Spring 2019

EECE 588
Lecture 5

Prof. Wonbin Hong

Scattering Area

- Note that under conjugate matching, only half of the intercepted power by the antenna is transferred to the load.
- The other half of the power is lost to Ohmic losses or scattered.
- To account for these power values, we can define scattering, loss and capture areas.
- Scattering Area:
  - The equivalent area, which when multiplied by the incident power density gives the total scattered to reradiated power.
  - Under complex conjugate matching condition, we have:

$$A_s = \frac{|V_T|^2}{8W_i}\left[\frac{R_r}{(R_L + R_r)^2}\right]$$

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Loss Area

- Loss Area:
  - The equivalent area, which when multiplied by the incident power density gives the total power lost in R_L (due to Ohmic and Dielectric losses in the antenna structure)
  - Under complex conjugate matching condition, we have:

$$A_L = \frac{|V_T|^2}{8W_i}\left[\frac{R_L}{(R_L + R_r)^2}\right]$$

Capture Area

- Capture Area:
  - The equivalent area, which when multiplied by the incident power density gives the total power captured, collected or intercepted by the antenna.
  - Under complex conjugate matching condition, we have:

$$A_c = \frac{|V_T|^2}{8W_i}\left[\frac{R_T + R_r + R_L}{(R_L + R_r)^2}\right]$$

  - Capture Area = Effective Area + Scattering Area + Loss Area
- Note that this relationship is valid in general (both conjugate and nonconjugate matching conditions).

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Aperture Efficiency

- Aperture Efficiency:
  - The ratio of the maximum effective area A_{em} of the antenna to its physical area A_p.

$$e_{ap} = \frac{A_{em}}{A_p} = \frac{\text{maximum effective area}}{\text{physical area}}$$

  - For aperture type antennas, such as waveguides, horns, and reflectors, the maximum effective area of the antenna cannot exceed the physical area but it can equal it.
  - $A_{em} <= A_p$ or $0 \leq e_{ap} \leq 1$
  - Note that for aperture antennas you can have aperture efficiencies as much as 100% but you can at most transfer half of this power to a load

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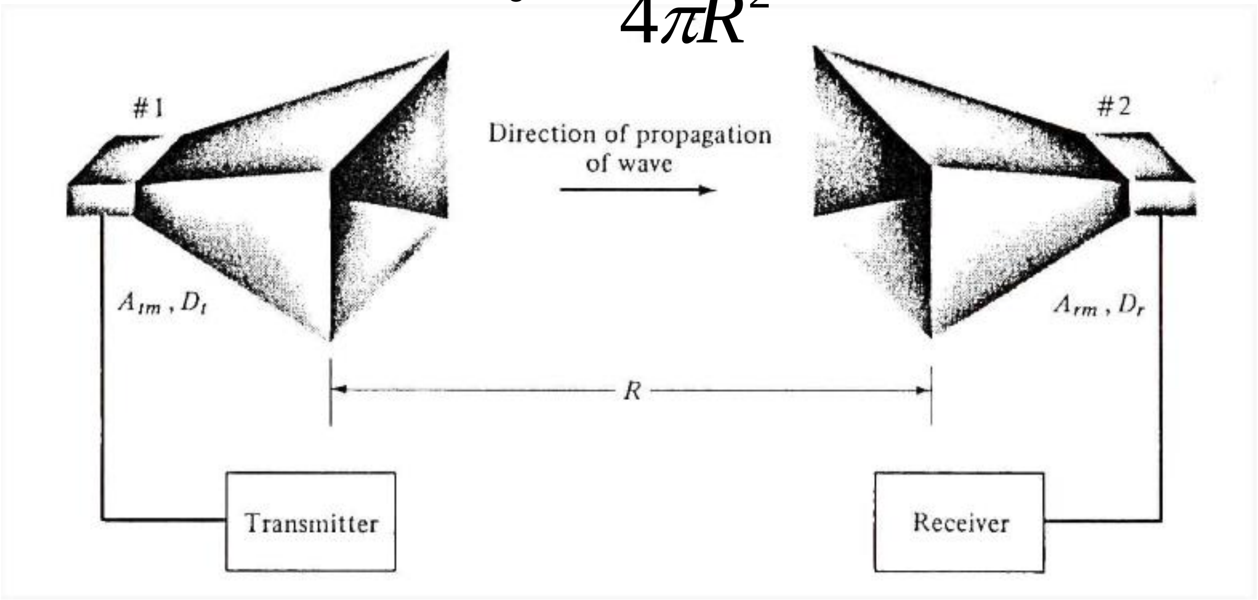
Maximum Directivity and Maximum Effective Area of the Antenna

- Consider the following situation
  - Antenna 1 is transmitter and 2 is receiver.
  - If A1 was isotropic, the power density at distance R would have been:

$$W_0 = \frac{P_t}{4\pi R^2}$$



Maximum Directivity and Maximum Effective Area of the Antenna

- However, A1 is a directive antenna. Therefore, the actual power density is:

$$W_t = W_0 D_t = \frac{P_t D_t}{4\pi R^2}$$

- The power received by antenna #2 is then:

$$P_r = W_t A_r = \frac{P_t D_t A_r}{4\pi R^2} \Rightarrow D_t A_r = \frac{P_r}{P_t}\left(4\pi R^2\right)$$

