2011 수학

1.

(a) $\lim_{x\to 0} \frac{8^x-4^x}{2-\sqrt{4-x}} = \lim_{x\to 0} \frac{(8^x-4^x)(2+\sqrt{4-x})}{(2-\sqrt{4-x})(2+\sqrt{4-x})} = \lim_{x\to 0}\left(\frac{8^x-4^x}{x}\right)\times 4$

$= 4\lim_{x\to 0}\left(\frac{8^x-1}{x} - \frac{4^x-1}{x}\right) = 4(\ln 8 - \ln 4) = \underline{4\ln 2}$

(b) Let $x-\frac{\pi}{2}=t$, then $\lim_{x\to\frac{\pi}{2}} \frac{\tan 2x}{x-\frac{\pi}{2}} = \lim_{t\to 0}\frac{\tan(2t+\pi)}{t} = \lim_{t\to 0}\frac{\tan 2t}{t} = \underline{2}$

2. $\ln y = x^n \ln x$, $\frac{y'}{y} = nx^{n-1}\ln x + x^{n-1} = x^{n-1}(n\ln x + 1)$

$y' = x^{x^n}\cdot x^{n-1}(n\ln x+1) = \underline{x^{x^n+n-1}(n\ln x+1)}$

3. Let $\cos x = t$, $-\sin x\, dx = dt$.

$\int \sin^3 x \cos^2 x\, dx = -\int (1-t^2)t^2\, dt = \int t^4 - t^2\, dt = \frac{1}{5}t^5 - \frac{1}{3}t^3 + C$

$\therefore \int \sin^3 x\cos^2 x\, dx = \underline{\frac{1}{5}\cos^5 x - \frac{1}{3}\cos^3 x + C}$

4. $s^2Y - sy(0) - y'(0) + 2sY - 2y(0) + Y = \frac{1}{s+1} = s^2Y + 2s - 3 + 2sY + 4 + Y$

$Y(s^2+2s+1) = \frac{1}{s+1} - (2s+1) = \frac{1-2s^2-3s-1}{s+1} \Rightarrow Y(s) = \frac{-2s^2-3s}{(s+1)^3} = \frac{A}{s+1} + \frac{B}{(s+1)^2} + \frac{C}{(s+1)^3}$

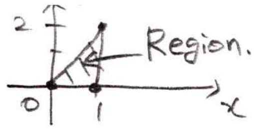
$As^2+2As+A+Bs+B+C = As^2+(2A+B)s+A+B+C = -2s^2-3s.$

$A=-2,\ B=1,\ C=1$

$Y(s) = \frac{-2}{s+1} + \frac{1}{(s+1)^2} + \frac{1}{(s+1)^3}$

$\therefore y(t) = \left(-2e^{-t} + te^{-t} + \frac{t^2}{2}e^{-t}\right)u(t)$

5. $L = xy,\quad M = x^2y^3$



$$\oint_C \{xy\,dx + x^2y^3dy\} = \iint_R 2xy^3 - x\,dxdy = \int_0^1\int_0^{2x} 2xy^3 - x\,dy\,dx$$

$$= \int_0^1 \left[\frac{x}{2}y^4 - xy\right]_0^{2x} dx = \int_0^1 8x^5 - 2x^2\,dx = \frac{8}{6} - \frac{2}{3} = \underline{\frac{2}{3}}$$

6. (a) $$[\underline{x_1}\ \underline{x_2}\ \underline{x_3}]\begin{bmatrix} a & c & 1 \\ b & d & -1 \\ 0 & -2 & e \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 1 & 1 & -1 \end{bmatrix}\begin{bmatrix} a & c & 1 \\ b & d & -1 \\ 0 & -2 & e \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$ =) inverse 곱해서 구해도 되지만 노가다가 더 빠름...

