광전자공학 Ch. 4
Multiple beam interference

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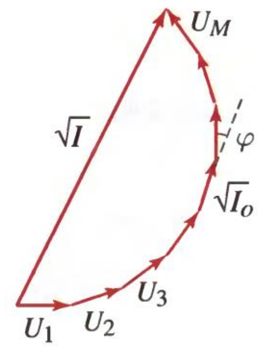
Multiple wave interference

$$U = U_1 + U_2 + \cdots + U_M$$

Interference of M Waves with Equal Amplitudes and Equal Phase Differences

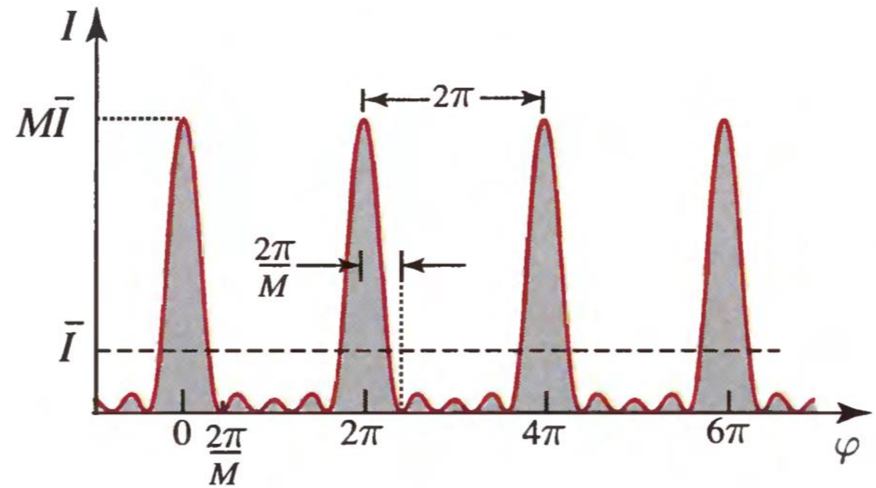


$$U_m = \sqrt{I_0}\,\exp[j(m-1)\varphi], \qquad m - 1, 2, \ldots, M\,.$$

$$U = \sqrt{I_0}\,\frac{1-\exp(jM\varphi)}{1-\exp(j\varphi)}$$

$$I = |U|^2 = I_0 \left| \frac{\exp(-jM\varphi/2) - \exp(jM\varphi/2)}{\exp(-j\varphi/2) - \exp(j\varphi/2)} \right|^2$$

$$\boxed{I = I_0 \frac{\sin^2(M\varphi/2)}{\sin^2(\varphi/2)}\,.}$$



Multiple wave interference

Interference of an Infinite Number of Waves of Progressively Smaller Amplitudes and Equal Phase Differences

$$U = \frac{\sqrt{I_0}}{1-h} = \frac{\sqrt{I_0}}{1-|h|e^{j\varphi}}$$

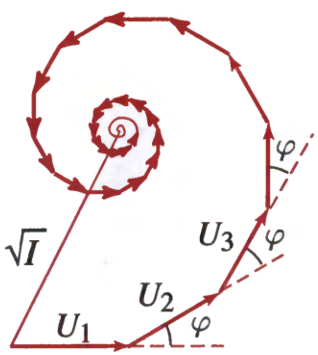
$$\boxed{I = \frac{I_{\max}}{1+(2\mathfrak{F}/\pi)^2 \sin^2(\varphi/2)}}$$

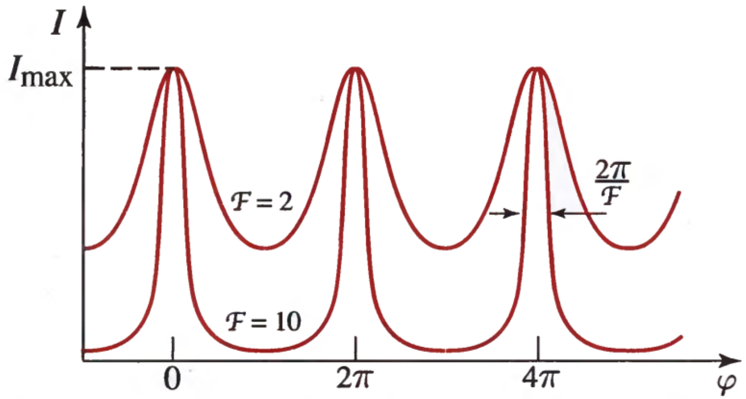
$$I = \frac{I_0}{(1-|h|)^2 + 4|h|\sin^2(\varphi/2)}$$

