광전자공학 Ch. 5

Thin optical components & Diffraction

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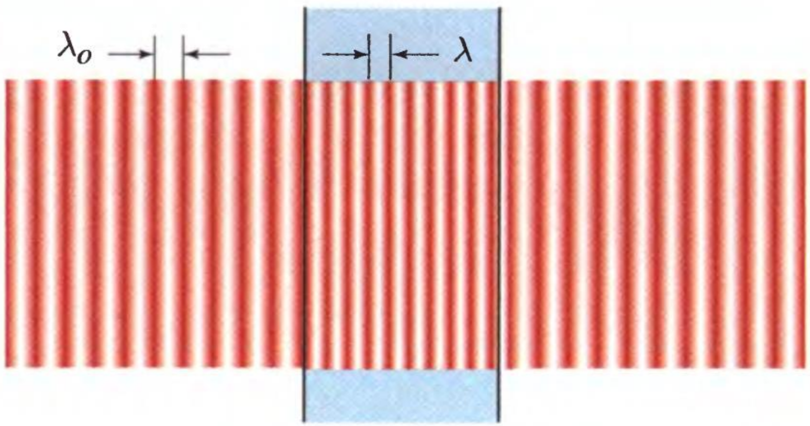
Thin optical elements

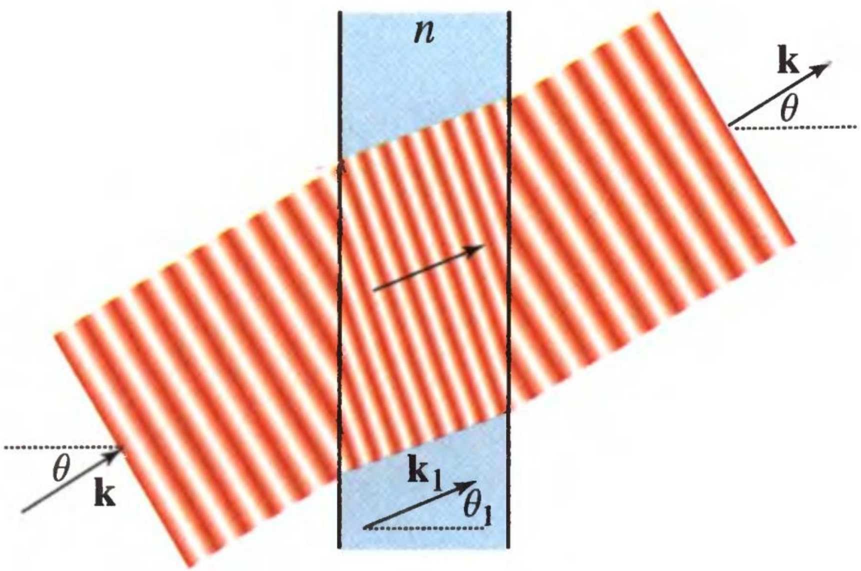
▶ Phase delay caused by thin optical elements



$$U(x,y,d)/U(x,y,0)$$

$$t(x,y) = \exp(-jnk_o d).$$

Transmittance of flat dielectric plate (normal incidence)



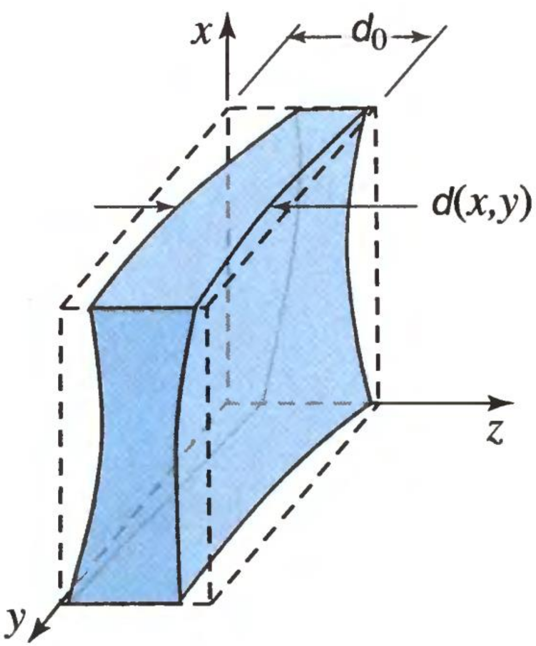
$$\exp(-j\mathbf{k}_1 \cdot \mathbf{r}) = \exp[-jnk_o(z\cos\theta_1 + x\sin\theta_1)]$$

$$t(x,y) = \exp(-jnk_o d\cos\theta_1)$$

Transmittance of flat dielectric plate (oblique incidence)

Thin optical elements

▶ Thin plate with variable thickness (refractive index = n)



Optical path length: $nd(x,y)+n_{air}(d_0-d(x,y))$

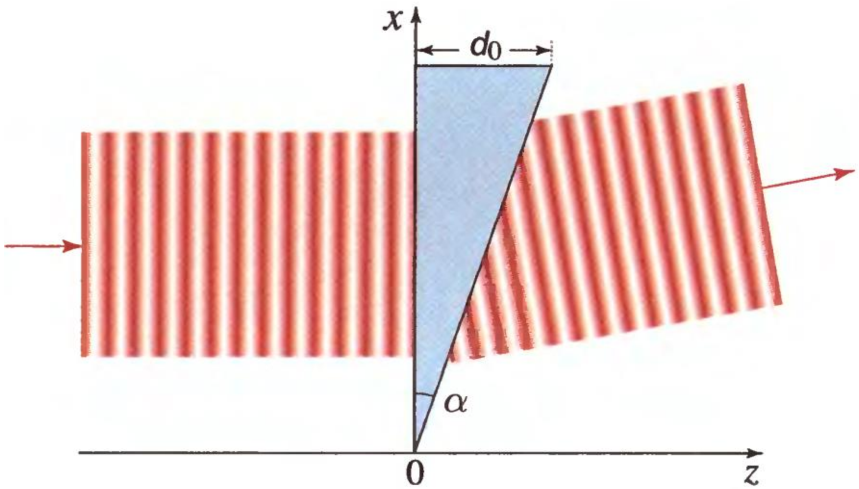
Phase delay:

$$t(x,y) \approx \exp[-jnk_o \mathbf{d}(x,y)]\exp[-jk_o(\mathbf{d}_0-\mathbf{d}(x,y))]$$

$$t(x,y) \approx h_0 \exp[-j(n-1)k_o \mathbf{d}(x,y)]$$

Transmittance of thin plate with variable thickness (slowly varying thickness)

