Spring 2019

EECE 588
Lecture 4

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Polarization

- Generally, we design our antennas to radiate the polarization we desire.
- However, life is not perfect. Therefore, your antenna inevitably radiates polarizations other than what you desire.
  - e.g., you design it to radiate vertical linear polarization and you get a little bit of horizontal as well.
- Now, we define the terms
  - Co-Pol component: the component of polarization that we desire.
  - Cross-Pol or X-Pol component: the component of the polarization that is orthogonal to the Co-Pol and hence undesired.
  - For each type of polarization, the two different versions are orthogonal to each other (e.g., horizontal and vertical, RHCP and LHCP, etc.).

Polarization

- Let's consider a plane wave traveling in -z direction (of course we assume the $e^{j\omega t}$ time dependence)

$$\vec{E}(z,t) = \hat{x}E_x(z,t) + \hat{y}E_y(z,t)$$

- We have:

$$E_x(z,t) = \mathrm{Re}\left[E_x^- e^{j(\omega t + kz)}\right] = \mathrm{Re}\left[E_{x0} e^{j(\omega t + kz + \varphi_x)}\right] = E_{x0}\cos(\omega t + kz + \varphi_x)$$

$$E_y(z,t) = \mathrm{Re}\left[E_y^- e^{j(\omega t + kz)}\right] = \mathrm{Re}\left[E_{y0} e^{j(\omega t + kz + \varphi_y)}\right] = E_{y0}\cos(\omega t + kz + \varphi_y)$$

- E_{x0} and E_{y0} are the magnitude of the x and y components.

Polarization

- To have linear polarization, we must have:

$$\Delta\varphi = \varphi_y - \varphi_x = n\pi, \quad n = 1, 2, 3, 4, \ldots$$

- Circular polarization:
  - We must have the same magnitudes and 90° phase shift between the two components

$$|E_x| = |E_y| \Rightarrow E_{x0} = E_{y0}$$

$$\Delta\varphi = \varphi_y - \varphi_x = \begin{cases} (\frac{1}{2} + 2n)\pi & \text{n} = 0, 1, 2, \ldots \quad \text{RHCP} \\ -(\frac{1}{2} + 2n)\pi & \text{n} = 0, 1, 2, \ldots \quad \text{LHCP} \end{cases}$$

Polarization

- Elliptical polarization can be obtained when:
  - The magnitudes of the two components are not the same and $\Delta\varphi = 90°$.
  - The phase difference is not 90° (also, $\Delta\varphi \neq 0$, π, 2π, etc.) irrespective of the magnitudes

$$E_{x0} \neq E_{y0} \textbf{ and } \Delta\varphi = \varphi_y - \varphi_x = \begin{cases} +\left(\dfrac{\pi}{2} + 2n\pi\right) & \textbf{CW} \\ -\left(\dfrac{\pi}{2} + 2n\pi\right) & \textbf{CCW} \end{cases}$$

$$E_{x0} \text{ and } E_{y0} \text{ arbitrary and } \Delta\varphi = \varphi_y - \varphi_x \neq \pm n\frac{\pi}{2} = \begin{cases} > 0 & \text{CW} \\ < 0 & \text{CCW} \end{cases}$$

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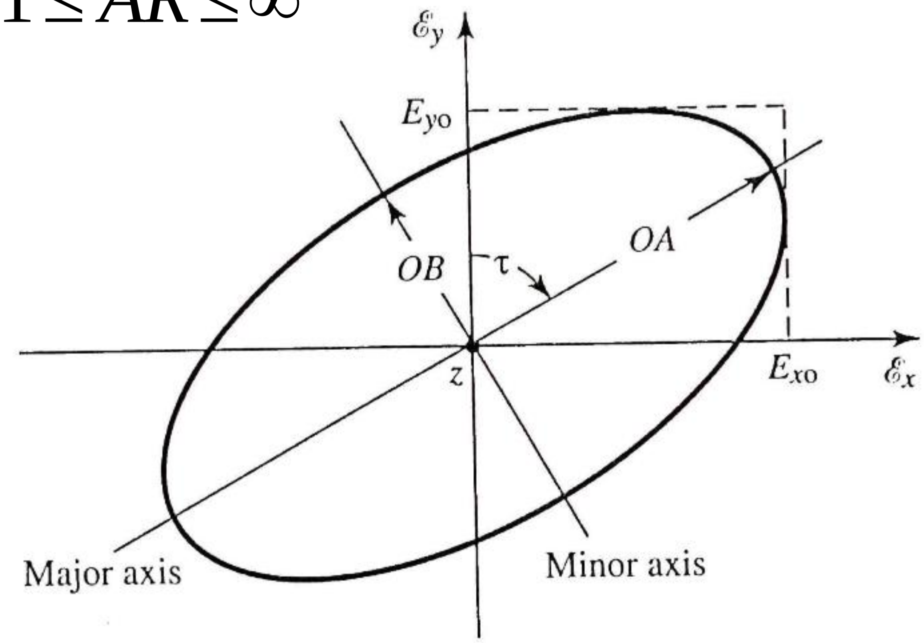
Polarization

