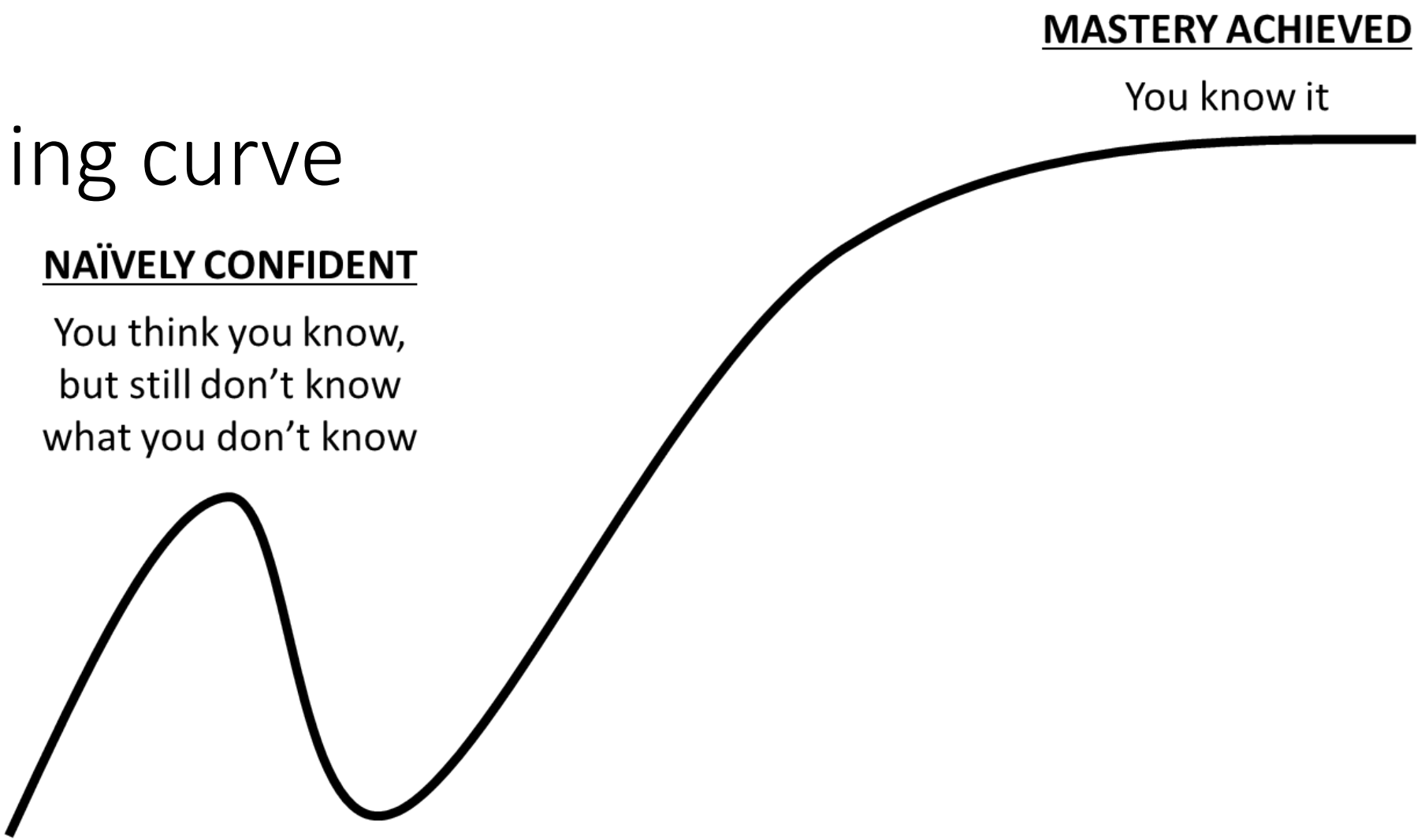
Statistics with Spa  ows II

Many models, matrices, and magic

Julia Schroeder

Julia.schroeder@imperial.ac.uk

Learning curve



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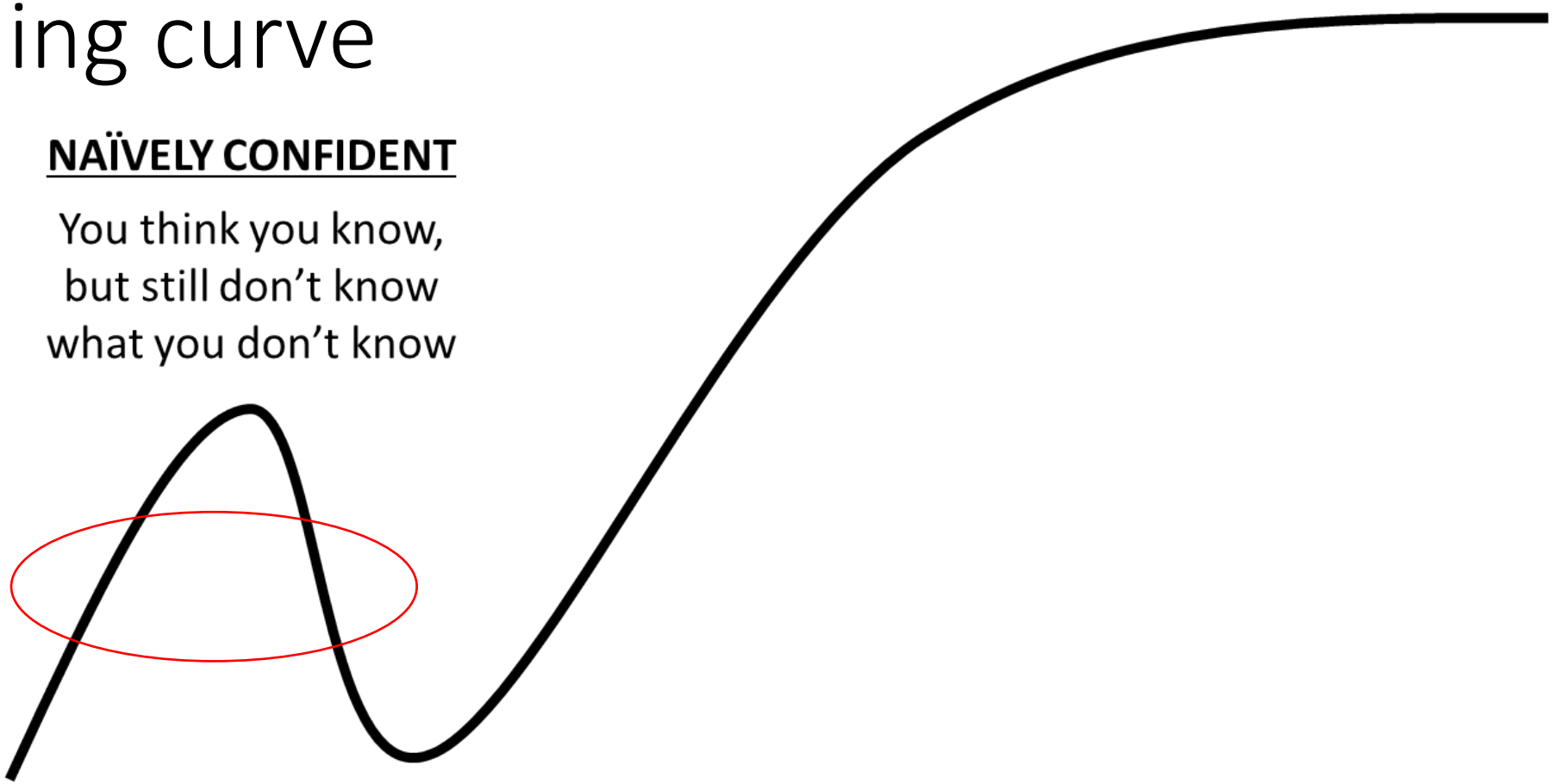

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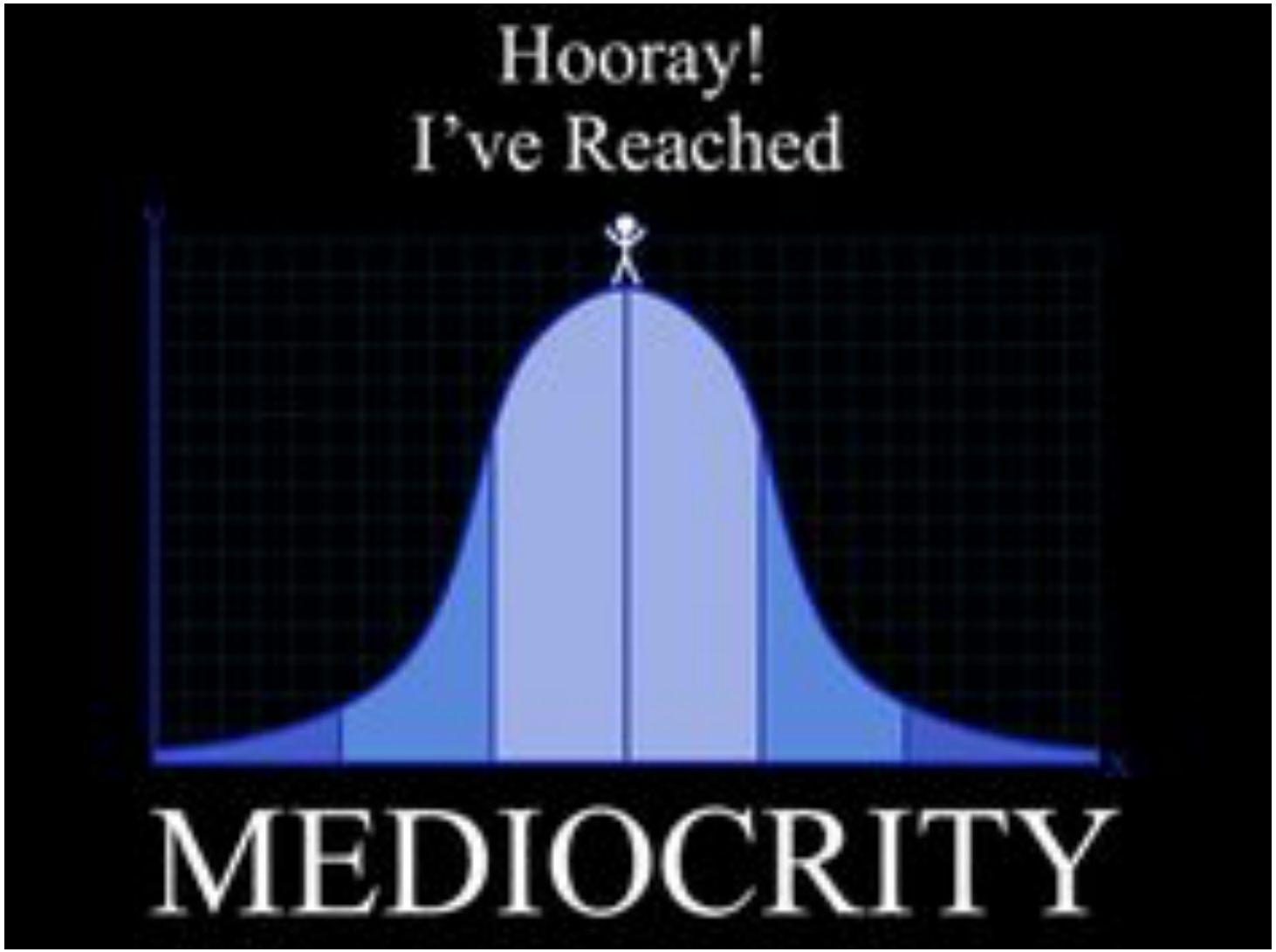
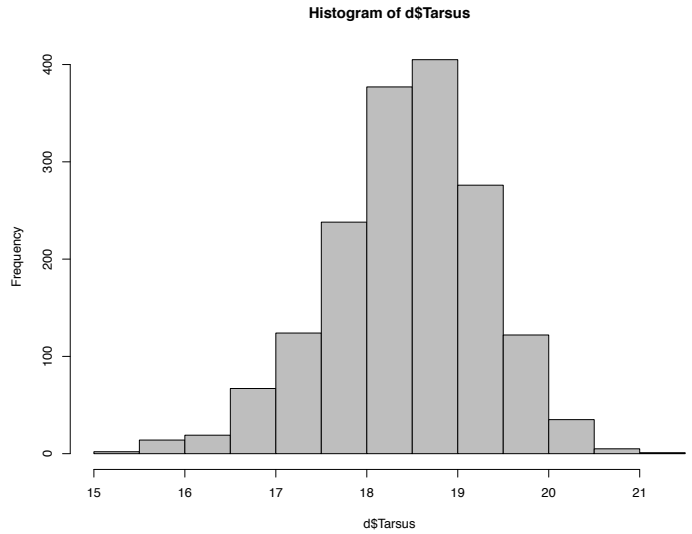


I never make the same mistake twice. I make it like five or six time, you know, just to be sure.

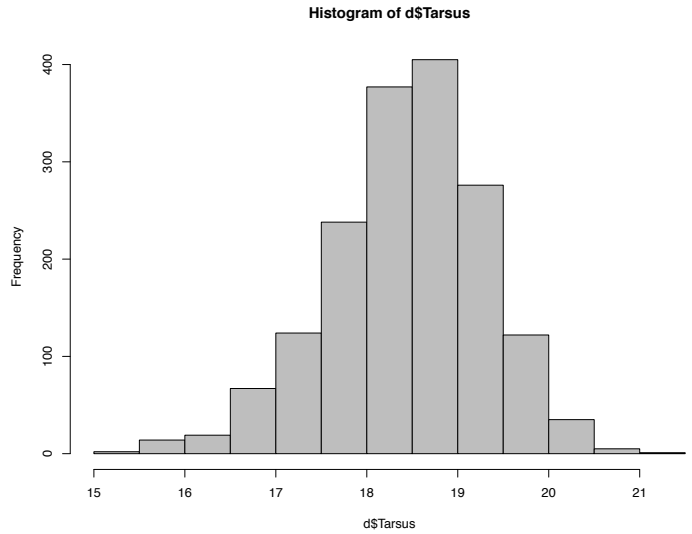
someecards
user card

Describing data distributions



Describing data distributions

- Centrality



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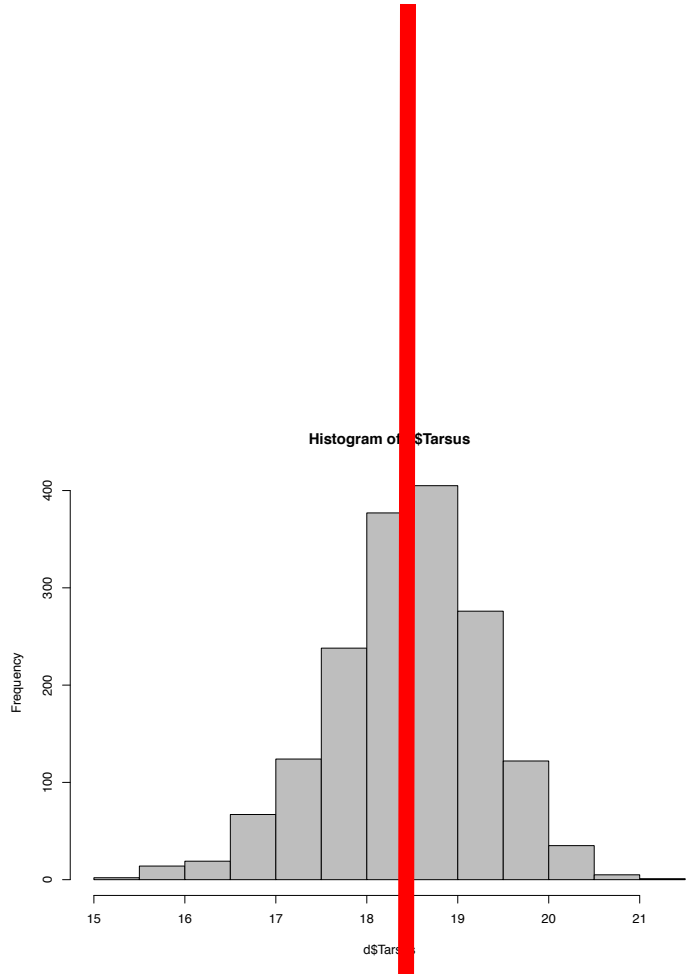
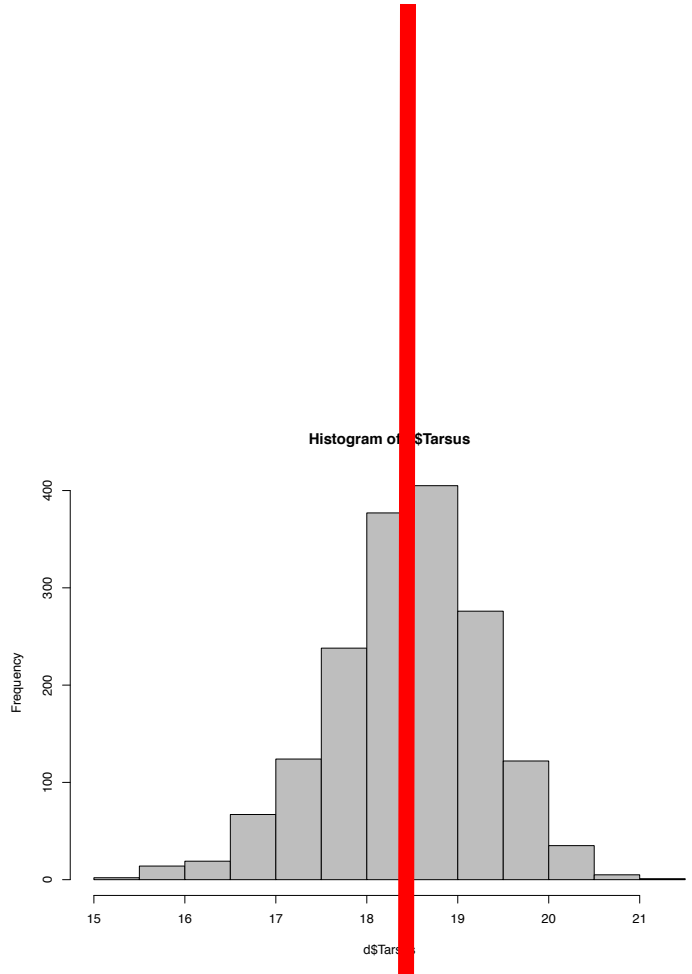


