Spring 2019

EECE 588
Lecture 9

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Power Density, Radiation Intensity, and Radiation Resistance

- The average radiation intensity can be calculated using:

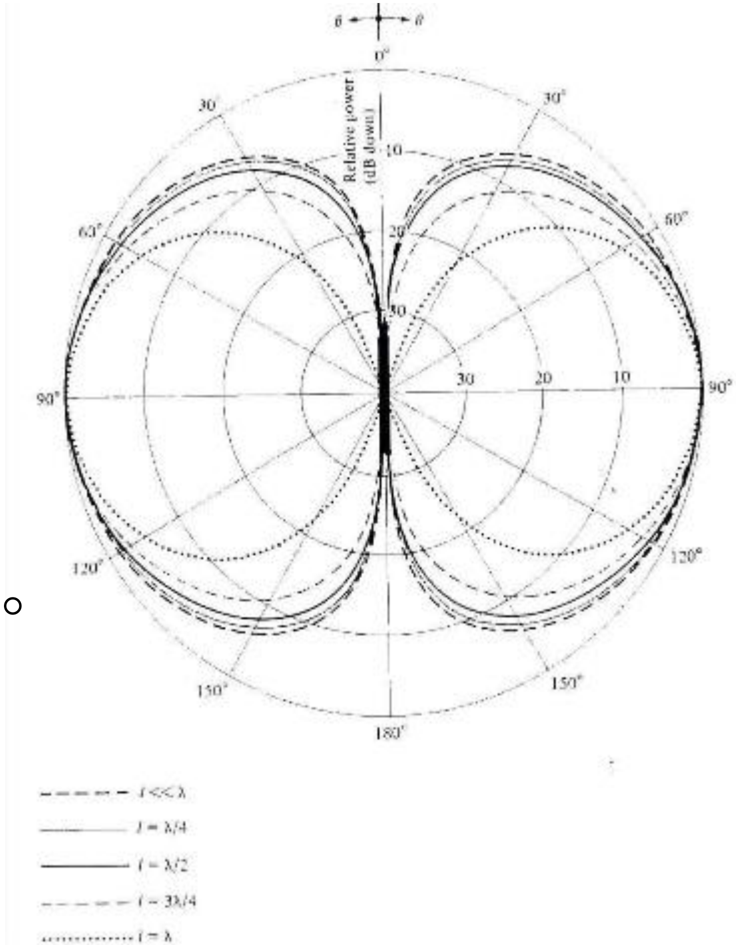
$$\vec{W}_{av} = \frac{1}{2}\mathrm{Re}\left[\vec{E}\times\vec{H}^*\right] = \frac{1}{2}\mathrm{Re}\left[\hat{\theta}E_\theta \times \hat{\varphi}H_\varphi^*\right] = \frac{1}{2}\mathrm{Re}\left[\hat{\theta}E_\theta \times \hat{\varphi}\frac{E_\theta^*}{\eta}\right]$$

$$\vec{W}_{av} = \hat{r}W_{av} = \hat{r}\frac{1}{2\eta}\left|E_\theta\right|^2 = \hat{r}\eta\frac{\left|I_0\right|^2}{8\pi^2 r^2}\left\{\cos\left(\frac{kl}{2}\cos\theta\right) - \cos\left(\frac{kl}{2}\right)\right\}^2 \Big/ \sin^2\theta$$

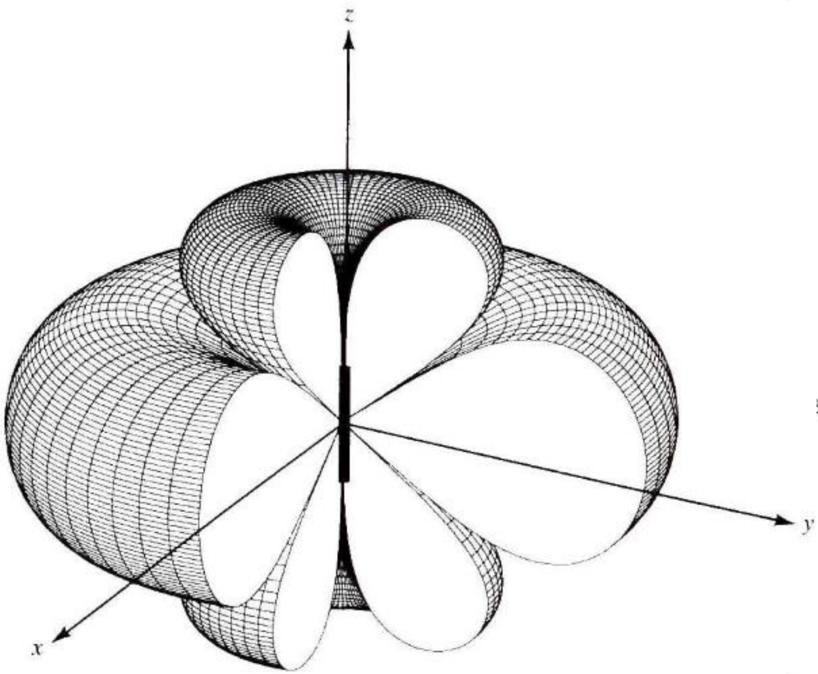
$$U = r^2 W_{av} = \eta\frac{\left|I_0\right|^2}{8\pi^2}\left[\frac{\cos\left(\frac{kl}{2}\cos\theta\right) - \cos\left(\frac{kl}{2}\right)}{\sin\theta}\right]^2$$

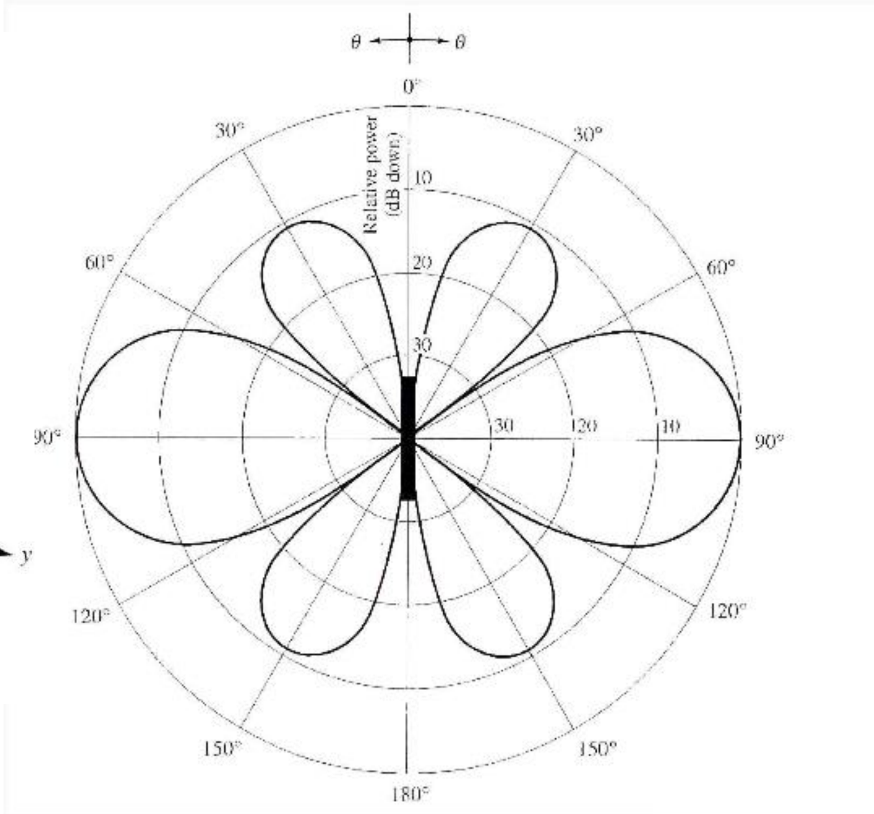
Radiation Patterns

- *l*<<λ ➔ 3dB Beam width=90°
- *l*=λ/4 ➔ 3dB Beam width=87°
- *l*=λ/2 ➔ 3dB Beam width=78°
- *l*=3λ/4 ➔ 3dB Beam width=64°
- *l*=λ ➔ 3dB Beam width=47.8°



Radiation Patterns for l=1.25λ





Radiated Power

$$P_{rad} = \oiint_S \vec{W}_{av}.d\vec{s} = \int_0^{2\pi}\int_0^{\pi} \hat{r}W_{av}.\hat{r}r^2 \sin\theta \; d\theta \; d\varphi = \int_0^{2\pi}\int_0^{\pi} W_{av}r^2 \sin\theta \; d\theta \; d\varphi$$

$$P_{rad} = \int_0^{2\pi}\int_0^{\pi} W_{av}r^2 \sin\theta \; d\theta \; d\varphi = \eta \frac{|I_0|^2}{4\pi}\int_0^{\pi} \frac{\left[\cos\left(\frac{kl}{2}\cos\theta\right) - \cos\left(\frac{kl}{2}\right)\right]^2}{\sin\theta} d\theta$$

$$P_{rad} = \eta \frac{|I_0|^2}{4\pi}\left[\begin{array}{l} C + \ln(kl) - C_i(kl) + \frac{1}{2}\sin(kl)\{S_i(2kl) - 2S_i(kl)\} + \\ \frac{1}{2}\cos(kl)\{C + \ln(kl/2) + C_i(2kl) - 2C_i(kl)\} \end{array}\right]$$

