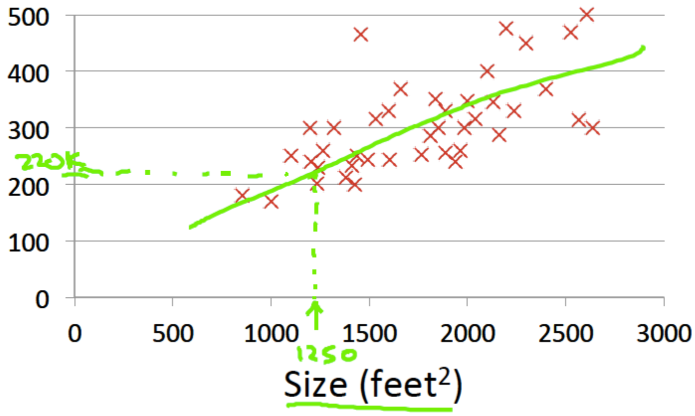
Housing Prices (Portland, OR)



Supervised Learning

Given the “right answer” for each example in the data.

Regression Problem

Predict real-valued output

Classification: Discrete-valued output

Andrew Ng

Training set of housing prices (Portland, OR)

Size in feet^2 (x)	Price ($) in 1000's (y)
2104	460
1416	232
1534	315
852	178
…	…

$m = 47$

Notation:

- **m** = Number of training examples
- **x**’s = “input” variable / features
- **y**’s = “output” variable / “target” variable

(x, y) – one training example

$(x^{(i)}, y^{(i)})$ – i^{th} training example

$x^{(1)} = 2104$

$x^{(2)} = 1416$

$y^{(1)} = 460$

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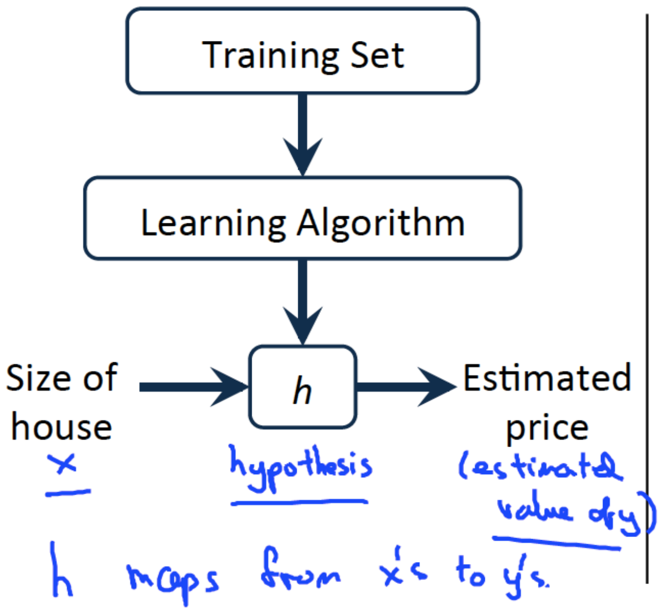

How do we represent h ?

$h_\theta(x) = \theta_0 + \theta_1 x$

Shorthand: $h(x)$

Linear regression with one variable. (x)
Univariate linear regression.
└ one variable

Andrew Ng

损失函数（衡量一个模型的好坏）

Training Set

Size in feet² (x)	Price ($) in 1000's (y)
2104	460
1416	232
1534	315
852	178
…	…

m = 47

Hypothesis: $h_\theta(x) = \theta_0 + \theta_1 x$

θ_i's: Parameters

How to choose θ_i's ?

$h_\theta(x) = \theta_0 + \theta_1 x$

$\theta_0 = 1.5$	$\theta_0 = 0$	$\theta_0 = 1$
$\theta_1 = 0$	$\theta_1 = 0.5$	$\theta_1 = 0.5$

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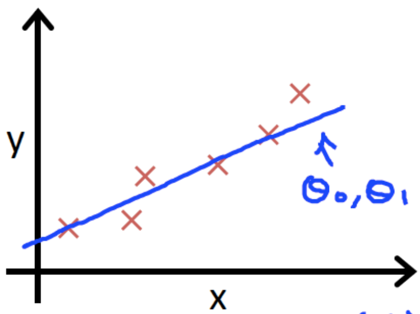


$(x^{(i)}, y^{(i)})$

training examples

$$\underset{\theta_0, \theta_1}{\text{minimize}} \ \frac{1}{2m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)} \right)^2$$

$$h_\theta(x^{(i)}) = \theta_0 + \theta_1 x^{(i)}$$

$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)} \right)^2$$

$$\underset{\theta_0, \theta_1}{\text{minimize}} \ \underbrace{J(\theta_0, \theta_1)}_{\text{Cost function}}$$

Squared error function

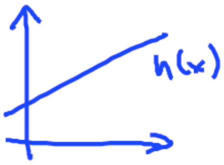
Idea: Choose θ_0, θ_1 so that $h_\theta(x)$ is close to y for our training examples (x, y)

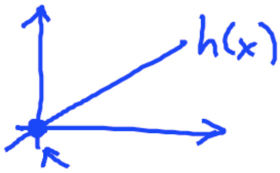
x, y

Andrew Ng

Simplified

Hypothesis: $h_\theta(x) = \theta_0 + \theta_1 x$	$h_\theta(x) = \theta_1 x$ $\theta_0 = 0$
Parameters: θ_0, θ_1	θ_1
Cost Function: $J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)}\right)^2$	$J(\theta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)}\right)^2$ $\theta_1 x^{(i)}$
Goal: $\underset{\theta_0, \theta_1}{\text{minimize}}\ J(\theta_0, \theta_1)$	$\underset{\theta_1}{\text{minimize}}\ J(\theta_1)$





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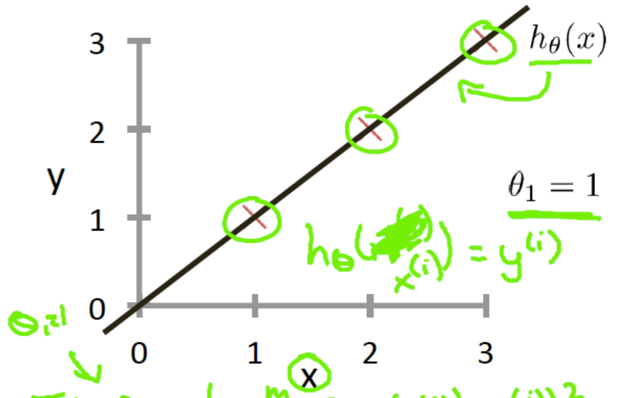
注：1/2m而不是1/m是为了方便计算（个人理解）。平方损失函数

