Advanced Optics (PHYS690)

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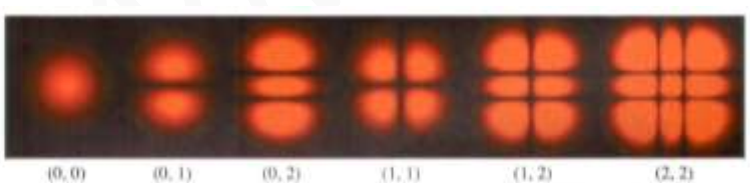
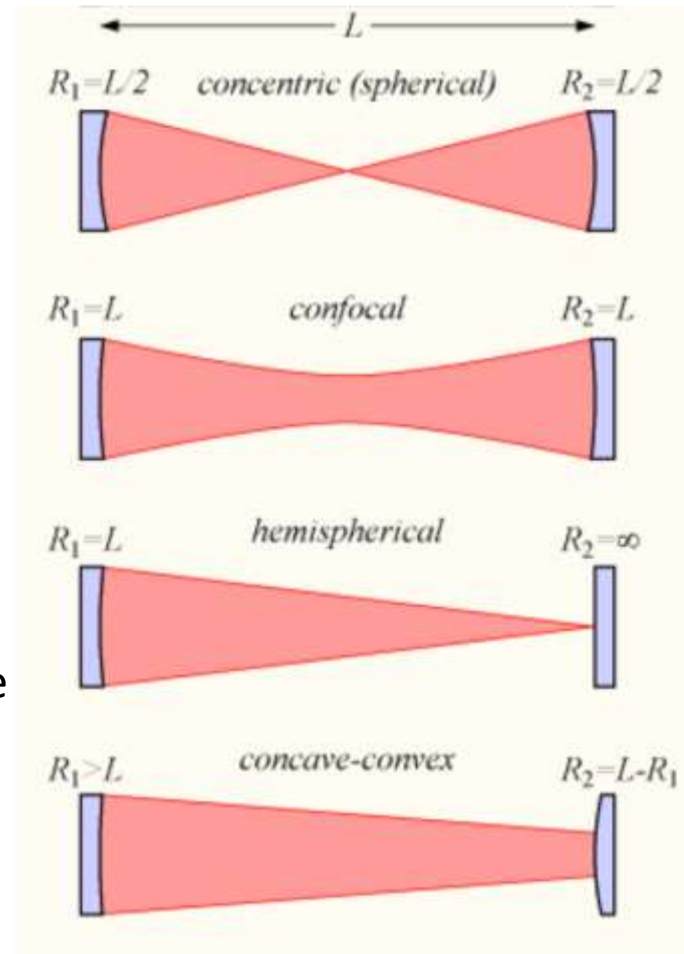
Resonator and Laser

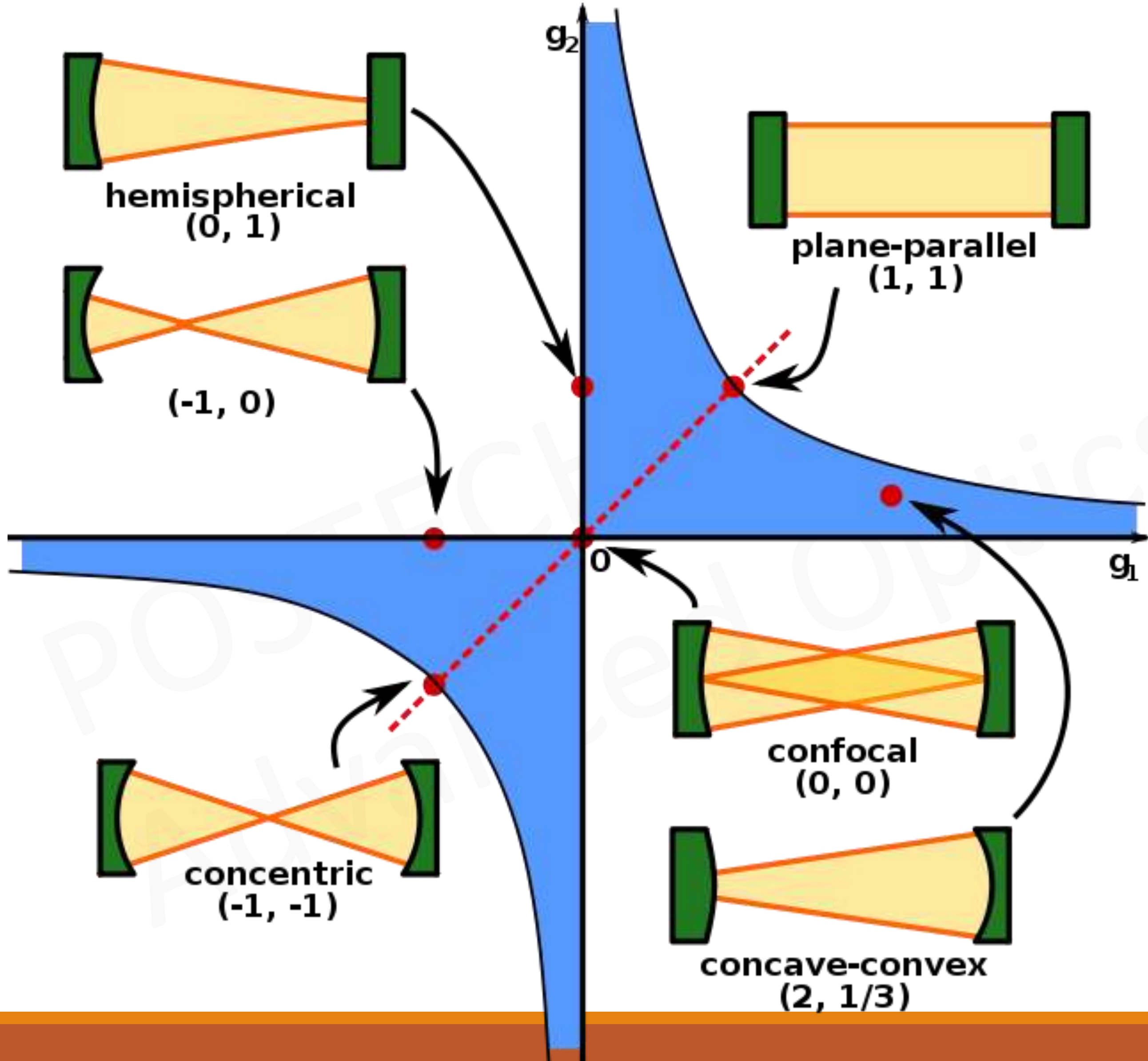
Resonator

- Resonator are the main ingredient of lasers.
- Used to increase the optical power associated to a mode.
- Boundary conditions implies the existence of “eigenmode s” and eigenfrequencies.





