Bayesian Causal Inference Workshop
Northwestern University School of Law

Baldus Data Analysis

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Setting for Causal Inference

- Causal Inference is concerned with what would have happened to case i's outcome variable, y_i, if it had received a different treatment level.
- Causal inference can also be considered as a special case of *prediction* under varying circumstances, only with much stricter assumptions than usual.
- Define T_c as the control, and T_t as the treatment, very generally defined.
- This is a highly active and highly controversial area of research.

A Critical Problem

- The manner in which the treatment is assigned is critical to experimental and observational studies.
- Suppose healthier people are more commonly given an effective treatment, then this exaggerates the actual effect.
- Suppose, alternatively, that healthier people are more commonly assigned to the control, then this mitigates the actual effect.
- In this context health is a confounding covariate.

Dealing with Confounders

- We want to compare treated and controlled units conditional on the confounding covariate.
- Simple solution: regress the outcome on two inputs: the treatment indicator and the confounding covariate (and any others).
- If the model is "correct," the coefficient on the treatment indicator estimates the average causal effect in the sample.
- This is an optimistic scenario with real data.

Omitted Variable Bias

- Suppose the "correct" model is given by:

$$y_i = \beta_0 + \beta_1 T_i + \beta_2 x_i + \epsilon_i,$$

  where T_i is the treatment and x_i is the confounding covariate, both for unit i.

- The model that ignores this confounding covariate is obviously:

$$y_i = \beta_0 + \beta_1 T_i + \epsilon_i.$$

- Define a third model that attempts to measure the effect of the treatment on on the confounding effect:

$$x_i = \gamma_0 + \gamma_1 T_i + \nu_i.$$

- Insert this specification into the correct equation:

$$\begin{aligned} y_i &= \beta_0 + \beta_1 T_i + \beta_2(\gamma_0 + \gamma_1 T_i + \nu_i) + \epsilon_i \\ &= \beta_0 + \beta_2\gamma_0 + (\beta_1 + \beta_2\gamma_1)T_i + \epsilon_i + \beta_2\nu_i. \end{aligned}$$

Omitted Variable Bias

- The heteroscedastic term $\beta_2\nu_i$ is often assumed away as with multilevel models.
- Now define $\beta_1^* = (\beta_1 + \beta_2\gamma_1)$, so that:

$$y_i = \beta_0 + \beta_1^* T_i + \beta_2\gamma_0 + \epsilon_i,$$

- We defined γ_1 as the coefficient relating the confounding covariate to the treatment.
- If this is zero then there is no confounding from x_i.
- Note that $\beta_2\gamma_0$ simply gives a baseline effect across both the treatment and control groups on the outcome variable (subtracting this from the outcome produces a "gain score" for modeling).

The Big Problem

- For a unit assigned the treatment, $T_i = 1$, the outcome y_i^1 observed and y_i^0 is the unobserved *counterfactual* outcome.
- Conversely, for a unit assigned the control, $T_i = 0$, the outcome y_i^0 observed and y_i^1 is the unobserved *counterfactual* outcome.
- The simple treatment effect is for unit $i$is:

$$TE_i = y_i^1 - y_i^0.$$

- The fundamental problem of causal inference is that only one of the right-hand-side terms can be realized, so TE_i can never be directly observed.
- Therefore we also cannot directly get the *average treatment effect*:

$$\overline{TE} = \frac{1}{n}\sum_{i=1}^{n}(y_i^1 - y_i^0).$$

Dealing With the Big Problem

- Randomization: the "gold standard" available only for controlled experiments.
- Close Substitutes: using similar treatments on the same subject, splitting units, making strong assumptions about controls on units (eg. the baseline equals the control outcome).
- Model Adjustment: statistical manipulation of the data and/or the model such that a pseudo-experimental treatment results and the interpretation of the outcome variable is changed.
- The latter approach often gives better *internal validity* (supportable claims about the sample) than *external validity* (supportable claims about the population from which it is drawn).

Assumption Number 1 for Causal Regression Modeling

- Conditional on the confounding variables used in the model, $\mathbf{X}$, the distribution of the potential outcomes across treatment conditions, T is the same.

$$y^0, y^1 \perp T | X,$$

- This is called ignorability ("selection on observables"), and it means that we control for the pretreatment variables that correlate with the treatment and the outcome.
- If the probability of treatment selection is equal, conditional on modeled confounding covariates, then ignorability holds.
- Therefore if ignorability holds in this sense (and we have the "correct" model), then causal inferences are valid without considering the treatment assignment process.

Assumption Number 2 for Causal Regression Modeling

- *SUTVA* (Rubin 1980): "stable unit treatment value assumption.
- Two components:
  - For each unit there is only one form of treatment that was not received,

$$T_c \cup T_t = \Omega.$$

  - Treatment for one unit does not affect other units potential outcomes.

$$T_i \perp T_j \; \forall i \neq j.$$

Data Issue Number 1 for Causal Regression Modeling

- Overlap describes the extent to which the *range* of the data is the same across groups.
- Poor overlap between the two groups means that there are data cases for which no counterfactuals exist, and therefore the model has to make out-of-sample claims about the treatment effect.
- Fortunately overlap is easy to test.
- Perfect overlap (generally from a controlled experiment) demands much less of the specified model.

Data Issue Number 2 for Causal Regression Modeling

- Balance is the degree to which the covariate levels match-up across paired cases.
- Imbalance limits the comparison of group means: $\bar{y}_1 - \bar{y}_0$ for some outcome of interest because the covariate distribution differ in the sample.
- Consider a *true* treatment effect of interest, ζ, which is tested by the simple regression:

$$y_{it} = \beta_0 + \beta_1 x_i + \zeta + \epsilon_i, \qquad y_{ic} = \beta_0 + \beta_1 x_i + \epsilon_i.$$

- Averaging over the outcome variable means that the estimate of the treatment effect is:

$$\hat{\zeta} = \bar{y}_t - \bar{y}_c = \beta(\bar{x}_t - \bar{x}_c).$$

- The magnitude of the bias is therefore the extent to which the distributions of x_t and x_c are different (particularly in their variance).

Matching

- Matching is family of procedures that attempts to associate pairs of cases in the data as if they were identical units in the experimental sense.
- Typically this means disposing of cases in the larger of the two groups, after matching all of the cases from the smaller group.
- Sometimes with highly discrete data and a small number of covariates perfect matching is possible.
- Usually this is impossible and matches are done on multivariate nearest-neighbor criteria (such as *Mahalanobis distance*) or one number summary criteria (such as *propensity scores*).

Common Matching Strategies

- Minimize the summed distance between all i, j cases D_{ij} according to:
  - Exact: $D_{ij} = \begin{cases} 0 \text{ if } x_i = x_j \\ \infty \text{ if } x_i \neq x_j \end{cases}$.
  - Mahalanobis: $D_{ij} = (x_i - x_j)' S^{-1} (x_i - x_j)$, where S is the sample variance/covariance matrix.
  - Propensity Score: $D_{ij} = |e_i - e_j|$, where e_k is unit k's probability of receiving the treatment given the observed covariates.

