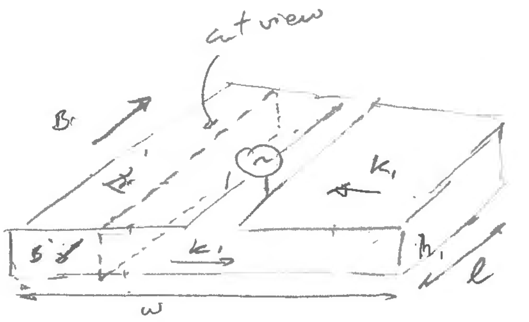
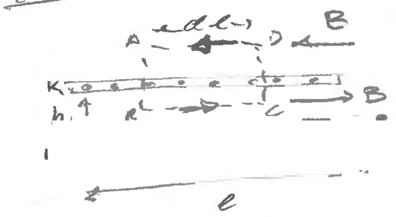
9.1 Flux Linkage.



1. Find Magnetic Field.

Cut view



$$\oint B\, d\vec{l} = \mu_0 I_{encl.} \Rightarrow I_{encl.} = K l.$$

$$\int_A^B \cancel{B\, d\vec{l}}^{\,0} + \int_B^C B\, dl + \int_C^D \cancel{B\, d\vec{l}}^{\,0} + \int_D^A B\, dl = \mu_0 K l$$

$$\int_B^C B\, dl + \int_D^A B\, dl = \mu_0 K l.$$

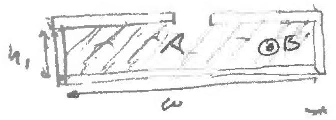
$$B\, dl + B\, dl = \mu_0 K l \Rightarrow \text{for } dl = l.$$

$$2 B \cancel{l} = \mu K \cancel{l} \Rightarrow$$

$$\Rightarrow B = \frac{\mu_0 K}{2} \text{ top sheet and } B = \frac{\mu_0 K}{2} \text{ bottom sheet}$$

$$B_T = \mu_0 K$$

2.



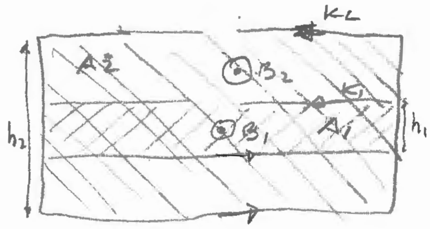
$$A = h_1 \cdot w, \quad I = K l, \quad B = \frac{\mu_0 K}{2};$$

$$\Phi_m = I L_1 \Rightarrow L_1 = \frac{1}{I} \Phi_m = \frac{1}{I} \int_S B\, ds = \frac{1}{\cancel{K} l} \frac{\mu_0 \cancel{K} (h_1 \cdot w)}{2}$$

$$L_1 = \frac{\mu_0 h_1 \cdot w}{2 l}$$

$$\mathcal{E}_1 = -\frac{\mu_0 h_1 w}{2 l} \frac{\partial I_1}{\partial t}$$

3.



$$A_1 \Rightarrow \Phi_{m_1} \Rightarrow L_1 = \frac{1}{I_1}\int_S B_1\, ds_1 + \frac{1}{I_2}\int B_2\, ds_1 =$$

$$= \frac{1}{\cancel{K_1}e}\,\frac{\mu_0 \cancel{K_1}}{2}(h_1 w) + \frac{1}{\cancel{K_2}e}\,\frac{\mu_0 \cancel{K_2}}{2}(h_1 w) = \frac{\mu_0 h_1 w}{e}$$

$$A_2 \Rightarrow \Phi_2 \Rightarrow L_2 = \frac{1}{I_2}\int_S B_2\, ds_2 + \frac{1}{I_1}\int_S B_1\, ds_1 =$$

$$= \frac{1}{\cancel{K_2}e}\,\frac{\mu_0 \cancel{K_2}}{2}(h_2 w) + \frac{1}{\cancel{K_1}e}\,\frac{\mu_0 \cancel{K_1}}{2}(h_1 w) =$$

$$= \frac{\mu_0 w}{e}(h_2 + h_1)$$

$$L_T = L_1 + L_2$$

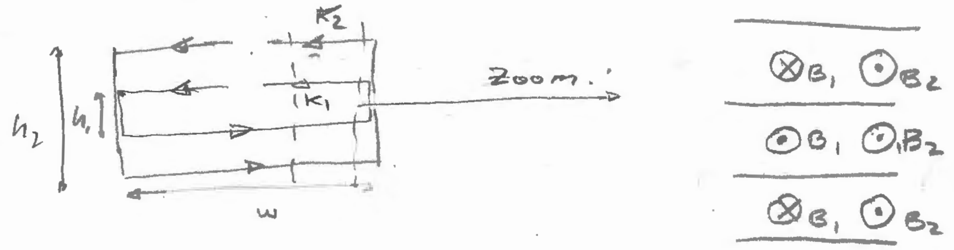
$$L_T = \frac{\mu_0 w}{e} h_1 + \frac{\mu_0 w}{e}(h_2 + h_1)$$

