Localization and Tracking for LDR-UWB Systems

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***Abstract*—Localization and Tracking (LT) algorithms for Low Data Rate (LDR) Ultra Wideband (UWB) systems developed within the Integrated Project PULSERS Phase II are reviewed and compared. In particular, two Localization Algorithms, designed for static networks with mesh topologies, and one Tracking Algorithm, designed for dynamic network with star topologies are described and/or compared. Each of the Localization Algorithms adopts a different approach, namely, a centralized non-parametric Weighted Least Squares approach (WLS), and a distributed Bayesian approach that relies on the cooperative maximization of the Log-Likelihood of range measurements (DMLL). The performance of these two alternatives are compared in a 3D indoor scenario under realistic ranging errors. The Tracking Algorithm is a fast non-parametric technique based on Multi-dimensional Scaling (MDS) and its performance is tested in a dynamic scenario. The proposed algorithms are practical and robust solutions addressing distinct network topologies and/or service requirements related to LDR-LT applications.**

I. Introduction

The algorithms here discussed were designed under the following general assumptions. The LDR network is formed amongst devices of three categories, namely, tags (T), ranging capable (RC) devices and anchor nodes (AN). The location of tags and RCs are unknown, but RCs but can measure its distance to other devices. In contrast, ANs are fixed nodes with known locations which are wired together and are able to perform ranging.

Localization systems are assumed to operate under static conditions, over meshed network topologies. In turn, Tracking systems are constrained by mobility (speed) and the need to minimize the amount of ranging information to be transmitted. Due to the above, the study of the localization and tracking systems will be discussed separately.

The paper is organized as follows. In section II the localization systems are described in details. Specifically, first the network architectures are introduced and following two localization algorithms are proposed.

In section III a non parametric approach based on multi-dimensional scaling (MDS) technique is proposed as a solution to the tracking problem. Concluding, in section IV the simulated performance for both localization and tracking algorithms are shown.

II. Localization Systems

In this section, we focus only on the localization systems which are assumed to operate under static conditions, over meshed network topologies. The network can be built in either a centralized (infrastructured) or decentralized architecture.

Under the first model, the network is equipped with a central unit, hereafter referred as "*LT-server*", which runs the algorithms and collects all the ranging information from the network. Such a scenario is suitable for large scale networks in which the devices cannot provide enough computational capability to perform local data processing.

The decentralized architecture, on the other hand, is advantageous in home/office scenarios, where the devices to be localized are more sophisticated and are able to perform the required data processing on their own.

Regardless of the model considered, localizing N nodes in the $\varepsilon-$embedded space means to estimate a set of coordinates $\hat{\mathbf{X}} \in \mathbb{R}^{N\times\varepsilon}$ that indicates the location of the devices upon a set of Euclidean distance measurements. In presence of a real scenario, such measurements might be affected by error and moreover, due to link blockages only a partial set might be available. Therefore, the localization algorithms need to handle *imperfect* and *incomplete* sets of distance measurements.

Denoting coordinates by $\mathbf{x}_n$ and the inner product by $\langle\cdot;\cdot\rangle$, the Euclidean distance between two nodes can be written as,

$$d_{n,m} = \mathcal{D}([\mathbf{x}_n, \mathbf{x}_m]) \triangleq \sqrt{\langle(\mathbf{x}_n - \mathbf{x}_m);(\mathbf{x}_n - \mathbf{x}_m)\rangle}. \quad (1)$$

The matrix containing imperfectly measured pairwise distances is denoted the Euclidean Distance Matrix (EDM) sample $\tilde{\mathbf{D}}$ and can present blank entries at various positions, if the related links are not measured or unavailable.

We denote as *connectivity matrix* $\mathbf{C}$ the binary-valued matrix, where $c_{n,m} = 1$ and $c_{n,m} = 0$ indicating the existence and the non-existence of a link between the devices v_n and v_m respectively. The term *completeness* defines the ratio ϱ of the number of available links and the number of links in a fully connected network. Mathematically, we have

$$\varrho = \frac{2}{N(N-1)} \cdot \sum_{i}^{N} \sum_{j=i+1}^{N} c_{ij}. \quad (2)$$

In this work, two complementary approaches are identified and developed. The first one, completely non parametric, is based on a weighted least squares optimization (WLS) [3], [2]. The second one, a Bayesian technique, assumes prior knowledge of ranging models and its parameters, and aims at maximizing the joint Log-likelihood of pair-wise measurements conditioned upon blind coordinates (DMLL) [13], [15].

