Spring 2019

EECE 588
Lecture 20

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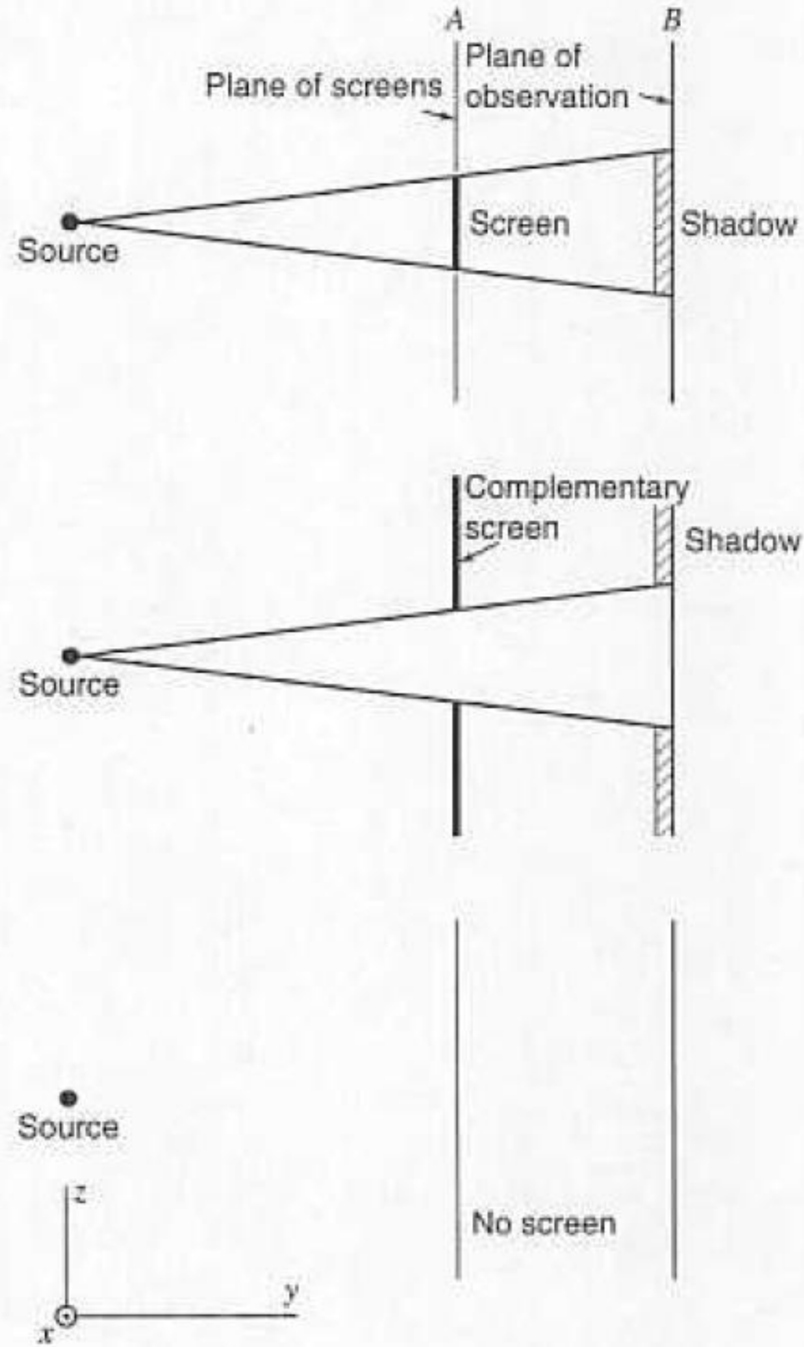
Babinet's Principle

- Babinet's (Ba-bi-nay's) principle:

  The field at any point behind a plane having a screen, if added to the field at the same point when the complementary screen is substituted, is equal to the field when there is no screen present.

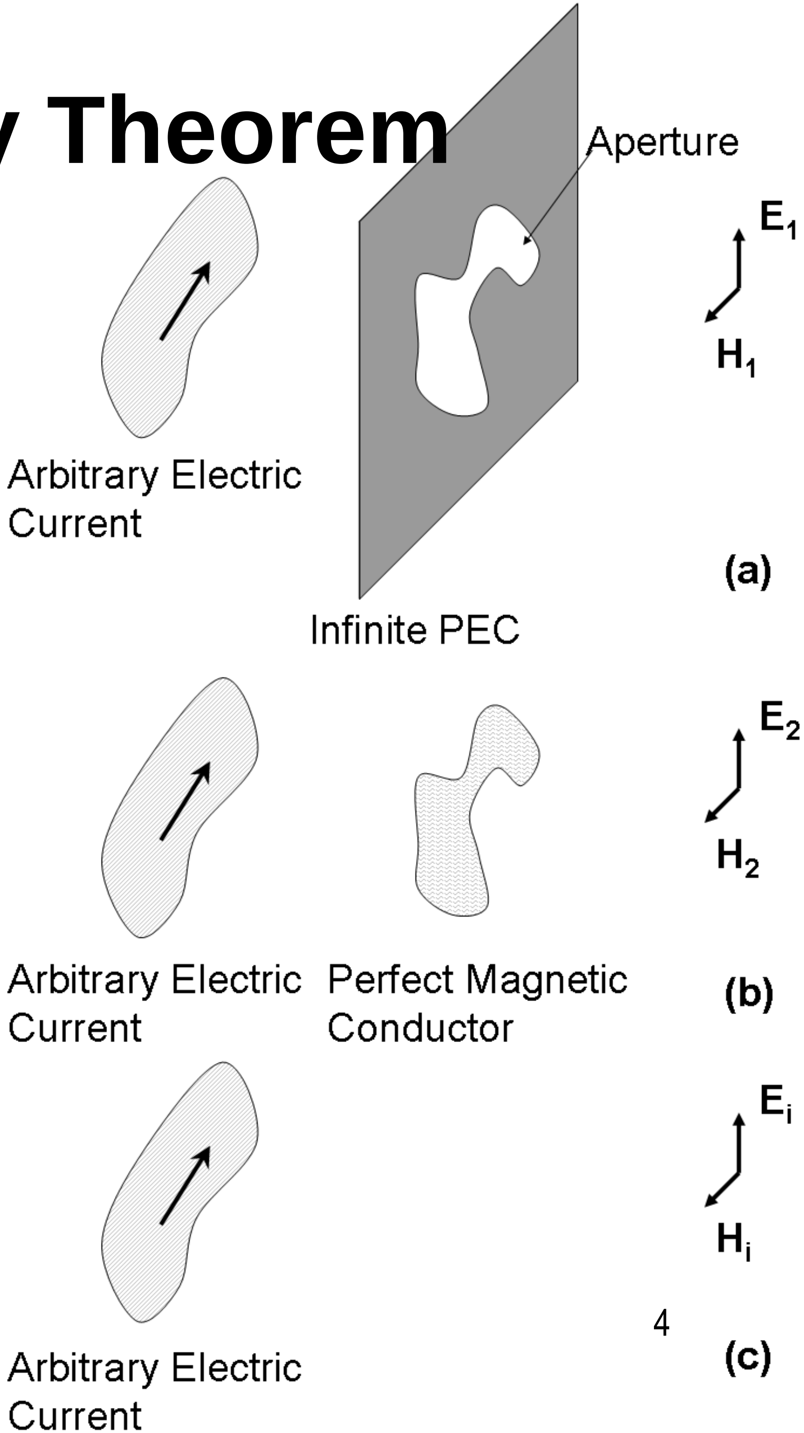


POSTECH

Babinet’s Principle

- Babinet’s principle is an Optical concept, where the polarization of the light is not considered.
- In EM, we have to consider the polarization of the electromagnetic wave.
- Babinat’s principle is generalized by Booker to take into account the vector nature of the electromagnetic field.
- We assume that the screen is an infinitesimally thin PEC plane and the complementary plane is a PMC plane.

