Tutorial on Learning from Demonstration

Part 2: Encoding Motions with Dynamical Systems

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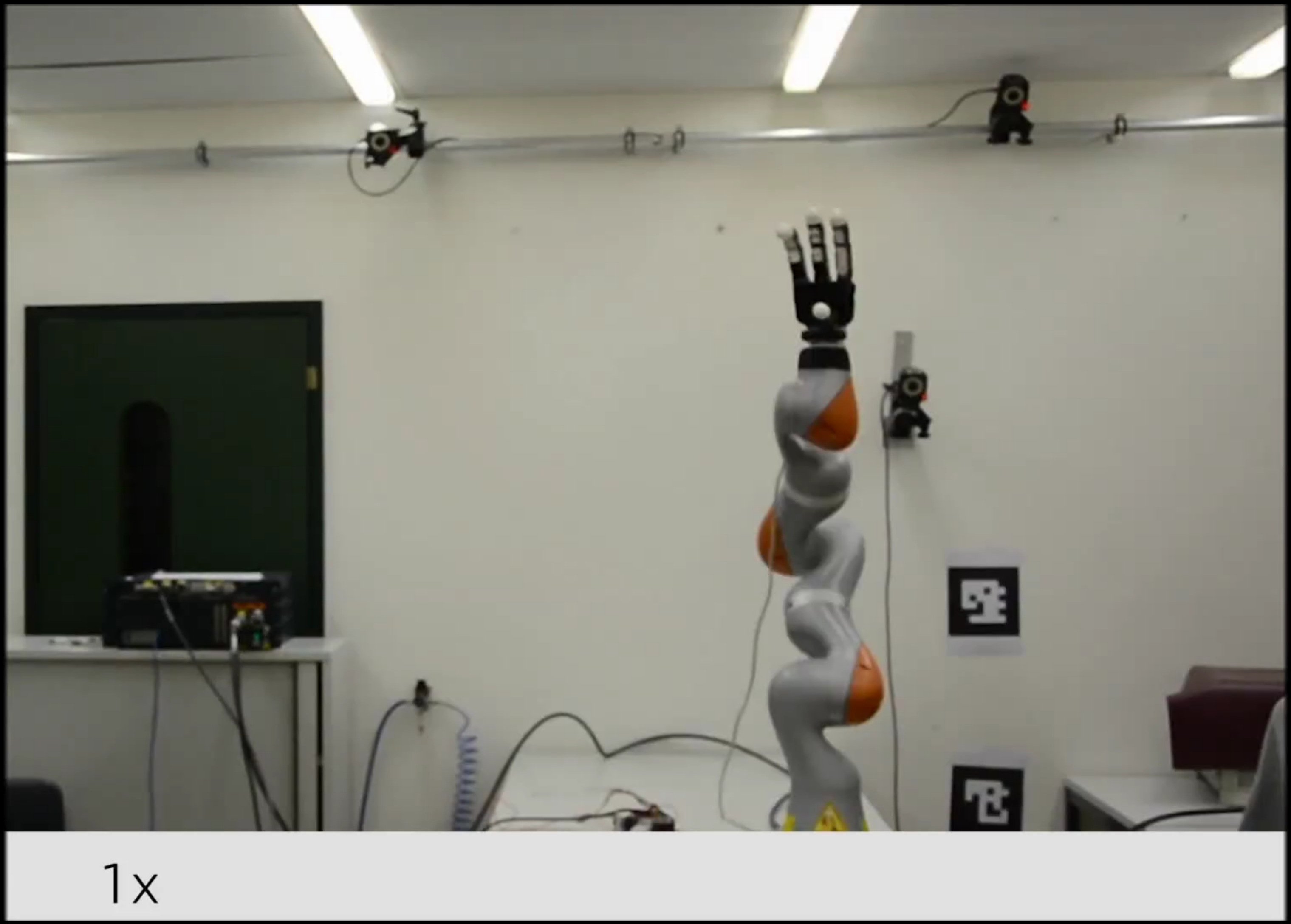
Dynamical systems – based control: fast adaptation

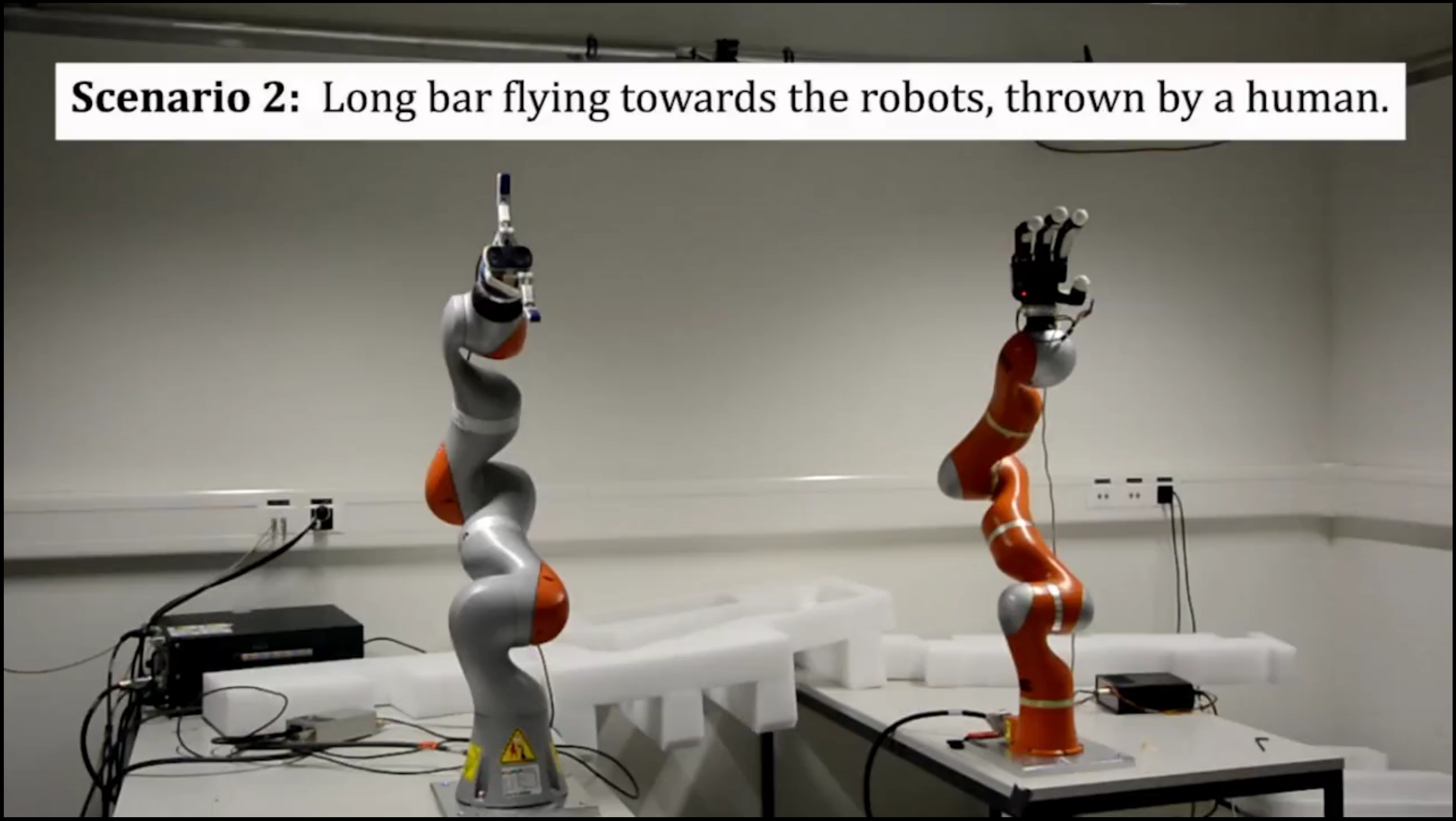
Kim, Shukla, Billard, IEEE Transaction on Robotics, 2014 *IEEE Transactions on Robotics, King-Sun Fu Memorial Best Paper Award 2014*

Dynamical systems – based control: fast adaptation



Mirrazzavi, Khoramshavi, Billard, IEEE Transaction on Robotics, 2016, *In Press*

Dynamical systems – based control: fast adaptation



Mirrazzavi, Figueroa, Billard, RSS 2016

Dynamical Systems

Ordinary Differential Equations

- ***Non-autonomous DS***, driving by a time-dependent ODE of the form

$$\dot{x} = f(x,t)$$

Popular approach to control.
Time-dependency is problematic if the system is delayed.

