11.2. Final Part II

Used the function created in the last problem (11.1.4) to calculate and return the values of Z_k, V_k, and I_k. As we know from HW 9.2.1, to obtain $V_n(z,t)$ from $\tilde{V}_n(z)$, we use

$\hat{V}_n(z,t) = Re[\tilde{V}_n(z) e^{j\omega t}]$, so apply this to our $\bar{V}_k$ and $\bar{I}_k$

we obtaining:

$$\hat{V}_k(t) = Re[\tilde{V}_k e^{j\omega t}] \quad \text{and} \quad \tilde{I}_k(t) = Re[\tilde{I}_k e^{j\omega t}]$$

Final_Part_II_code.m executes this code in part of the program.

1. Ploting $V_k(t)$ at $t=0$ in Final_Part_II_Figure_1.pdf, we see that at $t=0$, $V_k(t)$ and $I_k(t)$ follow V_k and I_k exactly, as at $t=0$, the $\cos(\omega t) = 1$. In addition, in the third subplot of Figure_1, we compare Z_k, with $\frac{V_k(t)}{I_k(t)}$, and see that both match very well.

Figure_1 presents the solution for $Z_L = \sqrt{L/C}$ for a matched impedance, as $Z_0 = \sqrt{L/C}$ as $\Delta z \to 0$. for the transmition line, and the amplitud of the waves do not see any reflection.

In Figure_2, the load impedance was changed to $Z_L = 5 * \sqrt{L/s}$ to represent a missmatch in impedance, where we see that the amplitud of V_k and $V_k(t)$ has increse to account for the forward and reflected waves.. In this case, we also see that $V_k(t)$, $I_k(t)$, and $Z_k(t)$ follow their counterparts V_k, I_k and Z_k

2. For $t = (2\pi/\omega)/4$, the $\cos(\omega t) = 0$ as the ω cancel, and the cosine function is evaluated at $\frac{2\pi}{4}$ or $90°$. In this case, as we can see in Figure_3, $V_k(t)$ is out of phase with V_k the corresponding $90°$, and the wave starts propagating at $N=0$ with a value of 0. The current plot follows similar pattern, and Z_k and $Z_k(t)$ still matches.

In Figure_4, Z_L was changed to create a missmatch ($Z_L = 5 * \sqrt{L/C}$) with the transmission line. Same as in Figure_2, the amplitude of $V_k(t)$ and $I_k(t)$ has changed to incorporate the reflected wave. It is of interest to note (same as in Figure_2) that the impedance of the transmission line drops to almost 0 for most of the transmission line, except at values of $\frac{\lambda}{2}$ from Z_L, where the source sees Z_L at this point (as it should) wich is $N = 372$ in our case.

Discrete vs exact solution

1. for $Z_L = \sqrt{L/C}$ and $Z_0 = \sqrt{L/C}$ as the characteristic impedance of the transmition line used for the exact solution.

Figures 5 an 6 show the discrete $V_k(t)$, $I_k(t)$ and Z_k vs the exact $V_Z(t)$, $I_Z(t)$ and $Z(t) = \frac{V_Z(t)}{I_Z(t)}$ for $t=0$ and $t = \left(\frac{2\pi}{\omega}\right)/4$. In this figures we can observe how well the discrete solution matches the exact solution when Z_L is matched to the transmition line impedance $Z_0 = \sqrt{L/C}$

2. For $Z_L \neq \sqrt{L/C}$ in our case $Z_L = 5 * \sqrt{L/C}$ we can observe in Figure 7 for $t=0$ that $I_k(t)$ and $I_z(t)$ matched quite well, but $V_z(t)$ deviates from $V_k(t)$ significantly. By contrast in Figure 8, when $t = \left(\frac{2\pi}{\omega}\right)/4$ $V_z(t)$ follows $V_k(t)$, while $I_z(t)$ deviates from $I_k(t)$.

This may be due to the impedance of the exact transmition line fixed at $Z_0 = \sqrt{L/C}$, that we can see in both figures that it remains at 1Ω, instead of being close to 0 and only seen 5Ω at $\frac{d}{2}$.

Both Figures, however, show an increase in Voltage and current amplitude due to the reflected wave.

Of interest is to observe Figures 10, 11 and 12, where the exact calculation was done to allow the impedance of the transmission line to change for each LC combination. In figure 10, we can see that the exact calculation of $V_z(t)$ and $I_z(t)$ change "frequency" as it progresses through the transmission line, refering to the velocity propagation of the wave changing at each Z_k stage. As we discussed in the piazza discussion of problem 10.2 about the change in the propagation velocity changing the wavelength, and the wave seems to compress as it travels along the transmission line. Figures 11 and 12 show the result of this with an impedance missmatch, at $t=0$ and $t = \left(\frac{2\pi}{\omega}\right)/4$.

In the cases shown in Figures 10, 11 and 12, the impedance of the exact solution $Z(t) = \frac{V_Z(t)}{I_Z(t)}$ matches the impedance of Z_k and $\frac{V_k(t)}{I_k(t)}$, even though the currents and voltages of both the exact and discrete approximation deviate from each other as N increases.

3. In the plots of Figure 9, we can observe how the error between $V_k(t)$ and $V_Z(t)$ decreases when changing w from 0.1 to 0.01 and 0.001.