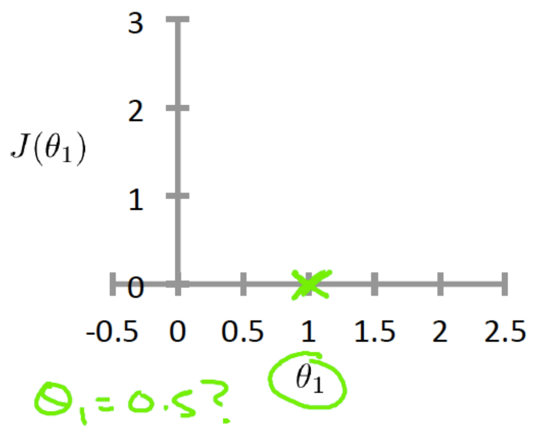
$h_\theta(x)$

(for fixed θ_1, this is a function of x)



$J(\theta_1)$

(function of the parameter θ_1)



$$J(\theta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)}\right)^2$$

$$= \frac{1}{2m} \sum_{i=1}^{m} \left(\theta_1 x^{(i)} - y^{(i)}\right)^2 = \frac{1}{2m}(0^2 + 0^2 + 0^2) = 0^2$$

$\theta_1 = 0.5$?

$J(1) = 0$

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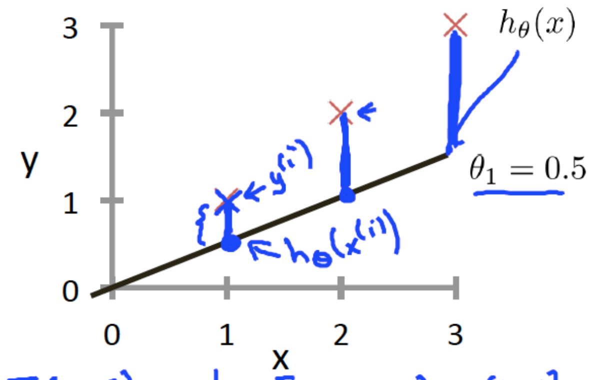
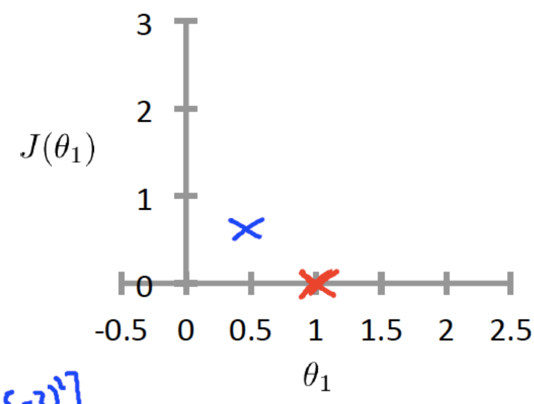
$h_\theta(x)$

(for fixed θ_1, this is a function of x)



$J(\theta_1)$

(function of the parameter θ_1)



$J(0.5) = \frac{1}{2m}\left[(0.5-1)^2 + (1-2)^2 + (1.5-3)^2\right]$

$= \frac{1}{2\times 3}(3.5) = \frac{3.5}{6} \approx 0.58$

$\theta_1 = 0$?

$J(0) = ?$

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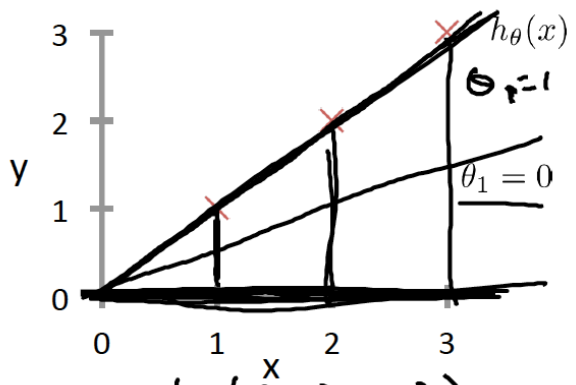
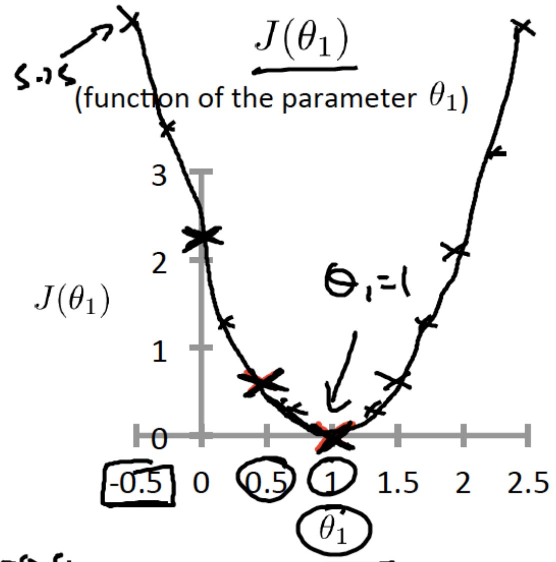
$h_\theta(x)$

(for fixed θ_1, this is a function of x)



$J(\theta_1)$

(function of the parameter θ_1)



$J(0) = \frac{1}{2m}(1^2 + 2^2 + 3^2)$

$= \frac{1}{6} \cdot 14 \approx 2.3$

$h(x) = -0.5x$

$h(x)$

$\underset{\theta_1}{\text{minimize}}\ J(\theta_1)$

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Hypothesis: $h_\theta(x) = \theta_0 + \theta_1 x$

