Advanced Optics (PHYS690)

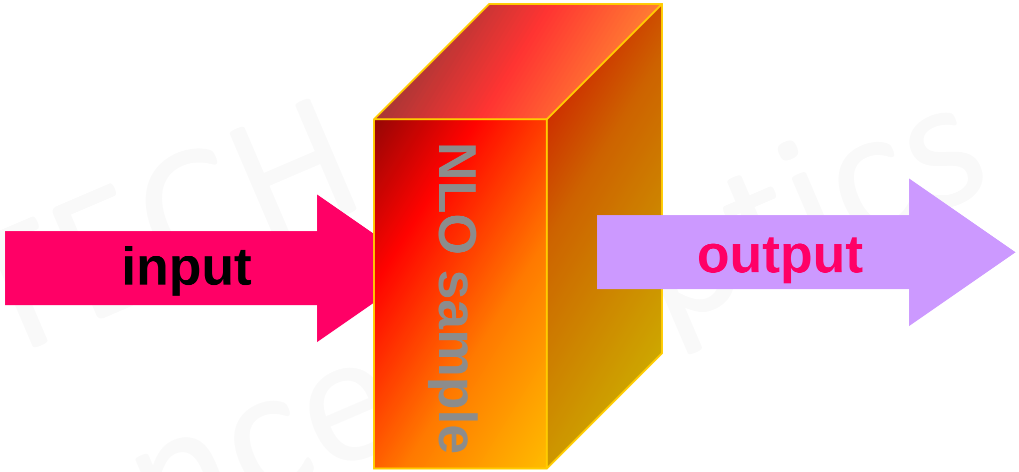
HEEDEUK SHIN

Lecture 20

POHANG UNIVERSITY OF SCIENCE AND TECHNOLOGY, KOREA

Question:

Is it possible to change the color of a monochromatic light?



Answer:

Not without a laser light

Nonlinear Optics

- Dielectric media characterized by a linear relation between polarization and E-field. Polarization: the total induced dipole moment per unit volume

$$P = \varepsilon_0 \chi E$$

- Media characterized by a nonlinear relation between *E* and *P*.

$$P = a_1 E + \frac{1}{2} a_2 E^2 + \frac{1}{6} a_3 E^3 + \ldots$$

$$= \varepsilon_0 \left(\chi E + \chi^{(2)} E^2 + \chi^{(3)} E^3 + \ldots \right)$$

$$= \epsilon_0 \chi E + 2\epsilon_0 d E^2 + \epsilon_0 \chi^{(3)} E^3 + \cdots$$

1st order **2nd order** **3rd order**

$$d = \frac{1}{4} a_2 = \frac{1}{2} \chi^{(2)}$$

d: second order nonlinear coefficient

Why polarization?

- From Maxwell's equations

$\nabla \cdot \vec{D} = \rho$	Gauss's law for electric field
$\nabla \cdot \vec{B} = 0$	Gauss's law for magnetic field
$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$	Ampere's law
$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$	Faraday's law

where

$\vec{D}$	electric displacement (C/m^2)
$\vec{E}$	electric field (V/m)
$\vec{B}$	magnetic field $(T$ or $V \cdot s/m^2)$
$\vec{H}$	magnetic field intensity (A/m)
ρ	volume charge density (C/m^3)
$\vec{J}$	current density (A/m^2)

μ_0: permeability
ϵ_0: permittivity

$$\mu_0 \epsilon_0 = \frac{1}{c^2}$$

- Free space wave equation

$$\nabla^2 \vec{E}(\vec{r}) + \mu_0 \epsilon_0 \frac{\partial^2}{\partial t^2} \vec{E}(\vec{r}) = 0$$

$$\vec{E}(\vec{r}) = \hat{n} E_0 e^{-j\vec{k}\cdot\vec{r}}$$

$$|\vec{k}| = c\omega$$

- In a dielectric medium

$$\nabla^2 \vec{E}(\vec{r}) + \mu_0 \epsilon_0 \epsilon \frac{\partial^2}{\partial t^2} \vec{E}(\vec{r}) = 0$$

$$|\vec{k}| = \frac{c}{n}\omega$$

$$n = \sqrt{\epsilon}$$

Why polarization?

- In a dielectric medium

$$\nabla^2 \vec{E}(\vec{r}) + \mu_0 \epsilon_0 \epsilon \frac{\partial^2}{\partial t^2} \vec{E}(\vec{r}) = 0 \qquad |\vec{k}| = \frac{c}{n}\omega \qquad n = \sqrt{\epsilon}$$

- Note that

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P} \qquad \vec{P} = \epsilon_0 \chi \vec{E}$$

$$\vec{D} = \epsilon_0 (1 + \chi) \vec{E} = \epsilon_0 \epsilon \vec{E}$$

- Then

$$\nabla^2 \vec{E}(\vec{r}) = -\mu_0 \epsilon_0 \epsilon \frac{\partial^2}{\partial t^2} \vec{E}(\vec{r}) = -\mu_0 \epsilon_0 \frac{\partial^2}{\partial t^2} \vec{E}(\vec{r}) - \mu_0 \epsilon_0 \chi \frac{\partial^2}{\partial t^2} \vec{E}(\vec{r})$$

$$\underbrace{\nabla^2 \vec{E}(\vec{r}) + \mu_0 \epsilon_0 \frac{\partial^2}{\partial t^2} \vec{E}(\vec{r})}_{\text{Propagating wave}} = -\mu_0 \frac{\partial^2}{\partial t^2} \vec{P}(\vec{r})$$

A source term

Propagating wave

Nonlinear wave equation I

- From Maxwell's equation and now considering

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P} \qquad \vec{P} = \epsilon_0 \chi \vec{E}$$

- We obtain a nonlinear wave equation

$$\nabla^2 \vec{E}(\vec{r}) + \mu_0 \epsilon_0 \frac{\partial^2}{\partial t^2} \vec{E}(\vec{r}) = -\mu_0 \frac{\partial^2}{\partial t^2} \vec{P}(\vec{r})$$

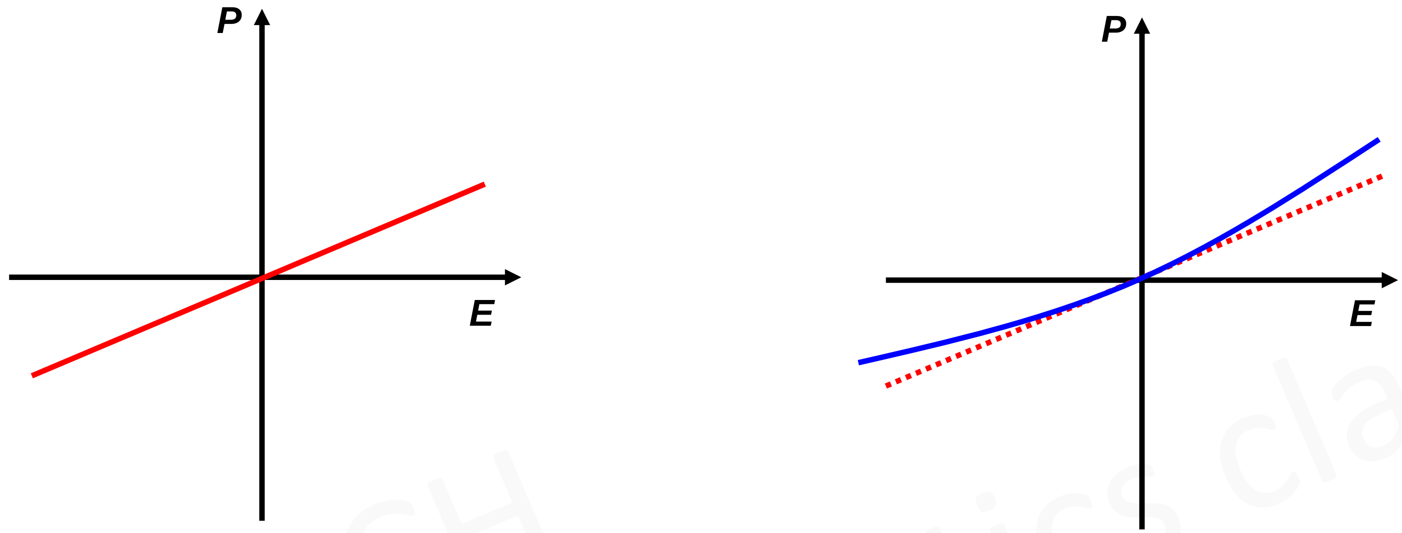
- where P is usually written as

$$P = \epsilon_0 \chi E + 2\epsilon_0 d E^2 + \epsilon_0 \chi^{(3)} E^3 + \cdots$$

$$P = P_L + P_{NL}$$

$$P_L = \epsilon_0 \chi E, \qquad P_{NL} = 2\epsilon_0 d E^2 + \epsilon_0 \chi^{(3)} E^3 + \cdots$$

In centrosymmetric media, d vanish, and the lowest order nonlinearity is of third order



P-*E* relation for (a) a linear dielectric medium, and (b) a nonlinear medium.

https://www.youtube.com/watch?v=anwl6OZ1UuQ

Typical values

$$d \sim 10^{-3} - 10^{3}\ [\frac{pm}{V}]$$

$$\chi^{(3)} \sim 10^{-25} - 10^{-8}\ [\frac{m^2}{V^2}]$$

$\chi^{(2)}$

Second order nonlinear effects

Second harmonic generation (SHG) I

- Assume higher order than second order are negligible

$$P_{NL} = 2\epsilon_0 d E^2$$

- Take

$$E = \frac{1}{2}\left(E(\omega)e^{i\omega t} + c.c.\right)$$

$$\mathcal{E}(t) = \mathrm{Re}\{E(\omega)\exp(j\omega t)\} = \tfrac{1}{2}[E(\omega)\exp(j\omega t) + E^*(\omega)\exp(-j\omega t)]$$

- Corresponding nonlinear polarization density is

$$P_{NL} = 2d\frac{1}{4}\left(E(\omega)e^{i\omega t} + c.c.\right)\left(E^*(\omega)e^{-i\omega t} + c.c.\right)$$

$$\mathcal{P}_{\mathrm{NL}}(t) = P_{\mathrm{NL}}(0) + \mathrm{Re}\{P_{\mathrm{NL}}(2\omega)\ \exp(j2\omega t)\}$$

$$P_{\mathrm{NL}}(0) = \mathrm{d}\, E(\omega)E^*(\omega)$$

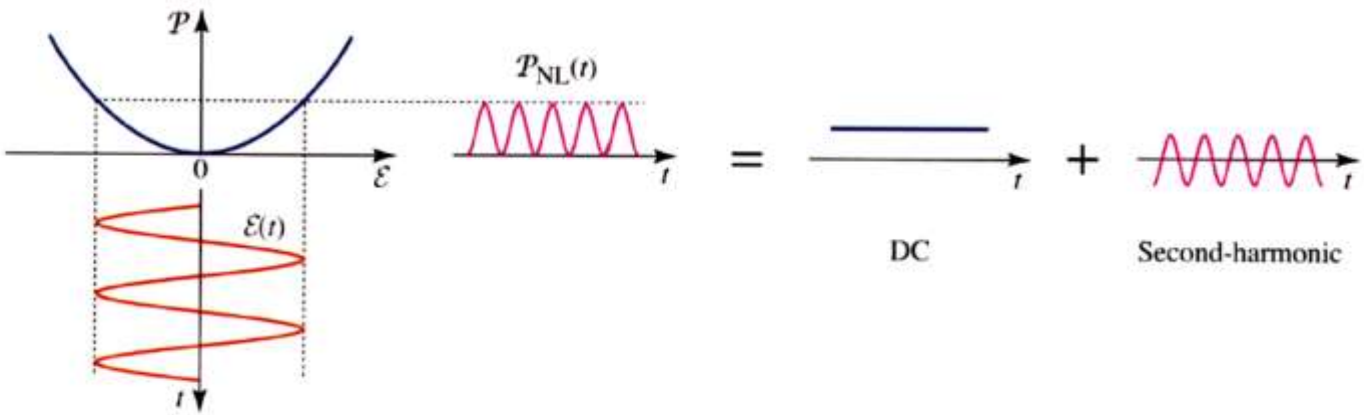
$$P_{\mathrm{NL}}(2\omega) = \mathrm{d}\, E^2(\omega). \longrightarrow E(2\omega)$$

Second harmonic generation (SHG) II

$$= dEE^* + d\left(EEe^{2i\omega t} + c.c.\right)$$

dc
optical rectification

2ω
2nd harm. generation



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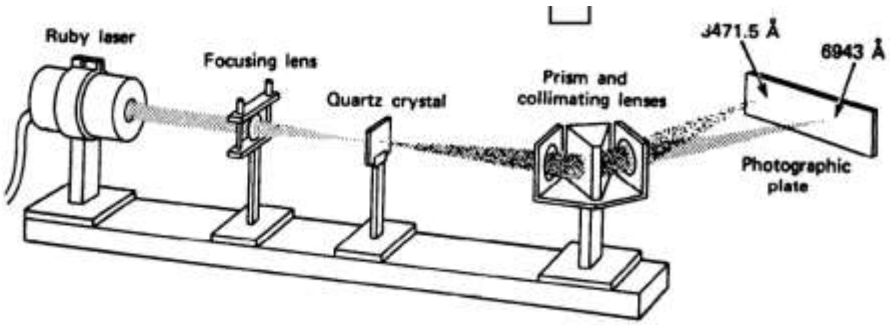
First SHG report



P.A. Franken, et al, Physical Review Letters 7, p. 118 (1961)

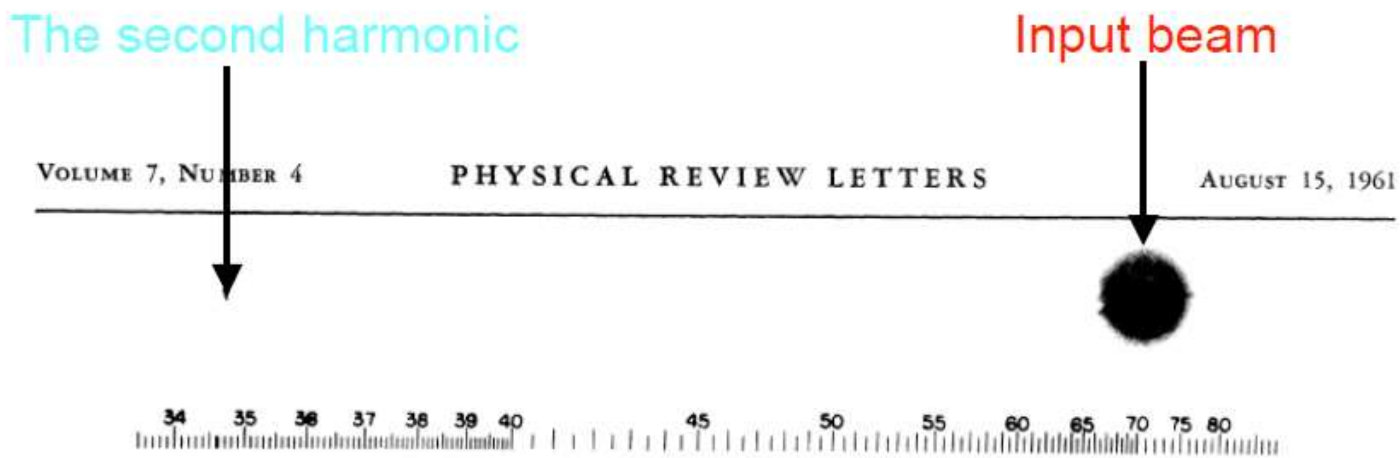


FIG. 1. A direct reproduction of the first plate in which there was an indication of second harmonic. The wavelength scale is in units of 100 A. The arrow at 3472 A indicates the small but dense image produced by the second harmonic. The image of the primary beam at 6943 A is very large due to halation.

