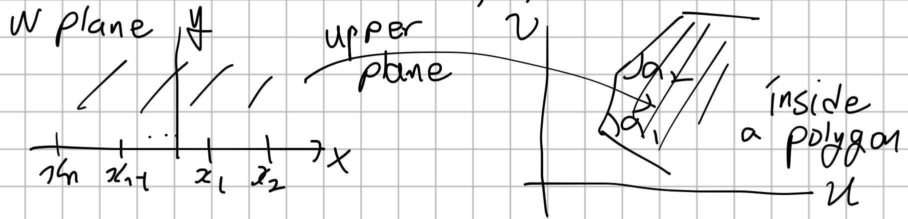
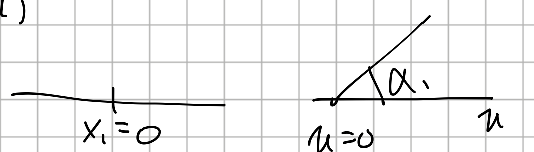
c) Schwartz-Christoffed Transformation

A general solution where the x-axis is beat so as to form a polygon with n angles $\alpha_1, \alpha_2, \cdots$ other mapped into



$$\frac{dw}{dz} = C_1 (z - x_1)^{\frac{\alpha_1}{\pi} - 1} \cdots (z - x_n)^{\frac{\alpha_n}{\pi} - 1} \quad \text{or} \quad w = C_1 \int dz \, (z - x_1)^{\frac{\alpha_1}{\pi} - 1} \cdots (z - x_n)^{\frac{\alpha_n}{\pi} - 1}$$

Ex 1)



$x_1 = 0$, single past line angle α_1

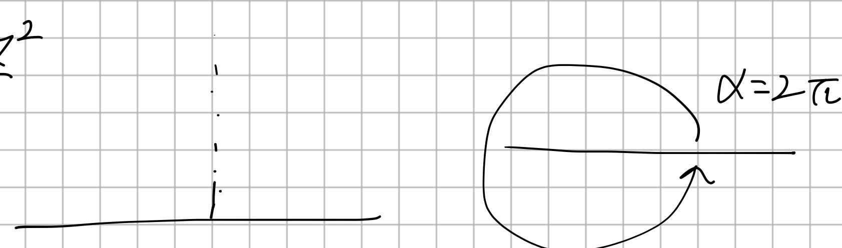
$$W = C_1 \int dz \, z^{\alpha_1/\pi - 1} = C_1 z^{\alpha_1/\pi} + C_2$$

magnification   displacement

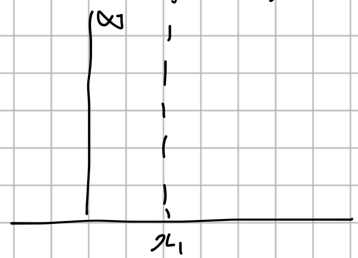
$\alpha < \pi$ angle compression
$\alpha > \pi$ angle expansion

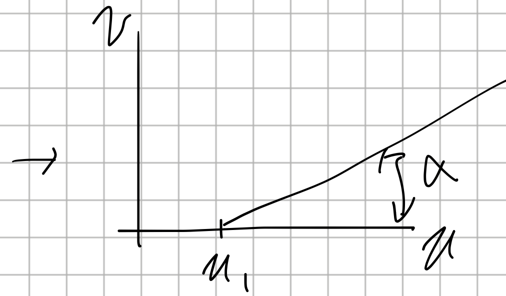
$w = z^{1/2}$

$w = z^2$



$W = (z - x_1)^{\alpha/\pi} + u_1$





$\parallel$ at ∞

Ex 2)

Potential distribution near the corner?

Conductor

another conducting plate at a large y

Conducting plate at $y=0$

equi-potential line :

