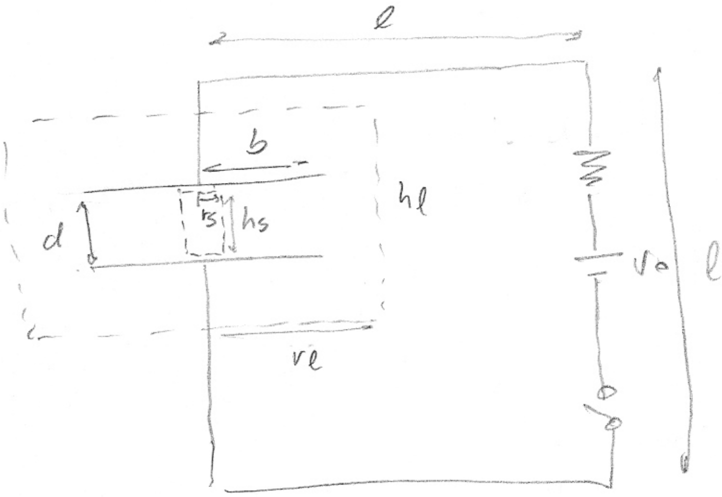
5.2 Poynting Theorem.



$hs < d$

$d < hl < l$

- 1. Ignoring fringing fields and calculating using Gauss law

$$\oint_S E\, da = \frac{Q_{enc}}{\varepsilon_0} \Rightarrow A_{cylinder} = \pi rs^2 \Rightarrow \sigma = \frac{Q}{\pi rs^2}$$

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 \pi rs^2}$$

Total Energy stored in E:

$$U_e = \frac{1}{2}\varepsilon_0 \int_V E^2\, dv = \frac{1}{2}\varepsilon_0 E^2 \pi rs^2 hs = \frac{1}{2}\varepsilon_0 \left(\frac{Q}{\varepsilon_0 \pi rs^2}\right)^2 \pi rs^2 hs =$$

$$U_e = \frac{1}{2}\frac{Q^2 hs}{\varepsilon_0 \pi rs^2}$$

From (6) $\rightarrow \int_V \left[\frac{\partial}{\partial t}\left(\frac{1}{2}\frac{B^2}{\mu_0}\right) + \underbrace{\frac{d}{dt}\overbrace{\left(\frac{1}{2}\varepsilon_0 E^2\right)}^{U_e}}_{\text{evaluate.}} + E\cdot J\right] dv.$

$$\frac{d}{dt}U_e = \frac{Q\, hs}{\varepsilon_0 \pi rs^2}\frac{dQ}{dt} \quad \text{with} \quad \frac{dQ}{dt} = I \Rightarrow \frac{d}{dt}U_e = \frac{Q\, hs\, I}{\varepsilon_0 \pi rs^2}$$ ✓

From Ampere's Law

$$\oint_C B \cdot ds = \mu_0 \iint \cancel{J}\cdot da + \mu_0 \varepsilon_0 \frac{d}{dt}\iint E\, da$$

0 → No current through a capacitor

Choose Amperian loop of radius ra for $ra < rs$

$$\oint_c B\,ds = B\,2\pi r_a \quad \text{and} \quad \mu_0 \varepsilon_0 \frac{d}{dt}\iint E\,da = \mu_0 \varepsilon_0 \frac{dE}{dt}\pi r_a^2$$

$$\frac{dE}{dt} = \frac{Q}{\varepsilon_0 \pi r_s^2}$$

$$B\,2\cancel{\pi}\cancel{r_a} = \mu_0 \frac{Q\cancel{\pi} r_a^{\cancel{2}}}{\pi r_s^2} \Rightarrow B = \frac{\mu_0 Q r_a}{2\pi r_s^2}$$

$$\vec{B} = \frac{\mu_0 Q r_a}{2\pi r_s^2}$$

$$U_m = \frac{1}{2\mu_0}\int_V B^2\,dV = \frac{1}{2\mu_0}\left(\frac{\mu_0 Q r_a}{2\pi r_s^2}\right)^2 \pi r_s^2 h_s = \frac{1}{8}\frac{\mu_0 Q^2 r_a^2 h_s}{\pi r_s^2}$$

$$\frac{d}{dt}U_m = \frac{1}{4}\frac{\mu_0 Q r_a^2 h_s}{\pi r_s^2}\frac{dQ}{dt} \Rightarrow \frac{dQ}{dt} = I$$

$$\frac{d}{dt}U_m = \frac{1}{4}\frac{\mu_0 Q I r_a^2 h_s}{\pi r_s^2} \quad ✓$$

No ohmic losses so $E \cdot J = 0$.

Eq. 6 becomes $\frac{1}{4}\frac{\mu_0 Q I r_a^2 h_s}{\pi r_s^2} + \frac{Q I h_s}{\varepsilon_0 \pi r_s^2} \Rightarrow Q(t) = It. \Rightarrow$

$$\Rightarrow \frac{I^2 h_s}{\pi r_s^2}\left(\frac{\mu_0 r_a^2}{4} + \frac{1}{\varepsilon_0}\right)t = -\frac{1}{\mu_0}\oint_S (E\times B)\,ds$$

2. For the large cylinder, replace h_s with d and r_s with b. However in this case the fringing field play a role, and added to the total work $\frac{dw}{dt}$ are.

$$\frac{I^2 d}{\pi b^2}\left(\frac{\mu_0 r_a^2}{4} + \frac{1}{\varepsilon_0}\right) + \frac{dw}{dt} = -\frac{1}{\mu_0}\oint_s (E\times B)\,ds.$$