2012 Qualifying Exam: Mathematics

Caution!!!
Use separate answer books for Problems 1-5 (Math.-A) and 6-7 (Math.-B).

Math.-A

Problem 1. (10 points) Find the limits.

(a) (5 points)

$$\lim_{x\to\infty} \left\{3^x + \left\{3^3\right\}^x\right\}^{\frac{1}{x}}$$

(b) (5 points)

$$\lim_{x\to 0} \frac{x + 2\sin(x)}{x\sqrt{x^2 + 2\sin(x) + 1} - \sqrt{\sin^2(x) - x + 1}}$$

Problem 2. (10 points) Differentiate

$$y = \frac{(x+2)^{3\ln x}}{(x^2+1)^{1/2}}.$$

Problem 3. (10 points) Find

$$\int e^x \cos x \, dx.$$

Problem 4. (10 points) Find the solution by utilizing Laplace transformation:

$$y^{(2)}(t) + 3y^{(1)}(t) + 2y(t) = e^{-2t},$$

where $y(0) = 0$ and $y^{(1)}(0) = 1$.

Problem 5. (10 points) Green's Formula says that

$$\oint_C \{Ldx + Mdy\} = \int\int_R \left(\frac{\partial M}{\partial x} - \frac{\partial L}{\partial y} \right) dxdy,$$

where C indicates the curve enclosing R, oriented counterclockwise. Let C be the circle $x^2 + y^2 = 4$, oriented counterclockwise. Use Green's Theorem to evaluate the following integral

$$\oint_C \{ \left(\cos(x^2) - y^3 \right) dx + x^3 dy \}$$

Caution!!!
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Math.-B

Problem 6. (30 points) Suppose that a linear system with input $\underline{x} \in \mathbb{R}^2$ and output $\underline{y} \in \mathbb{R}^3$ generates

$$\underline{y}_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad \text{and} \quad \underline{y}_2 = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$$

when it is driven by the input

$$\underline{x}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \text{and} \quad \underline{x}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix},$$

respectively. Answer the following questions.

(a) (5 points) Find the output of the system when the input is

$$\underline{v}_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad \text{and} \quad \underline{v}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix},$$

respectively.

(b) (5 points) Find the output of the system when the input is given by

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.$$

(c) (5 points) Find the set $\mathcal{N}$ of all possible inputs that result in the system output given by

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

(d) (5 points) Find the set $\mathcal{R}^{\perp}$ of all elements in $\underline{y} \in \mathbb{R}^3$ that cannot be obtained as the output of this system.

(e) (10 points) Find the maximum and the minimum values of $||\underline{y}||^2$, respectively, when the input satisfies $||\underline{x}||^2 = 1$.

Problem 7. (20 points) Answer the following questions.

(a) (5 points) Show that $y(t)$ is periodic if $x(t)$ is periodic and

$$y(t) = \int_{-\infty}^{\infty} h(t-\tau)x(\tau)d\tau$$

with $\int_{-\infty}^{\infty} |h(t)|d\tau < \infty$.

(b) (5 points) Show that

$$\int_{-\infty}^{\infty} x(t)y^*(t)dt = \int_{-\infty}^{\infty} X(f)Y^*(f)df,$$

where

$$X(f) = \int_{-\infty}^{\infty} x(t)e^{-\mathrm{j}2\pi \mathrm{f}\mathrm{t}}dt$$

and

$$Y(f) = \int_{-\infty}^{\infty} y(t)e^{-\mathrm{j}2\pi \mathrm{f}\mathrm{t}}dt$$

are the Fourier transforms of $x(t)$ and $y(t)$, respectively.

(c) (10 points) When

$$z(t) \triangleq \int_{0}^{T} x(t-\tau)y(\tau)d\tau$$

with

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{\mathrm{j}\frac{2\pi \mathrm{kt}}{\mathrm{T}}}$$

and

$$y(t) = \sum_{k=-\infty}^{\infty} b_k e^{\mathrm{j}\frac{2\pi \mathrm{kt}}{\mathrm{T}}}$$

for some $T > 0$, find c_k in terms of a_k and b_k to satisfy

$$z(t) = \sum_{k=-\infty}^{\infty} c_k e^{\mathrm{j}\frac{2\pi \mathrm{kt}}{\mathrm{T}}}.$$

