

Advanced Optics (PHYS690)

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Syllabus

- **Goal:** This course covers the various topics in the modern optics.
- **Topics:** Gaussian beam optics, LASER, polarization, electro-optics, nonlinear optics, Fourier optics, Guided wave optics, and other applications
- **Lecturer**
 - Prof. Heedeuk Shin (heedeukshin@postech.ac.kr)
 - Bldg 3, room 421
- **Grading**

- Homework	30%
- Mid term	30%
- Final	40%
- **Textbook:**
 - “Fundamentals of Photonics” by Saleh & Teich (Wiley, 2007)
 - “Nonlinear Optics” by Robert W. Boyd (Academic press, 2008)
 - “Introduction to Fourier Optics” by Goodman (McGraw-Hill, 1996)

Tuesday & Thursday

Officially 09:30 ~ 10:45,

How about 09:30 ~ 11:00?

Then no need to have make-up class

No class at 2/21 (광학회), 4/25(물리학회)



Syllabus

- **Schedule:**

1. Gaussian optics (2/21 No class)
2. Gaussian optics/LASER
3. LASER
4. Polarization optics
5. Guided-wave optics
6. Interference
7. Electro-optical effects and acousto-optical effects
8. Mid-term exam
9. Fourier optics
10. Fourier optics (4/25 No class)
11. Nonlinear Optics 2
12. Nonlinear Optics 3
13. Nonlinear Optics 4
14. Nonlinear Optics 5
15. Nonlinear Optics 6
16. Final exam



Today

Introduction to optics

Gaussian beam

Through a lens

ABCD Law

Knife edge technique

POSTECH
Advanced Optics class



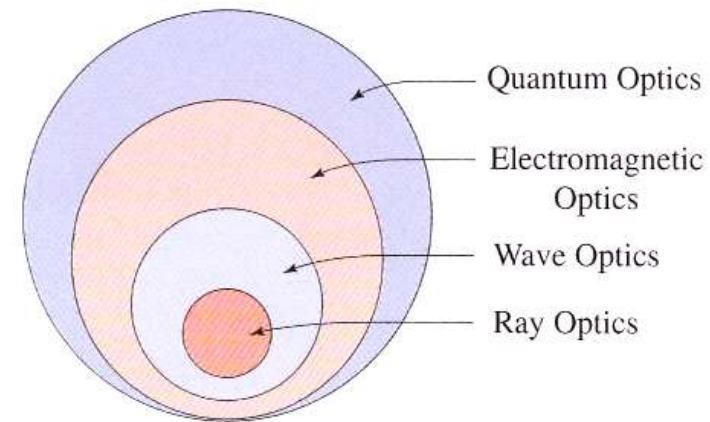
Introduction

POSTECH
Advanced Optics class

- **Optics** is the study field of the behavior and properties of light.
- Most optical phenomena can be understood by using the classical electromagnetics.



Archimedes' mirror used to burn Roman ships



Optics vs Photonics ?



Optics

versus

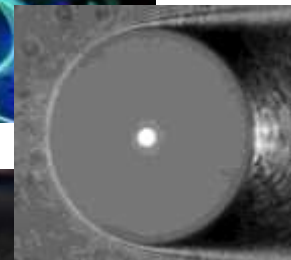
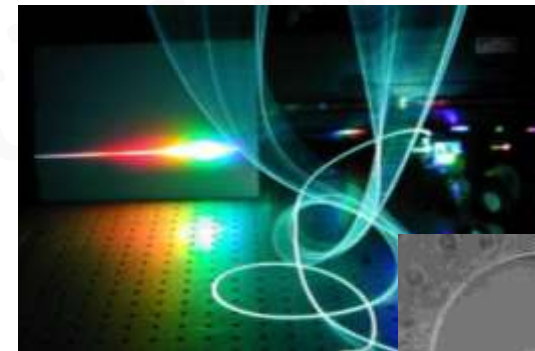
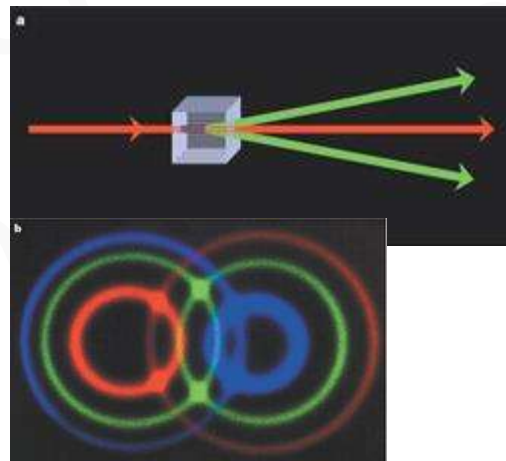
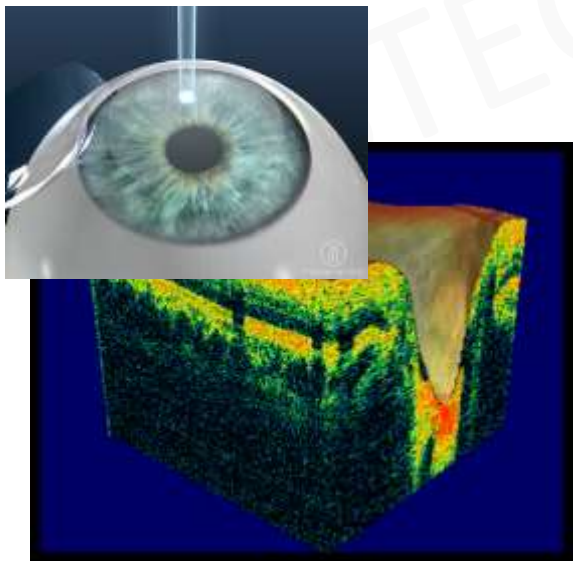


Photonics

- **Optics:** the study field of the behavior and properties of light.
- **Photonics:** the science of light (photon) generation, detection, and manipulation through emission, transmission, modulation, signal processing, switching, amplification, and detection/sensing.
- Photonics is closely related to Optics.
- Photonics is used to connote **applied research and development**.

The term *photonics* more specifically connotes:

- The particle properties of light,
- The potential of creating signal processing device technologies using photons,
- The practical application of optics, and
- An analogy to electronics.



Gaussian Beam optics

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Advanced Optics class

Gaussian beam



- A **Gaussian beam** is a beam of monochromatic electromagnetic radiation.
- Its transverse magnetic and electric field amplitude profiles are given by the Gaussian function
- The fundamental (or TEM_{00}) transverse Gaussian mode describes the intended output of most (but not all) lasers, as such a beam can be focused into the most concentrated spot.
- The mathematical expression for the electric field amplitude is a solution to the paraxial Helmholtz equation.

