광전자공학 Ch. 6
Optics of Solids
Absorption and Dispersion

Seung-Yeol Lee

KYUNGPOOK NATIONAL UNIVERSITY

EM waves in various materials

Source free Maxwell equation for general media

$$\nabla \cdot \mathbf{D} = 0$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t}$$

Optical characteristics of media is determined by constitutive relations!

$$\mathbf{D} = \varepsilon \mathbf{E} = \varepsilon_0 \mathbf{E} + \mathbf{P}$$

$$\mathbf{B} = \mu \mathbf{H} = \mu_0 \mathbf{H} + \mu_0 \mathbf{M}$$

Non-zero $\mathbf{P}$ and $\mathbf{M}$

$\mathbf{P}$: polarization density

$\mathbf{M}$: Magnetization density

It will be no more simple as vacuum..

KNU KYUNGPOOK NATIONAL UNIVERSITY

Material classification

Simple	Complicate
Homogeneous	Inhomogeneous
Non-dispersive	Dispersive
Linear	Nonlinear
Lossless	Lossy
Isotropic	Anisotropic
Spatially-nondispersive	Spatially-dispersive

For simplicity, we only consider about **P**

EM wave in material

1. Linear, isotropic, homogeneous, nondispersive media

$$\mathbf{P} = \varepsilon_0 \chi \mathbf{E} \qquad \mathbf{D} = \varepsilon_0 (1+\chi)\mathbf{E} = \varepsilon \mathbf{E} = \varepsilon_0 \varepsilon_r \mathbf{E}$$

$$\mathbf{B} = \mu \mathbf{H} = \mu_0 \mu_r \mathbf{H}$$

ε_r : relative permittivity, dielectric constant

μ_r : relative permeability

The wave equation can be written as,

$$\nabla^2 \mathbf{E} - \frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} = 0 \qquad c = \frac{c_0}{\sqrt{\varepsilon_r \mu_r}} = \frac{c_0}{n}$$

Ex. Pure water

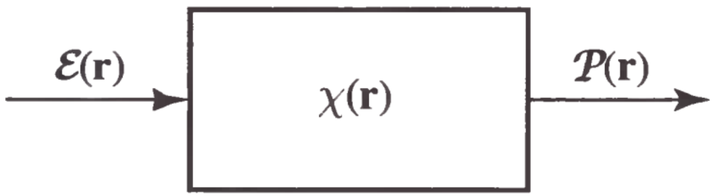
The velocity of light is simply reduced with a factor of n

KNU

KYUNGPOOK NATIONAL UNIVERSITY

Inhomogeneous media

2. Linear, isotropic, inhomogeneous, nondispersive media



The permittivity is space-dependent

$$\mathbf{D}(\mathbf{r}) = \varepsilon_0 (1 + \chi(\mathbf{r}))\mathbf{E}(\mathbf{r}) = \varepsilon(\mathbf{r})\mathbf{E}(\mathbf{r})$$

$$\nabla^2 \mathbf{E} + \nabla\left(\frac{\nabla \varepsilon(\mathbf{r})}{\varepsilon(\mathbf{r})} \cdot \mathbf{E}\right) - \mu_0 \varepsilon(\mathbf{r}) \frac{\partial^2 \mathbf{E}}{\partial t^2} = 0$$

Inhomogeneous wave equation (HW#1)

Slow varying $\varepsilon(\mathbf{r})$

Weak perturbation of n

$$\nabla^2 \mathbf{E} - \frac{\mathrm{n}^2(\mathbf{r})}{c_0^{\,2}} \frac{\partial^2 \mathbf{E}}{\partial t^2} = 0 \quad \Rightarrow \quad \nabla^2 \mathbf{E} - \frac{(\mathrm{n}+\Delta \mathrm{n})^2}{c_0^{\,2}} \frac{\partial^2 \mathbf{E}}{\partial t^2} = 0$$

Perturbed by Δn

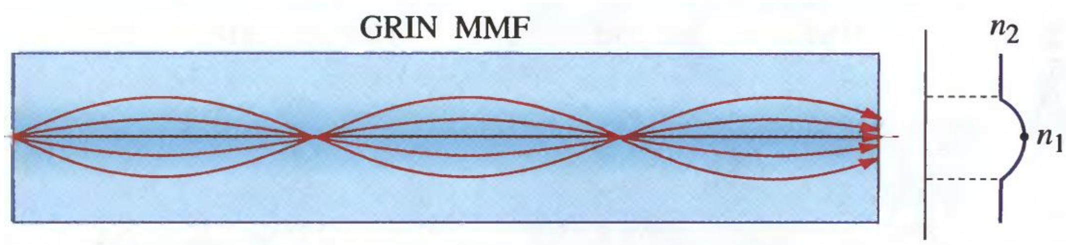
$$\rightarrow \boxed{\nabla^2 \mathbf{E} - \frac{\mathrm{n}^2}{c_0^{\,2}} \frac{\partial^2 \mathbf{E}}{\partial t^2}} = \boxed{\frac{1}{c_0^{\,2}} \frac{\partial^2 2n\Delta\mathrm{n}\mathbf{E}}{\partial t^2}}$$

homogeneous wave Eq.

