# APPENDIX 2

## TABLE OF TRANSFORMATIONS OF REGIONS

(See Sec. 41)

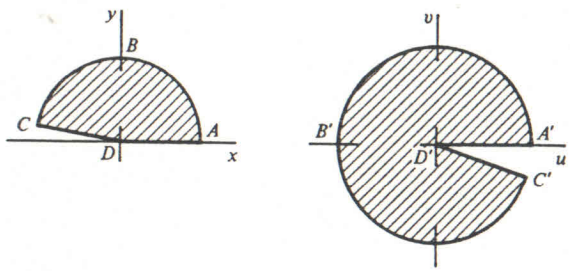


FIGURE 1.
$w = z^2$.

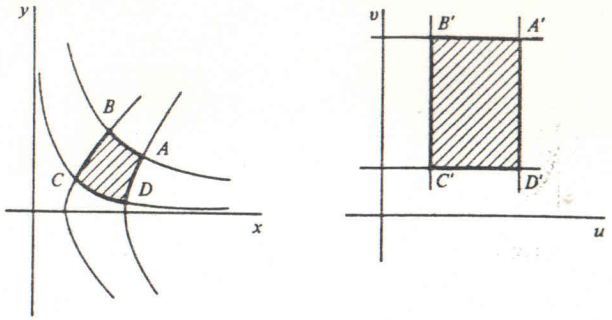


FIGURE 2.
$w = z^2$.

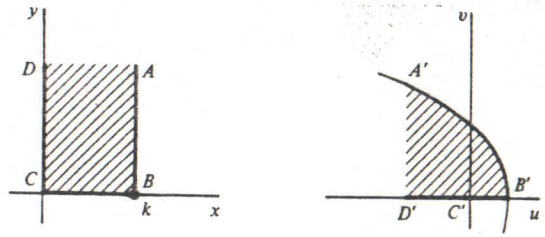


FIGURE 3.

$w = z^2$; $A'B'$ on parabola $\rho = \dfrac{2k^2}{1 + \cos\phi}$.

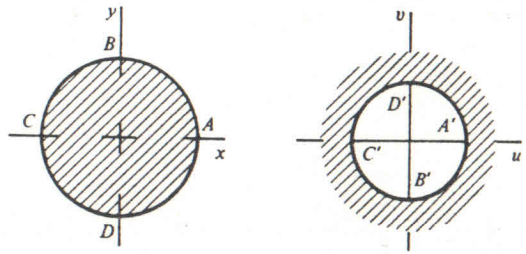


FIGURE 4.

$w = \dfrac{1}{z}$.

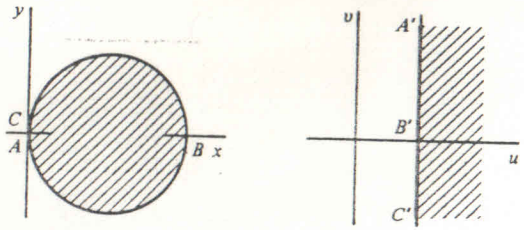


FIGURE 5.

$w = \dfrac{1}{z}$.

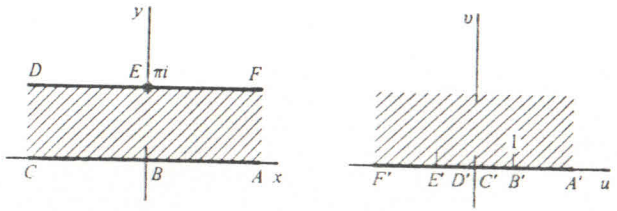


FIGURE 6.

$w = e^z$.

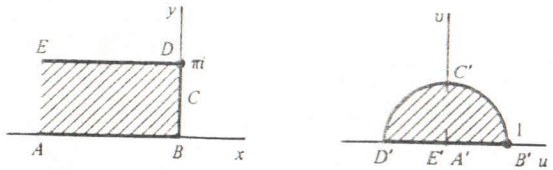


FIGURE 7.

