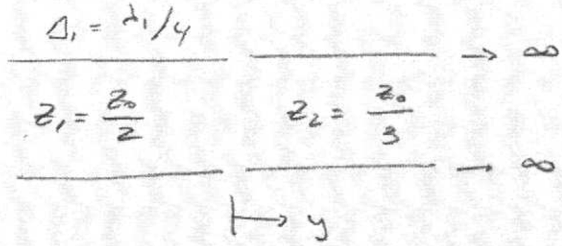
HW #10 Jesus Gil Gil

10.1 Impedance Transformation I



1. $\rho_1(0)$?
2. $Z_1(-\lambda_1/4)$:

10.1.1 Hand calculation

$$Z_n(y) = Z_n \frac{1 + \tilde{\rho}_n(y)}{1 - \tilde{\rho}_n(y)}$$

$\tilde{\rho}_2(y) = 0 \Rightarrow \infty$

$$Z_1 \frac{1 + \tilde{\rho}_1(0)}{1 - \tilde{\rho}_1(0)} = Z_2 \frac{1 + \rho_2(0)^{0}}{1 - \rho_2(0)_{0}} \Rightarrow \text{solve for } \tilde{\rho}_1(0)$$

$$\tilde{\rho}_1(0) = \frac{Z_2 - Z_1}{Z_2 + Z_1} = \frac{Z_2/Z_1 - 1}{Z_2/Z_1 + 1} = \frac{2/3 - 1}{2/3 + 1} = -\frac{1}{5}$$

$$\text{or } \frac{1}{5} \measuredangle 180^\circ \Rightarrow \tilde{\rho}_1(0) = \frac{1}{5} e^{j\pi}$$

$$Z_1(-\lambda_1/4) = Z_1 \frac{1 + \tilde{\rho}_1(-\lambda_1/4)}{1 - \tilde{\rho}_1(-\lambda_1/4)} ; \quad \tilde{\rho}_1(-\lambda_1/4) = -\frac{1}{5} e^{-2j\frac{2\pi}{\lambda_1}\frac{\lambda_1}{4}} = \frac{1}{5}$$

$$Z_1(-\lambda_1/4) = \frac{Z_0}{2} \frac{1 + 1/5}{1 - 1/5} = \frac{Z_0}{2}\frac{3}{2} = Z_0 \frac{3}{4}$$

10.1.2 Smith Chart

$$\frac{z_L}{z_1} = r + jx \quad ; \quad \frac{z_L}{z_1} = \frac{z_0/3}{z_0/2} = \frac{2}{3} \Rightarrow r = \frac{2}{3}, \quad x = 0$$

from smith chart.

at $\hat{\rho}_1(-\lambda_1/4) \Rightarrow r = 1.48 \quad x = 0$

$$z_1(-\lambda_1/4) = \frac{z_0}{2}(r + jx) = \frac{z_0}{2} 1.48 = z_0\, 0.74 \approx z_0 \frac{3}{4} \checkmark$$