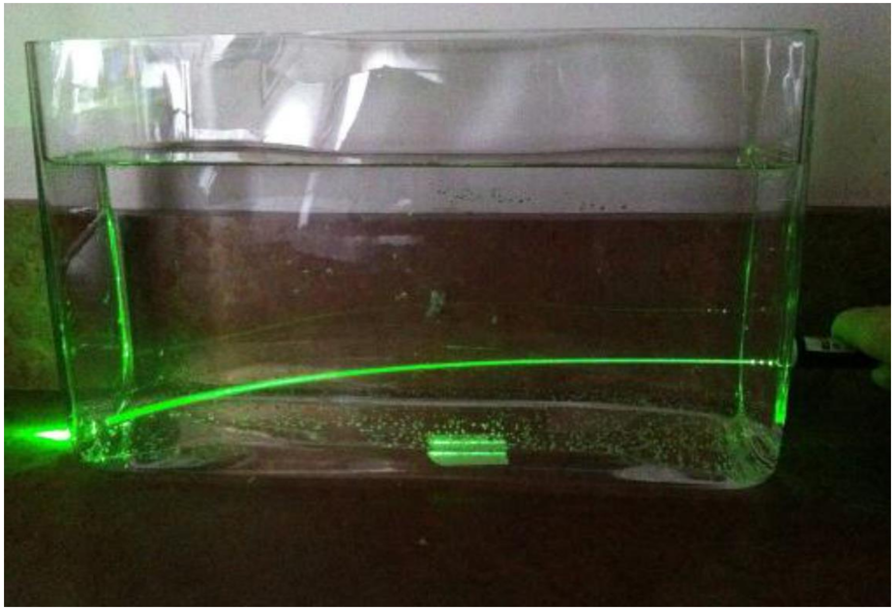
KYUNGPOOK NATIONAL UNIVERSITY

Inhomogeneous media

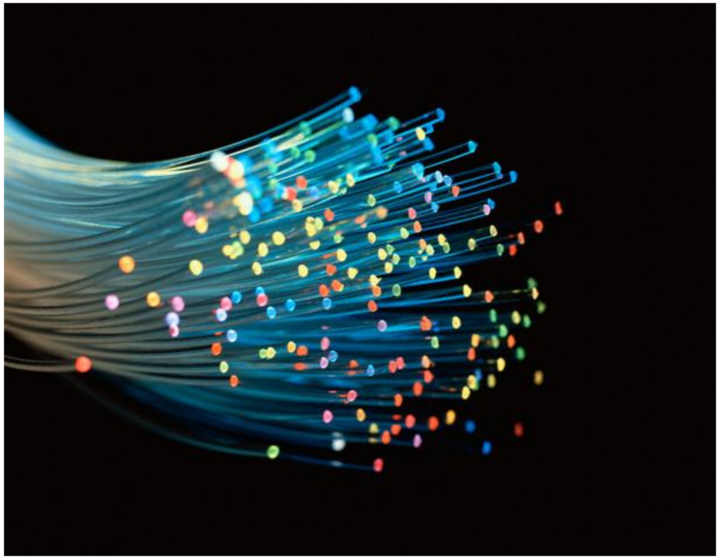
Ex. Graded index lens, Graded index fiber



Ex. Graded index formed by sugar water



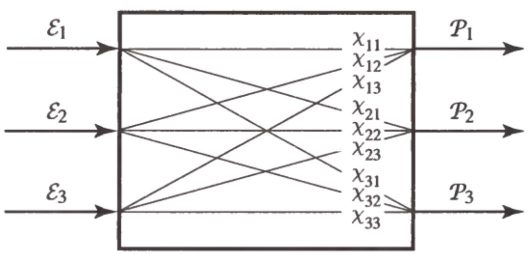
Optical fiber?



https://www.youtube.com/watch?v=QGnrJ1o3r0o

Anisotropic media

3. Linear, anisotropic, homogeneous, nondispersive media



The permittivity is dependent to E field vector

$$\begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}$$

In anisotropic material, Light propagates in different ways according to its polarization state

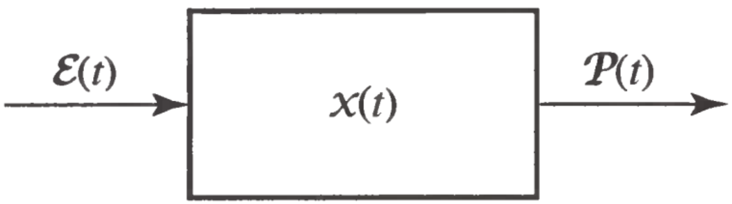
Ex. Liquid crystals, Calcite

These materials will be discussed in Ch. 6

Dispersive media

4. Linear, isotropic, homogeneous, dispersive media

In dispersive media, D is time-delayed according to the input signal of E.
(mainly caused by delayed response of molecule oscillation)



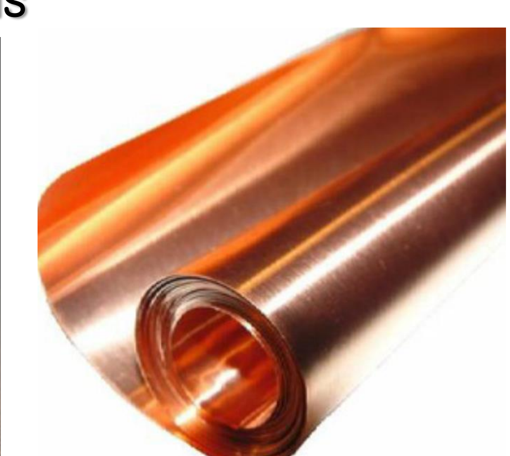
$$\mathbf{D}(t) = \varepsilon_0 \mathbf{E}(t) + \mathbf{P}(t)$$

$$= \varepsilon_0 \int_{-\infty}^{t} \overbrace{\boxed{(1+\chi(t-\tau))}}^{\varepsilon_r(t-\tau)} \mathbf{E}(\tau)\, d\tau$$