$w = e^z$.

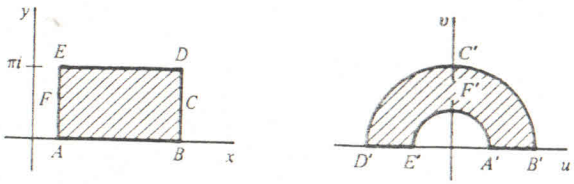


FIGURE 8.

$w = e^z$.

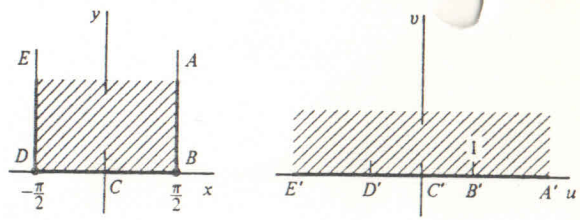


FIGURE 9.

$w = \sin z$.

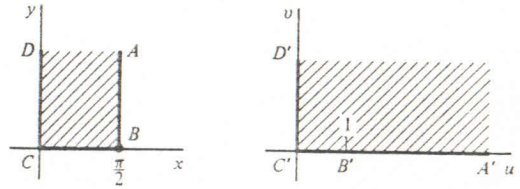


FIGURE 10.

$w = \sin z$.

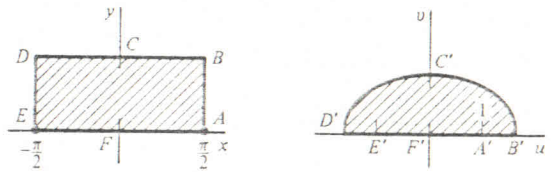


FIGURE 11.

$w = \sin z$; $BCD$ on line $y = k$, $B'C'D'$ on ellipse

$$\left(\frac{u}{\cosh k}\right)^2 + \left(\frac{v}{\sinh k}\right)^2 = 1.$$

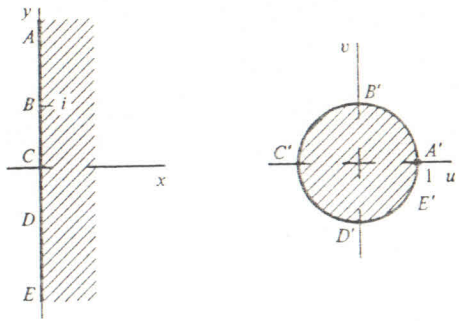


FIGURE 12.

$$w = \frac{z-1}{z+1}$$

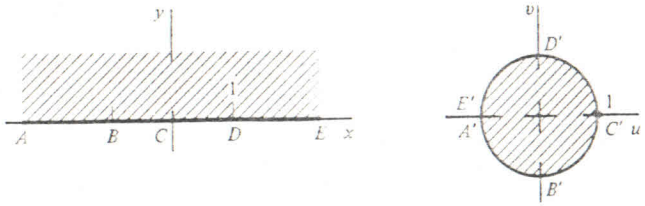


FIGURE 13.

$$w = \frac{i-z}{i+z}.$$

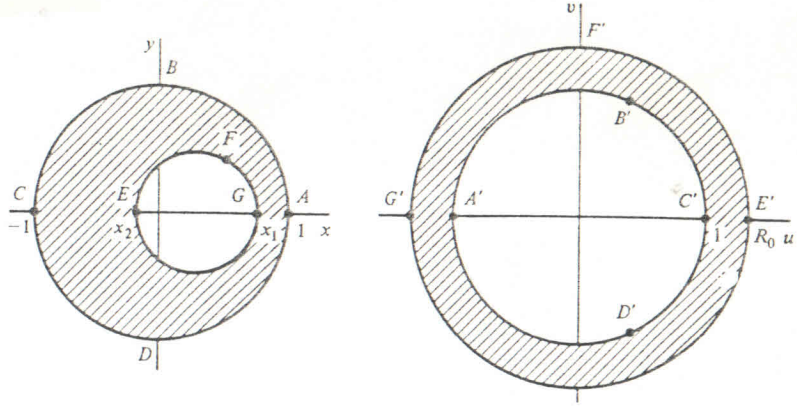


FIGURE 14.

