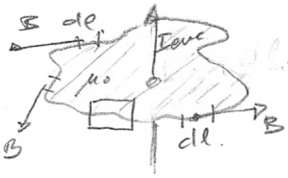
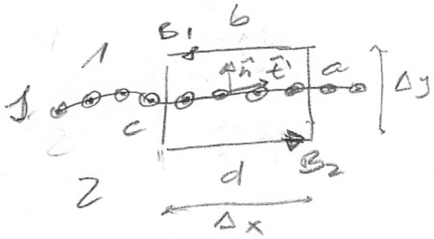
5.1. Boundary Value Derivation.

- from Ampere's Law $\rightarrow \oint_c B\cdot dl = \mu_0 I_{enc}$

$\nabla \times B = \mu_0 J$



- current flowing out of the page



stokes $\rightarrow \oint B\,dl = \int_a B\,dl + \int_b B\,dl + \int_c B\,dl + \int_d B\,dl$

as $\Delta y \rightarrow 0$ $\oint B\,dl = \int_b B_1\,dl + \int_d B_2\,dl$.

$\Rightarrow \int_b B_1\,dl + \int_d B_2\,dl = B_1(-\Delta x)\hat{t} + B_2(\Delta x)\hat{t} = (B_2 - B_1)\Delta x\,\hat{t}$

and $I_{enc} \Rightarrow J\,\Delta x\,(\hat{t}\times\hat{n})$ $\Rightarrow$ $(B_2 - B_1)\cancel{\vec{t}} = \cancel{\vec{t}}(\hat{n}\times J)$

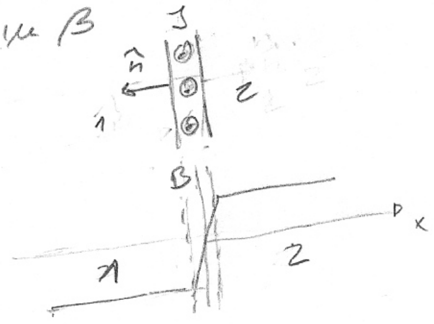
$(B_2 - B_1)\cancel{\Delta x}\,\hat{t} = J\cancel{\Delta x}\,(\hat{t}\times\hat{n})$

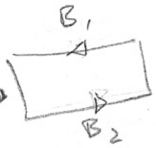
$\underline{n \times J}$

$\underline{J=0}$ $\Rightarrow \nabla\times B = 0$

if there is a current normal to $\hat{n}$, then there is a discontinuity in B

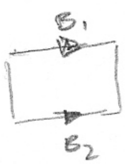
if $J=0$ $\Rightarrow B_{t2} = B_{t1}$





from before $\rightarrow$ $\int_a^b B\,dl = V(a) - V(b) = 0$

so as we go around they cancel out, but in our case for $J=0$



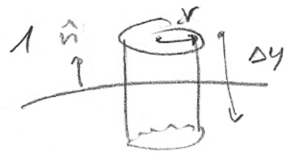
$V(a) - V(b) \neq 0$

$\hat{n}\times(B_2 - B_1) = \hat{n}\times\hat{n}\times J = -J$

$\hat{n}\times(B_1 - B_2) = J$

$B\int_a^b dl = V(a) - V(b) \Rightarrow B = -\nabla\psi$

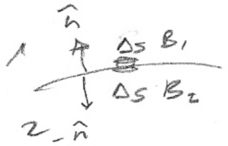
$\nabla \cdot B = 0$ from gauss law $\oint_S B \cdot ds = 0$



$\to \oint_S B \cdot ds = \int_{top} B ds + \int_{side} B ds + \int_{bottom} B ds$.

$\Delta y \to 0$ $\int_{side} B ds = 0$. $\Rightarrow \int_{top} B_1 ds + \int_{bottom} B_2 ds = 0$

as $r \to 0$ Area of top and botom = ΔS

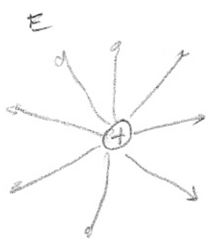


$\Rightarrow \hat{n} B_1 \Delta s + (-\hat{n}) B_2 \Delta s = 0$ as ΔS becomes neglegible.

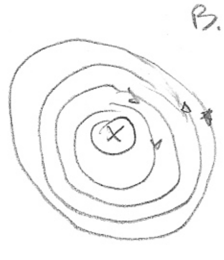
$\hat{n}(B_1 - B_2) = 0$

and the normal component of B across boundary is continuos

From the graphical point of view



There is nothing curly so $\nabla \times E = 0$ but the lines diverge from the charge so there must be some $\nabla \cdot E$



No divergence as the lines go around in circles. $\nabla \cdot B = 0$
Also the lines "curl" around so there must be $\nabla \times B$