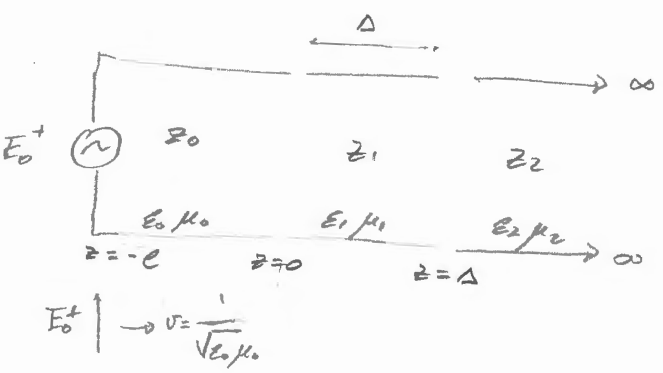
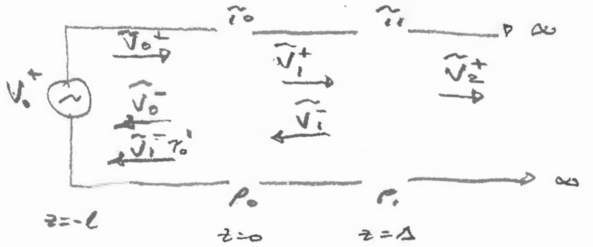
9.2 Multiple Impedances

9.2.1





$$\tilde{V}_n(z) = \tilde{V}_n^+ e^{-j\beta_n z} + \tilde{V}_n^- e^{+j\beta_n z} \Rightarrow V_n(z,t) = \mathrm{Re}\left[\tilde{V}_n(z)e^{j\omega t}\right]$$

V_0^+ is known and $V_2^- = 0$; Find $\tilde{V}_0^-(z)$, $\tilde{V}_1^+(z)$, $\tilde{V}_1^-(z)$ and $\tilde{V}_2^+(z)$

$$\beta_0 = \frac{\omega}{v_0} = \omega\sqrt{\varepsilon_0\mu_0} \quad ; \quad \beta_1 = \omega\sqrt{\varepsilon_1\mu_1}, \quad \beta_2 = \omega\sqrt{\varepsilon_2\mu_2}$$

$$\tilde{V}_0(z) = \tilde{V}_0^+ e^{-j\beta_0 z} + \tilde{V}_0^- e^{j\beta_0 z} \quad \text{and} \quad \rho_0 = \frac{Z_1 - Z_0}{Z_1 + Z_0}$$ from 5.7 (8)

$$\Rightarrow \tilde{V}_0^- = \tilde{V}_0^+ \rho_0 e^{j\beta_0 z} \quad \text{at } z=0 \Rightarrow \tilde{V}_0^-(z) = \tilde{V}_0^+\rho_0 = \tilde{V}_0^+\left(\frac{Z_1 - Z_0}{Z_1 + Z_0}\right)e^{j\beta_0 z}$$

$$\tilde{V}_1^+ = \tilde{V}_0^+ \tau_0 e^{-j\beta_1 z} \Rightarrow \text{for} \quad \tau_0 = \frac{2Z_1}{Z_1 + Z_0}$$ from 5.7 (9)

at $z=1$ $\quad \tilde{V}_1^+(z) = \tilde{V}_0^+ \tau_0 e^{-j\beta_1 z}$

$$\Rightarrow \text{at } z=\Delta \quad \tilde{V}_1^+(\Delta) = \tilde{V}_0^+ \tau_0 e^{-j\beta_1\Delta} = \tilde{V}_0^+\left(\frac{2Z_1}{Z_1+Z_0}\right)e^{-j\beta_1 z}$$

for $\tilde{V}_1^- = \tilde{V}_1^+ \rho_\Delta e^{j\beta_1\Delta}$ and $\rho_\Delta = \frac{Z_2 - Z_1}{Z_2 + Z_1}$ and at $z=\Delta$

$$\Rightarrow \tilde{V}_1^-(z) = \tilde{V}_0^+\left(\frac{2Z_1}{Z_1+Z_0}\right)\left(\frac{Z_2-Z_1}{Z_2+Z_1}\right)e^{j\beta_1 z}$$

at $z=\Delta \Rightarrow \tau = \frac{2Z_2}{Z_2+Z_1}$ and at $z=\Delta+1 \Rightarrow$

$$\Rightarrow \tilde{V}_2^+(z) = \tilde{V}_1^+ \tau e^{j\beta_2(\Delta+1)} = \tilde{V}_0^+\left(\frac{2Z_1}{Z_1+Z_0}\right)\left(\frac{2Z_2}{Z_2+Z_1}\right)e^{j\beta_2 z}$$