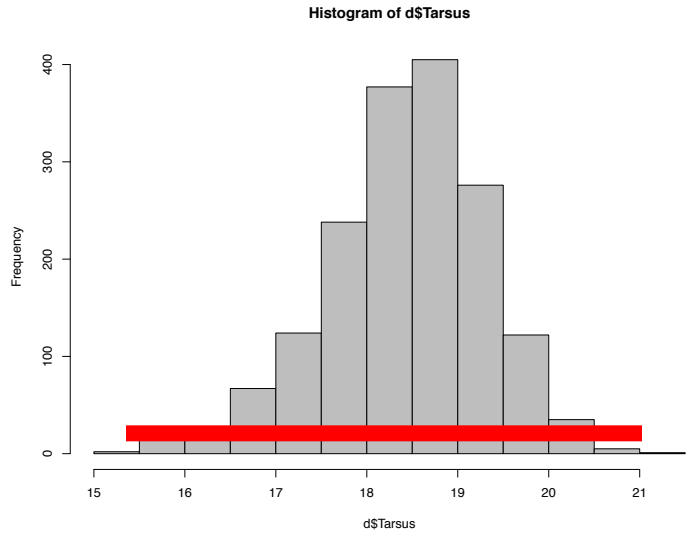
Describing data distributions

- Centrality
  - Mean $\overline{\mu}$
  - Mode
  - Median



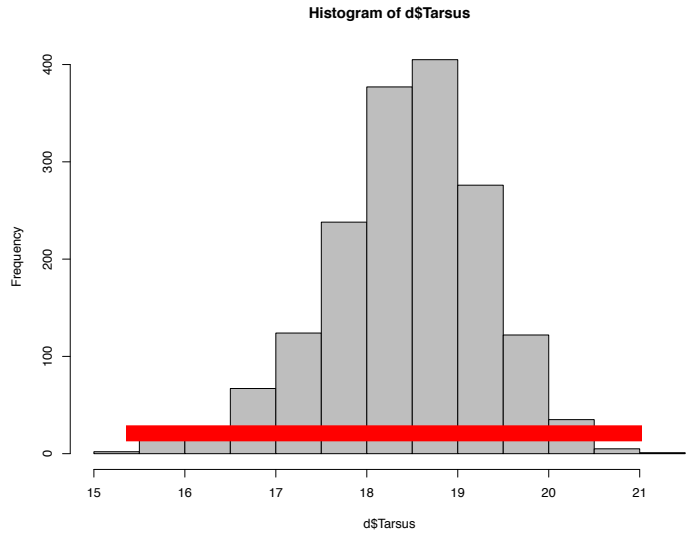
Describing data distributions

- Centrality
  - Mean $\overline{\mu}$
  - Mode
  - Median
- Spread



Describing data distributions

- Centrality
  - Mean $\overline{\mu}$
  - Mode
  - Median
- Spread
  - Range (min, max)



Describing data distributions

- Centrality
  - Mean $\overline{\mu}$
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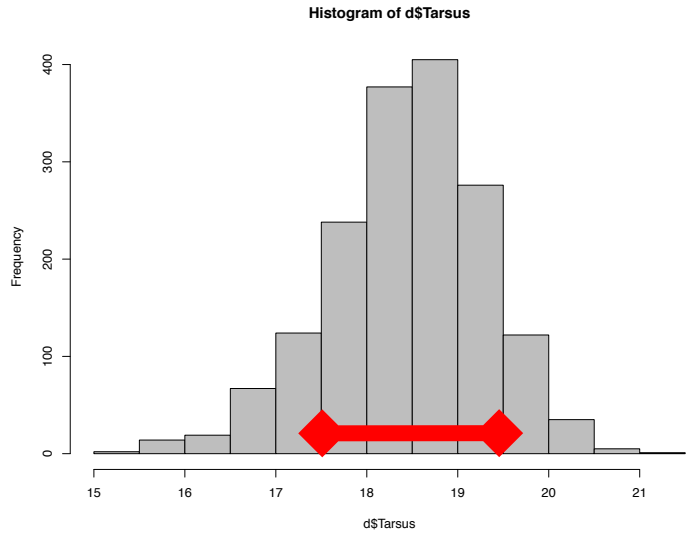
- Spread
  - Range (min, max)
  - Standard deviation σ $-$ 68.2%



Describing data distributions

- Centrality
  - Mean $\overline{\mu}$
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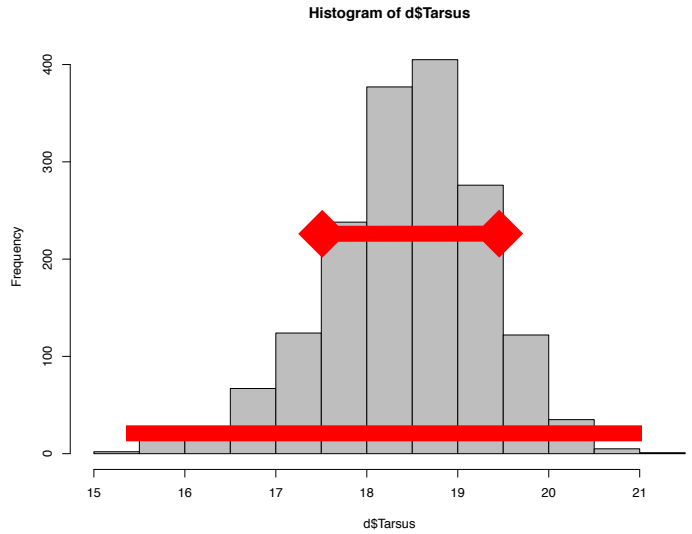
- Spread
  - Range (min, max)
  - Standard deviation $\sigma - 68.2\%$
  - Variance σ^2



Adding variances

$$\sigma^2_{Tarsus+Wing} = \sigma^2_{Tarsus} + \sigma^2_{Wing} + 2COV_{Wing,Tarsus}$$

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$$\sigma^2_{Tarsus+Wing} = \sigma^2_{Tarsus} + \sigma^2_{Wing}$$

If *Tarsus* and *Wing* are independent variables

Multiplying variances

$$10^2 \, \sigma^2_{Tarsus} = \sigma^2_{10Tarsus}$$

Linear models

- Basic building block of statistics
- Super useful
- Versatile
- Expandable

Fitting models to data

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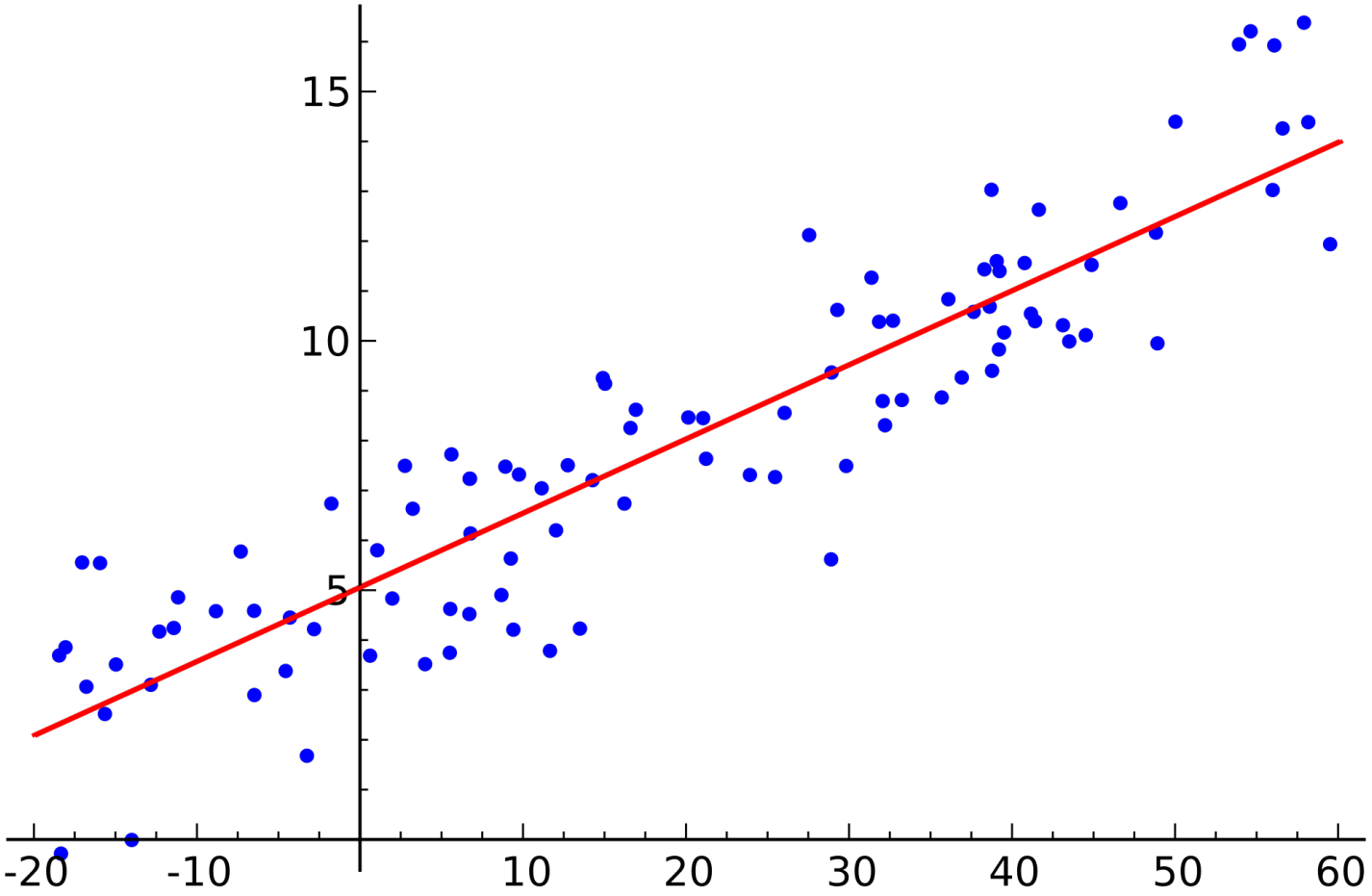
Fitting models to data



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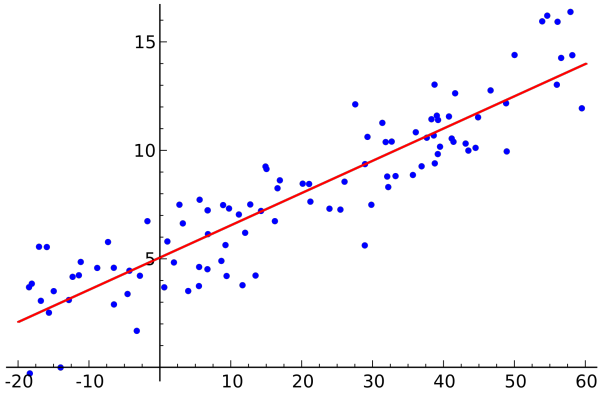

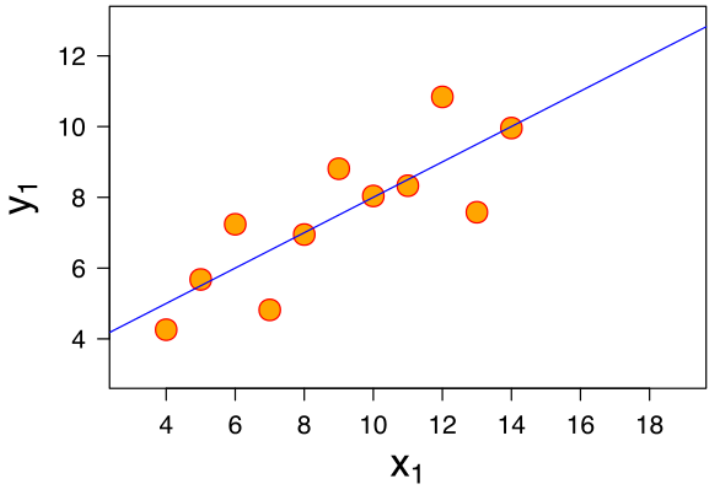

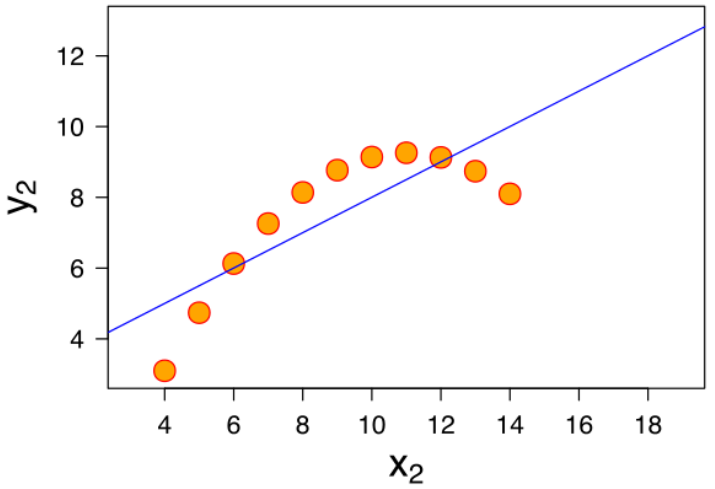

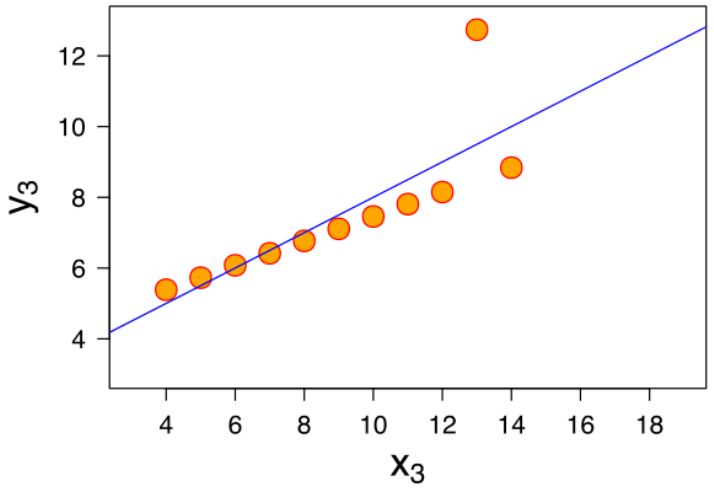

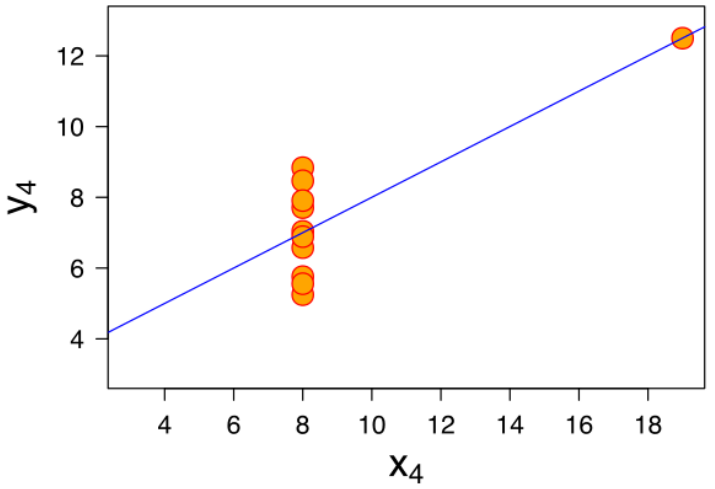

Linear models

$$y_i = b_0 + b_1 x_i + \varepsilon_i$$

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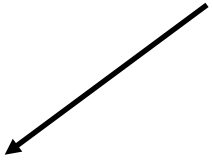
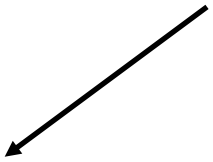


y
5
3
6
10
4

Data. Response variable,
e.g. sparrow body mass.

Linear models

$$y_i = b_0 + b_1 x_i + \varepsilon_i$$



y	i
5	1
3	2
6	3
10	4
4	5

Data. Response variable. Observation 1, 2, 3, etc.
e.g. sparrow body mass.

Linear models

$$y_i = b_0 + b_1 x_i + \varepsilon_i$$

y	i
5	1
3	2
6	3
10	4
4	5

Data. Response variable. Observation 1, 2, 3, etc.
e.g. sparrow body mass.

x	i
3	1
1	2
4	3
8	4
2	5

Data. Explanatory variable.
e.g. sparrow tarsus length.

Linear models

$$y_i = b_0 + b_1 x_i + \varepsilon_i$$

ε	i
?	1
?	2
?	3
?	4
?	5

Linear models

$$y_i = b_0 + b_1 x_i + \varepsilon_i$$

$b_1 = ?$

ε	i
?	1
?	2
?	3
?	4
?	5

Linear models

$$y_i = b_0 + b_1 x_i + \varepsilon_i$$

b_0 = ?

b_1 = ?

ε	i
?	1
?	2
?	3
?	4
?	5

Linear models

- *Vectors* vs **scalars**

$$y_i = \boldsymbol{b_0} + \boldsymbol{b_1} x_i + \varepsilon_i$$

y	i
5	1
3	2
6	3
10	4
4	5

$b_0 = ?$

$b_1 = ?$

x	i
3	1
1	2
4	3
8	4
2	5

ε	i
?	1
?	2
?	3
?	4
?	5

Linear models

$$y_i = \boldsymbol{b_0} + \boldsymbol{b_1} x_i + \boldsymbol{\varepsilon_i}$$

- Find solution: these parameter estimates (scalars) that minimise the left-over error residuals (vector)