POSTECH

Radiated Power

- C=0.5772

$$C_i(x) = -\int_x^\infty \frac{\cos y}{y} dy = \int_\infty^x \frac{\cos y}{y} dy \qquad S_i(x) = -\int_0^x \frac{\sin y}{y} dy$$

$$R_{rad} = \frac{2P_{rad}}{|I_0|^2} = \frac{\eta}{2\pi}[C + \ln(kl) - C_i(kl) + \frac{1}{2}\sin(kl)\{S_i(2kl) - 2S_i(kl)\} +$$

$$\frac{1}{2}\cos(kl)\{C + \ln(kl/2) + C_i(2kl) - 2C_i(kl)\}]$$

- Derivations for X_m are from Chapter 8 of text:

$$X_m = \frac{\eta}{4\pi}[2S_i(kl) + \cos(kl)[2S_i(kl) - S_i(2kl)]$$

$$-\sin(kl)[2C_i(kl) - C_i(2kl) - C_i\left(\frac{2ka^2}{l}\right)]]$$

Directivity

$$D_0 = 4\pi \frac{F(\theta,\varphi)_{\max}}{\int_0^{2\pi}\int_0^{\pi} F(\theta,\varphi)\sin\theta \, d\theta \, d\varphi}$$

- $U = B_0 F(\theta, \varphi)$

$$F(\theta,\varphi) = F(\theta) = \left[\frac{\cos\left(\frac{kl}{2}\cos\theta\right) - \cos\left(\frac{kl}{2}\right)}{\sin\theta}\right]^2 \qquad B_0 = \eta \frac{|I_0|^2}{8\pi^2}$$

$$D_0 = \frac{2F(\theta)|_{\max}}{\int_0^{\pi} F(\theta)\sin\theta d\theta} \quad \Rightarrow \quad D_0 = \frac{2F(\theta)|_{\max}}{Q}$$

$$Q = \{C + \ln(kl) - C_i(kl) + \frac{1}{2}\sin(kl)[S_i(2kl) - 2S_i(kl)]$$

$$+ \frac{1}{2}\cos(kl)[C + \ln(kl/2) + C_i(2kl) - 2C_i(kl)]\}$$

POSTECH

Input Resistance

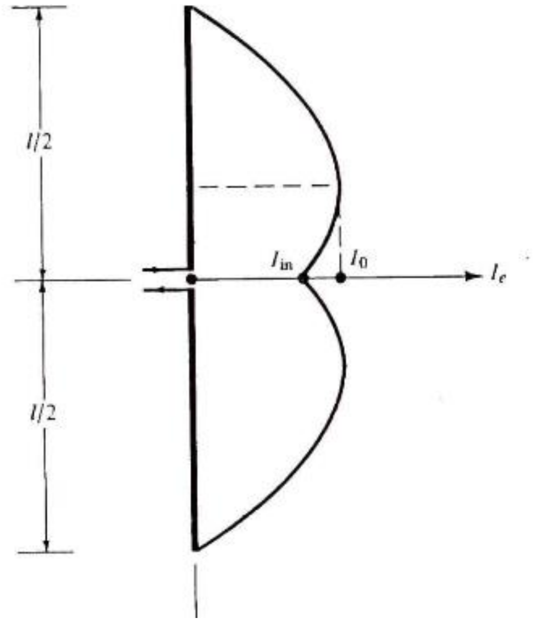
- We defined the input impedance as: the ratio of the voltage to the current and the input terminals of antennas.
- For a lossless antenna, the input resistance, which is the real part of the input impedance is related to the radiation resistance of the antenna.
- So far, we have calculated the radiation resistance of electrically small antennas as well as dipoles with sinusoidal current distribution. So far, we have used a definition, in which the radiation resistance is referred to the maximum current.
- For certain dipole antennas, this maximum does not occur at the input terminals (e.g., $l = \lambda$).

Input Resistance

- We can refer the radiation resistance to the input terminals of the antenna.
- Let's assume the antenna is lossless (i.e. $R_L = 0$).
- Then, we can equate the power at the input terminals of the antenna to the power at the current maximum.

$$\frac{|I_{in}|^2}{2} R_{in} = \frac{|I_0|^2}{2} R_r$$

$$R_{in} = \left[\frac{I_0}{I_{in}}\right]^2 R_r$$



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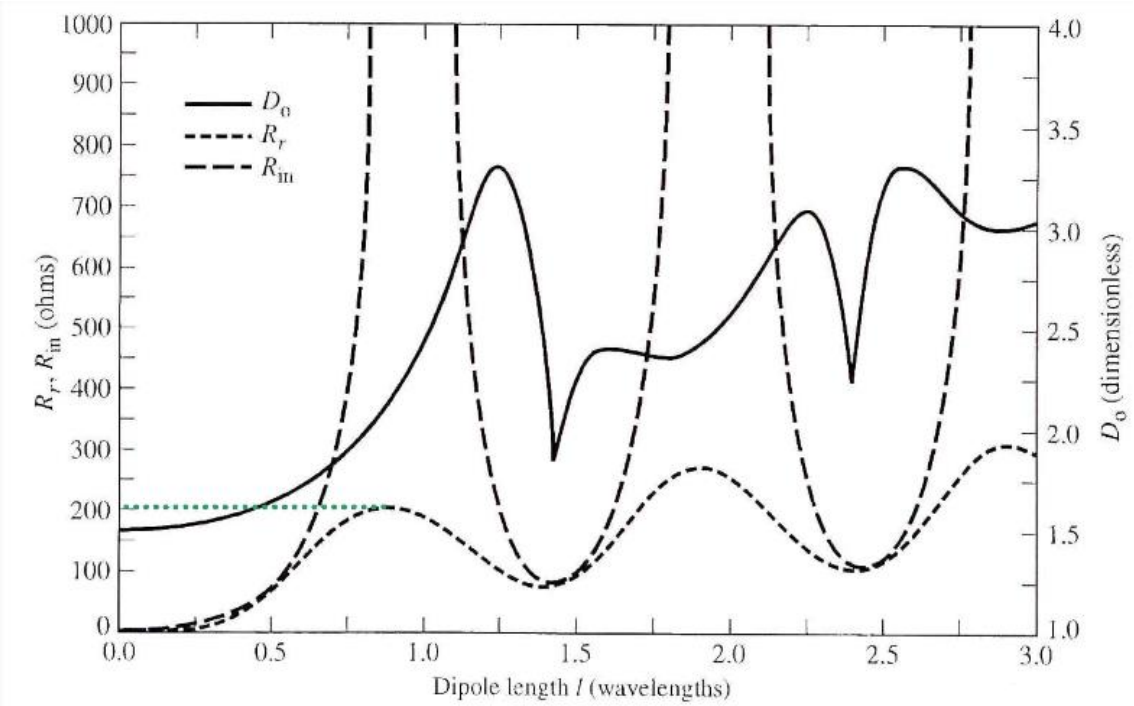
Input Resistance

- For a dipole of length l, the current at the input terminals (I_{in}) is related to the current maximum (I_0):

$$I_{in} = I_0 \sin(kl/2)$$

$$R_{in} = \frac{R_r}{\sin^2(kl/2)}$$



POSTECH

Half-Wavelength Dipole

$$E_\theta \approx j\eta \frac{I_0 e^{-jkr}}{2\pi r}\left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right] \qquad H_\varphi \approx j\frac{I_0 e^{-jkr}}{2\pi r}\left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right]$$

$$W_{av} = \eta \frac{|I_0|^2}{8\pi^2 r^2}\left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right]^2 \approx \eta \frac{|I_0|^2}{8\pi^2 r^2}\sin^3\theta \Rightarrow U = \eta \frac{|I_0|^2}{8\pi^2}\sin^3\theta$$

$$P_{rad} = \eta \frac{|I_0|^2}{4\pi}\int_0^\pi \frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}d\theta = \eta\frac{|I_0|^2}{8\pi}C_{in}(2\pi) \qquad C_{in}(2\pi) = \int_0^{2\pi}\left(\frac{1-\cos y}{y}\right)dy$$

