Week8 – Hall Conductivity

ECE 695-O Semiconductor Transport Theory

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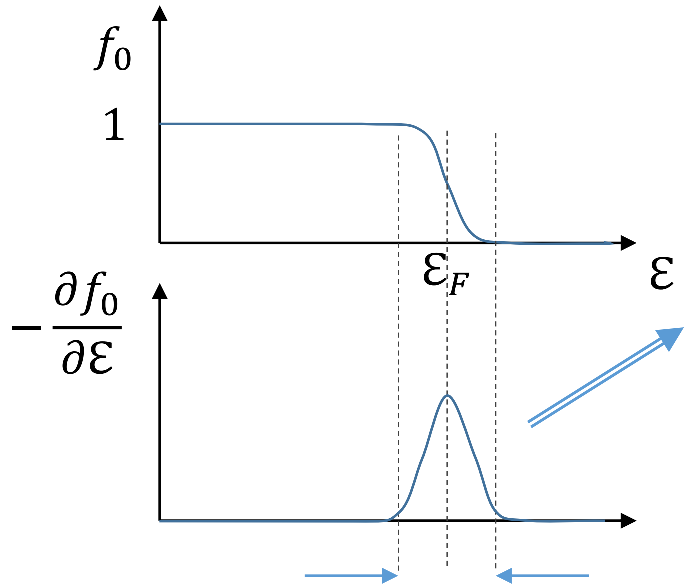
Contents

Conductivity in metals

- In metals, we cannot use the approximation, $f_0(1-f_0) \approx$ Maxwell-Boltzmann distribution, since it is degenerate case.
- We will use $f_0(1-f_0) = -k_B T \frac{\partial f_0}{\partial \mathcal{E}}$, the original form.



Thermal broadening of order of $k_B T$

We can approximate this function like this:

$$\frac{\partial f_0}{\partial \mathcal{E}} = -\delta(\mathcal{E} - \mathcal{E}_F)$$

If we integrate $\frac{\partial f_0}{\partial \mathcal{E}}$:

$$\int_{-\infty}^{\infty} \frac{\partial f_0}{\partial \mathcal{E}} d\mathcal{E} = f_0\Big|_{-\infty}^{\infty} = -1$$

So this is good approximation.

Conductivity in metals(2)

- Thus, for metals, we plug $\frac{\partial f_0}{\partial \mathcal{E}} = -\delta(\mathcal{E} - \mathcal{E}_F)$ into $\langle \tau \rangle$ expression then,

$$\langle \tau \rangle = \frac{\int \tau \ \mathcal{E}^{\frac{3}{2}} \frac{df_0}{d\mathcal{E}} d\mathcal{E}}{\int \mathcal{E}^{\frac{3}{2}} \ \frac{df_0}{d\mathcal{E}} \ d\mathcal{E}} = \frac{\int \tau \ \mathcal{E}^{\frac{3}{2}} \delta(\mathcal{E} - \mathcal{E}_F) d\mathcal{E}}{\int \mathcal{E}^{\frac{3}{2}} \delta(\mathcal{E} - \mathcal{E}_F) d\mathcal{E}}$$

$$\cong \frac{\tau(\mathcal{E}_F)\mathcal{E}_F^{3/2}}{\mathcal{E}_F^{3/2}} \cong \tau(\mathcal{E}_F)$$

.

- So, in metals, $\langle \tau \rangle$ near $\mathcal{E}_F$ is the dominant factor.
- In metal everything happens near Fermi level, and the states some energy under Fermi level do not have any empty spot to scatter in.
- So $\mu = \frac{q}{m^*} \tau(\mathcal{E}_F)$

Conductivity in metals(3) – a little more exact calculation

- For the students who may not be satisfied by the too simple result of approximating $\frac{\partial f_0}{\partial \mathcal{E}}$ into delta function, we can do more exact calculation.
- Let's consider the following integral form:

$$I = -\int_0^\infty G(\mathcal{E}) \frac{\partial f_0}{\partial \mathcal{E}} d\mathcal{E}$$

- Here, $G(\mathcal{E})$ is an arbitrary function of energy $\mathcal{E}$.
- And, we know

$$f_0 = f_0\left(\frac{\mathcal{E} + \mathcal{E}_c - \mathcal{E}_F}{k_B T}\right).$$

