Spring 2019

EECE 588
Lecture 17

Prof. Wonbin Hong

Mapping Between The Array Factor and $T_m(z)$ Polynomials

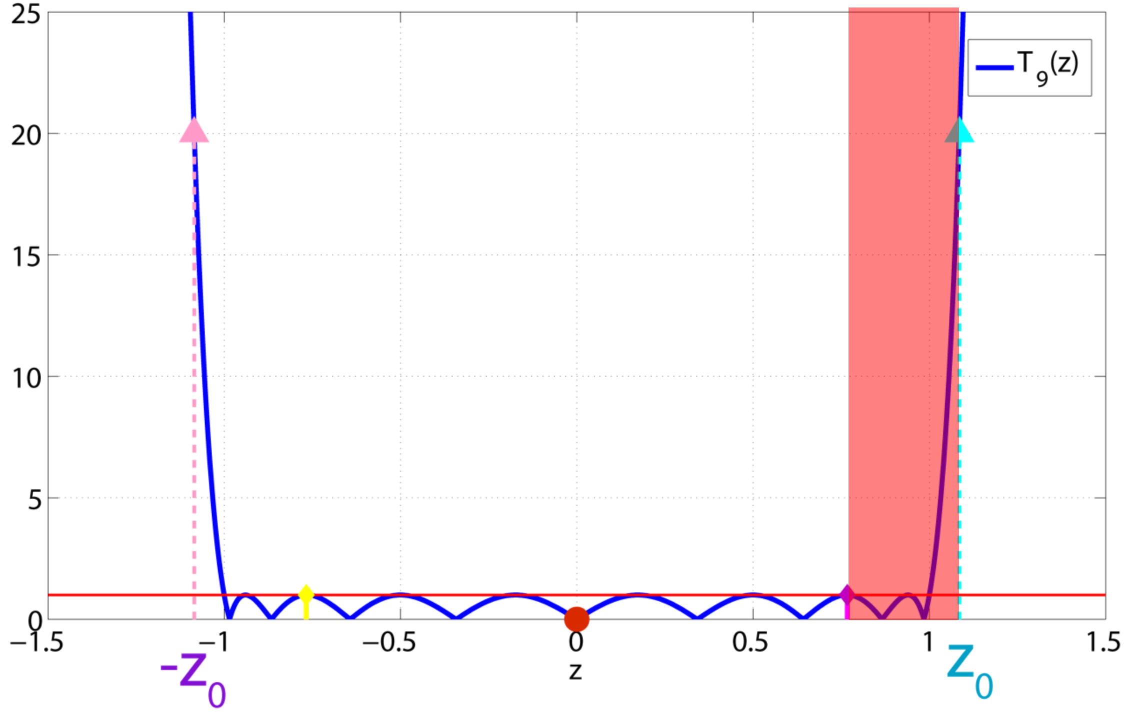
- Note that the process that we carried out was just a simple mapping between the expression for the array factor and the $T_m(z)$ polynomials.
- In the previous example, we had:

$$z = z_0 \cos u = z_0 \cos\left(\frac{\pi d}{\lambda}\cos\theta\right) = 1.085 \times \cos\left(\frac{\pi d}{\lambda}\cos\theta\right)$$

- Let us consider several cases:
  - d=λ/4, λ/2, 3λ/4, λ.

Mapping Between The Array Factor and $T_m(z)$ Polynomials

- d= λ/4:
  - In this case, the visible space (0≤θ≤π) is mapped to z values in the range of 0.7673 < z < 1.0851
  - The maximum value occurs for θ=90° and the minimum values occur for θ=180° and 0°.



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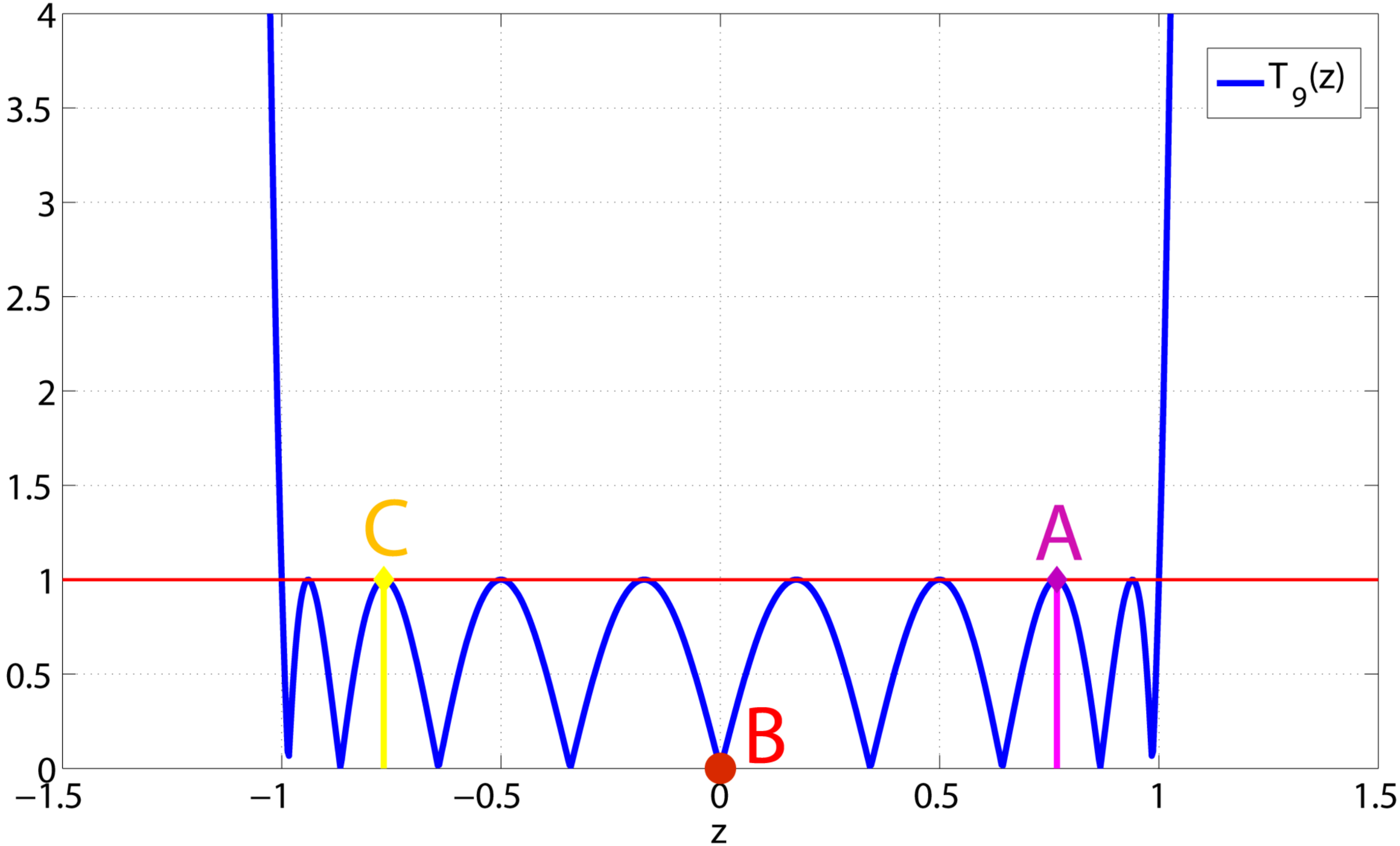
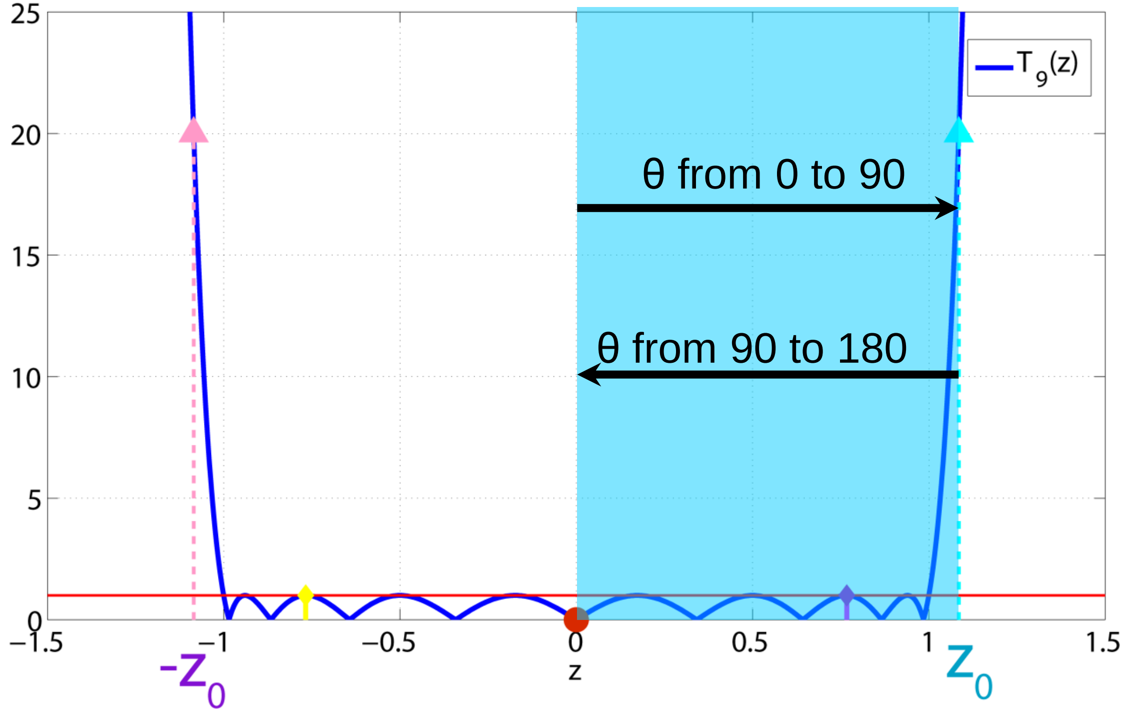
Mapping Between The Array Factor and $T_m(z)$ Polynomials