Stability

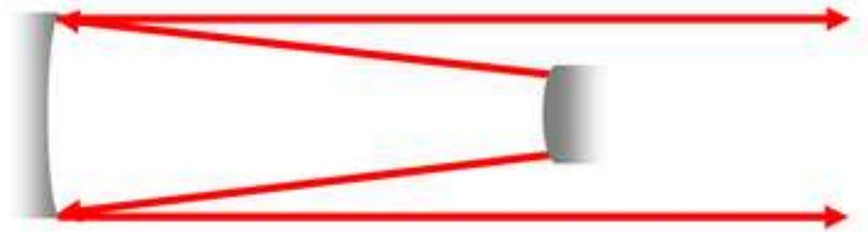


$$g = 1 - \frac{L}{R}$$

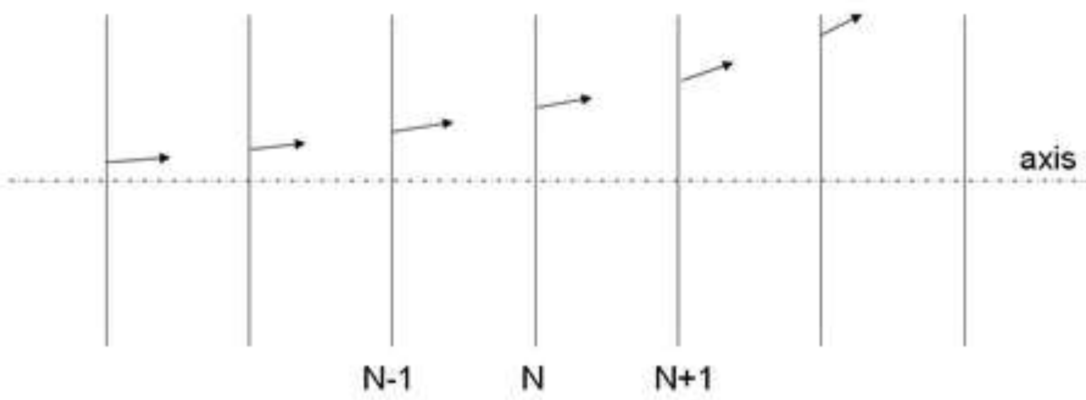
$$0 \leqslant \left(1 - \frac{L}{R_1}\right)\left(1 - \frac{L}{R_2}\right) \leqslant 1.$$

Unstable resonator

Unstable confocal resonator, $g_1 g_2 > 1$



For each round trip, the inclination of the ray increases, until it escapes from the resonator



Resonator: standing wave approach

- Take the case of a plane-parallel resonator configuration.
- Consider an optical wave with complex amplitude

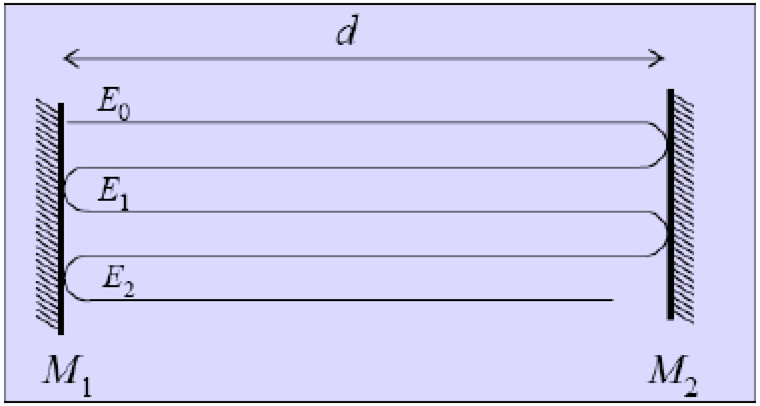
$$U(r,t) = U(\overrightarrow{r})e^{2i\pi\nu t}$$

- The boundaries conditions imposed by the planar mirror gives rise to a "quantification" of the wave vector

$$k_q = q\frac{\pi}{d} \qquad \nu_q = q\frac{c}{2d} \qquad \lambda = \frac{2d}{q}$$

The phase shift imparted by a round trip is

$$\phi = 2kd = q2\pi$$



(d is optical path length – includes index of refraction)

- The mode frequency separation is

Frequency spectral range (FSR) $\nu_f = \frac{c}{2d}$

Losses & resonance spectral width

- In a realistic resonator losses are present (mirror refection non unity, or due to the medium composing the resonator)

- Consider the loss per round trip to be

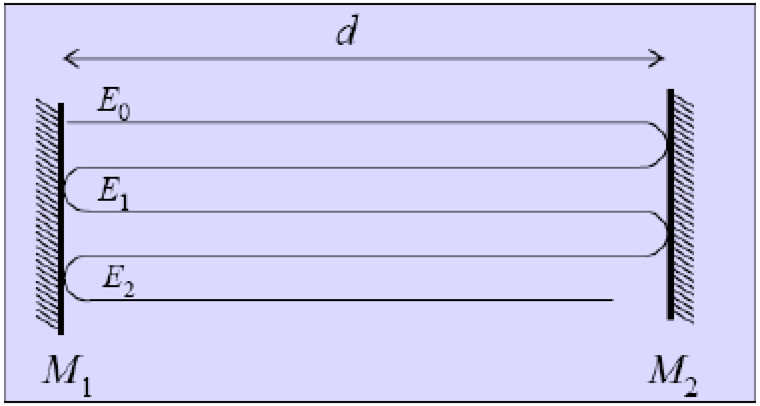
$$h = |r|e^{-i\phi} \qquad \phi = 2kd$$

- Then the complex amplitude summed over an infinite number of passes is

$$\begin{aligned} U &= U_0 + U_1 + U_2 + \ldots \\ &= U_0(1 + h + h^2 + \ldots) = \frac{U_0}{1-h} \end{aligned}$$

- For one-dimensional resonator this is

$$I = \frac{I_0}{|1-h|^2} = \frac{I_0}{1 + |r|^2 - 2|r|\cos\phi}$$



(d is optical path length – includes index of refraction)

