광전자공학 Ch. 3
Coherence and interference

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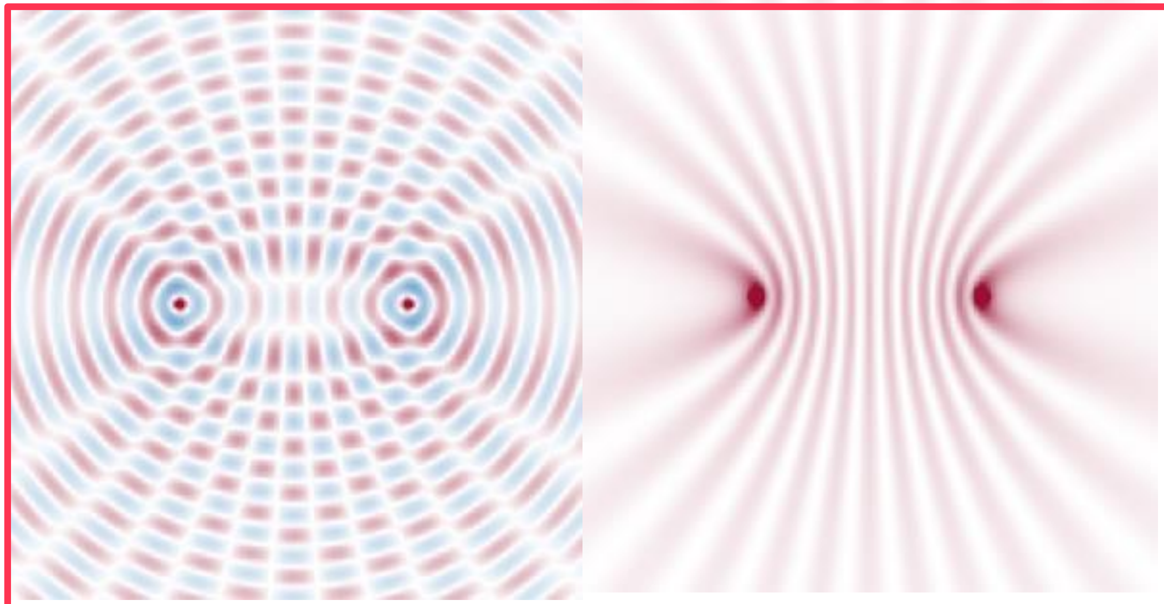
Interference

When two or more optical waves are simultaneously present in the same region of space and time, the total wavefunction is the sum of the individual wavefunctions.

Interference of two circular waves



Wavefunction Intensity

Linear superposition of light

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 + \mathbf{E}_3 \ldots$$

Usually satisfied, but not always satisfied when nolinear characteristics appears

Interference

Intensity of Interfered two waves

$$I = |\mathbf{E}|^2 = (\mathbf{E}_1 + \mathbf{E}_2)\cdot(\mathbf{E}^*_1 + \mathbf{E}^*_2) = |\mathbf{E}_1|^2 + |\mathbf{E}_2|^2 + 2\mathbf{E}_1\cdot\mathbf{E}_2\cos\varphi$$

$$\mathbf{E}_1 = \hat{\mathbf{e}}_1\sqrt{I_1}\exp(j\varphi_1),\quad \mathbf{E}_2 = \hat{\mathbf{e}}_2\sqrt{I_2}\exp(j\varphi_2)$$

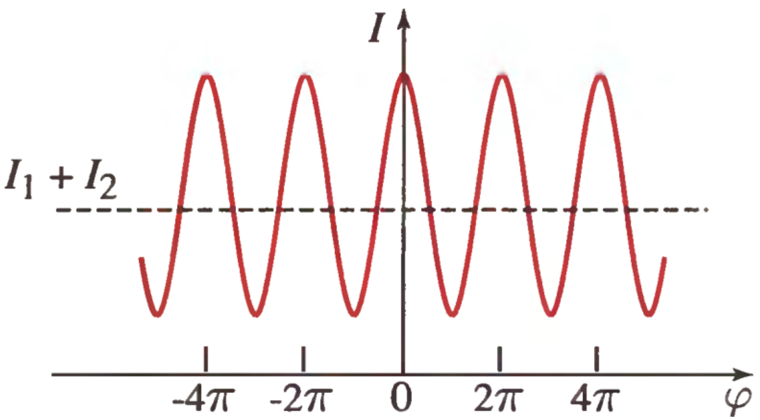
$$\varphi_1 = \mathbf{k}_1\cdot\mathbf{r} + \phi_1,\quad \varphi_2 = \mathbf{k}_2\cdot\mathbf{r} + \phi_2$$