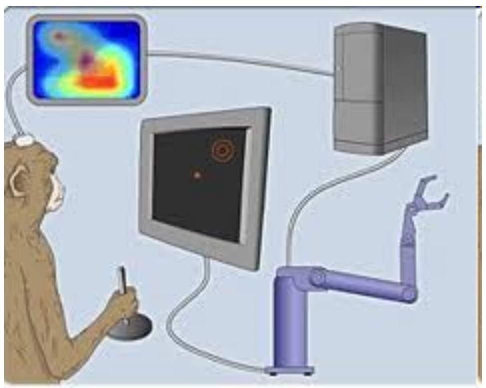
- ***Autonomous DS (ADS)***, driven by a time-invariant ODE of the form

$$\dot{x} = f(x)$$

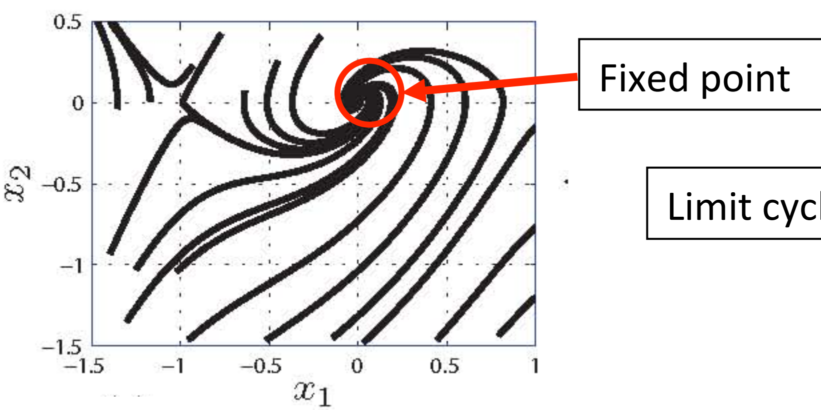
Lack of explicit time-dependency makes the system robust.

Usefulness of Dynamical Systems

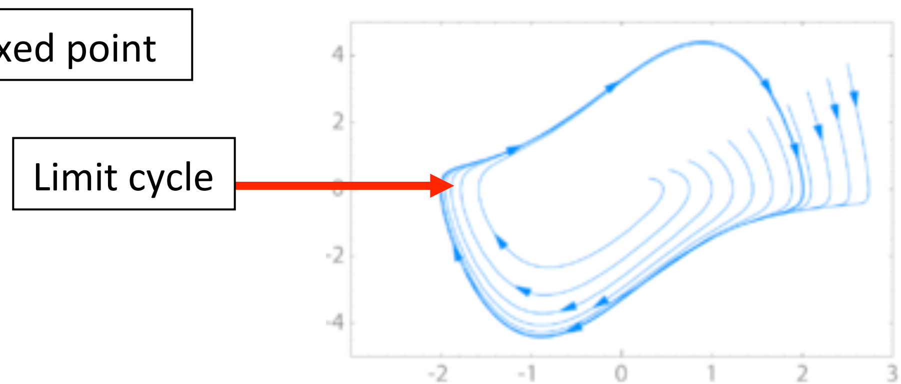
Dynamical systems drive vertebrates' control systems



Reaching uses fixed point ADS



Cyclic patterns underlie locomotion
(Central Pattern Generator)



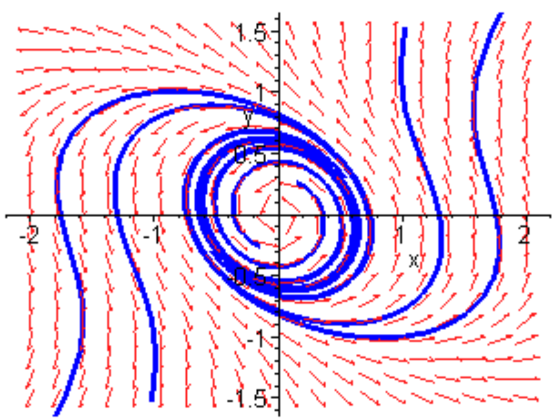
Dynamical Systems and Limit Cycles

• Limit cycles are particular cases where a non-linear DS oscillates.

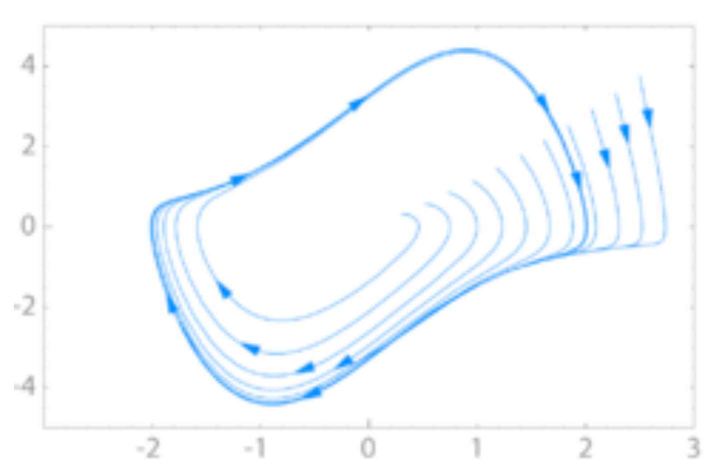
• *Stable oscillations* are such that the system converges to an oscillatory mode with fixed amplitude and frequency irrespective of initial condition.

• Such systems are particularly useful to model or control cyclic patterns as they show great robustness in the face of perturbations.

Unstable limit cycle



Stable limit cycle



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Usefulness of Dynamical Systems

Dynamical systems are used for modeling a variety of systems:

• Physics, Mechanical Engineering:

Modeling of weather, fluid dynamics, planet motion, etc.

• Computer Vision:

Prediction of crowd motion, tracking of limbs for motion capture

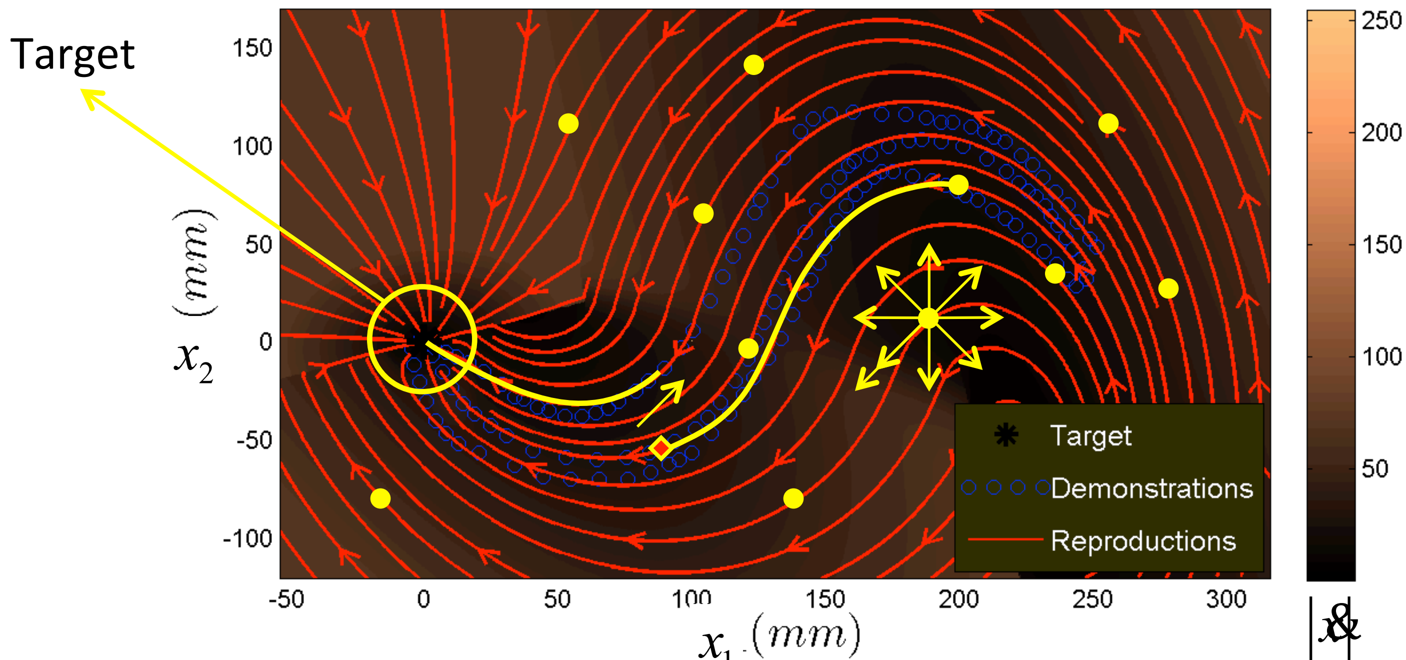
• Robot Control:

Stability of airplanes, joint & Cartesian control of biologically-inspired robots

Dynamical Systems for Point to Point Motion

Use a control law for end-effector motion driven by:

autonomous DS $\boxed{\dot{x} = f(x)}$ *with stable attractor* $\boxed{\dot{x}^* = f(x^*) = 0}$



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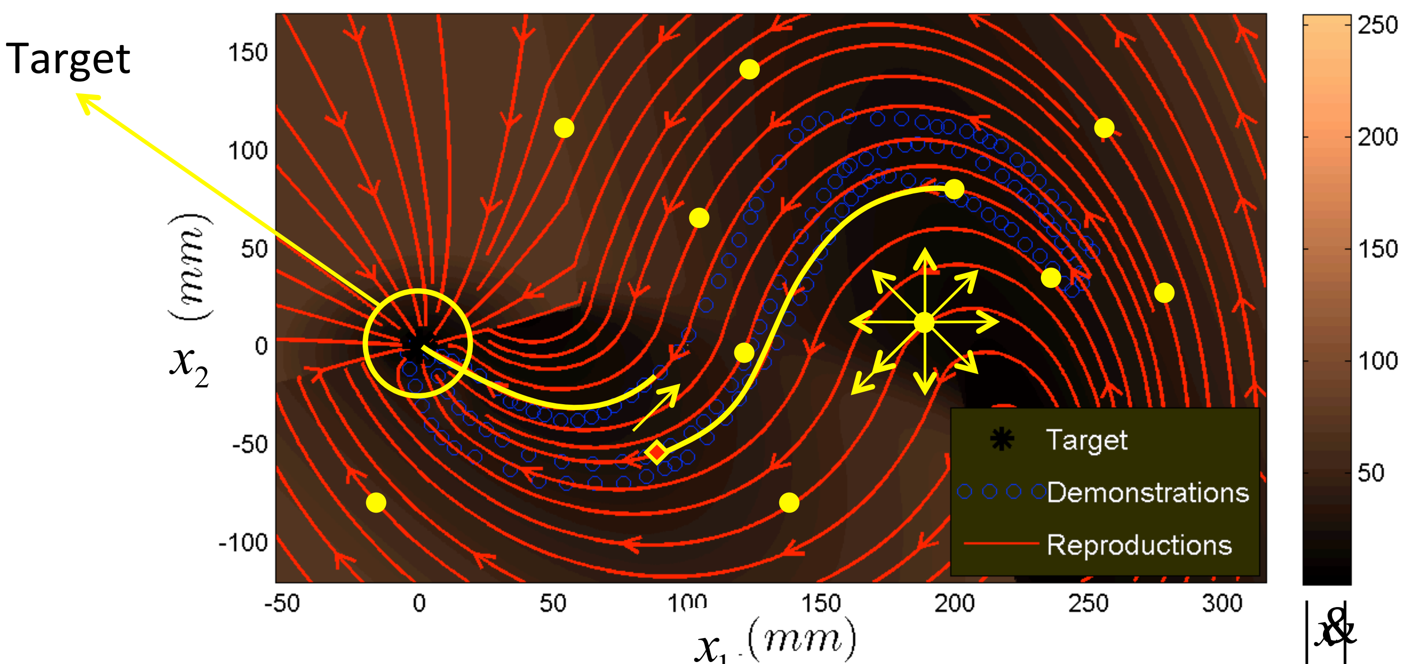
"Loose" definition of Stability of ADS

An autonomous dynamical system of the form $\dot{x} = f\left(x\right)$ is

– stable at x^* if $f\left(x^*\right) = 0.$

– asymptotically stable at x^* if $\lim\limits_{t \to \infty} x\left(t\right) = x^*$ and $f\left(x^*\right) = 0.$



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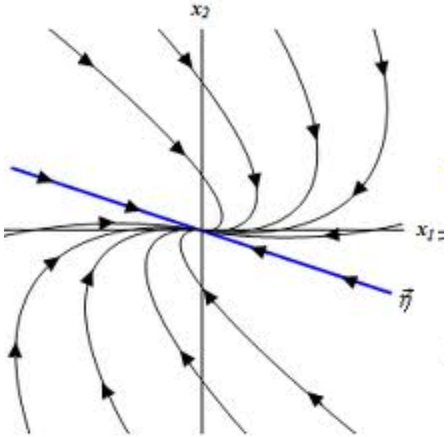
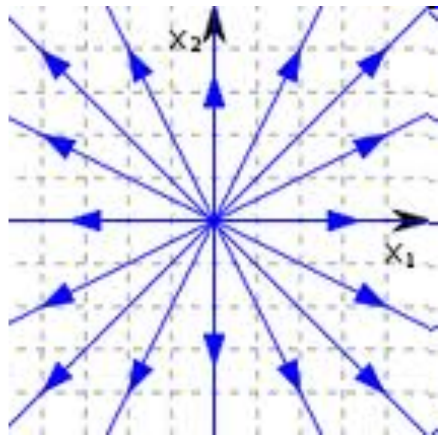
Characterizing linear dynamical systems

Stability can be assessed easily when f is linear:

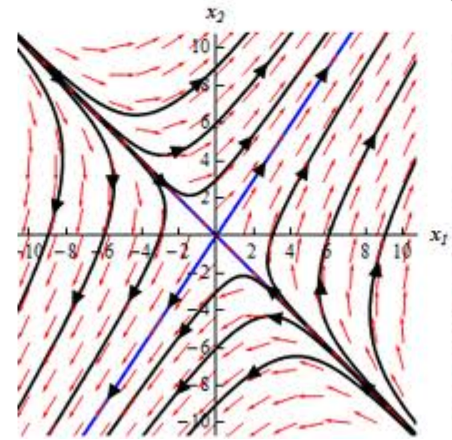
$\dot{x} = f(x) = Ax,\ x \in \mathbb{R}^N,\ A: N \times N$: study the eigenvalues of A.



Stable equilibrium point
All eigenvalues negatives



Unstable equilibrium point
All eigenvalues positives



Saddle point / One eigenvalue positive, one negative

What if *f* is non-linear?

Stability not easy to define: local linearization; numerical estimation of stability; analytical solution in special cases.

