2009 수학

1. $U(s)=\frac{1}{s+2}$ $\quad Y(s)=U(s)H(s)=\frac{1}{(s+2)^3(s+8)}=\frac{A}{s+2}+\frac{B}{(s+2)^2}+\frac{C}{(s+2)^3}+\frac{D}{s+8}$

$A(s+2)^2(s+8)+B(s+2)(s+8)+C(s+8)+D(s+2)^3$

$=A(s^2+4s+4)(s+8)+B(s^2+10s+16)+Cs+8C+D(s^3+2s^2+4s+8)$

$=As^3+8As^2+4As^2+32As+4As+32A+Bs^2+10Bs+16B+Cs+8C+Ds^3+2Ds^2+4Ds+8D$

$=s^3(A+D)+s^2(12A+B+2D)+s(36A+10B+C+4D)+(32A+16B+8C+8D)=1.$

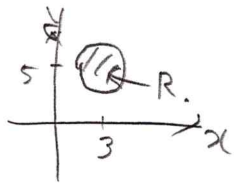
$$\left(\begin{array}{l} A+D=0 \\ 12A+B+2D=0 \\ 36A+10B+C+4D=0 \\ 32A+16B+8C+8D=1 \end{array}\right) \qquad \begin{array}{rl} & 288A+80B+8C+32D=0 \\ - & 32A+16B+8C+8D=1 \\ \hline & 256A+64B+24D=-1 \end{array}$$

귀찮.

$\therefore A=\frac{1}{216},\ B=-\frac{1}{36},\ C=\frac{1}{6},\ D=-\frac{1}{216}$

$\therefore y(t)=\left(\frac{1}{216}e^{-2t}-\frac{1}{36}te^{-2t}+\frac{1}{12}t^2e^{-2t}-\frac{1}{216}e^{-8t}\right)u_s(t)$

2. $\oint_C L\,dx+M\,dy=\iint_R \frac{\partial M}{\partial x}-\frac{\partial L}{\partial y}\,dxdy,\quad L=\frac{y}{x^2+y^2},\quad M=\frac{-x}{x^2+y^2},$



$\iint_R \frac{-(x^2+y^2)+2x^2}{(x^2+y^2)^2}-\frac{(x^2+y^2)-2y^2}{(x^2+y^2)^2}\,dxdy=\iint_R 0\,dxdy=\underline{0}.$

4. $\lim_{x\to\infty}\sqrt{x}(\sqrt{x+1}-\sqrt{x})=\lim_{x\to\infty}\frac{\sqrt{x}}{\sqrt{x+1}+\sqrt{x}}=\lim_{x\to\infty}\frac{1}{\sqrt{1+\frac{1}{x}}+1}=\underline{\frac{1}{2}}.$

5. Let $x=2\tan\theta$, $dx=2\sec^2\theta\,d\theta$ $\quad \int\frac{1}{x^2\sqrt{x^2+4}}dx=\int\frac{2\sec^2\theta}{4\tan^2\theta\cdot 2\sec\theta}d\theta=\int\frac{\sec\theta}{4\tan^2\theta}d\theta$

$=\int\frac{\frac{1}{\cos\theta}}{4\frac{\sin^2\theta}{\cos^2\theta}}d\theta=\frac{1}{4}\int\frac{\cos\theta}{\sin^2\theta}d\theta$, Let $\sin\theta=t$, $\cos\theta\,d\theta=dt$.

$\therefore \frac{1}{4}\int\frac{1}{t^2}dt=-\frac{1}{4t}+C=-\frac{1}{4\sin\theta}+C$, $x=\frac{2\sin\theta}{\sqrt{1-\sin^2\theta}}$ 이므로

$x^2-x^2\sin^2\theta=4\sin^2\theta$, $\sin^2\theta(x^2+4)=x^2$, $\sin\theta=\pm\sqrt{\frac{x^2}{x^2+4}}$