Note that although the WLS algorithm is preferably designed for centralized architectures and the Bayesian approach for decentralized ones, they are both flexible enough to be implemented in either a centralized or decentralized manner.

A. Non-parametric - Weighted Least Squares Optimization

The non parametric approach here proposed, is posed as a non linear weighted least squares optimization problem, inspired by the work done by Alfakih *et al.* [4] and Chu *et al.* [5]. While the former identified the need and meaning of a weighing matrix, the latter help to formulate the problem as a least square optimization. In our formulation, we merged the goodness of both, stating the localization problem as,

$$\min_{\hat{\mathbf{X}}\in\mathbb{R}^{\varepsilon\times N}} \left|\left|\mathcal{H}(\{\tilde{\mathbf{D}}\}) \circ \left((\mathcal{A}(\{\tilde{\mathbf{D}}\}) - (\hat{\mathbf{D}}^2)\right)\right|\right|_{\mathrm{F}}^2, \qquad \text{(P – 1)}$$

where $\mathcal{A}$ and $\mathcal{H}$, are functions of the input matrices $\{\tilde{\mathbf{D}}\}$ returning a single $\bar{\mathbf{D}}$, and a weighing matrix $\mathbf{H}$, respectively.

The contributions concern the definition of $\mathcal{A}$, the method to compute the weight matrix $\mathbf{H}$ and the derivation of closed form expression of the Jacobian used in the optimization. In [6] we discussed several alternatives for $\mathcal{A}$, finally choosing the expression below,

$$\mathcal{A}(\{\tilde{\mathbf{D}}\}) = \bar{\mathbf{D}}^2 = [\bar{d}^2_{n,m}], \tag{3a}$$

$$\bar{d}_{n,m} \triangleq \begin{cases} \dfrac{1}{K_{n,m}} \displaystyle\sum_{k=1}^{K_{n,m}} \tilde{d}_{n,m:k} & \textit{if } K_{n,m} \geqslant 1 \\ 0 & \textit{if } K_{n,m} = 0 \end{cases}, \tag{3b}$$

where $K_{n,m}$ is the number of samples available for each $d_{n,m}$.

Regarding the weighing matrix $\mathbf{H}$, its meaning in (P – 1) is to transfer the knowledge available from the statistics of the ranging into the optimization. In [4] it was only conjectured that the entries of $\mathbf{H}$ should be related to the confidence given to a distance measurement.

In [2] we proposed to assign to the $\mathbf{H}$ entries the probability that the sample mean $\bar{d}_{n,m}$ is within a certain range η from the true distance $d_{n,m}$, conditioned on the amount of offset $\rho_{n,m}$ on the observations. Defined $\{\tilde{d}_{n,m}\}$ as the set of $K_{n,m}$ measurements of considered distance, we may write

$$h_{n,m} \approx Q\left(-\frac{(\eta+\rho_{n,m})\cdot\sqrt{K_{n,m}}}{2\hat{\sigma}_{\tilde{d}_{n,m}}}\right) - Q\left(\frac{(\eta-\rho_{n,m})\cdot\sqrt{K_{n,m}}}{2\hat{\sigma}_{\tilde{d}_{n,m}}}\right), \tag{4}$$

where $d_{n,m}\hat{\sigma}^2_{\tilde{d}_{n,m}}$ is the sample-variance for $\tilde{d}_{n,m}$, and $Q(x)$ is the Gaussian Q-function.