$$w = \frac{z-a}{az-1};\ a = \frac{1+x_1x_2+\sqrt{(1-x_1^2)(1-x_2^2)}}{x_1+x_2};$$

$$R_0 = \frac{1-x_1x_2+\sqrt{(1-x_1^2)(1-x_2^2)}}{x_1-x_2}\ (a>1 \text{ and } R_0>1 \text{ when } -1<x_2<x_1<1).$$

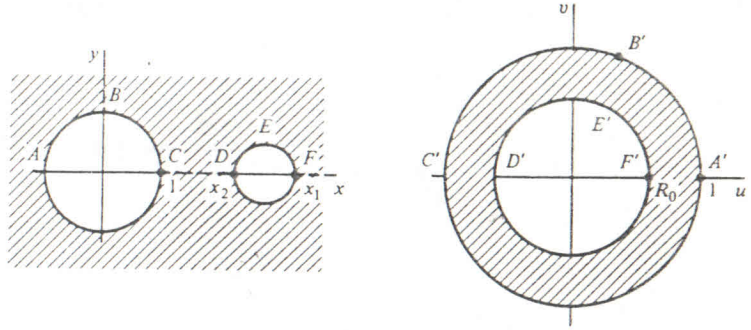


FIGURE 15.

$$w = \frac{z-a}{az-1};\ a = \frac{1+x_1x_2+\sqrt{(x_1^2-1)(x_2^2-1)}}{x_1+x_2};$$

$$R_0 = \frac{x_1x_2-1-\sqrt{(x_1^2-1)(x_2^2-1)}}{x_1-x_2}$$

$(x_2 < a < x_1 \text{ and } 0 < R_0 < 1 \text{ when } 1 < x_2 < x_1).$

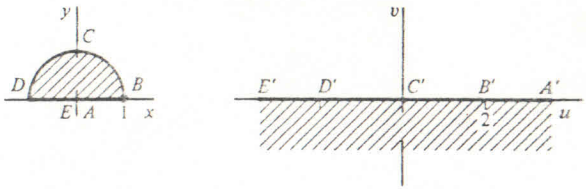


FIGURE 16.

$w = z + \dfrac{1}{z}$.

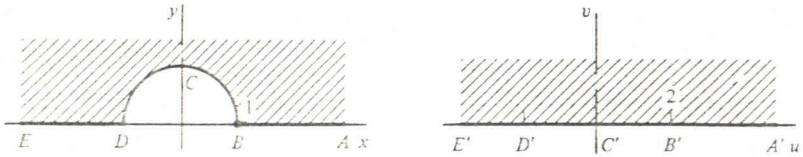


FIGURE 17.

$w = z + \dfrac{1}{z}$.

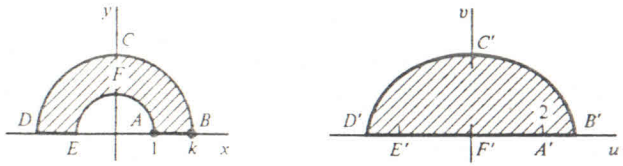


FIGURE 18.

$w = z + \dfrac{1}{z}$; $B'C'D'$ on ellipse $\left(\dfrac{ku}{k^2+1}\right)^2 + \left(\dfrac{kv}{k^2-1}\right)^2 = 1$.

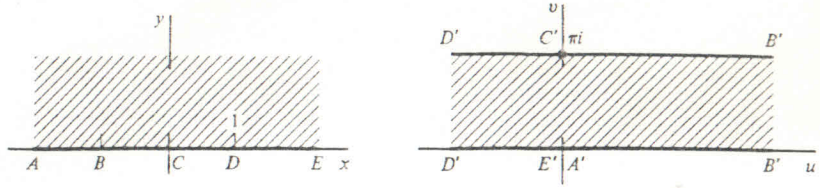


FIGURE 19.