Convolution in time domain = simple multiplication in frequency domain

$$\mathbf{D}(\omega) = \varepsilon(\omega)\mathbf{E}(\omega)$$

Ex. Colored glass, metals



KNU KYUNGPOOK NATIONAL UNIVERSITY

Nonlinear media

5. Nonlinear, isotropic, homogeneous, nondispersive media

In nonlinear media, D and E is no more linear function

$$\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P}$$

$$\nabla^2 \mathbf{E} - \mu_0 \varepsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2} = \mu_0 \frac{\partial^2 \mathbf{P}}{\partial t^2} \longleftarrow \mathbf{P} = \alpha_1 \mathbf{E} + \alpha_2 \mathbf{E}^2 \ldots$$

Most media contains nonlinear functionalities when E is very strong.
Because higher order coefficients are very small compared to linear one, nonlinear characteristics usually appear for very strong intensity.

Absorption and Dispersion

Lossy media (absorption)

We turn back to the simplest dielectric case

$\mathbf{D} = \varepsilon_0 \varepsilon_r \mathbf{E}$ Consider ε_r is a complex number of $\varepsilon' + j\varepsilon''$

A plane wave solution can be written as (propagating z),

$$\nabla^2 \mathbf{E} - \mu_0 \varepsilon_0 \left(\varepsilon' + j\varepsilon'' \right) \frac{\partial^2 \mathbf{E}}{\partial t^2} = 0 \quad \Rightarrow \quad \mathbf{E} = E_0 \exp(j\omega t - jkz)$$

$$k = \omega \sqrt{\mu_0 \varepsilon_0 \left(\varepsilon' + j\varepsilon'' \right)} = \left(\beta - j \frac{1}{2} \alpha \right)$$

A plane wave solution can be written as,

$$\mathbf{E} = E_0 \underbrace{\boxed{\exp\left(-\frac{1}{2} \alpha z \right)}}_{\text{Loss term}} \exp(j\omega t - \overbrace{\boxed{j\beta z}}^{\text{Propagation term}})$$

Weakly absorbing media

Weakly absorbing media,

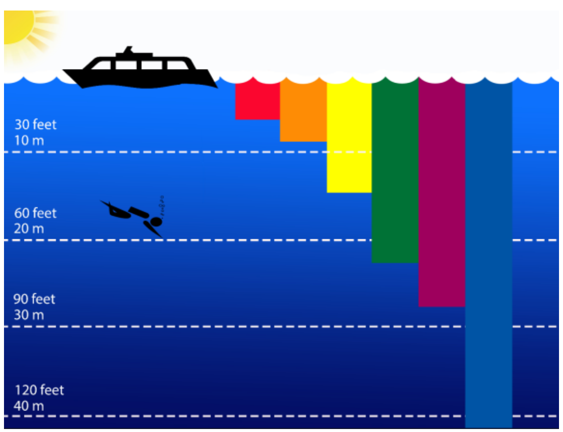
$$\varepsilon' + j\varepsilon'' \qquad |\varepsilon'| >> |\varepsilon''|$$

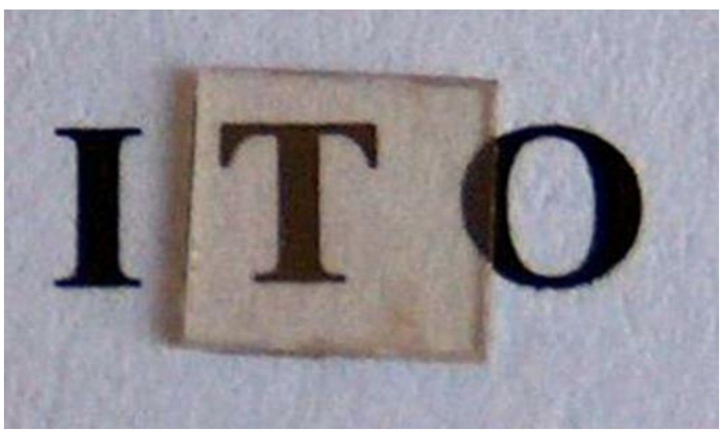
$$k = \omega\sqrt{\mu_0\varepsilon_0\left(\varepsilon' + j\varepsilon''\right)} \approx \omega\sqrt{\mu_0\varepsilon_0\varepsilon'}\left(1 + j\frac{\varepsilon''}{2\varepsilon'}\right)$$

$$\beta = \mathrm{n}k_0 = \sqrt{\varepsilon'}$$

$$\alpha = -k_0\varepsilon''/\sqrt{\varepsilon'}$$

Ex. Sea water, ITO





Strongly absorbing media

Strongly absorbing media,

$$\varepsilon' + j\varepsilon'' \qquad |\varepsilon'| << |\varepsilon''|$$

$$k = \omega\sqrt{\mu_0\varepsilon_0\left(\cancel{\varepsilon'} + j\varepsilon''\right)} \approx \omega\sqrt{\mu_0\varepsilon_0\left|\varepsilon''\right|}\,\frac{1+j}{\sqrt{2}}$$

