of recommended pairs under matching rule $\mathcal{M}$ and let A denote the matrix of agents' mutual acceptance probabilities

$$A \equiv \begin{pmatrix} \alpha(\theta_1^k, \theta_1^{-k}) & \cdots & \alpha(\theta_1^k, \theta_{N^{-k}}^{-k}) \\ \vdots & & \vdots \\ \alpha(\theta_{N^k}^k, \theta_1^{-k}) & \cdots & \alpha(\theta_{N^k}^k, \theta_{N^{-k}}^{-k}) \end{pmatrix} .$$

Formally, the matching outcome is defined as the componentwise multiplication (Hadamard product) of matrix A and M:

Definition 4. The matching outcome is defined by the matrix

$$A \odot M = \begin{bmatrix} \alpha(\theta_1^k, \theta_1^{-k})\Phi(\theta_1^k, \theta_1^{-k}) & \cdots & \alpha(\theta_1^k, \theta_{N^{-k}}^{-k})\Phi(\theta_1^k, \theta_{N^{-k}}^{-k}) \\ & \vdots & \\ \alpha(\theta_{N^k}^k, \theta_1^k)\Phi(\theta_{N^k}^k, \theta_1^{-k}) & \cdots & \alpha(\theta_{N^k}^k, \theta_{N^{-k}}^{-k})\Phi(\theta_{N^k}^k, \theta_{N^{-k}}^{-k}) \end{bmatrix} \equiv O(\cdot).$$

Matching outcomes are (i) assortative if $O(\cdot)$ has positive entries only along the main diagonal, (ii) weakly assortative if $O(\cdot)$ has positive entries along the main diagonal and to the right if and only if the entries below are also positive, and (iii) non-assortative otherwise.

If a matching outcome is assortative, this implies that lower types are matched with strictly lower types than higher types while the definition of weakly assortative implies that lower types can be matched with the same types as higher types. The definition is weak in the sense that it does not require that lower types accept with a higher probability than higher types. Other matching outcomes are called non-assortative and entail negative assortative outcomes where higher types are matched with strictly lower types than lower types.

Denote by $m(\theta_i^k, \theta_j^{-k}) = \alpha(\theta_i^k, \theta_j^{-k})\Phi(\theta_i^k, \theta_j^{-k})$ an entry of matrix $O(\mathcal{M})$. Each entry is therefore the mass of *matched* pairs that exit the market together in every period. For a given matching rule, an equilibrium induces at most one matching outcome since the mutual acceptance probabilities and steady state masses are pinned down in equilibrium.

To find the profit-maximizing matching rule and the associated matching outcome, I proceed in two steps. First, I fix a matrix of acceptance probabilities and determine the optimal feasible matching rule that implements the mutual acceptance probabilities. Second, supposing the optimal matching rule from step one is used to implement any chosen matrix of acceptance probabilities, I choose the matrix that yields the highest platform profits.

First, note that the platform finds it optimal to induce either full participation of a type or no participation.

Lemma 3. *It is without loss of generality to consider $\eta_i^k \in \{0, 1\}$.*

Suppose the platform charges type-dependent search fees, and type θ_i^k, who is indifferent between participating and not participating, participates with probability less than one. Then, the platform makes the same profit if type θ_i^k participates with probability one, the platform sometimes matches them to their outside option, and reduces their search fee such that they make the same payments in expectation. If the platform uses an advertising-based business model, the platform will strictly increase its profit by this procedure due to the concavity of advertising. Therefore, from now on I will focus on $\eta_i^k \in \{0, 1\}$ which allows to focus on the set of participating types. Then, suppose the platform induces a set $\hat{\Theta}^k$ for $k = A, B$ to participate.

In the following, I will transform the platform's profit-maximization problem into a linear program. For given search cost s_i^k, recall that the platform's objective is to maximize

$$\max_{\mathcal{M}} \sum_{k=A,B} \sum_{\theta_i^k \in \hat{\Theta}^k} \frac{(1-\delta)s_i^k}{1-\rho} f(\theta_i^k),$$

i.e., the platform maximizes the steady state mass of active agents with weight s_i^k. Note that the platform does not earn revenue from agents that are inactive or do not participate in the market in the first place. The maximization problem underlies a set of constraints. First, the matching rule must implement a steady state. The steady state condition (Equation 2) implies

$$\beta_i^k = f(\theta_i^k)\delta + (1-\delta) \sum_{\theta_j^{-k} \in \Theta^{-k}} \underbrace{\alpha(\theta_i^k, \theta_j^{-k})\Phi(\theta_j^{-k}|\theta_i^k)}_{=m(\theta_i^k,\theta_j^{-k})}. \tag{Steady State}$$

In the steady state, the inflow of agents of θ_i^k is equal to the mass of agents that become inactive in a period with probability δ and the mass of active agents that exit in matched pairs. In the steady state, the mass of agents of type θ_i^k can be restated as

$$f(\theta_i^k) = \frac{\beta_i^k - (1-\delta)\sum_j m(\theta_i^k, \theta_j^{-k})}{\delta}, \tag{Steady-State Mass}$$

and therefore, depends positively on the inflow, β_i^k, and negatively on the mass of matched pairs that include type θ_i^k. Second, the matching rule determines whether agents participate in the market and whether agents search according to the platform's recommendations. For participating agents, it must hold that the agent prefers participating in the market to accepting the outside option, i.e.

$$\omega_i^k \leq \frac{\delta\omega_i^k + (1-\delta)\left(-s_i^k + \sum_j \alpha(\theta_i^k, \theta_j^{-k})\phi(\theta_j^{-k}|\theta_i^k)\theta_i^k\theta_j^{-k}\right)}{\delta + (1-\delta)\left(\sum_j \alpha(\theta_i^k, \theta_j^{-k})\phi(\theta_j^{-k}|\theta_i^k)\right)} = V^C(\theta_i^k).$$

Since the match payoffs are supermodular, there exists a critical lowest type that an agent θ_i^k is willing to accept (Equation 4). Agent θ_i^k rejects (accepts) all types below (above) the critical lowest type. The incentive constraint for agent θ_i^k to follow the recommendation of the platform to (weakly) reject an agent θ_j^{-k} reads[15]

$$\theta_i^k\theta_j^{-k} \leq V^C(\theta_i^k).$$

By using the steady state condition, the participation and incentive constraints can be reformulated. Note that the denominator of the continuation value is equal to the probability that an agent exists, which is equal to $\beta_i^k/f(\theta_i^k)$ by Equation 2. Inserting into the continuation value and rearranging yields

$$\beta_i^k\omega_i^k \leq \delta f(\theta_i^k)\omega_i^k - (1-\delta)f(\theta_i^k)s_i^k + (1-\delta)\sum_j \underbrace{\alpha(\theta_i^k, \theta_j^{-k})\Phi(\theta_j^{-k}|\theta_i^k)}_{=m(\theta_i^k,\theta_j^{-k})}\theta_i^k\theta_j^{-k}, \tag{PC}$$

$$\beta_i^k\theta_i^k\theta_j^{-k} \leq \delta f(\theta_i^k)\omega_i^k - (1-\delta)f(\theta_i^k)s_i^k + (1-\delta)\sum_j \underbrace{\alpha(\theta_i^k, \theta_j^{-k})\Phi(\theta_j^{-k}|\theta_i^k)}_{=m(\theta_i^k,\theta_j^{-k})}\theta_i^k\theta_j^{-k}. \tag{IC}$$

[15]In mechanism design, this is often referred to as an obedience constraint because there is no private information throughout the model.

Lastly, the platform's matching rule must satisfy the feasibility constraints. Without loss of generality, let side B be of smaller or same size as side A. Then on side A, the sum over the mass of each recommended pair that includes type θ_i^A must be equal to the steady state mass of θ_i^A. On side B, the sum over the mass of each recommended pair that includes type θ_i^B and the mass of agents of type θ_i^B that are unmatched must be equal to the steady state mass of type θ_i^B

$$\sum_{\theta_j^{-k} \in \Theta^{-k}} \Phi(\theta_i^k, \theta_j^{-k}) + \mathbf{1}_{k=B} \Phi(\theta_i^k, \omega_i^k) = f(\theta_i^k), k = A, B. \tag{Feasibility}$$

As stated above, for given matrix A the above constraints and the objective function are all linear functions of the steady state masses, matched pairs, and recommended pairs. The steady-state mass in turn is also a linear functions of the mass of matched pairs. To complete the reformulation as linear program, it remains to include the indifference constraints for agents who mix when accepting type from the other market side, which implies that the respective incentive constraint must hold with equality. Appendix A.1 formally does so, leading to:

Lemma 4. *The platform's problem can be restated as a linear programming problem in the mass of matched and recommended pairs:* $\{m(\theta_i^k, \theta_j^{-k})\}, \{\Phi(\theta_i^k, \theta_j^{-k})\}_{ij}$.

Note that by Lemma 1, the solution to the linear program is an equilibrium as it fulfills Equation 2 and 5. Given a solution of the linear program — the auxiliary problem — the optimal matching rule to the original problem results from

$$\phi(\theta_j^{-k} | \theta_i^k) = \frac{\Phi(\theta_i^k, \theta_j^{-k})}{f(\theta_i^k)}.$$

Next, I show that the auxiliary problem has an optimal solution. I say that a matrix A of mutual acceptance probabilities can be implemented if there exists a matching mechanism $\mathcal{M}$ such that $\Big((f(\theta_i^k))_{\theta_i^k \in \Theta^k}, A, \eta\Big)$ is an equilibrium. Let $\mathcal{A}$ be the set of matrices A that can be implemented. By Proposition 2, $\mathcal{A}$ is non-empty. For every $A' \in \mathcal{A}$, construct a matrix A'' such that

$$\alpha'(\theta_i^k, \theta_j^{-k}) = \alpha''(\theta_i^k, \theta_j^{-k}) \text{ if } \alpha'(\theta_i^k, \theta_j^{-k}) \in \{0, 1\},$$
$$\alpha'(\theta_i^k, \theta_j^{-k}) = \alpha_{ij} \text{ otherwise},$$

where α_{ij} can take on any value in $[0, 1]$. I use $\alpha_{ij} \in [0, 1]$ whenever an agent is indifferent which implies that the same constraints in the auxiliary program must hold. Denote the resulting set of matrices as $\mathcal{A}^*$ and note that $\mathcal{A}^*$ is finite. Now, I can solve the linear program over the mass of matched and recommended pairs (ignoring acceptance probabilities). Solving this for all (finite) possible combinations of constraints yields a set of candidate solutions among which I choose the one that maximizes the platform's profit. To find the corresponding acceptance probabilities $\alpha_{ij} \in [0, 1]$ when the agent is indifferent, divide the matched pairs through the recommended ones

$$\alpha_{ij} = \frac{m(\theta_i^k, \theta_j^{-k})}{\Phi(\theta_i^k, \theta_j^{-k})}.$$

Formally, as $\mathcal{A}^*$ is finite, only a finite number of linear problems must be solved. Each linear program returns a set of candidate solutions and a value of the objective function. Fixing $A \in \mathcal{A}^*$, the linear program returns a value $\Pi(A)$, i.e., the profit level, and let $\mathcal{G} = \bigcup_{A \in \mathcal{A}^*} \Pi(A)$ be the set of profit levels for all linear programs with $A \in \mathcal{A}^*$ that implement an equilibrium.

Lemma 5. *The set $\mathcal{G}$ is non-empty and finite with $\Pi(A) < \infty$ for all $A \in \mathcal{A}^*$ and $-\infty < \Pi(A)$ for at least one $A \in \mathcal{A}^*$.*

Key to the proof is to show that the linear program for any given matrix $A \in \mathcal{A}$ is (i) not unbounded and (ii) not infeasible, i.e. the feasible region is non-empty. Given that both (i) and (ii) are satisfied, an optimal solution to the linear program exists and the linear program attains a finite optimal value (Dantzig, 1963).[16]

Theorem 1. *There exists an optimal solution.*

I proceed by showing that an optimal solution exists for any exogenous search costs s_i^k for all $\theta_i^k \in \Theta^k,\ k = A, B$. By Lemma 5, the maximum over set $\mathcal{G}$ is well-defined as $\mathcal{G}$ is finite and bounded such that an optimal solution exists. Next, I show that there exists an optimal solution if the platform chooses search costs s_i^k for all $\theta_i^k \in \Theta^k,\ k = A, B$. Through a series of Lemmas, I prove that the set $\mathcal{G}$ is compact-valued and upper hemicontinuous in the vector of search costs. This implies that the set $\max \mathcal{G}$ is upper semicontinous in the vector of search costs. Therefore, by an extension of the Weierstrass theorem a maximum exists.

To identify properties of the optimal solution, first consider two prominently studied matching rules. As discussed in Section 2, in decentralized matching-and-search markets agents are often assumed to meet according to a random matching technology. A natural question to consider is whether a platform that has access to extensive user data would commit to a random meeting technology as well.

Proposition 1. *Suppose $N^k \cdot N^{-k} > 1$. Then, random matching is generically suboptimal for exogenous search costs and endogenous search fees. Consider the class of functions: $\nu(s_i^k) = \kappa(s_i^k)^\alpha$ with $\kappa \in \mathbb{R}^+$ and $\alpha \in (0, 1)$. Random matching is generically suboptimal within this class of functions.*

The proposition shows that random matching is generically suboptimal for the platform if search costs are exogenous or type-dependent search fees are endogenously chosen.[17] For analytical convenience, I consider the class of concave revenue functions in the proof to determine a knife-edge solution.

[16]Existence follows from the fact that the constraint set is a convex polyhedron. Because the objective is linear and the constraint set is convex, any local extremum will be the global extremum. As the objective is linear, the extremum will be obtained at one of the extreme points of the constraint set, i.e., at the vertices of the polyhedron.

[17]Consider the following definition for *generically suboptimal*. The probability of the case in which random matching is optimal occurs with probability zero when the model parameters are randomly drawn from continuous intervals as defined in the proof.

Consider the nontrivial case in which there are different types to be matched. Under random matching, the conditional probability of meeting a type θ_i^k on side k is the same for all types $\theta_j^{-k} \in \Theta^{-k}$ on side $-k$ and corresponds to the proportion of type θ_i^k in the population. As shown in Appendix A.2, the probability of meeting a type θ_i^k is a function of the inflow, β_i^k, and the probability of exit, δ. In contrast, for given search costs, the optimal solution of the linear program is a function of these and internalizes changes in the search cost. Therefore, random matching is generically suboptimal for given search costs, although it may coincide with the optimal solution for knife-edge $s_i^k, \theta_i^k, \delta$, and β_i^k. This result extends to the case, in which the platform chooses a (linear) search fee. The platform does not choose random matching, but chooses a positive assortative matching rule that maximizes the agents match surplus and extracts all surplus via the search fee.

Proposition 1 highlights that a platform, which has increasing access to user data, does not commit to a random matching technology. Proposition 1 immediately implies that the platform values user data as access to data increases the platform's profit.

Corollary 1. *Suppose a platform has access to data about user types. The platform makes higher profits by using the data to discriminate users by conditioning the matching rule on user types instead of refraining from using user data.*

Second, consider the positive assortative matching rule (PAM) under the assumption that both sides are symmetric with respect to the inflow of new agents: $\beta_i^A = \beta_i^B$, their type space $\Theta^k = \Theta$ and outside options. Under symmetry, PAM matches agents if and only if they are of the same type on both sides of the market. In this particular case, PAM is of special interest in the literature as it maximizes total match surplus when the match utility is supermodular, where an agent's individual match surplus is defined as the difference between the expected match utility on the platform and the agent's outside option. Furthermore, the resulting matching outcome, i.e., the positive assortative matching outcome, is equivalent to the set of stable matchings (Roth and Sotomayor, 1992). That is, matches are individually rational, i.e., yield a utility greater than their outside option, and are pairwise stable, i.e., there exists no blocking pair of agent that would prefer to be matched to each other instead of the equilibrium matching. The next proposition shows under which circumstances the positive assortative matching rule (PAM) is not profit-maximizing under type-dependent search fees and advertising.

Proposition 2. *Suppose both market sides are symmetric.*

(i) PAM is profit-maximizing if the platform can charge arbitrary high type-dependent search fees. Conversely, for every type there exists a threshold $\overline{s}_i$ such that if $s_i < \overline{s}_i$, PAM is suboptimal.

(ii) There exists a threshold $\overline{\delta}$ such that if $\delta \leq \overline{\delta}$ and $\nu(\cdot)$ is concave, PAM is suboptimal.

When the platform commits to a (time-constant) deterministic matching rule such as PAM, agents will accept the recommended match in the first period. Therefore, all agents search for exactly one period, which results in a steady state population equal to the inflow for each type.

First, PAM is indeed profit-maximizing if the platform has pricing power. By charging (high) type-dependent fees, the platform can extract the full surplus from agents, i.e., the expected match value of an assortative match over the agent's outside option. In this case, the "search fee" is paid once, since agents search for only one period. The proposition, however, shows that if the platform cannot commit to high search fees, for example due to a (binding) price ceiling $\overline{s}$, then PAM is no longer optimal. Let $\overline{s}$ be such that s_i violates the condition in Proposition 2 for at least one type $\theta_i \in \Theta$. For the sake of exposition, assume that this is not the lowest type. Then the platform can no longer extract the full surplus from an agent of type θ_i. Then, PAM is not profit-maximizing, as the platform has an incentive to deviate to a matching rule under which type θ_i and the lowest type θ_1 meet with mass ε.The price ceiling $\overline{s}$ is such that whenever type θ_i and type θ_1, θ_i (weakly) rejects θ_1 under the new matching rule. This implies that type θ_i searches longer than one period such that the platform earns more from type θ_i. For example, fees for in-app purchases in Apple's App store are capped at 999.99\$, i.e., $\overline{s} = 999.99\$$. The estimated lifetime utility from a match and hence, potential willingness to pay for a partner could be well above 999.99\$. Traditional matchmakers charge over ten times the amount.[18] Alternatively, users may be reluctant to spend large sums online in one payment, such that the platform's pricing power might be limited as well.

Second, suppose the platform follows an advertising-based business model. If the return to advertising is concave and $\delta \leq \overline{\delta}$, then PAM is suboptimal. Under PAM agents search for only one period. Thus, a profit-maximizing platform would need to impose the highest feasible search cost per agent. With concave advertising returns, however, it becomes more profitable to reduce search costs and increase the mass of participating agents. Since $\delta > 0$ implies a loss in profits due to exogenous attrition that increases with longer search times, a high δ reduces the platform's willingness to trade off longer search durations for lower costs.

Proposition 2 raises the question of why we, as users, do not observe high search fees online, and why matching appears to be (anecdotally) worsening rather than improving. If the platform has pricing power and can perfectly identify users' types, Proposition 2 implies that it induces only one period of search and employs PAM to extract the full surplus from users. This raises the question: under what conditions does the platform have an incentive to induce more search?

In Section 4.2, I examine pricing under complexity constraints. I present an example with two user types, where the platform is limited to setting a single price, and show that under these conditions, the platform prefers not to use PAM. In Section 5, I demonstrate that even when the platform has full pricing power and can implement complex pricing schemes, it does not use PAM and instead relies on advertising—–provided it is sufficiently efficient. Furthermore, when users are overconfident, I show that the platform has an incentive to induce search by lowering fees for high types.

[18]See `https://www.nytimes.com/2024/02/13/business/dating-bounty-roy-zaslavskiy.html?unlocked_article_code=1.VU0.XqAb.q2iJT-p0bHz1&smid=nytcore-ios-share&referringSource=articleShare`

4.2 BINARY TYPES

Suppose now that market sides are symmetric. There are only two types on each side of the market and with slight abuse of notation denote the type set by $\Theta = \{\theta_h, \theta_l\}$ with $\theta_h > \theta_l$. Each type has an outside option of zero.[19] In the previous section, I showed that random matching is suboptimal for the platform, while PAM is optimal if the platform charges $s_h = \theta_h^2$ and $s_l = \theta_l^2$.

This section examines the case in which the platform is constrained in setting agents' search costs. In reality, a platform serves many types of users, which would require complex pricing schemes to extract each agent's surplus. I therefore consider a setting in which both types of agents face the same search cost designed by the platform, $s_h = s_l = s$. One possible interpretation is that both types use the basic service of a (freemium) platform. In this case, the platform is assumed to determine the amount of advertising shown to each agent using the basic service. Alternatively, if payments are involved, agents may choose among (discrete) pricing tiers, with all agents on the same tier paying the same amount—as is common on dating platforms. On job platforms, for example, firms often pay the same price per click when advertising a job in a given submarket. To determine how the matching outcome is affected by the platform-chosen matching rule, the analysis fully characterizes all possible matching outcomes in this example.

As in Section 4.1, I proceed in two steps. First, I characterize the optimal matching rule that implements the mutual acceptance probability matrices that are consistent with Equation 5. Given the first step, I find the optimal matrix of mutual acceptance probabilities that maximize the platform's profit. To identify the optimal matching rule for the platform, suppose for now that s is exogenous.

The first result, Lemma 11,[20] characterizes the optimal matching rule that implements the mutual acceptance probability matrices. With two types, the mutual acceptance matrix takes the following form

$$A = \begin{bmatrix} \alpha(\theta_h, \theta_h) & \alpha(\theta_h, \theta_l) \\ \alpha(\theta_h, \theta_l) & \alpha(\theta_l, \theta_l) \end{bmatrix},$$

where the mutual acceptance probability of the assortative matches are along the diagonal and the mutual acceptance probability of mismatches are off the diagonal. Trivially with one type, the mutual acceptance matrix consists only of one entry. With two types, only three possible matrices can be implemented as part of an equilibrium

$$A_{PAM} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, A_{WPAM} = \begin{bmatrix} 1 & \alpha'_{ij} \\ \alpha'_{ij} & 1 \end{bmatrix}, A_{NAM} = \begin{bmatrix} 1 & \alpha''_{ij} \\ \alpha''_{ij} & 0 \end{bmatrix}, \alpha'_{ij}, \alpha''_{ij} \in [0, 1].$$

Given the platform's matching rule, high type agents can either accept only other high types, or accept low types with positive probability. This results in three possible constellations of mutual acceptance probabilities and thus matching outcomes. If high type

[19]The following analysis qualitatively unaffected as long as the outside options are $\omega_l < \theta_l^2$ and $\omega_h < \theta_h\theta_l$. The platform's profit, however, is quantitatively affected as the platform can extract less rent from each agent.

[20]Lemma 11 and its proof can be found in Appendix B.

only accept high types, low types will always accept high and types, resulting in a positive assortative matching outcome — only agents of the same type accept each other (A_{PAM}). Depending on the matching rule if high types accept low types with positive probability, low types may accept low types, resulting in a weakly assortative matching outcome — high and low types mutually accept the same types of agents (A_{WPAM}). Alternatively, low types may reject low types, resulting in a non-assortative matching outcome — high types accept low types, but low types do not (A_{NAM}).

For each of the three possible matching outcomes, there exists an optimal matching rule that implements the outcome for a range of parameters (Lemma 11). The implementation of the matching outcomes depends crucially on feasibility. Given the total mass of agents that join, the ratio of new high to low type agents, $0 < \beta_h/\beta_l < \infty$, determines which outcome can be implemented, as the ratio affects the steady state population of both types. The positive assortative matching outcome can be implemented for all $0 < \beta_h/\beta_l < \infty$, whereas the weakly assortative and non-assortative outcomes cannot.

Given the existence of an optimal matching rule, which matrix A maximizes the platform's profit for fix search costs? The next proposition summarizes the results.

Proposition 3. *(i) Let $0 \leq s \leq \theta_l^2$. The platform implements A_{PAM} and the matching outcome, $\mathcal{O}_{PAM}$, is positive assortative if*

$$0 \leq \frac{\beta_h}{\beta_l} \leq \left(\frac{\beta_h}{\beta_l}\right)^{(a)}, \quad or \quad \left(\frac{\beta_h}{\beta_l}\right)^{(b)} \equiv \frac{(1-\delta)(\theta_h^2 - s)}{\theta_h(\theta_h - \theta_l) - s + \delta(\theta_h^2 - s)} \leq \frac{\beta_h}{\beta_l}.$$

The platform implements A_{WPAM} and the matching outcome, $\mathcal{O}_{WPAM}$, is weakly positive assortative if

$$\left(\frac{\beta_h}{\beta_l}\right)^{(a)} \leq \frac{\beta_h}{\beta_l} \leq \left(\frac{\beta_h}{\beta_l}\right)^{(b)}.$$

(ii) Let $\theta_l^2 \leq s \leq \theta_h\theta_l$. If $\beta_h \geq \beta_l$, the platform implements A_{WPAM} and the matching outcome is either weakly assortative, $\mathcal{O}_{WPAM}$, or non-assortative for large enough s, $\mathcal{O}_{NAM}$. If $\beta_h < \beta_l$, the platform implements A_{WPAM} and the matching outcome is weakly assortative, $\mathcal{O}_{WPAM}$, or only high types participate if s is large enough.
(iii) Lastly, if $\theta_h\theta_l \leq s \leq \theta_h^2$, low types do not participate on the platform. The mutual acceptance matrix and matching outcome is positive assortative.