③

The Complete Smith Chart

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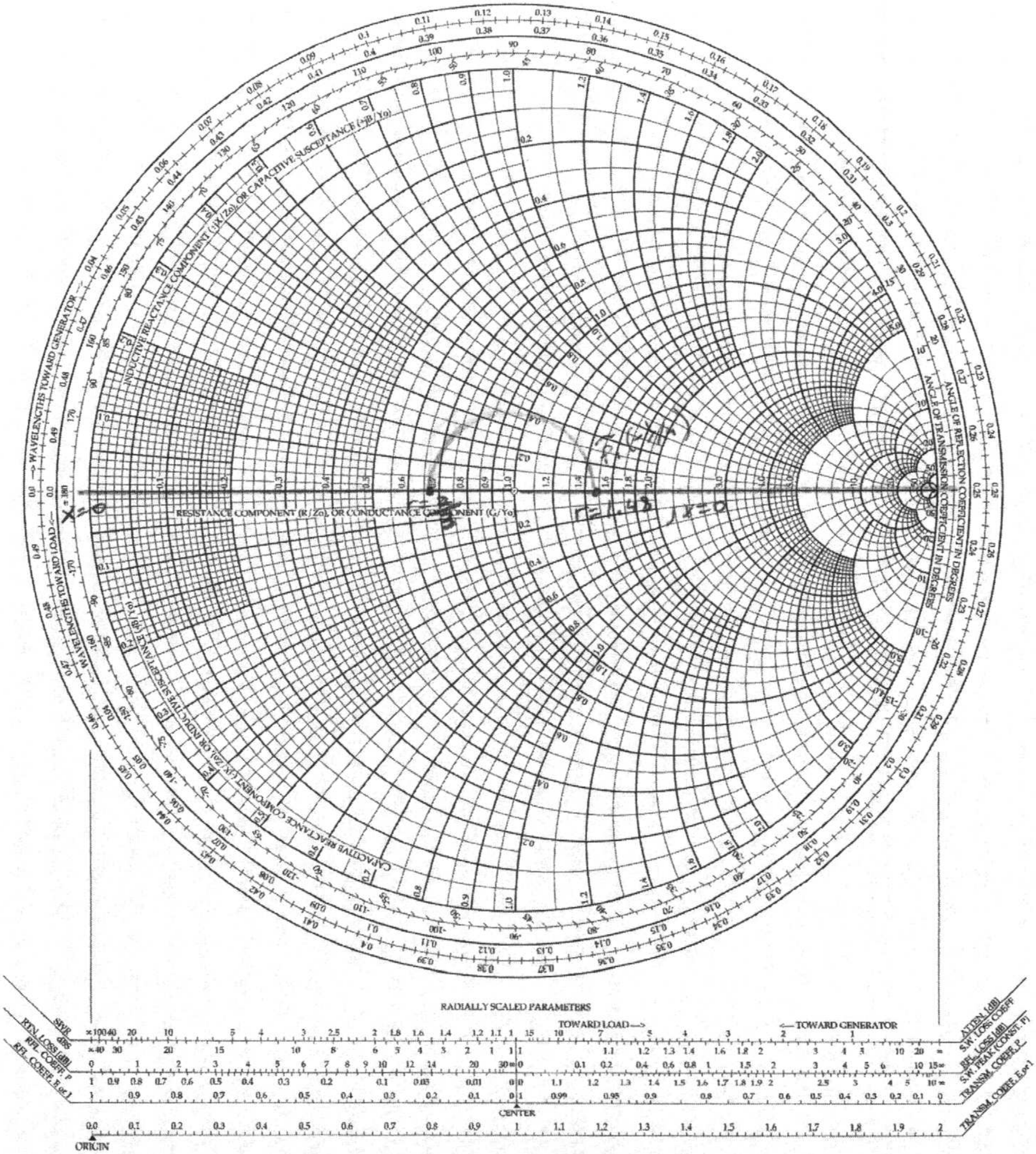
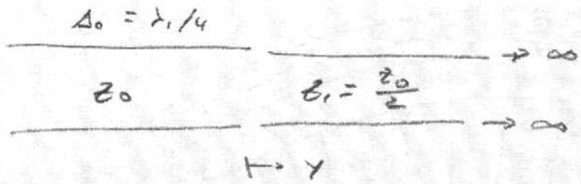


HW 10 Jesus Gil Gil

10.2 Impedance Transformation II



10.2.1 Hand Calculation

$$Z_n(y) = Z_n \frac{1+\tilde{\rho}_n(y)}{1-\tilde{\rho}_n(y)} \Rightarrow \tilde{\rho}_1(y) = 0 \Rightarrow \infty$$

$$Z_0 \frac{1+\tilde{\rho}_0(0)}{1-\tilde{\rho}_0(0)} = Z_1 \frac{1+\cancel{\tilde{\rho}_1(0)}^{\,0}}{1-\cancel{\tilde{\rho}_1(0)}_{\,0}} \Rightarrow \text{solve for } \tilde{\rho}_0(0)$$

$$\tilde{\rho}_0(0) = \frac{Z_1 - Z_0}{Z_1 + Z_0} = \frac{Z_1/Z_0 - 1}{Z_1/Z_0 + 1} = \frac{1/2 - 1}{1/2 + 1} = -\frac{1}{3} \text{ or } \frac{1}{3}\angle 180^\circ$$

$$Z_0(-\lambda/4) = Z_0 \frac{1+\tilde{\rho}_0(-\lambda/4)}{1-\tilde{\rho}_0(-\lambda/4)} \; ; \; \tilde{\rho}_0(-\lambda/4) = -\frac{1}{3} e^{-2j\frac{2\pi}{\lambda}\frac{\lambda}{4}} = \frac{1}{3}$$

$$Z_0(-\lambda/4) = Z_0 \frac{1+1/3}{1-1/3} = 2Z_0$$

10.2.2. Smith Chart

$$\frac{Z_1}{Z_0} = r + jx \; ; \; \frac{Z_1}{Z_0} = \frac{Z_0/2}{Z_0} = \frac{1}{2} \Rightarrow r = \frac{1}{2}, \; x = 0, \; \angle 180^\circ$$