Complementary Theorem



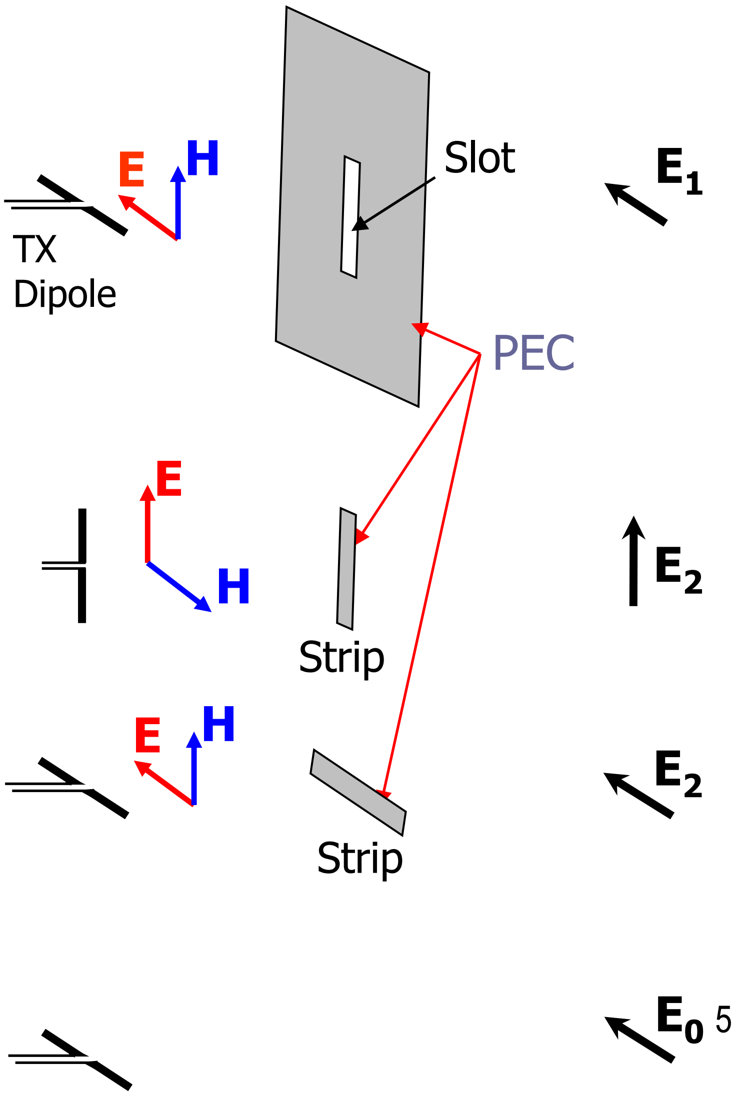
- Case a: Infinite PEC contains a number of apertures of arbitrary shapes.
  - This system is excited by electric sources located on one side of the sheet.
- Case b: Sources are identical to part a but all the apertures are replaced with PMCs.
- Complementary Theorem states that (in the source free region):
  - $\mathbf{E}_1+\mathbf{E}_2=\mathbf{E}_i$
  - $\mathbf{H}_1+\mathbf{H}_2=\mathbf{H}_i$
- $\mathbf{E}_i$ and $\mathbf{H}_i$ are the EM fields generated by $\mathbf{J}$ in the absence of all scatterers

POSTECH

What if we do not want to deal with PMCs?

- Well, lucky for us, we can use the duality between E & H fields to get rid of this problem.
- We will consider all screens to be PECs but we will switch the Electric and Magnetic fields.
- In the figure shown here, the Booker's extension of the Babinet's principle states that:

$$E_1+E_2=E_0$$



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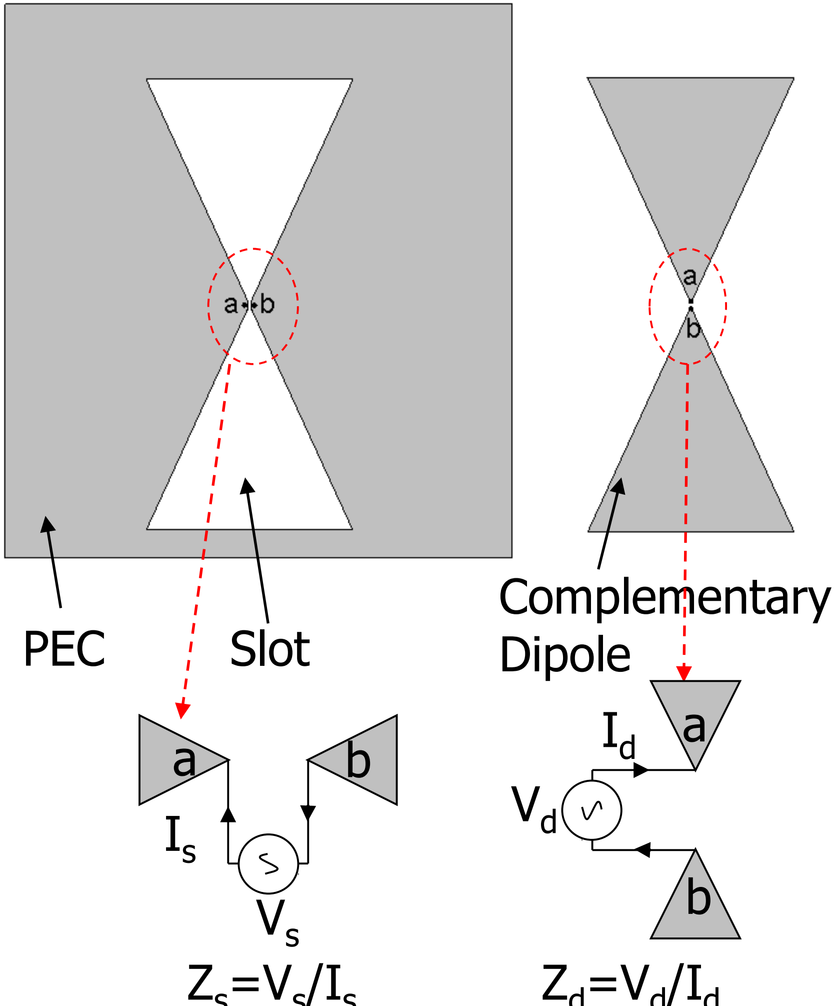
The Impedance of Complementary Screens

- A direct consequence of Babinet-Brook principle is the relationship that exist between the impedances of complementary antennas.
- The impedance of a slot (Z_s) and its complementary dipole structure (Z_d) are related together as follows:

$$Z_s Z_d = \frac{Z_0^{\ 2}}{4}$$

where Z_0 is the intrinsic impedance of the medium in which the antennas are present.

Z_0=377Ω for free space



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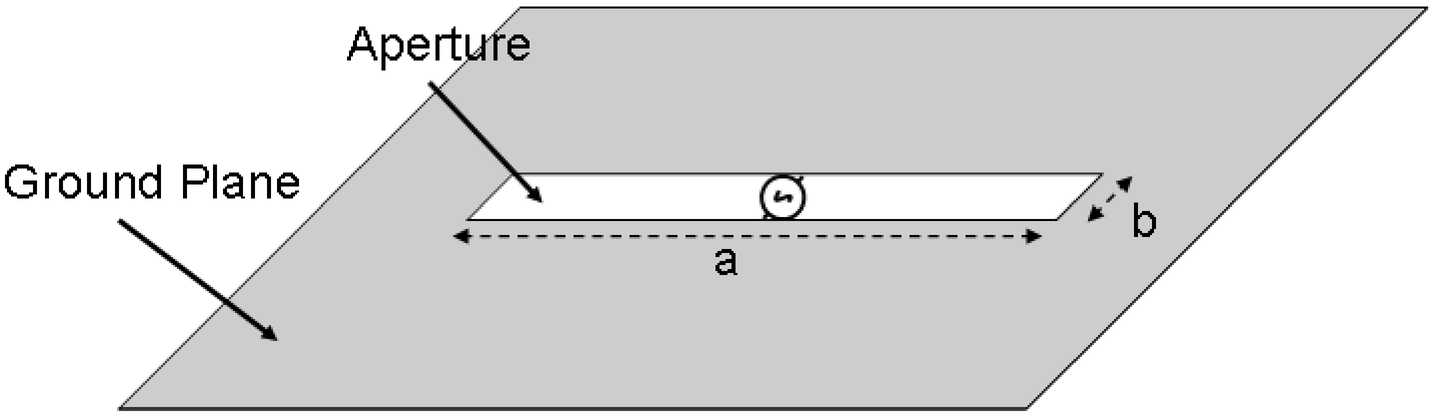
Slot Antennas

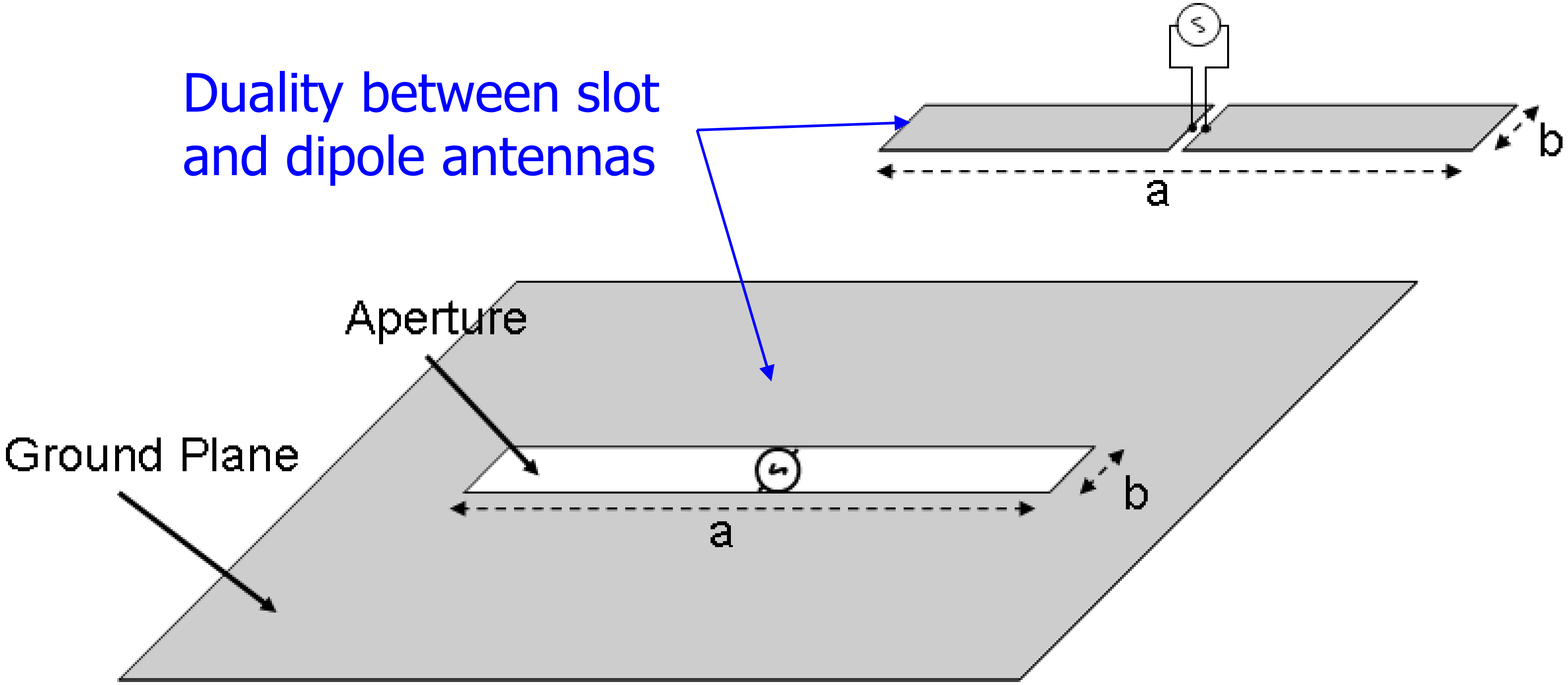
- Slot antennas are complementary structures of dipoles antennas.
- The most basic form of a slot antenna is a rectangular aperture in a ground plane as shown in this figure.
- Note that the ground plane extends to infinity (even though it is shown as finite).
- Many of the analysis techniques we learned about aperture antennas are applicable to slot antennas as well.



