Spring 2019

EECE 588
Lecture 2

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FIELD REGIONS

Antenna Field Regions

- The region surrounding an antenna is usually subdivided into three regions:
  - Reactive near field
  - Radiating near field (Fresnel)
  - Far-field (Fraunhofer)
- Reactive near field: **"that portion of the near field region immediately surrounding the antenna wherein the reactive field predominates"**
- The outer boundary of this region is taken to be:

$$R < 0.62\sqrt{D^3 / \lambda}$$

POSTECH

What Does Reactive Mean?

- So what does reactive mean?
  - Think of a capacitor or an inductor.
  - Between the two electrodes of a capacitor, we have an electric field and energy is stored in this field. After all this is the role of a capacitor!
  - Also, in a solenoid inductor, we have a region of relatively strong magnetic field where energy is stored in the magnetic field.
  - These are reactive fields. They are localized in a region and do not move or propagate; that is why we can store energy.
- Antennas are different devices from inductors and capacitors. In antennas, we WANT our fields to move away from us but inevitably, we get some stationary fields that do not move. These stationary fields are the strongest in the vicinity of the antenna.

Field Regions

- For a very short dipole or equivalent radiator, the outer boundary is commonly taken to exist at a distance of $\lambda/2\pi$ from the antenna.
- Radiating near field (Fresnel): **That region of the field of an antenna between the reactive near-field region and the far field region wherein radiation fields predominate and wherein the angular field distribution is dependent upon the distance from the antenna**.
- If the antenna's maximum dimension is not large compared to the wavelength this region may not exist.

Antenna Field Regions

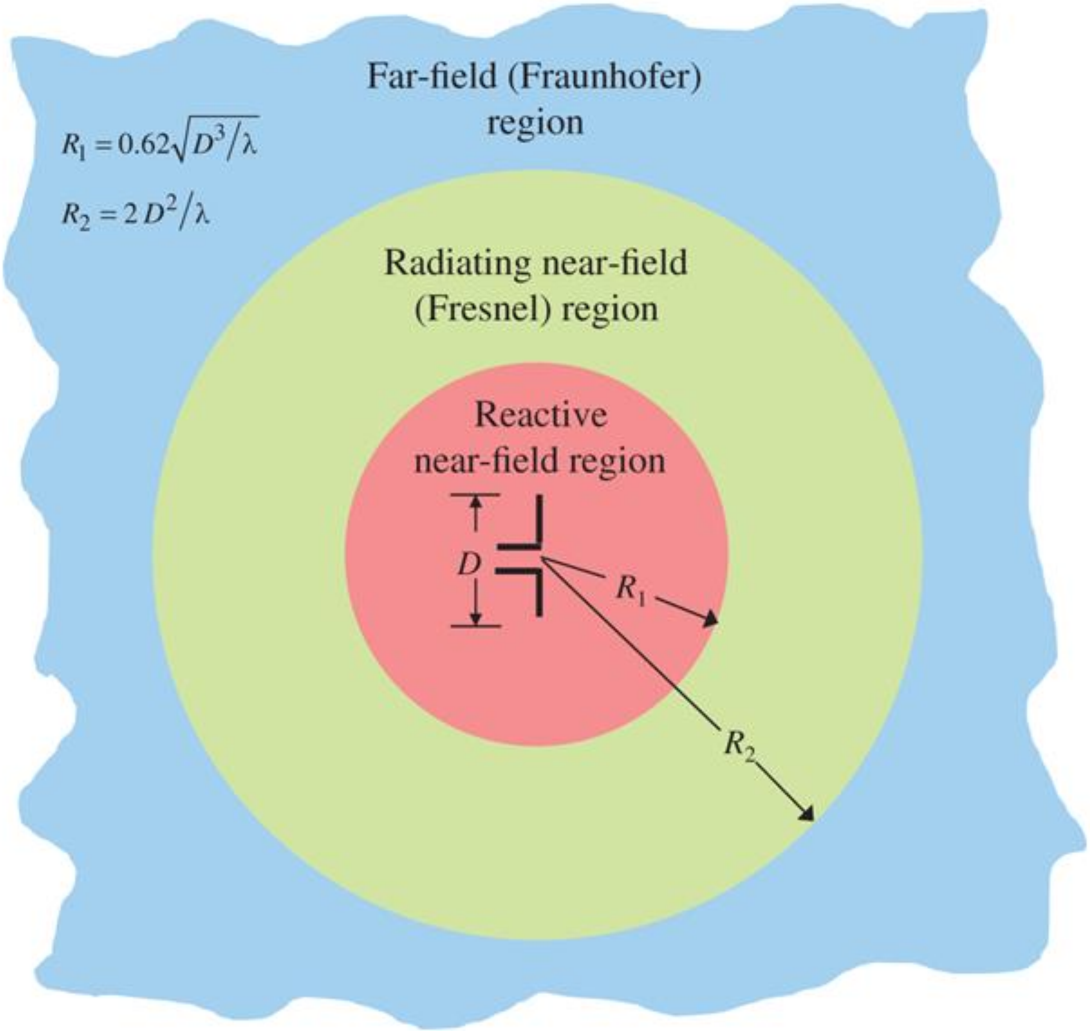
- The boundary of this region is considered to be in the following

$$0.62\sqrt{D^3/\lambda} < R < 2D^2/\lambda$$

- In the above equation, D is the maximum linear dimension of the antenna. Note that for these equations to be valid, D>>λ.
- In this region, the field pattern is in general a function of the radial distance and the radial field component is appreciable.
- Far Field region: That region of the field of an antenna where the angular field distribution is essentially independent of the distance between the observation point and the antenna.

