Advanced Optics (PHYS690)

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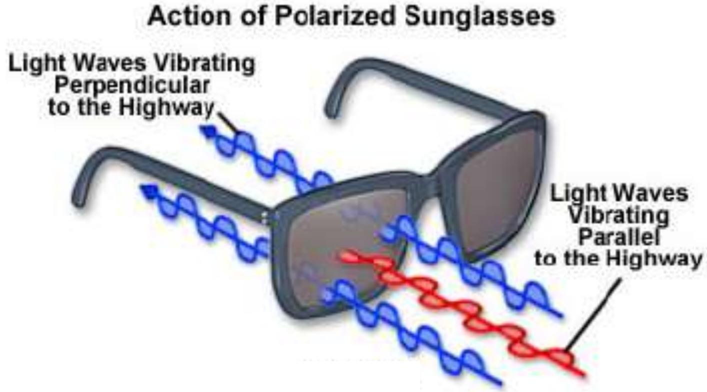
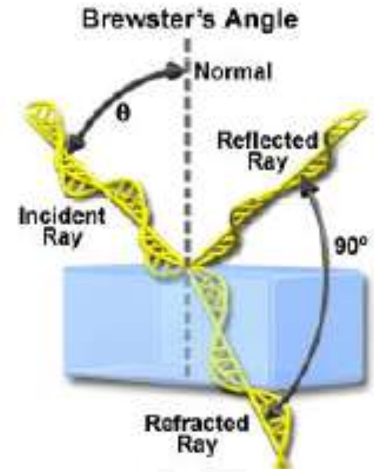
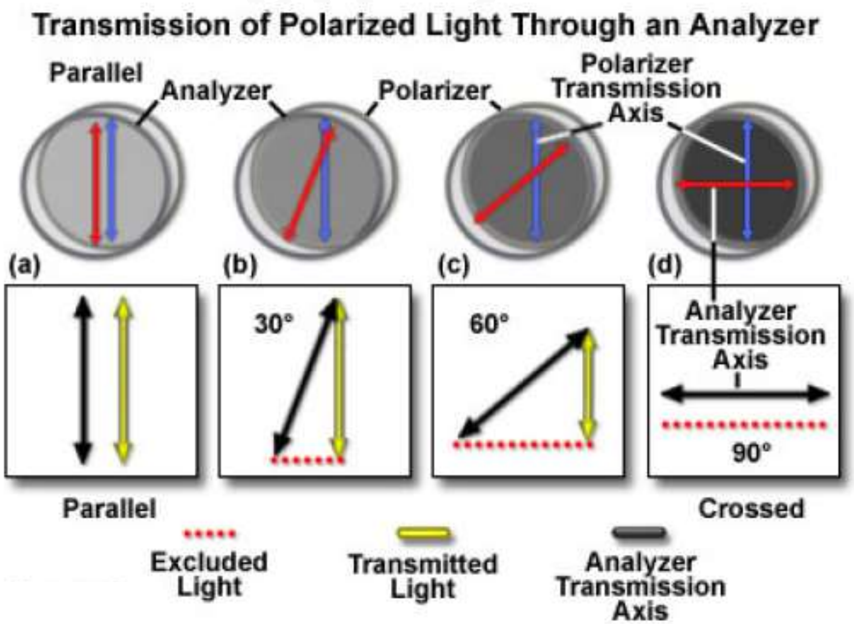
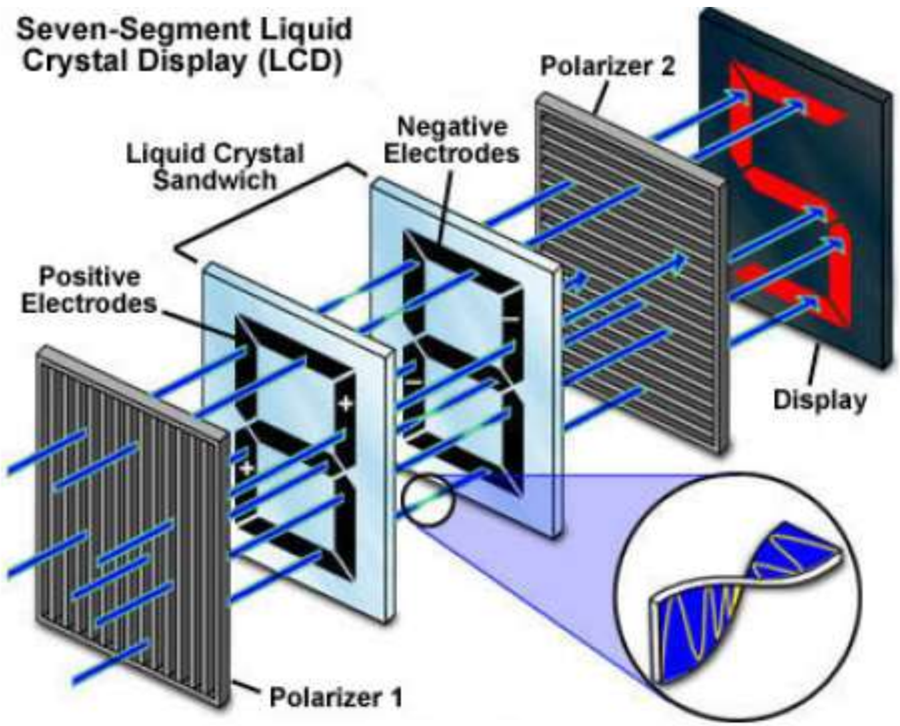
Polarization optics

Introduction








Seven-Segment Liquid
Crystal Display (LCD)
Polarizer 2
Negative
Electrodes
Liquid Crystal
Sandwich
Positive
Electrodes
Display
Polarizer 1

Polarization Optics

- Polarization of a wave is determined, by convention, by the E-field vec tor.

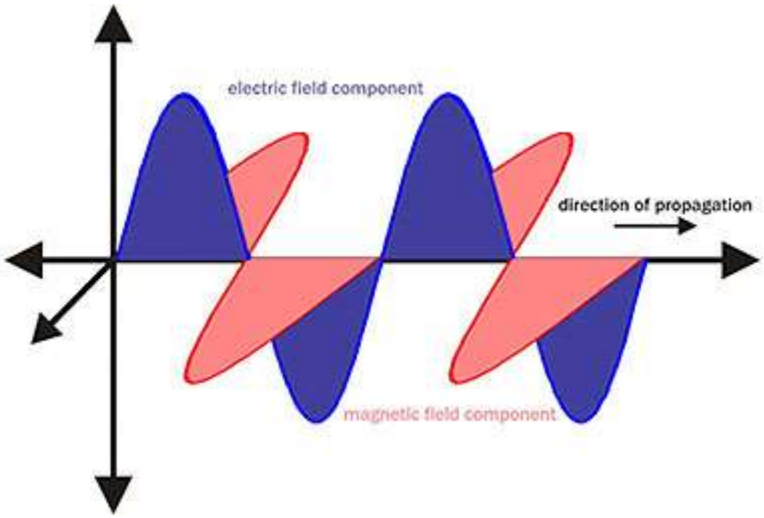
- Important role of polarization

– The amount of light reflected at the boun dary between two media depends on the p olarization of the incident wave.

– The amount of light absorbed by certain media is polarization dependent.

– Light scattering from matter is generally p olarization dependent.

– The refractive index of anisotropic mediu m depends on the polarization: different po larization travel a different velocity, this pro perty is used in MANY optical devices.



Polarization of light 1

- Consider a monochromatic wave

$$\vec{E} = \vec{A}\exp\left[i\omega\left(t - \frac{z}{c}\right)\right]$$

- Where the complex envelope is

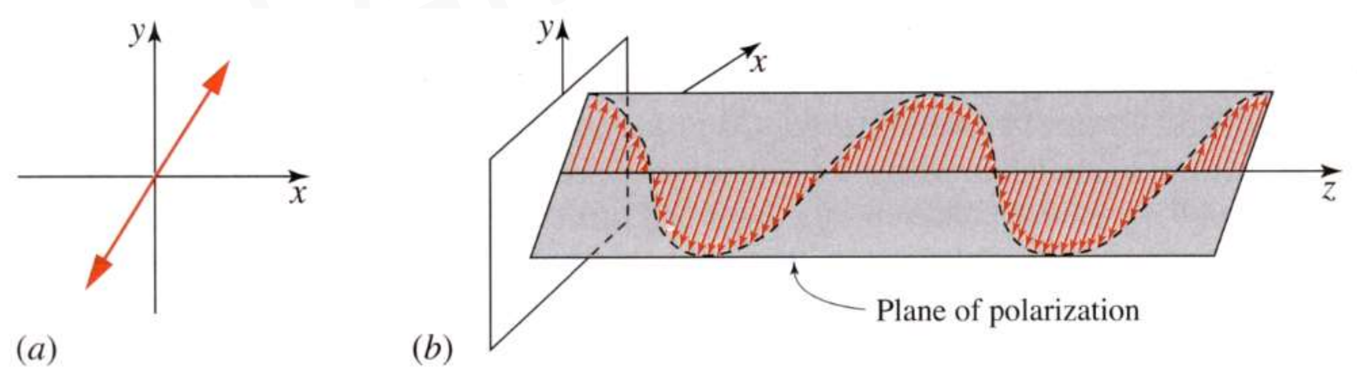
$$\vec{A} = A_x\hat{x} + A_y\hat{y}$$

- Note $a_{x,y}$ are complex numbers and take

$$Ax, y = a_{x,y}\exp(i\phi_{x,y})$$

- Then

$$\begin{aligned} E_x &= a_x\exp\left[i\omega\left(t - \frac{z}{c}\right) + \phi_x\right] \\ E_y &= a_y\exp\left[i\omega\left(t - \frac{z}{c}\right) + \phi_y\right] \end{aligned}$$



