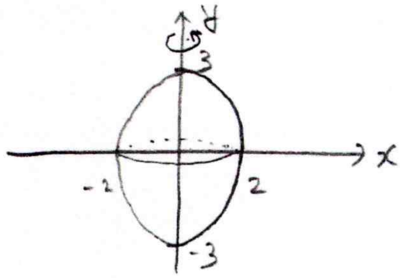
2016 수학

1. $x=2\cos t,\ y=3\sin t.$ $\Rightarrow$ $\dfrac{x^2}{4}+\dfrac{y^2}{9}=1$



Area : $\pi\cdot 2\cdot 3 = \underline{6\pi}$

Volume $V = 2\pi\int_0^3 g(y)^2\,dy = 2\pi\int_0^3 4-\frac{4}{9}y^2\,dy$

$= 2\pi\left[4y-\frac{4}{27}y^3\right]_0^3 = 2\pi(12-4) = \underline{16\pi}$

2. $F(s) = \dfrac{s}{s^2+1}$, $G(s) = \dfrac{1}{s^3}-\dfrac{1}{s^2}+\dfrac{2}{s}-\dfrac{2}{s+1} = \dfrac{s+1-s^2-s+2s^3+2s^2-2s^3}{s^3(s+1)}$

$= \dfrac{s^2+1}{s^3(s+1)}$.

$F(s)G(s) = \dfrac{1}{s^2(s+1)} = \dfrac{A}{s}+\dfrac{B}{s^2}+\dfrac{C}{s+1}$,

$1 = As(s+1)+B(s+1)+Cs^2 = As^2+As+Bs+B+Cs^2$

$=(A+C)s^2+(A+B)s+B.$ $\Rightarrow$ $B=1,\ A=-1,\ C=1$

$F(s)G(s) = \dfrac{1}{s^2}-\dfrac{1}{s}+\dfrac{1}{s+1}$

$\mathcal{L}^{-1}\{F(s)G(s)\} = f(t)*g(t) = \underline{(t-1+e^{-t})u_s(t)}$.

3. (1) characteristic equation : $|\lambda I-A|=0$.

$\lambda I-A = \begin{bmatrix} \lambda & -1 & 0 & 0 \\ 0 & \lambda & -1 & 0 \\ 0 & 0 & \lambda & -1 \\ 1 & -2 & 2 & \lambda-1 \end{bmatrix}$, $|\lambda I-A| = \lambda\left(\lambda(\lambda^2-\lambda+2)-2\right)-(-1)$

$= \underline{\lambda^4-\lambda^3+2\lambda^2-2\lambda+1=0}$

(2) Using Gauss-Jordan,

$$\left[\begin{array}{cccc|cccc} 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ -1 & 2 & -2 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \to \left[\begin{array}{cccc|cccc} -1 & 2 & -2 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{array}\right]$$

$$\downarrow$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 2 & -2 & 1 & -1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{array}\right] \leftarrow \left[\begin{array}{cccc|cccc} -1 & 0 & 0 & 0 & -2 & 2 & -1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{array}\right]$$

$$\therefore A^{-1} = \begin{bmatrix} 2 & -2 & 1 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

(3) $$A^2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 2 & -2 & 1 \\ -1 & 1 & 0 & -1 \end{bmatrix}, \quad A^3 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ -1 & 2 & -2 & 1 \\ -1 & 1 & 0 & -1 \\ 1 & -3 & 3 & -1 \end{bmatrix}.$$

$$A^5 = \begin{bmatrix} -1 & 1 & 0 & -1 \\ 1 & -3 & 3 & -1 \\ 1 & 5 & -1 & 2 \\ -2 & 5 & -5 & 1 \end{bmatrix}, \quad A^{10} = \begin{bmatrix} -1 & 1 & 0 & -1 \\ 1 & -3 & 3 & -1 \\ 1 & 5 & -1 & 2 \\ -2 & 5 & -5 & 1 \end{bmatrix} \begin{bmatrix} -1 & 1 & 0 & -1 \\ 1 & -3 & 3 & -1 \\ 1 & 5 & -1 & 2 \\ -2 & 5 & -5 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -9 & 8 & -1 \\ 1 & 20 & -7 & 7 \\ -1 & -9 & 6 & -6 \\ 0 & -37 & 15 & -12 \end{bmatrix}$$

