Faking Network Size

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Abstract

Users have an incentive to join large platforms, and hence platforms want to convince users that they have a large user base. I consider a monopolist platform's incentive and ability to signal its user base when there is uncertainty regarding the amount of potential users. The platform can signal using either fake accounts or prices. When — as in some real-world examples — users are naive regarding the platform's ability to use fake accounts, small platforms use them to profit from the artificial increase in demand. If users are sophisticated, larger platforms use fake profiles to differentiate themselves from smaller ones, and — in contrast to the case of naive consumers — platforms would benefit from a ban on such business practices.

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1 Introduction

Joining a new platform often involves uncertainty for an individual user who cannot be sure of its overall popularity. With uncertainty about a platform's network size, users need to form beliefs about the participation decisions of others. Based on the data platforms collect and the in-depth knowledge of their business model, platforms naturally have an informational advantage relative to individual users. Moreover, they have an incentive to let consumers believe that joining will enable them to reap large network effects. Given these incentives, this paper investigates a platform's ability to use "fake profiles" to its advantage both when users are sophisticated about and when they are unaware of the platform's ability to do so.

The use of fake profiles is a common phenomenon on digital platforms. Prominent cases are those of Dating platforms. For example, the Federal Trade Commission (FTC) sued the Match Group for using fake profiles to persuade users to upgrade to a paid subscription. Other dating platforms use company-created fake profiles to interact with users on the platform, giving them the impression of a real contact with users often being unaware about this practice.[1] Further examples include cryptocurrency exchange platforms, which are under investigation by the SEC for engaging in trading financial assets themselves to artificially inflate the trading volume (so-called wash trading). Recent studies show that about 70% of unregulated trades are subject to wash trading (Cong et al., 2023). The economic costs to users and platforms are substantial. If fake profiles induce users to hold incorrect belief about the platform, they may make inefficient participation decisions. Furthermore, creating fake profiles is costly to the platform without generating additional value.

Formally, I investigate how a monopoly platform uses multiple signals to convince users of its network size. In particular, users can learn from the price they observe, a (cheap-talk) message, and the network size. Users are uncertain about the distribution of stand-alone values provided by the platform, while the platform has private information about this fundamental. Given the information asymmetry, suboptimal membership fees and fake profiles set by the platform are both costly signals about the fundamental. Fake profiles can increase the perceived network size but do not generate network effects ex post.

Users observe the membership fee first and then decide whether to join the platform. Thereafter, users who joined observe the perceived network size and decide whether to exit prior to paying the membership fee (following a "free-trial period"). In contrast to most of the signaling literature, I analyze a game with multiple signaling instruments, continuous signals and a continuum of receivers, where receivers (users) care not only about the sender's (platform's) action, but also about the action of other receivers (users).

Absent fake profiles, the platform's only signaling instrument about its size is the price. If users cannot observe demand and pay the price upfront, prices must be distorted upwards above full information prices to credibly signal a high fundamental. Only in

[1] See https://www.ftc.gov/news-events/press-releases/2019/09/ftc-sues-owner-online-dating-service-matchcom-using-fake-love, last visited 01.09.2020); https://www.verbraucherzentrale.de/wissen/digitale-welt/onlinedienste/onlinedating-auf-diesen-portalen-flirten-fakeprofile-21848, last visited 01.09.2020 or https://www.faz.net/aktuell/wirtschaft/unternehmen/straftaten-schiessen-wegen-datingplattform-in-die-hoehe-18792428.html.

higher states can a platform set inefficiently high prices to optimally separate from lower states. Including the possibility of creating fake profiles, users' understanding thereof is crucial when evaluating the market outcome. Before users pay the price, they observe the perceived network size without being able to distinguish between real and fake profiles. Sophisticated users, however, are fully aware of the platform's practice, whereas naive users are unaware of the possible use of fake profiles or believe that fake profiles are forbidden and hence not used.

Sophisticated users anticipate the platform's incentives correctly and hence, discount the perceived network size by the expected amount of fake profiles. In that case, both fake profiles and high prices are costly in that they reduce profits taken demand as given, and hence are substitutes for signaling a high fundamental. Abstracting from existence issues, the platform can always fully differentiate itself from those with less users through costly signaling based either on inefficiently high prices or the use of costly fake profiles. I identify parameter conditions such that the latter separating equilibrium exists, whereas the former always exists. Given its existence, in the unique separating equilibrium the platform with the lowest fundamental sets its full information price and all other platforms need to create fake profiles and distort their prices. Otherwise, the unique equilibrium has the same properties as the unique equilibrium absent fake profiles.

In contrast, if users dogmatically believe that every profile is real, i.e. they never considered the possibility of creating fake profiles, the platform uses the cheap-talk message to communicate the expected network size upfront. Users blindly believe this message upfront and the corresponding network size later on. The platform can exploit this misperception by using fake profiles to signal an unrealistic high network size, and thus value, to the users. In equilibrium, the platform always prefers to lie and deceive users by pretending that their network size is larger than it actually is. With a bounded state space, the platform with the highest fundamental, however, cannot induce unrealistic high user beliefs. Hence, depending on the costs of fake profiles, platforms below a threshold lie by creating fake profiles and set a higher price, and those above induce the highest possible belief about the state. This results in pooling on the observable instruments, but differentiation on the unobservable instrument.

The results imply that a platform would like to commit to refrain from using fake profiles with sophisticated users. Fake profiles are a wasteful investment for the platform used for separation. Rather than observing that platforms commit on not using fake profiles, it can be observed that platforms actually hide the use of fake profiles in their terms and conditions. This, however, likely indicates that users are mainly naive in these markets, which renders fake profiles profitable. With sophisticated users the platform would benefit from a regulation that provides commitment for the platform's claim that the observed network size is the true network size. This would result in full information prices being incentive-compatible as lying by platforms with a low fundamental is detectable and punished by exiting and non-paying users. If platforms are not able to credibly commit, however, they can profit from fake profiles with sophisticated users: signaling via fake profiles can lower the overall signaling costs when compared to signaling via distortionary prices only.

Methodologically, I apply an adjusted version of the D1 criterion developed by Banks and Sobel (1987) to select the unique Perfect Bayesian Equilibrium. It is well known that in certain classes of games, the D1 criterion selects a unique equilibrium outcome, which is separating, whenever there is a single receiver (or multiple receivers whose decisions are strategically independent). I extend this result to strategically interdependent receivers by imposing a restriction on the coordination problem of users' entry decision off the equilibrium path.

The remainder of the paper proceeds as follows. The related literature is discussed in Section 1.1. Section 2 describes the model and discusses potential applications. Section 3 analyzes the model when users are sophisticated, whereas Section 4 provides the analysis for naive users. Section 6 concludes.

1.1 Related Literature

This paper is the first to introduce signaling into a model of platform adoption. As such it is related to models of platforms when there is incomplete information. Technically, it is related to the literature on signaling with multiple instruments. Since I allow for users who have incorrect beliefs, it is also related to papers on misleading consumers. I discuss these related papers below.

Platform Markets This paper belongs to the relatively sparse literature incorporating issues of incomplete information and asymmetric information (Halaburda et al., 2018a; Jullien and Pavan, 2019; Ke and Zhu, 2021; Kang and Muir, 2022) on platforms. Most models in the literature on platforms and two-sided markets assume complete information (Caillaud and Jullien, 2003; Rochet and Tirole, 2003, 2006; Armstrong, 2006; Halaburda et al., 2018b; Gal-Or, 2020). To the best of my knowledge, no paper has investigated asymmetric information between the platform and its users where the platform holds private information. The closest paper with respect to modelling the incomplete information is Jullien and Pavan (2019) who consider a platform market in which both users and platforms face uncertainty about participation decisions due to dispersion of information about their preferences. Especially given the growing importance of big data, I consider the more realistic case in which the platform has (superior) private information regarding its desirability to potential users. This, however, implies that the platform's choices act as a signal. Contrary to Jullien and Pavan (2019), I consider a platform with only one market side, or a platform on which both sides of the market are identical.

Signaling and Advertising As the monopoly platform has private information in my model, it is closely related to the literature on signaling (Kreps and Sobel, 1994), where signalling games with a continuum of states are studied by Mailath (1987) and in particular Mailath and von Thadden (2013). Fake profiles have not been studied in this context. Papers such as Kihlstrom and Riordan (1984), Milgrom and Roberts (1986), and Bagwell and Ramey (1988) that study the use of costly advertisement in combination with prices are conceptually closely related although they do not incorporate network effects. Fake profiles resemble persuasive advertisement, which is assumed to shift the willingness to pay of users (see Bagwell (2007) for an overview on advertisement). In a signaling model, Rhodes and Wilson (2018) analyzed false advertising used by firms to overstate the value of their products. False advertisement is only costly whenever it is punishable by a third-party. Buyers, nevertheless, may be affected by false advertisement in equilibrium. A key difference is how the amount of fake profiles is determined in my model, in which not only the cost function but also equilibrium prices and demand determine the amount fake profiles. Furthermore, the fact that fake profiles might not be observable to users influence their equilibrium amount.

The paper adds to the literature of signaling by identifying a novel channel — network effects — that makes signaling via price or fake profiles credible. Main channels in the literature on signaling are: 1) repeated purchases, 2) cost differences between qualities, and 3) information differences between users. Although learning by users bears similarities to repeated purchases, price signaling in my model even works absent learning. Due to the presence of network effects an increase in the price has two effects. First, the price has a direct effect on users' utility lowering their willingness to participate on the platform. It follows that additionally, the price also has an indirect effect on users' utility through the reduced participation decision of others, which further reduces their willingness to participate. Without network effects, users would not care about the state as their participation decision would be independent of those of other users.

Due to equilibrium multiplicity in signaling games, a wide range of papers focuses on appropriate equilibrium refinements (Cho and Kreps, 1987; Banks and Sobel, 1987; Cho and Sobel, 1990). As users exert positive externalities on each other, in my model the most prominent refinements in the literature fail to select a unique equilibrium. Therefore, I adopt a version of the D1 criterion for a continuum of states as in (Ramey, 1996) and impose a further (weak) restriction on the receivers' strategies off-path: that they are rationalizable (Bernheim, 1984; Pearce, 1984) given a common belief.

Consumer Naïveté The model investigates the effects of different types of user sophistication on the market outcome when users face fake profiles thereby adding to the literature on consumer naïveté. See Heidhues and Kőszegi (2018) for a survey on the growing literature on how consumer naïveté affects market outcomes. My paper is among the first to introduce consumer naïveté in platform markets. The only other paper in this context is Johnen and Somogyi (2021) who analyze and compare the sellers' and the platforms' incentive to hide parts of the price from naive consumers. They find that a platform has strong incentives to shroud additional fees if it increases perceived consumer surplus. Conceptually, my paper is closely related to work on consumer naivete in cheap talk models (Ottaviani and Squintani, 2006; Kartik et al., 2007; Chen, 2011) that analyze the impact of naive or credulous consumers who blindly believe the sender's message. In contrast to these papers, creating fake profiles is costly giving rise to signaling issues.

Manipulating Consumer Expectations More broadly, the paper is connected to the literature that studies the manipulation of consumer expectations, especially in network markets. Early contributions focus on the expectations of early adopters of a network good. More recently, the emergence of fake reviews for products on platforms is studied. Evidence of fake reviews on for example Amazon.com is provided by He et al. (2022) or Expedia.com and TripAdvisor.com by Mayzlin et al. (2014). Theoretic treatments of fake reviews can be found in Glazer et al. (2021) and Yasui (2020), where the most closely related paper is Knapp (2022). The author analyzes a cheap-talk game in which a reviewer of a good may create a truthful or fake review but abstracts from the platform setting with network effects. Similar to my paper, consumers differ in their understanding of the possibility of fake reviews (naive or sophisticated).

2 Model

I analyze a sender-receiver model with two types of players: a platform (sender) and a group of potential users (receivers) of mass one. The platform has private information about a fundamental $\theta \in [\underline{\theta}, \overline{\theta}] \subset \mathbb{R}$ that determines the users' distribution of stand-alone values r on the platform $F(r|\theta) \equiv F_\theta(r)$. The common prior about the fundamental is $\mu_0(\theta)$, a continuous probability distribution, and has full support on Θ.

The platform and users engage in the following four-period game. First, nature draws the fundamental $\theta \in \Theta$. The platform observes θ and sets a price p and message m, where the message space is restricted to the type space. Additionally, the platform may invest in fake profiles, where the amount of fake profiles is given by ξ. Then, upon observing the platform's message and price (p, m), as well as their own stand-alone value r_i, users decide whether or not to enter. Users who joined observe the perceived number of users and decide whether to exit. The perceived number will depend on the actual mass of users and fake profiles, in a way detailed below. Lastly, the platform collects fees.

Users: Payoff Users have a common outside option normalized to zero. They vary, however, in the utility they receive from joining the platform — their stand-alone value r_i. User i obtains utility

$$v_i = r_i + \beta n - p,$$

where the distribution of stand-alone values r, F_θ, is continuous with full and strictly positive support. Users benefit from positive network effects, β, and from the mass of users that stay on the platform, n, but pay price p.

Users: Actions and Beliefs Upon having learned about the true fundamental, the platform sets a price, sends a message, and determines the number of fake profiles. First, after observing a price-message pair, users update about the fundamental and form a belief $\mu(\theta|p, m)$ and, then after learning their individual stand-alone value, form a belief $\mu_1(\theta|p, m, r_i)$. Second, after joining the platform, users observe the perceived mass of users, which is a function of the mass of users who have joined and the mass of fake profiles. The corresponding belief is denoted by $\mu_2(\theta|r, p, m, \mathbb{I})$, where $\mathbb{I}$ denotes the information structure. Depending on the users' ability to learn about or observe fake profiles, they update about the fundamental based on the information structure $\mathbb{I} = \{\emptyset, \{[0,1], \mathbb{R}_0^+\}, [0,1] + \mathbb{R}_0^+\}$. Users may either not observe the network size at all, $\mathbb{I} = \emptyset$, observe the true network size and fake profiles separately, $\mathbb{I} = \{[0,1], \mathbb{R}_0^+\}$, or observe the sum of both $[0,1] + \mathbb{R}_0^+$, which may include fake profiles. Among these users, sophisticated users are aware of the possibility of fake profiles, while naive users blindly believe the message sent and take the network size at face value for values below or equal to one.[2]

[2] As the true network size is at most equal to a mass of one when all users enter, naive users take the network size at face value as long as it does not exceed a value of one. For values above one, naive users are free to hold any belief about the state, where I restrict attention to naive users holding the most pessimistic belief. Imposing any other belief such as the most optimistic belief, however, does not affect the equilibrium as long as the message space is restricted to the type space and users understand that the type space is bounded.

User i's entry strategy in the first period is a mapping $\sigma_i^1 : \mathbb{R} \times \Theta \to [0,1]$ from prices and messages to entry. For given price p and belief $\mu_1(\theta|\cdot)$, a user enters if their expected utility from entering is higher than their outside option. The aggregate entry decision of users depends on the distribution of r in society. Following entry, users update their beliefs to $\mu_2(\theta|\cdot)$ and decide whether to exit the platform; formally, their exit strategy in period two — given that the user entered in period one — is given by $\sigma_i^2 : \mathbb{R} \times \Theta \times \mathbb{I} \to [0,1]$.

Platform: Payoff and Actions The platform is a monopolist that chooses a price-message pair (p, m) with $p \in \mathbb{R}$ and $m \in M = \Theta$ and a number of fake profiles $\xi \in \mathbb{R}_0^+$. The platform's strategy maps the state space into prices, messages, and fake profiles $\sigma^P : \Theta \to \mathbb{R} \times \Theta \times \mathbb{R}_0^+$. The platform maximizes its profit with respect to prices and fake profiles

$$\max_{p,\xi} (p - c) n(\theta, \mu, p) - \gamma\xi,$$

where $n(\theta, \mu, p)$ is the mass of users that stay on the platform given the true fundamental, their belief about it, and price p. Let c denote the marginal cost of the platform to serve one user and γ the marginal cost of creating a fake profile.

Equilibrium Concept The equilibrium concept is a Perfect Bayesian Equilibrium (PBE) if all users are sophisticated. Strategies are optimal given beliefs at every information set. Beliefs of sophisticated users are updated via Bayes' rule whenever possible. At each information node, users optimize given their beliefs (sequentially rationality).

If users are naive, I use a Perception-Perfect Equilibrium (PPE). Naive users form their beliefs through the following rule. In the first period, naive users blindly believe in the fundamental stated by the observed message, and thus are point beliefs. Following entry, users take the network size at face value (for a mass below or equal to one). If the observed network size confirms the expected network size given first period belief and price, the belief remains the same. If the observed network size, $\hat{n}$, does not match the expected network size naive users revise their belief. The naive users' new belief must satisfy the following condition

$$\mu_2^N \equiv \{\theta' \in \Theta | n(\theta', \mu_1^N, p) = \hat{n}\}.$$

Naive users maximize expected utility given their beliefs.