Our final contribution is related to the optimization method. Due to complexity and time constraints, we made use of Levenberg-Marquardt's algorithm, suitable in non linear least squares problems. Using such an algorithm, we rewrote the cost function as the sum of squared multivariable functions in $\mathbb{R}^{N\dot{n}}$:

$$f(\hat{\mathbf{X}}) = 2\sum_{i=1}^{N-1}\sum_{j=i+1}^{N} (f_r)^2 . \tag{5}$$

where each subfunction is defined as:

$$f_r \triangleq h_{ij}\left(\bar{d}^2_{ij} - (\hat{\mathbf{x}}_i - \hat{\mathbf{x}}_j)^{\mathrm{T}}(\hat{\mathbf{x}}_i - \hat{\mathbf{x}}_j)\right). \tag{6}$$

Now it is possible to make the optimization efficient by computing the Jacobian $J_F(\hat{\mathbf{X}})$ as:

$$J_F(\hat{\mathbf{X}}) = \left[\frac{\partial f_r}{\partial \mathbf{x}_q}\right]_{r,q}, \tag{7}$$

whose generic term $\frac{\partial f_r}{\partial \mathbf{x}_q}$ is a row-element of dimension n and it is given by

$$\frac{\partial f_r}{\partial \mathbf{x}_q} = \begin{cases} -2h_{ij}\left[x_{i1}-x_{j1},\cdots,x_{in}-x_{jn}\right], & \textit{if } q=i \\ 2h_{ij}\left[x_{i1}-x_{j1},\cdots,x_{in}-x_{jn}\right], & \textit{if } q=j \\ \mathbf{0}_n, & \textit{otherwise} \end{cases} \tag{8}$$

with $\mathbf{0}_n$ indicating a row vector with n zeros.

Beyond the numerical advantages of having a Jacobian based minimization, this approach allows to design a distributed minimization. Indeed the cost function can be seen as the sum of sub-contributions coming from each node, therefore one can estimate its own location upon a partial or complete knowledge of the distances to its neighbors and their locations.

B. Parametric - Distributed Log-Likelihood Maximization

As a complementary alternative, we recall hereafter a positioning solution that benefits from the prior knowledge of UWB ranging errors statistics. This Distributed Maximum Log-Likelihood (DMLL) algorithm was initially introduced in [13] and proved in [15] to be flexible and sufficiently robust even in presence of Non-Line Of Sight (NLOS) links.

Assuming independence and symmetry, the Log-Likelihood of joint pair-wise measurements can be written as:

$$\Lambda = \sum_{i=1}^{N-1}\sum_{j=i+1}^{N} \ln p\left[\widetilde{d_{ij}}/d_{ij}\right] = \sum_{i=1}^{N-1}\sum_{j=i+1}^{N} \Lambda_{ij} \tag{9}$$

and can be maximized with respect to blind locations, by an iterative asynchronous and distributed gradient-ascent approach. At the k^{th} step, the update of v_i's coordinate $\widehat{x}_{i,k-1}$ can be realized each time this node communicates with one of its available neighbours v_j and receives $\hat{\mathbf{x}}_{j,k}$ from this neighbour, according to (similarly for the y and z coordinates):

$$\widehat{x}_{i,k} = \widehat{x}_{i,k-1} + \delta_{ij,k} \tag{10a}$$

$$\delta_{ij,k} = \beta_{ij,k}\left(\frac{\partial\Lambda_{ij}}{\partial d_{ij}}\right)_{\substack{\widetilde{d_{ij}} = \widetilde{d^*_{ij,k}} \\ d_{ij} = \widehat{d}_{ij,k-1}}} \left(\frac{\partial d_{ij}}{\partial x_i}\right)_{\substack{x_i = \widehat{x}_{i,k-1} \\ x_j = \widehat{x}_{j,k-1} \\ d_{ij} = \widehat{d}_{ij,k-1}}} \tag{10b}$$

with $\widetilde{d^*_{ij,k}}$ as a function of range measurements between v_i and v_j at time k.

The estimated distance $\widehat{d}_{ij,k} \triangleq \mathcal{D}(\hat{\mathbf{x}}_{i,k}, \hat{\mathbf{x}}_{j,k})$ between the two nodes is influenced by $\beta_{ij,k} = \alpha_{ij}\gamma_{ij,k}$, where α_{ij} is a confidence coefficient used to mitigate the ascent direction and $\gamma_{ij,k}$ a dynamic step, as discussed in [16] and [13].

