c) Force on an object in a charge system.

(point charges
conductors
dielectrics)

(1) The system is isolated

One of the parts of the system is allowed to make a small displacement, $d\xi$, under the influence of an electrical force acting on it.

The mechanical work done by the system:

$$dW_m = \vec{F}\cdot d\vec{\xi}$$

The internal energy of the system is also changed: dW

$$dW + dW_\xi = 0$$

Since the system is isolated

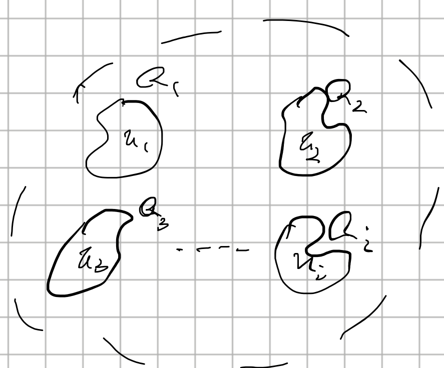
$$\vec{F}\cdot d\vec{\xi} = dW_m = -dW$$

$$\vec{F} = -\nabla_\xi W \Big|_{Q \text{ is fixed}}$$

(2) the system is fixed to a potential

(the system is connected to a battery, which supplies the energy.

$$dW + dW_m = \underset{\substack{\uparrow \\ \text{Work dones by the battery}}}{dW_b}$$



$$W = \frac{1}{2}\sum Q_j u_j \left(= \frac{1}{2}\int d^3x\, \rho(x)\, \Phi(x)\right)$$

$$\Rightarrow dW = \frac{1}{2}\sum u_j\, dQ_j$$

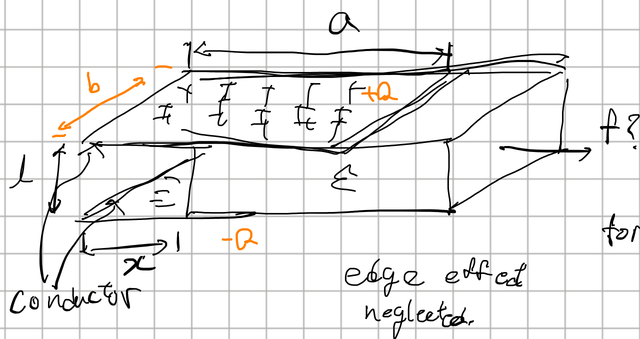
Work done by the battery to move dQ_j from zero potential to u_j

$$dW_b = \sum u_j dQ_j = \sum dW$$

$$dW_m = dW$$

$$\underset{(+)}{\vec{f}} = \nabla_\xi W \Big|_{U \text{ is fixed}}$$

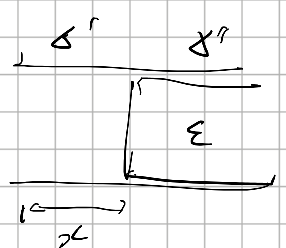
Ex)



force to hold the dielectric medium

(a) when the system is isolated (without battery)
(b) potential is fixed. (with battery)

(a)



· $\sigma' x b + \sigma''(a-x) b = Q$

· equipotential : $4\pi\sigma' l = \frac{4\pi\sigma''}{\varepsilon} l$

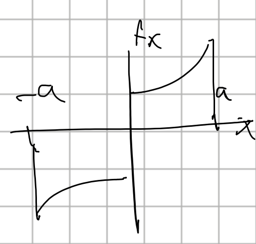
$$\sigma' = \frac{Q}{b}\,\frac{1}{x+\varepsilon(a-x)} \qquad \sigma'' = \varepsilon\sigma'$$

$$W = \overbrace{\frac{1}{8\pi} E_1^2 \Delta V_1}^{\text{region 1}} + \overbrace{\frac{\varepsilon E_2^2}{8\pi}\Delta V_2}^{\text{region 2}}$$

$$E_1 = 4\pi\sigma', \qquad E_2 = \frac{4\pi\sigma''}{\varepsilon}$$

$$= \frac{1}{2}\,\frac{4\pi l}{b}\,\frac{Q^2}{[x+(a-x)\varepsilon]}$$

$$f = -f_x = -\left(-\frac{\partial W}{\partial x}\right)_Q = \frac{2\pi l Q^2}{b}\cdot\frac{\varepsilon-1}{[\quad]^2}$$

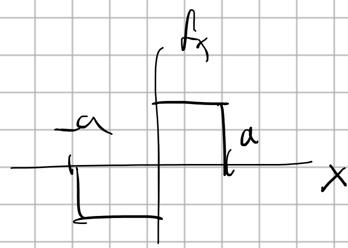


(b) $E_1 = E_2 = \frac{V}{l}$

$$W = \frac{1}{8\pi}\left(\frac{V}{l}\right)^2 bxl + \frac{1}{8\pi}\left(\frac{V}{l}\right)^2 \varepsilon bl(a-x)$$

$$= \frac{1}{8\pi}\left(\frac{V}{l}\right)^2 bl\left[x + \varepsilon(a-x)\right]$$

$$f = -f_x = -\left(\frac{\partial W}{\partial x}\right)_V = \frac{1}{8\pi}\frac{V^2 b}{l}(\varepsilon - 1)$$



