

APPROXIMATE INFERENCE FOR DETERMINANTAL POINT PROCESSES

Jennifer Gillenwater
Advised by Ben Taskar and Emily Fox
Joint work with Alex Kulesza

OUTLINE

OUTLINE

Motivation & Background

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1. Dimensionality Reduction

GILLENWATER, KULESZA, AND TASKAR (EMNLP 2012)

2. MAP Estimation

GILLENWATER, KULESZA, AND TASKAR (NIPS 2012)

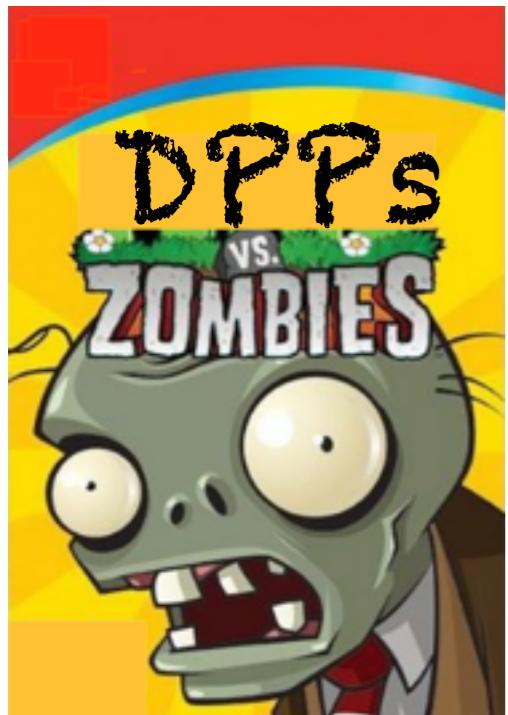
3. Likelihood Maximization

GILLENWATER, KULESZA, FOX, AND TASKAR (NIPS 2014)

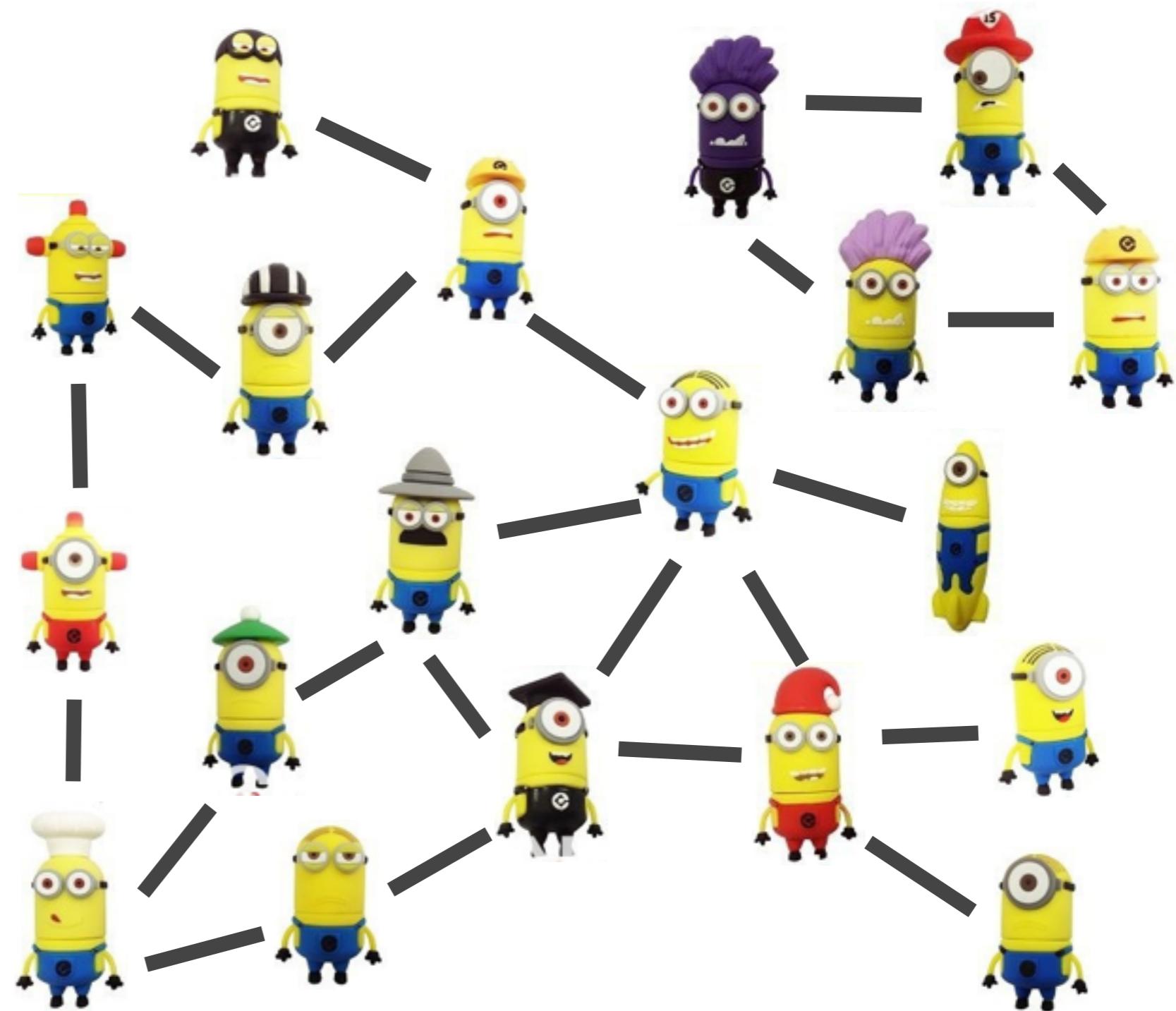
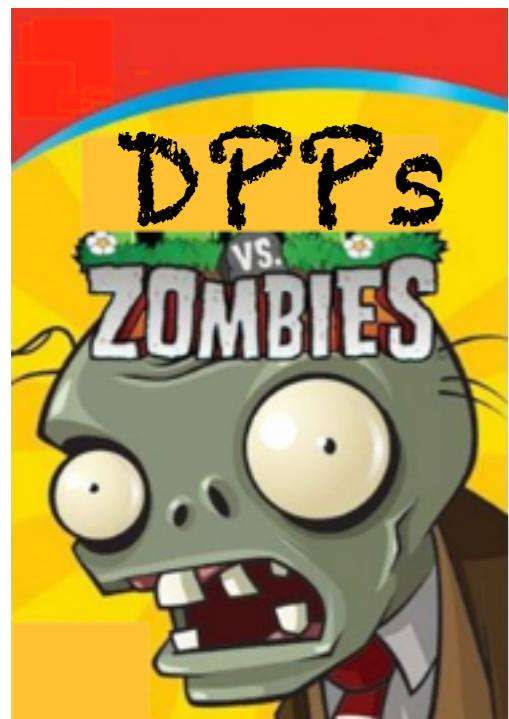
MOTIVATION & BACKGROUND

SOCIAL NETWORK MARKETING

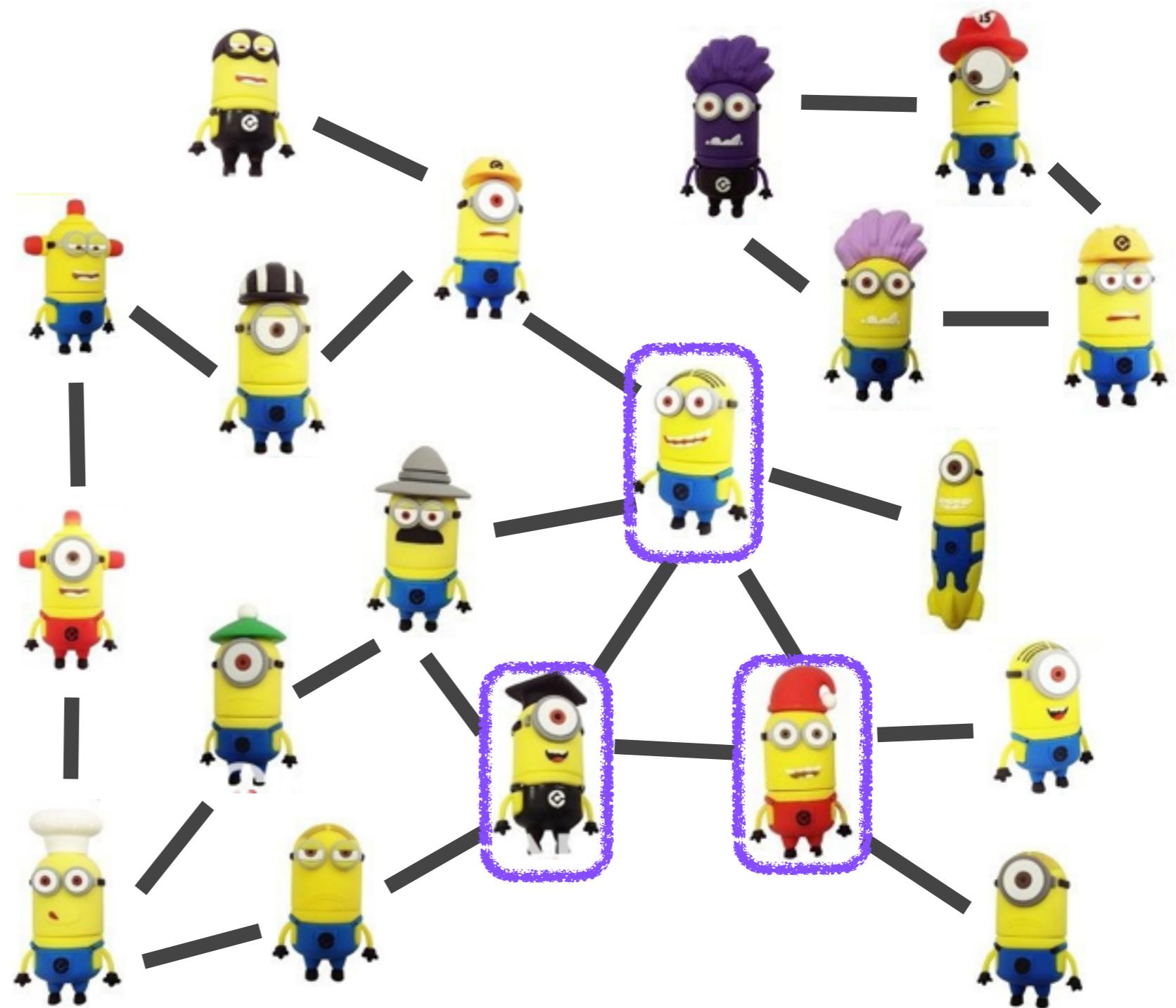
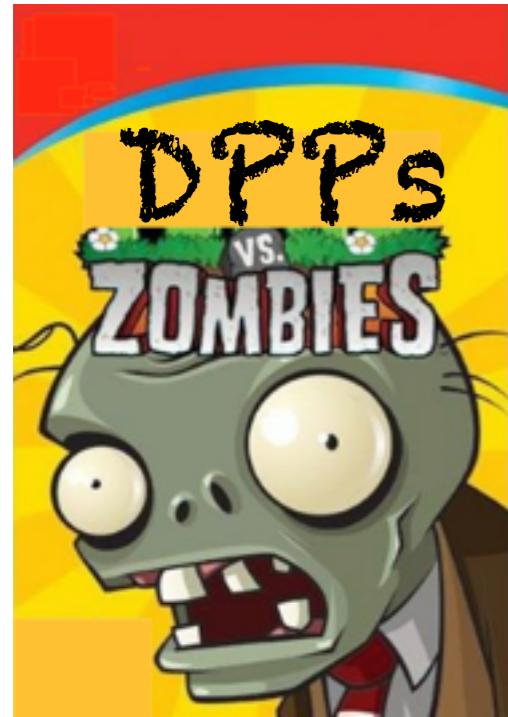
SOCIAL NETWORK MARKETING



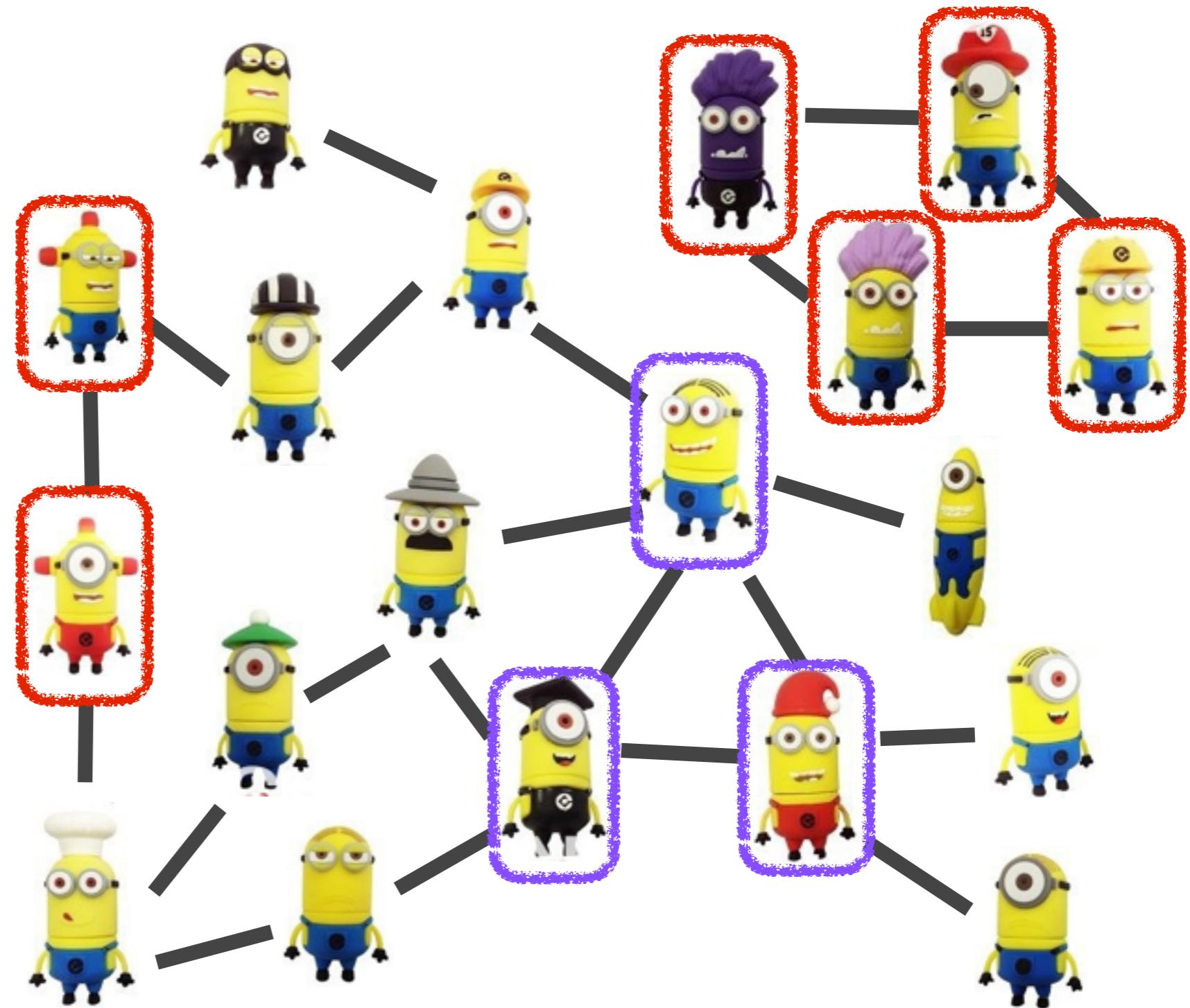
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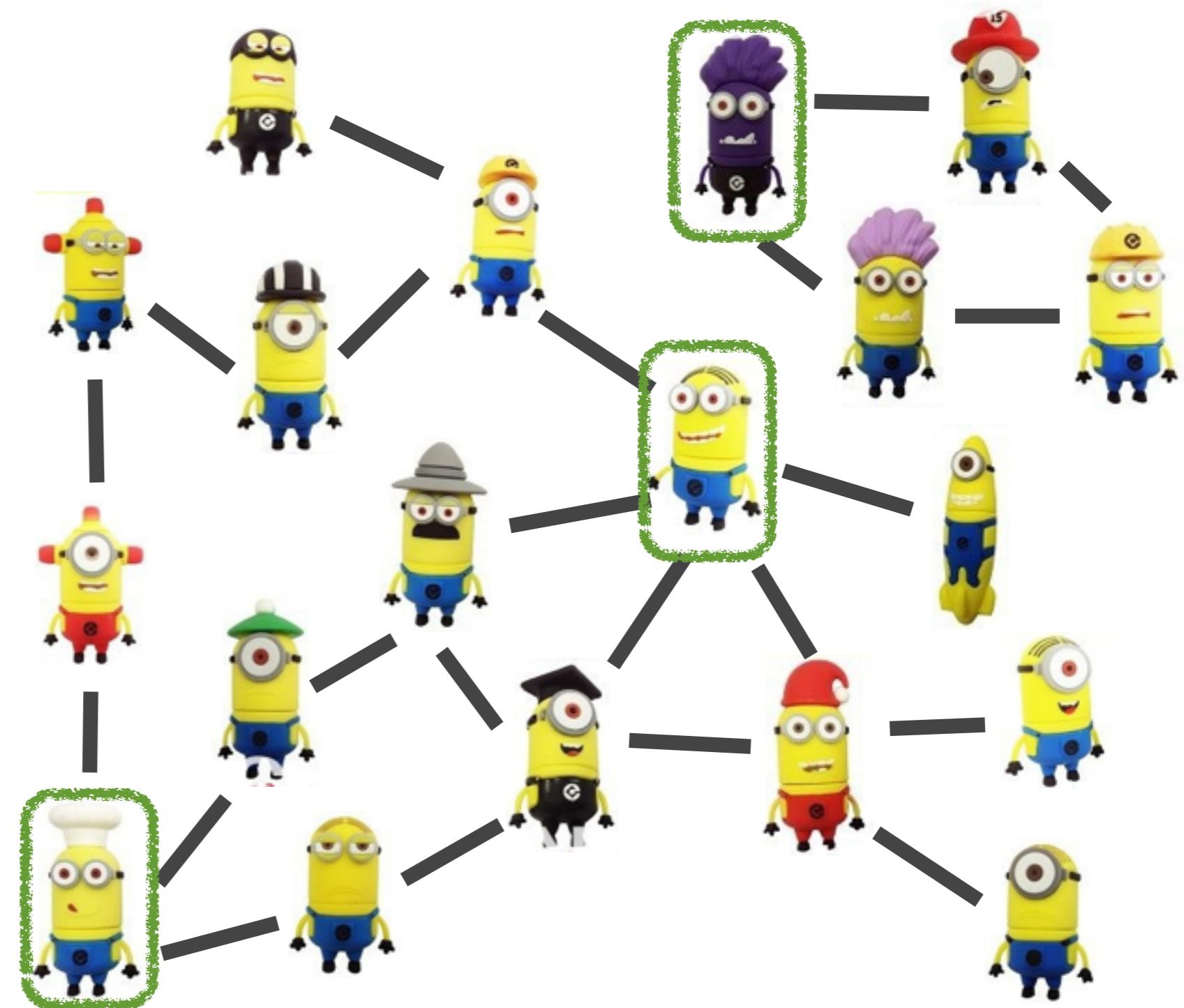
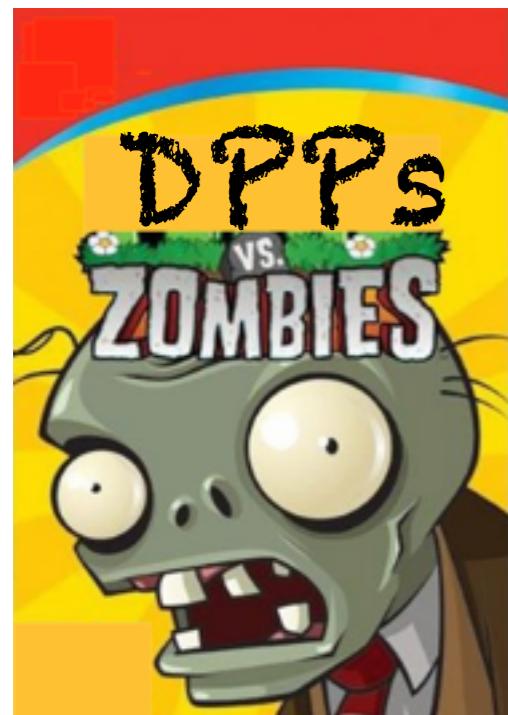
SOCIAL NETWORK MARKETING



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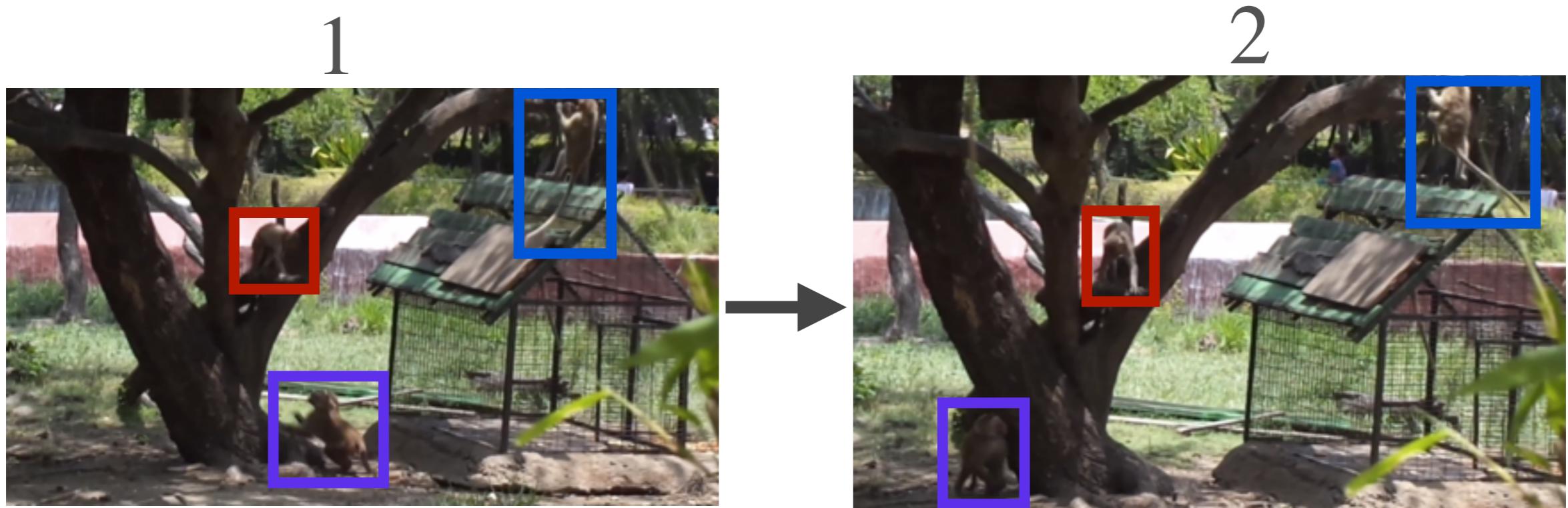
VIDEO TRACKING

VIDEO TRACKING

1



VIDEO TRACKING



VIDEOTRACKING

1



2



3



VIDEOTRACKING

1



2



3

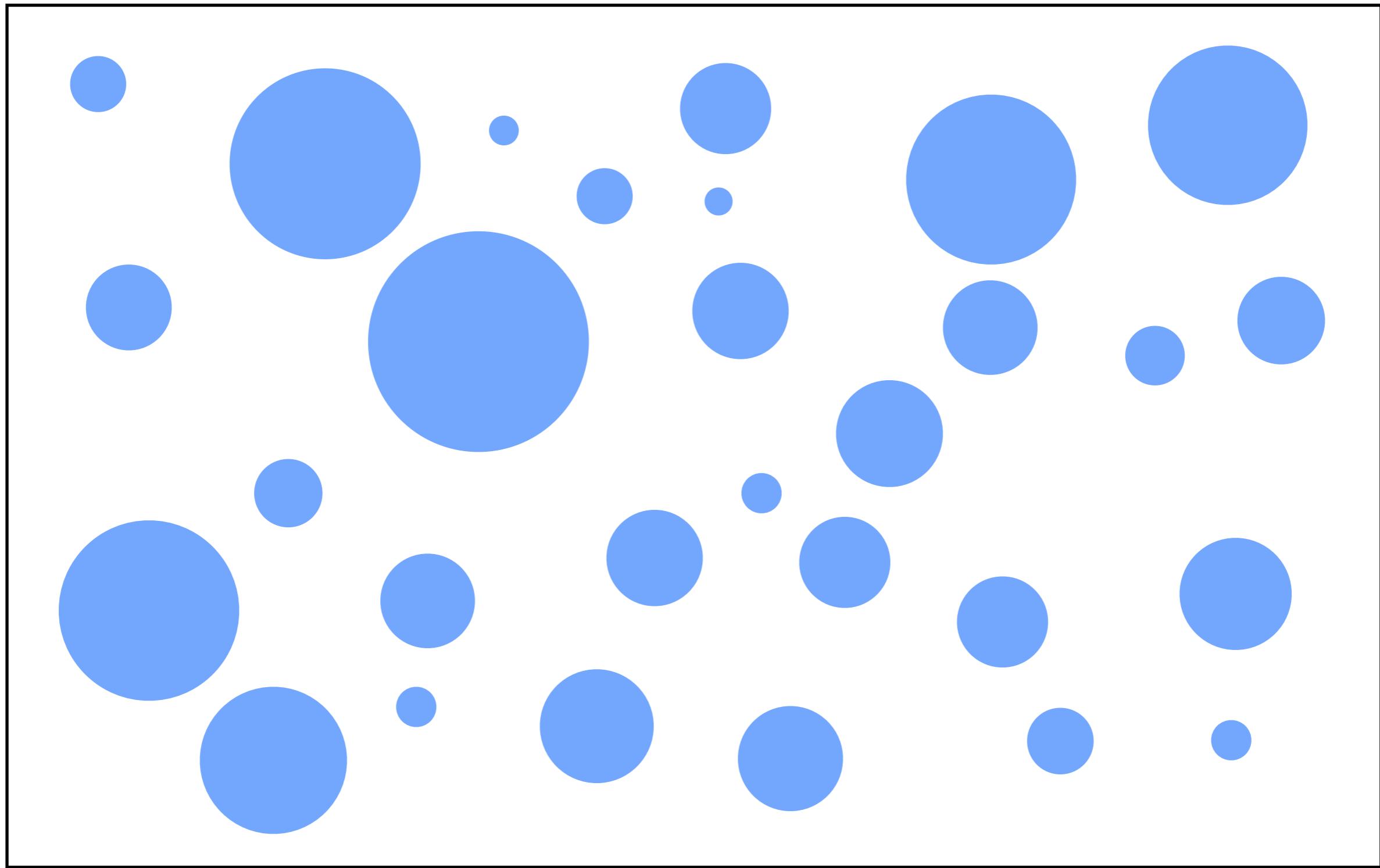


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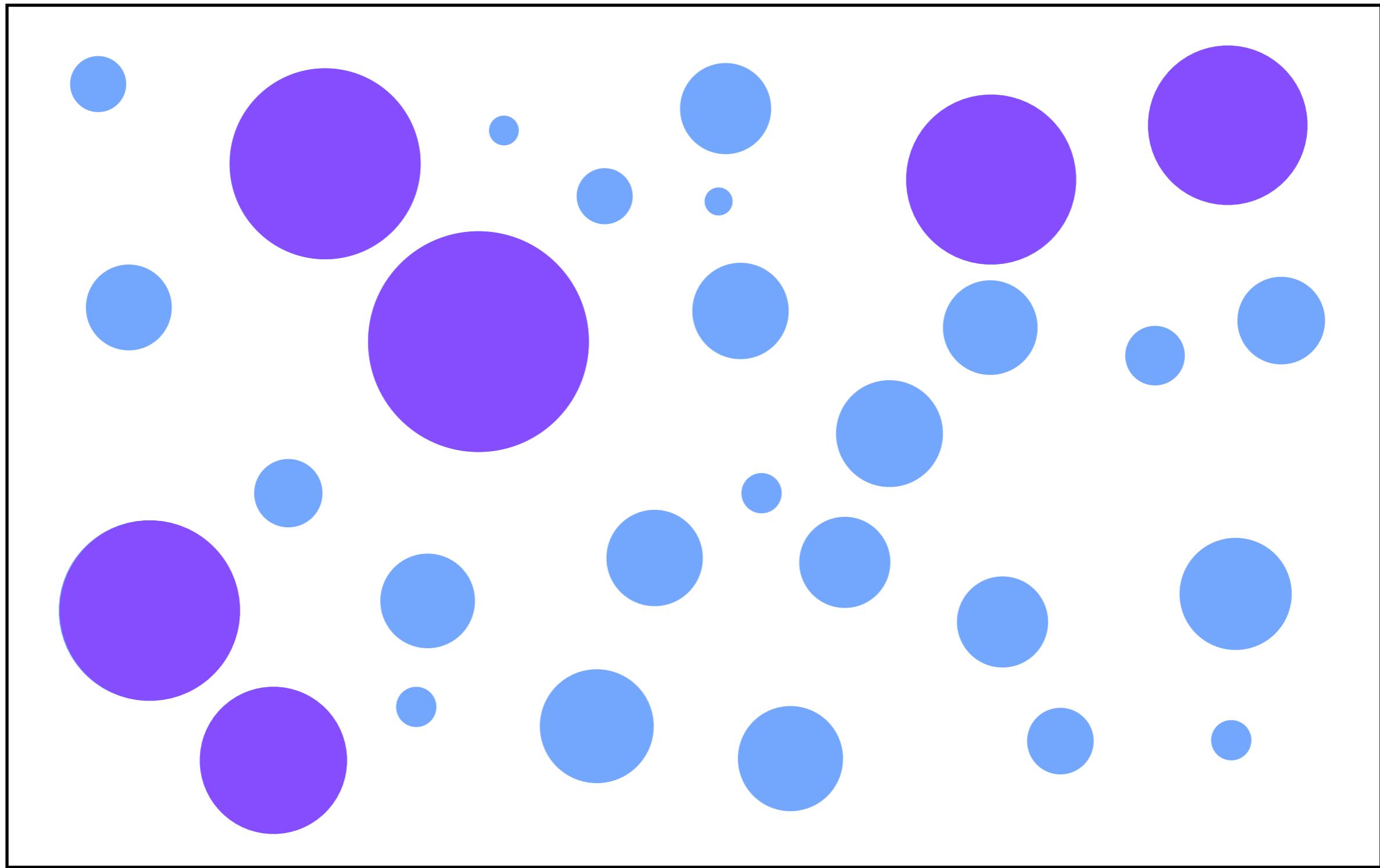


SUBSET SELECTION

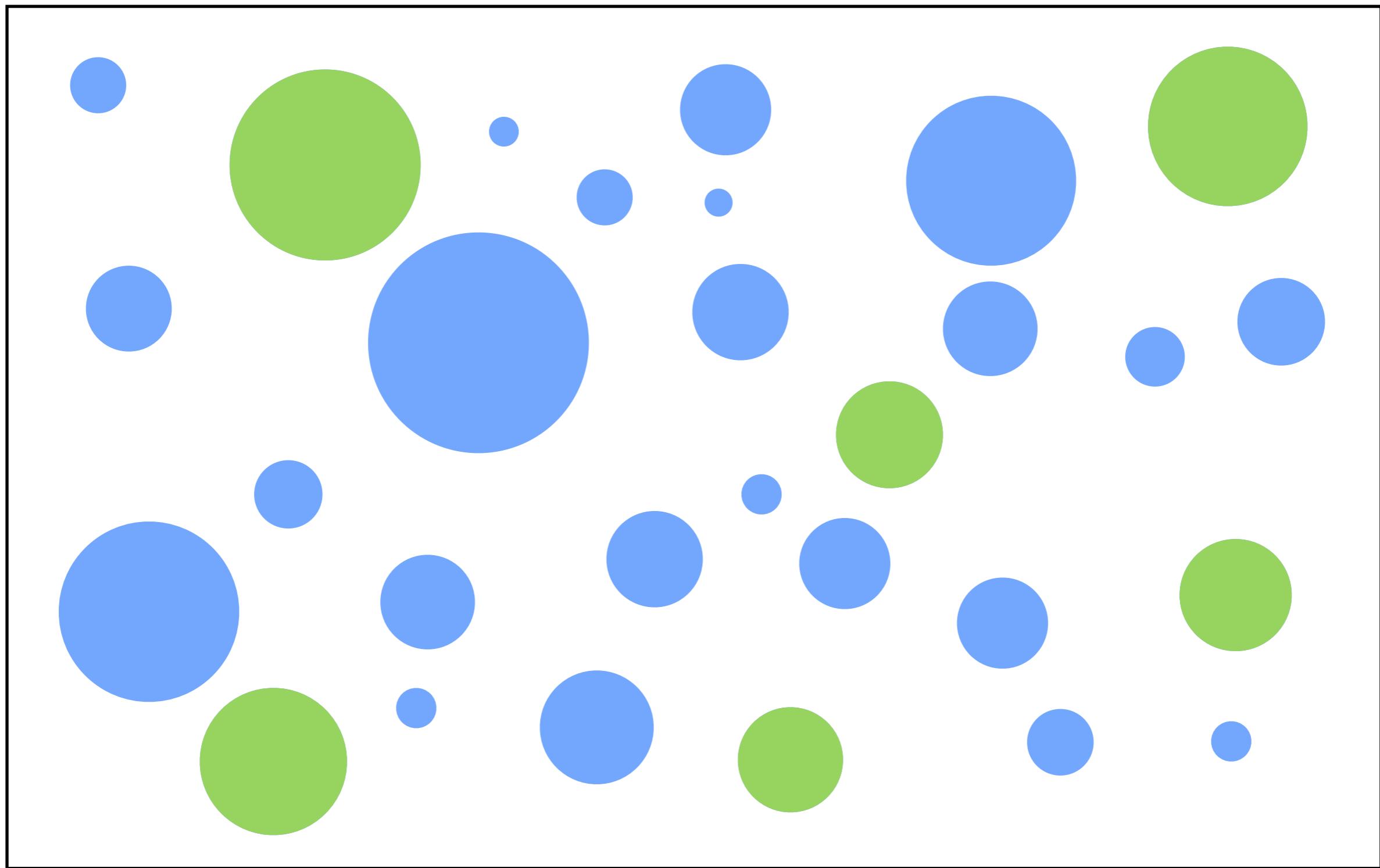
SUBSET SELECTION



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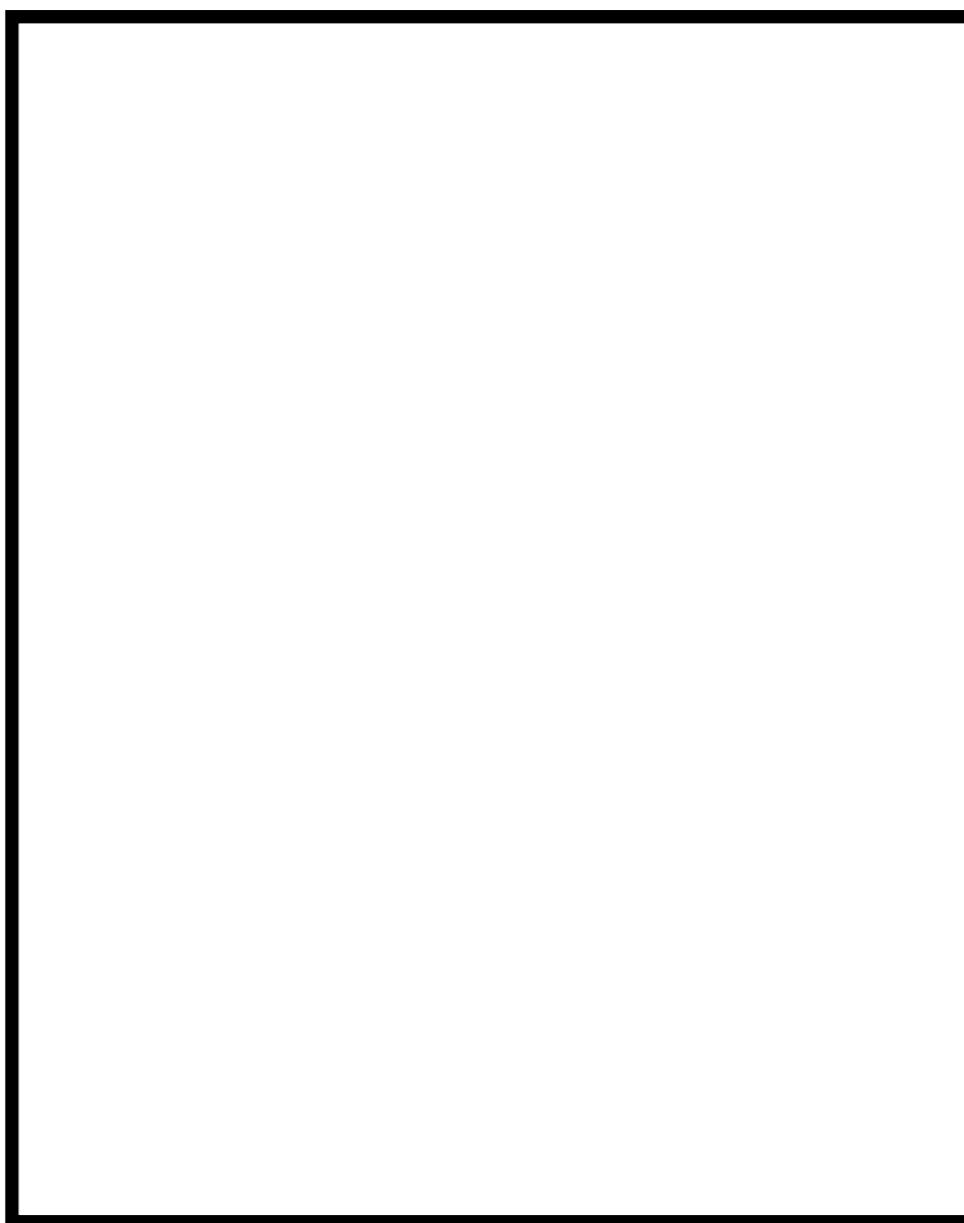
SUBSET SELECTION



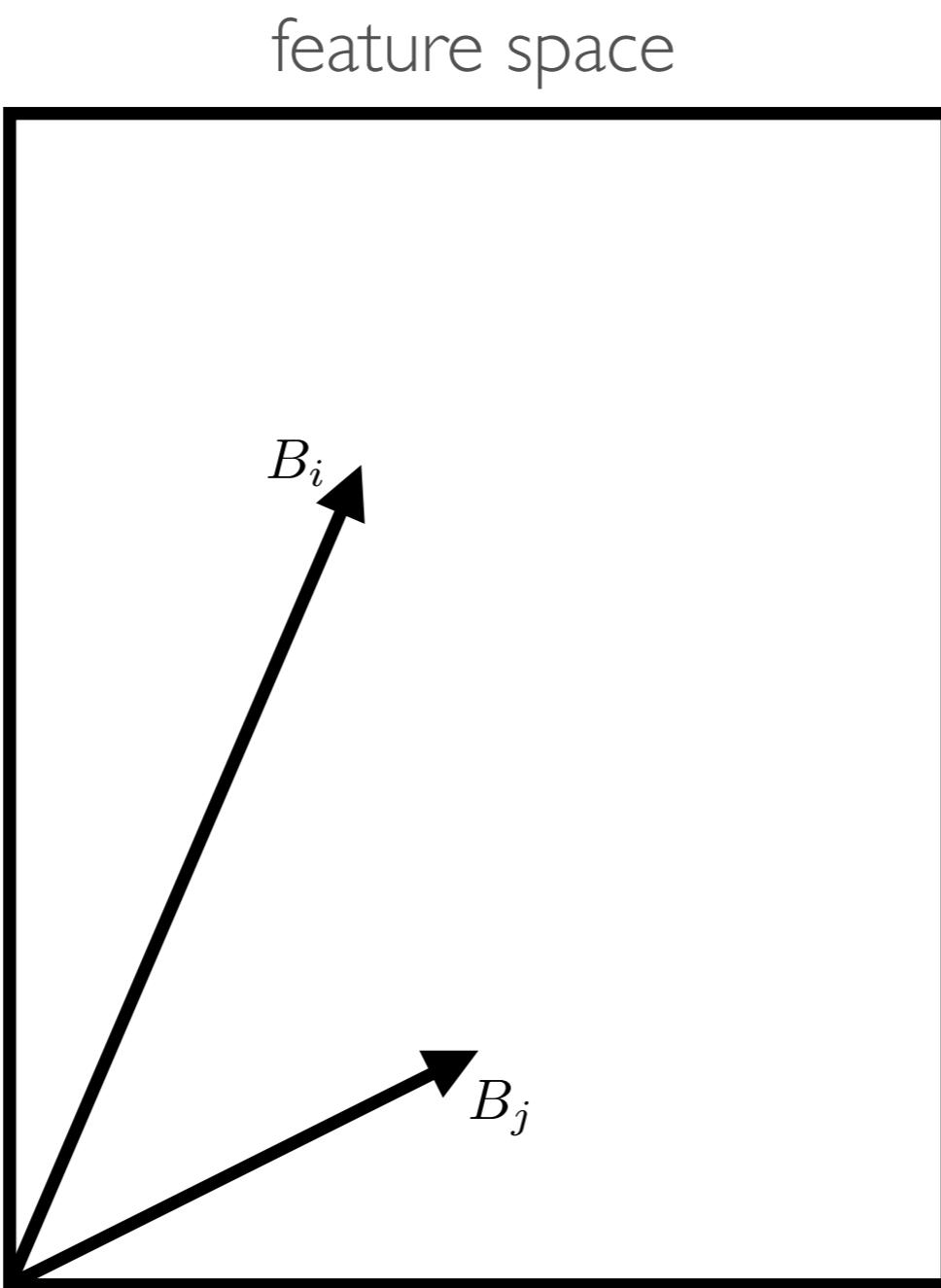
AREA AS SET-GOODNESS

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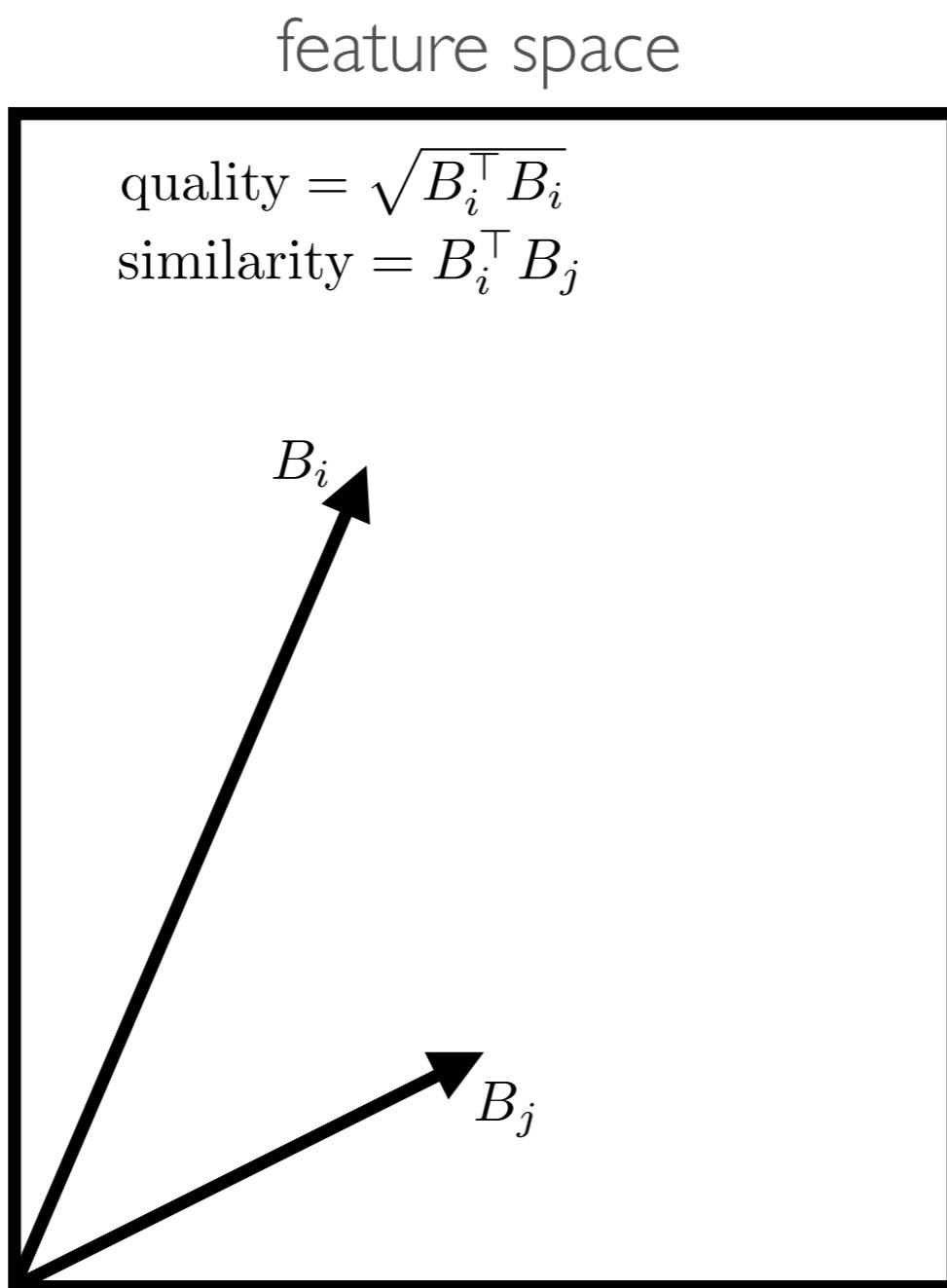
feature space



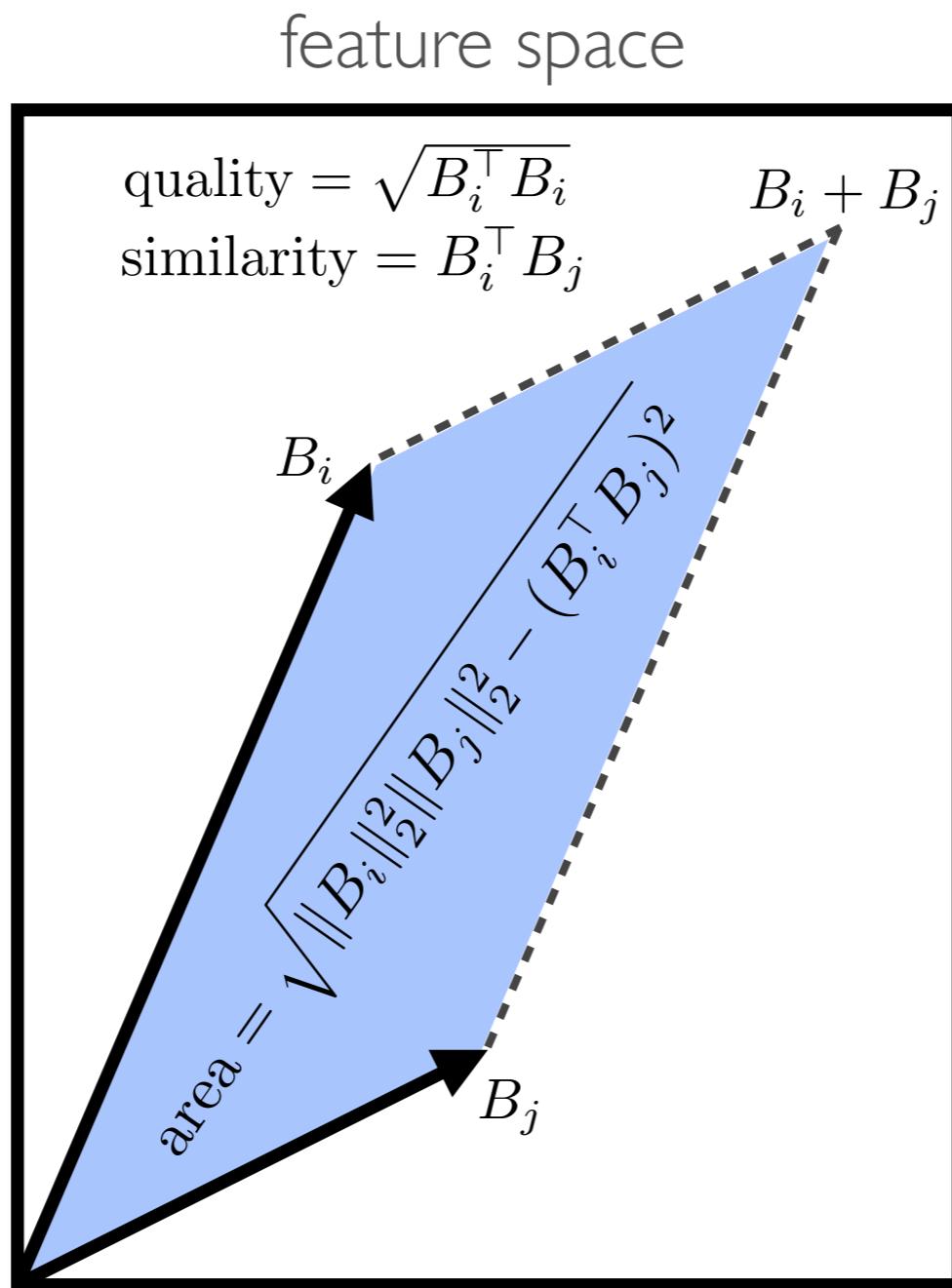
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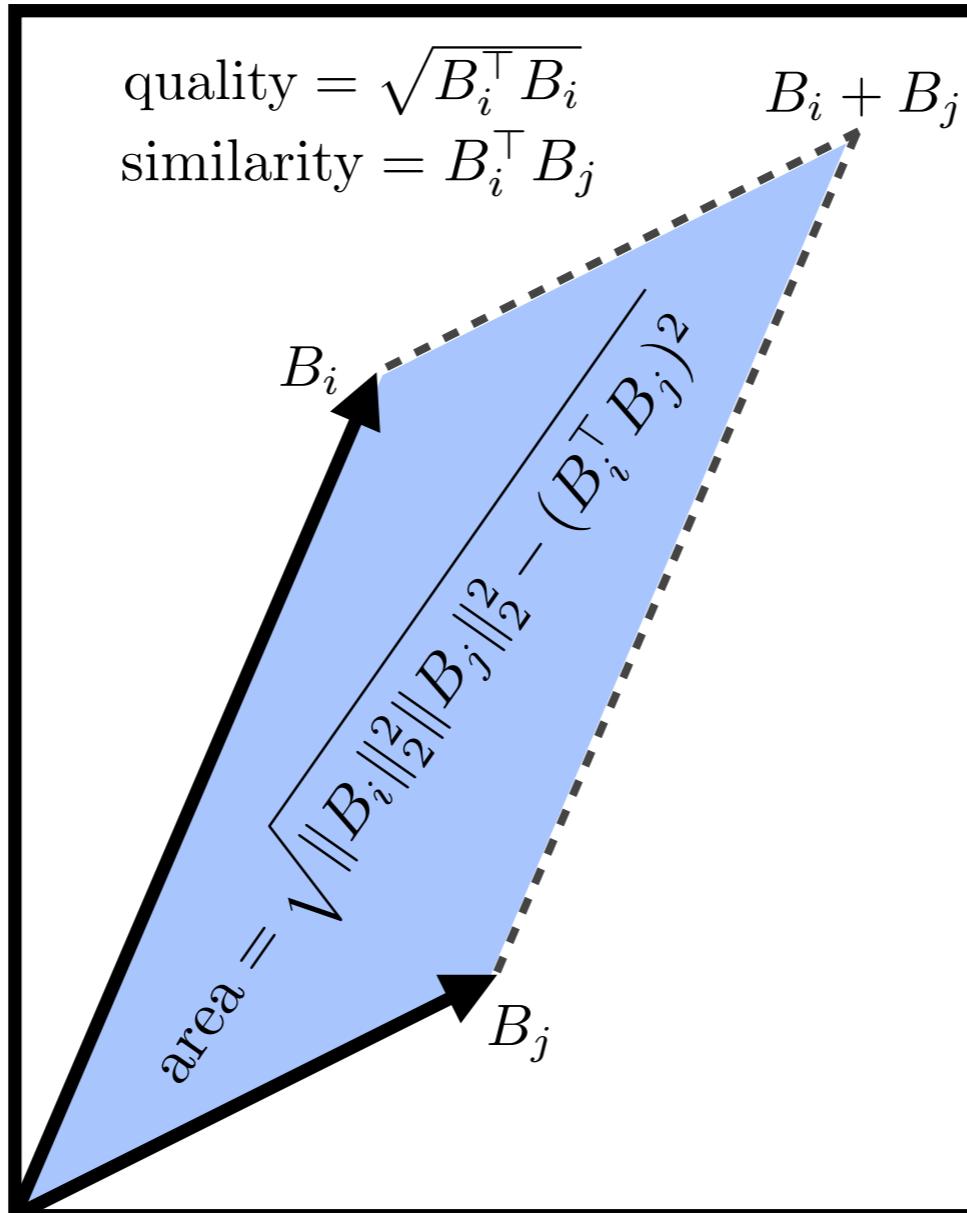
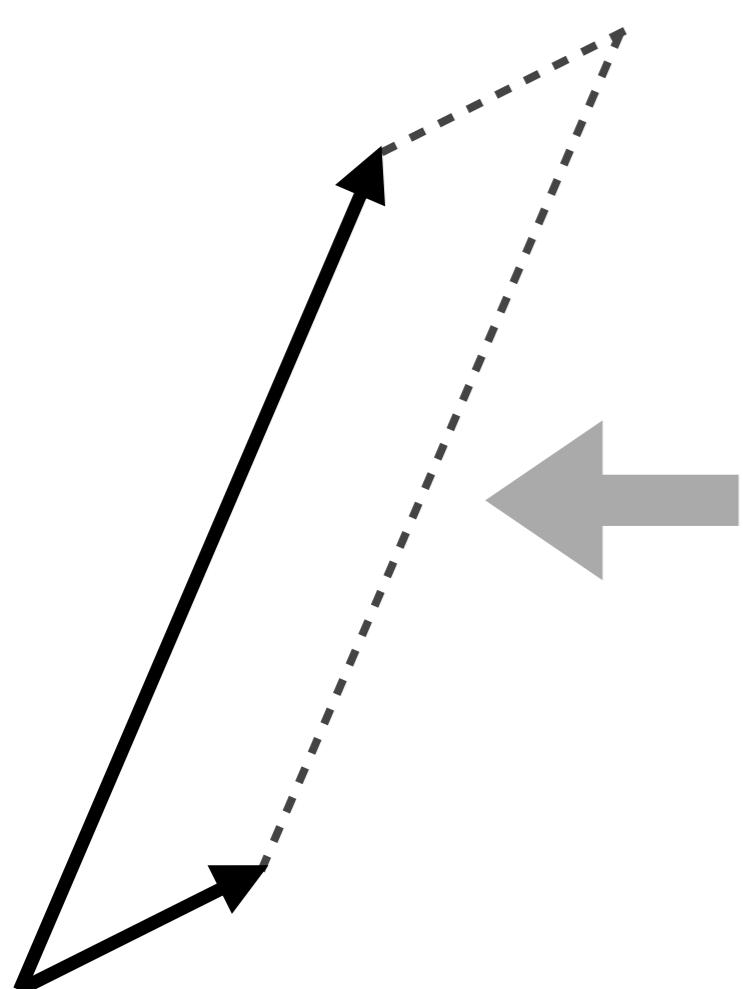
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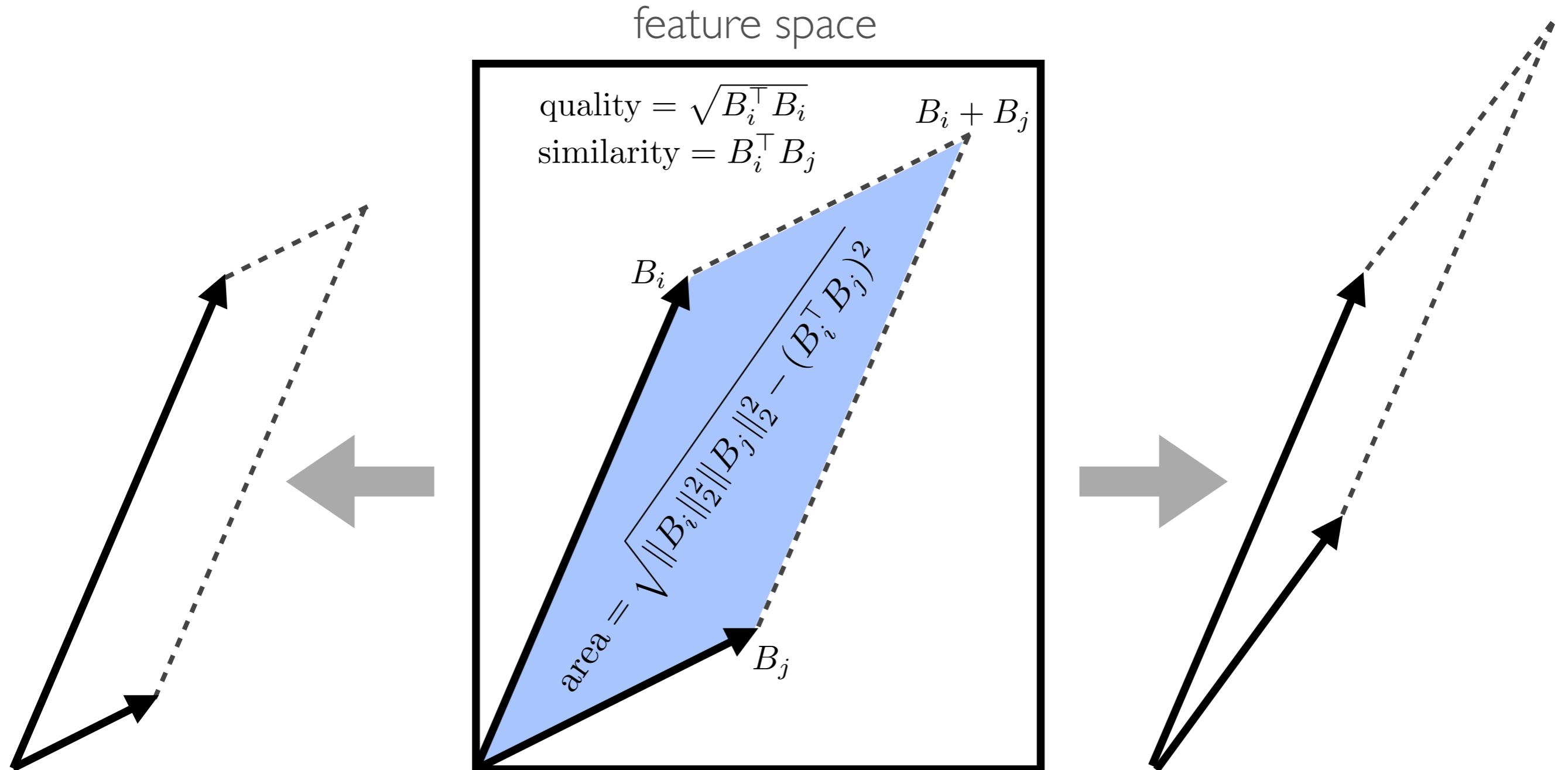
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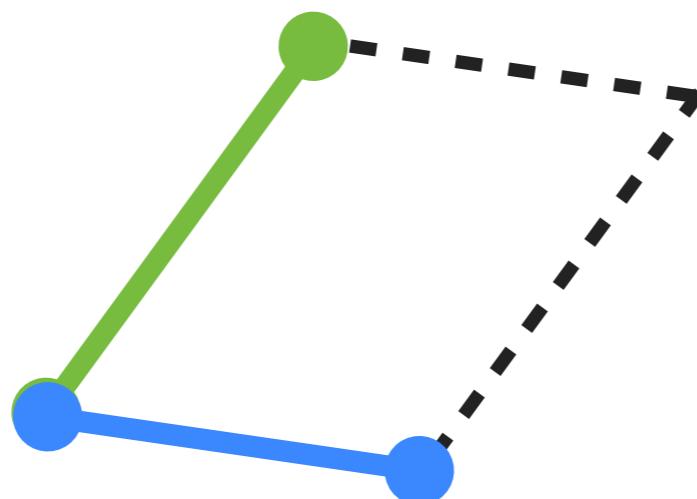