Physical Review Letters editor

Phase Matching

$$E(\omega_1) = A_1 \exp(-j\vec{k_1}\cdot\vec{r})$$

$$E(\omega_2) = A_2 \exp(-j\vec{k_2}\cdot\vec{r})$$

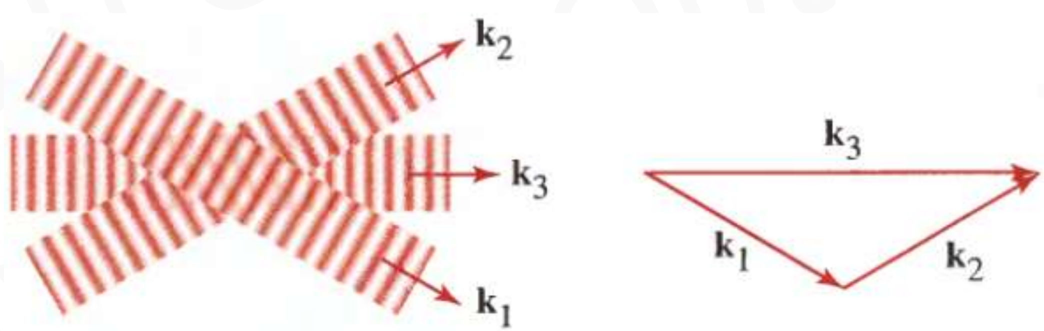
$$P_{NL}(\omega_3) = 2dE(\omega_1)E(\omega_2) = 2dA_1A_2 \exp(-j\vec{k_3}\cdot\vec{r})$$

where $\boxed{\omega_3 = \omega_1 + \omega_2}$

Frequency-Matching Condition

$\boxed{\vec{k}_3 = \vec{k}_1 + \vec{k}_2}$

Phase-Matching Condition



The phase-matching condition

Phase matching I

Consider a plane wave propagating along z, and write the paraxial Hemholtz equation

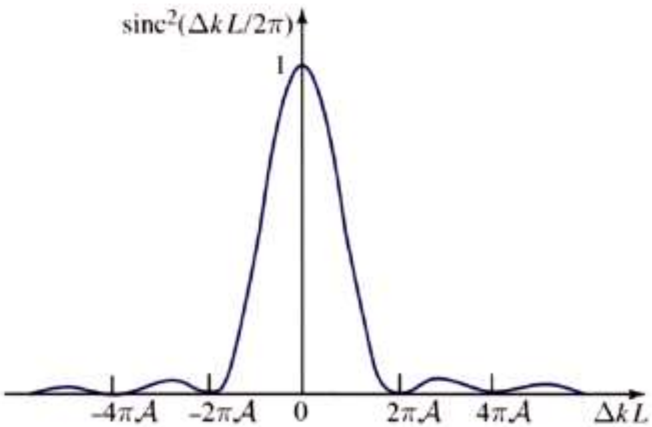
$$\left[\nabla_{\perp}^{2} E(2\omega) - 2ik\partial_z E(2\omega)\right]e^{-k(2\omega)z} = \mu_0 \partial_t^2 \left(dE^2(\omega)\right)e^{-k(\omega)z}$$

Can be approximated as $$\partial_z E(2\omega) = AdE^2(\omega)e^{\Delta kz}$$

where $\Delta k=k(2\omega)-k(\omega)$ so finally $$E(2\omega) \propto \frac{\sin \Delta kz}{\Delta kz}$$

need Δk=0 to achieve maximum E(2ω) electric field (this was not taken care in the 1961 experiment)

This is referred to as "phase matching"



Phase matching II

So we're creating light at $\omega_{sig} = 2\omega$.

So we're creating light at $k_{sig} = \dfrac{\omega_{sig}}{c_0} n(\omega_{sig}) = \dfrac{(2\omega)}{c_0} n(2\omega)$

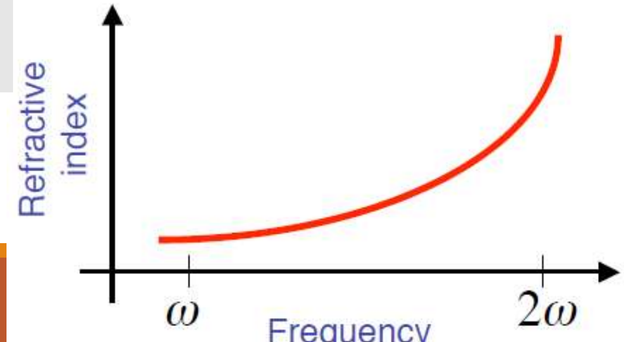
And the k-vector of the polarization is: $k_{pol} = 2\,k = 2\dfrac{\omega}{c_0} n(\omega)$

The phase-matching condition is: $k_{sig} = k_{pol}$

which will only be satisfied when: $n(2\omega) = n(\omega)$

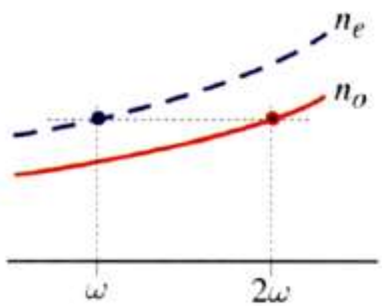
$$\omega_1 + \omega_2 = \omega_3, \qquad \omega_1 n_1 + \omega_2 n_2 = \omega_3 n_3,$$

Unfortunately, dispersion prevents this from ever happening!



Phase matching III

- Birefringent materials - different refractive indices for different polarizations.
- Ordinary and extraordinary refractive indices can be different by up to ~0.1 for SHG crystals.
- We can now satisfy the phase-matching condition.



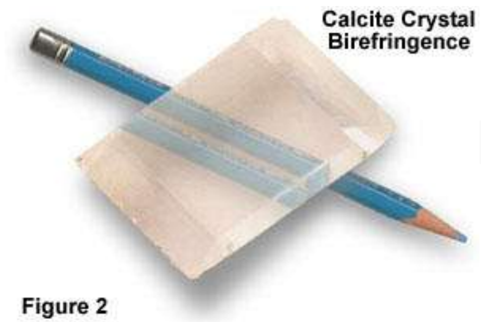
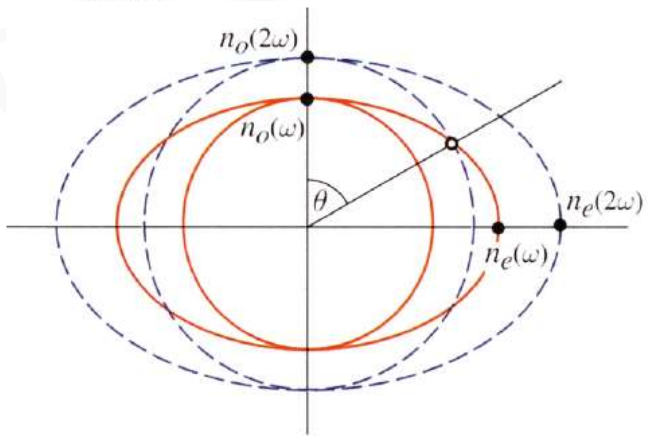


Figure 2

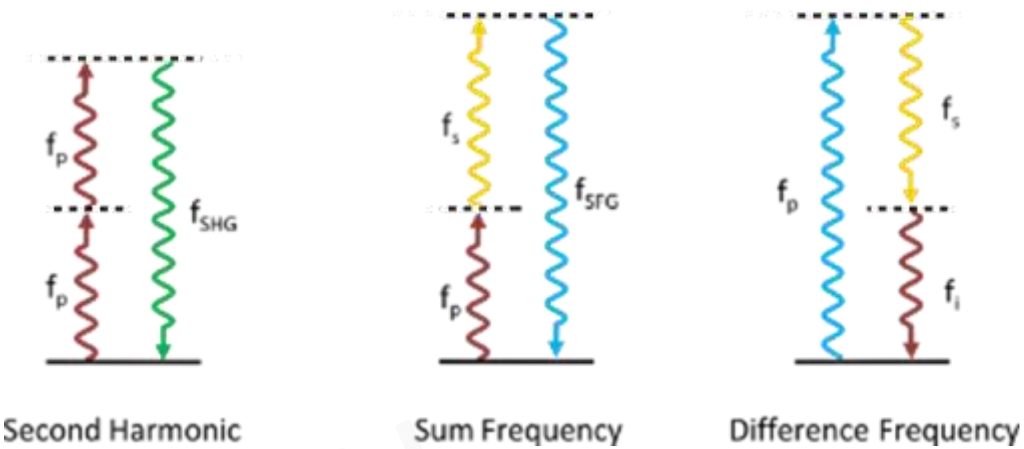


Use the extraordinary polarization for ω and the ordinary for 2ω.

$$n_o(2\omega) = n_e(\omega)$$

n_e depends on propagation angle, so we can tune for a given ω.
Some crystals have $n_e < n_o$, so the opposite polarizations work.

Phase Matching

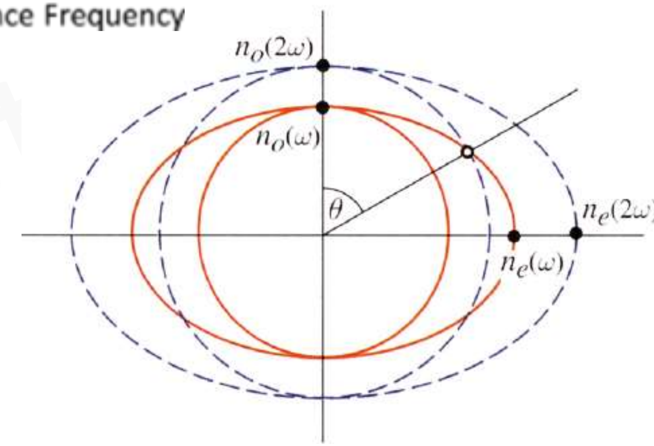


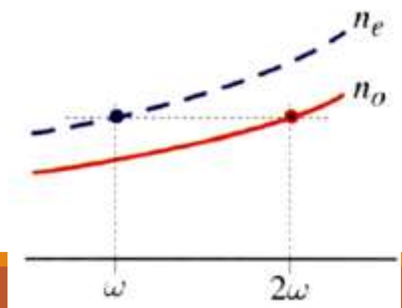
$$\boxed{\omega_3 = \omega_1 + \omega_2}$$

Frequency-Matching Condition

$$\boxed{\vec{k}_3 = \vec{k}_1 + \vec{k}_2}$$

Phase-Matching Condition





$$n_o(2\omega) = n_e(\omega)$$

Optical Rectification

$$= \underbrace{dEE^*}_{\text{dc}} + \underbrace{d\left(EEe^{2i\omega t} + c.c.\right)}_{2\omega}$$

dc
optical rectification

2ω
2nd harm. generation

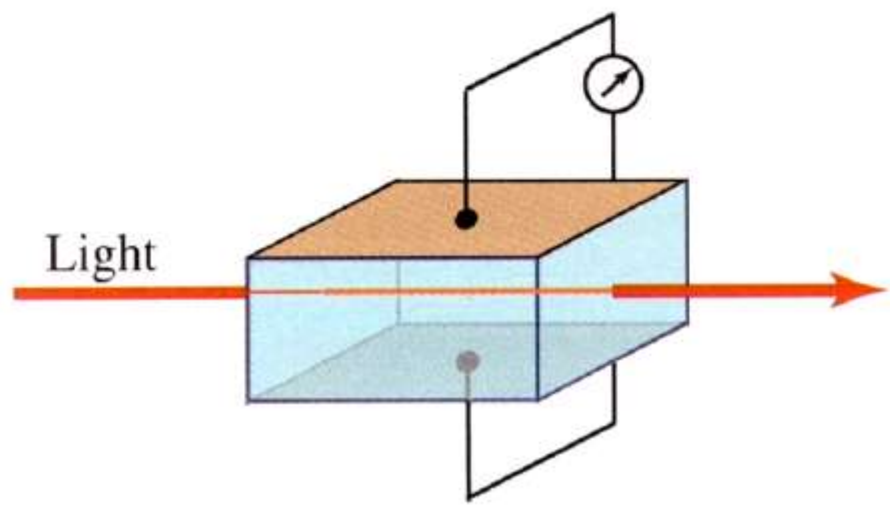
$$\mathcal{P}_{\mathrm{NL}}(t) = P_{\mathrm{NL}}(0) + \mathrm{Re}\{P_{\mathrm{NL}}(2\omega)\,\exp(j2\omega t)\}$$

$$P_{\mathrm{NL}}(0) = \mathrm{d}\,E(\omega)E^*(\omega)$$

$$P_{\mathrm{NL}}(2\omega) = \mathrm{d}\,E^2(\omega).$$

Generation of DC voltage

MW ➔ uV



Electro-Optic Effect

- E-field consists of harmonic component at optical frequency and zero fr equency.

$$\mathcal{E}(t) = E(0) + \mathrm{Re}\{E(\omega)\exp(j\omega t)\}$$

$$\mathcal{P}_{\mathrm{NL}}(t) = P_{\mathrm{NL}}(0) + \mathrm{Re}\{P_{\mathrm{NL}}(\omega)\exp(j\omega t)\} + \mathrm{Re}\{P_{\mathrm{NL}}(2\omega)\exp(j2\omega t)\},$$

