Tutorial on Learning from Demonstration

Part 5: Considering Model Uncertainty with Optimal Control

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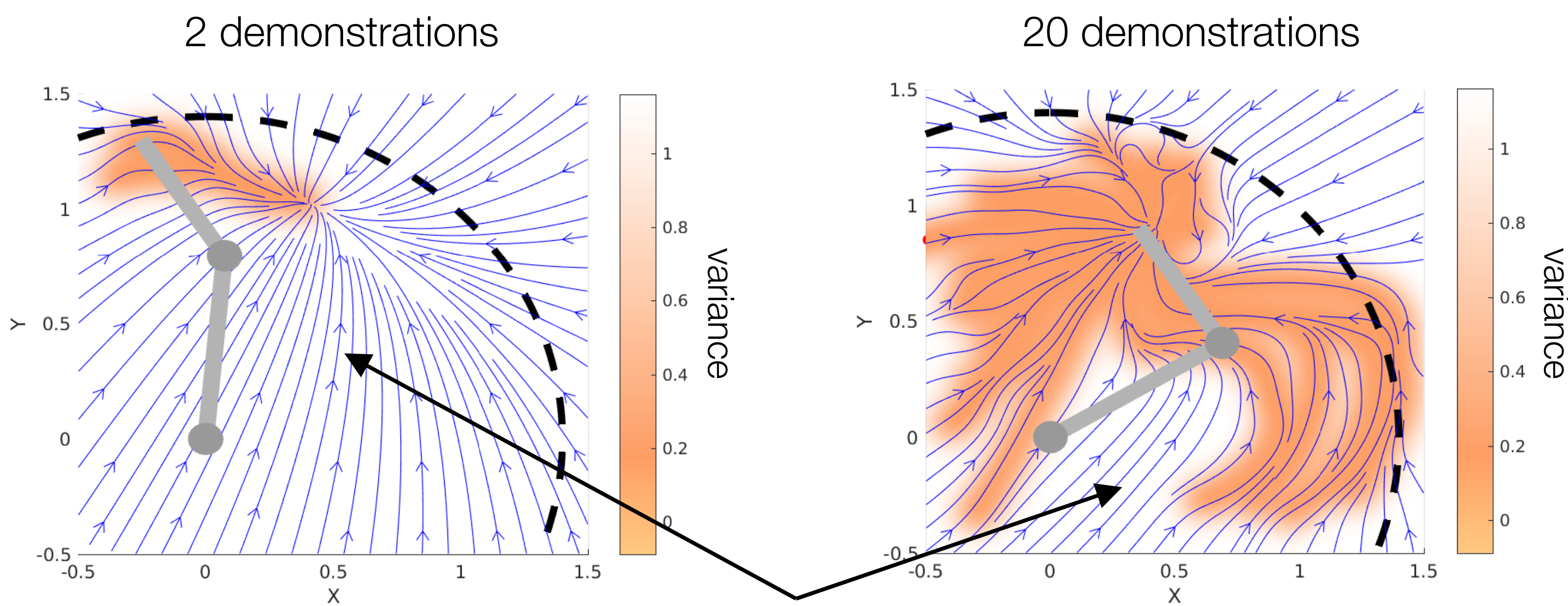
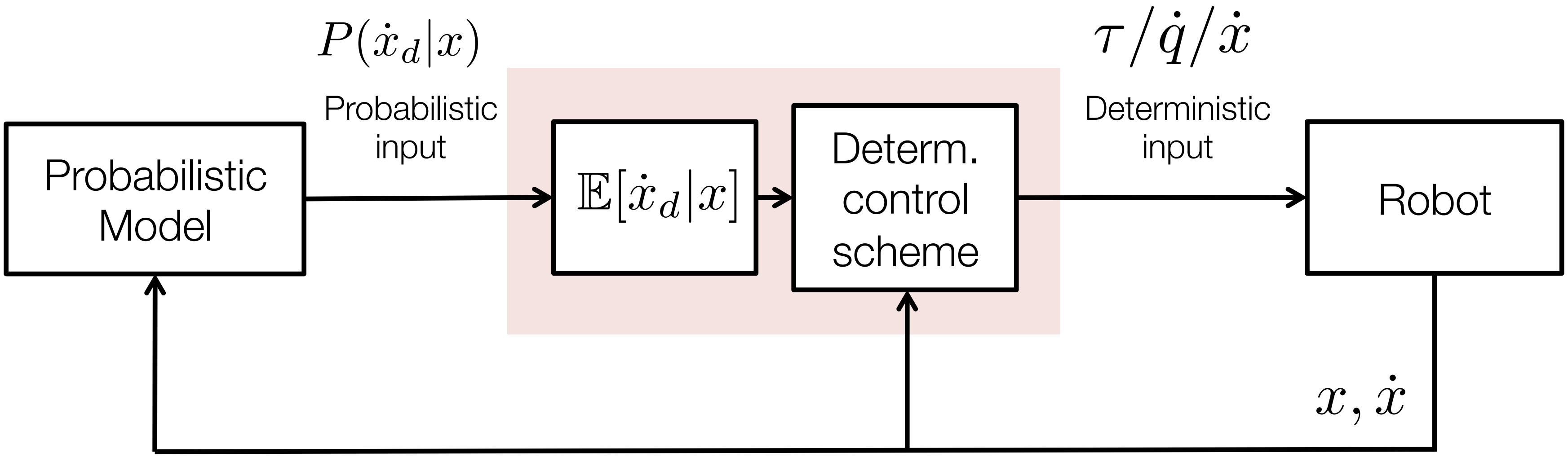
Generalization implies uncertainty

