광전자공학 Ch. 2 Part 1
Polarization of light

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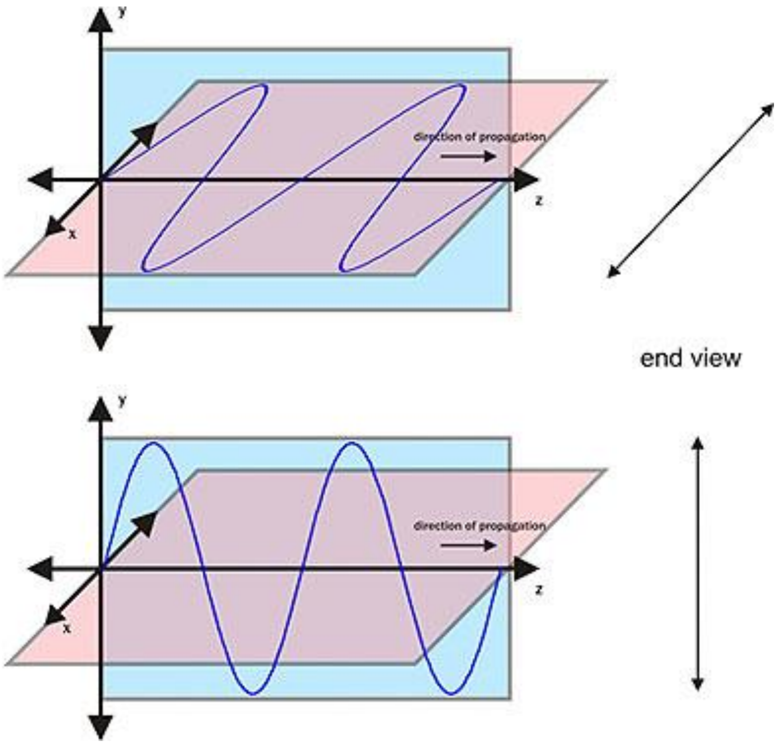
Polarization of light



Plane wave propagating to $\mathbf{k}$

$$U(r,t) = U_0 e^{j(\mathbf{k}\cdot\mathbf{r}-\omega t)}$$

$$\nabla\times\mathbf{H} = \varepsilon\frac{\partial\mathbf{E}}{\partial t} \longrightarrow \mathbf{k}\times\mathbf{H} = -\varepsilon\omega\mathbf{E}$$

$$\nabla\times\mathbf{E} = -\mu\frac{\partial\mathbf{H}}{\partial t} \longrightarrow \mathbf{k}\times\mathbf{E} = \mu\omega\mathbf{H}$$

$$\nabla\cdot\mathbf{H} = 0 \longrightarrow \mathbf{k}\cdot\mathbf{H} = 0$$

$$\nabla\cdot\mathbf{E} = 0 \longrightarrow \mathbf{k}\cdot\mathbf{E} = 0$$

1. E, H, and k are perpendicular to each others

2. There is a degree of freedom for choosing the direction of E

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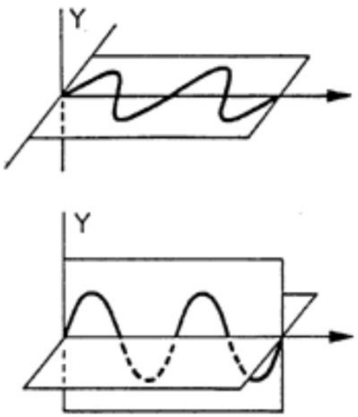
Linear polarization

Assume light is propagating through z-direction,

$$\mathbf{E}(z,t) = (A_x\hat{x} + A_y\hat{y})\exp\big(j(kz-\omega t)\big) = E_x(z,t)\hat{x} + E_y(z,t)\hat{y}$$