Finesse I

- The latter expression can be written

$$I = \frac{I_0}{|1-h|^2} = \frac{I_0}{1+|r|^2 - 2|r|\cos\phi}$$

$$I = \frac{I_{max}}{1 + (2\mathcal{F}/\pi)^2 \sin^2(\phi/2)}$$

where

Finesse

$$\mathcal{F} \equiv \frac{\pi\sqrt{|r|}}{1-|r|}.$$

$$I_{max} \equiv \frac{I_0}{(1-|r|)^2}.$$

$$\mathcal{F} \equiv v_{FSR}/\Delta v$$

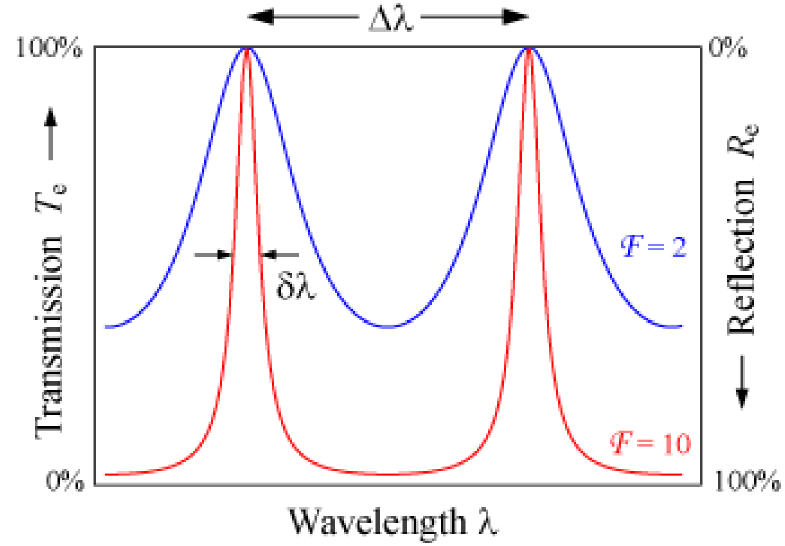
- Expliciting ϕ gives

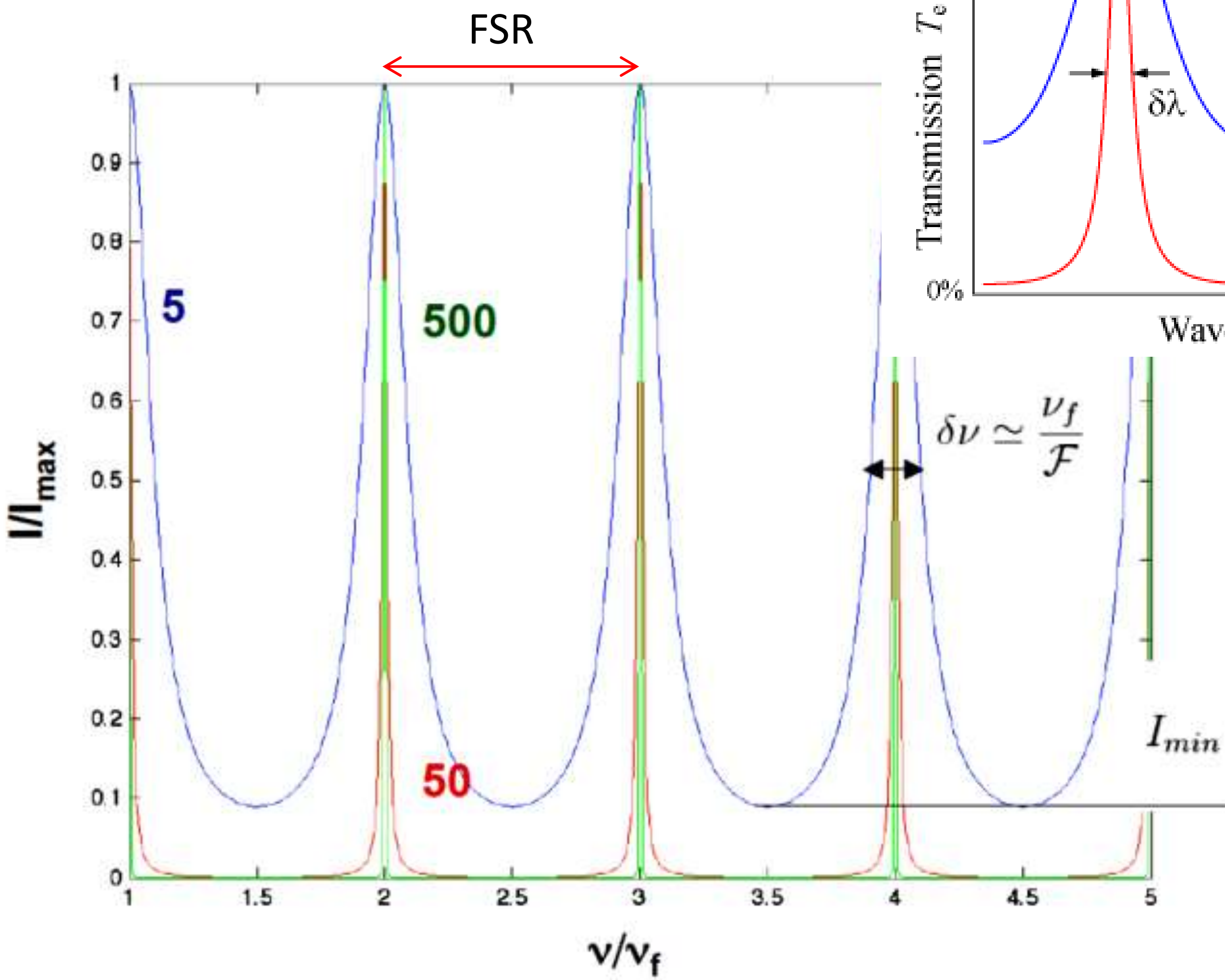
$$\phi = 2kd = 2\pi v/v_f$$

$$I = \frac{I_{max}}{1 + (2\mathcal{F}/\pi)^2 \sin^2(\pi\nu/\nu_f)}$$

Finesse the number of bounces a beam makes before leaking out or being absorbed.

Finesse II



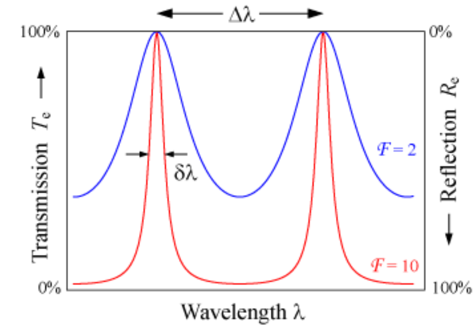


$$I_{min} = \frac{I_{max}}{1 + (2\mathcal{F}/\pi)^2}$$

Factors controlling finesse

Finesse the number of bounces a beam makes before leaking out or being absorbed.