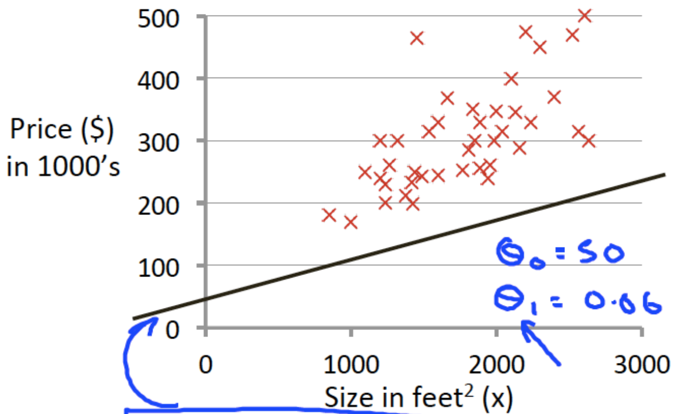
Parameters: θ_0, θ_1

Cost Function: $J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)}\right)^2$

Goal: $\underset{\theta_0, \theta_1}{\text{minimize}}\ J(\theta_0, \theta_1)$

$h_\theta(x)$

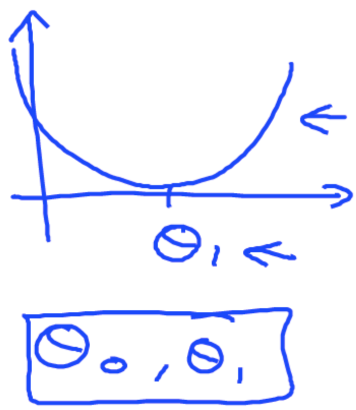
(for fixed θ_0, θ_1, this is a function of x)