Phase difference

$$\varphi = \varphi_2 - \varphi_1$$

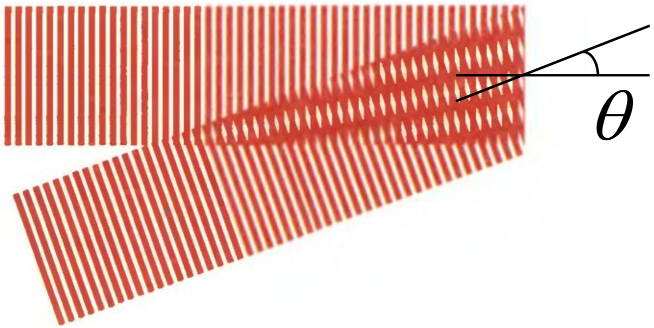
Interference equation

$$\boxed{I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\varphi\,,}$$



Interference of two oblique waves

Interference of two plane waves



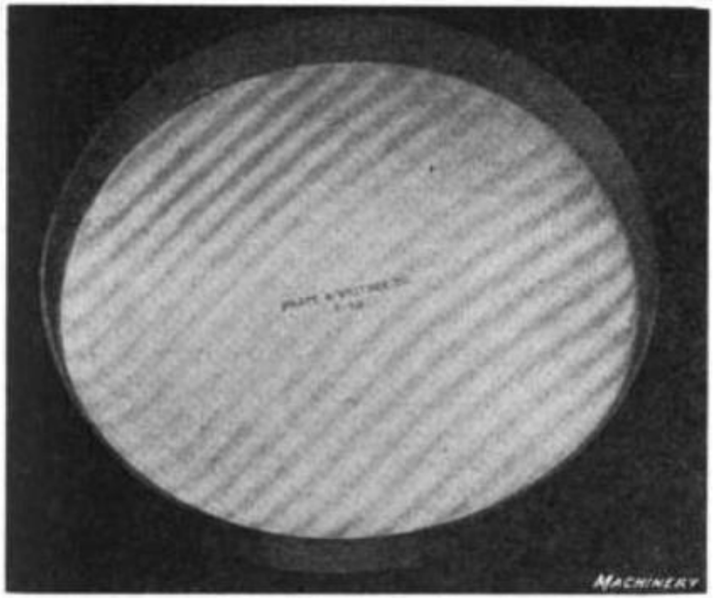
$$\mathbf{E}_1 = \sqrt{I_0}\,\hat{x}\exp(j(k_0 z - \omega t)),$$

$$\mathbf{E}_2 = \sqrt{I_0}\,\hat{x}\exp(j(k_0\cos\theta z + k_0\sin\theta x - \omega t))$$

$$I = 2I_0 + 2I_0\cos(k_0(1-\cos\theta)z + k_0\sin\theta x)$$

At z = 0

$$I = 2I_0 + 2I_0\cos(k_0\sin\theta x)$$

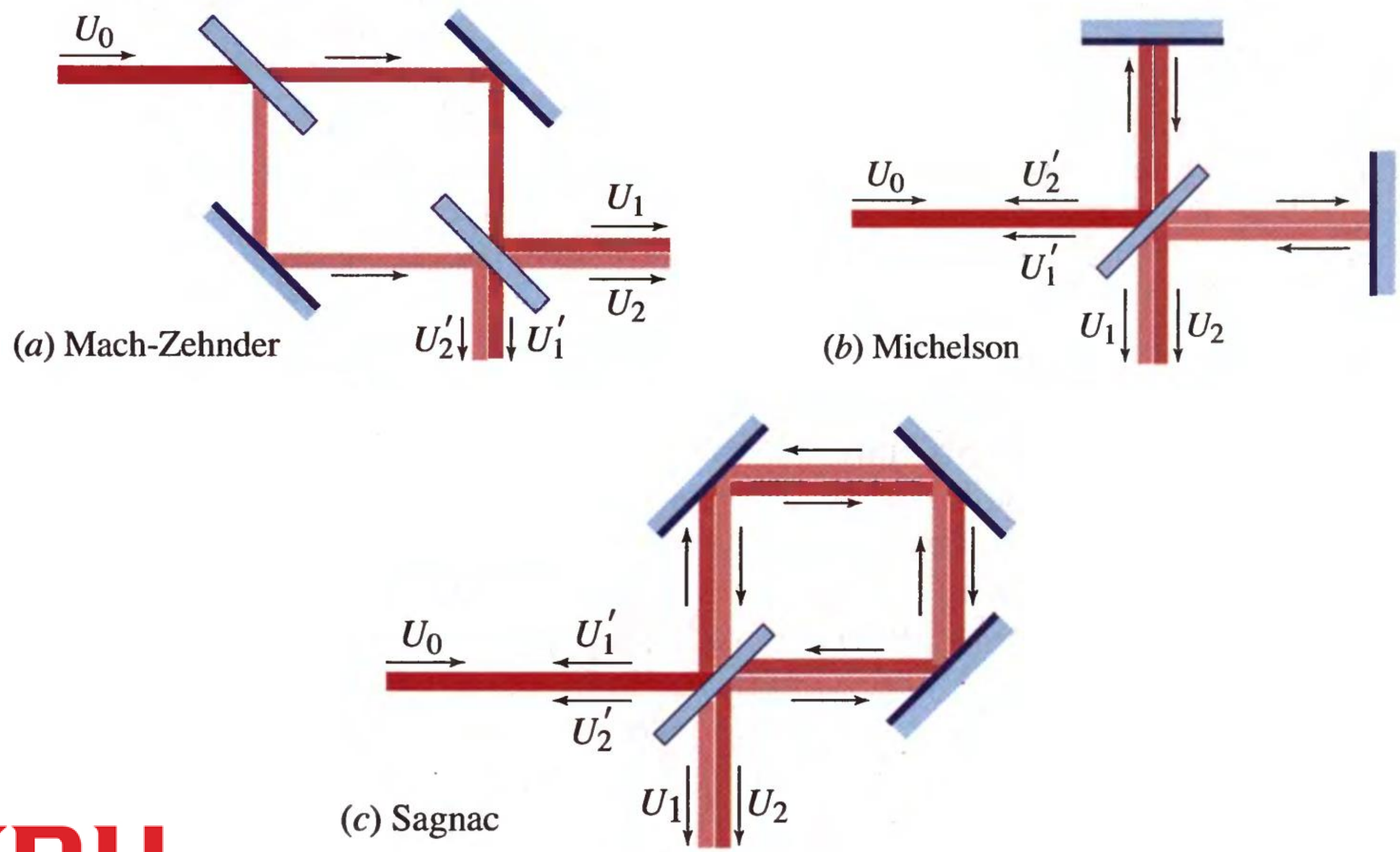


Period of interference pattern

$$k_0\sin\theta d = 2\pi \rightarrow d = \frac{\lambda_0}{\sin\theta}$$

Interferometer

An optical instrument that splits a wave into two waves using a beamsplitter, and recombines them for detecting the intensity of their superposition.