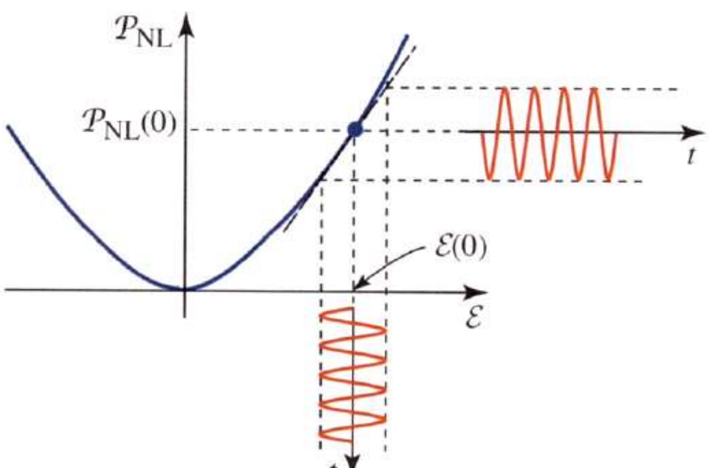
$$P_{\mathrm{NL}}(0) = \mathrm{d}\left[2E^2(0) + |E(\omega)|^2\right]$$

$$P_{\mathrm{NL}}(\omega) = 4\mathrm{d}\,E(0)E(\omega)$$

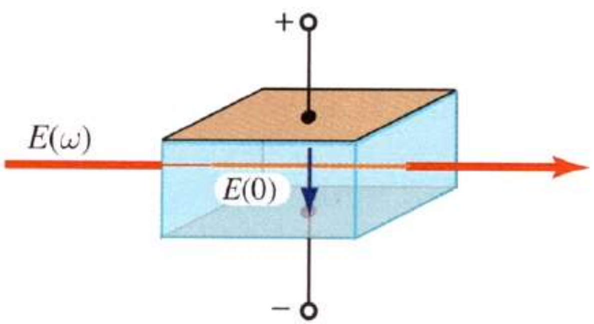
$$P_{\mathrm{NL}}(2\omega) = \mathrm{d}\,E^2(\omega),$$

$$|E(\omega)|^2 \ll |E(0)|^2$$

$$\Delta n = \frac{2\mathrm{d}}{n\epsilon_o}E(0).$$



Pockels effect

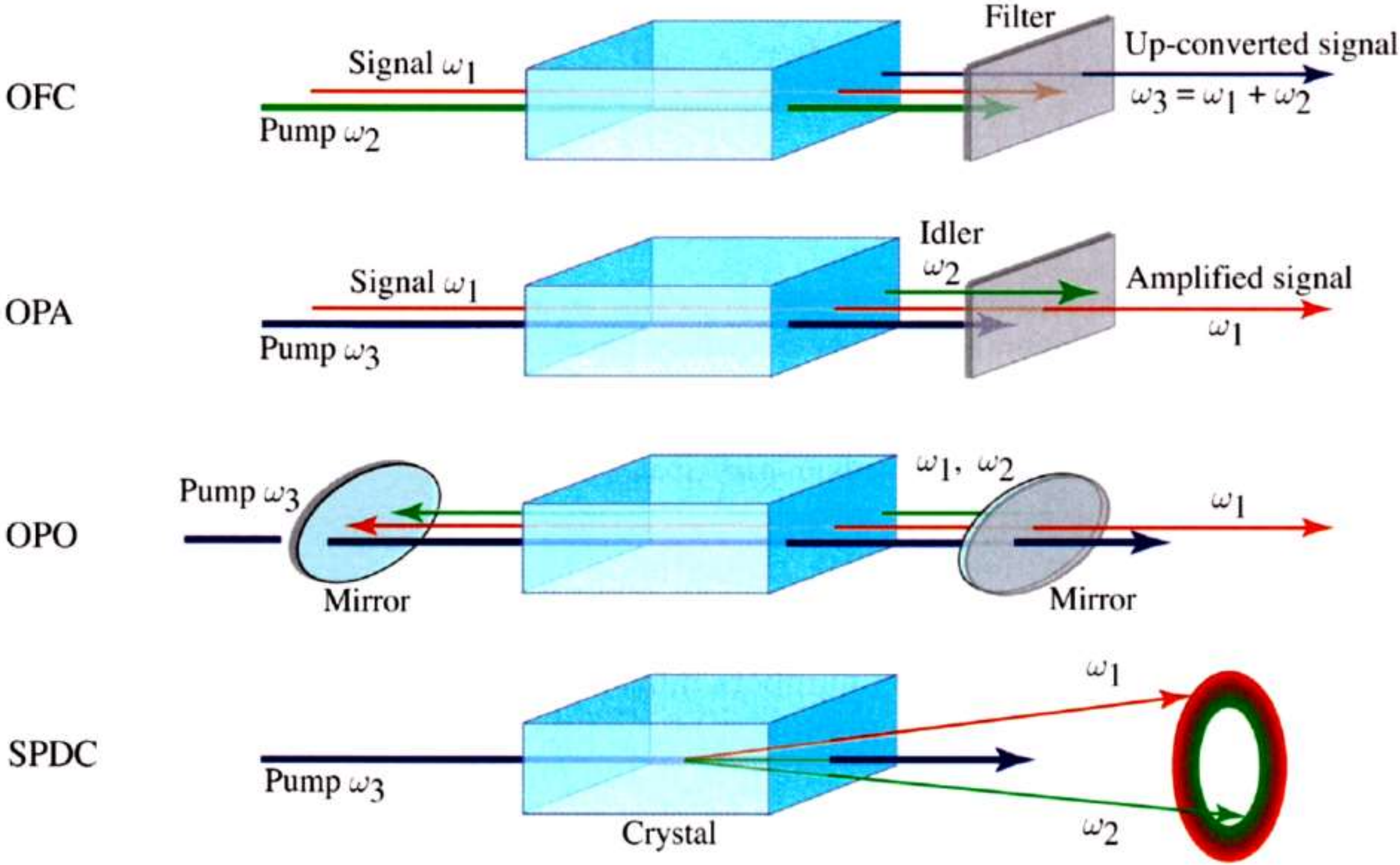


Three-wave mixing

- E-field

$$\mathcal{E}(t) = \mathrm{Re}\{E(\omega_1)\exp(j\omega_1 t) + E(\omega_2)\exp(j\omega_2 t)\}$$

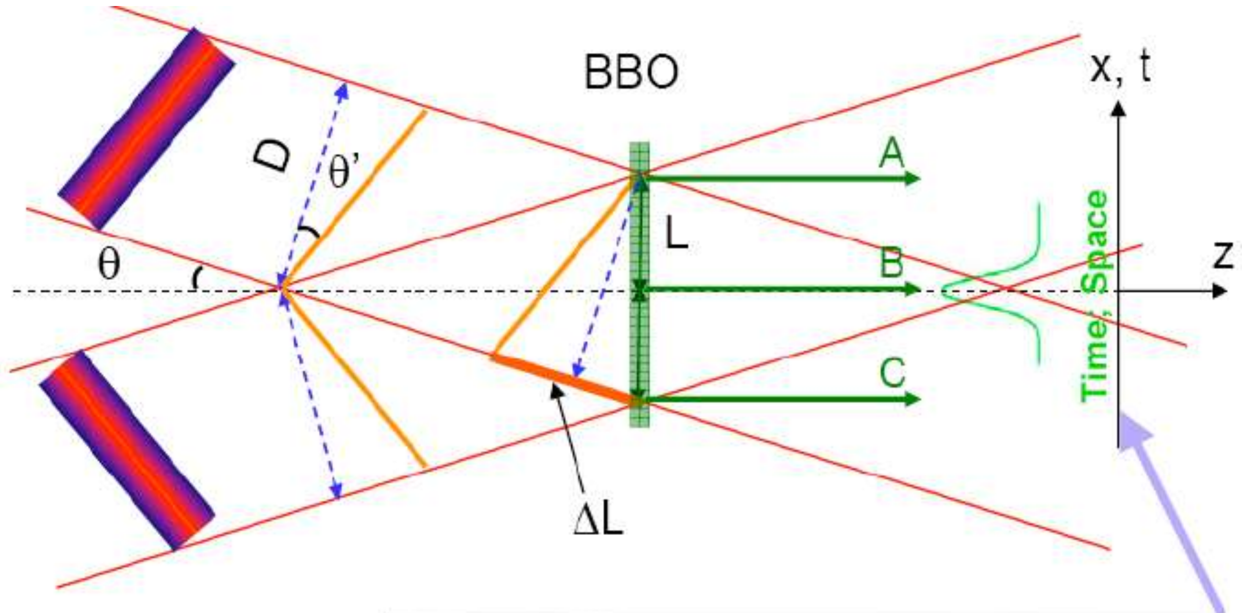
$$\begin{aligned}
P_{\mathrm{NL}}(0) &= \ \mathrm{d}\left[|E(\omega_1)|^2 + |E(\omega_2)|^2\right] \\
P_{\mathrm{NL}}(2\omega_1) &= \ \mathrm{d}\ E(\omega_1)E(\omega_1) \\
P_{\mathrm{NL}}(2\omega_2) &= \ \mathrm{d}\ E(\omega_2)E(\omega_2) \\
P_{\mathrm{NL}}(\omega_+) &= 2\mathrm{d}\ E(\omega_1)E(\omega_2) \\
P_{\mathrm{NL}}(\omega_-) &= 2\mathrm{d}\ E(\omega_1)E^*(\omega_2).
\end{aligned}$$

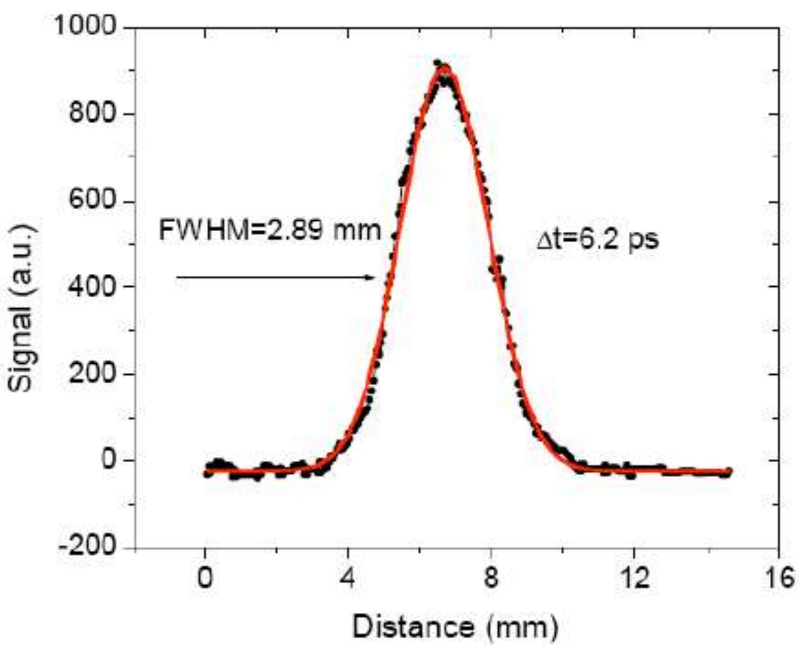


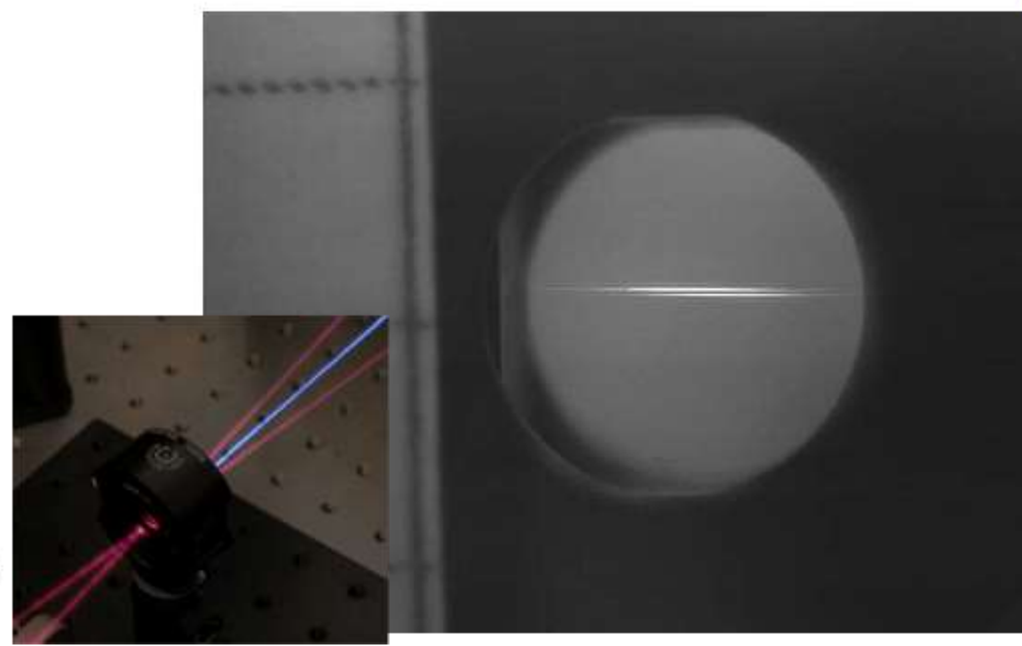
OFC
Signal ω_1
Pump ω_2
Filter
Up-converted signal
$\omega_3 = \omega_1 + \omega_2$
OPA
Signal ω_1
Pump ω_3
Idler
ω_2
Amplified signal
ω_1
OPO
Pump ω_3
Mirror
ω_1, ω_2
ω_1
Mirror
SPDC
Pump ω_3
Crystal
ω_1
ω_2

Application of noncollinear phase matching

- Noncollinear phase matching can be used to measure the duration of ultra-short pulse







$\chi^{(3)}$

Third order nonlinear effects

Third order nonlinear polarization

$$P_{NL}^{\delta} = 4\chi^{(3),\alpha\beta\gamma\delta} E_\alpha E_\beta E_\gamma$$

- Let's consider the simple case of third harmonic generation (THG).

$$E = E_0 \cos(\omega t)$$

- Then the third order polarization is

$$\begin{aligned} P_{NL} &= 4\chi^{(3)} E_0^3 \cos^3(\omega t) \\ &= 4\chi^{(3)} E_0^3 \left[\frac{1}{4}\cos(3\omega t) + \frac{3}{4}\cos(\omega t) \right] \end{aligned}$$

3rd order generation

Nonlinear contribution that affects the fundamental frequency

- The ω term in the polarization is of the form

$$\begin{aligned} P_{NL}(\omega) &= \chi^{(1)} E(\omega) + 3\chi^{(3)} |E(\omega)|^2 E(\omega) \\ &= \chi_{eff} E(\omega) \end{aligned}$$

Third order nonlinear effects

- The general form of the $\chi^{(3)}$ tensor is:

$$P_i^{(3)}(\omega) = \epsilon_0 \chi_{ijkl}^{(3)}(\omega : \omega_1, \omega_2, \omega_3) E_j(\omega_1) E_k(\omega_2) E_l(\omega_3)$$

- $\chi^{(2)}$ effects are only observed in materials without inversion symmetry.
- All materials have nonzero $\chi^{(3)}$ coefficients.
- The $\chi^{(3)}$ coefficient gives rise to an intensity dependent index of refraction.

$$\begin{aligned} P &= \chi^{(1)} E + \chi^{(3)} E^3 \\ &= \left(\chi^{(1)} + \chi^{(3)} |E|^2\right) E \end{aligned}$$

Optical Kerr effects

$$n = n_0 + n_2 I$$

- In Gaussian laser beams, the intensity is highest in the center of the beam.
- Hence the index of refraction is highest in the center.