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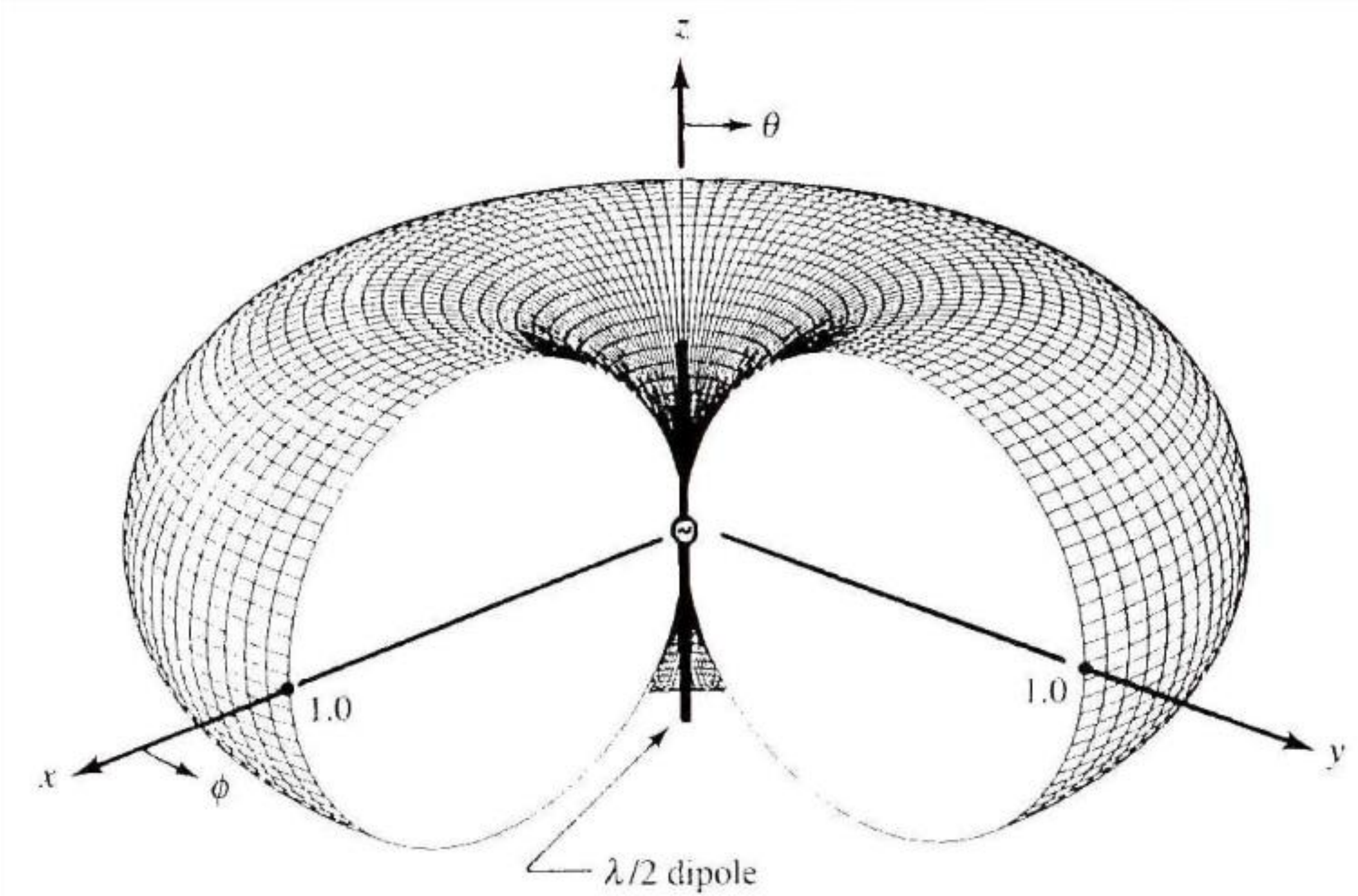
Radiation Patterns of a Half-Wavelength Dipole



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Half-Wavelength Dipole

- $C_{in}(2\pi)$=2.435.

$$D_0 = 4\pi U_{\max} / P_{rad} = 4\pi \frac{U|_{\theta=\pi/2}}{P_{rad}} = \frac{4}{C_{in}(2\pi)} \approx 1.643 = 2.15\,\mathrm{dBi}$$

$$A_{em} = \frac{\lambda^2}{4\pi} D_0 \rightarrow A_{em} = \frac{\lambda^2}{4\pi}(1.64) \approx 0.13\lambda^2$$

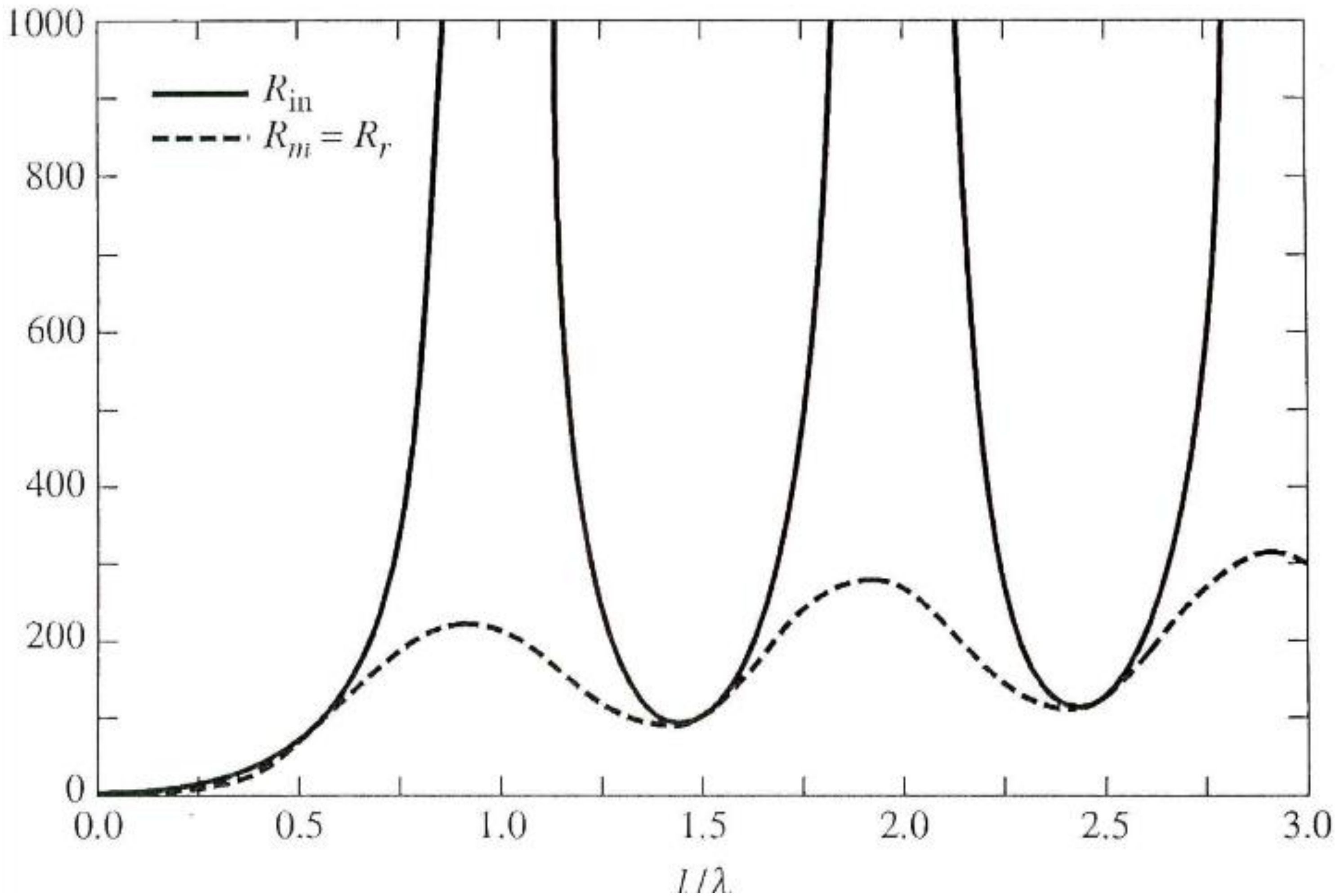
$$R_r \approx \frac{2P_{rad}}{|I_0|^2} = \frac{\eta}{4\pi} C_{in}(2\pi) = 30(2.43) \approx 73\ \Omega$$

- The input impedance of a half-wavelength dipole antenna is:

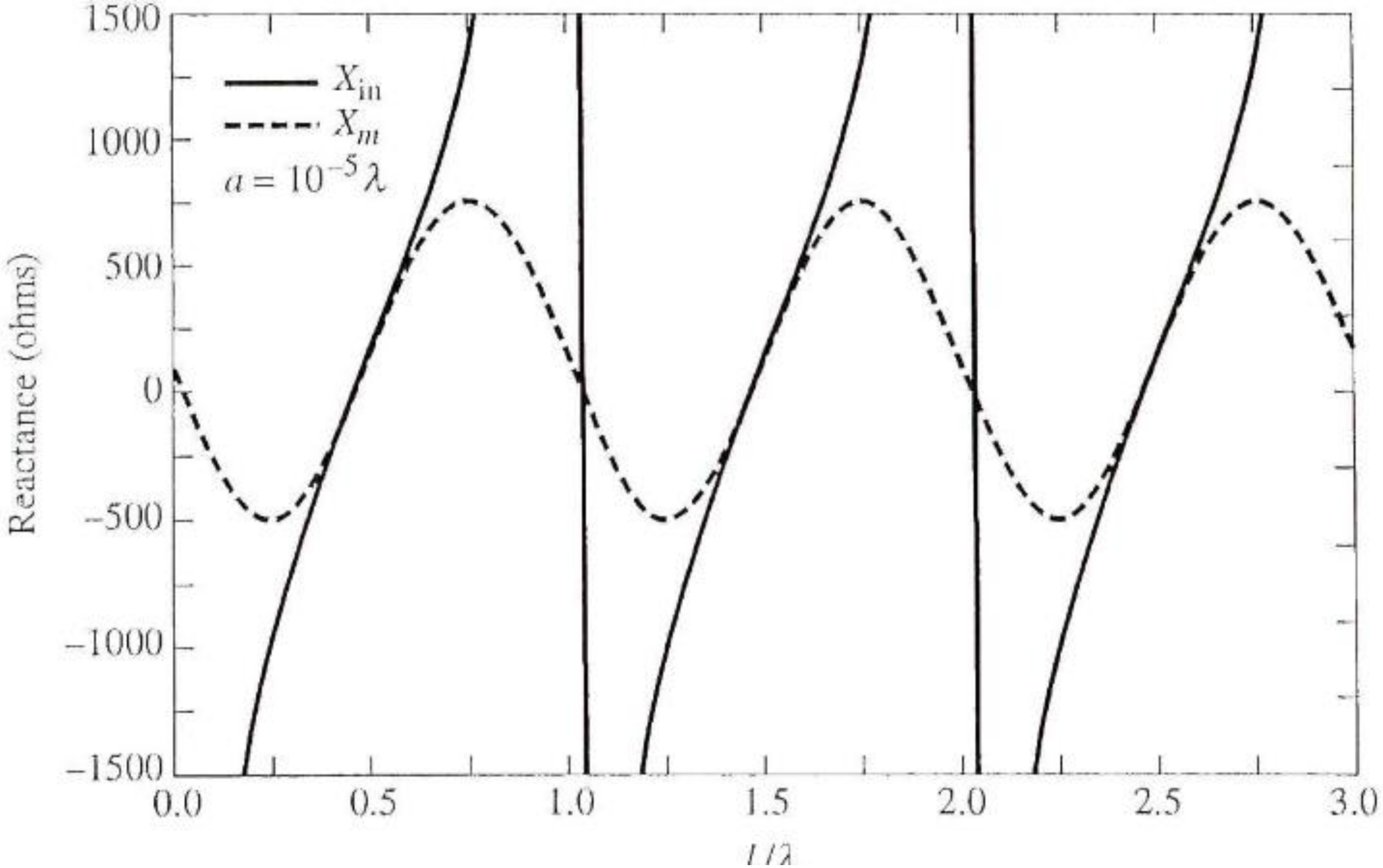
$$Z_{in} = 73 + j43\ \Omega$$

POSTECH

Dipole Antennas' Input Resistance



Dipole Antennas' Input Reactance



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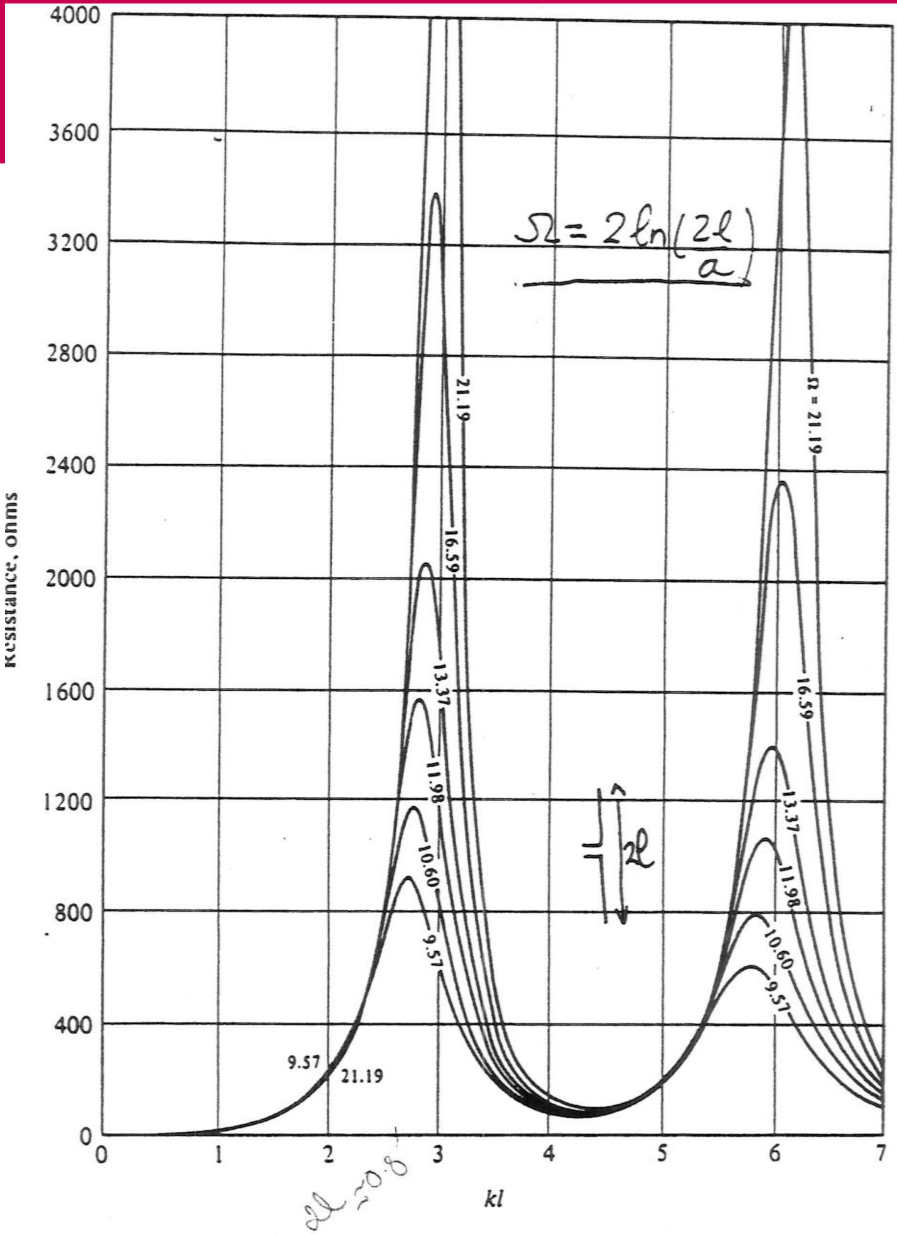


Fig. 7.15a Hallén's Curves of Resistance of a Center-Fed Cylindrical Dipole versus *kl* and Ω (Reprinted from E. Hallén, Cruft Laboratory Report No. 46, 1946, Courtesy of Harvard University.)

