Reflectivity of metallic films in the infrared

B. Carli
Unitá di ricerca CNR/GIFCO, Istituto di Fisica–Firenze, Via San Bonaventura, 13–50145 Florence, Italy
(Received 28 December 1976; revised manuscript received 22 April 1977)

The reflectivity of metallic films, as calculated from standard equations, is discussed in the case of infrared radiation. We derive the minimum thickness of a metallic film for infrared reflectors and show the reason that metallic films cannot be used to make Fabry-Perot filters in the infrared.

I. THEORY OF METALLIC FILMS

The reflectivity $\mathcal{R}$ and the transmissivity $\mathcal{T}$ of a metallic film, in the case of the geometry of Fig. 1, are given by the following formulas:

$$\mathcal{R}=\frac{\rho_{12}^2 e^{2v_2\eta}+\rho_{23}^2 e^{-2v_2\eta}+2\rho_{12}\rho_{23}\cos(\phi_{23}-\phi_{12}+2u_2\eta)}{e^{2v_2\eta}+\rho_{12}^2\rho_{23}^2 e^{-2v_2\eta}+2\rho_{12}\rho_{23}\cos(\phi_{23}+\phi_{12}+2u_2\eta)}, \tag{1.1}$$

$$\mathcal{T}=C\,\frac{\tau_{12}^2\tau_{33}^2}{e^{2v_2\eta}+\rho_{12}^2\rho_{23}^2 e^{-2v_2\eta}+2\rho_{12}\rho_{23}\cos(\phi_{23}+\phi_{12}+2u_2\eta)}, \tag{1.2}$$

where $C=n_3\cos\theta_3/n_1\cos\theta_1$ for TE waves and $C=n_1\cos\theta_3/n_3\cos\theta_1$ for TM waves. ρ_{ab} and ϕ_{ab} are the amplitude and the phase of the reflection coefficient at the boundary a, b; τ_{ab} is the amplitude of the transmission coefficient at the boundary a, b; u_2 and v_2 are, respectively, the real and the imaginary parts of the factor $\hat{n}_2\cos\theta_2$; η is equal to $2\pi h/\lambda_0$, where h is the thickness of the film, and λ_0 the free-space wavelength.

The complete expression of ρ_{ab}, ϕ_{ab}, and τ_{ab} is given by Born and Wolf.[1] We refer herewith to this well-known calculation and use the same symbols. More detailed considerations can be found in Hadley and Dennison.[2]

When $2v_e\eta$ tends to zero, expression (1.1) tends to

$$\mathcal{R}=\frac{\rho_{12}^2+\rho_{23}^2+2\rho_{12}\rho_{23}\cos(\phi_{23}-\phi_{12})}{1+\rho_{12}^2\rho_{23}^2+2\rho_{12}\rho_{23}\cos(\phi_{23}+\phi_{12})}=\rho_{13}^2, \tag{1.3}$$

i.e., to the reflectivity of the boundary between dielectric 1 and dielectric 3. $\mathcal{R}$ increases for intermediate values and for $2v_2\eta>1$ tends to a constant value

$$\mathcal{R}=\rho_{12}^2, \tag{1.4}$$

i.e., to the reflectivity of the boundary between dielectric 1 and the metal 2. This is usually the reflectivity that we observe in metallic films used as reflectors. The condition $2v_e\eta>1$, which is often used to ensure that the maximum value of reflectivity is reached, can also be written

$$h>\lambda_0/4\pi n\kappa=d, \tag{1.5}$$

where d is the "penetration depth."

For visible radiation conditions, (1.5) is easily satisfied by films obtained with evaporation techniques of the order of 100 Å in thickness, but in the far infrared and in the millimeter region ($\lambda_0=30-10^4$ μm) it imposes special requirements ($d\simeq 100-6000$ Å). In practice these requirements can be relaxed for some applications and it is useful to have a more complete criterion for reflecting layers at long wavelengths.

To this purpose we will discuss the meaning of expression (1.1) in the infrared region.

II. METALLIC FILMS IN THE FAR INFRARED

The refractive index $\hat{n}=n+in\kappa$ of a metal in the infrared is equal to (1)

$$\hat{n}=(\mu\sigma/\nu)^{1/2}+i(\mu\sigma/\nu)^{1/2},$$

where μ is the permeability of the material, σ is its electrical conductivity, and ν the frequency of the radiation. The quantity $\sqrt{(\mu\sigma/\nu)^{1/2}}$ is much greater than unity for a conductive material and far-infrared radiation; furthermore, it increases its value with increasing wavelength.

