2012 수학

1. (a) $\lim_{x\to\infty} \{3^x + 27^x\}^{\frac{1}{x}} = 27 \lim_{x\to\infty} \left(1+\left(\frac{1}{9}\right)^x\right)^{\frac{1}{x}} = 27$

(b) 0.

2. $\ln y = (3\ln x)(\ln(x+2)) - \frac{1}{2}\ln(x^2+1)$

$\frac{y'}{y} = \frac{3}{x}\ln(x+2) + \frac{3}{x+2}\ln x - \frac{x}{x^2+1}$

$\therefore y' = \frac{3(x+2)^{3\ln x}\ln(x+2)}{x(x^2+1)^{\frac{1}{2}}} + \frac{3(x+2)^{3\ln x}\ln x}{(x+2)(x^2+1)^{\frac{1}{2}}} - \frac{x(x+2)^{3\ln x}}{(x^2+1)^{\frac{3}{2}}}$

3. $\int e^x \cos x\,dx = e^x\cos x + \int e^x \sin x\,dx + C_1 = e^x\cos x + e^x \sin x - \int e^x \cos x\,dx + C_2$

$\therefore \underline{\int e^x \cos x\,dx = \frac{e^x\cos x + e^x \sin x}{2} + C}$

4. $s^2Y - sy(0) - y'(0) + 3sY - 3y(0) + 2Y = \frac{1}{s+2}$

$s^2Y - 1 + 3sY + 2Y = \frac{1}{s+2}$, $Y(s) = \frac{1}{(s+1)(s+2)} + \frac{1}{(s+1)(s+2)^2} = \frac{s+3}{(s+1)(s+2)^2}$

$\frac{s+3}{(s+1)(s+2)^2} = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{(s+2)^2}$, $As^2+4As+4A+Bs^2+3Bs+2B+Cs+C =$
$(A+B)s^2 + (4A+3B+C)s + 4A+2B+C = s+3$

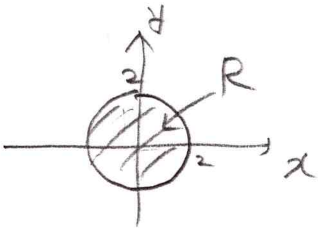
$4A+3B+C=1$
$4A+2B+C=3$

$B=-2 \quad A=2 \quad C=-1$

$Y(s) = \frac{2}{s+1} - \frac{2}{s+2} - \frac{1}{(s+1)^2}$

$\therefore \underline{y(t) = (2e^{-t} - 2e^{-2t} - te^{-t})u(t)}$

5. $\because$ $L(x,y) = \cos x^2 - y^3$, $M(x,y) = x^3$



$$\iint_R 3x^2+3y^2\,dxdy = 3\iint_R x^2+y^2\,dxdy, \quad \begin{matrix} x = r\cos\theta \\ y = r\sin\theta \end{matrix}$$

$$3\int_0^{2\pi}\int_0^2 r^3\,dr\,d\theta = 3\int_0^{2\pi} \frac{1}{4}r^4\Big|_0^2\,d\theta$$

$$= 12\cdot 2\pi = \underline{24\pi}$$

6. System matrix $H \in \mathbb{R}^{3\times 2}$, $H\underline{x} = \underline{y}$

(a) $H = \begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix}$, $\begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $\begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$

$\therefore H = \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \underline{\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}}$ $\qquad \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \underline{\begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}}$

(b) $\begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \underline{\begin{bmatrix} x_1+x_2 \\ 2x_1+x_2 \\ 3x_1+x_2 \end{bmatrix}}$

(c) $\begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$. $\Rightarrow$ $\begin{matrix} x_1+x_2=0 \\ 2x_1+x_2=0 \\ 3x_1+x_2=0 \end{matrix}$ $\therefore \underline{N = \left\{\begin{bmatrix} 0 \\ 0 \end{bmatrix}\right\}}$

(d)

7. (a) $y(t) = \int_{-\infty}^{\infty} h(t-\tau)x(\tau)d\tau = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau.$

$$x(t) = \sum_{n=-\infty}^{\infty} c_n e^{j\frac{2\pi}{T}nt}.$$

$$y(t) = \int_{-\infty}^{\infty} h(\tau)\sum_{n=-\infty}^{\infty} c_n e^{j\frac{2\pi}{T}n(t-\tau)} d\tau$$

$$= \sum_{n=-\infty}^{\infty} c_n e^{j\frac{2\pi}{T}nt} \cdot \int_{-\infty}^{\infty} h(\tau) e^{-j2\pi\tau\cdot\frac{n}{T}} d\tau = \sum_{n=-\infty}^{\infty} c_n H\left(\frac{n}{T}\right) e^{j\frac{2\pi}{T}nt}$$

$$= \sum_{n=-\infty}^{\infty} d_n e^{j\frac{2\pi}{T}nt}.$$

$\therefore$ $y(t)$는 coefficient $d_n = c_n H\left(\frac{n}{T}\right)$을 갖는 periodic signal.