Linear Polarized

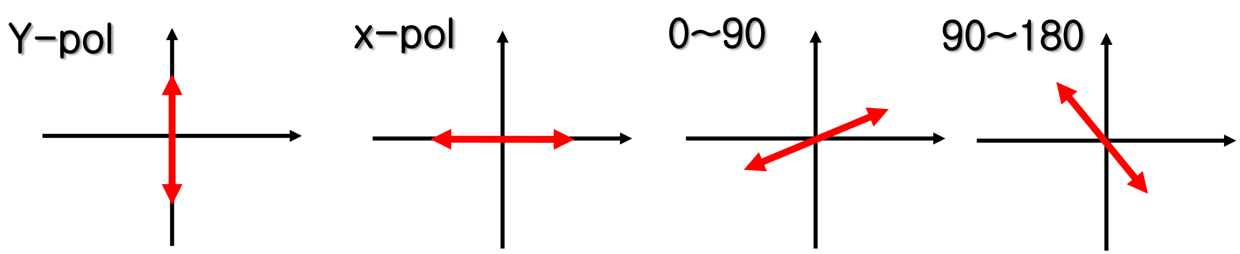
$$A_x = a_x e^{j\phi_x},\ A_y = a_y e^{j\phi_y}$$

$$E_x = a_x\cos(kz-\omega t+\phi_x)$$

$$E_y = a_y\cos(kz-\omega t+\phi_y)$$

Light is linearly polarized when,

$a_x = 0,$ $\quad a_y = 0,$ $\quad \varphi = 0,$ $\quad \varphi = \pi$

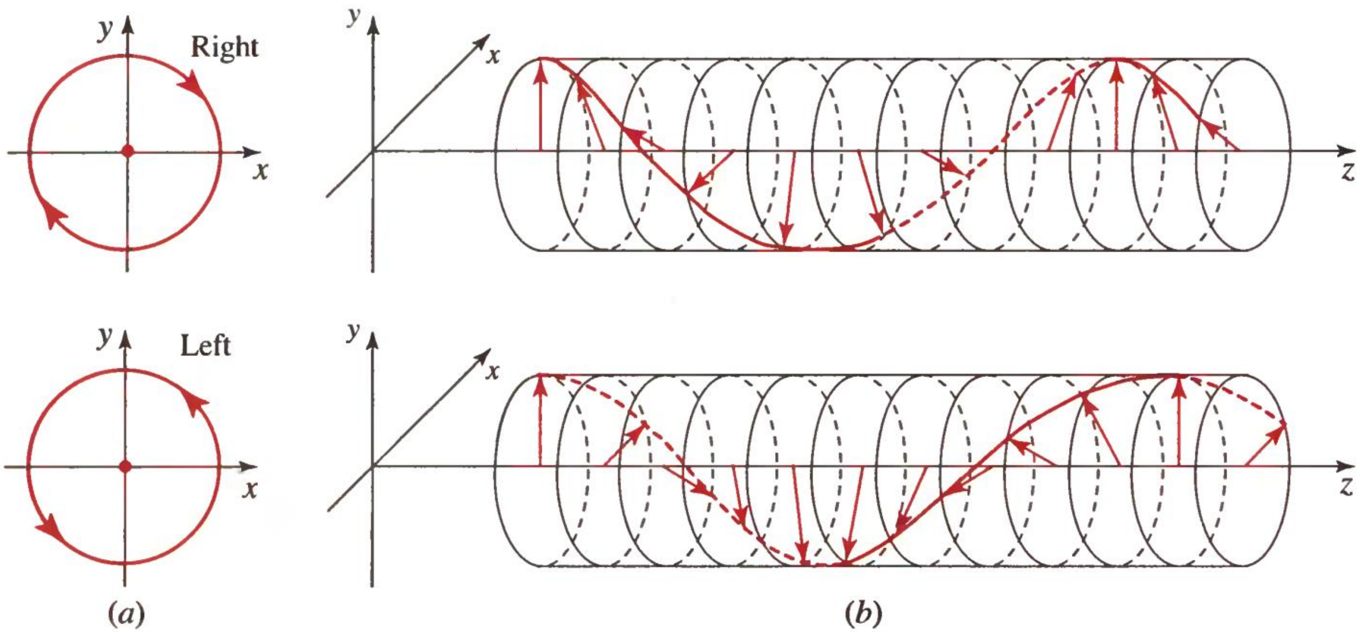


Circular polarization

Circular polarizations

$$\left|a_x\right| = \left|a_y\right|$$

and $\varphi = \pi / 2, 3\pi / 2$



Right-circularly polarization (RCP)