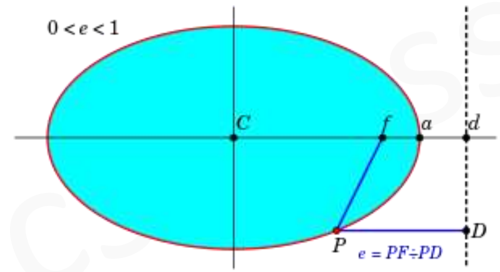
Polarization Ellipse

- Consider the real E-field

$$
\begin{aligned}
E_x &= a_x \cos\left[\omega\left(t - \frac{z}{c}\right) + \phi_x\right] \\
E_y &= a_y \cos\left[\omega\left(t - \frac{z}{c}\right) + \phi_y\right]
\end{aligned}
$$

- Parametric equation of an ellepse

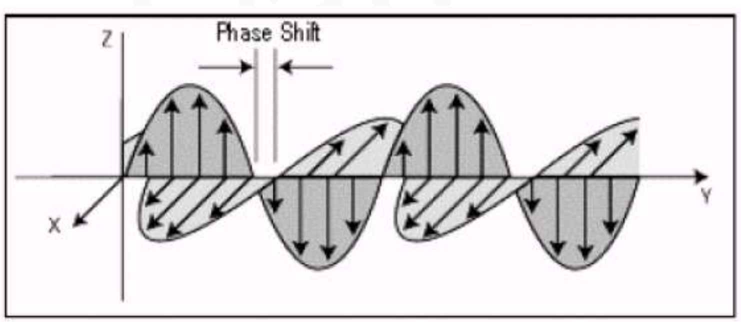
$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$$



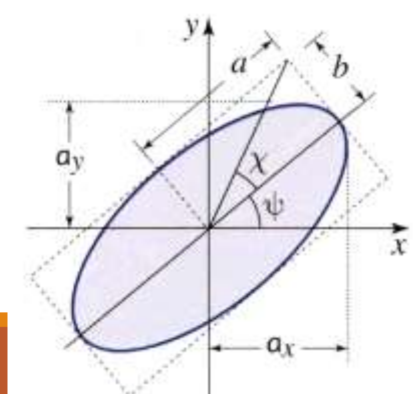
- More general case,

$$\frac{E_x^2}{a_x^2} + \frac{E_y^2}{a_y^2} - 2\cos\phi\frac{E_x E_y}{a_x a_y} = \sin^2\phi$$

where $\phi \equiv \phi_y - \phi_x$

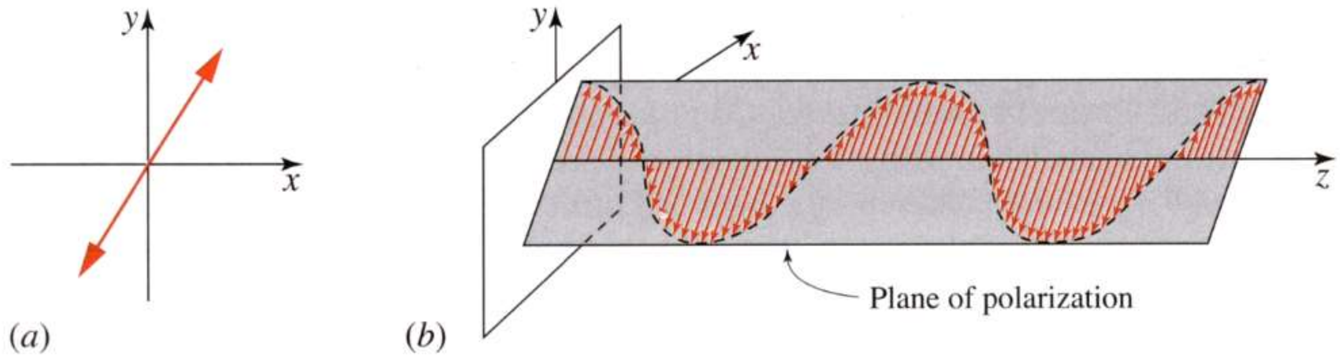


D. Elliptically Polarized Light

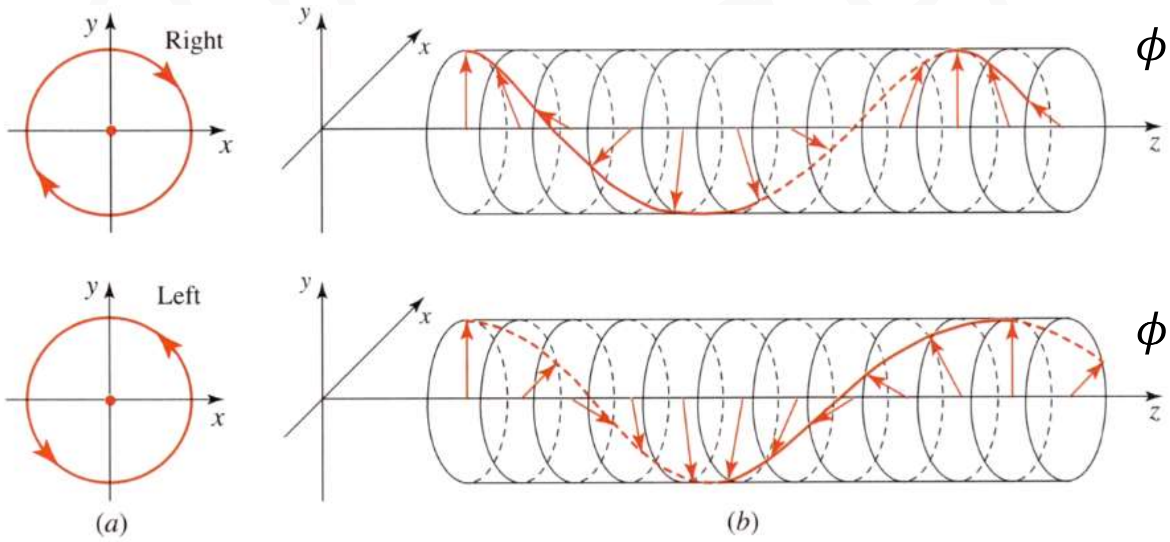


Linear and circular polarization

- Linearly polarized light (a_x or $a_y = 0, \phi = 0$ or π)



- Circularly polarized light ($a_x = a_y, \phi = \pm\pi/2$)



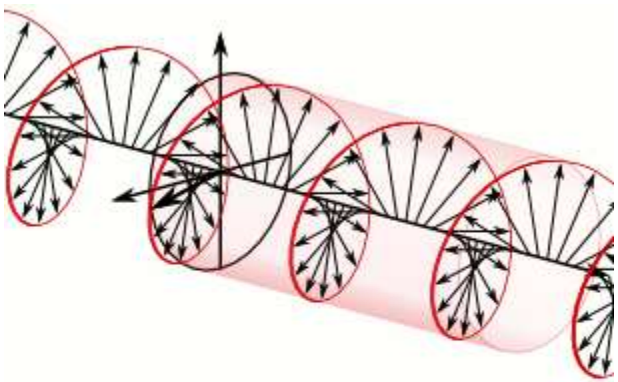
$\phi = \pi/2$

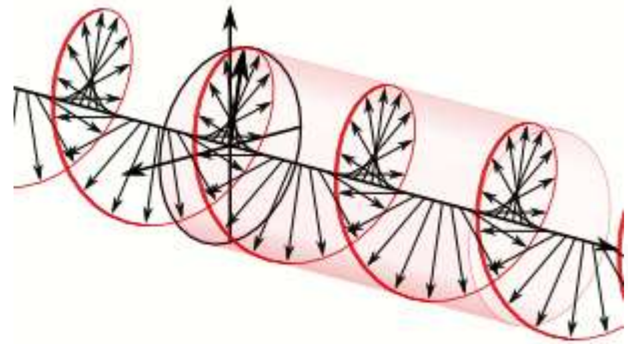
E_y is advanced by $\lambda/4$.

$\phi = -\pi/2$

E_y is delayed by $\lambda/4$.

Left/right handedness conventions





A right-handed/clockwise circularly p
olarized wave as defined from the p
oint of view of the source.
It would be considered left-handed/
anti-clockwise circularly polarized if
defined from the point of view of th
e receiver.

A left-handed/anti-clockwise circular
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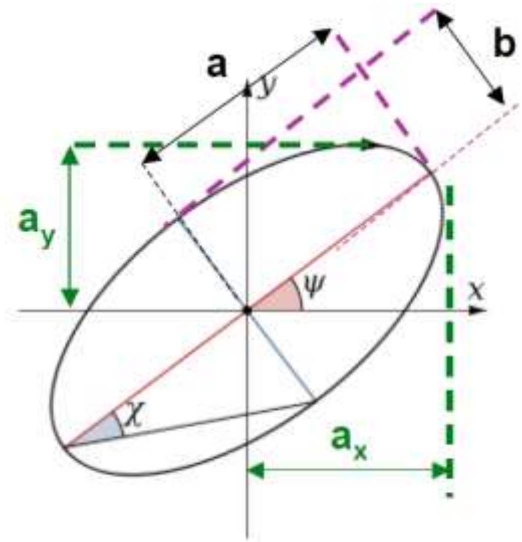
It would be considered right-handed
/clockwise circularly polarized if defi
ned from the point of view of the rec
eiver.