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Machine Learning Approaches to Modeling DS

Take a density based approach to modeling dynamical systems

$$\boxed{\dot{x} = f(x) \sim E\left\{p(\dot{x} \mid x)\right\}}$$

Several methods exist to estimate the regressive model:

- Gaussian Process Regression (GPR) and GPLVM
- Gaussian Mixture Regression (GMR)
- Locally Weighted Projected Regression (LWPR)

Optimization criteria do not take into account the issue of stability ➔ results in unstable estimates of the dynamics (spurious attractors, no global stability at the desired attractor)

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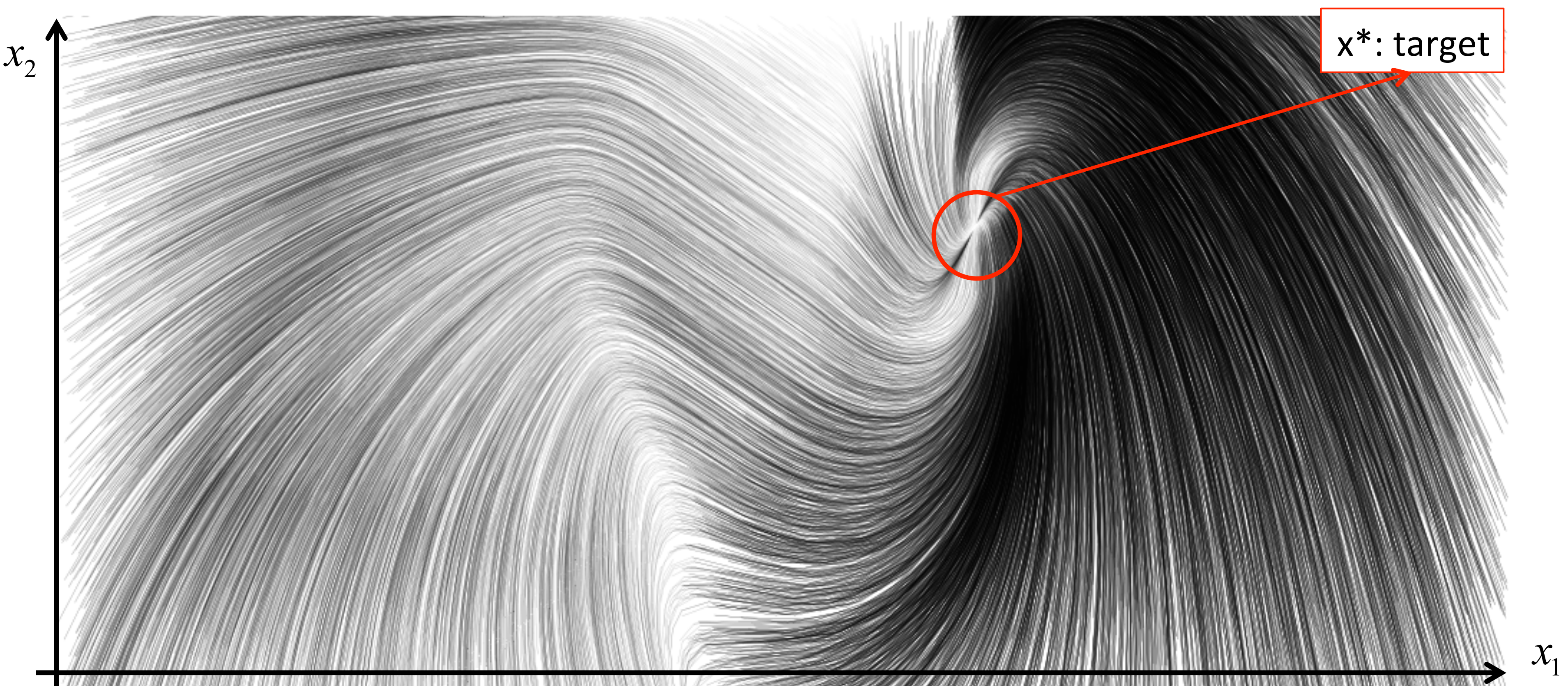
MLDEMO EXAMPLE

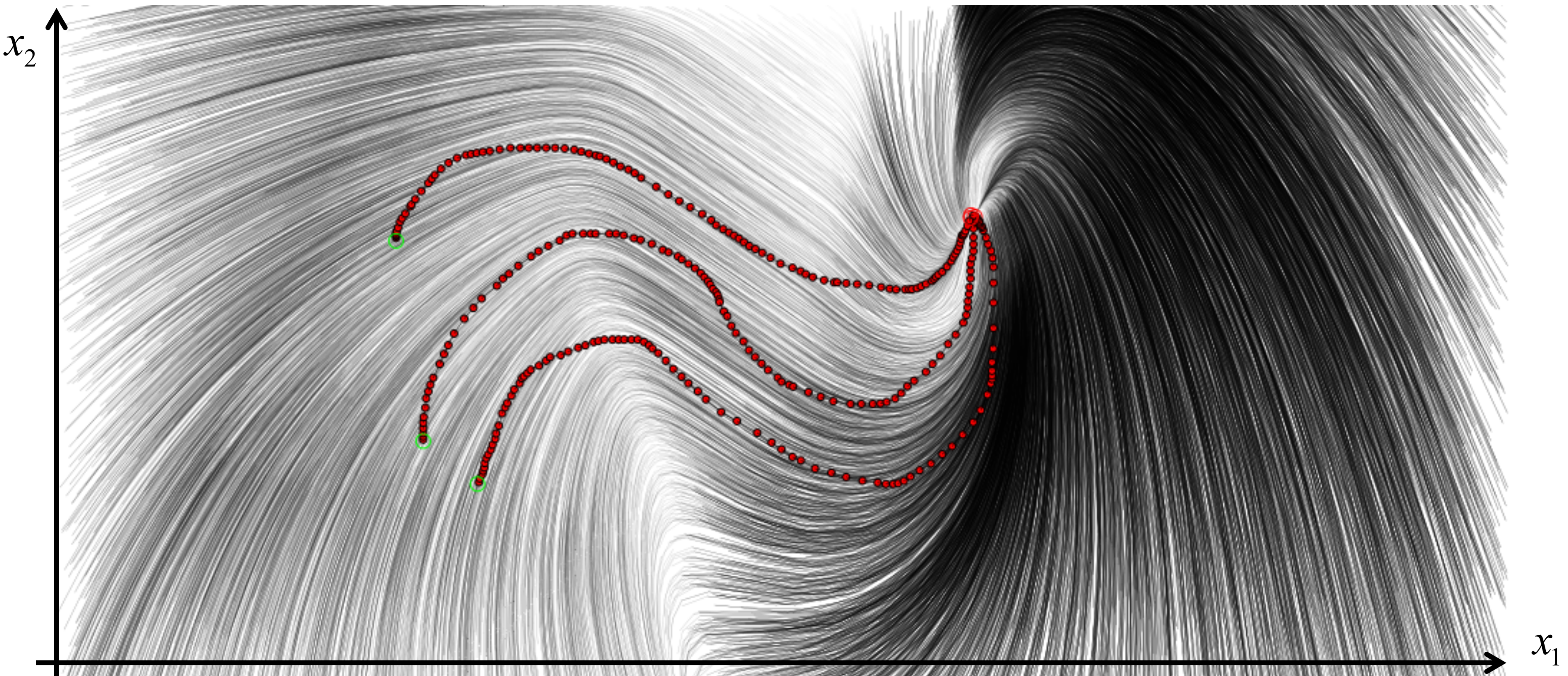
Learn a control law from examples

Time-invariant DS $\dot{x} = f(x)$ *with stable attractor* $\dot{x}^* = f(x^*) = 0.$



Learn a control law from examples

Make N observations of the state of the system $\left\{\dot{x}^i, x^i\right\}, i = 1 \dots N$.