- For elliptical polarization, the curve traced by the tip of the E vector for a fixed location is a tilted ellipse.
- We define Axial Ratio as

$$\mathrm{AR} = \frac{\text{major axis}}{\text{minor axis}} = \frac{OA}{OB}, 1 \le AR \le \infty$$



Polarization

- Using basic geometry:

$$OA = \frac{1}{\sqrt{2}}\left[E_{x0}^2 + E_{y0}^2 + \sqrt{E_{x0}^4 + E_{y0}^4 + 2E_{x0}^2 E_{y0}^2 \cos(2\Delta\varphi)}\right]^{1/2}$$

$$OB = \frac{1}{\sqrt{2}}\left[E_{x0}^2 + E_{y0}^2 - \sqrt{E_{x0}^4 + E_{y0}^4 + 2E_{x0}^2 E_{y0}^2 \cos(2\Delta\varphi)}\right]^{1/2}$$

- The tilt of the ellipse relative to the y axis is represented by:

$$\tau = \frac{\pi}{2} - \frac{1}{2}\tan^{-1}\left[\frac{2E_{x0}E_{y0}}{E_{x0}^2 - E_{y0}^2}\cos(\Delta\varphi)\right]$$

Polarization Loss Factor and Efficiency

- For a receiving antenna, if the antenna polarization and the polarization of the incident EM wave are not the same, we will have polarization mismatch.
- This will result in receiving less than the maximum possible power from the incoming wave.
- Let's assume that the E field of the incident field and that of the antenna are:

$$\vec{E}_i = \hat{\rho}_w E_i \qquad \vec{E}_a = \hat{\rho}_a E_a$$

- We define the Polarization Loss Factor (PLF) as:

$$PLF = \left|\hat{\rho}_w . \hat{\rho}_a\right|^2 = \left|\cos\psi\right|^2$$

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Polarization Loss Factor and Efficiency

- Antenna and the EM wave polarization matched, $PLF = 1$.
- Antenna and the EM wave polarization orthogonal, $PLF = 0$ and no power will be received by antenna.
- We can also define polarization efficiency as:
  - The ratio of the power received by an antenna from a given plane wave of arbitrary polarization to the power that would be received by the same antenna from a plane wave of the same power flux density and direction of propagation whose state of polarization has been adjusted for a maximum received power.

$$p_e = \frac{\left|\vec{\ell}_e \cdot \vec{E}_{inc}\right|^2}{\left|\vec{\ell}_e\right|^2 \cdot \left|\vec{E}_{inc}\right|^2}$$

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Example 2.11:

$$\underline{E}_w^i = \hat{a}_x E_0(x, y) e^{-jkz}$$

$$\underline{E}_a \simeq (\hat{a}_x + \hat{a}_y) E(r, \theta, \phi)$$

Solution:

$$\underline{E}_a = (\hat{a}_x + \hat{a}_y) E(r,\theta,\phi) = \left(\frac{\hat{a}_x + \hat{a}_y}{\sqrt{2}}\right)\sqrt{2}E(r,\theta,\phi)$$

$$\hat{\rho}_a = \frac{\hat{a}_x + \hat{a}_y}{\sqrt{2}}, \qquad \hat{\rho}_w = \hat{a}_x$$

$$PLF = |\hat{\rho}_w \cdot \hat{\rho}_a|^2 = \left|\hat{a}_x \cdot \left(\frac{\hat{a}_x + \hat{a}_y}{\sqrt{2}}\right)\right|^2 = \frac{1}{2}$$

$$PLF = \frac{1}{2} = 10\log_{10}\left(\frac{1}{2}\right) = -3 \text{ dB}$$

Example 2.12:

$$\underline{E}_a = \left(\hat{a}_\theta - j\hat{a}_\phi\right) E(r,\theta,\phi)$$

Solution:

$$\underline{E}_a = \left(\hat{a}_\theta - j\hat{a}_\phi\right) E(r,\theta,\phi)$$

$$= \frac{\left(\hat{a}_\theta - j\hat{a}_\phi\right)}{\sqrt{2}} \sqrt{2} E(r,\theta,\phi)$$

$$\underline{E}_a = \hat{\rho}_a \sqrt{2} E(r,\theta,\phi), \quad \hat{\rho}_a = \frac{\hat{a}_\theta - j\hat{a}_\phi}{\sqrt{2}}$$

