Pig in a Poke*

Jana Gieselmann† Mikael Mäkimattila‡
Teemu Pekkarinen§

26 March 2025

Abstract

We consider a market in which sellers privately choose vertical product qualities, consumers receive information about the chosen qualities according to a predetermined information structure, and the sellers then compete à la Bertrand given consumers' posterior beliefs. We characterize the set of possible outcomes for different market and information structures. When there is a single monopoly seller, the seller-optimal information structure fully reveals the quality and incentivizes welfare-maximizing quality production. Under competition, the seller-optimal symmetric information structure is coarse and incentivizes randomization in quality choice. This leads to vertical differentiation in the buyer's posterior beliefs resulting in inefficient allocation and quality provision.

Keywords: Quality, Information Design, Monopoly, Bertrand Competition.

JEL: D43, D82, L15.

---

*For helpful comments, we are grateful to Bengt Holmström, Andrew Koh, Daniel Luo, Stephen Morris, Pauli Murto, Vitalii Tubdenov, Juuso Välimäki, and seminar participants at Annual Meeting of the Finnish Economic Association 2025, Nordic Economic Theory Conference 2024, Summer Seminar of Finnish Economists 2024, and Yale Theory Breakfast.

†Department of Economics, DICE, Heinrich-Heine University Düsseldorf, gieselmann@dice.hhu.de

‡Department of Economics, Aalto University and Helsinki Graduate School of Economics, mikael.makimattila@aalto.fi

§Department of Economics, University of Bonn, teemu@uni-bonn.de

1. INTRODUCTION

According to a story, farmers in medieval marketplaces would sometimes deceive buyers when selling valuable pigs. They would put a worthless cat in a bag, but the buyer would still pay a high price for the animal, believing that there was a pig in the poke. The buyer would only notice this deception once they returned home and let the cat out of the bag. The information available to consumers in the market—whether the pigs were sold in pokes—affected farmers' incentives to bring a pig or a cat to market in the first place.

In this paper, we build a model in which sellers privately choose the quality of their products, unobserved by a single buyer. The buyer instead receives information about product quality through a predetermined information structure, and the sellers compete on prices based on the buyer's posterior beliefs. We examine the range of possible market outcomes under various market and information structures, and characterize the seller-optimal outcome along with the supporting information structure.

We find that in a monopoly setting, the seller-optimal information structure fully reveals the quality and incentivizes welfare-maximizing quality production (Proposition 1). When two sellers compete, the seller-optimal symmetric information structure is coarse, encouraging sellers to randomize over quality choices (Propositions 2 and 3). This leads to vertical differentiation in the buyer's posterior beliefs and results in inefficient allocation. These findings sharply contrast with standard economic theory, which typically suggests that competition enhances both quality and efficiency.

In Section 3, we introduce the model of a market where a single buyer purchases a good from either one seller or two competing sellers. Sellers privately choose the quality of their goods—either high or low—and compete on prices. Producing higher quality entails a higher (opportunity) cost for the seller. The buyer forms beliefs about product quality in three stages: ex ante (before receiving any information), post-experiment (after receiving signals from an information structure), and post-pricing (after observing sellers' prices). The information structure is symmetric and independent across sellers. We look for a Perfect Bayesian Equilibrium.

We start by considering a market with a single seller in Section 4.1. In this market, the seller can extract the entire expected social surplus from trade by choosing a price equal to the buyer's expected valuation of the good. The surplus is maximized when a high-quality product is traded with certainty. If the information structure is sufficiently informative, the seller will be incentivized to sell the high-quality product as deviations to a low-quality product are detected. As a result, the seller-optimal information structure is transparent and incentivizes the provision of high quality, which is also socially efficient.

In Section 4.2, we analyze a market with two competing sellers. We begin with a benchmark in which sellers' quality choices are exogenously randomized. In this setting, we show that sellers earn the highest profits in equilibrium under a *perfect news–sufficient news* information structure. Here, the buyer either receives a perfect signal about quality ("perfect news") or holds the worst belief still compatible with trade ("sufficient news").

When quality is endogenously chosen, randomization in equilibrium arises only if sellers are indifferent between high and low quality. To induce such indifference, the information structure must strike a balance: the information structure cannot be either too informative (otherwise the sellers would always choose high quality) or too uninformative (otherwise the sellers would always choose low quality). In both extremes, price competition drives profits to zero.

As in the case of exogenous quality, a coarse information structure generates vertical differentiation in the buyer's posterior beliefs. By randomizing the quality of the product, sellers can profit in the following two ways. First, when quality is probabilistic, buyers may form different posterior beliefs about each seller's product, allowing sellers to engage in price discrimination. Second, when quality is randomized and the information structure is only partially informative, buyers might prefer a low-quality product over a high-quality one at the same price due to limited information. As a result, when sellers privately choose quality and the information structure is coarse, they can earn strictly positive profits. Compared to a monopoly under a fully informative information structure, competition in this setting leads to lower quality and reduced efficiency—especially when the information structure is designed to maximize seller profits.

2. LITERATURE

This paper contributes to the literature on Bayesian persuasion and information design (Kamenica and Gentzkow, 2011). Roesler and Szentes (2017) analyze a monopoly setting where a buyer faces uncertainty about the product's valuation, and identify the buyer-optimal signal structure. In a duopoly setting, Armstrong and Zhou (2022) study a model with two horizontally differentiated sellers, where the buyer's valuation of each product is uncertain. Given the buyer's prior and the information structure, the sellers set their prices without observing the buyer's signals. Their analysis solves both buyer-optimal and seller-optimal information structures and shows that the seller-optimal information structure amplifies differentiation of the products. Our work differs in that we focus on a setting with endogenous vertical quality (valuation) and associated moral hazard concerns. As in Roesler and Szentes (2017), trade in the buyer-optimal information structure remains efficient in our model in both the monopoly and duopoly case. Similarly to Armstrong and Zhou (2022), the seller-optimal information structure is coarse in our model, but incentivizes randomization of the seller's quality provision.