VOLUME AS SET-GOODNESS

$$\text{area} = \sqrt{\|B_i\|_2^2 \|B_j\|_2^2 - (B_i^\top B_j)^2}$$

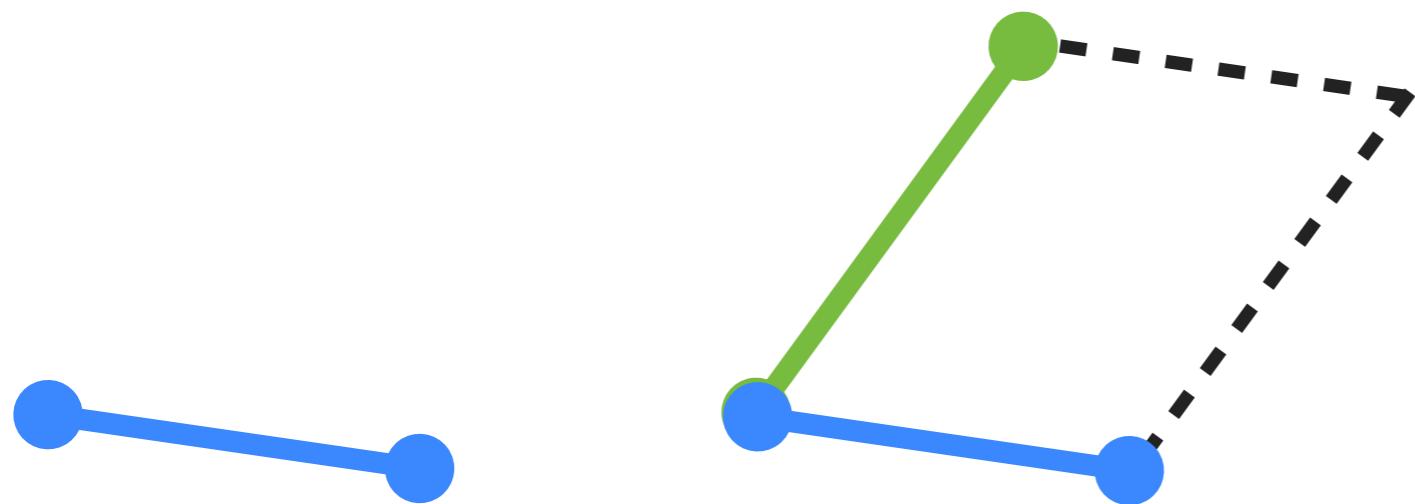
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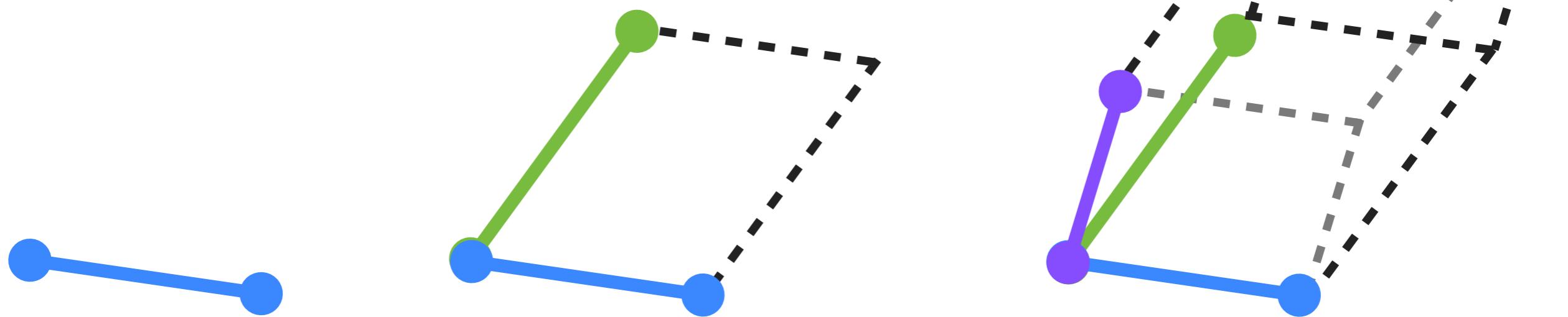
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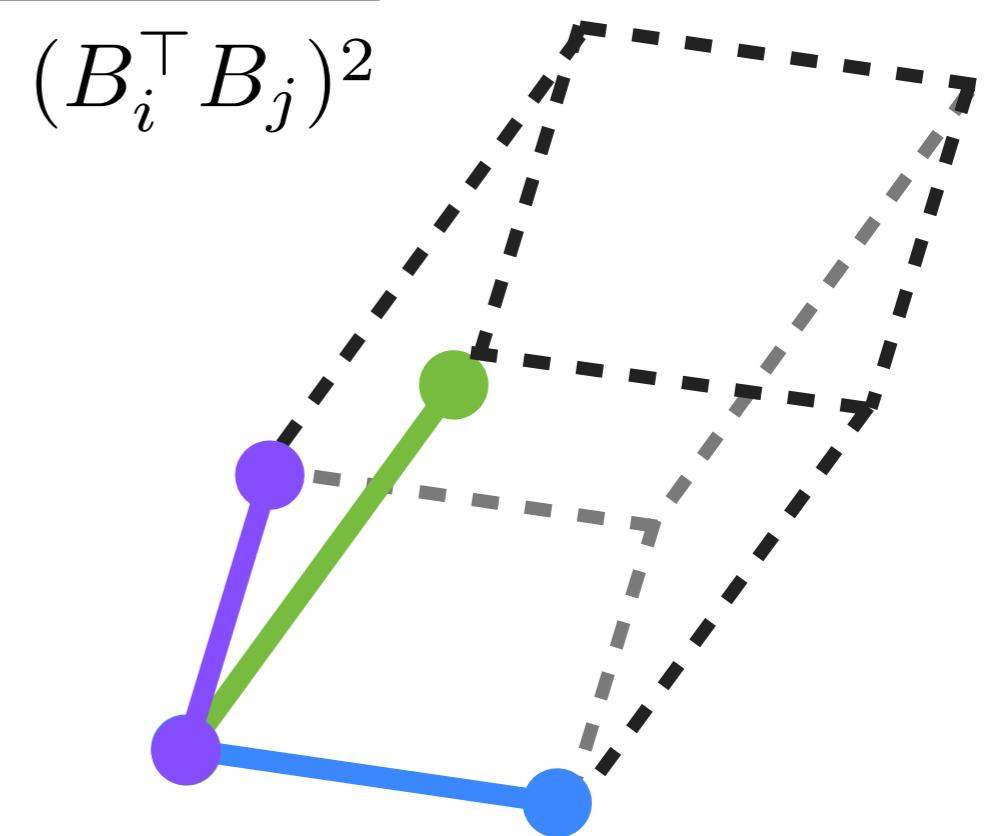
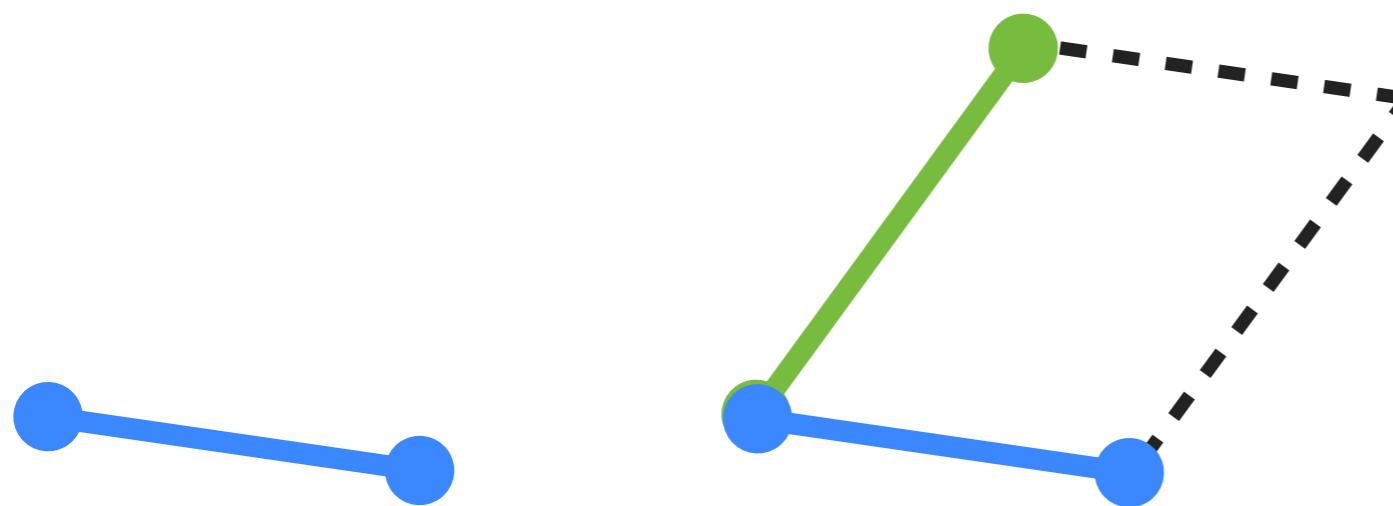
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$$\begin{aligned}\text{vol}(B) &= \text{height} \times \text{base} \\ &= \|B_1\|_2 \text{vol}(\text{proj}_{\perp B_1}(B_{2:N}))\end{aligned}$$

AREA AS A DET

$$\text{area} = \sqrt{\|B_i\|_2^2 \|B_j\|_2^2 - (B_i^\top B_j)^2}$$

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$$= \det \begin{pmatrix} \|B_i\|_2^2 & B_i^\top B_j \\ B_i^\top B_j & \|B_j\|_2^2 \end{pmatrix}^{\frac{1}{2}}$$

AREA AS A DET

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$$= \det \begin{pmatrix} \|B_i\|_2^2 & B_i^\top B_j \\ B_i^\top B_j & \|B_j\|_2^2 \end{pmatrix}^{\frac{1}{2}}$$

$$= \det \begin{pmatrix} \frac{B_i}{\|B_i\|_2} & \frac{B_j}{\|B_j\|_2} \\ \frac{B_i}{\|B_i\|_2} & \frac{-B_j}{\|B_j\|_2} \end{pmatrix}^{2^{-1}}$$

VOLUME AS A DET

$$\text{vol}(B_{\{i,j\}}) = \det \left(\begin{array}{c|c} B_i & \\ \hline B_j & \end{array} \begin{array}{c|c} & B_i \\ \hline & B_j \end{array} \right)^{2|1}$$

VOLUME AS A DET

$$\text{vol}(B_{\{i,j\}}) = \det \left(\begin{array}{c|c} \hline & B_i \\ \hline & B_j \\ \hline \end{array} \quad \begin{array}{c|c} \hline & B_i \\ \hline & B_j \\ \hline \end{array} \right)^{2|1}$$

$$\text{vol}(B) = \det \left(\begin{array}{c|c|c} \hline & B_1 & \\ \hline & \vdots & \\ \hline & B_N & \\ \hline \end{array} \quad \begin{array}{c|c|c} \hline & B_1 & \\ \hline & \ddots & \\ \hline & B_N & \\ \hline \end{array} \right)^{2|1}$$

VOLUME AS A DET

$$\text{vol}(B_{\{i,j\}}) = \det \left(\begin{array}{c|c} B_i & \\ \hline B_j & \end{array} \begin{array}{c|c} B_i & \\ \hline B_j & \end{array} \right)^{2|1}$$

$$\text{vol}(B) = \det \left(\begin{array}{c|c|c} B_1 & & \\ \vdots & & \\ B_N & & \end{array} \begin{array}{c|c|c} B_1 & & \\ \vdots & & \\ B_N & & \end{array} \right)^{2|1}$$

$$\text{vol}(B)^2 = \det(B^\top B) = \det(L)$$

COMPLEX STATISTICS

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COMPLEX STATISTICS



COMPLEX STATISTICS

$$\sum$$



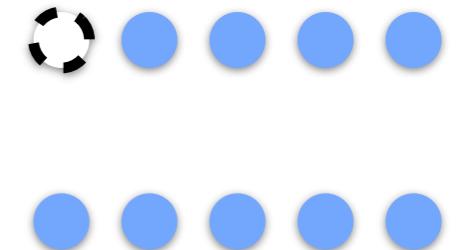
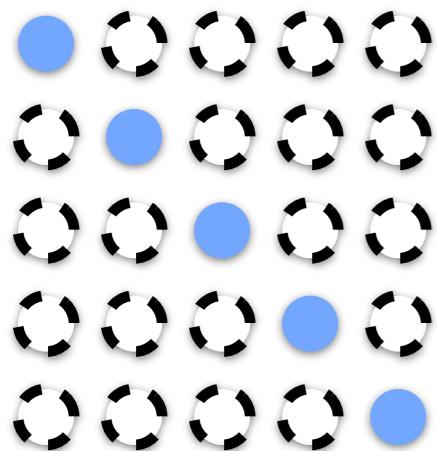
COMPLEX STATISTICS

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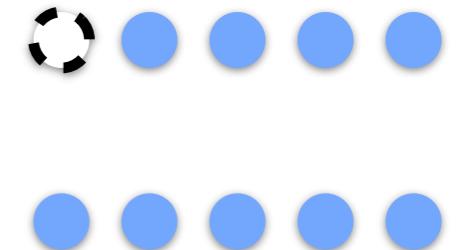
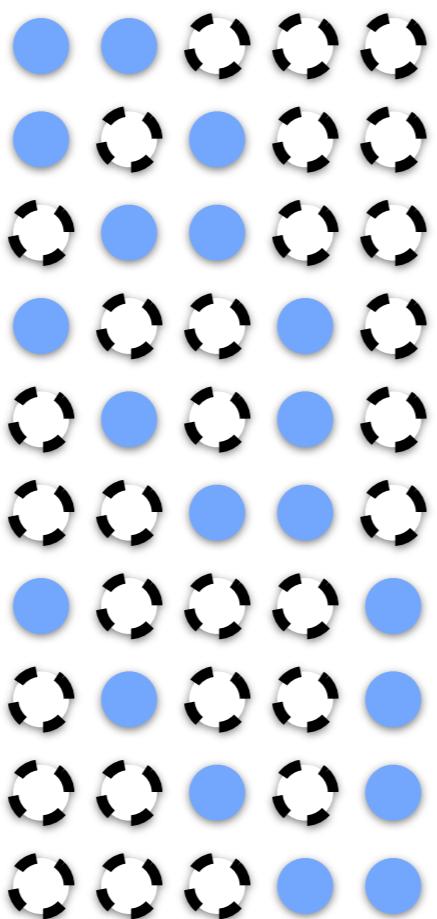
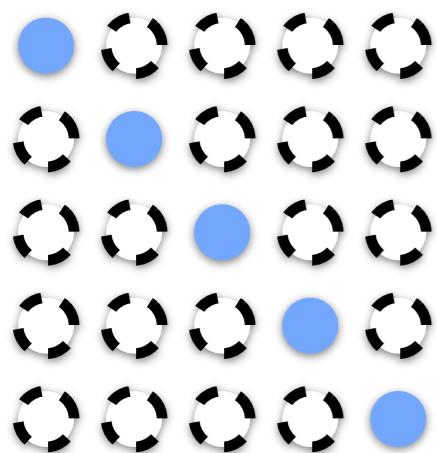
COMPLEX STATISTICS

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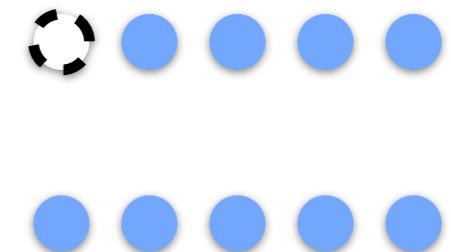
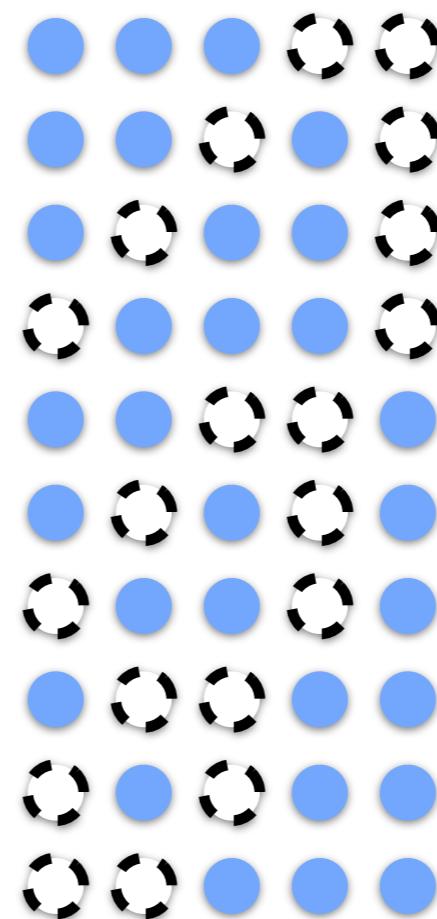
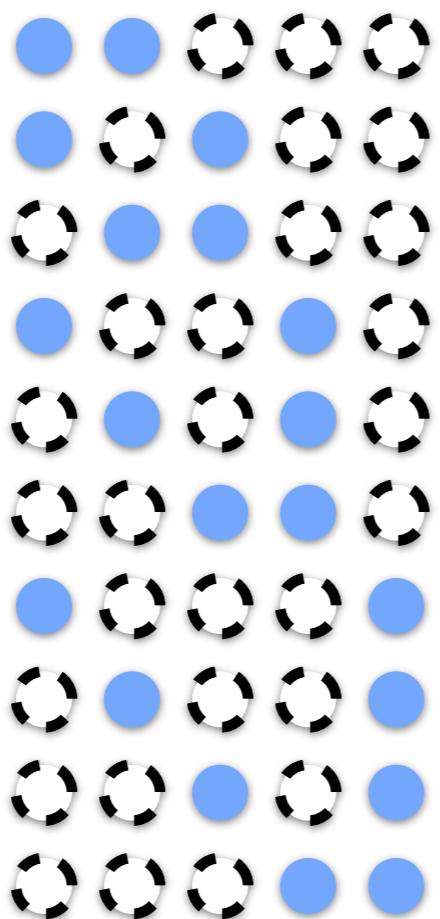
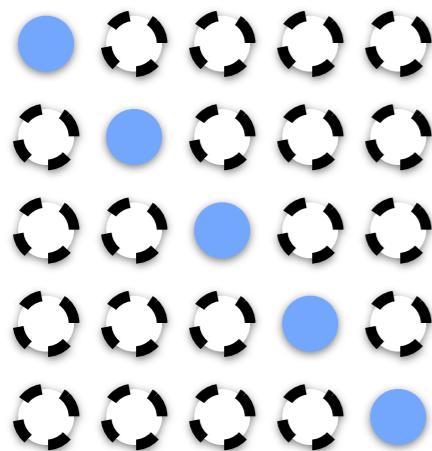
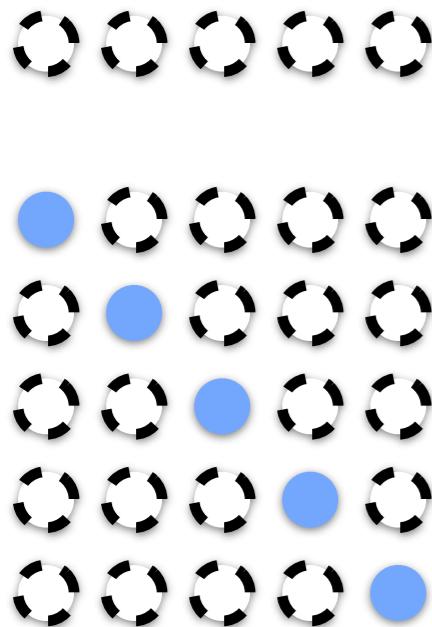
COMPLEX STATISTICS

$$\Sigma$$



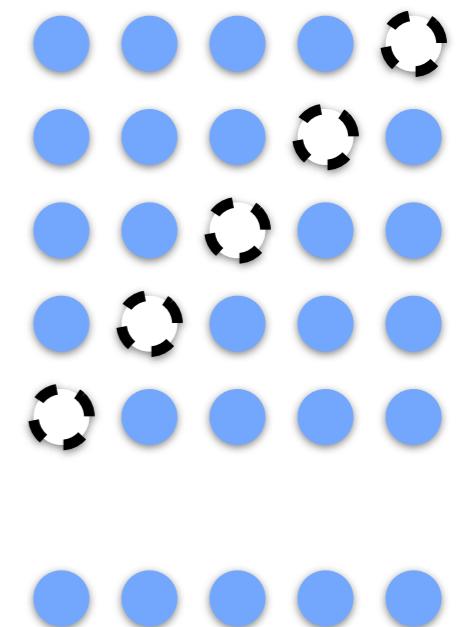
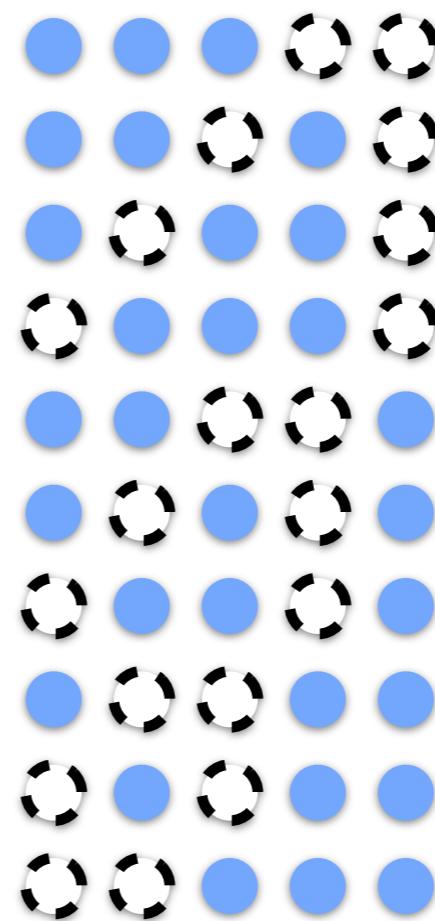
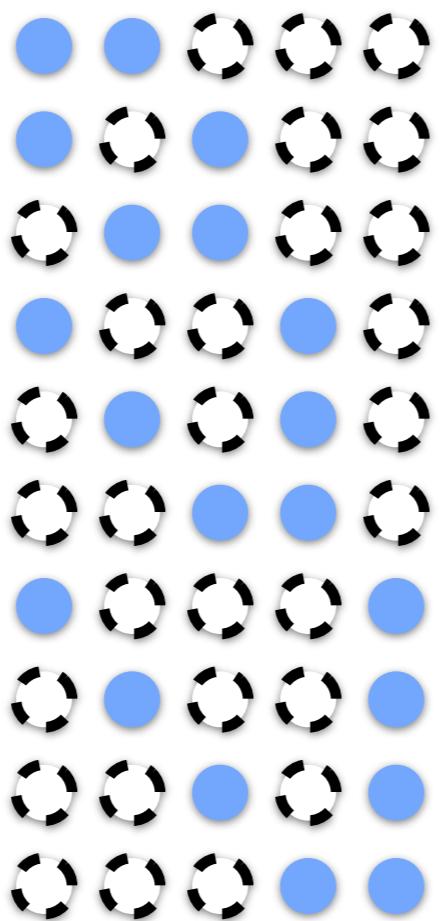
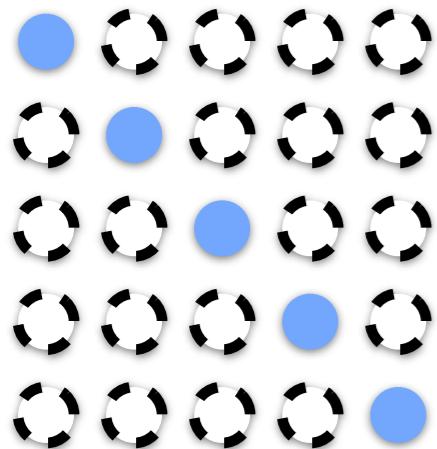
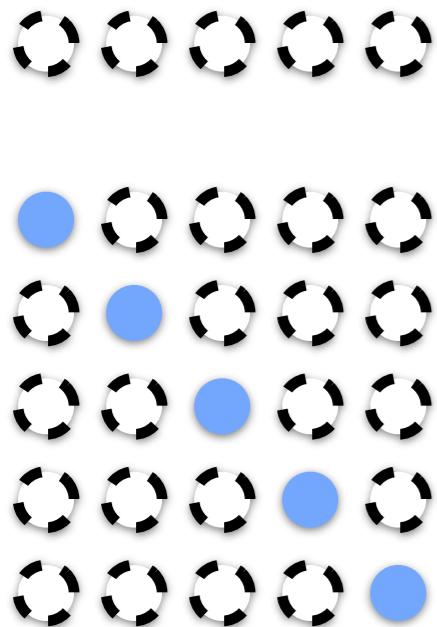
COMPLEX STATISTICS

$$\Sigma$$



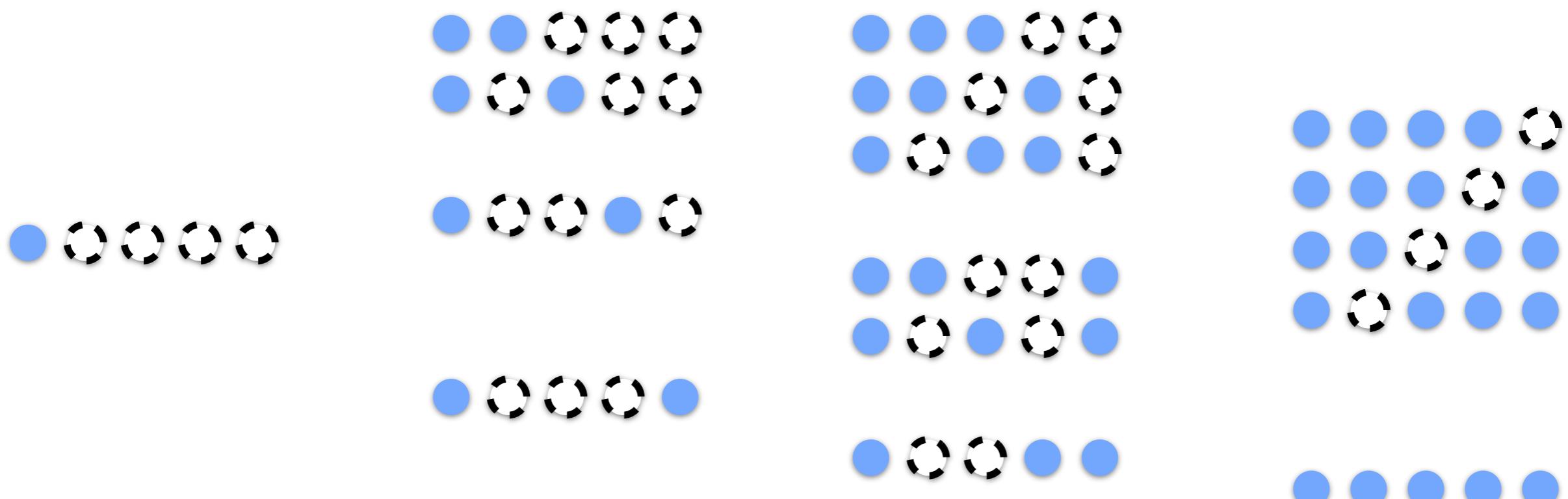
COMPLEX STATISTICS

$$\Sigma$$



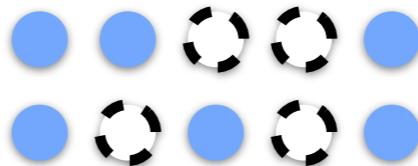
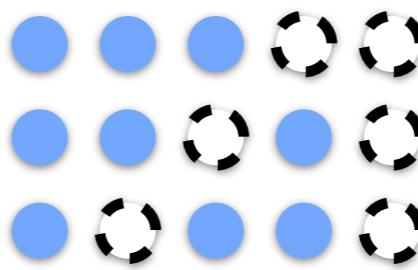
COMPLEX STATISTICS

Σ



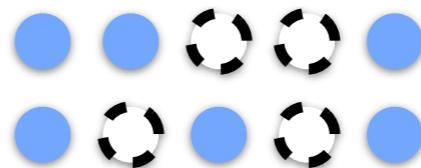
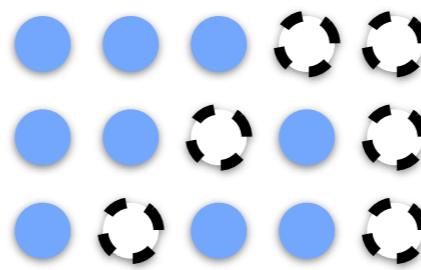
COMPLEX STATISTICS

$$\sum$$



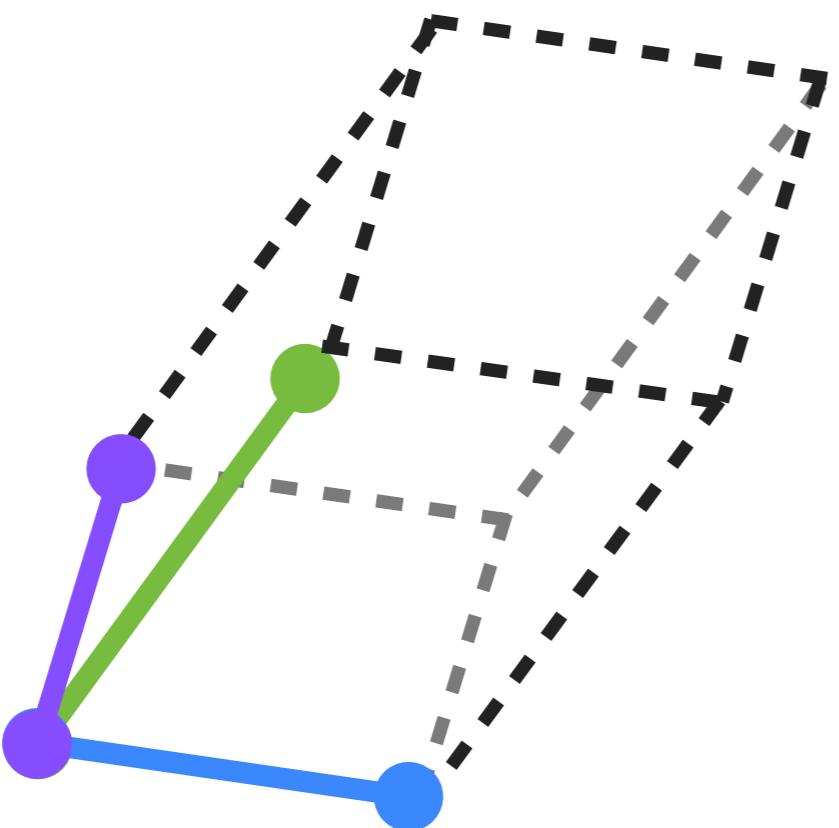
COMPLEX STATISTICS

N items $\Longrightarrow 2^N$ sets



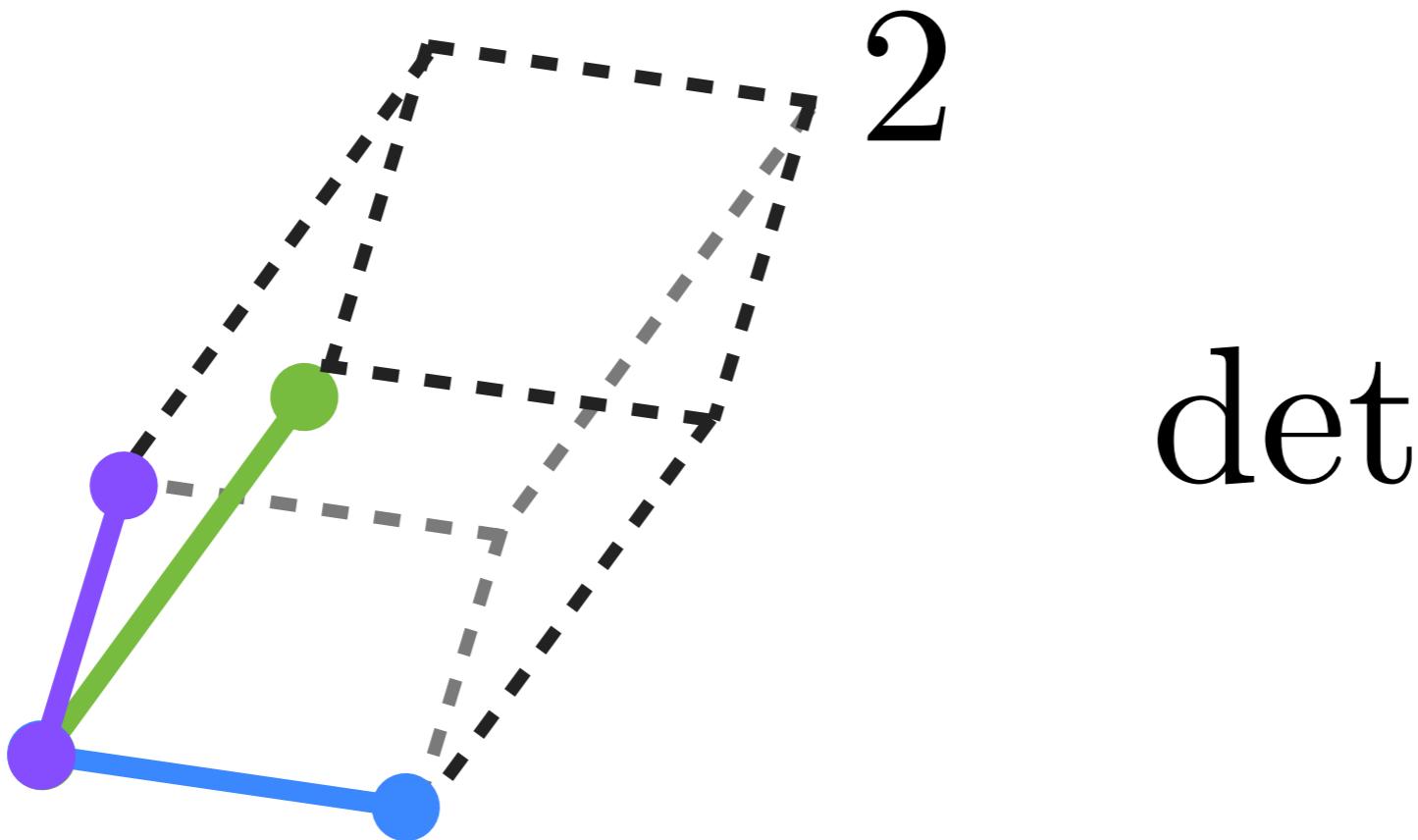
EFFICIENT COMPUTATION

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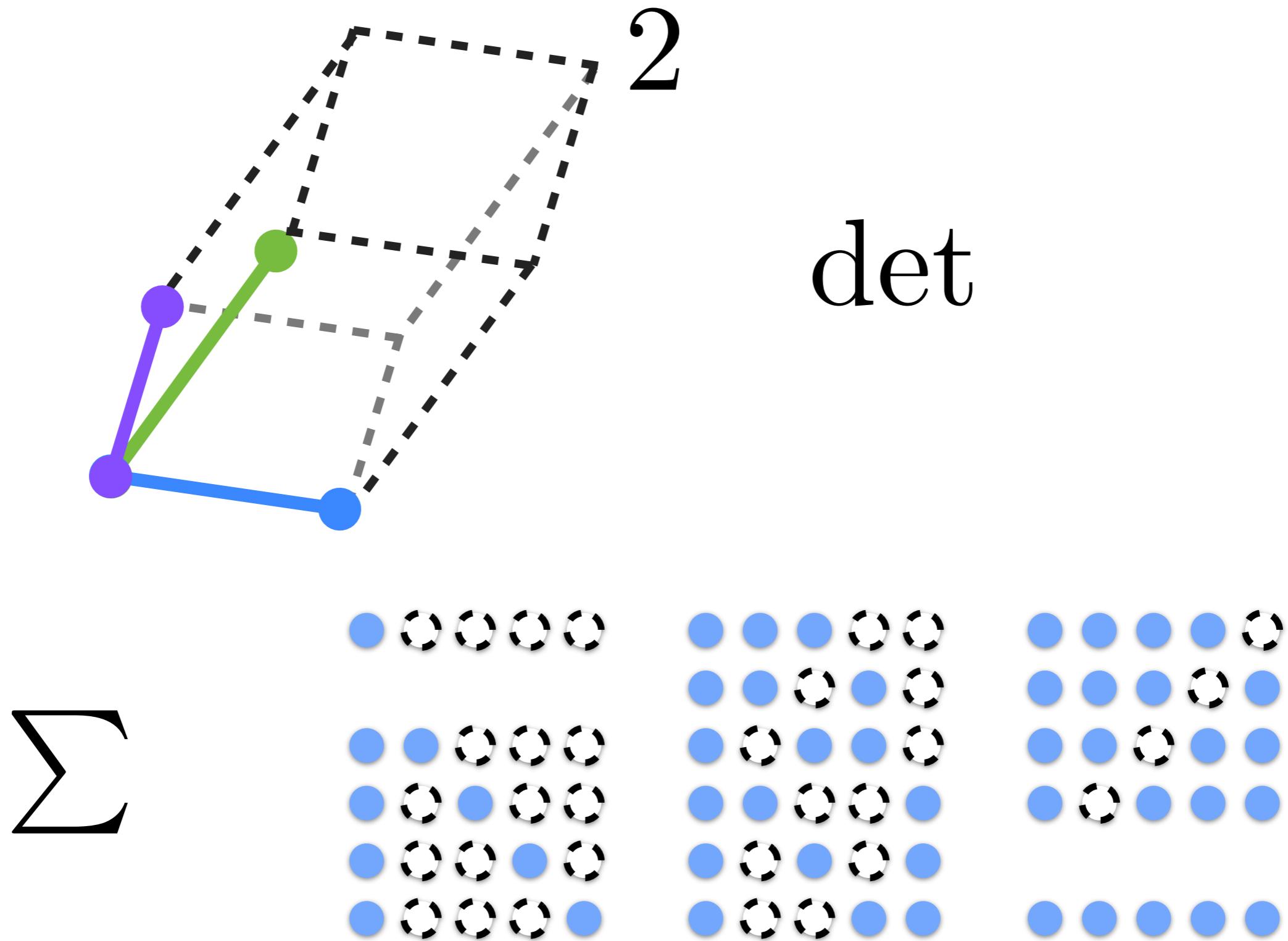


$$\det^{\frac{1}{2}}$$

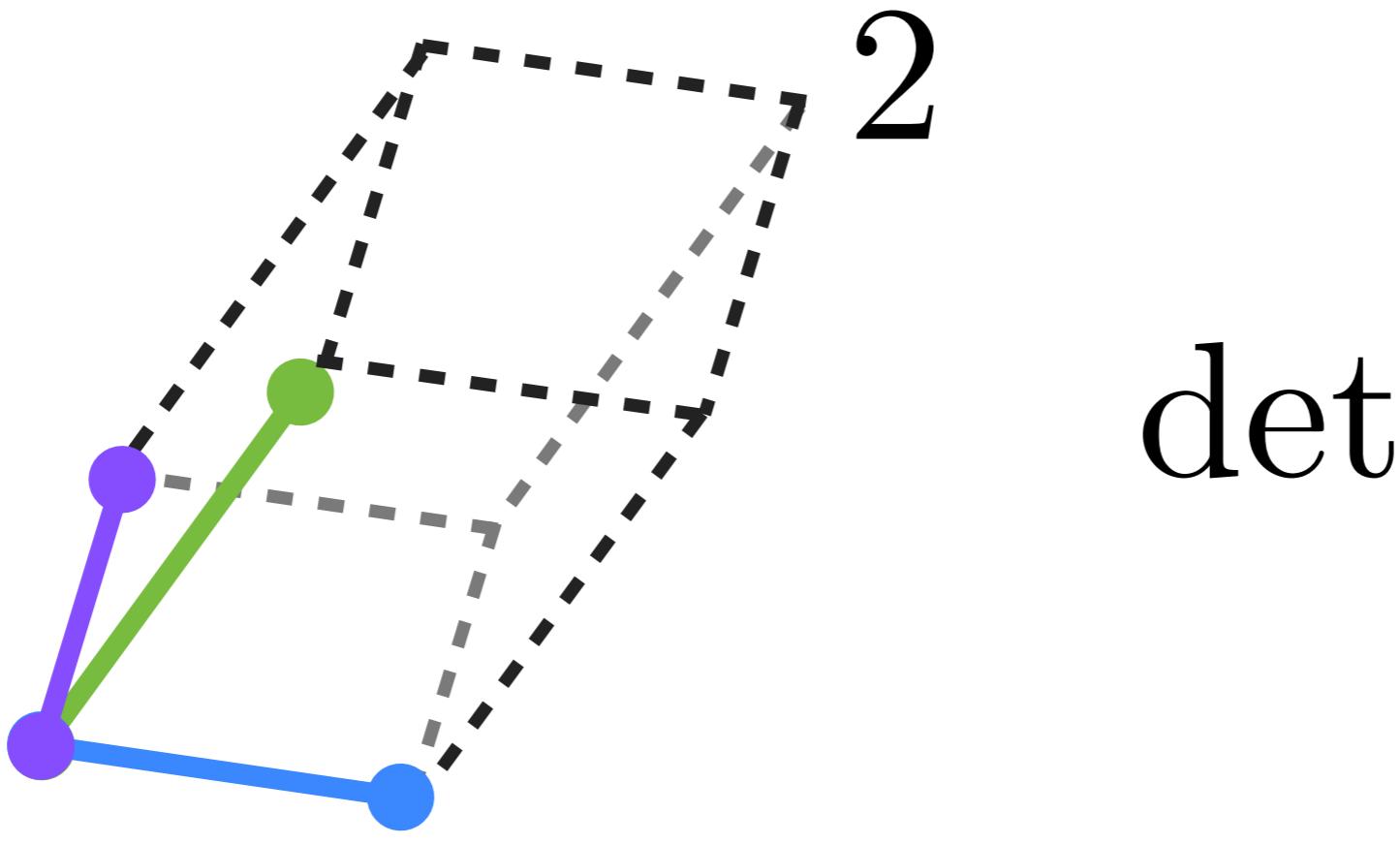
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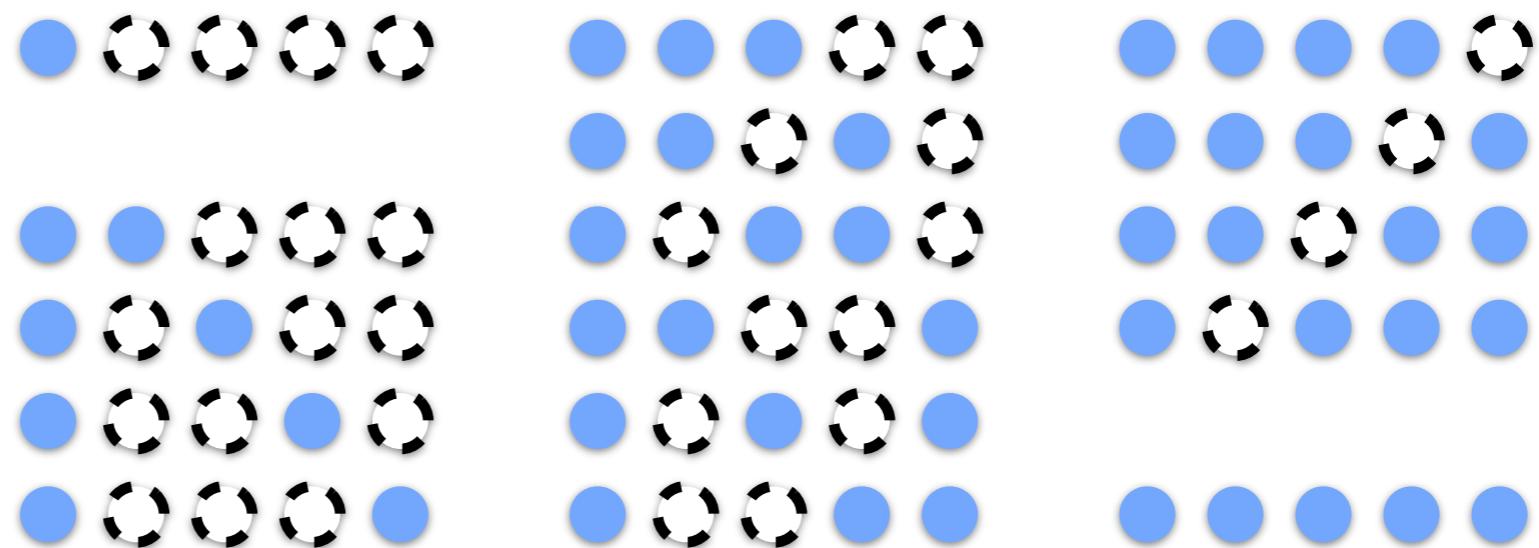
EFFICIENT COMPUTATION



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$O(N^3)$

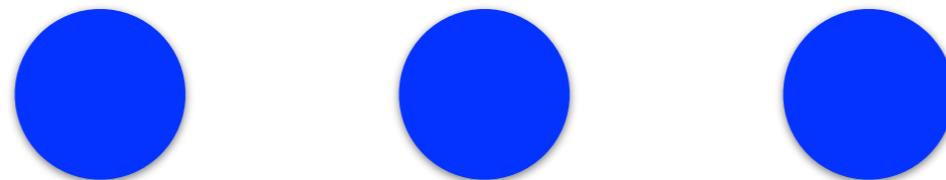
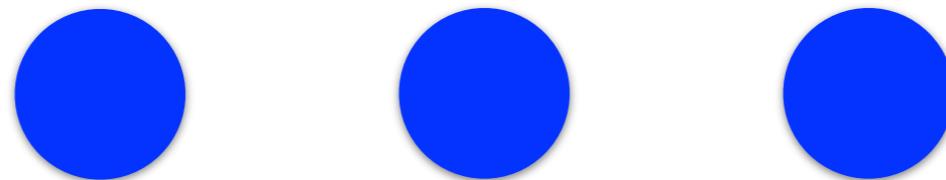
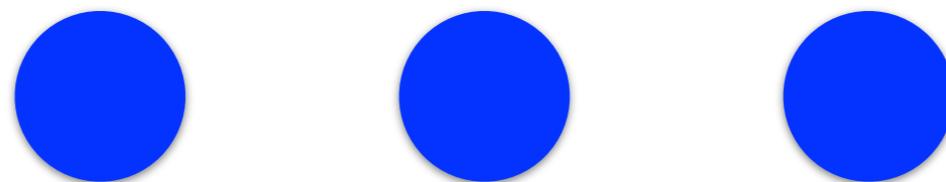