- Let $x = \frac{\mathcal{E}+\mathcal{E}_c-\mathcal{E}_F}{k_B T}$ then,

$$I = -\int_{\frac{\mathcal{E}_c-\mathcal{E}_F}{k_B T}}^\infty G(\mathcal{E}_F - \mathcal{E}_c + x k_B T) \frac{\partial f_0}{\partial x} dx$$

Conductivity in metals(4) – a little more exact calculation

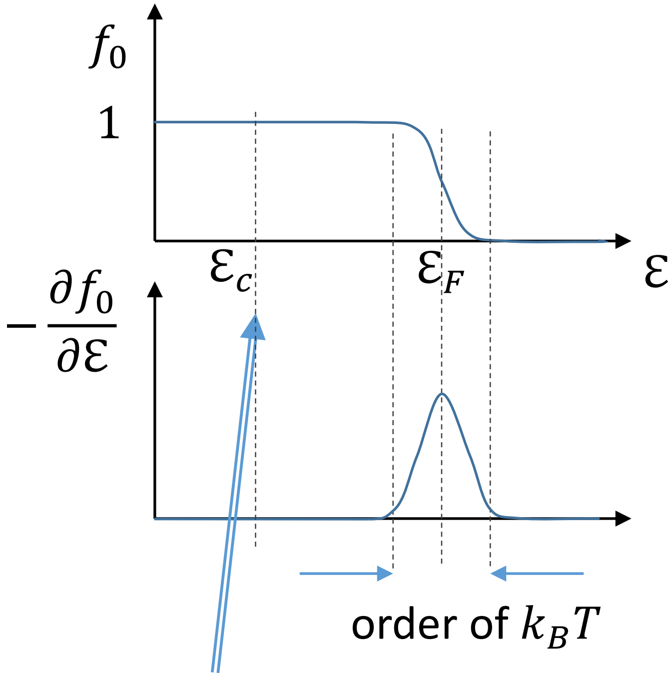
$$I = -\int_{\frac{\mathcal{E}_c-\mathcal{E}_F}{k_BT}}^{\infty} G(\mathcal{E}_F - \mathcal{E}_c + xk_BT)\frac{\partial f_0}{\partial x}dx$$

- $\frac{\mathcal{E}_c-\mathcal{E}_F}{k_BT}$ is a large negative number.
- Thus, we expand the lower bound of the integration to $-\infty$.

$$\Rightarrow I = -\int_{-\infty}^{\infty} G(\mathcal{E}_F - \mathcal{E}_c + xk_BT)\frac{\partial f_0}{\partial x}dx$$

- Let's consider the Taylor expansion of G with respect to x.

$$\begin{aligned}G(\mathcal{E}_F - \mathcal{E}_c + xk_BT) = G(\mathcal{E}_F - \mathcal{E}_c)\\ +G'(\mathcal{E}_F - \mathcal{E}_c)xk_BT\\ +\frac{1}{2}G''(\mathcal{E}_F - \mathcal{E}_c)(xk_BT\,)^2\\ +\cdots\end{aligned}$$



$\mathcal{E}_c$ is somewhere here since it is a metal. So electron band is heavily occupied and $\mathcal{E}_F - \mathcal{E}_c \gg k_BT$.

Conductivity in metals(5) – a little more exact calculation

- Dropping the higher order term,

$$G(\mathcal{E}_F - \mathcal{E}_c + xk_BT) \cong G(\mathcal{E}_F - \mathcal{E}_c) + G'(\mathcal{E}_F - \mathcal{E}_c)xk_BT + \frac{1}{2}G''(\mathcal{E}_F - \mathcal{E}_c)(xk_BT)^2$$

- And, the integral expression becomes

$$\Rightarrow I = -\int_{-\infty}^{\infty}\left\{G(\mathcal{E}_F - \mathcal{E}_c) + G'(\mathcal{E}_F - \mathcal{E}_c)xk_BT + \frac{1}{2}G''(\mathcal{E}_F - \mathcal{E}_c)(xk_BT)^2\right\}\frac{\partial f_0}{\partial x}dx$$

$$= -G(\mathcal{E}_F - \mathcal{E}_c)\boxed{\int_{-\infty}^{\infty}\frac{\partial f_0}{\partial x}dx} - k_BTG'(\mathcal{E}_F - \mathcal{E}_c)\boxed{\int_{-\infty}^{\infty}x\frac{\partial f_0}{\partial x}dx}$$

$= -1$
(do you recall the delta function approximation?)

