Spring 2019

EECE 588
Lecture 11

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Loop Antennas

- Simple and inexpensive like dipole antennas.
- Loop antennas come in many different shapes such as circle, square, triangle, rectangle, ellipse, etc.
- **A small loop antenna is equivalent to an infinitesimal magnetic dipole antenna whose axis is perpendicular to the loop plane.**
- **Remember that we mentioned that magnetic currents do not exist!**
- **Now, if magnetic currents did exist and if we had a magnetic Hertzian dipole, its fields would be identical to that of a small loop.**

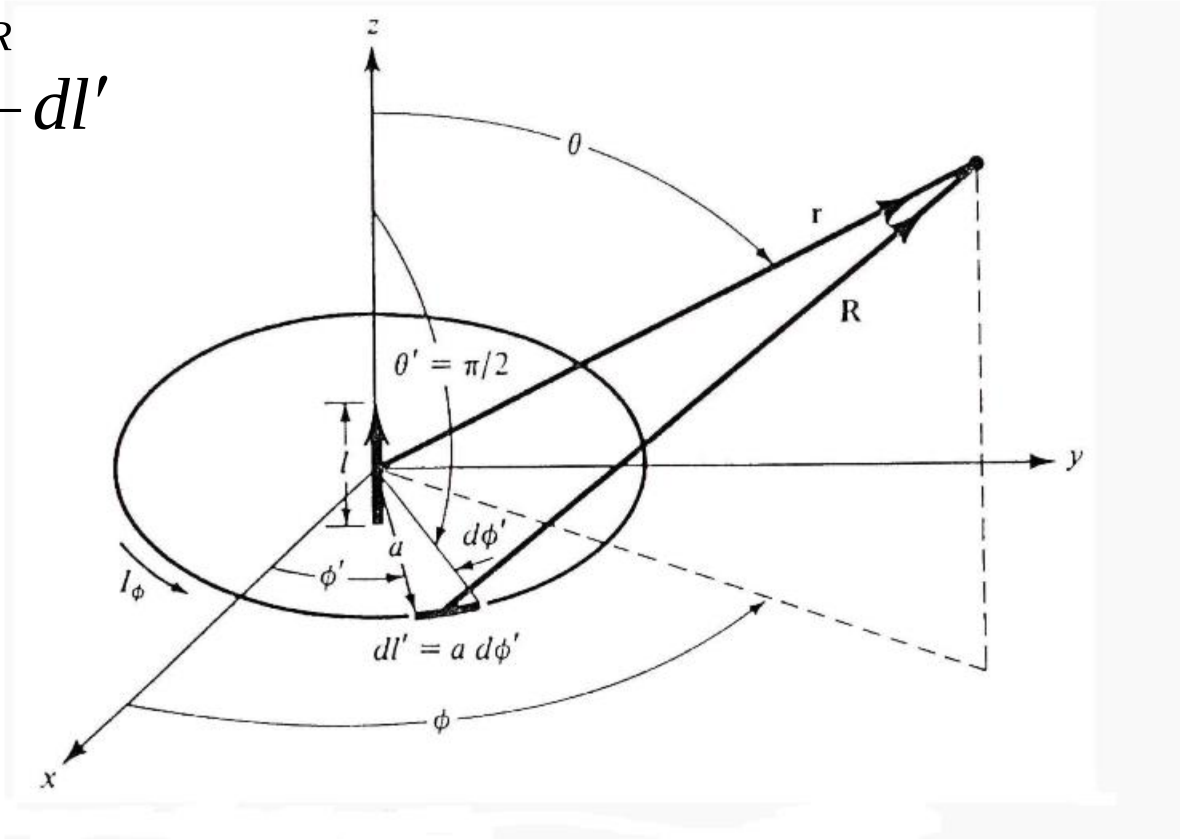
Infinitesimal Loop Antennas

- Similar to dipoles, we can have **"electrically small loops"** and **"electrically large loops".**
- Electrically small loops have circumference of less than $\lambda/10$.
- Electrically large loop antennas have a circumference close to λ.
- Electrically small loop antennas are very poor radiators.
  - They have small R_r and larger R_L ➔ Low radiation efficiency.
  - They are usually used in the receiving mode.
- If you open up an old AM radio (a transistor radio without a visible external antenna) chances are that you will see a loop with a ferrite core.

