Spring 2019

EECE 588
Lecture 7/8

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Linear Wire Antennas

- Linear or curved wire antennas are the most basic types of antennas that have been around for a long time.
- Even though these are some of the oldest types of antennas, they are in may cases the most effective types of antennas.

Infinitesimal Dipole Antennas

- An infinitesimal dipole antenna is an antenna whose length is much smaller than the operating wavelength.
  - i.e., $l << \lambda$.
- Let us assume that the dipole antenna is placed at the center of the coordinate system and is along the z axis.
- The length and the diameter of the wire of the dipole antenna are both smaller than a wavelength, i.e. $l << \lambda$ and $a << \lambda$.
- At first, we assume that the current along the dipole antenna is constant, i.e.,

$$\vec{I}(z') = \hat{z} I_0$$

POSTECH

Radiated Fields of the Infinitesimal Dipole Antennas

- To find the radiated fields of this antenna we will use the two step procedure.
- We will determine $\vec{A}$ and $\vec{F}$ and then $\vec{E}$ and $\vec{H}$.

$$\vec{A}(x,y,z) = \frac{\mu}{4\pi}\iiint_V \vec{I}_e(x',y',z')\frac{e^{-jkR}}{R}dv'$$

$$\vec{A}(x,y,z) = \frac{\mu}{4\pi}\int_C \vec{I}_e(x',y',z')\frac{e^{-jkR}}{R}dz'$$

$$I_e(x',y',z') = \hat{z}I_0 \qquad\qquad R = \left|r - r'\right|$$

POSTECH

Radiated Fields of the Infinitesimal Dipole Antennas

- In this case, we have

$$\vec{r} = x\hat{x} + y\hat{y} + z\hat{z} \qquad\qquad \vec{r} = x'\hat{x} + y'\hat{y} + z'\hat{z}$$

$$R = \sqrt{(x - x')^2 + (y - y')^2 + (z - z')^2}$$

- Since the antenna is infinitesimal, we have $x' = y' = z' = 0$. Therefore:

$$R = \sqrt{x^2 + y^2 + z^2} = r = \text{constant}$$

Radiated Fields of the Infinitesimal Dipole Antennas

- This way, we can obtain an expression for the vector magnetic potential.

$$\vec{A}(x,y,z) = \hat{z}\,\frac{\mu I_0}{4\pi r} e^{-jkr} \int_{-l/2}^{l/2} dz' = \hat{z}\,\frac{\mu I_0\, l}{4\pi r} e^{-jkr}$$

- Since in this case, we do not have $\vec{M}$, we can just proceed to calculate $\vec{E}$ and $\vec{H}$ from the above expression obtained for $\vec{A}$.
- Before doing that let us convert $\vec{A}$ from rectangular to spherical coordinate system:

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\varphi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\varphi & \sin\theta\sin\varphi & \cos\theta \\ \cos\theta\cos\varphi & \cos\theta\sin\varphi & -\sin\theta \\ -\sin\varphi & \cos\varphi & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

Radiated Fields of the Infinitesimal Dipole Antennas

- Therefore, we have:

$$A_r = A_z \cos\theta = \frac{\mu I_0 l e^{-jkr}}{4\pi r}\cos\theta$$

$$A_\theta = -A_z \sin\theta = -\frac{\mu I_0 l e^{-jkr}}{4\pi r}\sin\theta$$

$$A_\varphi = 0$$

- The magnetic field can then be easily calculated from:

$$\vec{H} = \frac{1}{\mu}\nabla\times\vec{A} \qquad \vec{H} = \hat{\varphi}\frac{1}{\mu r}\left\{\frac{\partial}{\partial r}\left(rA_\theta\right) - \frac{\partial A_r}{\partial\theta}\right\}$$

POSTECH

Radiated Fields of the Infinitesimal Dipole Antennas

- Therefore, we can obtain:

$$\vec{H} = \hat{\varphi} \frac{jkI_0 l \sin\theta}{4\pi r}\left[1 + \frac{1}{jkr}\right]e^{-jkr}$$

$$\vec{E} = \vec{E}_A = -j\omega\vec{A} - j\frac{1}{\omega\mu\varepsilon}\nabla(\nabla.\vec{A}) = \frac{1}{j\omega\varepsilon}\nabla\times\vec{H}$$

$$E_r = \eta\frac{I_0 l \cos\theta}{2\pi r^2}\left\{1 + \frac{1}{jkr}\right\}e^{-jkr}$$

$$E_\varphi = 0$$

$$E_\theta = j\eta\frac{kI_0 l \sin\theta}{4\pi r}\left\{1 + \frac{1}{jkr} - \frac{1}{(kr)^2}\right\}e^{-jkr}$$