Nonlinear index of refraction

- Real part of index is best described as a power series
- $n=n_1+n_2(P/A_{eff})$ $n = n_0 + n_2 I$
- For silica fiber, $n_2 \cong 2.6\times10^{-11}$ μm^2/mW

Interaction Length

$$L_{eff} = \frac{1-e^{-\alpha L}}{\alpha} \approx \frac{1}{\alpha} \qquad \text{If } L >> 1/\alpha$$

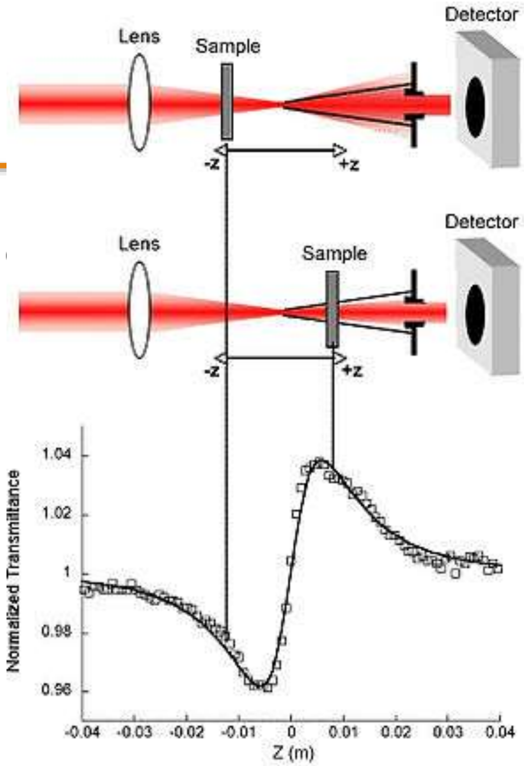
where α (in cm^{-1}) is the loss coefficient of the fiber. 0.1 dB/km=2.3×10^{-7} cm^{-1}.

Nonlinear parameter

$$\gamma = \frac{2\pi n_2}{\lambda A_{eff}} \qquad \beta_{NL} = \beta + \gamma P$$

- Propagation constant is power-dependent.

Self Focusing

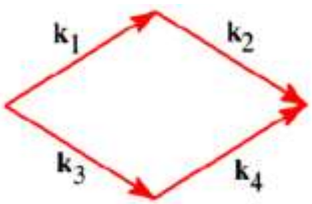


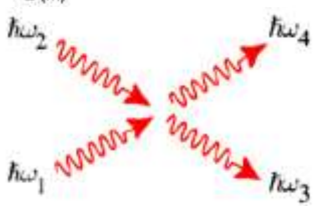
- From Fermat's principle we can estimate th

$$(n_0 + n_2 I) z_{SF} = \frac{n_0 z_{SF}}{\cos\theta_{SF}} \Rightarrow \theta_{SF} = \sqrt{\frac{2n_2 I}{n_0}}$$

$$z_{SF} = w_0 \theta_{SF} = w_0 \sqrt{\frac{n_0}{2n_2 I}}$$

4-wave mixing





It couples 4 E-fields together.

Similar to 3-wave mixing in 2nd order crystal (SHG is a degenerate case of 3-wave mixing), the process follow the conservation laws.

$$\omega_1 + \omega_2 = \omega_3 + \omega_4. \qquad \mathbf{k}_1 + \mathbf{k}_2 = \mathbf{k}_3 + \mathbf{k}_4.$$

$$P_{\mathrm{NL}}(\omega_2) = 6\chi^{(3)} E(\omega_3) E(\omega_4) E^*(\omega_1),$$

Self-phase modulation

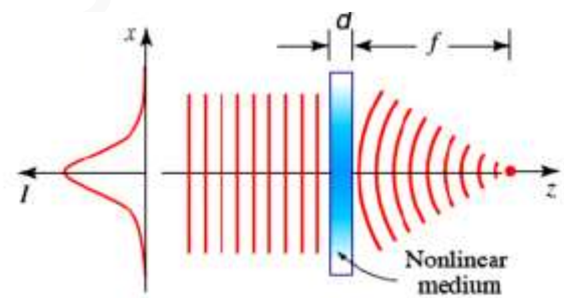
- Self-phase modulation is due to the intensity dependent refractive index

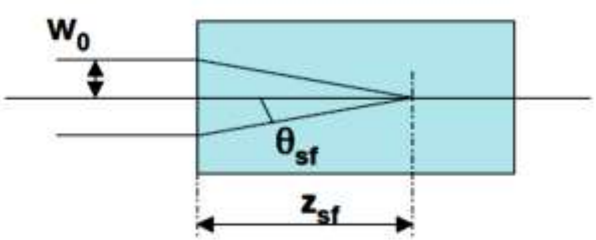
$$\varphi = -n(I)k_oL = 2\pi n(I)L/\lambda_o = -2\pi(n + n_2P/A)L/\lambda_o$$

- P and A are respective the beam power and cross-sectional area.
- The induces self phase modulation is $\Delta\varphi = -2\pi n_2 \frac{L}{\lambda_o A} P$

3rd order effects: Self Focusing

- Assume a beam with intensity larger at its center.
- When passing through a 3rd order medium, the me dium acts as an index graded element and results in focusing of the beam.
- In too long media, self focusing can locally leads to i ntensity above damage threshold.





Self-phase modulation in comm.

- Refractive index depends on optical intensity as (Kerr eect)

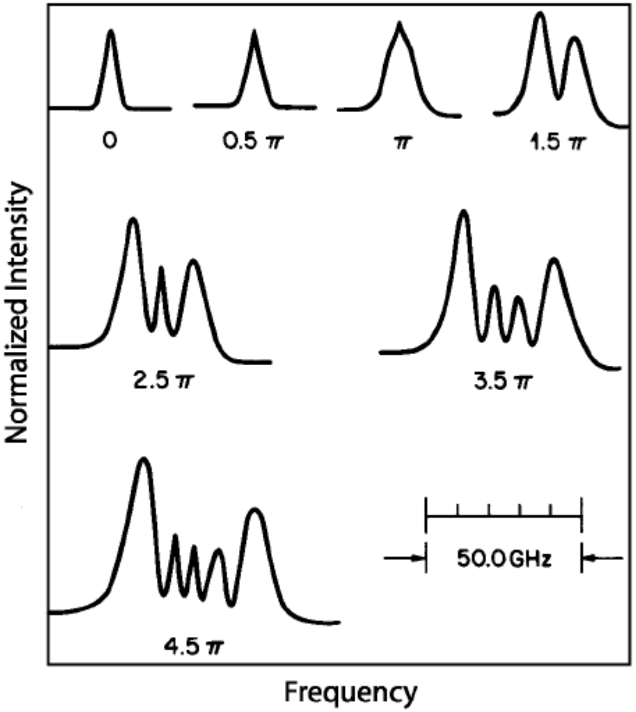
$$n(\omega, I) = n_0(\omega) + n_2 I(t)$$

- Intensity dependence leads to nonlinear phase shift

$$\phi_{NL}(t) = (2\pi/\lambda) n_2 I(t) L.$$

- An optical field modifies its own phase (SPM).
- Phase shift varies with time for pulses.
- Each optical pulse becomes chirped.

- As a pulse propagates along the fiber, its spectrum changes because of SPM.

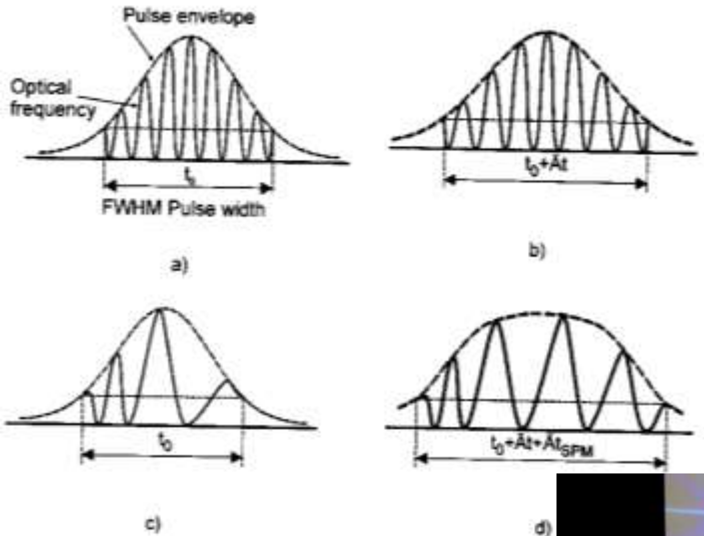


Phase modulation

Self-modulation: $\phi_{NL} = \gamma P L_{eff}$

Cross-modulation: $\phi_{NL} = 2\gamma P_{other} L_{eff}$

Effect of these phase changes is a frequency chirp (frequency changes du ring pulse), broadening pulse and reducing bit rate-length product.



Self-phase modulation effect:

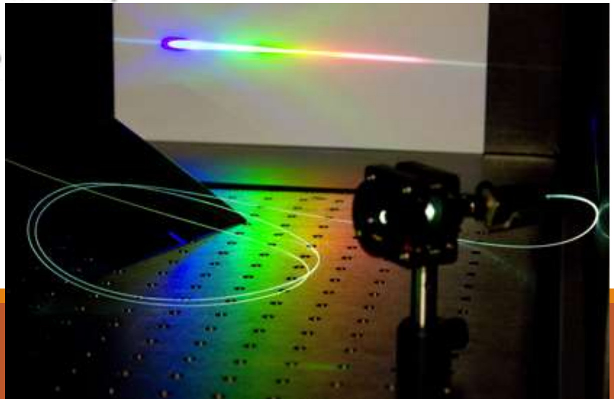
Spreading of chirped pulse:

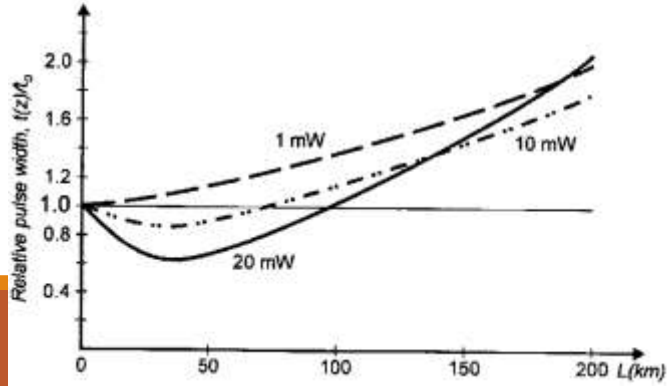
(a) Regular unchirped pulse entering the link;

(b) The same pulse distorted after traveling distance *L* along the fiber;

(c) Chirped pulse entering the link;

(d) Chirped pulse broadens after traveling distance *L*.





Cross-Phase Modulation

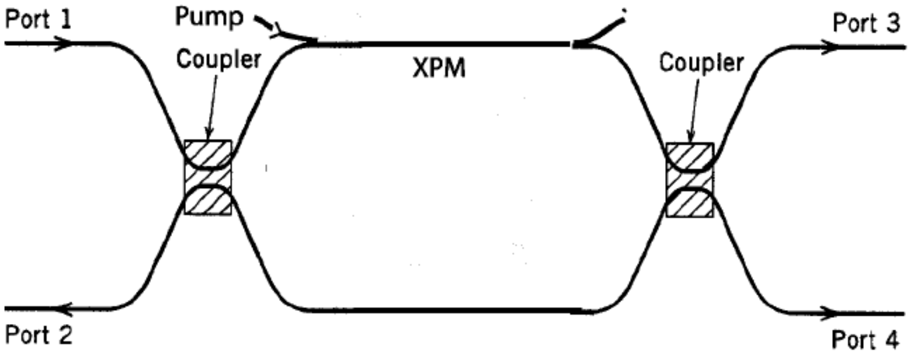
- Consider two optical fields propagating simultaneously.
- Nonlinear refractive index seen by one wave depends on the intensity of the other wave as

$$\Delta n_{\mathrm{NL}} = n_2(|A_1|^2 + b|A_2|^2)$$

- Total nonlinear phase shift in a fiber of length L:

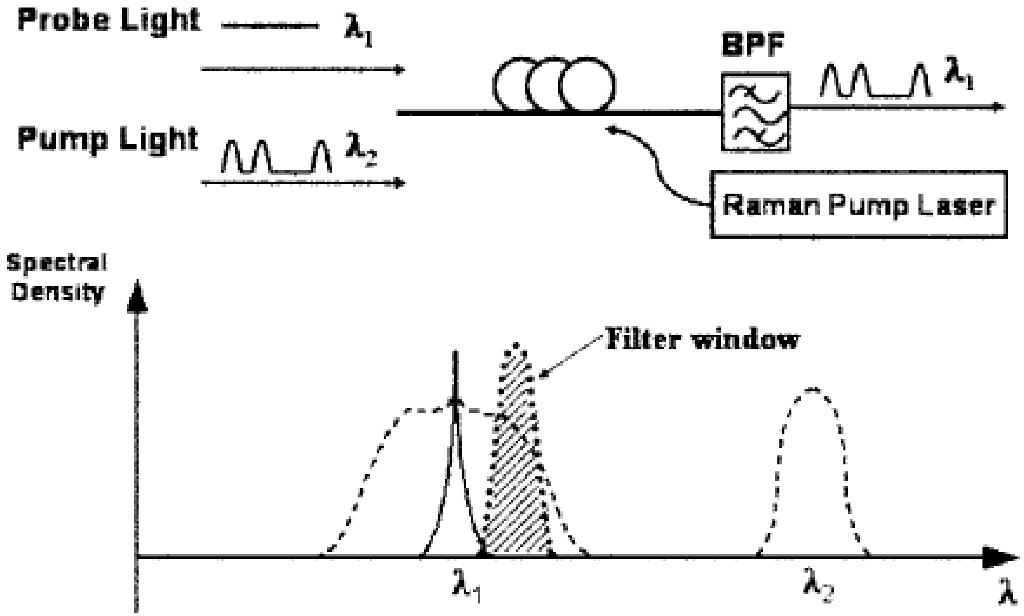
$$\phi_{\mathrm{NL}} = (2\pi L/\lambda) n_2 [I_1(t) + bI_2(t)]$$

- An optical beam modifies not only its own phase but also of other co-propagating beams (XPM).
- XPM induces nonlinear coupling among overlapping optical pulses.