Linear models - terminology

- Unicorn dataset – Runicorns.tx

```
> head(uni)
                    Unicorn Gender  Bodymass Hornlength Pregnant   Height Season Glizz
1     Beginda_Friday_McNutt Female  9.500673      6.515        0 3.758080 Spring     0
2 Carol_the_Cannon_Richards Female  9.860319      6.190        0 1.558938 Autumn     0
3    Alice_Dogface_McDonald Female 10.162390      6.330        0 8.190941 Spring     0
4                Diane_Gumbo Female 10.365228      5.505        0 1.584386 Autumn     1
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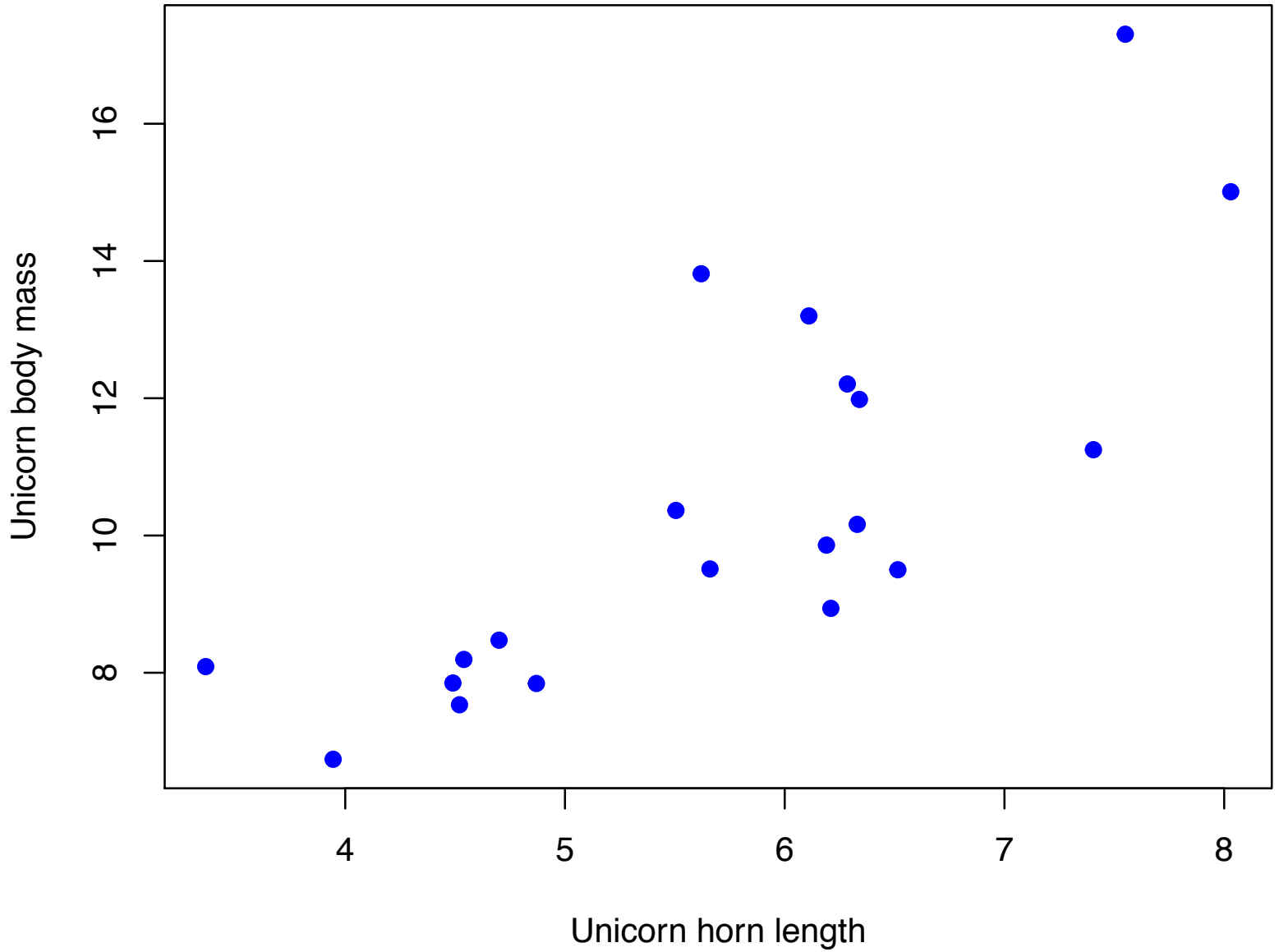
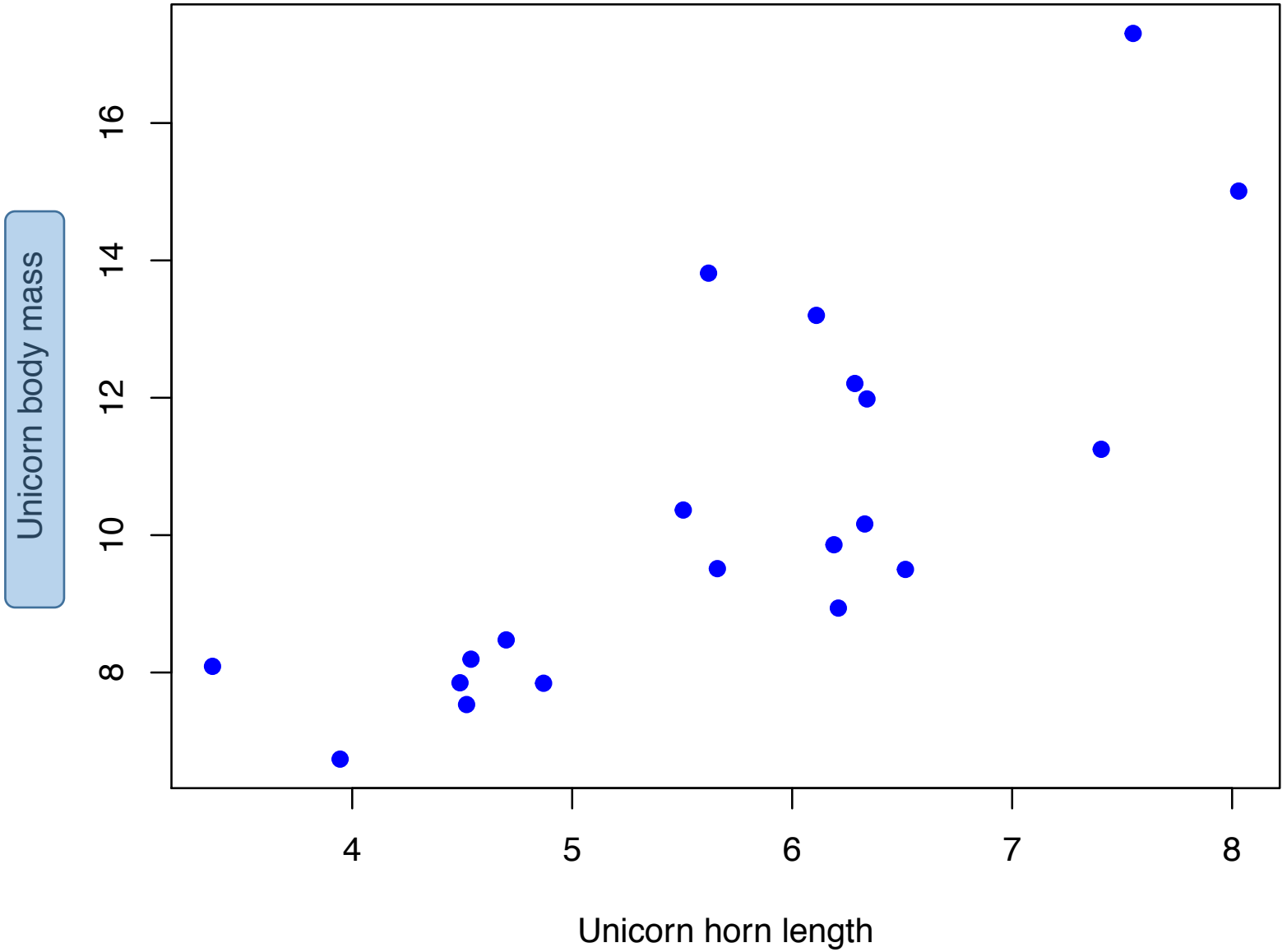
Linear models - terminology



Linear models - terminology

Y = response variable, body mass



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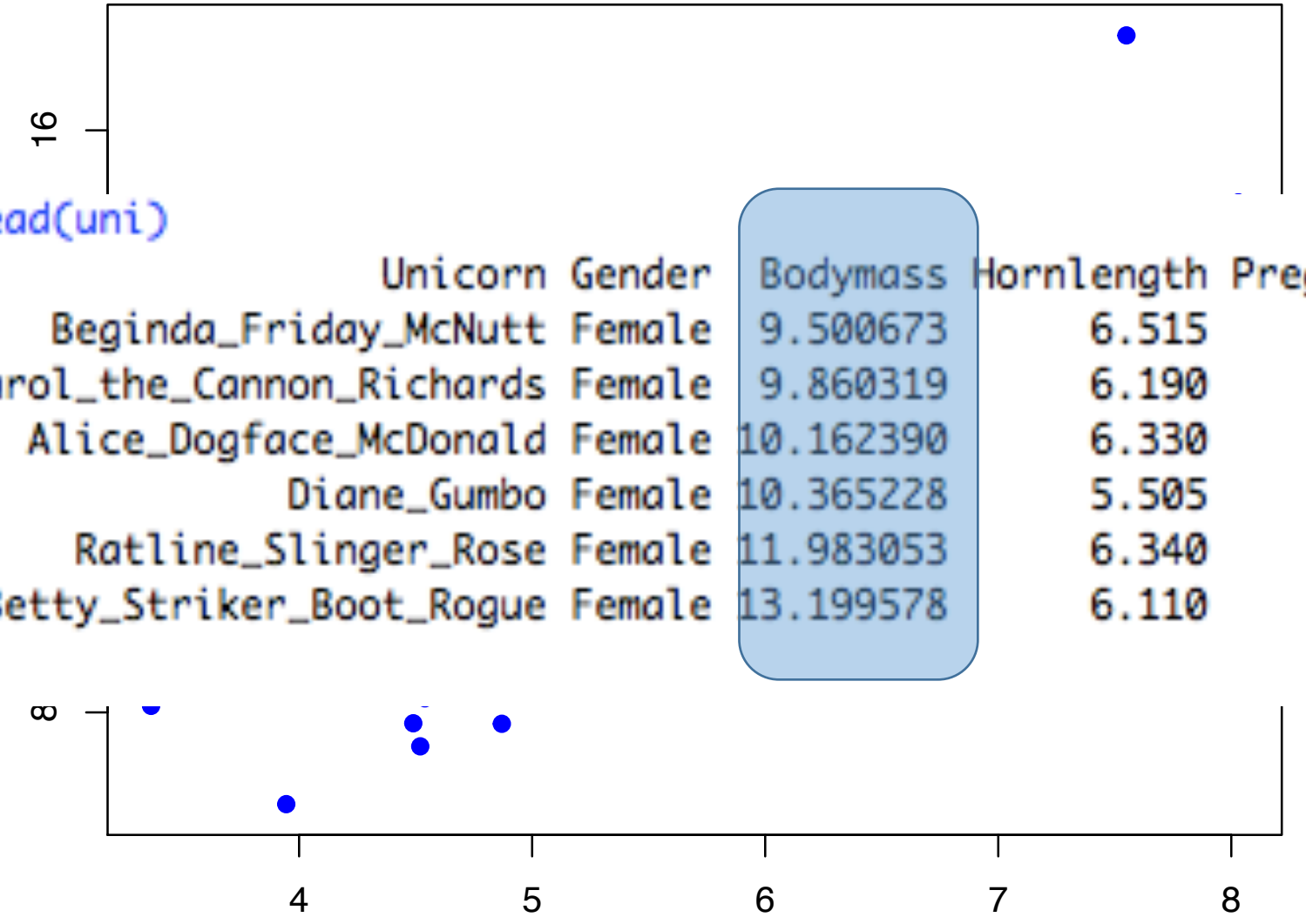


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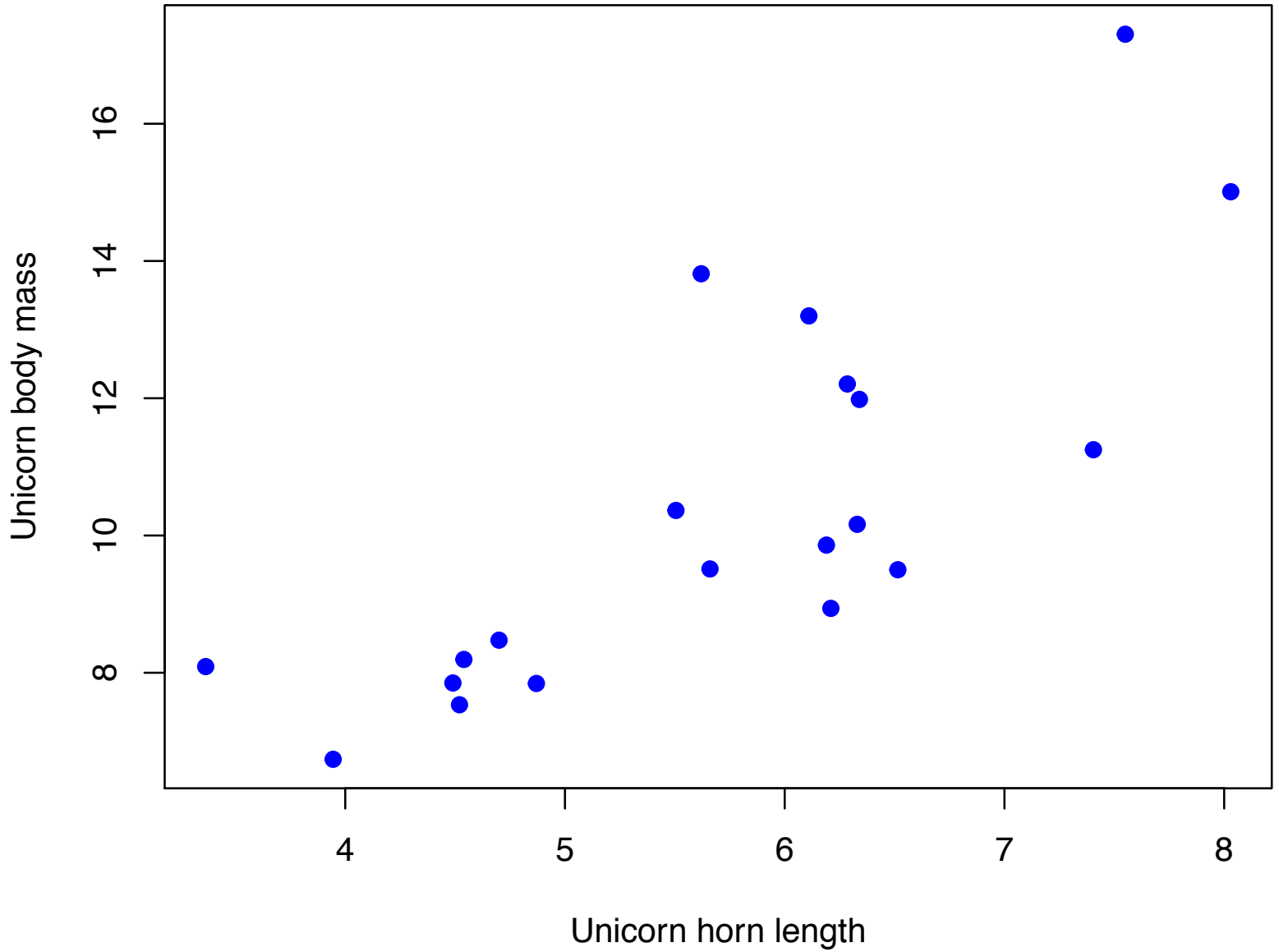
Linear models - terminology