Gaussian beam

- complex amplitude $U(\mathbf{r})$ of the Gaussian beam:

$$U(\mathbf{r}) = A_0 \frac{W_0}{W(z)} \exp \left[-\frac{\rho^2}{W^2(z)} \right] \exp[-jkz + j\zeta(z)] \exp \left[-jk \frac{\rho^2}{2R(z)} \right],$$

- $W(z) = W_0 \sqrt{1 + \left(\frac{z}{z_0} \right)^2}$: beam width
- $R(z) = z \left[1 + \left(\frac{z_0}{z} \right)^2 \right]$: radius of curvature
- $\zeta(z) = \tan^{-1} \frac{z}{z_0}$: phase retardation
- $W_0 = \sqrt{\frac{\lambda z_0}{\pi}}$: waist radius
- $z_0 = \frac{\pi n W_0^2}{\lambda_0}$: Rayleigh length

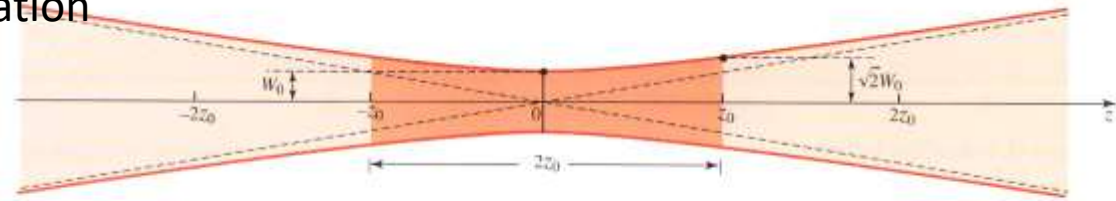
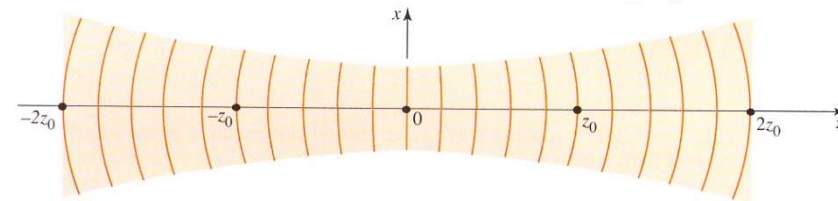


Figure 3.1-4 Depth of focus of a Gaussian beam.

$$I(\rho, z) = |U(\mathbf{r})|^2 = I_0 \left[\frac{W_0}{W(z)} \right]^2 \exp \left[-\frac{2\rho^2}{W^2(z)} \right],$$

Through a thin lens

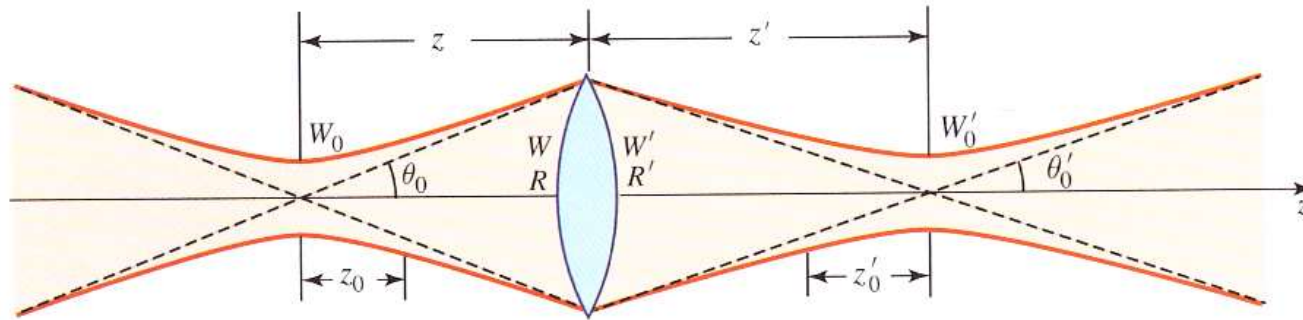


Figure 3.2-1 Transmission of a Gaussian beam through a thin lens.

- Waist radius
- Waist location
- Depth of focus
- Divergence angle
- Magnification

$$W'_0 = MW_0$$

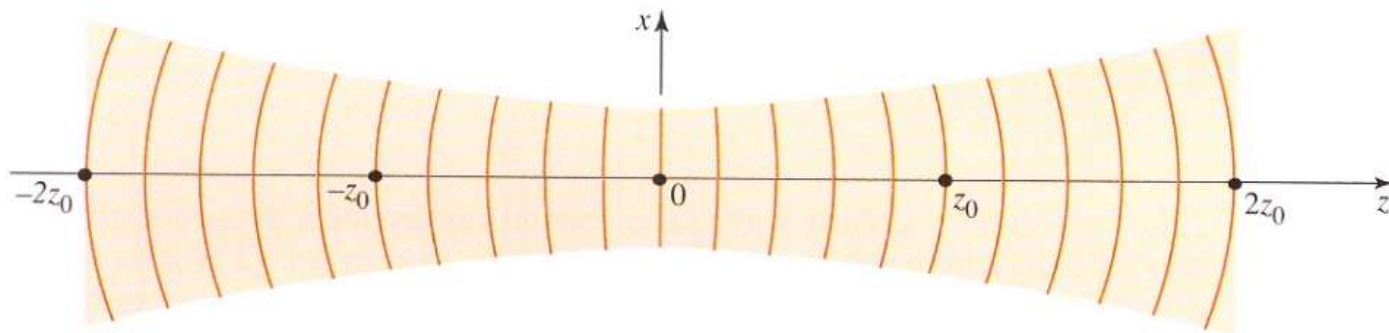
$$(z' - f) = M^2(z - f)$$

$$2z'_0 = M^2(2z_0)$$

$$2\theta'_0 = \frac{2\theta_0}{M}$$

$$M = \frac{M_r}{\sqrt{1+r^2}},$$

where $r = \frac{z_0}{z-f}$ and $M_r = \left| \frac{f}{z-f} \right|$: parameter transformation by a lens

