4.Fourier optics

Harmonic Analysis

- **Harmonic analysis**

Expansion of an arbitrary function of time $f(t)$ as a superposition of harmonic functions of time of different frequencies

$$\boxed{f(t) = \int_{-\infty}^{\infty} F(\nu) e^{+j2\pi\nu t} d\nu}$$

$F(\nu)$: Fourier transform of $f(t)$

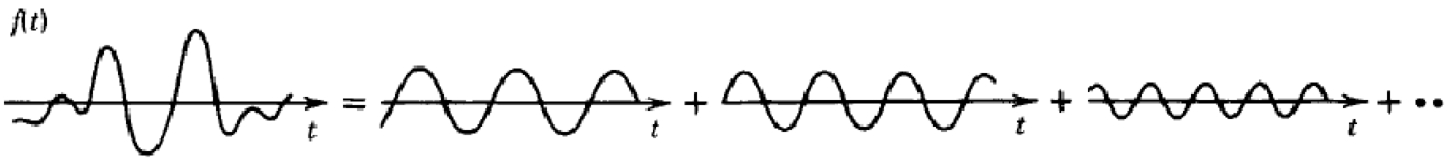


Figure 4.0-1 An arbitrary function $f(t)$ may be analyzed as a sum of harmonic functions of different frequencies and complex amplitudes.

If the response of the system to each harmonic function is known, the response to an arbitrary input function is readily determined by the use of harmonic analysis at the input and superposition at the output.

Fourier Optics

- Similarly, an arbitrary function *f*(*x*, *y*) may be written as a superposition of harmonic functions of *x* and *y*:

$$f(x,y) = \int_{-\infty}^{\infty} d\nu_x \int_{-\infty}^{\infty} d\nu_y \boxed{F(\nu_x,\nu_y)e^{-j2\pi(\nu_x x + \nu_y y)}}$$

Harmonic building block

$F(\nu_x,\nu_y)$: Complex amplitude

ν_x and ν_y : Spatial frequencies (cycles/mm)

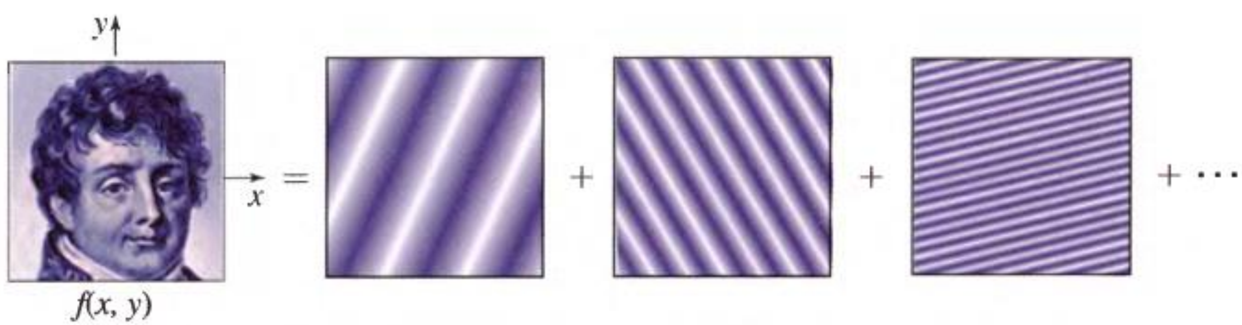
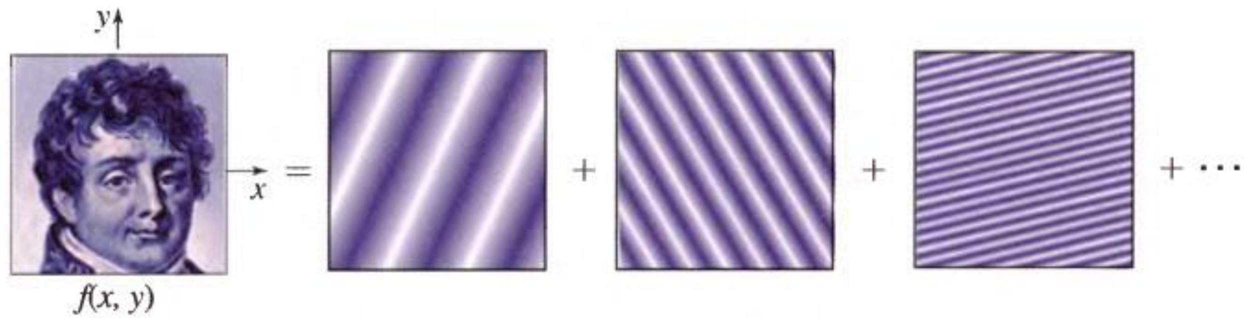
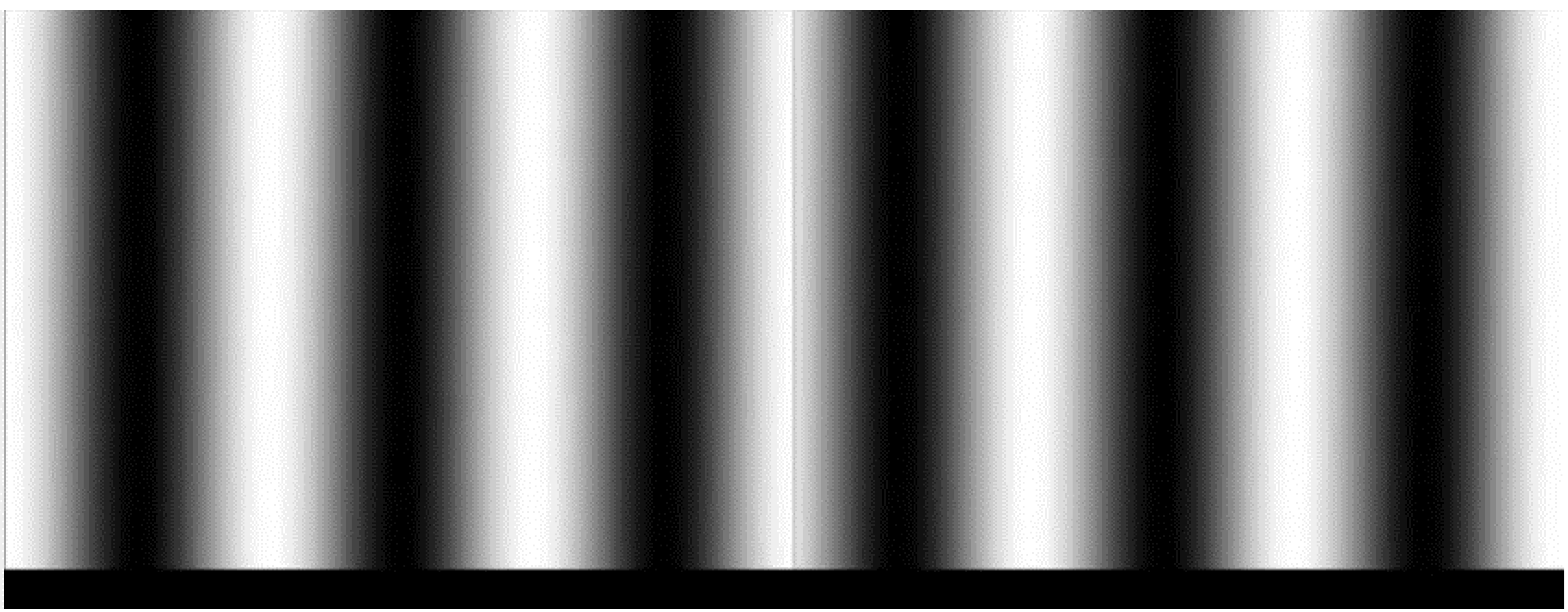


Figure 4.0-2 An arbitrary function $f(x, y)$ may be analyzed as a sum of harmonic functions of different spatial frequencies and complex amplitudes.

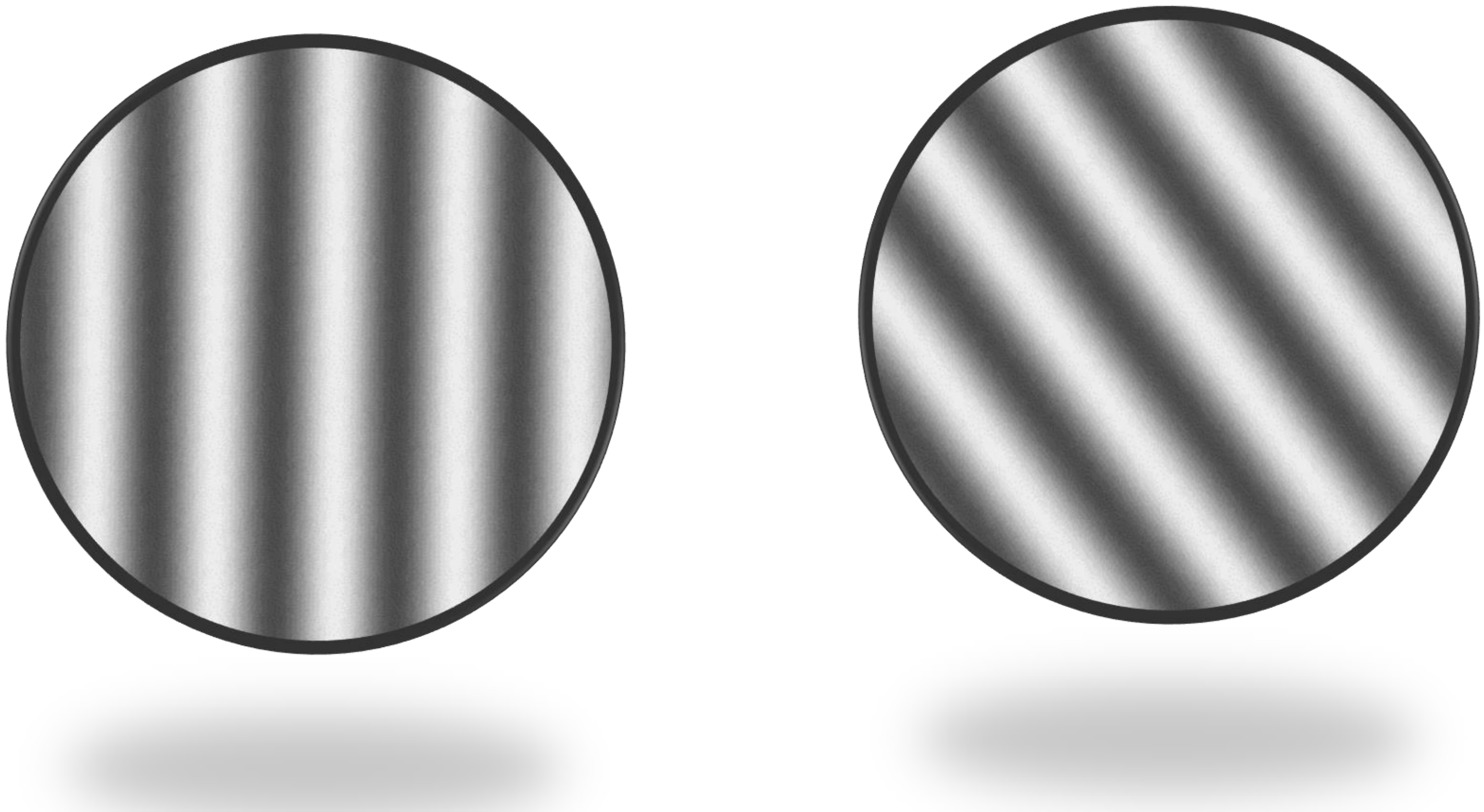


http://www.youtube.com/watch?v=D9ziTuJ3OCw



- http://www.youtube.com/watch?v=D9ziTuJ3OCw

Application 수행



$$\nabla^2 \tilde{\mathbf{E}} - \gamma_c^2 \tilde{\mathbf{E}} = 0$$

$$(\nabla^2 + k^2)\tilde{E}_x \hat{\boldsymbol{x}} + (\nabla^2 + k^2)\tilde{E}_y \hat{\boldsymbol{y}} + (\nabla^2 + k^2)\tilde{E}_z \hat{\boldsymbol{z}} = 0$$

$$(\nabla^2 + k^2)\tilde{E}_x(x, \mathrm{y}, \mathrm{z}) = 0$$

$$\boxed{(\nabla^2 + k^2) f(x, \mathrm{y}, \mathrm{z}) = 0 \quad \Rightarrow \quad \begin{gathered} f(x, \mathrm{y}, \mathrm{z}) = cos(k_x x + k_y y + k_z z) \\ k_x^2 + k_y^2 + k_z^2 = k^2 \end{gathered}}$$

$f(x, \mathrm{y}, \mathrm{z}) = f(z)$이면 $\quad \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) \rightarrow \frac{\partial^2}{\partial z^2}$

$$\boxed{\left(\frac{\partial^2}{\partial z^2} + k^2\right) f(\mathrm{z}) = 0 \quad \Rightarrow \quad f(\mathrm{z}) = cos(kz)}$$

$$f(x,y,z) = cos(k_x x + k_y y + k_z z)$$

숫자 k 와 $\vec{k}$는 다름

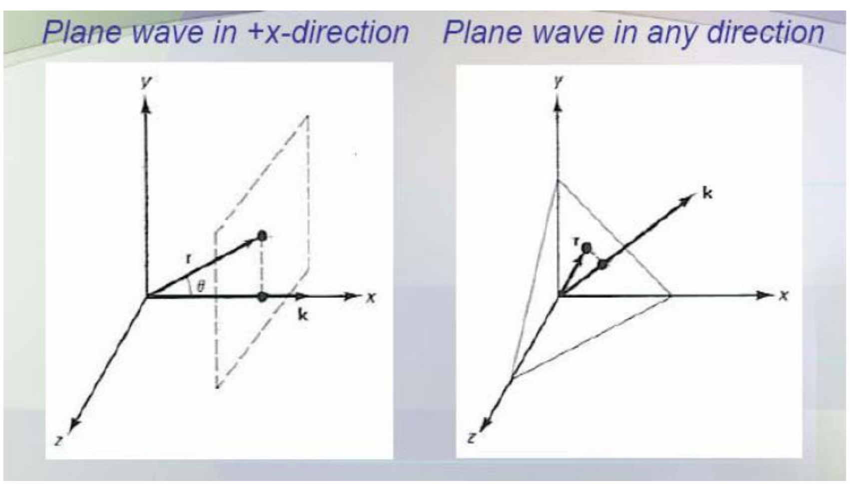
$\vec{k}$는

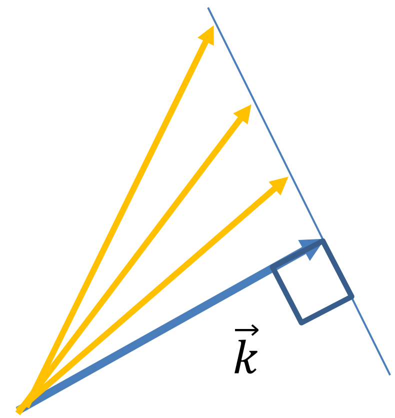
$$k_x^2+k_y^2+k_z^2 = k^2$$

만족하는 모든 벡터
즉, 벡터의 크기가 k인 임의의 벡터

$$k_x x + k_y y + k_z z$$
$$=(k_x\hat{\boldsymbol{x}} + k_y\hat{\boldsymbol{y}} + k_z\hat{\boldsymbol{z}}) \cdot (x\hat{\boldsymbol{x}} + y\hat{\boldsymbol{y}} + z\hat{\boldsymbol{z}})$$
$$=\vec{k} \cdot \vec{r}$$

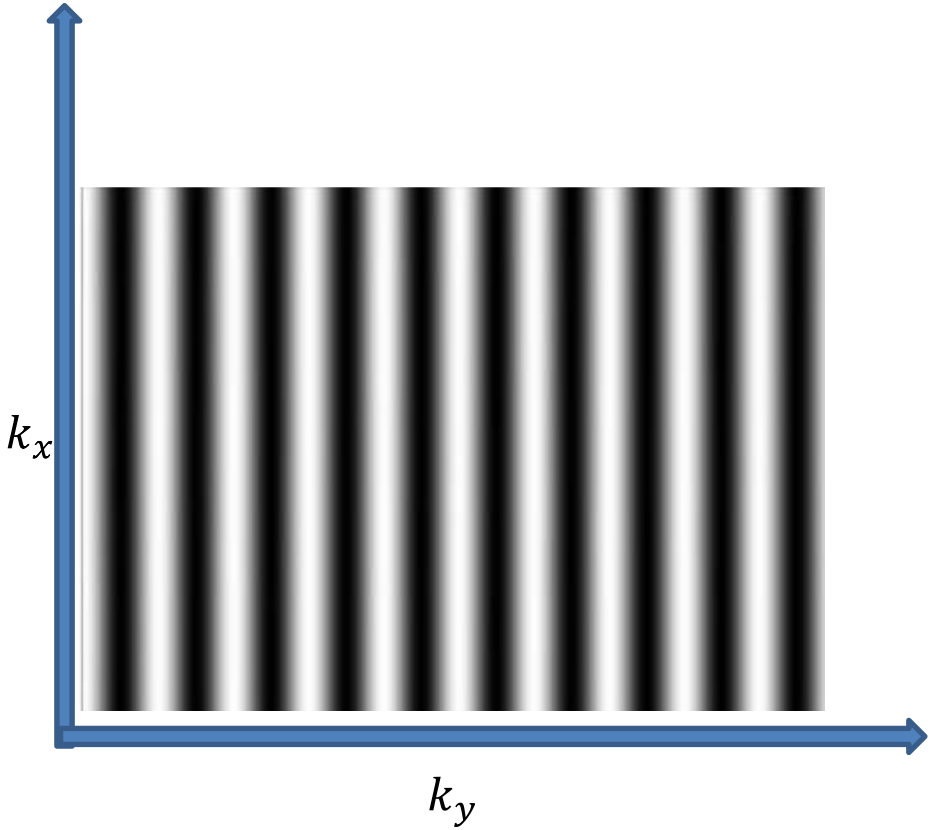
$$Ae^{j(k_x x+k_y y+k_z z\ )}=Ae^{j(\vec{k}\cdot\vec{r})}$$

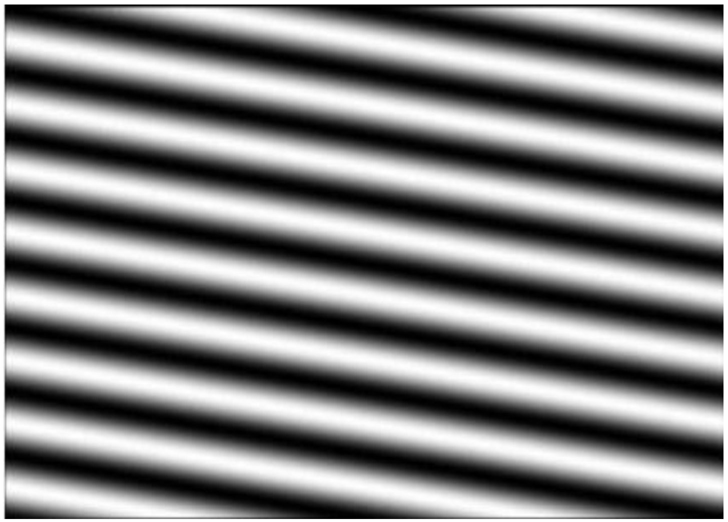




$\vec{k}$와 직교하는 선(면) 위의 점들은
$\vec{k}\cdot\vec{r}$ 값이 전부 같다: 동일 위상면

즉, $\vec{k}$는 파의 진행 방향을 가리키며 크기는 파수이다.

