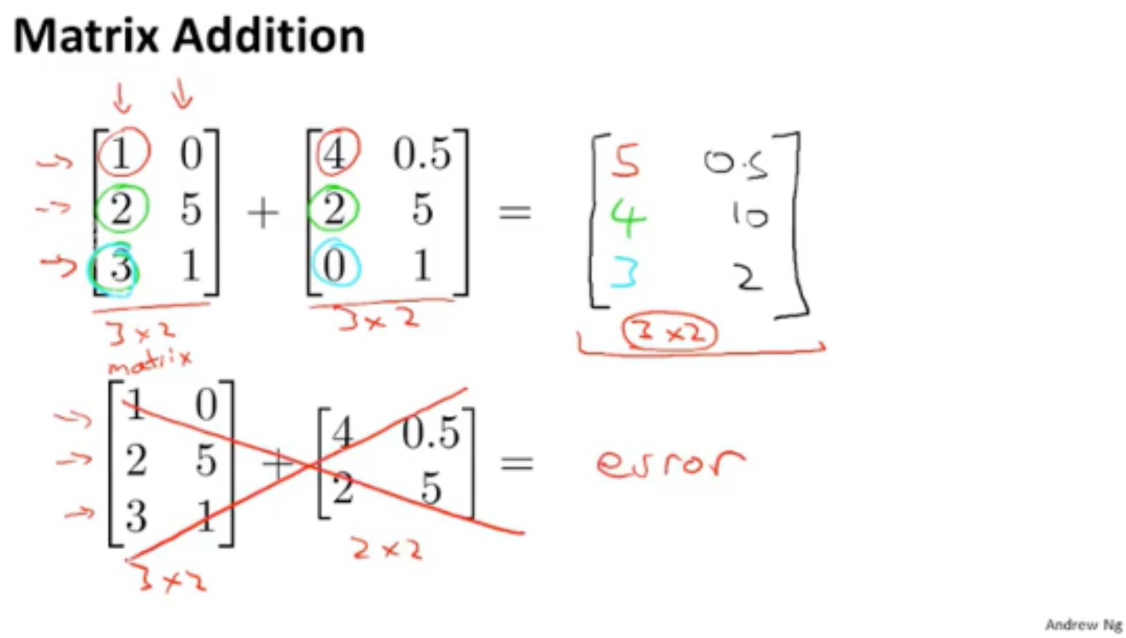
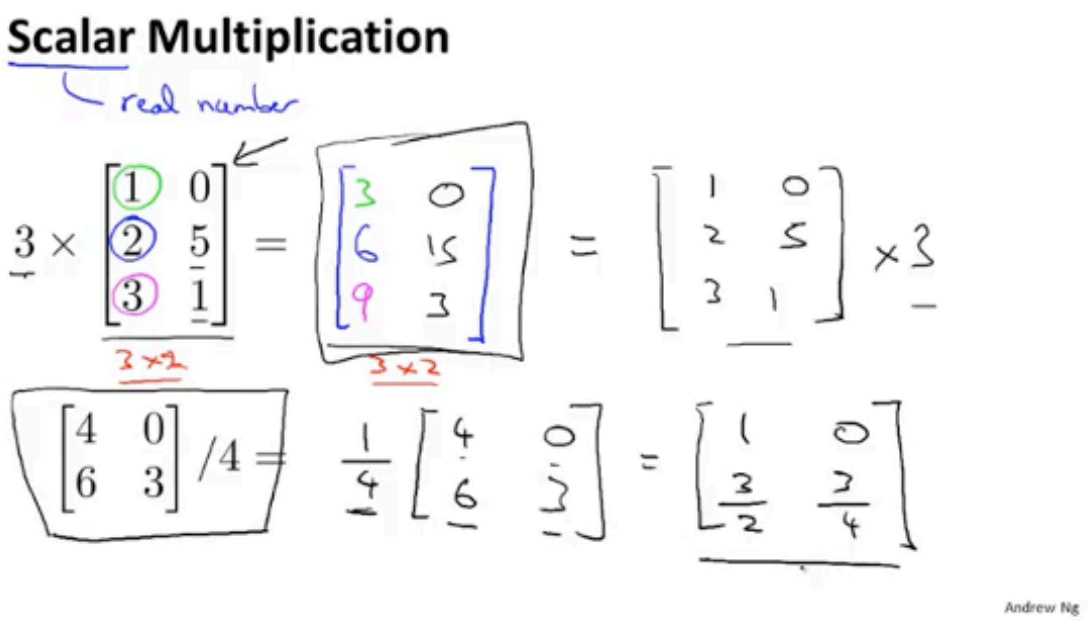
第三章 线性代数回顾