$= 0$
(since it is odd function (x) times even function ($\frac{\partial f_0}{\partial x}$))

$$-\frac{1}{2}(k_BT)^2G''(\mathcal{E}_F - \mathcal{E}_c)\boxed{\int_{-\infty}^{\infty}x^2\frac{\partial f_0}{\partial x}dx}$$

$= -\frac{\pi^2}{3}$

Conductivity in metals(6) – a little more exact calculation

- Thus,

$$\Rightarrow I = G(\mathcal{E}_F - \mathcal{E}_c) + \frac{(\pi k_B T)^2}{6} G''(\mathcal{E}_F - \mathcal{E}_c)$$

- This was an expression for a general function G and you can apply this to $\langle \tau \rangle$.
- Let's set $\mathcal{E}_c = 0$ for a convenience.

$$\langle \tau \rangle = \frac{\int \tau \ \mathcal{E}^{\frac{3}{2}} \frac{df_0}{d\mathcal{E}} d\mathcal{E}}{\int \mathcal{E}^{\frac{3}{2}} \ \frac{df_0}{d\mathcal{E}} \ d\mathcal{E}} = \frac{\tau(\mathcal{E}_F)\mathcal{E}_F^{3/2} + \frac{(\pi k_B T)^2}{6} \frac{d^2}{d\mathcal{E}^2}\left(\tau(\mathcal{E}) \ \mathcal{E}^{3/2}\right)\Big|_{\mathcal{E}=\mathcal{E}_F}}{\mathcal{E}_F^{3/2} + \boxed{\frac{(\pi k_B T)^2}{6} \frac{d^2}{d\mathcal{E}^2}\left(\mathcal{E}^{3/2}\right)\Big|_{\mathcal{E}=\mathcal{E}_F}}}$$

Ignore this since it is small

$$\cong \tau(\mathcal{E}_F) + \frac{(\pi k_B T)^2}{6} \frac{d^2}{d\mathcal{E}^2}\left(\tau(\mathcal{E})\right)\Big|_{\mathcal{E}=\mathcal{E}_F}$$

Conductivity in metals(7) – a little more exact calculation

- Let's consider the carrier density of metals. We know that in equilibrium, the carrier density can be expressed like

$$n = \int_0^\infty g(\mathcal{E}) f_0(\mathcal{E}) d\mathcal{E}$$

- Since we know that $g(\mathcal{E})$ in 3D is proportional to $\sqrt{\mathcal{E}}$, let's set $g(\mathcal{E}) = A\sqrt{\mathcal{E}}$ where A is a constant. Then,

$$n = \int_0^\infty A\mathcal{E}^{1/2} f_0(\mathcal{E}) d\mathcal{E} = \cancel{\frac{2}{3} A\, \mathcal{E}^{3/2} f_0(\mathcal{E})\Big|_0^\infty} - \frac{2}{3} A \int_0^\infty \mathcal{E}^{3/2} \frac{df_0}{d\mathcal{E}} d\mathcal{E}$$

=0 since $f_0(\mathcal{E})$ goes to zero at ∞

$$= -\frac{2}{3} A \boxed{\int_0^\infty \mathcal{E}^{3/2} \frac{df_0}{d\mathcal{E}} d\mathcal{E}} = \frac{2}{3} A \left\{ \mathcal{E}_F^{3/2} + \frac{(\pi k_B T)^2}{6} \frac{d^2}{d\mathcal{E}^2} \left(\mathcal{E}^{3/2} \right) \Big|_{\mathcal{E}=\mathcal{E}_F} \right\}$$

This appears in the denominator in the $\langle \tau \rangle$ expression.