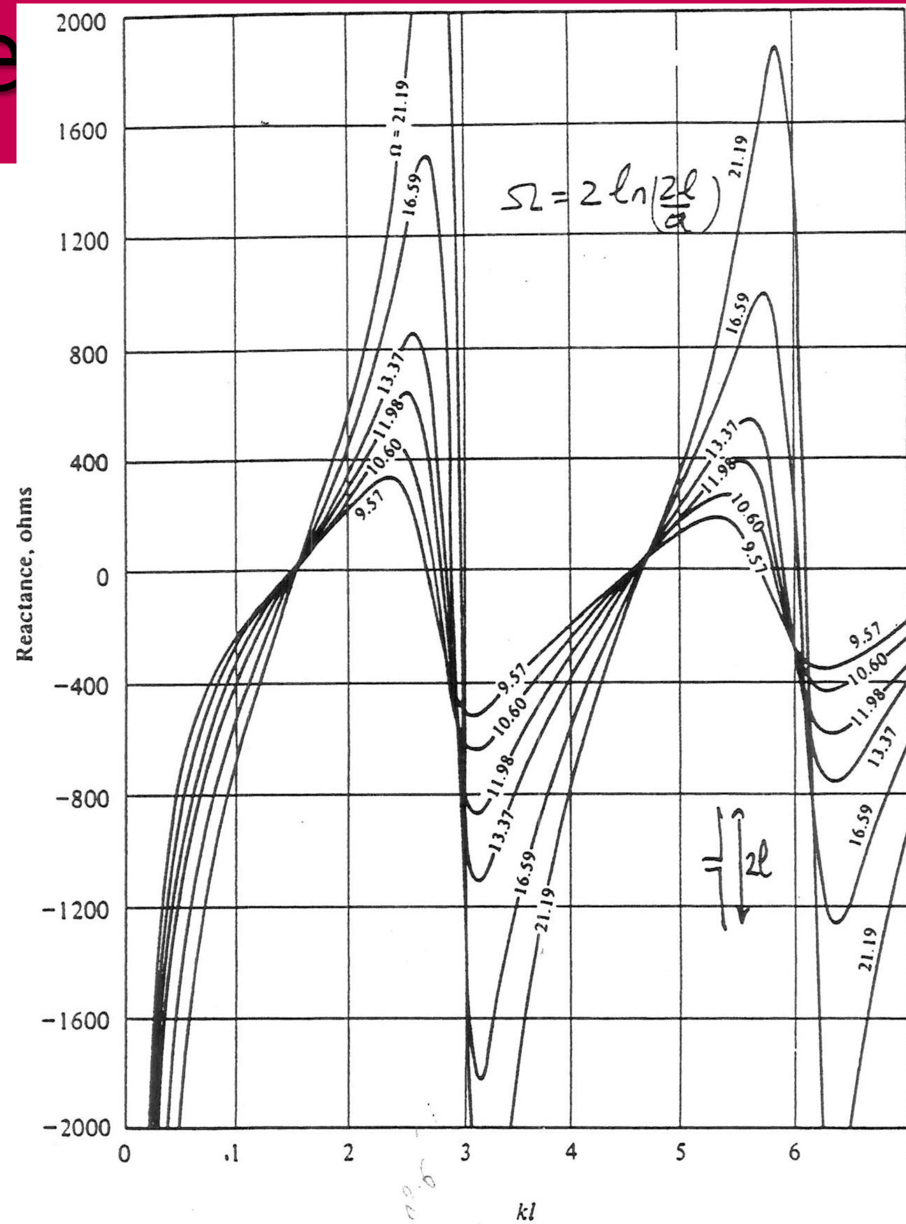


Fig. 7.15b Hallén's Curves of Reactance of a Center-Fed Cylindrical Dipole versus *kl* and Ω (Reprinted from E. Hallén, Cruft Laboratory Report No. 46, 1946, Courtesy of Harvard University.)

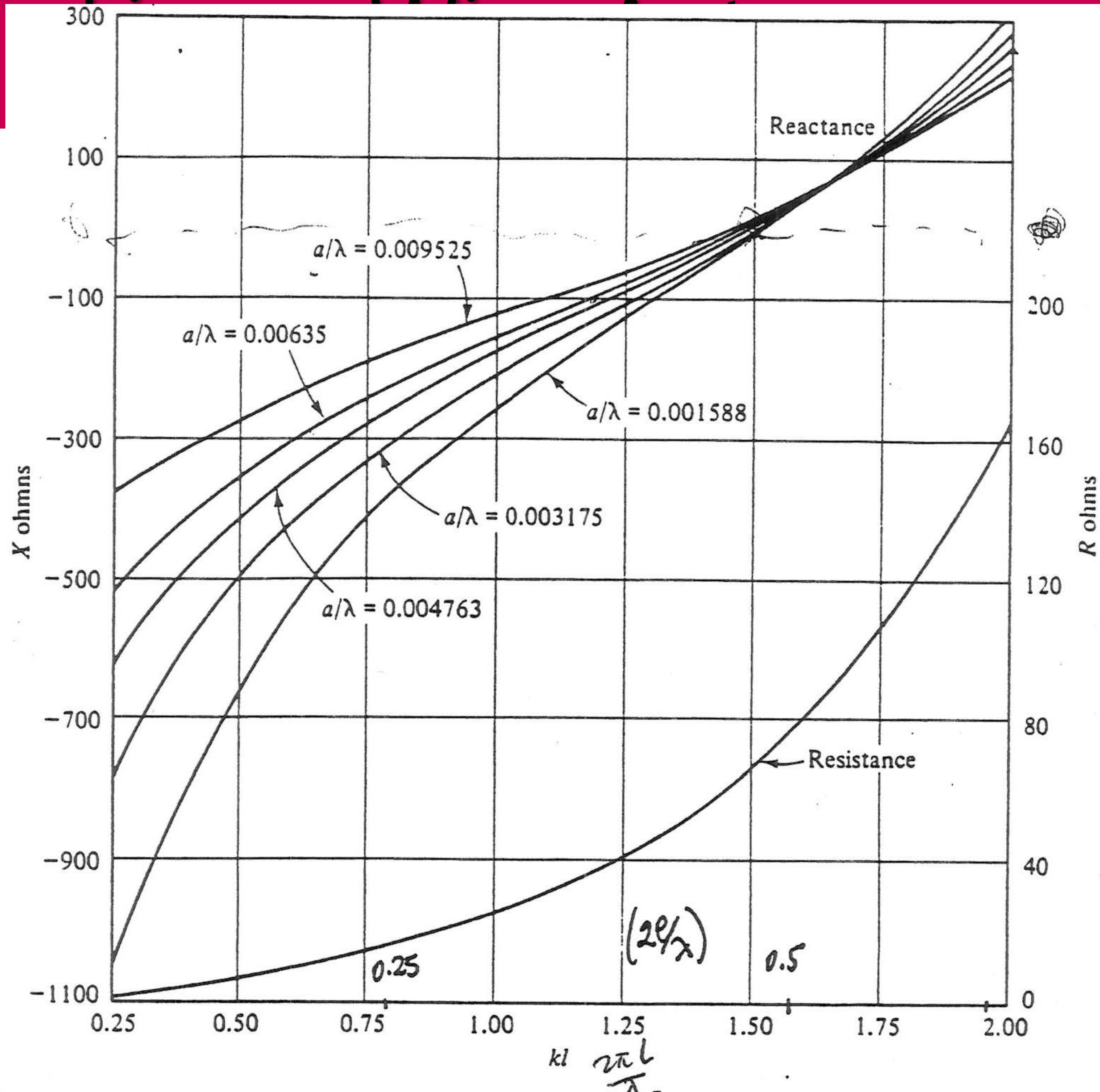


Fig. 7.11 The Resistance and Reactance of a Center-Fed Dipole versus kl and a/λ; Values Computed by the Induced EMF Method Using Equation 7.65