$$\tau_0' = \frac{V_L}{V^-} = \frac{2Z_1}{Z_1 - Z_0}$$

$$\tilde{V}_2^- = 0$$

Recalculate $\tilde{V}_0^- = \tilde{V}_0^-(0) + \tilde{V}_1^-(\Delta)\tau_0' = \tilde{V}_0^+\left(\frac{Z_1 - Z_0}{Z_1+Z_0}\right)e^{j\beta_0 z} + \tilde{V}_0^+\left(\frac{2Z_1}{Z_1+Z_0}\right)\left(\frac{Z_2-Z_1}{Z_2-Z_1}\right)\left(\frac{2Z_1}{Z_1-Z_0}\right)e^{j\beta_1 z}$

Find: $\dfrac{\tilde{V}_0^-(z)}{\tilde{V}_0^+(z)}$, $\dfrac{\tilde{V}_1^-(z)}{\tilde{V}_1^+(z)}$

$$\frac{\tilde{V}_0^-(z)}{\tilde{V}_0^+(z)} = \frac{\cancel{\tilde{V}_0^+}\left(\frac{Z_1-Z_0}{Z_1+Z_0}\right)e^{j\beta_0 z} + \cancel{\tilde{V}_0^+}\left(\frac{2Z_1}{Z_1+Z_0}\right)\left(\frac{Z_2-Z_1}{Z_2+Z_1}\right)\left(\frac{2Z_1}{Z_1-Z_0}\right)e^{j\beta_1 z}}{\cancel{\tilde{V}_0^+}\, e^{-j\beta_0 z}} =$$

$$= \frac{Z_1-Z_0}{Z_1+Z_0}e^{2j\beta_0 z} + \left(\frac{2Z_1}{Z_1+Z_0}\right)\left(\frac{Z_2-Z_1}{Z_2+Z_1}\right)\left(\frac{2Z_1}{Z_1-Z_0}\right)e^{jz(\beta_0+\beta_1)} =$$

$$= \rho_0 e^{2j\beta_0 z} + \tau_0 \rho_1 \tau_0' e^{jz(\beta_0+\beta_1)}$$

$$\frac{\tilde{V}_1^-(z)}{\tilde{V}_1^+(z)} = \frac{\cancel{\tilde{V}_0^+}\left(\cancel{\frac{2Z_1}{Z_1+Z_0}}\right)\left(\frac{Z_2-Z_1}{Z_2+Z_1}\right)e^{j\beta_1 z}}{\cancel{\tilde{V}_0^+}\left(\cancel{\frac{2Z_1}{Z_1+Z_0}}\right)e^{-j\beta_1 z}} = \left(\frac{Z_2-Z_1}{Z_2+Z_1}\right)e^{2j\beta_1 z} = \rho_1 e^{2j\beta_1 z}$$

9.2.2 . $V(z,t) = V^+[\cos(\omega t - \beta z) + \rho\cos(\omega t + \beta z)]$

1. $V(z,t) = A\cos\omega t\cos\beta z + B\sin\omega t\sin\beta z \Rightarrow$ find A and B

$V^+\cos(\omega t - \beta z) = V^+\cos\omega t\cos\beta z + V^+\sin\omega t\sin\beta t$

$V^+_\rho\cos(\omega t + \beta z) = V^+\rho\cos\omega t\cos\beta z - V^+_\rho\sin\omega t\sin\beta t.$