CW-CW (Maximum)

$$\text{CW:} \quad \hat{\rho}_a = \left(\frac{\hat{a}_\theta - j\hat{a}_\phi}{\sqrt{2}} \right), \quad \text{CW:} \quad \hat{\rho}_w = \left(\frac{\hat{a}_\theta + j\hat{a}_\phi}{\sqrt{2}} \right)$$

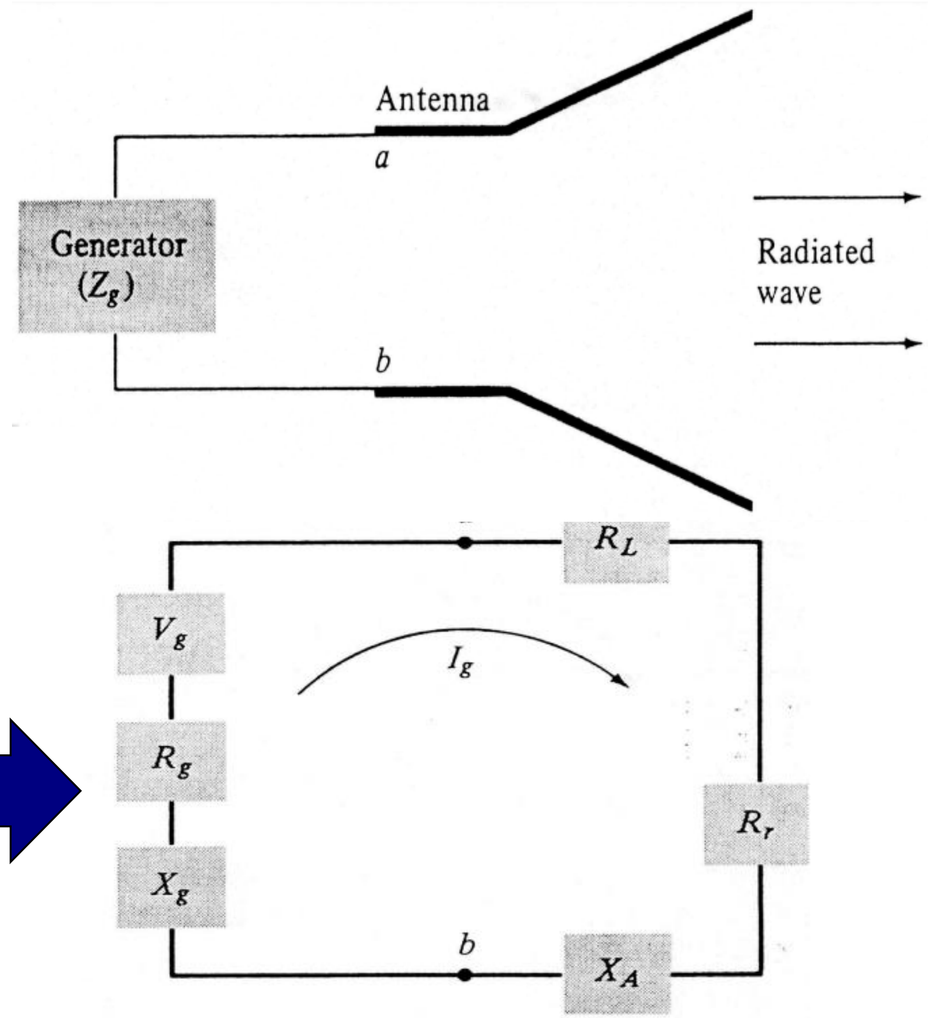
$$PLF = \left| \hat{\rho}_a \cdot \hat{\rho}_w \right|^2 = \left| \left(\frac{\hat{a}_\theta - j\hat{a}_\phi}{\sqrt{2}} \right) \left(\frac{\hat{a}_\theta + j\hat{a}_\phi}{\sqrt{2}} \right) \right|^2$$

$$PLF = \left| \frac{1+1}{2} \right|^2 = 1 = 0 \ \text{dB}$$

INPUT IMPEDANCE

- Input Impedance:
  - The impedance presented by an antenna at its terminals.
  - Ratio of the voltage to current at the input terminals of the antenna.
  - Ratio of the appropriate components of the electric to magnetic fields at the input terminals of the antenna

