전자기학/초고주파 분야

박사과정 자격시험 (2017.5.26)

1. **(20점)** A spherical charge distribution is defined by

$$\rho_V = \begin{cases} kr^2 & (C/m^3) \quad r<a \\ 0 & (C/m^3) \quad r>a \end{cases}$$

Where k is a constant. Using Gauss's law, determine the electric field for

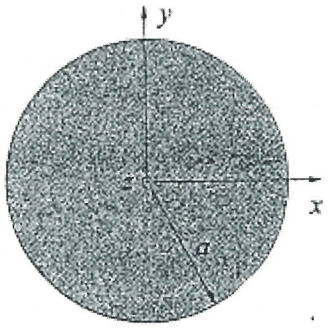
(a) r <a

(b) r > a

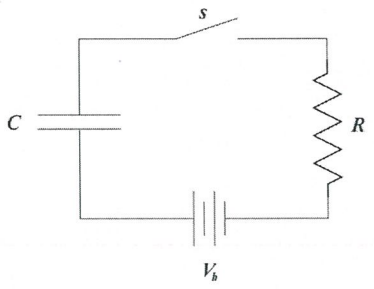
2. **(15점)** The cylindrical conductor shown in the Figure below carries a current density of $J = J_o \rho a_z$ A/m^3 where J_o is a constant. Use Ampere's law to determine

   (a) the vector magnetic field inside the conductor (ρ<a).

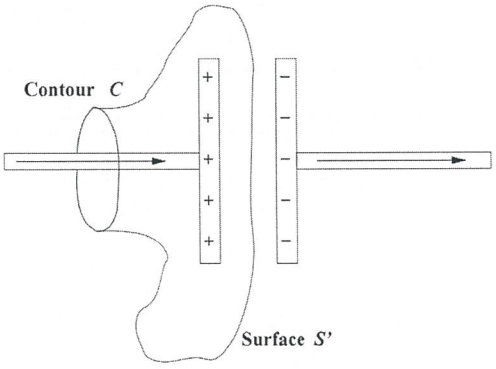
   (b) the vector magnetic field outside the conductor (ρ>a).



3. **(15점)** In linear circuits, we have learned that the same current I(t) flows through all the circuit elements. However we know this cannot be totally true – there is no way "real" current can flow across the plates of a capacitor!



Explain using plain words or mathematical expressions to justify how "real" current can flow through a capacitor.



4. (20 pts) A uniform plane wave with $\vec{E}=\hat{x}E_x$ propagates in a lossless simple medium $(\epsilon_r=4,\ \mu_r=1,\ \sigma=0)$ in the $+z$ direction. Assume that E_x is a sinusoidal with a frequency 100 (MHz) and a maximum value of 10^{-4} (V/m) at $t=0$ and $z=1/6$ (m). (Use π instead of 3.141592...)

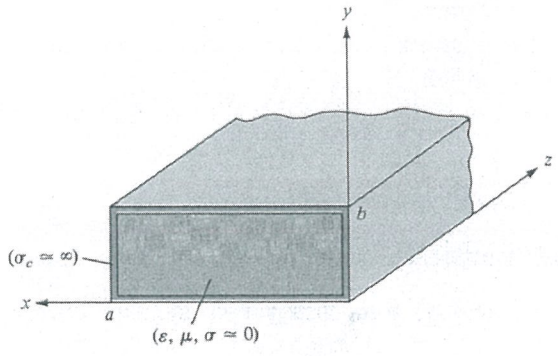
a) (5 pts) Write the instantaneous expression for $\vec{E}(z,t)$.

b) (5 pts) Write the instantaneous expression for $\vec{H}(z,t)$.

c) (10 pts) Determine the locations where E_x is a positive maximum when $t=10^{-8}$ (sec).

5. (10pts) Assume an infinitely long waveguide filled with an internal lossless medium (ϵ,μ), and its internal time-harmonic field propagates in $+z$ direction with propagation constant γ. Use time-harmonic Maxwell's equations in Cartesian coordinates in order to derive $E_x(x,y)$, $E_y(x,y)$, $H_x(x,y)$, $H_y(x,y)$ expressions in terms of E_z, H_z and $h^2=\gamma^2+k^2$, where $k=\omega\sqrt{\mu\epsilon}$ is a wavenumber in the waveguide.

6. [20pts] Consider the following rectangular waveguide for $a>b$. (Use the results in prob. 5)



(a) [10pts] Derive $H_z(x,y)$ for TE mode by solving the homogeneous vector Helmholtz equation and boundary conditions.

(b) [10pts] Derive the wave impedance Z_{TE} of the propagating TE modes in terms of cutoff frequency f_c and η, where η is the intrinsic impedance of the unbounded medium (ϵ,μ).