Spring 2019

EECE 588
Lecture 14

Prof. Wonbin Hong

Arrays

- So far, we have discussed single element antennas
- Of course our analysis has been limited to the simplest type of single-element antennas namely the dipole and loop.
- Usually the directivity of a single-element antenna is relatively small.
- In a lot of applications, we need high-gain antennas. This can only be accomplished by increasing the physical size of the antenna.
- This can be done in two forms:
  - Make the individual antenna larger.
  - Assemble bunch of single-element radiators in the form of arrays.

Antenna Arrays

- An antenna array is a combination of multiple antenna elements put together in a special geometrical arrangement.
- The antenna elements radiate the same signal with the exception of different magnitudes and phases.
- In an array of identical elements, there are a number of parameters that can be controlled to affect the radiation patterns of the antenna.
  - Geometrical configuration of the array.
  - Relative displacement between elements of the array.
  - Excitation amplitude and phase of each element.
  - Radiation patterns of the elements constituting the array.

Antenna Arrays

http://www.vla.nrao.edu/

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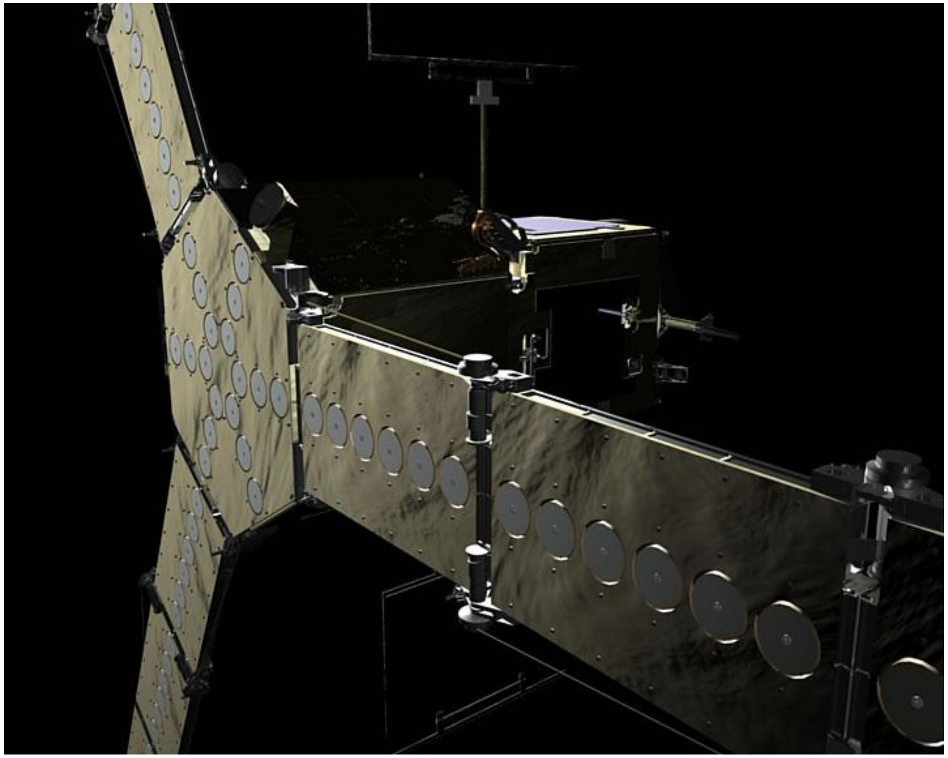
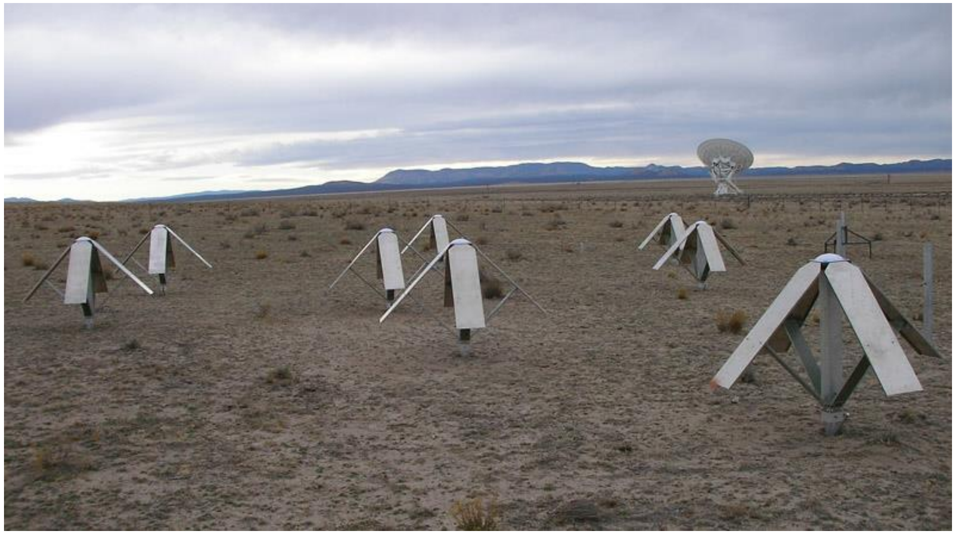
Antenna Arrays



