Spring 2019

# EECE 588
# Lecture 12

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# Circular Loops of Constant Current

- Now, we want to consider the radiation from a loop antenna with $C \cong \lambda$ but we still assume that the current is a uniform one.
- Note that this is inherently a false assumption.
- Even though this is a false assumption, the results will be helpful in our real calculations.

$$R = \sqrt{r^2 + a^2 - 2ar\sin\theta\cos\varphi'} \approx \sqrt{r^2 - 2ar\sin\theta\cos\varphi'} \quad \text{for } r >> a$$

- Using binomial expansion

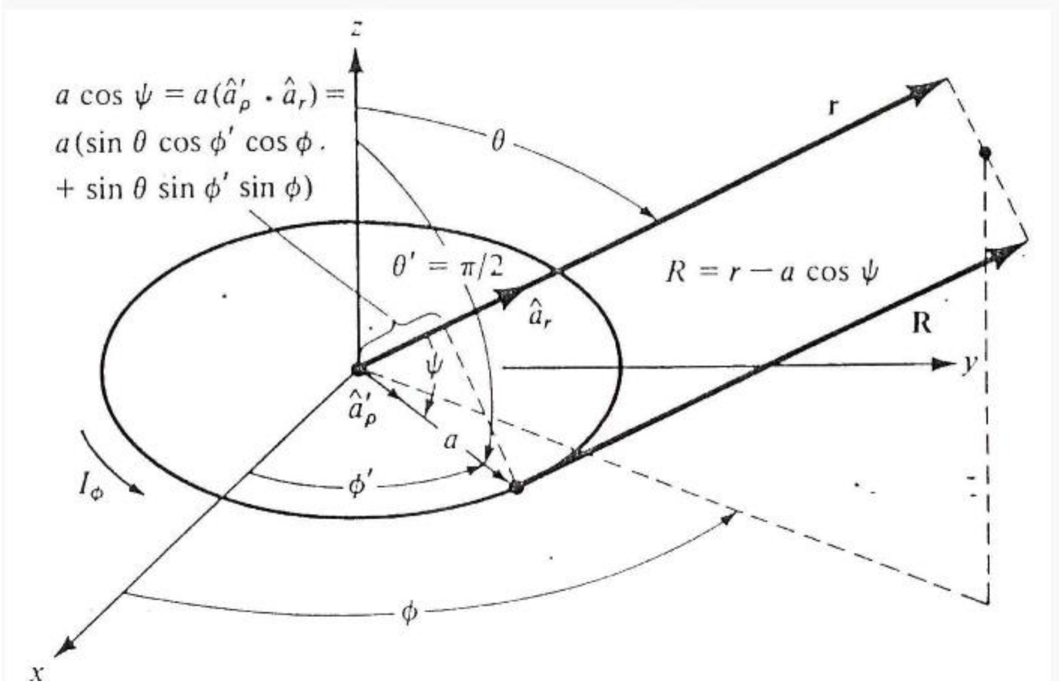
**Phase** $$\boxed{R \approx r\sqrt{1 - 2\frac{a}{r}\sin\theta\cos\varphi'} \approx r - a\sin\theta\cos\varphi' = r - a\cos\psi_o}$$

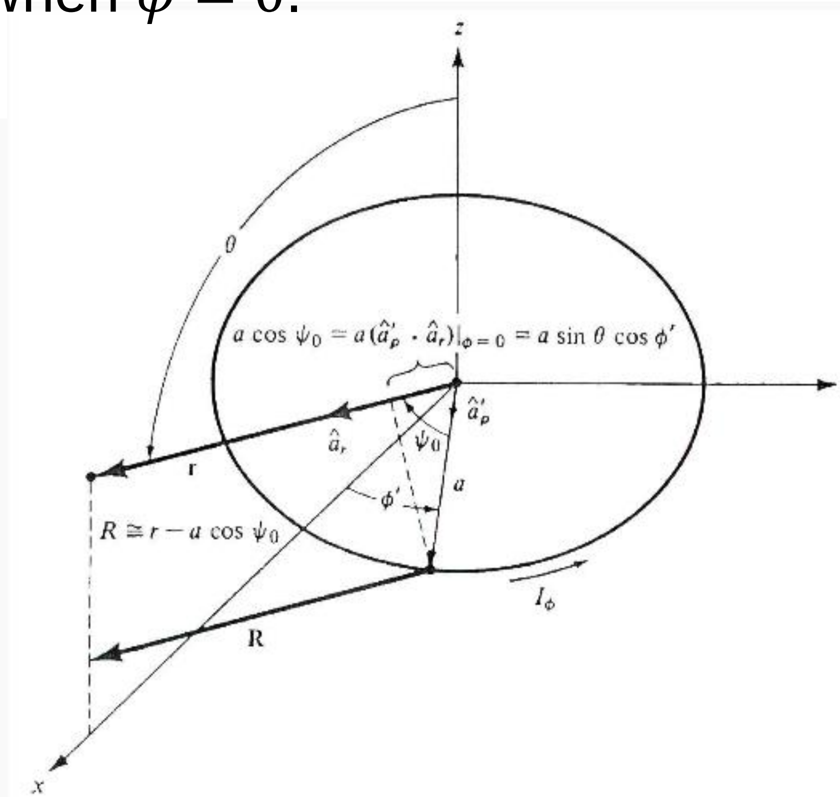
**Amplitude** $$\boxed{R \approx r}$$

POSTECH

# Circular Loops of Constant Current

- Note that angle $\psi$ is the angle between $r$ and $r'$.
- $\psi_0$ is the angle between $r$ and $r'$ when $\varphi = 0$.



# Circular Loops of Constant Current

$$A_\varphi = \frac{a\mu I_0}{4\pi}\int_0^{2\pi}\cos\varphi' \frac{e^{-jk\sqrt{r^2+a^2-2ar\sin\theta\cos\varphi'}}}{\sqrt{r^2+a^2-2ar\sin\theta\cos\varphi'}}d\varphi'$$

$$A_\varphi \approx \frac{a\mu I_0 e^{-jkr}}{4\pi r}\int_0^{2\pi}\cos\varphi' e^{jka\sin\theta\cos\varphi'}d\varphi'$$

$$A_\varphi \approx \frac{a\mu I_0 e^{-jkr}}{4\pi r}\left[\int_0^{\pi}\cos\varphi' e^{jka\sin\theta\cos\varphi'}d\varphi' + \int_\pi^{2\pi}\cos\varphi' e^{+jka\sin\theta\cos\varphi'}d\varphi'\right]$$

$$\varphi' = \varphi'' + \pi$$

$$A_\varphi \approx \frac{a\mu I_0 e^{-jkr}}{4\pi r}\left[\int_0^{\pi}\cos\varphi' e^{jka\sin\theta\cos\varphi'}d\varphi' - \int_0^{\pi}\cos\varphi'' e^{-jka\sin\theta\cos\varphi''}d\varphi''\right]$$

