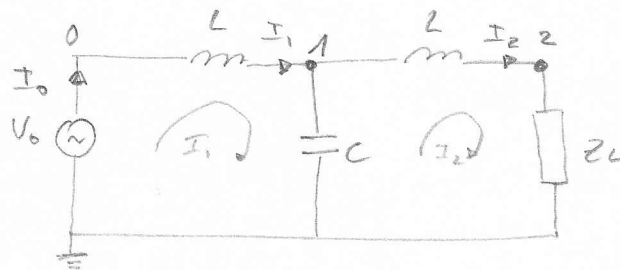


## 11.1 Part I



11.1.1. Apply Kirchhoff current and voltage law.:

voltage around each loop:

$$\text{1st loop: } \tilde{V}_0 - L \frac{d\tilde{I}_1}{dt} - \frac{1}{C} \int (\tilde{I}_1 - \tilde{I}_2) dt = 0 \Rightarrow \text{for } \frac{1}{C} \int (\tilde{I}_1 - \tilde{I}_2) dt = \tilde{V}_1$$

$$\tilde{V}_0 - L \frac{d\tilde{I}_1}{dt} - \tilde{V}_1 = 0 \quad \text{replacing } \frac{d}{dt} \text{ with } j\omega \Rightarrow$$

$$\tilde{V}_0 - j\omega L \tilde{I}_1 - \tilde{V}_1 = 0 \Rightarrow \tilde{V}_1 = \tilde{V}_0 - j\omega L \tilde{I}_1 \quad \checkmark$$

$$\text{2nd loop: } -\frac{1}{C} \int (\tilde{I}_2 - \tilde{I}_1) dt - L \frac{d\tilde{I}_2}{dt} - \tilde{I}_2 Z_L = 0$$

$$\text{for } \frac{1}{C} \int (\tilde{I}_2 - \tilde{I}_1) dt = -\tilde{V}_1 \text{ and } \tilde{I}_2 Z_L = \tilde{V}_2$$

$$\tilde{V}_1 - L \frac{d\tilde{I}_2}{dt} - \tilde{V}_2 = 0 \quad \text{replacing } \frac{d}{dt} \text{ with } j\omega$$

$$\tilde{V}_2 = \tilde{V}_1 - j\omega L \tilde{I}_2 \quad \checkmark$$

current at each node: 0 is not a node.

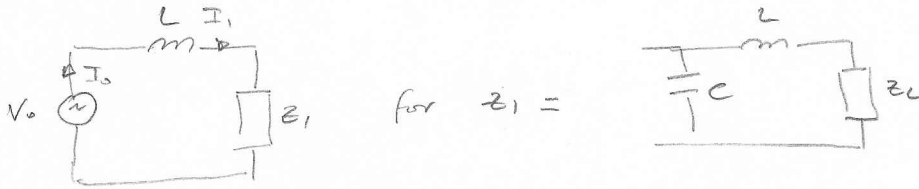
$$\text{Node 1: } \underbrace{\frac{1}{L} \int (\tilde{V}_1 - \tilde{V}_0) dt}_{-\tilde{I}_1} + \underbrace{\frac{1}{L} \int (\tilde{V}_1 - \tilde{V}_2) dt}_{\tilde{I}_2} + C \frac{d\tilde{V}_1}{dt} = 0 \Rightarrow -\tilde{I}_1 + \tilde{I}_2 + C \frac{d\tilde{V}_1}{dt} = 0$$

$$\Rightarrow \text{replacing } \frac{d}{dt} \text{ with } j\omega \Rightarrow \tilde{I}_2 = \tilde{I}_1 - j\omega C \tilde{V}_1 \quad \checkmark$$

Node 2:  $\frac{1}{Z} \int (\tilde{V}_2 - \tilde{V}_1) dt + \frac{\tilde{V}_2}{Z_L} = 0 \Rightarrow -\tilde{I}_2 + \frac{\tilde{V}_2}{Z_L} = 0$  (2)

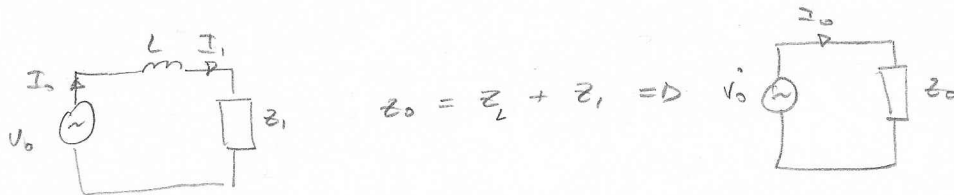
$$\tilde{I}_2 = \frac{\tilde{V}_2}{Z_L} \quad \checkmark$$

11.1.2  $\Rightarrow \tilde{V}_0 = V_0$  and  $\tilde{I}_1 = \frac{\tilde{V}_0}{Z_0}$  with  $\tilde{I}_1 = \tilde{I}_0$



$Z_1' = Z_L + j\omega L \Rightarrow Z_1 = \frac{1}{\frac{1}{Z_L} + \frac{1}{j\omega L + Z_L}}$

$\frac{1}{Z_1} = \frac{1}{Z_L} + \frac{1}{Z_1'}$ ;  $Z_1 = \frac{Z_1' Z_L}{Z_1' + Z_L} = \frac{(Z_L + j\omega L) \frac{1}{j\omega C}}{Z_L + j\omega L + \frac{1}{j\omega C}}$



$Z_0 = Z_1 + j\omega L = \frac{Z_L + j\omega L}{j\omega C Z_L - \omega^2 L C + 1} + j\omega L$

