Caution!!!
Use separate answer books for Problems 1-2 (Math.-A) and for 3-5 (Math.-B).

3. (15 points) Consider

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 2 & -2 & 1 \end{bmatrix}.$$

(1) (5 points) Find the characteristic equation.

(2) (5 points) Find the inverse of A.

(3) (5 points) Find A^{10}.

4. (15 points) Suppose that a linear system with input $x_i \in \mathcal{R}^2$ and output $y_i \in \mathcal{R}^3$ generates the output

$$y_1 = \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix} \text{ and } y_2 = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$$

when it is given by the input

$$x_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \text{ and } x_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix},$$

respectively.

(a) (10 points) Find the output of the system when the input is given by $\begin{bmatrix} u_1 & u_2 \end{bmatrix}^T$.

(b) (5 points) Find the set of all possible inputs that generates the zero output $\begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$.

5. (20 points) Let $f(t)$ be a function of period $T = 2$ with

$$f(t) = t^2 \text{ if } t \in [0, 2].$$

(a) (10 points) Find the Fourier series coefficients, c_n, of $f(t)$.
Remark: You must carefully consider the cases where $n = 0$ and $n > 0$.

(b) (10 points) Using the result in (a), compute

$$\sum_{n=1}^{\infty} \frac{1}{n^2}.$$