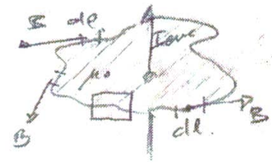
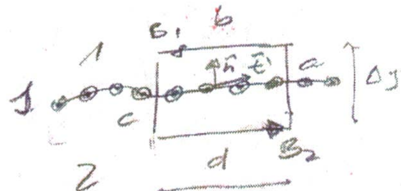
5.1. Boundary Value Derivation.

- from Ampere's Law $\rightarrow \oint_c B \cdot dl = \mu_0 I_{enc}$



$\nabla \times B = \mu_0 J$

- current flowing out of the page



stokes $\rightarrow \oint B\,dl = \int_a B\,dl + \int_b B\,dl + \int_c B\,dl + \int_d B\,dl$

as $\Delta y \rightarrow 0$ $\oint B\,dl = \int_b B_1\,dl + \int_d B_2\,dl$.

Scalar vector

$\Rightarrow \int_b B_1\,dl + \int_d B_2\,dl = B_1(-\Delta x)\hat{t} + B_2(\Delta x)\hat{t} = (B_2 - B_1)\Delta x\,\hat{t}$ X

and $I_{enc} \Rightarrow \underline{J\,\Delta x\,(\hat{t}\times\hat{n})}$ this is not obvious. diagram would help.

$(B_2 - B_1)\cancel{\Delta x}\,\hat{t} = J\,\cancel{\Delta x}\,(\hat{t}\times\hat{n}) \Rightarrow$ X $(B_2 - B_1)\cancel{\hat{t}} = \cancel{\hat{t}}(\hat{n}\times J)$

$\hat{t}(\hat{n}\times\vec{J})$ is a tensor.

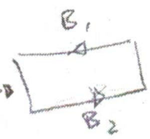
$\underline{n \times J}$

$\underline{J=0} \Rightarrow \nabla\times B = 0$

if $J=0 \Rightarrow B_{t2} = B_{t1}$

if there is a current normal to $\hat{n}$, then there is a discontinuity in B

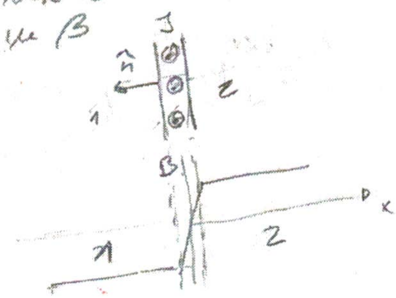
this is a major step that needs explanation.



from before $\rightarrow$

$\int_a^b B\,dl = V(a) - V(b) = 0$

odd notation



so as we go around they cancel out,

but in our case for $J=0$

$V(a) - V(b) \neq 0$

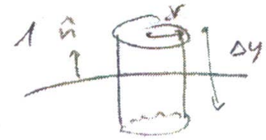
$\hat{n}\times(B_2 - B_1) = \hat{n}\times\hat{n}\times J = -J$

$\hat{n}\times(B_1 - B_2) = J$

$B\int_a^b dl = V(a) - V(b) \rightarrow B = -\nabla\psi$

will not work for arbitrary $\vec{B}$.

$\nabla \cdot B = 0$ from gauss law $\oint_s B \cdot ds = 0$ ✓



$\rightarrow \oint_s B \cdot ds = \int_{top} B\,ds + \int_{side} B\,ds + \int_{bottom} B\,ds$

$\Delta y \to 0 \quad \int_{side} B\,ds = 0 \; . \Rightarrow \int_{top} B_1\,ds + \int_{bottom} B\,ds = 0$

as $r \to 0$ Area of top and bottom $= \Delta S$ ✗

Area $= \pi r^2$

$(\vec{B}_{1n} - \vec{B}_{2n}) \Delta S = 0$

$\Rightarrow$ for arbitrarily small ΔS $\vec{B}_{1n} = \vec{B}_{2n}$

$\int \vec{B} \cdot d\vec{S} =$ Scalar

$\Rightarrow \hat{n} B_1 \Delta s + (-\hat{n}) \vec{B}_2 \Delta s = 0$

as ΔS becomes negligible.

you get $0 = 0$ when $\Delta S = 0$. Needs discussion

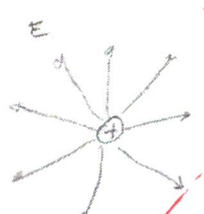
$\hat{n}(B_1 - B_2) = 0$ ✗

and the normal component of B across boundary is continuous

Put $\hat{n}$ in at end. you showed

$B_{1n} - B_{2n} = 0 \Rightarrow \hat{n} \cdot (\vec{B}_1 - \vec{B}_2) = 0$

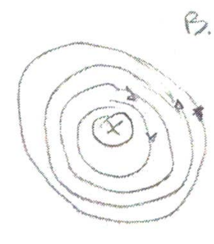
From the graphical point of view



There is nothing curly so $\nabla \times E = 0$

but the lines diverge from the charge so there must be some $\nabla \cdot E$

more generally, from a point

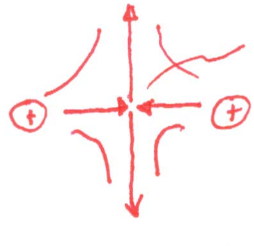


Also divergence as the lines go around in circles. $\nabla \cdot B = 0$

Also the lines "curl" around. so there must be $\nabla \times B$

✗

"Curly" fields do not imply $\vec{\nabla} \times \vec{B} = 0$

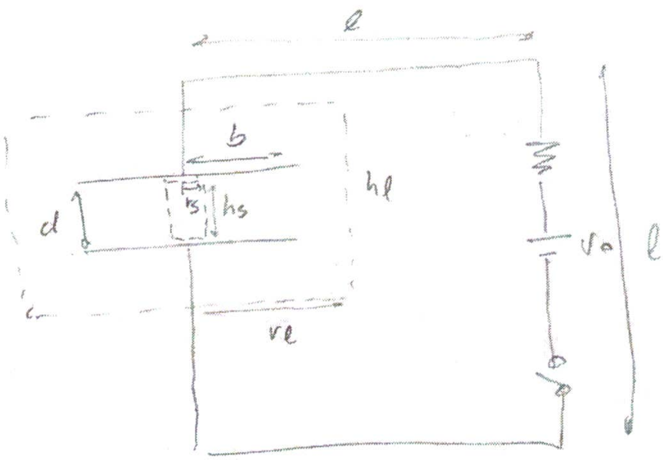


some lines diverge. $\Rightarrow \nabla \cdot \vec{E} \neq 0$?

$B \sim \frac{\hat{\phi}}{s}$

looks curly. $\nabla \times \vec{B} = 0$ for $s \neq 0$

5.2 Poynting Theorem.



$hs < d$

$d < hl < l$

- 1. Ignoring fringing fields and calculating using Gauss law

$$\oint_S E\, da = \frac{Q_{enc}}{\varepsilon_0} \Rightarrow \quad A_{cyl.area} = \pi r_s^2 \Rightarrow \sigma = \frac{Q}{\pi r_s^2}$$

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 \pi r_s^2}$$

Total Energy stored in E:

$$U_e = \frac{1}{2}\varepsilon_0 \int_V E^2\, dv = \frac{1}{2}\varepsilon_0 E^2 \pi r_s^2 h_s = \frac{1}{2}\varepsilon_0 \left(\frac{Q}{\varepsilon_0 \pi r_s^2}\right)^2 \pi r_s^2 h_s =$$

$$U_e = \frac{1}{2}\frac{Q^2 h_s}{\varepsilon_0 \pi r_s^2}$$

$$\text{From (6)} \rightarrow \int_V \left[\frac{\partial}{\partial t}\left(\frac{1}{2}\frac{B^2}{\mu_0}\right) + \underbrace{\frac{d}{dt}\overbrace{\left(\frac{1}{2}\varepsilon_0 E^2\right)}^{U_e}}_{\text{evaluate}} + E\cdot J\right] dv.$$

$$\frac{d}{dt}U_e = \frac{Q h_s}{\varepsilon_0 \pi r_s^2}\frac{dQ}{dt} \quad \text{with} \quad \frac{dQ}{dt} = I \Rightarrow \frac{d}{dt}U_e = \frac{Q h_s I}{\varepsilon_0 \pi r_s^2} \checkmark$$