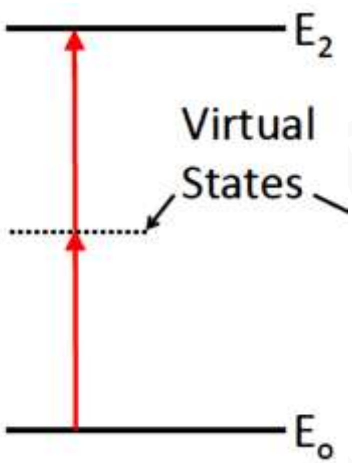
XPM-Based Wavelength Converter

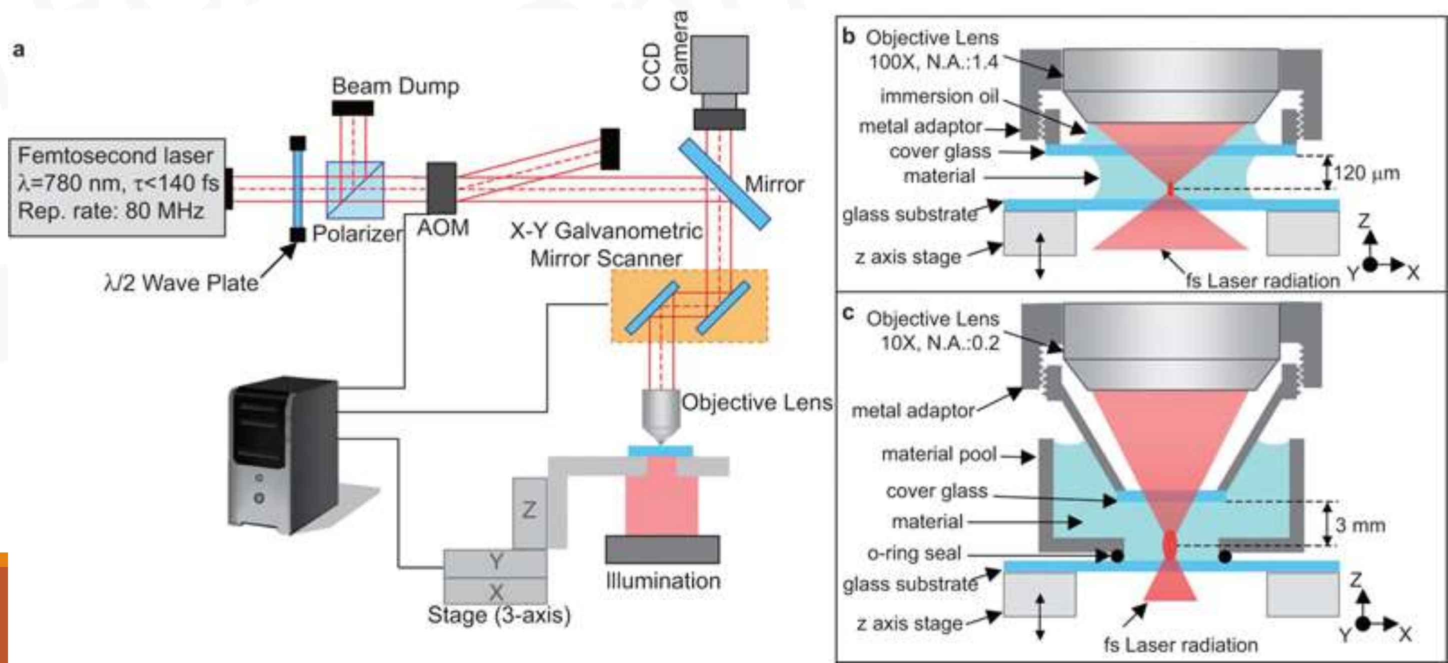
- WDM channel at λ_2 requiring conversion acts as a pump.
- A CW probe is launched at the desired wavelength λ_1.
- Probe spectrum broadens because of pump-induced XPM.
- An optical filter blocks pump and transfers data to probe.
- Raman amplification improves the device performance.

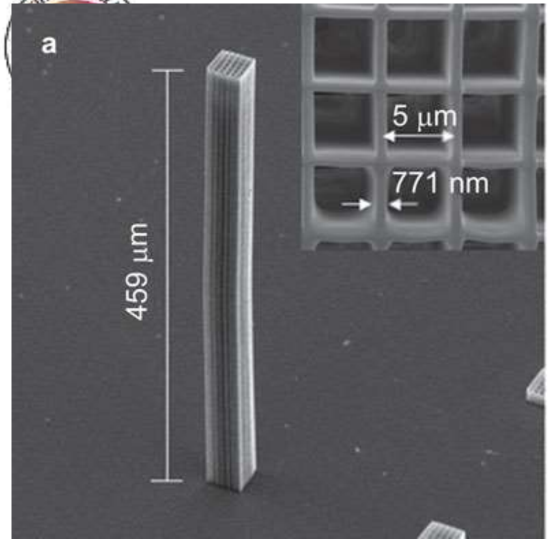


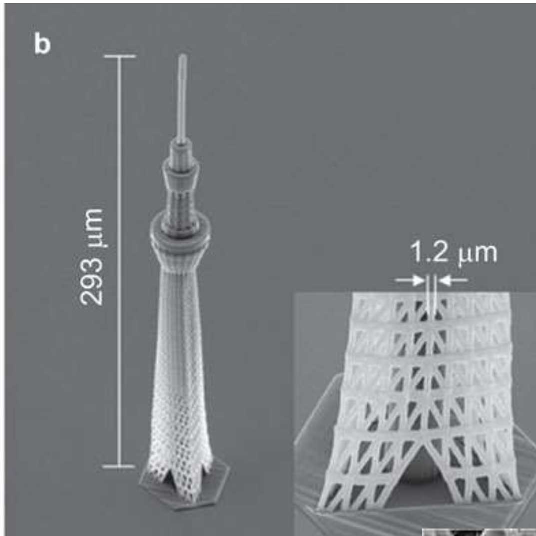
Two Photon Absorption

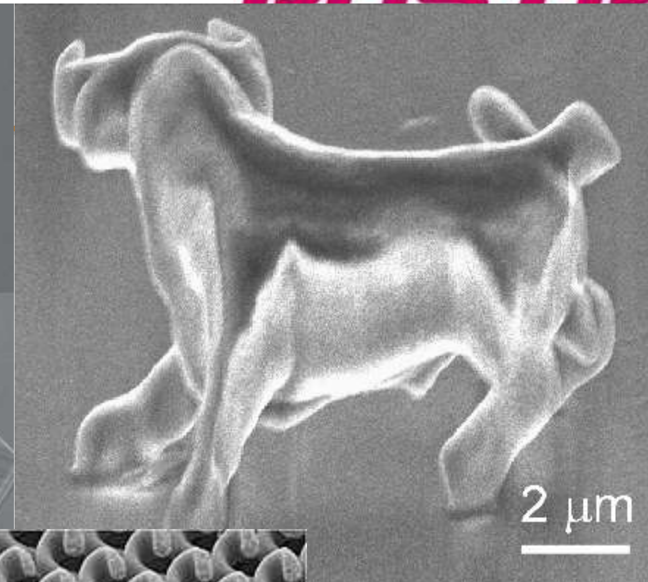
- Two photons absorbed simultaneously to bring atom to high-lying energy level.
- Absorption enhanced by enhanced by allowed transitions near sin gle photon resonance.
- Can limit the performance of high-powered lasers.

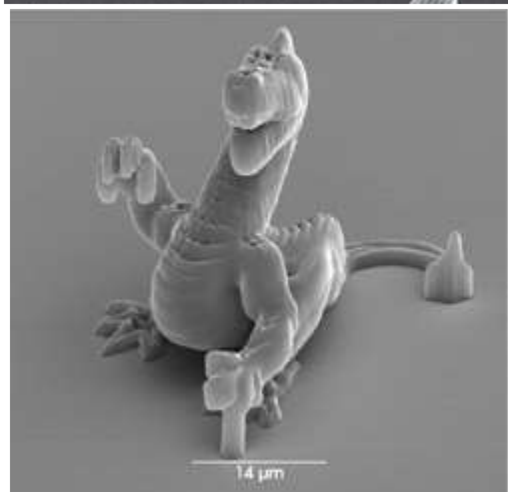


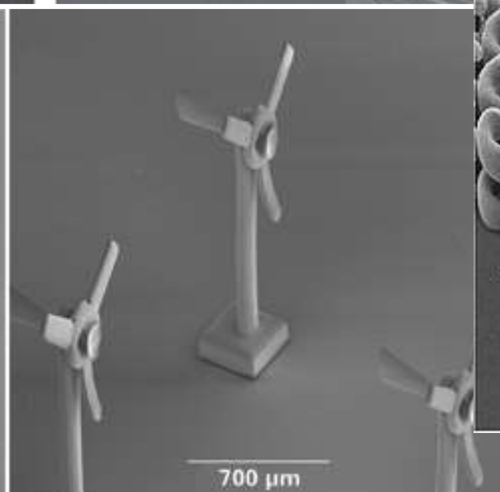


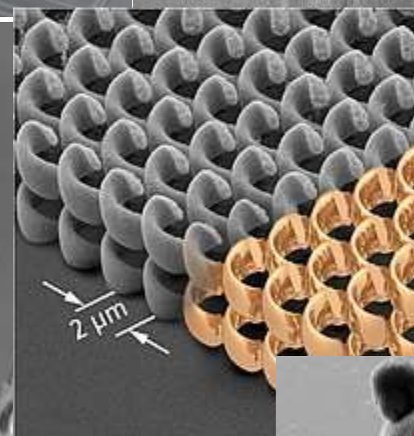

a
459 µm
5 µm
771 nm

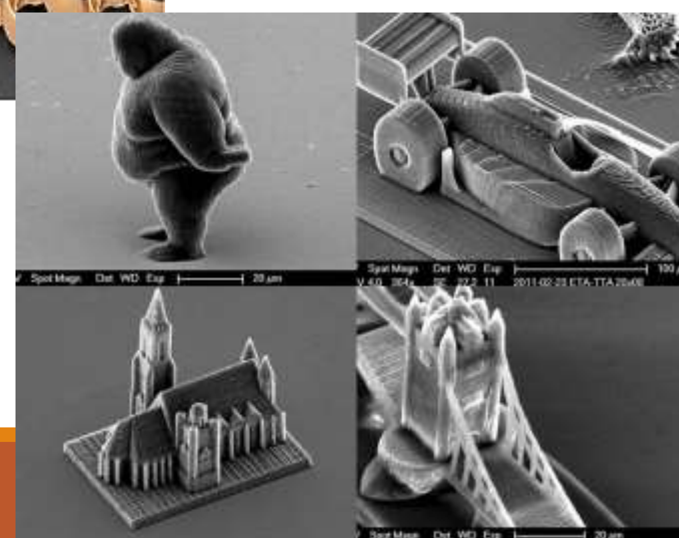

b
293 µm
1.2 µm

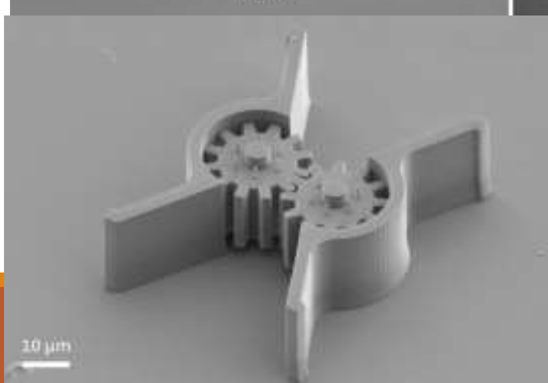

2 µm

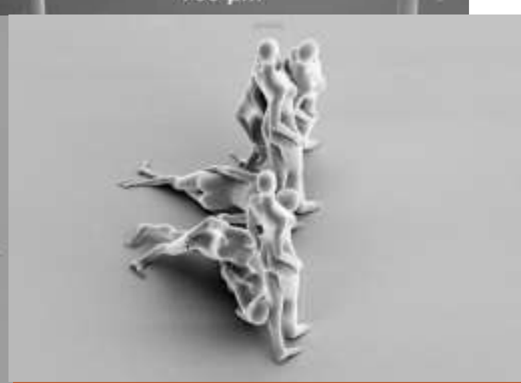

14 µm


700 µm

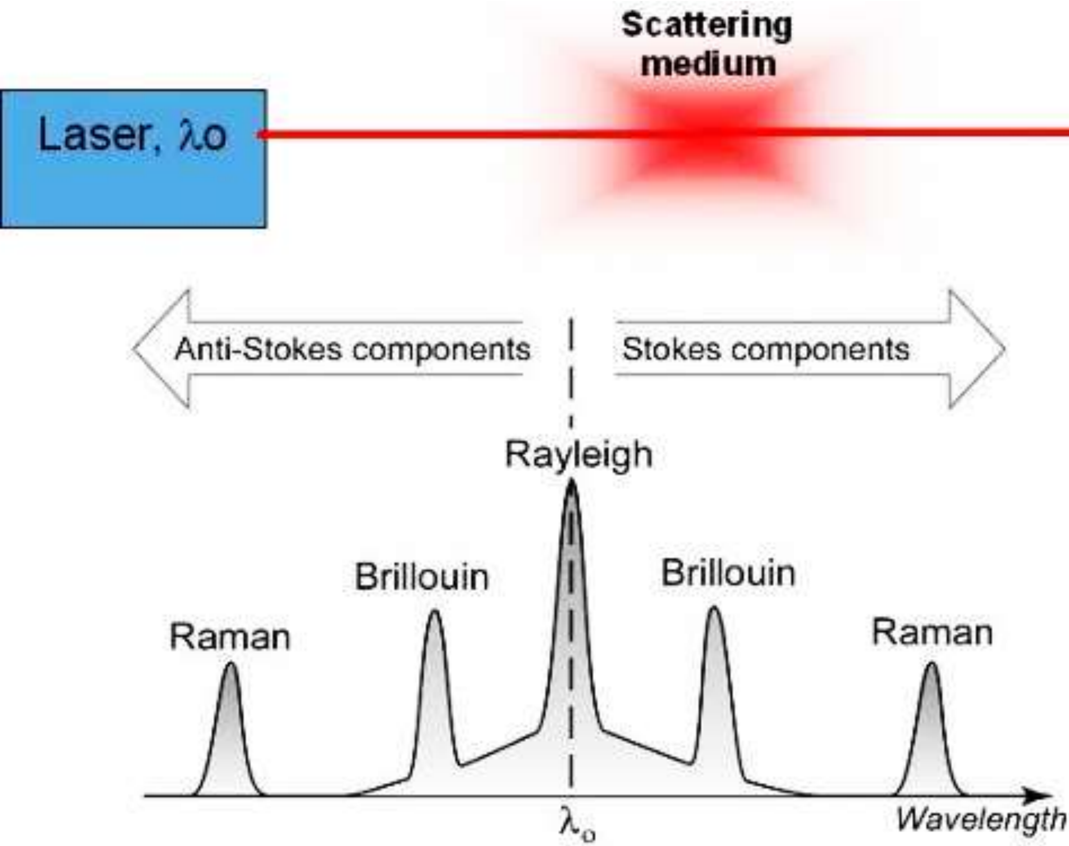

2 µm



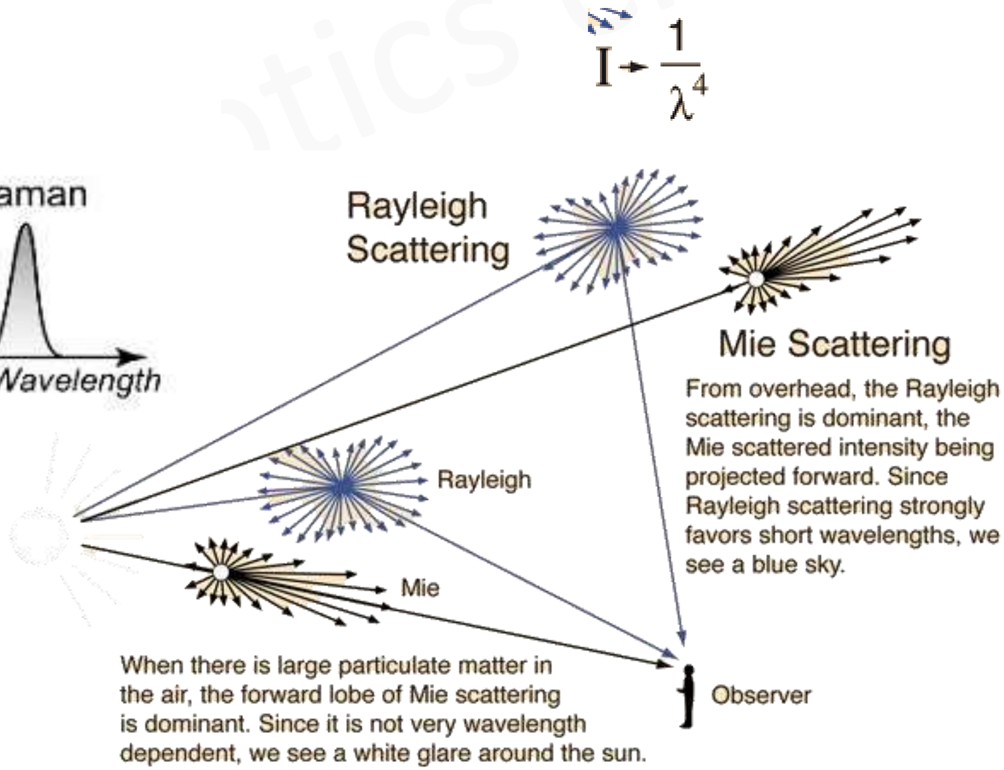

10 µm



Scattering



Scattering medium
Laser, λo
Anti-Stokes components
Stokes components
Rayleigh
Brillouin
Brillouin
Raman
Raman
λ_o
Wavelength



$I \rightarrow \frac{1}{\lambda^4}$
Rayleigh Scattering
Mie Scattering
From overhead, the Rayleigh scattering is dominant, the Mie scattered intensity being projected forward. Since Rayleigh scattering strongly favors short wavelengths, we see a blue sky.
Rayleigh
Mie
Observer
When there is large particulate matter in the air, the forward lobe of Mie scattering is dominant. Since it is not very wavelength dependent, we see a white glare around the sun.

