## EECE490U Spring 2017

# Mathematical Methods in Physical Electronics

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## **Topics (tentative)**

- Ch 1. Dirac Delta Function
- Ch 2. Vector Analysis
- Ch 3. Ordinary Differential Equations
- Ch 4. Partial Differential Equations
- Ch 5. Special Functions
- Ch 6. Green's Functions
- Ch 7. Integral Transforms

# **Model Equations**

#### **Maxwell's Equations in Classical Electrodynamics**

$$\nabla \times \mathbf{E}(\mathbf{r},t) = \frac{\partial}{\partial t} \mathbf{B}(\mathbf{r},t) \qquad \text{Faraday's Law}$$

$$\nabla \times \mathbf{H}(\mathbf{r},t) = \mathbf{J}(\mathbf{r},t) + \frac{\partial}{\partial t} \mathbf{D}(\mathbf{r},t) \qquad \text{(Generalized) Ampere's Law}$$

$$\nabla \cdot \mathbf{D}(\mathbf{r},t) = \rho(\mathbf{r},t) \qquad \text{Gauss Law (Poisson's Equation)}$$

$$\nabla \cdot \mathbf{B}(\mathbf{r},t) = 0 \qquad \text{No Magnetic Monopole}$$

$$\mathbf{F}(\mathbf{r},t) = q \big[ \mathbf{E}(\mathbf{r},t) + \mathbf{v} \times \mathbf{E}(\mathbf{r},t) \big] \qquad \text{Lorentz Force}$$

#### **Schödinger Equation in Quantum Mechanics**

$$i\hbar \frac{\partial}{\partial t} \Psi(\mathbf{r},t) = H\Psi(\mathbf{r},t)$$
 Schödinger Equation 
$$H = -\frac{\hbar^2}{2m} \nabla^2 + V(\mathbf{r},t)$$
 Hamiltonian Operator

### The Importance of Vector Calculus

# Original Maxwell's Equations (20 Scalar Equations)

$$e + \frac{df}{dx} + \frac{dg}{dy} + \frac{dh}{dz} = 0 \qquad (1) \quad \text{Gauss' Law}$$

$$\mu\alpha = \frac{dH}{dy} - \frac{dG}{dz}$$

$$\mu\beta = \frac{dF}{dz} - \frac{dH}{dx} \qquad (2) \quad \text{Equivalent to Gauss' Law for magnetism}$$

$$P = \mu \left( \gamma \frac{dy}{dt} - \beta \frac{dz}{dt} \right) - \frac{dF}{dt} - \frac{d\Psi}{dz}$$

$$Q = \mu \left( \alpha \frac{dz}{dt} - \gamma \frac{dx}{dt} \right) - \frac{dG}{dt} - \frac{d\Psi}{dy} \qquad (3) \quad \text{(with the Lorentz Force and Poisson's Law)}$$

$$R = \mu \left( \beta \frac{dx}{dt} - \alpha \frac{dy}{dt} \right) - \frac{dH}{dt} - \frac{d\Psi}{dz}$$

$$\frac{d\gamma}{dy} - \frac{d\beta}{dz} = 4\pi p' \qquad p' = p + \frac{df}{dt}$$

$$\frac{d\alpha}{dz} - \frac{d\gamma}{dx} = 4\pi p' \qquad q' = q + \frac{dg}{dt} \qquad (4) \quad \text{Ampère-Maxwell Law}$$

$$\frac{d\beta}{dx} - \frac{d\alpha}{dy} = 4\pi p' \qquad r' = r + \frac{dh}{dt}$$

$$P = -\xi p \quad Q = -\xi q \quad R = -\delta \qquad \text{Ohm's Law}$$

$$P = kf \quad Q = kg \quad R = kh \qquad \text{The electric elasticity equation } (\mathbf{E} = \mathbf{D}/\varepsilon)$$

$$\frac{de}{dt} + \frac{dp}{dx} + \frac{dq}{dy} + \frac{dq}{dz} = 0 \qquad \text{Continuity of charge}$$

# Heavyside's Maxwell's Equations (4 Vector Equations)

$$\nabla \times \mathbf{E}(\mathbf{r}, t) = \frac{\partial}{\partial t} \mathbf{B}(\mathbf{r}, t)$$

$$\nabla \times \mathbf{H}(\mathbf{r}, t) = \mathbf{J}(\mathbf{r}, t) + \frac{\partial}{\partial t} \mathbf{D}(\mathbf{r}, t)$$

$$\nabla \cdot \mathbf{D}(\mathbf{r}, t) = \rho(\mathbf{r}, t)$$

$$\nabla \cdot \mathbf{B}(\mathbf{r}, t) = 0$$

Oliver Heaviside has coined many important terms currently used for electromagnetic material parameters: admittance, conductance, impedance, etc.