Y-shaped symmetrical antenna: a view of the SMOS Y-shaped antenna arms and central hub.

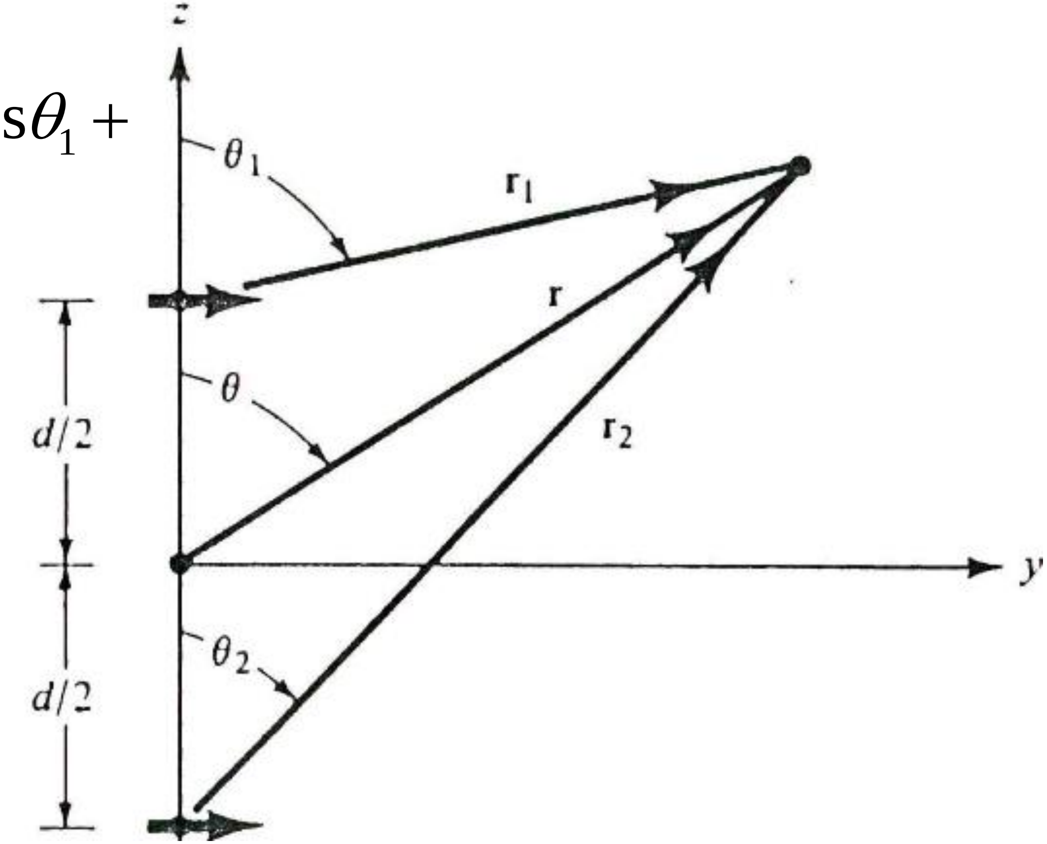
Antenna Arrays



Two Element Array

- Let's consider an array of two infinitesimal dipole antennas:
- The total E field radiated by the antenna is (in yz plane):

$$\vec{E} = \hat{\theta} j\eta \frac{kI_0 l}{4\pi}\left[\frac{e^{-j(kr_1-(\beta/2))}}{r_1}\cos\theta_1 + \frac{e^{-j(kr_2+(\beta/2))}}{r_2}\cos\theta_2\right]$$



Beta is the phase difference between the two excitations.