$\therefore$ cf: $\underline{\pm\frac{\sqrt{x^2+4}}{4x}+C}$

6. T T T T F F T T F(?) F F F T T F

7. (a) $|\lambda I - A| = \begin{vmatrix} \lambda-4 & -2 \\ 3 & \lambda+1 \end{vmatrix} = \lambda^2 - 3\lambda + 2 = 0 \Rightarrow \lambda_1 = 1 \quad \lambda_2 = 2.$

① $\lambda_1 = 1$ case

$\begin{bmatrix} -3 & -2 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow 3x_1 + 2x_2 = 0 \quad \therefore \underline{x_1} = \begin{bmatrix} 1 \\ -\frac{3}{2} \end{bmatrix}$

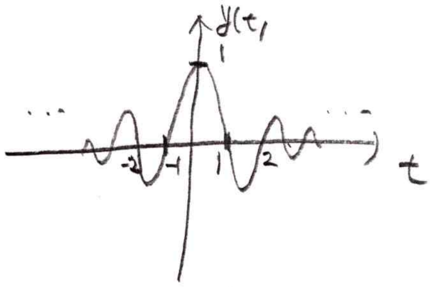
② $\lambda_2 = 2$ case

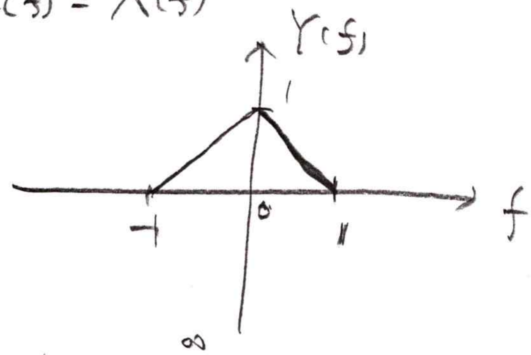
$\begin{bmatrix} -2 & -2 \\ 3 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow x_1 + x_2 = 0 \quad \therefore \underline{x_2} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

(b)

8. $x(t) = \text{sinc}\, t$

(a) $y(t) = \text{sinc}^2 t \longleftrightarrow Y(f) = \Lambda(f)$



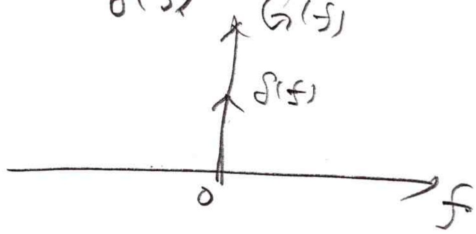


(b). $p(t) = \sum_{k=-\infty}^{\infty} \delta(t-k) \longleftrightarrow P(f) = \sum_{n=-\infty}^{\infty} \delta(f-n)$

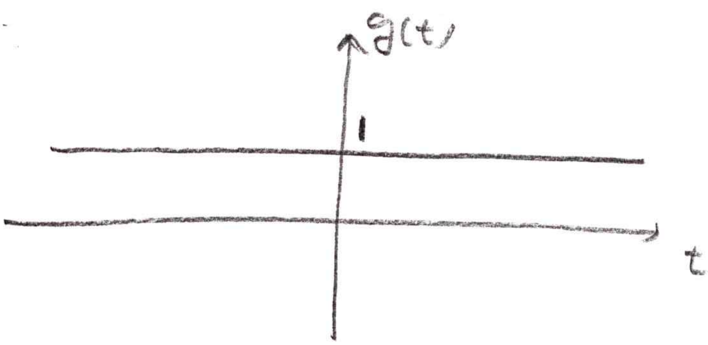
$g(t) = \sum_{k=-\infty}^{\infty} y(t-k) \longleftrightarrow G(f) = Y(f) \sum_{n=-\infty}^{\infty} \delta(f-n) = \sum_{n=-\infty}^{\infty} Y(f-n)$