$$I_{\max} = \frac{I_0}{(1-|h|)^2}$$

Finesse

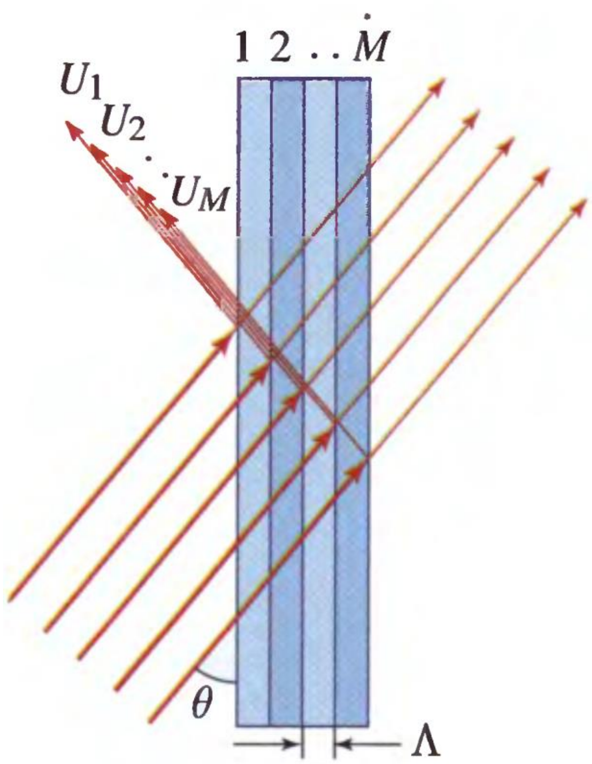
$$\mathfrak{F} = \frac{\pi\sqrt{|h|}}{1-|h|}$$





Bragg reflection

Multiple reflections in periodic dielectric slab



$$\varphi = k(2\Lambda \sin\theta)$$

$$\sin\theta = \frac{\lambda}{2\Lambda}$$ Bragg Angle

Specific angle of incident light will totally reflected

Bragg mirrors

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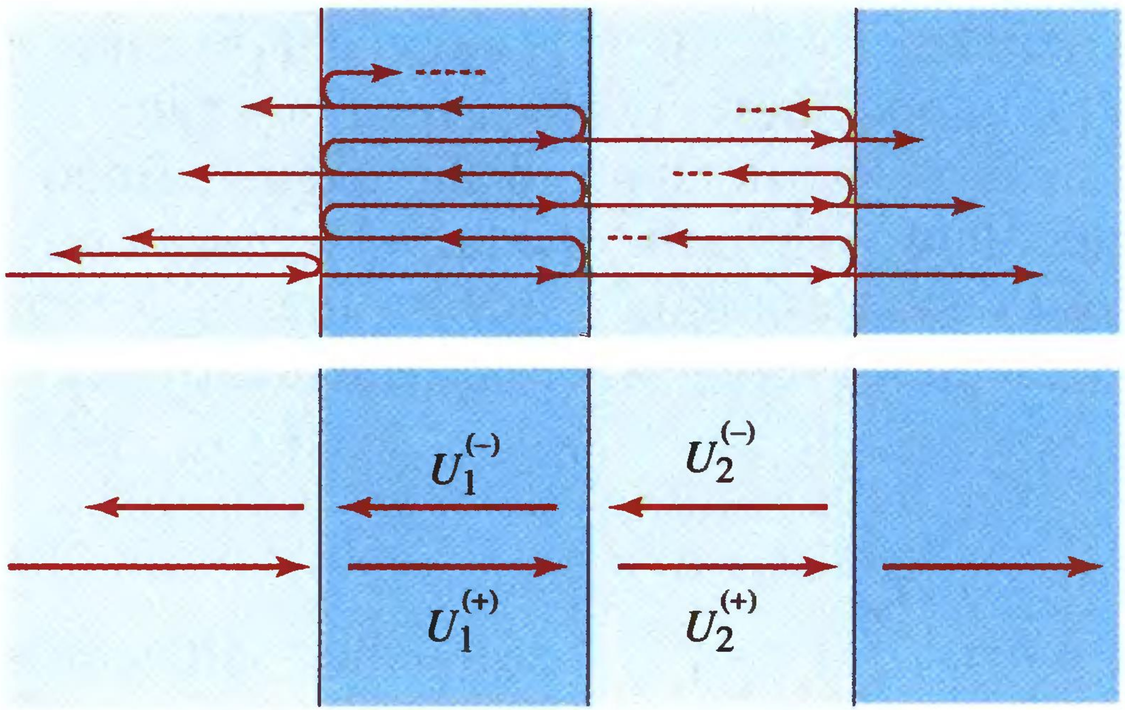
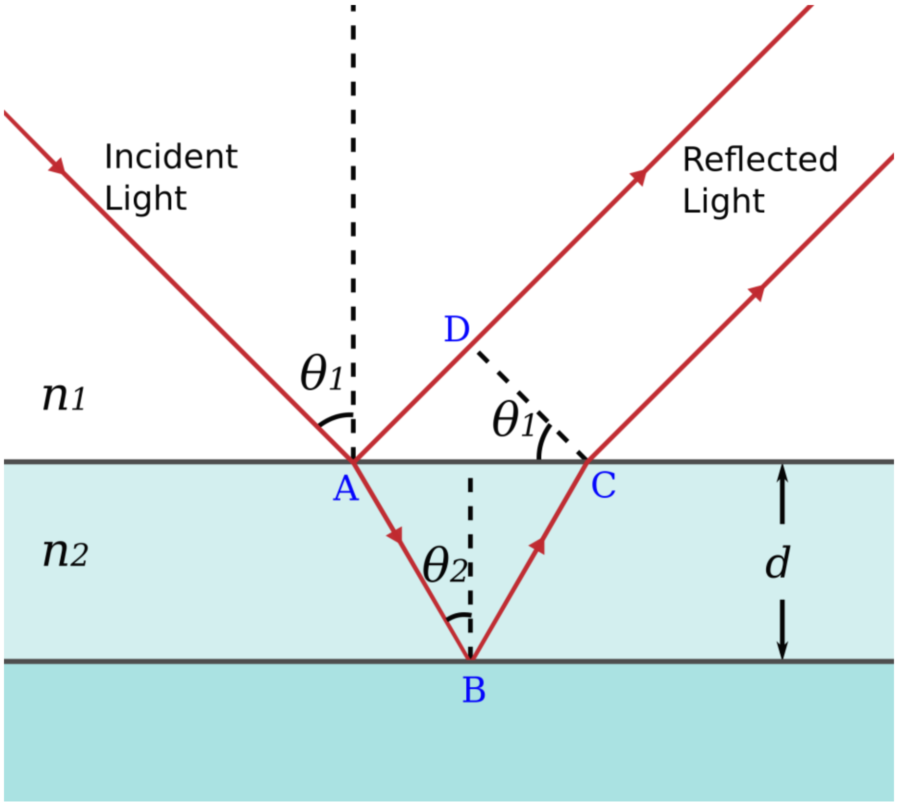
Multiple-reflections at interfaces