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Two Element Array

- In far field, we can write:

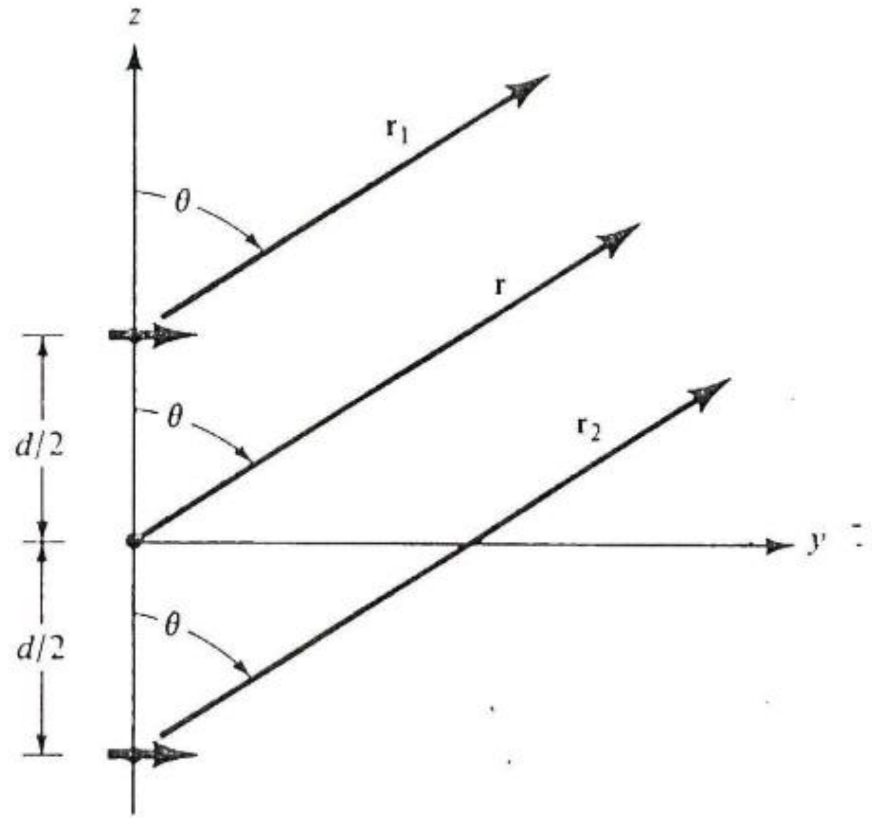
$$\theta_1 = \theta_2 = \theta$$

For Phase Variations

$$\boxed{\begin{aligned} r_1 &\approx r - \frac{d}{2}\cos\theta \\ r_2 &\approx r + \frac{d}{2}\cos\theta \end{aligned}}$$

For Amplitude Variations

$$\boxed{r_1 \approx r_2 \approx r}$$



(b) Far-field observations

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Two Element Array

- The field is then simplified as:

$$\vec{E} = \hat{\theta} j\eta \frac{kI_0 l e^{-jkr}}{4\pi r}\cos\theta\left[e^{+j(kd\cos\theta+\beta)/2} + e^{-j(kd\cos\theta+\beta)/2}\right]$$

$$\vec{E} = \boxed{\hat{\theta} j\eta \frac{kI_0 l e^{-jkr}}{4\pi r}\cos\theta} \times \boxed{\left(2\cos\left[\frac{1}{2}(kd\cos\theta+\beta)\right]\right)}$$

Field of a single element antenna

Array Factor

$$\vec{E}(\mathrm{Total}) = \left[\vec{E}(\text{single element at reference point})\right] \times [\text{array factor}]$$

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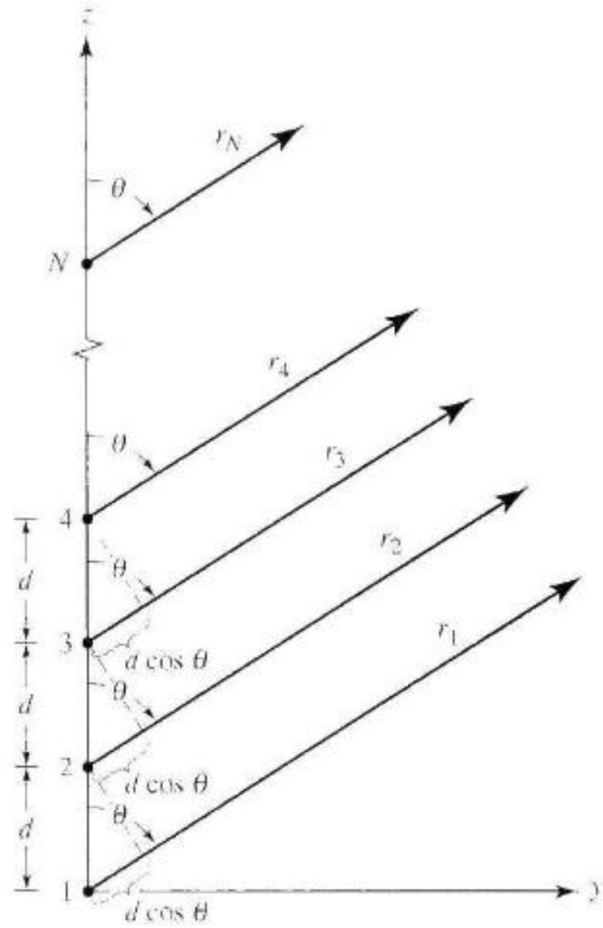
Linear Array: Uniform Amplitude and Spacing