QE: semiconductor [basic (60 points)]

physical constants

$q=1.6\times10^{-19}C$
$\varepsilon_0 = 8.8\times10^{-14}F/cm$
$\varepsilon_{r,Si} = 11.7$
$\varepsilon_{r,SiO2} = 3.9$
$kT=0.026eV$
$E_g^{Si} = 1.11\,eV$
$n_i = 1.5\times10^{10}\,/cm^3$

1. consider a silicon p-n junction with $N_a = 1\times10^{18}\,/cm^3$, $N_d = 3\times10^{18}\,/cm^3$;

   1) plot electron density (linear plot and semi-log plot) (5 points)
   2) plot (semi-log) hole density, N_a^- density, N_d^+ density and total charge density (15 points)

2. Fermi –Dirac distribution function ($N_a = 1\times10^{18}\,/cm^3$, silicon) ;

   1) plot function with T = 0[K] (5 points)
   2) plot function with T = 300 [K] and calculate values at $E = E_C$, $E = E_V$ (15 points)

3. consider a forward biased p-n junction ($N_a = 1\times10^{18}\,/cm^3$, $N_d = 1\times10^{20}\,/cm^3$, silicon) ;

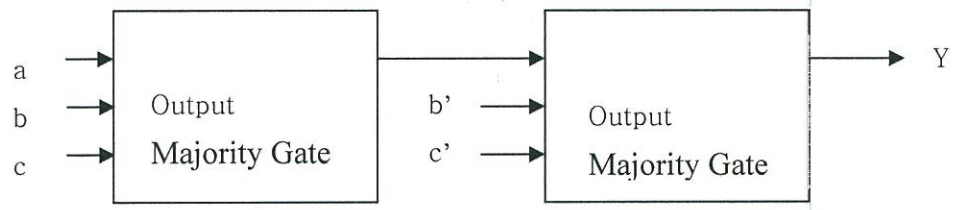
   1) plot a band diagram (including quasi-Fermi level) at V=0.5[V] (10 points)
   2) plot a band diagram (including quasi-Fermi level) at V=1.0[V] (10 points)

QE 2012 Computer Eng.

Digital

1. Using Boolean algebra, simplify: (10)
   F = x(y((yu')x')')'

2. 다음과 같이 2 개의 Majority Gate (다수의 입력이 1 일때만 출력이 1) 를 연결했을 때

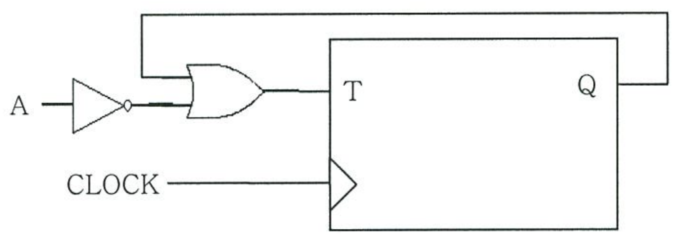


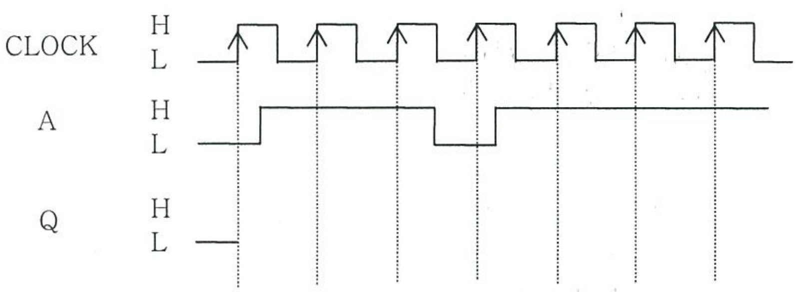
출력 Y 를 입력 a, b, c 의 함수로 표시하라. DO NOT USE KARNAUGH MAPS. (10)

3. Design a mod-66 counter using 2 4-bit binary counter chips and additional gate(s). Minimize the logic. (10)

4. A sequence detector has one input x and one output z. Any input sequence ending in 01 will produce an output z=1. Draw a Mealy state diagram for this detector. (10)

5. Given the circuit, complete the timing waveform. (10)





Microprocessors

1. What is programmer's model (programming model) in assembly coding? (10)

2. Explain paging mechanism utilizing two terms, page directory and page table. (15)

3. Explain how 'far call' is performed in Intel processors. (10)

4. Explain how (real mode) interrupt works in Intel processors. (15)

2012학년도 통신 및 신호 처리 분야 박사 자격 시험

1. (50 points)