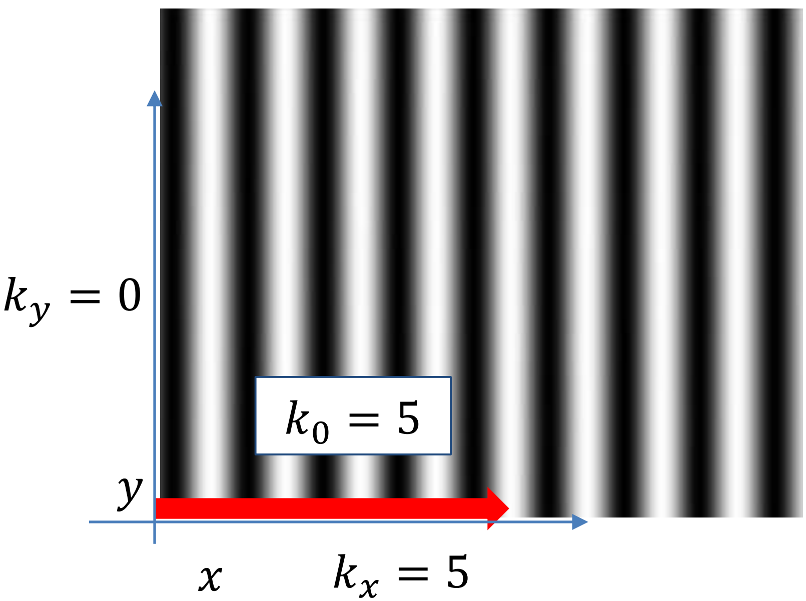

k_x
k_y



$$k_x^2 + k_y^2 = k_0^2$$

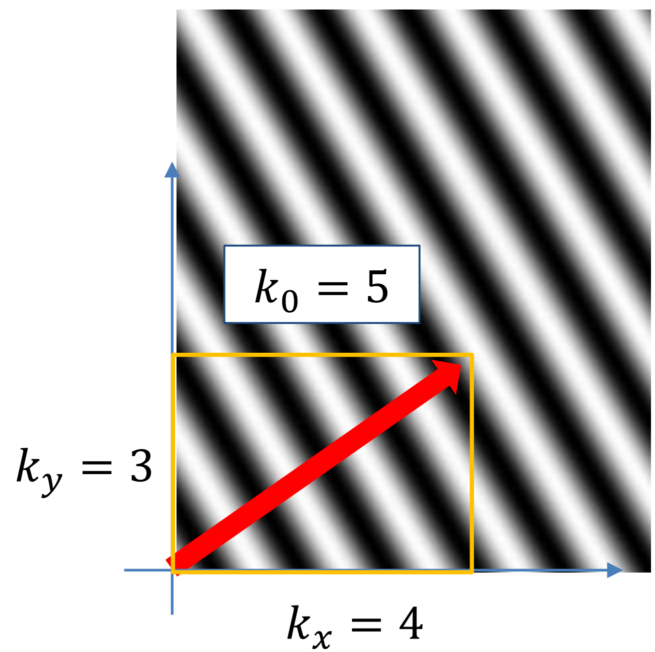
$k_0 = 5$

$k_x = 5$
$k_y = 0$



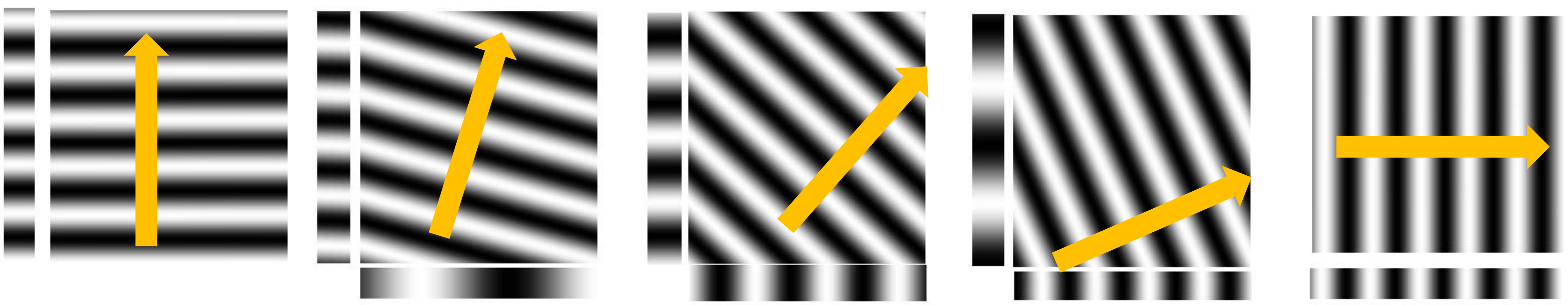
$k_0 = 5$

$k_x = 4$
$k_y = 3$



$$e^{j\,\overrightarrow{k_0}\cdot\vec{r}} = e^{j(4\hat{x}+3\hat{y})\cdot(x\hat{x}+y\hat{y})} = e^{j(4x+3y)}$$

Travelling $\cos(4x+3y)$

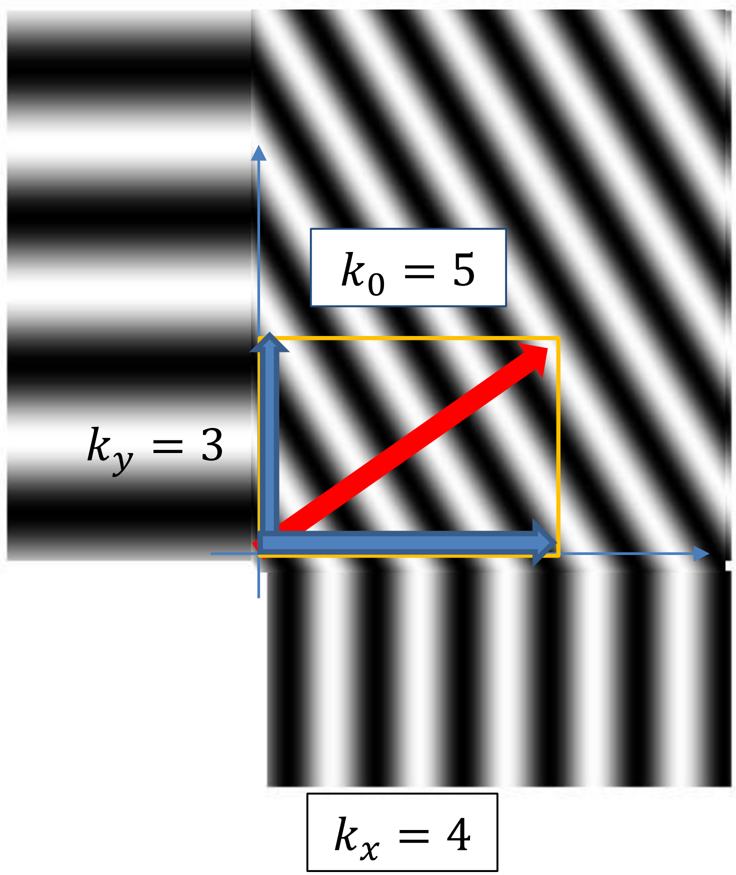


$$k_x^2 + k_y^2 = k_0^2$$

$$e^{j\,\overrightarrow{k_0}\cdot\vec{r}} = e^{j(4\hat{x}+3\hat{y})\cdot(x\hat{x}+y\hat{y})} = e^{j(4x+3y)}$$

$x = 0$

$\cos(3y - wt)$



$\cos(4x{+}3y - wt)$

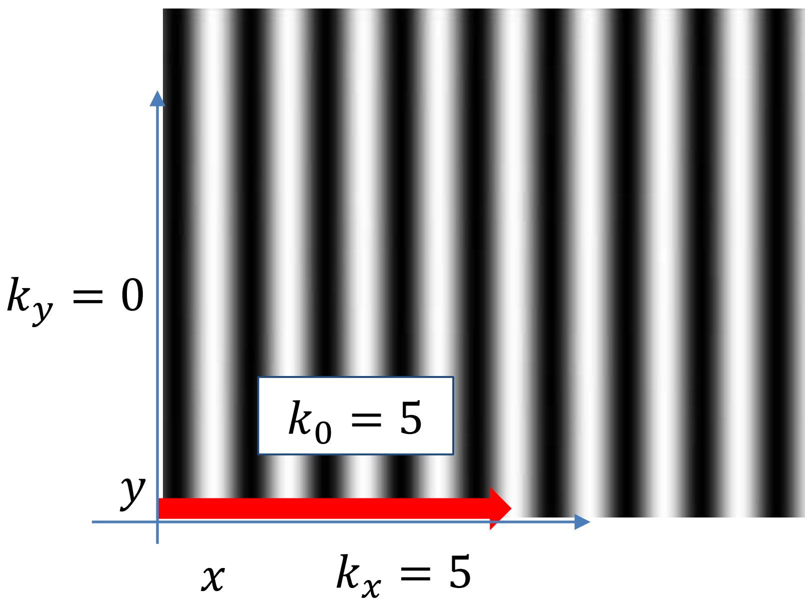
$y = 0$ $\cos(4x - wt)$

Travelling

$$k_x^2 + k_y^2 = k_0^2$$

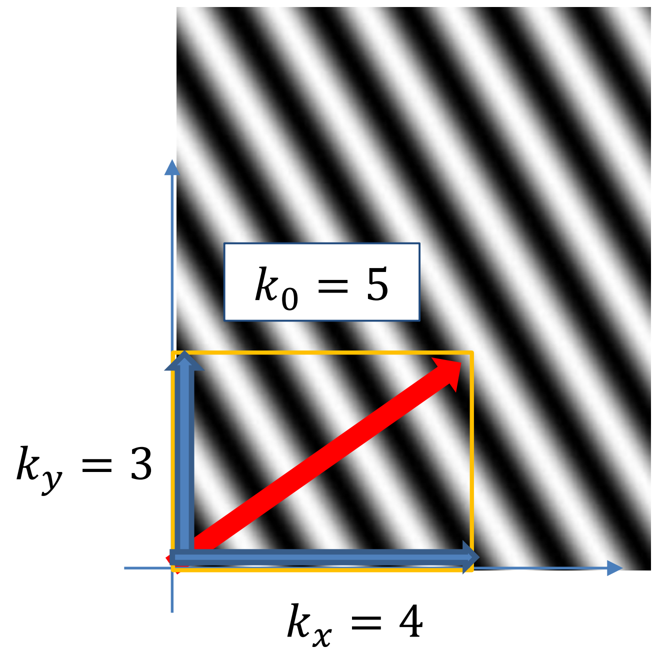
$k_0 = 5$

$k_x = 5$
$k_y = 0$



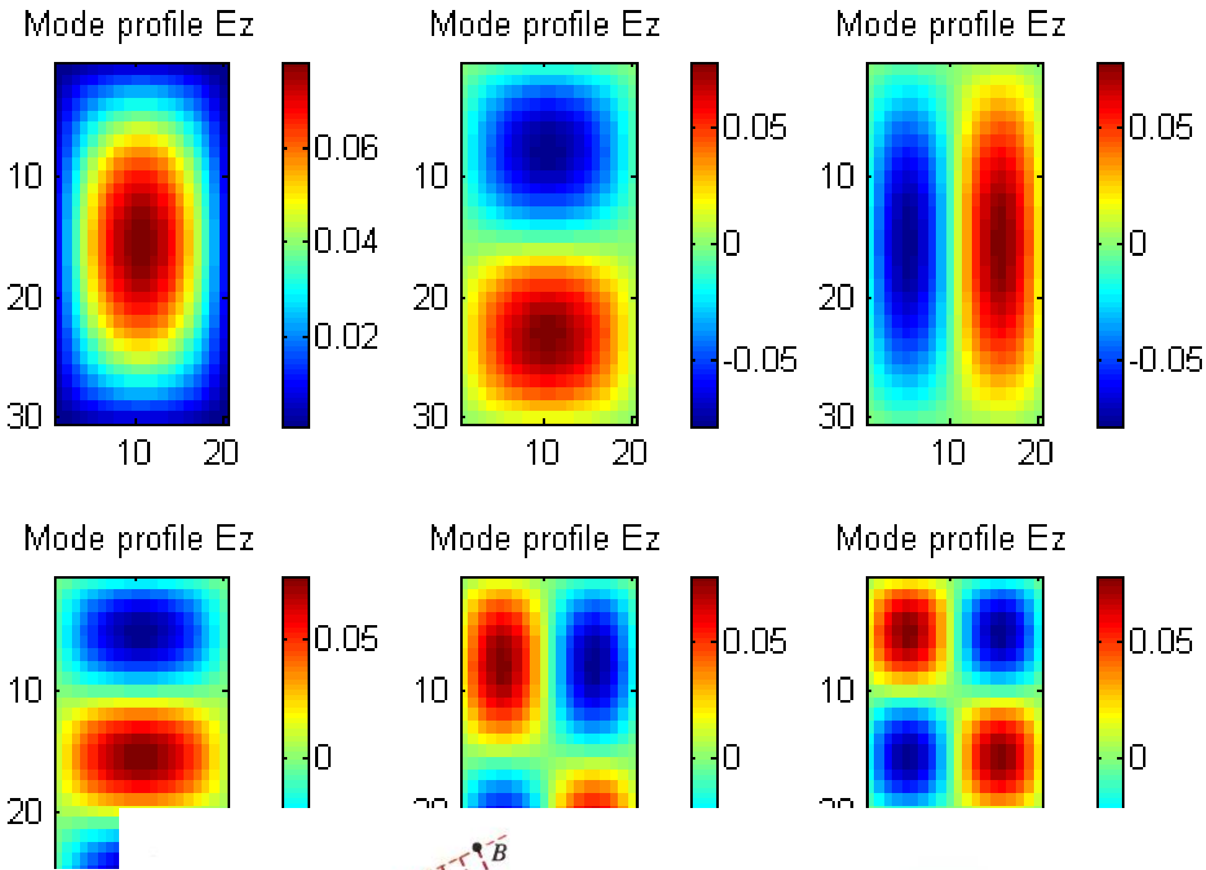
$k_0 = 5$

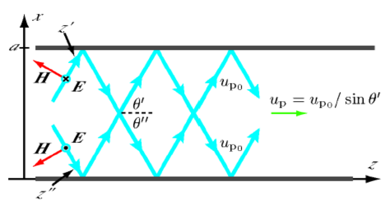
$k_x = 4$
$k_y = 3$



$$e^{j\,\overrightarrow{k_0}\cdot\vec{r}} = e^{j(4\hat{x}+3\hat{y})\cdot(x\hat{x}+y\hat{y})} = e^{j(4x+3y)}$$

Travelling $\cos(4x+3y)$





(b) TEM waves

Figure 8-27: The TE_{10} mode can be constructed as the sum of two TEM waves.

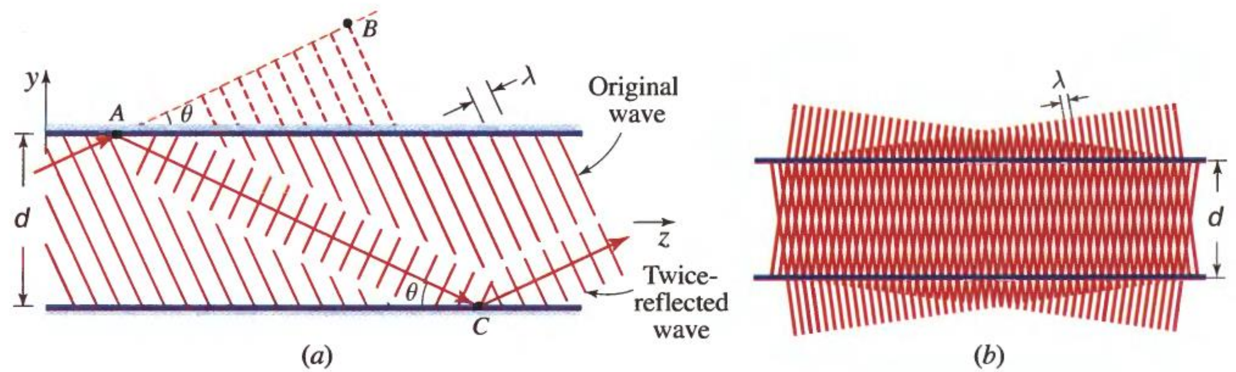
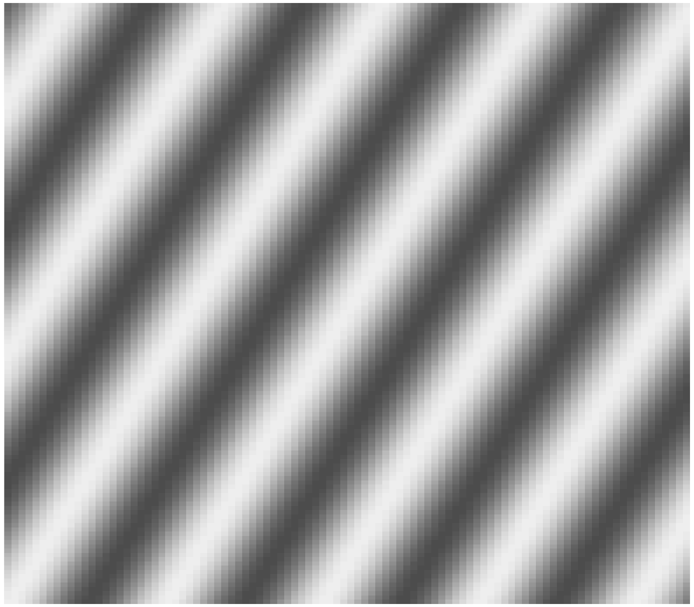


Figure 8.1-2 (*a*) Condition of self-consistency: as a wave reflects twice it duplicates itself. (*b*) At angles for which self-consistency is satisfied, the two waves interfere and create a pattern that does not change with z.



일반해

$$= E_0(\sin k_x x)(\sin k_y y)e^{-jk_z}$$

$$\boxed{\begin{array}{ll} \tilde{e}_z = 0 & \mathrm{x} = 0\,,\mathrm{a}\text{ 에서} \\ \tilde{e}_z = 0 & \mathrm{y} = 0\,,\mathrm{b}\text{ 에서} \end{array}}$$

$$k_x = \frac{m\pi}{a}, \qquad m = 1,2,3\ \ldots$$

$$k_y = \frac{n\pi}{b}, \qquad n = 1,2,3\ \ldots$$

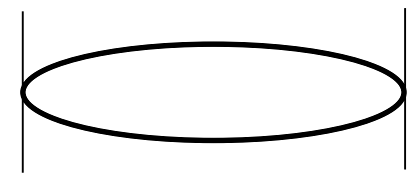
$$\left\{ \begin{array}{l} \tilde{E}_z(x,y,z) = \boxed{E_0 \sin(\frac{m\pi x}{a}) \sin(\frac{n\pi y}{b})\ e^{-jk_z}} \\ \widetilde{H}_z = 0 \end{array} \right.$$

$$= \tilde{e}_z(x,y)e^{-j\beta z}$$

TM mode

$(e^{-jk_x x} \pm e^{jk_x x})$

정상파 solution

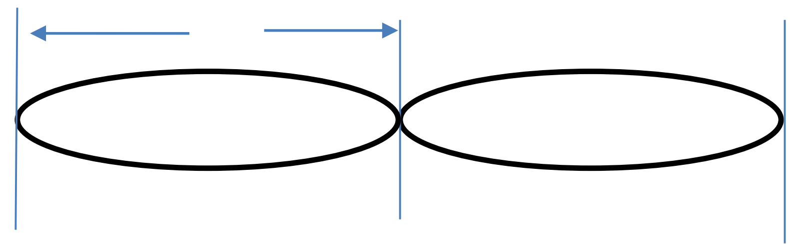


고무줄 예

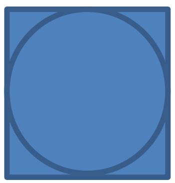
차단 주파수(cutoff frequency)

$$\mathrm{TM_{mn}} \qquad k_c^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$\mathrm{TM_{11}} \qquad k_c^2 = \left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{b}\right)^2$$



$$K_x = \frac{2\pi}{\lambda} \qquad \lambda = 2a$$



b

a

$$\omega^2 \mu\varepsilon = \left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{b}\right)^2$$

$$(2\pi f)^2 \mu\varepsilon = \left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{b}\right)^2$$

$$f_{11} = \frac{u_{p0}}{2}\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2}$$

$$\sqrt{\mu\varepsilon} = u_{p0}$$

물질 μ, ε 인 매질에서 TEM의 위상속도

$\mathrm{TEM_{mn}}$ 을 따라서,넣어주는 웨이브 가이드 보다 작은 파장이여야 한다.