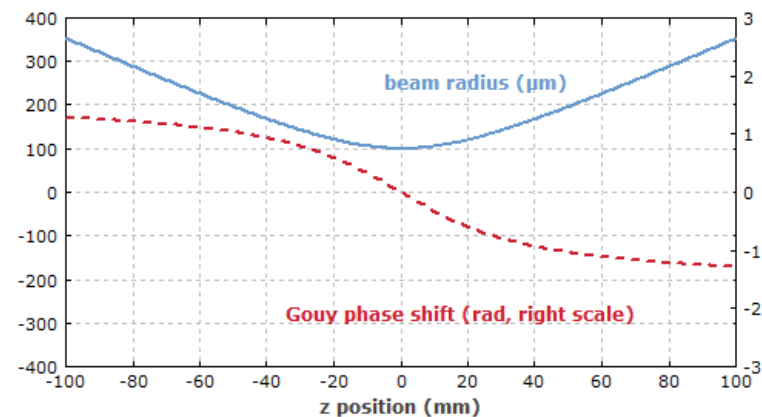
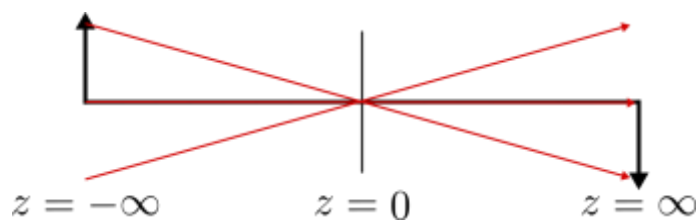


$$U(\mathbf{r}) = A_0 A_0 \frac{W_0}{W(z)} \exp\left[-\frac{\rho^2}{W^2(z)}\right] \exp[-jkz + j\zeta(z)] \exp\left[-jk \frac{\rho^2}{2R(z)}\right] \exp\left[-jkz - jk \frac{\rho^2}{2R(z)} + j\zeta(z)\right]$$

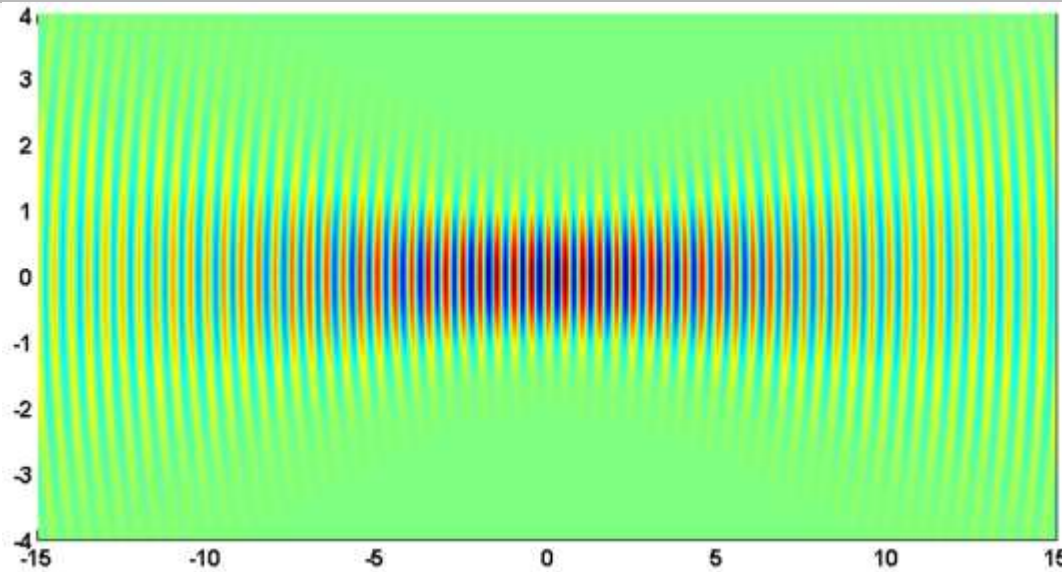
Plane wave

Wavefront bending

Gouy phase



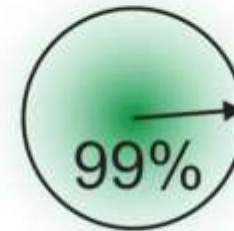
a axial phase shift when passing through its focus.



