Spring 2019

EECE 588
Lecture 19

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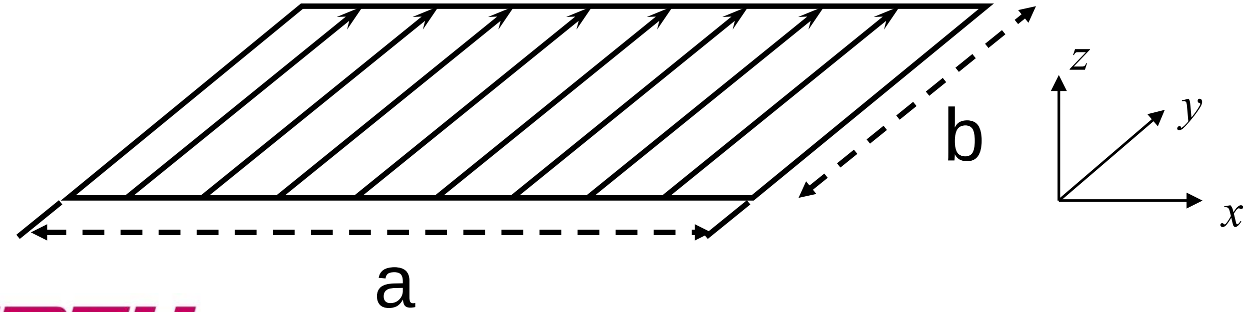
Uniform Aperture Distribution Over Space

Problem Formulation

- Uniform aperture distribution in space. In other words, no ground plane exists in this case.
- This case is similar to the previous case, except for the fact that no ground plane is present here.
- We can consider the aperture field distribution of:

$$\vec{E}_a = \hat{y}E_0 \qquad \vec{H}_a = -\hat{x}E_0 / \eta$$

- The next step is to replace this problem with an equivalent problem using equivalence theorem and find the equivalent electric and magnetic surface currents.

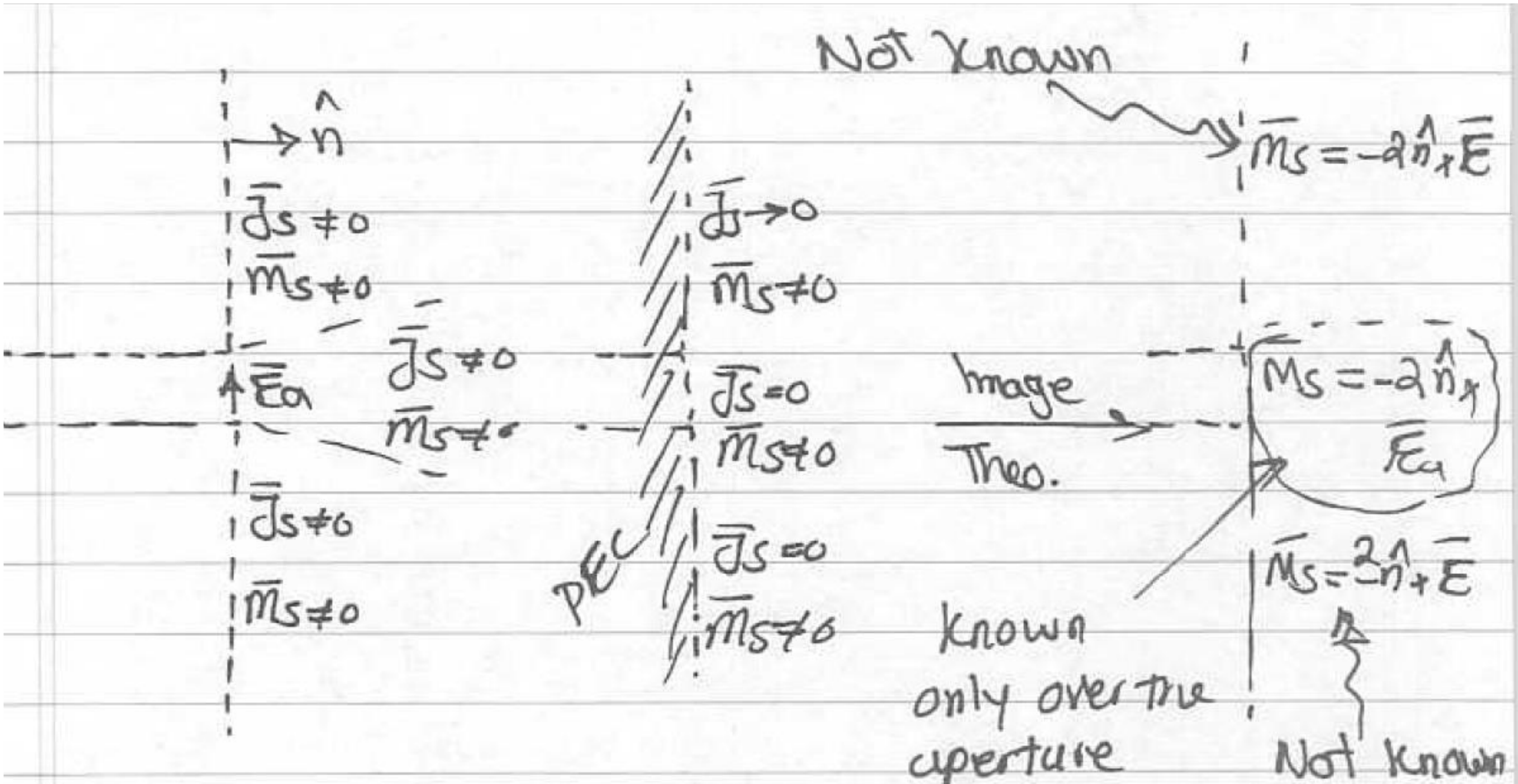


Problem Formulation

- Similar to the previous case, the equivalence theorem is used. However, in this case, no ground plane is present and consequently:

- Therefore both $\overrightarrow{J_s}$ and $\overrightarrow{M_s}$ exist everywhere.
- It is possible to replace the lower hemisphere with PEC or PMC and get rid of $\overrightarrow{J_s}$ or $\overrightarrow{M_s}$ but the other current component ($\overrightarrow{M_s}$ or $\overrightarrow{J_s}$) will be present.

