광전자공학 Ch. 2 Part 2
Reflection and Refraction

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Impedance of plane wave

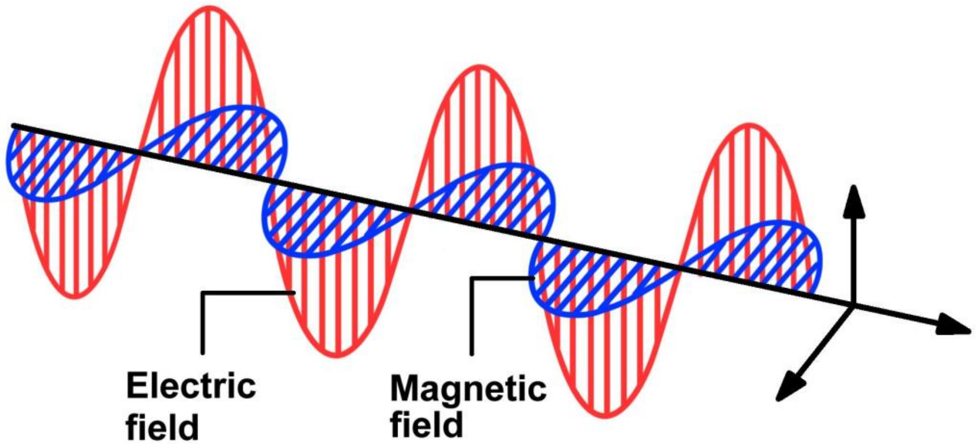
From $\mathbf{k}\times\mathbf{H} = -\varepsilon\omega\mathbf{E}$ or $\mathbf{k}\times\mathbf{E} = \mu\omega\mathbf{H}$

$$\eta = \frac{|\mathbf{E}|}{|\mathbf{H}|} = \frac{k}{\varepsilon\omega} = \frac{\mu\omega}{k} = \sqrt{\frac{\mu}{\varepsilon}}$$

Impedance of plane wave
(Magnitude ratio of E and H)

In free space,

$$\eta_0 = \frac{|\mathbf{E}|}{|\mathbf{H}|} = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 377\ \Omega$$



In nonmagnetic material,

$$\eta = \frac{|\mathbf{E}|}{|\mathbf{H}|} = \sqrt{\frac{\mu_0}{\varepsilon}} = \frac{\eta_0}{n}$$

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Energy carried by EM wave

Using Maxwell curl equations,

$$\mathbf{H}\cdot\nabla\times\mathbf{E} = -\mathbf{H}\cdot\frac{\partial\mathbf{B}}{\partial t} \qquad \mathbf{E}\cdot\nabla\times\mathbf{H} = \mathbf{J}\cdot\mathbf{E} + \mathbf{E}\cdot\frac{\partial\mathbf{D}}{\partial t}$$

Subtracting above equations,

$$\nabla\cdot\left(\mathbf{E}\times\mathbf{H}\right) = \mathbf{H}\cdot\nabla\times\mathbf{E} - \mathbf{E}\cdot\nabla\times\mathbf{H} = -\mathbf{H}\cdot\frac{\partial\mathbf{B}}{\partial t} - \mathbf{E}\cdot\frac{\partial\mathbf{D}}{\partial t} - \mathbf{J}\cdot\mathbf{E}$$

We define Poynting vector as $\mathbf{S} = \mathbf{E}\times\mathbf{H}$

$$\nabla\cdot\mathbf{S} = -\frac{\partial}{\partial t}\left(\frac{1}{2}\mu_0\mathbf{H}^2 + \frac{1}{2}\mu_0\mathbf{E}^2\right) - \mathbf{J}\cdot\mathbf{E}$$

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Energy Conservation

$$\boxed{\nabla\cdot\mathbf{S}}+\boxed{\mathbf{J}\cdot\mathbf{E}}=-\frac{\partial}{\partial t}\left(\frac{1}{2}\mu_0\mathbf{H}^2+\frac{1}{2}\mu_0\mathbf{E}^2\right)$$

Decrease of stored energy

Heat dissipation

EM wave radiation flux

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Poynting vector

Poynting vector indicates the energy flow direction of light

$$\mathbf{S} = \mathbf{E} \times \mathbf{H}$$

For a plane wave $\mathbf{E} = \mathbf{E}_0 \cos(\mathbf{k} \cdot \mathbf{r} - \omega t)$ $\mathbf{H} = \mathbf{H}_0 \cos(\mathbf{k} \cdot \mathbf{r} - \omega t)$

$$\mathbf{S} = \mathbf{E}_0 \times \mathbf{H}_0 \cos^2(\mathbf{k} \cdot \mathbf{r} - \omega t)$$