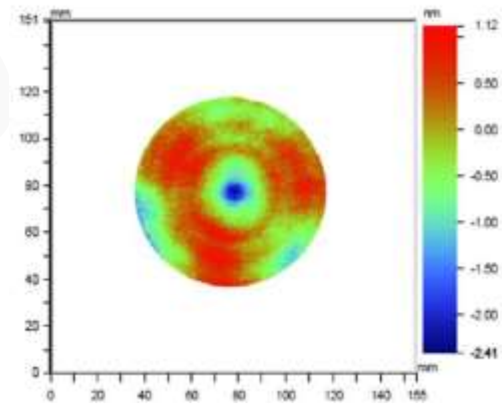
$$\mathcal{F} \equiv \frac{\nu_{FSR}}{\Delta\nu} = \frac{\lambda_{FSR}}{\Delta\lambda}$$



Q-factor $Q \stackrel{\text{def}}{=} \frac{f_r}{\Delta f} = \frac{\omega_r}{\Delta\omega},$

$$Q \stackrel{\text{def}}{=} 2\pi \times \frac{\text{Energy stored}}{\text{Energy dissipated per cycle}} = 2\pi f_r \times \frac{\text{Energy stored}}{\text{Power loss}}.$$

- Loss
- Scatter
- Micro-roughness
- Coating non-uniformity
- Ultimately, transmission of input mirrors



Photon lifetime $\Delta\nu = \frac{1}{2\pi\tau_p}$

High Q-factor resonators

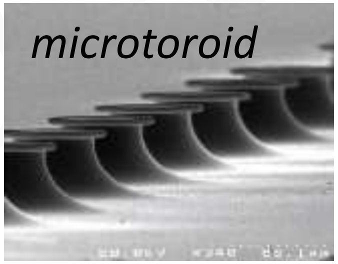
stabilization of lasers and high spectral purity microwave oscillators

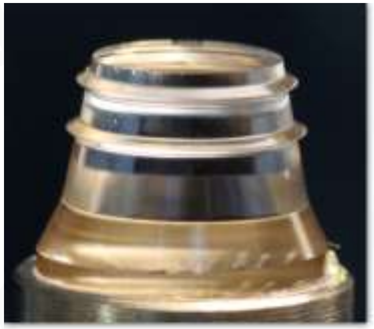
$$Q = \nu_0 T_{rt} \frac{2\pi}{\eta}$$

$\lambda = 1\ \mu\text{m}$, round trip time, $T_{rt} = 1$ ns, round trip loss, $\eta = 5\%$

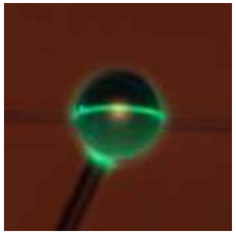
Q-factor: 3.8×10^7

10^8





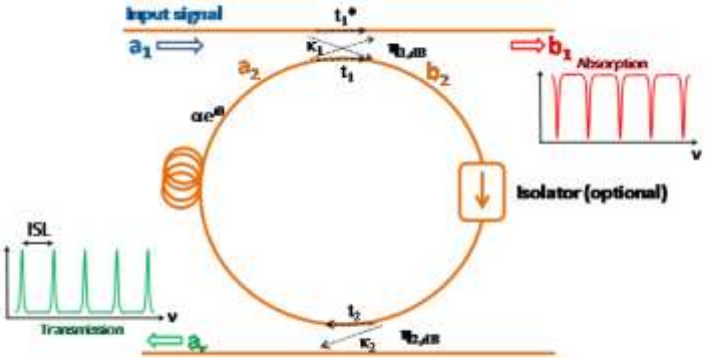
Crystalline Micro-resonators 10^9



Microsphere

10^{10}

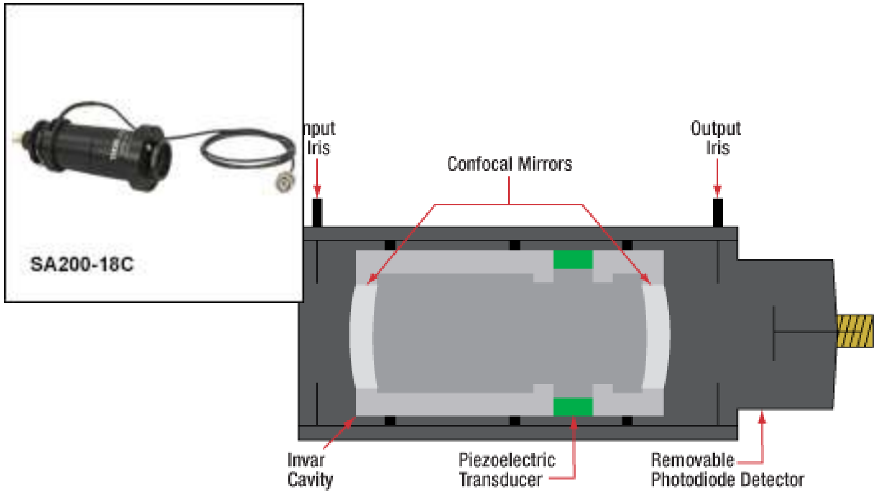
Fiber ring resonators 10^{10}



10^{10}

Saleh K, Fernandez A, Llopis O and Cibiel G
2013 Fiber ring resonators with Q factors in excess of 10^10 for time and frequency applications Proc. Int. Frequency Control Symp.
—European Frequency and Time Forum (IFCS–EFTF)

