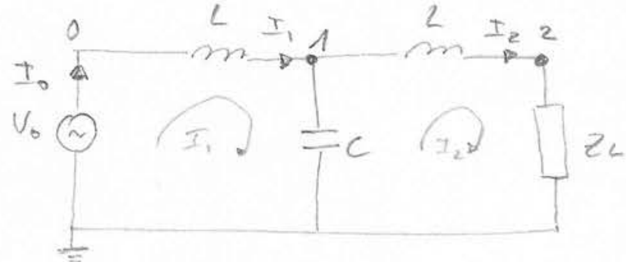
11.1 Part I



11.1.1. Apply Kirchhoff current and voltage law.:

voltage around each loop:

1st loop: $\tilde{V}_0 - L\dfrac{d\tilde{I}_1}{dt} - \dfrac{1}{C}\displaystyle\int(\tilde{I}_1 - \tilde{I}_2)\,dt = 0 \quad \Rightarrow \text{ for } \frac{1}{C}\int(\tilde{I}_1 - \tilde{I}_2)\,dt = \tilde{V}_1$

$$\tilde{V}_0 - L\frac{d\tilde{I}_1}{dt} - \tilde{V}_1 = 0 \quad \text{replacing } \frac{d}{dt} \text{ with } j\omega \Rightarrow$$

$$\tilde{V}_0 - j\omega L\tilde{I}_1 - \tilde{V}_1 = 0 \quad \Rightarrow \quad \tilde{V}_1 = \tilde{V}_0 - j\omega L\tilde{I}_1 \checkmark$$

2nd loop: $-\dfrac{1}{C}\displaystyle\int(\tilde{I}_2 - \tilde{I}_1)\,dt - L\dfrac{d\tilde{I}_2}{dt} - \tilde{I}_2 Z_L = 0$

$$\text{for } \frac{1}{C}\int(\tilde{I}_2 - \tilde{I}_1)\,dt = -V_1 \text{ and } \tilde{I}_2 Z_L = \tilde{V}_2$$

$$\tilde{V}_1 - L\frac{d\tilde{I}_2}{dt} - \tilde{V}^2 = 0 \quad \text{replacing } \frac{d}{dt} \text{ with } j\omega$$

$$\tilde{V}_2 = \tilde{V}_1 - j\omega L\tilde{I}_2 \checkmark$$

current at each Node: 0 is not a node.

Node 1: $\underbrace{\dfrac{1}{L}\displaystyle\int(\tilde{V}_1 - \tilde{V}_0)\,dt}_{-\tilde{I}_1} + \underbrace{\dfrac{1}{L}\displaystyle\int(\tilde{V}_1 - \tilde{V}_2)\,dt}_{\tilde{I}_2} + C\dfrac{d\tilde{V}_1}{dt} = 0 \Rightarrow -\tilde{I}_1 + \tilde{I}_2 + C\dfrac{d\tilde{V}_1}{dt} = 0$

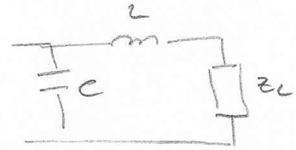
$$\Rightarrow \text{replacing } \frac{d}{dt} \text{ with } j\omega \Rightarrow \tilde{I}_2 = \tilde{I}_1 - j\omega C\tilde{V}_1 \checkmark$$

Node 2: $\underbrace{\frac{1}{L}\int(\tilde{V}_2-\tilde{V}_1)\,dt}_{-\tilde{I}_2} + \frac{\tilde{V}_2}{z_L} = 0 \Rightarrow -\tilde{I}_2 + \frac{\tilde{V}_2}{z_L} = 0$

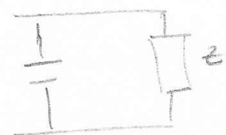
$$\tilde{I}_2 = \frac{\tilde{V}_2}{z_L} \checkmark$$

11.1.2 $\Rightarrow$ $\tilde{V}_0 = V_0$ and $\tilde{I}_1 = \frac{\tilde{V}_0}{z_0}$ with $\tilde{I}_1 = \tilde{I}_0$

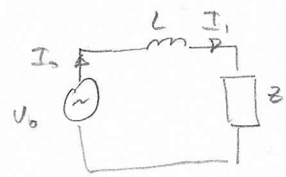
for $z_1 =$



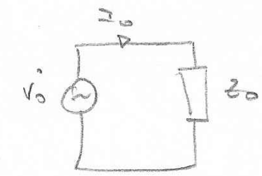
$z_1' = Z_L + j\omega L \Rightarrow z_1 =$



$$\frac{1}{z_1} = \frac{1}{Z_C} + \frac{1}{z_1'}\,; \qquad z_1 = \frac{z_1' Z_C}{z_1' + Z_C} = \frac{(z_L + j\omega L)\frac{1}{j\omega C}}{z_L + j\omega L + \frac{1}{j\omega C}}$$



$z_0 = z_L + z_1 \Rightarrow$



$$Z_0 = Z_1 + j\omega L = \frac{z_L + j\omega L}{j\omega C z_L - \omega^2 LC + 1} + j\omega L$$