From Ampere's Law

$$\oint_C B\cdot ds = \mu_0 \underbrace{\iint J\cdot da}_{0} + \mu_0 \varepsilon_0 \frac{d}{dt}\iint E\, da$$

0 → No current through a capacitor

Choose Amperian loop of radius r_a for $r_a < r_s$

$$\oint_c B\,ds = B\,2\pi r_a \quad \text{and} \quad \mu_0 \varepsilon_0 \frac{d}{dt}\iint E\,da = \mu_0\varepsilon_0 \frac{dE}{dt}\pi r_a^2$$

$$\frac{dE}{dt} = \frac{Q}{\varepsilon_0 \pi r_s^2}$$

$$B\,2\pi r_a = \mu_0 \frac{Q \pi r_a^2}{\pi r_s^2} \Rightarrow B = \frac{\mu_0 Q r_a}{2\pi r_s^2}$$

$$\vec{B} = \frac{\mu_0 Q r_a}{2\pi r_s^2}$$

$$U_m = \frac{1}{2\mu_0}\int_V B^2\,dV = \frac{1}{2\mu_0}\left(\frac{\mu_0 Q r_a}{2\pi r_s^2}\right)^2 \pi r_s^2 h_s = \frac{1}{8}\frac{\mu_0 Q^2 r_a^2 h_s}{\pi r_s^2}$$

$$\frac{d}{dt}U_m = \frac{1}{4}\frac{\mu_0 Q r_a^2 h_s}{\pi r_s^2}\frac{dQ}{dt} \Rightarrow \frac{dQ}{dt} = I$$

$$\frac{d}{dt}U_m = \frac{1}{4}\frac{\mu_0 Q I r_a^2 h_s}{\pi r_s^2} \checkmark$$

No ohmic losses so $E \cdot J = 0$.

Eq. 6 becomes $\frac{1}{4}\frac{\mu_0 Q I r_a^2 h_s}{\pi r_s^2} + \frac{Q I h_s}{\varepsilon_0 \pi r_s^2} \Rightarrow Q(t) = It. \Rightarrow$

$$\Rightarrow \frac{I^2 h_s}{\pi r_s^2}\left(\frac{\mu_0 r_a^2}{4} + \frac{1}{\varepsilon_0}\right)t = -\frac{1}{\mu_0}\oint_S (E\times B)\,ds$$

should compute

2. For the large cylinder, replace h_s with d and r_s with b. However in this case the fringing field plays a role, and add to the total work $\frac{dw}{dt}$ are.

$$\frac{I^2 d}{\pi b^2}\left(\frac{\mu_0 r_a^2}{4} + \frac{1}{\varepsilon_0}\right) + \frac{dw}{dt} = -\frac{1}{\mu_0}\oint_S (E\times B)\,ds$$

HW#5 Jesus Gil Gil

5.3. Wave Superposition.

$$E_x = A\cos(\omega t - kx + \delta)$$

$$E_x = A_1\cos(\omega t - kx + \delta_1) + A_2\cos(\omega t - kx + \delta_2)$$

$$E_x = A_3\cos(\omega t - kx + \delta_3)$$

$$\tilde{A}_3 e^{i(\omega t - kx)} = \tilde{A}_1 e^{i(\omega t - kx)} + \tilde{A}_2 e^{i(\omega t - kx)}$$

$$\tilde{A}_3 = \tilde{A}_1 + \tilde{A}_2 \rightarrow A_3 e^{i(\delta_3)} = A_1 e^{i(\delta_1)} + A_2 e^{i\delta_2} \quad \checkmark$$

$$A_3 = \sqrt{(A_3 e^{i\delta_3})(A_3 e^{-i\delta_3})} = \sqrt{(A_1 e^{i\delta_1} + A_2 e^{i\delta_2})(A_1 e^{-i\delta_1} + A_2 e^{-i\delta_2})} =$$

$$= \sqrt{A_1^2 + A_2^2 + A_1 A_2 (e^{i\delta_1} e^{-i\delta_2} + e^{-i\delta_1} e^{i\delta_2})} =$$

$$= \sqrt{A_1^2 + A_2^2 + A_1 A_2\, 2\cos(\delta_1 - \delta_2)} \quad \checkmark \quad \checkmark$$

$$\tan\delta_3 = \frac{A_3 \sin\delta_3}{A_3\cos\delta_3} = \frac{A_1\sin\delta_1 + A_2\sin\delta_2}{A_1\cos\delta_1 + A_2\cos\delta_2} \quad \checkmark$$

$$\delta_3 = \tan^{-1}\left(\frac{A_1\sin\delta_1 + A_2\sin\delta_2}{A_1\cos\delta_1 + A_2\cos\delta_2}\right) \quad \checkmark$$

6.2 Wave equation derivation

$$E = E_{0x}(x,t)\,\hat{x} + E_{0y}(x,t)\,\hat{y} + E_{0z}(x,t)\,\hat{z}$$

$$B = B_{0x}(x,t)\,\hat{x} + B_{0y}(x,t)\,\hat{y} + B_{0z}(x,t)\,\hat{z}$$

1. $E_y(x,t)$, $E_z(x,t)$, $B_y(x,t)$ and $B_z(x,t)$ each obey.

$$\frac{\partial^2 f}{\partial u^2} = \frac{1}{c}\frac{\partial^2 f}{\partial t^2} \Rightarrow u = x, y, t \text{ and } f = E_y, E_z, B_x, B_z.$$

Faraday's law $\Rightarrow \nabla \times E = -\dfrac{\partial B}{\partial t}$ Ampere's law $(J=0) \Rightarrow \nabla \times B = \dfrac{1}{c^2}\dfrac{\partial E}{\partial t}$

$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_{0x} & E_{0y} & E_{0z} \end{vmatrix} \qquad \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ B_{0x} & B_{0y} & B_{0z} \end{vmatrix}$$

$$\hat{x}(0) = -\frac{\partial B_{0x}}{\partial t} \quad (1)$$

$$\hat{y}\left(\frac{\partial E_{0z}}{\partial x}\right) = -\frac{\partial B_{0y}}{\partial t} \quad (2)$$

$$\hat{z}\left(\frac{\partial E_{0y}}{\partial x}\right) = -\frac{\partial B_{0z}}{\partial t} \quad (3)$$

$$\hat{x}(0) = \frac{1}{c^2}\frac{\partial E_{0x}}{\partial t} \quad (4)$$

$$\hat{y}\left(\frac{\partial B_{0z}}{\partial x}\right) = \frac{1}{c^2}\frac{\partial E_{0y}}{\partial t} \quad (5)$$

$$\hat{z}\left(\frac{\partial B_{0y}}{\partial x}\right) = \frac{1}{c^2}\frac{\partial E_{0z}}{\partial t} \quad (6)$$

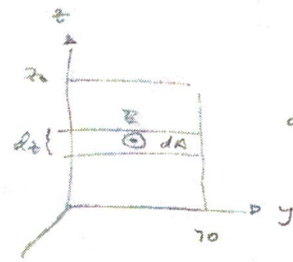
$$\frac{\partial}{\partial x}(3) \Rightarrow \frac{\partial^2 E_{0y}}{\partial x^2} = -\frac{\partial^2 B_{0z}}{\partial x\,\partial t} \quad ; \quad \frac{\partial}{\partial t}(5) \Rightarrow \frac{\partial^2 B_{0z}}{\partial t\,\partial x} = \frac{1}{c^2}\frac{\partial^2 E_{0y}}{\partial t^2}$$

$$\frac{\partial^2 E_{0y}}{\partial x^2} = -\frac{1}{c^2}\frac{\partial^2 E_{0y}}{\partial t^2} \quad ✓ \quad ✓$$

$$\frac{\partial}{\partial x}(5) \Rightarrow \frac{\partial^2 B_{0z}}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 E_{0y}}{\partial x\,\partial t} \quad ; \quad \frac{\partial}{\partial t}(3) \Rightarrow \frac{\partial^2 E_{0y}}{\partial x\,\partial t} = -\frac{\partial^2 B_{0z}}{\partial t^2}$$

$$\frac{\partial^2 B_{0z}}{\partial x^2} = -\frac{1}{c^2}\frac{\partial^2 B_{0z}}{\partial t^2} \quad ✓ \quad ✓$$

$$\Phi_E = \int E\, dA = \int_0^{z_0} E y_0\, dz = \int_0^{z_0} E_{0x} \cos(k_z z - \omega t) y_0\, dz =$$