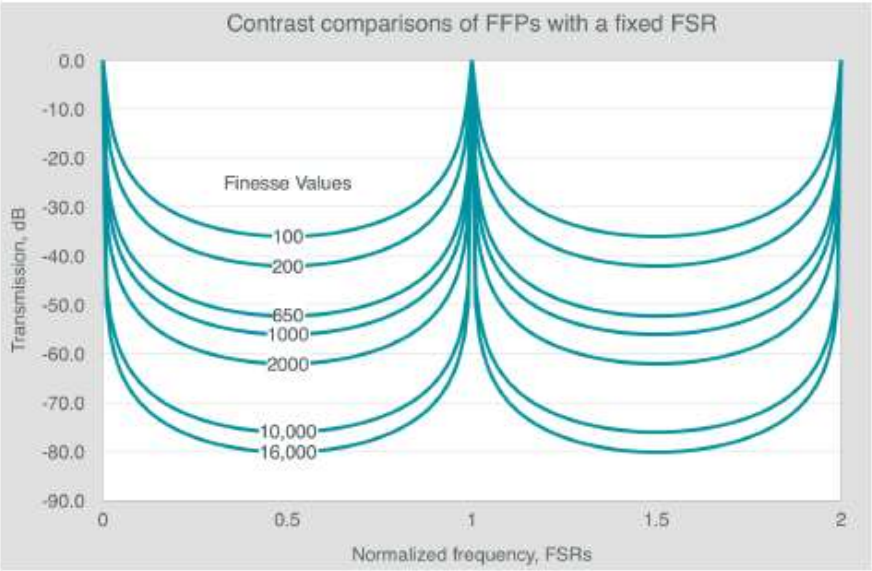
Different types of cavities



Item	SA210
Free Spectral Range (FSR)	10 GHz
Finesse	150 (Minimum) 180 (Typical)
Resolution	67 MHz
Cavity Length	7.5 mm
Mirror Substrate	UV Fused Silica[a]

Fiber Fabry-Perot Tunable Filter | FFP-TF

An all-fiber Fabry-Perot
super-cavity
in a robust, fast tuning
Telcordia qualified
package.



Key Features

All-fiber platform

- **High resolution** and **low loss design**
- **Super-cavity finesse**
- **Vibration** and **shock resistant**
- **Thermally stable**
- **Fast scanning** permits fast, accurate measurements

Ideal for OEM applications

- **Customizable** center wavelength, free spectral range, finesse & bandwidth
- **Center wavelength bands** from 800 to 2000 nm
- **Small** footprint
- **Low power** requirements
- **Telcordia GR 2883** qualified
- **Proven reliability** over decades of use

OEM Applications

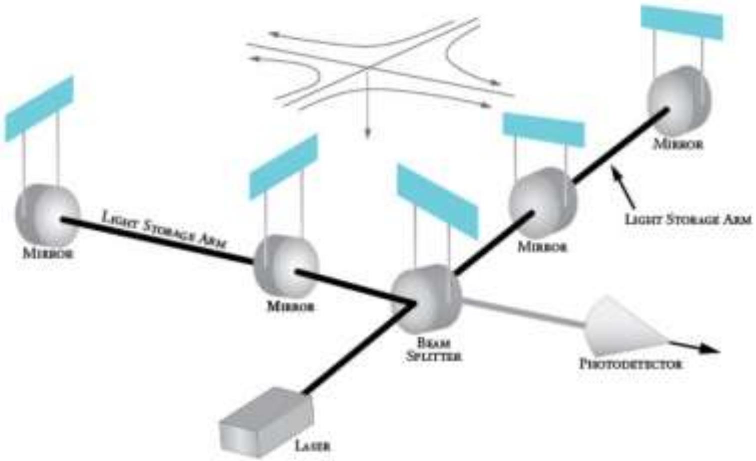
- **Optical Coherence Tomography** (see OCT datasheet)
- **Optical performance monitoring**
- **Spectrum analysis**
- **Tunable optical noise filtering**
- **Tunable channel drop for ultra DWDM**
- **Tunable sources**
- **Optical sensing**

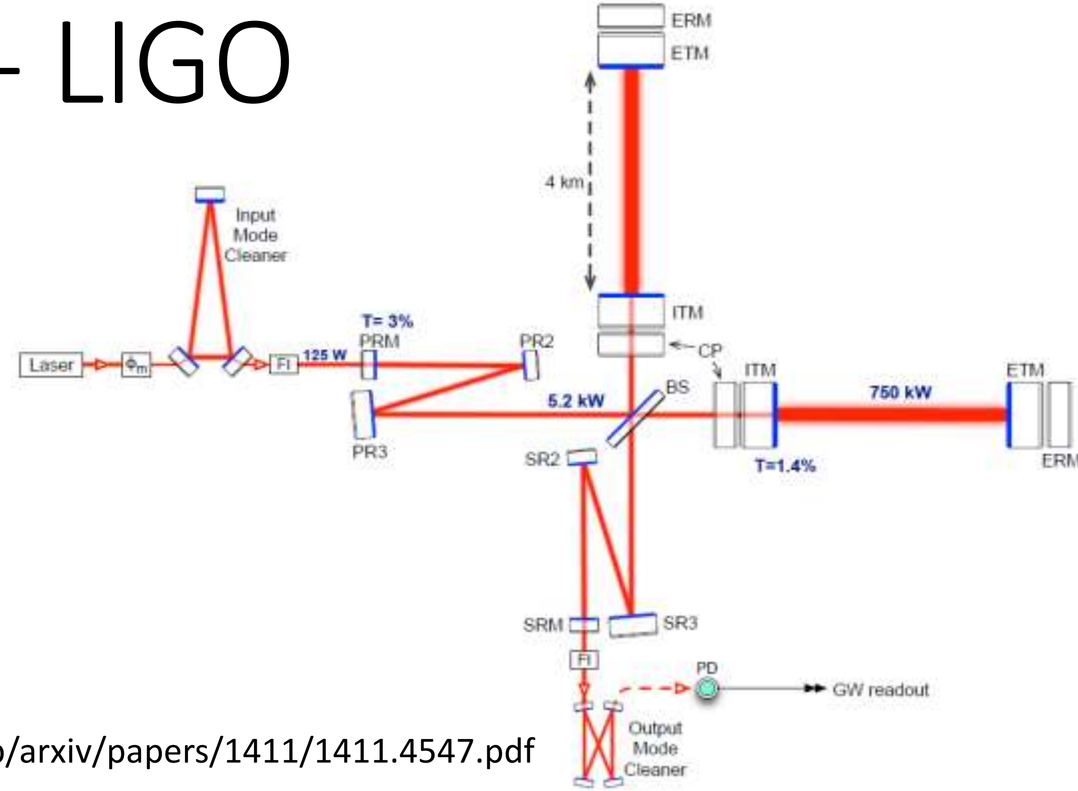
MICRON OPTICS

Fiber Fabry-Perot Tunable Filter | FFP-TF

Optical Properties	Standard[1] FFP-TFs				
Operating wavelength range	1520-1570 nm	1520-1570 nm	1520-1570 nm	1460-1620 nm	1460-1620 nm
Free spectral range[2]	15 THz (120 nm)	15 THz (120 nm)	15 THz (120 nm)	27.5 THz (220 nm)	27.5 THz (220 nm)
Finesse	500	1,000	2,000	2,000	10,000
Bandwidth, (FWHM or 3dB)[3]	30 GHz (240 pm)	15 GHz (120 pm)	7.5 GHz (60 pm)	13.8 GHz (110 pm)	2.8 GHz (22 pm)
Insertion loss	< 2.5 dB	< 3 dB	< 3 dB	< 3 dB	< 4 dB
Polarization dependent loss	< 0.2 dB				
Input power	50 mW	30 mW	15 mW	15 mW	3 mW
Electrical Properties					
Tuning voltage/FSR	< 12 V				

