Advanced Optics (PHYS690)

HEEDEUK SHIN

Lecture 19

POHANG UNIVERSITY OF SCIENCE AND TECHNOLOGY, KOREA

Electro-Optic modulator

Birefringence

the index ellipsoid:

$$\sum_{ij} \eta_{ij} x_i x_j = 1$$

is in the principal coordinate system:

$$\frac{x_1^2}{n_1^2} + \frac{x_2^2}{n_2^2} + \frac{x_3^2}{n_3^2} = 1$$

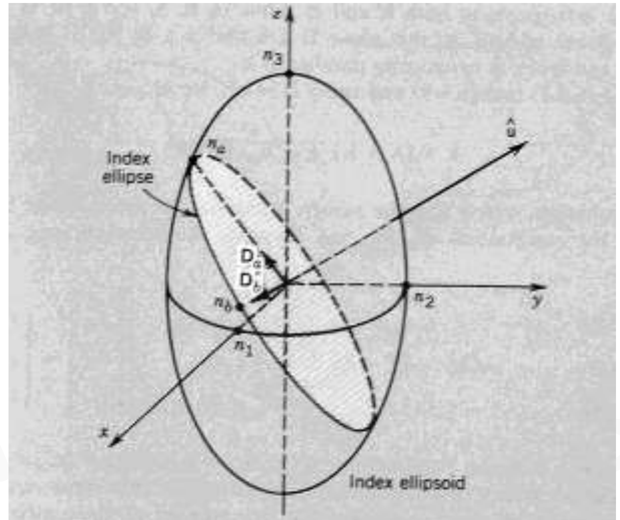


Figure 6.3-6 Normal modes determined from the index ellipsoid.

uniaxial crystals ($n_1 = n_2 \neq n_3$):

$$\frac{1}{n^2(\theta)} = \frac{\cos^2(\theta)}{n_0^2} + \frac{\sin^2(\theta)}{n_e^2}$$

$$n_a = n_0$$

$$n_b = n(\theta) \qquad n(0°) = n_0 \qquad n(90°) = n_e$$

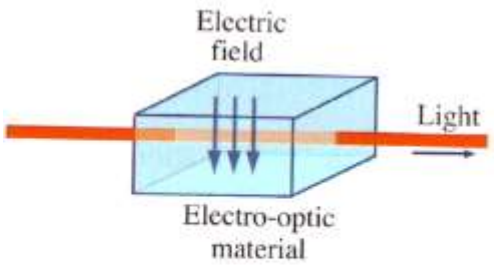
preferred crystals:

- $LiNbO_3$
- $LiTaO_3$
- KDP (KH_2PO_4)
- KD*P (KD_2PO_4)
- ADP ($NH_4H_2PO_4$)
- BBO (Beta-BaB_2O_4)

TABLE 8.4 Electro-optic constants (room temperature, λ_0 = 546.1 nm).

Material	r_{63} (units of 10^{-12} m/V)	n_o (approx.)	$V_{\lambda/2}$ (in kV)
ADP ($NH_4H_2PO_4$)	8.5	1.52	9.2
KDP (KH_2PO_4)	10.6	1.51	7.6
KDA (KH_2AsO_4)	~13.0	1.57	~6.2
KD*P (KD_2PO_4)	~23.3	1.52	~3.4

Electro-Optic Effect



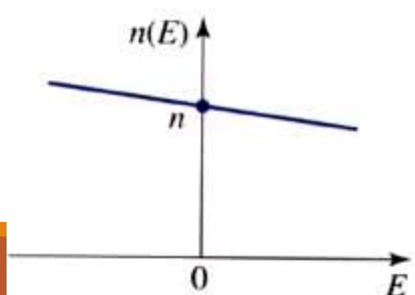
for certain materials n is a function of E,
as the variation is only slightly we can Taylor-expand n(E):

$$n(E) = n + a_1 E + \frac{1}{2} a_2 E^2 + \dots$$

linear electro-optic effect
(**Pockels effect**, 1893):

$$n(E) = n - \frac{1}{2} r \cdot n^3 E$$

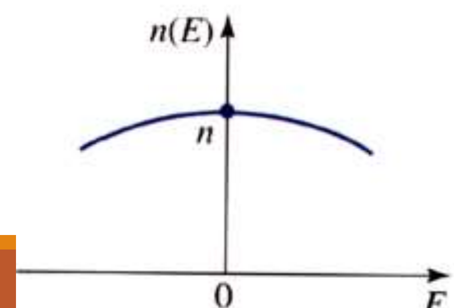
$$r = -2 \frac{a_1}{n^3}$$



quadratic electro-optic effect
(**Kerr effect**, 1875):

$$n(E) = n - \frac{1}{2} s \cdot n^3 E^2$$

$$s = -\frac{a_2}{n^3}$$



Kerr vs Pockels

the electric impermeability $\eta(E)$:

$$\eta = \frac{\varepsilon_0}{\varepsilon} = \frac{1}{n^2}$$

$$\Delta\eta(E) = \left(\frac{d\eta}{dn}\right)\cdot\Delta n = \left(\frac{-2}{n^3}\right)\cdot\left(-\frac{1}{2}r\cdot n^3 E - \frac{1}{2}s\cdot n^3 E^2\right) = r\cdot E + s\cdot E^2$$

...explains the choice of r and s.

Pockels effect:

typical values for r: 10^{-12} to 10^{-10} m/V

Δn for E=10^6 V/m : 10^{-6} to 10^{-4} (crystals)

Kerr effect:

typical values for s: 10^{-18} to 10^{-14} m^2/V^2

Δn for E=10^6 V/m : 10^{-6} to 10^{-2} (crystals)
10^{-10} to 10^{-7} (liquids)

Electro-Optic Effect

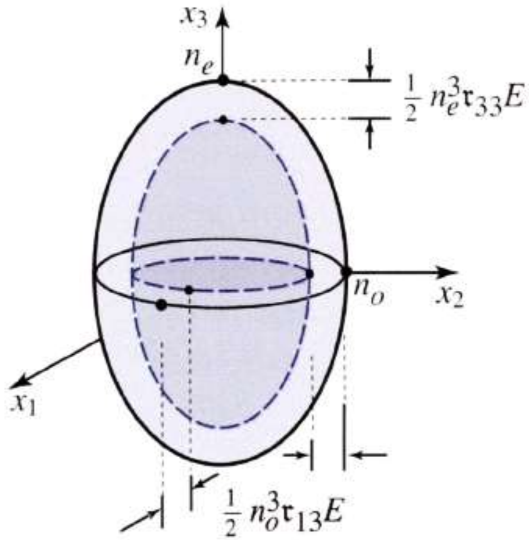
$$n_o(E) \approx n_o - \tfrac{1}{2} n_o^3 \mathfrak{r}_{13} E$$

$$n_e(E) \approx n_e - \tfrac{1}{2} n_e^3 \mathfrak{r}_{33} E.$$

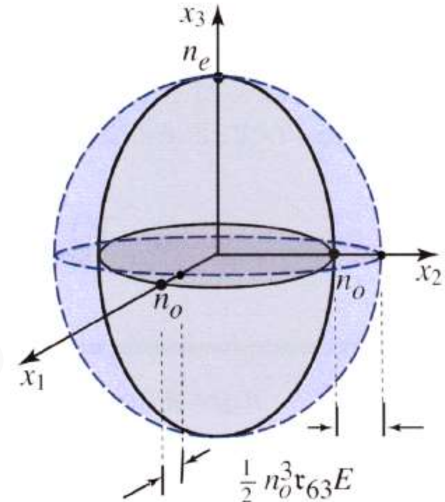
$$n_1(E) \approx n_o - \tfrac{1}{2} n_o^3 \mathfrak{r}_{63} E$$

$$n_2(E) \approx n_o + \tfrac{1}{2} n_o^3 \mathfrak{r}_{63} E$$

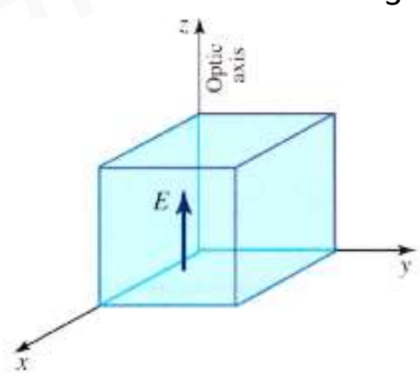
$$n_3(E) = n_e.$$



Modification of the index ellipsoid of a trigonal 3m crystal such as $LiNbO_3$.