$$\beta = k_0\sqrt{|\varepsilon''|}$$

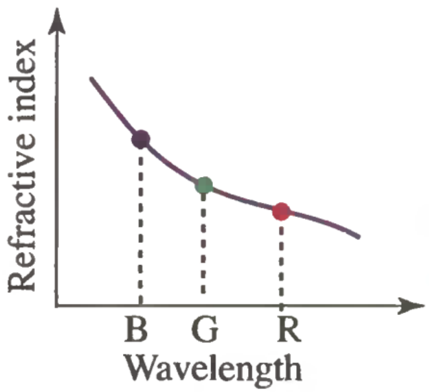
$$\alpha = 2k_0\sqrt{|\varepsilon''|}$$

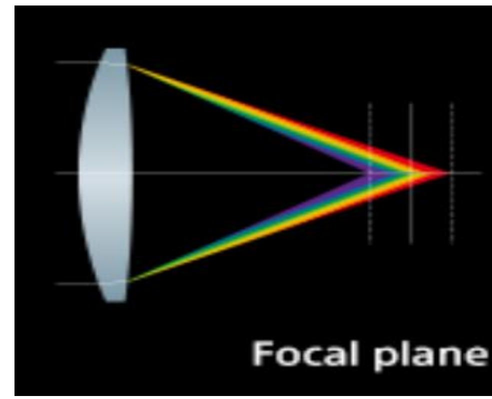
Ex. Metal, Si etc.

Dispersion

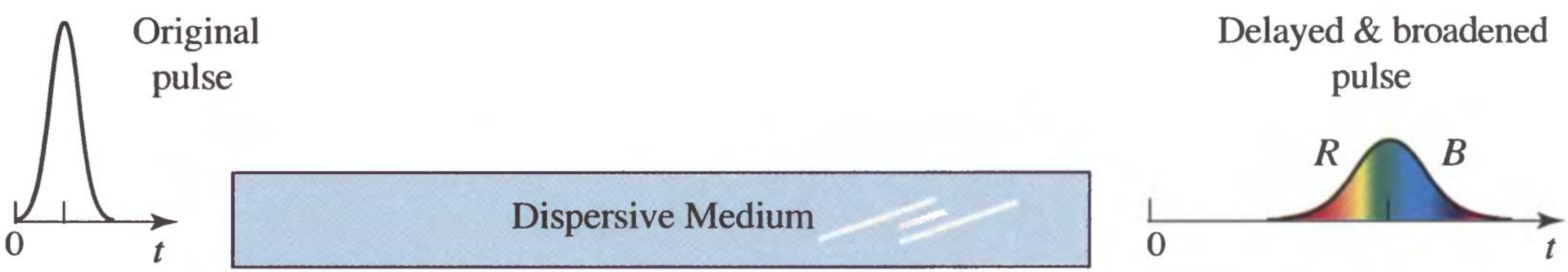
Dispersive materials have frequency-dependent permittivity!

$$\mathbf{D}(\omega) = \varepsilon(\omega)\mathbf{E}(\omega) = \varepsilon_0\left(1+\chi(\omega)\right)\mathbf{E}(\omega)$$



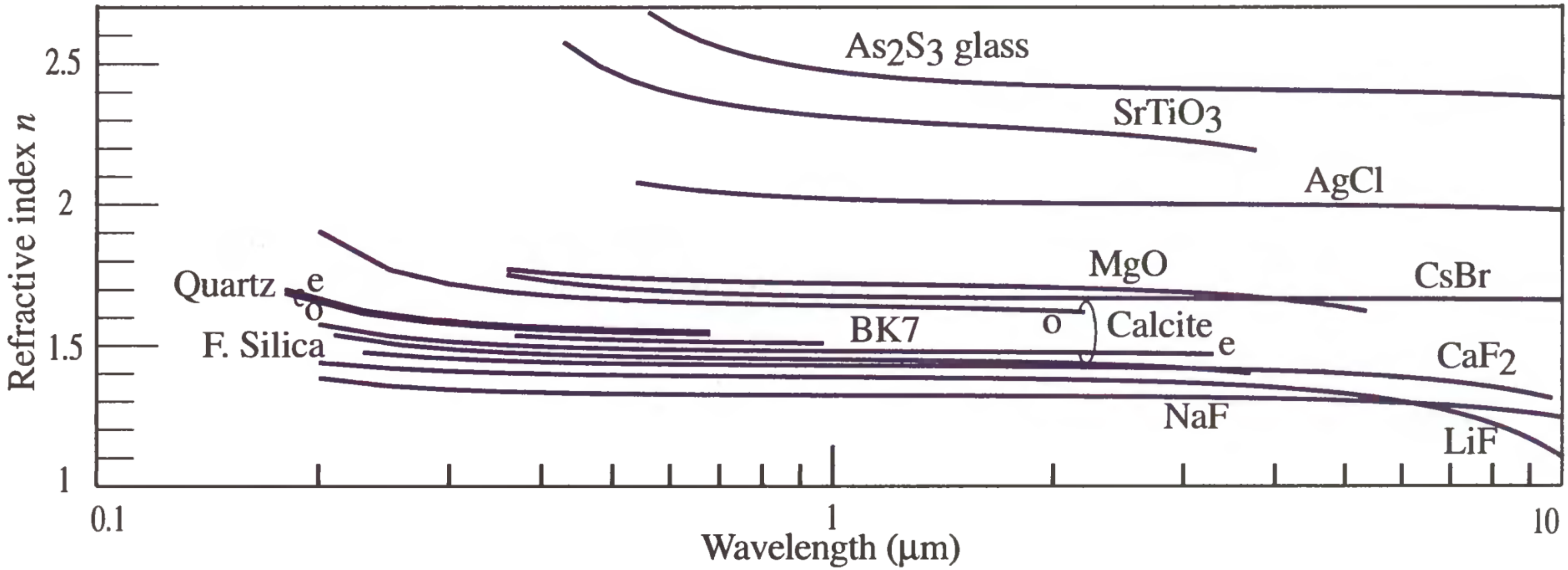


Pulse broadening by dispersion



Dispersion of dielectric material

Most materials have higher refractive index for blue wavelength



But why? (HW #2)