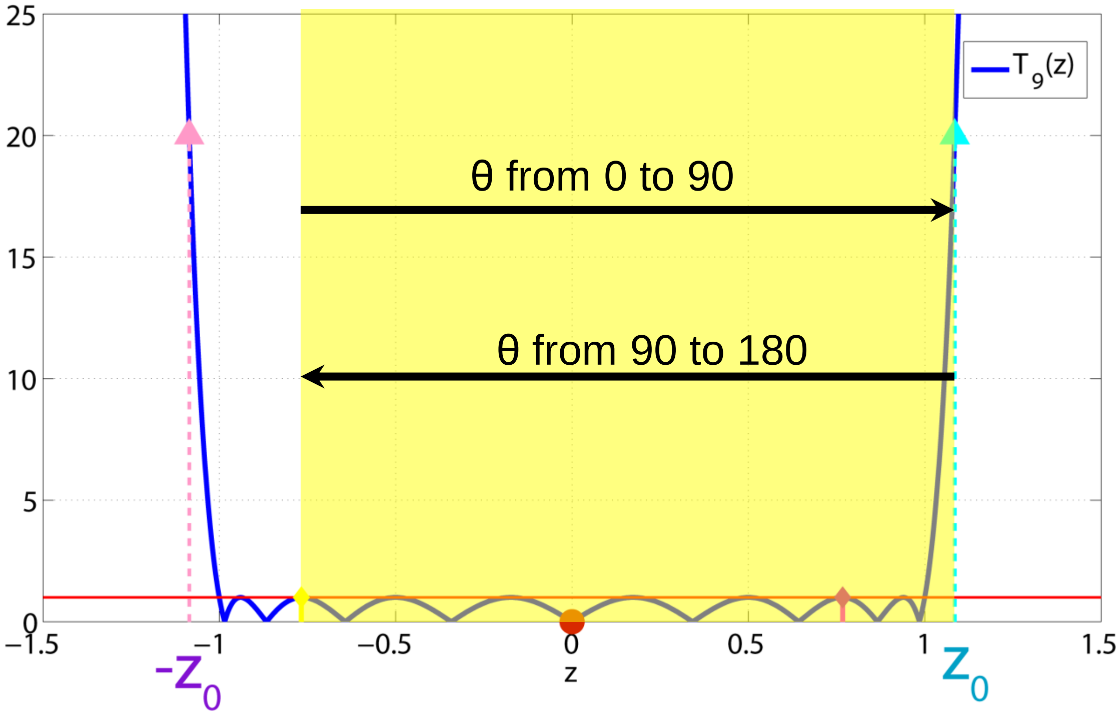
Mapping Between The Array Factor and $T_m(z)$ Polynomials

- d= λ/2:
  - In this case, the visible space (0≤θ≤π) is mapped to z values in the range of 0 < z < 1.0851.
  - The maximum value occurs for θ=90° and the minimum values occur for θ=180° and 0°.



Mapping Between The Array Factor and $T_m(z)$ Polynomials

- d= 3λ/4:
  - In this case, the visible space (0≤θ≤π) is mapped to z values in the range of -0.7673 < z < 1.0851.
  - The maximum value occurs for θ=90° and the minimum values occur for θ=180° and 0°.



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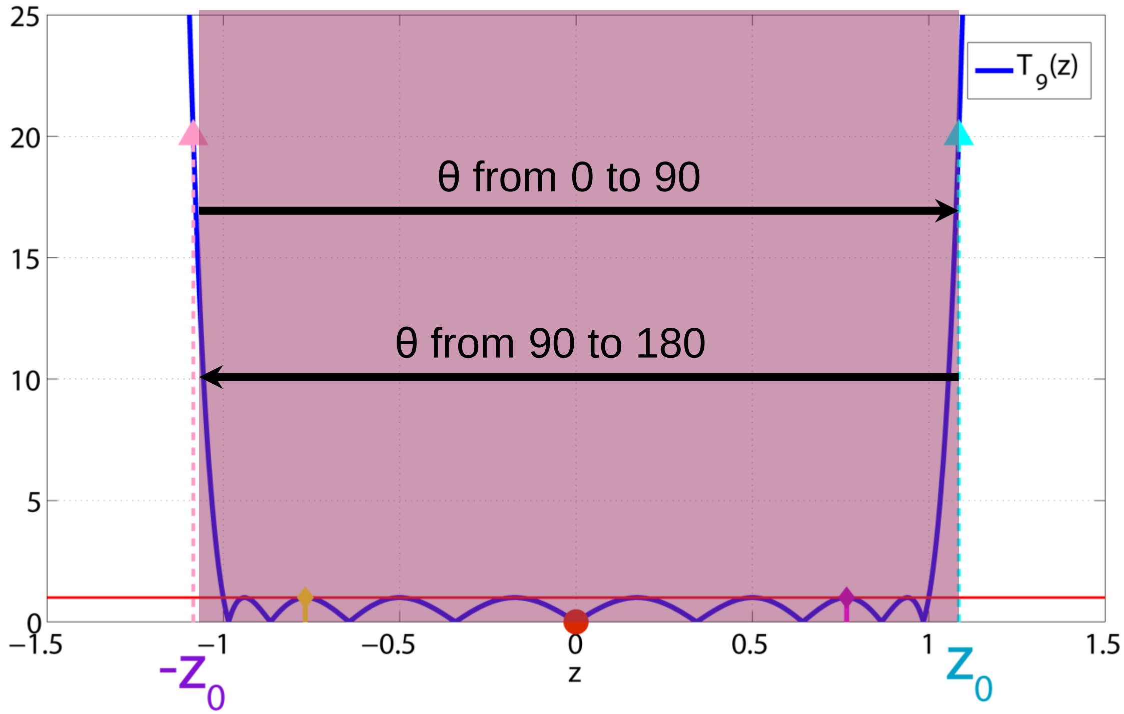
Mapping Between The Array Factor and $T_m(z)$ Polynomials