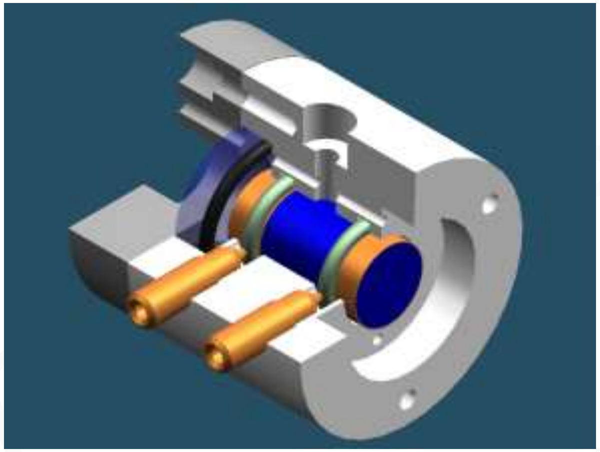


Modification of the index ellipsoid of a uniaxial tetragonal $\bar{4}2m$ crystal suc h as KDP.



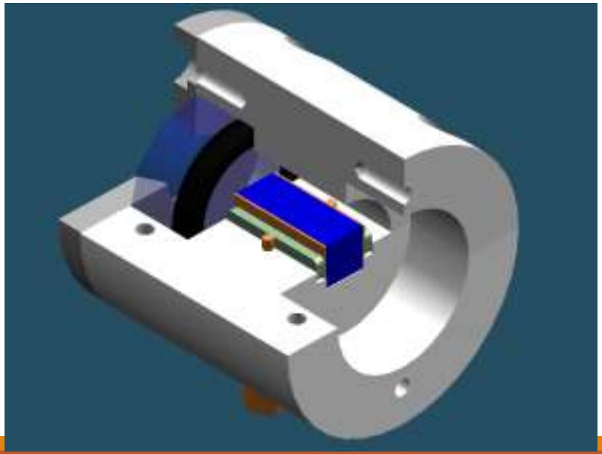
Pockels Cells

Construction



Longitudinal Pockels Cell (d=L)

- $V_\pi = \dfrac{\lambda}{r \cdot n^3}$
- V_π scales linearly with λ
- large apertures possible



Transverse Pockels Cell

- $V_\pi = \dfrac{d}{L} \dfrac{\lambda}{r \cdot n^3}$
- V_π scales linearly with λ
- aperture size restricted

from Linos Coorp.

Wave retarders

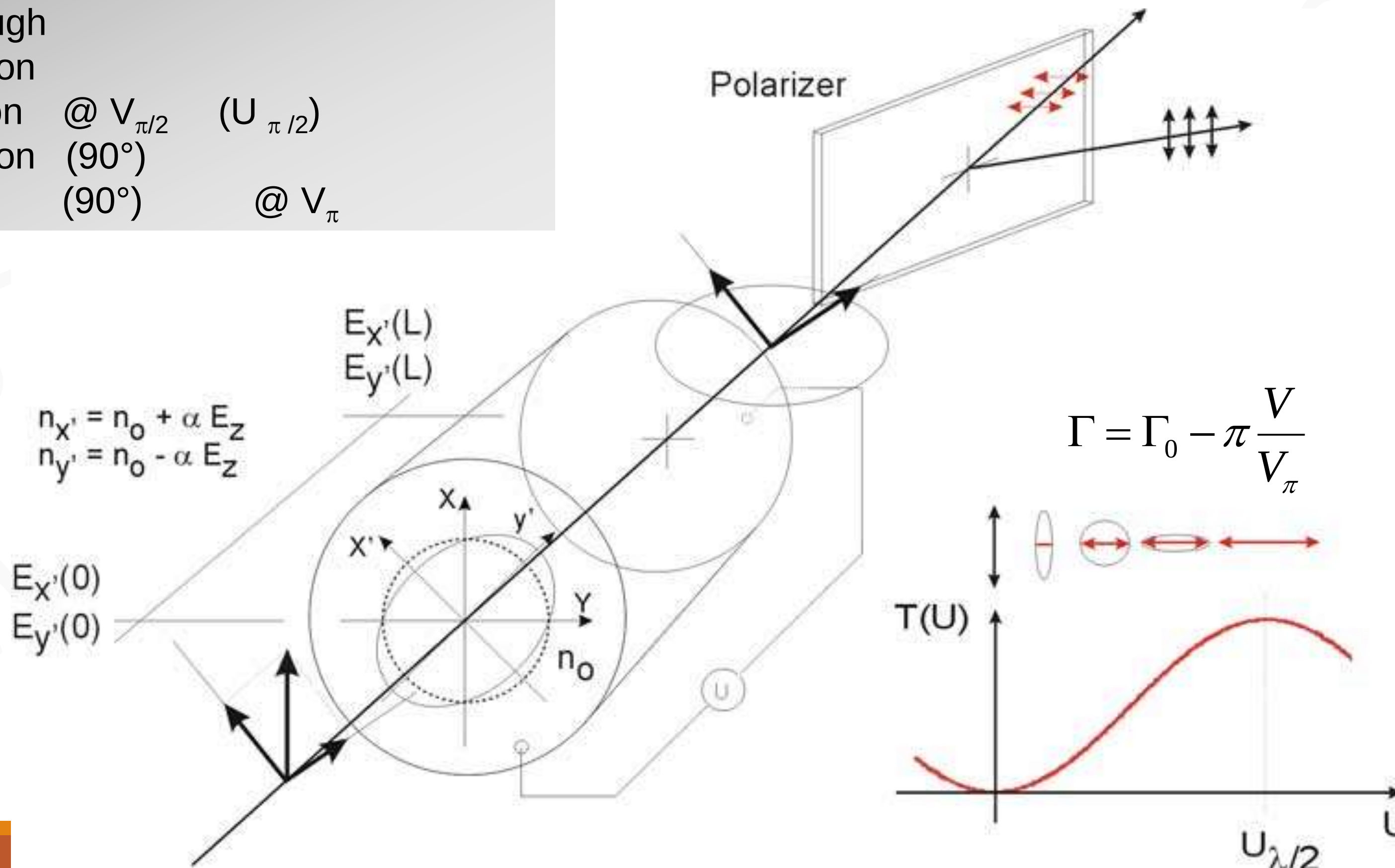
Pockels Cells

Pockels Cell can be used as dynamic wave retarders

Input light is vertical, linear polarized

with rising electric field (applied Voltage) the transmitted light goes through

- elliptical polarization
- circular polarization @ $V_{\pi/2}$ ($U_{\pi/2}$)
- elliptical polarization (90°)
- linear polarization (90°) @ V_{π}



$$\Gamma = \Gamma_0 - \pi \frac{V}{V_\pi}$$

Phase modulator

Phase modulation leads to **frequency modulation**

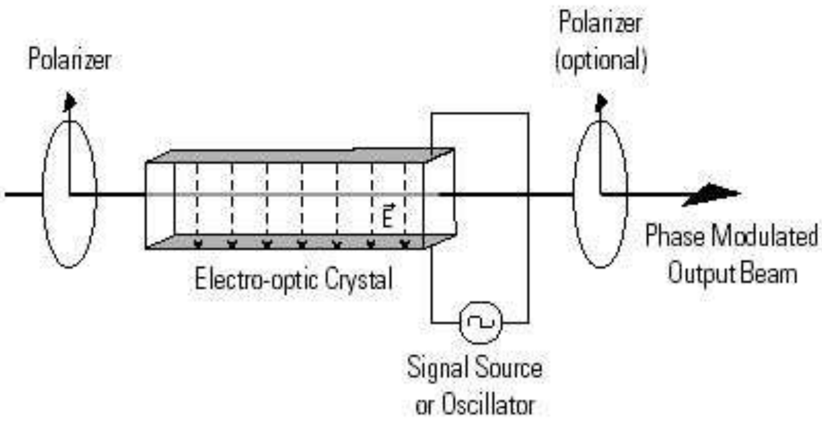
definition of frequency:

$$2\pi \cdot f(t) \equiv \frac{d\Phi(t)}{dt} = \omega \qquad [15]$$

with a phase modulation

$$2\pi \cdot f(t) \equiv \frac{d\Phi(t)}{dt} = \omega + \frac{d\phi(t)}{dt}$$

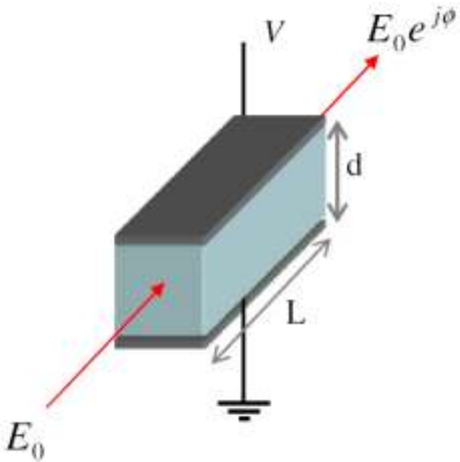
$$\phi(t) = m\sin(\Omega t)$$



$\Rightarrow$ frequency modulation at frequency Ω with 90° phase lag and peak to peak excursion of 2mΩ