Y = response variable, body mass

X = explanatory variable, horn length



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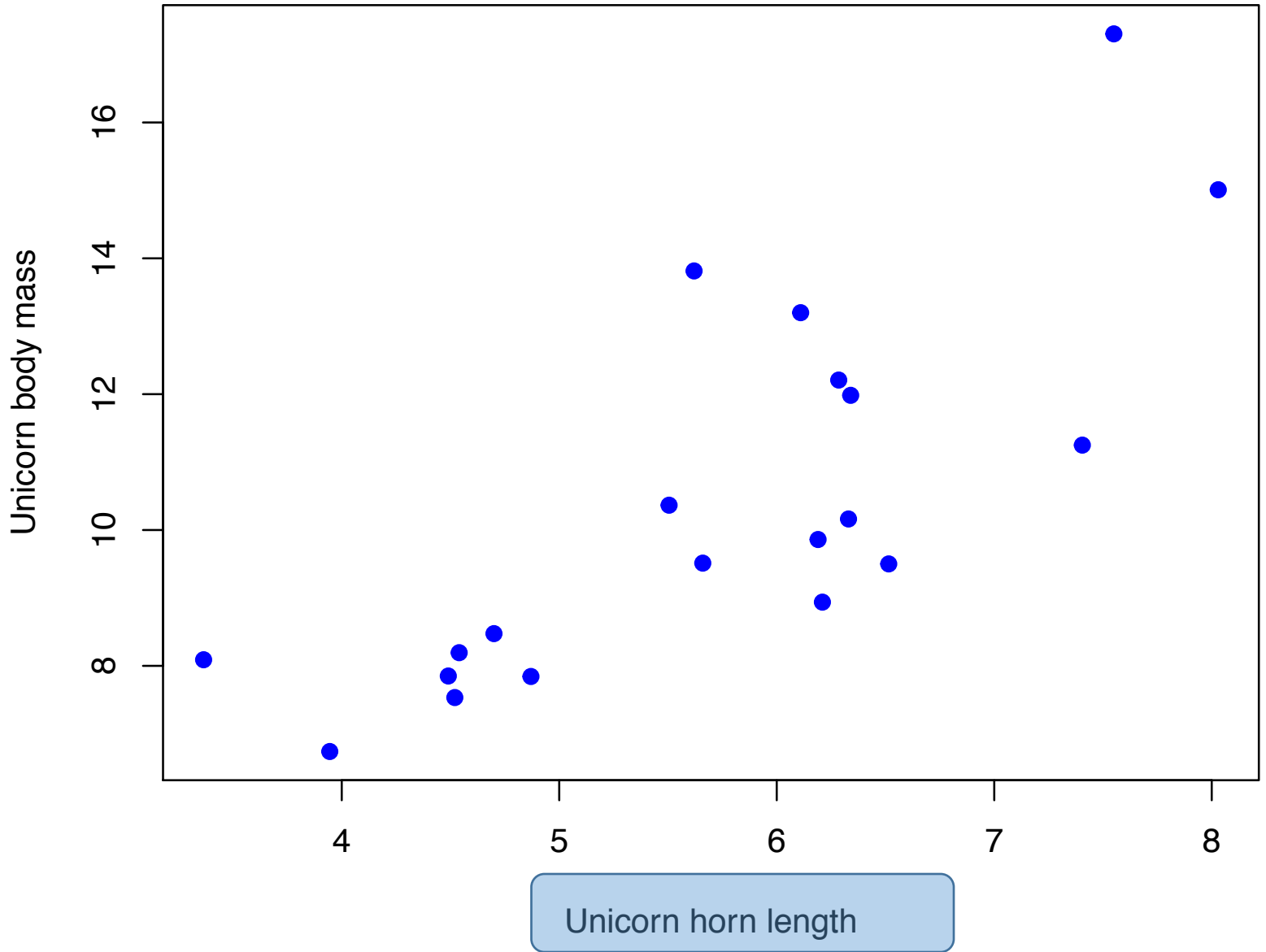

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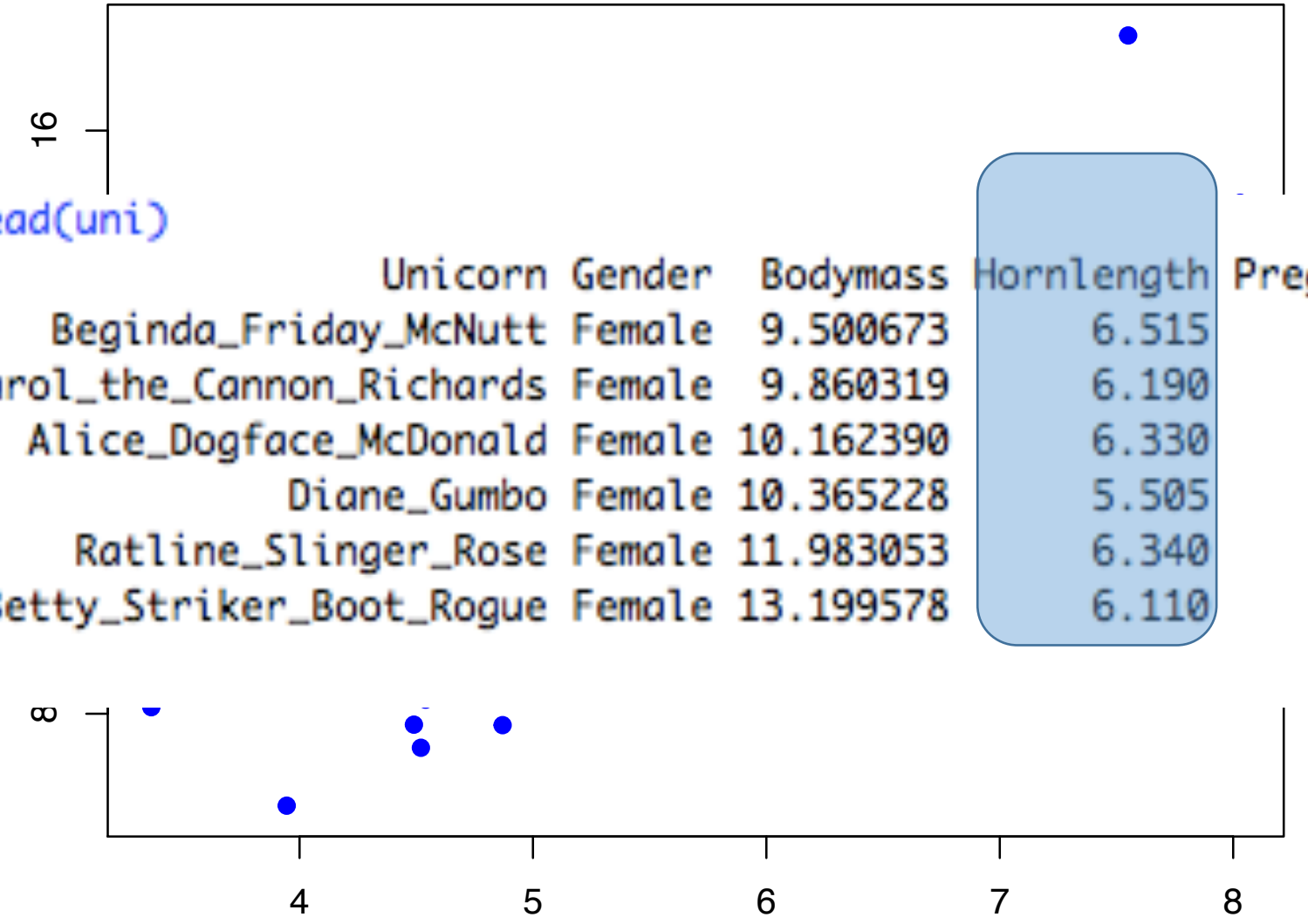


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Linear models

- *Vectors* vs **scalars**

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y	i
5	1
3	2
6	3
10	4
4	5

$b_0 = ?$

$b_1 = ?$

x	i
3	1
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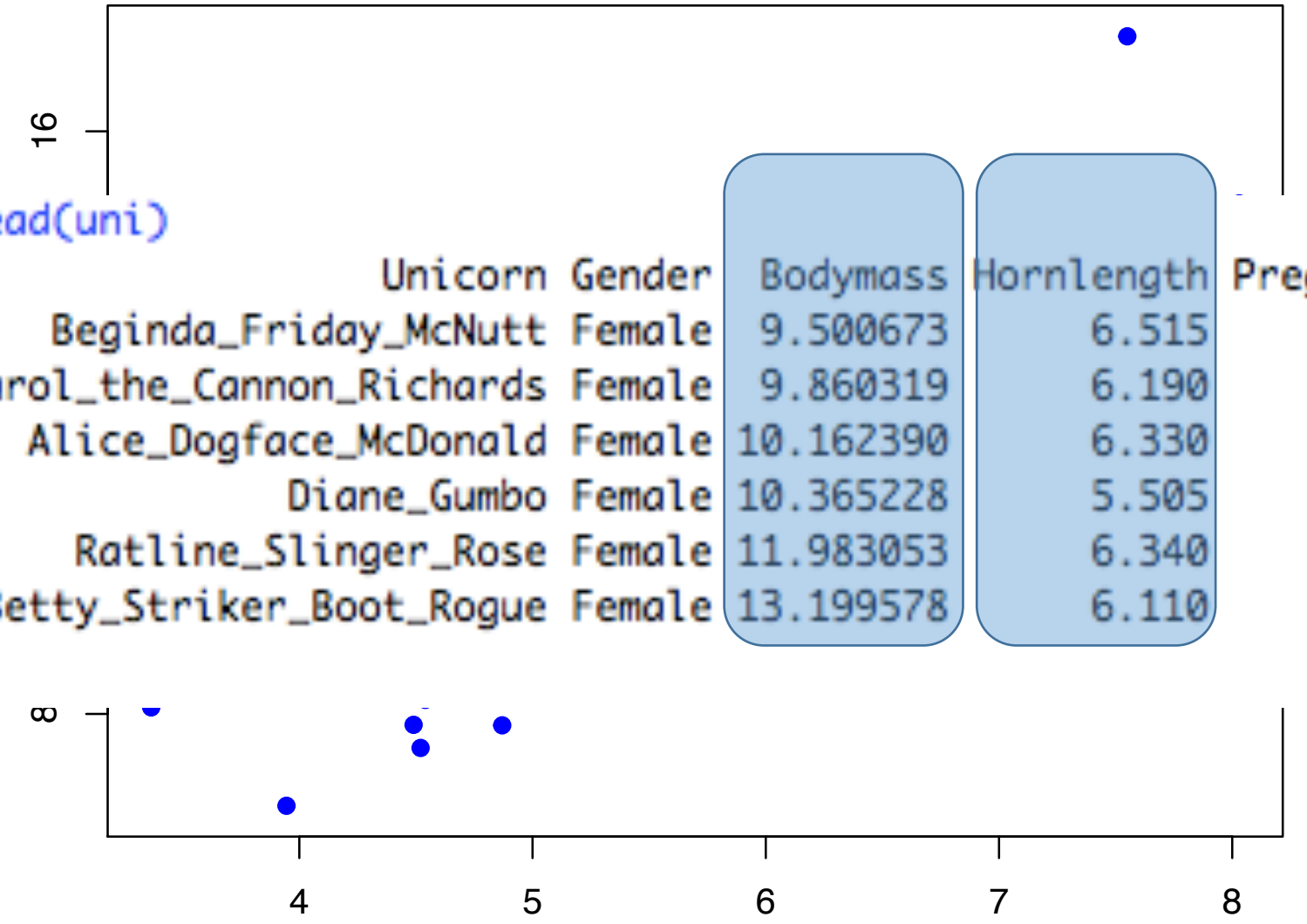


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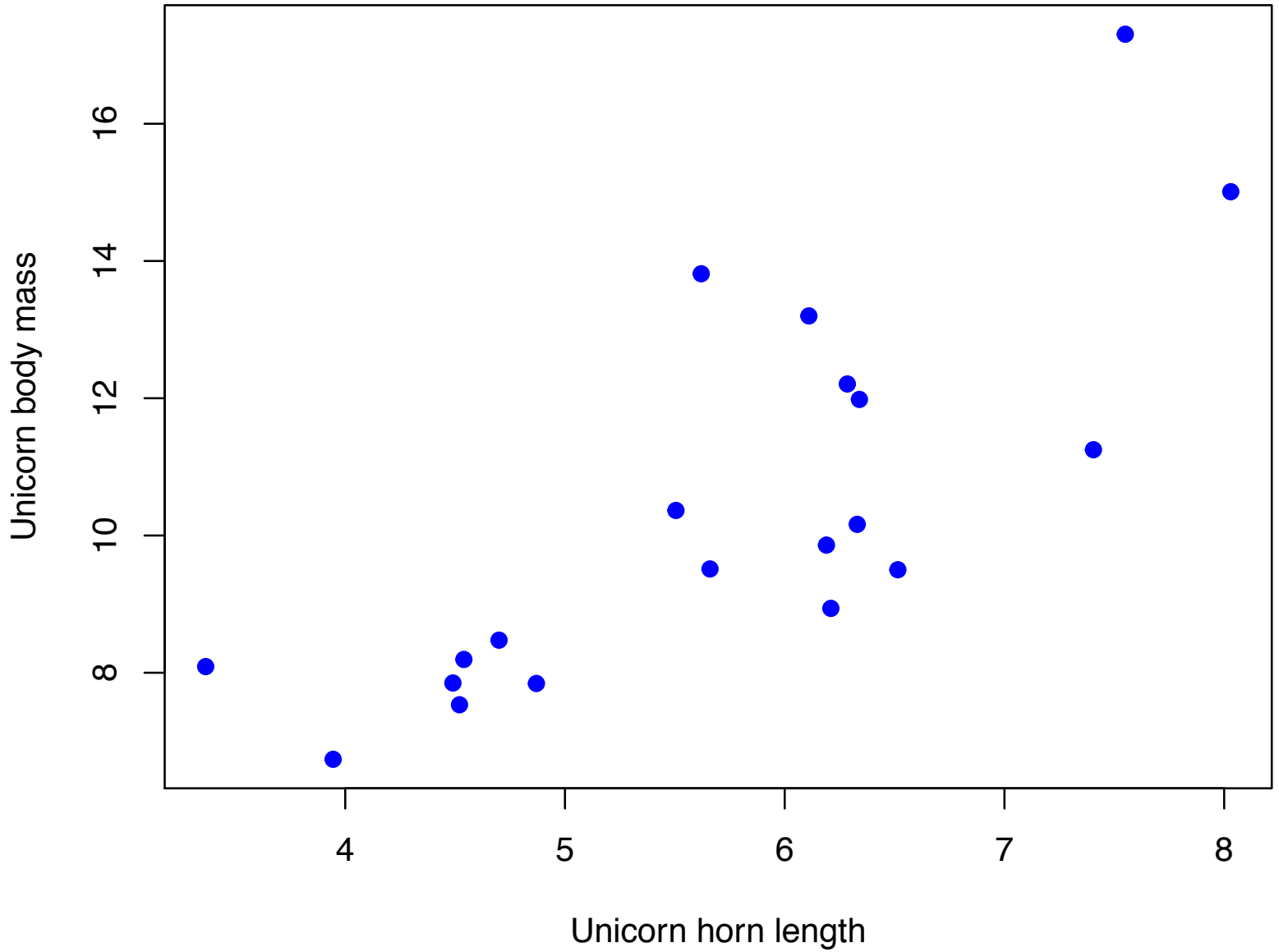

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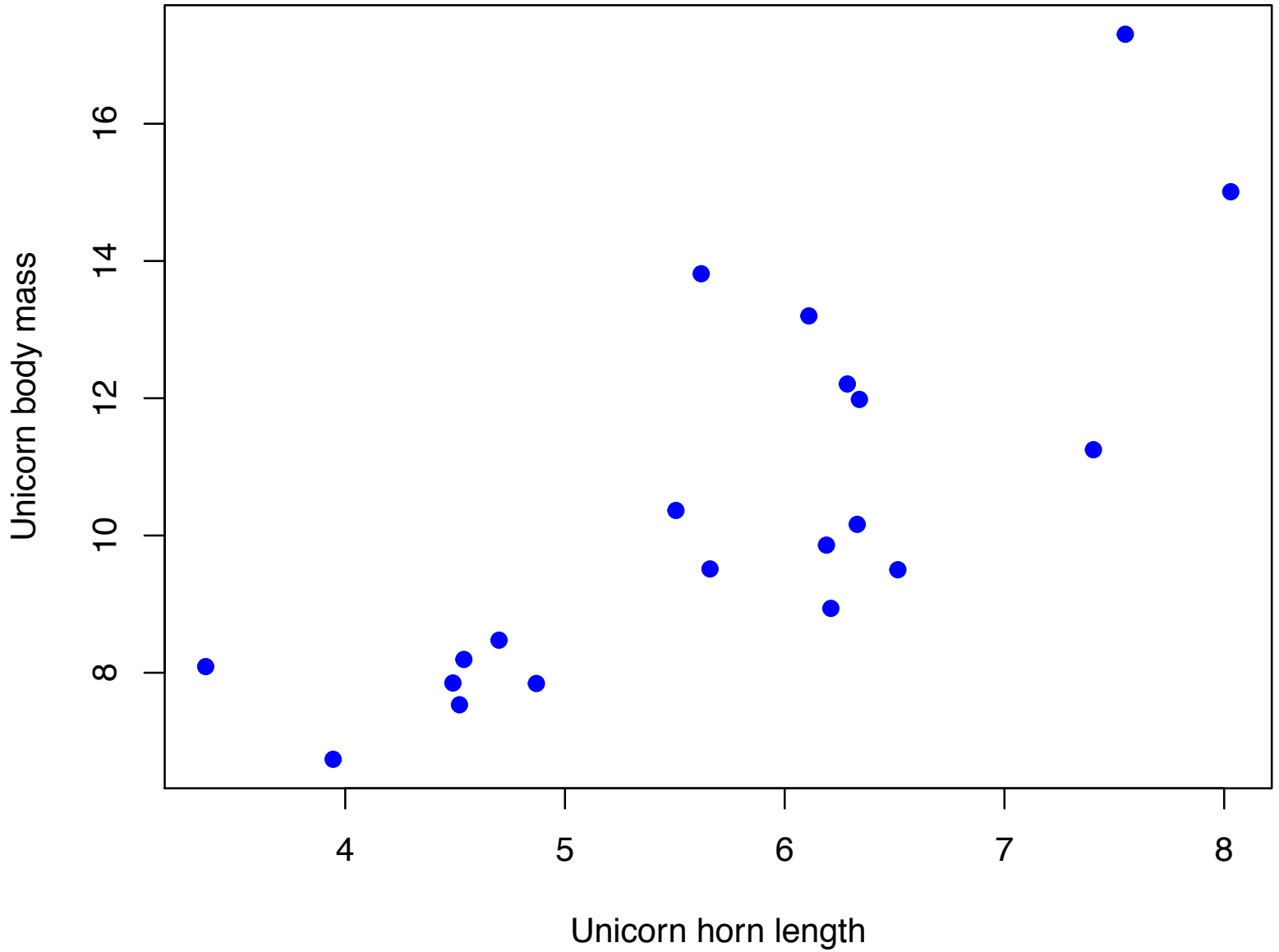
Y_i = OBSERVATIONS of body mass

Linear models - terminology



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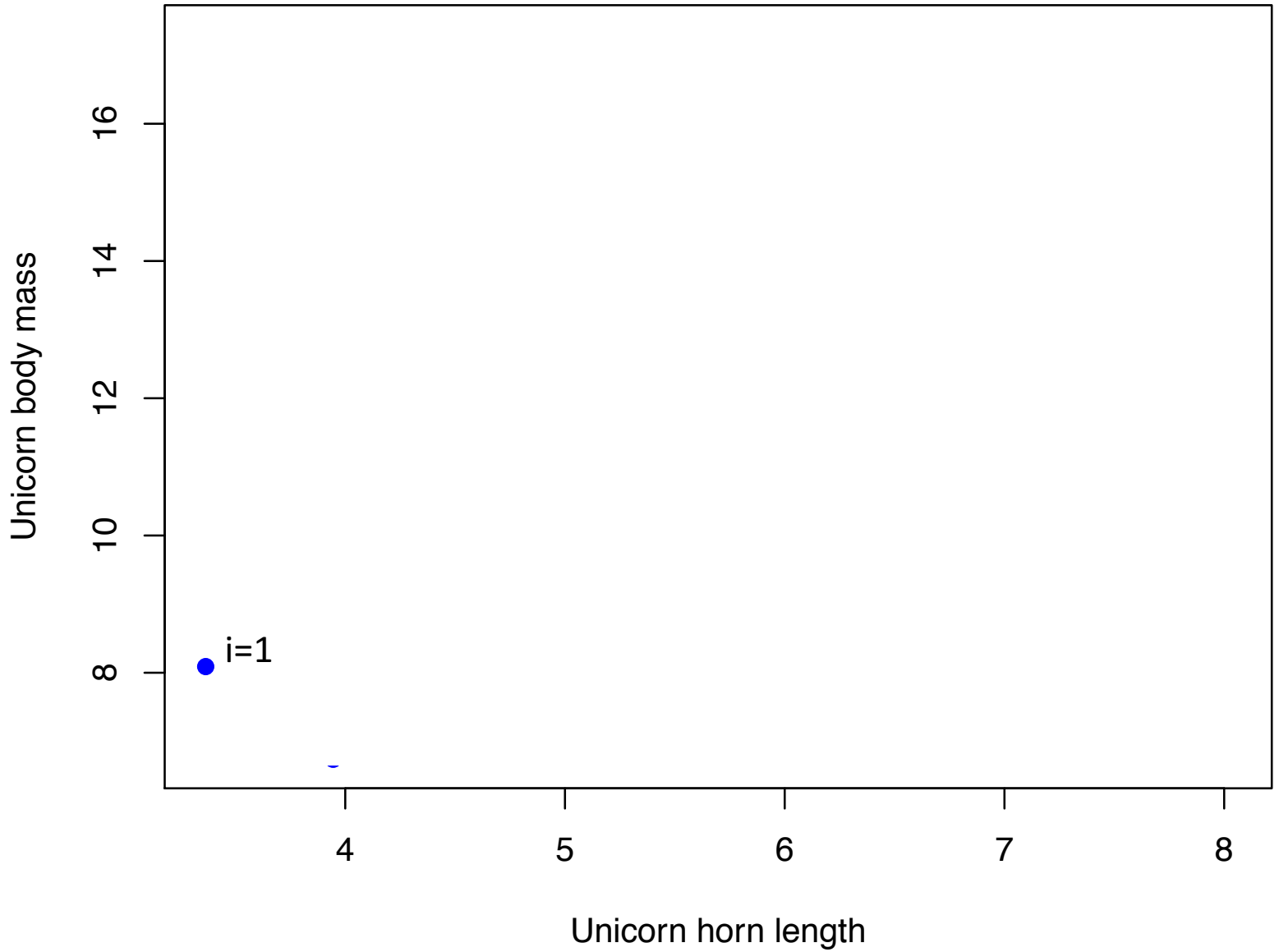

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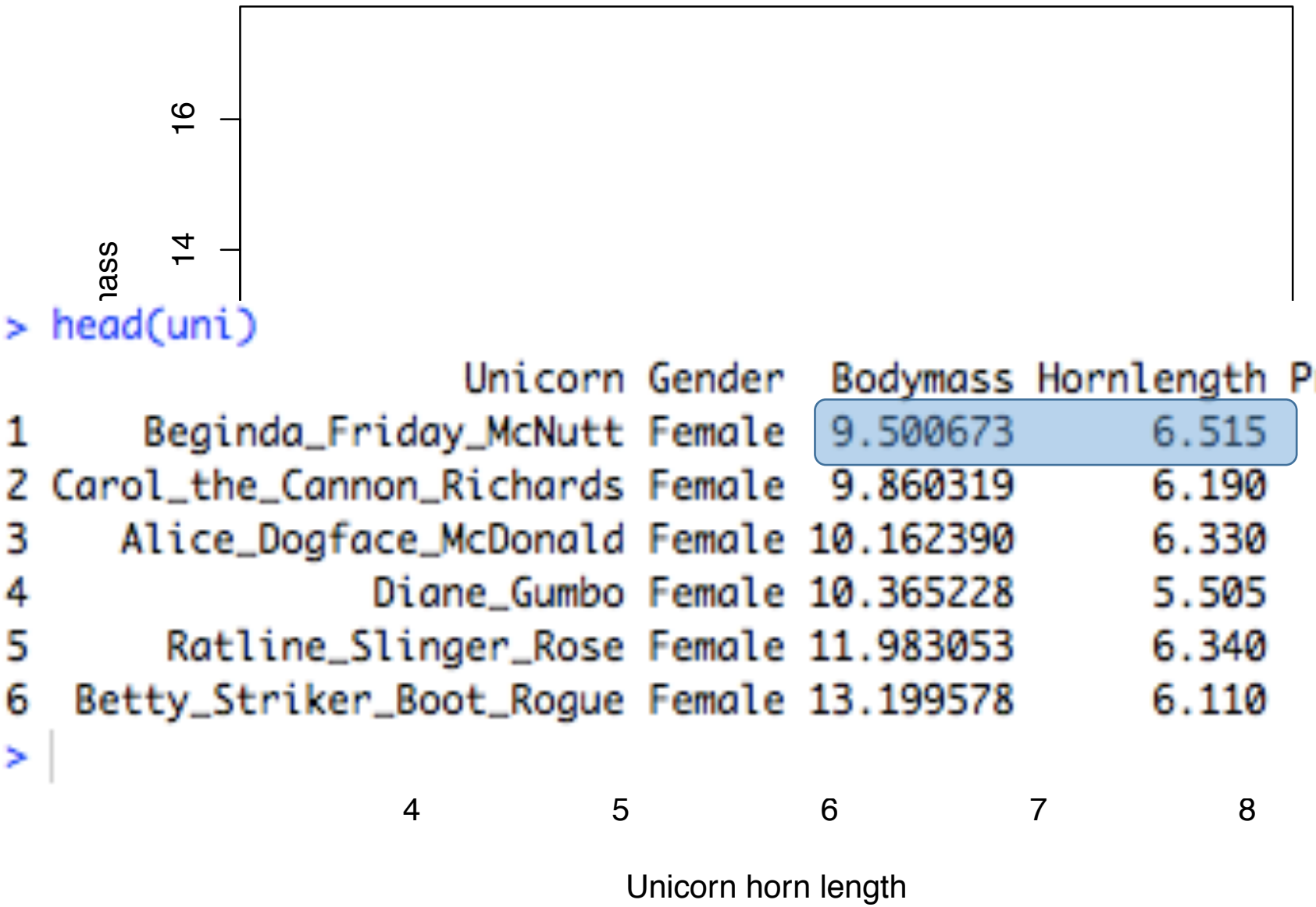