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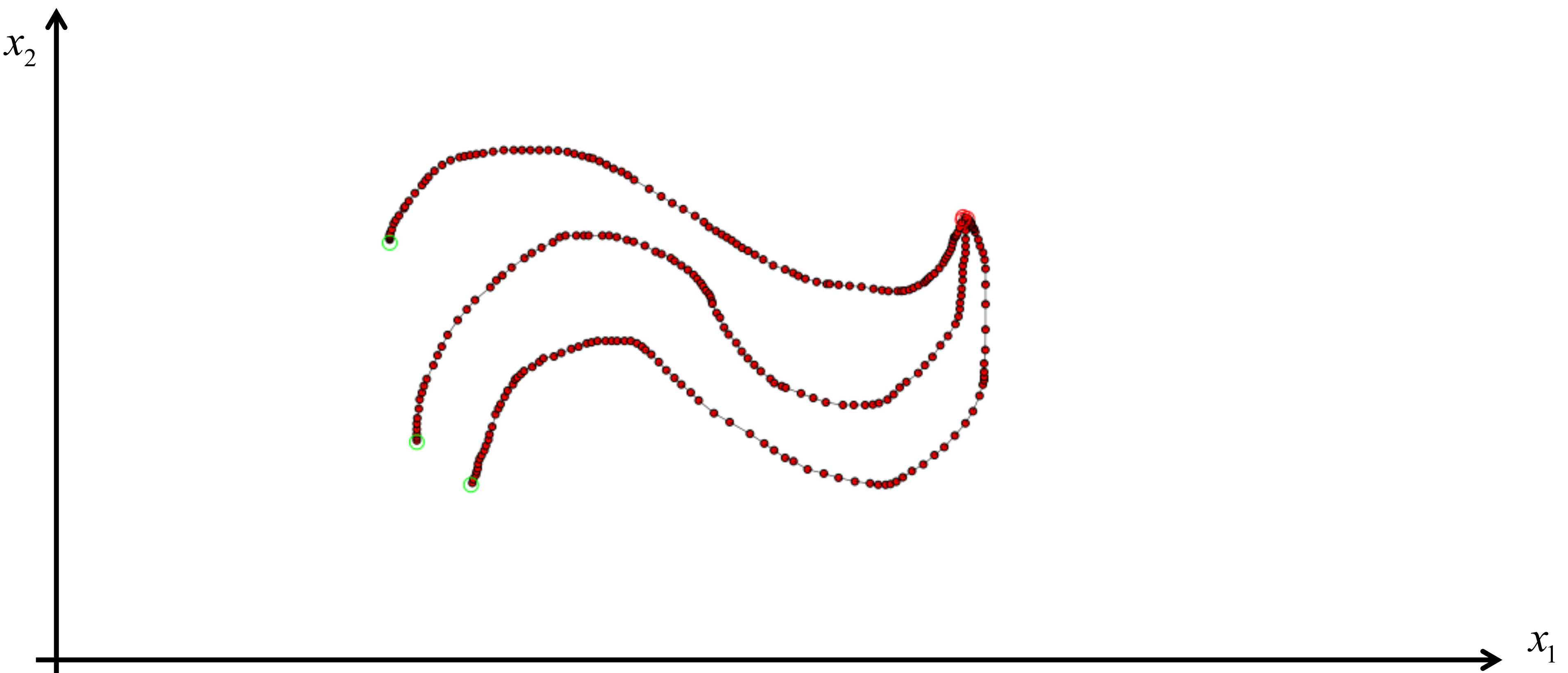
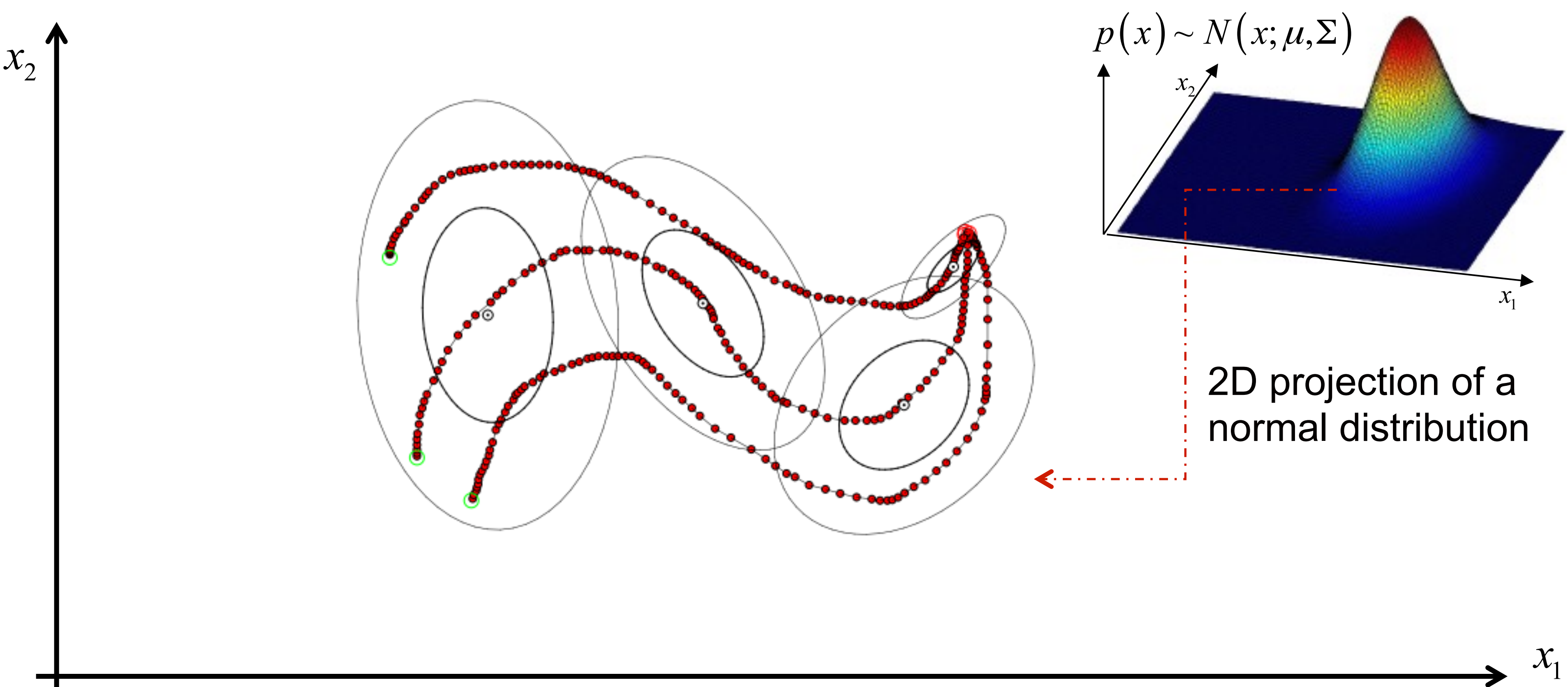


Learn a control law from examples

Make N observations of the state of the system $\left\{\dot{x}^i, x^i\right\}, i = 1 \ldots N.$

Probabilistic model: $p\left(\dot{x}, x\right)$ (e.g. mixture of Gaussians)