$\Rightarrow$ Fourier components:
power exists only at discrete optical frequen cies $\omega \pm$k Ω

Phase shift



$$\phi = k_0(S+\Delta S) = k_0(n_0+\Delta n)L = \frac{2\pi}{\lambda_0}\left(n_0 - \frac{1}{2}n_0^3 r_{eff} E\right)L$$

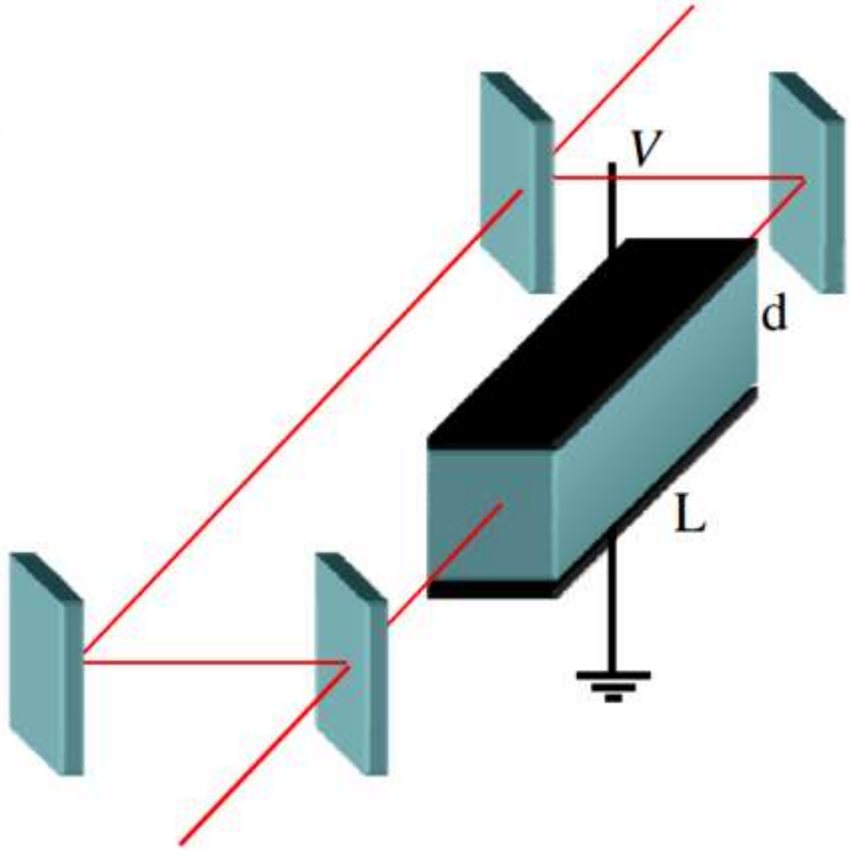
$$= \phi_0 - \frac{\pi}{\lambda_0} n_0^3 r_{eff} \frac{V}{d} L$$

$$\equiv \phi_0 - \pi \frac{V}{V_\pi}$$

$$V_\pi \equiv \frac{d}{L}\frac{\lambda_0}{n_0^3 r_{eff}}$$ "Half-wave voltage"

Transit-time limited bandwidth $\approx \dfrac{1}{T} = \dfrac{c}{n_0 L}$

Mach-Zender modulator



Phase-shifting Mach-Zender

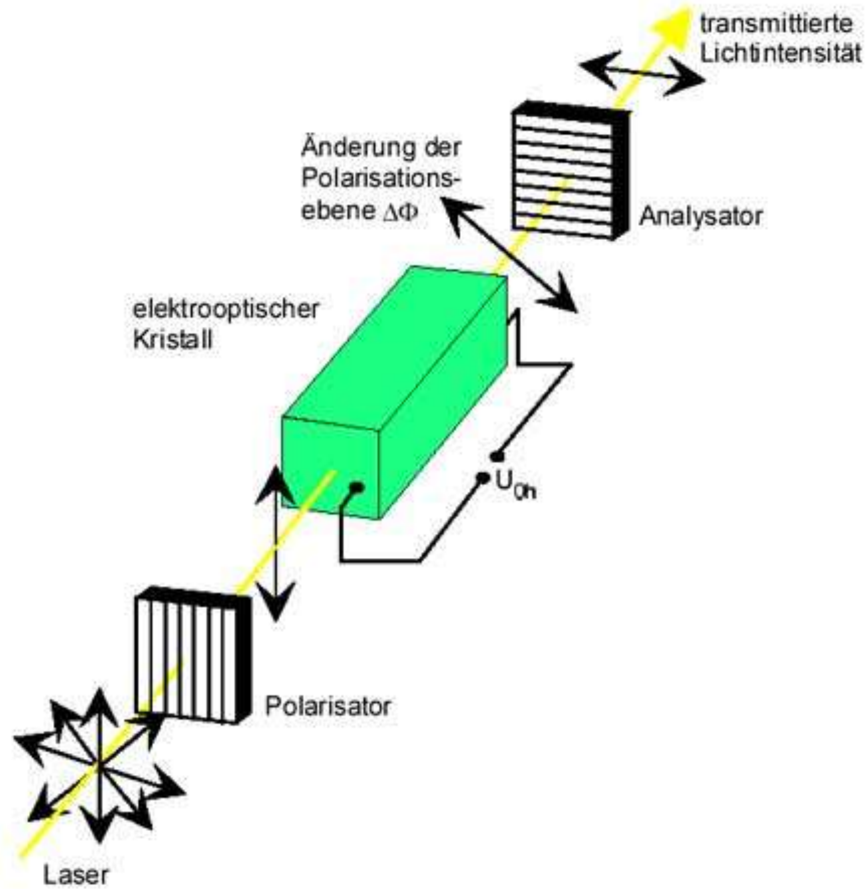
$$T = \frac{1}{2}(1 + \cos\phi)$$

$$= \cos^2\left(\frac{\phi_0}{2} - \frac{\pi}{2}\frac{V}{V_\pi}\right)$$

Amplitude modulator

- Polarizer

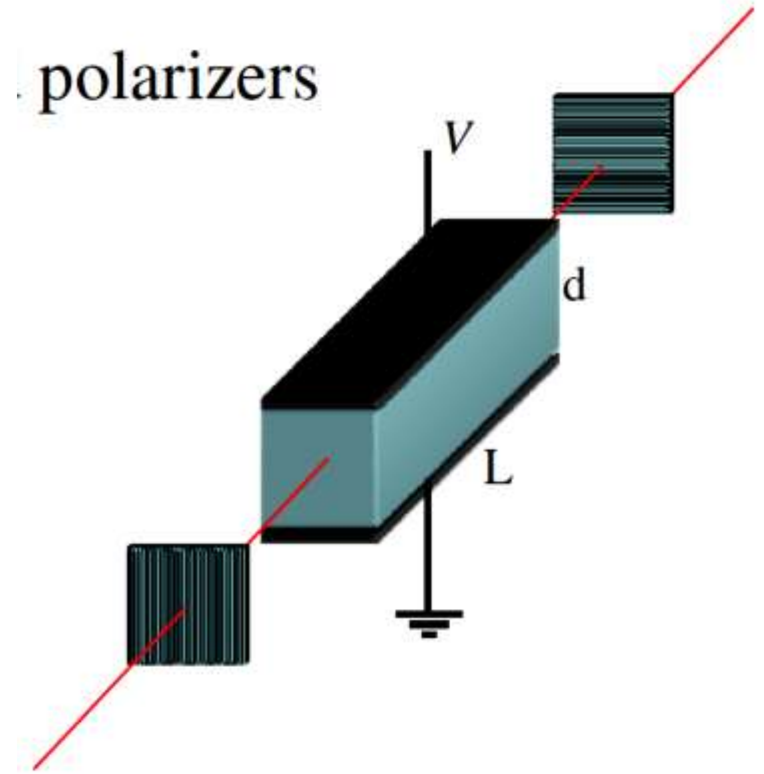
- Electro-Optic Crystal
  acts as a variable waveplate
- Analyser
  transmits only the component that has been rotated
  -> $\sin^2$ transmittance characteristic