Small Circular Loop

- To find the radiated fields of a loop antenna, we follow the procedure that we have seen so many times now.

$$\vec{A} = \frac{\mu}{4\pi}\int_C \vec{I}_e(\vec{r}')\frac{e^{-jkR}}{R}dl'$$



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Small Loop

- The current flowing in the loop is expressed as:

$$\vec{I}_e(x',y',z') = \hat{x}I_x(x',y',z') + \hat{y}I_y(x',y',z') + \hat{z}I_z(x',y',z')$$

- However, it makes sense to use the cylindrical coordinate systems:

$$\begin{bmatrix} I_x \\ I_y \\ I_z \end{bmatrix} = \begin{bmatrix} \cos\varphi' & -\sin\varphi' & 0 \\ \sin\varphi' & \cos\varphi' & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_\rho \\ I_\varphi \\ I_z \end{bmatrix}$$

- We express the radiated fields in spherical coordinate system. Therefore:

$$\hat{x} = \hat{r}\sin\theta\cos\varphi + \hat{\theta}\cos\theta\cos\varphi - \hat{\varphi}\sin\varphi$$

$$\hat{y} = \hat{r}\sin\theta\sin\varphi + \hat{\theta}\cos\theta\sin\varphi + \hat{\varphi}\cos\varphi$$

$$\hat{z} = \hat{r}\cos\theta - \hat{\theta}\sin\theta$$

Small Circular Loop

$$\vec{I}_e = \hat{r}\left[I_\rho \sin\theta\cos(\varphi-\varphi') + I_\varphi \sin\theta\sin(\varphi-\varphi') + I_z\cos\theta\right]$$
$$+\hat{\theta}\left[I_\rho \cos\theta\cos(\varphi-\varphi') + I_\varphi \cos\theta\sin(\varphi-\varphi') - I_z\sin\theta\right]$$
$$+\hat{\varphi}\left[-I_\rho \sin(\varphi-\varphi') + I_\varphi \cos(\varphi-\varphi')\right]$$

- For a circular loop, the current flows in along $\hat{\phi}$:

$$\vec{I}_e = \hat{r}I_\varphi \sin\theta\sin(\varphi-\varphi') + \hat{\theta}I_\varphi \cos\theta\sin(\varphi-\varphi') + \hat{\varphi}I_\varphi \cos(\varphi-\varphi')$$

$$R=\sqrt{(x-x')^2+(y-y')^2+(z-z')^2} \qquad x'=a\cos\varphi' \quad y'=a\sin\varphi' \quad z'=0$$

$$R=\sqrt{r^2+a^2-2ar\sin\theta\cos(\varphi-\varphi')}$$

Small Circular Loop

- This way, the ϕ component of $\vec{A}$ can be written in the following form:

$$A_\varphi = \frac{a\mu}{4\pi}\int_0^{2\pi} I_\varphi \cos(\varphi-\varphi')\frac{e^{-jk\sqrt{r^2+a^2-2ar\sin\theta\cos(\varphi-\varphi')}}}{\sqrt{r^2+a^2-2ar\sin\theta\cos(\varphi-\varphi')}}d\varphi'$$

- For small loop, we assume that the current is constant.
- Therefore, the fields will not be a function of ϕ (rotational symmetry).
- Hence choose $\phi = 0$:

$$A_\varphi = \frac{a\mu I_0}{4\pi}\int_0^{2\pi} \cos\varphi' \frac{e^{-jk\sqrt{r^2+a^2-2ar\sin\theta\cos\varphi'}}}{\sqrt{r^2+a^2-2ar\sin\theta\cos\varphi'}}d\varphi'$$

