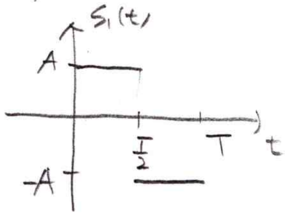
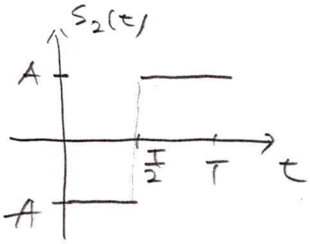
2014 통신

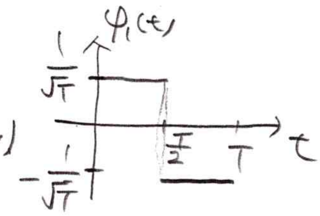
1. (a)

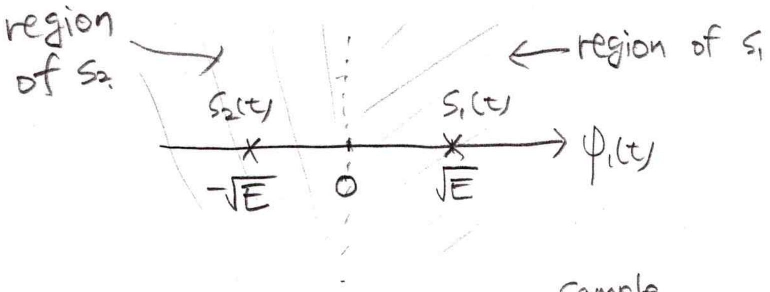




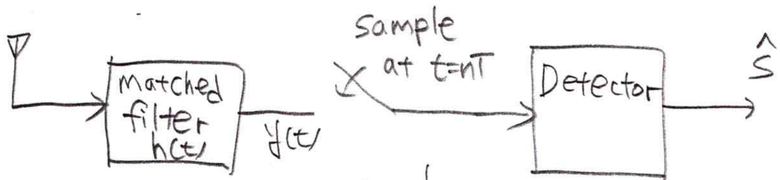
$$\int_0^T |s_1(t)|^2 dt = A^2T = E = \int_0^T |s_2(t)|^2 dt.$$

Let $\varphi_1(t) = \frac{s_1(t)}{A\sqrt{T}}$, then $\int_0^T |\varphi_1(t)|^2 dt = 1$ (unit energy)



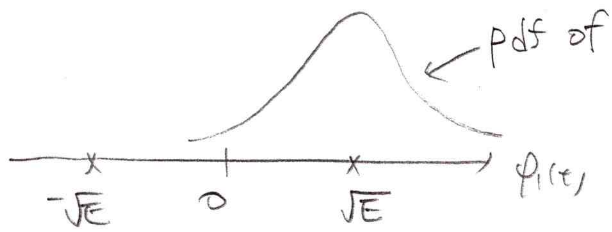


(b)



$$h(t) = \varphi_1(T-t) = \begin{cases} -\frac{1}{\sqrt{T}} & \text{if } 0 \le t < \frac{T}{2} \\ \frac{1}{\sqrt{T}} & \text{if } \frac{T}{2} \le t < T \\ 0 & \text{otherwise.} \end{cases}$$

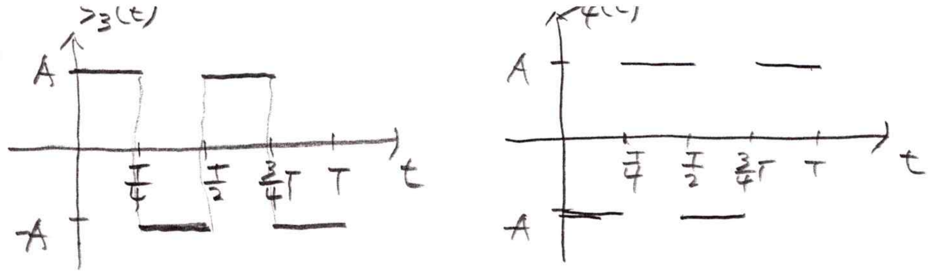
(c)



$$f_{y(t)|s(t)}(y(t)|s(t)=s_1(t)) = \frac{1}{\sqrt{\pi N_0}} e^{-\frac{(y(t)-\sqrt{E})^2}{N_0}}$$

(d) $P_e = Q\left(\frac{d}{2\sigma}\right) = Q\left(\frac{2\sqrt{E}}{\sqrt{2N_0}}\right) = Q\left(\sqrt{\frac{2E}{N_0}}\right)$

(e)

