광전자공학 Ch. 1
Propagation of light

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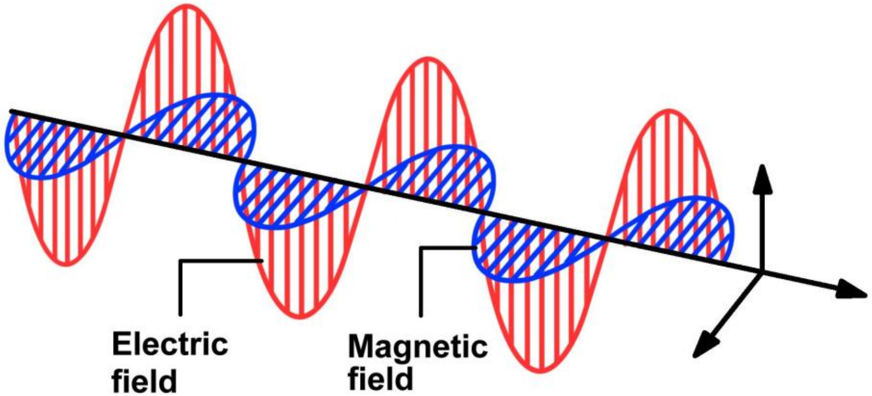
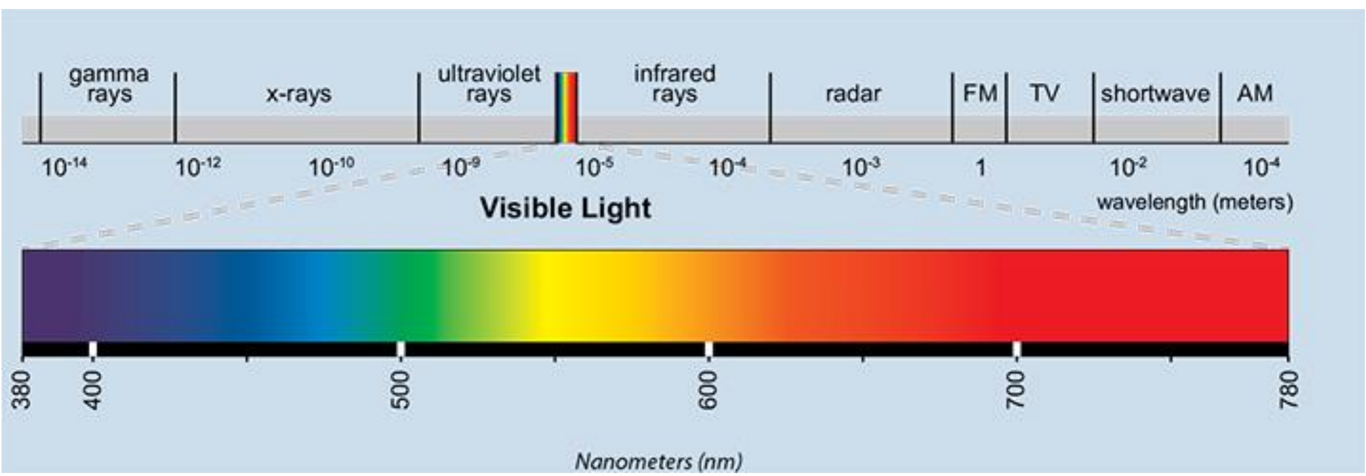
Electromagnetic wave



Light is an electromagnetic wave that includes infrared, visible light, and ultraviolet spectra.



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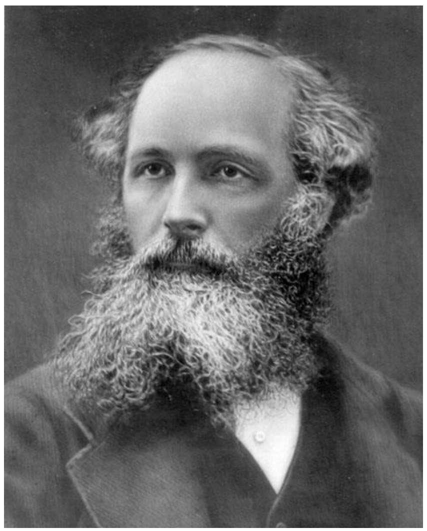
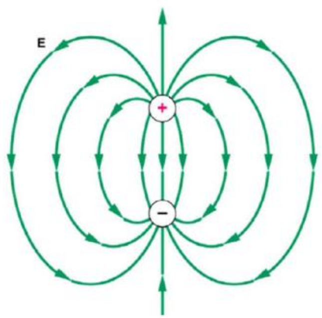
Maxwell's equation



James C. Maxwell
(1831–1879)