Equivalent Problem



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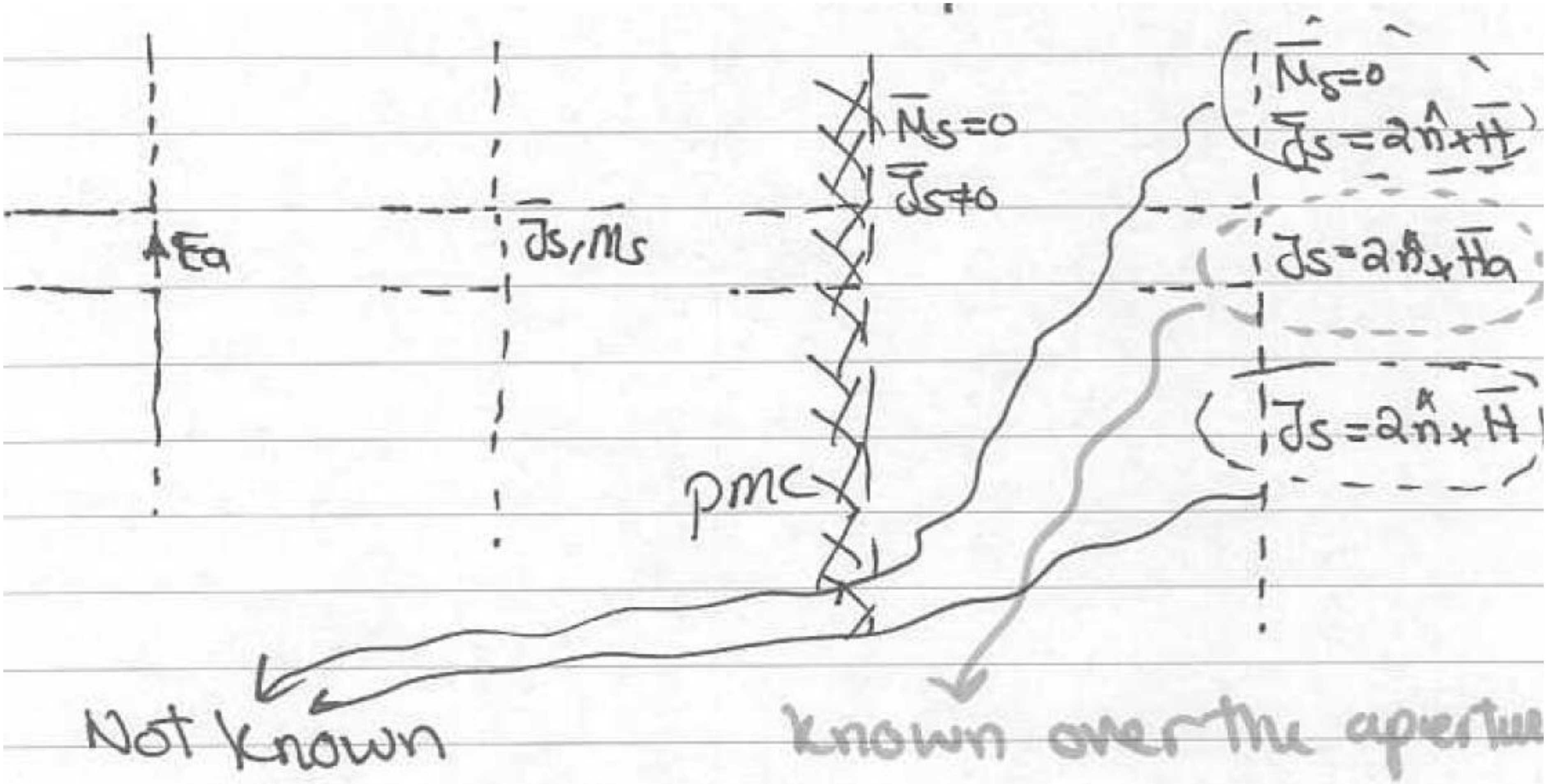
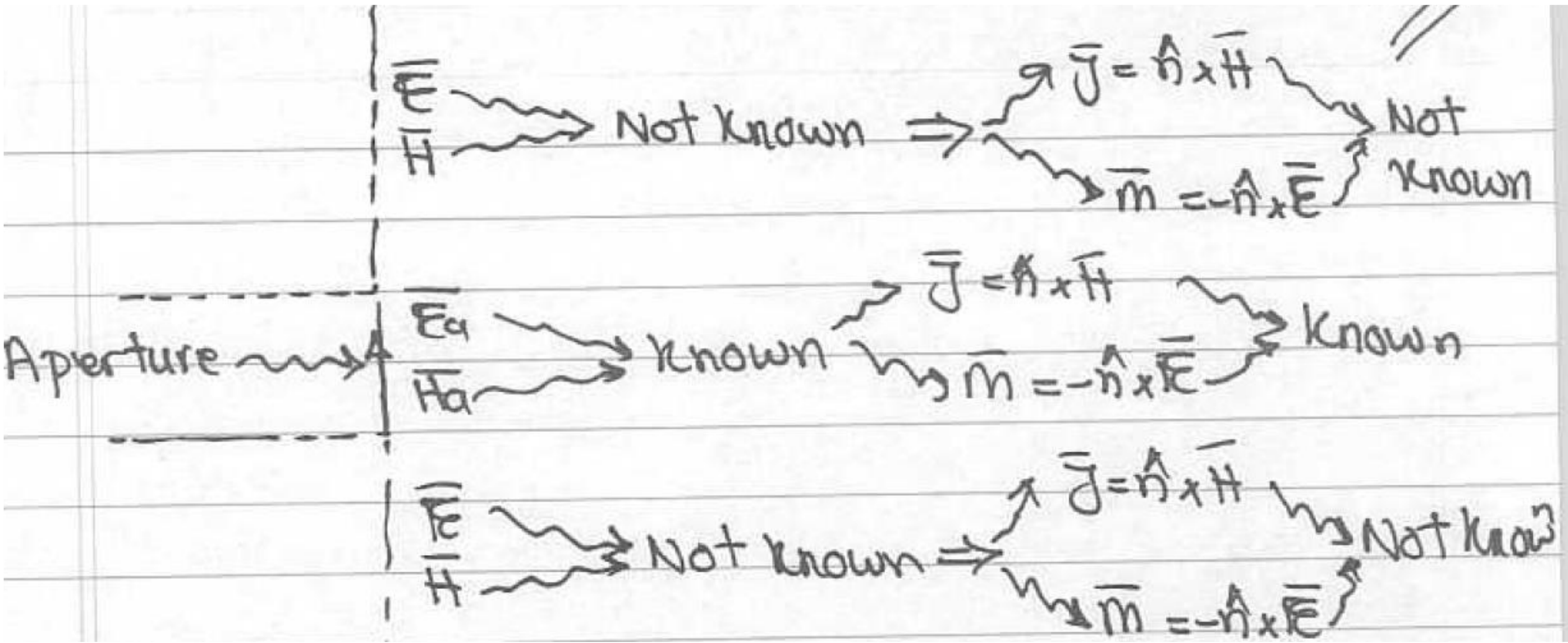
Equivalent Problem



Equivalent Problem

- The bottom line is that, we know the values of $\overrightarrow{J_s}$ or $\overrightarrow{M_s}$ only over the aperture.
- However, they exist <u>EVERYWHERE</u> on an infinite plane. So what should we do?



