2013 수학

1. $f(x,y) = f(1,0) + f_x(1,0)(x-1) + f_y(1,0)\,y + \frac{1}{2}\{ f_{xx}(1,0)(x-1)^2 + 2f_{xy}(1,0)(x-1)y + f_{yy}(1,0)\,y^2 \}$

$f_x(1,0) = 2xy\cos y \,\big|_{(1,0)} = 0.$ $\quad f_y(1,0) = x^2\cos y - x^2 y\sin y \,\big|_{(1,0)} = 1$

$f_{xx}(1,0) = 2y\cos y \,\big|_{(1,0)} = 0,$ $\quad f_{xy}(1,0) = 2x\cos y - 2xy\sin y \,\big|_{(1,0)} = 2$

$f_{yy}(1,0) = -x^2\sin y - x^2\sin y - x^2 y\cos y \,\big|_{(1,0)} = 0$

$$\therefore f(x,y) = f(1,0) + \begin{bmatrix} 0 & 1 \end{bmatrix}\begin{bmatrix} x-1 \\ y \end{bmatrix} + \frac{1}{2}\begin{bmatrix} x-1 & y \end{bmatrix}\begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} x-1 \\ y \end{bmatrix}$$

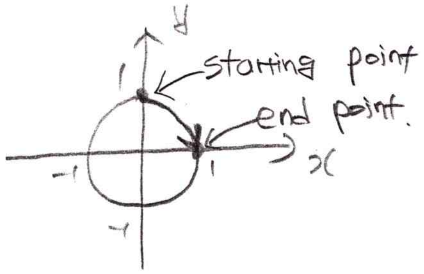
2. Let $B = T^{-1}AT$, then

$$\det(sI - B) = \det(sI - T^{-1}AT) = \det(sT^{-1}T - T^{-1}AT)$$
$$= \det(T^{-1}(sT - AT)) = \det(T^{-1}(sI-A)T)$$
$$= \det(T^{-1})\det(sI-A)\det(T) = \det(sI-A),$$

where $T^{-1}T = I$ (similarity transform matrix.)

$\therefore \det(sI-A) = \det(sI - T^{-1}AT)$

3.



Let $x = \cos\theta$, $y = \sin\theta$.

Then $\ell = \int_C 3x^2 y\,ds = \int_{\frac{\pi}{2}}^{0} 3\cos^2\theta\,\sin\theta\,d\theta$,

letting $t = \cos\theta$, we have

$\ell = -\int_1^0 3t^2\,dt = \int_0^1 3t^2\,dt = 1$

4. $s^2Y - sy(0) - y'(0) + Y = \frac{2}{s^2+1}$.

$s^2Y - 2s + Y = \frac{2}{s^2+1}$, $\quad Y(s^2+1) = 2s + \frac{2}{s^2+1}$.

$Y(s) = \frac{2s}{s^2+1} + \frac{2}{(s^2+1)^2}$,

$\frac{2}{(s^2+1)^2} = \frac{As+B}{s^2+1} + \frac{Cs+D}{(s^2+1)^2}$, $\quad (As+B)(s^2+1) + Cs + D = 2$,

$As^3 + As + Bs^2 + B + Cs + D = 2$, $\quad A=0,\ B=0,\ C=0,\ D=2.$

$\Rightarrow$ 안 풀림.

$\Rightarrow$ $tf(t) \xleftrightarrow{\mathcal{L}} -F'(s)$ 이용, $\quad f'(t) \xleftrightarrow{\mathcal{L}} \frac{1}{s}F(s)$ 이용.