Zapechelnyuk (2020) considers a setting where a monopolist chooses both quality and price, while a regulator (maximizing consumer surplus) commits to a rule for disclosing quality information to buyers. Our analysis continues from Zapechelnyuk's work by considering competitive interactions between sellers in the presence of endogenous quality choices.

There is a growing literature on information design with endogenous priors. Boleslavsky and Kim (2018) study a moral hazard setting in which a sender must simultaneously incentivize an agent's effort and persuade a receiver. Similarly, Rosar (2017) analyzes a test design problem where agents participate voluntarily, and the principal learns about their quality through the test outcomes. In this paper, we introduce information design with endogenous priors in a canonical Bertrand competition model with quality provision.

More broadly, within the information design literature, authors such as Bergemann et al. (2015) and Elliott et al. (2022) have examined models in which sellers, either monopolistic or competitive, are uncertain about the buyer's valuation. These studies complement our analysis, as we explore how different information structures influence market outcomes when *buyers* are uncertain and sellers strategically choose both quality and pricing.

3. MODEL

Consider a single buyer and at maximum two competing sellers. Each seller $i \in \{A, B\}$ privately chooses quality $q_i \in Q := \{0, 1\}$ of the good she takes to the market. Her (opportunity) cost of selling this good is $c \cdot q_i$ for some parameter $c \in (0, 1)$. The buyer has a unit demand and values a good with quality $q_i \in Q$ at q_i. The buyer receives information about the quality of the good through an experiment. After the experiment, the sellers compete in prices. The timing of the game is as follows.

Ex-Ante Stage At the ex-ante stage, sellers privately choose the quality of their good. The buyer's ex-ante belief that the quality of seller i is high, $q_i = 1$, is $\mu_0 \in [0, 1]$. The buyer's ex-ante belief is endogenously determined in the equilibrium.[1]

[1]Throughout the paper, we will focus on symmetric equilibria, so that μ_0 captures the buyer's ex-ante belief about each seller's good.

Experiment Stage The buyer receives information about the quality of the goods through an experiment. The buyer's post-experiment belief that the quality of seller i's good is high, $q_i = 1$, is denoted by $\mu_i \in [0, 1]$. The sellers observe the buyer's post-experiment beliefs.

An information structure is a pair of conditional post-experiment belief distributions, (F^0, F^1), where F^q is the distribution function of μ_i conditional on the true quality $q \in \{0, 1\}$ of seller $i \in \{A, B\}$. We restrict our attention to symmetric and independent information structures, so (F^0, F^1) captures all relevant information about the information structure.[2] For the information structure to be consistent with the ex-ante beliefs, the following condition must hold:

$$\mathbb{E}_{\mu_i \sim F}[\mu_i] = \int_0^1 \mu_i dF(\mu_i) = \mu_0,$$

where $F(\mu_i) := \mu_0 F^1(\mu_i) + (1 - \mu_0)F^0(\mu_i)$ is the unconditional distribution of the post-experiment belief. This is called Bayes plausibility by Kamenica and Gentzkow (2011).

Furthermore, each post-experiment belief μ_i follows from Bayes' rule. For example, if $\mu_0 \in (0, 1)$ and μ_i are distributed according to conditional densities (f^0, f^1) with full support, this requires

$$\mu_i = \frac{\mu_0 f^1(\mu_i)}{\mu_0 f^1(\mu_i) + (1 - \mu_0) f^0(\mu_i)}.$$

Pricing Stage After observing the post-experiment beliefs, the sellers engage in Bertrand competition to sell their goods by simultaneously choosing prices (p_A, p_B).

Purchase Stage After the sellers have chosen their prices, the buyer forms her post-pricing beliefs (μ_A^P, μ_B^P) and makes choices $x_i \in \{0, 1\}$ for $i \in \{A, B\}$ whether or not to buy a good from seller i. The ex-post payoffs of seller i and the buyer are, respectively,

$$\pi_i^S = (p_i - q_i c)x_i \qquad \text{and} \qquad u^B = \sum_i (q_i - p_i)x_i.$$

Equilibrium Concept The solution concept is perfect Bayesian equilibrium (PBE). We apply two refinements to the set of PBE.

[2] That is, random variables μ_A and μ_B are independent and identically distributed.

Definition 1 (**PBE with minimally informative prices**)**.** A profile of strategies and system of beliefs is a (weak) perfect Bayesian equilibrium with minimally informative prices if it satisfies the following:

(i) *(Perfect Bayesian Equilibrium)* The strategy profile is sequentially rational given the system of beliefs, and the system of beliefs is derived from the strategy profile using Bayes' rule whenever possible.

(ii) *(Cautiousness)* No seller chooses a price such that if the buyer were to buy her good at that price, the seller would make a negative profit.

(iii) *(Minimally Informative Prices)* The post-pricing beliefs coincide with the post-experiment beliefs for all prices $p \geq c$.

First, *PBE* dictates that the buyer's belief at each stage is consistent with equilibrium strategies.

Second, *cautiousness* requires that no seller chooses a price such that if the buyer were to buy her good at that price, the seller would make a negative profit. This criterion is standard in Bertrand games with heterogeneous costs or qualities; it rules out unreasonable equilibria in which a seller prices below her marginal cost since she knows she is not going to make a sale anyway. We call the criterion "cautiousness" following Bergemann and Välimäki (1996).

Third, the requirement of *minimally informative prices* rules out some of the equilibria where the buyer's off-path beliefs would dictate the equilibrium prices in an arbitrary way. In this way, we can concentrate on equilibria in which Bertrand competition works with minimal price signaling effects. For example, without the refinement, we could have equilibria in which after observing any other price than c, the buyer would believe that the seller has a low-quality good for sure. This could happen regardless of what the buyer learns from the experiment, and the buyer's off-path beliefs would then dictate that there could not be a sale in equilibrium with prices different than c or 0.

Discussion on Information Structures Let us elaborate on our concept of information structures. In information design models, experiments can be represented in two ways: either as a pair of a signal space and a mapping from the state space to the signal space, or as a distribution of posterior beliefs. We adopt the latter approach. When the ex-ante belief is extreme, $\mu_0 \in \{0, 1\}$, our notion of an information structure also captures the buyer's belief when he observes a signal that could only happen if a seller deviated from her equilibrium quality.