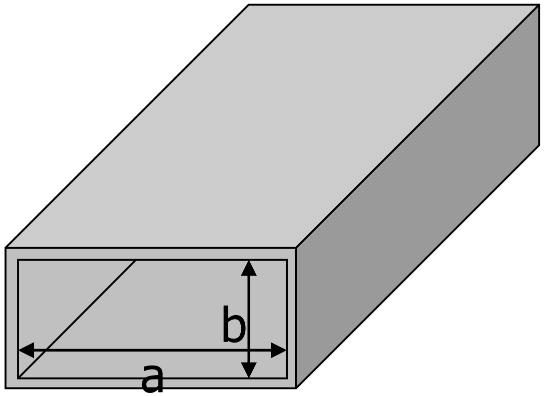
Equivalent Problem

- One simple solution is to assume that $\overrightarrow{E_a}$ and $\overrightarrow{H_a}$ exist only over the aperture and assume that they are zero everywhere else on the aperture plane.
- This simplifies the problem but is not the most accurate technique.
- However it provides reasonable accuracy for many applications and is much simpler than other alternative analytical techniques such as iterative techniques.
- <u>Solving this problem is similar to the previous case and it is going to be assigned as a Homework problem.</u>
  - The idea is to consider $\overrightarrow{J_s}$ and $\overrightarrow{M_s}$ over the aperture, and zero everywhere else, and find $\vec{A}$ and $\vec{F}$ and take it from there.

TE_{10} Aperture Distribution

TE_{10} Mode Distribution

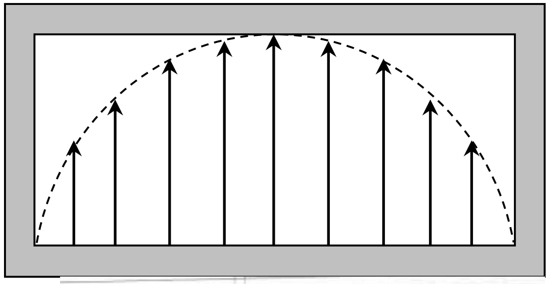
- TE_{10} is the first propagating mode in a rectangular waveguide. In a rectangular waveguide with inner dimensions of $a \times b$, the cut off frequency is given by:

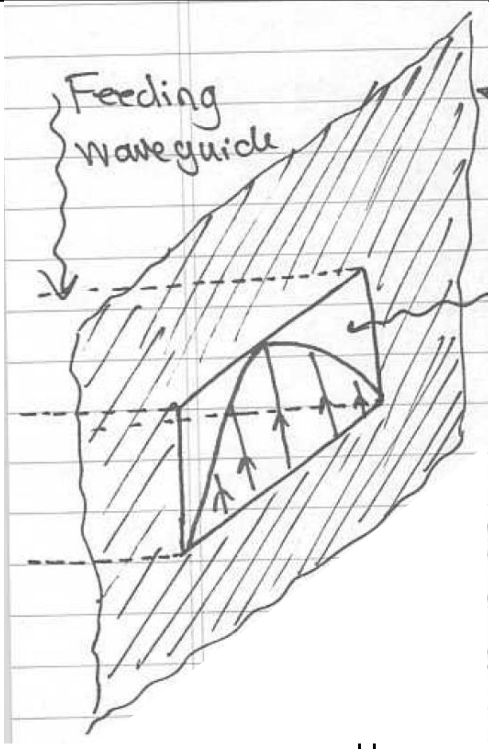


$$(f_c)_{mn} = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

- In a rectangular waveguide, it is not possible to have a uniform field distribution (as a result of the boundary conditions that mandate the tangential component of electric field must go to zero at the boundary between a dielectric and a PEC).

TE_{10} Mode Distribution





- Electric field distribution associated with TE_{10} mode distribution inside a rectangular waveguide.
- Obtaining the far field radiation of this antenna is pretty much similar to all the previous cases.
- We have to follows this procedure:
  1. Replace the problem with the equivalent problem.
  2. $\overrightarrow{J_s}$ exist everywhere.
  3. $\overrightarrow{M_s}$ exists only over the aperture, remember that $\hat{n} \times \vec{E} = 0$ over PEC.
  4. Replace the left half space with PEC ➔ $\overrightarrow{J_s}$ is short circuited.
  5. Using image theory, PEC can be removed and $\overrightarrow{M_s} = -2\hat{n} \times \overrightarrow{E_a}$ radiates in free space.

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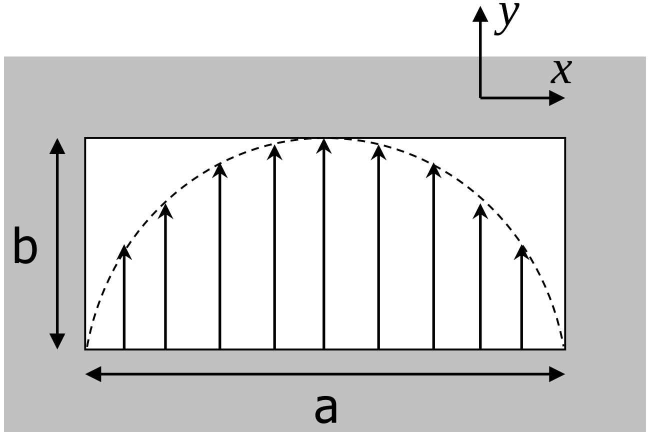
Homework Problem

- Consider the following electric field distribution in an aperture mounted on an infinite ground plane:

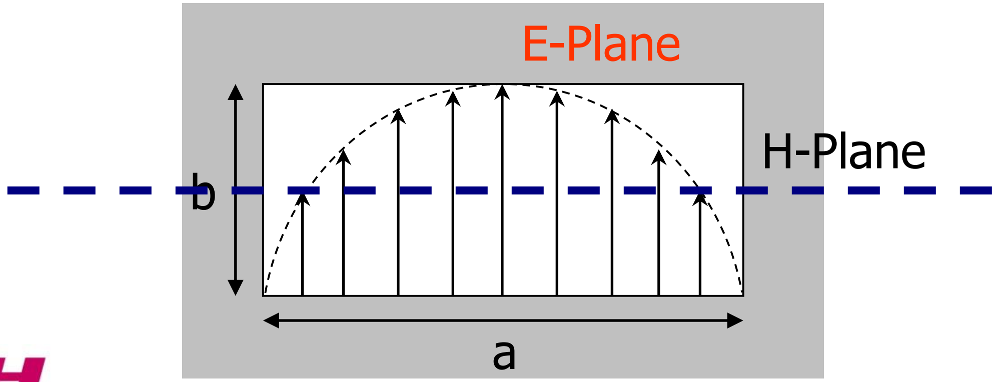
$$\vec{E}_a = \hat{y}E_0 \cos\left(\frac{\pi}{a}x'\right) \quad \begin{cases} -b/2 \le y' \le b/2 \\ -a/2 \le x' \le a/2 \end{cases}$$

- Find the radiated fields from this aperture in the far field.



Radiation Patterns

- The TE_{10} aperture field distribution results in a radiation pattern that has a wider beamwidth in the H-Plane.
- The tapering of the electric field in the H-plane results in broadening of the pattern in the H-Plane.
- However, for the same aperture dimensions, the E-Plane beamwidth does not change.
  - Note that the field components do not vary as a function of y (in the E-Plane) ➔The E-Plane pattern does not change.