$V(z,t) = V^+\cos(\omega t - \beta z) + V^+\rho\cos(\omega t + \beta z) =$

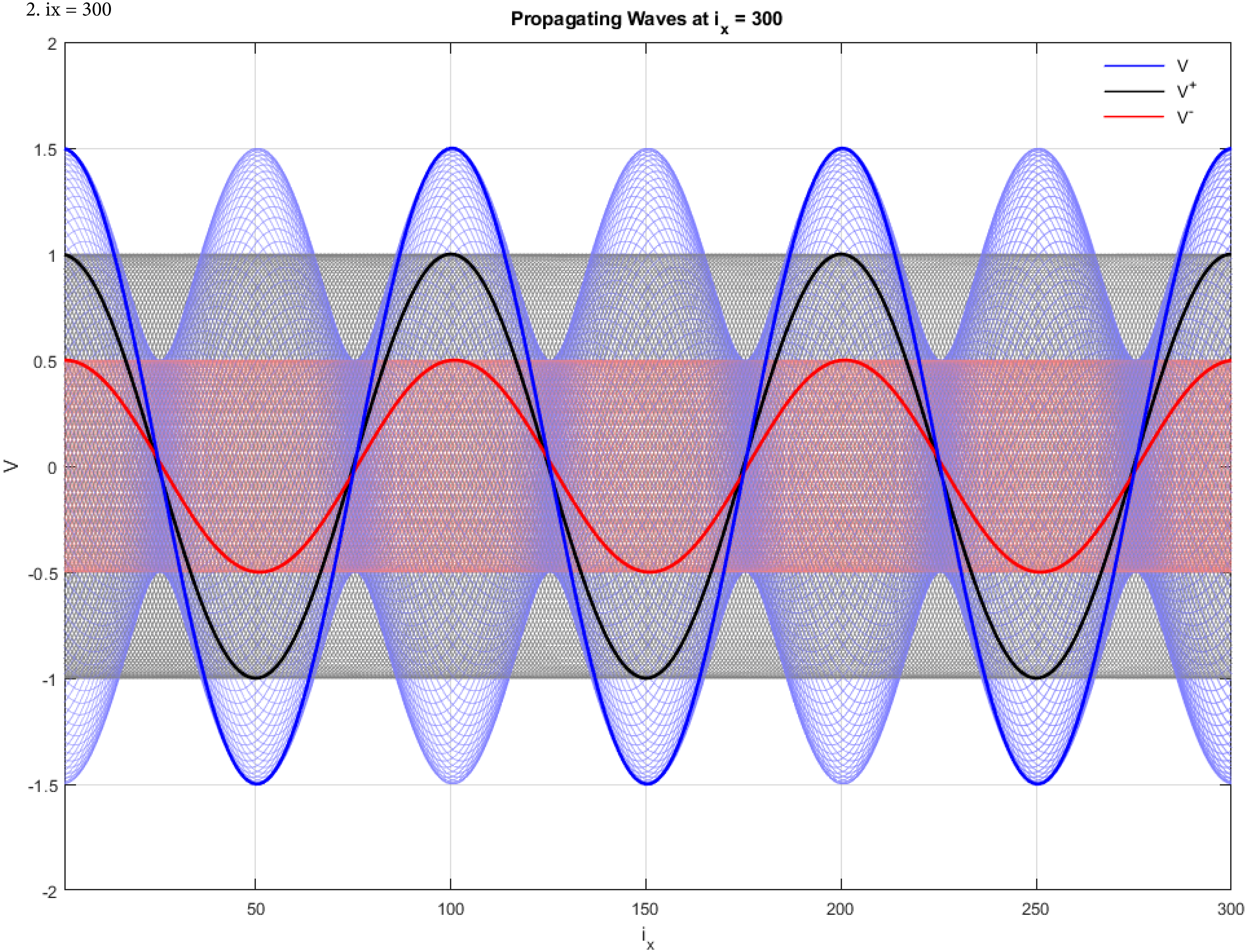
$= V^+\cos\omega t\cos\beta z + V^+\sin\omega t\sin\beta t + V^+\rho\cos\omega t\cos\beta z - V^+_\rho\sin\omega t\sin\beta z =$

$= V^+(1+\rho)\cos\omega t\cos\beta z + V^+(1-\rho)\sin\omega t\sin\beta z \Rightarrow$

$A = V^+(1+\rho)$ and $B = V^+(1-\rho) \Rightarrow$

$\Rightarrow A\cos\omega t\cos\beta z + B\sin\omega t\sin\beta z$

2. ix = 300


Propagating Waves at i_x = 300
V
V+
V-
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
50
100
150
200
250
300
i_x

3. By plotting a $V^- = 1/2$ traveling to the left, we are representing the reflected wave at $ix = 300$. When we add $V^+ = 1$ and $V^- = 0.5$ we can observe the $V_{max} = 1.5$ and $V_{min} = 0.5$ created in that wave (blue in my plot). From this V_{max} and V_{min}, we can obtain the Voltage Wave Standing Ratio (VSWR) by.

$$VSWR = \frac{V_{max}}{V_{min}} = \frac{1.5}{0.5} = 3$$

Likewise we could Obtain the magnitude of the reflection coefficient ρ by

$$|\rho| = \frac{s-1}{s+1} = \frac{2}{4} = 0.5$$

However, just from the plot is difficult to calculate θ_ρ. to obtain $\rho = |\rho| e^{j\theta_\rho}$, but an attempt looking at V_{min} from $ix=300$ looks like at $ix = 275$, or (0.25λ) and

