Spring 2019

EECE 588
Lecture 3

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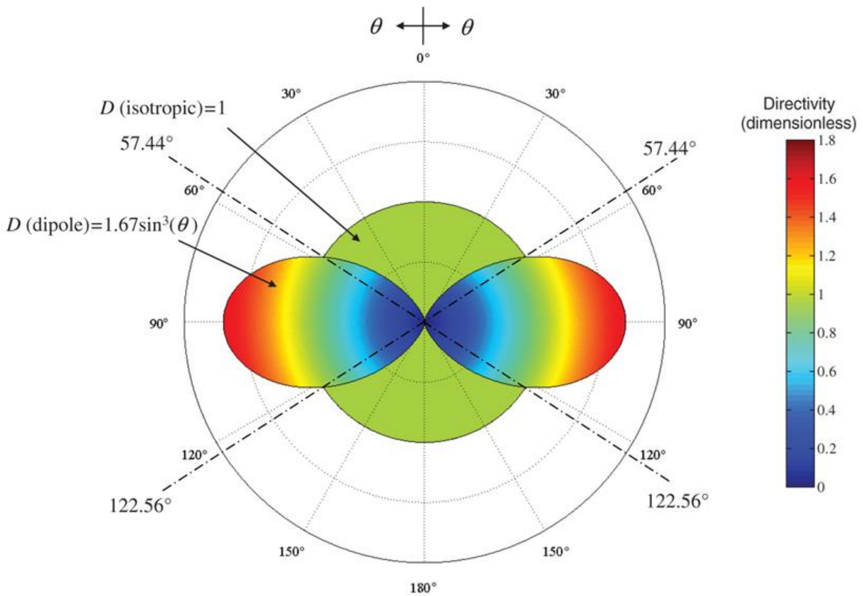
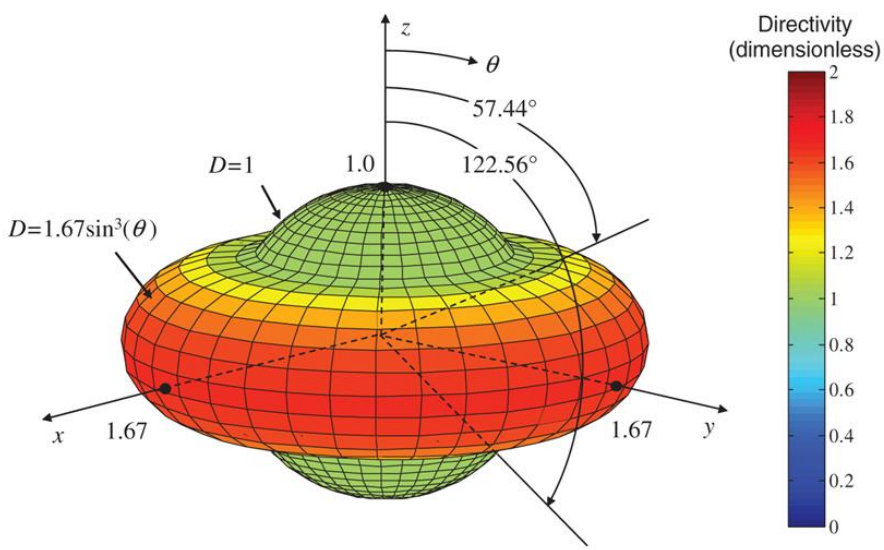
Directivity of a Half Wavelength Dipole

- For a dipole antenna whose axis is located along the z-axis and has a length of half wavelength is:

$$D = D_0 \sin^3\theta = 1.67 \sin^3\theta$$





General Equations for Directivity

- In general, the directivity of an antenna is a function of both θ and φ. In this case, we have

$$U = B_0 F(\theta,\varphi) \approx \frac{1}{2\eta}\left[\left|E_\theta^0(\theta,\varphi)\right|^2 + \left|E_\varphi^0(\theta,\varphi)\right|^2\right]$$

- We can find the total radiated power using:

$$P_{rad} = \oiint_\Omega U(\theta,\varphi)\, d\Omega = B_0 \int_0^{2\pi}\int_0^{\pi} F(\theta,\varphi)\sin\theta\, d\theta\, d\varphi$$

- This way, we can calculate the general expression for directivity

General Directivity Equations

$$D(\theta,\varphi) = 4\pi \frac{F(\theta,\varphi)}{\int_0^{2\pi}\int_0^{\pi} F(\theta,\varphi)\sin\theta \, d\theta \, d\varphi}$$

- Maximum Directivity is:

$$D_0 = 4\pi \frac{F(\theta,\varphi)\,|_{\max}}{\int_0^{2\pi}\int_0^{\pi} F(\theta,\varphi)\sin\theta \, d\theta \, d\varphi}$$

$$D_0 = \frac{4\pi}{\left[\int_0^{2\pi}\int_0^{\pi} F(\theta,\varphi)\sin\theta \, d\theta \, d\varphi\right] \Big/ F(\theta,\varphi)\,|_{\max}} = \frac{4\pi}{\Omega_A}$$

POSTECH

General Directivity Equations

- Ω_A is the beam solid angle. It is defined by:

$$\Omega_A = \frac{1}{F(\theta,\varphi)\,|_{\max}} \int_0^{2\pi}\int_0^{\pi} F(\theta,\varphi)\sin\theta\; d\theta\; d\varphi$$

$$= \int_0^{2\pi}\int_0^{\pi} F_n(\theta,\varphi)\sin\theta\; d\theta\; d\varphi \qquad F_n(\theta,\varphi) = \frac{F(\theta,\varphi)}{F(\theta,\varphi)\,|_{\max}}$$

- The beam solid angle is defined as "the solid angle through which all the power of the antenna would flow if its radiation intensity is constant (and equal to the maximum value of U) for all angles within Ω_A."

