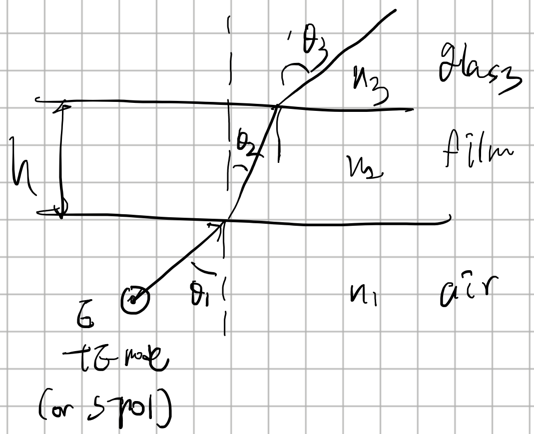
i) single interface

ii) two interfaces (single film)



$\beta = k_0 n_2 h \cos\theta_2$

$p_1 = n_1 \cos\theta_1$

$p_2 = n_2 \cos\theta_2$

$p_3 = n_3 \cos\theta_3$

$$\overleftrightarrow{M} = \begin{pmatrix} \cos\beta & \frac{-i}{p_2}\sin\beta \\ -ip_2\sin\beta & \cos\beta \end{pmatrix}$$

$$r_{13} = \frac{(\cos\beta - \frac{i}{p_2}\sin\beta\, p_3)p_1 - (-i p_2 \sin\beta + \cos\beta\, p_3)}{(\cos\beta - \frac{i}{p_2} p_3 \sin\beta) p_1 + (-i p_2 \sin\beta + p_3 \cos\beta)} = \frac{r_{12} + r_{23} e^{i2\beta}}{1 + r_{12} r_{23} e^{i2\beta}} \neq r_{12} + r_{23}$$

$$t_{13} = \frac{t_{12} t_{23} e^{i\beta}}{1 + r_{12} r_{23} e^{i2\beta}} \neq t_{12} t_{23}$$

$$R_\perp = |r_{13}|^2 = \frac{r_{12}^2 + r_{23}^2 + 2 r_{12} r_{23} \cos 2\beta}{1 + r_{12}^2 r_{23}^2 + 2 r_{12} r_{23} \cos 2\beta}$$

$$T_\perp = \frac{p_3}{p_1} |t_{13}|^2$$

Geometry

$H = n_0 h$: optical thickness

Let's find H for which the reflectivity has a max or min.

$$\frac{\partial R_\perp}{\partial H} = 0 \rightarrow \sin 2\beta = 0, \quad 2\beta = 2\frac{2\pi}{\lambda_0} n_2 h \cos\theta_2 = m\pi, \quad m = 0, 1, 2, 3 \ldots$$

$$\text{or } H = n_0 h = m\frac{\lambda_0}{4} \frac{1}{\cos\theta_2}$$

a) $m = 2m'$, or $\beta = m'\pi$, $m' = 0, 1, 2, \cdots$

$$R_{+} = \left(\frac{r_{12} + r_{23}}{1 + r_{12} r_{23}}\right)^2 = \left(\frac{n_1 \cos\theta_1 - n_3 \cos\theta_3}{n_1 \cos\theta_1 + n_3 \cos\theta_3}\right)^2$$

Snell's law $\Rightarrow$ no influences by the film

b) $2\beta = m\pi$, $m = 1, 3, 5, \cdots$ $(m = 2m' + 1)$

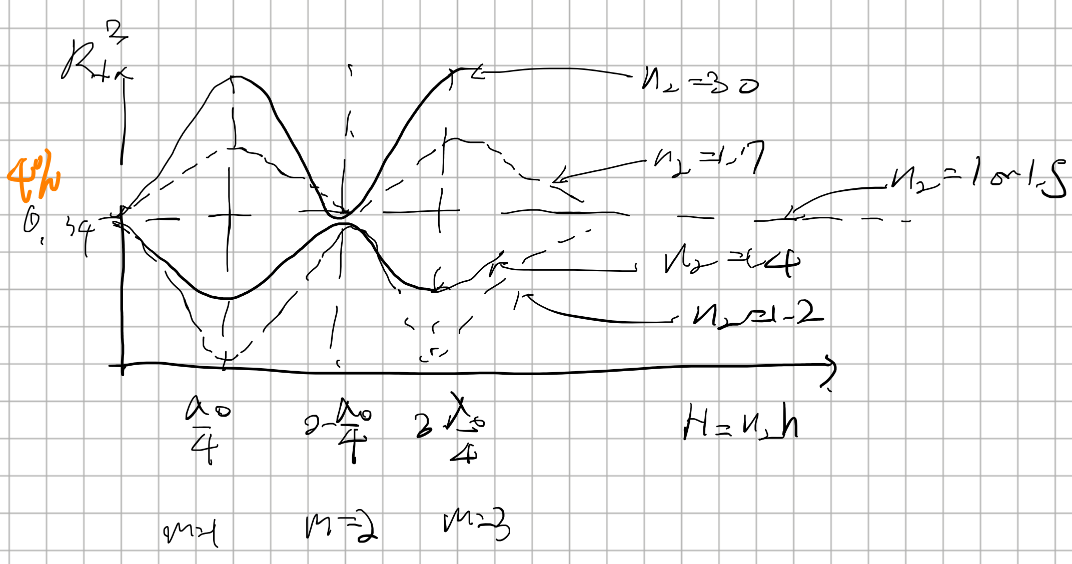
$$R_{+} = \left(\frac{r_{12} - r_{23}}{1 - r_{12} r_{23}}\right)^2$$