Duality Between Dipole and Slot Antennas

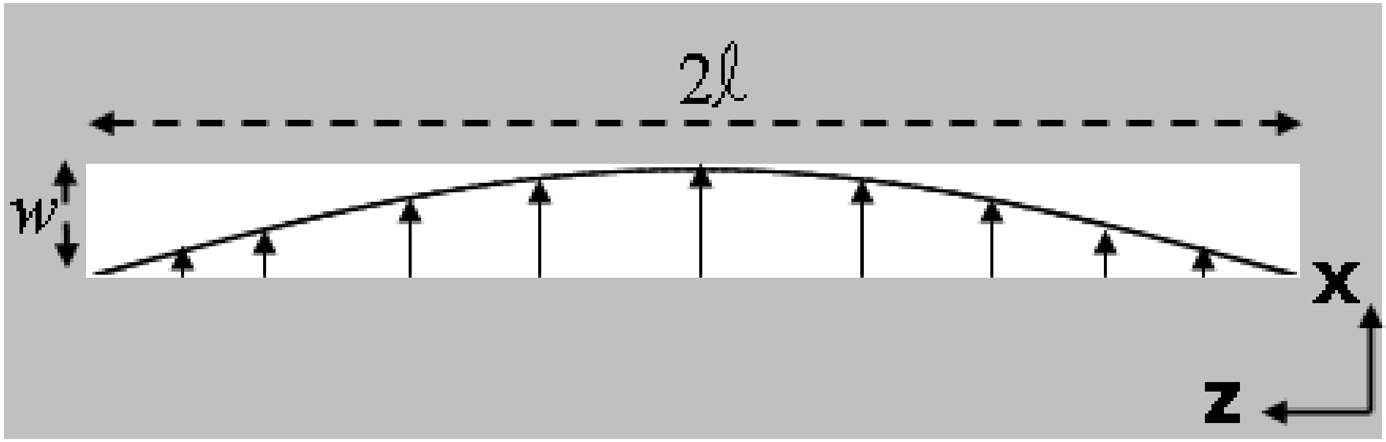
- Unlike the case we studied in aperture antennas, in a slot antenna, we cannot have a uniform distribution over the aperture.

Duality between slot and dipole antennas



Field Distribution Over The Aperture

- The first resonant mode of the slot antenna has a half-sinusoid aperture field distribution as shown in the figure below (TE_{10}).
- This happens when the length of the slot is approximately $\lambda/2$.
- To obtain the far field radiated fields of the antenna, use the equivalence theorem in conjunction with the image theory.
- The magnetic current $\vec{M}_s = -2\hat{n} \times \vec{E}_a$ exists over the aperture.

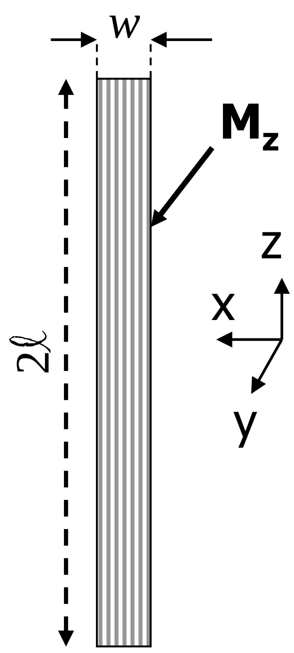


Far Field Calculation

- The first resonant mode of the slot antenna has a half-sinusoid E field distribution.

$$\vec{E}_x = \hat{x}\frac{V}{w}\sin(k(\ell - |z'|)) \quad -\ell < z' < \ell$$

$$\vec{M}_z = -2\hat{n}\times\vec{E} = \hat{z}\frac{2V}{w}\sin(k(\ell - |z'|))$$



$$\vec{F}_z = \frac{\varepsilon}{4\pi}e^{-jkr}\int_{-w/2}^{w/2}\int_{-\ell}^{\ell}\vec{M}_z e^{jkz'\cos\theta}dz'\,dx'$$

$$F_\theta = -\sin\theta\, F_z \quad H_\theta = -j\omega F_\theta \quad E_\varphi = -\eta\,\hat{r}\times\vec{H}_\theta$$

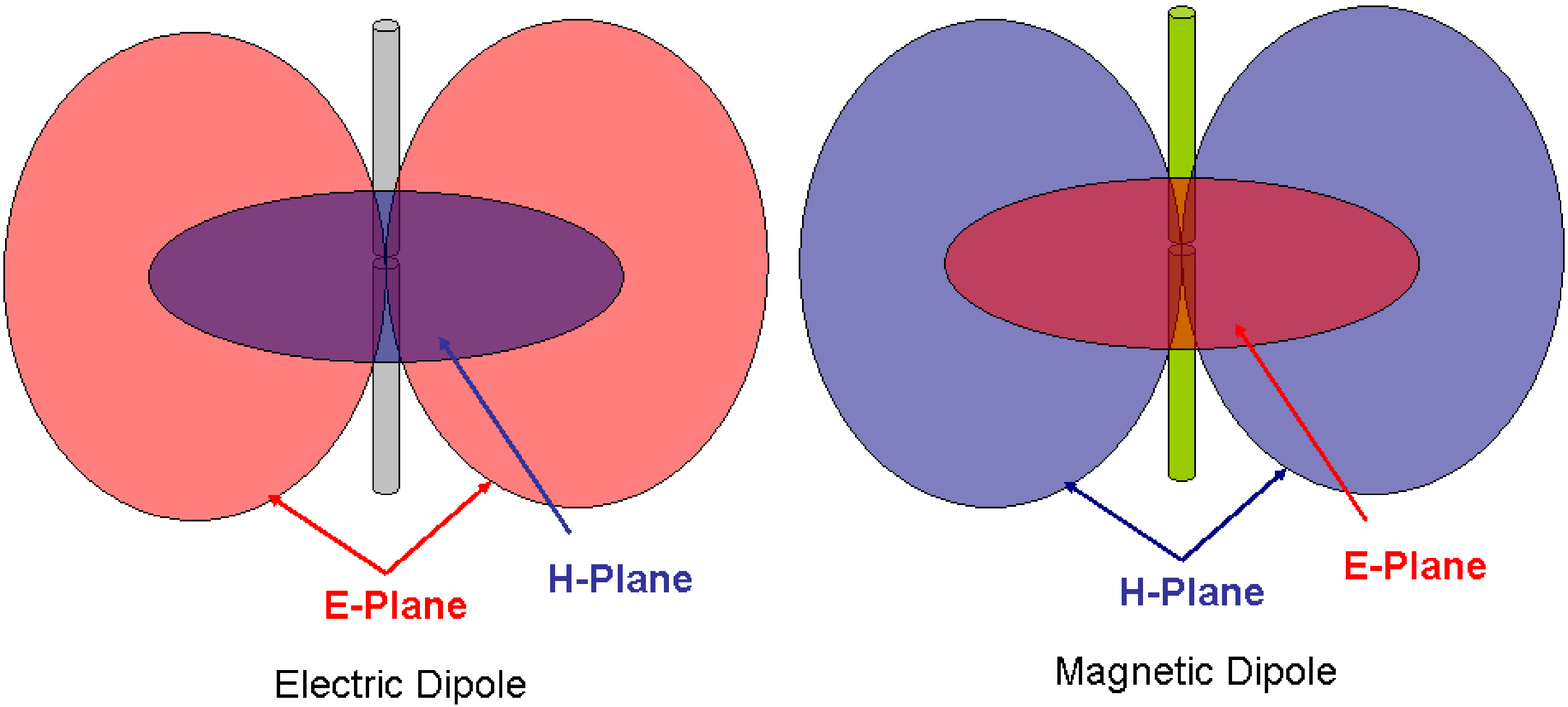
Far Field Calculation.