(*a*) Mach-Zehnder (*b*) Michelson (*c*) Sagnac

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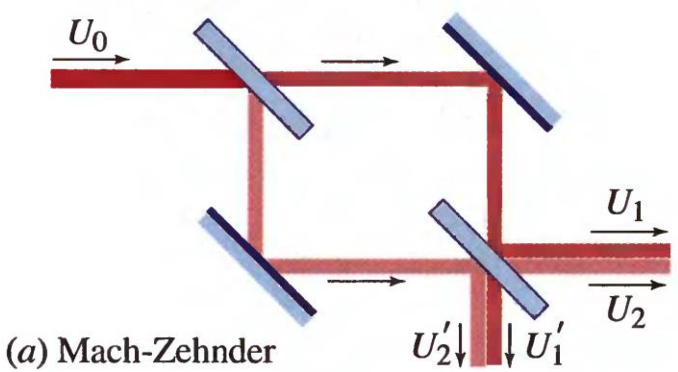
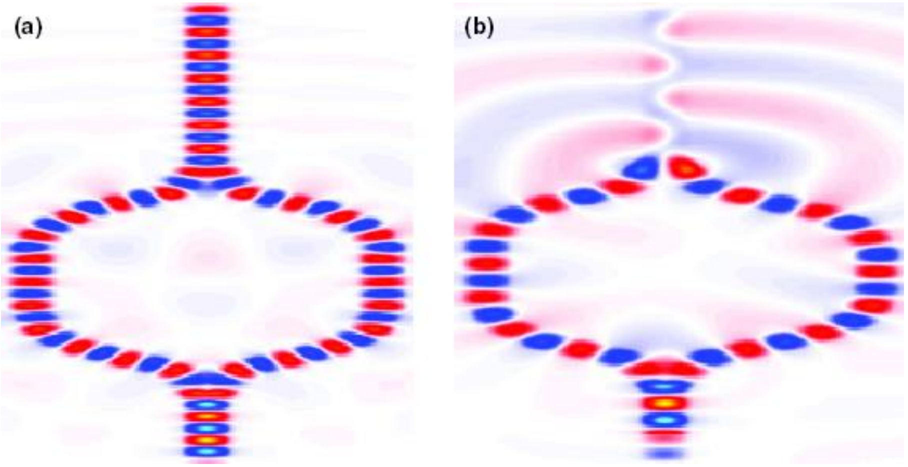
Mach-Zehnder interferometer

Uses two beamsplitters for splitting and recombining optical paths.





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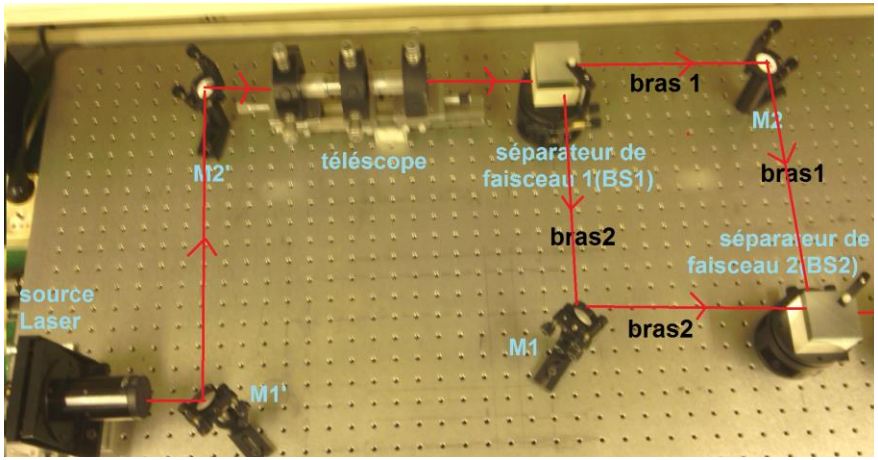
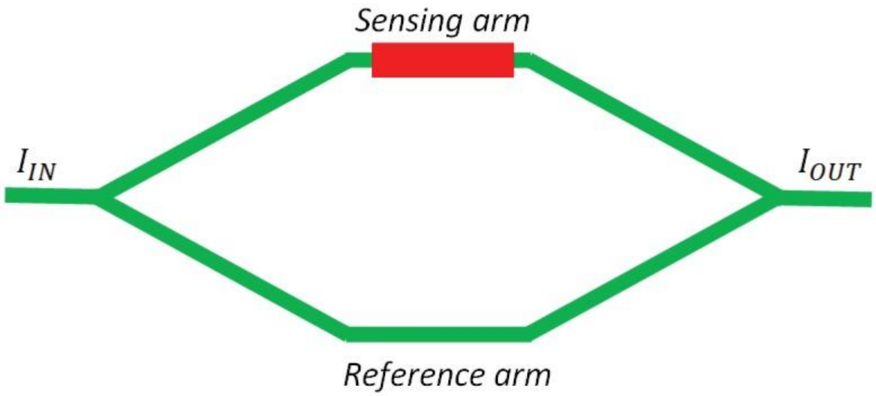
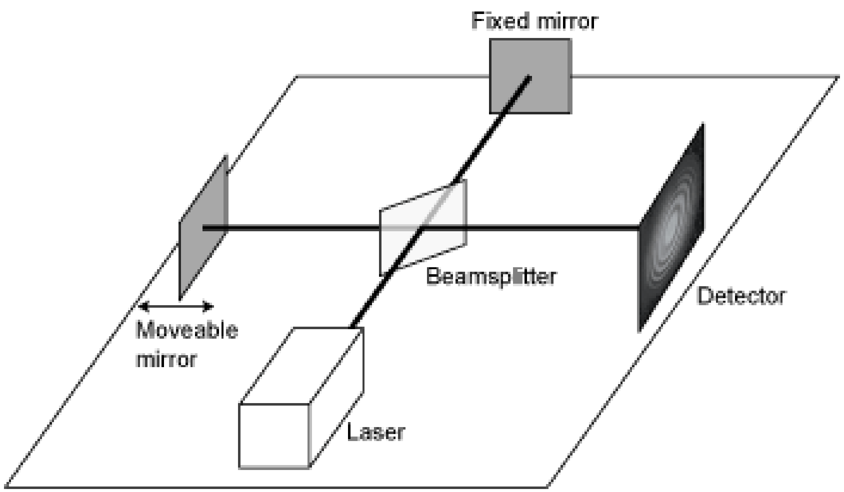