Conductivity in metals(8) – a little more exact calculation

- Thus,

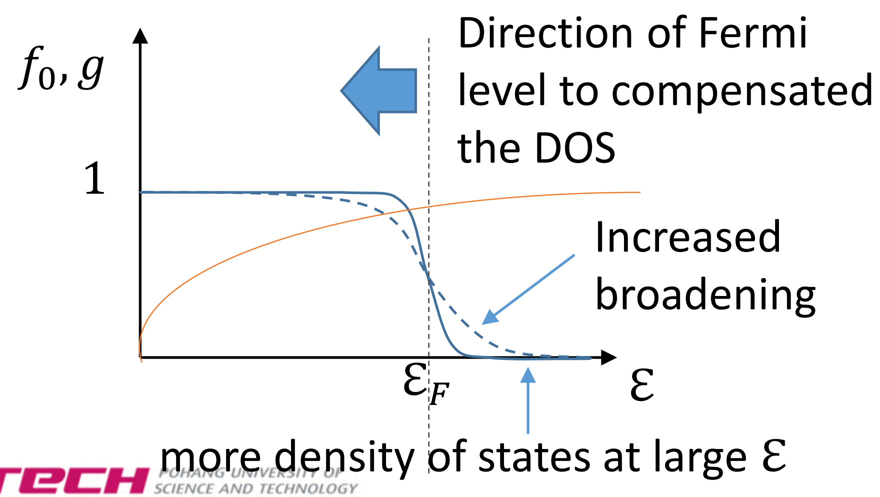
$$n(T) \cong \frac{2}{3}A\left\{\mathcal{E}_F^{3/2} + \frac{(\pi k_B T)^2}{6}\frac{d^2}{d\mathcal{E}^2}\left(\mathcal{E}^{3/2}\right)\Big|_{\mathcal{E}=\mathcal{E}_F}\right\}$$

- However, in metal, the carrier density is not a function of temperature and, this gives

$$n(0) = n(T)$$

$$\frac{2}{3}A\mathcal{E}_{F0}^{3/2} = \frac{2}{3}A\left\{\mathcal{E}_F^{3/2} + \frac{(\pi k_B T)^2}{6}\frac{d^2}{d\mathcal{E}^2}\left(\mathcal{E}^{3/2}\right)\Big|_{\mathcal{E}=\mathcal{E}_F}\right\}$$

$$\Rightarrow \mathcal{E}_F \approx \mathcal{E}_{F0}\left[1 - \frac{\pi^2}{12}\left(\frac{k_B T}{\mathcal{E}_{F0}}\right)^2\right]$$



Hall conductivity

- Now, we have to consider the case both E and B fields present.
- The solution of BTE under relaxation time approximation was

$$f = f_0 + \mathbf{v} \cdot \mathbf{G} \frac{\partial f_0}{\partial \mathcal{E}}$$

where

$$\mathbf{G} = -e\tau \left\{ \frac{\mathbf{E} + \frac{e\tau}{m^*} \mathbf{E} \times \mathbf{B} + \left(\frac{e\tau}{m^*}\right)^2 (\mathbf{E} \cdot \mathbf{B})\mathbf{B}}{1 + \left(\frac{e\tau}{m^*}\right)^2 \mathbf{B} \cdot \mathbf{B}} \right\}$$

for a spherical energy band.

- We will repeat the similar approach that we used before.
- The current is given as

$$\mathbf{J} = \sum \int e\mathbf{v}_n f_n \frac{d^3k}{4\pi^3} = \frac{e}{4\pi^3} \int \mathbf{v} f \, d^3k$$

$$= \frac{e}{4\pi^3} \int \mathbf{v}(\mathbf{v} \cdot \mathbf{G}) \frac{\partial f_0}{\partial \mathcal{E}} d^3k$$

Hall conductivity(2)

- When there was only electric field, $\mathbf{G} = -e\tau\mathbf{E}$,

$$\mathbf{J} = \frac{e^2 n}{m^*}\langle\tau\rangle\mathbf{E}$$

and you can see $\mathbf{E}$ comes out from the energy integral term $\langle\cdots\rangle = \int d\mathcal{E}$ since it is not a function of electron energy $\mathcal{E}$.