- Notice that the radiated fields of slot antennas are similar to those of dipoles but the electric field has changed polarization to $\mathbf{E}_{\boldsymbol{\varphi}}$ rather than $\mathbf{E}_{\boldsymbol{\theta}}$.

$$\begin{cases} E_{\varphi} = -j\dfrac{V}{\pi}\dfrac{\cos(k\ell\cos\theta)-\cos(k\ell)}{\sin\theta}\dfrac{e^{-jkr}}{r} \\ H_{\theta} = -E_{\varphi}/\eta \end{cases}$$

$$\text{if } \ell = \lambda/2 \Rightarrow \begin{cases} E_{\varphi} = -j\dfrac{V}{\pi}\dfrac{\cos(\pi/2\cos\theta)}{\sin\theta}\dfrac{e^{-jkr}}{r} \\ H_{\theta} = -E_{\varphi}/\eta \end{cases}$$

Radiation Patterns of Magnetic Dipoles and Electric Dipoles



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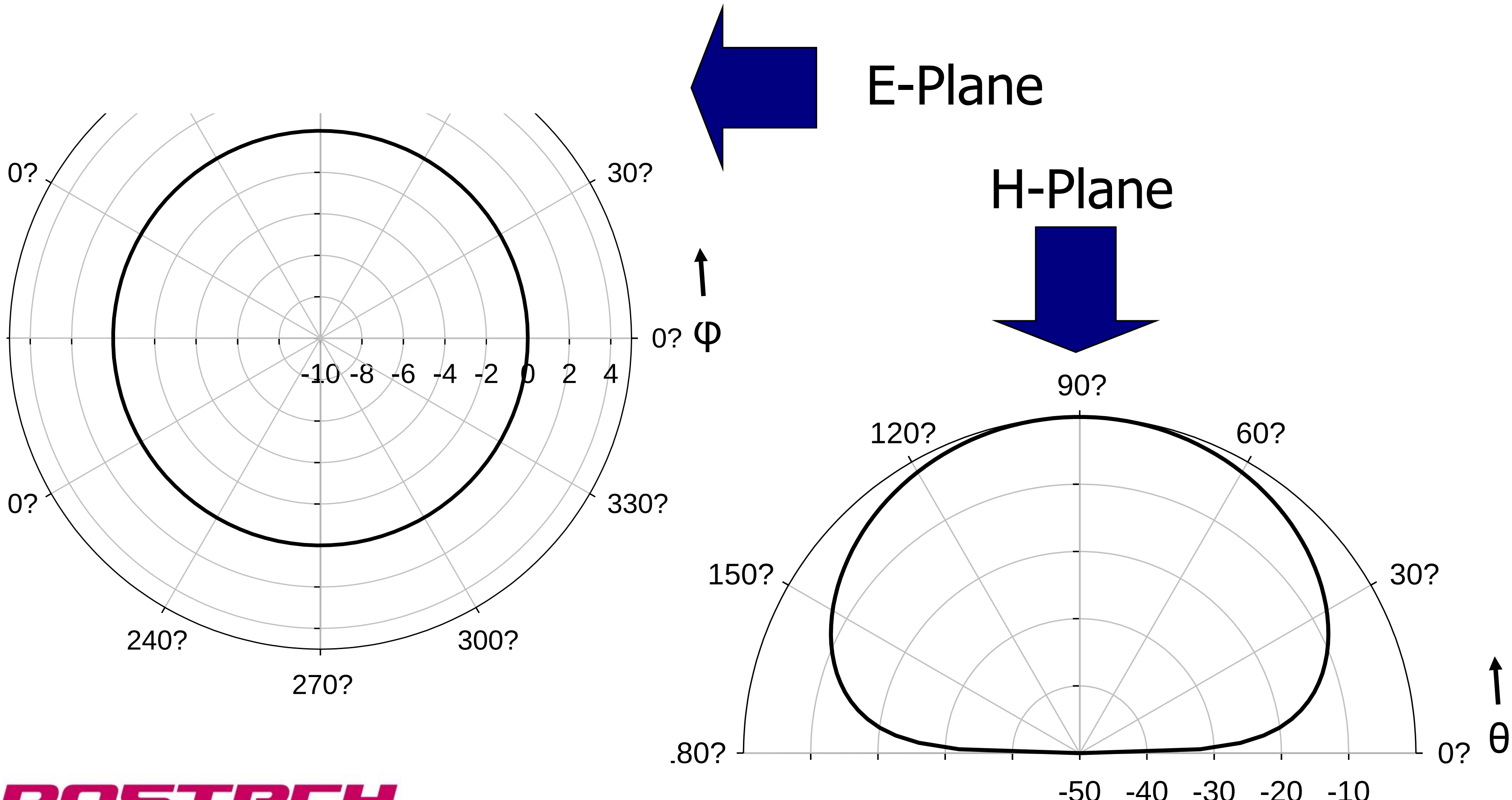
Radiation Patterns



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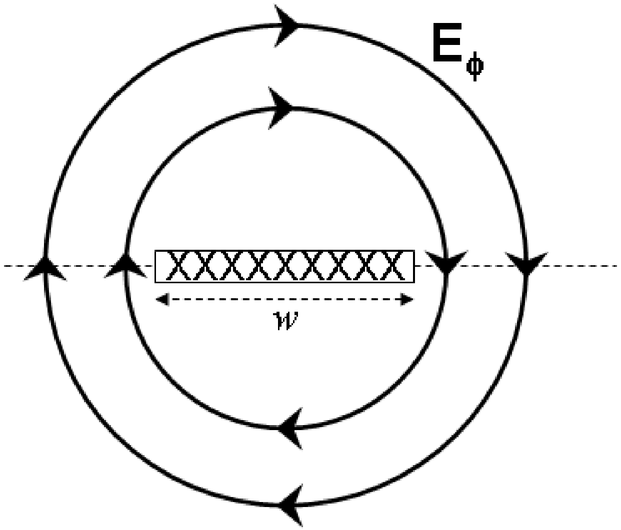
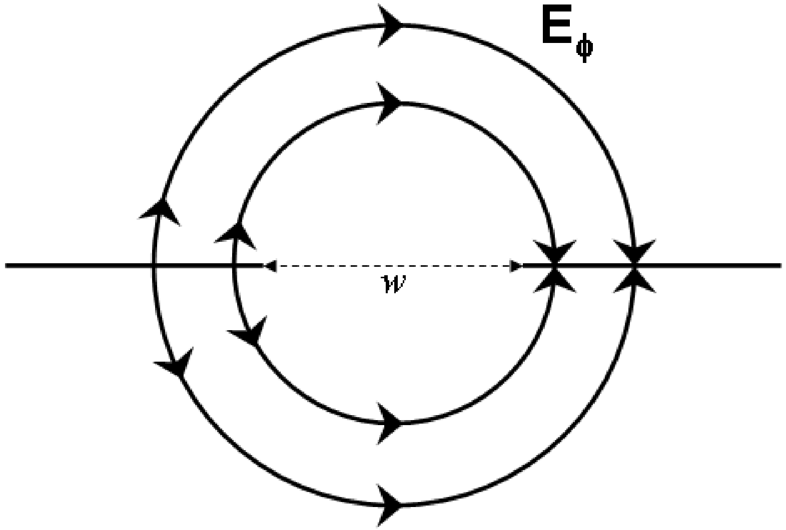
Magnetic Dipoles

- A slot antenna is not exactly identical to a magnetic dipole. In particular, the electric field directions are different as shown in the figure below.
- The electric field of a slot antenna reverses direction when we move from the upper half space to the lower half space.