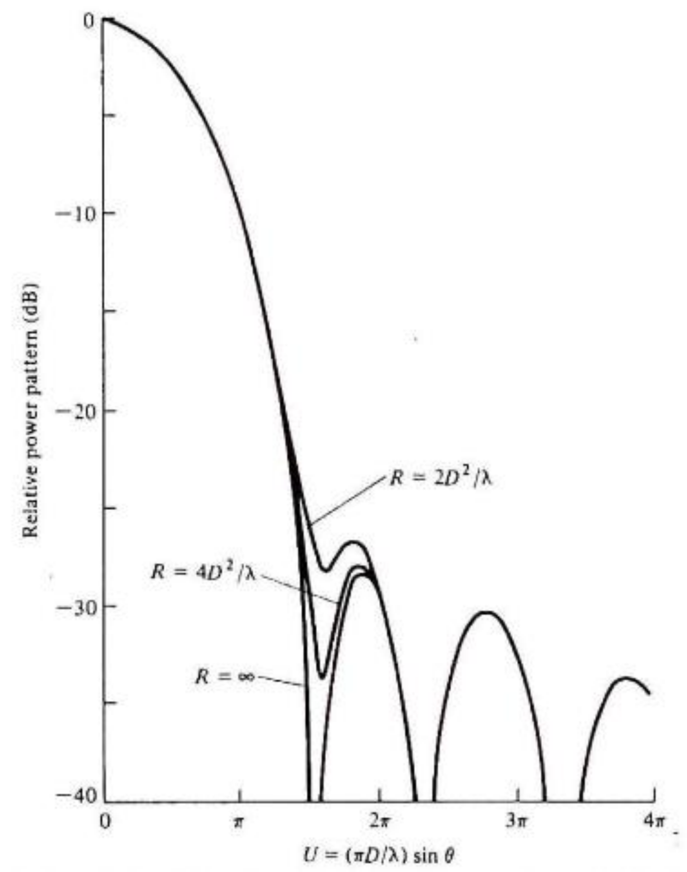
$$R > 2D^2/\lambda$$

Antenna Field Regions



Far Fields

- Far field:
  - Field components (E and H) are transverse to radial directions, i.e., normal to the direction of propagation.
  - Angular distribution is independent of the radial distance.
  - Note that the $R>2D^2/\lambda$ may not work for certain antennas but generally this is an accepted term.



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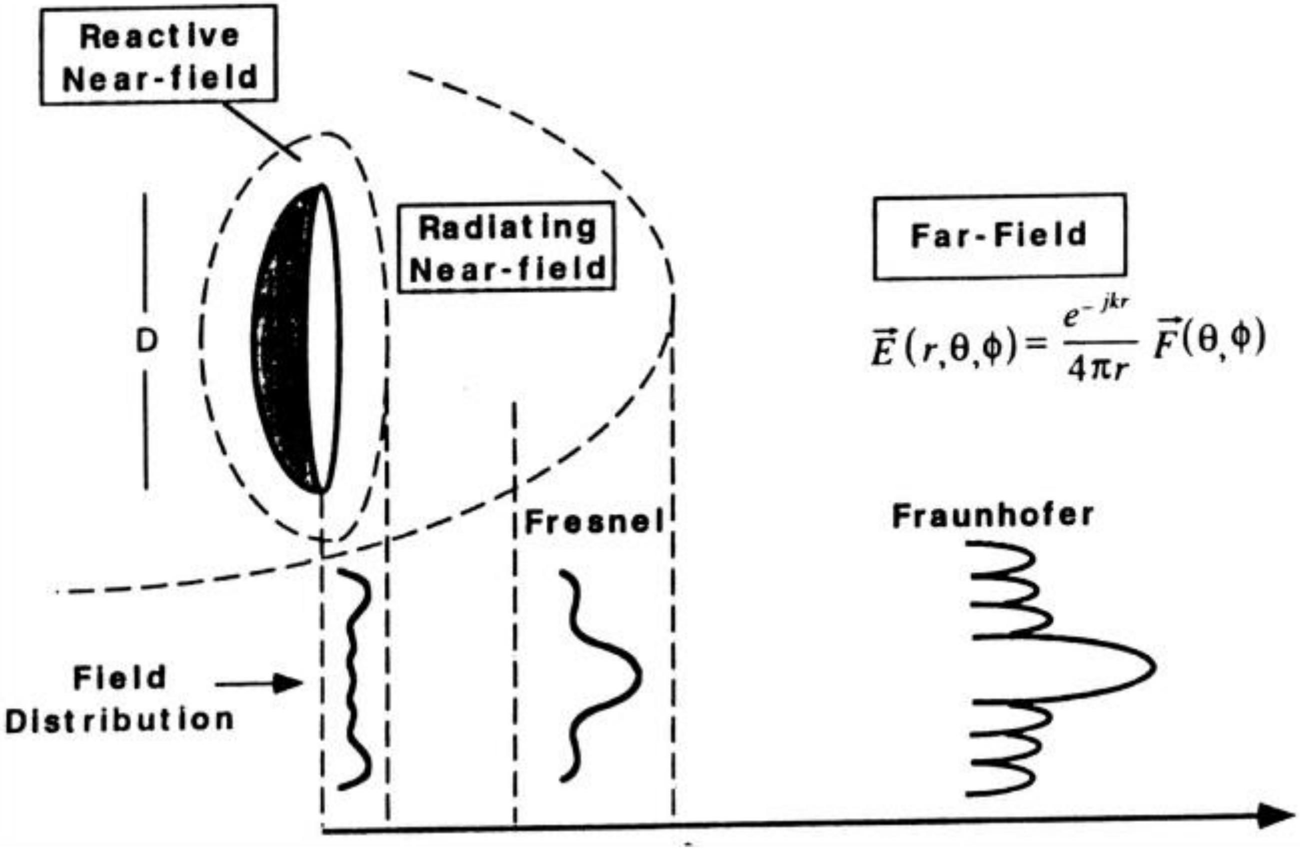
Spatial Field Distributions (Radiation Patterns) of the Antenna in the Near Field and Far Field