- Now, if antenna 2 is used as transmitter and 1 as receiver, and assuming the medium between the two antennas is linear, passive, isotropic, we will have:

$$D_r A_t = \frac{P_r}{P_t}\left(4\pi R^2\right)$$

Maximum Directivity and Maximum Effective Area of the Antenna

- Therefore, we will have:

$$\frac{D_t}{A_t} = \frac{D_r}{A_r}$$

- We can also write the same equation for maximum directivity and maximum effective areas:

$$\frac{D_{0t}}{A_{tm}} = \frac{D_{0r}}{A_{rm}}$$

- If antenna 1 is isotropic, we have, $D_{0t}=1$

$$A_{tm} = \frac{A_{rm}}{D_{0r}}$$

Maximum effective area of an isotropic source is equal to the ratio of the maximum effective area to the maximum directivity of any other source.

Maximum Directivity and Maximum Effective Area of the Antenna

- For an electrically small antenna, *l*<< λ, we do know that the effective area is 0.119λ^2 and the directivity is 1.5.
- Using these values, we can calculate the maximum effective area of the isotropic source:

$$A_{tm} = \frac{A_{rm}}{D_{0r}} = \frac{\lambda^2}{4\pi} \Rightarrow A_{rm} = D_{0r} A_{tm} = D_{0r} \frac{\lambda^2}{4\pi}$$

- Therefore,

$$\boxed{A_{em} = \frac{\lambda^2}{4\pi} D_0}$$

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Maximum Directivity and Maximum Effective Area of the Antenna

- This equation assumes that
  - there are no conduction or dielectric losses (e_{cd}=1).
  - the antenna is matched to the load.
  - Polarization of the impinging wave matches that of the antenna.
- If the losses are not zero:

$$A_{em} = e_{cd}\left(\frac{\lambda^2}{4\pi}\right)D_0$$

- In general

$$A_{em} = e_{cd}\left(1-|\Gamma|^2\right)\left(\frac{\lambda^2}{4\pi}\right)D_0\left|\hat{\rho}_w \cdot \hat{\rho}_a\right|^2$$

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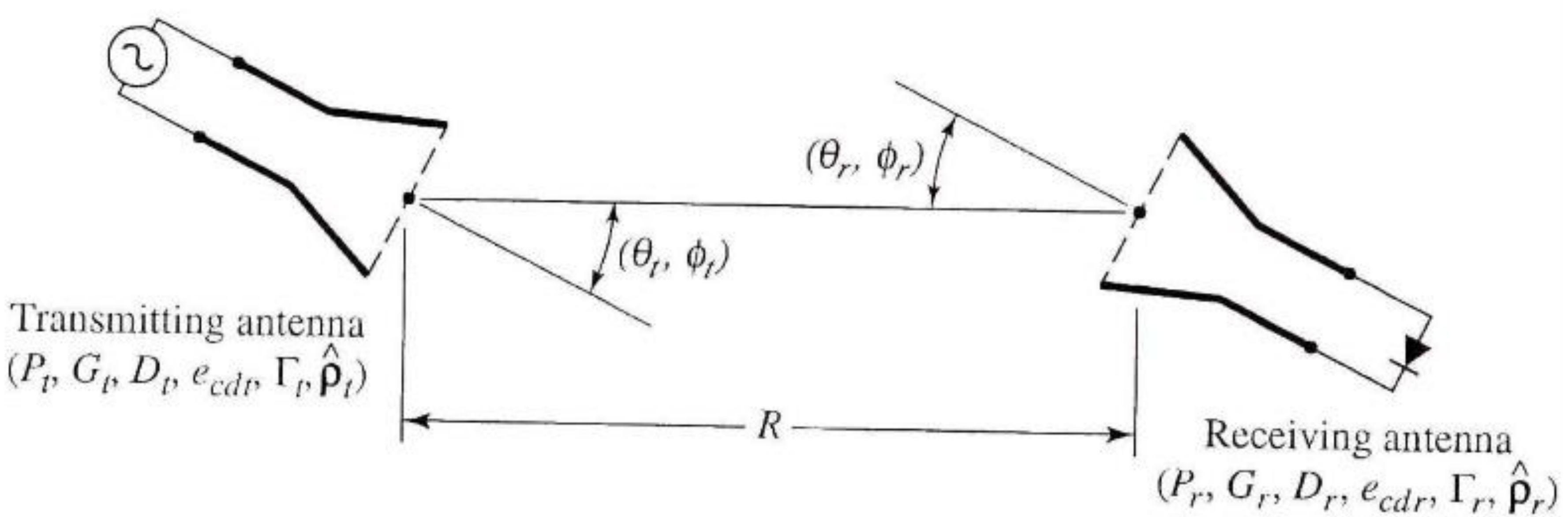
FRISS Transmission Equation and Radar Range Equation

- The FRISS transmission equation relates the power received to the power transmitted between two antennas separated by a distance $R>2D^2/\lambda$, where D is the largest dimension of either antenna.