Applications - LIGO





https://arxiv.org/ftp/arxiv/papers/1411/1411.4547.pdf

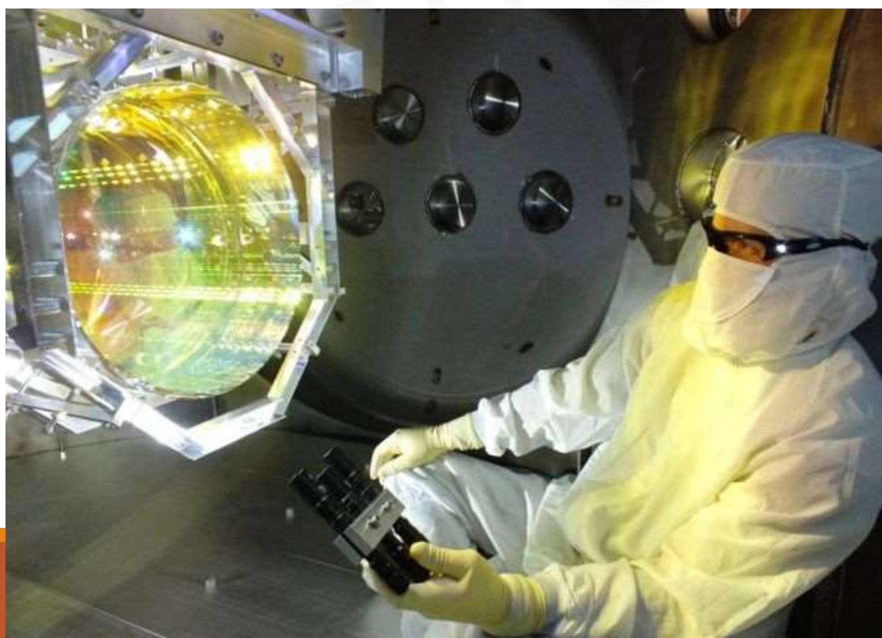


Table 1. Main parameters of the Advanced LIGO interferometers. PRC: power recycling cavity; SRC: signal recycling cavity.

Parameter	Value
Arm cavity length	3994.5 m
Arm cavity finesse	450
Laser type and wavelength	Nd:YAG, $\lambda = 1064$ nm
Input power, at PRM	up to 125 W
Beam polarization	linear, horizontal
Test mass material	Fused silica
Test mass size & mass	34cm diam. x 20cm, 40 kg
Beam radius ($1/e^2$), ITM / ETM	5.3 cm / 6.2 cm
Radius of curvature, ITM / ETM	1934 m / 2245 m
Input mode cleaner length & finesse	32.9 m (round trip), 500
Recycling cavity lengths, PRC / SRC	57.6 m / 56.0 m

Etalon

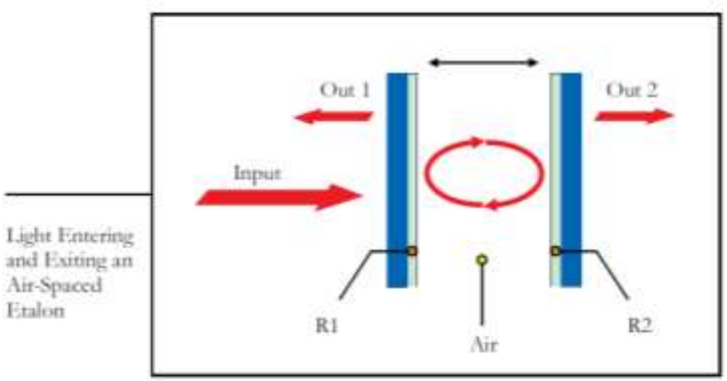
An etalon is an optical interferometer in which a beam of light undergoes multiple reflections between two reflecting surfaces, and whose resulting optical transmission is periodic in wavelength. Actually it is **narrowband wavelength filters** and **precise wavelength references**.

Air-spaced Etalons

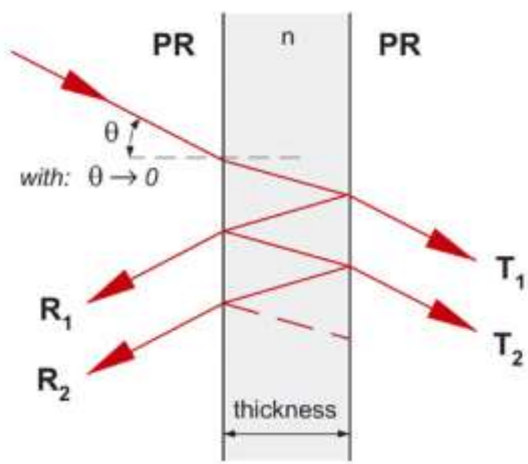
Two extremely parallel plates polished to very tight specification with an air gap between them.

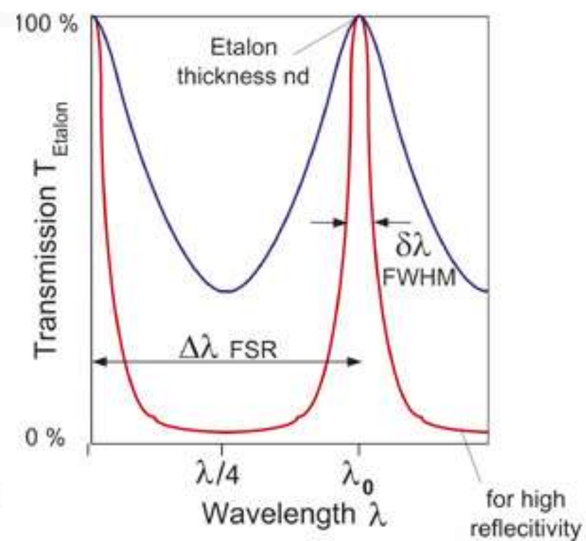
Solid Etalons

Partial reflection etalon coating on both sides of very good polished substrate.