A Note On Post-Treatment Covariates

- These are controversial in their use:
  - "...all posttreatment variables require careful evaluation and thought before they are used as covariates." (Greiner & Rubin 2011).
  - "...it is generally not a good idea to control for variables measured *after* the treatment. (Gelman & Hill 2007).
  - Posttreatment variables require adjustment of estimated treatment effects (Frangakis & Rubin 2002).
  - Pretreatment effects can be mitigated by posttreatment variables (Pearl 1995).
- In our data example, post-treatment variables are critical to the research question.

Problems with Causal Statements

- Causal vs. Correlational
- Reluctance to use causal
- Erroneous use of causal
- Other language problems
- Explanatory Variables = key causal variable(s)‘ $\bigcup$ control variables
- Ω for the key causal variable has at least two states: treatment and control

Realized Causal Effect

- Example: the Fourth Congressional District of New York, 1988
- Fraction of vote for Democratic incumbent: y_4^I
- Fraction of vote for hypothetical Democratic non-incumbent: y_4^N
- Realized Causal Effect of Incumbency: $y_4^I - y_4^N$
- Effect only defined in theory since the two quantities are not observable (counterfactual).

Fundamental Problem of Causal Inference

- No matter how perfect the research design...
- no matter how much data are collected...
- no matter how perceptive the field workers are...
- no matter how hard the research associates work...
- and no matter how much experimental control exists:
- we can *never* know causal inferences for certain.

The Systematic Component

- Random Causal Effect for unit i: $Y_i^I - Y_i^N$
- Mean Causal Effect:

$$
\begin{aligned}
MCE &= E(Y_i^I - Y_i^N) \\
&= E(Y_i^I) - E(Y_i^N) \\
&= \mu_i^I - \mu_i^N
\end{aligned}
$$

- Variance of the Causal Effect: $Var(Y_i^I - Y_i^N)$
- Qualitative Example: the fall of the Soviet Union and choice of government types.

More Definitions

- Causal Mechanisms: qualitative explainers.
- Multiple Causality: a plurality of causes, combinations of different explanatory variables, seen by many variables and fewer cases.
- Symmetric and Asymmetric Causality (direction of effect).

Two Assumptions Around the Fundamental Problem

- Unit Homogeneity:

$$\mu_I^i = \mu_I^j \forall \quad i, j$$
$$\mu_N^i = \mu_N^j \forall \quad i, j$$

- Conditional Independence: the values of the explanatory variables are not caused by levels of the outcome variables.

Rules for Constructing Causal Theories

- Construct Falsifiable Theories
- Build Theories That Are Internally Consistent
- Select Outcome Variables Carefully
- Maximize Concreteness
- State Theories in as Encompassing Ways as Feasible

Legal Background

- We are interested in the impact of the defendant's race in judge or jury decisions to impose the death penalty versus life in prison for convicted murders.
- Most studies focus on southern states, including Georgia.
- Before 1972 Georgia (plus other states) gave juries wide discretion in deciding whether to impose the death penalty on defendants convicted of death-eligible murder offenses.

Legal Background

- In *Furman v. Georgia*, the U.S. Supreme Court struck down this feature of Georgia's capital sentencing procedure and by implication invalidated the death penalty across the U.S.
- This 5-4 decision stated that capital sentencing based on the relatively unguided discretion of juries violates the "cruel and unusual punishment" clause of the 8th Amendment, because it permits juries to impose the irreversible sentence of death on some defendants while other juries can impose the sentence of life imprisonment under similar circumstances.
- Interesting, there was no majority opinion produced in the case.
- But Justice Stewart wrote "For, of all the people convicted of rapes and murders in 1967 and 1968, many just as reprehensible as these, the petitioners are among a capriciously selected random handful upon whom the sentence of death has in fact been imposed."

Legal Background

- After the Furman decision, Georgia, amended their death penalty statute to meet the new Furman guidelines, which were approved by the Supreme Court.
- After the defendant was convicted of a capital crime (the first part of the bifurcated trial proceeding), there is an second hearing at which the jury received additional evidence in aggravation and mitigation.
- In order for the defendant to be made eligible for the death penalty, the jury must first determine the existence at least one of ten aggravating factors.
- Passing this hurdle, the jury then evaluates all trial evidence including mitigating evidence and additional aggravating evidence.
- This is called a *non-weighing* scheme because the jury is not required to weigh the statutory aggravating factors against mitigating evidence before imposing a death sentence.

Study Background

- David C. Baldus, Charles Pulaski, and George Woodworth (eg. the Baldus study) looked at the potential disparity in the imposition of the death sentence in Georgia based on the race of the murder victim and the race of the defendant.
- This is actually two studies, the second one examining about 762 cases with a murder conviction in Georgia from March 1973 to December 1979.
- The data contains 160 variables, including legal background, crime description, and demographics.
- From the 1970 US Census 1,187,149/4,589,575 or about 26% of Georgia residents were black.
- The death penalty was imposed:

  22% cases of Black defendant, White victim
  8% cases of White defendant and White victim
  1% of cases of Black defendant and Black victim
  3% of cases of White defendant and Black victim

Study In the Legal Setting

- The Baldus study was cited in the US Supreme Court in McClesky v. Kemp (1987), in which a black defendant (McClesky) was sentenced to death for killing a white police officer in Georgia.
- The central argument was that the sentence violated the Equal Protection clause of the 14th Amendment, since statistically he stood was more likely to get the death penalty since the victim was white.
- The Court (5-4) rejected McClesky's argument, on the grounds that statistical trends did not effectively *prove* the existence of discrimination among the jury who decided his particular case (Justice Powell).
- Justice Powell later told his biographer that McCleskey was the biggest mistake in his career and that if he could to do it over again, he would rule the that death penalty always unconstitutional (Jeffries 1994).
- McClesky was executed in 1991.

Some Citations (Or Why This Example is Important In This Context)

- Imbens & Rubin, *New Palgrave Dictionary of Economics* 2008.
- Greiner & Rubin, *Review of Economics and Statistics* 2011.
- Petrie & Coverdill, *Social Problems* 2010.
- Angrist, Imbens, and Rubin, *Journal of the American Statistical Association* 1996.
- Hundreds of law review articles.

Data Manipulation: Potential Explanatory Variables

- Hispanic and "other" removed from cases for clarity ($n_r = 45$).
- `race`: 0=white ($n_w = 297$), 1=black ($n_b = 463$)
- `educatn`: 1=middle school or lower, 2=some high school, 3=high school degree
- `employm`: 0=unemployed, 1=employed
- `SES`: 0=not low wage, 1=low wage
- `married`: 0=unmarried, 1=married
- `num.chld`: defendant's number of children (1-9+)
- `military`: -1=not honorable or not general discharge, 0=no military, 1=honorable, general, or currently serving
- `pr.arrst`: number of prior arrests
- `pr.incrc`: record shows prior incarceration in Georgia
- `plea`: 0="not guilty," 1="guilty"

