

### 第三章 线性代数回顾

矩阵加法（需要维度相同）

注：看过高数的这部分应该不需要详解吧

#### Matrix Addition

$$\begin{array}{l} \rightarrow \begin{bmatrix} 1 & 0 \\ 2 & 5 \\ 3 & 1 \end{bmatrix} + \begin{bmatrix} 4 & 0.5 \\ 2 & 5 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 0.5 \\ 4 & 10 \\ 3 & 2 \end{bmatrix} \\ \text{3x2 matrix} \quad \text{3x2} \quad \text{3x2} \end{array}$$
  
$$\begin{array}{l} \rightarrow \begin{bmatrix} 1 & 0 \\ 2 & 5 \\ 3 & 1 \end{bmatrix} + \begin{bmatrix} 4 & 0.5 \\ 2 & 5 \end{bmatrix} = \text{error} \\ \text{3x2} \quad \text{2x2} \end{array}$$

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矩阵相乘

#### Scalar Multiplication

← real number

$$3 \times \begin{bmatrix} 1 & 0 \\ 2 & 5 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 6 & 15 \\ 9 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & 5 \\ 3 & 1 \end{bmatrix} \times 3$$
  
$$\begin{bmatrix} 4 & 0 \\ 6 & 3 \end{bmatrix} / 4 = \frac{1}{4} \begin{bmatrix} 4 & 0 \\ 6 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{3}{2} & \frac{3}{4} \end{bmatrix}$$

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## Combination of Operands

$$\begin{aligned}
 & \xrightarrow{\text{Scalar multiplication}} 3 \times \begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix} - \begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix} \xrightarrow{\text{Scalar division}} /3 \\
 & = \begin{bmatrix} 3 \\ 12 \\ 6 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \\ \frac{2}{3} \end{bmatrix} \quad \begin{array}{l} \text{matrix subtraction /} \\ \text{vector subtraction} \end{array} \\
 & = \begin{bmatrix} 2 \\ 12 \\ 10\frac{1}{3} \end{bmatrix} \quad \begin{array}{l} \text{matrix addition?} \\ \text{vector addition} \end{array}
 \end{aligned}$$

3x1 matrix  
3-dimensional vector

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## Example

$$\begin{array}{c}
 \begin{bmatrix} 1 & 3 \\ 4 & 0 \\ 2 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 \\ 5 \end{bmatrix} = \begin{bmatrix} 16 \\ 4 \\ 7 \end{bmatrix} \\
 \begin{array}{cc} 3 \times 2 & 2 \times 1 \end{array}
 \end{array}$$

$$\begin{aligned}
 1 \times 1 + 3 \times 5 &= 16 \\
 4 \times 1 + 0 \times 5 &= 4 \\
 2 \times 1 + 1 \times 5 &= 7
 \end{aligned}$$

## Details:

$$\begin{array}{c}
 \underline{A} \quad \times \quad \underline{x} \quad = \quad \underline{y} \\
 \begin{array}{c} \text{m} \times \text{n} \text{ matrix} \\ \text{(m rows, n columns)} \end{array} \quad \begin{array}{c} \text{n} \times 1 \text{ matrix} \\ \text{(n-dimensional vector)} \end{array} \quad \begin{array}{c} \text{m-dimensional} \\ \text{vector} \end{array}
 \end{array}$$

To get  $y_i$ , multiply  $A$ 's  $i^{\text{th}}$  row with elements of vector  $x$ , and add them up.

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House sizes:

$$\begin{pmatrix} 2104 \\ 1416 \\ 1534 \\ 852 \end{pmatrix}$$

Matrix

$$\begin{bmatrix} 1 & 2104 \\ 1 & 1416 \\ 1 & 1534 \\ 1 & 852 \end{bmatrix}$$

Have 3 competing hypotheses:

1.  $h_{\theta}(x) = -40 + 0.25x$
2.  $h_{\theta}(x) = 200 + 0.1x$
3.  $h_{\theta}(x) = -150 + 0.4x$

Matrix

$$\begin{bmatrix} -40 & 200 & -150 \\ 0.25 & 0.1 & 0.4 \end{bmatrix}$$

Matrix

$$\begin{bmatrix} 486 & 410 & 692 \\ 314 & 342 & 416 \\ 344 & 353 & 464 \\ 173 & 285 & 191 \end{bmatrix}$$

Prediction of 1st  $h_{\theta}$

Predictions of 2nd  $h_{\theta}$

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矩阵乘法不符合交换律

$$3 \times 5 = 5 \times 3$$

"Commutative"

Let  $A$  and  $B$  be matrices. Then in general,  
 $A \times B \neq B \times A$ . (not commutative.)

E.g.

$$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix} \quad \left| \quad \begin{array}{l} A \times B \\ m \times n \quad n \times m \end{array} \right.$$

$$\begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 2 & 2 \end{bmatrix} \quad \left| \quad \begin{array}{l} B \times A \\ n \times m \quad m \times n \end{array} \right.$$

$A \times B$  is  $m \times m$   
 $B \times A$  is  $n \times n$

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## Identity Matrix

Denoted  $I$  (or  $I_{n \times n}$ ).

Examples of identity matrices:

$$\begin{bmatrix} 1 \end{bmatrix}_{1 \times 1} \quad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2 \times 2} \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3 \times 3} \quad \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4 \times 4}$$

1 is identity.

$$1 \times z = z \times 1 = z$$

for any  $z$

Informally:

$$\begin{bmatrix} 1 & & & \\ & 1 & & \\ & & \ddots & \\ & & & 1 \end{bmatrix}$$

For any matrix  $A$ ,

$$A \cdot I = I \cdot A = A$$

$m \times n$   $n \times n$   $n \times m$   $m \times n$   $m \times n$

$$I_{n \times n}$$

Note:

$AB \neq BA$  in general

$$AI = IA \quad \checkmark$$

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矩阵的逆

1 = "identity."

$$3 \left[ \frac{1}{3} \right] = 1$$

$$12 \times \left( \frac{1}{12} \right) = 1$$

$$0 \left( \frac{1}{0} \right) \text{ undefined}$$

Not all numbers have an inverse.

**Matrix inverse:**

square matrix  
(# rows = # columns)

$A^{-1}$

If  $A$  is an  $m \times m$  matrix, and if it has an inverse,

$$A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\rightarrow A(A^{-1}) = A^{-1}A = I.$$

e.g.  $\underbrace{\begin{bmatrix} 3 & 4 \\ 2 & 16 \end{bmatrix}}_A \underbrace{\begin{bmatrix} 0.4 & -0.1 \\ -0.05 & 0.075 \end{bmatrix}}_{A^{-1}} = \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{A^{-1}A} = I_{2 \times 2}$

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矩阵的转置

## Matrix Transpose

Example:

$$\underline{A} = \begin{bmatrix} 1 & 2 & 0 \\ 3 & 5 & 9 \end{bmatrix}$$

$$\underline{A^T} = \begin{bmatrix} 1 & 3 \\ 2 & 5 \\ 0 & 9 \end{bmatrix}$$