POSTECH

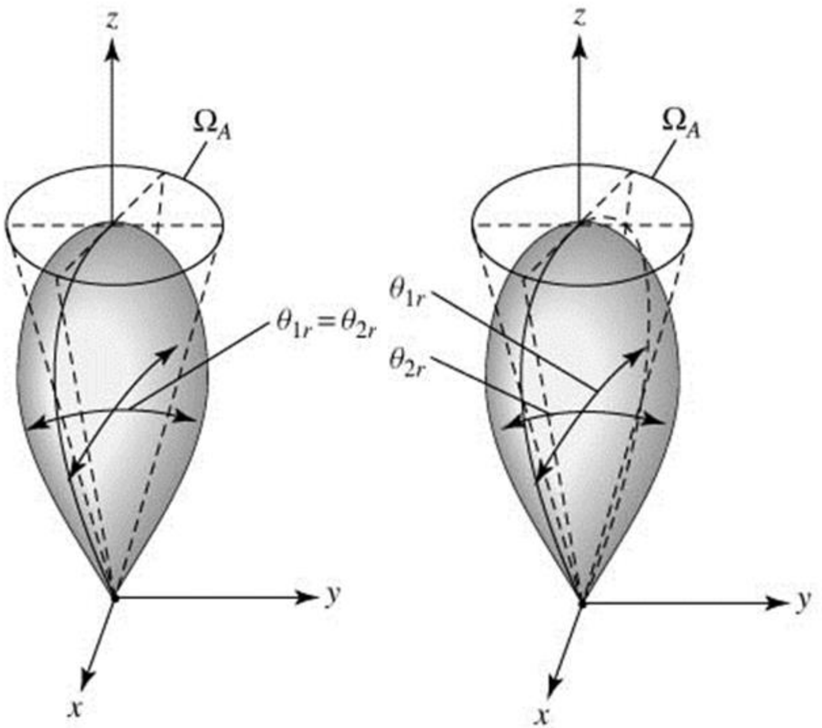
Directional Patterns

- For antennas with one narrow major lobe and very negligible minor lobes the beam solid angle is approximately equal to the product of the half-power beamwidths in two

$$D_0 = \frac{4\pi}{\Omega_A} \approx \frac{4\pi}{\Theta_{1r}\Theta_{2r}}$$

$$\Omega_A \approx \Theta_{1r}\Theta_{2r}$$

- Note that Θ_{1r} and Θ_{2r} are i



Directional Patterns

- If the beam widths are known in degrees, we will have

$$D_0 \approx \frac{4\pi(180/\pi)^2}{\Theta_{1d}\Theta_{2d}} = \frac{41253}{\Theta_{1d}\Theta_{2d}}$$

- For planar arrays, a better approximation is

$$D_0 \approx \frac{32400}{\Omega_A(\text{degrees})^2} = \frac{32400}{\Theta_{1d}\Theta_{2d}}$$

- These expressions are valid for radiation patterns with only one major lobe and extremely small minor lobes.

Example 2.7:

$$U = \begin{cases} B_o \cos\theta & \begin{array}{l} 0 \le \theta \le \pi/2 \\ 0 \le \phi \le 2\pi \end{array} \\ 0 & \begin{array}{l} \pi/2 \le \theta \le \pi \\ 0 \le \phi \le 2\pi \end{array} \end{cases}$$