Small Circular Loop

- There are techniques for evaluating this integral, which are elegant and accurate.
- Your book takes the easy way out, which is ok for our application (small loop).
- We express the following function by its Maclaurin series.

$$f = \frac{e^{-jk\sqrt{r^2+a^2-2ar\sin\theta\cos\varphi'}}}{\sqrt{r^2+a^2-2ar\sin\theta\cos\varphi'}}$$

$$f = f(0) + f'(0)a + \frac{1}{2!}f''(0)a^2 + \ldots + \frac{1}{(n-1)!}f^{(n-1)}(0)a^{n-1} + ..$$

Small Circular Loop

- Taking into account only the first two terms of the Maclaurin series, we will have:

$$f(0) = \frac{e^{-jkr}}{r} \qquad f'(0) = \left(\frac{jk}{r} + \frac{1}{r^2}\right) e^{-jkr} \sin\theta \cos\varphi'$$

$$f \approx \left(\frac{1}{r} + a\left(\frac{jk}{r} + \frac{1}{r^2}\right) \sin\theta \cos\varphi'\right) e^{-jkr}$$

$$A_\varphi \cong \frac{a\mu I_0}{4\pi} \int_0^{2\pi} \cos\varphi' \left[\frac{1}{r} + a\left(\frac{jk}{r} + \frac{1}{r^2}\right) \sin\theta \cos\varphi'\right] e^{-jkr} d\varphi'$$

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Small Circular Loop

- Taking into account only the first two terms of the Maclaurin series, we will have:

$$A_\varphi \approx \frac{a\mu I_0}{4} e^{-jkr} \sin\theta \left(\frac{jk}{r} + \frac{1}{r^2} \right)$$

- The r and θ components can be calculated in a similar manner.
- Turns out that they will be zero once you integrate them.

$$\vec{A} \approx \hat{\varphi} A_\varphi \to \vec{A} = \hat{\varphi} \frac{a^2 \mu I_0}{4} e^{-jkr} \left[\frac{jk}{r} + \frac{1}{r^2} \right] \sin\theta$$

$$\vec{A} = \hat{\varphi} j \frac{k a^2 \mu I_0 \sin\theta}{4r} e^{-jkr} \left[1 + \frac{1}{jkr} \right]$$

Small Circular Loop

- From this, we can calculate $\vec{E}$ and $\vec{H}$.

$$H_r = j\frac{ka^2 I_0 \cos\theta}{2r^2}\left[1+\frac{1}{jkr}\right]e^{-jkr}$$

$$H_\theta = -\frac{(ka)^2 I_0 \sin\theta}{4r}\left[1+\frac{1}{jkr}-\frac{1}{(kr)^2}\right]e^{-jkr}$$

$$H_\varphi = 0$$

$$E_r = E_\theta = 0$$

$$E_\varphi = \eta\frac{(ka)^2 I_0 \sin\theta}{4r}\left[1+\frac{1}{jkr}\right]e^{-jkr}$$

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Small Loop and Infinitesimal Magnetic Dipole

- If you calculate the fields of an infinitesimal magnetic dipole with current I_m and length, l, you will have:

$$E_r = E_\theta = H_\varphi = 0$$

$$E_\varphi = -j\frac{kI_m l\sin\theta}{4\pi r}\left[1+\frac{1}{jkr}\right]e^{-jkr}$$

$$H_r = \frac{I_m l\cos\theta}{2\pi\eta r^2}\left[1+\frac{1}{jkr}\right]e^{-jkr}$$

$$H_\theta = j\frac{kI_m l\sin\theta}{4\pi\eta r}\left[1+\frac{1}{jkr}-\frac{1}{(kr)^2}\right]e^{-jkr}$$

Small Loop and Infinitesimal Magnetic Dipole

- Therefore, provided that:

$$I_m l = jS\omega\mu I_0$$

The magnetic Hertzian dipole and small loop are completely equivalent to one another.