Variable retarder between crossed polarizers

$$\Gamma(V) = k_0[n_1(V) - n_2(V)]L$$
$$= k_0[n_1 - n_2]L - \frac{\pi}{\lambda_0}\left(n_1^3 r_1 - n_2^3 r_2\right)\frac{V}{d}L$$
$$= \Gamma_0 - \pi\frac{V}{V_\pi}$$
$$V_\pi = \frac{d}{L}\frac{\lambda_0}{\left(n_1^3 r_1 - n_2^3 r_2\right)}$$

$$T = \sin^2\left(\frac{\Gamma_0}{2} - \frac{\pi}{2}\frac{V}{V_\pi}\right)$$



Electro Optic Devices

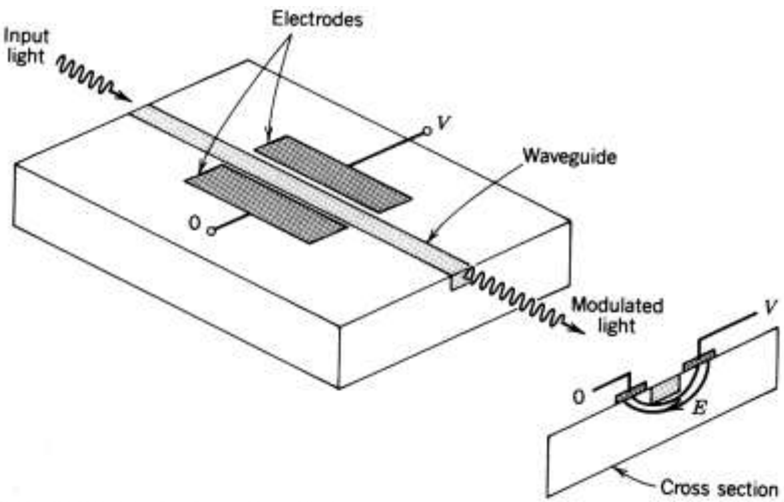


Figure 18.1-3 An integrated-optical phase modulator using the electro-optic effect.

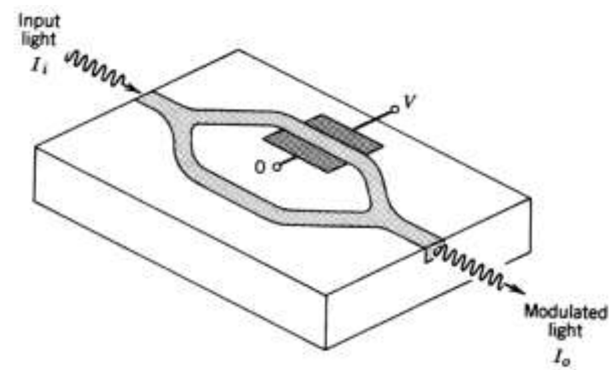


Figure 18.1-5 An integrated-optical intensity modulator (or optical switch). A Mach–Zehnder interferometer and an electro-optic phase modulator are implemented using optical waveguides fabricated from a material such as $LiNbO_3$.

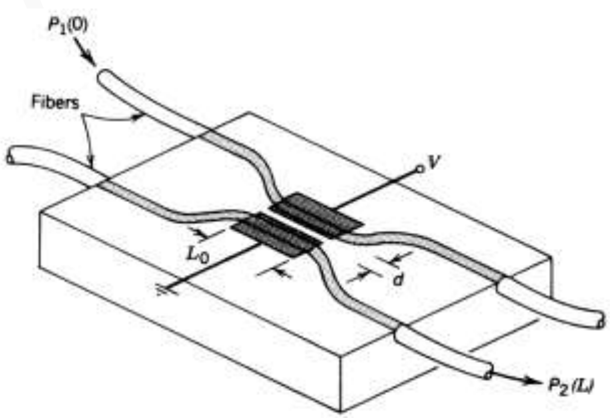


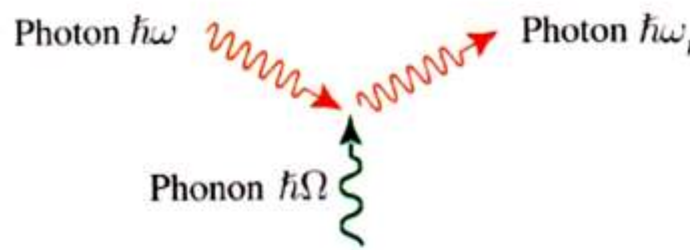
Figure 18.1-10 An integrated electro-optic directional coupler.

Acousto-Optic modulator

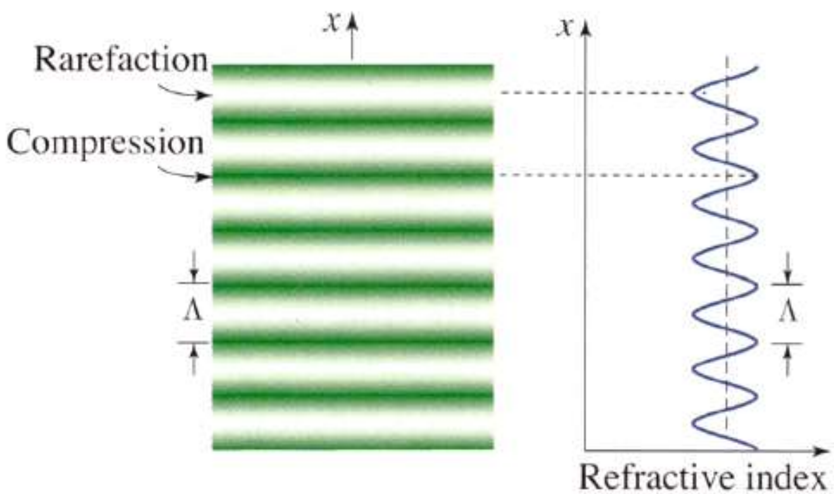
Acousto-optic modulator

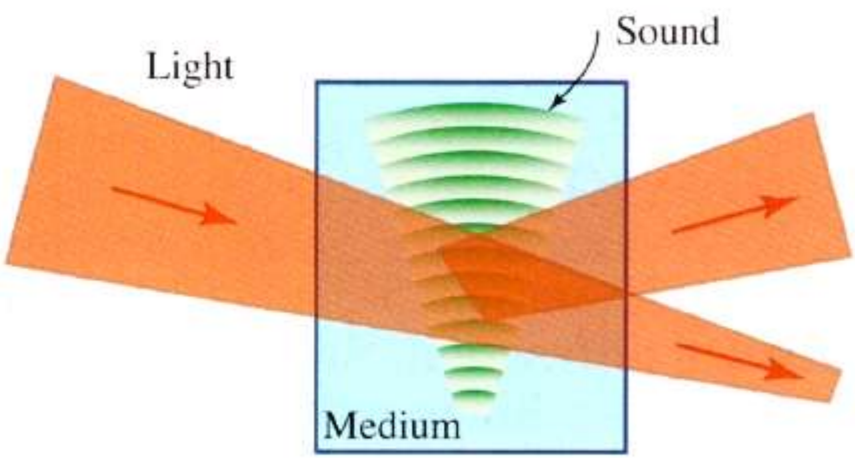
- The refractive index of an optical medium is altered by the presence of sound.
- Sound can control light.
- Acousto-optic effect.

Photon-phonon interactions



$$\hbar \mathbf{k}_r = \hbar \mathbf{k} + \hbar \mathbf{q}$$
$$\hbar \omega_r = \hbar \omega + \hbar \Omega$$





Photoelastic effect

- A strain is measure of deformation representing the displacement betw een particles in the body relative to a reference length.