As a tie-breaking rule, I impose that users enter only if they expect to stay: Whenever a user is indifferent between not joining the platform or joining the platform but leaving in period 3, I assume that the user does not enter.

Equilibrium Refinement For sophisticated users, off-path the set of equilibria is refined by adapting the D1 Criterion of Ramey (1996) to a signaling game in which the receivers strategically interact. Intuitively, under D1 users' out-of equilibrium beliefs put positive mass only to the types that are most likely to profit from a deviation from equilibrium. As users' participation decisions depend on the decision of other users and are thus not strategically independent, I impose a restriction on the coordination aspect of users' entry decision. For a given price, I suppose this induces a common receiver belief (as it does on the path of the play). Consumers take this common belief as given, and then resolve the coordination problem among themselves in the same way as they would if this belief was common knowledge. With common knowledge of the state, there is a unique rationalizable entry decision suggesting that the coordination problem should be resolved in exactly that way. The precise definition is given by Definition 4 in Appendix A.

For naive users, I do not use an equilibrium refinement. Beliefs (on and off-path) are naively given by the simple rule specified above.

Assumptions To analyze the game, I impose regularity conditions on the family of distributions $F_\theta(r)$ and the strength of the network effect β.

Assumption 1. *The distributions $F_\theta(r)$ are*

1. *twice differentiable in r and θ with density $f_\theta(r)$,*
2. *where the corresponding densities $f_\theta(r)$ are single-peaked in r, and*
3. *the distributions have a (weakly) increasing hazard rate $\lambda(r;\theta)$ in r, and*
4. *common support.*

The above assumption on F_θ ensures that the optimization problem of the platform is well-behaved under complete information and that there exists a unique (monopoly) price. The assumption on common support and single-peakedness can be relaxed to allow for the family of uniform distributions as well.[3]

Assumption 2. *(MLRP) For $\theta > \theta'$, f_θ likelihood dominates f_θ': $\frac{f_\theta(r)}{f_{\theta'}(r)}$ is an increasing function.*

The monotone likelihood ratio property implies that F_θ first-order stochastically dominates $F_{\theta'}$ and hazard rate $\lambda(p, n; \theta)$ is strictly monotonically decreasing in θ. First-order stochastic dominance implies that higher states lead to higher demand. The latter yields that the price elasticity increases with increasing θ holding participation constant. Hence, a higher state induces higher monopoly prices c.p. Lastly, to exclude multiplicity of (continuation) equilibria network effects β cannot be too strong.

Assumption 3. *(Network effects) $\beta \in \{\beta \in \mathbb{R}_+ : 1/2 - \beta \max_{\theta,r} f_\theta(r) > 0,\ \theta \in \Theta\}$.*

Networks effects must be small enough to avoid multiplicity of continuation equilibria, and hence, guarantees uniqueness of continuation equilibria.

[3] Assuming non-common support has the following implication for the analysis. After observing the stand-alone value, users with a high stand-alone value form a belief that puts zero probability on states that are not possible. This does not affect the analysis of separating equilibria on-path, but plays a role for incentive-compatibility as a deviation to a lower state cannot be credible to those users. During most of the analysis, however, the relevant incentive compatibility is for a low type to mimic a high type. In pooling equilibria beliefs are dispersed and the analysis remains unchanged in this case.

2.1 Preliminaries

Under the assumptions made for any price the platform has set, there exists a unique cutoff strategy for users, even if the information is incomplete. Each user has private information about their own reservation value. All users with reservation values above the cutoff participate in the platform, while users below the cutoff do not. The first lemma defines the cutoff.

Lemma 1. *(Unique cutoff) In any equilibrium, in which users hold a common belief upon observing (p, m), users use a cutoff strategy. The unique cutoff is given by*

$$r_c = p - \beta \int_{\Theta} (1 - F_\theta(r_c))\mu(\theta|r_c, p)d\theta, \tag{1}$$

which results in $n(\theta, \mu, p) = 1 - F_\theta(r_c)$ agents.

Proof. See Appendix B. □

This lemma implies that users also follow a unique cut-off strategy in an equilibrium in which some types pool, i.e. when there is incomplete separation. Users' beliefs are dispersed as although all users have a common prior, they draw inferences from their own r. As a result, after observing a price and their own reservation value, users hold different beliefs. To establish the lemma, however, it is sufficient to suppose that upon observing the price but not yet their standalone value, users hold a common belief. On path this must be fulfilled because all users rely on Bayes rule, whereas off-path the common belief assumption is imposed.

As a benchmark, the next lemma characterizes the full information benchmark which corresponds to the first-best solution in prices and user participation.

Lemma 2. *(First-best) Under full information, there exists a unique equilibrium. In this equilibrium, the platform's profit maximizing price $p^{FI}(\theta)$ satisfies*

$$p^{FI} - c = \frac{1 - F_\theta(r^{FI})}{f_\theta(r^{FI})}(1 - \beta f_\theta(r^{FI})), \tag{2}$$

where r^{FI} denotes the equilibrium cutoff given p^{FI}.

Proof. See Appendix B. □

It follows that the mark-up is always positive and hence, the price is always above marginal cost.

3 Price and Message as Signals

In this section, I will discuss a benchmark for analyzing the effectiveness of signaling on platforms, in which the presence of fake profiles does not impact demand. The benchmark assumes that users enter the platform and decide whether to stay without being able to observe the network size. In other words, the timing is as if user pay for their membership before joining the platform. Users cannot learn from the network size before their purchase decision, making fake profiles irrelevant. Hence, the participation and purchasing decision happen simultaneously represented by the information structure $\mathbb{I} = \emptyset$.

3.1 Sophisticated Users

Sophisticated users are rational and fully understand the signaling game. For those users, the price is the only credible signal and the message is ignored. Hence, I will suppress the message in the section below. For ease of exposition, the main part will focus on the construction of separating equilibria. The platform uses a one-to-one strategy $\tau : \Theta \to \mathbb{R}$ that maps the state to its chosen price and therefore, users hold a common belief on the path of play.

Definition 1. A separating equilibrium consist of the platform's strategy τ, users' strategy σ_i and beliefs, μ, such that:

1. For any $p \in \tau(\Theta), \mu(p) = \tau^{-1}(p)$,
2. For any $\theta \in \Theta, \tau(\theta) \in \arg\max_{p \in \mathbb{R}_+} \pi(\theta, \mu(p), p)$ (Incentive Compatibility).

The platform maximizes its profit with respect to the price given that users form their beliefs according to $\mu(\theta|p, m, r) = \tau^{-1}(p)$. With a slight abuse of notation $n(\theta, \tau^{-1}(p), p)$ denotes the network size based on the true state θ, the belief $\tau^{-1}(p)$ which is a Dirac measure, and the price. Therefore, when the platform increases its price, the effects on profit are two-fold. The first effect is the direct price effect on the mark-up and demand, whereas the second effect is the belief effect, i.e. a higher price potentially signals a higher state. The platform's pricing strategy is determined by

$$\{\tau(\theta)\} \equiv \arg\max_{p \in \tau \in ([\underline{\theta}, \overline{\theta}])} (p - c) n(\theta, \tau^{-1}(p), p).$$

Assumptions 1-3 ensure that the maximization problem is differentiable. In any separating equilibrium, rational users learn about the true state from the separating strategy. By Mailath (1987), given incentive-compatibility the problem is differentiable and the first-order condition can be used. The first-order condition given that in equilibrium beliefs are correct, i.e., $\tau^{-1}(p) = \theta$ yields

$$n(\theta, \theta, p) + (p - c) \left. \frac{\partial n(\theta, \tau^{-1}(p), p)}{\partial p} \right|_{\tau^{-1}(p) = \theta} + (\tau^{-1}(p))'(p - c) \left. \frac{\partial n(\theta, \tau^{-1}(p), p)}{\partial \tau^{-1}(p)} \right|_{\tau^{-1}(p) = \theta} = 0.$$

The separating strategy $\tau(\theta)$ is given by rewriting the differential equation

$$\tau'(\theta) = -\frac{(\tau - c) n_2(\theta, \theta, \tau)}{n(\theta, \theta, \tau) + (\tau - c) n_3(\theta, \theta, \tau)}. \tag{3}$$

Observe that setting $p = p^{FI}(\theta)$ from Equation 2 sets the denominator equal zero. Hence, setting the complete information prices for all types is not a solution. Prices must be distorted. Setting the initial value condition to $\tau(\underline{\theta}) = p^{FI}(\underline{\theta})$ (Riley condition), yields a unique solution to the differential equation that minimizes the level of costly signaling.

Proposition 1. *Suppose $\mathbb{I} = \emptyset$. Then there exists a unique separating equilibrium outcome in which the equilibrium price $p^{S,*}$ is given by Equation 3 with $\tau(\underline{\theta}) = p^{FI}(\underline{\theta})$.*

Proof. See Appendix B. ∎

The equilibrium refinement selects the unique separating equilibrium in which $\underline{\theta}$ sets $\tau(\underline{\theta}) = p^{FI}(\underline{\theta})$, i.e., the Riley condition Mailath (1987). Intuitively, once it is revealed that the fundamental is $\underline{\theta}$, the platform may deviate to set its full information price to maximize profits. Given the D1 criterion, the worst belief would be $\Theta^*(p(\underline{\theta})) = \{\underline{\theta}\}$ under which the platform facing state $\underline{\theta}$ wishes to deviate to obtain its full information profit.

Applying the D1 Criterion shows that the equilibrium outcome in Proposition 1 is unique. As users' strategies are given as a unique cut-off strategy also in the case of pooling equilibria, pooling equilibria are overruled by the desire of the highest type in the pool to separate. The single-crossing property in prices ensures that there exists a small movement in the price upwards for which the highest type wants to deviate. But, lower types would not set this type such that D1 beliefs are in the support of higher types only. The key is that if D1 beliefs put higher probability on high types the user response must increase for the highest type to face a profitable deviation and break the pooling equilibrium.

Proposition 2. *There exists no equilibrium, in which the platform in more than one state θ sets a price $p(\theta) = p'$. The equilibrium in Proposition 1 is the unique equilibrium.*

Proof. See Appendix B. □

In the unique equilibrium, the platform "burns money" to credibly communicate to its users taking the form of distorted prices. The price as signaling device is feasible as the marginal cost of a price increase depends on the demand curvature which in turn is influenced by the platform's true state. As shown in the Appendix, a price increase but response decrease satisfies the single-crossing property such that in higher states the platform is more willing to trade off price increases against response decreases. This link between the true fundamental and price is established by the network effects that arise on the platform. Without network effects, price signaling is never credible within the model framework. The results, therefore, resemble the well-known result in the signaling literature in which quality is unknown but firms have asymmetric marginal production costs. In that literature, firms must distort their prices as well.

The following corollary signs the signaling distortion.

Corollary 1. *The equilibrium price $p^{S,*}$ is increasing in θ and is always greater than the full information price.*

Proof. See Appendix B. □

To sign the pricing distortion, it is useful to recall that the full information price in Lemma 2 might be either increasing or decreasing in the state, but is always greater than c. In contrast, the equilibrium price under price signaling is always increasing in the state. Together with Equation 3 this implies that the price must be larger than the full information price (i.e. at $p^{S,*}$ the denominator of Equation 3 must be negative). Hence, signaling always takes the form of inflated prices.

This result provides a novel rationale for platforms that charge high prices, namely to signal their high quality. Many platforms offering "premium" services charge high prices to demonstrate that they can attract users with higher stand-alone values through their services.[4]

[4]For example in the dating industry, platforms such as eHarmony.com or ElitePartner advertise their

4 Price, Message and Fake Profiles as Signals

Hereafter, users can observe the network size after joining the platform, but are unable distinguish fake profiles from real profiles on the platform. Due to the timing of the game, signals are observed sequentially and beliefs are updated twice. It is crucial to consider the users' understanding of fake profiles. First, I consider sophisticated users who are aware of the possibility of fake profiles, but they are unable to distinguish them ad hoc. In a second step, I will analyze naive users who are unaware of the possibility to create fake profiles.

4.1 Sophisticated Users

The analysis of sophisticated users in which fake profiles are unobservable but feasible extends the analysis in Section 3.1 by adding an additional signaling instrument. Throughout I will focus on separating equilibria, where a separating equilibrium is defined as below

Definition 2. A separating equilibrium is one-to-one strategy in price and fake profiles $\rho : \Theta \to \mathbb{R}_+ \times \mathbb{R}_+,\ \theta \mapsto (p, \xi)$, users' strategy σ_i^1 and σ_i^2, and beliefs μ_1 and μ_2 such that:

1. For any $(p, \xi) \in \rho(\Theta), \mu_1(\cdot) = \rho^{-1}(p) = \mu_2(\cdot) = \rho^{-1}((p, \xi))$. (Belief Consistency)
2. For any $\theta, \theta' \in \Theta, \pi(\theta, \theta, \rho(\theta)) \geq \pi(\theta', \theta, \rho(\theta'))$. (Incentive Compatibility)

By construction, the platform faces a two-dimensional signaling problem as sophisticated users take both the price and the (expected) number of fake profiles as signal. In contrast to the one-dimensional signaling problem described in Section 3.1 it is not possible to utilize the first-order approach by Mailath (1987). Instead, one needs to deal with the incentive constraints explicitly. To do so, I focus on differentiable separating strategies.

The optimization problem can be formulated as the platform maximizing its profit given that users are able to infer the true state in equilibrium

$$\arg\max_{p,\xi} (p - c)n(\theta, \theta, p) - \gamma(\xi)$$

subject to incentive compatibility

$$\pi(\theta, \theta, \rho(\theta)) \geq \pi(\theta, \theta', \rho(\theta')), \forall \theta, \theta' \in \Theta,$$

given by

$$\begin{aligned} &(p(\theta) - c)n(\theta, \theta, p(\theta)) - \gamma(\xi(\theta)) \\ &\geq (p(\theta') - c)n(\theta, \theta', p(\theta')) - \gamma\left[\xi(\theta') + (n(\theta', \theta', p(\theta')) - n(\theta, \theta', p(\theta')))\right]. \end{aligned} \tag{IC}$$

high quality services in comparison to other dating sites such as Match.com. ElitePartner, a dating site for academics, offers to create an account for free, but to take any action on the platform, users need to sign up for their membership which ranges between 70 Euro/month (6 months contract) to 35 Euro/month (24 month contract).

A deviation in the equilibrium strategy $\rho(\theta)$ consists of a deviation in price $p(\theta)$ alongside a change in fake profiles $\xi(\theta)$. Although fake profiles bear similarities to the concept of advertising, note that the incentive constraints are different. As users only observe $n+\xi$, a deviation to price-fake profile pair (p', ξ') perfectly reveals the state. Instead, the platform needs to adjust its fake profiles by the difference in demand when deviating to induce belief θ'. Deviations to a price-fake profile pair (p', ξ') without adjusting ξ' is trivially unprofitable as the platform does not gain users by a more favorable belief.

Turning to the analysis of the incentive constraint, the IC must bind. Setting the IC slack would imply that the platform could decrease the difference between $\xi(\theta)$ and $\xi(\theta')$ and save costs. Rearranging yields

$$\gamma[\xi(\theta') - \xi(\theta)] =$$
$$(p(\theta') - c)n(\theta, \theta', p(\theta')) - (p(\theta) - c)n(\theta, \theta, p(\theta)) - \gamma\left[n(\theta', \theta', p(\theta')) - n(\theta, \theta', p(\theta'))\right]. \tag{IC*}$$

which pins down the fake profile strategy of type θ as a function of the price $p(\theta)$. Note that I restrict fake profiles to be non-negative throughout the analysis. Another possibility is to pin down the pricing strategy as a function of fake profiles. Hence, it is possible to construct a continuum of separating equilibria as separation can be achieved either via the price or fake profiles. The platform's preferred separating equilibrium is stated in the following theorem.

Theorem 1. *The unique separating equilibrium is characterized as follows:*

1) If costs γ are sufficiently small

$$\underbrace{(p-c)n_2(\theta, \theta, p)}_{\textit{Signaling benefit of extra fake profile}} > \underbrace{\gamma n_1(\theta, \theta, p)}_{\textit{Cost of extra fake profile}}, \forall \theta \in \Theta. \tag{I}$$

signalling takes the form of either

a) Price signaling: zero fake profiles are used and prices are set to

$$p'(\theta) = -\frac{(p(\theta) - c)n_2(\theta, \theta, p(\theta))}{(p(\theta) - c)n_3(\theta, \theta, p(\theta)) + n(\theta, \theta, p(\theta))}., \tag{4}$$

with the inital value condition $p(\underline{\theta}) = p^{FI}(\underline{\theta})$, or

b) Price and fake profile signaling: fake profiles are given by

$$\gamma\xi = (p(\theta) - c)n(\theta, \theta, p(\theta)) - (p(\underline{\theta}) - c)n(\underline{\theta}, \underline{\theta}, p(\underline{\theta})) - \int_{\underline{\theta}}^{\theta} (p(t) - c - \gamma)n_1(t, t, p(t))dt, \tag{5}$$

and prices maximize equilibrium profits

$$p(\theta) - c - \gamma = -\frac{n_1(\theta, \theta, p)}{n_{13}(\theta, \theta, p)}, \textit{ for } \theta > \underline{\theta}. \tag{6}$$

2) Suppose cost γ are sufficiently large, zero fake profiles are used and full information prices are set.