To further clarify the concept, we distinguish two experiments of special interest. The first one, a fully informative experiment, perfectly reveals the quality, as if the game were one of perfect information.

Definition 2 (**Fully informative experiment**). Experiment (F^0, F^1) is called *fully informative* if

$$F^0(\mu_i) = 1 \text{ for all } \mu_i \in [0,1], \qquad F^1(\mu_i) = \begin{cases} 0, & \text{for } \mu_i < 1, \\ 1, & \text{for } \mu_i = 1. \end{cases}$$

The natural counterpart of a fully informative experiment is a fully uninformative experiment. A fully uninformative experiment never changes the buyer's belief, so the game is as if there were no experiment at all.

Definition 3 (**Fully uninformative experiment**). Experiment (F^0, F^1) is called fully uninformative if $F^0 \equiv F^1$.

We call an information structure partially informative or coarse if it is neither fully informative nor fully uninformative.

Lastly, we assume that sellers are able to observe the buyer's post-experiment beliefs. This assumption is particularly suitable in applications where buyers obtain information through certification processes.

4. ANALYSIS

In this section, we first analyze the single seller model. Then we move to a model with two competing sellers. In both models we analyze two cases: (i) quality is exogenously given and (ii) quality is endogenously chosen by the seller(s).

4.1 SINGLE SELLER

Let us start by considering the situation of a single seller in the market, say, seller A. The seller sets her price after observing the realized post-experiment belief of the buyer μ_A.

First, consider pricing in equilibrium if a post-experiment belief $\mu_A \geq c$ is realized such that the seller can set a price above c and still make a sale. As the buyer is willing to buy the good as long as $\mu_A - p_A \geq 0$ and the seller's profit is strictly increasing in p_A, the seller sets price $p_A = \mu_A$ in equilibrium to make the buyer indifferent.[3] Given our equilibrium concept with minimally

[3]We assume that the buyer always makes the purchase when indifferent between purchasing or not.

informative prices, $p_A = \mu_A$ is the greatest price at which the buyer is still willing to buy the good. For prices higher than μ_A in equilibrium, the buyer is not willing to purchase the good and the seller makes no sale. For a price below μ_A in equilibrium, the seller would have a strict incentive to deviate to $p_A = \mu_A$ as the buyer's belief is invariant to the change in price.

Next, consider pricing in equilibrium if post-experiment belief $\mu_A < c$ is realized. The seller cannot make a sale at a price greater than c. This is due to an adverse selection problem: only a low-quality seller would be willing to sell at a price less than c. Setting a price less than c would reveal to the buyer that the seller has a low-quality good, and the buyer would not be willing to buy the good. This implies that a high-quality good cannot be traded in the market and there cannot be a sale at a positive price.[4] The seller's expected profit after a posterior realization below c is then 0.

Exogenous Quality For the sake of comparison, we will thoroughly analyze a situation in which quality is exogenously given: the quality is high with exogenous prior probability μ_0, and the seller privately observes it but cannot affect it. Suppose the buyer's post-experiment belief μ_A exceeds c. The seller sets price $p_A = \mu_A$ in equilibrium and the buyer buys with probability one. The expected cost of the seller is $\mu_A c$ and the expected profit is $\mu_A - \mu_A c$. Suppose that the post-experiment belief μ_A is below c. This implies that there is no trade at a positive price and the seller's expected profit is zero.

The expected profit of the seller as a function of the posterior realization μ_A is $\mathbb{1}_{\mu_A \geq c}\mu_A(1-c)$ as depicted in Figure 1. Given ex-ante belief μ_0, the expected profit of the seller is a convex combination of the seller's profit for different post-experiment belief realizations. For prior $\mu_0 \geq c$, a fully informative experiment and a fully uninformative experiment yield the same profit in expectation: $\mu_0(1-c)$. For prior $\mu_0 < c$, the experiment must be sufficiently informative for the seller to make a positive profit. For example, if the experiment is fully uninformative, the buyer's willingness to pay is lower than any price at which a high-quality seller would be willing to sell, so the market unravels.

As can be seen in Figure 1, the seller's expected profit is maximized at a post-experiment belief of $\mu_A = 1$. Therefore, if quality is exogenous, the seller obtains the maximum profit if quality is always high and thus, the buyer's post-experiment belief is equal to one.

[4] After post-experiment belief realization $\mu_A < c$, pricing in equilibrium can be as follows: the seller sets price $p_A = c$ regardless of her quality. The buyer's post-pricing belief is $\mu_A^P = \mu_A$ after any price $p_A \geq c$ and $\mu_A^P = 0$ after any price $p_A < c$. In equilibrium, the buyer always rejects trade, $x_A = 0$.

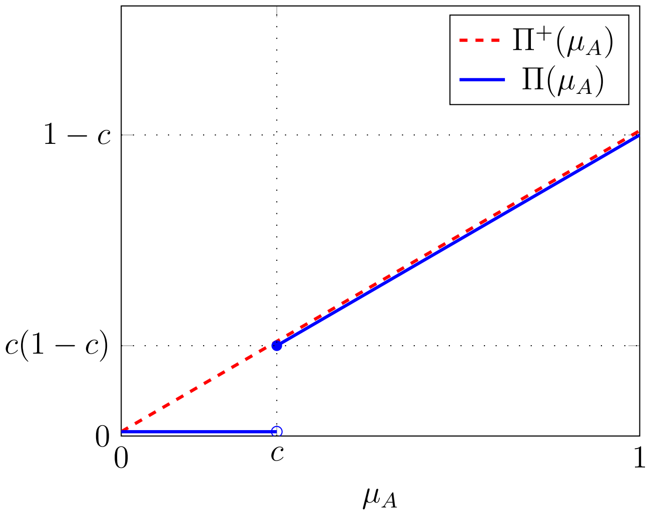

Figure 1: Single seller: Expected profit $\Pi(\mu_A)$ as a function of posterior μ_A and the concave closure $\Pi^+(\mu_A)$.

