rates in the following queuing systems? Defend your position in each case.
(a) cafeteria in your school
(b) barbershop
(c) hardware store
(d) dentist's office
(e) college class
(f) movie theater

Problems*

13-10 The Schmedley Discount Department Store has approximately 300 customers shopping in its store between 9 A.M. and 5 P.M. on Saturdays. In deciding how many cash registers to keep open each Saturday, Schmedley's manager considers two factors: customer waiting time (and the associated waiting cost) and the service costs of employing additional checkout clerks. Checkout clerks are paid an average of $8 per hour. When only one is on duty, the waiting time per customer is about 10 minutes (or $\frac{1}{6}$ hour); when two clerks are on duty, the average checkout time is 6 minutes per person; 4 minutes when three clerks are working; and 3 minutes when four clerks are on duty.

Schmedley's management has conducted customer satisfaction surveys and has been able to estimate that the store suffers approximately $10 in lost sales and goodwill for every *hour* of customer time spent waiting in checkout lines. Using the information provided, determine the optimal number of clerks to have on duty each Saturday to minimize the store's total expected cost.

13-11 The Rockwell Electronics Corporation retains a service crew to repair machine breakdowns that occur on an average of $\lambda = 3$ per day (approximately Poisson in nature).

The crew can service an average of $\mu = 8$ machines per day, with a repair time distribution that resembles the exponential distribution.
(a) What is the utilization rate of this service system?
(b) What is the average downtime for a machine that is broken?
(c) How many machines are waiting to be serviced at any given time?
(d) What is the probability that more than one machine is in the system? Probability that more than two are broken and waiting to be repaired or being serviced? More than three? More than four?

13-12 From historical data, Harry's Car Wash estimates that dirty cars arrive at the rate of 10 per hour all day Saturday. With a crew working the wash line, Harry figures that cars can be cleaned at the rate of one every 5 minutes. One car at a time is cleaned in this example of a single-channel waiting line.

Assuming Poisson arrivals and exponential service times, find the
(a) average number of cars in line.
(b) average time a car waits before it is washed.
(c) average time a car spends in the service system.
(d) utilization rate of the car wash.
(e) probability that no cars are in the system.

13-13 Mike Dreskin manages a large Los Angeles movie theater complex called Cinema I, II, III, and IV. Each of the four auditoriums plays a different film; the schedule is set so that starting times are staggered to avoid the large crowds that would occur if all four movies started at the same time. The theater has a single ticket booth and a cashier who can maintain an average service rate of 280 movie patrons per hour. Service times are assumed to follow an exponential distribution. Arrivals on a typically active day are Poisson distributed and average 210 per hour.

To determine the efficiency of the current ticket operation, Mike wishes to examine several queue operating characteristics.
(a) Find the average number of moviegoers waiting in line to purchase a ticket.
(b) What percentage of the time is the cashier busy?
(c) What is the average time that a customer spends in the system?
(d) What is the average time spent waiting in line to get to the ticket window?
(e) What is the probability that there are more than two people in the system? More than three people? More than four?

13-14 A university cafeteria line in the student center is a self-serve facility in which students select the food items they want and then form a single line to pay the cashier. Students arrive at a rate of about four per minute according to a Poisson distribution. The single cashier ringing up sales takes about 12 seconds per customer, following an exponential distribution.
(a) What is the probability that there are more than two students in the system? More than three students? More than four?
(b) What is the probability that the system is empty?
(c) How long will the average student have to wait before reaching the cashier?
(d) What is the expected number of students in the queue?
(e) What is the average number in the system?
(f) If a second cashier is added (who works at the same pace), how will the operating characteristics computed in parts (b), (c), (d), and (e) change? Assume that customers wait in a single line and go to the first available cashier.

*Note: means the problem may be solved with QM for Windows; means the problem may be solved with Excel QM; and means the problem may be solved with QM for Windows and/or Excel QM.

13-15 The wheat harvesting season in the American Midwest is short, and most farmers deliver their truckloads of wheat to a giant central storage bin within a two-week span. Because of this, wheat-filled trucks waiting to unload and return to the fields have been known to back up for a block at the receiving bin. The central bin is owned cooperatively, and it is to every farmer's benefit to make the unloading/storage process as efficient as possible. The cost of grain deterioration caused by unloading delays, the cost of truck rental, and idle driver time are significant concerns to the cooperative members. Although farmers have difficulty quantifying crop damage, it is easy to assign a waiting and unloading cost for truck and driver of $18 per hour. The storage bin is open and operated 16 hours per day, 7 days per week, during the harvest season and is capable of unloading 35 trucks per hour according to an exponential distribution. Full trucks arrive all day long (during the hours the bin is open) at a rate of about 30 per hour, following a Poisson pattern.

To help the cooperative get a handle on the problem of lost time while trucks are waiting in line or unloading at the bin, find the

(a) average number of trucks in the unloading system.
(b) average time per truck in the system.
(c) utilization rate for the bin area.
(d) probability that there are more than three trucks in the system at any given time.
(e) total daily cost to the farmers of having their trucks tied up in the unloading process.