$$f_{mn} = \frac{u_{po}}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

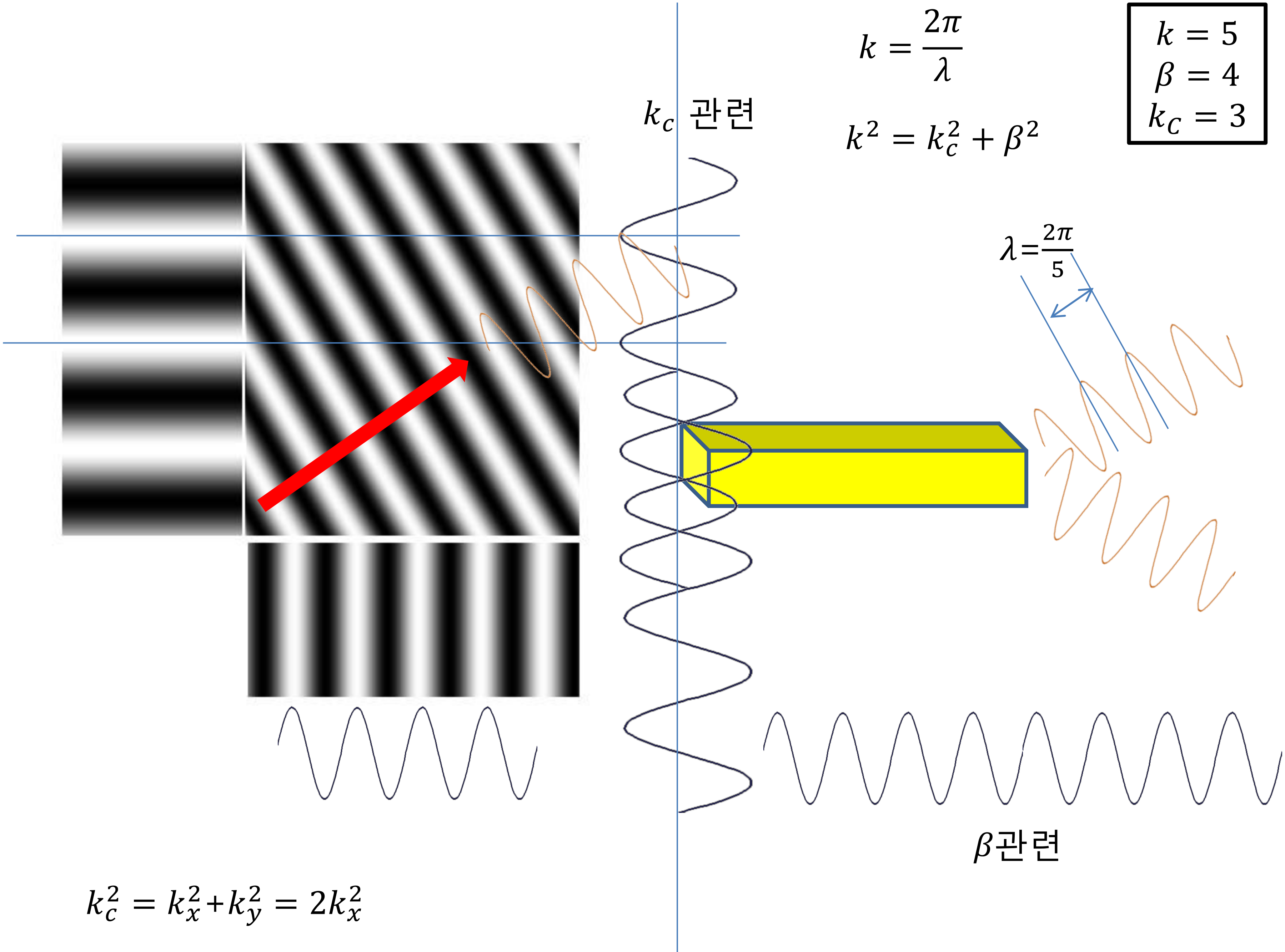
$f > f_{mn}$ 즉, 주파수가 차단주파수보다 높아야 전파된다

입사파(ω)와 관련된 파장 $\omega = ck$

$$k = \frac{2\pi}{\lambda}$$

$$k^2 = k_c^2 + \beta^2$$

$$\boxed{\begin{aligned} k &= 5 \\ \beta &= 4 \\ k_c &= 3 \end{aligned}}$$



$$k_c^2 = k_x^2 + k_y^2 = 2k_x^2$$

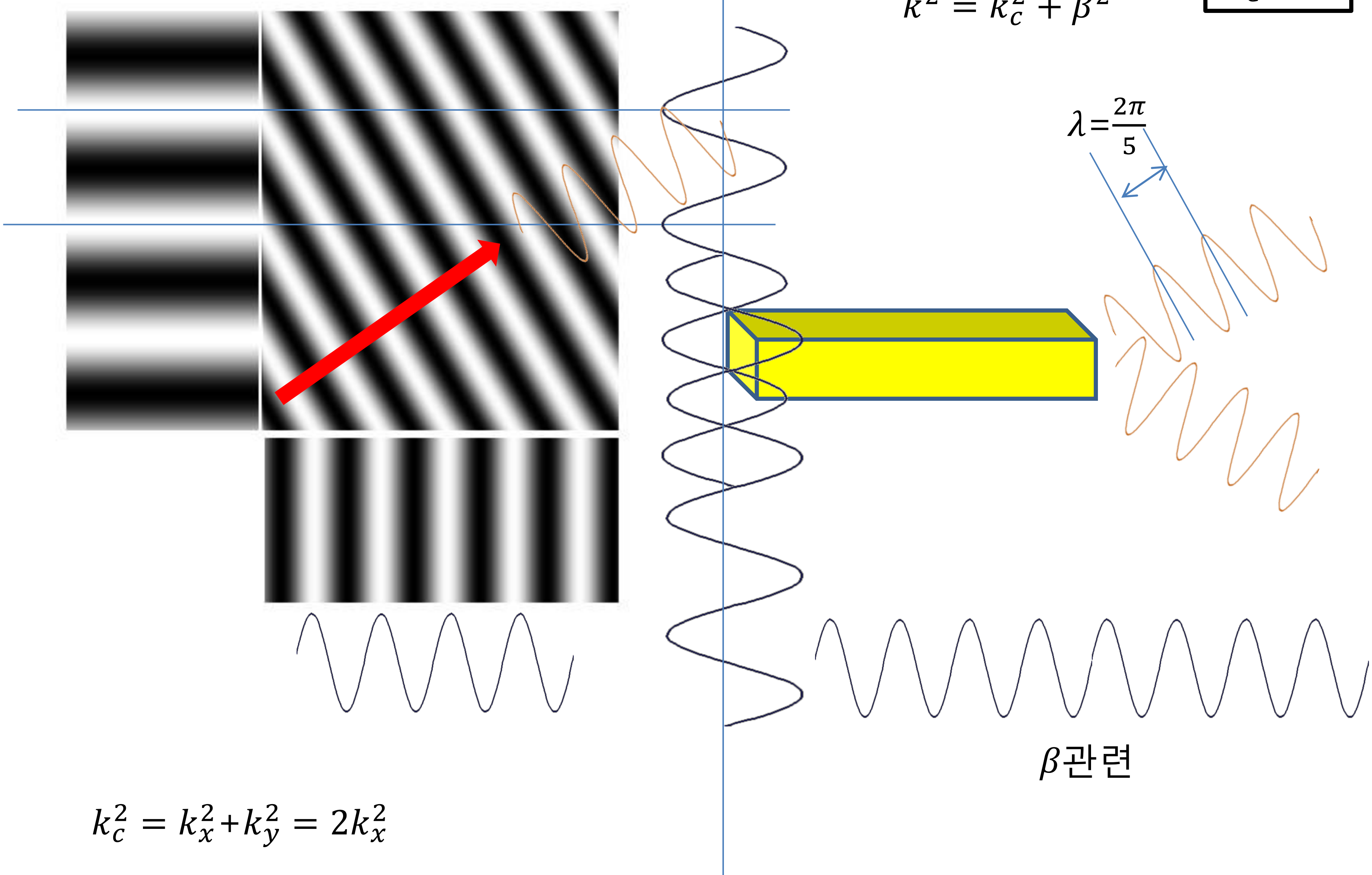
입사파(ω)와 관련된 파장 $\omega = ck$

$$k = \frac{2\pi}{\lambda}$$

$$k^2 = k_c^2 + \beta^2$$

$$\boxed{\begin{aligned} k &= 5 \\ \beta &= 4 \\ k_c &= 3 \end{aligned}}$$

k_c 관련



β관련

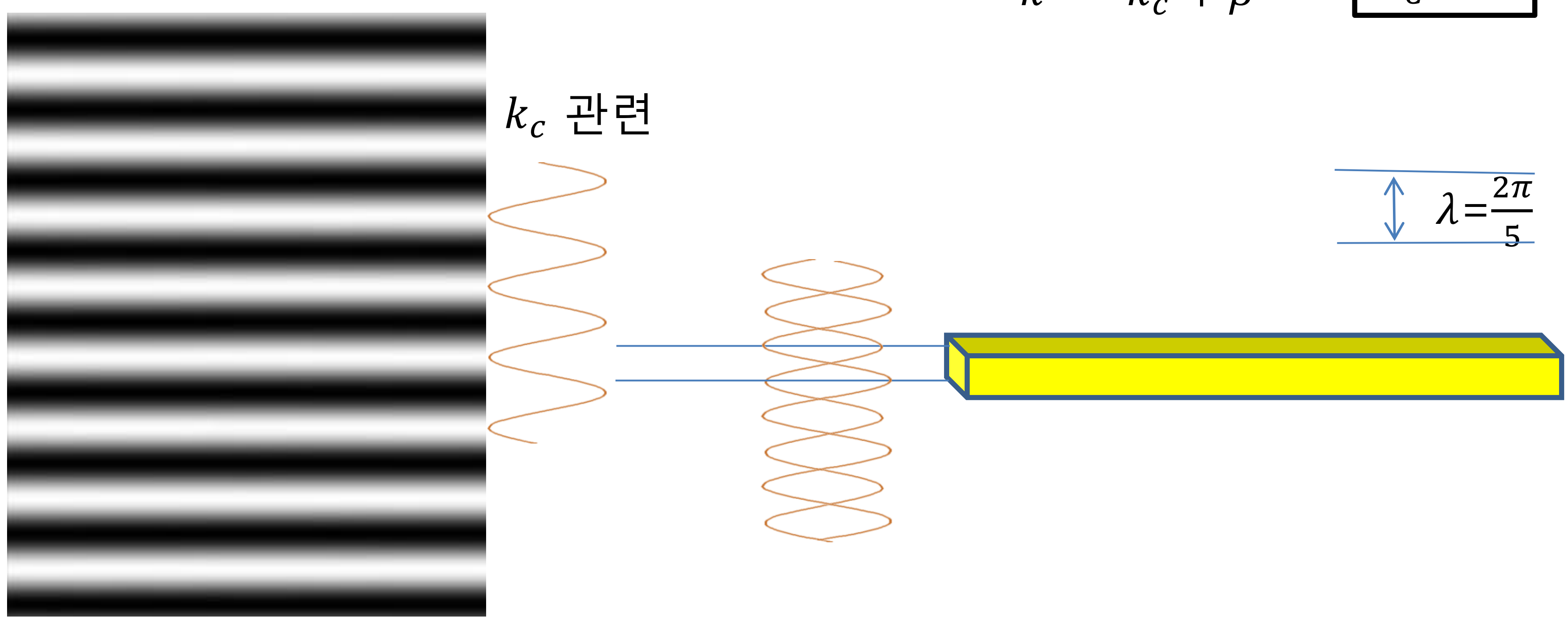
$$k_c^2 = k_x^2 + k_y^2 = 2k_x^2$$

입사파(ω)와 관련된 파장 $\omega = ck$

$$k = \frac{2\pi}{\lambda}$$

$$k^2 = k_c^2 + \beta^2$$

$$\boxed{\begin{aligned} k &= 5 \\ \beta &= 0 \\ k_c &= 5 \end{aligned}}$$



$\beta = 0$ 관련

정확히는

$$k_c^2 = k_x^2 + k_y^2 = 2k_x^2$$

입사파장은, $a\sqrt{2}$ cut−off freq. ($\neq 2a$)

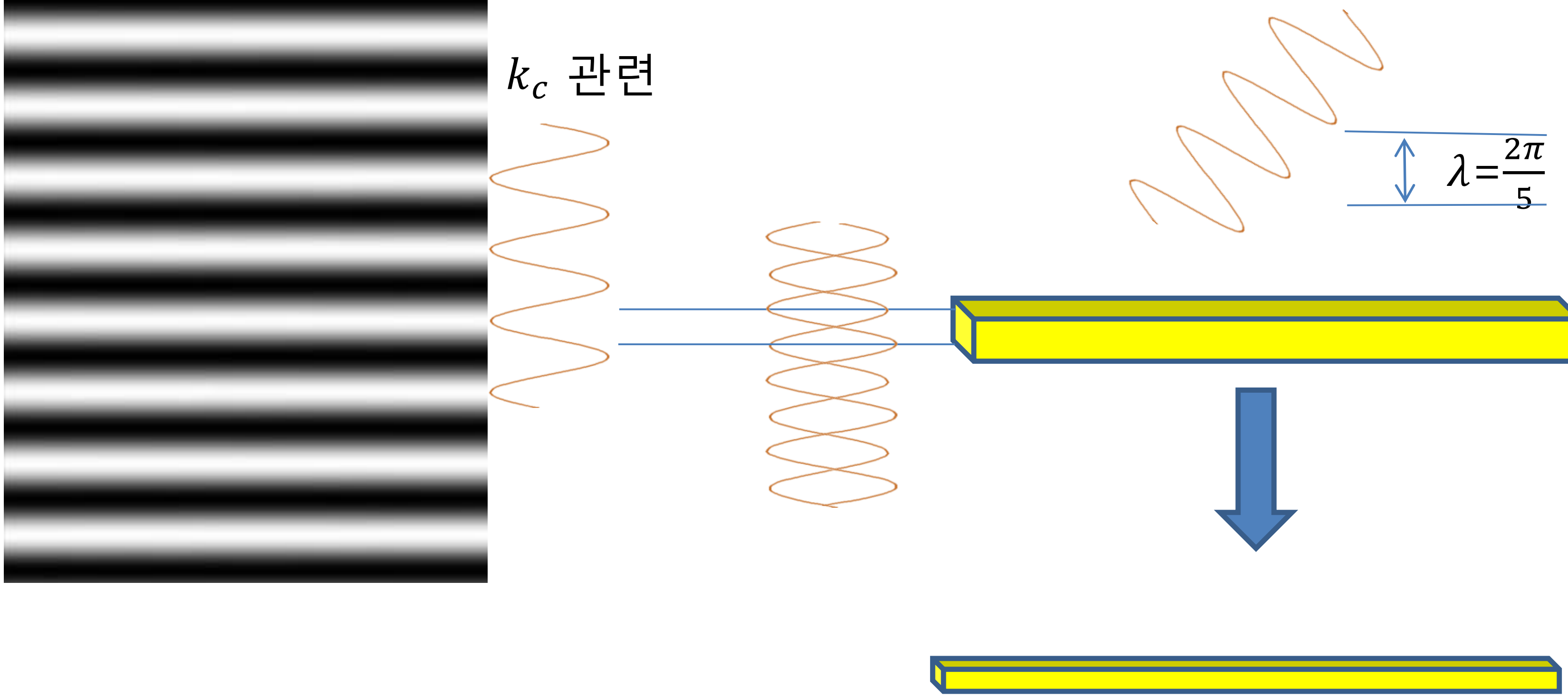
$$k_x = \frac{2\pi}{2a} \quad k_c = \frac{2\pi}{\frac{2a}{\sqrt{2}}} = \frac{2\pi}{\sqrt{2}a}$$

입사파(ω)와 관련된 파장 $\omega = ck$

$$k = \frac{2\pi}{\lambda}$$

$$k^2 = k_c^2 + \beta^2$$

$$\boxed{\begin{aligned} k &= 5 \\ \beta &= 0 \\ k_c &= 5 \end{aligned}}$$

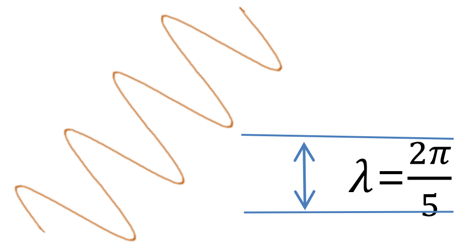


입사파(ω)와 관련된 파장 $\omega = ck$

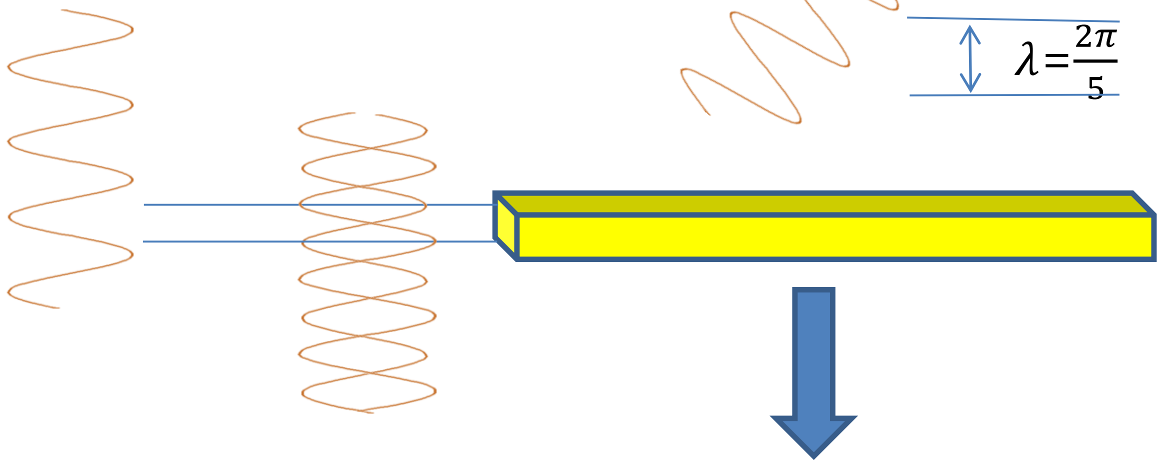
$$k = \frac{2\pi}{\lambda}$$

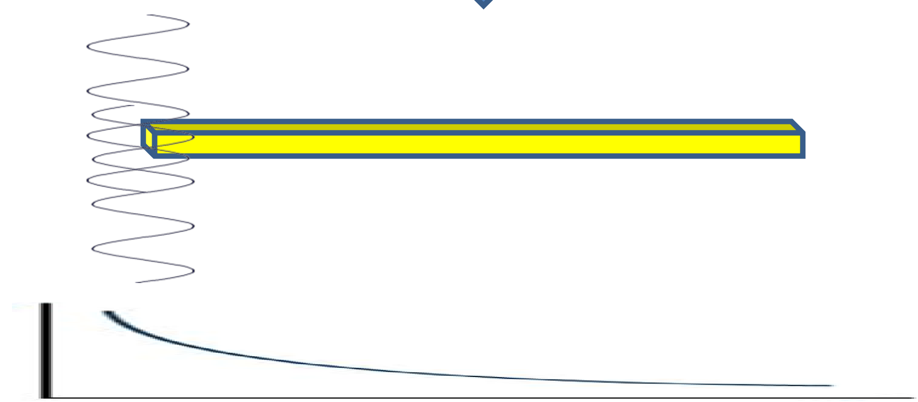
$$\boxed{\begin{aligned} k &= 5 \\ \beta &= 0 \\ k_c &= 5 \end{aligned}}$$

$$k^2 = k_c^2 + \beta^2$$



k_c 관련





$k_c = 7$

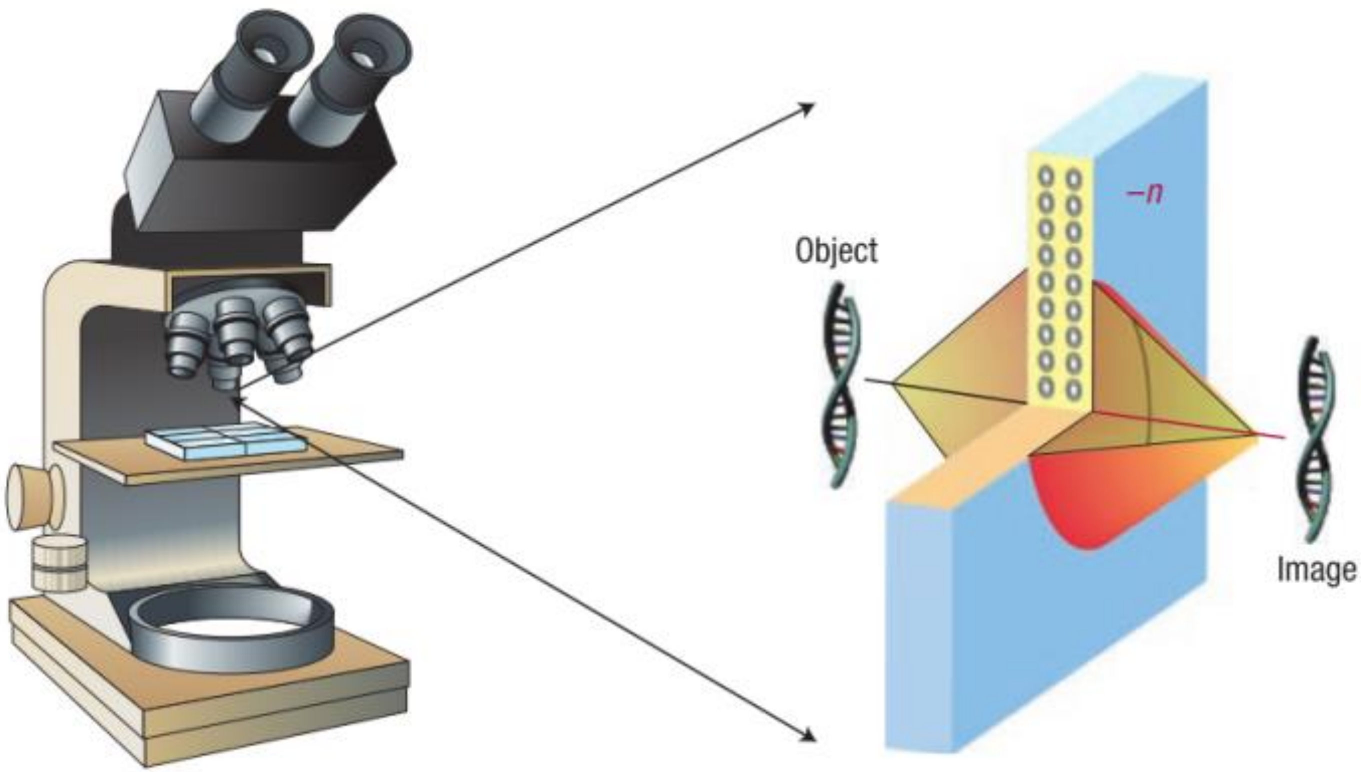
$\beta = j\sqrt{24}$

$$e^{j\beta z} = e^{-\sqrt{24}z}$$

지수 감소 통과하지 못함, $k^2 = k_c^2$
따라서, $k^2 = k_c^2$ 경우 보다
더 큰 진동수(더 단파장) 빛만 통과

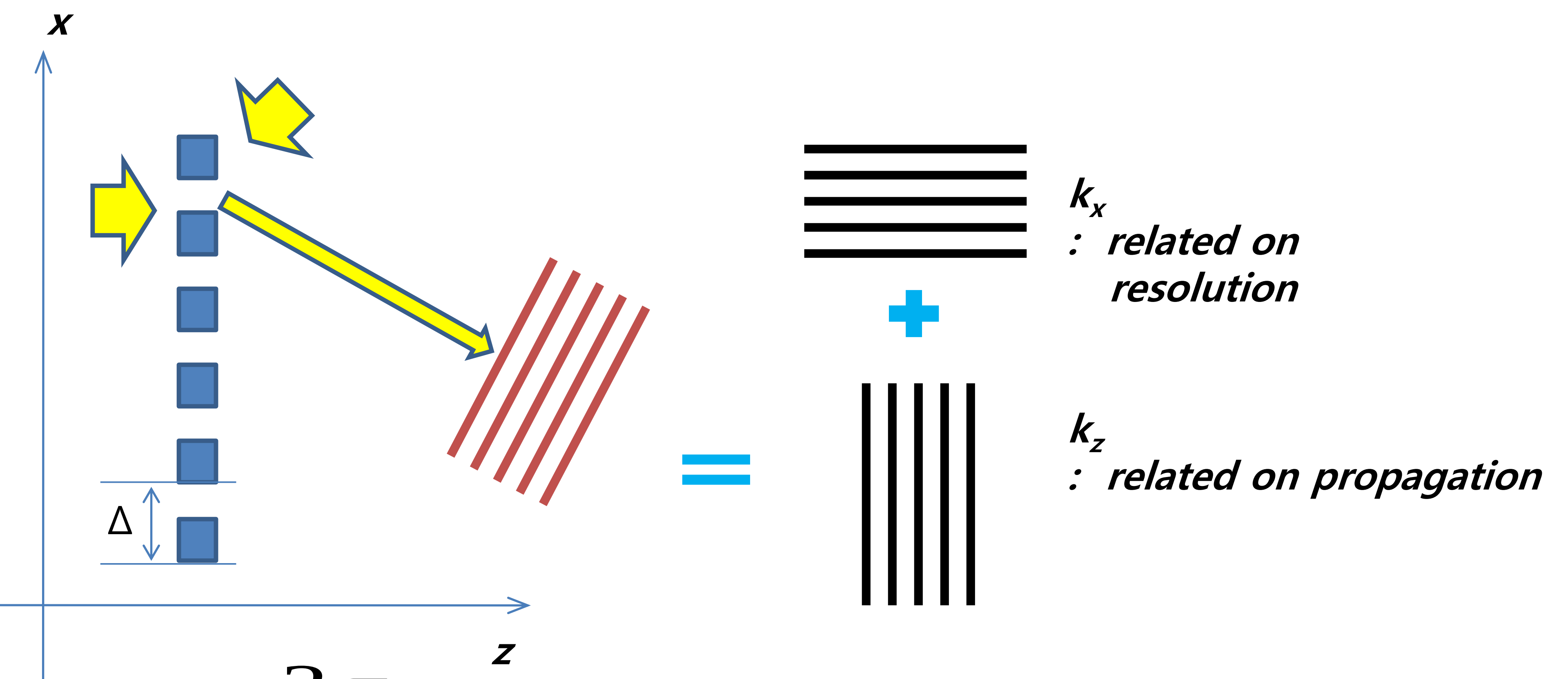
Application of NIM(Perfect lens)