The strain (relative displacement)

$S_0 = \frac{\partial(x-X)}{\partial X}$ where X is the reference position of material points

$$s(x,t) = S_0 \cos(\Omega t - qx) \qquad q = 2\pi/\Lambda$$ wavenumber

The strain creates a proportional perturbation of the refractive index as,

$$\Delta n(x,t) = -\tfrac{1}{2}\mathfrak{p} n^3 s(x,t)$$

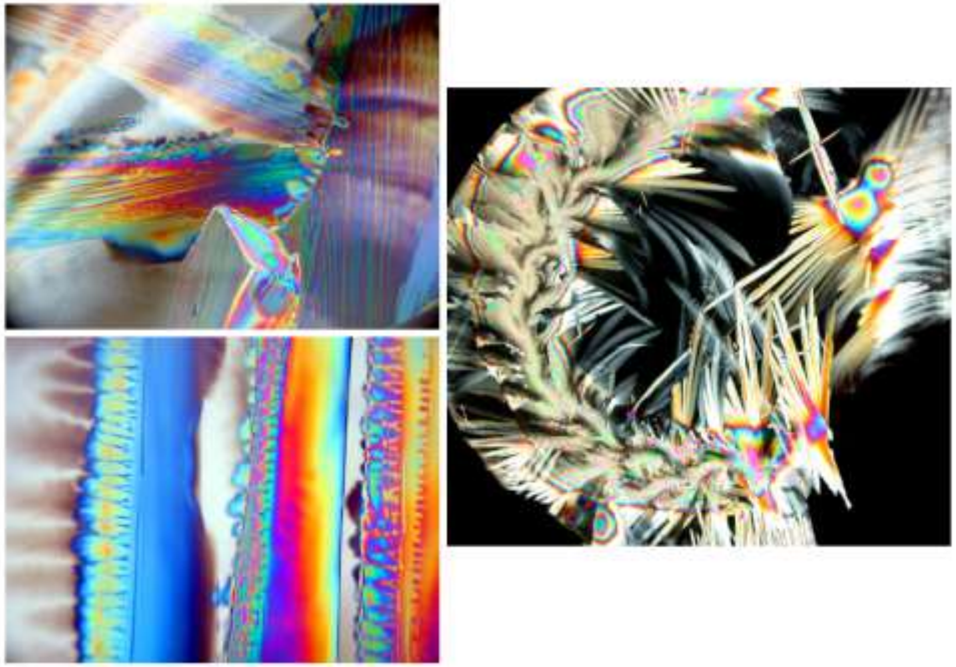
$\mathfrak{p}$ Phenomenological dimensionless coefficient known as the photo-elastic constant.

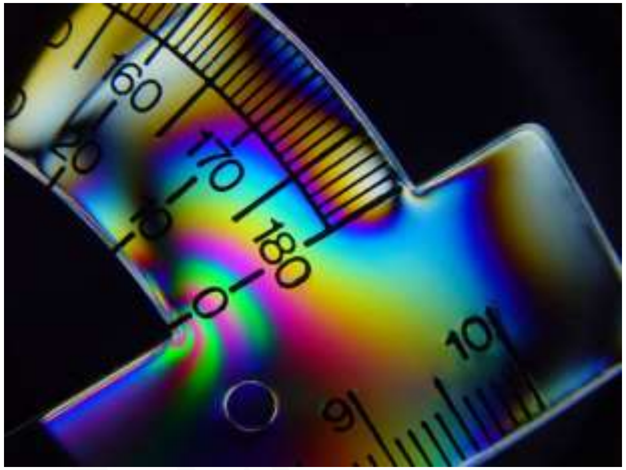
The medium has a time-varying inhomogeneous refractive index as

$$n(x,t) = n - \Delta n_0 \cos(\Omega t - qx), \qquad \Delta n_0 = \tfrac{1}{2}\mathfrak{p} n^3 S_0.$$

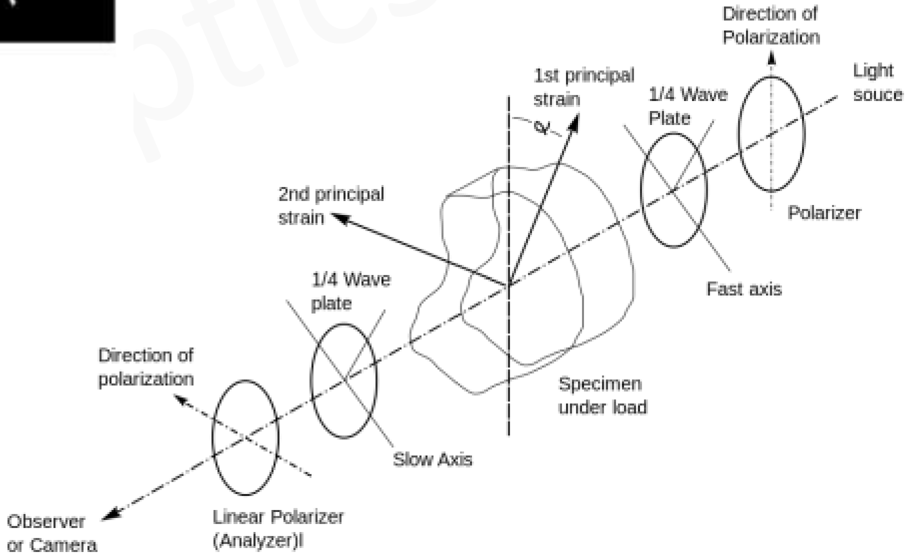
Photoelasticity

- NASA scientist Peter Wasilewski painting with ice and light: Peter.J.Wasilewski.1@gsfc.nasa.gov





Stress induced birefringence



Photoelastic effect

$$\eta_{ij}(s_{kl}) \approx \eta_{ij}(0) + \sum_{kl} \mathfrak{p}_{ijkl} s_{kl}, \qquad i, j, l, k = 1, 2, 3,$$

- The electric impermeability tensor characterizes the optical properties of an anisotropic medium.

$$\mathfrak{p}_{ijkl} = \partial \eta_{ij} / \partial s_{kl}$$

- Strain-optic tensor or photoelastic tensor

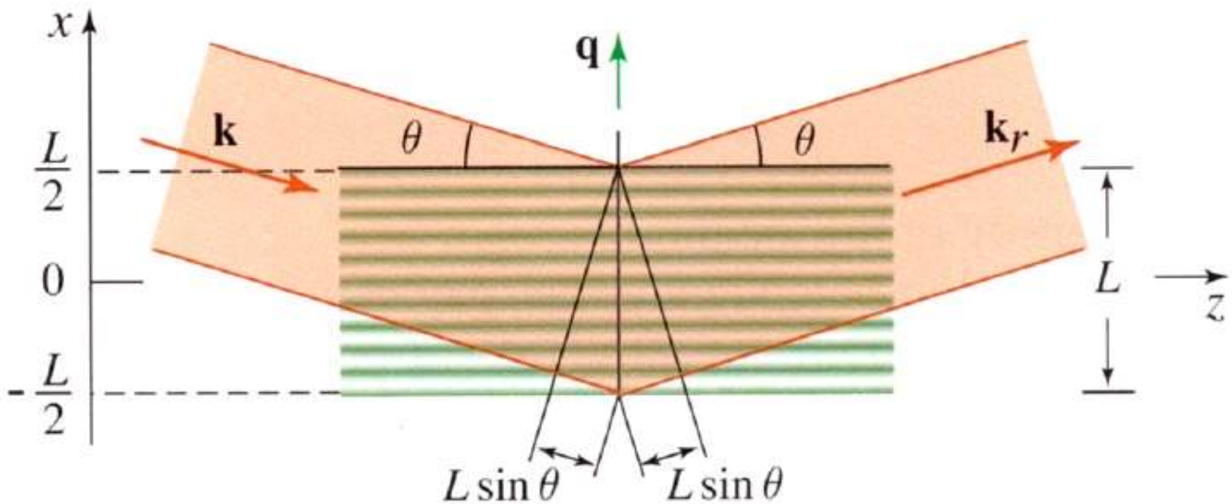
$$\mathfrak{p}_{IK} = \begin{bmatrix} \mathfrak{p}_{11} & \mathfrak{p}_{12} & \mathfrak{p}_{12} & 0 & 0 & 0 \\ \mathfrak{p}_{12} & \mathfrak{p}_{11} & \mathfrak{p}_{12} & 0 & 0 & 0 \\ \mathfrak{p}_{11} & \mathfrak{p}_{12} & \mathfrak{p}_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathfrak{p}_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & \mathfrak{p}_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & \mathfrak{p}_{44} \end{bmatrix}$$