Type 1: Principle of Air-Space Etalon



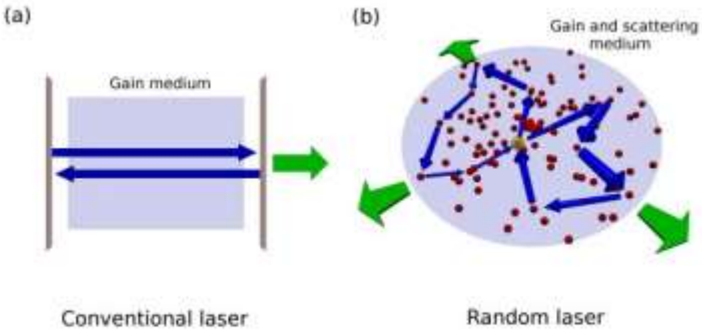


$$T_{Etalon} = \frac{(1-R)^2}{1+R^2-2R\cos(4\pi\frac{nd}{\lambda})}$$

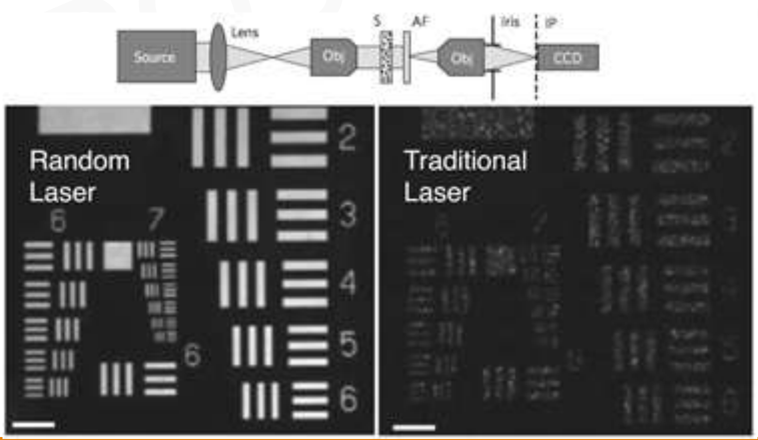
Random laser

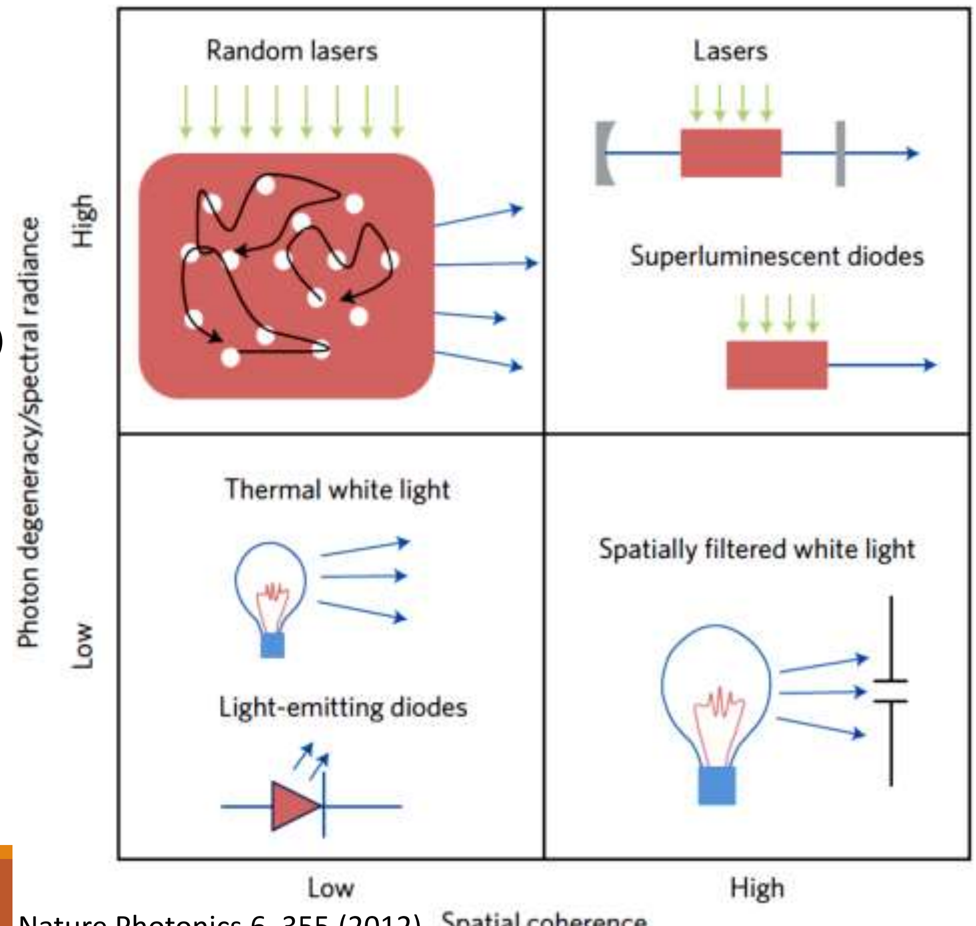
A **random laser** is a laser that uses a highly disordered gain medium.

A random laser uses **no optical cavity** but the remaining principles of op eration remain the same as for a conventional laser.



Nature 406, 132 (2000)





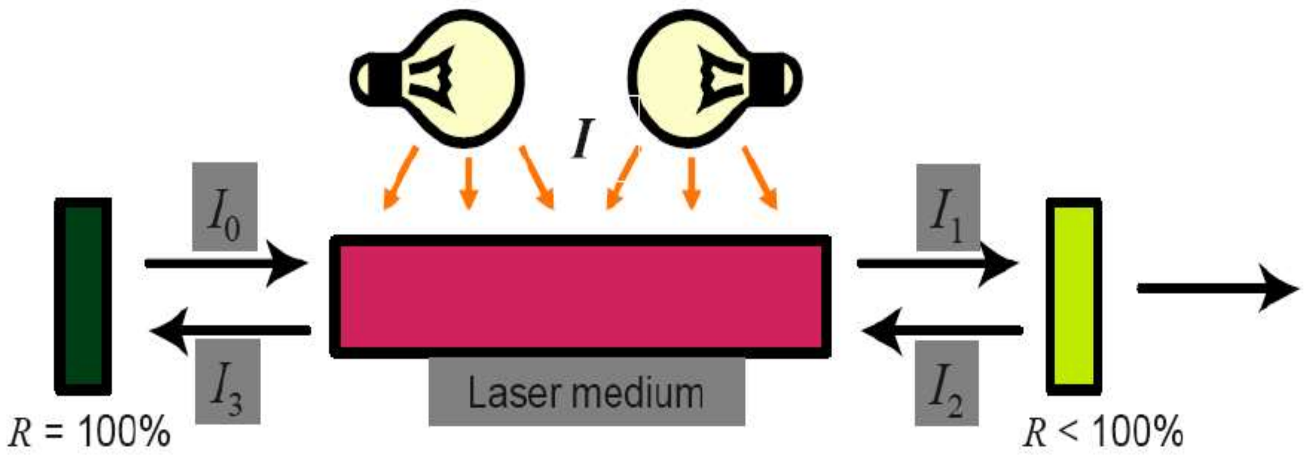
Nature Photonics 6, 355 (2012)