POSTECH

# Circular Loops of Constant Current

**We know** $$\pi j^n J_n(z) = \int_0^{\pi} \cos(n\varphi) e^{jz\cos\varphi} d\varphi$$

$J_n(z)$ is the Bessel function of the first kind of order $n$

$$A_\varphi \approx \frac{a\mu I_0 e^{-jkr}}{4\pi r} \left\{ \pi j J_1(ka\sin\theta) - \pi j J_1(-ka\sin\theta) \right\}$$

$$J_n(z) = \sum_{m=0}^{\infty} \frac{(-1)^m (z/2)^{n+2m}}{m!(n+m)!}$$

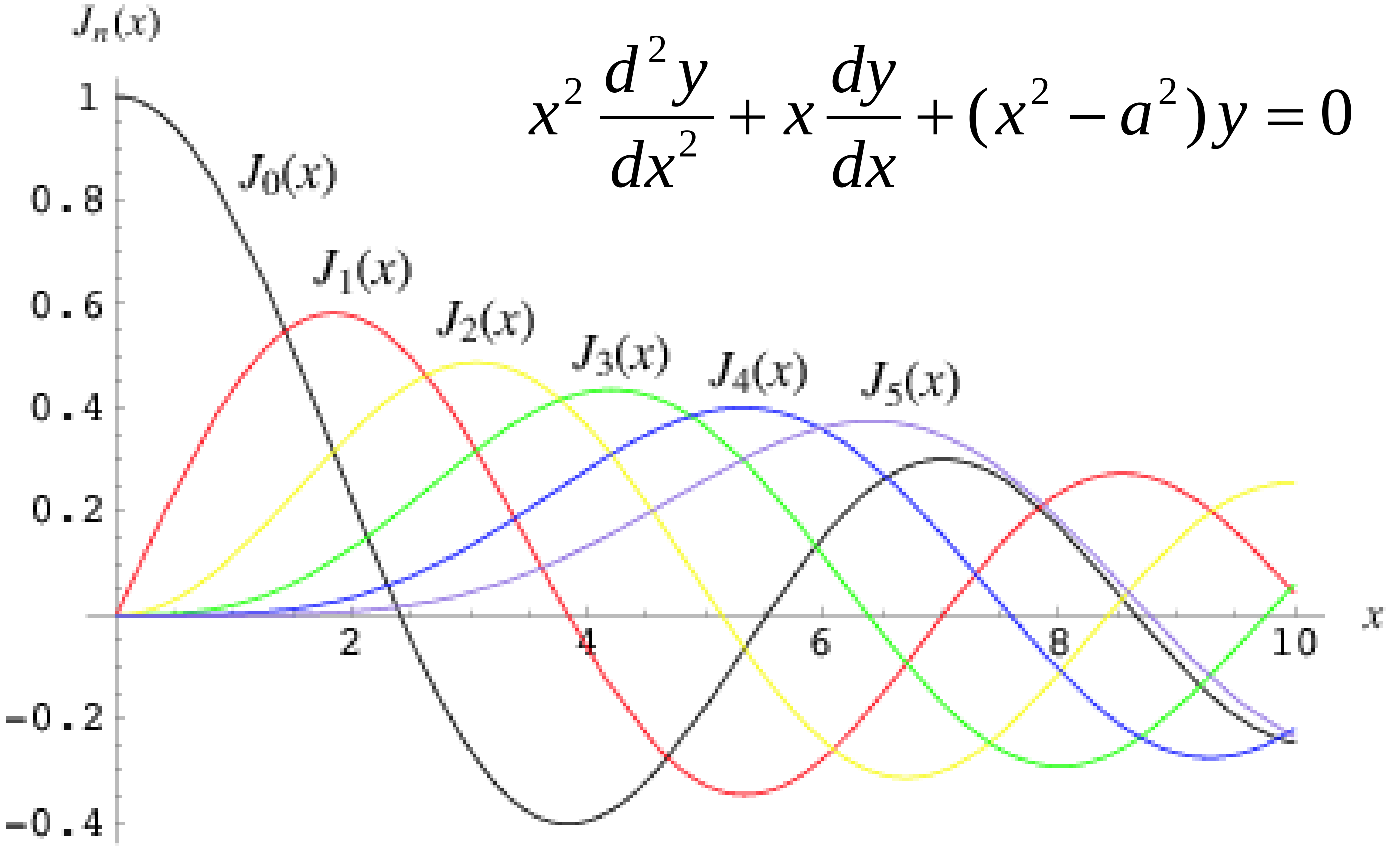
$$J_n(-z) = (-1)^n J_n(z) \quad \Rightarrow \quad A_\varphi \approx j \frac{a\mu I_0 e^{-jkr}}{2r} J_1(ka\sin\theta)$$

POSTECH

# Circular Loops of Constant Current

- Note that the Bessel functions of the first kind $J_a(x)$ are solutions to the following differential equation:

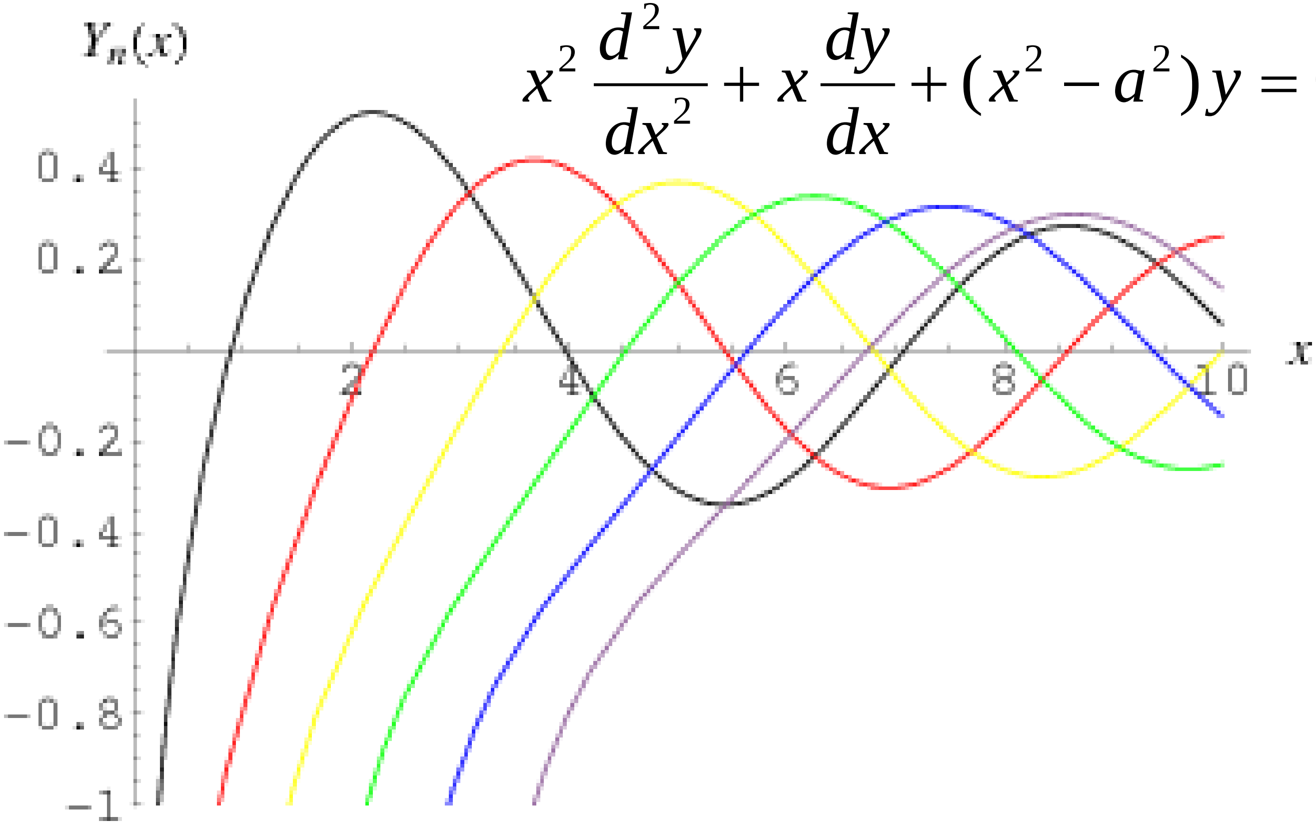
$$x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + (x^2 - a^2) y = 0$$