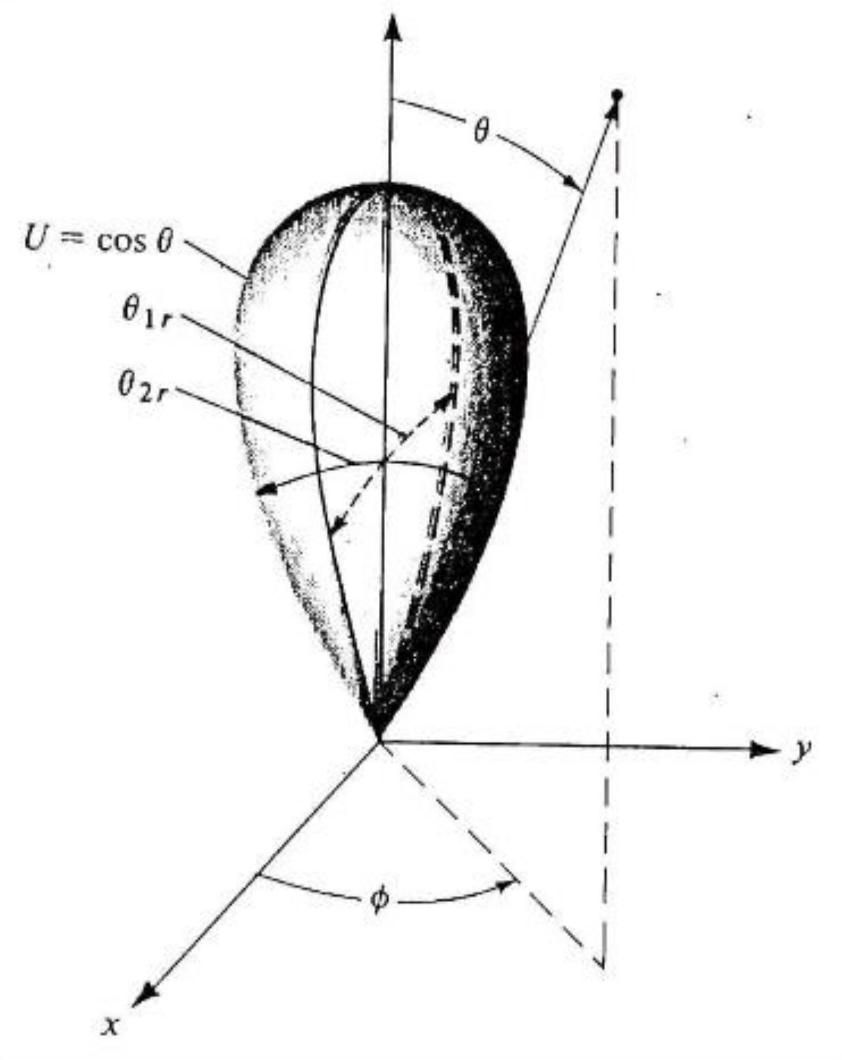
Solution:

$$P_{rad} = \int_0^{2\pi}\int_0^{\pi/2} U \sin\theta d\theta d\phi = B_o \int_0^{2\pi}\int_0^{\pi/2} \cos\theta \sin\theta d\theta d\phi$$

$$= 2\pi B_o \int_0^{\pi/2} \cos\theta \sin\theta d\theta = 2\pi\left(\frac{1}{2}\right)B_o$$

$$P_{rad} = \pi B_o$$

Radiation Intensity Pattern



$$U(\theta,\phi) = \cos\theta$$

$$0 \le \theta \le 90^o$$

$$0 \le \phi \le 360^o$$

Fig. 2.15

$$D_0(\text{exact}) = \frac{4\pi U_{\max}}{P_{rad}} = \frac{4\pi B_0}{\pi B_0} = 4 = 6.02 \text{ dB}$$

Approximate:

To find the HPBW, you set

$$\cos\theta_h = 0.5 \Rightarrow \theta_h = \cos^{-1}(0.5)$$

$$\theta_h = \frac{\pi}{3} \text{ radians} = 60^\circ$$

Because of the symmetry of the pattern

$$\Theta_1 = \Theta_{2r} = 2\pi/3 \text{ radians} = 120^\circ$$

Using the previous results, we get the following approximate directivities:

$$D_0\left(\text{Kraus}\right) \simeq \frac{4\pi}{\left(2\pi/3\right)^2} = \frac{9}{\pi} = 2.86 = 4.56\ \text{dB}$$

$$\left(-28.5\%\ \text{Error}\right)$$

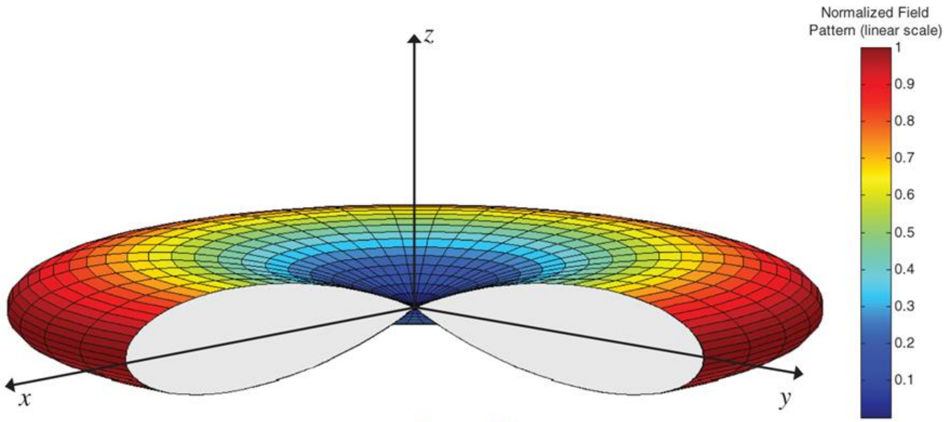
$$D_0\left(\text{T\&P}\right) \simeq \frac{22.181}{2\left(2\pi/3\right)^2} = 2.53 = 4.03\ \text{dB}$$