Michelson interferometer

Uses one beamsplitter for splitting and recombining optical paths.



Michelson-Morley experiment (1887)

(The most famous failed experiment in history)

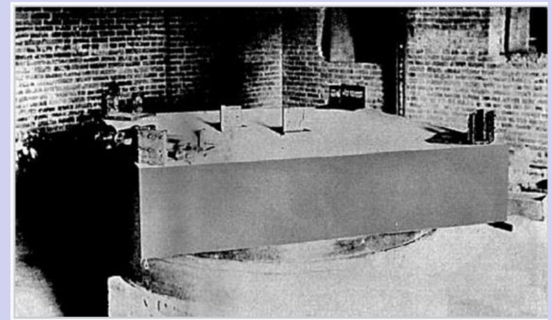
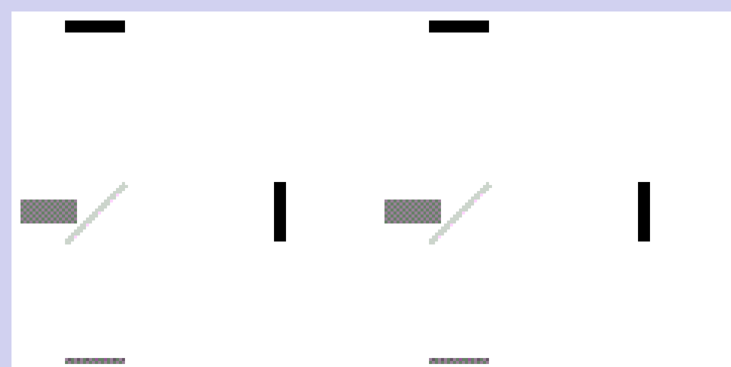
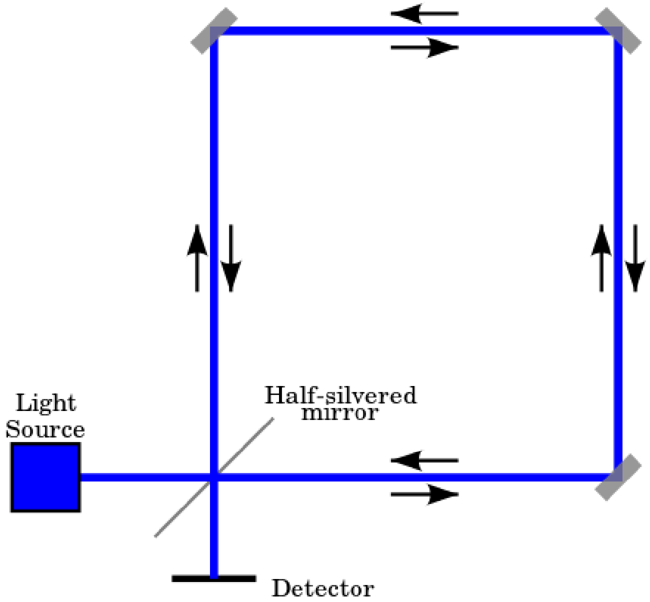


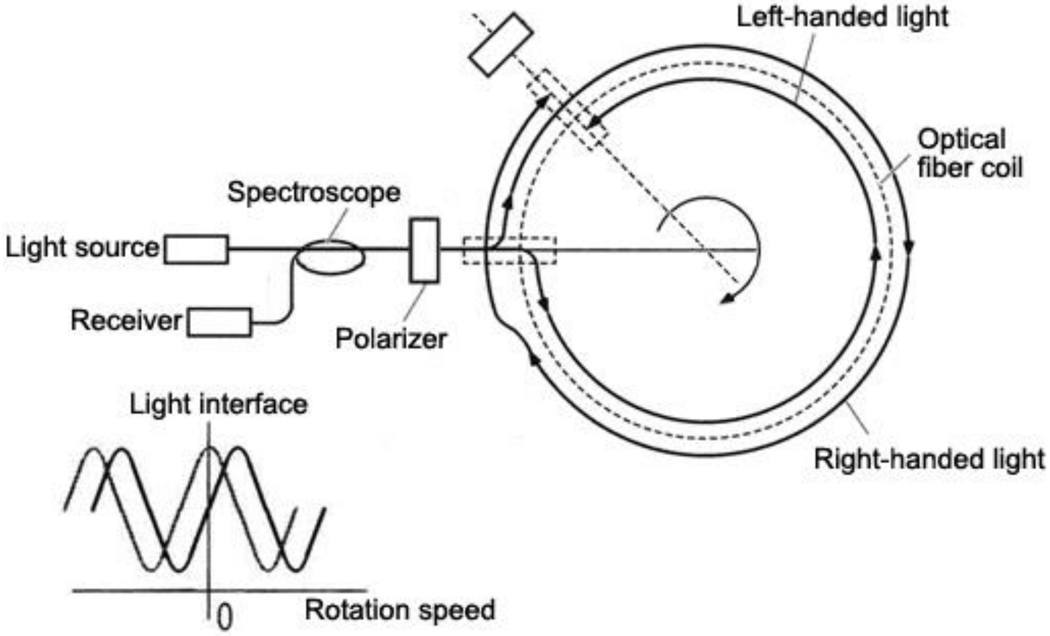
Figure 1. Michelson and Morley's interferometric setup, mounted on a stone slab that floats in an annular trough of mercury.