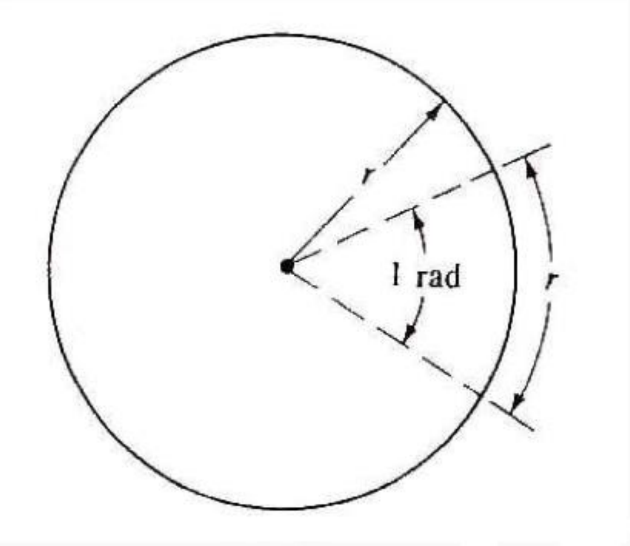


Summary

- Reactive near field:
  - Fields are essentially quasi-static in nature.
  - Fields have components along the radial direction.
- Radiative near field:
  - Fields are propagating.
  - Angular field distribution changes as a function of distance from the antenna.
  - Fields are not completely transverse to the direction of propagation.
- Far field:
  - Fields are transverse to the direction of propagation
  - Angular field distribution is not a function of distance

Radian and Steradian

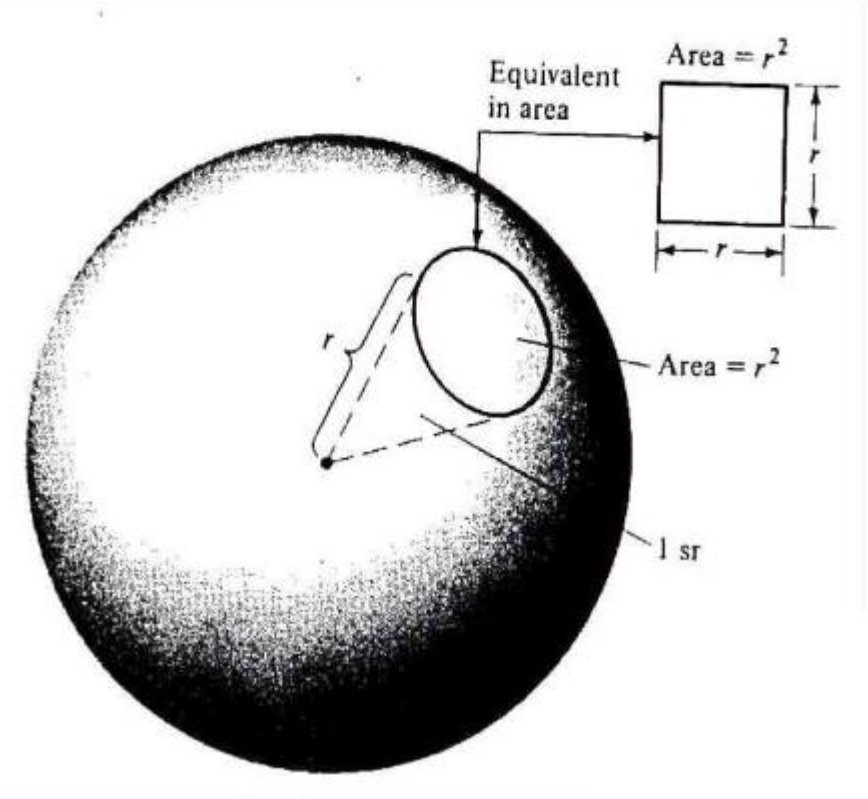
- Radian is a unit for measuring an angle in a plane
- 1 Rad = plane angle whose vertex is at the center of a circle of radius r that is subtended by an arc with arc length r.
- The circumference of a circle with radius r is:
  - $C = 2\pi r$ => there are 2π radians in full circles.