First, consider the maximum rent that the platform can extract when the positive assortative matrix, A_{PAM}, is implemented. A high type agent is willing to search the longest for a match with another high type. In this case, the maximum rent the platform can extract from a high type agent is proportional to $\theta_h(\theta_h - \theta_l)$, which is the value of its own type times the *match premium.* The match premium is the gain from being in a match with a high type instead of leaving with a low type. If the platform were to extract more rent, high types would start accepting low types as well, and thus only search for one period. Conversely, if high types always reject low types, the maximum rent the platform can extract from low types is proportional to θ_l^2.

Due to feasibility constraints, the platform is constrained by the ratio of high to low types when choosing the matching rule. The platform can extract the rent from both types — as described above — at

$$\left(\frac{\beta_h}{\beta_l}\right)^{(a)} = \frac{(1-\delta)(\theta_l^2 - s)(s + \delta(\theta_h^2 - s))}{(\theta_h(\theta_h - \theta_l) - s - \delta(\theta_h^2 - s))(s + \delta(\theta_l^2 - s))}, \tag{6}$$

At this "optimal" ratio, high types are just indifferent between accepting and rejecting low types, while low types are just indifferent between participating or not, which results in

$$\phi(\theta_h|\theta_h) = \frac{s + \delta(\theta_h\theta_l - s)}{(1-\delta)\theta_h(\theta_h - \theta_l)}, \tag{7}$$

$$\phi(\theta_l|\theta_l) = \frac{s}{\theta_l^2}. \tag{8}$$

Due to feasibility constraints, the incentive and participation constraints cannot generally bind at the same time while implementing a positive assortative matching outcome. As the ratio increases, relatively more high type agents enter compared to low type agents. In this case, high types inevitably meet high types more often, so the platforms makes the participation constraint binding for low types. The platform must increase the probability of a high type meeting a high type such that high types are left with a rent greater than $\theta_h\theta_l$. As the ratio decreases, relatively few high type agents enter compared to low type agents. The platform makes the incentive constraint binding for high types, leaving a positive rent for low types by increasing the probability of a low type meeting a low type. In both cases, the platform potentially forgoes a significant amount of rent when moving away from the "optimal" ratio.

Second, consider the maximum rent that the platform can extract when the weakly positive assortative matrix, A_{WPAM}, is implemented. Suppose the ratio of high to low types is greater than in Equation 6. Then, the platform can commit to a matching rule in which high types randomize over accepting and rejecting low types, while low types remain indifferent between participating and their outside option. The expected match utility of high types decreases, while the expected match utility of low types increases. For a ratio of high to low types greater than in Equation 6, implementing A_{WPAM} yields a higher profit than A_{PAM}. When implementing A_{PAM}, the platform must increase the meeting probability of assortative pairs as the ratio ${}^{\beta_h}/_{\beta_l}$ increases, otherwise low types will no longer be willing to participate. This implies, however, that the platform forgoes rent from high types. Inducing high types to accept mismatches with positive probability, $\alpha(\theta_h, \theta_l) > 0$, leads to a longer search of low types as they receive a higher expected match utility. Extending the search of low types, implies that there are more low types on the platform, so the platform can also extend the search time of high types.

Third, consider the maximum rent that the platform can extract when the negative assortative matrix, A_{NAM}, is implemented. High types accept both types with positive probability, while low types reject low types and only enter in (mis-)matches with high types. The rent extracted from low types is then proportional to $\theta_l(\theta_h - \theta_l)$, the value of their own type times the *match premium*. The platform, however, never finds it profitable to implement A_{NAM} when it can implement A_{WPAM} as the platform can extract all rent from low types in the latter case, whereas it can only extract the rent premium in the former case. Lastly, if search costs are large, the platform can implement A_{WPAM}, but match low types only to high types if feasible. This in turn results in a non-assortative matching outcome albeit mutual acceptance would be weakly assortative.

For a given inflow of high and low types, $\frac{\beta_h}{\beta_l}$, Proposition 3 presents the matching outcomes that the platform prefers to implement. The assortativity of the matching outcomes is non-monotonic in the ratio of high to low types. For example, the platform can implement the positive assortative matching outcome in markets in which one type dominates. In contrast, the platform implements mismatch in relatively balanced markets.

Corollary 2. *The platform strategically lowers the quality of (recommended) matches. The platform's matching creates two economic inefficiencies: delayed matching and mismatched pairs.*

In other words, the platform recommends mismatches to agents when feasible, i.e., the platform fosters *mismeetings* to delay agent's matches. By delaying matches, the platform increases the payments that it collects from agents per period. Extending users' search, such as in one-sided (matching) markets, has also been shown by for example, Hagiu and Jullien (2011). In addition to mismeetings, the platform also fosters actual mismatches by inducing users to leave in mismatched (inefficient) pairs. Without allowing side payments, mismatches are a way of shifting utility from high to low types to incentivize low type participation.

Next, consider the inefficiencies measured as (i) the amount of mismatch compared to the socially optimal matching and (ii) the length of search for agents. Let the (welfare) loss from mismatch be given by

$$\mathcal{W} = \sum_{(\theta_i,\theta_j)\in\Theta\times\Theta} \alpha(\theta_i,\theta_j)\Phi(\theta_i,\theta_j)(\theta_i\theta_j - \theta_i^2),$$

i.e., the sum over the mass of mismatches times the difference in match utilities between the mismatches and the assortative matches. The expected usage time of an agent is given by their stopping time

$$\mathcal{T}(\theta_i) = \frac{1}{\delta + (1-\delta)\sum_{j=h,l}\alpha(\theta_i,\theta_j)\phi(\theta_j|\theta_i)},$$

such that the total length of search is $\mathcal{T} = \mathcal{T}(\theta_h) + \mathcal{T}(\theta_l)$.

Proposition 4. *(i) If the platform implements A_{PAM} together with matching outcome $\mathcal{O}_{PAM}$, mismatch is $\mathcal{W}_{PAM} = 0$ and $\mathcal{T}(\theta_i)$ is decreasing in s and δ.*
(ii) If the platform implements A_{WPAM} together with matching outcome $\mathcal{O}_{WPAM}$, mismatch is $\mathcal{W}_{WPAM}$ is increasing in s if $\beta_l > \beta_h$ and in- or decreasing in s otherwise as well as decreasing in δ for $s \leq \theta_l^2$ and in- or decreasing in δ otherwise. $\mathcal{T}(\theta_i)$ is decreasing in s and δ.
(iii) If the platform implements A_{WPAM} together with matching outcome $\mathcal{O}_{NAM}$, mismatch is $\mathcal{W}_{NAM} = -\beta_l(\theta_h - \theta_l)^2$ and $\mathcal{T}(\theta_i) = 1$.

By definition, welfare loss is zero under positive assortative matching, as it maximizes total surplus. As search cost or friction δ increases—both of which lower agents' continuation values—the platform must raise assortativity and decrease agents' search time to keep low types participating and high types rejecting low types. In the weakly assortative case, assortativity rises with δ, reducing mismatches as long as $s \leq \theta_l^2$. Since the mass of assortative matches varies with s, the mass of mismatches may increase or decrease depending on whether β_h or β_l is larger. In contrast, welfare loss in the non-assortative case is unaffected by search cost or δ, and the platform induces only one period of search.

4.3 ADVERTISING AND SEARCH FEE

Search Fee If $\nu(s) = s$, the platform charges a linear search fee. It maximizes profit by choosing the fee, given a matching matrix A and the optimal rule that implements it. According to Proposition 3, the platform considers three cases where both types participate: (i) implementing A_{PAM} with the positive assortative outcome $\mathcal{O}(A_{PAM})$, (ii) implementing A_{WPAM} with the weakly positive assortative outcome $\mathcal{O}(A_{WPAM})$, and (iii) implementing A_{WPAM} with the non-assortative outcome $\mathcal{O}(A_{PAM})$. Additionally, there is case (iv) where only high types participate. The next proposition states that the platform can implement any of the four cases depending on the exogenous parameters.

Proposition 5. *For a range of parameters, the platform chooses*
(i) s to maximize $\Pi(A_{PAM})$ s.t. $s \in [0, \theta_l^2] : {}^{\beta_h}/_{\beta_l} \leq ({}^{\beta_h}/_{\beta_l})^{(a)}$ and implements $\mathcal{O}_{PAM}$.
(ii) s to maximize $\Pi(A_{WPAM})$ s.t. $s \in [0, \overline{s}] : ({}^{\beta_h}/_{\beta_l})^{(a)} \leq {}^{\beta_h}/_{\beta_l} \leq ({}^{\beta_h}/_{\beta_l})^{(b)}$ and implements $\mathcal{O}_{WPAM}$.
(iii) $s = \theta_h\theta_l$ and implements $\mathcal{O}_{NAM}$.
(iv) $s = \theta_h^2$ and excludes low types from participating.

The proposition characterizes the platform's optimal solution when s is a uniform search fee paid by agents. When ${}^{\beta_h}/_{\beta_l}$ is relatively low, the platform chooses to implement A_{PAM}. In this case, its profit is bounded by

$$\Pi_{PAM} < \frac{2(1-\delta)}{1-\rho}\left(\beta_h\theta_h(\theta_h - \theta_l) + \beta_l\theta_l^2\right),$$

which corresponds to the maximum surplus the platform can extract as $\delta \to 0$, when high types are indifferent between accepting or rejecting low types, and low types are indifferent between participating or opting out. As in Proposition 3 if ${}^{\beta_h}/_{\beta_l} \geq ({}^{\beta_h}/_{\beta_l})^{(a)}$, the platform can implement A_{WPAM} and outcome $\mathcal{O}_{WPAM}$.

For ${}^{\beta_h}/_{\beta_l} \geq 1$, the platform can implement the non-assortative outcome. As agents only search for one period, the profit given $\mathcal{O}_{NAM}$ is maximized if the search fee is set as high as possible. Thus, the platform chooses $s = \theta_h\theta_l$ yielding a profit of

$$\Pi_{NAM} = \frac{2(1-\delta)}{1-\rho}\left(\beta_h + \beta_l\right)\theta_h\theta_l.$$

With increasing ${}^{\beta_h}/_{\beta_l}$, the platform finds it profitable to charge the highest possible search fee $s = \theta_h^2$ to extract the full surplus from high types, while excluding low types from participating on the platform. This holds as with increasing ${}^{\beta_h}/_{\beta_l}$, the share of revenue from high types grows larger and hence, it becomes more profitable exploiting only one type of users. The platform makes a profit of

$$\Pi_{\theta_h} = \frac{2(1-\delta)}{1-\rho}\beta_h\theta_h^2.$$

Advertising Now, suppose the platform sells the attention of its users to advertisers. The platform decides on the advertising intensity, which is related to the search cost that users experience. Let $\nu(s)$ be the revenue per unit of search cost to users. Recall that $\nu(s)$ is an increasing, concave function of search cost s with $\nu(0) = 0$. This assumption excludes functions that are convex, i.e., under which the platform could prefer an advertising intensity that induces users to stay for only one period, thereby significantly reducing the mass of active users. The platform maximizes its profit with respect to s

$$\max_s \frac{2\nu(s)(1-\delta)}{1-\rho}\left(\sum_{\theta_i\in\Theta} f(\theta_i^k)(s)\right),$$

where the mass of agents of type θ_i is given by Lemma 3 subject to the conditions in Proposition 3. The platform chooses $s = s^A$ such that

$$\frac{\nu(s^A)}{\nu'(s^A)} = -\frac{\sum_k\sum_i f(\theta_i^k)}{\partial\sum_k\sum_i f(\theta_i^k)/\partial s}\,|s = s^A\,. \tag{9}$$

Under the above condition, the marginal cost of an increase in search cost, given by the semi-elasticity of demand on the right-hand side, is equal to the marginal benefit of an increase in search cost, given by the semi-elasticity of advertising revenue.

Proposition 6. *If $\frac{\nu(s)}{\nu'(s)} \geq s$ for $s \in [0, \theta_h^2]$, the platform chooses search costs that are lower or equal than a uniform search fee. Furthermore, if ${}^{\nu(\theta_h\theta_l)}/_{\nu(\theta_h^2)} \geq {}^{\beta_h}/_{\beta_h+\beta_l}$, i.e. $\nu(\cdot)$ is sufficiently concave at high search costs, the platform finds it profitable to never exclude low types from the search process.*

5. EXPLANATIONS

If $\nu(s_i^k) = s_i^k$, the optimal contract is a set of personalized search fees. The platform maximizes the total match surplus as in Appendix A.2 and extracts the surplus from each agent via the fee. Considering the simplified model from Section 4.2, the platform commits to the positive assortative matching rule and personalized fees ($s_h = \theta_h^2, s_l = \theta_l^2$) (see Proposition 2). Under the positive assortative matching rule, agents meet their match in the first period. The platform's profit is

$$\Pi^{PAM} = \frac{2(1-\delta)}{(1-\rho)}(\beta_h\theta_h^2 + \beta_l\theta_l^2).$$

5.1 ADVERTISEMENT

Advertisement plays a key role in the digital economy. More specifically, in the light of the application to dating and job search platforms, a substantial share of these platforms rely on advertisement as a source of revenue, see Appendix C for an overview of dating and job search apps that show advertisement. In the following example, I highlight that a (partly) advertising-based business model can outperform profits generated by personalized prices demonstrated by the following example.

Example 1. Consider the concave function $\nu(s) = \kappa s^{\alpha}$ for $\alpha = \frac{1}{2}$. Figure 2 plots the function for different values of κ. Furthermore, assume that $\beta_h < \beta_l$ and let the value of a high type, $\theta_h = 2$, be twice as large as the value of a low type, $\theta_l = 1$. Denote the ${}^{1-\delta}/_{1-\rho} = \gamma$.

For $\beta_h < \beta_l$, the platform either implements A_{PAM} or A_{WPAM}. To maximize advertising profits, the platform chooses $s^A \in [0, \theta_l^2]$ to solve

$$\frac{\beta_h}{\beta_l} = \frac{(1-\delta)(\theta_l^2 - s^A)(s + \delta(\theta_h^2 - s^A))}{(\theta_h(\theta_h - \theta_l) - s^A - \delta(\theta_h^2 - s^A))(s^A + \delta(\theta_l^2 - s^A))},$$

at which the agents' search time under A_{PAM} and A_{WPAM} coincides. Furthermore recall that if the condition is satisfied, the agents' search time is maximized as low types are indifferent between participating or not and high types are indifferent between accepting and rejecting low types (and reject with probability one). The platform's advertising profit is

$$\Pi^A = 2\gamma\kappa\sqrt{s^A}\left(\frac{\beta_h\theta_h(\theta_h - \theta_l)}{s^A + \delta(\theta_h^2 - s^A)} + \frac{\beta_l\theta_l^2}{s^A + \delta(\theta_l^2 - s^A)}\right).$$

For the chosen parameters, s^A is equal to $\theta_l^2 = 1$ if $\beta_h = 0$ and strictly larger than zero for β_h approaching β_l. The profits for $\beta_h = 0$ are

$$\Pi^A(\beta_h = 0) = \gamma\kappa\sqrt{\theta_l^2}\beta_l = \gamma\kappa\beta_l,$$
$$\Pi^{PD}(\beta_h = 0) = \gamma\beta_l\theta_l^2 = \gamma\beta_l,$$

which coincide for $\kappa = 1$. Thus, for $\kappa > \underline{\kappa} = 1$, advertising profits are larger than the profits of the optimal contract for some $\beta_l > 0$. Now, let β_h approach β_l, the profits are

$$\Pi^A(\beta_h = 0) = 2\gamma\kappa\sqrt{s^A}\left(\frac{\beta_h\theta_h(\theta_h - \theta_l)}{s^A + \delta(\theta_h^2 - s^A)} + \frac{\beta_l\theta_l^2}{s^A + \delta(\theta_l^2 - s^A)}\right),$$
$$\Pi^{PD}(\beta_h \to \beta_l) = 4\gamma\beta_l(\theta_h^2 + \theta_l^2).$$

Then, there exists a $\kappa > \overline{\kappa}$ such that advertising profits are larger than the profits of the optimal contract for all $\beta_h \in [0, \beta_l)$. For the values in this example, $\overline{\kappa} \approx {}^3/_2$.

For general revenue functions $\nu(s)$, an advertisement-based business model generates higher profits than charging personalized prices if advertisement revenue is sufficiently efficient compared to its nuisance:

$$\frac{\nu(s)}{s} \geq \frac{\beta_h\theta_h^2 + \beta_l\theta_l^2}{s(\mathcal{T}(\theta_h) + \mathcal{T}(\theta_l))},$$

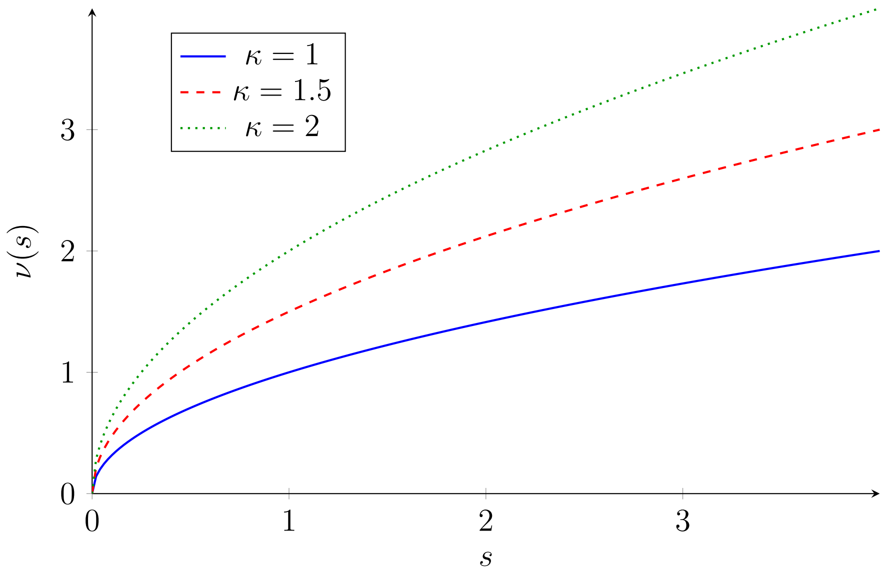

Figure 2: $\nu(s) = \kappa\sqrt{s}$ for different κ

where the numerator is the full surplus that can be extracted from agents under PAM with personalized fees and the denominator is the total amount of search cost that agent's pay while searching under advertising. Note that if the market is extremely unbalanced, i.e. if only high types are in the market, advertising is less profitable as long as $\nu(\theta_h^2)/\theta_h^2 \leq 1$.

5.2 OVERCONFIDENCE

Up to this point, the model has assumed that agents behave rationally and have a correct expectation about their own type. In the following, I will introduce a fraction of overconfident agents, i.e., agents who perceive themselves to be of a higher type than they actually are. In the simplest example, an overconfident low type perceives itself as a high type. Overconfidence is a widely documented bias in the psychology and behavioral economics literature.[21]

Especially in dating markets and labor markets overconfidence is thought to be prevalent for example, when it comes to a person's own attractiveness or ability. In dating markets, both women and men prefer attractive over unattractive profiles regardless of their own attractiveness (Egebark et al., 2021). Bruch and Newman (2018, 2019) analyze the structure of online dating markets in US cities and provide suggestive evidence for the fact that the majority of users contacts a partner who is more desirable than they are instead of contacting a partner who is as desirable than they are. One possible explanation is documented by Greitemeyer (2020), that is, more unattractive people are unaware

[21]Ample evidence suggests that on average agents overestimate their ability, traits and prospects. Such overconfidence has been documented in laboratory experiments by Burks et al. (2013); Dubra (2015); Charness et al. (2018). Additionally, there is empirical evidence that consumers are overoptimistic regarding future self-control when signing up for a gym membership (Della Vigna and Malmendier, 2006), workers overpredict their own productivity (Hoffman and Burks, 2020), and some CEOs are overoptimistic regarding their firm's performance (Malmendier and Tate, 2005, 2008).

of their (un-)attractiveness from a psychological perspective. Similarly in labor markets, Spinnewijn (2015) and Mueller et al. (2021) find that the unemployed overestimate how quickly they will find a job and are persistently overconfident about their desirability to firms. In line with the empirical evidence, Dargnies et al. (2019) document in a laboratory experiment that agents who are overconfident are less likely to accept earlier job offers in a matching market.

Following this evidence, consider the following simple extension to the model in Section 4.2. There exists a symmetric share of λ overconfident users on each side of the market. An overconfident user has type θ_l, but persistently believes to have type θ_h, i.e. is stubborn and does not learn their true type. Denote the overconfident type by $\hat{\theta}_l$. Other agents correctly identify overconfident types as low types. Following Definition 3, an overconfident type chooses their strategy confidently believing in their misperceived type. As a result of overestimating their own type, they, however, are overoptimistic about the likelihood of being accepted by other. As before, users incur search costs and become inactive with probability δ.[22]

As overconfidence has been identified in empirical and experimental setting, I suppose that the platform can perfectly identify overconfident users as well. The platform chooses matching rule $\mathcal{M}$, which consists of $\phi(\cdot|\theta_i)$ for $\theta_i \in \{\theta_l, \theta_h, \hat{\theta}_l\}$, and search costs (s_h, s_l). As a benchmark, suppose the platform induces only one period of search by charging $(s_h = \theta_h^2, s_l = \theta_l^2)$ and choosing the positive assortative matching rule in which high types only meet each other and (true) low types, which includes overconfident types, only meet each other. The platform's profits are

$$\Pi_{PAM}^{OC} = \frac{2(1-\delta)}{1-\rho}(\beta_h\theta_h^2 + \beta_l(1-\lambda)\theta_l^2 + \beta_l\lambda\theta_h^2).$$

To show that the platform can improve on this profit, let the platform induce search by inducing high types to reject low types. The matching rule and search costs must satisfy the participation constraint of low types and the incentive constraint of high types

$$\theta_h\theta_l \leq \frac{(1-\delta)(-s + \phi(\theta_h|\theta_h)\theta_h^2)}{\delta + (1-\delta)\phi(\theta_h|\theta_h)}, \tag{IC-θ_h}$$

$$0 \leq \frac{(1-\delta)(-s + \phi(\theta_l|\theta_l)\theta_l^2)}{\delta + (1-\delta)\phi(\theta_l|\theta_l)}. \tag{10}$$

Given both constraints are satisfied, the participation constraint of high types and the incentive constraint of low types (to reject low types) are satisfied as well. Next, consider the acceptance behavior of an overconfident type. Given their perception of the game, rejecting low types is perceived optimal if

$$\theta_h\theta_l \leq \frac{(1-\delta)(-s + \phi(\theta_h|\theta_h)\theta_h^2)}{\delta + (1-\delta)\phi(\theta_h|\theta_h)}, \tag{PIC-$\hat{\theta}_h$}$$

[22]Note that δ can have an additional interpretation in the presence of overconfident users. If overconfident users do not find a match, δ can be interpreted as the probability that an overconfident agent leaves due to growing dissatisfaction with the platform.

which coincides with the incentive constraint of high types. Similarly, they face the same perceived participation constraint. The payoff from participation is

$$-\frac{s}{\delta} < 0,$$

since overconfident users reject low types, but high types never accept overconfident types. This leads them to search until they exogenously exit with probability δ.

Remark. Overconfident users search too intensively.