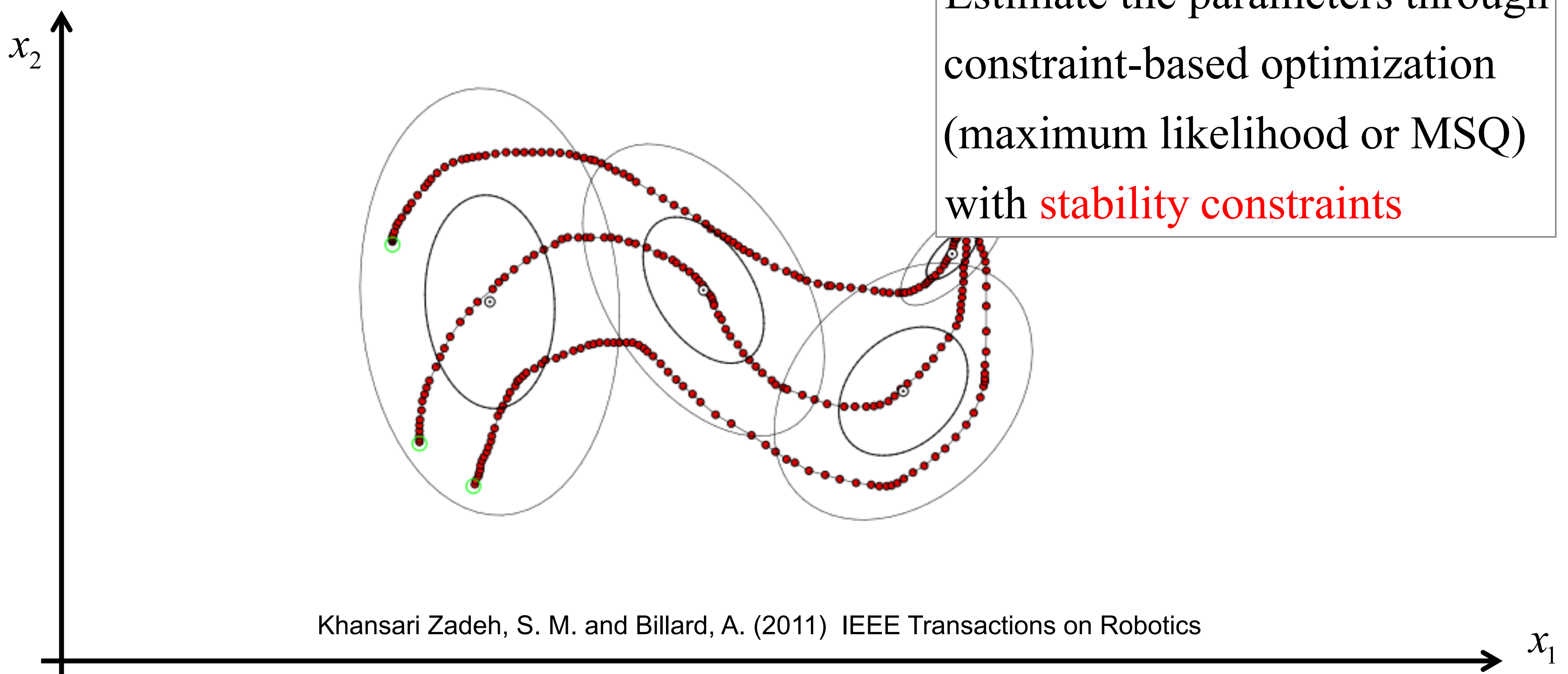
Learn a control law from examples

Make N observations of the state of the system $\left\{\dot{x}^i, x^i\right\}, i = 1 \dots N.$

Probabilistic model: $p\left(\dot{x}, x\right)$ (e.g. mixture of Gaussians)

Compute $f\left(x\right) = E\left\{p\left(\dot{x} \mid x\right)\right\}$ (closed form solution)

Estimate the parameters through constraint-based optimization (maximum likelihood or MSQ) with stability constraints



Khansari Zadeh, S. M. and Billard, A. (2011) IEEE Transactions on Robotics

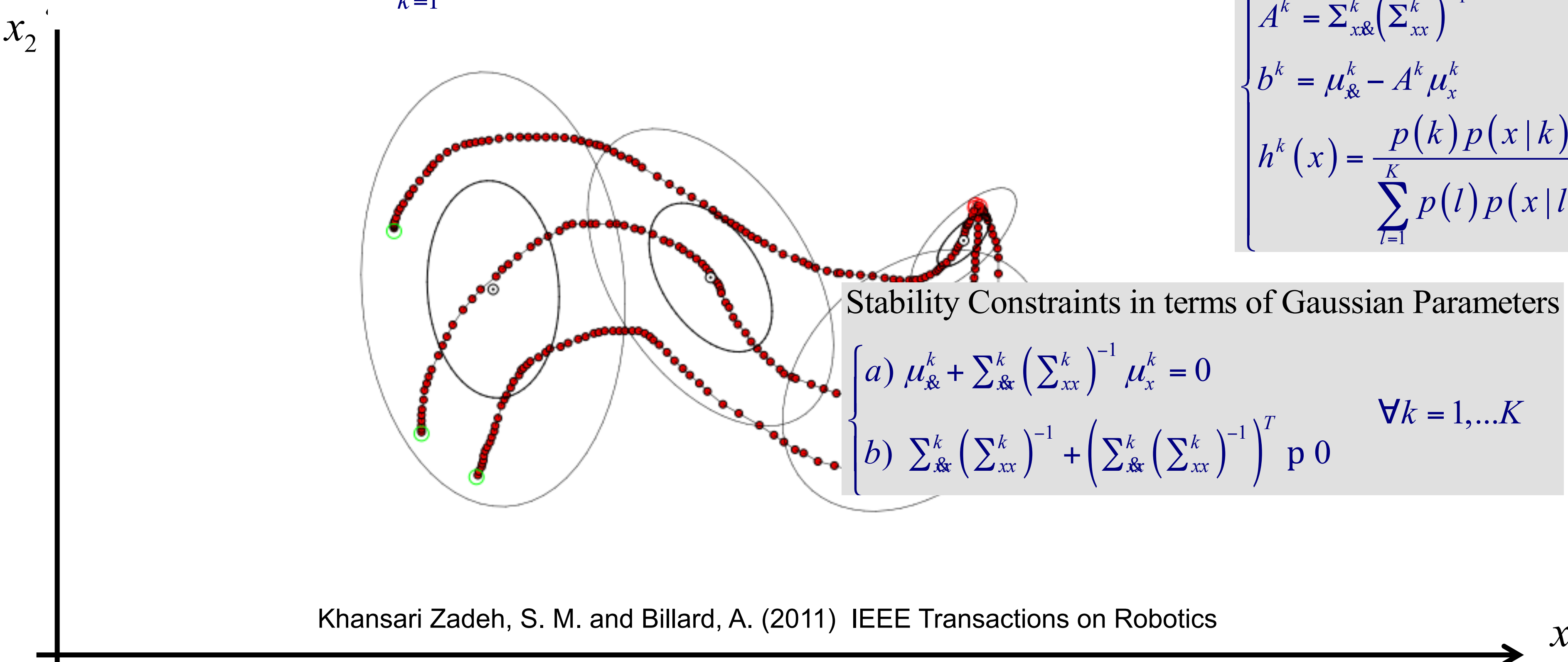
Learn a control law from examples

Make N observations of the state of the system $\left\{\dot{x}^i, x^i\right\}, i = 1 \ldots N.$

Learn $p(\dot{x}, x) = \sum_{k=1}^{K} w_k N\left(\dot{x}, x; \mu^k, \Sigma^k\right)$: joint density (mixture of Gaussians)

Compute $f(x) = \sum_{k=1}^{K} h^k(x)\left(A^k x + b^k\right)$ (closed form for f)

$$\begin{cases} A^k = \Sigma^k_{x\dot{x}}\left(\Sigma^k_{xx}\right)^{-1} \\ b^k = \mu^k_{\dot{x}} - A^k \mu^k_x \\ h^k(x) = \dfrac{p(k)\, p(x \mid k)}{\sum_{l=1}^{K} p(l)\, p(x \mid l)} \end{cases}$$



Stability Constraints in terms of Gaussian Parameters

$$\begin{cases} a)\ \mu^k_{\dot{x}} + \Sigma^k_{\dot{x}}\left(\Sigma^k_{xx}\right)^{-1} \mu^k_x = 0 \\ b)\ \Sigma^k_{\dot{x}}\left(\Sigma^k_{xx}\right)^{-1} + \left(\Sigma^k_{\dot{x}}\left(\Sigma^k_{xx}\right)^{-1}\right)^T \prec 0 \end{cases} \quad \forall k = 1, \ldots K$$