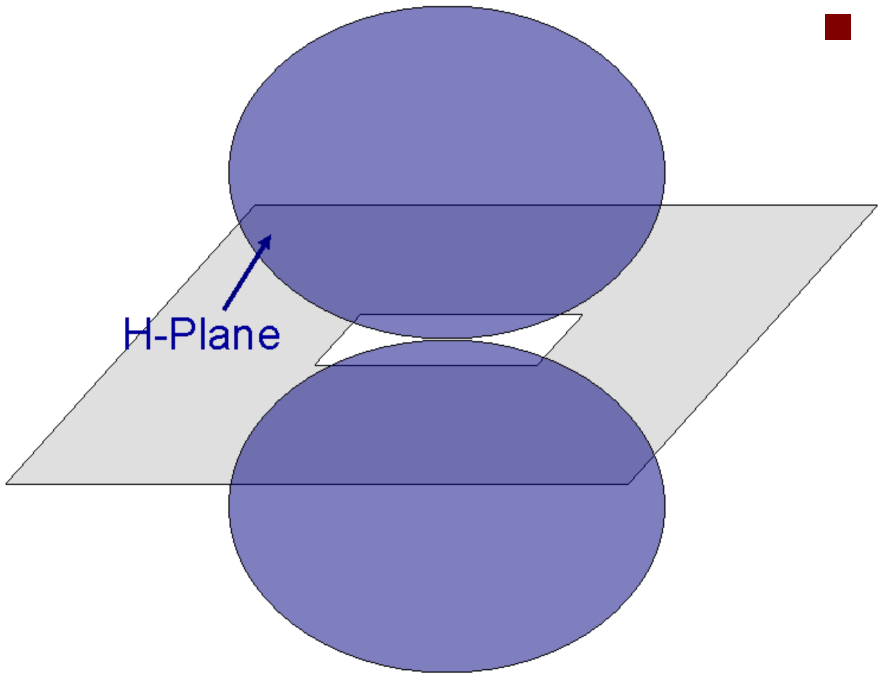


Magnetic Dipole

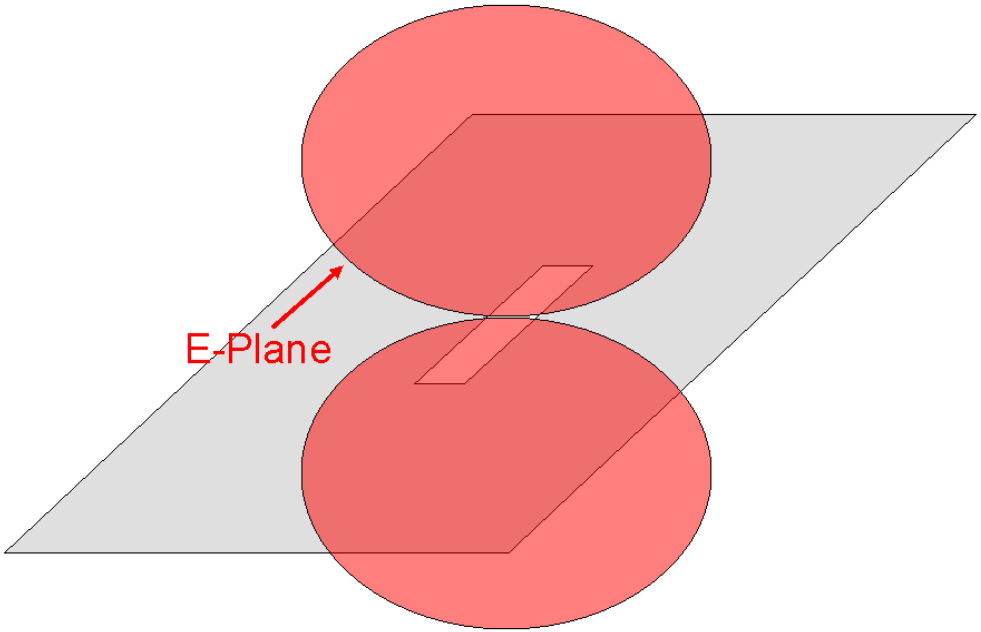


Slot Antenna

Radiation Patterns of Slot

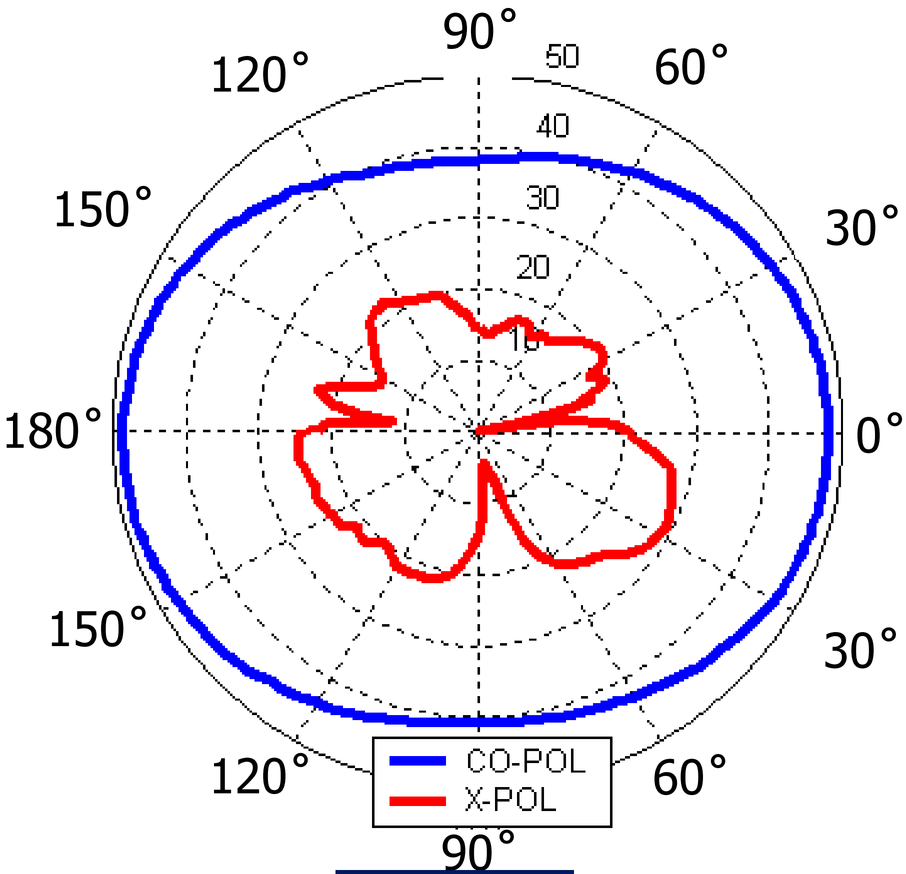


- In reality, the radiation patterns in the E-Plane also go to zero at grazing angles.

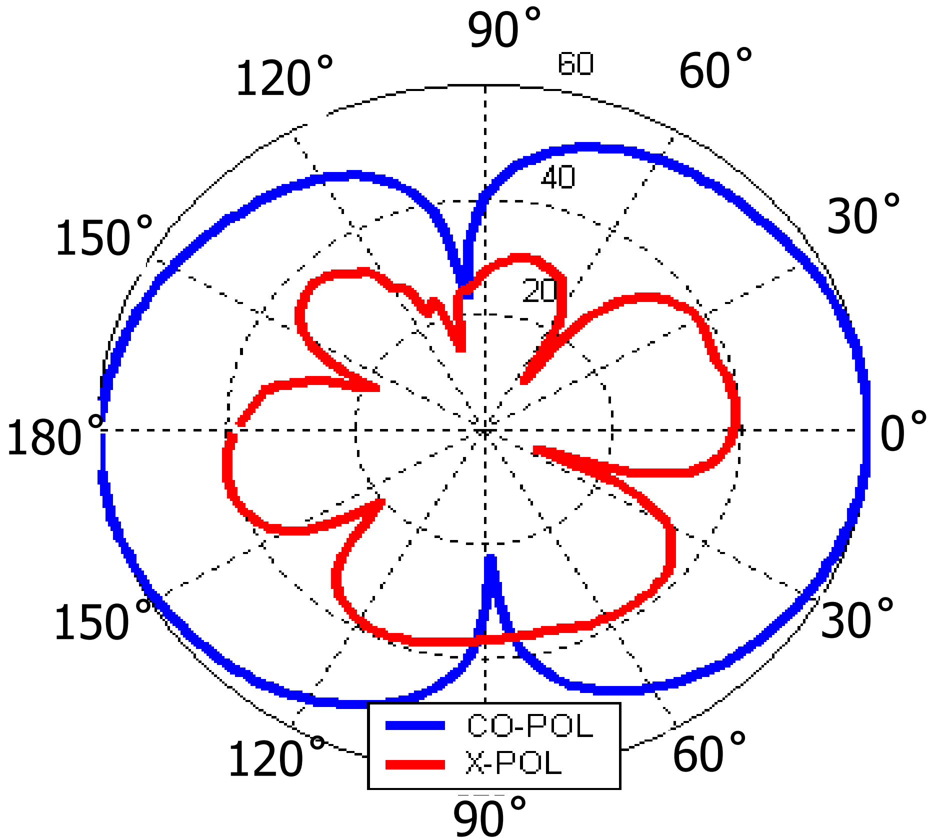


- For slot antennas with finite ground plane, the null in the E-plane is usually deeper than that at the H-plane.

Measured Radiation Patterns of a Slot Antenna



H-Plane



E-Plane

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Radiated Power and Radiation Resistance

- The radiated power of the slot antenna can be calculated by integrating the Poynting vector over a closed spherical surface.
- The radiation resistance (conductance) can also be calculated using the expression for the radiated power. Note that it can also be obtained using the Babinet's principle.