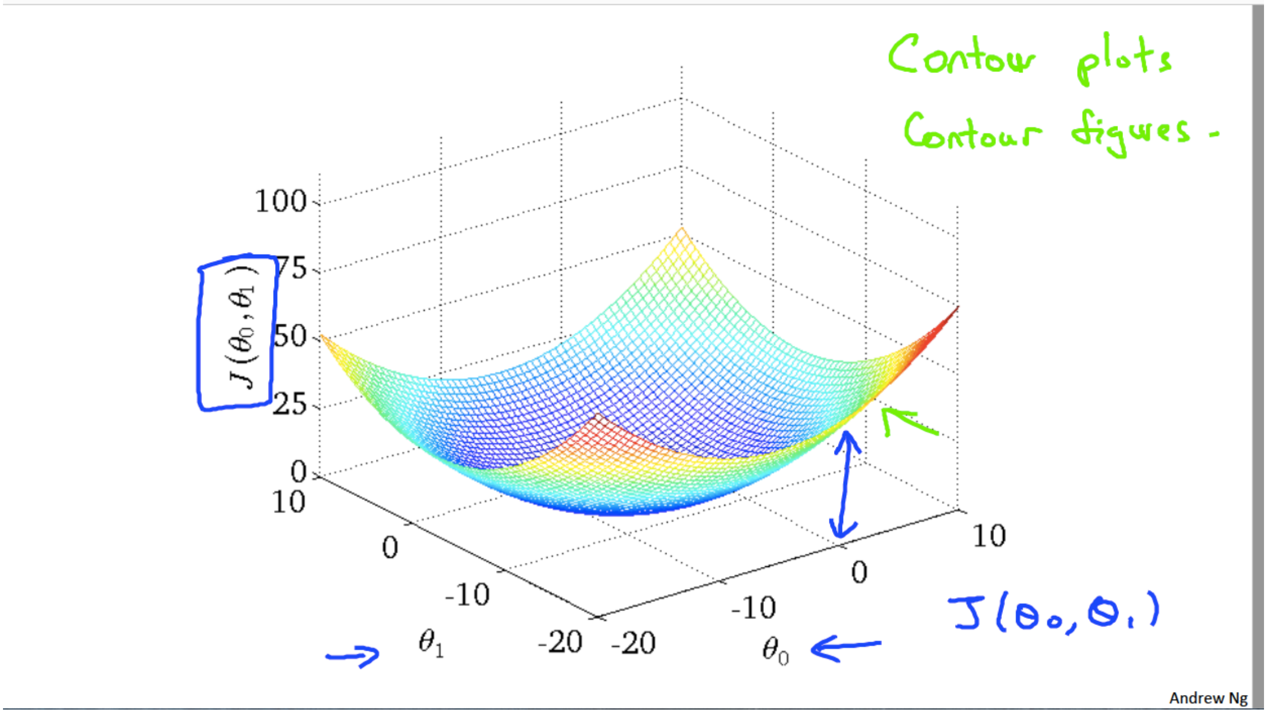


$\boxed{h_\theta(x) = 50 + 0.06x}$

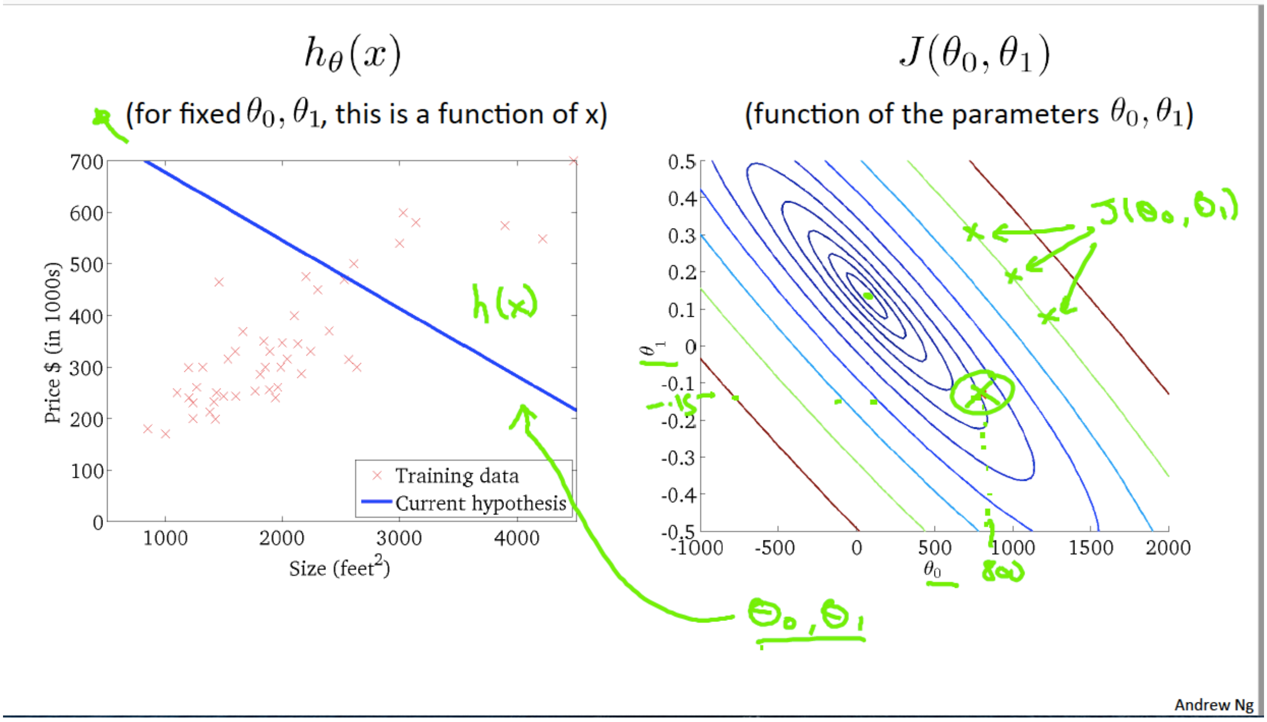
$J(\theta_0, \theta_1)$

(function of the parameters θ_0, θ_1)

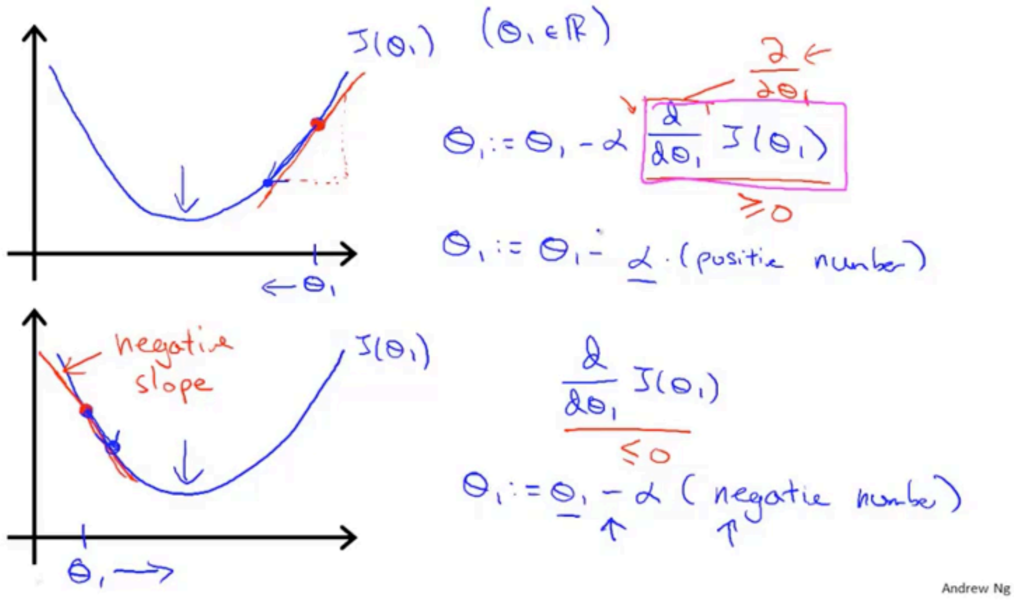




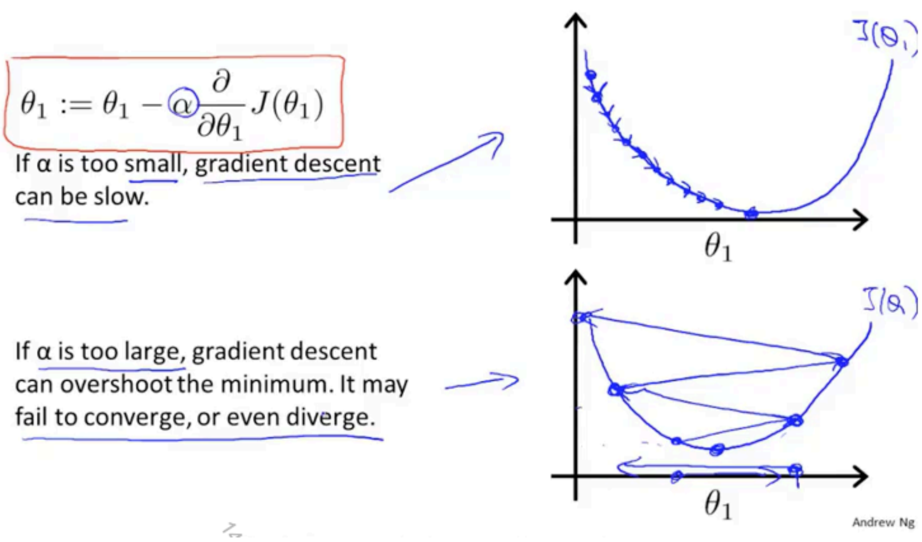
注：可采用梯度下降算法寻找最低点（即最小损失函数）



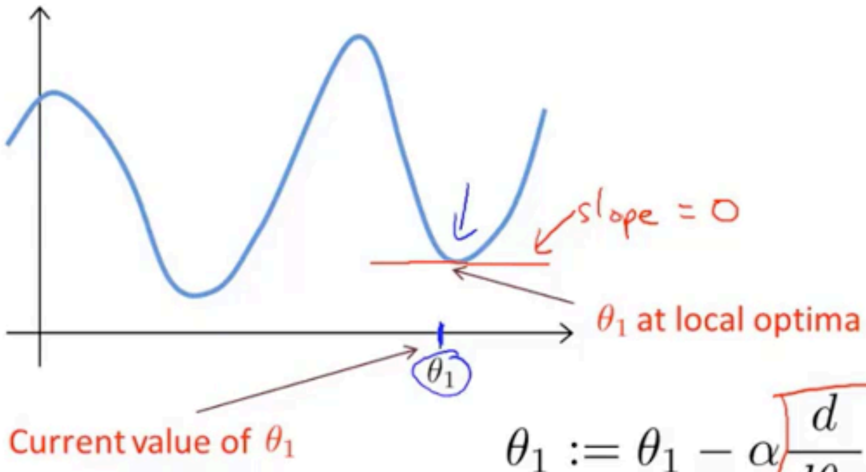
梯度下降算法寻找最小代价函数



当a过小或者过大的时候对梯度下降的影响



当代价函数处于局部最小时（即斜率为零）梯度下降不会改变原来参数的值



$$\theta_1 := \theta_1 - \alpha \underbrace{\boxed{\frac{d}{d\theta_1} J(\theta_1)}}_{=0}$$

