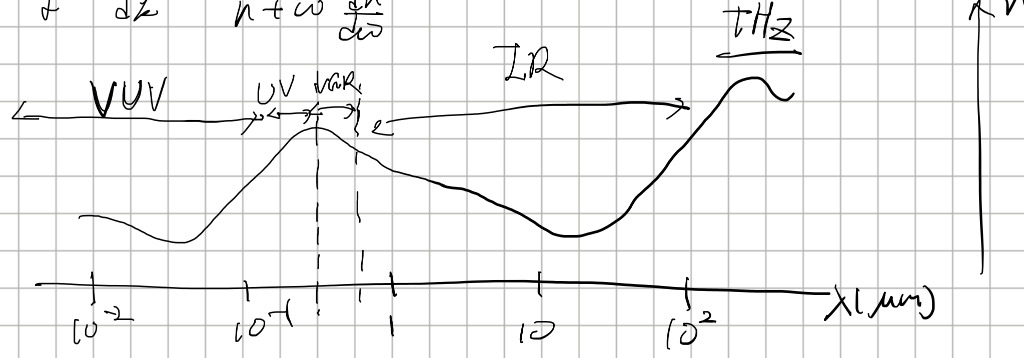
$$v_g = \frac{d\omega}{dk} = \frac{c}{n + \omega \frac{dn}{d\omega}}$$



In visible region, as $\omega\uparrow$, $n\uparrow$, $\frac{dn}{d\omega} > 0 \rightarrow v_g \downarrow$

$$v_{g,\text{blue}} < v_{g,\text{red}}$$

4.2 Electro magnetic plane waves in unbounded dielectric media

Current free $J = 0$

Charge free $\rho = 0$

$$\nabla \times \vec{H} - \frac{1}{c}\frac{\partial \vec{D}}{\partial t} = 0$$

$$\nabla \times \vec{E} + \frac{1}{c}\frac{\partial \vec{B}}{\partial t} = 0$$

$$\nabla^2 \begin{pmatrix} \vec{E} \\ \vec{H} \end{pmatrix} - \frac{1}{c}\frac{\partial^2}{\partial t^2}\begin{pmatrix} \vec{E} \\ \vec{H} \end{pmatrix} = 0$$

one of typical solution

$$\vec{E} = \vec{E_0}\, e^{i(\vec{k}\cdot\vec{r} - \omega t)}$$

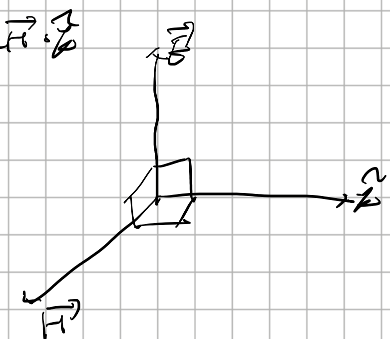
$$\vec{H} = \vec{H_0}\, e^{i(\vec{k}\cdot\vec{r} - \omega t)}$$

$$\vec{E} = -\sqrt{\frac{\mu}{\varepsilon}}\, \hat{k} \times \vec{H}$$

$$\vec{H} = \sqrt{\frac{\varepsilon}{\mu}}\, \hat{k} \times \vec{E}, \quad \hat{k} = \frac{\vec{k}}{k}$$

① $\vec{E}\cdot\hat{k} = 0 = \vec{H}\cdot\hat{k}$

$\vec{E}\cdot\vec{H} = 0$



(cf. dipole radiation)

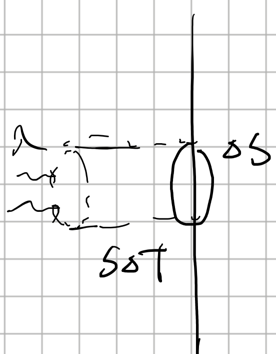
② $\sqrt{\varepsilon}\,|\vec{E}| = \sqrt{\mu}\,|\vec{H}|$