4. $x_i \in \mathbb{R}^2$, $y_i \in \mathbb{R}^3$ 이므로 system matrix H: 3×2 matrix. $(H\underline{x} = \underline{y})$

Let $H = \begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix}$, $\begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}$, $\begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix}\begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$

$\begin{pmatrix} 2a+b=2 \\ a+b=1 \end{pmatrix}$ $\begin{pmatrix} 2c+d=1 \\ c+d=1 \end{pmatrix}$ $\begin{pmatrix} 2e+f=1 \\ e+f=3 \end{pmatrix}$

$a=1$ $b=0$ $c=0$ $d=1$ $e=-2$ $f=5$

$\therefore H = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -2 & 5 \end{bmatrix}$

(a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -2 & 5 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \underbrace{\begin{bmatrix} u_1 \\ u_2 \\ -2u_1+5u_2 \end{bmatrix}}$: output.

(b) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -2 & 5 \end{bmatrix}\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow a_1 = a_2 = 0$ $\therefore \underbrace{\left\{\begin{bmatrix} 0 \\ 0 \end{bmatrix}\right\}}$.

5. (a) (i) n=0 인 경우 $C_0 = \frac{1}{T}\int_0^T f(t)\,dt = \frac{1}{2}\int_0^2 t^2 dt = \underline{\frac{4}{3}}$

(ii) n>0 인 경우 $C_n = \frac{1}{T}\int_0^T f(t) e^{-j\frac{2\pi}{T}nt} dt = \frac{1}{2}\int_0^2 t^2 e^{-j\pi n t} dt$,

using integral by parts,

$$2C_n = \frac{t^2 e^{-j\pi n t}}{-j\pi n}\bigg|_0^2 + \frac{2}{j\pi n}\int_0^2 t e^{-j\pi n t} dt = \frac{4}{-j\pi n} + \frac{2}{j\pi n}\int_0^2 t e^{-j\pi n t} dt$$

$$= \frac{4}{-j\pi n} + \frac{2}{j\pi n}\left[\frac{t e^{-j\pi n t}}{-j\pi n}\bigg|_0^2 + \frac{1}{j\pi n}\int_0^2 e^{-j\pi n t} dt\right]$$

$$= \frac{4}{-j\pi n} + \frac{4}{\pi^2 n^2} - \left(\frac{1}{j\pi n}\right)^2 \cdot e^{-j\pi n t}\bigg|_0^2 = -\frac{4}{j\pi n} + \frac{4}{\pi^2 n^2}$$

$$\therefore \underline{C_n = \frac{2}{\pi^2 n^2} + j\frac{2}{\pi n}}$$

(b) $f(t) = t^2 = \sum_{n=-\infty}^{\infty} C_n e^{j\frac{2\pi}{T}nt} = \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty}\left(\frac{2}{\pi^2 n^2} + j\frac{2}{\pi n}\right) e^{j\pi n t} + \frac{4}{3}$

For $t=2$, $4 - \frac{4}{3} = \sum_{n=-\infty}^{-1}\left(\frac{2}{\pi^2 n^2} + j\frac{2}{\pi n}\right) + \sum_{n=1}^{\infty}\left(\frac{2}{\pi^2 n^2} + j\frac{2}{\pi n}\right)$

$$= 2\sum_{n=1}^{\infty}\frac{2}{\pi^2 n^2} = \frac{4}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2} = \frac{8}{3}$$

$$\therefore \underline{\sum_{n=1}^{\infty}\frac{1}{n^2} = \frac{2}{3}\pi^2}$$

2016 통신

1.

