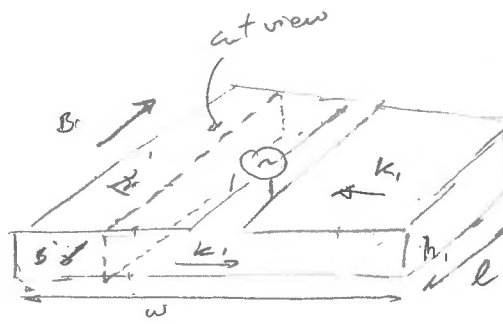
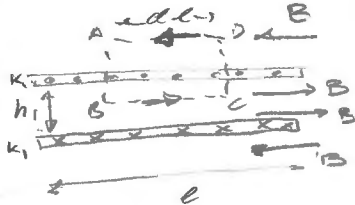


## 9.1 Flux Linkage.

1. Find Magnetic Field.

cut view



$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{\text{encl.}} \Rightarrow I_{\text{encl.}} = k\ell$$

$$\int_A^B \vec{B} \cdot d\vec{\ell} + \int_B^C \vec{B} \cdot d\vec{\ell} + \int_C^D \vec{B} \cdot d\vec{\ell} + \int_D^A \vec{B} \cdot d\vec{\ell} = \mu_0 k\ell$$

$$\int_B^C \vec{B} \cdot d\vec{\ell} + \int_D^A \vec{B} \cdot d\vec{\ell} = \mu_0 k\ell$$

$$B\ell + B\ell = \mu_0 k\ell \Rightarrow \text{for } \ell = \ell$$

$$2B\ell = \mu_0 k\ell \Rightarrow$$

$$\Rightarrow B = \frac{\mu_0 k}{2} \text{ top sheet and } B = \frac{\mu_0 k}{2} \text{ bottom sheet}$$

$$B_T = \mu_0 k$$

2.



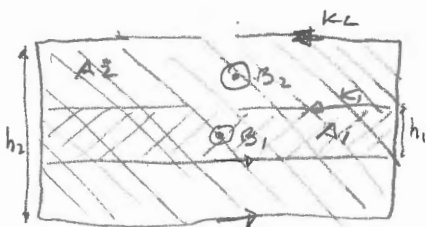
$$A = h_1 \cdot w, I = k\ell, B = \frac{\mu_0 k}{2}$$

$$\Phi_m = I L_1 \Rightarrow L_1 = \frac{1}{I} \Phi_m = \frac{1}{I} \int_S B \cdot d\vec{s} = \frac{1}{k\ell} \frac{\mu_0 k}{2} (h_1 \cdot w)$$

$$L_1 = \frac{\mu_0 h_1 \cdot w}{2\ell}$$

$$\mathcal{E}_1 = - \frac{\mu_0 h_1 \cdot w}{2\ell} \frac{\partial I_1}{\partial t}$$

3.



$$A_1 \Rightarrow \Phi_1 \Rightarrow L_1 = \frac{1}{I_1} \int_s B_1 ds_1 + \frac{1}{I_2} \int_s B_2 ds_1 =$$

$$= \frac{1}{\frac{1}{2}e} \frac{\mu_0 \kappa_1}{2} (h_1 \omega) + \frac{1}{\frac{1}{2}e} \frac{\mu_0 \kappa_2}{2} (h_1 \omega) = \frac{\mu_0 h_1 \omega}{e}$$

$$A_2 \Rightarrow \Phi_2 \Rightarrow L_2 = \frac{1}{I_2} \int_s B_2 ds_2 + \frac{1}{I_1} \int_s B_1 ds_1 =$$

$$= \frac{1}{\frac{1}{2}e} \frac{\mu_0 \kappa_2}{2} (h_2 \omega) + \frac{1}{\frac{1}{2}e} \frac{\mu_0 \kappa_1}{2} (h_1 \omega) =$$

