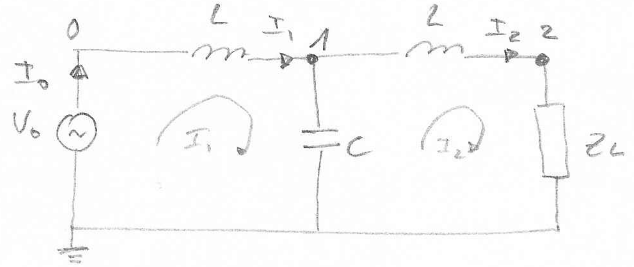
11.1 Part I



11.1.1. Apply Kirchhoff current and voltage law.:

Voltage around each loop:

1st loop: $\tilde{V}_0 - L\dfrac{d\tilde{I}_1}{dt} - \dfrac{1}{C}\int(\tilde{I}_1 - \tilde{I}_2)\,dt = 0 \Rightarrow$ for $\dfrac{1}{C}\int(\tilde{I}_1 - \tilde{I}_2)\,dt = \tilde{V}_1$

$\tilde{V}_0 - L\dfrac{d\tilde{I}_1}{dt} - \tilde{V}_1 = 0$ replacing $\dfrac{d}{dt}$ with $j\omega \Rightarrow$

$\tilde{V}_0 - j\omega L\tilde{I}_1 - \tilde{V}_1 = 0 \Rightarrow \tilde{V}_1 = \tilde{V}_0 - j\omega L\tilde{I}_1$ ✓

2nd loop: $-\dfrac{1}{C}\int(\tilde{I}_2 - \tilde{I}_1)\,dt - L\dfrac{d\tilde{I}_2}{dt} - \tilde{I}_2 Z_L = 0$

for $\dfrac{1}{C}\int(\tilde{I}_2 - \tilde{I}_1)\,dt = -V_1$ and $\tilde{I}_2 Z_L = \tilde{V}_2$

$\tilde{V}_1 - L\dfrac{d\tilde{I}_2}{dt} - \tilde{V}^2 = 0$ replacing $\dfrac{d}{dt}$ with $j\omega$

$\tilde{V}_2 = \tilde{V}_1 - j\omega L\tilde{I}_2$ ✓

current at each Node: 0 is not a node.

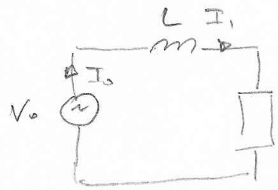
Node 1: $\underbrace{\dfrac{1}{L}\int(\tilde{V}_1 - \tilde{V}_0)\,dt}_{-\tilde{I}_1} + \underbrace{\dfrac{1}{L}\int(\tilde{V}_1 - \tilde{V}_2)\,dt}_{\tilde{I}_2} + C\dfrac{d\tilde{V}_1}{dt} = 0 \Rightarrow -\tilde{I}_1 + \tilde{I}_2 + C\dfrac{d\tilde{V}_1}{dt} = 0$

$\Rightarrow$ replacing $\dfrac{d}{dt}$ with $j\omega \Rightarrow \tilde{I}_2 = \tilde{I}_1 - j\omega C\tilde{V}_1$ ✓

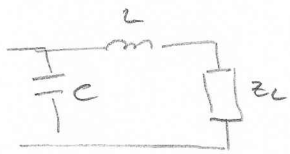
Node 2: $\underbrace{\frac{1}{L}\int(\tilde{V}_2-\tilde{V}_1)\,dt}_{-\tilde{I}_2}+\frac{\tilde{V}_2}{z_L}=0 \Rightarrow -\tilde{I}_2+\frac{\tilde{V}_2}{z_L}=0$

$$\tilde{I}_2=\frac{\tilde{V}_2}{z_L} \quad ✓$$

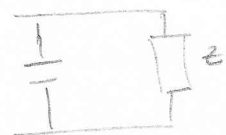
11.1.2 $\Rightarrow \tilde{V}_0=V_0$ and $\tilde{I}_1=\frac{\tilde{V}_0}{z_0}$ with $\tilde{I}_1=\tilde{I}_0$



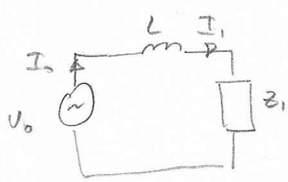
for z_1 =



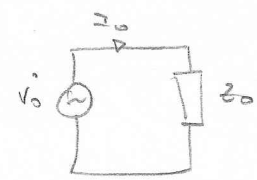
$z_1' = z_L + j\omega L \Rightarrow z_1 =$



$\frac{1}{z_1}=\frac{1}{z_C}+\frac{1}{z_1'}$; $\quad z_1=\frac{z_1'\,z_C}{z_1'+z_C}=\frac{(z_L+j\omega L)\frac{1}{j\omega C}}{z_L+j\omega L+\frac{1}{j\omega C}}$



$z_0 = z_L + z_1 \Rightarrow$



$$z_0 = z_1 + j\omega L = \frac{z_L+j\omega L}{j\omega C z_L-\omega^2 LC+1}+j\omega L$$

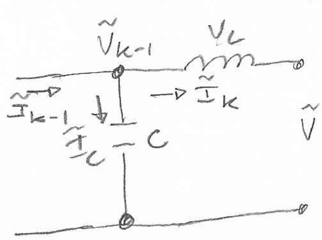
11.1.3

$\tilde{I}_k = I_{k-1} - j\omega C\tilde{V}_{k-1}$ and $\tilde{V}_k=\tilde{V}_{k-1}-j\omega L\tilde{I}_k$