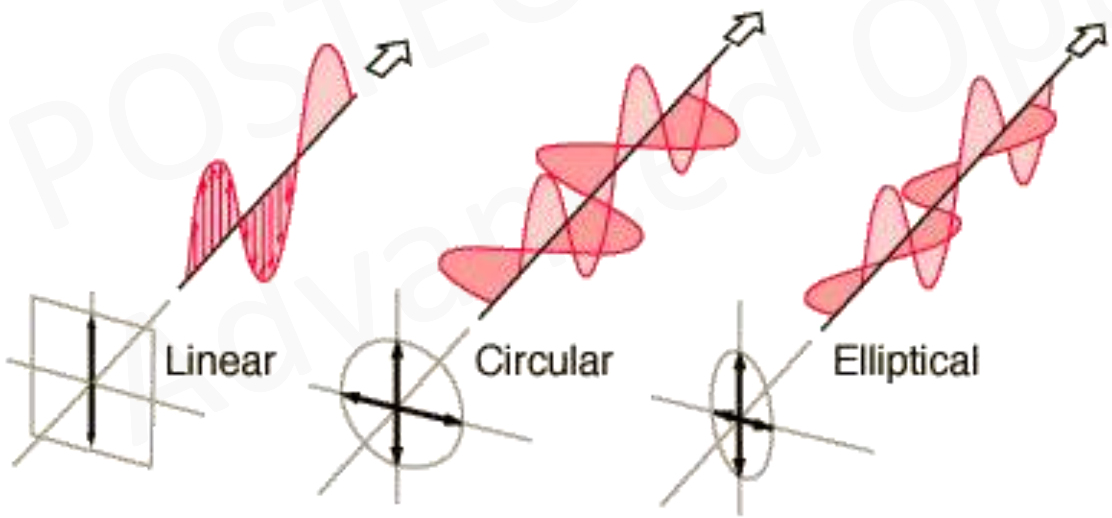
General form - Ellipse

- Polarization ellipse is defined by

$$\tan(2\psi) = \frac{2r}{1-r^2}\cos\phi$$
$$\sin(2\chi) = \frac{2r}{1+r^2}\sin\phi$$
$$r \equiv \frac{a_y}{a_x}$$

$$\phi \equiv \phi_y - \phi_x$$





General form - Ellipse

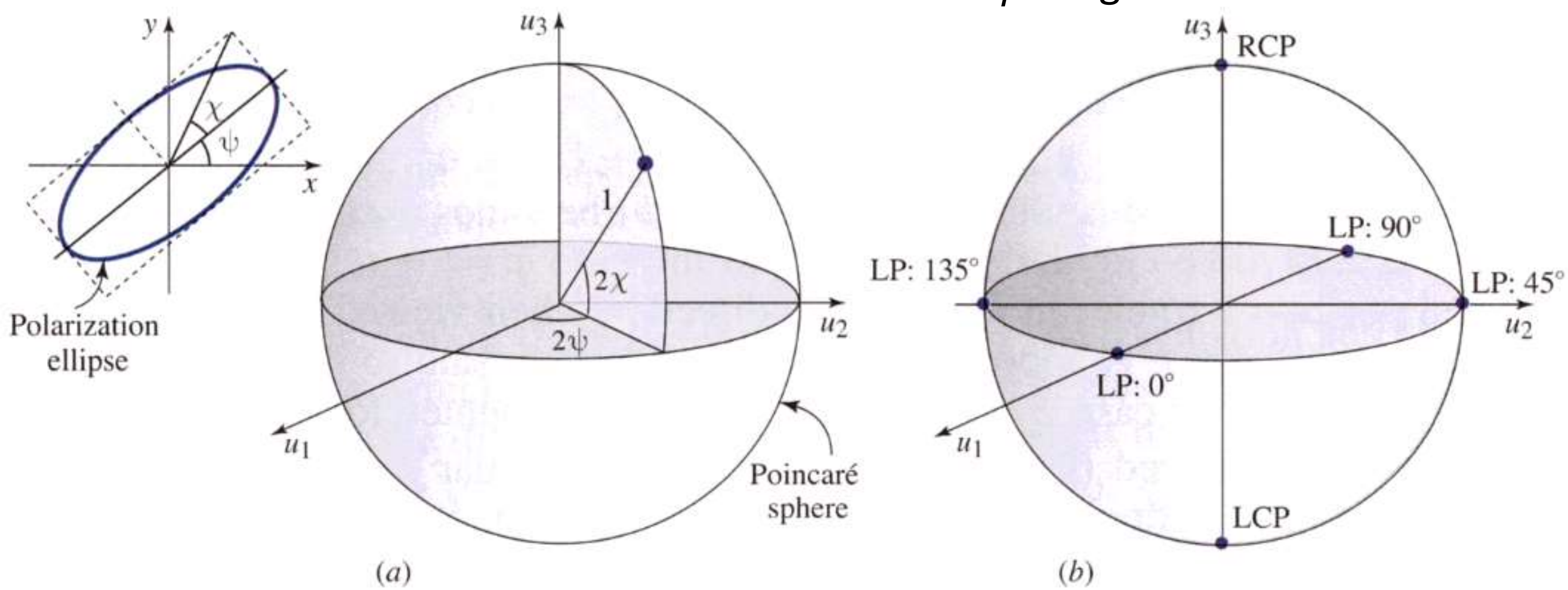
- The state of polarization of a light wave can be decribed by two real par ameters: $R = a_x/a_y$, and $\phi = \phi_y - \phi_x$

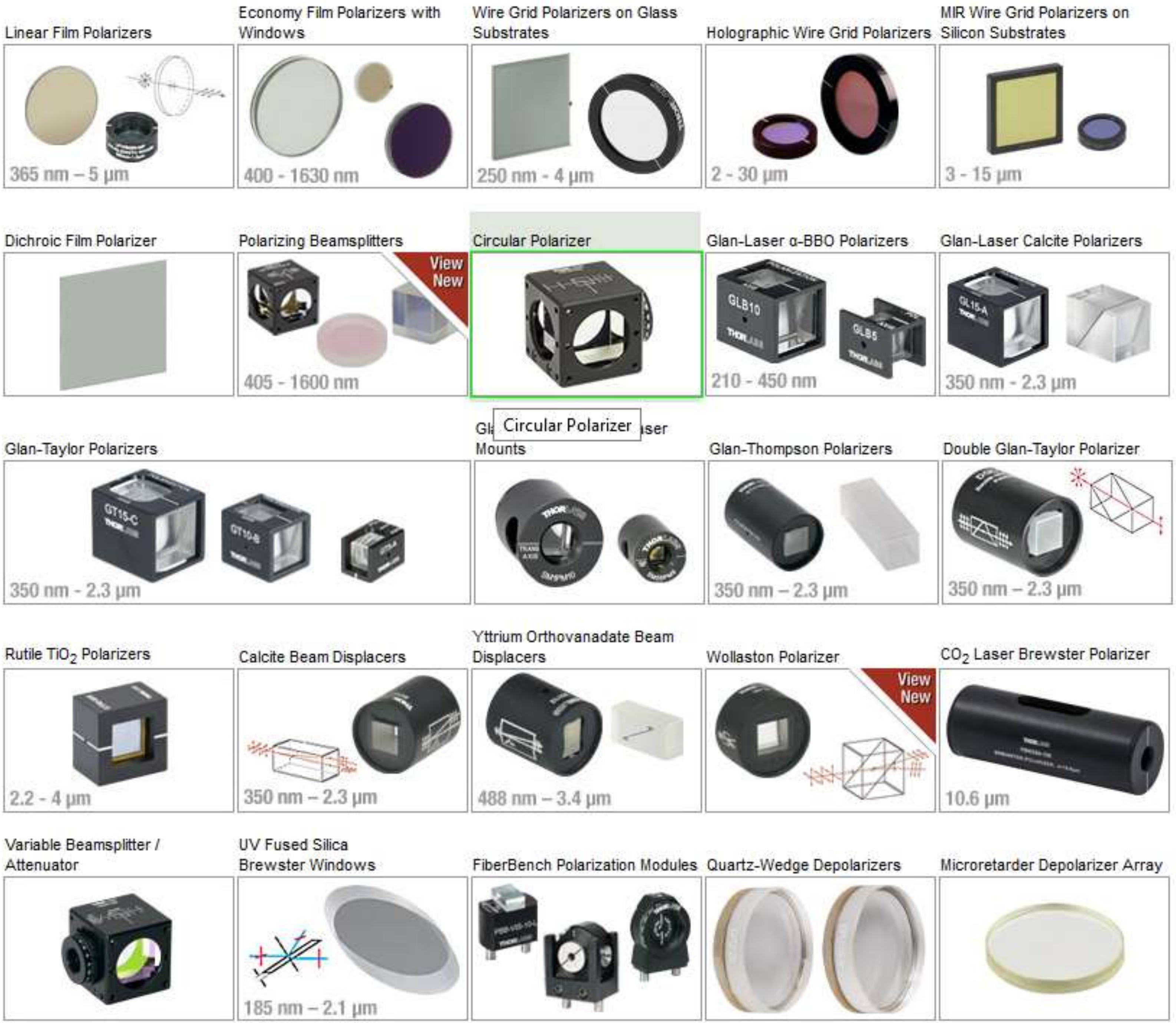
- Complex polarization ratio: $R\ \exp(j\phi)$