POSTECH

Radian and Steradian

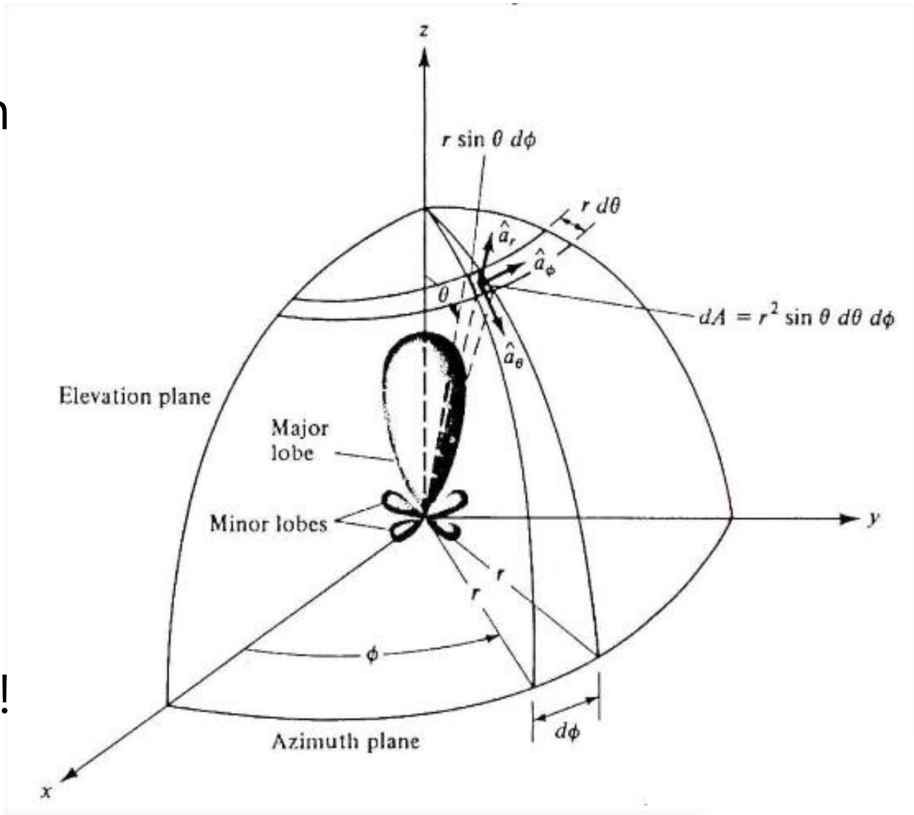
- The measure of a solid angle is Steradian.
- What is a Solid Angle?
  - The **solid angle**, Ω, is the angle in three-dimensional space that an object subtends at a point.
- 1 Str = the solid angle with its vertex at the center of a sphere of radius r which is subtended by a spherical surface area equal to r^2.



In a closed sphere, we have 4π Sr

Radian and Steradian

- The infinitesimal area dA on the surface of the sphere with radius r is given by:
  - $dA = r^2 \sin\theta \, d\theta \, d\varphi \; (m^2)$
  - $d\Omega = \frac{dA}{r^2} = \sin\theta \, d\theta \, d\varphi$ (Sr)
- If you do not remember how dA is calculated, please revisit your undergrad EM course. These calculations should be second nature to you by now!



Example 2.1:

For a sphere of radius *r*, find the solid angle Ω_A (*in square radians/steradians*) of a spherical cap on the surface of the sphere over the north-pole region defined by spherical angles of

$$0 \le \theta \le 30^o,\ 0 \le \phi \le 360^o$$

Refer to Figures 2.1 and 2.7. Do this:

a. exactly.

b. using $\Omega_A \approx \Delta\Theta_1 \cdot \Delta\Theta_2$ where $\Delta\Theta_1$ and $\Delta\Theta_2$ are two perpendicular angular separations of the spherical cap passing through the north pole.

Compare the two.

