Advanced Optics (PHYS690)

HEEDEUK SHIN

POHANG UNIVERSITY OF SCIENCE AND TECHNOLOGY, KOREA

Laser

Light
Amplification by
Stimulated
Emission of
Radiation

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- Laser history
- Gain media
- Light atom interaction
- Einstein coefficient
- Pumping

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Laser

- Laser: a devices that produce intense beams of light.
- Monochromatic (pure color or wavelength)
- Coherent (fixed phase relationship)
- Highly collimated (low divergence)
- Small spot with a brightness which exceeds that of the sun.

- The first amplifier based on discrete energy levels (quantum amplifier) was the MASER (Microwave Amplification by Stimulated Emission of Radiation), which was invented by Gordon, Townes and Zeiger 1954.
- The basic operating principles of the laser were put forth by Charles Townes and Arthur Schalow from the Bell Telephone Laboratories in 1958, and the first actual laser, based on a pink ruby crystal, was demonstrated in 1960 by Theodor Maiman at Hughes Research Laboratories.

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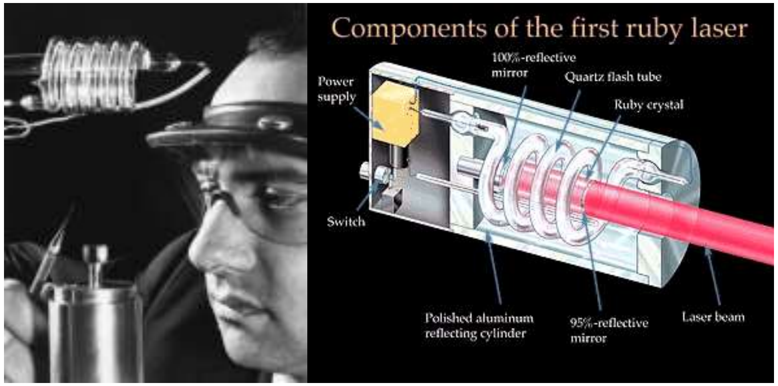
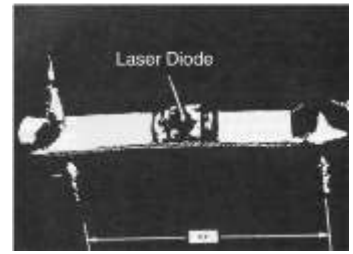
World's first working lasers