Endogenous Quality Next, we study the equilibrium outcome when quality is endogenously chosen by the seller. We ask first: which information structures (F^0, F^1) induce the seller to always produce high quality? Suppose the seller produces high quality with probability one. Bayes plausibility requires

$$F^1(\mu_A) = \begin{cases} 0, & \text{for } \mu_A < 1, \\ 1, & \text{for } \mu_A = 1. \end{cases} \tag{1}$$

Bayes plausibility does not restrict the selection of F^0, but to induce good quality in the equilibrium, the information structure must be such that the payoff from choosing high quality, $1-c$, exceeds the expected payoff from deviating to low quality, that is,

$$1 - c \geq \int_c^1 \mu_A dF^0(\mu_A). \tag{2}$$

The expected payoff from deviating to low quality is determined by the buyer's valuation μ_A minus the seller's opportunity cost, which is zero if low quality is produced.

Proposition 1
With a single seller, the seller makes the largest profit in an equilibrium in which high quality is produced and trade takes place with probability 1. This equilibrium is supported by an information structure that satisfies (1) *and* (2).

As the social surplus is largest if the seller produces high quality and the seller extracts the entire social surplus, the seller makes the largest profit if high quality is produced. An equilibrium in which high quality is produced exists if the information structure satisfies conditions (1) and (2). One information structure that satisfies conditions (1) and (2) is the fully informative one. A fully informative experiment perfectly reveals the quality, so that $\mu_A = 0$ if low quality is chosen. The expression then simplifies to $1 - c \geq 0$, which is fulfilled by assumption.

To complete the analysis, we ask which information structures (F^0, F^1) induce the seller to always produce low quality or be indifferent between producing high and low quality. If the seller always produces low quality, Bayes plausibility requires $F^0(\mu_A) = 1$ for all $\mu_A \geq 0$. Bayes plausibility does not restrict the conditional distribution F^1. Low quality is produced in equilibrium if the payoff from choosing low quality, 0, is greater or equal to the expected payoff from choosing high quality

$$0 \geq \int_c^1 \mu_A(1-c)dF^1(\mu_A).$$

To satisfy the condition, the probability of a posterior $\mu_A \geq c$ realizing conditional on high quality must be zero. For instance, a fully uninformative information structure supports an equilibrium in which low quality is always chosen.

Lastly, let us analyze which information structures (F^0, F^1) make the seller indifferent between producing high or low quality and can therefore endogenize an interior ex-ante belief $\mu_0 \in (0,1)$. In this case, high quality should yield the same expected payoff to the seller as low quality,

$$\int_c^1 (\mu_A - c)dF^1(\mu_A) = \int_c^1 \mu_A dF^0(\mu_A).$$

The left-hand side is equal to the seller's expected payoff from high quality and the right-hand side is the seller's expected payoff from low quality. Note that the unconditional distribution $F := \mu_0 F^1 + (1-\mu_0)F^0$ pins down conditional distributions F^1 and F^0 when $\mu_0 \in (0,1)$. For the seller to be indifferent between producing high and low quality, the experiment must be sufficiently informative so that high quality leads to a more favorable posterior distribution that compensates for the higher cost. The experiment cannot be too informative; e.g., if the experiment is fully informative, it is more profitable to choose high quality.

4.2 TWO SELLERS

In this section, we turn to the model in which two sellers compete in the market. First, we derive the seller-optimal equilibrium and information structure when quality is exogenous and then proceed to the endogenous quality model. Recall that we restrict the analysis to symmetric equilibria as well as symmetric and independent information structures.[5]

Both sellers compete and set their prices after observing the realized post-experiment beliefs. If the post-experiment beliefs are such that $\mu_i > \mu_j \geq c$, the sellers' goods are perceived to be vertically differentiated. Hence, seller i can charge a premium of $\mu_i - \mu_j$ over seller j's price. In equilibrium, seller i sets price $\mu_i - \mu_j + c$ and makes the sale, earning an expected profit of $\mu_i(1 - c) - \mu_j + c$, while seller j sets price c and earns no profit. As in the single seller case, no seller sets a price below marginal cost of the high-quality product, as this would reveal that the seller's quality is low. If $\mu_i = \mu_j \geq c$, the sellers' goods are perceived to have the same quality in expectation. In this case, sellers compete à la Bertrand to prices equal to marginal cost of the high quality good and each seller makes *positive* expected profit. The sum of sellers' expected profits is equal to $c(1 - c)$.

If $\mu_i \geq c > \mu_j$, the buyer's expected valuation for the good of seller j is lower than any price at which a high-quality seller would be willing to sell, so seller i acts as a monopolist in the market and charges the monopoly price. If both posterior beliefs are below marginal cost, $c > \mu_i > \mu_j$, no seller makes a sale at a positive price.

4.2.1 Exogenous Quality

Suppose the quality of each seller is high with exogenous probability μ_0 independent of the other seller's quality. Each seller privately observes her quality but cannot affect it. Let $f_{\hat{\mu}} := \Pr(\mu_i = \hat{\mu})$ denote the ex-ante probability that the buyer's post-experiment belief μ_i is $\hat{\mu}$. The next proposition characterizes the seller optimal information structure for any given probability μ_0.

Proposition 2
Assume that there are two sellers and quality is exogenous with prior μ_0. In any optimal information structure, $\mu_i \in \{0, c, 1\}$ with probability one and

(i) if $\mu_0 \leq \frac{1}{2-c}$, then

$$(f_0, f_c, f_1) = (1 - \mu_0(2 - c), \mu_0, (1 - c)\mu_0)\,, \ \textit{and}$$

[5] Note that the seller-optimal equilibrium and information structure refer to the equilibrium in which the sellers make the highest profits among the class of symmetric equilibria with symmetric and independent information structures.