$$\varphi = -\pi / 2$$

Left-circularly polarization (LCP)

$$\varphi = \pi / 2$$

Elliptical polarization

$$E_x = a_x \cos(kz - \omega t + \phi_x)$$
$$E_y = a_y \cos(kz - \omega t + \phi_y)$$

$$\frac{E_x}{a_x} = \cos(kz - \omega t), \quad \frac{E_y}{a_y} = \cos(kz - \omega t + \varphi) = \cos(kz - \omega t)\cos\varphi - \sin(kz - \omega t)\sin\varphi$$

$$\sin(kz - \omega t) = \frac{E_x}{a_x}\frac{\cos\varphi}{\sin\varphi} - \frac{E_y}{a_y}\frac{1}{\sin\varphi}$$

$$\cos^2(kz - \omega t) + \sin^2(kz - \omega t) = \left(\frac{E_x}{a_x}\frac{\cos\varphi}{\sin\varphi} - \frac{E_y}{a_y}\frac{1}{\sin\varphi}\right)^2 + \left(\frac{E_x}{a_x}\right)^2 = 1$$

$$\left(\frac{E_x}{a_x}\right)^2 + \left(\frac{E_y}{a_y}\right)^2 - 2\frac{E_x}{a_x}\frac{E_y}{a_y}\cos\varphi = \sin^2\varphi \quad \longleftarrow \quad \varphi = \phi_y - \phi_x$$

Elliptical polarization

$$\left(\frac{E_x}{a_x}\right)^2+\left(\frac{E_y}{a_y}\right)^2-2\frac{E_xE_y}{a_xa_y}\cos\Delta=\sin^2\Delta$$

$$\begin{pmatrix}E_{x'}\\E_{y'}\end{pmatrix}=\begin{pmatrix}\cos\psi & \sin\psi\\-\sin\psi & \cos\psi\end{pmatrix}\begin{pmatrix}E_x\\E_y\end{pmatrix}$$

For certain ψ, Equation can be written as,

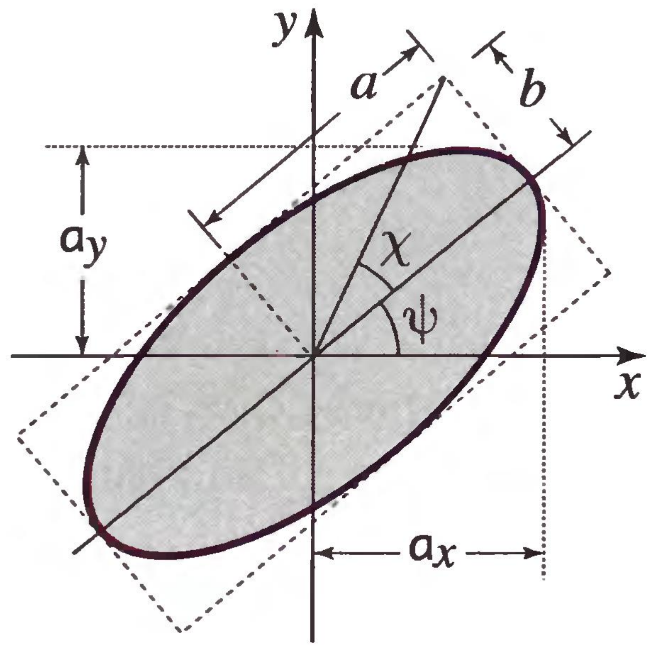
$$\left(\frac{E_x}{a}\right)^2+\left(\frac{E_y}{b}\right)^2=1$$