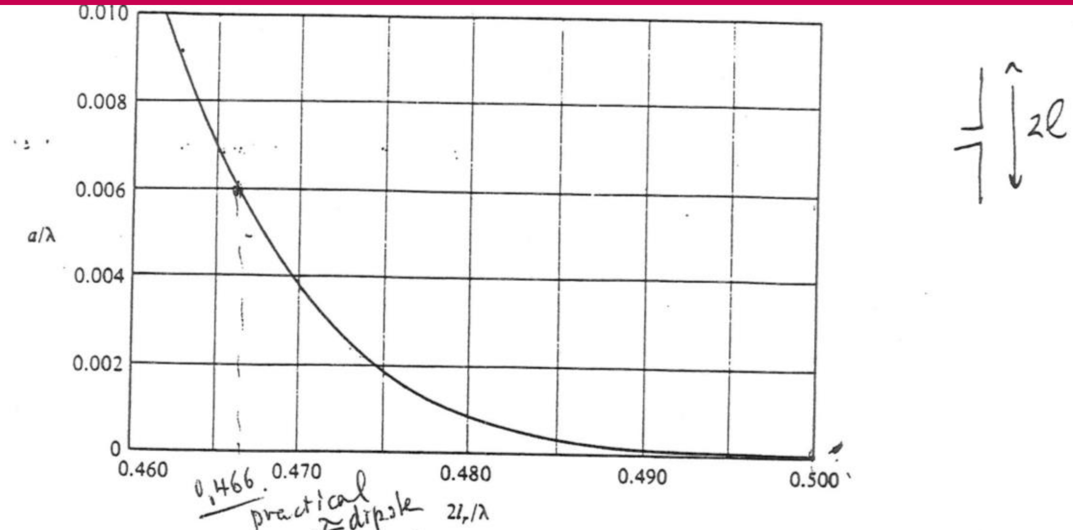


Fig. 7.13 Resonant Length versus Radius for Center-Fed Cylindrical Dipoles

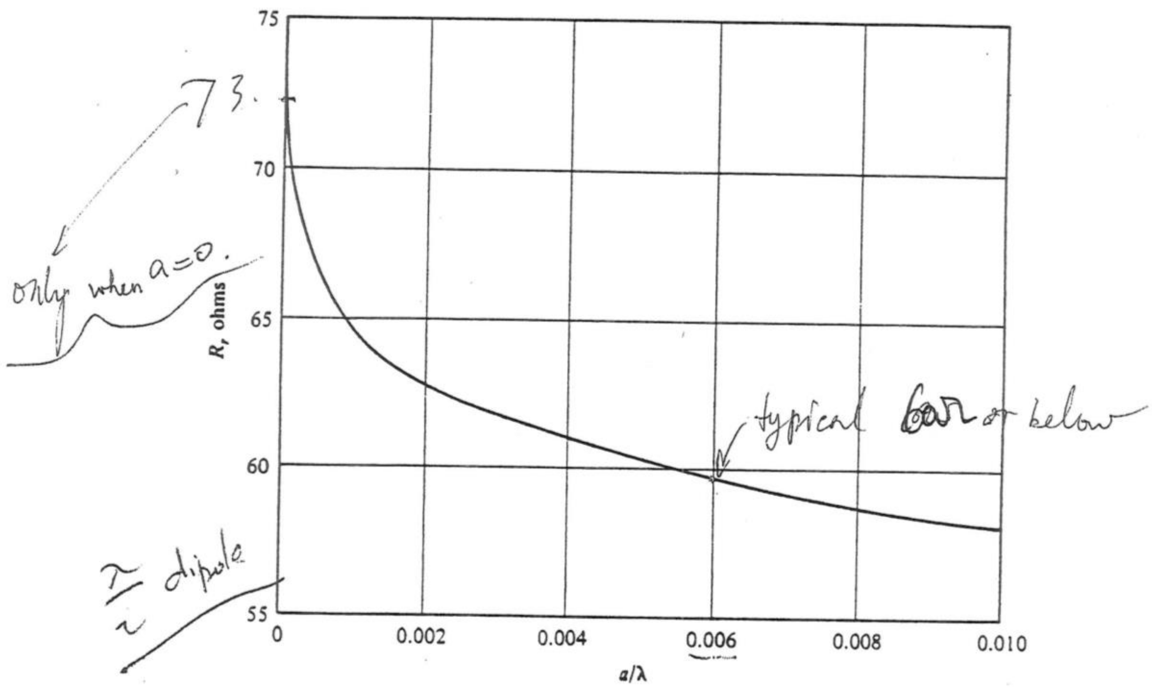


Fig. 7.14 Resonant Resistance versus Radius for Center-Fed Cylindrical Dipoles

Image Theory

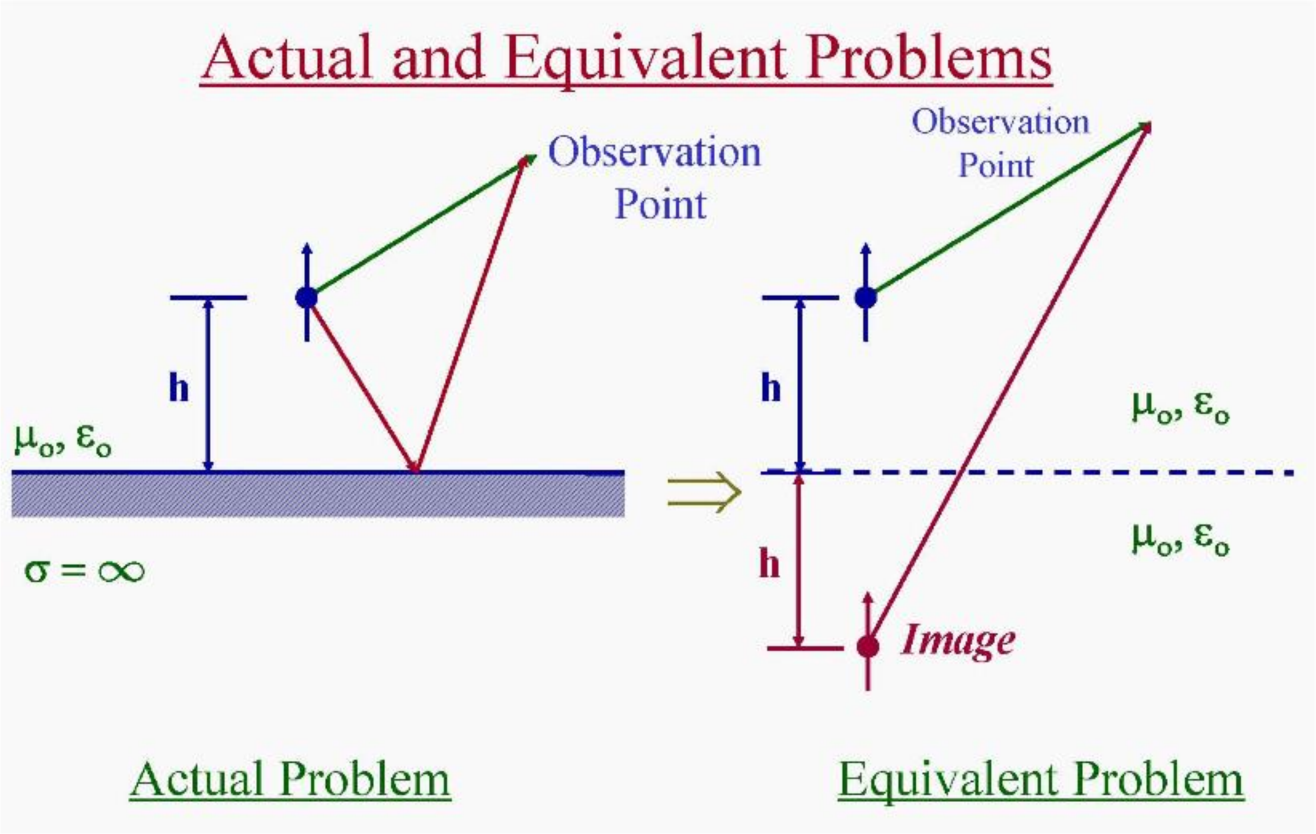


Image Theory: Perfect Electric Conductor (PEC)