Thin prism



When alpha is small,

$$t(x,y) = h_0 \exp[-j(n-1)k_o \alpha x]$$

Plane wave incidence

$$U(x,y,z) = A\exp(-jk_0 z) \qquad z<0$$

Outgoing wave (paraxial approximation)

$$U(x,y,z) = Ah_0 \exp(-j(n-1)\alpha k_0 x)\exp(-jk_z z) \qquad z>d$$

Outgoing wave have wavevector of $\mathbf{k} = \big((n-1)\alpha k_0, 0, k_z\big)$

$$k_z = \sqrt{k_0{}^2 - k_x{}^2} \approx k_0 \qquad \sin\theta_d \approx \theta_d \approx (n-1)\alpha$$

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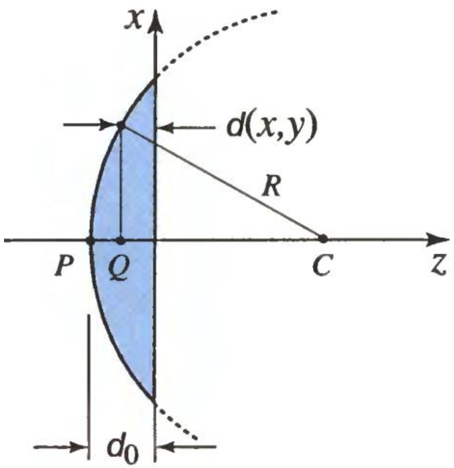
Thin lens



$$\mathsf{d}(x,y) = \mathsf{d}_0 - \left[R - \sqrt{R^2 - (x^2+y^2)}\right]$$

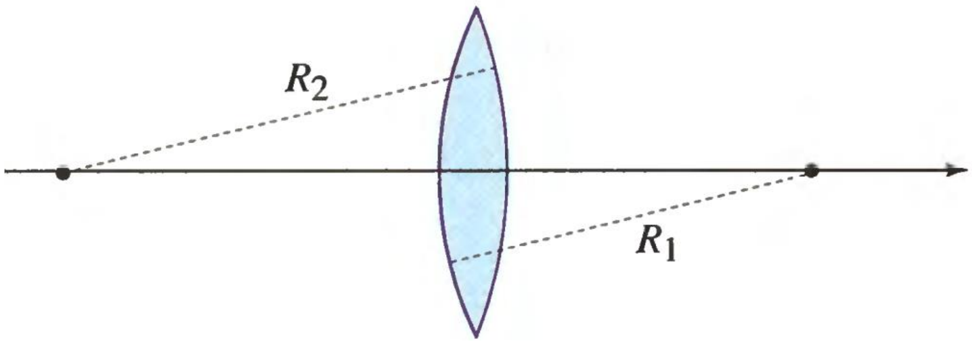
$$\sqrt{R^2 - (x^2+y^2)} \approx R\left(1 - \frac{x^2+y^2}{2R^2}\right)$$

$$\mathsf{d}(x,y) \approx \mathsf{d}_0 - \frac{x^2+y^2}{2R}$$

Transmittance of thin planoconvex lens

$$\mathsf{t}(x,y) \approx h_0 \exp\left[jk_o \frac{x^2+y^2}{2f}\right] \qquad f = \frac{R}{n-1}$$

Thin lens



A (double) convex lens

$$t_{R_1}(x,y) = h_1 \exp(jk_0 \frac{x^2+y^2}{2f_1})$$

$$t_{R_2}(x,y) = h_2 \exp(jk_0 \frac{x^2+y^2}{2f_2})$$

$$t(x,y) = h_1 h_2 \exp(jk_0 \frac{(x^2+y^2)}{2}\left(\frac{1}{f_1}+\frac{1}{f_2}\right)) = h_0 \exp(jk_0 \frac{(x^2+y^2)}{2f})$$

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} = (n-1)\left(\frac{1}{R_1} - \frac{1}{(-R_2)}\right)$$

Same fomula with ray-optic Assumption!

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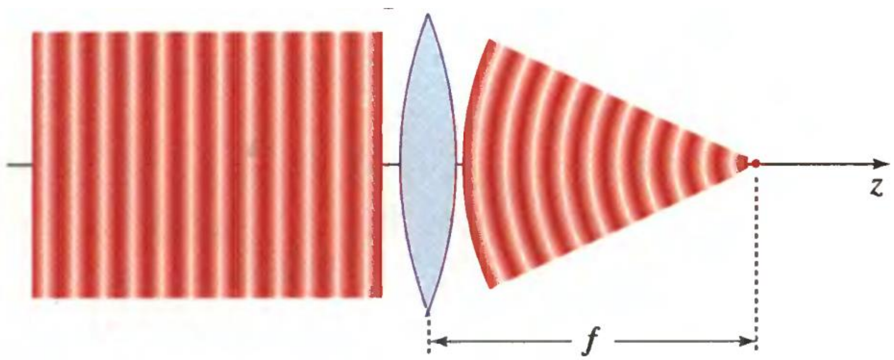
Thin lens

Plane wave incidence

$$U(x,y,z) = A\exp(-jk_0 z) \qquad z<0$$

Outgoing wave (paraxial approximation) Wavefront Φ

$$U(x,y,z) \approx Ah_0 \exp\left(-jk_0 \frac{-x^2}{2f}\right)\exp(-jk_0 z) = C\exp\left(-j\boxed{k_0\left(\frac{-x^2}{2f}+z\right)}\right) \qquad z>d$$



Wavevector at position x_0

$$\nabla\Phi = \mathbf{k} = \left(-k_0 x_0 / f, 0, k_0\right)$$

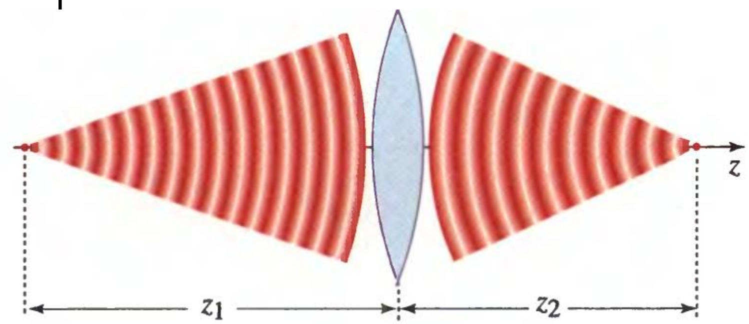
It focused to focal point!