$w = \operatorname{Log} \dfrac{z-1}{z+1}$; $z = -\coth \dfrac{w}{2}$.

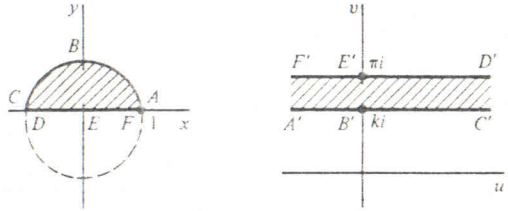


FIGURE 20.

$w = \operatorname{Log} \dfrac{z-1}{z+1}$; $ABC$ on circle $x^2 + y^2 - 2y \cot k = 1$.

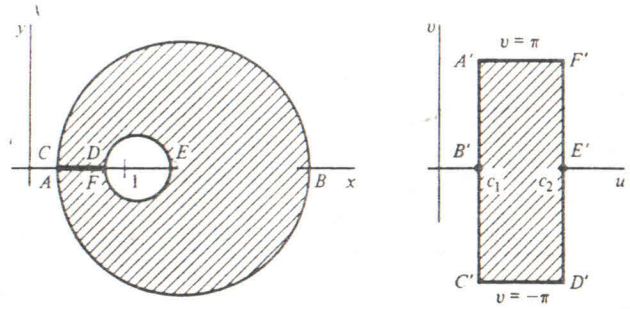


FIGURE 21.

$w = \operatorname{Log} \dfrac{z+1}{z-1}$; centers of circles at $z = \coth c_n$,

radii: $\operatorname{csch} c_n (n = 1, 2)$.

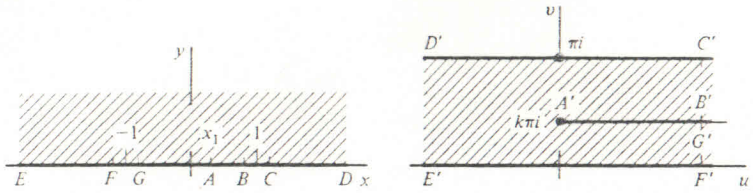


FIGURE 22.

$$w = k \operatorname{Log} \frac{k}{1-k} + \operatorname{Log} 2(1-k) + i\pi - k \operatorname{Log} (z+1) - (1-k) \operatorname{Log} (z-1),$$

$x_1 = 2k - 1$.


FIGURE 23.

$$w = \left(\tan \frac{z}{2}\right)^2 = \frac{1 - \cos z}{1 + \cos z}.$$

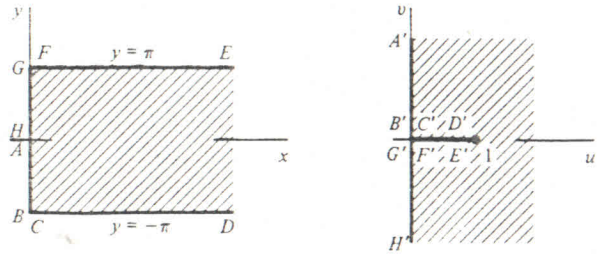


FIGURE 24.

$$w = \coth \frac{z}{2} = \frac{e^z + 1}{e^z - 1}.$$

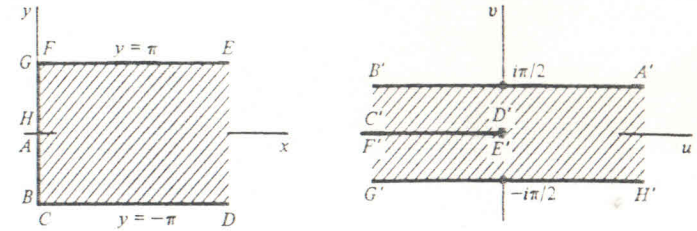


FIGURE 25.

