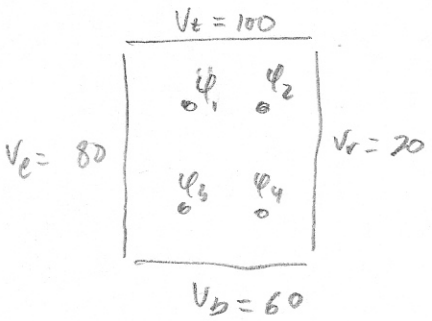
HW #2

2.2 Laplace Equation in two dimensions – Numerical

assume $x_0, y_0 = 1$

$\varphi_1\left(\frac{1}{3}, \frac{2}{3}\right)$ $\varphi_2\left(\frac{2}{3}, \frac{2}{3}\right)$ $\varphi_3\left(\frac{1}{3}, \frac{1}{3}\right)$ $\varphi_4\left(\frac{2}{3}, \frac{1}{3}\right)$



using formulas from 2.1

$\varphi_1\left(\frac{1}{3}, \frac{2}{3}\right)$

$$\varphi_e\left(\frac{1}{3}, \frac{2}{3}\right) = \sum_{n=1}^{10} -\frac{4*80}{n\pi} \frac{1}{\sinh n\pi} \sin\left(\frac{2}{3}n\pi\right) \sinh\left(-n\pi\left(\frac{1}{3}-1\right)\right) = 25.35$$

$$\varphi_b\left(\frac{1}{3}, \frac{2}{3}\right) = \sum_{n=1}^{10} -\frac{4*60}{n\pi} \frac{1}{\sinh n\pi} \sin\left(\frac{1}{3}n\pi\right) \sinh\left(n\pi\left(\frac{2}{3}-1\right)\right) = 7.66$$

$$\varphi_t\left(\frac{1}{3}, \frac{2}{3}\right) = \sum_{n=1}^{10} \frac{4*100}{n\pi} \frac{1}{\sinh n\pi} \sin\left(\frac{1}{3}n\pi\right) \sinh\left(\frac{2}{3}n\pi\right) = 44.44$$

$$\varphi_r\left(\frac{1}{3}, \frac{2}{3}\right) = \sum_{n=1}^{10} \frac{4*20}{n\pi} \frac{1}{\sinh n\pi} \sin\left(\frac{2}{3}n\pi\right) \sinh\left(\frac{1}{3}n\pi\right) = 2.22$$

$$\varphi_1 = \varphi_e + \varphi_b + \varphi_t + \varphi_r = 25.35 + 7.66 + 44.44 + 2.22 = \underline{79.67}$$

$\varphi_2\left(\frac{2}{3}, \frac{2}{3}\right)$ => same way

$\varphi_e\left(\frac{2}{3}, \frac{2}{3}\right) = 8.89$, $\varphi_b\left(\frac{2}{3}, \frac{2}{3}\right) = 6.67$, $\varphi_t\left(\frac{2}{3}, \frac{2}{3}\right) = 31.7$

$\varphi_r\left(\frac{2}{3}, \frac{2}{3}\right) = 6.34$

$$\varphi_2 = 8.89 + 6.67 + 31.9 + 6.34 = \underline{53.59}$$

$\varphi_3\left(\frac{1}{3}, \frac{1}{3}\right)$ => same way

$\varphi_e\left(\frac{1}{3}, \frac{1}{3}\right) = 35.55$, $\varphi_b\left(\frac{1}{3}, \frac{1}{3}\right) = 26.67$, $\varphi_t\left(\frac{1}{3}, \frac{1}{3}\right) = 12.76$, $\varphi_r\left(\frac{1}{3}, \frac{1}{3}\right) = 2.55$

$$\varphi_3 = 35.55 + 26.67 + 12.76 + 2.55 = \underline{77.51}$$

$\varphi_4\left(\frac{2}{3}, \frac{1}{3}\right)$ => Same way

$\varphi_e\left(\frac{2}{3}, \frac{1}{3}\right) = 10.19$ $\varphi_b\left(\frac{2}{3}, \frac{1}{3}\right) = 19.02$ $\varphi_t\left(\frac{2}{3}, \frac{1}{3}\right) = 11.1$ $\varphi_r\left(\frac{2}{3}, \frac{1}{3}\right) = 8.89$

$$\varphi_4 = 10.19 + 19.02 + 11.1 + 8.89 = \underline{49.22}$$