$$P_{rad} = \oint \frac{1}{2} \mathrm{Re}\left\{\vec{E} \times \vec{H}^*\right\} \cdot d\vec{s} = 0.609 \frac{2V^2}{\pi\eta} = \frac{1}{2} V^2 G_{rad}$$

$$G_{rad} = \frac{1}{R_{rad}} = 0.00206 \Rightarrow R_{rad} = 486\,\Omega$$

$$\text{Remember} \quad Z_s Z_d = \eta^2 / 4$$

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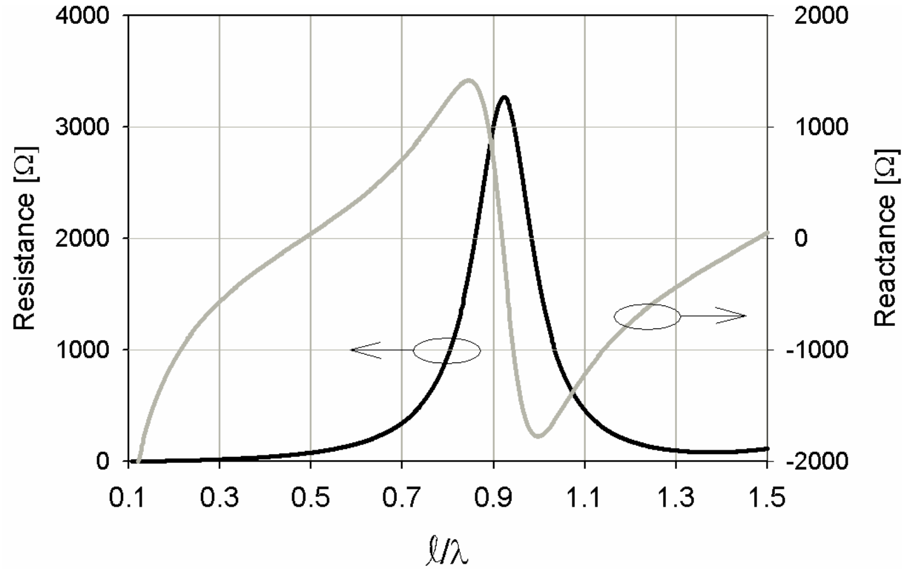
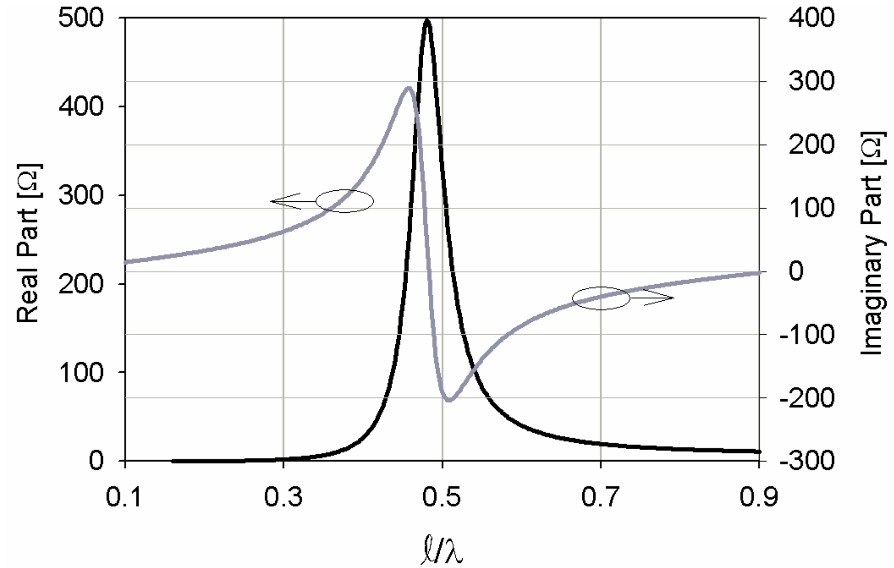
Input Impedance of Slot Antennas

- The input impedance of a center-fed slot antenna can be obtained from the impedance of a center-fed dipole antenna with the aid of the Babinet's principle.
- At its first resonance, the slot antenna impedance resembles the impedance of parallel RLC circuit.





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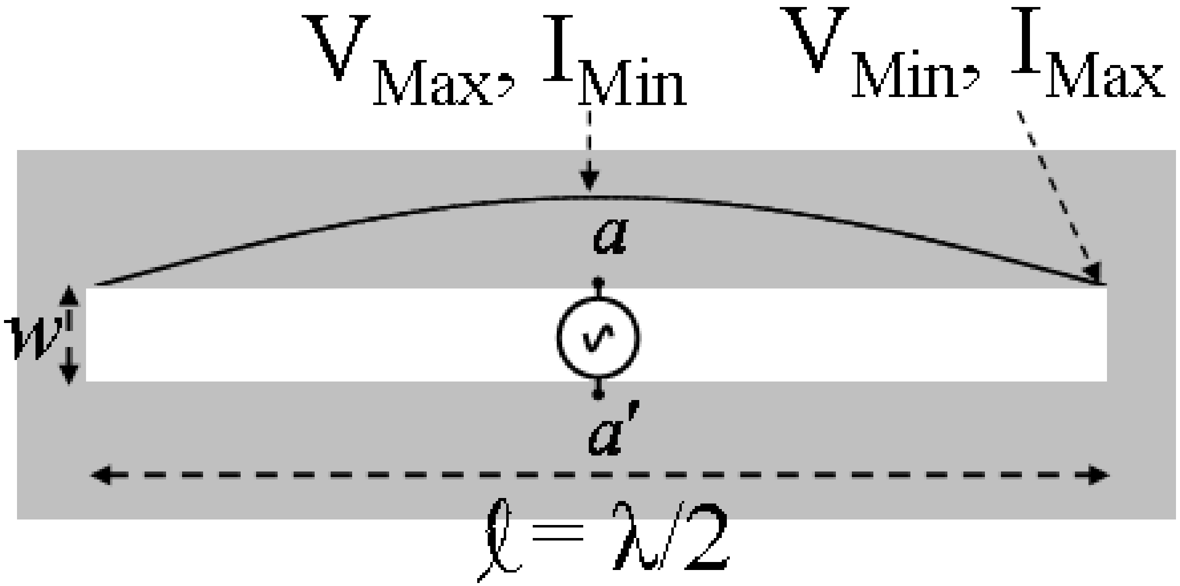
Feeding Slot Antennas

- To feed slot antennas, a voltage difference between the two edges of a slot must be created.
- This can theoretically be done by connecting a source to the two points on opposite edges (a and a') of a slot antenna as shown in figure below.



Feeding Slot Antennas

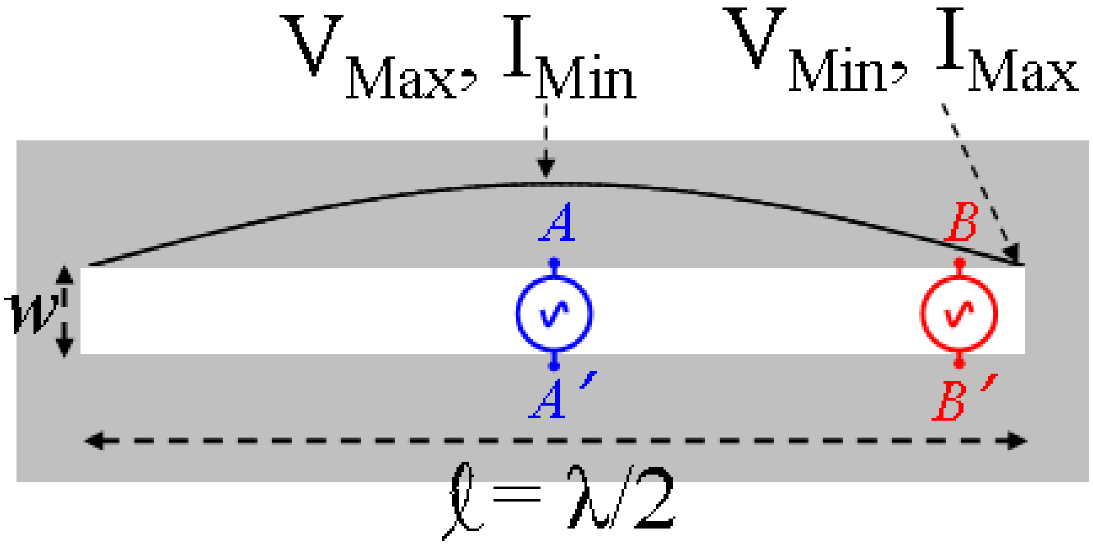
- Note that the potential difference (voltage) is defined as

$$V = -\int \vec{E}.d\vec{l}$$

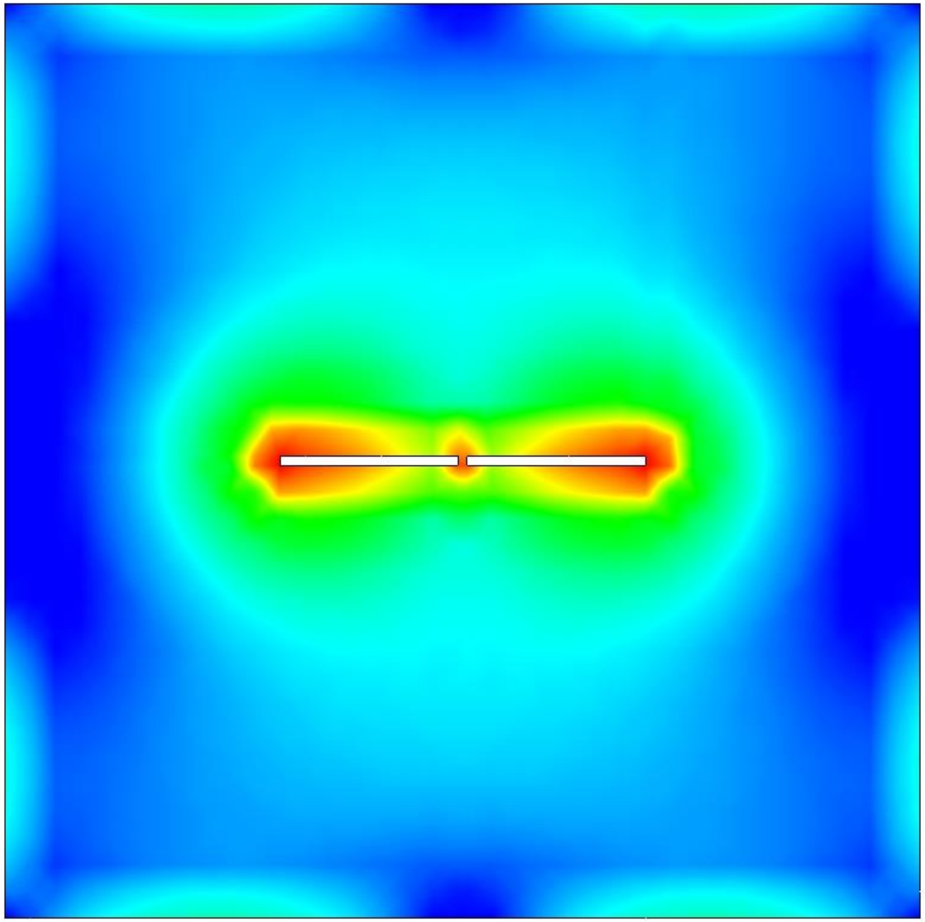
- Therefore, the voltage distribution over the aperture will be similar to the E field distribution (since there is no change in y direction).
- This means that the potential difference (between any two points on the horizontal edges of the slot) is maximum at the center and it is zero at the edges of the slot.
- The electric current flowing in the ground plane of the slot antenna is maximum at the edges of the aperture and zero at the center of the aperture.