$\quad\; = y(t) * \sum_{k=-\infty}^{\infty} \delta(t-k)$

$= \delta(f)$



$\therefore\; g(t) = \mathcal{F}^{-1}\{\delta(f)\} = 1 \;,\; \forall t$

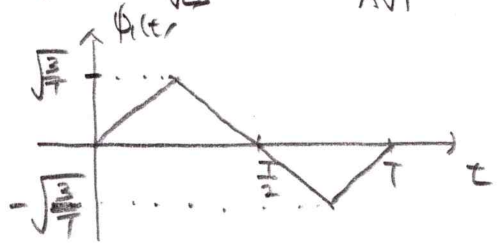


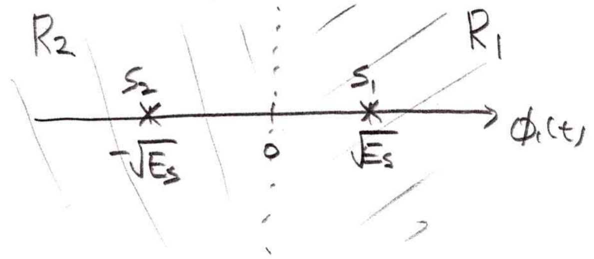
2009 통신

1.

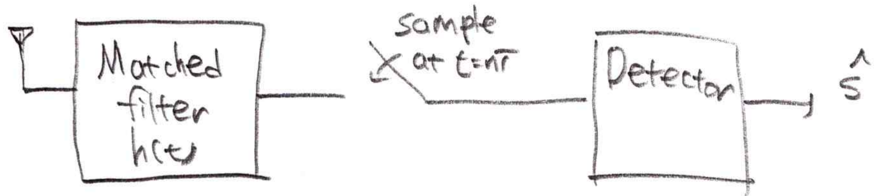
(a) $\int_0^T |s_1(t)|^2 dt = 4\int_0^{\frac{T}{4}} \frac{16A^2}{T^2} t^2 dt = \frac{64A^2}{T^2}\cdot\frac{1}{3}\cdot\frac{T^3}{64} = \frac{1}{3}A^2T = E_s.$

$\phi_1(t) = \frac{1}{\sqrt{E_s}} s_1(t) = \frac{\sqrt{3}}{A\sqrt{T}} s_1(t)$





(b)



$h(t) = \phi_1(T-t)$

(c) $f_{Y|S}(y(t)|s(t)=s_1(t)) = \frac{1}{\sqrt{\pi N_0}} e^{-\frac{(y-\sqrt{E_s})^2}{N_0}}$

$f_{Y|S}(y(t)|s(t)=s_2(t)) = \frac{1}{\sqrt{\pi N_0}} e^{-\frac{(y+\sqrt{E_s})^2}{N_0}}$

(d) $P_e = \frac{1}{2}\times Q\left(\frac{d}{2\sigma}\right) + \frac{1}{2}\times Q\left(\frac{d}{2\sigma}\right) = Q\left(\frac{2\sqrt{E_s}}{\sqrt{2N_0}}\right) = Q\left(\sqrt{\frac{2E_s}{N_0}}\right)$

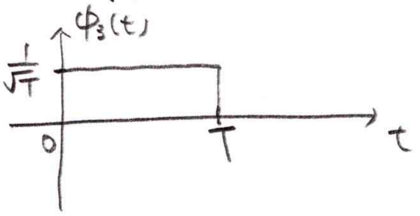
(e) Chernoff inequality of Q-function: $Q(x) \le \frac{1}{2}e^{-\frac{x^2}{2}}$

$\therefore Q\left(\sqrt{\frac{2E_s}{N_0}}\right) \le \frac{1}{2}e^{-\frac{E_s}{N_0}}$

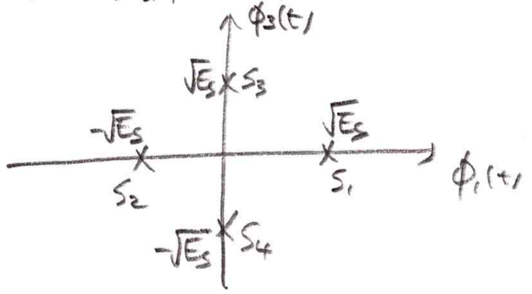
(f) $\int_0^T |s_3(t)|^2 dt = B^2 T = E_s \quad \Rightarrow \quad B^2 T = \frac{1}{3} A^2 T, \quad \underline{B = \frac{1}{\sqrt{3}} A}$