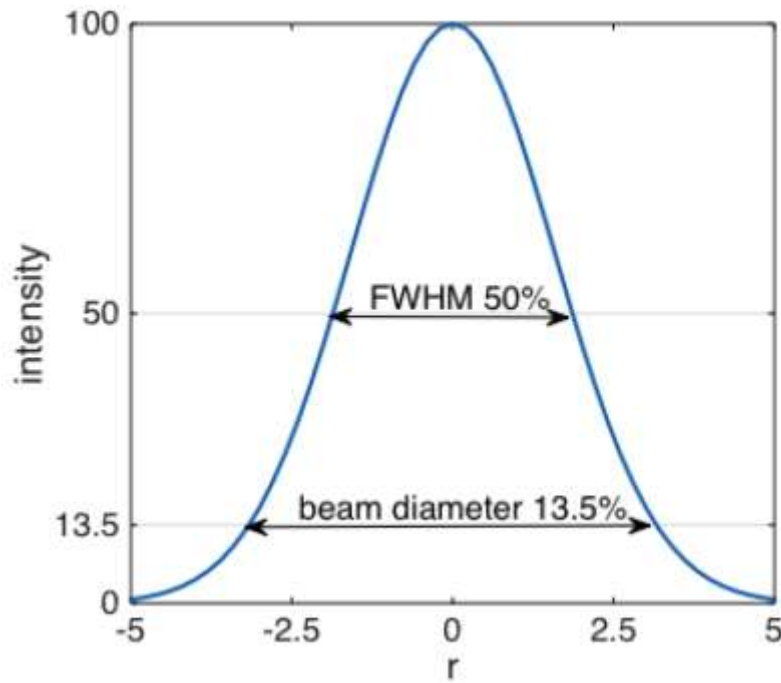
Beam power

$$P = \int_0^{\infty} I(r, z) 2\pi r dr$$

$$P = \frac{1}{2} I_0 \pi w_0^2 \text{ [Watt]}$$



$$r = 1.5w(z)$$



Intensity drops by factor e^{-2}

$$1/e^2 \sim 0.135 = 13.5\%$$

$$I(r, z) = I_0 \left[\frac{w_0}{w(z)} \right]^2 e^{-\frac{2r^2}{w(z)^2}}$$

Beam Focusing

- Focusing a Gaussian beam with a lens at the beam waist

$$W'_0 = \frac{W_0}{\sqrt{1+(z_0/f)^2}}$$

$$z' = \frac{f}{1+(f/z_0)^2}$$

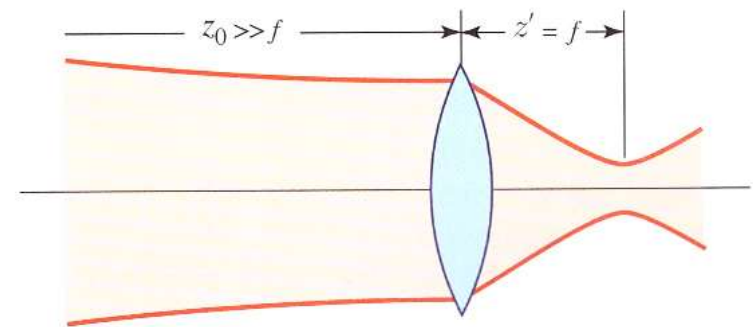
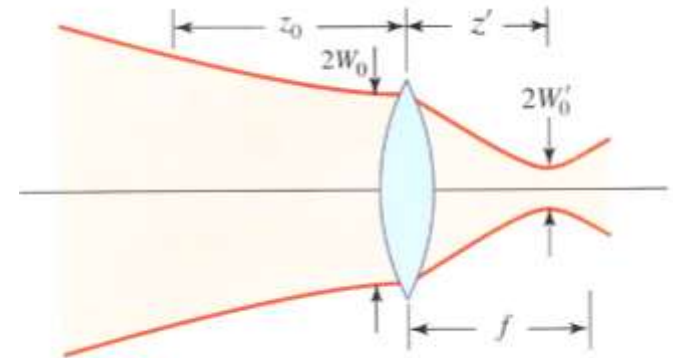
- When $z_0 \gg f$, using $2z_0 = \frac{2\pi W_0^2}{\lambda}$

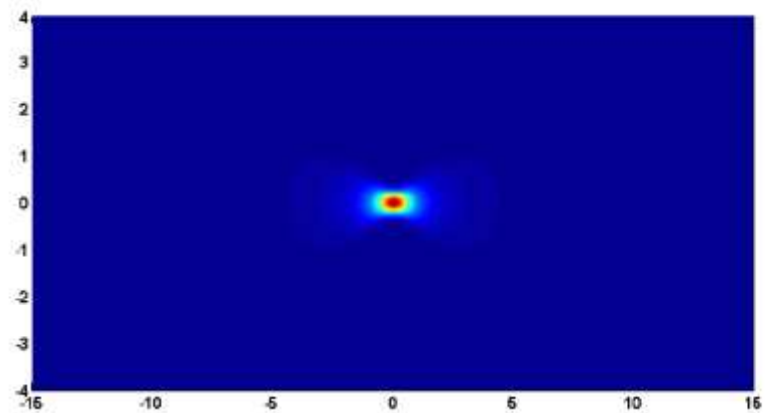
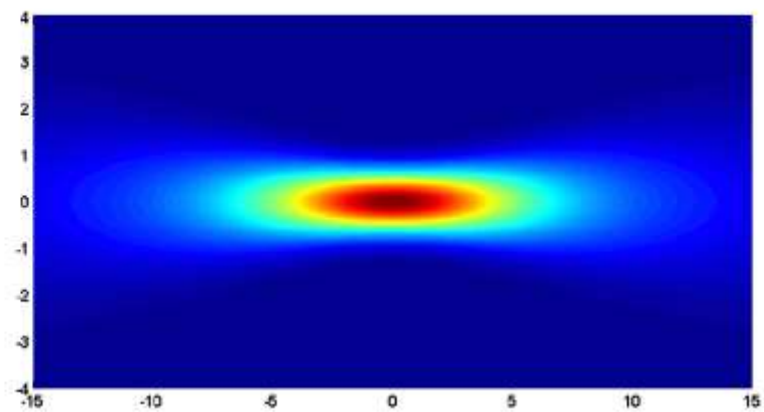
$$W'_0 \approx \frac{f}{z_0} W_0 = \frac{\lambda}{\pi W_0} f = \theta_0 f$$

$$z' \approx f$$

$$D = 2W_0$$

$$W'_0 \approx \frac{4}{\pi} \lambda F_{\#}, \text{ where } F_{\#} = \frac{f}{D}: \text{F-number of the lens}$$

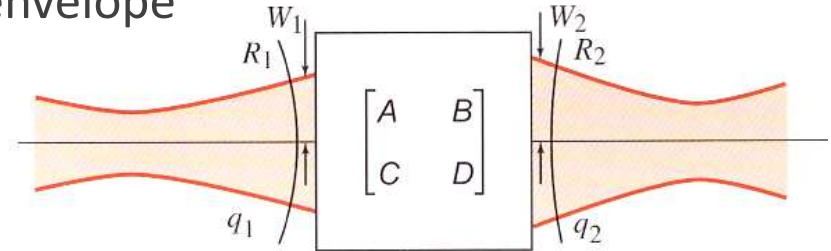




ABCD law

- amplitude and phase of the complex envelope

$$\frac{1}{q(z)} = \frac{1}{R(z)} - j \frac{\lambda}{\pi W^2(z)}$$



- **q- parameter**

- The Gaussian beam propagation can be described with matrix optics.

$$q_2(z) = \frac{Aq_1 + B}{Cq_1 + D}$$

$$R(z) = z \left[1 + \left(\frac{z_0}{z} \right)^2 \right]$$

- Free space propagation ABCD

$$\begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} \Rightarrow q_2 = q_1 + d \quad (\because q = z + jz_0)$$

- Thin lens ABCD

$$\begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix}$$

- Example of ABCD law

q_1

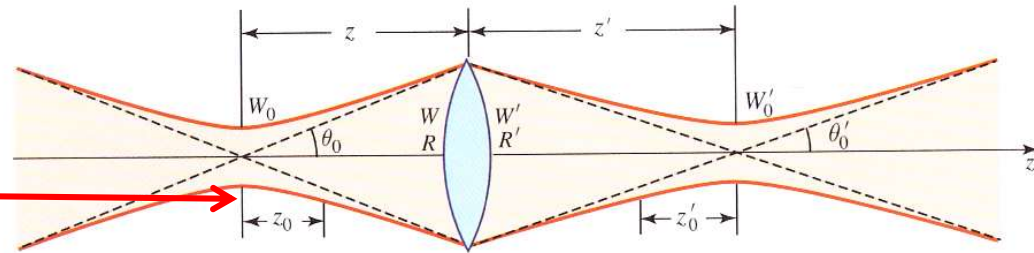


Figure 3.2-1 Transmission of a Gaussian beam through a thin lens.

- Where is the waist and how big is it?