- Let us consider the following antenna array.
- Each dot represents a point source or an isotropic radiator.
- The elements are spaced at a distance of *d* apart.
- The elements are fed with the same current magnitude but a progressive phase difference of β.



Linear Array

- The array factor for this antenna can be written as:

$$AF = 1 + e^{+j(kd\cos\theta+\beta)} + e^{+2j(kd\cos\theta+\beta)} + \ldots. + e^{+j(N-1)(kd\cos\theta+\beta)}$$

$$AF = \sum_{n=1}^{N} e^{j(n-1)(kd\cos\theta+\beta)}$$

$$AF = \sum_{n=1}^{N} e^{j(n-1)\psi}$$

$$\psi = (kd\cos\theta + \beta)$$

Linear Array (2)

- The array factor for this antenna can be written as:

$$(AF)e^{j\psi} = e^{j\psi} + e^{j2\psi} + e^{j3\psi} + \ldots + e^{j(N-1)\psi} + e^{jN\psi}$$

$$(AF) = 1 + e^{j\psi} + e^{j2\psi} + \ldots + e^{j(N-1)\psi}$$

$$AF\left(e^{j\psi} - 1\right) = -1 + e^{jN\psi}$$

$$AF = \frac{e^{jN\psi} - 1}{e^{j\psi} - 1} = e^{j[(N-1)/2]\psi}\left[\frac{e^{j(N/2)\psi} - e^{-j(N/2)\psi}}{e^{j(1/2)\psi} - e^{-j(1/2)\psi}}\right] = e^{j[(N-1)/2]\psi}\left[\frac{\sin\left(\frac{N}{2}\psi\right)}{\sin\left(\frac{1}{2}\psi\right)}\right]$$

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Linear Array (3)

- Maximum value of this function is N. Therefore we normalize it to N.

$$AF = \frac{1}{N}\left[\frac{\sin\left(\frac{N}{2}\psi\right)}{\sin\left(\frac{1}{2}\psi\right)}\right]$$

- Here, we have assumed that the array's center is at the origin.
- For small values of psi.

$$AF = \left[\frac{\sin\left(\frac{N}{2}\psi\right)}{\frac{N}{2}\psi}\right]$$

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Linear Array – The Fourier Transform Approach

- Consider a continuous current filament on the z-axis:

$$I_e(z') = 1 \qquad\qquad 0 < z < l$$

- The radiated fields of this antenna can be calculated from:

$$E_\theta = j\eta \frac{ke^{-jkr}}{4\pi r} \sin\theta \left[\int_0^l I_e(z') e^{jkz' \cos\theta} dz' \right]$$

- The term in brackets is the space factor that we discussed in Lecture 3.
- Now, let's assume that we sample the current $I_e(z')$ in space (similar to what we do with continuous time-domain signals and analog to digital converters)

Linear Array – The Fourier Transform Approach (2)

- The sampling function is:

$$Comb(z') = \sum_{n=-\infty}^{\infty} \delta(z' - nd)$$

where n is an integer and the spacing between the delta functions is d.

- The space factor is then:

$$\int_0^l I_e(z') e^{jkz' \cos\theta} dz' = \int_0^l \sum_{n=-\infty}^{\infty} \delta(z' - nd)\, I_e(z') e^{jkz' \cos\theta} dz' =$$

$$\sum_{n=-\infty}^{\infty} \int_0^l \delta(z' - nd) e^{jkz' \cos\theta} dz' = \sum_{n=0}^{N-1} e^{jknd \cos\theta} = \sum_{n=1}^{N} e^{jk(n-1)d \cos\theta}$$

- Notice that $N = \mathrm{int}(l/d)$

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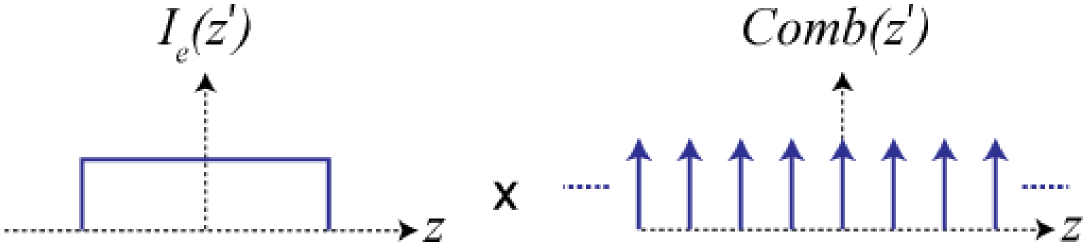
Linear Array – The Fourier Transform Approach (3)

- So, the array factor is the discrete version of the space factor for a continuous electric current distribution.



