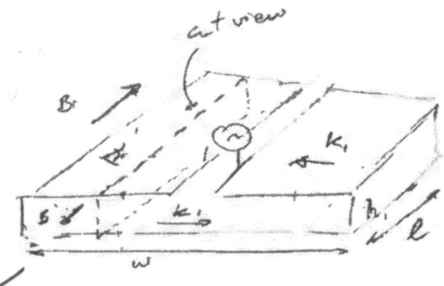
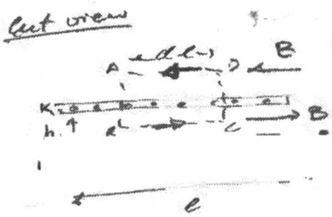
9.1 "Flux Linkage.

1. Find Magnetic Field.





$$\oint B\, d\vec{l} = \mu_0 I_{encl.} \Rightarrow I_{encl.} = kl.$$

$$\int_A^B B\, d\vec{l}^{\,0} + \int_B^C B\, dl + \int_C^D B\, d\vec{l}^{\,0} + \int_D^A B\, dl = \mu_0 k l$$

$$\int_B^C B\, dl + \int_D^A B\, dl = \mu_0 k l.$$

$$B dl + B dl = \mu_0 k l \quad \Rightarrow \text{for } dl = l.$$

$$2B\cancel{l} = \mu_0 k \cancel{l} \Rightarrow$$

$$\Rightarrow B = \frac{\mu_0 k}{2} \text{ top sheet and } B = \frac{\mu_0 k}{2} \text{ bottom sheet}$$

$$B_T = \mu_0 k$$

2.

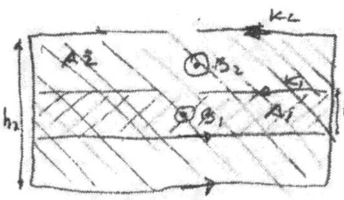
$$A = h_1 \cdot w, \quad I = kl, \quad B = \frac{\mu_0 k}{2} \; \mu_0 K$$

$$\Phi_m = I L_1 \Rightarrow L_1 = \frac{1}{I} \Phi_m = \frac{1}{I} \int_S B\, ds = \frac{1}{\cancel{k} l} \frac{\mu_0 \cancel{k}}{\cancel{2}} (h_1 \cdot w)$$

$$L_1 = \frac{\mu_0 h_1 \cdot w}{\cancel{2} l}$$

$$\varepsilon_1 = -\frac{\mu_0 h_1 w}{\cancel{2} l} \frac{\partial I_1}{\partial t}$$

3.



B_2 will be smaller than B_1. Field due to inner duct is zero outside of it

$B_1 = 2\mu_0 K_1$

$B_2 = \mu_0 K_2$

$$A_1 \Rightarrow \Phi_{m_1} \Rightarrow L_1 = \frac{1}{I_1}\int_s B_1\, ds_1 + \frac{1}{I_2}\int B_2\, ds_1 =$$

$$= \frac{1}{\cancel{K_1}\ell}\,\frac{\mu_0 \cancel{K_1}}{2}(h_1 w) + \frac{1}{\cancel{K_2}\ell}\,\frac{\mu_0 \cancel{K_2}}{2}(h_1 w) = \frac{\mu_0 h_1 w}{\ell}$$

$$A_2 \Rightarrow \Phi_2 \Rightarrow L_2 = \frac{1}{I_2}\int_s B_2\, ds_2 + \frac{1}{I_1}\int_s B_1\, ds_1 =$$

$$= \frac{1}{\cancel{K_2}\ell}\,\frac{\mu_0 \cancel{K_2}}{2}(h_2 w) + \frac{1}{\cancel{K_1}\ell}\,\frac{\mu_0 K_1}{2}(h_1 w) =$$