A time-average Poynting vector can be expressed as

$$\langle \mathbf{S} \rangle = \bar{\mathbf{S}} = \frac{1}{2} \mathbf{E}_0 \times \mathbf{H}_0 = \frac{1}{2} \bar{\mathbf{E}} \times \bar{\mathbf{H}}^*$$

Upper-bar indicates a complex phasor notation

For isotropic media, $\mathbf{S}$ has the same direction as $\mathbf{k}$

Snell's law

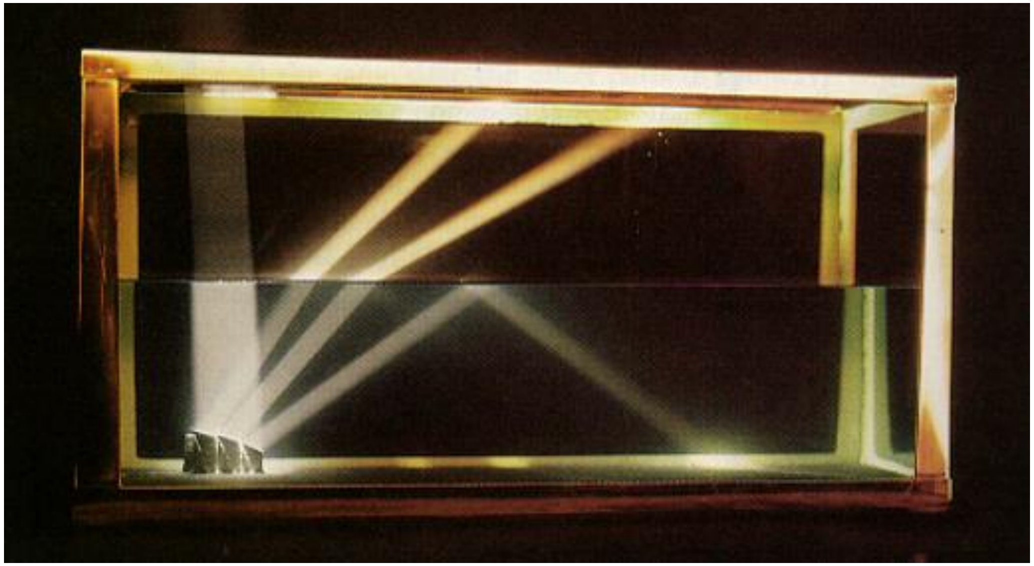
스넬의 법칙

$$n_i \sin\theta_i = n_r \sin\theta_r$$

1. 빛을 광선의 관점으로 해석하면:
   페르마의 원리로 증명 가능

2. 빛을 파동의 관점으로 해석하면:
   Phase matching condition 으로 해석 가능

빌레브로르트 스넬



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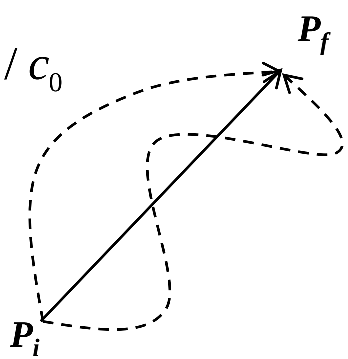
Fermat's Principle

헤론 (c. 10~c. 70)

빛은 두 지점 간의 최단거리의 경로를 통과하여 지나간다.

