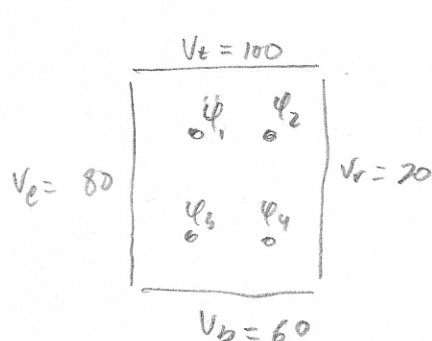


2.2 Laplace Equation in two dimensions - Numerical

assume $x_0, y_0 = 1$

$$\phi_1\left(\frac{1}{3}, \frac{2}{3}\right) \quad \phi_2\left(\frac{2}{3}, \frac{2}{3}\right) \quad \phi_3\left(\frac{1}{3}, \frac{1}{3}\right) \quad \phi_4\left(\frac{2}{3}, \frac{1}{3}\right)$$

Should be $n = 1, 3, \dots$. I think that you mentioned that you realized this later.

As discussed, this is how you would verify the "Correct Potentials" column in the textbook.

$$\phi_1\left(\frac{1}{3}, \frac{2}{3}\right) = \sum_{n=1}^{10} \frac{4 \times 80}{n\pi} \frac{1}{\sinh n\pi} \sin\left(\frac{2}{3} n\pi\right) \sinh\left(-n\pi\left(\frac{1}{3} - 1\right)\right) = 25.35$$

$$\phi_b\left(\frac{1}{3}, \frac{2}{3}\right) = \sum_{n=1}^{10} \frac{4 \times 60}{n\pi} \frac{1}{\sinh n\pi} \sin\left(\frac{1}{3} n\pi\right) \sinh\left(n\pi\left(\frac{2}{3} - 1\right)\right) = 7.66$$

$$\phi_t\left(\frac{1}{3}, \frac{2}{3}\right) = \sum_{n=1}^{10} \frac{4 \times 100}{n\pi} \frac{1}{\sinh n\pi} \sin\left(\frac{1}{3} n\pi\right) \sinh\left(\frac{2}{3} n\pi\right) = 44.44$$

$$\phi_r\left(\frac{1}{3}, \frac{2}{3}\right) = \sum_{n=1}^{10} \frac{4 \times 20}{n\pi} \frac{1}{\sinh n\pi} \sin\left(\frac{2}{3} n\pi\right) \sinh\left(\frac{1}{3} n\pi\right) = 2.22$$

$$\phi_1 = \phi_c + \phi_b + \phi_t + \phi_r = 25.35 + 7.66 + 44.44 + 2.22 = \underline{\underline{79.67}}$$

$$\phi_2\left(\frac{2}{3}, \frac{2}{3}\right) \Rightarrow \text{same way}$$

$$\phi_c\left(\frac{2}{3}, \frac{2}{3}\right) = 8.89, \quad \phi_b\left(\frac{2}{3}, \frac{2}{3}\right) = 6.67, \quad \phi_t\left(\frac{2}{3}, \frac{2}{3}\right) = 31.7$$

$$\phi_r\left(\frac{2}{3}, \frac{2}{3}\right) = 6.34$$

$$\phi_2 = 8.89 + 6.67 + 31.7 + 6.34 = \underline{\underline{53.59}}$$

$$\phi_3\left(\frac{1}{3}, \frac{1}{3}\right) \Rightarrow \text{same way}$$

$$\phi_c\left(\frac{1}{3}, \frac{1}{3}\right) = 35.55, \quad \phi_b\left(\frac{1}{3}, \frac{1}{3}\right) = 26.67, \quad \phi_t\left(\frac{1}{3}, \frac{1}{3}\right) = 12.76, \quad \phi_r\left(\frac{1}{3}, \frac{1}{3}\right) = 2.55$$

$$\phi_3 = 35.55 + 26.67 + 12.76 + 2.55 = \underline{\underline{77.51}}$$

$$\phi_4\left(\frac{2}{3}, \frac{1}{3}\right) \Rightarrow \text{same way}$$

$$\phi_c\left(\frac{2}{3}, \frac{1}{3}\right) = 10.19, \quad \phi_b\left(\frac{2}{3}, \frac{1}{3}\right) = 19.02, \quad \phi_t\left(\frac{2}{3}, \frac{1}{3}\right) = 11.1, \quad \phi_r\left(\frac{2}{3}, \frac{1}{3}\right) = 8.89$$

$$\phi_4 = 10.19 + 19.02 + 11.1 + 8.89 = \underline{\underline{49.22}}$$