$$\left(B = \frac{\sqrt{E_s}}{\sqrt{T}} \right)$$

$$\phi_3(t) = \frac{1}{\sqrt{E_s}} = \frac{1}{B\sqrt{T}} s_3(t)$$



(g) $\{s_i(t)\}_{i=1}^{4} = C_1$



$$P_e \le P(s_1 \to s_2) + P(s_1 \to s_3) + P(s_1 \to s_4)$$
$$= Q\left(\sqrt{\frac{2E_s}{N_0}}\right) + Q\left(\sqrt{\frac{E_s}{N_0}}\right) + Q\left(\sqrt{\frac{E_s}{N_0}}\right)$$

$$\therefore \underline{P_e \le 2Q\left(\sqrt{\frac{E_s}{N_0}}\right) + Q\left(\sqrt{\frac{2E_s}{N_0}}\right)}$$

2009 제어

제어필수

1. (a) $e(t) = L\frac{di(t)}{dt} + Ri(t)$

$M\frac{d^2y(t)}{dt^2} = Mg - \frac{i^2(t)}{y(t)}$

$\therefore \begin{cases} \frac{di(t)}{dt} = -\frac{R}{L}i(t) + \frac{1}{L}e(t) \\ \frac{d^2y(t)}{dt^2} = g - \frac{i^2(t)}{My(t)} \end{cases}$

(b) $\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = g - \frac{x_3^2(t)}{Mx_1(t)} \\ \dot{x}_3(t) = -\frac{R}{L}x_3(t) + \frac{1}{L}e(t) \end{cases}$ $\Leftarrow$ Nonlinear system 이므로 system matrices로 표현 불가능하다

(c) $x_{10} = y_0$, $x_{20} = \frac{dy_0}{dt} = 0$, differential equation 에서 $0 = g - \frac{x_{30}^2}{My_0}$ 이므로

$x_{30} = \sqrt{Mgy_0}$

$\therefore X_0 = \begin{bmatrix} y_0 \\ 0 \\ \sqrt{Mgy_0} \end{bmatrix}$

(d) $\Delta\dot{x}_1(t) = \Delta x_2(t)$

$\Delta\dot{x}_2(t) = -\frac{2x_{30}}{My_0}\Delta x_3(t) + \frac{x_{30}^2}{My_0^2}\Delta x_1(t)$

$\Delta\dot{x}_3(t) = -\frac{R}{L}\Delta x_3(t) + \frac{1}{L}\Delta e(t)$

$x_{30} = \sqrt{Mgy_0}$ 이므로 $-\frac{2x_{30}}{My_0} = -\frac{2}{My_0}\sqrt{Mgy_0} = -2\sqrt{\frac{g}{My_0}}$,

$\frac{x_{30}^2}{My_0^2} = \frac{Mgy_0}{My_0^2} = \frac{g}{y_0}$.

$$\therefore \begin{bmatrix} \Delta\dot{x}_1(t) \\ \Delta\dot{x}_2(t) \\ \Delta\dot{x}_3(t) \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ \frac{g}{y_0} & 0 & -2\sqrt{\frac{g}{My_0}} \\ 0 & 0 & -\frac{R}{L} \end{bmatrix}}_{A} \begin{bmatrix} \Delta x_1(t) \\ \Delta x_2(t) \\ \Delta x_3(t) \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ \frac{1}{L} \end{bmatrix}}_{B} \Delta e(t)$$

제어 선택

1. Differential equation

$$\begin{cases} \dfrac{di_a(t)}{dt} = -\dfrac{R_a}{L_a} i_a(t) - \dfrac{1}{L_a} e_b(t) + \dfrac{1}{L_a} e_a(t) \\ T_m = K_i \, i_a(t) \\ e_b(t) = K_b \omega_m(t) = K_b \dfrac{d\theta_m(t)}{dt} \\ \dfrac{d^2\theta_m(t)}{dt^2} = -\dfrac{B_m}{J_m} \dfrac{d\theta_m(t)}{dt} - \dfrac{1}{J_m} T_L + \dfrac{1}{J_m} T_m \end{cases}$$