# Circular Loops of Constant Current

- This equation has another solution called the Bessel function of the second kind or the Neumann function $Y_a(x)$:

$$x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + (x^2 - a^2) y = 0$$



# Circular Loops of Constant Current

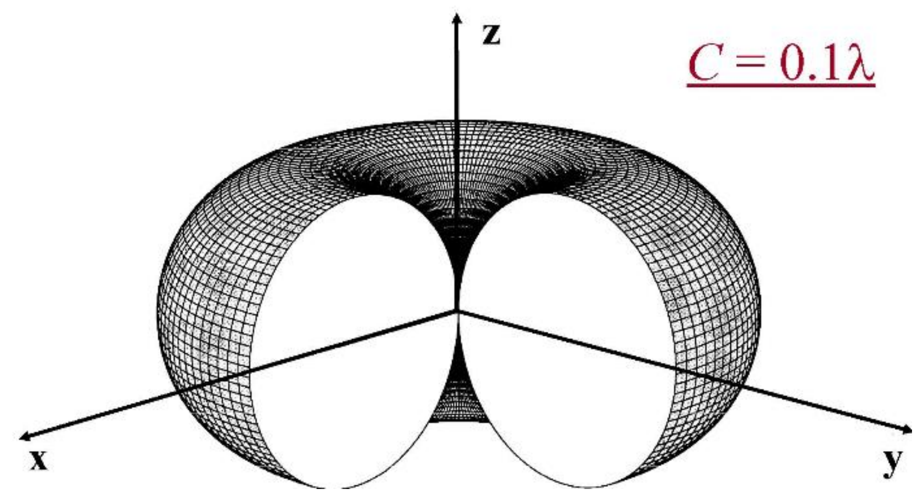
$$E_r \approx E_\theta = 0$$

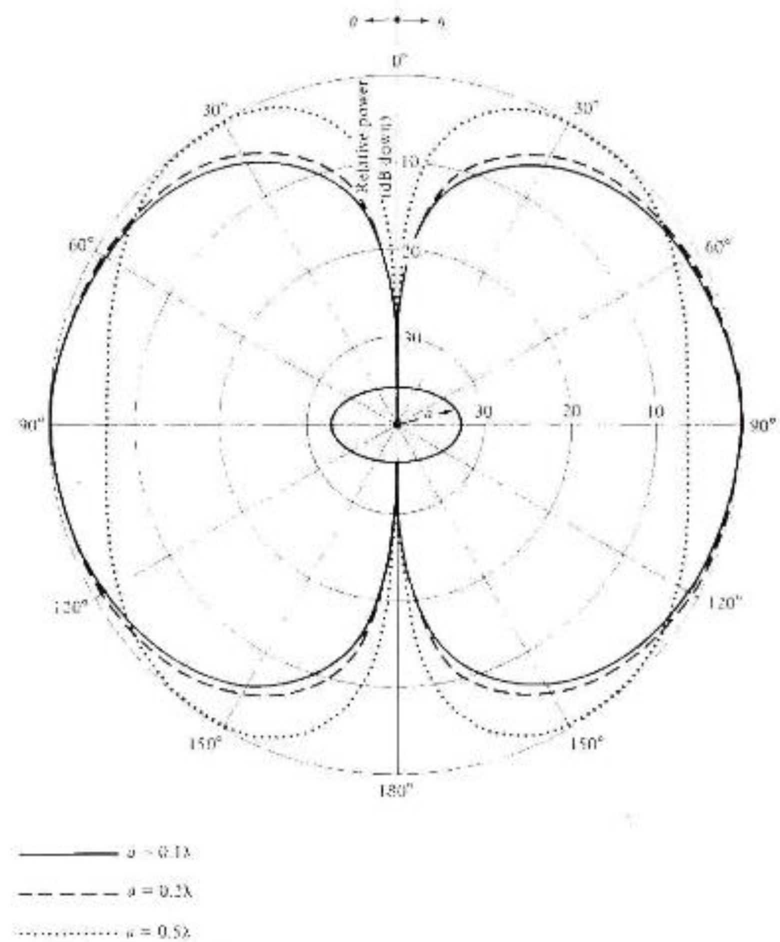
$$E_\varphi \approx \frac{ak\eta I_0 e^{-jkr}}{2r} J_1(ka\sin\theta)$$

$$H_r \approx H_\varphi = 0$$

$$H_\theta = -E_\varphi / \eta = -\frac{akI_0 e^{-jkr}}{2r} J_1(ka\sin\theta)$$

# Circular Loops of Constant Current

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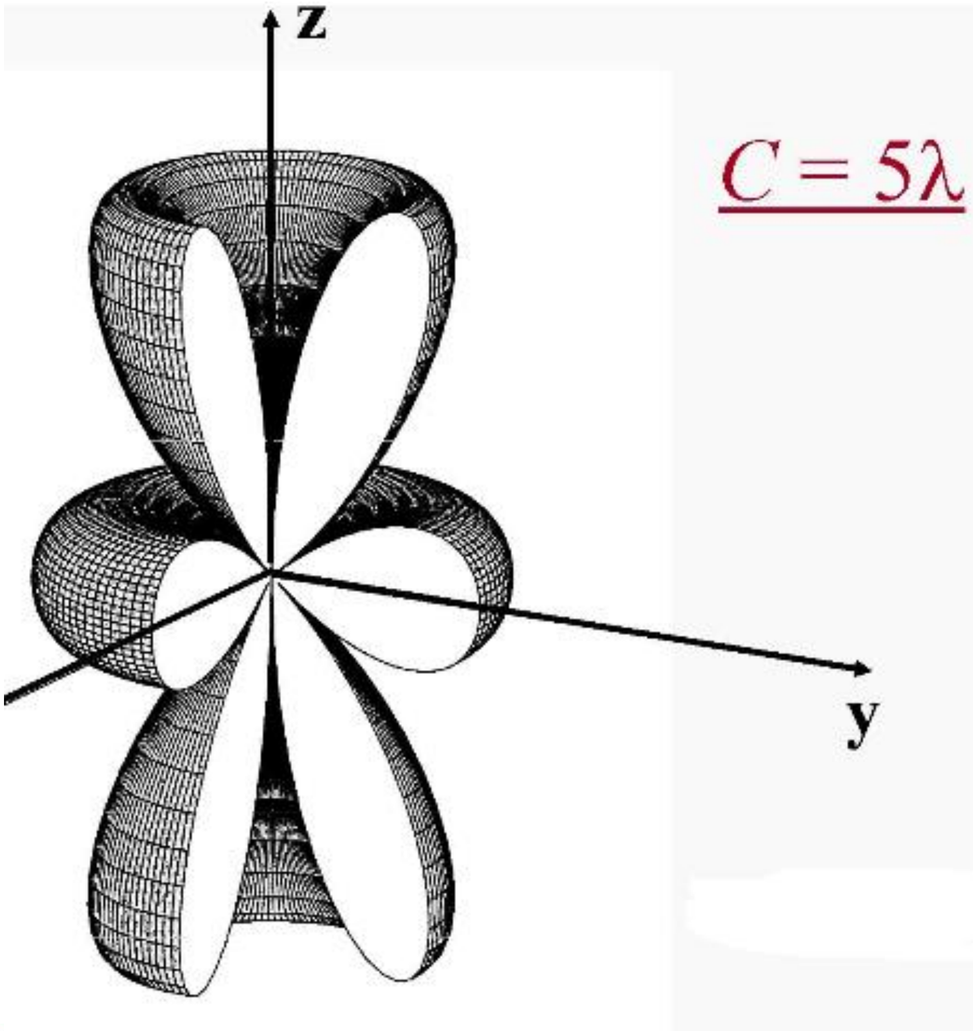
# Circular Loops of Constant Current