The first order ascent in (10a) represents a trade-off for maintaining reasonable complexity and acceptable performance. At each step k of the main optimization procedure, the linear search for $\gamma_{ij,k}$ [13] is one of the simplest alternates to the steepest-descent/ascent method, guaranteeing a "reasonable progress" on the performance surface.

Moreover, hierarchical updates can be performed. Typically, the mobile nodes with the highest number of anchor nodes in their neighbourhood, or with the highest number of neighbours, can be updated preferentially. At this point, after network initialisation (e.g. topology discovery), a list of nodes to be updated in priority can be proposed.

In (10a), $\widetilde{d}^{*}_{ij,k}$ is incorporated at each step of the optimization procedure. Just like in (3b), it could be the average $\bar{d}_{i,j}$ of K_{ij} preliminary range measurements performed before optimizing. The exchange of $\hat{\mathbf{x}}_{j,k}$, can be advantageously coupled with cooperative ranging transactions so that new measurements can be incorporated while optimizing. In such a case, $\widetilde{d}^{*}_{ij,k}$ is the average of the $k'_{ij} \leqslant k$ range measurements available up to the k^{th} optimization step.

Finally, as a Bayesian approach, DMLL requires a priori statistical models of the observed data. A first option (DMLL CI) consists in conditioning the measured distance on the real distance d_{ij} and the channel configuration $C_{h_{ij}}$, assuming the preliminary identification of each pair-wise channel $\hat{C}_{h_{ij}}$:

$$\frac{\partial \Lambda_{ij}}{\partial d_{ij}} = \left[\frac{\partial \ln p_{C_{h_{ij}}} \left(\tilde{d}_{ij} | d_{ij}, C_{h_{ij}} \right)}{\partial d_{ij}} \right]_{C_{h_{ij}} = \hat{C}_{h_{ij}}} \tag{11}$$

Another strategy (DMLL No CI) relies on a mixture model $p[\tilde{d}_{ij}/d_{ij}]$ that assumes contributions from distinct channel configurations (IV-A)), thus conditioning *in fine* the measured distance on d_{ij}, like in the initial formulation (9):

$$\frac{\partial \Lambda_{ij}}{\partial d_{ij}} = \frac{\partial \ln \Big(\sum_{C_{h_{ij}}} W_{C_{h_{ij}}} p_{C_{h_{ij}}} \left(\tilde{d}_{ij} | d_{ij}, C_{h_{ij}} \right) \Big)}{\partial d_{ij}} \tag{12}$$

III. Tracking System

The combination of mobility as a feature, and localization as a need or application, gives rise to the problem of target tracking in sensor networks. Here we consider the target tracking application in which a small number of ANs are placed at known, fixed, privileged locations, providing the infrastructure necessary to track the several Ts on the environment.

This affects the network architecture, transforming it into a star topology between each AN and all the Ts. This new topology, compared to the meshed one used for the localization scenario, over a sufficiently large number of tags, reduces the connectivity and consequently the amount of information trough the network, with direct benefit on the refreshing rate of the estimated trajectories. The main challenges in the problem outlined above are that targets may have different dynamics, and be in large numbers. In our formulation we consider the problem of tracking multiple targets, each one characterized by different dynamics, i.e.describing independent, continuous and differentiable trajectories with non-stationary statistics, in the presence of imperfect and incomplete ranging information.

A. Non Parametric Tracking Approach

Next, we introduce the MDS-based tracking algorithm first proposed in [10]. Since the location estimates of the tracked target does only rely on the current observations, the performance is immune to the non-stationarity of the target's dynamics, although the complexity of the algorithm is not.