- 반사의 원리를 설명**하였으나, 굴절의 원리를 정확히 설명할 수 없었다.**

$$t = nl / c_0$$



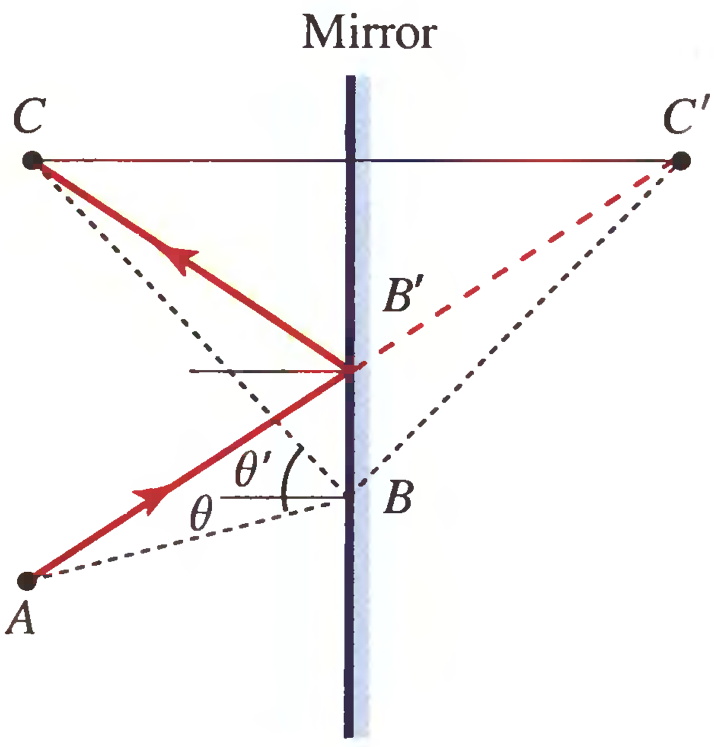
광경로란, **굴절률과 거리의 곱**의 개념으로, 매질에 관계없이 단위 시간동안 빛은 같은 거리의 광경로를 지나간다.

피에르 드 페르마 (1601~1665)

빛은 두 지점 간의 가장 짧은 광경로, 즉 최단시간의 경로를 통과하여 지나간다.

Fermat's Principle

Fermat's principle in Mirror reflection

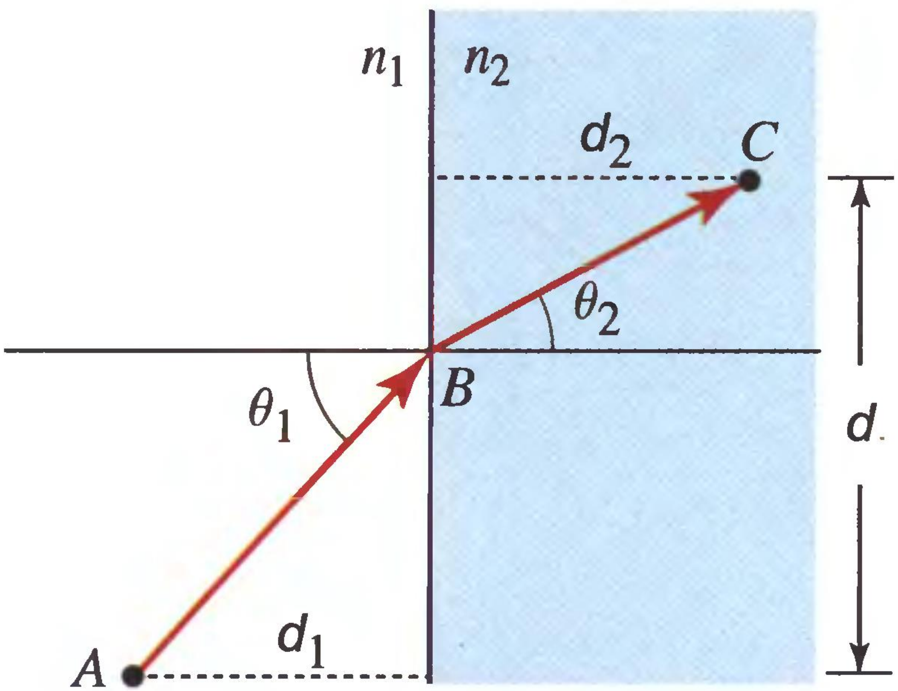


$$\overline{AB'C} = \overline{AB'C'} < \overline{ABC'}$$

Light choose the way having the shortest optical pathlength!