# Circular Loops of Constant Current

$$P_{rad} = \oiint_S \vec{W}_{av} . d\vec{s} = \pi (a\omega\mu)^2 |I_0|^2 / 4\eta \int_0^{\pi} J_1^2(ka\sin\theta) \sin\theta \, d\theta$$

$$1/2 \int_0^{\pi} J_1^2(ka\sin\theta) \sin\theta \, d\theta = Q_{11}^{(1)}(ka) = \frac{1}{ka} \sum_{m=0}^{\infty} J_{2m+3}(2ka)$$

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# Large Loop Approximation (a≥λ/2)

$$\int_0^{\pi} J_1^2(ka\sin\theta)\,\sin\theta\,d\theta = \frac{1}{ka}\int_0^{2ka} J_2(x)dx \approx \frac{1}{ka}$$

$$P_{rad} \approx \pi(a\omega\mu)^2|I_0|^2/(4\eta ka)$$

$$U|_{\max} = \frac{(a\omega\mu)^2|I_0|^2}{8\eta} J_1(ka\sin\theta)\Big|_{ka\sin\theta=1.84} = \frac{(a\omega\mu)^2|I_0|^2}{8\eta}(0.582)^2$$

$$\boxed{R_{rad} = \frac{2P_{rad}}{|I_0|^2} = 60\pi^2\left(\frac{C}{\lambda}\right)^2}$$

$$\boxed{D_0 = 0.677\left(\frac{C}{\lambda}\right)}$$

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# Intermediate Loop Approximation ($\lambda/6\pi \le a < \lambda/2$)

$$R_{rad} = \frac{2P_{rad}}{|I_0|^2} = \eta\pi(ka)^2 Q_{11}^{(1)}(ka)$$

$$D_0 = \frac{4\pi U_{\max}}{P_{rad}} = \frac{F_m(ka)}{Q_{11}^{(1)}(ka)}$$

$$F_m(ka) = J_1^2(ka\sin\theta)\Big|_{\max} = \begin{cases} J_1^2(1.84) = 0.339 & ka > 1.84 \\ J_1^2(ka) & ka < 1.84 \end{cases}$$

# Small Loop Approximation ($a<\lambda/6\pi$)

$$J_1(ka\sin\theta) \approx \frac{ka\sin\theta}{2}$$

With this approximation, the experssions will be those we obtained for the electrically small loop.

