Communications and Signal Processing
2017 Doctoral Qualifying Exam

Caution!!!
Use a separate answer booklet for Problem 1.

Problem 1. (50 points) Suppose that a real-valued message signal $m(t)$ with bandwidth B modulates a carrier with frequency $f_c \gg B$ and phase θ to generate a transmit signal

$$x(t) = m(t)\cos(2\pi f_c t + \theta).$$

This signal propagates through a multipath channel and is received as

$$y(t) = \sum_{l=1}^{L} \alpha_l x(t - \tau_l) + N(t),$$

where α_l and τ_l are the gain and delay of the lth path, respectively, and $N(t)$ is an ambient noise. Let α and ϕ be the magnitude and phase of the complex number $\sum_{l=1}^{L} \alpha_l e^{-\mathrm{j}2\pi f_c \tau_l}$, respectively, i.e.,

$$\alpha e^{\mathrm{j}\phi} = \sum_{l=1}^{L} \alpha_l e^{-\mathrm{j}2\pi f_c \tau_l}.$$

Answer the following questions.

(a) (10 points) Ignoring $N(t)$, find the impulse response $h(t)$ of the multipath channel.

(b) (20 points) Show that $x(t) = \Re\{m(t)e^{\mathrm{j}(2\pi f_c t + \theta)}\}$, where $\Re\{\cdot\}$ denotes the real part of the argument.

(d) (20 points) When $m(t - \tau_l)$ can be well approximated for $l = 1, 2, ..., L$, by $m(t - \tau_0)$ for some τ_0, show that

$$y(t) \approx \alpha m(t - \tau_0)\cos(2\pi f_c t + \theta + \phi) + N(t).$$

Caution!!!
Use a separate answer booklet for Problem 2.

Problem 2. (50 points) Consider a pulse amplitude modulation (PAM) communication system with transmit baseband signal

$$x(t) = \sum_n s[n]g(t - nT),$$

where $s[n]$ denotes a PAM symbol, $g(t)$ represents the pulse shaping filter, and $T > 0$ is a constant.

Suppose that, after matched filtering and analog-to-digital converter operations at a receiver, the discrete-time output signal of the nth sample at the receiver is given by

$$y[n] = s[n] + v[n], \tag{1}$$

where $v[n]$ denotes the real-valued additive white Gaussian noise with mean zero and variance σ^2, i.e., $v[n] \sim \mathcal{N}(0, \sigma^2)$.

Answer the following questions.

(a) (10 points) Design a pulse shaping filter $g(t)$ such that $g(t) * g(-t)$ satisfies the Nyquist zero-ISI criterion. (Hint: Just write any $g(t)$. You don't need to derive the necessary and sufficient condition that every such $g(t)$ needs to satisfy.)

(b) (5 points) When the binary antipodal data symbol, i.e., $s[n] \in \mathcal{C}_1 = \{1, -1\}$ with $\mathbb{P}[s[n] = 1] = \mathbb{P}[s[n] = -1] = \frac{1}{2}$, is used, what is the transmission rate in (bits/sec) of this system?

(c) (5 points) In Problem 2-(b), explain the maximum likelihood (ML) detection rule, given the matched filter output in Eq. (1).

(d) (15 points) In Problem 2-(c), compute the symbol error rate in terms of $Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-t^2/2} \mathrm{d}t$.

(e) (5 points) When the 4-ary PAM symbol, i.e., $s[n] \in \mathcal{C}_2 = \{3, 1, -1, -3\}/\sqrt{5}$ with equal probability, is used, what is the bit transmission rate of this communication system?

(f) (5 points) In Problem 2-(e), under the premise that the receiver performs ML detection based on the matched filter output in Eq. (1), what is the symbol error rate? (Please express it in terms of the $Q(\cdot)$ function).

(g) (5 points) Compare and discuss the transmission rates and the symbol error rates of the BPSK and 4-ary PAM signal sets.