When light illuminates the thin-film layer, there may be multiple reflections, and we have to consider the superposed value.



Thin-film interference



$$OPD = n_2(\overline{AB} + \overline{BC}) - n_1(\overline{AD})$$

Where,

$$\overline{AB} = \overline{BC} = \frac{d}{\cos(\theta_2)}$$

$$\overline{AD} = 2d\tan(\theta_2)\sin(\theta_1)$$

Using Snell's Law, $n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$

$$OPD = n_2\left(\frac{2d}{\cos(\theta_2)}\right) - 2d\tan(\theta_2)n_2\sin(\theta_2)$$

$$OPD = 2n_2 d\left(\frac{1-\sin^2(\theta_2)}{\cos(\theta_2)}\right)$$

$$OPD = 2n_2 d\cos(\theta_2)$$

$$2n_2 d\cos(\theta_2) = m\lambda$$

Simply,

$$2k_z d = 2\pi m \quad \Rightarrow \quad 2n_2 k_0 \cos(\theta_2) d = 2\pi m$$

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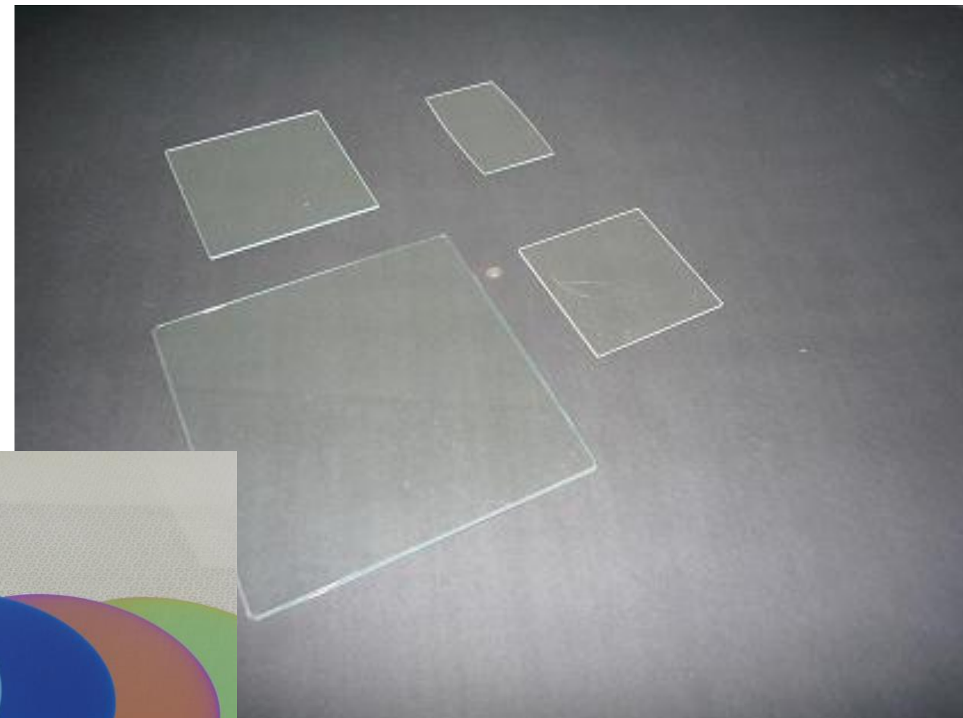
Thin-film interference

Why only thin-film interference? No thick-film interference??