Proposition 7. *(Overconfidence) Let $\lambda^* \equiv \frac{\beta_h}{\beta_l}\frac{\delta\theta_h\theta_l}{(1-\delta)\theta_h^2-\theta_h\theta_l}$. For $\lambda < \lambda^*$, the platform maximizes profits by setting $(s_h = \theta_h^2, s_l = \theta_l^2)$ and inducing only one period of search. The platform's profit is Π^{OC}_{PAM}. For $\lambda \geq \lambda^*$, the platform maximizes profits by setting $(s_h = \theta_h(\theta_h - \theta_l) - \delta/1-\delta\theta_h\theta_l, s_l = \theta_l^2)$ and inducing search from overconfident users. The platform's profit is*

$$\Pi^{OC}_S = \frac{2(1-\delta)}{1-\rho}\left(\beta_h(\theta_h(\theta_h-\theta_l) - \frac{\delta}{1-\delta}\theta_h\theta_l) + \beta_l(1-\lambda)\theta_l^2 + \frac{\beta_l\lambda(\theta_h(\theta_h-\theta_l) - \frac{\delta}{1-\delta}\theta_h\theta_l)}{\delta}\right).$$

Anecdotes from Dating Apps, such as Tinder, provide evidence for the fact that less than 10% of users account for a disproportional amount of revenue.[23] On Tinder, an average user spends around 30$ in in-app purchases and subscriptions, whereas "heavy" users would spend 10 times the amount.

Consider the following example to illustrate that in markets with many low types, already a small percentage of overconfident users can be sufficient to achieve higher profits.

Example 2. Let $\beta_h = \frac{1}{4}, \beta_l = \frac{3}{4}, \delta = 1/10, \theta_h = 2, \theta_l = 1$. Then, $\lambda \geq 4.2\%$. For low values of δ, a relatively small percentage of overconfident users is necessary to substantially increase the platforms profit. Note that δ is directly related to the stopping time of overconfident users, i.e. overconfident users search for ten periods before they exit. More generally, consider the following comparative statics.

Corollary 3. *λ^* increases in δ, and $\frac{\beta_h}{\beta_l}$.*

Intuitively, the necessary share of overconfident users decreases if δ becomes small as overconfident users search for more periods. If the ratio $\frac{\beta_h}{\beta_l}$ increases, i.e. there are more high types than low types in the market, the platform needs to rely more on overconfident users. The reason is that given that the platform lowers the search fee for high types to exploit overconfident users, they become less profitable. Hence, with more high types, there must be more overconfident types to offset the loss from high types.

[23]See https://uxdesign.cc/how-tinder-drives-over-1-6-billion-in-revenue-8006e718e761 and the referenced podcast therein, https://open.spotify.com/episode/1ZfL2Mq1nONzyVKKerynvZ?si=UBlpCunARLW8jPfNNYK4dw.

6. CONCLUSION

On matching platforms, the misalignment of incentives between users and the platform becomes more problematic as platforms collect more data and develop more predictive algorithms. This paper presents a model in which a platform has perfect information about its users' types and matches them to its advantage. In contrast, random matching corresponds to the case where the platform has no information about its users' types. The platform benefits from more information about its users' types: Random matching is strictly suboptimal.

Both sorting and search time have implications for real-world markets. The platform's algorithm can support the socially optimal matching. But even absent exogenous search costs and search frictions, the algorithm can also foster non-assortative matching outcomes in fully symmetric markets resulting in mismatch. Additionally, it increases users' search time by recommending unsuitable matches. While mismatch has a negative impact on productivity and long-term unemployment in labor markets (Şahin et al., 2014; McGowan and Andrews, 2015), assortative mating in marriage markets is a driver of household inequality (Pestel, 2017; Eika et al., 2019; Almar et al., 2023). Therefore, if policies aim to reduce mismatch — as in labor markets — policymakers should be concerned about matching platforms that employ the business models described above. Rather than relying on platforms to reduce search frictions, the platform's algorithm is a potential source of additional mismatch. In contrast, dating apps can make a positive contribution to reducing household inequality.

Empirical evidence on online matching and search platforms is mixed. For example, in dating markets Hitsch et al. (2010) show that matches are approximately efficient and stable. The authors, however, rely on data before the advent of large dating apps. In contrast, more recent evidence, such as Sharabi and Dorrance-Hall (2024), finds that people who meet online are less satisfied in their marriages. In labor markets, Kroft and Pope (2014) shows that Craigslist has no effect on the unemployment rate. Similarly, Gürtzgen et al. (2021) provide evidence that online searches do not affect employment stability or wage outcomes, but instead increase the proportion of unsuitable candidates in job applications.

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A. APPENDIX

A.1 LINEAR PROGRAMMING FORMULATION

The linear programming formulation of the platform's problem in Lemma 4 is given in the following. For $\alpha(\theta_i^k, \theta_j^{-k}) \in \{0,1\}$, the platform's optimization problem can be represented by the following (mixed integer) linear program:

$$\max_{\{\Phi(\cdot), m(\cdot)\}_{ij}^k} \sum_{k=A,B} \sum_{\theta_i^k \in \Theta^k} \frac{(1-\delta)\nu(s_i^k)}{1-\rho} f(\theta_i^k), \tag{11}$$

subject to participation constraints

$$\beta_i^k \omega_i^k \leq f(\theta_i^k)(\delta\omega_i^k - (1-\delta)s_i^k) + (1-\delta)\sum_j m(\theta_i^k, \theta_j^{-k})\theta_i^k \theta_j^{-k}, \forall \theta_i^k \in \Theta^k,\ k = A, B, \tag{12}$$

incentive constraints

$$\begin{aligned} \beta_i^k \theta_i^k \theta_j^{-k} + \alpha(\theta_i^k, \theta_j^{-k})(-\beta_i^k \theta_i^k \theta_j^{-k}) \leq & f(\theta_i^k)(\delta\omega_i^k - (1-\delta)s_i^k) + (1-\delta)\sum_j m(\theta_i^k, \theta_j^{-k})\theta_i^k \theta_j^{-k} \\ \leq & \left(\frac{\beta_i^k}{\delta}\theta_i^k \theta_j^{-k} - \beta_i^k \theta_i^k \theta_j^{-k}\right)(1 - \alpha(\theta_i^k, \theta_j^{-k})) + \beta_i^k \theta_i^k \theta_j^{-k}, \\ & \forall \theta_i^k \in \Theta^k,\ k = A, B, \end{aligned} \tag{13}$$

feasibility and steady state constraints

$$\sum_{\theta_j^{-k} \in \Theta^{-k}} \Phi(\theta_i^k, \theta_j^{-k}) + \mathbf{1}_{k=B}\Phi(\theta_i^k, \omega_i^k) = f(\theta_i^k), \forall \theta_i^k \in \Theta^k,\ k = A, B, \tag{14}$$

$$f(\theta_i^k) = \frac{\beta_i^k - (1-\delta)\sum_j m(\theta_i^k, \theta_j^{-k})}{\delta}, \forall \theta_i^k \in \Theta^k,\ k = A, B, \tag{15}$$

and constraints on the matched and recommended pairs $\forall(\theta_i^k, \theta_j^{-k}) \in \Theta^k \times \Theta^{-k}$. First, the mass of recommended and matched pairs must be non-negative and the mass of matched pairs cannot be greater than the mass of recommended pairs

$$\Phi(\theta_i^k, \theta_j^{-k}) \geq 0, m(\theta_i^k, \theta_j^{-k}) \geq 0, \tag{16}$$

$$m(\theta_i^k, \theta_j^{-k}) \leq \Phi(\theta_i^k, \theta_j^{-k}). \tag{17}$$

Second, the mass of matched pairs must be smaller than the largest possible mass of the agents, i.e. the mass that arises when agents only exit upon becoming inactive β_i^k/δ, times the acceptance probability and larger than the mass of recommended pairs minus the largest possible mass times the probability of a rejection

$$m(\theta_i^k, \theta_j^{-k}) \leq \frac{\min\{\beta_i^k, \beta_j^{-k}\}}{\delta}\alpha(\theta_i^k, \theta_j^{-k}), \tag{18}$$

$$m(\theta_i^k, \theta_j^{-k}) \geq \Phi(\theta_i^k, \theta_j^{-k}) - \frac{\min\{\beta_i^k, \beta_j^{-k}\}}{\delta}(1 - \alpha(\theta_i^k, \theta_j^{-k})). \tag{19}$$

This ensures that the mass of matched pairs must be smaller than the mass of recommended pairs and for $\alpha(\theta_i^k, \theta_j^{-k}) = 0$ the mass of matched pairs cannot be greater than zero. To accommodate for mixed acceptance probabilities of agents, consider an agent of type θ_m^k that is indifferent between accepting and rejecting a type θ_s^{-k}. Hence, θ_m^k could randomize over the acceptance probability towards type θ_s^{-k}: $\sigma_k(\theta_m^k, \theta_s^{-k}) \in (0, 1)$. Conceptually, this imposes indifference or equality on some constraints rather than inequalities in the original formulation above. For any pair $(\theta_m^k, \theta_s^{-k}) \in \Theta^k \times \Theta^{-k}$ for which $\alpha(\theta_m^k, \theta_s^{-k}) \in (0, 1)$, the adjusted incentive constraints are

$$\beta_m^k \theta_m^k \theta_s^{-k} = f(\theta_m^k)(\delta\omega_m^k - (1-\delta)s_m^k) + (1-\delta)\sum_j m(\theta_m^k, \theta_j^{-k})\theta_m^k \theta_j^{-k}, \text{ for } \theta_m^k, \tag{20}$$

$$\beta_s^{-k} \theta_m^k \theta_s^{-k} \geq f(\theta_s^{-k})(\delta\omega_s^{-k} - (1-\delta)s_s^{-k}) + (1-\delta)\sum_j m(\theta_m^k, \theta_j^{-k})\theta_m^k \theta_j^{-k}, \text{ for } \theta_s^k, \tag{21}$$

where θ_m^k is indifferent between accepting and rejecting θ_s^{-k} and θ_s^{-k} (weakly) accepts θ_m^k. The constraints on the mass of recommended and matched pairs are

$$m(\theta_m^k, \theta_s^{-k}) \leq \frac{\min\{\beta_m^k, \beta_s^{-k}\}}{\delta}, \text{ for } (\theta_m^k, \theta_s^{-k}), \tag{22}$$

$$m(\theta_m^k, \theta_j^{-k}) \leq \Phi(\theta_m^k, \theta_s^{-k}), \text{ for } (\theta_m^k, \theta_s^{-k}). \tag{23}$$

The linear program can be summarized in the subsequent lemma.

Lemma 6 (Linear Program)**.** *Fix any mutual acceptance matrix A. The platform's maximization problem yields the same profit as linear programming problem with objective function in Equation 11 subject to constraints Equation 12 through 16 for any $\alpha(\theta_i^k, \theta_j^{-k}) \in \{0, 1\}$, and for any pair $(\theta_m^k, \theta_s^{-k}) \in \Theta^k \times \Theta^{-k}$ for which $\alpha(\theta_s^k, \theta_m^{-k}) \in (0, 1)$, replace Equation 13 for θ_m^k by Equation 20 and replace Equation 13 for θ_s^k by Equation 21 and replace Equations 18 to 19 for $(\theta_m^k, \theta_s^{-k})$ by Equations 22 to 23.*

Note on Standard Form of a Linear Program To abbreviate future arguments, I relate the linear program to the standard form of a linear program. The matrix notation is

$$\begin{aligned} &\max xc^T, \\ &s.t. Hx \leq b, x \geq 0, \end{aligned}$$

where $c \in \mathbb{R}^n$. The variable vector $x \in X \subset \mathbb{R}^n$ consists of n variables, i.e., the mass of recommended and matched pairs, and is an element of the compact set X as each mass takes a value in $[0, \beta_i/\delta]$. The m inequalities are given by matrix $H \in \mathbb{R}^{m \times n}$. Equalities, such as the feasibility constraints, can be expressed as two opposite inequalities. Vector $b \in \mathbb{R}^m$ captures the right-hand side of the inequalities. $\mathcal{P} \equiv \{x \in \mathbb{R}^n | Hx \leq b\}$ is the feasible region given by the inequality constraints.

A.2 BENCHMARKS

This section analyzes two polar cases, in which the intermediary has full information about agent's types and is able to extract the full rent from the matching output or the intermediary has no information about agent's types and must match agents at random.

Socially-Optimal Matching The first benchmark constitutes the case in which the intermediary (or a social planner) provides the socially-optimal matching under the premise that agent's types can be identified perfectly. The intermediary or social planner maximizes the sum of total matching outputs given that agents only search for one period. The matching output function is supermodular, i.e. types of both sides are complements. The socially-optimal matching is the solution to the linear program

$$\max_{M} \sum_{k=A,B} \sum_{\theta_j^{-k} \in \Theta^{-k}} \sum_{\theta_i^k \in \Theta^k} (\theta_i^k \theta_j^{-k} - \omega_i^k) m(\theta_i^k, \theta_j^{-k}) \tag{24}$$

subject to feasibility

$$\sum_{\theta_j^{-k} \in \Theta^{-k}} m(\theta_i^k, \theta_j^{-k}) \leq \beta_i^k, \forall \theta_i^k \in \Theta^k, \tag{25}$$

$$\sum_{\theta_i^k \in \Theta^k} m(\theta_i^k, \theta_j^{-k}) \leq \beta_j^{-k}, \forall \theta_j^{-k} \in \Theta^{-k}, \tag{26}$$

$$m(\theta_i^k, \theta_j^{-k}) \geq 0, \forall (\theta_i^k, \theta_j^{-k}) \in \Theta^k \times \Theta^{-k}. \tag{27}$$

The linear program follows the optimal assignment problem by Koopmans and Beckmann (1957) and Shapley and Shubik (1971). Both agents that form the match $(\theta_i^k, \theta_j^{-k})$ receive the output $\theta_i^k \cdot \theta_j^{-k}$.

Remark. If markets are fully symmetric, the socially optimal matching is $m(\theta_i^k, \theta_j^{-k}) = \beta_i^k$ if $\theta_i^k = \theta_j^{-k}$. The outcome is said to exhibit positive assortative matching.

If market sides are fully symmetric, $\beta_i^A = \beta_i^B$, the solution to the linear program is attained with $m(\theta_i^k, \theta_j^{-k}) \in \{0, \beta_i^k\}$, that is a pair is either matched with probability one or not matched. Although the linear program permits partial or fractional matching of agents, Dantzig (1963) showed that the maximum value of the objective is attained with probabilities in $\{0, 1\}$.

For symmetric populations of agents, optimality requires that no individual remains unmatched, such that the feasibility constraints must hold with equality. Otherwise, the social planner can increase welfare by assigning an unmatched agent to another unmatched agent as the value of their match is greater than zero. The objective is maximized if $m(\theta_i^k, \theta_j^{-k}) = \beta_i^k$ when $\theta_i^k = \theta_j^{-k}$ by applying the rearrangement inequality (Hardy et al., 1952).

Random Matching The second benchmark is a random matching market. For example, if an intermediary has no information (data) about agents' types, and thus cannot condition on any observables, the intermediary's matching rule incorporates random meetings between agents. A random matching market may also reflect offline meetings between agents that are not intermediated by any platform.

A random matching market is a tuple $(\hat{\Theta}^k, f(\theta_i^k))_{k=A,B}$ with parameters (s_i^k, δ).The analysis builds on the model of Lauermann and Nöldeke (2014).[24]

The total mass of agents on side k is $\overline{f}^k = \sum_{\theta_i^k \in \Theta^k} f(\theta_i^k)$. Since each agent can meet at most one agent per unit of time, the total mass of meetings is given by $\min\{\overline{f}^A, \overline{f}^B\}$. Given that meetings are random, the fraction of meetings that involve type θ_i^k on side k and type θ_j^{-k} on side $-k$ is then

$$\frac{f(\theta_i^k) f(\theta_j^{-k}) \min\{\overline{f}^k, \overline{f}^{-k}\}}{\overline{f}^k \cdot \overline{f}^{-k}}.$$

If $\overline{f}^k > \overline{f}^{-k}$, then the mass of agents on side k that meet their outside option is $\Phi(\theta_i^k, \omega_i^k) = \frac{\overline{f}^k - \overline{f}^{-k}}{\overline{f}^k}$. The probability to meet type θ_j^{-k} on side $-k$ conditional on being an agent of any type on side k is

$$\phi(\theta_j^{-k}) = \frac{f(\theta_j^{-k})}{\overline{f}^{-k}} \frac{\min\{\overline{f}^k, \overline{f}^{-k}\}}{\overline{f}^k},$$

where the probability that type θ_i^k on side k exits the search process in a match with type θ_j^{-k} is

$$\mu(\theta_i^k, \theta_j^{-k}) = \frac{(1-\delta)\alpha(\theta_i^k, \theta_j^{-k})\phi(\theta_j^{-k})}{\delta + (1-\delta)\sum_{\theta_j^{-k}} \alpha(\theta_i^k, \theta_j^{-k})\phi(\theta_j^{-k})},$$

where $\mu(\theta_i^k, \omega_i^k) = 1 - \sum_{\theta_j^{-k}} \mu(\theta_i^k, \theta_j^{-k})$ is the probability that type θ_i^k remains unmatched.

Let $(f(\theta_i^k), \alpha(\theta_i^k, \theta_j^{-k})_{ij})_{k=A,B}$ be a steady state. Then M with entries given by

$$m(\theta_i^k, \theta_j^{-k}) = \frac{\alpha(\theta_i^k, \theta_j^{-k}) f(\theta_i^k) f(\theta_j^{-k}) \min\{\overline{f}^k, \overline{f}^{-k}\}}{\overline{f}^k \cdot \overline{f}^{-k}}. \tag{28}$$

is the unique matching outcome induced by the steady state under random matching. Vice versa, if M is a steady state matching outcome then $f(\theta_i^k), \alpha(\theta_i^k, \theta_j^{-k})$ is given by

$$f(\theta_i^k) = \frac{\beta_i^k}{\delta} \mu(\theta_i^k, \omega_i^k), \tag{29}$$

$$\alpha(\theta_i, \theta_j) = m(\theta_i^k, \theta_j^{-k}) \frac{\overline{f}^k \cdot \overline{f}^{-k}}{f(\theta_i^k) f(\theta_j^{-k}) \min\{\overline{f}^k, \overline{f}^{-k}\}}, \tag{30}$$

[24]In contrast to Lauermann and Nöldeke (2014), agents may face explicit search cost s_i^k in addition to δ.

where $\alpha(\theta_i^k, \theta_j^{-k}) \leq 1$ for all $(\theta_i^k, \theta_j^{-k}) \in \hat{\Theta}^k \times \hat{\Theta}^{-k}$ and $m(\theta_i^k, \omega_i^k)$ is the probability of ending up with one's outside option. Matching M is an **equilibrium** matching if

$$m(\theta_i^k, \theta_j^{-k}) = \begin{cases} 0 & \text{if } \theta_i^k \theta_j^{-k} < V^C(\theta_i^k) \text{ or } \theta_i^k, \theta_j^{-k} < V^C(\theta_j^{-k}) \\ \frac{f(\theta_i^k) f(\theta_j^{-k}) \min\{\overline{f}^k, \overline{f}^{-k}\}}{\overline{f}^k \cdot \overline{f}^{-k}} & \text{if } \theta_i^k \theta_j^{-k} > V^C(\theta_i^k) \text{ and } \theta_i^k \theta_j^{-k} > V^C(\theta_j^{-k}) \end{cases}$$

holds for all $(\theta_i^k, \theta_j^{-k}) \in \hat{\Theta}^k \times \hat{\Theta}^{-k}$.

B. APPENDIX: PROOFS

B.1 MULTIPLE TYPES

Proof of Lemma 1 and 2 in the text.

Proof of Lemma 3. If $\eta_i^k < 1$ and $\Phi(\theta_i^k, \omega_i^k) \geq 0$ are optimal for any $\theta_i^k \in \Theta^k$, then $\eta_i^k = 1$ and $\Phi'(\theta_i^k, \omega_i^k,)$ are also optimal such that

$$\Phi(\theta_i^k, \theta_j^{-k}) = \Phi'(\theta_i^k, \theta_j^{-k}), \tag{31}$$

$$(1 - \eta_i^k) f(\theta_i^k) + \Phi(\theta_i^k, \omega_i^k) = \Phi'(\theta_i^k, \omega_i^k), \tag{32}$$

for all $\theta_i^k \in \Theta^k$ and $\theta_j^{-k} \in \Theta^{-k}$. For given $\eta_i^k < 1$ and matching rule $\mathcal{M}$, Equation 31 and 32 determine the new matching rules for $\eta_i^k = 1$.

Now consider the participation for type θ_i^k. In equilibrium, the participation constraint must be binding for agents to find it optimal to randomize in their participation decision. Suppose the participation constraint is binding, then it can be rewritten as

$$(1 - \delta) s_i^k = (1 - \delta) \sum_j \alpha(\theta_i^k, \theta_j^{-k}) \phi(\theta_j^{-k} | \theta_i^k) \left(\theta_i^k \theta_j^{-k} - \omega_i^k\right).$$

As the masses are the same by Equation 31, the total surplus extracted by the platform remains the same as optimality requires that the participation constraint continues to bind. Multiplying with the total mass of agents of type θ_i^k if $\eta_i^k < 1$ yields

$$(1 - \delta) \eta_i^k f(\theta_i^k) s_i^k = \underbrace{(1 - \delta) \sum_j \alpha(\theta_i^k, \theta_j^{-k}) \Phi(\theta_j^{-k} | \theta_i^k) \left(\theta_i^k \theta_j^{-k} - \omega_i^k\right)}_{\text{Total Surplus}}.$$

Similarly, when multiplying with the total mass of agents of type θ_i^k if $\eta_i^k = 1$ yields

$$(1 - \delta) f'(\theta_i^k) s_i^{k,'} = (1 - \delta) \sum_j \alpha(\theta_i^k, \theta_j^{-k}) \Phi(\theta_j^{-k} | \theta_i^k) \left(\theta_i^k \theta_j^{-k} - \omega_i^k\right).$$

Therefore, the total surplus extracted is the same in both cases by construction. Thus, if the platform charges a search fee both cases yield the same surplus.

In the case of advertising note that $f'(\theta_i^k)$ must increase if η_i^k increases, i.e. the steady-state mass increases if more agents participate everything else equal. Rewrite equation 31 as

$$\eta_i^k f(\theta_i^k)\phi(\theta_j^{-k}|\theta_i^k) = f'(\theta_i^k)\phi'(\theta_j^{-k}|\theta_i^k)$$

Therefore, to fulfill the equality in Equation 31 $\phi'(\theta_j^{-k}|\theta_i^k)$ must decrease to decrease the right-hand side. This implies that $s_i^{k,'} < s_i^k$ and therefore, the platform profit increases in the advertising case due to the concavity of $\nu(s_i^k)$. $\square$

Proof of Lemma 5 As defined in the Section 4.1, the set $\mathcal{G}$ is the set of profit levels following from all linear programs with $A \in \mathcal{A}^*$. I show that the set $\mathcal{G}$ is (i) non-empty with $\Pi(A) < \infty$ for all $A \in \mathcal{A}^*$ and $-\infty < \Pi(A)$ for at least one $A \in \mathcal{A}^*$ and (ii) finite.

To define set $\mathcal{G}$, recall the following definitions from the text. (i) Define a subset $\mathcal{A}^* \subset \mathcal{A}$, where $\mathcal{A}$ are the mutual acceptance matrices that can be implemented by a matching mechanism $\mathcal{M}$. Construct $\mathcal{A}^*$ through the following procedure: For every $A' \in \mathcal{A}$, construct a matrix A'' such that

$$\begin{aligned}
&\alpha'(\theta_i^k, \theta_j^{-k}) = \alpha''(\theta_i^k, \theta_j^{-k}) \text{ if } \alpha'(\theta_i^k, \theta_j^{-k}) \in \{0,1\},\\
&\alpha'(\theta_i^k, \theta_j^{-k}) = \alpha_{ij} \text{ otherwise,}
\end{aligned}$$

where α_{ij} is a variable in $[0,1]$. (ii) For each $A \in \mathcal{A}^*$, the linear program is given by Lemma 6. The value of the objective is given by $\Pi(A)$. Then, (iii) $\mathcal{G} = \bigcup_{A\in\mathcal{A}^*} \Pi(A)$.

(a) $\mathcal{G}$ is non-empty.