Fermat's Principle

Fermat's principle in Snell's law



Find the minimum value of

$$OPL = n_1 \overline{AB} + n_2 \overline{BC}$$

$$OPL = n_1 d_1 \sec\theta_1 + n_2 d_2 \sec\theta_2$$

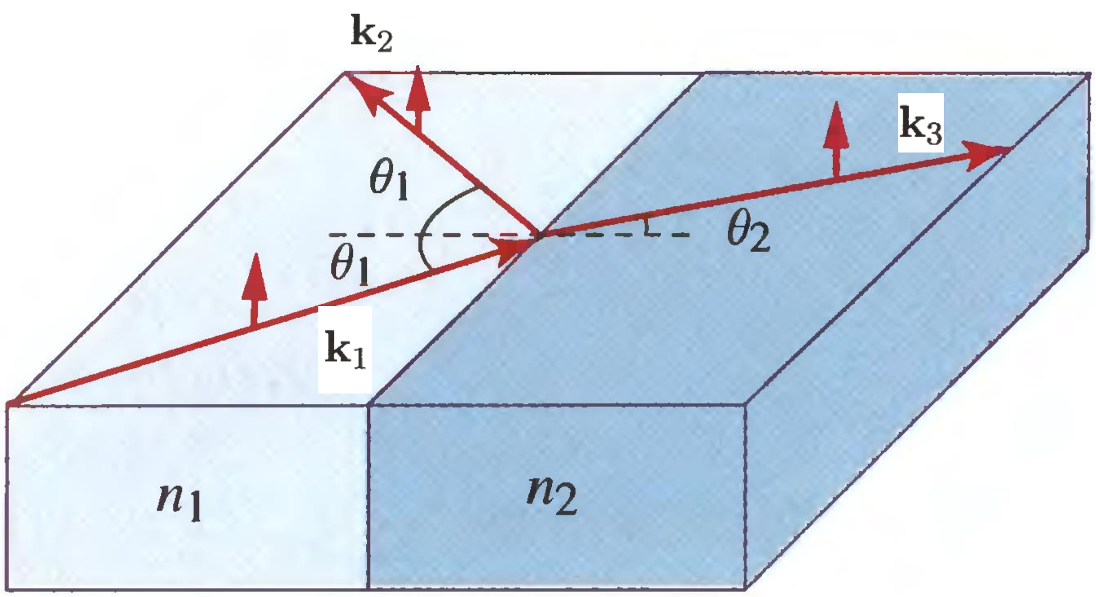
$$d = d_1 \tan\theta_1 + d_2 \tan\theta_2 \quad \text{(Constant)}$$

Use $\dfrac{d\theta_2}{d\theta_1} = -\dfrac{d_1 \sec^2\theta_1}{d_1 \sec^2\theta_2}$

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

Phase matching condition

At any point of boundary, wavefront must be continuous, in order to satisfy the EM boundary condition.



$$k_{1x} = k_{2x}$$

$$\|$$

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$\mathbf{k}_1 \cdot \mathbf{r} = \mathbf{k}_2 \cdot \mathbf{r} = \mathbf{k}_3 \cdot \mathbf{r} \qquad \text{for all } \mathbf{r} = (x, y, 0)$$

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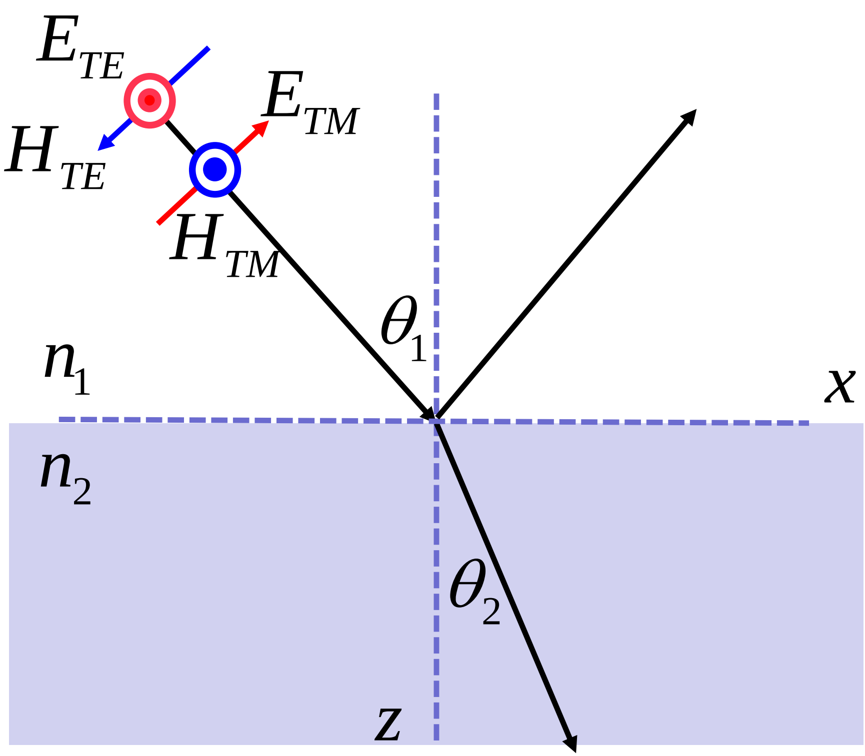
Reflection and Refraction