Data Manipulation: Potential Explanatory Variables

- `defense`: 1=retained, 2=appointed
- `dp.sght`: did prosecution seek death penalty, yes=2, no=1
- `jdge.dec`: did judge take death penalty issue away from jury, 0=unknown, 1=yes, 0=no
- `pen.phse`: was there a penalty trial, 1=yes, 0=no
- `did.appl`: did the defendant appeal, 1=yes, 0=no
- `out.appl`: 1=conviction and dp affirmed, 2=conviction affirmed dp changed to life, 3=conviction reversed, 4=conviction and life affirmed, 5=conviction only reversed, 6=conviction affirmed life modified, 9=no appeal
- `vict.age`: 1=12 or less, 0=13 or more
- `vict.sex`: 1=male, 2=female
- `vict.rel`: 0=non-family, 1=family
- `vict.st1`: 1=police or judicial official, 0=otherwise
- `specialA`: 1=special/cruel circumstances, 0=otherwise

Data Manipulation: Potential Explanatory Variables

- `methodA`: 1=gun, 2=knife, 3=blunt object, 4=beating, 5=fractures, 8=hand strangulation, 10=rope/garrote, 14=drowning, 21=buried alive, 24=other
- `num.kill`: 1, 2, or 3.
- `num.prps` number of co-perpetrators in addition to defendant
- `def.age`: defendant's age according to: $1(<= 16), 2(17 - 20), 3(21 - 25), 4(26 - 35)$, $5(36 - 50), 6(> 50)$
- `aggrevat`: one or more aggravated method of killing
- `bloody`: bloody murder involved
- `fam.lov`: family or lover dispute
- `insane`: insanity defense used
- `mitcir`: one or more mitigating circumstances
- `num.depr`: number of depraved circumstances in murder
- `rape`: rape involved

Data Manipulation, Restriction, Matching, and Outcome Variable

- `sentence`: 0=life sentence(325), 1=death penalty (127)
- Pre/Post-Furman Breakdown:

	Not-DP	DP
Pre	112	44
Post	494	112

- `victim.rac`: victim's race (white=454, black=287), a key variable that we will manipulate by having white victims only

Details on Matching Here

- We will match 287 out of 454 white cases to the existing 287 black cases using Mahalanobis distance matching with `Match` in `R`.
- For `R` details, see: Sekhon, Jasjeet S. 2011. "Multivariate and Propensity Score Matching Software with Automated Balance Optimization." *Journal of Statistical Software* 42(7): 1-52. `http://www.jstatsoft.org/v42/i07/`.
- Alternatively, see: Imbens, Guido. 2004. "Matching Software for Matlab and Stata." `http://elsa.berkeley.edu/ imbens/estimators.shtml`.

Checking Overlap For Our Match

```
WHITE
      race        educatn             SES              num.chld          defense           vict.sex         pr.incrc          aggrevat            mitcir
 Min.   :0   Min.   :1.00   Min.   :0.000   Min.   :0.00   Min.   :1.00   Min.   :1.0   Min.   :1.00   Min.   :0.000   Min.   :0.000
 1st Qu.:0   1st Qu.:1.00   1st Qu.:0.000   1st Qu.:0.00   1st Qu.:1.00   1st Qu.:1.0   1st Qu.:2.00   1st Qu.:0.000   1st Qu.:0.000
 Median :0   Median :2.00   Median :0.000   Median :1.00   Median :2.00   Median :1.0   Median :2.00   Median :0.000   Median :1.000
 Mean   :0   Mean   :2.01   Mean   :0.466   Mean   :1.23   Mean   :1.61   Mean   :1.2   Mean   :1.76   Mean   :0.101   Mean   :0.719
 3rd Qu.:0   3rd Qu.:3.00   3rd Qu.:1.000   3rd Qu.:2.00   3rd Qu.:2.00   3rd Qu.:1.0   3rd Qu.:2.00   3rd Qu.:0.000   3rd Qu.:1.000
 Max.   :0   Max.   :3.00   Max.   :1.000   Max.   :9.00   Max.   :2.00   Max.   :2.0   Max.   :2.00   Max.   :1.000   Max.   :1.000

BLACK
      race        educatn             SES              num.chld          defense           vict.sex          pr.incrc         aggrevat            mitcir
 Min.   :1   Min.   :1.00   Min.   :0.000   Min.   :0.00   Min.   :1.00   Min.   :1.00   Min.   :1.0   Min.   :0.000   Min.   :0.000
 1st Qu.:1   1st Qu.:1.00   1st Qu.:0.000   1st Qu.:0.00   1st Qu.:1.00   1st Qu.:1.00   1st Qu.:1.0   1st Qu.:0.000   1st Qu.:0.000
 Median :1   Median :2.00   Median :1.000   Median :1.00   Median :2.00   Median :1.00   Median :2.0   Median :0.000   Median :1.000
 Mean   :1   Mean   :1.88   Mean   :0.607   Mean   :1.56   Mean   :1.69   Mean   :1.21   Mean   :1.7   Mean   :0.157   Mean   :0.618
 3rd Qu.:1   3rd Qu.:2.00   3rd Qu.:1.000   3rd Qu.:2.00   3rd Qu.:2.00   3rd Qu.:1.00   3rd Qu.:2.0   3rd Qu.:0.000   3rd Qu.:1.000
 Max.   :1   Max.   :3.00   Max.   :1.000   Max.   :9.00   Max.   :2.00   Max.   :2.00   Max.   :2.0   Max.   :1.000   Max.   :1.000
```

Index of `http://jgill.wustl.edu.edu/Bayesian.Causal.Workshop`

Doc/	Figures/
baldus.full.jags.dat	baldus.full.winbugs.dat
baldus.null.jags	baldus.null.jags.dat
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baldus3.jags	baldus3.jags.inits
baldus3.winbugs.dat	baldus3.winbugs.inits

Analysis on Full Dataset with Race Included (code)

```
model  {
    for (i in 1:N) {
        logit(p[i]) <- beta[1] + beta[2]*race[i]     + beta[3]*educatn[i]  + beta[4]*SES[i]
                               + beta[5]*num.chld[i] + beta[6]*defense[i]  + beta[7]*vict.sex[i]
                               + beta[8]*pr.incrc[i] + beta[9]*aggrevat[i] + beta[10]*mitcir[i]
        sentence[i] ~ dbern(p[i])
    }
    beta[1]  ~ dnorm(0.0,0.01)
    beta[2]  ~ dnorm(0.0,0.01)
    beta[3]  ~ dnorm(0.0,0.01)
    beta[4]  ~ dnorm(0.0,0.01)
    beta[5]  ~ dnorm(0.0,0.01)
    beta[6]  ~ dnorm(0.0,0.01)
    beta[7]  ~ dnorm(0.0,0.01)
    beta[8]  ~ dnorm(0.0,0.01)
    beta[9]  ~ dnorm(0.0,0.01)
    beta[10] ~ dnorm(0.0,0.01)
}
```

Analysis on Full Dataset with Race Included (data in JAGS format)

```
N <- 358
race <-
c(1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1,...)
educat <-
c(2, 2, 3, 1, 2, 1, 1, 3, 2, 2, 2, 1, 1, 2, 2, 1, 3, 3, 2, 3,...)
SES <-
c(1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 0, 1, 1, 1, 0, 1, 1, 1,...)
num.prs <-
c(0, 1, 0, 0, 1, 1, 0, 2, 1, 3, 0, 0, 0, 2, 7, 2, 0, 0, 3, 2,...)
:
:
```