$$\left(-36.75\%\ \text{Error}\right)$$

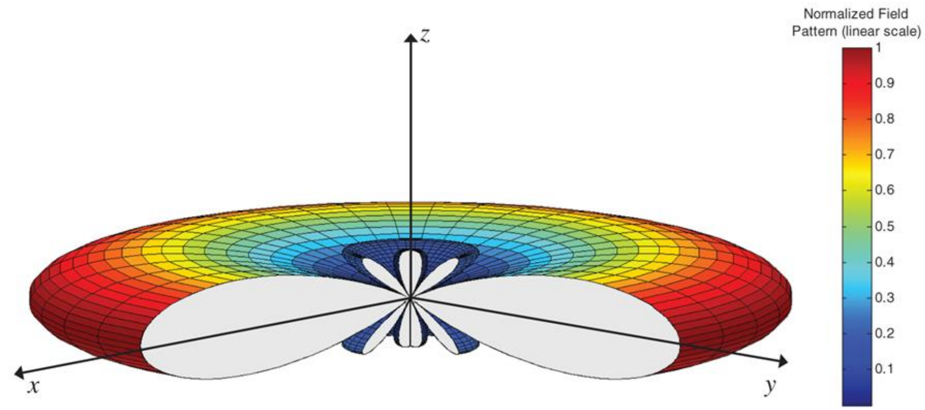
Omnidirectional Patterns

- As we will see in coming weeks, some antennas have an omni-directional radiation pattern. Omnidirectional patterns can be approximated with

$$U = |sin^n\theta| \text{ for } 0 \le \theta \le \pi, 0 \le \varphi \le 2\pi$$



$$U \approx \left|\sin^n(\theta)\right| \begin{cases} 0 \le \theta \le \pi \\ 0 \le \phi \le 2\pi \end{cases}$$



$$U \approx \left|\sin^n(\theta)\right| \begin{cases} 0 \le \theta \le \pi \\ 0 \le \phi \le 2\pi \end{cases}$$

Omnidirectional Patterns

- For omnidirectional antennas, we can use the following approximate formulas for calculating the directivity:
- McDonald [Based on array]:

$$D_0 \approx \frac{101}{HPBW\,(\text{degrees}) - 0.0027\,[HPBW\,(\text{degrees})]^2}$$

- Pozar [Based on curve fitting]:

$$D_0 \approx -172.4 + 191\sqrt{0.818 + 1/HPBW(\text{deg})}$$

Omindirectional Patterns

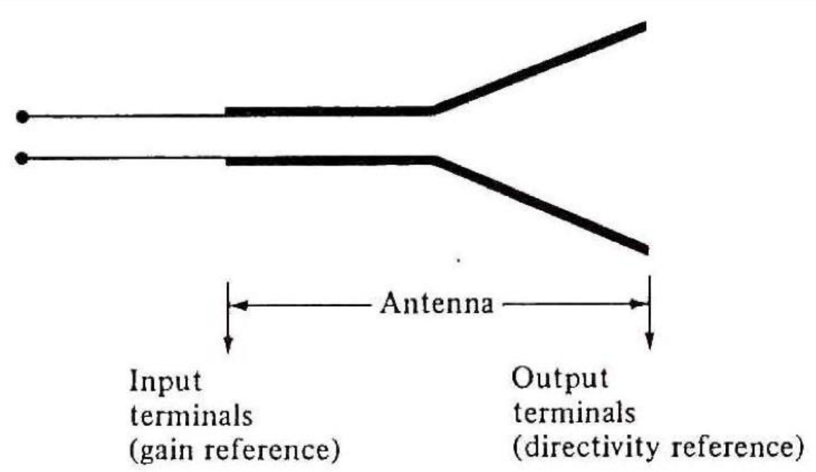
- These approximate formulas are good for rapid calculations of directivities based on some measurement results, etc.
- CAUTION:
  - Don't focus on these approximate formulas, etc. Make sure that you understand the concept rather than memorizing formulas
  - Note that all of the formulas we have discussed so far can be obtained directly from the definitions and hence from the concepts

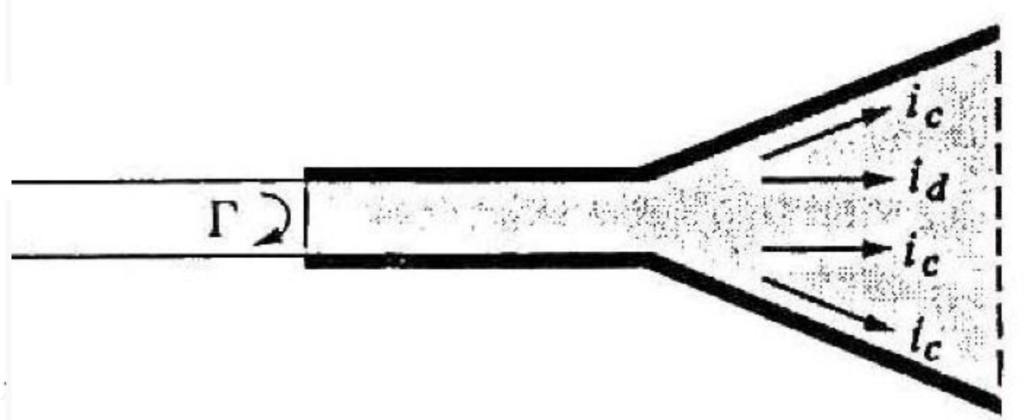
Antenna Efficiency

- What is antenna efficiency?
  - What is the role of an antenna?
    - From a system level point of view, we want all the power that is sent to the antenna to be radiated by the antenna.
  - So, antenna efficiency is a measure of how effectively an antenna does its job. i.e., how effectively it radiates the power that we send to its input port.
- So, what are some factors that you can think of that result in a reduced antenna efficiency?

