Spring 2019

EECE 588
Lecture 18

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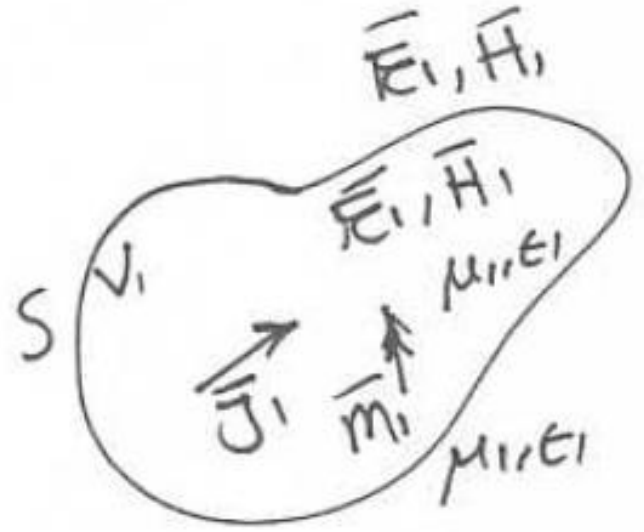
Uniqueness Theorem

- A field in a lossy region is uniquely specified by the source within the region plus the tangential component of the magnetic field over the boundary, or the tangential component of the electric field over the boundary or the former over some part of the boundary and the latter over the remaining part of the boundary.

- Consider the following situation:
  - J_1 and M_1 are radiating in an unbounded space (μ_1, ε_1). The sources are bounded by an imaginary surface S.
  - We can replace this problem with an equivalent problem.

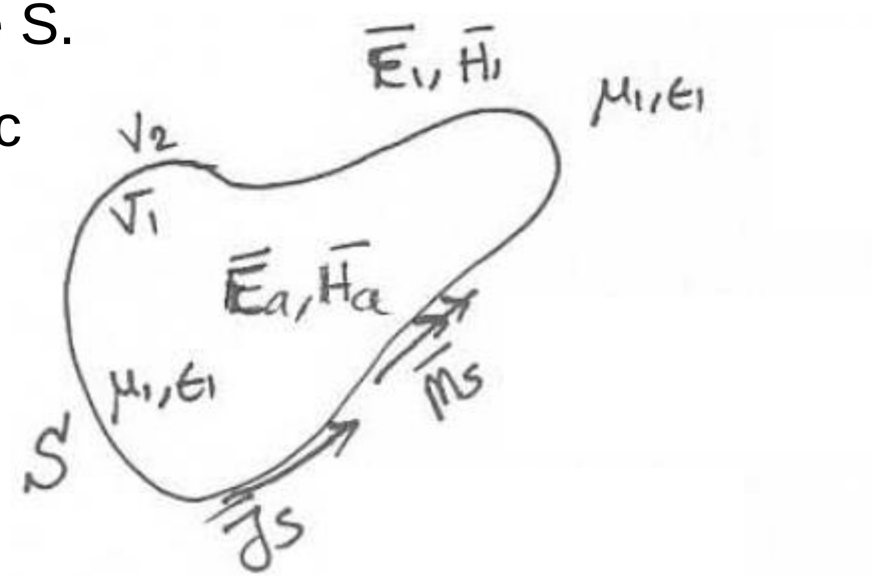


Equivalence

- We can remove $\mathbf{J_1}$ and $\mathbf{M_1}$ and place the surface currents $\mathbf{J_s}$ and $\mathbf{M_s}$ on S that satisfy the boundary conditions.
- Note that J_s and M_s radiate in the unbounded space with material properties of (μ_1, ε_1).
- The currents produce the original fields ($\mathbf{E_1}$, $\mathbf{H_1}$) outside S and they produce $\mathbf{E_a}$ and $\mathbf{H_a}$ inside S.
- $\mathbf{E_a}$ and $\mathbf{H_a}$ are <u>arbitrary</u> electric and magnetic fields.

$$\vec{M}_s = -\hat{n}\times(\vec{E}_1 - \vec{E}_a)$$

$$\vec{J}_s = \hat{n}\times(\vec{H}_1 - \vec{H}_a)$$

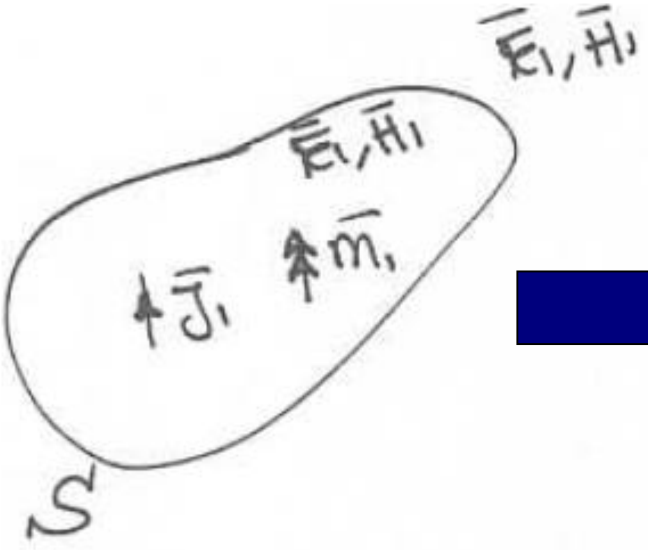


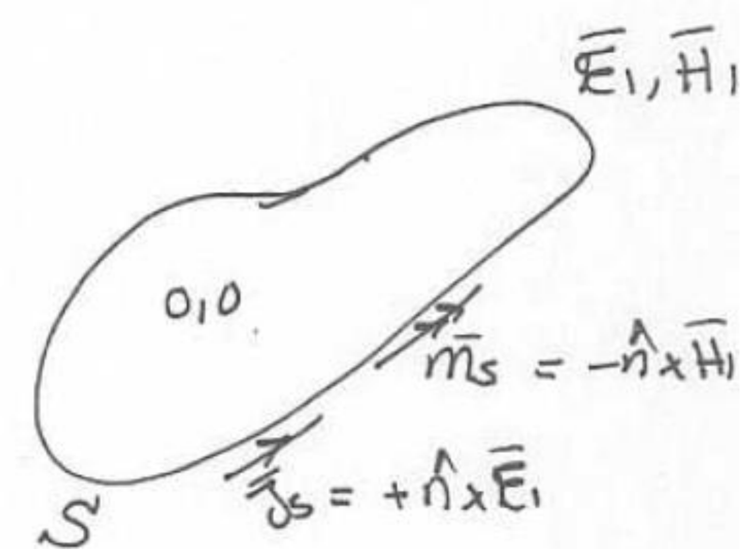
Equivalence Principle

- Since $\mathbf{E_a}$ and $\mathbf{H_a}$ are arbitrary, we can choose them to be zero.
- This means that:

$$\vec{M}_s = -\hat{n} \times (\vec{E}_1 - \vec{E}_a) \quad \Rightarrow \quad \vec{M}_s = -\hat{n} \times \vec{E}_1$$

$$\vec{J}_s = \hat{n} \times (\vec{H}_1 - \vec{H}_a) \quad \Rightarrow \quad \vec{J}_s = \hat{n} \times \vec{H}_1$$





Equivalence Principle

- If V_1 (the volume inside S) is filled with PEC, $\mathbf{J}_s$ is short circuited and only $\mathbf{M}_s$ exist.
- However, in this case, the magnetic current $\mathbf{M}_s$ is radiating in the presence of a PEC body.
- This means that you can NO LONGER use the free space Green's function. i.e., the following formula cannot be used ANYMROE:

$$\vec{\mathrm{F}} = \frac{\varepsilon}{4\pi} \iiint_{Vol} \vec{\mathrm{M}}(r') \frac{e^{-jk|r-r'|}}{|r-r'|} dv'$$

  - But you will see that this technique is not totally useless.