Duality between small loop and small dipole

$$H_r = j\frac{ka^2 I_0\cos\theta}{2r^2}\left[1+\frac{1}{jkr}\right]e^{-jkr}$$

$$H_\theta = -\frac{(ka)^2 I_0\sin\theta}{4r}\left[1+\frac{1}{jkr}-\frac{1}{(kr)^2}\right]e^{-jkr}$$

$$H_\varphi = 0$$

$$E_r = E_\theta = 0$$

$$E_\varphi = \eta\frac{(ka)^2 I_0\sin\theta}{4r}\left[1+\frac{1}{jkr}\right]e^{-jkr}$$

$$E_r = \eta\frac{I_0 l\cos\theta}{2\pi r^2}\left\{1+\frac{1}{jkr}\right\}e^{-jkr}$$

$$E_\theta = j\eta\frac{kI_0 l\sin\theta}{4\pi r}\left\{1+\frac{1}{jkr}-\frac{1}{(kr)^2}\right\}e^{-jkr}$$

$$E_\varphi = 0$$

$$H_r = H_\theta = 0$$

$$\vec{H} = \hat{\varphi}\frac{jkI_0 l\sin\theta}{4\pi r}\left[1+\frac{1}{jkr}\right]e^{-jkr}$$

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Power Density and Radiation Resistance

$$P_{rad} = \eta\left(\frac{\pi}{12}\right)(ka)^4|I_0|^2$$

$$R_{rad} = \eta\left(\frac{\pi}{6}\right)(ka)^4 = 20\pi^2(C/\lambda)^4 \cong \underbrace{31171\left(\frac{S^2}{\lambda^4}\right)}$$

Valid for loops of other shapes as well.

Multi Turn Loops:

$$R_{rad} = 20\pi^2(C/\lambda)^4 N^2 \cong 31171 N^2\left(\frac{S^2}{\lambda^4}\right)$$

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Directivity

- You can calculate the directivity of the loop using the procedures that we discussed.
- However, without doing any calculations, tell me what is the directivity of a small loop?
- Why?

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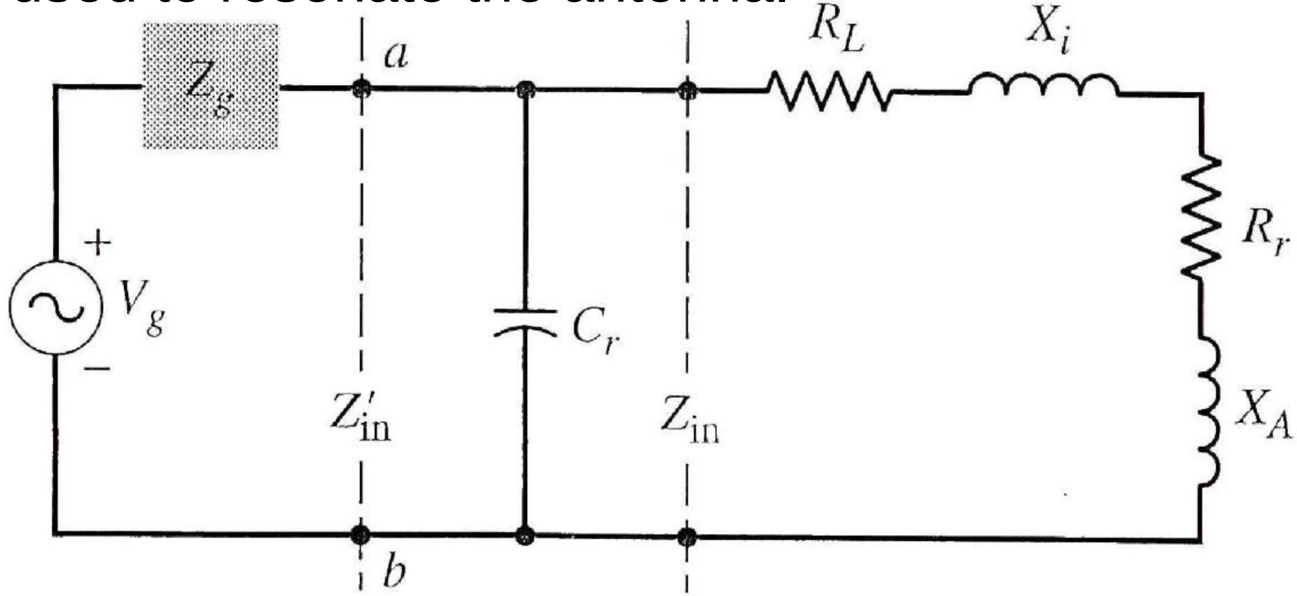
Equivalent Circuit in Transmitting Mode