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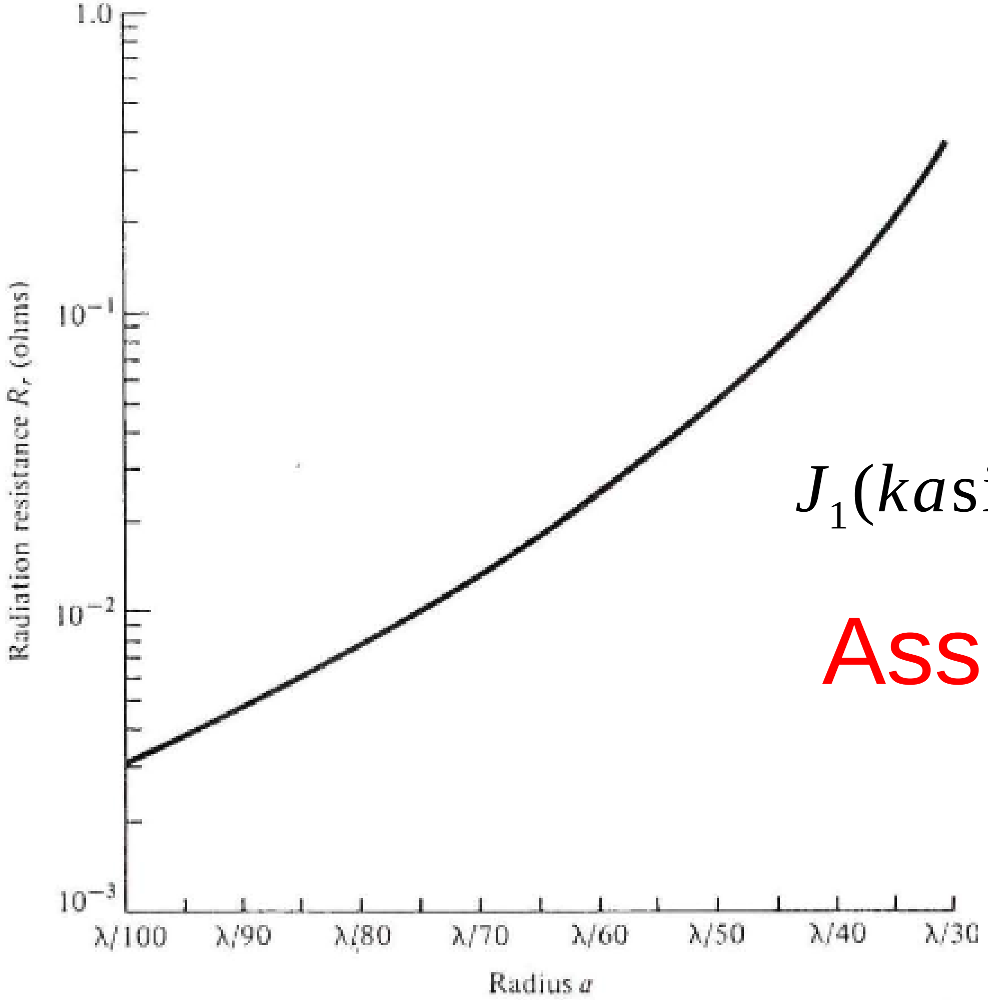
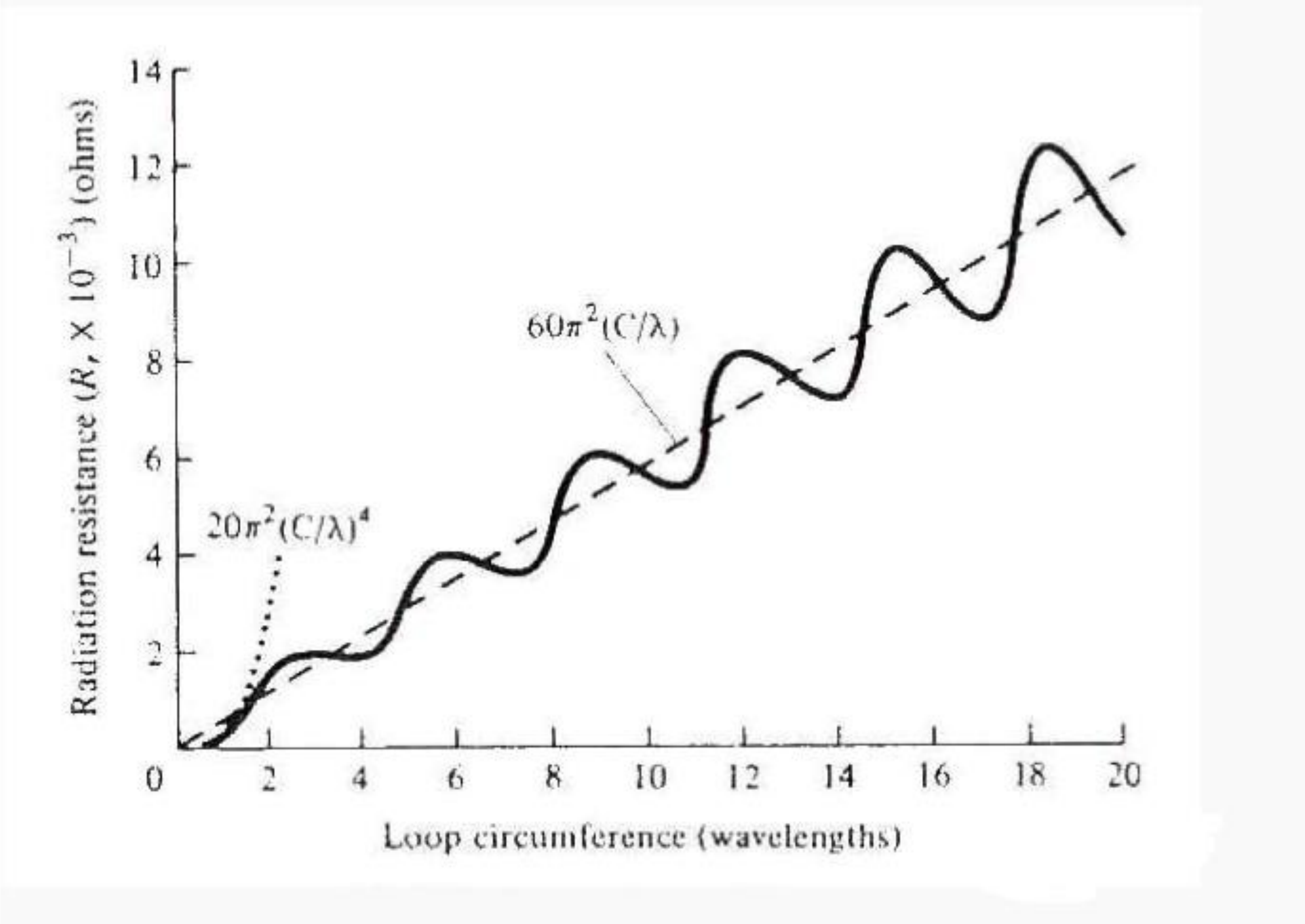
# Radiation Resistance for a Constant Current Loop Antenna




$$J_1(ka\sin\theta) \approx \frac{ka\sin\theta}{2}$$

Assumption

# Radiation Resistance of a Circular Loop with Constant Current

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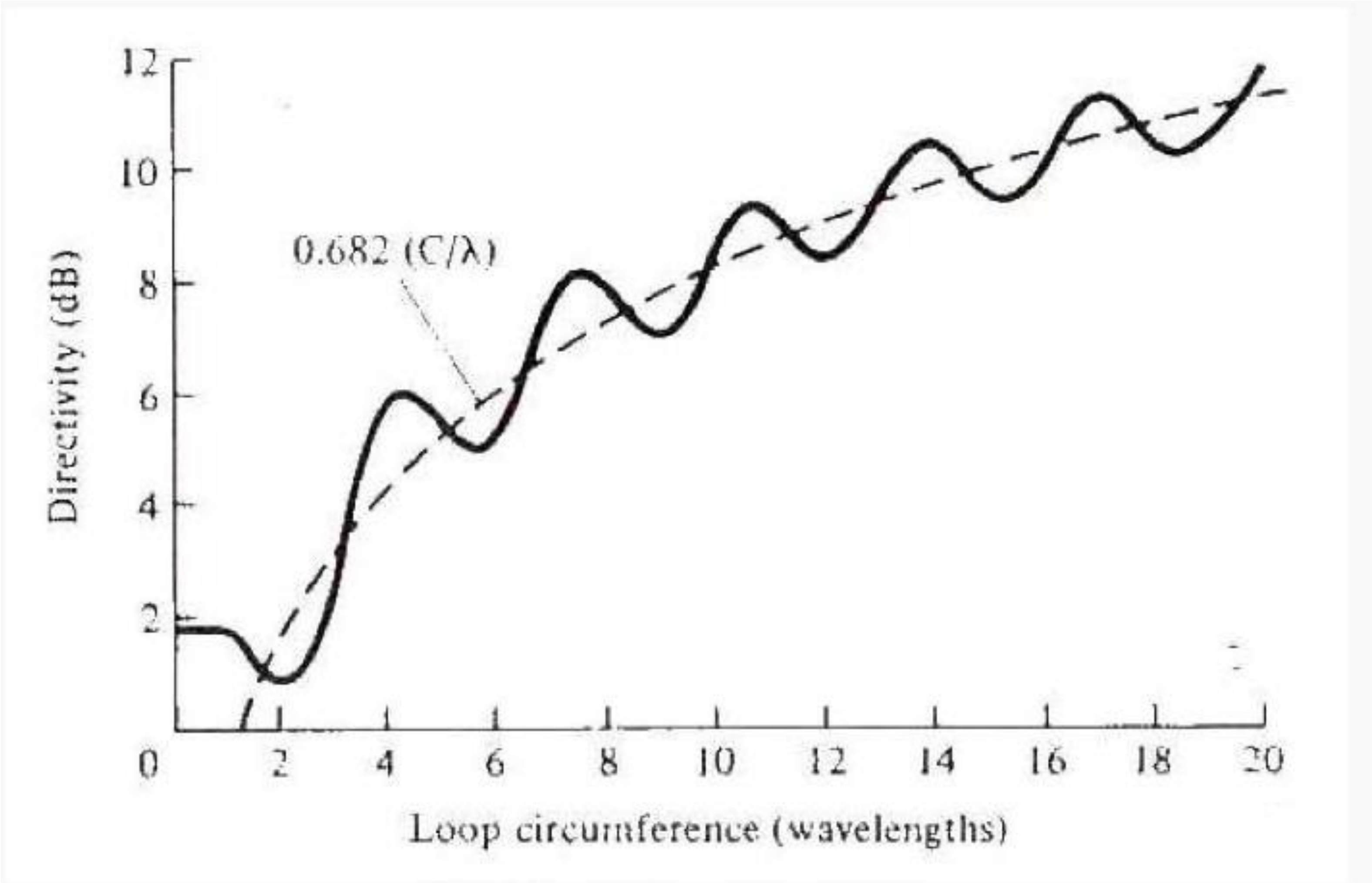
# Directivity of a Circular Loop with Constant Current