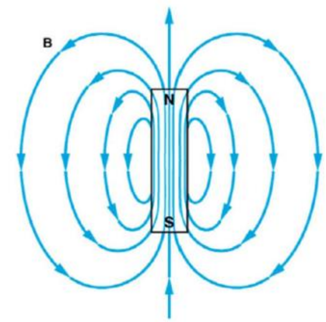
1. Gauss's law

$$\nabla \cdot \mathbf{D} = \rho$$



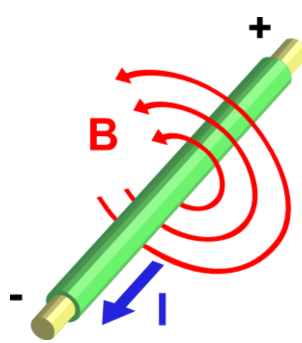
2. Gauss's law for magnetism

$$\nabla \cdot \mathbf{B} = 0$$



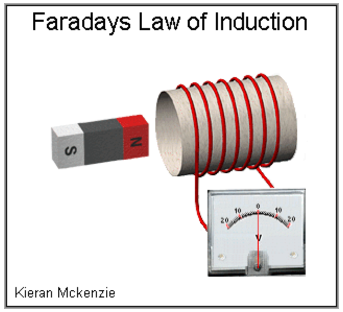
3. Ampère-Maxwell's law

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$



4. Faraday's induction law

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$



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Wave equation

Derivation of "Wave equation" from Maxwell Equations (in vacuum)

$$\nabla\times\mathbf{H}=\mathbf{J}+\frac{\partial\mathbf{D}}{\partial t}\xrightarrow[(\mathbf{J}=0,\ \mathbf{D}=\varepsilon_0\mathbf{E})]{}\nabla\times\mathbf{H}=\varepsilon_0\frac{\partial\mathbf{E}}{\partial t}$$

$$\nabla\times\mathbf{E}=-\frac{\partial\mathbf{B}}{\partial t}\xrightarrow[(\mathbf{B}=\mu_0\mathbf{H})]{}\nabla\times\mathbf{E}=-\mu_0\frac{\partial\mathbf{H}}{\partial t}$$

$\varepsilon_0 : 8.854\times10^{-12}$ (F/m) Permittivity of vacuum

$\mu_0 : 4\pi\times10^{-7}$ (H/m) Permeability of vacuum

$\nabla\cdot\mathbf{E}=0$ (charge free)

$$\nabla\times(\nabla\times\mathbf{E})=-\mu_0\frac{\partial\nabla\times\mathbf{H}}{\partial t}=-\mu_0\varepsilon_0\frac{\partial^2\mathbf{E}}{\partial t^2}\longleftarrow\nabla\times(\nabla\times\mathbf{E})=\nabla(\cancel{\nabla\cdot\mathbf{E}})-\nabla^2\mathbf{E}$$

$$\boxed{\nabla^2\mathbf{E}-\mu_0\varepsilon_0\frac{\partial^2\mathbf{E}}{\partial t^2}=0}$$

Wave equation

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Wave equation

$$\boxed{\nabla^2 \mathbf{E} - \frac{1}{c_0{}^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} = 0}$$

Wave equation for E field

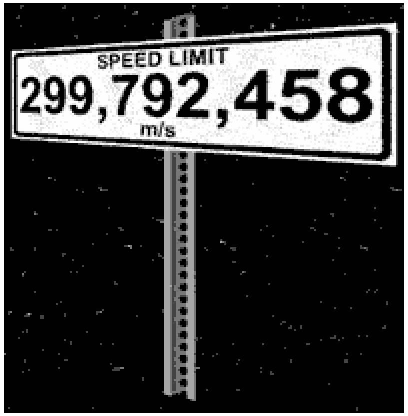
$$\boxed{\nabla^2 \mathbf{H} - \frac{1}{c_0{}^2}\frac{\partial^2 \mathbf{H}}{\partial t^2} = 0}$$

Wave equation for E field

$$\mu_0 \varepsilon_0 = \frac{1}{c_0{}^2}$$

$$c_0 \approx 2.99783 \times 10^8 \,\mathrm{m/s}$$

Speed of light (in vacuum)



Wave equation in medium

In homogeneous linear medium (such as glass, water, pure dielectric materials)

$$\mathbf{D} = \varepsilon\mathbf{E} = \varepsilon_0\varepsilon_r\mathbf{E}$$

Relative permittivity & permeability

$$\mathbf{B} = \mu\mathbf{H} = \mu_0\mu_r\mathbf{H}$$

Wave equation is now changed to

$$\nabla^2\mathbf{E} - \frac{1}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2} = 0 \qquad \frac{1}{c^2} = \frac{\mu_r\varepsilon_r}{c_0{}^2} = \frac{\mathrm{n}^2}{c_0{}^2} \longrightarrow c = \frac{c_0}{\mathrm{n}}$$

Speed of light is reduced

$$\boxed{\mathrm{n} = \sqrt{\varepsilon_r\mu_r}}$$

Refractive index of material!

Most materials are nonmagnetic in visible light range. Therefore, $\mathrm{n} = \sqrt{\varepsilon_r}$

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Refractive index of materials

Various materials (or different conditions of the same materials) have various refractive indexes.