Definition TE-pol and TM-pol



Plane of incidence : x-z plane

TE-pol (s-pol)

$$\mathbf{E} = E_0 \exp(jk_1 \sin\theta_1 x + jk_1 \cos\theta_1 z)\hat{y}$$
$$= E_0 \hat{y} \exp(j(k_x x + k_z z))$$
$$\mathbf{H} = -\frac{j}{\omega\mu_0}\nabla\times\mathbf{E} = -\frac{j}{\omega\mu_0}\left(-\frac{\partial E_y}{\partial z}\hat{x} + \frac{\partial E_y}{\partial x}\hat{z}\right)$$
$$= -\frac{E_0 n_1}{\eta_0}\left(\cos\theta_1\hat{x} - \sin\theta_1\hat{z}\right)\exp(j(k_x x + k_z z))$$

TM-pol (p-pol)

$$\mathbf{H} = H_0 \hat{y} \exp(j(k_x x + k_z z))$$
$$\mathbf{E} = \frac{j}{\omega\varepsilon}\nabla\times\mathbf{H} = \frac{j}{\omega\varepsilon}\left(-\frac{\partial H_y}{\partial z}\hat{x} + \frac{\partial H_y}{\partial x}\hat{z}\right)$$
$$= \frac{\eta_0 H_0}{n_1}\left(\cos\theta_1\hat{x} - \sin\theta_1\hat{z}\right)\exp(j(k_x x + k_z z))$$

Reflection and Refraction

Reflection & Refraction at interface

TE-pol (s-pol)

Ey

$$E_i = \hat{y}E_0 \exp(j(k_x x + k_{z1} z))$$

$$E_r = \hat{y}r_s E_0 \exp(j(k_x x - k_{z1} z))$$

$$E_t = \hat{y}t_s E_0 \exp(j(k_x x + k_{z2} z))$$

Hx

$$H_i = -\frac{n_1 E_0}{\eta_0} \cos\theta_1 \hat{x} \exp(j(k_x x + k_z z))$$

$$H_r = -\frac{-n_1 r_s E_0}{\eta_0} \cos\theta_1 \hat{x} \exp(j(k_x x - k_z z))$$

$$H_t = -\frac{n_2 t_s E_0}{\eta_0} \cos\theta_2 \hat{x} \exp(j(k_x x + k_{z2} z))$$

$$1 + r_s = t_s \qquad 1 - r_s = t_s \frac{n_2 \cos\theta_2}{n_1 \cos\theta_1}$$

$$r_s = \frac{n_1 \cos\theta_1 - n_2 \cos\theta_2}{n_1 \cos\theta_1 + n_2 \cos\theta_2}$$

$$t_s = \frac{2 n_1 \cos\theta_1}{n_1 \cos\theta_1 + n_2 \cos\theta_2}$$

Reflection and Refraction

Reflection & Refraction at interface

TM-pol (p-pol)

Ex

$$E_i = \hat{x}E_0 \exp(j(k_x x + k_{z1} z))$$

$$E_r = \hat{x}E_0 r_p \exp(j(k_x x - k_{z1} z))$$

$$E_t = \hat{x}E_0 t_p \exp(j(k_x x + k_{z2} z))$$

Hy

$$H_i = \frac{n_1 E_0}{\eta_0} \sec\theta_1 \hat{y} \exp(j(k_x x + k_z z))$$

$$H_r = \frac{n_1 E_0}{\eta_0} r_p \sec\theta_1 \hat{y} \exp(j(k_x x - k_z z))$$

$$H_t = \frac{n_2 E_0}{\eta_0} t_p \sec\theta_2 \hat{y} \exp(j(k_x x + k_{z2} z))$$

$$1 + r_p = t_p \qquad 1 - r_p = t_p \frac{n_2 \sec\theta_2}{n_1 \sec\theta_1}$$

$$r_s = \frac{n_1 \sec\theta_1 - n_2 \sec\theta_2}{n_1 \sec\theta_1 + n_2 \sec\theta_2}$$