I will show that for any $A \in \mathcal{A}^*$, there exists an optimal value $\Pi(A) < \infty$ to the linear program. To do so, fix $A \in \mathcal{A}^*$ and consider the linear program as defined in Lemma 6 in Appendix A.1. To prove that an optimal solution exists, I show that: (i) the objective of the linear program is bounded, i.e., the linear program is not unbounded, and (ii) the feasible region of the variable vector, $\mathcal{P}$, is non-empty for a range of parameters. From both it follows that there exists an optimal solution by Dantzig (1963); Bertsimas and Tsitsiklis (1997).

(i) First, I show that the objective is bounded for all linear programs for fix $A \in \mathcal{A}^*$. For a maximization problem to be bounded there must exists a constant $C \in \mathbb{R}$ such that for all feasible $x \in \mathbb{R}^n$ $c^T x \leq C$ holds. The objective is bounded as

$$\sum_{k=A,B} \sum_{\theta_i^k \in \Theta^k} \frac{(1-\delta)s_i^k}{(1-\rho)} f(\theta_i^k) < \sum_{k=A,B} \sum_{\theta_i^k \in \Theta^k} \frac{(1-\delta)s_i^k}{(1-\rho)} \frac{\beta_i^k}{\delta} \equiv C. \tag{33}$$

This implies that $\Pi(A) < \infty$ for all $A \in \mathcal{A}^*$.

(ii) Second, I show that the feasible region is non-empty. The feasible region is defined by the set $\mathcal{P} = \{x \in \mathbb{R}^n : Hx \leq b\}$. For any $A \in \mathcal{A}^*$, there exists a matching rule under which the constraints are not inconsistent for a range of parameters. This follows from the fact that $\mathcal{A}^* \subset \mathcal{A}$ and the definition of $\mathcal{A}$ implies that $A \in \mathcal{A}$ if and only if there exists an exogenous matching rule for which an equilibrium with mutual acceptance matrix A exists. By Lemma 2 there exists at least one equilibrium that can be implemented by a matching mechanism, hence, $\mathcal{A}^*$ is non-empty. Therefore, the feasible region is non-empty for a range of parameters for each linear program for fix $A \in \mathcal{A}^*$.

Then, by strong duality (Dantzig, 1963), it follows that the linear program attains an optimal solution for any $A \in \mathcal{A}^*$. The optimal value to the linear program, $\Pi(A)$, is finite and $\mathcal{G}$ is non-empty.

(b) $\mathcal{G}$ is finite.

As $\mathcal{G} = \bigcup_{A\in\mathcal{A}^*} \Pi(A)$ and $\mathcal{A}^*$ is finite by construction, $\mathcal{G}$ is also finite as the profit level of a given linear program is a singleton. As each linear program for fix $A \in \mathcal{A}^*$ is bounded, the profit level takes on either a (finite) optimal value if an optimal solution exists or the value is undefined if the linear program is infeasible for given parameters. $\square$

I prove Theorem 1 through a series of lemma. Recall that $s_i^k \in [0, \overline{u}] \equiv \mathcal{S}$ and denote the vector of search costs by $(s_1^k, ..., s_N^k)^{k=A,B} \equiv \mathbf{s}$.

Lemma 7. *Let the vector of search costs* $\mathbf{s}$ *be given. There exists an optimal solution with* $\Pi^* \equiv \max_{\mathbf{s}} \mathcal{G}(\mathbf{s})$.

Proof. By Lemma 5, the set $\mathcal{G}$ is finite and non-empty for any given $s_i^k \in \mathbb{R}_+$. Hence for given vector $\mathbf{s}$, $\mathcal{G}$ has a maximum element and $\Pi^* = \max \mathcal{G}$ is well-defined and has a finite value. $\square$

Now, let the platform choose the vector of search costs $\mathbf{s}$. I prove that there exists an optimal solution $\Pi^{*,s} \equiv \max_{\mathbf{s}} \Pi^*(\mathbf{s})$.

First, observe that if $s_i^k \geq \max_{\theta_j^{-k}}\{\theta_i^k \cdot \theta_j^{-k} - \omega_i^k\}$ for all $\theta_i^k \in \Theta^k$, no agent participates and the equilibrium profit is zero. Therefore, to make positive profits $s_i^k \leq \max_{\theta_j^{-k}}\{\theta_i^k \cdot \theta_j^{-k} - \omega_i^k\}$ for at least one $\theta_i^k \in \Theta^k$ such that the set of participating types $\hat{\Theta}^k$ is non-empty. Recall that $\mathcal{G}(\mathbf{s})$ is the set of profit levels induced through all linear programs that have a feasible solution for given $\mathbf{s}$. In slight abuse of notation, define $\mathcal{G}(\mathbf{s})$ as a correspondence from $\mathbf{s}$ to such profit levels $\Pi(\mathbf{s})$

$$\mathcal{G}(\mathbf{s}) : \mathcal{S}^{|\Theta^k|\times|\Theta^{-k}|} \rightrightarrows \mathbb{R}_0^+.$$

which assigns to each point $\mathbf{s}$ of $\mathcal{S}^{|\Theta^k|\times|\Theta^{-k}|}$ a finite subset $\mathcal{G}(\mathbf{s})$ of $\mathbb{R}_0^+$. The correspondence is compact-valued as $\mathcal{G}(\mathbf{s})$ is a compact (finite) subset of $\mathcal{C}$ for all $\mathbf{s} \in \mathcal{S}^{|\Theta^k|\times|\Theta^{-k}}$. In the following, I will show that the correspondence is upper hemicontinuous in $\mathbf{s}$ on $\mathcal{S}^{|\Theta^k|\times|\Theta^{-k}}$.

To do so, recall the matrix notation of the linear program in Appendix A.1:

$$\max_{x\in X} xc^T \equiv \Pi_A(\mathbf{s}),$$
$$s.t. H_A x \leq b_A, x \geq 0.$$

Denote by subscript A, the profit level and constraint set of the linear program for given matrix $A \in \mathcal{A}^*$. In Lemma 5, I have shown that a linear program for a fixed $A \in \mathcal{A}^*$

has a solution for some $\mathbf{s} \in \mathcal{S}^{|\Theta^k|\times|\Theta^{-k}}$. Additionally, whenever the linear program has a solution, it has an optimal solution. The value of the linear program, $\Pi_A(s)$, is thus finite on a set $\mathcal{J}_A \equiv \{\mathbf{s} \in \mathcal{S}^{|\Theta^k|\times|\Theta^{-k}|} | -\infty < \Pi_A(\mathbf{s}) < \infty\}$, where $\mathcal{J}_A \subseteq \mathcal{S}^{|\Theta^k|\times|\Theta^{-k}}$. The set is compact.[25]

Lemma 8. *The value of the objective $\Pi_A(s)$ of a linear program for given matrix $A \in \mathcal{A}^*$ is upper hemicontinuous in* $\mathbf{s}$ *on* $\mathcal{J}_A$.

Proof. Fix $A \in \mathcal{A}^*$, and consider the associated linear program from Lemma 6. For given $A \in \mathcal{A}^*$, $\mathbf{s}$ changes vector c continuously, as each entry, $\nu(s_i^k)$ or 0, is continuous in s_i^k. Furthermore $\mathbf{s}$ changes matrix H_A continuously as s_i^k linearly enters as a coefficient in the incentive and participation constraints. The optimal value of the linear program is given by

$$\Pi_A(\mathbf{s}) \equiv \sup_{x \in \mathbb{R}^n} \{c(\mathbf{s})x | H_A(\mathbf{s})x \leq b_A, x \geq 0\},$$

which is finite on $\mathcal{J}_A$. In slight abuse of notation, denote the correspondence from $\mathbf{s}$ to the optimal value of the linear program by

$$\Pi_A(\mathbf{s}) : \mathcal{S}^{|\Theta^k|\times|\Theta^{-k}|} \rightrightarrows \mathbb{R}_0^+.$$

Next, consider set of primal feasible solutions of the linear program that defines objective Π, which is given by the correspondence

$$\mathbf{s} \to P_A(\mathbf{s}) \equiv \{x | H_A(\mathbf{s})x \leq b, x \geq 0\}.$$

First, I show that the set of (primal) feasible solutions of the linear program is upper hemicontinuous in $\mathbf{s}$. Consider the following definition: $P_A(\mathbf{s})$ is upper hemicontinuous at $\mathbf{s}$ on $\mathcal{J}_A$ if

$$\mathbf{s} = \lim_{n\to\infty} \mathbf{s}_n, \; x_n \in P_A(\mathbf{s}_n), \text{ and } x = \lim_{n\to\infty} x_n,$$

implies that $x \in P_A(\mathbf{s})$.[26] To see that $P_A(\mathbf{s})$ is upper hemicontinuous, suppose that $\{\mathbf{s}_n\}_n \in \mathcal{J}_A$ and $\mathbf{s} = \lim_{n\to\infty} \mathbf{s}_n$. Let $\{x_n\}_n$ be a sequence such that for all n, $x_n \in P_A(\mathbf{s})$: $H_A(\mathbf{s}_n)x_n \leq b_A$, and $x = \lim_{n\to\infty} = x_n$. Since by the continuity of $H_A(\cdot)$ and independence of b_A in $\mathbf{s}$

$$||H_A(\mathbf{s}_n) - H_A(\mathbf{s})|| \to 0, \; ||x_n - x|| \to 0, \text{ and } ||b_A - b_A|| = 0,$$

[25]The set $\mathcal{J}_A$ contains all $\mathbf{s} \in \mathcal{S}^{|\Theta^k|\times|\Theta^{-k}|}$ for which the value of the linear program is finite. In other words, the linear program must be bounded and feasible for those $\mathbf{s}$. By Lemma 5, the linear program is bounded. The linear program is feasible for some $\mathbf{s}$ if all constraints can be met, i.e. the feasible region $\mathcal{P}$ is non-empty. Suppose for contradiction that $\mathcal{J}_A$ is not compact. Now, take any sequence $\mathbf{s}_n \to \mathbf{s}$, for which the feasible region is non-empty for all $\mathbf{s}_n$. For the limit point $\mathbf{s}$ not to be in set $\mathcal{J}_A$, the feasible region must be empty for $\mathbf{s}$, and hence, at least one inequality must be violated strictly. But then, as the linear constraints are continuous in $\mathbf{s}$, the constraints must also be violated for $\mathbf{s}_n$ close enough to $\mathbf{s}$, a contradiction.

[26]This definition follows Wets (1985). Furthermore, let $||H|| = \sup_{x \in X} ||Hx||$

it follows that $H_A x \leq b_A$ and $x \geq 0$, which yields $x \in P_A(\mathbf{s})$. This implies that $P_A(\mathbf{s})$ is in fact upper hemicontinuous in $\mathbf{s}$ on $\mathcal{J}_A$.

Next, I show that this implies that $\Pi_A(\mathbf{s}) = c(\mathbf{s})x$ is upper hemicontinuous in $\mathbf{s}$ on $\mathcal{J}_A$. Suppose that $\{\mathbf{s}_n\}_n \in \mathcal{J}_A$ and $\mathbf{s} = \lim_{n\to\infty} \mathbf{s}_n$. Let $\{\Pi_n\}_n$ be a sequence such that for all n, $\Pi_n \in \Pi_A(\mathbf{s})$, and $\Pi = \lim_{n\to\infty} \Pi_n$. Since by the continuity of $c(\cdot)$

$$||c(\mathbf{s}_n) - c(\mathbf{s})|| \to 0,$$

and the upper hemicontinuity of $P_A(\mathbf{s})$ on $\mathcal{J}_A$

$$||x_n - x|| \to 0$$

it follows that $\Pi \in \Pi_A(\mathbf{s})$. This implies that $\Pi_A(\mathbf{s})$ is in fact upper hemicontinuous in $\mathbf{s}$ on $\mathcal{J}_A$. $\square$

Recall that $\mathcal{G}(\mathbf{s}) = \bigcup_{A\in\mathcal{A}^*} \Pi_A(\mathbf{s})$ is the finite union over the equilibrium profit levels of each linear program.

Lemma 9. $\mathcal{G}(\mathbf{s})$ *is upper hemicontinuous in* $\mathbf{s}$ *on* $\mathcal{S}^{|\Theta^k|\times|\Theta^{-k}|}$.

Proof. Recall that for each $\Pi_A(\mathbf{s})$ the value $\Pi_A(\mathbf{s})$ is finite on $\mathcal{J}_A$ and empty on $\mathcal{S}^{|\Theta^k|\times|\Theta^{-k}|} \setminus \mathcal{J}_A$. I prove the lemma by induction over the equilibria associated with the finite set $\mathcal{A}^*$. Let there be $\overline{K}$ equilibria, which can be implemented by the linear programs and consider the correspondence $\mathcal{G}_K(\mathbf{s}) = \bigcup_{\{A_1,\ldots,A_K\}} \Pi_A(\mathbf{s})$ that includes K out of $\overline{K}$ equilibria. By induction, I will consider $\mathcal{G}_K$ to include increasingly more equilibria.

Base case: Let $\mathcal{G}_1$ be the correspondence that includes only the trivial equilibrium from Lemma 2 with $A_1 \in \mathcal{A}^*$. Note that $\mathcal{S}^{|\Theta^k|\times|\Theta^{-k}|} = \mathcal{J}_{A_1}$ as the trivial equilibrium is a solution to the corresponding linear program for each $\mathbf{s} \in \mathcal{S}^{|\Theta^k|\times|\Theta^{-k}|}$. Hence, the statement follows from Lemma 8.

Induction step: The induction hypothesis states that $\mathcal{G}_K(\mathbf{s}) = \bigcup_{\{A_1,\ldots,A_K\}} \Pi_A(\mathbf{s})$ is upper hemicontinuous on $\mathcal{S}^{|\Theta^k|\times|\Theta^{-k}|}$. Note that by the induction step, K includes the trivial equilibrium. It remains to show that $\mathcal{G}_K(\mathbf{s}) \cup \Pi_{A_{K+1}}(\mathbf{s})$ is upper hemicontinuous in $\mathbf{s}$ on $\mathcal{S}^{|\Theta^k|\times|\Theta^{-k}|}$.

Recall that the correspondence $\mathcal{G}_K(\mathbf{s}) \cup \Pi_{A_{K+1}}(\mathbf{s})$ is upper hemicontinuous at $\mathbf{s}_0 \in \mathcal{S}^{|\Theta^k|\times|\Theta^{-k}|}$, if for any open set $V \subseteq \mathbb{R}_0^+$ with $\mathcal{G}_K(\mathbf{s}_0) \cup \Pi_{A_{K+1}}(\mathbf{s}_0) \subseteq V$, there exists an open neighborhood $U(\mathbf{s}_0) \subseteq \mathcal{S}^{|\Theta^k|\times|\Theta^{-k}|}$ such that if $\mathbf{s} \in U(\mathbf{s}_0)$, then $\mathcal{G}_K(\mathbf{s}) \cup \Pi_{A_{K+1}}(\mathbf{s}) \subseteq V$.

Let $\mathbf{s}_0 \in \mathcal{S}^{|\Theta^k|\times|\Theta^{-k}|}$ and V be an open set with $\mathcal{G}_K(\mathbf{s}_0) \cup \Pi_{A_{K+1}}(\mathbf{s}_0) \subseteq V$. Suppose first that $\Pi_{A_{K+1}}$ is empty at $\mathbf{s}_0$. Since $\mathcal{G}_K(\mathbf{s}_0) \cup \Pi_{A_{K+1}}(\mathbf{s}_0) \subseteq V$, it follows that $\mathcal{G}_K(\mathbf{s}_0) \subseteq V$ and $\Pi_{A_{K+1}}(\mathbf{s}_0) \subseteq V$ by assumption (where V is the union of an open set and the empty set). By the upper hemicontinuity of $\mathcal{G}_K(\mathbf{s})$, there exists a neighborhood U_K of $\mathbf{s}_0$ such that $\mathcal{G}_K(\mathbf{s}_0) \subseteq V$ for all $\mathbf{s} \in U_K$. Additionally, there exists a neighborhood U_{K+1} of $\mathbf{s}_0$ such that $\Pi_{A_{K+1}}(\mathbf{s}_0) = \emptyset \subseteq V$ for all $\mathbf{s} \in U_{K+1}$ (by the compactness of $\mathcal{J}_{A_{K+1}}$. Let $U = U_K \cap U_{K+1}$. Then, for any $\mathbf{s} \in U$, both $\mathcal{G}_K(\mathbf{s}) \subseteq V$ and $\Pi_{A_{K+1}}(\mathbf{s}) \subseteq V$ such that $\mathcal{G}_K(\mathbf{s}) \cup \Pi_{A_{K+1}}(\mathbf{s}) \subseteq V$.

Let both $\mathcal{G}_K(\mathbf{s})$ and $\Pi_{A_{K+1}}(\mathbf{s})$ be non-empty at $\mathbf{s}_0$. Since $\mathcal{G}_K(\mathbf{s}_0) \cup \Pi_{A_{K+1}}(\mathbf{s}_0) \subseteq V$, it follows that $\mathcal{G}_K(\mathbf{s}_0) \subseteq V$ and $\Pi_{A_{K+1}}(\mathbf{s}_0) \subseteq V$. As both $\mathcal{G}_K(\mathbf{s}_0)$ and $\Pi_{A_{K+1}}(\mathbf{s}_0)$ are upper hemicontinuous for $\mathbf{s}_0$, it holds that: There exists a neighborhood U_K of $\mathbf{s}_0$ such that

$\mathcal{G}_K(\mathbf{s}_0) \subseteq V$ for all $\mathbf{s} \in U_K$ and U_{K+1} of $\mathbf{s}_0$ such that $\Pi_{A_{K+1}}(\mathbf{s}_0) \subseteq V$ for all $\mathbf{s} \in U_{K+1}$. Then, for any $\mathbf{s} \in U$, both $\mathcal{G}_K(\mathbf{s}) \subseteq V$ and $\Pi_{A_{K+1}}(\mathbf{s}) \subseteq V$ such that $\mathcal{G}_K(\mathbf{s}) \cup \Pi_{A_{K+1}}(\mathbf{s}) \subseteq V$.

Therefore, $\mathcal{G}(\mathbf{s})$ is upper hemicontinuous in $\mathbf{s}$ on $\mathcal{S}^{|\Theta^k|\times|\Theta^{-k}|}$. $\square$

Lemma 10. *The function $\Pi^*(\mathbf{s})$ is upper semi-continuous in $\mathbf{s}$ on $\mathcal{S}^{|\Theta^k|\times|\Theta^{-k}|}$.*

Proof. The function Π^* is upper-semicontinuous if for every point $\mathbf{s} \in \mathcal{S}^{|\Theta^k|\times|\Theta^{-k}|}$, $\Pi(\mathbf{s}) \geq \limsup \Pi(\mathbf{s}_n)$ for every sequence $\{\mathbf{s}_n\}_n \subset \mathcal{S}^{|\Theta^k|\times|\Theta^{-k}|}$ satisfying $\lim_{n\to\infty} \mathbf{s}_n = \mathbf{s}$.

Let $\lim_{n\to\infty} \mathbf{s}_n = \mathbf{s}$, and define $\Pi_n^* = \max \mathcal{G}(\mathbf{s}_n)$, so that $\Pi_n^* \in \mathcal{G}(\mathbf{s}_n)$ for all n. Since for each $\mathbf{s}_n$ $\mathcal{G}(\cdot)$ is finite by Lemma 5 and the sequence $\{\Pi_n^*\}$ is bounded, it has a convergent subsequence by the Bolzano-Weierstrass theorem: $\Pi_{n_k}^* \to \Pi'$ for some $\Pi' \in \mathbb{R}_0^+$. Then, as $\Pi_{n_k}^* \in \mathcal{G}(\mathbf{s}_{n_k})$, $\mathbf{s}_{n_k} \to \mathbf{s}$, and $\Pi_{n_k}^* \to \Pi'$, the upper hemiconituity of $\mathcal{G}(\mathbf{s})$ implies that any limit point of $\Pi_{n_k}^*$ belongs to $\mathcal{G}(\mathbf{s})$, i.e. $\Pi' \in \mathcal{G}(\mathbf{s})$. Therefore, $\Pi' \leq \max G(\mathbf{s})$. Since $\Pi_{n_k}^* \to \Pi'$, this implies:

$$\lim_{n\to\infty} \sup \Pi_n = \lim_{n\to\infty} \sup \max \mathcal{G}(\mathbf{s}_n) \leq \max \mathcal{G}(\mathbf{s}).$$

$\square$

Intuitively, Π^* is continuous in $\mathbf{s}$ except for jump points (discontinuities). At a jump point, the definition of upper semicontinuity requires that the function is only allowed to jump "up". By the upper hemicontinuity, I already know that the limit point — when taking a sequence of $\mathbf{s}$ — is still in $\max \mathcal{G}(\mathbf{s})$. Additionally, due to the definition of Π^* as $\Pi^* = \max \mathcal{G}(\mathbf{s})$, the limit point can only jump "up".

Proof of Theorem 1 By Lemma 10, $\max \mathcal{G} = \Pi^*(\mathbf{s})$ is upper semi-continuous in $\mathbf{s}$ and compact-valued. Thus, there exists a maximum by Weierstrass extreme value theorem on the compact set $\mathcal{S}^{|\Theta^k|\times|\Theta^{-k}|}$. $\square$

Proof of Proposition 1 The proof proceeds by considering the cases where search costs are exogenous and where search costs are chosen as search fee or advertising.

Case 1: Exogenous Search Cost First, suppose search costs are exogenously given. Let the parameters be drawn uniformly from the following sets: $\theta_i^k \in \Theta^k = [\underline{\theta}, \overline{\theta}] \subseteq \mathbb{R}_+$, $\beta_i^k \in [0, \overline{\beta}]$, $\delta \in (0,1]$, $\omega_i^k \in \Omega = [0, \overline{\omega}]$, and $s_i^k \in [0, \overline{u}]$. An outcome is said to be generically suboptimal if the set of parameter values for which it is optimal has measure zero in the relevant parameter space.

For given $A \in \mathcal{A}^*$, an optimal solution is a matching rule for which the objective function of the linear program in Appendix A.1 attains its maximum value. Recall from Appendix A.1 that the platform solves

$$\max_{\{\Phi(\cdot), m(\cdot)\}_{ij}^k} \sum_{k=A,B} \sum_{\theta_i^k \in \Theta^k} \frac{(1-\delta)\nu(s_i^k)}{1-\rho} f(\theta_i^k),$$

subject to participation constraints (Equation 12), incentive constraints and 13), feasibility constraints (Equation 14) and steady-state constraints (Equation 15). Using the steady state conditions from Equation 15 to substitute for $f(\theta_i^k)$ yields

$$\max_{\{\Phi(\cdot),m(\cdot)\}_{ij}^k} \sum_{k=A,B} \sum_{\theta_i^k \in \Theta^k} \frac{(1-\delta)\nu(s_i^k)}{1-\rho} \frac{\beta_i^k - (1-\delta)\sum_j m(\theta_i^k, \theta_j^{-k})}{\delta}.$$

Both feasibility and steady-state constraints must be binding in the optimal solution. Additionally, at least one participation or incentive constraint must be binding in the optimal solution. Suppose otherwise, then the platform can decrease at least one $m(\theta_i^k, \theta_j^{-k})$ such that one constraint is binding and thereby increase its profits.