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FRISS Transmission Equation and Radar Range Equation

- Assuming that the TX antenna is isotropic, we can find its power density:

$$W_0 = e_t \frac{P_t}{4\pi R^2}$$

  - P_t is the power input at the transmitting antenna.
  - e_t is the radiation efficiency of the transmitting antenna.

- If the antenna is nonisotropic, the power density in the direction θ_t, φ_t can be written as:

$$W_t = \frac{P_t G_t(\theta_t, \varphi_t)}{4\pi R^2} = e_t \frac{P_t D_t(\theta_t, \varphi_t)}{4\pi R^2}$$

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FRISS Transmission Equation and Radar Range Equation

- $G_t(\theta_t, \varphi_t)$ is the gain and $D_t(\theta_t, \varphi_t)$ the directivity of the transmitting antenna in thes direction θ_t, φ_t
- We have:

$$A_r = e_r D_r(\theta_r, \varphi_r)\left(\frac{\lambda^2}{4\pi}\right)$$

- The power received by the receiving antenna can be calculated using:

$$P_r = e_r D_r(\theta_r, \varphi_r)\frac{\lambda^2}{4\pi} W_t = e_t e_r \frac{\lambda^2 D_t(\theta_t, \varphi_t) D_r(\theta_r, \varphi_r) P_t}{(4\pi R)^2} \left|\hat{\rho}_t \cdot \hat{\rho}_r\right|^2$$

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Another Way of Looking at the FRISS Transmission Formul

- $G_t(\theta_t, \varphi_t)$ is the gain and $D_t(\theta_t, \varphi_t)$ the directivity of the transmitting antenna in the direction θ_t, φ_t.
- We have:

$$P_r = \frac{P_t}{4\pi R^2} G_t(\theta_t, \varphi_t) A_{er} \qquad \boxed{G_r(\theta_r, \varphi_r) = \frac{4\pi A_{er}}{\lambda^2}}$$

$$P_r = \frac{P_t}{4\pi R^2} G_t(\theta_t, \varphi_t) \frac{G_r(\theta_r, \varphi_r)\lambda^2}{4\pi}$$

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Another Way of Looking at the FRISS Transmission Formula

- If the antennas are not polarization and impedance matched:

$$\frac{P_r}{P_t} = e_{cdt} e_{cdr} \left(1-|\Gamma_t|^2\right)\left(1-|\Gamma_r|^2\right)\left(\frac{\lambda}{4\pi R}\right)^2 D_t(\theta_t,\varphi_t) D_r(\theta_r,\varphi_r) |\hat{\rho}_r \cdot \hat{\rho}_t|^2$$

- Let's look at this a bit differently:

$$P_r = \frac{P_t\left(1-|\Gamma_t|^2\right)}{4\pi R^2} \times D_t(\theta_t,\varphi_t) e_{cdt} \times |\hat{\rho}_r \cdot \hat{\rho}_t|^2 \times A_{er}(\theta_t,\varphi_t)\left(1-|\Gamma_r|^2\right)$$

- The relationship between Gain and Effective area is:

$$G_r(\theta_r,\varphi_r) = e_{cdr} D_r(\theta_r,\varphi_r) = \frac{4\pi A_{er}(\theta_t,\varphi_t)}{\lambda^2}$$

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Another Way of Looking at the FRISS Transmission Formula

- For reflection and polarization matched antennas aligned along the direction of maximum radiation, we can write:

$$\frac{P_r}{P_t} = \left(\frac{\lambda}{4\pi R}\right)^2 G_{0t} G_{0r}$$

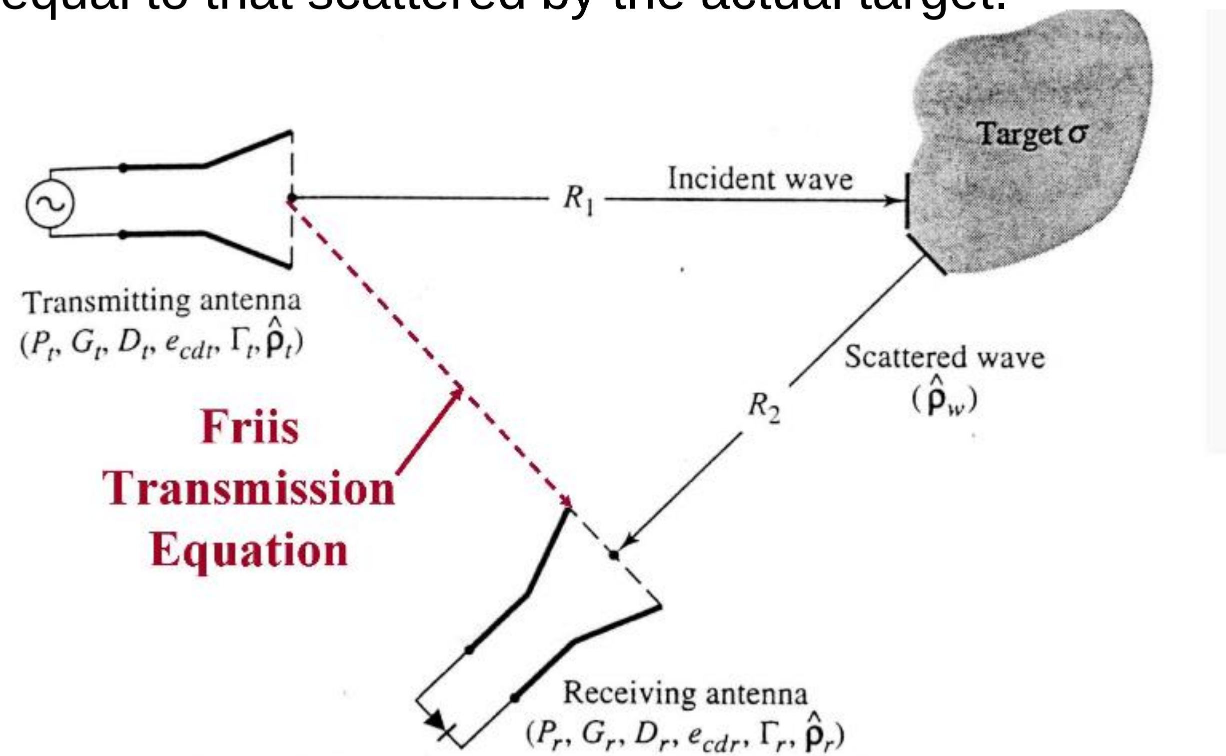
- Also note that:

$$L_{FS} = \left(\frac{\lambda}{4\pi R}\right)^2$$

- is called the free space loss.

Radar Range Equation

- Radar cross section or echo area:
  - The area intercepting that amount of power which, when scattered isotropically, produces at the receiver a density which is equal to that scattered by the actual target.



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Radar Range Equation

- What does this mean in English?
  - (1) Assume that you have a target and shine it with an EM wave and get an echo back at TX location.
  - (2) Now, assume that at the location of the target, you have an imaginary area that intercepts the incident power.
  - (3) Assume that this power is reradiated isotropically and part of it is received at the original transmitter location.
  - Now, the question is how big this surface must be in order to receive the same power at the transmitter location in cases 1 and 3 above.
- In equation form:

$$\lim_{R\to\infty}\left[\frac{\sigma W_i}{4\pi R^2}\right]=W_S$$

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Radar Range Equation

$$\sigma = \lim_{R \to \infty} \left[4\pi R^2 \frac{W_S}{W_i} \right] = \lim_{R \to \infty} \left[4\pi R^2 \frac{\left|E^S\right|^2}{\left|E^i\right|^2} \right]$$

- σ = Radar Cross Section (m^2).
- R = Observation distance from target (m).
- W_i = Incident power density (W/m^2).
- W_S = Scattered power density (W/m^2).
- E^i (E^S) = incident (scattered) electric field (V/m).
- H^i (H^S) = incident (scattered) magnetic field (A/m).

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Radar Range Equation

- Now let us derive radar equation.
- The power density of the transmitter at the location of the target is:

$$W_t = \frac{P_t}{4\pi R_1^2} G_t(\theta_t, \varphi_t)\left(1 - |\Gamma_t|^2\right)$$

- The total captured power at the target location is:

$$P_c = \sigma W_t$$

- This power is then reradiated isotrpically and the received power at the receiver position is:

$$P_r = \frac{P_c}{4\pi R_2^2} A_{er}\left(1 - |\Gamma_r|^2\right)$$

Radar Range Equation

- Now if we combine all these basic equations, we can have:

$$\frac{P_r}{P_t} = e_{cdt} e_{cdr} (1 - |\Gamma_t|^2)(1 - |\Gamma_r|^2) \sigma \frac{D_t(\theta_t, \varphi_t) D_r(\theta_r, \varphi_r)}{4\pi} \times \left(\frac{\lambda}{4\pi R_1 R_2} \right)^2 |\hat{\rho}_t \cdot \hat{\rho}_r|^2$$

- For reflection and polarization matched antennas aligned along the direction of maximum radiation:

$$\frac{P_r}{P_t} = \sigma \frac{G_{0t} G_{0r}}{4\pi} \left[\frac{\lambda}{4\pi R_1 R_2} \right]^2$$

POSTECH

Radar Cross Section of Antennas

- Radar Cross Section (RCS) is a far field parameter.
  - Used to characterize the scattering properties of radar targets.
- Monostatic RCS or Backscattering:
  - When the radar transmitter and receiver are located in the same location.
- Bistatic RCS:
  - When the TX and RX are at different locations.
- RCS is a very complex parameter to analyze and obtain.
- In Stealth and low observable applications, RCS is the parameter that you want to minimize.