SOLUTION:

a. Exact: Using (2-2), we can write that

$$\Omega_A = \int_0^{360^o}\int_0^{30^o} d\Omega = \int_0^{2\pi}\int_0^{\pi/6} \sin\theta d\theta d\phi = \int_0^{2\pi} d\phi \int_0^{\pi/6} \sin\theta d\theta$$

$$= 2\pi\left[-\cos\theta\right]\Big|_0^{\pi/6} = 2\pi\left[-0.867+1\right]$$

$$\Omega_A = 2\pi\left(0.133\right) = 0.83566 \ \ sterads$$

b. Approximate:

$$\Omega_A \approx \Delta\Theta_1 \cdot \Delta\Theta_2 \overset{\Delta\Theta_1=\Delta\Theta_2}{=} (\Delta\Theta_1)^2 = \frac{\pi}{3}\left(\frac{\pi}{3}\right) = \frac{\pi^2}{9} = 1.09662 \ sterads$$

It is apparent that the approximate beam solid angle is about 31.23% in error.

POSTECH

Radiation Power Density

- Recall from your undergraduate EM that EM waves carry power.
- The power density of an EM wave can be calculated by Poyinting Vector:
  - **W**=**E**×**H**
  - $\vec{W}$ is the instantaneous Poyinting vector (W/m^2)
  - $\vec{E}$ = Instantaneous electric-field intensity (V/m)
  - $\vec{H}$ = Instantaneous magnetic field intensity (A/m)
- Note that the Poyinting vector is the POWER DENSITY. To obtain the total power we need to integrate the vector over a surface

$$P = \oiint_S \vec{W} . d\vec{s} = \oiint_S \vec{W} . \hat{n} ds$$

Radiation Power Density: Time Harmonic Situations

- In time varying fields, we are not very much concerned about the instantaneous power. We are interested in average power.
- In time harmonic situations of the form $e^{j\omega t}$, we define the complex fields $\vec{E}$ and $\vec{H}$

$$\vec{E}(x,y,z,t) = \mathrm{Re}[\vec{\mathbf{E}}(x,y,z)e^{j\omega t}]$$

$$\vec{H}(x,y,z,t) = \mathrm{Re}[\vec{\mathbf{H}}(x,y,z)e^{j\omega t}]$$

- Note that:
- Therefore,

$$\mathrm{Re}[\vec{\mathrm{E}}e^{j\omega t}] = 1/2[\vec{\mathrm{E}}e^{j\omega t} + \vec{\mathrm{E}}^{*}e^{-j\omega t}]$$

$$\vec{W} = \vec{E}\times\vec{H} = 1/2\,\mathrm{Re}[\vec{\mathrm{E}}\times\vec{\mathrm{H}}^{*}] + 1/2\,\mathrm{Re}[\vec{\mathrm{E}}\times\vec{\mathrm{H}}e^{2j\omega t}]$$

POSTECH

Radiation Power Density: Time Harmonic Situations

- The first term of this equation is not a function of time.
- The second one is essentially a sinusoidal function with the frequency of 2ω.
- Averaging these two terms over a single period results in:

$$\vec{\mathbf{W}}_{\mathbf{av}}(x,y,z)=\frac{1}{T}\int_0^T \vec{W}(x,y,z,t)\,dt=1/2\,\mathrm{Re}[\vec{\mathbf{E}}\times\vec{\mathbf{H}}^*] \quad (W/m^2)$$

- Note that in our phasor representation for $\vec{E}$ and $\vec{H}$, we are using peak values.
- If RMS values are used, the 1/2 factor in front of the Poyinting equation should be omitted.

POSTECH

Radiation Power Density: Time Harmonic Situations

- Average power radiated by the antenna can be calculated using the following equation:

$$P_{rad} = \oiint_S \vec{\mathrm{W}}_{\mathrm{rad}} . d\vec{s} = \oiint_S \vec{\mathrm{W}}_{\mathrm{av}} . \hat{n} ds = \frac{1}{2} \oiint_S \mathrm{Re}[\vec{\mathrm{E}} \times \vec{\mathrm{H}}^*] . \overrightarrow{ds}$$

- Note that S is a closed surface which completely encloses the antenna.
- In its simplest form S can be a sphere but this is not a requirement.

POSTECH

- The radial component of the radiated power density of an antenna is given by the following equation:

$$\vec{W}_{rad} = \hat{r}W_r = \hat{r}A_0 \frac{\sin\theta}{r^2} \ (W/m^2)$$

- A_0 is the peak value of the power density. Determine the total radiated power.
- Solution: Simply integrate the above equation over a closed sphere of radius r.

$$\underline{W}_{ave} = \underline{W}_{rad} = \hat{a}_r A_0 \frac{\sin\theta}{r^2}$$

$$P_{ave} = P_{rad} = \oiint_S \underline{W}_{ave} \cdot d\underline{s}$$

$$= \int_0^{2\pi}\int_0^{\pi}\left[\hat{a}_r A_0 \frac{\sin\theta}{r^2}\right]\cdot\left[\hat{a}_r r^2 \sin\theta d\theta d\phi\right]$$

$$P_{ave} = P_{rad} = A_0 \int_0^{2\pi}\int_0^{\pi}\sin^2\theta d\theta d\phi = 2\pi A_0 \int_0^{\pi}\sin^2\theta d\theta$$

$$P_{rad} = A_0\pi^2$$

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Radiation Intensity

- Radiation Intensity in a given direction is defined as the "power radiated from an antenna per unit solid angle"
- The radiation intensity is a far field parameter.
- Radiation intensity can be obtained by multiplying the radiation density by the square of distance:

$$U = r^2 W_{rad}$$

- Note that we have:

$$P_{rad} = \oint\!\!\oint_S W_{rad} r^2 \sin\theta \, d\theta \, d\varphi = \oint\!\!\oint_S U \, d\Omega$$