Ruby laser, 1960



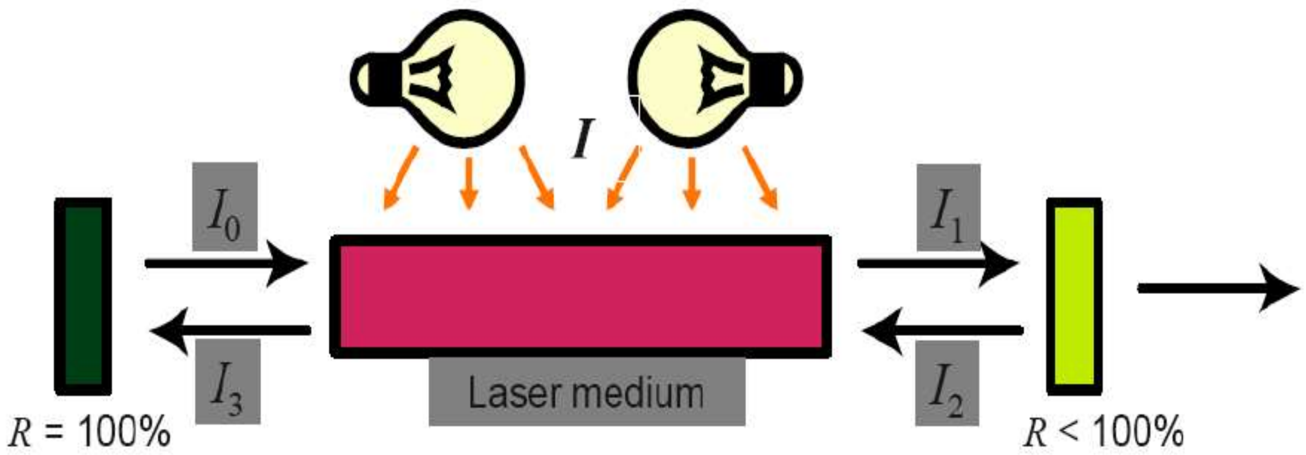
GaAs semiconductor diode laser, 1962

HeNe laser, 1961
First gas & cw laser

Optical fiber laser, 1961

Laser

- Laser medium
- Pumping
- Resonator: laser oscillator or cavity



Laser Gain Media

Important characteristics of laser gain media

Solid, a gas or liquid

How population inversion can be achieved? (pumping schemes)

What the spectroscopic parameters are?

i.e. upperstate lifetime, T_1, and linewdith $\Delta f_{\mathrm{FWHM}} = \frac{2}{T_2}$

What is the cross-section for stimulated emission?

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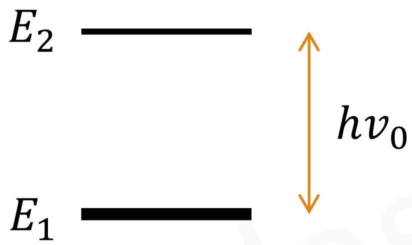
Light-matter interaction

- Consider an atom and consider two of its energy levels to be E_1 and E_2 (assume $E_1 < E_2$).

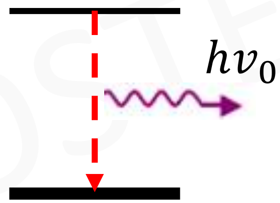


- Chose ν_0 such that

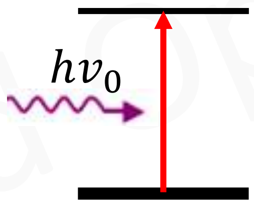
$$h\nu_0 = E_2 - E_1$$

the photon energy matches the energy-level difference.

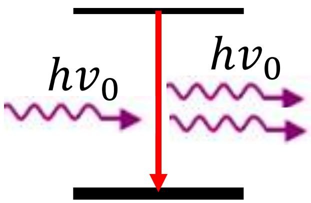
- Three types of mechanism are possible:



– Spontaneous emission



– Absorption



– Stimulated emission

- Measure of the probability of absorption or emission of light by an atom
- The **Einstein A coefficient**: the rate of spontaneous emission of light
- The **Einstein B coefficients**: the absorption and stimulated emission of light

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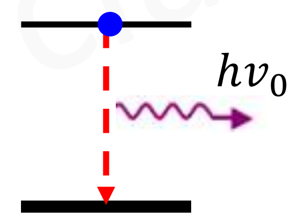
Einstein coefficients I

– Spontaneous emission

- Atom is initially in "excited" state E2
- Atom decays spontaneously and add the energy *hv* to the optical mode.
- The process is independent of the number of photon already in the optical mode, but dependent on the number of excited atoms.
- The number of atoms of the upper level: N_2
- The number of atoms of the lower level: N_1



The change $d\,N_2$ of the population N_2 within a time interval dt

$$\mathrm{d}N_2 = -A_{21}N_2\mathrm{d}t$$

A_{21} is the *Einstein coefficient of spontaneous emission.*

The population of the upper level decays exponentially

$$N_2(t) = N_2(0)\,\mathrm{e}^{-A_{21}t} = N_2(0)\,\mathrm{e}^{-t/\tau_{\mathrm{sp}}}$$

τ_{sp} *spontaneous lifetime*

$$A_{21} = 1/\tau_{\mathrm{sp}}$$

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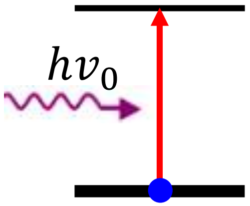
Einstein coefficients II

– Absorption



- Atom is initially in state E1.
- Process is induced by a photon: the photon is annihilated and the atom go into excited state E2.
- The change $d\ N_1$ of the ground state within a time interval dt

$$\mathrm{d}N_1 = -B_{12}\rho(\nu_0)N_1\mathrm{d}t$$

where B_{12} is the *Einstein coefficient of absorption* and ρ is the spectral energy density of radiation at frequencies around ν_0.