(a) $m(-t)$: $\int_{-\infty}^{\infty} m(-t) e^{-j2\pi f t} dt = -\int_{-\infty}^{\infty} m(\tau) e^{-j2\pi(-f)\tau} d\tau = -M(-f)$

$\therefore \underline{m(-t) \overset{F}{\longleftrightarrow} -M(-f)}$

$m^*(t)$: $\int_{-\infty}^{\infty} m^*(t) e^{-j2\pi f t} dt = \left\{ \int_{-\infty}^{\infty} m(t) e^{-j2\pi(-f)t} dt \right\}^* = M^*(-f)$.

$\therefore \underline{m^*(t) \overset{F}{\longleftrightarrow} M^*(-f)}$

$m^*(-t)$: $\int_{-\infty}^{\infty} m^*(-t) e^{-j2\pi f t} dt = -\int_{-\infty}^{\infty} m^*(\tau) e^{j2\pi f \tau} d\tau = -\left\{ \int_{-\infty}^{\infty} m(\tau) e^{-j2\pi f \tau} d\tau \right\}^*$

$= -M^*(f)$.

$\therefore \underline{m^*(-t) \overset{F}{\longleftrightarrow} -M^*(f)}$

(b) $M^*(-f) = \left\{ \int_{-\infty}^{\infty} m(t) e^{j2\pi f t} dt \right\}^* = \int_{-\infty}^{\infty} m^*(t) e^{-j2\pi f t} dt = \int_{-\infty}^{\infty} m(t) e^{-j2\pi f t} dt$

$= M(f)$ $\quad \therefore \underline{M(f) = M^*(-f), \ \forall f}$

(c) $y(t) = \alpha \cos\left(2\pi f_c t + \beta \int_{-\infty}^{t} m(\tau) d\tau\right)$

$\frac{d}{dt} y(t) = -\alpha \sin\left(2\pi f_c t + \beta \int_{-\infty}^{t} m(\tau) d\tau\right) \cdot \left(2\pi f_c + \beta m(t)\right)$ (differentiator)

(Envelope detector)

$\underline{g(t) = -\alpha\beta m(t) + 2\pi f_c \rightarrow A m(t) + B \quad (A, B : \text{Constant})}$

DC 성분 (B) 제거 후 gain ($\frac{1}{A}$) 곱해서 m(t) recover.

2. (a)

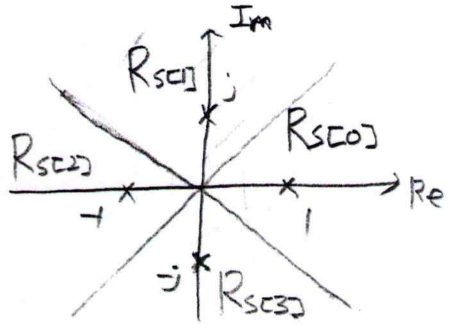
If we sample $g(t)$ at $t=nT$, $n\in\mathbb{Z}$, we have

$$g(nT)=\begin{cases}\frac{1}{\sqrt{\frac{2}{3}T}}, & n=0\\ 0, & n\neq 0.\end{cases} \Rightarrow \frac{1}{T}\sum_{k=-\infty}^{\infty}G\left(f-\frac{k}{T}\right)=\text{constant},\ \forall f.$$

$\therefore$ Zero ISI Nyquist criterion is satisfied,

$g(t)$ is Nyquist pulse shape.

(b)



← Voronoi region.