(a) $$\begin{pmatrix} a+2b=1 \\ 2a+3b=1 \\ a+b=0 \end{pmatrix}\quad \underline{\underline{a=-1 \\ b=1}}\qquad \begin{pmatrix} c+2d-2=1 \\ c+d+2=0 \end{pmatrix}\begin{pmatrix} c+2d=3 \\ c+d=-2 \end{pmatrix}\quad \underline{\underline{c=-7 \\ d=5}}$$

$-1+e=0 \qquad \underline{e=1}$

(b) $Y = HX$, H: system matrix,

in (a), $XP = V$. $\therefore\ YP = HXP = HV = Y_0$.

$$Y_0 = \begin{bmatrix} 1 & 3 & 2 \\ 5 & 7 & 2 \\ 4 & 4 & -4 \end{bmatrix}\begin{bmatrix} -1 & -7 & 1 \\ 1 & 5 & -1 \\ 0 & -2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 4 & 0 \\ 2 & -4 & 0 \\ 0 & 0 & -4 \end{bmatrix}$$

(c) Gauss-Jordan method

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 \end{array}\right] \to \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & -2 & 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 \end{array}\right] \to \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & -2 & 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$\downarrow$$

$$[I \vdots V^{-1}] = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 1 & 0 & \frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 \end{array}\right]$$

$$\therefore V^{-1} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

(d) $H\underline{x} = \lambda \underline{x}$ ⇒ eigenvector 를 구하는데, 먼저 H를 알아야 함

(b)에서 $YP = \begin{bmatrix} 2 & 4 & 0 \\ 2 & -4 & 0 \\ 0 & 0 & -4 \end{bmatrix} = HV$ 이므로

$$H = \begin{bmatrix} 2 & 4 & 0 \\ 2 & -4 & 0 \\ 0 & 0 & -4 \end{bmatrix} \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 0 \\ -1 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$\det(H - \lambda I) = \begin{vmatrix} 3-\lambda & -1 & 0 \\ -1 & 3-\lambda & 0 \\ 0 & 0 & 4-\lambda \end{vmatrix} = (4-\lambda)(\lambda^2 - 6\lambda + 9 - 1) = 0$$

$$\Rightarrow (\lambda-4)(\lambda-2)(\lambda-4) = 0$$

① $\lambda = 2$ case

$$\begin{bmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_1 = x_2$, $x_3 = 0$

$\therefore \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \underline{X_1}$

② $\lambda = 4$ case

$$\begin{bmatrix} -1 & -1 & 0 \\ -1 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\underline{X_2} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$ $\underline{X_3} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$$\therefore S = \{ \underline{x} \in R^3 \mid \underline{x} = k_1 \underline{X_2} + k_2 \underline{X_3},\ k_1, k_2 \in R \}$$

(e) $\underline{y} = H\underline{x}$

$$\underline{y} = \begin{bmatrix} 3 & -1 & 0 \\ -1 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix} \underline{x}$$

7.

(a) sum of two periodic signal is <u>not</u> a periodic signal generally.

(반례) Let $x_1(t) = \sin t$, $x_2(t) = \sin 2\pi t$.

$x_1(t)$의 주기는 $2\pi = T_1$, $x_2(t)$의 주기 $T_2 = 1$.

$y(t) = x_1(t) + x_2(t)$ 일때 $y(t)$의 주기 T_y 가 존재하기 위해서는

$k_1 T_1 = k_2 T_2$ 인 $k_1 \in \mathbb{N}$, $k_2 \in \mathbb{N}$ 가 존재해야 함.

But, $2\pi k_1 = k_2$, $2\pi = \frac{k_2}{k_1}$ 는 rational number가 아니므로

$k_1 \in \mathbb{N}$ and $k_2 \in \mathbb{N}$ 존재하지 않으므로 $y(t)$ is not a periodic signal.