POSTECH

Radiation Intensity

- $\vec{U}$=radiation intensity (W/unit solid angle)
- $\overrightarrow{W}_{rad}$=radiation power density (W/m^2)
- We will see that the far field of an antenna can be expressed as:

$$\vec{E}(r,\theta,\varphi) = \vec{E}^{\circ}(\theta,\varphi)\frac{e^{-jkr}}{r}$$

- Therefore:

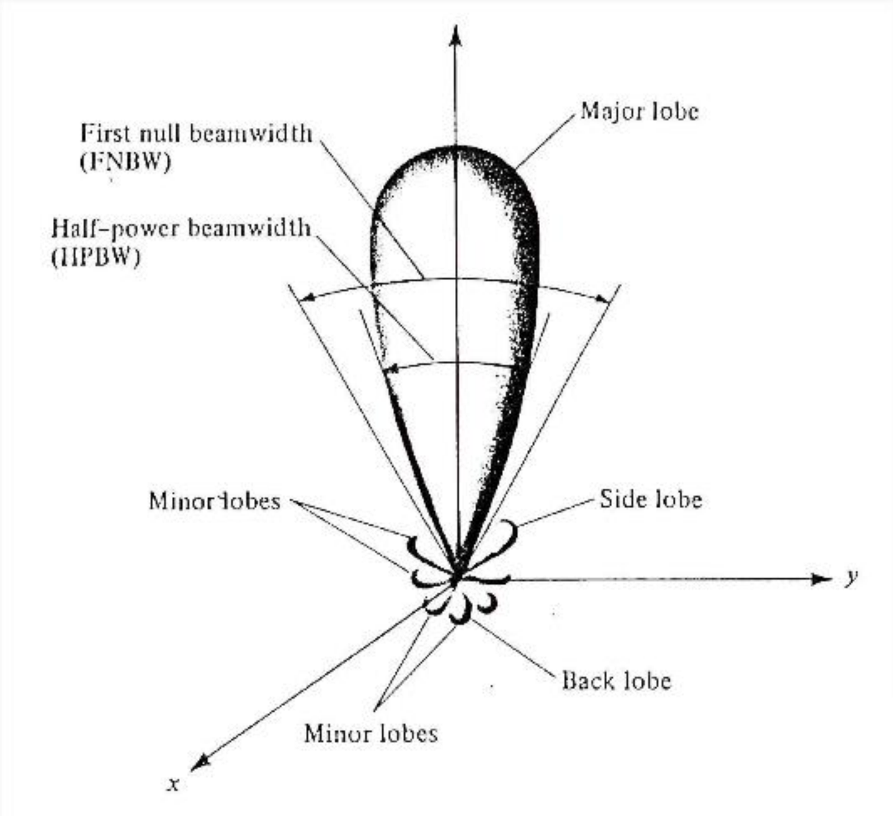
$$U(\theta,\varphi) = \frac{r^2}{2\eta}|\vec{E}(r,\theta,\varphi)|^2 \approx \frac{r^2}{2\eta}\left[|E_\theta(r,\theta,\varphi)|^2 + |E_\varphi(r,\theta,\varphi)|^2\right]$$

$$\approx \frac{1}{2\eta}\left[|E_\theta^{\circ}(r,\theta,\varphi)|^2 + |E_\varphi^{\circ}(r,\theta,\varphi)|^2\right]$$