- Space factor is the Fourier transform of the current distribution.
- An array is a sampled version of the current distribution (multiplication in space domain)
- Therefore, the array factor is the convolution of the Fourier transform of the current distribution (i.e., the space factor) and that of the sampling function.

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Linear Array – The Fourier Transform Approach (4)

- For simplicity, let's consider a symmetric current distribution:

$$I_e(z') = 1 \qquad -l < z < l$$

considering that $\Omega = -k\cos\theta$, the space factor is:

$$\int_{-\infty}^{\infty} I_e(z') e^{-j\Omega z'} dz' = 2l \frac{\sin \Omega l}{\Omega l} = 2l \operatorname{sinc}(\Omega l)$$

Also, the Fourier transform for the sampling function can be expressed as:

$$FT\{Comb(z')\} = \sum_{n=-\infty}^{\infty} \frac{1}{d} \delta(\Omega - n\Omega_0)$$

and $\Omega_0 = \frac{2\pi}{d}$

Linear Array – The Fourier Transform Approach (5)

- The array factor is the convolution of these two Fourier transforms:

$$AF = 2l\,\mathrm{sinc}(\Omega l) \ * \sum_{n=-\infty}^{\infty} \frac{1}{d}\delta(\Omega - n\Omega_0)$$

$$= \frac{2l}{d}\sum_{n=-\infty}^{\infty} \mathrm{sinc}\,\{(\Omega - n\Omega_0)l\}$$

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Linear Array – The Fourier Transform Approach (6)

- If we calculate the array factor directly in the Ω domain, we get:

$$AF = \frac{\sin\left(\frac{N\Omega d}{2}\right)}{\sin\left(\frac{\Omega d}{2}\right)}$$

- Keep in mind that despite the fact that they look different these array factor functions are identical.

$$AF\ (direct) = \frac{\sin\left(\frac{N\Omega d}{2}\right)}{\sin\left(\frac{\Omega d}{2}\right)}$$

$$AF\ (convolution) = \frac{2l}{d} \sum_{n=-\infty}^{\infty} \operatorname{sinc}\{(\Omega - n\Omega_0)l\}$$