- d= λ:
- In this case, the visible space (0≤θ≤π) is mapped to z values in the range of -0.7673 < z < 1.0851.
  - The maximum value occurs for θ=90° and the minimum values occur for θ=180° and 0°.



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Element Spacing in Dolph-Chebychev Arrays

- Generally, we would like to use the maximum possible spacing between elements without having grating lobes or side lobes that exceed a minimum.
- In DT arrays, the ripples of the Chebychev function are all of the same magnitude.
- Therefore, the only way that the side lobe levels can increase the minimum desired value is to have a wide visible region that goes below z < -1.
- To ensure that this does not happen, we must find the d value that corresponds to z=-1.

$$z = z_0 \cos\left(\frac{\pi d}{\lambda}\cos\theta\right) = -1 \rightarrow d_{\max} = \frac{\lambda}{\pi}\cos^{-1}\left(\frac{-1}{z_0}\right)$$

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Element Spacing in Dolph-Chebychev Arrays

- To ensure that no other lobe exist with a magnitude larger than the side lobes, the element spacing must follow this condition:

$$d_{\max} \le \frac{\lambda}{\pi} \cos^{-1}\left(\frac{-1}{z_0}\right)$$

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Beamwidth and Directivity

- For large DT arrays that are scanned not too close to end-fire and have SLL's in the -20 to -60 dB range, HPBW and directivity are found by introducing a beam broadening factor (R. S. Elliot) as:

$$f = 1 + 0.636\left\{\frac{2}{R_0}\left(\sqrt{\left(\cosh^{-1} R_0\right)^2 - \pi^2}\right)\right\}^2$$

- R_0 is the ratio of major lobe to side lobe (voltage) level.

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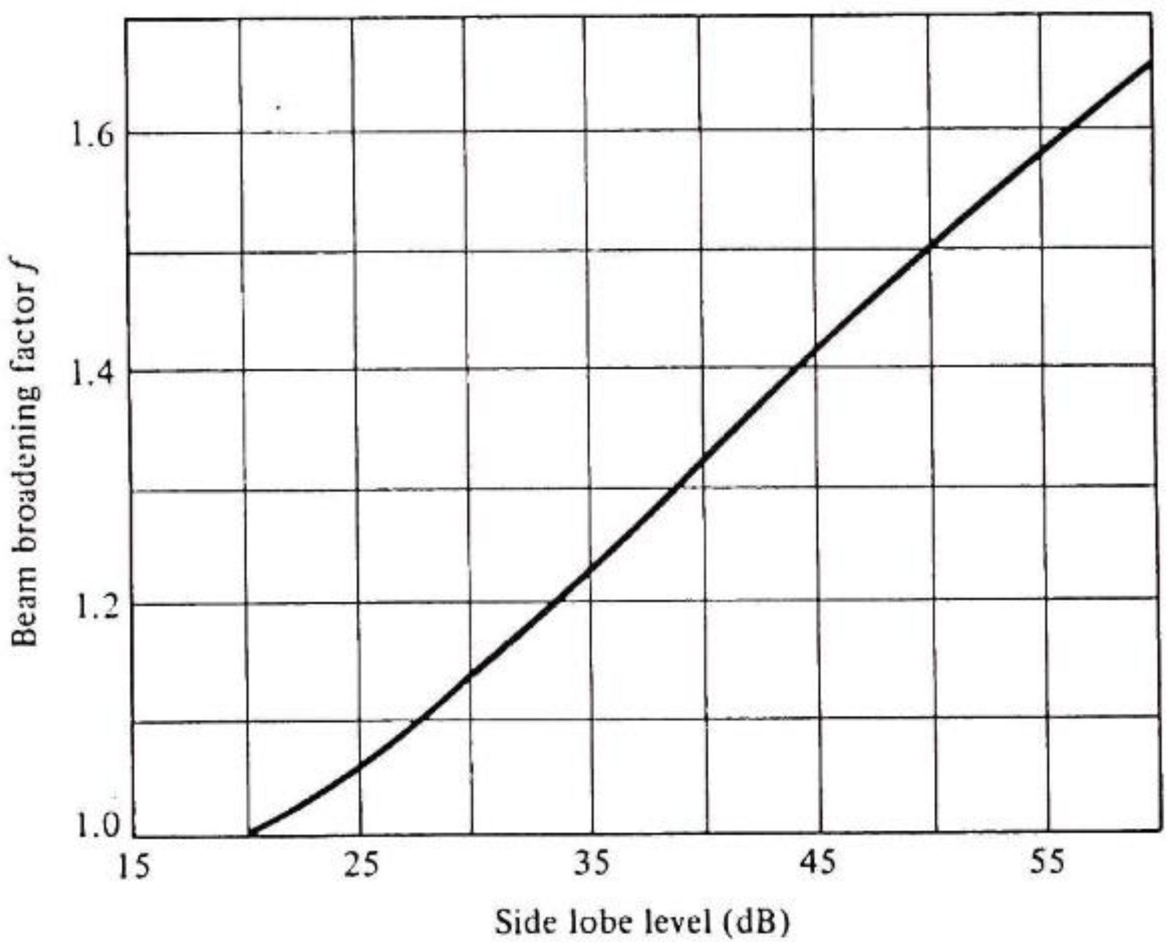
Beamwidth and Directivity