Analysis on Full Dataset with Race Included (data in `WinBUGS` format)

```
list( sentence = c( 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0,...),
      race = c( 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1,...),
      educatn = c( 2, 2, 3, 1, 2, 1, 1, 3, 2, 2, 2, 1, 1, 2, 2, 1, 3, 3, 2, 3,...),
      SES = c( 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 0, 1, 1, 1, 0, 1, 1, 1, 0,...),...
      N = 358)
```

Analysis on Full Dataset with Race Included (commands)

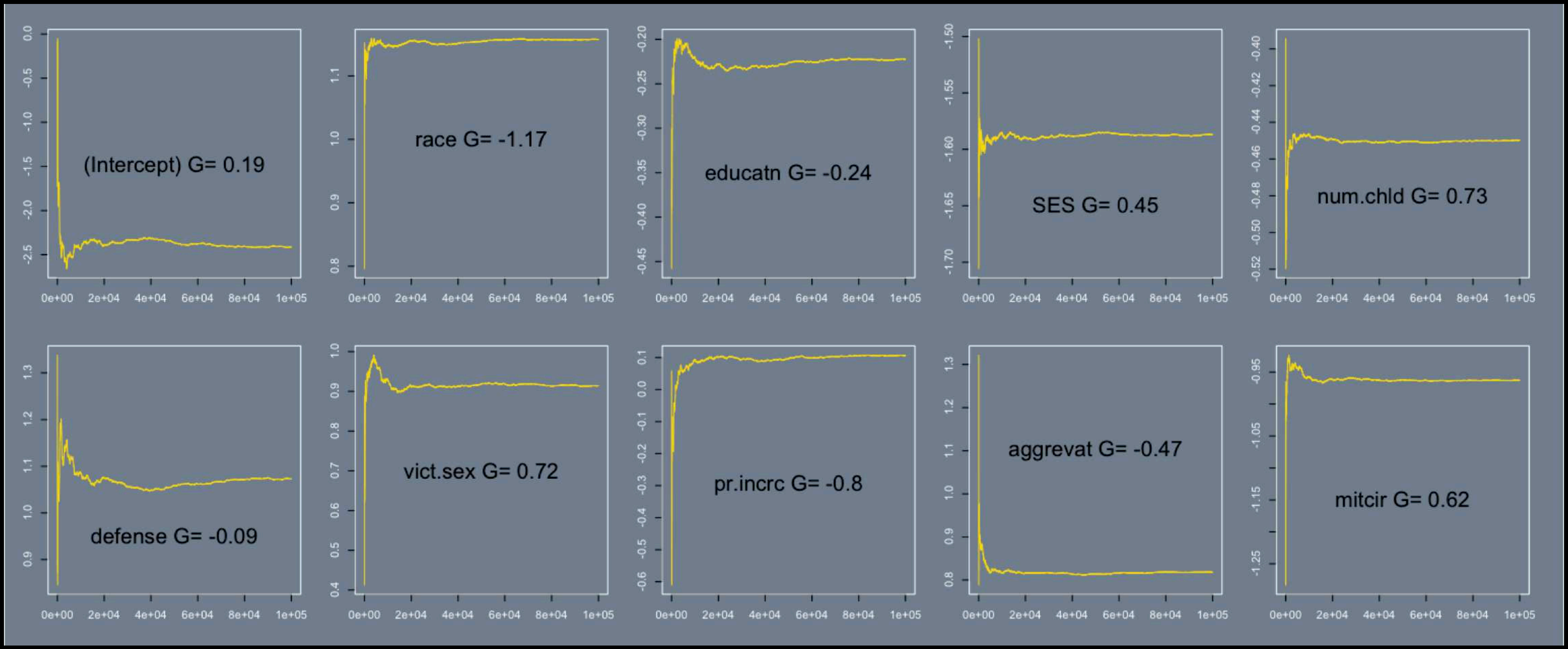
```
load dic
model in "baldus.short.jags"
data in "baldus.short.jags.dat"
compile
inits in "baldus.short.jags.inits"
initialize
update 100000
monitor deviance
monitor beta
update 100000
coda *
exit
```

Analysis on Full Dataset with Race Included (results)

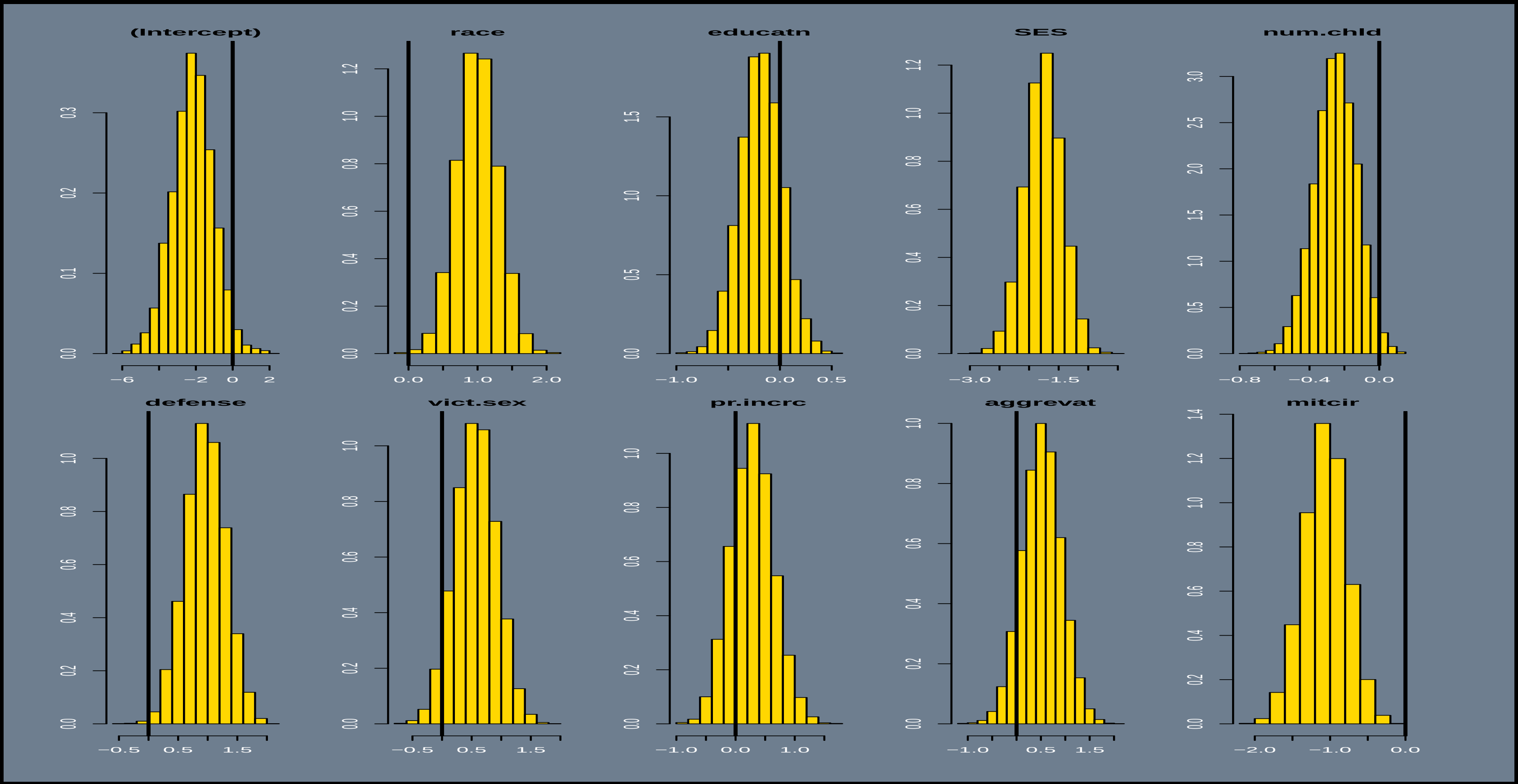
```
               Coef StdErr  t-score p-value
(Intercept)  -2.4136 1.1201 -2.1548  0.0156
race          1.1572 0.3097  3.7372  0.0001
educatn      -0.2219 0.2110 -1.0513  0.1466
SES          -1.5869 0.3043 -5.2154  0.0000
num.chld     -0.4497 0.1239 -3.6308  0.0001
defense       1.0732 0.3275  3.2766  0.0005
vict.sex      0.9144 0.3539  2.5837  0.0049
pr.incrc      0.1063 0.3454  0.3077  0.3791
aggrevat      0.8181 0.4259  1.9206  0.0274
mitcir       -0.9629 0.2877 -3.3464  0.0004
--------
    Null deviance: 1414.1  on 355  degrees of freedom
Residual deviance: 672.90  on 346  degrees of freedom
DIC: 346.3678
```