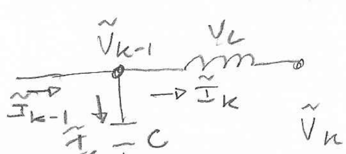
11.1.3

$\tilde{I}_k = I_{k-1} - j\omega C \tilde{V}_{k-1}$ and $\tilde{V}_k = \tilde{V}_{k-1} - j\omega L \tilde{I}_k$

$\tilde{I}_2 = \tilde{I}_1 - j\omega C \tilde{V}_1$ and $\tilde{V}_2 = \tilde{V}_1 - j\omega L \tilde{I}_2$

$\tilde{I}_1 = \tilde{I}_0 - j\omega \cancel{C} \tilde{V}_0$ and $\tilde{V}_1 = \tilde{V}_0 - j\omega L I_1$

$\tilde{I}_1 = \tilde{I}_0$



$\tilde{V}_k = \tilde{V}_{k-1} - j\omega L \tilde{I}_k$ is obvious, as V_k will be equal to V_{k-1} minus the voltage drop in the inductor L denoted by $V_L = j\omega L \tilde{I}_k$.

$\Rightarrow$

Likewise, $\tilde{I}_k$ is the total current arriving at the node $\tilde{I}_{k-1}$ minus the current drawn by the capacitor $I_c = j\omega C \tilde{V}_{k-1}$

Therefore the generalitization is correct:

$$I_k = I_{k-1} - j\omega C \tilde{V}_{k-1} \quad \text{and} \quad \tilde{V}_k = \tilde{V}_{k-1} - j\omega L \tilde{I}_k \quad \checkmark$$

In time domain with a Z_L matched impedance, and no reflecting wave:

$$\tilde{V}_o(t) = V_o \cos\omega t = V_o e^{j\omega t} \quad \text{and} \quad \tilde{I}_o(t) = \frac{\tilde{V}_o(t)}{Z_o}$$

$$\tilde{V}_1(t) = \tilde{V}_o(t) - L\frac{d\tilde{I}_1(t)}{dt} \quad \text{and} \quad \tilde{I}_1(t) = \tilde{I}_o(t) - C\frac{dV_1(t)}{dt}$$

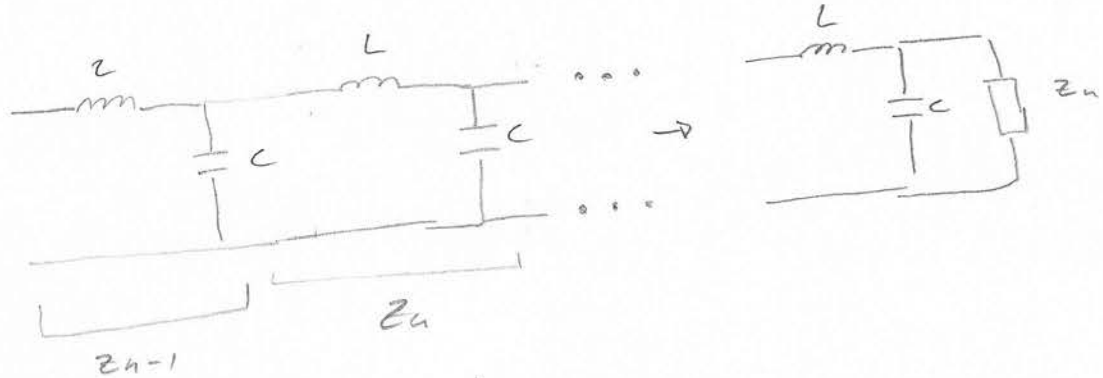
$$\tilde{V}_2(t) = \tilde{V}_1(t) - L\frac{dI_2(t)}{dt} \quad \text{and} \quad I_2(t) = I_1(t) - C\frac{dV_2(t)}{dt}$$

$$\tilde{V}_k(t) = \tilde{V}_{k-1}(t) - L\, d\tilde{I}_k(t)$$

$$\tilde{I}_k(t) = \tilde{I}_{k-1}(t) - C\frac{dV_k(t)}{dt}$$

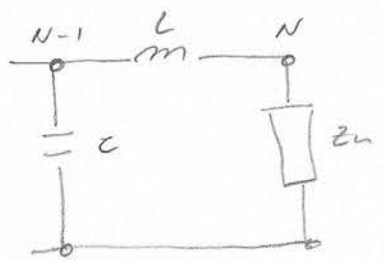
11.1.4

1.



$$z_{n-1} = \frac{z_n \frac{1}{j\omega C}}{z_n + \frac{1}{j\omega C}} + j\omega L \qquad \text{For } Z_N = Z_L,\ Z_n = Z_N + j\omega L$$

or



$$z_{n-1} = \frac{(z_n + j\omega L)\frac{1}{j\omega C}}{z_n + j\omega L + \frac{1}{j\omega C}} =$$

$$= \frac{z_n + j\omega L}{z_n j\omega C + j\omega C j\omega L + 1} \Rightarrow$$

$$z_{n-1} = \frac{z_n + j\omega L}{z_n j\omega C - \omega^2 C L + 1}$$

and $z_0 = j\omega L + z_1$

2. $\hat{I}_k = \hat{I}_{k-1} - j\omega C \hat{V}_{k-1}$ for $k = 0, 1, \dots, N$ and $\hat{I}_1 = \frac{\hat{V}_0}{z_0}$

3. $\hat{V}_k = \hat{V}_{k-1} - j\omega L \hat{I}_k$