$$= \frac{\mu_0 w}{\ell}(h_2 + h_1)$$

$$L_T = L_1 + L_2$$

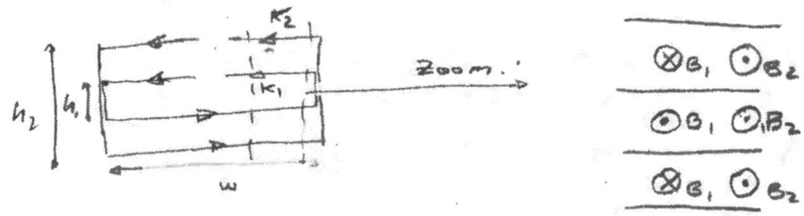
$$L_T = \frac{\mu_0 w}{\ell} h_1 + \frac{\mu_0 w}{\ell}(h_2 + h_1)$$

$$L_T = \frac{\mu_0 w}{\ell}(2h_1 + h_2)$$

$$\mathcal{E} = \mathcal{E}_1 + \mathcal{E}_2$$

$$\mathcal{E} = -\frac{\mu_0 w}{\ell}(2h_1 + h_2)\frac{\partial I}{\partial t}$$

4.



For $\Phi_{m_1} \Rightarrow$ $L_1 = \frac{1}{I_1}\int_S B_1\, ds_1 + \frac{1}{I_2}\int B_2\, ds_1 =$

$$= \frac{1}{\cancel{K_1}\, l}\,\frac{\mu_0 \cancel{K_1}}{2}(h_1 w) + \frac{1}{\cancel{K_2}\, l}\,\frac{\mu_0 \cancel{K_2}}{2}(h_1 w) = \frac{\mu_0 h_1 w}{l}$$

$$\mathcal{E}_1 = -\frac{\mu_0 h_1 w}{l}\left(\frac{\partial I_1}{\partial t} + \frac{\partial I_2}{\partial t}\right) = -\frac{\mu_0 h_1 w}{l}\,\frac{\partial I}{\partial t}$$

For $\Phi_{m_2} \Rightarrow$ $L_2 = \frac{1}{I_2}\int_S B_2\, ds_2 + \frac{1}{I_1}\int_S B_1\, ds_1 - \frac{1}{I_1}\int_S B_1\,(ds_2 - ds_1) =$

$$= \frac{1}{\cancel{K_2}\, l}\,\frac{\mu_0 \cancel{K_2}}{2}(h_2 w) + \frac{1}{\cancel{K_1}\, l}\,\frac{\mu_0 \cancel{K_1}}{2}(h_1 w) - \frac{1}{\cancel{K_1}\, l}\,\frac{\mu_0 \cancel{K_1}}{2}(h_2 - h_1)\, w =$$

$$= \frac{\mu_0 w\,(h_2 + h_1)}{l} - \frac{\mu_0 w\,(h_2 - h_1)}{2 l} = \frac{\mu_0 w}{l}\left[(h_2 + h_1) - \frac{1}{2}(h_2 - h_1)\right]$$

$$\mathcal{E}_2 = -\left[\frac{\mu_0 w}{l}(h_2 + h_1)\left(\frac{\partial I_2}{\partial t} + \frac{\partial I_1}{\partial t}\right) - \frac{1}{2}\,\frac{\mu_0 w}{l}(h_2 - h_1)\frac{\partial I_1}{\partial t}\right]$$

$$\mathcal{E}_2 = -\frac{\mu_0 w}{l}(h_2 + h_1)\frac{\partial I}{\partial t} + \frac{1}{2}\,\frac{\mu_0 w}{l}(h_2 - h_1)\frac{\partial I_1}{\partial t}$$

$$\mathcal{E} = \mathcal{E}_1 + \mathcal{E}_2$$

$$\mathcal{E} = -\left[\frac{\mu_0 h_1 w}{l} + \frac{\mu_0 w}{l}(h_2 + h_1)\right]\frac{\partial I}{\partial t} + \frac{1}{2}\,\frac{\mu_0 w}{l}(h_2 - h_1)\frac{\partial I_1}{\partial t} =$$

$$= -\left[\frac{\mu_0 w}{l}(2h_1 + h_2)\frac{\partial I}{\partial t} - \frac{1}{2}\,\frac{\mu_0 w}{l}(h_2 - h_1)\frac{\partial I_1}{\partial t}\right]$$

See solns.