Laser

Light
Amplification by
Stimulated
Emission of
Radiation

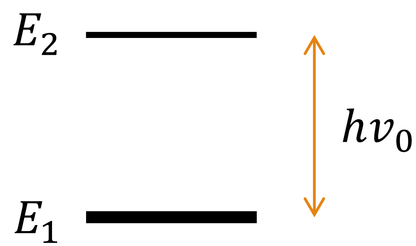
Laser

- Laser medium
- Pumping
- Resonator: laser oscillator or cavity



Light-matter interaction

- Consider an atom and consider two of its energy levels to be E_1 and E_2 (assume $E_1 < E_2$).

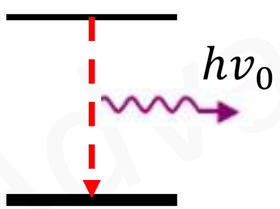


- Chose ν_0 such that

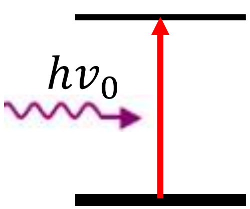
$$h\nu_0 = E_2 - E_1$$

the photon energy matches the energy-level difference.

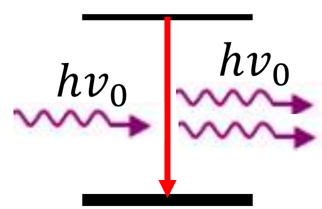
- Three types of mechanism are possible:



– Spontaneous emission



– Absorption

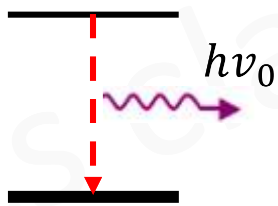


– Stimulated emission

Einstein coefficients I

- Measure of the probability of absorption or emission of light by an atom
- The **Einstein A coefficient**: the rate of spontaneous emission of light
- The **Einstein B coefficients**: the absorption and stimulated emission of light

– Spontaneous emission



- The number of atoms of the upper level: N_2
- The number of atoms of the lower level: N_1

The change $d\,N_2$ of the population N_2 within a time interval dt

$$\mathrm{d}N_2 = -A_{21} N_2 \mathrm{d}t$$

A_{21} is the ***Einstein coefficient of spontaneous emission***.

The population of the upper level decays exponentially

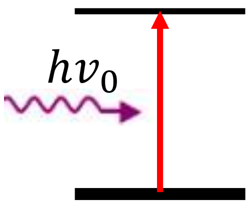
$$N_2(t) = N_2(0)\,\mathrm{e}^{-A_{21}t} = N_2(0)\,\mathrm{e}^{-t/\tau_{sp}}$$

τ_{sp} *spontaneou s lifetime*

$$A_{21} = 1/\tau_{sp}$$

Einstein coefficients II

– Absorption

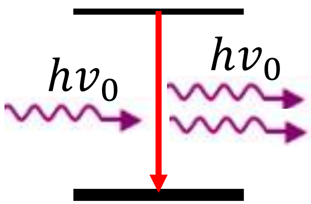


- The change $d\ N_1$ of the ground state within a time interval dt

$$\mathrm{d}N_1 = -B_{12}\rho(\nu_0)N_1\mathrm{d}t$$

where B_{12} is the *Einstein coefficient of absorption* and ρ is the spectral energy density of radiation at frequencies around ν_0.

– Stimulated emission



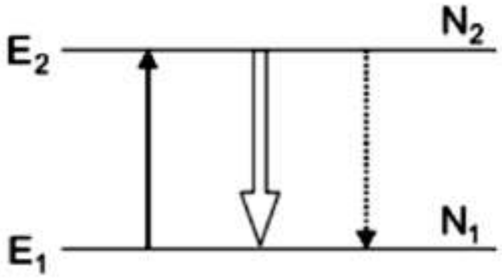
- The change $d\ N_2$ of N_2 within a time interval dt

$$\mathrm{d}N_2 = -B_{21}\rho(\nu_0)N_2\mathrm{d}t$$

where B_{21} is the *Einstein coefficient of stimulated emission.*

Einstein coefficients III

The Einstein Relations



- The rate of change of the population N_1 due to absorption is given by

$$(\mathrm{d}N_1/\mathrm{d}t)_{\mathrm{abs}} = -B_{12}\,\rho(\nu_0)\,N_1$$

- The rate of change of the population N_2 due to stimulated emission is given by

$$(\mathrm{d}N_2/\mathrm{d}t)_{\mathrm{stim}} = -B_{21}\,\rho(\nu_0)\,N_2$$

- The rate of change of the population N_2 due to spontaneous emission is given by

$$(\mathrm{d}N_2/\mathrm{d}t)_{\mathrm{sp}} = -A_{21}\,N_2$$

Einstein coefficients IIII

- In thermal equilibrium
- The ratio N_2/N_1 is a constant.
- The absorption rate has to be equal to the emission rate.

$$(\mathrm{d}N_1/\mathrm{d}t)_{\mathrm{abs}} = (\mathrm{d}N_2/\mathrm{d}t)_{\mathrm{sp}} + (\mathrm{d}N_2/\mathrm{d}t)_{\mathrm{stim}}$$

$$B_{12}\rho(\nu_0)N_1 = A_{21}N_2 + B_{21}\rho(\nu_0)N_2$$

- From this equation, we can determine the spectral energy density

$$\rho(\nu_0) = \frac{A_{21}/B_{21}}{(B_{21}/B_{12})N_1/N_2 - 1}$$

- The ratio N_1/N_2 can be determined by the Boltzmann factor.

$$N_2/N_1 = \mathrm{e}^{-h\nu_0/kT}$$

Planck's radiation law

$$\rho(\nu) = \frac{8\pi\nu^2}{c^3}\frac{h\nu}{\mathrm{e}^{h\nu/kT} - 1}$$

$$B_{21} = B_{12},$$

$$A_{21} = \frac{8\pi\nu^2}{c^3}h\nu B_{21}$$