$$= \frac{\mu_0 \omega}{e} (h_2 + h_1)$$

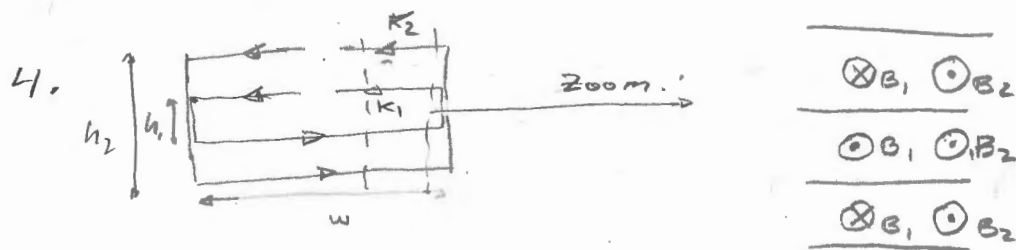
$$L_T = L_1 + L_2$$

$$L_T = \frac{\mu_0 \omega}{e} h_1 + \frac{\mu_0 \omega}{e} (h_2 + h_1)$$

$$L_T = \frac{\mu_0 \omega}{e} (2h_1 + h_2)$$

$$E = E_1 + E_2$$

$$E = - \frac{\mu_0 \omega}{e} (2h_1 + h_2) \frac{\partial I}{\partial t}$$



$$\text{For } \Phi_{m1} \Rightarrow L_1 = \frac{1}{I_1} \int_s B_1 ds_1 + \frac{1}{I_2} \int B_2 ds_1 =$$

$$= \frac{1}{\frac{1}{2}e} \frac{\mu_0 K_1}{2} (h_1 w) + \frac{1}{\frac{1}{2}e} \frac{\mu_0 K_2}{2} (h_1 w) = \frac{\mu_0 h_1 w}{e}$$

$$\mathcal{E}_1 = - \frac{\mu_0 h_1 w}{e} \left( \frac{\partial I_1}{\partial t} + \frac{\partial I_2}{\partial t} \right) = - \frac{\mu_0 h_1 w}{e} \frac{\partial I}{\partial t}$$

$$\text{For } \Phi_{m2} \Rightarrow L_2 = \frac{1}{I_2} \int B_2 ds_2 + \frac{1}{I_1} \int B_1 ds_1 - \frac{1}{I_1} \int B_1 (ds_2 - ds_1) =$$

$$= \frac{1}{\frac{1}{2}e} \frac{\mu_0 K_2}{2} (h_2 w) + \frac{1}{\frac{1}{2}e} \frac{\mu_0 K_1}{2} (h_1 w) - \frac{1}{\frac{1}{2}e} \frac{\mu_0 K_1}{2} (h_2 - h_1) w =$$

$$= \frac{\mu_0 w (h_2 + h_1)}{e} - \frac{\mu_0 w (h_2 - h_1)}{2e} = \frac{\mu_0 w}{e} \left[ (h_2 + h_1) - \frac{1}{2} (h_2 - h_1) \right]$$

$$\mathcal{E}_2 = - \left[ \frac{\mu_0 w}{e} (h_2 + h_1) \left( \frac{\partial I_2}{\partial t} + \frac{\partial I_1}{\partial t} \right) - \frac{1}{2} \frac{\mu_0 w}{e} (h_2 - h_1) \frac{\partial I_1}{\partial t} \right]$$

$$\mathcal{E}_2 = - \frac{\mu_0 w}{e} (h_2 + h_1) \frac{\partial I}{\partial t} + \frac{1}{2} \frac{\mu_0 w}{e} (h_2 - h_1) \frac{\partial I_1}{\partial t}$$

$$\mathcal{E} = \mathcal{E}_1 + \mathcal{E}_2$$

$$\mathcal{E} = - \left[ \frac{\mu_0 h_1 w}{e} + \frac{\mu_0 w}{e} (h_2 + h_1) \right] \frac{\partial I}{\partial t} + \frac{1}{2} \frac{\mu_0 w}{e} (h_2 - h_1) \frac{\partial I_1}{\partial t} =$$

$$= - \left[ \frac{\mu_0 w}{e} (2h_1 + h_2) \frac{\partial I}{\partial t} - \frac{1}{2} \frac{\mu_0 w}{e} (h_2 - h_1) \frac{\partial I_1}{\partial t} \right]$$

### 5. Flux Linkage:

When current flows through a conductor, it induces a magnetic field around to the current direction, following the right hand rule.

This magnetic field can affect other conductors or circuits nearby, creating a link. This link created by the magnetic field of the original circuit/conductor affects the second circuit, changing the voltage, Inductance, magnetic field, and other parameters of the second circuit/conductor (and vice versa). This flux linkage creates a mutual inductance coupling, that can be calculated using Faraday's Law, by integrating the magnetic flux created by the first conductor/circuit, due to the current in that conductor, over the area of the second conductor or circuit.