The signals of a quaternary system are defined by

$$s_i(t) = (2i-5)\sqrt{\frac{2E_o}{T}}\cos(2\pi f_c t), \qquad 0 \le t \le T, \qquad i = 1, 2, \ldots, 4$$

where E_o is the energy of the signal with the lowest amplitude. Assume that each signal is transmitted equally likely and f_cT is an integer. The received signal is given by

$$x(t) = s_i(t) + w(t), \qquad 0 \le t \le T$$

where $w(t)$ is a sample function of a white Gaussian noise process of zero mean and power spectral density $N_0/2$. Justify all your answers.

(a) Determine the impulse response $h(t)$ of a filter matched to $s_3(t)$ such that $h(t)$ has unit energy, and sketch it as a function of time.

(b) Plot the signal constellation for the system and compute its average energy E.

(c) Find the mean and variance of the matched filter output at time $t = T$ due to the input $x(t)$, given that $s_3(t)$ was transmitted.

(d) Under the maximum-likelihood (ML) detection, find the average probability P_s of symbol error in terms of E/N_0 by using the Q-function

$$Q(x) = \int_x^\infty \frac{1}{2\pi} e^{-z^2/2} dz.$$

(e) Compute the conditional bit error probability $P_b(s_3)$, given that $s_3(t)$ was transmitted. Here, we employ the following mapping:

$$s_1(t) \leftrightarrow 01 \qquad s_2(t) \leftrightarrow 11 \qquad s_3(t) \leftrightarrow 10 \qquad s_4(t) \leftrightarrow 00$$

2. (50 points)

(a) Suppose a finite-duration sequence x[n] has {1, -1, 2} and another finite-duration input sequence h[n] is represented by {1, -2, 0, 2}. The 5-point DFT of x[n] and h[n] is denoted by X[k] and H[k], respectively. Find and sketch the inverse DFT of Y[k]=H[k]X[k].

(b) A signal consists of a linear combination of three sinusoids of frequencies of 4.00 kHz, 4.08 kHz, and 4.20 kHz. The signal is sampled at a rate of 32 kHz and L samples are collected. The L samples are then windowed by a Hamming window and the DFT of the result is computed. What is the minimum value of L in order that the DFT be capable of clearly resolving the three sinusoids that are present?

(Selection)

<u>QE Semiconductor : Electronic devices (40 points)</u>

1. (MOSFET) Fig. 1 shows an n-MOSFET with uniformly doped substrate.
   - **a)** (7 pts) Sketch the transverse electric field along the y direction when $V_{GS}-V_{TH}>0$, $V_D=V_S=V_B=GND$. Assume that the thickness of the inversion layer is 1/3 of that of the depletion layer. y(0) is just the interface between the gate oxide and the substrate.
   - **b)** (8 pts) Draw the channel potential along the x direction from the source to the drain when $V_{GS}=V_{DS}= V_{DD}$, $V_S=V_B=GND$. Assume a long channel device.
   - **c)** (10 pts) Describe how to improve the subthreshold swing (S) value in a short-channel planar device.

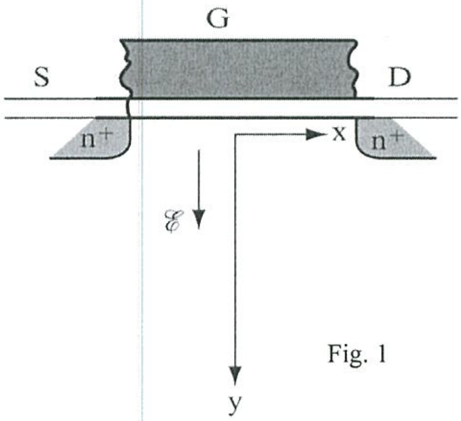


Fig. 1

2. (BJT) Consider a p^+np BJT with graded doping in the base region, as shown in Fig. 2. For an exponential grading profile, that is, $N(x_n)=Nd-Na=N(0)\exp[-ax_n/W_b]$,
   - **a)** (5 pts) Derive an expression for the built-in electric field.
   - **b)** (5 pts) Sketch the equilibrium band diagram of the base region.
   - **c)** (5 pts) Would the electric field in the base assist or oppose the electron flow? Provide the reason.