Radar Cross Section of Antennas

- RCS is generally a function of the following variables:
  - Polarization of the incident wave.
  - Angle of incidence.
  - Angle of observation.
  - Geometry of the target.
  - Electrical properties of the target.
  - Frequency of operation.
- RCS units are m^2 (Square meters) and sometimes in dBsm (Decibels per square meters)

Table 2.2 RCS OF SOME TYPICAL TARGETS

	Typical RCSs [22]	
Object	RCS (m^2)	RCS (dBsm)
Pickup truck	200	23
Automobile	100	20
Jumbo jet airliner	100	20
Large bomber *or* commercial jet	40	16
Cabin cruiser boat	10	10
Large fighter aircraft	6	7.78
Small fighter aircraft *or* four-passenger jet	2	3
Adult male	1	0
Conventional winged missile	0.5	−3
Bird	0.01	−20
Insect	0.00001	−50
Advanced tactical fighter	0.000001	−60

Antenna Noise Temperature

- Every object with a physical temperature above absolute zero (0 K = -273° C) radiates energy.
- The amount of energy radiated is represented by an equivalent temperature, brightness temperature, T_B.

$$T_B(\theta,\phi) = \epsilon(\theta,\phi)T_m = (1 - |\Gamma|^2)T_m$$

- T_B=brightness temperature (K).
- ε = emissivity (dimensionless)
- T_m=physical temperature (K)
- $\Gamma(\theta,\phi)$ = reflection coefficient of the surface for the polarization of the wave.

- From: Microwave Remote Sensing by Ulaby, Moore, and Fung

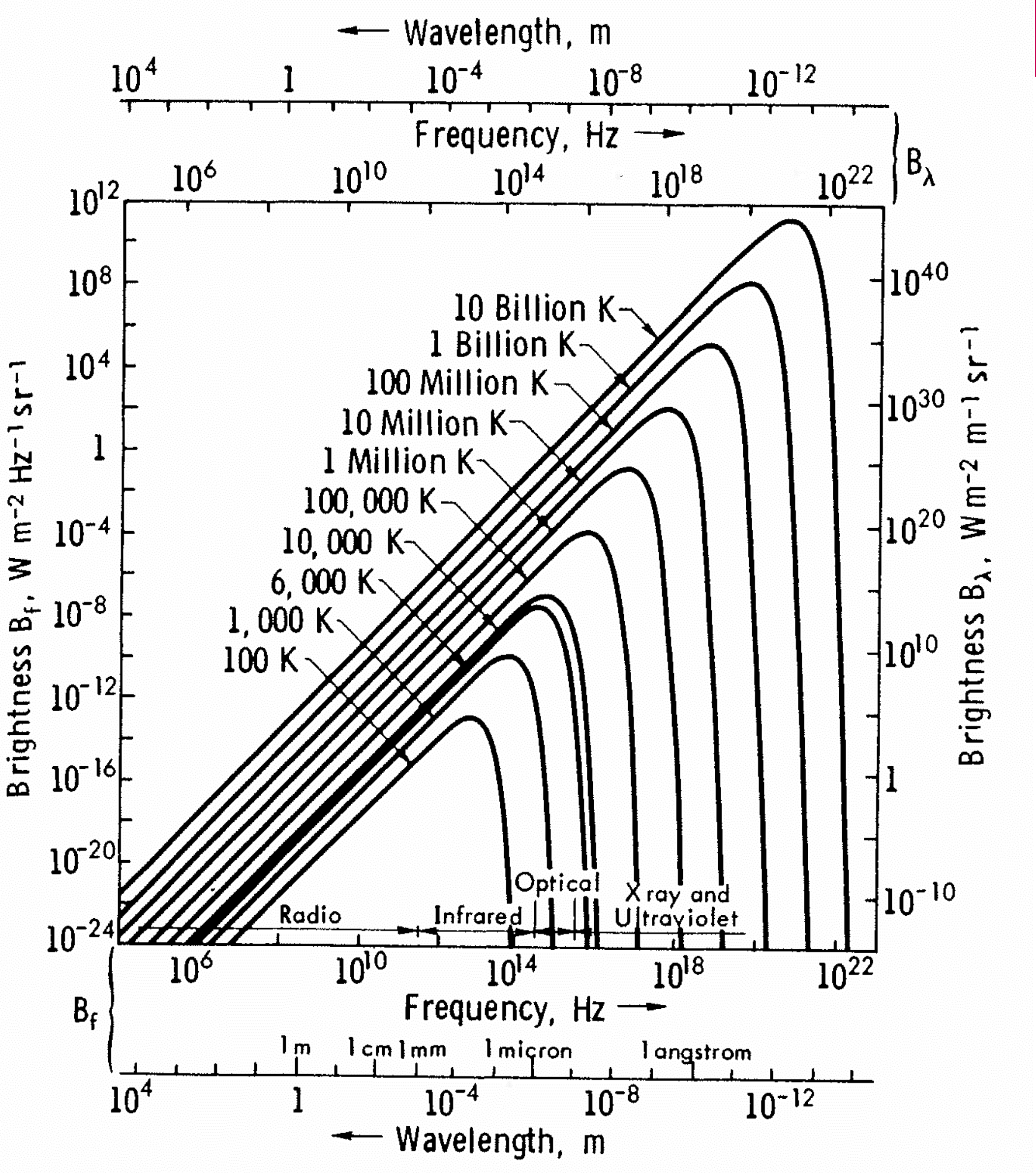


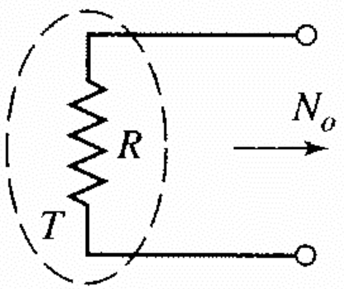
Fig. 4.4 Planck radiation-law curves (adapted from Kraus, 1966).