$dA = y_0 dz$

$$= -\frac{E_{0x} y_0}{k} \left[\sin(\omega t - k_z z_0) - \sin(\omega t) \right]$$

$$\frac{\partial \Phi_E}{\partial t} = \frac{\partial}{\partial t} \left[-\frac{E_{0x} y_0}{k} \left[\sin(\omega t - k_z z_0) - \sin(\omega t) \right] \right] =$$

$$= -E_{0x} y_0 \frac{\omega}{k} \left[\cos(\omega t - k_z z_0) - \cos(\omega t) \right] =$$

$$= E_{0x} y_0 \frac{\omega}{k} \left[\cos(\omega t) - \cos(\omega t - k_z z_0) \right] \rightarrow \text{note that } \frac{\omega}{k} = c$$

$$\frac{1}{c} \cdot \frac{\partial \Phi_E}{dt} = E_{0x} y_0 \left[\cos(\omega t) - \cos(\omega t - k_z z_0) \right].$$

$$c \cdot \oint B\, dl = E_{0x} y_0 \left[\cos(\omega t) - \cos(\omega t - k_z z_0) \right]$$

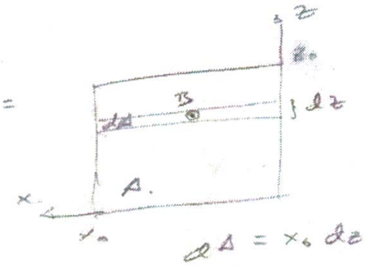
$$c \oint B\, dl = \frac{1}{c} \frac{\partial \Phi_E}{\partial t} \Rightarrow \oint B\, dl = \frac{1}{c^2} \frac{\partial \Phi_E}{\partial t} \quad ✓$$

$d\vec{A} = -\hat{y}\, dA$ not sure how it worked out. I don't see why they matched.

$$\oint E\, dl = x_0 E_{0x} \cos(\omega t) - x_0 E_{0x} \cos(\omega t - k_z z_0) =$$

$$= x_0 E_{0x} \left[\cos(\omega t) - \cos(\omega t - k_z z_0)\right] \checkmark$$

$$\Phi_B = \int B \cdot dA = \int_0^{z_0} -\frac{E_{0x} k_z}{\omega} \cos(k_z z - \omega t)\, x_0\, dz =$$



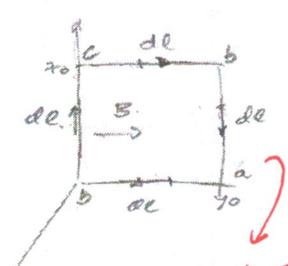
$$= \frac{E_{0x} x_0}{\omega} \left[\sin(\omega t - k_z z_0) - \sin(\omega t)\right]$$

$$-\frac{d\Phi_B}{dt} = -\frac{d}{dt}\left[\frac{E_{0x} x_0}{\omega}\left[\sin(\omega t - k_z z_0) - \sin(\omega t)\right]\right] =$$

$$= - x_0 E_{0x}\left[\cos(\omega t - k_z z_0) - \cos(\omega t)\right] = x_0 E_{0x}\left[\cos(\omega t) - \cos(\omega t - k_z z_0)\right] \checkmark$$

$$\oint E\, dl = -\frac{d\Phi_B}{dt} \checkmark$$

3. Show that B satisfies Ampere's law for $J = 0$ $\quad \oint B \cdot dl = \frac{1}{c^2}\frac{d\Phi_E}{dt}$



Similar issue here with $\hat{n}$

Same as before.

$$\oint B \cdot dl = \int_a^b B\, dl + \int_b^c \overset{0}{\cancel{B\, dl}} + \int_c^d B\, dl + \int_d^a \overset{0}{\cancel{B\, dl}} =$$

$$= \int_{y_0}^{0} B\, dy + \int_0^{y_0} B\, dy = \int_{y_0}^{0} -\frac{E_{0x} k_z}{\omega}\cos(k_z \cdot 0 - \omega t)\, dy +$$

$$+ \int_0^{y_0} -\frac{E_{0x} k_z}{\omega}\cos(k_z z_0 - \omega t)\, dy =$$

$$= E_{0x} y_0 \frac{k}{\omega}\cos(\omega t) - E_{0x} y_0 \frac{k}{\omega}\cos(\omega t - k_z z_0) =$$

$$= E_{0x} y_0 \frac{k}{\omega}\left[\cos(\omega t) - \cos(\omega t - k_z z_0)\right] \qquad \text{Note that } \frac{k}{\omega} = \frac{1}{c}$$

HW#6 Jesus Gil Gil ①

6.1 Faraday's Law and Ampere's Law

$$E = E_{ox} \cos(k_z z - \omega t)\hat{x}$$

1. Find B using $\nabla \times E = -\dfrac{\partial B}{\partial t}$

Find the curl of E

$$\nabla \times E = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} \Rightarrow$$

$$\nabla \times E = \left(\underset{0}{\cancel{\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}}} \right)\hat{x} + \left(\frac{\partial E_x}{\partial z} - \underset{0}{\cancel{\frac{\partial E_z}{\partial x}}} \right)\hat{y} + \left(\underset{0}{\cancel{\frac{\partial E_y}{\partial x}}} - \underset{0}{\cancel{\frac{\partial E_x}{\partial y}}} \right)\hat{z} \Rightarrow$$

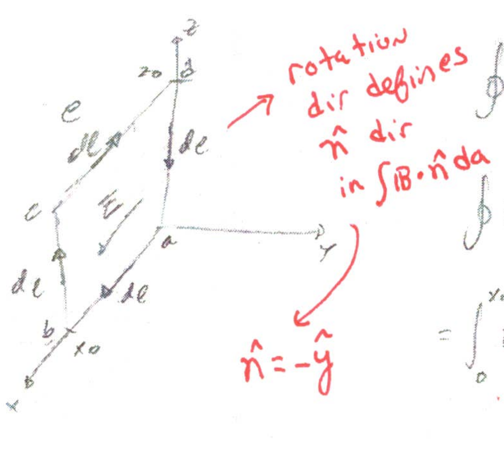
$$\nabla \times E = \frac{\partial E_x}{\partial z}\hat{y} = \frac{\partial}{\partial z} E_{ox} \cos(k_z z - \omega t)\hat{y} = -E_{ox} k_z \sin(k_z z - \omega t)\hat{y}$$

$$-\frac{\partial B}{\partial t} = -E_{ox} k_z \sin(k_z z - \omega t)\hat{y}$$

$$\int \frac{d}{dt} B\, dt = \int E_{ox} k_z \sin(k_z z - \omega t)\hat{y}\, dt$$

$$B = -\frac{E_{ox} k_z}{\omega} \cos(k_z z - \omega t)\hat{y}$$ ✓

2. Show that E satisfies Faraday's law.



$$\oint E\, dl = -\frac{\partial \Phi_B}{\partial t}$$

$$\oint E\, dl = \int_a^b E\, dl + \underset{0}{\cancel{\int_b^c E\, dl}} + \int_c^d E\, dl + \underset{0}{\cancel{\int_d^a E\, dl}} =$$

$$= \int_0^{x_0} E\, dx + \int_{x_0}^0 E\, dx = \int_0^{x_0} E_{ox} \cos(k_z \underset{0}{\cancel{z}} - \omega t)\, dx + \int_{x_0}^0 E_{ox} \cos(k_z z_0 - \omega t)\, dx$$

$\Rightarrow$

$$\frac{\partial}{\partial x}(2) \Rightarrow \frac{\partial^2 E_{0z}}{\partial x^2} = -\frac{\partial^2 B_{0y}}{\partial x\,\partial t} \qquad \frac{\partial}{\partial t}(6) \Rightarrow \frac{\partial^2 B_{0y}}{\partial x\,\partial t} = \frac{1}{c^2}\frac{\partial^2 E_{0z}}{\partial t^2}$$

$$\frac{\partial^2 E_{0z}}{\partial x^2} = -\frac{1}{c^2}\frac{\partial^2 E_{0z}}{\partial t^2} \quad \checkmark$$

$$\frac{\partial}{\partial x}(6) \quad \frac{\partial^2 B_{0y}}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 E_{0z}}{\partial x\,\partial t} \qquad \frac{\partial}{\partial t}(2) \Rightarrow \frac{\partial^2 E_{0z}}{\partial x\,\partial t} = -\frac{\partial^2 B_{0y}}{\partial t^2}$$

$$\frac{\partial^2 B_{0y}}{\partial x^2} = -\frac{1}{c}\frac{\partial^2 B_{0y}}{\partial t^2} \quad \checkmark$$

2. $E_x(x,t) = B_x(x,t) = 0$?