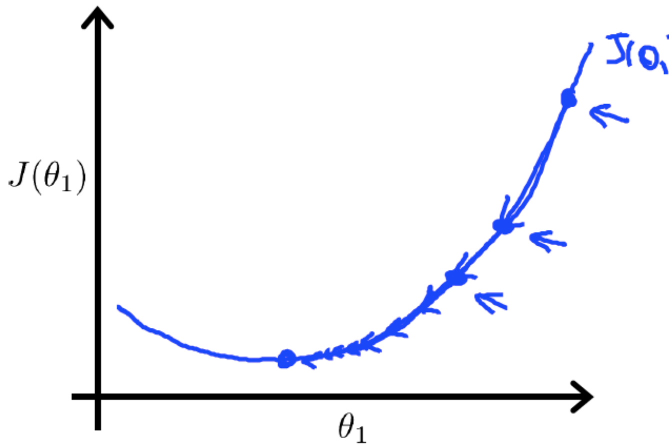
$\theta_1 := \theta_1 - \alpha \cdot 0$

$\theta_1 := \theta_1$

Gradient descent can converge to a local minimum, even with the learning rate α fixed.

$$\theta_1 := \theta_1 - \alpha \boxed{\frac{d}{d\theta_1} J(\theta_1)}$$

As we approach a local minimum, gradient descent will automatically take smaller steps. So, no need to decrease α over time.



Gradient descent algorithm

repeat until convergence {

$$\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1)$$

$$(\text{for } j = 1 \text{ and } j = 0)$$

}

Linear Regression Model

$$h_\theta(x) = \theta_0 + \theta_1 x$$

$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)}\right)^2$$

$$\frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1) = \frac{\partial}{\partial \theta_j} \frac{1}{2m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)}\right)^2$$

$$= \frac{\partial}{\partial \theta_j} \frac{1}{2m} \sum_{i=1}^{m} \left(\theta_0 + \theta_1 x^{(i)} - y^{(i)}\right)^2$$

$$j = 0 : \frac{\partial}{\partial \theta_0} J(\theta_0, \theta_1) = \frac{1}{m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)}\right)$$

$$j = 1 : \frac{\partial}{\partial \theta_1} J(\theta_0, \theta_1) = \frac{1}{m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)}\right) \cdot x^{(i)}$$

Gradient descent algorithm

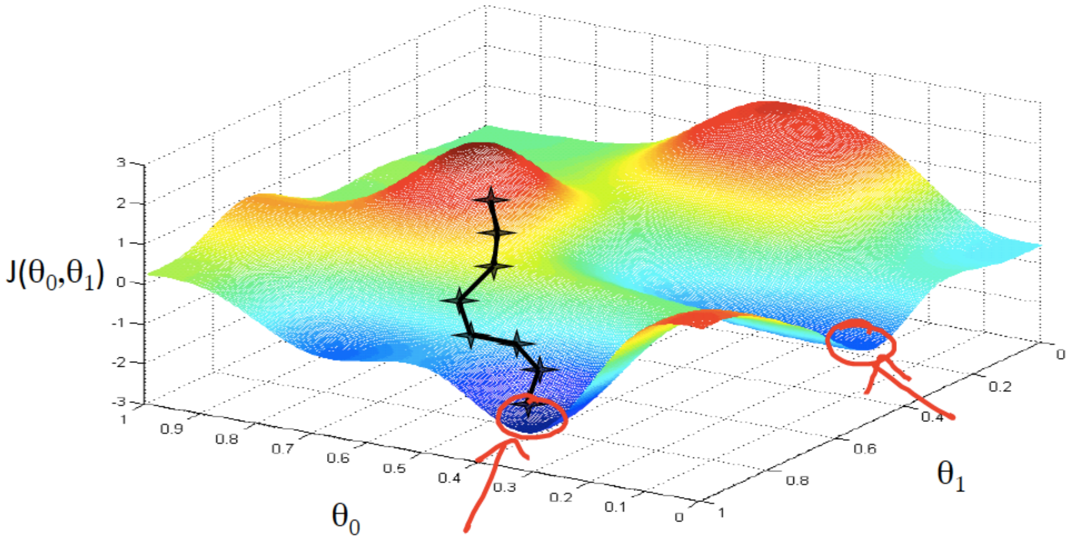
repeat until convergence {

$$\theta_0 := \theta_0 - \alpha \overbrace{\boxed{\frac{1}{m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)}\right)}}^{\frac{\partial}{\partial \theta_0} J(\theta_0, \theta_1)}$$

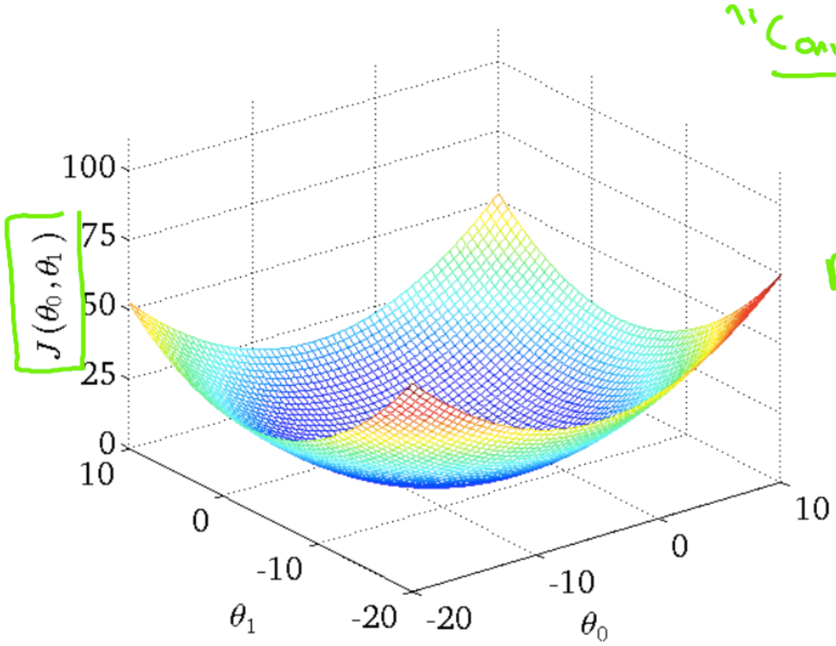
$$\theta_1 := \theta_1 - \alpha \underbrace{\boxed{\frac{1}{m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)}\right) \cdot x^{(i)}}}_{\frac{\partial}{\partial \theta_1} J(\theta_0, \theta_1)}$$