The unique D1 equilibrium is the least-cost separating equilibrium, so either a) or b). It maximizes the platform's payoffs over the set of separating equilibria.

Proof. See Appendix. □

There are two main differences compared to the analysis of advertisement. First, in Equation IC*, fake profiles require that the additional term $\gamma\,[n(\theta,\theta',p(\theta')) - n(\theta',\theta',p(\theta'))]$ is present. This reduces, ceteris paribus, the slope of the fake profile function in equilibrium which must be created to ensure incentive-compatibility. Intuitively, when the platform with a low fundamental wants to mimic a platform with a high fundamental, the latter has an advantage of an already larger network size. Second, the price in a model with advertisement would be independent of the costs γ which appear as a mark-up on the right-hand side of Equation 4. Therefore, the platform can shift a part of the marginal costs of creating fake profiles to its users.

4.2 Naive Users

This section turns to the analysis of naive users. Users are assumed to be misspecified about fake profiles as they do not take the possibility into account that fake profiles can be created. Therefore, naive users take the network size on a platform at face value, i.e. real profiles are equal to the profiles they see. As users cannot see the network size upfront, they observe the platform's message $m \in \Theta$ and take this as a literal statement about its network size. Given a price p, the platform's message $m \in [\underline{\theta}, \overline{\theta}]$ can be interpreted as sending a message about the feasible network size $m \in [n(\underline{\theta}, \underline{\theta}, p), n(\overline{\theta}, \overline{\theta}, p)]$.

The notion of naive users is motivated by wrong legal beliefs, as users may believe that fake profiles are simply illegal or impossible to create.[5] A majority of users that is new to those platforms is surprised afterwards about the use of fake profiles. In other cases, users form beliefs about these practices in traditional markets, where the use of fake profiles or similar practices is forbidden, and take over their beliefs to online markets or digital platforms. Fake profiles are often legal or the creation of fake profiles is legal as long as firms disclose their use in the terms and conditions. In the terms and conditions, however, the phrases are well-hidden (see Section x). Hence, if users are naive with respect to the possibility of fake profiles, they take any demand at face value that they deem feasible. This approach is combined with the notion of credulity of users. The platform's message about the network size is technically cheap-talk, but users blindly believe in the message as they take the potential network size at face value. In online markets, this message could be the announcement of membership statistics or advertisement about the network size. Ottaviani and Squintani (2006) and Kartik et al. (2007) define the notion of credulous users for cheap-talk games. In the model at hand, the credulity stems from the naivete about the network size.

The equilibrium analysis is greatly simplified due to the fact that users hold point-belief as users put probability one onto the state that the platform announces (which corresponds to a mass of users on the platform). Hence, all subsequent beliefs in this section are point-beliefs, i.e. a Dirac measure on θ', where I denote $\delta_{\theta'} = \theta'$. To analyze

[5] See Armstrong and Vickers (2012) who make a similar argument towards naivete with respect to hidden prices.

the equilibrium of the game with naive users, I need to define the platform's strategy and equilibrium concept. [6]

Definition 3. A sender's strategy ν is a LSHP (low types separate and high types pool) strategy if, for any price p, there exists a $\tilde{\theta} \in [\underline{\theta}, \overline{\theta}]$ such that:

1. For all $\theta < \tilde{\theta}, \nu(\theta) \in \{m(\theta) | m \in \Theta \setminus \{\overline{\theta}\}\}$, with $\nu(\theta) \neq \nu(\theta') \forall \theta \neq \theta'$.
2. For all $\theta \geq \tilde{\theta}, \nu(\theta) = \overline{\theta}$.

Given a price p, a perceived separating equilibrium consists of

1. A LSHP strategy on messages $\nu(\theta)$.
2. User beliefs $\mu(m) = m$.
3. A fake profile strategy $\xi(m) = n(m, m, p) - n(\theta, m, p)$.

Note that the fake profile strategy takes the shape of matching the difference in real demand and is part of the equilibrium. This definition, however, is not restrictive as the platform does so optimally.[7]

Lemma 3. *The platform optimally sets fake profiles equal to* $\xi(\phi^*(m)) = n(\phi^*(m), \phi^*(m), p) - n(\theta^*, \phi(m), p)$ *such that* m *and* ξ *induce the same belief* $\phi^*(m)$.

Proof. See Appendix B. □

Users hold two relevant beliefs for the platform, once during their entry decision $\mu(\theta|p, m, r) = m$ and once during their exit decision $\mu(\theta|p, m, r, n)$. If the announced message (or announced demand) differs from the actual observed demand on the platform, these two beliefs are not the same.

The platform's maximization problem is

$$\arg\max_{\{m \in \Theta, p \in \mathbb{R}\}} (p - c)n(\theta, m, p) - \gamma \left(n(m, m, p) - n(\theta, m, p)\right)$$

The first-order conditions result in

$$p - c + \gamma = -\frac{n(\theta, m, p)}{n_3(\theta, m, p)} + \gamma \frac{n_3(m, m, p)}{n_3(\theta, m, p)} \tag{7}$$

$$(p - c + \gamma) n_2(\theta, m, p) = \gamma \left(n_1(m, m, p) + n_2(m, m, p)\right). \tag{8}$$

The first equation determines the optimal price given a chosen message m. If the message is equal to the true state, the optimal price is the full information price. The second equation determines the choice of the optimal message. For a given price, the left-hand side is the marginal benefit of fake profiles, i.e. the increase in users' beliefs, and the right-hand side are the marginal costs.

[6] This definition is based on Kartik (2009) who defines a strategy about a message as a LSHP strategy in the context of a cheap-talk game.

[7] In this context, the use of fake profiles can be seen as the cost of lying when announcing a higher state than the true state θ. This connects to the literature of cheap-talk with lying costs (Kartik, 2009).

The first-order conditions are only applicable for $m \geq \theta$. If $m < \theta$, the platform does not set any fake profiles as the number of fake profiles is bounded from below by zero.

If Condition I holds, setting $m^* = \theta$, i.e. every type reveals its true type to the naive users, does not satisfy the first-order condition as the marginal benefit from lying upwards is greater than its cost. Hence, if costs are sufficiently small, every type uses an inflated message $m > \theta$ such that no complete separation (from the platform's perspective) is possible. Due to the bounded state space, the highest type runs out of claims to make, such that some high types will pool on the highest possible message.

The platform does not fully separate on the observable instruments, but separates on the (unobservable) fake profile strategy. Users, however, hold separating beliefs, hence, a point belief after seeing a price and message which is why the equilibrium is termed *perceived* separating equilibrium. As *full* separation is not feasible under low costs, there exists a cutoff-state for which the platform pools on the highest message if the fundamental is larger than the cutoff and separates if the fundamental is smaller.

The cutoff is determined by the state for which the platform chooses the highest message first. To solve for the cutoff type $\tilde{\theta} < \overline{\theta}$, the indifference condition is examined. Let the profit of the platform in the *indifferent* state $\tilde{\theta}$ be

$$\pi(\tilde{\theta}, m, \xi(\tilde{\theta}, m), p) \equiv \pi(\tilde{\theta}, m),$$

which solves $\overline{\theta} = \arg\max_{m \in \Theta} \pi(\tilde{\theta}, m)$:[8]

$$\left(-\frac{n(\tilde{\theta}, \overline{\theta}, p)}{n_3(\tilde{\theta}, \overline{\theta}, p)} + \gamma \frac{n_3(\overline{\theta}, \overline{\theta}, p)}{n_3(\tilde{\theta}, \overline{\theta}, p)} \right) n_2(\tilde{\theta}, \overline{\theta}, p) = \gamma \left(n_1(\overline{\theta}, \overline{\theta}, p) + n_2(\overline{\theta}, \overline{\theta}, p) \right). \tag{9}$$

Theorem 2. *The unique equilibrium with naive users is characterized as follows:*

1) *If costs γ are sufficiently small $p^{FI}(\underline{\theta}) n_2(\underline{\theta}, \underline{\theta}, p) - c > \gamma n_1(\underline{\theta}, \underline{\theta}, p)$.*

   a) *Low types separate with $m > \theta$ and $p^{N,*}(m)$ given by Equations 7&8 and high types pool on $m = \overline{\theta}$ if $\overline{\theta} > \tilde{\theta} > \underline{\theta}$, or*

   b) *All types pool on $m = \overline{\theta}$ if $\tilde{\theta} = \underline{\theta}$.*

2) *Suppose cost γ are sufficiently large, zero fake profiles are used and full information prices are set.*

Proof. See Appendix B. □

Each type sets a higher price and even the lowest type creates fake profiles. Fake profiles increase up to the indifferent type and decrease after, because high types "run out of claims to make" as the type space is bounded. Depending on the interval Θ, this type could be already the lowest type, which results in a complete "pooling" equilibrium. Note that if the marginal costs from creating the first fake profiles are zero, marginal benefit always exceeds marginal costs such that all types pool on the highest message and there is never an equilibrium without fake profiles. Hence, there is never an equilibrium without

[8]In contrast to Kartik (2009), this condition is easier to solve as the "pooling" in this equilibrium does not feature a pooled belief among those types that are pooled.

fake profiles as $MB(\theta, \theta) > MC(\theta, \theta) = 0$. With constant costs, $p^{FI}(\underline{\theta})n_2(\underline{\theta}, \underline{\theta}, p) - c > \gamma n_1(\underline{\theta}, \underline{\theta}, p)$ is needed to achieve this result.

To exploit the consumer's naivete, a bound is imposed on the feasible strategies. In contrast to the fake profile strategy in Section 4.1 which was not bounded, the consumer naivete makes the restriction of the state space binding which in turn influences the strategy space.

From a welfare perspective, participation on the platform is distorted. The new indifferent user does not benefit from the use of fake profiles as he pays an inflated price for a non-existing network size. But, users who had entered in the original game might benefit indirectly from fake profiles if they value network effects via β enough. Due to excessive entry by users, the real network size increases on the platform which might offset the higher prices for some users.

In equilibrium, consumers surplus and welfare might increase compared to the full information benchmark. Naive users benefit from an increase in the real network size compared to the full information benchmark. A monopolist platform typically under-provides a network good from a welfare perspective, i.e. chooses a too small network size. The use of fake profiles with naive users, however, increases not only the total but also the real network size. In contrast, prices increase compared to the full information benchmark. If network effects are relatively strong consumers surplus increases.

4.3 Regulation

This section analyzes possible remedies and regulation to deal with the use of fake profiles. More specifically, consider a ban of fake profiles, labelling fake profiles on the platform, mandatory disclosure of fake profiles upfront. It is assumed that the regulation is publicly known and users are educated and informed about the policy. The analysis considers the first three policies first and will show that all three will lead to the same unique market outcome. Lastly, it will be shown that educating users about the use of fake profiles is not sufficient to prevent the use of fake profiles.

Banning Fake Profiles How does a ban of fake profiles impact the market outcome? Suppose the ban of fake profiles is public and users are informed about the policy. Sophisticated users will deduce that whenever they join a platform, they will observe the real network size. Hence, after joining sophisticated users are in a subgame of complete information in which the state is known. The unique equilibrium is summarized in Proposition x.

Labelling Fake Profiles Suppose through labeling fake profiles, users can perfectly identify fake profiles and determine the real network size. Again, in the last period sophisticated users face a subgame of complete information (see Proposition x). If fake profiles cannot serve as a signal due to perfect identification, no fake profiles are used by the platform. This follows directly from the fact that fake profiles are costly, but do not yield a positive benefit to the platform at this stage. Naive users behave as above.

Mandatory Disclosure Lastly, consider that platform must mandatory disclose their use of fake profiles upfront (either stating their use or abstention). Under signaling with price and fake profiles, or price only, the profit of a platform is always lower than the full information profit which is first-best as signaling is costly. Through mandatory disclosure the platform can choose to refrain from fake profiles credibly, which induces sophisticated users to deduce, again, that they will observe the real network size on the platform. As will be shown in the following proposition, this will enable the platform to achieve its full information profit

Analysis All policies result in perfect knowledge of users about the real network size on the platform after joining, such that fake profiles cannot influence their perception. Users are sophisticated and take the price as the only costly signal. Hence, the platform maximizes its full information profit

$$\max_p (p-c)n(\theta,\theta,p), \text{ subject to } n(\theta,\mu_2(\cdot)=\theta,p) \leq n(\theta,\mu_1(\cdot)=\hat{\theta},p).$$

As users' participation decisions are made after observing the price after the first period, the constraint imposes an upper bound on demand in the last period. The optimal prices are given by the first-order condition

$$n(\theta,\theta,p)+(p-c)\frac{\partial n(\theta,\theta,p)}{\partial p}=0$$

resulting in $p=p^{FI}(\theta)$, the full information benchmark price given that $\theta \leq \hat{\theta}$.

To see that $p^{FI}(\theta)$ is an incentive-compatible separating strategy, suppose that the platform in state θ sets a price $p^{FI}(\theta') < p^{FI}(\theta)$ for $\theta' < \theta$. This influences demand in two ways: first, a price decrease leads to more demand holding all else constant and second, a price decrease influences the believed state $\hat{\theta}$ and leads to a lower expected state. This in turn, decreases demand all else constant. Suppose first that demand overall increases and more user enter than in equilibrium. Then, in the last period users observe the realized demand given price and belief $(p^{FI}(\theta'),\theta')$ and the true state θ. As too many users entered given belief θ', users exit again such that

$$n(\theta,\theta,p^{FI}(\theta')=1-F_\theta(p^{FI}(\theta')-\beta n(\theta,\theta,p^{FI}(\theta'))).$$

As the price and realized demand $n(\theta,\theta,p^{FI}(\theta'))$ are not profit maximizing in state θ, the platform does not face a profitable deviation. A similar argument can be constructed if demand overall decreases. Then, following the deviating price $p^{FI}(\theta')$ too few users join the platform than optimal.

Proposition 3. *Suppose the government regulates platforms by either banning fake profiles, or inducing them to label fake profiles, or mandatory disclosing fake profiles. If users are aware of these policies, there exists a unique equilibrium that is separating and first-best. In equilibrium, the platform sets the full information price and zero fake profiles are used.*

Proof. See Appendix A. □

This result stresses the importance of observing the network size before paying the membership fee. Compared to Section 3.1 in which users paid the membership fee upfront, the platform can increase its profit by offering a free-trial period before collecting the (one-time) membership fee.[9] This "free-trial" period is desirable from a platform's perspective as full information prices are incentive compatible. The platform does not incur a loss in profit due to wasteful signaling. In equilibrium, both users and the platform are better off compared to the Section 3.1. In comparison to the signaling literature, the learning-from-demand-stage resembles repeat purchases for regular products or a full warranty/money-back guarantee.

The effect of educating naive users about the possibility of fake profiles are ambiguous. Users and the platform might be both worse off. If naive users are educated and no other action on fake profiles are taken, the platform might still use fake profiles for signaling as in Theorem 1. Due to the consumer sophistication, the platform still needs to engage in costly signaling. Comparing the equilibrium outcome in Theorem 1 and 2, the platform makes losses when moving from the latter to the first. With naive users, the platform makes higher profits than under full information, whereas with sophisticated users the platform makes lower profit than under full information. The effect on users depends on the prices and network effects. Naive users benefit from an increase in the real network size compared to the equilibrium in Theorem 1. Additionally, prices might also increase compared to the latter equilibrium.

5 Discussion

Convincing Users to Upgrade into a Premium Subscription The Dating platform "Match.com" presumably utilized third-party fake profiles to persuade (male) users into a paid subscription.[10] Following the model, users are able to sign up to the platform for free. Initially, undecided users, who did not pay for the membership, received emails from potentially interested users. In the stylized version, users are assumed to view the total perceived network size $\tilde{n}$. To interact with the other users, they needed to upgrade their free trial. The platform's network size included fake profiles. The platform allegedly used the third-party fake profiles ξ to direct messages towards non-paying users which lead them to upgrade to a premium membership and pay p. Due to the platform's intentional use of fake profiles, i.e. identifying the third-party fake profiles, directing those to non-paying users but keeping those away from paying members, it is plausible to assume that the platform incurred small (effort) costs γ.