More generalization → higher uncertainty

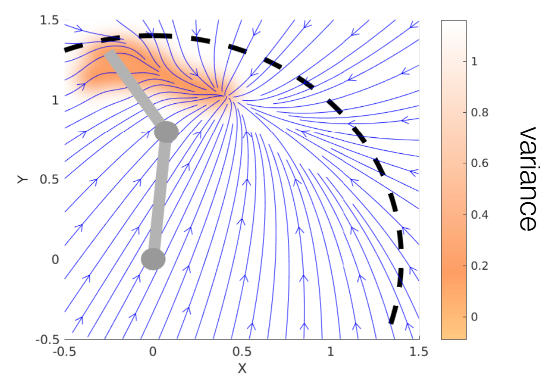


How should the robot
behave under uncertainty?

From probabilistic models to robot control



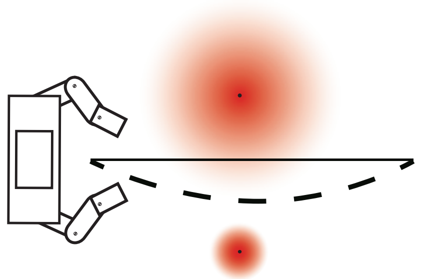
Discards model variance!



Uncertainty influences control

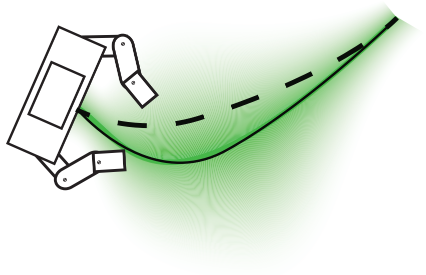
Intuitive examples

Avoiding uncertain obstacles



Uncertainty *increases* obs. avoiding penalty

Following uncertain trajectory



Uncertainty *decreases* tracking penalty

<u>Evidence from neuroscience</u>

Humans exhibit uncertainty-dependent behavior in the context of *stochastic optimal control* by adapting...

- ...compliance under stochastic disturbances [Braun 2011]
- ...trajectory/plans under task/partner uncertainty [Grau-Moya 2013]

D. Braun, A. Nagengast and D. Wolpert, **Risk-sensitivity in sensorimotor control**, Frontiers in Human Neuroscience 2011

J. Grau-Moya, P. Ortega and D. Braun, **Risk-sensitivity in Bayesian sensorimotor integration**, PLOS comp Biol 2013

Uncertainty-dependent optimal control

Approaches

Machine learning ⇔ Robot control

- Adding a variance-dependent term in the cost [Mitrovic/Vijayakumar 2010]
- Adding a measure of probability of collision [Mitrovic/Vijayakumar 2014]
- Generalized binary saturating cost [Deisenroth/Fox/Rasmussen 2013]
- Risk-sensitive stochastic optimal control [Kuindersma/Grupen/Barto 2013]

Stochastic optimal control as a tool for synthesizing uncertainty-dependent behavior based on learned models