Thevenin Equivalent Circuit



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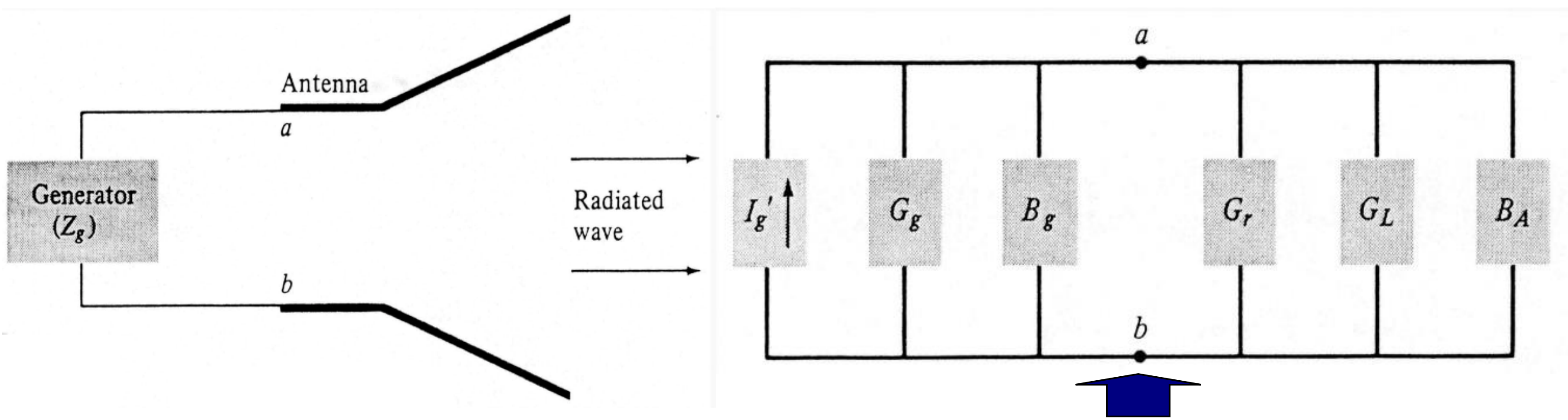
Input Impedance

- The ratio of voltage (V) to current (I) at the input terminals of the antenna ($a - b$) is the input impedance of the antenna.

$$Z_A = R_A + jX_A$$



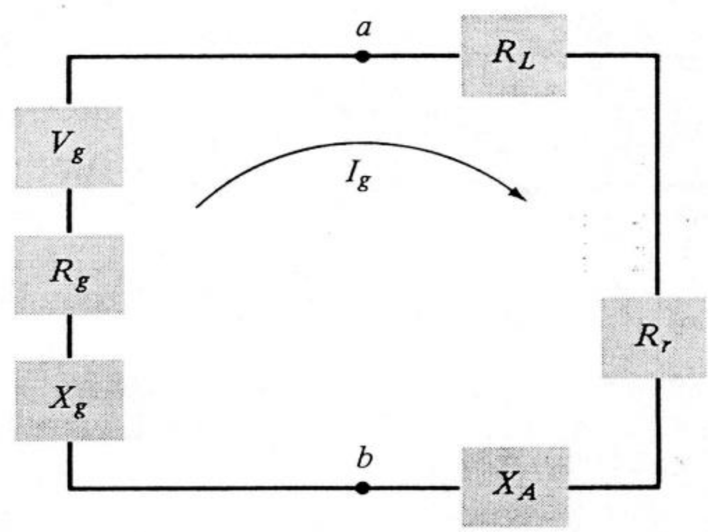
Norton Equivalent Circuit

Input Impedance

- Z_A =Antenna Input Impedance at terminals $a - b$ (in Ohms)
- R_A =Antenna Input Resistance at terminals $a - b$ (in Ohms)
- X_A =Antenna Input Reactance at terminals $a - b$ (in Ohms)
- Note that $R_A = R_r + RL$
  - R_r =Radiation resistance of the antenna.
  - R_L =Loss resistance of the antenna.
- When the antenna is used in the transmitting mode, generally the antenna is connected to some sort of a generator that can be represented with either its Thevenin or Norton equivalent circuits.