Manipulating the Network Size: Wash Trades One of the largest cryptocurrency exchange platforms ("Binance") is under investigation by the SEC for "manipulative trading that artificially inflated the platform's trading volume". They engaged in so-called Wash trading. More precisely, another associated company ("Sigma Chain") owned by the same entity ("Zhao") as the crypto exchange platform manipulated the platform's trading volume by selling and buying the same financial assets, therefore artificially inflating the

[9] The model abstracts from discounting between periods. Otherwise, the firm must be sufficiently patient or the free-trial period must be sufficiently short.

[10] https://www.ftc.gov/news-events/press-releases/2019/09/ftc-sues-owner-online-dating-service-matchcom-using-fake-love

platform's volume.[11] Furthermore, the U.S. based affiliate of "Binance" called "BAM Trading Services" is accused of misleading investors about non-existent trading controls on Binance.US. Wash trading is prohibited in offline (financial) markets, e.g. in the US by the Commodity Exchange Act.[12] For example, the Intercontinental Exchange (ICE) takes measures to prevent self-trade to comply with regulations.[13]

In this application, the platform is a cryptocurrency trading platform and its users are potential investors both buying and selling assets on the platform. Network effects take the form of caring for liquidity. A platform with a large network, i.e. a high trading volume, has more liquid assets and is more credible. Fake profiles are financial assets that are self-traded by the platform and hence inflate the network size.

Manipulating the Network Size: Dating Platforms Other dating platforms use company-created fake profiles; a list of several dating sites using this practice has been published by the Verbraucherzentrale Bayern (Center for Consumer Advise Bavaria) in Germany. These platforms employ paid workers to create profiles, and interact with users on the platform, giving them the impression of a real contact.[14] It is not commonly known that platforms themselves create fake users to possibly stimulate demand, although it is legal to do so as long as it is mentioned in the terms and conditions. There are companies that specialize in providing employees as chat moderators to these platforms.[15] These chat moderators set up fake profiles and engage in conversations with the users of the platform pretending to be a real profile.

Furthermore, the UK Consumer and Markets Authority (CMA) confirms in its report about the online dating industry that dating platforms may use "pseudo profiles" or provider-generated profiles that could possibly mislead consumers. The CMA states that if these fake profiles are not disclosed as such, it may be in breach with the "Consumer Protection from Unfair Trading Regulations".[16] In another industry report issued by the Australian Competition and Consumer Commission (ACCC), the ACCC acknowledges that fake profiles generated by providers exist, but stress that this issue lies beyond the scope of their investigation mandate.[17] This shows that the use of fake profiles might be more common than initially expected and might not be restricted to the examples given above.

Evidence that chat bots might have been used by dating platforms exists for the dating

[11] https://www.sec.gov/news/press-release/2023-101

[12] https://www.law.cornell.edu/uscode/text/7/chapter-1

[13] https://www.theice.com/publicdocs/futures/IFEU_Self_Trade_Prevention_FAQ.pdf

[14] See https://www.verbraucherzentrale.de/wissen/digitale-welt/onlinedienste/onlinedating-auf-diesen-portalen-flirten-fakeprofile-21848, last visited 01.09.2020.

[15] For example, Cloudworkers or Agentur da Chatdeife are companies that employ freelancers to work for and on one or more social-community platforms. See also https://www.spiegel.de/wirtschaft/service/singleboersen-ein-moderator-von-fake-profilen-spricht-ueber-seinen-job-a-1113937.html. and https://www.ndr.de/fernsehen/sendungen/panorama_die_reporter/Undercover-als-Chatschreiberin-Abzocke-Flirtportal,sendung1098906.html for an interview (in German) and https://www.marieclaire.fr/,dating-assistant,750821.asp for an article (in French).

[16] See https://assets.publishing.service.gov.uk/media/5b114a8040f0b634abe911e7/compliance_statement.pdf.

[17] See https://www.accc.gov.au/system/files/927_ICPEN%20Dating%20Industry%20Report_D09.pdf.

site "Ashley Madison". Ashley Madison was subject to a large data leak by hackers.[18] The dating site used "chat hostesses" before 2011 to engage men, which coincides with the notion of fake profiles in this context. After 2011, however, it is reported that they stopped employing "chat hostesses". Instead, the dating platform allegedly used chat bots to deceive users to spend money on the platform. Although one might think that chats bots are easier to be identified, this might not have been the case. Users seem to have spent a reasonable amount of money on communicating with chat bots.

Lastly, there is evidence on dating platforms that use methods to create a similar effect as with platform-generated fake profiles. The CMA investigated the case of Venntro Media Group Ltd, a company that operates several dating sites. To inflate the network size on their dating sites, Venntro cross-registered their members on various sites and not only the site they originally signed up for.[19]

Launching Strategy for Start-Ups Upon launching a new platform, founders often generate artificial demand (or supply) to onboard producers (or consumers). This practice is documented in the business and management literature, e.g. (Schirrmacher et al., 2017) or Reillier and Reillier (2017). Evans and Schmalensee (2016) describe the practice as 'self-supply". In case studies by Schirrmacher et al. (2017) some platforms self-supplied in the beginning at launch to influence participants' beliefs, whereas one platform simulated fake demand.

6 Conclusion

Especially on Dating platforms the use of fake profiles is heavily relied upon. Suggestive evidence from a data leak of the platform "Ashley Madison" shows that fake profiles were used excessively. Most of the female users were in fact fake profiles. The data, however, included credit card transactions (mostly from men) indicating that many users spend a lot of money on the platform even though the chance of encountering a real women was surprisingly low.

Economic papers exploring the regulation of platform markets are scarce, although policy papers such as Fletcher et al. (2021) investigate common issues on platform that may need to be regulated. For fake profiles, one suggested policy is banning fake profiles. Similarly, one could consider mandatory disclosure policies. Voluntary and mandatory disclosure has been discussed by scholars such as Grossman (1981), Mathios (2000), or Fishman and Hagerty (2003).

In a classical model with rational users and voluntary disclosure all but the lowest type should disclose their type and state that they would not use fake profiles. In my model a platform would like to commit to refrain using fake profiles with sophisticated users as they are costly which would indicate that if users are sophisticated voluntary disclosure on fake profiles should be observed in online markets. Instead their actual use is stated in the terms in conditions. There is often no evidence that platforms commit on not using fake profiles. Voluntary disclosure might fail for two reasons in platform markets. First,

[18]See https://financialpost.com/fp-tech-desk/inside-ashley-madison-calls-from-crying-spouses-fake-profiles-and-the-hack-that-changed-everything?__lsa=b245-a155.

[19]See https://www.gov.uk/government/news/online-dating-giant-vows-clearer-path-to-love.

disclosure may fail due to credibility issues as users might also create fake profiles and it the source of creation might be hard to distinguish. Second, the presence of naive users eliminated the incentives to voluntary disclose the own type.

Combining suggestive evidence and the failure to observe voluntary disclosure in these markets may indicate that users are mainly naive. This speaks in favor of consumer protection policies against practices that influence network effects such as a ban of fake profiles or mandatory disclosure.

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Appendix A

Definition of Equilibrium Refinement for Sophisticated Users

Definition 4. D1 Criterion & Rationalizability
Denote the equilibrium strategy profile by $\Sigma = ((p^*, m^*), r^*(p^*, m^*))$, where $r(\cdot)$ denotes the equilibrium cut-off mapping. The equilibrium profit of a platform of type θ is $\pi^*(\theta, \Sigma)$.

For a given price p, an arbitrary non-empty subset of sender type space $\tilde{\Theta} \subseteq \Theta$, and a non-empty subset of the other receivers action spaces $\tilde{\mathbf{Y}}_{-i}$ let

$$\mathrm{BR}_i(\tilde{\Theta}, \tilde{\mathbf{Y}}_{-i}) = \cup_{\rho_i \sim \Delta(\tilde{\Theta} \times \tilde{\mathbf{Y}}_{-i})} \underset{y_i \in [0,1]}{\arg\max}\, \mathbb{E}_{(\theta, \mathbf{y}_{-i}) \sim \rho_i}[u_i(\theta, y_i, \mathbf{y}_{-i}, p)] \forall i$$

be the set of user i's best responses to p for some belief ρ_i over sender type and the other receivers action pairs with support contained in $\tilde{\Theta} \times \tilde{\mathbf{Y}}_{-i}$. For an arbitrary non-empty subset of sender type space $\tilde{\Theta} \subseteq \Theta$ and $k \in \{0, 1, 2, ...\}$ let

$$Y_i^k(\tilde{\Theta}) = \mathrm{BR}_i(\tilde{\Theta}, \tilde{\mathbf{Y}}_{-i}^{k-1}(\tilde{\Theta})), \text{ and } Y_i^\infty(\tilde{\Theta}) = \cap_{k \in \{0,1,2,...\}} Y_i^k(\tilde{\Theta}) \forall i$$

be the set of rationalizable actions given $\tilde{\Theta}$ for receiver i. Denote by $\mathcal{R}^\infty(\tilde{\Theta}, p)$ the set of rationalizable receiver action profiles for given p and $\tilde{\Theta}$.

For a given out-of equilibrium price p and for each type θ, find all rationalizable action profiles $\alpha \in \mathcal{R}^\infty(\Theta, p)$ by users that would cause θ to deviate from equilibrium. For $\theta \in \Theta$, p, and equilibrium profile Σ,

$$D_\theta = \{\alpha \in \mathcal{R}^\infty(\Theta, p) : \pi^*(\theta, \Sigma) < \mathbb{E}_{r \sim \alpha} \pi(\theta, p, r)\},$$

is the set of receiver rationalizable actions for which type θ is strictly better-off deviating towards p, and

$$D_\theta^0 = \{\alpha \in \mathcal{R}^\infty(\Theta, p) : \pi^*(\theta, \Sigma) = \mathbb{E}_{r \sim \alpha} \pi(\theta, p, r)\},$$

is the set of receiver rationalizable actions for which type θ is indifferent between deviating towards p and setting equilibrium price p^*. If for some type θ there exists another type θ' such that

$$D_\theta \cup D_\theta^0 \subset D_{\theta'},$$

then (θ, p) may be pruned from the game. The set of types that cannot be deleted is denoted by $\Theta^*(p)$. A PBE violates D1 if there exists a type and action (θ, p) such that

$$\min_{\alpha \in \mathcal{R}^\infty(\Theta^*(p), p)} \pi(\theta, p, r) > \pi^*(\theta, \Sigma) \text{ for some } \theta \in \Theta^*(p). \tag{D1}$$

Discussion Ramey (1996) shows that under the following assumptions the unique D1 equilibrium is separating. The set of types is given by non-degenerate interval $[\underline{\theta}, \overline{\theta}]$, where prior beliefs are given by a continuous probability distribution $\mu(\theta)$ with full support. Signals are $p \in \mathbb{R}^k$ and the (single) receiver's response r is chosen from the real line. Payoff functions are continuously differentiable; the sender's payoff π increases in the receiver's response. The receiver's utility function u is strictly quasi-concave in its action

r for each signal and type. The receiver's payoff is maximized in r by $r^*(\theta, p)$, which is strictly increasing in θ. Furthermore, $r^*(\theta, p)$ is uniformly bounded above and for $k = 1, .., n : \lim_{p_k \to +/-\infty} \pi(\theta, r, p) = -\infty$. Enger's Incentive Montonicity Condition holds for the k signals (weaker condition than the multi-dimensional single-crossing property).

In the model at hand the assumptions on the receiver's payoff functions and actions are not fulfilled. A receiver's response given price p is binary and depends on the belief over the other receivers' actions. The sender, however, does not care about the action of a single receiver, but cares about the aggregate action taken by the receivers. The receivers' payoffs are instead quasi-concave (linear) in the aggregate response. Additionally, the aggregate response $n^*(\theta, p)$ is strictly increasing in θ due to the assumptions on F_θ.

Appendix B

Proofs

Proof of Lemma 1 First, consider the case of complete information, in which the assumption that users hold a common belief upon observing (p, m) is trivially fulfilled. Users play sequentially rationalizable strategies, i.e. they play a best-response to a symmetric cutoff of other. To establish the unique cutoff $r_c \in [\underline{r}, \overline{r}] \subseteq [-\infty, \infty]$, I will iterate on the best-responses of users once from above starting at $r_0 = \overline{r}$ and once from below starting at $r_0 = \underline{r}$.

Step (i) Iteration starting from $r_0 = \overline{r}$.

Consider the best response of an agent i given an arbitrary state θ, price and message pair (p, m) and the action profile of the other agents. The first iteration given the symmetric cutoff $r_0 = \overline{r}$ yields

$$BR_i^1(\{\theta\}, r_0 = \overline{r}) = \begin{cases} 1 & \text{if } r_i \geq p \\ 0 & \text{if } r_i < p. \end{cases}$$

In the first iteration, agents with a reservation value of $r_1 \equiv p$ or higher will always enter even if no one enters the platform (independent of their beliefs). Iterated elimination of not best responses yields a cutoff value of r_i in the $i+1$'th iteration given by

$$r_{i+1} = p - \beta(1 - F_\theta(r_i))$$

This sequence is bounded by $r \in [\beta - p, p]$ and strictly decreasing by the assumptions made in Section 2. Hence, the sequence converges to its limit

$$r_c = \lim_{i\to\infty} r_{i+1} = \lim_{i\to\infty} p - \beta(1 - F_\theta(r_i)) = p - \beta(1 - \lim_{i\to\infty} F_\theta(r_i)) = p - \beta(1 - F_\theta(r_c)).$$

The last inequality follows from the fact that the probability function is assumed to be continuous. Then, the condition

$$\overline{r}_c = p - \beta(1 - F_\theta(\overline{r}_c))$$

Step (ii) Iteration starting from $r_0 = \underline{r}$.

The first iteration given the symmetric cutoff $r_0 = \underline{r}$ yields

$$BR_i^1(\{\theta\}, r_0 = \underline{r}) = \begin{cases} 1 & \text{if } r_i \geq p - \beta \\ 0 & \text{if } r_i < p - \beta. \end{cases}$$

In the first iteration, agents with a stand-alone value below $r_1 \equiv p - \beta$ will never enter even if all others join the platform (independent of their beliefs). The cutoff value of r_i in the $i+1$'th iteration is given by

$$r_{i+1} = p - \beta(1 - F_\theta(r_i)),$$

as the sequence is bounded by $r \in [\beta - p, p]$ and strictly increasing,it converges to its limit

$$\underline{r}_c = p - \beta(1 - F_\theta(\overline{r}_c)).$$

Step (iii) Show that $\overline{r}_c$ and $\underline{r}_c$ coincide.

Given Assumption 3, for any $p \in \mathbb{R}_+$ there exists one and only one solution to the equation

$$r + \beta(1 - F_\theta(r)) = p, \tag{10}$$

as r is increasing in r with slope one, whereas $\beta(1 - F_\theta(r))$ is decreasing in r with slope smaller than one. Hence, the left-hand side is strictly increasing in r. Then, $r_c \equiv \{r \in [\underline{r}, \overline{r}] : r + \beta(1 - F_\theta(r)) = p\}$ characterizes the unique cutoff which is the unique sequentially rationalize user profile given the action-pair (p, m) by the platform.

Second, consider the case of incomplete information under the assumption that users hold a common belief upon observing (p, m). Given the equilibrium definition of PBE, there are two relevant types of equilibria — separating and pooling equilibria. In both equilibria, the common belief assumption is fulfilled again. In a separating equilibrium, users hold point-beliefs after observing (p, m), whereas users' Bayesian update in a pooling equilibrium after observing (p, m) is equal to their common (full support) prior. The proof proceeds as follows. First, I will show that if users play cutoff strategies, there exists a unique cutoff. Second, I will show that users will play a cutoff strategy in any equilibrium.

Step (i) Unique cutoff.

Suppose there exist two cutoffs defined by

$$\underline{r}_c + \beta \int_\Theta (1 - F_\theta(\underline{r}_c)) d\mu(\theta | p, m, \underline{r}_c) = p, \text{where} \tag{11}$$

$$\mu(\theta | p, m, \underline{r}_c) = \frac{\mu(\theta | p.m) f_\theta(\underline{r}_c | \theta)}{\int_{\tilde{\theta} \in \Theta} \mu(\tilde{\theta} | p, m) f_\theta(\underline{r}_c | \tilde{\theta}) d\tilde{\theta}}, \tag{12}$$

and

$$\overline{r}_c + \beta \int_\Theta (1 - F_\theta(\overline{r}_c)) d\mu(\theta | p, m, \overline{r}_c) = p, \text{where} \tag{13}$$

$$\mu(\theta | p, m, \overline{r}_c) = \frac{\mu(\theta | p.m) f_\theta(\overline{r}_c | \theta)}{\int_{\tilde{\theta} \in \Theta} \mu(\tilde{\theta} | p, m) f_\theta(\overline{r}_c | \tilde{\theta}) d\tilde{\theta}}. \tag{14}$$

For the sake of contradiction, suppose that the cutoff differ, e.g. $\underline{r}_c < \overline{r}_c$. Denote $X(r_1, r_2) = \int_\Theta (1 - F_\theta(r_1)) d\mu(\theta | r_2)$.