(ii) if $\mu_0 > \frac{1}{2-c}$, then

$$(f_0, f_c, f_1) = \left(0, \frac{1-\mu_0}{1-c}, \frac{\mu_0 - c}{1-c}\right).$$

Given any exogenous prior μ_0, Proposition 2 shows that the seller-optimal Bayes-plausible information structure has mass only on post-experiment beliefs $\mu_i \in \{0, c, 1\}$. We call this a **perfect news–sufficient news** information structure: either the buyer learns a seller's quality perfectly, corresponding to beliefs $\mu_i \in \{0, 1\}$, or she gets "sufficient news" $\mu_i = c$—sufficient in the sense that $\mu_i = c$ is the lowest belief such that seller i can still make a sale. Therefore, with a *sufficient* belief c and a price offer c, there is still trade in equilibrium. Consequently, the buyer's belief matches the lowest possible price that a high-quality seller could ever ask for.

For any $c \in (0, 1)$, the seller-optimal information structure is partially informative—it is neither fully informative nor fully uninformative. The sellers' equilibrium pricing and profits are the following. If the realized post-experiment belief about a seller's good is zero, the seller makes zero profit. If the realized post-experiment belief is c, the seller posts a price of c. In this case, the seller makes the sale if the realized posterior of the other seller is zero. The sellers share the market if both post-experiment beliefs are c. Even though the sellers are not differentiated in the latter case, they make a positive profit in expectation: as lowering the price below c leads the buyer to believe that the quality of the good is low, it allows a low-quality seller to price above true cost. If the realized post-experiment belief is one and zero or c for the other seller, the seller posts price one and makes positive profit. Note that in this case, the seller's good is of high quality with probability one. If both sellers have a post-experiment belief realization of one, they compete to marginal costs c and make zero profit.

Figure 2 illustrates the proof of Proposition 2. The blue curves show the expected sum of sellers' profits $\Pi_{\mu_j}(\mu_i)$ as a function of seller i's post-experiment belief μ_i (horizontal axis) for given seller j's belief realization μ_j.

The graph on the left shows the case in which $\mu_j \geq c$. If $\mu_i < c$, the belief about the quality of seller i's good is so low that the market equilibrium is as if seller j had monopoly power. Seller j sells at price μ_j, and makes expected profit $\mu_j(1-c)$. If $\mu_i \geq c$, seller i competes with seller j, and the amount of (perceived) vertical differentiation determines how much profit the sellers make in total.

The graph on the right shows the case in which $\mu_j < c$. The buyer would never buy the good from seller j, so seller j does not bring competition to the market. The belief about the quality of seller j's good is so low that the

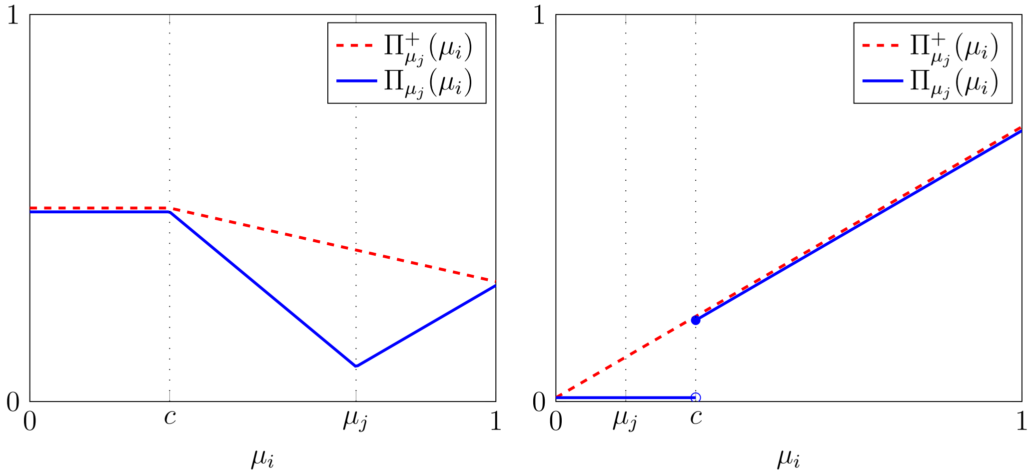

Figure 2: The expected sum of profits Π_{μ_j} for the two sellers as a function of posterior realization μ_i given posterior realization $\mu_j \geq c$ (left-hand side) and $\mu_j < c$ (right-hand side). $\Pi^+_{\mu_j}$ is the concave closure of Π_{μ_j}.

situation is as if seller i has monopoly power. Thus, the profit of seller i in Figure 2 compares to Figure 1.

As the blue curve Π_{μ_j} is convex on interval $[0, c]$, a mean-preserving spread of post-experiment beliefs on open interval $(0, c)$ to the end points of the interval always increases the expected sum of sellers profits. Similarly, a mean-preserving spread of post-experiment beliefs on open interval $(c, 1)$ to the end points of the interval always increases the expected sum of sellers profits. Since this is true for any μ_j with $i, j \in \{A, B\}$, restricting the posterior support to $\{0, c, 1\}$ is always optimal for the sellers for any exogenous prior μ_0. Finding the seller-optimal information structure is then reduced to a simple program of maximizing the sellers profit subject to choosing the masses on beliefs zero, c and one under the condition that Bayes plausibility is satisfied (see Appendix).

To maximize total surplus in the exogenous quality case, the buyer must buy the high quality good whenever at least one seller offers it. If there is a positive probability that the buyer is not certain about the sellers' qualities, there is also a positive probability that a low quality good is traded in the market even though a high quality good would be available.

Remark 1. With two sellers and exogenous quality, an information structure that maximizes total surplus must have $\mu_i \in \{0, 1\}$ with probability 1.

By Proposition 2 and Remark 1, the socially optimal information structure and seller-optimal information structure only coincide in special cases $\mu_0 = 0$

and $\mu_0 = 1$. This is different from the single seller case where the seller-optimal information structure is socially efficient. To understand why the seller-optimal information structure with two sellers has positive mass of c (sufficient news), consider the following argument. When the buyer remains uncertain about the qualities and only receives sufficient news, the sellers make a positive profit in expectation because a costless low-quality good can be traded in the market even if a high-quality good is available. Here, information asymmetry prevents competition from driving the low-quality sellers' profits to zero, as they understand that decreasing their price would reveal their low quality.