Table 1. Index of Refraction of Various Materials.	
Material	**Index of Refraction**
Vacuum	1.0000
Air	1.0003
Water (pure)	1.3330
Seawater (35 ppt)	1.3394
Ethyl alcohol	1.361
Sugar Ssolution (80% sugar)	1.49
Glass (soda lime)	1.510
Bromine (liquid)	1.661
Ruby	1.760
Diamond	2.417

Heat shimmer

Mirage

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Plane wave

$$\nabla^2 u - \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} = 0$$

u can be any component of either **E** or **H**

Laplacian in Cartesian coordinate,

$$\nabla^2 u = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) u$$

Assume that light is propagating through z-direction,

$$\nabla^2 u - \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} = 0 \rightarrow \frac{\partial^2 u}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} = 0$$

Solutions of above partial differential equation

$$u = \boxed{u_1 \cos\left(kz - \omega t + \varphi_1\right)} + \boxed{u_2 \cos\left(kz + \omega t + \varphi_2\right)}$$

Forward propagation   Backward propagation

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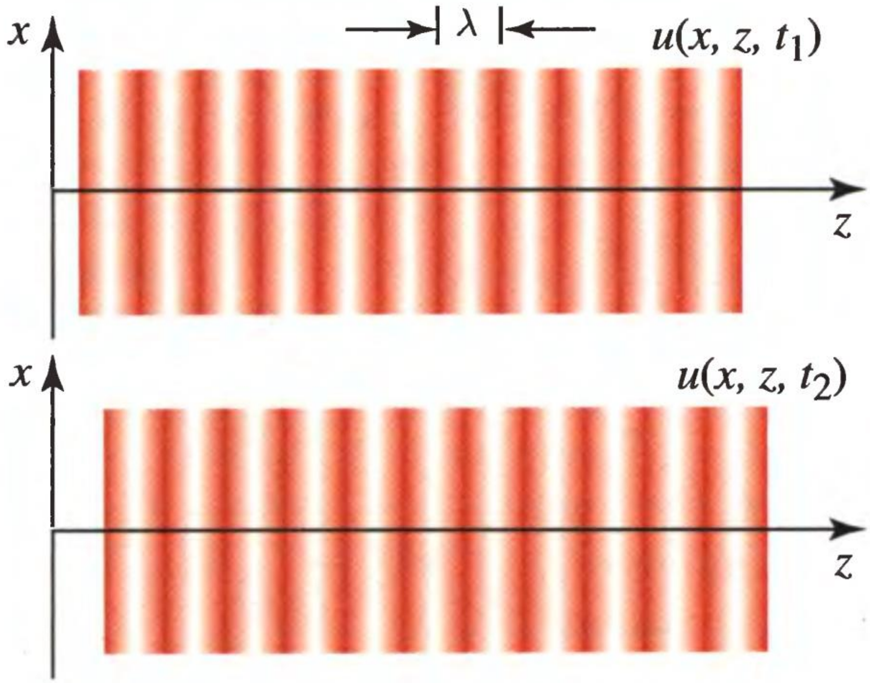
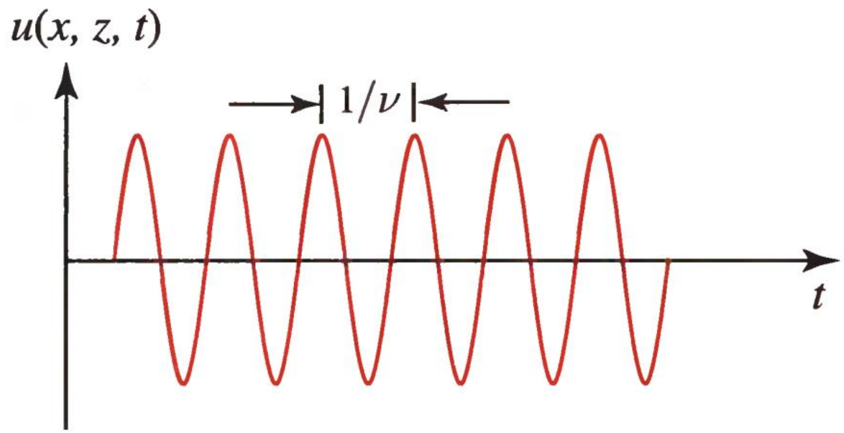
Plane wave

Propagation characteristics of plane wave $u(x,t) = u_0 \cos\left(kz - \omega t + \phi_0\right)$