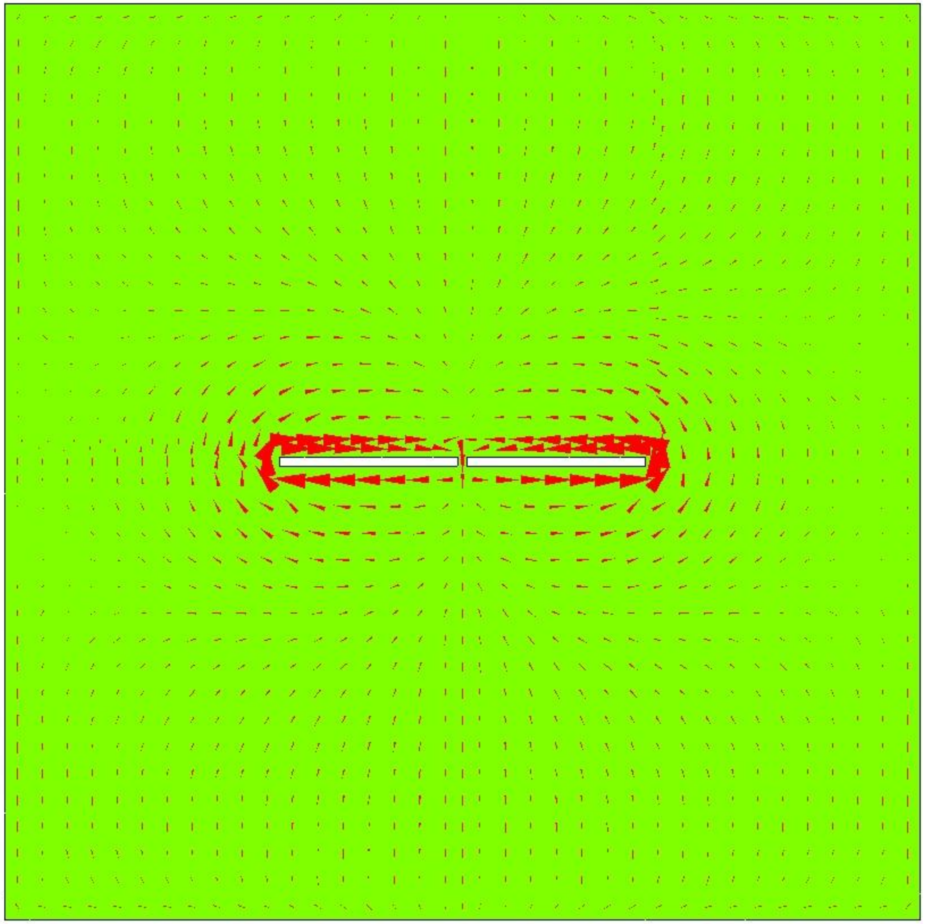
Feeding Slot Antennas

- The input impedance of the antenna (AS SEEN FROM THE TWO TERMINALS OF THE SOURCE) is then maximum when the source terminals are connected to the center of the slot antenna (*A* -*A'*) and it goes to zero as the feed approaches the edges of the slot.
- The input impedance at (*B* -*B'*) is then lower than that at (*A* -*A'*).



Current Distribution of a Slot Antenna





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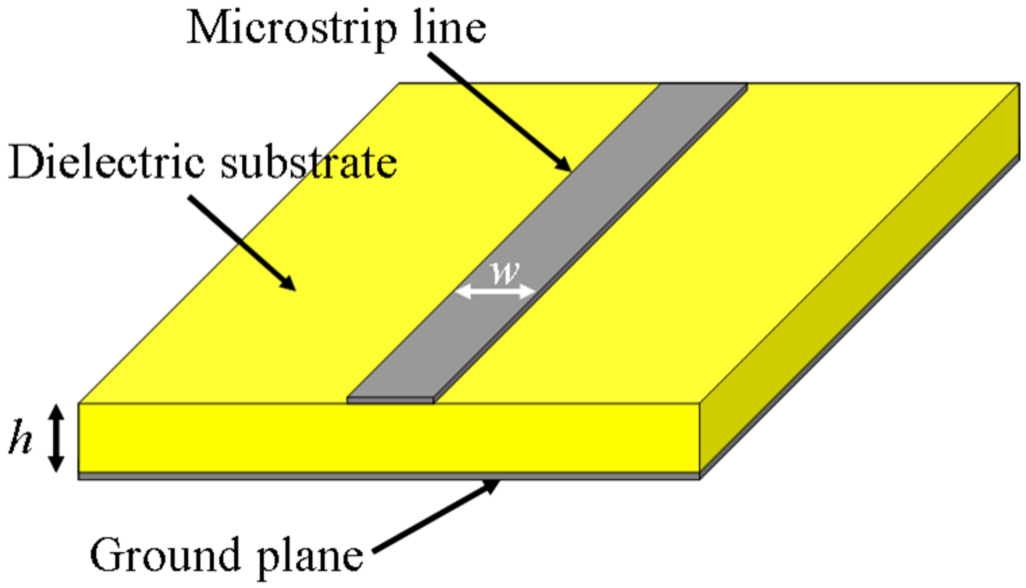
Feeding Slot Antennas

- Now, how do we REALLY feed slot antennas??
- In real life, we do not use simple sources as was shown in the previous slides. Usually, some kind of transmission line is used. It can be in the form of a coaxial cable, a microstrip line, or a coplanar waveguide.
- Microstrip Lines or Coplanar Waveguides (CPW) are used more than other kinds of transmission lines to feed slot antennas.

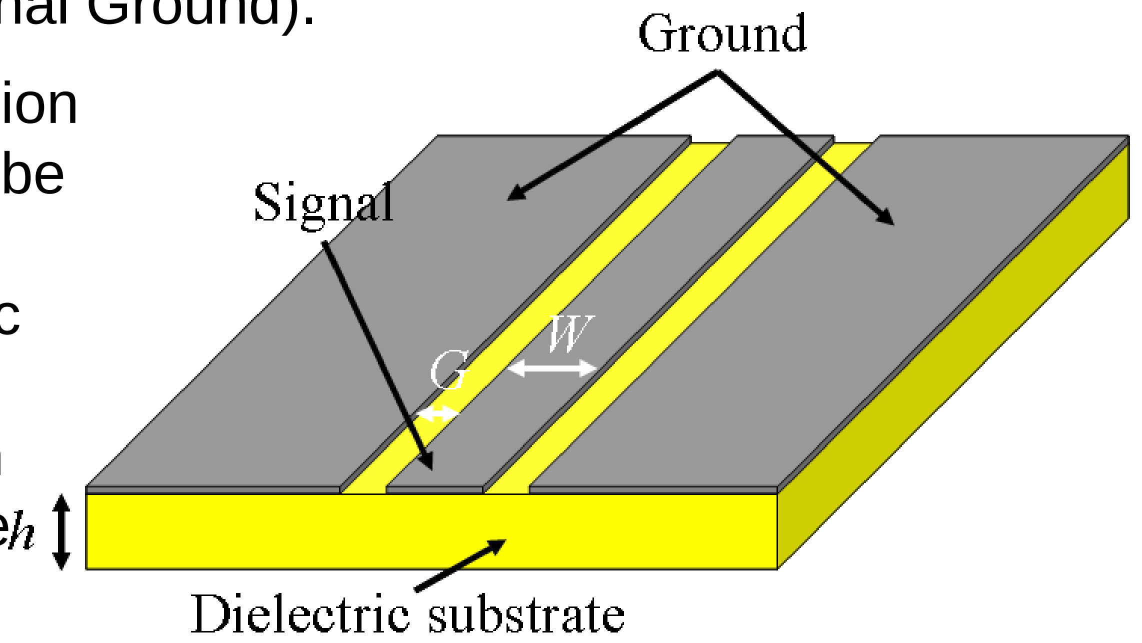
Microstrip Lines

- A microstrip line consists of a narrow strip of metal printed on one side of a dielectric substrate, while the other side is entirely covered with a ground plane (metal).
- The impedance and other characteristics of microstrip lines are covered extensively in Microwave Engineering books.



