Communications and Signal Processing
2015 Doctoral Qualifying Exam

Caution!!!
Use a separate answer booklet for Problem 1.

Problem 1. (50 points)

Caution!!!
Use a separate answer booklet for Problem 2.

Problem 2. (50 points) Suppose that a received signal consisting of K 4-QAM signals is modeled by

$$Y(t) = \mathrm{Re}\left\{ \sum_{k=1}^{K} \sqrt{2P_k} d_k e^{\mathrm{j}(2\pi f_k t + \theta_k)} \right\} + N(t), \text{ for } 0 \le t < T,$$

where $\mathrm{Re}\{\cdot\}$ denotes the real part, P_k's are positive numbers, $d_k \in \{e^{\mathrm{j}\frac{\pi}{4}}, e^{\mathrm{j}\frac{3\pi}{4}}, e^{\mathrm{j}\frac{5\pi}{4}}, e^{\mathrm{j}\frac{7\pi}{4}}\}$ is a 4-QAM data symbol for $k = 1, 2, \cdots, K$, $f_k > 0$ is the kth carrier frequency, θ_k is the kth carrier phase, and $N(t)$ is real-valued additive white Gaussian noise with two-sided power spectral density $N_0/2$. When $f_k T \gg 1, \forall k$, answer the following questions.

(a) (5 points) Rewrite $e^{\mathrm{j}\theta}$ in terms of $\sin\theta$ and $\cos\theta$.

(b) (5 points) Rewrite $\mathrm{Re}\{x\}$ in terms of x and x^*, where x^* is the conjugation of the complex number x.

(c) (10 points) Find the minimum value of $|f_k - f_{k'}|$ such that

$$\frac{1}{2T} \int_0^T e^{\mathrm{j}(2\pi f_k t + \theta_k)} e^{-\mathrm{j}(2\pi f_{k'} t + \theta_{k'})} dt$$

is exactly equal to zero.

(d) (10 points) Show that

$$\frac{1}{2T} \int_0^T e^{-\mathrm{j}(2\pi f_k t + \theta_k)} e^{-\mathrm{j}(2\pi f_{k'} t + \theta_{k'})} dt$$

is approximately equal to zero.

(e) (10 points) Using the results in (c) and (d), show that

$$\frac{1}{T} \int_0^T Y(t) e^{-\mathrm{j}(2\pi f_{k'} t + \theta_{k'})} dt$$

is approximately equal to $\sqrt{P_{k'}/2}\, d_{k'}$, when $N_0 = 0$.

(f) (5 points) Find the mean and the variance of the real part of

$$N_{k'} \triangleq \frac{1}{T} \int_0^T N(t) e^{-\mathrm{j}(2\pi f_{k'} t + \theta_{k'})} dt.$$

(g) (5 points) Justify that the real and the imaginary parts of $N_{k'}$ defined in (f) are approximately independent and identically distributed.