POINT PROCESSES

$$\mathcal{Y} = \{1, \dots, N\}$$

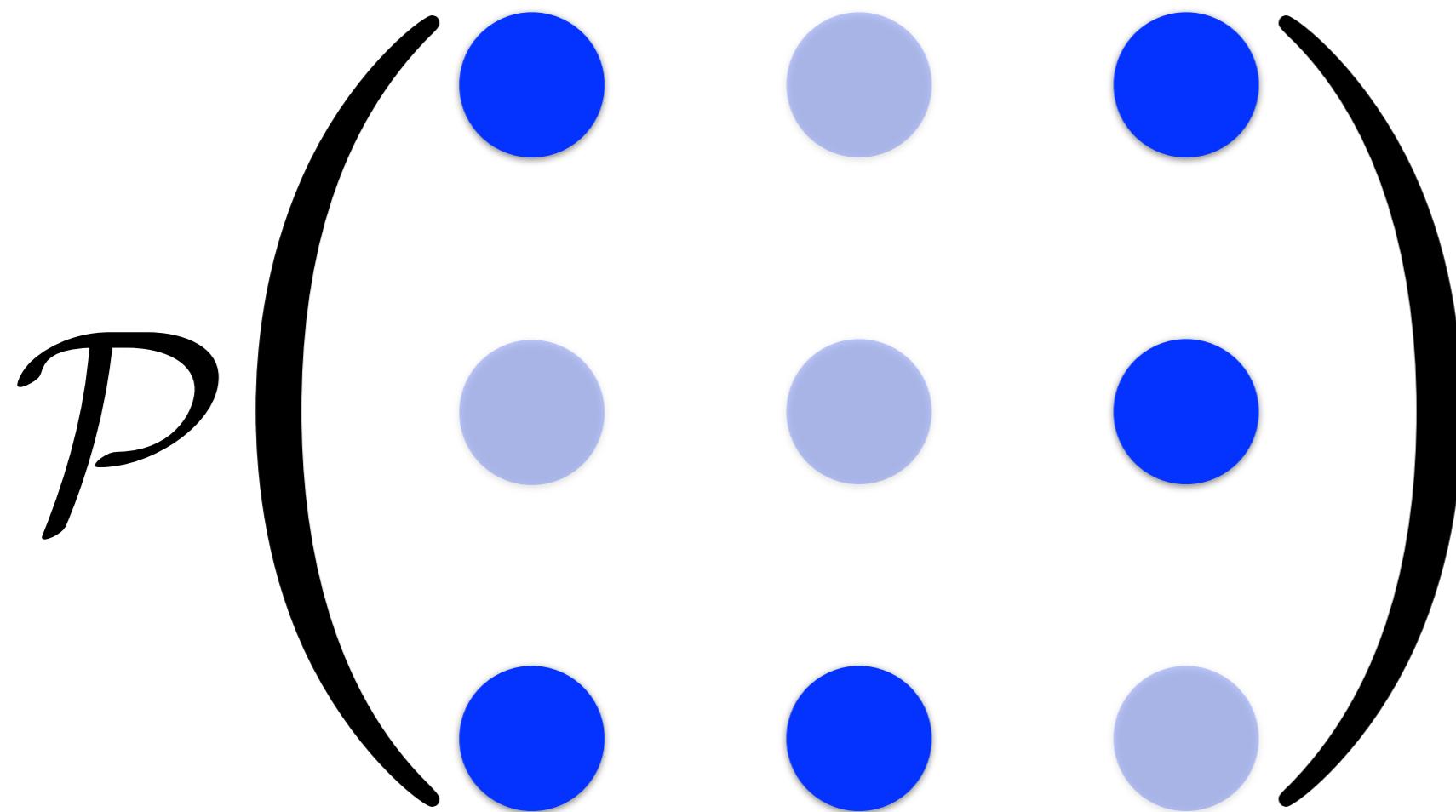
POINT PROCESSES

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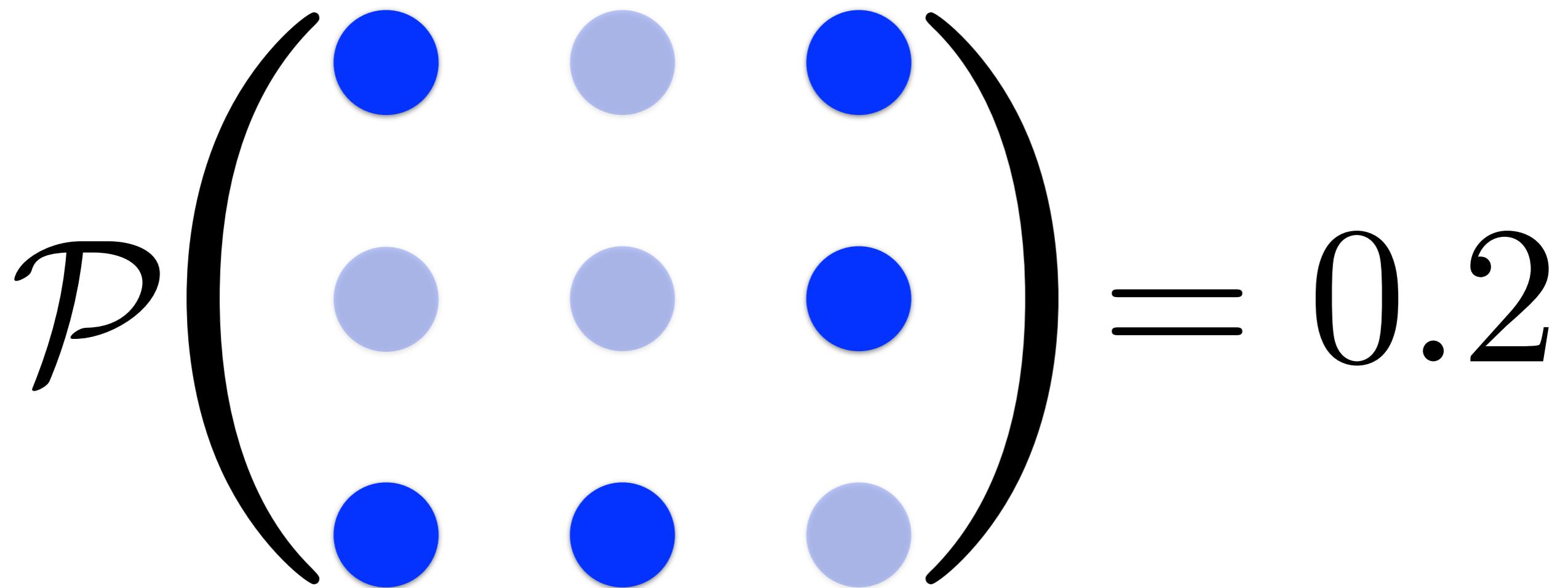
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DETERMINANTAL

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$$\mathcal{P}(\{2,3,5\}) \propto$$

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$$L_{11}\;\;\; L_{12}\;\;\; L_{13}\;\;\; L_{14}\;\;\; L_{15}$$

$$L_{21}\;\;\; L_{22}\;\;\; L_{23}\;\;\; L_{24}\;\;\; L_{25}$$

$$L_{31}\;\;\; L_{32}\;\;\; L_{33}\;\;\; L_{34}\;\;\; L_{35}$$

$$L_{41}\;\;\; L_{42}\;\;\; L_{43}\;\;\; L_{44}\;\;\; L_{45}$$

$$L_{51}\;\;\; L_{52}\;\;\; L_{53}\;\;\; L_{54}\;\;\; L_{55}$$

DETERMINANTAL

$$\mathcal{P}(\{2, 3, 5\}) \propto$$

L_{11}	L_{12}	L_{13}	L_{14}	L_{15}
L_{21}	L_{22}	L_{23}	L_{24}	L_{25}
L_{31}	L_{32}	L_{33}	L_{34}	L_{35}
L_{41}	L_{42}	L_{43}	L_{44}	L_{45}
L_{51}	L_{52}	L_{53}	L_{54}	L_{55}

DETERMINANTAL

$$L_{22} \quad L_{23} \quad L_{25}$$

$$\mathcal{P}(\{2,3,5\}) \propto \qquad \qquad L_{32} \quad L_{33} \quad L_{35}$$

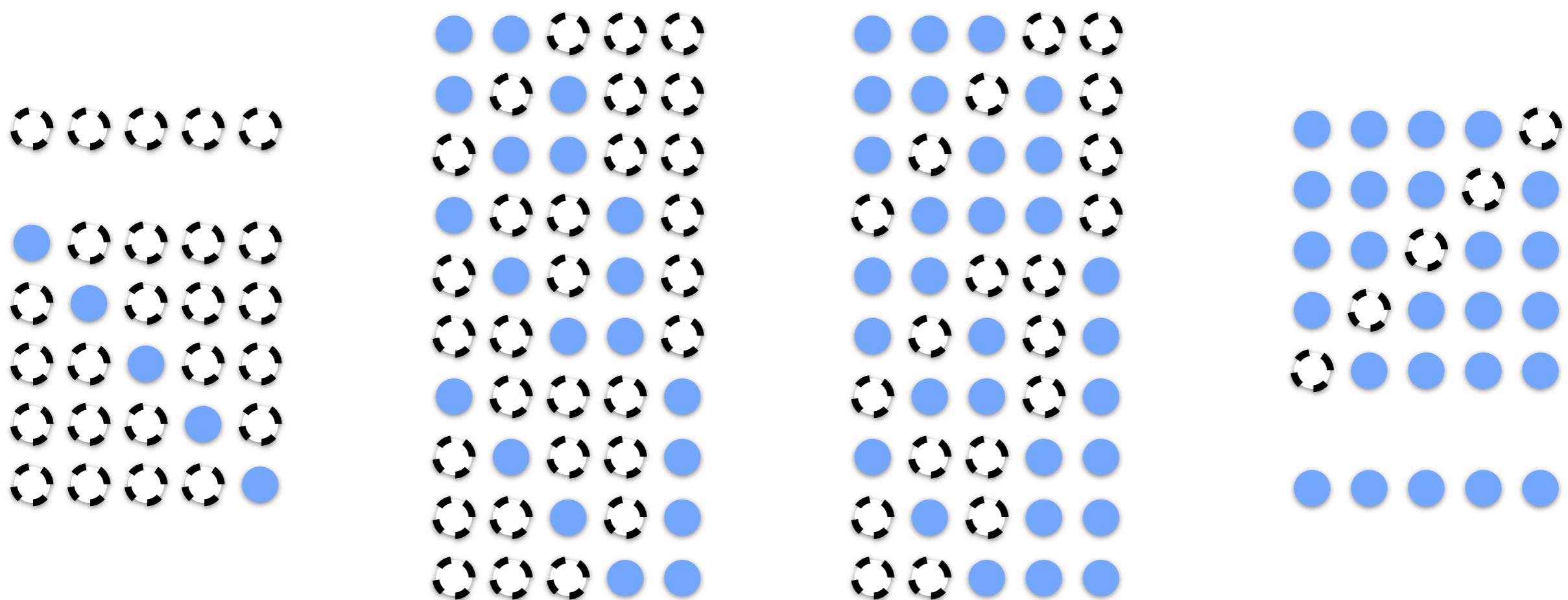
$$L_{52} \quad L_{53} \quad L_{55}$$

DETERMINANTAL

$$\mathcal{P}(\{2, 3, 5\}) \propto \det \begin{pmatrix} L_{22} & L_{23} & L_{25} \\ L_{32} & L_{33} & L_{35} \\ L_{52} & L_{53} & L_{55} \end{pmatrix}$$

DETERMINANTAL

$$\mathcal{P}(\{2, 3, 5\}) = \det \begin{pmatrix} L_{22} & L_{23} & L_{25} \\ L_{32} & L_{33} & L_{35} \\ L_{52} & L_{53} & L_{55} \end{pmatrix}$$



DETERMINANTAL

$$\mathcal{P}(\{2, 3, 5\}) = \det \begin{pmatrix} L_{22} & L_{23} & L_{25} \\ L_{32} & L_{33} & L_{35} \\ L_{52} & L_{53} & L_{55} \end{pmatrix} \frac{\det(L + I)}{\det(L + I)}$$

EFFICIENT INFERENCE

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Normalizing: $\mathcal{P}_L(\mathbf{Y} = Y)$

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Marginalizing: $\mathcal{P}(Y \subseteq \mathbf{Y})$

Conditioning: $\mathcal{P}_L(\mathbf{Y} = B \mid A \subseteq \mathbf{Y})$

$\mathcal{P}_L(\mathbf{Y} = B \mid A \cap \mathbf{Y} = \emptyset)$

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Normalizing: $\mathcal{P}_L(\mathbf{Y} = Y)$

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Sampling: $\mathbf{Y} \sim \mathcal{P}_L$

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Normalizing: $\mathcal{P}_L(\mathbf{Y} = Y)$

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$\mathcal{P}_L(\mathbf{Y} = B \mid A \cap \mathbf{Y} = \emptyset)$

Sampling: $\mathbf{Y} \sim \mathcal{P}_L$

$O(N^3)$

1. DIMENSIONALITY REDUCTION

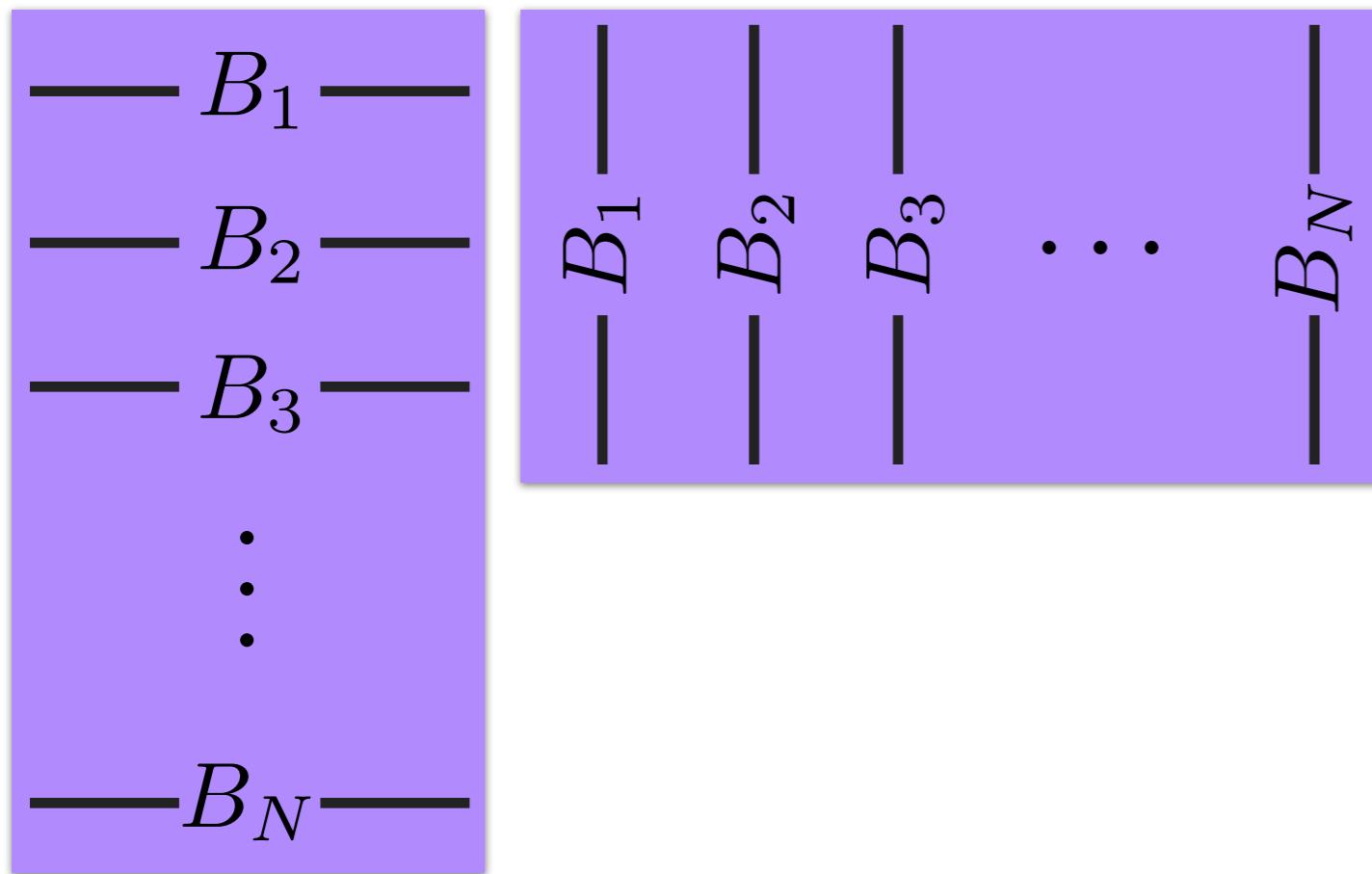
DUAL KERNEL

KULESZA AND TASKAR (NIPS 2010)

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L



DUAL KERNEL

KULESZA AND TASKAR (NIPS 2010)

L

$$\begin{array}{c} \boxed{B_1} \\ \boxed{B_2} \\ \boxed{B_3} \\ \vdots \\ \boxed{B_N} \end{array} = \begin{array}{c} \boxed{B_1} \\ \boxed{B_2} \\ \boxed{B_3} \\ \dots \\ \boxed{B_N} \end{array} = N \times N$$

DUAL KERNEL

KULESZA AND TASKAR (NIPS 2010)

C

$$\begin{array}{c} \boxed{\begin{array}{ccccccc} \hline & B_1 & & & & & \\ \hline & B_2 & & & & & \\ \hline & B_3 & & \cdots & & & \\ \hline & B_N & & & & & \end{array}} & = & \boxed{N \times N} \\ & & & & & & \end{array}$$
$$\begin{array}{c} \boxed{\begin{array}{c} \hline B_1 \hline \\ \hline B_2 \hline \\ \hline B_3 \hline \\ \vdots \\ \hline B_N \hline \end{array}} \end{array}$$

DUAL KERNEL

KULESZA AND TASKAR (NIPS 2010)

C

$$\begin{array}{c} \boxed{\cdots} \\ \begin{array}{c} \boxed{B_1} \\ \boxed{B_2} \\ \boxed{B_3} \\ \vdots \\ \boxed{B_N} \end{array} \end{array} = \boxed{\quad}$$

DUAL KERNEL

KULESZA AND TASKAR (NIPS 2010)

C

$$\begin{array}{c} \boxed{\begin{array}{ccccccc} \hline & B_1 & \hline & B_2 & \hline & B_3 & \hline & \cdots & \hline & B_N & \hline \end{array}} \\ \times \\ \boxed{\begin{array}{c} \hline B_1 \hline \\ \hline B_2 \hline \\ \hline B_3 \hline \\ \vdots \\ \hline B_N \hline \end{array}} \end{array} = \boxed{D \times D}$$

DUAL INFERENCE

DUAL INFERENCE

$$L = \textcolor{violet}{V} \Lambda V^\top$$

$$C = \hat{V} \Lambda \hat{V}^\top$$

DUAL INFERENCE

$$L = V \Lambda V^\top \quad \longleftrightarrow \quad V = B^\top \hat{V} \Lambda^{-\frac{1}{2}} \quad C = \hat{V} \Lambda \hat{V}^\top$$

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Normalizing $\sum_Y \det(L_Y) \quad O(\textcolor{blue}{D}^3)$

DUAL INFERENCE

$$L = V \Lambda V^\top \quad \begin{array}{c} \swarrow \\ V = B^\top \hat{V} \Lambda^{-\frac{1}{2}} \\ \searrow \end{array} \quad C = \hat{V} \Lambda \hat{V}^\top$$

Normalizing $\sum_Y \det(L_Y)$ $O(D^3)$

Marginalizing & Conditioning $O(D^3 + D^2 k^2)$

DUAL INFERENCE

$$L = V \Lambda V^\top \quad \xleftrightarrow{V = B^\top \hat{V} \Lambda^{-\frac{1}{2}}} \quad C = \hat{V} \Lambda \hat{V}^\top$$

Normalizing $\sum_Y \det(L_Y)$ $O(D^3)$

Marginalizing & Conditioning $O(D^3 + D^2 k^2)$

Sampling $\mathbf{Y} \sim \mathcal{P}_L$ $O(ND^2 k)$

EXPONENTIAL N

EXPONENTIAL N

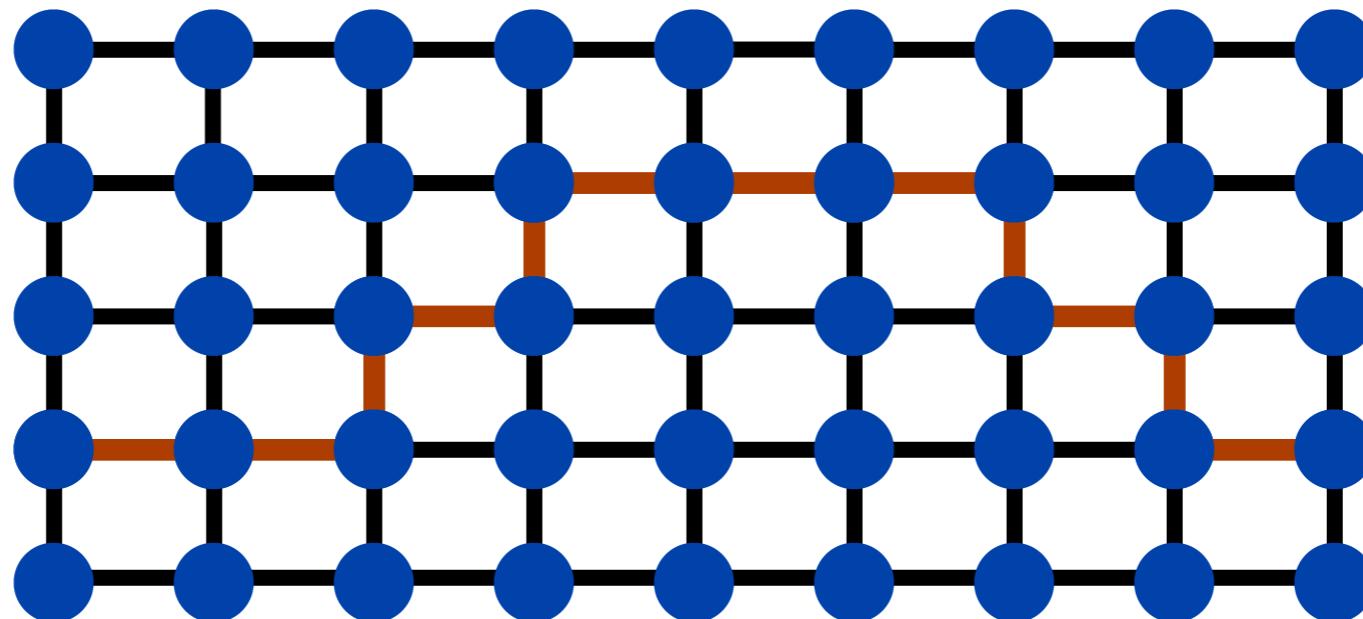
We want to select a diverse set of parses.

$$N = O(\{\text{sentence length}\}^{\{\text{sentence length}\}})$$

EXPONENTIAL N

We want to select a diverse set of parses.

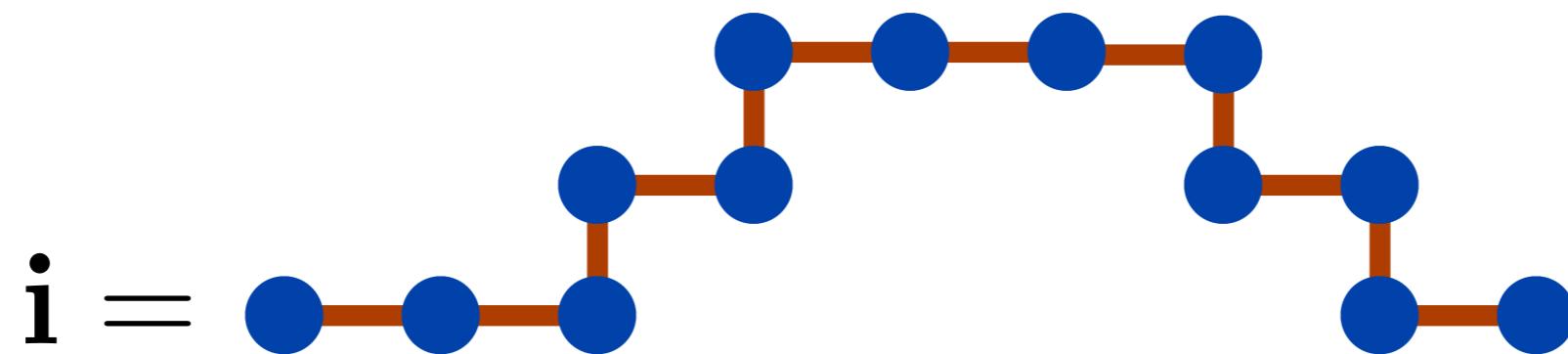
$$N = O(\{\text{sentence length}\}^{\{\text{sentence length}\}})$$



$$N = O(\{\text{node degree}\}^{\{\text{path length}\}})$$

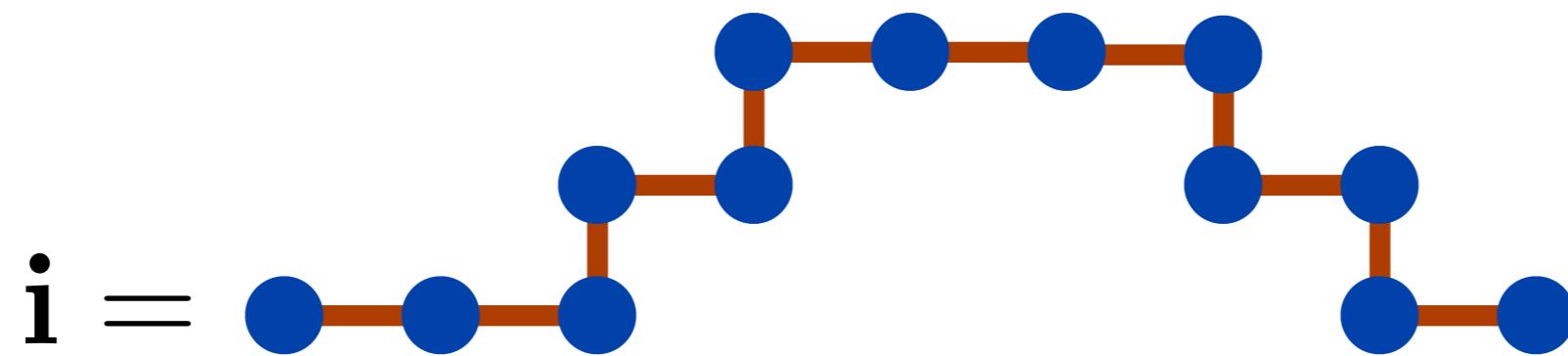
STRUCTURE FACTORIZATION

KULESZA AND TASKAR (NIPS 2010)



STRUCTURE FACTORIZATION

KULESZA AND TASKAR (NIPS 2010)

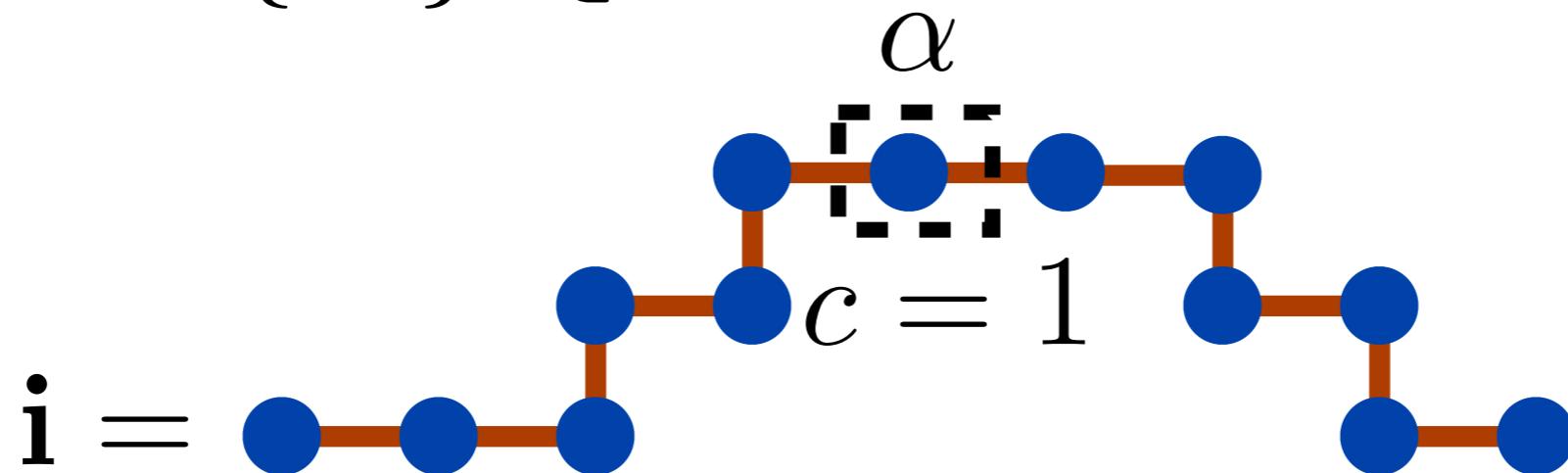


$$B_{\mathbf{i}} = q(\mathbf{i})\phi(\mathbf{i})$$

STRUCTURE FACTORIZATION

KULESZA AND TASKAR (NIPS 2010)

$$\mathbf{i} = \{i_\alpha\}_{\alpha \in F}$$

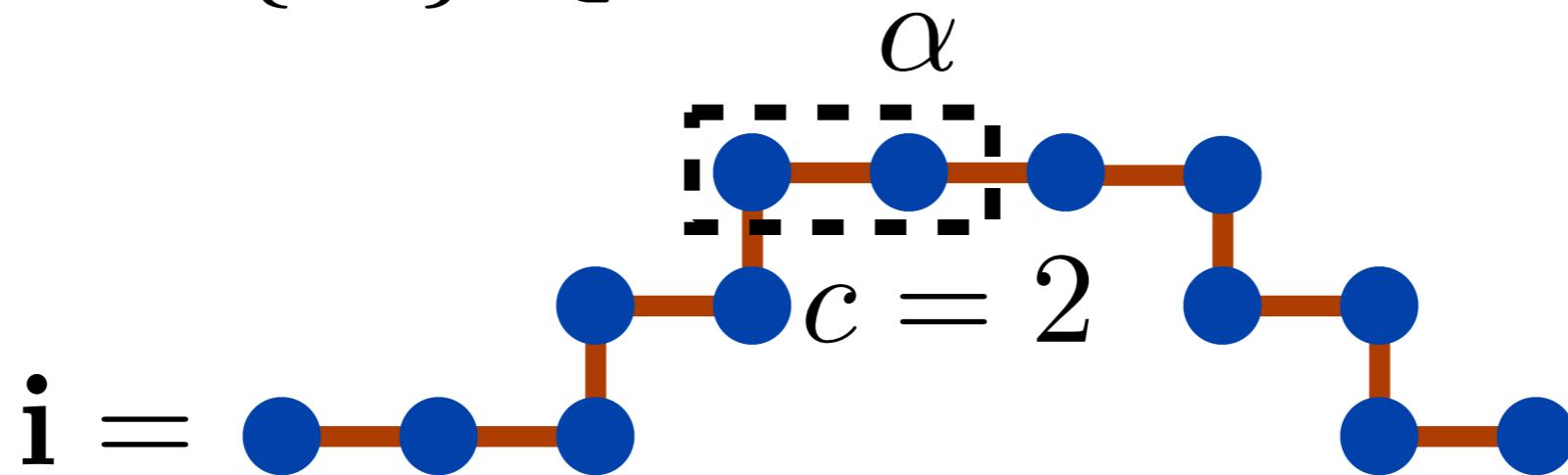


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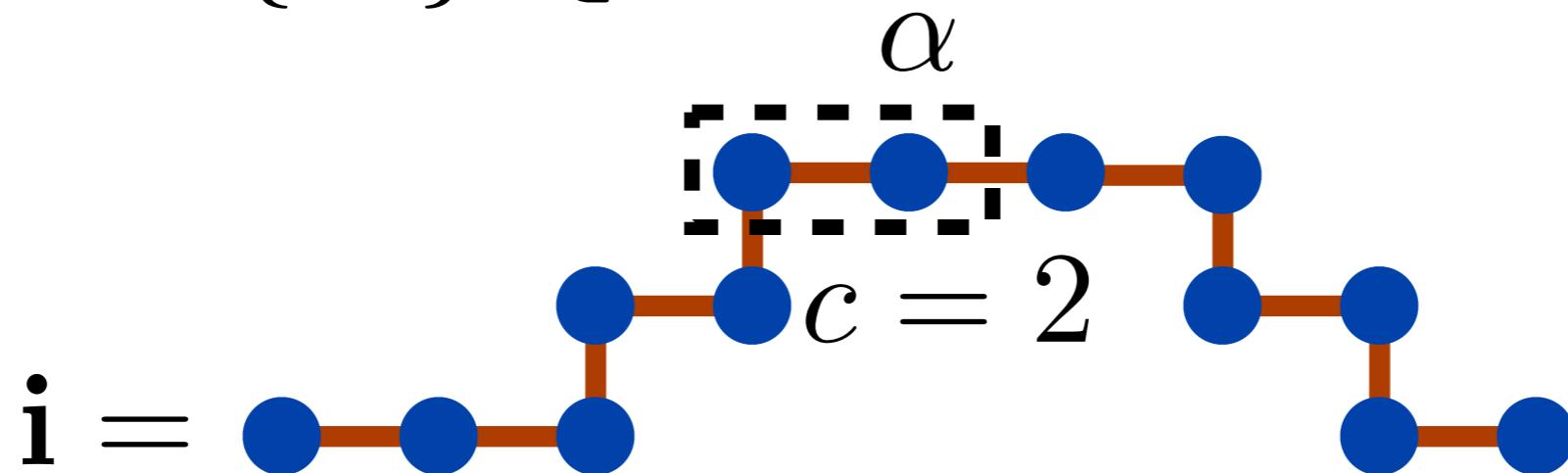


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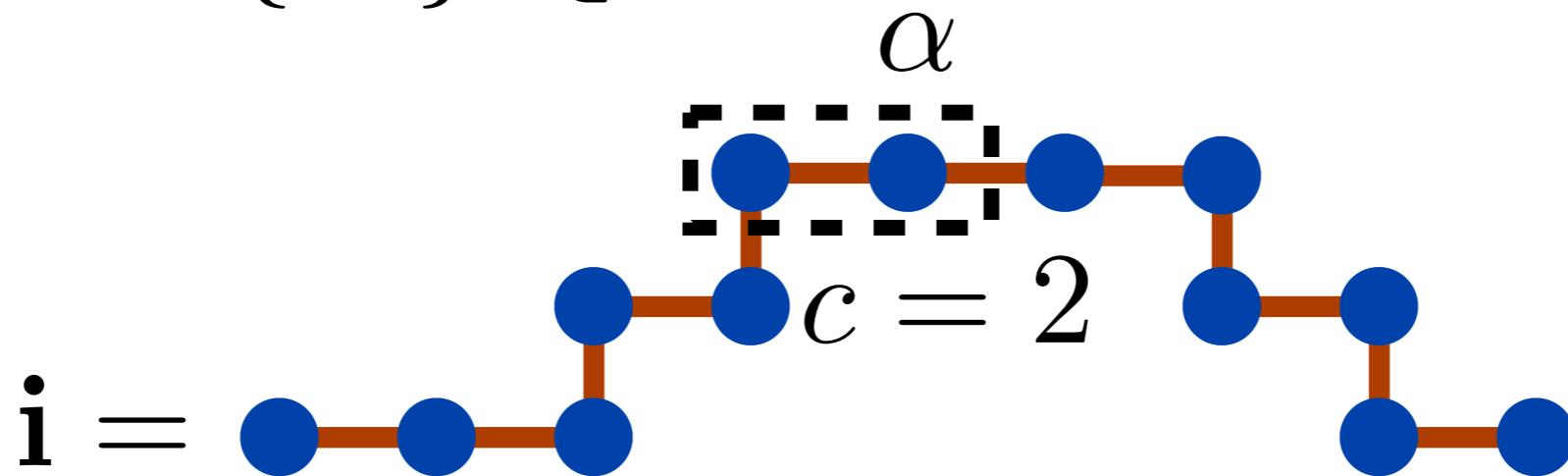


$$B_{\mathbf{i}} = \left[\prod_{\alpha \in F} q(i_\alpha) \right] \phi(\mathbf{i})$$

STRUCTURE FACTORIZATION

KULESZA AND TASKAR (NIPS 2010)

$$\mathbf{i} = \{i_\alpha\}_{\alpha \in F}$$

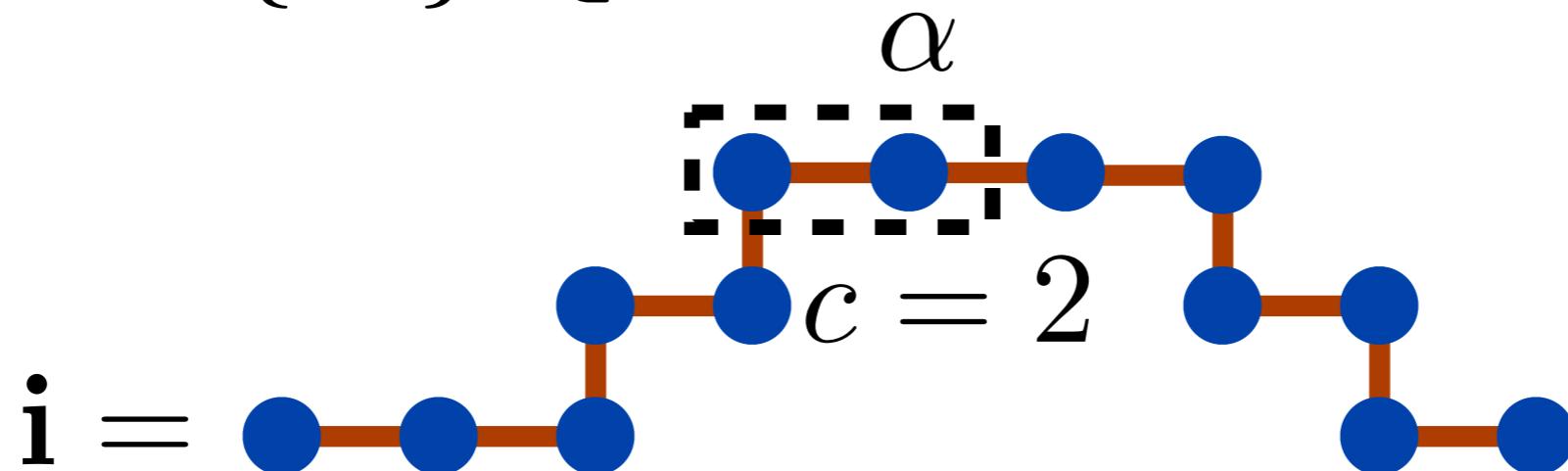


$$B_{\mathbf{i}} = \left[\prod_{\alpha \in F} q(i_\alpha) \right] \left[\sum_{\alpha \in F} \phi(i_\alpha) \right]$$

STRUCTURE FACTORIZATION

KULESZA AND TASKAR (NIPS 2010)

$$\mathbf{i} = \{i_\alpha\}_{\alpha \in F}$$

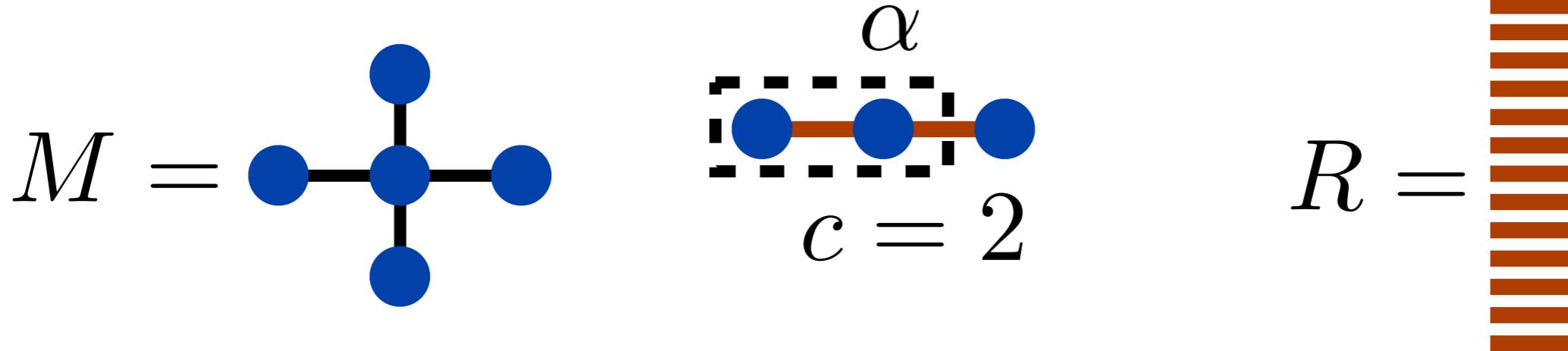


$$B_{\mathbf{i}} = \left[\prod_{\alpha \in F} q(i_\alpha) \right] \left[\sum_{\alpha \in F} \phi(i_\alpha) \right]$$

$$\mathbf{Y} \sim \mathcal{P}_L \quad O(\textcolor{blue}{N} \textcolor{teal}{D}^2 \textcolor{green}{k})$$

STRUCTURE FACTORIZATION

KULESZA AND TASKAR (NIPS 2010)

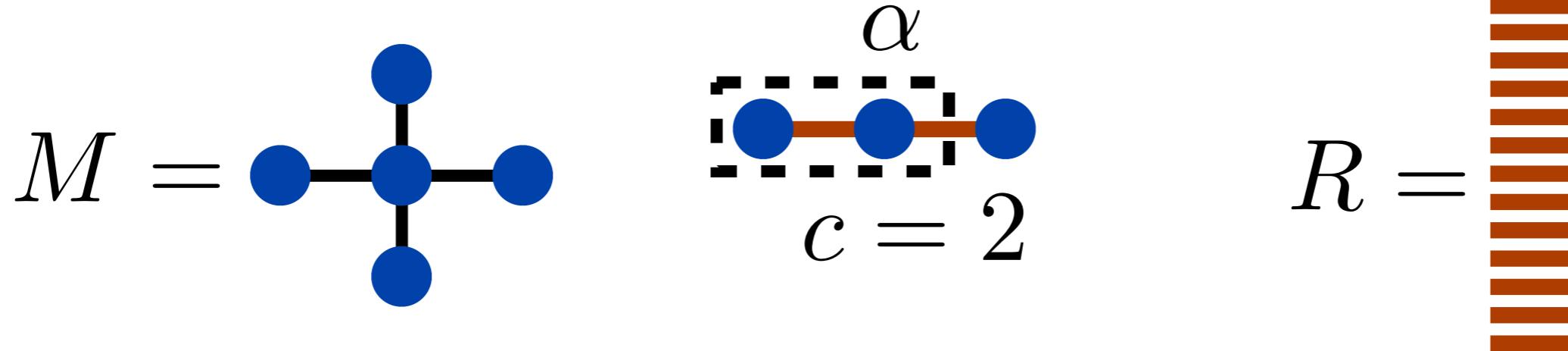


$$B_{\mathbf{i}} = \left[\prod_{\alpha \in F} q(i_\alpha) \right] \left[\sum_{\alpha \in F} \phi(i_\alpha) \right]$$

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STRUCTURE FACTORIZATION

KULESZA AND TASKAR (NIPS 2010)



$$B_{\mathbf{i}} = \left[\prod_{\alpha \in F} q(i_\alpha) \right] \left[\sum_{\alpha \in F} \phi(i_\alpha) \right]$$

$$\mathbf{Y} \sim \mathcal{P}_L \quad O(\textcolor{blue}{D}^2 \textcolor{green}{k}^3 + \textcolor{blue}{D} \textcolor{green}{k}^2 M^c R)$$

$$M^c R = 4^2 * 12 = 192 \ll N = 4^{12} = 16,777,216$$

LARGE FEATURE SETS

LARGE FEATURE SETS

$N = \# \text{ of items}$

$D = \# \text{ of features}$

	Large	Exponential
Small	dual	dual + structure

LARGE FEATURE SETS

$N = \# \text{ of items}$

$D = \# \text{ of features}$	Small	Large
Large	?	?
Large	dual	dual + structure
Large	Exponential	

RANDOM PROJECTIONS

GILLENWATER, KULESZA, AND TASKAR (EMNLP 2012)

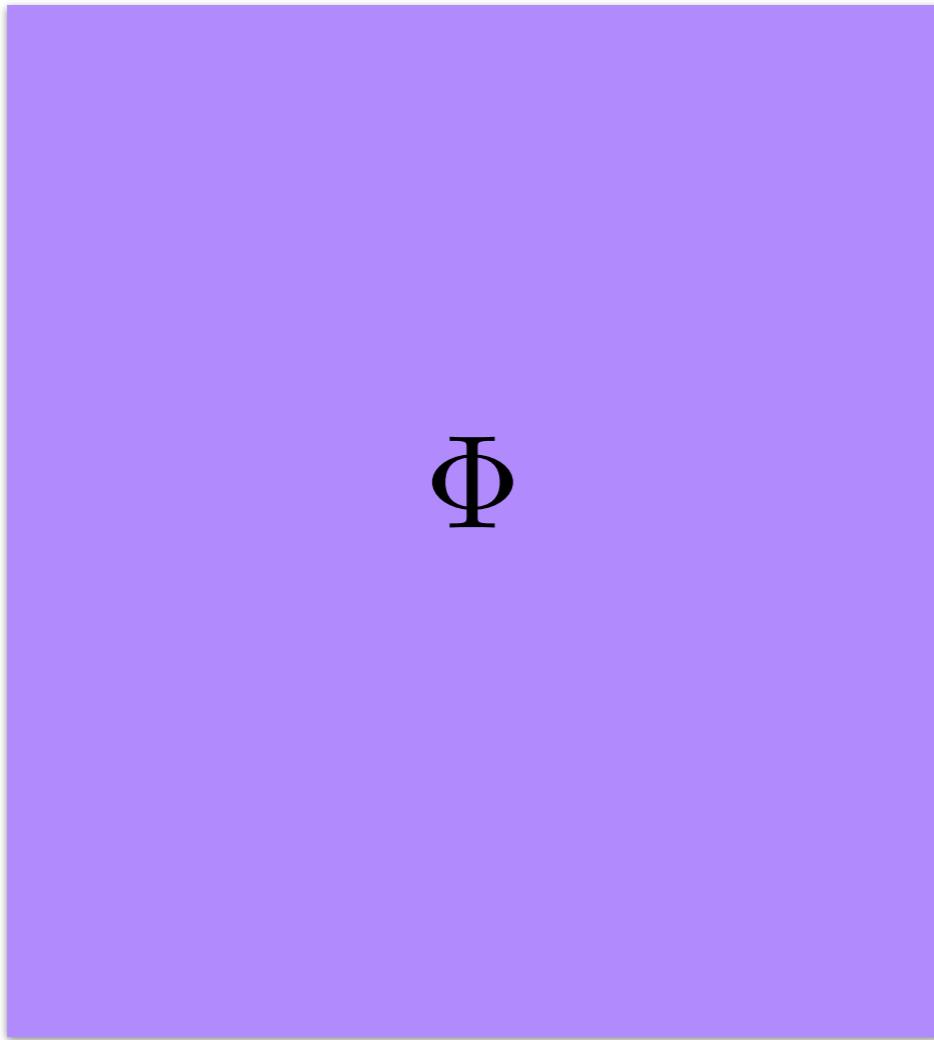
RANDOM PROJECTIONS

GILLENWATER, KULESZA, AND TASKAR (EMNLP 2012)

D

N

Φ



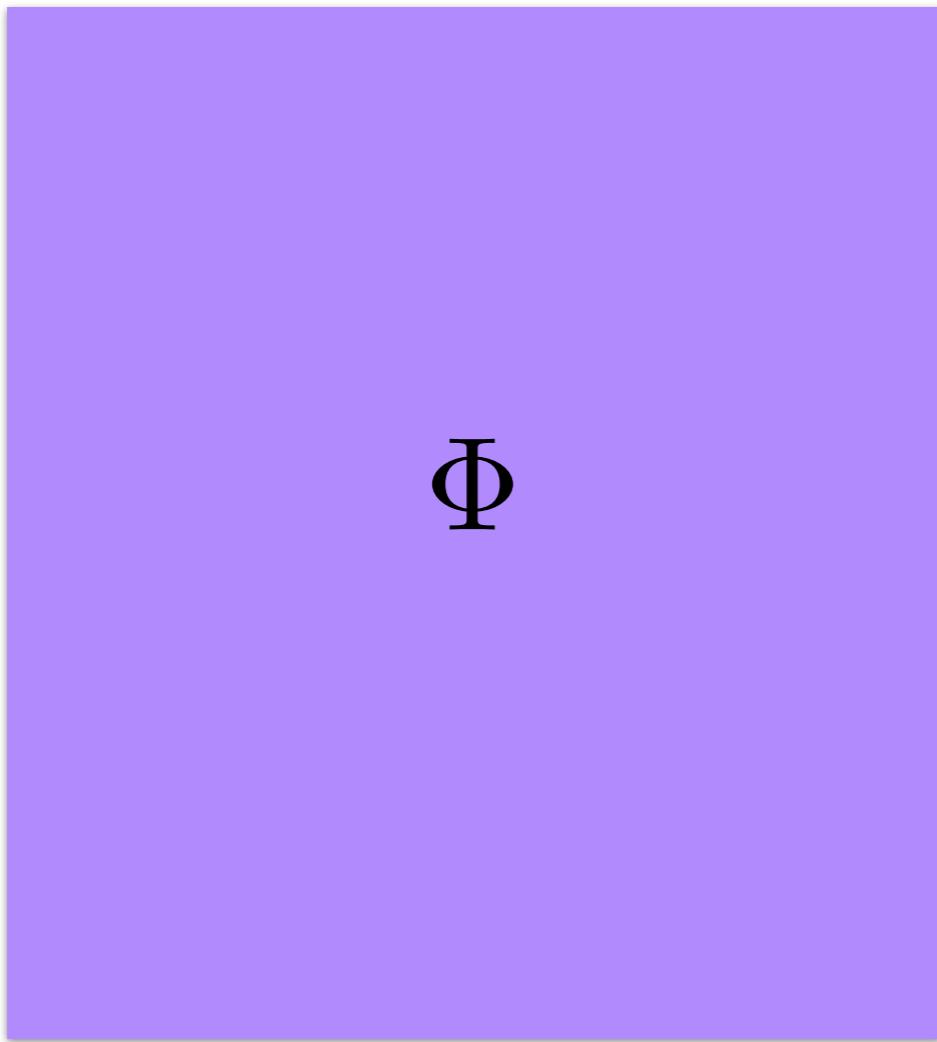
RANDOM PROJECTIONS

GILLENWATER, KULESZA, AND TASKAR (EMNLP 2012)

D

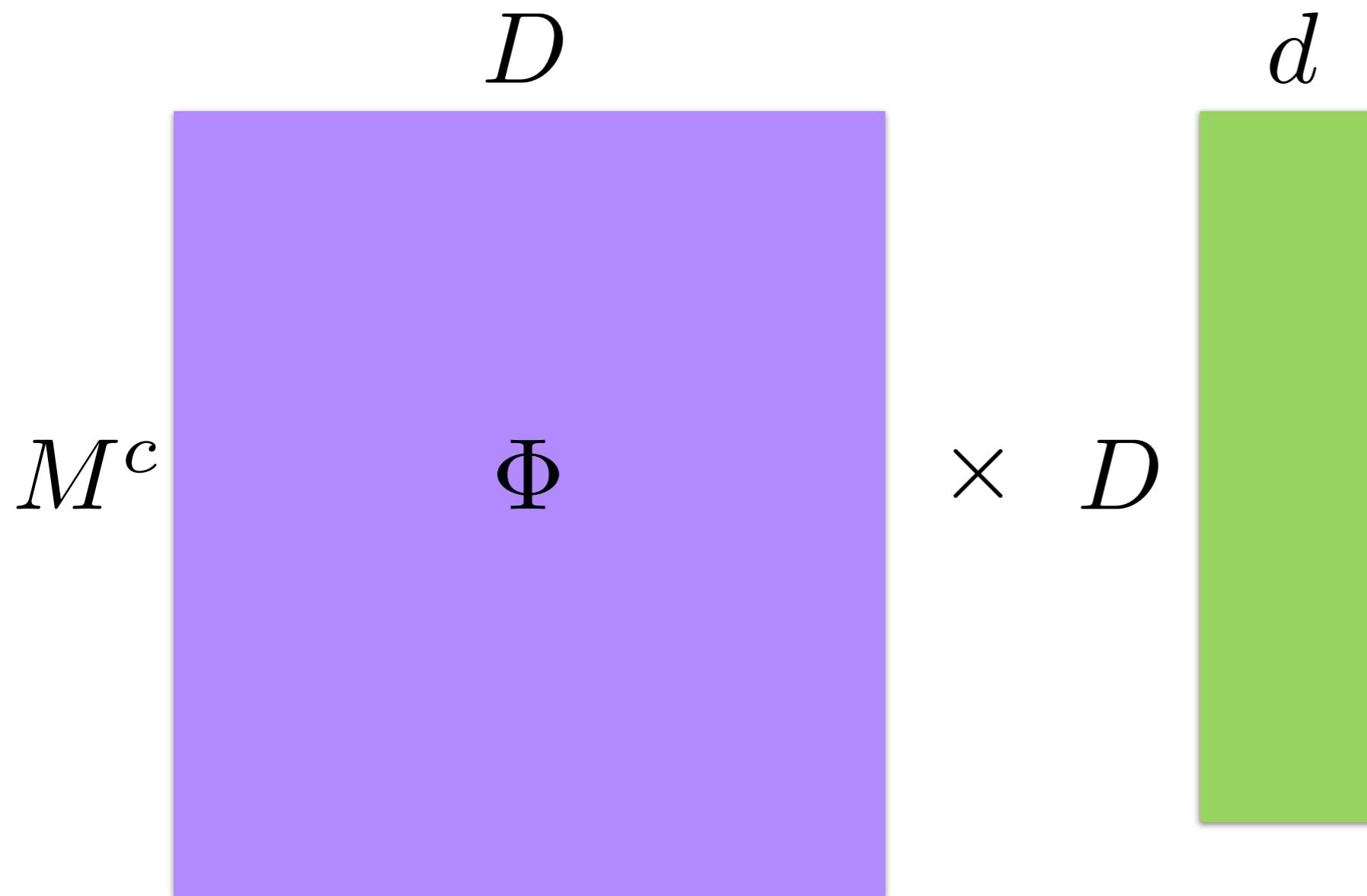
M^c

Φ



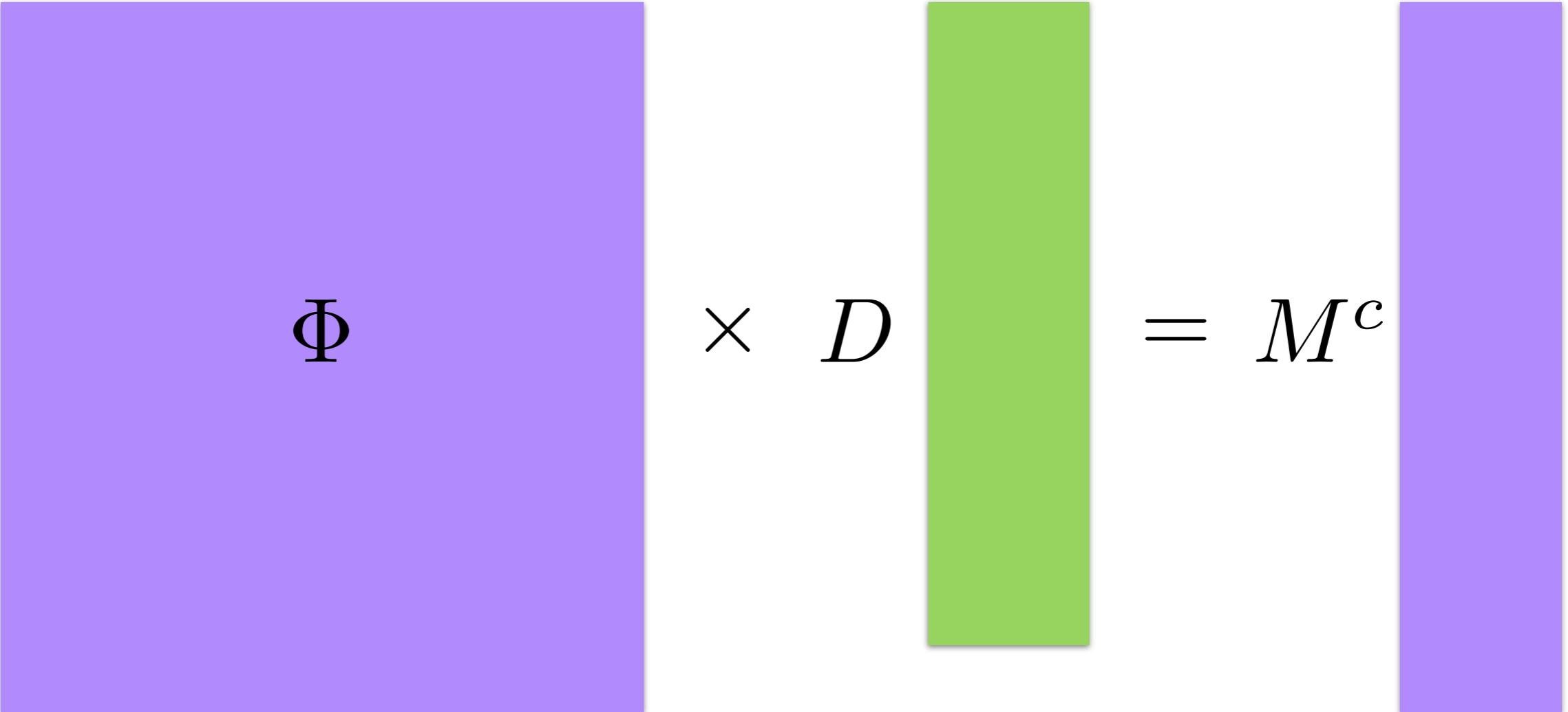
RANDOM PROJECTIONS

GILLENWATER, KULESZA, AND TASKAR (EMNLP 2012)