(Output formatted to look like standard GLM results.)

Running Means of Marginal Posterior Distributions from MCMC Output



Histograms of Marginal Posterior Distributions from MCMC Output

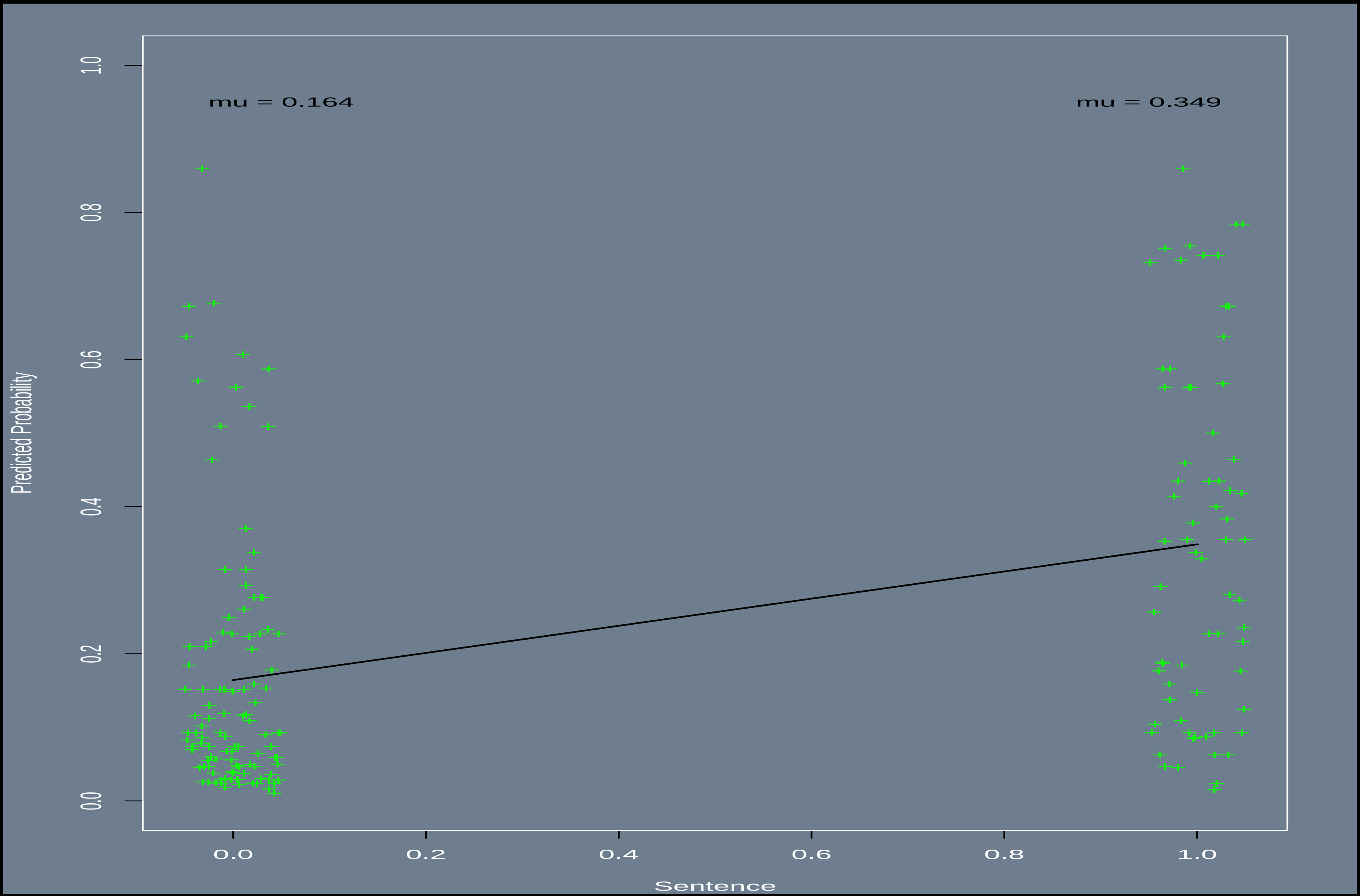


Basic Causal Methodology for These Results

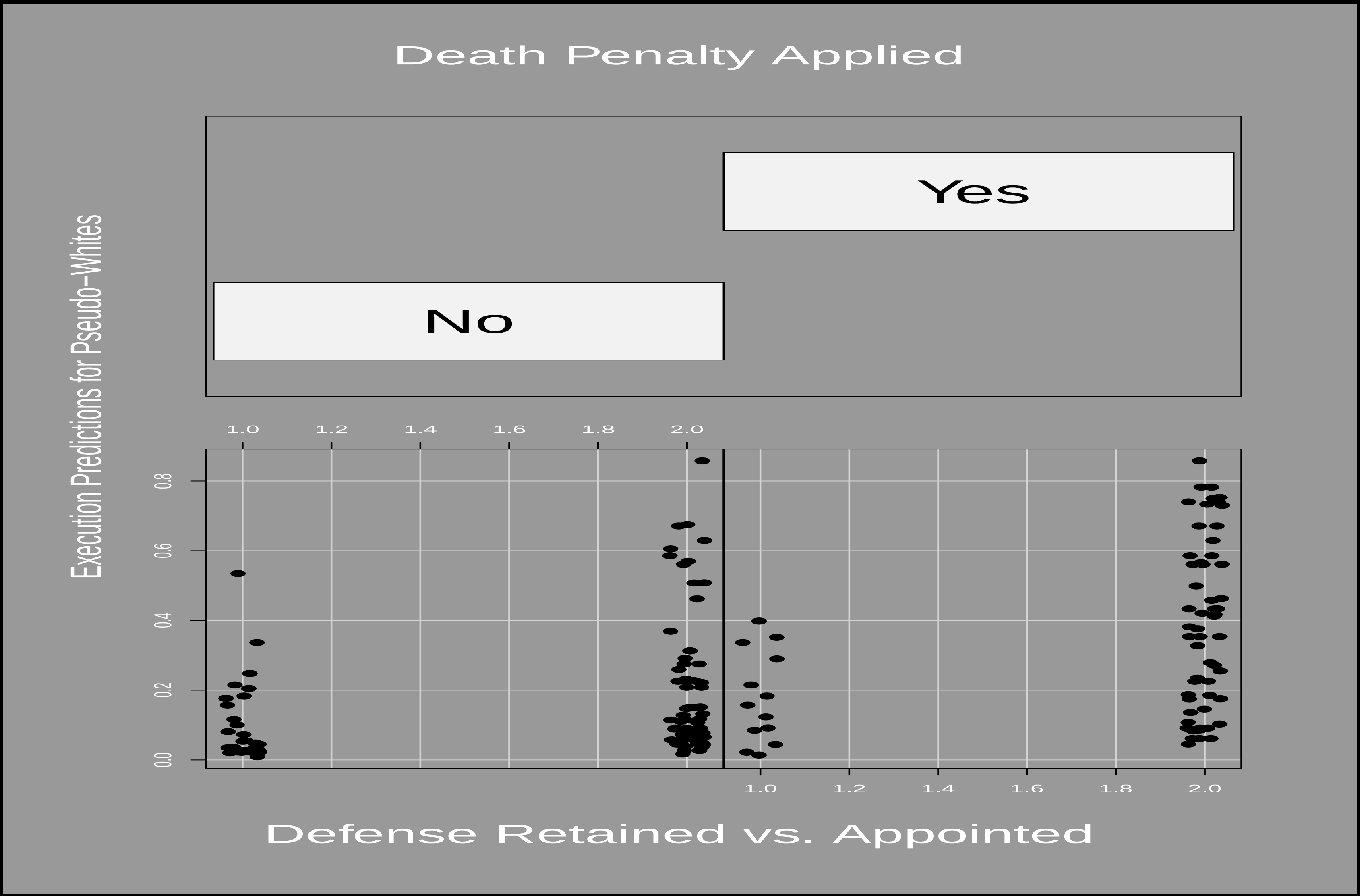
- Subset the data to give blacks only as defendant's.
- Change the race variable from all 1's to all 0's to create pseudo-white cases from the black cases.
- Therefore these case have the same covariates but now "look white" in the data.
- Calculate the expected outcomes from $g^{-1}(\mathbf{X}\boldsymbol{\beta})$ for these manipulated data.
- Compare with actual outcomes for black cases:

	Other	Death Penalty