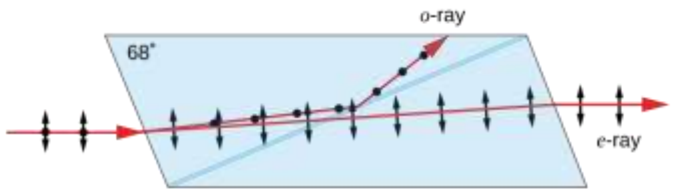
χ: How close to circular shape
0°: linear polarization
90°: circular polarization

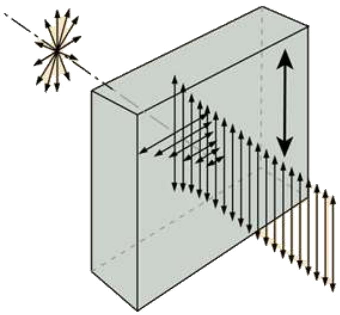
- Poincare sphere using 2χ and 2ψ

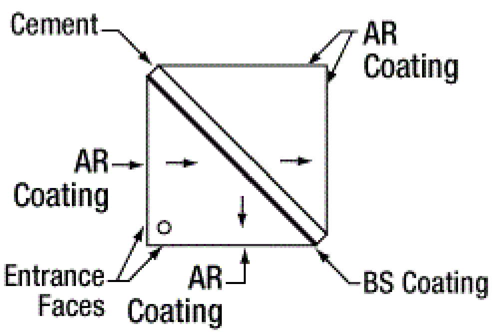
ψ: Angle from horizontal line

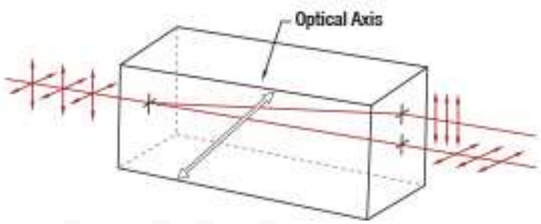


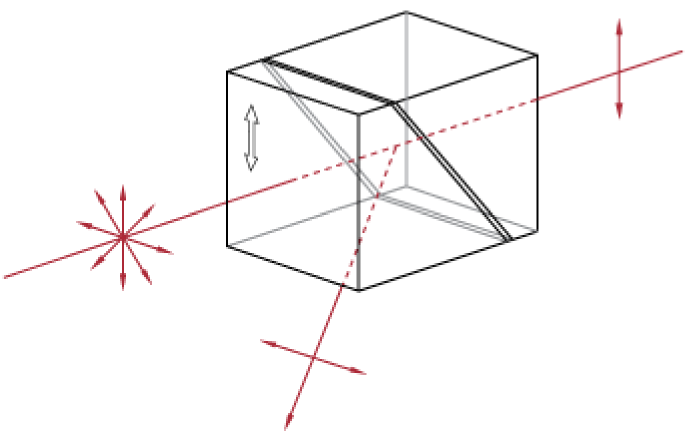

Linear Film Polarizers
365 nm – 5 µm
Economy Film Polarizers with Windows
400 - 1630 nm
Wire Grid Polarizers on Glass Substrates
250 nm - 4 µm
Holographic Wire Grid Polarizers
2 - 30 µm
MIR Wire Grid Polarizers on Silicon Substrates
3 - 15 µm
Dichroic Film Polarizer
Polarizing Beamsplitters
View New
405 - 1600 nm
Circular Polarizer
Glan-Laser α-BBO Polarizers
GLB10
GLB5
THORLABS
210 - 450 nm
Glan-Laser Calcite Polarizers
GL15-A
350 nm - 2.3 µm
Glan-Taylor Polarizers
GT15-C
GT10-B
THORLABS
350 nm - 2.3 µm
Gl
Circular Polarizer
ser
Mounts
THORLABS
SM1PM10
Glan-Thompson Polarizers
350 nm – 2.3 µm
Double Glan-Taylor Polarizer
350 nm – 2.3 µm
Rutile TiO_2 Polarizers
2.2 - 4 µm
Calcite Beam Displacers
350 nm – 2.3 µm
Yttrium Orthovanadate Beam Displacers
488 nm – 3.4 µm
Wollaston Polarizer
View New
CO_2 Laser Brewster Polarizer
THORLABS
10.6 µm
Variable Beamsplitter / Attenuator
UV Fused Silica Brewster Windows
185 nm – 2.1 µm
FiberBench Polarization Modules
THORLABS
Quartz-Wedge Depolarizers
Microretarder Depolarizer Array











Manual Fiber Polarization Controllers

Jones vector

- Polarization state can be described by a vector

$$\vec{\boldsymbol{E}} = \hat{\boldsymbol{x}}E_x + \hat{\boldsymbol{y}}E_y$$

$$E_x = E_{0x}\exp(j(kz - wt + \varphi_x))$$
$$E_y = E_{0y}\exp(j(kz - wt + \varphi_y))$$

Initial phase

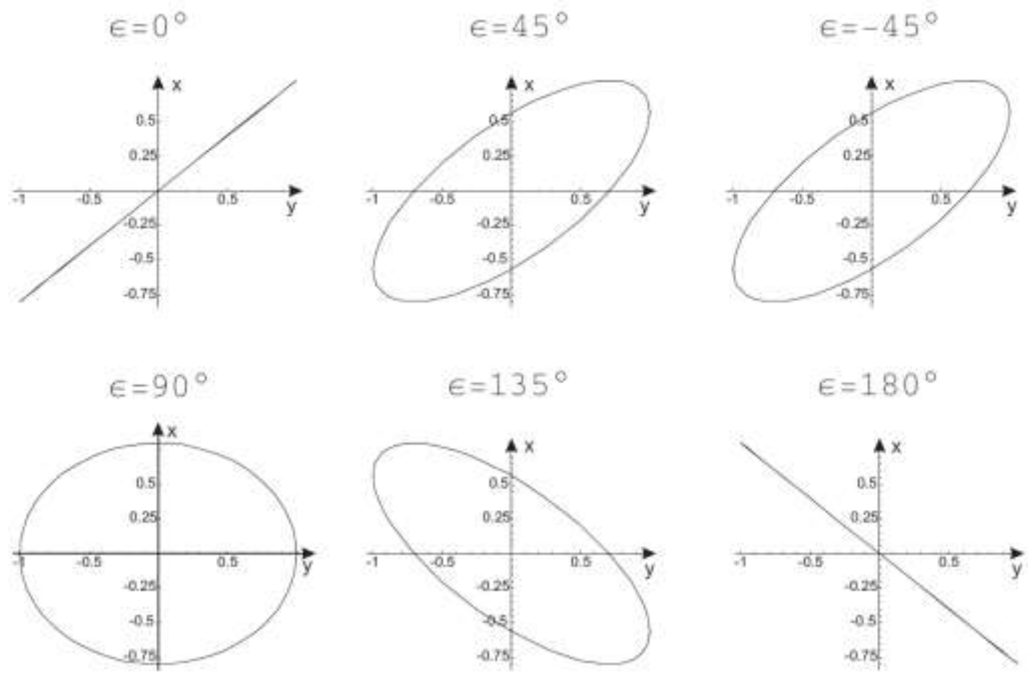
$$\vec{\boldsymbol{E}} = [\hat{\boldsymbol{x}}E_{0x}\exp(j\varphi_x) + \hat{\boldsymbol{y}}E_{0y}\exp(j\varphi_y)]\exp\big(j(kz - wt)\big) = \widetilde{\boldsymbol{E}}_0\exp\big(j(kz - wt)\big)$$

$$\widetilde{\boldsymbol{E}}_0 = \begin{bmatrix} E_{0x}\exp(j\varphi_x) \\ E_{0y}\exp(j\varphi_y) \end{bmatrix}$$

Then real parts

$$E_x = E_{0x}\cos(\omega t)$$

$$E_y = E_{0y}\cos(\omega t + \epsilon)$$



Jones vector

- Polarization state can be described by a vector

$$\overrightarrow{J} = \begin{bmatrix} A_x \\ A_y \end{bmatrix}$$

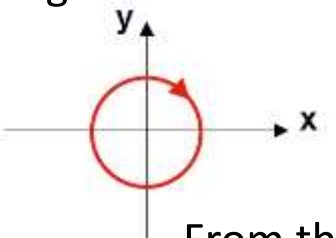
- The vector components (ratio of modulus, and difference of arguments) define the polarization.

$$\overrightarrow{J} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad \overrightarrow{J} = \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} \qquad \overrightarrow{J} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ i \end{bmatrix} \qquad \overrightarrow{J} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -i \end{bmatrix}$$

Linear polarization
- Horizontal direction

Linear polarization
- Angle θ from horizontal line

Circular polarization
- Right handed



From the point of view of the receiver.

Circular polarization
- Left handed

linear, general

$$\begin{bmatrix} \sin(\alpha) \\ \cos(\alpha) \end{bmatrix}$$

Jones vector (Ellipse)

elliptical, principal axes parallel to x,y axes	$\begin{bmatrix} A \\ \pm iB \end{bmatrix}$
elliptical, general	$\begin{bmatrix} A \\ B \pm iC \end{bmatrix}$

Jones matrix

- A device that acts on polarization can be described by a Jones matrix.