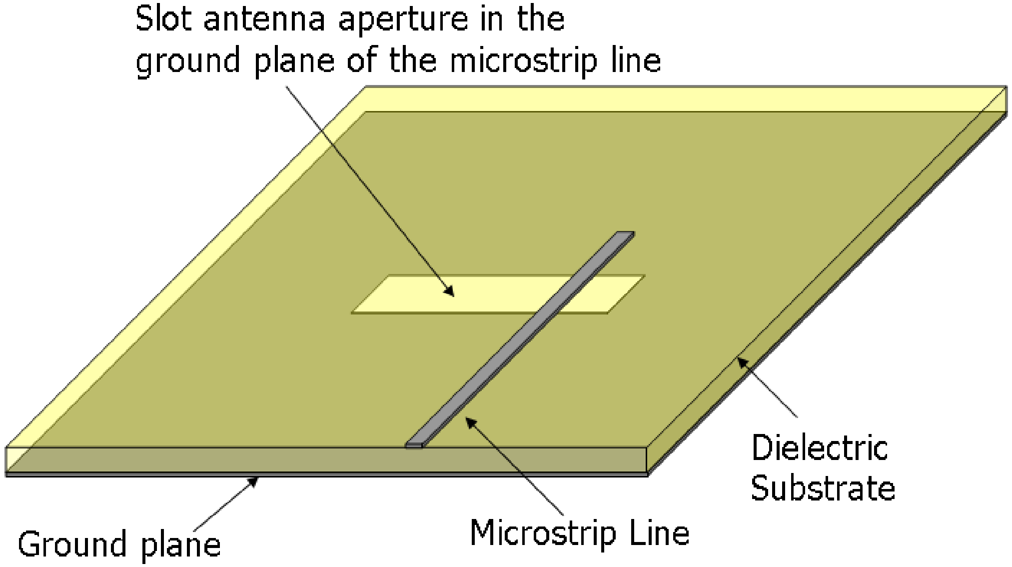
Coplanar Waveguides

- A coplanar waveguide is a uniplanar structure.
- It consists of a narrow signal trace and two infinitely wide ground planes located on each side of the signal trace.
- Some times (in the RFIC context mostly) CPW lines are called GSG (for Ground Signal Ground).
- To act as a transmission line, CPW lines must be excited in the ODD mode (i.e., the electric field in one slot is 180˚out of phase with the electric field in the other slot).



Microstrip-Fed Slot Antennas

- In microstrip-fed slot antennas, the slot is etched in the ground plane of the microstrip line as shown in the figure below.
- At the point where microstrip line crosses the slot antenna, energy is coupled to the antenna and it is as if a voltage source is placed at that location.
- After the microstrip-slot crossing, the microstrip line can be short circuited to the ground or it can be left open.



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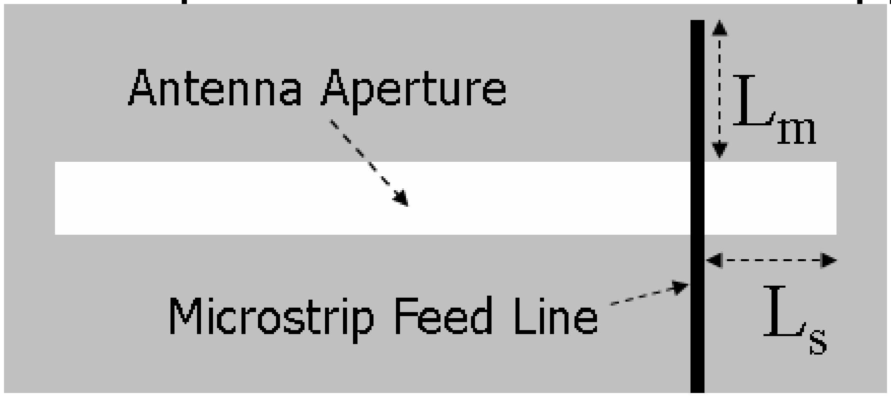
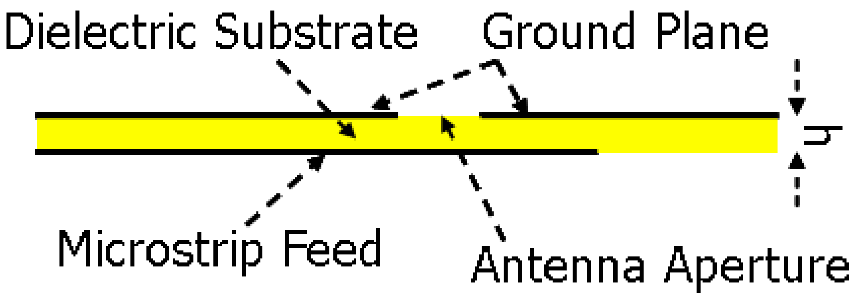
Microstrip-Fed Slot Antennas

- The location of the microstrip line feed, L_s, the characteristic impedance of the line, and the length of the open circuited stub, L_m, are all parameters that affect the input impedance of the antenna as seen from the microstrip line.
- If the microstrip line is located at the center, the input impedance is high.
- As the line moves towards the edge of the slot, the input impedance decreases and approaches zero at the edge.





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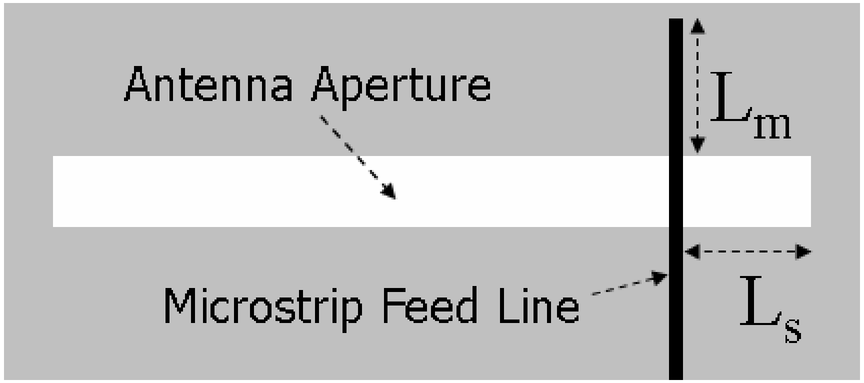
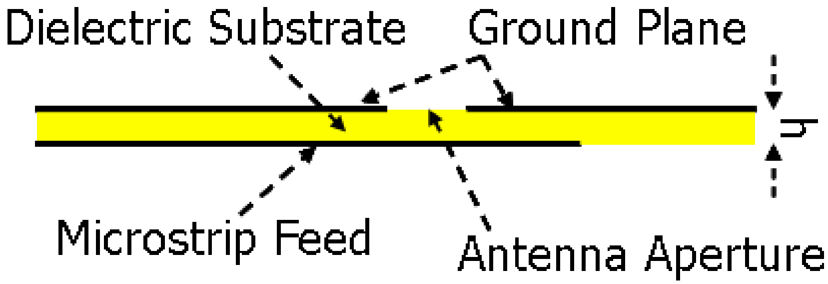
Microstrip-Fed Slot Antennas

- The length of the open circuited stub, L_m, can be chosen appropriately to cancel the input reactance of the antenna and achieve a good impedance match.
- Usually $L_m \approx \lambda/4$.
- By just choosing the feed location, L_s, and the length of the open circuited stub, L_m, appropriately, you can match the input impedance of the antenna to a wide range of source impedances (very easily in the range of 20 Ω-200 Ω).





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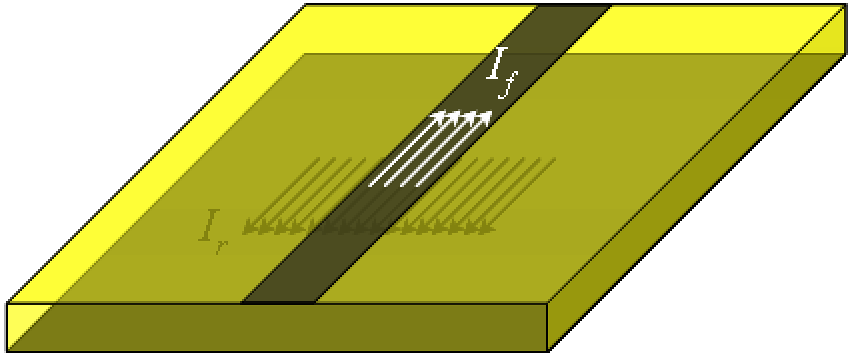
Equivalent Circuit Model of A Microstrip-Fed Slot Antenna

- Note the electric current distribution in a microstrip line and its ground plane.
- Forward current exists on the microstrip and the current returns in the ground plane of the TX line.



The forward and reverse currents flowing in a microstrip line and its ground plane.

- If a slot is cut in the ground plane of a microstrip line, the return electric current has to travel around the slot.

WHAT DOES IT MEAN?

Equivalent Circuit Model of A Microstrip-Fed Slot Antenna

- The equivalent circuit model for the slot antenna in the ground plane of a microstrip line is a series impedance in a transmission line.
- Considering the electric current distribution shown, it is easy to see why a series impedance representation is used.