Thin lens

When Paraboloidal wave (source at −z1) is incident light

$$U(x,y,0)=\frac{A}{z_1}\exp(-jk_0 z_1)\exp\left(-jk_0\frac{x^2}{2z_1}\right) \qquad z=0$$

$$t(x,y)=h_0\exp\left(jk_0\frac{x^2}{2f}\right)$$



Outgoing wave

$$U(x,y,z)=B\exp(-jk_0 z)\exp\left(-jk_0\frac{x^2}{2}\boxed{\left(\frac{1}{z_1}-\frac{1}{f}\right)}\right) \qquad z>0$$

$$\frac{1}{z_2}$$

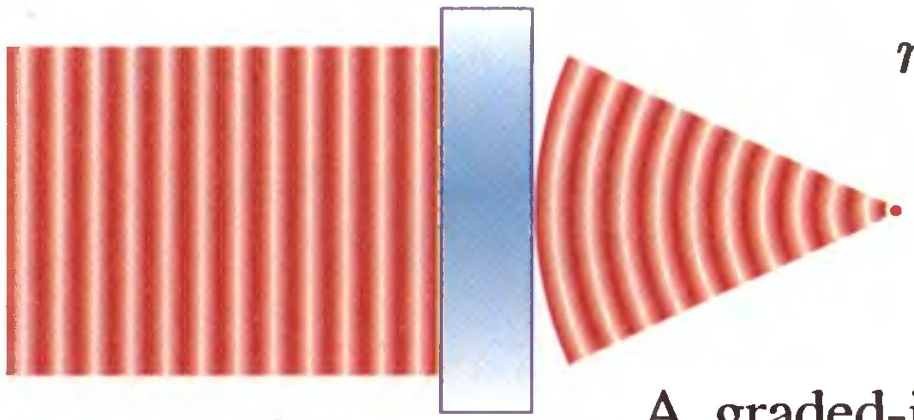
$$\frac{1}{z_1}+\frac{1}{z_2}=\frac{1}{f}$$

Imaging equation
(Derived with Wave Opt.)

Graded-index optical element

Phase of transmittance can be adjusted not only varying thickness but also varying refractive index

$$t(x,y) = \exp\left[-jn(x,y)k_o d_0\right]$$



$$n(x,y) = n_0\left[1 - \tfrac{1}{2}\alpha^2(x^2+y^2)\right]$$

A graded-index plate

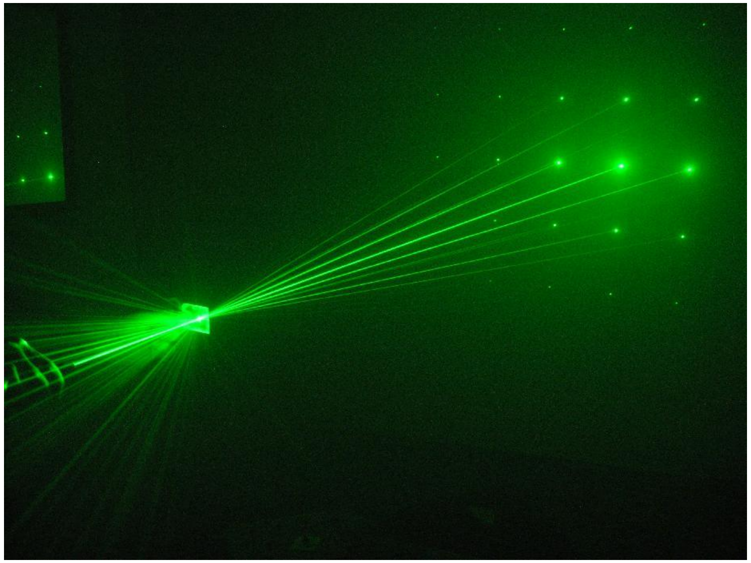
When both of thickness and refractive indexed are varying,

$$t(x,y) \approx h_0 \exp[-j(n(x,y)-1)k_0 d(x,y)]$$

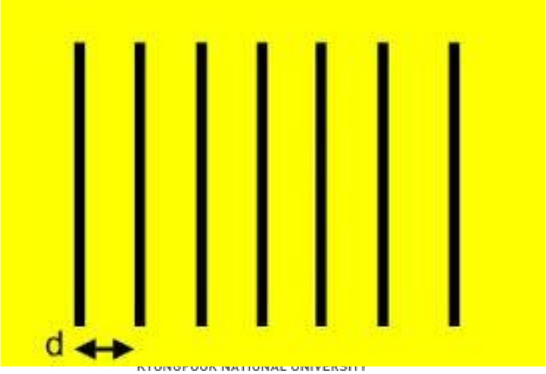
Diffraction gratings

Diffraction gratings: Periodic slits or apertures that can diffract light

Light diffraction



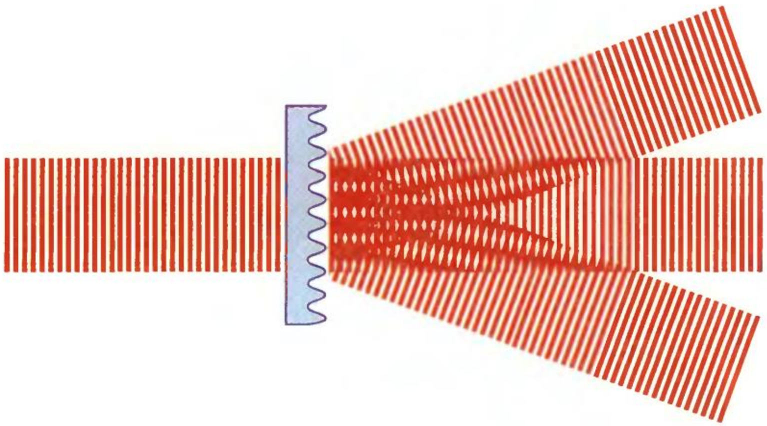
Diffraction from DVD



Diffraction gratings

Diffraction gratings

Angle diffracted from the diffraction grating



When $\lambda \ll \Lambda$ is small (paraxial approximation)

$$\theta_q = \theta_i + q\frac{\lambda}{\Lambda}$$

In general case,

$$\sin\theta_q = \sin\theta_i + q\frac{\lambda}{\Lambda}$$

Arbitrary periodic function (period of Λ) can be expressed as,

$$t(x) = \sum_{n=-\infty}^{\infty} C_n e^{-j\frac{2\pi n}{\Lambda}x}$$

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Diffraction gratings

Incident plane wave $U(x,z) = A\exp(-j(k_0 \sin\theta_{in} x + k_0 \cos\theta_{in} z))$