5. Flux linkage:

When current flows through a conductor, it induces a magnetic field normal to the current direction, following the right hand rule. This magnetic field can affect other conductors or circuits nearby, creating a link. This link created by the magnetic field of the original circuit/conductor affects the second circuit, changing the voltage, Inductance, magnetic field, and other parameters of the second circuit/conductor (and vice versa). This flux linkage creates a mutual inductance coupling, that can be calculated using Faraday's law, by integrating the magnetic flux created by the first conductor/circuit, due to the current in that conductor, over the area of the second conductor or circuit.

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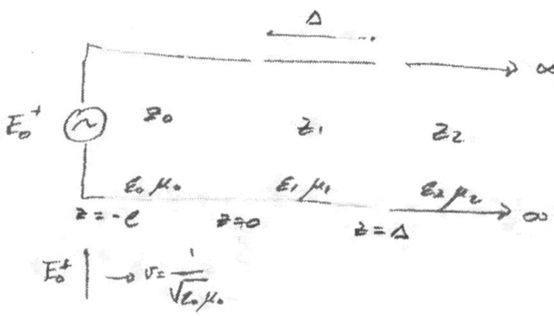
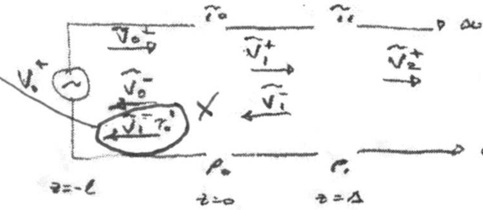
& using method of part 4. neglects the coupling between the two ducts.

9.2 Multiple Impedances

In steady state, all reflections are accounted for in V_0^-

9.2.1





$$\tilde{V}_n(z) = \tilde{V}_n^+ e^{-j\beta_n z} + \tilde{V}_n^- e^{+j\beta_n z} \Rightarrow V_n(z,t) = Re[\tilde{V}_n(z) e^{j\omega t}]$$

V_0^+ is known and $V_2^- = 0$; Find $\tilde{V}_0^-(z)$, $\tilde{V}_1^+(z)$, $\tilde{V}_1^-(z)$ and $\tilde{V}_2^+(z)$

$$\beta_0 = \frac{\omega}{v_0} = \omega\sqrt{\varepsilon_0\mu_0} \quad ; \beta_1 = \omega\sqrt{\varepsilon_1\mu_1}, \quad \beta_2 = \omega\sqrt{\varepsilon_2\mu_2}$$

$$\tilde{V}_0(z) = \tilde{V}_0^+ e^{-j\beta_0 z} + \tilde{V}_0^- e^{j\beta_0 z} \quad \text{and} \quad \rho_0 = \frac{z_1 - z_0}{z_1 + z_0} \quad \text{from 5.7 (8)}$$

$$\Rightarrow \tilde{V}_0^- = \tilde{V}_0^+ \rho_0 e^{j\beta_0 z} \quad \text{at } z=0 \Rightarrow \tilde{V}_0^-(z) = \tilde{V}_0^+ \rho_0 = \tilde{V}_0^+ \left(\frac{z_1 - z_0}{z_1 + z_0}\right) e^{j\beta_0 z} \checkmark$$

$$\checkmark \quad \tilde{V}_1^+ = \tilde{V}_0^+ \tau_0 e^{-j\beta_1 z} \Rightarrow \text{for } \tau_0 = \frac{2 z_1}{z_1 + z_0} \quad \text{from 5.7 (9)}$$