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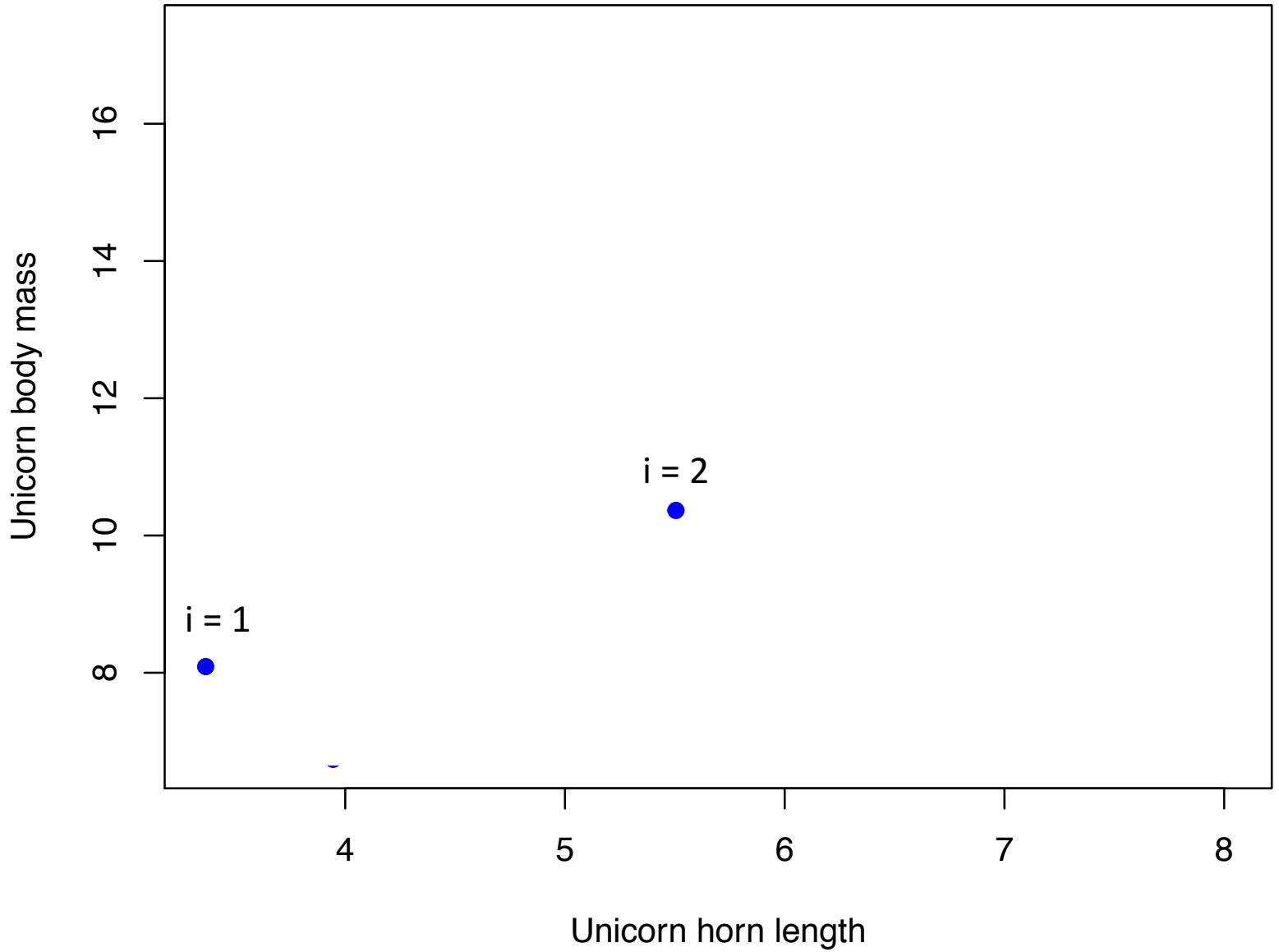

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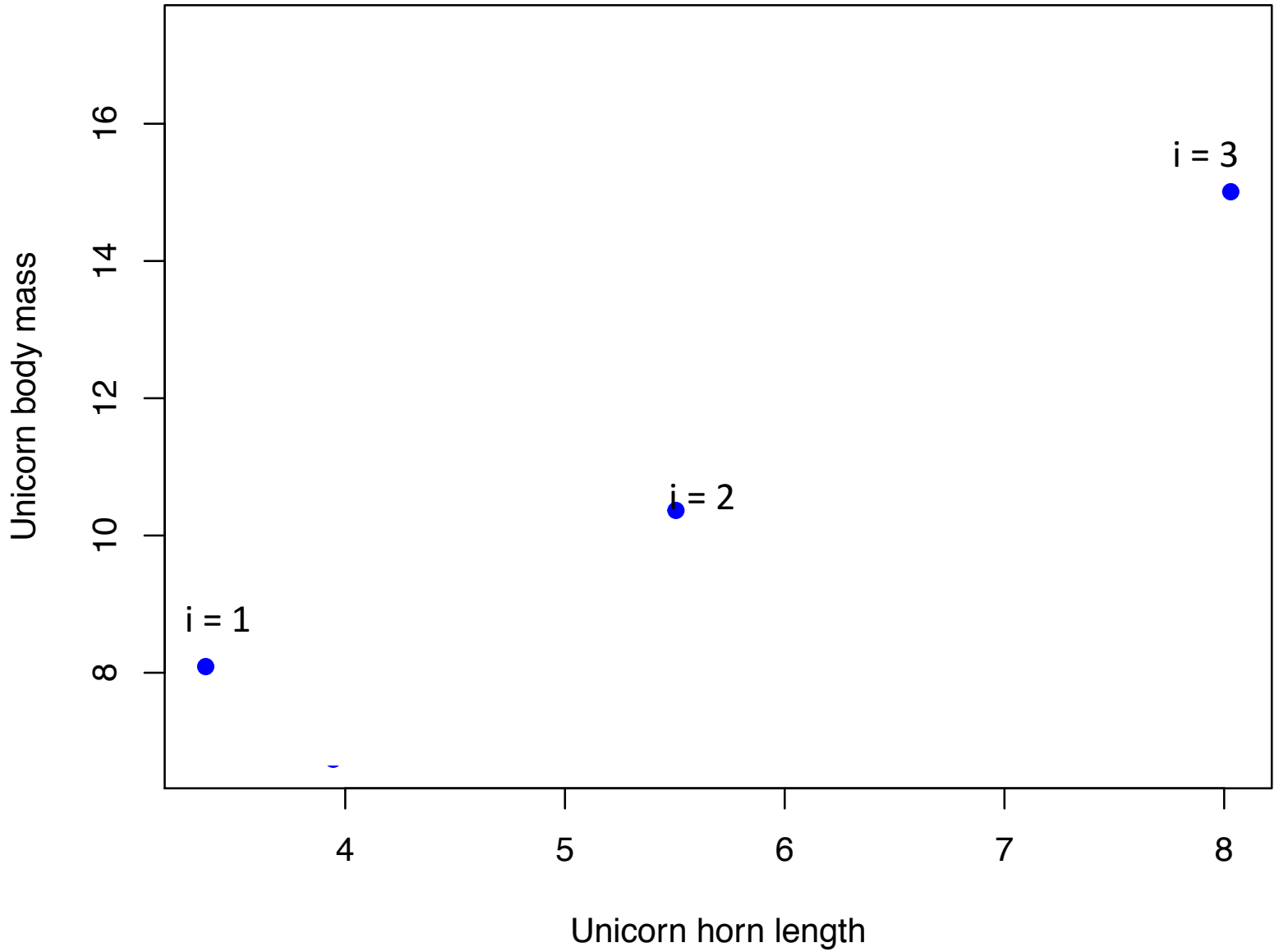

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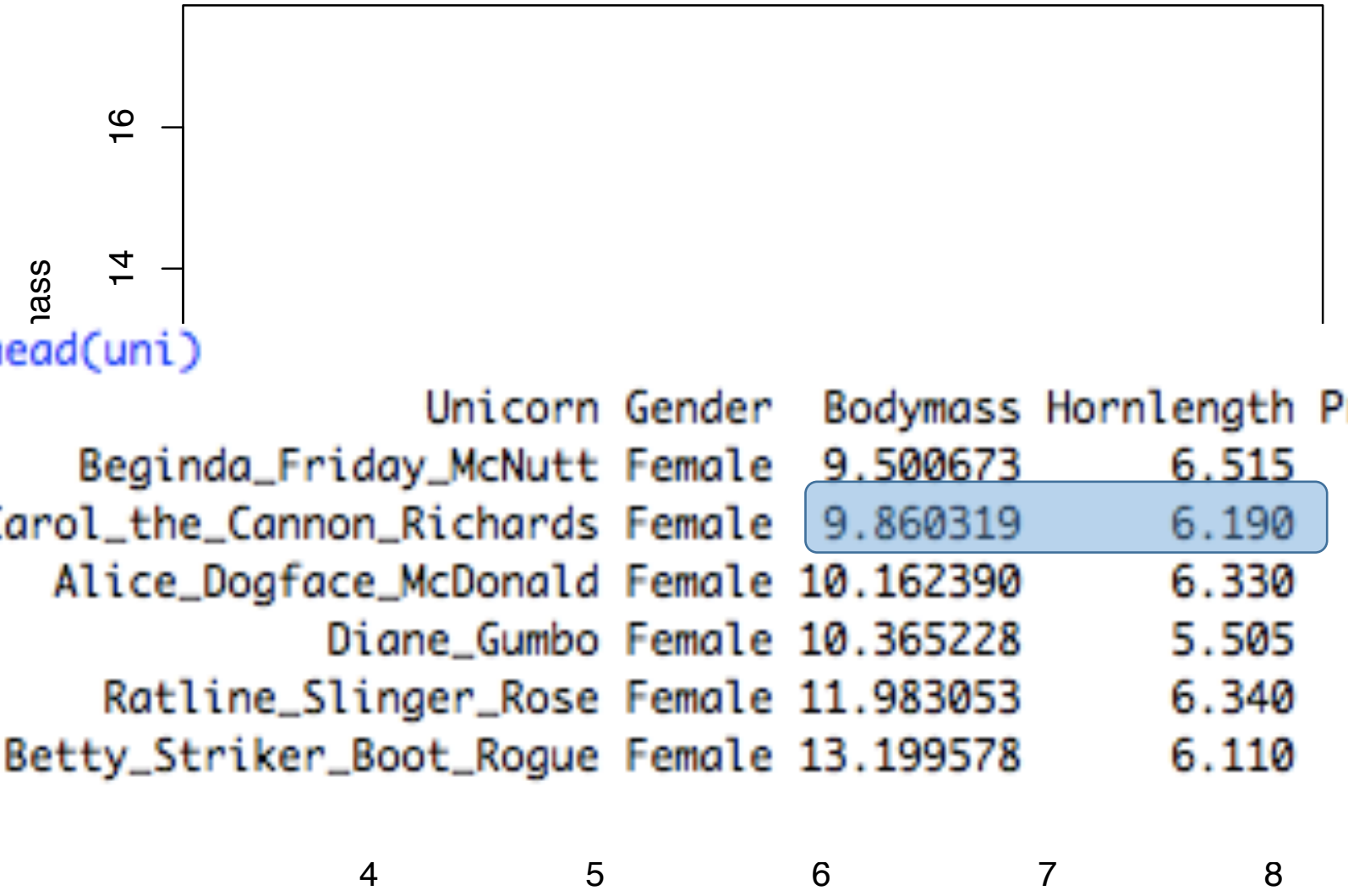


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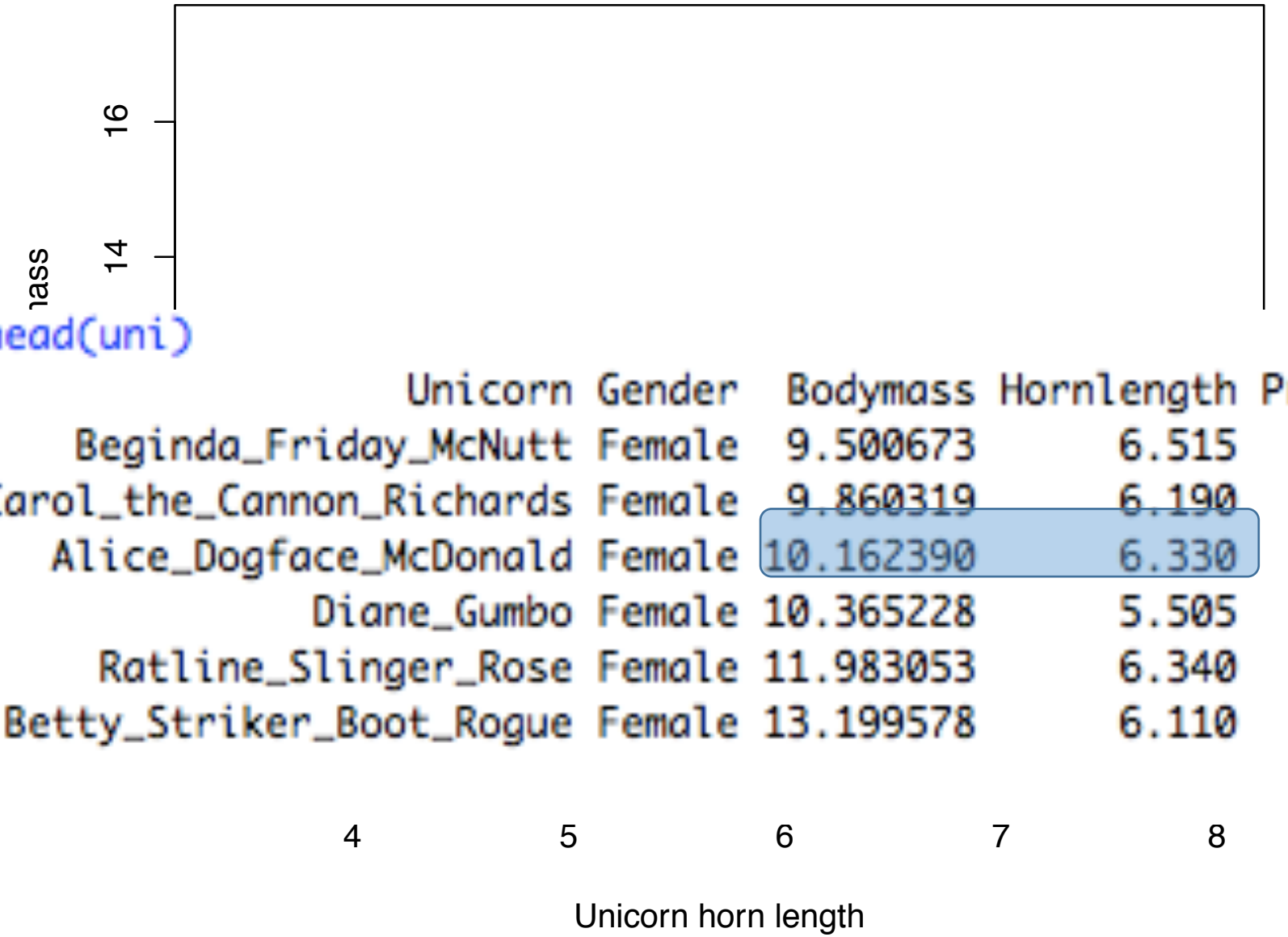


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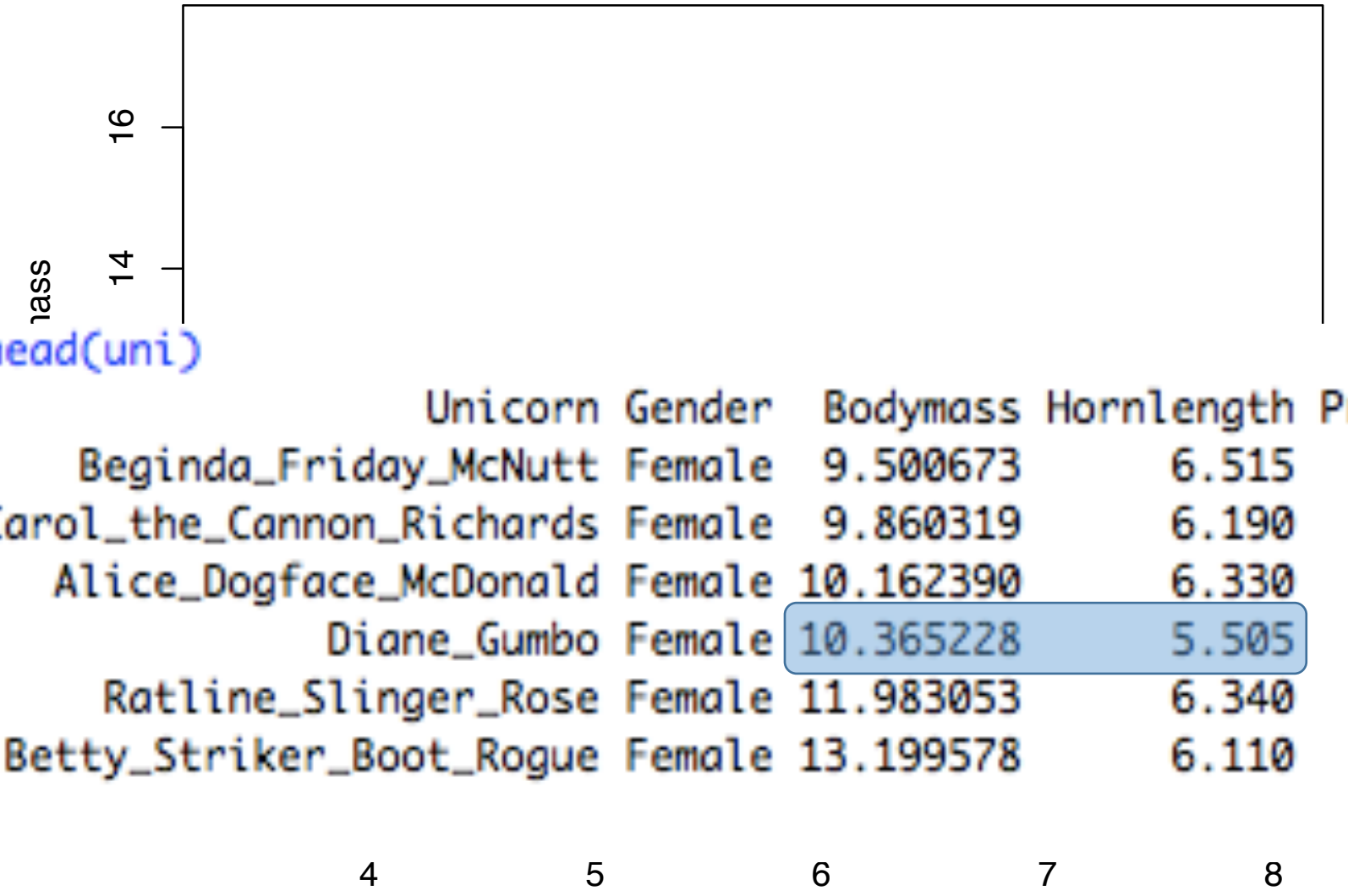


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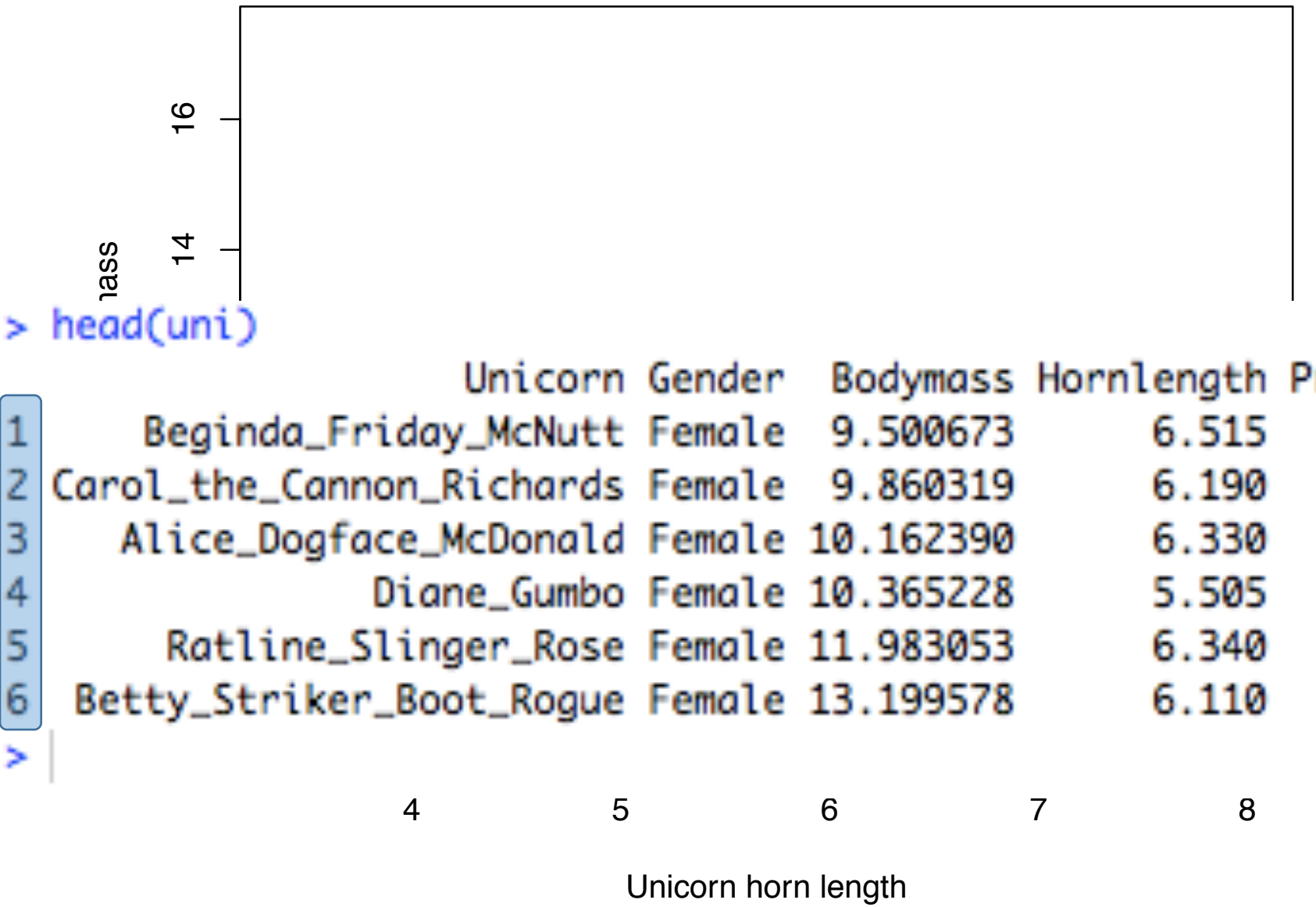


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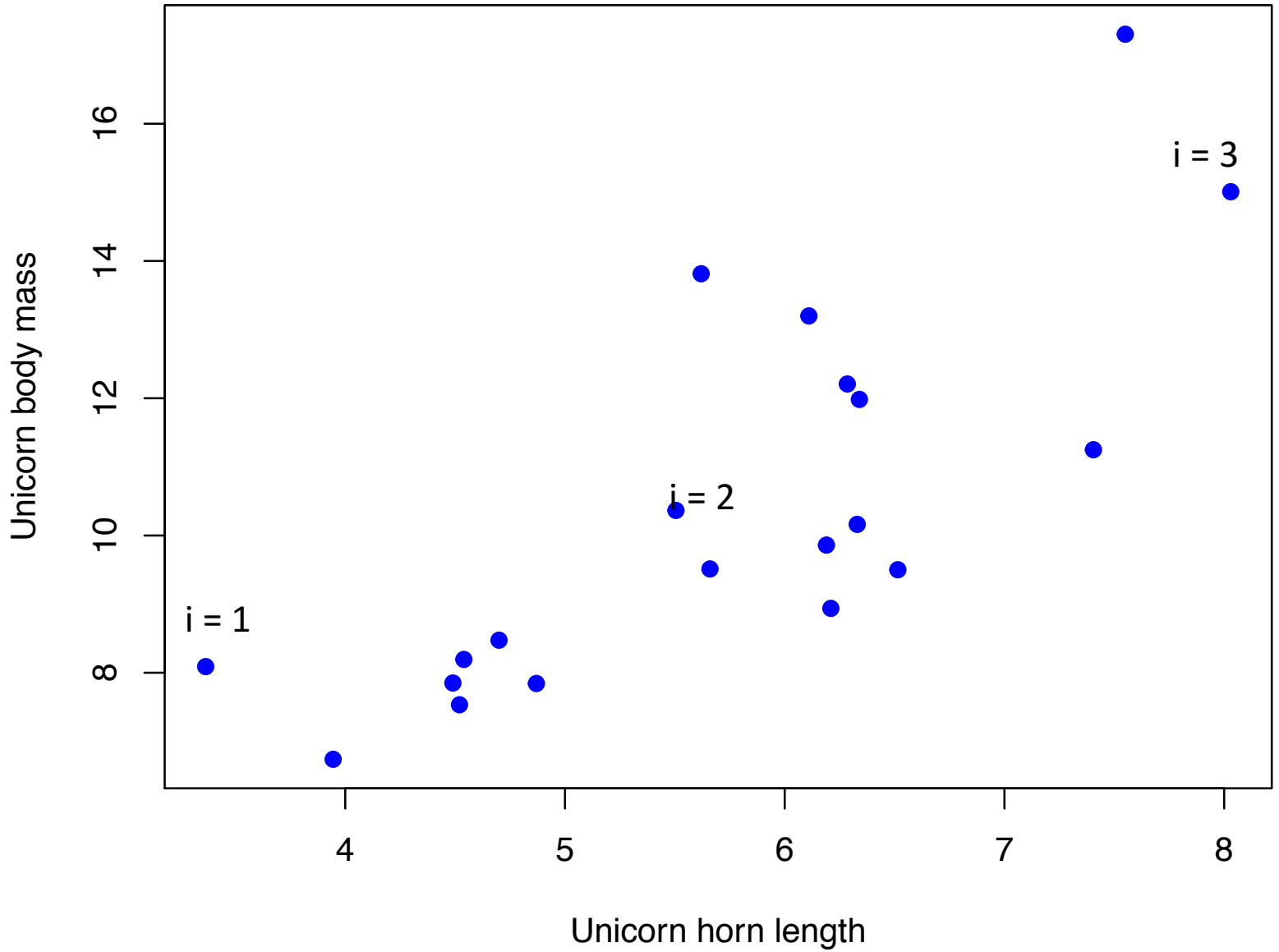

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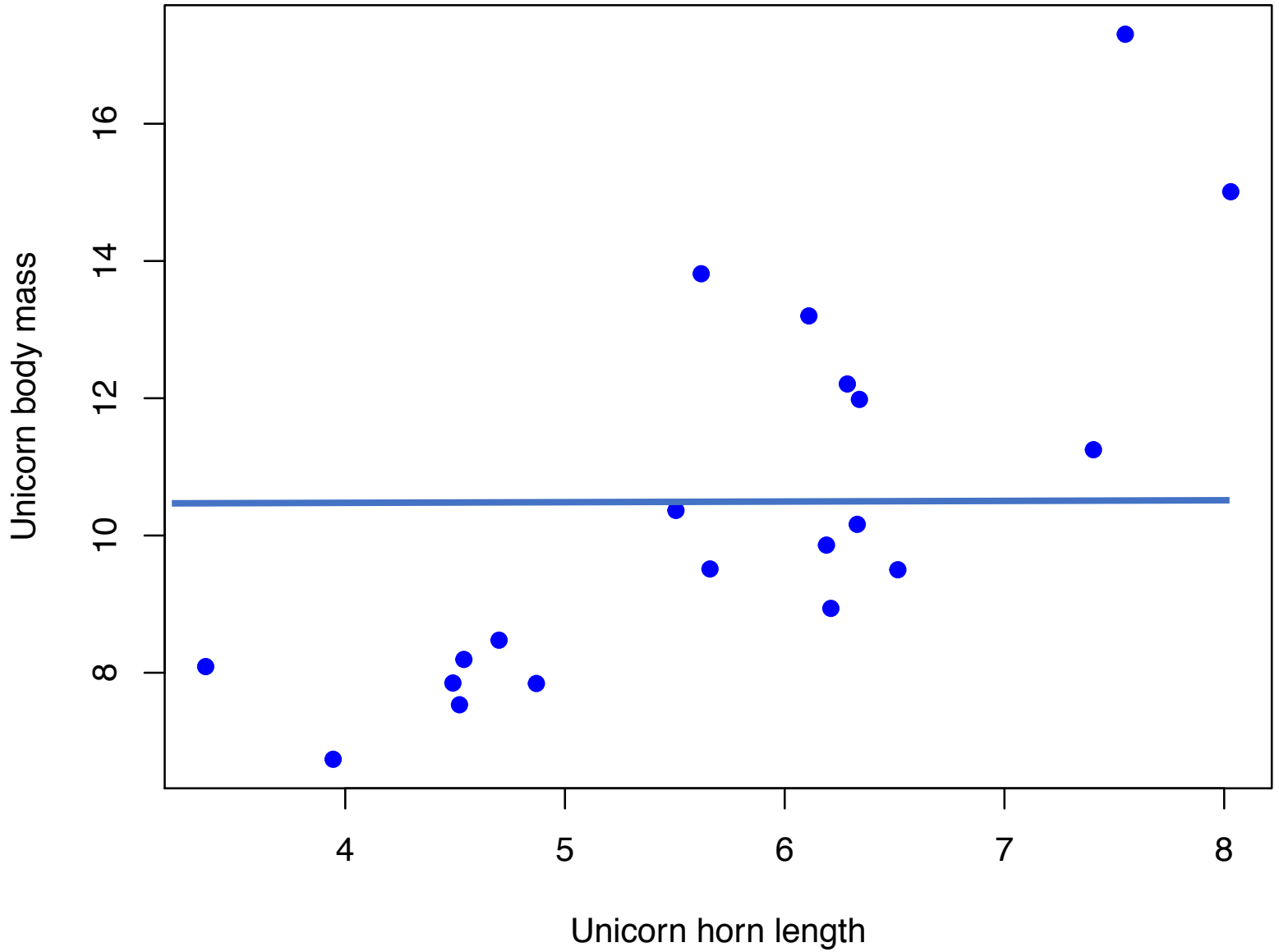

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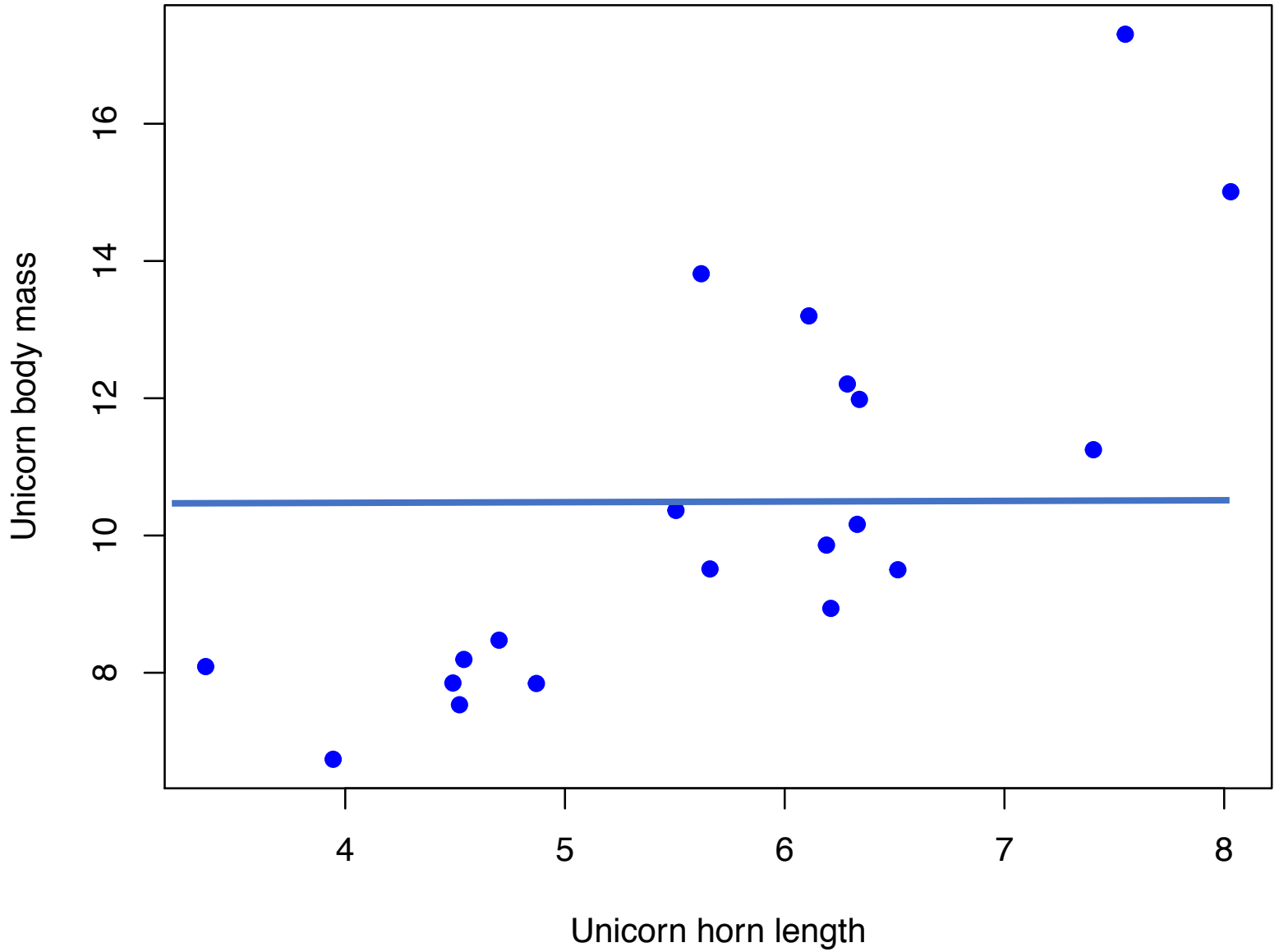

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$$y_i = b_0 + b_1 x_i + \varepsilon_i$$