Outgoing wave (after passing the gratings)

$$U(x,z) = \sum_{n=-\infty}^{\infty} C_n A\exp(-j(k_0 \sin\theta_{in} + \frac{2\pi n}{\Lambda})x - jk_z z)$$

The diffraction angle can be driven as

$$\sin\theta_{out} = \sin\theta_{in} + \frac{2\pi n}{\Lambda k_0} = \sin\theta_{in} + n\frac{\lambda}{\Lambda}$$

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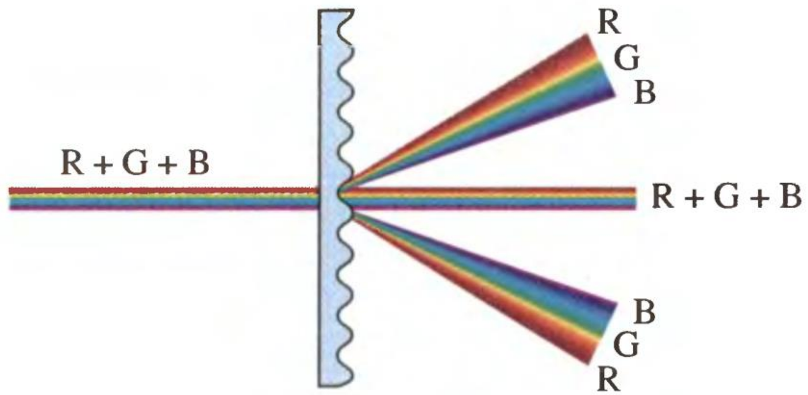
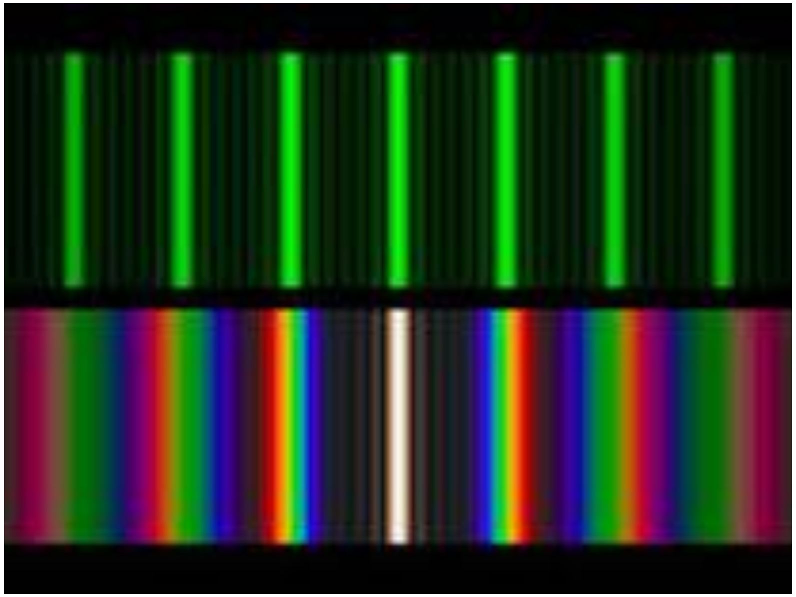
Diffraction gratings

Diffraction angle depends on period of the gratings and wavelength of incident light



Monochromatic light

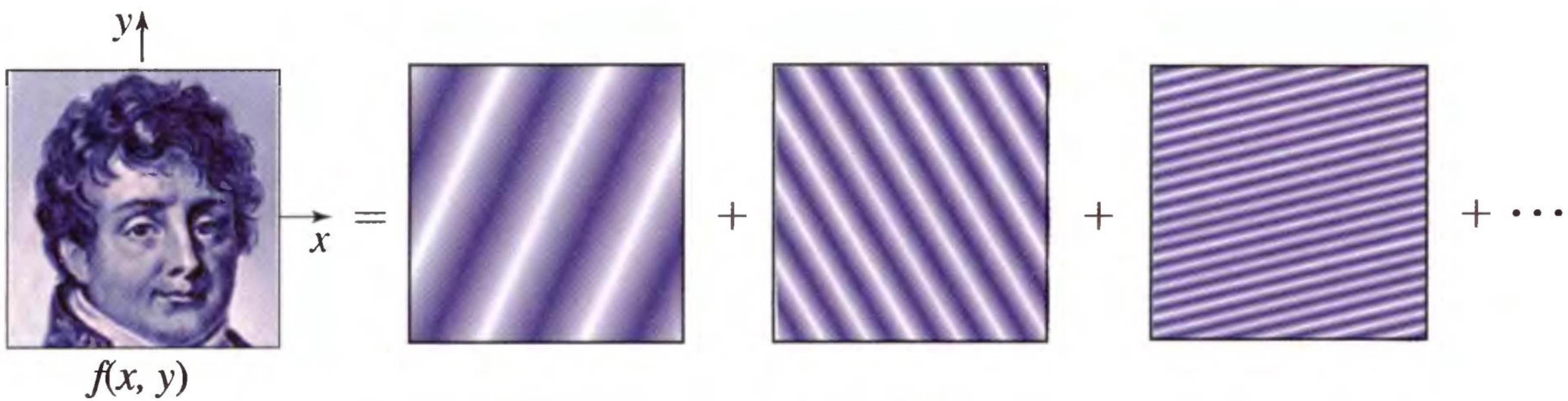


White light

2D-Fourer transform

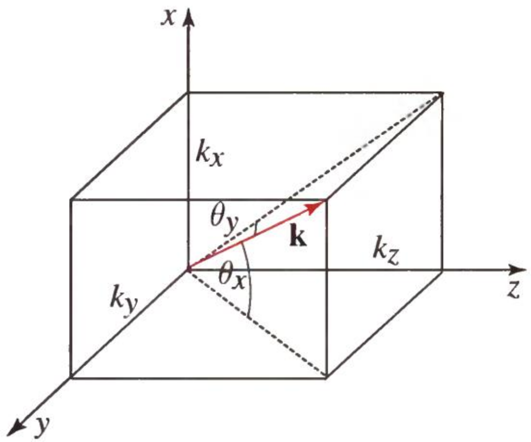
1D Fourier transform: Any arbitrary function can be expressed by the sum of infinite sin, cos waves having different frequencies.

2D Fourier transform: Any Image (2D-function) can be expressed by the sum of infinite sin, cos waves having different frequencies.