Thin-film

Thick-film



http://www.vaporpulse.com

Thin-film

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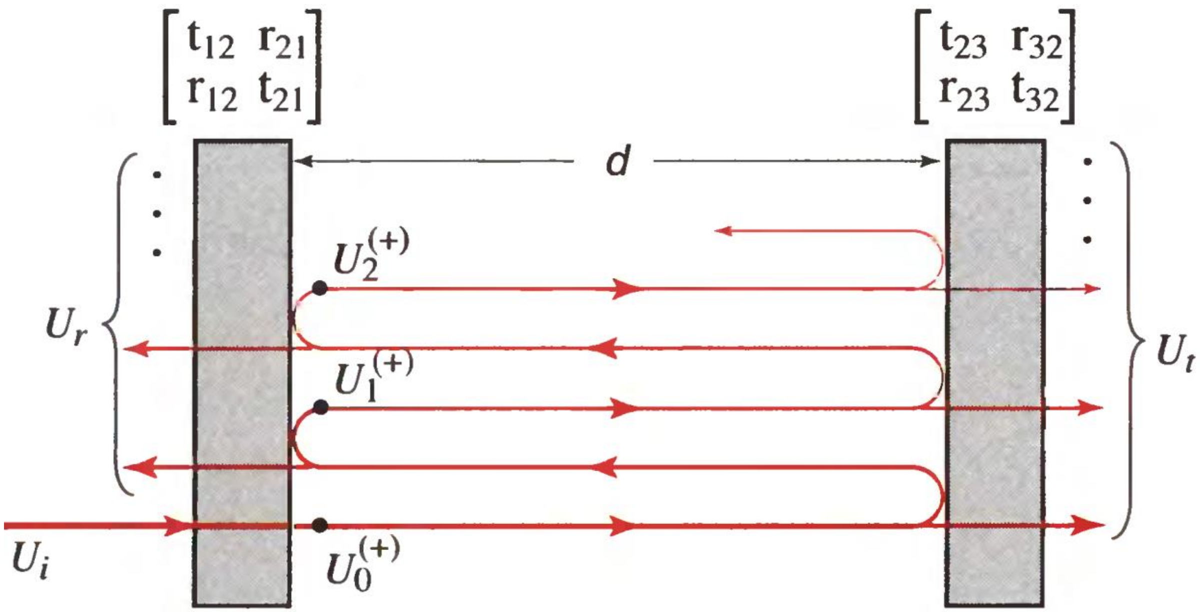
Airy's formula

Two cascade systems distanced with homogeneous media.
-Multiple reflections are analytically calculated.



$$t_{13} = \frac{t_{12} t_{23} \exp(-j\varphi)}{1 - r_{21} r_{23} \exp(-j2\varphi)},$$

$$r_{13} = r_{12} + \frac{t_{12} t_{21} r_{23} \exp(-j2\varphi)}{1 - r_{21} r_{23} \exp(-j2\varphi)}$$

$$\varphi = n k_o d$$

Thin-film resonance condition

$$2\varphi = 2\pi m$$

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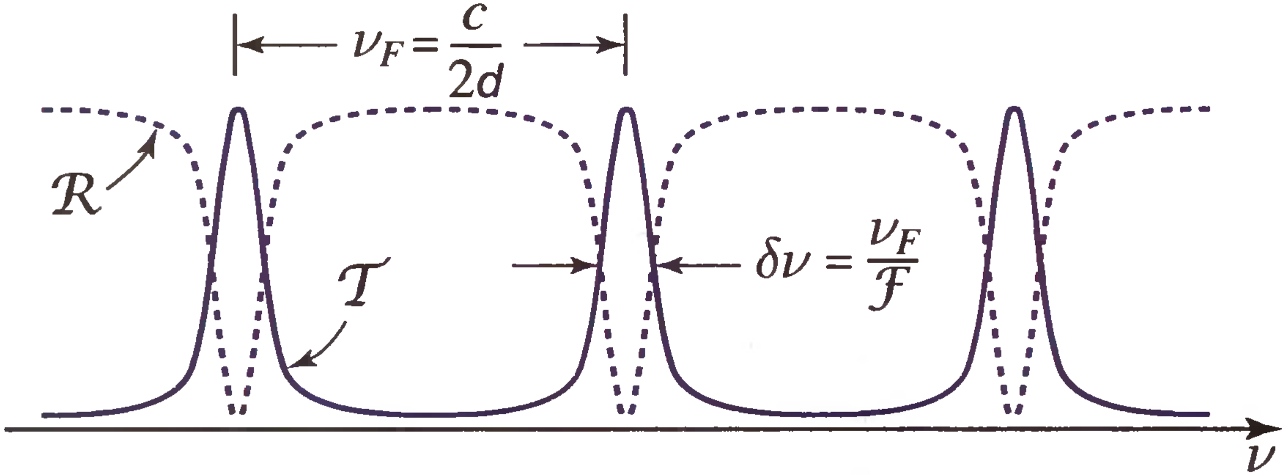
Fabry-Perot Etalon

Two mirror system can act as a resonator.

$$\mathcal{T} = |\mathrm{t}|^2 = \frac{|\mathrm{t}_1\mathrm{t}_2|^2}{|1 - \mathrm{r}_1\mathrm{r}_2\exp(-j2\varphi)|^2}$$

$$\mathcal{T} = \frac{\mathcal{T}_{\max}}{1 + (2\mathcal{F}/\pi)^2\sin^2\varphi}$$



Maximum transmission

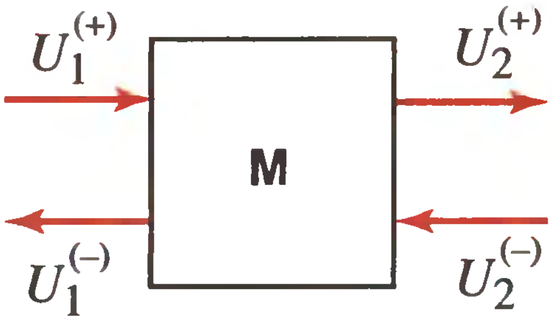
$$\mathcal{T}_{\max} = \frac{|\mathrm{t}_1\mathrm{t}_2|^2}{(1 - |\mathrm{r}_1\mathrm{r}_2|)^2}$$