矩阵加法（需要维度相同）

注：看过高数的这部分应该不需要详解吧



矩阵相乘



Combination of Operands

$$\boxed{3 \times \begin{bmatrix}1\\4\\2\end{bmatrix}} + \begin{bmatrix}0\\0\\5\end{bmatrix} - \boxed{\begin{bmatrix}3\\0\\2\end{bmatrix} /3}$$

Scalar multiplication

Scalar division

$$= \begin{bmatrix}3\\12\\6\end{bmatrix} + \begin{bmatrix}0\\0\\5\end{bmatrix} - \begin{bmatrix}1\\0\\\frac{2}{3}\end{bmatrix}$$

matrix subtraction / vector subtraction

matrix addition / vector addition

$$= \begin{bmatrix}2\\12\\10\frac{1}{3}\end{bmatrix}$$

3×1 matrix

3-dimensional vector

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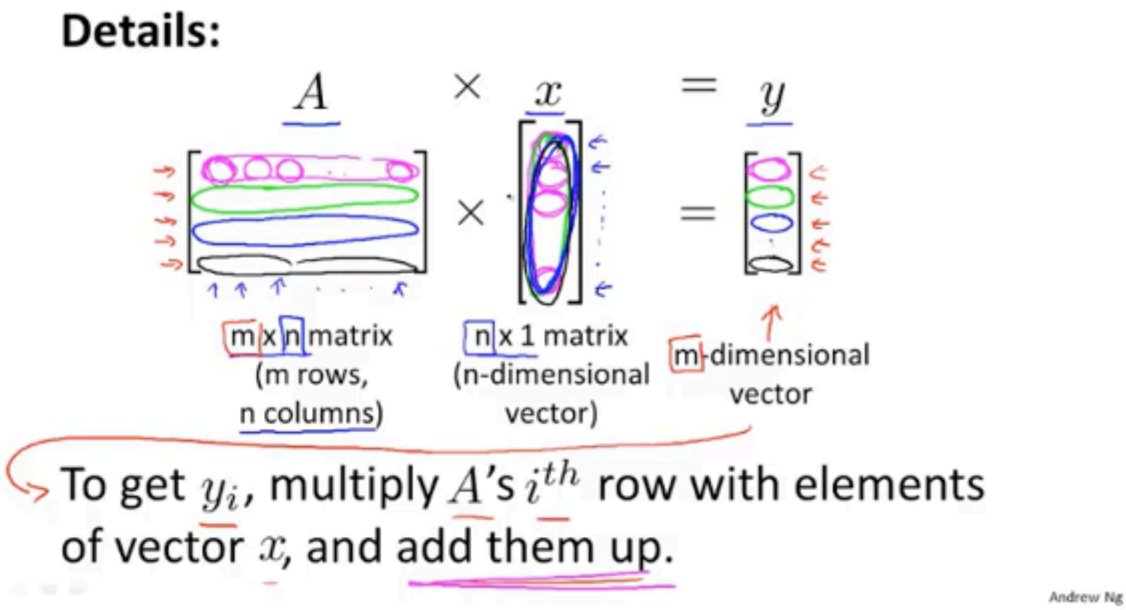
Example

$$\underbrace{\begin{bmatrix}1 & 3\\4 & 0\\2 & 1\end{bmatrix}}_{3\times 2} \underbrace{\begin{bmatrix}1\\5\end{bmatrix}}_{2\times 1} = \begin{bmatrix}16\\4\\7\end{bmatrix}$$