Spatial frequency of light

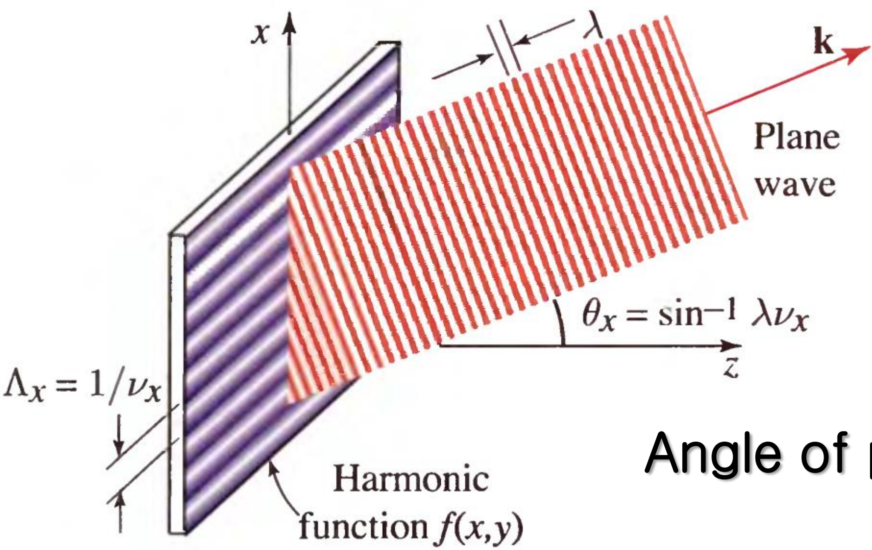
A Plane wave $U(x, y, z) = A \exp[-j(k_x x + k_y y + k_z z)]$



express spatial wavelength in x-y plane

$$\lambda_x = \frac{2\pi}{k_x} = \frac{1}{\nu_x} \quad , \quad \lambda_y = \frac{2\pi}{k_y} = \frac{1}{\nu_y}$$

$$\theta_x = \sin^{-1} \lambda\nu_x, \quad \theta_y = \sin^{-1} \lambda\nu_y.$$



If $k_x \ll k$ and $k_y \ll k$,

$$\theta_x \approx \lambda\nu_x, \quad \theta_y \approx \lambda\nu_y.$$

(Paraxial approximation)

Angle of plane wave is related to spatial frequency

Light transmittance

Transmitted wavefunction is <u>multiplication</u> of transmittance and incident wavefunction

$$U(x,y,d) = t(x,y)U(x,y,0)$$

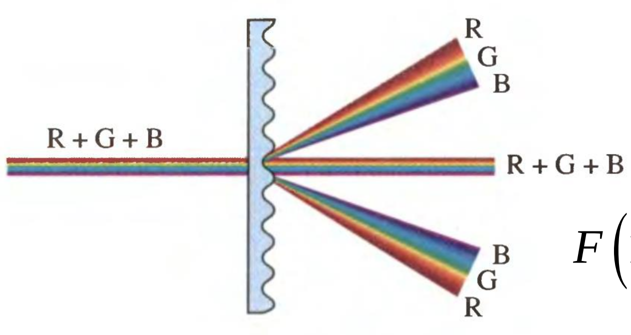
Using Fourier-transform,

$$F\left(U(x,y,d)\right) = F\left(t(x,y)\right) * F\left(U(x,y,0)\right)$$

Angular spectrum of transmitted light is convolution of Fourier transformed transmittance and angular spectrum of incident light.

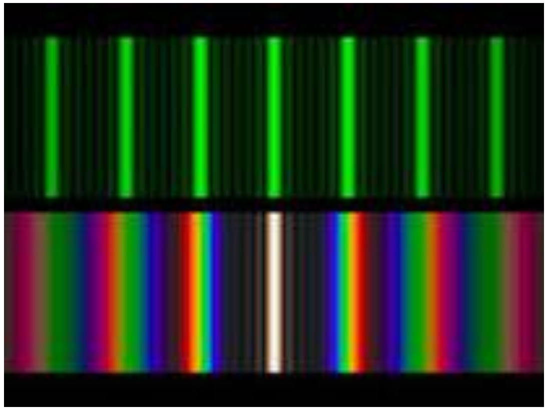
Diffraction – revisited

- An element with a transmittance that varies as $1 + \cos(2\pi\nu_y y)$ behaves as a diffraction grating (see Exercise 2.4-5); the incident wave is bent into right and left components, and a portion of it travels straight through.



$$F\left(U(x,y,0)\right) = \delta(k_x,k_y)$$

$$F\left(1+\cos\left(k_{y0}y\right)\right) = \delta(k_x,k_y) + \frac{1}{2}\delta\left(k_x,k_y - k_{y0}\right) + \frac{1}{2}\delta\left(k_x,k_y + k_{y0}\right)$$



$$F\left(U(x,y,d)\right) = F\left(t(x,y)\right) * F\left(U(x,y,0)\right)$$

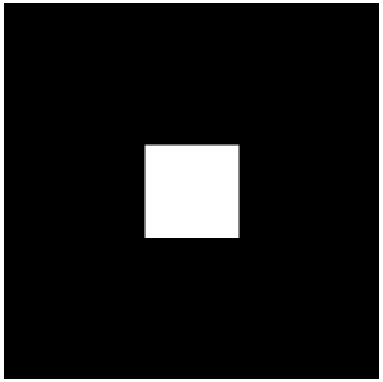
$$= \delta(k_x,k_y) + \frac{1}{2}\delta\left(k_x,k_y - k_{y0}\right) + \frac{1}{2}\delta\left(k_x,k_y + k_{y0}\right)$$

$$\approx \delta\left(\frac{\theta_x}{\lambda},\frac{\theta_y}{\lambda}\right) + \frac{1}{2}\delta\left(\frac{\theta_x}{\lambda},\frac{\theta_y}{\lambda} - \frac{\theta_{y0}}{\lambda}\right) + \frac{1}{2}\delta\left(\frac{\theta_x}{\lambda},\frac{\theta_y}{\lambda} + \frac{\theta_{y0}}{\lambda}\right)$$

Fraunhofer Diffraction

Consider an aperture function of
$$t(x,y)=\begin{cases}1 & \text{for open area}\\ 0 & \text{for blocked area}\end{cases}$$

A square aperture



If incident wave is normal plane wave,

$$F\left(U(x,y,0)\right)=\delta(k_x,k_y)$$

Angular spectrum of transmitted light is a Fourier transformed aperture function

$$F\left(U(x,y,d)\right)=F\left(t(x,y)\right)*F\left(U(x,y,0)\right)=F\left(t(x,y)\right)$$