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# Circular Loop with Non Uniform Current Distribution

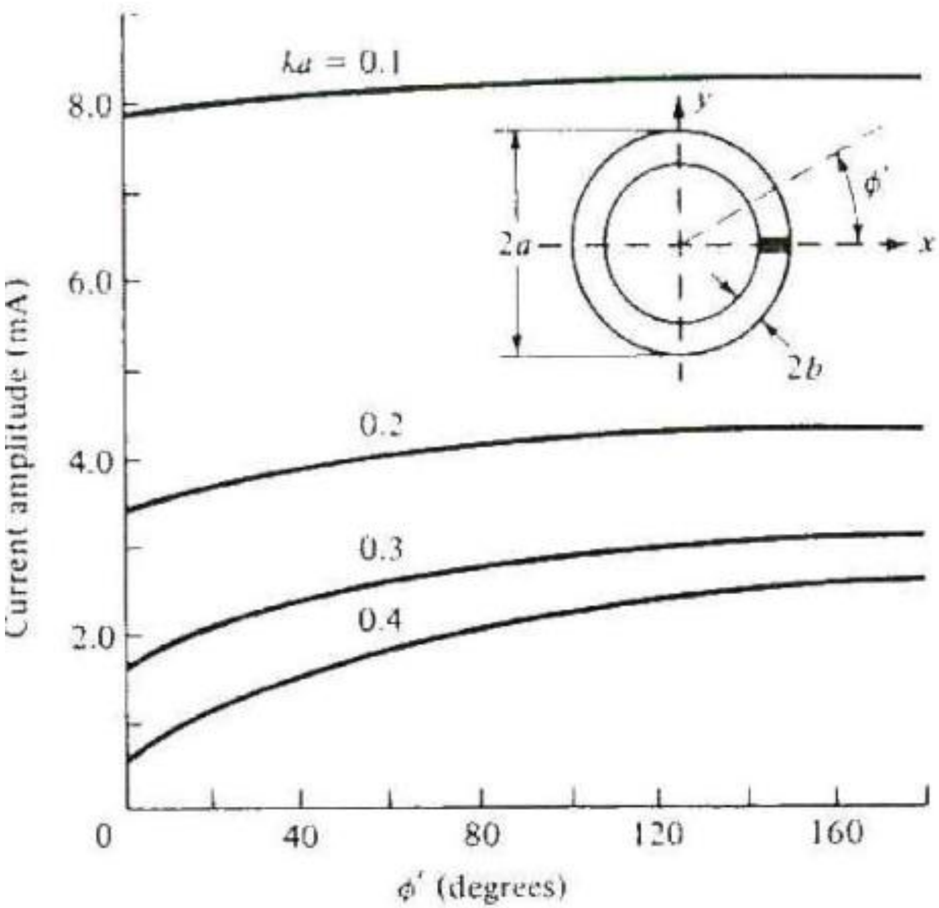
- As the dimensions of the loop increase the current variations along the circumference of the loop must be taken into account.
- Generally, a common assumption for the current distribution is a sinusoidal variation.
- This assumption is good. However, it is not valid close to the feed point.
- A better approximation is representing the current distribution by a Fourier Series.

# Circular Loop with Non Uniform Current Distribution

$$I(\varphi') = I_0 + 2\sum_{n=1}^{M} I_n \cos(n\varphi')$$

- $\phi'$ is measured from the feed point of the loop along the circumference.
- Using this approximation, the problem can be solved analytically.
- This, however, is cumbersome and we do not attempt it here.

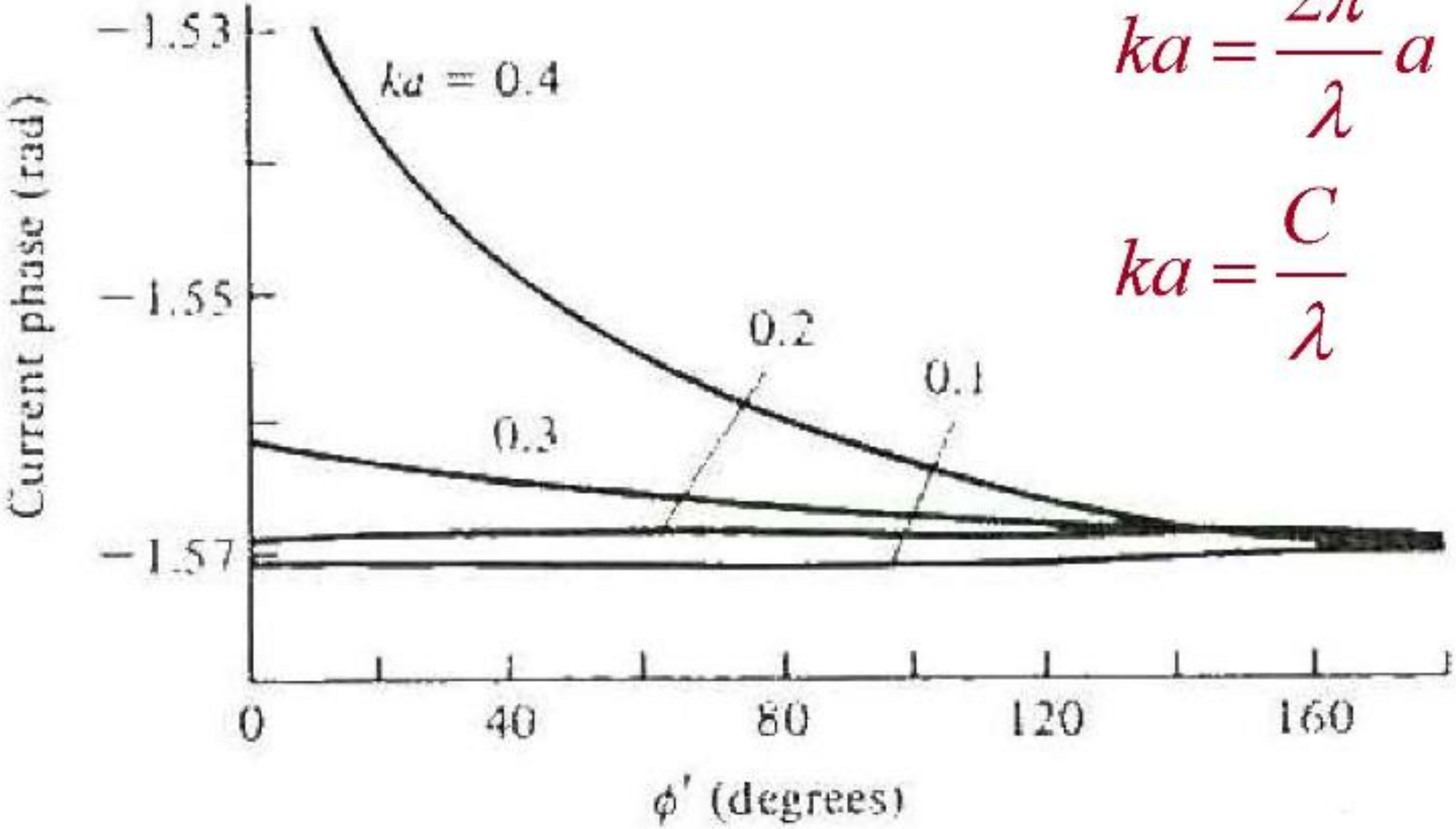
# Magnitude of the Current Distribution as a Function of Location along the Loop