Lemma 4. *$X(r_1, r_2)$ is strictly decreasing in r_1 and (weakly) increasing in r_2.*

The first part follows directly from the fact that $1 - F_\theta(r_1)$ is decreasing in r_1 for all θ and hence, $\int_\Theta (1 - F_\theta(r_1)) d\mu(\theta|r_2)$ is decreasing in r_1 as well holding $\mu(\theta|r_2)$ fixed.

For the second part, note that for $r' > r''$, $\mu(\theta|p, m, r')$ has first-order stochastic dominance over $\mu(\theta|p, m, r'')$ due Assumption 2. Assumption 2 states that the family of densities $\{f_\theta(r) \equiv f(r|\theta)\}$ is assumed to have the monotone likelihood ratio property. By Milgrom (1981) (Proposition 2) a family of densities has the MLRP iff $r' > r''$ implies that r' is more favorable than r'' meaning that $\mu(\cdot|r')$ dominates $\mu(\cdot|r'')$.

Recall that $1 - F_\theta(r_1)$ is bounded by $[0, 1]$ and is strictly monotone increasing in θ by Assumption 1 and 2. Then, $\int_\Theta (1 - F_\theta(r_1)) d\mu(\theta|r') \geq \int_\Theta (1 - F_\theta(r_1)) d\mu(\theta|r'')$ holds as r' is more favorable than r'', where it holds with equality whenever $1 - F_\theta(r_1) \in \{0, 1\}$.

In equilibrium, both the left-hand side of Equation 11 and 13 must be equal to p. Therefore, $\underline{r}_c + \beta X (= \overline{r}_c + \beta \int_\Theta (1 - F_\theta(\overline{r}_c)) d\mu(\theta|p, m, \overline{r}_c)$ must hold with $\underline{r}_c < \overline{r}_c$. Note that in equilibrium, both the left-hand side of Equation 11 and 13 must be equal to p. Therefore, $\underline{r}_c + \beta X (= \overline{r}_c + \beta \int_\Theta (1 - F_\theta(\overline{r}_c)) d\mu(\theta|p, m, \overline{r}_c)$ must hold with $\underline{r}_c < \overline{r}_c$. Note that In equilibrium, both the left-hand side of Equation 11 and 13 must be equal to p. Therefore, $\underline{r}_c + \beta X(\underline{r}_c, \underline{r}_c) = \overline{r}_c + \beta X(\overline{r}_c, \overline{r}_c)$ needs hold with $\underline{r}_c < \overline{r}_c$; however,

$$\underline{r}_c + \beta X(\underline{r}_c, \underline{r}_c) < \overline{r}_c + \beta X(\overline{r}_c, \underline{r}_c) \leq \overline{r}_c + \beta X(\overline{r}_c, \overline{r}_c),$$

where the first inequality follows from Assumption 3 for any given θ and the second inequality follows from the lemma above. This contradicts the initial assumption, thus, $\underline{r}_c = \overline{r}_c$.

Step (ii) Users play cutoff strategies in any equilibrium.

Define $\tilde{r} \equiv inf\{r_i : u(r_i, p) \geq 0\}$ to be the user with the lowest r_i of the set of users that have a non-negative utility from joining the platform. Similarly, let $\tilde{\tilde{r}} \equiv sup\{r_i : u(r_i, p) \leq 0\}$ be the users with the largest r_i of the set of users that have a negative or zero utility from joining the platform.

Therefore, it needs to hold that $\tilde{\tilde{r}} + \beta X(\tilde{\tilde{r}}, \tilde{\tilde{r}}) \leq p \leq \tilde{r} + \beta X(\tilde{r}, \tilde{r})$ for $\tilde{r} > \tilde{\tilde{r}}$ by definition. Imposing that $X(r_1, r_2)$ *strictly* increases in r_2, I can use the previous argument from Step (i) to show that there agents play cutoff strategies. As long as $\tilde{r}, \tilde{\tilde{r}} \in (\underline{r}, \overline{r})$ and hence $X(r_1, r_2)$ *strictly* increases in r_2, the following holds

$$\tilde{\tilde{r}} + \beta X(\tilde{\tilde{r}}, \tilde{\tilde{r}}) < \tilde{\tilde{r}} + \beta X(\tilde{\tilde{r}}, \tilde{r}) < \tilde{r} + \beta X(\tilde{r}, \tilde{r})$$

Therefore, $\tilde{r} = \tilde{\tilde{r}}$ and users follow a cutoff strategy.

Then, the condition

$$r_c = p - \beta \int_\Theta (1 - F_\theta(r_c)) d\mu(\theta|p, m, r_c), \tag{15}$$

characterizes the unique cutoff. Note that if $\mu(\theta|r_c) = \delta_{\tilde{\theta}}$, i.e. the belief concentrates on state θ with probability 1 (a.s.), this condition is the same as under complete information. Observe that

Remark. If belief $\mu(\theta|p, m, r)$ has first-order stochastic dominance over a belief $\mu'(\theta|p, m, r)$, $\int_\Theta (1 - F_\theta(r_c)) \mu(\theta|r_c) d\theta$ increases and therefore, the cutoff r_c decreases.

The result follows directly from the Equation 15 and Lemma 4.

Proof Lemma 2 Under complete information equilibrium demand is determined by the unique solution to

$$n^* = Pr(r + \beta n^* - p \geq 0) = 1 - F_\theta(p - \beta n^*),$$
$$\Leftrightarrow n^* = 1 - F_\theta(r^*)$$

given that Assumption 3. It is possible to rewrite this condition as $G(n^*; p) = 1 - F_\theta(p - \beta n^*) - n^*$. The implicit function theorem implies that a function g exists such that $n^* = g(p)$. Implicit differentiation yields

$$-f_\theta(p - \beta n^*) + \beta f_\theta(p - \beta n^*)\frac{\partial n}{\partial p} - \frac{\partial n}{\partial p} = 0$$
$$\frac{\partial n}{\partial p} = \frac{-f_\theta(p - \beta n^*)}{1 - \beta f_\theta(p - \beta n^*)}.$$

The platform faces the optimization problem $\max_p (p - c)n(p)$ and yields

$$1 - F_\theta(r^*) + (p - c)\frac{-f_\theta(r^*)}{1 - \beta f_\theta(r^*)} = 0,$$

which can be rewritten as

$$p - c = \underbrace{\frac{1 - F_\theta(r^*)}{f_\theta(r^*)}}_{\eta(\theta,p)} \underbrace{(1 - \beta f_\theta(r^*))}_{>0},$$

where $\eta(\theta, p)$ is the users price elasticity. Given Assumption 1 the hazard rate defined by $\lambda \equiv \frac{1}{\eta}$ is decreasing. Thus, the first-order condition solves for a unique price $p^*(\theta)$. The second-order condition is

$$-\left[2\frac{f_\theta(r^*)}{1 - \beta f_\theta(r^*)} + (p - c)\frac{f'_\theta(r^*)}{(1 - \beta f_\theta(r^*))^3}\right].$$

At p^* it holds that

$$-\frac{1}{1 - \beta f_\theta(r^*)}\left[2f_\theta(r^*) + f'_\theta(r^*)\frac{1 - F_\theta(r^*)}{f_\theta(r^*)(1 - \beta f_\theta(r^*))}\right]$$
$$\Leftrightarrow -\frac{1}{1 - \beta f_\theta(r^*)}\left[\frac{2(1 - \beta f_\theta(r^*))[f_\theta(r^*)]^2 + f'_\theta(r^*)(1 - F_\theta(r^*))}{f_\theta(r^*)}\right] < 0$$

Note that the term in rectangular brackets is positive if $2(1 - \beta f_\theta(r^*)) \geq 1$ which holds if $1/2 \geq \beta f_\theta(r^*)$. The denominator is always positive, however, the numerator must also be positive due to the assumption that the hazard rate is increasing in r given $2(1 - \beta f_\theta(r^*)) \geq 1$. To see this, take the first derivative of the hazard rate with respect to r

$$\lambda'(r) = \frac{[f_\theta(r^*)]^2 + f'_\theta(r^*)(1 - F_\theta(r^*))}{[1 - F_\theta(r^*)]^2} > 0, \text{ by Assumption 1.}$$

Lastly, to show that the equilibrium price can be increasing/decreasing or constant in state θ, note first that the hazard rate $\lambda(r,\theta)$ is strictly decreasing in θ by the MLRP property. It follows that

$$\frac{\partial\lambda(r,\theta)}{\partial\theta}=\frac{f^{1,\theta}(r)(1-F_\theta(r))+f_\theta(r)F^{1,\theta}(r)}{(1-F_\theta)^2}<0,$$

which allows to bound $f^{1,\theta}(r)<\frac{f_\theta(r)}{1-F_\theta(r)}(-F^{1,\theta}(r))$.

Taking the derivative of the first-order condition with respect to θ yields

$$\begin{aligned}\frac{\partial p}{\partial\theta}&=\frac{-\beta f^{1,\theta}(r)f_\theta(r)(1-F_\theta(r))-(f^{1,\theta}(r)(1-F_\theta(r))+f_\theta(r)F^{1,\theta}(r))(1-\beta f_\theta(r))}{f_\theta(r)^2}\\&=\frac{-f^{1,\theta}(r)(1-F_\theta(r))-f_\theta(r)F^{1,\theta}(r)(1-\beta f_\theta(r))}{f_\theta(r)^2}\\&>\beta F^{1,\theta}(r),\end{aligned}$$

where $F^{1,\theta}(r)<0$. Hence, the sign cannot be determined.

Proof of Proposition 1

Proof. The proof from Mailath (1987), or more precisely Mailath and von Thadden (2013) who provide new sufficient conditions for when incentive-compatibility implies differentiability and hence, the differential equation in Equation 3.

To apply the results from Mailath and von Thadden (2013), I need to check whether the regularity conditions are fulfilled. Denote the platform's profit function by $\pi(\theta,\hat{\theta},p)$, where the first position denotes the true state, the second position the user's point belief on a state $\hat{\theta}$ and the last position the price. First, the platform's profit is twice differentiable: $\pi(\theta,\mu,p)$ is C^2 on $\Theta^2\times\mathbb{R}$. The two main assumptions in Mailath and von Thadden (2013) (Assumption 1 and 2) state that $\pi(\theta,\theta,p)$ has a unique solution under full information (first-best). Second, Mailath and von Thadden (2013) require that the second-order condition in the first-best solution is negative. Both are fulfilled by Lemma 2. And lastly, the derivative of $\pi(\theta,\theta,p)$ in p is bounded from below, whenever the second-order condition is non-negative, i.e. there exists an $\varepsilon>0$ such that for all $(\theta,p)\in\Theta\times\mathbb{R}$

$$\pi_{33}(\theta,\theta,p)\geq 0\Rightarrow|\pi_3(\theta,\theta,p)|>\varepsilon.$$

Note that $\pi_{33}\geq 0$ if $(p-c)\leq -2^{f}/_{f'}(1-\beta f)^2$, then π_3 is

$$\begin{aligned}|(p-c)-\frac{f}{1-\beta f}+1-F|&\geq|2\frac{f^2}{f'}(1-\beta f)+1-F|\\&=|2f^2(1-\beta f)+(1-F)f'|>\varepsilon>0,\end{aligned}$$

for $\varepsilon=\inf_r|2f^2(1-\beta f)+(1-F)f'|$. Similarly to the Proof of Lemma 2, this follows from the fact that the hazard rate is strictly increasing in r, hence its derivative is positive for all r, and given that Assumption 3 holds.

Additionally, the condition on belief monotonicity, $\pi_2(\theta,\mu,p)=(p-c)n_2(\theta,\hat{\theta},p)>0$ for $p>c$, is fulfilled, i.e. the platform always has an incentive to manipulate beliefs in a way that users believe that the state is higher than it actually is. In Mailath (1987) one regularity condition is that $\pi_{13}(\theta, \hat{\theta}, p) \neq 0$, i.e. never changes sign, which is not necessarily fulfilled in this model. Note that there might exist a p such that $\pi_{13}(\theta, \hat{\theta}, p) = 0$.

Additional conditions are the initial value condition $\tau(\underline{\theta}) = p^{FI}(\theta)$ and the single-crossing condition. To see that the single-crossing condition is fulfilled, denote profits by

$$\pi = (p - c)(1 - F_\theta(r_c))$$

The (strict) single-crossing property is satisfied: $MRS(\theta, p, r) = -\frac{\partial \pi(\cdot)}{\partial p} / \frac{\partial \pi(\cdot)}{\partial -r}$ is a strictly decreasing function of θ. The marginal rate of substitution trades off increases in the price and increases in the users' response. As an increase in the user response is given by a decrease in r, the MRS is taken with respect to $-r$. Take the respective derivatives and rearrange

$$-\frac{\partial \pi(\theta,p,r)/\partial p}{\partial \pi(\theta,p,r)/\partial -r} = -\frac{(p-c)\left[\frac{-f_\theta(r)}{1-\beta \partial/\partial r \int_\Theta 1-F_\theta(r)d\mu(\theta)}\right] + 1 - F_\theta(r)}{(p-c)f_\theta(r)} \tag{16}$$

$$= \frac{1}{1-\beta \partial/\partial r \int_\Theta 1-F_\theta(r)d\mu(\theta)} - \frac{1}{(p-c)}\frac{1-F_\theta(r)}{f_\theta(r)}, \tag{17}$$

where the last term depends on θ and corresponds to the inverse hazard rate. The inverse hazard rate is strictly increasing in θ, such that the whole expression is strictly decreasing in θ. The first term integrates over all $\theta \in \Theta$ such that an increase in θ has no effect on this term.

To prove the result, it needs to be shown that incentive-compatibility implies differentiability and vice versa. First, let $\tau(\theta)$ be one-to-one and incentive-compatible, then differentiability of $\tau(\theta)$ is implied by Theorem 4 and Theorem 5 of Mailath and von Thadden (2013). And, the equilibrium strategy $\tau(\theta)$ satisfies the differential equation as shown in the text.

Second, $\tau(\theta)$ is continuous on Θ and satisfies the differential equation and $\frac{\pi_3(\theta,\delta_{\hat{\theta}},p)}{\pi_2(\theta,\delta_{\hat{\theta}},p)}$ is increasing in θ. As

$$\underbrace{\tau'(\theta)}_{>0}\underbrace{\pi_2(\theta,\hat{\theta},\tau(\theta))}_{>0}\underbrace{\frac{d}{d\hat{\theta}}\left\{\frac{\pi_3(\theta,\hat{\theta},\tau(\theta))}{\pi_2(\theta,\hat{\theta},\tau(\theta))}\right\}}_{\geq 0} \geq 0 \tag{18}$$

for all $\theta, \hat{\theta} \in \Theta$, τ is incentive-compatible due to Theorem 6 of Mailath and von Thadden (2013).

First note that from Equation 18, it is possible to determine the sign of $\tau'(\theta)$, as the only incentive-compatible solution is given if $\tau'(\theta) > 0$. Second, given the initial value condition, this equilibrium is the unique separating equilibrium (Mailath, 1987).

□

Proof of Proposition 2

Proof. The key element to this proof is the single-crossing condition as shown in the previous proof. Given single-crossing, I show that there exists a small increase in price and the users' response that gives higher types scope to separate from lower types. Formally, the upcoming lemma provides a price-response pair for which a higher type would like to deviate whereas lower types do not. Note that both on-path and off-path responses are given by the cutoff in Equation 1 as the unique continuation equilibrium in monotone strategies and the unique rationalizable profile (for given beliefs) coincide. In Definition 4, I restrict attention to users playing rationalizable strategies off-path. This has the advantage that there exists a unique cutoff also off-path following any price and given beliefs. Due to continuity, the cutoffs are $r \in [r(p, \delta_{\underline{\theta}}), r(p, \delta_{\overline{\theta}})]$. For given, beliefs the cutoff is unique, but there might be different beliefs inducing the same cutoff.

Lemma 5. *Choose any price p', users' response r', and type θ'. For any type $\theta < \theta'$, there exists $h \in \mathbb{R}_+$ and λ, such that $0 < \lambda < \varepsilon$ implies*

$$\pi(\theta', p_\lambda, r_\lambda) > \pi(\theta', p', r') \tag{A1}$$

$$\pi(\tilde{\theta}, p_\lambda, r_\lambda) < \pi(\tilde{\theta}, p', r'), \ \forall \tilde{\theta} \leq \theta, \tag{A2}$$

where $(p_\lambda, r_\lambda) = (p' + h\lambda, r' - \lambda)$.

