# Communications and Signal Processing
# 2015 Doctoral Qualifying Exam

**Caution!!!**
**Use a separate answer booklet for Problem 1.**

**Problem 1.** (50 points) A voice signal $m(t)$ has the continuous-time Fourier transform $M(f)$ defined as

$$M(f) = \int_{-\infty}^{\infty} m(t)e^{-\mathrm{j}2\pi ft}dt.$$

When this voice signal is modulated as

$$x(t) = \cos\left(2\pi f_c t + \beta\int_{-\infty}^{t} m(\tau)d\tau\right)$$

and transmitted using a radio wave, answer the following questions.

(a) (15 points) Find $m(-t), m^*(t)$, and $m^*(-t)$ in terms of $M(f)$, where the superscript $^*$ denotes the conjugation.

(b) (15 points) Using the fact that $m(t)$ is a real-valued signal, show that $M(f) = M^*(-f), \forall f$.

(d) (20 points) When the received signal is modeled by $y(t) = \alpha x(t)$, differentiate $y(t)$ with respect to $t$ and explain how to recover $m(t)$ from $y(t)$ by using a differentiator and an envelope detector.

**Caution!!!**
**Use a separate answer booklet for Problem 2.**

**Problem 2. (50 points)** Consider a communication system that employs complex pulse amplitude modulation or quadrature amplitude modulation (QAM). When the complex envelope of the transmitted signal is modeled by

$$x(t) = \sum_n s[n]g(t - nT),$$

where $s[n]$ denotes the 4-QAM data symbol, i.e., $s[n] \in \mathcal{C}_1 = \{1, j, -1, -j\}$, $g(t)$ represents the pulse shaping filter, and $T$ is the symbol duration, answer the following questions.

(a) (15 points) Suppose that the pulse shaping filter is given by

$$g(t) = \frac{p(t/T)}{\sqrt{\frac{2}{3}T}}$$

where $p(t) = 1 - |t|$, for $|t| \leq 1$ and $p(t) = 0$ for $|t| > 1$. Show that $g(t)$ is a Nyquist pulse shape.

(b) (15 points) Assuming a square-root Nyquist pulse $g(t)$, matched filtering, and analog-to-digital converter (ADC) operations at a receiver, we can model the discrete output signal of the $n$th sample at a receiver by

$$y[n] = s[n] + v[n],$$

where $s[n] \in \mathcal{C}_1 = \{1, j, -1, -j\}$ and $v[n]$ denotes the complex AWGN with zero-mean and variance of $\sigma^2$, i.e., $v[n] \sim \mathcal{N}_{\mathbb{C}}(0, \sigma^2)$. Explain the maximum likelihood (ML) detection rule.

(c) (20 points) Suppose that the transmitter sends the QAM symbol using different constellation points, i.e., $s[n] \in \mathcal{C}_2 = \{1 + j, 1 - j, -1 + j, -1 - j\}/\sqrt{2}$, and the receiver uses the ML detection. Compare which constellation set is better between $\mathcal{C}_1$ and $\mathcal{C}_2$ in terms of the symbol error rate.