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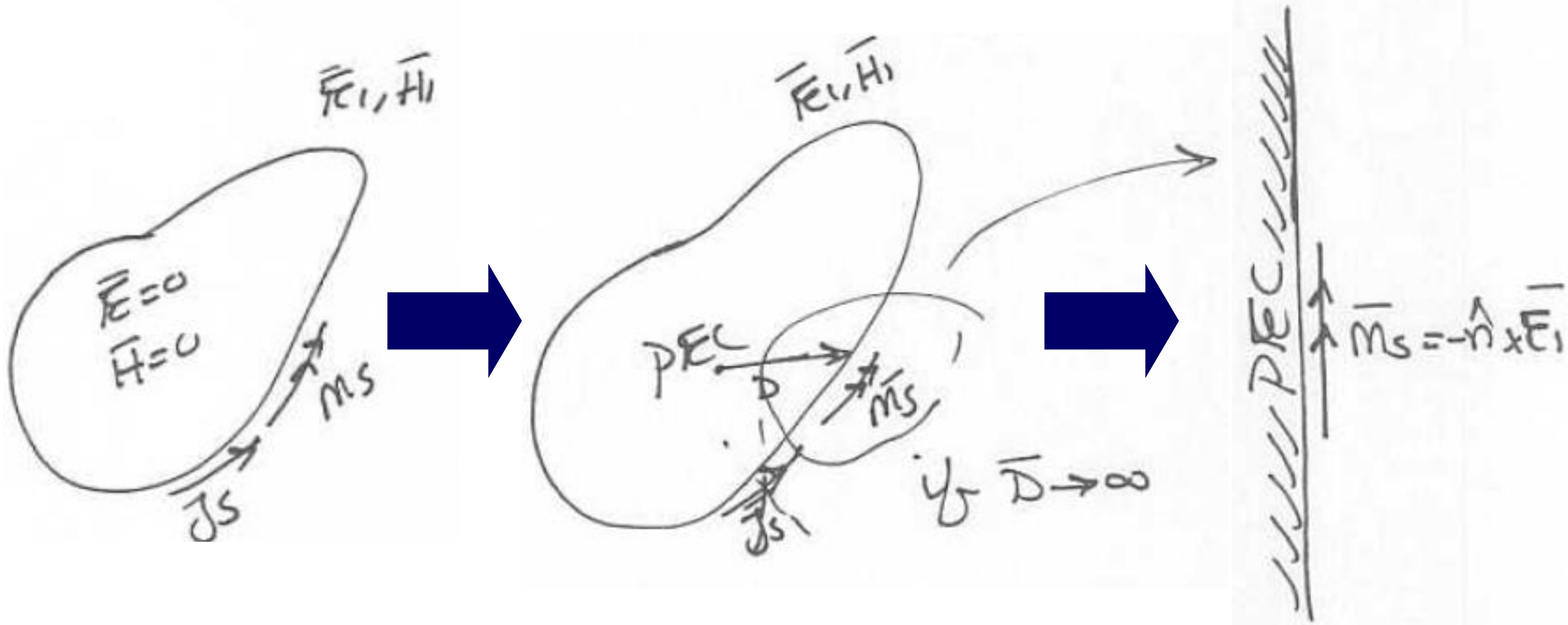
Equivalence Principle

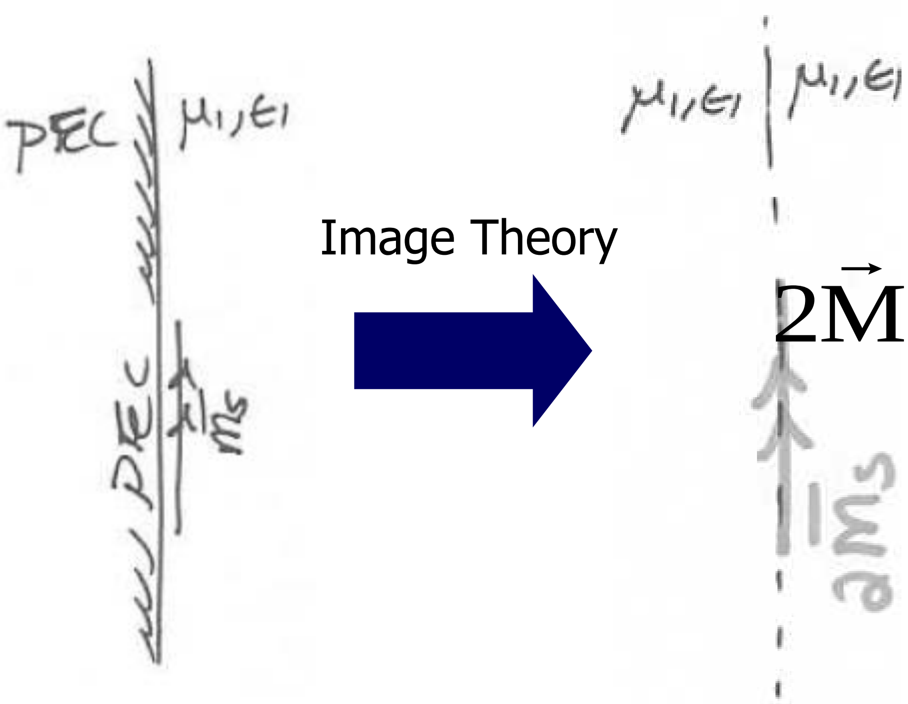
- An example of such a case is shown in this figure:



- If the radius of curvature (D) is very large, we can approximate the surface with a flat surface (e.g., the earth surface).