RANDOM PROJECTIONS

GILLENWATER, KULESZA, AND TASKAR (EMNLP 2012)

$$M^c \begin{matrix} D \\ \Phi \end{matrix} \times \begin{matrix} d \\ D \end{matrix} = M^c$$


RANDOM PROJECTIONS

GILLENWATER, KULESZA, AND TASKAR (EMNLP 2012)

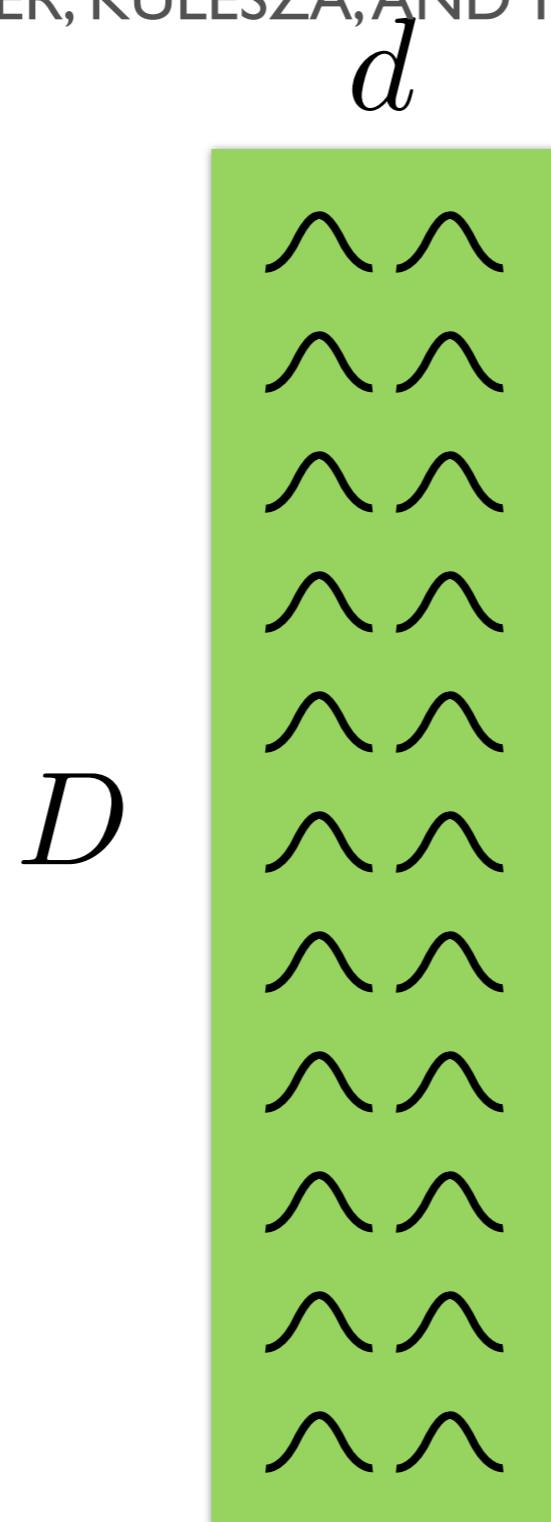
d

D



RANDOM PROJECTIONS

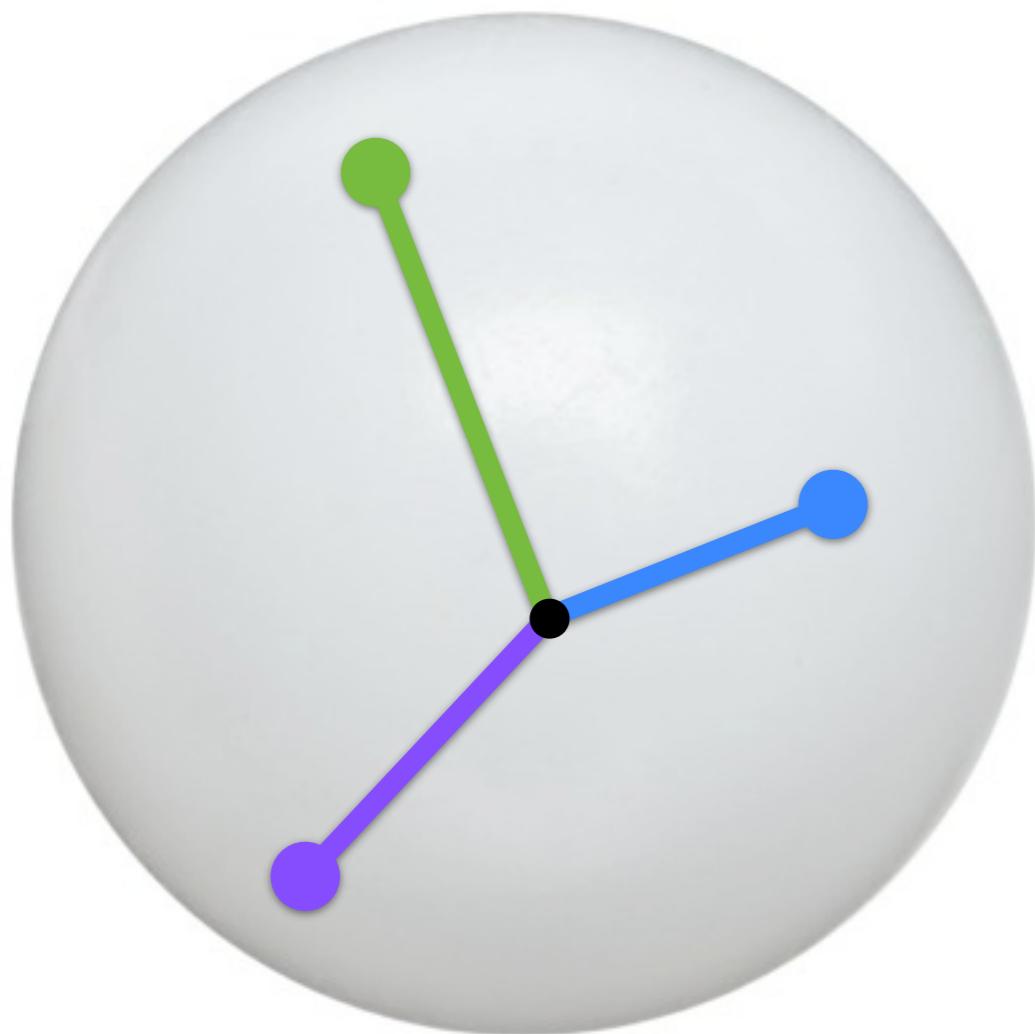
GILLENWATER, KULESZA, AND TASKAR (EMNLP 2012)



VOLUME PRESERVATION

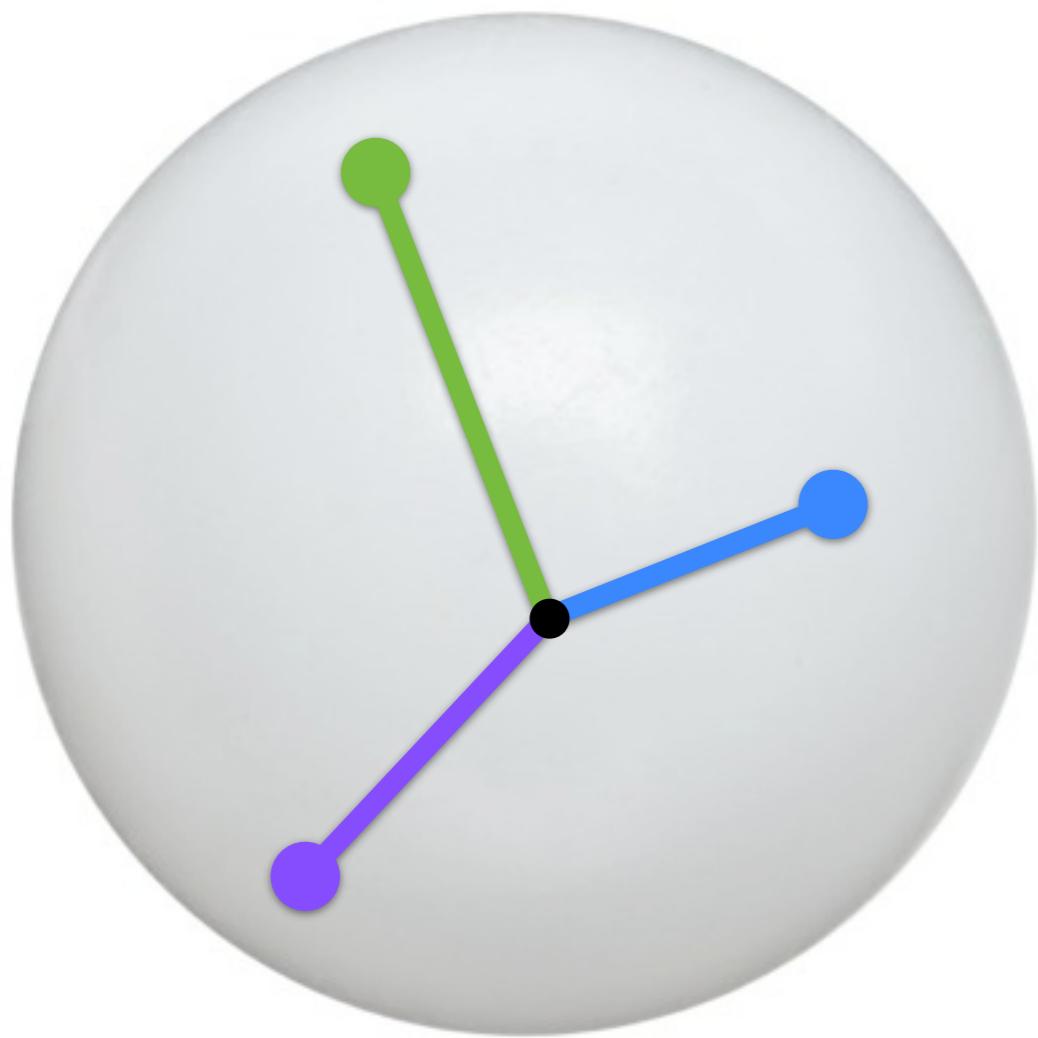
VOLUME PRESERVATION

JOHNSON AND LINDENSTRAUSS (1984)

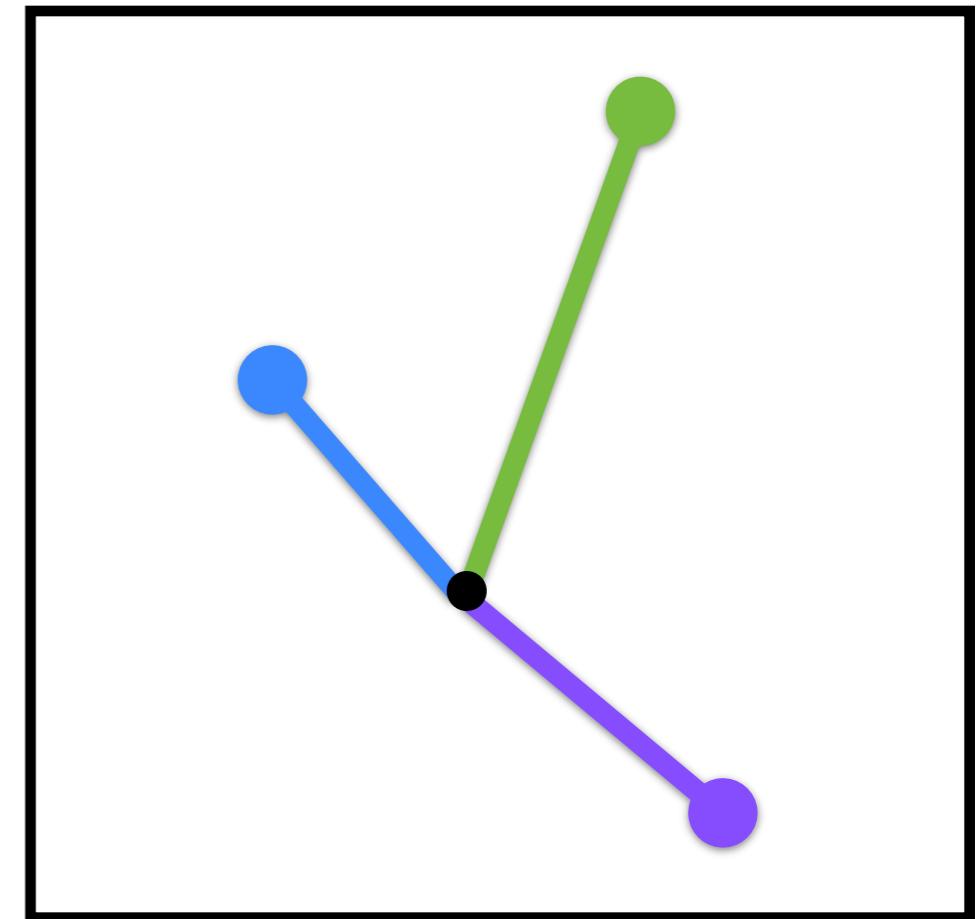


VOLUME PRESERVATION

JOHNSON AND LINDENSTRAUSS (1984)

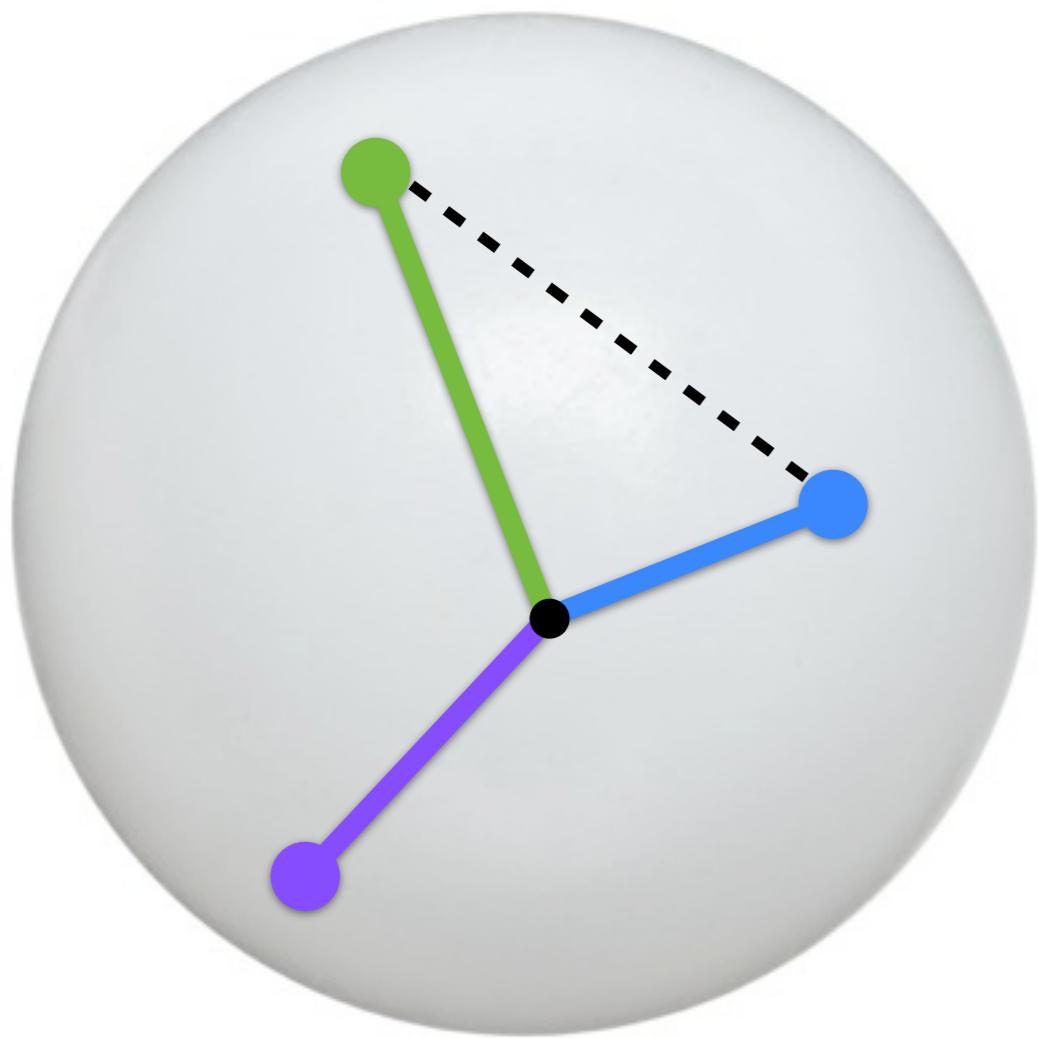


$\log N$

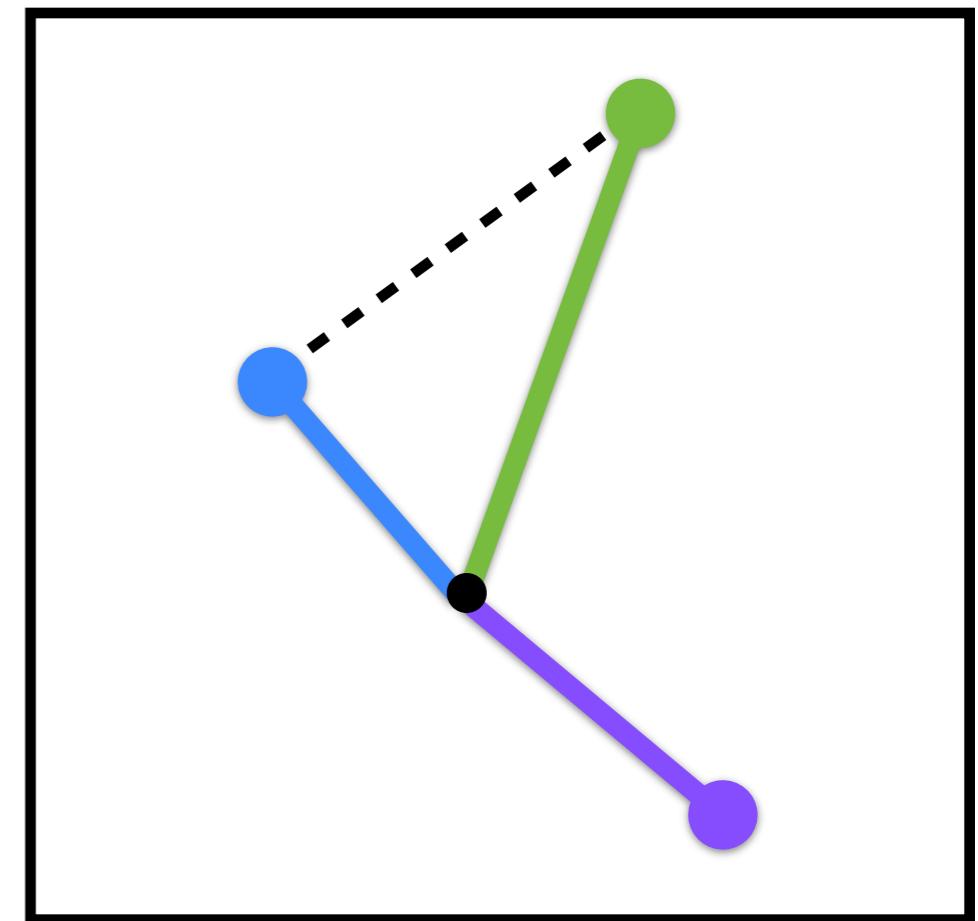


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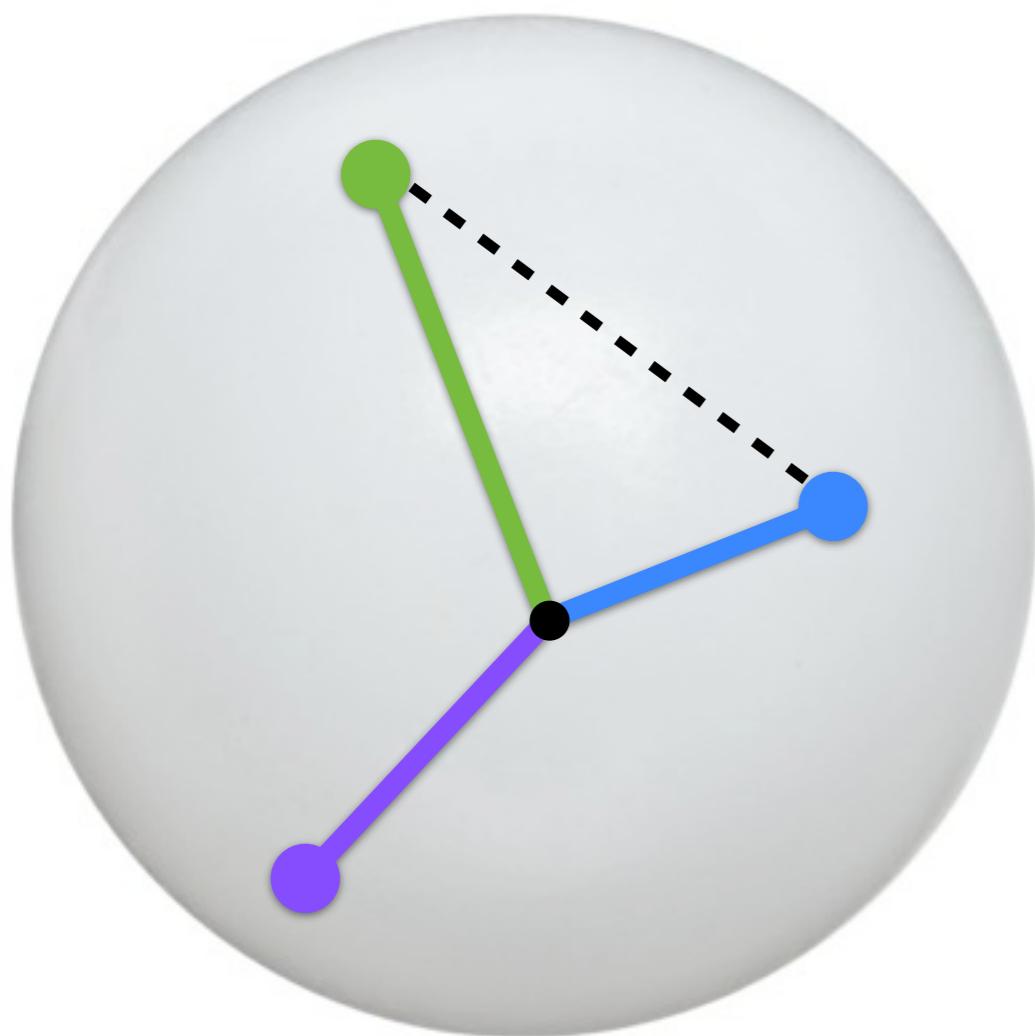


$\log N$

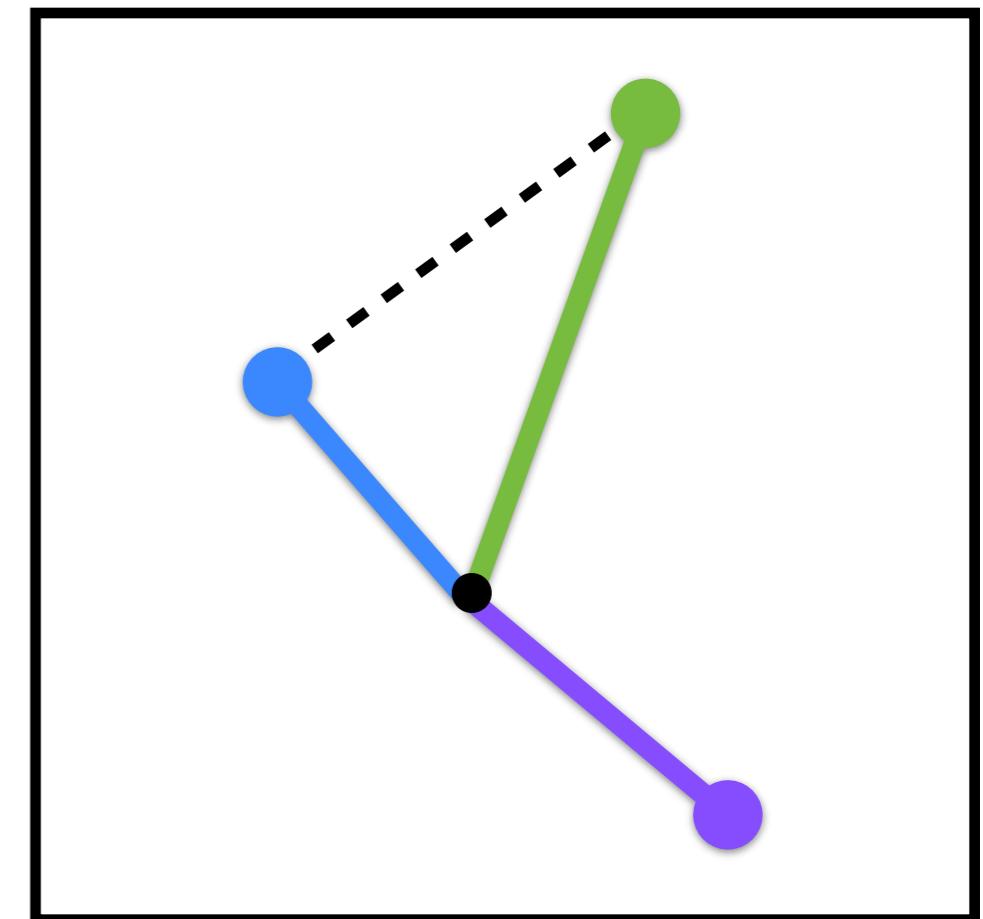


VOLUME PRESERVATION

MAGEN AND ZOUZIAS (2008)

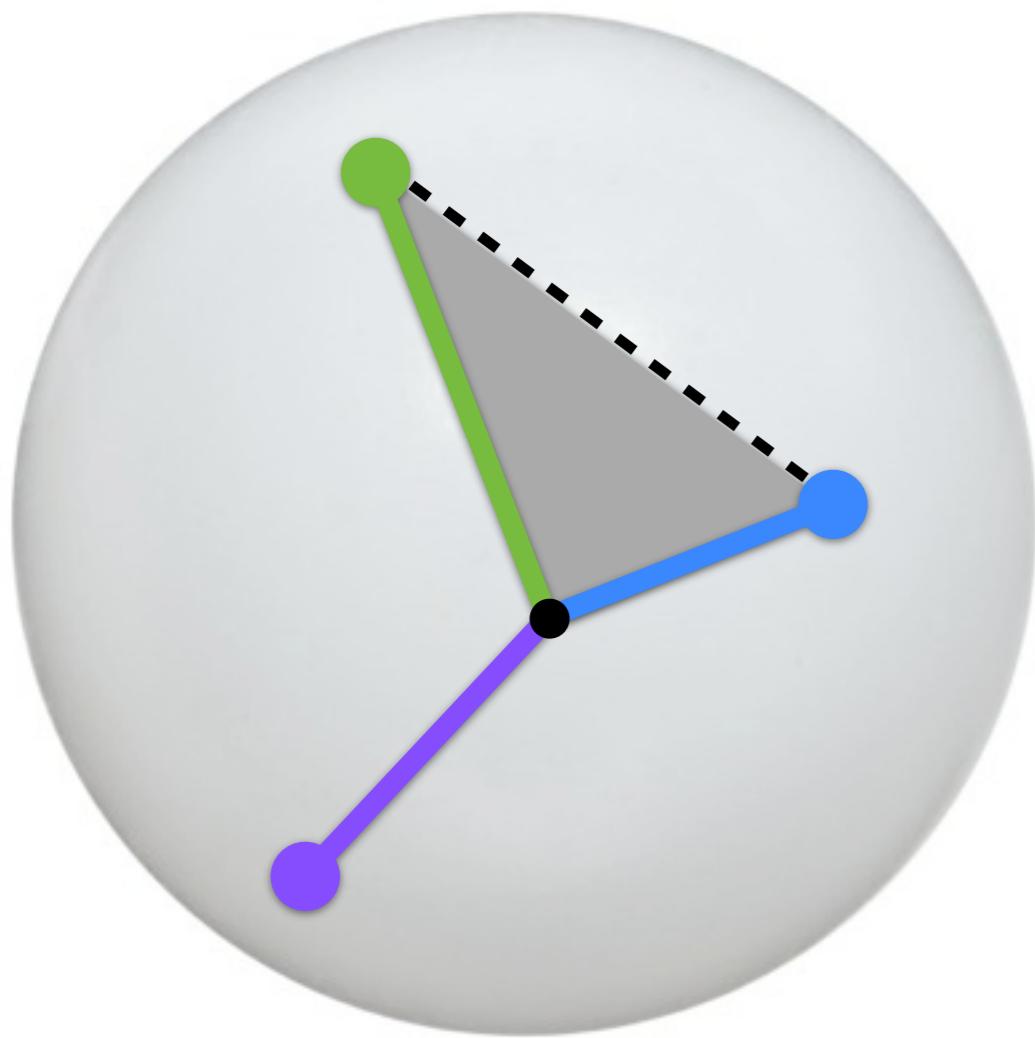


$\log N$

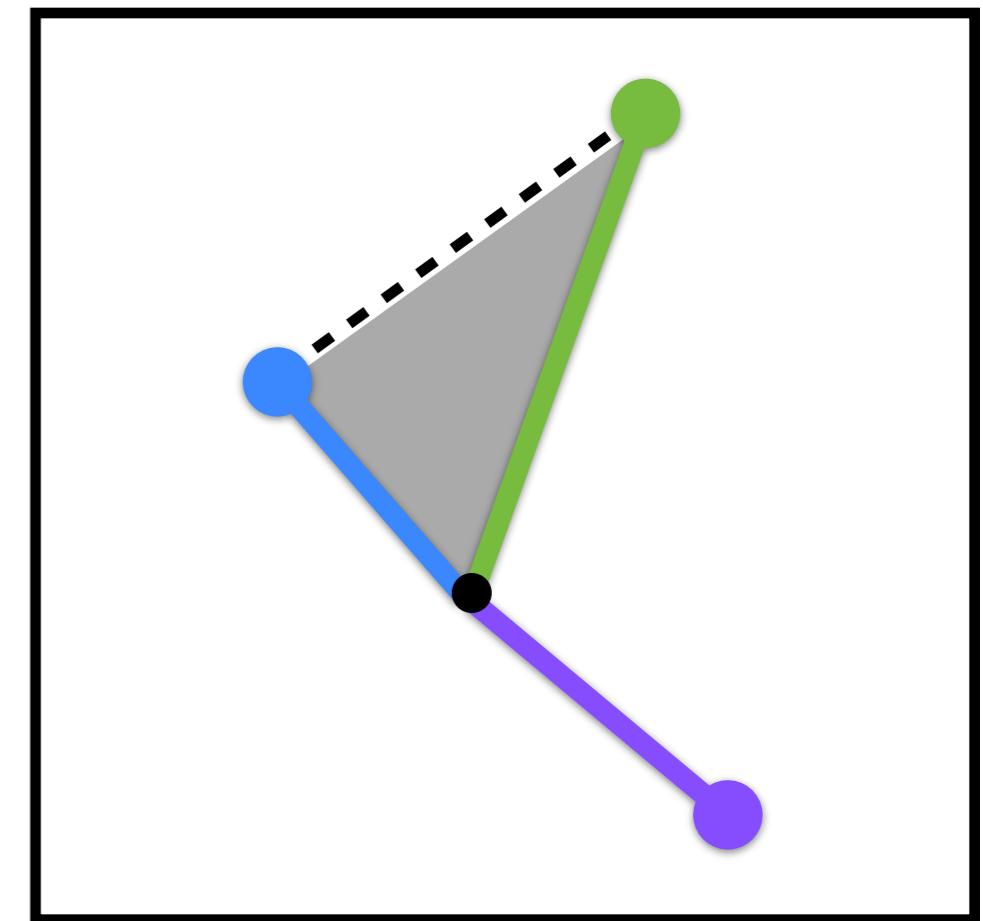


VOLUME PRESERVATION

MAGEN AND ZOUZIAS (2008)



$\log N$



DPP PRESERVATION

GILLENWATER, KULESZA, AND TASKAR (EMNLP 2012)

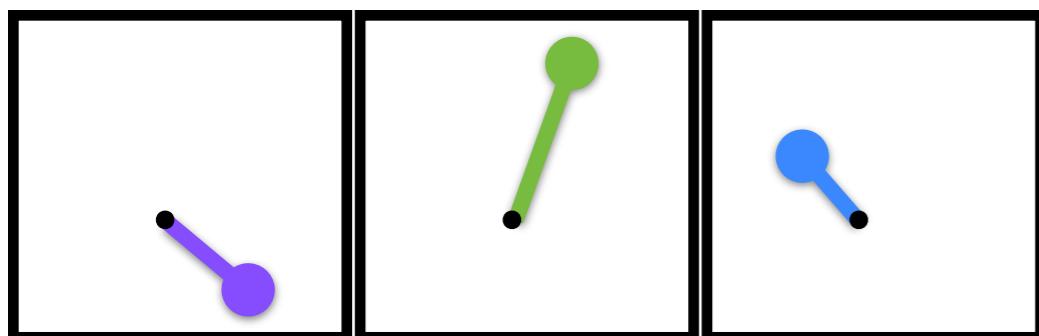
DPP PRESERVATION

GILLENWATER, KULESZA, AND TASKAR (EMNLP 2012)

$$k = 1$$



\approx



DPP PRESERVATION

GILLENWATER, KULESZA, AND TASKAR (EMNLP 2012)

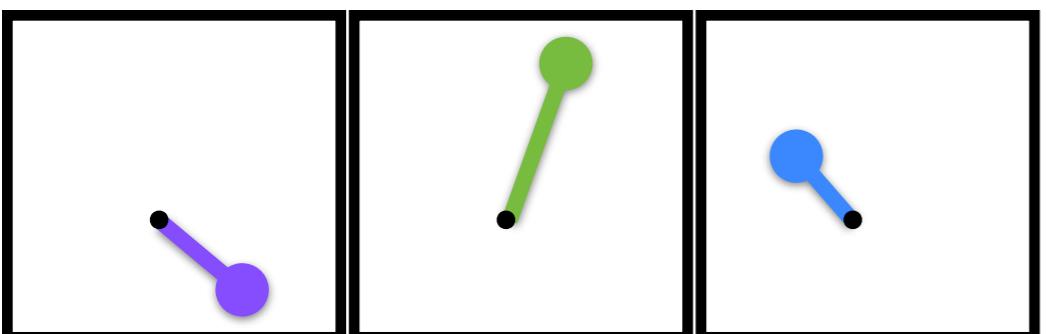
$$k = 1$$



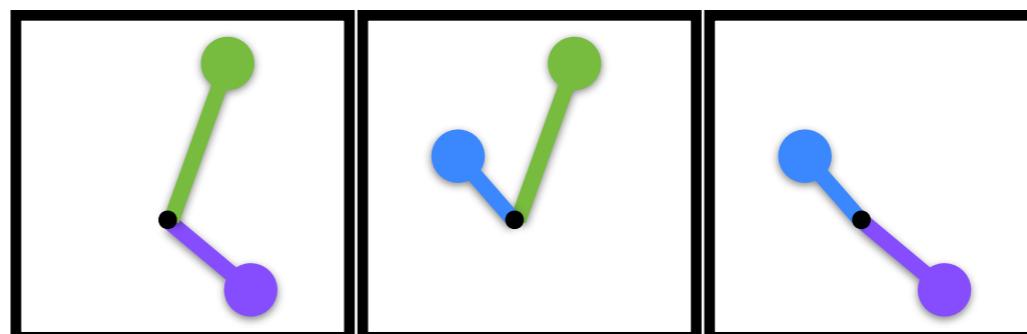
$$k = 2$$



\approx



\approx



DPP PRESERVATION

GILLENWATER, KULESZA, AND TASKAR (EMNLP 2012)

$k = 1$



$k = 2$



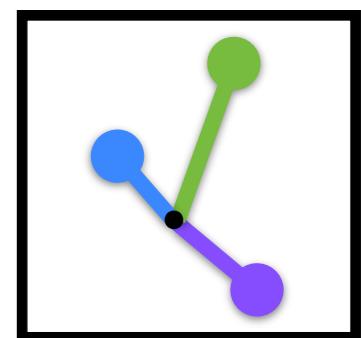
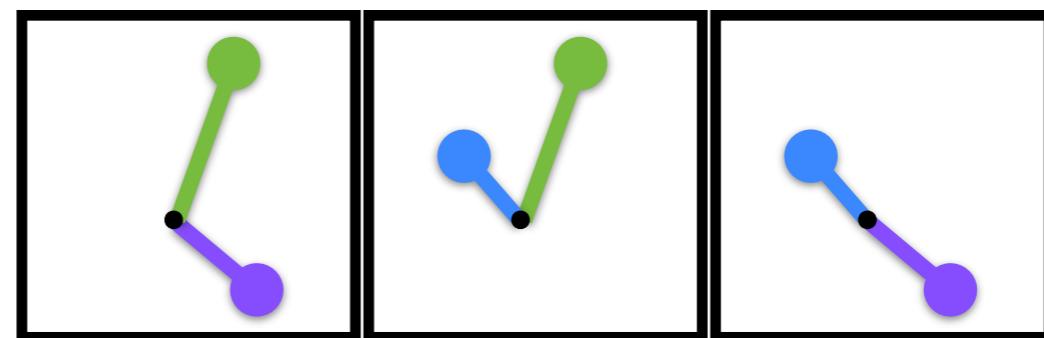
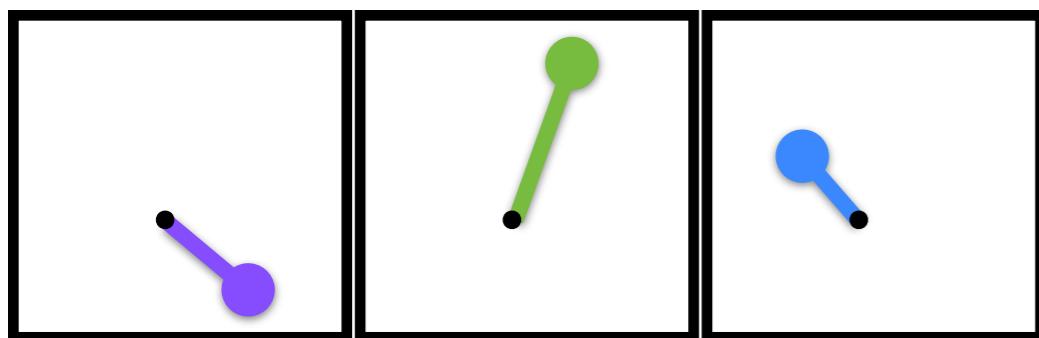
$k = 3$



\approx

\approx

\approx



DPP PRESERVATION

GILLENWATER, KULESZA, AND TASKAR (EMNLP 2012)

DPP PRESERVATION

GILLENWATER, KULESZA, AND TASKAR (EMNLP 2012)

$$d = O\left(\max\left\{\frac{k}{\epsilon}, \frac{\log(1/\delta) + \log(N)}{\epsilon^2} + k\right\}\right)$$

DPP PRESERVATION

GILLENWATER, KULESZA, AND TASKAR (EMNLP 2012)

$$d = O\left(\max\left\{\frac{k}{\epsilon}, \frac{\log(1/\delta) + \log(N)}{\epsilon^2} + k\right\}\right)$$

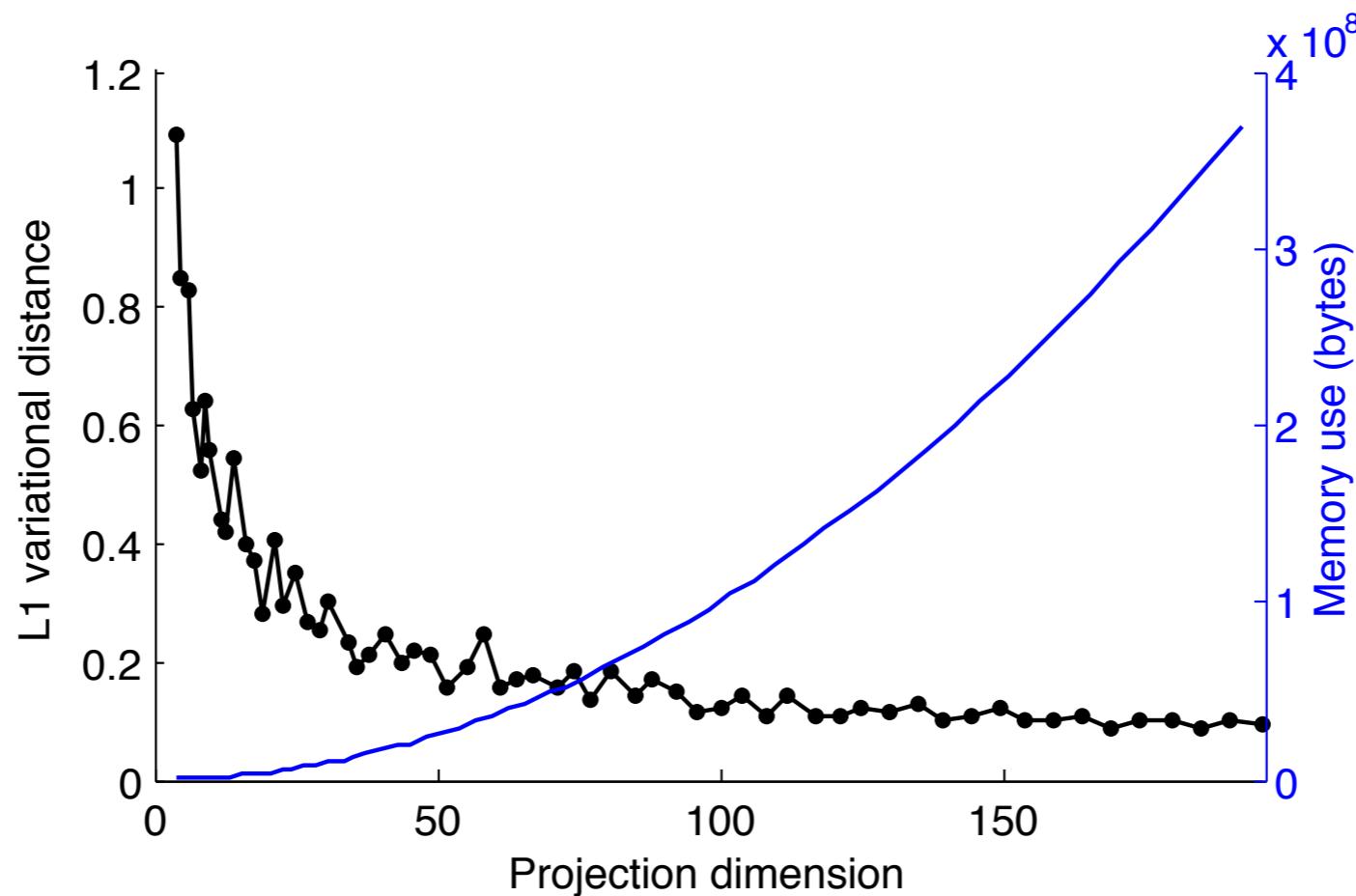
$$\text{w.p. } 1 - \delta : \|\mathcal{P}^k - \tilde{\mathcal{P}}^k\|_1 \leq e^{6k\epsilon} - 1$$

DPP PRESERVATION

GILLENWATER, KULESZA, AND TASKAR (EMNLP 2012)

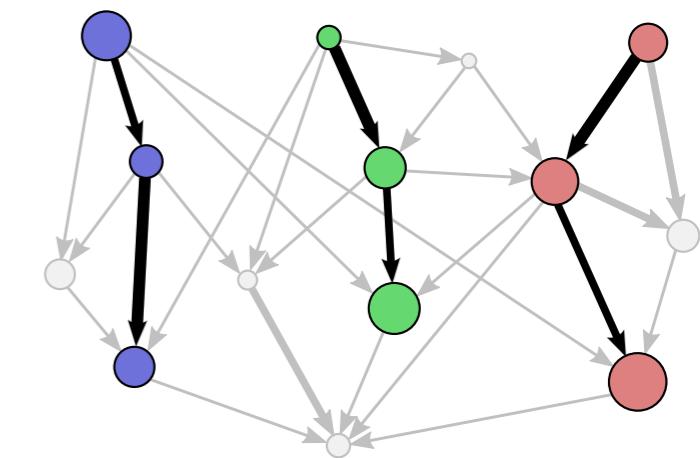
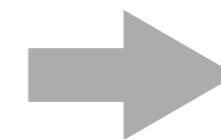
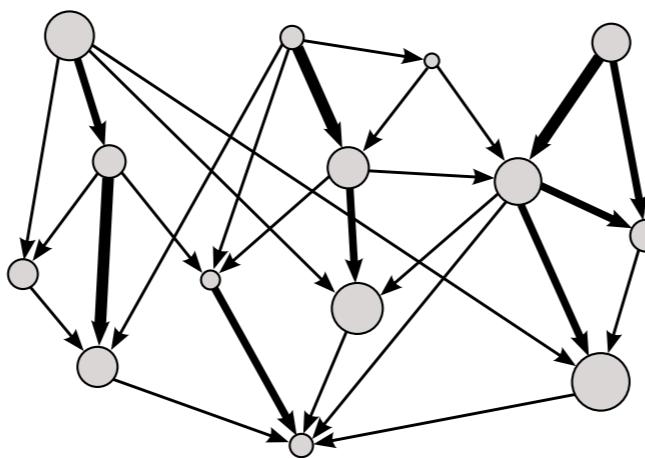
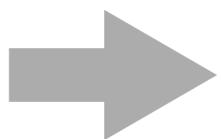
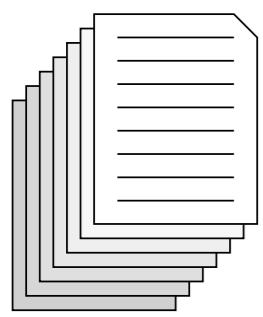
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w.p. $1 - \delta$: $\|\mathcal{P}^k - \tilde{\mathcal{P}}^k\|_1 \leq e^{6k\epsilon} - 1$

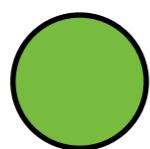
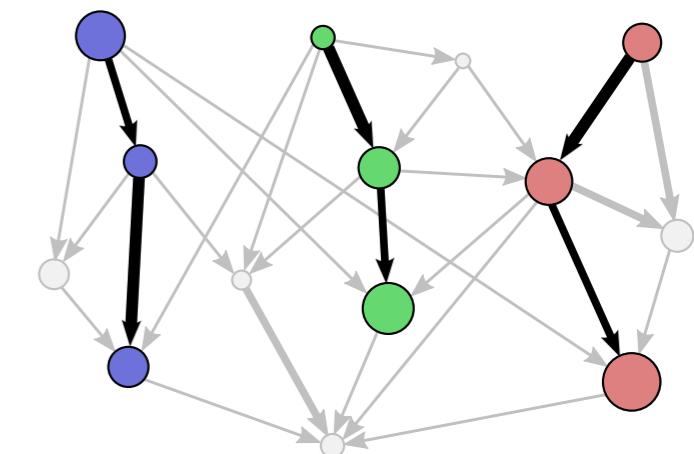
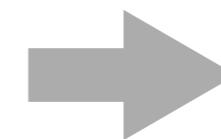
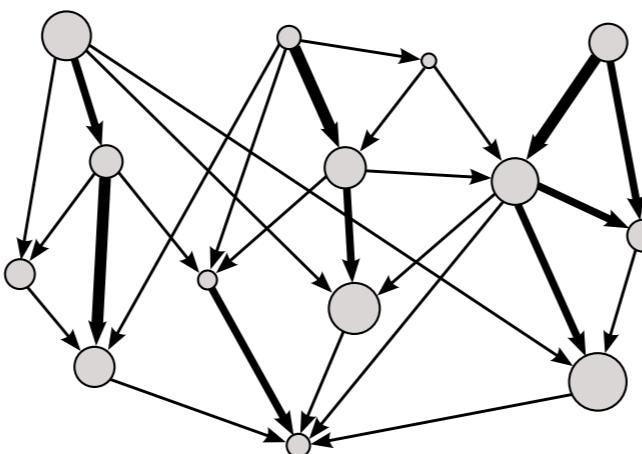
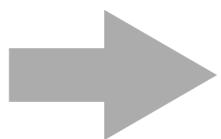
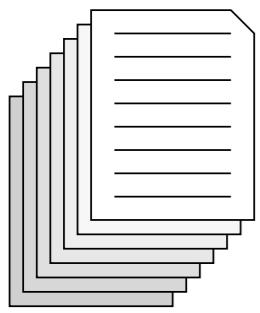


NEWS THREADING

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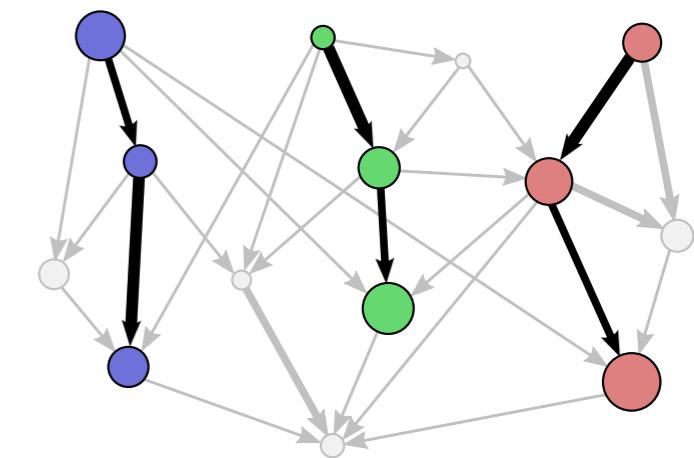
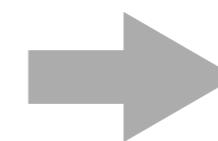
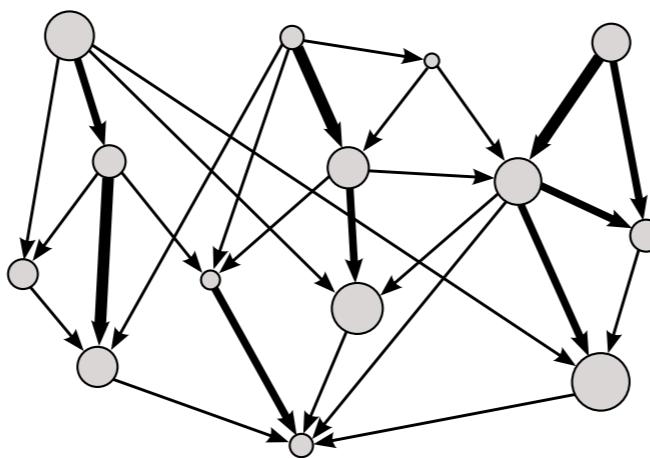
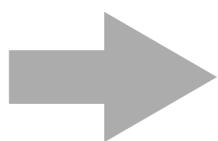
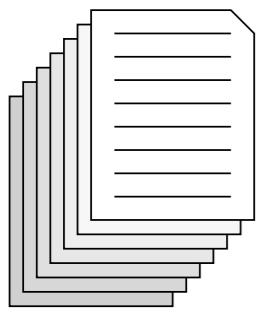


NEWS THREADING

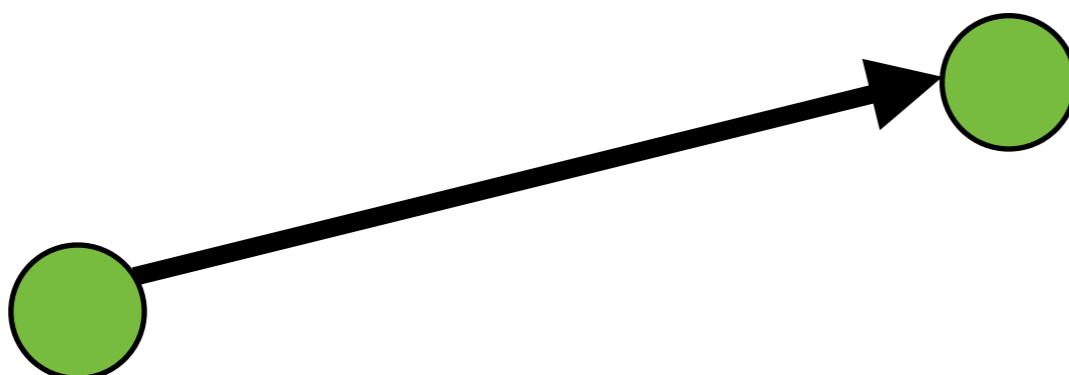


March 28: Health officials confirm
Ebola outbreak in Guinea's capital

NEWS THREADING

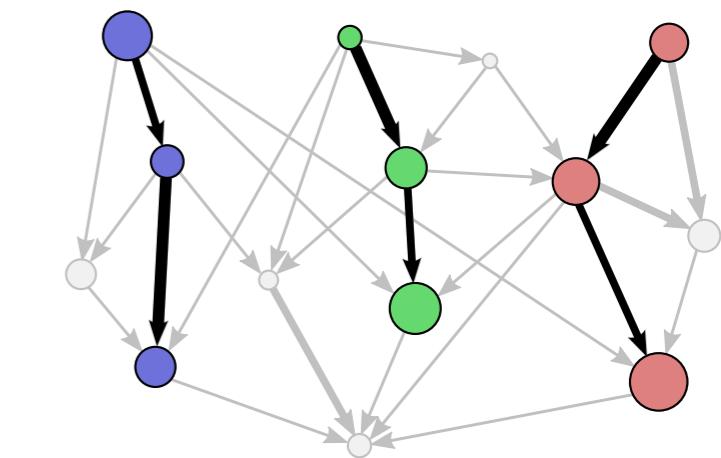
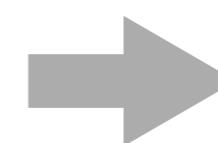
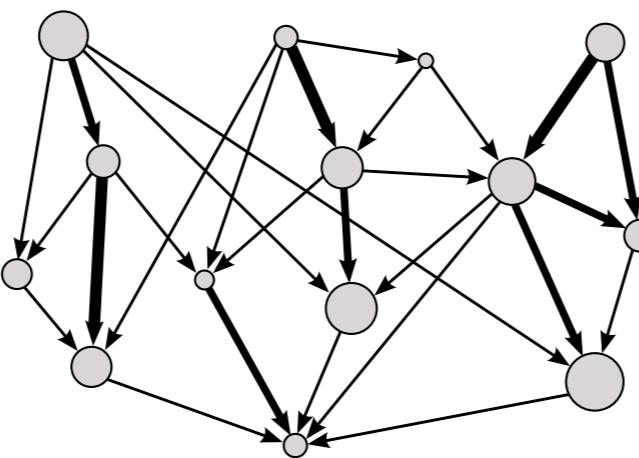
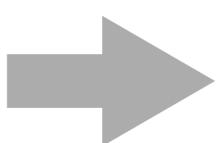
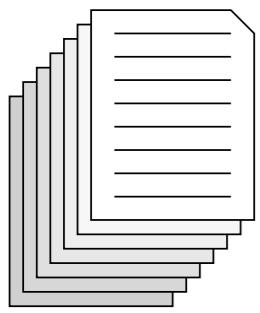