Wavenumber, angular frequency

$$k = \frac{2\pi}{\lambda}, \quad \omega = \frac{2\pi}{T} = 2\pi\nu$$

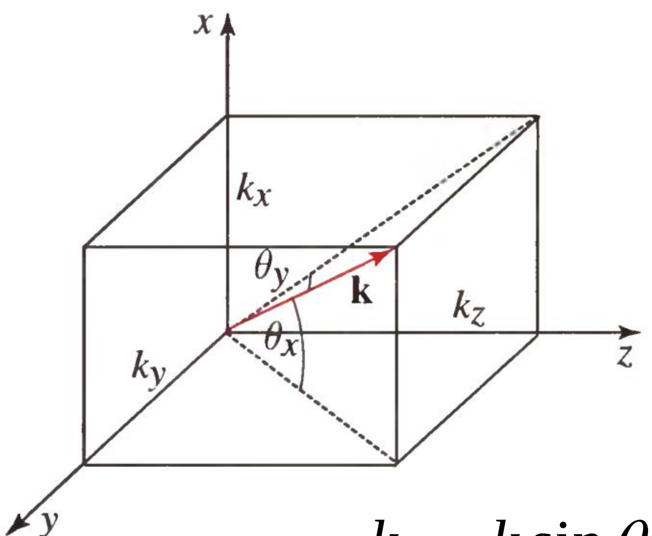
$$\frac{\omega^2}{k^2} = \frac{\lambda^2}{T^2} = \lambda^2\nu^2 = c^2$$

$$k = nk_0, \ \lambda = \lambda_0 / n$$

Plane wave in 3D space

Plane wave propagating through arbitrary direction of $\mathbf{k} = (k_x, k_y, k_z)$

$$u(r,t) = u_0 \cos\left(\mathbf{k}\cdot\mathbf{r} - \omega t + \varphi_0\right) = u_0 \cos\left(k_x x + k_y y + k_z z - \omega t + \varphi_0\right)$$