Recall that $\{m^{RM}(\theta_i^k, \theta_j^{-k})\}_{ij}^k$ is the vector of masses of matched pairs under random matching. Then, $m^{RM}(\theta_i^k, \theta_j^{-k}) = 0$ if $\alpha(\theta_i^k, \theta_j^{-k}) = 0$ and

$$m^{RM}(\theta_i^k, \theta_j^{-k}) = \frac{\alpha(\theta_i^k, \theta_j^{-k})\beta_i^k \mu(\theta_i^k, \omega_i^k)\beta_j^{-k}\mu(\theta_j^{-k}, \omega_j^{-k})}{\left(\sum_{\theta_i^k} \beta_i^k \mu(\theta_i^k, \omega_i^k)\right) \cdot \left(\sum_{\theta_j^{-k}} \beta_j^{-k}\mu(\theta_j^{-k}, \omega_j^{-k})\right)}$$

if $\alpha(\theta_i^k, \theta_j^{-k}) \in (0,1]$ (see Appendix A.2).This is a function of the inflow vector $(\beta_1^k, ..., \beta_{N^k}^k)_k$, δ and the probability of a type θ_i^k being matched to their outside option ω_i^k $(\mu(\theta_i^k, \omega_i^k))$. Observe that for given $A \in \mathcal{A}^*$, $m^{RM}(\theta_i^k, \theta_j^{-k})$ is independent of s_i^k. Fix $A \in \mathcal{A}^*$. Given $\{m^{RM}(\theta_i^k, \theta_j^{-k})\}_{ij}^k$, I show that the participation and incentive constraints are generically non-binding. Rearranging and using the steady state condition yields the following constraints

$$\beta_i^k \left(\theta_i^k \theta_j^k - \omega_i^k + \frac{(1-\delta)}{\delta} s_i^k\right) \leq (1-\delta)\sum_j m^{RM}(\theta_i^k, \theta_j^{-k}) \left(\theta_i^k \theta_j^{-k} - \omega_i^k + \frac{(1-\delta)}{\delta} s_i^k\right), \tag{34}$$

$$\beta_i^k \frac{(1-\delta)}{\delta} s_i^k \leq (1-\delta)\sum_j m^{RM}(\theta_i^k, \theta_j^{-k}) \left(\theta_i^k \theta_j^{-k} - \omega_i^k + \frac{(1-\delta)}{\delta} s_i^k\right). \tag{35}$$

Suppose $\beta_i^k, \omega_i^k, \theta_i^k, \theta_j^{-k}$ and δ are drawn uniformly from their continuous intervals. Note that each constraint for a type θ_i^k is a linear equation in s_i^k. Hence, for given $\{m^{RM}(\theta_i^k, \theta_j^{-k})\}_{ij}^k$, there exists at most one s_i^k per participation or incentive constraint of type θ_i^k such that the constraint is binding. This implies that if s_i^k is drawn uniformly from a continuous interval, the set of parameters for which the constraint is binding has measure zero. Therefore integrating over the cases for which at least one constraint is binding, the corresponding set of parameters has measure zero as well. Hence, for each $A \in \mathcal{A}^*$, the constraints are generically non-binding. Lastly, since $\mathcal{A}^*$ is finite, this concludes the proof that random matching is generically suboptimal for exogenously given search costs. Next, consider the case where the platform chooses the vector of search costs.

Case 2: Endogenous Search Cost (Search Fee) Consider the case, in which the platform sets a linear search fee and earns $\nu(s_i^k) = s_i^k$ for all $\theta_i^k = \Theta^k, k = A, B$. The platform maximizes the total match surplus and fully extracts the surplus through the search fee. The platform solves the maximization problem in Equation 24 subject to Equation 25, 26, and 27 from Appendix A.2. Given the solution to this problem, $\{m^{PAM}(\theta_i^k, \theta_j^{-k})\}_{ij}^k$, the platform sets s_i^k to fully extract each type's surplus:

$$\beta_i^k \frac{(1-\delta)}{\delta} s_i^k = (1-\delta) \sum_j m^{PAM}(\theta_i^k, \theta_j^{-k}) \left(\theta_i^k \theta_j^{-k} - \omega_i^k + \frac{(1-\delta)}{\delta} s_i^k \right).$$

The optimal matching rule that maximizes total match surplus follows the procedure: Starting with the highest possible type on side A, each agent is matched to the highest possible type on side B. If there are not enough high types remaining on side B, the algorithm proceeds in descending order of type on side B until all agents of the highest possible type on side A are matched. The process continues in descending order with the next highest type on side A, each time matching to the next available remaining types on side B. Once all agents on B have been matched, any remaining agents on side A are assigned to their outside option. The optimal matching rule hence always differs from random matching. To see this, observe that higher types receive better recommendations under the above procedure, whereas each type receive the same recommendations under random matching.

Case 3: Endogenous Search Cost (Advertising) Now consider the case in which the platform earns a revenue of $\nu(s_i^k)$ when charging search costs s_i^k. I, again, examine random matching that satisfies the feasibility constraints outlined in Equation 14. For any $A \in \mathcal{A}^*$, and using the steady-state conditions to substitute for $f^{RM}(\theta_i^k)$, the platform's objective under random matching becomes the following maximization problem:

$$\max_{\mathbf{s}} \sum_{k=A,B} \sum_{\theta_i^k \in \Theta^k} \frac{(1-\delta)\nu(s_i^k)}{1-\rho} \underbrace{\frac{\beta_i^k - (1-\delta)\sum_j m^{RM}(\theta_i^k, \theta_j^{-k})}{\delta}}_{=f^{RM}(\theta_i^k)}. \tag{36}$$

subject to the participation and incentive constraints in Equation 34 and 35.

To maximize $\mathbf{s}$, observe first that $\nu'(s_i^k) > 0$ as $\nu(s_i^k)$ is strictly increasing in s_i^k. This implies that the platform has an incentive to increase the search costs as much as possible given the constraints. Therefore for $A \in \mathcal{A}^*$, the optimal solution is to choose s_i^k such that for each type $\theta_i^k \in \Theta^k, k = A, B$ either the participation or the relevant incentive constraint — induced by A — is binding. Note that the random matching vector satisfies the feasibility condition, and as random matching is independent of s_i^k feasibility remains to be satisfied.

Next, I show that the platform has an incentive to deviate from the above solution. First, suppose $A' \in \mathcal{A}^*$ consists only of entries equal to one, so all agents accept any match in the first period. Incentive constraints are slack, and the platform chooses $\mathbf{s}$ to make participation constraints binding. Under random matching, the platform can at most charge the expected value of a match. By deviating to positive assortative matching—the solution under a linear search fee—the platform can raise search costs and profits, since $\nu(s_i^k)$ is strictly increasing in s_i^k.

Second, consider any matrix in $A'' \in \mathcal{A}^* \setminus \{A'\}$. In this case, at least one type rejects another type with positive probability. As the match utility with the lowest type is the lowest, this implies that at least one type is willing to reject the lowest type. Consider the pair of types $(\theta_1^k, \theta_R^{-k})$ for which type $\theta_R^{-k} \in \Theta^{-k}$ is willing to reject the lowest type θ_1^k on the other market side ($\alpha(\theta_1^k, \theta_R^{-k}) = 0$). Recall that each type must be accepted by at least one other type on the opposite market side to be willing to participate, thus consider pairs $(\theta_1^k, \theta_A^{-k})$ and $(\theta_A^k, \theta_R^{-k})$ for which $\alpha(\theta_1^k, \theta_A^{-k}) = 1$ and $\alpha(\theta_A^k, \theta_R^{-k}) = 1$.

For fix $A'' \in \mathcal{A}^* \setminus \{A'\}$, I will show that the platform's profit can be improved by changing the matching rules for types $\theta_1^k, \theta_A^k, \theta_A^{-k}$ and θ_R^{-k} as well as adjusting their search costs. The platform will choose the mass of recommended pairs

$$\Phi'(\theta_1^k, \theta_R^{-k}), \Phi'(\theta_1^k, \theta_A^{-k}), \Phi'(\theta_A^k, \theta_R^{-k}), \Phi'(\theta_A^k, \theta_A^{-k}),$$

and the mass of matched pairs $m'(\cdot,\cdot) = \alpha(\cdot,\cdot)\Phi'(\cdot,\cdot)$ as detailed below. For all other types, the platform chooses the mass of recommended pairs such that

$$\begin{aligned}
\Phi'(\theta_i^k, \theta_j^{-k}) &= \Phi^{RM}(\theta_i^k, \theta_j^{-k}), \ \forall \theta_i^k \in \Theta^k \setminus \{\theta_1^k, \theta_A^k\}, \\
\Phi'(\theta_i^k, \theta_j^{-k}) &= \Phi^{RM}(\theta_i^k, \theta_j^{-k}), \ \forall \theta_j^{-k} \in \Theta^{-k} \setminus \{\theta_R^{-k}, \theta_A^{-k}\}.
\end{aligned}$$

Without loss of generality, suppose that the total mass of all types on market side A is smaller or equal than the total mass of all types on market side B: $\sum_{\theta_i^A \in \Theta^A} f^{RM}(\theta_i^A) \leq \sum_{\theta_i^B \in \Theta^B} f^{RM}(\theta_i^B)$. Then, for market side A, the platform chooses the mass of types that are recommended to their outside option such that

$$\Phi'(\theta_i^k, \omega_i^k) = \Phi^{RM}(\theta_i^k, \omega_i^k), \forall \theta_i^k \in \Theta^k \setminus \{\theta_1^k, \theta_A^k\}, k = A.$$

The mass of recommended pairs and the mutual acceptance probabilities remain the same as under random matching. As a result, the mass of matched pairs—defined as the product of these two terms—is also unchanged. Therefore, the participation and incentive constraints for all other types continue to hold. In addition, the feasibility constraint in Equation (14) and the steady-state constraint in Equation (15) are still satisfied.

For $\varepsilon \in [-\min\{\beta_i - m^{RM}(\cdot,\cdot)\}, \min\{m^{RM}(\cdot,\cdot)\}]$, the platform chooses $m^{RM}(\theta_1^k, \theta_A^{-k}) - m'(\theta_1^k, \theta_A^{-k}) = \varepsilon$ and $m^{RM}(\theta_A^k, \theta_R^{-k}) - m'(\theta_A^k, \theta_R^{-k}) = \varepsilon$, i.e. the platform changes the mass of the two matched pairs by ε. Substituting the change into the steady state condition (Equation 15) for type θ_1^k and type θ_R^{-k} yields

$$\begin{aligned}
f^{RM}(\theta_1^k) + \frac{1-\delta}{\delta}\varepsilon =& \frac{1}{\delta}\left(\beta_1^k - (1-\delta)\left(-\varepsilon + \sum_{\Theta^{-k}} m^{RM}(\theta_1^k, \theta_j^{-k})\right)\right), \\
f^{RM}(\theta_R^{-k}) + \frac{1-\delta}{\delta}\varepsilon =& \frac{1}{\delta}\left(\beta_R^{-k} - (1-\delta)\left(-\varepsilon + \sum_{\Theta^{k}} m^{RM}(\theta_i^k, \theta_R^{-k})\right)\right).
\end{aligned}$$

Therefore, by decreasing (increasing) the mass of the two matched pairs, increases (decreases) the steady state mass by $\frac{1-\delta}{\delta}\varepsilon$ compared to the steady state mass under random matching. Substituting $\Phi^{RM}(\theta_1^k, \theta_A^{-k}) - \Phi'(\theta_1^k, \theta_A^{-k}) = \varepsilon$ and $\Phi^{RM}(\theta_A^k, \theta_R^{-k}) - \Phi'(\theta_A^k, \theta_R^{-k}) = \varepsilon$

into the feasibility constraints of type θ_1^k and type θ_R^{-k} yields

$$f^{RM}(\theta_1^k) + \frac{1-\delta}{\delta}\varepsilon = \Phi'(\theta_1^k, \theta_R^{-k}) - \varepsilon + \mathbf{1}_{k=A}\Phi^{RM}(\theta_1^k, \omega_1^k) + \sum_{\Theta^{-k}\setminus\{\theta_R^{-k}\}} \Phi^{RM}(\theta_1^k, \theta_j^{-k}), \tag{37}$$

$$f^{RM}(\theta_R^{-k}) + \frac{1-\delta}{\delta}\varepsilon = \Phi'(\theta_1^k, \theta_R^{-k}) - \varepsilon + \mathbf{1}_{k=A}\Phi^{RM}(\theta_R^{-k}, \omega_R^{-k}) + \sum_{\Theta^{k}\setminus\{\theta_1^{k}\}} \Phi^{RM}(\theta_i^k, \theta_R^{-k}), \tag{38}$$

which implies that $\Phi'(\theta_1^k, \theta_R^{-k}) - \Phi^{RM}(\theta_1^k, \theta_R^{-k}) = \frac{1-\delta}{\delta}\varepsilon + \varepsilon = \varepsilon/\delta$. It remains to determine $\Phi'(\theta_A^k, \theta_A^{-k}) - \Phi^{RM}(\theta_A^k, \theta_A^{-k})$ and $m'(\theta_A^k, \theta_A^{-k}) - m^{RM}(\theta_A^k, \theta_A^{-k})$. To do so, consider two cases: either $\alpha(\theta_A^k, \theta_A^{-k}) = 0$ or $\alpha(\theta_A^k, \theta_A^{-k}) = 1$.

In the first case, $\alpha(\theta_A^k, \theta_A^{-k}) = 0$, I can exchange θ_A^k for θ_1^k and θ_A^{-k} for θ_R^{-k} in Equation 37 and 38 above. Then, it follows that $\Phi'(\theta_A^k, \theta_A^{-k}) - \Phi^{RM}(\theta_A^k, \theta_A^{-k}) = \varepsilon/\delta$ and $m'(\theta_A^k, \theta_A^{-k}) = 0$.

In the second case, $\alpha(\theta_A^k, \theta_A^{-k}) = 1$, the platform can set $\Phi'(\theta_A^k, \theta_A^{-k}) - \Phi^{RM}(\theta_A^k, \theta_A^{-k}) = m'(\theta_A^k, \theta_A^{-k}) - m^{RM}(\theta_A^k, \theta_A^{-k}) = \varepsilon$. Since the platform decreases (increases) the mass of the matched pair $(\theta_A^k, \theta_R^{-k})$ but increases (decreases) the mass of the matched pair $(\theta_A^k, \theta_A^{-k})$ by the same amount, this implies that the steady state mass of type θ_A^k is unchanged compared to the steady state masses under random matching. Additionally, feasibility continues to be satisfied as the platform shift mass ε from one recommended pair to the other. Similarly, the steady state mass of type θ_A^{-k} is the same as under random matching and the steady state constraint as well as feasibility constraint remain satisfied.

Next determine the change in search costs for types $\theta_1^k, \theta_A^k, \theta_A^{-k}$ and θ_R^{-k}. Note that for the newly chosen mass of recommended and matched pairs $(\Phi'(\cdot,\cdot), m'(\cdot,\cdot))$, the originally binding participation or incentive constraint is no longer binding. Since, however, the right-hand side of the participation or incentive constraints (see Equation 34 and 35) are linearly increasing in $m(\cdot,\cdot)$ and the left-hand side of the constraints are ordered due to the supermodularity of the match utility, the platform can choose a new search cost $\tilde{s}_i^k$ such that the constraint becomes binding again. Let the platform choose $\tilde{s}_1^k, \tilde{s}_A^k, \tilde{s}_R^{-k}, \tilde{s}_A^{-k}$ such that originally binding participation incentive constraint of each type is binding again.

Using Equations 34 and 35 and $m^{RM}(\theta_1^k, \theta_A^{-k}) - m'(\theta_i^k, \theta_i^{-k}) = \varepsilon$ and $m^{RM}(\theta_A^k, \theta_R^{-k}) - m'(\theta_A^k, \theta_R^{-k}) = \varepsilon$, the difference between $s_1^k - \tilde{s}_1^k$ and $s_R^{-k} - \tilde{s}_A^{-k}$ is given by

$$(1-\delta)\underbrace{\frac{\beta_1^k - (1-\delta)\sum_j m^{RM}(\theta_1^k, \theta_j^{-k})}{\delta}}_{f(\theta_1^k)}(s_1^k - \tilde{s}_1^k) = \varepsilon(\theta_1^k\theta_A^{-k} - \omega_1^k + \frac{1-\delta}{\delta}\tilde{s}_1^k). \tag{39}$$

Observe that the right-hand side is positive for $\varepsilon > 0$ as $\theta_1^k\theta_A^{-k} - \omega_1^k > 0$ due to the fact that both types mutually accept each other. Then, it follows that $\tilde{s}_1^k$ must be smaller than s_1^k for $\varepsilon > 0$. Additionally, by the steady state constraint, the factor on the left-hand side is equal to $(1-\delta)f^{RM}(\theta_1^k)$. Similarly, using $m^{RM}(\theta_A^k, \theta_R^{-k}) - m'(\theta_A^k, \theta_R^{-k}) = \varepsilon$ and taking the difference, $s_R^{-k} - \tilde{s}_A^{-k}$ is given by

$$(1-\delta)f^{RM}(\theta_R^{-k})(s_R^{-k} - \tilde{s}_R^{-k}) = \varepsilon(\theta_A^k\theta_R^{-k} - \omega_R^{-k} + \frac{1-\delta}{\delta}\tilde{s}_R^{-k}). \tag{40}$$

Observe that the right-hand side is again positive, so that $s_R^{-k} > \tilde{s}_A^{-k}$ for $\varepsilon > 0$.

Next, if $\alpha(\theta_A^k, \theta_A^{-k}) = 0$, the difference between the search costs for types θ_A^k and θ_A^{-k} can be derived as above

$$(1-\delta) f^{RM}(\theta_A^k)(s_A^k - \tilde{s}_A^k) = \varepsilon(\theta_A^k \theta_R^{-k} - \omega_A^k + \frac{1-\delta}{\delta}\tilde{s}_A^k), \tag{41}$$

$$(1-\delta) f^{RM}(\theta_A^{-k})(s_A^{-k} - \tilde{s}_A^{-k}) = \varepsilon(\theta_1^k \theta_A^{-k} - \omega_A^{-k} + \frac{1-\delta}{\delta}\tilde{s}_A^{-k}). \tag{42}$$

Again, it holds that $s_A^k > \tilde{s}_A^k$ and $s_A^{-k} > \tilde{s}_A^{-k}$ for $\varepsilon > 0$.

If $\alpha(\theta_A^k, \theta_A^{-k}) = 1$, recall that the platform sets: $m^{RM}(\theta_1^k, \theta_A^{-k}) - m'(\theta_i^k, \theta_i^{-k}) = \varepsilon$ $m^{RM}(\theta_A^k, \theta_R^{-k}) - m'(\theta_A^k, \theta_R^{-k}) = \varepsilon$ and $m^{RM}(\theta_A^k, \theta_A^{-k}) - m'(\theta_A^k, \theta_A^{-k}) = -\varepsilon$. Using Equations 34 and 35, the difference of $s_A^{-k} - \tilde{s}_A^{-k}$ is given by

$$(1-\delta) f^{RM}(\theta_A^{-k})(s_A^{-k} - \tilde{s}_A^{-k}) = \varepsilon(\theta_1^k \theta_A^{-k} - \omega_A^{-k} + \frac{1-\delta}{\delta}\tilde{s}_A^{-k}) - \varepsilon(\theta_A^k \theta_A^{-k} - \omega_A^{-k} + \frac{1-\delta}{\delta}\tilde{s}_A^{-k}). \tag{43}$$

Since $\theta_A^k > \theta_1^k$, the right-hand side is negative, so that $s_A^{-k} < \tilde{s}_A^{-k}$ for $\varepsilon > 0$. Similarly, the difference of $s_A^k - \tilde{s}_A^k$ is given by

$$(1-\delta) f^{RM}(\theta_A^k)(s_A^k - \tilde{s}_A^k) = \varepsilon(\theta_A^k \theta_R^{-k} - \theta_A^k \theta_A^{-k}), \tag{44}$$

where the right-hand side is non-negative if $\theta_R^{-k} \geq \theta_A^{-k}$.

To determine whether the deviation is profitable, consider the difference in profits between the deviation profits and random matching profits (Equation 36). In the first case, when $\alpha(\theta_A^k, \theta_A^{-k}) = 0$, the steady state mass of all four types $(\theta_1^k, \theta_A^k, \theta_R^{-k}, \theta_A^{-k})$ increases (decreases) by ${}^{(1-\delta)\varepsilon}/_{\delta}$ while their search costs decrease (increase). The difference in profits is therefore

$$\sum_{\theta_i^k \in \left\{\theta_1^k, \theta_A^k, \theta_R^{-k}, \theta_A^{-k}\right\}} \left[(\nu(\tilde{s}_i^k) - \nu(s_i^k)) f^{RM}(\theta_i^k) + \nu(\tilde{s}_i^k)\frac{(1-\delta)\varepsilon}{\delta}\right]. \tag{45}$$

Differentiating Equation 45 with respect to ε_i^k, and evaluating the condition at $\varepsilon = 0$, yields:

$$\sum_{\theta_i^k \in \left\{\theta_1^k, \theta_A^k, \theta_R^{-k}, \theta_A^{-k}\right\}} \nu'(\tilde{s}_i^k)|_{\tilde{s}_i^k = s_i^k} \frac{\partial \tilde{s}_i^k}{\partial \varepsilon}|_{\varepsilon=0} f^{RM}(\theta_i^k) + \frac{1-\delta}{\delta}\nu(\tilde{s}_i^k)|_{\tilde{s}_i^k = s_i^k}. \tag{46}$$

To determine the partial derivative of the search costs with respect to ε, I totally differentiate Equations 39, 40, 41, and 42. Evaluating the derivative at $\varepsilon = 0$ yields:

$$
\begin{aligned}
\frac{\partial \tilde{s}_1^k}{\partial \varepsilon}|_{\varepsilon=0} &= -\frac{\theta_1^k\theta_A^{-k} - \omega_1^k + \frac{1-\delta}{\delta}s_1^k}{(1-\delta)f^{RM}(\theta_1^k)},\\
\frac{\partial \tilde{s}_A^k}{\partial \varepsilon}|_{\varepsilon=0} &= -\frac{\theta_A^k\theta_R^{-k} - \omega_A^k + \frac{1-\delta}{\delta}s_A^k}{(1-\delta)f^{RM}(\theta_A^k)},\\
\frac{\partial \tilde{s}_R^{-k}}{\partial \varepsilon}|_{\varepsilon=0} &= -\frac{\theta_A^k\theta_R^{-k} - \omega_R^{-k} + \frac{1-\delta}{\delta}s_R^{-k}}{(1-\delta)f^{RM}(\theta_R^{-k})},\\
\frac{\partial \tilde{s}_A^{-k}}{\partial \varepsilon}|_{\varepsilon=0} &= -\frac{\theta_1^k\theta_A^{-k} - \omega_A^{-k} + \frac{1-\delta}{\delta}s_A^{-k}}{(1-\delta)f^{RM}(\theta_A^{-k})}.
\end{aligned}
$$

For analytical convenience, consider the class of concave functions $\nu(s_i^k) = \kappa(s_i^k)^\alpha$ for $\kappa \in \mathbb{R}^+$ and $\alpha \in (0,1)$ from now on. Substituting $\nu(s_i^k) = \kappa(s_i^k)^\alpha$, $\nu'(s_i^k) = \kappa\alpha(s_i^k)^{\alpha-1}$, and the partial derivatives above into Equation 46 yields

$$
\begin{aligned}
D \equiv\ & \underbrace{\alpha(s_1^k)^{\alpha-1}\left(-\frac{\theta_1^k\theta_A^{-k} - \omega_1^k + \frac{1-\delta}{\delta}s_1^k}{(1-\delta)}\right) + \frac{1-\delta}{\delta}(s_1^k)^\alpha}_{=d(\theta_1^k)}\\
& + \underbrace{\alpha(s_A^k)^{\alpha-1}\left(-\frac{\theta_A^k\theta_R^{-k} - \omega_A^k + \frac{1-\delta}{\delta}s_A^k}{(1-\delta)}\right) + \frac{1-\delta}{\delta}(s_A^k)^\alpha}_{d(\theta_A^k)}\\
& + \underbrace{\alpha(s_A^{-k})^{\alpha-1}\left(-\frac{\theta_1^k\theta_A^{-k} - \omega_A^{-k} + \frac{1-\delta}{\delta}s_A^{-k}}{(1-\delta)}\right) + \frac{1-\delta}{\delta}(s_A^{-k})^\alpha}_{=d(\theta_A^{-k})}\\
& + \underbrace{\alpha(s_R^{-k})^{\alpha-1}\left(-\frac{\theta_A^k\theta_R^{-k} - \omega_R^{-k} + \frac{1-\delta}{\delta}s_R^{-k}}{(1-\delta)}\right) + \frac{1-\delta}{\delta}(s_R^{-k})^\alpha}_{=d(\theta_R^{-k})}.
\end{aligned}
$$

For the deviation to be profitable, the expression must be non-zero when being evaluated at $\varepsilon = 0$. First, observe that D is continuous in α and $D > 0$ if $\alpha = 0$. Second, I will argue that D has at most one root in α. To do so, examine the terms for θ_i^k:

$$
d(\theta_1^k) = \alpha(s_1^k)^{\alpha-1}\left(-\frac{\theta_1^k\theta_A^{-k} - \omega_1^k + \frac{1-\delta}{\delta}s_1^k}{(1-\delta)}\right) + \frac{1-\delta}{\delta}(s_1^k)^\alpha.
$$

Differentiating with respect to α results in

$$
\begin{aligned}
\frac{\partial d(\theta_i^k)}{\partial \alpha} &= ((s_1^k)^{\alpha-1} + \alpha(s_1^k)^{\alpha-1}\ln s_1^k)\left(-\frac{\theta_1^k\theta_A^{-k} - \omega_1^k + \frac{1-\delta}{\delta}s_1^k}{(1-\delta)}\right) + \frac{1-\delta}{\delta}(s_1^k)^\alpha \ln s_1^k,\\
&= (s_1^k)^{\alpha-1}\left(-\frac{\theta_1^k\theta_A^{-k} - \omega_1^k + \frac{1-\delta}{\delta}s_1^k}{(1-\delta)}(1+\alpha\ln(s_1^k)) + \frac{1-\delta}{\delta}s_i^k\ln(s_i^k)\right).
\end{aligned}
$$

Now, observe that $(s_1^k)^{\alpha-1}$ is strictly increasing in α, whereas the expression in brackets changes sign at most once since it is linear in α. This implies that $\frac{\partial d(\theta_i^k)}{\partial \alpha}$ changes sign at most once, in which case it is positive for some $\alpha < \alpha'$ and negative for $\alpha > \alpha'$. Similarly, this holds for the equivalent expressions, $d(\cdot)$, for each type $\theta_A^k, \theta_A^{-k}, \theta_R^{-k}$. Then, since the function D is continuous in α, $D > 0$ for $\alpha = 0$, and D is increasing in α for $\alpha < \alpha''$ and decreasing for $\alpha > \alpha''$, it follows that D has at most one root.