$1\times 1 + 3\times 5 = 16$

$4\times 1 + 0\times 5 = 4$

$2\times 1 + 1\times 5 = 7$


Details:
A × x = y
m x n matrix
(m rows,
n columns)
n x 1 matrix
(n-dimensional
vector)
m-dimensional
vector
To get y_i, multiply A's i^{th} row with elements
of vector x, and add them up.
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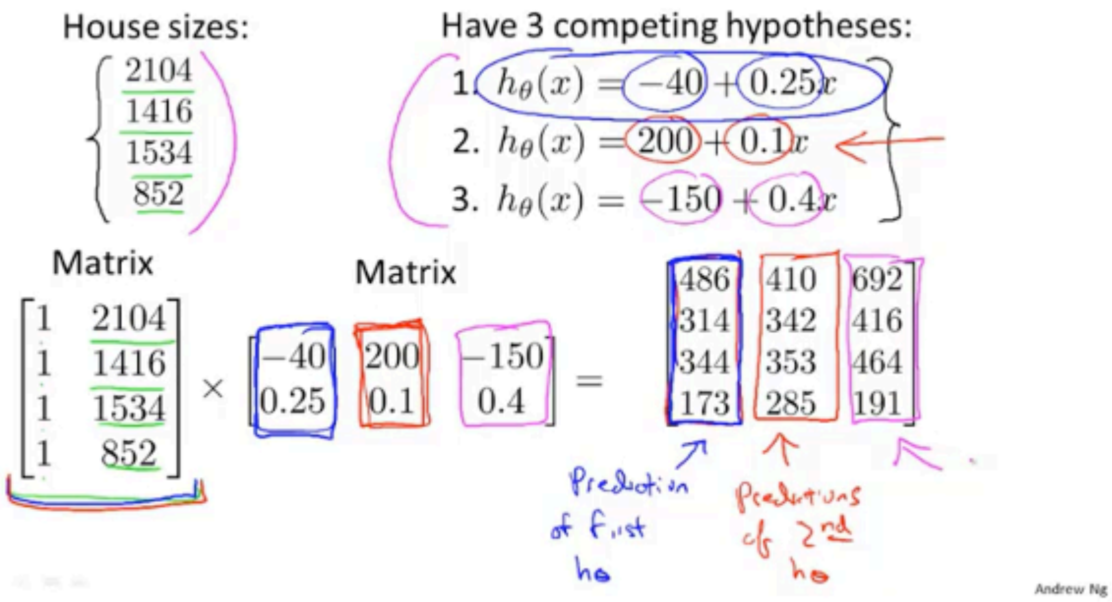
House sizes:
2104
1416
1534
852
Have 3 competing hypotheses:
1. $h_\theta(x) = -40 + 0.25x$
2. $h_\theta(x) = 200 + 0.1x$
3. $h_\theta(x) = -150 + 0.4x$
Matrix
Matrix
$\begin{bmatrix} 1 & 2104 \\ 1 & 1416 \\ 1 & 1534 \\ 1 & 852 \end{bmatrix} \times \begin{bmatrix} -40 & 200 & -150 \\ 0.25 & 0.1 & 0.4 \end{bmatrix} = \begin{bmatrix} 486 & 410 & 692 \\ 314 & 342 & 416 \\ 344 & 353 & 464 \\ 173 & 285 & 191 \end{bmatrix}$
Prediction of first h_θ
Predictions of 2nd h_θ
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矩阵乘法不符合交换律

$3 \times 5 = 5 \times 3$ "Commutative"

Let A and B be matrices. Then in general,
$A \times B \neq B \times A$. (not commutative.)