The cooperative, as mentioned, uses the storage bin only two weeks per year. Farmers estimate that enlarging the bin would cut unloading costs by 50% next year. It will cost $9,000 to do so during the off-season. Would it be worth the cooperative's while to enlarge the storage area?

13-16 Ashley's Department Store in Kansas City maintains a successful catalog sales department in which a clerk takes orders by telephone. If the clerk is occupied on one line, incoming phone calls to the catalog department are answered automatically by a recording machine and asked to wait. As soon as the clerk is free, the party that has waited the longest is transferred and answered first. Calls come in at a rate of about 12 per hour. The clerk is capable of taking an order in an average of 4 minutes. Calls tend to follow a Poisson distribution, and service times tend to be exponential. The clerk is paid $10 per hour, but because of lost goodwill and sales, Ashley's loses about $50 per hour of customer time spent waiting for the clerk to take an order.

(a) What is the average time that catalog customers must wait before their calls are transferred to the order clerk?
(b) What is the average number of callers waiting to place an order?
(c) Ashley's is considering adding a second clerk to take calls. The store would pay that person the same $10 per hour. Should it hire another clerk? Explain.

13-17 Automobiles arrive at the drive-through window at a post office at the rate of 4 every 10 minutes. The average service time is 2 minutes. The Poisson distribution is appropriate for the arrival rate and service times are exponentially distributed.

(a) What is the average time a car is in the system?
(b) What is the average number of cars in the system?
(c) What is the average time cars spend waiting to receive service?
(d) What is the average number of cars in line *behind* the customer receiving service?
(e) What is the probability that there are no cars at the window?
(f) What percentage of the time is the postal clerk busy?
(g) What is the probability that there are exactly two cars in the system?

13-18 For the post office in Problem 13-17, a second drive-through window is being considered. A single line would be formed and as a car reached the front of the line it would go to the next available clerk. The clerk at the new window works at the same rate as the current one.

(a) What is the average time a car is in the system?
(b) What is the average number of cars in the system?
(c) What is the average time cars spend waiting to receive service?
(d) What is the average number of cars in line *behind* the customer receiving service?
(e) What is the probability that there are no cars in the system?
(f) What percentage of the time are the clerks busy?
(g) What is the probability that there are exactly two cars in the system?

13-19 Juhn and Sons Wholesale Fruit Distributors employ one worker whose job is to load fruit on outgoing company trucks. Trucks arrive at the loading gate at an average of 24 per day, or 3 per hour, according to a Poisson distribution. The worker loads them at a rate of 4 per hour, following approximately the exponential distribution in service times.

Determine the operating characteristics of this loading gate problem. What is the probability that there will be more than three trucks either being loaded or waiting? Discuss the results of your queuing model computation.

13-20 Juhn believes that adding a second fruit loader will substantially improve the firm's efficiency. He estimates that a two-person crew, still acting like a single-server system, at the loading gate will double the loading rate from 4 trucks per hour to 8 trucks

per hour. Analyze the effect on the queue of such a change and compare the results with those found in Problem 13-19.

13-21 Truck drivers working for Juhn and Sons (see Problems 13-19 and 13-20) are paid a salary of $10 per hour on average. Fruit loaders receive about $6 per hour. Truck drivers waiting in the queue or at the loading gate are drawing a salary but are productively idle and unable to generate revenue during that time. What would be the *hourly* cost savings to the firm associated with employing two loaders instead of one?

13-22 Juhn and Sons Wholesale Fruit Distributors (of Problem 13-19) are considering building a second platform or gate to speed the process of loading their fruit trucks. This, they think, will be even more efficient than simply hiring another loader to help out the first platform (as in Problem 13-20).

Assume that workers at each platform will be able to load 4 trucks per hour each and that trucks will continue to arrive at the rate of 3 per hour. Find the waiting line's new operating conditions. Is this new approach indeed speedier than the other two considered?

13-23 Bill First, general manager of Worthmore Department Store, has estimated that every hour of customer time spent waiting in line for the sales clerk to become available costs the store $100 in lost sales and goodwill. Customers arrive at the checkout counter at the rate of 30 per hour, and the average service time is 3 minutes. The Poisson distribution describes the arrivals and the service times are exponentially distributed. The number of sales clerks can be 2, 3, or 4, with each one working at the same rate. Bill estimates the salary and benefits for each clerk to be $10 per hour. The store is open 10 hours per day.

(a) Find the average time in the line if 2, 3, and 4 clerks are used.
(b) What is the total time spent waiting in line each day if 2, 3, and 4 clerks are used?
(c) Calculate the total of the daily waiting cost and the service cost if 2, 3, and 4 clerks are used. What is the minimum total daily cost?