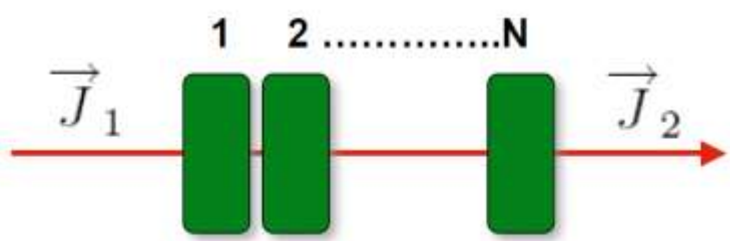
$$\vec{J}_2 = T\vec{J}_1 \qquad \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x_i \\ y_i \end{bmatrix} = \begin{bmatrix} x_i' \\ y_i' \end{bmatrix}$$

- It can be used to describe polarization state while propagating through polarization optical components.

$$T = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \qquad T = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \qquad T = \begin{bmatrix} 1 & 0 \\ 0 & e^{-i\Gamma} \end{bmatrix}$$

Polarizer   Rotator   Retarder

$$T = T_N \ldots T_2 T_1$$



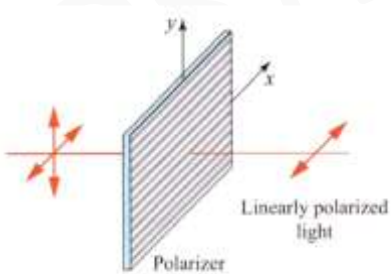
Polarization retarder

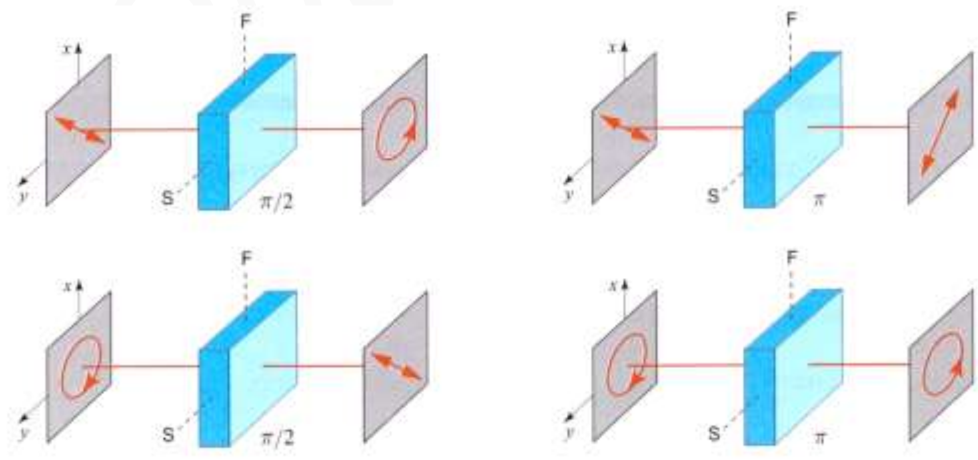
- Waveplate can be used to manipulate the polarization of an incoming w ave.

$$T = \begin{bmatrix} 1 & 0 \\ 0 & e^{-i\Gamma} \end{bmatrix}$$

- The phase shift between the two optical directions is

$$\Gamma = \frac{2\pi\,\Delta n\,L}{\lambda}$$





(a) Quarter-wave retarder (b) Half-wave retarder

Jones matrix

Component	Jones matrix
Horizontal (P) polarizer [PP]	$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$
Vertical (S) polarizer [PS]	$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$
Phase retarder	$\begin{bmatrix} e^{i\epsilon_x} & 0 \\ 0 & e^{i\epsilon_y} \end{bmatrix}$
Rotator	$\begin{bmatrix} \cos(\beta) & -\sin(\beta) \\ \sin(\beta) & \cos(\beta) \end{bmatrix}$

Examples

Examples

$$\begin{bmatrix} e^{-i*\pi/2} & 0 \\ 0 & e^{i*\pi/2} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -i \\ 0 \end{bmatrix}$$

Phase retarder (half wave plate)

$$\begin{bmatrix} \cos(\beta) & -\sin(\beta) \\ \sin(\beta) & \cos(\beta) \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \cos(\beta) \\ \sin(\beta) \end{bmatrix}$$

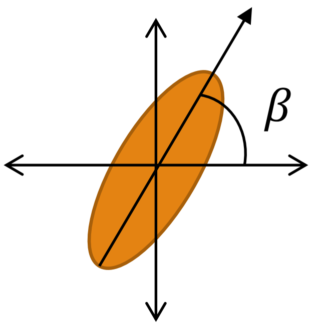
Coordinate rotator

Half wave plate with an arbitrary angle

$$HW(\beta) = \begin{bmatrix} \cos(\beta) & -\sin(\beta) \\ \sin(\beta) & \cos(\beta) \end{bmatrix} \begin{bmatrix} e^{-i\,\pi/2} & 0 \\ 0 & e^{i\,\pi/2} \end{bmatrix} \begin{bmatrix} \cos(-\beta) & -\sin(-\beta) \\ \sin(-\beta) & \cos(-\beta) \end{bmatrix}$$

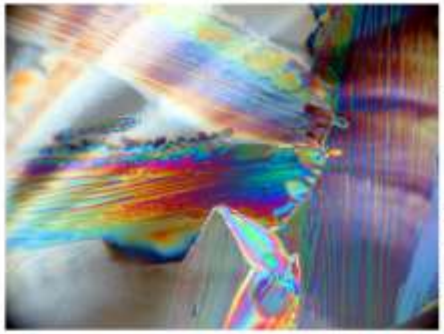
$$HW[45°]P = \begin{bmatrix} 0 & -i \\ -i & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -i \end{bmatrix}$$

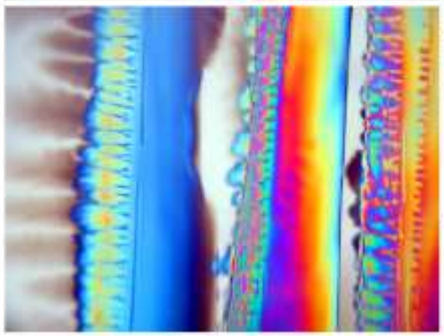
General case: $\begin{bmatrix} \cos(2\beta) \\ \sin(2\beta) \end{bmatrix}$

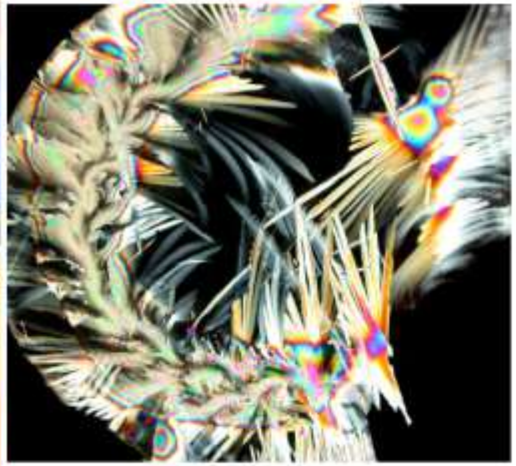


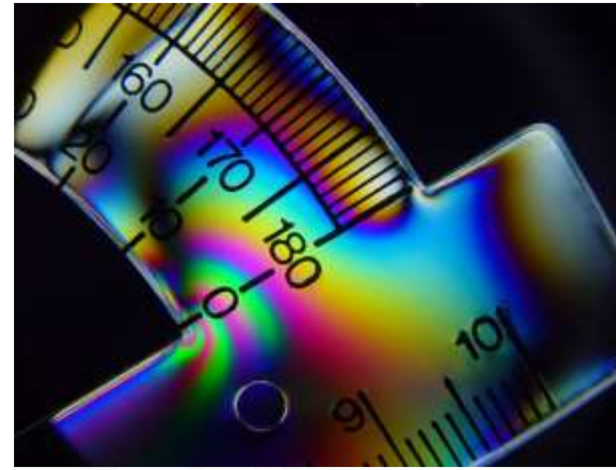
Photoelasticity

- NASA scientist Peter Wasilewski painting with ice and light: Peter.J.Wasilewski.1@gsfc.nasa.gov

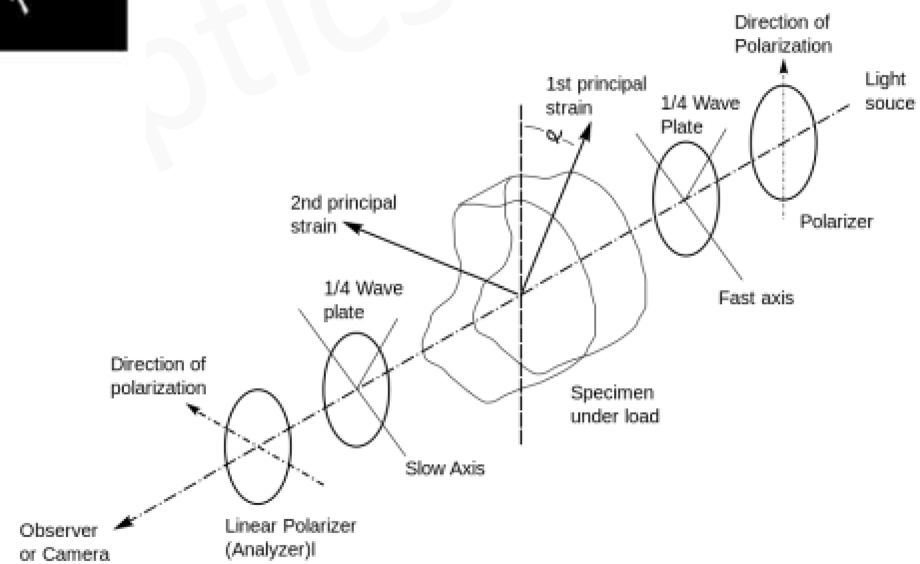








Stress induced birefringence



Polarization in quantum optics

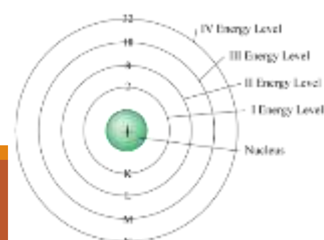
- **What is the polarization state of a single photon?**

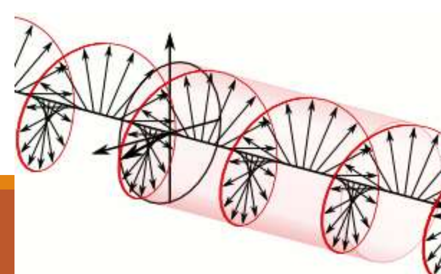
Circular polarization?   Linear polarization?   Or any polarization state?