Equivalence Principle

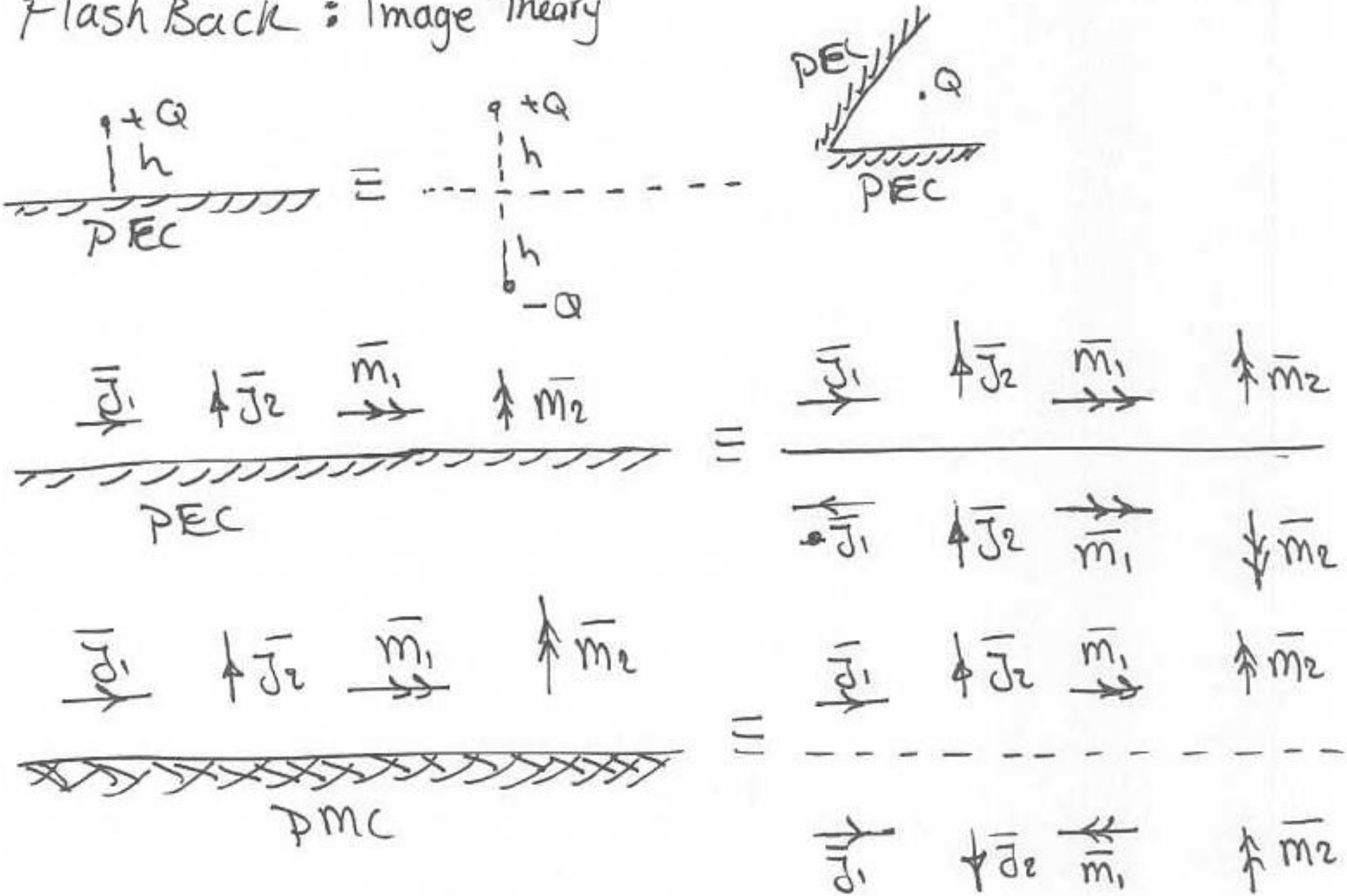
- The next step is to use the image theory:



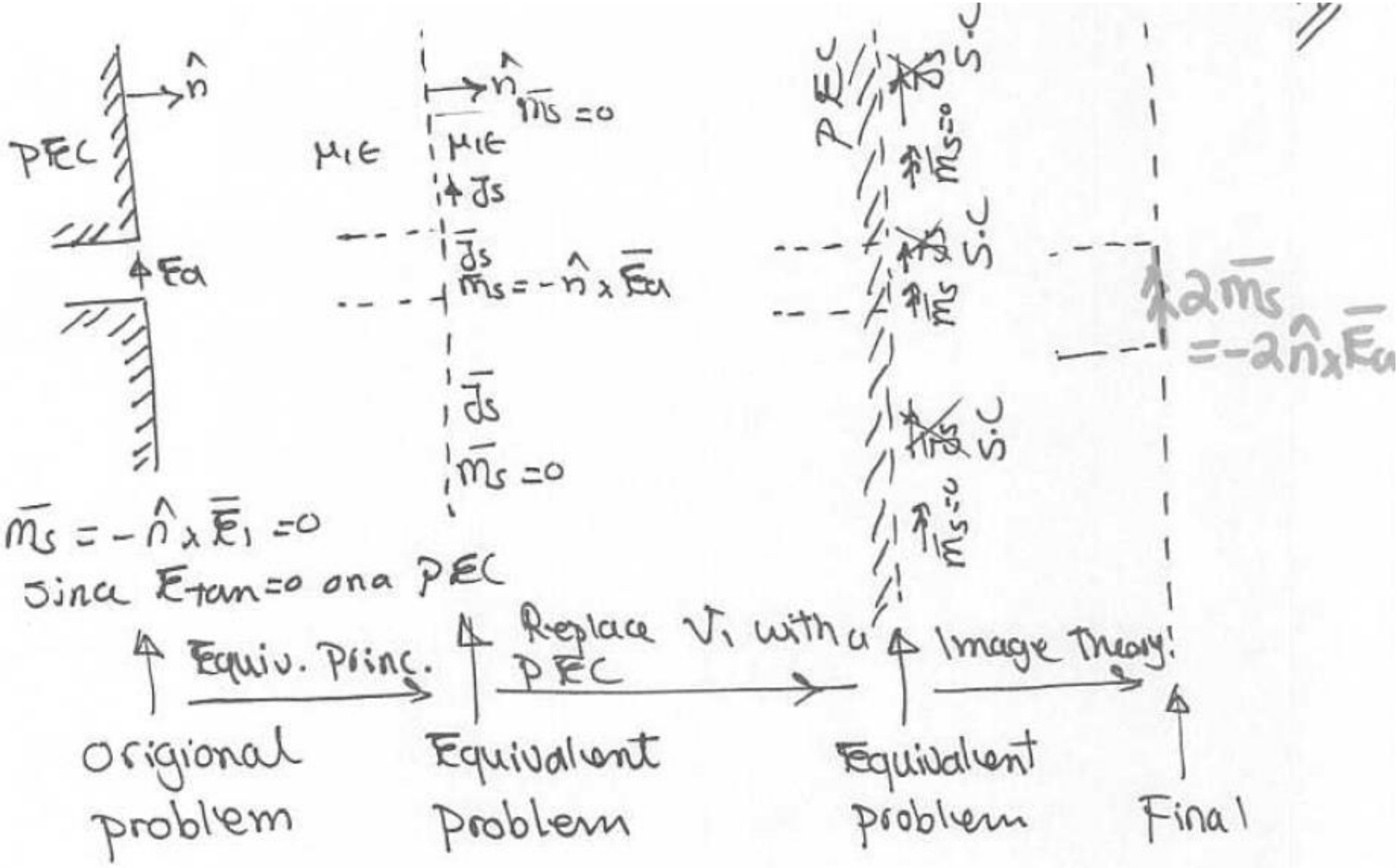
$$2\vec{M}_s = -2\hat{n} \times \vec{E}$$

- So you can use image theory to remove the PEC body and replace the magnetic current with 2**M**$_s$.