In the second case, when $\alpha(\theta_A^k, \theta_A^{-k}) = 1$, the steady state mass of types $(\theta_1^k, \theta_R^{-k})$ increases (decreases) by $\frac{(1-\delta)\varepsilon}{\delta}$ while the steady state mass of types θ_A^k and θ_A^{-k} remains unchanged. The difference in profits is therefore

$$\sum_{\theta_i^k \in \left\{\theta_1^k, \theta_R^{-k}\right\}} \left[(\nu(\tilde{s}_i^k) - \nu(s_i^k)) f^{RM}(\theta_i^k) + \nu(\tilde{s}_i^k) \frac{1-\delta}{\delta} \varepsilon \right] + \sum_{\theta_i^k \in \{\theta_A^k, \theta_A^{-k}\}} \left[(\nu(\tilde{s}_i^k) - \nu(s_i^k)) f^{RM}(\theta_i^k) \right]. \tag{47}$$

Differentiating Equation 47 with respect to ε and evaluating the condition at $\varepsilon = 0$, yields

$$\sum_{\theta_i^k \in \left\{\theta_1^k, \theta_R^{-k}\right\}} \left[\nu'(\tilde{s}_i^k)|_{\tilde{s}_i^k = s_i^k} \frac{\partial \tilde{s}_i^k}{\partial \varepsilon}|_{\varepsilon=0} f^{RM}(\theta_i^k) + \frac{1-\delta}{\delta} \nu(\tilde{s}_i^k)|_{\tilde{s}_i^k = s_i^k} \right] \tag{48}$$

$$+ \sum_{\theta_i^k \in \{\theta_A^k, \theta_A^{-k}\}} \left[\nu'(\tilde{s}_i^k)|_{\tilde{s}_i^k = s_i^k} \frac{\partial \tilde{s}_i^k}{\partial \varepsilon}|_{\varepsilon=0} f^{RM}(\theta_i^k) \right]. \tag{49}$$

Again, the expression must be non-zero for the deviation to be profitable. To determine the partial derivatives of $\tilde{s}_A^k$ and $\tilde{s}_A^{-k}$ with respect to ε, I totally differentiate Equations 43 and 44:

$$\frac{\partial \tilde{s}_A^k}{\partial \varepsilon}|_{\varepsilon=0} = \frac{\theta_A^k(\theta_A^{-k} - \theta_R^{-k})}{(1-\delta) f^{RM}(\theta_A^k)} > 0 \text{ if } \theta_A^{-k} > \theta_R^{-k},$$
$$\frac{\partial \tilde{s}_A^{-k}}{\partial \varepsilon}|_{\varepsilon=0} = \frac{\theta_A^{-k}(\theta_A^k - \theta_1^k)}{(1-\delta) f^{RM}(\theta_A^{-k})} > 0.$$

Substituting $\nu(s_i^k) = \kappa(s_i^k)^\alpha$, $\nu'(s_i^k) = \kappa\alpha(s_i^k)^{\alpha-1}$, and the partial derivatives above into Equation 49 yields

$$D_2 \equiv \alpha(s_1^k)^{\alpha-1} \left(-\frac{\theta_1^k \theta_A^{-k} - \omega_1^k + \frac{1-\delta}{\delta} s_1^k}{(1-\delta)} \right) + \frac{1-\delta}{\delta} (s_1^k)^\alpha + \underbrace{\alpha(s_A^k)^{\alpha-1} \left(\frac{\theta_A^k(\theta_A^{-k} - \theta_R^{-k})}{(1-\delta)} \right)}_{d_2(\theta_A^k)}$$
$$+ \underbrace{\alpha(s_A^{-k})^{\alpha-1} \left(\frac{\theta_A^{-k}(\theta_A^k - \theta_1^k)}{(1-\delta)} \right)}_{=d_2(\theta_A^{-k})} + \alpha(s_R^{-k})^{\alpha-1} \left(-\frac{\theta_A^k \theta_R^{-k} - \omega_R^{-k} + \frac{1-\delta}{\delta} s_R^{-k}}{(1-\delta)} \right) + \frac{1-\delta}{\delta} (s_R^{-k})^\alpha.$$

Examining the two new terms shows that $d_2(\theta_A^{-k})$ is strictly increasing in α, and $d_2(\theta_A^k)$ is strictly increasing in α if $\theta_A^{-k} > \theta_R^{-k}$, and decreasing otherwise. Again, D_2 has at most one root.

Suppose $\beta_i^k, \omega_i^k, \theta_i^k, \theta_j^{-k}$ and δ are drawn uniformly from their continuous intervals. Then, there exists at most one α for which $D = 0$ (or $D_2 = 0$). Let α be drawn uniformly from $(0, 1)$, then random matching is generically suboptimal as such α is drawn with measure zero. $\square$

Proof of Proposition 2 Since market sides are fully symmetric, for brevity I drop the superscript k of each type θ_i^k. In this case, the positive assortative matching rule is defined as $\phi(\theta_i|\theta_j) = 1$ if and only if $i = j$ and results in steady-state mass $f(\theta_i) = \beta_i$ for any $\theta_i \in \Theta$.

Case 1: Search Fee

(a) "If" direction: PAM is optimal if the platform sets $s_i = \theta_i^2 - \omega_i$ for all $\theta_i \in \Theta$. As shown in Appendix A.2, PAM maximizes total match surplus across all agents. By choosing $s_i = \theta_i^2 - \omega_i$, the platform can extract each agent's match surplus as no agent is willing to pay more, thereby maximizing the platform's profit.

(b) "Only if" direction: Suppose, for contradiction, that PAM is profit-maximizing even if $s_i < \overline{s}_i$ for some $\theta_i \in \Theta \setminus \{\theta_1\}$ and

$$\overline{s}_i = \min\left\{\theta_i^2 - \omega_i, \theta_i^2 - \frac{\theta_i\theta_1 - \delta\omega_i}{1-\delta}\right\},$$

where the first entry is smaller than the second entry if $\omega_i > \theta_i\theta_1$. The platform's profit under PAM is strictly less than

$$\Pi^{PAM} < \frac{2(1-\delta)}{1-\rho}\left(\sum_{\theta_j \in \Theta\setminus\{\theta_i\}} \beta_j(\theta_j^2 - \omega_j) + \beta_i\overline{s}_i\right).$$

Next, observe that if the platform uses PAM with probability one in the next period, then type θ_i would reject the lowest type θ_1 in the (zero-probability) event they meet, since

$$\max\{\theta_i\theta_1, \omega_i\} < \delta\omega_i + (1-\delta)(\theta_i^2 - s_i). \tag{50}$$

Consider a deviation from PAM in which all types other than θ_1 and θ_i continue to only meet each other ($\Phi^D(\theta_j, \theta_j) = \beta_j$), but θ_1 and θ_i meet each other with mass $\epsilon \in (0, {\min\{\beta_1,\beta_i\}}/{\delta}]$ ($\Phi^D(\theta_i, \theta_1) = \epsilon$). Simultaneously, reduce the search fee of type θ_1 from $s_1 = \theta_1^2 - \omega_1$ to some s_1', which I will specify below. Then, I will show that there exists an $\epsilon > 0$ and a corresponding s_1' such that the resulting matching rule is feasible, incentive compatible, and strictly improves the platform's profit.

To check feasibility, substitute the steady state conditions in Equation 15 into the feasibility constraints in Equation 14:

$$\frac{\beta_i}{\delta + (1-\delta)\phi(\theta_i|\theta_i)} = \frac{\beta_i\phi(\theta_i|\theta_i)}{\delta + (1-\delta)\phi(\theta_i|\theta_i)} + \epsilon,$$
$$\frac{\beta_1}{\delta + (1-\delta)\phi(\theta_1|\theta_1)} = \frac{\beta_1\phi(\theta_1|\theta_1)}{\delta + (1-\delta)\phi(\theta_1|\theta_1)} + \epsilon.$$

Solving for the new (conditional) matching probabilities under the deviation, denoted ϕ^D, gives:

$$\phi^D(\theta_i|\theta_i) = \frac{\beta_i - \epsilon\delta}{\beta_i + (1-\delta)\epsilon}, \ \phi^D(\theta_1|\theta_1) = \frac{\beta_1 - \epsilon\delta}{\beta_1 + (1-\delta)\epsilon}. \tag{51}$$

For any ${}^{\min\{\beta_1,\beta_i\}}/_{\delta} > \epsilon > 0$, these probabilities are strictly less than one and larger than zero. Set the new search fee of type θ_1 to

$$s_1' = \frac{\beta_1 - \epsilon\delta}{\beta_1 + (1-\delta)\epsilon}(\theta_1^2 - \omega_1)$$

Next, I verify that the condition in Equation 50 for type θ_i remains satisfied. Under PAM, if $s_i < \overline{s}_i$, then the inequalities are slack. Since matching probabilities are continuous in ϵ, there exists a small $\epsilon > 0$ such that the condition remains non-binding or becomes just binding. Thus, type θ_i continues to reject matches with θ_1, and their search behavior does not change for sufficiently small ϵ.

Now consider the participation constraint of type θ_1 (Equation 12). Under PAM, its participation constraint is binding when $s_1 = \theta_1^2 - \omega_1$. Since θ_1 now meets type θ_i with positive probability, continuing to charge $s_1 = \theta_1^2 - \omega_1$ would violate the constraint. By lowering the search fee to s_1' as defined above, the constraint remains binding. The platform's profit given the new matching rule and search fees is

$$\begin{aligned}
\Pi^D &= \frac{2(1-\delta)}{1-\rho}\left(\frac{\beta_i s_i}{\delta + (1-\delta)\phi^D(\theta_i|\theta_i)} + \frac{\beta_1 s_1'}{\delta + (1-\delta)\phi^D(\theta_1|\theta_1)} + \sum_{j\neq 1,i}\beta_j(\theta_j^2 - \omega_j)\right), \\
&= \frac{2(1-\delta)}{1-\rho}\left((\beta_i + \epsilon)s_i + \frac{(\beta_1+\epsilon)(\beta_1 - \epsilon\delta)}{\beta_1 + (1-\delta)\epsilon}(\theta_1^2 - \omega_1) + \sum_{j\neq 1,i}\beta_j(\theta_j^2 - \omega_j)\right),
\end{aligned}$$

The deviation is profitable if $\Pi^D - \Pi^{PAM} > 0$, that is if

$$s_i - \frac{\varepsilon\delta}{\beta_1 + (1-\delta)\varepsilon}(\theta_1^2 - \omega_1) > 0,$$

which holds for $0 < \varepsilon < \frac{\beta_1 s_i}{\delta(\theta_1 - \omega_1) - (1-\delta)s_i}$.

Suppose for contradiction, that PAM is profit-maximizing even if $s_1 < \overline{s}_1$ for type θ_1

$$\begin{cases} \overline{s}_1 = \theta_1^2 - \omega_1 - \frac{\beta_i}{\beta_1}(\theta_i\theta_1 - \omega_i) & \text{if } \theta_i\theta_1 > \omega_i \\ \overline{s}_1 = \theta_1^2 - \omega_1 & \text{if } \theta_i\theta_1 \leq \omega_i. \end{cases}$$

Consider a deviation from PAM in which all types other than θ_1 and some type θ_i continue to only meet each other ($\Phi^D(\theta_j, \theta_j) = \beta_j$). For type θ_1 and θ_i, the platform chooses a new matching rule and search fee s_i' such that θ_i is indifferent between accepting and rejecting θ_1 and θ_1 is indifferent between participating or not given $s_1 < \overline{s}_1$

$$\begin{aligned}
\max\{\theta_i\theta_1, \omega_i\} &= \frac{\delta\omega_i + (1-\delta)(-s_i + \phi(\theta_i|\theta_i)\theta_i^2)}{\delta + (1-\delta)\phi(\theta_i|\theta_i)}, \\
\omega_1 &= \frac{\delta\omega_1 + (1-\delta)(-s_1 + \phi(\theta_1|\theta_1)\theta_1^2)}{\delta + (1-\delta)\phi(\theta_1|\theta_1)}.
\end{aligned}$$

This results in matching rules $\phi(\theta_i|\theta_i) = \frac{(1-\delta)s_i+\delta(\theta_i\theta_1-\omega_i)}{(1-\delta)\theta_i(\theta_i-\theta_1)}$ if $\theta_i\theta_1 > \omega_i$ or $\phi(\theta_i|\theta_i) = \frac{s_i}{\theta_i^2-\omega_i}$ if $\theta_i\theta_1 \leq \omega_i$, and $\phi(\theta_1|\theta_1) = \frac{s_1}{\theta_1^2-\omega_1}$. Given the matching rule and $s_1 < \theta_1^2 - \omega_1$, feasibility requires that s_i is chosen such that $f(\theta_i)(1-\phi(\theta_i|\theta_i)) = f(\theta_1)(1-\phi(\theta_1|\theta_1))$, where $f(\theta_k) = \frac{\beta_k}{\delta+(1-\delta)\phi(\theta_k|\theta_k)}$ for $k = 1, i$.

Then, the platform's deviation profit is larger than the profit under PAM when $s_1 < \overline{s}_1$. For $\delta \to 0$, it holds that

$$\beta_i \min\{\theta_i(\theta_i - \theta_1), \theta_i^2 - \omega_i\} + \beta_1(\theta_1^2 - \omega_1) \geq \beta_i(\theta_i^2 - \omega_i) + \beta_1 s_1,$$

where the inequality is strict if $\theta_i\theta_1 \leq \omega_i$ or if $\theta_i\theta_1 > \omega_i$ and $s_1 < \overline{s}_1$. This implies that there also exists a small movement to $\delta > 0$ where the inequality still holds.

Case 2: Advertisement

Suppose $\nu(\cdot)$ is strictly increasing, concave and fulfills the conditions in the proposition. Given PAM, the platform's profit is equal to

$$\Pi^{PAM} = \frac{2(1-\delta)}{1-\rho} \sum_{\theta_i \in \Theta} \nu(\theta_i^2 - \omega_i)\beta_i$$

where search costs are set to $s_i = \theta_i - \omega_i$ to maximize profits given PAM. Consider the a deviation in which all types other than type θ_1 and some type θ_i continue to meet each other:

$$\Phi'(\theta_j, \theta_j) = \Phi^{PAM}(\theta_j, \theta_j) = \beta_j, \forall \theta_j \in \Theta \setminus \{\theta_1, \theta_i\}.$$

For type θ_1 and θ_i choose the mass of recommended and matched pairs

$$\{\Phi'(\theta_1, \theta_1), \Phi'(\theta_1, \theta_i), \Phi'(\theta_i, \theta_i) m'(\theta_1, \theta_1), m'(\theta_1, \theta_i), m'(\theta_i, \theta_i)\}$$

such that $\beta_1 - \Phi'(\theta_1, \theta_1) = \beta_1 m'(\theta_1, \theta_1) = \varepsilon$ and $\beta_i - \Phi'(\theta_i, \theta_i) = \beta_i - m'(\theta_i, \theta_i) = \varepsilon$ for $\varepsilon \in (0, \min\{\beta_1, \beta_i\}]$. The new matching rule must satisfy the feasibility constraints (Equation 14) and steady state conditions (Equation 15) below

$$\beta_1 + \frac{1-\delta}{\delta}\varepsilon = \frac{1}{\delta}\left(\beta_1 - (1-\delta)m'(\theta_1, \theta_1)\right), \tag{52}$$

$$\beta_1 + \frac{1-\delta}{\delta}\varepsilon = \Phi'(\theta_1, \theta_1) + \Phi'(\theta_1, \theta_i), \tag{53}$$

$$\beta_i + \frac{1-\delta}{\delta}\varepsilon = \frac{1}{\delta}\left(\beta_i - (1-\delta)m'(\theta_i, \theta_i)\right), \tag{54}$$

$$\beta_i + \frac{1-\delta}{\delta}\varepsilon = \Phi'(\theta_i, \theta_i) + \Phi'(\theta_1, \theta_i). \tag{55}$$

It follows that $\Phi'(\theta_1, \theta_i) = \frac{\varepsilon}{\delta}$. To ensure that type θ_i rejects type θ_1 under the new matching rule (so that $m'(\theta_1, \theta_i) = 0$), while type θ_1 participates, the platform chooses $(\tilde{s}_1, \tilde{s}_i)$ such that

$$\beta_i \left(\max\{0, \theta_i\theta_1 - \omega_i\} + \frac{(1-\delta)}{\delta}\tilde{s}_i\right) = (1-\delta)(\beta_i - \varepsilon)\left(\theta_i^2 - \omega_i + \frac{(1-\delta)}{\delta}\tilde{s}_i\right),$$

and

$$\beta_1 \left(\frac{(1-\delta)}{\delta} \tilde{s}_1 \right) = (1-\delta)(\beta_1 - \varepsilon) \left(\theta_1^2 - \omega_1 + \frac{(1-\delta)}{\delta} \tilde{s}_1 \right),$$

hold, which results in

$$\tilde{s}_1 = \frac{(\beta_1 - \varepsilon)(\theta_1^2 - \omega_1)\delta}{\varepsilon + \delta(\beta_1 - \varepsilon)};$$

$$\tilde{s}_i = \begin{cases} \frac{(\beta_i - \varepsilon)(\theta_i^2 - \omega_i)\delta}{\varepsilon + \delta(\beta_i - \varepsilon)} & \text{if } \omega_i \geq \theta_i \theta_1 \\ \frac{\delta}{1-\delta} \frac{\beta_i \theta_i(\theta_i - \theta_1) - (\theta_i - \omega_i)(\varepsilon + \delta(\beta_i - \varepsilon)}{\varepsilon + \delta(\beta_i - \varepsilon)} & \text{otherwise .} \end{cases}$$

The deviation profit is therefore:

$$\Pi^D = \frac{2(1-\delta)}{1-\rho} \left(\nu(\tilde{s}_1) \left(\beta_1 + \frac{1-\delta}{\delta} \varepsilon \right) + \nu(\tilde{s}_i) \left(\beta_i + \frac{1-\delta}{\delta} \varepsilon \right) + \sum_{\theta_j \in \Theta \setminus \{\theta_1, \theta_i\}} \nu(\theta_j^2 - \omega_j)\beta_j \right).$$

Then for $(s_1 = \theta_1^2 - \omega_1, s_i = \theta_i^2 - \omega_i)$, the deviation is profitable if $\Pi^D - \Pi^{PAM} > 0$:

$$D \equiv \nu(\tilde{s}_1) \left(\beta_1 + \frac{1-\delta}{\delta} \varepsilon \right) - \nu(s_1)\beta_1 + \nu(\tilde{s}_i) \left(\beta_i + \frac{1-\delta}{\delta} \varepsilon \right) - \nu(s_i)\beta_i \geq 0. \tag{56}$$

Rewriting the conditions yields

$$(1-\delta)(\nu(\tilde{s}_1) + \nu(\tilde{s}_i))\varepsilon \geq \delta[\beta_1(\nu(s_1) - \nu(\tilde{s}_1)) + \beta_i(\nu(s_i) - \nu(\tilde{s}_i))]$$

Rearranging for δ gives the following condition:

$$\delta \leq \frac{(\nu(\tilde{s}_1) + \nu(\tilde{s}_i))\varepsilon}{\beta_1(\nu(s_1) - \nu(\tilde{s}_1)) + \beta_i(\nu(s_i) - \nu(\tilde{s}_i)) + (\nu(\tilde{s}_1) + \nu(\tilde{s}_i))\varepsilon} \equiv \overline{\delta},$$

where $\overline{\delta} \in (0,1)$ for $\varepsilon > 0$. $\square$

B.2 BINARY TYPES

Lemma 11. *For $\delta \to 0$, the optimal matching rule that implements*
(a) A_{PAM} is

$$\begin{bmatrix} \frac{s}{\theta_h(\theta_h-\theta_l)} & 1-\frac{s}{\theta_h(\theta_h-\theta_l)} \\ 1-\frac{\beta_h(\theta_h(\theta_h-\theta_l)-s)}{\beta_h\theta_h(\theta_h-\theta_l)+(\beta_l-\beta_h)s} & \frac{\beta_h(\theta_h(\theta_h-\theta_l)-s)}{\beta_h\theta_h(\theta_h-\theta_l)+(\beta_l-\beta_h)s} \end{bmatrix}, \text{ if } \frac{\beta_h}{\beta_l} \leq \frac{\theta_l^2-s}{\theta_h(\theta_h-\theta_l)-s}, \tag{57}$$

or otherwise,

$$\begin{bmatrix} \frac{\beta_h s}{\beta_l\theta_l^2+(\beta_h-\beta_l)s} & 1-\frac{\beta_h s}{\beta_l\theta_l^2+(\beta_h-\beta_l)s} \\ 1-\frac{s}{\theta_l^2} & \frac{s}{\theta_l^2} \end{bmatrix}, \text{ if } \frac{\beta_h}{\beta_l} \geq \frac{\theta_l^2-s}{\theta_h(\theta_h-\theta_l)-s}, \tag{58}$$

where at equality both matrices coincide. $\mathcal{O}(A_{PAM})$ is positive assortative.

(b) A_{WPAM} is

$$\begin{bmatrix} \frac{s}{\theta_h(\theta_h-\theta_l)} & 1-\frac{s}{\theta_h(\theta_h-\theta_l)} \\ 1-\frac{(\beta_l(\theta_h^2-s)-\beta_h(\theta_h(\theta_h-\theta_l)-s))s}{\theta_l(\theta_h-\theta_l)(\beta_h(\theta_h(\theta_h-\theta_l)-s)+\beta_l(\theta_h\theta_l-s))} & \frac{(\beta_l(\theta_h^2-s)-\beta_h(\theta_h(\theta_h-\theta_l)-s))s}{\theta_l(\theta_h-\theta_l)(\beta_h(\theta_h(\theta_h-\theta_l)-s)+\beta_l(\theta_h\theta_l-s))} \end{bmatrix}, \tag{59}$$

$$\text{if } \frac{(\theta_l^2-s)}{\theta_h(\theta_h-\theta_l)-s)} \leq \frac{\beta_h}{\beta_l} \leq \frac{(\theta_h^2-s)}{\theta_h(\theta_h-\theta_l)-s)}, \text{ and } \mathcal{O}(A_{WPAM}) \text{ is weakly assortative.}$$

A_{WPAM} is

$$\begin{bmatrix} \frac{\beta_h-\beta_l}{\beta_h} & 1-\frac{\beta_h-\beta_l}{\beta_h} \\ 1 & 0 \end{bmatrix}, \text{ if } \beta_h \geq \beta_l \text{ and } \frac{\beta_h-\beta_l}{\beta_h}\theta_h(\theta_h-\theta_l) \leq s \leq \theta_h\theta_l, \tag{60}$$

or

$$\begin{bmatrix} 0 & 1 \\ 1-\frac{\beta_h-\beta_l}{\beta_l} & \frac{\beta_h-\beta_l}{\beta_l} \end{bmatrix}, \text{ if } \beta_h \leq \beta_l \text{ and } s \leq \beta_l^2\theta_l^2+\beta_h\theta_l(\theta_h-\theta_l). \tag{61}$$

$\mathcal{O}(A_{WPAM})$ is non-assortative.