In the infrared we have, therefore, $n\simeq n\kappa\gg 1$, which implies that $\cos\theta_2\simeq 1$ and $u_2+iv_2=\hat{n}\cos\theta_2\simeq\hat{n}$. We now assume that the metallic film has a thickness much smaller than the wavelength and expand the terms of (1.1) to the second order for $1/n\kappa\ll 1$ and $2n\kappa\eta\ll 1$. For TE waves we obtain

$$\rho_{12}=1-\frac{n_1\cos\theta_1}{n\kappa}+\frac{1}{2}\left(\frac{n_1\cos\theta_1}{n\kappa}\right)^2,$$

$$\rho_{23}=1-\frac{n_3\cos\theta_3}{n\kappa}+\frac{1}{2}\left(\frac{n_3\cos\theta_3}{n\kappa}\right)^2,$$

$$e^{2v_2\eta}=1+2n\kappa\eta+2n^2\kappa^2\eta^2,$$

$$e^{-2v_2\eta}=1-2n\kappa\eta+2n^2\kappa^2\eta^2,$$

$$\cos(\phi_{23}\pm\phi_{12}+2u_2\eta)=1-\left(\frac{n_3\cos\theta_3}{n\kappa}\pm\frac{n_1\cos\theta_1}{n\kappa}+2n\kappa\eta\right)^2\Big/2.$$

Substituting in (1.1),

$$\mathcal{R}_\perp=\left(\frac{2f+n_3\cos\theta_3-n_1\cos\theta_1}{2f+n_3\cos\theta_3+n_1\cos\theta_1}\right)^2, \tag{2.1}$$

where $f=n^2\kappa^2\eta$.

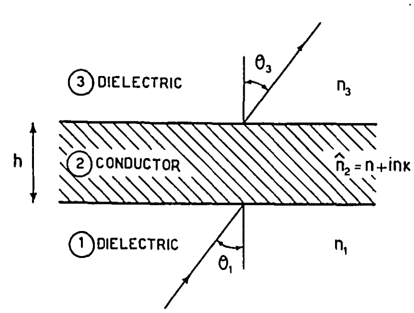

FIG. 1. Illustrating a plane wave incident on a metallic film of thickness h.

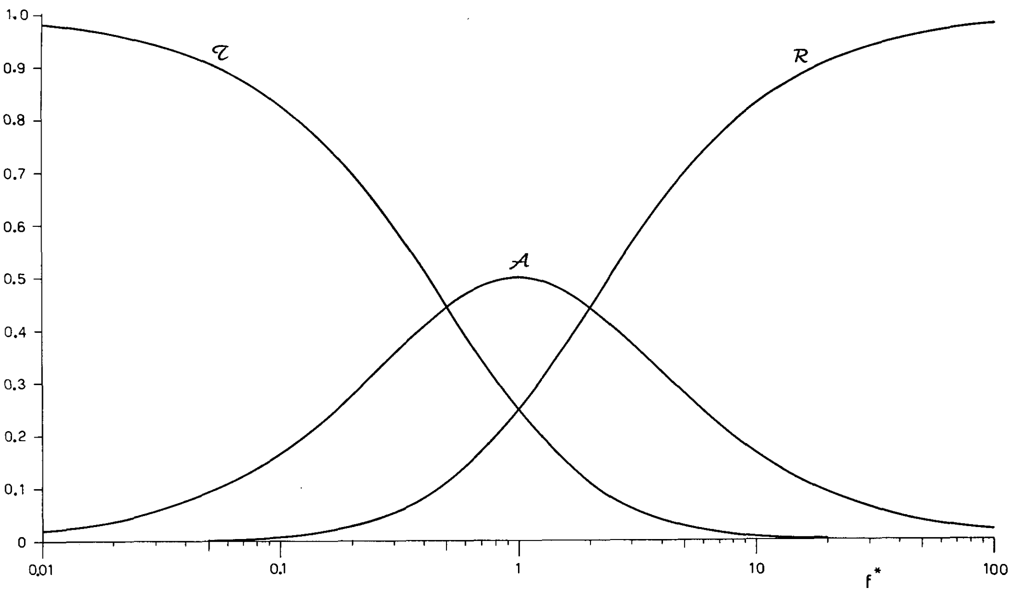

FIG. 2. Transmissivity, reflectivity, and absorption of a metallic film as a function of f^*.

Passing from Gaussian units to mks units,

$$f=\frac{60\pi[\text{ohms}]}{r[\text{ohms}]} \quad . \tag{2.2}$$

The quantity $r=1/h\sigma$ is the resistance of a square piece of the metallic film, measured between opposite sides of the square. It is independent of the size of the square and is therefore used to express the properties of the film. f is inversely proportional to the resistance r, normalized to a resistance of $60\ \pi\Omega = 188.5\ \Omega$.