Finesse

$$\mathcal{F} = \frac{\pi\sqrt{|\mathrm{r}_1\mathrm{r}_2|}}{1 - |\mathrm{r}_1\mathrm{r}_2|}$$

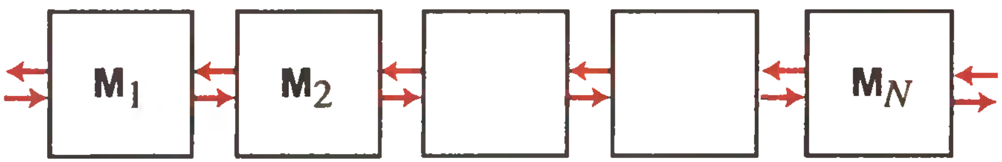
Transfer & scattering matrix

Transfer matrix : input–to–output layer relation



$$\begin{bmatrix} U_2^{(+)} \\ U_2^{(-)} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} U_1^{(+)} \\ U_1^{(-)} \end{bmatrix}$$

Benefit: Cascaded system can be simply calculated



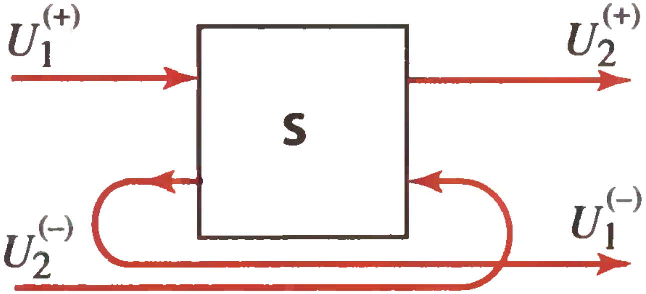
$$\mathbf{M} = \mathbf{M}_N \ldots \mathbf{M}_2 \mathbf{M}_1$$

Drawback: parameter A B C D, do not contain physical insights.

Applications : ray transfer, thin–film system etc.

Transfer & scattering matrix

Scattering matrix : matrix given by reflection, transmission coefficients



$$\begin{bmatrix} U_2^{(+)} \\ U_1^{(-)} \end{bmatrix} = \begin{bmatrix} t_{12} & r_{21} \\ r_{12} & t_{21} \end{bmatrix} \begin{bmatrix} U_1^{(+)} \\ U_2^{(-)} \end{bmatrix}$$

Benefit: Each elements have direct physical meaning.

Drawback: cascading calculation is more difficult compared to transfer matrix.

Applications : transmission line, microwave circuits, scattering system.

Transfer & scattering matrix

Conversion of transfer matrix to scattering matrix and vice versa,

$$\mathbf{M} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \frac{1}{\mathrm{t}_{21}} \begin{bmatrix} \mathrm{t}_{12}\mathrm{t}_{21} - \mathrm{r}_{12}\mathrm{r}_{21} & \mathrm{r}_{21} \\ -\mathrm{r}_{12} & 1 \end{bmatrix},$$

$$\mathbf{S} = \begin{bmatrix} \mathrm{t}_{12} & \mathrm{r}_{21} \\ \mathrm{r}_{12} & \mathrm{t}_{21} \end{bmatrix} = \frac{1}{D} \begin{bmatrix} AD - BC & B \\ -C & 1 \end{bmatrix}.$$

For lossless, reciprocal system,

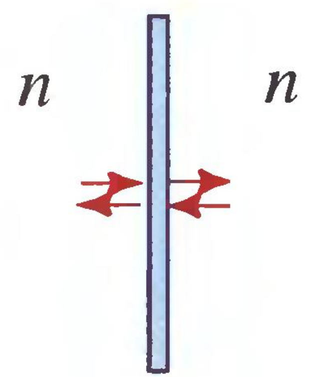
$$|\mathrm{t}|^2 + |\mathrm{r}|^2 = 1 \qquad \mathrm{t}/\mathrm{r} = -(\mathrm{t}/\mathrm{r})^* \qquad \mathrm{r}_{21} = \mathrm{r}_{12} \equiv \mathrm{r} \qquad \mathrm{t}_{21} = \mathrm{t}_{12} \equiv \mathrm{t}$$

$$\arg\{\mathrm{t}\} - \arg\{\mathrm{r}\} = \pm\pi/2$$

$$\mathbf{S} = \begin{bmatrix} \mathrm{t} & \mathrm{r} \\ \mathrm{r} & \mathrm{t} \end{bmatrix}, \qquad \mathbf{M} = \begin{bmatrix} 1/\mathrm{t}^* & \mathrm{r}/\mathrm{t} \\ \mathrm{r}^*/\mathrm{t}^* & 1/\mathrm{t} \end{bmatrix}$$