Beamwidth and Directivity

- The Half-Power Beamwidth is obtained from:
  - Calculating the beamwidth of a uniform array (with the same N and d).
  - Multiplying the beamwidth of this array by the beam broadening factor calculated previously.
- This BBF can be used to calculate the directivity of DT arrays scanned near the broadside:

$$D_0 = \frac{2R_0^2}{1+(R_0^2-1)\,f\,\dfrac{\lambda}{L+d}}$$

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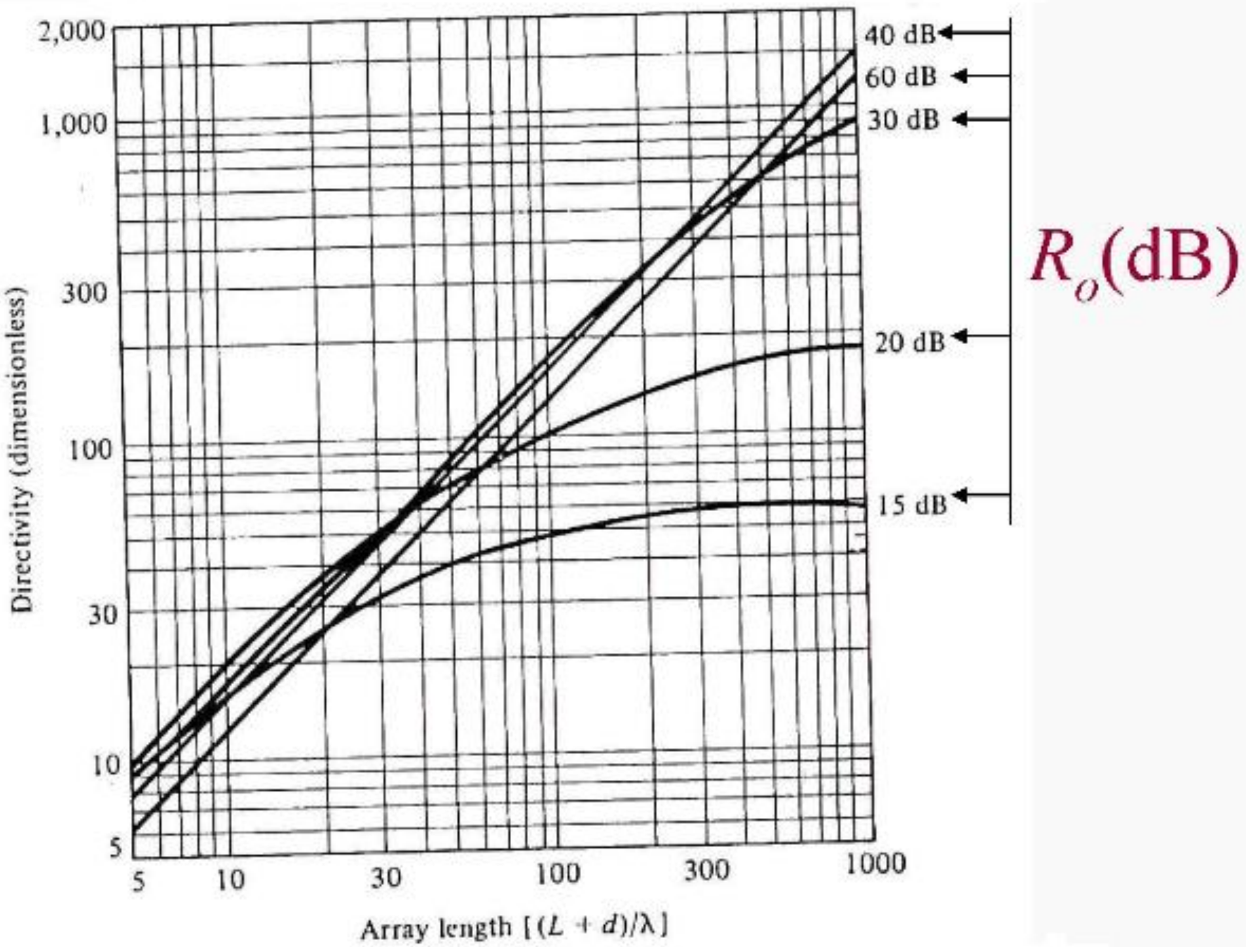
Beamwidth and Directivity



(b) Directivity

Amplitude Distribution of Dolph-Tschebyscheff Array For Different Sidelobe Levels