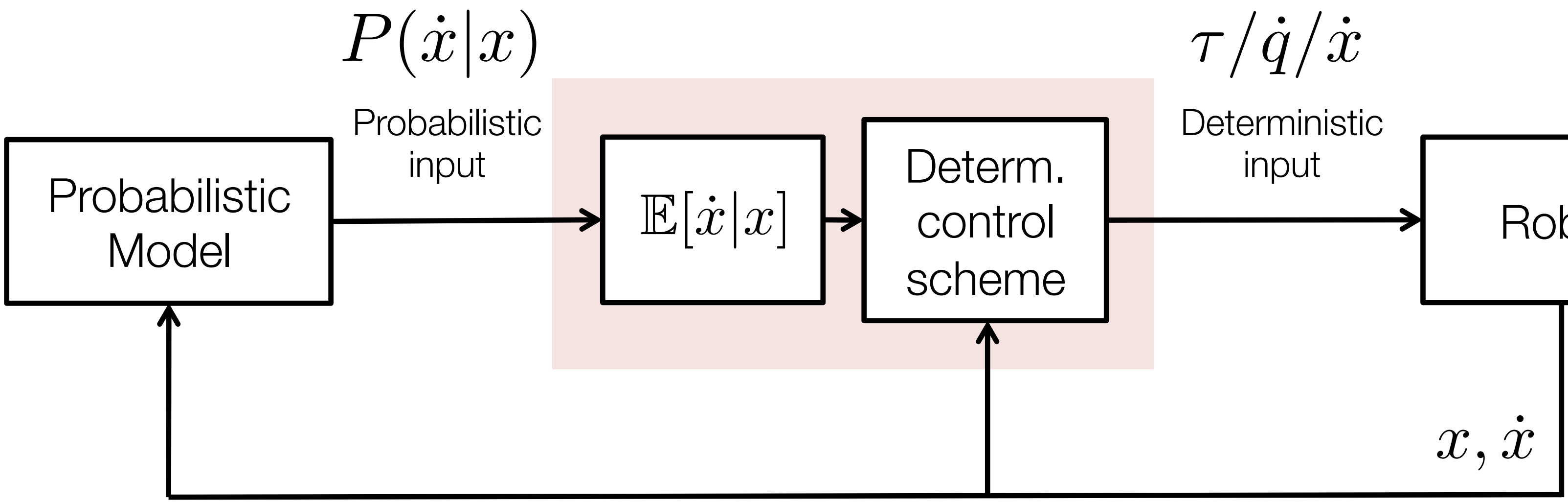
J. Mueller and G. Sukhatme , **Risk-aware trajectory generation with application to safe quadrotor landing** IROS 2014

M. Deisenroth, D. Fox, and C. Rasmussen , **Gaussian Processes for Data-Efficient Learning in Robotics and Control** PAMI 2013

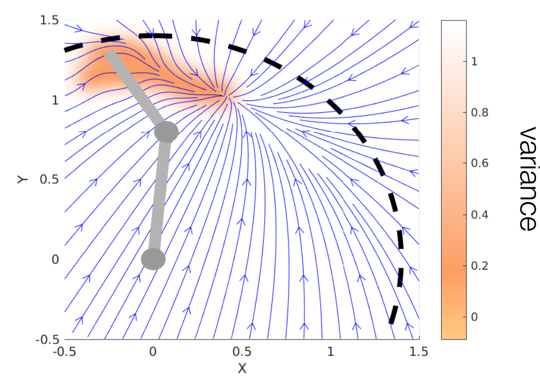
D. Mitrovic, S. Klanke and S. Vijayakumar, **Adaptive optimal feedback control with learned internal dynamics models** Springer 2010