Antenna Efficiency

- Reflections: If the antenna is not matched to the transmission line feeding it, the power sent to the antenna will be reflected back towards the source. Of course, this is not desirable
- Antennas are generally metallic structures. Finite conductivity means Ohmic losses.
- We might have dielectrics in the antenna as well. Finite tan δ means dielectric losses





Antenna Efficiency

- In general, the overall efficiency of the antenna can be written as:

$$e_0 = e_r e_c e_d$$

- e_0 = total efficiency (dimensionless)
- e_r = reflection efficiency = $1 - |\Gamma|^2$ (dimensionless)
  - Γ is the reflection coefficient at the input terminals of the antenna
  - How can you calculate Γ?
- e_c = conduction efficiency (dimensionless)
- e_d = dielectric efficiency (dimensionless)
- Note that generally, calculating e_c and e_d is difficult. We can determine them experimentally but we cannot separate them from one another

POSTECH

Antenna Efficiency

- Therefore, we usually write the efficiency of the antenna as

$$e_0 = e_{cd}\left(1 - |\Gamma|^2\right)$$

- $e_{cd} = e_c e_d$

GAIN

- An extremely useful measure describing the performance of an antenna is its gain.
- The gain and directivity are closely related to one another but in real system level calculations, we DO use the gain and not the directivity
- Directivity tells you about the potential of the antenna and gains tells you about how it actually performs
- Gain is defined as: "the ratio of the intensity in a given direction to the radiation intensity that would have been obtained if the power accepted by the antenna were radiated isotropically"
- Note that in the above equation, we are taking into account the power ACCEPTED by the antenna (reflections don't matter)

GAIN

$$Gain = 4\pi \frac{\text{radiation Intensity}}{\text{total input power}} = 4\pi \frac{U(\theta,\varphi)}{P_{in}}$$

- Relative gain is defined as "the ratio of the power gain in a given direction to the power gain of a reference antenna in its referenced direction"
- In most case, we use an isotropic antenna as the reference
- If not specified, the direction of maximum radiation is assumed
- Note that $P_{rad} = e_{cd} P_{in}$
- Also note that $P_{in} = (1 - |\Gamma|^2)\, P_{inc}$, where P_{inc} is the power incident on the input port of the antenna

GAIN

- According to IEEE standards, gain does not include losses arising from impedance mismatches or polarization mismatches.
- Now if you have an antenna with a very large gain but large mismatch losses, that defeats the purpose. i.e., we cannot radiate this power efficiently
- However, the IEEE definition of gain does make sense. After all, we want to see the potential of the radiator. In other words, the best you can do with this radiator is …
- In system level calculations, however, you MUST take into account reflection and polarization mismatches

GAIN

- From the definition of gain, we have:

$$G(\theta,\varphi) = e_{cd}\left[4\pi \frac{U(\theta,\varphi)}{P_{rad}}\right]$$

- Therefore, gain and directivity are related together through:

$$G(\theta,\varphi) = e_{cd} D(\theta,\varphi)$$

$$G_0 = G(\theta,\varphi)\big|_{\max} = e_{cd} D(\theta,\varphi)\big|_{\max} = e_{cd} D(\theta,\varphi)$$

POSTECH

GAIN

- We can define absolute gain of the antenna by taking into account reflection losses

$$G_{abs}(\theta,\varphi) = (1-|\Gamma|^2)G(\theta,\varphi) =$$

$$e_r e_{cd} D(\theta,\varphi) = e_0 D(\theta,\varphi)$$

- Partial gain: "that part of the radiation intensity corresponding to a given polarization divided by the total radiation intensity that would be obtained if the power accepted by the antenna were radiated isotropically."

GAIN

- In a spherical coordinate system, we have

$$G_0 = G_\theta + G_\varphi$$

$$G_\theta = \frac{4\pi U_\theta}{P_{in}} \qquad G_\theta = \frac{4\pi U_\varphi}{P_{in}}$$

- $U_{\theta,\ \varphi}$ = radiation intensity in a given direction contained $E_{\theta,\varphi}$