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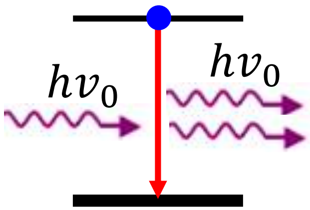
Einstein coefficients III

– Stimulated emission



- Atom is initially in “excited” state E2.
- The optical mode contains a photon.
- Atom may be induced to emit another photon into the same mode.
- This is the inverse of the absorption process.
- The presence of a photon in the mode stimulates the emission of a “clo ne” photon.
- The change $d\ N_2$ of N_2 within a time interval dt

$$\mathrm{d}N_2 = -B_{21}\rho(\nu_0)N_2\mathrm{d}t$$

where B_{21} is the *Einstein coefficient of stimulated emission.*

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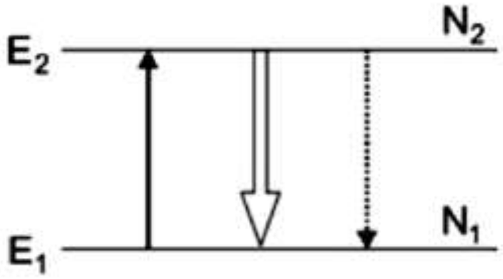
Einstein coefficients IV

The Einstein Relations



- The rate of change of the population N_1 due to absorption is given by

$$(\mathrm{d}N_1/\mathrm{d}t)_{\mathrm{abs}} = -B_{12}\,\rho(\nu_0)\,N_1$$

- The rate of change of the population N_2 due to stimulated emission is given by

$$(\mathrm{d}N_2/\mathrm{d}t)_{\mathrm{stim}} = -B_{21}\,\rho(\nu_0)\,N_2$$

- The rate of change of the population N_2 due to spontaneous emission is given by

$$(\mathrm{d}N_2/\mathrm{d}t)_{\mathrm{sp}} = -A_{21}N_2$$

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Einstein coefficients V

- In thermal equilibrium
- The ratio N_2/N_1 is a constant.
- The absorption rate has to be equal to the emission rate.

$$(\mathrm{d}N_1/\mathrm{d}t)_{\mathrm{abs}} = (\mathrm{d}N_2/\mathrm{d}t)_{\mathrm{sp}} + (\mathrm{d}N_2/\mathrm{d}t)_{\mathrm{stim}}$$

$$B_{12}\rho(\nu_0)N_1 = A_{21}N_2 + B_{21}\rho(\nu_0)N_2$$

- From this equation, we can determine the spectral energy density

$$\rho(\nu_0) = \frac{A_{21}/B_{21}}{(B_{21}/B_{12})N_1/N_2 - 1}$$

- The ratio N_1/N_2 can be determined by the Boltzmann factor.

$$N_2/N_1 = \mathrm{e}^{-h\nu_0/kT}$$

Planck's radiation law

$$\rho(\nu) = \frac{8\pi\nu^2}{c^3}\frac{h\nu}{\mathrm{e}^{h\nu/kT} - 1}$$

$$B_{21} = B_{12},$$

$$A_{21} = \frac{8\pi\nu^2}{c^3}h\nu B_{21}$$

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$B_{21} = B_{12}$

- The probability density of spontaneous emission of an atom is

$$p_{sp} = \frac{c}{V}\sigma(\upsilon)$$

- The process of absorption is governed by same law as in spontaneous emission with n photons in the optical mode.

$$P_{ab} = \frac{nc}{V}\sigma(\upsilon)$$

This is the probability of absorption of one photon from a mode with n photons.

