Caution!!!
Use a separate answer booklet for Problem 2.

Problem 2. (50 points) Consider a communication system that employs complex pulse amplitude modulation or quadrature amplitude modulation (QAM). When the complex envelope of the transmitted signal is modeled by

$$x(t) = \sum_n s[n] g(t - nT),$$

where $s[n]$ denotes the 4-QAM data symbol, i.e., $s[n] \in \mathcal{C}_1 = \{1, j, -1, -j\}$, $g(t)$ represents the pulse shaping filter, and T is the symbol duration, answer the following questions.

(a) (15 points) Suppose that the pulse shaping filter is given by

$$g(t) = \frac{p(t/T)}{\sqrt{\frac{2}{3}T}}$$

where $p(t) = 1 - |t|$, for $|t| \leq 1$ and $p(t) = 0$ for $|t| > 1$. Show that $g(t)$ is a Nyquist pulse shape.

(b) (15 points) Assuming a square-root Nyquist pulse $g(t)$, matched filtering, and analog-to-digital converter (ADC) operations at a receiver, we can model the discrete output signal of the nth sample at a receiver by

$$y[n] = s[n] + v[n],$$

where $s[n] \in \mathcal{C}_1 = \{1, j, -1, -j\}$ and $v[n]$ denotes the complex AWGN with zero-mean and variance of σ^2, i.e., $v[n] \sim \mathcal{N}_{\mathbb{C}}(0, \sigma^2)$. Explain the maximum likelihood (ML) detection rule.

(c) (20 points) Suppose that the transmitter sends the QAM symbol using different constellation points, i.e., $s[n] \in \mathcal{C}_2 = \{1 + j, 1 - j, -1 + j, -1 - j\}/\sqrt{2}$, and the receiver uses the ML detection. Compare which constellation set is better between $\mathcal{C}_1$ and $\mathcal{C}_2$ in terms of the symbol error rate.