④ Energy of a magnetic dipole in an external field

$$\vec{F} = \nabla(\vec{m}\cdot\vec{B}) \Rightarrow U = \int(-\vec{F})\cdot d\vec{x} = -\int \nabla(\vec{m}\cdot\vec{B})\cdot d\vec{x}$$
$$= -\vec{m}\cdot\vec{B}$$

employed in that treatment of magnetic effect on atomic energy level.

Zeeman effect:

splitting of atomic sublevels

Fine structure: spin interacting with the magnetic fied induced by nucleus seen by in the electron's frame

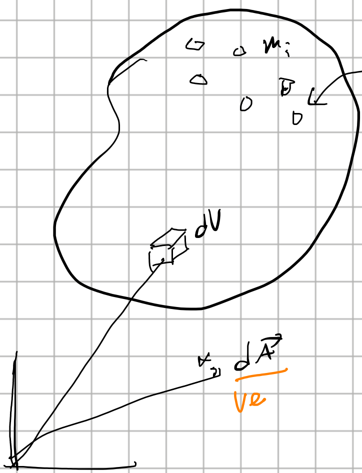
Hyper fine structure: magnetic moment of a nucleus in the magnetic field produced by electrons

B) Response of a material to magnetic field (macroscopic description)

- response in solid are macroscopic
- In a microscopic description, all the sources are assumed to be known, which is not generally

$$\nabla \cdot B_{micro} = 0 \rightarrow \nabla\cdot B_{macro} = 0$$

$$\nabla\times B_{micro} = \frac{4\pi}{c} J_{micro} \longrightarrow \nabla\times B_{macro} = ?$$



localized sources

intrinsic or localized current sources

$\rightarrow$ intrinsic or induced magnetic moments

$\vec{M} = \sum_i N_i \langle m_i \rangle$: magnetization magnetic dipole moments / unit volume

N_i : average number of atoms or molecules of type i

$\langle m_i \rangle$: average magnetic moment of type i

$J(x)$: macroscope current by free charges.

$$\Delta \vec{A} = \frac{1}{c} \frac{\vec{J}(\vec{x}')\Delta V}{|\vec{x}-\vec{x}'|} + \frac{\vec{M}(x') \times (\vec{x}-\vec{x}')}{|\vec{x}-\vec{x}'|^3} \Delta V$$

$$\vec{A}(\vec{x}) = \frac{1}{c}\int d^3x' \Big[\frac{\vec{J}(\vec{x}')}{|\ |} + c \underbrace{\frac{\vec{M}\times(\vec{x}-\vec{x}')}{|\ |^3}}_{\vec{M}\times\nabla'\left(\frac{1}{|\ |}\right)} \Big] = \frac{1}{c}\int d^3x' \Big[\frac{J}{|\ |} + \frac{c\,\nabla'\times M}{|\ |} \Big]$$

$$\nabla' \times \left(\frac{\vec{M}}{|\ |} \right) = \frac{\nabla'\times M}{|\ |} + \nabla'(\ )\times\vec{M}$$

Volume integral, Gauss' theorem, M is localized.

$$0 = \int d^3x' \frac{\nabla'\times M}{|\ |} - \int d^3x' \, \vec{M} \times \nabla' \left(\frac{1}{|\ |}\right)$$

$\vec{J}_M = c\, \nabla \times M$ plays a role of a current

: effective current density due to magnetization

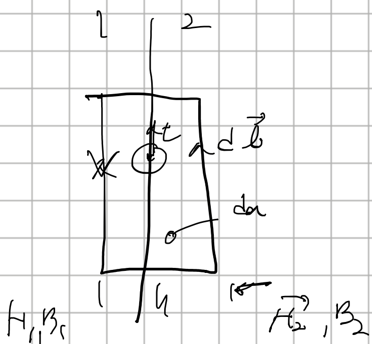
$(\text{cf. } \rho_p = -\nabla \cdot \vec{P})$

Boundary Conditions

$$\nabla \times \vec{H} = \frac{4\pi}{c} J \xrightarrow{\text{Stokes theorem}} \hat{n} \times (\vec{H}_2 - \vec{H}_1) = \frac{4\pi}{c} \vec{K}$$

$$\nabla \cdot \vec{B} = 0 \xrightarrow[\text{Gauss theorem}]{} (\vec{B}_2 - \vec{B}_1) \cdot \hat{n} = 0$$

Surface current density
$[J/\text{length}] = \text{statamp}/\text{cm}^3$



$$\int_A (\nabla \times \vec{H}) \cdot d\vec{a} = \frac{4\pi}{c} \int \vec{J} \cdot d\vec{a}$$

$$\oint \vec{H} \cdot d\vec{l} = \frac{4\pi}{c} \int \vec{J} \cdot \hat{t}\, da$$

$$d\vec{l} = dl\, (\hat{t} \times \hat{n}), \ \& \quad h \to 0$$

$$\Delta L\, (\vec{H}_2 - \vec{H}_1) \cdot (\hat{t} \times \hat{n}) = \frac{4\pi}{c} \vec{K} \cdot \hat{t}\, \Delta L$$

$$\to \hat{n} \times (\vec{H}_2 - \vec{H}_1) = \frac{4\pi}{c} \vec{K}$$

<u>$K = 0$</u>

$$\vec{B}_2 \cdot \hat{n} = \vec{B}_1 \cdot \hat{n}$$

$$\mu_2 H_2 \hat{n} = \mu_1 H_1 \hat{n} \quad , \quad \hat{n} \times H_2 = \hat{n} \times H_1$$

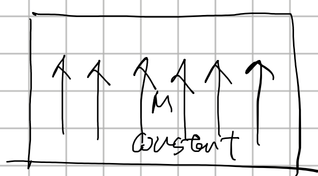
$$H_2 \cdot \hat{n} = \frac{\mu_1}{\mu_2} \vec{H}_1 \cdot \hat{n}$$

The surface of a very highly-permeable is like the surface of a conduction for electric field.