Stimulated Raman Scattering (SRS)

- Interaction of light with an acoustic phonon in the material.

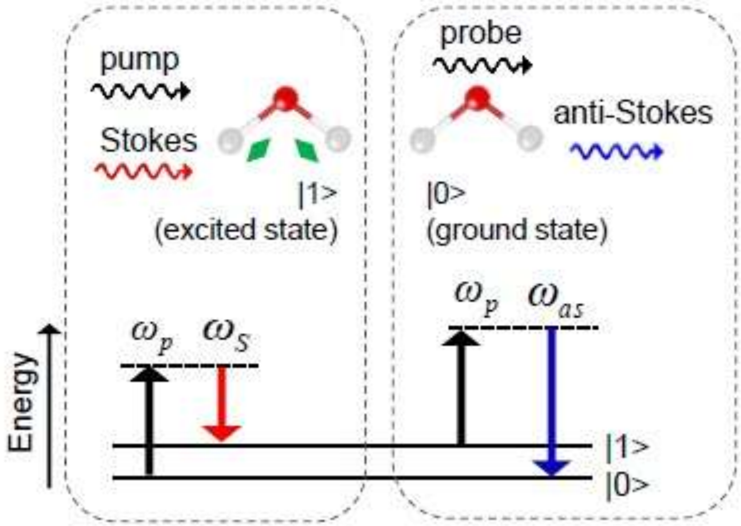
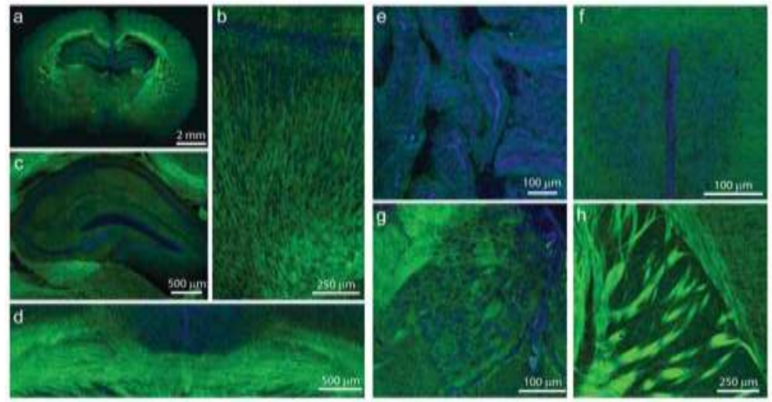
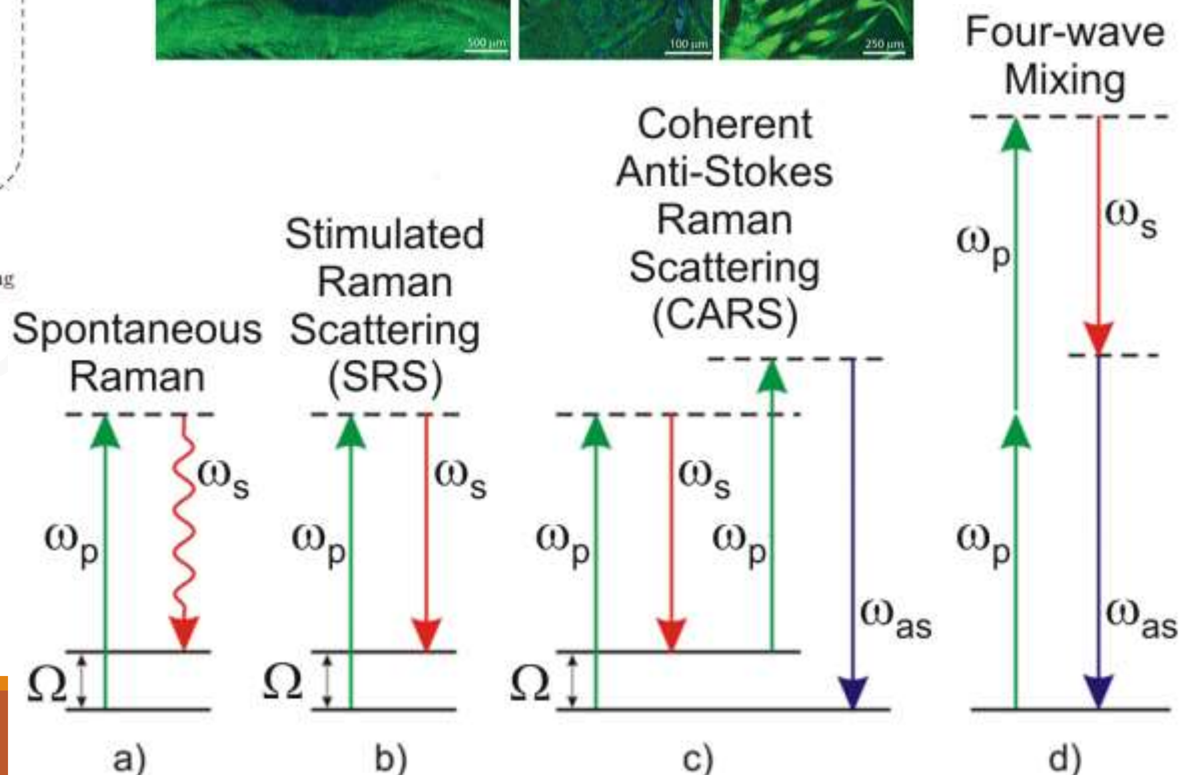


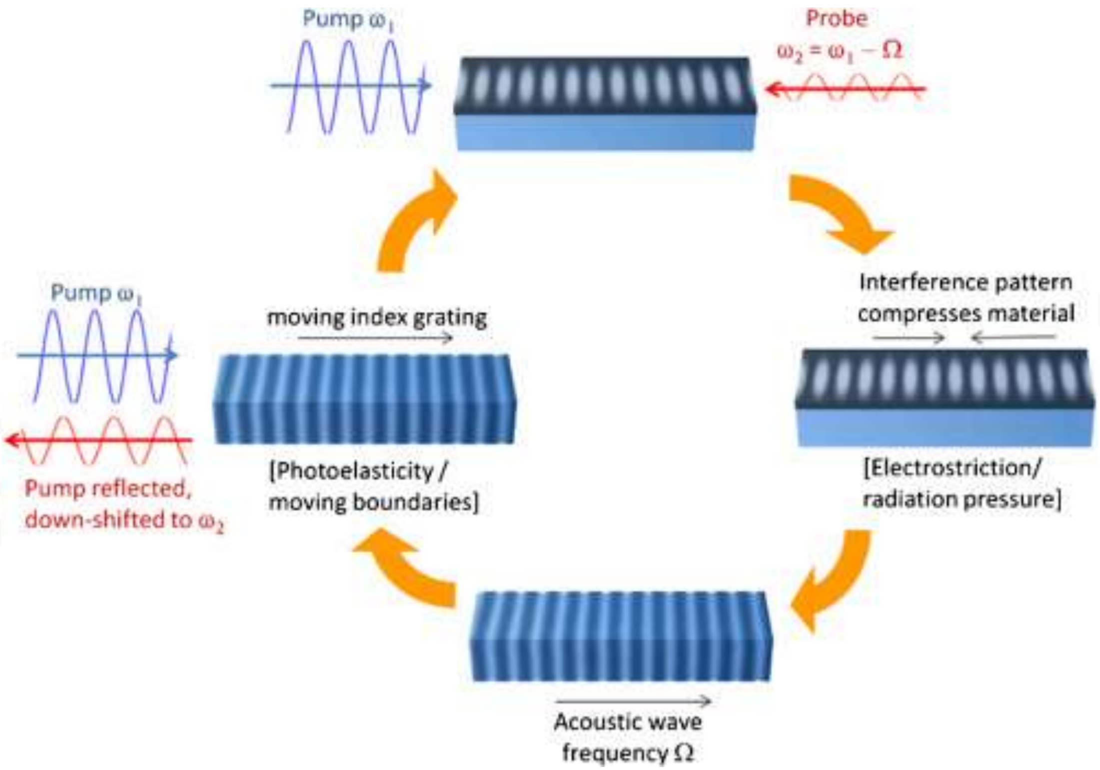
Figure 1: Energy level diagram for coherent anti-Stokes Raman scattering





Stimulated Brillioun Scattering (SBS)

- Interaction of light with an acoustic phonon in the material.
- The scattered wave is always(?) backwards propagating.



Concluding Remarks

- Nonlinear effects are sources of noise in many systems and not easy to understand.
- Don’t be afraid of it.
- **Nonlinearities can be managed thorough proper system design.**
- **By combining two or more nonlinear effects, unprecedented behavior properties and applications can be designed.**

Nonlinear wave equation II

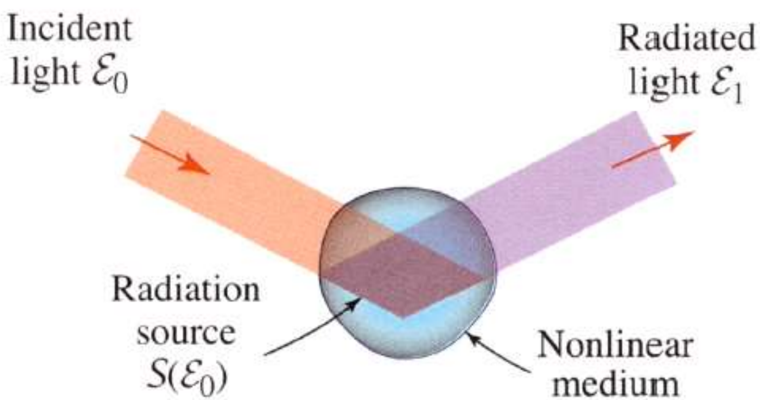
- Using $c = c_0 / n$

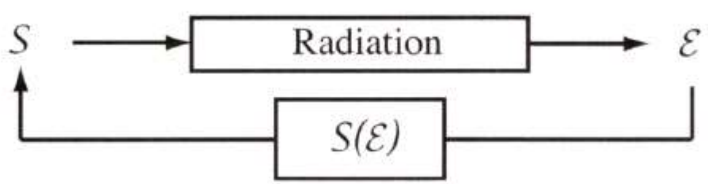
  $n^2 = 1 + \chi$

- The nonlinear wave equation can be rewritten

$$\nabla^2 \vec{E}(\vec{r}) + \mu_0 \epsilon_0 \epsilon \frac{\partial^2}{\partial t^2} \vec{E}(\vec{r}) = -\mu_0 \frac{\partial^2}{\partial t^2} \vec{P}_{NL}(\vec{r})$$

- The fist Born approximation. An incident optical field $\mathcal{E}_0$ creates a sourc e $\mathcal{S}(\mathcal{E}_0)$, which radiates an optical field $\mathcal{E}_1$.





Coupled- Wave Equations for Second-Harmonic Generation.

$$\frac{da_1}{dz} = -jga_3a_1^* \exp(-j\Delta kz)$$

$$\frac{da_3}{dz} = -j\frac{g}{2}a_1a_1 \exp(-j\Delta kz)$$

where $\Delta k = k_3 - 2k_1$ $\qquad g^2 = 4\hbar\omega^3\eta^3 d^2$

perfect phase matching $\Delta k = 0$

$$\frac{da_1}{dz} = -jga_3a_1^*$$

$$\frac{da_3}{dz} = -j\frac{g}{2}a_1a_1$$

Coupled Equations (Second-Harmonic Generation)

the solution

$$a_1(z) = a_1(0)\sec h\,\frac{g a_1(0) z}{\sqrt{2}}$$

$$a_3(z) = -\frac{j}{\sqrt{2}}\,a_1(0)\tan h\,\frac{g a_1(0) z}{\sqrt{2}}$$

Consequently, the photon flux densities

$$\phi_1(z) = \phi_1(0)\sec h^2\,\frac{\gamma z}{2}$$

$$\phi_3(z) = -\frac{1}{2}\phi_1(0)\tan h^2\,\frac{\gamma z}{2}$$

Efficiency

complex amplitude $$S(2\omega) = 4\mu_o\omega^2 \, dE(\omega)E(\omega)$$

- SHG intensity

$$I(2\omega) \propto |\mathcal{S}(2\omega)|^2 \propto |I(\omega)|^2 \propto d^2 \propto L^2$$

- Efficiency of second-harmonic generation

$$I(\omega) = P/A$$

$$\eta_{\mathrm{SHG}} = I(2\omega)/I(\omega) \propto L^2 I(\omega)$$

The efficiency of second-harmonic generation for an interaction region of length L is

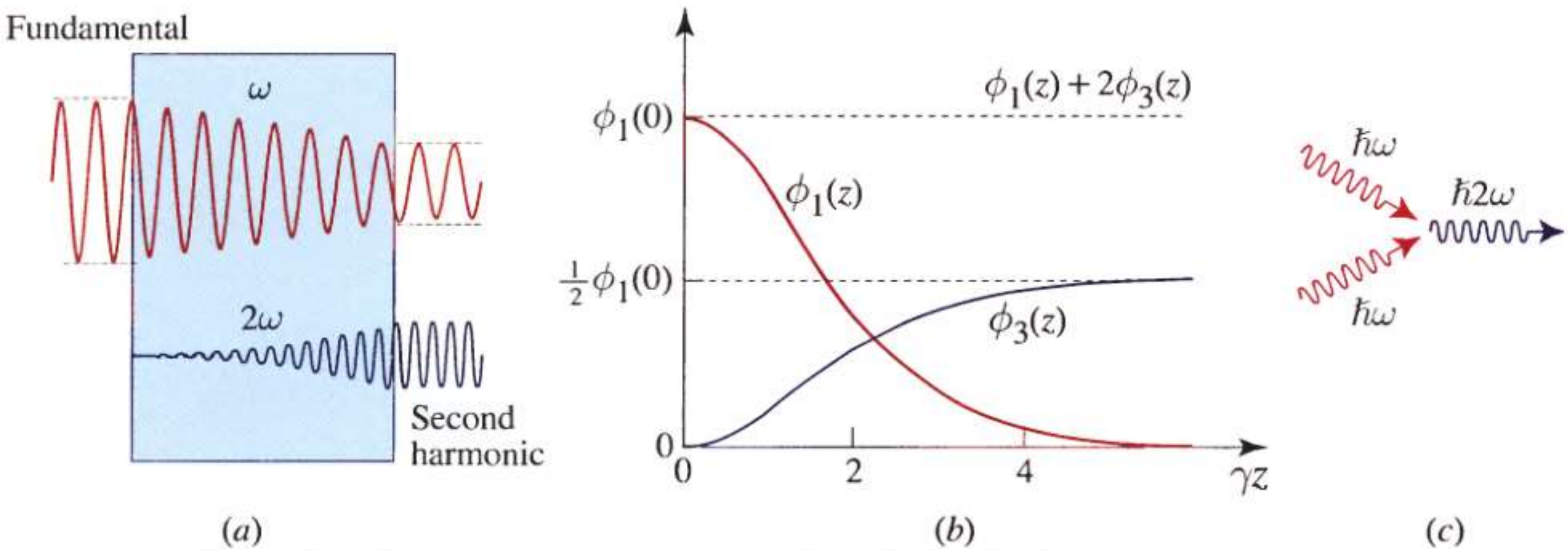
$$\frac{I_3(L)}{I_1(0)} = \frac{\hbar\omega_3\phi_3(L)}{\hbar\omega_1\phi_1(L)} = \frac{2\phi_3(L)}{\phi_1(0)} = \tanh^2\frac{\gamma L}{2}$$

For large γL (long cell, large input intensity, or large nonlinear parameter), the efficiency approaches one. This signifies that all the input power (at frequency w) has been transformed into power at frequency 2w; all input photons of frequency w are converted into half as many photons of frequency 2w.