Flash Back : Image Theory



An Example: Open Ended Waveguide



An Example: Open Ended Waveguide

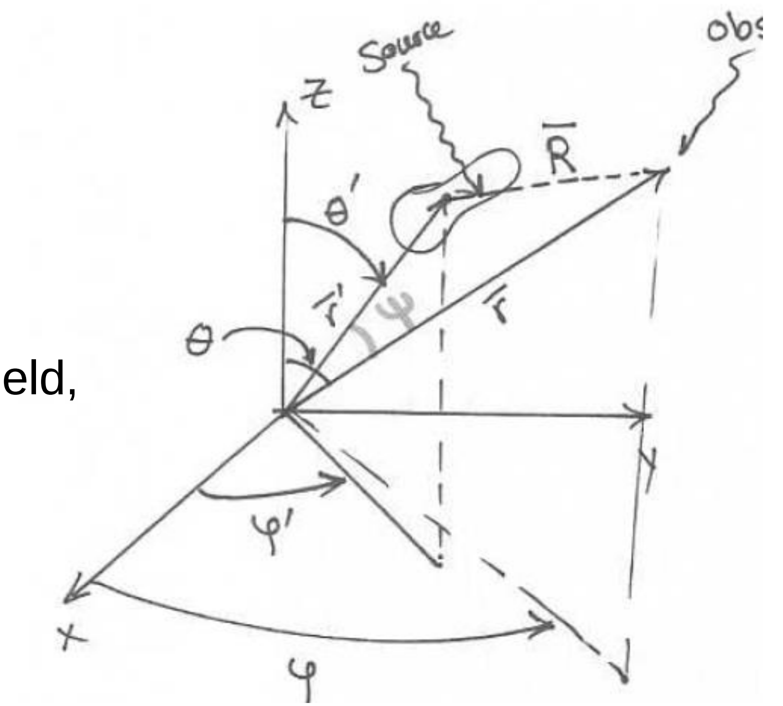
- The waveguide problem is simplified to a magnetic current radiating in free space.
- Since the final magnetic current $\vec{\mathrm{M}} = -2\hat{\mathrm{n}} \times \vec{\mathrm{E}}_{\mathrm{a}}$ is radiating in free space, we can use the Free Space Green's function. i.e.:

$$\vec{\mathrm{F}} = \frac{\varepsilon}{4\pi} \iiint_{\mathrm{Vol}} \vec{\mathrm{M}}(r') \frac{e^{-jk|r-r'|}}{|r-r'|} dv'$$

- Far field approximation:
  - For calculating phase variation in the far field, R can be approximated as:

$$R = |\vec{R}| = |\vec{r} - \vec{r}'|$$

$$\approx r - r' \cos\psi$$



Far Field Approximation

- Far field approximation:
  - For calculating amplitude variations in the far field, R can be approximated as r

$$R \approx r$$

- Primed coordinates are used to refer to the source coordinates and the unprimed coordinates refer to the observation points.
  - Note that the volume integral is reduced to a surface integral over aperture.

$$\vec{\mathrm{A}} = \frac{\mu}{4\pi} \iint_S \vec{\mathrm{J}}_s(r') \frac{e^{-jk(r-r')}}{|r-r'|} ds' \approx \frac{\mu e^{-jkr}}{4\pi r} \vec{\mathrm{N}}$$

$$\vec{\mathrm{F}} = \frac{\varepsilon}{4\pi} \iint_S \vec{\mathrm{M}}_s(r') \frac{e^{-jk|r-r'|}}{|r-r'|} ds' \approx \frac{\varepsilon e^{-jkr}}{4\pi r} \vec{\mathrm{L}}$$

$$\vec{\mathrm{N}} = \iint_S \vec{\mathrm{J}}_s(r') e^{jkr'\cos\psi} ds'$$

$$\vec{\mathrm{L}} = \iint_S \vec{\mathrm{M}}_s(r') e^{jkr'\cos\psi} ds'$$

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Far Field Approximation

- In the general case where both $\vec{J}$ and $\vec{M}$ are radiating:

$$E_r \approx 0 \qquad H_r \approx 0$$

$$E_\theta \approx -\frac{jke^{-jkr}}{4\pi r}(L_\varphi + \eta N_\theta) \qquad H_\theta \approx \frac{jke^{-jkr}}{4\pi r}(N_\varphi - \frac{L_\theta}{\eta})$$

$$E_\varphi \approx \frac{jke^{-jkr}}{4\pi r}(L_\theta - \eta N_\varphi) \qquad H_\varphi \approx -\frac{jke^{-jkr}}{4\pi r}(N_\theta + \frac{L_\varphi}{\eta})$$

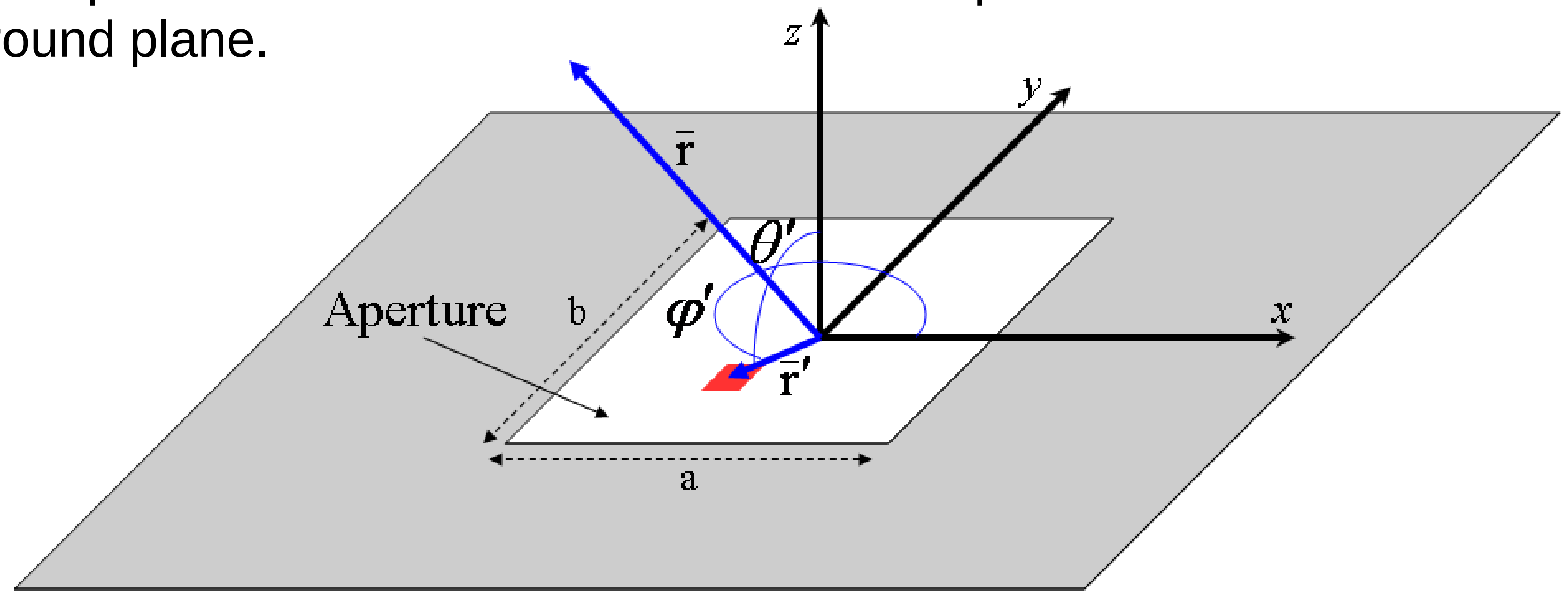
- L_θ, N_θ, L_φ, and N_φ are the θ and φ components of **N** and **L**.