E.g. $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$

$\neq$

$\begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 2 & 2 \end{bmatrix}$

$\underset{m \times n}{A} \times \underset{n \times m}{B}$

$A \times B$ is $m \times m$

$B \times A$ is $n \times n$

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Identity Matrix

1 is identity. $\boxed{1 \times z = z \times 1 = z}$ for any z

Denoted I (or $\boxed{I_{n \times n}}$).
Examples of identity matrices:

$\underset{1 \times 1}{[1]}$ $\underset{2 \times 2}{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}$ $\underset{3 \times 3}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}$ $\underset{4 \times 4}{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}$

Informally: $\begin{bmatrix} 1 & & & 0 \\ & 1 & & \\ & & 1 & \\ 0 & & & \ddots \; 1 \end{bmatrix}$

For any matrix A,
$$\underset{m \times n}{A} \cdot \underset{n \times n}{\boxed{I}} = \underset{m \times m}{\boxed{I}} \cdot \underset{m \times n}{A} = \underset{m \times n}{A}$$

$I_{n \times n}$

Note:
$AB \neq BA$ in general
$AI = IA$ ✓

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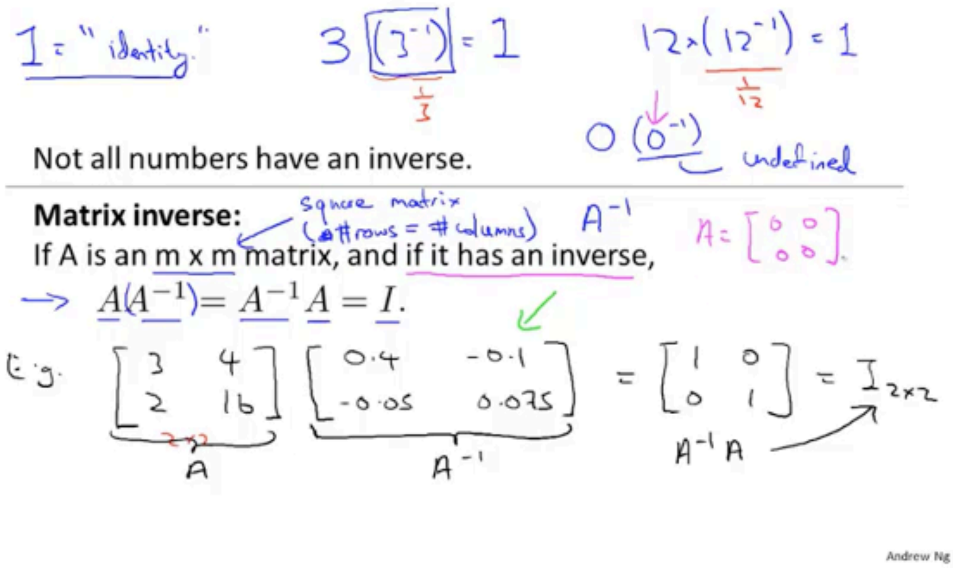
矩阵的逆


1 = "identity"
3 (3⁻¹) = 1
12 × (12⁻¹) = 1
Not all numbers have an inverse.
0 (0⁻¹) undefined
Matrix inverse:
square matrix (#rows = #columns)
If A is an m x m matrix, and if it has an inverse,
A(A⁻¹) = A⁻¹A = I.
E.g.
A⁻¹
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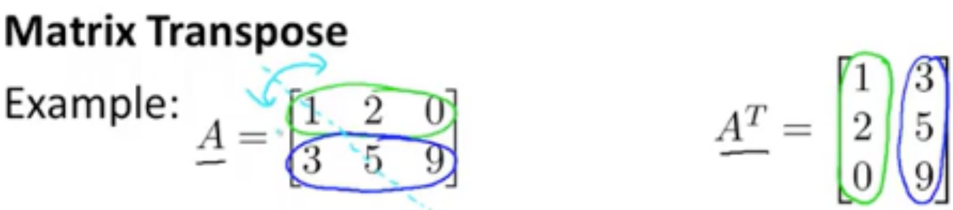
矩阵的转置


Matrix Transpose
Example:
A =
Aᵀ =