- R_r=Radiation Resistance.
- R_L=loss resistance of loop conductor.
- X_A=External inductive reactance of loop antenna.
- X_i=internal high-frequency reactance of loop conductor.
- C_r is a capacitor used to resonate the antenna.



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Transmitting Mode Equivalent Circuit

- The external and internal inductances can be calculated using the following formulas.
- Circular loop of radius a and wire radius b:

$$L_A = \mu_0 a\left[\ln\left(\frac{8a}{b}\right) - 2\right]$$

- Square loop with side a and wire radius b:

$$L_A = 2\mu_0 \frac{a}{\pi}\left[\ln\left(\frac{a}{b}\right) - 0.774\right]$$

- Internal inductance (taking into account skin depth) for a single turn:

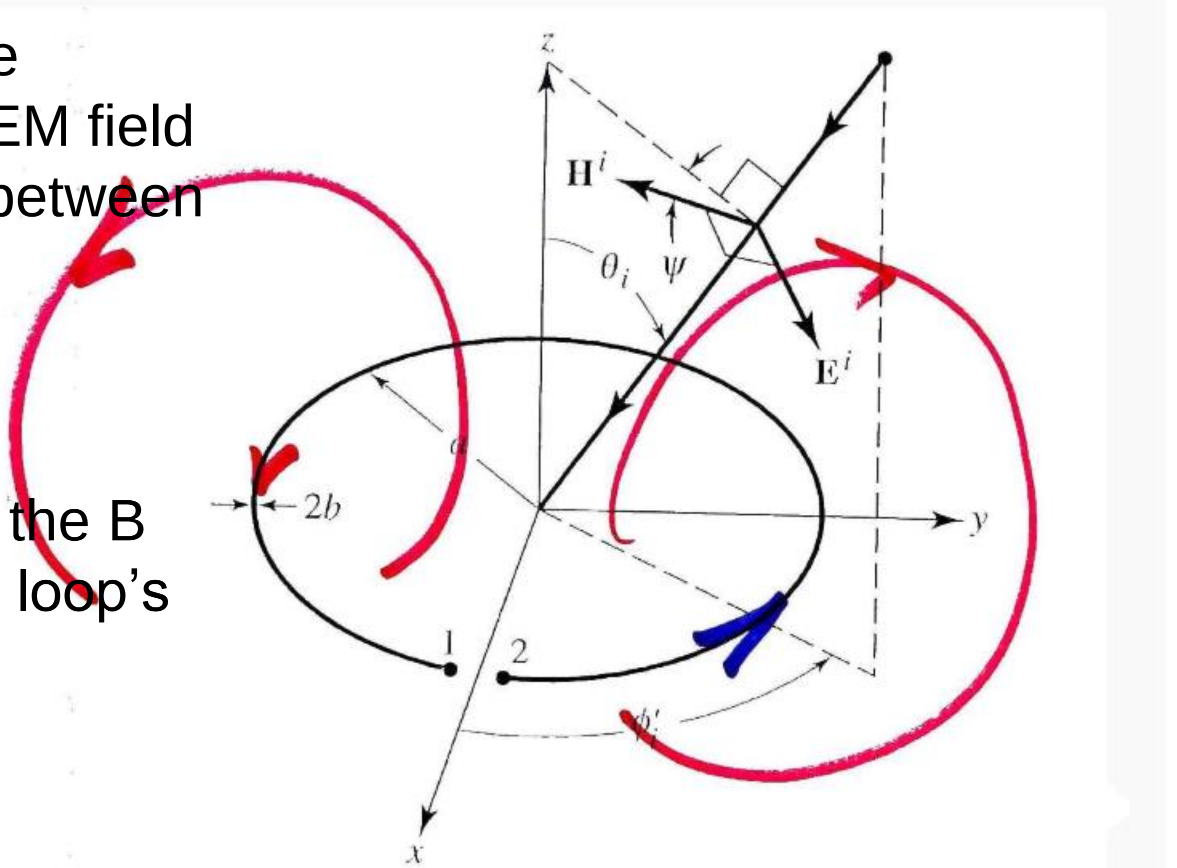
$$L_i = \frac{l}{\omega P}\sqrt{\frac{\omega\mu_0}{2\sigma}} = \frac{a}{\omega b}\sqrt{\frac{\omega\mu_0}{2\sigma}}$$