- From the semiclassical theory, it is circularly polarized because photons are boson.

- Bosons have spins whose values are integer numbers, eg. +1, 0, -1, but p hotons have no mass and no longitudinal wave (No 0 spin).

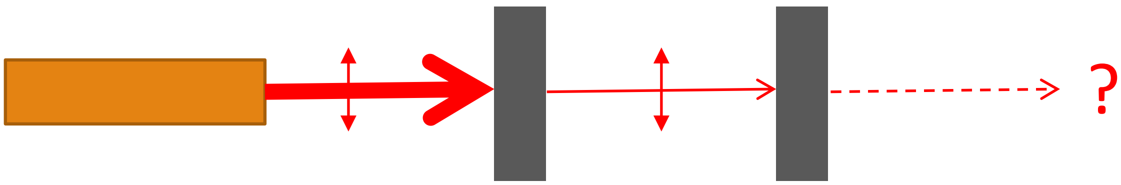
- Spins are same with angular momentum (aka, polarization) so that a sing le photon has Right-handed (+1) or Left-handed (-1) polarization.





In reality

- A linearly polarized laser beam.



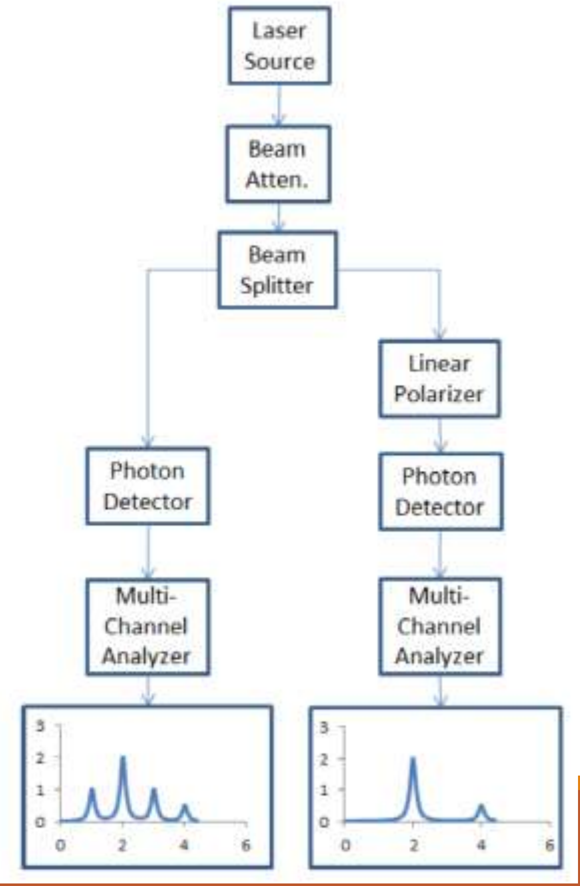
- The first attenuator doesn't change polarization direction.
- The second attenuator which is identical to the first one lets one photon per second pass through it.
- What is the polarization of single photon?

> Manuscript Submitted for Special Issue of IEEE J. Sel. Topics in Quantum Electronics on Superconducting QE<

Is a Single Photon Always Circularly Polarized? A Proposed Experiment Using a Superconducting Microcalorimeter Photon Detector

Alan M. Kadin, *Senior Member, IEEE*, and Steven B. Kaplan, *Senior Member, IEEE*

$$\begin{pmatrix}1\\0\end{pmatrix} = \frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}\begin{pmatrix}1\\j\end{pmatrix} + \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-j\end{pmatrix}\right]$$



Q & A

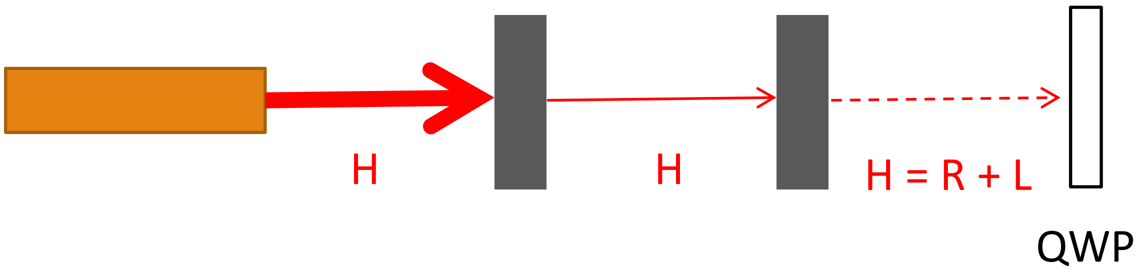
$$\leftrightarrow \;=\; \circlearrowright \;+\; \circlearrowleft$$

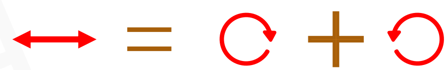
$$\begin{pmatrix}1\\0\end{pmatrix} = \frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}\begin{pmatrix}1\\j\end{pmatrix} + \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-j\end{pmatrix}\right]$$

- Is this mean the split of a single photon?

- Weird…

Experimental proof

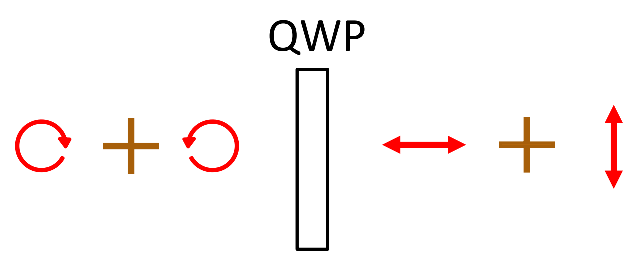




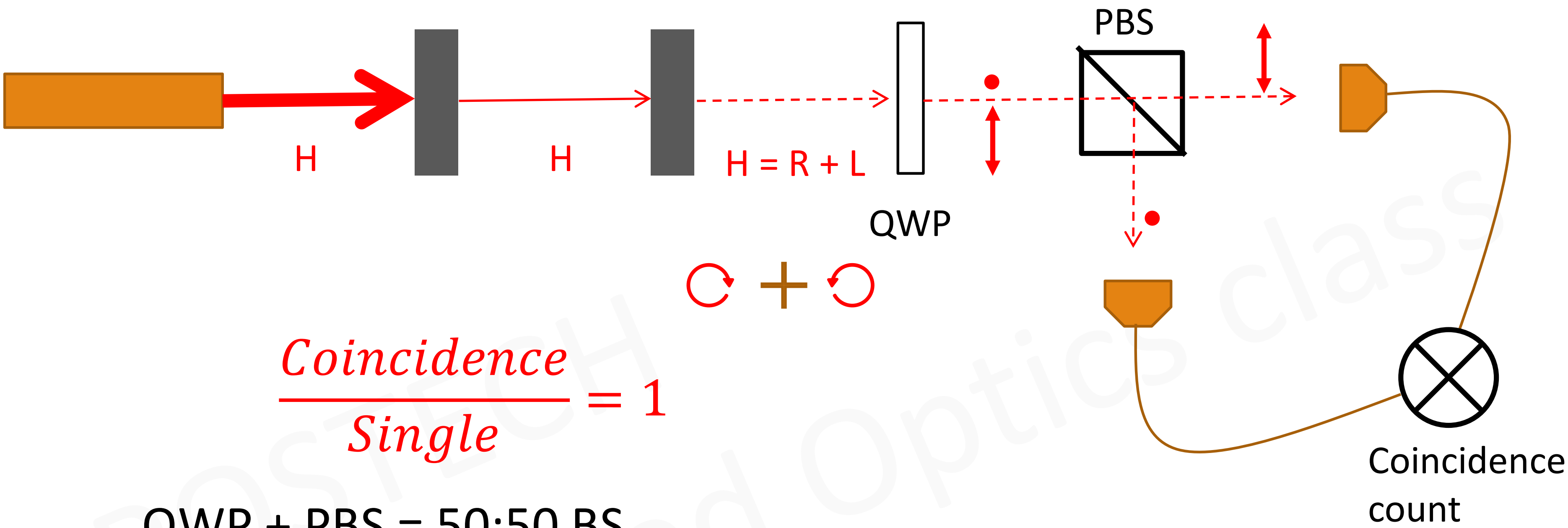
Quarter-wave plate with fast axis at angle 45 w.r.t the horizontal axis

$$\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} e^{j\pi/4} \begin{bmatrix} 1 & 0 \\ 0 & -j \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

Right circular ➔ Horizontal
Left circular ➔ Vertical



Experimental proof



$$\frac{Coincidence}{Single} = 1$$

QWP + PBS = 50:50 BS

Hanbury Brown and Twiss Experiment

$$\frac{Coincidence}{Single} = 0$$

Circular polarization? Linear polarization? Or any polarization state?