$\phi(y) = \frac{V}{L} y$

field line

$w = f(z) = z^{1/2}$

$\phi(u,v) = \frac{V}{L}(2uv)$

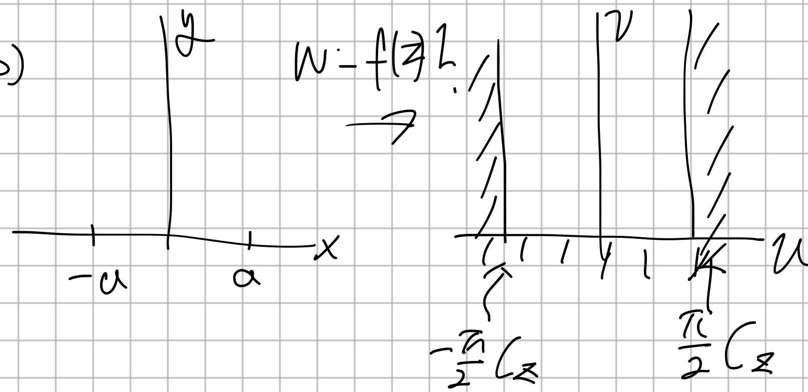
$W^2 = z$

$(u+iv)^2 = x+iy$

$u = \sqrt{\left(x + \sqrt{x^2+y^2}\right)/2}$

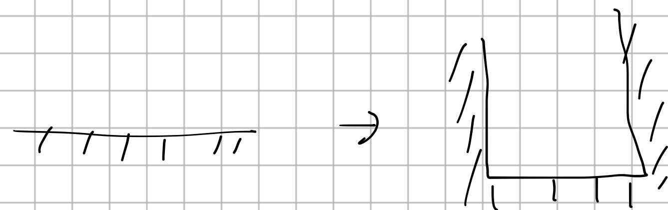
$v = \frac{y}{2u} = \sqrt{u^2 - x}$

Ex3)



$$\frac{dw}{dz} = C_1 (z+a)^{\frac{1}{\pi}\left(\frac{\pi}{2}\right)-1} (z-a)^{\frac{1}{\pi}\left(\frac{\pi}{2}\right)-1}$$

$$= C_1 (z+a)^{-1/2} (z-a)^{-1/2}$$

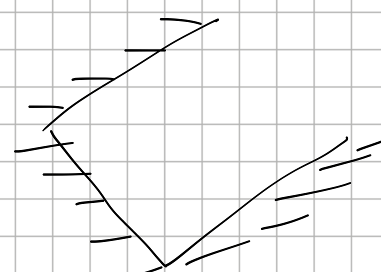


$$W = C_1 \int dz\, (z-a)^{-1/2} (z+a)^{-1/2}$$

$$= C_1 \int dz \frac{1}{\sqrt{z^2-a^2}}, \quad C_1 = i C_z$$

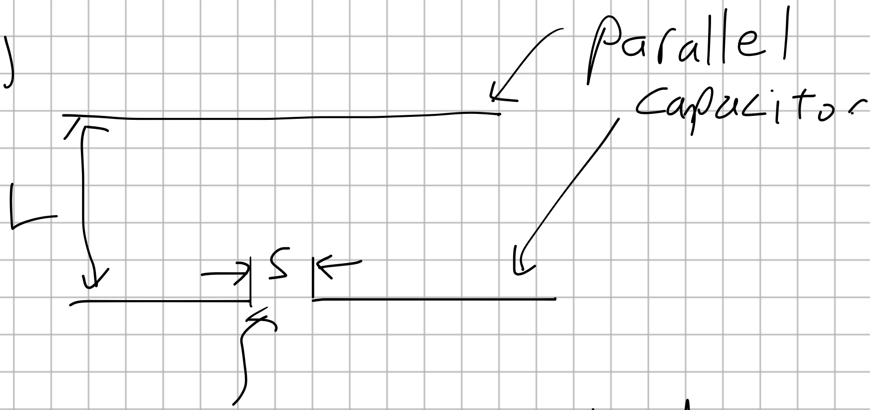
$$= C_z \int dz \frac{1}{\sqrt{a^2-z^2}} = C_z \sin^{-1}(z/a)$$

$$z = a \sin\left(\frac{w}{C_z}\right)$$



C_z has to be determind to satisfy geometry

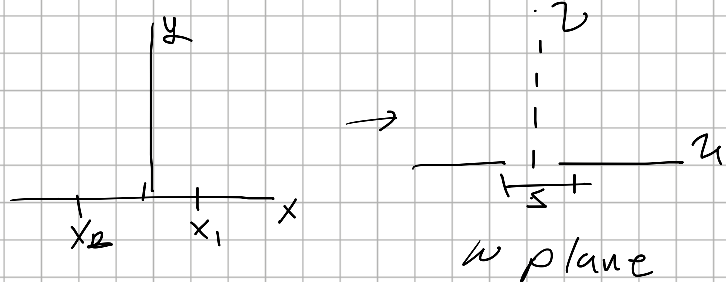
Ex 4)



