

Problems*

- 15-7 Find the inverse of each of the following matrices:

(a) $\begin{bmatrix} 0.9 & -0.1 \\ -0.2 & 0.7 \end{bmatrix}$

(b) $\begin{bmatrix} 0.8 & -0.1 \\ -0.3 & 0.9 \end{bmatrix}$

(c) $\begin{bmatrix} 0.7 & -0.2 \\ -0.2 & 0.9 \end{bmatrix}$

(d) $\begin{bmatrix} 0.8 & -0.2 \\ -0.1 & 0.7 \end{bmatrix}$

- 15-8 Ray Cahnman is the proud owner of a 1955 sports car. On any given day, Ray never knows whether his car will start. Ninety percent of the time it will start if it started the previous morning, and 70% of the time it will not start if it did not start the previous morning.
- Construct the matrix of transition probabilities.
 - What is the probability that it will start tomorrow if it started today?
 - What is the probability that it will start tomorrow if it did *not* start today?
- 15-9 Alan Resnik, a friend of Ray Cahnman, bet Ray \$5 that Ray's car would not start five days from now (see Problem 15-8).
- What is the probability that it will not start five days from now if it started today?
 - What is the probability that it will not start five days from now if it did not start today?
 - What is the probability that it will start in the long run if the matrix of transition probabilities does not change?
- 15-10 Over any given month, Dress-Rite loses 10% of its customers to Fashion, Inc., and 20% of its market to Luxury Living. But Fashion, Inc., loses 5% of its market to Dress-Rite and 10% of its market to Luxury Living each month; and Luxury Living loses 5% of its market to Fashion, Inc., and 5% of its market to Dress-Rite. At the present time, each of these clothing stores has an equal share of the market. What do you think the market shares will be next month? What will they be in three months?
- 15-11 Draw a tree diagram to illustrate what the market shares would be next month for Problem 15-10.
- 15-12 Goodeating Dog Chow Company produces a variety of brands of dog chow. One of their best values is the 50-pound bag of Goodeating Dog Chow. George Hamilton, president of Goodeating, uses a very old machine to load 50 pounds of Goodeating Chow automatically into each bag. Unfortunately, because the machine is old, it occasionally over- or under-fills the bags. When the machine is *correctly* placing 50 pounds of dog chow into each bag, there is a 0.10

probability that the machine will only put 49 pounds in each bag the following day, and there is a 0.20 probability that 51 pounds will be placed in each bag the next day. If the machine is currently placing 49 pounds of dog chow in each bag, there is a 0.30 probability that it will put 50 pounds in each bag tomorrow and a 0.20 probability that it will put 51 pounds in each bag tomorrow. In addition, if the machine is placing 51 pounds in each bag today, there is a 0.40 probability that it will place 50 pounds in each bag tomorrow and a 0.10 probability that it will place 49 pounds in each bag tomorrow.

- If the machine is loading 50 pounds in each bag today, what is the probability that it will be placing 50 pounds in each bag tomorrow?
 - Resolve part (a) when the machine is only placing 49 pounds in each bag today.
 - Resolve part (a) when the machine is placing 51 pounds in each bag today.
- 15-13 Resolve Problem 15-12 (Goodeating Dog Chow) for five periods.
- 15-14 The University of South Wisconsin has had steady enrollments over the past five years. The school has its own bookstore, called University Book Store, but there are also three private bookstores in town: Bill's Book Store, College Book Store, and Battle's Book Store. The university is concerned about the large number of students who are switching to one of the private stores. As a result, South Wisconsin's president, Andy Lange, has decided to give a student three hours of university credit to look into the problem. The following matrix of transition probabilities was obtained:

	UNIVERSITY	BILL'S	COLLEGE	BATTLE'S
UNIVERSITY	0.6	0.2	0.1	0.1
BILL'S	0	0.7	0.2	0.1
COLLEGE	0.1	0.1	0.8	0
BATTLE'S	0.05	0.05	0.1	0.8

At the present time, each of the four bookstores has an equal share of the market. What will the market shares be for the next period?