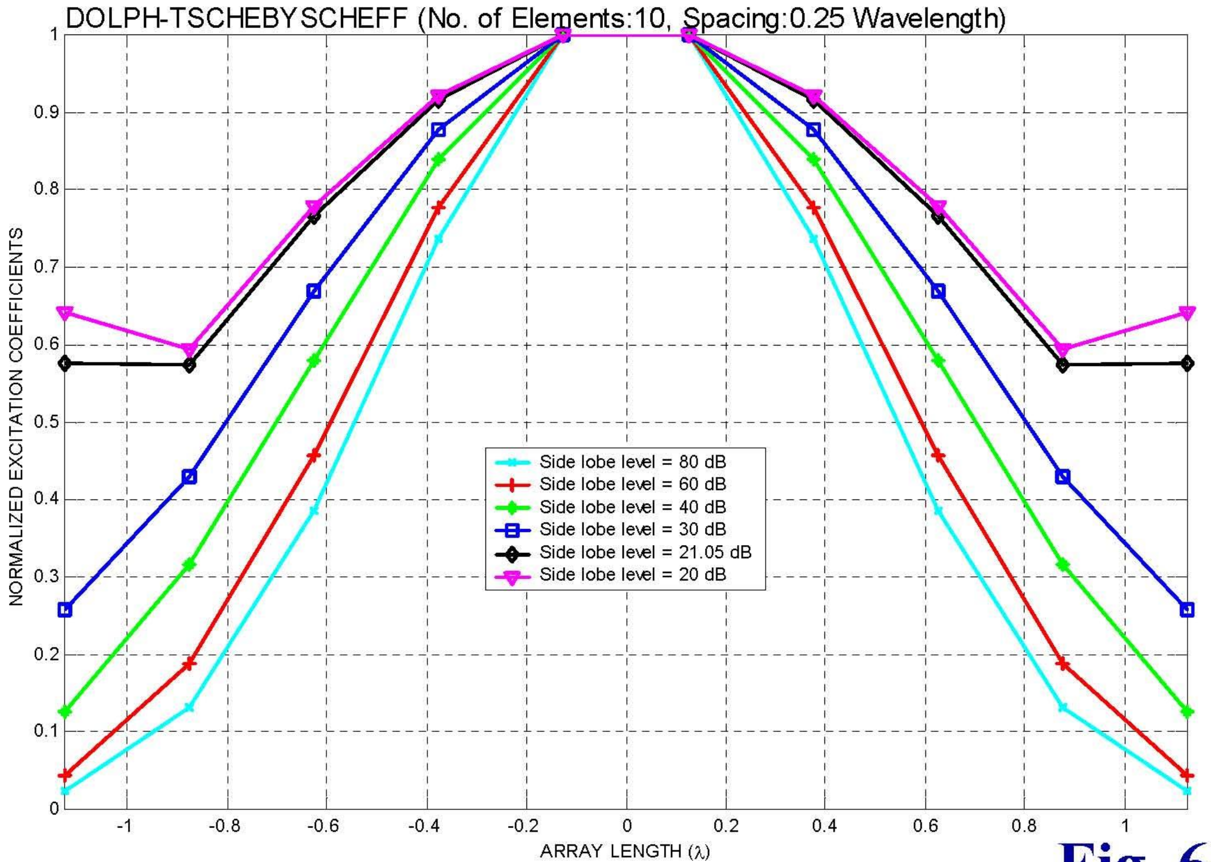


Fig. 6.25

Directivity and HPBW of Dolph-Tschebyscheff Array Vs. Sidelobe Level

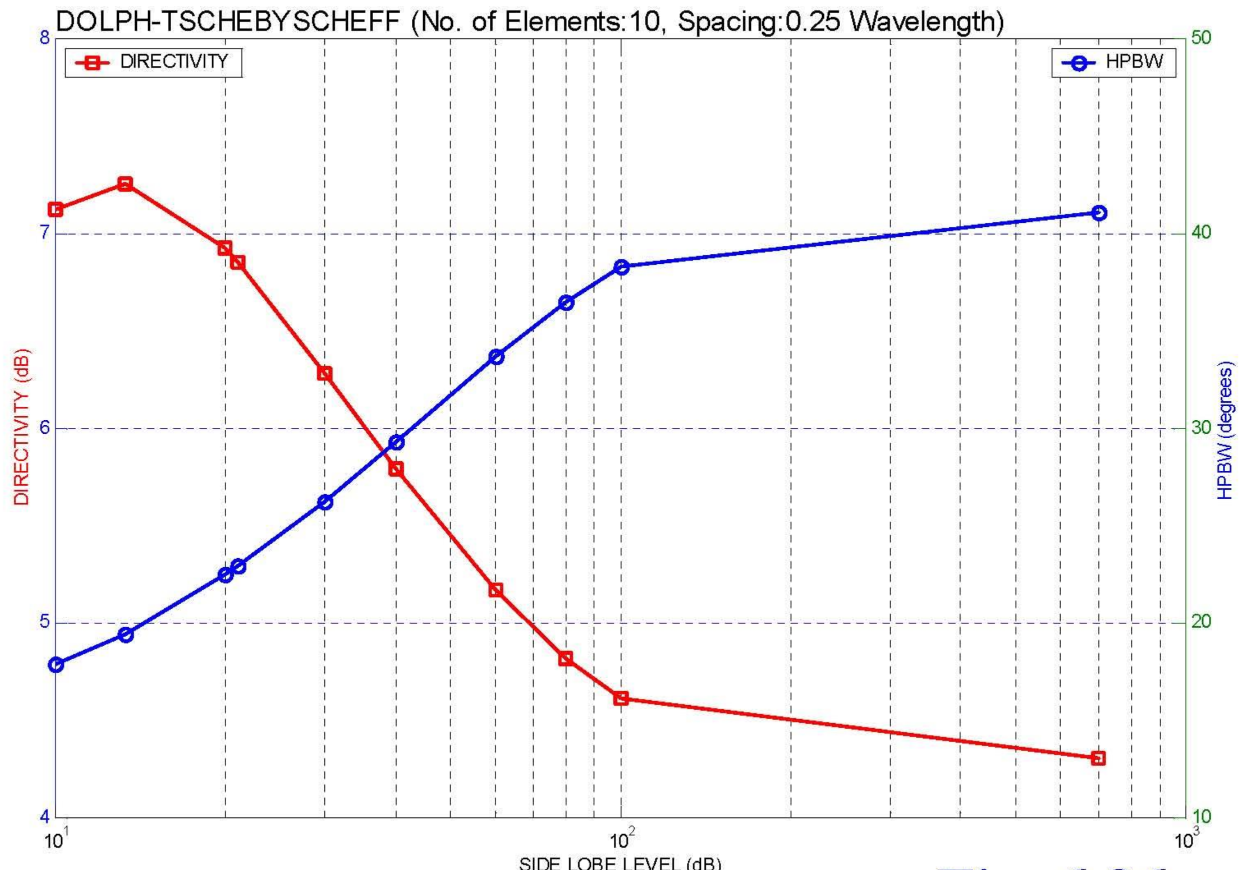


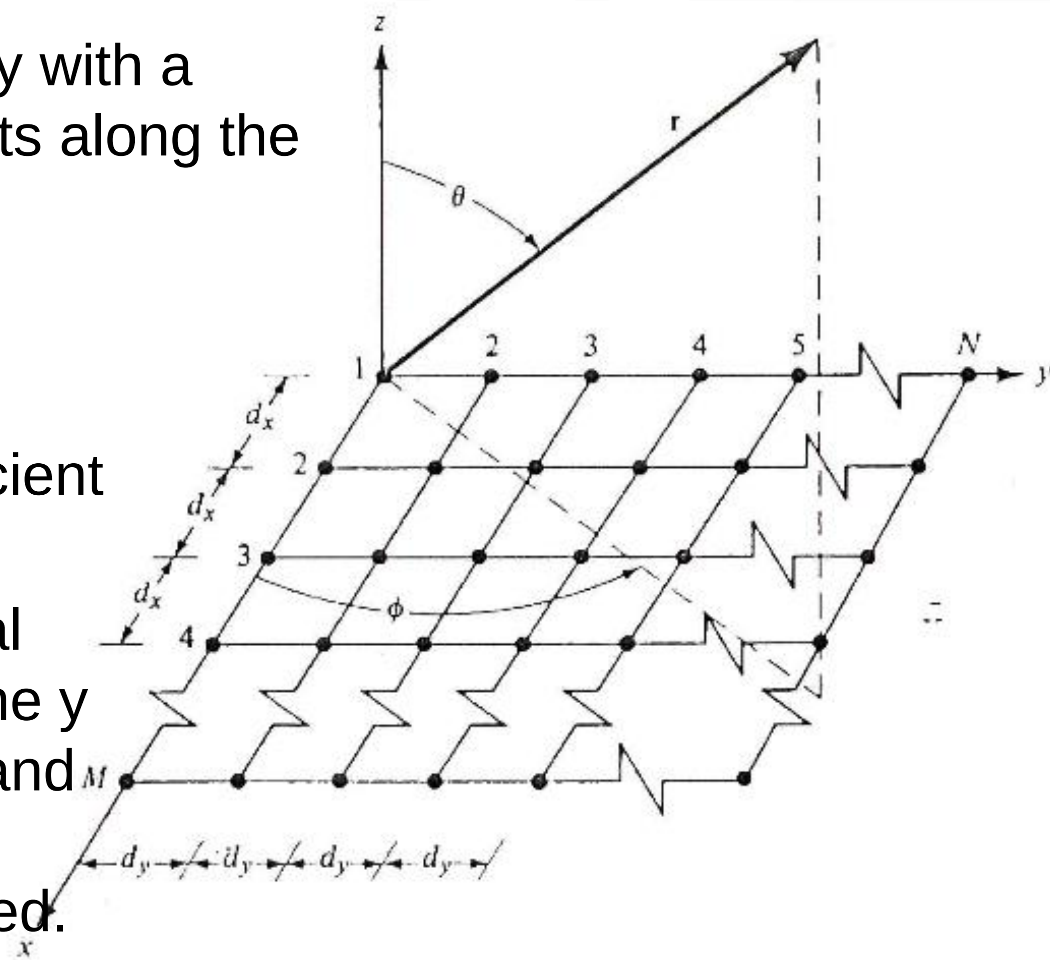
Fig. 6.26

Planar Arrays

Planar Arrays

- In a one dimensional array with a total number of M elements along the x axis, the array factor is:

- I_{m1} is the excitation coefficient of each element.
- If, N such one dimensional arrays are placed along the y direction with spacing d_y and phase shift of β_y a two dimensional array is formed.