}

update θ_0 and θ_1 simultaneously



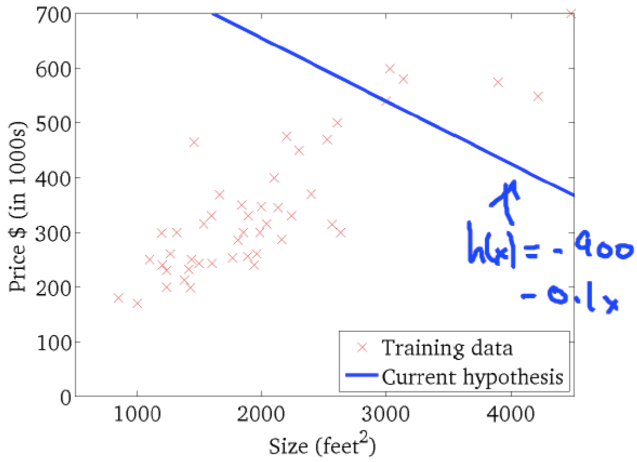
"Convex function"



Bowl-shaped

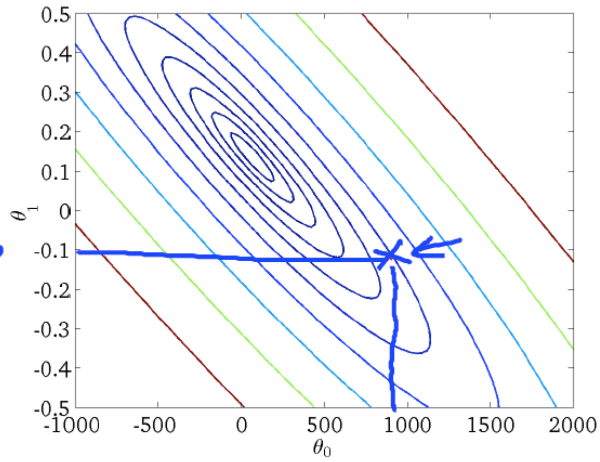
$h_\theta(x)$

(for fixed θ_0, θ_1, this is a function of x)



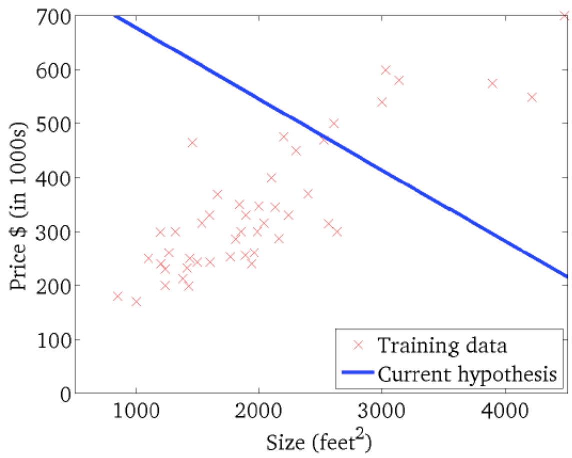
$J(\theta_0, \theta_1)$

(function of the parameters θ_0, θ_1)



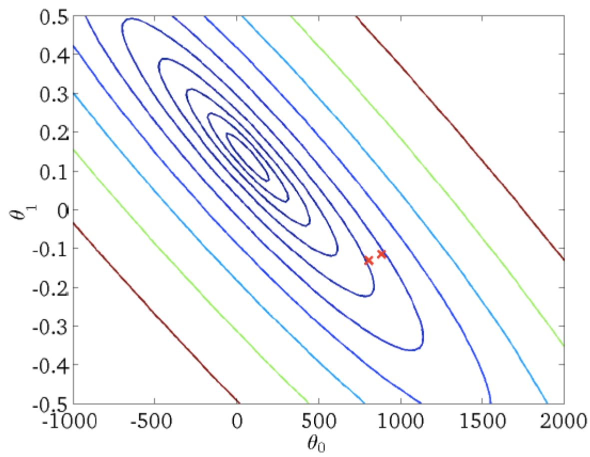
$h_\theta(x)$

(for fixed θ_0, θ_1, this is a function of x)



$J(\theta_0, \theta_1)$

(function of the parameters θ_0, θ_1)



$h_\theta(x)$

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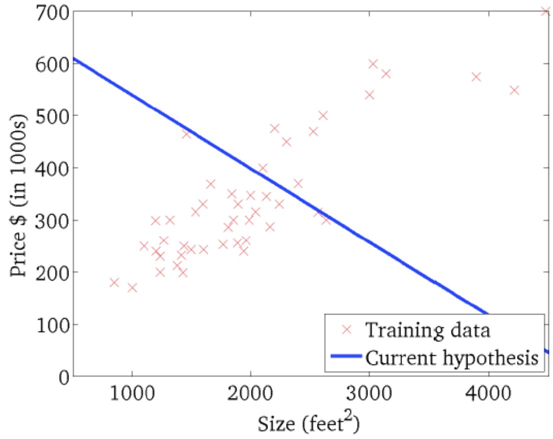