Input Impedance

- Using simple analysis, one can calculate



$$I_g = \frac{V_g}{Z_t} = \frac{V_g}{(R_r + R_L + R_g) + j(X_A + X_g)}$$

$$|I_g| = \left|\frac{V_g}{Z_t}\right| = \frac{|V_g|}{\sqrt{(R_r + R_L + R_g)^2 + (X_A + X_g)^2}}$$

$$P_r = \frac{1}{2} R_r |I_g|^2 = \frac{|V_g|^2}{2}\left[\frac{R_r}{(R_r + R_L + R_g)^2 + (X_A + X_g)^2}\right]$$

$$P_L = \frac{1}{2} R_L |I_g|^2 = \frac{|V_g|^2}{2}\left[\frac{R_L}{(R_r + R_L + R_g)^2 + (X_A + X_g)^2}\right]$$

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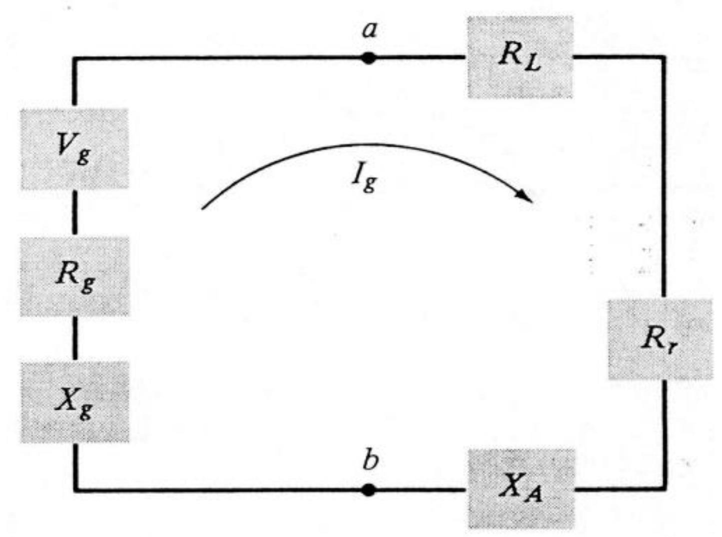
Input Impedance

$$P_g = \frac{|V_g|^2}{2}\left[\frac{R_g}{(R_r + R_L + R_g)^2 + (X_A + X_g)^2}\right]$$



- To achieve maximum power transfer, we must then have:
  - $R_r + RL = R_g$
  - $X_A = -X_g$
- Max. power transfer results in:

$$P_r = \frac{|V_g|^2}{2}\left[\frac{R_r}{4(R_r + R_L)^2}\right] = \frac{|V_g|^2}{8}\left[\frac{R_r}{(R_r + R_L)^2}\right]$$

$$P_L = \frac{|V_g|^2}{8}\left[\frac{R_L}{(R_r + R_L)^2}\right]$$

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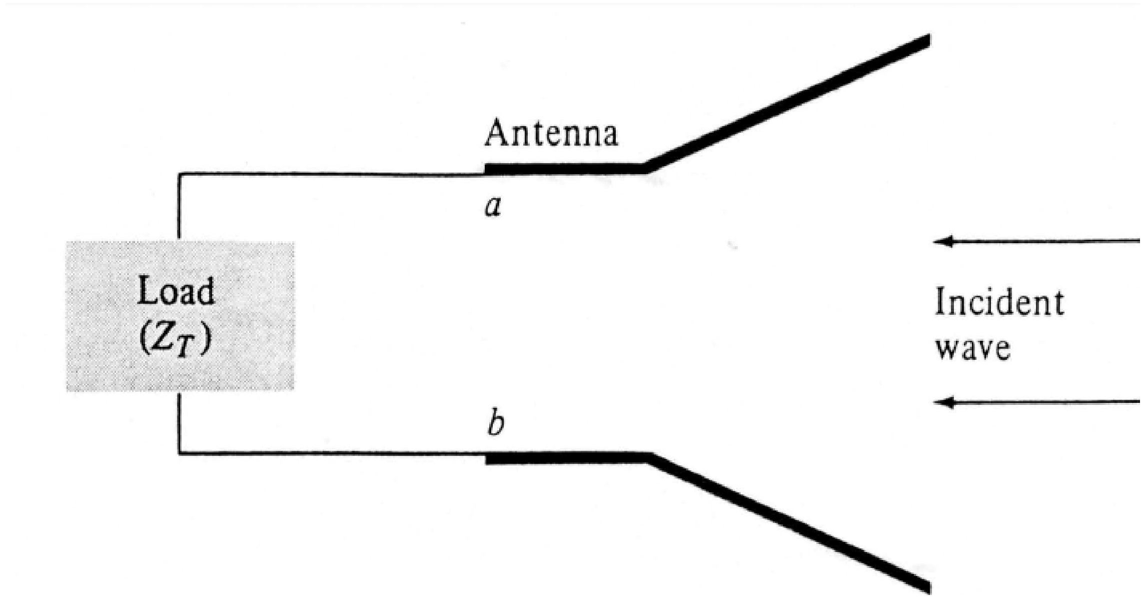
Antenna in Transmitting Mode: Summary

- Basically what we have learnt so far is as follows:
  - Under conjugate matching, a maximum of ½ of the power available from the source is transferred to the antenna.
    - NO SURPRISE HERE. We have seen this before.
    - Under receiving mode, however this has interesting physical interpretations.
  - The rest of this power is absorbed in the generator resistance.
  - Note that the generator represents your actual generator + all the feeding and matching circuits, etc. that you have.
  - Some part of the power transferred to the antenna is radiated (the power dissipated in Rr) and other part of it is dissipated as heat absorbed in R_L.