Power Density and Radiation Resistance

- The complex Poynting vector can be calculated from:

$$\vec{W} = \frac{1}{2}\left(\vec{E} \times \vec{H}^*\right) = \frac{1}{2}\left(\hat{r}E_r + \hat{\theta}E_\theta\right) \times \left(\hat{\varphi}H_\varphi^*\right) = \frac{1}{2}\left(\hat{r}E_\theta H^*_\varphi - \hat{\theta}E_r H_\varphi^*\right)$$

- The various components of $\vec{W}$ are found from:

$$W_r = \frac{\eta}{8}\left|\frac{I_0 l}{\lambda}\right|^2 \frac{\sin^2\theta}{r^2}\left\{1 - \frac{j}{(kr)^3}\right\}$$

$$W_\theta = j\eta \frac{k|I_0 l|^2 \cos\theta \sin\theta}{16\pi^2 r^3}\left[1 + \frac{1}{(kr)^2}\right]$$

Power Density and Radiation Resistance

- The complex power moving in the radial direction is obtained using:

$$P = \oiint_S \vec{W} . d\vec{s} = \int_0^{2\pi}\int_0^{\pi} \left(\hat{r}W_r + \hat{\theta}W_\theta\right) . \hat{r} r^2 \sin\theta \; d\theta \; d\varphi$$

$$P = \int_0^{2\pi}\int_0^{\pi} W_r r^2 \sin\theta \; d\theta \; d\varphi = \eta \frac{\pi}{3}\left|\frac{I_0 l}{\lambda}\right|^2 \left[1 - \frac{j}{(kr)^3}\right]$$

- Note that the W_θ component does not contribute to this integral.

Power Density and Radiation Resistance

- We also can write this equation as:

$$P = \frac{1}{2}\oiint_S \left(\vec{E} \times \vec{H}^*\right) . d\vec{s} = \eta\left(\frac{\pi}{3}\right)\left|\frac{I_0 l}{\lambda}\right|^2 \left[1 - j\frac{1}{(kr)^3}\right]$$

$$= P_{rad} + j2\omega\left(\tilde{W}_m - \tilde{W}_e\right)$$

- Note that

P = Power in radial direction.

P_{rad} = Time averaged power radiated.

$\tilde{W}_m$ = Time averaged magnetic energy density (in radial direction).

$\tilde{W}_e$ = Time averaged electric energy density (in radial direction).

$2\omega\left(\tilde{W}_m - \tilde{W}_e\right)$ = Time - average imaginary (reactive) power (in radial direction).

POSTECH

Power Density and Radiation Resistance

- Therefore, the total real radiated power is:

$$P_{rad} = \eta\left(\frac{\pi}{3}\right)\left|\frac{I_0 l}{\lambda}\right|^2$$

- and, we have:

$$2\omega\left(\tilde{W}_m - \tilde{W}_e\right) = -\eta\left(\frac{\pi}{3}\right)\left|\frac{I_0 l}{\lambda}\right|^2 \frac{1}{(kr)^3}$$

- Note that radial electric energy is larger than the radial magnetic energy.
- The imaginary power is r dependent and approaches zero as r approaches infinity.

POSTECH

Power Density and Radiation Resistance

- The radiation resistance of the antenna can then be found (from its definition) as:

$$P_{rad} = \eta\left(\frac{\pi}{3}\right)\left|\frac{I_0 l}{\lambda}\right|^2 = \frac{1}{2}\,|\,I_0\,|^2\;R_r$$