$$\begin{pmatrix} x_1(t) = i_a(t) \\ x_2(t) = \omega_m(t) \\ x_3(t) = \theta_m(t) \end{pmatrix} \Rightarrow \begin{pmatrix} \dot{x}_1(t) = -\dfrac{R_a}{L_a} x_1(t) - \dfrac{K_b}{L_a} x_2(t) + \dfrac{1}{L_a} e_a(t) \\ \dot{x}_2(t) = -\dfrac{B_m}{J_m} x_2(t) - \dfrac{1}{J_m} T_L + \dfrac{K_i}{J_m} x_1(t) \\ \dot{x}_3(t) = x_2(t) \end{pmatrix}$$

$$y(t) = \theta_m(t) = x_3(t)$$

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{bmatrix} = \begin{bmatrix} -\dfrac{R_a}{L_a} & -\dfrac{K_b}{L_a} & 0 \\ \dfrac{K_i}{J_m} & -\dfrac{B_m}{J_m} & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} \dfrac{1}{L_a} \\ 0 \\ 0 \end{bmatrix} e_a(t) + \begin{bmatrix} 0 \\ -\dfrac{1}{J_m} \\ 0 \end{bmatrix} T_L$$

$$y(t) = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}$$

$$\therefore \dot{x}(t) = \begin{bmatrix} -10 & -10 & 0 \\ 10 & -0.1 & 0 \\ 0 & 1 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix} u(t)$$

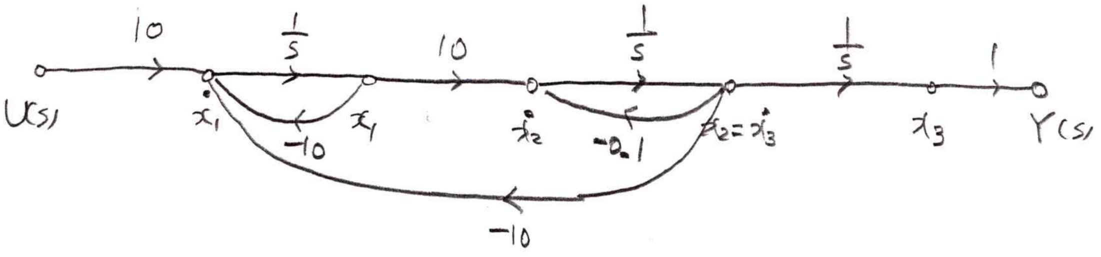
$$y(t) = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} x(t)$$

2. $H(s) = C(sI-A)^{-1}B$

$$= \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} s+10 & 10 & 0 \\ -10 & s+0.1 & 0 \\ 0 & -1 & s \end{bmatrix}^{-1} \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix}$$

To solve $(sI-A)^{-1}$, use Gauss-Jordan method? ⇒ 망함..

⇒ signal flow graph 그리기.



Mason's rule로 H(s) 구한다.

$$H(s) = \frac{Y(s)}{U(s)} = \frac{\frac{100}{s^3}}{1 + \frac{10}{s} + \frac{0.1}{s} + \frac{100}{s^2} + \frac{1}{s^2}} = \frac{100}{s^3 + 10.1s^2 + 101s}$$

3. Controllable canonical form.

$$\dot{\bar{x}}(t) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -101 & -10.1 \end{bmatrix} \bar{x}(t) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 100 & 0 & 0 \end{bmatrix} \bar{x}(t)$$

4. $P = SM$, $S = [B \quad AB \quad A^2B] = \begin{bmatrix} 10 & -100 & 0 \\ 0 & 100 & -1010 \\ 0 & 0 & 100 \end{bmatrix}$

$$M = \begin{bmatrix} 101 & 10.1 & 1 \\ 10.1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 10 & -100 & 0 \\ 0 & 100 & -1010 \\ 0 & 0 & 100 \end{bmatrix} \begin{bmatrix} 101 & 10.1 & 1 \\ 10.1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} = \underline{\begin{bmatrix} 0 & 1 & 10 \\ 0 & 100 & 0 \\ 100 & 0 & 0 \end{bmatrix} = P}$$