Sagnac interferometer

Uses the rotation of the optical system to make a difference in optical pathlength.

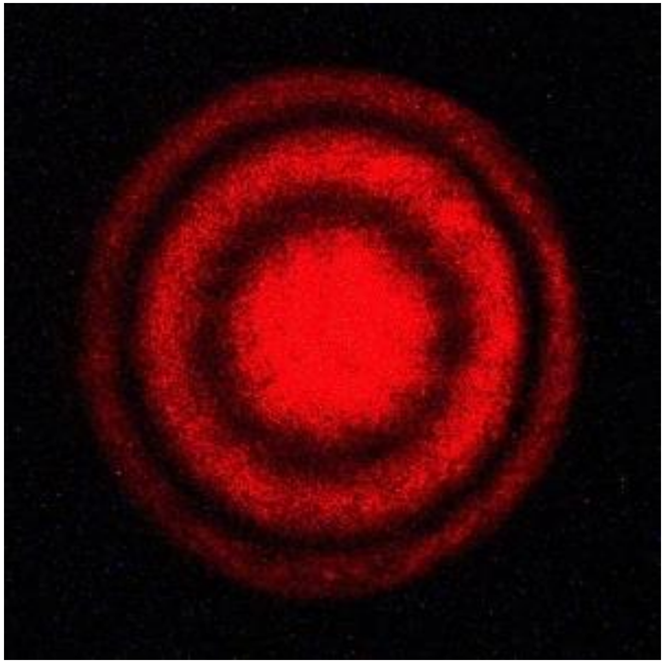




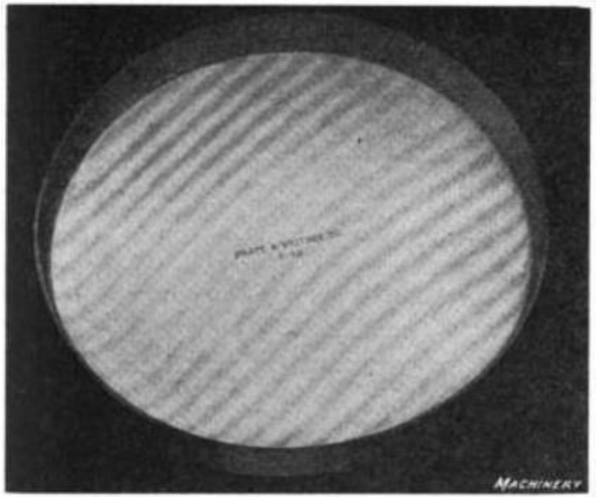
Can be used for fiber-optic gyroscope

Interference fringe pattern

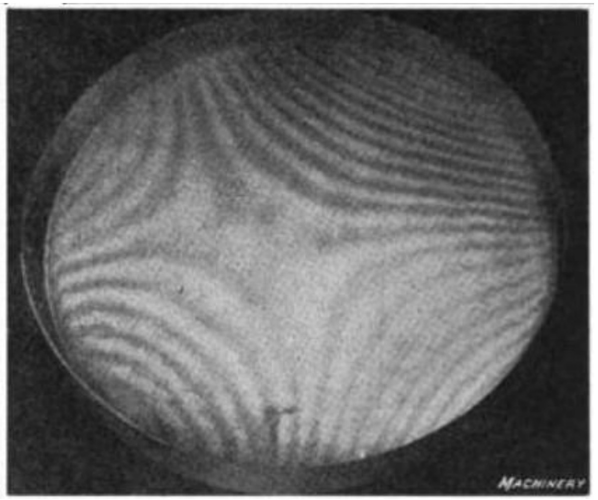
Common interference patterns shown by interferometer system