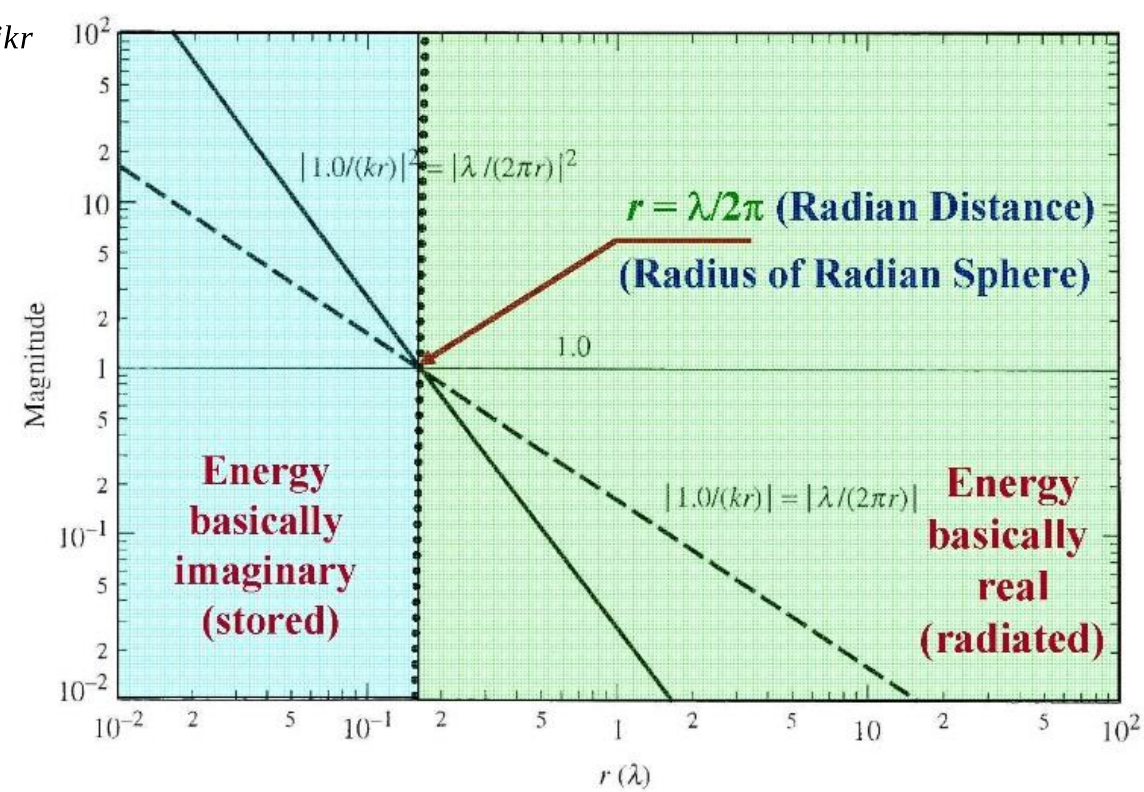
$$R_r = \eta\left(\frac{2\pi}{3}\right)\left(\frac{l}{\lambda}\right)^2 = 80\pi^2\left(\frac{l}{\lambda}\right)^2$$

Radian Distance and Radian Sphere

$$E_\theta = j\eta \frac{kI_0 l \sin\theta}{4\pi r}\left\{1+\frac{1}{jkr}-\frac{1}{(kr)^2}\right\}e^{-jkr} \qquad E_r = \eta \frac{I_0 l \cos\theta}{2\pi r^2}\left\{1+\frac{1}{jkr}\right\}e^{-jkr}$$

$$\vec{H} = \hat{\varphi}\frac{jkI_0 l \sin\theta}{4\pi r}\left[1+\frac{1}{jkr}\right]e^{-jkr}$$

- $r = \lambda/2\pi$ or $kr = 1$ is referred to as the radian sphere.
- Note that for $kr << 1$ the second and third terms dominate.
- At $kr >> 1$ the power is real.



Near Field Region

- For very small distances, i.e., $kr << 1$ →
- Note that for $kr << 1$, the terms with higher powers of r in the denominator dominate.
- Note that the $\vec{E}$ and $\vec{H}$ are 90° out of phase.
- Therefore, no real power flows in this region.

$$\left.\begin{aligned}
E_r &= -j\eta \frac{I_0 l e^{-jkr}}{2\pi k r^3}\cos\theta \\
E_\theta &\approx -j\eta \frac{I_0 l e^{-jkr}\sin\theta}{4\pi k r^3} \\
E_\varphi &= H_\theta = H_r = 0 \\
H_\varphi &= \frac{I_0 l e^{-jkr}}{4\pi r^2}\sin\theta
\end{aligned}\right\} kr << 1$$

$$\vec{W}_{av} = \frac{1}{2}\mathrm{Re}\left[\vec{E}\times\vec{H}^*\right] = 0$$

POSTECH

Intermediate Field Range ($kr > 1$)

- In the intermediate range $kr > 1$ the E_θ and H_φ start to become in phase and hence carry real average outward power.

$$E_r = \eta \frac{I_0 l \cos\theta}{2\pi r^2}\left\{1 + \frac{1}{jkr}\right\}e^{-jkr}$$

$$\vec{H} = \hat{\varphi}\frac{jkI_0 l \sin\theta}{4\pi r}\left[1 + \frac{1}{jkr}\right]e^{-jkr}$$

$$E_\theta = j\eta \frac{kI_0 l \sin\theta}{4\pi r}\left\{1 + \frac{1}{jkr} - \frac{1}{(kr)^2}\right\}e^{-jkr}$$

$$\left.\begin{aligned} E_r &\approx \eta \frac{I_0 l e^{-jkr}}{2\pi r^2}\cos\theta \\ E_\theta &\approx j\eta \frac{kI_0 l e^{-jkr}}{4\pi r}\sin\theta \\ E_\varphi &= H_\theta = H_r = 0 \\ H_\varphi &= j\frac{kI_0 l e^{-jkr}}{4\pi r}\sin\theta \end{aligned}\right\} kr > 1$$

In this region, we have power propagation but E_r's magnitude is still not very small

POSTECH

Far Field Region ($kr >> 1$)

- In this range, $kr >> 1$, $E_r = 0$.
- The ratio of E_θ to H_φ is the wave impedance.

$$Z_w = \frac{E_\theta}{H_\varphi} \approx \eta$$

$$\eta = 377\Omega$$

$$\left.\begin{aligned} E_\theta &\approx j\eta \frac{kI_0 l e^{-jkr}}{4\pi r}\sin\theta \\ E_r &\approx E_\varphi = H_\theta = H_r = 0 \\ H_\varphi &\approx j\frac{kI_0 l e^{-jkr}}{4\pi r}\sin\theta \end{aligned}\right\} kr >> 1$$

- η is the free space impedance.