Khansari Zadeh, S. M. and Billard, A. (2011) IEEE Transactions on Robotics

Learn a control law from examples

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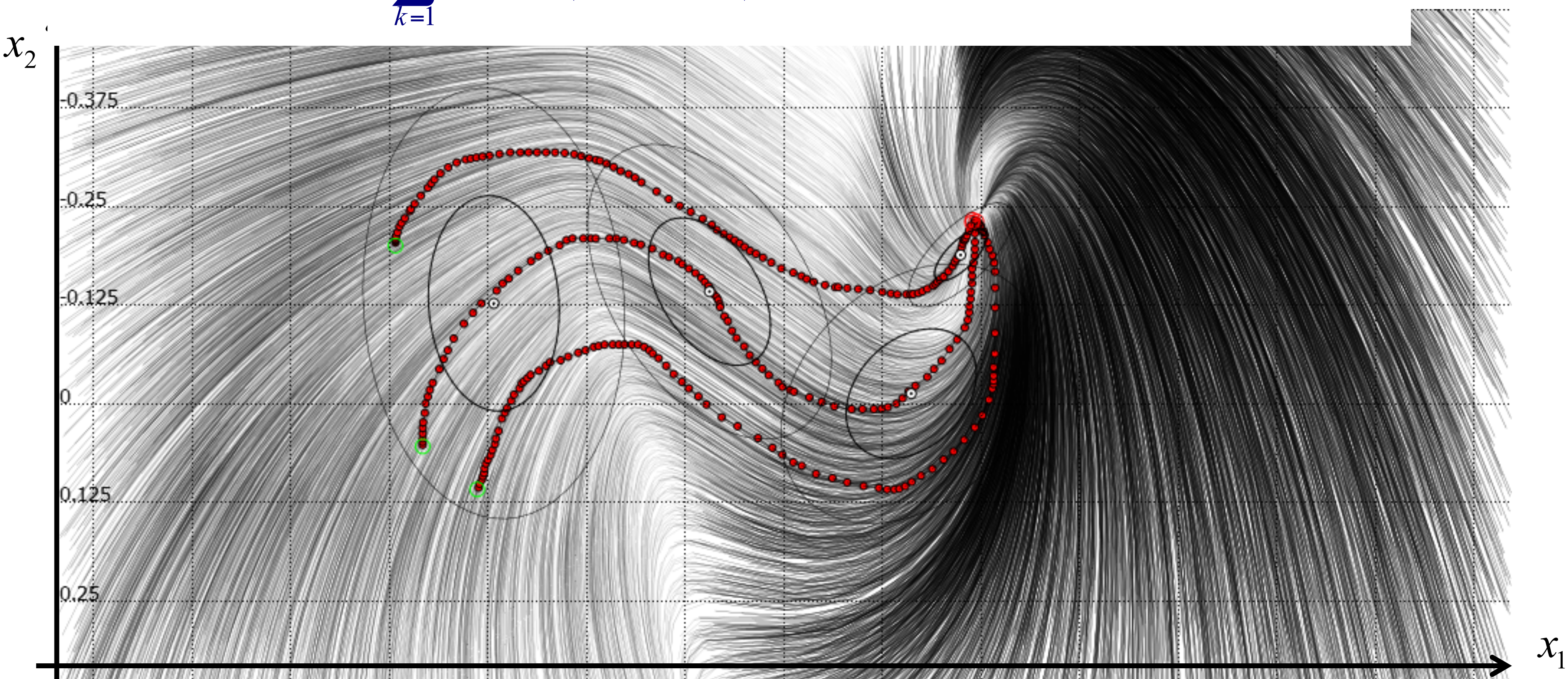


Teaching Hitting Motions

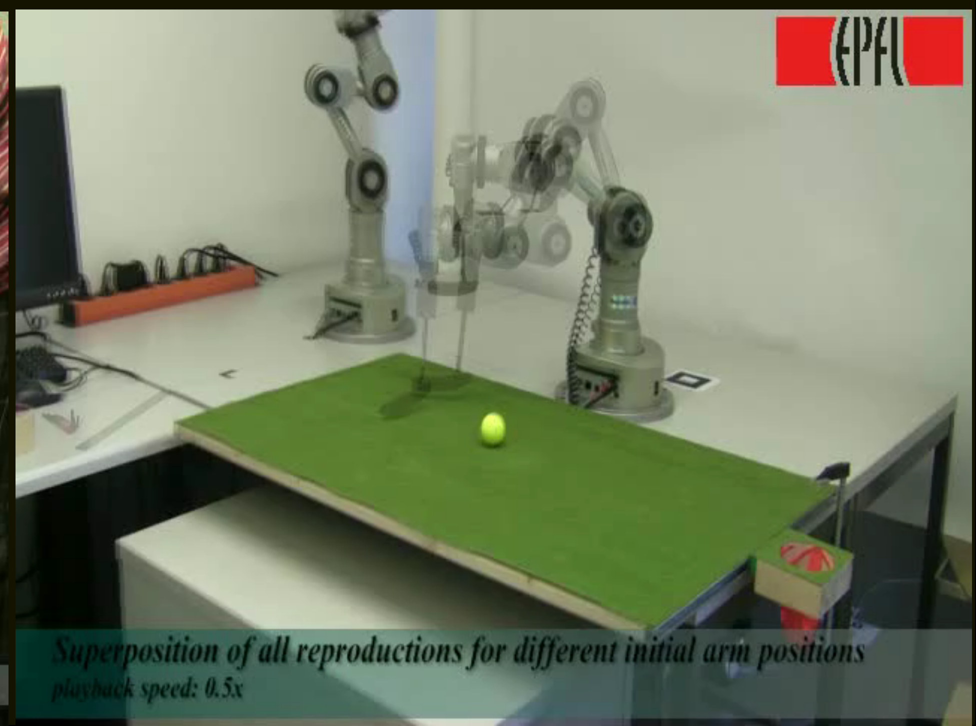
Autonomous dynamical system $\dot{x} = f(x)$ with a stable attractor $\cancel{\dot{x}^* = f(x^*)} = 0.$

Add a modulatory term to control for orientation and velocity at target.

$$\boxed{\dot{x} = E(x)\,g(x), \quad E(x) = \frac{f(x)}{\|f(x)\|}}$$

Orientation at target

Velocity at target



Robust Obstacle Avoidance

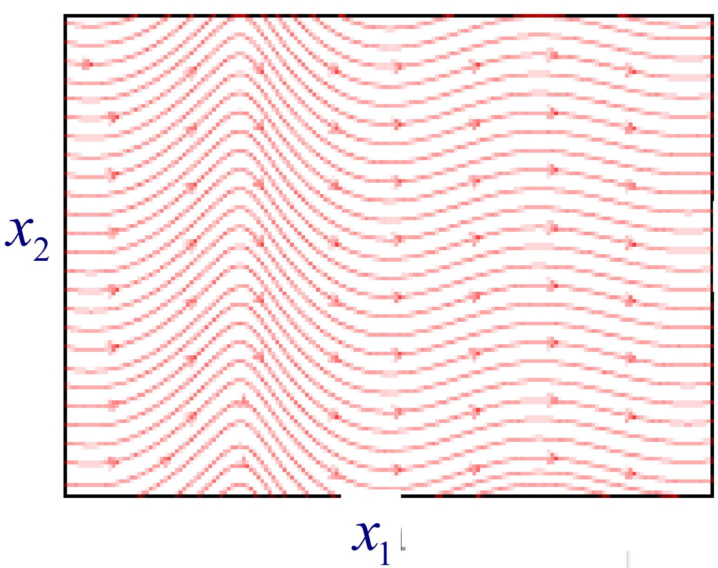
Start with an estimate of $\dot{x} = f(x)$ with stable attractor $\dot{x}^* = f(x^*) = 0$