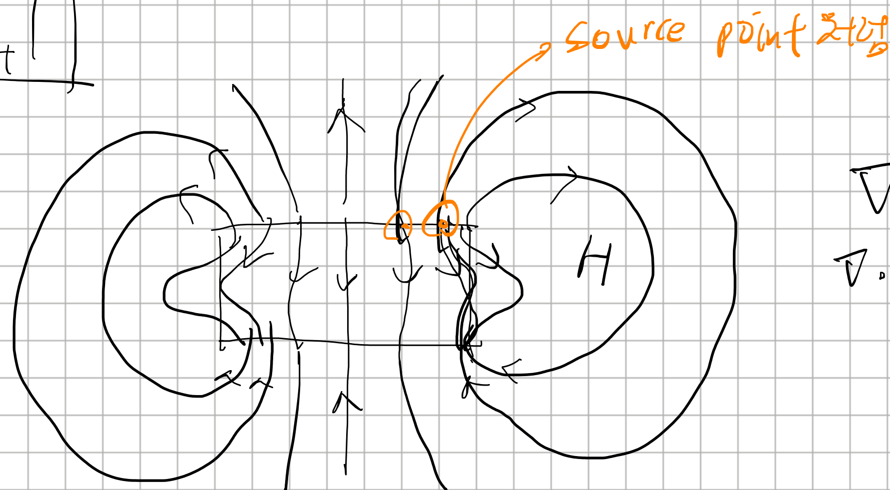
cf. no surface s charge

$$\vec{D_2}\cdot\hat{n} = \vec{D_1}\cdot\hat{n}, \quad (\vec{E_2}-\vec{E_1})\times\hat{n} = 0$$

$$E_{2t} = \varepsilon_1/\varepsilon_2\, E_{1t}, \quad \varepsilon_1 \gg \varepsilon_2$$

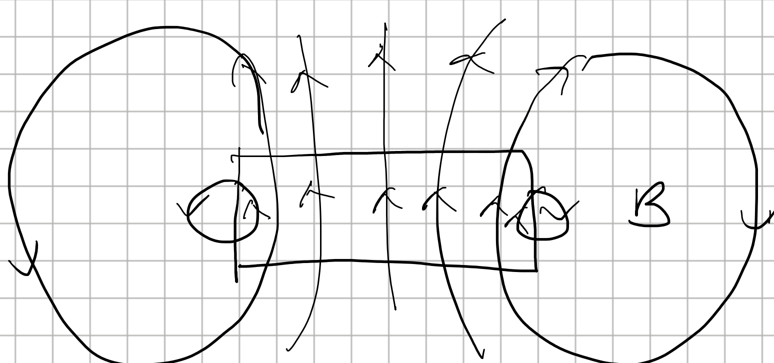


$H = B$ outside



$$\nabla\times\vec{H} = 0$$

$$\nabla\cdot\vec{H} = -4\pi\nabla\cdot\vec{M}$$

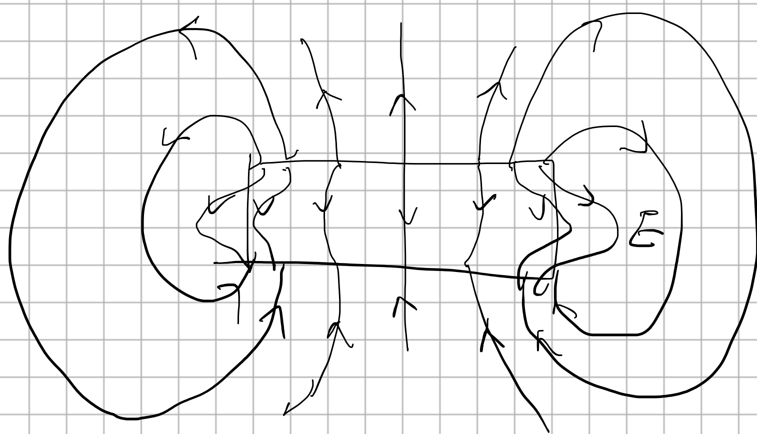


$$\nabla\times\vec{B} = 4\pi\nabla\times M$$

$$\nabla\cdot B = 0$$

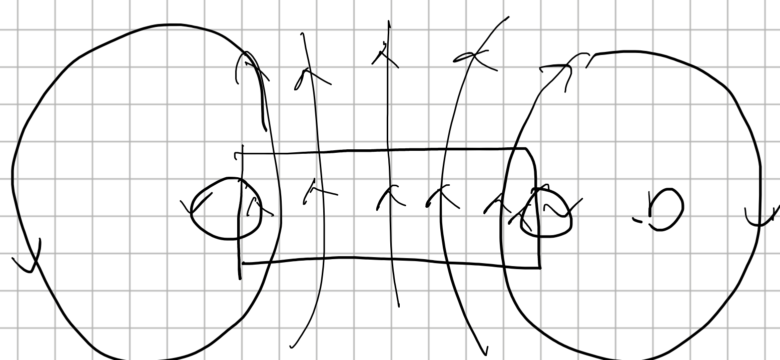
cf.

$D = E$ outside.



$$\nabla \cdot \vec{E} = -4\pi \nabla \cdot \vec{p}$$

$$\nabla \times \vec{E} = 0$$



$$\nabla \cdot D = 0$$

$$\nabla \times \vec{D} = -4\pi \nabla \times \vec{p}$$