- The probability density of stimulated emission is same law that governs spontaneous emission and absorption

$$P_{st} = \frac{nc}{V}\sigma(\upsilon)$$

$$P_{ab} = P_{st} \qquad B_{21} = B_{12}$$

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Gain coefficient I

- Consider an atom located in an optical field of flux φ. The probability of stimulated emission is

$$W_i = \varphi\sigma(\nu)$$

- If N_1 and N_2 are respectively the number of atoms in the lower and upper energy level then

– The average density of absorbed photons (number of photon per unit time per unit volume) is $N_1 W_i$

– The average density of stimulated photons is $N_2 W_i$

- The net number of photon gained is

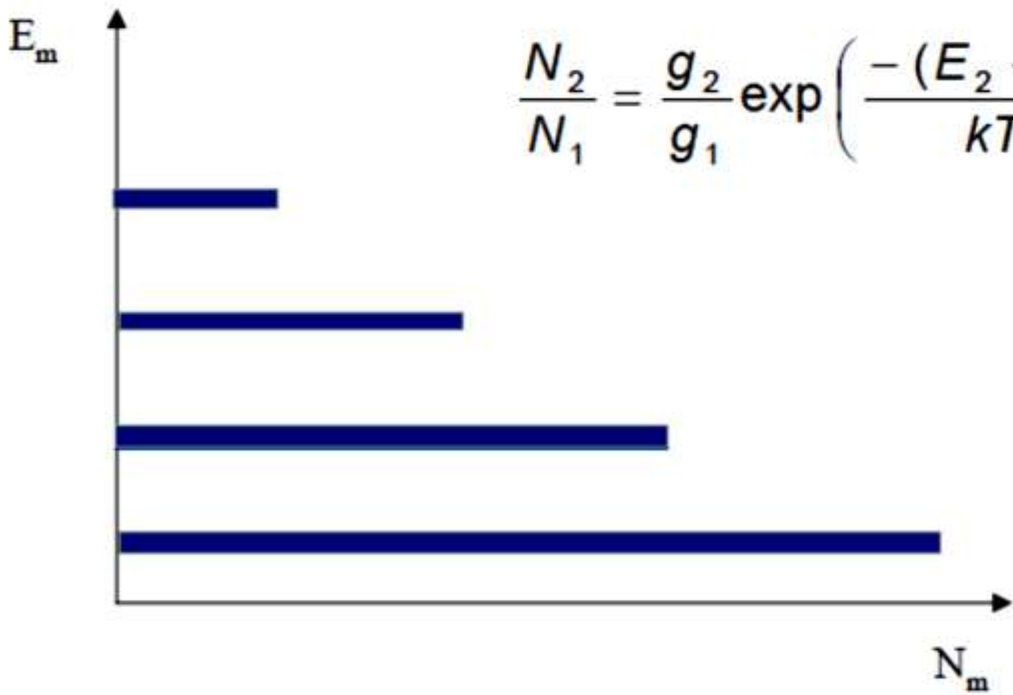
$$N = (N_2 - N_1)W_i$$

N is the population density difference.
– $N>0$ **population inversion** (more atoms in excited states): medium can act as an amplifier.
– $N<0$ medium act as an absorber.
– $N=0$ medium is transparent.

Population inversion

- Under thermal equilibrium conditions, the lower energy levels are populated first, and are always more populated than the higher levels.
- If the laser really was a simple two level system, what could you deduce about the "colour" of the absorbed and the stimulated photons?

