2010 수학

1. $s^2Y - sy(0) - y'(0) + 4Y = \frac{s}{s^2+9}$

$s^2Y - 2s + 4Y = \frac{s}{s^2+9}$ $\qquad Y(s)(s^2+4) = 2s + \frac{s}{s^2+9}$

$Y(s) = 2\cdot\frac{s}{s^2+4} + \frac{s}{(s^2+4)(s^2+9)}$,

$\frac{s}{(s^2+4)(s^2+9)} = \frac{As+B}{s^2+4} + \frac{Cs+D}{s^2+9} \Rightarrow As^3+9As+Bs^2+9B+Cs^3+Ds^2+4Cs+4D$

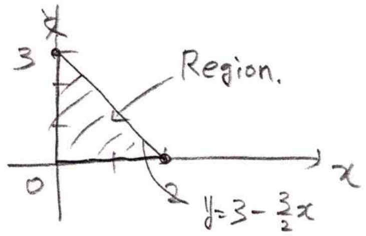
$= (A+C)s^3 + (B+D)s^2 + (9A+4C)s + 9B+4D = s.$

$A+C=0$
$9A+4C=1$ $\qquad A=\frac{1}{5},\ C=-\frac{1}{5},\ B=0,\ D=0$

$Y(s) = 2\cdot\frac{s}{s^2+4} + \frac{1}{5}\cdot\frac{s}{s^2+4} - \frac{1}{5}\cdot\frac{s}{s^2+9}$

$\therefore y(t) = \left(\frac{11}{5}\cos 2t - \frac{1}{5}\cos 3t\right)u(t)$

2.



$L = 3x^2+4xy+y^2$
$M = 5x+2x^2.$

$\iint_R 4x+5-2y-4x\,dxdy = \iint_R 5-2y\,dxdy = \int_0^2\int_0^{3-\frac{3}{2}x} 5-2y\,dy\,dx.$

$= \int_0^2 \left[5y-y^2\Big|_0^{3-\frac{3}{2}x}\right]dx = \int_0^2 15-\frac{15}{2}x-\frac{9}{4}x^2+9x-9\,dx$

$= \int_0^2 -\frac{9}{4}x^2+\frac{3}{2}x+6\,dx = \left[-\frac{3}{4}x^3+\frac{3}{4}x^2+6x\Big|_0^2\right]$

$= -\frac{3}{4}\cdot 8 + \frac{3}{4}\cdot 4 + 12 = -6+3+12 = 9$

3. (a) $\lim_{x \to 1} \frac{x^{\frac{1}{4}}-1}{x^{\frac{1}{3}}-1}$, letting $x-1=t$, $\lim_{t \to 0} \frac{(t+1)^{\frac{1}{4}}-1}{(t+1)^{\frac{1}{3}}-1} = \lim_{t \to 0} \frac{\frac{1}{4}(t+1)^{-\frac{3}{4}}}{\frac{1}{3}(t+1)^{-\frac{2}{3}}} = \frac{3}{4}$

(b) $\lim_{x \to 0} \frac{\sin 5x}{5x} \cdot \frac{5}{3} = \frac{5}{3}$

4. $\ln y = x^3 \ln(\sin x)$, $\frac{y'}{y} = 3x^2 \ln(\sin x) + \frac{x^3 \cos x}{\sin x}$

$\therefore y' = (\sin x)^{x^3} \left(3x^2 \ln(\sin x) + \frac{x^3 \cos x}{\sin x} \right)$

5. $\int \frac{3x+6}{x^2+5x+4} dx = \int \frac{3x+6}{(x+1)(x+4)} dx = \int \frac{A}{x+1} + \frac{B}{x+4} dx$

$A+B=3$
$4A+B=6$ $\quad A=1, \; B=2$

$\therefore \int \frac{1}{x+1} + \frac{2}{x+4} dx = \ln(x+1) + 2\ln(x+4) + C$

6. (a) F

(b) F

(c) F

(d) F

(e) T

17. (a) $|\lambda I - A| = \begin{vmatrix} \lambda-1-\sin^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \lambda-1-\cos^2\theta \end{vmatrix} = \lambda^2 - \lambda - \lambda\cos^2\theta - \lambda + 1 + \cos^2\theta - \lambda\sin^2\theta$
$+ \sin^2\theta + \sin^2\theta\cos^2\theta - \cos^2\theta\sin^2\theta$