Transfer & scattering matrix

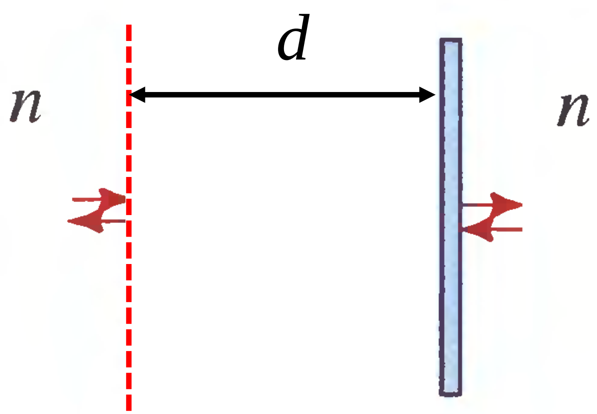
For example, at partially-reflective mirror



$$\mathbf{S} = \begin{bmatrix} |\mathrm{t}| & j|\mathrm{r}| \\ j|\mathrm{r}| & |\mathrm{t}| \end{bmatrix}, \quad |\mathrm{t}|^2 + |\mathrm{r}|^2 = 1$$

$$\mathbf{M} = \frac{1}{|\mathrm{t}|}\begin{bmatrix} 1 & j|\mathrm{r}| \\ -j|\mathrm{r}| & 1 \end{bmatrix}$$

Partially-reflective mirror distance with d

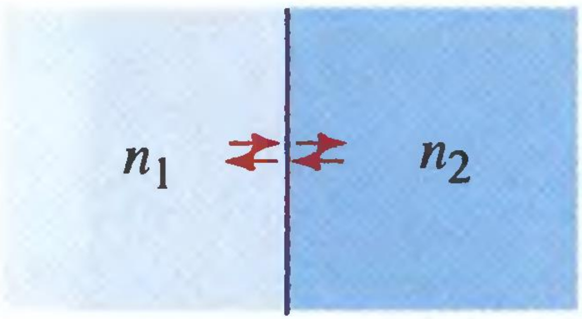


$$\mathbf{S}_1 = \begin{bmatrix} e^{j\varphi} & 0 \\ 0 & e^{j\varphi} \end{bmatrix} \qquad \mathbf{S}_2 = \begin{bmatrix} |t| & j|r| \\ -j|r| & |t| \end{bmatrix}$$

$$\mathbf{M} = \frac{1}{|t|}\begin{bmatrix} 1 & j|r| \\ -j|r| & 1 \end{bmatrix}\begin{bmatrix} e^{-j\varphi} & 0 \\ 0 & e^{j\varphi} \end{bmatrix} = \frac{1}{|t|}\begin{bmatrix} e^{-j\varphi} & j|r|e^{j\varphi} \\ -j|r|e^{-j\varphi} & e^{j\varphi} \end{bmatrix}$$

Transfer & scattering matrix

For example, at a single dielectric boundary

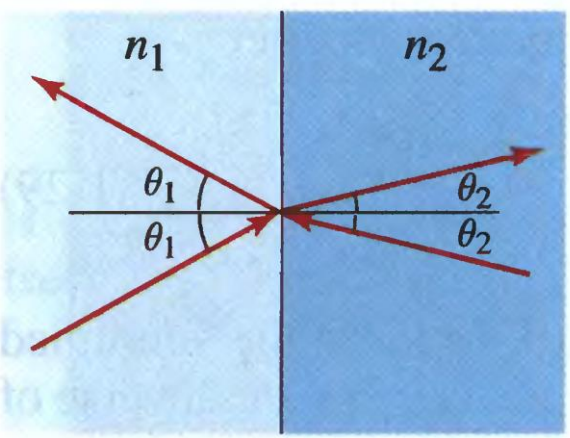


Using Fresnel equations,

$$\mathbf{S} = \begin{bmatrix} t_{12} & r_{21} \\ r_{12} & t_{21} \end{bmatrix} = \frac{1}{n_1+n_2}\begin{bmatrix} 2n_1 & n_2-n_1 \\ n_1-n_2 & 2n_2 \end{bmatrix}$$

$$\mathbf{M} = \frac{1}{2n_2}\begin{bmatrix} n_2+n_1 & n_2-n_1 \\ n_2-n_1 & n_2+n_1 \end{bmatrix}$$

For off-axis case,



$$\mathbf{S} = \begin{bmatrix} t_{12} & r_{21} \\ r_{12} & t_{21} \end{bmatrix} = \frac{1}{\tilde{n}_1+\tilde{n}_2}\begin{bmatrix} 2a_{12}\tilde{n}_1 & \tilde{n}_2-\tilde{n}_1 \\ \tilde{n}_1-\tilde{n}_2 & 2a_{21}\tilde{n}_2 \end{bmatrix}$$

$$\mathbf{M} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \frac{1}{2a_{21}\tilde{n}_2}\begin{bmatrix} \tilde{n}_1+\tilde{n}_2 & \tilde{n}_2-\tilde{n}_1 \\ \tilde{n}_2-\tilde{n}_1 & \tilde{n}_1+\tilde{n}_2 \end{bmatrix}.$$

TE: $\tilde{n}_1 = n_1\cos\theta_1$, $\tilde{n}_2 = n_2\cos\theta_2$, $a_{12} = a_{21} = 1$,