$\frac{2}{(s^2+1)^2} = -\frac{1}{s}\left(\frac{2}{s^2+1}\right)' = \frac{1}{s}\left(\frac{-2}{s^2+1}\right)' \leftrightarrow \frac{d}{dt}(2t\sin t) = 2\sin t + 2t\cos t.$

$\therefore\ y(t) = (2\cos t + 2\sin t + 2t\cos t)\,u(t)$

5. Let $t = 2n-1$, we have

$\lim_{t\to\infty}\left(\frac{t+1}{t}\right)^{\frac{t+1}{2}} = \lim_{t\to\infty}\left(1+\frac{1}{t}\right)^{t\cdot\frac{1}{2}} \cdot \left(1+\frac{1}{t}\right)^{\frac{1}{2}}$

$= \left\{\lim_{t\to\infty}\left(1+\frac{1}{t}\right)^t\right\}^{\frac{1}{2}} = e^{\frac{1}{2}} = \sqrt{e}$

6.

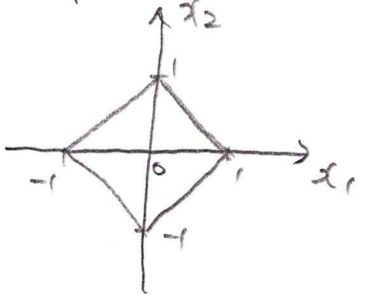
(a) <p=1 case.>

$\|\underline{x}\|_1 = |-3| + |4j| = 3 + 4 = 7.$

<p=2 case>

$\|\underline{x}\|_2 = \sqrt{|-3|^2 + |4j|^2} = \sqrt{9+16} = 5.$

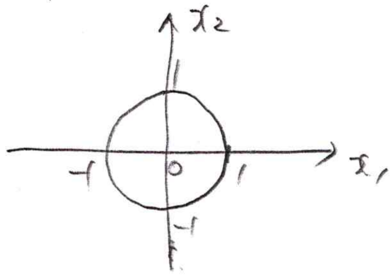
(b) <p=1 case>



$\underline{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ 일 때,

$\|\underline{x}\|_1 = |x_1| + |x_2| = 1.$

<p=2 case>



$\|\underline{x}\|_2 = \sqrt{x_1^2 + x_2^2} = 1 \Rightarrow x_1^2 + x_2^2 = 1.$

(c) i) $|x_1| > |x_2|$ case

$\lim_{p\to\infty} \|\underline{x}\|_p = \lim_{p\to\infty} (|x_1|^p + |x_2|^p)^{\frac{1}{p}} = \lim_{p\to\infty} |x_1| \left(1 + \frac{|x_2|^p}{|x_1|^p}\right)^{\frac{1}{p}} = \underline{|x_1|}$

ii) $|x_1| = |x_2|$ case.

$\lim_{p\to\infty} \|\underline{x}\|_p = \lim_{p\to\infty} |x_1| \left(1 + \frac{|x_2|^p}{|x_1|^p}\right)^{\frac{1}{p}} = |x_1| \lim_{p\to\infty} (2)^{\frac{1}{p}} = \underline{|x_1| = |x_2|}.$

iii) $|x_2| > |x_1|$ case

$\lim_{p\to\infty} \|\underline{x}\|_p = \lim_{p\to\infty} |x_2| \left(1 + \frac{|x_1|^p}{|x_2|^p}\right)^{\frac{1}{p}} = \underline{|x_2|}.$

(d) For $m \geq 1$, $\underline{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{bmatrix}$, $\lim_{p\to\infty} \|\underline{x}\|_p = \max\{|x_1|, \cdots, |x_m|\} \cdot \lim_{p\to\infty} \left(1 + \sum_{\substack{k=1, \\ k \neq \arg\max_i |x_i|}}^{m} \frac{|x_k|^p}{\max\{|x_i|\}^p}\right)^{\frac{1}{p}}$

$= \underline{\max\{|x_1|, \cdots, |x_m|\} = \|\underline{x}\|_\infty}$

7. (a) Analysis: $S(f) = \int_{-\infty}^{\infty} s(t) e^{-j2\pi f t} dt$, $(-B < f < B)$

Synthesis: $s(t) = \int_{-\infty}^{\infty} S(f) e^{j2\pi f t} df = \int_{-B}^{B} S(f) e^{j2\pi f t} df$, $\forall t$.