$$\text{at } z=1 \quad \tilde{V}_1^+(z) = \tilde{V}_0^+ \tau_0 e^{-j\beta_1 z}$$

$$\Rightarrow \text{at } z=\Delta \quad \tilde{V}_1^+(\Delta) = \tilde{V}_0^+ \tau_0 e^{-j\beta_1\Delta} = \tilde{V}_0^+ \left(\frac{2 z_1}{z_1 + z_0}\right) e^{-j\beta_1 z}$$

$$\text{for } \tilde{V}_1^- = \tilde{V}_1^+ \rho_\Delta e^{j\beta_1\Delta} \quad \text{and} \quad \rho_\Delta = \frac{z_2 - z_1}{z_2 + z_1} \quad \text{and at } z=\Delta$$

$$\Rightarrow \tilde{V}_1^-(z) = \tilde{V}_0^+ \left(\frac{2 z_1}{z_1 + z_0}\right)\left(\frac{z_2 - z_1}{z_2 + z_1}\right) e^{j\beta_1 z}$$

$$\text{at } z=\Delta \Rightarrow \tau = \frac{2 z_2}{z_2 + z_1} \quad \text{and at } z=\Delta+1 \Rightarrow$$

$$\Rightarrow \tilde{V}_2^+(z) = \tilde{V}_1^+ \tau e^{j\beta_2(\Delta+1)} = \tilde{V}_0^+ \left(\frac{2 z_1}{z_1 + z_0}\right)\left(\frac{2 z_2}{z_2 + z_1}\right) e^{j\beta_2 z}$$

$$\tilde{V}_2^- = 0 \qquad \tau_0' = \frac{V_L}{V^-} = \frac{2 z_1}{z_1 - z_0}$$

$$\text{Recalculate } \tilde{V}_0^- = \tilde{V}_0^-(0) + \tilde{V}_1^-(\Delta)\tau_0' = \tilde{V}_0^+ \left(\frac{z_1 - z_0}{z_1 + z_0}\right) e^{j\beta_0 z} + \tilde{V}_0^+ \left(\frac{2 z_1}{z_1 + z_0}\right)\left(\frac{z_2 - z_1}{z_2 - z_1}\right)\left(\frac{2 z_1}{z_1 - z_0}\right) e^{j\beta_1 z}$$

See solns for general procedure for steady state.

Find $\dfrac{\tilde{V}_0^-(z)}{\tilde{V}_0^+(z)}$, $\dfrac{\tilde{V}_1^-(z)}{\tilde{V}_1^+(z)}$