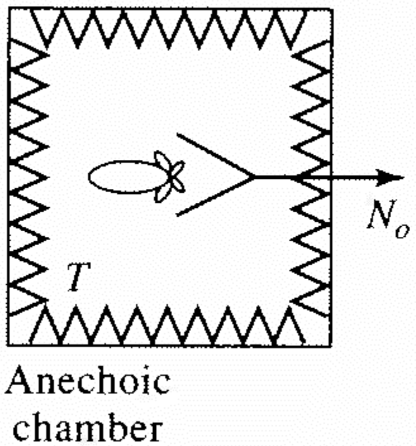
Antenna Noise Temperature

- Consider the three examples shown in this figure:
  - The simple resistor at temperature T produces an available noise output power of:

$$N_o = kTB$$

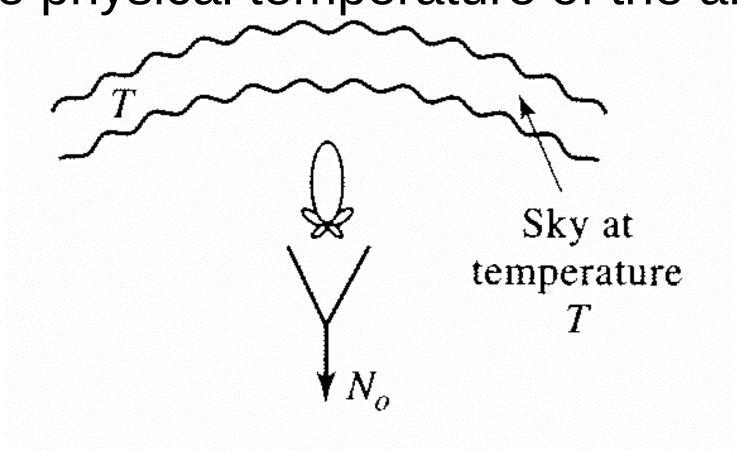
  - B is the system bandwidth and k is the Boltzmann's constant.
  - For the antenna in an anechoic chamber: the anechoic chamber appears as a perfectly absorbing enclosure and is in thermal equilibrium with the antenna.
  - If the antenna is impedance matched, the terminals of the antenna are indistinguishable from the terminals of the resistor.