(b) $p(t) = \sum_{n=-\infty}^{\infty} s(t-nT) = s(t) * \sum_{n=-\infty}^{\infty} \delta(t-nT)$

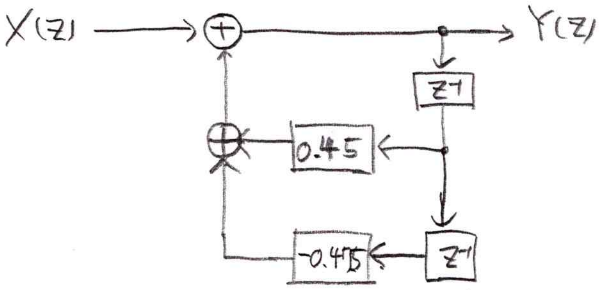
$P(f) = S(f) \cdot \frac{1}{T} \sum_{k=-\infty}^{\infty} \delta(f - \frac{k}{T}) \quad \left(= \frac{1}{T} \sum_{k=-\infty}^{\infty} S(\frac{k}{T}) \right)$

(c)

2013 통신.

1. $H(z) = \dfrac{1}{(1-0.5z^{-1})(1+0.95z^{-1})}$.

(a) $H(z) = \dfrac{1}{1+0.45z^{-1}-0.475z^{-2}}$



(Mason's rule 쓰면 H(z) 구해진다.)

(b) $H(z) = \dfrac{a}{1-0.5z^{-1}} + \dfrac{b}{1+0.95z^{-1}}$.

$\begin{cases} a+b=1, \\ 0.95a-0.5b=0. \end{cases} \Rightarrow \begin{cases} a+b=1 \\ 1.9a-b=0 \end{cases} \quad \therefore a = \dfrac{10}{29},\ b = \dfrac{19}{29}$

$H(z) = \dfrac{\frac{10}{29}}{1-0.5z^{-1}} + \dfrac{\frac{19}{29}}{1+0.95z^{-1}} \Rightarrow \underline{h[n] = \dfrac{10}{29}(0.5)^n u[n] + \dfrac{19}{29}(-0.95)^n u[n]}$.

(c) $z = e^{jw}$ (Frequency response 구하기 위해서) 대입.

$H(e^{jw}) = |H(e^{jw})| \angle H(e^{jw})$,

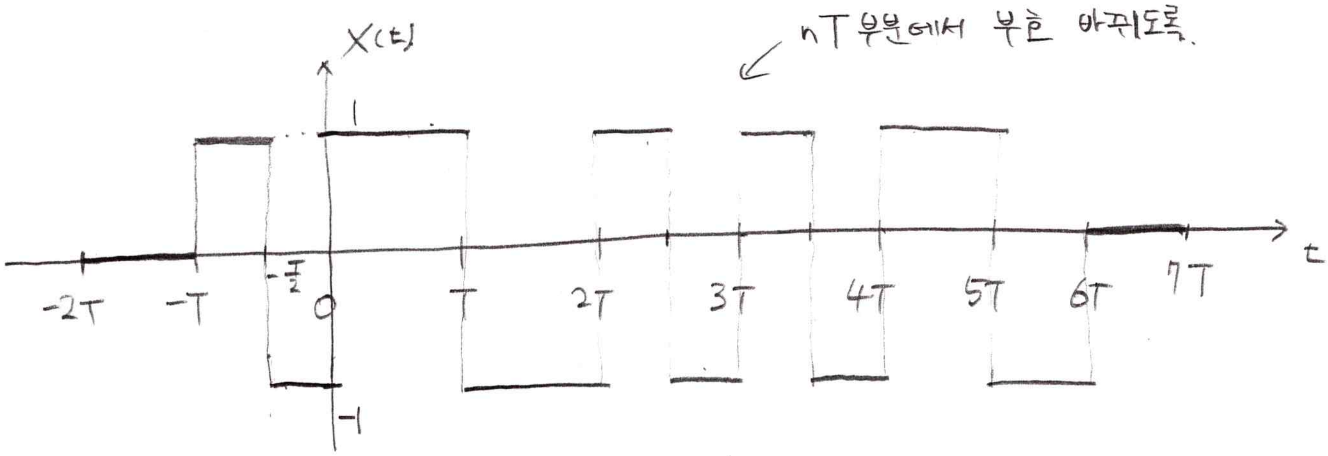
$|H(e^{jw})|$ (magnitude response) : $Re + jIm$로 분리한 후 $\sqrt{Re^2 + Im^2}$.