Uniform Aperture Distribution

- Let us consider the following aperture field distribution:

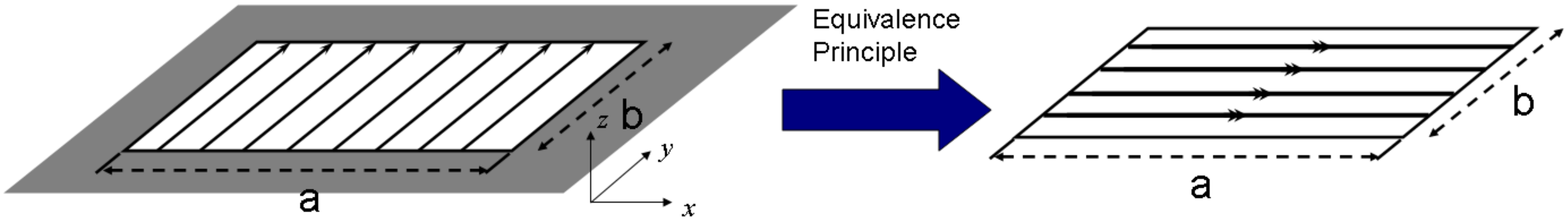
$$\vec{E}_a = \hat{y}E_0 \quad \begin{cases} -a/2 \le x' \le a/2 \\ -b/2 \le y' \le b/2 \end{cases}$$

- The aperture distribution is uniform and the aperture is in an infinite ground plane.



Uniform Aperture Distribution

- Note that if the aperture if physically in the infinite ground plane, it is not possible to have a uniform electric field distribution because of the boundary conditions.
- Therefore, this problem is more mathematical rather than practical. However, the implications are very important.
- The equivalence principle can be used to convert this problem to a simple problem of magnetic current distribution in space.



$$\overline{Ms} = -2\hat{n} \times \overline{E} = 2E_0\hat{x}$$

Uniform Aperture Distribution

$$\vec{\mathrm{F}}=\frac{\varepsilon}{4\pi}\iint_S \vec{\mathrm{M}}_S(r')\frac{e^{-jk|r-r'|}}{|r-r'|}ds' \approx \frac{\varepsilon e^{-jkr}}{4\pi r}\boxed{\iint_S \vec{\mathrm{M}}_S(r')e^{jkr'\cos\psi}ds'}\ \mathbf{L}$$

$$\mathrm{L}_\theta=\iint_S\{\mathrm{M}_x\cos\theta\cos\varphi+\mathrm{M}_y\cos\theta\sin\varphi-\mathrm{M}_z\sin\theta\}e^{jkr'\cos\psi}ds'$$

$$\mathrm{L}_\phi=\iint_S\{-\mathrm{M}_x\sin\varphi+\mathrm{M}_y\cos\varphi\}e^{jkr'\cos\psi}ds'$$

- Use $\vec{\mathrm{M}}_s=2\mathrm{E}_0\hat{\mathrm{x}}$ in the above equations.

$$\mathrm{L}_\theta=\int_{-b/2}^{b/2}\int_{-a/2}^{a/2}\mathrm{M}_x\cos\theta\cos\varphi\, e^{jk(x'\sin\theta\cos\varphi+y'\sin\theta\sin\varphi)}dx'dy'$$

$$=\cos\theta\cos\varphi\boxed{\left\{\int_{-b/2}^{b/2}\int_{-a/2}^{a/2}\mathrm{M}_x e^{jk(x'\sin\theta\cos\varphi+y'\sin\theta\sin\varphi)}dx'dy'\right\}}$$

2D Space Factor

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Uniform Aperture Distribution

$$\mathrm{L}_\theta = 2abE_0\left\{\cos\theta\cos\varphi\left(\frac{\sin X}{X}\right)\left(\frac{\sin Y}{Y}\right)\right\}$$

$$X = \frac{ka}{2}\sin\theta\cos\varphi \quad Y = \frac{kb}{2}\sin\theta\sin\varphi$$

$$\mathrm{L}_\varphi = -2abE_0\left\{\sin\varphi\left(\frac{\sin X}{X}\right)\left(\frac{\sin Y}{Y}\right)\right\}$$

- Using L_θ and L_φ expressions, we have:

$$\mathrm{E}_\theta = j\frac{abkE_0e^{-jkr}}{2\pi r}\left\{\sin\varphi\left(\frac{\sin X}{X}\right)\left(\frac{\sin Y}{Y}\right)\right\} \qquad \mathrm{H}_\theta = -\frac{\mathrm{E}_\varphi}{\eta}$$

$$\mathrm{E}_\varphi = j\frac{abkE_0e^{-jkr}}{2\pi r}\left\{\cos\theta\cos\varphi\left(\frac{\sin X}{X}\right)\left(\frac{\sin Y}{Y}\right)\right\} \quad \mathrm{H}_\varphi = \frac{\mathrm{E}_\theta}{\eta}$$

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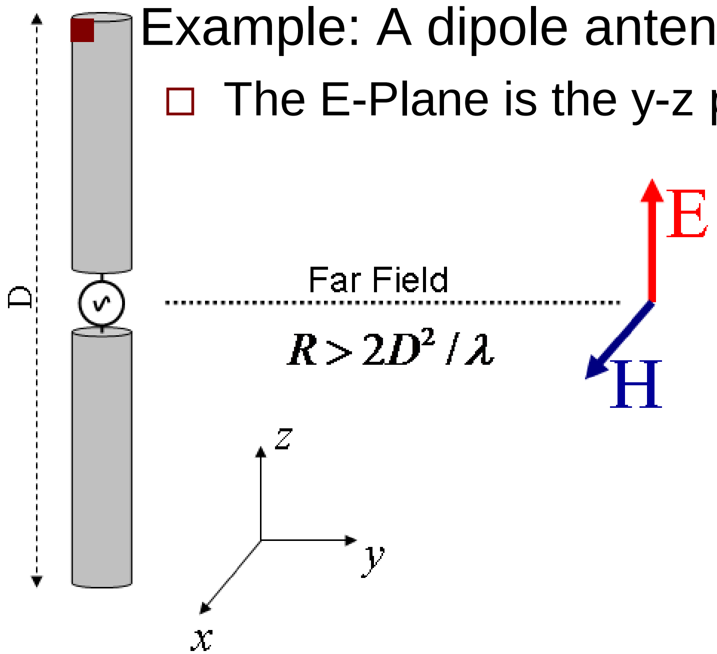
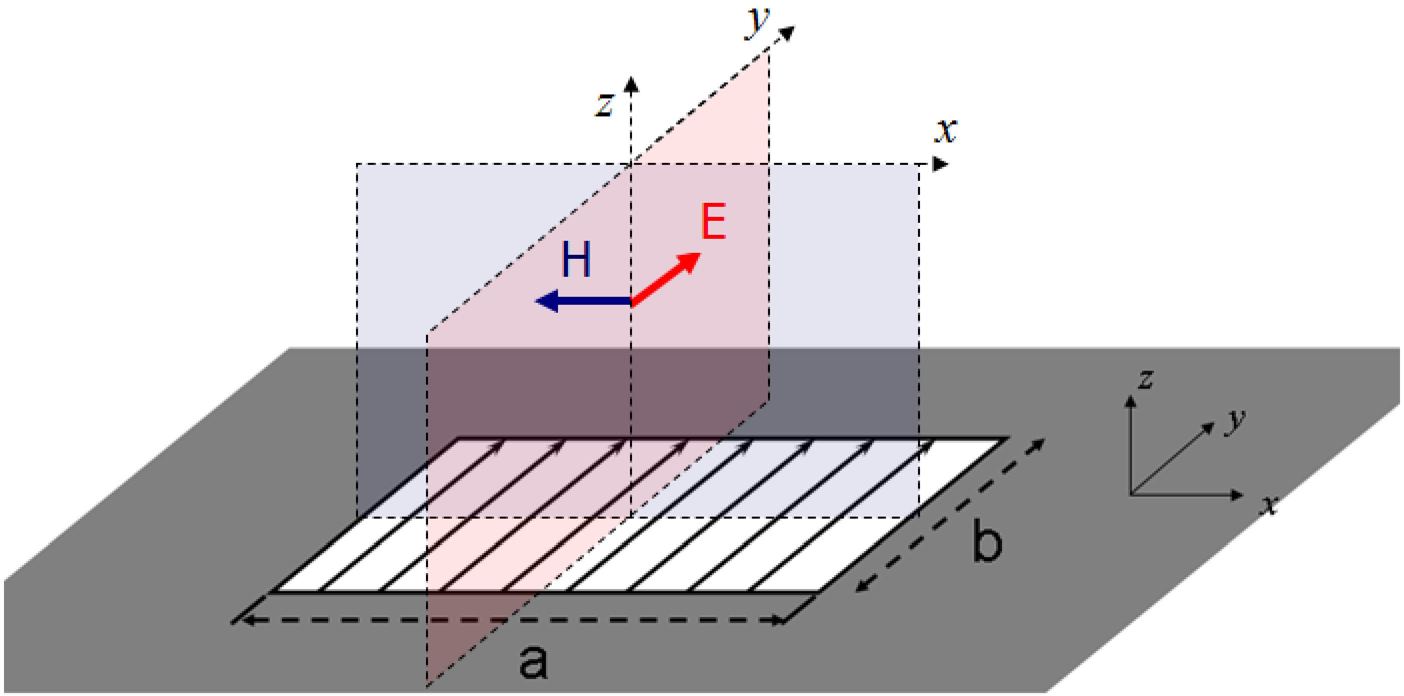
Radiation Pattern