Yes, $\nabla \times E = 0\,\hat{x}$ and $\nabla \times B = 0\,\hat{x}$. The E and B fields as the wave propagates in the $\hat{x}$ direction, the E and B fields are normal to $\hat{x}$ and therefore, B_x and E_x are 0.

techically

$$\frac{\partial B_x}{\partial t} = 0$$

$$\frac{\partial B_x}{\partial x} = 0$$

3. $\nabla^2 E = \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2}$ and $\nabla^2 B = \frac{1}{c^2}\frac{\partial^2 B}{\partial t^2}$

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$\nabla \times \nabla \times E = -\frac{\partial}{\partial t}(\nabla \times B) \Rightarrow \nabla \times B = \frac{1}{c^2}\frac{\partial E}{\partial t} \text{ for } J = 0$$

$$-\nabla^2 E + \nabla(\underbrace{\cancel{\nabla \cdot E}}_{0}) = -\frac{\partial}{\partial t}\left(\frac{1}{c^2}\frac{\partial E}{\partial t}\right) \Rightarrow \nabla \cdot E = 0 \text{ for } \rho = 0$$

No sources.

$$\nabla^2 E = \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} \Rightarrow \nabla^2 \text{ is } \frac{\partial^2}{\partial x^2} \text{ (in our case)}$$

$$\frac{\partial^2 E}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} \quad \checkmark$$

$$\nabla \times B = \frac{1}{c^2}\frac{\partial E}{\partial t} \Rightarrow \nabla \times \nabla \times B = \frac{1}{c^2}\frac{\partial}{\partial t}(\nabla \times E) \Rightarrow \nabla \times E = -\frac{\partial B}{\partial t}.$$

$$\Rightarrow -\nabla^2 B + \nabla(\underbrace{\cancel{\nabla \cdot B}}_{0}) = \frac{1}{c^2}\frac{\partial}{\partial t}\left(-\frac{\partial B}{\partial t}\right) \Rightarrow \nabla^2 B = \frac{1}{c^2}\frac{\partial^2 B}{\partial t^2}$$

$$\frac{\partial^2 B}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 B}{\partial t^2} \quad \checkmark$$

6.3 Wave equation Solutions

1. $E = E_{0x} \cos(k_z z - \omega t + \delta_x)\hat{x}$ satisfies $\nabla^2 E = \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2}$

* $\nabla^2 E = \frac{\partial^2 E}{\partial z^2} = \frac{\partial^2}{\partial z^2} E_{0x}\cos(k_z z - \omega t + \delta_x)\hat{x} =$

$= -k_z^2 E_{0x}\cos(k_z z - \omega t + \delta_x)\hat{x}$

$\frac{\partial^2 E}{\partial t^2} = \frac{\partial^2}{\partial t^2} E_{0x}\cos(k_z z - \omega t + \delta_x)\hat{x} =$

$= -\omega^2 E_{0x}\cos(k_z z - \omega t + \delta_x)\hat{x}$

for $\frac{1}{c^2} = \frac{k^2}{\omega^2}$

$-k_z^2 E_{0x}\cos(k_z z - \omega t + \delta_x)\hat{x} = -\frac{k^2}{\cancel{\omega^2}}\cancel{\omega^2} E_{0x}\cos(k_z z - \omega t + \delta_x)\hat{x}$ ✓

✓

* $B = B_{0x}\cos(k_z z - \omega t + \delta'_x)\hat{x} + B_{0y}\cos(k_z z - \omega t + \delta'_y)\hat{y}$

$\nabla^2 B = \frac{\partial^2 B}{\partial z^2} = \frac{\partial^2}{\partial z^2}[B_{0x}\cos(k_z z - \omega t + \delta'_x)\hat{x} + B_{0y}\cos(k_z z - \omega t + \delta'_y)y' =$

$= -k^2 B_{0x}\cos(k_z z - \omega t + \delta'_x)\hat{x} - k^2 B_{0y}\cos(k_z z - \omega t + \delta'_y)y' =$

$= -k^2[B_{0x}\cos(k_z z - \omega t + \delta'_x)\hat{x} + B_{0y}\cos(k_z z - \omega t + \delta'_y)\hat{y}]$

$\frac{\partial^2 B}{\partial t^2} = \frac{\partial^2}{\partial t^2}[B_{0x}\cos(k_z z - \omega t + \delta'_x)\hat{x} + B_{0y}\cos(k_z z - \omega t + \delta'_y)\hat{y}] =$

$= -\omega^2[B_{0x}\cos(k_z z - \omega t + \delta'_x)\hat{x} + B_{0y}\cos(k_z z - \omega t + \delta'_y)\hat{y}]$

✓

For $\frac{1}{c^2} = \frac{k^2}{\omega^2}$ same as before:

$-k^2[B_{0x}\cos(k_z z - \omega t + \delta'_x)\hat{x} + B_{0y}\cos(k_z z - \omega t + \delta'_y)\hat{y} = -\frac{k^2}{\cancel{\omega^2}}\cancel{\omega^2}[B_{0x} \ldots]$ ✓

- k_z and ω are related by $c = \frac{\omega}{k}$
- E is linearly polarized & B is circularly polarized. In the case of E & B both having the same polarization, their magnitudes (E_{0x} & B_{0x}) are related by:

$E_{0x} = B_{0y} \cdot c$ according to the solution in 6.1 using Faraday's law.

In our case E & B are different traveling waves

- In the case of B circular polarized, the relation between B_{0x} and B_{0y} will give a circular polarized is $B_{0x} = B_{0y}$, and elliptical polarized if $B_{0x} \neq B_{0y}$. and the difference between δ'_x & δ'_y will give us left-hand or right-hand polarization and the angle of polarization

2. For Both B & E be consistent with Maxwell equations,

$B_{0x} = 0 \Rightarrow B = B_{0y} \cos(k_z z - \omega t + \delta'_y)\hat{y}$

Then $E_{0x} = B_{0y} c$, $c = \frac{\omega}{k}$ and $\delta_x = \delta'_y$

3. ~~Redo~~ $E = E_{0y} \cos(k_z z - \omega t + \delta_y)\hat{y}$

$B_{0y} = 0 \Rightarrow B = B_{0x} \cos(k_z z - \omega t + \delta'_x)\hat{x}$

Same as before $E_{0y} = c B_{0x}$, $k = \frac{\omega}{c}$ and $\delta_y = \delta'_x$

4. $k = k_x\hat{x} + k_y\hat{y} + k_z\hat{z}$ $\Rightarrow B = \frac{1}{c}\hat{k} \times E$

$$\hat{k} \times E = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ k_x & k_y & k_z \\ E_x & 0 & 0 \end{vmatrix} = \hat{y}(k_z E_x) + \hat{z}(\cancel{-k_y E_x})^{0}$$

(2) $\hat{k} \times \bar{E} = k_z E_{0x} \cos(k_z z - \omega t + \delta_x)\hat{y}$