$\angle H(e^{jw})$ (phase response) : $Re + jIm$로 분리한 후 $\tan^{-1}\left(\dfrac{Im}{Re}\right)$

계산 복잡하니까 skip..

2.

(a)



(b) $[X_0[1], X_1[1]] = [0, -T]$

$[X_0[2], X_1[2]] = [T, 0]$

$[X_0[3], X_1[3]] = [T, 0]$

$[X_0[4], X_1[4]] = [0, T]$

$[X_0[5], X_1[5]] = [0, -T]$

$\Leftarrow$ 각 구간마다 $X(t)$ 값과 $S_0(t)$ 또는 $S_1(t)$ 대입해서 적분

(c) $N(t)$: AWGN 이므로 mean, variance 구하고, 보통 i.i.d. 이므로 이 문제에서 N_0와 N_1의 covariance를 구해서 uncorrelated 되어있는지 확인. (Gaussian noise: Uncorrelated $\underset{\text{iff}}{\Leftrightarrow}$ indep.)

① Mean of $N_0[n]$, $N_1[n]$

Randomness는 $N(t)$에 존재.

$$E[N_0[n]] = E\left[\int_{nT}^{(n+1)T} N(t)S_0(t-nT)dt\right] = \int_{nT}^{(n+1)T} E[N(t)]S_0(t-nT)dt = 0.$$

마찬가지로 $E[N_1[n]] = 0$

② Variance of $N_0[n]$, $N_1[n]$.

$E[N_0[n]]=0$ 이므로 $Var(N_0[n]) = E[N_0^2[n]]$

$$E[N_0^2[n]] = E\left[\int_{nT}^{(n+1)T} N(t_0)S_0(t_0-nT)\,dt_0 \int_{nT}^{(n+1)T} N(t_1)S_0(t_1-nT)dt_1\right]$$

$$= \int_{nT}^{(n+1)T}\int_{nT}^{(n+1)T} E[N(t_0)N(t_1)]S_0(t_0-nT)S_0(t_1-nT)\,dt_0dt_1$$

$$= \int_{nT}^{(n+1)T}\int_{nT}^{(n+1)T} \frac{N_0}{2}\delta(t_0-t_1)S_0(t_0-nT)S_0(t_1-nT)dt_0dt_1 = \frac{N_0}{2}\int_{nT}^{(n+1)T}\{S_0(t_0-nT)\}^2dt_0$$

$$= \frac{N_0T}{2},$$ 마찬가지로 $E[N_1^2[n]] = \frac{N_0T}{2}$.

$$\therefore f_{N_0[n]}(n_0) = \frac{1}{\sqrt{\pi N_0T}}e^{-\frac{n_0^2}{N_0T}}, \quad f_{N_1[n]}(n_1) = \frac{1}{\sqrt{\pi N_0T}}e^{-\frac{n_1^2}{N_0T}}$$

③ Covariance of $N_0[n]$, $N_1[n]$.

$$Cov(N_0[n], N_1[n]) = E[N_0[n]N_1[n+m]] = E\left[\int_{nT}^{(n+1)T} N(t)S_0(t-nT)\,dt \int_{(n+m)T}^{(n+m+1)T} N(t')S_1(t'-(n+m)T)\right]$$

$$= \int_{nT}^{(n+1)T}\int_{(n+m)T}^{(n+m+1)T} E[N(t)N(t')]S_0(t-nT)S_1(t'-(n+m)T)\,dt\,dt'$$

$= 0$, So $N_0[n]$, $N_1[n]$ are statistically independent.

$$\therefore f_{N_0[n],N_1[n]}(n_0,n_1) = f_{N_0[n]}(n_0)\, f_{N_1[n]}(n_1)$$

$$= \frac{1}{\pi N_0 T} e^{-\frac{n_0^2+n_1^2}{N_0 T}}$$

(d) $T \le t \le 2T$ 구간에서 $Y(t) = N(t) - \sqrt{P}$.