Hint, if you understand Lorentz resonance model and Kramers-Kronig relation, you may find the answer.

Measuring the material dispersion

Abbe number : indicate the broad region of dispersion covering whole visible range (parameter for lens, camera, etc.)

$$V = \frac{n_g - 1}{n_b - n_r}$$

Blue, green, and red wavelengths for 486.1 nm 587.6 nm and 656.3 nm

Material dispersion at specific wavelength $\frac{dn}{d\lambda}$

$\frac{dn}{d\lambda} < 0$ Normal dispersion

$\frac{dn}{d\lambda} > 0$ Anomalous dispersion

KNU

KYUNGPOOK NATIONAL UNIVERSITY

The Kramers-Kronig relation

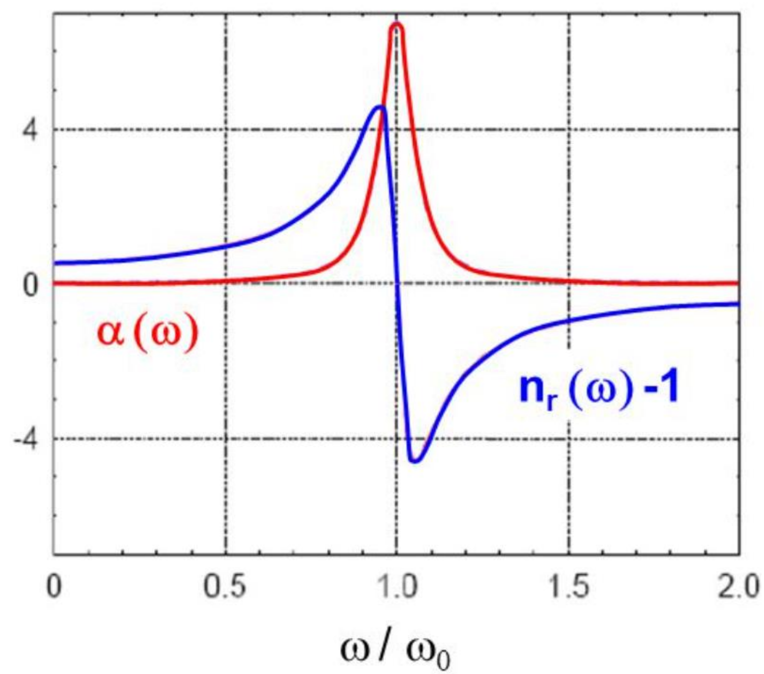
Material absorption and dispersion are intimately related

If there is a linear complex function $\chi(\omega)$ where causality is satisfied,

The real and imaginary part of $\chi(\omega) = \chi'(\omega) + j\chi''(\omega)$ must satisfy the following relation.

$$\chi'(\omega) = \frac{2}{\pi}\int_0^\infty \frac{s\chi''(\omega)}{s^2 - \omega^2}ds$$

$$\chi''(\omega) = \frac{2}{\pi}\int_0^\infty \frac{\omega\chi'(\omega)}{s^2 - \omega^2}ds$$