- 15-15 Andy Lange, president of the University of South Wisconsin, is concerned with the declining business at the University Book Store. (See Problem 15-14 for details.) The students tell him that the prices are simply too high. Andy, however, has decided not to lower the prices. If the same conditions exist, what long-run market shares can Andy expect for the four bookstores?

*Note: means the problem may be solved with QM for Windows; means the problem may be solved with Excel QM; and means the problem may be solved with QM for Windows and/or Excel QM.

15-16 Hervis Rent-A-Car has three car rental locations in the greater Houston area: the Northside branch, the West End branch, and the Suburban branch. Customers can rent a car at any of these places and return it to any of the others without any additional fees. However, this can create a problem for Hervis if too many cars are taken to the popular Northside branch. For planning purposes, Hervis would like to predict where the cars will eventually be. Past data indicate that 80% of the cars rented at the Northside branch will be returned there, and the rest will be evenly distributed between the other two. For the West End branch, about 70% of the cars rented there will be returned there, and 20% will be returned to the Northside branch and the rest will go to the Suburban branch. Of the cars rented at the Suburban branch, 60% are returned there, 25% are returned to the Northside branch, and the other 15% are dropped off at the West End. If there are currently 100 cars being rented from Northside, 80 from West End, and 60 from the suburban branch, how many of these will be dropped off at each of the car rental locations?

15-17 A study of accounts receivables at the A&W Department Store indicates that bills are either current, one month overdue, two months overdue, written off as bad debts, or paid in full. Of those that are current, 80% are paid that month, and the rest become one month overdue. Of the one month overdue bills, 90% are paid, and the rest become two months overdue. Those that are two months overdue will either be paid (85%) or be listed as bad debts. If the sales each month average \$150,000, determine how much the company expects to receive of this amount. How much will become bad debts?

15-18 The cellular phone industry is very competitive. Two companies in the greater Lubbock area, Horizon and Local Cellular, are constantly battling each other in an attempt to control the market. Each company has a one-year service agreement. At the end of each year, some customers will renew, while some will switch to the other company. Horizon customers tend to be loyal, and 80% renew, while 20% switch. About 70% of the Local Cellular customers renew with them and about 30% switch to Horizon. If there are currently 100,000 Horizon customers this year, and 80,000 Local Cellular customers, how many would we expect each company to have next year?

15-19 The personal computer industry is very fast moving and technology provides motivation for customers to upgrade with new computers every few years. Brand loyalty is very important and companies try to do things to keep their customers happy. However, some current customers will switch to a different company. Three particular brands, Doorway, Bell, and Kumpaq, hold the major shares of the market. People who own Doorway computers will buy another Doorway in their next purchase 80% of the

time, while the rest will switch to the other companies in equal proportions. Owners of Bell computers will buy Bell again 90% of the time, while 5% will buy Doorway and 5% will buy Kumpaq. About 70% of the Kumpaq owners will make Kumpaq their next purchase while 20% will buy Doorway and the rest will buy Bell. If each brand currently has 200,000 customers who plan to buy a new computer in the next year, how many computers of each type will be purchased?

15-20 In Section 15.7, we investigated an accounts receivable problem. How would the paid category and the bad debt category change with the following matrix of transition probabilities?

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0.7 & 0 & 0.2 & 0.1 \\ 0.4 & 0.2 & 0.2 & 0.2 \end{bmatrix}$$

15-21 Professor Green gives two-month computer programming courses during the summer term. Students must pass a number of exams to pass the course, and each student is given three chances to take the exams. The following states describe the possible situations that could occur:

1. *State 1*: pass all of the exams and pass the course
2. *State 2*: do not pass all of the exams by the third attempt and flunk the course
3. *State 3*: fail an exam in the first attempt
4. *State 4*: fail an exam in the second attempt

After observing several classes, Professor Green was able to obtain the following matrix of transition probabilities:

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0.6 & 0 & 0.1 & 0.3 \\ 0.3 & 0.3 & 0.2 & 0.2 \end{bmatrix}$$

At the present time, there are 50 students who did not pass all exams on the first attempt, and there are 30 students who did not pass all remaining exams on the second attempt. How many students in these two groups will pass the course, and how many will fail the course?