Equivalent Circuit of The Antenna Under Receiving Mode

- In the receiving mode the incident EM wave induces a voltage V_T at the terminals of the antenna.



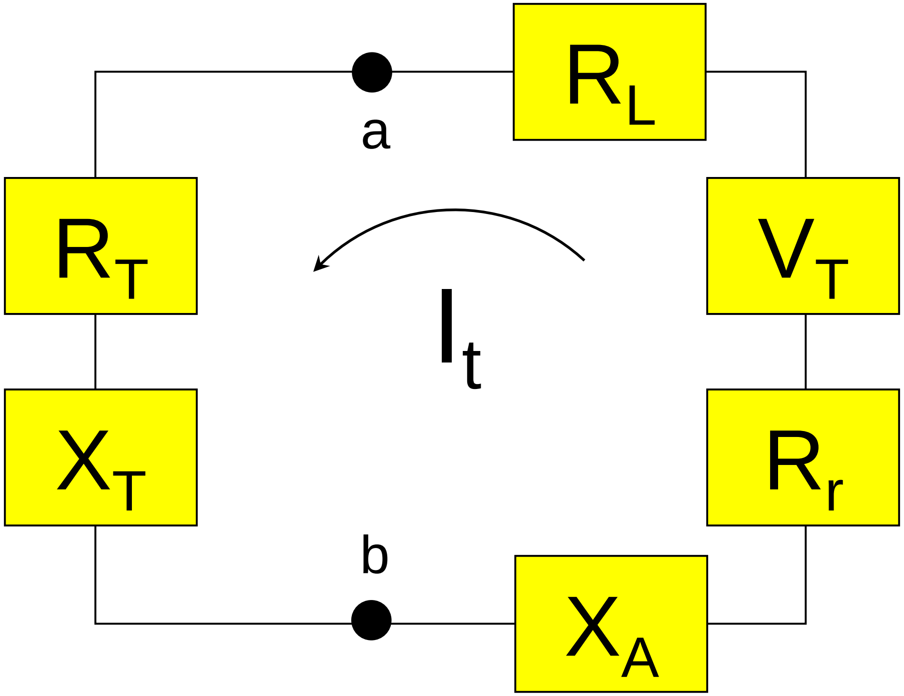
Equivalent Circuit of the Receiving Antenna

- Under complex conjugate matching conditions:

$$P_T = \frac{|V_T|^2}{8}\left[\frac{R_T}{(R_T + R_L)^2}\right] = \frac{|V_T|^2}{8R_T}$$

$$P_r = \frac{|V_T|^2}{8}\left[\frac{R_r}{(R_r + R_L)^2}\right]$$

$$P_L = \frac{|V_T|^2}{8}\left[\frac{R_L}{(R_T + R_L)^2}\right]$$



Collected Power $$P_c = \frac{1}{2}V_T I_T^* = \frac{1}{2}V_T\left[\frac{V_T^*}{2(R_r + R_L)}\right] = \frac{|V_T|^2}{4}\left(\frac{1}{R_r + R_L}\right)$$

Equivalent Circuit of the Receiving Antenna

- $\mathrm{P_r}$ delivered to R_r is called scattered power.
- Under conjugate matching condition:
  - Half of the collected power is transferred to the load.
  - The remaining half is reradiated through R_r or lost in R_L.
  - If $R_L = 0$, then of all the power collected by the antenna, only half of it will be delivered to the load and the other half will be re-radiated by the antenna.
  - This is very interesting because even though you believe that your antenna is a receiver antenna it will reradiate half of its received power.