at $\tilde{\rho}_0(-\lambda/4) \Rightarrow r = 2, \; x = 0$

$$Z(-\lambda/4) = Z_0(r + jx) = 2Z_0$$

②

The Complete Smith Chart

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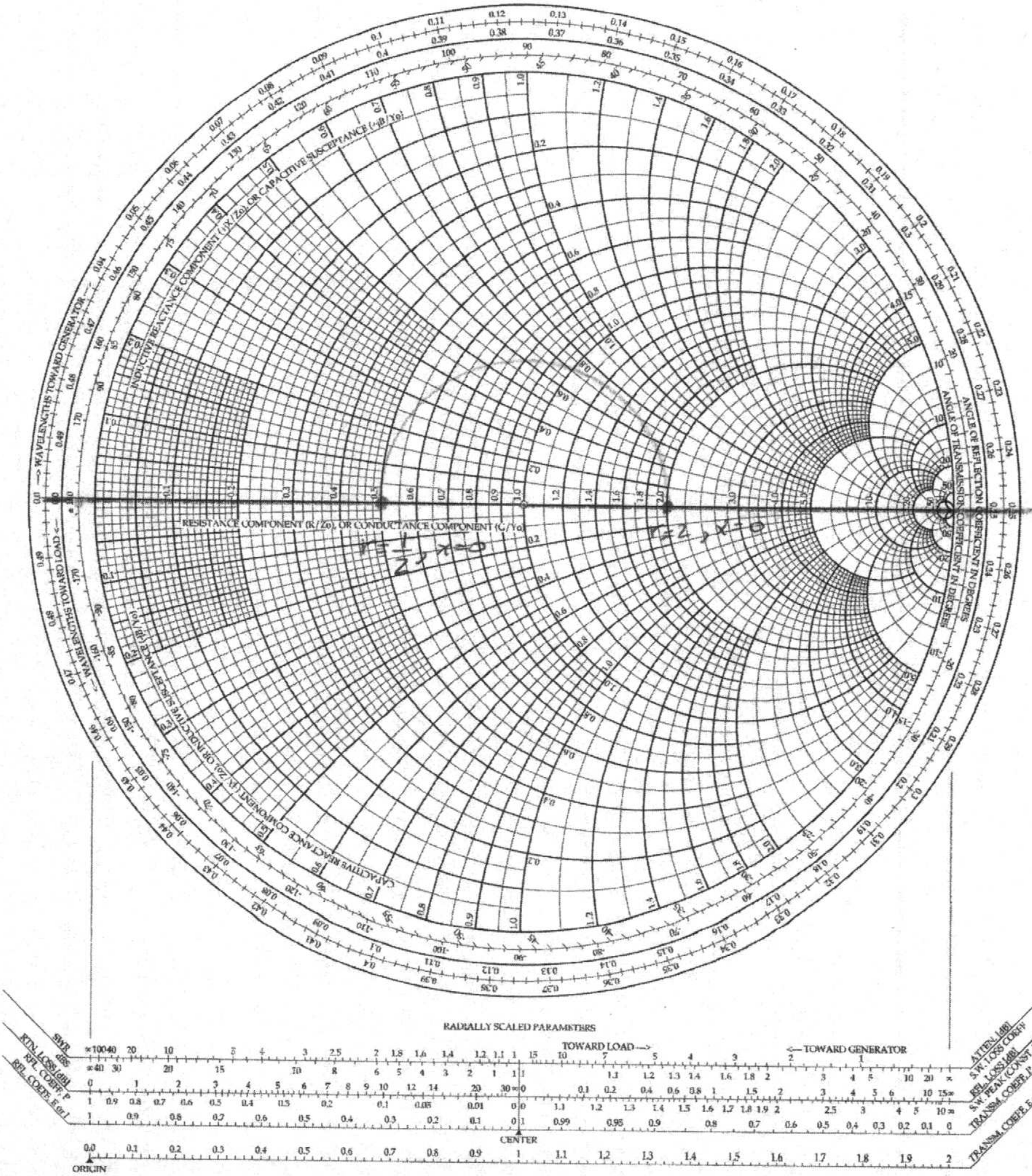
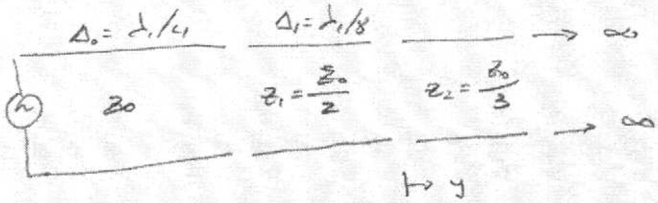


