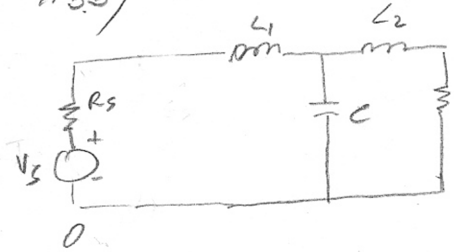
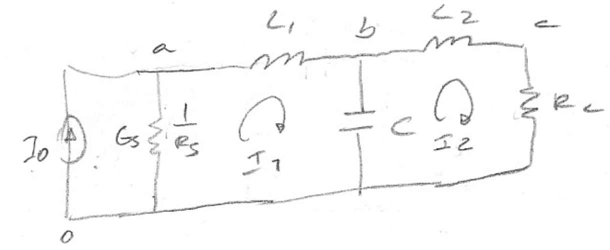
HW # 7 Jesus Gil Gil

7.2 Problems 4.3b, 4.3c, 4.3d & 4.6e.

4.3b)



$\Rightarrow$



loop

$$\frac{1}{R_S}(I_0 - I_1) - L_1\frac{dI_1}{dt} - \frac{1}{C}\int (I_1 - I_2)\,dt = 0$$

$$\frac{1}{C}\int (I_1 - I_2)\,dt - L_2\frac{dI_2}{dt} - R_L I_2 = 0$$

Node.

a)

$$I_0 - \frac{V_a}{1/R_s} - \frac{1}{L_1}\int (V_a - V_b) = 0$$

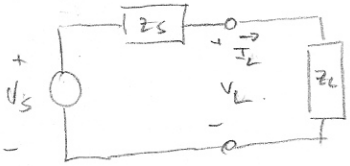
b)

$$\frac{1}{L_1}\int (V_a - V_b) - C\frac{dV_b}{dt} - \frac{1}{L_2}\int (V_b - V_c) = 0$$

c)

$$\frac{1}{L_2}\int (V_b - V_c) - \frac{V_c}{R_L} = 0$$

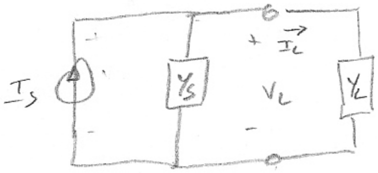
4.3 c)



$\Rightarrow \quad V_L = Z_L I_L \quad ; \quad I_L = I_s$

$V_L = V_s - Z_s I_L \Rightarrow I_L = \frac{V_s - V_L}{Z_s}$

$V_L = Z_L I_L \Rightarrow I_L = \frac{V_s}{Z_s} - \frac{Z_L I_L}{Z_s} \Rightarrow I_L = \frac{V_s}{Z_L + Z_s}$



$I_s = V_s Y_s$ (10)

$V_L = \frac{I_L}{Y_L}$; $I_s = V_L Y_L + V_L Y_s \Rightarrow$

$\Rightarrow I_s = V_L (Y_L + Y_s) \Rightarrow V_L = \frac{I_s}{Y_L + Y_s}$

From Thevenin $\Rightarrow I_L = \frac{V_s}{Z_L + Z_s} \Rightarrow I_L = I_s$

$I_s = \frac{V_s}{Z_L + Z_s} \Rightarrow$ Solving for $Z_L = \frac{V_s}{I_s} - Z_s \Rightarrow$

$\Rightarrow Z_L = \frac{V_L}{I_L} \Rightarrow \frac{V_L}{I_L} = \frac{V_s}{I_s} - Z_s$ ✓

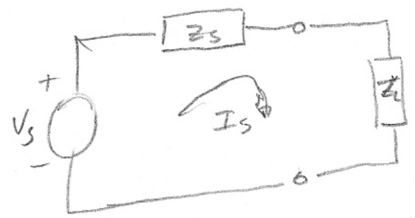
From Norton $V_L = \frac{I_s}{Y_L + Y_s} \Rightarrow V_L = V_s$

$V_s = \frac{I_s}{Y_L + Y_s} \Rightarrow$ solving for $Y_L = \frac{I_s}{V_s} - Y_s$

$\Rightarrow Y_L = \frac{I_L}{V_L} \Rightarrow \frac{I_L}{V_L} = \frac{I_s}{V_s} - Y_s \Rightarrow$ inverting. $\Rightarrow$

$\frac{V_L}{I_L} = \frac{V_s}{I_s} - Z_s$ ✓

4.3 d)



$Z_S = R_S + jX_S$

$Z_L = R_L + jX_L$

$$I_S = \frac{V_S}{Z_S + Z_L} = \frac{V_S}{(R_S + jX_S) + (R_L + jX_L)} = \frac{V_S}{(R_S + R_L) + j(X_S + X_L)}$$

$$|I_S| = \frac{|V_S|}{\sqrt{(R_S+R_L)^2 + (X_S+X_L)^2}} \Rightarrow P_L = |I_L|^2 R_L = \frac{|V_S|^2 R_L}{(R_S+R_L)^2 + (X_S+X_L)^2}$$

$$\frac{d}{dX_L}\left(\frac{|V_S|^2 R_L}{(R_S+R_L)^2 + (X_S+X_L)^2}\right) = \frac{-2(X_L+X_S) R_L V_S^2}{[(R_S+R_L)^2 + (X_S+X_L)^2]^2}$$