2. Magnetostatics

2-1 Review

A Biot – Savart's law

B Magnetic potential

2-2 Magento statics

A Magnetic Dipole
B Magnetization
C Boundary value problems
D Magnetism
E Magnetostatic Energy

A) basic experimental observation

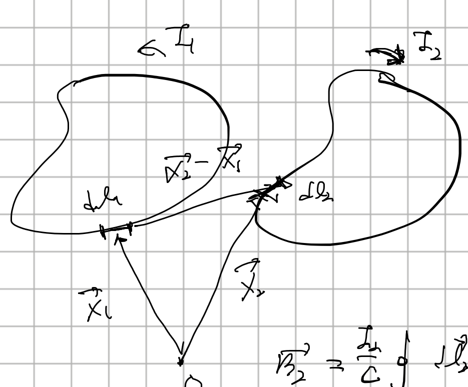
Oested : deflection of a compass near a current-flowing wire

Biot & Savart : first studied the magnetic field by a straight wire

Ampere has made more elegant & thorough measurement

$$\vec{F}_{12} = \frac{I_1 I_2}{c^2} \oint_{C_1} \oint_{C_2} \frac{d\vec{l}_1 \times (d\vec{l}_2 \times \vec{x}_{12})}{|\vec{x}_{12}|^2}$$

$c = 3 \times 10^{10}$ cm/sec



$$\vec{B}_2 = \frac{I_2}{c} \oint_{C} d\vec{l}_2 \times \frac{(\vec{x}_1 - \vec{x}_2)}{|\vec{x}_1 - \vec{x}_2|}$$

$$\vec{B}_{12} = \frac{I}{c} \oint d\vec{l}_1 \times \vec{B}_2(\vec{x}_1)$$

Basic Properties

① inverse square law $= \frac{1}{r^2}$

② $\vec{F}_{12} = -\vec{F}_{21}$ (anti-symmetric)

$$\vec{F}_{12} = \frac{I_1 I_2}{c^2} \oint\oint \frac{d\vec{l}_1 \times (d\vec{l}_2 \times \vec{x}_{12})}{|\vec{x}_{12}|^3}$$

$$= \frac{I_1 I_2}{c^2} \oint\oint \left(d\vec{l}_2 \frac{d\vec{l}_1 \cdot \vec{x}_{12}}{|\ |^3} - \frac{d\vec{l}_1 \cdot d\vec{l}_2}{|\ |^3} \vec{x}_{12} \right)$$

$$= -\frac{I_1 I_2}{c^2} \oint\oint \frac{dl_1 \cdot dl_2}{|\ |^3} \vec{x}_{12} = -\vec{F}_{21}$$

$$= -\oint\oint d\vec{l}_2\, d\vec{l}_1 \cdot \nabla\left(\frac{1}{|\ |}\right) = -\oint dl_2 \left[\frac{1}{|\ |}\right]_A^A = 0$$

③ F_{12} depends on the shape & relative orientation of the loops

④ unit

$[m] = g$, $[l] = cm$, $[F] = dyne$

$[charge] = \sqrt{dyne}\text{-}cm$ (cf. $F = \frac{q_1 q_2}{r^2}$)

$[current] = \sqrt{dyne}\ cm/sec$

$$F_{12} = \frac{I_2}{c} \int d\vec{l}_2 \times B_2(x_1)$$

$$dyne = \frac{\sqrt{dyne}\text{-}cm}{sec} \cdot \frac{1}{\frac{cm}{sec}} \cdot cm\, [B]$$

$$[B] = \sqrt{dyne}/cm = gauss$$

cf. $F = qE$

$$dyne = \sqrt{dyne} \cdot cm\, [E]$$

$$[E] = \sqrt{dyne}/cm$$

$[E] = [B]$ in CGS unit

cf. MKS system

$$dF = dI \times B$$

$$Newton = Ampere \cdot m \cdot [B]$$

$$[B] = \frac{N}{A\text{-}m} = \frac{J}{A\text{-}m^2} = \frac{Weber}{m^2} = Tesla.$$

⑤ Continuous distribution of current

$$I\,dl \to J\, da\, dl = J\, d^3x$$

current density: current / unit area

$$\vec{B}(\vec{x}) = \frac{1}{c}\int d^3x'\, \vec{J}(\vec{x}') \times \frac{(\vec{x}-\vec{x}')}{|\vec{x}-\vec{x}'|}$$

cf. $\vec{E}(\vec{x}) = \int dx'\, \rho(\vec{x}')\frac{(\vec{x}-\vec{x}')}{|\vec{x}-\vec{x}'|}$

$$\vec{F} = \frac{1}{c}\int d^3x\, \vec{J}(\vec{x}) \times \vec{B}(\vec{x})$$

(when $J(\vec{x})$ is embeded in external $B(\vec{x})$)

cf. $\vec{F} = \int d^3x\, \rho(\vec{x})\vec{E}(\vec{x})$