From Faraday's law $\nabla \times E = -\frac{\partial B}{\partial t}$

$$\nabla \times E = \hat{y}\,\frac{\partial E_x}{\partial z} = -k_z E_{ox} \sin(k_z z - \omega t + \delta_x)\,\hat{y} \quad \text{integrate to get } B$$

$$B = -\int -k_z E_{ox} \sin(k_z z - \omega t + \delta_x)\,\hat{y}\,dt = \frac{1}{\omega}\,\underbrace{k_z E_{ox} \cos(k_z z - \omega t + \delta_x)\,\hat{y}}_{\hat{k} \times E}$$

for $k = |k|\,\hat{k}_z$

$$B = \frac{1}{c}\,\hat{k} \times E$$

(3) Same for $E = E_{oy} \cos(k_z z - \omega t + \delta_y)\,\hat{y}$

$$k \times E = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ k_x & k_y & k_z \\ 0 & E_{oy} & 0 \end{vmatrix} = -\hat{x}(k_z E_{oy}) + \cancelto{0}{\hat{z}(k_x E_{oy})}$$

From Faraday's law $\nabla \times E \Rightarrow k \times E = -\frac{\partial B}{\partial t}$

$$\nabla \times E = -\hat{x}\,\frac{\partial E_x}{\partial z} = k_z E_{oy} \sin(k z - \omega t + \delta_y)\,\hat{x} \quad \text{integrate to get } B.$$

$$B = -\int k_z E_{oy} \sin(k z - \omega t + \delta_y)\,\hat{x}\,dt = \frac{1}{\omega}\,\underbrace{(-k_z E_{oy} \cos(k z - \omega t + \delta_y)\,\hat{x})}_{\hat{k} \times E}$$

for $k = |k|\,\hat{k}_z$

$$B = \frac{1}{c}\,\hat{k} \times E$$ ✓

6.4. Complex Form.

$$E = Re[\hat{E} e^{-i(\omega t - k\cdot r)}] \qquad k = k_x \hat{x} + k_y \hat{y} + k_z \hat{z}$$

$$B = Re[\hat{B} e^{-i(\omega t - k\cdot r)}] \qquad r = x\hat{x} + y\hat{y} + z\hat{z}$$

where $\tilde{E}$ & $\tilde{B}$ are complex constants as components.

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$\nabla \times E = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = \hat{x}\left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right) + \left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right) +$$

$$+ \hat{z}\left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right) =$$

$$= \tilde{E} e^{-i(\omega t - kr)}[(k_y - k_z)\hat{x} + (k_z - k_x)\hat{y} + (k_x - k_y)\hat{z}]$$

Integrate over time to get B.

$$B = -\int \tilde{E} e^{-i(\omega t - kr)}[(k_y - k_z)\hat{x} + (k_z - k_x)\hat{y} + (k_x - k_y)\hat{z}]\, dt =$$

$$= \frac{1}{\omega}\tilde{E} e^{-i\left(\frac{2\omega t - 2kr + \pi}{2}\right)}[(k_y - k_z)\hat{x} + (k_z - k_x)\hat{y} + (k_x - k_y)\hat{z}]$$

$$\tilde{E} = E e^{i\delta} \Rightarrow \text{absorbe } \frac{\pi}{2} \Rightarrow \hat{E} = E e^{i(\delta - \frac{\pi}{2})}$$

$$= \frac{1}{\omega}\tilde{E} e^{-i(\omega t - kr)}[(k_y - k_z)\hat{x} + (k_z - k_x)\hat{y} + (k_x - k_y)\hat{z}]$$

$$\hat{k} \times \hat{E} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ k_x & k_y & k_z \\ E_x & E_y & E_z \end{vmatrix} = \hat{x}(k_y E_z - k_z E_y) + \hat{y}(k_z E_x - k_x E_z) + \hat{z}(k_x E_y - k_y E_x)$$

for $E_x, E_y, E_z = Re[\tilde{E} e^{i(\omega t - (k_x + k_y + k_z) r}]$

$$= \tilde{E} e^{-i(\omega t - kr)} [(k_y - k_z)\hat{x} + (k_z - k_x)\hat{y} + (k_x - k_y)\hat{z}]$$

from $\tilde{B} = \frac{1}{\omega} \underbrace{\tilde{E} e^{-i(\omega t - kr)} [(k_y - k_z)\hat{x} + (k_z - k_x)\hat{y} + (k_x - k_y)\hat{z}]}_{\hat{k} \times \tilde{E}}$ ✓

using $k = |k| \hat{k}$

$$\tilde{B} = \frac{k}{\omega} \hat{k} \times \tilde{E} = \frac{1}{c} \hat{k} \times \tilde{E}$$ ✓

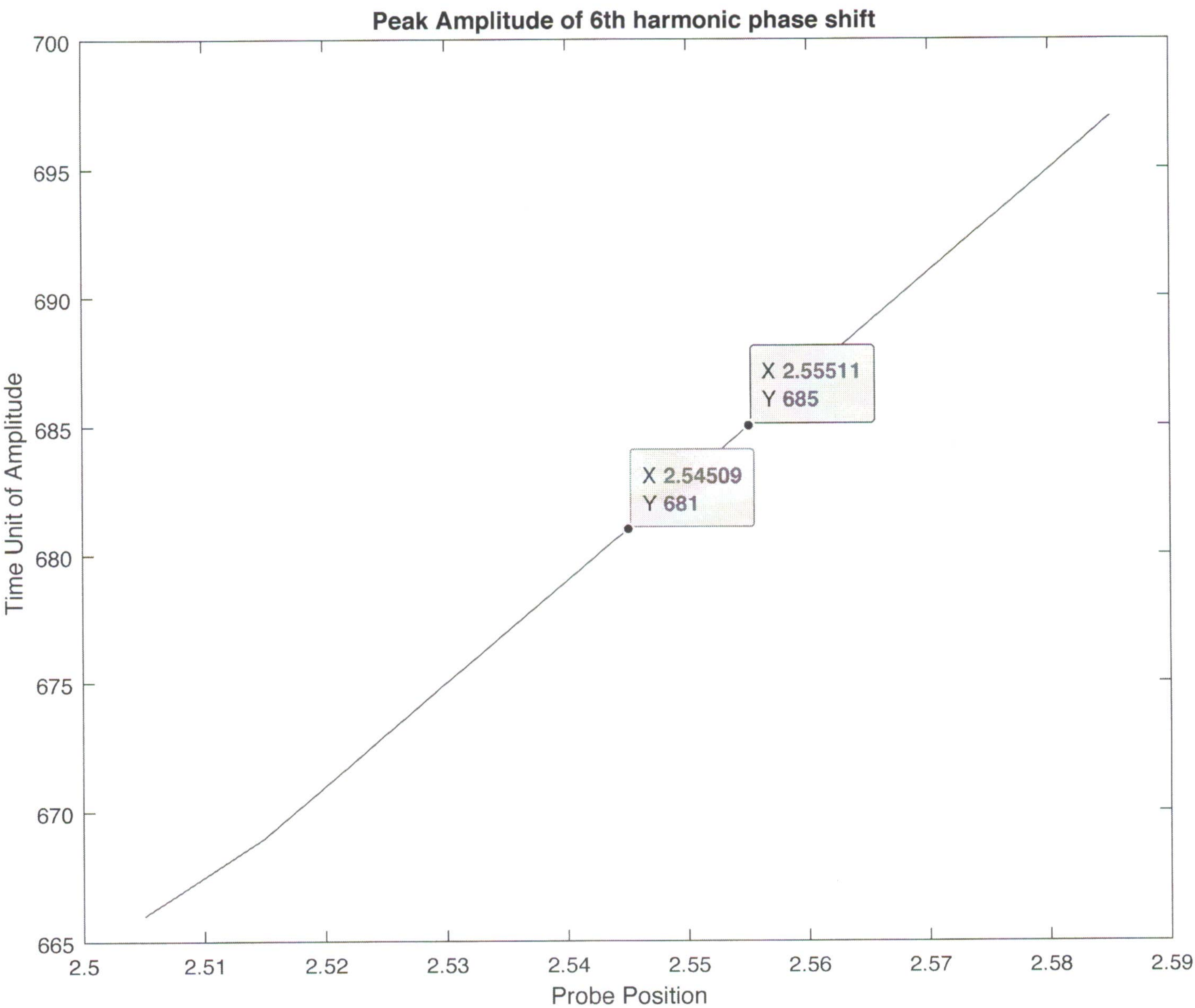
for $\tilde{E} = E e^{i\delta}$ and $\tilde{B} = B e^{i\delta}$ and substituting above.