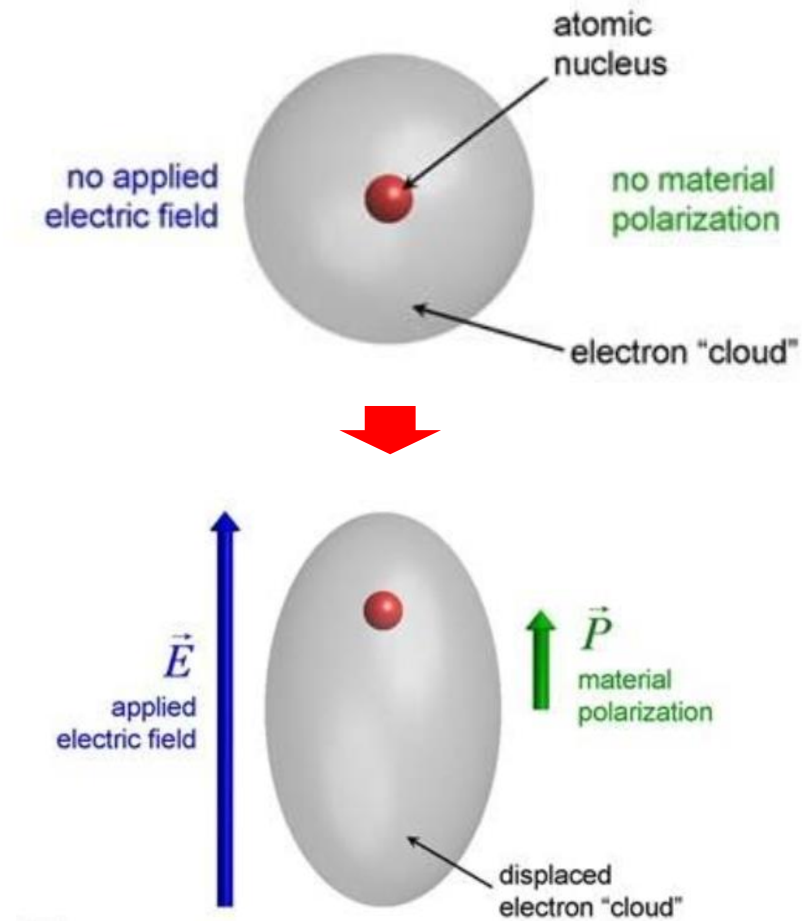


A dispersive material must be absorptive

KYUNGPOOK NATIONAL UNIVERSITY

The Lorentz oscillator model

The atom of certain material consists of nucleus and electron cloud.



External electric field may displace the electron cloud to have a polarization field.

Lorentz oscillator model simplify this phenomenon into the second order differential equation,

$$\frac{d^2\mathbf{P}(t)}{dt^2} + \sigma\frac{d\mathbf{P}(t)}{dt} + \omega_0{}^2\mathbf{P}(t) = \omega_0{}^2\varepsilon_0\chi_0\mathbf{E}(t)$$

Determined by material

$$\chi_0 = \frac{Ne^2}{\varepsilon_0 m_e \omega_0{}^2}$$

https://www.youtube.com/watch?v=MBH5-oHqzp4

The Lorentz oscillator model

$$\frac{d^2\mathbf{P}(t)}{dt^2} + \sigma \frac{d\mathbf{P}(t)}{dt} + \omega_0{}^2\mathbf{P}(t) = \omega_0{}^2\varepsilon_0\chi_0\mathbf{E}(t)$$

Substitute, $\mathbf{P}(t) = \mathbf{P}\exp(j\omega t),\ \mathbf{E}(t) = \mathbf{E}\exp(j\omega t)$

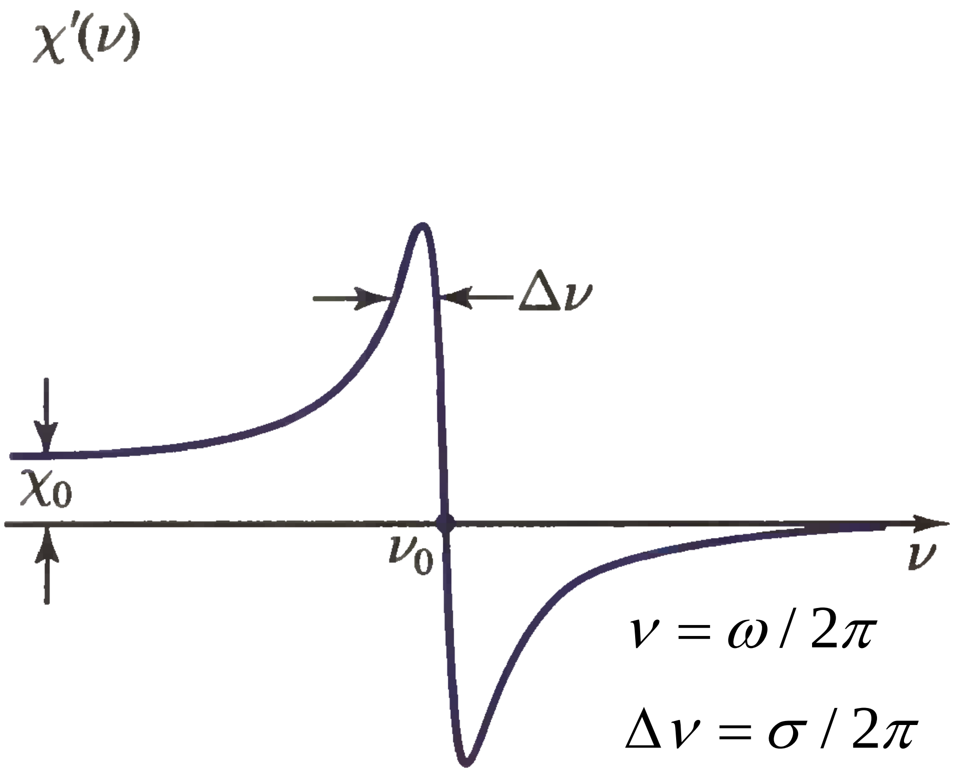
$$\left(-\omega^2 + j\omega\sigma + \omega_0{}^2\right)\mathbf{P} = \omega_0{}^2\varepsilon_0\chi_0\mathbf{E}$$