$= \lambda^2 - \lambda(1+\cos^2\theta+1+\sin^2\theta) + 1 + \cos^2\theta + \sin^2\theta$

$= \lambda^2 - 3\lambda + 2$

$\therefore$ $a=1, \; b=-3, \; c=2$

(b) $|\lambda I - A| = \lambda^2 - 3\lambda + 2 = 0 \Rightarrow \lambda_1 = 1,\ \lambda_2 = 2.$

$\lambda_1 = 1$ case

$$\begin{bmatrix} -\sin^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & -\cos^2\theta \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

$-\sin^2\theta\, x_1 + \cos\theta\sin\theta\, x_2 = 0$

$\cos\theta\sin\theta\, x_1 - \cos^2\theta\, x_2 = 0$

$$\therefore \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} = \underline{X_1}$$

$\lambda_2 = 2$ case

$$\begin{bmatrix} 1-\sin^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & 1-\cos^2\theta \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\cos^2\theta\, x_1 + \cos\theta\sin\theta\, x_2 = 0$

$\cos\theta\sin\theta\, x_1 + \sin^2\theta\, x_2 = 0$

$$\therefore \begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix} = \underline{X_2}$$

$\|X_1\| = \|X_2\| = 1$, $\langle X_1, X_2 \rangle = 0$ 이므로 X_1, X_2 are orthonormal eigenvector

(c) 케일리-해밀턴 정리에 의하면

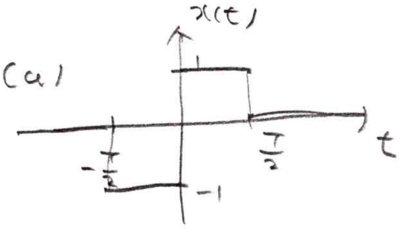
$A^2 - (1+\sin^2\theta + 1 + \cos^2\theta)A + (1 + \cos^2\theta + \sin^2\theta + \cos^2\theta\sin^2\theta - \cos^2\theta\sin^2\theta)I =$

$A^2 - 3A + 2I = 0$, $A^2 = 3A - 2I$, $A^3 = 3A^2 - 2A$.

$A^3 - 3A^2 + 2A = A^3 - 3(3A - 2I) + 2A = A^3 - 7A + 6I = 0$

$\therefore \underline{d = -7,\ e = 6}$

8. (a)



$$X(f) = -\int_{-\frac{T}{2}}^{0} e^{-j2\pi ft}\,dt + \int_{0}^{\frac{T}{2}} e^{-j2\pi ft}\,dt = \frac{1}{j2\pi f} e^{-j2\pi ft}\Big|_{-\frac{T}{2}}^{0} - \frac{1}{j2\pi f} e^{-j2\pi ft}\Big|_{0}^{\frac{T}{2}}$$

$$= \frac{1-e^{j\pi fT}}{j2\pi f} - \frac{e^{-j\pi fT}-1}{j2\pi f} = \frac{1}{j\pi f} - \frac{1}{j2\pi f}\left(e^{j\pi fT} + e^{-j\pi fT}\right) = \underline{\frac{1}{j\pi f}(1-\cos \pi fT)}$$

(b) $$\sum_{n=-\infty}^{\infty} x(t-nT) = \sum_{k=-\infty}^{\infty} a_k e^{j\frac{2\pi}{T}tk}, \quad a_k = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x(t) e^{-j2\pi\frac{k}{T}t}\,dt = \frac{1}{T}X\left(\frac{k}{T}\right),\ a_0 = 0$$

$$\therefore \underline{\sum_{n=-\infty}^{\infty} x(t-nT) = \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \frac{1}{j\pi k}(1-\cos \pi k)\, e^{j\frac{2\pi}{T}kt}}$$

(c) $$(1-\cos \pi k) = \begin{cases} 0 & , k: \text{even} \\ 2 & , k: \text{odd.} \end{cases}$$

$$\sum_{n=-\infty}^{\infty} x(t-nT) = \frac{2}{j\pi}\Big(\frac{1}{1}e^{j\frac{2\pi}{T}t} + \frac{1}{3}e^{j\frac{2\pi}{T}\cdot 3t} + \frac{1}{5}e^{j\frac{2\pi}{T}5t} + \cdots$$

$$+ \left(-\frac{1}{1}e^{-j\frac{2\pi}{T}t} - \frac{1}{3}e^{-j\frac{2\pi}{T}\cdot 3t} - \frac{1}{5}e^{-j\frac{2\pi}{T}\cdot 5t} \cdots\right)\Big)$$

$$t = \frac{T}{4} \Rightarrow \quad 1 = \frac{2}{\pi}\left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots + 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots\right)$$

$$= \frac{4}{\pi}\left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots\right)$$

$$\therefore \underline{1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots = \frac{\pi}{4}}$$

2010 통신

1.