- So, if we plug $\mathbf{G} = -e\tau\left\{\dfrac{\mathbf{E} + \frac{e\tau}{m^*}\mathbf{E}\times\mathbf{B} + \left(\frac{e\tau}{m^*}\right)^2(\mathbf{E}\cdot\mathbf{B})\mathbf{B}}{1 + \left(\frac{e\tau}{m^*}\right)^2 \mathbf{B}\cdot\mathbf{B}}\right\}$ into the current expression then,

$$\Rightarrow \mathbf{J} = \frac{e^2 n}{m^*}\left\langle\frac{\tau}{1+\omega_c^2\tau^2}\right\rangle\mathbf{E} + \frac{e^3 n}{m^{*2}}\left\langle\frac{\tau^2}{1+\omega_c^2\tau^2}\right\rangle(\mathbf{E}\times\mathbf{B}) + \frac{e^4 n}{m^{*3}}\left\langle\frac{\tau^3}{1+\omega_c^2\tau^2}\right\rangle(\mathbf{E}\cdot\mathbf{B})\mathbf{B}$$

- You may wonder where the denominator$\left(1 + \left(\frac{e\tau}{m^*}\right)^2 \mathbf{B}\cdot\mathbf{B}\right)$ goes but, it is still in the average bracket.(Why?) And $\omega_c = \frac{e|\mathbf{B}|}{m^*}$.

Hall conductivity(3)

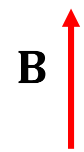
- Assuming low magnetic field so that we can say $\omega_c \tau \ll 1$ and $|(\mathbf{E}\cdot\mathbf{B})\mathbf{B}| \ll 1$ then,

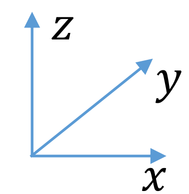
$$\Rightarrow \mathbf{J} = \frac{e^2 n}{m^*}\langle\tau\rangle\mathbf{E} + \frac{e^3 n}{m^{*2}}\langle\tau^2\rangle(\mathbf{E}\times\mathbf{B})$$

- Let's assume a magnetic field $\mathbf{B} = B_z\hat{\mathbf{z}}$.

$$\Rightarrow J_x = \frac{e^2 n}{m^*}\langle\tau\rangle E_x + \cancel{\frac{e^3 n}{m^{*2}}\langle\tau^2\rangle E_y B_z}$$

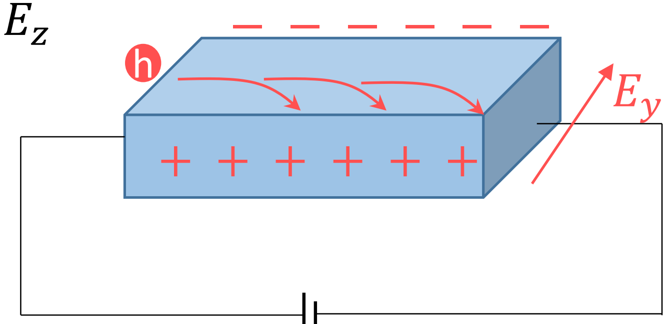
B_z is small

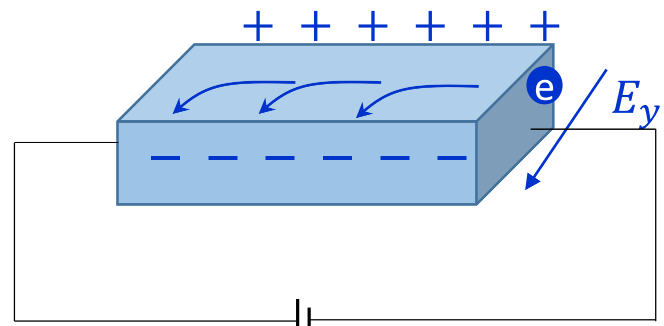




$$J_y = \frac{e^2 n}{m^*}\langle\tau\rangle E_y - \frac{e^3 n}{m^{*2}}\langle\tau^2\rangle E_x B_z$$

$$J_z = \frac{e^2 n}{m^*}\langle\tau\rangle E_z$$





Hall conductivity(3)

- According to the set-up $J_y = J_z = 0$ and, this gives $E_z = 0$.