$b_0 = ?$

$b_1 = ?$

ε	i
?	1
?	2
?	3
?	4
?	5

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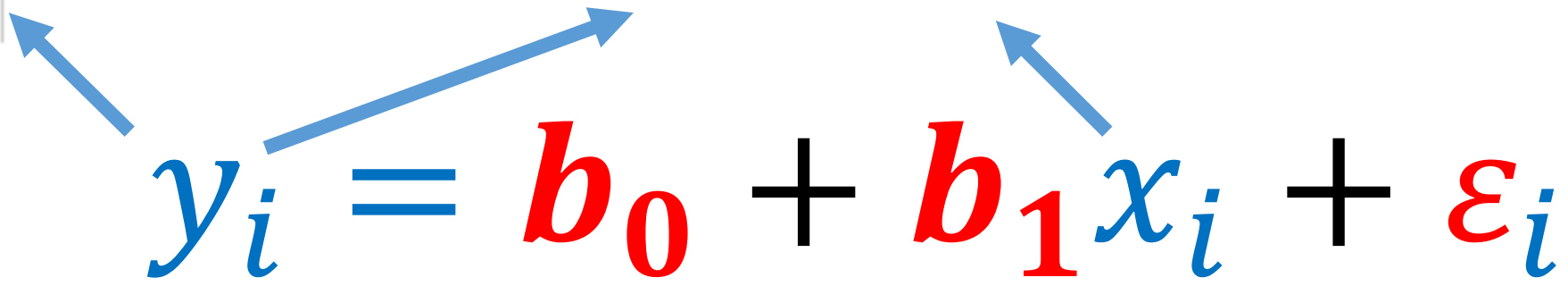

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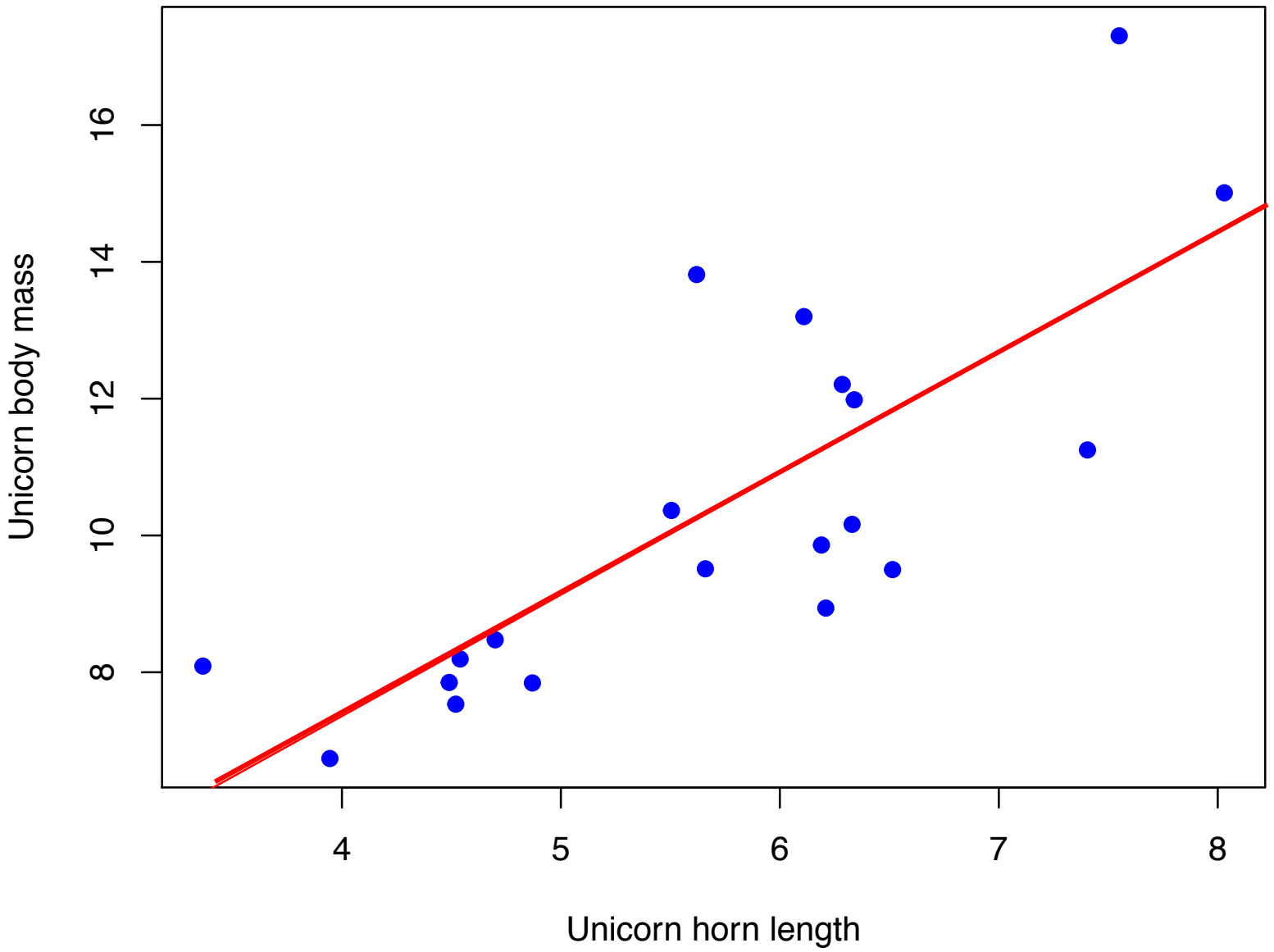

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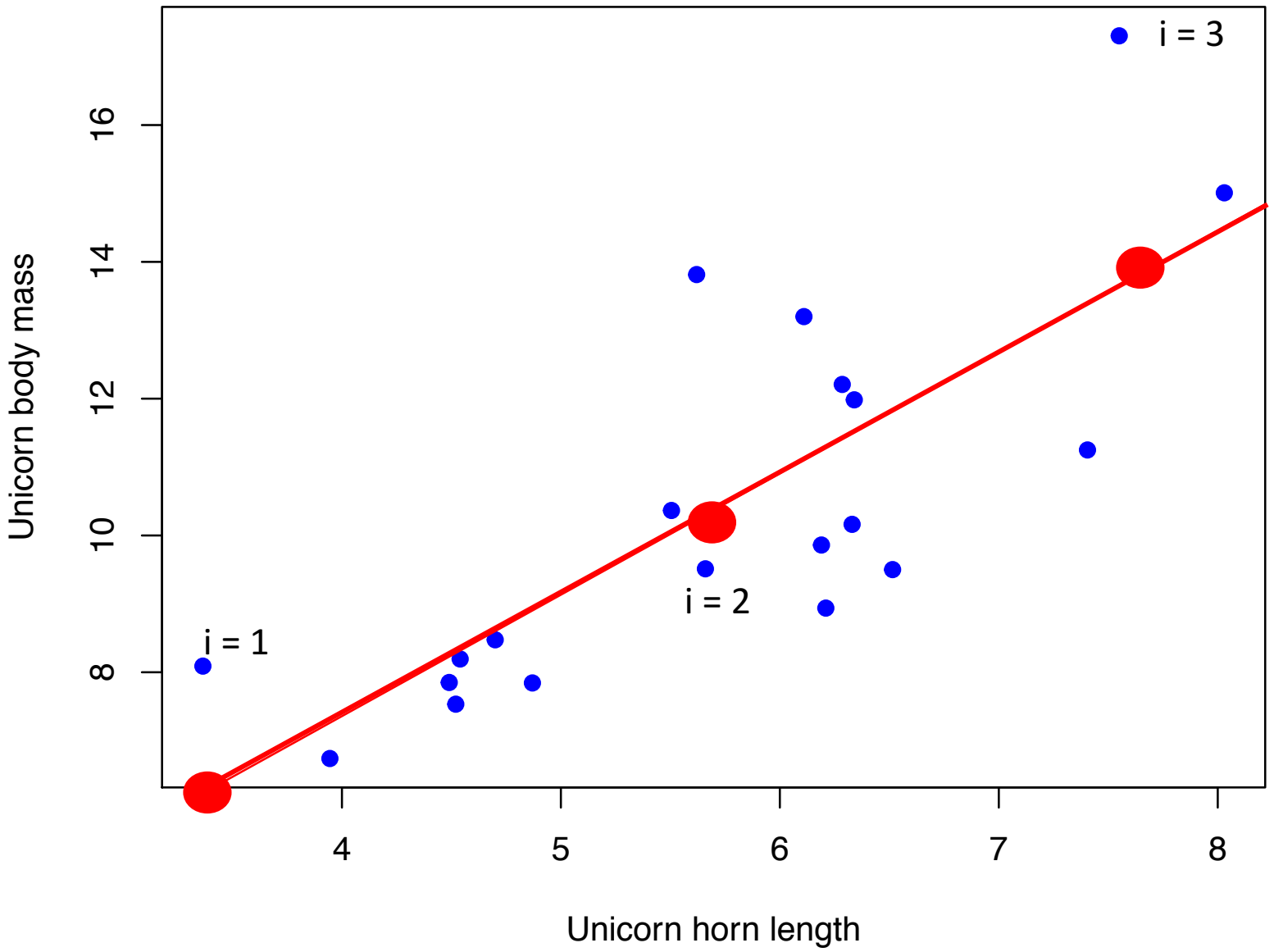

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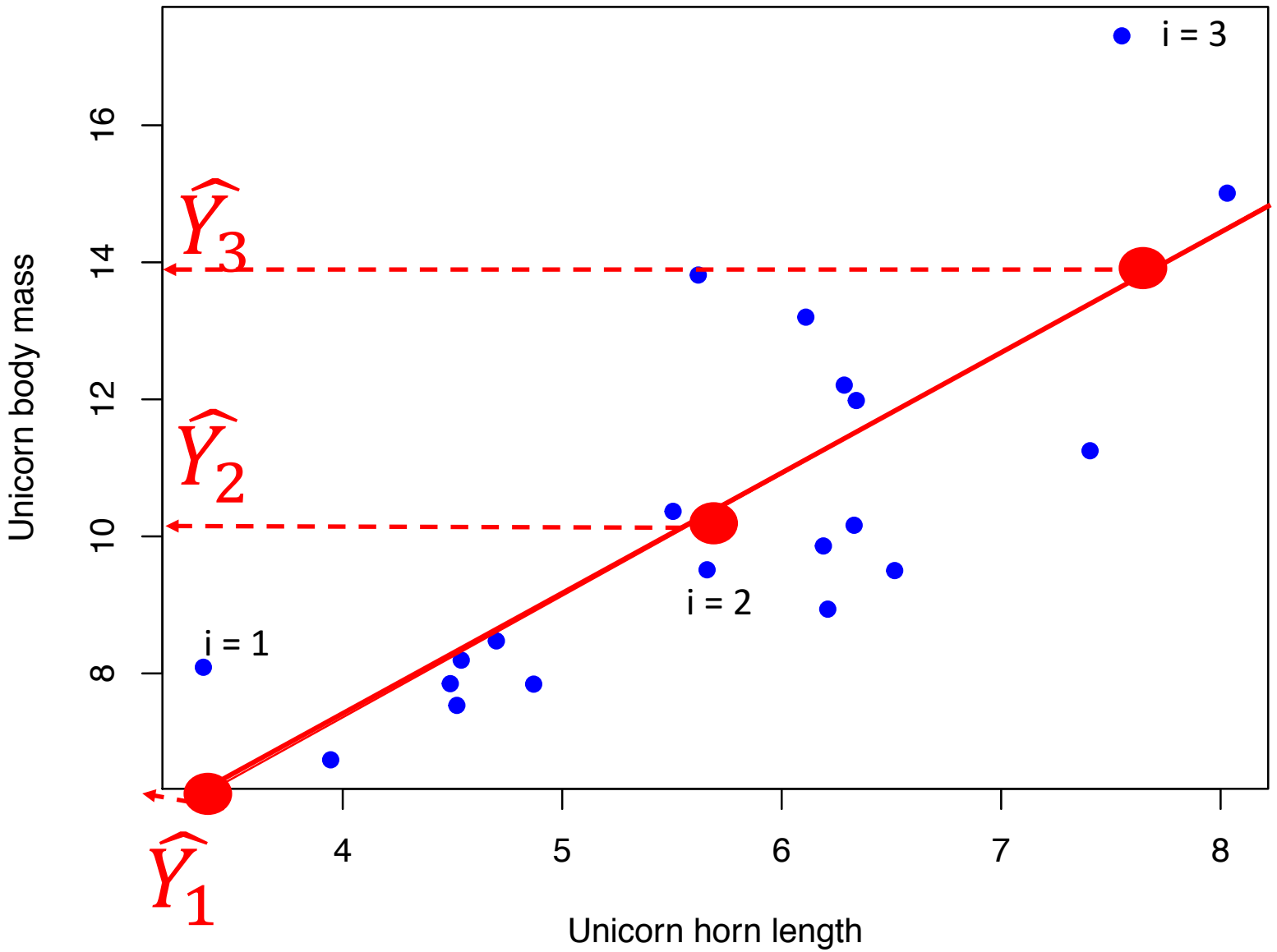

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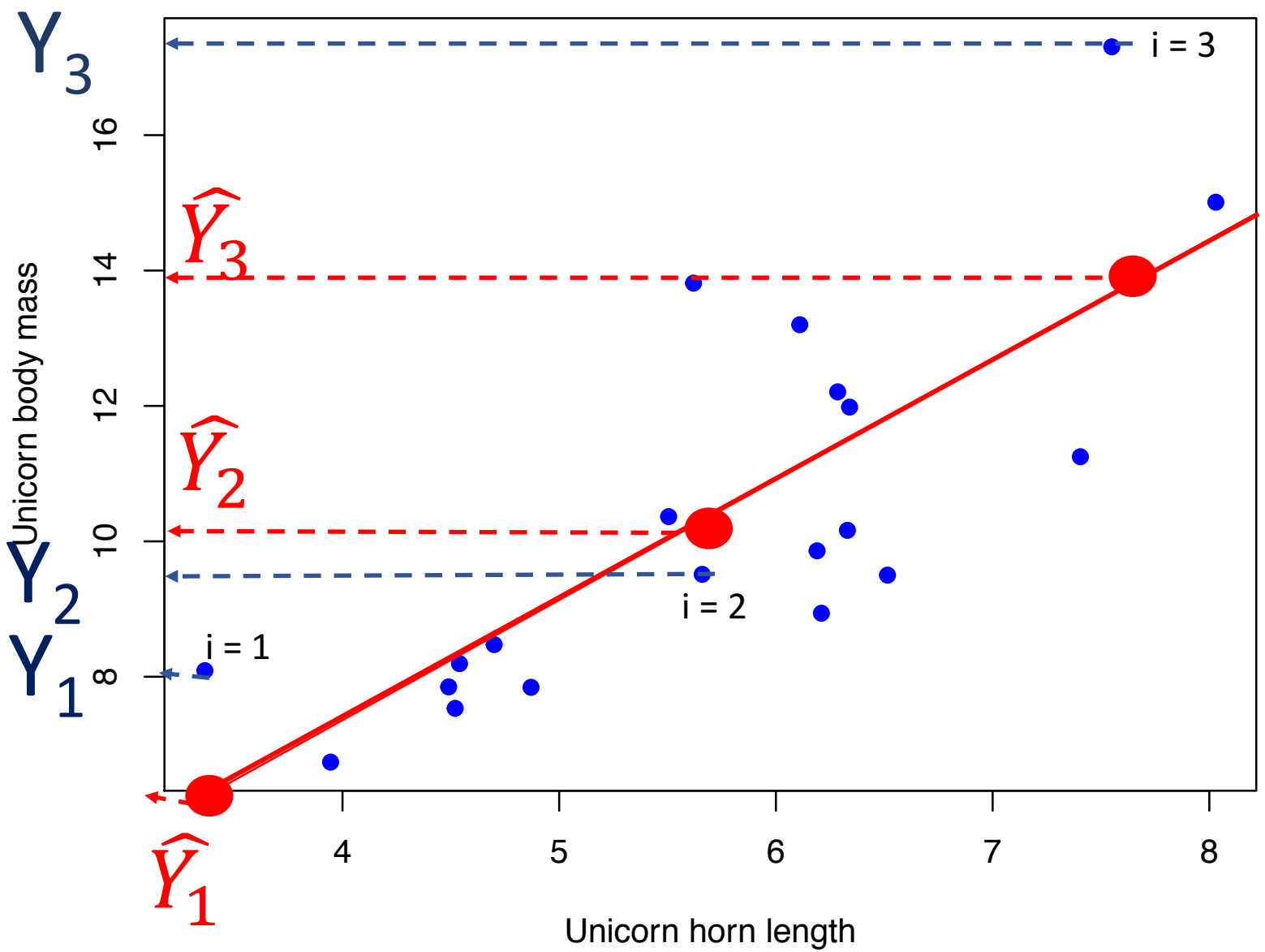

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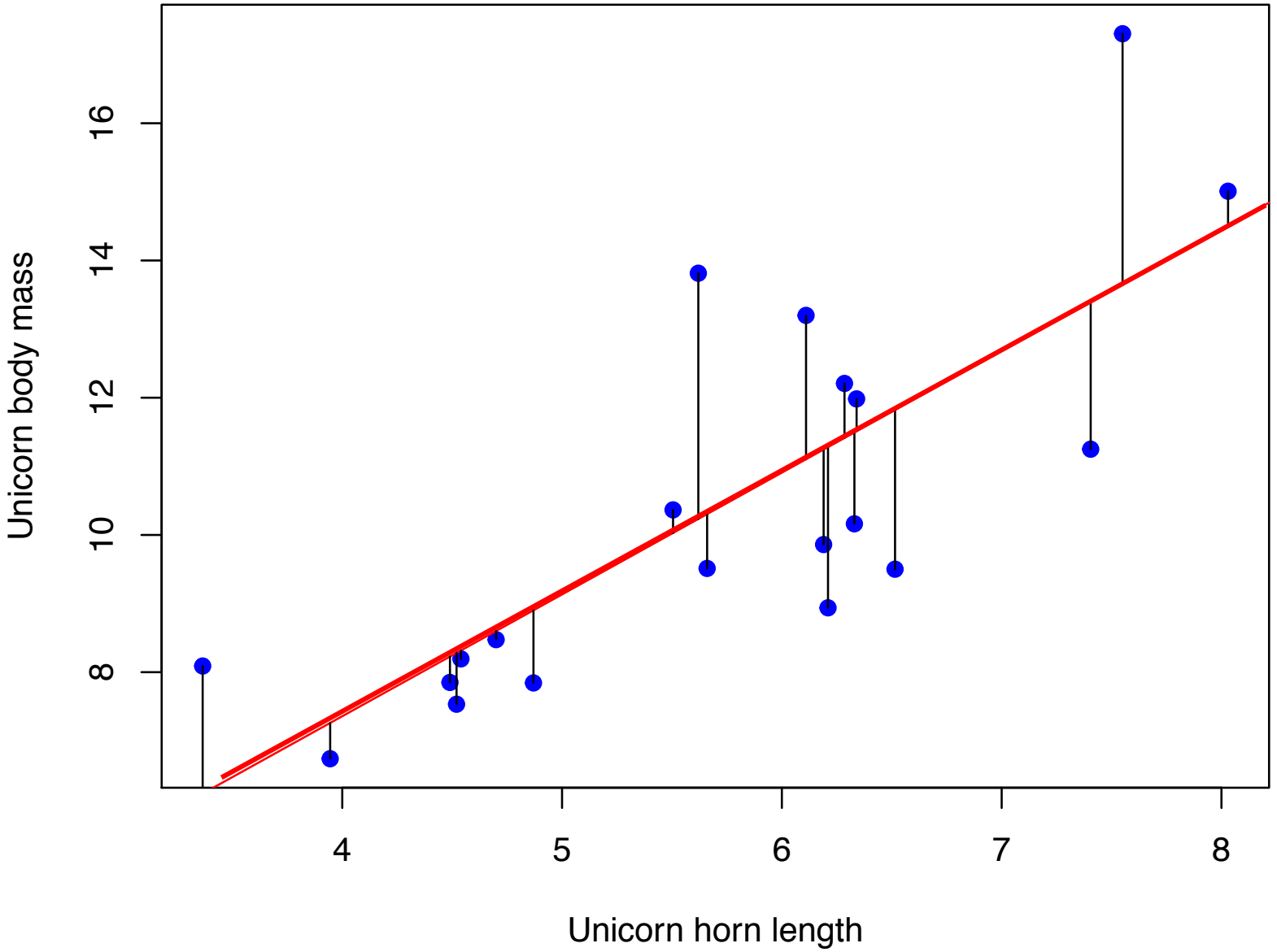

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ε_i = ERROR, residuals

Linear model predictors

- Continuous

Linear model predictors

- Continuous
- Categorical

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$$y_i = b_0 + b_1 x_i + \varepsilon_i$$

y	i
5	1
3	2
6	3
10	4
4	5

$b_0 = ?$

$b_1 = ?$

