Caution!!!
Use separate answer books for Problems 1-5 (Math.-A) and for 6-8 (Math.-B).

6. (15 points) When 2×2 matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{bmatrix} 1 & 2 \\ -2 & -4 \end{bmatrix},$$

answer the following questions.

(a) (5 points) Find the eigenvalues and corresponding eigenvectors of $\mathbf{A}$.

(b) (10 points) Find the minimum and the maximum values of

$$f(\underline{x}) = \frac{\underline{x}^T \mathbf{A} \underline{x}}{\underline{x}^T \underline{x}},$$

where $\underline{x} \neq \underline{0}$.

7. (15 points) For a given $A = \begin{bmatrix} 1 & 2 \\ -2 & -3 \end{bmatrix}$, compute

(1) (5 points) $\sin(A)$.

(2) (5 points) $\cos(A)$.

(3) (5 points) Show $\cos^2(A) + \sin^2(A) = I$.

8. (20 points) Let a be a given integer. The function $f(t)$ of period π is defined by $f(t) = \sin(at)$ for $t \in [0, \pi]$. Find its Fourier consine series.

Remark: You must carefully consider the cases where a is even or odd.