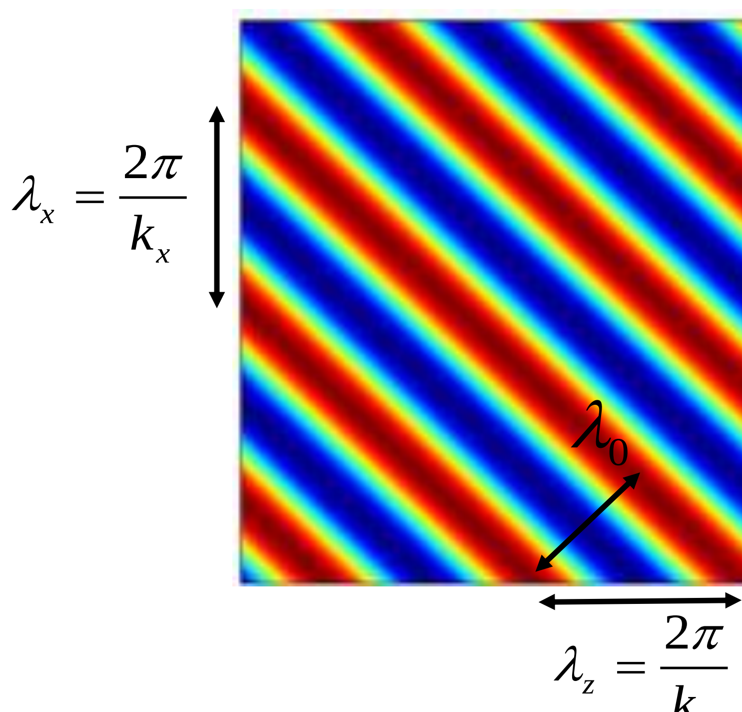


$$k_x = k \sin\theta \cos\phi$$

$$k_y = k \sin\theta \sin\phi$$

$$k_z = k \cos\theta$$

Oblique plane wave



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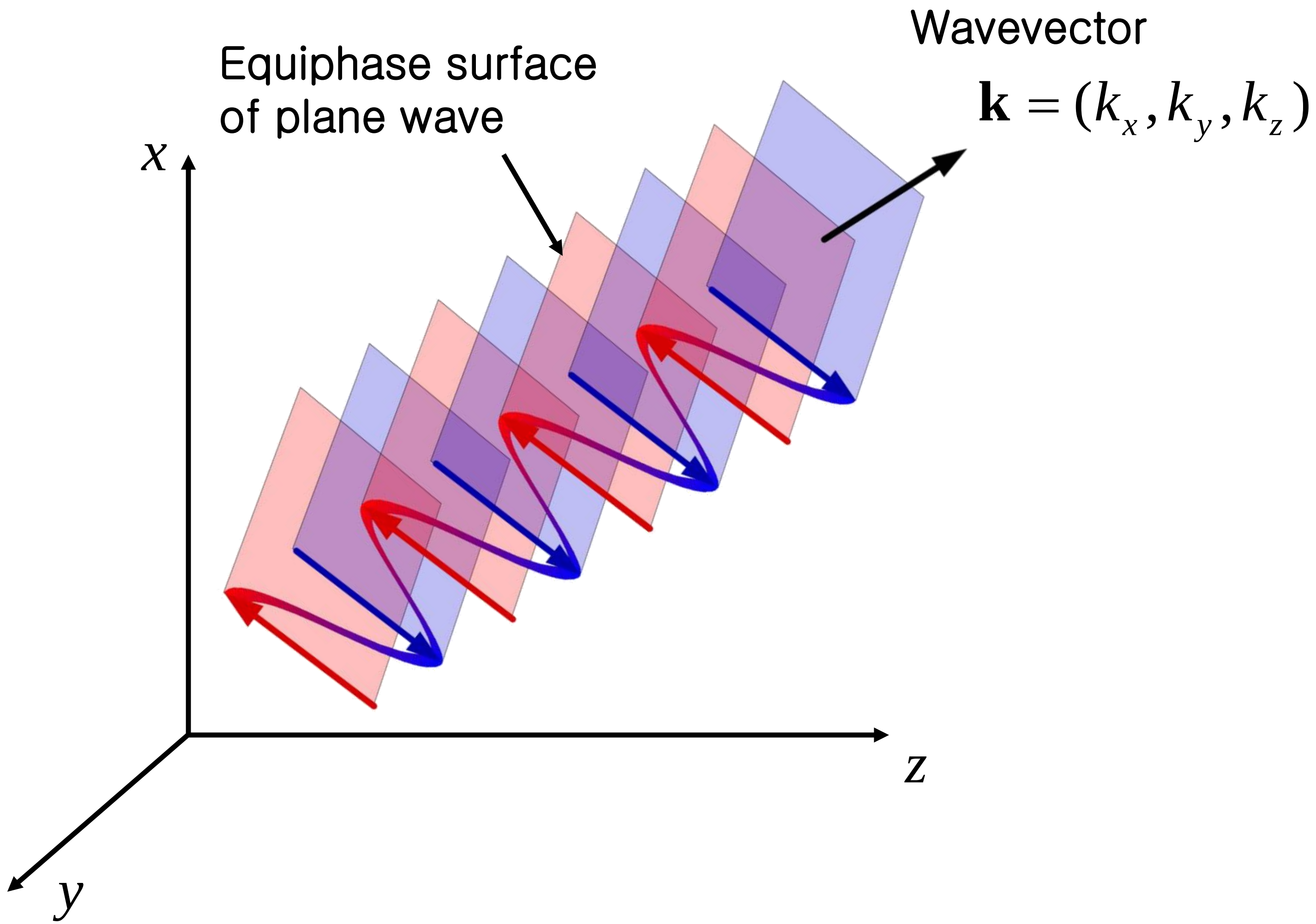
Plane wave in 3D space



Complex phasor notation

$$u_0 \cos(\mathbf{k}\cdot\mathbf{r} - \omega t + \varphi_0) = \mathrm{Re}\left(\boxed{u_0 e^{j\varphi_0}} e^{j(\mathbf{k}\cdot\mathbf{r} - \omega t)}\right)$$

$$U_0 = u_0 e^{j\varphi_0}$$ Phasor notation

Using phasor notation, wave equation can be changed to

$$\nabla^2 U - \frac{1}{c^2}\frac{\partial^2 U}{\partial t^2} = 0 \rightarrow \boxed{\nabla^2 U + k^2 U = 0}$$ Helmholtz equation

Solving Helmholtz equation is same problem as solving wave equation, when light is monochromatic wave.

Monochromatic wave can be expressed as, $U(r,t) = U_0 e^{j(\mathbf{k}\cdot\mathbf{r} - \omega t)}$

Ideal sinusoidal wave which has pure single frequency

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Monochromatic wave

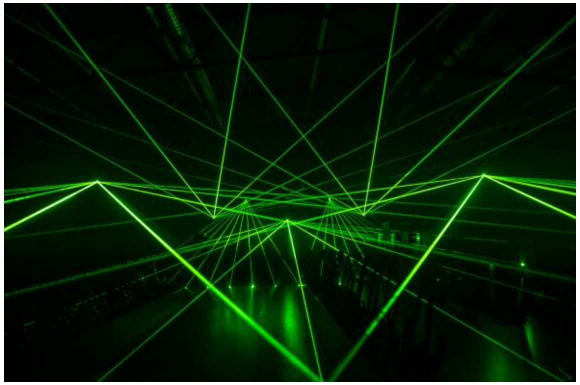
- Monochromatic wave: light with purely sin, cos wavefront