$$\frac{N_2}{N_1} = \frac{g_2}{g_1} \exp\left(\frac{-(E_2 - E_1)}{kT}\right)$$



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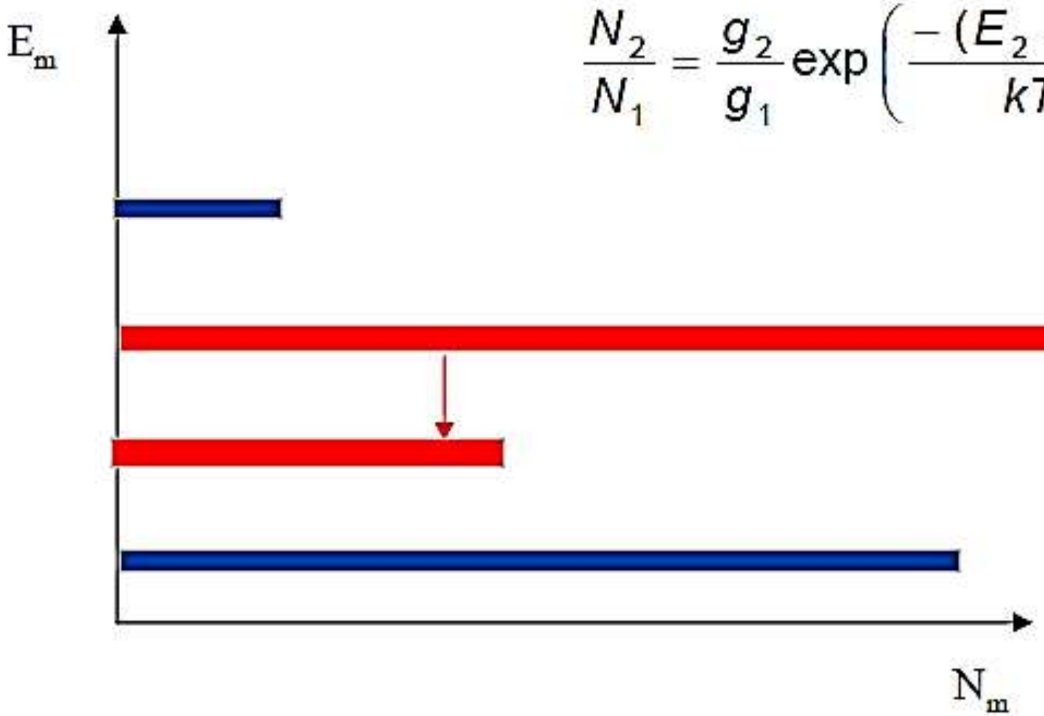

Pumping

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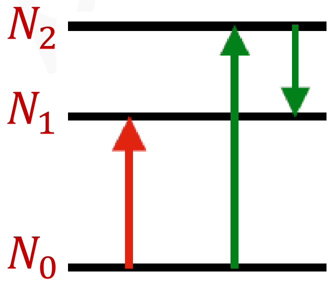
- To provide N >0 (Population inversion), we need an external pump that excites the atom.
- External pumping is achieved via radiative or non radiative effects:

– Optical pump,

– Chemical reaction

– Electrical process

- Pump should provide pumping to excite the needed state (directly or in directly).



- Pumping dynamics is described by the **rate equations**: which provide th e change of population densities N_1 and N_2.

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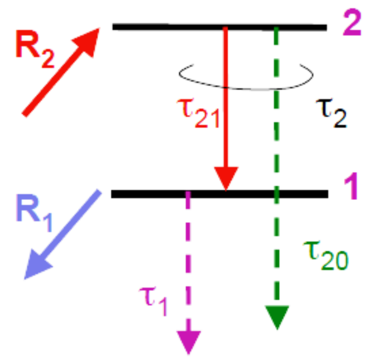
Pumping - rate equations

- When pumping is provided the rate of increase of population densities is

$$\frac{dN_2}{dt} = R_2 - \frac{N_2}{\tau_2}$$

$$\frac{dN_1}{dt} = -R_1 - \frac{N_1}{\tau_1} + \frac{N_2}{\tau_{21}}$$



R1: rate of pumping atoms out of state 1
R2: rate of pumping atoms into state 2
“rate” are per unit volume per second

- Steady-state condition is

$$N_0 = R_2\tau_2\left(1 - \frac{\tau_1}{\tau_{21}}\right) + R_1\tau_1$$

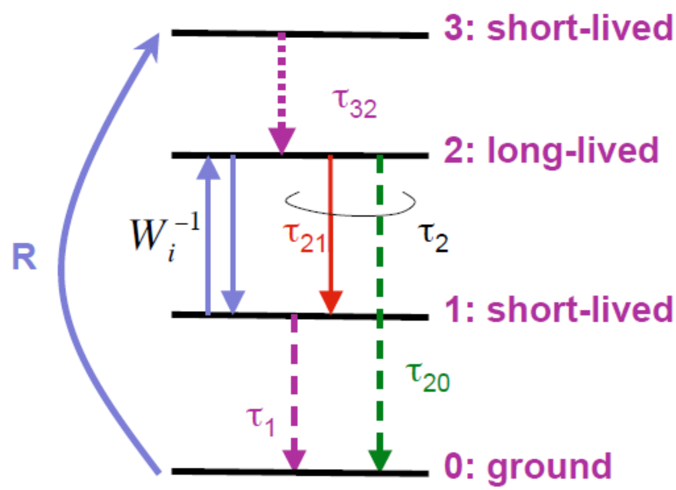
Steady-state population difference

- To have a large gain
  - Large R1 and R2
  - Long τ2
  - Short τ1 if R1<τ2/τ21R2