- $\vec{E}$ and $\vec{H}$ components are normal to one another and to the propagation direction.
- The r variations are separable from those of θ and φ components.

POSTECH

Far Field Region

- Note, the shape of the radiation pattern is not a function of r.
- The radiated fields form a Transverse Electromagnetic (TEM) field.
- The impedance of this TEM field is the same as the intrinsic impedance of the medium in which the TEM wave is propagating, i.e. free space.
- As we mentioned before and as we will see again as well, this relationship is applicable in the far field region of all antennas of finite dimensions.

Directivity

$$\vec{W}_{av} = \frac{1}{2}\mathrm{Re}\left[\vec{E} \times \vec{H}^*\right] = \hat{r}\frac{\eta}{2}\left|\frac{kI_0 l}{4\pi}\right|\frac{\sin^2\theta}{r^2}$$

$$P_{rad} = \oiint_S \vec{W}_{av}.d\vec{s} = \eta\left(\frac{\pi}{3}\right)\left|\frac{I_0 l}{\lambda}\right|^2$$

$$U = r^2 W_{av} = \frac{\eta}{2}\left|\frac{kI_0 l}{4\pi}\right|^2 \sin^2\theta = \frac{r^2}{\eta}\,|E_\theta(r,\theta,\varphi)|^2$$

$$U_{\max} = \frac{\eta}{2}\left|\frac{kI_0 l}{4\pi}\right|^2 \quad \Rightarrow \quad D = 4\pi\frac{U_{\max}}{P_{rad}} = \frac{3}{2} = 1.75\,\mathrm{dBi}$$

Directivity

- The maximum effective area of the antenna can be calculated:

$$A_{em} = \frac{\lambda^2}{4\pi} D_0 = \frac{3\lambda^2}{8\pi}$$

POSTECH

Summary

Procedure for Determining Antenna Radiation Characteristics in Far Field

1. Specify electric and/or magnetic current densities [physical or equivalent (*see Chapter* 3, *Figure* 3.1)].
2. Determine vector potential components A_θ, A_ϕ and/or F_θ, F_ϕ using (3-46)-(3-54) in far field.
3. Find far-zone $\underline{E}$ and $\underline{H}$ radiated fields ($E_\theta, E_\phi, H_\theta, H_\phi$) using (3-58a)-(3-58b).

$$Summary\ of\ Antenna\ Characteristics$$

$$In\ Far-Field\ (Cont'd)$$

4. Form either:

$$a.\ \underline{\mathrm{W}}_{rad}(r,\theta,\phi)=\underline{W}_{av}(r,\theta,\phi)=\frac{1}{2}\mathrm{Re}\left[\underline{E}\times\underline{H}^*\right]$$

$$\simeq\frac{1}{2}\mathrm{Re}\left[\left(\hat{a}_\theta E_\theta+\hat{a}_\phi E_\phi\right)\times\left(\hat{a}_\theta H_\theta^*+\hat{a}_\phi H_\phi^*\right)\right]$$

$$\underline{\mathrm{W}}_{rad}(r,\theta,\phi)=\hat{a}_r\frac{1}{2}\left[\frac{\left|E_\theta\right|^2+\left|E_\phi\right|^2}{\eta}\right]=\hat{a}_r\frac{1}{r^2}\left|f(\theta,\phi)\right|^2$$

or

$$b.\ U(\theta,\phi)=r^2W_{rad}(r,\theta,\phi)=\left|f(\theta,\phi)\right|^2$$

Summary of Antenna Characteristics In Far-Field (Cont'd)

5. Determine either:

$$a.\ P_{rad} = \int_0^{2\pi}\int_0^{\pi} W_{rad}(r,\theta,\phi)\ r^2 \sin\theta d\theta d\phi$$

or

$$b.\ P_{rad} = \int_0^{2\pi}\int_0^{\pi} U(\theta,\phi)\sin\theta d\theta d\phi$$

6. Find Directivity using:

$$D(\theta,\phi) = \frac{U(\theta,\phi)}{U_o} = \frac{4\pi U(\theta,\phi)}{P_{rad}}$$

$$D_o = D_{\max} = D(\theta,\phi)|_{\max} = \frac{U(\theta,\phi)|_{\max}}{U_o} = \frac{4\pi U(\theta,\phi)|_{\max}}{P_{rad}}$$