For small γL (small device length L, small nonlinear parameter d, or small input photon flux density $\phi_1(0)$), the argument of the tanh function is small and therefore the approximation tanhx=x may be used. The efficiency of second-harmonic generation is then

$$\frac{I_3(L)}{I_1(0)} = 2\eta_0^3\omega^2\frac{d^2}{n^3}\frac{L^2}{A}P$$

Second-harmonic generation



- Two photons of frequency w combine to make one photon of frequency 2w.
- As the photon flux density $\phi_1(z)$ of the fundamental wave decreases, the p hoton flux density $\phi_3(z)$ of the second-harmonic wave increases.
- Since photon numbers are conserved, the sum $\phi_1(z)+2\phi_3(z)= \phi_1(0)$ is a const ant.

Effect of Phase Mismatch

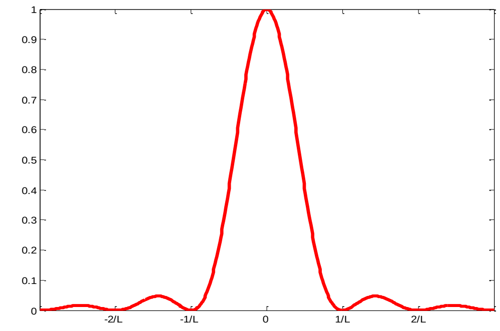
$$\boxed{\begin{aligned}\frac{da_1}{dz} &= -jga_3a_1^*\exp(-j\Delta kz)\\ \frac{da_3}{dz} &= -j\frac{g}{2}a_1a_1\exp(-j\Delta kz)\end{aligned}}\qquad \Delta k \neq 0$$

Solution

$$a_3(L) = -j\frac{g}{2}a_1^2(0)\int_0^L \exp(-j\Delta kz')dz' = -(\frac{g}{2\Delta k})a_1^2(0)[\exp(-j\Delta kL)-1]$$

Efficiency

$$\frac{I_3(L)}{I_1(0)} = \frac{2\phi_3(L)}{\phi_1(0)} = \frac{1}{2}g^2L^2\phi_1(0)\sin c^2\frac{\Delta kL}{2\pi}$$



Phase matching I

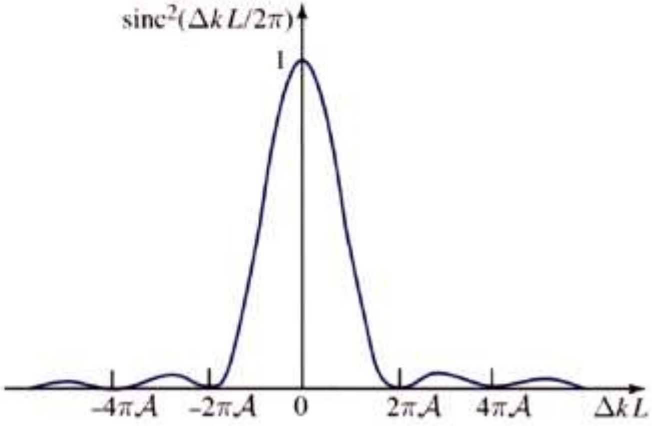
Consider a plane wave propagating along z, and write the paraxial Hemh oltz equation

$$\left[\cancel{\nabla_\perp^2 E(2\omega)} - 2ik\partial_z E(2\omega)\right]e^{-k(2\omega)z} = \mu_0 \partial_t^2 \left(dE^2(\omega)\right)e^{-k(\omega)z}$$

Can be approximated as $$\partial_z E(2\omega) = AdE^2(\omega)e^{\Delta kz}$$

where $\Delta k = k(2\omega) - k(\omega)$ so finally $$E(2\omega) \propto \frac{\sin \Delta kz}{\Delta kz}$$

need Δk=0 to achieve maximum E(2ω) electric field (this was not taken c are in the 1961 experiment)

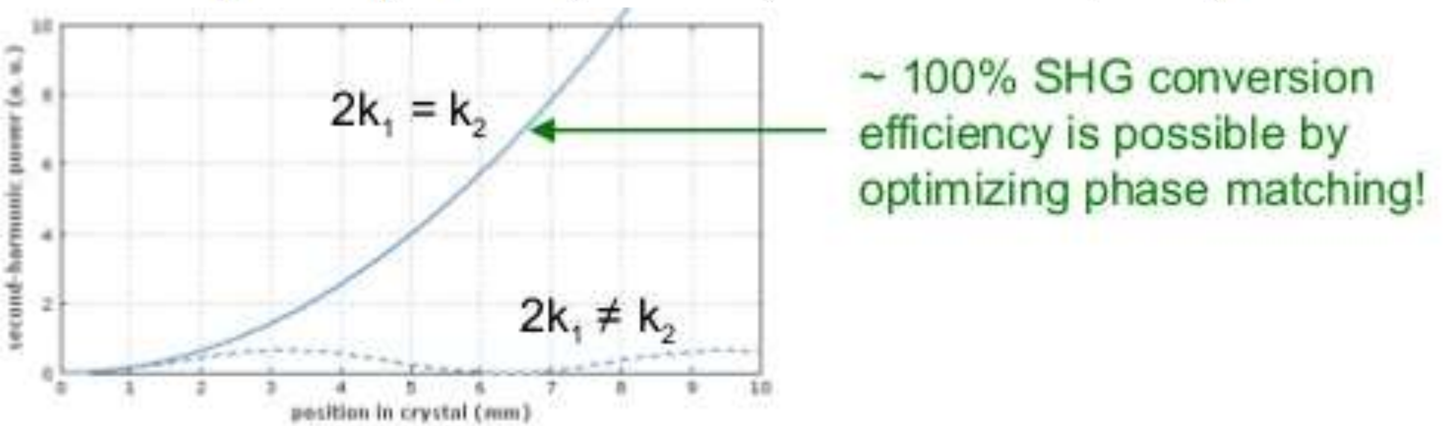


This is referred to as “phase matching”

Quasi-phase matching

- Only when $2k_1 = k_2$, SHG will be efficient.
- Only when phase matching is achieved, these contributions add up con structively, and a high power conversion efficiency is achieved.
- The direction of energy transfer changes periodically according to the c hange in the phase relation between the interacting waves.
- The energy then oscillates between the waves rather than being transf erred in a constant direction.
- The effect on the power conversion is illustrated.

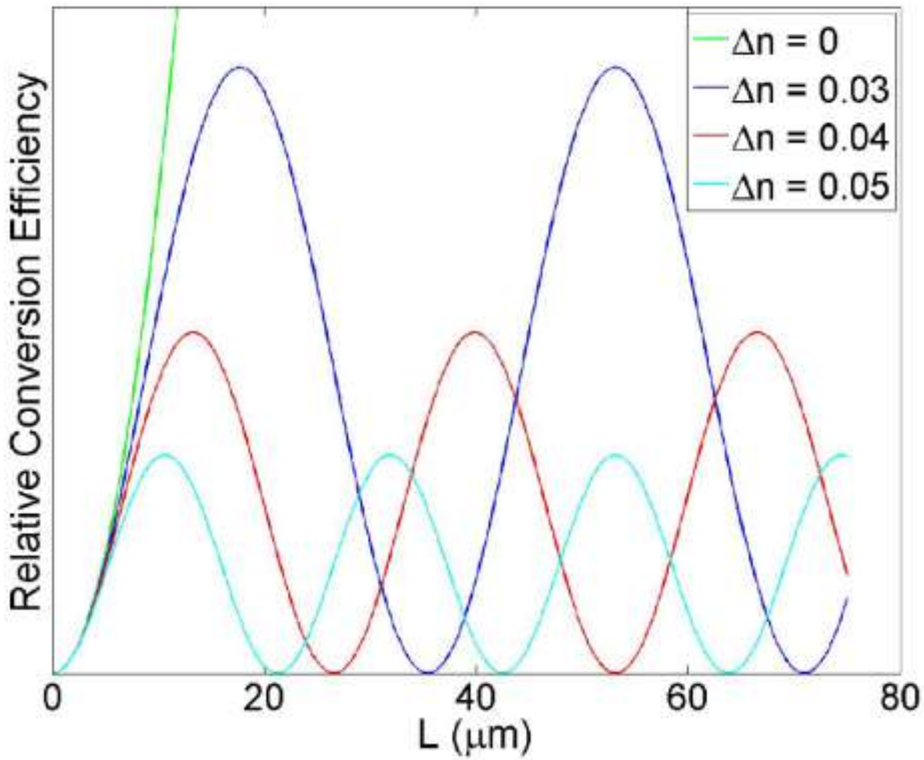
$$\frac{I_{2\omega_1}}{I_{\omega_1}} = \frac{A_2^* A_2}{A_1^* A_1} = \left(\frac{2\omega_1 d_{eff}}{n_2 c \Delta k}\right)^2 |A_1|^2 \sin^2\left(\frac{\Delta k L}{2}\right)$$

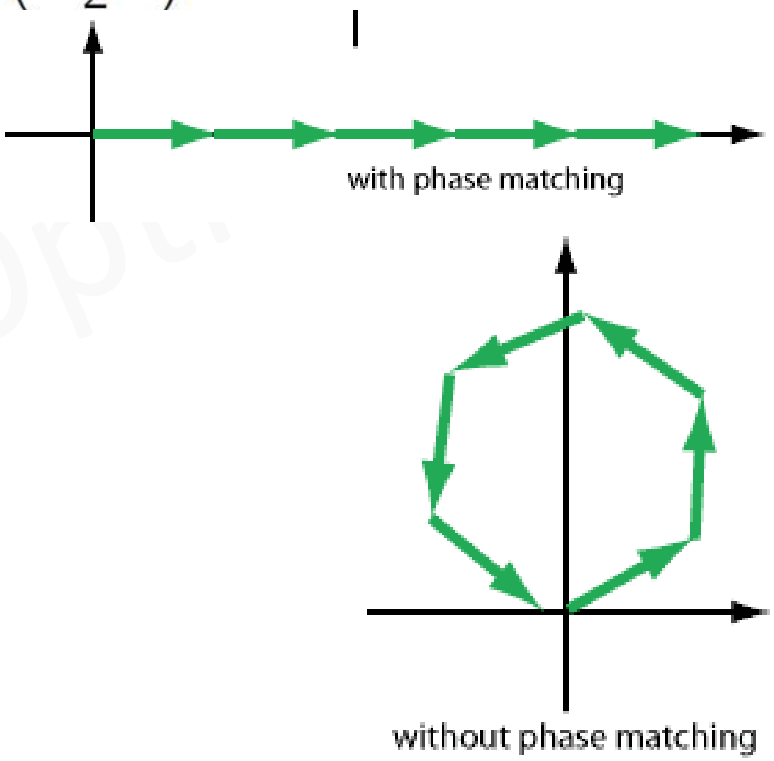


Conversion efficiency

- Wavevector mismatch Δk may cause significant power reduction of gen erated parametric wave.