(1) $\int_0^T |s_i(t)|^2 dt = E_s$. $\Rightarrow$ $\phi_i(t) = \frac{1}{\sqrt{E_s}} s_i(t)$ 이면 $\int_0^T |\phi_i(t)|^2 dt = 1$ 이므로

$s_i(t) = \sqrt{E_s}\,\phi_i(t)$ $\quad \therefore \underline{A = \sqrt{E_s}}$

(2) For example,

$\phi_1(t) = \sqrt{\frac{2}{T}} \cos 2\pi f_1 t \quad (0 \le t \le T)$ $\quad (f_1 = \frac{1}{T})$

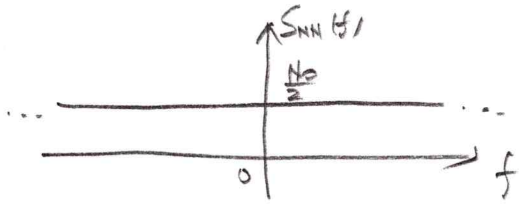
$\phi_2(t) = \sqrt{\frac{2}{T}} \cos 2\pi (2f_1) t$

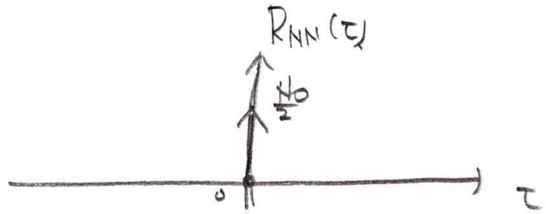
$\phi_3(t) = \sqrt{\frac{2}{T}} \sin 2\pi f_1 t$ $\quad \Rightarrow \{\phi_i(t)\}_{i=1}^{4}$ are orthonormal.

$\phi_4(t) = \sqrt{\frac{2}{T}} \sin 2\pi (2f_1) t$.

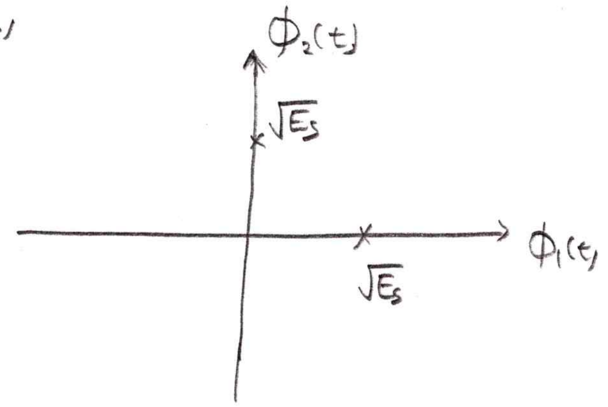
(3) Let $N(t)$: AWGN process, PSD of $N(t) = S_{NN}(f) = \frac{N_0}{2}$, $\forall f$

$R_{NN}(\tau) = \mathcal{F}^{-1}\{S_{NN}(f)\} = \frac{N_0}{2}\delta(\tau)$.





(4)



$$P_e = Q\left(\frac{d}{2\sigma}\right) = Q\left(\frac{\sqrt{2E_s}}{\sqrt{2N_0}}\right) = \underline{Q\left(\sqrt{\frac{E_s}{N_0}}\right)}.$$

(5) $\{\phi_i(t)\}_{i=1}^{4}$ 에서도 (4)의 경우와 symbol 간의 d_{min}은 같다.

$$\therefore P_e \le (L-1)\,Q\left(\sqrt{\frac{E_s}{N_0}}\right) = \underline{3Q\left(\sqrt{\frac{E_s}{N_0}}\right)} \quad \text{(union bound)}$$

(6) L=4인 경우, 2bit가 할당되므로 $E_s = 2E_b$. (E_b: 1bit 당 energy)

$$\therefore \underline{P_e \le 3Q\left(\sqrt{\frac{2E_b}{N_0}}\right)} \quad \text{(union bound)}$$