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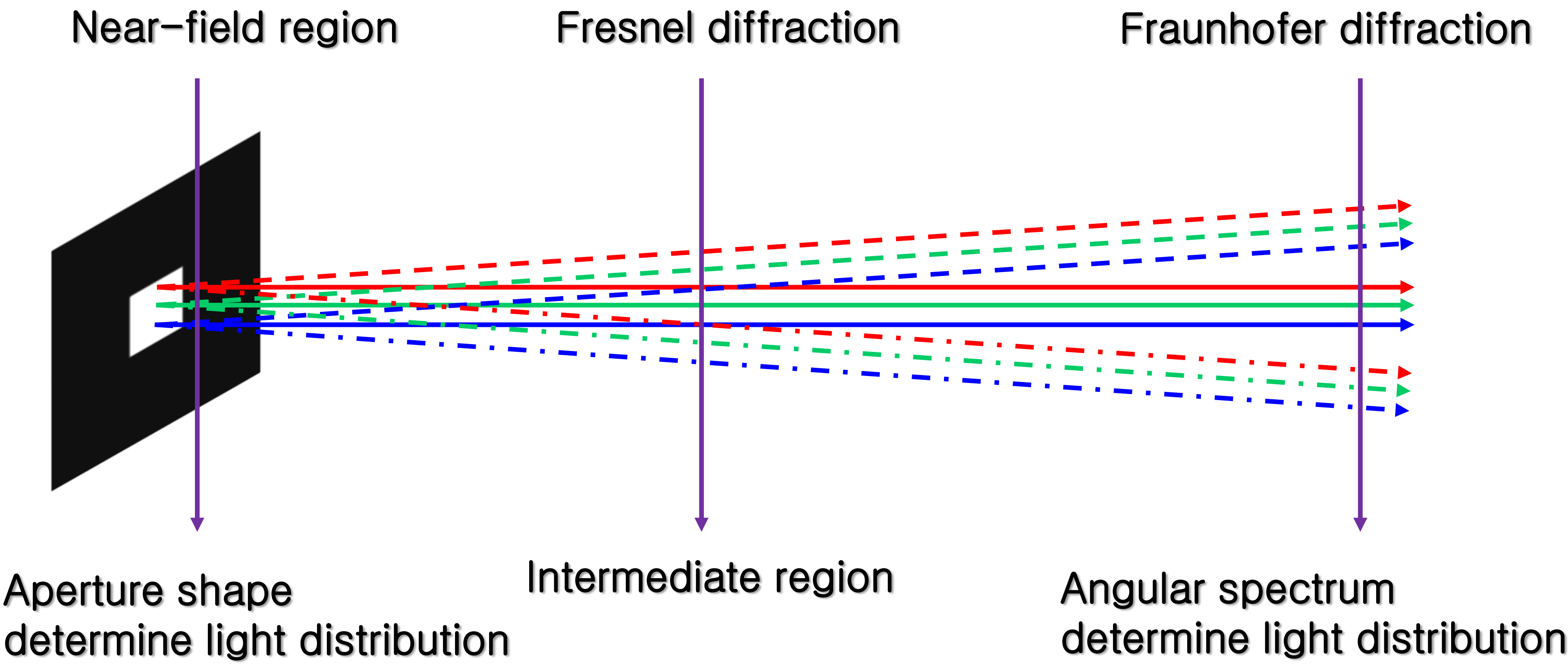
Fraunhofer Diffraction

If observing distance is very far away from the aperture,

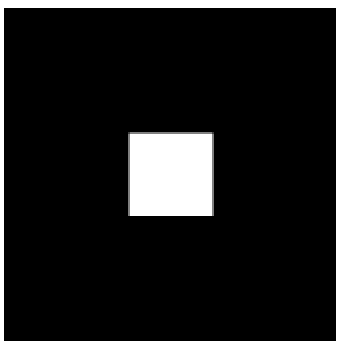


Field distribution finally became its angular spectrum

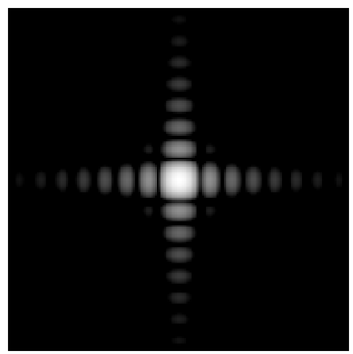
Fraunhofer Diffraction

In Fraunfoher diffraction zone, field distribution follow its angular spectrum, which is given as

$$U(x,y,d) \propto h_0 T(\theta_x,\theta_y) = h_0 T(\lambda \frac{x}{d}, \lambda \frac{y}{d}), \quad T(\nu_x,\nu_y) = F(t(x,y))$$



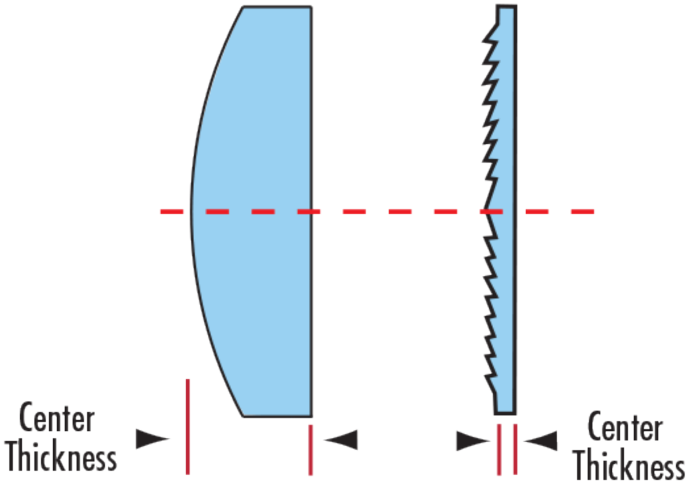
Aperture



Fraunfoher diffraction

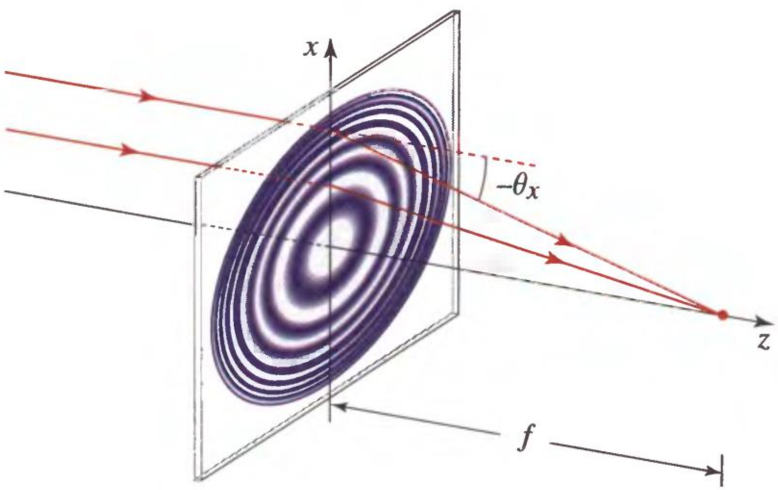
Can you guess the shape of initial aperture?

