

Math 818 Report

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1 Important Equations

Symbol	Meaning	Value
#in	Inline math appearing near equation #	
#der	Equation derived from equation #	
\hbar	Reduced Planck's constant	$1.054\,571\,817 \times 10^{-34} \text{ J} \cdot \text{s}$
e	Charge of Electron	$1.602 \times 10^{-19} \text{ C}$

Table 1: Important Notation

1.1 Oscillator Equations

Starting with the Hamiltonian for the oscillator (an LC circuit)

$$H_{LC} = \frac{Q^2}{2C} + \frac{1}{2}C\omega_r^2\Phi^2, \quad (2)$$

the corresponding quantized obserables are

$$\hat{\Phi} = \Phi_{\text{zpf}}(\hat{a}^\dagger + \hat{a}), \quad \hat{Q} = iQ_{\text{zpf}}(\hat{a}^\dagger - \hat{a}), \quad (4)$$

with coefficients

$$\Phi_{\text{zpf}} = \sqrt{\frac{\hbar Z_r}{2}}, \quad Q_{\text{zpf}} = \sqrt{\frac{\hbar}{2Z_r}} \quad (4\text{in})$$

In these terms the Hamiltonian is defined

$$\hat{H}_s = \hbar\omega_r \left(\hat{a}^\dagger \hat{a} + \frac{1}{2} \right) \quad (5)$$

Throughout this project, the following matrix representation of the ladder operators will be used

$$\hat{a}_{nm} = \begin{cases} \sqrt{i} & \text{when } n = m - 1 \\ 0 & \text{otherwise} \end{cases} \quad (5\text{ex})$$

1.2 Transmon

The Hamiltonian for the transmon is

$$\hat{H} = \sqrt{8E_C E_J} \hat{b}^\dagger \hat{b} - \frac{E_C}{12} (\hat{b}^\dagger + \hat{b})^4 \quad (27)$$

where the corresponding ladder operators \hat{b} and \hat{b}^\dagger can be related to \hat{a} and \hat{a}^\dagger via Eq.(24-25) and

$$\hat{n} = \frac{\hat{Q}}{2e}, \quad \hat{\varphi} = \frac{2e}{\hbar} \hat{\Phi} \quad (22\text{in})$$

to obtain

$$\hat{b} = \frac{1}{2} \left(\frac{\hat{\varphi}}{\varphi_0} - \frac{\hat{n}}{n_0} \right), \quad \hat{b}^\dagger = \frac{1}{2} \left(\frac{\hat{\varphi}}{\varphi_0} + \frac{\hat{n}}{n_0} \right) \quad (25-26\text{der})$$

where

$$\varphi_0 = \left(\frac{2E_C}{E_J} \right)^{1/4}, \quad n_0 = \frac{i}{2} \left(\frac{E_J}{2E_C} \right)^{1/4} \quad (25-26\text{der-a})$$

1.3 Transmons Coupled Via Resonator

The Hamiltonian for a pair of transmons coupled by a resonator is given by

$$\hat{H} = \hat{H}_{q1} + \hat{H}_{q2} + \hbar\omega_r \hat{a}^\dagger \hat{a} + \sum_{i=1}^2 \hbar g_i (\hat{a}^\dagger \hat{b}_i + \hat{a} \hat{b}_i^\dagger) \quad (138)$$

1.4 Master Equation

The master equation for the harmonic oscillator is given by

$$\dot{\rho} = -i[\hat{H}_s, \rho] + \kappa(\bar{n}_\kappa + 1)\mathcal{D}[\hat{a}]\rho + \kappa\bar{n}_\kappa\mathcal{D}[\hat{a}^\dagger]\rho, \quad (70)$$

where the damping factor κ is defined according to

$$\kappa = 2\pi\lambda(\omega_r)^2 = Z_{tml}\omega_r^2 \frac{C_\kappa^2}{C_r},$$

and the dissipation operator is

$$\mathcal{D}[A]B = ABA^\dagger - \frac{1}{2}\{A^\dagger A, B\} \quad (71)$$

where A and B are operators, and $\{A, B\}$ is the anti-commutator of A and B . The equivalent form for the transmon is given by

$$\dot{\rho} = -i[\hat{H}_q, \rho] + \gamma(\bar{n}_\gamma + 1)\mathcal{D}[\hat{b}]\rho + \gamma\bar{n}_\gamma\mathcal{D}[\hat{b}^\dagger]\rho, \quad (79)$$

where \bar{n}_γ and γ are constants.

1.5 Misc Parameters

The equation for the qubit frequency is given by

$$\omega_q = \sqrt{8E_C E_J} - E_C, \quad (27in)$$

where E_J is the Josephson energy and E_C is the charging energy.