Coaxial-circular



Linear lines



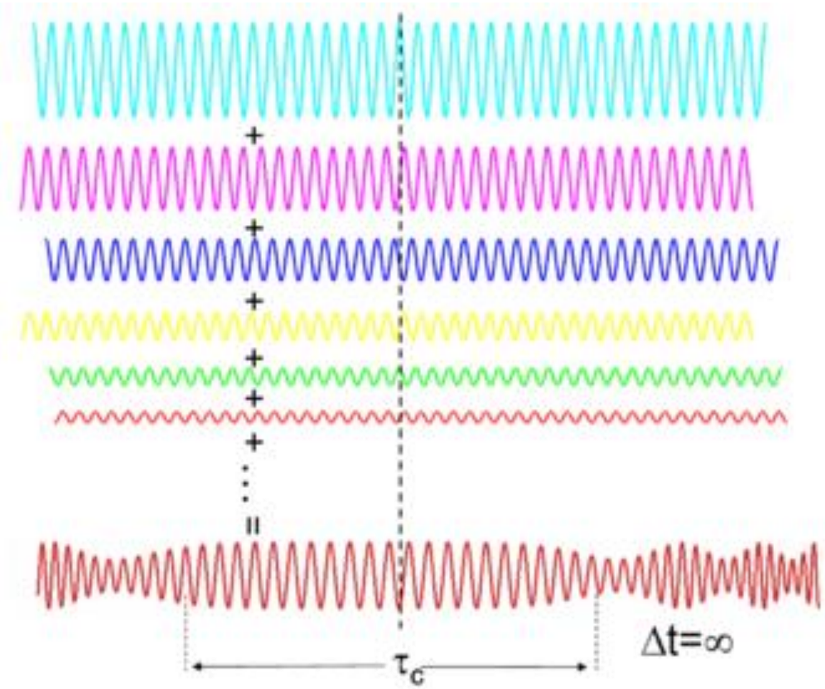
Hyperbolic curves

Partial coherence

In ideal case, optical field was considered as perfect(infinite) sinusoidal wave

In actual case, light amplitude and phase vary with time in a random fashion

ideal coherent wave



partial coherent wave

Coherence function

General interference equation

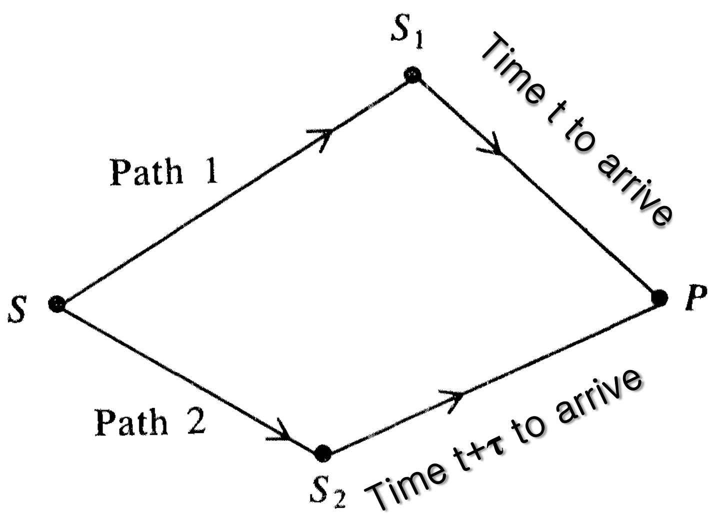
$$I = \left\langle |\mathbf{E}|^2 \right\rangle = \left\langle (\mathbf{E}_1 + \mathbf{E}_2) \cdot (\mathbf{E}^*{}_1 + \mathbf{E}^*{}_2) \right\rangle = \left\langle |\mathbf{E}_1|^2 + |\mathbf{E}_2|^2 + 2\,\mathrm{Re}\left(\mathbf{E}_1 \cdot \mathbf{E}^*{}_2\right) \right\rangle$$

$$I = I_1 + I_2 + 2\,\mathrm{Re}\left\langle \mathbf{E}_1 \cdot \mathbf{E}^*{}_2 \right\rangle$$

The third term of above equation can be generally given as

$$2\,\mathrm{Re}(\Gamma_{12}(\tau)), \quad \Gamma_{12}(\tau) = \left\langle E_1(t) E^*{}_2(t+\tau) \right\rangle$$

For simplicity, assume that polarization of two beams are same, so vectorial characteristics are ignored.



Degree of partial coherence

Define degree of partial coherence $$\gamma_{12}(\tau) = \frac{\Gamma_{12}(\tau)}{\sqrt{I_1 I_2}}$$

Rewrite general interference equation $$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\,\gamma_{12}(\tau)$$

$$\left|\gamma_{12}(\tau)\right| = 1 \quad \text{(completely coherent)}$$

$$0 < \left|\gamma_{12}(\tau)\right| < 1 \quad \text{(partially coherent)}$$

$$\left|\gamma_{12}(\tau)\right| = 1 \quad \text{(completely incoherent)}$$

Coherence time

Self-coherence
Coherence of the light source itself

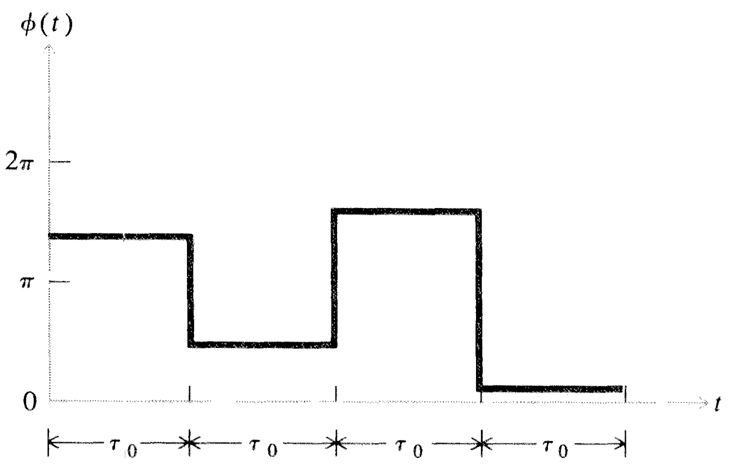
$$\gamma(\tau) = \frac{\langle E(t)E^*(t+\tau)\rangle}{\langle |E|^2 \rangle}$$