POSTECH

GAIN

- Note that we ALWAYS present the gain and directivity values in dB

$$G(\mathrm{dB}) = 10\log_{10}(G(\mathrm{dimensionless}))$$

$$D(\mathrm{dB}) = 10\log_{10}(D(\mathrm{dimensionless}))$$

Example 2.10 (Resonant, lossless λ/2 dipole):

$$Z_{in} = 73, \;\; Z_c = 50, \;\; \text{lossless} \;\; \Rightarrow e_{cd} = 1$$

$$U = B_0 \sin^3 \theta$$

Solution:

$$D_0 = 4\pi \frac{U_{\max}}{P_{rad}}, \;\; G_0 = e_0 D_0$$

$$U_{\max} = B_0 \sin^3 \theta \Big|_{\max} = B_0$$

$$P_{rad} = \int_0^{2\pi}\int_0^{\pi} U(\theta,\phi)\sin\theta d\theta d\phi$$

$$= B_0 \int_0^{2\pi} \underbrace{\left[\int_0^{\pi} \sin^3\theta d\theta\right]}_{3\pi/8} d\phi$$

$$P_{rad} = B_0(2\pi)(3\pi/8) = B_0\left(\frac{3\pi^2}{4}\right)$$

$$D_0 = 4\pi\left[\frac{B_0}{B_0\left(3\pi^2/4\right)}\right] = \frac{16}{3\pi} = 1.697$$

$$G_0 = e_{cd} D_0 = (1)(1.697) = 1.967 = 2.297 \text{ dB}$$

$$e_r = \left(1 - |\Gamma_{in}|^2\right)$$

$$|\Gamma_{in}| = \left|\frac{Z_{in} - Z_c}{Z_{in} + Z_c}\right| = \left|\frac{73 - 50}{73 + 50}\right| = 0.187$$

$$e_r = \left(1 - |0.187|^2\right) = 1 - 0.035 = 0.965$$

$$e_0 = e_r e_{cd} = 0.965(1) = 0.965$$

$$e_0(\text{dB}) = 10\log_{10}\left[0.965(1)\right] = \underbrace{10\log_{10}(0.965)}_{-0.155 \text{ dB}} + \underbrace{10\log_{10}(1)}_{0 \text{ dB}}$$

$$G_{abs} = e_o D_o = 0.965(1.697) = 1.638 = 2.242 \text{ dB}$$

$$=(2.297-0.155) = 2.242 \text{ dB}$$

Beam Efficiency

- For an antenna with its major lobe directed along z-axis ($\theta = 0°$) the beam efficiency (BE) is defined by

$$\text{BE} = \frac{\text{Power transmitted (received) within cone angle } \theta_1}{\text{Power transmitted (received) by the antenna}}$$

- Note that beam efficiency is dimensionless
- θ_1 is the half-angle of the cone within which the percentage of the total power is to be found

$$\mathbf{BE} = \frac{\int_0^{2\pi}\int_0^{\theta_1} U(\theta,\varphi)\sin\theta\, d\theta\, d\varphi}{\int_0^{2\pi}\int_0^{\pi} U(\theta,\varphi)\sin\theta\, d\theta\, d\varphi}$$

POSTECH

Beam Efficiency

- What does beam efficiency indicate?
  - It pretty much tells us how much of the power is carried out in the main beam (or a percentage of the main beam) as opposed to the minor lobes

Bandwidth

- The bandwidth of an antenna is a range of frequencies over which the performance of the antenna is within acceptable limits.
  - What does this mean?
  - An antenna has a number of radiation and electrical parameters such as: Input impedance, polarization, radiation patterns, direction of maximum radiation, gain, radiation efficiency, etc.
  - Consequently, definition of bandwidth is tricky. While over a certain frequency range some of these might not vary significantly, others may vary considerably.