15-22 Hicourt Industries is a commercial printing outfit in a medium-sized town in central Florida. Its only competitors are the Printing House and Gandy Printers. Last month, Hicourt Industries had approximately 30% of the market for the printing business in the area. The Printing House had 50% of the market, and Gandy Printers had 20% of the market. The association of printers, a locally run association, had recently determined how these three printers and smaller printing operations not involved in the

commercial market were able to retain their customer base. Hicourt was the most successful in keeping its customers. Eighty percent of its customers for any one month remained customers for the next month. The Printing House, on the other hand, had only a 70% retention rate. Gandy Printers was in the worst condition. Only 60% of the customers for any one month remained with the firm. In one month, the market share had significantly changed. This was very exciting to George Hicourt, president of Hicourt Industries. This month, Hicourt Industries was able to obtain a 38% market share. The Printing House, on the other hand, lost market share. This month, it only had 42% of the market share. Gandy Printers remained the same; it kept its 20% of the market. Just looking at market share, George concluded that he was able to take 8% per month away from the Printing House. George estimated that in a few short months, he could basically run the Printing House out of business. His hope was to capture 80% of the total market, representing his original 30% along with the 50% share that the Printing House started off with. Will George be able to reach his goal? What do you think the long-term market shares will be for these three commercial printing operations? Will Hicourt Industries be able to run the Printing House completely out of business?

- 15-23 John Jones of Bayside Laundry has been providing cleaning and linen service for rental condominiums on the Gulf coast for over 10 years. Currently, John is servicing 26 condominium developments. John's two major competitors are Cleanco, which currently services 15 condominium developments, and Beach Services, which performs laundry and cleaning services for 11 condominium developments.

Recently, John contacted Bay Bank about a loan to expand his business operations. To justify the loan, John has kept detailed records of his customers and the customers that he received from his two major competitors. During the past year, he was able to keep 18 of his original 26 customers. During the same period, he was able to get 1 new customer from Cleanco and 2 new customers from Beach Services. Unfortunately, John lost 6 of his original customers to Cleanco and 2 of his original customers to Beach Services during the same year. John has also learned that Cleanco has kept 80% of its current customers. He also knows that Beach Services will keep at least 50% of its customers. For John to get the loan from Bay Bank, he needs to show the loan officer that he will maintain an adequate share of the market. The officers of Bay Bank are concerned about the recent trends for market share, and they have decided not to give John a loan unless he will keep at least 35% of the market share in the long run. What types of equilibrium market

shares can John expect? If you were an officer of Bay Bank, would you give John a loan?

- 15-24 Set up both the vector of state probabilities and the matrix of transition probabilities given the following information:

Store 1 currently has 40% of the market; store 2 currently has 60% of the market.

In each period, store 1 customers have an 80% chance of returning; 20% of switching to store 2.

In each period, store 2 customers have a 90% chance of returning; 10% of switching to store 1.

- 15-25 Find $\pi(2)$ for Problem 15-24.
- 15-26 Find the equilibrium conditions for Problem 15-24. Explain what it means.
- 15-27 As a result of a recent survey of students at the University of South Wisconsin, it was determined that the university owned bookstore currently has 40% of the market. (See Problem 15-14.) The other three bookstores, Bill's, College, and Battle's, each split the remaining initial market share. Given that the state probabilities are the same, what is the market share for the next period given the initial market shares? What impact do the initial market shares have on each store next period? What is the impact on the steady state market shares?
- 15-28 Sandy Sprunger is part owner in one of the largest quick-oil-change operations for a medium-sized city in the Midwest. Currently, the firm has 60% of the market. There are a total of 10 quick lubrication shops in the area. After performing some basic marketing research, Sandy has been able to capture the initial probabilities, or market shares, along with the matrix of transition, which represents probabilities that customers will switch from one quick lubrication shop to another. These values are shown in the table on the next page:
- Initial probabilities, or market share, for shops 1 through 10 are 0.6, 0.1, 0.1, 0.1, 0.05, 0.01, 0.01, 0.01, 0.01, and 0.01.
- (a) Given these data, determine market shares for the next period for each of the 10 shops.
- (b) What are the equilibrium market shares?
- (c) Sandy believes that the original estimates for market shares were wrong. She believes that shop 1 has 40% of the market, and shop 2 has 30%. All other values are the same. If this is the case, what is the impact on market shares for next-period and equilibrium shares?
- (d) A marketing consultant believes that shop 1 has tremendous appeal. She believes that this shop will retain 99% of its current market share; 1% may switch to shop 2. If the consultant is correct, will shop 1 have 90% of the market in the long run?
- 15-29 During a recent trip to her favorite restaurant, Sandy (owner of shop 1) met Chris Talley (owner of shop 7) (see Problem 15-28). After an enjoyable lunch,