$$L_T = \frac{\mu_0 w}{e}(2h_1 + h_2)$$

$$\mathcal{E} = \mathcal{E}_1 + \mathcal{E}_2$$

$$\mathcal{E} = -\frac{\mu_0 w}{e}(2h_1 + h_2)\frac{\partial I}{\partial t}$$

4.



For Φ_{m_1} $\Longrightarrow$ $L_1 = \frac{1}{I_1}\int_s B_1\, ds_1 + \frac{1}{I_2}\int B_2\, ds_1 =$

$$= \frac{1}{\cancel{K_1}\ell}\,\frac{\mu_0 K_1}{2}(h_1 w) + \frac{1}{\cancel{K_2}\ell}\,\frac{\mu_0 K_2}{2}(h_1 w) = \frac{\mu_0 h_1 w}{\ell}$$

$$\mathcal{E}_1 = -\frac{\mu_0 h_1 w}{\ell}\left(\frac{\partial I_1}{\partial t} + \frac{\partial I_2}{\partial t}\right) = -\frac{\mu_0 h_1 w}{\ell}\,\frac{\partial I}{\partial t}$$

For Φ_{m_2} $\Longrightarrow$ $L_2 = \frac{1}{I_2}\int_s B_2\, ds_2 + \frac{1}{I_1}\int_s B_1\, ds_1 - \frac{1}{I_1}\int_s B_1\,(ds_2 - ds_1) =$

$$= \frac{1}{\cancel{K_2}\ell}\,\frac{\mu_0 \cancel{K_2}}{2}(h_2 w) + \frac{1}{\cancel{K_1}\ell}\,\frac{\mu_0 \cancel{K_1}}{2}(h_1 w) - \frac{1}{\cancel{K_1}\ell}\,\frac{\mu_0 \cancel{K_1}}{2}(h_2 - h_1)\, w =$$

$$= \frac{\mu_0 w\,(h_2 + h_1)}{\ell} - \frac{\mu_0 w\,(h_2 - h_1)}{2\ell} = \frac{\mu_0 w}{\ell}\left[(h_2 + h_1) - \frac{1}{2}(h_2 - h_1)\right]$$

$$\mathcal{E}_2 = -\left[\frac{\mu_0 w}{\ell}(h_2 + h_1)\left(\frac{\partial I_2}{\partial t} + \frac{\partial I_1}{\partial t}\right) - \frac{1}{2}\,\frac{\mu_0 w}{\ell}(h_2 - h_1)\frac{\partial I_1}{\partial t}\right]$$

$$\mathcal{E}_2 = -\frac{\mu_0 w}{\ell}(h_2 + h_1)\frac{\partial I}{\partial t} + \frac{1}{2}\,\frac{\mu_0 w}{\ell}(h_2 - h_1)\frac{\partial I_1}{\partial t}$$

$$\mathcal{E} = \mathcal{E}_1 + \mathcal{E}_2$$

$$\mathcal{E} = -\left[\frac{\mu_0 h_1 w}{\ell} + \frac{\mu_0 w}{\ell}(h_2 + h_1)\right]\frac{\partial I}{\partial t} + \frac{1}{2}\,\frac{\mu_0 w}{\ell}(h_2 - h_1)\frac{\partial I_1}{\partial t} =$$

$$= -\left[\frac{\mu_0 w}{\ell}(2h_1 + h_2)\frac{\partial I}{\partial t} - \frac{1}{2}\,\frac{\mu_0 w}{\ell}(h_2 - h_1)\frac{\partial I_1}{\partial t}\right]$$

5. Flux linkage:

When current flows through a conductor, it induces a magnetic field normal to the current direction, following the right hand rule. This magnetic field can affect other conductors or circuits nearby, creating a link. This link created by the magnetic field of the original circuit/conductor affects the second circuit, changing the voltage, inductance, magnetic field, and other parameters of the second circuit/conductor (and vice versa). This flux linkage creates a mutual inductance coupling, that can be calculated using Faraday's law, by integrating the magnetic flux created by the first conductor/circuit, due to the current in that conductor, over the area of the second conductor or circuit.