S. Kuindersma, R. Grupen and A. Barto, **Variational Bayesian optimization for runtime risk-sensitive control** RSS 2013

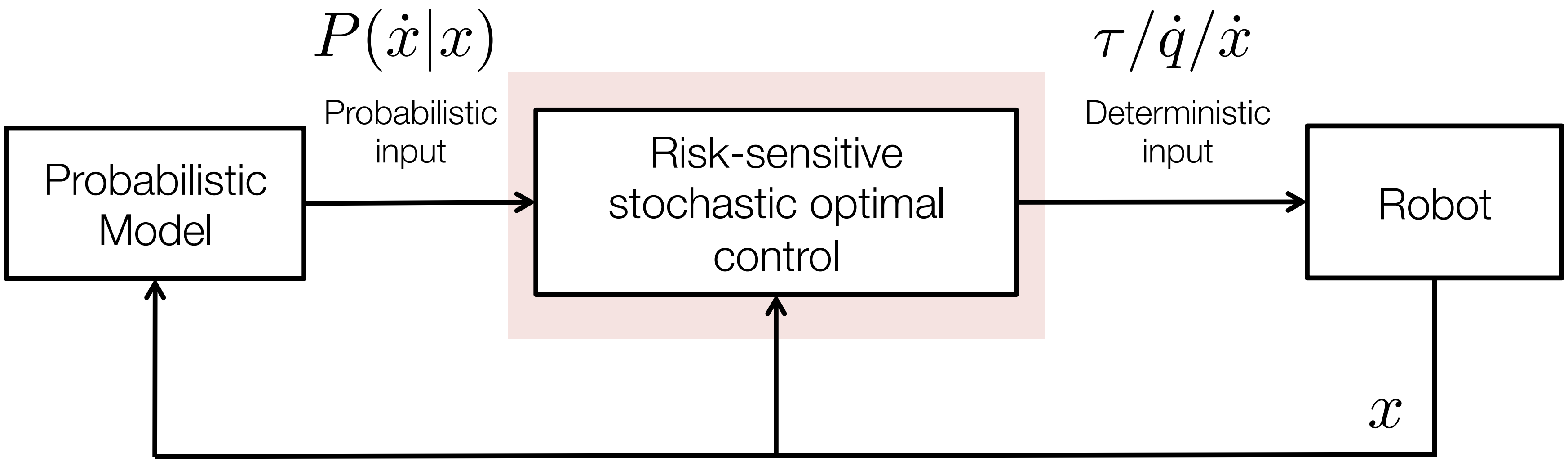
From probabilistic models to robot control



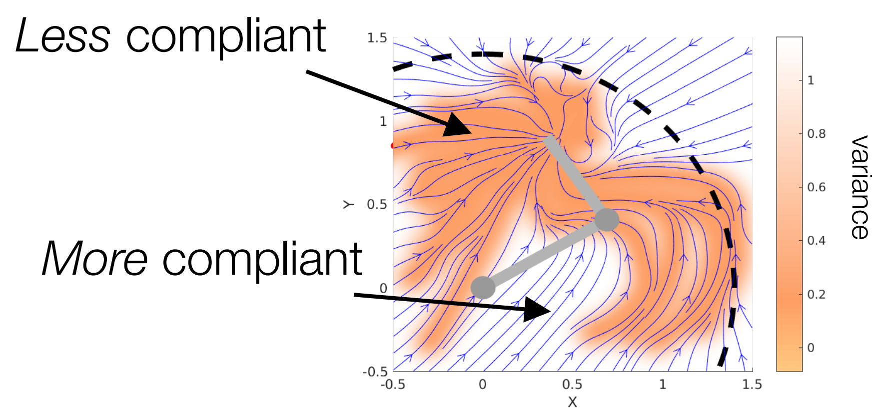
Discards model variance!



From probabilistic models to robot control



Adapts robot behavior to uncertainty level



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Optimal control

Problem formulation

- System dynamics $\boldsymbol{\xi}_{k+1} = \boldsymbol{f}(\boldsymbol{\xi}_k, \boldsymbol{u}_k)$
- Initial state $\boldsymbol{\xi}_0$
- Cost (finite horizon T)

$$J(\boldsymbol{\xi}_{0\ldots T}, \boldsymbol{u}_{0\ldots T-1}) = h_f(\boldsymbol{\xi}_T) + \sum_{k=0}^{T-1} h_k(\boldsymbol{\xi}_k, \boldsymbol{u}_k)$$

- Optimization criterion

$$\min_{\boldsymbol{u}_{0\cdots T-1}} \quad J$$

Stochastic optimal control

Problem formulation

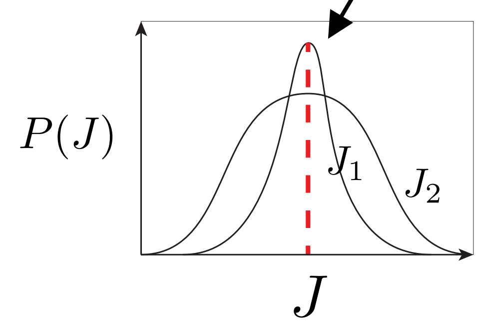
- System dynamics $\boldsymbol{\xi}_{k+1} = \boldsymbol{f}(\boldsymbol{\xi}_k, \boldsymbol{u}_k) + \boldsymbol{\varepsilon}_k \qquad \boldsymbol{\varepsilon}_k \sim \mathcal{N}(0, \Sigma_k)$
- Initial state $\boldsymbol{\xi}_0$
- *Random* Cost (finite horizon T)

$$J(\boldsymbol{\xi}_{0\dots T}, \boldsymbol{u}_{0\dots T-1}) = h_f(\boldsymbol{\xi}_T) + \sum_{k=0}^{T-1} h_k(\boldsymbol{\xi}_k, \boldsymbol{u}_k)$$

- Optimization criterion

$$\min_{\boldsymbol{u}_{0\dots T-1}} \quad \mathbb{E}[J]$$

Neglects cost variance!