(b) $$\int_{-\infty}^{\infty}\left(\int_{-\infty}^{\infty} x(t_1)e^{-j2\pi f t_1} dt_1 \int_{-\infty}^{\infty} y^*(t_2)e^{j2\pi f t_2} dt_2\right) df$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x(t_1)y^*(t_2)\left[\int_{-\infty}^{\infty} e^{-j2\pi f(t_1-t_2)} df\right] dt_1 dt_2$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x(t_1)y^*(t_2)\,\delta(t_1-t_2)\,dt_1 dt_2$$

$$= \int_{-\infty}^{\infty} x(t)y^*(t)\,dt.$$

$$\therefore \int_{-\infty}^{\infty} x(t)y^*(t)dt = \int_{-\infty}^{\infty} X(f)Y^*(f)\,df$$

(c)

$$z(t) = \int_0^T \sum_{k_1=-\infty}^{\infty} a_{k_1} e^{j\frac{2\pi k_1}{T}(t-\tau)} \sum_{k_2=-\infty}^{\infty} b_{k_2} e^{j\frac{2\pi k_2 \tau}{T}} d\tau$$

$$= \sum_{k_1=-\infty}^{\infty} \sum_{k_2=-\infty}^{\infty} a_{k_1} b_{k_2} e^{j\frac{2\pi k_1 t}{T}} \int_0^T e^{j\frac{2\pi}{T}(k_2-k_1)\tau} d\tau$$

$$\Rightarrow \int_0^T e^{j\frac{2\pi}{T}\tau(k_2-k_1)} d\tau = \frac{1}{\frac{2\pi}{T}(k_2-k_1)}\left[e^{j2\pi(k_2-k_1)} - 1\right]$$

$$= \begin{cases} 0, & \text{if } k_1 \neq k_2 \\ T, & \text{if } k_1 = k_2 \end{cases}$$

$$\therefore z(t) = \sum_{k_1=-\infty}^{\infty} \sum_{k_2=-\infty}^{\infty} a_{k_1} b_{k_2} e^{j\frac{2\pi}{T}kt} \int_0^T e^{j\frac{2\pi}{T}(k_2-k_1)\tau} d\tau$$

$$= \sum_{k=-\infty}^{\infty} T a_k b_k e^{j\frac{2\pi}{T}kt} = \sum_{k=-\infty}^{\infty} c_k e^{j\frac{2\pi k t}{T}}$$

$$\therefore \underline{c_k = T a_k b_k}$$

2012 통신

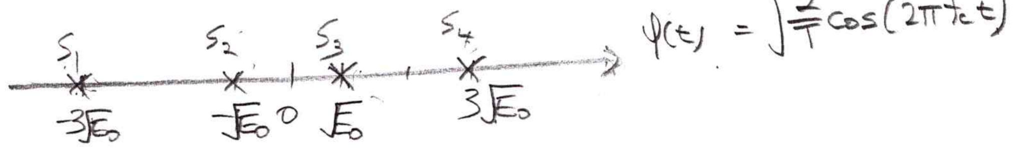
1.

(a) $s_3(t) = \sqrt{\frac{2E_0}{T}} \cos(2\pi f_c t)$

$$\int_0^T |s_3(t)|^2\, dt = \frac{2E_0}{T} \cdot \int_0^T \frac{1+\cos(4\pi f_c t)}{2}\, dt = \underline{E_0}$$

$$\underline{h(t) = \sqrt{\frac{2}{T}} \cos(2\pi f_c (T-t))}$$

(b)



$$\underline{E} = \frac{1}{4}(9E_0 + E_0 + E_0 + 9E_0) = \underline{5E_0}$$

(c) $N\left(\sqrt{E_0}, \frac{N_0}{2}\right) \Rightarrow$ mean $= \sqrt{E_0}$, variance $= \frac{N_0}{2}$.

(d) $p = Q\left(\frac{d}{2\sigma}\right) = Q\left(\frac{2\sqrt{E_0}}{\sqrt{2N_0}}\right) = Q\left(\sqrt{\frac{2E_0}{N_0}}\right)$

$$P_e = \frac{1}{4}(p + 2p + 2p + p) = \frac{3}{2}p = \underline{\frac{3}{2} Q\left(\sqrt{\frac{2E_0}{N_0}}\right)}$$

(e) Since Gray coding method was employed,

1-bit error probability $= p$.