The algorithm is formulated on the following incomplete EDM which collects all the AN-AN and AN-T ranging information at each sampling instance,

$$\tilde{\mathbf{D}} = [d_{i,j}(t)] = \begin{bmatrix} \mathbf{D}_{\mathrm{A}^2} & \tilde{\mathbf{D}}_{\mathrm{AT}}(t) \\ \tilde{\mathbf{D}}^{\mathrm{T}}_{\mathrm{AT}}(t) & \mathbf{0} \end{bmatrix}, \tag{13}$$

At any time t, the MDS techniques allows one to recover the location $\mathbf{Y}(t)$ of all sensors (up to rigid transformations) from the complete Gram matrix $\mathbf{G}(t)$. Mathematically

$$\mathbf{Y}(t) = [\mathbf{V}(t)]_{1:n} \cdot [\mathbf{\Lambda}(t)]^{\frac{1}{2}}_{1:n}, \tag{14}$$

$$\mathbf{G}(t) = \mathbf{V}(t) \cdot \mathbf{\Lambda}(t) \cdot \mathbf{V}(t)^{\mathrm{T}}, \tag{15}$$

where a complete Gram-matrix $\mathbf{G}(t)$ can be obtained from the incomplete $\tilde{\mathbf{D}}$ using the Nyström approximation [11],

$$\mathbf{G}(t) \approx \begin{bmatrix} \mathbf{G}_{\mathrm{A}} & \mathbf{G}_{\mathrm{T}}(t) \\ \mathbf{G}_{\mathrm{T}}(t) & \mathbf{G}_{\mathrm{T}}(t)^{\mathrm{T}} \cdot \mathbf{G}_{\mathrm{A}}^{-1} \cdot \mathbf{G}_{\mathrm{T}}(t) \end{bmatrix}, \tag{16}$$

Provided that the coordinates $\mathbf{X}_{\mathrm{A}}(t)$ of at least $A > \varepsilon$ anchor nodes are known, the solution $\mathbf{Y}(t)$ – subjected to rigid motion – can be re-oriented via the Procrustes transformation [12].

1) Jacobian Eigenspace Adaptation: As indicated by the notation above, tracking using the afore mentioned MDS technique requires repetitive eigen-decomposition of the Gram-matrix. In order to circumvent this problem, a well-known iterative procedure, *i.e.* Jacobian algorithm [7], is used to eigen-decompose a matrix $\mathbf{B}$ as below:

$$\mathbf{B}_{k+1} \longleftarrow \mathbf{R}(i_k, j_k, \theta_k) \cdot \mathbf{B}_k \cdot \mathbf{R}(i_k, j_k, \theta_k)^{\mathrm{T}}, \tag{17}$$

where $\mathbf{R}(i_k, j_k, \theta_k)$ are the Givens rotation matrices. The approximate eigenvector matrix of $\mathbf{B}$ after K_{E} iterations is,

$$\mathbf{V} = \prod_{k=1}^{K_{\mathrm{E}}} \prod_{(i_k, j_k)} \mathbf{R}(i_k, j_k, \theta_k). \tag{18}$$

In [8] the convergence and stability properties a Jacobian-like eigen-decomposition capable of finding a single matrix $\mathbf{R}$ that jointly (approximately) diagonalizes a set of matrices $\mathcal{B} \triangleq \{\mathbf{B}_1, ..., \mathbf{B}_M\}$ are proved. In this Jacobian-like joint-diagonalization technique, one iterates the expression

$$\mathcal{B}_{k+1} = \mathbf{R}(i_k, j_k, \theta_k) \cdot \mathcal{B}_k \cdot \mathbf{R}(i_k, j_k, \theta_k)^{\mathrm{T}} \tag{19}$$

where $\mathbf{R}(i_k, j_k, \theta_k)$ is the solution of

$$\min_{\theta(i,j)} \sum_{m=1}^{M} \mathrm{off} \left(\mathbf{R}(i,j,\theta) \cdot \mathbf{B}_m \cdot \mathbf{R}(i,j,\theta)^{\mathrm{T}} \right), \tag{20}$$

where

$$\mathrm{off}\,(\mathbf{B}) \triangleq \sum_{i \neq j} \|b_{i,j}\|^2. \tag{21}$$

