Math 818
Report

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1 Important Equations

Symbol	Meaning	Value
#in	Inline math appearing near equation #	
#der	Equation derived from equation #	
$\hbar$	Reduced Planck's constant	$1.054\,571\,817 \times 10^{-34}\,\mathrm{J \cdot s}$
e	Charge of Electron	$1.602 \times 10^{-19}\,\mathrm{C}$

Table 1: Important Notation

1.1 Oscillator Equations

Starting with the Hamiltonian for the oscillator (an LC circuit)

$$H_{LC} = \frac{Q^2}{2C} + \frac{1}{2}C\omega_r^2\Phi^2 \ , \tag{2}$$

the corresponding quantized obserables are

$$\hat{\Phi} = \Phi_{\text{zpf}}(\hat{a}^\dagger + \hat{a}) \ , \qquad \hat{Q} = iQ_{\text{zpf}}(\hat{a}^\dagger - \hat{a}) \ , \tag{4}$$

with coefficients

$$\Phi_{\text{zpf}} = \sqrt{\frac{\hbar}{2Z_r}} \ , \qquad Q_{\text{zpf}} = \sqrt{\frac{\hbar}{2Z_r}} \tag{4in}$$

In these terms the Hamiltonian is defined

$$\hat{H}_s = \hbar\omega_r\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right) \tag{5}$$

Throughout this project, the following matrix representation of the ladder operators will be used

$$\hat{a}_{nm} = \begin{cases} \sqrt{i} & \text{when} \quad n = m - 1 \\ 0 & \quad \text{otherwise} \end{cases} \tag{5ex}$$

1.2 Transmon

The Hamiltonian for the transmon is

$$\hat{H} = \sqrt{8E_C E_J}\hat{b}^\dagger\hat{b} - \frac{E_C}{12}(\hat{b}^\dagger + \hat{b})^4 \tag{27}$$

where the corresponding ladder operators $\hat{b}$ and $\hat{b}^\dagger$ can be related to $\hat{a}$ and $\hat{a}^\dagger$ via Eq.(24-25) and

$$\hat{n} = \frac{\hat{Q}}{2e} \ , \qquad \hat{\varphi} = \frac{2e}{\hbar}\hat{\Phi} \tag{22in}$$

to obtain

$$\hat{b} = \frac{1}{2}\left(\frac{\hat{\varphi}}{\varphi_0} - \frac{\hat{n}}{n_0}\right) \ , \qquad \hat{b}^\dagger = \frac{1}{2}\left(\frac{\hat{\varphi}}{\varphi_0} + \frac{\hat{n}}{n_0}\right) \tag{25-26der}$$

where

$$\varphi_0 = \left(\frac{2E_C}{E_J}\right)^{1/4} \ , \qquad n_0 = \frac{i}{2}\left(\frac{E_J}{2E_C}\right)^{1/4} \tag{25-26der-a}$$

1.3 Transmons Coupled Via Resonator

The Hamiltonian for a pair of transmons coupled by a resonator is given by

$$\hat{H} = \hat{H}_{q1} + \hat{H}_{q2} + \hbar\omega_r \hat{a}^\dagger \hat{a} + \sum_{i=1}^{2} \hbar g_i (\hat{a}^\dagger \hat{b}_i + \hat{a}\hat{b}_i^\dagger) \tag{138}$$

1.4 Master Equation

The master equation for the harmonic oscillator is given by

$$\dot{\rho} = -i[\hat{H}_s, \rho] + \kappa(\bar{n}_\kappa + 1)\mathcal{D}[\hat{a}]\rho + \kappa\bar{n}_\kappa \mathcal{D}[\hat{a}^\dagger]\rho \ , \tag{70}$$

where the damping factor κ is defined according to

$$\kappa = 2\pi\lambda(\omega_r)^2 = Z_{tml}\omega_r^2 \frac{C_\kappa^2}{C_r},$$

and the dissipation operator is

$$\mathcal{D}[A]B = ABA^\dagger - \frac{1}{2}\{A^\dagger A, \ B\} \tag{71}$$

where A and B are operators, and $\{A, B\}$ is the anti-commutator of A and B. The equivalent form for the transmon is given by

$$\dot{\rho} = -i[\hat{H}_q, \rho] + \gamma(\bar{n}_\gamma + 1)\mathcal{D}[\hat{b}]\rho + \gamma\bar{n}_\gamma \mathcal{D}[\hat{b}^\dagger]\rho \ , \tag{79}$$

where $\bar{n}_\gamma$ and γ are constants.