Where,

$$\tan 2\psi=\frac{2r}{1-r^2}\cos\varphi,\qquad r=\frac{a_y}{a_x},$$

$$\sin 2\chi=\frac{2r}{1+r^2}\sin\varphi,\qquad \varphi=\varphi_y-\varphi_x$$

Polarization ellipse



Condition for Linear polarizations

$$\frac{a_xa_y}{a_x^{\ 2}+a_y^{\ 2}}\sin\varphi=0$$

Condition for Circular polarizations

$$\frac{a_xa_y}{a_x^{\ 2}+a_y^{\ 2}}\sin\varphi=\pm1$$

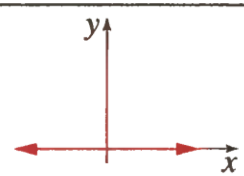
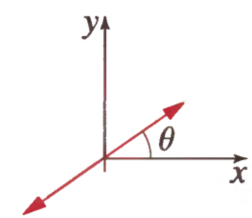
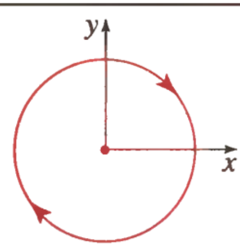
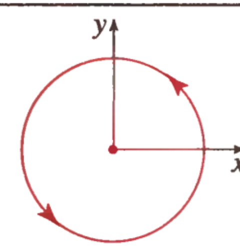
Jones matrix expression

$$\mathbf{E}(z,t) = (A_x \hat{x} + A_y \hat{y}) \exp\left(j(kz - \omega t) \right)$$

Each orthogonal polarization vector can be written as simple matrix form.

$$\mathbf{J} = \begin{pmatrix} A_x \\ A_y \end{pmatrix} = \begin{pmatrix} a_x e^{j\phi_x} \\ a_y e^{j\phi_y} \end{pmatrix} = \begin{pmatrix} a_x \\ a_y e^{j\varphi} \end{pmatrix}$$

Some representative polarizations can be expressed as,

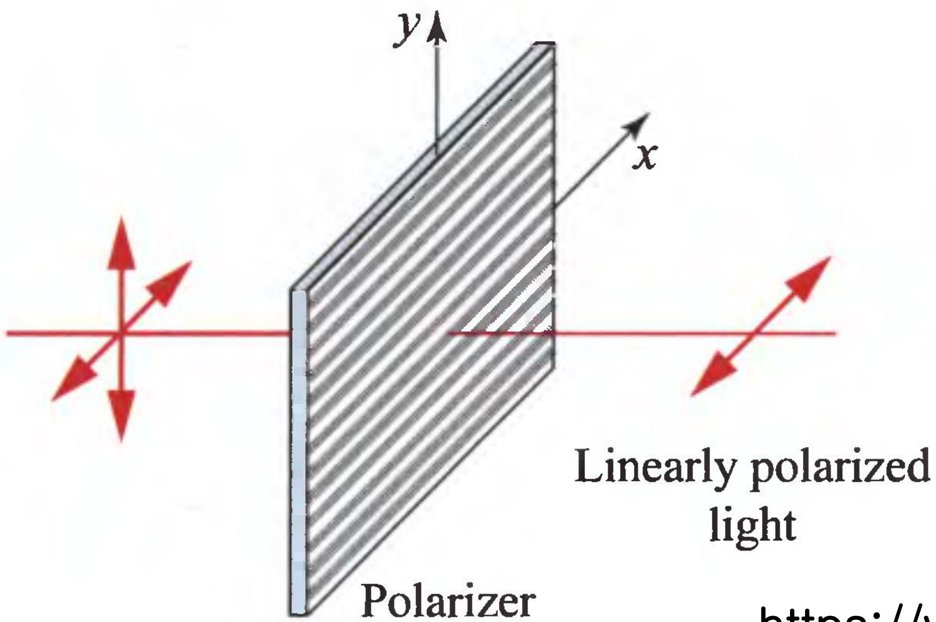
LP in x direction	$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$		LP at angle θ	$\begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}$	
RCP	$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ j \end{bmatrix}$		LCP	$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -j \end{bmatrix}$	

Linear polarizer

A Linear polarizer
: an optical device that only transmit certain linear polarization.

$$\mathbf{T} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

Linear Polarizer
Along x Direction



https://www.youtube.com/watch?v=QgA6L2n476Y

Wave retarders (waveplate)

Wave retarder
: Give a phase delay to certain polarization respect to its orthogonal polarization.