$$J_y = \frac{e^2 n}{m^*}\langle\tau\rangle E_y - \frac{e^3 n}{m^{*2}}\langle\tau^2\rangle E_x B_z = 0$$

$$\Rightarrow E_x = \frac{\langle\tau\rangle m^*}{\langle\tau^2\rangle e}\frac{E_y}{B_z}$$

$$\Rightarrow J_x = \frac{e^2 n}{m^*}\langle\tau\rangle E_x = en\frac{\langle\tau\rangle^2}{\langle\tau^2\rangle}\frac{E_y}{B_z}$$

- We define so-called Hall coefficient such as

$$\Rightarrow R_H = \frac{E_y}{J_x B_z} = \frac{\langle\tau^2\rangle}{en\langle\tau\rangle^2}$$

and as you have seen from the previous picture, if $R_H > 0$, p-type and, if $R_H < 0$, n-type.

- Hall mobility: $\mu_H = R_H\sigma = \frac{\langle\tau^2\rangle}{en\langle\tau\rangle^2}ne\mu = \mu\frac{\langle\tau^2\rangle}{\langle\tau\rangle^2}$
- Hall factor: $r_H = \frac{\langle\tau^2\rangle}{\langle\tau\rangle^2}$

Thermal Contribution to Electrical Current

- Although we have been discussing the cases with E-field only, we defined the electro thermal field such as

$$\boldsymbol{\mathcal{F}} = \mathbf{E} + \frac{T}{e}\nabla_{\mathbf{r}}\left(\frac{\mathcal{E}-\mathcal{E}_F}{T}\right)$$

- When there is no B field (to simplify the problem),

$$\phi = -\mathbf{v}\cdot\mathbf{G} = \frac{1}{\hbar}\nabla_{\mathbf{k}}\mathcal{E}\cdot(e\tau\boldsymbol{\mathcal{F}})$$

$$= \frac{\tau}{\hbar}\nabla_{\mathbf{k}}\mathcal{E}\cdot\left[\nabla_{\mathbf{r}}(\mathcal{E}-\mathcal{E}_F) - \frac{\mathcal{E}-\mathcal{E}_F}{T}\nabla_{\mathbf{r}}T\right] + \frac{e}{\hbar}\nabla_{\mathbf{k}}\mathcal{E}\cdot\mathbf{E}$$

- Then, the current is

$$J_x = -\frac{e}{4\pi^3}\int v_x\phi\frac{\partial f_0}{\partial\mathcal{E}}d^3k$$

$$= \frac{e}{4\pi^3}\left[-\int e\tau E_x v_x^2\frac{\partial f_0}{\partial\mathcal{E}}d^3k + \int\tau\left\{\frac{\mathcal{E}-\mathcal{E}_F}{T}\frac{\partial T}{\partial x} - \frac{\partial(\mathcal{E}-\mathcal{E}_F)}{\partial x}\right\}v_x^2\frac{\partial f_0}{\partial\mathcal{E}}d^3k\right]$$

Thermal Contribution to Electrical Current(2)

$$J_x = \frac{e}{4\pi^3}\left[-\int e\tau E_x\, v_x^2 \frac{\partial f_0}{\partial \mathcal{E}} d^3k + \int \tau \left\{\frac{\mathcal{E}-\mathcal{E}_F}{T}\frac{\partial T}{\partial x} - \frac{\partial(\mathcal{E}-\mathcal{E}_F)}{\partial x}\right\} v_x^2 \frac{\partial f_0}{\partial \mathcal{E}} d^3k\right]$$

These are functions of $\mathcal{E}$.

Note: If we think $\mathcal{E} = \mathcal{E}'(\mathbf{k}) + \mathcal{E}_c(\mathbf{r})$, $\frac{\mathcal{E}-\mathcal{E}_F}{T} = \frac{\mathcal{E}'}{T} + \frac{\mathcal{E}_c-\mathcal{E}_F}{T}$. Here, $\frac{\mathcal{E}_c-\mathcal{E}_F}{T}$ is independent of $\mathbf{k}$. In addition, $\mathcal{E}'$ is not a function of $\mathbf{r}$ and $\mathcal{E}_c - \mathcal{E}_F$ has spatial dependence.