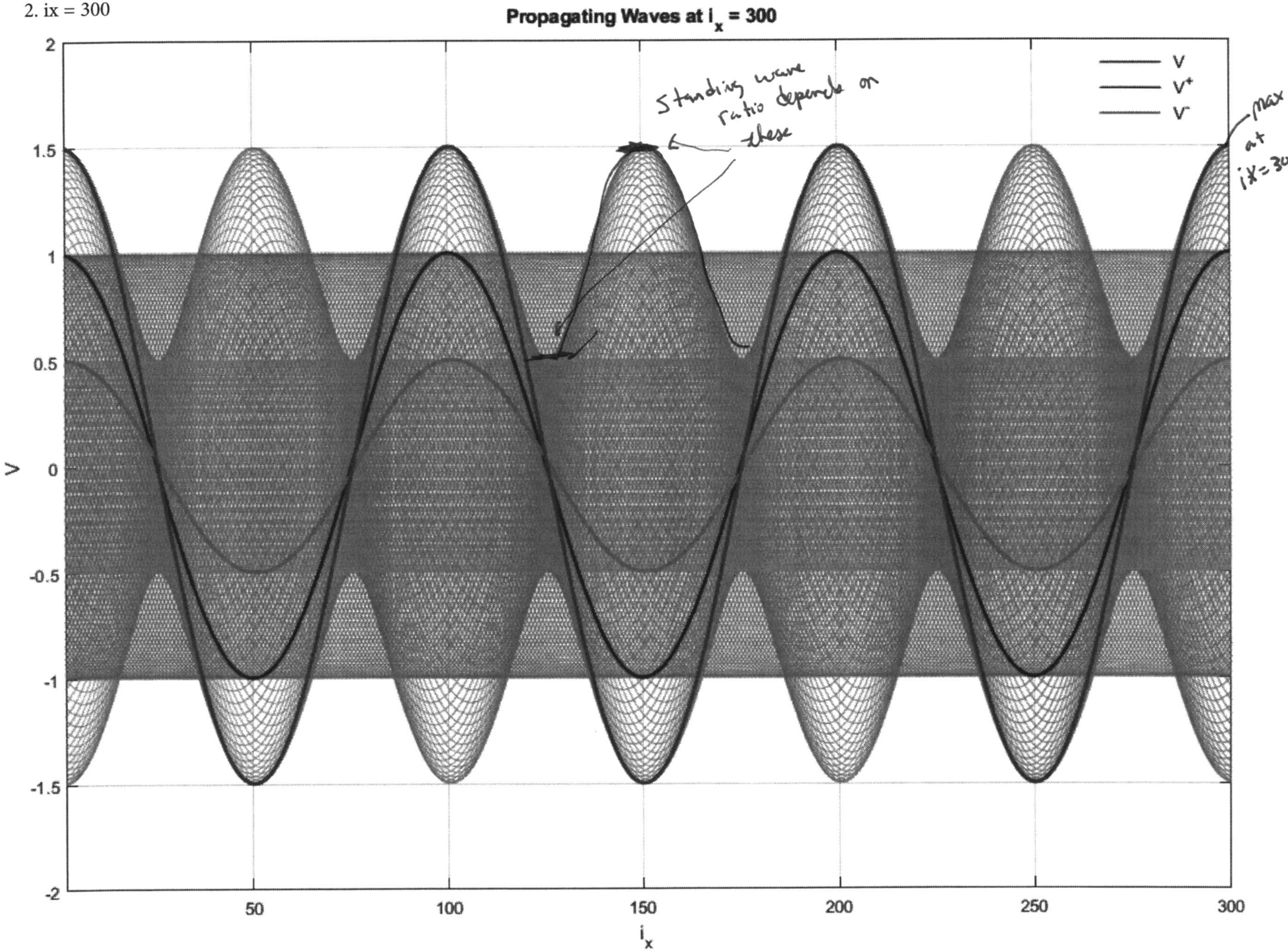
$$\frac{\tilde{V}_0^-(z)}{\tilde{V}_0^+(z)} = \frac{\cancel{\tilde{V}_0^+}\left(\frac{Z_1-Z_0}{Z_1+Z_0}\right)e^{j\beta_0 z} + \cancel{\tilde{V}_0^+}\left(\frac{2Z_1}{Z_1+Z_0}\right)\left(\frac{Z_2-Z_1}{Z_2+Z_1}\right)\left(\frac{2Z_1}{Z_1-Z_0}\right)e^{j\beta_1 z}}{\cancel{\tilde{V}_0^+}\,e^{-j\beta_0 z}} =$$

$$= \frac{Z_1-Z_0}{Z_1+Z_0}e^{2j\beta_0 z} + \left(\frac{2Z_1}{Z_1+Z_0}\right)\left(\frac{Z_2-Z_1}{Z_2+Z_1}\right)\left(\frac{2Z_1}{Z_1-Z_0}\right)e^{jz(\beta_0+\beta_1)} =$$

$$= \rho_0 e^{2j\beta_0 z} + \tau_0 \rho_1 \tau_0' e^{jz(\beta_0+\beta_1)}$$

$$\frac{\tilde{V}_1^-(z)}{\tilde{V}_1^+(z)} = \frac{\cancel{\tilde{V}_0^+}\left(\cancel{\frac{2Z_1}{Z_1+Z_0}}\right)\left(\frac{Z_2-Z_1}{Z_2+Z_1}\right)e^{j\beta_1 z}}{\cancel{\tilde{V}_0^+}\left(\cancel{\frac{2Z_1}{Z_1+Z_0}}\right)e^{-j\beta_1 z}} = \left(\frac{Z_2-Z_1}{Z_2+Z_1}\right)e^{2j\beta_1 z} = \rho_1 e^{2j\beta_1 z}$$