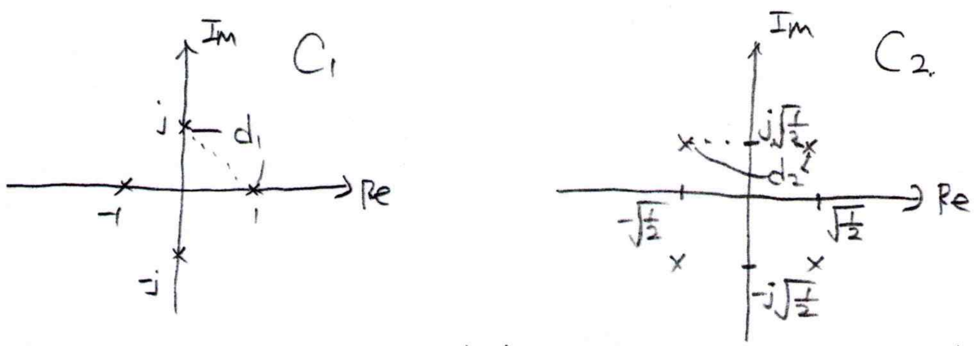
Suppose that $s[n]\in C_1$ is equally likely, the ML detection rule is

$$\hat{s}_{ML}=\underset{s[n]\in C_1}{\arg\max}\ f_{y[n]|s[n]}(y[n]|s[n])=\underset{s[n]\in C_1}{\arg\max}\ k\exp\left(-\|y[n]-s[n]\|^2\right)$$

$$=\underset{s[n]\in C_1}{\arg\min}\ \|y[n]-s[n]\|^2.$$ So, it becomes a Minimum Distance (MD) detection rule.

Then we can define Voronoi region like above constellation.
If we receive a symbol in $R_{s[l]}$ $(l=0,1,2,3)$, we can state that the $s[l]$ was transmitted.

(c)



Since C_1, C_2 have equal symbol energy and equal minimum distance between symbols, the Symbol Error Rate of C_1 and C_2 is equal.

2016 제어.

제어 필수

1.

1) $V(t) = L\frac{di_L(t)}{dt} + Ri_R(t)$, $Ri_R(t) = \frac{1}{C}\int_{-\infty}^{t} i_C(t)dt$, $i_C(t) = RC\frac{di_R(t)}{dt}$.

$$i_L(t) = i_R(t) + i_C(t) = i_R(t) + RC\frac{di_R(t)}{dt}$$

$$V(t) = L\frac{d}{dt}\left(i_R(t) + RC\frac{di_R(t)}{dt}\right) + Ri_R(t)$$

$$\therefore \underline{V(t) = LRC\frac{d^2}{dt^2}i_R(t) + L\frac{d}{dt}i_R(t) + Ri_R(t)}$$

2) $V(s) = LRCs^2 I_R(s) + LsI_R(s) + RI_R(s)$

$$= I_R(s)(LRCs^2 + Ls + R)$$

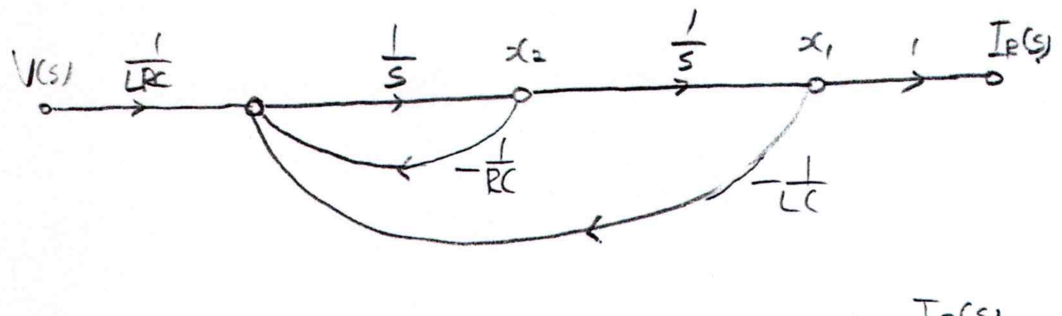
$$\therefore \underline{\frac{I_R(s)}{V(s)} = \frac{1}{LRCs^2 + Ls + R}}$$ (initial condition = 0 으로 가정)