- So the thermal part (2nd term of left hand) shows another $\mathcal{E}$ depedence.
- We defined a transport integral such as

$$K_n = -\frac{1}{4\pi^3}\int d^3k\ \tau\ v_x^2\ \mathcal{E}'^{\boldsymbol{n-1}} \frac{\partial f_0}{\partial \mathcal{E}}$$

- Then the current can be expressed using the transport integral such as

$$J_x = e^2 K_1 E_x - \frac{e}{T} K_2 \frac{\partial T}{\partial x} - e\left[\frac{\mathcal{E}_c-\mathcal{E}_F}{T}\frac{\partial T}{\partial x} - \frac{\partial(\mathcal{E}_c-\mathcal{E}_F)}{\partial x}\right] K_1$$

Thermal Contribution to Electrical Current(3)

$$K_1 = \frac{n}{m^*}\langle\tau\rangle$$

The same as the E-field only case

$$K_2 = \frac{n}{m^*}\langle\tau\mathcal{E}'\rangle$$

- $\mathcal{E}_c - \mathcal{E}_F$ is a function of carrier density and temperature of location. Thus,

$$\frac{\partial(\mathcal{E}_c - \mathcal{E}_F)}{\partial x} = \frac{\partial(\mathcal{E}_c - \mathcal{E}_F)}{\partial n}\frac{\partial n}{\partial x} + \frac{\partial(\mathcal{E}_c - \mathcal{E}_F)}{\partial T}\frac{\partial T}{\partial x}$$

- And this gives,

$$\Rightarrow J_x = e^2K_1E_x - \frac{e}{T}K_2\frac{\partial T}{\partial x} - e\left[\frac{\mathcal{E}_c - \mathcal{E}_F}{T}\frac{\partial T}{\partial x} - \frac{\partial(\mathcal{E}_c - \mathcal{E}_F)}{\partial x}\right]K_1$$

$$= K_1e^2E_x - e\left[\frac{\mathcal{E}_c - \mathcal{E}_F}{T}K_1 - \frac{\partial(\mathcal{E}_c - \mathcal{E}_F)}{\partial T}K_1 + \frac{1}{T}K_2\right]\frac{\partial T}{\partial x}$$

$$-eK_1\frac{\partial(\mathcal{E}_c - \mathcal{E}_F)}{\partial n}\frac{\partial n}{\partial x}$$

Thermal Contribution to Electrical Current(4)

- From this, we can write down the current as following.

$$J_x = |e|\mu_n\, n\, E_x - e\, \mu_n\, n\, C_n \frac{\partial T}{\partial x} - eD_n \frac{\partial n}{\partial x}$$

where mobility : $\mu_n = \frac{|e|\langle\tau\rangle}{m^*}$

diffusivity : $D_n = \frac{n}{m^*}\frac{\partial(\mathcal{E}_F - \mathcal{E}_c)}{\partial n}$

Thermal coefficient : $C_n = \frac{1}{e}\left[\frac{\mathcal{E}_c - \mathcal{E}_F}{T} - \frac{\partial(\mathcal{E}_c - \mathcal{E}_F)}{\partial T} + \frac{1}{T}\frac{\langle\tau\mathcal{E}'\rangle}{\langle\tau\rangle}\right]$

- $C_n \frac{\partial T}{\partial x}$ term shows that if there is a temperature gradient, although the carriers are randomly moving, there is a diffusion from the hot side to the cold side.

Thermal Contribution to Electrical Current(5)

- Special case:

i) $\nabla_{\mathbf{r}} n \neq 0$, $\nabla_{\mathbf{r}} T = 0$ and, $\mathbf{J} = 0$