  Light emitted from Laser source is nearly ideal monochromatic wave

Sunlight

LED

Laser



polychromatic ⟷ monochromatic

- Monochromatic light = delta function spectrum

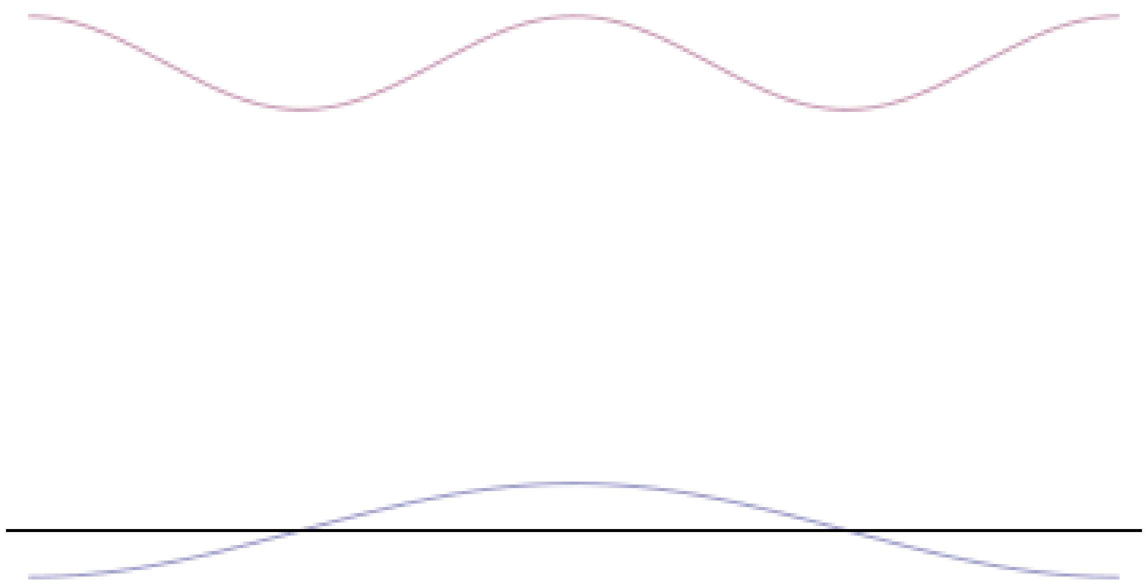
purely sin, cos wavefront ⟷ Narrow spectral width

Fourier transform relation

How to make light pulse?

A light pulse can be expressed as a sum of continuous waves

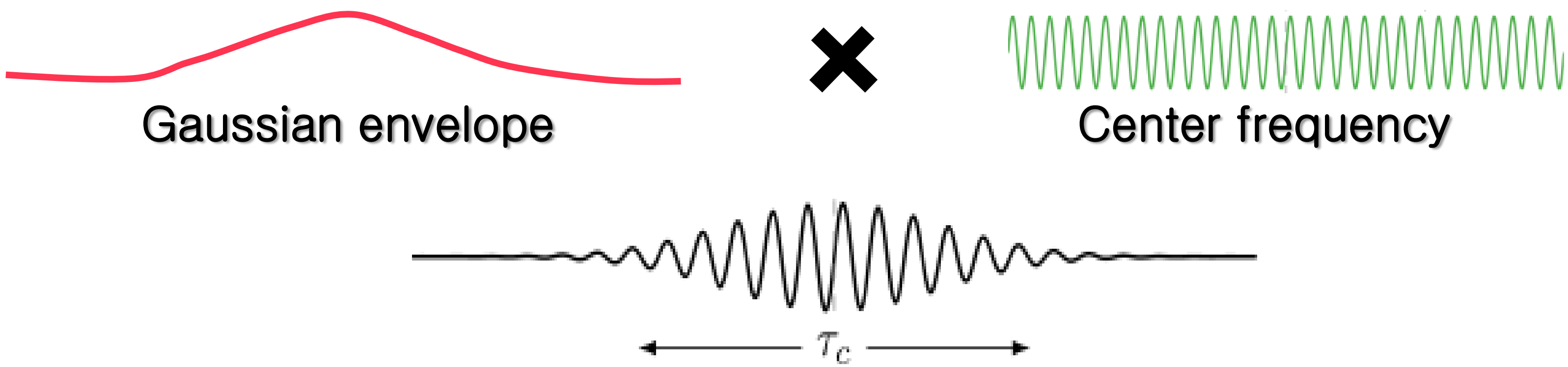
$$E(t) = \sum_{\Omega} A_{\Omega} \exp(-j\beta(\omega_0 + \Omega)z)\exp(j(\omega_0 + \Omega)t)$$



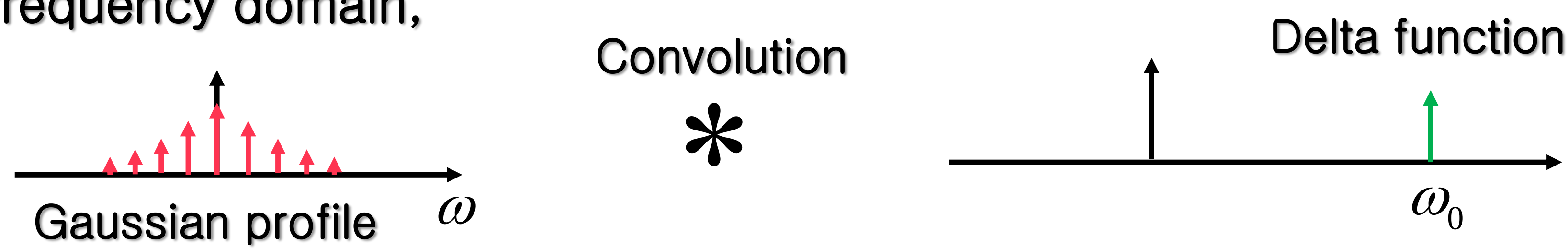
How to make light pulse?