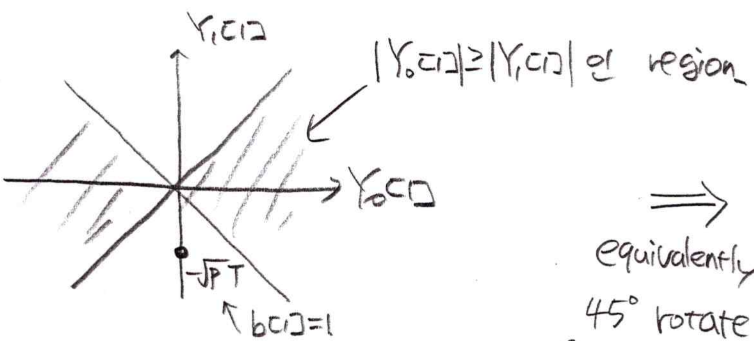
So, $E[Y_0[1]] = 0$, $E[Y_1[1]] = -\sqrt{P}\,T$,

$Var(Y_0[1]) = \frac{N_0 T}{2}$, $Var(Y_1[1]) = \frac{N_0 T}{2}$.

$Cov(Y_0[1], Y_1[1]) = 0$.

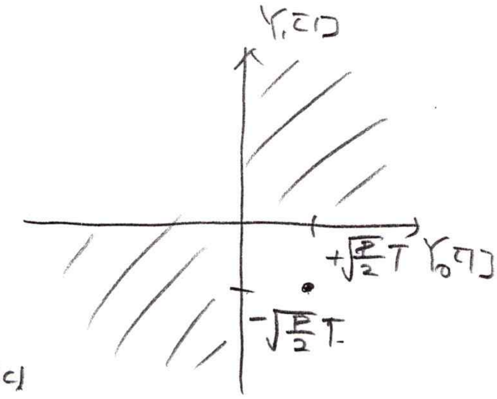
$$\therefore f_{Y_0[1],Y_1[1]}(y_0, y_1) = \frac{1}{\pi N_0 T} e^{-\frac{n_0^2+(n_1+\sqrt{P}T)^2}{N_0 T}}$$

(e)



$\Longrightarrow$

equivalently, 45° rotate.
(circularly symmetric)



Letting

$$Q\left(\frac{d}{2\sigma}\right) = Q\left(\sqrt{\frac{PT}{N_0}}\right) = p, \quad Pr(|Y_0[1]| \ge |Y_1[1]| \mid b[1]=1) = p + p - p^2 = 2p(1-p).$$

$$\therefore Pr(|Y_0[1]| \ge |Y_1[1]| \mid b[1]=1) = 2Q\left(\sqrt{\frac{PT}{N_0}}\right)\left\{1 - Q\left(\sqrt{\frac{PT}{N_0}}\right)\right\}$$

필수 2013 제어

1. a) $L(j\omega) = \dfrac{K}{j\omega(j\omega+2)(j\omega+10)} = \dfrac{K}{(-\omega^2+2j\omega)(j\omega+10)}$

$$= \frac{K}{-j\omega^3-10\omega^2-2\omega^2+20j\omega} = \frac{K}{-12\omega^2+j(20\omega-\omega^3)}.$$