$$ka = \frac{2\pi}{\lambda} a$$

$$ka = \frac{C}{\lambda}$$

# Phase of the Current Distribution as a Function of Location along the Loop



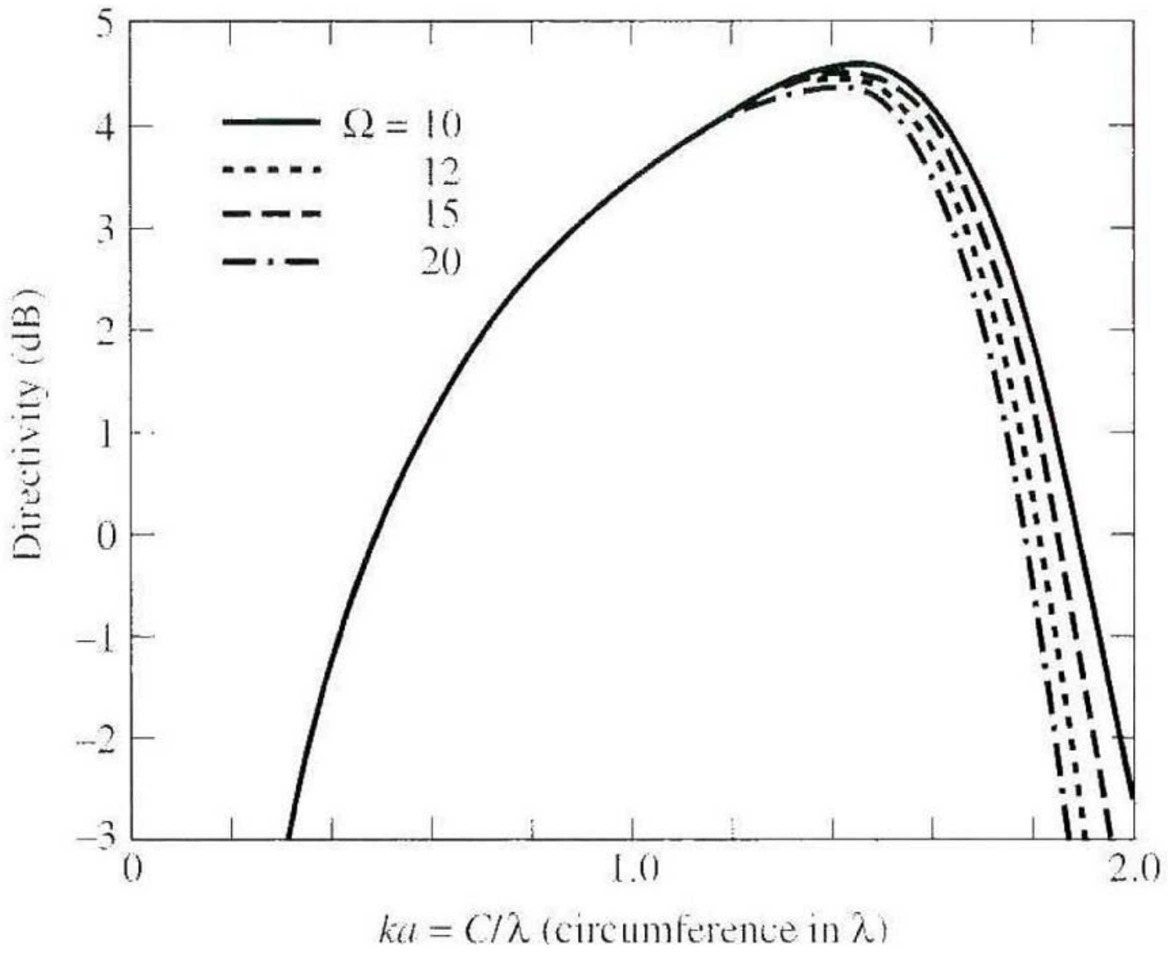

$$ka = \frac{2\pi}{\lambda} a$$

$$ka = \frac{C}{\lambda}$$

# Directivity of the Loop for θ=0°

$$\Omega = 2\ln\left(\frac{2\pi a}{b}\right)$$

- $2a$: diameter of the loop.
- $2b$: diameter of the wire of the loop.

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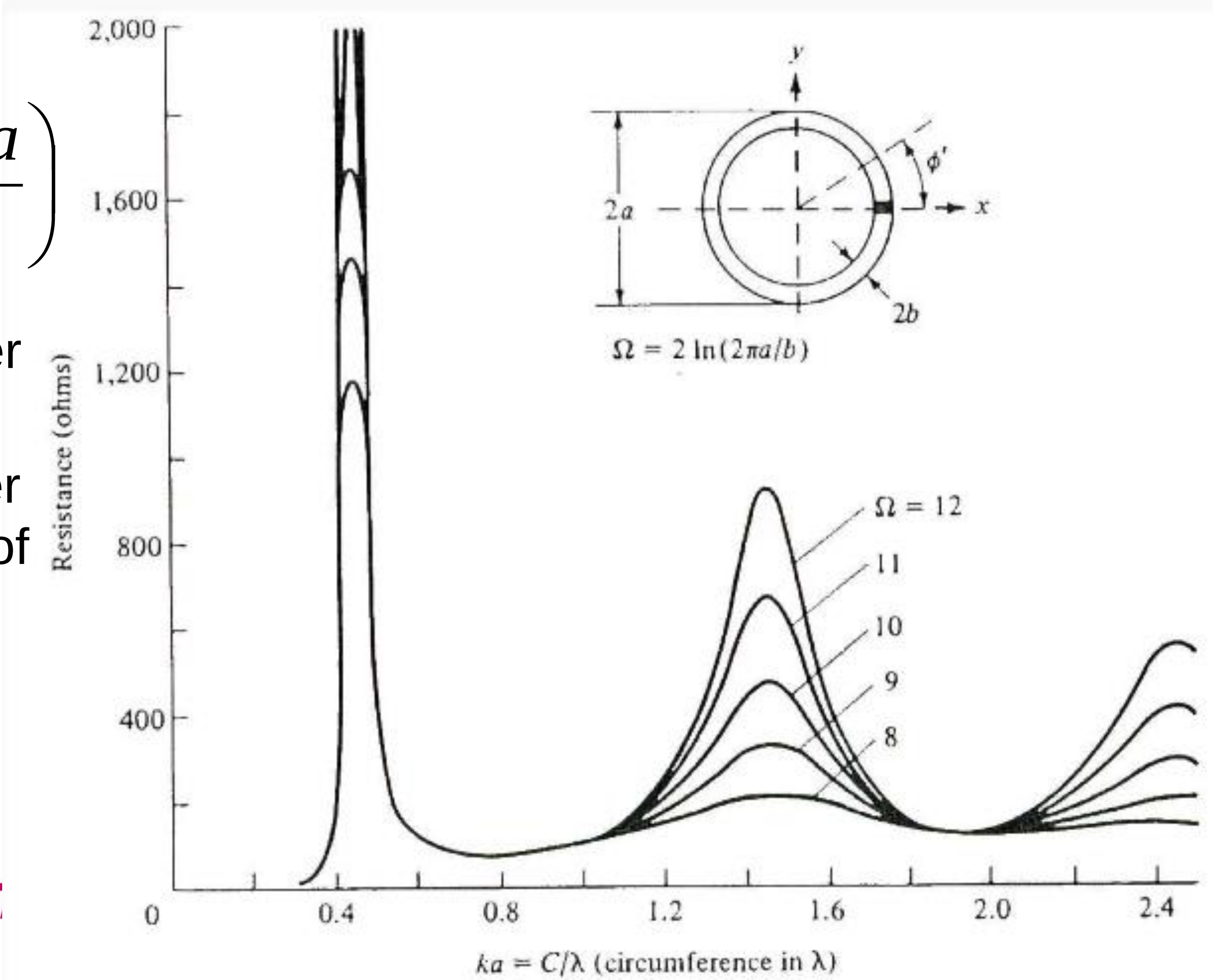
# Input Impedance of a Circular Loop