The proof of this lemma follows Ramey (1996), but is simplified as there is only one signal p for which the one-dimensional single-crossing condition holds. With multiple signals, Ramey (1996) shows that invoking a weaker condition than the multi-dimensional single-crossing condition is sufficient.

Take $\theta < \theta'$ and let $x \in \mathbb{R}$ be such that $x \geq MRS(\theta, p', r')$. Note that $x \neq MRS(\theta', p', r')$ and $\{MRS(\theta', p', r')\}$, $\{x\}$ are closed, convex sets as they are a singleton. Hence, it is possible to apply Minkowski's hyperplane separation theorem (potentially something easier here, e.g. by just constructing an h such that this fits), which implies the existence of $h \in \mathbb{R}$, $h \neq 0$, such that

$$h \cdot MRS(\theta', p', r') < 1 < h \cdot x, \tag{19}$$

which implies that $h > 0$. Suppose $(p_\lambda, r_\lambda) = (p' + h\lambda, r' - \lambda)$ for $\lambda > 0$, i.e., a small increase in price and a small increase in demand (a small decrease in the cutoff). To determine whether $\pi(\tilde{\theta}, p_\lambda, r_\lambda) - \pi(\tilde{\theta}, p', r') \lesseqgtr 0$, define

$$\zeta(\lambda, \tilde{\theta}) = \frac{-\pi(\tilde{\theta}, p_\lambda, r_\lambda) + \pi(\tilde{\theta}, p', r - \lambda)}{\pi(\tilde{\theta}, p', r - \lambda) - \pi(\tilde{\theta}, p', r')},$$

and then

$$\pi(\tilde{\theta}, p_\lambda, r_\lambda) - \pi(\tilde{\theta}, p', r') = \left[\pi(\tilde{\theta}, p', r - \lambda) - \pi(\tilde{\theta}, p', r')\right]\left[1 - \zeta(\lambda, \tilde{\theta})\right].$$

To determine the sign of $\zeta(\lambda, \tilde{\theta})$, observe that

$$\lim_{\lambda \to 0} \zeta(\lambda, \tilde{\theta}) = \lim_{\lambda \to 0} \frac{-\pi(\tilde{\theta}, p_\lambda, r_\lambda) + \pi(\tilde{\theta}, p', r - \lambda)}{\pi(\tilde{\theta}, p', r - \lambda) - \pi(\tilde{\theta}, p', r')} = -\frac{h\pi_p(\tilde{\theta}, p', r')}{\pi_r(\tilde{\theta}, p', r')} = h \cdot MRS(\tilde{\theta}, p', r'),$$

by l'Hospital rule and note that $\partial\pi(\cdot)/\partial r < 0$, so it is written as $-\pi_r(\cdot)$. Hence, it is possible to extend $\zeta(\cdot)$ continuously to $\lambda = 0$. Define

$$\zeta(\lambda, \tilde{\theta}) = \begin{cases} \frac{-\pi(\tilde{\theta}, p_\lambda, r_\lambda) + \pi(\tilde{\theta}, p', r - \lambda)}{\pi(\tilde{\theta}, p', r - \lambda) - \pi(\tilde{\theta}, p', r')} & \text{if } \lambda > 0 \\ -\frac{h\pi_p(\tilde{\theta}, p', r')}{\pi_r(\tilde{\theta}, p', r')} & \text{if } \lambda = 0 \end{cases}$$

In $\lambda \in \mathbb{R}_{>0}$ $\zeta(\lambda, \tilde{\theta})$ is differentiable as a composition of differentiable functions, however, the function is not differentiable in $\lambda = 0$ as $MRS(\tilde{\theta}, p', r') \neq 0$. For $\lambda > 0$ the function is strictly decreasing in λ

$$\frac{-h\pi_p(\tilde{\theta}, p', r')(\pi(\tilde{\theta}, p', r-\lambda) - \pi(\tilde{\theta}, p', r')) - (-\pi(\tilde{\theta}, p_\lambda, r_\lambda) + \pi(\tilde{\theta}, p', r-\lambda))(\pi_r(\tilde{\theta}, p', r'))}{(\pi(\tilde{\theta}, p', r-\lambda) - \pi(\tilde{\theta}, p', r'))^2} < 0$$

From Equation (19), it follows that $\zeta(\lambda = 0, \theta') < 1$ and $\zeta(\lambda, \theta') < 1$ as well, such that $\pi(\theta', p_\lambda, r_\lambda) - \pi(\theta', p', r') > 0$. Furthermore, $\zeta(\lambda = 0, \tilde{\theta}) > 1$ and hence $\zeta(\lambda, \tilde{\theta}) > 1$ for λ sufficiently small ($\lambda < \varepsilon$), such that $\pi(\tilde{\theta}, p_\lambda, r_\lambda) - \pi(\tilde{\theta}, p', r') < 0$, which needed to be shown.

Lemma 6. *In an equilibrium in which $p^*(\theta) = p'$ is set by more than one type, the highest type of the pool θ' can set price p_λ to break the equilibrium. For p_λ, there exists $r \in \mathcal{R}^\infty(\Theta^*(p_\lambda), p_\lambda)$ such that $\min_{r \in \mathcal{R}^\infty(\Theta^*(p_\lambda), p_\lambda)} \pi(\theta', r, p_\lambda) \leq \pi^*(\theta', \Sigma)$.*

Now, consider an equilibrium candidate in which $p^*(\theta) = p'$ for more than one θ. Let $\theta' = sup\{\theta | p^*(\theta) = p'\}$ be the highest type in the pool and $r' = r(p')$ be the user response to observing price p' in equilibrium. Since $\{\theta | p^*(\theta) = p'\}$ is non-degenerate and $\mu_1(\theta | r, p')$ has full support on the closure of the set $\{\theta | p^*(\theta) = p'\}$, receivers place strictly positive probability on the set $cl\{\theta | p^*(\theta) = p'\} - \{\theta'\}$.

Given users use rationalizable strategies off-path, Lemma 1 provides a unique cutoff for given beliefs. Then, $n(\theta', p', r') < n(\theta', p', r^*)$, where the cutoff for the highest type in the pool is lower if users believe $\mu(\theta') = \delta_{\theta'}$ (r^*) than the cutoff r'

$$r' = p' - \beta \int_\Theta (1 - F_\theta(r_c))\mu_1(\theta | r', p) d\theta,$$
$$r^* = p' - \beta(1 - F_{\theta'}(r^*)),$$

such that $r^* < r'$ as the users place strictly positive probability on lower types other than θ'.

The rest of the proof follows Ramey (1996). Take a type θ sufficiently close to the highest type of the pool θ' that yields $r' > r(p', \delta_\theta)$. Given θ, there exist small moves upwards in price and receiver response (p_λ, r_λ) supplied by Lemma 5 that satisfies Equation A1 and Equation A2. By taking λ sufficiently small $r_\lambda > r(p_\lambda, \theta)$.

For p_λ, the user response is either such that Equation A1 and Equation A2 are satisfied, or if $\pi(\tilde{\theta}, p_\lambda, r) \geq \pi(\tilde{\theta}, p', r')$ for $\tilde{\theta}$ resulting in $r > r_\lambda$, then because of the single-crossing property $\pi(\theta', p_\lambda, r) > \pi(\theta', p', r')$ such that θ' has a stricter incentive to deviate. In both cases, $D_{\tilde{\theta}} \cup D^0_{\tilde{\theta}} \subset D_{\theta'}$ holds, i.e. for types $\tilde{\theta}$ there are less rationalizable strategy profiles for which it can improve. Then, D1 criterion requires the support of $\mu^*(\theta | p_\lambda)$ to be in $[\theta, \overline{\theta}]$, i.e. $\Theta^*(p_\lambda) = [\theta, \overline{\theta}]$ with $\theta' \in [\theta, \overline{\theta}]$. By Equation 1, it must be that $r(p_\lambda, \delta_\theta) > r(p_\lambda, \mu(\Theta^*(p_\lambda))$, and Equation A1 implies that θ' has a profitable deviation breaking the equilibrium.

□

Proof of Theorem 1 I prove the following theorem.

Theorem 3. *The unique separating equilibrium is characterized as follows:*

1) If costs γ are sufficiently small

$$\underbrace{(p-c)n_2(\theta,\theta,p)}_{\text{Signaling benefit of extra fake profile}} > \underbrace{\gamma n_1(\theta,\theta,p)}_{\text{Cost of extra fake profile}}, \forall \theta \in \Theta. \quad (20)$$

signalling takes the form of either

a) Price signaling: zero fake profiles are used and prices are set to

$$p'(\theta) = -\frac{(p(\theta)-c)n_2(\theta,\theta,p(\theta))}{(p(\theta)-c)n_3(\theta,\theta,p(\theta))+n(\theta,\theta,p(\theta))}. \quad (21)$$

Or

b) Price and fake profile signaling. Suppose $\theta \in [\underline{\theta}^*, \overline{\theta}^*]$, where $[\underline{\theta}^*, \overline{\theta}^*] \subset [\underline{\theta}, \overline{\theta}]$. Fake profiles are given by

$$\gamma\xi = (p(\theta)-c)n(\theta,\theta,p(\theta)) - (p(\underline{\theta})-c)n(\underline{\theta},\underline{\theta},p(\underline{\theta})) - \int_{\underline{\theta}}^{\theta}(p(t)-c-\gamma)n_1(t,t,p(t))dt, \quad (22)$$

and prices maximize equilibrium profits

$$p^{PF} \equiv p(\theta) - c - \gamma = -\frac{n_1(\theta,\theta,p)}{n_{13}(\theta,\theta,p)}. \quad (23)$$

To ensure existence, $\frac{\partial p^{PF}}{\partial\theta} > 0$, i.e. p^{PF} must be strictly increasing. Let $[\underline{\theta}^*, \overline{\theta}^*]$ be such that $p^{FB} \leq p^{FP} \leq p^{\dagger}$ holds for all $\theta \in [\underline{\theta}^*, \overline{\theta}^*]$, where $p^{\dagger}$ is given by the differential equation $p'(\theta) = -\frac{(p(\theta)-c)n_2(\theta,\theta,p(\theta))-\gamma n_1(\theta,\theta,p(\theta))}{(p(\theta)-c)n_3(\theta,\theta,p(\theta))+n(\theta,\theta,p(\theta))}$. A necessary condition for existence is that $f^{1,\theta}(r(p^{FP})) > 0$. As a sufficient condition, the second-order condition must be fulfilled: $-\frac{n_1(\theta,\theta,p)}{n_{13}(\theta,\theta,p)}n_{133}(\theta,\theta,p) + 2n_{13}(\theta,\theta,p) < 0$.

2) Suppose cost γ are sufficiently large, zero fake profiles are used and full information prices are set.

The unique D1 equilibrium is given by b) if it exists and a) otherwise. If Condition I is not fulfilled, the unique D1 equilibrium is 2). The unique D1 equilibrium is least-cost separating, i.e. maximizes the platform's payoffs over the set of separating equilibria.

Proof. Suppose first that γ is sufficiently small and Θ is chosen such that a marginal increase in fake profiles is potentially profitable for all types. First, I will construct and prove the existence of the platform's preferred equilibrium. I focus on differentiable strategies of the platform.[20] Given differentiability, the equilibrium separating strategy must satisfy the incentive constraints locally.

[20] In the one-dimensional signaling problem, Mailath (1987) show that incentive-compatibility results differentiablity (and vice versa).

Part I a) Suppose first that zero fake profiles are used in equilibrium, i.e. $\xi(\theta) = 0$. The incentive constraints for $\theta' > \theta$ read

$$(p(\theta) - c)n(\theta, \theta, p(\theta)) \geq (p(\theta') - c)n(\theta, \theta', p(\theta')) - \gamma\left[(n(\theta', \theta', p(\theta')) - n(\theta, \theta', p(\theta')))\right]$$
$$(p(\theta') - c)n(\theta', \theta', p(\theta')) \geq (p(\theta) - c)n(\theta', \theta, p(\theta))$$

It is important to note that the incentive constraints for deviations up- and downwards are asymmetric. If type θ' deviates towards θ, it does not need to produce fake profile to match the difference in demand. Instead, it would need to create negative fake profiles to actually match the demand of the lower type. This, however, is not feasible and hence reveals type θ' in the last stage (after entry): $\mu_2(\cdot) = \delta_{\theta'}$. At entry users nevertheless hold belief $\mu_1(\cdot) = \delta_\theta$ after observing $p(\theta)$ which results in entry of $n(\theta', \theta, p(\theta))$. As the entered mass of users is smaller than the mass of users that would have liked to enter under true state θ' , the demand after entry is binding for the platform.

For close θ to θ', the incentive constraints are

$$p'(\theta)[(p(\theta) - c)n_3(\theta, \theta, p(\theta)) + n(\theta, \theta, p(\theta))] + (p(\theta) - c)n_2(\theta, \theta, p(\theta)) - \gamma n_1(\theta, \theta, p(\theta)) \leq 0$$
$$p'(\theta)[(p(\theta) - c)n_3(\theta, \theta, p(\theta)) + n(\theta, \theta, p(\theta))] + (p(\theta) - c)n_2(\theta, \theta, p(\theta)) \geq 0$$

The second IC must be binding, whereas the first IC is slack. If the first IC would be binding, the second IC would not be fulfilled and higher types would prefer to set prices of lower types (which are less distorted from first-best). The resulting differential equation is

$$p'(\theta) = -\frac{(p(\theta) - c)n_2(\theta, \theta, p(\theta))}{(p(\theta) - c)n_3(\theta, \theta, p(\theta)) + n(\theta, \theta, p(\theta))}$$

All signaling happens via prices. Hence, prices are maximally distorted to exclude deviations from smaller types. The separating strategy is the same as in Proposition 1. The equilibrium is separating as $p'(\theta) \neq 0$ and $n(\theta, \theta, p(\theta)) \neq n(\theta', \theta', p(\theta')), \forall\theta \neq \theta'$ which can easily be verified. The separating strategy is (locally) incentive compatible as it satisfies the differential equation.

Given that the equilibrium pricing strategy fulfills local incentive compatibility, global incentive compatibility is ensured by the single-crossing condition in Equation 17 meaning that higher types are more willing to trade off an increase in price over a decrease in demand. For deviations from a higher type to a lower type's price, the argument is as in Proposition 1.

Global incentive compatibility for deviations from a lower type to a higher type's price is fulfilled (even easier as before). To see this consider the following. If a deviation of $p(\theta)$ to $p(\theta')$ as well of $p(\theta')$ to $p(\theta'')$ for $\theta'' > \theta' > \theta$ is unprofitable (due to local IC), then due to single-crossing a deviation of θ from $p(\theta)$ to $p(\theta'')$ is also unprofitable. Additionally, when deviating from $p(\theta)$ to $p(\theta')$ to induce belief θ', type θ need to create additional fake profiles which are included in the incentive compatibility constraint. A deviation of θ from $p(\theta)$ to $p(\theta'')$ requires even more fake profiles as $n(\theta'', \theta'', p(\theta'')) - n(\theta, \theta'', p(\theta'')) > n(\theta', \theta', p(\theta')) - n(\theta, \theta', p(\theta'))$ making a global deviation even less profitable.