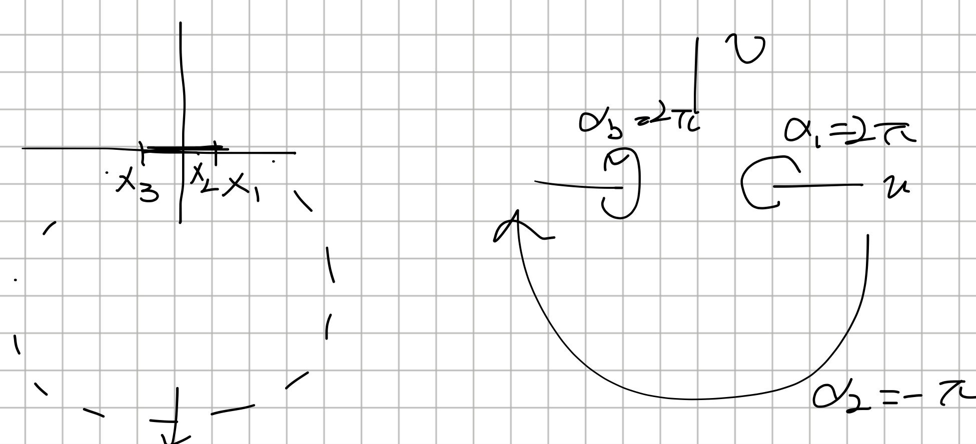
Slot with a width of S

$S \ll L$

Potential near the slot?



The plane between x1&x2 is stretched downward until x2 is at v=-
& x-axis folds back on it self, leaving the gap



$x_1 \to u_1,\ x_3 \to u_2 \qquad x_2 \to v = -\infty$

$$w = C_1 \int (z - x_1)^{\frac{2\pi}{\pi}-1}\, z^{\frac{-\pi}{\pi}-1}\, (z + x_1)^{\frac{2\pi}{\pi}-1}\, dz$$

$\alpha_1 = 2\pi,\ \alpha_2 = -\pi,\ \alpha_3 = 2\pi$

$x_3 = -x_1$

$$\Rightarrow w = w_1 \int dz\, \frac{z^2 - x_1}{z^2} = C_1\left(z + \frac{x_1^2}{z}\right) + C_2$$

For $z = \pm x_1 + i0$, $w = \pm S/2 + i0$

$$\pm S/2 = C_1(\pm x_1 \pm x_1) + C_2$$

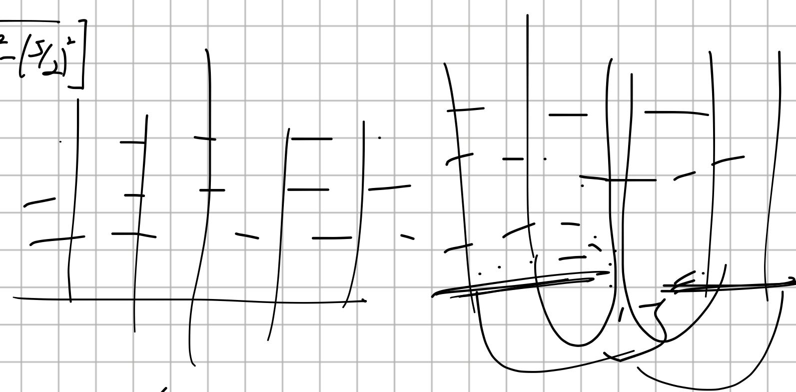
$$C_2 = 0,\quad C_1 = S/4x_1$$

$$w = S/4\left(\frac{z}{x_1} + \frac{x_1}{z}\right) = z + \frac{(S/4)^2}{z}, \quad \text{if } C_1 = 1,\ x_1 = S/4$$

$$z = \frac{1}{2}\left[w + \sqrt{w^2 - (S/2)^2}\right]$$

cf. check

z	w
$+\infty$	$+\infty$
x_1	$S/4 \cdot 2$
$x_1/2$	$S/4(2 + \frac{1}{2})$
$+0$	$+\infty$
-0	$-\infty$
$-\frac{1}{2}x_1$	$S/4(-2 - \frac{1}{2})$
$-x_1$	$S/4(-2)$
$-\infty$	$-\infty$



$$\phi(y) = E_0 y$$

$$\phi(u, v) = E_0\, \mathrm{Im}(z)$$

$$= E_0\, \mathrm{Im}\left[\frac{1}{2} w + \sqrt{w^2 - \left(\frac{S}{2}\right)^2}\right]$$

$$= E_0/2\left[v + \frac{(uv)^2}{-(uv)^2}\left(u^2 - v^2\left(\frac{S}{2}\right)^2\right)\right]$$