Similarly,

$$\mathcal{T}_\perp=\frac{4n_1 n_3 \cos\theta_1 \cos\theta_3}{(2f+n_3\cos\theta_3+n_1\cos\theta_1)^2} \tag{2.3}$$

and

$$\mathcal{A}_\perp = 1-\mathcal{R}_\perp-\mathcal{T}_\perp=\frac{8fn_1\cos\theta_1}{(2f+n_3\cos\theta_3+n_1\cos\theta_1)^2} \quad . \tag{2.4}$$

For TM waves,

$$\mathcal{R}_\parallel=\left(\frac{2f\cos\theta_1\cos\theta_3+n_3\cos\theta_1-n_1\cos\theta_3}{2f\cos\theta_1\cos\theta_3+n_3\cos\theta_1+n_1\cos\theta_3}\right)^2 ,$$

$$\mathcal{T}_\parallel=\frac{4n_1 n_3\cos\theta_1\cos\theta_3}{(2f\cos\theta_1\cos\theta_3+n_3\cos\theta_1+n_1\cos\theta_3)^2} \tag{2.5}$$

$$\mathcal{A}_\parallel=\frac{8fn_1\cos\theta_1\cos^2\theta_3}{(2f\cos\theta_1\cos\theta_3+n_3\cos\theta_1+n_1\cos\theta_3)^2} \quad .$$

A similar result can be found in (2).

For $n_1=n_3=n^*$ and $\theta_1=\theta_3=\theta^*$, these formulas can be written

$$\mathcal{R}=\left(\frac{f^*}{f^*+1}\right)^2 , \quad \mathcal{T}=\left(\frac{1}{f^*+1}\right)^2 , \quad \text{and} \quad \mathcal{A}=\frac{2f^*}{(f^*+1)^2} , \tag{2.6}$$

where for TE waves $f^*=f/n^*\cos\theta^*$ and for TM waves $f^*=f\cos\theta^*/n^*$.

In the case of normal incidence and for $n^*=1$, the same result is obtained with transmission line theory (see Appendix).

In Fig. 2 reflectivity, transmissivity, and absorption are plotted as a function of f^*. An interesting feature of these curves is their symmetry relative to the value of $f^*=1$.

From (2.2) and (2.6) it follows that the minimum thickness necessary to obtain a reflectivity $\mathcal{R}$ with a metal of conductivity σ is

$$h_{\min}=(1+\sqrt{\mathcal{R}})\sqrt{\mathcal{R}}/(1-\mathcal{R})\,60\pi\sigma , \tag{2.7}$$

and for $\mathcal{R}\simeq 1$

$$h_{\min}=1/(1-\mathcal{R})30\pi\sigma .$$

As an example, the value of $h_{\min}$ as a function of $(1-\mathcal{R})$ for different metals is shown in Fig. 3. For simplicity, as the conductivity of metals depends very little on frequency in the far infrared, the bulk dc conductivity has been used to calculate these curves. The bulk dc conductivity will give a correct value of $\mathcal{R}$ in the submillimeter region, but will in general give optimistic values of reflectivity in the near-middle infrared.

It is important to recall that expression (2.7) is valid only for $h \ll d$, and therefore the curves of Fig. 3 can be used only within this limit. The penetration depth d for silver and the other metals is marked in the figure for some infrared wavelengths and defines, as a function of the wavelength, the interval at which the curves are valid.

For $h \gg d$ the reflectivity becomes equal to $\mathcal{R}_{\max}=\rho_{12}^2$ [see (1.4)] and the value of $(1-\mathcal{R}_{\max})$ is also marked in the figure.

For example, from Fig. 3 one can deduce that with a silver film at 1 mm wavelength it is possible to reach a maximum reflectivity $\mathcal{R}_{\max}=1-(1.4\times10^{-3})$ for $h \gg d$