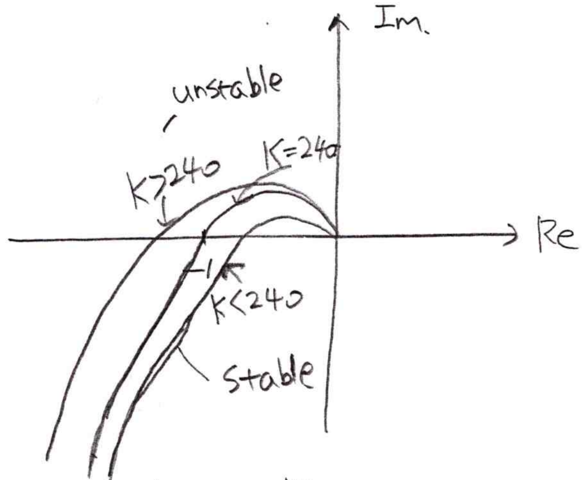
Gain margin을 구하기 위해서 $L(j\omega)$의 imaginary part $=0$ 이 되도록 하는 ω_p를 구한다.

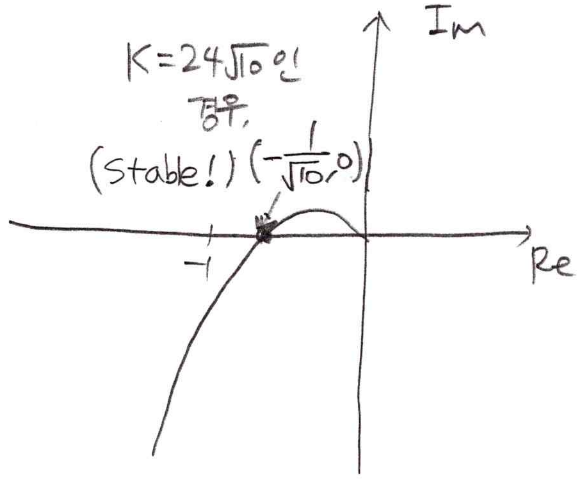
$\Rightarrow\ 20\omega-\omega^3=0,\quad \underline{\omega_p=\sqrt{20}}$

$$|L(j\omega_p)| = \frac{12K\omega_p^2}{144\omega_p^4+\omega_p^2(20-\omega_p^2)^2} = \frac{K}{240}$$

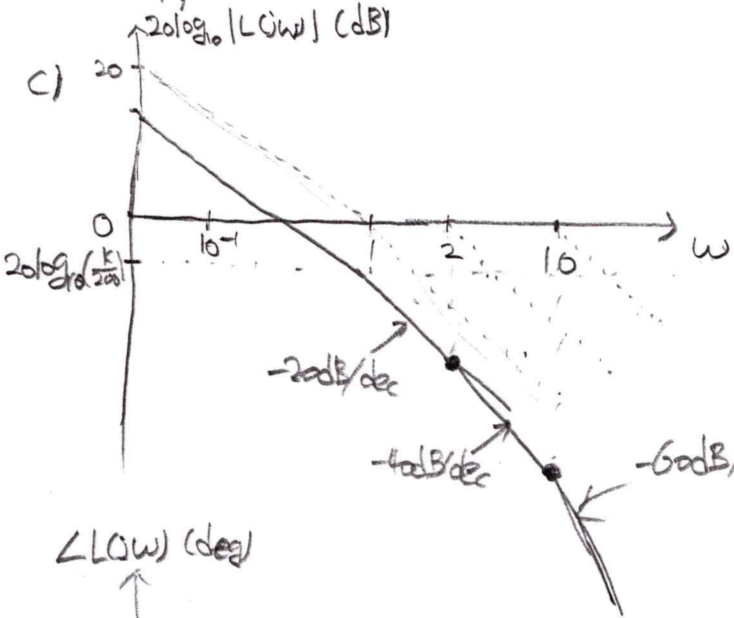
$20\log_{10}|L(j\omega_p)| = -10 = 20\log_{10}\left(\frac{K}{240}\right),\quad \frac{K}{240}=\frac{1}{\sqrt{10}}\quad \therefore \underline{K=24\sqrt{10}}$

b)





c)



$$L(j\omega) = \frac{\frac{K}{200}}{j\omega(1+j\frac{\omega}{2})(1+j\frac{\omega}{10})}$$

d) ⇐ 그림이 정확하지 않지만, magnitude plot 에서 0 dB 인 ω 에서의 $\angle L(j\omega)$ 값이 phase margin, $\angle L(j\omega)$가 $-180°$인 ω에서의 magnitude가 gain margin.