Bragg reflection

Photo-elastic effect

$$n(x) = n - \Delta n_0 \cos(qx - \varphi), \;\; \varphi = \Omega t$$

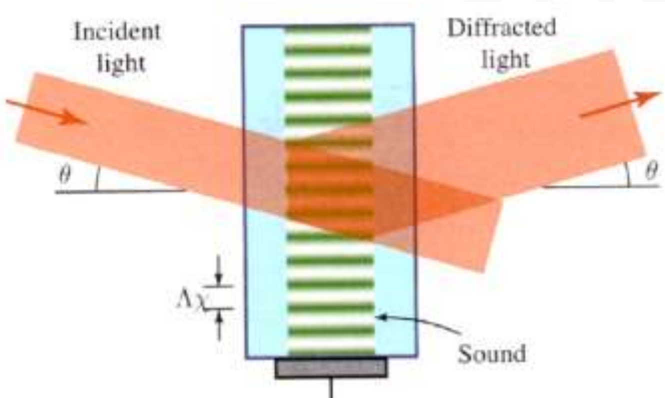
$$\Delta n = -\frac{1}{2} p\, n_0^3\, S_0 \cos(\Omega t - \vec{K} \cdot \vec{r})$$

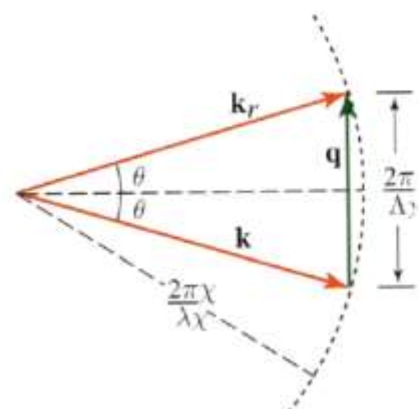


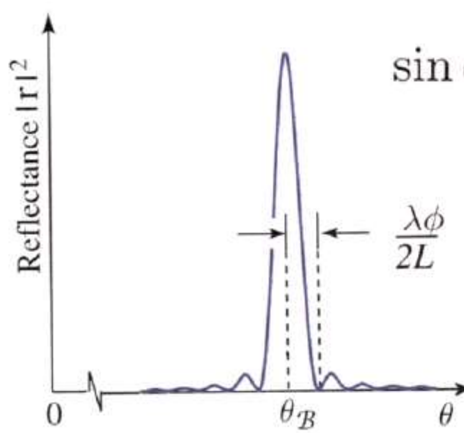
$$\sin\theta = \frac{\lambda}{2\Lambda}$$

$$\Lambda = v_s / f$$

Bragg condition

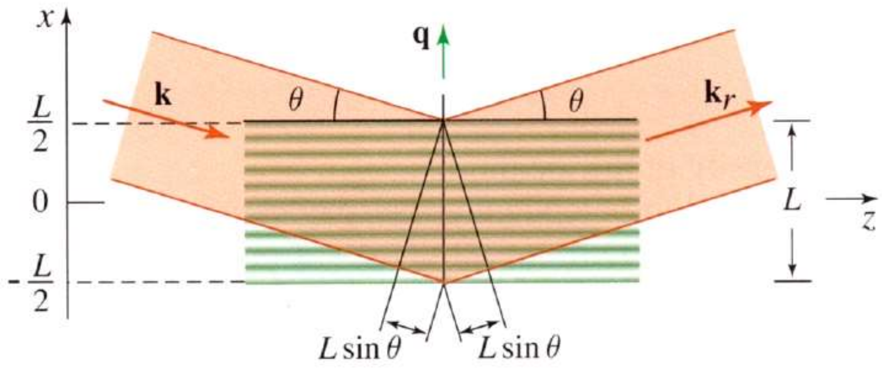






$$\sin\theta_{\mathcal{B}} = \frac{\lambda}{2\Lambda}$$

Reflection



Amplitude reflectance

$$r_{\pm} = \pm j r_0 \ \operatorname{sinc}\left[(2k\sin\theta \mp q)\frac{L}{2\pi}\right] e^{\pm j\Omega t}$$

$$\operatorname{sinc}(x) \equiv \sin(\pi x)/(\pi x)$$

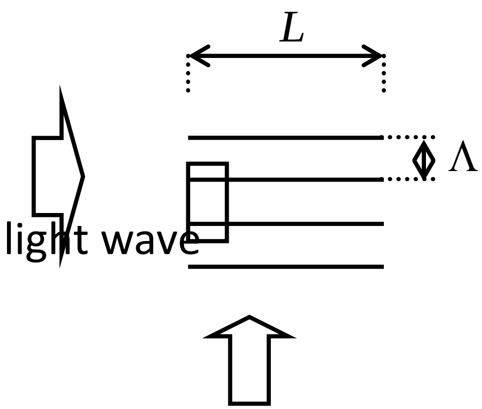
$$\mathcal{R} = 2\pi^2 n^2 \frac{L^2\Lambda^2}{\lambda_o^4}\mathcal{M} I_s.$$

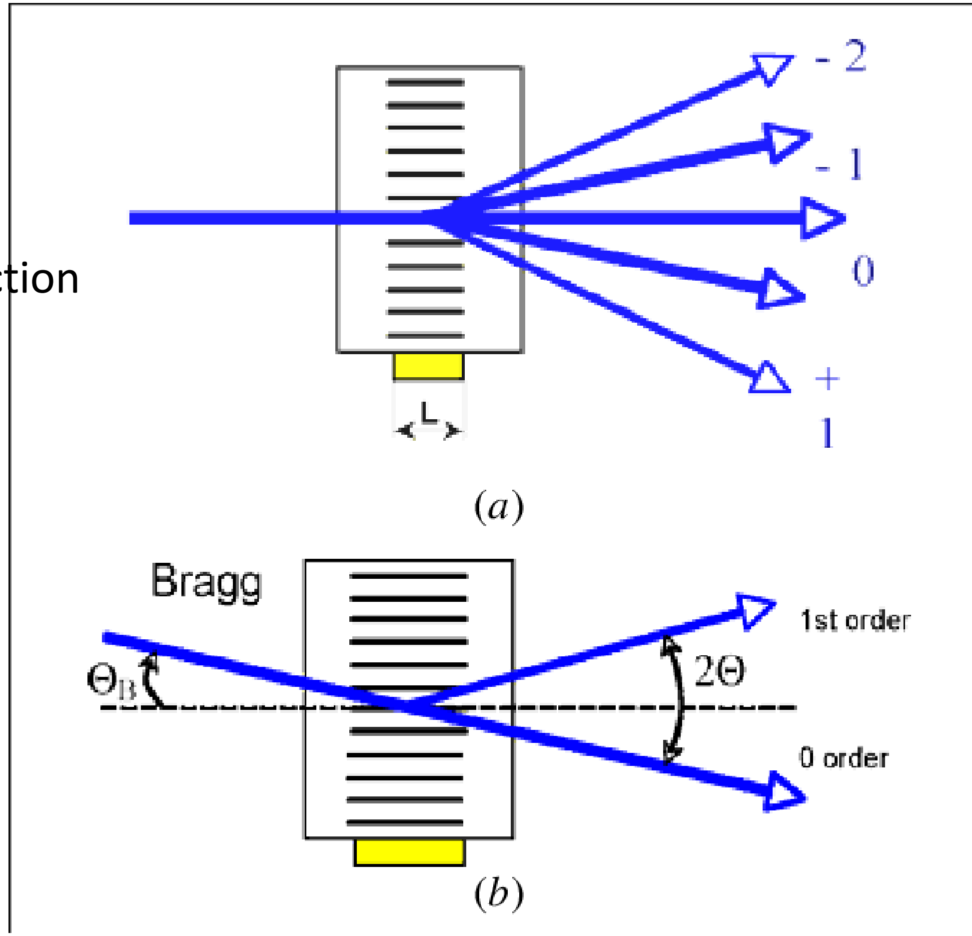
Power reflectance

$$\mathcal{M} = \frac{\mathfrak{p}^2 n^6}{\varrho v_s^3}$$

Acousto-Optic effect

Bragg diffraction & Raman-Nath diffraction





Dimensionless parameter : $Q = \frac{4\pi\theta_B}{\Phi} = \frac{2\pi\lambda L}{n\Lambda^2}$ > 1 : Bragg diffraction