August 8: World Health Organization
declares Ebola epidemic an
international health emergency

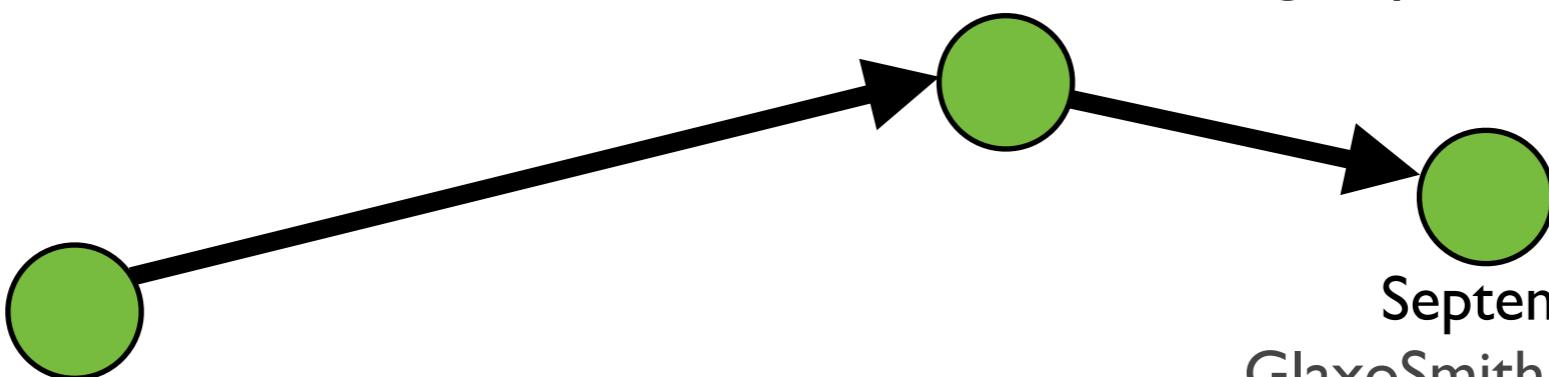


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NEWS THREADING



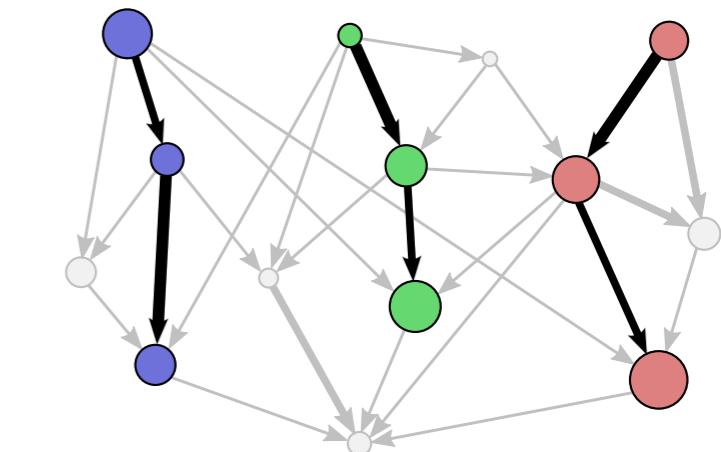
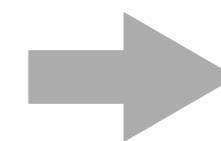
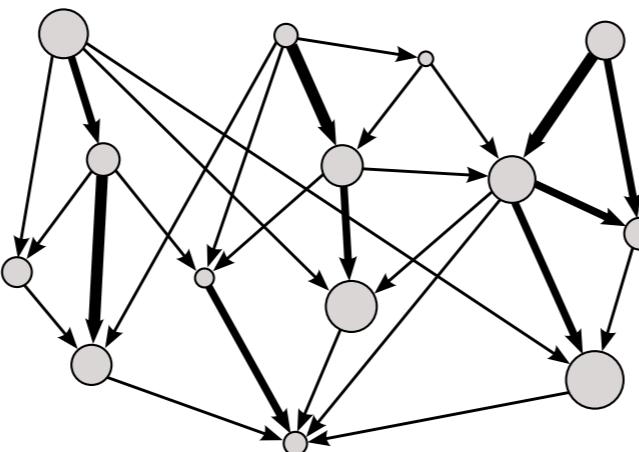
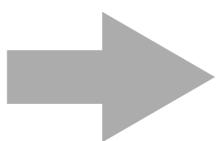
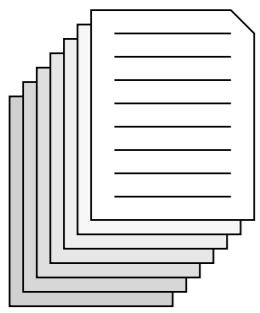
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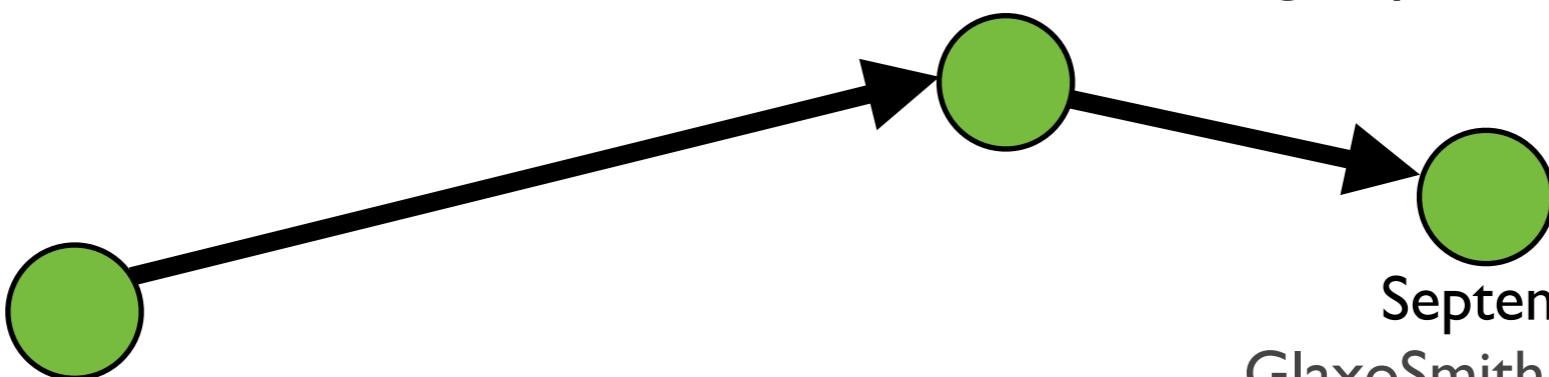
NEWS THREADING



$M \approx 35,000$

10^{360}

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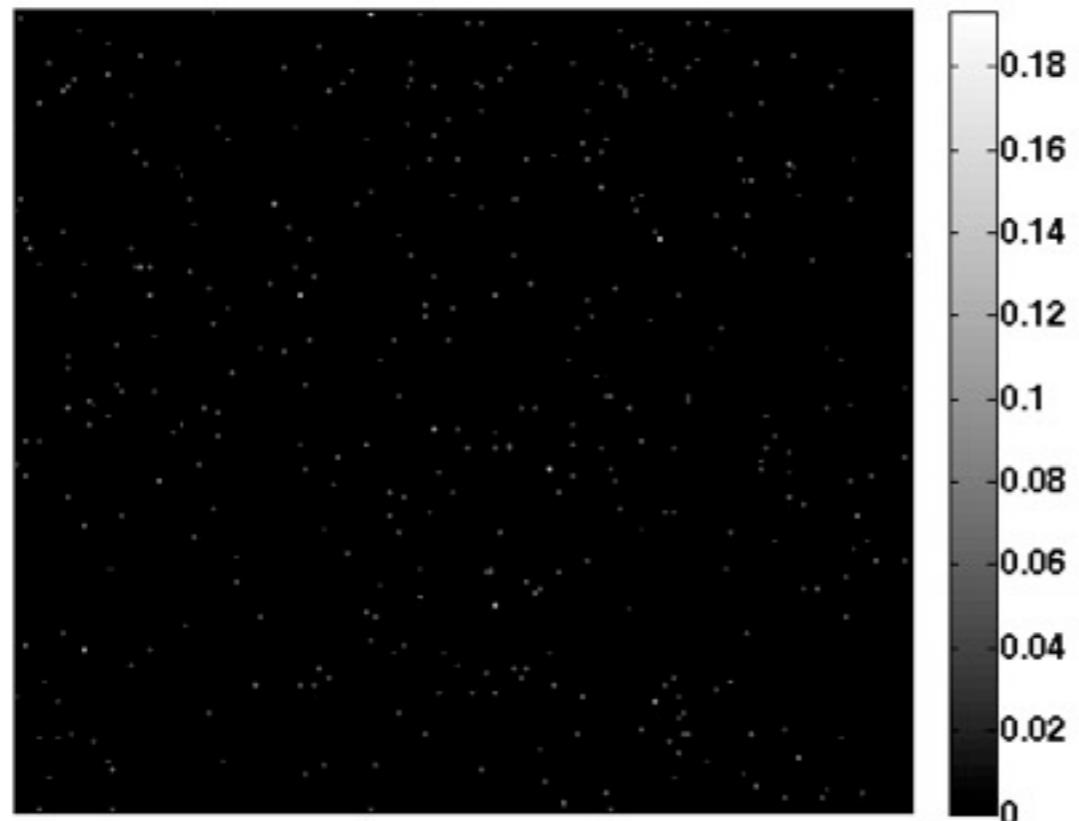
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PROJECTING NEWS FEATURES

PROJECTING NEWS FEATURES

$\phi(\mathbf{i})$



$D = 36,356$

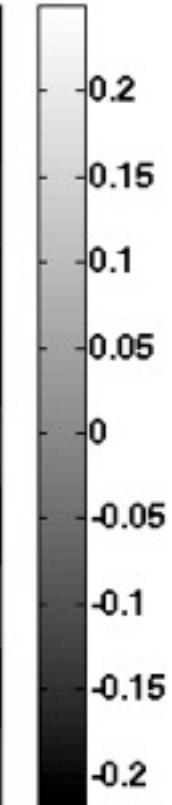
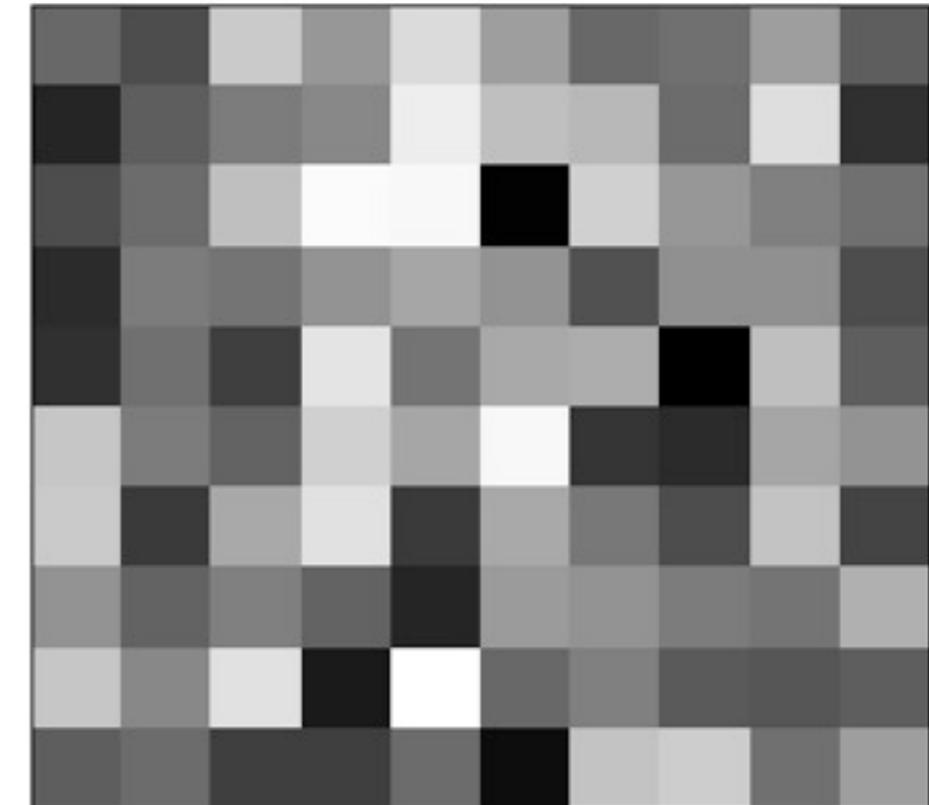
PROJECTING NEWS FEATURES

$\phi(\mathbf{i})$



$D = 36,356$

$G\phi(\mathbf{i})$

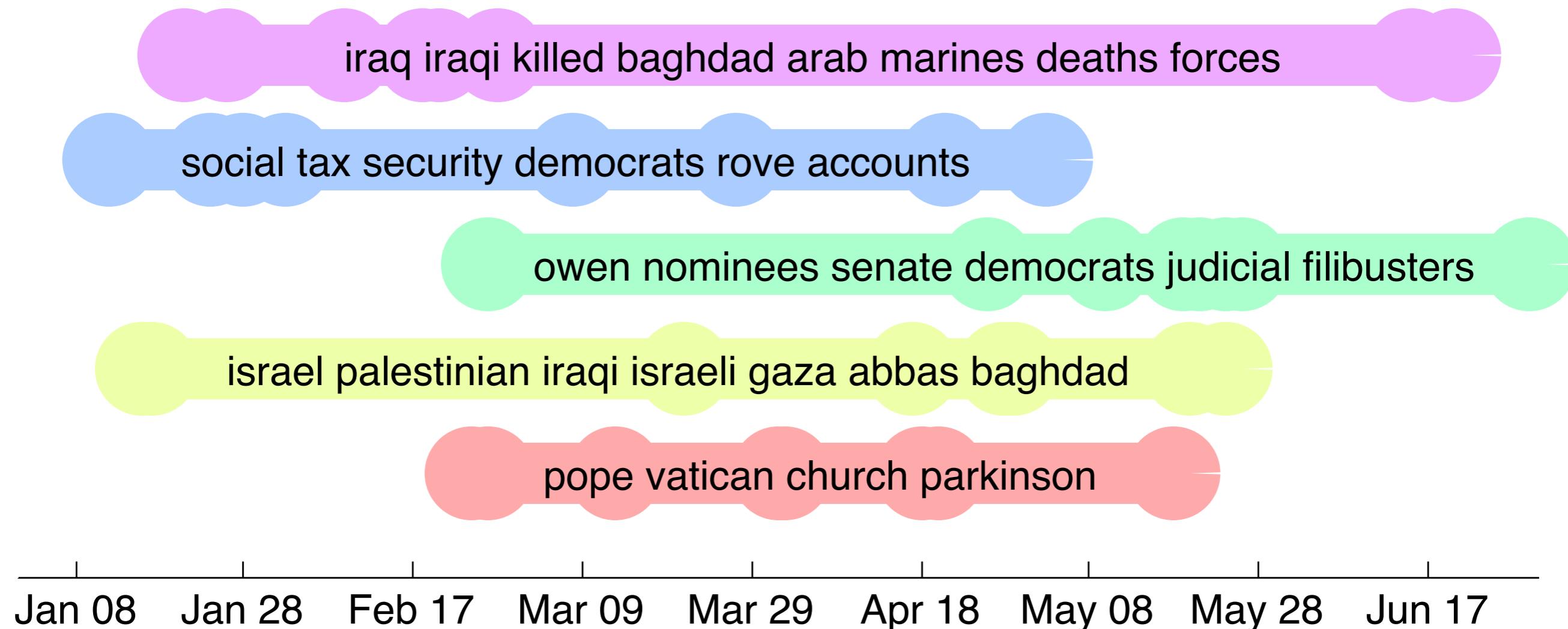


G
→

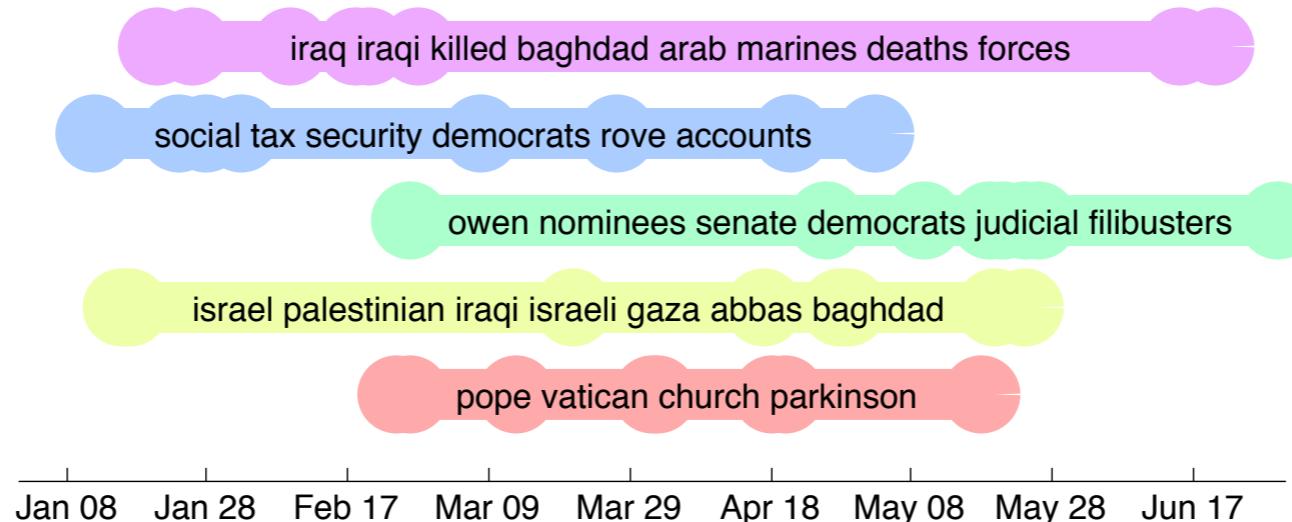
$d = 50$

DPP THREADS

DPP THREADS



DPP THREADS



Feb 24: Parkinson's Disease Increases Risks to Pope

Feb 26: Pope's Health Raises Questions About His Ability to Lead

Mar 13: Pope Returns Home After 18 Days at Hospital

Apr 01: Pope's Condition Worsens as World Prepares for End of Papacy

Apr 02: Pope, Though Gravely Ill, Utters Thanks for Prayers

Apr 18: Europeans Fast Falling Away from Church

Apr 20: In Developing World, Choice [of Pope] Met with Skepticism

May 18: Pope Sends Message with Choice of Name

PROPOSED WORK

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- Survey algorithms for large-scale eigendecomps

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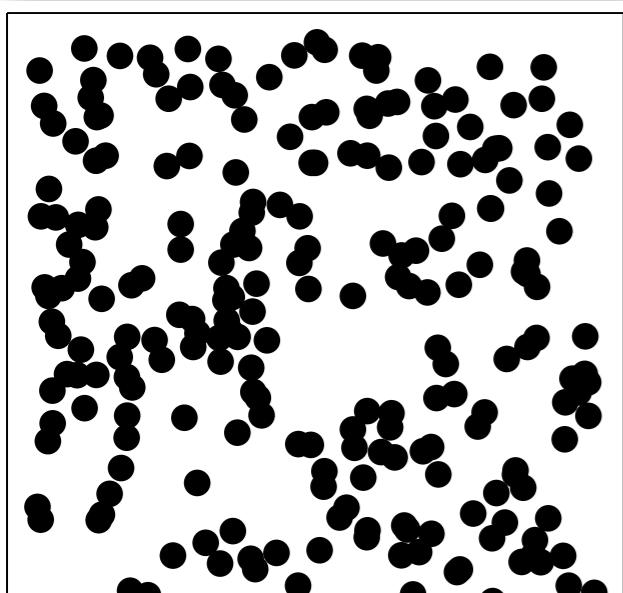
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2. MAP ESTIMATION

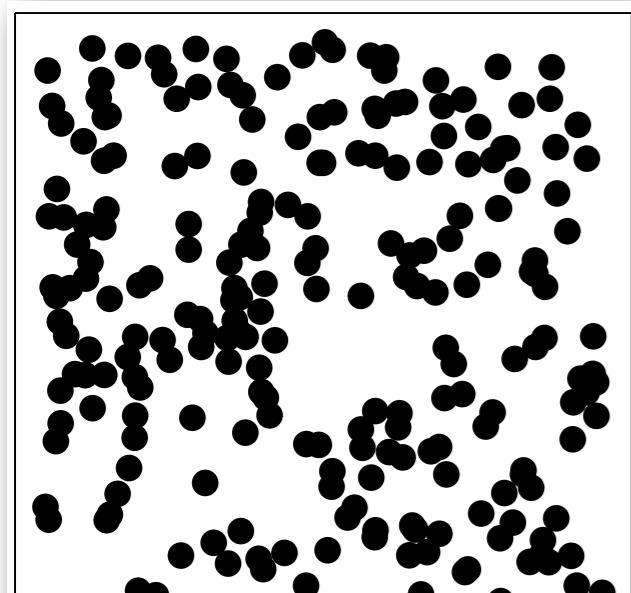
HARDNESS OF MAP

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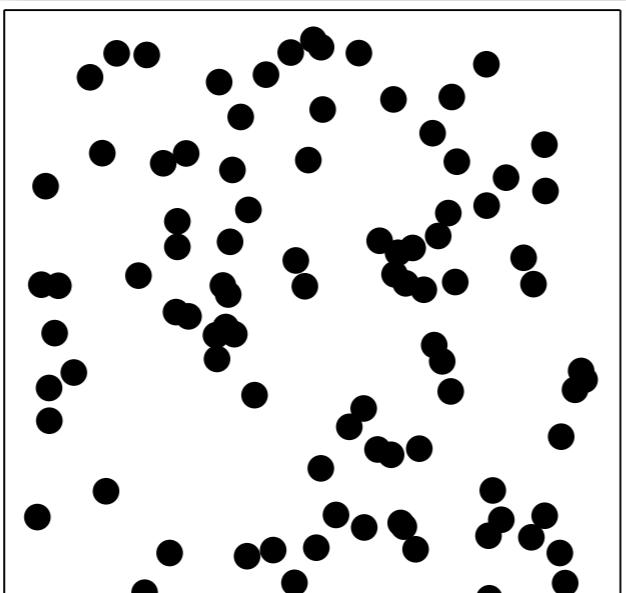


All points

HARDNESS OF MAP

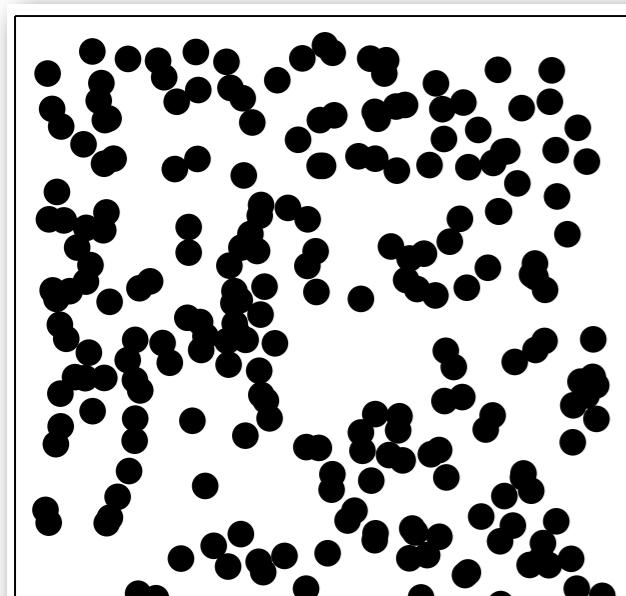


All points

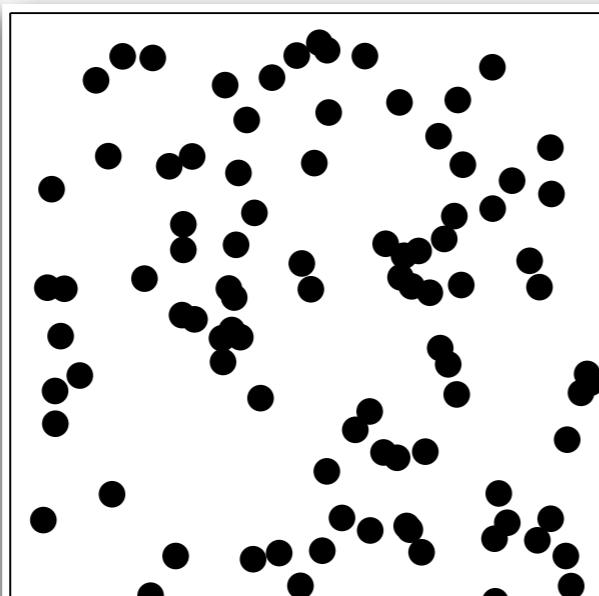


Independent sample

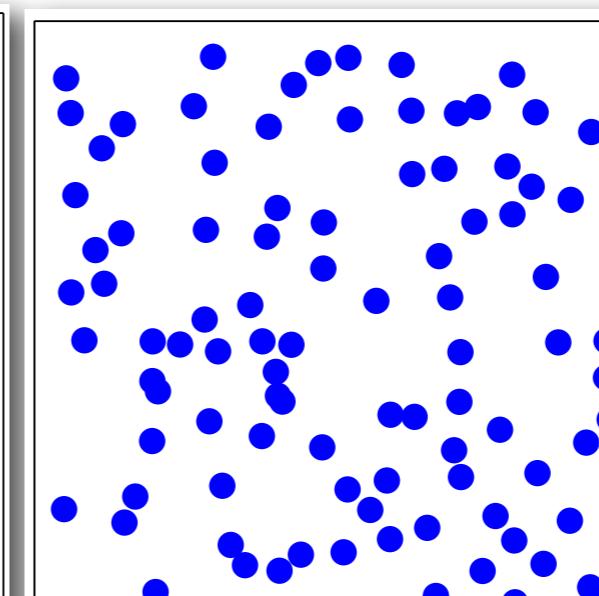
HARDNESS OF MAP



All points

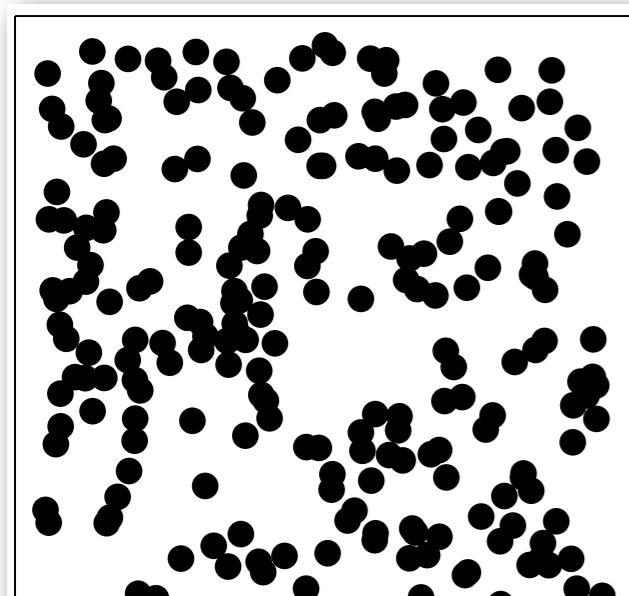


Independent sample

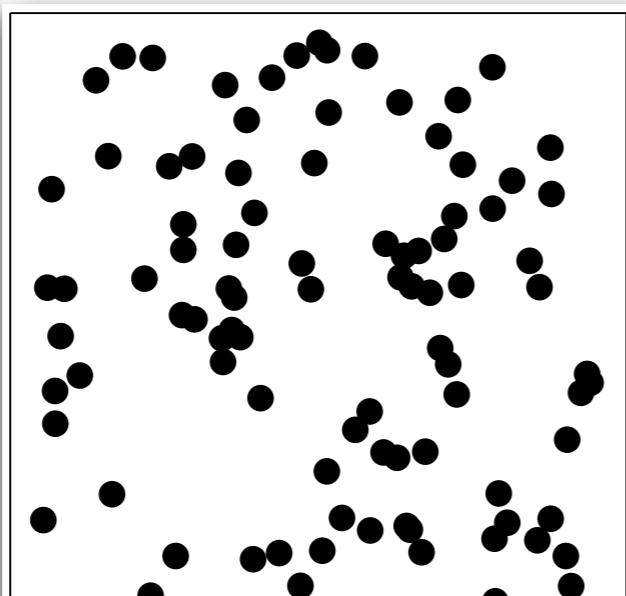


DPP sample

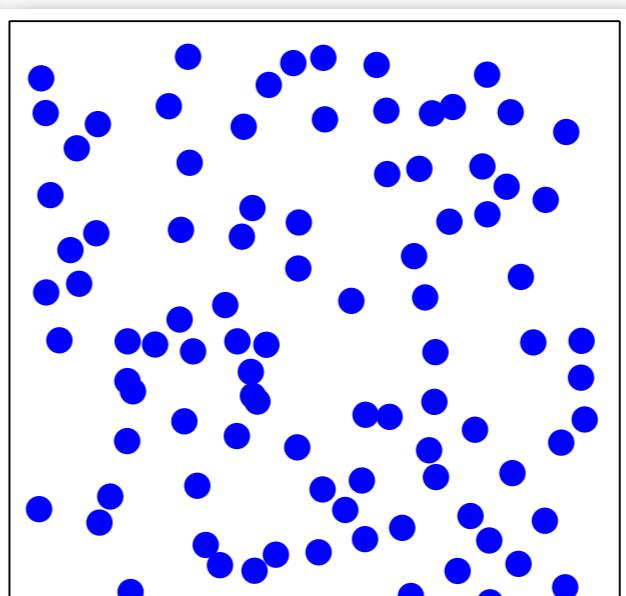
HARDNESS OF MAP



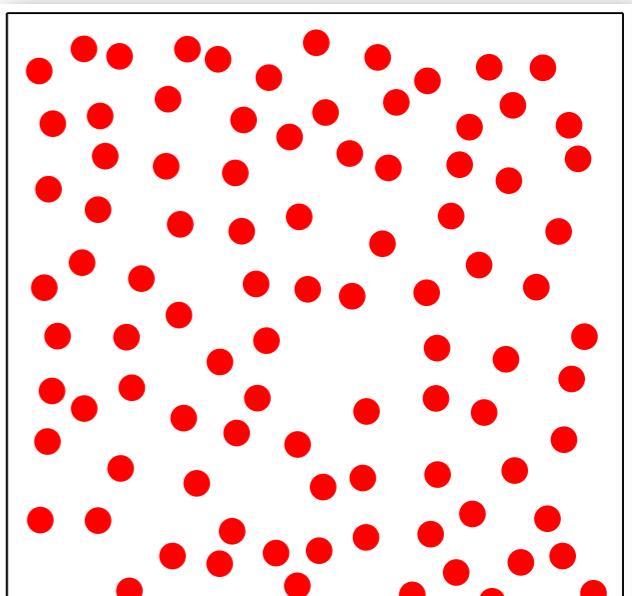
All points



Independent sample

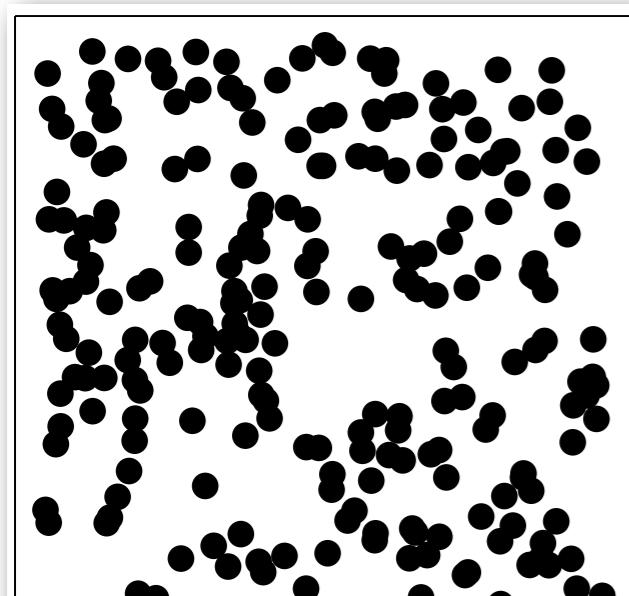


DPP sample

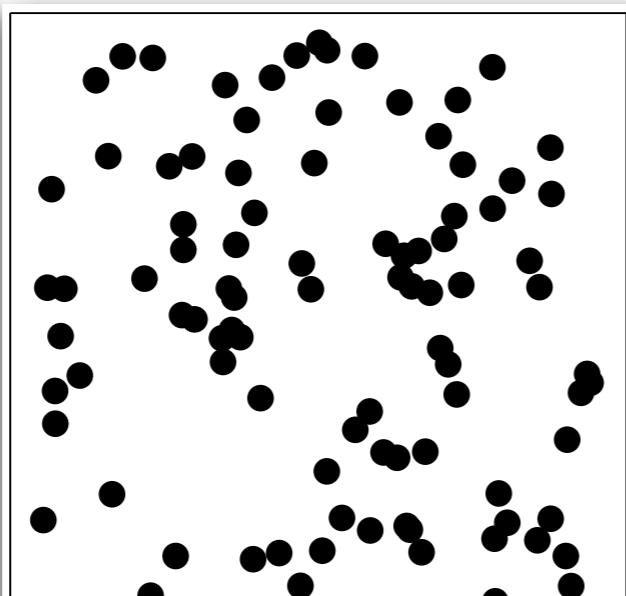


DPP (approx) MAP

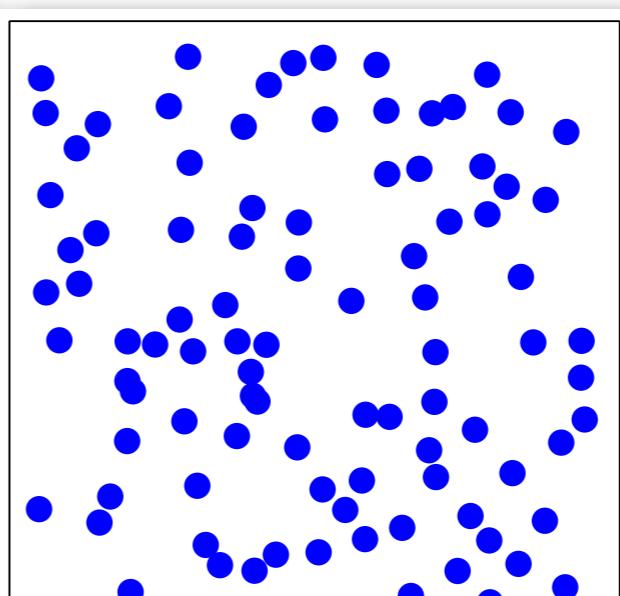
HARDNESS OF MAP



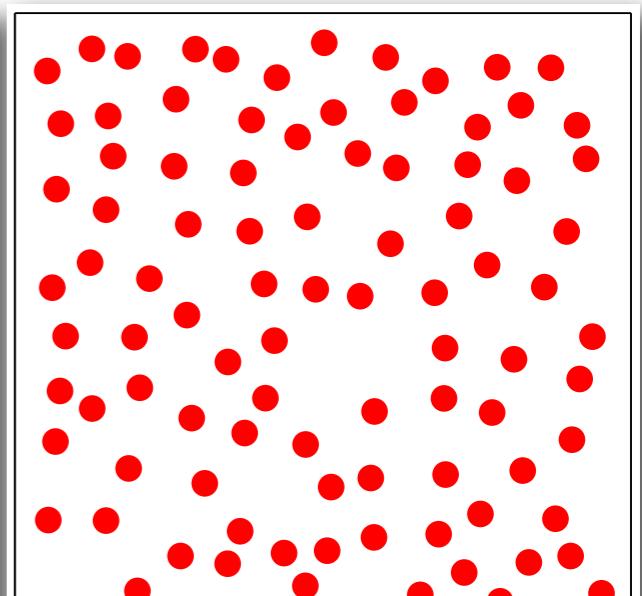
All points



Independent sample



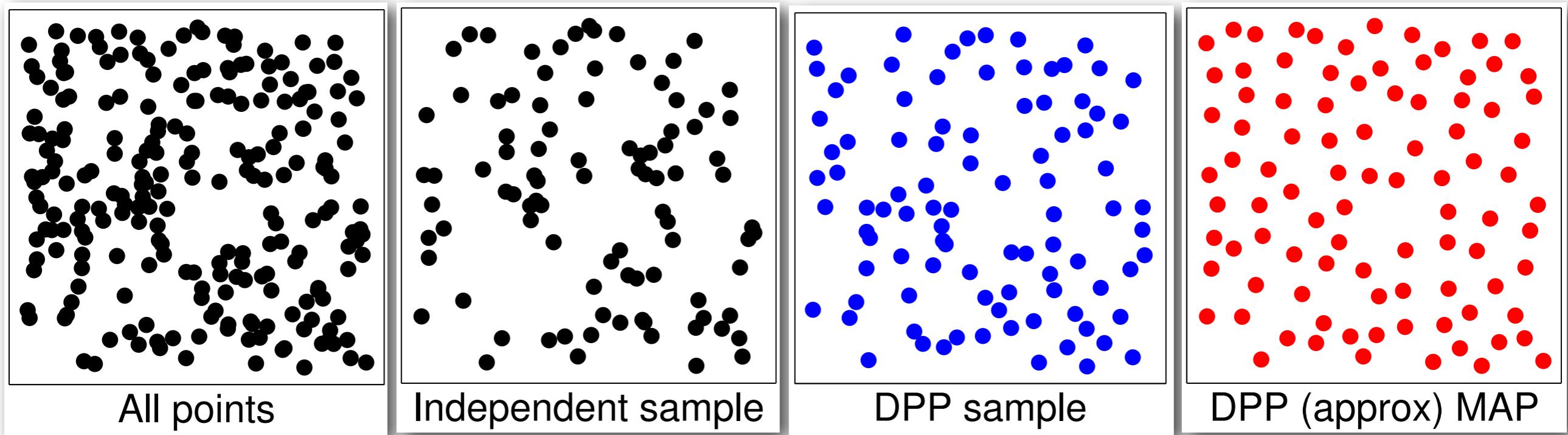
DPP sample



DPP (approx) MAP

$$Y = \arg \max_{Y': Y' \subseteq \mathcal{Y}} \det(L_{Y'})$$

HARDNESS OF MAP

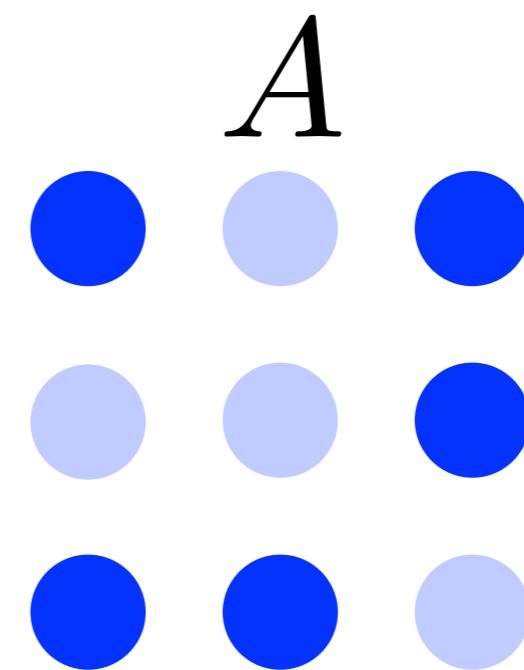


$$Y = \arg \max_{Y': Y' \subseteq \mathcal{Y}} \det(L_{Y'})$$

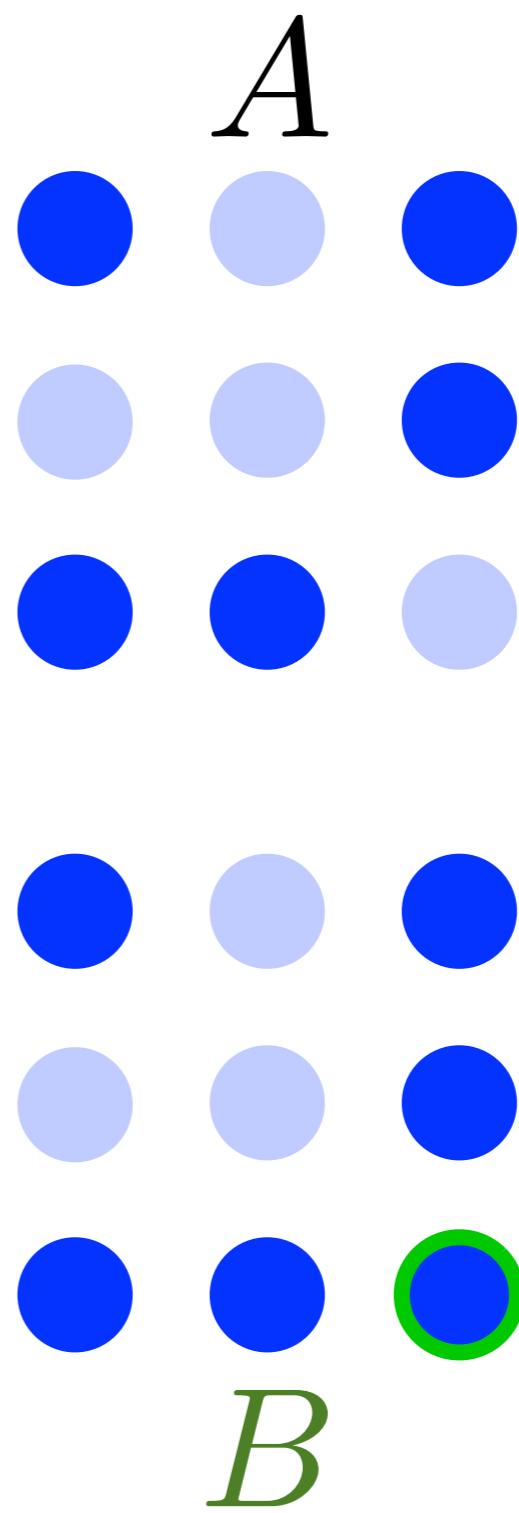
NP-hard, no PTAS: $\det(L_{\hat{Y}}) \geq \left(\frac{8}{9} + \epsilon\right) \det(L_{Y^*})$

SUBMODULARITY

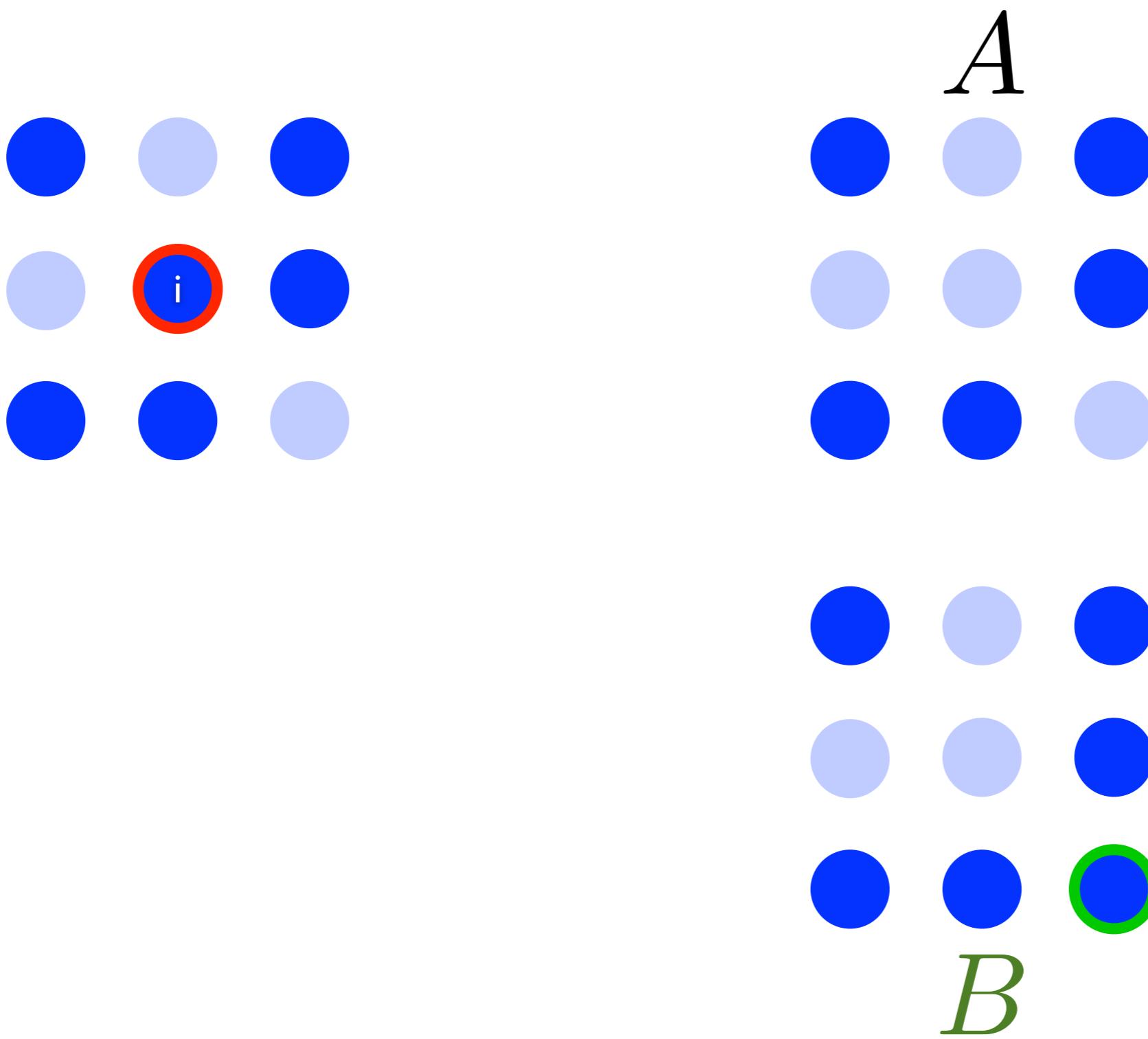
SUBMODULARITY



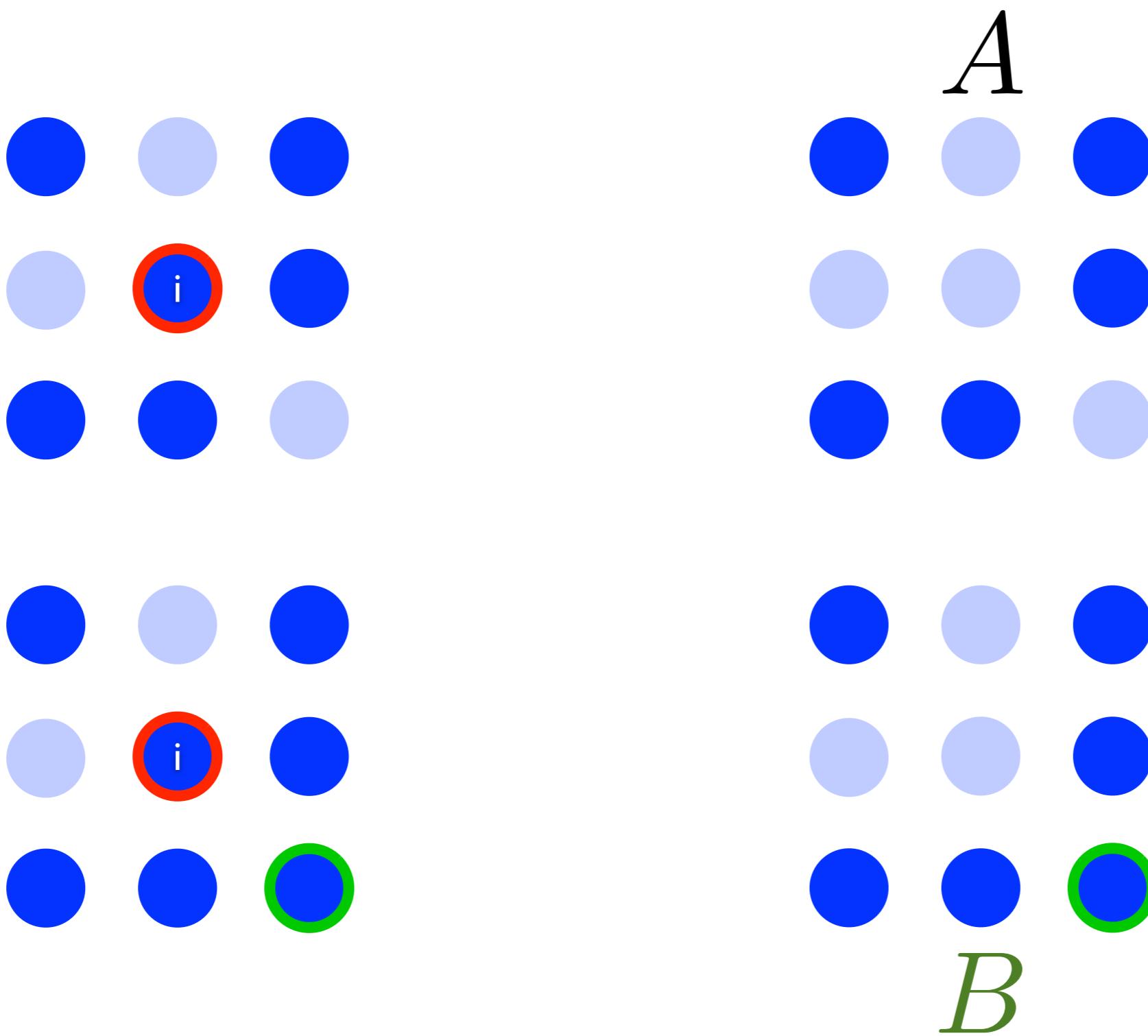
SUBMODULARITY



SUBMODULARITY



SUBMODULARITY



SUBMODULARITY

$$f\left(\begin{array}{c} \text{blue} \\ \text{light blue} \\ \text{light blue} \\ \text{red circle labeled } i \\ \text{blue} \end{array}\right) - f\left(\begin{array}{c} \text{blue} \\ \text{light blue} \\ \text{light blue} \\ \text{light blue} \\ \text{blue} \end{array}\right) \geq$$

A

$$f\left(\begin{array}{c} \text{blue} \\ \text{light blue} \\ \text{light blue} \\ \text{red circle labeled } i \\ \text{blue} \end{array}\right) - f\left(\begin{array}{c} \text{blue} \\ \text{light blue} \\ \text{light blue} \\ \text{light blue} \\ \text{green circle highlighted} \end{array}\right)$$

B

LOG-SUBMODULARITY

$$f\left(\begin{array}{ccc} \textcolor{blue}{\bullet} & \textcolor{lightblue}{\bullet} & \textcolor{blue}{\bullet} \\ \textcolor{lightblue}{\bullet} & \textcolor{red}{\text{---}}^{\textcolor{red}{i}} \textcolor{blue}{\bullet} & \textcolor{blue}{\bullet} \\ \textcolor{blue}{\bullet} & \textcolor{blue}{\bullet} & \textcolor{lightblue}{\bullet} \end{array}\right) - f\left(\begin{array}{ccc} \textcolor{blue}{\bullet} & \textcolor{lightblue}{\bullet} & \textcolor{blue}{\bullet} \\ \textcolor{lightblue}{\bullet} & \textcolor{lightblue}{\bullet} & \textcolor{blue}{\bullet} \\ \textcolor{blue}{\bullet} & \textcolor{blue}{\bullet} & \textcolor{lightblue}{\bullet} \end{array}\right) \geq$$

A

$$f\left(\begin{array}{ccc} \textcolor{blue}{\bullet} & \textcolor{lightblue}{\bullet} & \textcolor{blue}{\bullet} \\ \textcolor{lightblue}{\bullet} & \textcolor{red}{\text{---}}^{\textcolor{red}{i}} \textcolor{blue}{\bullet} & \textcolor{blue}{\bullet} \\ \textcolor{blue}{\bullet} & \textcolor{blue}{\bullet} & \textcolor{green}{\bullet} \end{array}\right) - f\left(\begin{array}{ccc} \textcolor{blue}{\bullet} & \textcolor{lightblue}{\bullet} & \textcolor{blue}{\bullet} \\ \textcolor{lightblue}{\bullet} & \textcolor{lightblue}{\bullet} & \textcolor{blue}{\bullet} \\ \textcolor{blue}{\bullet} & \textcolor{blue}{\bullet} & \textcolor{green}{\bullet} \end{array}\right)$$

B

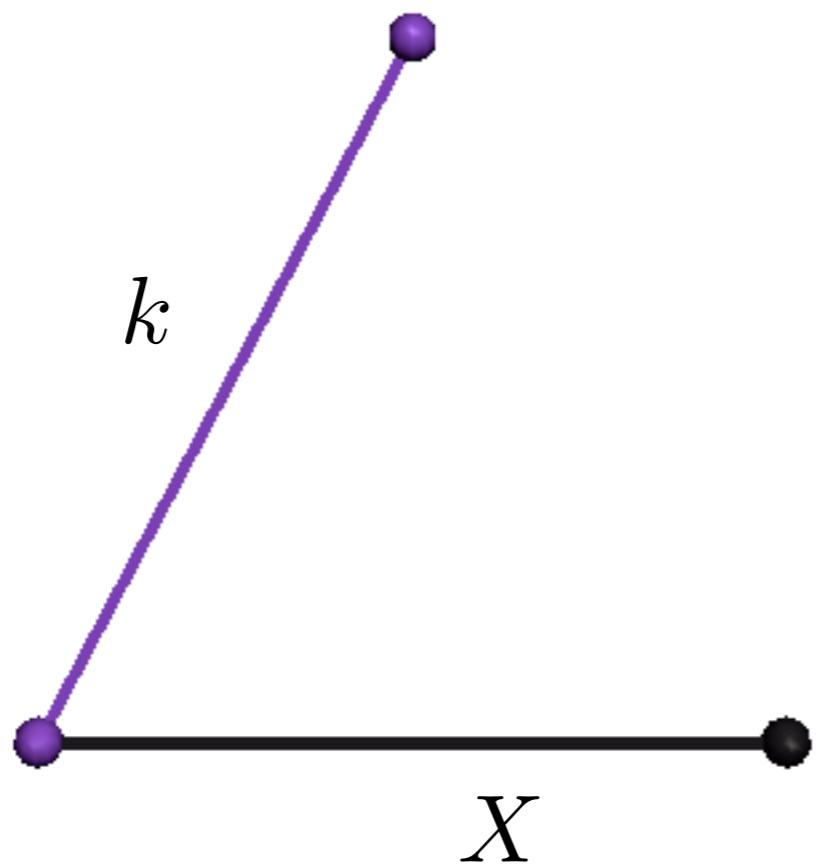
LOG-SUBMODULARITY

$$\frac{f\left(\begin{array}{ccc} \text{blue} & \text{light blue} & \text{blue} \\ \text{light blue} & \text{red circle labeled } i & \text{blue} \\ \text{blue} & \text{blue} & \text{light blue} \end{array}\right)}{f\left(\begin{array}{ccc} \text{blue} & \text{light blue} & \text{blue} \\ \text{light blue} & \text{light blue} & \text{blue} \\ \text{blue} & \text{blue} & \text{light blue} \end{array}\right)} \geq \frac{f\left(\begin{array}{ccc} \text{blue} & \text{light blue} & \text{blue} \\ \text{light blue} & \text{red circle labeled } i & \text{blue} \\ \text{blue} & \text{blue} & \text{green circle} \end{array}\right)}{f\left(\begin{array}{ccc} \text{blue} & \text{light blue} & \text{blue} \\ \text{light blue} & \text{light blue} & \text{blue} \\ \text{blue} & \text{blue} & \text{green circle} \end{array}\right)}$$