4.2.2 Endogenous Quality

In this part of the analysis, we study the implications of allowing sellers to choose their qualities privately, making the prior μ_0 endogenous. We show that in the seller-optimal equilibrium, the sellers randomize their qualities, which results in strictly positive profits. To induce randomization in qualities, the information structure must make the sellers indifferent between choosing high or low quality. If sellers do not randomize their quality, the buyer will always have beliefs $\mu_A = \mu_B = 1$ or $\mu_A = \mu_B = 0$ and consequently, price competition will drive the sellers' profits to zero.

Proposition 3
Assume that the quality is chosen by the sellers. In equilibrium, sellers make strictly positive profits for all $c \in (0, 1)$ if and only if the quality is probabilistic. For each $c \in (0, 1)$, there exists an information structure such that the sellers randomize quality in equilibrium. Such an information structure is partially informative.

By randomizing the quality, the sellers can make profits in the following two ways. First, when quality is probabilistic, it is possible that the buyer receives different posterior beliefs about the sellers' qualities, and hence the sellers can price discriminate. Second, when the sellers randomize quality and the information structure is only partially informative, low-quality sellers can make profits when the buyer receives the same posterior beliefs about the sellers' goods. In other words, it is possible that a seller sells a low-quality good at a positive price, even though there is a high-quality good for sale as well. This makes the allocation inefficient, in general.

Example 1. Fix the information structure to be the perfect news–sufficient news. When the probability of sufficient news is $(1-c)\left(1+\frac{\sqrt{1+c^2}-1}{c}\right)$ independent of the true quality, the sellers randomize quality in equilibrium and make a strictly positive profit for all $c \in (0, 1)$ (shown in the proof of Proposition 3). In addition, the outcome is inefficient since a high-quality product is not traded with certainty.

If we compare these observations to the monopoly case, where the monopolist always offers the high-quality good and the buyer accepts the offer, we observe that competition (increasing the number of sellers to two) actually decreases quality and efficiency in a market whose information structure is designed to maximize profit.

Remark 2. When the quality is chosen privately by the sellers and the information structure maximizes sellers' profit, competition decreases the overall quality in the market, resulting in inefficiency.

This insight is in stark contrast with the conventional wisdom that competition improves quality. In our model, price competition drives the profits of selling homogeneous goods of known quality to zero, and consequently the sellers find it profitable to conceal the quality of their goods in order to differentiate their products from each other. The differentiation of the goods can be done only by randomizing the quality, which further makes it possible to generate differentiated expectations of the buyer's valuation of the goods. If the choice of quality is not random in equilibrium, an information structure that satisfies Bayes plausibility cannot generate uncertain posterior beliefs.

Since endogenous quality creates inefficiency in trade, sellers cannot get maximal surplus from the trade. The following lemma gives an upper bound for the sellers' profits. The idea of the lemma is to look at a situation in which the sellers' quality is exogenous, but the exogenous probability of high quality μ_0 is optimized for the sellers together with the information structure.

Lemma 1
Suppose that the sellers can commit to a (symmetric, independent) quality distribution that maximizes their total profits. The uniquely optimal information structure is such that the support of the posterior beliefs is $\{c, 1\}$ and its associated probabilities are

$$(f_c, f_1) = \left(\frac{1}{2} + \frac{c}{4}, \frac{1}{2} - \frac{c}{4}\right)$$

with the optimal probability of high quality at

$$\mu_0 = \frac{1}{2} + \frac{c}{4} + \frac{c^2}{4}.$$

The profits conditional on having a high-quality or a low-quality good are

$$\pi^1 = \frac{(1-c)(2-c)(2+c)}{4(c^2+c+2)} \qquad \text{and} \qquad \pi^0 = \left(\frac{1}{2} + \frac{c}{4}\right)\frac{c}{2}, \tag{3}$$

respectively, resulting in the expected profits of

$$\pi = \mu_0 \pi^1 + (1-\mu_0)\pi^0 = \frac{1}{32}(1-c)(2+c)(4+c^2). \tag{4}$$

This lemma gives an upper bound for the two sellers' total equilibrium profit with endogenous quality, as Lemma 1 solves a relaxed version of the full endogenous problem: a problem in which interior $\mu_0 \in (0,1)$ can be achieved without the expected profit from high and low quality being equal.

The seller-optimal information structure in Lemma 1 either fully discloses the high-quality good or gives a noisy signal about quality. Full disclosure has a smaller probability than the noisy signal ex ante. This is due to competitive motives: a seller makes a profit only if she is the only one who has the fully revealing signal or if she has a low quality good and can sell it at a price c.

In Figure 3, we plot the profit from a low-quality good and high-quality good as well as the expected profit given the optimal information structure of Lemma 1. The expected profits are decreasing in c. With a low c, the expected profits of the sellers conditional on having a high-quality good are high. If cost c is high, the expected profit of a seller conditional on having a low-quality good is relatively high. This implies that, given the information structure of Lemma 1 but without commitment, at low cost c, the sellers would like to deviate to high quality instead of randomizing. At high cost of quality, they would like to deviate to low quality instead of randomizing.

We can find a cost level c^* such that the profits from high and low quality coincide. At this point, the sellers are indifferent between producing high- or low-quality. Therefore, at cost level c^*, the solution to the problem of finding the optimal exogenous μ_0 together with the information structure satisfies the indifference condition and therefore gives also the optimal equilibrium with endogenous quality. However, this is the case for only a single value of cost.

Proposition 4
The sellers' profits with endogenous quality can achieve the upper bound given by expression (4) in Lemma 1 only for a single value of cost c.