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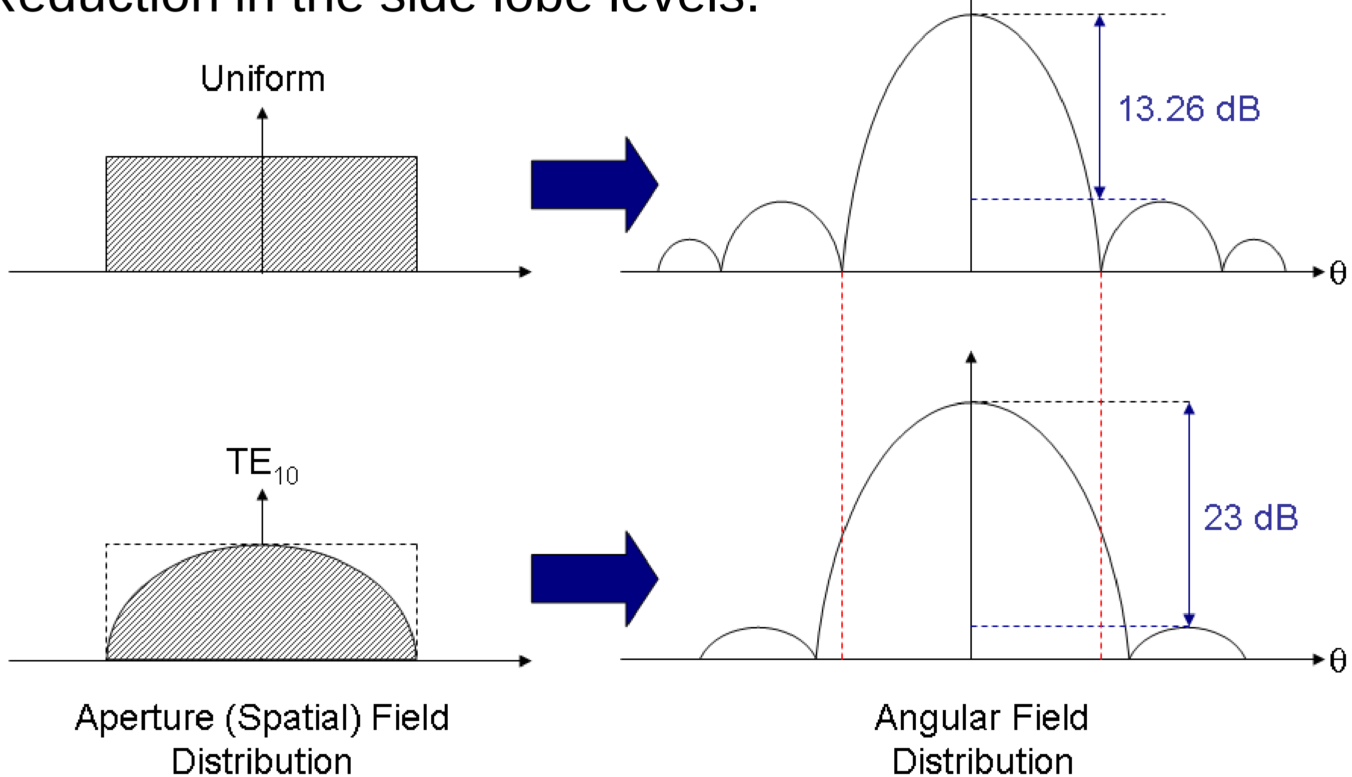
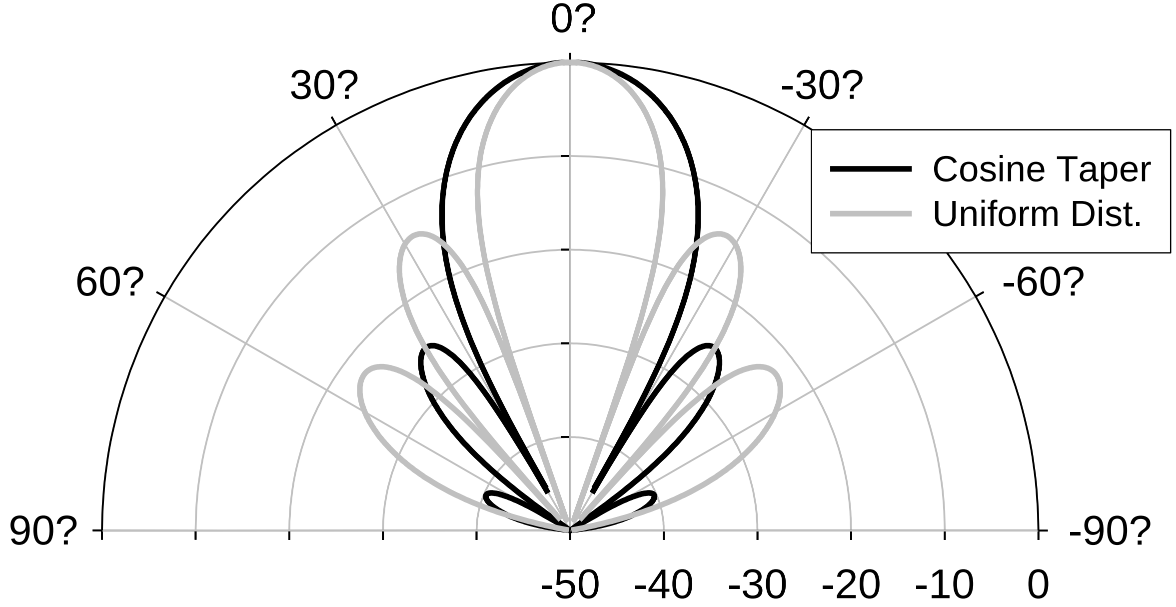
Radiation Patterns

- Tapering the aperture field distribution in the H-Plane results in:
  - Broadening the pattern in the H-Plane radiation patterns.
  - Reduction in the side lobe levels.



Radiation Patterns

- Comparison between the radiation patterns of an aperture with Cosine Taper and Uniform aperture distributions.
- Both apertures are mounted on an infinite ground and the aperture dimensions are $a = 3\lambda$ and $b = 2\lambda$ with the exception that one has uniform distribution and the other has a cosine.



