Chapter 7
GREEN'S FUNCTIONS

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Math/Physics
"Self-Taught" Education
Green's Functions
Math for EM Waves

Lecture 23

7.1 Green's Function : Superposition of Impulse Responses

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In physics and electrical engineering, the Green's function is one of the most important mathematical tools. There are many different mathematical approaches to define the Green's function. However, here we take a **physics approach** which is very simple and clear:

ex) $L = (\nabla^2 + k^2)$

$$Lf(x) = s(x) \tag{7.1}$$

where L is a **linear** differential operator, $f(x)$ is a classical or quantum mechanical field, and $s(x)$ is a source function.

For a linear system, we can use the **superposition principle** along with the **sifting property** of the Dirac delta function:

$$\boxed{s(x) = \int dx' \, s(x')\delta(x - x')} \tag{7.2}$$

field point

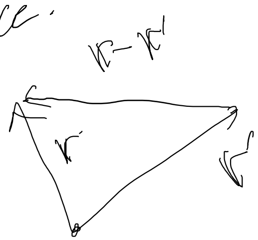
This means that we can first solve the **impulse response** or the **Green's Function** $g(x, x')$,

source point

$$\boxed{Lg(x, x') = -\delta(x - x')} \tag{7.3}$$

and then we can find the solution*,

add all source.



$$\boxed{f(x) = \int dx' \, g(x')\delta(x - x')} \tag{7.4}$$

This is just a **particular solution**. For the complete solution, we have to need to add a **homogeneous solution**.