Risk-sensitive optimal control

Problem formulation

- System dynamics $\boldsymbol{\xi}_{k+1} = \boldsymbol{f}(\boldsymbol{\xi}_k, \boldsymbol{u}_k) + \boldsymbol{\varepsilon}_k \qquad \boldsymbol{\varepsilon}_k \sim \mathcal{N}(0, \Sigma_k)$
- Initial state $\boldsymbol{\xi}_0$
- *Random* Cost (finite horizon T)

$$J(\boldsymbol{\xi}_{0\ldots T}, \boldsymbol{u}_{0\ldots T-1}) = h_f(\boldsymbol{\xi}_T) + \sum_{k=0}^{T-1} h_k(\boldsymbol{\xi}_k, \boldsymbol{u}_k)$$

- Optimization criterion

$$\min_{\boldsymbol{u}_{0\cdots T-1}} \; \theta^{-1} \log \mathbb{E}[\exp\{\theta J\}] \approx \boxed{\mathbb{E}[J] + \theta Var[J]}$$

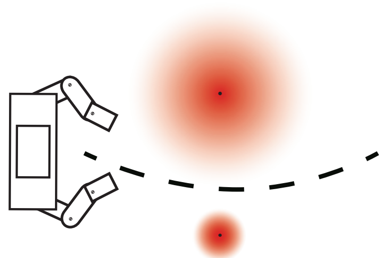
Risk-sensitive optimal control

Uncertainty as a *positive* or *negative* influence?

$$\min_{\boldsymbol{u}_{0\cdots T-1}} \theta^{-1} \log \mathbb{E}[\exp\{\theta J\}] \approx \boxed{\mathbb{E}[J] + \theta Var[J]}$$