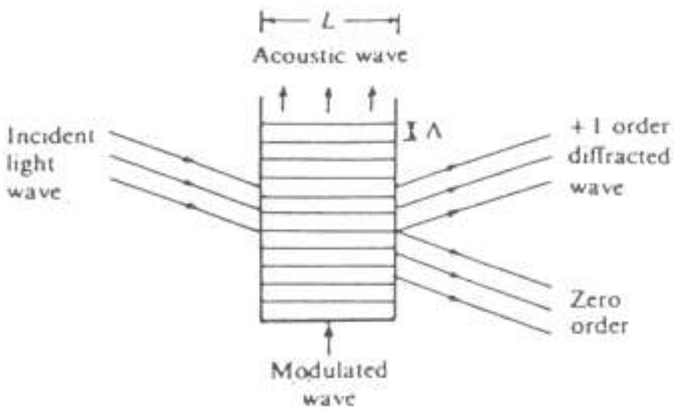
< 1 : Raman-Nath diffraction

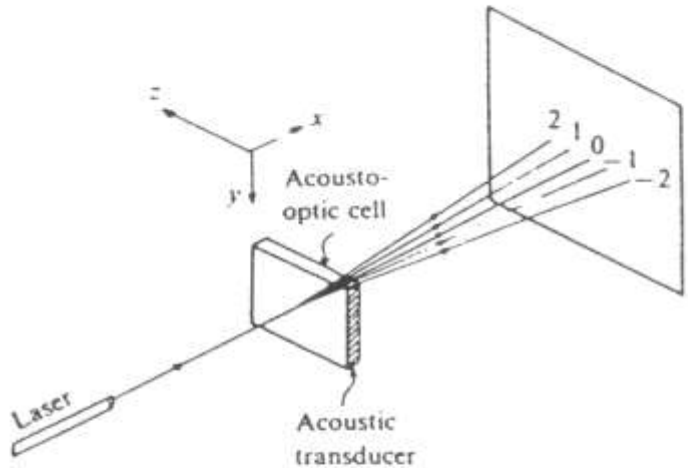
Example) Water, n=1.33, W=6MHz (v_s=1,500 m/s), l=632.8 nm

$$\Lambda = v_s/\Omega = 250\,\mu\text{m}$$

$$L \ll \Lambda^2 n/(2\pi\lambda) \approx 2\,\text{cm}: \text{Raman-Nath Regime}$$

$$\gg 2\,\text{cm} \qquad : \text{Bragg Regime}$$



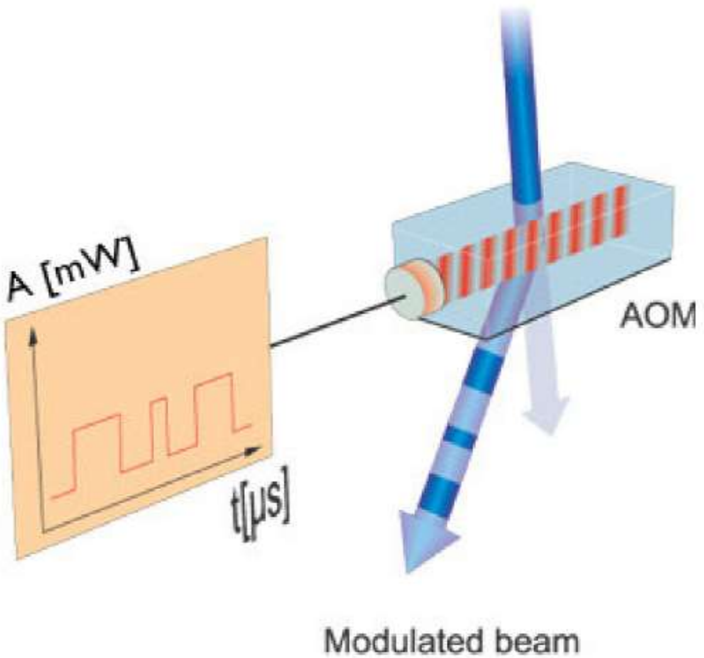


Bragg diffraction :
Single order diffraction

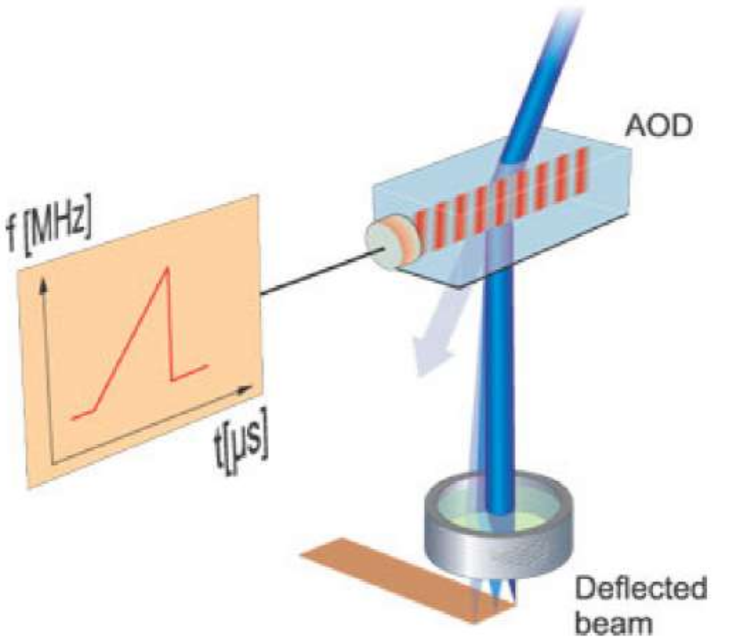
Raman-Nath diffraction :
Multiple order diffraction

Applications

Modulator

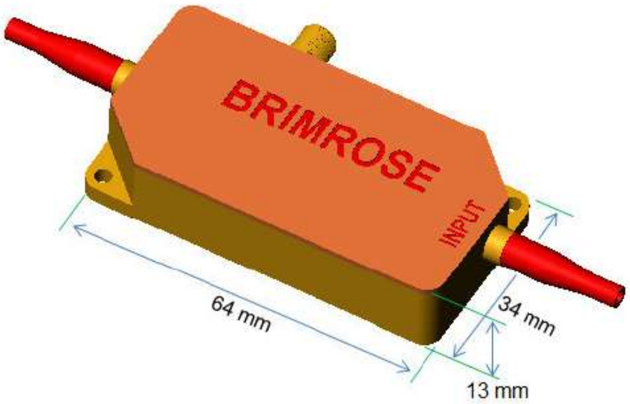


Deflector



$$\theta = \sin^{-1}\frac{\lambda}{2\Lambda} = \sin^{-1}\frac{f\lambda}{2v_s} \approx \frac{\lambda}{2v_s}f$$

Fiber pigtailed AOM



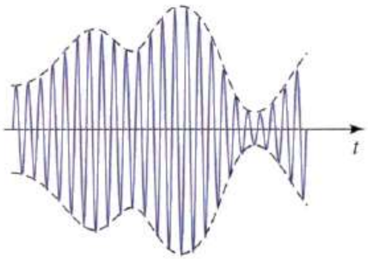
Modulator

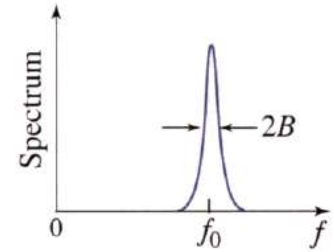
Frequency shifter

Wavelength of Operation	1550 nm
Optical Range	±25 nm
Maximum Optical Power	230 mW
Frequency Shift	+75 MHz or -75 MHz
Frequency Shift Range (3dB)	5 MHz
Beam Diameter Inside the Crystal	0.4 mm
Rise Time	140 nsec
Digital Modulation Bandwidth	5 Mhz
Acoustic Velocity (m/sec)	2.5E+3
Maximum RF Power (Watt)	<0.5 W
RF Connector	SMB
Extinction Ratio	>50 dB
Input Impedance	50 ohms
V.S.W.R.	2.1:1
Optical Polarization	Linear
Case Type	2 Port Fiber Optically Pigtailed
Type of Fiber, Port 1 and 2	8 μm core, 125 μm cladding SM PM
Fiber Connector Type	FC
Polishing of the Fiber End	APC
Fiber Length	1 m
Fiber Jacket	900 μm OD
Back Reflection*	40 dB
Total Insertion Loss**	3.0 - 4.0 dB at the center wavelength
PER	16 - 19 dB