$$\therefore P_b(s_3) = p + p + Q\left(\frac{2d}{2\sigma}\right) = \underline{2Q\left(\sqrt{\frac{2E_0}{N_0}}\right) + Q\left(\sqrt{\frac{8E_0}{N_0}}\right)}$$

$$\underline{\approx 2Q\left(\sqrt{\frac{2E_0}{N_0}}\right)}$$

2012 제어

제어 필수

1. a) $f(t) = K(y_1(t) - y_2(t))$, $\quad f(t) - B\frac{dy_2(t)}{dt} = M\frac{d^2y_2(t)}{dt^2}$

$$Ky_1(t) - Ky_2(t) - B\frac{dy_2(t)}{dt} = M\frac{d^2y_2(t)}{dt^2}$$

$$\therefore \underline{\frac{d^2y_2(t)}{dt^2} = -\frac{B}{M}\frac{dy_2(t)}{dt} - \frac{K}{M}y_2(t) + \frac{K}{M}y_1(t)}$$

b) $x_1(t) = y_2(t)$, $x_2(t) = \dot{x}_1(t) = \frac{dy_2(t)}{dt}$

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{B}{M} \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{M} \end{bmatrix} f(t)$$

$$y_2(t) = \begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

c) $$\frac{Y_2(s)}{F(s)} = \begin{bmatrix} 1 & 0 \end{bmatrix}\left(\begin{bmatrix} s & -1 \\ 0 & s+\frac{B}{M} \end{bmatrix}^{-1}\right)\begin{bmatrix} 0 \\ \frac{1}{M} \end{bmatrix},$$

$$\begin{bmatrix} s & -1 \\ 0 & s+\frac{B}{M} \end{bmatrix}^{-1} = \frac{1}{s^2+\frac{B}{M}s}\begin{bmatrix} s+\frac{B}{M} & 1 \\ 0 & s \end{bmatrix} = \begin{bmatrix} \frac{1}{s} & \frac{1}{s^2+\frac{B}{M}s} \\ 0 & s+\frac{B}{M} \end{bmatrix}$$

$$\therefore \frac{Y_2(s)}{F(s)} = \begin{bmatrix} \frac{1}{s} & \frac{1}{s^2+\frac{B}{M}s} \end{bmatrix}\begin{bmatrix} 0 \\ \frac{1}{M} \end{bmatrix} = \underline{\frac{1}{Ms^2+Bs}}$$

d) $$Y_2(s) = \frac{1}{s^2(s+1)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+1}, \quad As^2+As+Bs+B+Cs^2 = (A+C)s^2+(A+B)s+B = 1$$

$$A = -1 \quad B = 1 \quad C = 1$$

$$Y_2(s) = -\frac{1}{s} + \frac{1}{s^2} + \frac{1}{s+1} \qquad \therefore \underline{y_2(t) = (-1+t+e^{-t})u(t)}$$

e) $f(t) = u(t) = ky_1(t) - ky_2(t)$.

$$ky_1(t) = ky_2(t) + u(t) = (-k + kt + ke^{-t} + 1)u(t)$$

$$\therefore y_1(t) = (-1 + t + e^{-t} + \tfrac{1}{k})u(t), \qquad \underline{u(t) = f(t) = \left(\frac{1}{t-1+e^{-t}+\frac{1}{k}}\right)y_1(t)}$$

. 제어 선택

1. $\dot{x}(t) = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} [-k_1 \ -k_2] \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ p \end{bmatrix} v(t)$

$\dot{x}(t) = \begin{bmatrix} 2 & 1 \\ -k_1 & 1-k_2 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ p \end{bmatrix} v(t)$

1) $\begin{vmatrix} \lambda-2 & -1 \\ k_1 & \lambda-1+k_2 \end{vmatrix} = \lambda^2 - \lambda + k_2\lambda - 2\lambda + 2 - 2k_2 + k_1$

$= \lambda^2 + (k_2-3)\lambda + k_1 - 2k_2 + 2 = \lambda^2 + 4\lambda + 8 = 0.$

$k_2 - 3 = 4, \quad k_1 - 2k_2 + 2 = 8 \quad \therefore \underline{k_2 = 7, \ k_1 = 20}$

2) $\lim_{s\to 0}\left(sV(s) - sY(s)\right) = \lim_{s\to 0} Y(s) = \frac{1}{s} \qquad \lim_{s\to 0} H(s) = 1.$ ($H(s)$: transfer function)

$H(s) = [1 \quad 1] \begin{bmatrix} s-2 & -1 \\ 20 & s+6 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ p \end{bmatrix}, \quad \begin{bmatrix} s-2 & -1 \\ 20 & s+6 \end{bmatrix}^{-1} = \frac{1}{s^2+4s+8} \begin{bmatrix} s+6 & 1 \\ -20 & s-2 \end{bmatrix}$

$H(s) = \frac{1}{s^2+4s+8}\left([s-14 \quad s-1] \begin{bmatrix} 0 \\ p \end{bmatrix} \right) = \frac{ps-p}{s^2+4s+8}$

$\lim_{s\to 0} \frac{ps-p}{s^2+4s+8} = 1$ 이므로 $\underline{p = -8}$

3) $v(t) = \sin(t + \frac{1}{4}\pi), \quad H(j\omega) = \frac{8 - j8\omega}{8 - \omega^2 + j4\omega}$

$y_s(t) = |H(j\omega)| \sin(t + \frac{1}{4}\pi + \angle H(j\omega))$