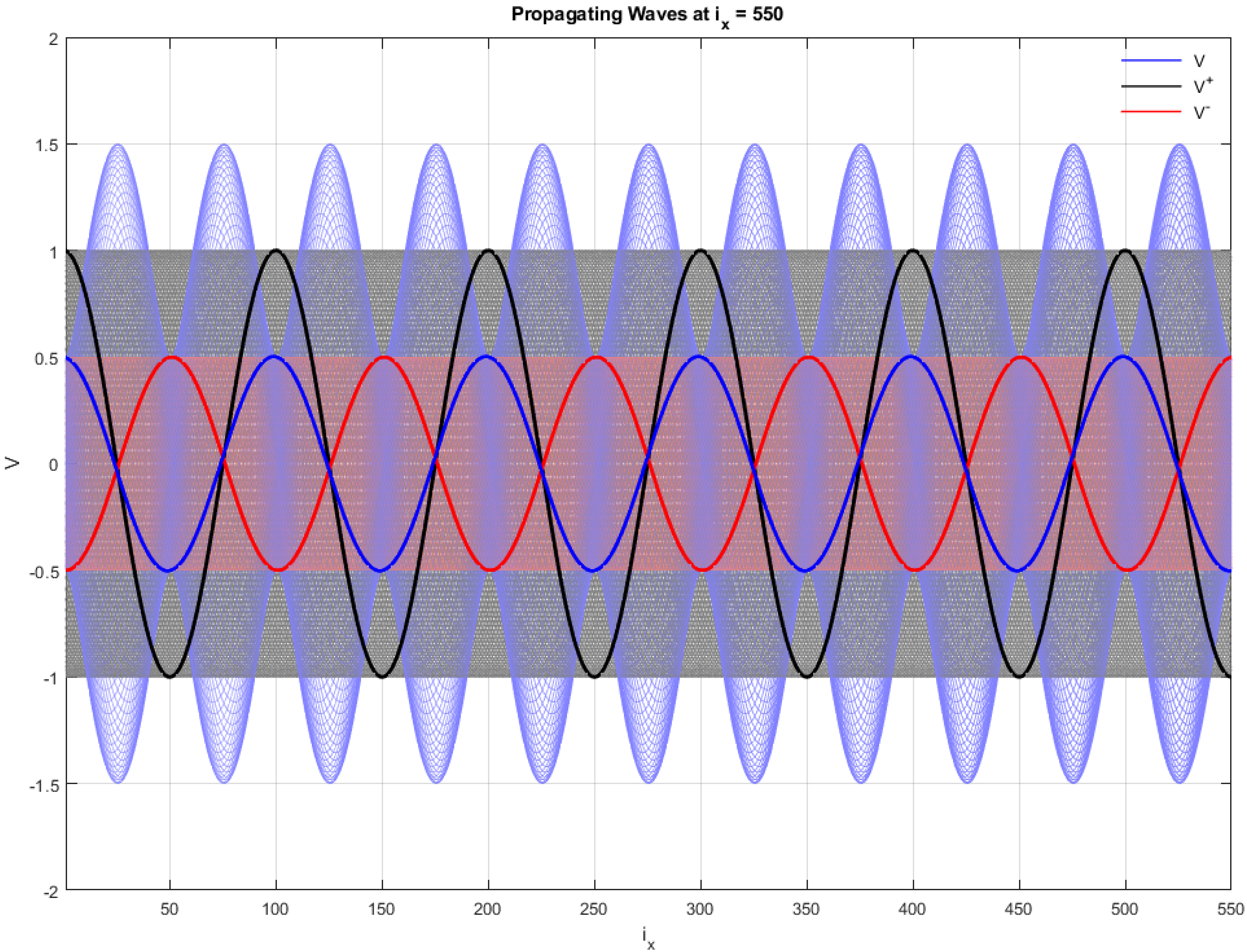
$$\theta_\rho = 2\beta(0.25\lambda) - \pi \quad ; \text{ for } \beta = \frac{\omega}{c} = \frac{2\pi f}{c} = \frac{2\pi}{\lambda}$$

$$\theta_\rho = 4\pi(0.25) - \pi = 0$$

This confirms the lack of phase difference in V^+ and V^- accross the plot, and:

$$\rho = |\rho| = \frac{1}{2}$$

4. ix = 550



Propagating Waves at i_x = 550
V
V^+
V^-
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
50
100
150
200
250
300
350
400
450
500
550
i_x

4. From the plot, and as before in 3.

$$VSWR = \frac{V_{max}}{V_{min}} = \frac{1.5}{0.5} = 3$$

$$|\rho| = \frac{S-1}{S+1} = 0.5$$

however, with $ix = 550$, θ_ρ has changed, and V_{min} is at $ix = 500$ or (0.5λ), therefore

$$\theta_\rho = 2\beta(0.5\lambda) - \pi = \pi$$

which seems about right when comparing V^+ and V^- that show about $180°$ difference.