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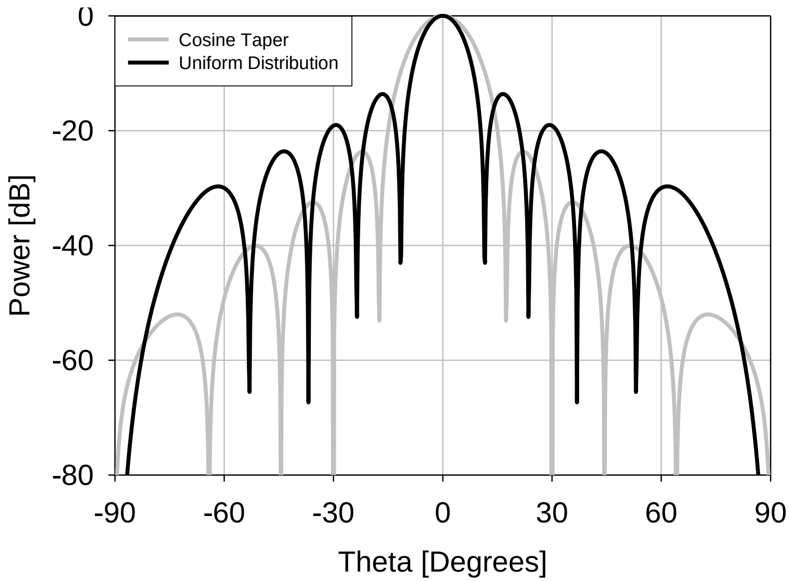
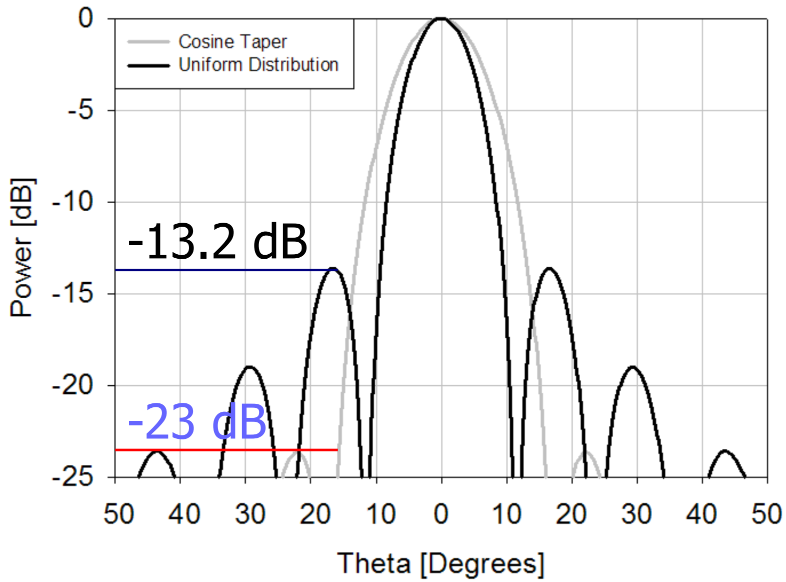
Radiation Patterns

- Comparison between the radiation patterns of an aperture with Cosine Taper and uniform aperture distributions. Both apertures are mounted on an infinite ground and the aperture dimensions are $b = 2\lambda$ and $a = 5\lambda$.





Directivity

- Broadening the radiation pattern results in reduction in the Directivity of the antenna. The directivity of the TE_{10} mode can be calculated as:

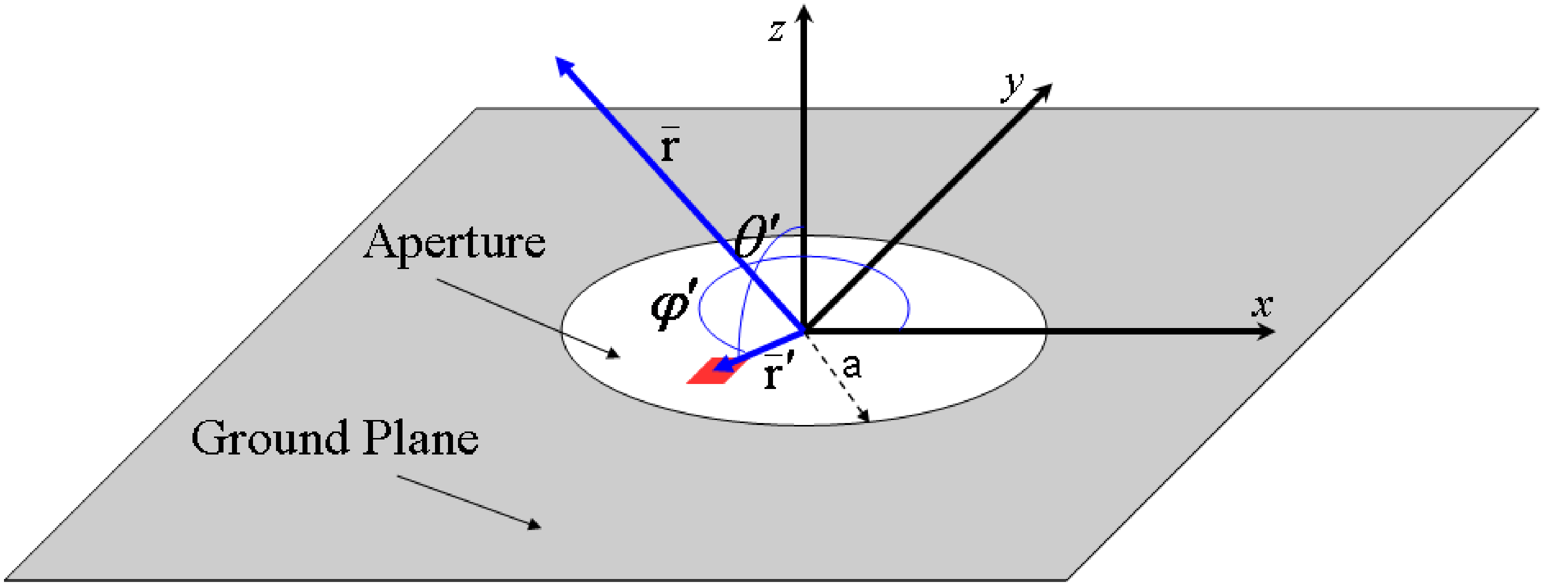
$$\textbf{Directivity} = \begin{cases} 4\pi \dfrac{ab}{\lambda^2} & \textbf{uniform field dist.} \\ 0.81 \times 4\pi \dfrac{ab}{\lambda^2} & \textbf{cosine taper } (\mathbf{TE_{10}}) \end{cases}$$

- One advantage of tapering is the reduction in side lobe level that occurs (from -13.26 dB to -23 dB).

Circular Apertures

Circular Apertures

- Treatment of circular apertures is not much different from the rectangular apertures.
- Equivalence principle in conjunction with image theory must be used to formulate the problem.



Circular Apertures

- After formulating the problem, the problem simplifies to a magnetic current of $\overrightarrow{M_s} = -2\hat{n} \times \overrightarrow{E_a}$ existing only over the aperture.
- Vector electric potential $\vec{F}$ should be calculated in the far field and the $\vec{E}$ and $\vec{H}$ fields simply follow using equations provided in slide #14.
- We will consider two different cases:
  - Uniform aperture distribution (easy).
  - TE_{11} Mode Distribution (more realistic).