Applicable to super optical imaging



Same explanation can be given to lithography

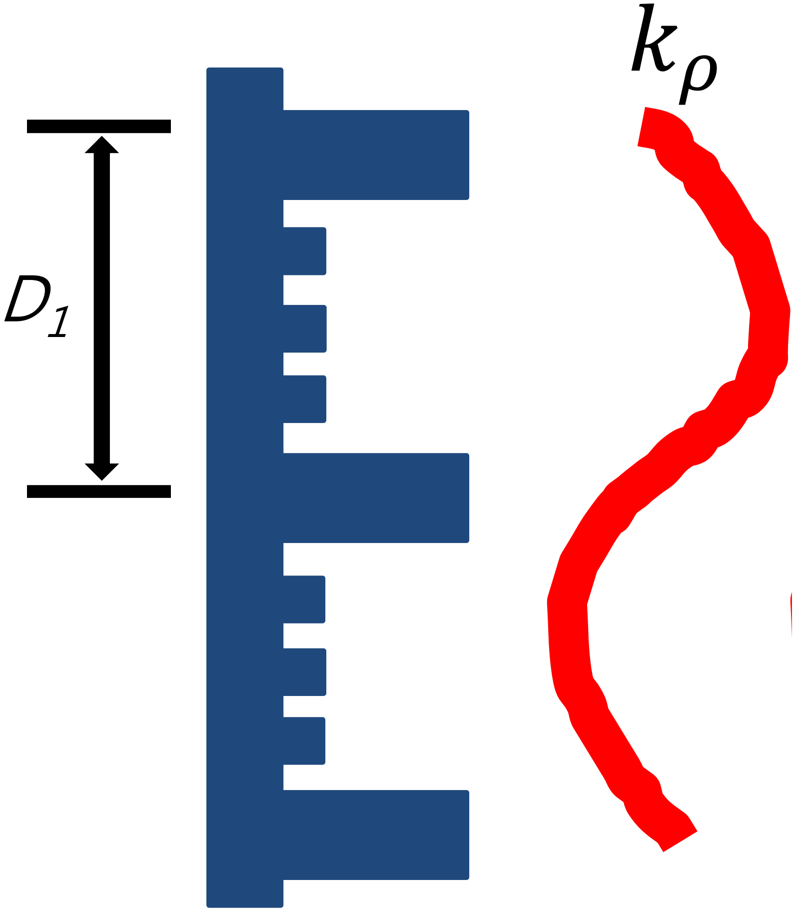
Diffraction limit



$$k_x \approx \frac{2\pi}{\Delta}$$

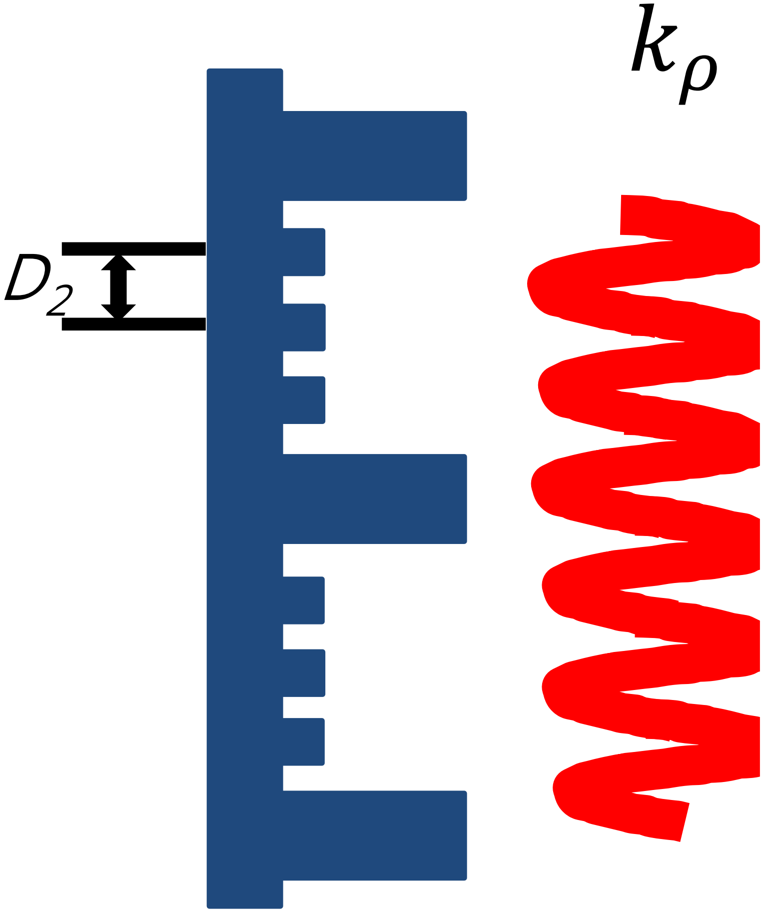
Δ slit detail :

Diffraction limit



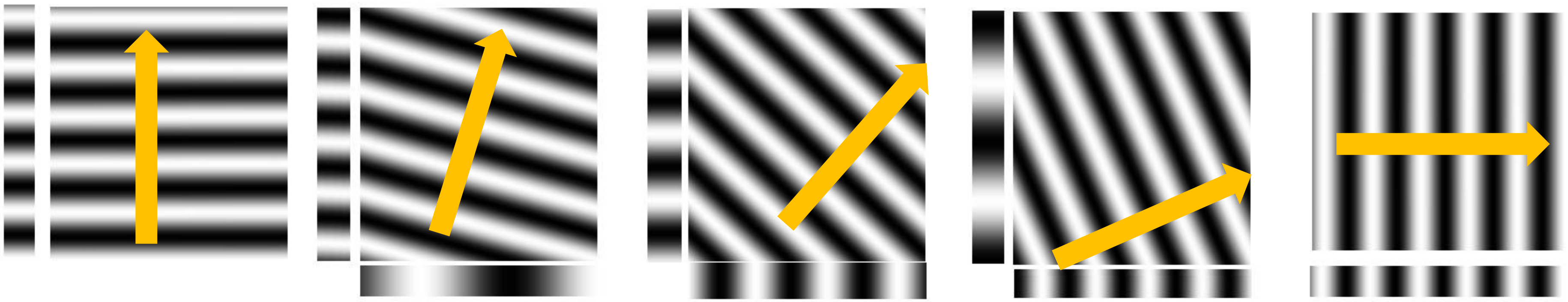
$k = \frac{\omega}{c}$, $\quad k_x^2 + k_y^2 + k_z^2 = \left(\frac{\omega}{c}\right)^2 \xrightarrow{k_x^2 + k_y^2 = k_\rho^2} k_\rho^2 + k_z^2 = \left(\frac{\omega}{c}\right)^2$

Diffraction limit

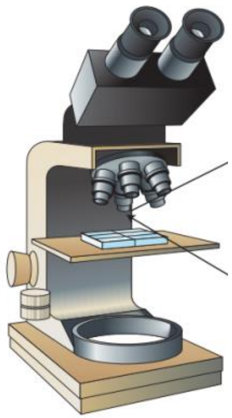


$$k_\rho^2 + k_z^2 = \left(\frac{\omega}{c}\right)^2$$

Decay $k_\rho = \frac{\omega}{c} = \frac{2\pi}{\lambda}$

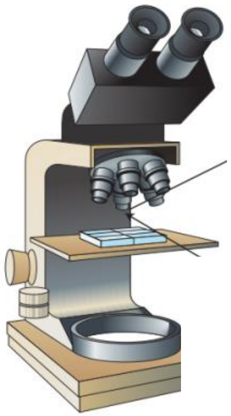


회절 한계

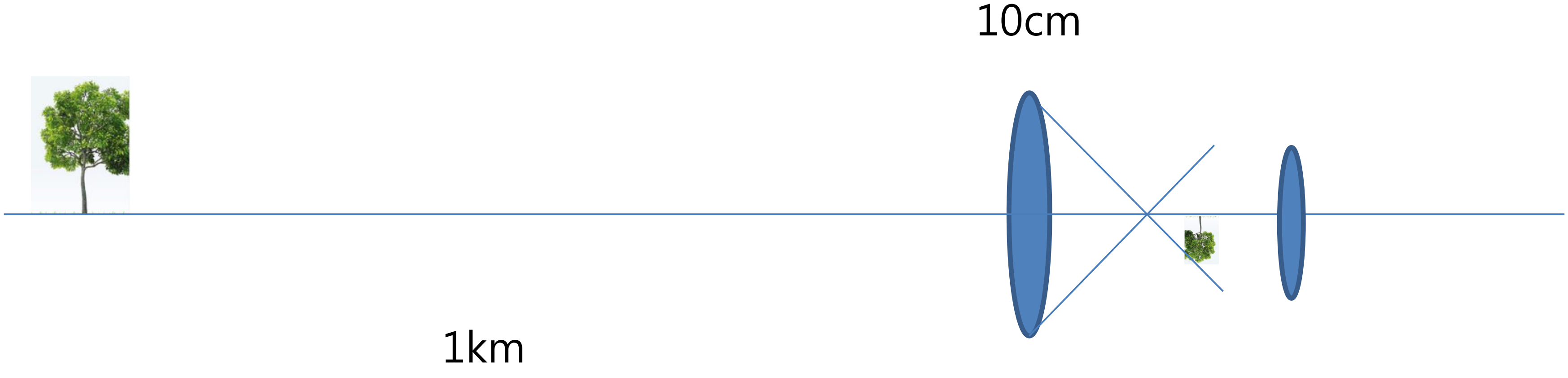


대물렌즈
대안렌즈

회절 한계

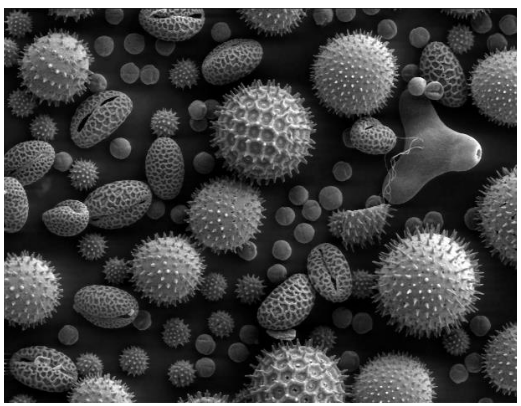


회절 한계

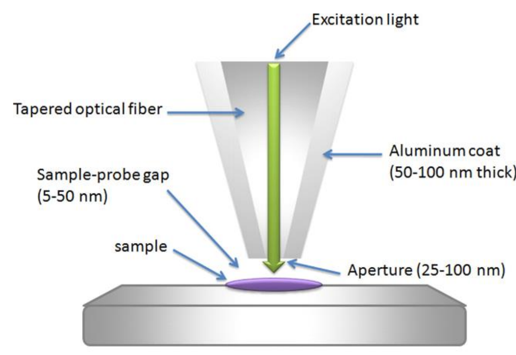


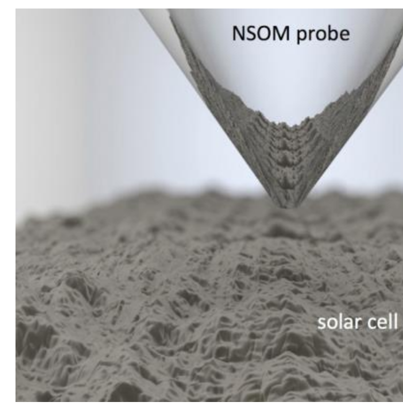
회절 한계

SEM(Scanning Electron Microscopy)

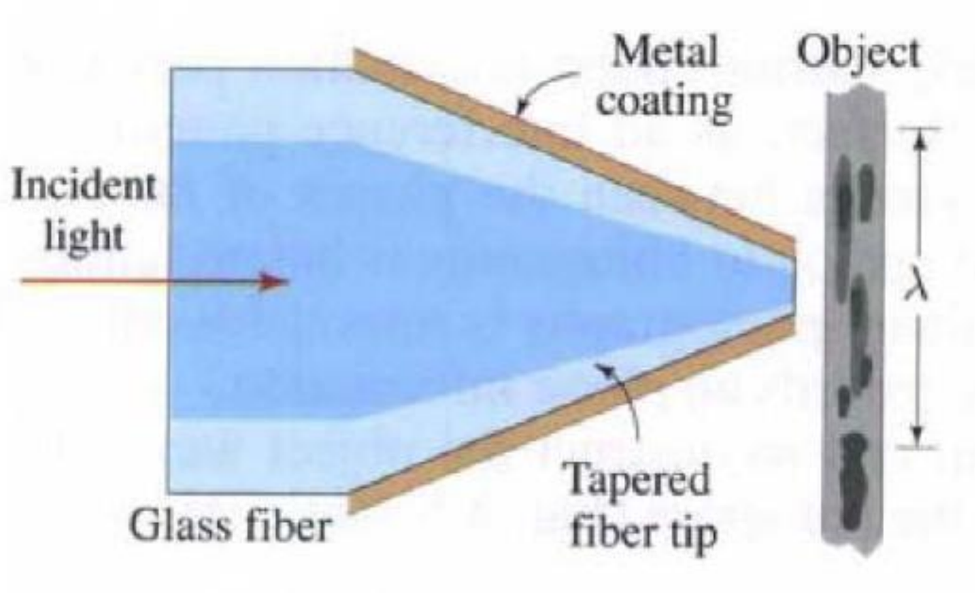


NSOM(Near-field Scanning Optical Microscopy)





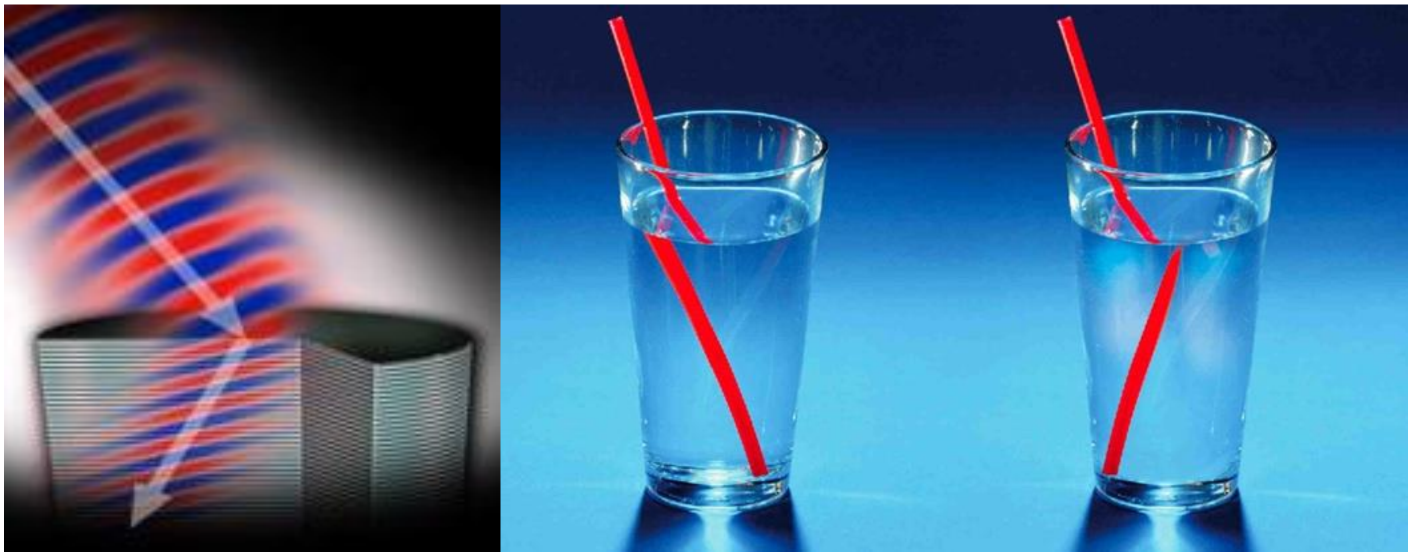
Near-Field Imaging



NSOM (Near-field Scanning Optical Microscopy) or SNOM (Scanning Near-field Optical Microscopy)

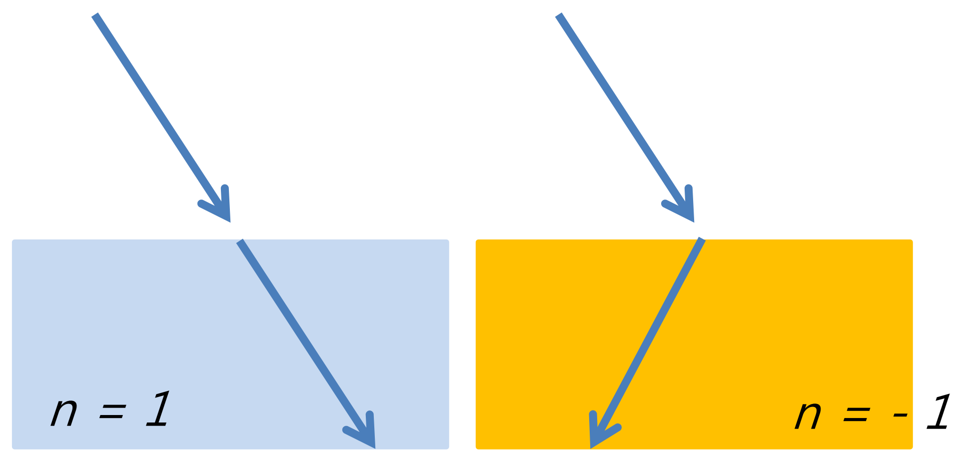
→ To resolve a subwavelength-scale features

Negative refractive index materials



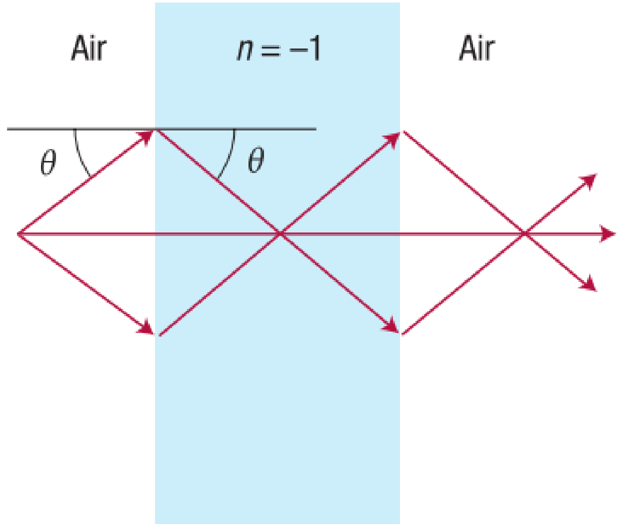
Snell's law

$$n = \frac{\sin(\theta_1)}{\sin(\theta_2)}$$

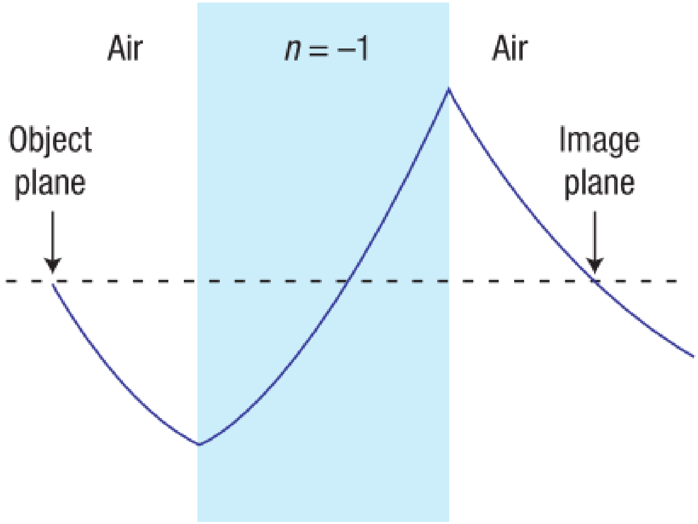


Beyond diffraction limit

If negative refractive index materials exist ?