Fresnel lens

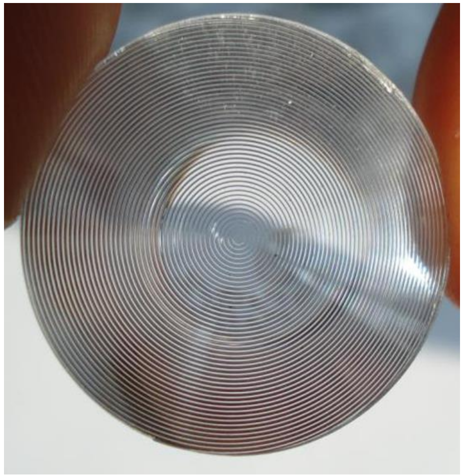


$$f(x,y) = \exp[j\pi(x^2+y^2)/\lambda f]$$

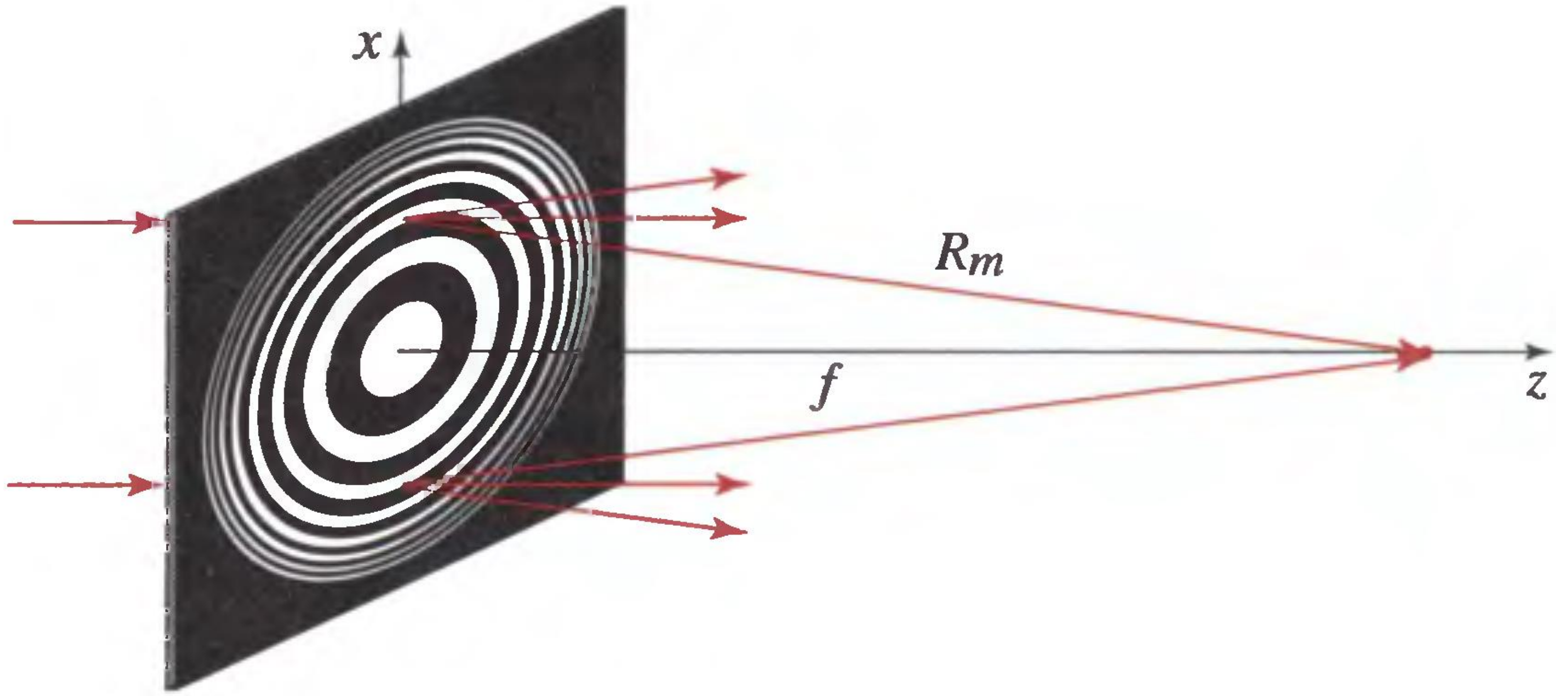
$$\theta_x = \sin^{-1}(\lambda\partial\phi/\partial x) = \sin^{-1}(-x/f)$$



Fresnel lens



Fresnel zone plate



Much easier to make compared to Frenel lens
However, multifocus appears and focusing efficiency is low

multiple focal lengths equal to $\infty, \pm f, \pm f/2, \ldots$.

