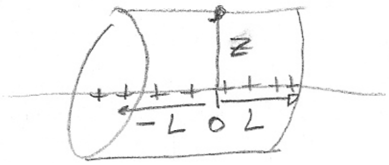
HW 1.3 a)

$$Q = \lambda 2L$$

From Griffiths $\rightarrow$ $E(z) = \dfrac{k\,2\lambda L}{z\sqrt{z^2+L^2}} = \boxed{\dfrac{kQ}{z\sqrt{z^2+L^2}}}$



Gauss Law $\rightarrow$ $E = \dfrac{Q}{\varepsilon_0 \text{ Area}}$ $\Rightarrow$ For cylinder.

$$\Phi = E\int_S dA = E(2\pi z(2L)) = \frac{Q}{\varepsilon_0}$$

$$\boxed{E(z) = \frac{Q}{4\pi\varepsilon_0 z\cdot L} = \frac{kQ}{z\cdot L}}$$

$$E(z) = \frac{kQ}{z\sqrt{z^2+L}} = \frac{kQ}{z\cdot L\cdot\sqrt{1+z^2/L^2}} \;;\; E(z) = \text{Gauss Law}\cdot\frac{1}{\sqrt{1+\frac{z^2}{L^2}}} \quad \text{for } L=z \;\; \frac{1}{\sqrt{2}}$$

Do Taylor on $f(L) = \dfrac{1}{\sqrt{1+\frac{z^2}{L^2}}} = \dfrac{L}{\sqrt{z^2+L^2}}$ evaluating at $L=0$

$$f(L) = \frac{L}{\sqrt{z^2}} - \frac{1}{2}\frac{L^3}{(z^2)^{3/2}} + \frac{3L^5\sqrt{z^2}}{8z^6} - \frac{5}{16}\frac{L^7\sqrt{z^2}}{z^2} \quad \text{for } z=L$$

$f(L) = 1 - \frac{1}{2} + \frac{3}{8} - \frac{5}{16} \Rightarrow 0.56 \Rightarrow$ increasing the order to 10

$f(L) \approx 0.802$, and see how it aproaches $\frac{1}{\sqrt{2}}$ (exact for $z=L$) as the order increases.

From HW1.2 $\Rightarrow$ $E(z)$ for $\lambda = 1E^{-9}$ and $L = z = 1$

$$E(z) = 12.7163 \text{ N/C}$$

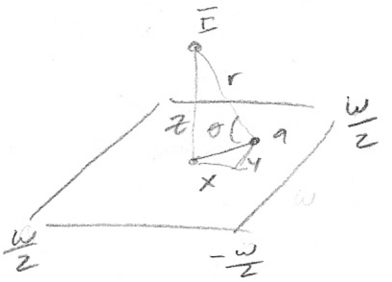
from Gauss Law $\Rightarrow$ $E(z) = 17.98$ with the same parameters.

using the taylor aproximation of 0.802 $\Rightarrow$

$$E(z) = 17.98 * 0.802 = 14.42 \text{ N/C}$$

This solution is still far off, even with an order 10 Taylor Series aproximation, but getting closer.

HW 1.36



$$r = \sqrt{x^2+y^2+z^2}$$

$$\sin\theta = \frac{z}{d} = \frac{z}{\sqrt{x^2+y^2+z^2}}$$

$$GL \Rightarrow E = \frac{\sigma}{2\epsilon_0}$$

$$E = \frac{\sigma r}{4\pi\epsilon_0}\int_{-w/2}^{w/2}\int_{-w/2}^{w/2}\frac{1}{\sqrt{x^2+y^2+z^2}}\,dx\,dy = \frac{\sigma}{\pi\epsilon_0}\tan^{-1}\left(\frac{w^2}{4z\sqrt{\frac{w^2}{2}+z^2}}\right) \quad \text{ok ✓}$$

so do Taylor series on $2\pi\tan^{-1}\left(\frac{w^2}{4z\sqrt{\frac{w^2}{2}+z^2}}\right)$ for $w=0$

for order 2 $\Rightarrow \dfrac{\pi\sqrt{z^2}\,w^2}{2z^3} - \dfrac{(\pi\sqrt{z^2})\,w^4}{8z^5}$

For $\sigma = -1E{-9}\,C$, $z=1$ and $w=2$ or $2\cdot z$

$$GL \rightarrow E = \frac{\sigma}{2\epsilon_0} = \frac{\sigma}{2\epsilon_0} = \frac{1E^{-9} * 4}{2\epsilon_0} \cong 56 \text{ N/C}$$

$$E_2(z) = \frac{\sigma}{\pi\epsilon_0}\tan^{-1}\left(\frac{w^2}{4z}\,\frac{1}{\sqrt{z^2+\frac{w^2}{2}}}\right) = 1079 \text{ N/C}$$

$$\text{Taylor} \Rightarrow \frac{\pi\sqrt{z^2}\,w^2}{2z^3} = 2\pi$$

$GL * \text{Taylor} \Rightarrow 56 * 2\pi = 355$ N/C still too far, may need additional Taylor orders.