Propagating waves



Evanescent waves

Imaging by planar lens
Not necessary

- **single 'optical axis'**
- **curved surface**

Overcome diffraction limit by amplification of evanescent near field
e^{inkt}

Fourier Optics

- Consider a plane wave: $U(x, y, z) = A \exp[-j(k_x x + k_y y + k_z z)]$

In the $z = 0$ plane,

$$\boxed{U(x, y, 0) = f(x, y) = A \exp[-j2\pi(\nu_x x + \nu_y y)]}$$

$$f = \frac{\omega}{2\pi} \quad \text{analogy}$$

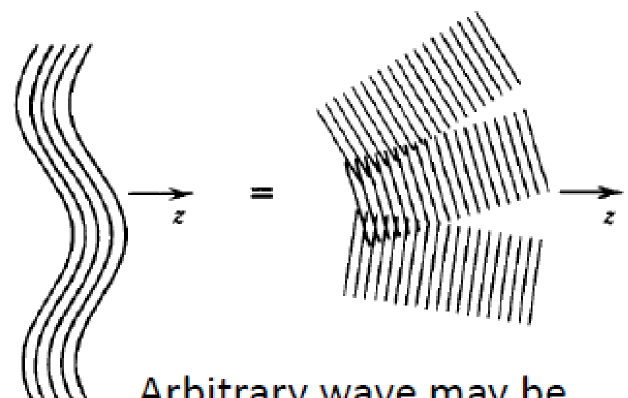
where $\left.\begin{aligned} \nu_x &= k_x/2\pi \\ \nu_y &= k_y/2\pi \end{aligned}\right\}$ Spatial frequencies

k_x and k_y : Spatial frequencies (radians/mm)

$U(x, y, z) \longleftrightarrow f(x, y) = U(x, y, 0)$

One-to-one correspondence

$(k_x^2 + k_y^2 + k_z^2 = \omega^2/c^2 = (2\pi/\lambda)^2)$



Arbitrary wave may be analyzed as a sum of plane

Spatial Harmonic Functions and Plane Waves

- Consider a plane wave: $U(x, y, z) = A \exp[-j(k_x x + k_y y + k_z z)]$

$$\theta_x = \sin^{-1}(k_x/k)$$
$$\theta_y = \sin^{-1}(k_y/k)$$

$$\rightarrow \boxed{\begin{aligned}\theta_x &= \sin^{-1} \lambda \nu_x \\ \theta_y &= \sin^{-1} \lambda \nu_y\end{aligned}}$$

Spatial frequencies and angles

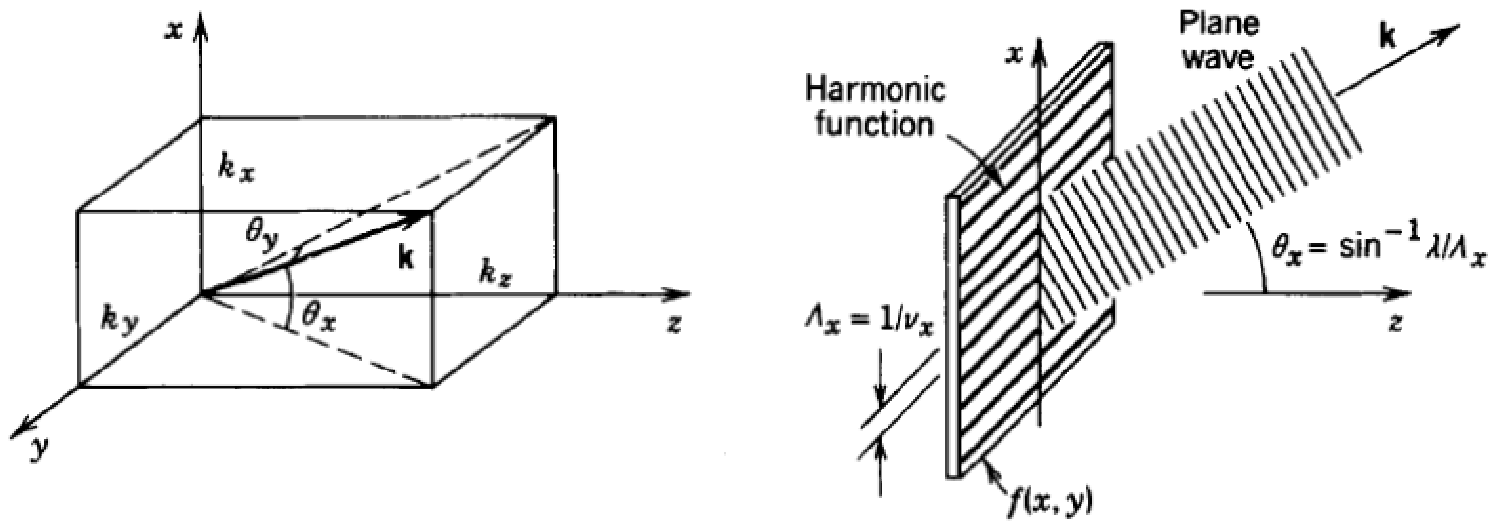
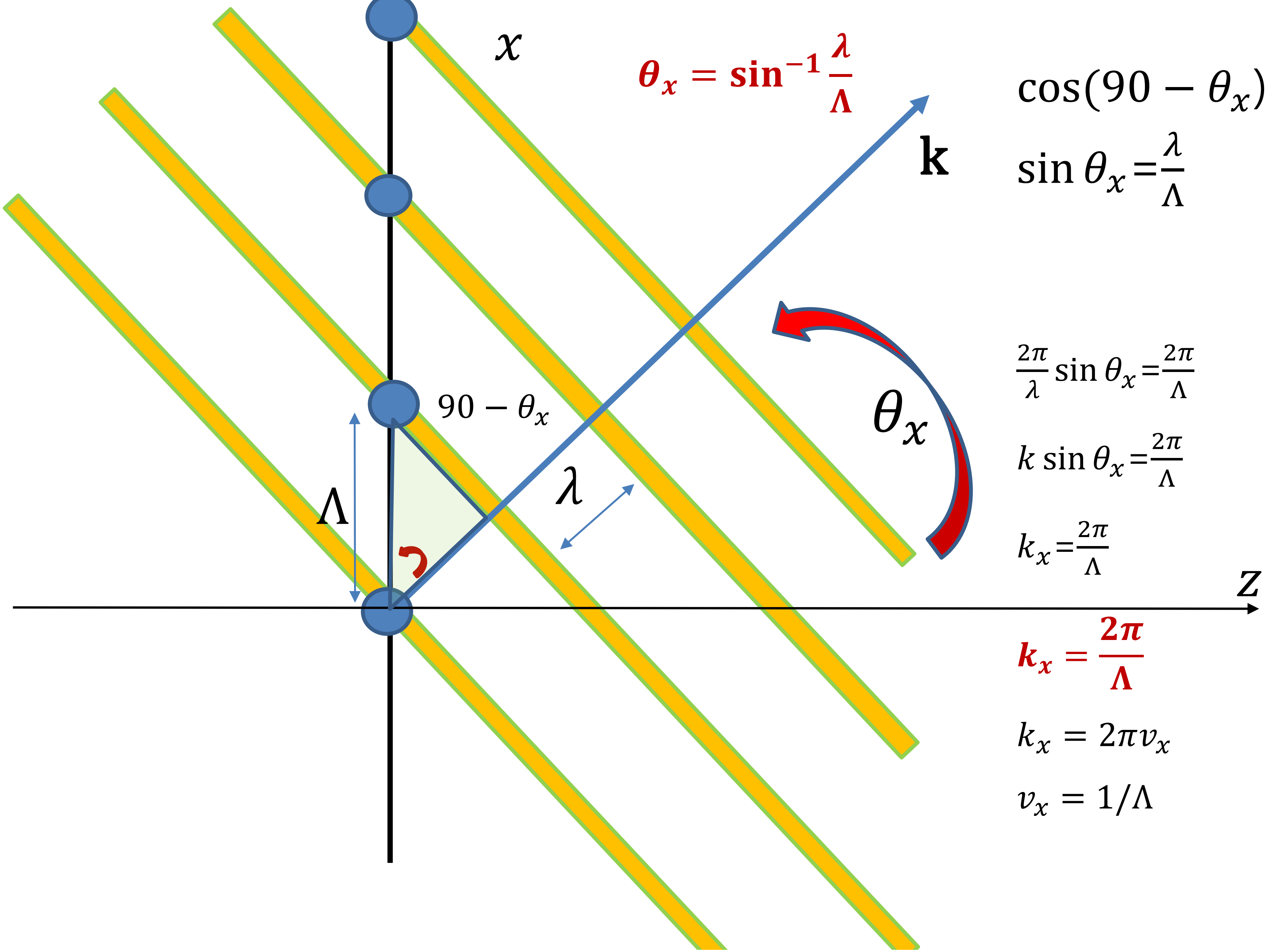


Figure 4.1-1 A harmonic function of spatial frequencies ν_x and ν_y at the plane $z = 0$ is consistent with a plane wave traveling at angles $\theta_x = \sin^{-1} \lambda \nu_x$ and $\theta_y = \sin^{-1} \lambda \nu_y$.



$$\boldsymbol{\theta_x = \sin^{-1}\frac{\lambda}{\Lambda}}$$

$$\cos(90 - \theta_x) = \frac{\lambda}{\Lambda}$$

$$\sin\theta_x = \frac{\lambda}{\Lambda}$$

$$\frac{2\pi}{\lambda}\sin\theta_x = \frac{2\pi}{\Lambda}$$

$$k\sin\theta_x = \frac{2\pi}{\Lambda}$$

$$k_x = \frac{2\pi}{\Lambda}$$

$$\boldsymbol{k_x = \frac{2\pi}{\Lambda}}$$

$$k_x = 2\pi v_x$$

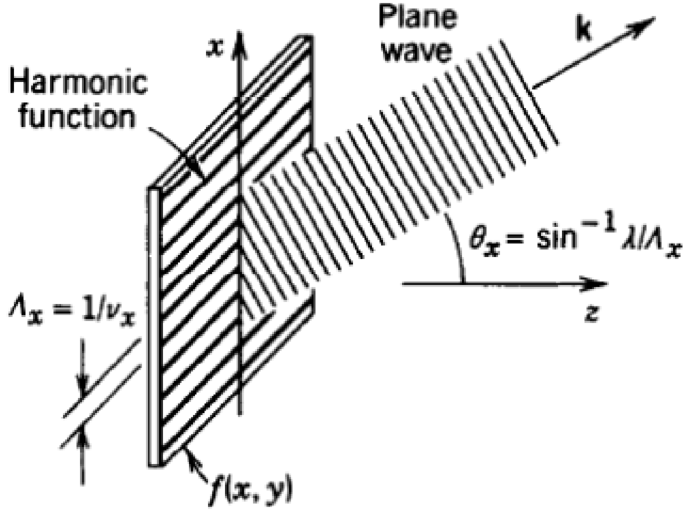
$$v_x = 1/\Lambda$$

$$k_z = \pm\sqrt{k^2 - k_x^2 - k_y^2}, \qquad k = 2\pi/\lambda. \tag{4.1-3}$$

Spatial Harmonic Functions and Plane Waves

- Recognizing $\Lambda_x = 1/\nu_x$ and $\Lambda_y = 1/\nu_y$ as the periods of the harmonic functions in the *x* and *y* directions,

$$\boxed{\begin{aligned} \theta_x &= \sin^{-1}(\lambda/\Lambda_x) \\ \theta_y &= \sin^{-1}(\lambda/\Lambda_y) \end{aligned}}$$



The angles of propagation is governed by the ratios of the wavelength of light to the period of the harmonic function in each direction

If $k_x \ll k$ and $k_y \ll k$, so that the wavevector **k** is paraxial, the angles θ_x and θ_y are small ($\sin\theta_x \approx \theta_x$ and $\sin\theta_y \approx \theta_y$) and

$$\boxed{\begin{aligned} \theta_x &\approx \lambda\nu_x \\ \theta_y &\approx \lambda\nu_y . \end{aligned}}$$

Spatial frequencies and angles (Paraxial approximation)

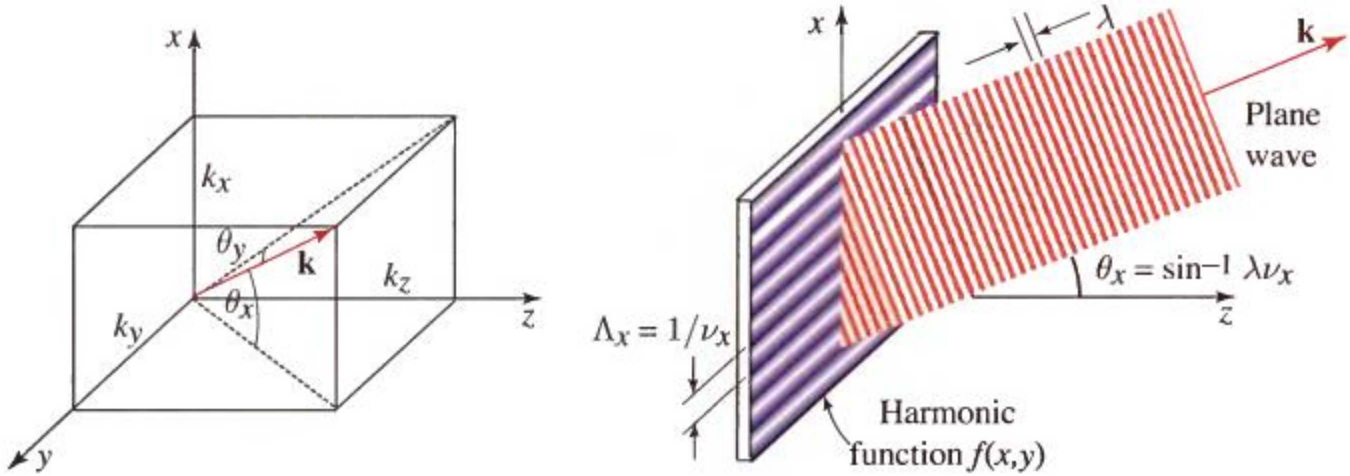


Figure 4.1-1 A harmonic function of spatial frequencies ν_x and ν_y at the plane $z = 0$ is consistent with a plane wave traveling at angles $\theta_x = \sin^{-1} \lambda\nu_x$ and $\theta_y = \sin^{-1} \lambda\nu_y$.

Given one, the other can be readily determined, provided the wavelength λ is known: the harmonic function $f(x, y)$ is obtained by sampling at the $z = 0$ plane, $f(x, y) = U(x, y, 0)$. Given the harmonic function $f(x, y)$, on the other hand, the wave $U(x, y, z)$ is constructed by using the relation $U(x, y, z) = f(x, y) \exp(-jk_z z)$ with

$$k_z = \pm\sqrt{k^2 - k_x^2 - k_y^2}, \qquad k = 2\pi/\lambda. \tag{4.1-3}$$

A condition for the validity of this correspondence is that $k_x^2 + k_y^2 < k^2$, so that k_z is real. This condition implies that $\lambda\nu_x < 1$ and $\lambda\nu_y < 1$, so that the angles θ_x and θ_y defined by (4.1-1) exist. The $+$ and $-$ signs in (4.1-3) represent waves traveling in the forward and backward directions, respectively. We shall be concerned with forward waves only.

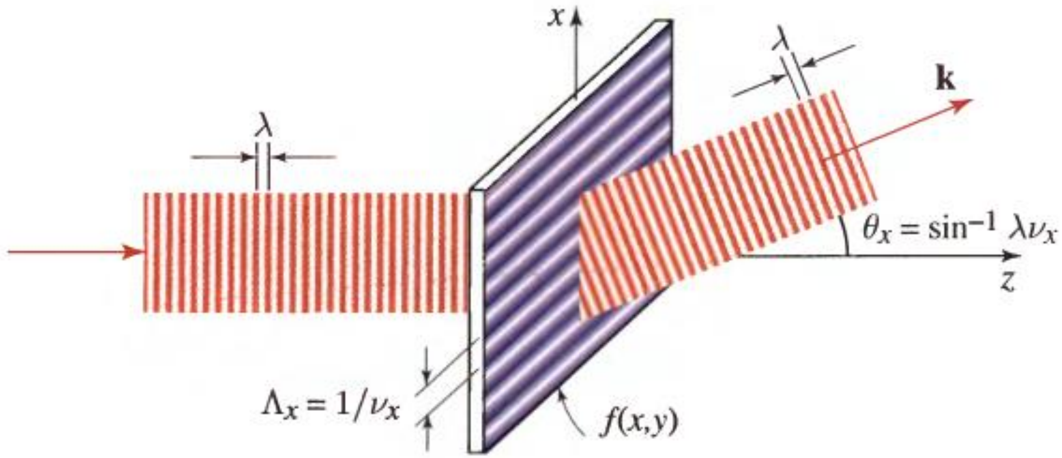
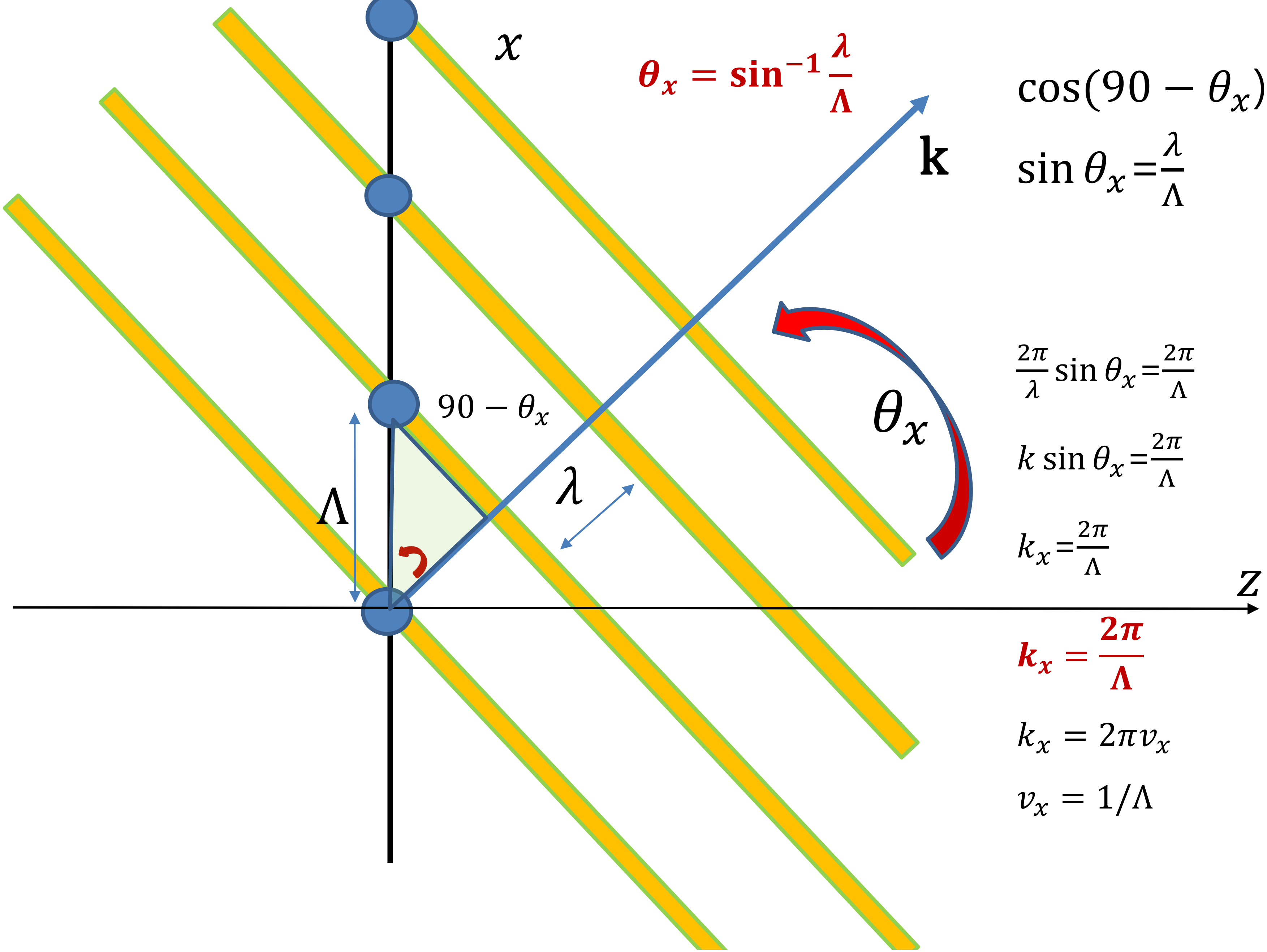


Figure 4.1-2 A thin element whose complex amplitude transmittance is a harmonic function of spatial frequency ν_x (period $\Lambda_x = 1/\nu_x$) bends a plane wave of wavelength λ by an angle $\theta_x = \sin^{-1}\lambda\nu_x = \sin^{-1}(\lambda/\Lambda_x)$. The blue color is used to indicate that the element is a *phase* grating (changing only the phase of the wave).



$$\theta_x = \sin^{-1}\frac{\lambda}{\Lambda}$$

$$\cos(90-\theta_x) = \frac{\lambda}{\Lambda}$$

$$\sin\theta_x = \frac{\lambda}{\Lambda}$$

$$\frac{2\pi}{\lambda}\sin\theta_x = \frac{2\pi}{\Lambda}$$

$$k\sin\theta_x = \frac{2\pi}{\Lambda}$$

$$k_x = \frac{2\pi}{\Lambda}$$

$$\boldsymbol{k_x = \frac{2\pi}{\Lambda}}$$

$$k_x = 2\pi v_x$$

$$v_x = 1/\Lambda$$

$$k_z = \pm\sqrt{k^2 - k_x^2 - k_y^2}, \qquad k = 2\pi/\lambda. \tag{4.1-3}$$

Spatial Harmonic Functions and Plane Waves

- One-to-one correspondence between the plane wave $U(x, y, z)$ and $f(x, y)$

→ Given the wave $U(x, y, z)$

$$\boxed{f(x, y) = U(x, y, 0)}$$

→ Given the harmonic function $f(x, y)$

$$\boxed{U(x, y, z) = f(x, y)\exp(-jk_z z)}$$

with $k_z = \pm\left(k^2 - k_x^2 - k_y^2\right)^{1/2}, \qquad k = 2\pi/\lambda$

Therefore, the one-to-one correspondence is valid with the assumptions: $k_x^2 + k_y^2 < k^2$ and forward propagating waves

More generally, if $f(x, y)$ is a superposition integral of harmonic functions,

$$f(x,y) = \iint_{-\infty}^{\infty} F(\nu_x, \nu_y) \exp\left[-j2\pi(\nu_x x + \nu_y y)\right] d\nu_x \, d\nu_y, \tag{4.1-4}$$

with frequencies (ν_x, ν_y) and amplitudes $F(\nu_x, \nu_y)$, the transmitted wave $U(x, y, z)$ is the superposition of plane waves,

$$U(x,y,z) = \iint_{-\infty}^{\infty} F(\nu_x, \nu_y) \exp\left[-j(2\pi\nu_x x + 2\pi\nu_y y)\right] \exp(-jk_z z) d\nu_x \, d\nu_y, \tag{4.1-5}$$

with complex envelopes $F(\nu_x, \nu_y)$ where $k_z = \sqrt{k^2 - k_x^2 - k_y^2} = 2\pi\sqrt{\lambda^{-2} - \nu_x^2 - \nu_y^2}$. Note that $F(\nu_x, \nu_y)$ is the Fourier transform of $f(x, y)$ (see Appendix A, Sec. A.3.

Since an arbitrary function may be Fourier analyzed as a superposition integral of the form (4.1-4), the light transmitted through a thin optical element of arbitrary transmittance may be written as a superposition of plane waves (see Fig. 4.1-3), provided that $\nu_x^2 + \nu_y^2 < \lambda^{-2}$.

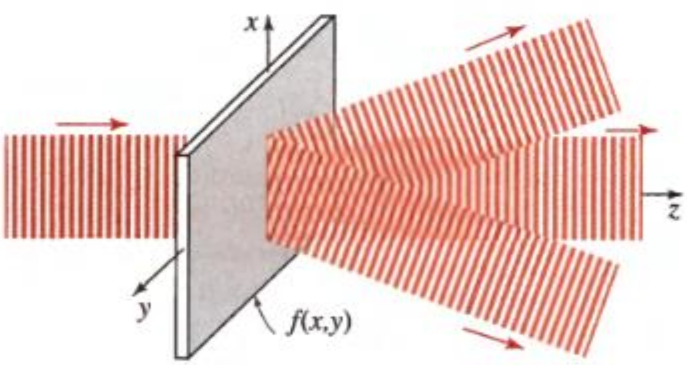


Figure 4.1-3 A thin optical element of amplitude transmittance $f(x, y)$ decomposes an incident plane wave into many plane waves. The plane wave traveling at the angles $\theta_x = \sin^{-1}\lambda\nu_x$ and $\theta_y = \sin^{-1}\lambda\nu_y$ has a complex envelope $F(\nu_x, \nu_y)$, the Fourier transform of $f(x, y)$.

