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Verification of Zeno's Paradoxes

Parmenides, a philosopher in Greece, thought that motion, change, and plurality were illusions. Zeno, Parmenides’ disciple, made 9 paradoxes to support Parmenides’ claim. The paradoxes can be divided into two main categories, paradoxes of motion and paradoxes of plurality. This paper will discuss about 4 paradoxes for the paradoxes of motion, “The Achilles and Tortoise”, “The Dichotomy”, “The Arrow, and The Stadium.” Each paradox supports the divided by continuity about space and time.

The Achilles and Tortoise is a case of continuous space and continuous time. This paradox said that Achilles can’t catch up with the tortoise. The Dichotomy is a case of continuous space and discrete time. This paradox said that Achilles can’t reach the tortoise’s departed position or even start. We can solve the two paradoxes by the sum of infinite series. The Arrow is a case of discrete space and continuous time. This paradox said that the flying arrow is actually stop. British philosopher Bertrand Russell (1872-1970) suggest a theory for solving the paradox. However, it is not an exact solution. The Stadium is a case of discrete space and discrete time. It is an only the paradox which has no solution. It makes some complicate situation. People usually think that Zeno’s paradox means that the Achilles and Tortoise, and it was already solved. However, Zeno’s paradoxes about motion are not perfectly wrong.

The Achilles and Tortoise paradox said that Achilles fastest runner in Greece can’t catch up with the tortoise. Suppose Achilles and the tortoise run a race. Also, let italicized and bold A (***A***) is Achilles, and italicized and bold T (***T***) is the tortoise. It is unfair if they start from same line, because basically a human faster than a tortoise and ***A*** is fastest runner in Greece. Therefore, ***T*** departs first in front of ***A***. Let T0 is ***T***’s original starting position, T1 is the first passage point, T2 is the second passage point and so on. When race start, ***A*** arrives T0, then ***T*** arrives T1. When ***A*** arrives T1, ***T*** arrives T2. This process will repeat infinitely, then ***A*** can’t catch up ***T***(Salmon, 1998, p. 130).

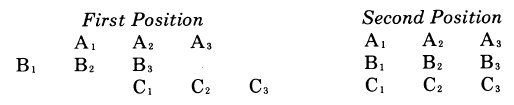
Achilles can’t reach the position which the tortoise departed or even start by the Dichotomy paradox. ***A*** should arrive at the half of T0 before arriving T0. After that, ***A*** should arrive the half of the rest, and so on. This process will repeat infinitely, then ***A*** can’t pass T0. Actually ***A*** can’t even get started. ***A*** should arrive at the half of T0 before arriving T0. Before that, ***A*** should arrive the half of that, and so on. Again, this process will repeat infinitely, then ***A*** can’t even get started (Salmon, 1998, p. 130).

The two paradoxes were solved by the sum of infinite series. While Achilles is chasing turtles, time can be infinitely divided into non-zero intervals. Therefore, Achilles have to move infinite spaces throughout an infinite time. With a simple sum, Achilles can never catch up with the tortoise. However, if the sum of infinite time intervals and the sum of infinite space intervals have converged value “Limit,” Achilles can catch up with the tortoise. We can solve the two paradoxes by the sum of an integer series.

The arrow can’t move by the Arrow paradox. When the arrow moves, it occupies a space equal to itself. During the instant, a minimal and indivisible-element of time, it cannot move. If the arrow moves during the instant, it has to be in one place at one part of the instant, and in a different place at another part of the instant. It means that the arrow occupies a space lager than itself, but anything can’t bigger than itself. Therefore, the arrow stops each instant. As British philosopher Bertrand Russell (1872-1970) says, "It is never moving, but in some miraculous way the change of position has to occur between the instants, that is to say, not at any time whatever (Salmon, 1998, p. 131).”

The Arrow paradox has some unsatisfactory solution. It is a problem of instantaneous motion and instantaneous rest. Zeno thinks that the arrow only can get instantaneous rest at the instant. We can imagine instantaneous motion by definition of instantaneous velocity. However, it has some problem. It can’t explain that infinitesimal distances move within infinitesimal times, how arrow occupies the space during infinitesimal time, how long is an infinitesimal time, and *etc*. Therefore, Russell made “at-at theory” to explain motion by mathematical function.

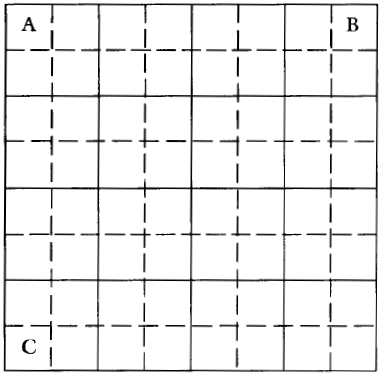
The Stadium doesn’t have any solution. It is an only paradox which has no solution. Consider three rows of A (A1 A2 A3), B (B1 B2 B3), and C (C1 C2 C3) like the first position of figure 1.



*Figure 1*. Three rows change to second position from first position while a time unit (Salmon, 1998, p.131)

A remains at rest, B moves from left to right, and C moves from right to left. The velocity of B and C are same. In the process of changing from the first position to the second position, C1 passes twice as many B's as A's. Zeno concluded that “double the time is equal to half (Salmon, 1998, p. 131).” It looks like the relative velocity problem, but it is not at all. This paradox supposes that space and time has the atomistic property, so they composed by space-atoms and time-atoms of non-zero size respectively. Now, suppose that the As remain at rest, but the Bs move to the right at the rate of one space-atom per one time-atom while the Cs move to the left with the same speed, some of the Cs move without ever passing some of the Bs. C1 starts at the right of B2 and it stops at the left of B2, but there is no time-atom to C1 pass B2. There is no time to pass each other.

Discrete space and continuous time situation causes the critical problem. German mathematician Hermann Weyl (1885-1955) suggested quantize space, figure 2.



*Figure 2*. Hermann Weyl suggested quantize space (Salmon, 1998, p. 146)

It is a space that made of tiles separated by solid lines. If assume a right triangle ABC, line AB and line AC are 4 tiles. Also line BC is 4 tiles. However, we can know that line BC is much longer than the others by the Pythagorean Theorem. It means that if space-atom and time-atom are small enough, discontinuous space and discontinuous motion of time is difficult to distinguish with continuous motion (Salmon, 1998, p. 147).

In physics, the Planck length (1.61624 \* 10-35m) and the Planck time (5.39106 \* 10-44s) are the minimum units of space and time respectively. This refers to the minimum unit of measurement that is physically meaningful, rather than referring to each non-zero size elements when space and time are atomic structures. In classical physics, the description of motion does not depend on space and time continuity or discontinuity. In modern physics, errors of classical physics appear, but there are no phenomena related to space and time continuity or discontinuity. However, just as Planck discovered the quantization of energy and solved the problem of blackbody radiation, ensuring continuity and discontinuity of space and time could be the key to unresolved physical phenomena.

The Zeno's paradoxes are not only about revisiting the existence of movement, but also thinking about the appearance of space and time. The appearance of space and time is a very important question in thinking about the root of the movement. The continuous case is a popular belief, but there is no evidence to confirm that. Therefore, we should consider the discrete case. In the discrete case, there is a problem of equal intervals, but different lengths, such as the quantized space proposed by Hermann Weyl. It is a very interesting subject, so it has a value to do further study.

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