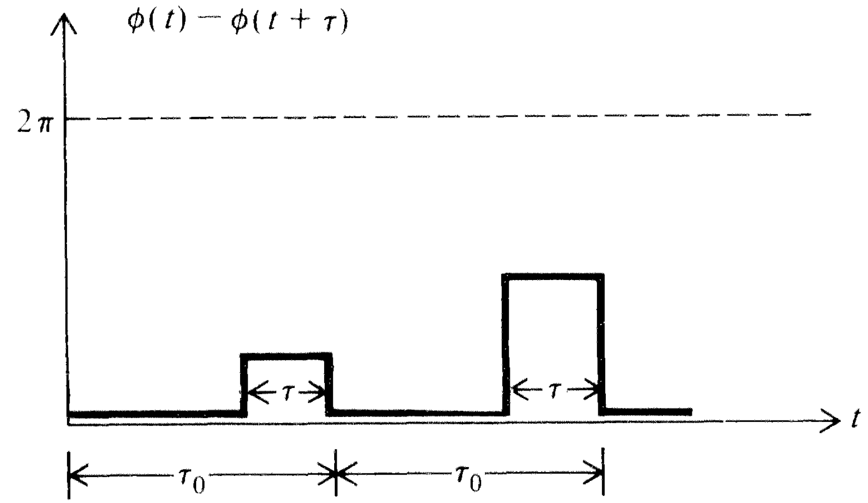
Assuming a quasi-monochromatic light
Where phase is abruptly changing

$$E(t) = E_0 e^{-i\omega t}\, e^{i\phi(t)}$$

$$\gamma(\tau) = \langle e^{i\omega\tau}\, e^{i[\phi(t)-\phi(t+\tau)]}\rangle$$

$$= e^{i\omega\tau} \lim_{T\to\infty} \frac{1}{T}\int_0^T e^{i[\phi(t)-\phi(t+\tau)]}\, dt$$



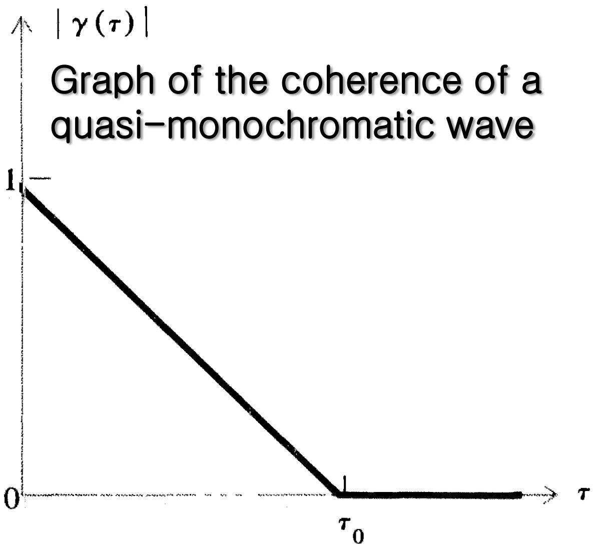


Coherence time

$$\frac{1}{\tau_0}\int_0^{\tau_0} e^{i[\phi(t)-\phi(t+\tau)]}\,dt = \frac{1}{\tau_0}\int_0^{\tau_0-\tau} dt + \frac{1}{\tau_0}\int_{\tau_0-\tau}^{\tau_0} e^{i\Delta}\,dt$$

$$= \frac{\tau_0-\tau}{\tau_0} + \cancel{\frac{\tau}{\tau_0}e^{i\Delta}}$$

Evolve to zero because delta is random phase



$$|\gamma(\tau)| = 1 - \frac{\tau}{\tau_0} \qquad \tau < \tau_0$$

$$= 0 \qquad \tau \geqslant \tau_0$$

Here we define τ_0 as a coherence time.

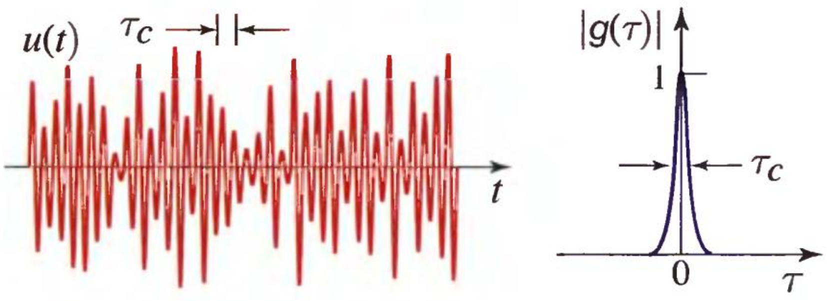
Coherent length is defined as,

$$c\tau_0 = l_c$$

In realistic, random phase function do not appear constantly, but also randomly.

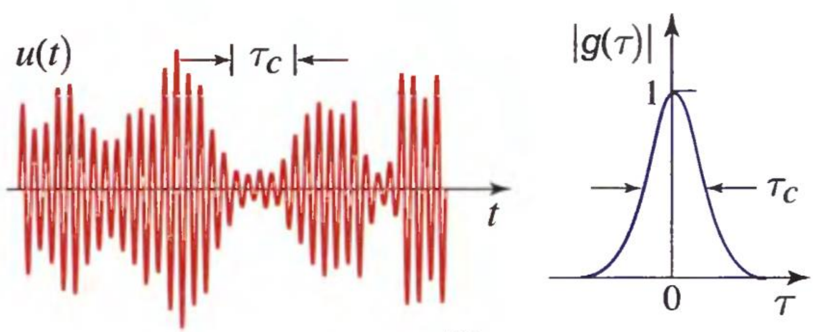
Coherence function

In realistic, random phase function do not appear constantly, but also randomly.