The optimum angles that solves the minimization problem (20) was later discovered in a closed-form expression in [9]. As noted in [10], these angles can be used inside the joint-diagonalization algorithm as an eigenspectrum tracking algorithm. Considering two Gram matrices $\mathbf{G}(t)$ and $\mathbf{G}(t+T)$ corresponding to consecutive observations, and assuming the matrix $\mathbf{V}(t)$ of eigenvectors for $\mathbf{G}(t)$ known. Then, $\mathbf{V}(t+T)$ can be computed by feeding the Jacobian algorithm with $\mathbf{B} = \mathbf{V}(t) \cdot \mathbf{G}(t+T) \cdot \mathbf{V}^{\mathrm{T}}(t)$. This leads to the following solution for the eigen-decomposition of $\mathbf{G}(t+T)$,

$$\mathbf{V}(t+T) = \mathbf{V}(t) \cdot \prod_{k=1}^{K_{\mathrm{E}}} \prod_{(i_k, j_k)} \mathbf{R}(i_k, j_k, \theta_k). \tag{22}$$

With this approach, the number of iterations K_{E} required to update $\mathbf{V}(t)$ into $\mathbf{V}(t+T)$ is related to the dynamics of the target. The conclusion that can be foreseen from the results is that $\bar{K}_{\mathrm{E}}$ is not strongly dependent on ν which, in turn, imply that the complexity of the MDS-based algorithm is mainly determined by the number of targets, and only slightly affected by the target dynamics.

IV. Simulation Results and Comparisons

The scenario for localization consists of a network of $A = 8$ ANs placed at the corners of a cube with 20-meter long edges and location perfectly known, while the $N - A = 10$ RCs are placed random inside the room (uniformly distributed). The tracking scenario replaces the RCs by the Ts.

In the localization context, we consider the connectivity bounded within R_{MAX}, referred to as the *connectivity range*. In the tracking scenario all the AN-T links are assumed available.

The performance of the algorithm are measured with two metrics namely, the Root-Measn-Square-Error (RMSE) and the average percentage of nodes with positioning error less than a given error ξ. While the former returns an overall error, the latter indicates a distribution of the positioning error at each node. The RMSE is computed as follows:

$$\zeta^{(\varrho)} \triangleq \left\| [\mathbf{X}]_{\text{UL}:(N-A)\times\eta} - [\hat{\mathbf{X}}^{(\varrho)}]_{\text{UL}:(N-A)\times\eta} \right\|_{\text{F}} / \sqrt{N-A}, \quad (23)$$

A. Ranging model for UWB measurements

The ranging transactions are based on TWR schemes, modeled as:

$$\tilde{d}_{ij} = d'_{ij} + n_{ij} \quad (24)$$

where d'_{ij} is the distance travelled between v_i and v_j, n_{ij} is a noise term whose variance $\sigma^2_{n,ij}$is function of protocol durations and detection noise.

A distinction is made here between the Euclidean distance d, and the biased distance d'. Indeed, d' can be viewed as a random variable, in a mixture based model, whose pdf is conditioned upon d and a particular channel configuration, $C_h = \{LOS, NLOS, NLOS^2\}$ as proposed in [17] and [14].

$$p_{C_h}(d'|\{d, C_h\}) = \frac{G_{C_h} e^{-\frac{(d'-d)^2}{2d^2\sigma^2_{C_h}}}}{d\sigma_{C_h}\sqrt{2\pi}} + \frac{E_{C_h}\lambda_{C_h} 1_{d'>d} e^{-\frac{\lambda_{C_h}(d'-d)}{d}}}{d} \quad (25)$$

where $\{G_{C_h}, \sigma_{C_h}\}$ and $\{E_{C_h}, \lambda_{C_h}\}$ are the weights and parameters of Gaussian and Exponential mixture components, $1_{x>y} = 1$ whenever $x > y$ and 0 otherwise.

In our simulations, d' has been generated according to (25), and the experimental parameters proposed in [13] and [15]. Regarding n_{ij} in (24), a gamma distribution is used. Note that for the sake of tractability, DMLL assumes an underlying Gaussian model instead. An analytical expression for the pdf of $\tilde{d}$ under this simplified assumption can be found in [13].

B. Performance of the Localization Algorithms

Figure 1 compares the performance for the localization algorithms in both LOS and Mixed scenarios against the completeness and under imperfect ranging.

It highlights how the WLS algorithm is more robust to incompleteness for measurements non affected by strong bias. On the other hand, the DMLL algorithm provides better performance in case of Mixed channel configuration.