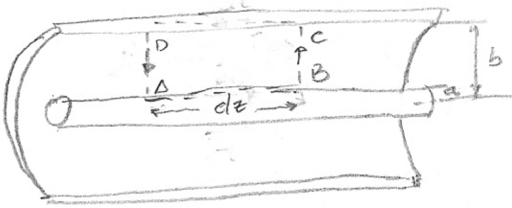
$$\frac{-2(X_L+X_S) R_L V_S^2}{[(R_S+R_L)^2 + (X_S+X_L)^2]^2} = 0$$ and solving $\Rightarrow X_L + X_S = 0$ or $X_L = -X_S$

$$P_L = \frac{|V_S|^2 R_L}{(R_S+R_L)^2} \Rightarrow \frac{dP_L}{dR_L} = \frac{-(R_L - R_S)|V_S|^2}{(R_L+R_S)^3} = 0$$ and solving $\Rightarrow R_L = R_S$

For max Power $\Rightarrow X_L = -X_S$ and $R_L = R_S$

$$Z_L = R_L + jX_L = R_S - jX_S = Z_S^*$$

4.6.e



$\to z$ wave propagation. $\sum_i v_i = 0$

$$V_z(t) - RI(t) - L_i \frac{dI(t)}{dt} - \frac{1}{C}\int I(t)\,dt - L_e \frac{dI(t)}{dt} - V_{z+dz}(t) = 0$$

Voltages.

$$V_z = V_{DA} = -\int_A^D E\,dl \; ; \; V_{z+dz} = V_{CB} = -\int_B^C - E\,dl \quad ; V_{BA} = 0 \; ; V_{CD} = 0$$

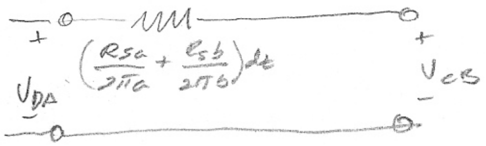
Impedance

$V_{DA} = 0 \; ; \; V_{CB} = 0 \quad ; \quad A = 2\pi r$

$$V_{BA} + V_{DC} = -\int_A^B E\,dz - \int_C^D E\,dz = -I\left[\underbrace{\int_A^B \frac{dz}{\sigma A}}_{R_{BA}} + \underbrace{\int_C^D \frac{dz}{\sigma A}}_{R_{DC}}\right]$$

For high frequency: 4.5 (11) $R_{(hf)} = \frac{R_s}{2\pi r_o}$ per unit length (dz)

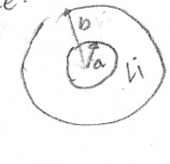
$$V_{BA} + V_{DC} = I\left(\frac{R_{sa}}{2\pi a} + \frac{R_{sb}}{2\pi b}\right) dz.$$



Inductance

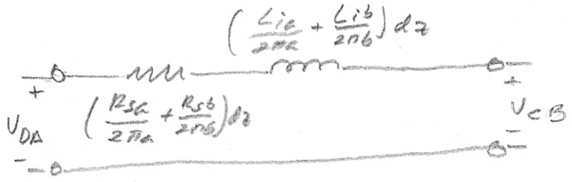
$$\oint E \cdot dl = -\frac{\partial}{\partial t}\int_s B\,ds \quad ; \quad ds = dz\,dr \quad ; \quad \Delta V = V_{CD} - V_{DA} = -\frac{d\Phi}{dt}$$

re.



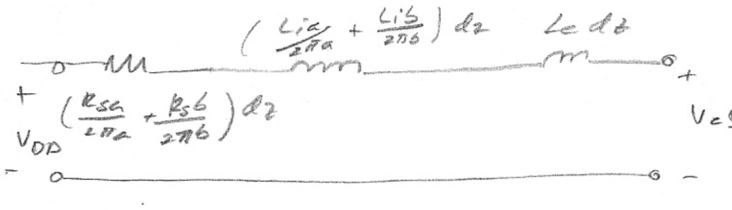
$$\Phi_i = \int_0^a B\,dr\,dz + \int_0^b B\,dr\,dz \Rightarrow L_i = \left[\int B\,ds\right]/I$$

$$\Phi_i = \left[\frac{L_{ia}}{2\pi a}dz + \frac{L_{ib}}{2\pi b}dz\right] I \Rightarrow -\frac{d\Phi}{dt} = -\left[\frac{L_{ia}}{2\pi a} + \frac{L_{ib}}{2\pi b}\right] dz \frac{dI(t)}{dt}$$



external Inductance

$L_e = \left[\int B\, ds\right] / I$ $\quad$ L_e is arbitrary so $\quad \Delta V_{L_e} = -\frac{d\Phi_{Le}}{dt} = -\,dz\, L_e \frac{dI(t)}{dt}$.



capacitance

from eq. 1.9(4) $\Rightarrow$ $C = \frac{2\pi\varepsilon}{\ln\left(\frac{b}{a}\right)} \cdot dz$.