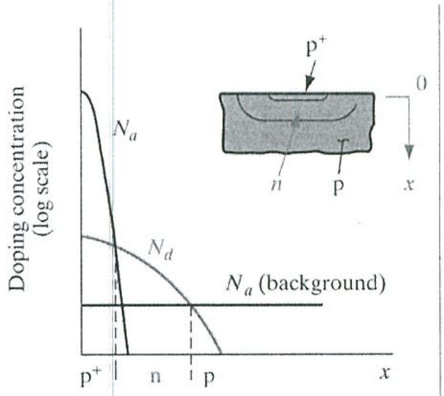


Fig. 2

(Selection)

<u>QE Semiconductor : Optical devices (40 pts)</u>

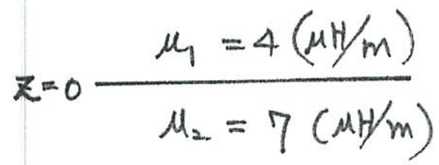
[1] Describe and explain the guided and radiation modes of a dielectric optical waveguide, using the ray optics theory. [20 pts]

[2] Discuss the band structure and density of states of graphene in comparison with those of conventional semiconductors with parabolic band structures. Also give some examples of the graphene applications for optical devices. [20 pts]

전자기학/초고주파 분야
2012 박사과정자격시험

1. [5점] Use the analogy between electrostatics and magnetostatics to find the defining equation for the magnetic field intensity $\overline{H}$ in a magnetic material with magnetization vector $\overline{M}$. The corresponding equation for the electric flux density is given by, $\overline{D} = \epsilon_0 \overline{E} + \overline{P}$.

2. [10점] Assuming that there is no surface current at the boundary, find $\overrightarrow{B_2}$ in medium 2 if $\overrightarrow{B_1} = \hat{x}2 + \hat{y}3 + \hat{z}4$ (mT).

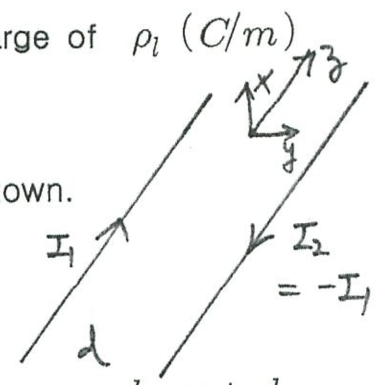


3. [15 점] (a) Define the electric field intensity $\overline{E}$.

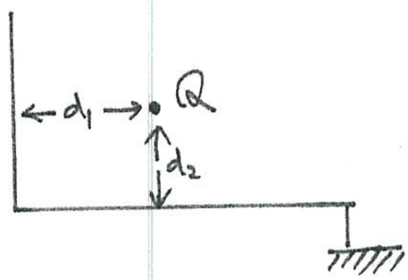
(b) Using Gauss's law, find $\overline{E}$ due to an infinite uniform sheet of charge ρ_s (C/m^2) located in free space.

(c) Using Gauss's law, find $\overline{E}$ due to an infinite line charge of ρ_l (C/m) located in free space.

4. [5점] Find the force between two long parallel wires shown.



5. [15 점] A positive point charge Q is located at distances d_1 and d_2, respectively, from the grounded perpendicular conducting half-planes, as shown. Use the method of images to determine the force on Q caused by the charges induced on the planes.



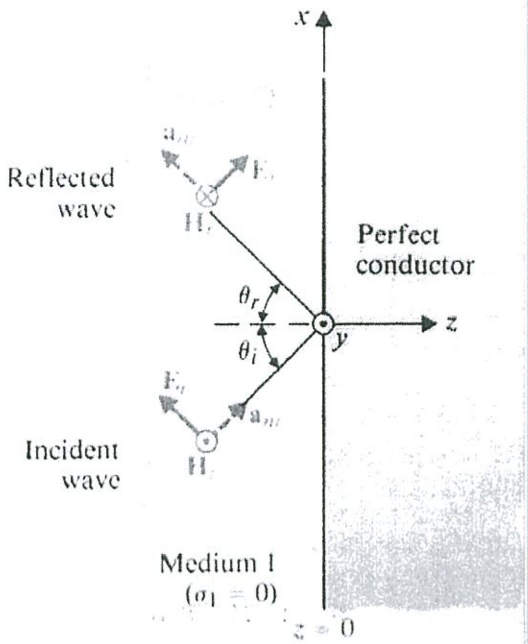
6. [25점] In a simple (linear, isotropic and homogeneous), nonconducting source-free medium

(a) [10점] Write down the time-harmonic Maxwell's equation in the medium.