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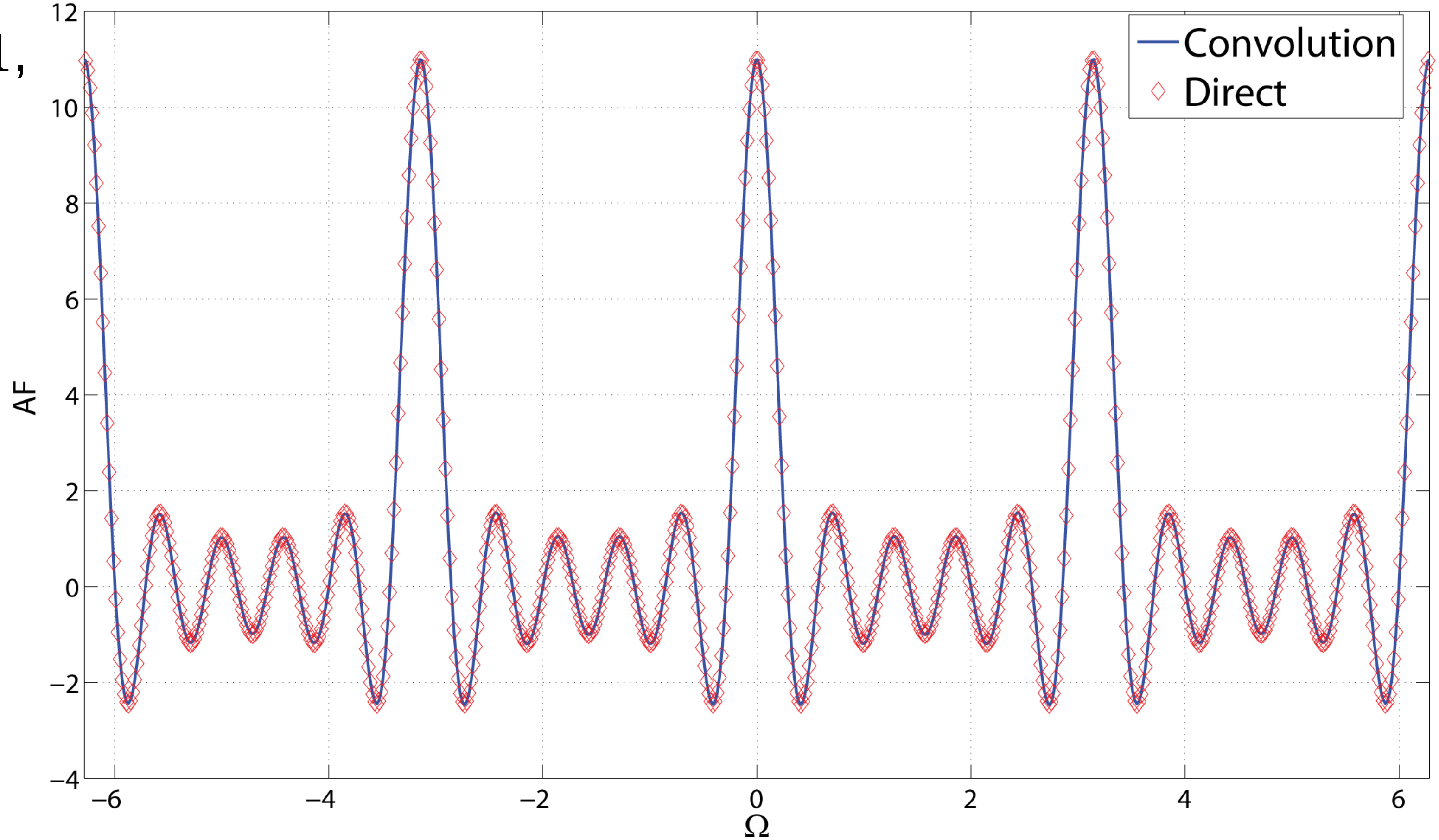
Linear Array – The Fourier Transform Approach (7)

- $l = 11$, $d = 2$



Linear Array – The Fourier Transform Approach (8)

- What is the significance of this approach?

Later on in this lecture, we will discuss the issue of tapering in arrays and different array factor coefficients. This approach becomes increasingly important in gaining an understanding and a feeling about the effects of tapering on the performance of the arrays.

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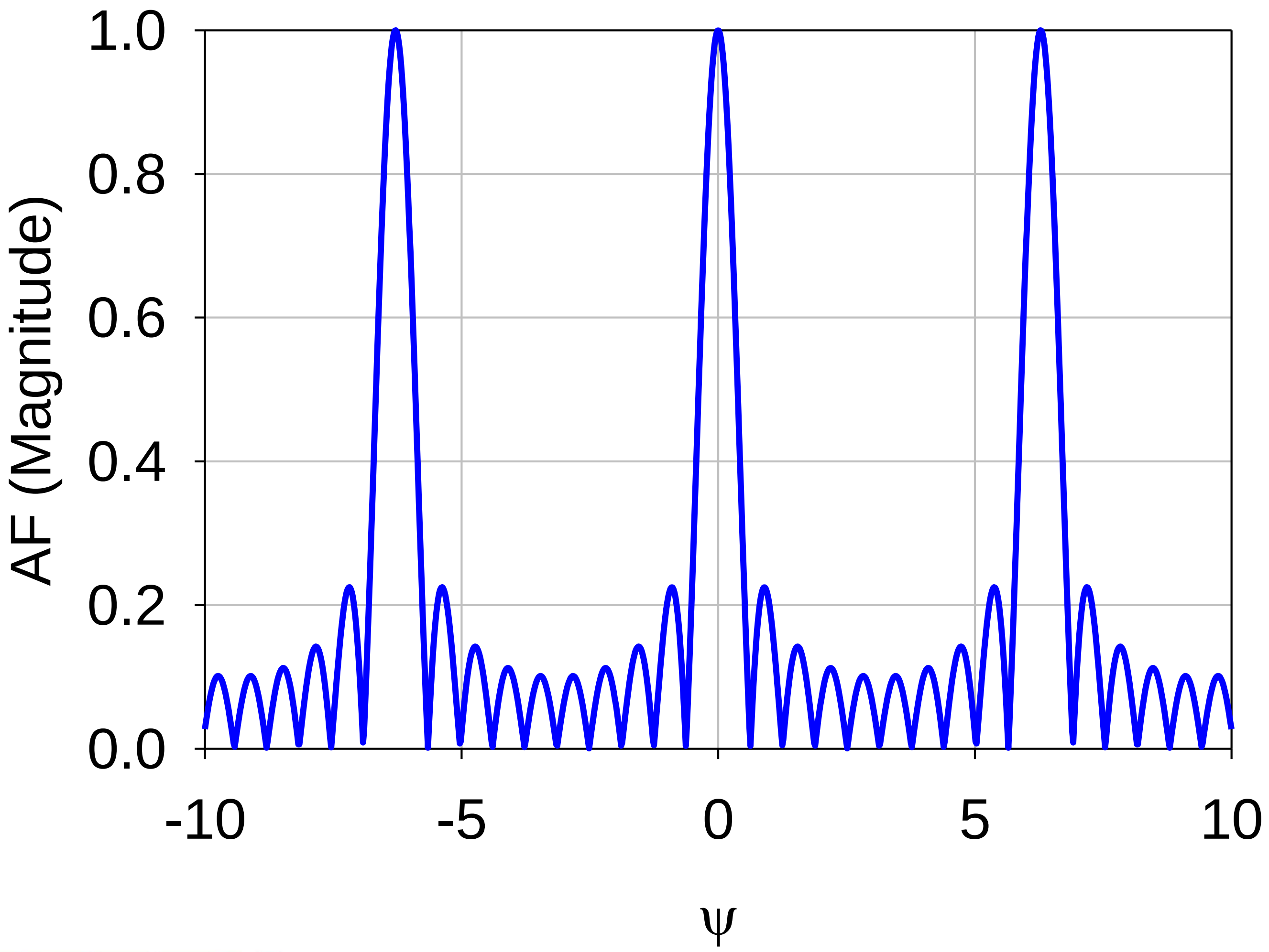
Array Factor for N=10