$\frac{d^2 R_{+}}{dH^2}$ $>$ $\rightarrow$ min, $m=1$ $n_1 < n_2 < n_3$

$<$ $\rightarrow$ max $n_2 < n_1$ or $n_2 > n_3$ for $n_1 < n_3$

For normal incidence $R_{\perp} = (\quad)^2 = \left(\frac{n_1 n_3 - n_2^2}{n_1 n_3 + n_2^2}\right)^2$

$R_{+} = 0$ for $n_2^2 = n_1 n_3 \simeq 1.23$ for $n_1 = 1$, $n_3 = 1.52$ glass



4.4 EM waves in conducting media

free electrons in a conducting medium their macro scopic response: σ (conducting)

$$J = \sigma E$$

A) propagation

$$\vec{J} = \sigma \vec{E}, \quad \vec{D} = \varepsilon \vec{E}, \quad \vec{B} = \mu \vec{H}$$

$$\nabla \times \vec{H} - \frac{\varepsilon}{c}\frac{\partial \vec{E}}{\partial t} = \frac{4\pi}{c}\sigma \vec{E}$$

$$\nabla \times E + \frac{\mu}{c}\frac{\partial \vec{H}}{\partial t} = 0$$

$$\nabla \cdot \vec{E} = \frac{4\pi}{\varepsilon}\rho$$

$$\nabla \cdot \vec{H} = 0$$

$$-\frac{\varepsilon}{c}\frac{\partial}{\partial t}\nabla \cdot E = \frac{4\pi}{c}\sigma \overbrace{\frac{4\pi}{\varepsilon}\rho}^{\partial E}$$

$$\frac{\partial \rho}{\partial t} + \frac{4\pi\sigma}{\varepsilon}\rho = 0, \quad \rho = \rho_0 e^{-t/\tau}, \quad \tau = \frac{\varepsilon}{4\pi\sigma}$$

$$\nabla \times (\nabla \times \vec{E}) + \frac{\mu}{c}\frac{\partial}{\partial t}(\nabla \times H) = 0, \quad \nabla \cdot E = 0, \; E = E_0 e^{-i\omega t}$$

$$\nabla^2 \vec{E} + \underbrace{\frac{\omega^2}{c^2}\mu\left(\varepsilon + i\frac{4\pi\sigma}{\omega}\right)}_{k^2}\vec{E} = 0$$

$$[\nabla^2 + k^2]\vec{E} = 0$$ the same equation as in dielectric media

$\hat{\varepsilon} = \varepsilon + \underbrace{i\frac{4\pi\sigma}{\omega}}$ the con

$\hat{n} = \sqrt{\hat{\varepsilon}\mu} = n(1+i\beta)$

$E = e^{i(\vec{k}\cdot\vec{r}-\omega t)} = E_0 \underbrace{e^{-\frac{\omega}{c}n\beta\hat{k}\cdot\vec{r}}}_{\text{attenuation.}} \; e^{i\omega[\frac{n}{c}\hat{k}\cdot\vec{r}-t]}$

intensity $I \propto E^2 \propto e^{-\underbrace{2\frac{\omega}{c}n\beta}_{\alpha}\hat{k}\cdot\vec{r}} = e^{-\alpha z}$ $\quad \hat{k} = \hat{z}$

$\delta = \frac{1}{\alpha}$ absorption length (skin depth)

$= \frac{\lambda_0}{2\pi n\beta} \sim \frac{1}{\sqrt{8\pi\mu\sigma\omega}} \sim 6\,nm$