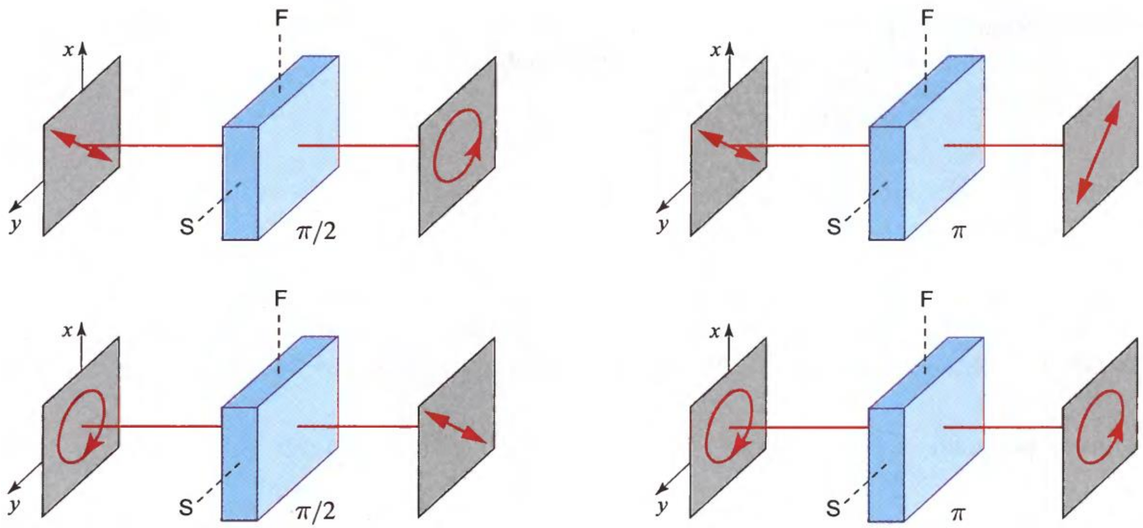
$$\mathbf{T} = \begin{bmatrix} 1 & 0 \\ 0 & e^{-j\Gamma} \end{bmatrix}$$

Half -wave retarder

$$\mathbf{T} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Quarter-wave retarder

$$\mathbf{T} = \begin{bmatrix} 1 & 0 \\ 0 & -j \end{bmatrix}$$



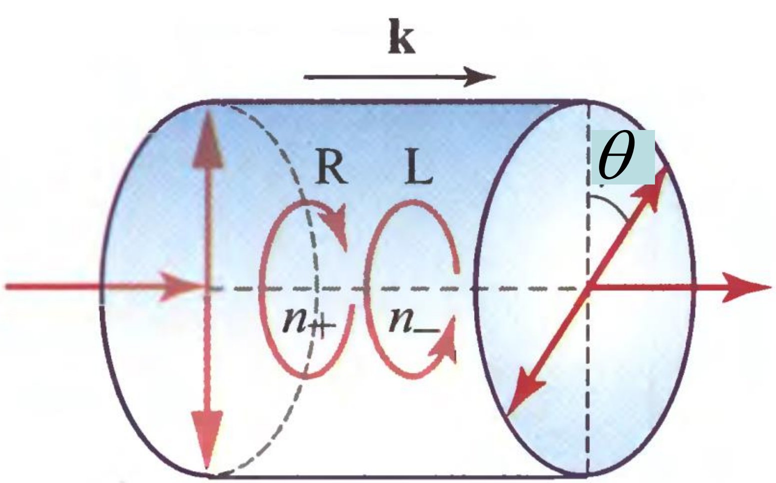
Polarization rotator

Rotate the plane of polarization by a particular angle

Made by optically active media
(will be introduced later)