POSTECH

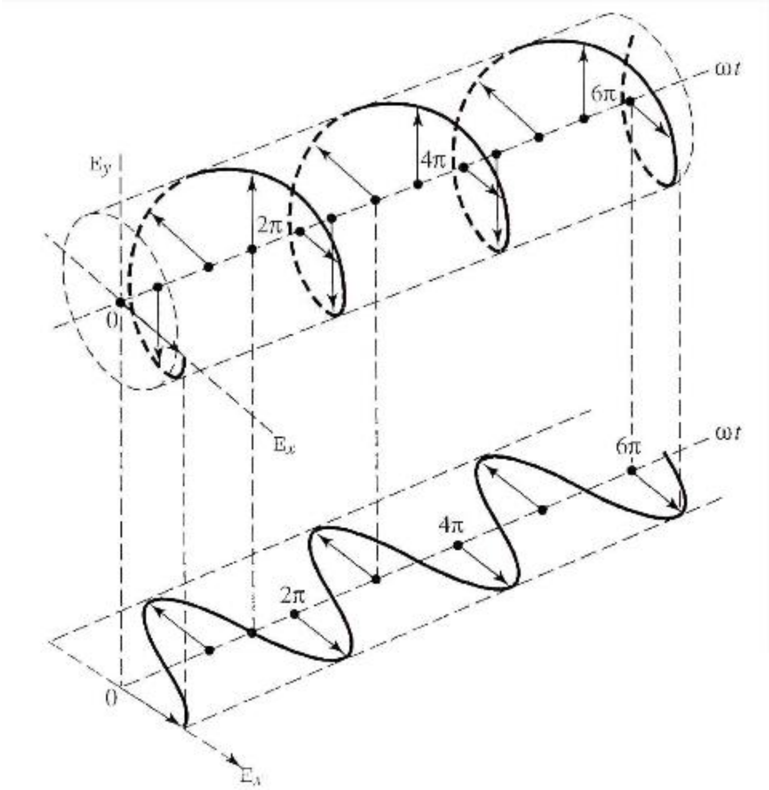
Bandwidth

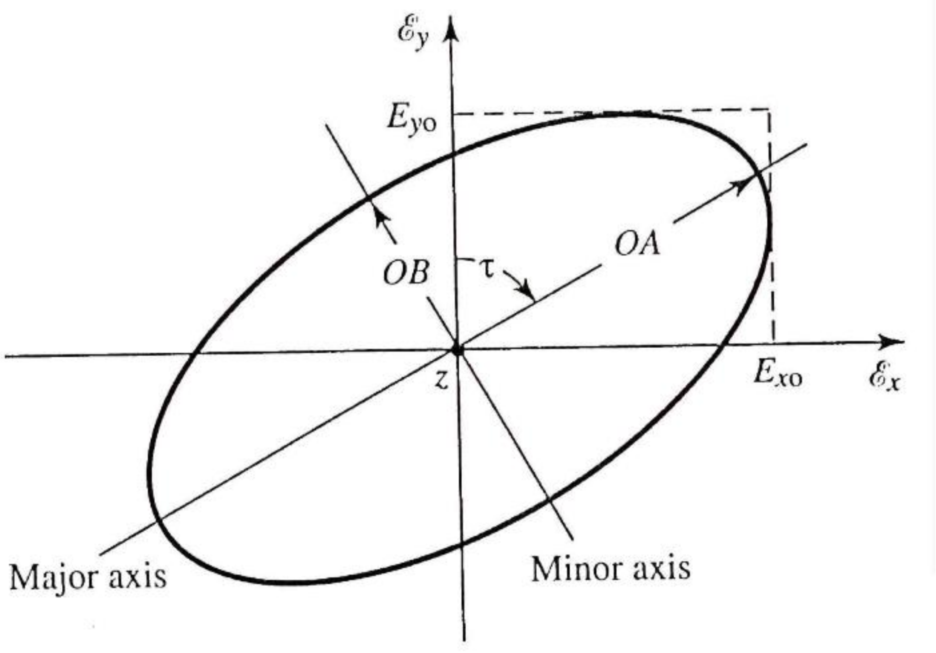
- Generally, we express the bandwidth of the antenna in either of two forms:
  - For narrow band antennas, as a percentage of the center frequency of operation.
    - e.g., f_1 and f_2 ➔ BW [%] = $(f_2-f_1)/\{(f_2+f_1)/2\}*100$
  - For wideband antennas as the ratio of the upper and lower frequencies of operation.
    - e.g., f_1 and $f_2=10\ f_1$ ➔ BW is 10:1

Polarization

- Polarization of the antenna in a given direction is defined as "the polarization of the wave transmitted (radiated) by the antenna"
- When direction is not stated, we always assume direction of maximum radiation
- Polarization of a radiated wave is defined as "that property of an electromagnetic wave describing the time varying direction and relative magnitude of the electric field vector."
- Consider the instantaneous electric field vector varying as a function of time. Now fix your observation location and monitor the time variation of the tip of this electric field vector. The figure traced as a function of time by the tip of this vector and the sense in which it is traced (e.g. rotates) specifies the polarization of the wave