(b)

$$h(t) * \left\{ \sum_{k=-K}^{K} a_k e^{j\frac{2\pi k t}{T}} \right\} = \int_{-\infty}^{\infty} h(\tau) \sum_{k=-K}^{K} a_k e^{+j\frac{2\pi k (t-\tau)}{T}} d\tau$$

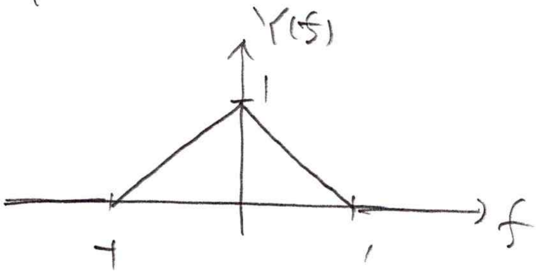
$$= \sum_{k=-K}^{K} a_k e^{+j\frac{2\pi k t}{T}} \int_{-\infty}^{\infty} h(\tau) e^{-j\frac{2\pi k \tau}{T}} d\tau = \sum_{k=-K}^{K} a_k e^{+j\frac{2\pi k t}{T}} H\left(\frac{k}{T}\right)$$

$$= \sum_{k=-K}^{K} a_k H\left(\frac{k}{T}\right) e^{j\frac{2\pi k t}{T}} = \sum_{k=-K}^{K} b_k e^{j\frac{2\pi k t}{T}}.$$

$\therefore$ periodic function with Fourier coefficient $b_k = a_k H\left(\frac{k}{T}\right)$

(c) $x(t) = \text{sinc}\, t$.

$Y(f) = X(f) * X(f) = \text{rect}(f) * \text{rect}(f) = \Lambda(f)$



$$Y(f) = \begin{cases} 1 - |f|, & \text{for } -1 \le f \le 1 \\ 0, & \text{otherwise} \end{cases}$$

2011 통신

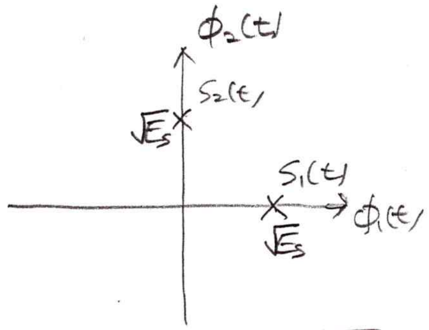
1. 2014 통신 참고!

2. a) $s_1(t) = A\cos 2\pi f_1 t$, $\int_0^T A^2\cos^2(2\pi f_1 t)\,dt = \frac{A^2}{2}T = E_s$, $A = \sqrt{\frac{2E_s}{T}}$

$s_1(t) = \sqrt{\frac{2E_s}{T}}\cos 2\pi f_1 t$, $(0 \le t < T)$ $s_2(t) = \sqrt{\frac{2E_s}{T}}\cos 2\pi f_2 t$ $(0 \le t < T)$ $(f_1 \ne f_2)$

b) $\phi_1(t) = \sqrt{\frac{2}{T}}\cos 2\pi f_1 t\ (0 \le t < T)$

$\phi_2(t) = \sqrt{\frac{2}{T}}\cos 2\pi f_2 t\ (0 \le t < T)$



c) $P_e = \frac{1}{2}\times Q\left(\frac{\sqrt{2E_s}}{\sqrt{2N_0}}\right) + \frac{1}{2}\times Q\left(\frac{\sqrt{2E_s}}{\sqrt{2N_0}}\right) = \underline{Q\left(\sqrt{\frac{E_s}{N_0}}\right)}$ //

d) $\{\phi_i(t)\}_{i=1}^{M}$ are all orthonormal basis, so minimum distance between two symbols are the same as $\sqrt{2E_s}$

$\therefore \underline{P_e \le (M-1)\,Q\left(\sqrt{\frac{E_s}{N_0}}\right)}$ //

2011 제어
제어필수
1. (a)

$$u(t)\cos\theta - Mg\sin\theta = ML\ddot{\theta}$$

$$\therefore \ddot{\theta} = \frac{1}{ML}u(t)\cos\theta - \frac{g}{L}\sin\theta$$

(b) $x_1(t) = \theta(t)$
$x_2(t) = \dot{\theta}(t)$

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = -\frac{g}{L}\sin x_1(t) + \frac{u(t)}{ML}\cos x_1(t) \end{cases}$$