$$t_s = \frac{2 n_1 \sec\theta_1}{n_1 \sec\theta_1 + n_2 \sec\theta_2}$$

Fresnel Equations

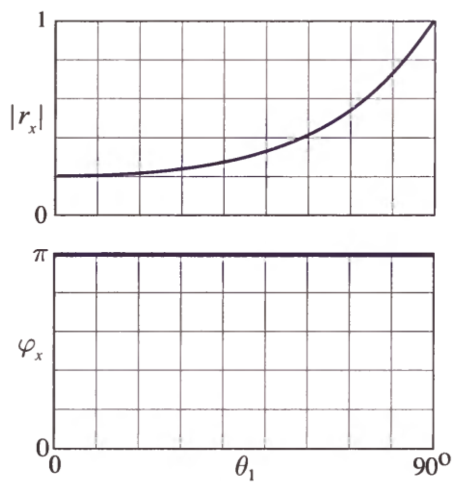
Reflection & Refraction at interface

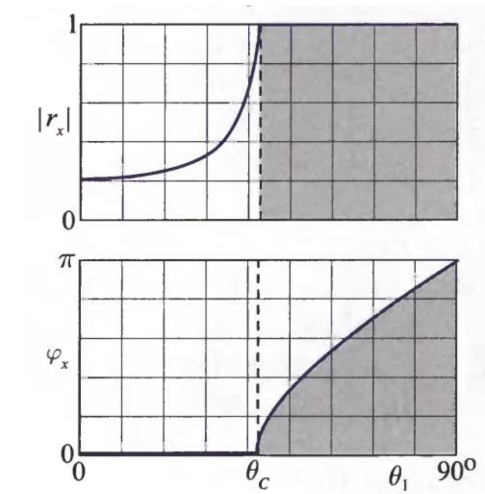
n1 < n2 | n1 > n2

TE-pol (s-pol)

$$r_s = \frac{n_1 \cos\theta_1 - n_2 \cos\theta_2}{n_1 \cos\theta_1 + n_2 \cos\theta_2}$$

$$t_s = \frac{2n_1 \cos\theta_1}{n_1 \cos\theta_1 + n_2 \cos\theta_2}$$

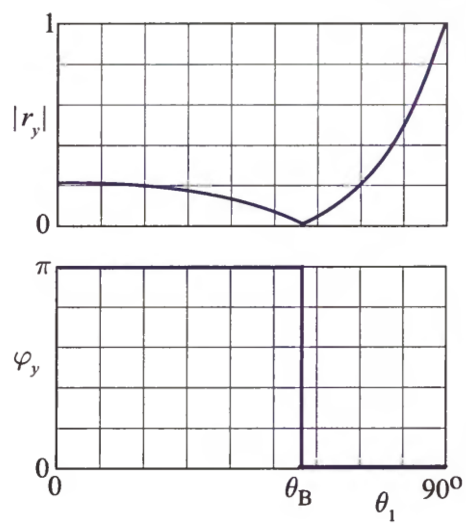


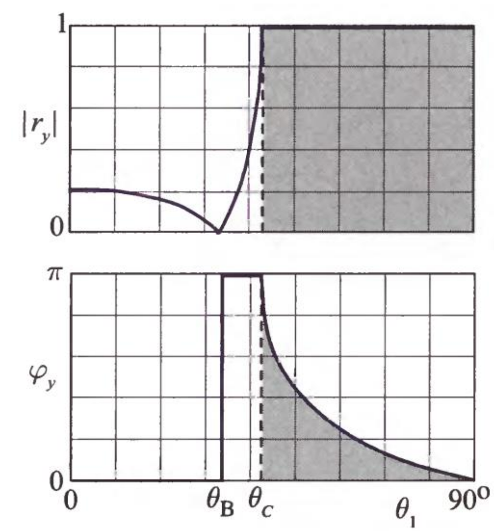


TM-pol (p-pol)

$$r_p = \frac{n_1 \sec\theta_1 - n_2 \sec\theta_2}{n_1 \sec\theta_1 + n_2 \sec\theta_2}$$

$$t_p = \frac{2n_1 \sec\theta_2}{n_1 \sec\theta_1 + n_2 \sec\theta_2}$$





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Critical angle

Only shown for internal reflection (n1 > n2)

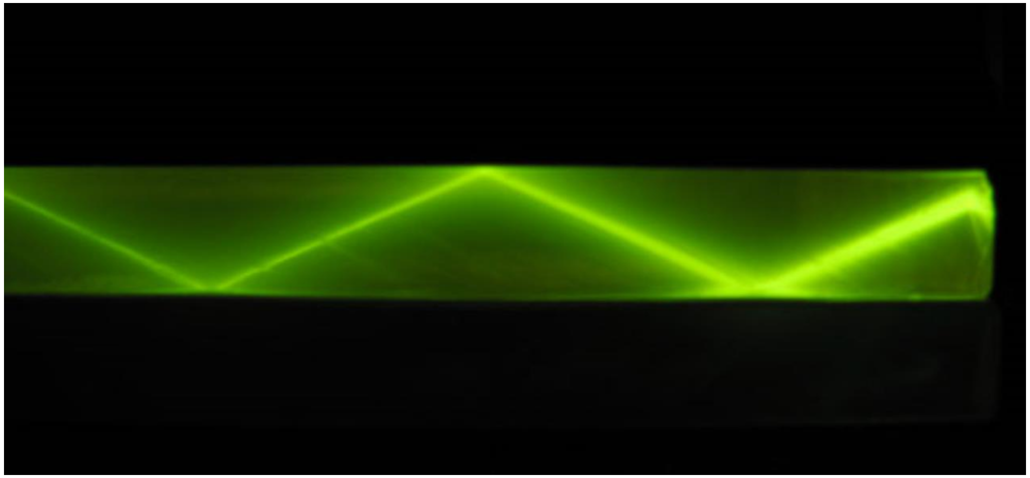
If $k_x = n_1 k_0 \sin\theta_1 \geq n_2 k_0$