- Principal planes of radiation:
  - E-Plane: The plane containing the electric field vector and direction of maximum radiation.
  - H-Plane: The plane containing the magnetic field vector and the direction of maximum radiation.
- Example: A dipole antenna as shown in the figure.
  - The E-Plane is the y-z plane and the H-plane is the x-z plane.



Radiation Pattern

- For the aperture antenna under consideration, the principal planes of radiation are shown here.
  - E-Plane: y-z plane ($\phi = \pi/2$).
  - H-Plane: x-z plane ($\phi = 0$).



Radiation Pattern

- In E-plane, the expression for electric field simplifies to:

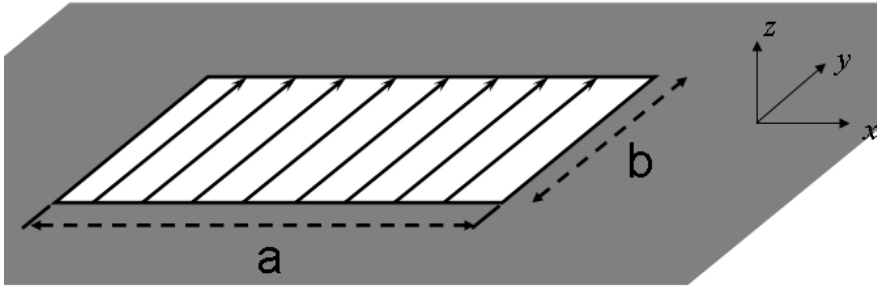
$$\mathrm{E}_\theta = j\frac{abkE_0 e^{-jkr}}{2\pi r}\left\{\frac{\sin\left(\frac{kb}{2}\sin\theta\right)}{\frac{kb}{2}\sin\theta}\right\} \quad \mathrm{E}_r = \mathrm{E}_\varphi = 0$$

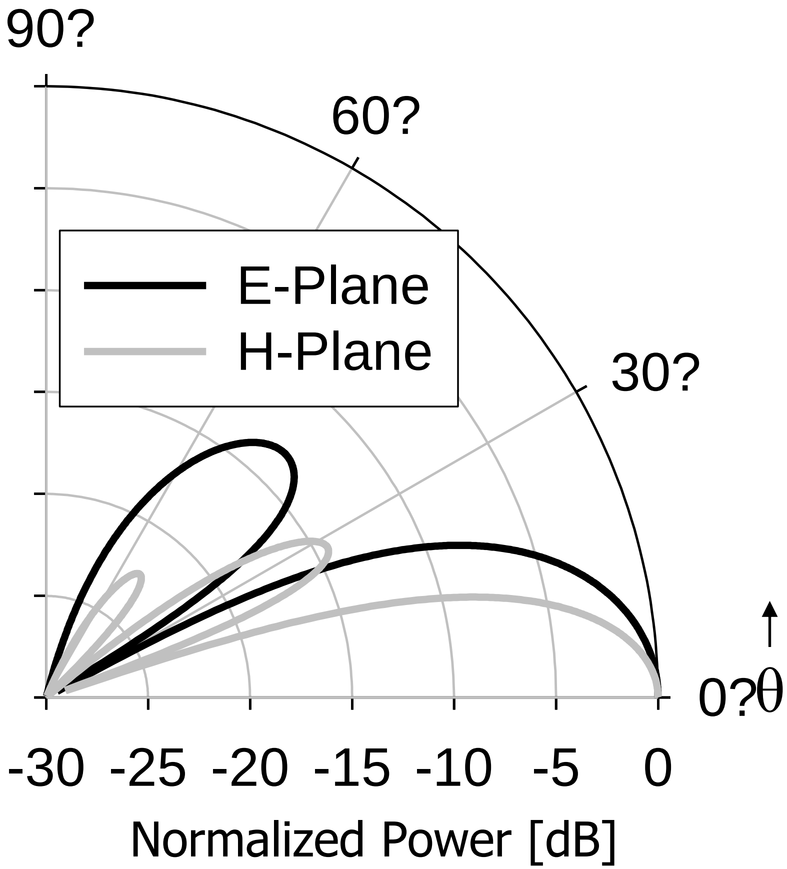
- In H-plane, the expression for electric field simplifies to:

$$\mathrm{E}_\varphi = j\frac{abkE_0 e^{-jkr}}{2\pi r}\left\{\cos\theta\frac{\sin\left(\frac{ka}{2}\sin\theta\right)}{\frac{ka}{2}\sin\theta}\right\} \quad \mathrm{E}_r = \mathrm{E}_\theta = 0$$

Radiation Pattern

- Radiation patterns of an aperture antenna mounted on an infinite ground plane.
- The antenna has a uniform aperture field distribution.
- The dimensions are $a = 3\lambda$, $b = 2\lambda$.