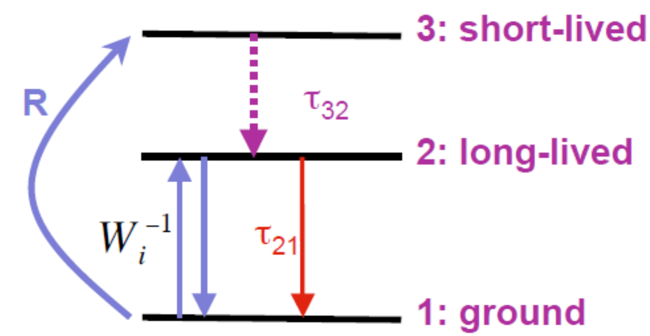
• Upper level should be pumped strongly and decay slowly.
• Lower state should be depumped strongly so it quickly disposes its population.

Pumping schemes

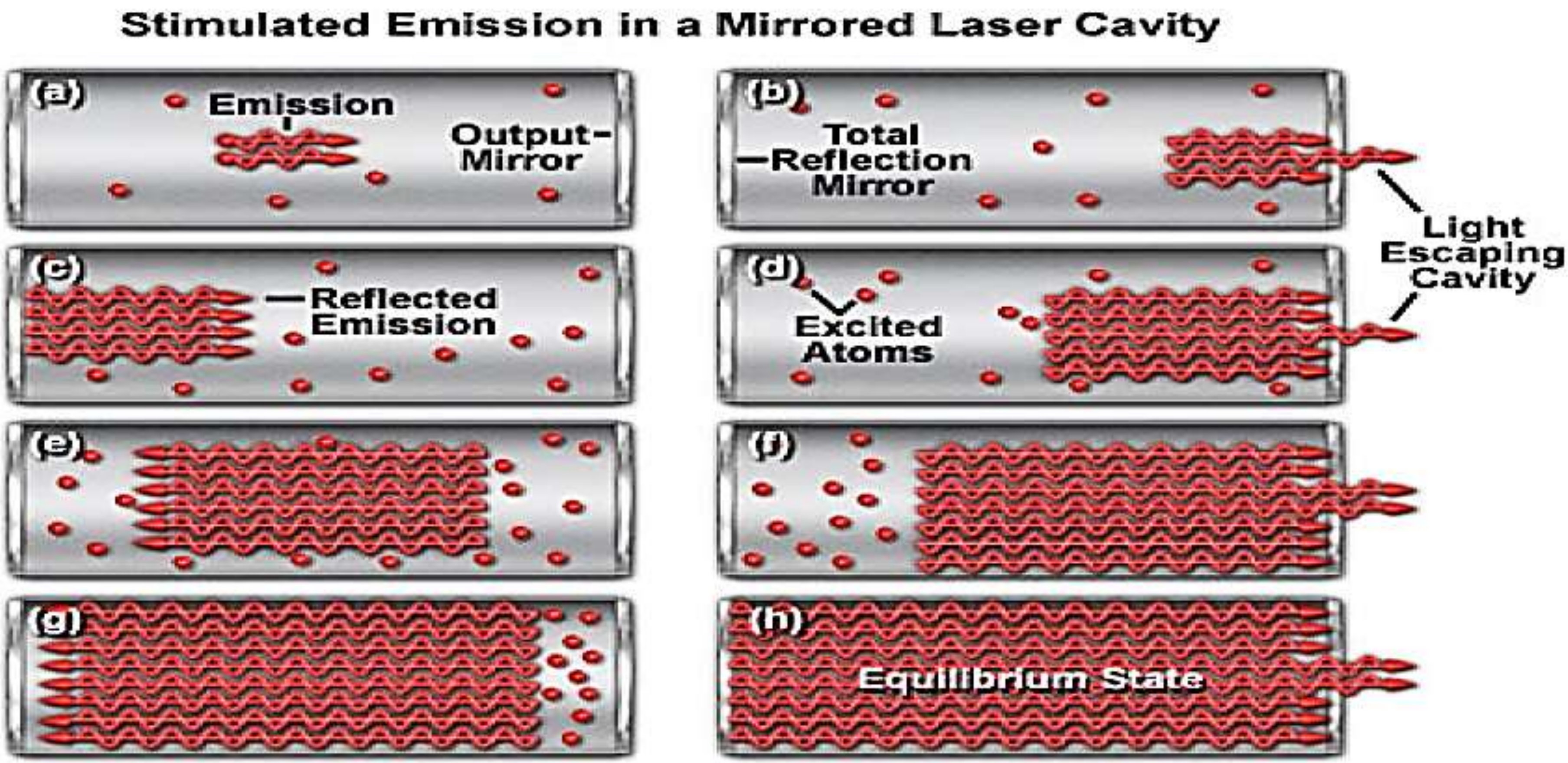
Four level pumping



Three level pumping

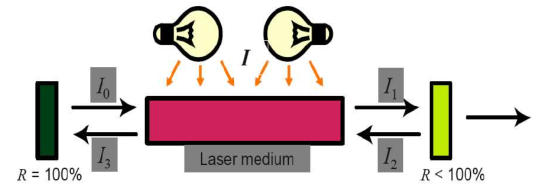


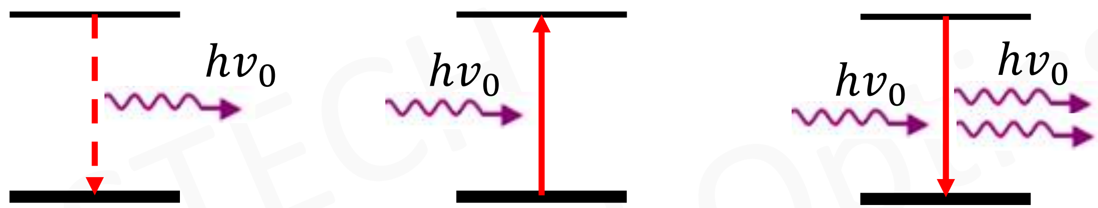
HW: Why cannot two-level system be used for a gain medium?


Stimulated Emission in a Mirrored Laser Cavity
(a)
Emission
Output-
Mirror
(b)
Total
Reflection
Mirror
Light
Escaping
Cavity
(c)
Reflected
Emission
(d)
Excited
Atoms
(e)
(f)
(g)
(h)
Equilibrium State

Light
Amplification by
Stimulated
Emission of
Radiation

- Laser gain medium
- Pumping
- Resonator: laser oscillator or cavity





– Spontaneous emission – Absorption – Stimulated emission

$$B_{21} = B_{12},$$

$$A_{21} = \frac{8\pi\nu^2}{c^3} h\nu B_{21}$$

- The **Einstein A coefficient**: the rate of spontaneous emission of light
- The **Einstein B coefficients**: the absorption and stimulated emission of light

population inversion (more atoms in excited states): medium can act as an amplifier.

- Upper level should be pumped strongly and decay slowly.
- Lower state should be depumped strongly so it quickly disposes its population.

$$I = \frac{I_0}{|1-h|^2} = \frac{I_0}{1+|r|^2 - 2|r|\cos\phi}$$

$$I = \frac{I_{max}}{1+(2\mathcal{F}/\pi)^2 \sin^2(\phi/2)}$$

$$\mathcal{F} \equiv \frac{\pi\sqrt{|r|}}{1-|r|}. \qquad I_{max} \equiv \frac{I_0}{(1-|r|)^2}.$$

$$Q \stackrel{\text{def}}{=} \frac{f_r}{\Delta f} = \frac{\omega_r}{\Delta\omega}, \qquad \mathcal{F} \equiv \frac{v_{FSR}}{\Delta v} = \frac{\lambda_{FSR}}{\Delta\lambda} \qquad Q = v_0 T_{rt} \frac{2\pi}{\eta}$$

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