This result implies that the perfect news–sufficient news information structure in Lemma 1 is hardly ever the seller-optimal information structure with endogenous quality. At low levels of c, perfect news comes with too high a probability for the sellers to ever be willing to choose low quality. At high

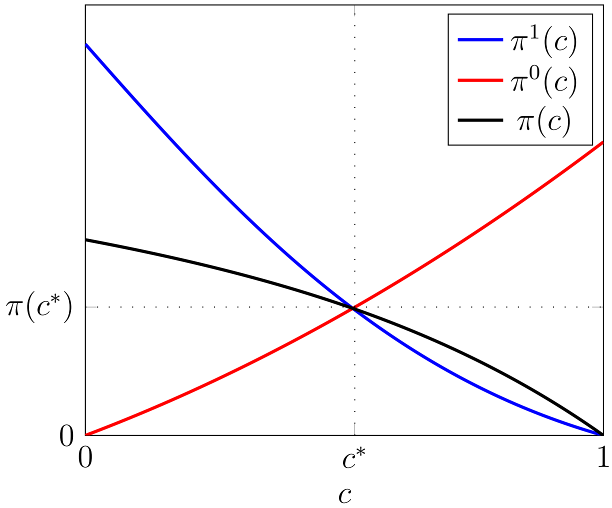

Figure 3: Expected profits π, profits conditional of having a high-quality good π^1, and profits conditional of having a low-quality good π^0 as functions of costs c in the optimal information structure of Lemma 1.

levels of c, perfect news comes with too low a probability for the sellers to ever be willing to choose high quality. The indifference condition changes the optimal information structure in almost all cases, and the sellers would benefit from committing to a certain (optimal) quality randomization.

5. CONCLUSION

Our article provides insights into how the interaction between information and market structure shapes prices and quality provision in a market. When focusing on equilibria in which sellers make the largest profits, we show that while quality provision in a monopolistic market is socially optimal, it is suboptimal in a competitive market. A monopoly seller maximizes profit when the information structure fully reveals product quality, leading to the production of high-quality goods. In contrast, competing sellers achieve their highest profits when the information structure is coarse, incentivizing randomization in quality choice.

Our research connects to the regulation and functioning of marketplaces. If a regulator oversees a marketplace where sellers offer products to buyers, they may want to design the information structure to achieve welfare-maximizing quality provision, which may require high transparency. However, if the marketplace is operated by a profit-maximizing organization that charges commissions from sellers, it has an incentive to structure the information in a way that maximizes seller profits. In this case, with competing sellers, quality provision is not welfare-maximizing. Examples include online marketplaces and wholesale markets, such as agricultural markets. In particular, online platforms have access to granular information and control over how it is disseminated to buyers.

Additionally, our theory can explain why certification is often incomplete or does not fully guarantee quality in many markets. Competition may naturally lead to certification practices that obscure product quality rather than fully guarantee it. Incomplete information, in this context, benefits competing sellers.

APPENDIX: PROOFS

PROOF OF PROPOSITION 2

Proof. Take arbitrary unconditional cumulative distribution function of posteriors F and say that μ_A, μ_B are each (independently) distributed according to F.

Given posterior realization μ_j for seller j, let us write the expected sum of profits for the two sellers as a function of posterior realization μ_i for seller $i \neq j$:

$$\Pi_{\mu_j}(\mu_i) = \mathbb{1}_{\max\{\mu_A,\mu_B\}\geq c} \max\{\mu_A, \mu_B\}(1-c) - \max\{0, \min\{\mu_A, \mu_B\} - c\}.$$

For any fixed μ_j, $\Pi_{\mu_j}(\mu_i)$ is convex in μ_i on interval $[0, c]$, as it is constant on $[0, c)$ and $\Pi_{\mu_j}(c) \geq \lim_{\mu_i \to c^-} \Pi_{\mu_j}(\mu_i)$. Furthermore, $\Pi_{\mu_j}(\mu_i)$ is convex on interval $[c, 1]$, as it is linearly decreasing on $[c, \max\{c, \mu_j\}]$ and linearly increasing on $[\max\{c, \mu_j\}, 1]$. Then also $\mathbb{E}_{\mu_j \sim F}[\Pi_{\mu_j}(\mu_i)]$ is convex in μ_i on intervals $[0, c]$ and $[c, 1]$ as a (weighted) sum of convex functions is also convex.

Consequently, following Kamenica and Gentzkow (2011), a mean-preserving spread of posteriors $\mu_i \in (0, c)$ (performed independently of μ_j) to the end points of the interval weakly increases the sellers' total expected profit. Similarly, a mean-preserving spread of posteriors $\mu_i \in (c, 1)$ to the end points of the interval weakly increases the sellers' total profit. Furthermore, if the original information structure F assigns a positive probability to posterior realizations on open intervals $(0, c)$ and $(c, 1)$, these mean-preserving spreads strictly increase the sellers' total profit. Analogous mean-preserving spreads of posteriors μ_j also weakly increase the sellers' total expected profit. This procedure produces a new (symmetric, independent and mean-preserving) information structure with support $\{0, c, 1\}$ that produces a weakly greater profit for the sellers than the original information structure (strictly greater if the original information structure F assigns a positive probability to posterior realizations on open intervals $(0, c)$ and $(c, 1)$). Hence, the sellers' profit is maximized with an information structure such that $\mu_i \in \{0, c, 1\}$ with probability 1 for both $i \in \{A, B\}$.

Letting $f_{\hat{\mu}} := \Pr[\mu_i = \hat{\mu}]$ denote the ex-ante probability that the buyer's posterior about the good of any seller i is $\hat{\mu}$, the problem of finding the seller-optimal information structure has now been reduced to one of finding f_0, f_c and f_1 to maximize the sellers' expected profit:

$$\max_{f_0, f_c, f_1 \in [0,1]} (f_c^2 + 2f_0 f_c)c(1-c) + 2f_1(1-f_1)(1-c) \tag{5}$$

$$\text{s.t. } \mu_0 = f_c c + f_1 \tag{6}$$

$$f_0 + f_c + f_1 = 1 \tag{7}$$

Note that if $\mu_A = \mu_B = 0$ or $\mu_A = \mu_B = 1$, the sellers do not make profit. They make positive profit if $\max\{\mu_A, \mu_B\} = c$, which happens with probability $f_c^2 + 2f_0 f_c$ and in which case the expected profit is $c(1-c)$. They also make positive profit if $\mu_i = 1$ and $\mu_j < 1$ for $i, j \in \{A, B\}$, $i \neq j$, which happens with probability $2f_1(1-f_1)$ and in which case the expected profit is $1-c$. Therefore, the sellers' total expected profit given f_0, f_c and f_1 is given by (5). Constraint (6) is a Bayes plausibility constraint and constraint (7) is a feasibility constraint; they can be plugged to the objective to make f_c the only decision variable:

$$\max_{f_c \in \mathbb{R}} (1-c)\Big((2\mu_0 - f_c)c f_c + 2(1-\mu_0)\mu_0\Big) \tag{8}$$

$$\text{s.t. } f_c \geq 0 \tag{9}$$

$$f_c \leq \frac{\mu_0}{c}, \tag{10}$$

$$f_c \leq \frac{1-\mu_0}{1-c} \tag{11}$$

Here, the constraints ensure that the solution satisfies $f_0, f_c, f_1 \in [0, 1]$. If the constraints are ignored, the unique solution to the problem is $f_c = \mu_0$. This solution satisfies the constraints if $\mu_0 \leq \frac{1}{2-c}$; otherwise it is uniquely optimal to set $f_c = \frac{1-\mu_0}{1-c}$. Given f_c, we can then find f_c and f_1 from (6) and (7) to obtain Proposition 2. □

PROOF OF PROPOSITION 3

Suppose that the quality of the goods is endogenous choice of the sellers, and the information structure has the following three-point support: $\{0, c, 1\}$. The seller-optimal information structure is given by the following maximization problem:

$$\max_{\boldsymbol{f}\in\Delta(S)} \mu_0\pi^P + (1-\mu_0)\pi^C$$

such that

$$\pi^P = \pi^C \tag{I}$$

and

$$\mu_0 = cf_c + f_1, \tag{BP}$$

where

$$\pi^P = \frac{f_1}{\mu_0}(f_0+f_c)(1-c) \qquad \text{and} \qquad \pi^C = \frac{1-c}{1-\mu_0}\left[(f_0+f_c)^2 - f_0^2\right]\frac{c}{2}.$$

The condition (I) is the indifference condition that ensures that the sellers do not have a profitable deviation from randomizing their quality, and condition (BP) is the Bayes plausibility. The solution to this optimization problem gives a lower bound for the sellers' profits with endogenous quality.

We will present an information structure under which the sellers randomize quality in equilibrium and derive a positive profit for all $c \in (0,1)$. Fix the prior to be $\mu_0 = c$. Using the Bayes Plausibility condition $f_1 = c(1-f_c)$ and the fact that $f_0 + f_c + f_1 = 1$, we observe that the indifference condition is satisfied with $(f_0, f_c, f_1) = (f_0^*, f_c^*, f_1^*)$ such that

$$f_c^* = (1-c)\left(1 + \frac{\sqrt{1+c^2}-1}{c}\right)$$

and $f_1^* = c(1-f_c^*)$ and $f_0^* = 1 - f_c^* - f_1^*$. To see that $f_c^* \in [0,1]$ for all $c \in [0,1]$, consider f_c^* as a function of c. It is continuous and strictly decreasing in c such that $\lim_{c\to 0} f_c^* = 1$ and $\lim_{c\to 1} f_c^* = 0$.

Then the total profit (the sum of individual seller's profits) is

$$2\frac{1}{c}(1-c)^2\left(c+\sqrt{1+c^2}\right)\left(1-\sqrt{1+c^2}-c\left(1-c-\sqrt{1+c^2}\right)\right) > 0.$$

In other words, with this information structure, positive profit can always be generated.

Lastly, if $\mu_0 \in \{0,1\}$, then any information structure that satisfies Bayes Plausibility can generate only posteriors of 0 or 1. In both of these cases, the price competition drives the profits to zero.

PROOF OF PROPOSITION 4

In the proof of Proposition 2 we showed that the perfect news–sufficient news is the unique sellers' profit maximizing information structure when the probability of high quality is exogenous. In Lemma 1 we extended the result by optimizing the prior so that the sellers' profits are maximized. Therefore, if the solution in Lemma 1 happens to satisfy the indifference condition, it gives the uniquely optimal equilibrium and information structure. Since in formulas 3, $\pi^1(c)$ is continuous and strictly decreasing in c and $\pi^0(c)$ continuous and strictly increasing c so that $\pi^1(0) > \pi^0(0)$ and $\pi^1(1) < \pi^0(c)$, there exists a single value of cost $c^* \in (0, 1)$ where $\pi^1(c^*) = \pi^0(c^*)$. This is illustrated in Figure 3.

Lastly, since the upper bound of endogenous-quality profits given in Lemma 1 is unique, adding the indifference condition to the sellers' profit maximization problem with the endogenous quality keeps the sellers' profits strictly lower than the upped bound whenever the indifference condition is binding. As argued above, the indifference condition is binding for all $c \neq c^*$, and therefore the upper bound can be achieved only with a single value of cost c^* when the quality is endogenous.

REFERENCES

Armstrong, Mark and Jidong Zhou, "Consumer information and the limits to competition," *American Economic Review*, 2022, *112* (2), 534–577.

Bergemann, Dirk and Juuso Välimäki, "Learning and strategic pricing," *Econometrica: Journal of the Econometric Society*, 1996, pp. 1125–1149.

_ , Benjamin Brooks, and Stephen Morris, "The limits of price discrimination," *American Economic Review*, 2015, *105* (3), 921–957.

Boleslavsky, Raphael and Kyungmin Kim, "Bayesian persuasion and moral hazard," *Available at SSRN 2913669*, 2018.

Elliott, Matthew, Andrea Galeotti, Andrew Koh, and Wenhao Li, "Market segmentation through information," 2022.

Kamenica, Emir and Matthew Gentzkow, "Bayesian persuasion," *American Economic Review*, 2011, *101* (6), 2590–2615.

Roesler, Anne-Katrin and Balázs Szentes, "Buyer-optimal learning and monopoly pricing," *American Economic Review*, 2017, *107* (7), 2072–2080.

Rosar, Frank, "Test design under voluntary participation," *Games and Economic Behavior*, 2017, *104*, 632–655.

Zapechelnyuk, Andriy, "Optimal quality certification," *American Economic Review: Insights*, 2020, *2* (2), 161–176.