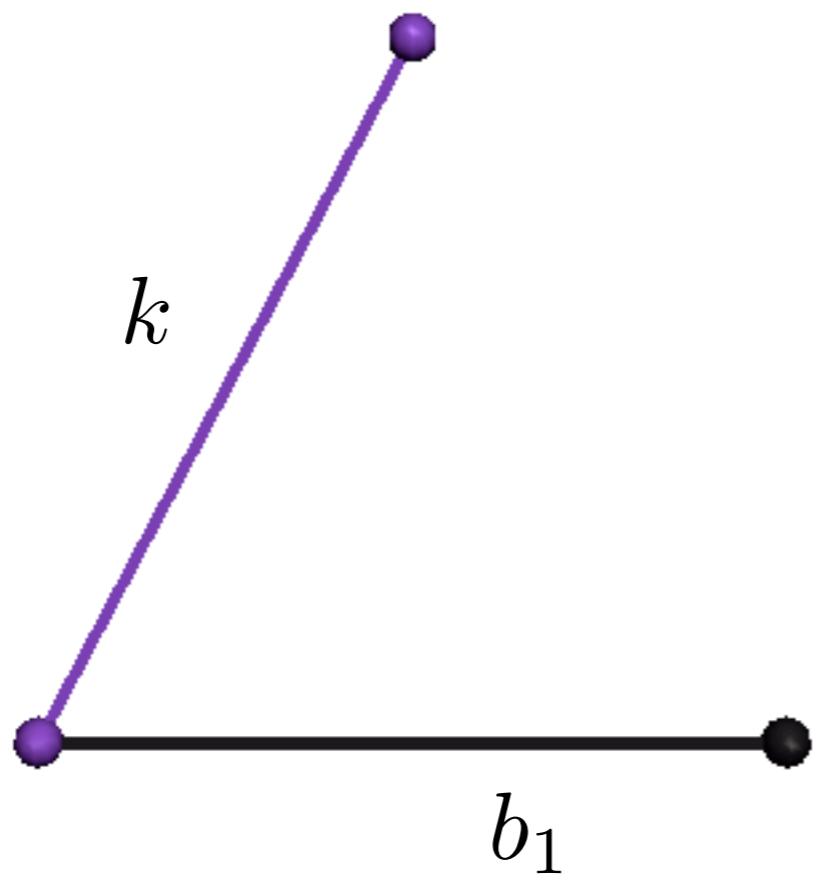
A

B

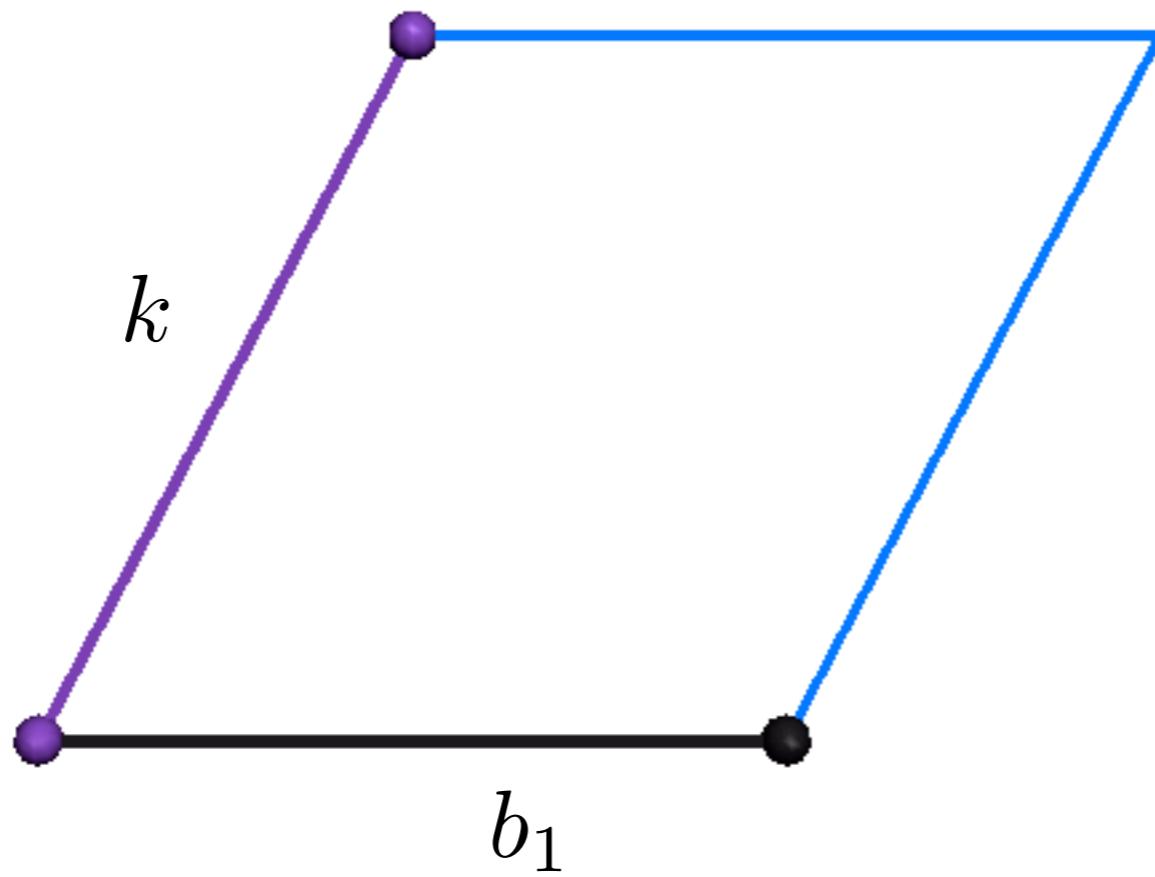
$$\frac{\text{vol}(X \cup \{k\})}{\text{vol}(X)}$$



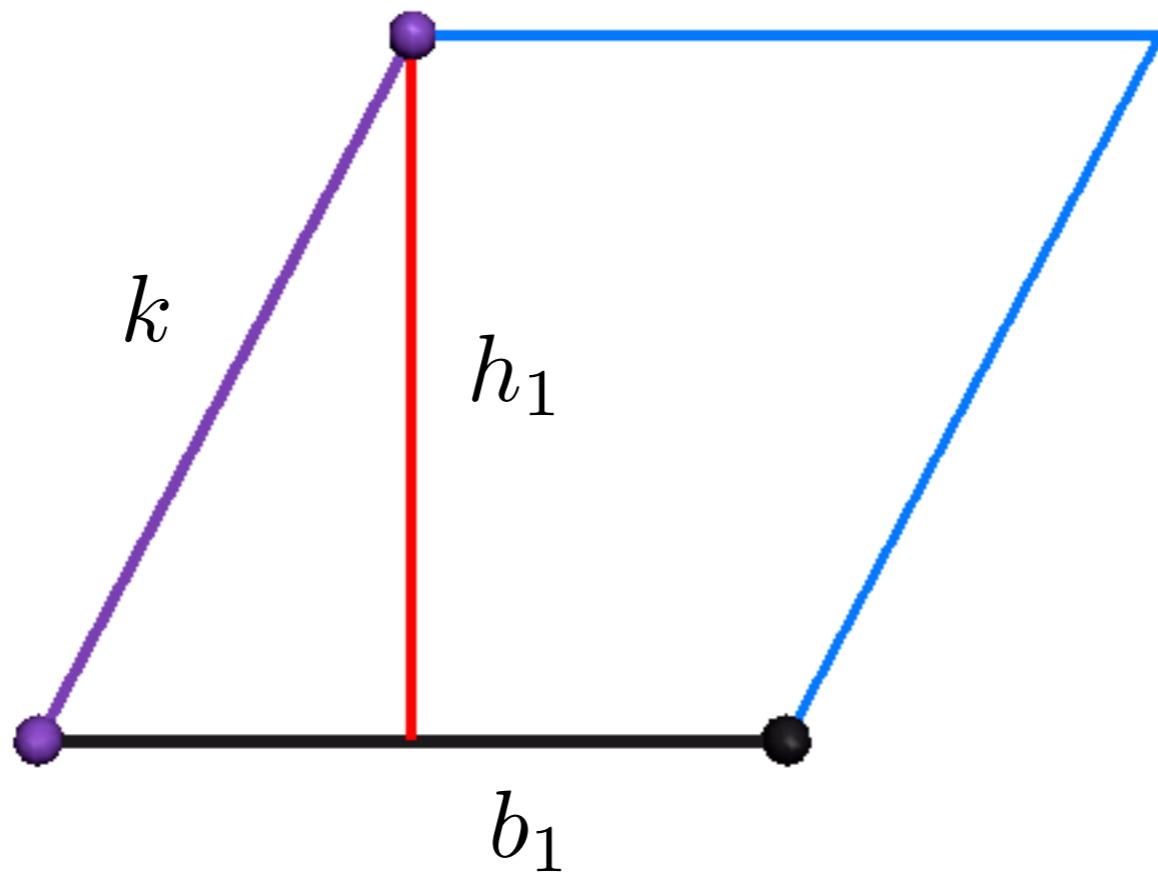
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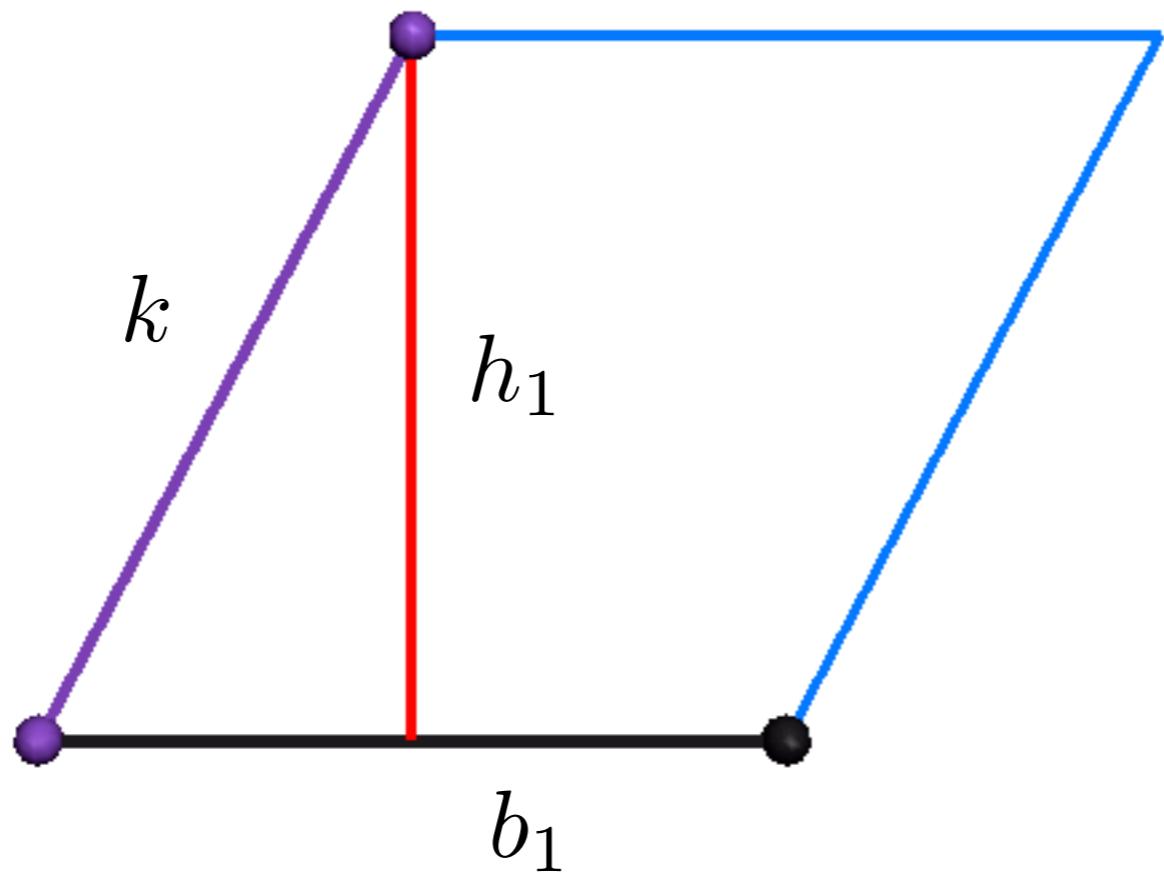
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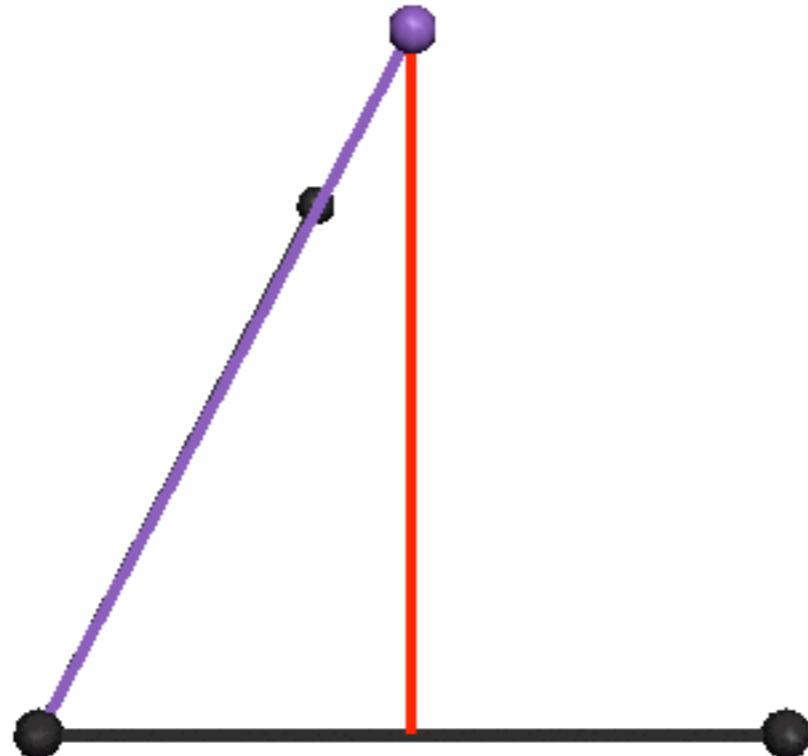
$$\frac{\text{vol}(X \cup \{k\})}{\text{vol}(X)}$$



$$\frac{\text{vol}(X \cup \{k\})}{\text{vol}(X)} = \frac{b_1 h_1}{b_1} = h_1$$

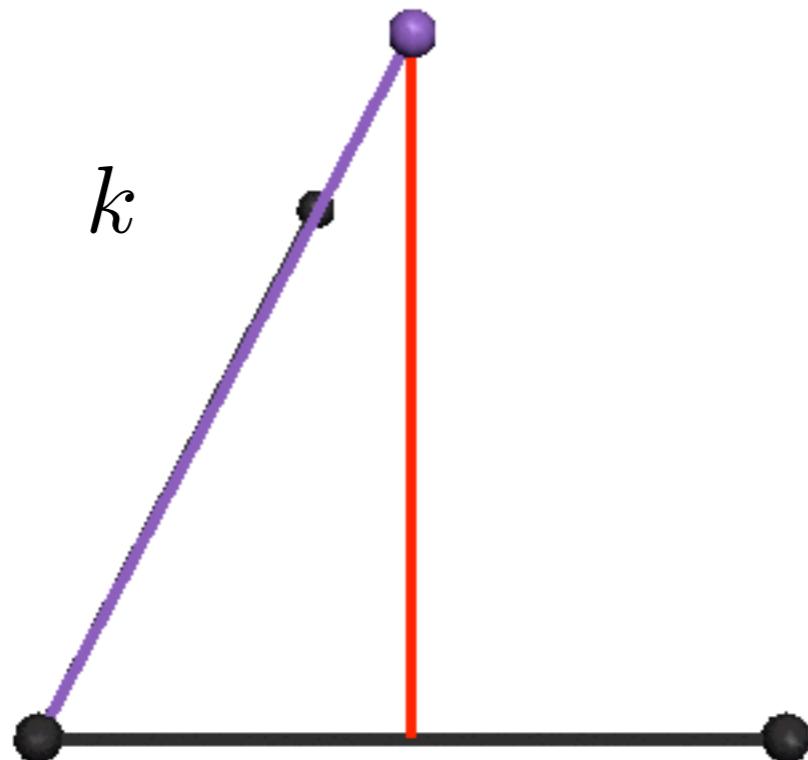


$$\frac{\text{vol}(X \cup \{k\})}{\text{vol}(X)} = \frac{b_1 h_1}{b_1} = h_1$$



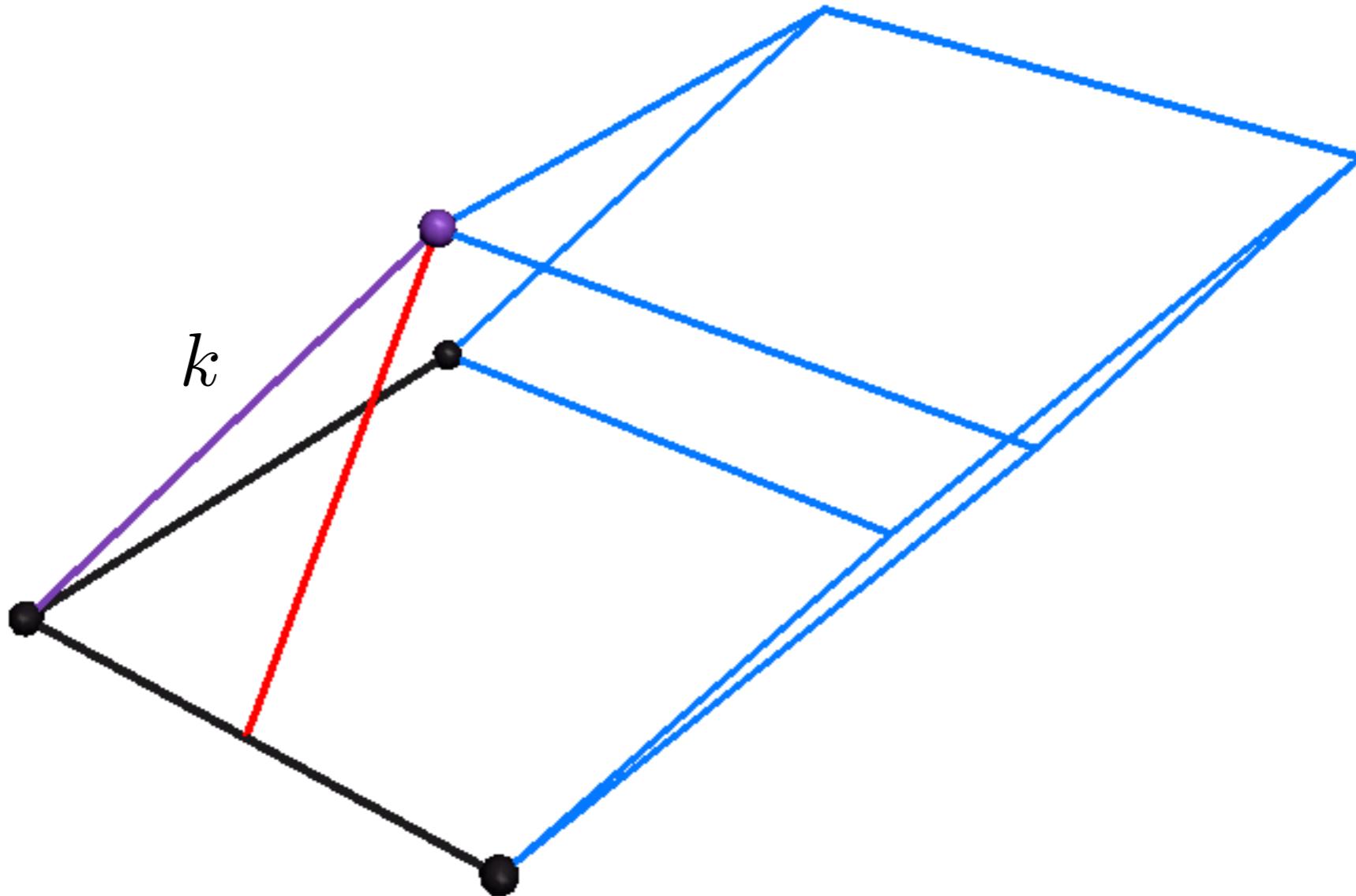
$$\frac{\text{vol}(Y \cup \{k\})}{\text{vol}(Y)}$$

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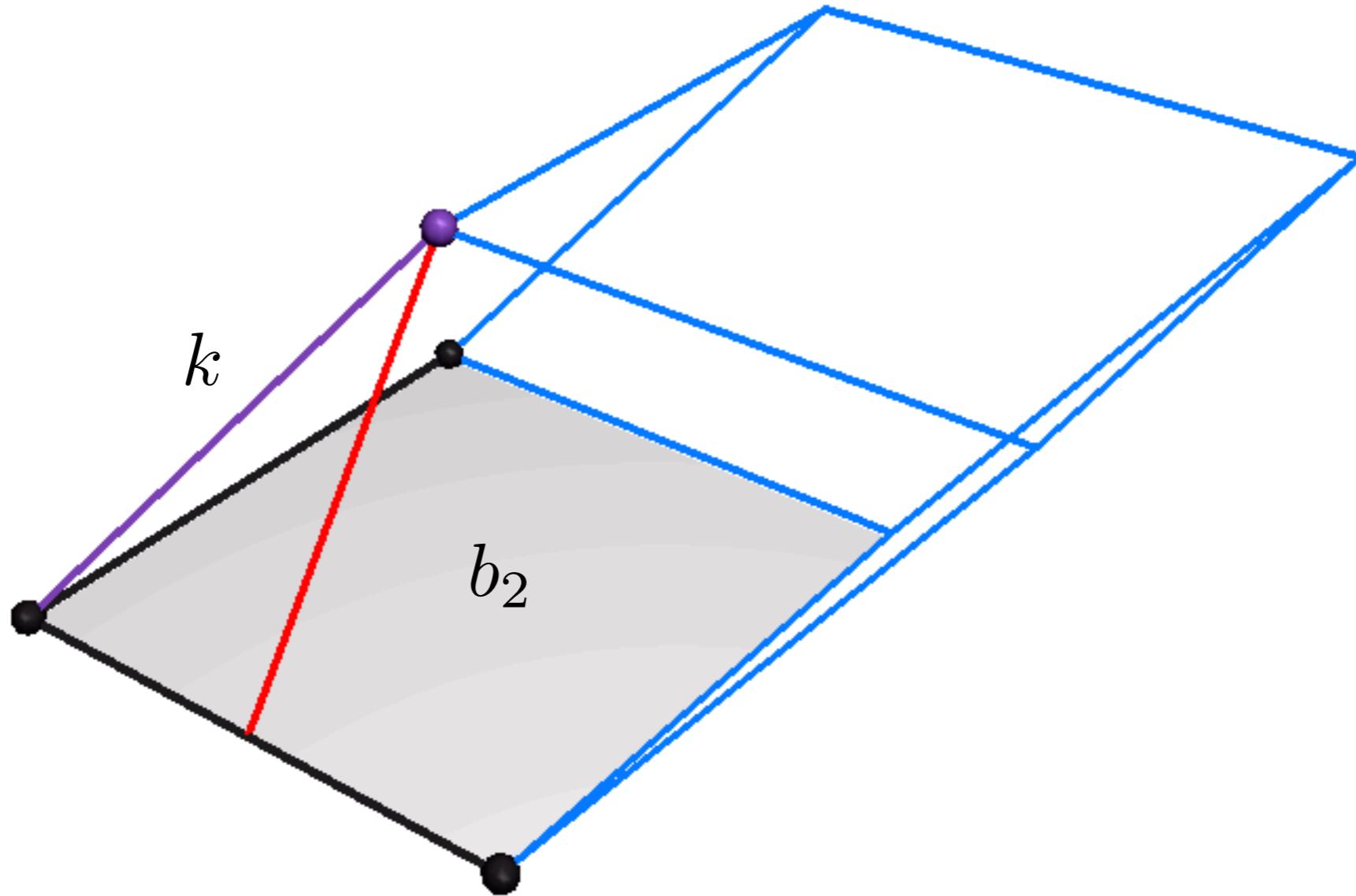
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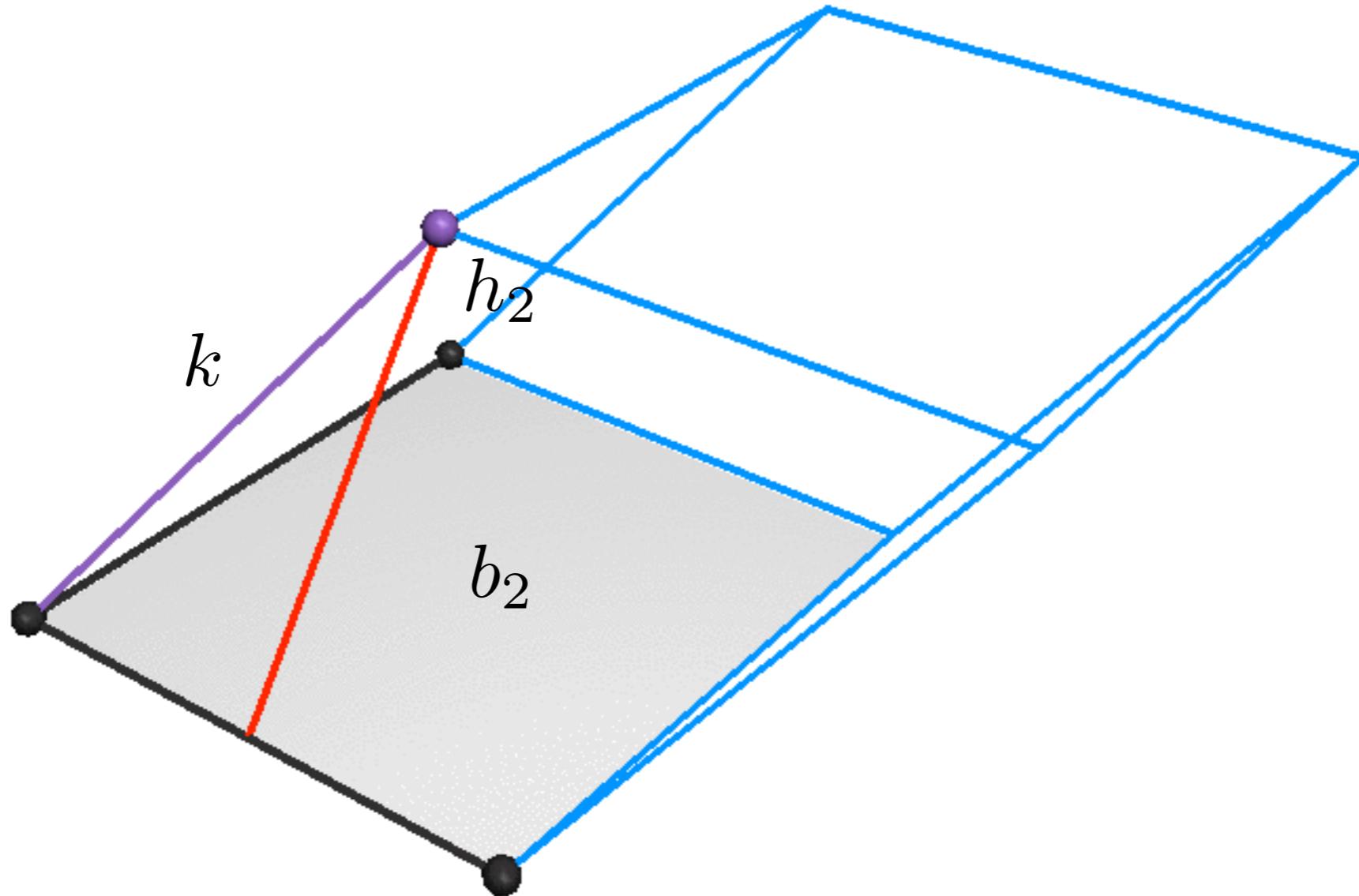
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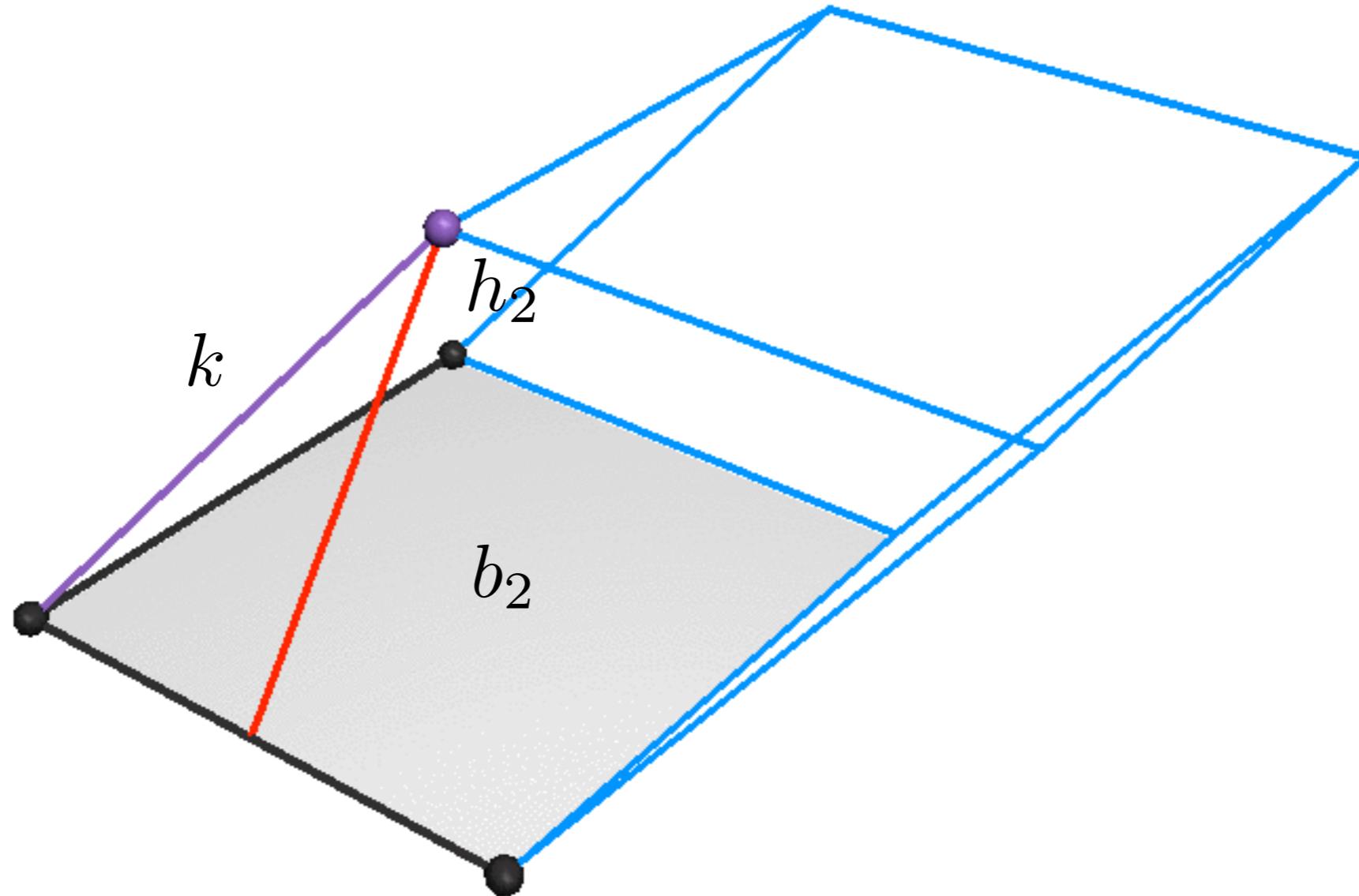
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$$\frac{\text{vol}(X \cup \{k\})}{\text{vol}(X)} = \frac{b_1 h_1}{b_1} = h_1$$



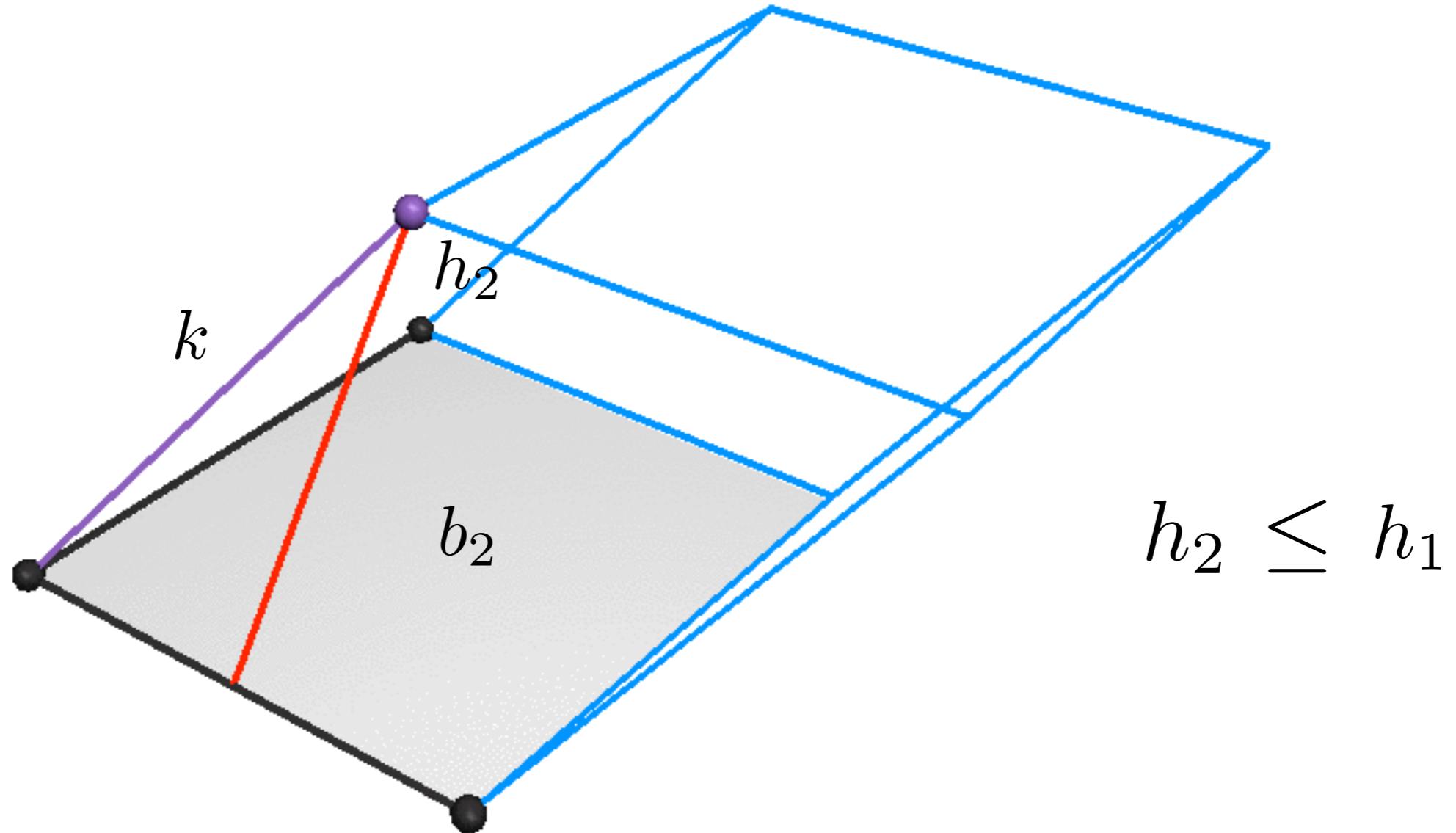
$$\frac{\text{vol}(Y \cup \{k\})}{\text{vol}(Y)}$$

$$\frac{\text{vol}(X \cup \{k\})}{\text{vol}(X)} = \frac{b_1 h_1}{b_1} = h_1$$



$$\frac{\text{vol}(Y \cup \{k\})}{\text{vol}(Y)} = \frac{b_2 h_2}{b_2} = h_2$$

$$\frac{\text{vol}(X \cup \{k\})}{\text{vol}(X)}$$



$$h_2 \leq h_1$$

$$\frac{\text{vol}(Y \cup \{k\})}{\text{vol}(Y)}$$

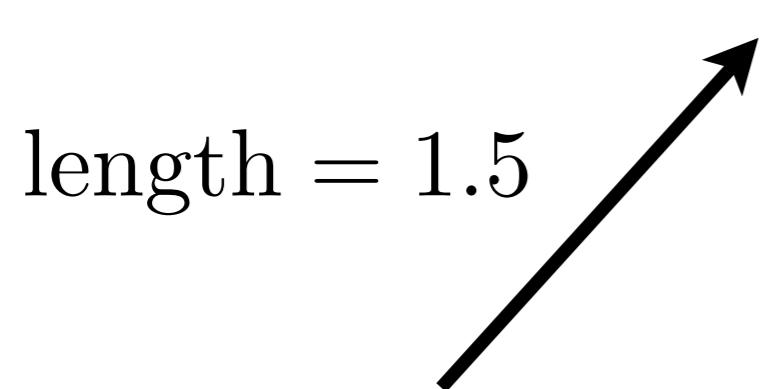
MONOTONICITY

MONOTONICITY

$$X \subseteq Y \implies f(X) \leq f(Y)$$

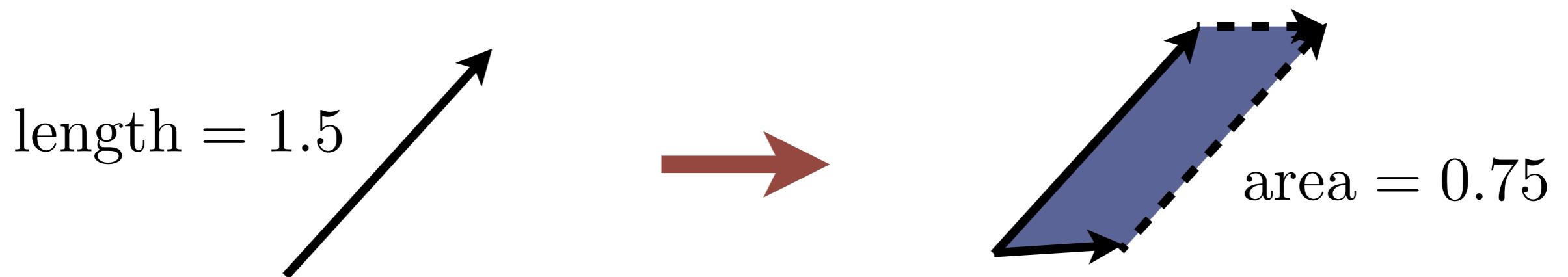
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MONOTONICITY

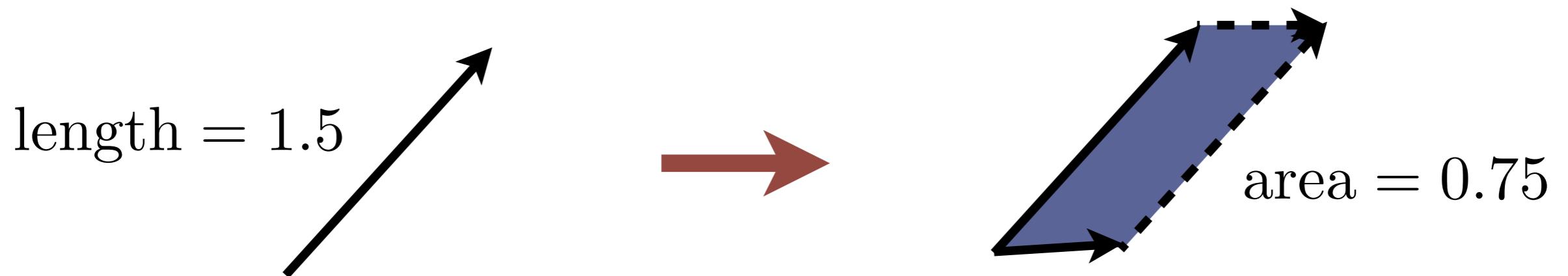
$$X \subseteq Y \implies f(X) \leq f(Y)$$



MONOTONICITY

$$X \subseteq Y \implies f(X) \leq f(Y)$$

\det is non-monotone



PRIOR WORK

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Monotone:

“greedy” $(1 - 1/e)$ -approx
Nemhauser et al. (1978)

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Non-monotone + constraints:

“multilinear” $1/4$ -approx sans constraints,
various (lesser) guarantees dependent on constraint type
Chekuri et al. (2011)

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Non-monotone + constraints:

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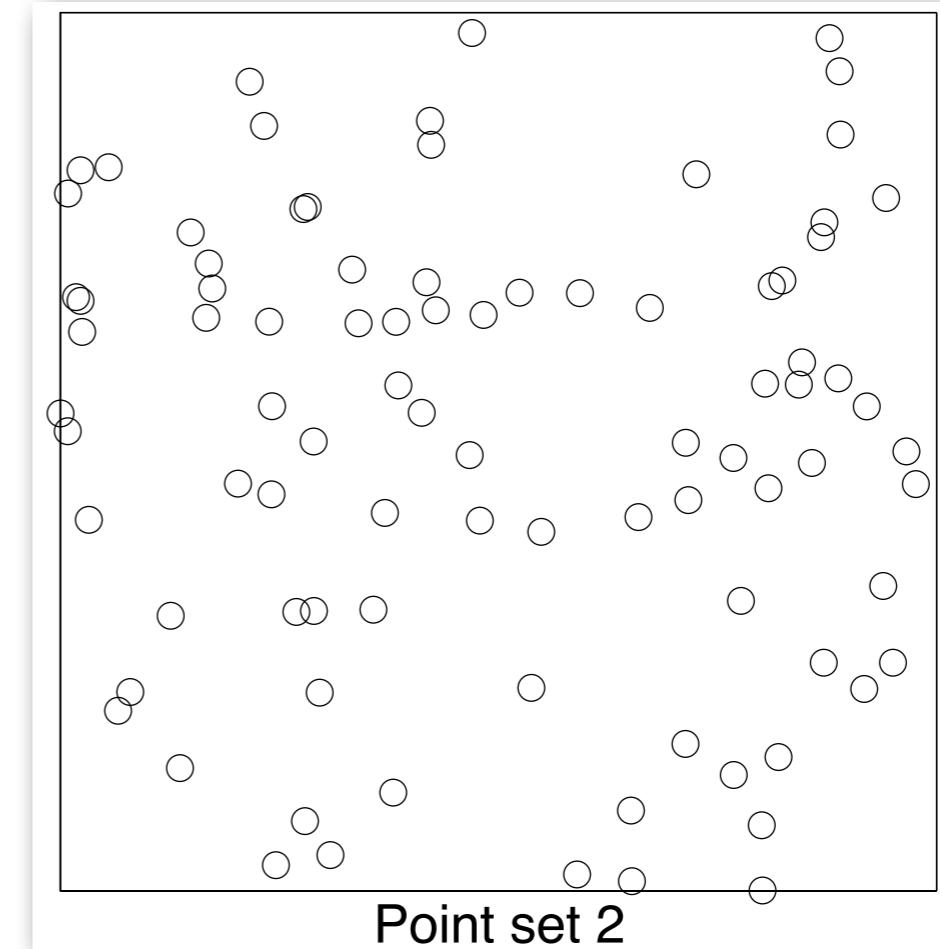
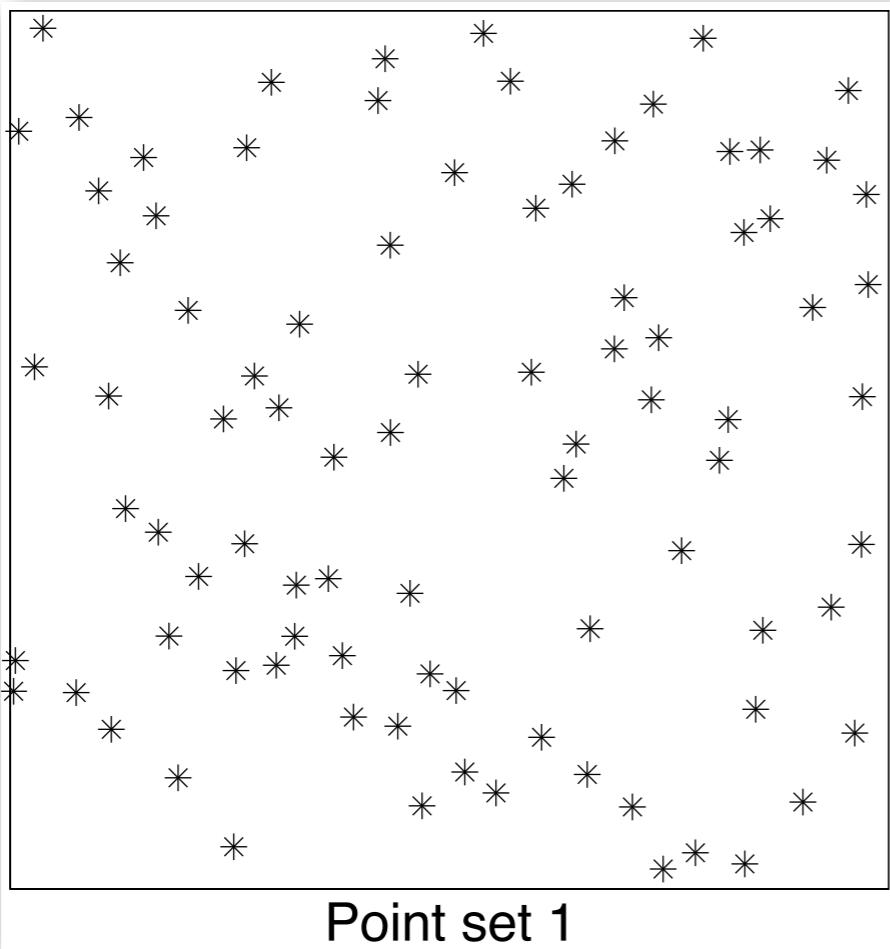
Chekuri et al. (2011)