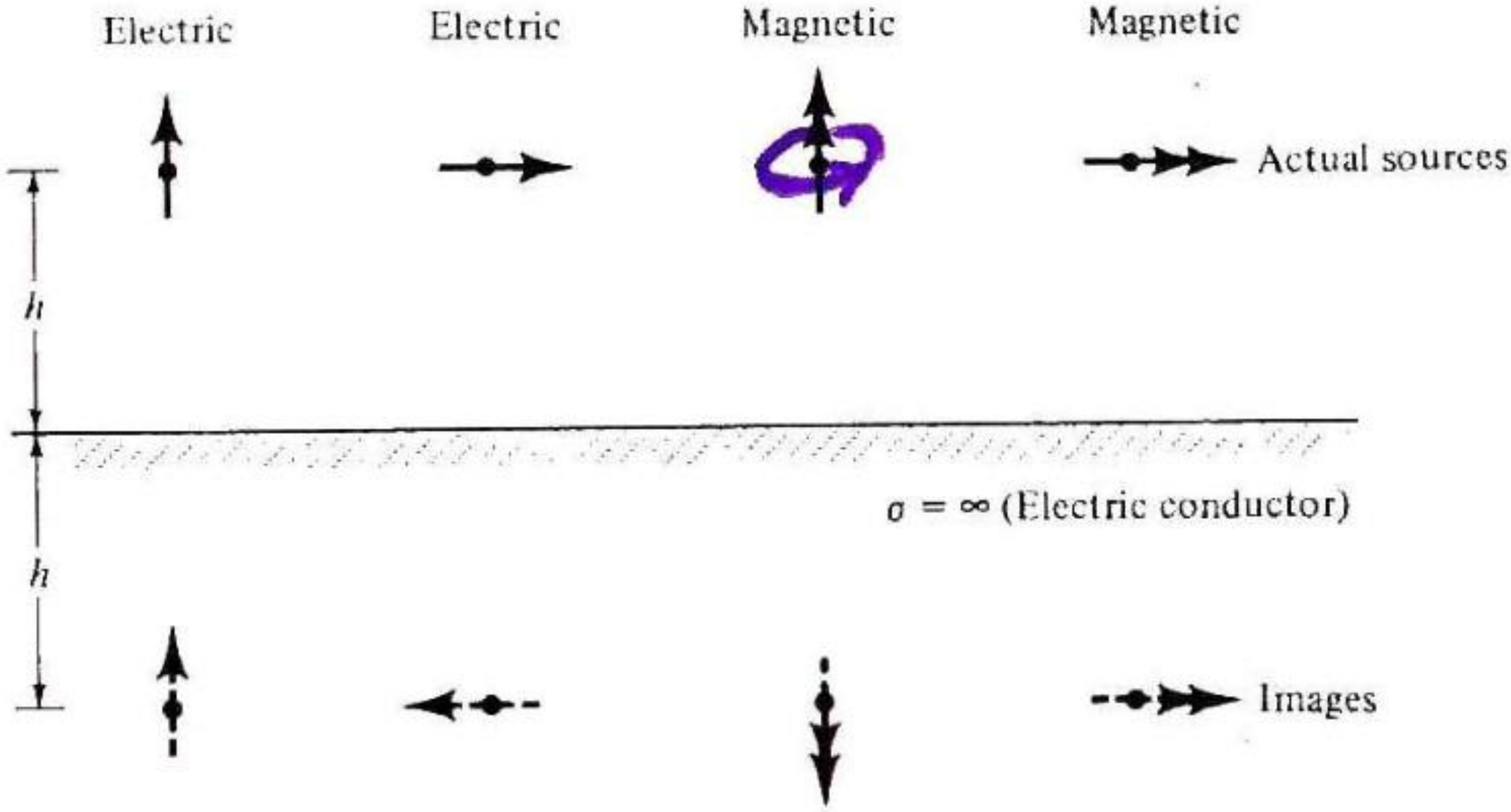
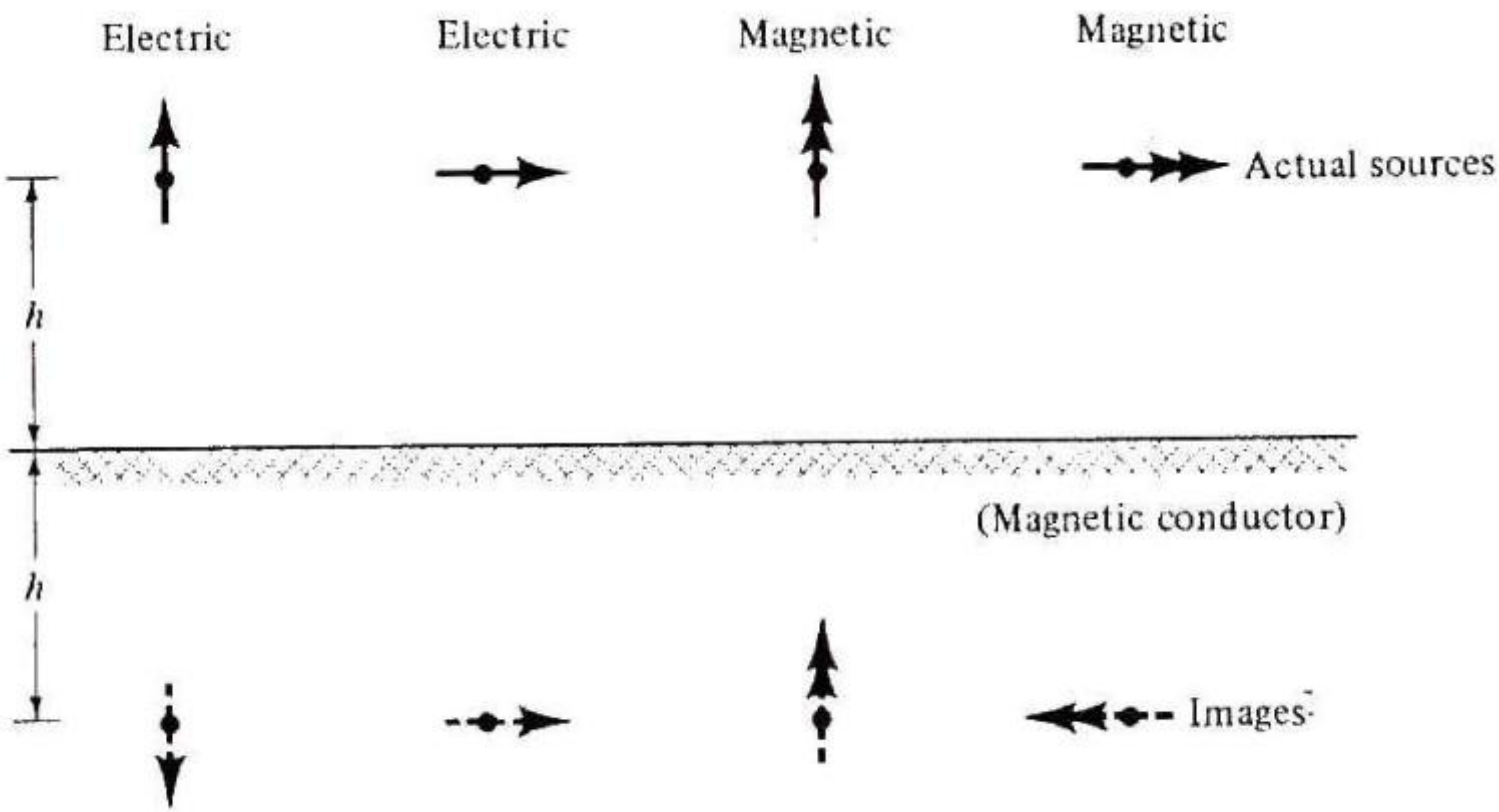
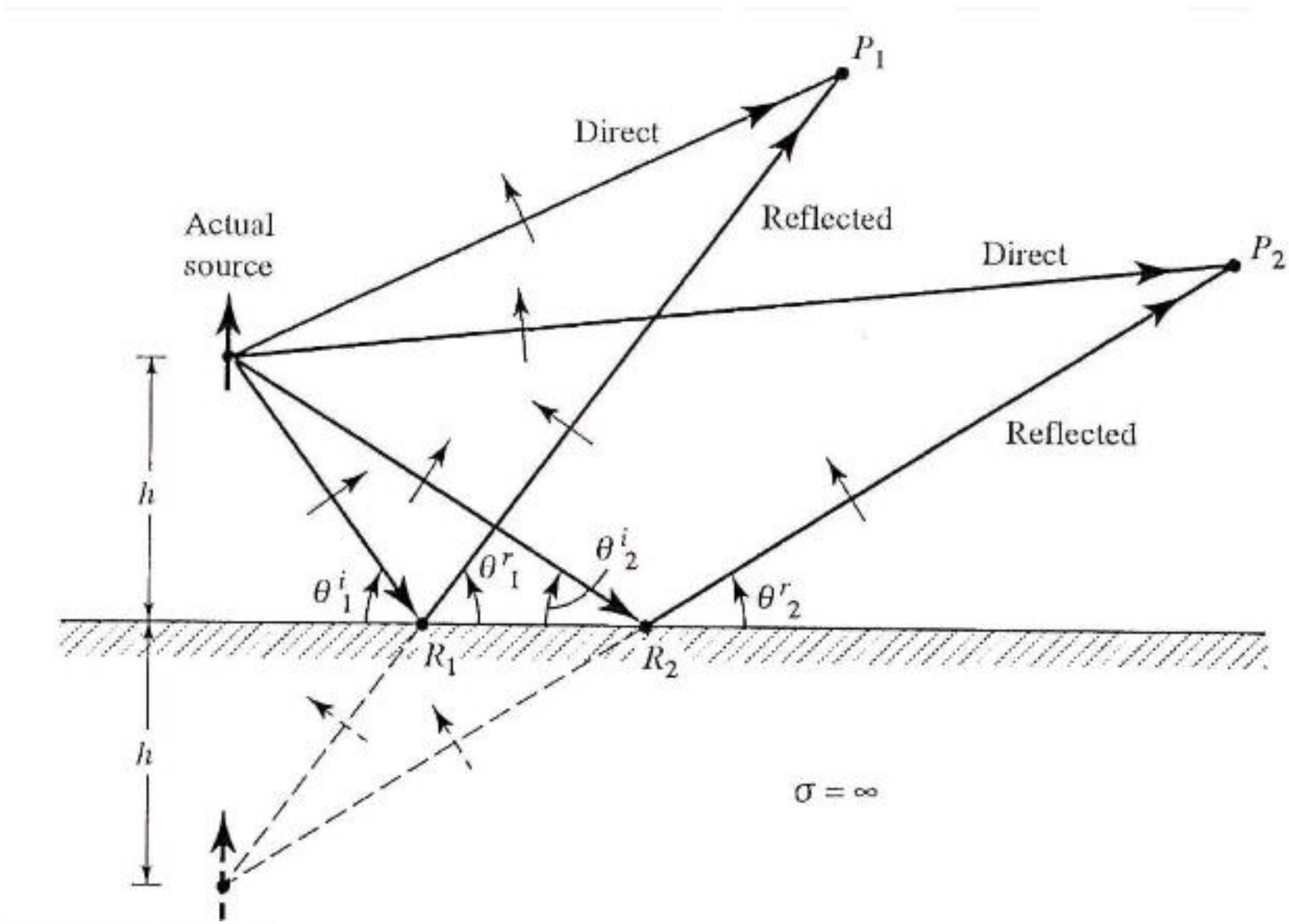