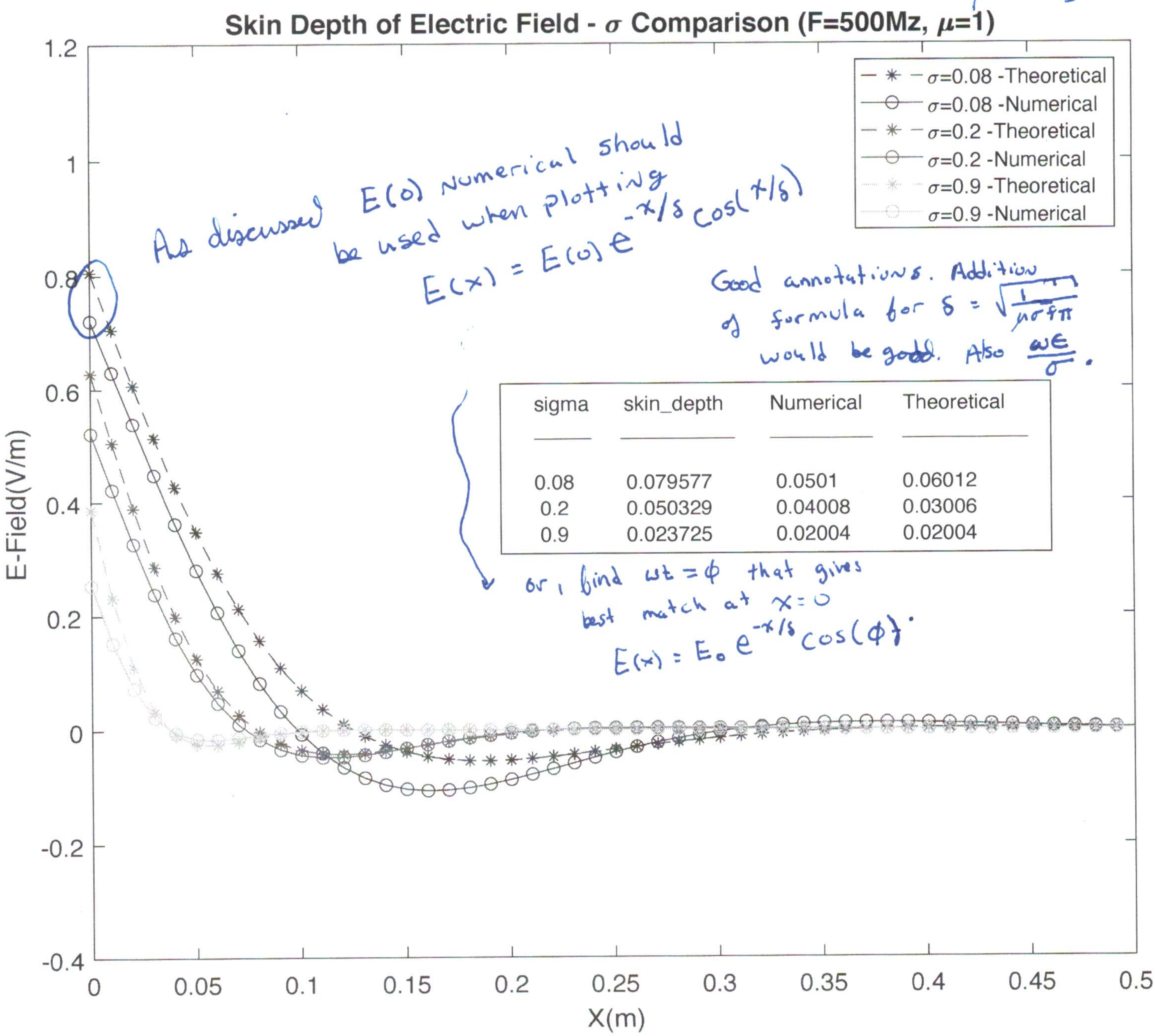
v. good.

$$B \cancel{e^{i\delta}} = \frac{1}{c} \hat{k} \times E \cancel{e^{i\delta}} \Rightarrow B = \frac{1}{c} \hat{k} \times E$$

meaning that the magnitudes of B and E are related by

$$B = \frac{1}{c} E$$


Peak Amplitude of 6th harmonic phase shift
Time Unit of Amplitude
Probe Position
700
695
690
685
680
675
670
665
2.5
2.51
2.52
2.53
2.54
2.55
2.56
2.57
2.58
2.59
X 2.55511
Y 685
X 2.54509
Y 681



sigma	skin_depth	Numerical	Theoretical
0.08	0.079577	0.0501	0.06012
0.2	0.050329	0.04008	0.03006
0.9	0.023725	0.02004	0.02004

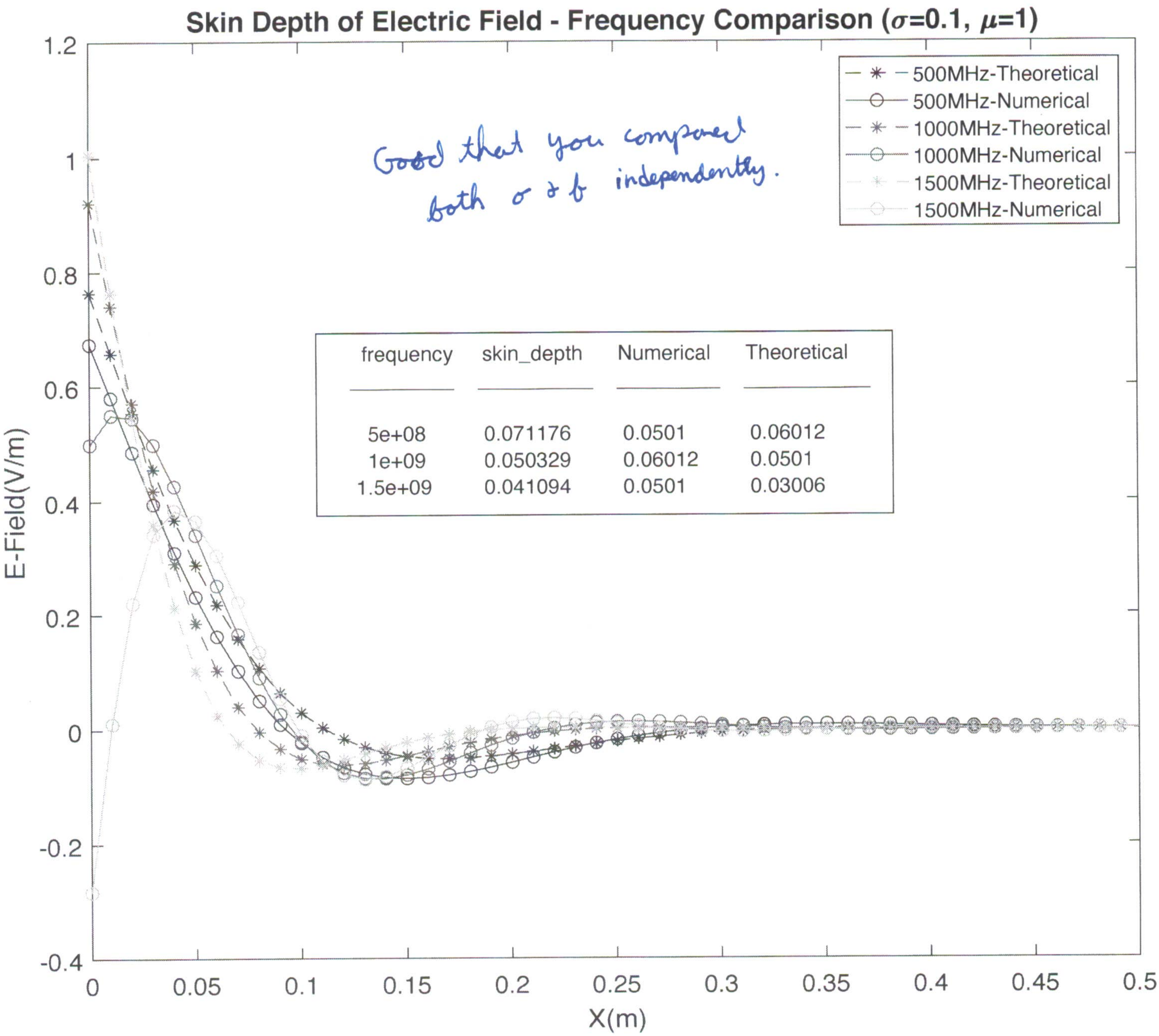
? $\mu_r = 1$. I think

As discussed E(0) Numerical should be used when plotting $E(x) = E(0)\, e^{-x/\delta} \cos(x/\delta)$