(평형상태일 경우)

$$0 = \frac{1}{ML}u(t)\cos\theta - \frac{g}{L}\sin\theta$$

$$\rightarrow u(t) = Mg\tan\theta$$

(c) Linear 한 시스템이 아니기 때문에 input - output 간의 linear relationship을 정의할 수 없으므로 transfer function can't be found.
But, by using linearization method, we can approximately find the transfer function.

$$\Delta\dot{x}_2(t) = \frac{\Delta u(t)}{ML}\cos\theta_0 - \frac{u_0}{ML}\Delta x_1(t)\sin\theta_0 - \frac{g}{L}\Delta x_1(t)\cos\theta_0$$

$$= \frac{1}{ML}\cos\theta_0 \cdot \Delta u(t) - \left(\frac{1}{ML}\sin\theta_0 \cdot Mg\tan\theta_0 + \frac{g}{L}\cos\theta_0\right)\Delta x_1(t)$$

$$= \frac{1}{ML}\cos\theta_0 \cdot \Delta u(t) - \frac{g}{L}\sec\theta_0 \cdot \Delta x_1(t)$$

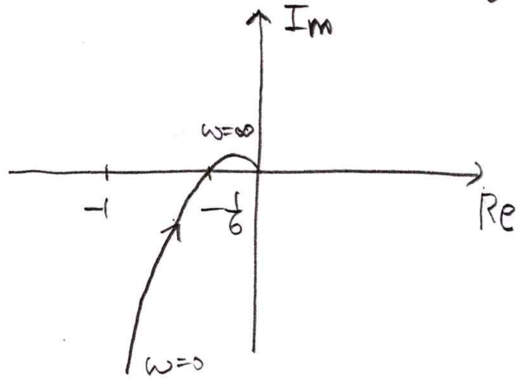
$$\Delta\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ -\frac{g}{L}\sec\theta_0 & 0 \end{bmatrix}\Delta x(t) + \begin{bmatrix} 0 \\ \frac{1}{ML}\cos\theta_0 \end{bmatrix}\Delta u(t)$$

2.

a) $L(s)|_{T=0} = \frac{1}{s(s+1)(s+2)} \Rightarrow L(j\omega) = \frac{1}{j\omega(j\omega+1)(j\omega+2)}$

① $\omega = 0 \Rightarrow L(j\omega) = \infty \angle -90°$

② $\omega = \infty \Rightarrow L(j\omega) = 0 \angle -270°$



③ Gain Margin

$L(j\omega) = \frac{1}{(j\omega-\omega^2)(j\omega+2)} = \frac{1}{-\omega^2+2j\omega-j\omega^3-2\omega^2} = \frac{1}{-3\omega^2+j(2\omega-\omega^3)}$

$Im\{L(j\omega)\}=0$ 이 되는 $\omega_p = \sqrt{2}$.

그 때의 $L(j\omega) = -\frac{1}{6}$.

Gain margin : $-20\log_{10}\left(\frac{5}{6}\right) = 20(\log_{10}6 - \log_{10}5) = 2.33\,dB$

b) $L(j\omega) = \frac{e^{-j\omega T}}{j\omega(j\omega+1)(j\omega+2)}$

① $|L(j\omega)| = 1$ 이 되는 ω_g 찾는다.

$|L(j\omega)| = \frac{1}{\sqrt{9\omega^4+(2\omega-\omega^3)^2}} = 1 \Rightarrow \omega_g = 0.447\ldots$

② $\omega_g = 0.447$ 일 때의 phase 구한다.

$\angle L(j\omega_g) = -90° - \tan^{-1}(\omega) - \tan^{-1}\left(\frac{\omega}{2}\right) - \omega T$

$= -126.7° - 0.447T$

③ 이 때 phase 가 $-180°$ 일 때 marginally stable 하므로,

$0.447T = 53.3°$, $53.3° = 53.3 \times \frac{\pi}{180} = 0.3\pi\,rad.$

$\therefore T = 0.67\pi\,s$