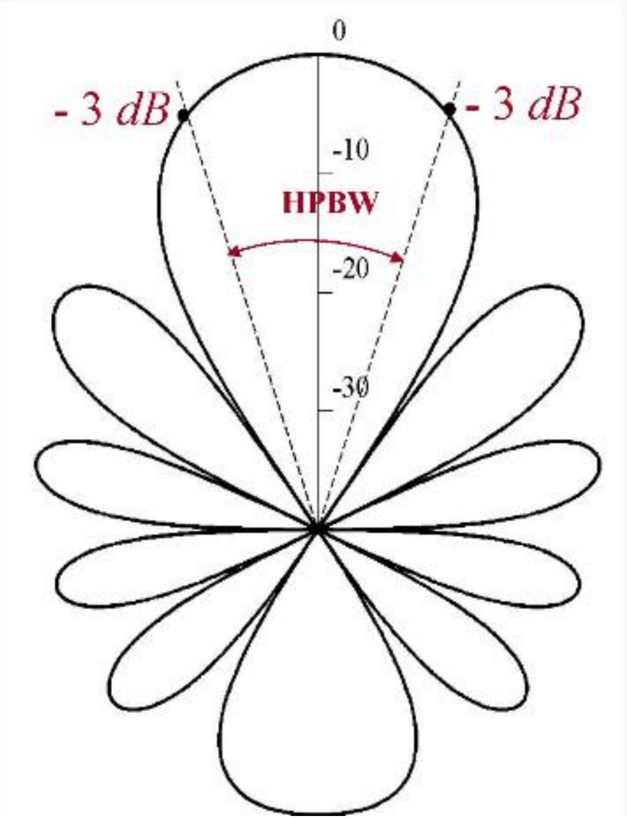
Beamwidth

- The beamwidth of a pattern is defined as the angular separation between two identical points on opposite side of the pattern maxima.
- In an antenna, there are a number of beamwidths.
- Half Power Beamwidth is defined by IEEE as "In a plane containing the direction of the maximum of a beam, the angle between the two directions in which the radiation intensity is one-half value of the maximum value"
- First Null Beamwidth is defined as angular separation between the first nulls of the pattern.

Beamwidth



First null beamwidth
(FNBW)
Half-power beamwidth
(HPBW)
Major lobe
Minor lobes
Side lobe
Back lobe
Minor lobes
x
y



0
- 3 dB
- 3 dB
-10
HPBW
-20
-30

Example 2.4

The normalized radiation intensity of an antenna is represented by

$$U(\theta) = \cos^2(\theta)\cos^2(3\theta),$$

$$(0 \le \theta \le 90^\circ,\quad 0^o \le \phi \le 360^o)$$

The three- and two-dimensional plots of this plotted in a linear scale, are shown in Figure 2.10. Find the:

a. *HPBW* (*in radians and degrees*)

b. *FNBW* (*in radians and degrees*)

Solution:

a. Since the $U(\theta)$ represents the *power* pattern, to find the half-power beamwidth you set the function equal to half of its maximum, or

$$U(\theta)|_{\theta=\theta_h} = \cos^2(\theta)\cos^2(3\theta)|_{\theta=\theta_h} = 0.5 \quad \Rightarrow \quad \cos\theta_h \cos 3\theta_h = 0.707$$

$$\theta_h = \cos^{-1}\left(\frac{0.707}{\cos 3\theta_h}\right)$$

Since the equation is nonlinear, after few iterations it is found that

$$\theta_h \approx 0.25 \; radians = 14.32^\circ$$

Since the function $U(\theta)$ is symmetrical about the maximum at θ=0, then the HPBW is

$$HPBW = 2\theta_h \approx 0.50 \; radians = 28.65^\circ$$

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To find the first-null beamwidth (FNBW), you set the equal to zero, or

$$U(\theta)|_{\theta=\theta_n} = \cos^2(\theta)\cos^2(3\theta)|_{\theta=\theta_n} = 0$$

This leads to two solutions for θ_n.

$$\cos\theta_n = 0 \quad \Rightarrow \quad \theta_n = \cos^{-1}(0) = \frac{\pi}{2}\ radians = 90^o$$

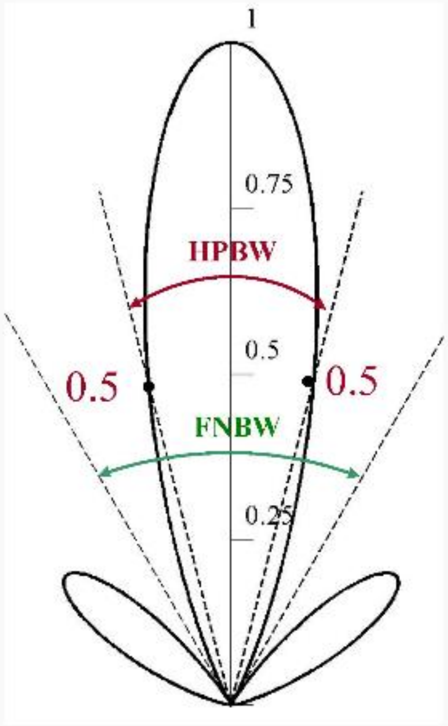
$$\cos 3\theta_n = 0 \quad \Rightarrow \quad \theta_n = \frac{1}{3}\cos^{-1}(0) = \frac{\pi}{6}\ radians = 30^o$$

The one with the smallest value leads to the *FNBW*. Because of the symmetry of the pattern, the *FNBW* is

$$FNBW = 2\theta_n = \frac{\pi}{3}\ radians = 60^o$$

POSTECH

HPBW and FNBW of Radiation Intensity U



Linear Scale

$$U(\theta,\phi) = \cos^2\theta\cos^2 3\theta$$

HPBW $= 28.65^o$

FNBW $= 60^o$

Fig. 2.11(b)

Directivity

- The directivity of an antenna is defined as "the ratio of the radiation intensity in a given direction from the antenna to the radiation intensity averaged over all directions"
- Note that the average radiation intensity is equal to the total power radiated by the antenna divided by 4π; WHY?
- Also, you can see that the directivity of a non isotropic source is equal to the ratio of its radiation intensity in a given direction over that of an isotropic source

$$D = \frac{U}{U_0} = \frac{4\pi U}{P_{rad}}$$

POSTECH

Directivity

- If the direction is not specified, it implies the direction of maximum radiation intensity (maximum directivity) expressed as

$$D_{\max} = D_0 = \frac{U_{\max}}{U_0} = \frac{4\pi U_{\max}}{P_{rad}}$$

- Note that Directivity is dimensionless
- What is the directivity of an isotropic source?
- How do we define directivity for antennas that have orthogonal polarization components?