One explanation is that, in our simulations the DMLL algorithm assumes Gaussian noise as an approximation to actual Gamma occurrences. Hence, the assumed model becomes less judicious as biases are not the main contributors to ranging errors. This highlight the sensitivity that Bayesian, and more generally parametric approaches have to underlying models. The strong performance degradation of the WLS in Mixed channel, is due to the fact that no bias correction procedure is applied, therefore the algorithm "judges" the distance measurements based only on the noise.

The second result shows the percentage of nodes that are localized in LOS and Mixed channel condition with a given accuracy. Only the WLS and DMLL CI are considered since they presented the best performance.

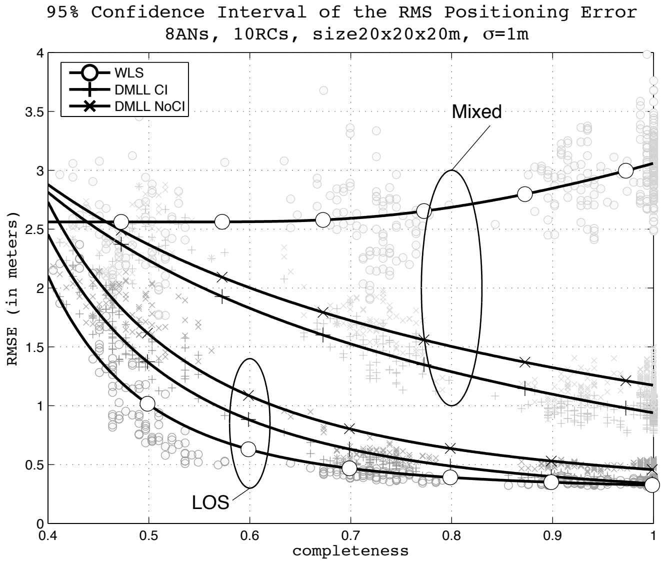

Fig. 1. Comparison of the WLS, DMLL CI and DMLL NoCI algorithms with respect to the incompleteness. The background points indicates the samples within the 95% confidence interval. The lines are representing the typical behavior.

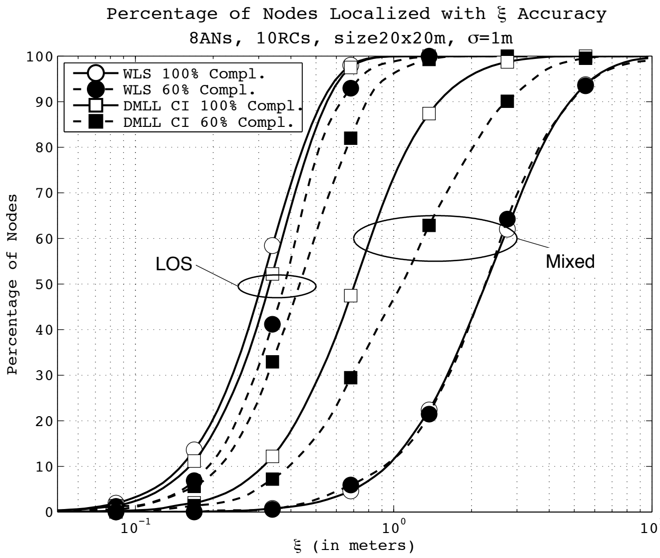

Fig. 2. Comparison of the WLS and DMLL CI algorithms with respect to a given accuracy.

Figure 2 shows clearly that positioning errors are confined within a smaller interval in the LOS situations. It is consolidated that the WLS has more robustness to incompleteness than the DMLL, while the latter has a better performance in the presence of bias.

C. Tracking in a Dynamic Scenario

Here below the results for the MDS-based tracking algorithm are briefly summarized compared against an Extended Kalman Filter (EKF).

The complexity for both the algorithms has been deeply investigated in [10]. Since a tracing problem is considered, we compare the algorithms' performance against a dynamic metric.