2. ix = 300


Propagating Waves at i_x = 300
V
V
V^+
V^-
i_x
Standing wave ratio depends on these
max at ix = 300

9.2.2 . $V(z,t) = V^+[\cos(\omega t - \beta z) + \rho \cos(\omega t + \beta z)]$

1. $V(z,t) = A \cos \omega t \cos \beta z + B \sin \omega t \sin \beta z$ $\Rightarrow$ find A and B

$V^+ \cos(\omega t - \beta z) = V^+ \cos \omega t \cos \beta z + V^+ \sin \omega t \sin \beta z$

$V^+_\rho \cos(\omega t + \beta z) = V^+\rho \cos \omega t \cos \beta z - V^+_\rho \sin \omega t \sin \beta z$.

$V(z,t) = V^+ \cos(\omega t - \beta z) + V^+_\rho \cos(\omega t + \beta z) =$

$= V^+ \cos \omega t \cos \beta z + V^+ \sin \omega t \sin \beta z + V^+_\rho \cos \omega t \cos \beta z - V^+_\rho \sin \omega t \sin \beta z =$

$= V^+(1+\rho) \cos \omega t \cos \beta z + V^+(1-\rho) \sin \omega t \sin \beta z \Rightarrow$

$A = V^+(1+\rho)$ and $B = V^+(1-\rho) \Rightarrow$

$\Rightarrow A \cos \omega t \cos \beta z + B \sin \omega t \sin \beta z$

$\therefore$ this is origin of the term standing wave ratio.

4. From the plot, and as before in 3.

$$VSWR = \frac{V_{max}}{V_{min}} = \frac{1.5}{0.5} = 3$$

$$|\rho| = \frac{S-1}{S+1} = 0.5$$

however, with $ix = 550$, θ_ρ has changed, and V_{min} is at $ix = 500$ or (0.5λ), therefore