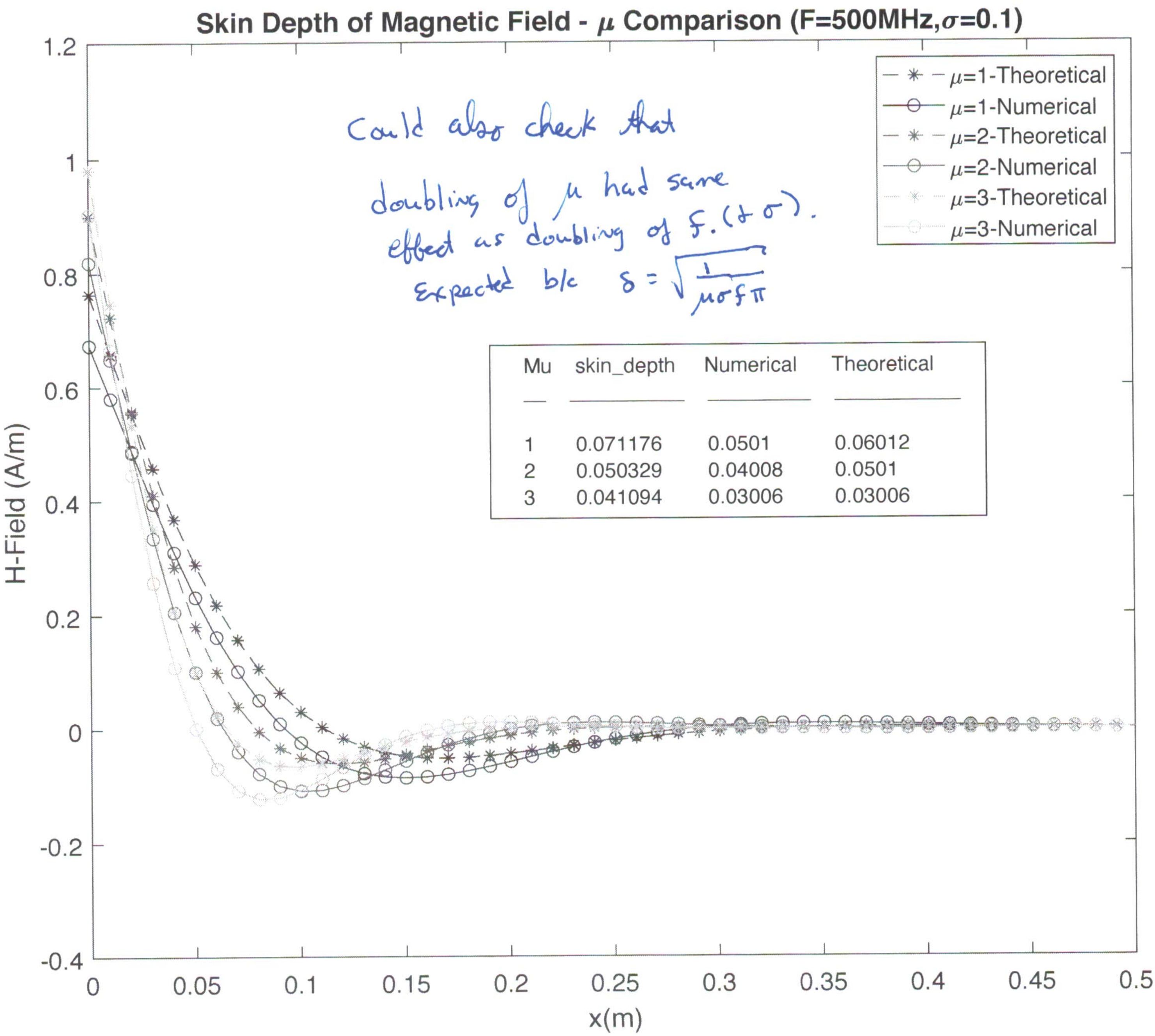
Good annotations. Addition of formula for $\delta = \sqrt{\frac{1}{\mu \sigma f \pi}}$ would be good. Also $\frac{\omega \epsilon}{\sigma}$.

or, find $\omega t = \phi$ that gives best match at $x = 0$

$E(x) = E_0\, e^{-x/\delta} \cos(\phi)$.



Good that you compared both σ & f independently.



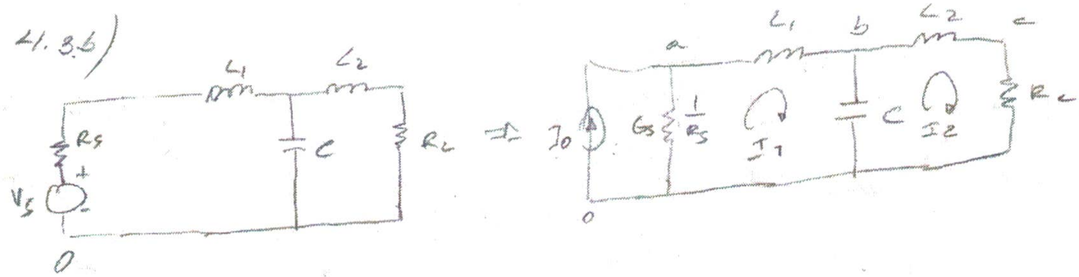
Could also check that doubling of μ had same effect as doubling of f. (& σ).
Expected b/c $\delta = \sqrt{\frac{1}{\mu \sigma f \pi}}$

Mu	skin_depth	Numerical	Theoretical
1	0.071176	0.0501	0.06012
2	0.050329	0.04008	0.0501
3	0.041094	0.03006	0.03006

HW#7 Jesus Gil Gil

7.2 Problems 4.3b, 4.3c, 4.3d & 4.6e

4.3b)



Loop

$$\frac{1}{R_s}(I_0 - I_1) - L_1 \frac{dI_1}{dt} - \frac{1}{C}\int (I_1 - I_2)\,dt = 0$$

$$\frac{1}{C}\int (I_1 - I_2)\,dt - L_2 \frac{dI_2}{dt} - R_L I_2 = 0$$

Node

$V_a G_s$ or V_a/R_s

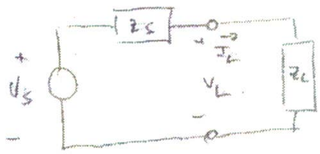
a)

$$I_0 - \frac{V_a}{\cancel{1/R_s}} - \frac{1}{L_1}\int (V_a - V_b) = 0$$

b) $$\frac{1}{L_1}\int (V_a - V_b) - C\frac{dV_b}{dt} - \frac{1}{L_2}\int (V_b - V_c) = 0$$

c) $$\frac{1}{L_2}\int (V_b - V_c) - \frac{V_c}{R_L} = 0$$

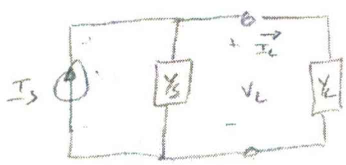
4.3 c)