Polarization





Fix location and variable time

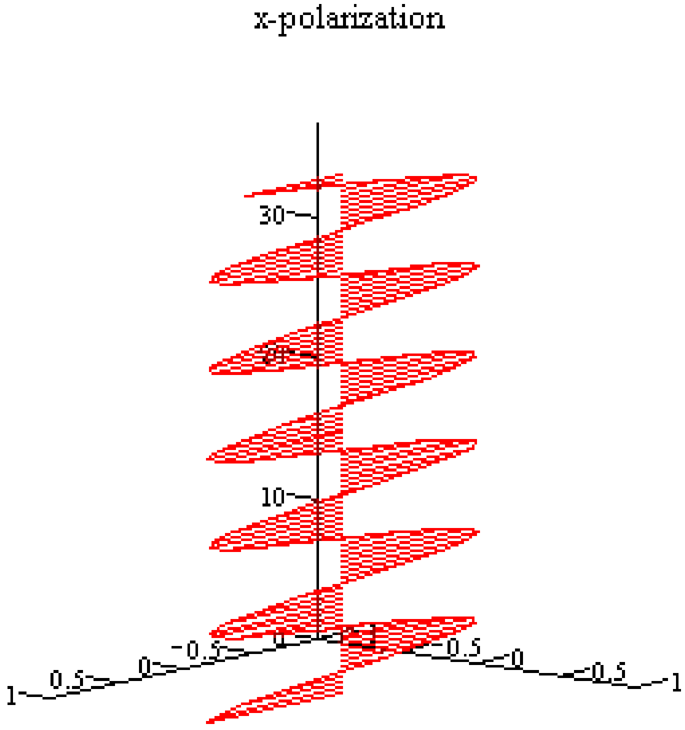
Polarization

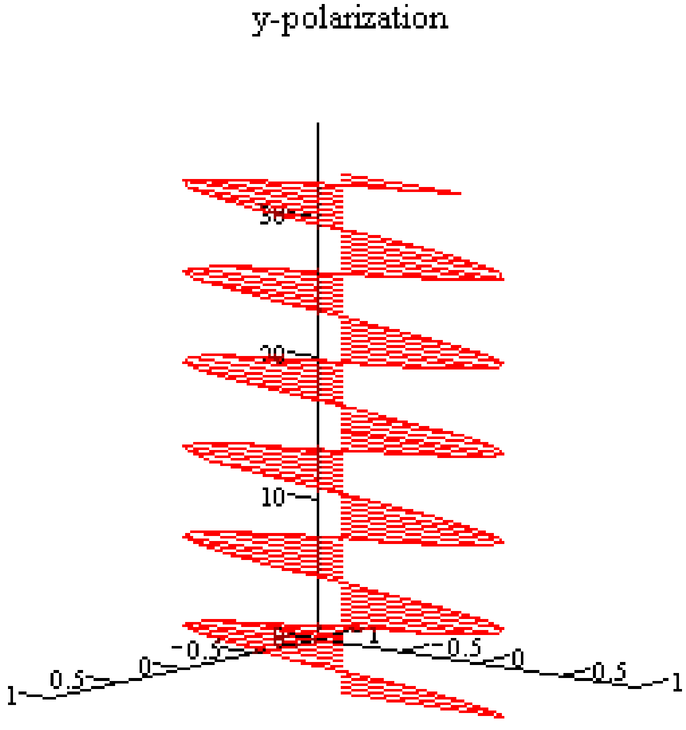
- Polarization of an antenna is ALWAYS defined in its transmitting mode
  - Basically, in the far field we have a plane wave and the polarization of the antenna is the same as that of the transmitted wave.
- If we have an antenna that is used as a receiver, its polarization would be the same as the
  - Polarization of a plane wave, incident from a given direction and having a given power flux density, which results in maximum available power at the antenna terminals
- For non-reciprocal antennas, the polarization in transmit and receive modes may be different but 99.9% of the antennas we work with are reciprocal antennas.

Polarization

- Different Types of Polarization:
  - Linear
    - Horizontal
    - Vertical
  - Circular
    - Right Handed Circular Polarization (RHCP)
    - Left Handed Circular Polarization (LHCP)
  - Elliptical
    - Right Handed Elliptical Polarization (RHEP)
    - Left Handed Elliptical Polarization (LHEP)
- Note that Elliptical polarization is the most general case and the linear and circular polarizations are especial cases of elliptical polarizations.

Polarization

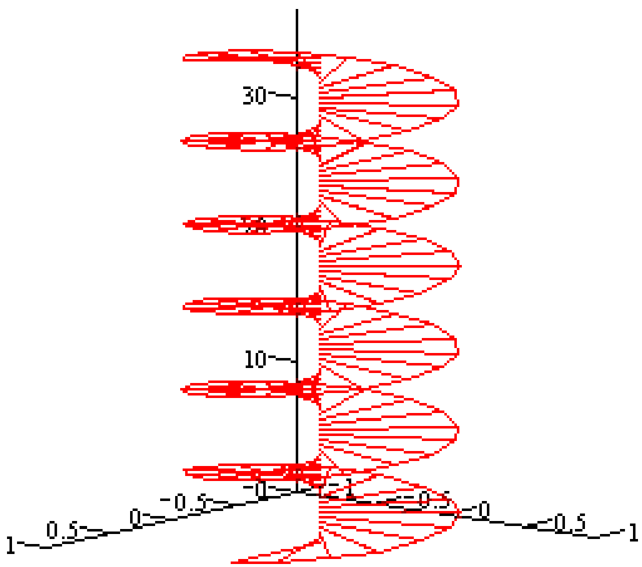




POSTECH

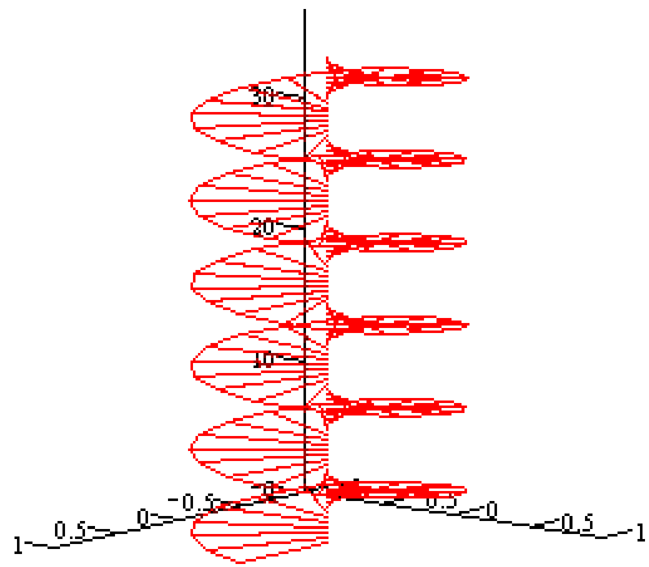
Polarization

LHC-polarization



X_L, Y_L, Z_L

RHC-polarization



X_R, Y_R, Z_R