POSTECH

Directivity

- The partial directivity of the antenna for a given polarization in a given direction is defined as "that part of the radiation intensity corresponding to a given polarization divided by the total radiation intensity averaged over all directions"
- Therefore, "the total directivity is the sum of the partial directivities for any two orthogonal polarizations"

$$D_0 = D_\theta + D_\varphi$$

$U_{\theta(\phi)}$: Radiation intensity contained in the $\theta(\phi)$ field component.

$(P_{rad})_{\theta(\phi)}$: Radiated power due to the the $\theta(\phi)$ field component.

$$D_\theta = \frac{4\pi U_\theta}{(P_{rad})_\theta + (P_{rad})_\varphi} \qquad D_\varphi = \frac{4\pi U_\varphi}{(P_{rad})_\theta + (P_{rad})_\varphi}$$

Example 2.5:

$$\underline{W}_{rad} = \hat{a}_r A_0 \frac{\sin\theta}{r^2}$$

Solution:

$$P_{rad} = \pi^2 A_0$$

$$U = r^2 W_{rad} = A_0 \sin\theta$$

$$U_{\max} = U\big|_{\max} = A_0 \sin\theta\big|_{\theta=\pi/2} = A_0$$

$$D_0 = \frac{4\pi U_{\max}}{P_{rad}} = \frac{4\pi(1)A_0}{\pi^2 A_0} = 1.27 = \underbrace{1.038\ dB}$$

$$D = D_0 \sin\theta = 1.27 \sin\theta$$

Example 2.6:

$$\underline{W}_{rad} = \hat{a}_r A_0 \frac{\sin^2\theta}{r^2}$$

Solution:

$$P_{rad} = \frac{8\pi}{3}$$

$$U = r^2 W_{rad} = A_0 \sin^2\theta$$

$$U_{\max} = U\big|_{\max} = A_0 \sin^2\theta\big|_{\theta=\pi/2} = A_0$$

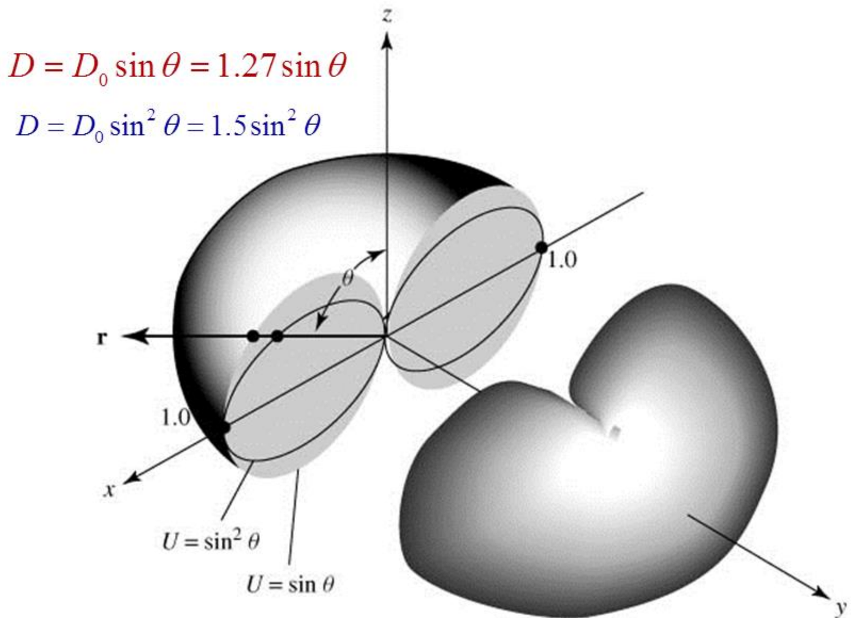
$$D_0 = \frac{4\pi U_{\max}}{P_{rad}} = \frac{4\pi A_0}{8\pi/2} = 1.5 = \underbrace{1.761\,dB}$$

$$D = D_0 \sin^2\theta = 1.5\sin^2\theta$$

POSTECH

Directivity

- Both patterns are omni-directional
- The radiation pattern of $U = \sin^2\theta$ is more directional in elevation pattern
- Directivity is a measure of how directive the antenna is
- Note that narrower radiation patterns result in larger directivities



The relative radiation intensities of $U = A_0 \sin\theta$ and $U = A_0 \sin^2\theta$

POSTECH

Directivity of a Half Wavelength Dipole

- For a dipole antenna whose axis is located along the z-axis and has a length of half wavelength is:

$$D = D_0 \sin^3\theta = 1.67\sin^3\theta$$