(b) [15점] Derive the homogeneous vector Helmholtz's equation.

(Hint: $\nabla\times\nabla\times\vec{A}=\nabla(\nabla\cdot\vec{A})-\nabla^2\vec{A}$)

7. [25점] A plane wave in the free space is incident on conducting boundary in the following figure.



(a) [5점] Determine whether the above configuration is perpendicular or parallel polarization.

(b) [20점] Write down the mathematical expression for the reflected electric field $\vec{E_r}(x,z)$ and magnetic field $\vec{H_r}(x,z)$ in phasor form when the plane wave is incident on the above boundary.

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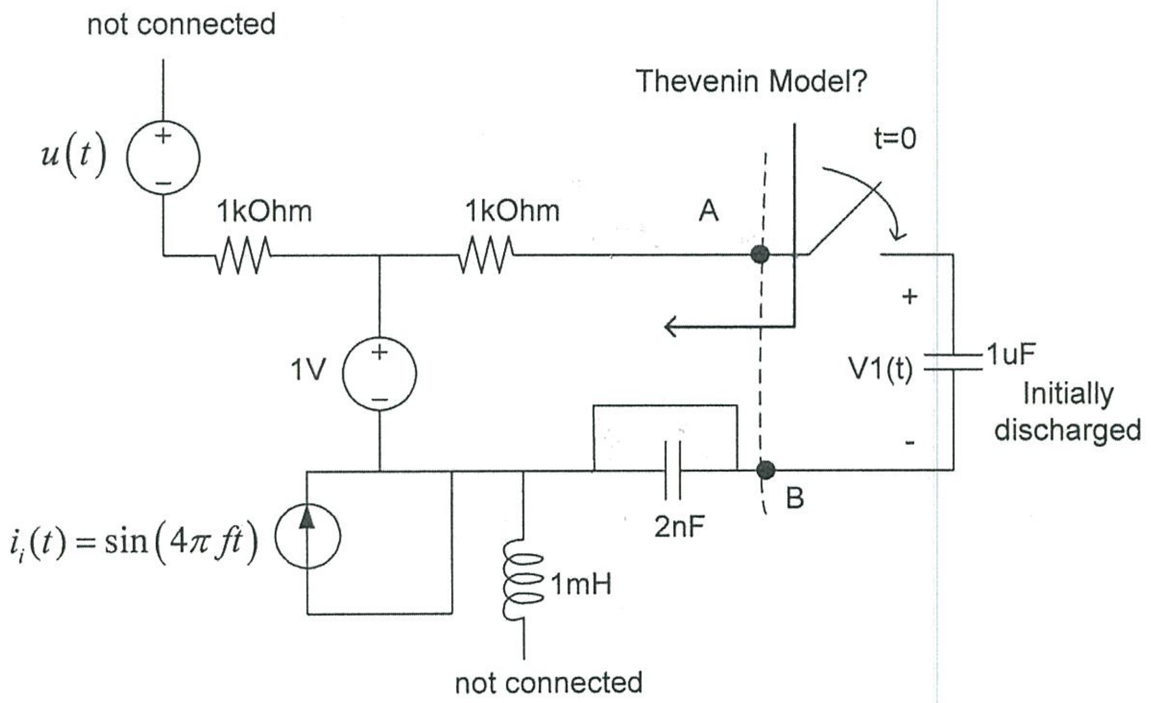
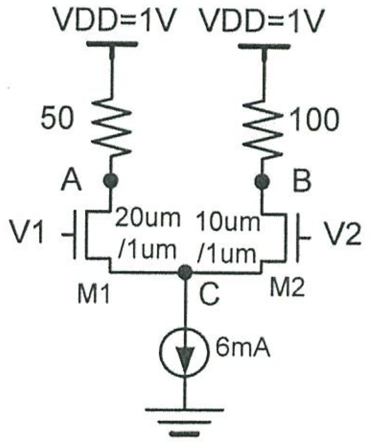
[1] 다음의 회로도에서, V1(t) 값이 언제 0.5V가 되는지를 찾으시오. u(t)는 unit step function. 모든 케퍼시터는 초기에 완전 방전되었음 (15점)



(1) AB점선의 왼쪽회로의 Theven 등가회로를 찾고 그리시오 (5점).