Far Field Radiation

- In the E-Plane ($\varphi = \pi/2$ or y-z plane)

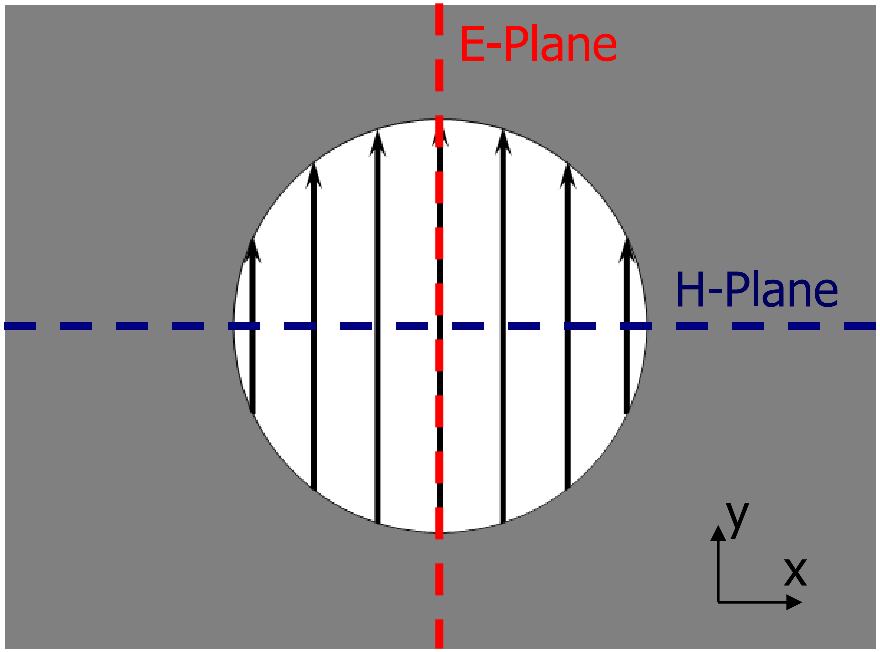
$$E_\theta = j\frac{ka^2E_0e^{-jkr}}{r}\left\{\frac{\mathrm{J}_1(ka\sin\theta)}{ka\sin\theta}\right\} \quad E_r = E_\varphi = 0$$

- $J_p(x)$ = Bessel function of the first kind of order p.
- For uniform aperture distribution in the H-Plane:

$$E_\varphi = j\frac{ka^2E_0e^{-jkr}}{r}$$

$$\cos\theta\left[\frac{\mathrm{J}_1(ka\sin\theta)}{ka\sin\theta}\right]$$

$$E_r = E_\theta = 0$$



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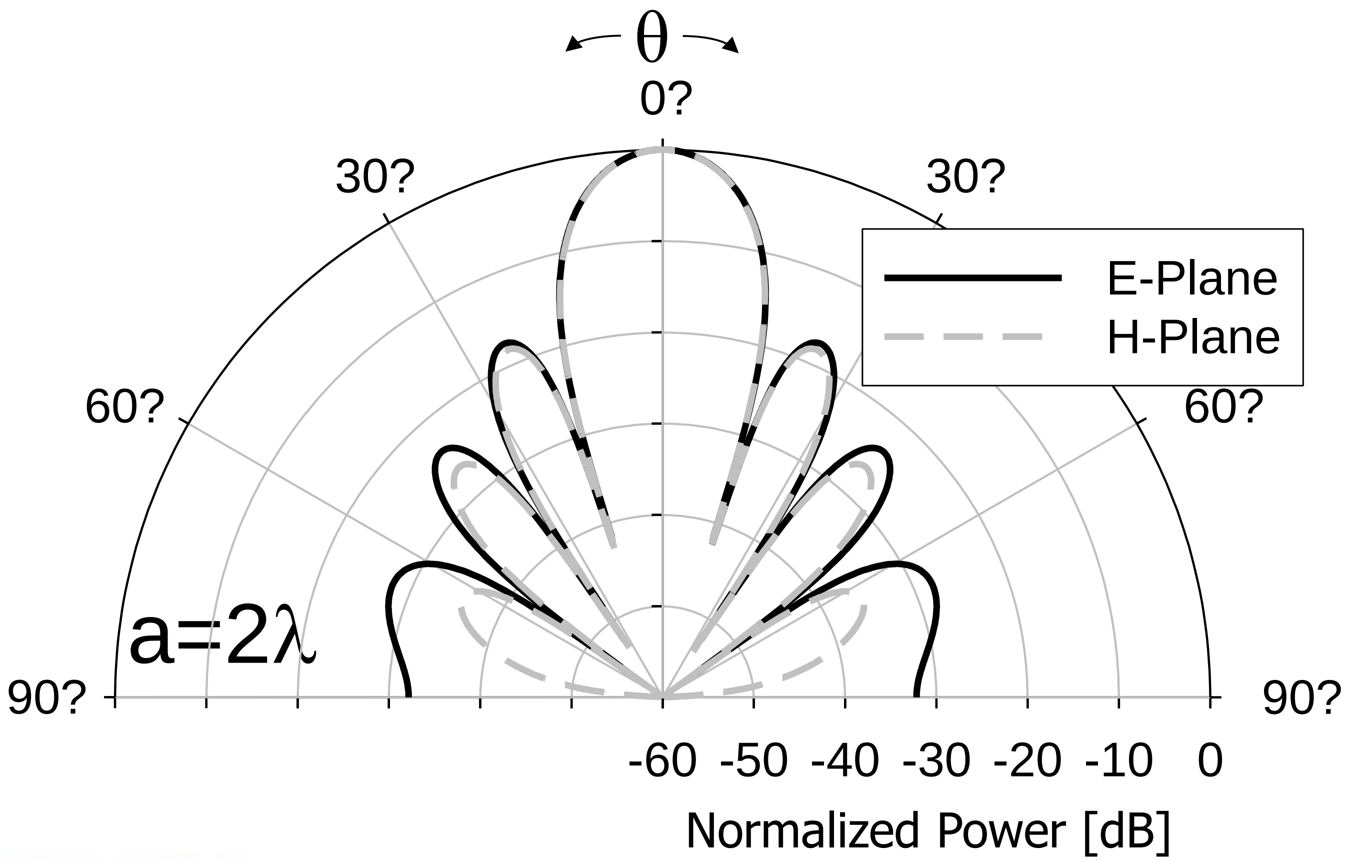
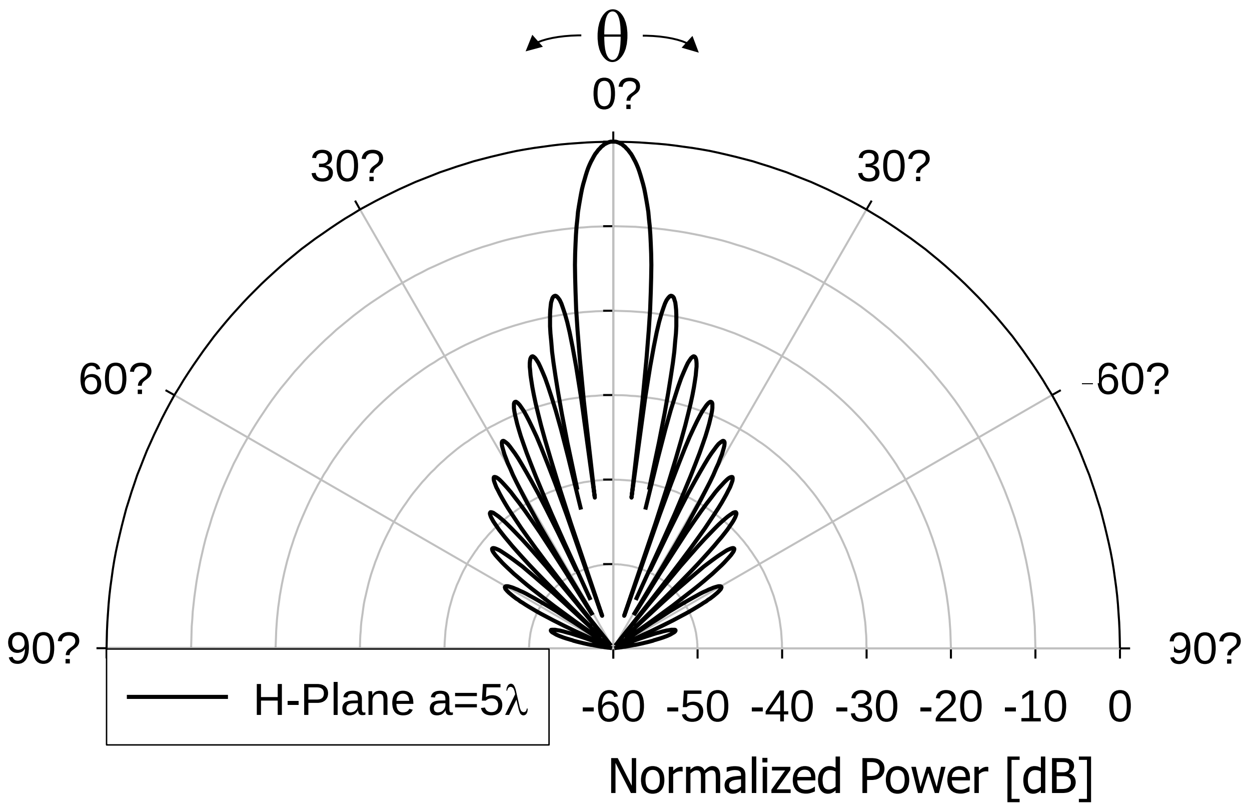
Radiation Patterns of an Aperture with Uniform Electric Field Distribution