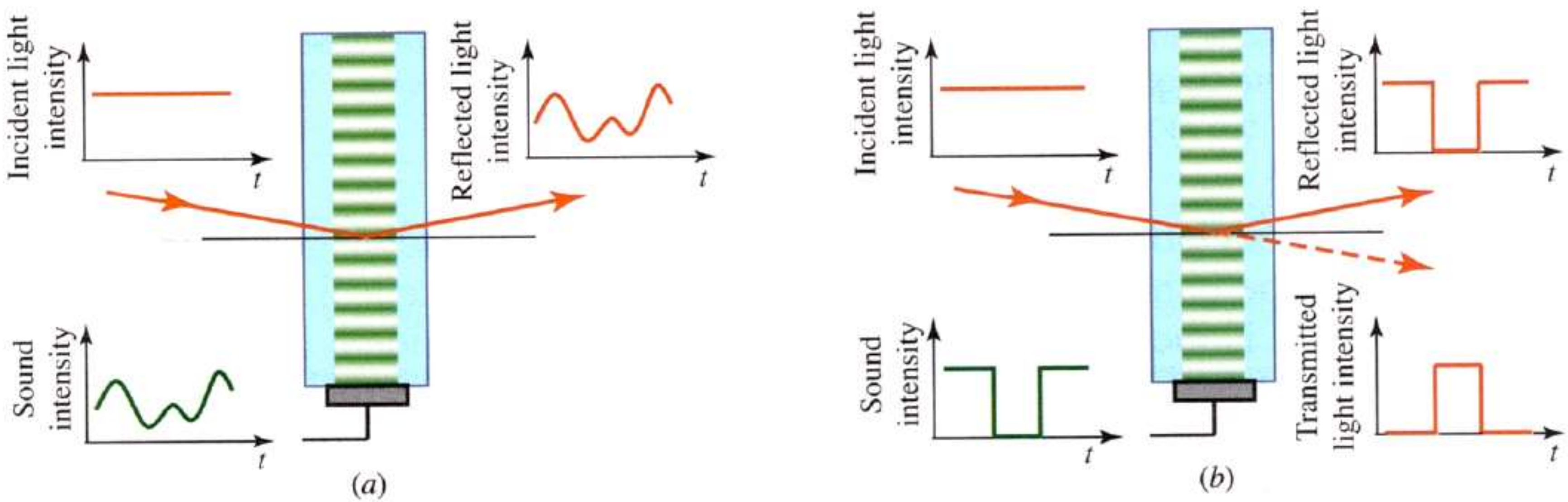
Modulators

- The amplitude of an acoustic wave of frequency f_0
- A function of time by amplitude modulation
- A signal of bandwidth B.





$$\theta = \sin^{-1}\frac{\lambda}{2\Lambda} = \sin^{-1}\frac{f\lambda}{2v_s} \approx \frac{\lambda}{2v_s}f$$

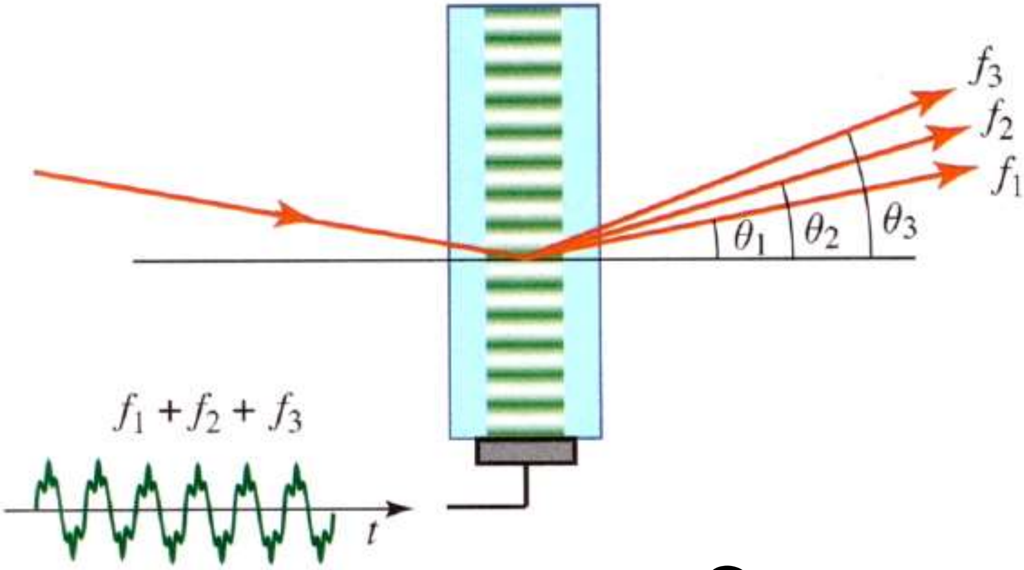


Maximum rate of modulation

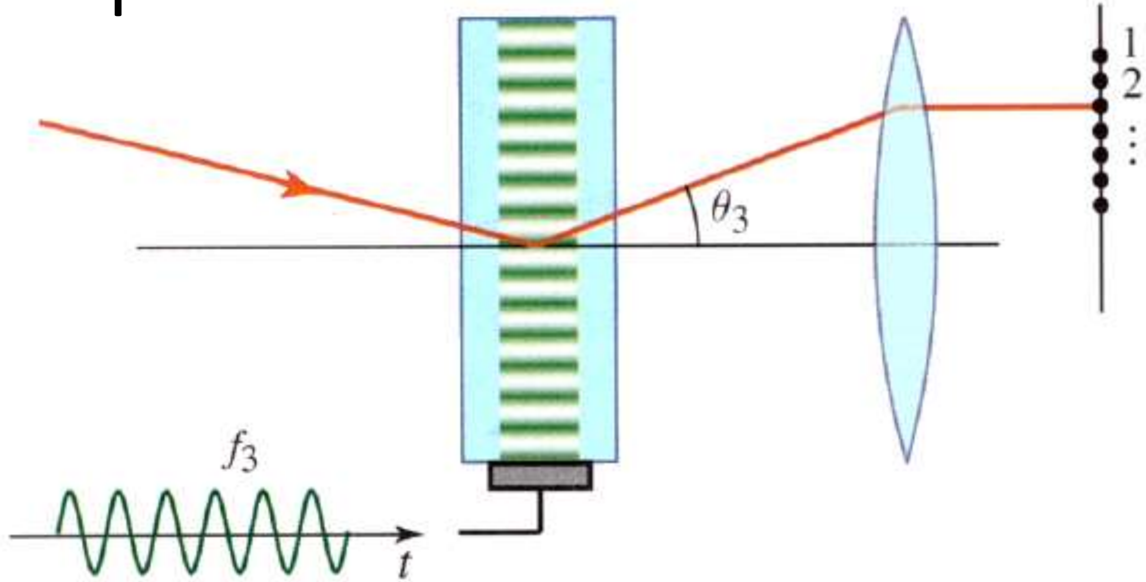
- T is the transit time of sound across the waist of the light beam.
- It takes time T to change the amplitude of the sound wave at all points in the light-sound interaction region.
- The maximum rate of modulation is $1/T$ Hz
- Beam width: D

$$B = \frac{1}{T}, \qquad T = \frac{D}{v_s}, \qquad B = v_s \frac{\delta\theta}{\lambda} = \frac{v_s}{D},$$

Spectrum analyzer



Space switches

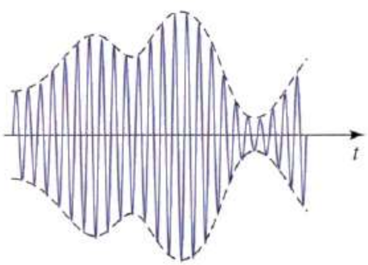


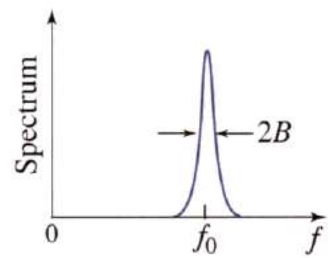
Frequency shifters

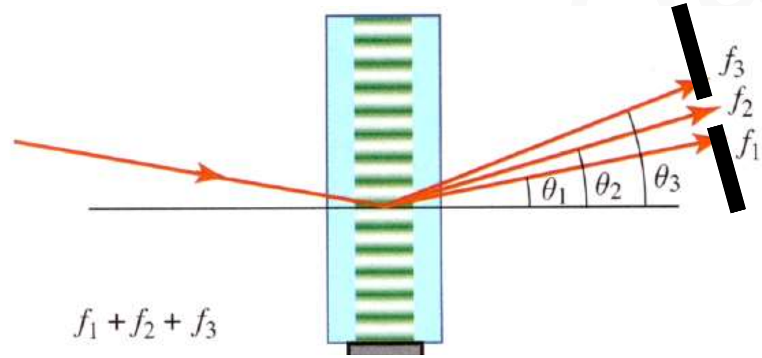
$$\mathrm{r}_{\pm} = \pm j \mathrm{r}_0 \ \mathrm{sinc}\left[(2k\sin\theta \mp q)\frac{L}{2\pi}\right] e^{\pm j\Omega t}$$

$$\boldsymbol{E_{refl} \propto E_{in} e^{\pm j\Omega t} = E_0 e^{j(\omega \pm \Omega)t}}$$

Tunable Acousto-Optic Filters







Optical isolators