2016 QE Math. (Part 1)

1. (25 pt) You have an ellipse defined by $x = 2\cos t$, $y = 3\sin t$. Find the area enclosed by the ellipse. Obtain the volume obtained by rotating the ellipse around the y- axis.

2. (25 pt) Let

$$\begin{aligned} f(t) &= (\cos t)\, u_s(t) \\ g(t) &= \left(\frac{1}{2}t^2 - t + 2 - 2e^{-t}\right) u_s(t) \end{aligned}$$

where $u_s(t)$ is a unit step function. Find the convolution $f * g(t)$. Using the Laplace transform is absolutely necessary here.