- Radiation patterns of a circular aperture radius of $a = 2\lambda$.



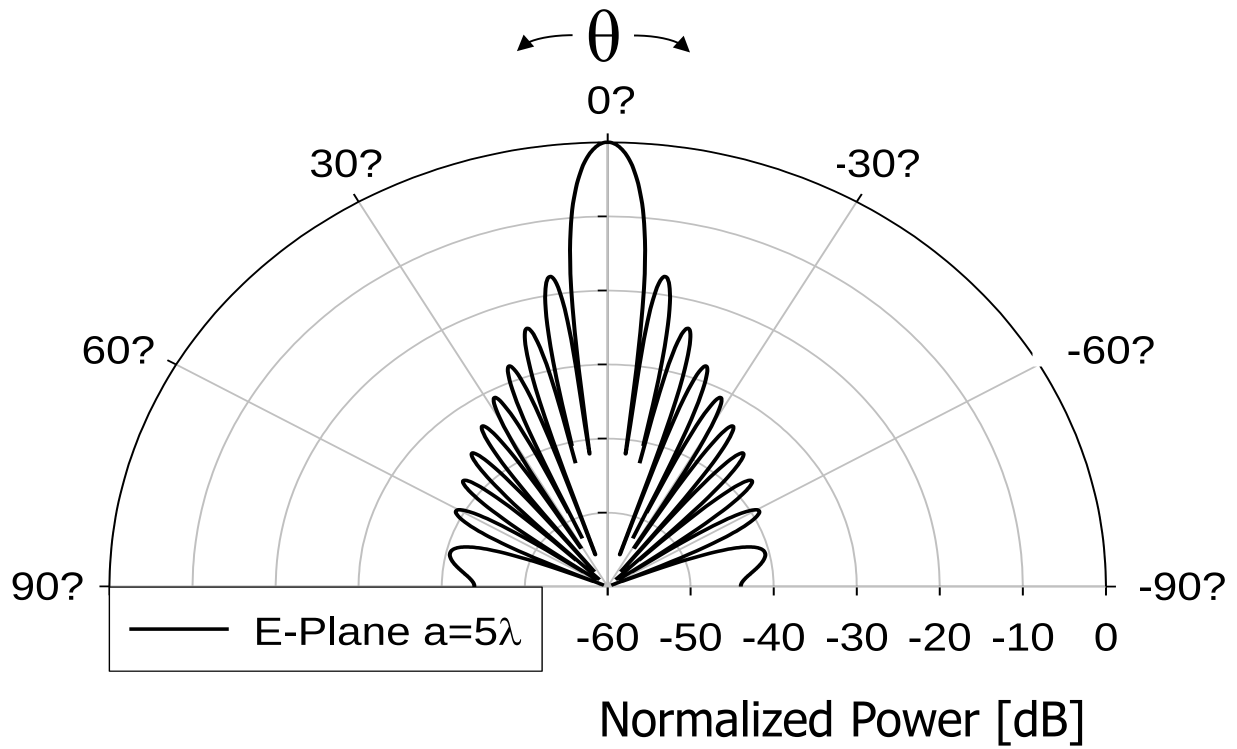
Radiation Patterns: H-Plane

- Radiation patterns of a circular aperture radius of $a = 5\lambda$.



Radiation Patterns: E-Plane

- Radiation patterns of a circular aperture radius of $a = 5\lambda$.



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Directivity

- For a circular aperture with uniform aperture distribution, we have:

$$D = \frac{4\pi A_{em}}{\lambda^2}$$

- But the maximum effective area and the physical area are the same for uniform aperture distribution. Therefore:

$$D = \frac{4\pi}{\lambda^2}\pi a^2 = \left(\frac{C}{\lambda}\right)^2$$

- The side lobe level for a circular aperture with uniform aperture distribution ($a \gg \lambda$) is -17.6 dB in the E-Plane and H-Plane.

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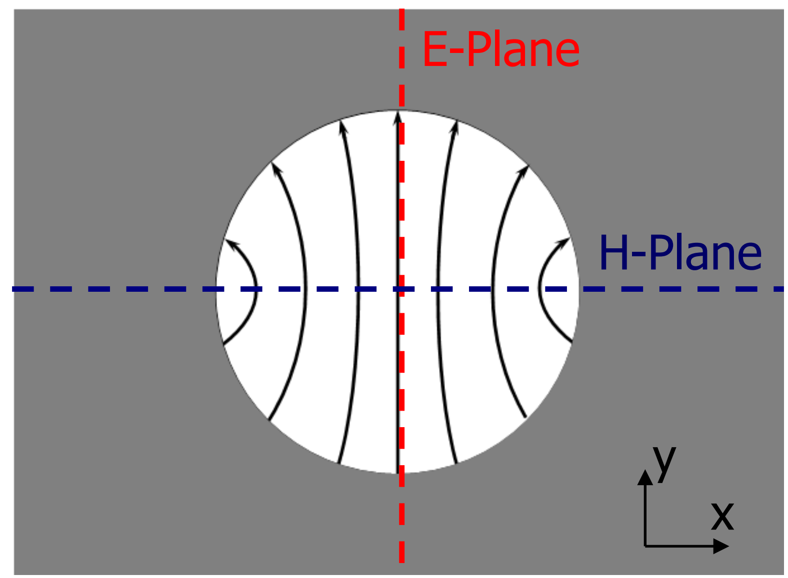
TE_{11} Mode Distribution

- TE_{11} mode is the dominant mode in circular waveguides and has an electric field distribution as shown in the figure below.
- In many applications such as circular horn antennas, a circular waveguide (with TE_{11} mode distribution) is used to feed the antenna.
- A simple way of obtaining TE_{11} aperture distribution is to have an open ended circular waveguide.