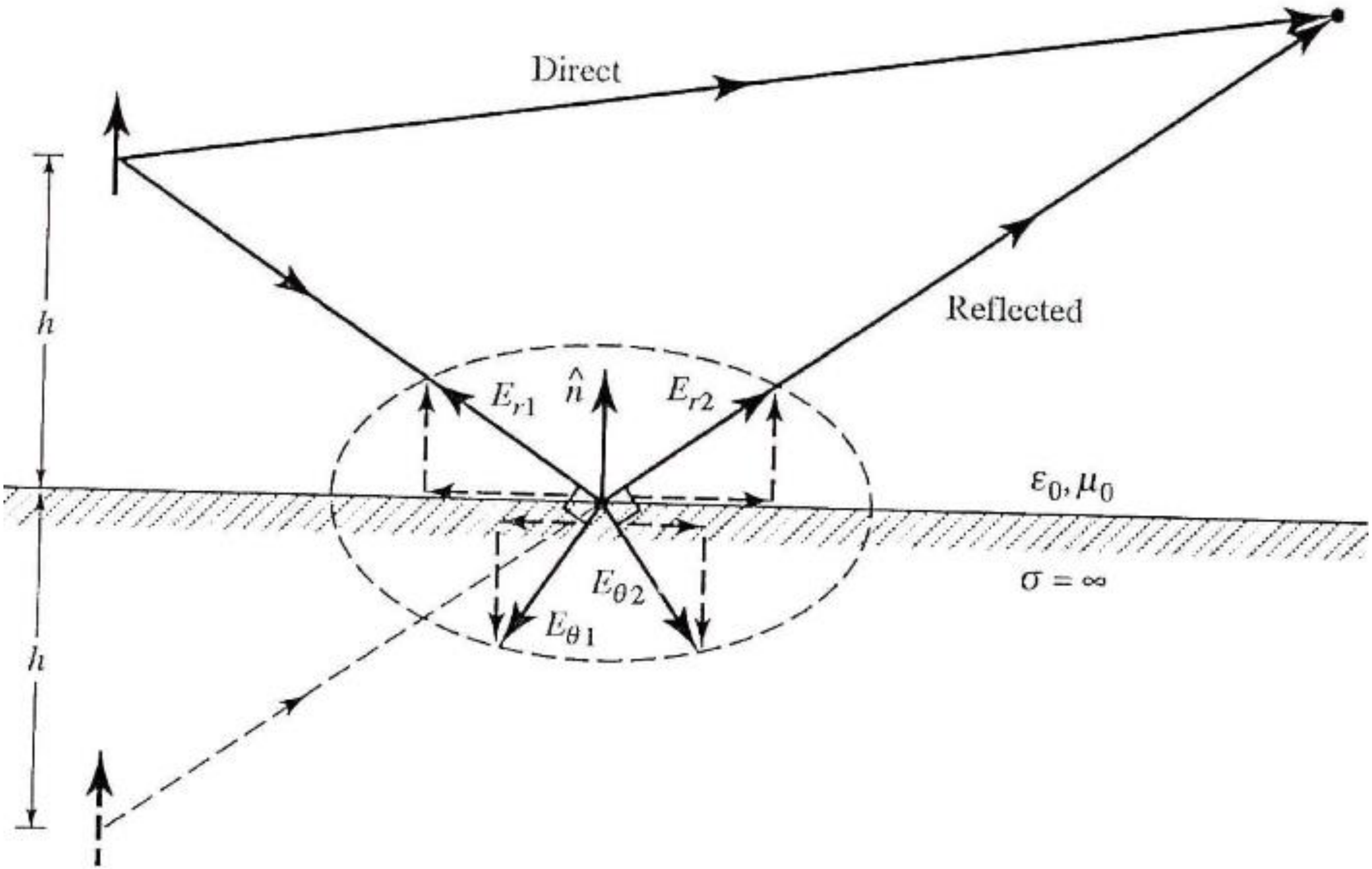
Image Theory: Perfect Magnetic Conductor



Vertical Electric Dipole and Image Theory (PEC)



Field Components at Point of Reflection



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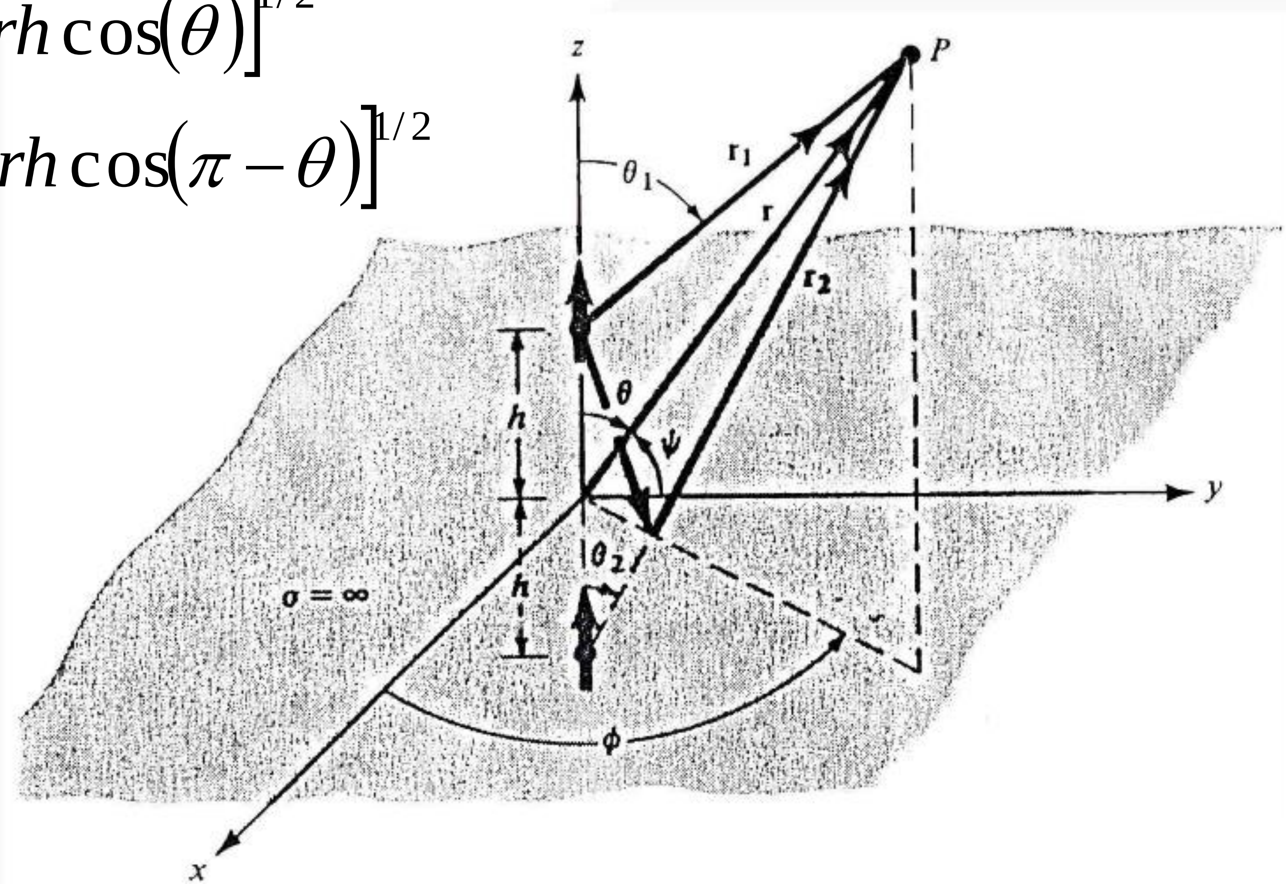
Analyzing the Vertical Electric Dipole Above PEC

- We use the image theory in conjunction with superposition:

$$r_1 = \left[r^2 + h^2 - 2rh\cos(\theta)\right]^{1/2}$$

$$r_2 = \left[r^2 + h^2 - 2rh\cos(\pi - \theta)\right]^{1/2}$$



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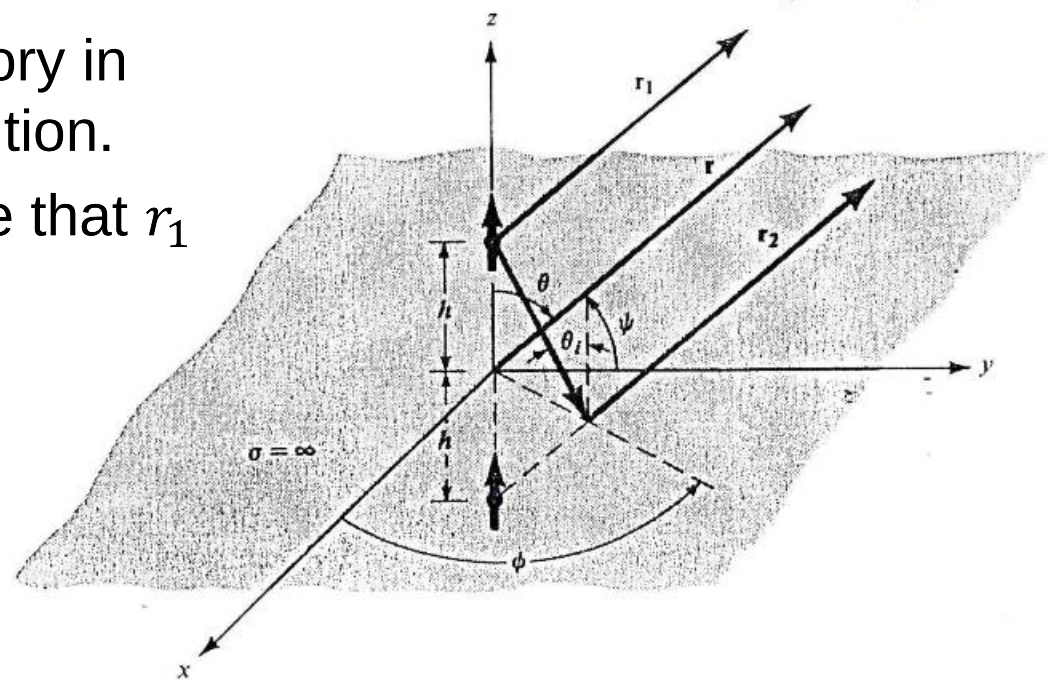
Analyzing the Vertical Electric Dipole Above PEC

- We will use the image theory in conjunction with superposition.
- In far field, we can assume that r_1 and r_2 are in parallel:



$$r_1 \approx r - h\cos(\theta)$$

$$r_2 \approx r + h\cos(\theta)$$

$$E_\theta^d = j\eta \frac{klI_0 e^{-jkr_1}}{4\pi r_1} \sin\theta_1 \qquad E_\theta^r = j\eta \frac{klI_0 e^{-jkr_2}}{4\pi r_2} \sin\theta_2$$

POSTECH