Summary of Antenna Characteristics In Far – Field (Cont'd)

7. Form Normalized Power Amplitude Pattern:

$$U_n(\theta,\phi) = \frac{U(\theta,\phi)}{U_{\max}}$$

8. Determine Radiation and Input Resistance:

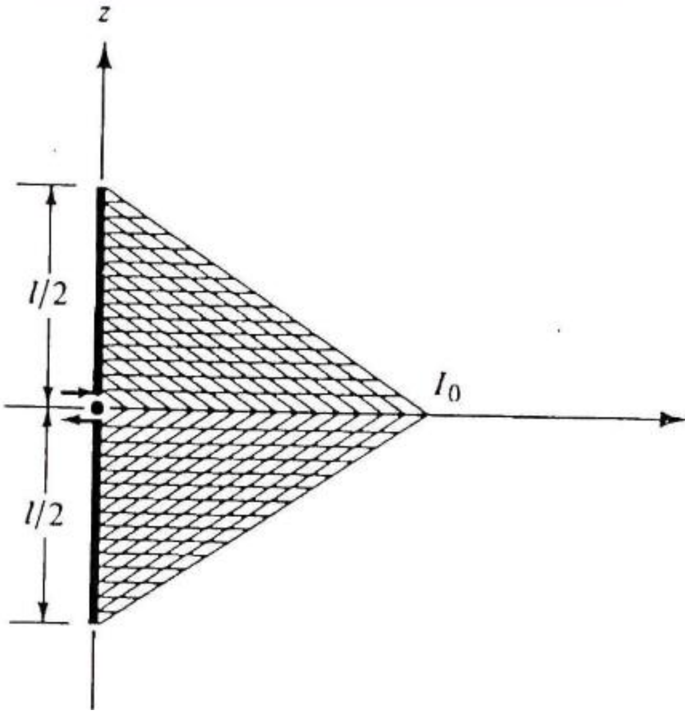
$$R_r = \frac{2P_{rad}}{|I_o|^2}; \qquad R_{in} = \frac{R_r}{\sin^2\left(\frac{k\ell}{2}\right)}$$

9. Determine Maximum Effective Area.

$$A_{em} = \frac{\lambda^2}{4\pi} D_o$$

Small Dipole

- The electric current distribution on an electrically small antenna is not uniform (unless a top hat loaded dipole is used).
- A better approximation for the current distribution on electrically small antennas is a triangular distribution.
- Note that the current at the open end of the dipole antenna must be zero hence the triangular distribution.

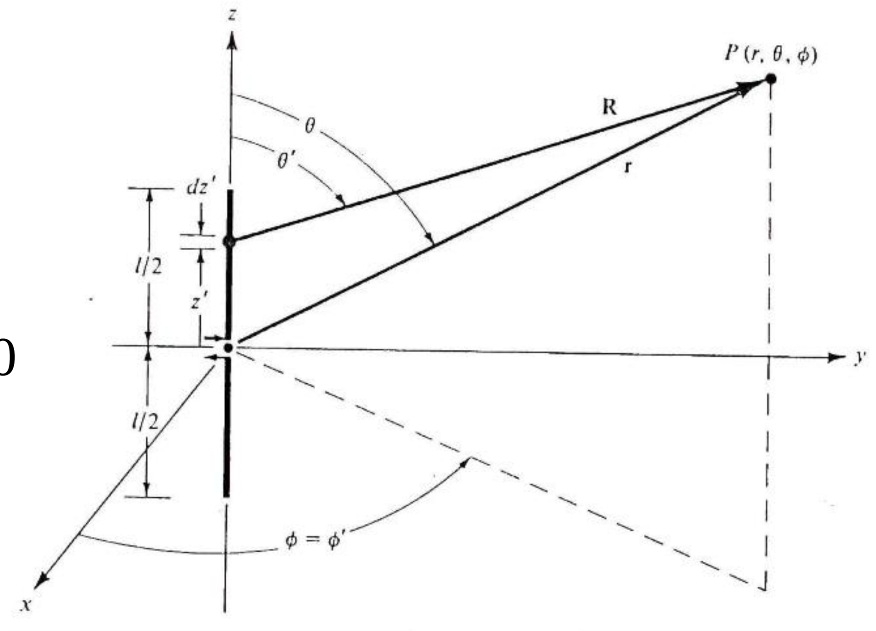


POSTECH

Small Dipole

- This distribution can be expressed mathematically as:

$$\vec{I}_e(x',y',z') = \begin{cases} \hat{z} I_0 (1 - \frac{2}{l} z'), & 0 \le z' \le l/2 \\ \hat{z} I_0 (1 + \frac{2}{l} z'), & -l/2 \le z' \le 0 \end{cases}$$