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Planar Arrays

- The array factor of this planar array is:

$$AF = \sum_{n=1}^{N} I_{1n} \left\{ \sum_{m=1}^{M} I_{m1} e^{j(m-1)(kd_x \sin\theta \cos\phi + \beta_x)} \right\} e^{j(n-1)(kd_y \sin\theta \sin\phi + \beta_y)}$$

$$AF = S_{xm} S_{yn}$$

$$S_{xm} = \sum_{m=1}^{M} I_{m1} e^{j(m-1)(kd_x \sin\theta \cos\phi + \beta_x)}$$

$$S_{yn} = \sum_{n=1}^{N} I_{n1} e^{j(n-1)(kd_y \sin\theta \sin\phi + \beta_y)}$$

Planar Arrays

- For uniform array excitation, the 2-D AF is:

$$AF_n(\theta,\varphi)=\left\{\frac{1}{M}\frac{\sin\left(\dfrac{M}{2}\psi_x\right)}{\sin\left(\dfrac{\psi_x}{2}\right)}\right\}\left\{\frac{1}{N}\frac{\sin\left(\dfrac{N}{2}\psi_y\right)}{\sin\left(\dfrac{\psi_y}{2}\right)}\right\}$$

$$\psi_x = kd_x \sin\theta\cos\varphi + \beta_x$$

$$\psi_y = kd_y \sin\theta\sin\varphi + \beta_y$$

Planar Arrays

- For a rectangular array, the major lobe and the grating lobes of S_{xm} and S_{yn} are found from:

$$kd_x \sin\theta \cos\varphi + \beta_x = \pm 2m\pi \qquad m = 0, 1, 2, \ldots.$$

$$kd_y \sin\theta \sin\varphi + \beta_y = \pm 2n\pi \qquad n = 0, 1, 2, \ldots.$$

- To ensure that we have only one main beam directed towards θ_0, φ_0, we must have:

$$\beta_x = -kd_x \sin\theta_0 \cos\varphi_0$$

$$\beta_y = kd_y \sin\theta_0 \sin\varphi_0$$

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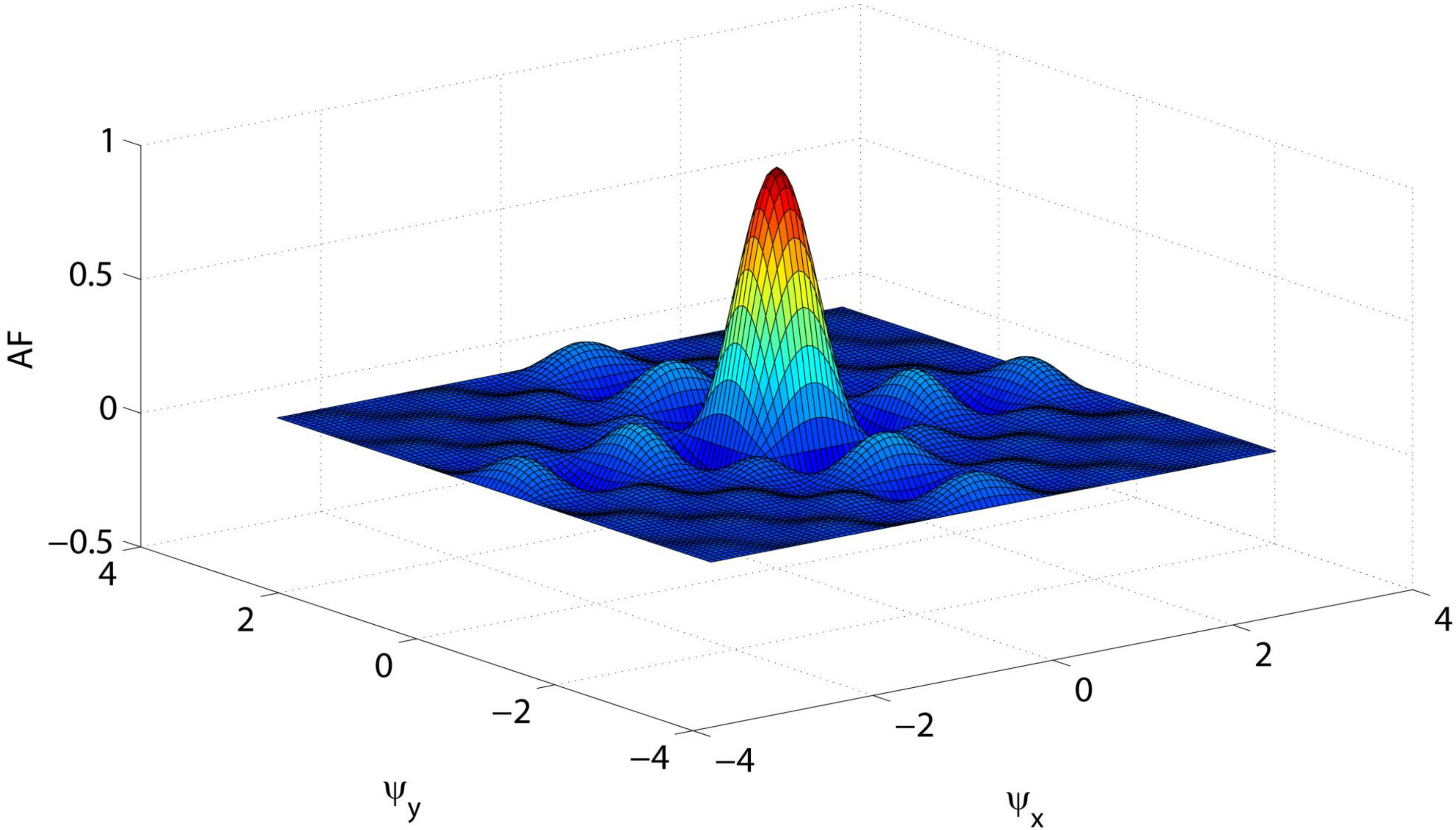
Planar Arrays

- If this is the case, the principal maximum and the directions of grating lobes are found from:

$$kd_x\left(\sin\theta\cos\varphi - \sin\theta_0\cos\varphi_0\right) = \pm 2m\pi \quad m = 0,1,2,\ldots$$
$$kd_y\left(\sin\theta\sin\varphi - \sin\theta_0\sin\varphi_0\right) = \pm 2n\pi \quad n = 0,1,2,\ldots$$

Two-Dimensional Array Factor (Uniform Element Excitation)



