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SDS 383 D - Statistical Modeling 2
Homework

Exercise 1

$X_1, \dots X_n \sim Bernoulli(p), \quad h(p) \sim Beta(\alpha, \beta)$

$$\begin{aligned} p|X_1, \dots X_N \propto & f(x_1, \dots x_n|p) \cdot h(p) \\ \propto & p^{\sum x_i}(1-p)^{n-\sum x_i} \cdot p^{\alpha-1}(1-p)^{\beta-1} \\ \propto & p^{\sum x_i+\alpha-1}(1-p)^{\beta+n-\sum x_i-1} \end{aligned}$$

$$p|X_1, \dots X_N \sim Beta(\sum x_i + \alpha, \beta + n - \sum x_i)$$

Exercise 2

$X_1, \dots X_N \overset{iid}{\sim} Cat(p), \quad p = (p_1, \dots p_K), \quad f(x|p) = \prod_{i=1}^{n} \prod_{k=1}^{K} p_k^{I(x_i=k)}$
$p \sim Dirichlet(\alpha_1, \dots \alpha_K)$

$$\pi(p) = \frac{\Gamma(\sum_k \alpha_k)}{\prod_K \Gamma(\alpha_k)} \prod_{k=1}^{K} p^{\alpha_k-1} I(\sum_{k=1}^{K-1} p_k \leq 1, p_k \geq 0)$$

$$\begin{aligned} p|X_{1:n} \propto & f(x_{1:n}|p) \cdot \pi(p) \\ \propto & \prod_{i=1}^{n} \prod_{k=1}^{K} p_k^{I(x_i=k)} \cdot \prod_{k=1}^{K} p_k^{\alpha_k-1} \\ \propto & \prod_{k=1}^{K} p_K^{n_k+\alpha_k-1}, \quad \text{where } n_k = \sum_{i=1}^{N} I(x_i = k) \\ p|X_{1:n} \sim & Dir(n_1 + \alpha_1, \dots n_k + \alpha_k) \end{aligned}$$