TM: $\tilde{n}_1 = n_1\sec\theta_1$, $\tilde{n}_2 = n_2\sec\theta_2$, $a_{12} = \cos\theta_1/\cos\theta_2 = 1/a_{21}$

Airy's formula using T-matrix

Two cascade systems distanced with homogeneous media.

$$\mathbf{M} = \begin{bmatrix} 1/t_1^* & r_1/t_1 \\ r_1^*/t_1^* & 1/t_1 \end{bmatrix} \begin{bmatrix} \exp(-j\varphi) & 0 \\ 0 & \exp(j\varphi) \end{bmatrix} \begin{bmatrix} 1/t_2^* & r_2/t_2 \\ r_2^*/t_2^* & 1/t_2 \end{bmatrix}$$

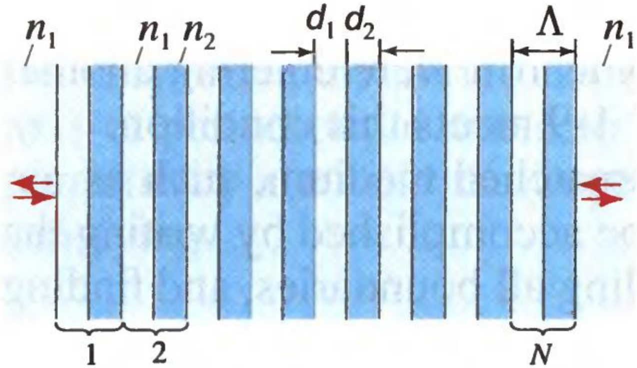
$$\mathbf{M} = \begin{bmatrix} 1/t_1^* & r_1/t_1 \\ r_1^*/t_1^* & 1/t_1 \end{bmatrix} \begin{bmatrix} 1/t_2^* \exp(-j\varphi) & r_2/t_2 \exp(-j\varphi) \\ r_2^*/t_2^* \exp(j\varphi) & 1/t_2 \exp(j\varphi) \end{bmatrix}$$

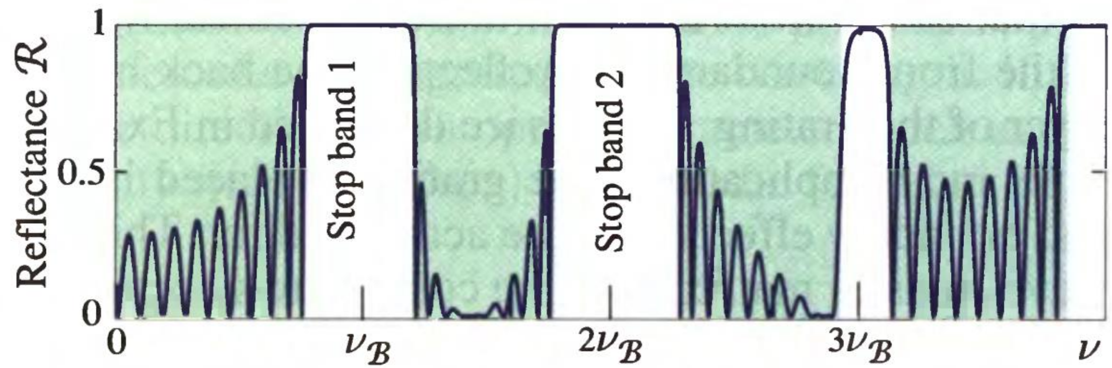
$$\mathbf{M}_{22} = \frac{1}{t_{total}} = -r_1 r_2 \exp(-j\varphi)/t_1 t_2 + \exp(j\varphi)/t_1 t_2 = \frac{1 - r_1 r_2 \exp(-j2\varphi)}{t_1 t_2 \exp(-j\varphi)}$$

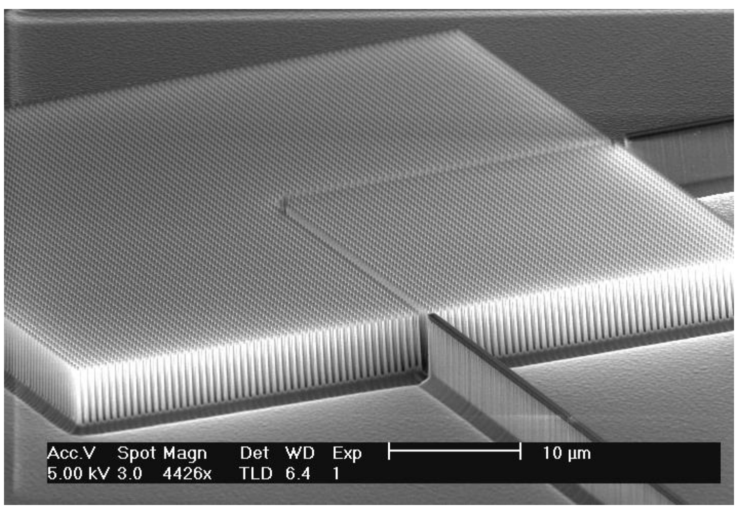
$$t = \frac{t_1 t_2 \exp(-j\varphi)}{1 - r_1 r_2 \exp(-j2\varphi)}$$