(2) V1(t)는 언제 0.5V가 되는가? (10점)

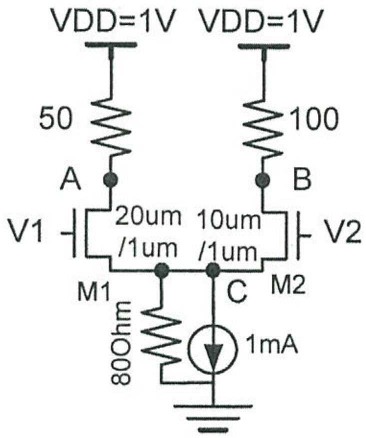
[2] 그림의 NMOS 트렌지스터는 다음과 같은 파라미터값을 가진다: $\mu_n C_{ox}$ =0.01A/V^2, V_{TH}=0.2V. 체널 길이 모듈레이션(Channel Length Modulation)과 바디 이펙트(Body Effect)를 무시하고, 다음의 회로를 (1), (2), (3), 지시문에 따라 분석하시오. (35점).



(1) V_1=V_2=0.8V일때, 문제의 회로의 DC 동작점(V_A, V_B, V_C)들을 구하시오. (10점).

(2) V_1= 0.8+v_a , V_2= 0.8-2v_a 일 때, 차등 소신호 출력 v_{ab} (A와B노드사이의 차등 소신호 전압)을 v_a의 식으로 나타내시오. (10점)

(3) 주어진 그림의 커런트 소스를 다음의 그림처럼 바꾸었다: (80옴 저항과 1mA커런트 소스의 병렬연결). 소신호 차등 입력 (V_1= 0.8+0.5v_{diff}, V_2= 0.8-0.5v_{diff})에 대해서 차등 전압 이득 (=v_{ab}/v_{diff}) 을 구하시오 (15점).



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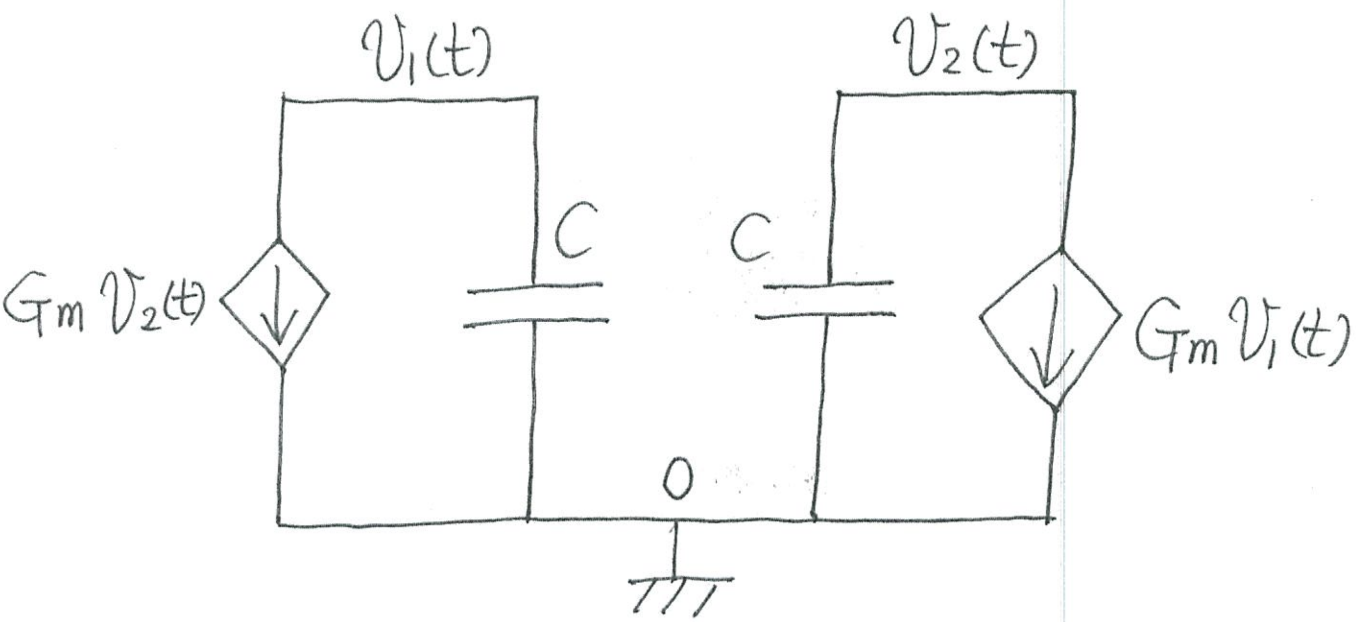
[3] 다음 소신호 등가회로는 디지털 인버터를 서로 연결한 경우(cross-coupled inverter)를 나타낸다. 초기조건은 다음과 같다.