Risk-averse

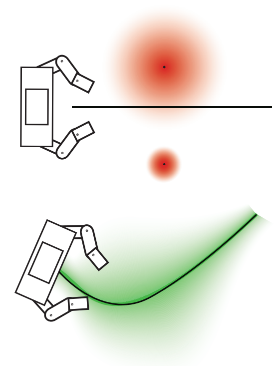
$\theta > 0$



Uncertainty *increases* overall cost

Risk-neutral

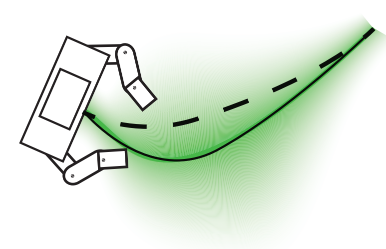
$\theta = 0$



Ignore uncertainty

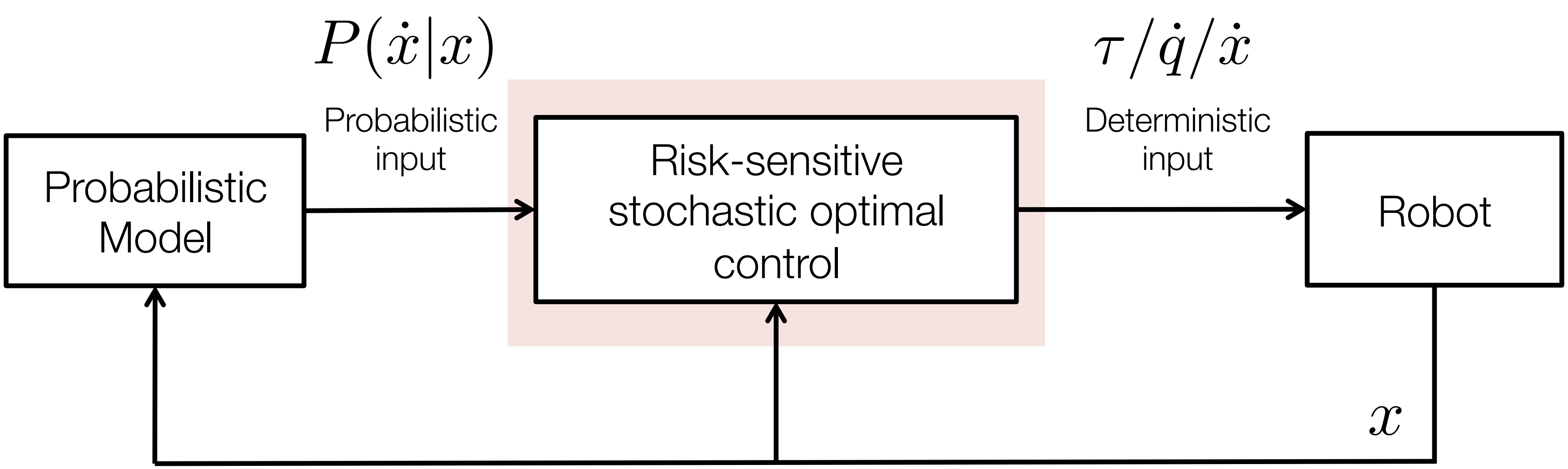
Risk-seeking

$\theta < 0$

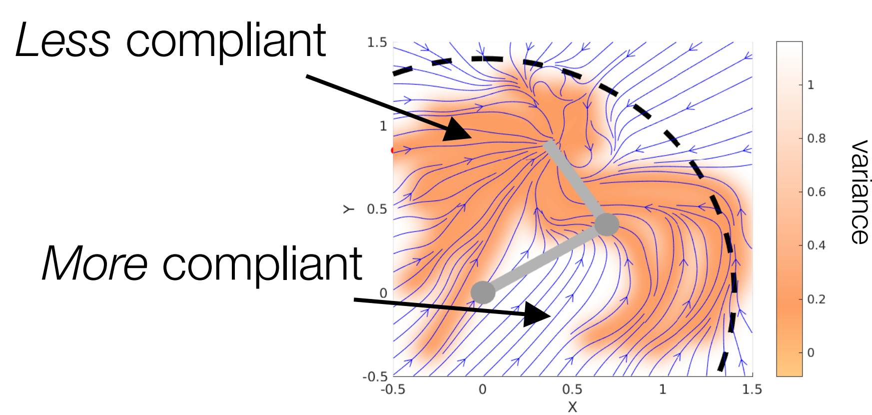


Uncertainty *decreases* overall cost

Exercise 4: uncertainty-dependent compliance through optimal control

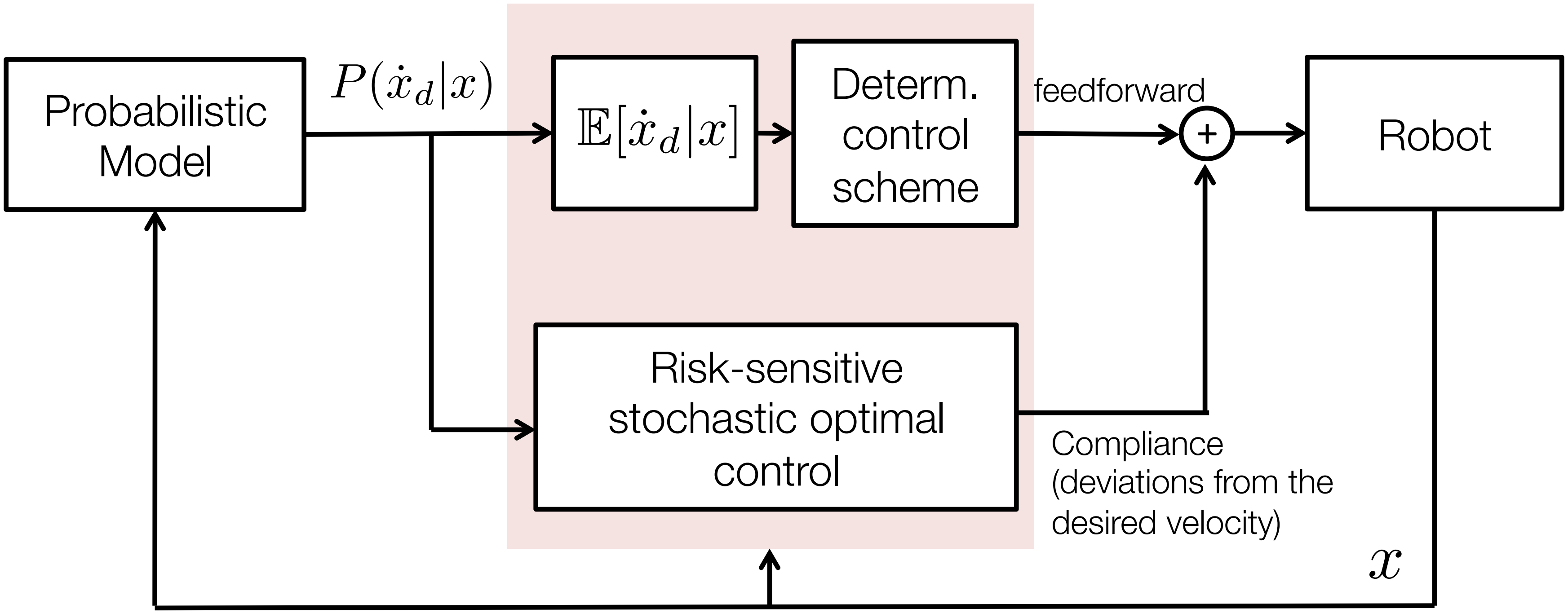


Adapts robot behavior to uncertainty level



Exercise 4: uncertainty-dependent compliance through optimal control

Simplified setting -> Regulation around desired trajectory



Exercise 4: uncertainty-dependent compliance through optimal control

Problem formulation

- System dynamics $\boldsymbol{\xi}_{k+1} = \boldsymbol{f}(\boldsymbol{\xi}_k, \boldsymbol{u}_k) + \boldsymbol{\varepsilon}_k \qquad \boldsymbol{\varepsilon}_k \sim \mathcal{N}(0, \Sigma_k)$

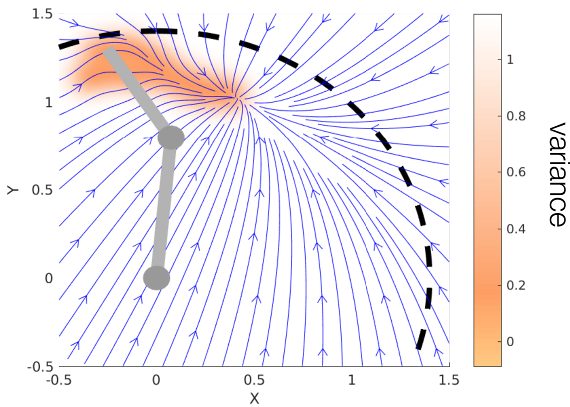
Second order *error* dynamics:

$$\boldsymbol{\xi} = \begin{bmatrix} x_d - x \\ \dot{x}_d - \dot{x} \end{bmatrix} \qquad \boldsymbol{\xi}_{k+1} = A_k \boldsymbol{\xi}_k + B_k \boldsymbol{u}_k + \varepsilon_k \qquad \Sigma = \begin{bmatrix} 0 & 0 \\ 0 & Var[\dot{x}_d] \end{bmatrix}$$

DS – Manipulator dynamics
(linear approx around simulated trjectory)

- Initial state

$$\boldsymbol{\xi}_0 = \begin{bmatrix} 0 \\ \dot{x}_{d,0} - \dot{x}_0 \end{bmatrix}$$



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Exercise 4: uncertainty-dependent compliance through optimal control

Problem formulation

- Cost

$$J = \sum_{k=0}^{T} \boldsymbol{\xi}_k^{\mathsf{T}} Q \boldsymbol{\xi}_k + \boldsymbol{u}_k^{\mathsf{T}} R \boldsymbol{u}_k$$

Trade-off between accuracy/effort

- Optimization criterion

$$\min_{\boldsymbol{u}_{0\cdots T-1}} \theta^{-1} \log \mathbb{E}[\exp\{\theta J\}] \approx \boxed{\mathbb{E}[J] + \theta Var[J]}$$

Exercise 4: uncertainty-dependent compliance through optimal control

Solution

(Linear-quadratic regulation problem)

$$\boldsymbol{u}_k = -L_k \boldsymbol{\xi}_k = -\begin{bmatrix} K_k(x_d - x) \\ D_k(\dot{x}_d - \dot{x}) \end{bmatrix}$$ *Stiffness* / *Damping*