Actual:	0.6123	0.3876
Prediction Mean:	0.7804	0.2197

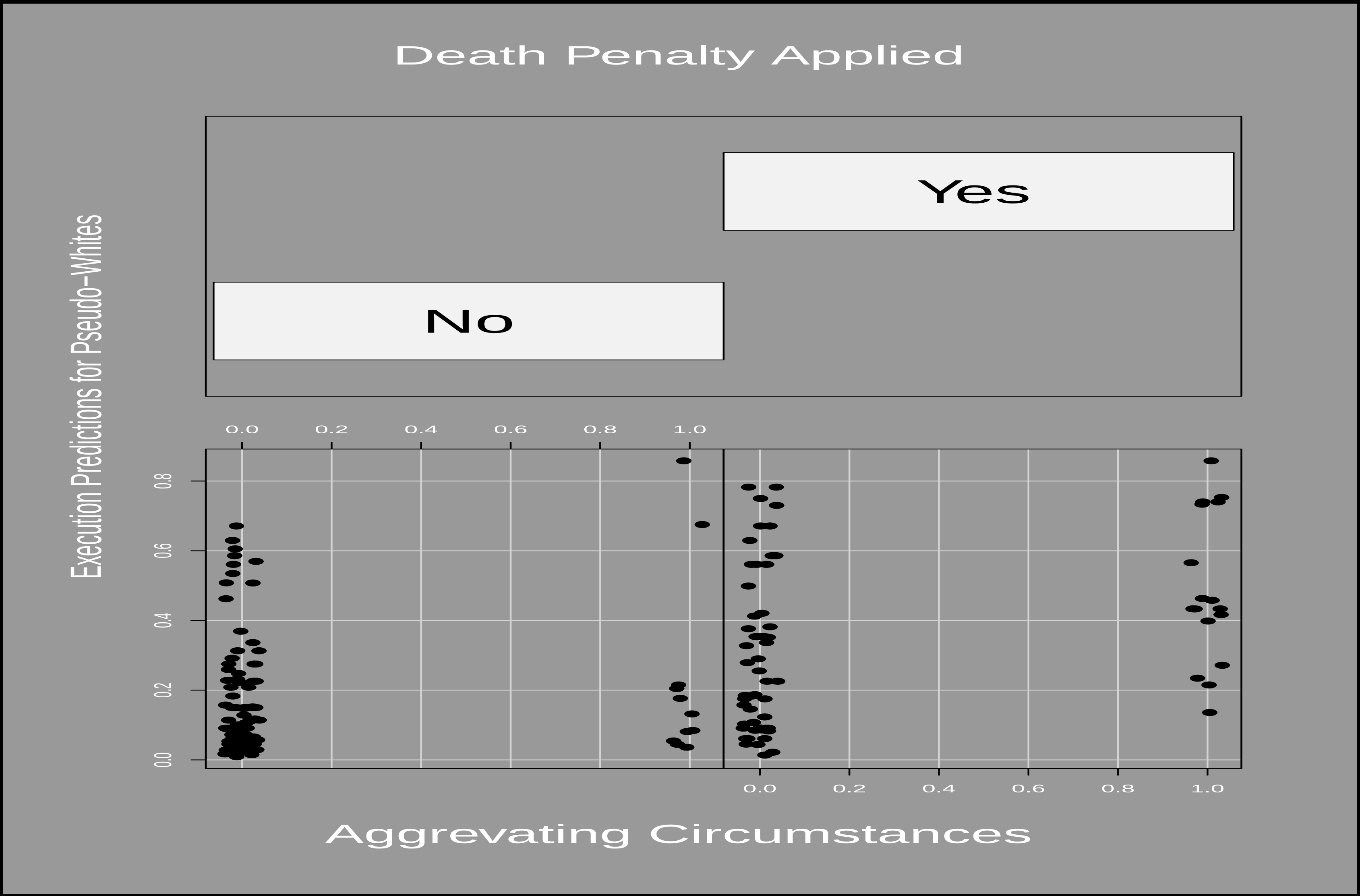
Actual Versus Predicted from the Manipulated Model



Coplot Analysis



Coplot Analysis



Coplot Analysis

Considering (Post-Treatment) Procedural Factors in the Causal Model

	Penalty Phase?		Judge Decides?	
DP Sought?	No	Yes	No	Yes
No	232	8	225	15
Yes	27	185	187	25

	Judge Decides?	
Penalty Phase?	No	Yes
No	225	34
Yes	187	6

A Hierarchical Form of the Causal Model

- `dp.sght`: coded 1/2.
- `jdge.dec`: coded 0/1.
- `pen.phse`: coded 0/1.
- New random effects specification: `dp.sght[i]+jdge.dec[i]*pen.phse[i]`, which takes on three values: 1/2/3.
- Designed to increase risk across categories.
- There are many ways to make such a specification.