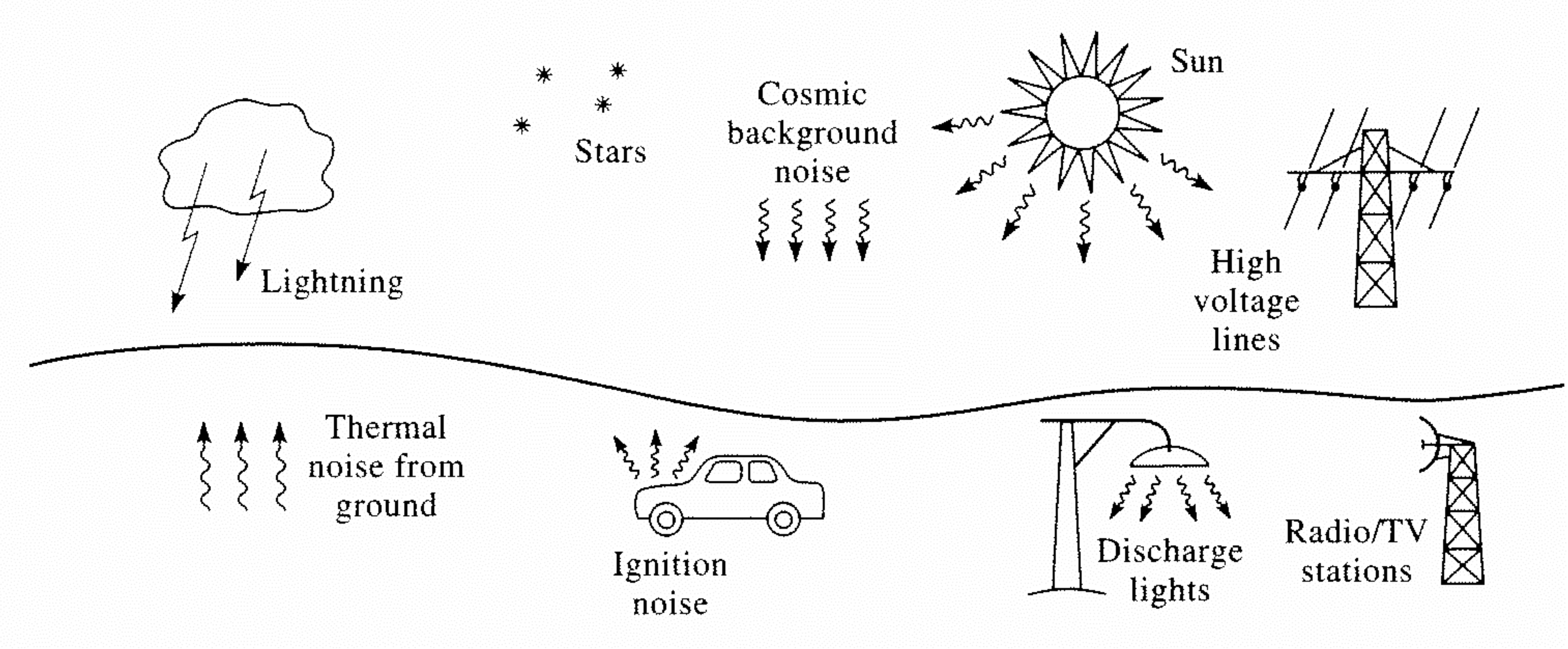
Antenna Noise Temperature

- In this case, the antenna produces the same output noise power as the resistor.
- If the same antenna is now directed towards sky and the main beam of the antenna is narrow enough to see a uniform region with a physical temperature of T, then the antenna again appears as a resistor with a physical temperature of T.
    - This is true regardless of the radiation efficiency of the antenna provided that the physical temperature of the antenna is also T.



Antenna Noise Temperature

- In reality, the environment that the antenna sees is much more complex and the antenna can pick up noise power from a variety of different sources.



Antenna Noise Temperature

- We define the background noise temperature (or brightness temperature), T_B, as the equivalent temperature of a resistor required to produce the same noise power as the actual environment seen by the antenna.
- Typical brightness temperatures at low microwave frequencies:
  - $T_B = 300$ K for ground
  - $T_B = 3 - 5$ K for sky towards zenith
  - $T_B = 100 - 150$ K towards horizon
- The overhead sky background temperature of 3-5K is the cosmic background radiation, which is remnant of the big bang at the creation of the universe.

Antenna Noise Temperature

- If you take a highly directive antenna and point it upward at an empty region of space, you measure a noise temperature of 3-5K.
  - Arno Penzias and Robert Woodrow Wilson won the Nobel Prize in Physics in 1978 for measuring the cosmic background radiation.
- The background noise temperature increases as the antenna is pointed toward the horizon because of the greater atmosphere thickness.

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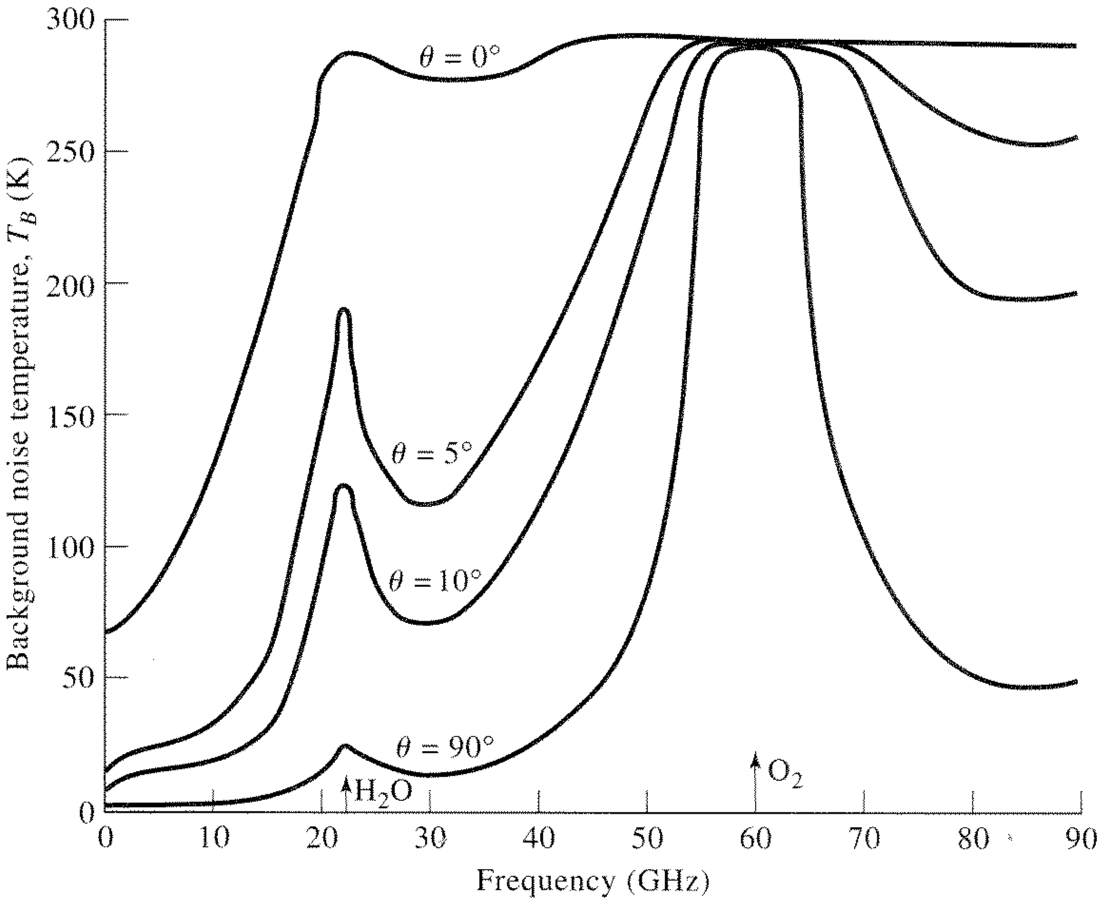
Background Noise Temperature

- Notice that the background radiation goes down with elevation angle.
- Notice the resonance peaks of H_2O and O_2.
  - 22 GHz and 60 GHz.
- These are associated with the absorption because of the resonances of Water and Oxygen.
  - Loss increases, absorption increases, and brightness temperature also increases.
- Useful in remote sensing.