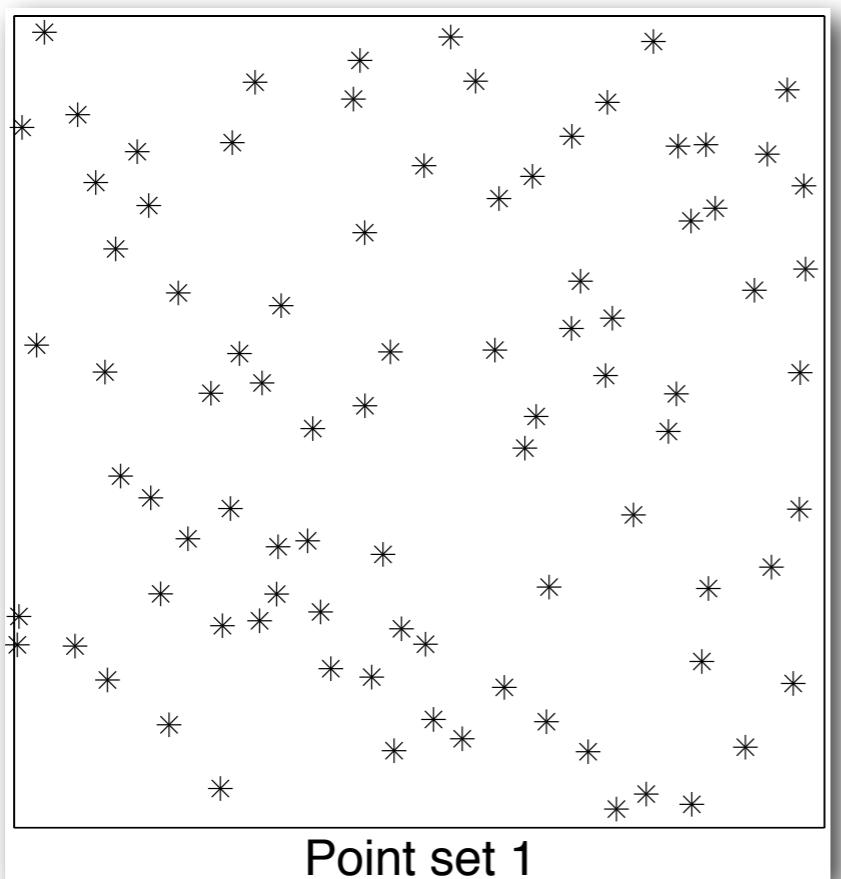
$$\arg \max_Y \det(L_Y) \longrightarrow$$

$$\arg \max_{Y \in S} \det(L_Y)$$

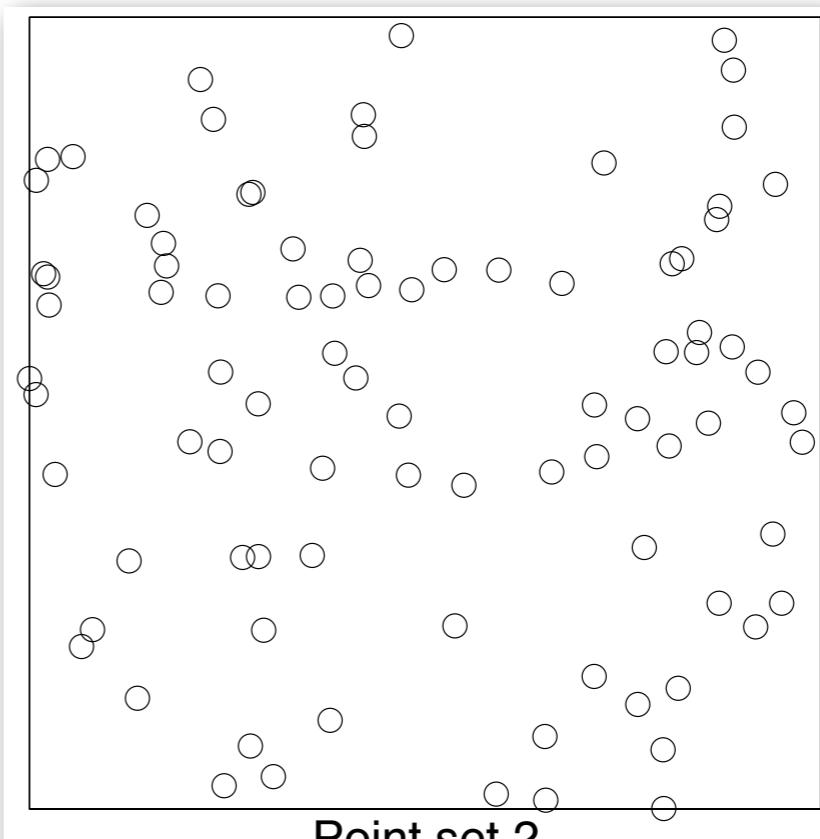
where S is a solvable polytope

IMAGE COMPARISON WITH CONSTRAINTS

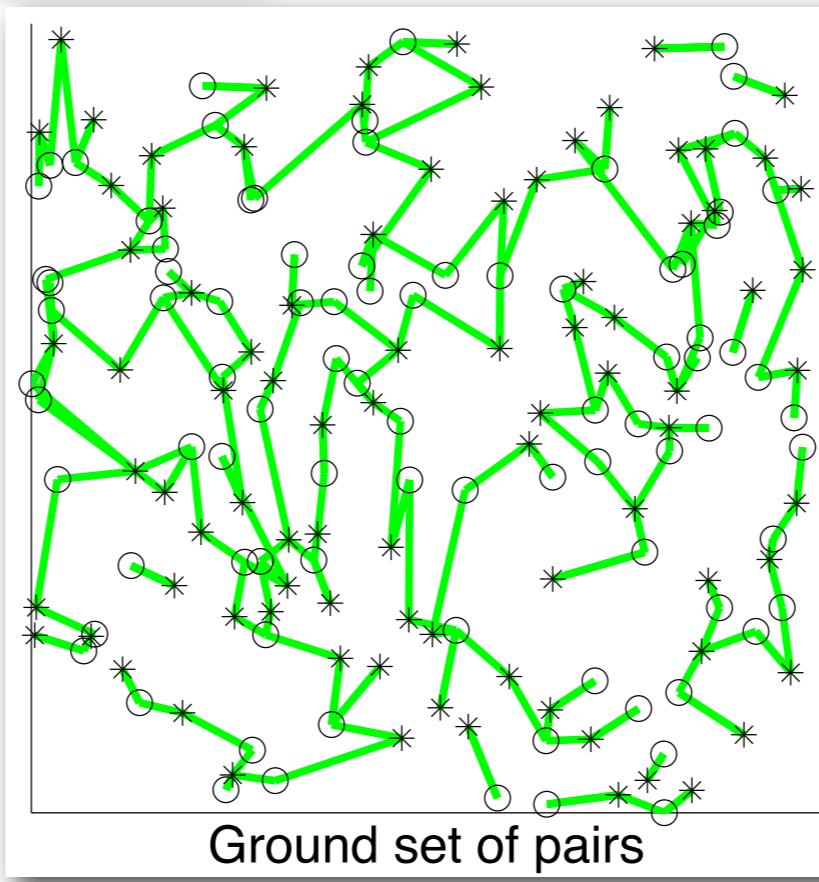




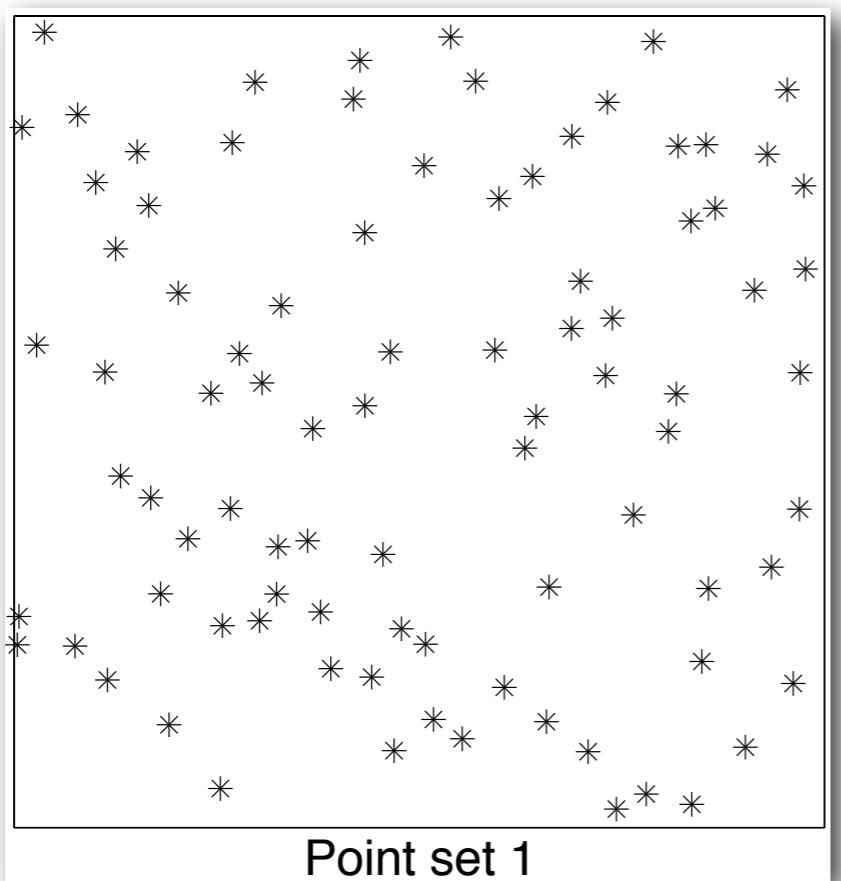
Point set 1



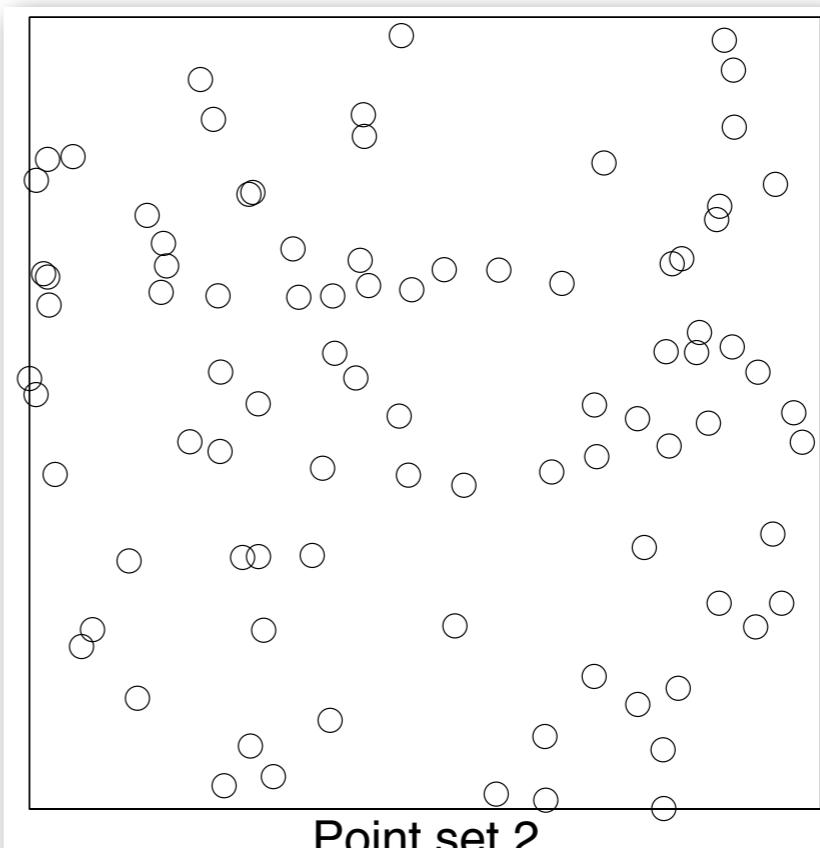
Point set 2



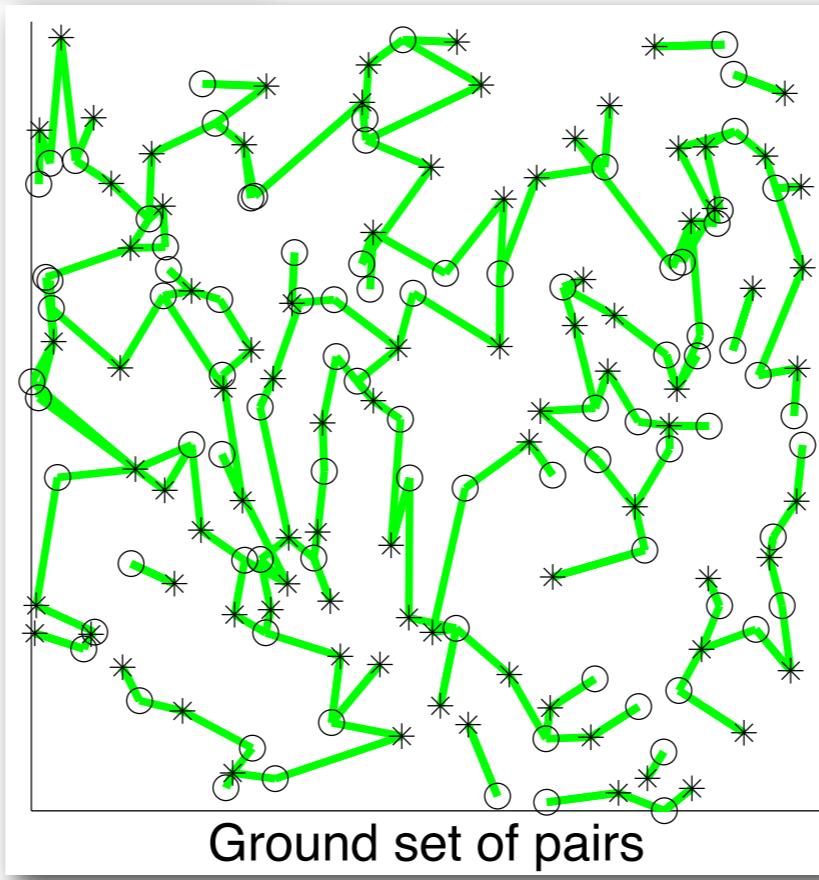
Ground set of pairs



Point set 1

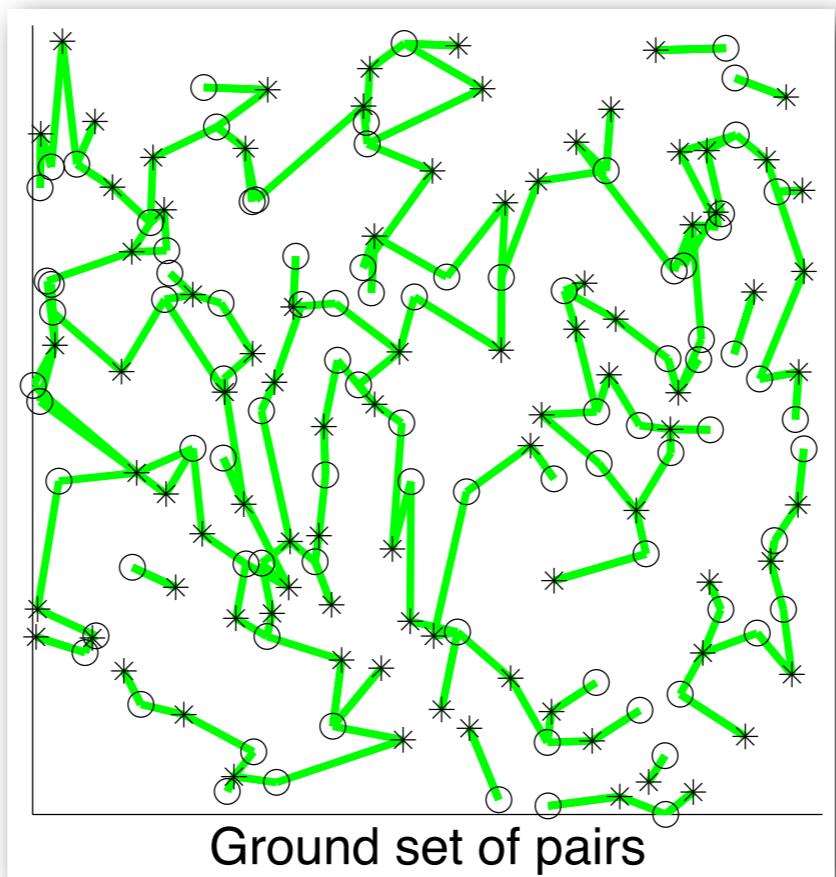


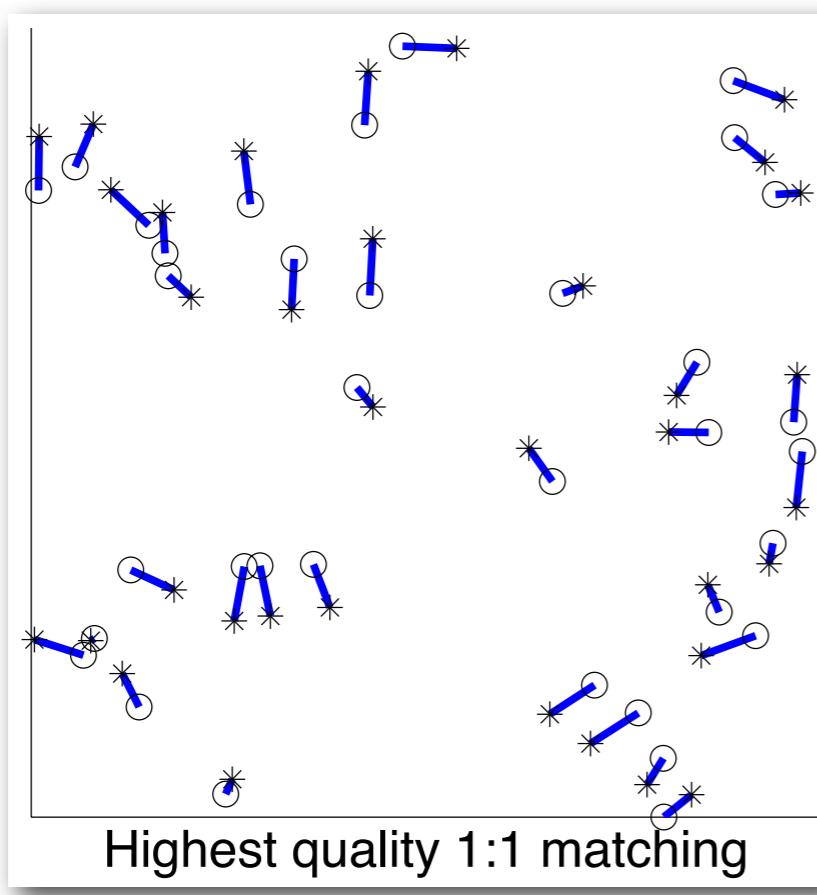
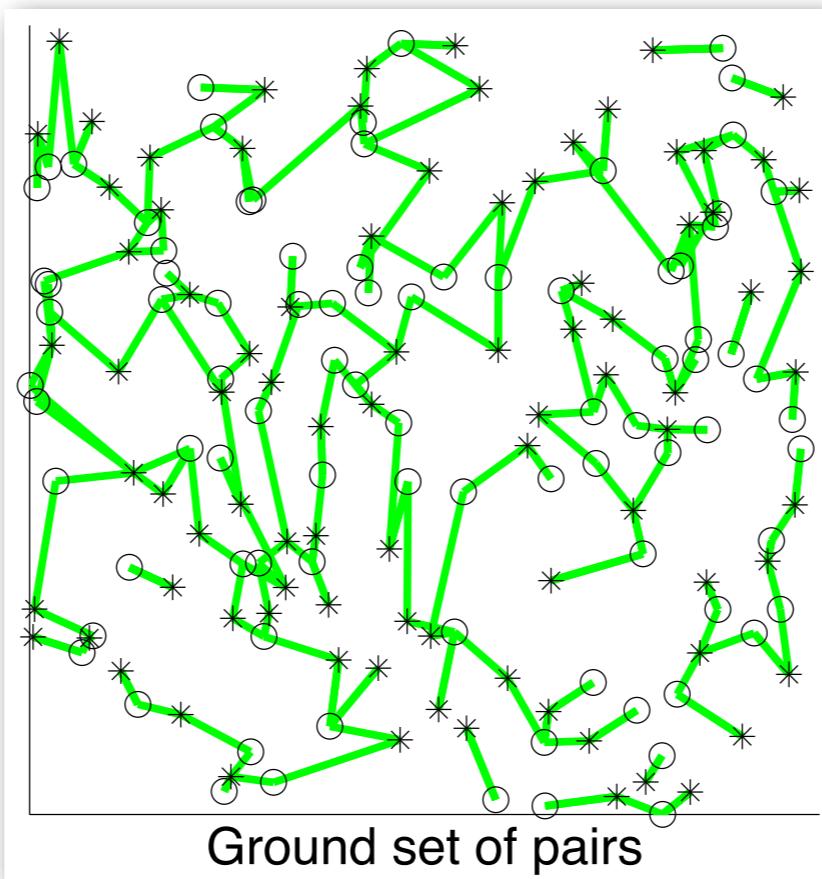
Point set 2

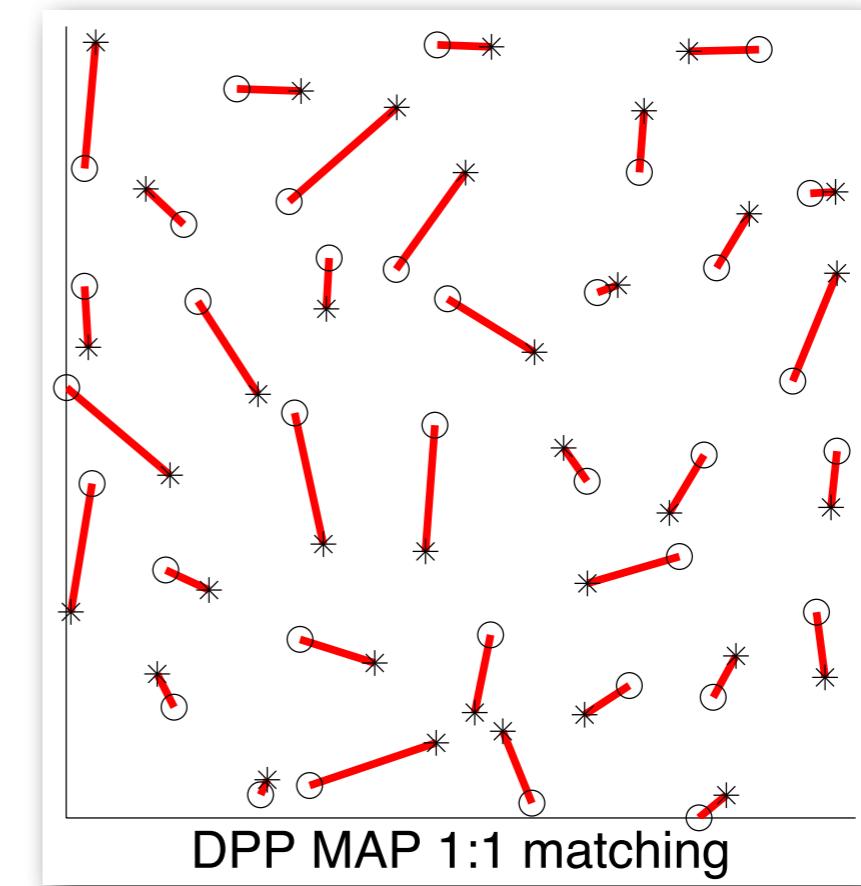
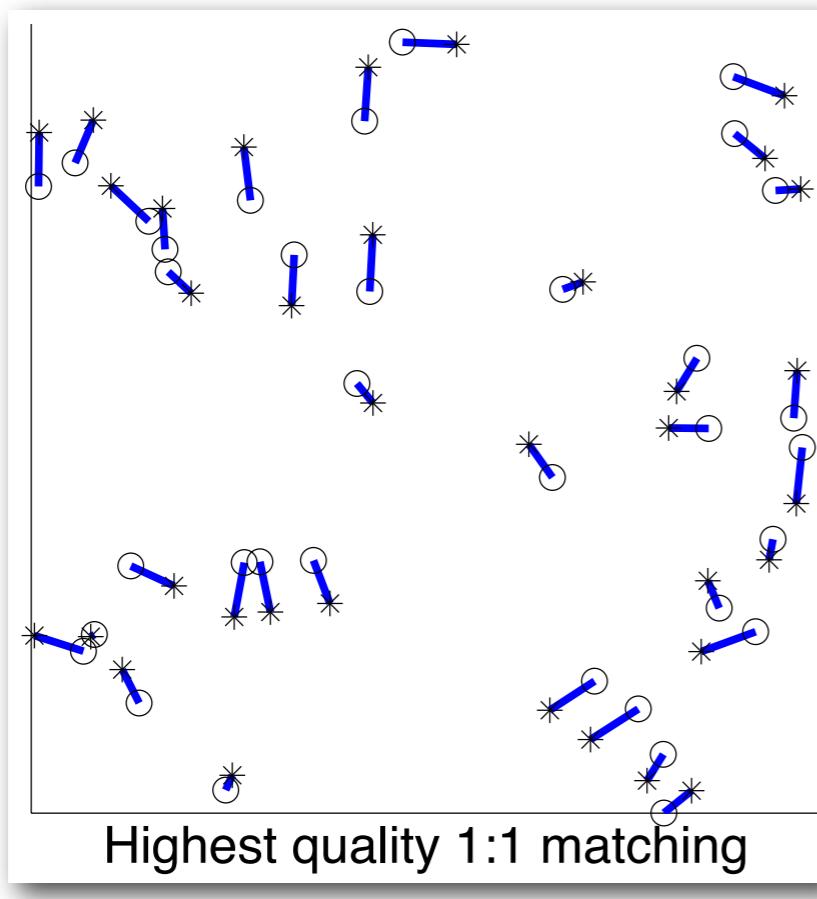
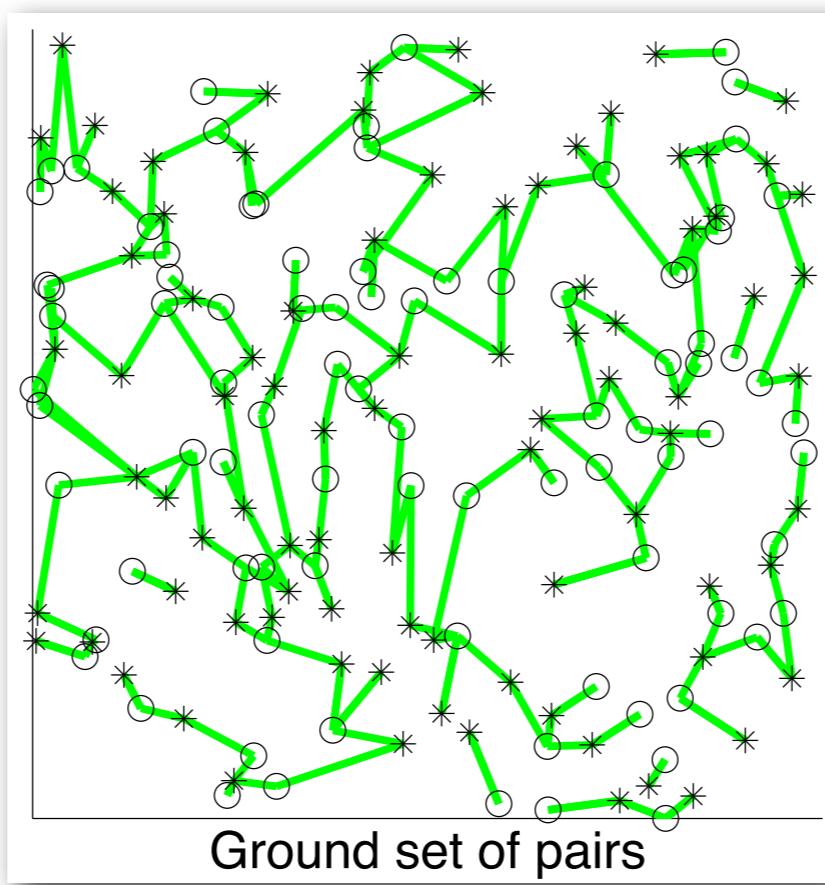


Ground set of pairs

$Y \in \text{matching polytope}$



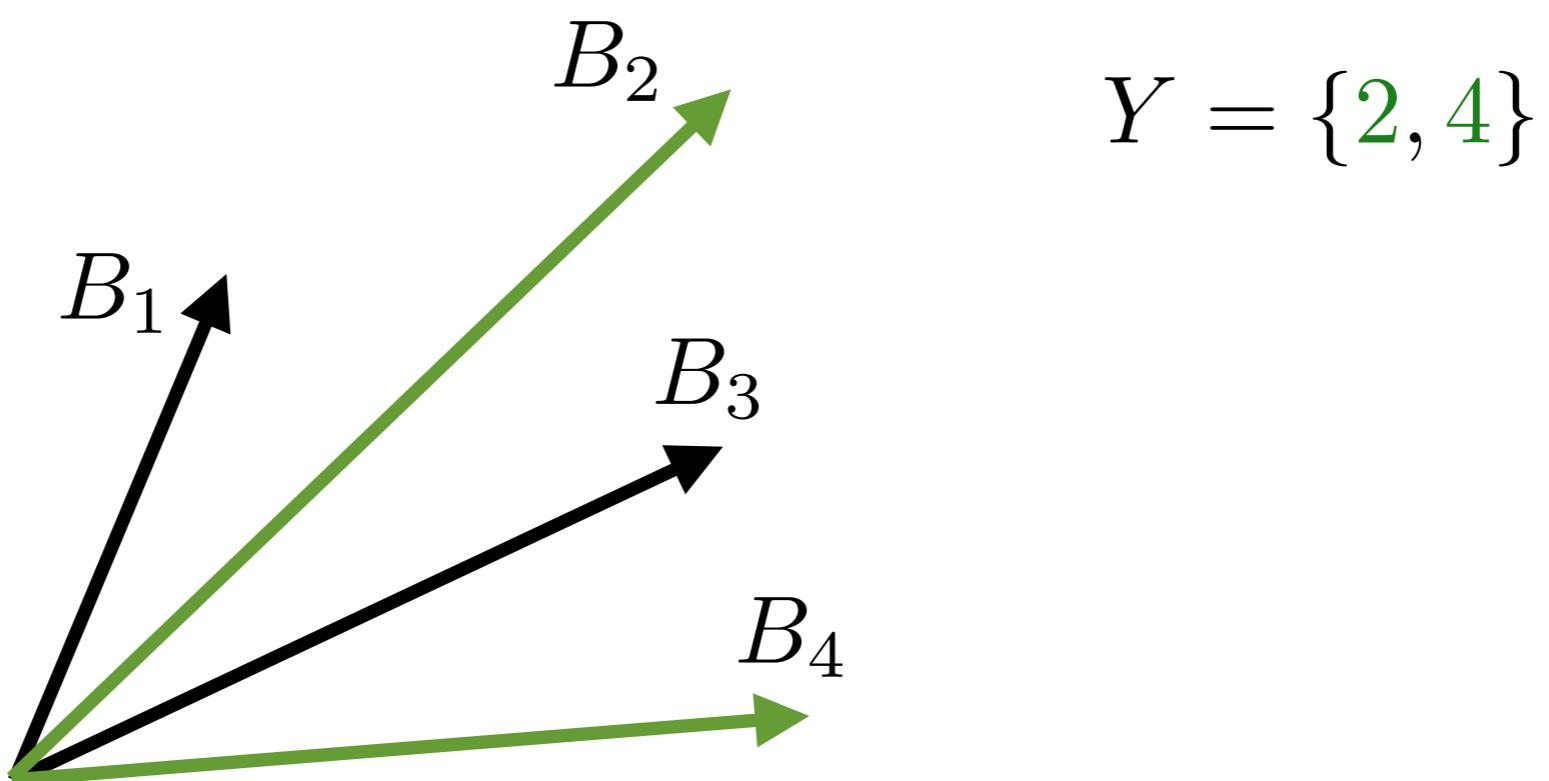




CHEKURI ET AL. 2011

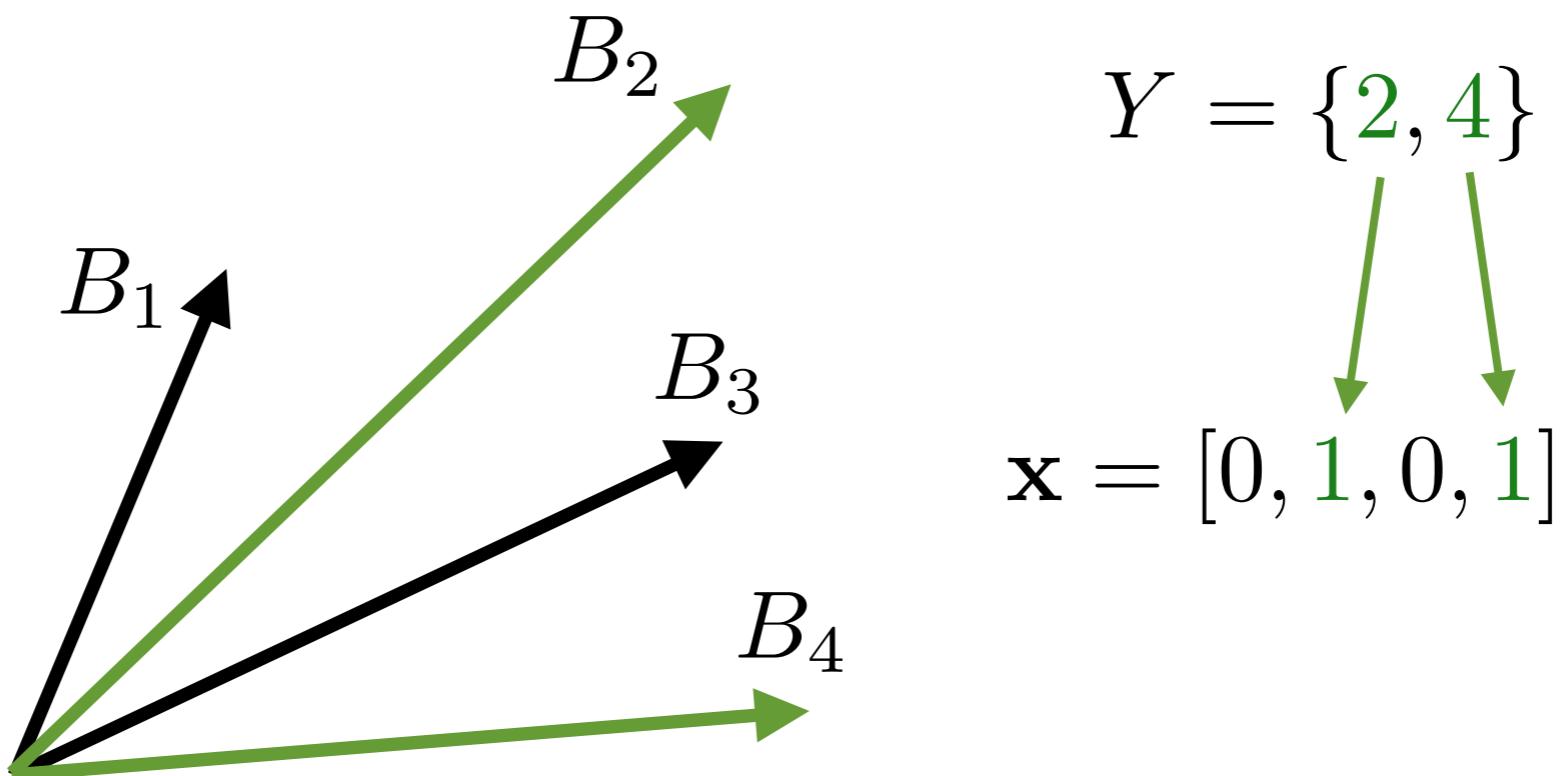
CHEKURI ET AL. 2011

Step 1: Relax inclusion-exclusion

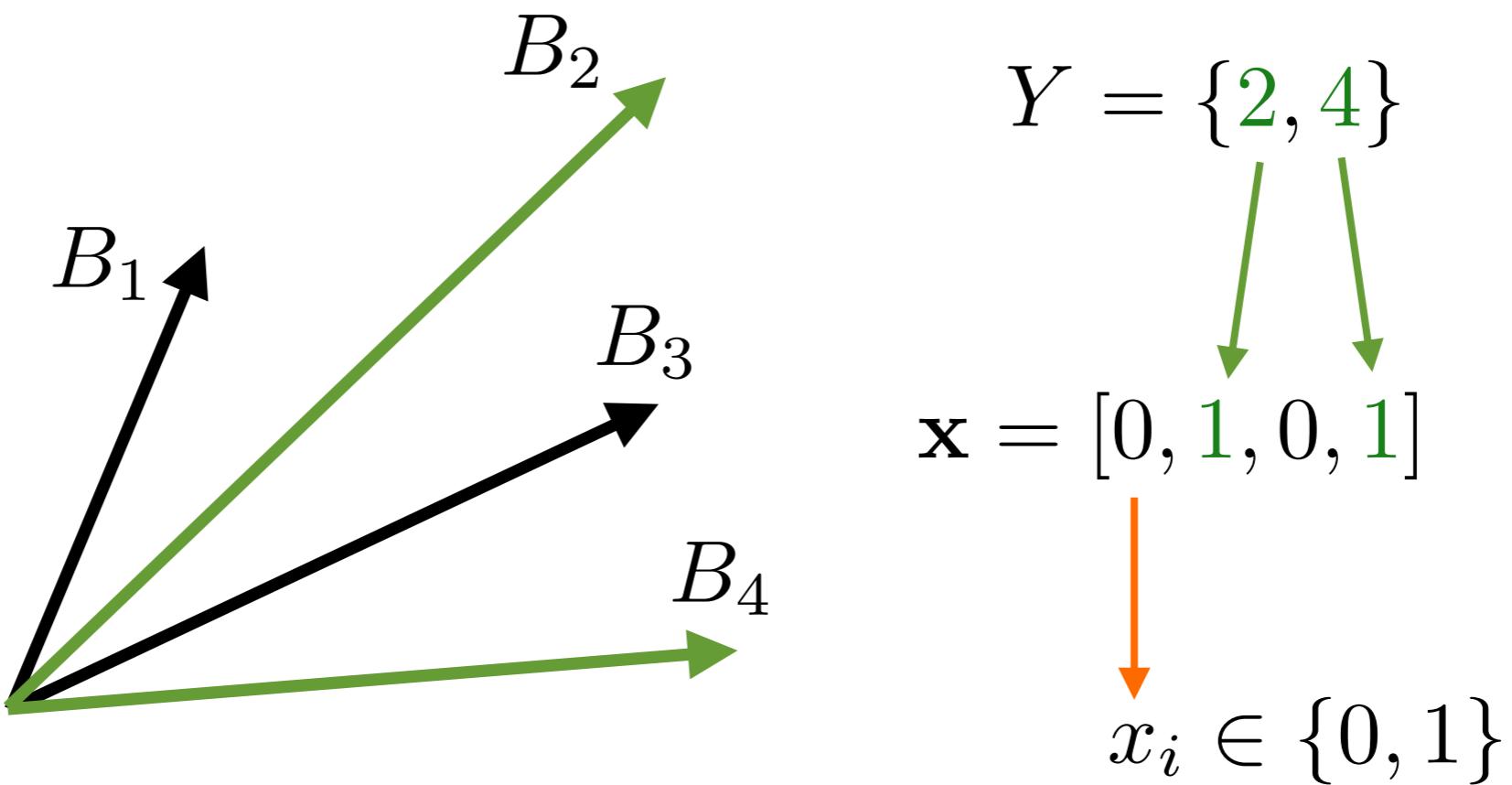


CHEKURI ET AL. 2011

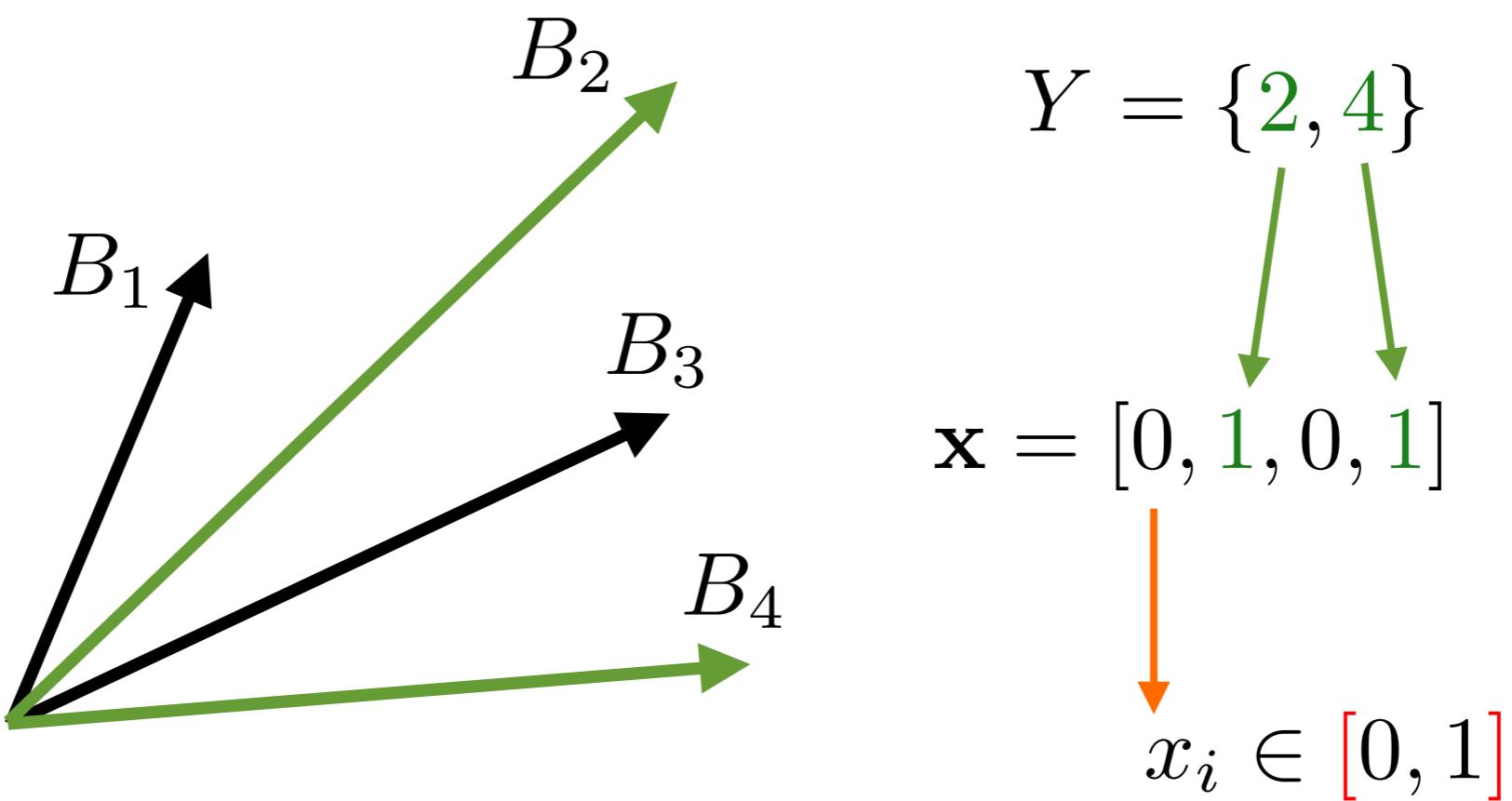
Step 1: Relax inclusion-exclusion



Step 1: Relax inclusion-exclusion



Step 1: Relax inclusion-exclusion



Step 2: Extend objective

$$F(\mathbf{x}) = E_{\mathbf{x}}[\log f(Y)] \quad \text{multilinear extension}$$



log-submodular, like $\det(L_Y)$

Step 2: Extend objective

$$\begin{aligned} F(\mathbf{x}) &= E_{\mathbf{x}}[\log f(Y)] \quad \text{multilinear extension} \\ &= \sum_Y \prod_{i \in Y} x_i \prod_{i \notin Y} (1 - x_i) \log f(Y) \end{aligned}$$

Step 2: Extend objective

$$F(\mathbf{x}) = E_{\mathbf{x}}[\log f(Y)] \quad \text{multilinear extension}$$

$$= \sum_Y \left[\prod_{i \in Y} x_i \prod_{i \notin Y} (1 - x_i) \right] \log f(Y)$$

$$\downarrow \\ p_Y$$

Step 2: Extend objective

$$F(\mathbf{x}) = E_{\mathbf{x}}[\log f(Y)] \quad \text{multilinear extension}$$

$$= \boxed{\sum_Y} \prod_{i \in Y} x_i \prod_{i \notin Y} (1 - x_i) \log f(Y)$$

\downarrow
 2^N subsets

Step 2: Extend objective

$$F(\mathbf{x}) = E_{\mathbf{x}}[\log f(Y)] \quad \text{multilinear extension}$$

$$= \boxed{\sum_Y} \prod_{i \in Y} x_i \prod_{i \notin Y} (1 - x_i) \log f(Y)$$

\downarrow
 2^N subsets

Step 3: Optimize using gradient-based methods $\frac{\partial F(\mathbf{x})}{\partial \mathbf{x}}$

Step 2: Extend objective

$$F(\mathbf{x}) = E_{\mathbf{x}}[\log f(Y)] \quad \text{multilinear extension}$$

$$= \boxed{\sum_Y} \prod_{i \in Y} x_i \prod_{i \notin Y} (1 - x_i) \log f(Y)$$

\downarrow
 2^N subsets

Step 3: Optimize using gradient-based methods $\frac{\partial F(\mathbf{x})}{\partial \mathbf{x}}$

Step 4: If unconstrained, solution will already be integer;
else, round solution: $x_i \in [0, 1] \rightarrow x_i \in \{0, 1\}$

Step 2: Extend objective

$$F(\mathbf{x}) = E_{\mathbf{x}}[\log f(Y)] \quad \text{multilinear extension}$$

$$= \boxed{\sum_Y} \prod_{i \in Y} x_i \prod_{i \notin Y} (1 - x_i) \log f(Y)$$

\downarrow
 2^N subsets \implies Monte Carlo required

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SOFTMAX EXTENSION

GILLENWATER, KULESZA, AND TASKAR (NIPS 2012)

Multilinear: $F(\mathbf{x}) = \sum_Y \prod_{i \in Y} x_i \prod_{i \notin Y} (1 - x_i) \log f(Y)$

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Softmax: $\tilde{F}(\mathbf{x}) = \log \sum_Y \prod_{i \in Y} x_i \prod_{i \notin Y} (1 - x_i) f(Y)$

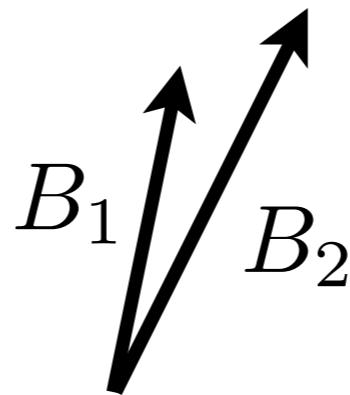
SOFTMAX EXTENSION

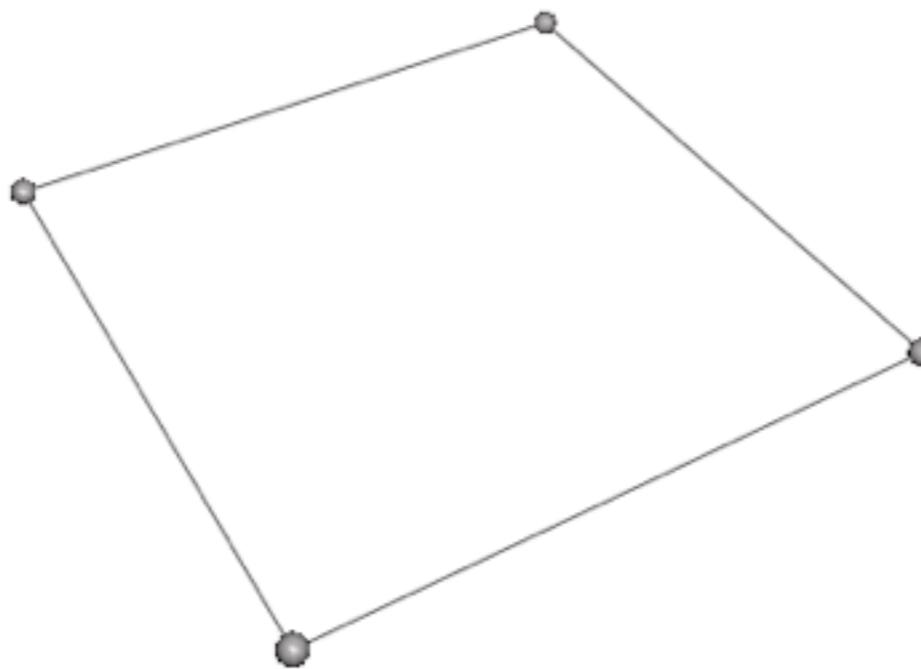
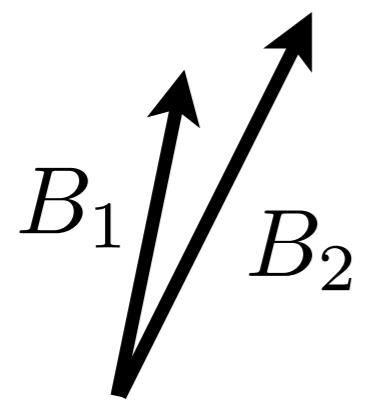
GILLENWATER, KULESZA, AND TASKAR (NIPS 2012)

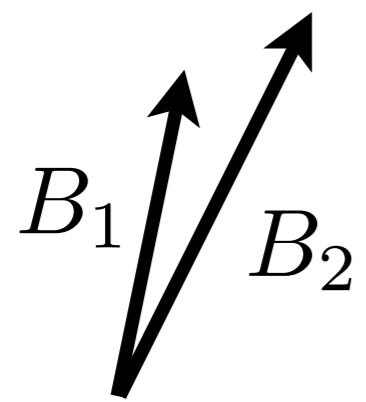
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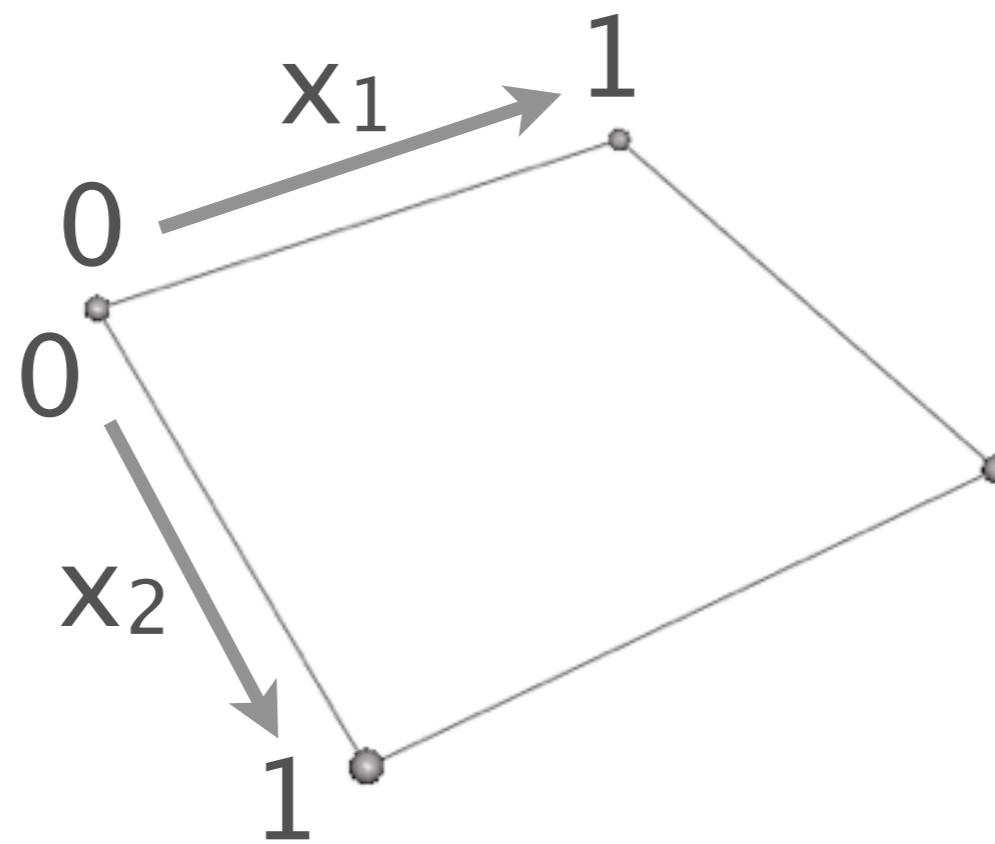
$$N = 2$$

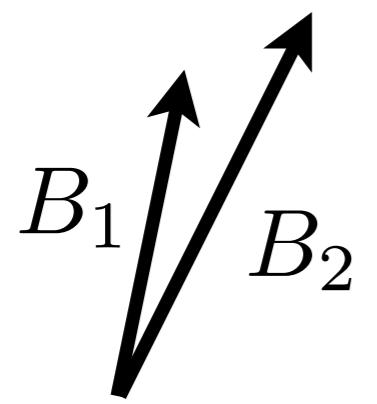




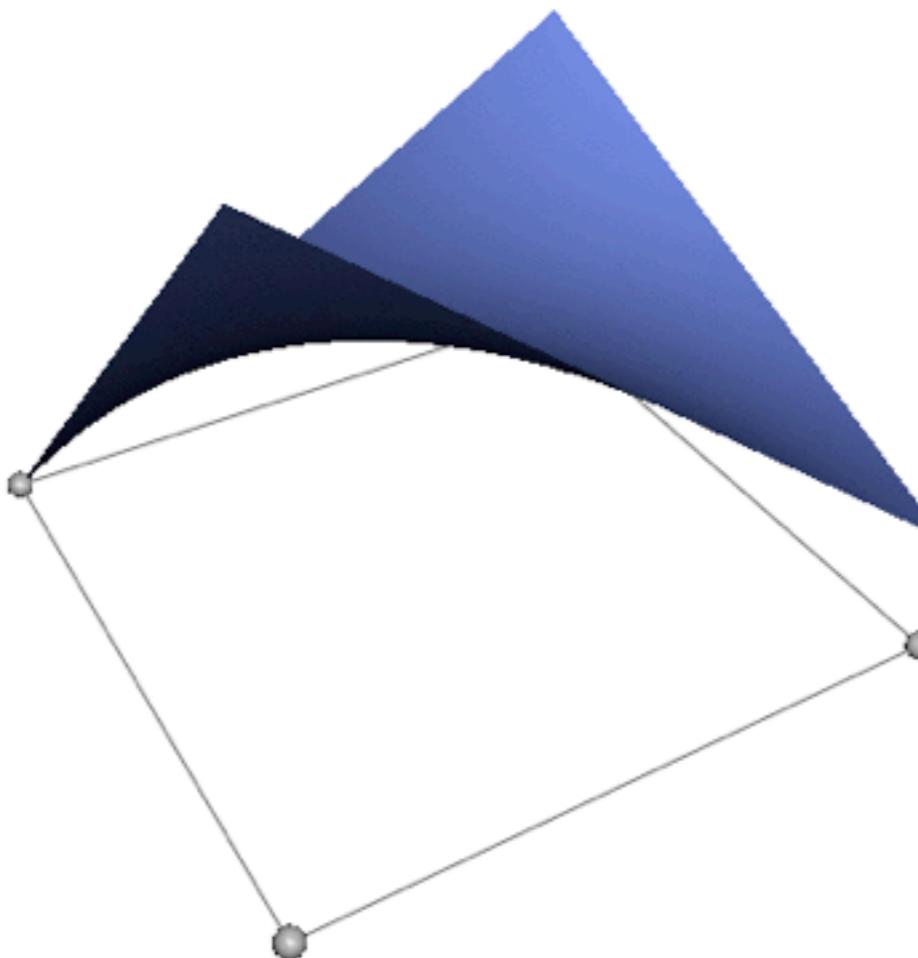


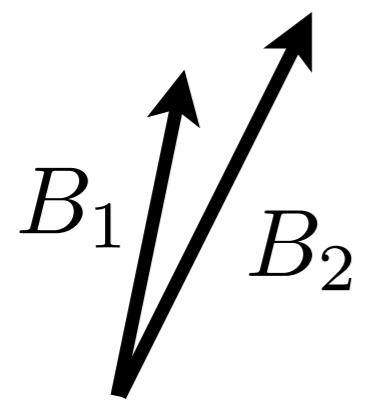
Relaxed domain



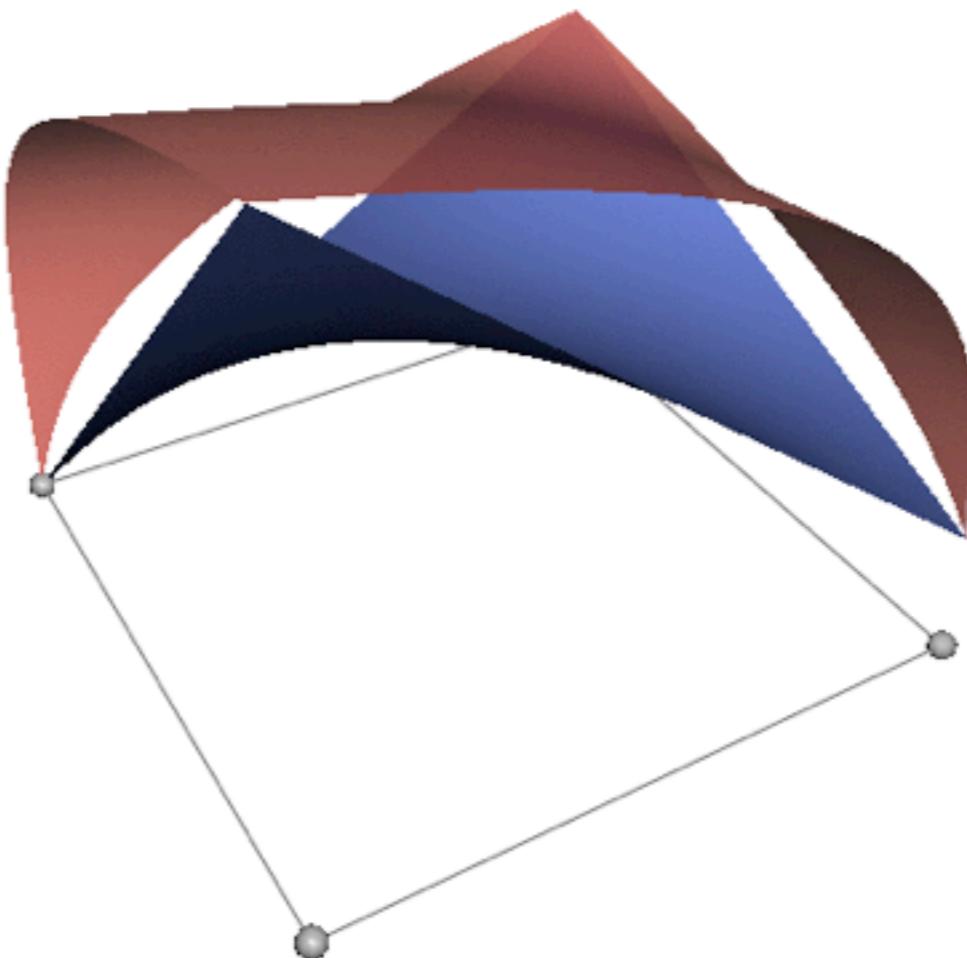


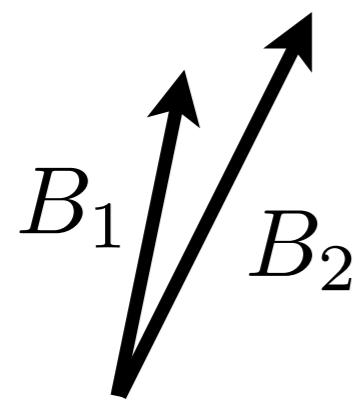
Multilinear extension



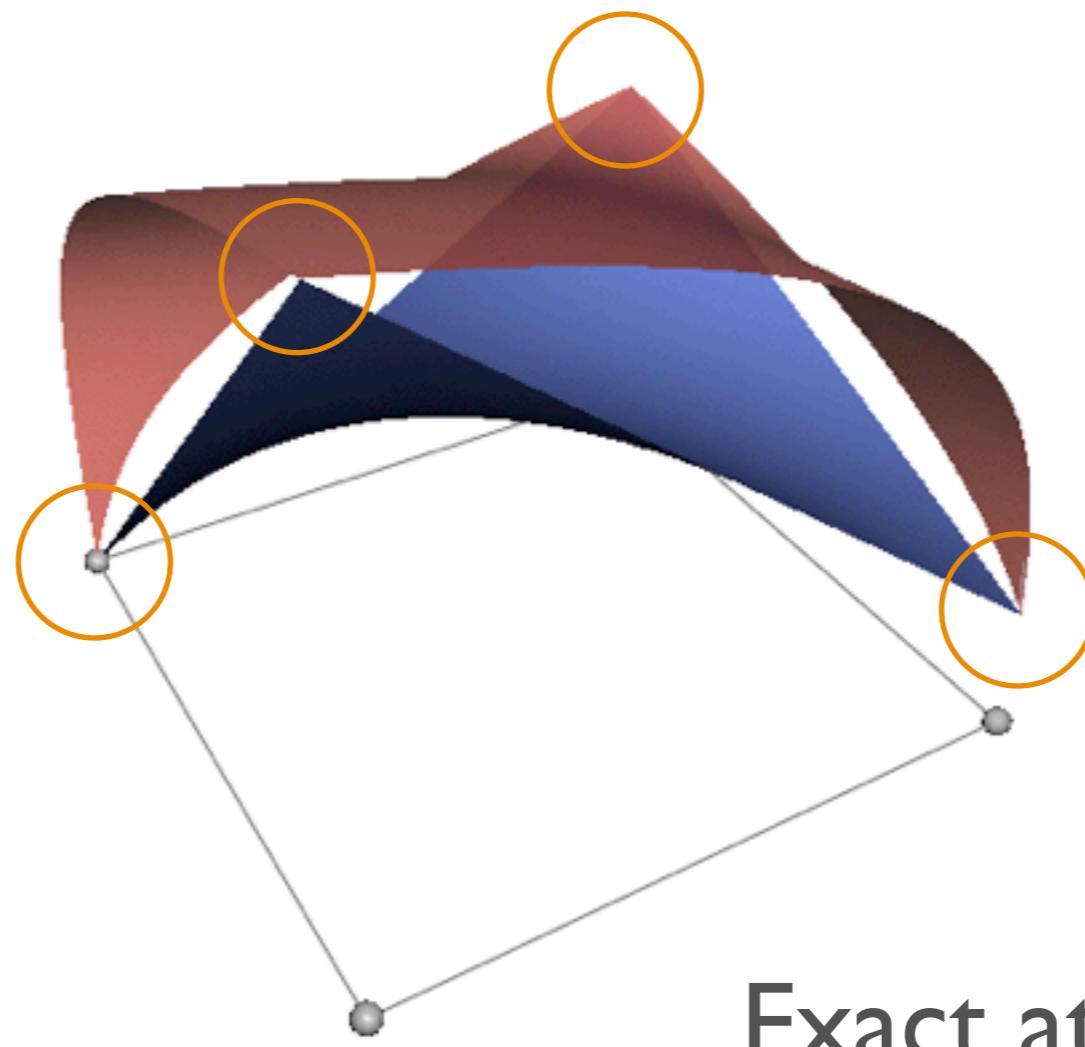


Softmax extension
Multilinear extension





Softmax extension
Multilinear extension



SOFTMAX EXTENSION

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$$\tilde{F}(\mathbf{x}) = \log \sum_Y \prod_{i \in Y} x_i \prod_{i \notin Y} (1 - x_i) f(Y)$$

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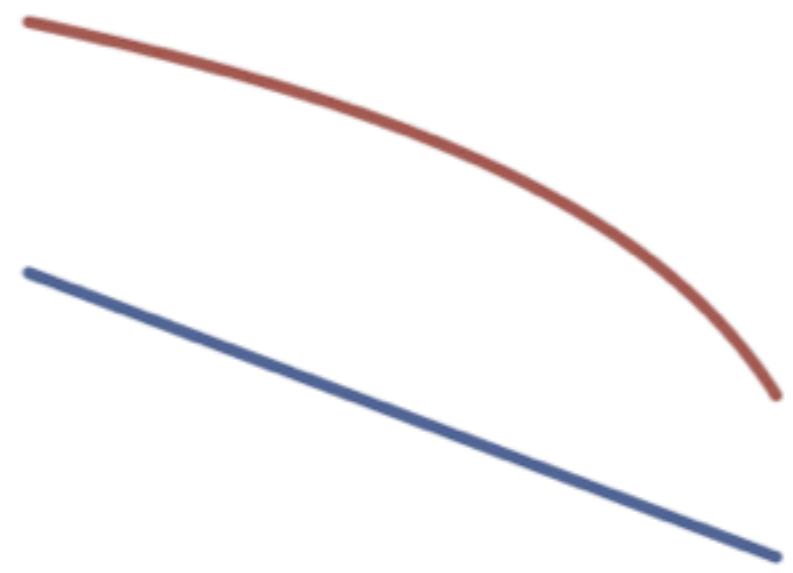
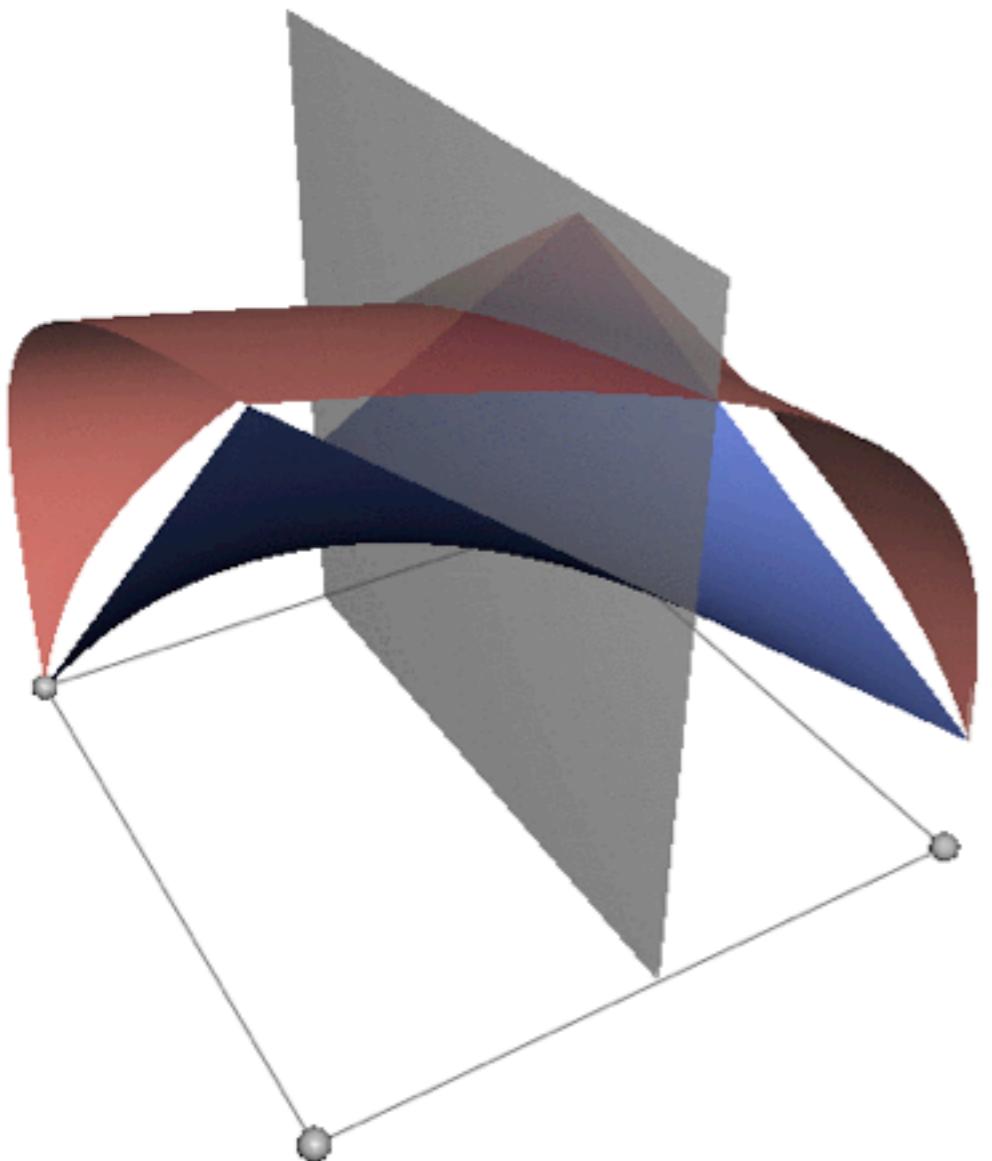
Theorem:

Efficiently computable for $f(Y) = \det(L_Y)$

$$O(N^3)$$

$$\tilde{F}(\mathbf{x}) = \log \det(\text{diag}(\mathbf{x})(L - I) + I)$$

Concave in all-positive/all-negative directions



APPROXIMATION GUARANTEE

Theorem: Concavity in all-positive directions

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+ Submodularity \longrightarrow

$$\text{LOCAL OPT of } \tilde{F} \geq \frac{1}{4} \max_{\mathbf{x}} \tilde{F}(\mathbf{x})$$

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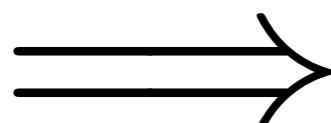
$$\text{LOCAL OPT of } \tilde{F} \geq \frac{1}{4} \max_{\mathbf{x}} \tilde{F}(\mathbf{x}) \geq \frac{1}{4} \max_Y \log \det(L_Y)$$

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Theorem: In the unconstrained case, LOCAL OPT will be integer (no rounding necessary).