2010 제어

제어필수

1. a) $\det(\lambda I - A) = 0$, $|\lambda I - A| = \begin{vmatrix} \lambda & -1 & 0 \\ 0 & \lambda & -1 \\ 6 & 11 & \lambda+6 \end{vmatrix} = \lambda(\lambda^2+6\lambda+11)+6$
$= \lambda^3+6\lambda^2+11\lambda+6=0.$

$$\begin{array}{r|rrrr} -1 & 1 & 6 & 11 & 6 \\ & & -1 & -5 & -6 \\ \hline & 1 & 5 & 6 & 0 \end{array}$$

$(\lambda+1)(\lambda+2)(\lambda+3)=0$ $\therefore \lambda_1=-1 \quad \lambda_2=-2 \quad \lambda_3=-3.$

① $\lambda_1=-1$ case

$: \begin{bmatrix} -1 & -1 & 0 \\ 0 & -1 & -1 \\ 6 & 11 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ $\quad x_1+x_2=0$, $x_2+x_3=0$, $6x_1+11x_2+5x_3=0.$ $\quad \therefore \underline{x_1} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$

② $\lambda_2=-2$ case

$: \begin{bmatrix} -2 & -1 & 0 \\ 0 & -2 & -1 \\ 6 & 11 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ $\quad 2x_1+x_2=0$, $2x_2+x_3=0$, $6x_1+11x_2+4x_3=0$ $\quad \therefore \underline{x_2} = \begin{bmatrix} 1 \\ -2 \\ 4 \end{bmatrix}$

③ $\lambda_3=-3$ case

$: \begin{bmatrix} -3 & -1 & 0 \\ 0 & -3 & -1 \\ 6 & 11 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ $\quad 3x_1+x_2=0$, $3x_2+x_3=0$, $6x_1+11x_2+3x_3=0$ $\quad \therefore \underline{x_3} = \begin{bmatrix} 1 \\ -3 \\ 9 \end{bmatrix}$

$$\therefore P = \begin{bmatrix} 1 & 1 & 1 \\ -1 & -2 & -3 \\ 1 & 4 & 9 \end{bmatrix}$$

$\bar{A} = P^{-1}AP$ 이므로, P^{-1}을 Gauss-Jordan method를 통해서 구한다.

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ -1 & -2 & -3 & 0 & 1 & 0 \\ 1 & 4 & 9 & 0 & 0 & 1 \end{array}\right] \rightarrow \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & -1 & -2 & 1 & 1 & 0 \\ 0 & 3 & 8 & -1 & 0 & 1 \end{array}\right] \rightarrow \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & -1 & -2 & 1 & 1 & 0 \\ 0 & 0 & 2 & 2 & 3 & 1 \end{array}\right]$$

$$\downarrow$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & \frac{5}{2} & \frac{1}{2} \\ 0 & 1 & 0 & -3 & -4 & -1 \\ 0 & 0 & 1 & 1 & \frac{3}{2} & \frac{1}{2} \end{array}\right] \leftarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & \frac{5}{2} & \frac{1}{2} \\ 0 & -1 & 0 & 3 & 4 & 1 \\ 0 & 0 & 2 & 2 & 3 & 1 \end{array}\right] \leftarrow \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 0 & -\frac{3}{2} & -\frac{1}{2} \\ 0 & -1 & 0 & 3 & 4 & 1 \\ 0 & 0 & 2 & 2 & 3 & 1 \end{array}\right]$$

$$\therefore P^{-1} = \begin{bmatrix} 3 & \frac{5}{2} & \frac{1}{2} \\ -3 & -4 & -1 \\ 1 & \frac{3}{2} & \frac{1}{2} \end{bmatrix}$$

사실, $A = P\bar{A}P^{-1} = P\Lambda P^{-1} \Rightarrow \Lambda = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix} = \bar{A}$ 으로 바로 구하면 됨..

b) $\dot{\bar{x}}(t) = P^{-1}AP\bar{x}(t) + P^{-1}Bu(t)$

$y(t) = CP\bar{x}(t)$

$$\frac{Y(s)}{U(s)} = CP(sI - P^{-1}AP)^{-1}P^{-1}B = CP(P(sI-P^{-1}AP))^{-1}B$$

$$= CP(sP-AP)^{-1}B = C((sP-AP)P^{-1})^{-1}B = C(sI-A)^{-1}B.$$

즉, similarity transform을 하기 전과 후의 transfer function은 같으므로 system은 similarity transform에 invariant 하다.