10.3 Impedance Transformation III



10.3.1

$\tilde{\rho}_2(0) = 0 \Rightarrow \infty \quad \Rightarrow \quad z_2(0) = z_1(0)$

$z_1 \dfrac{1+\tilde{\rho}_1(0)}{1-\tilde{\rho}_1(0)} = z_2 \dfrac{1+\cancel{\tilde{\rho}_2(0)}^{\,0}}{1-\cancel{\tilde{\rho}_2(0)}_{\,0}} \Rightarrow$ solve for $\tilde{\rho}_1(0) \Rightarrow \tilde{\rho}_1(0) = \dfrac{z_2/z_1 - 1}{z_2/z_1 + 1} = -\dfrac{1}{5}$

or $\frac{1}{5} \angle 180^\circ \Rightarrow \tilde{\rho}_1(-\lambda/8) = -\frac{1}{5} e^{-2j\beta y} = -\frac{1}{5} e^{-2j\frac{2\pi}{\lambda}\frac{\lambda}{8}} = -\frac{1}{5} e^{-j\frac{\pi}{2}} = j\frac{1}{5}$

$z_1(-\lambda_1/8) = \dfrac{z_0}{2} \dfrac{1+\tilde{\rho}_1(-\lambda/8)}{1-\tilde{\rho}_1(-\lambda/8)} = \dfrac{z_0}{2} \dfrac{1+j\,1/5}{1-j\,1/5} = z_0\left(\dfrac{6}{13} + j\dfrac{5}{26}\right) = \underline{z_0(0.46 + j0.19)}$ ✓

$z_0(-\Delta_1) = z_1(\Delta_1)$ ✓

$z_0 \dfrac{1+\tilde{\rho}_0(-\Delta_1)}{1-\tilde{\rho}_0(-\Delta_1)} = z_1 \dfrac{1+\tilde{\rho}_1(-\Delta_1)}{1-\tilde{\rho}_1(-\Delta_1)}$ solving for $\tilde{\rho}_0(\Delta_1)$ and $z_1 = \frac{z_0}{2}$

$\tilde{\rho}_0(-\Delta_1) = \dfrac{1-3\tilde{\rho}_1(-\Delta_1)}{\tilde{\rho}_1(-\Delta_1) - 3} = \dfrac{1 - \frac{3}{5}j}{j\frac{1}{5} - 3} = -\dfrac{39}{113} + \dfrac{20}{113}j = -0.345 + j0.177$

$\tilde{\rho}_0(-\Delta_1) = \tilde{\rho}_0 e^{2j\beta_0(-\Delta_1)}$

$\Rightarrow \tilde{\rho}_0 = \tilde{\rho}_0(-\Delta_1) e^{-2j\beta_0(-\Delta_1)}$

$\tilde{\rho}_0(-\Delta_1) \Rightarrow |\tilde{\rho}| = 0.388 \angle 152.85^\circ \approx 153^\circ$

$\tilde{\rho}_0(-\Delta_1 - \Delta_0) = \tilde{\rho}_0 e^{2j\beta_0 y} = 0.388\, e^{j\frac{153^\circ \pi}{180}} e^{-2j\frac{2\pi}{\lambda}\frac{3\lambda}{8}} = -0.18 - 0.35j$

$\tilde{\rho}_0(-\Delta_1 - \Delta_0) \Rightarrow |\rho| = 0.388 \angle -117^\circ$

$z_0(-\Delta_1 - \Delta_0) = z_0 \dfrac{1+\tilde{\rho}_0(-\Delta_1-\Delta_0)}{1-\tilde{\rho}_0(-\Delta_1-\Delta_0)} = z_0 \dfrac{1+(-0.18-0.35j)}{1-(-0.18-0.35j)} = \underline{z_0(0.57 - 0.46j)}$ ✗

10.3.2. Smith chart

$$\frac{z_L}{z_1} = r + jx \Rightarrow \frac{z_L}{z_1} = \frac{z_0/3}{z_0/2} = \frac{2}{3} \Rightarrow r = \frac{2}{3}, \; x = 0$$

Move Towards the source $\frac{1}{8}\lambda \rightarrow r = 0.94 \quad x = 0.37$

$$z_1(-\lambda/4) = \frac{z_0}{2}(0.94 + j\,0.37) = z_0(0.47 + 0.185j) \checkmark$$

Move towards the source $\frac{1}{4}\lambda \Rightarrow r = 0.94 \quad x = -0.4$

$$z_{sour} = z_0(0.94 - 0.4j) \quad \times$$

(3)

The Complete Smith Chart

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