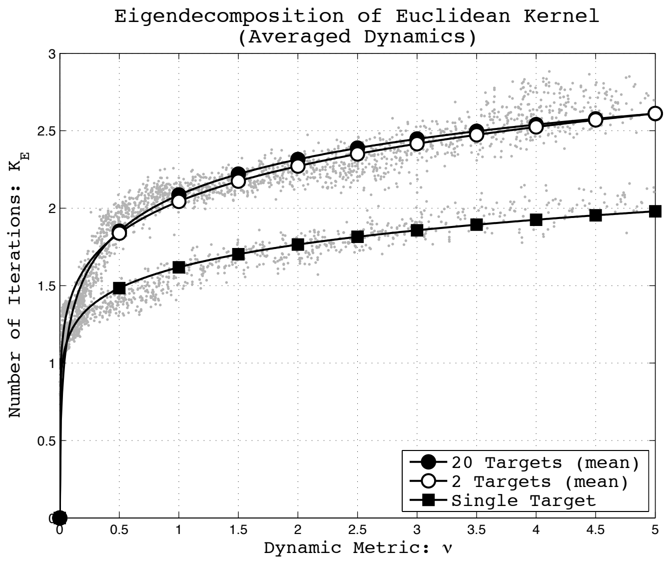

Fig. 3. Average number of iterations $\bar{K}_{\mathrm{E}}$ of the Jacobian eigen-decomposition algorithm against the average dynamic metric $\bar{\nu}$.

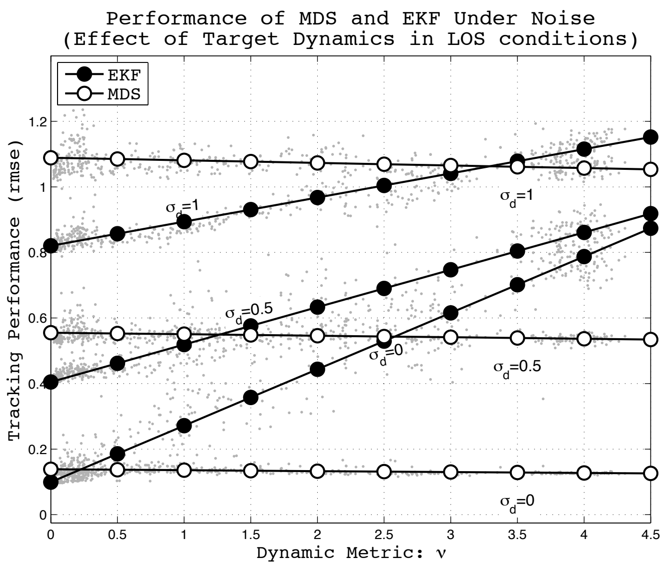

Fig. 4. Performance of the MDS-based and EFK tracking algorithms as a function of the average target dynamics with perfect and noisy ranging information.

Making use of a polar coordinate system, we choose our metric as the velocity associated with the discrete observation of the target's motion, namely:

$$\vec{\boldsymbol{v}} = \frac{d\vec{\boldsymbol{r}}}{dt} + \vec{\boldsymbol{r}} \cdot \frac{d\theta}{dt}. \tag{26}$$

The relationship between the average number of iterations $\bar{K}_{\mathrm{E}}$ and the dynamic metric ν in systems with a single, 2 and 20 targets is shown in figure 3. The curves are obtained using perfect ranging information so as to isolate the effect of the target dynamics on the tracking algorithms from the noise.

Figure 4 shows the error performances of the two tracking techniques in a single-target scenario with ranging subject to errors, so as described in IV-A for the LOS scenario.

The figure illustrates the fact the Kalman Filter performs poorly if tracked targets move in a sufficiently dynamic manner, $i.e.$, with trajectories characterized by pseudo-random processes of non-stationary statistics. Therefore we can summarize the main conclusion of our study, namely, that a subspace-based tracking algorithm is advantageous, from both complexity and performance points of view, compared to KF-based methods. Furthermore, the fact that the particular eigenspace-based tracking technique here considered incorporates no noise-filtering capabilities, and the significantly lower complexity associated with this method, suggest that even better results can be achieved if the eigenspace approach is coupled with an *a posteriori* smoothing.

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