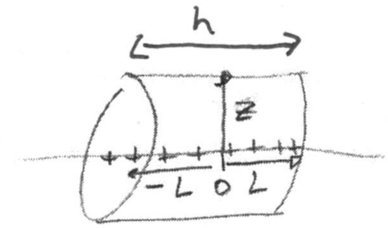
HW 1.3 a)

$Q = \lambda 2L$



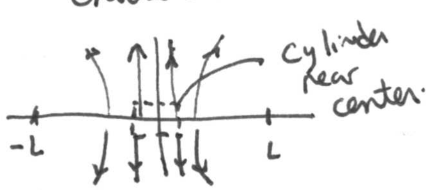
From Griffiths $\rightarrow E(z) = \dfrac{k\,2\lambda L}{z\sqrt{z^2+L^2}} \boxed{\dfrac{kQ}{z\sqrt{z^2+L^2}}}$

Gauss Law $\rightarrow E = \dfrac{Q}{\varepsilon_0 \text{ Area}} \Rightarrow$ For cylinder.

$h \ll L$ for Gauss's law to apply

$\Phi = E\int_s dA = E(2\pi z(2L)) = \dfrac{Q}{\varepsilon_0}$ X

$\boxed{E(z) = \dfrac{Q}{4\pi\varepsilon_0 z \cdot L}} = \dfrac{kQ}{z \cdot L}$



$E(z) = \dfrac{kQ}{z\sqrt{z^2+L}} = \dfrac{kQ}{z \cdot L \cdot \sqrt{1+z^2/L^2}}$; $E(z) = \text{Gauss Law} \cdot \dfrac{1}{\sqrt{1+\frac{z^2}{L^2}}}$ for $L=z$ $\dfrac{1}{\sqrt{2}}$ ✓

Do Taylor on $f(L) = \dfrac{1}{\sqrt{1+\frac{z^2}{L^2}}}$ Evaluating at $\frac{z}{L}=0$

$f = z = L$

$f(L) = 1 - \frac{1}{2}\left(\frac{z}{L}\right)^2 + \frac{3}{4}\left(\frac{z}{L}\right)^4 - \frac{5}{16}\left(\frac{z}{L}\right)^6$ ✓

$f(L) = 1 - \frac{1}{2} + \frac{3}{8} - \frac{5}{16} \Rightarrow 0.56 \Rightarrow$ increasing the order to 10

$f(L) \approx 0.802$, and see how it aproaches $\frac{1}{\sqrt{2}}$ (exact for $z=L$) as the order increases.

From HW1.2 $\Rightarrow E(z)$ for $\lambda = 1E^{-9}$ and $L = z = 1$

$E(z) = 12.7163$ N/C

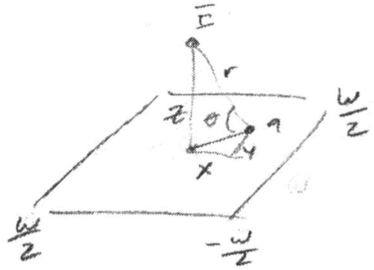
from Gauss Law $\Rightarrow E(z) = 17.98$ with the same parameters

using the taylor aproximation of $0.802 \Rightarrow$

$E(z) = 17.98 * 0.802 = 14.42$ N/C

This solution is still far off, even with an order 10 Taylor Series aproximation, but getting closer.

HW 1.36



$r = \sqrt{x^2+y^2+z^2}$

$\sin\theta = \frac{z}{d} = \frac{z}{\sqrt{x^2+y^2+z^2}}$

$GL \Rightarrow E = \frac{\sigma}{2\epsilon_0}$ $\qquad E = \frac{\sigma r}{4\pi\epsilon_0}\int_{-\frac{w}{2}}^{\frac{w}{2}}\int_{-\frac{w}{2}}^{\frac{w}{2}} \frac{1}{\sqrt{x^2+y^2+z^2}}\,dx\,dy = \frac{\sigma}{\pi\epsilon_0}\tan^{-1}\left(\frac{w^2}{4z\sqrt{\frac{w^2}{2}+z^2}}\right)$ ok ✓

so do Taylor series on $2\pi\tan^{-1}\left(\frac{w^2}{4z\sqrt{\frac{w^2}{2}+z^2}}\right) = 2\pi\tan^{-1}\left(\frac{w^2}{4z^2}\frac{1}{\sqrt{1+\frac{1}{2}\frac{w^2}{z^2}}}\right)$

for order 2 $\Rightarrow \frac{\pi}{2}\frac{w^2}{z^2} - \frac{\pi}{8}\left(\frac{w}{z}\right)^4 + \frac{7\pi}{192}\left(\frac{w}{z}\right)^6$ ✓

For $\sigma = -1E{-9}\,C$, $z = 1$ and $w = 1$

$GL \to E = \frac{\sigma}{2\epsilon_0} = \frac{\sigma}{2\epsilon_0} = \frac{1E^{-9}}{2\epsilon_0} \approx 56\ N/C$

$E_z(z) = \frac{\sigma}{\pi\epsilon_0}\tan^{-1}\left(\frac{w^2}{4z}\frac{1}{\sqrt{z^2+\frac{w^2}{2}}}\right) = 414\ N/C$

Taylor $\Rightarrow$ aproximates to $1.29 \Rightarrow$ GL · taylor $\Rightarrow 56 \times 1.29 = 72$

for $w = 2$ and $z = 1$

$E_z(z) = 1079\ NC$ and taylor $\Rightarrow 7.33 \Rightarrow$ GL · Taylor $= 410\ NC$

for $w = 3$ and $z = 1$

$E_z(z) = 1575\ NC$ and taylor $\Rightarrow 65 \Rightarrow$ GL · Taylor $= 3686$

Can use Wolfram Alpha or
$f(z') \simeq f(0) + z'\frac{\partial f}{\partial z'}\Big|_{z'=0} + \cdots$

could Not find the series when evaluating for $\frac{z}{w}$:

$f(z') = 2\pi\tan^{-1}\left(\frac{\sqrt{2}}{4\frac{z}{w}\sqrt{1+2\left(\frac{z}{w}\right)^2}}\right)$ I may have solve this wrong..