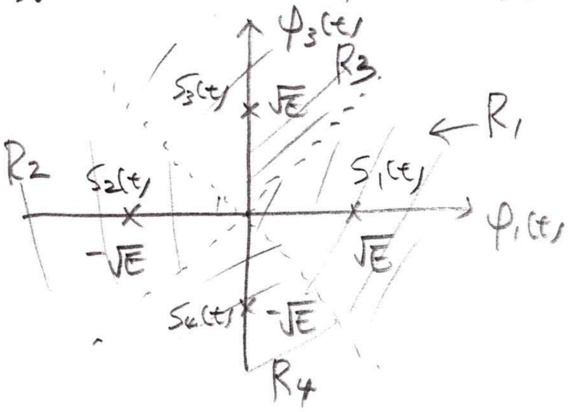


$E = \int_0^T |s_3(t)|^2 dt = \int_0^T |s_4(t)|^2 dt = A^2 T$, letting $\varphi_3(t) = \frac{s_3(t)}{A\sqrt{T}}$, then

$\int_0^T |\varphi_3(t)|^2 dt = 1.$

Let's check if $\varphi_1(t)$ and $\varphi_3(t)$ are orthonormal basis. Since

$\int_0^T \varphi_1(t)\varphi_3(t)dt = 0$ and $\int_0^T |\varphi_1(t)|^2 dt = \int_0^T |\varphi_3(t)|^2 dt = 1$, they are orthonormal basi



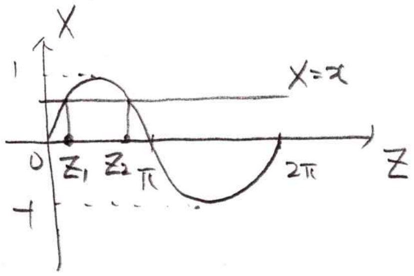
(f) Since $M=4$-ary signaling, we have

$$P_e \le (M-1)\,Q\left(\frac{d}{2\sigma}\right) = 3Q\left(\frac{\sqrt{2E}}{\sqrt{2N_0}}\right) = \underline{3Q\left(\sqrt{\frac{E}{N_0}}\right)}$$

union bound

2.

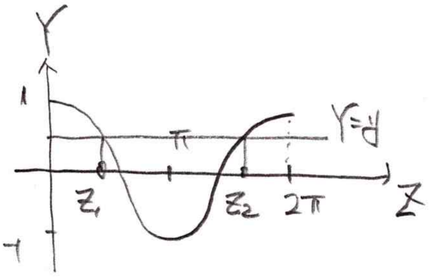
(a)



$$f_X(x) = f_Z(z_1)\cdot\left|\frac{dz_1}{dx}\right| + f_Z(z_2)\cdot\left|\frac{dz_2}{dx}\right|$$

$$= \frac{1}{2\pi}\cdot\left|\frac{d\sin^{-1}x}{dx}\right| + \frac{1}{2\pi}\cdot\left|\frac{d\sin^{-1}x}{dx}\right|$$

$$\therefore f_X(x) = \begin{cases} \dfrac{1}{\pi\sqrt{1-x^2}}, & \text{for } x \in (-1,1) \\ 0, & \text{otherwise.} \end{cases}$$



$$f_Y(y) = f_Z(z_1)\left|\frac{dz_1}{dy}\right| + f_Z(z_2)\left|\frac{dz_2}{dy}\right|$$

$$= \frac{1}{2\pi}\left|\frac{d\cos^{-1}y}{dy}\right| + \frac{1}{2\pi}\left|\frac{d\cos^{-1}y}{dy}\right|$$

$$\therefore f_Y(y) = \begin{cases} \dfrac{1}{\pi\sqrt{1-y^2}}, & \text{for } y \in (-1,1) \\ 0, & \text{otherwise} \end{cases}$$

(b) $E[X] = \int_{-1}^{1} x f_X(x)\,dx = \frac{1}{\pi}\int_{-1}^{1} \frac{x}{\sqrt{1-x^2}}\,dx$, $x = \sin\theta$ 일 때 $dx = \cos\theta\, d\theta$.

$$\therefore E[X] = \frac{1}{\pi}\int_{\frac{3\pi}{2}}^{\frac{\pi}{2}} \sin\theta\, d\theta = 0.$$

마찬가지로 $E[Y]=0$. Then,

$$Cov(X,Y) = E[XY] = E[\sin Z \cos Z] = \frac{1}{2\pi}\int_0^{2\pi} \sin z \cos z\, dz = 0.$$

$\therefore$ X and Y are uncorrelated.

(c) X,Y가 statistically independent 하기 위해서는 Y로 어떤 conditioning을 해도 X의 확률은 같아야 한다. $P(X=0\mid Y=-1)$ 에서는 $Z=\pi$인 경우 이므로 1이 되지만 $P(X=0\mid Y=0)$ 에서는 $Y=0$ 이기 위한 $Z=\frac{\pi}{2}, \frac{3}{2}\pi$ 이고 그 때 $X=0$이 될 확률은 0 이다.

즉 $P(X=0\mid Y=-1) \neq P(X=0\mid Y=0)$, $\therefore$ X,Y are dependent