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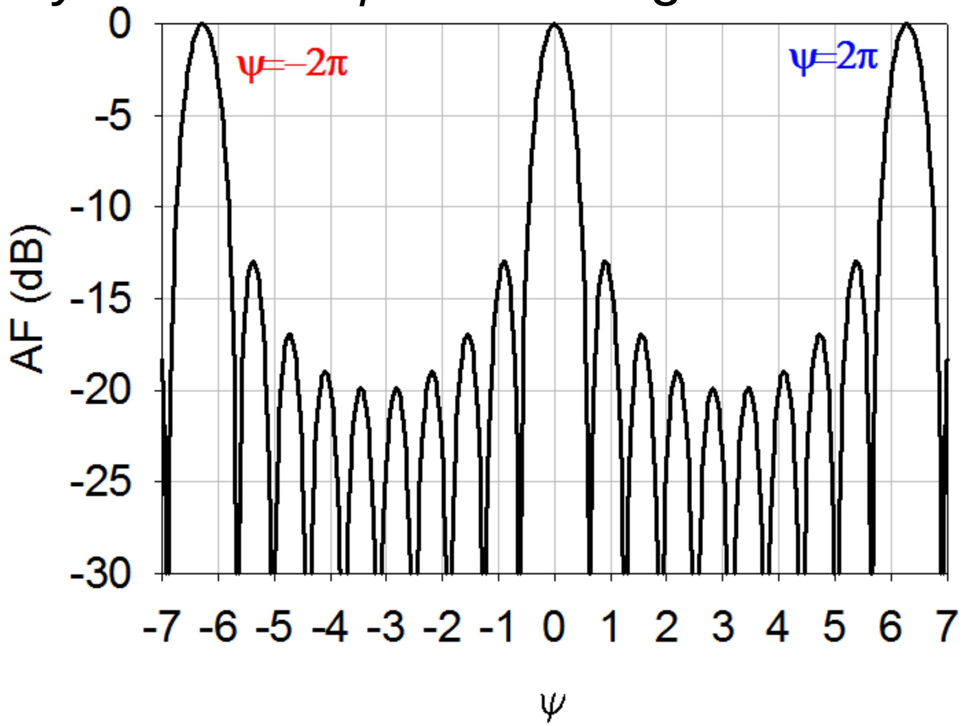
Array Factor for N=10 (2)