선택

1. ① completely controllable at t_0

: State space의 임의의 $X(t_0)$와 X_1에서, $X(t_0)$ state를 t_1 ($t_1 > t_0$, t_1 is finite)에서의 state X_1으로 transfer 하는 input $u[t_0, t_1]$이 존재할 때 t_0에서 controllable 하다.

② completely observable at t_0

: t_0에서 임의의 state X_0에 대해서 input $u[t_0, t_1]$와 output $y[t_0, t_1]$을 time interval $[t_0, t_1]$, ($t_1 > t_0$, t_1 is finite)에 대해서 아는 것으로 state X_0를 결정할 수 있을 때, dynamical equation은 t_0에서 observable 하다.

③ BIBO stable

: 임의의 bounded input에 대해 bounded output이 나오는 relaxed system은 BIBO stable 하다.

④ Asymptotically stable

2. 1) State transition matrix $\phi(t) = \mathcal{L}^{-1}\{(sI-A)^{-1}\}$,

$$sI-A = \begin{bmatrix} s & -1 \\ 2 & s+3 \end{bmatrix}, \quad (sI-A)^{-1} = \begin{bmatrix} \frac{s+3}{(s+1)(s+2)} & \frac{1}{(s+1)(s+2)} \\ \frac{-2}{(s+1)(s+2)} & \frac{s}{(s+1)(s+2)} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{2}{s+1} - \frac{1}{s+2} & \frac{1}{s+1} - \frac{1}{s+2} \\ \frac{-2}{s+1} + \frac{2}{s+2} & \frac{-1}{s+1} + \frac{2}{s+2} \end{bmatrix}$$

$$\therefore \phi(t) = \begin{bmatrix} 2e^{-t} - e^{-2t} & e^{-t} - e^{-2t} \\ -2e^{-t} + 2e^{-2t} & -e^{-t} + 2e^{-2t} \end{bmatrix} u(t)$$

2) $\dot{x}(t) = Ax(t) + Bu(t) \xrightarrow{\mathcal{L}} sX(s) - x(0) = AX(s) + BU(s),$

$$X(s) = (sI-A)^{-1}x(0) + (sI-A)^{-1}BU(s)$$

$$(sI-A)^{-1}BU(s) = \begin{bmatrix} \frac{1}{s(s+1)} - \frac{1}{s(s+2)} \\ \frac{-1}{s(s+1)} + \frac{2}{s(s+2)} \end{bmatrix} = \begin{bmatrix} \frac{1}{2}\cdot\frac{1}{s} - \frac{1}{s+1} + \frac{1}{2}\cdot\frac{1}{(s+2)} \\ \frac{1}{s+1} - \frac{1}{s+2} \end{bmatrix}$$

$$\therefore x(t) = \phi(t)x(0) + \begin{bmatrix} \frac{1}{2} - e^{-t} + \frac{1}{2}e^{-2t} \\ e^{-t} - e^{-2t} \end{bmatrix} u(t)$$

$y(t) = [2 \ 0]x(t)$, $\lim_{t\to\infty} y(t) = 1$ (exponential term 들은 모두 0 으로 수렴)

$\therefore \underline{y_{ss}(t) = 1}$

3) 2) 에서 $U(s) = \mathcal{L}\left\{\frac{1}{2}\sin t + \frac{\sqrt{3}}{2}\cos t\right\} = \frac{1}{2(s^2+1)} + \frac{\sqrt{3}s}{2(s^2+1)}$ 로 놓고

같은 과정으로...