HW # 8

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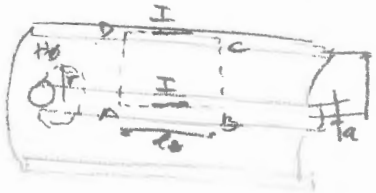
8.1. Inductance Calculation



$$H_\phi = \frac{I}{2\pi r}$$

$$\int_s B\,ds = \int_0^a B\,ds + \int_a^b B\,ds \Rightarrow \begin{cases} \dfrac{\mu I r}{2\pi a^2} & 0 < r < a \\ \dfrac{\mu I}{2\pi r} & a < r < b \end{cases}$$

$$\int_s B\,ds = \int_0^a \frac{\mu I r}{2\pi a^2}\,dr\,dz + \int_a^b \frac{\mu I}{2\pi r}\,dr\,dz = \mu I\left[\frac{1}{4\pi} + \frac{1}{2\pi}\ln\frac{b}{a}\right]dz$$

$$L = \frac{1}{I}\int_s B\,ds = \frac{\mu}{2\pi}\left[\frac{1}{2} + \ln\frac{b}{a}\right]dz$$

$$\int_s B\,ds = \int_{(a-d)}^{a} B\,ds + \int_a^b B\,ds \Rightarrow \begin{cases} \dfrac{\mu I r}{2\pi a^2} & a-d < r < a \\ \dfrac{\mu I}{2\pi r} & a < r < b \end{cases}$$

$$\int_s B\,ds = \int_{a-d}^{a} \frac{\mu I r}{2\pi a^2}\,dr\,dz + \int_a^b \frac{\mu I}{2\pi r}\,dr\,dz =$$

$$= \mu I\left[\frac{2ad - d^2}{4a^2\pi} + \frac{1}{2\pi}\ln\frac{b}{a}\right]dz = \mu I\left[\frac{2ad\left(1 - \frac{d}{2a}\right)}{4a^2\pi} + \frac{1}{2\pi}\ln\frac{b}{a}\right]dz$$

for $\frac{d}{a} \ll 1 \Rightarrow$ $= \mu I\left[\frac{2ad}{4a^2\pi} + \frac{1}{2\pi}\ln\frac{b}{a}\right]dz = \frac{\mu I}{2\pi}\left[\frac{d}{a} + \ln\frac{b}{a}\right]dz$

$$L = \frac{1}{I}\int_s B\,ds = \frac{\mu}{2\pi}\left[\frac{d}{a} + \ln\frac{b}{a}\right]dz$$

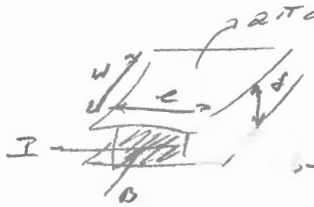
2. External inductance

$$B = \frac{\mu I}{2\pi r} \qquad a < r < b$$

$$\int_S B \cdot ds = \int_a^b \frac{\mu I}{2\pi r} dr\, dz = \frac{\mu I}{2\pi} \ln\frac{b}{a}\, dz$$

$$L = \frac{1}{I}\int_S B \cdot ds = \frac{\mu}{2\pi} \ln\frac{b}{a}\, dz$$

3.



$$\Rightarrow L = \mu \frac{d}{w} \ell = \mu \frac{b-a}{w} \ell$$

for cylinder $w = 2\pi$

$$\int_S B\, ds = \int_0^{\ell}\int_a^b \frac{\mu I}{2\pi r} dr\, dz = \frac{\mu I}{2\pi} \ln\frac{b}{a}\, \ell$$

for $\frac{d}{a} \ll 1 \Rightarrow \ln\frac{b}{a} \approx b - a$

$$L = \frac{\mu}{2\pi}(b-a)\,\ell.$$