Free-space transfer function

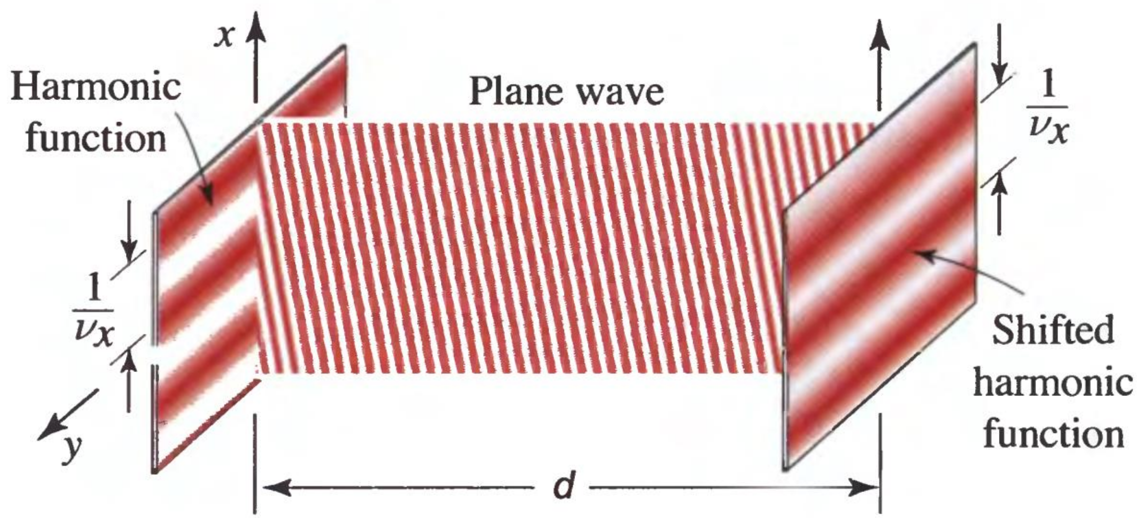
Transmittance of free-space for plane-wave (distance of d)

$$\exp(-jk_z d) \qquad k_z = \sqrt{k^2 - k_x^2 - k_y^2} = 2\pi\sqrt{\lambda^{-2} - \nu_x^2 - \nu_y^2}$$

Transmittance of free-space is given as a function of spatial frequency

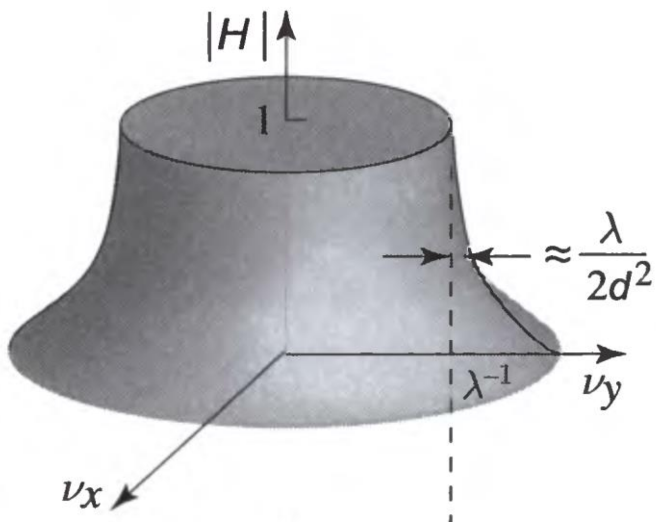
$$H(\nu_x, \nu_y) = \exp\left(-j2\pi d\sqrt{\lambda^{-2} - \nu_x^2 - \nu_y^2}\right)$$

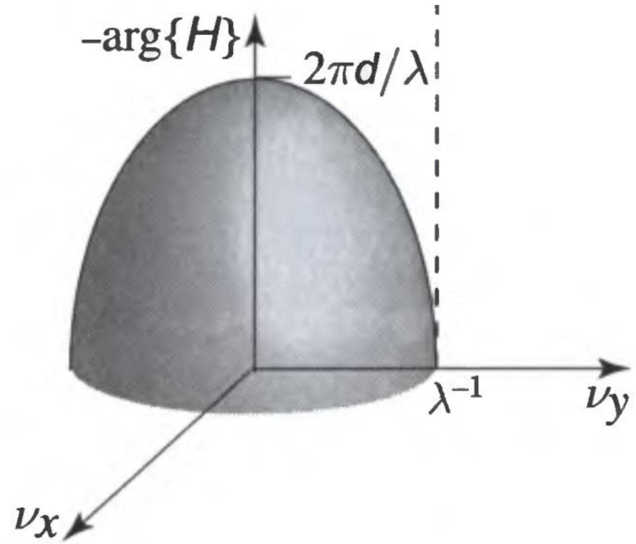
Transfer function of free space



Free-space transfer function

Amplitude and Phase of free-space transfer function





$k_x{}^2 + k_y{}^2 < k^2$ Propagating wave $\lambda_T > \lambda$

$k_x{}^2 + k_y{}^2 > k^2$ Evanescent wave $\lambda_T < \lambda$

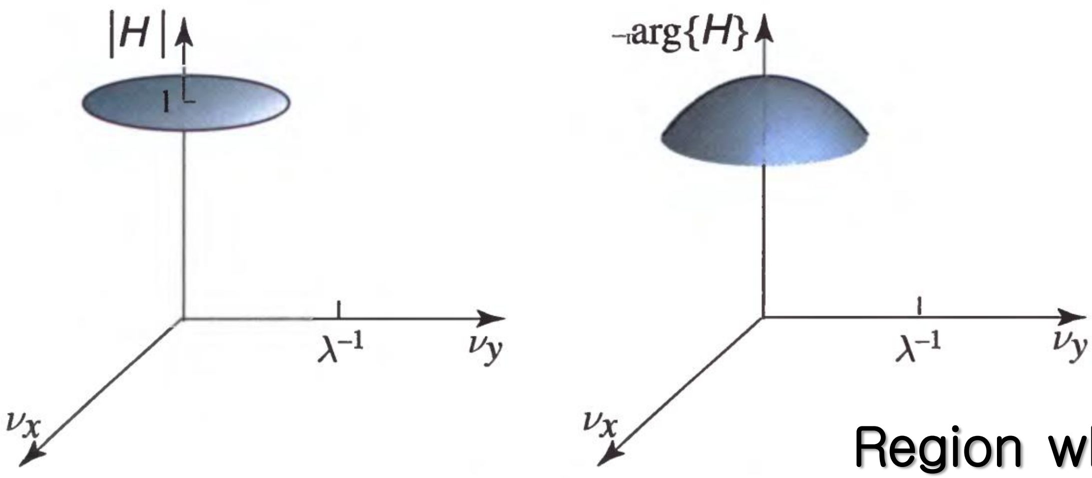
Fresnel approximation

If we can approximate

$$2\pi d\sqrt{\lambda^{-2}-\nu_x^2-\nu_y^2}=2\pi\frac{d}{\lambda}\sqrt{1-\theta^2}=2\pi\frac{d}{\lambda}\left(1-\frac{\theta^2}{2}\right)$$

$$H(\nu_x,\nu_y)\approx H_0\exp\left[j\pi\lambda d\left(\nu_x^2+\nu_y^2\right)\right]$$

Transfer Function of Free Space
(Fresnel Approximation)



$$\frac{\theta^4 d}{4\lambda}\ll 1$$

Region where we can approximate Hemicircular arg(H) to paraboloid