v1(t=0) = vc + 0.5 vd , v2(t=0) = vc – 0.5 vd (20점)

(1) v1(t)와 v2(t)를 구하기 위한 시간영역 연립미분 방정식을 구하시오. (5점)

(2) (1)에서 구한 시간영역 연립미분방정식을 Laplace 변환 형태로 바꾸어, V1(s)와 V2(s) 식을 각각 구하시오. 이 때 t=0에서의 초기조건을 고려하시오. (8점) Hint: vc, 0.5 vd, s+(Gm/C), s-(Gm/C) 의 식으로만 표시하시오.

(3) (2)의 결과를 이용하여 t>0 에 대한 시간영역 v1(t)와 v2(t)의 식을 구하시오. (7점)



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[4] 정상상태(steady state)에서 동작하는 다음 회로에 대해 물음에 답하시오. (30점)

V1(t) = 100 sin (500 t) [V], L1=10 Henry, L2 = 0.1 Henry, M=1 Henry, R1=5K Ohm, R2= 50 Ohm, t in sec.

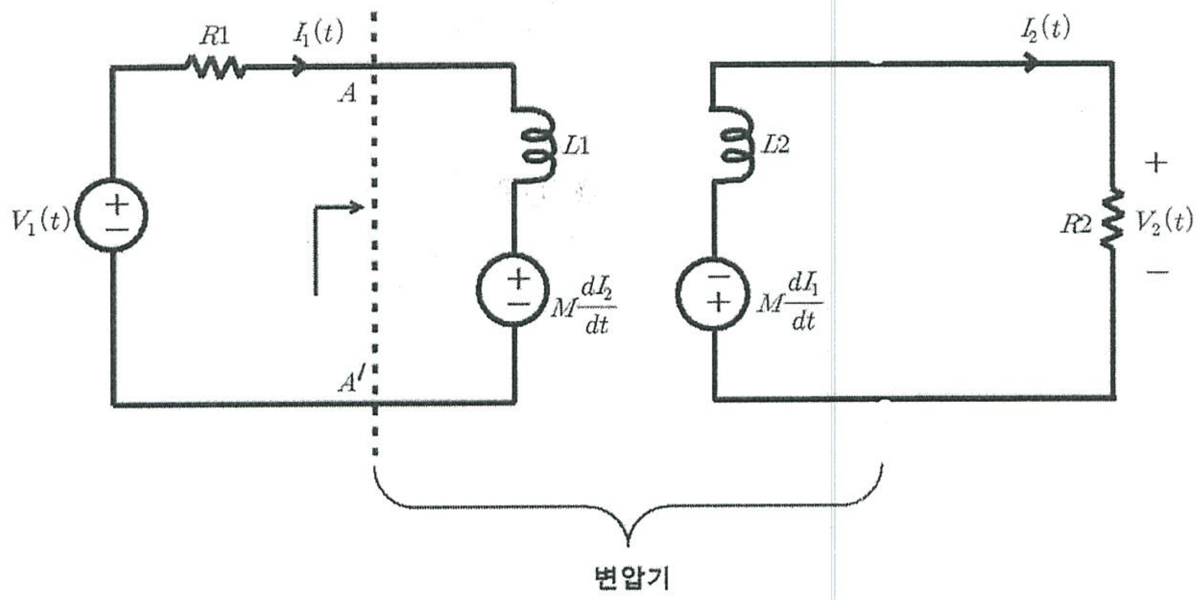
Hints: $\tan^{-1}(1)=0.785\text{rad}=45^\circ$, $\tan^{-1}(2)=1.107\text{rad}=63.4^\circ$,
sqrt(0.4)=0..632, sqrt(1/2)=0.707

(1) A-A′ 에서 오른쪽으로 들여다 본 등가 임피던스를 인덕터(L) 한 개와 저항(R) 한 개로 구성된 등가회로로 표시하시오. (8점) Note M = sqrt(L1 x L2)
(힌트: d/dt를 jw로 바꾸고, VAA′을 I1의 식으로 표시한 후, I1/VAA′을 구하시오.)