(a)

Coherence time is short,
Sinusoidal phase breaks more frequently



(b)

Coherence time is long,
Sinusoidal phase breaks more seldomly

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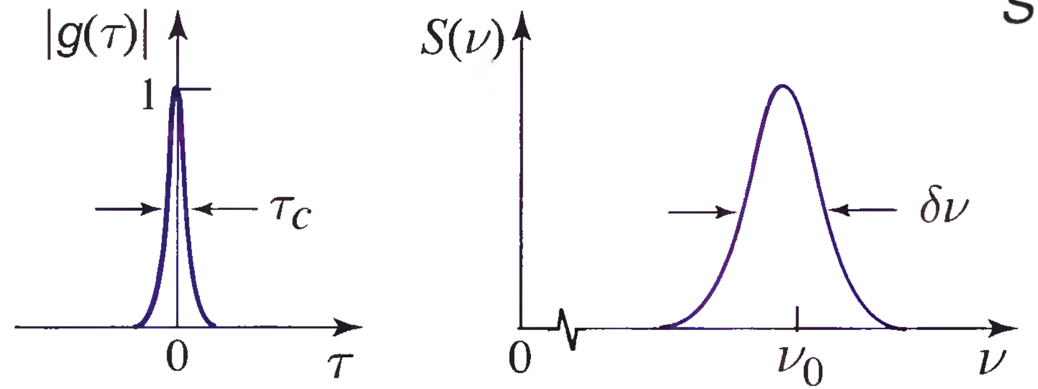
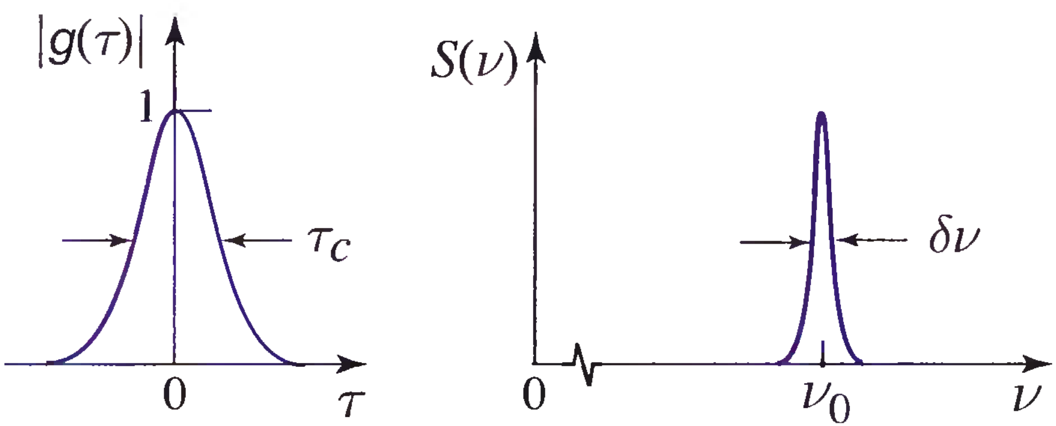
Power spectral density

Power spectral density is defined as a Fourier transform of coherence function

$$S(\nu) = \int_{-\infty}^{\infty} G(\tau) \exp(-j2\pi\nu\tau)\, d\tau.$$



Short coherence time means broad spectral width



long coherence time means narrow spectral width

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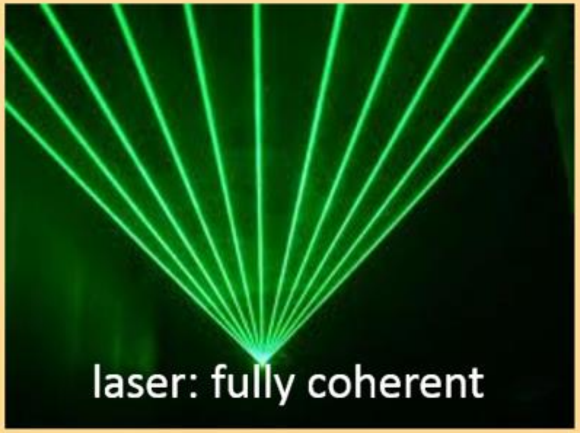
Spectral width of various sources

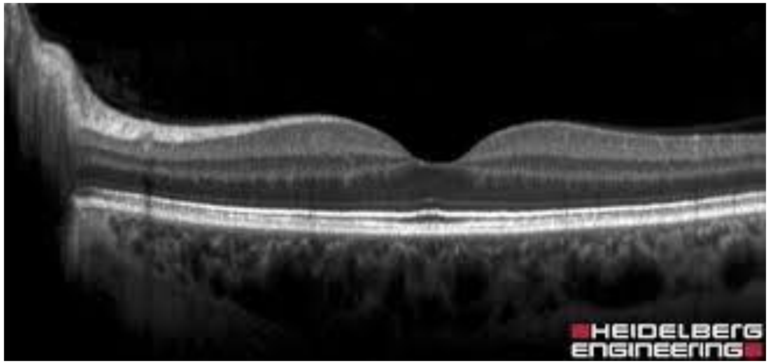
Table 11.1-2 Spectral widths of a number of light sources together with their coherence times and coherence lengths in free space.

Source	$\Delta\nu_c$ (Hz)	$\tau_c = 1/\Delta\nu_c$	$l_c = c\tau_c$
Filtered sunlight ($\lambda_o = 0.4$–$0.8\ \mu$m)	3.74×10^{14}	2.67 fs	800 nm
Light-emitting diode ($\lambda_o = 1\ \mu$m, $\Delta\lambda_o = 50$ nm)	1.5×10^{13}	67 fs	20 μm
Low-pressure sodium lamp	5×10^{11}	2 ps	600 μm
Multimode He–Ne laser ($\lambda_o = 633$ nm)	1.5×10^{9}	0.67 ns	20 cm
Single-mode He–Ne laser ($\lambda_o = 633$ nm)	1×10^{6}	1 μs	300 m



Applications using interferences

Optical Coherence Tomography (OCT)



For measuring the reflectance and depth of each of its boundaries. It makes use of a partially coherent light source of short coherence length and a Michelson interferometer.