There is no fast, slow axis
(normal modes are RCP and LCP)

$$\mathbf{T} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$



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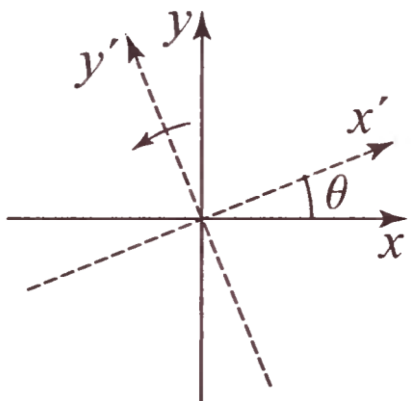
Coordinate transform

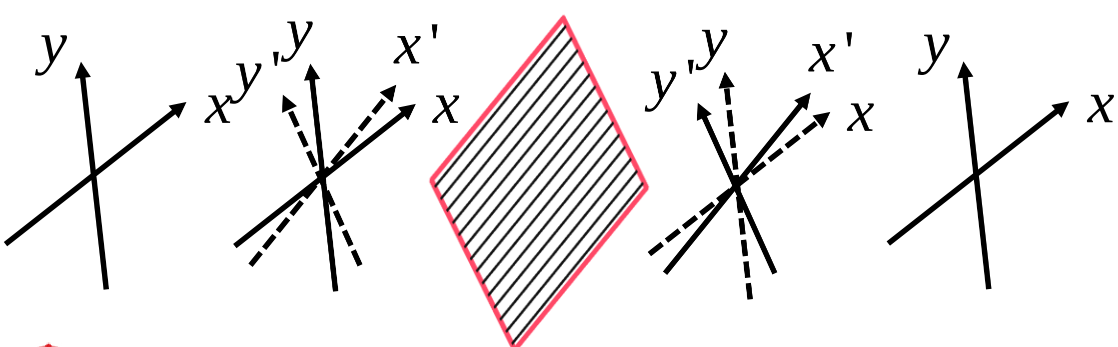
Sometimes we need to rotate the coordinate to match with the slow(fast) axis of the waveplates.



$$\mathbf{J}' = \mathbf{R}(\theta)\,\mathbf{J}$$

$$\mathbf{R}(\theta) = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

For example, passing through linear polarizer with a transmission axis making an angle of θ from x-axis

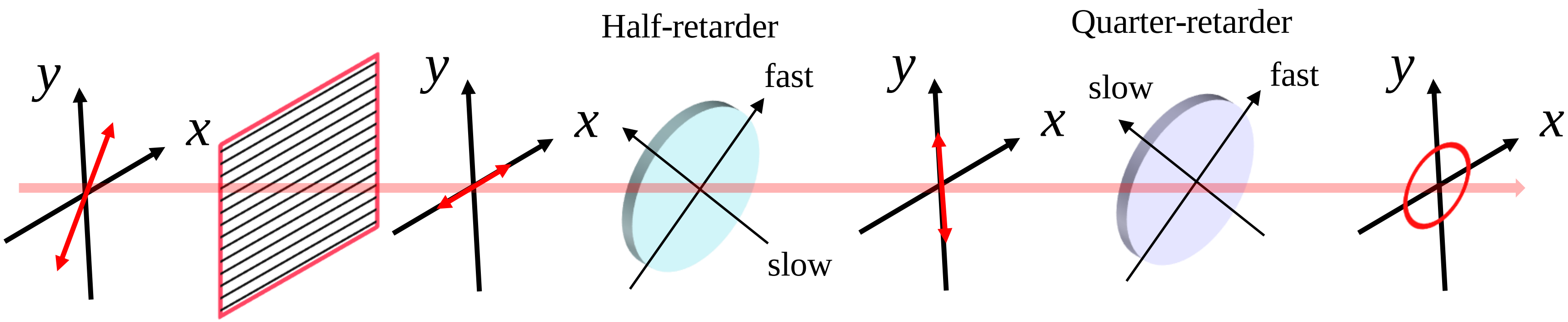


$$\mathbf{T} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}$$

Cascade analysis of Jones matrix

Optical polarization passing through cascaded polarization devices can be analyzed.