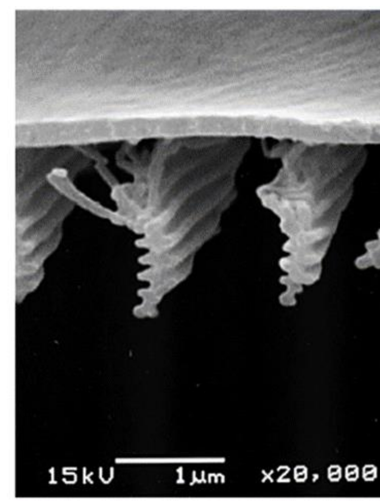
Photonic crystals

A photonic crystal is a periodic arrangement of a dielectric material that exhibits strong interaction with light





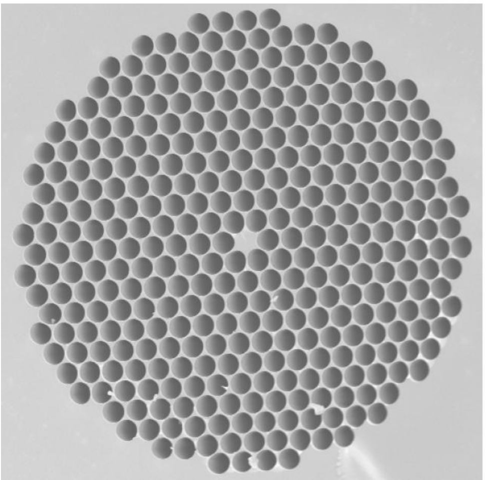




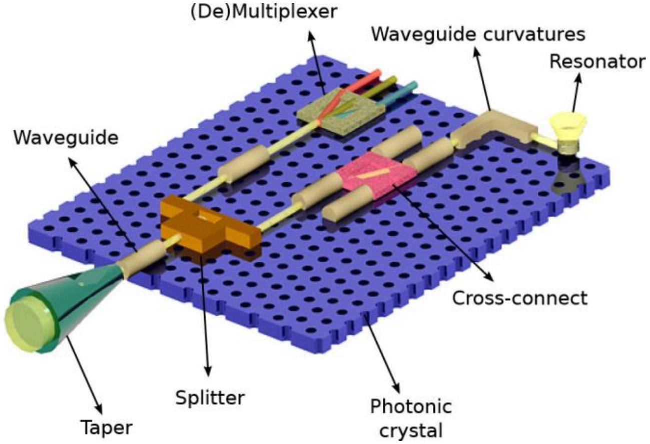
Zijlstra, van der Drift, De Dood, and Polman (DIMES, FOM)

Photonic crystals applications

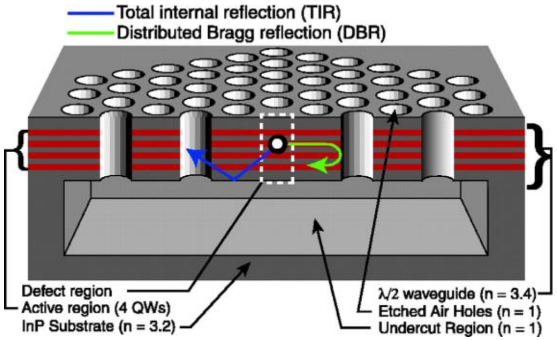
Photonic crystal fiber



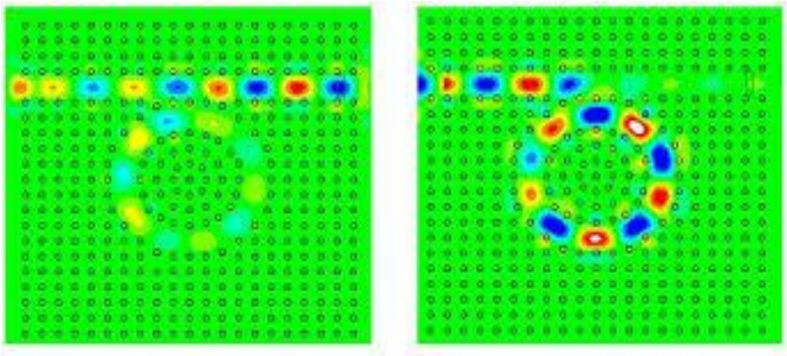
Integrated optics



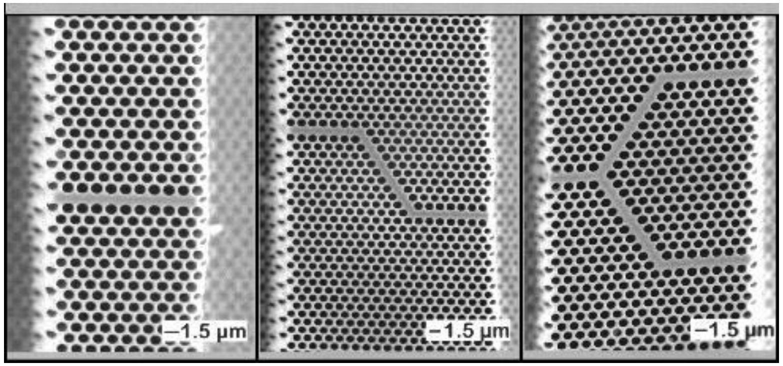
Microcavity & laser



Photonic crystal ring resonator



Photonic crystal waveguides



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