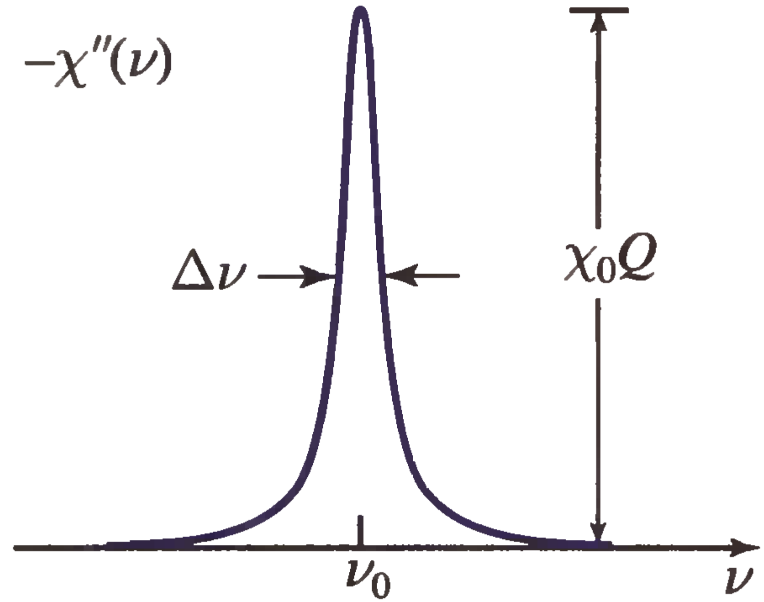
$$\mathbf{P}(\omega) = \varepsilon_0\chi(\omega)\mathbf{E}(\omega) = \varepsilon_0\chi_0 \frac{\omega_0{}^2}{\left(\omega_0{}^2 - \omega^2 + j\omega\sigma\right)}\mathbf{E}(\omega)$$

KNU KYUNGPOOK NATIONAL UNIVERSITY

The Lorentz oscillator model

Dispersion of almost dielectrics can be modeled by sum of Lorentz oscillator models.





$$\nu = \omega / 2\pi$$

$$\Delta\nu = \sigma / 2\pi$$

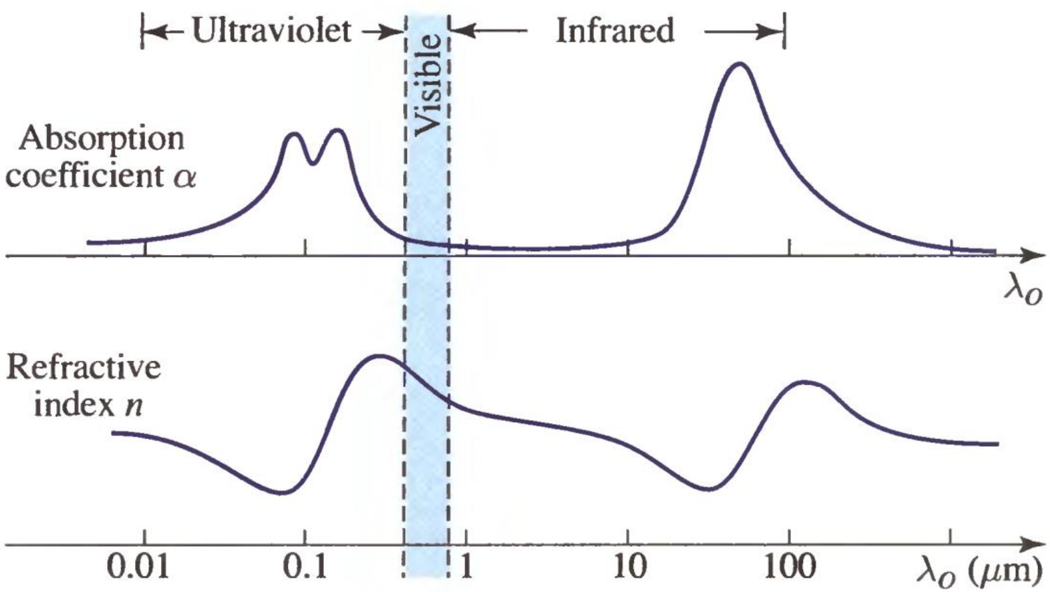
$$Q = \nu_0 / \Delta\nu$$

$$\alpha(\nu) \approx -\left(\frac{2\pi\nu}{n_0 c_o}\right)\chi''(\nu)$$

$$n(\nu) \approx n_0 + \frac{\chi'(\nu)}{2n_0}.$$

The Sellmeier Equation

Materials have various types of resonances at different range of frequencies.



$$n^2 \approx 1 + \sum_i \chi_{0i} \frac{\nu_i^2}{\nu_i^2 - \nu^2} = 1 + \sum_i \chi_{0i} \frac{\lambda^2}{\lambda^2 - \lambda_i^2}.$$

Low loss dielectrics does not have resonances at visible light range.

In usual, highly dispersive material is highly loss.

Can you know how normal & anomalous dispersion defined?

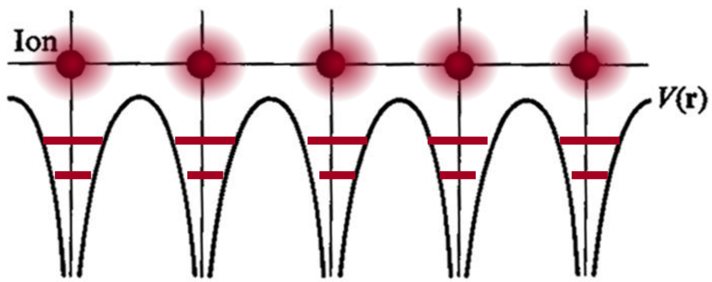
KNU KYUNGPOOK NATIONAL UNIVERSITY

Dispersion in conductive media

In conductive media like metals, electron clouds are not bounded near the nucleus. -> Free electrons instead of bounded electrons