제어선택

1. 1) $[B \quad AB] = \begin{bmatrix} b_1 & 2b_1+b_2 \\ b_2 & b_2 \end{bmatrix}$가 full rank가 되어야 controllable.

$$\begin{bmatrix} b_1 & 2b_1+b_2 \\ b_2 & b_2 \end{bmatrix} \rightarrow \begin{bmatrix} b_1 & 2b_1+b_2 \\ 0 & -b_2-\frac{b_2^2}{b_1} \end{bmatrix} \Rightarrow b_1 \neq 0,\ b_2 + \frac{b_2^2}{b_1} \neq 0,\ b_2 \neq 0$$

$\begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} c_1 & c_2 \\ 2c_1 & c_1+c_2 \end{bmatrix}$가 full rank가 되어야 observable.

$$\begin{bmatrix} c_1 & c_2 \\ 2c_1 & c_1+c_2 \end{bmatrix} \rightarrow \begin{bmatrix} c_1 & c_2 \\ 0 & c_1-c_2 \end{bmatrix} \Rightarrow c_1 \neq 0,\quad c_1 \neq c_2$$

2) $\dot{x}(t) = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix}[-k_1 \ \ -k_2]x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix}v(t)$

$$= \begin{bmatrix} 2 & 1 \\ -k_1 & 1-k_2 \end{bmatrix}x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix}v(t)$$

$y(t) = [1 \quad 0]x(t)$

$$\begin{vmatrix} \lambda-2 & -1 \\ k_1 & \lambda+k_2-1 \end{vmatrix} = \lambda^2 + (k_2-3)\lambda + 2 - 2k_2 + k_1 = \lambda^2 + 4\lambda + 4 = 0$$

$\therefore k_2 = 7,\ k_1 = 16 \qquad \therefore \underline{K = [16 \quad 7]}$

3) $\dot{x}(t) = \begin{bmatrix} 2 & 1 \\ -16 & -6 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} v(t)$

$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(t)$

$\frac{Y(s)}{V(s)} = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} s-2 & -1 \\ 16 & s+6 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $\begin{bmatrix} s-2 & -1 \\ 16 & s+6 \end{bmatrix}^{-1} = \frac{1}{s^2+4s+4} \begin{bmatrix} s+6 & 1 \\ -16 & s-2 \end{bmatrix}$

$= \frac{1}{(s+2)^2} \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} s+6 & 1 \\ -16 & s-2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{1}{(s+2)^2} \begin{bmatrix} s+6 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{1}{(s+2)^2}$.

$Y(s) = \frac{1}{(s+2)^2} \cdot \frac{1}{s^2+1} = \frac{A}{s+2} + \frac{B}{(s+2)^2} + \frac{Cs+D}{s^2+1}$

$A(s+2)(s^2+1) + B(s^2+1) + (Cs+D)(s+2)^2 = As^3 + 2As^2 + As + 2A + Bs^2 + B$
$+ Cs^3 + 4Cs^2 + 4Cs + Ds^2 + 4Ds + 4D$
$= (A+C)s^3 + (2A+B+4C+D)s^2$
$+ (A+4C+4D)s + (2A+B+4D) = 1$

$\left.\begin{array}{l} A+C=0 \\ 2A+B+4C+D=0 \\ A+4C+4D=0 \\ 2A+B+4D=1 \end{array}\right\} \Rightarrow A = \frac{4}{25},\ B = \frac{1}{5},\ C = -\frac{4}{25},\ D = \frac{3}{25}$

$Y(s) = \frac{4}{25}\left(\frac{1}{s+2}\right) + \frac{1}{5}\left(\frac{1}{s+2}\right)^2 - \frac{4}{25}\left(\frac{s}{s^2+1}\right) + \frac{3}{25}\left(\frac{1}{s^2+1}\right)$

$y(t) = \left(\frac{4}{25}e^{-2t} + \frac{1}{5}te^{-2t} - \frac{4}{25}\cos t + \frac{3}{25}\sin t\right)u(t)$

$\therefore y_s(t) = \left(-\frac{4}{25}\cos t + \frac{3}{25}\sin t\right)u(t)$

4)