- Using this current distribution, the vector magnetic potential can be obtained as:

$$\vec{A}(x,y,z) = \frac{\mu}{4\pi}\left[\hat{z} \int_{-l/2}^{0} I_0 \left(1 + \frac{2}{l} z'\right) \frac{e^{-jkR}}{R} dz' + \hat{z} \int_{0}^{l/2} I_0 \left(1 - \frac{2}{l} z'\right) \frac{e^{-jkR}}{R} dz' \right]$$

Small Dipole

- Since the length of the antenna is very small, there is little difference between R and r:

$$R = \left|\vec{r} - \vec{r}'\right| = \left|\vec{r} - \hat{z}z'\right| \approx r$$

- Using this approximation, $\vec{A}$ may be calculated as:

$$\vec{A} = \hat{z}A_z = \hat{z}\frac{1}{2}\left[\frac{\mu I_0 l e^{-jkr}}{4\pi r}\right] \Rightarrow \left.\begin{aligned} E_\theta &\approx j\eta\frac{kI_0 l e^{-jkr}}{8\pi r}\sin\theta \\ E_r &\approx E_\varphi = H_r = H_\theta = 0 \\ H_\varphi &\approx j\frac{kI_0 l e^{-jkr}}{8\pi r}\sin\theta \end{aligned}\right\} kr >> 1$$

POSTECH

Small Dipole

- Parameters of this antenna can be calculated similar to the infinitesimal dipole. In particular, the radiation resistance is:

$$R_r = \frac{2P_{rad}}{|I_0|^2} = 20\pi^2 \left(\frac{l}{\lambda} \right)^2$$

- Note that the radiation pattern of this antenna and that with uniform current distribution are the same.
- This means that the directivity of these are also same.

POSTECH

Region Separation

- Our main problem in calculating closed form expressions that are valid everywhere for any practical antenna rises from the inability to perform the following integration:

$$\vec{A}(x,y,z) = \frac{\mu}{4\pi}\int_C \vec{I}\left(x',y',z'\right)\frac{e^{-jkR}}{R}dl'$$

$$R = \sqrt{\left(x-x'\right)^2 + \left(y-y'\right)^2 + (z-z')^2}$$

- If a thin dipole antenna of finite length of l is symmetrically positioned about the z axis, we will have:

$$x' = y' = 0 \qquad\qquad R = \sqrt{x^2 + y^2 + (z-z')^2}$$

Thin Dipole

POSTECH

Region Separation

- This expression can be expanded as:

$$R=\sqrt{\left(x^2+y^2+z^2\right)+\left(z'^2-2zz'\right)}=\sqrt{r^2+\left(z'^2-2rz'\cos\theta\right)}$$

- Use binomial expansion to expand R as:

$$R=r-z'\cos\theta+\frac{1}{r}\left(\frac{z'^2}{2}\sin^2\theta\right)+\frac{1}{r^2}\left(\frac{z'^3}{2}\cos\theta\sin^2\theta\right)+\dots.$$

$$\boxed{(1+x)^{1/2}=1+\frac{x}{2}-\frac{x^2}{8}+\frac{x^3}{16}-\dots.}$$

Binomial Expansion

POSTECH

Far Field (Fraunhofer) Region

- The most convenient simplification that we can make to this expression other than $R \approx r$ is to keep the first two terms of the Binomial expansion:

$$R \approx r - z' \cos\theta$$

- If we take this approximation, the most significant neglected term is the third term whose maximum value is:

$$\frac{1}{r}\left(\frac{z'^2}{2}\sin^2\theta\right)_{\max} = \frac{z'^2}{2r} \quad \text{when } \theta = \frac{\pi}{2}$$

POSTECH

Far Field (Fraunhofer) Region

- Generally a phase error of 22.5° or $\pi/8$ is not very detrimental in the analytical formulation.

$$\frac{k(z')^2}{2r} \le \frac{\pi}{8}$$

- For $-l/2 \le z' \le l/2$ the expression becomes

$$r \ge 2\left(\frac{l^2}{\lambda}\right)$$

- Basically, to maintain the maximum phase error of an antenna equal to or less than 22.5°, the observation distance r must equal or be greater than $2l^2/\lambda$, where l is the largest dimensions of the antenna.