(2) (1)의 결과를 이용하여, $I_1(t)$ 식을 구하시오. (7점)

(3) V2(t) 식과 V2(t)의 rms (root mean square) 전압값을 각각 구하시오. (7점)

(4) V1(t) 가 공급한 평균 power, R1이 소모한 평균 power, R2가 소모한 평균 power를 각각 Watt 단위로 구하시오.(8점)



POHANG UNIVERSITY OF SCIENCE AND TECHNOLOGY

Department of Electrical Engineering

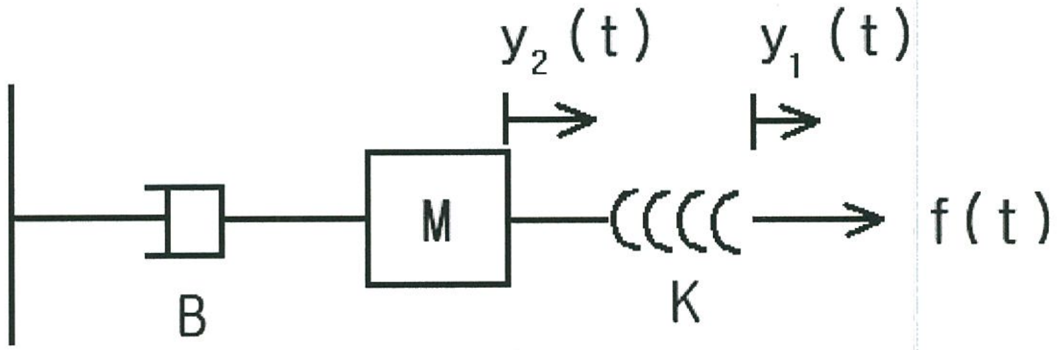
Automatic Control Qualifying Exam

Aug. 2012

(제어필수는 반드시 풀고 제어선택과 전력전자 중 한 분야를 선택해서 푸시오)

제어필수 (Control requirement) (50 points)

1. Consider a Mechanical system with translational motion shown below.
   f(t) is an external force which is an unit step input.



a) Find the differential equations of the system.
b) Establish a state equations of the system.
c) Find the transfer function of the system.
d) Let M=B=K=1, and let all initial conditions be zero. find the solution.
e) can you implement the unit step input f(t) in terms of $y_1(t)$?

전력전자 문제 (선택)
Power Electronics (Selection)

1. (20 points)

1) Explain the operation principle of the boost converter and derive the input-output voltage relation in the discontinuous conduction mode. (10 points)

2) Explain the unipolar PWM in the full-bridge dc-to-dc converter and show its advantages compared to the bipolar PWM. Derive the rms output voltage of the bipolar PWM and unipolar PWM, respectively. (10 points)

2. Consider the space vector modulation technique for the voltage source inverter with the dc-link voltage V_d. (30 points)

1) Derive the space vector modulation and then express the duty ratios of two nonzero space vectors and a zero vector. (20 points)

2) Explain the advantages of the space vector modulation comparing with the sinusoidal PWM (SPWM). (10 points)

제어 문제 (선택)
Control (Selection)

1. $\dot{x}(t) = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t), \quad y(t) = [1 \quad 1]x(t).$

Let $u(t) = -Kx(t) + pv(t) = -[k_1 \quad k_2]x(t) + pv(t)$.

1) Find the state feedback gain $K = [k_1 \quad k_2]$ such that the eigenvalues of the closed loop system are $-2 \pm j2$. (20)

2) Find p such that the steady state error $(\lim_{t\to\infty}(v(t) - y(t)))$ of the closed loop system with K of 1) is zero when $v(t)$ is an unit step function. (15)

3) Find the steady state output $y_s(t)$ of the closed loop system with K of 1) and p of 2) when $v(t) = \sin(t + \frac{1}{4}\pi)$. (15)