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & z_2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix} \begin{bmatrix} 1 & z_1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 - \frac{z_2}{f} & z_1 + z_2 - \frac{z_1 z_2}{f} \\ \frac{-1}{f} & 1 \end{bmatrix}$$

$$q_2(z) = \frac{Aq_1 + B}{Cq_1 + D} = \left[\frac{1}{R_2(z_2)} - j \frac{\lambda}{\pi W_2^2(z_2)} \right]^{-1}$$

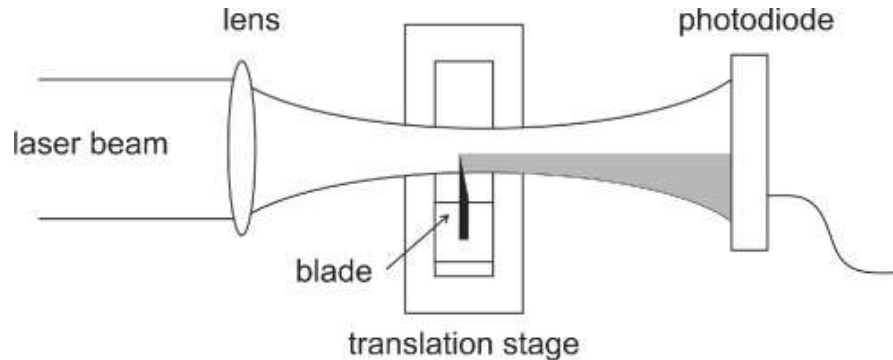
$$\frac{1}{W_2^2(z_2)} = \frac{1}{W_1^2(z_1)} \left(1 - \frac{z_1}{f} \right)^2 + \left(\frac{\pi W_1^2(z_1)}{f \lambda} \right)^2$$

$$z_2 = f + \frac{(z_1 - f)f^2}{(z_1 - f)^2 + \left(\frac{\pi W_1^2(z_1)}{\lambda} \right)^2}$$

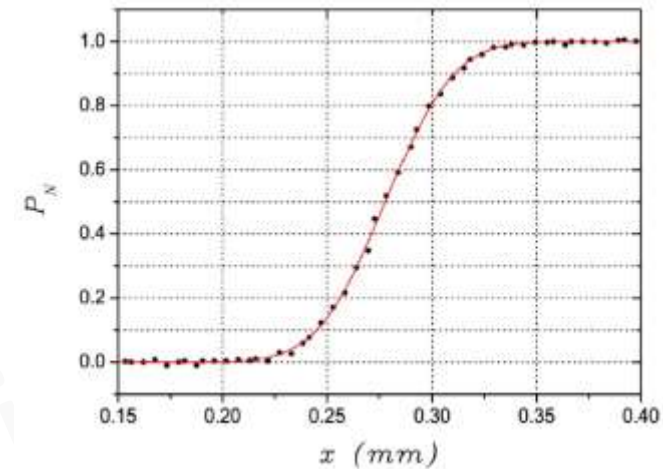
Minimum spot size does not occur in the lens focal plane.

Knife edge technique

- Measures of Gaussian beam waist at position z



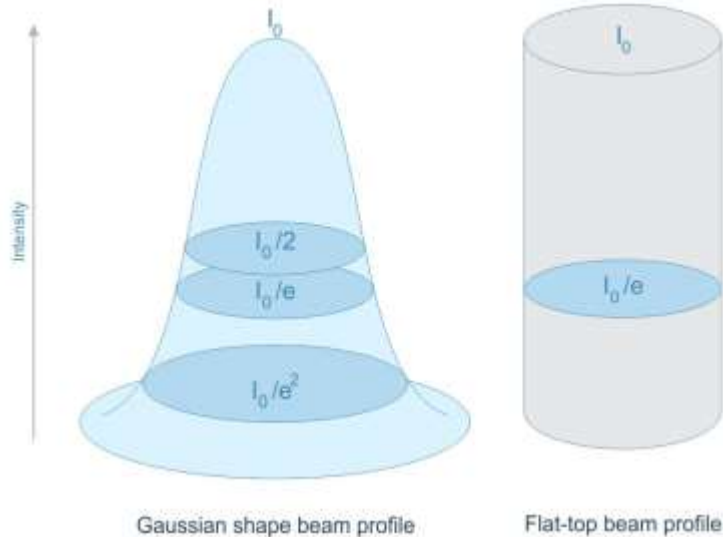
Applied Optics 48, 393 (2009)



$$X_{90\%-10\%} = 1.28W$$

Examples

- Verdi series laser



Verdi	G10
Wavelength (nm)	532 ± 2
Pulse Format	CW
Spectral Purity (%)	>99
Output Power (W)	10^2
Spatial Mode	TEM_{00}
Beam Quality	<1.1
Beam Circularity ³	1.0 ± 0.1
Beam Waist Diameter (mm)(FW, $1/e^2$)	$2.25 \pm 10\%$
Beam Divergence (mrad)(FW, $1/e^2$)	<0.5
Beam Waist Location ⁴ (m)	± 0.5
Beam Pointing Stability ⁵ ($\mu\text{rad}/^\circ\text{C}$)	<2
Horizontal Beam Position Tolerance ⁶ (mm)	$\pm <1.0$
Vertical Beam Position Tolerance ⁶ (mm)	$\pm <1.0$
Polarization Ratio	Linear, $>100:1$
Polarization Direction	Vertical, $\pm 5^\circ$
Noise (% rms)(10 Hz to 100 MHz)	<0.02
Power Stability ⁷ (%)(pk-pk)	$\pm <1$
Warm-Up Time (minutes)	<10
CDRH Compliant	Yes

Examples

- Verdi series laser
- Power: 10 W
- Area: $\pi\left(\frac{D}{2}\right)^2 = \pi\left(\frac{2.25 \text{ [mm]}}{2}\right)^2$
- fs laser

System Specifications

Average Power (W)	10 W
Wavelength (nm)	1040
Pulse Repetition Rate (MHz)	80
Pulse Duration ¹ (fs)	140
Noise ² (%)	<0.25
Power Stability ³ (%)	±0.5
M ²	<1.2
Beam Diameter (mm)	1.2 (±0.2)
Ellipticity	0.8 to 1.2
Astigmatism (%)	<10
Polarization	100:1 Horizontal