- Expectation $\min_{\boldsymbol{u}_{0\cdots T-1}} \mathbb{E}[J]$

$$K_k = R^{-1}B'(BR^{-1}B' + \Pi_{k+1}^{-1})^{-1}A$$

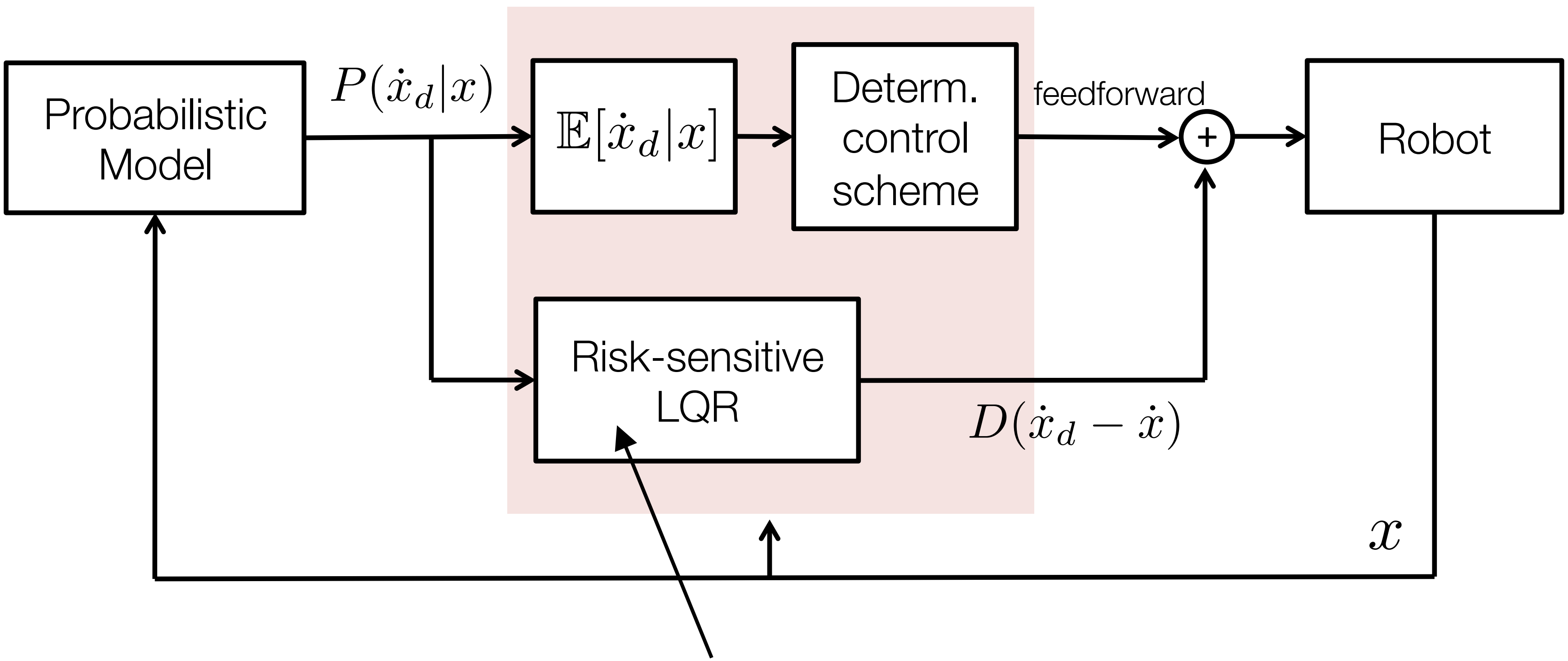
$$\Pi_k = Q_k + A'(BR^{-1}B' + \Pi_{k+1}^{-1})^{-1}A \,, \qquad \Pi_T = Q_T$$

- Risk-sensitive solution $\min_{\boldsymbol{u}_{0\cdots T-1}} \theta^{-1} \log \mathbb{E}[\exp\{\theta J\}] \approx \boxed{\mathbb{E}[J] + \theta Var[J]}$

$$K_k = R^{-1}B'(BR^{-1}B' - \theta\Sigma_k + \Pi_{k+1}^{-1})^{-1}A$$

$$\Pi_k = Q_k + A'(BR^{-1}B' - \theta\Sigma_k + \Pi_{k+1}^{-1})^{-1}A \,, \qquad \Pi_T = Q_T$$

Exercise 4: uncertainty-dependent compliance through optimal control



Compute at each time step → Model predictive control

Only use the solution at simulation time 0

Summary

- Uncertainty encodes valuable information!

- Risk-sensitive optimal control is a convenient tool for control problems with *uncertain* probabilistic models (LfD)

- Assess uncertainty‘s influence in different ways depending on risk-sensitivity parameter.

Open issues and ongoing research

- Consider feedforward + feedback control in the optimization => uncertainty-dependent *trajectories* and *compliance*.

- Systematic way to select risk-sensitivity parameters.

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