$$w = \operatorname{Log} \coth \frac{z}{2}.$$

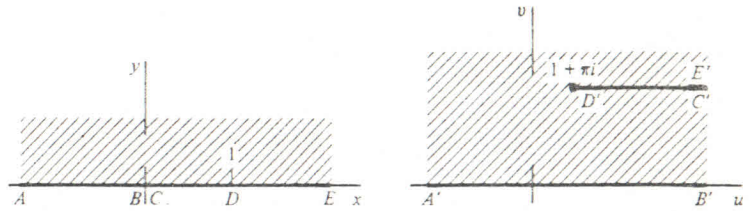


FIGURE 26.
$w = \pi i + z - \operatorname{Log} z$.

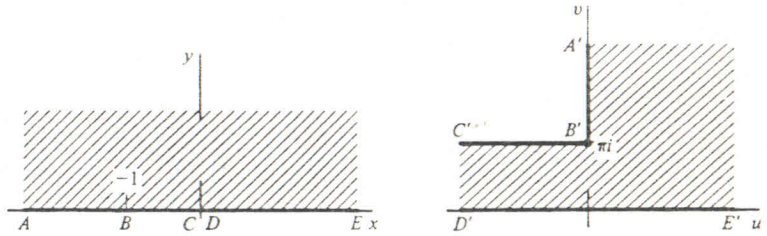


FIGURE 27.

$$w = 2(z+1)^{1/2} + \operatorname{Log} \frac{(z+1)^{1/2} - 1}{(z+1)^{1/2} + 1}.$$

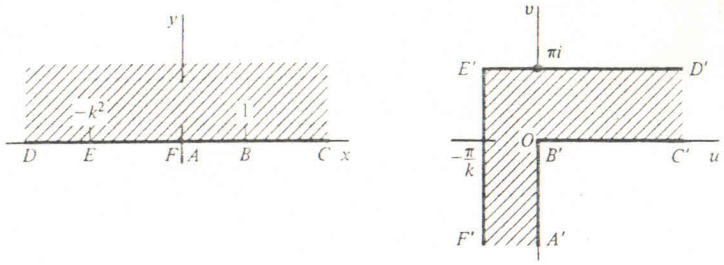


FIGURE 28.

$$w = \frac{i}{k} \operatorname{Log} \frac{1+ikt}{1-ikt} + \operatorname{Log} \frac{1+t}{1-t}; \; t = \left(\frac{z-1}{z+k^2}\right)^{1/2}$$

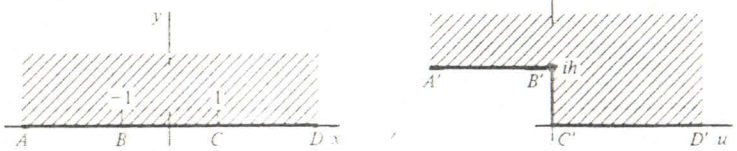


FIGURE 29.

$$w = \frac{h}{\pi} [(z^2-1)^{1/2} + \cosh^{-1} z].^*$$

* See Exercise 4, Sec. 98.

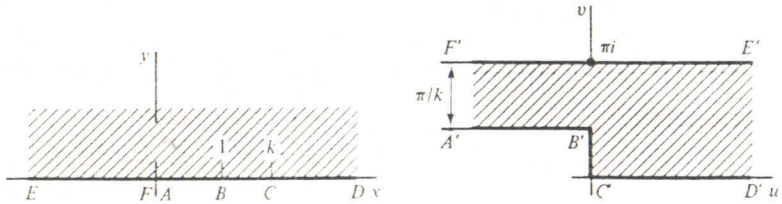


FIGURE 30

$$w = \cosh^{-1}\left(\frac{2z-k-1}{k-1}\right) - \frac{1}{k}\cosh^{-1}\left[\frac{(k+1)z-2k}{(k-1)z}\right].$$

# INDEX