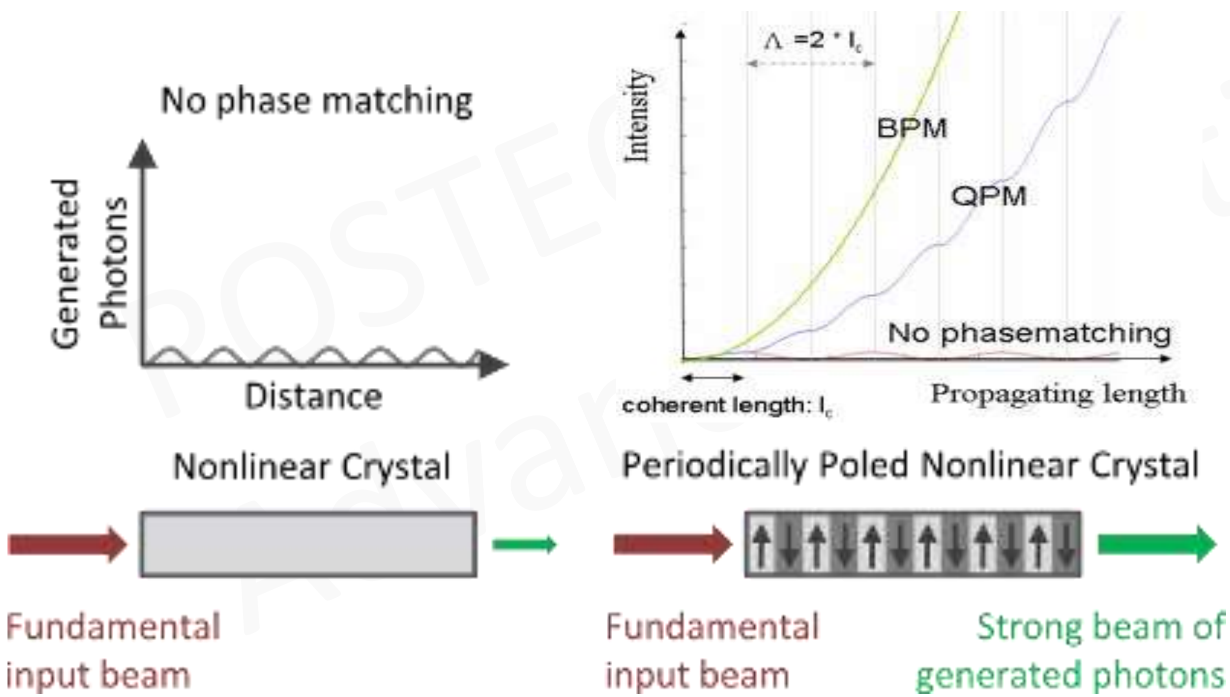
$$\frac{I_{2\omega_1}}{I_{\omega_1}} = \frac{A_2^* A_2}{A_1^* A_1} = \left(\frac{2\omega_1 d_{eff}}{n_2 c \Delta k}\right)^2 |A_1|^2 \sin^2\left(\frac{\Delta k L}{2}\right)$$

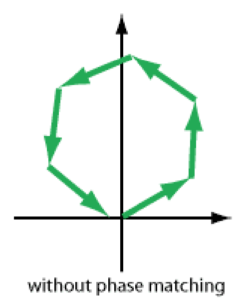


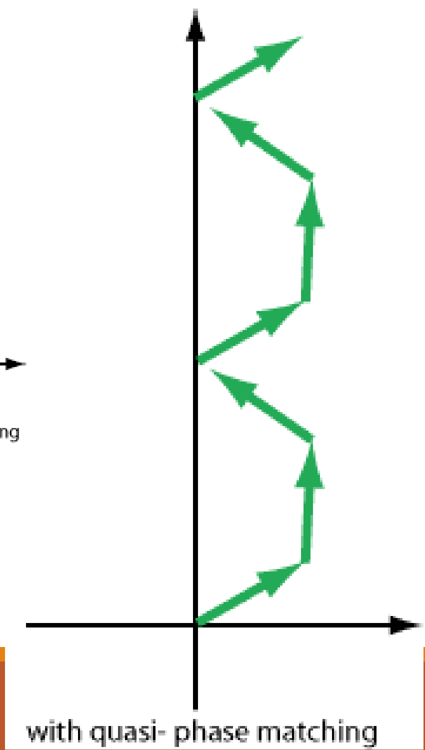


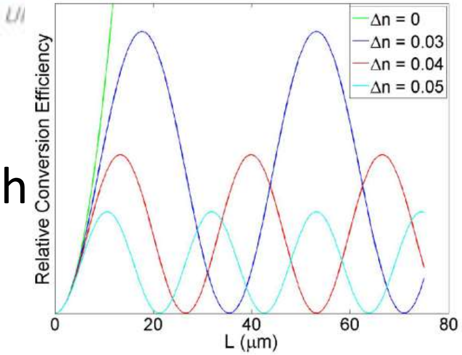
Quasi-phase matching

- Perfect phase matching can be difficult to achieve.
- Alternate way for high efficient generation is **periodic nonlinearity**.
- Such periodicity introduces an opposite phase bringing back the phases of the distributed radiation elements into better alignment.









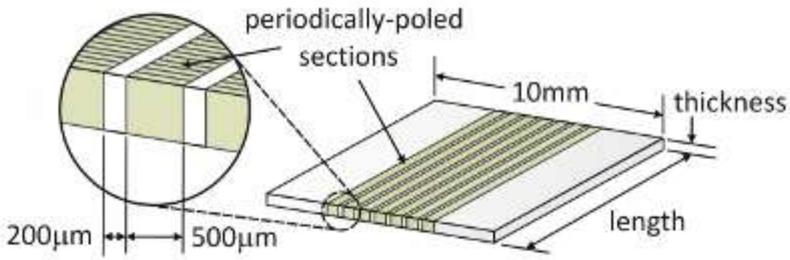
- Consider constructively adding distance, coherence length

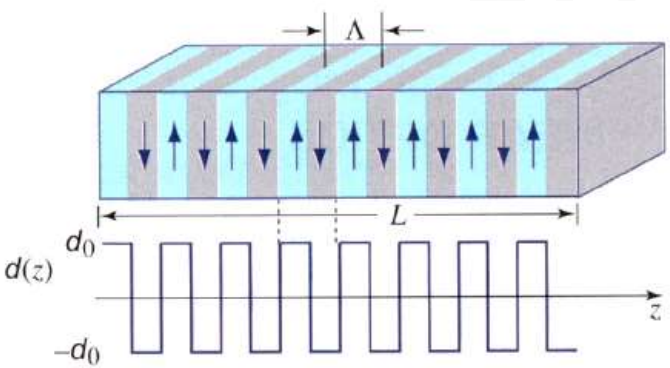
$$\text{the coherence length } l_c = \pi/\Delta k,$$

- It is useful to think of the periodic structure as adding a grating vector of magnitude $2m\pi/\Lambda$ to the interaction.

- The grating allows momentum to be conserved in the nonlinear process.

$$\Delta k - \frac{2\pi m}{\Lambda} = 0 \qquad \Lambda < 10\mu m$$

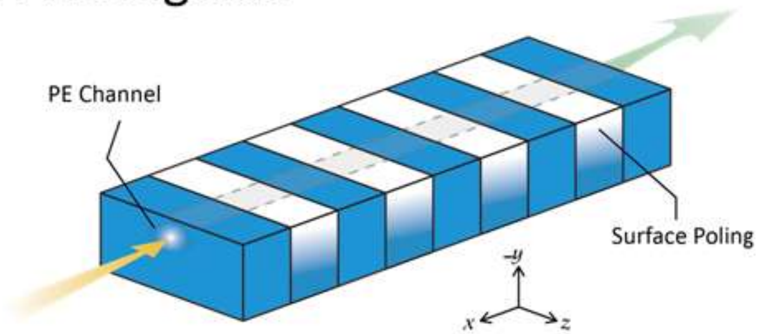


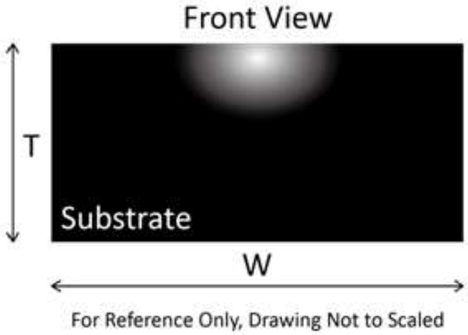


Crystal	Periodically poled $LiNbO_3$
Coercive Field	21 kV/mm
Maximum Poling Depth	1 mm
Maximum Crystal Length	50 mm
d_{eff}	17 pm/V
Transparency Range	0.4 - 5.5μm

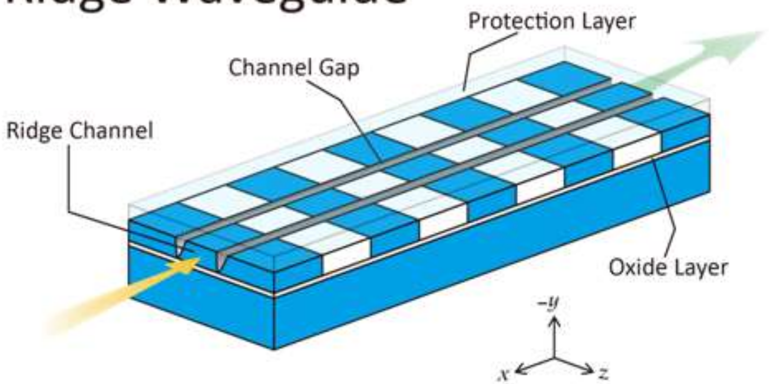
PP crystal waveguides

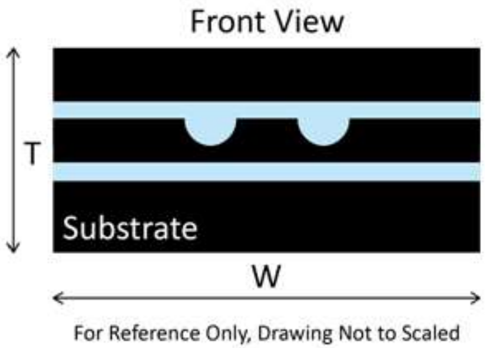
PE Waveguide





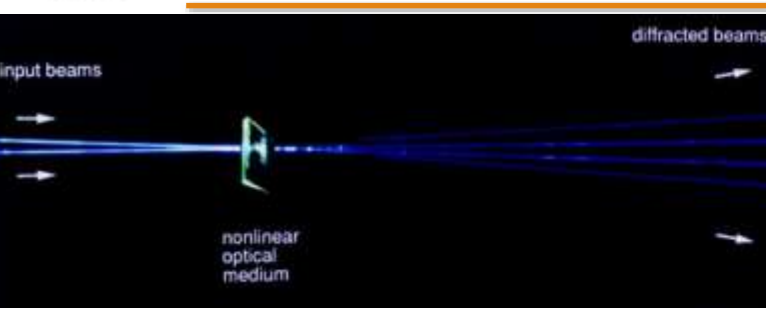
Ridge Waveguide

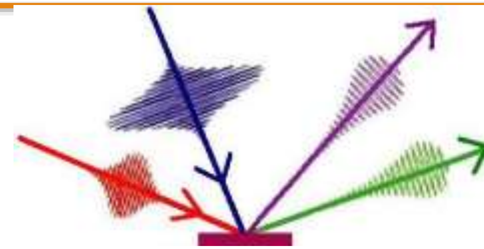


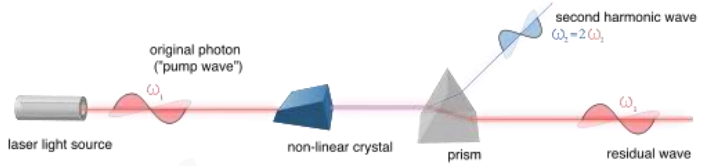


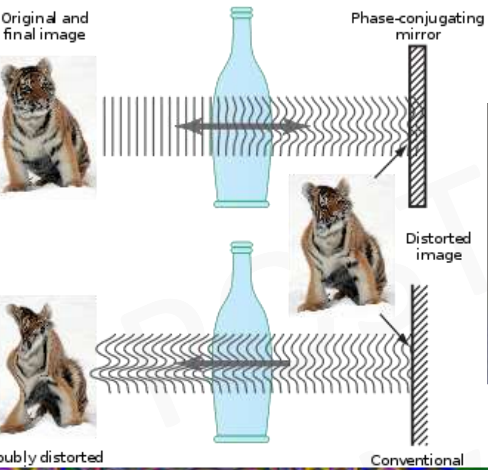
	PE WG	Ridge WG
Fundamental Wavelength for SHG (nm)	750~2100	780 ~ 1580
Power Handling (mW)	~200	>1000

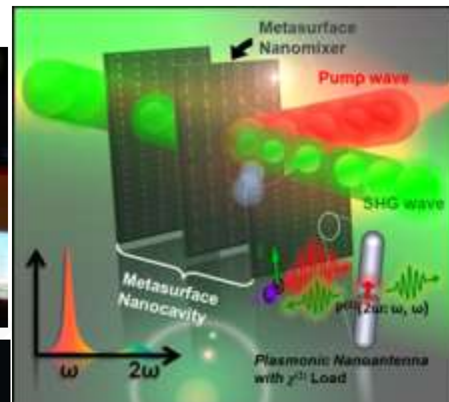
Google images

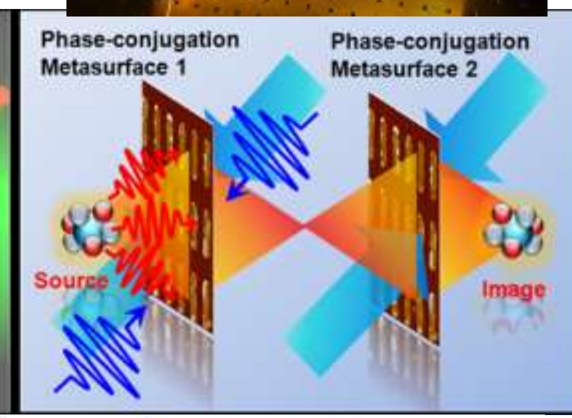


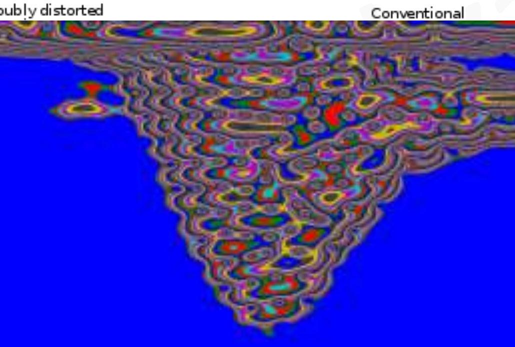


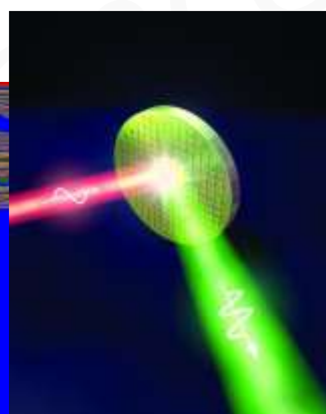


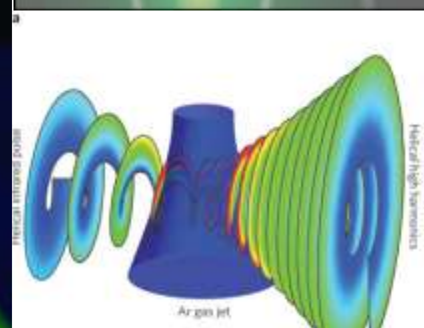


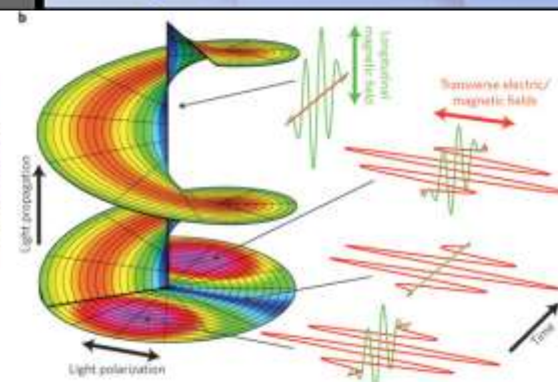












Optical phase conjugation (OPC)

- OPC is a totally degenerated case of the 4-wave mixing and is sometime refer to as degenerate 4-wave mixing:

$$\omega_1 = \omega_2 = \omega_3 = \omega_4 = \omega.$$

- Consider two plane waves with $\mathbf{k}_4 = -\mathbf{k}_3,$

$$E_3(\mathbf{r}) = A_3 \exp(-j\mathbf{k}_3 \cdot \mathbf{r}), \qquad E_4(\mathbf{r}) = A_4 \exp(-j\mathbf{k}_4 \cdot \mathbf{r}),$$

- The corresponding polarization is

$$P_{NL} = 6\chi^{(3)} A_3 A_4 E_1^*(\mathbf{r})$$

- And thus the E-field is

$$E_2 = 6\chi^{(3)} A_3 A_4 E_1^*(\mathbf{r})$$

- The E-field E_2 is conjugate of field E_1.
- The wave E_3 and E_4 are called pumps.