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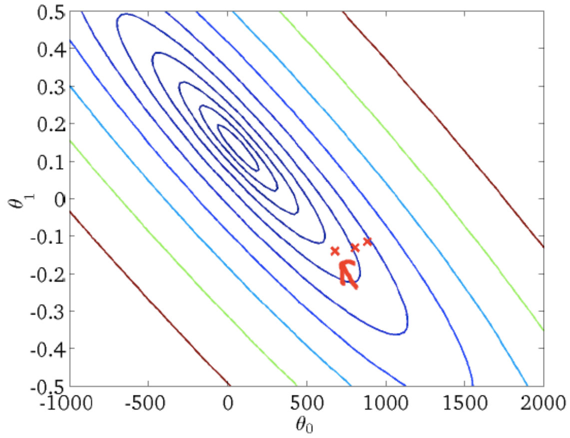
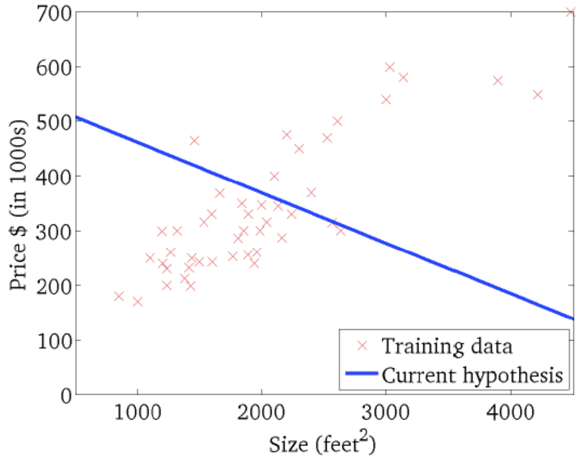


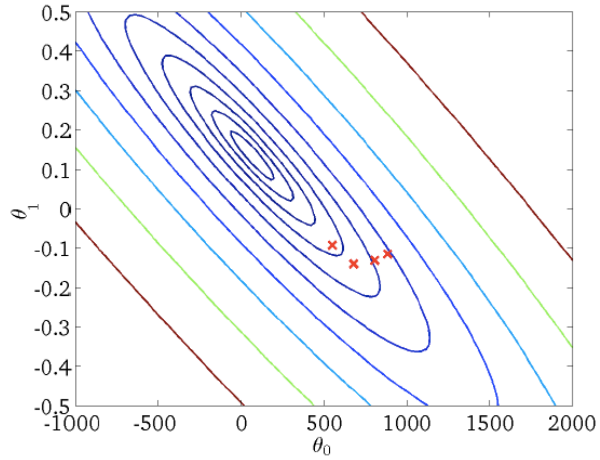
$h_\theta(x)$

(for fixed θ_0, θ_1, this is a function of x)



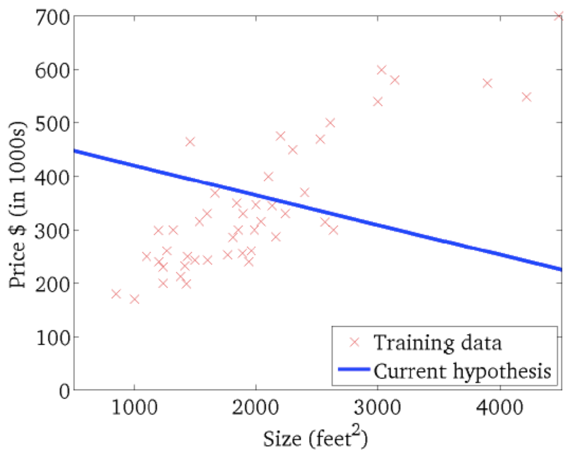
$J(\theta_0, \theta_1)$

(function of the parameters θ_0, θ_1)



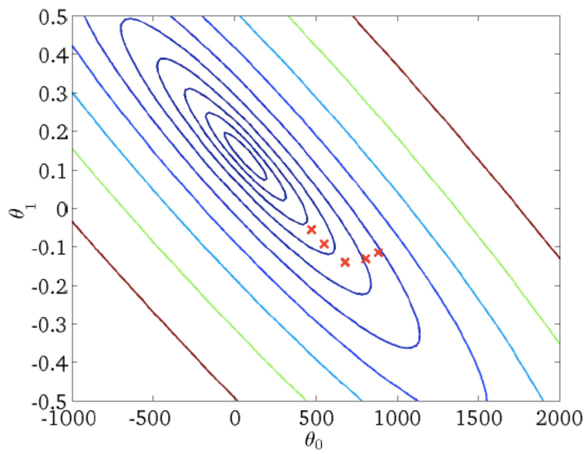
$h_\theta(x)$

(for fixed θ_0, θ_1, this is a function of x)



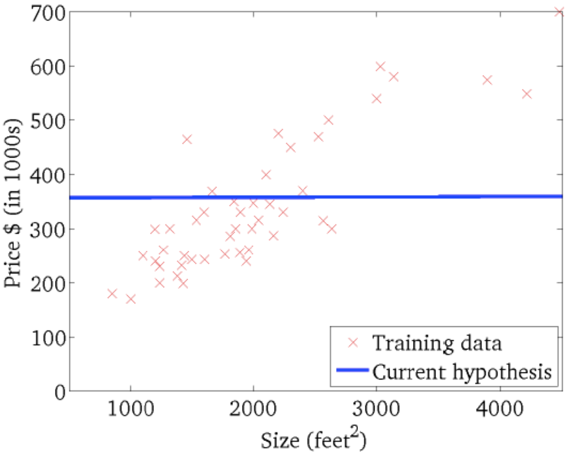
$J(\theta_0, \theta_1)$

(function of the parameters θ_0, θ_1)



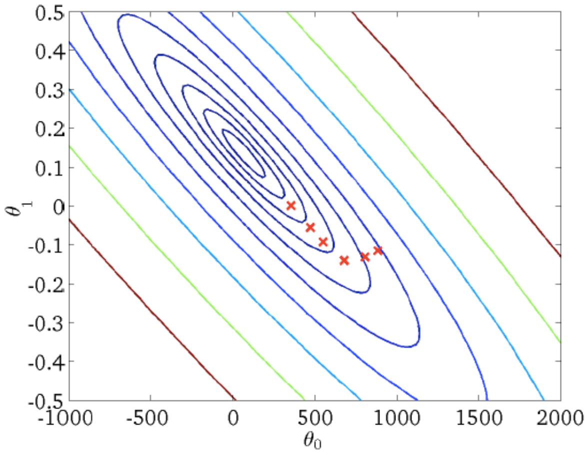
$h_\theta(x)$

(for fixed θ_0, θ_1, this is a function of x)



