Communications and Signal Processing
2013 Doctoral Qualifying Exam

Caution!!!
Use a separate answer book for Problem 1.

Problem 1. (50 points) A causal and stable LTI system has a system function:

$$H(z) = 1/(1 - 0.5z^{-1})(1 + 0.95z^{-1}).$$

(a) Draw the block diagram of the system.

(b) Find the impulse response of this system.

(c) Sketch the frequency response of the system.

Caution!!!
Use a separate answer book for Problem 2.

Problem 2. (50 points) The real-valued observed signal $Y(t)$ at a receiver is modeled by

$$Y(t) = \sqrt{P}X(t) + N(t)$$

where $P > 0$ is the signal power, $X(t)$ is the desired component given by

$$X(t) = s_0(t+T) + s_1(t) + \sum_{n=1}^{N} s(b[n]; t - nT), \tag{1}$$

$b[n] \in \{0, 1\}$ is a binary data symbol for $n = 1, 2, \cdots, N$, and $N(t)$ is real-valued additive white Gaussian noise with two-sided power spectral density $N_0/2$.

In Equation (1), $s_0(t)$ is given by

$$s_0(t) \triangleq \begin{cases} 1, & \text{for } 0 < t \le \frac{T}{2} \\ -1, & \text{for } \frac{T}{2} < t \le T \\ 0, & \text{elsewhere,} \end{cases}$$

$s_1(t)$ is given by

$$s_1(t) \triangleq \begin{cases} 1, & \text{for } 0 < t \le T \\ 0, & \text{elsewhere,} \end{cases}$$

$s(0; t)$ is chosen as either $s_0(t)$ or $-s_0(t)$, and $s(1; t)$ is chosen as either $s_1(t)$ or $-s_1(t)$, in such a way that the sign of the signal $Y(t)$ changes at every nT for $n = 1, 2, \cdots, N$. Answer the following questions.

(a) (15 points) When $N = 5$ and $[b[1], b[2], \cdots, b[5]] = [1, 0, 0, 1, 1]$, sketch $X(t)$ for $-2T < t \le 7T$.

(b) (10 points) In (a), find $[X_0[n],\ X_1[n]]$ for $n = 1, 2, \cdots, 5$, where

$$X_0[n] \triangleq \int_{nT}^{(n+1)T} X(t) s_0(t - nT)\, dt$$

and

$$X_1[n] \triangleq \int_{nT}^{(n+1)T} X(t) s_1(t - nT)\, dt.$$

(c) (10 points) Find the joint probability density function $f_{N_0[n],N_1[n]}(n_0, n_1)$ of $[N_0[n],\ N_1[n]]$, where

$$N_0[n] \triangleq \int_{nT}^{(n+1)T} N(t)s_0(t - nT)\ dt$$

and

$$N_1[n] \triangleq \int_{nT}^{(n+1)T} N(t)s_1(t - nT)\ dt.$$

(d) (5 points) In (a), find the joint probability density function $f_{Y_0[1],Y_1[1]}(y_0, y_1)$ of $[Y_0[1],\ Y_1[1]]$, where

$$Y_0[1] \triangleq \int_{T}^{2T} Y(t)s_0(t - T)\ dt$$

and

$$Y_1[1] \triangleq \int_{T}^{2T} Y(t)s_1(t - T)\ dt.$$

(e) (10 points) Using the results in (a)-(d), show that

$$\Pr\left(|Y_0[1]| \geq |Y_1[1]| \bigm| b[1] = 1\right) = 2Q\left(\sqrt{\frac{PT}{N_0}}\right)\left\{1 - Q\left(\sqrt{\frac{PT}{N_0}}\right)\right\},$$

where the Q-function is defined as

$$Q(x) \triangleq \int_{x}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}}\ dy.$$