Antenna Radiation Efficiency

- The conduction and dielectric losses of an antenna are very difficult to compute.
- In most cases, these losses are measured or calculated numerically using numerical Electromagnetic Simulations
- At any rate and even with measurements, the losses are not easy to separate.
  - i.e., we can figure out how much loss is there but not easy to attribute how much of that is due to metal and how much of it is due to dielectric losses.

$$e_{cd} = \left[\frac{R_r}{R_r + R_L} \right]$$

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Antenna Radiation Efficiency

- The DC resistance of a metal rod is calculated as:

$$R_{dc} = \frac{\ell}{\sigma A} \qquad (\Omega)$$

- Recall from undegraduate EM that the skin depth can be calculated using:

$$\delta = \sqrt{2/\omega\mu_0\sigma}$$

- If the skin depth is significantly smaller than the smallest diagonal cross section of the rod, the current is confined to a thin layer near the conductor surface.
- One can define a high-frequency resistance {based on uniform current distribution}

$$R_{hf} = \frac{\ell}{P} R_s = \frac{\ell}{P}\sqrt{\frac{\omega\mu_0}{2\sigma}} \qquad (\Omega)$$

Antenna Vector Effective Length and Equivalent Areas

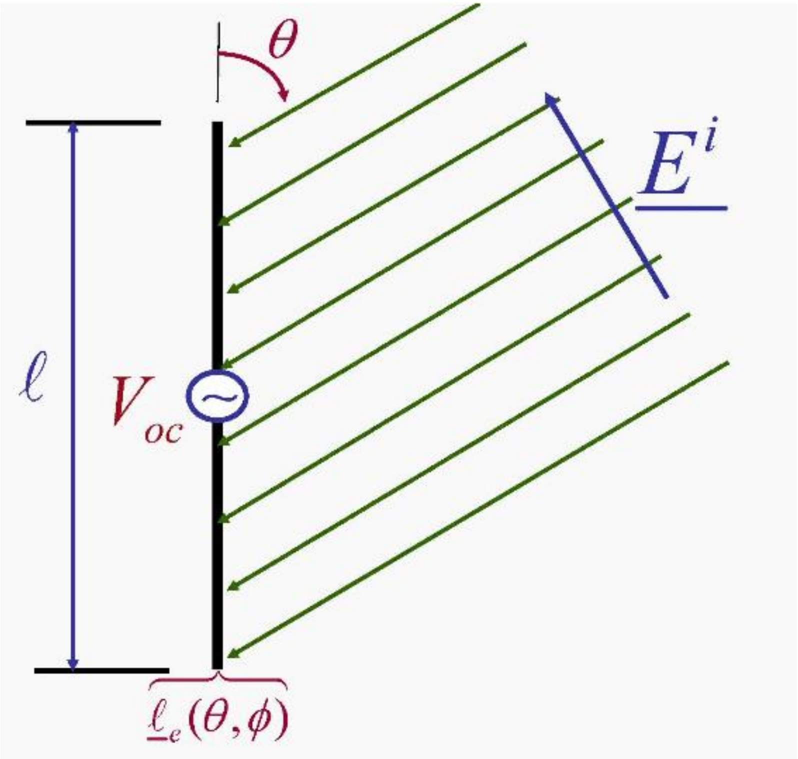
- Receiving antennas are used to collect electromagnetic waves and to extract power from them.
  - It does not matter what type of antenna we are talking about: They can be in the form of a wire, horn, apertures, arrays, dielectric rods, etc.
- To quantify the antennas' capabilities in collecting power from impinging EM waves we define:
  - Vector Effective Length.
  - Various Effective Areas.
- These quantities are used to describe the receiving characteristics of antennas.
  - These quantities are specially useful in system level analysis of antennas (e.g., in link budget calculations).

Vector Effective Length

- The vector effective length of an antenna is a quantity used to determine the voltage induced on the open-circuited terminals of the antenna when a wave impinges upon it.
- The vector effective length of an antenna is usually a complex vector quantity:

$$\bar{\ell}_e(\theta,\varphi)=\ell_\theta(\theta,\varphi)\hat{\theta}+\ell_\varphi(\theta,\varphi)\hat{\varphi}$$

- Note that effective length is sometimes referred to as effective height.



Vector Effective Length

- The vector effective length of antenna is a far field quantity.
- It is related to the far field $\overrightarrow{E_a}$ radiated by the antenna with current I_{in} in its terminals.

$$\vec{E}_a = \hat{\theta}E_\theta + \hat{\varphi}E_\varphi = -j\eta \frac{kI_{in}}{4\pi r}\vec{\ell}_e e^{-jkr}$$

- The effective length represents the antenna in its transmitting and receiving modes.
- This is particularly useful in relating the open circuit voltage V_{oc} of receiving antennas:

$$V_{oc} = \vec{\ell}_e . \vec{E}_i$$

Vector Effective Length

- Effective length of a linearly polarized antenna receiving a plane wave in a given direction is defined as:
  - The ratio of the magnitude of the open-circuit voltage developed at the terminals of the antenna to the magnitude of the electric field strength in the direction of the antenna polarization.
- One can also think of the equivalent length of the antenna as:
  - Consider a thin straight conductor.
  - Now, orient this conductor perpendicular to the direction of the incoming EM wave and parallel to the antenna polarization.
  - Assume that this conductor has a uniform current distribution with a magnitude equal to that at the antenna terminals and producing the same far field strength as the antenna in that direction.
  - The length of this conductor would be the vector effective length of the antenna.

