2015 Qualifying Exam: Mathematics

Caution! Use separate answer books fo Math.-A and Math.-B.

Math.-A

Problem 1. (10 points) Find the limits.

$$\lim_{x\to 0} \frac{\sqrt{1+\sin^2 x^2} - \cos^3 x^2}{x^3 \tan x}, \quad \lim_{x\to 0} \frac{\int_0^x \sin(xt^3)dt}{x^5}$$

Problem 2. (10 points) Determine the following limits if $\lim_{x\to 0^+} f(x) = A$ and $\lim_{x\to 0^-} f(x) = B$

$$\lim_{x\to 0^-} f(x^2-x), \ \lim_{x\to 0^-} \Big(f(x^2)-f(x)\Big), \ \lim_{x\to 0^+} f(x^3-x), \ \lim_{x\to 0^-} \Big(f(x^3)-f(x)\Big), \ \lim_{x\to 1^-} f(x^2-x)$$

Problem 3. (10 points) Find

$$\int \sin^5 x \cos^4 x \, dx.$$

Problem 4. (10 points) Find the solution by utilizing Laplace transformation:

$$y^{(2)}(t) + 3y^{(1)}(t) + 2y(t) = e^{-2t}, \ t > 0,$$

where $y(0^-) = -2$ and $y^{(1)}(0^-) = 1$.

Problem 5. (10 points) Green's Formula says that

$$\oint_C \{Ldx + Mdy\} \ = \ \int\int_R \left(\frac{\partial M}{\partial x} - \frac{\partial L}{\partial y}\right) dxdy,$$

where C indicates the curve enclosing R, oriented counterclockwise. Let R be the region bounded by the counterclockwise rechtangle with vertices $(1,1)$, $(3,1)$, $(3,2)$, and $(1,2)$. Compute

$$\oint_C \{xydx + xydy\}$$