Equivalent Circuit in Receiving Mode

- The loop antenna is often used as a receiving antenna or as probe to measure magnetic flux density.
- In receiving mode, the magnetic field of the EM field will induce a voltage between the terminals 1-2:

$$V_{oc} = j\omega\pi a^2 B_z^i$$

- This is assuming that the B field is uniform on the loop's area.



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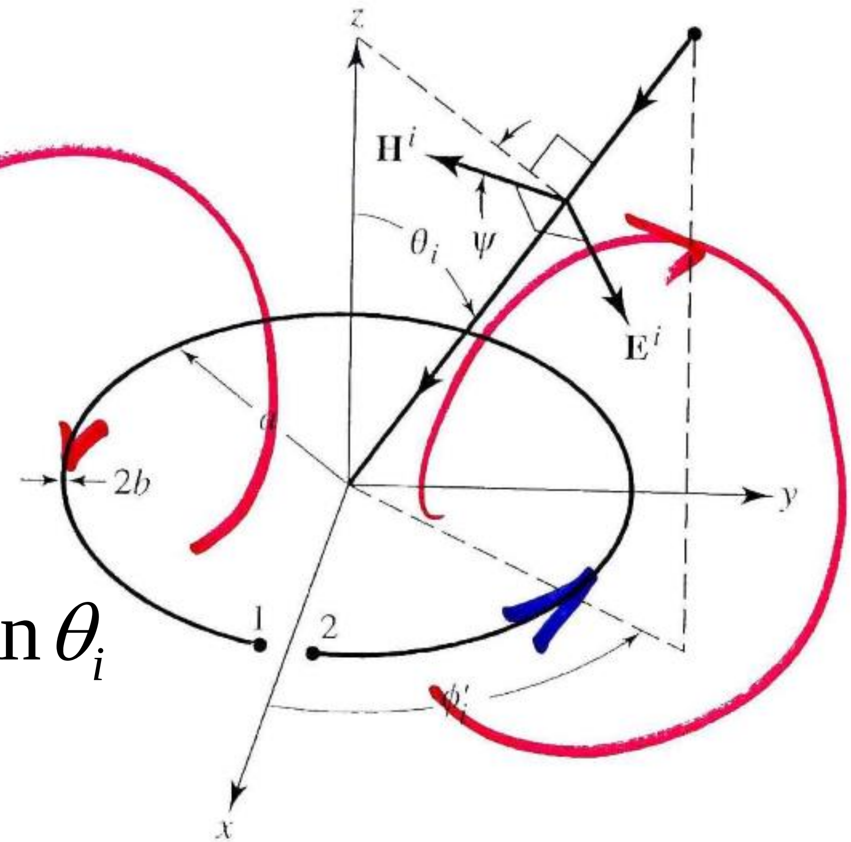
Equivalent Circuit in Receiving Mode

$$V_{oc} = j\omega\pi a^2 \mu_0 H^i \cos\psi_i \sin\theta_i = jk_0\pi a^2 E^i \cos\psi_i \sin\theta_i$$