Antenna Equivalent Areas

- For an antenna we can define a number of equivalent areas.
- These equivalent areas are used to describe the power capturing capability of the antenna when a wave impinges upon it.
- The most important of these is the “Effective Area” or “Effective Aperture”
  - The ratio of the available power at the terminals of a receiving antenna to the power flux density of a plane wave incident on the antenna from that direction (the wave is polarization matched to the antenna).
  - If the direction is not specified, we automatically assume the direction of maximum radiation.

Effective Area

- This is written in equation form as:

$$A_e = \frac{P_T}{W_i} = \frac{|I_T|^2 R_T / 2}{W_i}$$

  - A_e =Effective area (m^2).
  - P_T =Power delivered to the load (W).
  - W_i =Power density of incident wave (W/m^2)
- The effective aperture is the area which when multiplied by the incident power density gives the power delivered to the load.

Equivalent Circuit of the Receiving Antenna

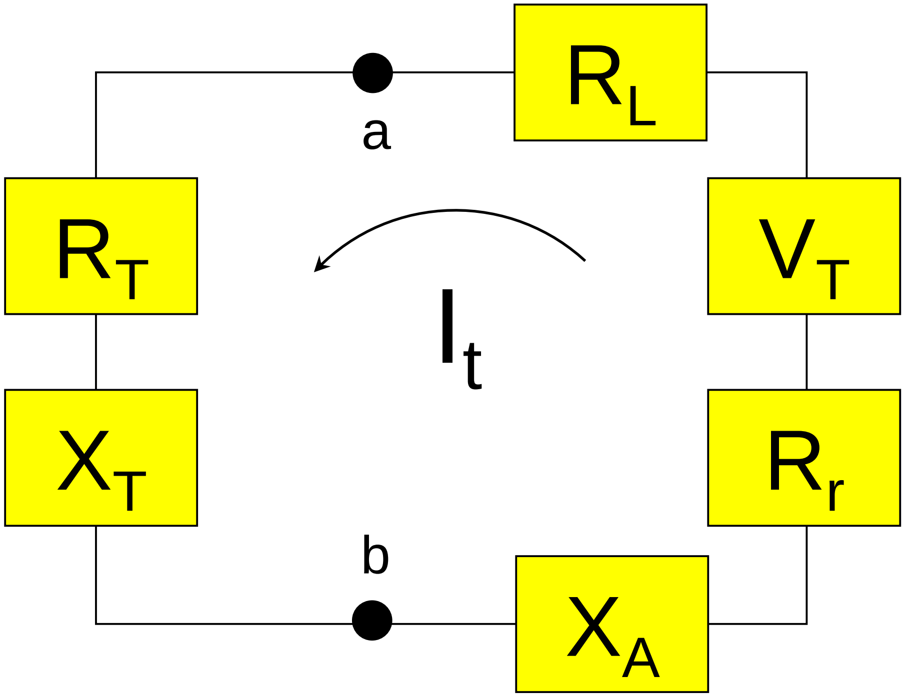
- Using this equivalent circuit, we can write:

$$A_e = \frac{|V_T|^2}{2W_i}\left[\frac{R_T}{(R_r + R_L + R_T)^2 + (X_A + X_T)^2}\right]$$

- Conjugate matching, $R_r + RL = RT$ and $X_A = -XT$

$$A_{em} = \frac{|V_T|^2}{8W_i}\left[\frac{R_T}{(R_L + R_r)^2}\right] = \frac{|V_T|^2}{8W_i}\left[\frac{1}{R_L + R_r}\right]$$

- All of the power that is intercepted by the antenna will not be delivered to the load.



Scattering Area

- Note that under conjugate matching, only half of the intercepted power by the antenna is transferred to the load.
- The other half of the power is lost to Ohmic losses or scattered.
- To account for these power values, we can define scattering, loss and capture areas.
- Scattering Area:
  - The equivalent area, which when multiplied by the incident power density gives the total scattered to reradiated power.
  - Under complex conjugate matching condition, we have:

$$A_s = \frac{|V_T|^2}{8W_i}\left[\frac{R_r}{(R_L + R_r)^2}\right]$$

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