Far Field Region

- The usual simplification that we use for fall field calculations is:

$$R \approx r - z' \cos\theta \qquad \text{for phase terms}$$

$$R \approx r \qquad \text{for amplitude terms}$$

$$A_z = \frac{\mu}{4\pi} \int_{-l/2}^{l/2} I_e(z') \frac{e^{-jkR}}{R} dz' \qquad A_z \approx \frac{\mu}{4\pi} \int_{-l/2}^{l/2} I_e(z') \frac{e^{-jk(r - z'\cos\theta)}}{r} dz'$$

$$A_z \approx \frac{\mu e^{-jkr}}{4\pi r} \int_{-l/2}^{l/2} I_e(z') e^{+jkz'\cos\theta} dz'$$

POSTECH

Radiating Near-Field (Fresnel) Region

- If the observation distance is shorter than $2D^2/\lambda$, the phase difference will be larger than 22.5°, which may not be acceptable in certain instances.
- Therefore, we need to retain more terms in the binomial expansion. Let's start with the third term:

$$R \approx r - z'\cos\theta + \frac{2}{r}\left(\frac{z'^2}{2}\sin^2\theta\right)$$

- To find the distance at which the maximum phase error is 22.5°, we need to find the maximum value of the fourth term. Then follow a similar situation as before. The result is (see the treatment in pp. 169 of text):

$$r \geq 0.62\sqrt{\frac{l^3}{\lambda}}$$

Reactive Near Field

- If the observation distance is shorter than the boundary of the fresnel region, we are in the reactive near field:

$$r < 0.62\sqrt{\frac{l^3}{\lambda}}$$

Finite Length Dipole with Sinusoidal Current Distribution

- The procedure we developed in the previous slides can be used to analyze any linear wire antenna provided that its current distribution is already known.
- Now, let's consider a dipole antenna with sinusoidal current distribution:

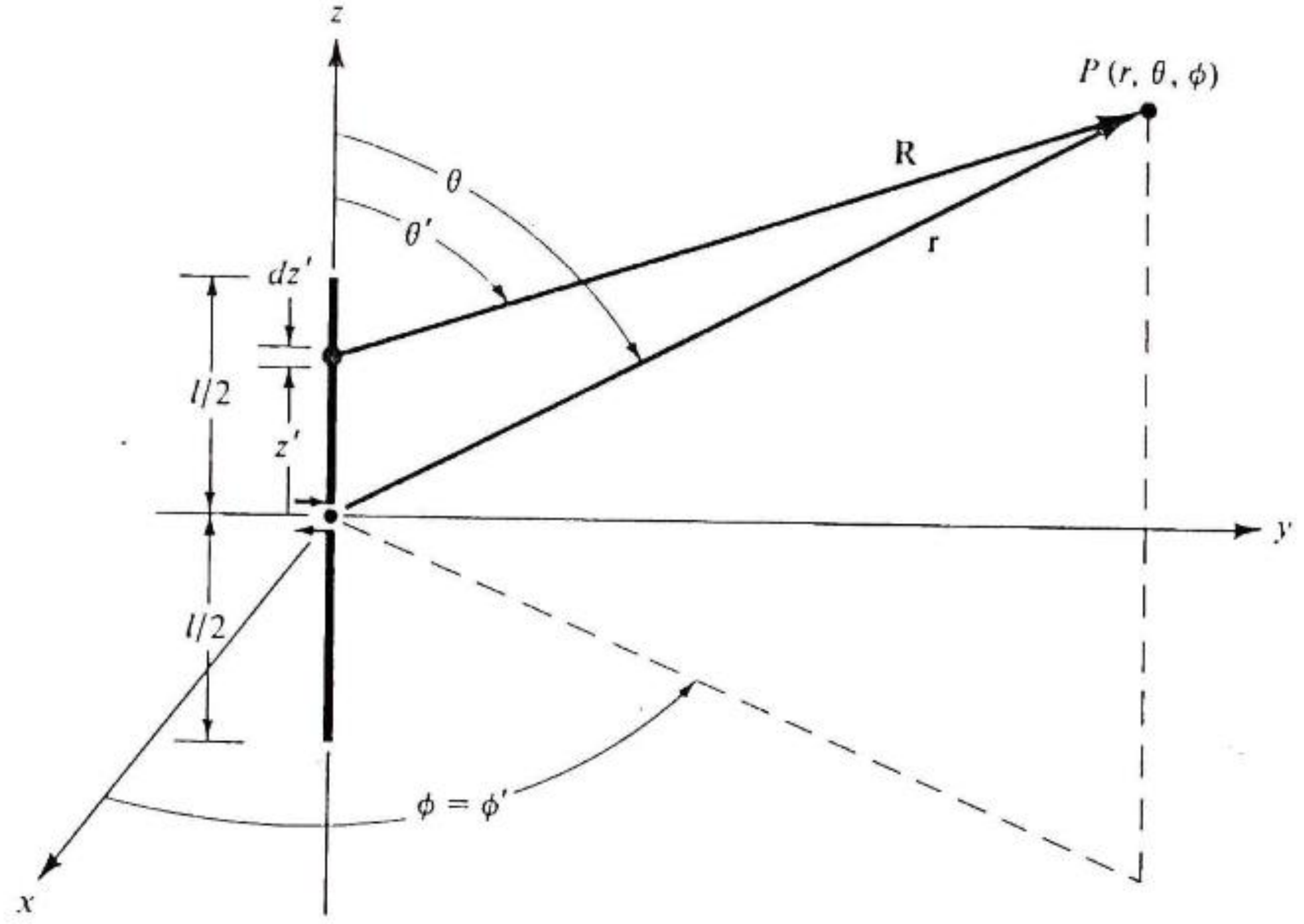
$$\vec{I}_e(0,0,z') = \begin{bmatrix} \hat{z} I_0 \sin\left[k\left(\frac{l}{2} - z'\right)\right] & 0 \le z' \le l/2 \\ \hat{z} I_0 \sin\left[k\left(\frac{l}{2} + z'\right)\right] & -l/2 \le z' \le 0 \end{bmatrix}$$