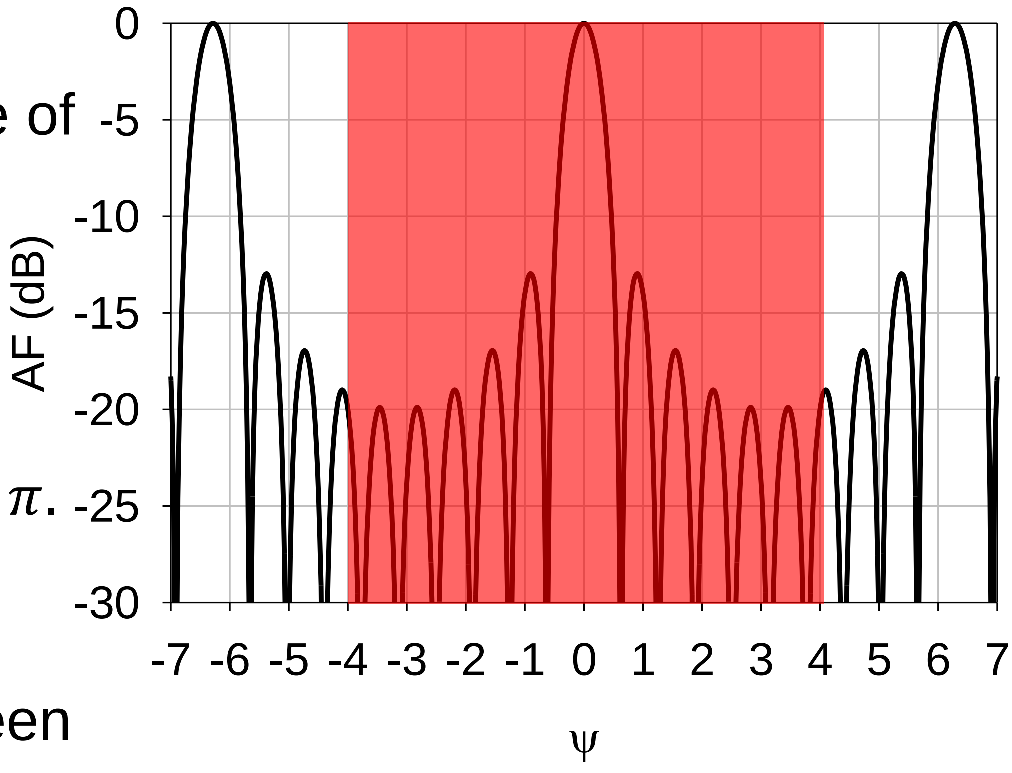
- Note that $\psi = kd\ \cos\theta + \beta$ but θ can only assume values in the range of $0°$ to π.
- Even though this mathematical function can be plotted for values of ψ from $-\infty$ to ∞ only values of ψ in the range of $kd + \beta$ to $-kd + \beta$ correspond to real angles.

- This is called the visible range
- Values of ψ falling out of this range correspond to imaginary angles.



Array Factor Mapping to Radiation Patterns

- Let's assume that $\beta = 0$.
- In this case, the visible region will be in the range of $-kd < \psi < kd$.
- The exact range of this depends on the spacing between the elements d.
- e.g., if $d = \lambda$, $-\pi < \psi < \pi$.
- This will determine your radiation pattern. Since a unique relationship between AF and θ is established now.



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Array Factor Mapping to Radiation Patterns (2)

- Note that the $(AF)_n$ function attains its maximum for $\psi = 0$.
- Therefore, you can safely use the approximated value of AF instead of the exact value in most cases.
- Array nulls can be determined from:

$$\sin(\frac{N}{2}\psi) = 0 \Rightarrow \frac{N}{2}\psi\Big|_{\theta=\theta_n} = \pm n\pi \Rightarrow \theta_n = \cos^{-1}\left[\frac{\lambda}{2\pi d}\left(-\beta \pm \frac{2n}{N}\pi\right)\right]$$

$$n = 1, 2, 3, \ldots.$$

$$n \neq N, 2N, 3N, \ldots.$$

- The maximum of the AF occurs for: $m = 0, 1, 2, \ldots.$

$$\frac{\psi}{2} = \frac{1}{2}(kd\cos\theta + \beta)\,|_{\theta=\theta m} = \pm m\pi \Rightarrow \theta_m = \cos^{-1}\left[\frac{\lambda}{2\pi d}(-\beta \pm 2m\pi)\right]$$

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Array Factor Mapping to Radiation Patterns (3)

- The 3 dB point for the array factor can be calculated as:

$$\frac{N}{2}\psi = \frac{N}{2}\left(kd\cos\theta + \beta\right) = \pm 1.39$$

$$\theta_h = \cos^{-1}\left[\frac{\lambda}{2\pi d}\left(-\beta \pm 2.782/N\right)\right]$$

$$\theta_h = \frac{\pi}{2} - \sin^{-1}\left[\frac{\lambda}{2\pi d}\left(-\beta \pm 2.782/N\right)\right]$$

- Half-power BW of the array can be calculated from:

$$\Theta_h = 2\left|\theta_m - \theta_h\right|$$