$\left(n\beta \sim \sqrt{\frac{2\pi\mu\sigma}{\omega}} \quad \text{if } \sigma \gg \omega\right)$

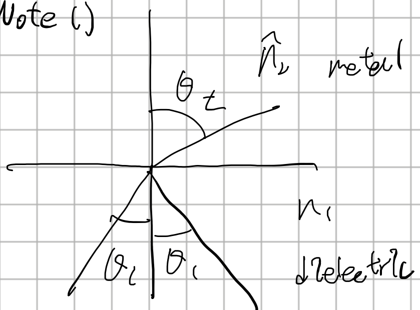
B) Reflection & Refraction

Base equations are the same

boundary conditions are the same

→ the same formula can be used except that the index of refraction is complex

(Note 1)



Snell's law

$$\hat{n}_2 \sin\theta_t = n_1 \sin\theta_i$$

$$R_\parallel = \frac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)} A_\parallel$$

$$R_\perp = \frac{-\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)} A_\perp$$

$\hat{n}_2$ complex

θ_t complex

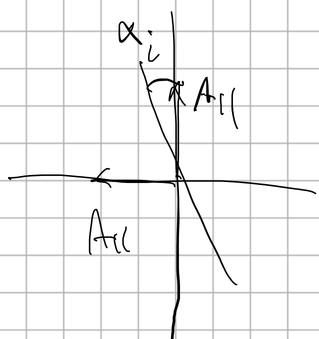
$R_\parallel / A_\parallel$, $R_\perp / A_\perp$ complex

↓

extra phase introduced

linear pol ⇄ elliptical pol.

$$r_\parallel = \frac{R_\parallel}{A_\parallel} = \rho_\parallel e^{i\phi_\parallel}, \quad r_\perp = \rho_\perp e^{i\phi_\perp}$$



$$\tan\alpha_i = \frac{A_\perp}{A_\parallel}$$

$$\tan\alpha_r = \frac{R_\perp}{R_\parallel} = -\frac{\cos(\theta_i - \theta_t)}{\cos(\theta_i + \theta_t)} \tan\alpha_i$$

$$= \rho e^{i\Delta} \tan\alpha_i$$

$$\rho = \rho_\perp / \rho_\parallel, \quad \Delta = \phi_\parallel - \phi_\perp$$

If $\delta = \pi/2$, $\rho \tan\alpha_i = 1$

$$\tan\alpha_r = \frac{R_\perp}{R_\parallel} = -i$$ → linear pol ⟷ circular pol.

Note 2.

Measurement of the amplitude and phase of reflected light from a conductor

$\rightarrow$ n and β determined.

$$\frac{1-\rho e^{i\Delta}}{1+\rho e^{i\Delta}} = -\frac{\cos\theta_i \cos\theta_t}{\sin\theta_i \sin\theta_t} = -\frac{\sqrt{\hat{n}^2 - \sin^2\theta_i}}{\sin\theta_i \tan\theta_i}$$

$$\rho = \tan\psi, \quad \hat{n}^2 = n^2(1-\beta^2) + \sin^2\beta$$

$$n^2(1-\beta^2) = \sin^2\theta_i \left\{ 1 + \frac{\tan\theta_i (\cos^2 2\psi - \sin^2 2\psi \sin^2\Delta)}{(1+\sin(2\psi)\cos\Delta)^2} \right\}$$

$$2n^2\beta = \frac{\sin^2\theta_i \tan^2\theta_i \sin 4\psi \sin\Delta}{(\qquad\qquad)}$$

Note 3.

For normal incidence

$$R = \left|\frac{\hat{n}-1}{\hat{n}+1}\right|^2 = \frac{n^2(1+\beta^2)+1-2n}{n^2(1+\beta^2)+1+2n} \sim \frac{2\cdot\frac{2\pi\sigma}{\omega}+1-2\sqrt{\frac{2\pi\sigma}{\omega}}}{2\cdot\frac{2\pi\sigma}{\omega}+1+2\sqrt{\frac{2\pi\sigma}{\omega}}}$$

$$\simeq 1-2\sqrt{\frac{\omega'}{2\pi\sigma}}$$

$$\sim \begin{cases} 0.96 & Ag \\ 0.90 & Al \end{cases} \quad \text{at } \lambda = 600\,nm$$

opaque.

cf. $n \sim n\beta \sim \sqrt{\frac{2\pi\mu\sigma'}{\omega}}$

for $\frac{4\pi\sigma\mu}{\omega} \gg 1$