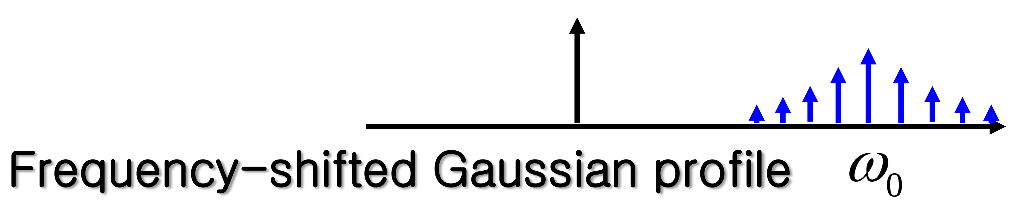
A light pulse can be expressed as a sum of continuous waves

In time domain, $E(t) = A(t)\exp(j\omega_0 t)$



In frequency domain,





The group velocity

Phase velocity : propagation speed of light phase

$$c = \frac{\omega}{k} = \frac{c_0}{n} \qquad n: \text{ refractive index}$$

Group velocity : propagation speed of light pulse envelope

$$v = \frac{d\omega}{dk} = \frac{c_0}{N} \qquad N: \text{ group index}$$

Group velocity and phase velocity

https://www.youtube.com/watch?v=hqwa1nktc_E

Zero Group velocity

https://www.youtube.com/watch?v=v9DPzMoWpc0

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The group velocity

A light pulse can be expressed as a sum of continuous waves

$$E(t) = \sum_{\Omega} A_{\Omega} \exp(-jk_{(\omega=\omega_0+\Omega)} z) \exp(j(\omega_0 + \Omega)t)$$

$$k_{(\omega=\omega_0+\Omega)} \approx k_{(\omega=\omega_0)} + \Omega \frac{dk}{d\omega}$$

Phase velocity $\frac{1}{c} = \frac{k_{(\omega=\omega_0)}}{\omega} = \frac{n}{c_0}$

$$E(t) = \sum_{\Omega} A_{\Omega} \exp(j\Omega(t - \frac{dk}{d\omega} z)) \exp(j\omega_0 (t - \frac{k_{(\omega=\omega_0)}}{\omega_0} z))$$

Group velocity $\frac{1}{v} = \frac{dk}{d\omega} = \frac{N}{c_0}$

$$N = c_0 \frac{dk}{d\omega} = c_0 \frac{dk}{d\lambda_0} \frac{d\lambda_0}{d\omega}$$

$$= -\frac{d}{d\lambda_0}\left(\frac{2\pi n}{\lambda_0}\right)\frac{\lambda_0}{\omega} = n - \lambda \frac{dn}{d\lambda}$$

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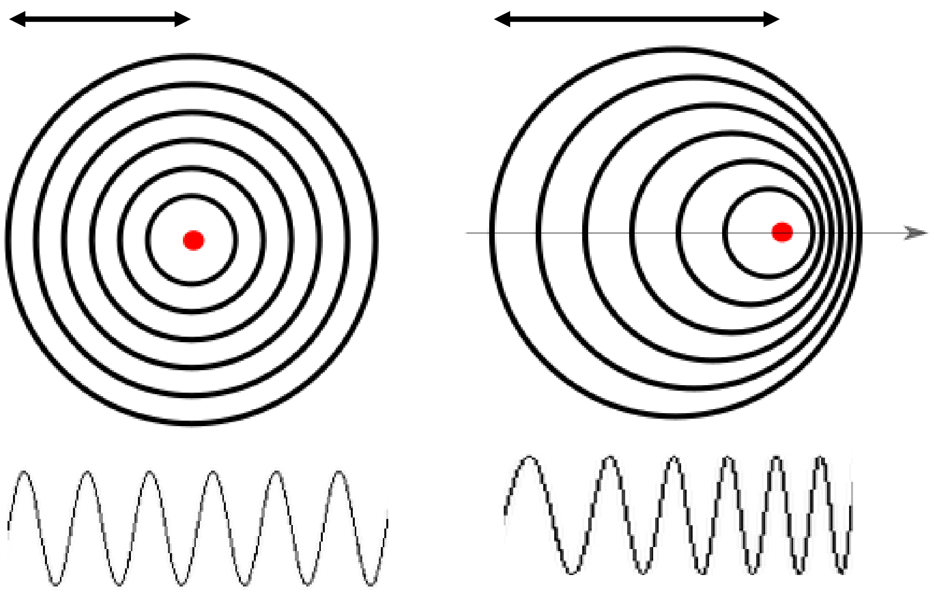
Doppler effect