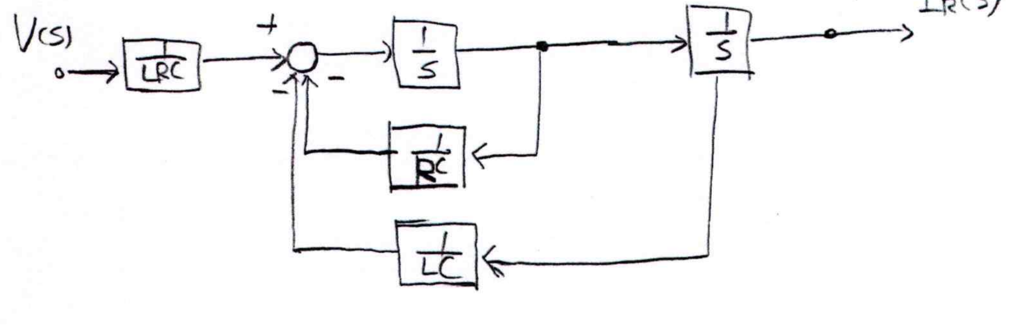
3) $x_1(t) = i_R(t)$, $x_2(t) = \dot{i}_R(t)$, $y(t) = i_R(t) = x_1(t)$

4)

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{1}{LC} & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{LRC} \end{bmatrix} V(t)$$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$ ⟸ state space model.





· 제어 선택

1. ∘ controllable at t_0

: t_0에서의 임의의 state $X(t_0)$를 원하는 상태 $X(t_1)$으로 이동시키는 input $u[t_0, t_1]$ ($t_1 > t_0$, $t_1 < \infty$) 이 존재

∘ Fundamental matrix $F(t)$

: Nonhomogeneous equation $\dot{x} = Ax$를 만족하는 vector x를 $\psi_1, \psi_2, \cdots$ 라고 할 때, linearly independent한 ψ_i, ψ_j 들로 $(i \neq j)$ 이루어진 matrix $F(t)$.

∘ State transition matrix $\Phi(t, t_0)$

: $\Phi(t, t_0) x(t_0) = x(t)$ $\Rightarrow$ $\Phi(t, t_0) = x(t) x^{-1}(t_0)$. 즉 state $x(t_0)$를 state $x(t)$로 만드는 matrix $\Phi(t, t_0)$

∘ BIBO stable.

: Initial condition = 0 인 relaxed system에서, 임의의 bounded input에 대해 bounded output 이 나오면 BIBO stable.

2. 1) $\bar{x}(t) = Px(t)$. , $\dot{\bar{x}}(t) = P\dot{x}(t)$, 원래의 state space를 변형하면

$$P^{-1}\dot{\bar{x}}(t) = AP^{-1}\bar{x}(t) + Bu(t), \quad y(t) = CP^{-1}\bar{x}(t)$$

$$\Rightarrow \dot{\bar{x}}(t) = PAP^{-1}\bar{x}(t) + PBu(t), \quad y(t) = CP^{-1}\bar{x}(t)$$

$\therefore$ $\bar{A} = PAP^{-1}$, $\bar{B} = PB$, $\bar{C} = CP^{-1}$

2) $$[\bar{B}\ \bar{A}\bar{B}\ \cdots\ \bar{A}^n\bar{B}] = [PB\ \ PAB\ \cdots\ PA^nB]$$
$$= P[B\ \ AB\ \cdots\ A^nB]$$

P는 nonsingular matrix 이므로

$\mathrm{rank}([\bar{B}\ \bar{A}\bar{B} \cdots \bar{A}^n\bar{B}]) = \mathrm{rank}([B\ \ AB \cdots A^nB])$.

$\therefore$ 두 system의 controllable은 iff 관계 성립한다.