Fidelity HP

High Power Femtosecond Fiber Laser



Laser beam quality factor

- M^2 is the formalism for a laser beam quality factor.
- M^2 indicates how close it is to being a fundamental-mode Gaussian beam.
- The focused spot diameter of a Gaussian beam is defined by

$$d_{00} = \frac{4\lambda f}{\pi D_{00}}$$

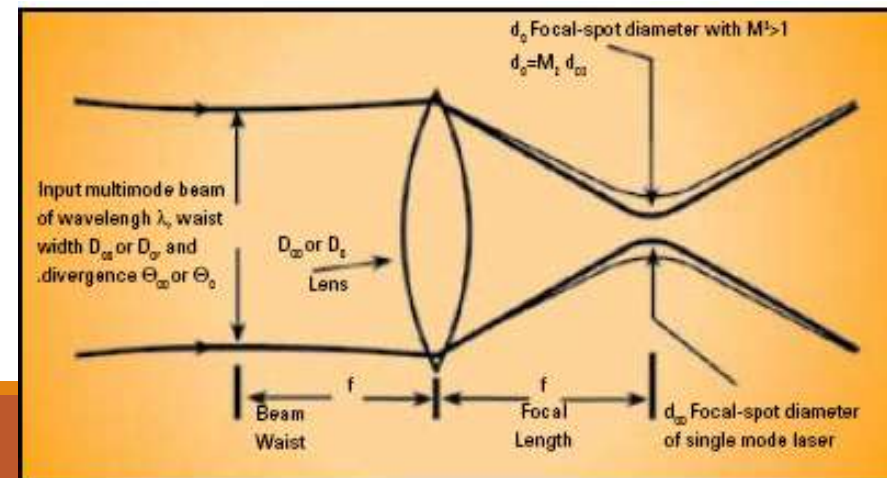
where d_{00} : the ideal focused spot diameter

f : the focal length of the focusing lens

D_{00} : the input beam waist

- When a multimode beam is focused,

$$d_0 = M^2 \frac{4\lambda f}{\pi D_0}$$



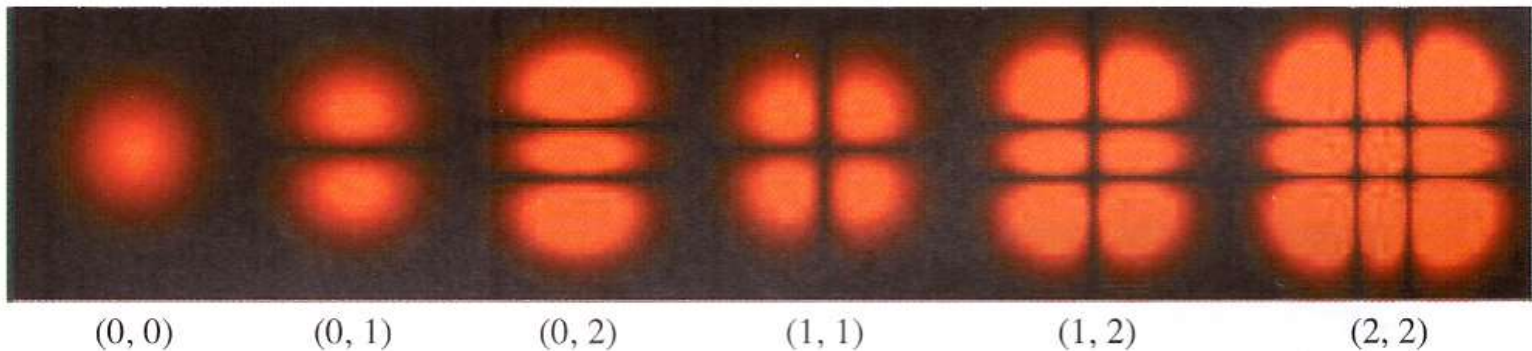
Other solutions of Helmholtz eq.

- Another solution of the paraxial Helmholtz equation:
- Hermite-Gaussian beam

$$U_{l,m}(x, y, z) = A_{l,m} \left[\frac{W_0}{W(z)} \right] G_l \left[\frac{\sqrt{2}x}{W(z)} \right] G_m \left[\frac{\sqrt{2}y}{W(z)} \right] \\ \times \exp \left[-jkz - jk \frac{x^2 + y^2}{2R(z)} + j(l + m + 1)\zeta(z) \right]$$

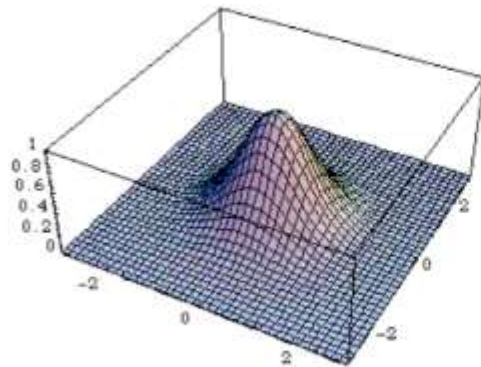
where $G_l(u) = H_l(u) \exp(\frac{-u^2}{2})$, $l = 0, 1, 2, \dots$

Hermite-Gaussian function of order l , and $A_{l,m}$ is a constant.

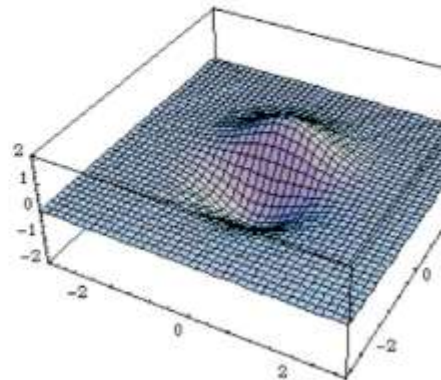


Hermit-Gaussian

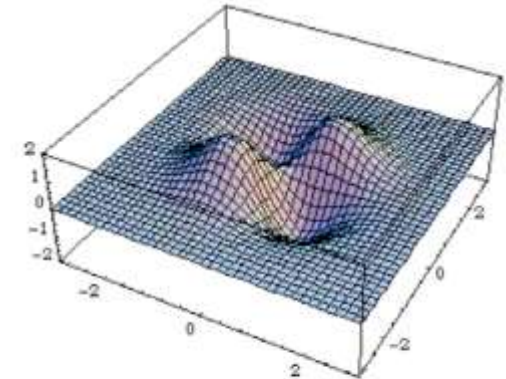
Complex amplitude (arb. units)



