Solid state physics 2018 Problem Set #5. Due Apr. 11

1. Consider a one-dimensional crystal, where each atom of mass M has two neighbors: one at a distance d away and the other at $a - d$ away ($d<a$) in equilibrium. The two neighboring atoms are connected by springs of constant K (near) and G (far).
   a) Write down the equation of motion for the atoms in the n-th unit cell.
   b) Write down the harmonic potential energy in terms of the displacement of atoms from their equilibrium positions, and show that you have the same equation of motion for an atom as in a) when the force is equal to the minus of the derivative of the potential energy with respect to the displacement of the atom.
   c) Obtain the dispersion relations.

2. For a neutron scattering with lattices, the momentum conservation gives $\boldsymbol{K}= \boldsymbol{k} - \boldsymbol{k}'$ (normal process), where $\boldsymbol{K}$ is the phonon wave vector, and $\boldsymbol{k}$ and $\boldsymbol{k}'$ the wave vector for the incident and scattered neutron, or $K^2=k^2+k'^2-2kk'\cos\theta$. The energy conservation gives $\hbar\omega= E - E'$, each of which is the corresponding energy for the particles.

a) Express $\hbar^2K^2/2m$ as a function of $\hbar\omega$, E, E' and θ.

b) Evaluate $d\omega/dK$ at $\theta=0°$ and ω->0, and show that its value is $\pm v$, the incident neutron velocity.