In free space, $\varepsilon = 1$, $\mu = 1$, $|\vec{E}| = |\vec{H}|$

③ $\vec{S} = \frac{c}{4\pi}\vec{E}\times\vec{H} = \frac{c}{4\pi}EH\hat{z} = \frac{c}{\sqrt{\varepsilon\mu}}\left(\frac{\varepsilon}{4\pi}E^2\right)\hat{z} = v\,u\,\hat{z}$

$$u = \frac{\vec{E}\cdot\vec{D}}{8\pi} + \frac{\vec{B}\cdot\vec{H}}{8\pi} = \frac{1}{8\pi}(\epsilon E^2 + \mu H^2) = \frac{\varepsilon E^2}{4\pi}$$

④ Radiation Pressure



$$(\text{pressure}) = \frac{(\text{force})}{(\text{Area})} = \frac{(\text{momentum change})}{\delta T}\cdot\frac{1}{\delta S}$$

$$= \frac{E^2}{4\pi c}(c\,\delta T\,\delta S)\cdot\frac{1}{\delta T}\cdot\frac{1}{\delta S}$$

$$= \frac{E^2}{4\pi} = u \quad \text{energy density}$$

⑤ $i\vec{k}\times\vec{H} - \frac{1}{c}\epsilon(-i\omega)\vec{E} = 0$, $\quad i\vec{k}\times\vec{E} + \frac{1}{c}\mu(-i\omega)\vec{H} = 0$

$\rightarrow \left(-k^2 + \mu\epsilon\frac{\omega^2}{c^2}\right)\vec{E} = 0$

$k = \frac{\omega}{c}\sqrt{\mu\varepsilon} = n\frac{\omega}{c}$ dispersion relation of EM wave in free space

⑥ Polarization of EM wave

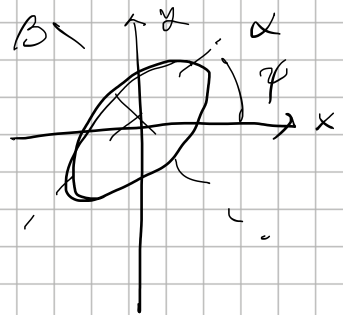
the direction of electric field

$E_x = a_1\cos(\tau + \delta_1)$, $\quad \tau = \omega t - \vec{k}\cdot\vec{r}$

$E_y = a_2\cos(\tau + \delta_2)$, $\quad E_z = 0$

Trajectory of the end point of electric field

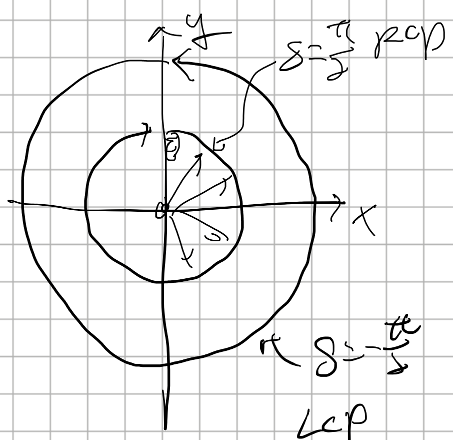
$$\left(\frac{E_x}{a_1}\right)^2+\left(\frac{E_y}{a_2}\right)^2-2\frac{E_x}{a_1}\cdot\frac{E_y}{a_2}\cdot\cos\delta=\sin^2\delta \qquad : \text{ellipse}$$



⇒ elliptically polarized $a_1, a_2, \delta=\delta_2-\delta_1$

i) $\delta=\delta_2-\delta_1=m\pi,\quad m=0,\pm1,\pm2,\cdots$

$$\frac{E_y}{E_x}=\frac{a_2\cos(\tau+\delta_1+m\pi)}{a_1\cos(\tau+\delta_1)}=(-1)^m\frac{a_2}{a_1}$$



⇒ linearly polarized.

ii) $a_1=a_2=a,\quad \delta=\delta_2-\delta_1=\frac{\pi}{2}a,\quad m=\pm1,\pm3,\pm5\cdots$

$$E_x^2+E_y^2=a^2$$