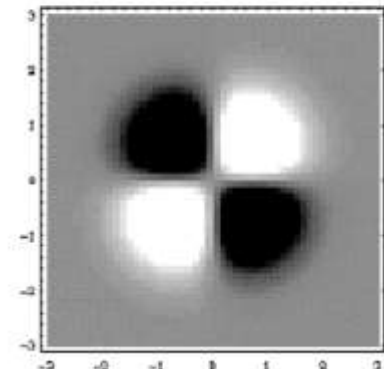
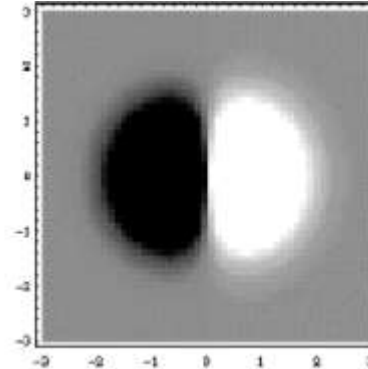
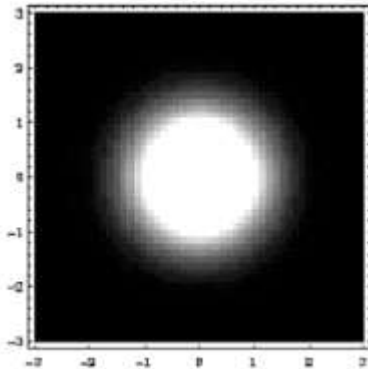
[0, 0] Hermite-Gaussian



[0, 1] Hermite-Gaussian



[1, 1] Hermite-Gaussian



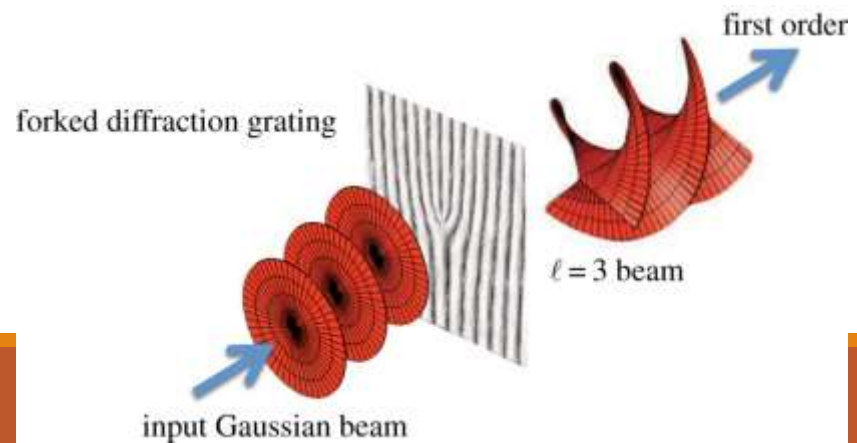
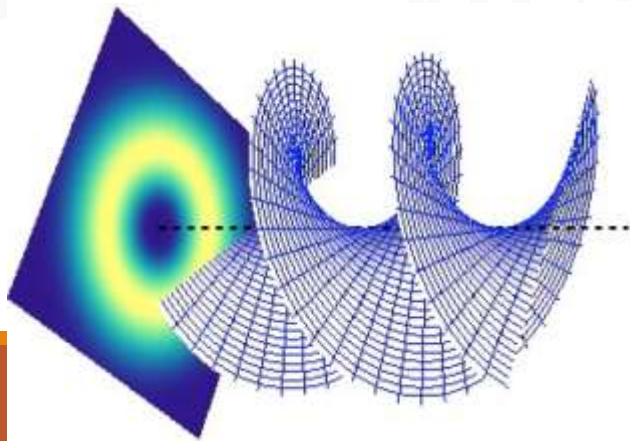
Other solutions of Helmholtz eq.

- Laguerre-Gaussian beam

$$U_{l,m}(\rho, \phi, z) = A_{l,m} \left[\frac{W_0}{W(z)} \right]^l \left(\frac{\rho}{W(z)} \right)^l L_m^l \left(\frac{2\rho^2}{W^2(z)} \right) \exp \left(-\frac{\rho^2}{W^2(z)} \right) \times \exp \left[-jkz - jk \frac{\rho^2}{2R(z)} - jl\phi + j(l + 2m + 1)\zeta(z) \right]$$

where $L_m^l(u)$: the generalized Laguerre polynomial function

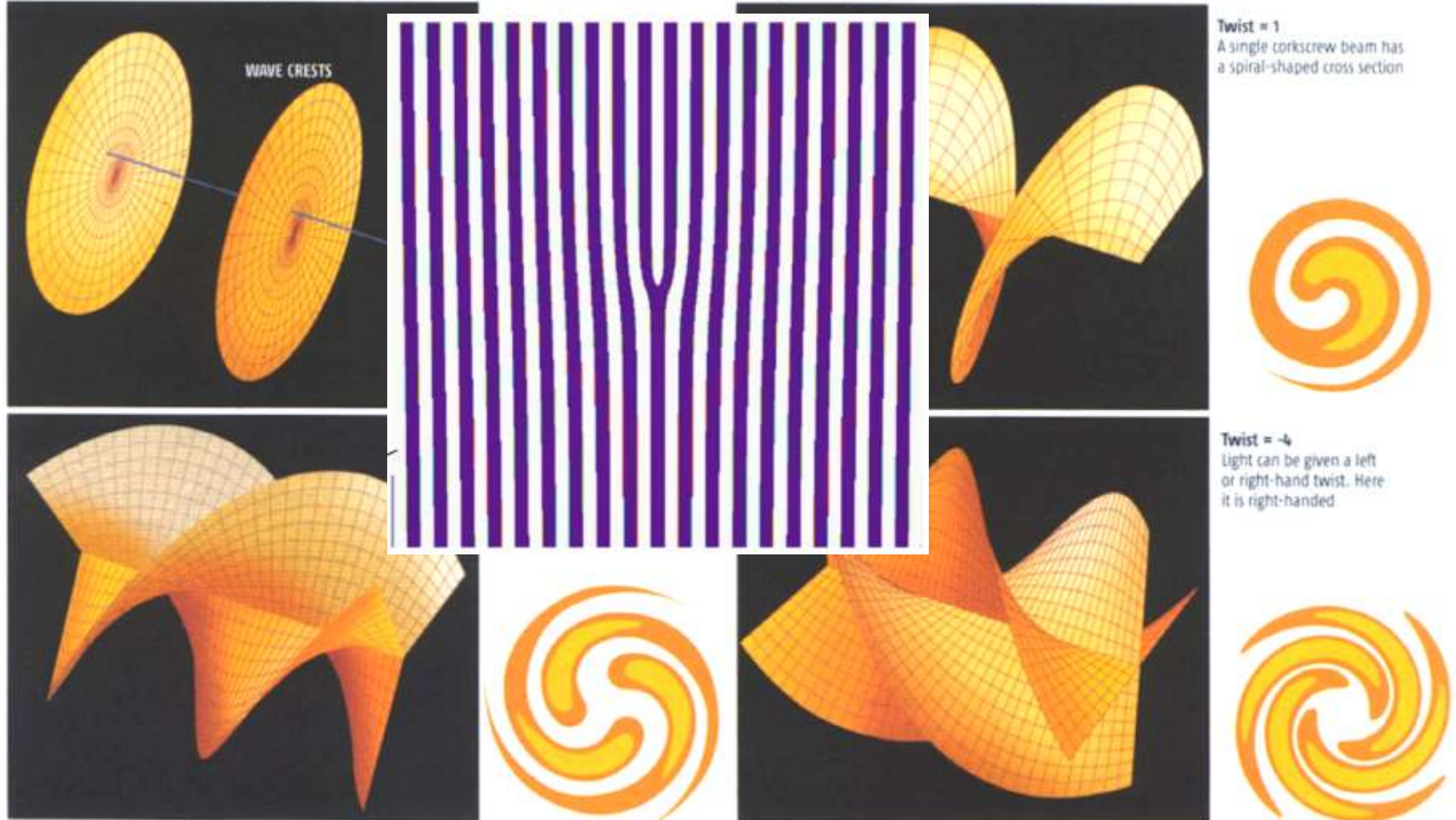
Orbital angular momentum (OAM)