Spatial Spectral Analysis

- Consider a plane wave of unity amplitude traveling in the *z* direction is transmitted through a thin optical element with complex amplitude transmittance $f(x,y) = \exp[-j2\pi(\nu_x x + \nu_y y)]$

→ $U(x, y, 0) = f(x, y)$

→ $\boxed{U(x, y, z) = f(x, y)\exp(-jk_z z)}$

A plane wave with a wavevector at angles

$$\boxed{\begin{aligned}\theta_x &= \sin^{-1}\lambda\nu_x \\ \theta_y &= \sin^{-1}\lambda\nu_y\end{aligned}}$$

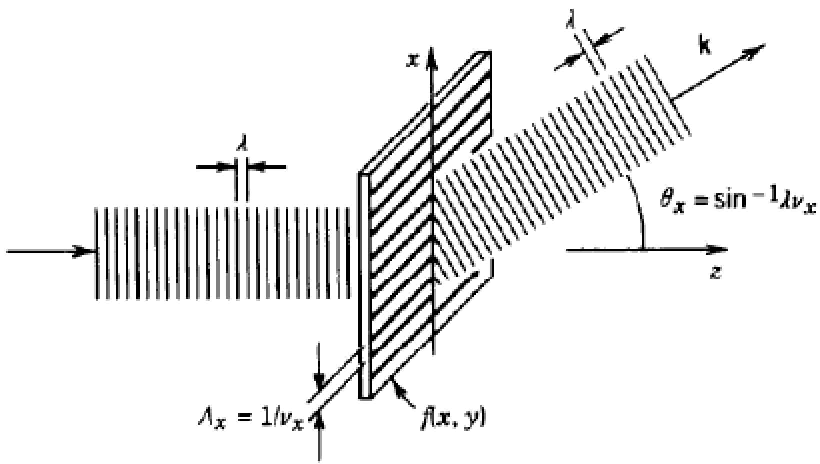
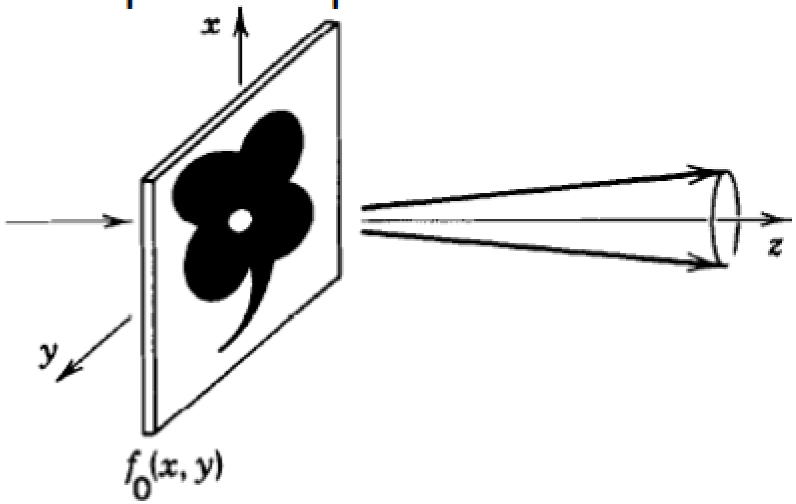


Figure 4.1-2 A thin element whose amplitude transmittance is a harmonic function of spatial frequency ν_x (period $\Lambda_x = 1/\nu_x$) bends a plane wave of wavelength λ by an angle $\theta_x = \sin^{-1}\lambda\nu_x = \sin^{-1}(\lambda/\Lambda_x)$.

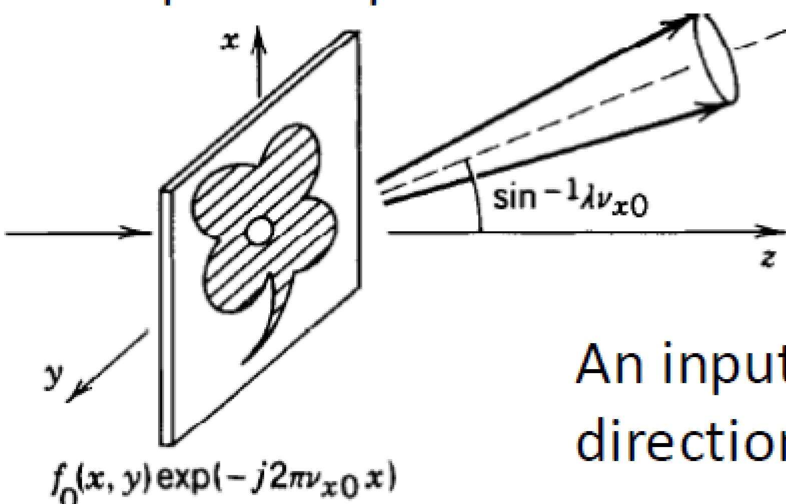
Amplitude Modulation

Complex amplitude transmittance $f_0(x, y)$.



If its Fourier transform extends over width $\pm\Delta\nu_x$ and $\pm\Delta\nu_y$, the incident plane wave will be deflected by $\pm\sin^{-1}(\lambda\,\Delta\nu_x)$ and $\pm\sin^{-1}(\lambda\,\Delta\nu_y)$ in the x and y directions

Complex amplitude transmittance $f_0(x, y)\exp[-j2\pi(\nu_{x0}x + \nu_{y0}y)]$.



$(\Delta\nu_x \ll \nu_{x0} \quad \Delta\nu_y \ll \nu_{y0})$

Slowly varying amplitude

$\boxed{F_0(\nu_x - \nu_{x0}, \nu_y - \nu_{y0})}$

An input plane wave will be deflected to directions centered about the angles

$\theta_{x0} = \sin^{-1}\lambda\nu_{x0} \qquad \theta_{y0} = \sin^{-1}\lambda\nu_{y0}$

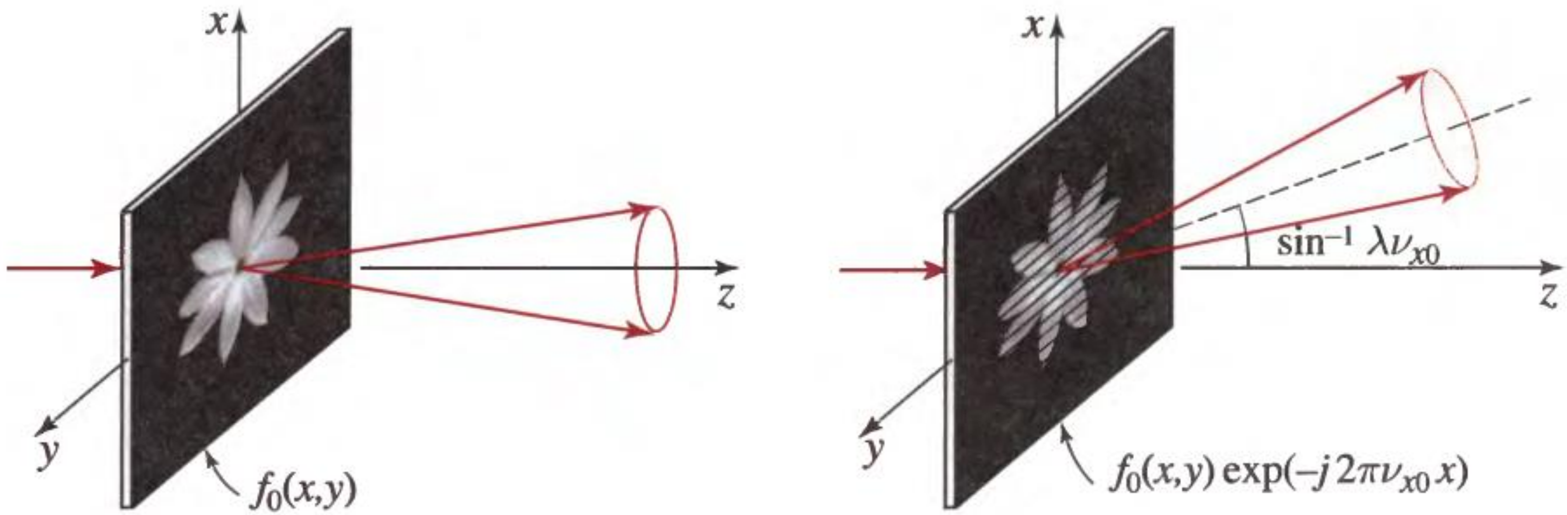


Figure 4.1-4 Deflection of light by the transparencies $f_0(x, y)$ and $f_0(x, y)\exp(-j2\pi\nu_{x0}x)$. The "carrier" harmonic function $\exp(-j2\pi\nu_{x0}x)$ acts as a prism that deflects the wave by an angle $\theta_{x0} = \sin^{-1}\lambda\nu_{x0}$.

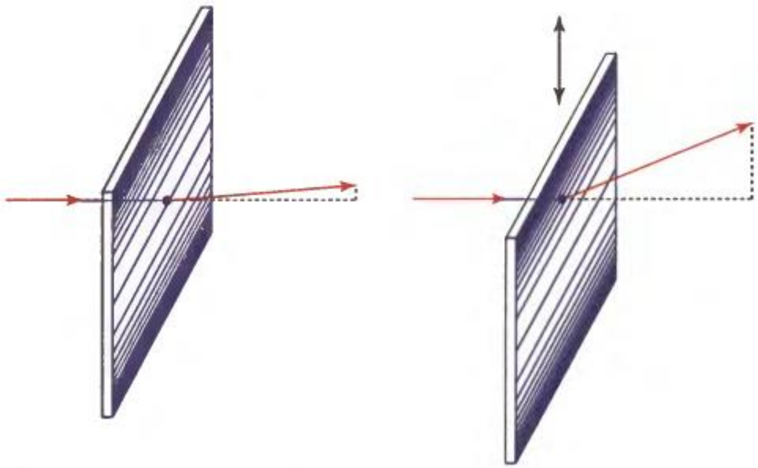


Figure 4.1-6 Using a frequency-modulated transparency to scan an optical beam.

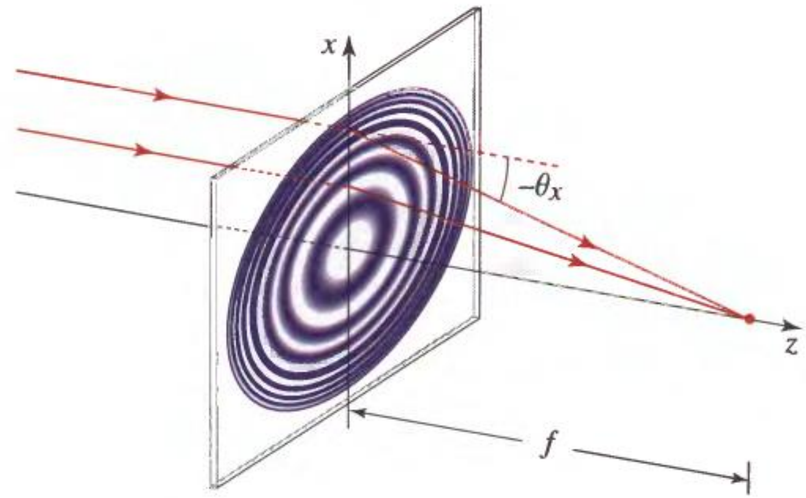


Figure 4.1-7 A transparency with transmittance $f(x, y) = \exp(j\pi x^2/\lambda f)$ bends the wave at position x by an angle $\theta_x \approx -x/f$ so that it acts as a cylindrical lens with focal length f.

D. Huygens–Fresnel Principle

The **Huygens–Fresnel principle** states that each point on a wavefront generates a spherical wave (Fig. 4.1-13). The envelope of these secondary waves constitutes a new wavefront. Their superposition constitutes the wave in another plane. The system's impulse response function for propagation between the planes $z = 0$ and $z = d$ is

$$h(x, y) \propto \frac{1}{r} \exp(-jkr), \qquad r = \sqrt{x^2 + y^2 + d^2}. \tag{4.1-21}$$

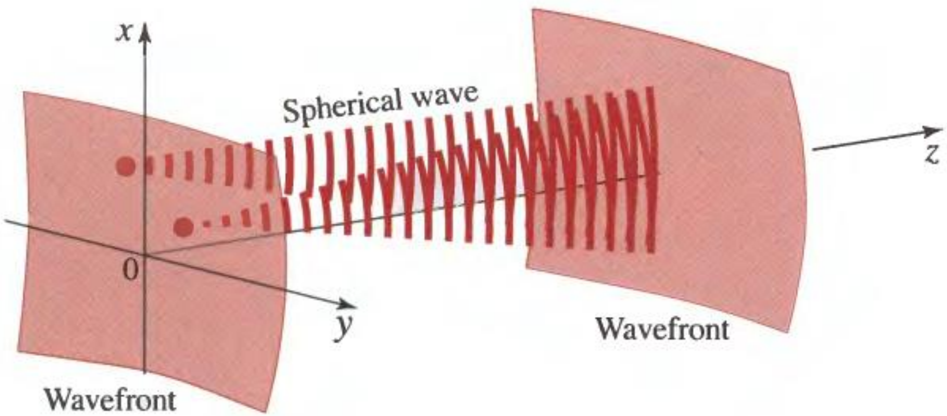


Figure 4.1-13 The Huygens–Fresnel principle. Each point on a wavefront generates a spherical wave.

Fourier Transform Using a Lens

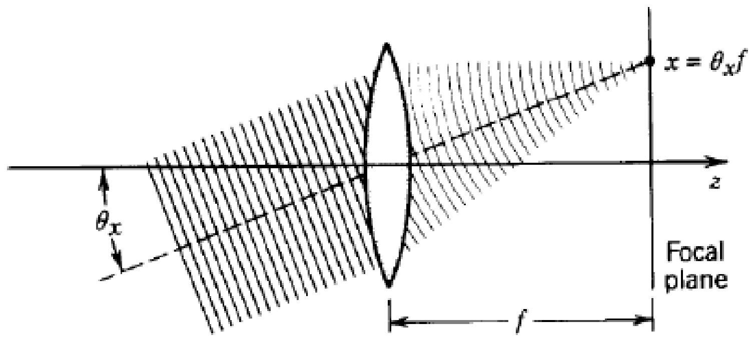


Figure 4.2-2 Focusing of a plane wave into a point. A direction (θ_x, θ_y) is mapped into a point $(x, y) = (\theta_x f, \theta_y f)$.

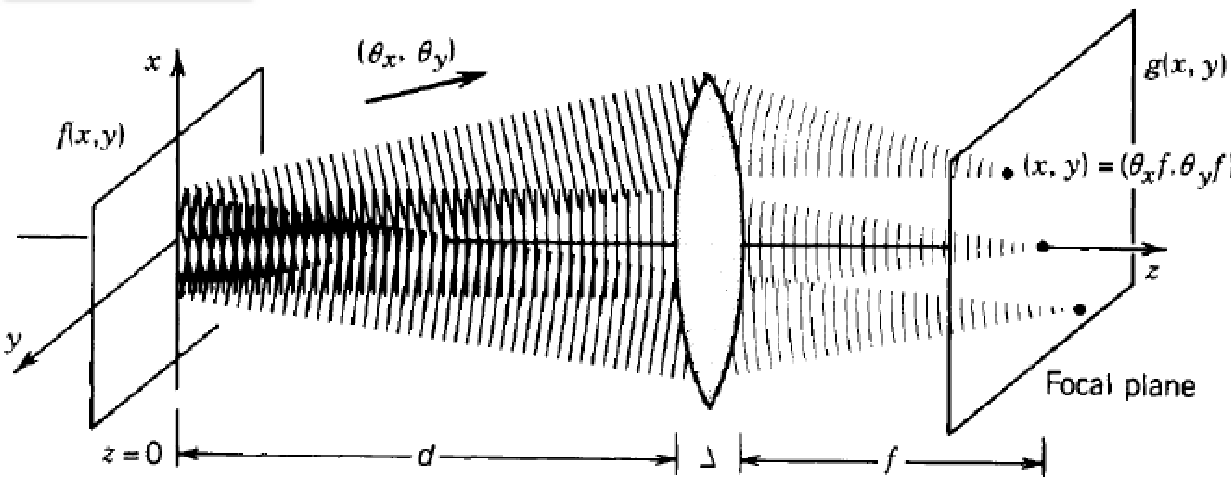
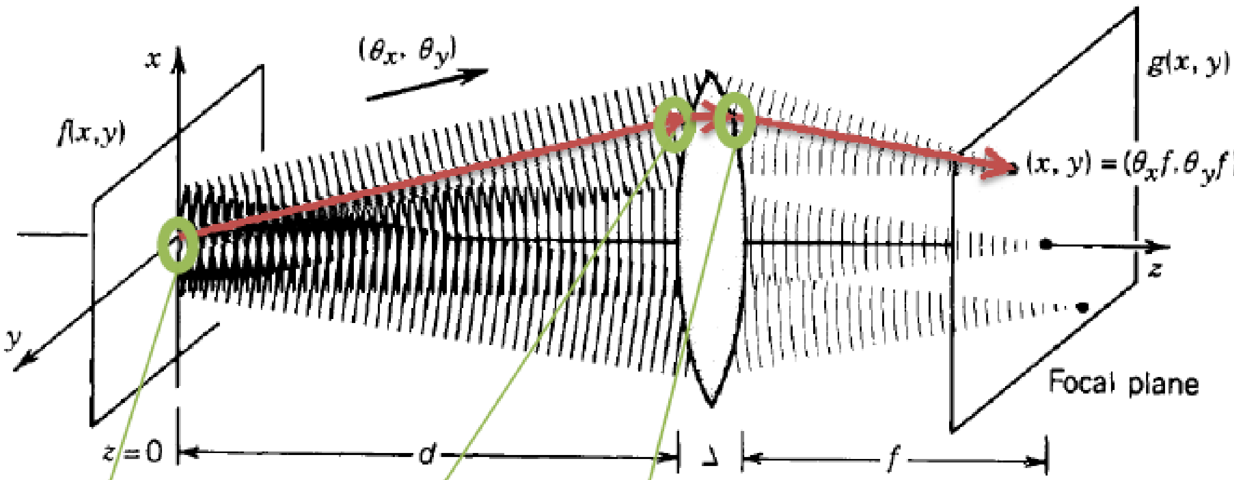


Figure 4.2-3 Focusing of the plane waves associated with the harmonic Fourier components of the input function $f(x, y)$ into points in the focal plane. The amplitude of the plane wave with direction $(\theta_x, \theta_y) = (\lambda \nu_x, \lambda \nu_y)$ is proportional to the Fourier transform $F(\nu_x, \nu_y)$ and is focused at the point $(x, y) = (\theta_x f, \theta_y f) = (\lambda f \nu_x, \lambda f \nu_y)$.

Fourier Transform Using a Lens



$$U(x,y,0) = F(\nu_x, \nu_y) \exp[-j2\pi(\nu_x x + \nu_y y)]$$

$$\boxed{\mathcal{H}(\nu_x, \nu_y)} F(\nu_x, \nu_y) \exp[-j2\pi(\nu_x x + \nu_y y)]$$

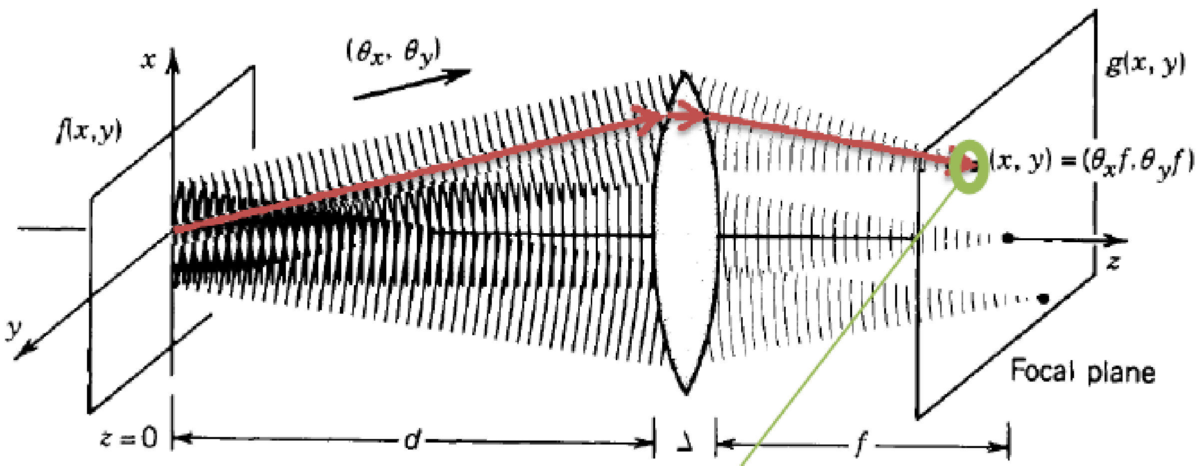
Free space transfer function

where $\mathcal{H}(\nu_x, \nu_y) = \mathcal{H}_0 \exp[j\pi\lambda d(\nu_x^2 + \nu_y^2)]$

$$\mathcal{H}_0 \boxed{\exp\left(j\pi \frac{x^2 + y^2}{\lambda f}\right)} \exp\left[j\pi\lambda d\left(\nu_x^2 + \nu_y^2\right)\right] F(\nu_x, \nu_y) \exp\left[-j2\pi(\nu_x x + \nu_y y)\right]$$

Lens transfer function

Fourier Transform Using a Lens



$$U(x, y, d + \Delta + f) = h_0 \iint_{-\infty}^{\infty} U(x' y', d + \Delta) \exp\left[-j\pi \frac{(x - x')^2 + (y - y')^2}{\lambda f} \right] dx' \, dy'$$

$$\int \exp[\, j2\pi(x - x_0)x'/\lambda f \,] \, dx' = \lambda f \delta(x - x_0)$$

$$U(x, y, d + \Delta + f) = h_0(\lambda f)^2 A(\nu_x, \nu_y) \delta(x - x_0) \delta(y - y_0)$$

Fourier Transform Using a Lens

$$U(x, y, d + \Delta + f) = h_0(\lambda f)^2 A(\nu_x, \nu_y)\delta(x - x_0)\delta(y - y_0)$$

Integrate over all the plane waves

$$\boxed{g(x, y) = h_l \exp\left[j\pi \frac{(x^2 + y^2)(d - f)}{\lambda f^2}\right] F\left(\frac{x}{\lambda f}, \frac{y}{\lambda f}\right)}$$

where $h_l = \mathcal{H}_0 h_0 = (j/\lambda f)\exp[-jk(d + f)]$

if $d = f$:

$$\boxed{g(x, y) = h_l F\left(\frac{x}{\lambda f}, \frac{y}{\lambda f}\right)}$$

Fourier transform using a lens

Note that this result is obtained by assuming the Fresnel approximation

B. Fourier Transform Using a Lens

The plane-wave components that constitute a wave may also be separated by use of a lens. A thin spherical lens transforms a plane wave into a paraboloidal wave focused to a point in the lens focal plane (see Sec. 2.4 and Exercise 2.4-3). If the plane wave arrives at small angles θ_x and θ_y, the paraboloidal wave is centered about the point $(\theta_x f, \theta_y f)$, where f is the focal length (see Fig. 4.2-2). The lens therefore maps each direction (θ_x, θ_y) into a single point $(\theta_x f, \theta_y f)$ in the focal plane and thus separates the contributions of the different plane waves.