(c) A_{NAM} is

$$\begin{bmatrix} \frac{\beta_h-\beta_l}{\beta_h} & 1-\frac{\beta_h-\beta_l}{\beta_h} \\ \frac{s}{\theta_l(\theta_h-\theta_l)} & 1-\frac{s}{\theta_l(\theta_h-\theta_l)} \end{bmatrix}, \text{ if } 1 \leq \frac{\beta_h}{\beta_l} \leq \frac{\theta_h(\theta_h-\theta_l)}{\theta_h(\theta_h-\theta_l)-s}, \tag{62}$$

or

$$\begin{bmatrix} \frac{s}{\theta_h(\theta_h-\theta_l)} & 1-\frac{s}{\theta_h(\theta_h-\theta_l)} \\ 1-\frac{\beta_l(\theta_h^2-\theta_l^2-s)-\beta_h(\theta_h(\theta_h-\theta_l)-s)}{(\theta_h-\theta_l)\theta_l\beta_l} & \frac{\beta_l(\theta_h^2-\theta_l^2-s)-\beta_h(\theta_h(\theta_h-\theta_l)-s)}{(\theta_h-\theta_l)\theta_l\beta_l} \end{bmatrix}, \tag{63}$$

$$\text{if } \frac{\theta_h(\theta_h-\theta_l)}{\theta_h(\theta_h-\theta_l)-s} \leq \frac{\beta_h}{\beta_l} \leq \frac{\theta_h^2-\theta_l^2-s}{\theta_h(\theta_h-\theta_l)-s}, \text{ and } \mathcal{O}(A_{NAM}) \text{ is non-assortative.}$$

Proof of Lemma 11 The proof proceeds as follows. Fixing each matrix of mutual acceptance probabilities, I solve for the optimal matching rule by using the auxiliary problem from Appendix A.1. The linear program in the binary case is given by

$$\max \frac{2(1-\delta)s}{1-\rho}\left(f(\theta_h)+f(\theta_l)\right),$$

subject to feasibility and steady state conditions

$$f(\theta_h) = \Phi(\theta_h,\theta_h)+\Phi(\theta_h,\theta_l), \tag{64}$$
$$f(\theta_l) = \Phi(\theta_l,\theta_l)+\Phi(\theta_h,\theta_l), \tag{65}$$
$$\beta_h = f(\theta_h)\delta+(1-\delta)(\alpha(\theta_h,\theta_h)\Phi(\theta_h,\theta_h)+\alpha(\theta_h,\theta_l)\Phi(\theta_h,\theta_l)), \tag{66}$$
$$\beta_l = f(\theta_l)\delta+(1-\delta)(\alpha(\theta_l,\theta_l)\Phi(\theta_l,\theta_l)+\alpha(\theta_h,\theta_l)\Phi(\theta_h,\theta_l)), \tag{67}$$

as well as the respective participation and incentive constraints.

(a) A_{PAM}**:**

A_{PAM} induces the following constraints: A high type must be willing to continue searching after meeting a low type and the low type must be willing to participate. The transformed incentive and participation constraints take the following form

$$\beta_h(\delta\theta_h\theta_l+(1-\delta)s) \leq (1-\delta)\Phi(\theta_h|\theta_h)(\delta\theta_h^2+(1-\delta)s), \tag{68}$$
$$\beta_l(1-\delta)s \leq (1-\delta)\Phi(\theta_l|\theta_l)(\delta\theta_l^2+(1-\delta)s). \tag{69}$$

By Theorem 1 an optimal solution exists. In the binary case, the optimal solution can easily be checked. As the platform maximizes the steady state mass, it chooses $\Phi(\theta_h,\theta_h)$ and $\Phi(\theta_l,\theta_l)$ to be as small as possible without violating the constraints. Here, $\Phi(\theta_h,\theta_h)$ and $\Phi(\theta_l,\theta_l)$ are minimal when Equation 68 and Equation 69 bind resulting in

$$\Phi^{(a)}(\theta_h,\theta_h) = \frac{\beta_h((1-\delta)s+\delta\theta_h\theta_l)}{(1-\delta)((1-\delta)s+\delta\theta_h^2)},$$
$$\Phi^{(a)}(\theta_l,\theta_l) = \frac{\beta_l s}{(1-\delta)s+\delta\theta_l^2}.$$

Both the incentive and participation constraint, however, can only bind at the same time whenever

$$\left(\frac{\beta_h}{\beta_l}\right)^{(a)} = \frac{(1-\delta)(\theta_l^2-s)(s+\delta(\theta_h^2-s))}{(\theta_h(\theta_h-\theta_l)-s-\delta(\theta_h^2-s))(s+\delta(\theta_l^2-s))},$$

due to the feasibility constraints, Equation 64 and 65.

The steady state mass can be calculated by inserting $\Phi^{(a)}(\theta_h,\theta_h)$ and $\Phi^{(a)}(\theta_l,\theta_l)$ into

$$f(\theta_h) = \frac{\beta_h-(1-\delta)\Phi(\theta_h,\theta_h)}{\delta},$$
$$f(\theta_l) = \frac{\beta_l-(1-\delta)\Phi(\theta_l,\theta_l)}{\delta}.$$

The optimal matching rule is then given by $\phi(\theta_i|\theta_i) = \frac{\Phi(\theta_i,\theta_i)}{f(\theta_i)}$ for $i = h, l$.

If $\frac{\beta_h}{\beta_l} > (\frac{\beta_h}{\beta_l})^{(a)}$, only the participation constraint can be binding such that $\Phi(\theta_l,\theta_l) = \Phi^{(a)}(\theta_l,\theta_l)$. Inserting $\Phi(\theta_l,\theta_l) = \Phi^{(a)}(\theta_l,\theta_l)$ into the feasibility constraint of the low types yields $\Phi(\theta_h,\theta_l)$, which in turn determines $\Phi(\theta_h,\theta_h)$ by inserting it into the feasibility constraint of the high type. If $\frac{\beta_h}{\beta_l} < (\frac{\beta_h}{\beta_l})^{(a)}$, only the incentive constraint of the high type can be binding such that $\Phi(\theta_h,\theta_h) = \Phi(\theta_h,\theta_h)^{(a)}$ and the steps above can be repeated respectively.

(b) A_{WPAM}:

(b.1) A_{WPAM} induces the following constraints: A high type must be indifferent between searching and accepting low types

$$\beta_h(\delta\theta_h\theta_l + (1-\delta)s) = (1-\delta)\left(\Phi(\theta_h|\theta_h)(\delta\theta_h^2 + (1-\delta)s) + \alpha(\theta_h,\theta_l)\Phi(\theta_h,\theta_l)(\delta\theta_h\theta_l + (1-\delta)s)\right).$$

which holds for $\phi(\theta_h|\theta_h) = \frac{(1-\delta)s+\delta\theta_h\theta_l}{(1-\delta)\theta_h(\theta_h-\theta_l)}$. Additionally, low types must be willing to participate

$$\beta_l(1-\delta)s \leq (1-\delta)\left(\Phi(\theta_l,\theta_l)(\delta\theta_l^2 + (1-\delta)s) + \alpha(\theta_h,\theta_l)\Phi(\theta_h,\theta_l)(\delta\theta_h\theta_l + (1-\delta)s)\right).$$

From $\phi(\theta_h|\theta_h) = \frac{(1-\delta)s+\delta\theta_h\theta_l}{(1-\delta)\theta_h(\theta_h-\theta_l)}$ it follows

$$\begin{aligned}\Phi^{(b)}(\theta_h,\theta_h) &= \phi(\theta_h|\theta_h)\underbrace{\frac{\beta_h}{\delta + (1-\delta)(\phi(\theta_h|\theta_h) + \alpha(\theta_h,\theta_l)(1-\phi(\theta_h|\theta_h)))}}_{=f(\theta_h)} \\ &= \frac{\beta_h((1-\delta)s + \delta\theta_h\theta_l)}{(1-\delta)(\alpha(\theta_h,\theta_l)(\theta_h(\theta_h-\theta_l) - \delta\theta_h^2 - (1-\delta)s) + \delta\theta_h^2(1-\delta)s)}\end{aligned}$$

Then, $\Phi^{(b)}(\theta_h,\theta_l)$ follows by inserting $\Phi^{(b)}(\theta_h,\theta_h)$ in Equation 64, i.e.,

$$\frac{\beta_h\left(\theta_h\theta_l - (1-\delta)\theta_h^2 + (1-\delta)s\right)}{(1-\delta)(\alpha(\theta_h,\theta_l)(\delta\theta_h^2 - \delta s - \theta_h^2 + \theta_h\theta_l + s) - \delta\theta_h^2 + \delta s - s)}.$$

Furthermore, $\Phi^{(b)}(\theta_l,\theta_l)$ follows from feasibility of the low type by inserting $\Phi^{(b)}(\theta_h,\theta_l)$ into Equation 65.

The low type is indifferent between participating and not participating if

$$\alpha^{WPAM} \equiv \left\{\alpha(\theta_h,\theta_l) : \beta_l s = \Phi^{(b)}(\theta_l,\theta_l)(\delta\theta_l^2 + (1-\delta)s) + \alpha(\theta_h,\theta_l)\Phi^{(b)}(\theta_h,\theta_l)(\delta\theta_h\theta_l + (1-\delta)s)\right\}.$$

For $\delta \to 0$, I get

$$\alpha^{WPAM} = \frac{s\left(\beta_h(\theta_h(\theta_h-\theta_l) - s) - \beta_l(\theta_l^2 - s)\right)}{(\theta_h(\theta_h-\theta_l) - s)\left(\beta_h\theta_l(\theta_h-\theta_l) + \beta_l\theta_l^2 + (\beta_h-\beta_l)s\right)}. \tag{70}$$

The mutual acceptance probability is then given by the above. For $\delta \to 0$, to ensure that $\alpha^{WPAM} \leq 1$ and $\phi(\theta_l|\theta_l) \geq 0$, the conditions in the lemma must hold.

(b.2) Additionally for $\beta_h \geq \beta_l$, the platform can implement A_{WPAM} by always matching low types with high types, i.e. $\phi(\theta_h|\theta_l) = 1$. This implies that low types search for only one period, such that $f(\theta_l) = \Phi(\theta_h, \theta_l) = \beta_l$. The high types' incentive constraint for $\alpha(\theta_h, \theta_l) = 1$ is

$$\beta_h(\delta\theta_h\theta_l + (1-\delta)s) \geq (1-\delta)\left(\Phi(\theta_h|\theta_h)(\delta\theta_h^2 + (1-\delta)s) + \Phi(\theta_h, \theta_l)(\delta\theta_h\theta_l + (1-\delta)s)\right),$$

and from the feasibility constraint (Equation 64), it follows that $\Phi(\theta_h, \theta_h) = \beta_h - \beta_l$. The incentive constraint of high types is satisfied if

$$s \geq \frac{\beta_h - (1-\delta)\beta_l}{(1-\delta)\beta_h}\theta_h(\theta_h - \theta_l) - \frac{\delta}{(1-\delta)}\theta_h^2$$

The participation constraint of low types is satisfied

$$\beta_l(1-\delta)s \leq (1-\delta)\beta_l(\delta\theta_h\theta_l + (1-\delta)s),$$

if $s \leq \theta_h\theta_l$.

Lastly for $\beta_h \leq \beta_l$, the platform can implement A_{WPAM} by always matching high types to low types, i.e. $\phi(\theta_l|\theta_h) = 1$. This implies that high types search for only one period, such that $f(\theta_h) = \Phi(\theta_h, \theta_l) = \beta_h$. Low types must be willing to participate

$$\beta_l(1-\delta)s \leq (1-\delta)\left(\Phi(\theta_l, \theta_l)(\delta\theta_l^2 + (1-\delta)s) + \beta_h(\delta\theta_h\theta_l + (1-\delta)s)\right).$$

If the participation constraint is satisfied, low types also search for only one period, such that $f(\theta_l) = \beta_l$. Therefore, $\Phi(\theta_l, \theta_l) = \beta_l - \beta_h$. Thus, the participation constraint is satisfied if

$$s \leq \beta_l^2\theta_l^2 + \beta_h\theta_l(\theta_h - \theta_l),$$

and low types do not reject low types if

$$s \geq \frac{\delta\theta_l(\beta_h(1-\delta)\theta_h - \beta_l\theta_l)}{(1-\delta)(\beta_l - (1-\delta)\beta_h)},$$

which equals zero for $\delta \to 0$.

(c) A_{NAM}**:**

(c.1) A_{NAM} can be implemented if

$$\beta_h((1-\delta)s + \delta\theta_h\theta_l) \geq (1-\delta)\Phi(\theta_h|\theta_h)((1-\delta)s + \delta\theta_h^2) + (1-\delta)\Phi(\theta_h, \theta_l)((1-\delta)s + \delta\theta_h\theta_l),$$
$$\beta_l((1-\delta)s + \delta\theta_l^2) \leq (1-\delta)\Phi(\theta_h|\theta_l)((1-\delta)s + \delta\theta_h\theta_l).$$

As high types accept both high and low types and search for only one period, the steady state mass of high types is equal to their inflow: $f(\theta_h) = \beta_h$. The platform's profit from high types is, therefore, independent of the matching rule. To maximize profits, the platform minimizes $\Phi(\theta_h, \theta_l)$ such that

$$\Phi(\theta_h, \theta_l) = \frac{\beta_l((1-\delta)s + \delta\theta_l^2)}{(1-\delta)((1-\delta)s + \delta\theta_h\theta_l)},$$

and the incentive constraint of the low type binds. $\Phi(\theta_h, \theta_h) = \beta_h - \beta_l$ follows from the feasibility constraints (Equation 64), where $\Phi(\theta_h, \theta_h)$ and $\Phi(\theta_h, \theta_l)$ must be such that the incentive constraint of the high type is fulfilled, which is true if

$$1 \leq \frac{\beta_h}{\beta_l} \leq \frac{((1-\delta)s + \delta\theta_l^2)\,\theta_h(\theta_h - \theta_l)}{(\theta_h(\theta_h - \theta_l) - (1-\delta)s - \delta\theta_h^2)\,((1-\delta)s + \delta\theta_h\theta_l)}.$$

For $\delta \to 0$ this results in

$$1 \leq \frac{\beta_h}{\beta_l} \leq \frac{\theta_h(\theta_h - \theta_l)}{\theta_h(\theta_h - \theta_l) - s}.$$

(c.2) A_{NAM} can be implemented if a high type is indifferent between accepting and rejecting a low type, while a low type is willing to reject low types. Again as in part (b), $\phi(\theta_h|\theta_h) = \frac{(1-\delta)s+\delta\theta_h\theta_l}{(1-\delta)\theta_h(\theta_h-\theta_l)}$ must hold to ensure the indifference constraint of high types. Then for $\alpha(\theta_h, \theta_l) \in [0, 1]$, $\Phi^{(c)}(\theta_h, \theta_h) = \Phi^{(b)}(\theta_h, \theta_h)$ and $\Phi^{(c)}(\theta_h, \theta_l) = \Phi^{(b)}(\theta_h, \theta_l)$. Inserting into the incentive constraint of the low type, the low type rejects low types if

$$\beta_l((1-\delta)s + \delta\theta_l^2) \leq (1-\delta)\alpha(\theta_h, \theta_l)\Phi^{(c)}(\theta_h, \theta_l)((1-\delta)s + \delta\theta_h\theta_l),$$

which holds with equality for

$$\alpha^{NAM} = \frac{\beta_l s}{(\beta_h - \beta_l)(\theta_h(\theta_h - \theta_l) - s)} \tag{71}$$

if $\delta \to 0$. It holds that $\alpha^{NAM} > 0$ generally, and $\alpha^{NAM} \leq 1$ if $\frac{\beta_h}{\beta_l} \geq \frac{\theta_h(\theta_h-\theta_l)}{\theta_h(\theta_h-\theta_l)-s}$. Additionally, $\phi(\theta_h|\theta_l) = \frac{(\beta_h-\beta_l)(\theta_h^2-\theta_l^2-s)}{\beta_l\theta_l(\theta_h-\theta_l)}$, which is larger than zero if $\beta_h \geq \beta_l$ and smaller than one if $\frac{\beta_h}{\beta_l} \leq \frac{\theta_h^2-\theta_l^2-s}{\theta_h(\theta_h-\theta_l)-s}$. $\square$

Proof of Proposition 3 Next, I determine the platform's preferred outcome. First, let $s \leq \theta_l^2$.

(i) For $\frac{\beta_h}{\beta_l} \leq \left(\frac{\beta_h}{\beta_l}\right)^{(a)}$, the profit when implementing A_{PAM} (Equation 57) is

$$\Pi^{(a.1)} = \frac{2\nu(s)(1-\delta)}{1-\rho}\left(\frac{2\beta_h\theta_h(\theta_h-\theta_l) + (\beta_l - \beta_h)(s + \delta(\theta_h^2 - s))}{s + \delta(\theta_h^2 - s)}\right).$$

For $\frac{\beta_h}{\beta_l} \geq \left(\frac{\beta_h}{\beta_l}\right)^{(a)}$, the platform can either implement A_{PAM} (Equation 58) or A_{WPAM} (Equation 59). The profits are

$$\Pi^{(a.2)} = \frac{2\nu(s)(1-\delta)}{1-\rho}\left(\frac{2\beta_l\theta_l^2 + (\beta_h - \beta_l)(s + \delta(\theta_l^2 - s))}{s + \delta(\theta_l^2 - s)}\right)$$

and

$$\Pi^{(b.1)} = \frac{2\nu(s)(1-\delta)}{1-\rho}\frac{(2\beta_h\theta_h^2\theta_l - (\beta_h-\beta_l)(2\theta_l^2 - s - \theta_l s) - \delta(\beta_h-\beta_l)(\theta_h-\theta_l)(s+\theta_h\theta_l))}{(\theta_h+\theta_l)(s+\delta(\theta_h\theta_l - s))},$$

where the difference is positive

$$\Pi^{(b.1)} - \Pi^{(a.1)} \geq 0.$$

Thus for $\left(\frac{\beta_h}{\beta_l}\right)^{(a)} \leq \frac{\beta_h}{\beta_l} \leq \frac{(1-\delta)\theta_h^2 - s}{\theta_h(\theta_h-\theta_l)-s+\delta(\theta_h^2-s)}$ the platform implements A_{WPAM} and A_{PAM} if $\frac{\beta_h}{\beta_l} \geq \frac{(1-\delta)\theta_h^2-s}{\theta_h(\theta_h-\theta_l)-s+\delta(\theta_h^2-s)}$.

It remains to compare the profit in equilibrium (b) when implementing A_{WPAM} against the profit from equilibrium (c) when implementing A_{NAM}. Note that for $s \leq \theta_l^2$, the profit when implementing A_{WPAM} is maximized in (b.1) as agents in both equilibria in (b.2) only search for one period. The profit in (c) is

$$\Pi^{(c.1)} = \frac{2\nu(s)(1-\delta)}{1-\rho}\left(\beta_h + \frac{\beta_l\theta_l(\theta_h-\theta_l)}{s+\delta(\theta_h\theta_l-s)}\right)$$

or

$$\Pi^{(c.2)} = \frac{2\nu(s)(1-\delta)}{1-\rho}$$
$$\left(\frac{s(\beta_h-\beta_l)\theta_h(\theta_h-\theta_l) + s\beta_l\theta_l(\theta_h-\theta_l) + \delta(\beta_h\theta_h(\theta_h-\theta_l)(\theta_h\theta_l-s) + \beta_l(\theta_h-\theta_l)^2(s+\theta_h\theta_l))}{(s+\delta(\theta_h^2-s))(s+\delta(\theta_h\theta_l-s))}\right).$$

Then, it holds that $\Pi^{(b)} \geq \Pi^{(c.1)}, \Pi^{(c.2)}$.
(ii) Let $\theta_l^2 \leq s \leq \theta_h\theta_l$. Then, the platform can only implement A_{WPAM} or A_{NAM}. Alternatively, the platform can exclude low types from participating. Recall that $\Pi^{(c.1)}$ and $\Pi^{(c.2)}$ are strictly dominated by $\Pi^{(b.1)}$. Therefore, the platform implements either A_{WPAM} in Equation 59, 60, or 61. If $\beta_h \geq \beta_l$, the platform can either implement A_{WPAM} in Equation 59 or 61. If $\beta_h < \beta_l$, the platform can either implement A_{WPAM} in Equation 59 or 60. In this case, however, for too large s no low type is willing to participate such that the platform excludes low types. Note that at $s = \theta_h\theta_l$, the matching outcome is non-assortative if $\beta_h \geq \beta_l$, whereas only high types participate if $\beta_h < \beta_l$.
(iii) Let $\theta_h\theta_l \leq s \leq \theta_h^2$. If search costs are larger than $\theta_h\theta_l$, low types are no longer willing to participate. To maximize surplus from high types, the platform sets $\phi(\theta_h|\theta_h) = 1$. □

Proof of Proposition 4 (i) The platform implements A_{PAM} together with the matching rule as in Lemma 11 (a). As the positive assortative matching outcome maximizes match productivity, the welfare loss from mismatch is zero. For $\frac{\beta_h}{\beta_l} \leq \left(\frac{\beta_h}{\beta_l}\right)^{(a)}$, agents' expected search time is

$$\mathcal{T}(\theta_h) = \frac{\theta_h(\theta_h-\theta_l)}{s+\delta(\theta_h^2-s)},$$
$$\mathcal{T}(\theta_l) = \frac{\beta_h(\theta_h(\theta_h-\theta_l) + (\beta_l-\beta_h)s + \delta(\beta_l-\beta_h)(\theta_h^2-s)}{\beta_l(s+\delta(\theta_h^2-s))}.$$

Observe that $\mathcal{T}(\theta_h)$ is decreasing in s and δ. Differentiating $\mathcal{T}(\theta_l)$ with respect to s and δ yields

$$\begin{aligned}\frac{\partial \mathcal{T}(\theta_l)}{\partial s} &= -\frac{\beta_h(1-\delta)\theta_h(\theta_h-\theta_l)}{\beta_l(s+\delta(\theta_h^2-s))^2} < 0,\\ \frac{\partial \mathcal{T}(\theta_l)}{\partial \delta} &= -\frac{\beta_h\theta_h(\theta_h-\theta_l)(\theta_h^2-s)}{\beta_l(s+\delta(\theta_h^2-s))^2} < 0,\end{aligned}$$

i.e. $\mathcal{T}(\theta_h)$ is decreasing in s and δ as well.

For $\frac{\beta_h}{\beta_l} \geq \frac{(1-\delta)\theta_h^2-s}{\theta_h(\theta_h-\theta_l)-s+\delta(\theta_h^2-s)}$, agents' expected search time is

$$\begin{aligned}\mathcal{T}(\theta_h) &= \frac{\beta_l\theta_l^2+(\beta_h-\beta_l)(s+\delta(\theta_l^2-s))}{\beta_h(s+\delta(\theta_l^2-s)},\\ \mathcal{T}(\theta_l) &= \frac{\theta_l^2}{s+\delta(\theta_l^2-s)}.\end{aligned}$$

Observe that $\mathcal{T}(\theta_l)$ is decreasing in s and δ. Differentiating $\mathcal{T}(\theta_h)$ with respect to s and δ yields

$$\begin{aligned}\frac{\partial \mathcal{T}(\theta_h)}{\partial s} &= -\frac{\beta_l(1-\delta)\theta_l^2}{\beta_h(s+\delta(\theta_l^2-s))^2} < 0,\\ \frac{\partial \mathcal{T}(\theta_h)}{\partial \delta} &= -\frac{\beta_l(\theta_l^2-s)\theta_l^2}{\beta_h(s+\delta(\theta_l^2-s))^2} < 0,\end{aligned}$$

i.e. $\mathcal{T}(\theta_h)$ is decreasing in s and δ as well.