Beams with angular momentum

CORKSCREW LIGHT

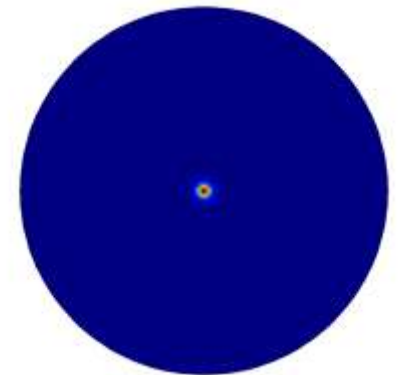
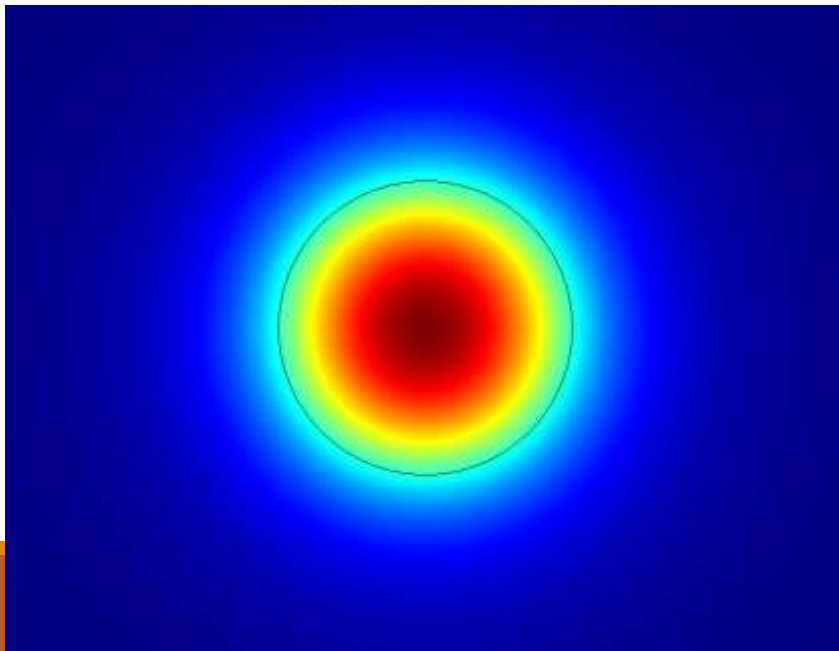
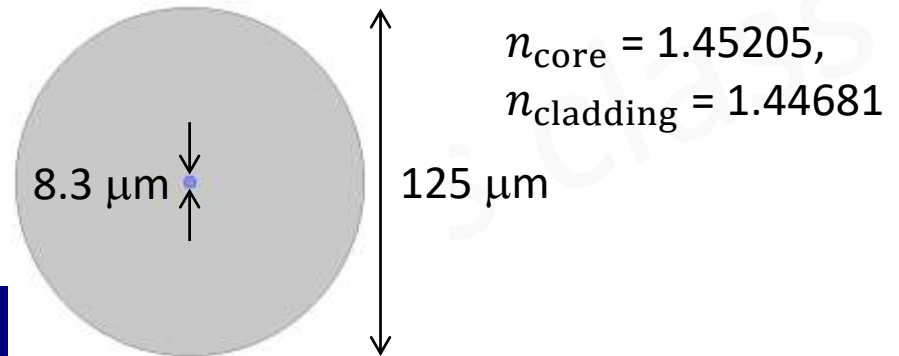
By giving laser photons orbital angular momentum, the wavefronts of light become twisted. To see the twist, researchers interfere the twisted light with normal laser light



<http://www.physics.gla.ac.uk/Optics/projects/AM/>

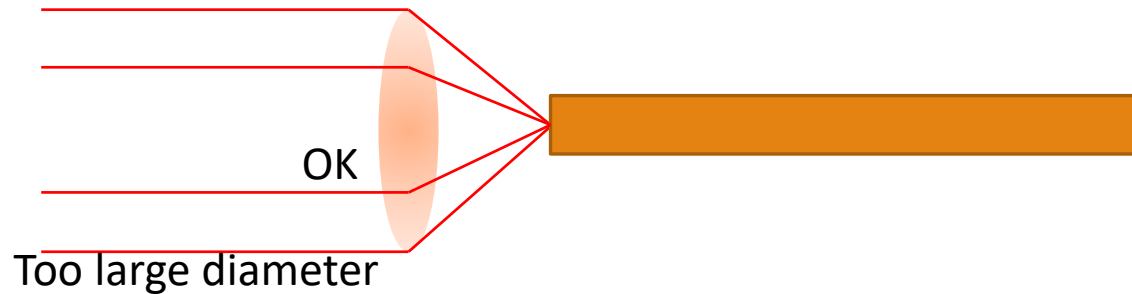
Optical fiber mode

- Modes in optical fiber
 - Definition of mode: self-consistent electric field distributions in waveguides, optical resonators or in free space.
 - In other words, its transverse intensity profile or the distribution of light energy across the fiber.
- What determines modes?
 - Boundary conditions



Coupling light to fiber

- NA of lens < NA of fiber



- Mode size matching

$$\text{MFD} = \phi_{\text{spot}} = \frac{4\lambda f}{\pi D}, \text{ or } f = \frac{\pi D(\text{MFD})}{4\lambda}$$