- ψ_i is the angle between the direction of the magnetic field of the incident plane wave and the plane of incidence.
  - Here, the plane of incidence is defined by the $\vec{k}$ vector and a vector normal to the surface of the loop.



$$\vec{\ell}_e = \hat{\varphi}\ell_e = \hat{\varphi} jk_0\pi a^2 \cos\psi_i \sin\theta_i$$

$$= \hat{\varphi} jk_0 S \cos\psi_i \sin\theta_i$$

Circular Loops of Constant Current

- Now, we want to consider the radiation from a loop antenna with $C \cong \lambda$ but we still assume that the current is a uniform one.
- Note that this is inherently a false assumption.
- Even though this is a false assumption, the results will be helpful in our real calculations.

$$R = \sqrt{r^2 + a^2 - 2ar\sin\theta\cos\varphi'} \approx \sqrt{r^2 - 2ar\sin\theta\cos\varphi'} \quad \text{for } r >> a$$

- Using binomial expansion

Phase

$$\boxed{R \approx r\sqrt{1 - 2\frac{a}{r}\sin\theta\cos\varphi'} \approx r - a\sin\theta\cos\varphi' = r - a\cos\psi_o}$$

Amplitude

$$\boxed{R \approx r}$$

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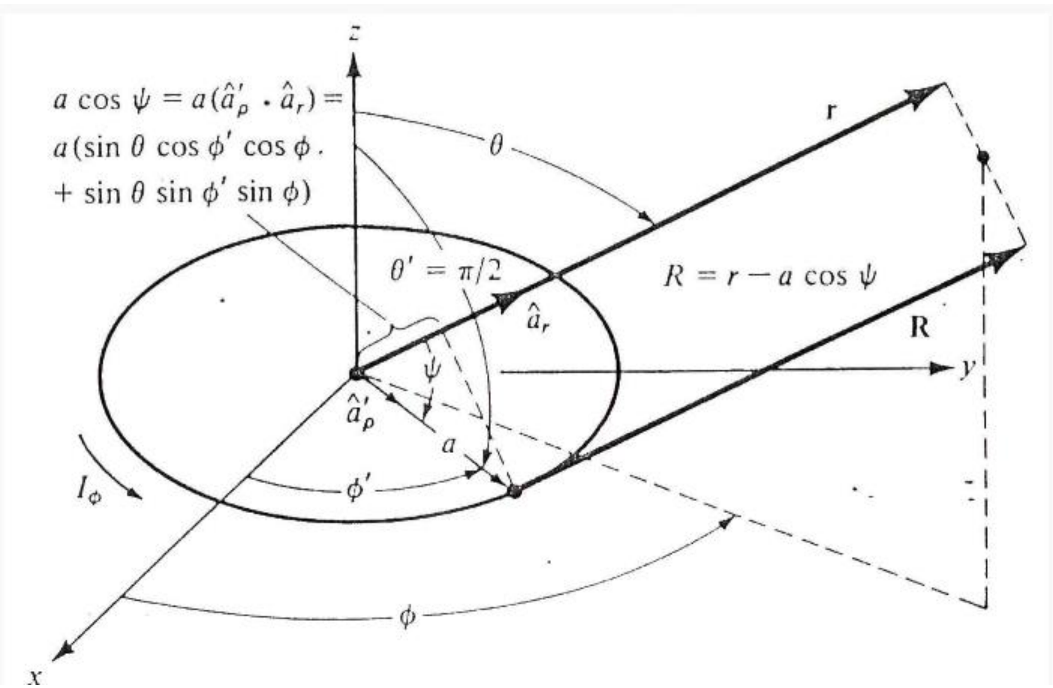
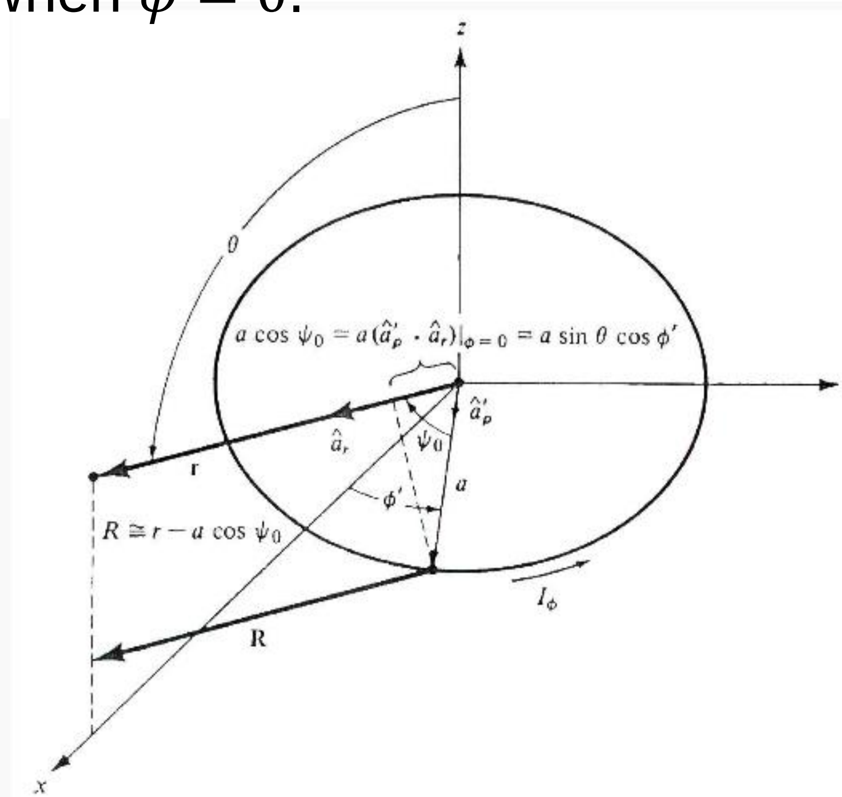
Circular Loops of Constant Current

- Note that angle ψ is the angle between r and r'.
- ψ_0 is the angle between r and r' when $\varphi = 0$.





Circular Loops of Constant Current

$$A_\varphi = \frac{a\mu I_0}{4\pi}\int_0^{2\pi}\cos\varphi' \frac{e^{-jk\sqrt{r^2+a^2-2ar\sin\theta\cos\varphi'}}}{\sqrt{r^2+a^2-2ar\sin\theta\cos\varphi'}}d\varphi'$$

$$A_\varphi \approx \frac{a\mu I_0 e^{-jkr}}{4\pi r}\int_0^{2\pi}\cos\varphi' e^{jka\sin\theta\cos\varphi'}d\varphi'$$

$$A_\varphi \approx \frac{a\mu I_0 e^{-jkr}}{4\pi r}\left[\int_0^{\pi}\cos\varphi' e^{jka\sin\theta\cos\varphi'}d\varphi' + \int_\pi^{2\pi}\cos\varphi' e^{+jka\sin\theta\cos\varphi'}d\varphi'\right]$$

$$\varphi' = \varphi'' + \pi$$

$$A_\varphi \approx \frac{a\mu I_0 e^{-jkr}}{4\pi r}\left[\int_0^{\pi}\cos\varphi' e^{jka\sin\theta\cos\varphi'}d\varphi' - \int_0^{\pi}\cos\varphi'' e^{-jka\sin\theta\cos\varphi''}d\varphi''\right]$$

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