Optics of anisotropic media

- A dielectric medium is said to be anisotropic if its optical properties depends on the direction.

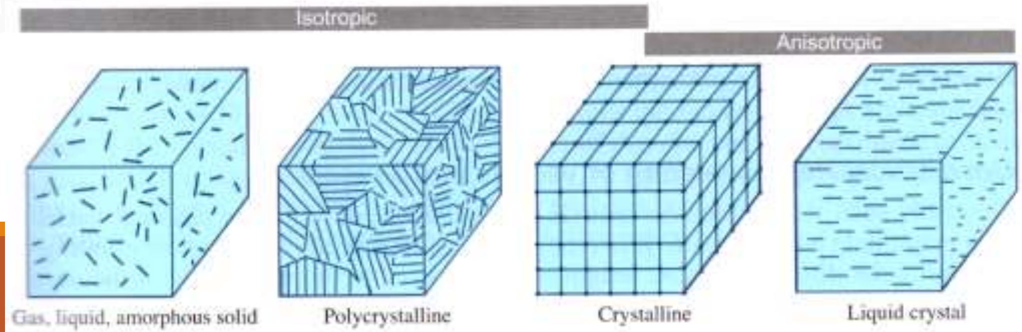
- Anisotropy depends on the crystal structure.

– If molecules randomly located in space and are isotropic (or oriented along r andom directions), the medium is isotropic (gases, liquid and amorphous solid s)

– If the structure takes the form of disjoined crystalline grains randomly orient ed, the crystal is a polycrystalline and is in general anisotropic

– If the molecules are organized in space according to a regular periodic patte rn and oriented in the same direction, as in crystals, the medium is in general anisotropic.

– If the molecules are anisotropic and their orientations are not totally rando m, the medium is anisotropic, even if their positions are totally random.



Permittivity Tensor

- Electric flux density D is a linear combination of the three components of the electric field.

$$D_i = \sum_i \epsilon_{ij} E_j$$

ϵ_{ij} 3 x 3 array of nine coefficients

$\boldsymbol{\epsilon}$**:** Electric permittivity tensor

Principal axes

Boyd's 'nonlinear optics'

- The elements of the permittivity tensor depend on the choice of coordinate system.
- In certain systems, however, the tensor is diagonal i.e.

$$\begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} = \epsilon_o \begin{bmatrix} n_x^2 & 0 & 0 \\ 0 & n_y^2 & 0 \\ 0 & 0 & n_z^2 \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}$$

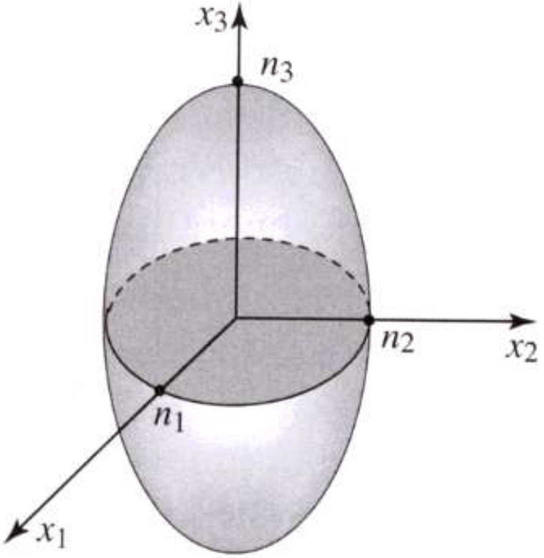
- This coordinate system defines the principal axes associated to the crystal.
- The corresponding refractive indexes are known as principal indexes.

Biaxial, Uniaxial & isotropic crystal

- Crystals with three different principal refractive indexes are referred to as **biaxial crystals**.

$$n_1 \neq n_2 \neq n_3$$

- Crystal with two different principal refractive indexes are referred to as **uniaxial crystals.**
- For uniaxial crystals, the refractive indexes are $n_1 = n_2 = n_\mathrm{o}$, and $n_3 = n_\mathrm{e}$ where "o" stands for ordinary axis and "e" for extraordinary axis.
- If $n_\mathrm{e} > n_\mathrm{o}$ the crystal is said to be a positive uniaxial crystal.



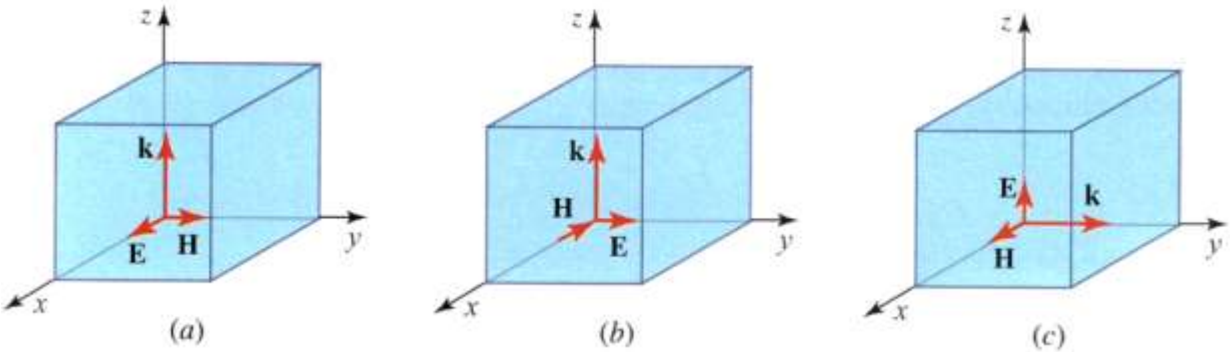
Index ellipsoid

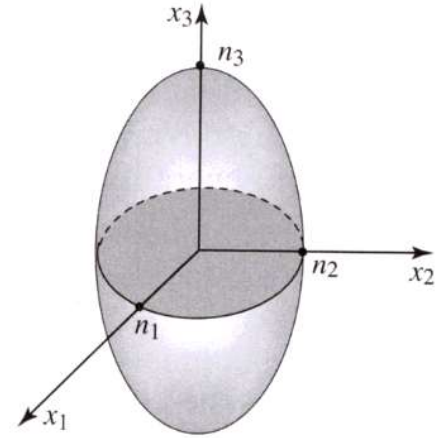
- The coordinates (x_1, x_2, x_3) are the principal axes.
- The values (n_1, n_2, n_3) are the principal refractive indexes of the crystal.

$$\frac{{x_1}^2}{{n_1}^2} + \frac{{x_2}^2}{{n_2}^2} + \frac{{x_3}^2}{{n_3}^2} = 1$$

Propagation

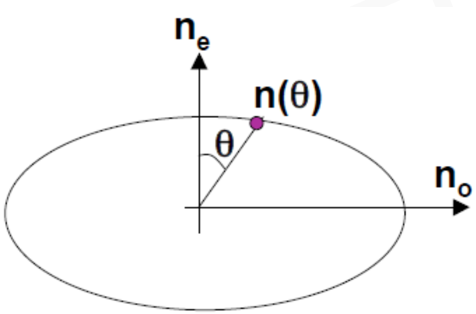
- Along a principal axis





Index ellipsoid

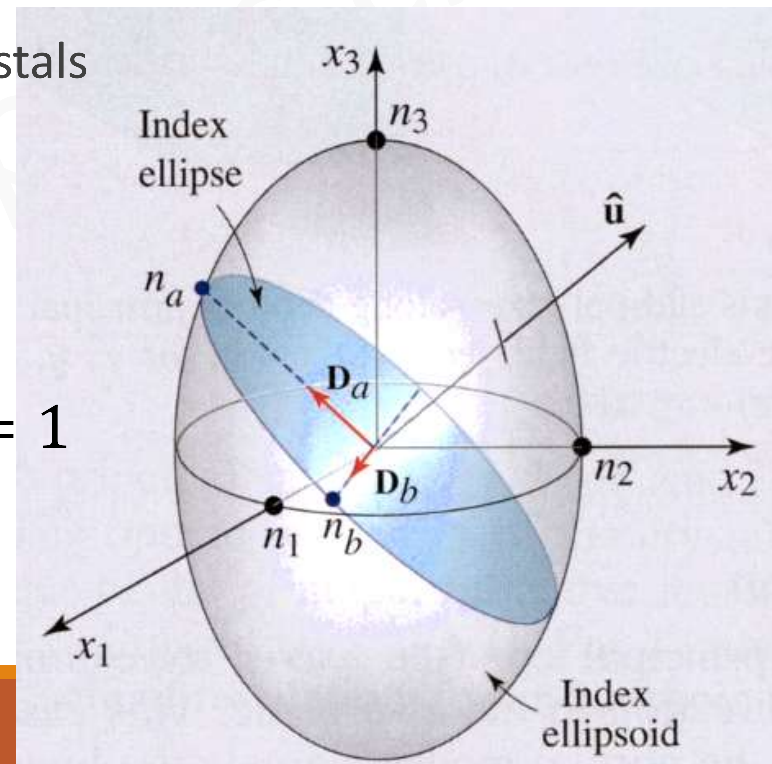
- Along an arbitrary direction in uniaxial crystals