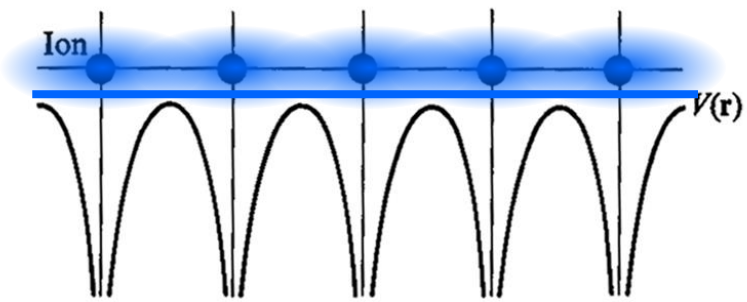
- **Dielectric – Lorentz model (electrons are bound to atom core)**

$$\varepsilon(\omega) = 1 + \frac{\omega_p^2}{\omega_0^2 - \omega^2 - i\Gamma\omega}$$



- **Metal – Drude model (electrons are not bound to atom core; free-electron)**

$$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2 + i\Gamma\omega}$$



KYUNGPOOK NATIONAL UNIVERSITY

The Drude model

In conductive media like metals, electron clouds are not bounded near the nucleus. -> Free electrons instead of bounded electrons

$$\nabla \times \mathbf{H} = j\omega \mathbf{D} + \mathbf{J}, \longleftarrow \quad \mathbf{J} = \sigma \mathbf{E},$$

$$\nabla \times \mathbf{H} = j\omega\boxed{(\varepsilon + \frac{\sigma}{j\omega}}) \mathbf{E}$$

Effective permittivity of metal

For low frequency, J is proportional to E instantaneously, But in optical region, J is time-delayed due to the relaxation time of electron,

$$\sigma = \frac{\sigma_0}{1 + j\omega\tau}$$

KNU KYUNGPOOK NATIONAL UNIVERSITY

The Drude model

For optical frequency,

$$\varepsilon_m = \varepsilon + \frac{\sigma_0}{j\omega(1+j\omega\tau)} \approx \varepsilon_0\left(1-\frac{{\omega_p}^2}{\omega^2}\right)$$

It can also be modeled as removing the restoring (and damping) parameters from Lorentz model

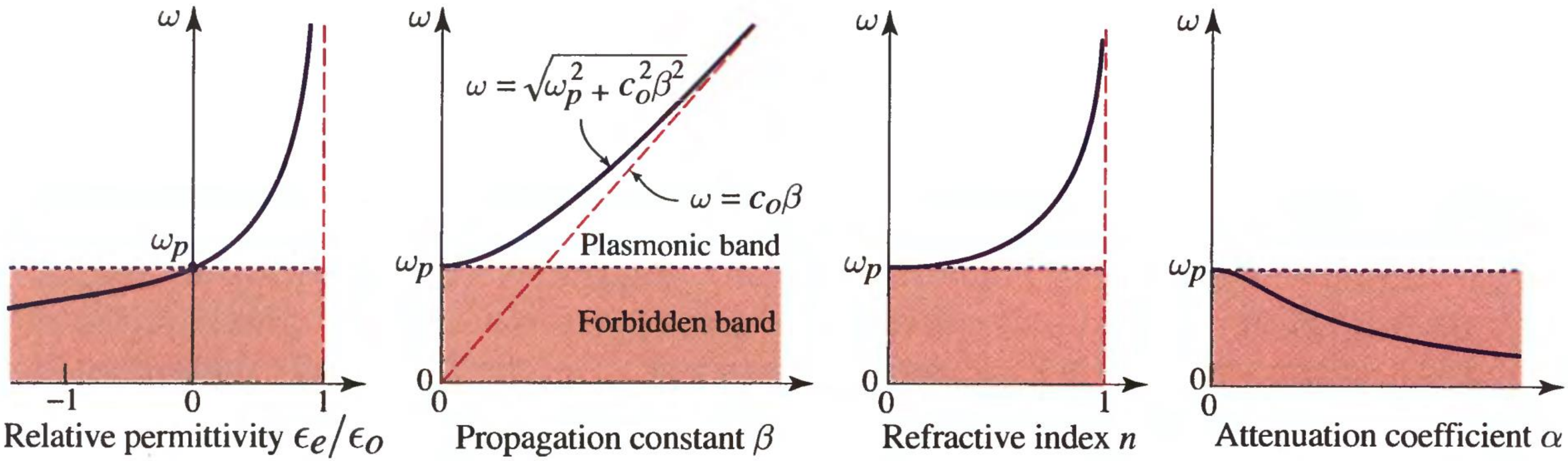
$$\frac{d^2\mathbf{P}(t)}{dt^2} + \sigma\frac{d\mathbf{P}(t)}{dt} + {\omega_0}^2\mathbf{P}(t) = {\omega_0}^2\varepsilon_0\chi_0\mathbf{E}(t)$$

$$\mathbf{P}(\omega) = \varepsilon_0\chi(\omega)\mathbf{E}(\omega) = -\varepsilon_0\frac{{\omega_p}^2}{\omega^2}\mathbf{E}(\omega)$$

Plasma frequency

$$\omega_p = \sqrt{\frac{\sigma_0}{\varepsilon_0\tau}} = \sqrt{\frac{Ne^2}{\varepsilon_0 m_e}}$$

The Drude model



1. Metal reflects light far below the plasma frequency.
2. Metal oscillates with light (and strongly absorb) near the plasma frequency.
3. Extremely short UVs and X-rays can easily pass through the most of metals.
4. In doped semiconductor, plasma frequency is laid on infrared region.