$\tilde{I}_2 = \tilde{I}_1 - j\omega C\tilde{V}_1$ and $\tilde{V}_2=\tilde{V}_1-j\omega L\tilde{I}_2$

$\tilde{I}_1 = \tilde{I}_0 - j\omega \cancel{C}\tilde{V}_0$ and $\tilde{V}_1=\tilde{V}_0-j\omega L I_1$

$\tilde{I}_1 = \tilde{I}_0$



$\tilde{V}_k=\tilde{V}_{k-1}-j\omega L\tilde{I}_k$ is obvious, as V_k will be equal to V_{k-1} minus the voltage drop in the inductor L denoted by $V_L=j\omega L\tilde{I}_k$.

$\Rightarrow$

Likewise, $\tilde{I}_k$ is the total current arriving at the node $\tilde{I}_{k-1}$ minus the current drawn by the capacitor $I_c = j\omega C \tilde{V}_{k-1}$

Therefore the generalitization is corret:

$$I_k = I_{k-1} - j\omega C \tilde{V}_{k-1} \quad \text{and} \quad \tilde{V}_k = \tilde{V}_{k-1} - j\omega L \tilde{I}_k \quad \checkmark$$

In time domain with a Z_L matched impedance, and no reflecting wave:

$$\tilde{V}_0(t) = V_0 \cos \omega t = V_0 e^{j\omega t} \quad \text{and} \quad \tilde{I}_0(t) = \frac{\tilde{V}_0(t)}{Z_0}$$

$$\tilde{V}_1(t) = \tilde{V}_0(t) - L \frac{d\tilde{I}_1(t)}{dt} \quad \text{and} \quad \tilde{I}_1(t) = \tilde{I}_0(t) - C \frac{dV_1(t)}{dt}$$

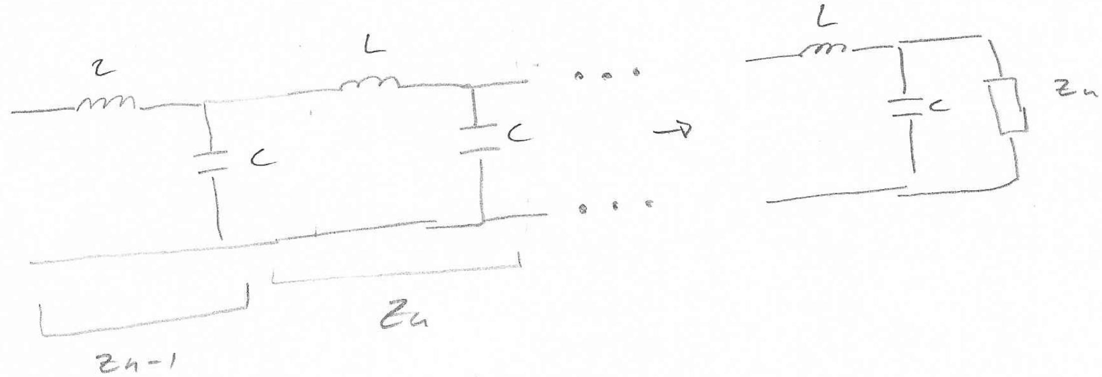
$$\tilde{V}_2(t) = \tilde{V}_1(t) - L \frac{dI_2(t)}{dt} \quad \text{and} \quad I_2(t) = I_1(t) - C \frac{dV_2(t)}{dt}$$

$$\tilde{V}_k(t) = \tilde{V}_{k-1}(t) - L\, d\tilde{I}_k(t)$$

$$\tilde{I}_k(t) = \tilde{I}_{k-1}(t) - C \frac{dV_k(t)}{dt}$$

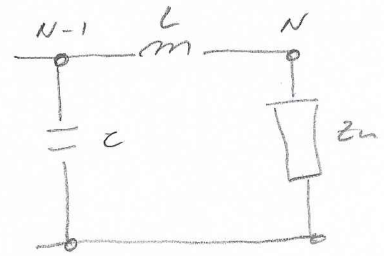
11.1.4

1.



$$Z_{n-1} = \frac{Z_n \frac{1}{j\omega C}}{Z_n + \frac{1}{j\omega C}} + j\omega L \qquad \text{For } Z_N = Z_L,\ Z_n = Z_N + j\omega L$$

or



$$Z_{n-1} = \frac{(Z_n + j\omega L)\frac{1}{j\omega C}}{Z_n + j\omega L + \frac{1}{j\omega C}} =$$

$$= \frac{Z_n + j\omega L}{Z_n j\omega C + j\omega C j\omega L + 1} \Rightarrow$$

$$Z_{n-1} = \frac{Z_n + j\omega L}{Z_n j\omega C - \omega^2 C L + 1}$$

and $Z_0 = j\omega L + Z_1$

2. $\tilde{I}_k = \tilde{I}_{k-1} - j\omega C \tilde{V}_{k-1}$ for $k = 0, 1, \dots, N$ and $\tilde{I}_1 = \frac{\tilde{V}_0}{Z_0}$

3. $\tilde{V}_k = \tilde{V}_{k-1} - j\omega L \tilde{I}_k$