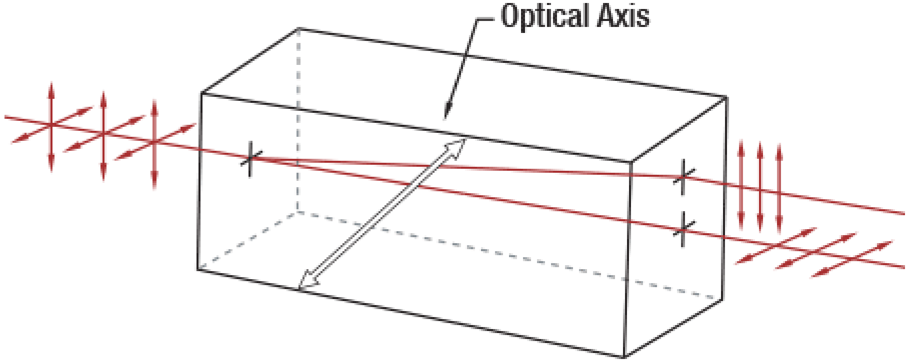
Index ellipse

$$\frac{x_1{}^2}{n_1{}^2} + \frac{x_2{}^2}{n_2{}^2} + \frac{x_3{}^2}{n_3{}^2} = 1$$

$$\frac{1}{n^2(\theta)} = \frac{\cos^2\theta}{n_o^2} + \frac{\sin^2\theta}{n_e^2}$$



Poynting vector



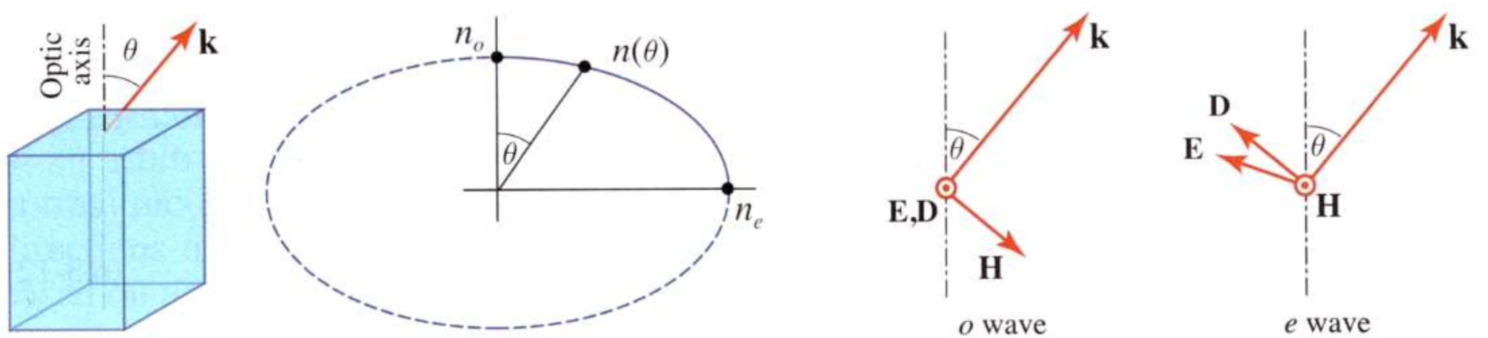
From Maxwell's equations,

$$\vec{k} \times \vec{H} = -\omega \vec{D}$$

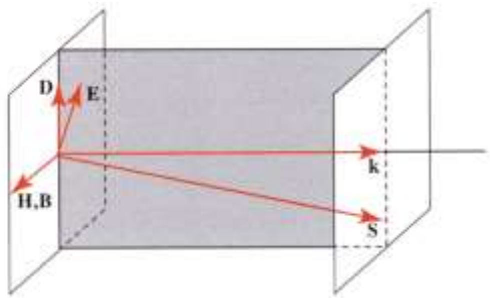
$$\vec{k} \times \vec{E} = \omega \mu \vec{H}$$

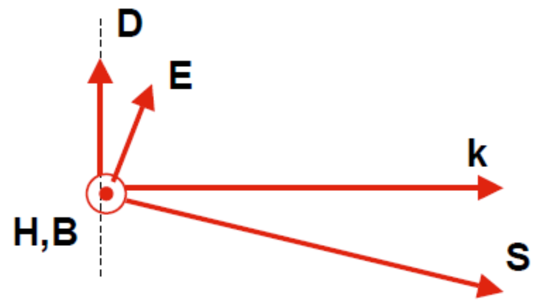
$$\vec{S} = \frac{1}{2} \vec{E} \times \vec{H}^*$$

- Optical wave characterized by k, E, D, H, and B with power flow given by S.
- D, E, k, S are in the same plane ⊥ to (B and H)
- D ⊥ k and H
- H ⊥ k and E
- S ⊥ E and H

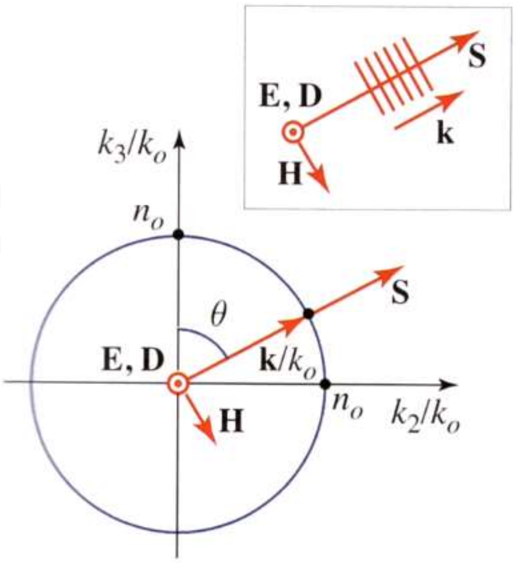


Poynting vector

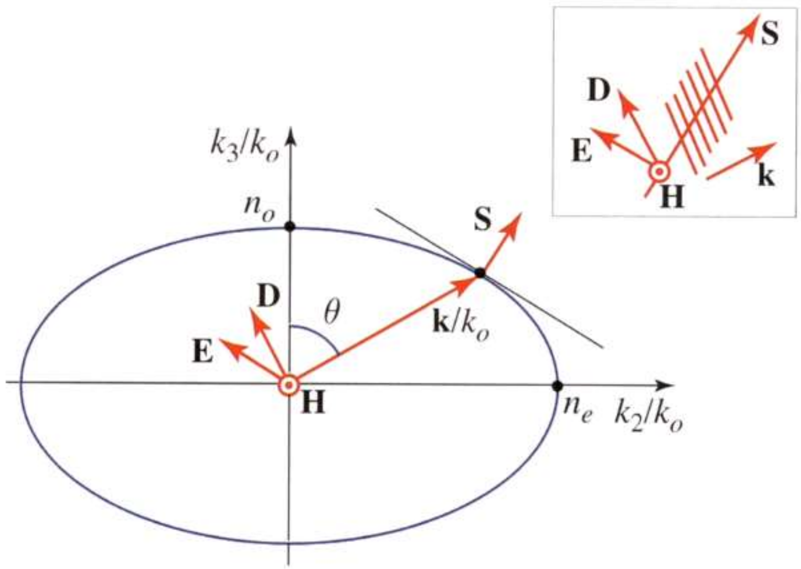




Wave front & wave vector

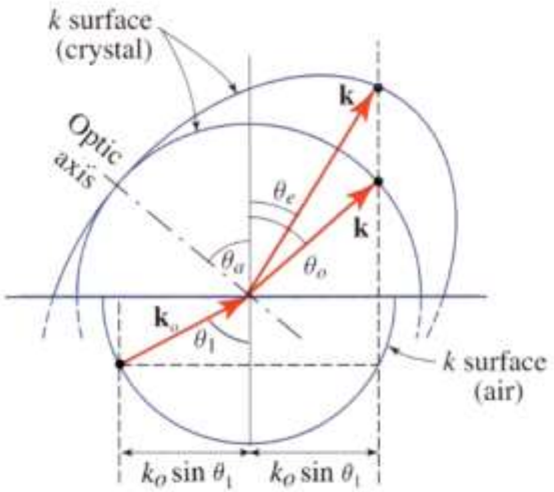


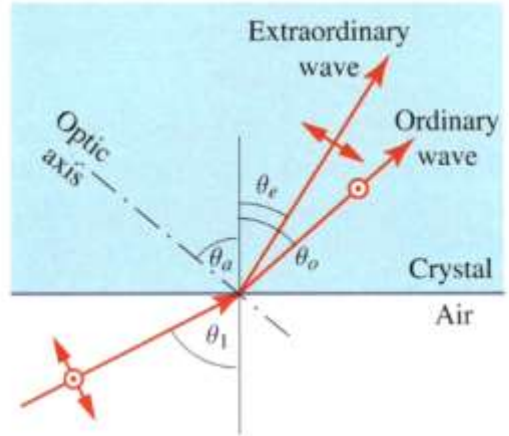
(*a*) Ordinary



(*b*) Extraordinary

Birefringence crystals





$$\sin\theta_1 = n_0 \sin\theta_0$$
$$\sin\theta_1 = n(\theta_a + \theta_e)\sin\theta_e$$