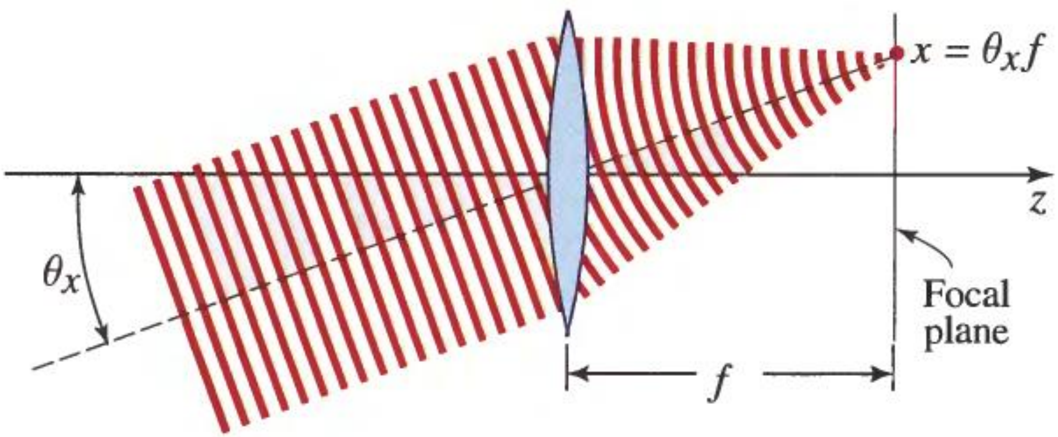


Figure 4.2-2 Focusing of a plane wave into a point. A direction (θ_x, θ_y) is mapped into a point $(x, y) = (\theta_x f, \theta_y f)$. (see Exercise 2.4-3.)

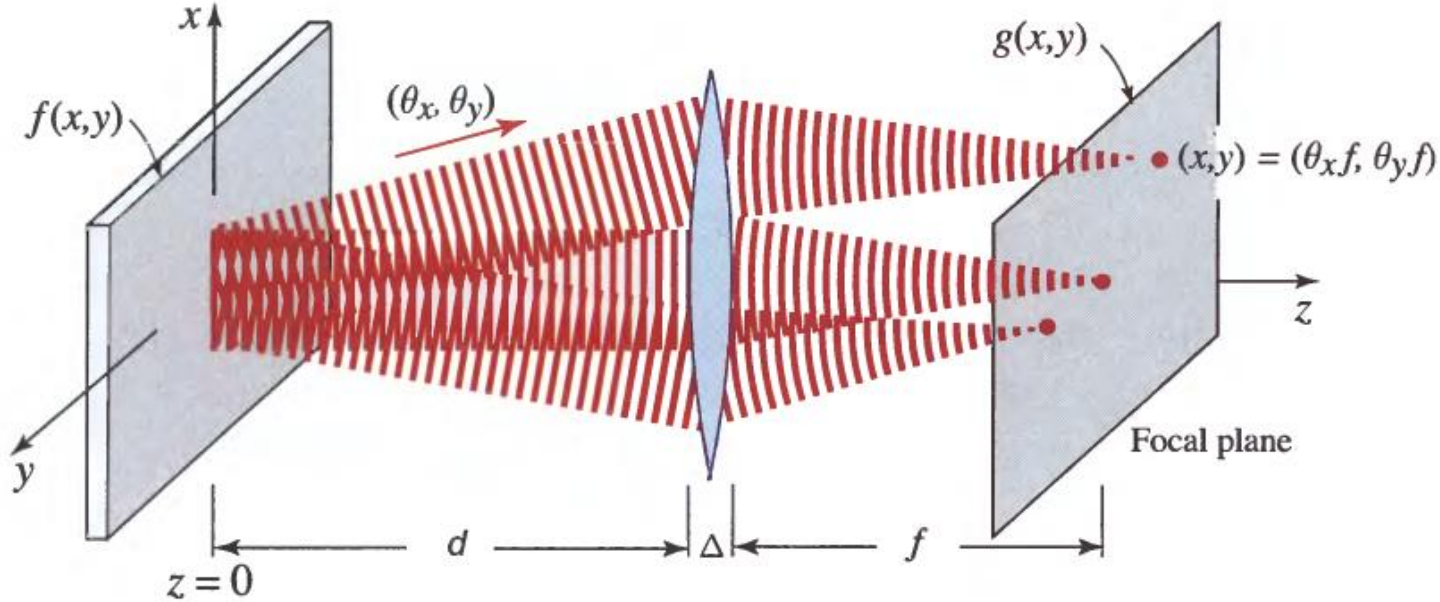


Figure 4.2-3 Focusing of the plane waves associated with the harmonic Fourier components of the input function $f(x, y)$ into points in the focal plane. The amplitude of the plane wave with direction $(\theta_x, \theta_y) = (\lambda\nu_x, \lambda\nu_y)$ is proportional to the Fourier transform $F(\nu_x, \nu_y)$ and is focused at the point $(x, y) = (\theta_x f, \theta_y f) = (\lambda f \nu_x, \lambda f \nu_y)$.

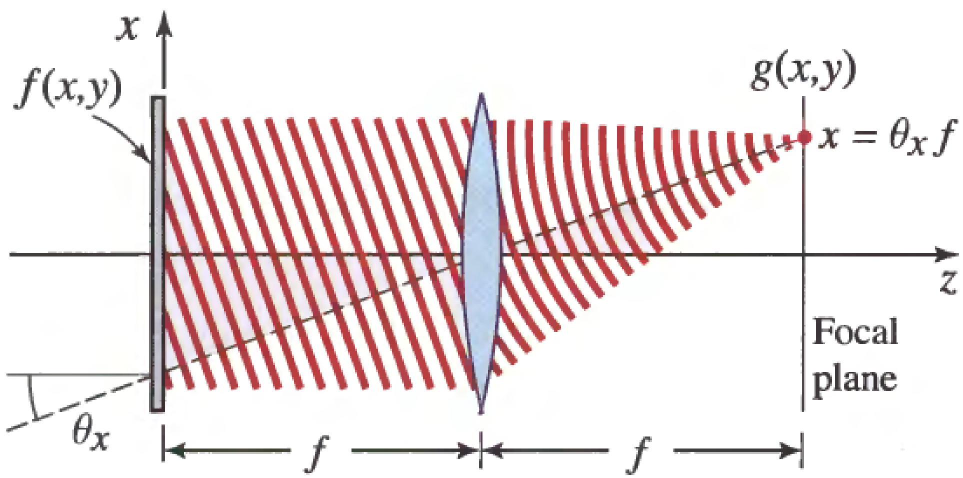


Figure 4.2-4 The 2-f system. The Fourier component of $f(x, y)$ with spatial frequencies ν_x and ν_y generates a plane wave at angles $\theta_x = \lambda\nu_x$ and $\theta_y = \lambda\nu_y$ and is focused by the lens to the point $(x, y) = (f\theta_x, f\theta_y) = (\lambda f\nu_x, \lambda f\nu_y)$ so that $g(x, y)$ is proportional to the Fourier transform $F(x/\lambda f, y/\lambda f)$.

B. Wave-Optics of a 4-f Imaging System

Consider now the two-lens imaging system illustrated in Fig. 4.4-3. This system, called the **4-*f* system**, serves as a focused imaging system with unity magnification, as can be easily verified by ray tracing.

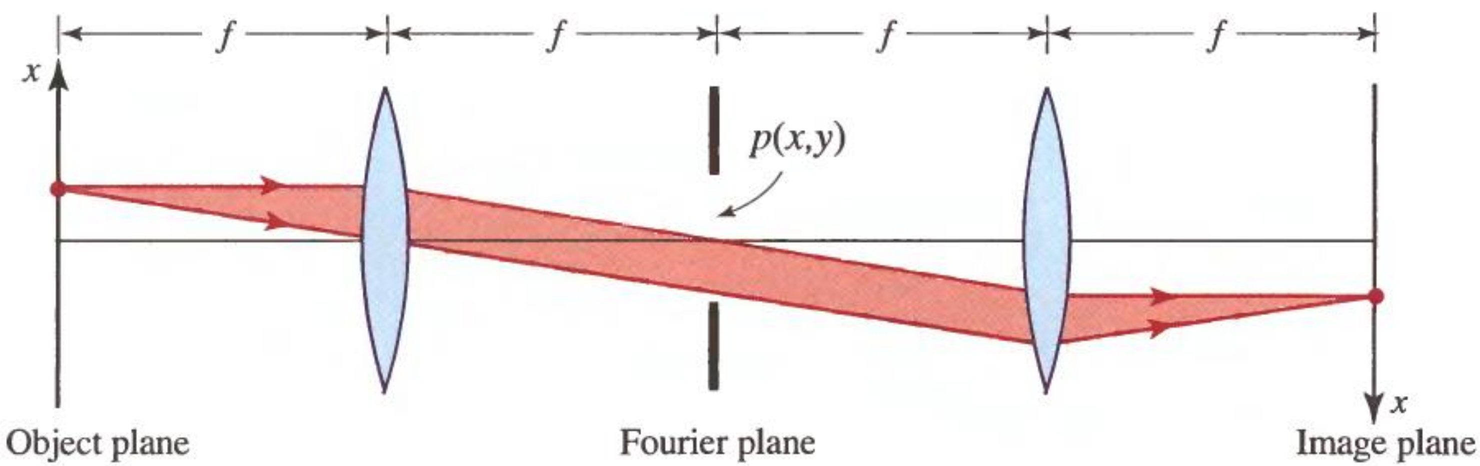


Figure 4.4-3 The 4-f imaging system. If an inverted coordinate system is used in the image plane, the magnification is unity.

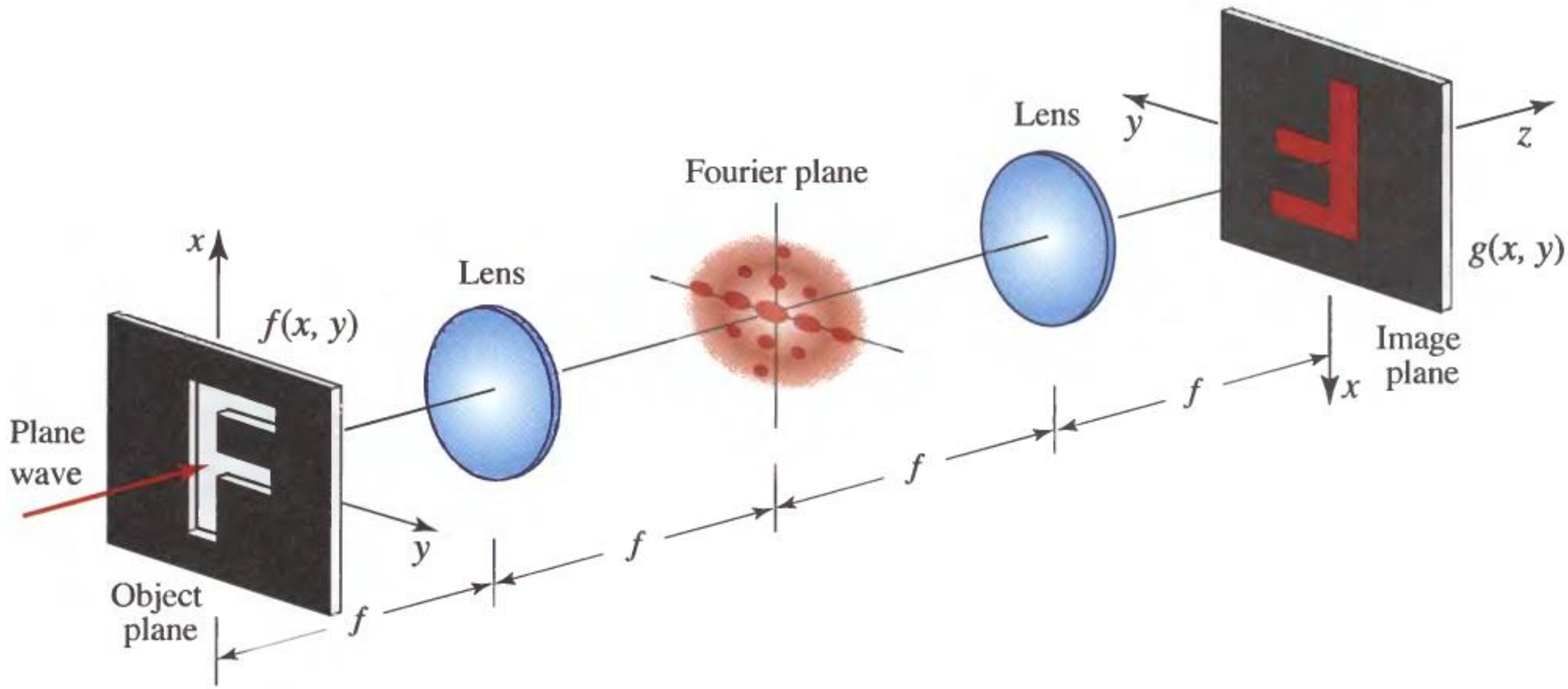


Figure 4.4-4 The 4-f imaging system performs a Fourier transform followed by an inverse Fourier transform, so that the image is a perfect replica of the object.

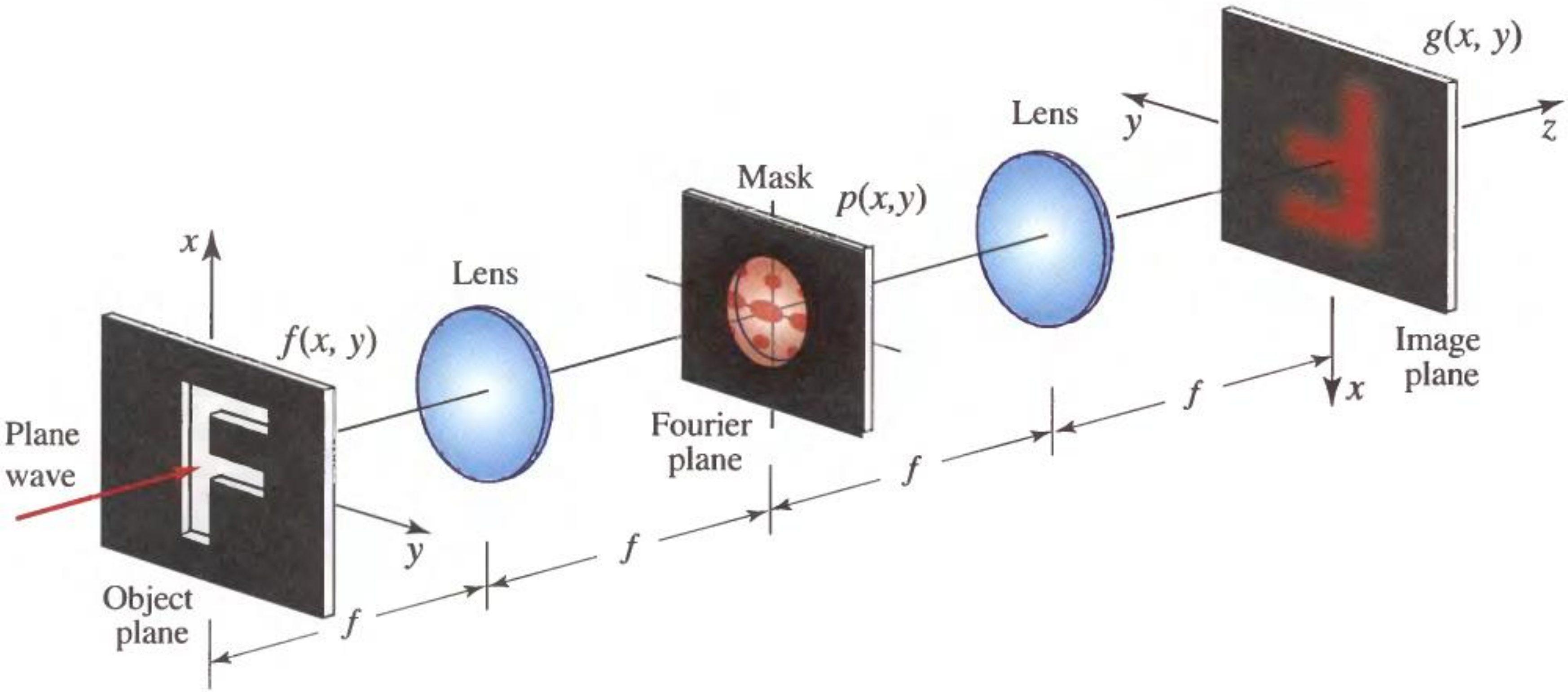


Figure 4.4-5 Spatial filtering. The transparencies in the object and Fourier planes have complex amplitude transmittances $f(x, y)$ and $p(x, y)$. A plane wave traveling in the z direction is modulated by the object transparency, Fourier transformed by the first lens, multiplied by the transmittance of the mask in the Fourier plane, and inverse Fourier transformed by the second lens. As a result, the complex amplitude in the image plane $g(x, y)$ is a filtered version of $f(x, y)$. The system has a transfer function $H(\nu_x, \nu_y) = p(\lambda f \nu_x, \lambda f \nu_y)$.

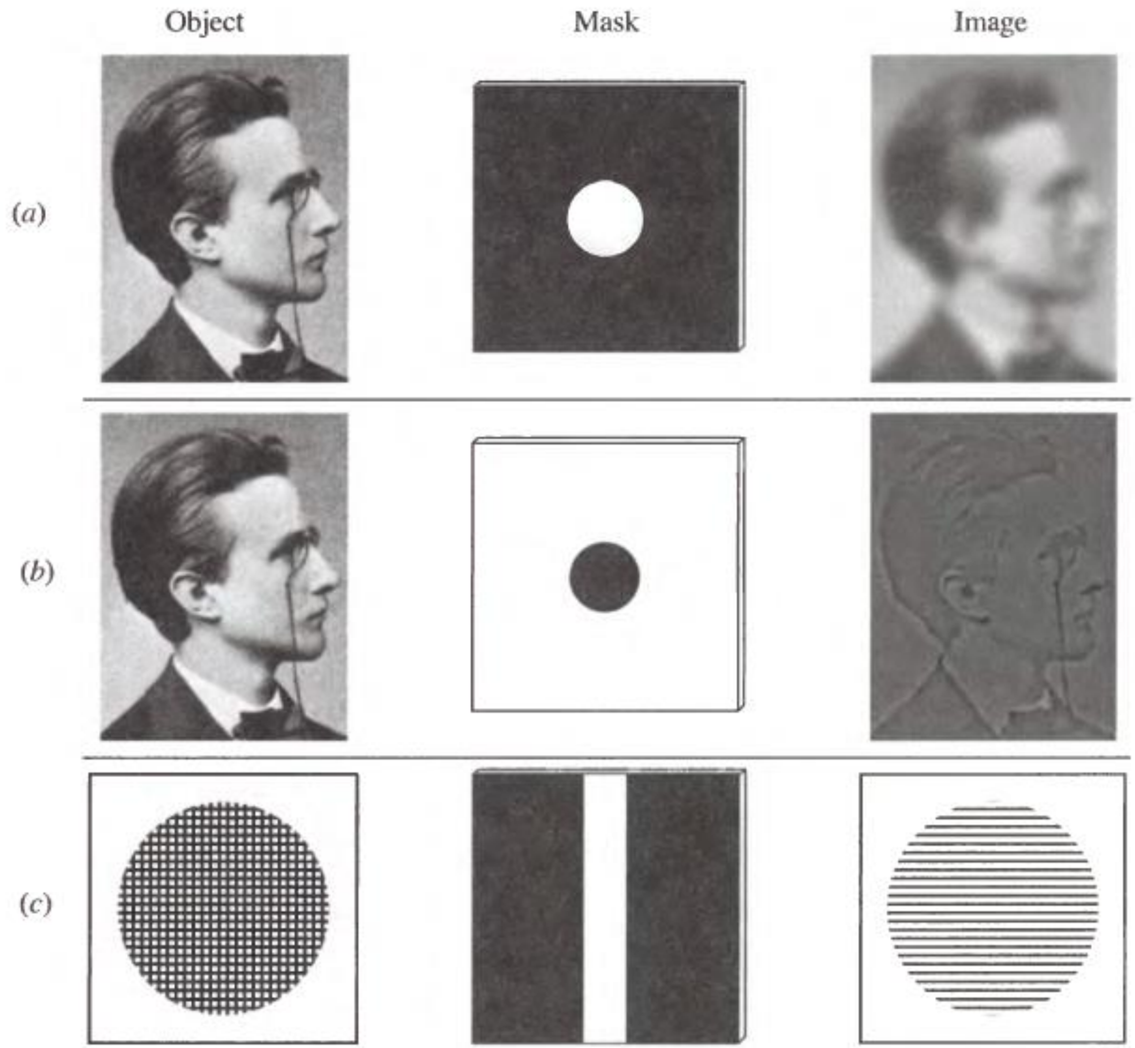


Figure 4.4-6 Examples of object, mask, and filtered image for three spatial filters: (*a*) low-pass filter; (*b*) high-pass filter; (*c*) vertical-pass filter. Black means the transmittance is zero and white means the transmittance is unity.