13-24 Billy's Bank is the only bank in a small town in Arkansas. On a typical Friday, an average of 10 customers per hour arrive at the bank to transact business. There is one single teller at the bank, and the average time required to transact business is 4 minutes. It is assumed that service times can be described by the exponential distribution. Although this is the only bank in town, some people in the town have begun using the bank in a neighboring town about 20 miles away. A single line would be used, and the customer at the front of the line would go to the first available bank teller. If a single teller at Billy's is used, find

(a) the average time in the line.
(b) the average number in the line.
(c) the average time in the system.
(d) the average number in the system.
(e) the probability that the bank is empty.

13-25 Refer to the Billy's Bank situation in Problem 13-24. Billy is considering adding a second teller (who would work at the same rate as the first) to reduce the waiting time for customers, and he assumes that this will cut the waiting time in half. If a second teller is added, find

(a) the average time in the line.
(b) the average number in the line.
(c) the average time in the system.
(d) the average number in the system.
(e) the probability that the bank is empty.

13-26 For the Billy's Bank situation in Problems 13-24 and 13-25, the salary and benefits for a teller would be $12 per hour. The bank is open 8 hours each day. It has been estimated that the waiting time cost per hour is $25 per hour in the line.

(a) How many customers would enter the bank on a typical day?
(b) How much total time would the customers spend waiting in line during the entire day if one teller were used? What is the total daily waiting time cost?
(c) How much total time would the customers spend waiting in line during the entire day if two tellers were used? What is the total waiting time cost?
(d) If Billy wishes to minimize the total waiting time and personnel cost, how many tellers should be used?

13-27 Customers arrive at an automated coffee vending machine at a rate of 4 per minute, following a Poisson distribution. The coffee machine dispenses a cup of coffee in exactly 10 seconds.

(a) What is the average number of people waiting in line?
(b) What is the average number in the system?
(c) How long does the average person wait in line before receiving service?

13-28 The average number of customers in the system in the single-channel, single-phase model described in Section 13.4 is

$$L = \frac{\lambda}{\mu - \lambda}$$

Show that for $m = 1$ server, the multichannel queuing model in Section 13.5,

$$L = \frac{\lambda\mu\left(\frac{\lambda}{\mu}\right)^m}{(m-1)!(m\mu - \lambda)^2} P_0 + \frac{\lambda}{\mu}$$

is identical to the single-channel system. Note that the formula for P_0 (Equation 13-13) must be utilized in this highly algebraic exercise.

13-29 One mechanic services 5 drilling machines for a steel plate manufacturer. Machines break down on an average of once every 6 working days, and breakdowns tend to follow a Poisson distribution. The mechanic can handle an average of one repair job per day. Repairs follow an exponential distribution.
(a) How many machines are waiting for service, on average?
(b) How many are in the system, on average?
(c) How many drills are in running order, on average?
(d) What is the average waiting time in the queue?
(e) What is the average wait in the system?

13-30 A technician monitors a group of five computers that run an automated manufacturing facility. It takes an average of 15 minutes (exponentially distributed) to adjust a computer that develops a problem. The computers run for an average of 85 minutes (Poisson distributed) without requiring adjustments. What is the
(a) average number of computers waiting for adjustment?
(b) average number of computers not in working order?
(c) probability the system is empty?
(d) average time in the queue?
(e) average time in the system?

13-31 The typical subway station in Washington, D.C., has 6 turnstiles, each of which can be controlled by the station manager to be used for either entrance or exit control—but never for both. The manager must decide at different times of the day just how many turnstiles to use for entering passengers and how many to be set up to allow exiting passengers.

At the Washington College Station, passengers enter the station at a rate of about 84 per minute between the hours of 7 and 9 A.M. Passengers exiting trains at the stop reach the exit turnstile area at a rate of about 48 per minute during the same morning rush hours. Each turnstile can allow an average of 30 passengers per minute to enter or exit. Arrival and service times have been thought to follow Poisson and exponential distributions, respectively. Assume riders form a common queue at both entry and exit turnstile areas and proceed to the first empty turnstile.

The Washington College Station manager does not want the average passenger at his station to have to wait in a turnstile line for more than 6 seconds, nor does he want more than 8 people in any queue at any average time.
(a) How many turnstiles should be opened in each direction every morning?
(b) Discuss the assumptions underlying the solution of this problem using queuing theory.

13-32 The Clear Brook High School band is holding a car wash as a fundraiser to buy new equipment. The average time to wash a car is 4 minutes, and the time is exponentially distributed. Cars arrive at a rate of one every 5 minutes (or 12 per hour), and the number of arrivals per time period is described by the Poisson distribution.
(a) What is the average time for cars waiting in the line?
(b) What is the average number of cars in the line?
(c) What is the average number of cars in the system?
(d) What is the average time in the system?
(e) What is the probability there are more than three cars in the system?

13-33 When additional band members arrived to help at the car wash (see Problem 13-32), it was decided that two cars should be washed at a time instead of just the one. Both work crews would work at the same rate.
(a) What is the average time for cars waiting in the line?
(b) What is the average number of cars in the line?
(c) What is the average number of cars in the system?
(d) What is the average time in the system?

Internet Homework Problems

See our Internet home page, at [illegible], for additional homework problems, Problems 13-34 to 13-38.