POSTECH

Dipole with Sinusoidal Current Distribution

- In this current distribution we assume that the antenna is center fed and the current vanishes at the end points of the antenna.
- Generally, we cannot obtain simple closed form expressions for the radiated fields of antennas that are valid in everywhere.
  - For a dipole antenna, closed form solutions that are valid everywhere do exists.
  - See Stratton (Electromagnetic Theory 1941).
- Therefore, we limit ourselves to the far field observation regions, where closed form expressions are easier to obtain. Although for some antennas, even those are not simple to calculate.

Dipole with Sinusoidal Current Distribution



Dipole with Sinusoidal Current Distribution

- The finite length dipole antenna can be subdivided into a number of infinitesimal dipoles of length $\Delta z'$.
- As $N \to \infty$, $\Delta z' \to dz'$
- For an infinitesimal dipole antenna of length dz', positioned along the z-axis at z', the $\vec{E}$ and $\vec{H}$ field components in the far field were calculated before:

$$dE_\theta \approx j\eta \frac{kI_e(x', y', z')e^{-jkR}}{4\pi R} \sin\theta \; dz'$$

$$dE_r \approx dE_\varphi = dH_r = dH_\theta = 0$$

$$dH_\varphi \approx j \frac{kI_e(x', y', z')e^{-jkR}}{4\pi R} \sin\theta \; dz'$$

Dipole with Sinusoidal Current Distribution

- We use the far field approximation and obtain dE_θ as:

$$dE_\theta \approx j\eta \frac{kI_e(x',y',z')e^{-jkr}}{4\pi r} \sin\theta \, e^{+jkz'\cos\theta} \, dz'$$

- The total field is then calculated by integrating:

$$E_\theta = \int_{-l/2}^{l/2} dE_\theta = \underbrace{j\eta \frac{ke^{-jkr}}{4\pi r} \sin\theta}_{\text{Element Factor}} \underbrace{\left[\int_{-l/2}^{l/2} I_e(x',y',z') e^{+jkz'\cos\theta} \, dz' \right]}_{\text{Space Factor}}$$

Element Factor **Space Factor**

POSTECH

Dipole with Sinusoidal Current Distribution

- For this antenna, the element factor is equal to the field of a unit length infinitesimal dipole located at the origin of the coordinate system.
- Generally
  - Element factor depends on the type of current and its direction of flow.
  - Space factor is a function of the current distribution along the source.
    - Total field = (element factor) × (space factor)

Dipole with Sinusoidal Current Distribution

$$E_\theta \approx j\eta \frac{ke^{-jkr}}{4\pi r}\sin\theta \left\{ \int_{-l/2}^{0} \sin\left[k\left(\frac{l}{2}+z'\right)\right] e^{+jkz'\cos\theta} dz' + \int_{0}^{l/2} \sin\left[k\left(\frac{l}{2}-z'\right)\right] e^{+jkz'\cos\theta} dz' \right\}$$

- Each of the integrals can be calculated using:

$$\int e^{\alpha x}\sin(\beta x+\gamma)dx = \frac{e^{\alpha x}}{\alpha^2+\beta^2}\left[\alpha\sin(\beta x+\gamma)-\beta\cos(\beta x+\gamma)\right]$$

$$\alpha = \pm jk\cos\theta \quad \beta = \pm k \quad \gamma = kl/2$$

POSTECH

Dipole with Sinusoidal Current Distribution

$$E_\theta \approx j\eta \frac{I_0 e^{-jkr}}{2\pi r}\left[\frac{\cos\left(\dfrac{kl}{2}\cos\theta\right)-\cos\left(\dfrac{kl}{2}\right)}{\sin\theta}\right]$$

$$H_\varphi = \frac{E_\theta}{\eta} \approx j\frac{I_0 e^{-jkr}}{2\pi r}\left[\frac{\cos\left(\dfrac{kl}{2}\cos\theta\right)-\cos\left(\dfrac{kl}{2}\right)}{\sin\theta}\right]$$