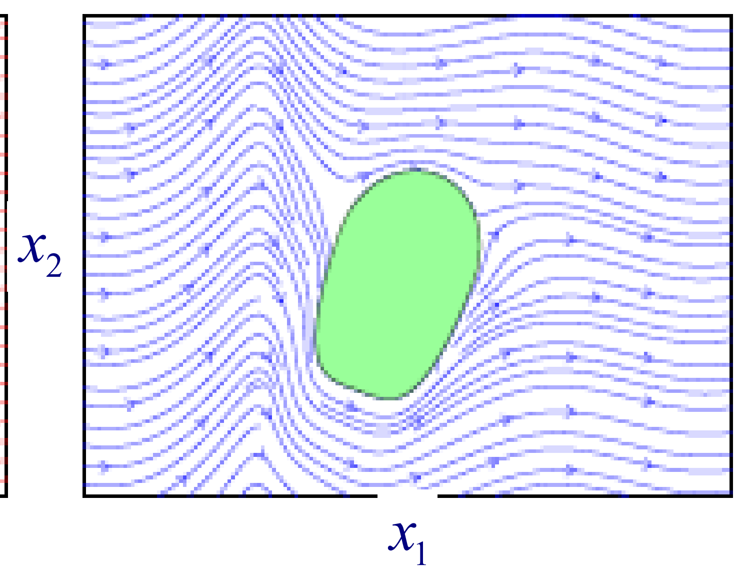
Add a modulatory term to represent the effect of the obstacle

$$\dot{x} = \boxed{M(x - x^o)}\, f(x)$$

Modulation due to the presence of obstacle

In the obstacle's frame of reference





S.M. Khansari-Zadeh and Billard: *Autonomous Robots, 2012*

Robust Obstacle Avoidance

Start with an estimate of $\dot{x} = f(x)$ with stable attractor $\dot{x}^* = f(x^*) = 0$

Add a modulatory term to represent the effect of the obstacle

$$\dot{x} = \boxed{M(x - x^o)} f(x)$$

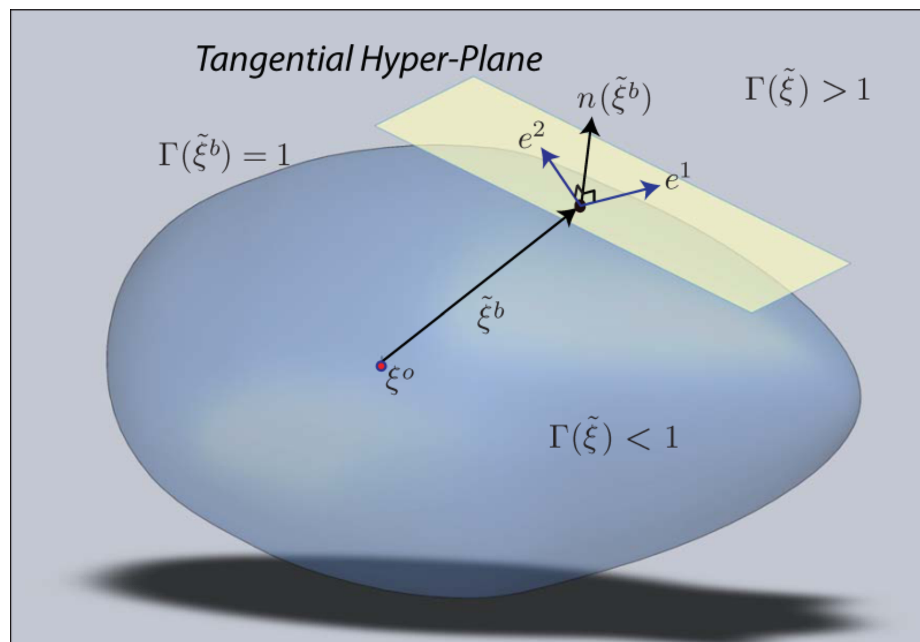
$$\boxed{M^k(\xi) = E^k(\xi)\, D^k(\xi)\, \left(E^k(\xi)\right)^{-1}}$$

In the obstacle's frame of reference

$$E(\tilde{\xi}) = \left[\, n(\tilde{\xi}) \;\; e^1 \;\; \cdots \;\; e^{d-1} \,\right]$$

$$D(\tilde{\xi}) = \begin{bmatrix} \lambda^1 & & 0 \\ & \ddots & \\ 0 & & \lambda^d \end{bmatrix}$$

$$\begin{cases} \lambda^1(\tilde{\xi}) = 1 - \frac{1}{|\Gamma(\tilde{\xi})|} \\ \lambda^i(\tilde{\xi}) = 1 + \frac{1}{|\Gamma(\tilde{\xi})|} & 2 \le i \le d \end{cases}$$



Robust Obstacle Avoidance

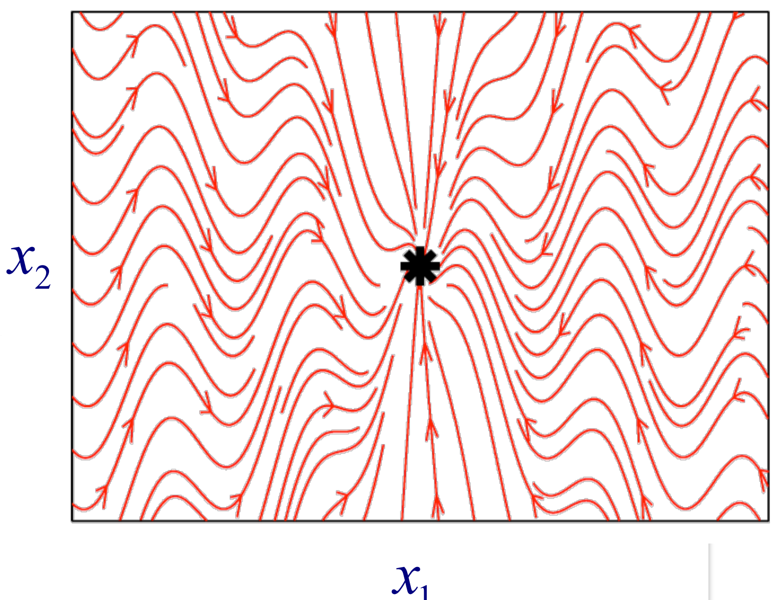
Start with an estimate of $\dot{x} = f(x)$ with stable attractor $\dot{x}^* = f(x^*) = 0$

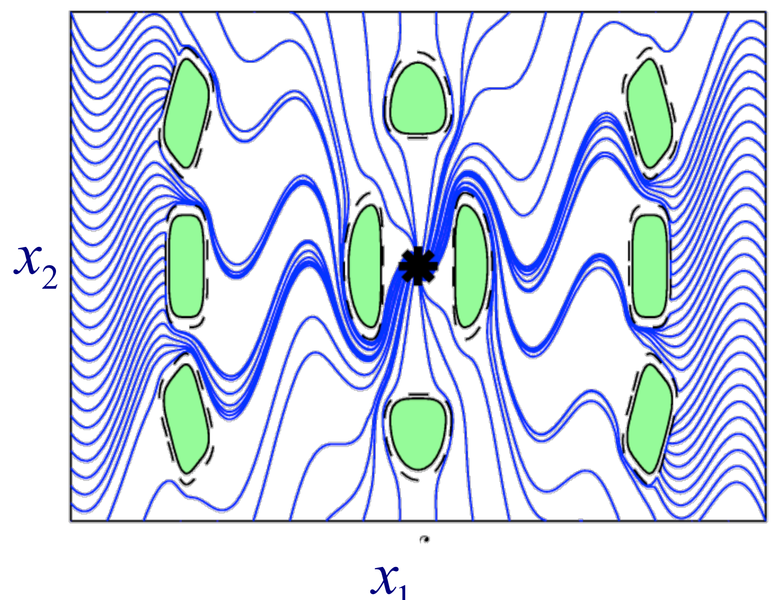
Add a modulatory term to represent the effect of the obstacle

$$\dot{x} = \boxed{\prod_{i=1}^{nm\ \text{obstacles}} M_i\left(x - x^i\right)} f(x)$$

Modulation due to the presence of each obstacle

In each obstacle's frame of reference





Start exercises