$$k_{z2} = k_0\sqrt{n_2^{\ 2} - \left(n_1 \sin\theta_1\right)^2} = -jn_2k_0\sqrt{\left(\frac{n_1}{n_2}\sin\theta_1\right)^2 - 1} = -j\kappa$$

$$E_t = tE_0\exp(-j(k_x x - j\kappa z)) = tE_0\exp(-\kappa x)\exp(-jk_x x)$$

Evanescent field

Total internal reflection



Critical angle

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

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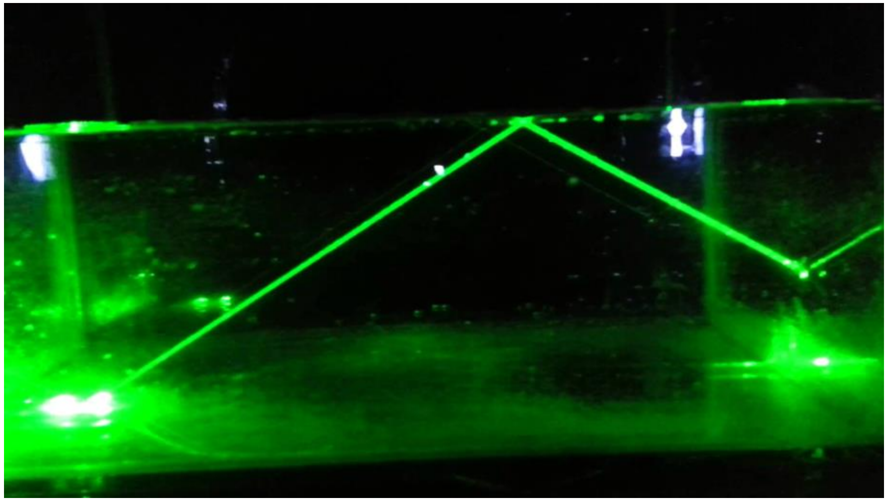
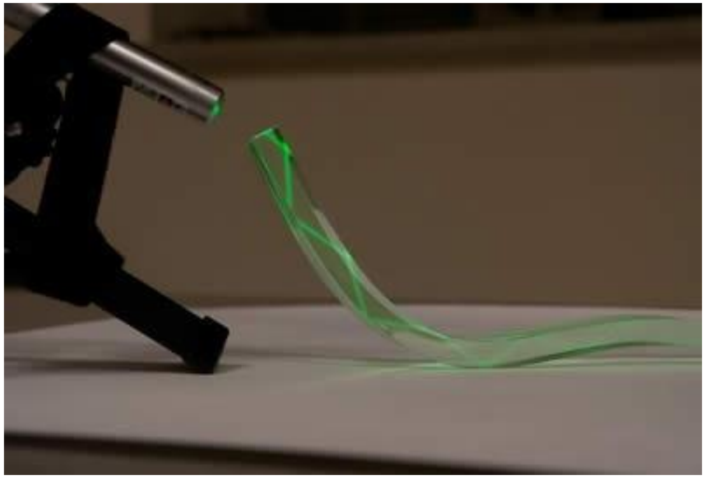
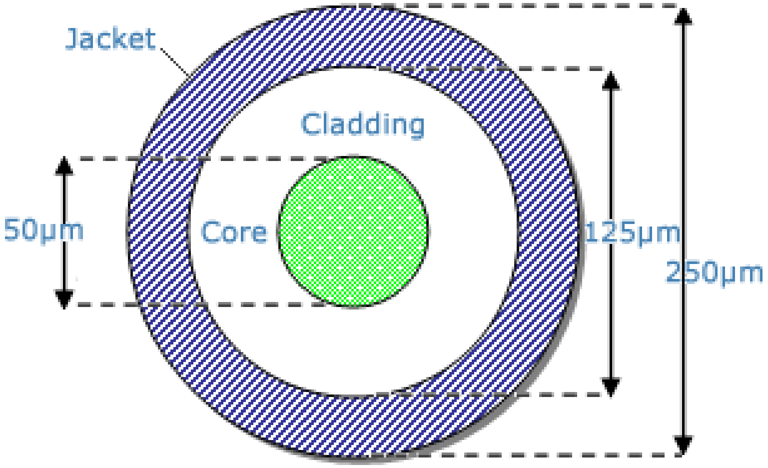
Total internal reflection