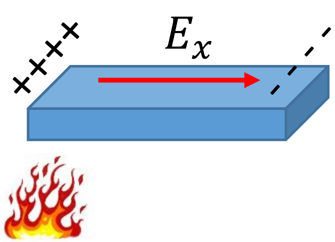
$$J_x = |e|\mu_n\, n\, E_x - e\,\mu_n\, n\, C_n \frac{\partial T}{\partial x} - eD_n \frac{\partial n}{\partial x}$$

$$= |e|\mu_n\, n\, E_x - eD_n \frac{\partial n}{\partial x} = 0$$

$$\Rightarrow E_x = -\frac{D_n}{\mu_n}\frac{\partial n}{\partial x}$$

→ The built-in field by doping gradient

ii) $\nabla_{\mathbf{r}} T \neq 0$, $\nabla_{\mathbf{r}} n = 0$ and, $\mathbf{J} = 0$



$$J_x = |e|\mu_n\, n\, E_x - e\,\mu_n\, n\, C_n \frac{\partial T}{\partial x} - eD_n \frac{\partial n}{\partial x}$$

$$= |e|\mu_n\, n\, E_x - e\,\mu_n\, n\, C_n \frac{\partial T}{\partial x} = 0$$

$$\Rightarrow E_x = -C_n \frac{\partial T}{\partial x}$$

→ A built-in E-field by temperature gradient

Thermal Contribution to Electrical Current(6)

- Let's define thermoelectric power (amount of electric field induced by thermal gradient) as

$$\alpha_n = \frac{E_x}{\partial T/_{\partial x}}$$
$$= -C_n$$
$$= \frac{1}{e}\left[\frac{\mathcal{E}_c - \mathcal{E}_F}{T} - \frac{\partial(\mathcal{E}_c - \mathcal{E}_F)}{\partial T} + \frac{1}{T}\frac{\langle \tau \mathcal{E}' \rangle}{\langle \tau \rangle}\right]$$
$$= \frac{\langle \tau \mathcal{E}' \rangle + \left[(\mathcal{E}_c - \mathcal{E}_F) - T\frac{\partial(\mathcal{E}_c - \mathcal{E}_F)}{\partial T}\right]\langle \tau \rangle}{-|e|\langle \tau \rangle T}$$

- For a non-degenerate semiconductor with $\tau = A\mathcal{E}'^{-S}$.

$$\langle \tau \mathcal{E}' \rangle = \frac{Ak_BT}{(k_BT)^S}\frac{\Gamma\left(\frac{7}{2}-s\right)}{\Gamma\left(\frac{5}{2}\right)}$$

FYI, $\langle \tau \rangle = \frac{A}{(k_BT)^S}\frac{\Gamma\left(\frac{5}{2}-s\right)}{\Gamma\left(\frac{5}{2}\right)}$

$$= \frac{Ak_BT}{(k_BT)^S}\frac{\left(\frac{5}{2}-s\right)\Gamma\left(\frac{5}{2}-s\right)}{\Gamma\left(\frac{5}{2}\right)} = \left(\frac{5}{2}-s\right)k_BT\langle \tau \rangle$$

Thermal Contribution to Electrical Current(7)

- This gives

$$\alpha_n = \frac{\langle \tau \mathcal{E}' \rangle + \left[(\mathcal{E}_c - \mathcal{E}_F) - T \frac{\partial(\mathcal{E}_c - \mathcal{E}_F)}{\partial T} \right] \langle \tau \rangle}{-|e| \langle \tau \rangle T}$$

$$= \frac{\left(\frac{5}{2} - s \right) k_B T \langle \tau \rangle + \left[(\mathcal{E}_c - \mathcal{E}_F) - T \frac{\partial(\mathcal{E}_c - \mathcal{E}_F)}{\partial T} \right] \langle \tau \rangle}{-|e| \langle \tau \rangle T}$$

$$\cong -\frac{k_B}{|e|} \left[\left(\frac{5}{2} - s \right) + \frac{\mathcal{E}_c - \mathcal{E}_F}{k_B T} \right]$$

- In the case of metal, there is no $\mathcal{E}_c$ so,

$$\alpha_n = \frac{\langle \tau \mathcal{E}' \rangle + \left[-\mathcal{E}_F + T \frac{\partial(\mathcal{E}_F)}{\partial T} \right] \langle \tau \rangle}{-|e| \langle \tau \rangle T}$$

$$\cong \frac{\tau(\mathcal{E}_F) \mathcal{E}_F + [-\mathcal{E}_F] \tau(\mathcal{E}_F)}{-|e| \langle \tau \rangle T} = 0$$