Checking Overlap For Our Match On the New Variables

```
WHITE
    dp.sght          jdge.dec          pen.phse
 Min.   :1.00   Min.   :0.0000   Min.   :0.000
 1st Qu.:1.00   1st Qu.:0.0000   1st Qu.:0.000
 Median :1.00   Median :0.0000   Median :0.000
 Mean   :1.44   Mean   :0.0393   Mean   :0.427
 3rd Qu.:2.00   3rd Qu.:0.0000   3rd Qu.:1.000
 Max.   :2.00   Max.   :1.0000   Max.   :1.000

BLACK
   dp.sght          jdge.dec          pen.phse
 Min.   :1.00   Min.   :0.0000   Min.   :0.000
 1st Qu.:1.00   1st Qu.:0.0000   1st Qu.:0.000
 Median :2.00   Median :0.0000   Median :1.000
 Mean   :1.56   Mean   :0.0506   Mean   :0.539
 3rd Qu.:2.00   3rd Qu.:0.0000   3rd Qu.:1.000
 Max.   :2.00   Max.   :1.0000   Max.   :1.000
```

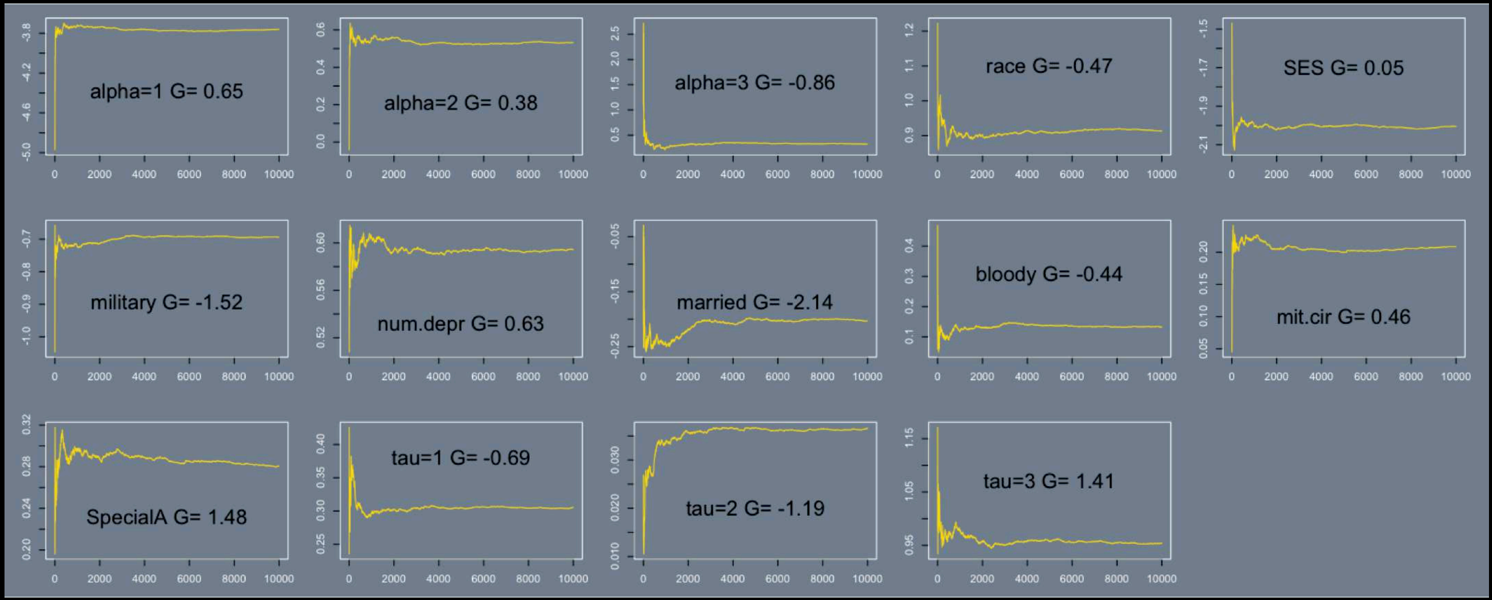
A Hierarchical Form of the Causal Model

```
model  {
    for (i in 1:N) {
            logit(p[i]) <- alpha[dp.sght[i]+jdge.dec[i]*pen.phse[i]]
                              + beta[1]*race[i]     + beta[2]*educatn[i]  + beta[3]*SES[i]
                              + beta[4]*num.chld[i] + beta[5]*defense[i]  + beta[6]*vict.sex[i]
                              + beta[7]*pr.incrc[i] + beta[8]*aggrevat[i] + beta[9]*mitcir[i]
            sentence[i] ~ dbern(p[i])
    }
    alpha[1]  ~ dnorm(0.0,0.01)
    alpha[2]  ~ dnorm(0.0,0.01)
    alpha[3]  ~ dnorm(0.0,0.01)
    beta[1]  ~ dnorm(0.0,0.01)
    beta[2]  ~ dnorm(0.0,0.01)
    beta[3]  ~ dnorm(0.0,0.01)
    beta[4]  ~ dnorm(0.0,0.01)
    beta[5]  ~ dnorm(0.0,0.01)
    beta[6]  ~ dnorm(0.0,0.01)
    beta[7]  ~ dnorm(0.0,0.01)
    beta[8]  ~ dnorm(0.0,0.01)
    beta[9] ~ dnorm(0.0,0.01)
}
```

JAGS Commands

```
load dic
model in "baldus2.jags"
data in "baldus.full.jags.dat"
compile
inits in "baldus2.jags.inits"
initialize
update 500000
monitor deviance
monitor alpha
monitor beta
update 500000
coda *
exit
```

Running Means of Marginal Posterior Distributions from MCMC Output



Results From the Second Model

	Coef	StdErr	95% Lower	95% Upper
DIC	223.3088			
alpha=1	-4.8638	1.4178	-7.6427	-2.0849
alpha=2	0.1851	1.3085	-2.3796	2.7498
alpha=3	6.4990	6.3871	-6.0198	19.0178
race	1.4161	0.4076	0.6171	2.2151
educatn	-0.3372	0.2686	-0.8637	0.1894
SES	-2.2316	0.4145	-3.0441	-1.4191
num.chld	-0.2904	0.1769	-0.6370	0.0563
defense	0.2048	0.4227	-0.6236	1.0332
vict.sex	0.9936	0.4813	0.0504	1.9369
pr.incrc	-0.1245	0.4150	-0.9379	0.6889
aggrevat	0.5575	0.5821	-0.5834	1.6984
mitcir	0.2668	0.3749	-0.4681	1.0017

Prediction Mean: 0.25121

Prediction Means Versus Actual for Blacks

- Compare with actual outcomes for black cases:

	Other	Death Penalty
Actual:	0.6124	0.3876
Model 1 Prediction Mean:	0.7803	0.2197
Model 2 Prediction Mean:	0.7488	0.2512

Another Hierarchical Form of the Causal Model

- Add another non-nested hierarchy according to:

```
logit(q[i]) <- tau[1]*aggrevat[i] + tau[2]*num.kill[i] + tau[3]*rape[i]
specialA[i] ~ dbern(q[i])
```

- Using

```
          aggrevat num.kill   rape
Min.         0.000     1.00 0.0000
1st Qu.      0.000     1.00 0.0000
Median       0.000     1.00 0.0000
Mean         0.157     1.09 0.0787
3rd Qu.      0.000     1.00 0.0000
Max.         1.000     3.00 1.0000
```

- Can we get a more reliable version of these extra effects?