Total Internal Reflection





Key principle for designing optical fiber



Brewster angle

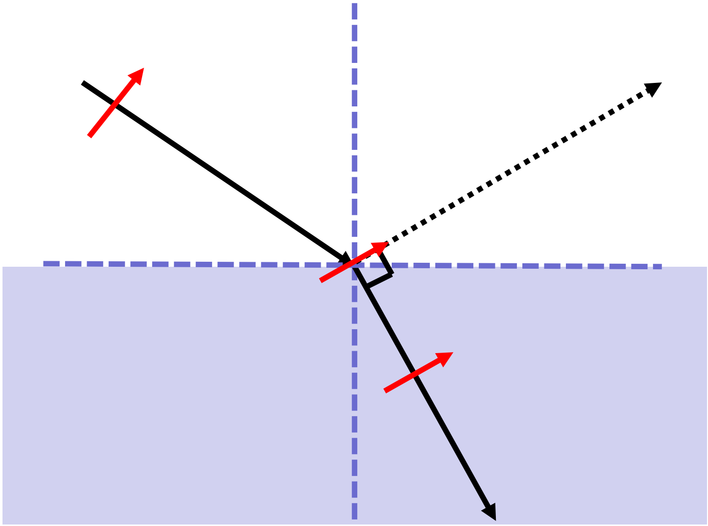
Only shown for TM polarization

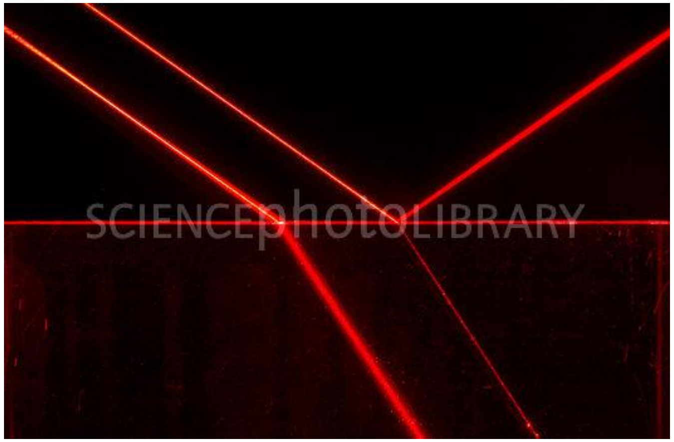
$$r_p = \frac{n_1 \sec\theta_1 - n_2 \sec\theta_2}{n_1 \sec\theta_1 + n_2 \sec\theta_2} = 0$$, when $n_1 \sec\theta_1 - n_2 \sec\theta_2 = 0$

$$\frac{\cos\theta_2}{\cos\theta_1} = \frac{n_2}{n_1} = \frac{\sin\theta_1}{\sin\theta_2}$$

$$\theta_1 + \theta_2 = 90^\circ$$

$$\tan\theta_1 = \frac{n_2}{n_1}$$





There is no reflection at Brewster angle for TM pol

Power reflectance & transmittance

Use complex Poynting vector and take z-component

$$\bar{\mathbf{S}} = \frac{1}{2}\bar{\mathbf{E}} \times \bar{\mathbf{H}}^* \qquad S_z = \frac{1}{2}\left(E_x H_y^{\ *} - E_y H_x^{\ *}\right)$$

TE-pol (s-pol)

$$S_{zi} = \frac{k_0 \left|E_0\right|^2}{\omega \mu_0} n_1 \cos\theta_1 \qquad S_{zr} = -\frac{k_0 \left|E_0\right|^2}{\omega \mu_0} \left|r\right|^2 n_1 \cos\theta_1 \qquad S_{zt} = \frac{k_0 \left|E_0\right|^2}{\omega \mu_0} \left|t\right|^2 n_2 \cos\theta_2$$

Power reflectance & transmittance

$$\mathbf{R} = -\frac{S_{zr}}{S_{zi}} = \left|r\right|^2 \qquad \mathbf{T} = \frac{S_{zt}}{S_{zi}} = \frac{n_2 \cos\theta_2}{n_1 \cos\theta_1}\left|t\right|^2$$

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