Holography

- Holography: a technique that allows the light scattered from an object to be recorded and later reconstructed so that it appears as if the object is in the same position relative to the recording medium as it was when recorded.
- Hologram: transparency containing a coded record of the optical wave

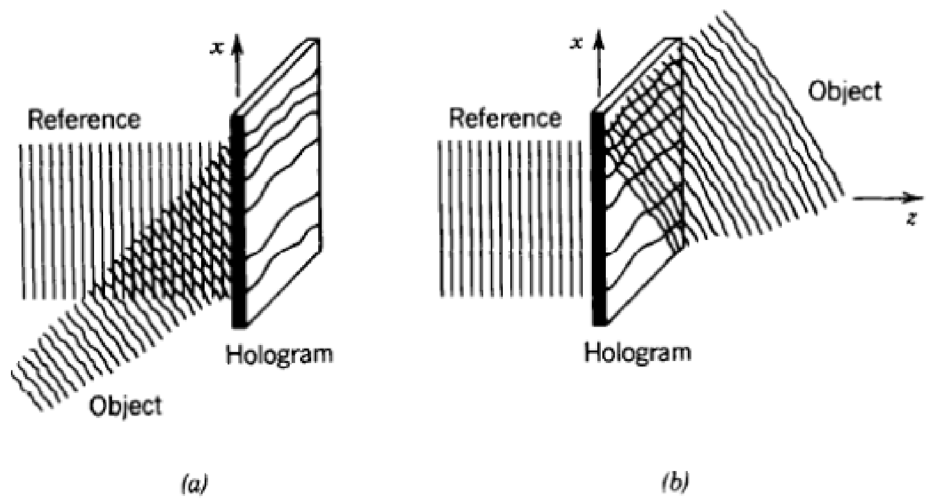


Figure 4.5-1 (*a*) A hologram is a transparency on which the interference pattern between the original wave (object wave) and a reference wave is recorded. (*b*) The original wave is reconstructed by illuminating the hologram with the reference wave.

The Holographic Code

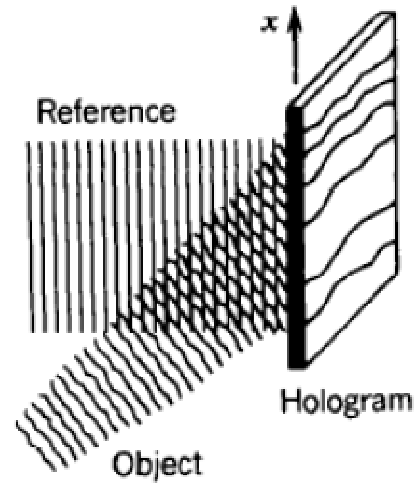
- Mixing the object wave U_0 with a known reference wave U_r

  → Recording their interference pattern

$$t \propto |U_o + U_r|^2 = |U_r|^2 + |U_o|^2 + U_r^* U_o + U_r U_o^*,$$

$$= I_r + I_o + U_r^* U_o + U_r U_o^*,$$

$$= I_r + I_o + 2(I_r I_o)^{1/2} \cos[\arg\{U_r\} - \arg\{U_o\}],$$



Complex amplitude transmittance of a hologram

Coded information related to the phase of the object wave

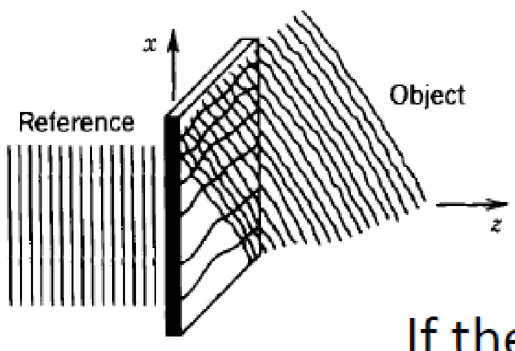
Coded information related to the magnitude of the object wave

The Holographic Code

- To decode the information in the hologram and to reconstruct the object wave

  → <u>Illuminate the hologram by the reference wave U_r</u>

$$\boxed{U = t U_r \propto \underbrace{U_r I_r + U_r I_o}_{} + I_r U_o + U_r^2 U_o^*}$$



The reference wave modulated by the sum of the intensities

The desired object wave (if I_r is constant)

If the reference wave is a plane wave propagating along the z axis, $U_r(x, y) = I_r^{1/2}$ at z=0

$$\boxed{U(x, y) \propto I_r + I_o(x, y) + \boxed{I_r^{1/2} U_o(x, y)} + I_r^{1/2} U_o^*(x, y)}$$

Separate the third term!!!

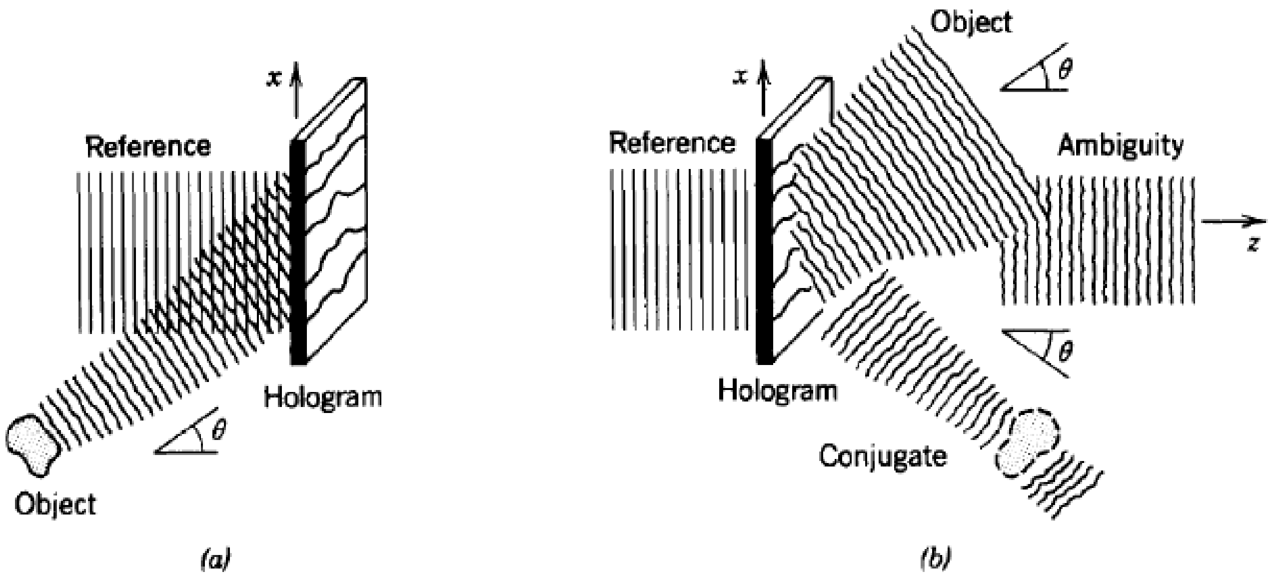
Off-Axis Holography

- Assume that the object wave has a complex amplitude:

$$U_o(x, y) = f(x, y)\exp(-jk\sin\theta\, x)$$

Then, $U(x, y) \propto I_r + |f(x, y)|^2 + I_r^{1/2} f(x, y)\exp(-jk\sin\theta\, x)$

$$+ I_r^{1/2} f^*(x, y)\exp(+jk\sin\theta\, x).$$



This gives a natural angular separation!

Figure 4.5-4 Hologram of an off-axis object wave: (a) recording; (b) reconstruction. The object wave is separated from both the reference and conjugate waves.

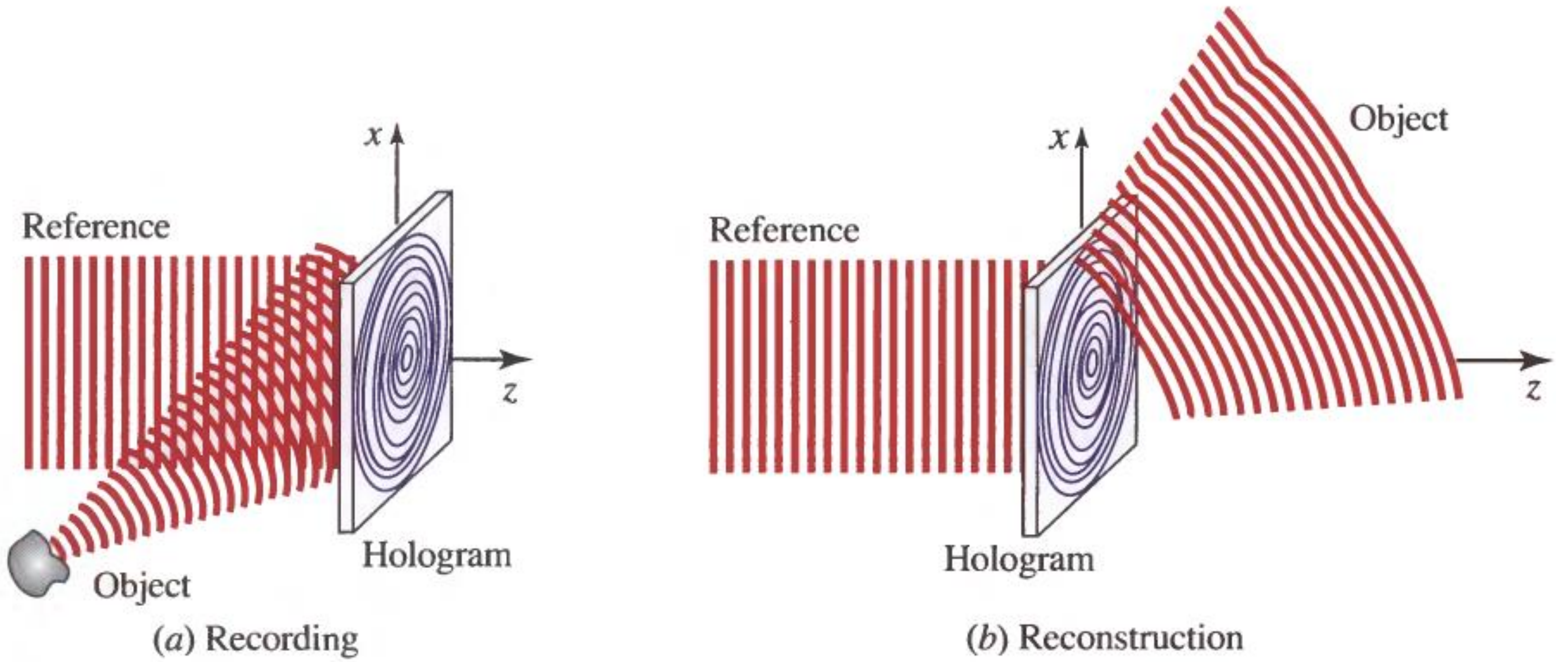


Figure 4.5-1 (*a*) A hologram is a transparency on which the interference pattern between the original wave (object wave) and a reference wave is recorded. (*b*) The original wave is reconstructed by illuminating the hologram with the reference wave.

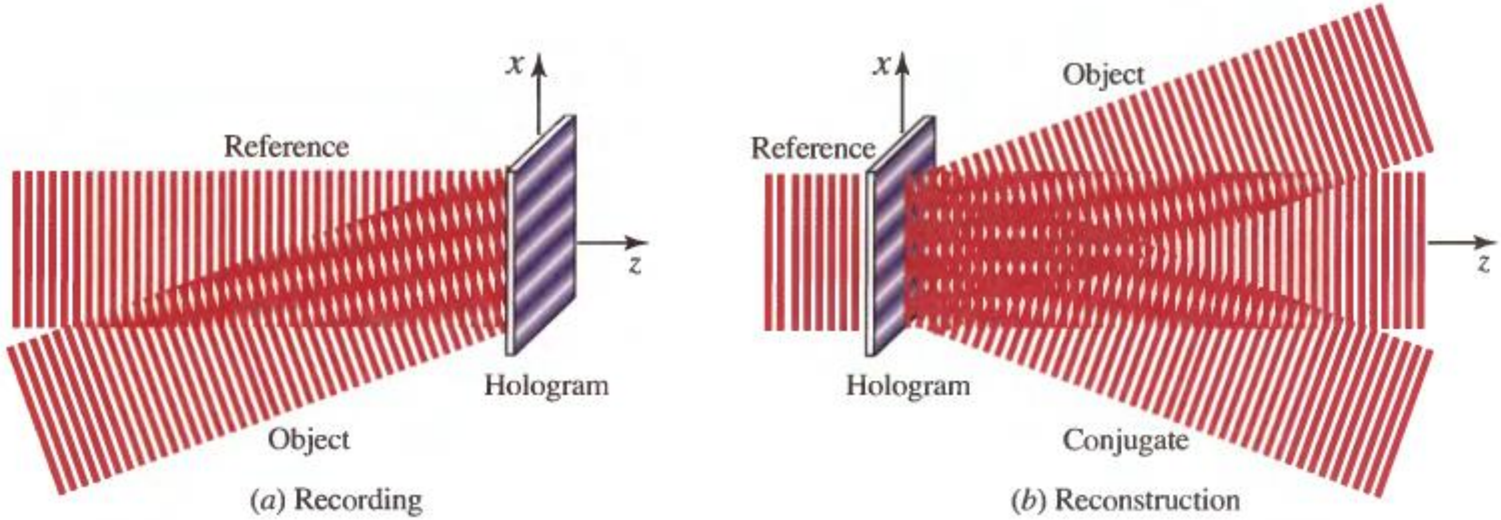


Figure 4.5-2 The hologram of an oblique plane wave is a sinusoidal diffraction grating.

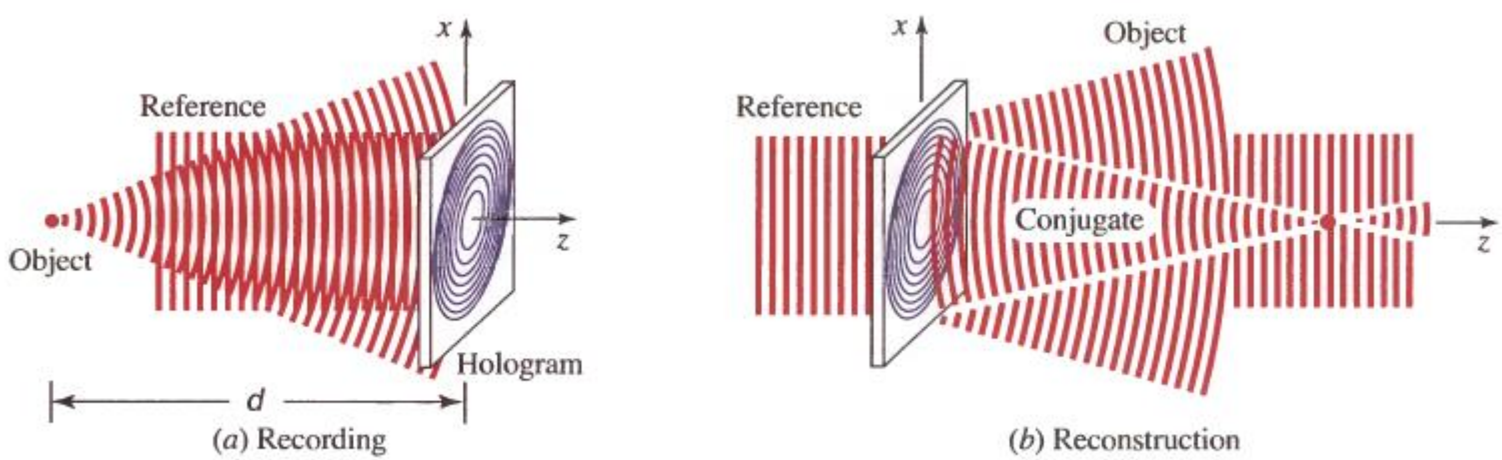


Figure 4.5-3 Hologram of a spherical wave originating from a point source. The conjugate wave forms a real image of the point.

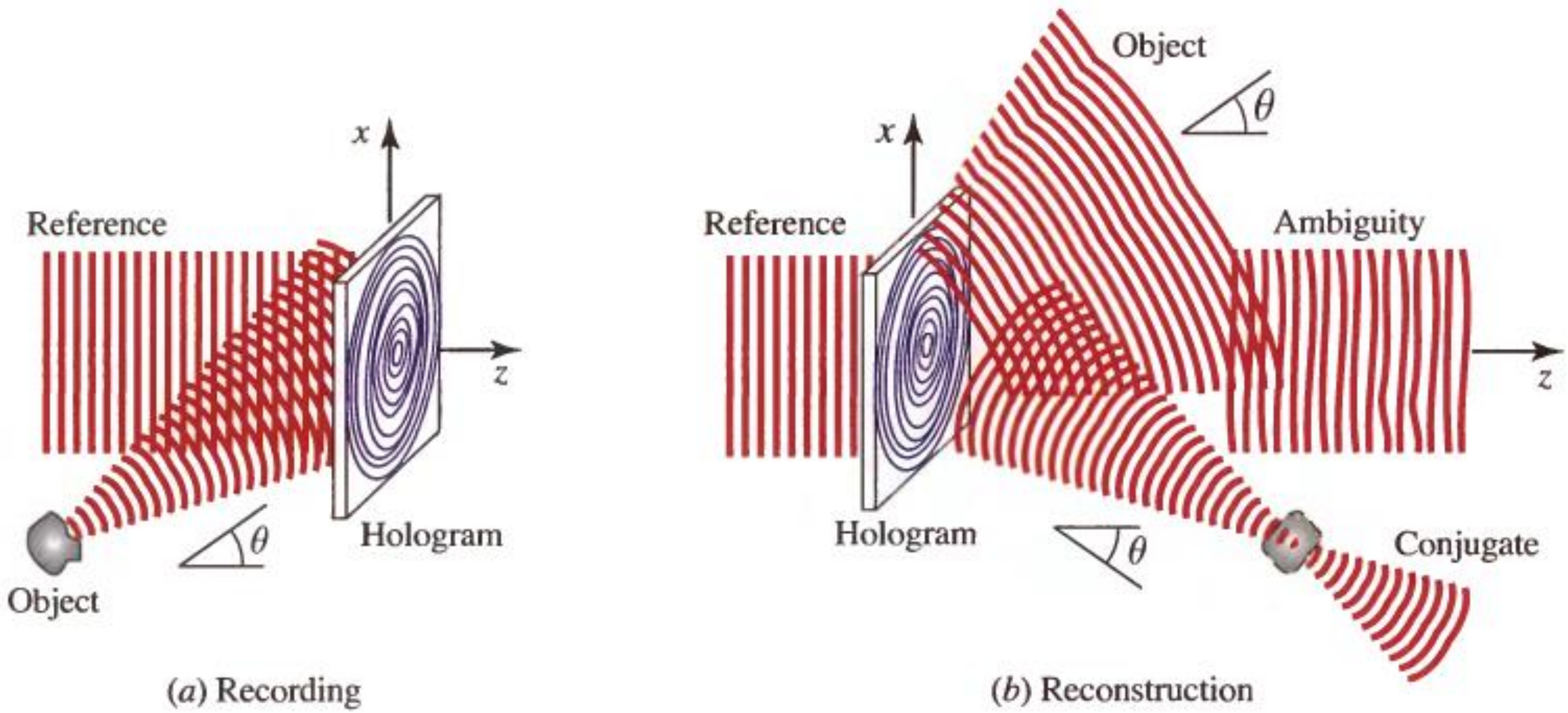


Figure 4.5-4 Hologram of an off-axis object wave. The object wave is separated from both the reference and conjugate waves.

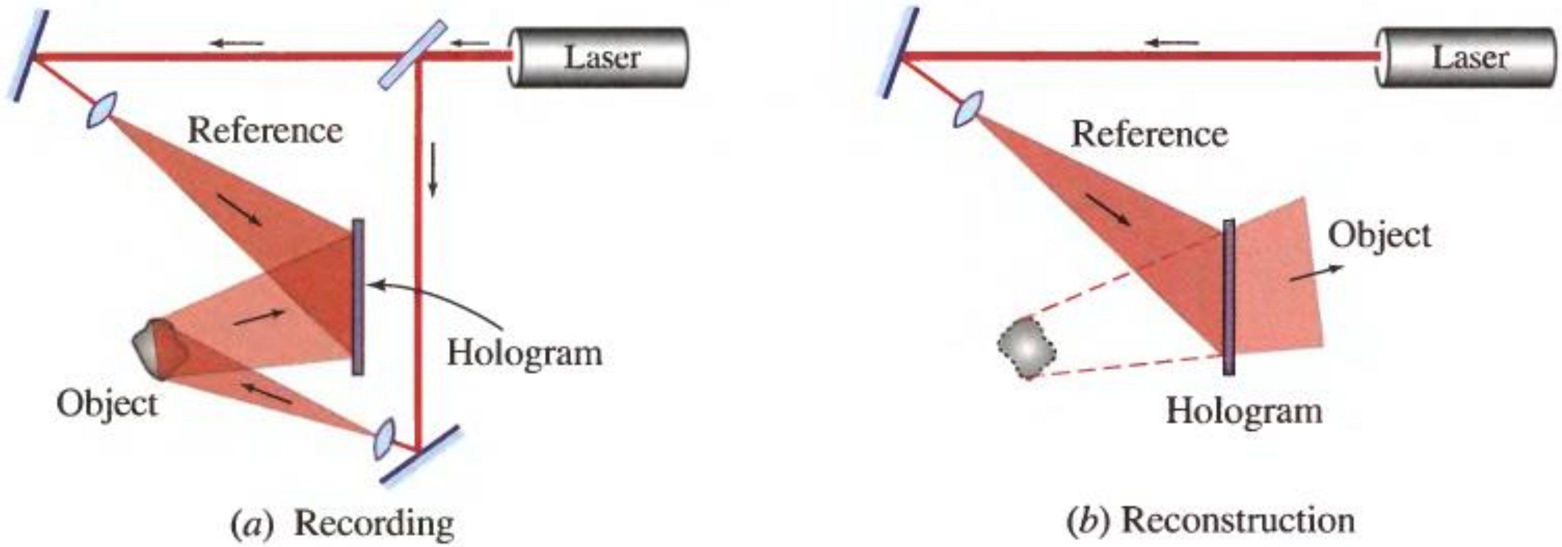


Figure 4.5-7 Holographic recording and reconstruction.