$\Rightarrow \quad V_L = Z_L I_L \quad ; \quad I_L = I_S$

$V_L = V_S - Z_S I_L \Rightarrow I_L = \dfrac{V_S - V_L}{Z_S}$

$V_L = Z_L I_L \quad \Rightarrow I_L = \dfrac{V_S}{Z_S} - \dfrac{Z_L I_L}{Z_S} \Rightarrow I_L = \dfrac{V_S}{Z_L + Z_S}$ ✓



$I_S = V_S Y_S$ (10)

$V_L = \dfrac{I_L}{Y_L} \quad ; \quad I_S = V_L Y_L + V_L Y_S \Rightarrow$

$\Rightarrow I_S = V_L (Y_L + Y_S) \Rightarrow V_L = \dfrac{I_S}{Y_L + Y_S}$

From thevenin $\Rightarrow I_L = \dfrac{V_S}{Z_L + Z_S}$ ✓ $\Rightarrow I_L = I_S$

$I_S = \dfrac{V_S}{Z_L + Z_S} \Rightarrow$ Solving for $Z_L = \dfrac{V_S}{I_S} - Z_S \Rightarrow$

$\Rightarrow Z_L = \dfrac{V_L}{I_L} \Rightarrow \underline{\dfrac{V_L}{I_L} = \dfrac{V_S}{I_S} - Z_S}$ ✓ .

From Norton $\quad V_L = \dfrac{I_S}{Y_L + Y_S} \quad \Rightarrow \quad V_L = V_S$

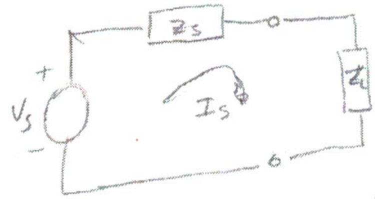
$V_S = \dfrac{I_S}{Y_L + Y_S} \Rightarrow$ solving for $Y_L = \dfrac{I_S}{V_S} - Y_S$

$\Rightarrow Y_L = \dfrac{I_L}{V_L} \Rightarrow \dfrac{I_L}{V_L} = \dfrac{I_S}{V_S} - Y_S \Rightarrow$ inverting. $\Rightarrow$

$\underline{\dfrac{V_L}{I_L} = \dfrac{V_S}{I_S} - Z_S}$ ✓

usually write V_L in terms of V_S (as you did w/ I_L)

4.3 d)



$Z_S = R_S + jX_S$

$Z_L = R_L + jX_L$

$$I_S = \frac{V_S}{Z_S + Z_L} = \frac{V_S}{(R_S + jX_S) + (R_L + jX_L)} = \frac{V_S}{(R_S + R_L) + j(X_S + X_L)}$$

$$|I_S| = \frac{|V_S|}{\sqrt{(R_S+R_L)^2 + (X_S+X_L)^2}} \Rightarrow P_L = |I_L|^2 R_L = \frac{|V_S|^2 R_L}{(R_S+R_L)^2 + (X_S+X_L)^2}$$

$$\frac{d}{dX_L}\left(\frac{|V_S|^2 R_L}{(R_S+R_L)^2 + (X_S+X_L)^2}\right) = \frac{-2(X_L+X_S) R_L V_S^2}{[(R_S+R_L)^2 + (X_S+X_L)^2]^2}$$

$$\frac{-2(X_L+X_S) R_L V_S^2}{[(R_S+R_L)^2 + (X_S+X_L)^2]^2} = 0$$ and solving $\Rightarrow X_L + X_S = 0$ ✓

or $X_L = -X_S$

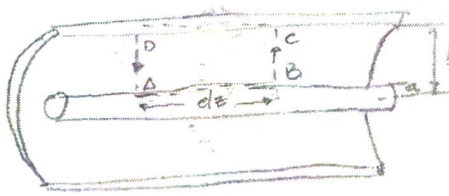
$$P_L = \frac{|V_S|^2 R_L}{(R_S+R_L)^2} \Rightarrow \frac{dP_L}{dR_L} = \frac{-(R_L - R_S)|V_S|^2}{(R_L+R_S)^3} = 0$$ and solving $\Rightarrow$ $R_L = R_S$

For max Power $\Rightarrow$ $X_L = -X_S$ and $R_L = R_S$

$Z_L = R_L + jX_L = R_S - jX_S = Z_S^*$

Could be min power.

4.6.e



→ z wave propagation $\quad \sum_i V_i = 0$

$$V_z(t) - RI(t) - L_i\frac{dI(t)}{dt} - \frac{1}{C}\int I(t)\,dt - L_e\frac{dI(t)}{dt} - V_{z+dz}(t) = 0$$

Voltage

$$V_z = V_{DA} = -\int_A^D E\,d\ell\,; \quad V_{z+dz} = V_{CB} = -\int_B^C -E\,d\ell \quad ; V_{BA} = 0\ ; V_{CD} = 0$$

Impedance

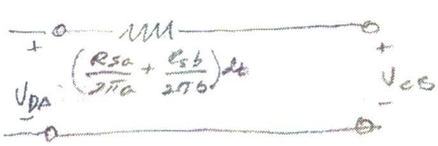
$$A = 2\pi r$$

$$V_{DA} = 0\ ; \ V_{CB} = 0 \ ;$$

$$V_{BA} + V_{DC} = -\int_A^B E\,dz - \int_C^D E\,dz = -I\Big[\underbrace{\int_A^B \frac{dz}{\sigma A}}_{R_{BA}} + \underbrace{\int_C^D \frac{dz}{\sigma A}}_{R_{DC}}\Big]$$

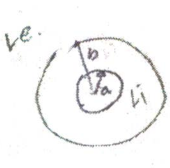
For high frequency: 4.5 (11) $R_{(hf)} = \dfrac{R_S}{2\pi r_0}$ per unit length (dz)

$$V_{BA} + V_{DC} = I\left(\frac{R_{sa}}{2\pi a} + \frac{R_{sb}}{2\pi b}\right)dz.$$ See Piazza for derivation



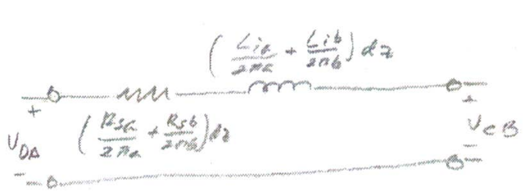
Inductance

$$\oint E\cdot d\ell = -\frac{\partial}{\partial t}\int_S B\,ds \ ; \quad ds = dz\,dr \ ; \ \Delta V = V_{CD} - V_{DA} = -\frac{d\Phi}{dt}$$



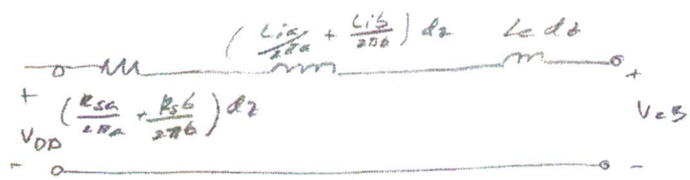
$$\Phi_i = \int_0^a B\,dr\,dz + \int_0^b B\,dr\,dz \Rightarrow L_i = \left[\int B\,ds\right]/I$$

$$\Phi_i = \left[\frac{L_{ia}}{2\pi a}dz + \frac{L_{ib}}{2\pi b}dz\right]I \Rightarrow -\frac{d\Phi}{dt} = -\left[\frac{L_{ia}}{2\pi a} + \frac{L_{ib}}{2\pi b}\right]dz\,\frac{dI(t)}{dt}$$

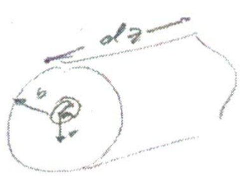


external Inductance

$L_e = \left[\int B\, ds\right]/I$ L_e is arbitrary so $\Delta V_{L_e} = -\frac{d\Phi_e}{dt} = -dz\, L_e \frac{dI(t)}{dt}$.



capacitance



from eq. 1.9(4) $\Rightarrow$ $C = \frac{2\pi\varepsilon}{\ln\left(\frac{b}{a}\right)}\cdot dz$.