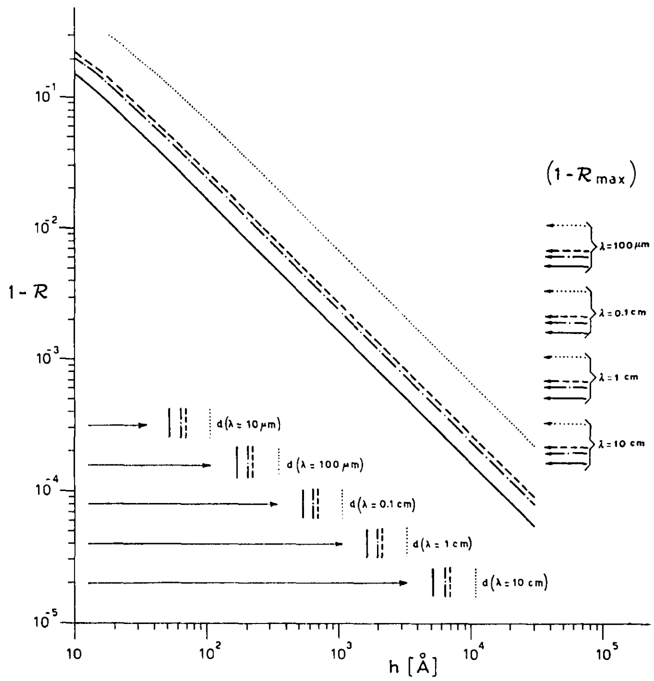

FIG. 3. The minimum thickness necessary to obtain a reflectivity $\mathcal{R}$ as a function of $(1-\mathcal{R})$. The four curves refer to four different materials: solid curve, silver; dash-dot-dash, gold; dashed, aluminium; and dotted, nickel. The values of the penetration depth and of $(1-\mathcal{R}_{max})$ are marked, respectively at the bottom and on the right of the figure, for the four materials at several wavelengths.

= 570 Å; but, if a reflectivity of say 98% is satisfactory, the thickness requirement can be reduced to $h = 80$ Å.

III. INTERFERENCE FILTERS

The formulas derived in Sec. II can be used to understand why in the far infrared, interference filters (Fabry-Perot-type) require the use of meshes and do not work if made with metallic films.

The reason is found in the fact that in the far infrared, a metallic film with high reflectivity

$$\mathcal{R} = [f^*/(1+f^*)]^2 , \tag{3.1}$$

that is with $f^* > 1$, has an absorption

$$\mathcal{A} = 2f^*/(1+f^*)^2 \tag{3.2}$$

which is greater than the transmissivity

$$\mathcal{T} = 1/(1+f^*)^2 . \tag{3.3}$$

This makes the efficiency of Fabry-Perot filters very low. In fact, from Born and Wolf,[3] the peak transmission of a Fabry-Perot with reflecting surfaces which have reflectivity $\mathcal{R}$ and absorption $\mathcal{A}$, is equal to

$$\mathcal{T} = \left(1 - \frac{\mathcal{A}}{1-\mathcal{R}}\right)^2 . \tag{3.4}$$

Substituting (3.1) and (3.2) in (3.4),

$$\mathcal{T} = 1/(1+2f^*) .$$

Therefore, a good Fabry-Perot filter, which requires a high reflectivity ($f^* \gg 1$), has necessarily a very low peak transmission.

IV. CONCLUSIONS

From the formulas which describe the optical properties of metallic films in the infrared, we have derived an expression relating reflectivity and the thickness of metallic films. The results, summarized in Fig. 3, can be useful to specify the thickness of the reflecting layer required by infrared mirrors.

Finally we have verified that, because of their relatively large absorption, metallic films cannot be exploited to make Fabry-Perot filters in the far infrared.

APPENDIX

The problem of perpendicular incidence of a plane wave on a metallic film with a resistance per square equal to r is equivalent to a transmission line problem, where the transmission line has an impedence k (equal to the vacuum impedence, i.e., 377 Ω) and a localized load of impedence $Z = r$.

The total impedence Z' present at the point of the load is equal to the parallel of Z and k:

$$Z' = Zk/(Z+k) .$$

The "reflectivity" at this point is equal to[4]

$$\mathcal{R} = \left|\frac{k-Z'}{k+Z'}\right|^2 = \frac{x^2}{(x+2)^2} ,$$

with $x = k/Z$, and the transmissivity and the absorption are equal to

$$\mathcal{T} = (1-\mathcal{R})\,\frac{Z}{Z+k} = \frac{4}{(x+2)^2} ,$$

$$\mathcal{A} = (1-\mathcal{R})\,\frac{k}{Z+k} = \frac{4x}{(x+2)^2} .$$

It is straightforward to verify that $x = 2f$ and that these formulas are equivalent to those given in Sec. II.

[1] M. Born and E. Wolf, *Principles of Optics*, 5th ed. (Pergamon, Oxford, 1975), Sec. 13.4.

[2] L. N. Hadley and D. M. Dennison, "Reflection and Transmission Interference Filters, Part I, Theory," J. Opt. Soc. Am. 37, 451 (1947).

[3] M. Born and E. Wolf, *Principles of Optics*, 5th ed. (Pergamon, Oxford, 1975), Sec. 7.6.

[4] J. Millman and H. Taub, *Pulse Digital and Switching Waveforms* (McGraw-Hill, Tokyo, 1965), Sec. 3.15.