Existence: with off-path beliefs, $\mu^{off} = \delta_{\underline{\theta}}$

Part I b) Now I will construct an equilibrium candidate for the platform's preferred equilibrium when fake profiles are created in equilibrium, i.e. $\xi(\theta) > 0$. The incentive constraints for $\theta' > \theta$ are: First, for deviations upwards

$$(p(\theta) - c)n(\theta, \theta, p(\theta)) - \gamma\xi(\theta) \geq (p(\theta') - c)n(\theta, \theta', p(\theta')) - \gamma\left[\xi(\theta') + (n(\theta', \theta', p(\theta')) - n(\theta, \theta', p(\theta')))\right],$$

second, for deviations downwards given that $\gamma\left[\xi(\theta) - (n(\theta', \theta, p(\theta)) - n(\theta, \theta, p(\theta)))\right] > 0$

$$(p(\theta') - c)n(\theta', \theta', p(\theta')) - \gamma\xi(\theta') \geq (p(\theta) - c)n(\theta', \theta, p(\theta)) - \gamma\left[\xi(\theta) - (n(\theta', \theta, p(\theta)) - n(\theta, \theta, p(\theta)))\right],$$

and third, if $\gamma\left[\xi(\theta) - (n(\theta', \theta, p(\theta)) - n(\theta, \theta, p(\theta)))\right] \leq 0$

$$(p(\theta') - c)n(\theta', \theta', p(\theta')) - \gamma\xi(\theta') \geq (p(\theta) - c)n(\theta', \theta, p(\theta)).$$

I will show that it suffices to impose the incentive constraint only for nearby types. For close θ to θ', the ICs are

$$p'(\theta)[(p(\theta) - c)n_3(\theta, \theta, p(\theta)) + n(\theta, \theta, p(\theta))] + (p(\theta) - c)n_2(\theta, \theta, p(\theta)) - \gamma\xi'(\theta) \leq \gamma n_1(\theta, \theta, p(\theta)) \tag{24}$$

$$p'(\theta)[(p(\theta) - c)n_3(\theta, \theta, p(\theta)) + n(\theta, \theta, p(\theta))] + (p(\theta) - c)n_2(\theta, \theta, p(\theta)) - \gamma\xi'(\theta) \geq \gamma n_1(\theta, \theta, p(\theta)). \tag{25}$$

Setting the first IC to bind, implies that the second IC binds and vice versa. This results in a differential equation of the fake profile strategy as a function of prices.

$$\gamma\xi'(\theta) = p'(\theta)[(p(\theta) - c)n_3(\theta, \theta, p(\theta)) + n(\theta, \theta, p(\theta))] + (p(\theta) - c)n_2(\theta, \theta, p(\theta)) - \gamma n_1(\theta, \theta, p(\theta)).$$

The platform's preferred equilibrium includes as few fake profiles as possible. Additionally, equilibrium reasoning yields that once the lowest type $\underline{\theta}$ is identified as such, it cannot do better than setting zero fake profiles. This implies the following initial value condition for the differential equation: $\xi(\underline{\theta}) = 0$. For a given price $p(\theta)$, the differential equation can be solved via the Fourier method and yields the unique solution (up to a constant):

$$\gamma\xi(\theta) = (p(\theta) - c)n(\theta, \theta, p(\theta)) - (p(\underline{\theta}) - c)n(\underline{\theta}, \underline{\theta}, p(\underline{\theta})) - \int_{\underline{\theta}}^{\theta} (p(t) - c - \gamma)n_1(t, t, p(t))dt.$$

Equilibrium profit as a function of price is given by

$$\begin{aligned}\Pi(\theta) &= (p(\theta) - c)n(\theta, \theta, p(\theta)) - \gamma\xi(\theta) \\ &= (p(\underline{\theta}) - c)n(\underline{\theta}, \underline{\theta}, p(\underline{\theta})) + \int_{\underline{\theta}}^{\theta} (p(t) - c - \gamma)n_1(t, t, p(t))dt\end{aligned}$$

To get the platform's preferred equilibrium, prices must be chosen such that

$$\arg\max_{p(\theta)} (p(\underline{\theta}) - c)n(\underline{\theta}, \underline{\theta}, p(\underline{\theta})) + \int_{\underline{\theta}}^{\theta} (p(t) - c - \gamma)n_1(t, t, p(t))dt, \forall\theta \in \Theta.$$

which yields

$$p^{FP} \equiv \begin{cases} p(\underline{\theta}) - c = \frac{1 - F_{\underline{\theta}}(r)}{f_{\underline{\theta}}(r)}(1 - \beta f_{\underline{\theta}})(r) & \theta = \underline{\theta} \\ p(\theta) - c - \gamma = -\frac{F^{1,\theta}(r)}{f^{1,\theta}(r)}(1 - \beta f_{\theta}(r)) & \theta \in (\underline{\theta}, \overline{\theta}]. \end{cases}$$

This approach is equivalent to Wilson (1985). Wilson (1985) solves a multi-dimensional signaling problem of a firm that can use price and dissipative advertisement to signal its quality. To do so, he sets up the Lagrangian of the firm's profit maximization problem subject to the incentive constraint (for nearby smaller types). Key in the analysis is that dissipative advertising does not affect user's utility directly (just as fake profiles) so that the Lagrange parameter must be equal to one. Wilson (1985), however, does not provide necessary or sufficient assumptions for his approach.

Lastly, to check global incentive compatibility note that prices and fake profiles are single-crossing. The single-crossing conditions are

$$MRS(\theta, p, r) = -\frac{1}{1 - \beta \partial/\partial r \int_{\Theta} 1 - F_\theta(r) d\mu(\theta)} - \frac{1}{(p-c)} \frac{1 - F_\theta(r)}{f_\theta(r)},$$
$$MRS(\theta, \xi, r) = \frac{\gamma}{(p-c) f_\theta(r)},$$

where $MRS(\theta, p, r)$ is strictly decreasing in θ and $MRS(\theta, \xi, r)$ is strictly decreasing in θ if $f^{1,\theta}(r) > 0$. Note that for mimicking a higher type, a lower type needs to create even more fake profiles to balance out the difference in demand. This is not reflected in the $MRS(\theta, \xi, r)$ but works in its favor.

Hence, as long as $p(\theta) - c - \gamma = -\frac{F^{1,\theta}(r)}{f^{1,\theta}(r)}(1 - \beta f_\theta(r))$ is solved for a price-response pair resulting in a positive mark-up ($p - c - \gamma > 0$) and a r such that $f^{1,\theta} > 0$, both signals meet the strict single-crossing condition. In the next paragraph, I will show that this is the only equilibrium candidate with positive fake profiles. Given its existence, global incentive compatibility is ensured for deviations from lower to higher types.

Assume from now on $p - c - \gamma > 0$ and $f^{1,\theta} > 0$. To take all possible deviations into account, global incentive compatibility becomes

$$\begin{aligned}
&(p(\underline{\theta}) - c) n(\underline{\theta}, \underline{\theta}, p(\underline{\theta})) + \int_{\underline{\theta}}^{\theta} (p(t) - c - \gamma) n_1(t, t, p(t)) dt \geq \\
&(p(\theta') - c) n(\theta, \theta', p(\theta')) \\
&- [(p(\theta') - c) n(\theta', \theta', p(\theta')) - (p(\underline{\theta}) - c) n(\underline{\theta}, \underline{\theta}, p(\underline{\theta})) - \int_{\underline{\theta}}^{\theta'} (p(t) - c - \gamma) n_1(t, t, p(t)) dt],
\end{aligned}$$

which reduces to

$$\int_{\theta'}^{\theta} (p(t) - c - \gamma) n_1(t, t, p(t)) dt \geq \int_{\theta'}^{\theta} (p(\theta') - c - \gamma) n_1(t, \theta', p(\theta')) dt.$$

A sufficient condition is

$$(\theta - \theta')[(p(t) - c - \gamma) n_1(t, t, p(t)) - (p(\theta') - c - \gamma) n_1(t, \theta', p(\theta')) dt] \geq 0.$$

For $\theta < \theta'$, $[(p(t) - c - \gamma) n_1(t, t, p(t)) - (p(\theta') - c - \gamma) n_1(t, \theta', p(\theta')) dt] \leq 0$ (and vice versa) which holds if

$$p'(\theta)(-F^{1,\theta}(r)) + (p(\theta) - c - \gamma)(-f^{1,\theta}(r)) < 0.$$

The inequality holds if $p'(\theta) > 0$ and if for $\theta < \theta'$ $r(\theta', p(\theta')) > r(\theta, p(\theta))$, that is, the price effect dominates the belief effect. The effect on the response is signed by

$$\frac{p'(\theta) + F^{1,\theta}(r)}{1 - \beta f_\theta(r)},$$

where $p'(\theta) > 0$ but $F^{1,\theta}(r) < 0$ for all r. As $f^{1,\theta}(r) > 0$, the belief effect is vanishing for increasing r: $F^{1,\theta}(r) \to 0$. Therefore, if the belief effect is small enough locally, it is also small enough globally.

Lastly, to guarantee that the non-negativity constraint on the fake profile strategy is not violated, the following condition must hold: For all $\theta \in [\underline{\theta}, \overline{\theta}]$,

$$p^{FB} \leq p^{FP} \leq p^{\dagger}, \tag{26}$$

where $p^{\dagger}$ is given by the differential equation $p'(\theta) = -\frac{(p(\theta)-c)n_2(\theta,\theta,p(\theta)) - \gamma n_1(\theta,\theta,p(\theta))}{(p(\theta)-c)n_3(\theta,\theta,p(\theta)) + n(\theta,\theta,p(\theta))}$. In the corner cases, it is known that

$$\gamma\xi'(\theta, p^{FB}(\theta)) > 0, \text{ and } \gamma\xi'(\theta, p^{\dagger}(\theta)) = 0.$$

Hence, the equilibrium candidate exists for a range of $\theta \in [\underline{\theta}^*, \overline{\theta}^*]$ for which the Inequality in 26 holds.

Platform's preferred equilibrium given Condition I The platform's preferred equilibrium is the equilibrium candidate that maximizes the platform's profit. Focus for now on the equilibrium candidate in Part Ib). For $\theta = \underline{\theta}$, the profit function is concave under the assumptions (see Lemma 2), hence the only profit-maximizing price is the first-best price. For $\theta \in (\underline{\theta}, \overline{\theta}]$, the profit function with respect to p is not necessarily single-peaked. The necessary and sufficient conditions are given by the first-order condition

$$(p - c - \gamma)n_{13}(\theta, \theta, p) + n_1(\theta, \theta, p) = 0,$$

and the second-order condition

$$(p - c - \gamma)n_{133}(\theta, \theta, p) + 2n_{13}(\theta, \theta, p).$$

For any price $p \in \mathbb{R}$, there exists a unique cutoff r by Lemma 1. However, if n_{13} changes sign (n_1 is strictly positive), there might be two price-response pairs solving the first-order condition. For a unique interior solution n_{13} must be strictly (positive) negative, i.e. price increases have a negative effect on demand and more so for higher types. I will consider two cases, either n_{13} never changes sign or n_{13} changes sign once. To do so, reformulate both conditions in terms of the underlying distribution

$$(p - c - \gamma)\frac{-f^{1,\theta}(r)}{1 - \beta f_\theta(r)} - F^{1,\theta}(r) = 0$$

Case 1): First, suppose that the density has no peak, i.e. is constant. Together with MLRP, this implies $f^{1,\theta}(r) < 0, \forall r$ (as exhibited by the uniform distribution on $[0, \theta]$). Then, the solution to the FOC is $p - c - \gamma < 0$, which is a local minimum if

$$-\frac{F^{1,\theta}(r)}{f^{1,\theta}(r)} \frac{\partial f_\theta(r)/\partial r\partial\theta + f^{1,\theta}(r)f'(r)\beta/1-\beta f_\theta(r)}{1 - \beta f_\theta(r)} + 2\frac{-f^{1,\theta}(r)}{1 - \beta f_\theta(r)} > 0.$$

As the density is constant, ${}^{\partial f_\theta(r)}/_{\partial r \partial \theta} = 0$ and ${}^{\partial f_\theta(r)}/_{\partial r} = 0$ and hence the SOC is always positive. In that case, the profit function is convex and the profit-maximizing price is a corner-solution. As the profit-maximizing solution must still adhere to incentive-compatibility prices are chosen as high as possible given IC. The profit-maximizing solution is then given by Part Ia.

Case 2): Suppose now, that the density is single-peaked and hence $f^{1,\theta}(r)$ changes sign only once. If the density is right-skewed, $f^{1,\theta}(r)$ is strictly increasing in r. This is implied by any distribution of the canonical exponential family with $A''(\theta) \geq 0$. For small values of r $f^{1,\theta}(r)$ is negative and turn positive for larger values of r. By the MLRP assumption which implies a monotonically decreasing hazard rate in θ, it is possible to bound $f^{1,\theta}(r)$ by $f^{1,\theta}(r) < \frac{f_\theta(r)}{1-F_\theta(r)}(-F^{1,\theta}(r)$.

Then, the first-order condition exhibits one or two solutions:

a) Two solutions: One solution exists with $p - c - \gamma < 0$ when $f^{1,\theta} < 0$ and one with $p - c - \gamma > 0$ when $f^{1,\theta} > 0$. If

$$SOC := -\frac{F^{1,\theta}(r)}{f^{1,\theta}(r)} \frac{{}^{\partial f_\theta(r)}/_{\partial r \partial \theta} + {}^{f^{1,\theta}(r) f'(r) \beta}/_{1-\beta f_\theta(r)}}{1 - \beta f_\theta(r)} + 2\frac{-f^{1,\theta}(r)}{1 - \beta f_\theta(r)} < 0,$$

for $f^{1,\theta} > 0$ and $SOC > 0$ for $f^{1,\theta} < 0$, the profit function is relatively well-behaved such that $p - c - \gamma$ is the local and global maximum. To see that the interior maximum would also be the global maximum note that with increasing p, r increases further as well such that $-f^{1,\theta}(r)$ and thus the FOC gets more negative (implying decreasing profits).

b) One solutions: As $f^{1,\theta}(r)$ is bounded from above, only one solution at $p* - c - \gamma < 0$ with $f^{1,\theta} < 0$ might exist. In that case, the FOC is always positive after $p*$. Hence, the profit is increasing from then on. The solution $p*$ is a local minimum and the profit-maximizing prices are again a corner solution and given by Part I a) (same argument as Case 1)).

If the density is left-skewed, $f^{1,\theta}(r)$ is strictly decreasing in θ, i.e. $f^{2,\theta}(r) < 0$. Only one solution can exist for $p - c - \gamma > 0$ with $f^{1,\theta} > 0$ (as with increasing r, the FOC would need to solve for a price with an even higher mark-up, however $f^{1,\theta} < 0$ for increasing r which would result in a negative mark-up).

Part II If the inequality of Condition I is reversed for all $\theta \in \Theta$, fake profiles are sufficiently costly for all types. As the cost of fake profiles are common knowledge, users anticipate that fake profiles cannot be used in equilibrium. Then, the equilibrium is the same as characterized in Proposition 3 and is unique by D1 (see proof of Proposition 3).

Part III: Uniqueness by D1 Suppose Condition I holds. Uniqueness for the case that Condition I is reversed, is given by Proposition 3. To prove uniqueness via the D1 criterion, I need to consider the candidate equilibria from Theorem 1 in Part Ia) and Ib) and exclude possible pooling equilibria.

The platform may "pool" completely, i.e. on price and observable demand $(n(\theta, p) + \xi(\theta))$, or on the price only. If types pool on observable demand and set different prices, the equilibrium is separating and so are beliefs. Hence, this case is covered by the analysis of separating equilibria before.

Proposition 4. *There exist no (pooling) equilibrium in which a) more than one type θ sets a price p' and observable demand $n' = n(\theta, p') + \xi(\theta)$, where $\xi(\theta) \neq \xi(\theta')$ for $\theta \neq \theta'$, b) more than one type θ sets a price p'.*

The proof follows the proof of Proposition 1 closely by considering the incentives to deviate from the pooled price in the first period. The entry decisions and beliefs at $\mu_1(\cdot)$ are characterized as in Proposition. By Lemma 5 there still exists an upward movement in price for a type of the pool that makes it possible to separate from the pool thereby increasing the mass of entering users. This lemma in unaffected by the possibility of fake profiles as the strict single-crossing property holds for the price as a signal alone ($MRS(\theta, p, r)$ is strictly decreasing in θ).[21] Ramey (1996) notes that "With multiple signals, such separating movements remain possible as long as the MRS of any one signal is strictly decreasing in type at every point of the space of signals and responses." As a deviation in the price is sufficient to separate again, the argument for uniqueness by D1 part of the proof of Proposition 1 applies here as well.

Next, I will establish which separating equilibrium is the unique PBE of the game by D1.

Case 1) The equilibrium in Part Ia) exists, whereas the conditions for the equilibrium in Part 1b) are not met. Note that the equilibrium in Part Ia) exists under the assumptions in Section 2. The equilibrium candidate in Part Ia) is the separating equilibrium with least-costly signaling as pinned down by the initial value condition (Riley condition). Hence, the candidate equilibrium for the unique equilibrium is the one that maximizes the platform's profits.

Case 2) The equilibrium in Part Ia) exists and the conditions for the equilibrium in Part 1b) are met as well. Whenever the candidate equilibrium in Part Ib) exists, the platform's profit is higher than in the equilibrium in Ia). This is because the equilibrium in Ia) can be characterized as a corner solution of the platform's profit maximization problem. If, however, the conditions for the equilibrium in Part 1b) are met, the platform's profit-maximization problem is characterized by an interior solution.