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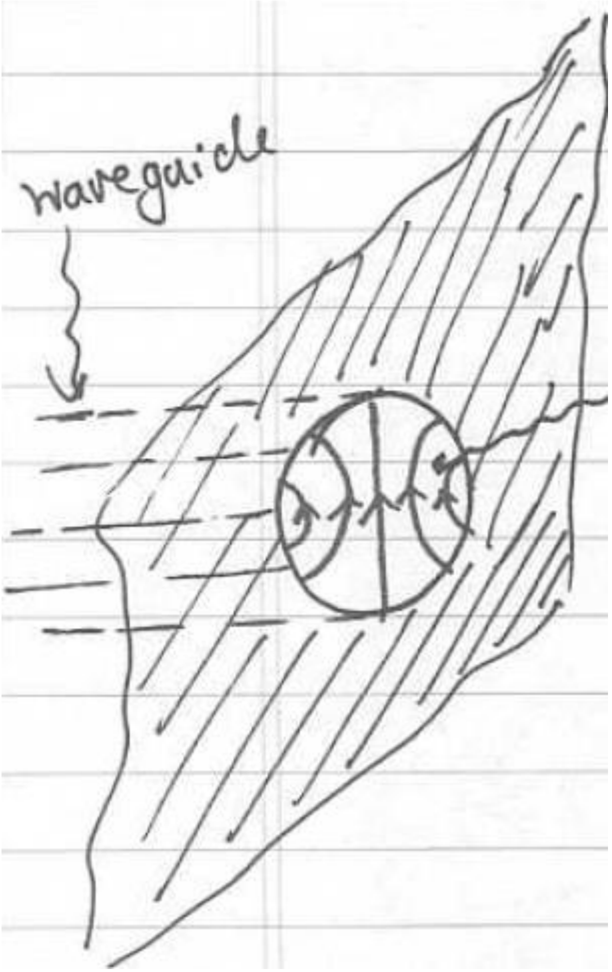
TE_{11} Mode Distribution

- Treatment of this problem is almost identical to the rectangular aperture in an infinite ground plane.
- Two main differences are:
  1. The problem must be solved in cylindrical coordinate system rather than the rectangular coordinate system used before.
  2. The expression for the aperture electric field (magnetic current) is different.

$$
\left.\begin{aligned}
\vec{E}_a &= \hat{\rho} E_\rho + \hat{\varphi} E_\varphi \\
E_\rho &= E_0 J_1(\chi'_{11}\rho'/a)\sin\varphi'/\rho' \\
E_\varphi &= E_0 J'_1(\chi'_{11}\rho'/a)\cos\varphi'
\end{aligned}\right\}
\begin{aligned}
&\rho' \le a \\
&\chi'_{11} = 1.841 \\
&J' = \frac{\partial}{\partial \rho'} J
\end{aligned}
$$



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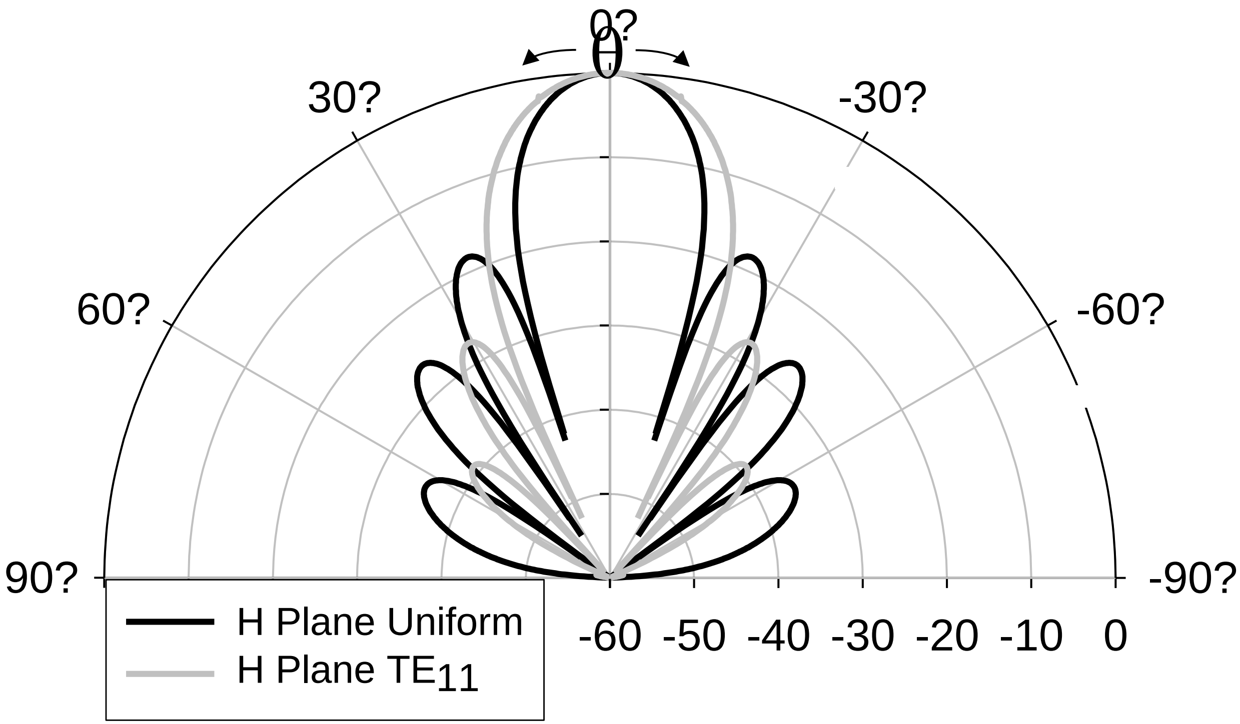
Radiation Patterns

- The aperture field is tapered in the H-Plane ➔ The pattern in the H-Plane is broadened.
- However, the side lobe level also decreases.



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Directivity

- The directivity of the circular aperture antenna with TE_{11} mode distribution is:

$$D = \begin{cases} 10.5\pi\left(\dfrac{a}{\lambda}\right)^2 & \mathrm{TE}_{11}\ \text{mode distribution.} \\ 4\pi^2\left(\dfrac{a}{\lambda}\right)^2 & \text{Uniform field dist.} \end{cases}$$

- Side lobe levels are:
  - -26.2 dB in H-Plane
  - -17.6 dB in E-Plane
- Note that these are valid for $a \gg \lambda$.