T

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\frac{1}{2}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\frac{1}{2}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -j \end{bmatrix}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1-j & 1+j \\ 1+j & 1-j \end{bmatrix}$$

J

$$\begin{bmatrix} \cos\phi \\ \sin\phi \end{bmatrix} \qquad \begin{bmatrix} \cos\phi \\ 0 \end{bmatrix} \qquad \begin{bmatrix} 0 \\ \cos\phi \end{bmatrix} \qquad \frac{1}{2}\begin{bmatrix} 1+j \\ 1-j \end{bmatrix}\cos\phi$$

$$= \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -j \end{bmatrix}\cos\phi$$

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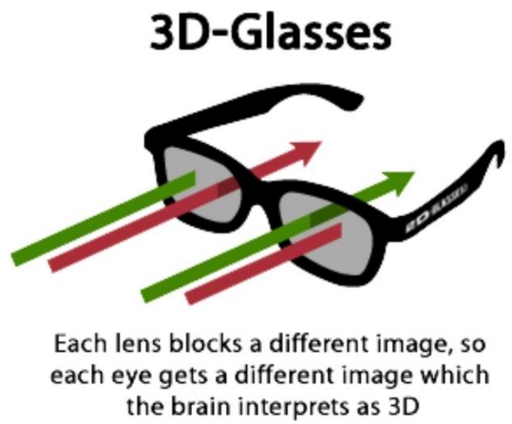
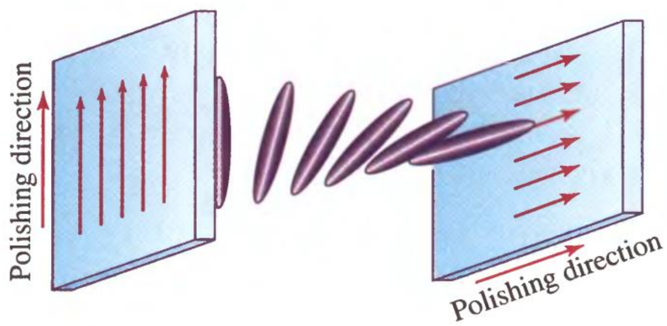
Application of polarized light

3D movie with polarization glasses





Liquid crystal display

Sunglasses with polarization filtering

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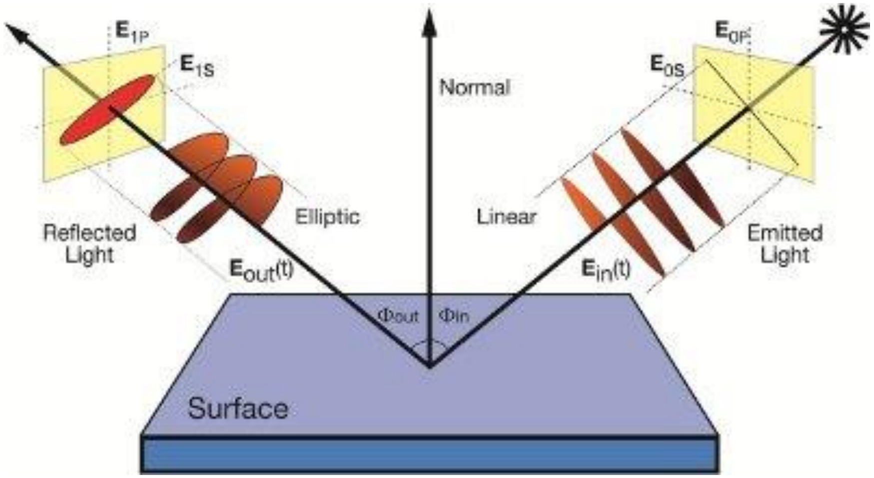
Application of polarized light

Ellipsometry



Optical technique for investigating the dielectric properties (complex refractive index or dielectric function) of thin films. Ellipsometry measures the change of polarization upon reflection or transmission and compares it to a model.