Data for Problem 15-28

FROM	To									
	1	2	3	4	5	6	7	8	9	10
1	0.60	0.10	0.10	0.10	0.05	0.01	0.01	0.01	0.01	0.01
2	0.01	0.80	0.01	0.01	0.01	0.10	0.01	0.01	0.01	0.03
3	0.01	0.01	0.70	0.01	0.01	0.10	0.01	0.05	0.05	0.05
4	0.01	0.01	0.01	0.90	0.01	0.01	0.01	0.01	0.01	0.02
5	0.01	0.01	0.01	0.10	0.80	0.01	0.03	0.01	0.01	0.01
6	0.01	0.01	0.01	0.01	0.01	0.91	0.01	0.01	0.01	0.01
7	0.01	0.01	0.01	0.01	0.01	0.10	0.70	0.01	0.10	0.04
8	0.01	0.01	0.01	0.01	0.01	0.10	0.03	0.80	0.01	0.01
9	0.01	0.01	0.01	0.01	0.01	0.10	0.01	0.10	0.70	0.04
10	0.01	0.01	0.01	0.01	0.01	0.10	0.10	0.05	0.00	0.70

Sandy and Chris had a heated discussion about market share for the quick-oil-change operations in their city. Here is their conversation:

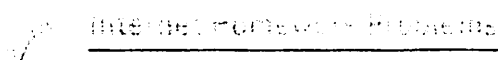
Sandy: My operation is so superior that after someone changes oil at one of my shops, they will never do business with anyone else. On second thought, maybe 1 person out of 100 will try your shop after visiting one of my shops. In a month, I will have 99% of the market, and you will have 1% of the market.

Chris: You have it completely reversed. In a month, I will have 99% of the market, and you will only have 1% of the market. In fact, I will treat you to a meal at a restaurant of your choice if you are right. If I am right, you will treat me to one of those big steaks at David's Steak House. Do we have a deal?

Sandy: Yes! Get your checkbook or your credit card. You will have the privilege of paying for two very expensive meals at Anthony's Seafood Restaurant.

- Assume that Sandy is correct about customers visiting one of her quick-oil-change shops. Will she win the bet with Chris?
- Assume that Chris is correct about customers visiting one of his quick-oil-change shops. Will he win the bet?
- Describe what would happen if both Sandy and Chris are correct about customers visiting their quick-oil-change operations.

15-30 The first quick-oil-change store in Problem 15-28 retains 73% of its market share. This represents a probability of 0.73 in the first row and first column of the matrix of transition probabilities. The other probability values in the first row are equally distributed across the other stores (namely, 3% each). What impact does this have on the steady-state market shares for the quick-oil-change stores?



Internet Homework Problems

See our Internet home page, at <http://www.mhhe.com/stevenson>, for additional homework problems, Problems 15-31 to 15-34.

Case Study

Rental Trucks

Jim Fox, an executive for Rental Trucks, could not believe it. He had hired one of the town's best law firms, Folley, Smith, and Christensen. Their fee for drawing up the legal contracts was over \$50,000. Folley, Smith, and Christensen had made one important omission from the contracts, and this blunder would more than likely cost Rental Trucks millions of dollars.

For the hundredth time, Jim carefully reconstructed the situation and pondered the inevitable.

Rental Trucks was started by Robert (Bob) Renton more than 10 years ago. It specialized in renting trucks to businesses and private individuals. The company prospered, and Bob increased his net worth by millions of dollars. Bob was a legend