When light source is moving, frequency of received light can be changed

$$c\Delta t = m\lambda \qquad (c+u)\Delta t = m\lambda'$$



The number m must be same,

$$\frac{\lambda'}{(c+u)} = \frac{\lambda}{c}$$

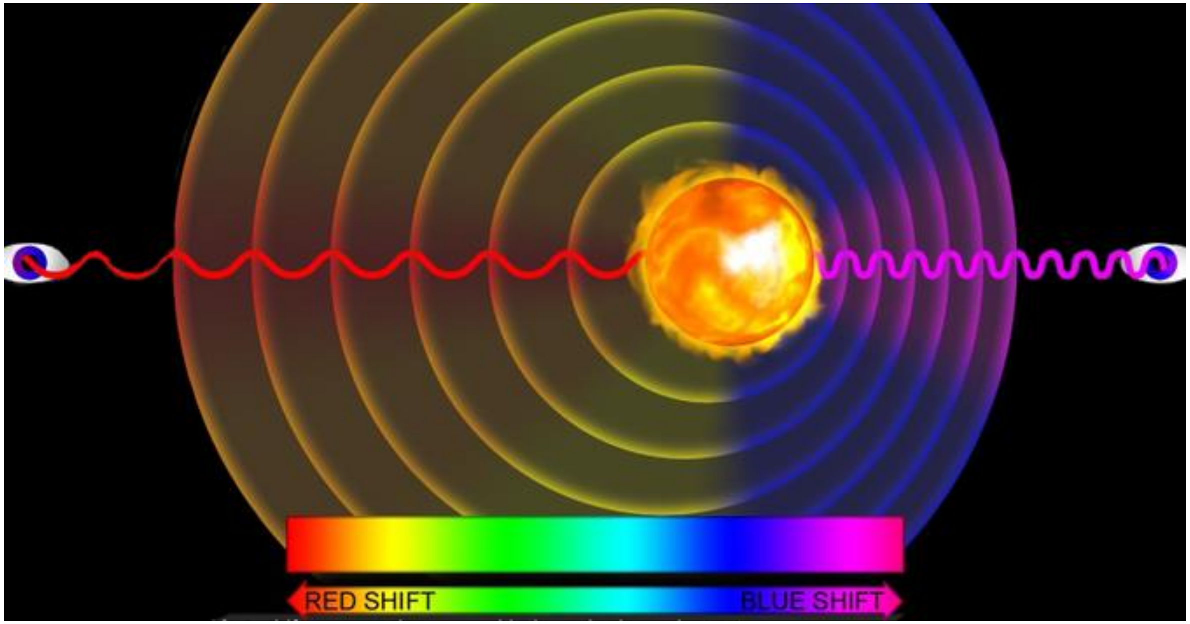
Material does not changed, so

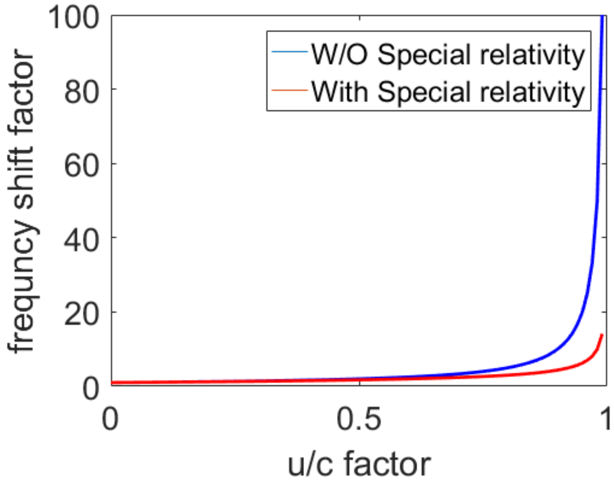
$$\omega' = \frac{2\pi c_0}{\lambda'}, \quad \omega' = \frac{\omega}{1+u/c}$$

This is satisfied when u is much slower than c
(special relativity does not applied)

Doppler effect

When special relativity is applied





$$\omega' = \omega\sqrt{\frac{1-u/c}{1+u/c}}$$

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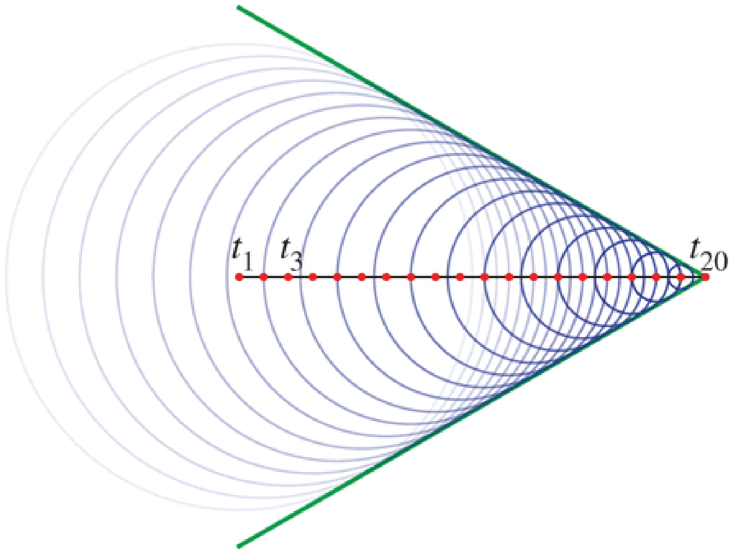
Cherenkov radiation



In dielectric material, phase velocity of light is smaller than c0

Therefore, fast radiative particle can exceed the phase velocity of light

The characteristic blue glow of an underwater nuclear reactor is due to Cherenkov radiation