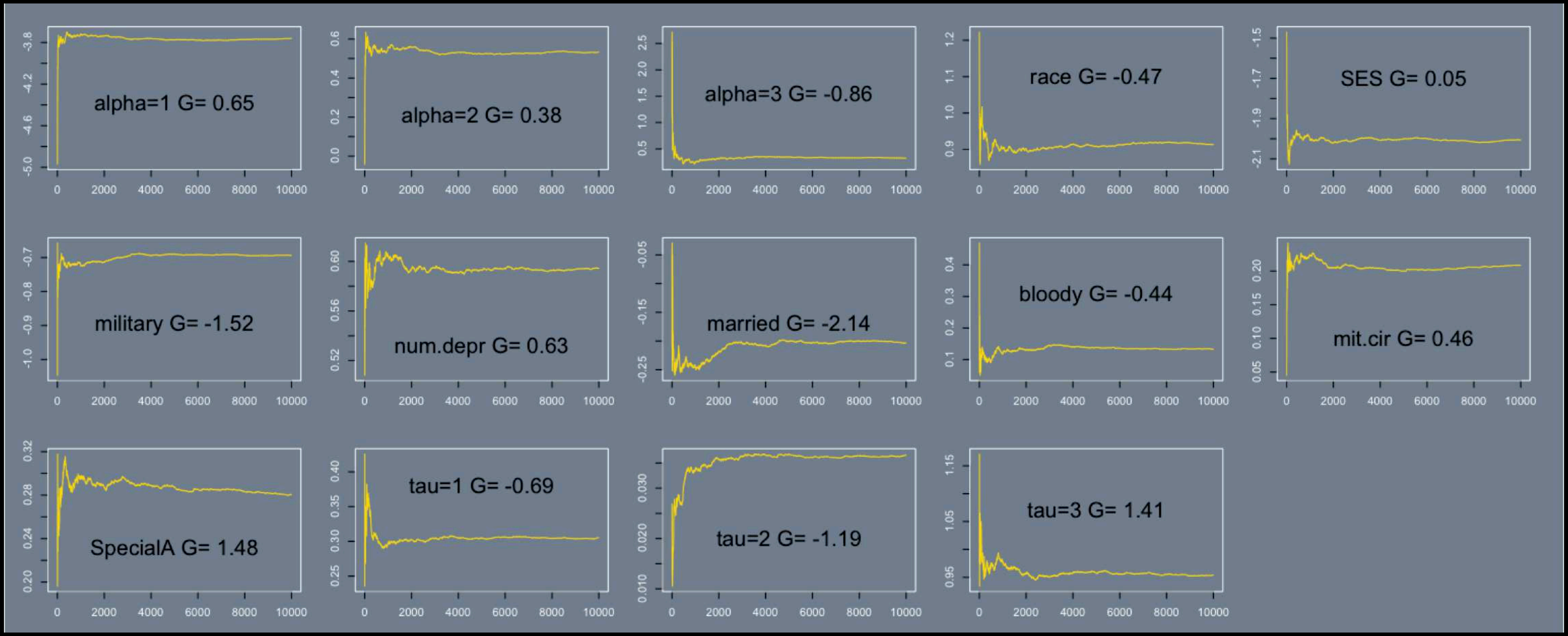
Expanding the Causal Model

```
model  {
  for (i in 1:N) {
    logit(p[i]) <- alpha[dp.sght[i]+jdge.dec[i]*pen.phse[i]] + beta[1]*race[i] + beta[2]*SES[i]
                   + beta[3]*military[i] + beta[4]*num.depr[i] + beta[5]*married[i]
                   + beta[6]*bloody[i] - beta[7]*mitcir[i] + beta[8]*specialA[i]
    sentence[i] ~ dbern(p[i])
    logit(q[i]) <- tau[1]*aggrevat[i] + tau[2]*num.kill[i] + tau[3]*rape[i]
    specialA[i] ~ dbern(q[i])
  }
  alpha[1] ~ dnorm(0.0,0.3)
  alpha[2] ~ dnorm(0.0,0.3)
  alpha[3] ~ dnorm(0.0,0.3)
  beta[1]  ~ dt(0,1,1)
  beta[2]  ~ dt(0,1,1)
  beta[3]  ~ dt(0,1,1)
  beta[4]  ~ dgamma(1.0,0.1)
  beta[5]  ~ dt(0,1,1)
  beta[6]  ~ dt(0,1,1)
  beta[7]  ~ dgamma(1.0,0.1)
  beta[8]  ~ dgamma(1.0,0.1)
  tau[1]   ~ dgamma(1.0,0.1)
  tau[2]   ~ dgamma(1.0,0.1)
  tau[3]   ~ dgamma(1.0,0.1)
}
```

JAGS Commands

```
load dic
model in "baldus3.jags"
data in "baldus.full.jags.dat"
compile
inits in "baldus3.jags.inits"
initialize
update 10000
monitor deviance
monitor alpha
monitor beta
monitor tau
update 10000
coda *
exit
```

Running Means of Marginal Posterior Distributions from MCMC Output



Results From the Third Model

	Coef	StdErr	95% Lower	95% Upper
DIC	700.8964			
alpha=1	-3.7602	0.5997	-4.9356	-2.5847
alpha=2	0.5326	0.3624	-0.1777	1.2430
alpha=3	0.3256	1.6778	-2.9629	3.6141
race	0.9130	0.3662	0.1953	1.6307
SES	-2.0052	0.3936	-2.7766	-1.2338
num.chld	-0.6934	0.2827	-1.2475	-0.1393
num.depr	0.5943	0.1879	0.2261	0.9625
married	-0.2030	0.3518	-0.8925	0.4865
bloody	0.1328	0.3992	-0.6496	0.9153
mit.cir	0.2089	0.1721	-0.1284	0.5463
SpecialA	0.2804	0.2191	-0.1490	0.7098
tau=1	0.3054	0.2157	-0.1174	0.7282
tau=2	0.0366	0.0334	-0.0288	0.1020
tau=3	0.9541	0.4349	0.1017	1.8065

Prediction Means Versus Actual for Blacks

- Compare with actual outcomes for black cases:

	Other	Death Penalty
Actual:	0.6124	0.3876
Model 1 Prediction Mean:	0.7803	0.2197
Model 2 Prediction Mean:	0.7488	0.2512
Model 3 Prediction Mean:	0.5935	0.4065