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Antenna Temperature

- The microwave energy radiated by the different sources is intercepted by antennas.
  - This appears at the terminal of the antenna as an antenna temperature.
  - The temperature appearing at the terminals of an antenna is given by:

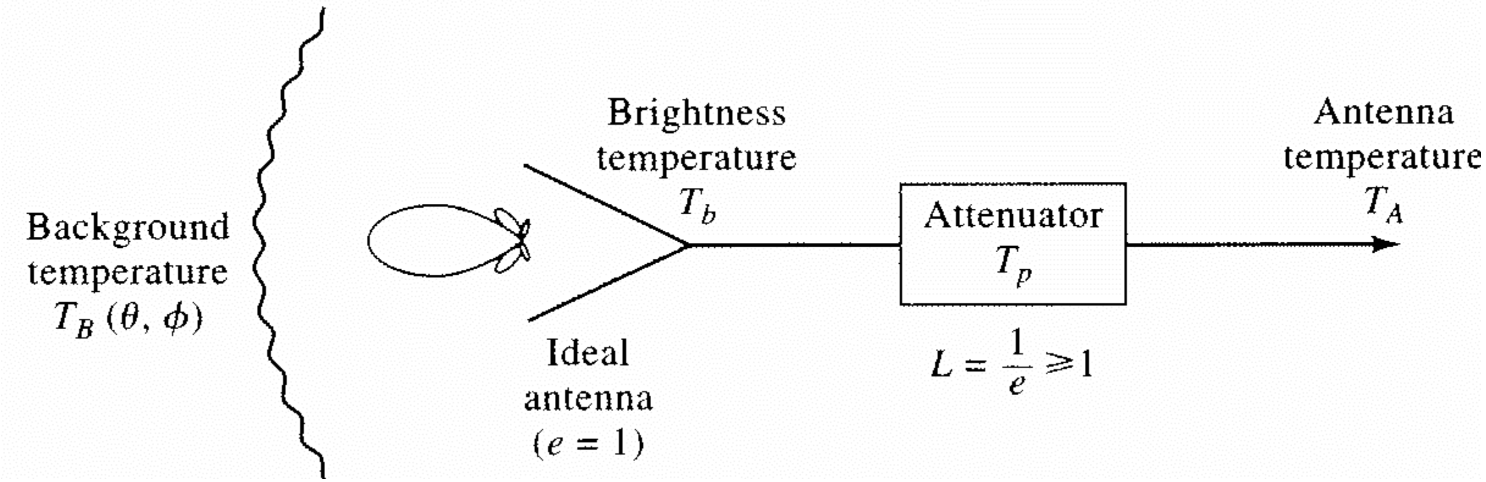
$$T_b = \frac{\int_0^{2\pi}\int_0^{\pi} T_B(\theta,\phi) D(\theta,\phi) \sin\theta \, d\theta \, d\phi}{\int_0^{2\pi}\int_0^{\pi} D(\theta,\phi) \sin\theta \, d\theta \, d\phi}$$

  - $T_B(\theta,\phi)$ is the distribution of the background temperature and of the antenna.

Antenna Temperature

- T_b is the antenna brightness temperature.
- $D(\theta, \phi)$ = directivity pattern of the antenna.
- If the antenna had dissipative loss, its radiation efficiency (e_{cd}) will be less than 1.
  - This way, the power available at the terminals of a receiving antenna is educed by e_{cd} from that intercepted by the antenna.
  - If the efficiency is less than 1, thermal noise will be generated by resistive losses in the antenna as well and this will increase the noise temperature of the antenna.
- A lossy antenna at physical temperature T_p viewing a background noise temperature distribution T_B can be represented by the system described in the next slide.

Antenna Temperature



- The lossy antenna is modeled as an ideal antenna with $e_{cd} = 1$, followed by an attenuator having a power loss factor of $L \geq 1$, at physical temperature T_p.
- Notice that the radiation efficiency of the antenna is related to L using:

$$L = \frac{1}{e_{cd}}$$

Antenna Temperature

- The brightness temperature of the antenna is given by:

$$T_A = \frac{T_b}{L} + \frac{(L-1)}{L} T_p = e_{rad} T_b + (1 - e_{rad}) T_p$$

- Notice that the term $(L-1)/L\ T_p$ is the noise temperature of an attenuator with an attenuation factor of L.
  - For derivation of this term refer to p. 90-91 of "Microwave and RF design of wireless systems" by D. M. Pozar.
- This temperature is references at the output of the antenna.