Constrained: No guarantees, but in practice pipage
 $\max_{Y \in S}$ rounding and thresholding work well.

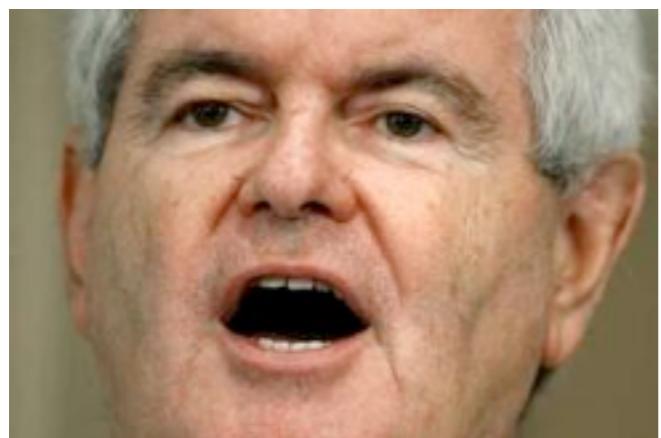
MATCHED SUMMARIZATION

MATCHED SUMMARIZATION

20 Republican primary debates

MATCHED SUMMARIZATION

20 Republican primary debates



MATCHED SUMMARIZATION

20 Republican primary debates



Average of 179 quotes per candidate

MATCHED SUMMARIZATION







‣ **R1 (taxes): No tax on interest, dividends, or capital gains.**



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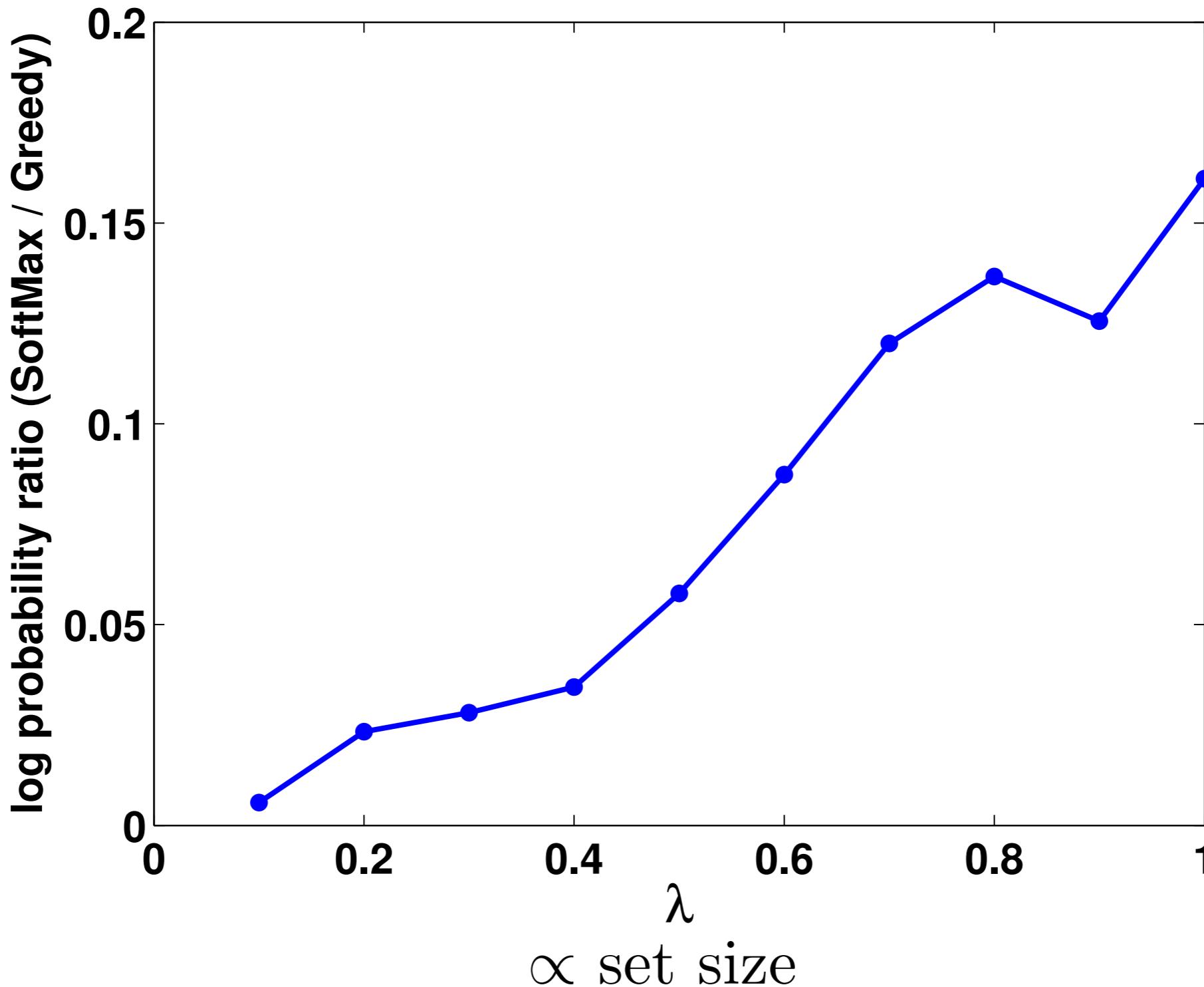
Matched summary

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PERFORMANCE



PROPOSED WORK

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- Update experiments with faster algorithms and testing with random restarts

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- k-DPP MAP estimation algorithm based on marginals, plus extension to structured k-DPPs

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3. LIKELIHOOD MAXIMIZATION

LIKELIHOOD

LIKELIHOOD

Y_1 ● ● ○ ○ ○

Y_2 ○ ● ● ● ○

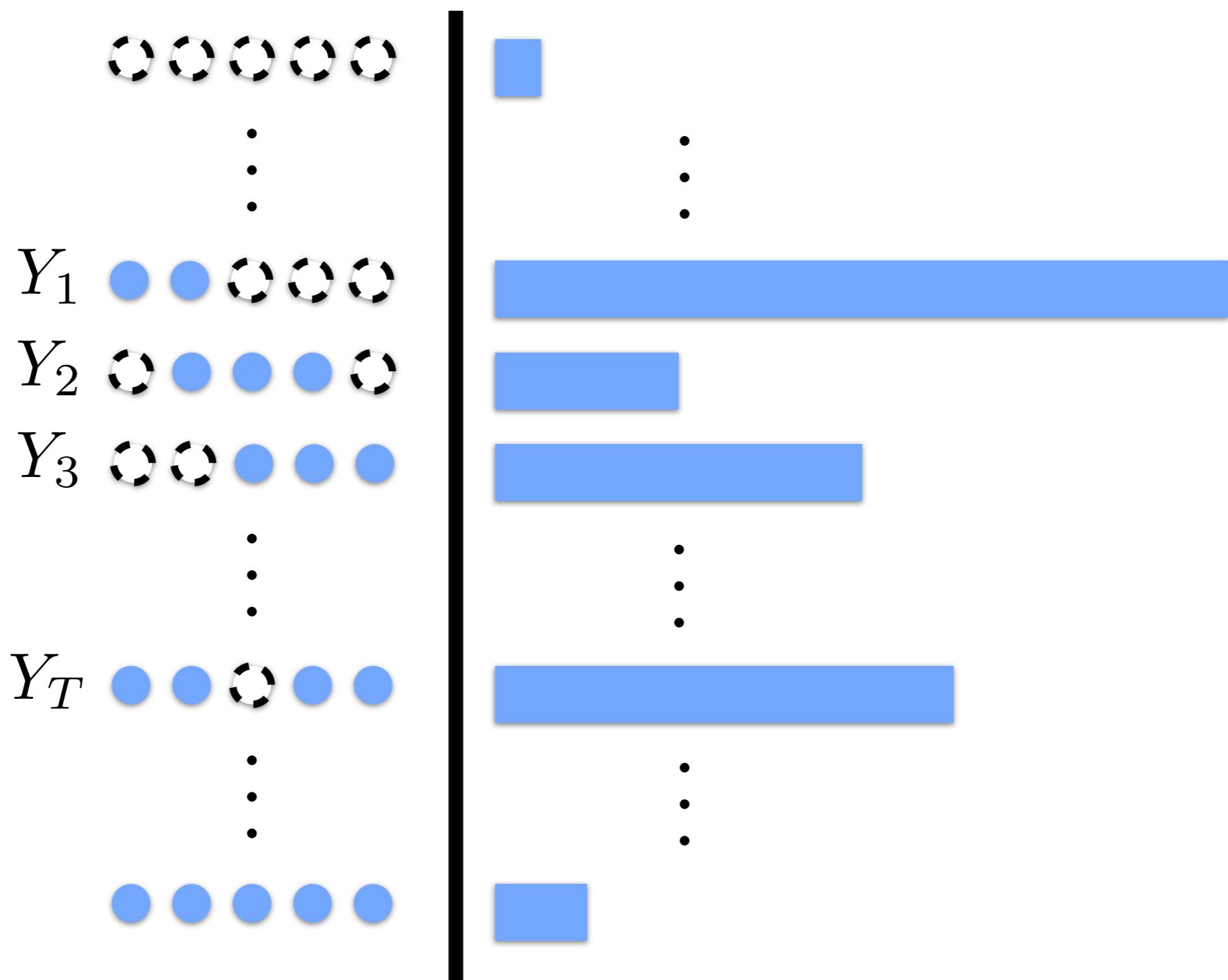
Y_3 ○ ○ ● ● ●

⋮
⋮

Y_T ● ● ○ ● ●

LIKELIHOOD

\mathcal{P} is a DPP



NON-CONCAVITY

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$$\mathcal{L}(L) = \sum_{t=1}^T [\log \det(L_{Y_t}) - \log \det(L + I)]$$

NON-CONCAVITY

$$\begin{aligned}\mathcal{L}(L) &= \sum_{t=1}^T [\log \det(L_{Y_t}) - \log \det(L + I)] \\ &\max_{L: L \succeq 0} \mathcal{L}(L)\end{aligned}$$

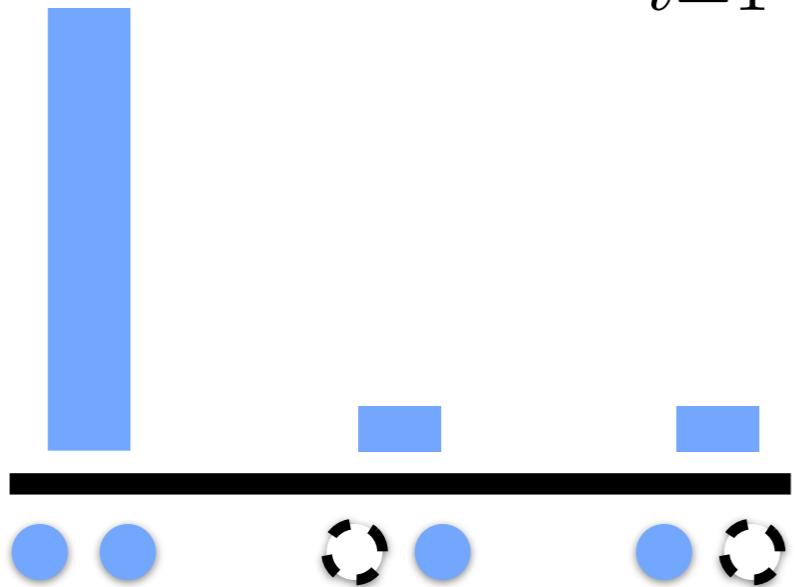
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NON-CONCAVITY

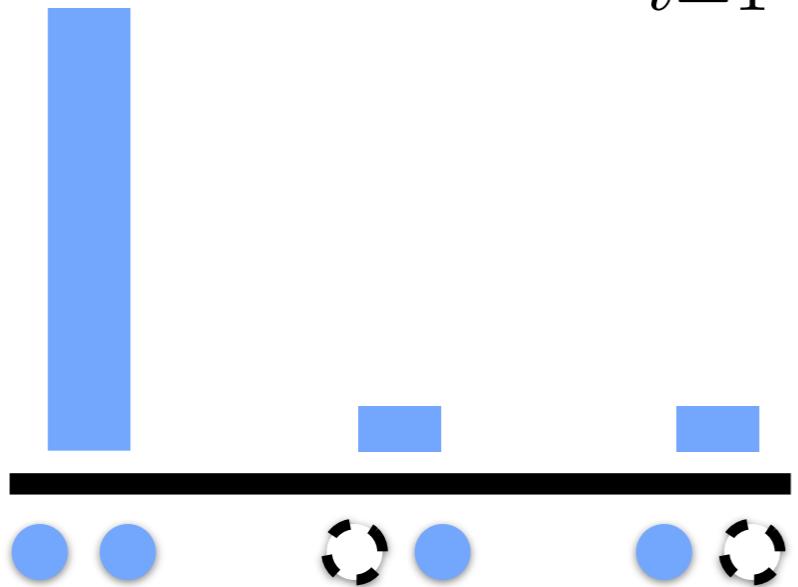
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$$L = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$$

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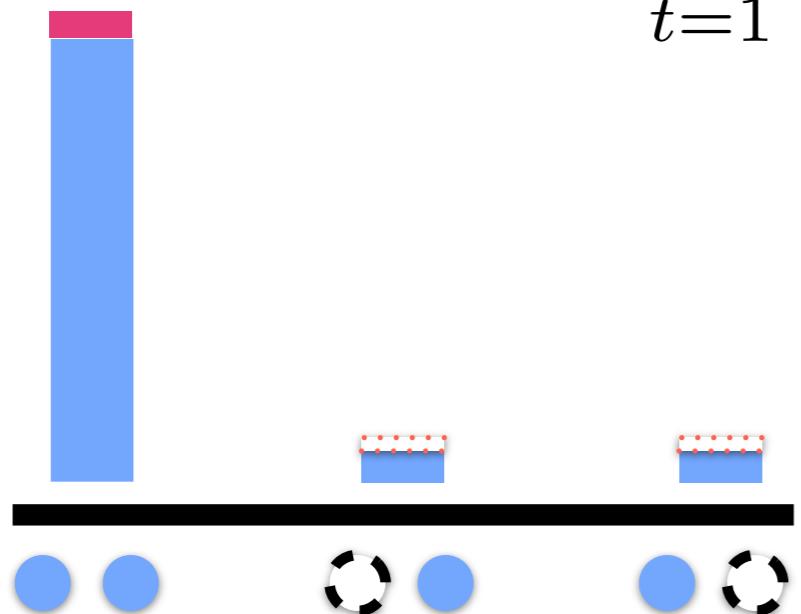


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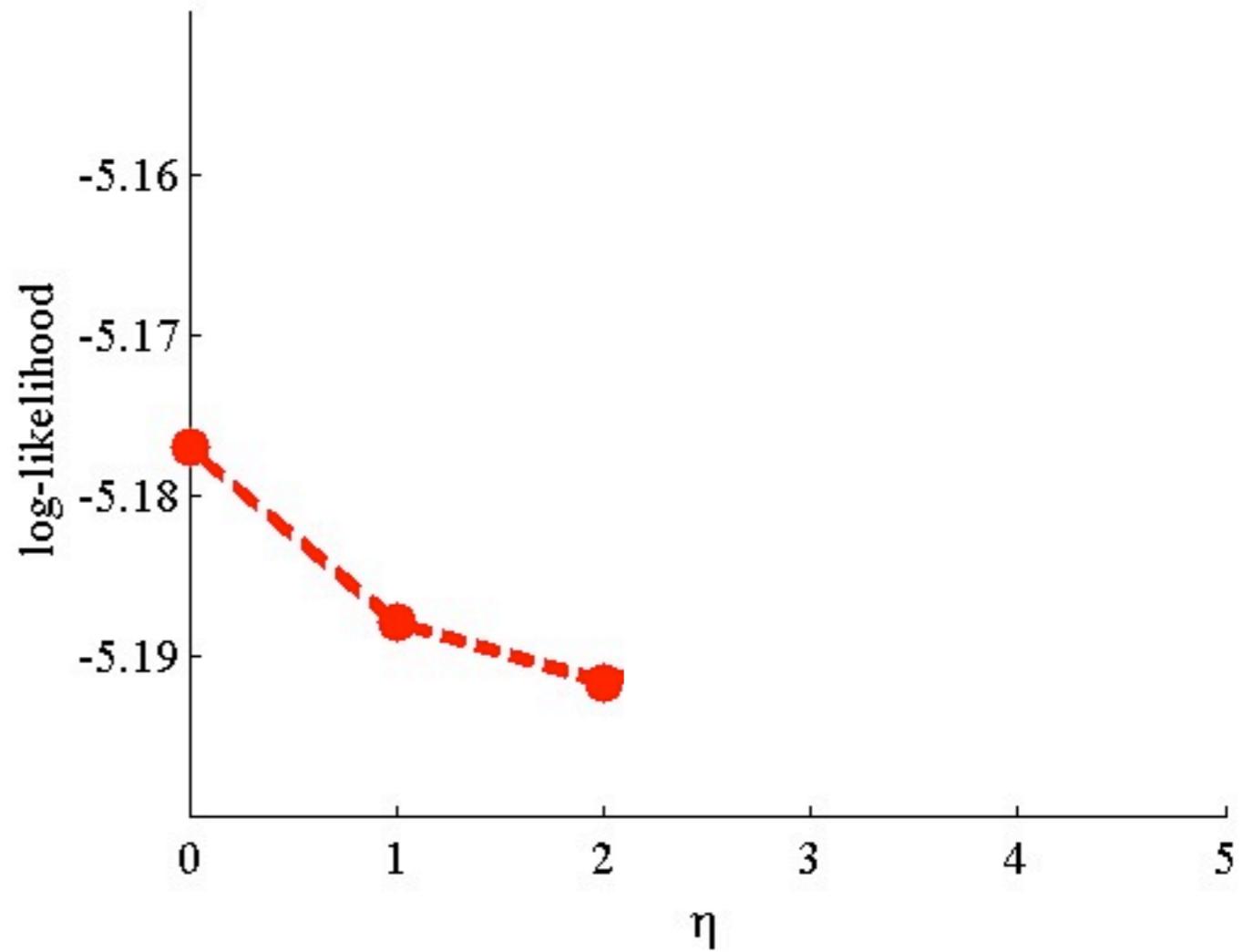
$$+ \eta \begin{bmatrix} 0.1 & 0.5 \\ 0.5 & 0.1 \end{bmatrix}$$

NON-CONCAVITY

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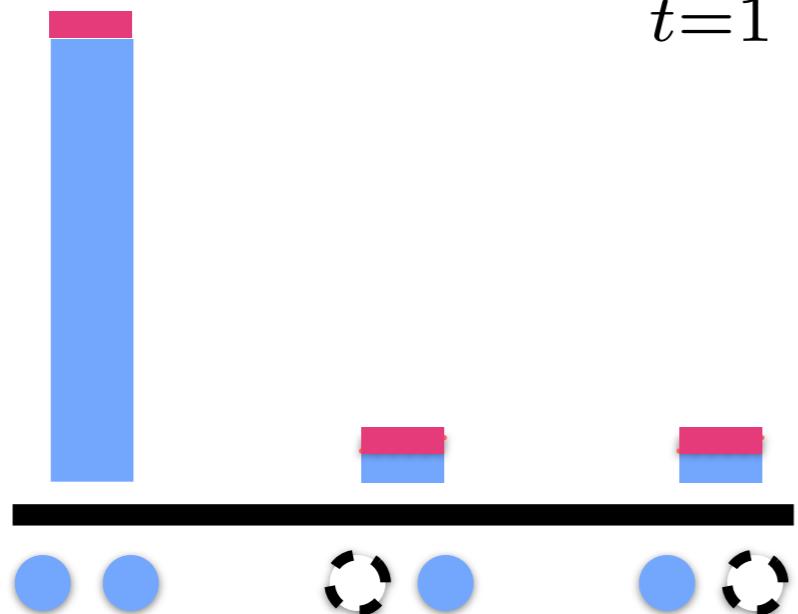


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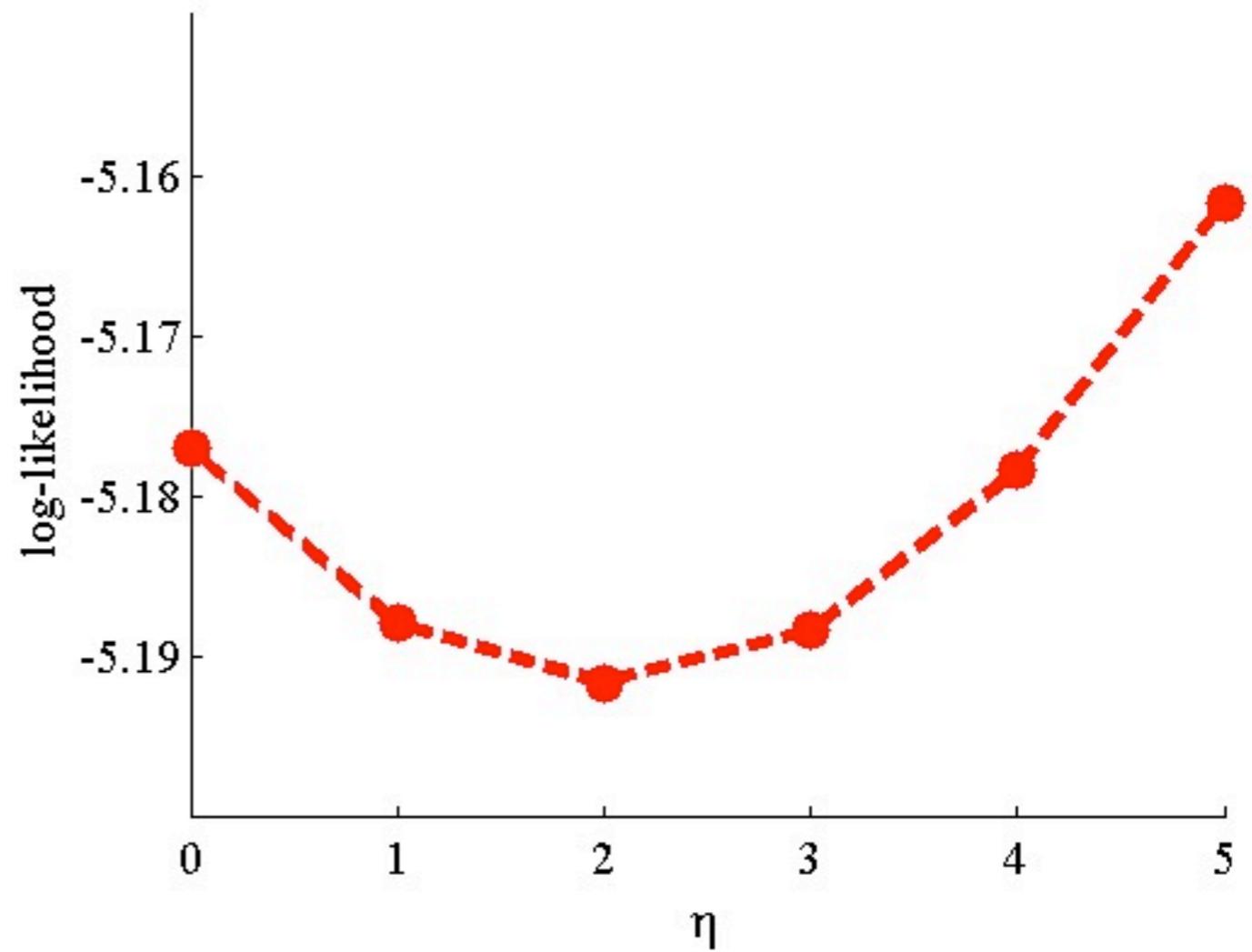


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K PARAMETERIZATION

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$$L = V \text{diag}(\lambda) V^\top$$

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$$L = V \text{diag}(\lambda) V^\top$$

$$K = V \text{diag}\left(\frac{\lambda}{1+\lambda}\right) V^\top$$

K PARAMETERIZATION

$$L = \textcolor{blue}{V} \mathrm{diag}(\textcolor{violet}{\lambda}) V^\top$$

$$K = \textcolor{blue}{V} \mathrm{diag}\big(\frac{\lambda}{1+\textcolor{violet}{\lambda}}\big) \textcolor{blue}{V}^\top$$

$$\mathcal{L}(K) = \sum_{t=1}^T \log |\det(K - I_{\overline{Y}_t})|$$

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$$\max_{\begin{array}{c} K: K \succeq 0, \\ I-K \succeq 0 \end{array}} \mathcal{L}(K)$$

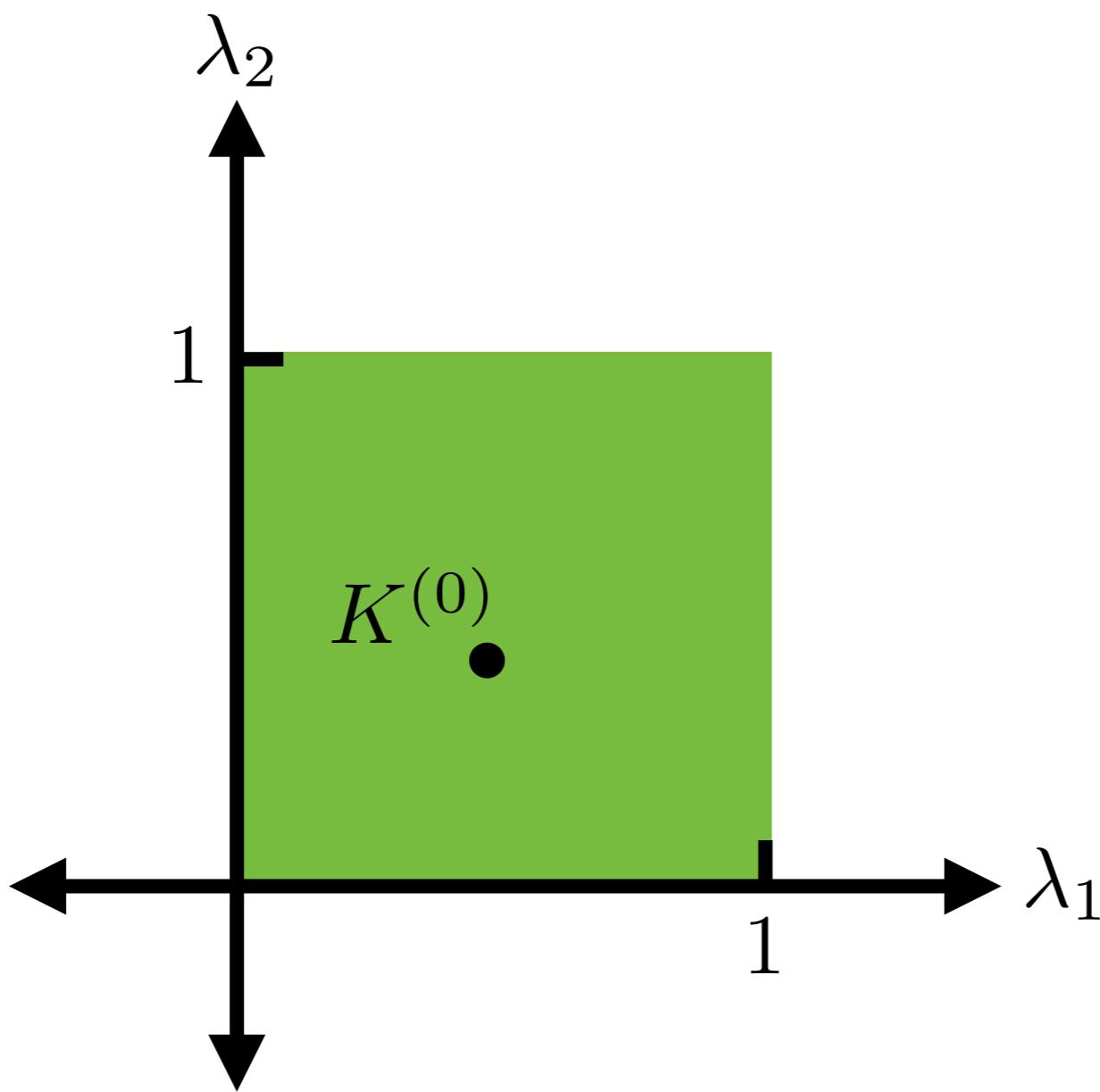
GRADIENT ASCENT

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$$\frac{\partial \mathcal{L}(K)}{\partial K} = \sum_{t=1}^T (K - I_{\bar{Y}_t})^{-1}$$

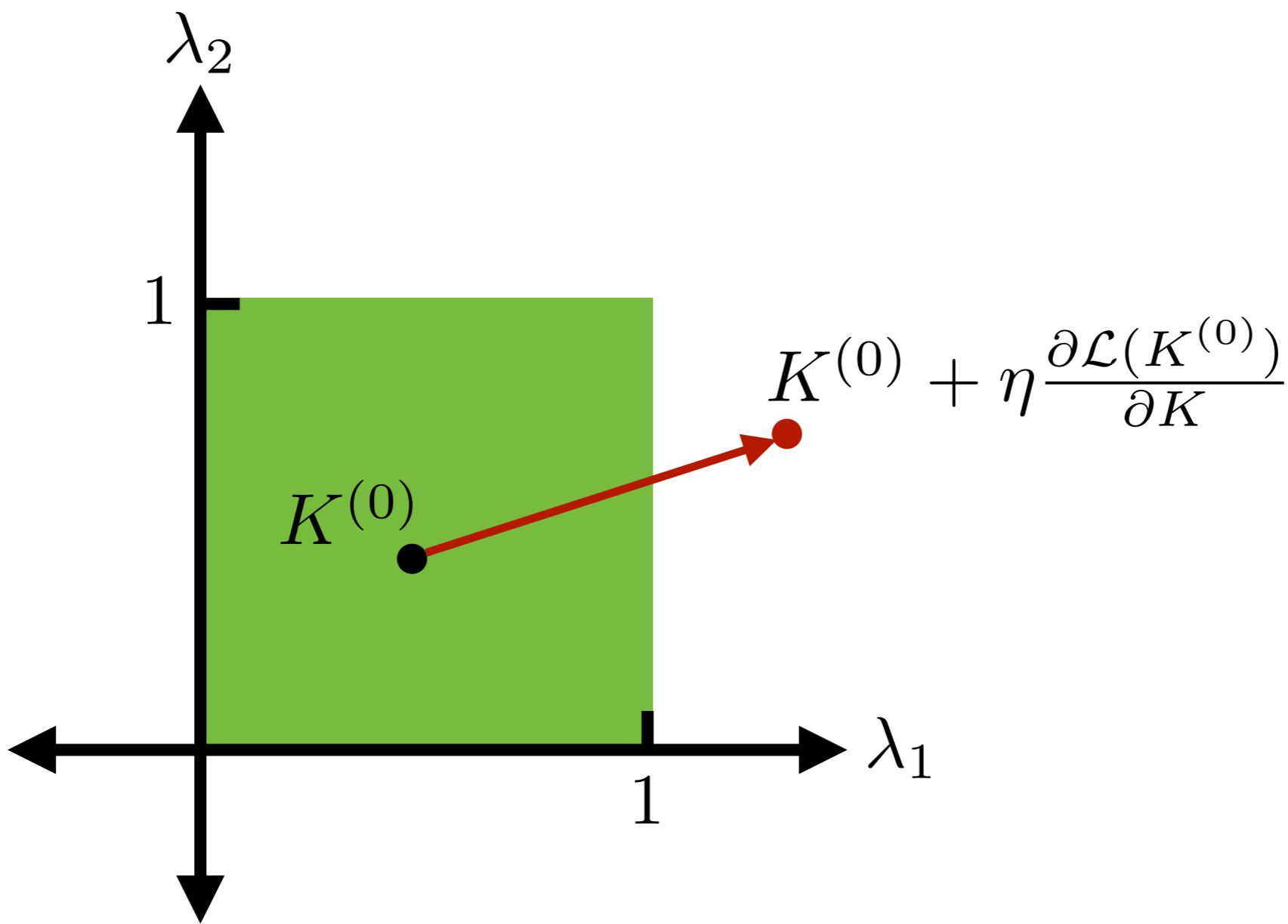
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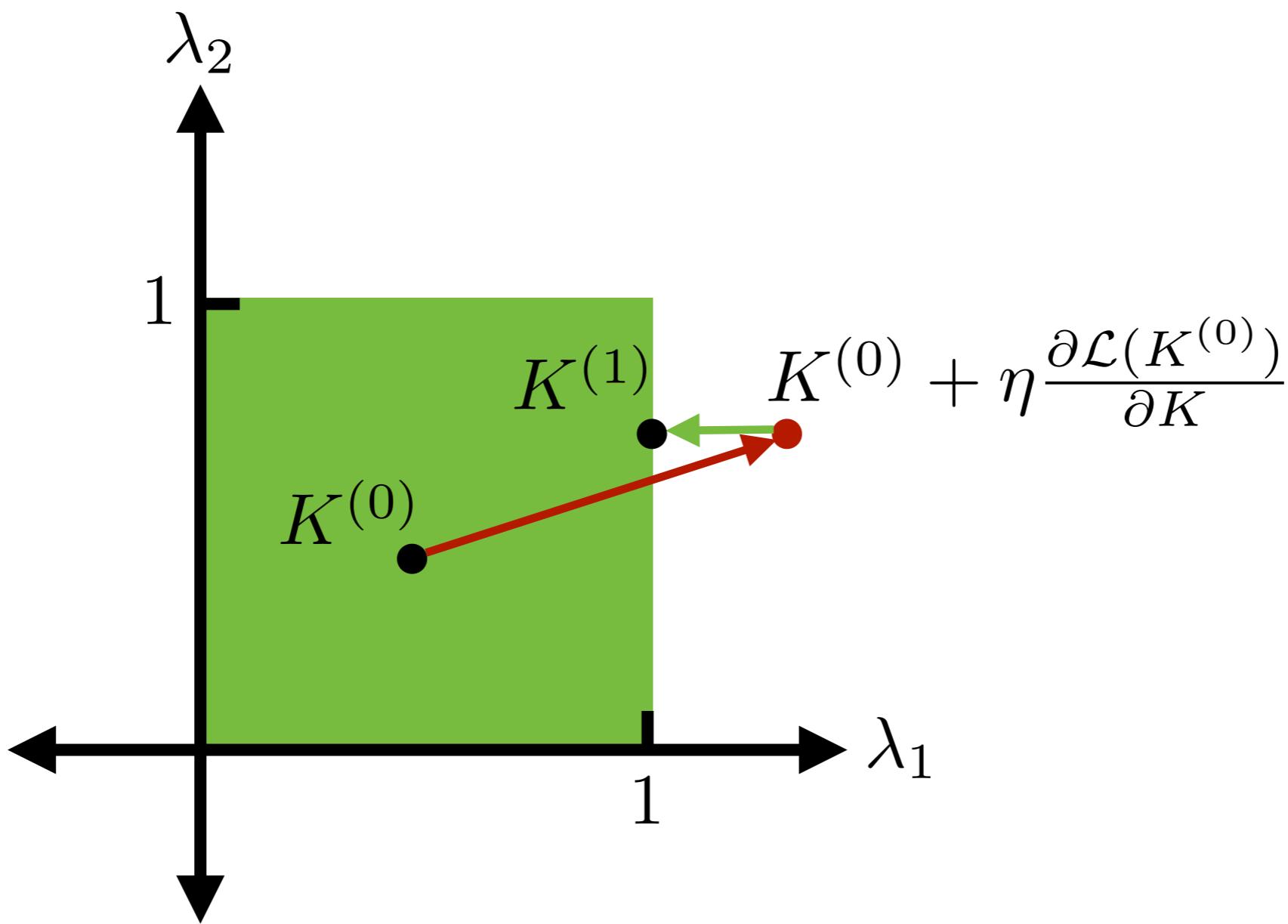
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EXPECTATION-MAXIMIZATION

GILLENWATER, KULESZA, FOX, AND TASKAR (NIPS 2014)

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$$K = V \Lambda V^\top$$

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$$K = \mathcal{V} \Lambda \mathcal{V}^\top$$

$$K^J = \mathcal{V}^J \text{diag}(\mathbf{1}) (\mathcal{V}^J)^\top$$

EXPECTATION-MAXIMIZATION

GILLENWATER, KULESZA, FOX, AND TASKAR (NIPS 2014)

$$K = \textcolor{blue}{V} \Lambda \textcolor{blue}{V}^\top$$

$$K^{\textcolor{red}{J}} = \textcolor{blue}{V}^{\textcolor{red}{J}} \text{diag}(\mathbf{1}) (\textcolor{blue}{V}^{\textcolor{red}{J}})^\top$$

$$\mathcal{P}(Y) = \frac{\det(L_Y)}{\det(L + I)} = |\det(K - I_{\bar{Y}})|$$

EXPECTATION-MAXIMIZATION

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$$K = \textcolor{blue}{V} \Lambda \textcolor{blue}{V}^\top$$

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$$\begin{aligned} \mathcal{P}(Y) &= \frac{\det(L_Y)}{\det(L + I)} = |\det(K - I_{\bar{Y}})| \\ &= \sum_{\textcolor{red}{J}: \textcolor{red}{J} \subseteq \{1, \dots, N\}} \mathcal{P}_{K^{\textcolor{red}{J}}}(Y) \prod_{j:j \in \textcolor{red}{J}} \lambda_j \prod_{j:j \notin \textcolor{red}{J}} (1 - \lambda_j) \end{aligned}$$

EXPECTATION-MAXIMIZATION

GILLENWATER, KULESZA, FOX, AND TASKAR (NIPS 2014)

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Hidden variable $\textcolor{red}{J}$

EXPECTATION-MAXIMIZATION

GILLENWATER, KULESZA, FOX, AND TASKAR (NIPS 2014)

$$K = \mathcal{V} \Lambda \mathcal{V}^\top$$

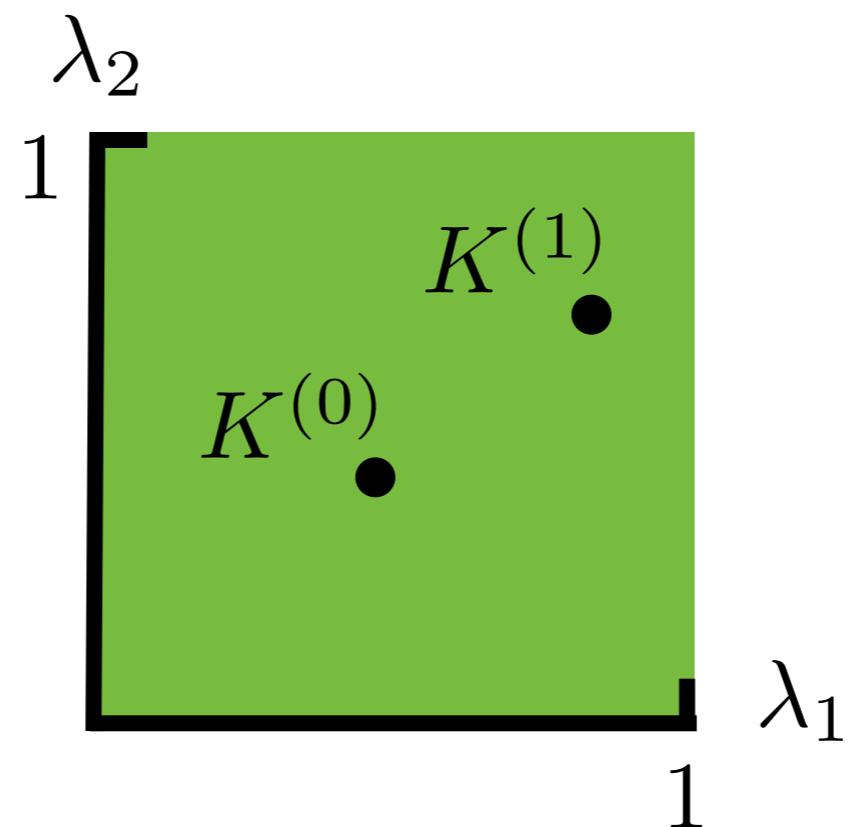
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Jensen's inequality  $F(\mathcal{V}, \Lambda) \leq \mathcal{L}(\mathcal{V}, \Lambda)$

EXPECTATION-MAXIMIZATION

GILLENWATER, KULESZA, FOX, AND TASKAR (NIPS 2014)



Jensen's inequality → $F(\textcolor{blue}{V}, \textcolor{green}{\Lambda}) \leq \mathcal{L}(\textcolor{blue}{V}, \textcolor{green}{\Lambda})$

PRODUCT RECOMMENDATION

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~ 30,000 Amazon baby registries

PRODUCT RECOMMENDATION

~ 30,000 Amazon baby registries

13 categories

PRODUCT RECOMMENDATION

~ 30,000 Amazon baby registries

13 categories



furniture

PRODUCT RECOMMENDATION

~ 30,000 Amazon baby registries

13 categories



furniture



carseats

PRODUCT RECOMMENDATION

~ 30,000 Amazon baby registries

13 categories



furniture



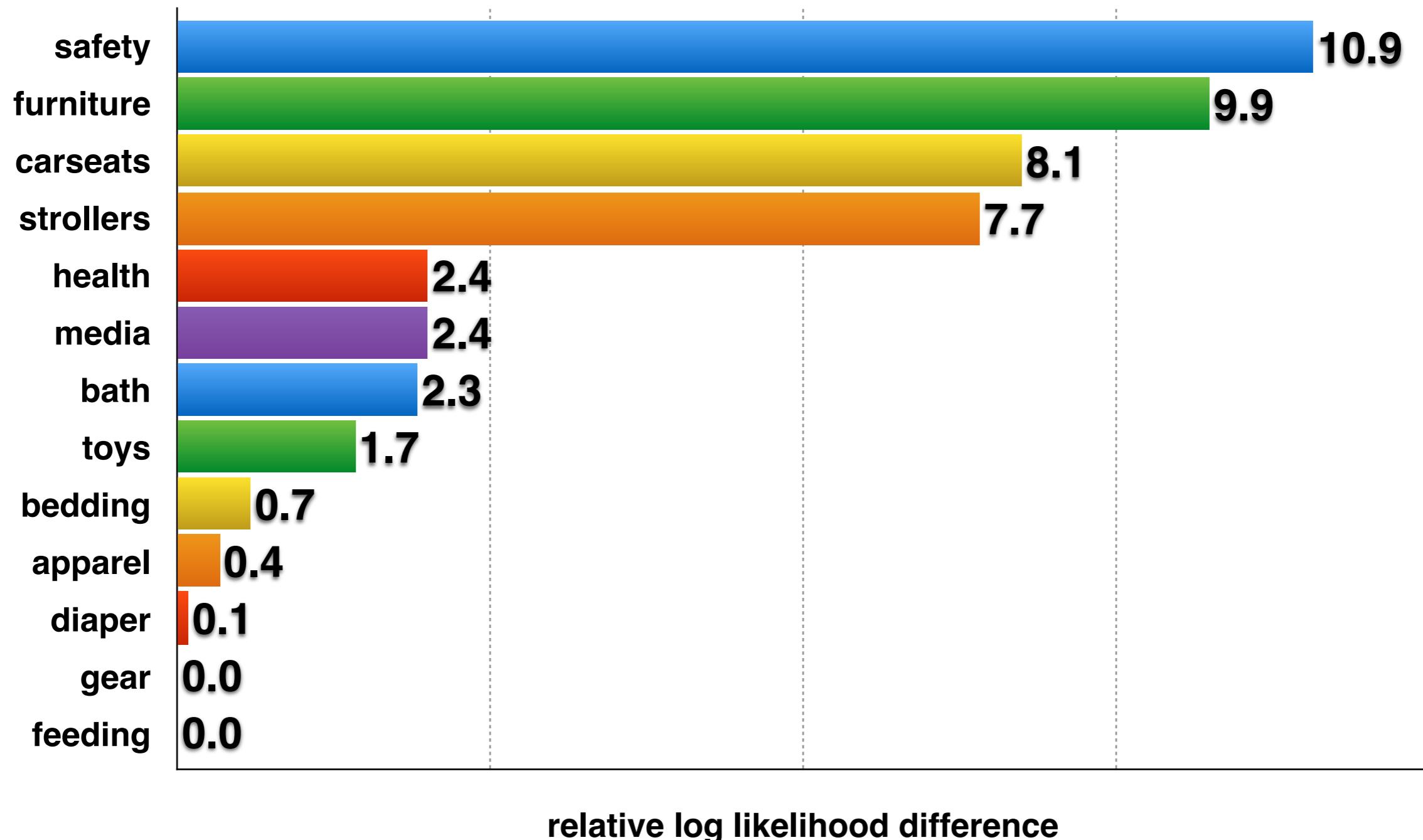
carseats



toys

EM VS PROJECTED GRADIENT

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“SAFETY” SELECTION

“SAFETY” SELECTION

Graco Sweet Slumber
Sound Machine



Boppy Noggin Nest
Head Support



Cloud b Twilight
Night Light



Braun ThermoScan
Lens Filters



Aquatopia Bath
Thermometer Alarm



Britax EZ-Cling
Sun Shades



TL Care Organic
Cotton Mittens



Regalo Easy Step
Walk Thru Gate



VTech Comm.
Audio Monitor



Infant Optics
Video Monitor

“SAFETY” SELECTION

Graco Sweet Slumber Sound Machine



Motorola Video Monitor



Cloud b Twilight Night Light



Summer Infant Video Monitor



Aquatopia Bath Thermometer Alarm



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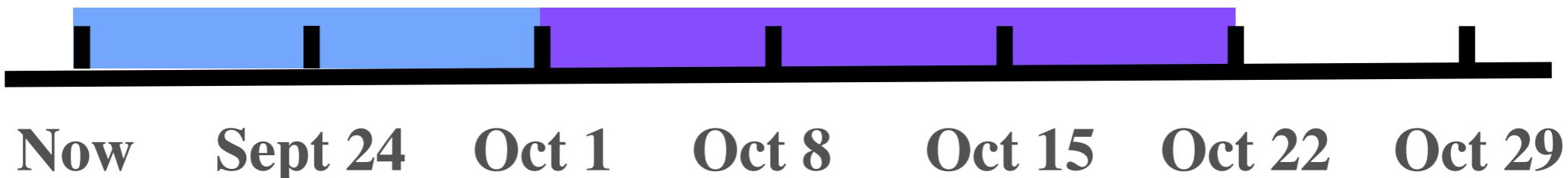
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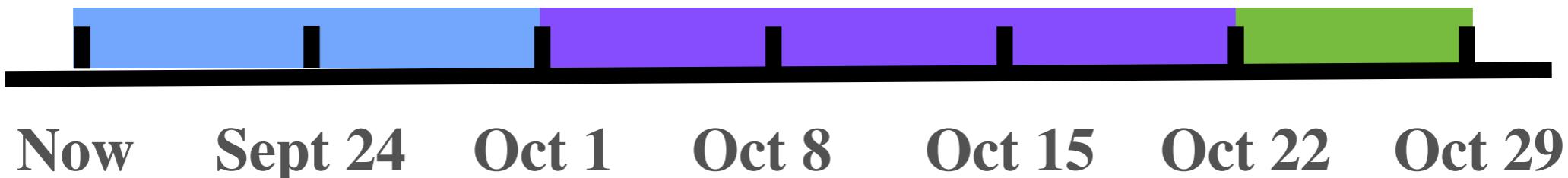
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