$$\Omega = 2\ln\left(\frac{2\pi a}{b}\right)$$

- $2a$: diameter of the loop.
- $2b$: diameter of the wire of the loop.

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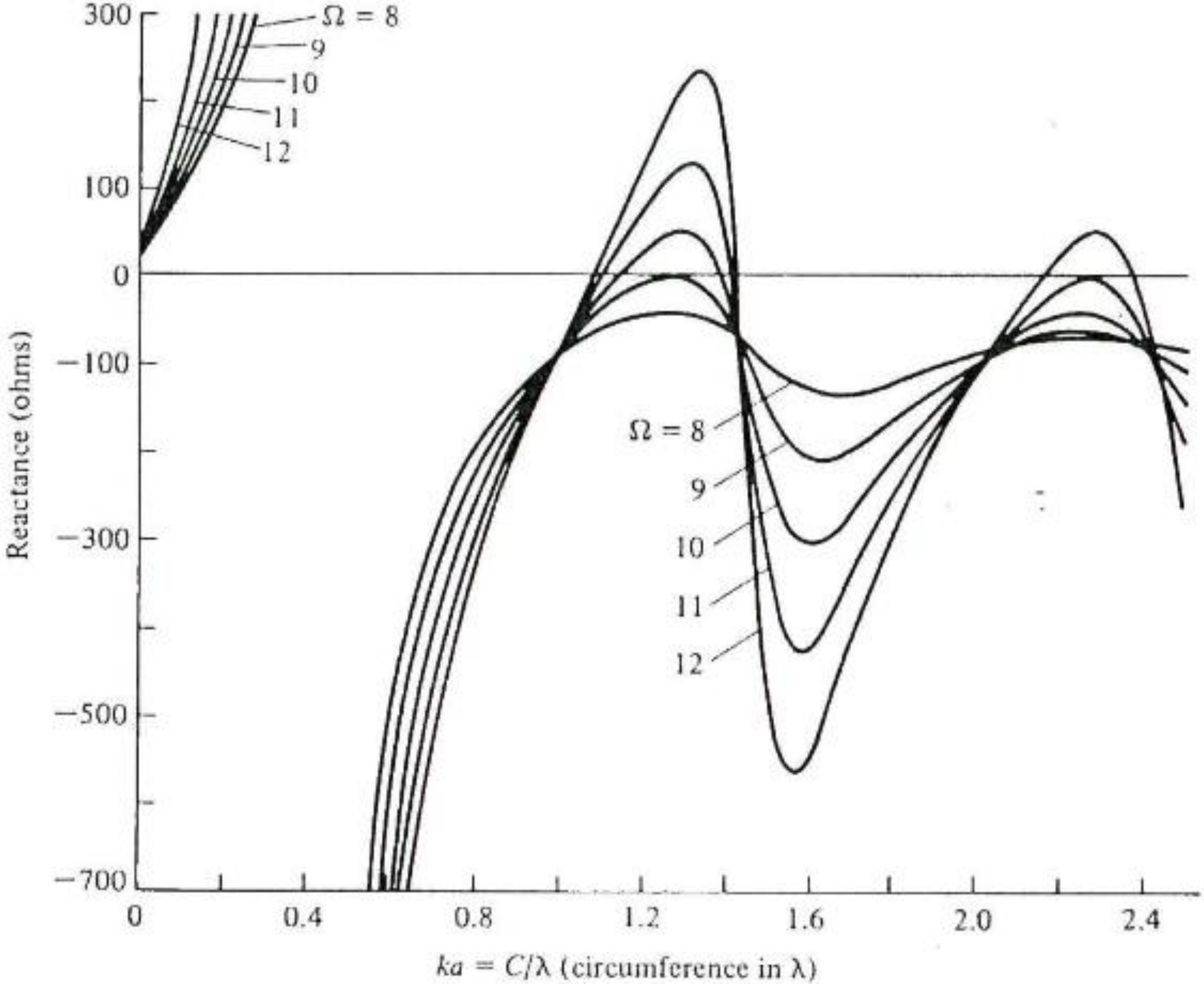
# Input Impedance of a Circular Loop



# Ferrite Loops

- Ferrite materials can be placed in the loop to increase the magnetic flux density of the loop.
- This will increase radiation resistance and hence the efficiency of the antenna.

$$\frac{R_f}{R_r} = \left(\frac{\mu_{ce}}{\mu_0}\right)^2$$

- $R_f$=radiation resistance of ferrite loop.
- $R_r$=radiation resistance of air core loop.
- $\mu_{ce}$=effective permeability of the ferrite core.
- $\mu_0$= free space permeability.

# Ferrite Loops

- For a single turn small ferrite loop:

$$R_f = 20\pi^2\left(\frac{C}{\lambda}\right)^4\left(\frac{\mu_{ce}}{\mu_0}\right)^2 = 20\pi^2\left(\frac{C}{\lambda}\right)^4\mu_{cer}^2$$

$$R_f = 20\pi^2\left(\frac{C}{\lambda}\right)^4\left(\frac{\mu_{ce}}{\mu_0}\right)^2 N^2 = 20\pi^2\left(\frac{C}{\lambda}\right)^4\mu_{cer}^2 N^2$$

- Relative effective permeability of the ferrite core is related to the relative intrinsic permeability of the unbounded ferrite material:

# Ferrite Loops

- D is the demagnetization factor which is a function of the shape of the core.