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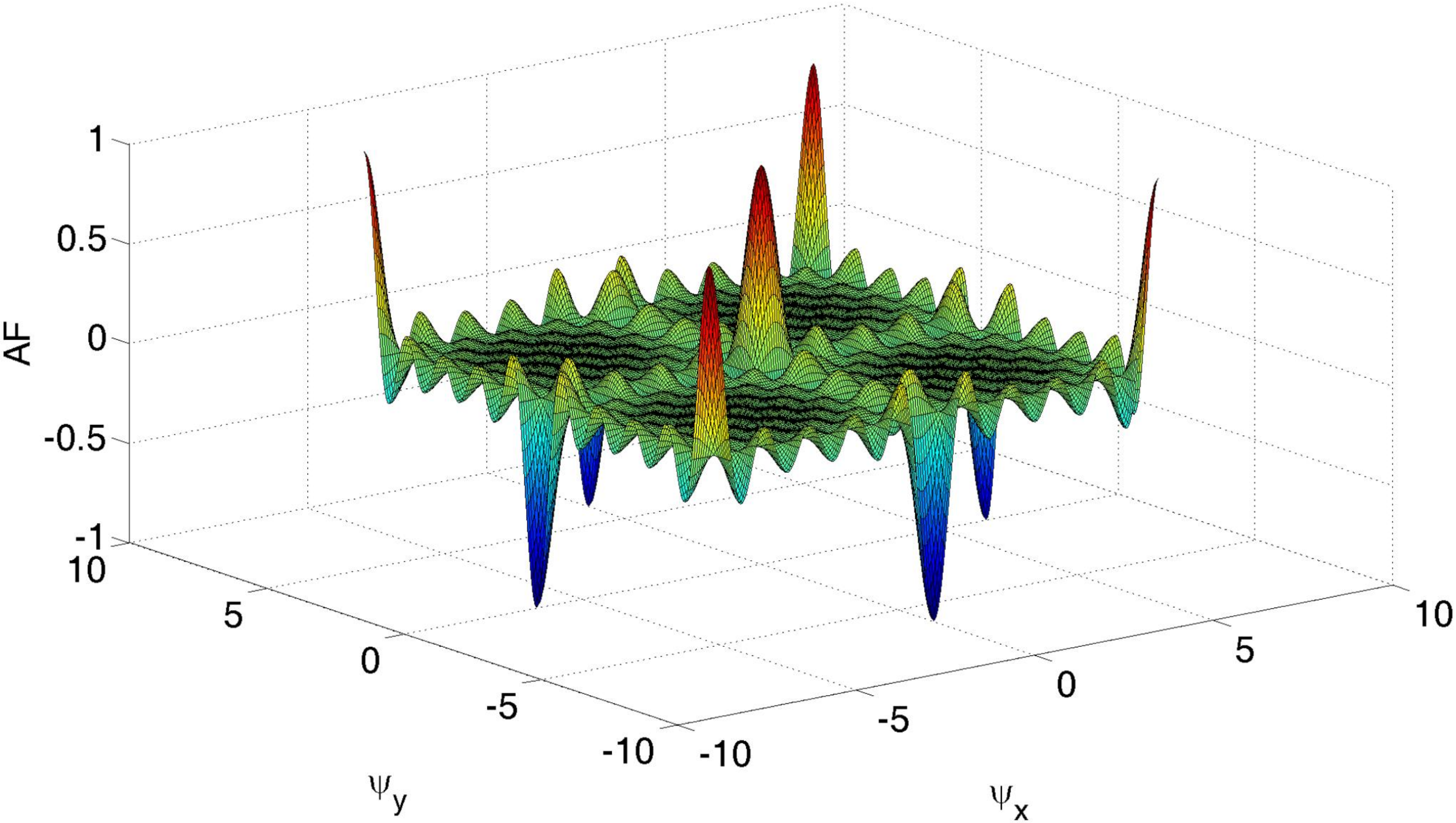
Two-Dimensional Array Factor (Uniform Element Excitation)



Two-Dimensional Array Factor (Uniform Distribution)

- Let's assume that the array is a broadside array. i.e., $\beta_x=\beta_y=0$
- This way,
  - $\psi_x=kd_x \sin\theta \cos\varphi$
  - $\psi_y=kd_y \sin\theta \sin\varphi$
- The maximum range of ψ_x is $|\psi_x|<kd_x$ and $|\psi_y|<kd_y$.
- However, these are not independent from one another.
- You can show that (if $d_x=d_y=d$)
  - $\psi^2_x+ \psi^2_y \leq k^2d^2$
  - This represents the region inside a circle with the radius of kd on the ψ_x-ψ_y surface.
- If $d_x \neq d_y$:
  - $(\psi_x/d_x)^2+ (\psi_y/d_y)^2 \leq k^2$.
- This represents the region inside an ellipse.

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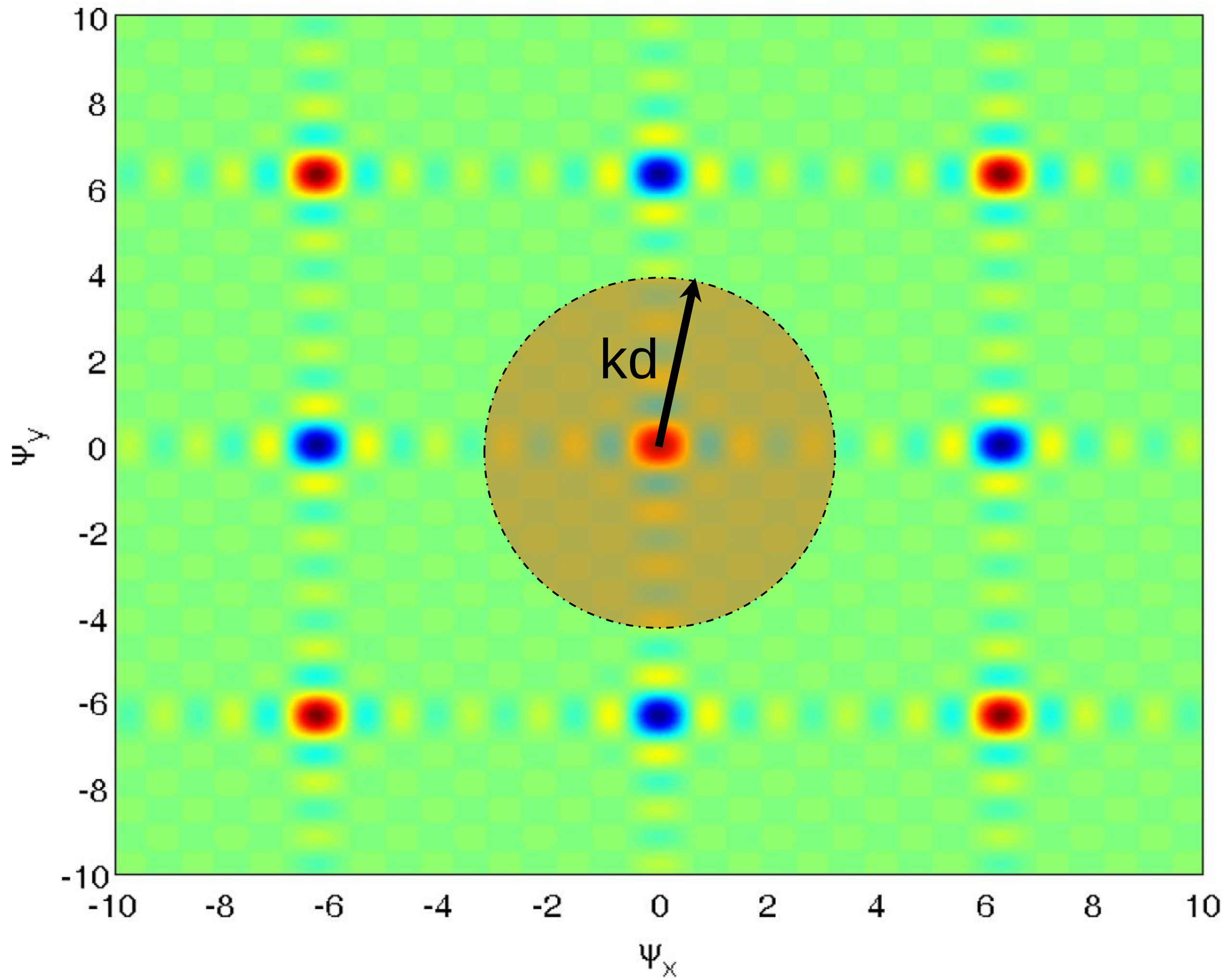
Two-Dimensional Array Factor (Uniform Distribution)