$J(\theta_0, \theta_1)$

(function of the parameters θ_0, θ_1)



$h_\theta(x)$

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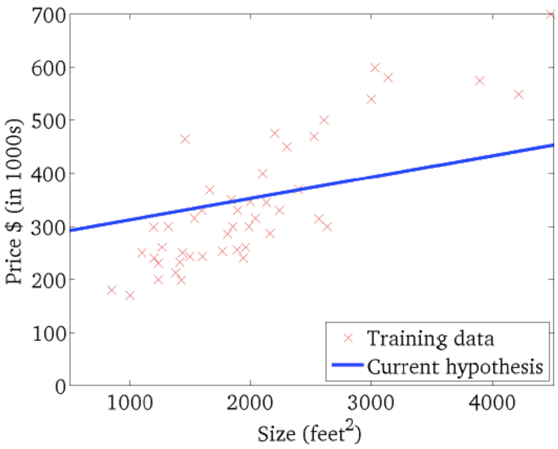
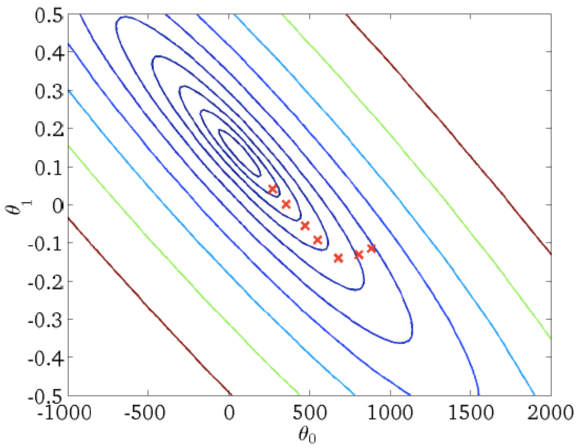


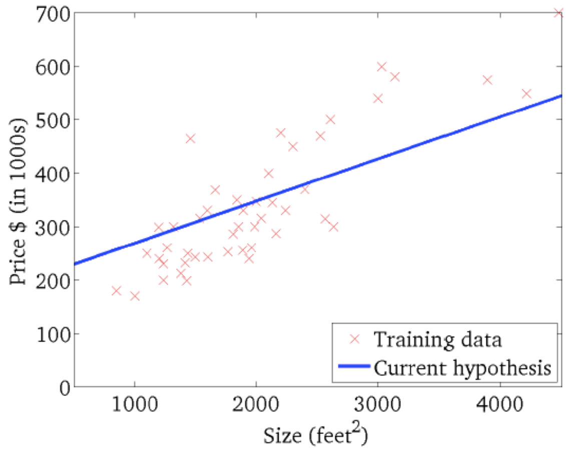
$J(\theta_0, \theta_1)$

(function of the parameters θ_0, θ_1)



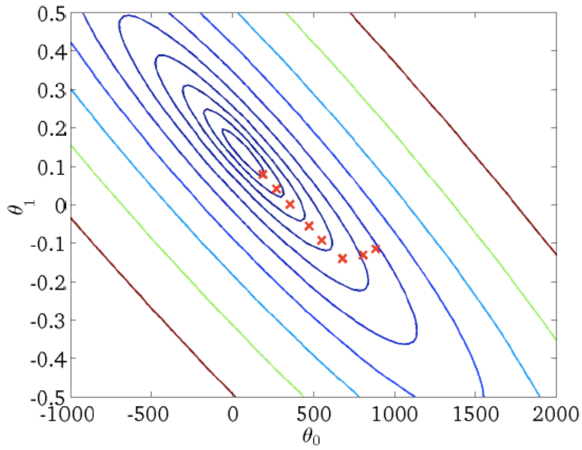
$h_\theta(x)$

(for fixed θ_0, θ_1, this is a function of x)



$J(\theta_0, \theta_1)$

(function of the parameters θ_0, θ_1)



$h_\theta(x)$

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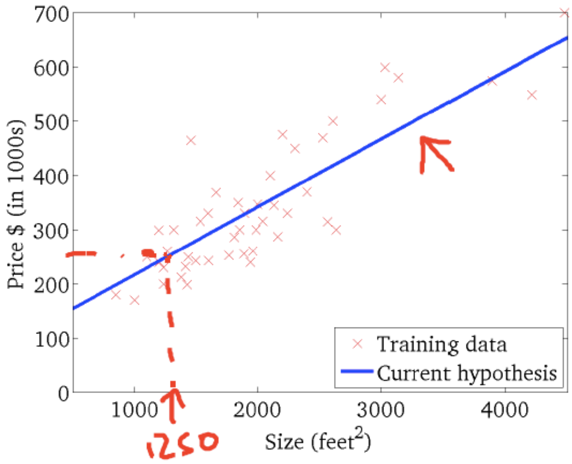
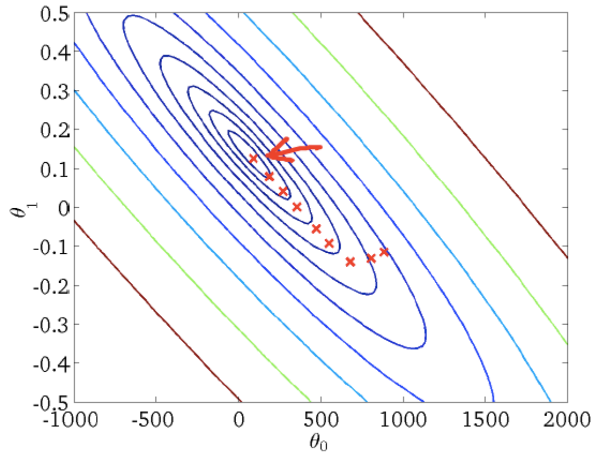


$J(\theta_0, \theta_1)$

(function of the parameters θ_0, θ_1)



"Batch" Gradient Descent

"Batch": Each step of gradient descent uses all the training examples.

$$\rightarrow \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)} \right)$$

以上是'batch'梯度下降法的过程。