(ii) The platform implements A_{WPAM} together with the matching rule as in Lemma 11 (b.1). The welfare loss from mismatches is

$$\mathcal{W}_{WPAM} = \alpha_{WPAM}\Phi^{(b)}(\theta_h,\theta_l)(\theta_h-\theta_l)^2,$$

and agents' expected search time is

$$\begin{aligned}\mathcal{T}(\theta_h) &= \frac{\beta_h-(1-\delta)(\Phi^{(b)}(\theta_h\theta_h)+\alpha_{WPAM}\Phi^{(b)}(\theta_h,\theta_l))}{\beta_h\delta},\\ \mathcal{T}(\theta_l) &= \frac{\beta_l-(1-\delta)(\Phi^{(b)}(\theta_l\theta_l)+\alpha_{WPAM}\Phi^{(b)}(\theta_h,\theta_l))}{\beta_l\delta},\end{aligned}$$

where

$$\begin{aligned}\Phi^{(b)}(\theta_h,\theta_h) &= \frac{\beta_h\theta_h\theta_l-(\beta_h-\beta_l)(1-\delta)(\theta_l^2-s)}{(1-\delta)(\theta_h^2-\theta_l^2)},\\ \Phi^{(b)}(\theta_l,\theta_l) &= \frac{\beta_h\theta_h\theta_l-(\beta_h-\beta_l)(1-\delta)(\theta_h^2-s)}{(1-\delta)(\theta_h^2-\theta_l^2)},\end{aligned}$$

which are both increasing (decreasing) in s if $\beta_h > \beta_l$ ($\beta_h < \beta_l$) and increasing in δ. Note that $\Phi^{(b)}(\theta_h,\theta_l)$ followed from feasibility (see proof of Lemma 11) and α_{WPAM} is set to fulfill the low types' participation constraint. Using the implicit function theorem and differentiating the participation constraint with respect to s yields

$$\begin{aligned}&\beta_l-(1-\delta)(\alpha_{WPAM}\Phi^{(b)}(\theta_h,\theta_l)+\Phi^{(b)}(\theta_l,\theta_l))=\\&\frac{\partial\Phi^{(b)}(\theta_l,\theta_l)}{\partial s}(\delta\theta_l^2+(1-\delta)s)+\frac{\partial\alpha_{WPAM}\Phi^{(b)}(\theta_h,\theta_l)}{\partial s}(\delta\theta_h^2+(1-\delta)s),\end{aligned}$$

where the left-hand side corresponds to $\delta f(\theta_l)>0$ and $\Phi^{(b)}(\theta_l,\theta_l)$ is increasing in s if $\beta_h>\beta_l$ and decreasing otherwise. Thus, it follows that $\alpha_{WPAM}\Phi^{(b)}(\theta_h,\theta_l)$ must be increasing in s if $\beta_l>\beta_h$ and either in-or decreasing for $\beta_l<\beta_h$ (depending on the parameter values). Using the implicit function theorem and differentiating the participation constraint with respect to δ yields

$$\begin{aligned}0=&\frac{\partial\Phi^{(b)}(\theta_l,\theta_l)}{\partial\delta}(\delta\theta_l^2+(1-\delta)s)+\Phi^{(b)}(\theta_l,\theta_l)(\theta_l^2-s)\\&+\frac{\partial\alpha_{WPAM}\Phi^{(b)}(\theta_h,\theta_l)}{\partial\delta}(\delta\theta_h^2+(1-\delta)s)+\alpha_{WPAM}\Phi^{(b)}(\theta_h,\theta_l)(\theta_h\theta_l-s),\end{aligned}$$

As $\Phi^{(b)}(\theta_l,\theta_l)$ is increasing in δ, $\alpha_{WPAM}\Phi^{(b)}(\theta_h,\theta_l)$ must be decreasing in δ for $s\leq\theta_l^2$. For δ for $s>\theta_l^2$, $\alpha_{WPAM}\Phi^{(b)}(\theta_h,\theta_l)$ can be either in- or decreasing in δ.

It follows that $\mathcal{W}_{WPAM}$ is increasing in s if $\beta_l>\beta_h$ and either in-or decreasing for $\beta_l<\beta_h$ (depending on the parameter values). Furthermore, $\mathcal{W}_{WPAM}$ is decreasing in δ for $s\leq\theta_l^2$ and either in- or decreasing for $s>\theta_l^2$.

Differentiating $\mathcal{T}(\cdot)$ with respect to s and δ yields

$$\begin{aligned}\frac{\partial\mathcal{T}(\theta_h)}{\partial s}&=-\frac{(1-\delta)\theta_h\theta_l\left(\beta_l\left((\theta_h-\theta_l)\delta+\theta_l\right)+(\theta_h-\theta_l)(1-\delta)\beta_h\right)}{\left((\theta_h\theta_l-s)\delta+s\right)^2\beta_h(\theta_h+\theta_l)}<0,\\\frac{\partial\mathcal{T}(\theta_l)}{\partial s}&=-\frac{(1-\delta)\theta_h\theta_l\left(\beta_h(\theta_h-\theta_l)(1-\delta)+\beta_l\left((\theta_h-\theta_l)\delta+\theta_l\right)\right)}{\left((\theta_h\theta_l-s)\delta+s\right)^2(\theta_h+\theta_l)}<0,\\\frac{\partial\mathcal{T}(\theta_h)}{\partial\delta}&=-\frac{\theta_h^2\theta_l\left(\beta_h\theta_l(\theta_h-\theta_l)+\beta_l(\theta_l^2-s)\right)}{(\delta\theta_h\theta_l+s(1-\delta))^2\beta_h(\theta_h+\theta_l)}<0,\\\frac{\partial\mathcal{T}(\theta_l)}{\partial\delta}&=-\frac{\theta_h^2\theta_l\left(\beta_h\theta_l(\theta_h-\theta_l)+\beta_l(\theta_l^2-s)\right)}{(\delta\theta_h\theta_l+s(1-\delta))^2(\theta_h+\theta_l)}<0.\end{aligned}$$

That is, $\mathcal{T}(\cdot)$ is decreasing in s and δ.

(iii) The platform implements A_{WPAM} by the matching rule as in Lemma 11 (b.2). The matching outcome is non-assortative. The welfare loss from mismatch is

$$\mathcal{W}_{NAM}=-2\beta_l(\theta_h-\theta_l)^2,$$

and agents search for one period only.

□

Proof of Proposition 5 The proof follows the structure of Proposition 3. Note all matching outcomes in Proposition 3 are implemented when choosing the search fee except the positive assortative matching outcome for $\frac{\beta_h}{\beta_l} \geq \left(\frac{\beta_h}{\beta_l}\right)^{(b)}$.

(i) Suppose the platform implements A_{PAM} for $\frac{\beta_h}{\beta_l} \leq \left(\frac{\beta_h}{\beta_l}\right)^{(a)}$. Recall that for $s = \theta_l^2$, $\left(\frac{\beta_h}{\beta_l}\right)^{(a)} = 0$ and thus A_{PAM} can never be implemented if there is a positive inflow of both types. The platform maximizes its profit with respect to s under the constraint that $s \in [0, \theta_l^2]$ and the condition $\frac{\beta_h}{\beta_l} \leq \left(\frac{\beta_h}{\beta_l}\right)^{(a)}$ is still fulfilled.

(ii) Suppose the platform implements A_{WPAM} for $\left(\frac{\beta_h}{\beta_l}\right)^{(a)} \leq \frac{\beta_h}{\beta_l} \leq \left(\frac{\beta_h}{\beta_l}\right)^{(b)}$. There exists an $\theta_h(\theta_h - \theta_l) > \overline{s} > \theta_l^2$ such that if $s > \overline{s}$ A_{WPAM} can never be implemented if there is a positive inflow of both types. The platform maximizes its profit with respect to s under the constraint that $s \in [0, \overline{s}]$ and the condition $\left(\frac{\beta_h}{\beta_l}\right)^{(a)} \leq \frac{\beta_h}{\beta_l} \leq \left(\frac{\beta_h}{\beta_l}\right)^{(b)}$ is still fulfilled.

(iii) Suppose the platform implements A_{NAM} for $\beta_h > \beta_l$. Then, agents only search for one period. Therefore, the platform increases the search fee as much as possible. By Proposition 3, the upper limit is given by $s = \theta_h\theta_l$.

(iv) Lastly, the platform can exclude low types from participating. To maximize profits, the platform extract the surplus from high types by setting $s = \theta_h^2$. The platform does so for sufficiently high $\frac{\beta_h}{\beta_l}$.

The platform does not implement the positive assortative matching outcome for $\frac{\beta_h}{\beta_l} \geq \left(\frac{\beta_h}{\beta_l}\right)^{(b)}$. Recall that profits are

$$\frac{2(1-\delta)}{1-\rho}\left(\frac{2\beta_l\theta_l^2 s}{(1-\delta)s + \delta\theta_l^2} + (\beta_h - \beta_l)s\right).$$

Note that both terms are strictly increasing in s such that the platform would choose $s = \theta_l^2$ resulting in

$$\frac{2(1-\delta)}{1-\rho}\left(\beta_h\theta_l^2 + \beta_l\theta_l^2\right).$$

As A_{NAM} can be implemented for $\beta_h > \beta_l$ with $s = \theta_h\theta_l$, the profit from A_{NAM} is always strictly larger than the profit from A_{PAM} for $\frac{\beta_h}{\beta_l} \geq \left(\frac{\beta_h}{\beta_l}\right)^{(b)}$. $\square$

Proof of Proposition 6 For the proof of the first sentence, I first examine the first-order condition in Equation 9. Observe that $\frac{\nu(s)}{\nu'(s)}$ is strictly increasing in s due to the concavity of $\nu(s)$. Additionally, from Proposition 4 it follows that $\partial f(\theta_i^k)/\partial s < 0$, i.e. $f(\theta_i^k)$ is strictly decreasing in s. From the proof of Proposition 4, it follows that $\partial^2 f(\theta_i^k)/\partial s^2 > 0$. Thus, the right-hand side of Equation 9 decreases in s and the left-hand side increases in s which implies the statement.

To prove the second sentence, note that the profit for charging $s = \theta_h\theta_l$ and implementing $\mathcal{O}_N AM$ is

$$\frac{2(1-\delta)}{(1-\rho)}\nu(\theta_h\theta_l)(\beta_h+\beta_l),$$

and for charging $s = \theta_h^2$ and excluding low types is

$$\frac{2(1-\delta)}{(1-\rho)}\nu(\theta_h^2)(\beta_h).$$

Then, if $\nu(\cdot)$ is sufficiently concave, the platform prefers to charge a lower price for a discrete increase in demand:

$$\frac{\nu(\theta_h\theta_l)}{\nu(\theta_h^2)} \geq \frac{\beta_h}{\beta_h+\beta_l}(<1).$$

□

Proof of Proposition 7 To characterize the profit-maximizing solution with overconfident users, note first that it is optimal for the platform to have all three types participate. Otherwise, the platform can always increase profits by including the formerly excludes type by charging a positive fee and matching them to each other. Consider the feasible mutual acceptance matrices of the form

$$\begin{bmatrix} \alpha(\theta_h,\theta_h) & \alpha(\theta_h,\theta_l) & \alpha(\theta_h,\hat{\theta}_l) \\ \alpha(\theta_l,\theta_h) & \alpha(\theta_l,\theta_l) & \alpha(\theta_l,\hat{\theta}_l) \\ \alpha(\hat{\theta}_l,\theta_h) & \alpha(\hat{\theta}_l,\theta_l) & \alpha(\hat{\theta}_l,\hat{\theta}_l). \end{bmatrix}$$

As overconfident users perceive to have the same continuation value as high types, $V^C(\theta_h)$, they follow the same acceptance strategy. That is, overconfident users accept high types with probability one and low types with probability $\alpha \in [0,1]$ if and only if high types do. Furthermore, overconfident users are accepted by high (low) types with positive probability if and only if high (low) types accept low types with positive probability.

The feasible mutual acceptance matrices are

$$A_1 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}, A_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, A_3 = \begin{bmatrix} 1 & \alpha' & \alpha' \\ \alpha' & 1 & \alpha' \\ \alpha' & \alpha' & (\alpha')^2 \end{bmatrix}, A_4 = \begin{bmatrix} 1 & \alpha'' & \alpha'' \\ \alpha'' & 0 & 0 \\ \alpha'' & 0 & (\alpha'')^2 \end{bmatrix},$$

for $\alpha' \in (0,1]$ and $\alpha'' \in (0,1]$. The profit from implementing A_1 is given in the text preceding Proposition 7. Observe that implementing $A_2 - A_4$ can induce search for more than one period for at least one type.

To implement A_2–A_4, the incentive constraint ensuring that high types reject low types with positive probability must hold. The platform then maximizes revenue from both high and low types by maximizing match surplus and extracting it through the search fee conditional on leaving a rent of $\theta_h\theta_l$ to high types. From Appendix A.2, match surplus is maximized under positive assortative matching—that is, when the platform implements A_2. Moreover, agents must search for only one period; otherwise, surplus is lost due to $\delta > 0$.

Revenue from overconfident agents is maximized under $A_2 - A_4$ when they search for $\frac{1}{\delta}$ periods—i.e., no one they match with accepts them, and s_h is maximized. Under A_2, this is exactly the case: overconfident types are rejected, they search for $\frac{1}{\delta}$ periods, and the platform captures the match surplus from high types through s_h, i.e. s_h is maximal. Thus, it follows that the relevant constraints are given by

$$\theta_h\theta_l \leq \frac{(1-\delta)(-s+\phi(\theta_h|\theta_h)\theta_h^2)}{\delta+(1-\delta)\phi(\theta_h|\theta_h)}, \tag{IC-θ_h}$$

$$\theta_h\theta_l \leq \frac{(1-\delta)(-s+\phi(\theta_h|\theta_h)\theta_h^2)}{\delta+(1-\delta)\phi(\theta_h|\theta_h)}, \tag{PIC-$\hat{\theta}_l$}$$

$$0 \leq \frac{(1-\delta)(-s+\phi(\theta_l|\theta_l)\theta_l^2)}{\delta+(1-\delta)\phi(\theta_l|\theta_l)}. \tag{PC-θ_l}$$

From the steady state constraints, Equation 15, the platform's profit maximization problem can be written as

$$\frac{\beta_h s_h}{\delta+(1-\delta)\phi(\theta_h|\theta_h)} + \frac{\beta_l(1-\lambda)}{\delta+(1-\delta)\phi(\theta_l|\theta_l)} + \frac{\beta_l\lambda}{\delta},$$

subject to feasibility constraints, Equation 14, and the three incentive and participation constraints above. It can easily be verified that $s_l = \theta_l^2$ and $s_h = \theta_h(\theta_h - \theta_l) - {}^{d}/_{1-\delta}\theta_h\theta_l$ and $\phi(\theta_h|\theta_h) = 1, \phi(\theta_l|\theta_l) = 1$ and $\phi(\hat{\theta}_l|\hat{\theta}_l) = 1$ maximize the platform's profit and satisfy all constraints with equality. The platform's profit, Π_S^{OC}, is given in Proposition 7, where λ^* is derived by setting $\Pi_S^{OC} = \Pi_{PAM}^{OC}$ and solving for λ. $\square$

C. APPENDIX: TABLES

Application	App Price	Subscriptions	One-Time Purchases
Tinder	Free	Tinder Gold (1 Week): $13.99 – 18.99 Tinder Gold (1 Month): $14.99 – 24.99 Tinder Plus (1 Month): $9.99	1 Boost: $3.99 – 7.99 3 Super Likes: $9.99 5 Super Likes: $4.99
Bumble	Free	Bumble Premium (1 Year) $129.99 – 169.99	5 Spotlights + Compliments $24.99 – 29.99 15 Spotlights + Compliments $44.99 – 59.99 30 Spotlights + Compliments $79.99 – 99.99
Hinge	Free	Hinge+ Subscription (1 Week): $16.99 Hinge Subscription (1 Month): $29.99 – 34.99 Membership (1 Month): $19.99 Hinge Subscription (1 Week): $14.99 HingeX Subscription: $24.99	Bundle of three Roses: $9.99 Bundle of twelve Roses: $29.99 Boost: $9.99 – 19.99
Match	Free	Match (1 Month): $19.99 – 42.99 Match (3 Months): $74.99 Match (6 Months): $129.99 Standard (1 Month): $44.99 Basic (1 Months): $44.99 Platinum (1 Week): $29.99	1 Top Spot: $2.99 Top Spot 10-Pack: $19.99 Boost 1-Pack: $5.99
Hily	Free	Hily Premium (1 Week): $14.99 Profile boost (1 Week): $5.99 – 9.99 Premium+ (1 Week): $24.99 Hily Elixir (1 Week): $19.99	1 Unblur: $4.99 5 Unblur: $12.99
Plenty of Fish	Free	Upgrade (1 Month): $19.99 Upgrade (3 Months): $38.99 Premium Membership (1 Month): $29.99	1 Token: $1.99 5 Tokens: $8.99 10 Tokens: $17.99
Badoo	Free	Badoo Premium (1 Week): $5.99 – 8.99 Super Powers (1 Week): $2.99 Super Powers (1 Months): $11.99	Pack of 100 Credits: $1.99 – 3.99
Coffee Meets Bagel	Free	Premium (1 Month): $14.99 – 34.99 Premium (3 Months): $74.99 Premium (6 Months): $71.99 Platinum (1 Month): $46.99 Platinum (3 Month): $99.99	200 Coffee Beans: $2.99 400 Coffee Beans: $4.99 3000 Coffee Beans: $24.99
Raya	Free	Membership (1 Month): $24.99 Membership (6 Month): $113.99 Raya+ Membership: $49.99	30 Extra Likes: $10.99 Skip the Wait: $7.99 5 Skip the Waits: $29.99 1 Direct Request: $4.99 3 Direct Requests: $12.99 12 Direct Requests: $49.99
MeetMe	Free	MeetMe (1 Month): $7.99 MeetMe (3 Months): $17.99 MeetMe+ (1 Month): $7.99	Pack of 200 Credits: $1.99 Pack of 500 Credits: $1.99 – 4.99 Pack of 1800 Credits: $14.99 Pack of 14500 Credits: $99.99 Pack of 3200 Credits: $24.99

Table 1: A Selection of Dating Apps in the US Apple Store

Application	App Price	Subscriptions	One-Time Purchases
Tinder	Free	Tinder Gold (1 Week): 13,99 € Tinder Gold (1 Month): 8,99 – 27,49 € Tinder Platinum (1 Month): 32,99 €	1 Boost: 7,99 – 9,99 € 3 Super-Likes: 11,99 € 5 Super-Likes: 5,99 – 9,99 €
Bumble	Free	Bumble Premium (1 Week): 14,99 – 19,99 € Bumble Boost (1 Week): 5,99 – 6,99 € Bumble Premium (1 Month): 34,99 €	
Hinge	Free	Hinge+ Sub (1 Week): 14,99 € Hinge+ Sub (1 Month): 24,99 € HingeX Sub (1 Week): 24,99 €	Bundle of twelve Roses: 24,99 € Bundle of three Roses: 7,99 € One Superboost: 14,99 € One Boost: 7,99 €
LOVOO	Free	Lovoo Premium (1 Month): 11,99 – 24,99 €	300 Credits: 5,99 € 500 Credits: 4,99 € 550 Credits: 7,99 € 3000 Credits: 19,99 € 5 Icebreaker: 5,99 € Unbegrenzte Likes: 1,19 €
Badoo	Free	Badoo Premium (1 Week): 5,99 – 7,99 € Badoo Premium (1 Month): 19,99 €	100 Badoo Credits: 1,99 – 4,99 € 550 Badoo Credits: 12,99 € Super Powers (1 Woche): 2,99 € Super Powers (1 Monat): 8,99 € Super Powers (1 Woche): 2,99 €
Parship	Free	Premium lite (6 Month): 209,99 – 229,99 € Premium classic (1 Year): 224,99 – 499,99 € Premium Comfort: 249,99 €	Parship Premium: 9,99 €
OkCupid	Free	OkCupid Premium (1 Month): 15,99 – 32,99 € OKCupid Premium (3 Month): 65,99 €	1 Boost: 1,99 – 7,99 € 2 Superlikes: 7,99 €
Raya	Free	Membership (1 Month): 18,99 € Membership (6 Month): 83,99 € Raya+ Membership (1 Month): 44,99 €	Skip the Wait 7,99 € 3 Direct Requests 12,99 € 1 Direct Request 4,99 € 30 Extra Likes 10,99 € 5 Skip the Waits 29,99 € 1 Travel Plan 9,99 €
LoveScout24	Free	Lovescout24 (1 Month): 39,99 € Mobile Plus (1 Month): 9,99 € Mobile Plus (1 Week): 4,99 € Lovescout24 (1 Week): 9,99 € Lovescout24 (3 Month): 89,99 €	1 Booster: 1,99 € Wer sucht mich?: 1,99 € Boost: 1,99 € Dateroulette: 2,99 € Favouriten-Funk: 1,99 €
ElitePartner	Free	ElitePartner Premium Go: 3,99 – 19,99 € Premium plus (1 Year): 399,99 € Premium basic (6 Months): 279,99 € Premium comfort (2 years) : 599,99 €	

Table 2: A Selection of Dating Apps in the German Apple Store

App Name	Price	Contains Ads	Prices of In-App Purchases	Number of Installations
German Store				
happn	Free	Yes	0.59 – 274.99 €	100M+
Badoo	Free	Yes	0.39 – 244.99 €	100M+
Tinder	Free	Yes	0.29 – 324.99 €	100M+
SweetMeet	Free	Yes	0.59 – 219.99 €	50M+
Bumble	Free	No	0.29 – 314.99 €	50M+
BLOOM	Free	Yes	1.49 – 299.00 €	10M+
OkCupid	Free	Yes	0.71 – 194.99 €	10M+
Zoosk	Free	Yes	0.50 – 434.99 €	10M+
Mamba	Free	Yes	0.50 – 294.99 €	10M+
Boo	Free	Yes	0.46 – 218.85 €	10M+
US Store				
happn	Free	Yes	$0.49 – 224.99	100M+
Badoo	Free	Yes	$0.49 – 239.99	100M+
Tinder	Free	Yes	$0.49 – 299.99	100M+
SweetMeet	Free	Yes	$0.99 – 199.99	50M+
Bumble	Free	No	$0.49 – 259.99	50M+
BLOOM	Free	Yes	$1.99 – 349.00	10M+
OkCupid	Free	Yes	$0.99 – 179.99	10M+
Zoosk	Free	Yes	$0.49 – 399.99	10M+
Mamba	Free	Yes	$0.99 – 264.99	10M+
Boo	Free	Yes	$1.00 – 269.99	10M+

Table 3: Most Popular Dating Apps in the German and US Google Play Store

App Name	App Price	In-App Purchases	Price	Adds	In-App Purchases	No. of Downloads
US Apple Store			US Android Store			
LinkedIn	Free	Career (1 Month): $29,99 – 39,99 Business (1 Month): $69,99	Free	Yes	$7.49 – 839.88	1B+
Indeed	Free	None	Free	Yes	none	100M+
Glassdoor	Free	None	Free	No	none	10M+
ZipRecruiter	Free	None	Free	No	none	10M+
Monster	Free	None	Free	No	none	5M+
German Apple Store			German Android Store			
LinkedIn	Free	Essentials (1 Month): 9,99 € Career (1 Month): 29,99 – 39,99 € Business (1 Month): 69,99 €	Free	Yes	7,00 – 839,88 €	1B+
Indeed	Free	None	Free	Yes	none	100M+
Glassdoor	Free	None	Free	Yes	none	10M+
Stepstone	Free	None	Free	Yes	none	10M+
Monster	Free	None	Free	Yes	none	5M+
Costs for Recruiters						
LinkedIn	The standard account is free. Premium accounts cost between 40 – 125 €/$ (See above)					
Indeed	There is an option for free listings. Costly adds are charged per click, with a minimum of 5 €/$ per day					
Glassdoor	No information					
Monster	Two Options: Monster+ Standard: Pay per Click and Monster+ Pro: 749€/$299 per month					
Stepstone	Multiple tiers: “Campus” 199 €,“Select”: 329€, “Pro”: 1399 €, Pro Plus: 1699 €, Pro Ultimate: 2399 €					
Zip Recruiter	Pricing depends on the number of job ads. Ads are charged per day and per add:“Standard”: $16, “Premium”: $24 Plans are charged per ad and per month: “Standard”: $299, “Premium”: $419, “Pro”: $719					

Table 4: Selected Job Platforms in the German and US App Store