Lemma 7. *If the separating equilibrium in Ib) exists, it is the unique D1 equilibrium and maximizes the platform's profit over the set of separating equilibria.*

The proof follows Ramey (1996) (Theorem 6) closely. Let γ_S be the equilibrium from Part Ib) with pricing and fake profile strategy $\rho_S = (p_S(\theta), \xi_S(\theta))$ and user response $r_S(p_S(\theta), \theta)$ and denote equilibrium profits by $\pi_S(\theta)$. Suppose γ is a D1 equilibrium, which must be separating, with $\hat{\pi}(\theta) < \pi_S(\theta)$ for some type θ, i.e. the equilibrium described in Ia). Define θ' as the minimum of the set of types that maximizes $\pi_S(\theta) - \hat{\pi}(\theta)$. Denote by ρ' the equilibrium strategy of type θ' in equilibrium γ_s and let $r' < r_S(\rho', \theta')$ be given by $\pi(\rho', r', \theta') = \hat{\pi}(\theta')$. For $\theta < \theta'$ it holds that

$$\pi(\rho', r_S(\rho', \theta'), \theta) - \hat{\pi}(\theta) \leq \pi_S(\theta) - \hat{\pi}(\theta) < \pi_S(\theta') - \hat{\pi}(\theta').$$

The first inequality holds as incentive compatibility for type θ is fulfilled in equilibrium. The second inequality follows from the fact that type θ' is a type for which the difference

[21] Ramey (1996) imposes a weaker condition on signals such that separating movements remain possible even though none of the signals have a strictly decreasing MRS.

$\pi_S(\cdot) - \hat{\pi}(\cdot)$ is maximized. Rearranging yields,

$$-\hat{\pi}(\theta) < -\hat{\pi}(\theta') + \pi_S(\theta') - \pi(\rho', r_S(\rho', \theta'), \theta) = -\hat{\pi}(\theta') + \pi(\rho', r_S(\rho', \theta'), \theta') - \pi(\rho', r_S(\rho', \theta'), \theta)$$

Next, I want to show that for $\theta < \theta'$ and $r' < r_S$

$$\pi(\rho', r_S(\rho', \theta'), \theta') - \pi(\rho', r_S(\rho', \theta'), \theta) < \pi(\rho', r', \theta') - \pi(\rho', r', \theta).$$

The inequality holds as long as $f^{1,\theta}(r) > 0$. Recall that $p^{FP} < p^{IC,\gamma} < p^{IC}$, and $f^{1,\theta}(r(p^{FP})) > 0$ and so $f^{1,\theta}(r(p^{IC})) > 0$. Hence, for deviations from the price in equilibrium Ia) to the price in equilibrium IIb), the necessary property is fulfilled. To see this, note

$$(p' - c)(1 - F_{\theta'}(r_S)) - (p' - c)(1 - F_\theta(r_S)) < (p' - c)(1 - F_{\theta'}(r')) - (p' - c)(1 - F_\theta(r')),$$
$$F_\theta(r_S) - F_{\theta'}(r_S) < F_\theta(r') - F_{\theta'}(r'),$$

as the difference is monotone

$$F^{1,\theta}(r_S) > F^{1,\theta}(r'),$$

which must be increasing in r, i.e. $f^{1,\theta}(r) > 0$.

Then, the inequality becomes

$$-\hat{\pi}(\theta) < -\hat{\pi}(\theta') + \pi(\rho', r_S(\rho', \theta'), \theta') - \pi(\rho', r_S(\rho', \theta'), \theta) < -\hat{\pi}(\theta') + \pi(\rho', r', \theta') - \pi(\rho', r', \theta),$$

rearranging yields

$$\pi(\rho', r', \theta) - \hat{\pi}(\theta') < \pi(\rho', r', \theta') - \hat{\pi}(\theta') = 0. \tag{27}$$

As the last inequality holds for all $\theta < \theta'$, the D1 criterion implies that users put positive probability on type θ' or higher. Hence, user's response to the off-path deviation to ρ' is $r(\rho') \geq r_S(\rho', \theta')$ which induces θ' to deviate.

□

Proof Proposition 3

Proof. The first-order condition for optimality is given above. It is clear that the unique solution to the first FOC is $p^* = p^*(\theta)$ and $\xi = 0$ to the second FOC. To establish equilibrium existence, suppose that the platform randomizes over messages in equilibrium.

Now, fix the equilibrium strategy $\tau(\theta) = p^*(\theta)$; this is separating as $\frac{\partial p^*(\theta)}{\partial \theta} > 0$ and differentiable. For any $p \in \tau(\Theta)$, let the equilibrium beliefs be $\phi(p) = \tau^{-1}(p)$. To prove that this construction is a separating equilibrium, it needs to be shown that for all θ, $\tau(\theta) \in \arg\max_{p \in \mathbb{R}} \pi(\theta, \phi(p), p)$.

Due to the perfect identification and sophistication of users the problem result again in $\theta, \tau(\theta) \in \arg\max_{p \in \mathbb{R}} \pi(\theta, \theta, p)$. This implies the FOC above resulting in $\tau(\theta) = p(\theta)$ showing that $p^*(\theta)$ is incentive compatible.

Beliefs out-of equilibrium are determined by the D1 Criterion. A deviation in the price is attributed to the type that is most likely to have deviated. If the single-crossing property is satisfied, it has been shown that with a finite set of types the D1 Criterion selects a unique equilibrium which is separating (Cho and Kreps, 1987; Cho and Sobel, 1990). Ramey (1996) additionally shows that the D1 Criterion also selects a unique equilibrium if types are drawn from a compact interval. In contrast to Ramey (1996) user do not play an interior best-response to the platform's actions but follow a cut-off strategy instead, which violates an assumption in Ramey (1996). Nevertheless, it is possible to show that (1) the proposed separating equilibrium survives the D1 Criterion and (2) no pooling equilibrium exists.

First, I need to show that the proposed equilibrium involving the complete information prices and zero fake profiles survives D1. Note that creating a positive amount of fake profiles is never profitable if users can perfectly identify them as they will be able to figure out the true state in any case. Hence, it is sufficient to consider deviations from the equilibrium prices to show that the equilibrium candidate survives the D1 Criterion.

Lemma 8. *For every $p' \in \mathbb{R}_+$, there is a unique rationalizable strategy for each user resulting in $r \in \mathcal{R}^\infty(\Theta^*(p'), p')$ such that $\min_{r \in \mathcal{R}^\infty(\Theta^*(p'),p')} \pi(\theta, r, p') \leq \pi^*(\theta)$ for all $\theta \in \Theta$.*

Denote the equilibrium profit by $\pi^*(\theta) = (p^*(\theta) - c)n(\theta, p^*(\theta))$. Consider an arbitrary set of D1 types $\Theta^*(p')$ for a given out-of equilibrium price p'. Then no type has an incentive to deviate from the equilibrium, complete information price. Take price p' and type θ: If the mixed best response α given the set of D1 types results in entry larger than entry under complete information, users will leave once they join the platform. Demand in the last period is $n(\theta, p')$. On the other hand, less users than under complete information may enter depending on the set of D1 types. Therefore, $\pi(\theta, \alpha, p') \leq \pi(\theta, p') \leq \pi^*(\theta, p^*(\theta)), \forall \theta \in \Theta$, where the last inequality follows from the fact that $p^*(\theta)$ maximizes the complete information profit.

Separating equilibria, in which at least one type does not set the complete information price do not exist due to the results provided by Mailath (1987). Hence, the proposed separating equilibrium is the unique separating equilibrium outcome. Note that all assumptions needed for Mailath (1987) are fulfilled in this model. Furthermore, observe that the game with rational users essentially only involves one signal (price) for separation as fake profiles do not serve as a signal with rational users. Otherwise, the result by Mailath (1987) could not be applied.

Second, to ensure that this equilibrium in the unique outcome, I need to exclude pooling equilibria. Assume that some types pool at a given action. Then, it can be shown that the highest type pooled at a given action wants to separate from the lower types in the pool (see Proposition 2). The same argument applies here as well. As user entry is lower in a pooling equilibrium, which restricts the platform's demand in the third, the highest type has an incentive to deviate as in Proposition 2 to increase entry and demand. □

Proof Lemma 3

Proof. Given the timing of the game, users "update" their beliefs twice. In period 2, users observe the message adjust their belief to be equal to the message $\mu(\theta|p, m, r) = m$. In period 3, users observe the perceived amount of users $\tilde{n} = n + \xi$ and adjust their belief to $\mu(\theta|p, m, r, \tilde{n})$. In general, the platform profits from more users and a higher point-belief θ': $\frac{\partial \pi(\theta,\theta',p)}{\partial n} = p - c > 0$, $\frac{\partial \pi(\theta,\theta',p)}{\partial \theta'} = (p - c)\frac{\partial n(\theta,\theta',p)}{\partial \theta'} \geq 0$. Suppose $\mu(\theta|p, m, r)\theta'$ and

$\mu(\theta|p,m,r,\tilde{n}) = \theta''$. I will show that

$$\frac{\partial \pi(\theta,\theta'',p)}{\partial \theta''}|_{\theta'>\theta''} > 0, \quad \frac{\partial \pi(\theta,\theta'',p)}{\partial \theta''}|_{\theta'\leq\theta''} \leq 0.$$

After the users' entry decisions, the maximal amount of users on the platform is restricted. Denote the number of entered users by n^{en} given θ' and the number of users in equilibrium given θ'' in period 3 by n^*. Then, the amount of users that stay are $\min\{n^*, n^{en}\}$.

Given that the true state is θ and the believed state is θ', $n(\theta,\theta',p)$ enter

$$\text{(actual demand) } n(\theta,\theta',p) = 1 - F_\theta(p - \beta n(\theta',\theta',p)), \text{ where}$$
$$\text{(believed demand) } n(\theta',\theta',p) = 1 - F_{\theta'}(p - \beta n(\theta',\theta',p)).$$

Given that the true state is θ and the believed state is θ'', $n(\theta,\theta'',p) = n(\theta,\theta',p)$ stay if $\theta' \leq \theta''$ and $n(\theta,\theta'',p) < n(\theta,\theta',p)$ if $\theta' > \theta''$. In the first case, the utility of all users on the platform is positive. In the second case, $\frac{\partial n(\theta,\theta',p)}{\partial \theta'}$ is known from the previous analysis. Holding θ' fixed, the platform does not choose to induce $\theta' > \theta''$, as it would loose some costumers. If $\theta' \leq \theta''$, let the profit be

$$\Pi(\theta,\theta',\theta'',p) = (p-c)n(\theta,\theta',p) - \gamma(\xi(\theta'')),$$

where the amount of fake profiles depend on the believed state in period 3. Given θ', increasing θ'' has a negative effect. Hence, the platform chooses the corner solution and sets $\theta' = \theta''$. □

Proof of Theorem 2

Proof. The proof is as follows.

1) Consider the case in which costs γ are sufficiently small, i.e. $p^{FI}(\underline{\theta})n_2(\underline{\theta},\underline{\theta},p) - c > \gamma n_1(\underline{\theta},\underline{\theta},p)$.

First, I will show that the indifferent type is given as the unique solution to Equation 9. To get Equation (9), substitute first-order condition (7) into first-order condition (8). The indifferent type is then given by the following equation evaluated at $m = \overline{\theta}$

$$\left(-\frac{n(\theta,m,p)}{n_3(\theta,m,p)} + \gamma\frac{n_3(m,m,p)}{n_3(\theta,m,p)}\right) n_2(\theta,m,p) - \gamma\left(n_1(m,m,p) + n_2(m,m,p)\right)|_{m=\overline{\theta}} = 0,$$

which results in Equation (9). Substituting the respective derivatives into the equation above yields

$$\left(\frac{1-F_\theta(r)}{f_\theta(r)}(1-\beta f_\theta(r) + \gamma\frac{f_{\overline{\theta}}(r)}{f_\theta(r)}\frac{1-\beta f_\theta(r)}{1-\beta f_{\overline{\theta}}(r)}\right)\frac{f_\theta(r)(-\beta F^{1,\theta}(r))}{1-\beta f_\theta(r)} = \gamma\left((-F^{1,\overline{\theta}}(r)) + f_{\overline{\theta}}(r)\frac{(-\beta F^{1,\overline{\theta}}(r))}{1-\beta f_{\overline{\theta}}(r)}\right)$$

Note that all derivatives and demand itself are a function of the same r as the combination of (m,p) determines r. The difference lies only in the true type. Let $\Lambda(\overline{\theta}) = \frac{f_{\overline{\theta}}(r)}{1-\beta f_{\overline{\theta}}(r)}$, so that rearranging yields

$$1 - F_\theta(r) = \gamma\left[-\Lambda(\overline{\theta}) + (\frac{1}{\beta} + \Lambda(\overline{\theta}))\frac{F^{1,\overline{\theta}}(r)}{F^{1,\theta}(r)}\right] \tag{28}$$

Now the indifferent type $\tilde{\theta} \in \Theta$ is the solution to the above equation.

Lemma 9. *The solution to Equation 28 is unique if it exists. The unique solution is defined as* $\tilde{\theta} = \left\{\theta \in \Theta | 1 - F_\theta(r) = \gamma \left[-\Lambda(\overline{\theta}) + (1 + \Lambda(\overline{\theta}))\frac{F^{1,\overline{\theta}}(r)}{F^{1,\theta}(r)}\right]\right\}$*. The solution exists if* $F^{2,\theta} > 0$ *and*

$$\frac{1 - F_{\underline{\theta}}(r)}{-\Lambda(\overline{\theta}) + (1 + \Lambda(\overline{\theta}))\frac{F^{1,\overline{\theta}}(r)}{F^{1,\underline{\theta}}(r)}} \equiv \underline{\gamma} \leq \gamma \leq \overline{\gamma} \equiv 1 - F_{\overline{\theta}}(r(\overline{\theta}, p)).$$

First, recall that $1 - F_\theta(r)$ is a strictly increasing function in θ due to first-order stochastic dominance. In a graph that would plot θ against the left- and right-hand side of the equation, $1 - F_\theta(r)$ has a positive intercept and is concave (convex) if $F^{2,\theta} > (<)0$. The left-hand side has a negative intercept as $\Lambda(\overline{\theta})$ is strictly positive and does not depend on θ. If $F^{2,\theta}(r) > 0$, then $\frac{F^{1,\overline{\theta}}(r)}{F^{1,\theta}(r)}$ is increasing up to 1 at $\theta = \overline{\theta}$. If $F^{2,\theta}(r) < 0$, the right-hand side is strictly decreasing, and hence, always negative. In this case, the two functions never intersect and there is no solution.

Consider $F^{2,\theta} > 0$, if $\gamma = 1 - F_{\overline{\theta}}(r(\overline{\theta}, p))$ the highest type $\overline{\theta}$ is indifferent. All types below separate on their true types as messages and full information price. If $\gamma = \underline{\gamma}$, the indifferent type will be $\tilde{\theta} < \underline{\theta}$ and all types pool on the highest message.

If both existence conditions are met, the solution is unique as both sides of the equation are strictly increasing function. Furthermore, the left-hand side is larger than the right-hand side at $\underline{\theta}$ if $\gamma \geq \underline{\gamma}$. As the left-hand side is concave and right-hand side is either concave or convex, both functions intersect exactly once.

Off-path beliefs are defined as follows: 1) Users are also credulous off-path, and 2) take network size for face value. If network size is larger than the maximum of one (i.e. through fake profiles), users still take it for face value. This assumption could be justified by an exogenous shock on demand, however, the equilibrium characterization does not rely on this assumption.

Case a) Interior solution.

Let the indifferent type be $\tilde{\theta}$. For $\theta < \tilde{\theta}$ the equilibrium strategy is given by Equation (8), which is separating. Note that the relevant separating strategy is defined with respect to m as users are unaware of $\xi(\theta)$ and ignore p as a signal. That is, $m \neq m'$ for $\theta \neq \theta'$. Fixing p, incentive compatibility is fulfilled as $m = \arg\max \pi(\theta, m, p)$. The platform does not have an incentive to deviate off-path as for each θ it maximizes its profit.

For $\theta > \tilde{\theta}$, types set $m = \overline{\theta}$. This is pooling on the highest action. Types do not have an incentive to deviate off-path either. As users believe in the message upfront a maximum of $n(\overline{\theta}, \overline{\theta}, p)$ users joins the platform which restricts the possibility to improve off-path through fake profiles. Given that $n(\overline{\theta}, \overline{\theta}, p)$ enter, types act optimally. Even if the platform creates more fake profiles, not more users will enter and the platform incurs only additional cost.

Case b) Corner solution.

If γ is sufficiently small $\gamma < \underline{\gamma}$, or $F^{2,\theta} < 0$, the marginal benefit from creating fake profiles is always greater than its costs for all $\theta \in \Theta$. The last paragraph of the previous section applies analogously.

c) Uniqueness

The equilibrium is unique as the platform maximizes its profit in equilibrium given the restrictions it incurs. Furthermore, it is the unique profit maximium. The first-order condition 8 yields a unique optimal message and the first-order condition 7 yields a unique price. Other equilibria at different combinations of (m, p, ξ), which are non-profit maximizing, can be ruled out. The platform can copy off-path what it would do in the profit-maximizing equilibrium to overrule the equilbrium.

2) Suppose cost are sufficiently large, i.e. $\gamma > \overline{\gamma}$. All types set $m = \theta$ as well. This is separating and incentive-compatible as in the proof of Proposition 3. There are no incentives to deviate off-path either as types maximize their (unique) profit.

$\square$