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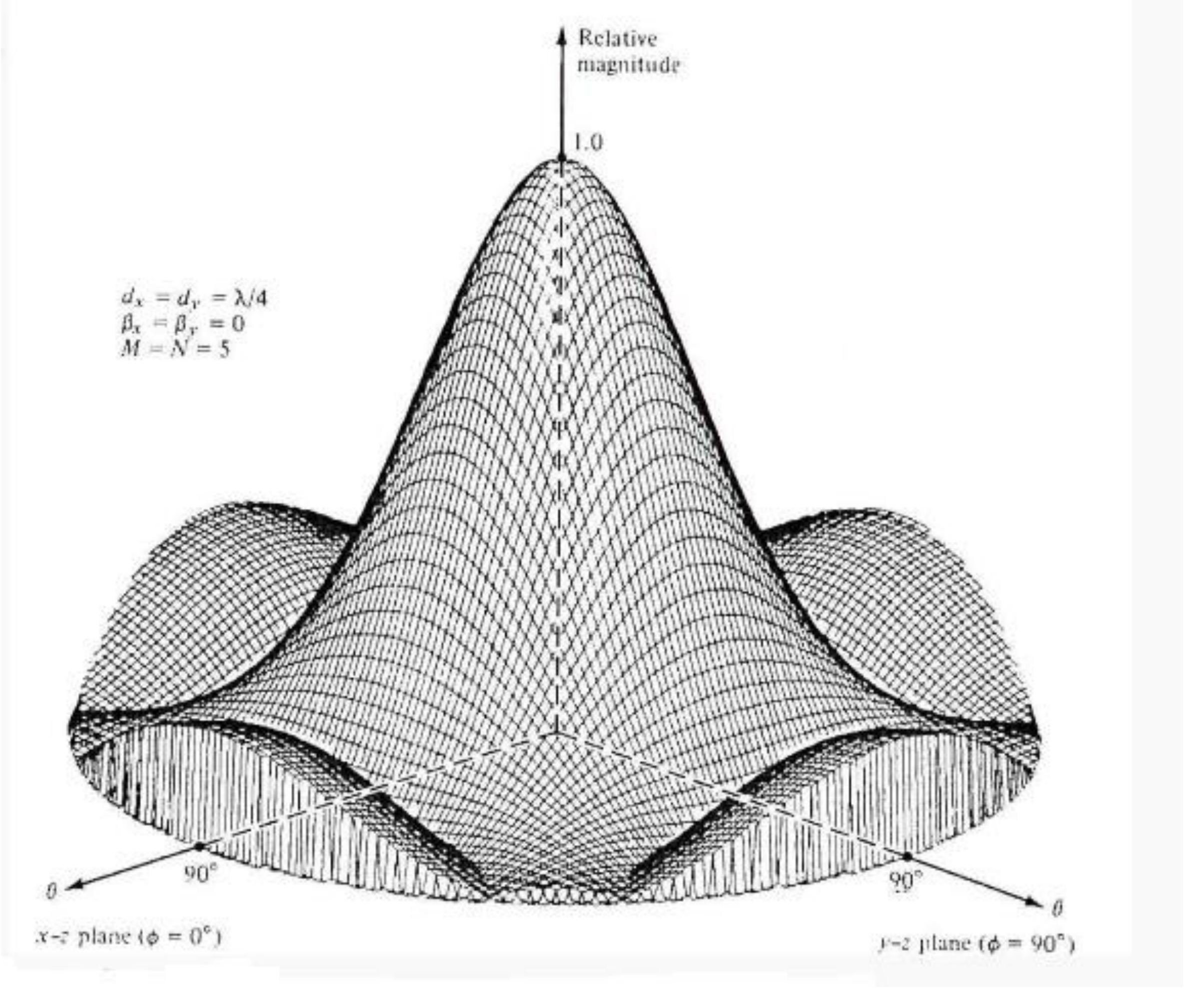
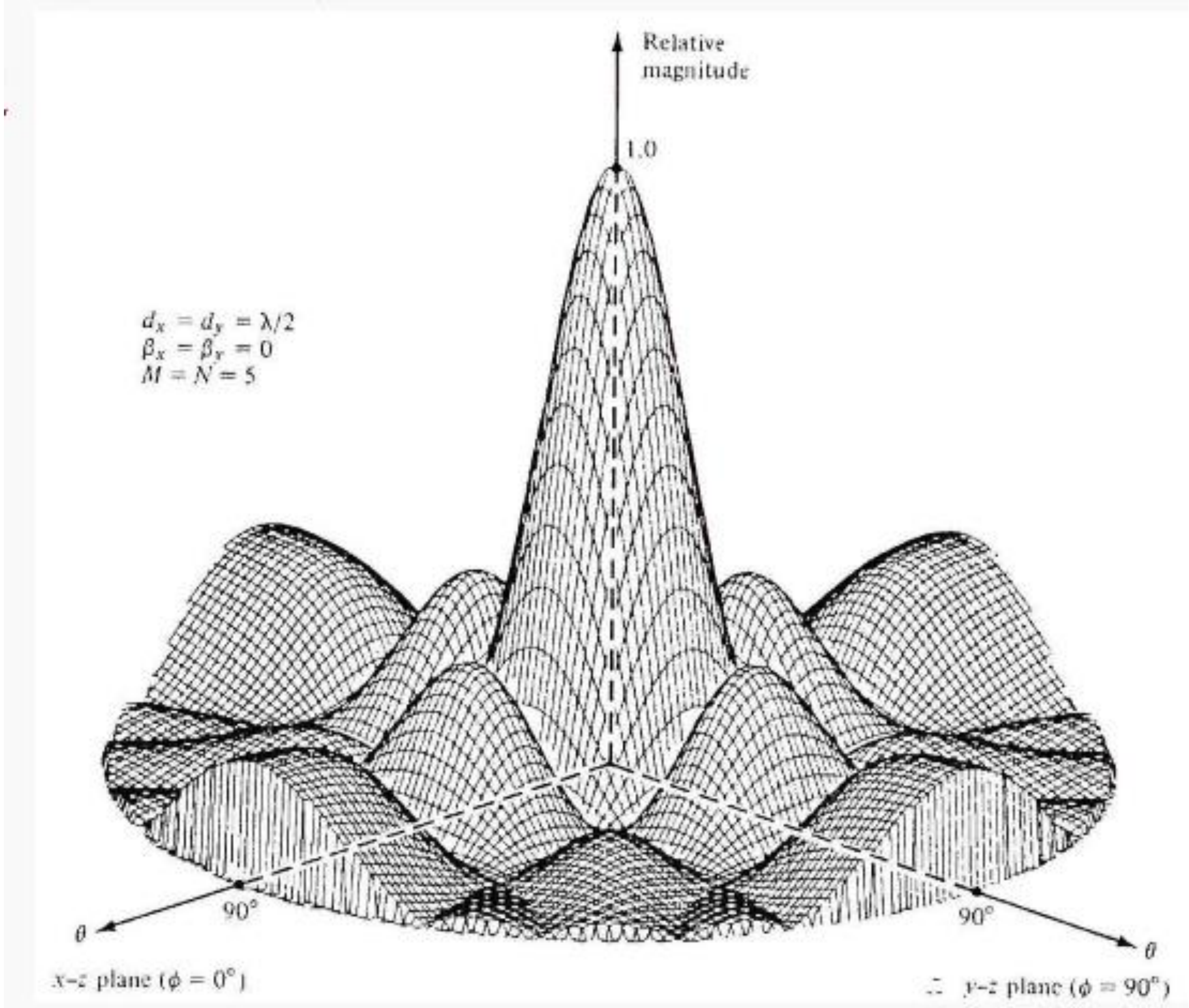
Array Pattern of an Array with d=λ/4 and Uniform Distribution



Array Factor of an Array with d=λ/2 and Uniform Distribution



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