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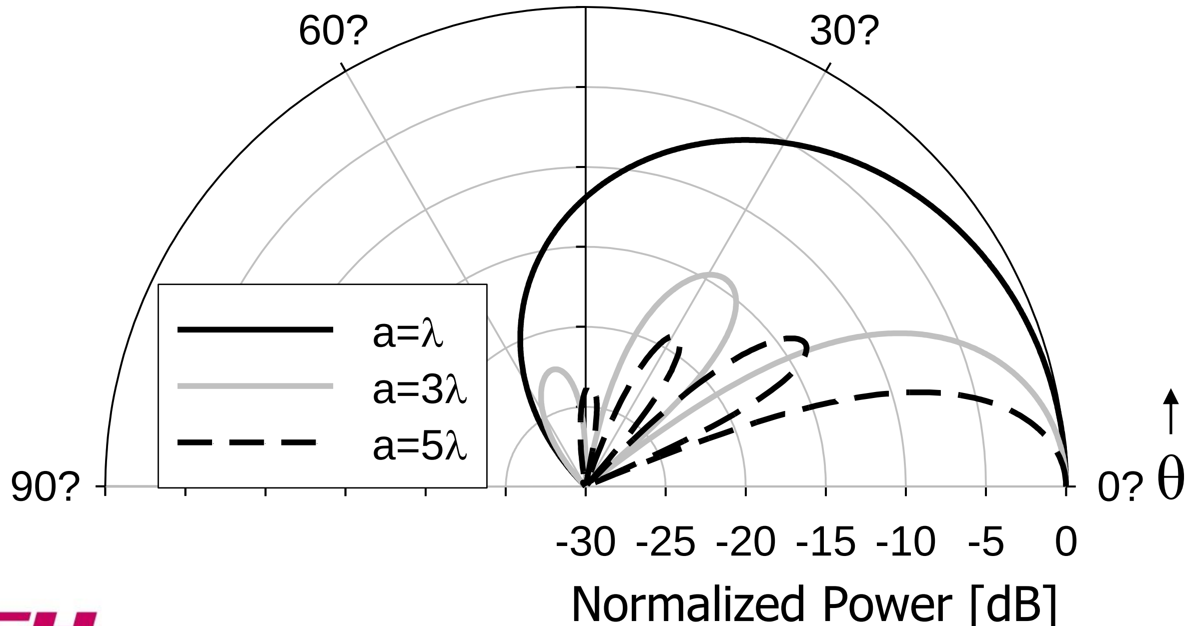
Radiation Patterns

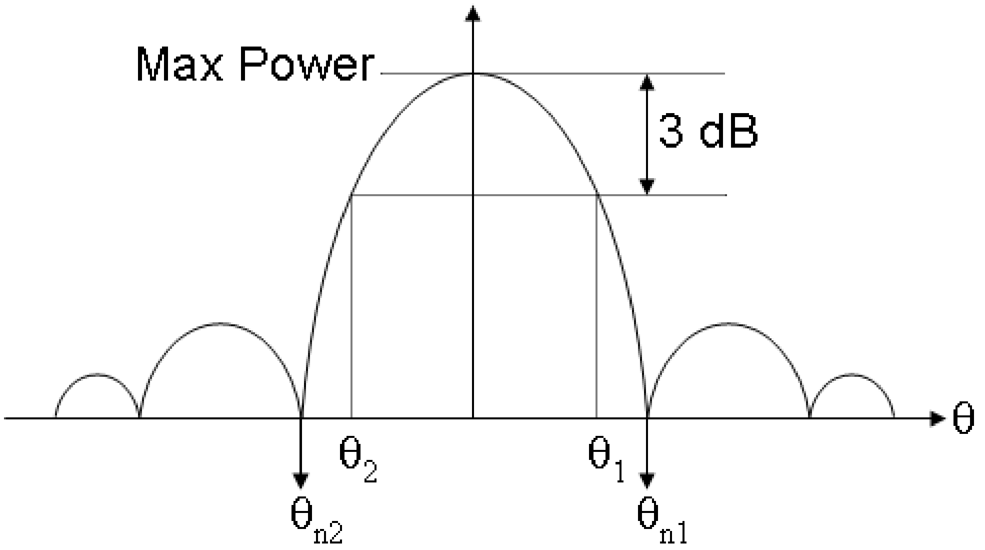
- The H-plane radiation pattern of an aperture antenna mounted on an infinite ground plane with uniform aperture distribution.
- $b = \lambda$ and values of “a” are variable.
- Note that as “a” increases, the antenna becomes more directive and the number of side lobes increase.



Radiation Patterns

- Half power beam width: In a plane containing the direction of the maximum radiation, the angle between the two directions in which the radiation intensity is one half the maximum value of the beam.
- Using the generic radiation pattern shown below as reference: HPBW = $\theta_1 + \theta_2$.
- First Null Beam Width (FNBW) can also be obtained from the expressions of the radiation patterns.

- Using the generic pattern shown here, FNBW =$\theta_{n1} + \theta_{n2}$.



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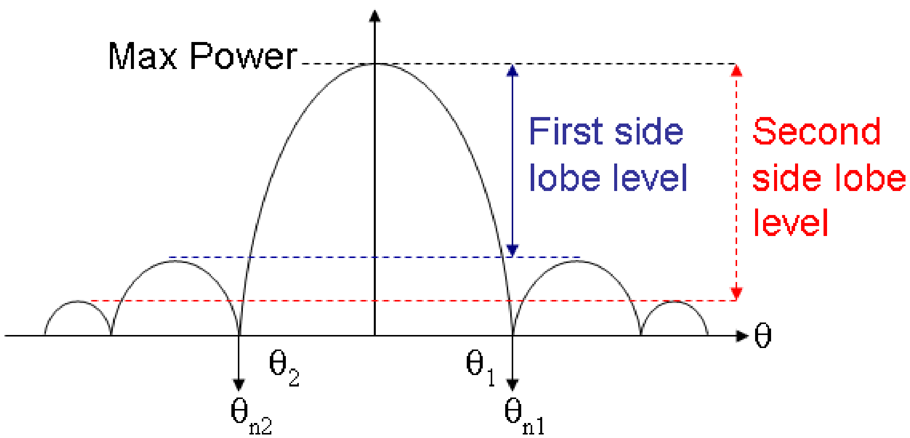
Beamwidths

- For uniform aperture distribution and for large values of a and b:

$$\text{HPBW} = \begin{cases} \dfrac{50.6^\circ}{b/\lambda} & \text{E-Plane} \\ \dfrac{50.6^\circ}{a/\lambda} & \text{H-Plane} \end{cases} \qquad a,b \gg \lambda$$

- Similarly FNBW is:

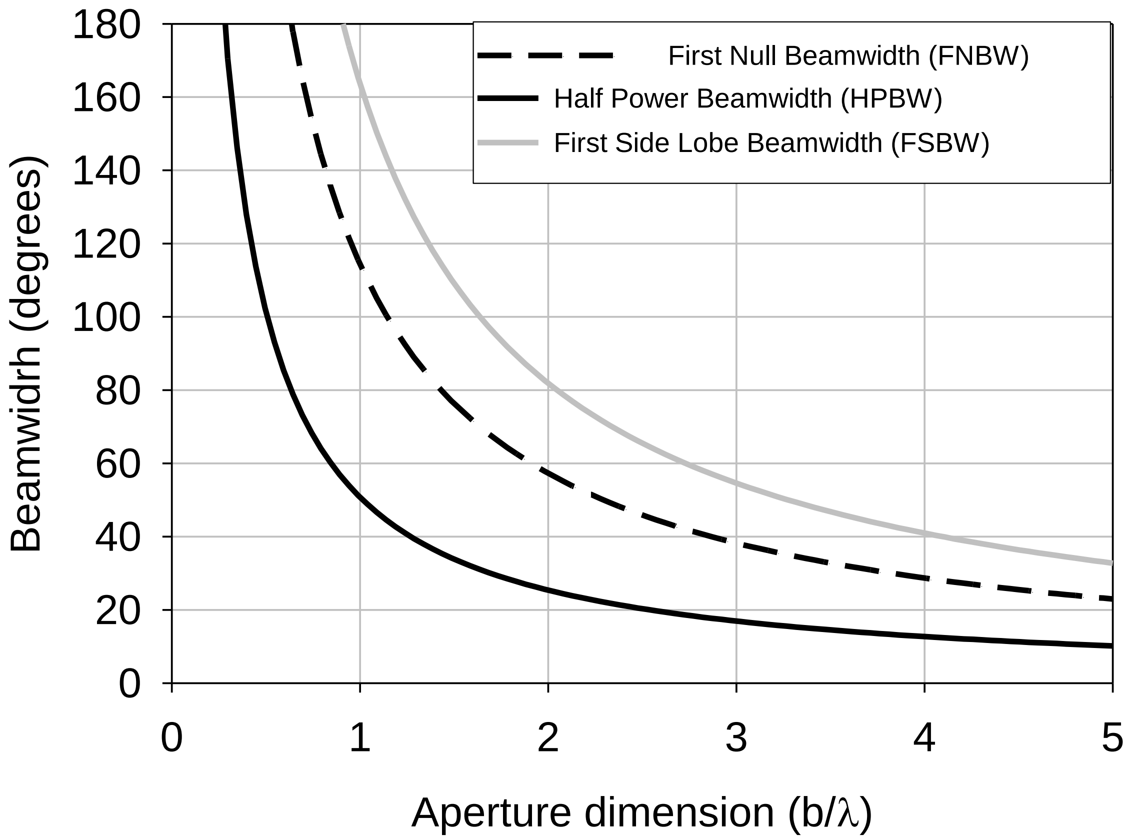
$$\text{FNBW} = \begin{cases} \dfrac{114.6^\circ}{b/\lambda} & \text{E-Plane} \\ \dfrac{114.6^\circ}{a/\lambda} & \text{H-Plane} \end{cases} \qquad a,b \gg \lambda$$



- The first side lobe level is 13.26 dB down the main lobe for <u>uniform</u> aperture distribution.

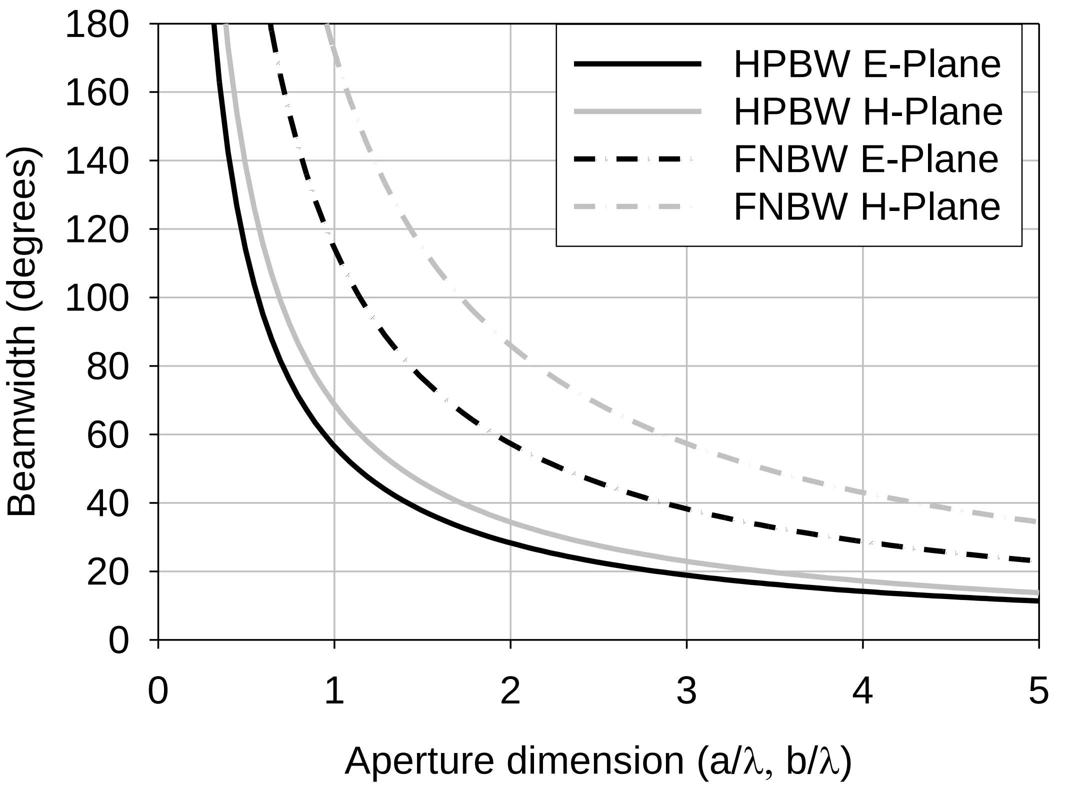
Beamwidths

- E-Plane beamwidths of an aperture antenna with uniform electric field distribution mounted on an infinite ground plane.



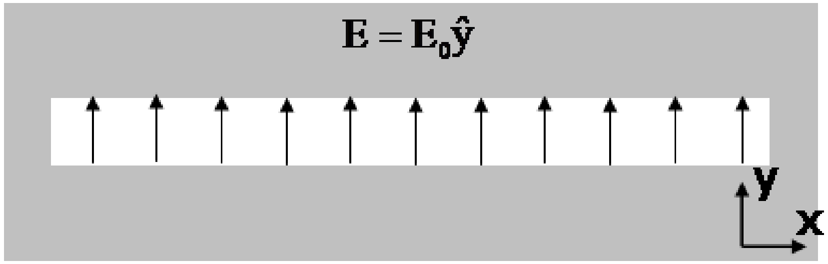
Beamwidths

- Half power beamwidth as a function of aperture dimensions.



Directivity

- Directivity can be calculated using the definition of directivity and the far field electric field expressions.
- However, since an infinite ground plane exists, a simpler technique for calculating directivity can be used.
- Assuming that the magnetic field exist over aperture, the radiated power can be calculates as:
- The next step is to calculate the maximum value of radiation intensity.



Directivity

- Radiation intensity is the power radiated from an antenna per unit solid angle.

$$U = r^2 W_{rad} \qquad W_{rad} = \text{radiation density} = \vec{\mathrm{E}} \times \vec{\mathrm{H}}$$

- U_{max} occurs at $\theta = 0°$.

$$U_{\max} = \left(\frac{ab}{\lambda}\right)^2 \frac{|E_0|^2}{2\eta}$$

- The definition of directivity is: the ratio of radiation intensity to the radiation intensity averaged over all directions.

$$D = \frac{U}{U_0} = \frac{4\pi U}{P_{rad}} \Rightarrow D = \frac{4\pi}{\lambda^2} ab$$

Directivity

- The physical area of the aperture A_p is $a \times b$, therefore, the directivity of the aperture can be written as:

$$D = \frac{4\pi}{\lambda^2} A_p$$

- Aperture efficiency is defined as the ratio of the maximum effective area of the aperture to the physical area of the aperture.

$$\eta_{ap} = \frac{A_{em}}{A_p}$$

- For uniform aperture distribution, aperture efficiency is 100%.
- Note that maximum effective area of any antenna is related to its maximum directivity:

$$D_m = \frac{4\pi}{\lambda^2} A_{em}$$