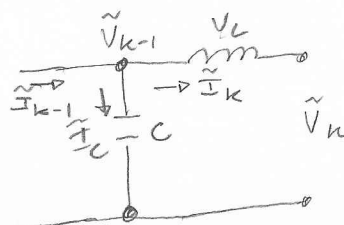
11.1.3

$\tilde{I}_k = \tilde{I}_{k-1} - j\omega C \tilde{V}_{k-1}$  and  $\tilde{V}_k = \tilde{V}_{k-1} - j\omega L \tilde{I}_k$

$\tilde{I}_2 = \tilde{I}_1 - j\omega C \tilde{V}_1$  and  $\tilde{V}_2 = \tilde{V}_1 - j\omega L \tilde{I}_2$

$\tilde{I}_1 = \tilde{I}_0 - j\omega C \tilde{V}_0$  and  $\tilde{V}_1 = \tilde{V}_0 - j\omega L \tilde{I}_1$

$\tilde{I}_1 = \tilde{I}_0$



$\tilde{V}_k = \tilde{V}_{k-1} - j\omega L \tilde{I}_k$  is obvious, as  $V_k$  will be equal to  $V_{k-1}$  minus the voltage drop in the inductor  $L$  denoted by  $V_L = j\omega L \tilde{I}_k$ .

$\Rightarrow$

(3)

Like wise,  $\tilde{I}_k$  is the total current arriving at the node  $\tilde{I}_{k-1}$  minus the current drawn by the capacitor  $I_c = j\omega C \tilde{V}_{k-1}$ .

Therefore the generalization is correct:

$$I_k = I_{k-1} - j\omega C \tilde{V}_{k-1} \quad \text{and} \quad \tilde{V}_k = \tilde{V}_{k-1} - j\omega L \tilde{I}_k \quad \checkmark$$

In time domain with a  $Z_0$  matched impedance, and no reflecting wave:

$$\tilde{V}_0(t) = V_0 \cos \omega t = V_0 e^{j\omega t} \quad \text{and} \quad \tilde{I}_0(t) = \frac{\tilde{V}_0(t)}{Z_0}$$

$$\tilde{V}_1(t) = \tilde{V}_0(t) - L \frac{d\tilde{I}_1(t)}{dt} \quad \text{and} \quad \tilde{I}_1(t) = \tilde{I}_0(t) - C \frac{d\tilde{V}_1(t)}{dt}$$

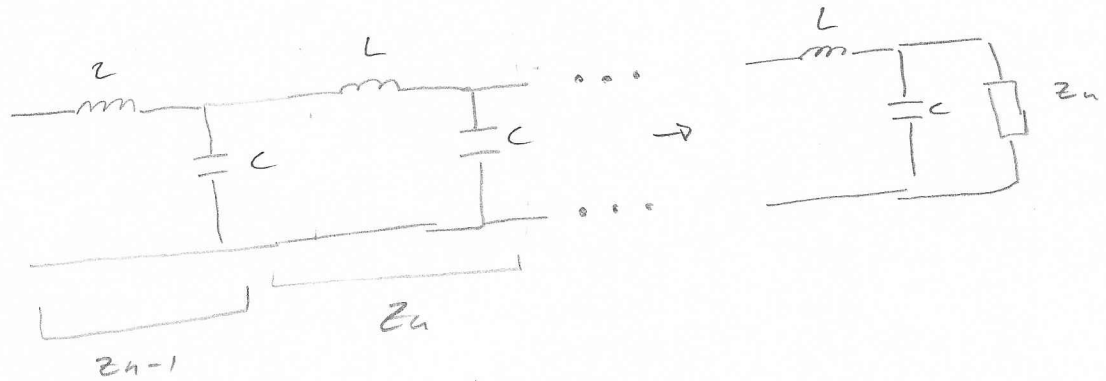
$$\tilde{V}_2(t) = \tilde{V}_1(t) - L \frac{d\tilde{I}_2(t)}{dt} \quad \text{and} \quad \tilde{I}_2(t) = \tilde{I}_1(t) - C \frac{d\tilde{V}_2(t)}{dt}$$

$$\tilde{V}_k(t) = \tilde{V}_{k-1}(t) - L \frac{d\tilde{I}_k(t)}{dt}$$

$$\tilde{I}_k(t) = \tilde{I}_{k-1}(t) - C \frac{d\tilde{V}_k(t)}{dt}$$

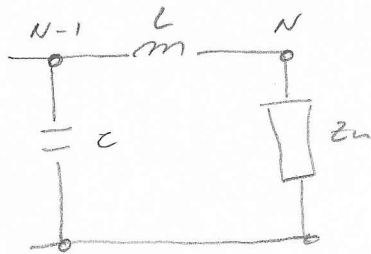
11.1.4

1.



$$z_{n-1} = \frac{z_n \frac{1}{j\omega C} + j\omega L}{z_n + \frac{1}{j\omega C}} \quad \text{For } z_N = z_L, \quad z_n = z_N + j\omega L$$

or



$$z_{n-1} = \frac{(z_n + j\omega L) \frac{1}{j\omega C}}{z_n + j\omega L + \frac{1}{j\omega C}} =$$

$$= \frac{z_n + j\omega L}{z_n j\omega C + j\omega C j\omega L + 1} \Rightarrow$$

$$z_{n-1} = \frac{z_n + j\omega L}{z_n j\omega C - \omega^2 C L + 1}$$

$$\text{and } z_0 = j\omega L + z_1$$

$$2. \quad \tilde{I}_k = \tilde{I}_{k-1} - j\omega C \tilde{V}_{k-1} \quad \text{for } k=0, 1, \dots, N \quad \text{and } \tilde{I}_1 = \frac{\tilde{V}_0}{z_0}$$

$$3. \quad \tilde{V}_k = \tilde{V}_{k-1} - j\omega L \tilde{I}_k$$