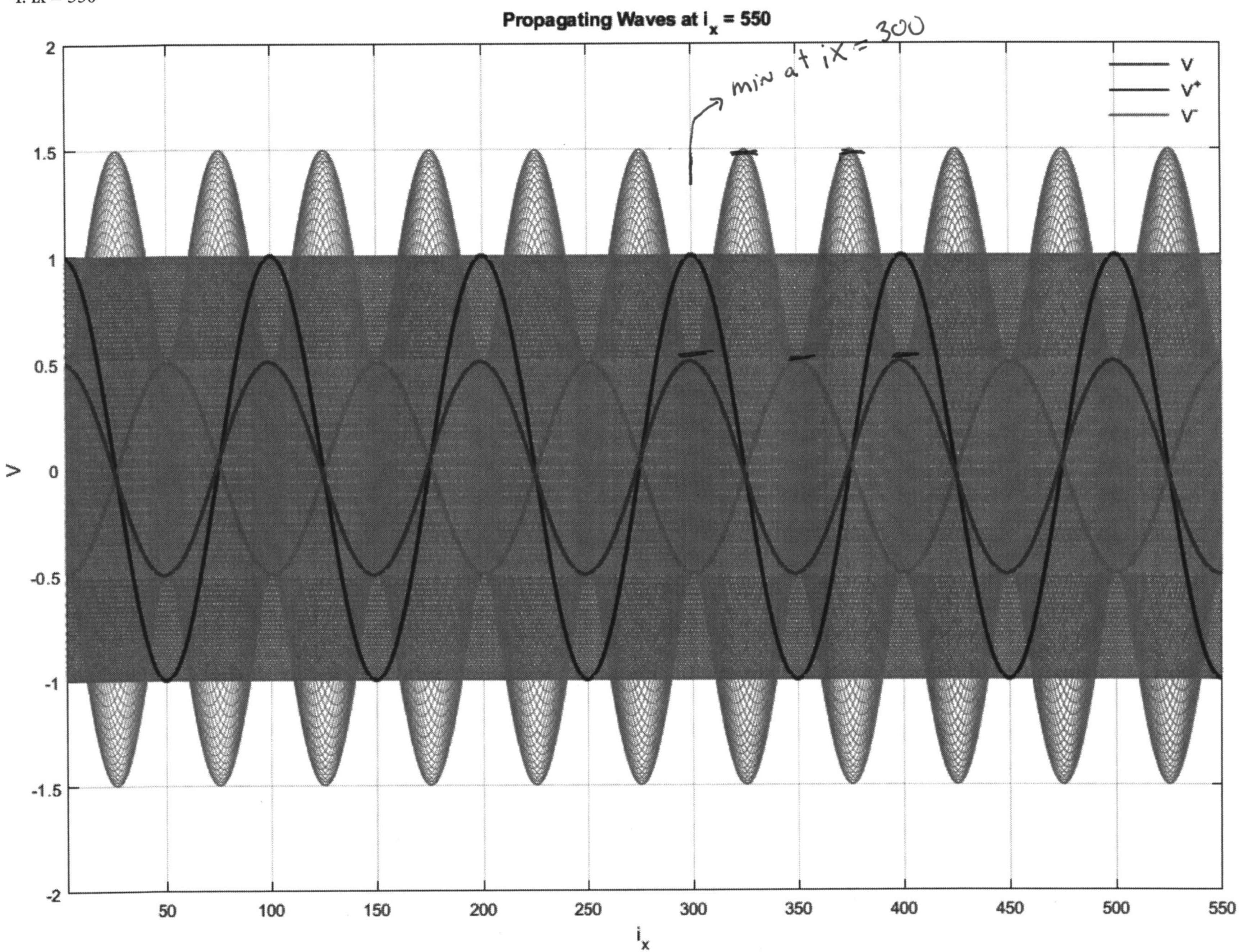
$$\theta_\rho = 2\beta(0.5\lambda) - \pi = \pi$$

which seems about right when comparing V^+ and V^- that show about 180° difference.

Note also that at $ix = 300$ VSWR envelope is at a minimum. In previous plot, it was at maximum.

$\Rightarrow$ increase in length by multiple of $\lambda/2$ shifts VSWR from max to min.

4. ix = 550


Propagating Waves at i_x = 550
min at iX = 300
V
V⁺
V⁻
V
i_x
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
50
100
150
200
250
300
350
400
450
500
550

3. By ploting a $V^- = 1/2$ traveling to the left, we are replicating the reflected wave at $ix = 300$. When we add $V^+ = 1$ and $V^- = 0.5$ we can observe the $V_{max} = 1.5$ and $V_{min} = 0.5$ created in that wave (blue in my plot). From this V_{max} and V_{min}, we can obtain the Voltage Wave Standing Ratio (VSWR) by.

$$VSWR = \frac{V_{max}}{V_{min}} = \frac{1.5}{0.5} = 3$$

Likewise we could obtain the magnitude of the reflection coefficient ρ by

$$|\rho| = \frac{S-1}{S+1} = \frac{2}{4} = 0.5$$

However, just from the plot is difficult to calculate θ_ρ. to obtain $\rho = |\rho| e^{j\theta_\rho}$, but an attempt looking at V_{min} from $ix = 300$ looks like at $ix = 275$, or (0.25λ) and

$$\theta_\rho = 2\beta(0.25\lambda) - \pi \quad ; \text{ for } \beta = \frac{\omega}{c} = \frac{2\pi f}{c} = \frac{2\pi}{\lambda}$$

$$\theta_\rho = 4\pi(0.25) - \pi = 0$$

This confirms the lack of phase difference in V^+ and V^- accross the plot, and:

$$\rho = |\rho| = \frac{1}{2}$$