x	i
blue	1
red	2
blue	3
blue	4
yellow	5

ε	i
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- → GENERAL linear models

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Linear model predictors

- Continuous
- Categorical
- Interactions between variables
- Squared terms

$$y_i = b_0 + b_1 x_{i1} + b_2 x_{i1}^2 + b_3 x_{i2} + b_4 x_{i1} x_{i2} + \varepsilon_i$$

Process

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Process

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What to include?

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What to include? **HYPOTHESIS**

- Does the horn of the unicorns help them to eat more so they are heavier?

$Body\ mass \sim Hornlength$

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What to include? **HYPOTHESIS**

- Maybe heavier unicorns are able to grow larger horns?
  - Age?
  - Sex?
  - Size?
  - Nutrients in diet?
  - Ecology (shorter horns in denser forests, longer horns on grasslands)
  - Predator presence

Process – common problems

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How to decide which model is good?

?

Different ways to compare models

- Information criterion (AIC, BIC, DIC)
- Log-likelihood

Model selection

- Common sense!
- Information criterion (AIC, BIC, DIC)
- Likelihood ratio test
- Step wise deletion

Model selection

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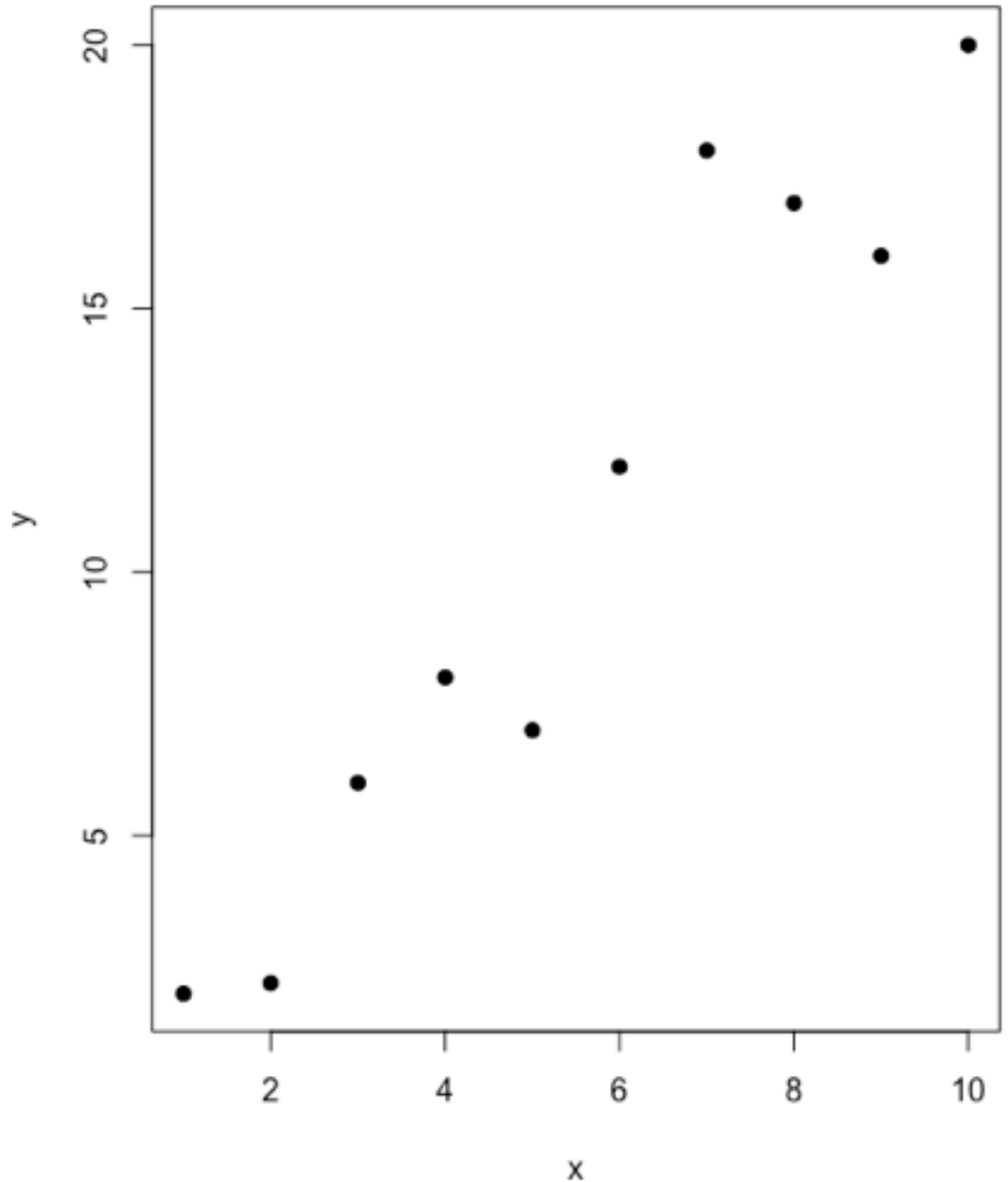

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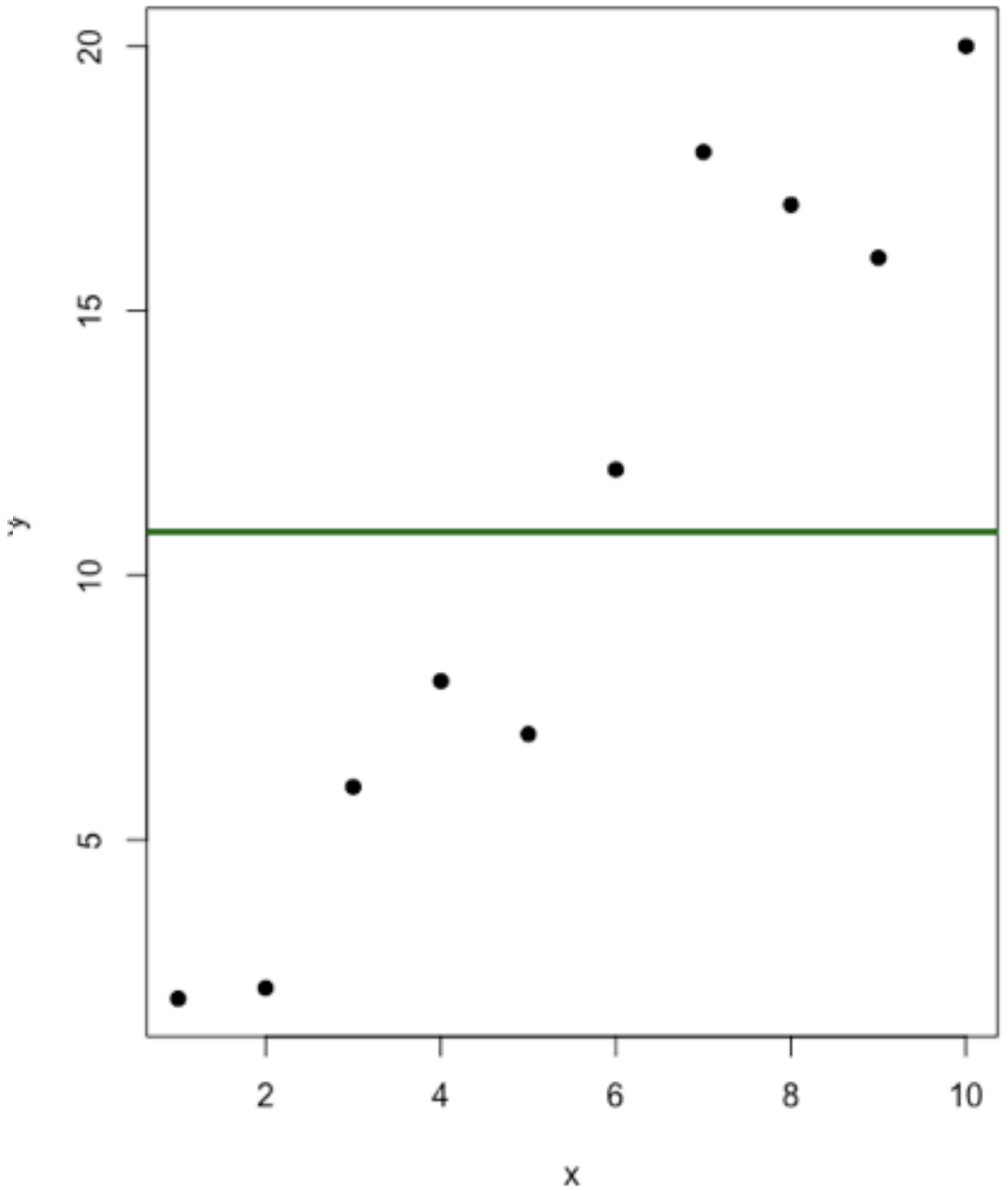
$$y_i = \beta_0$$



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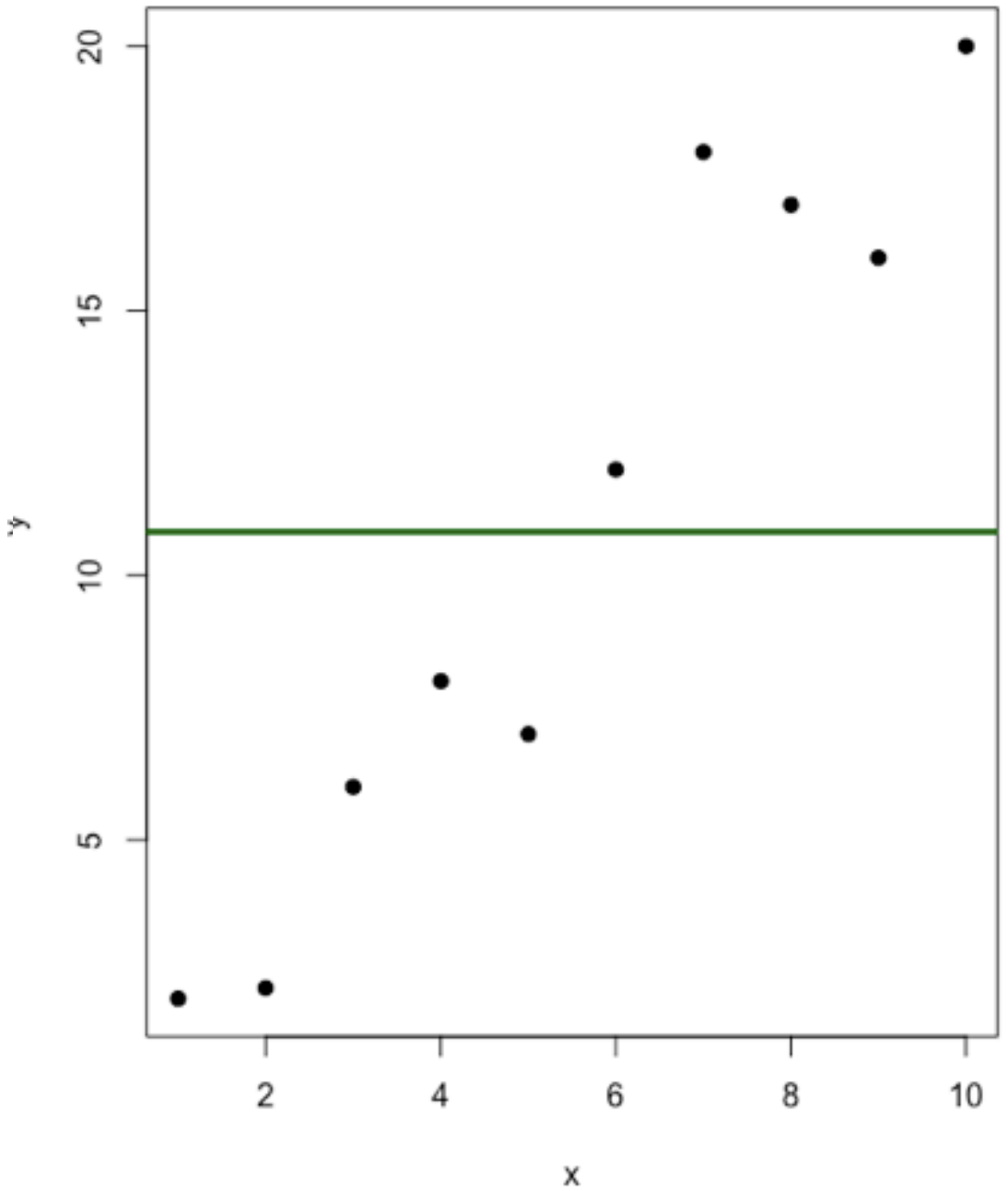

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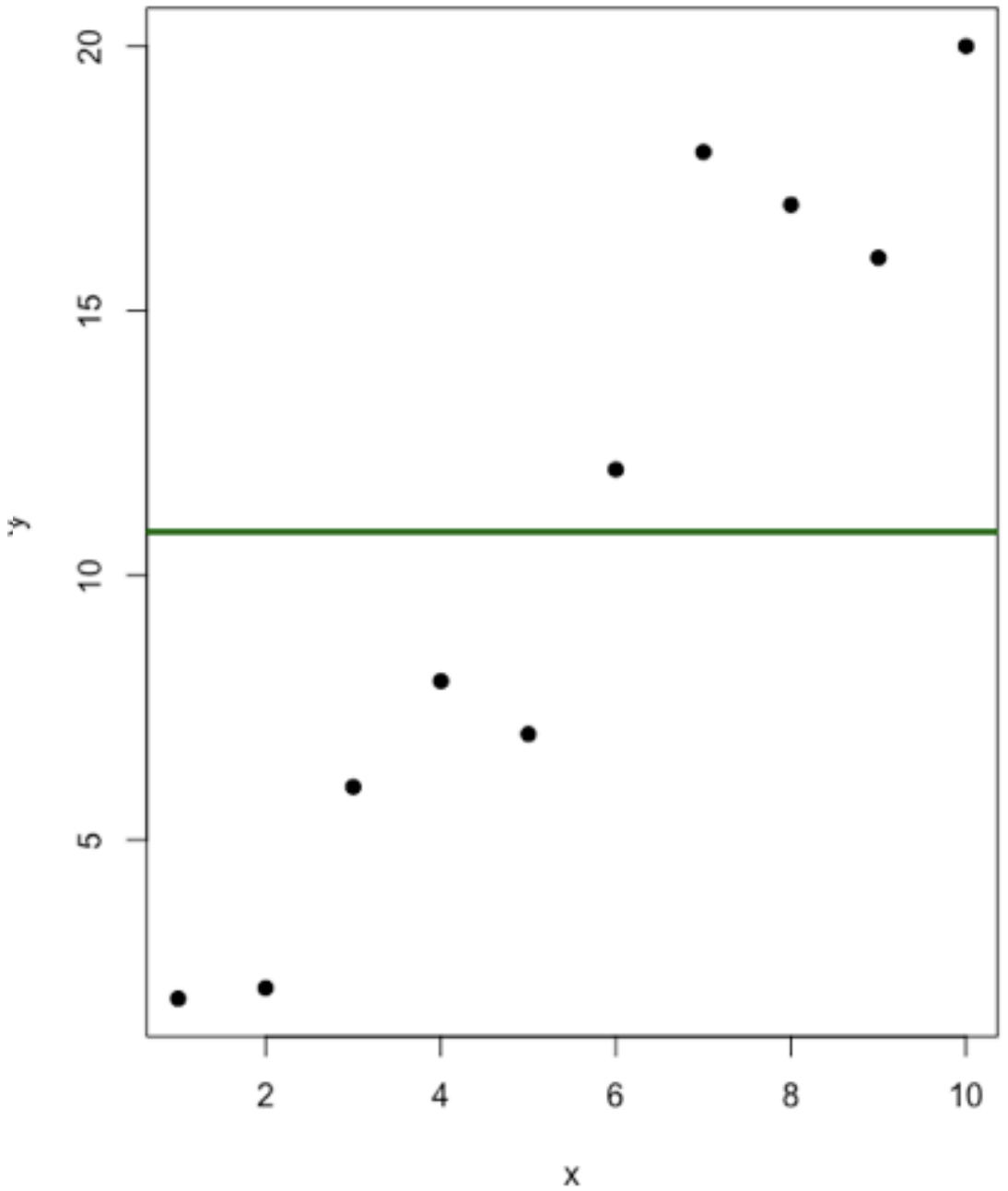

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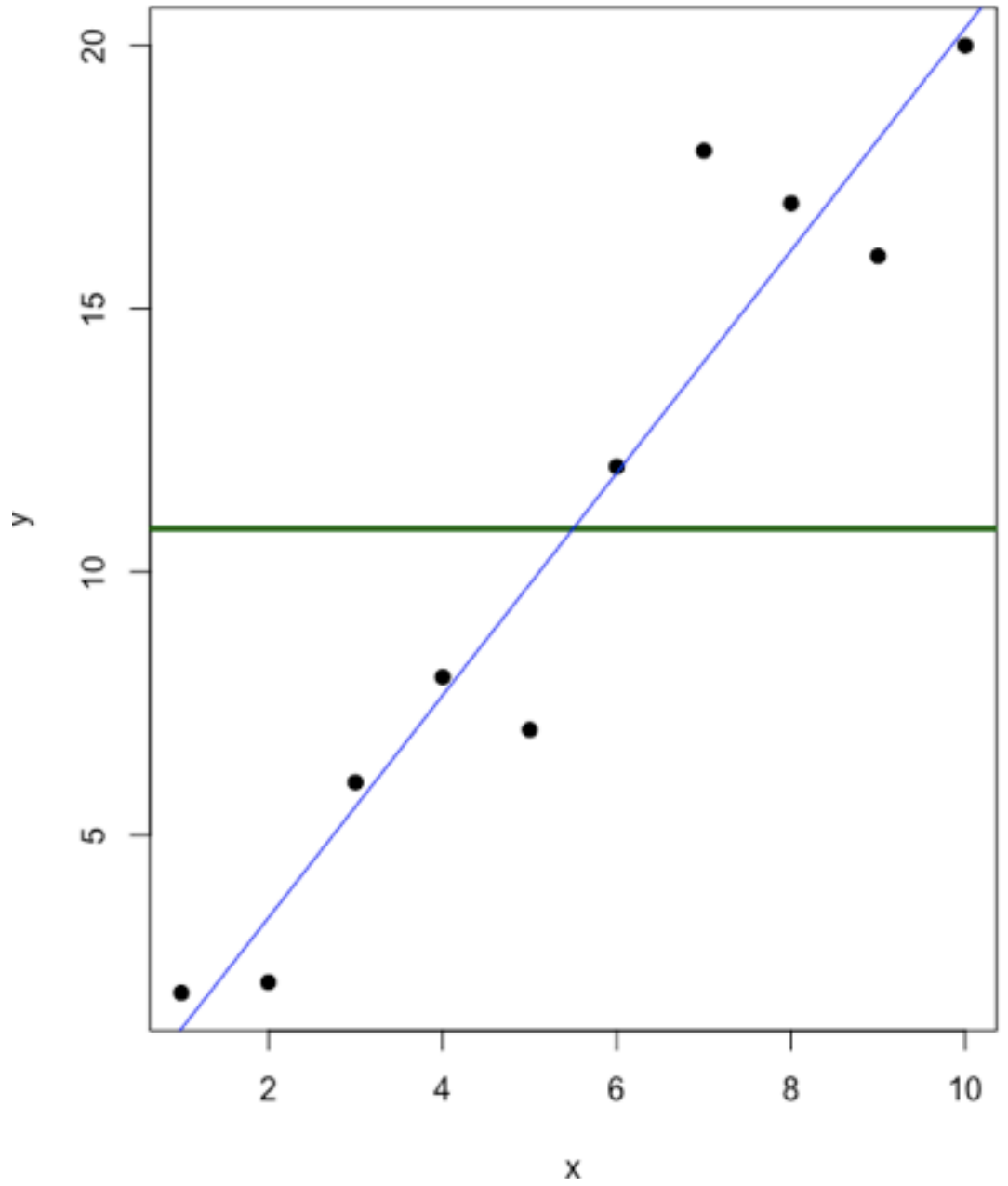

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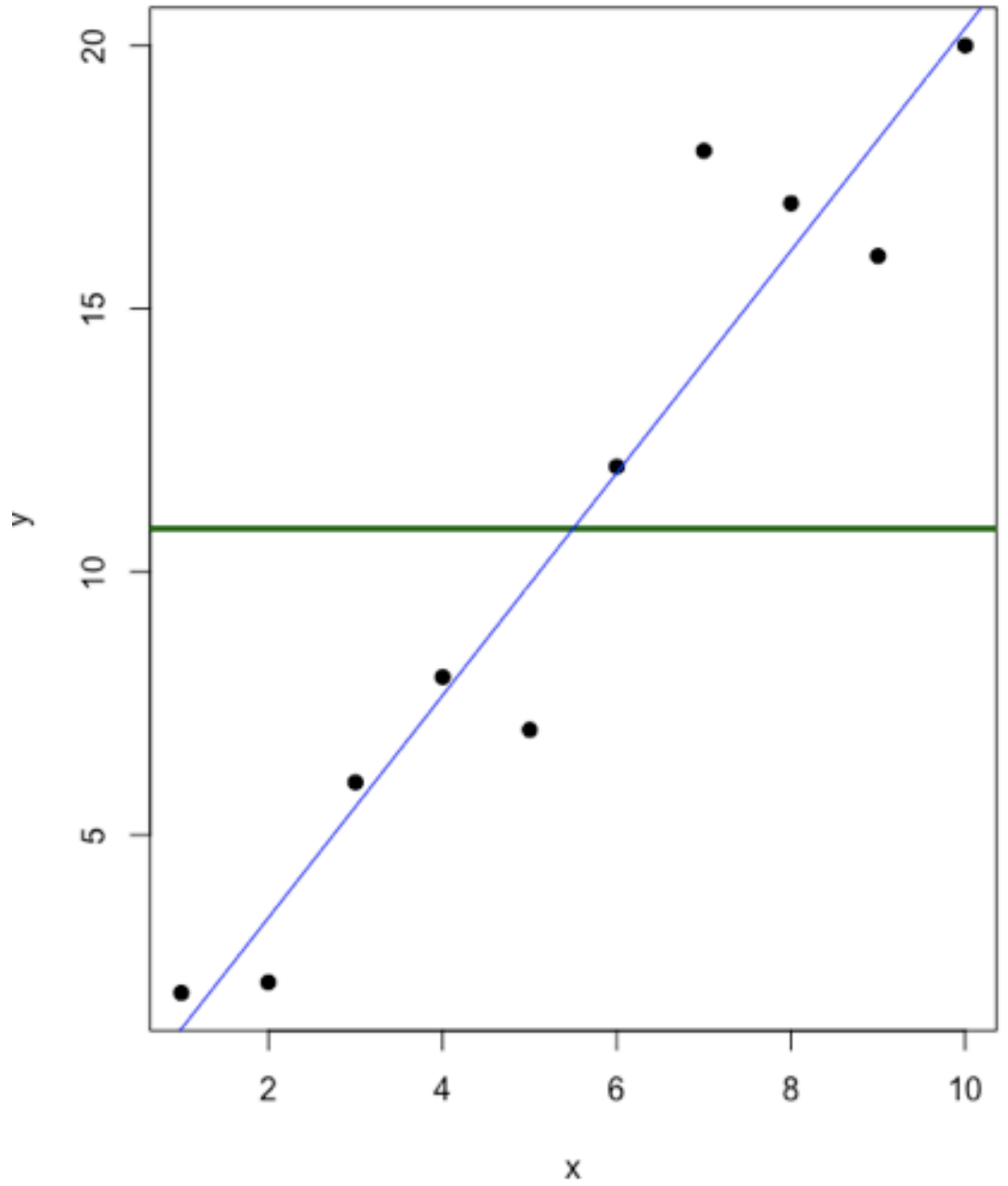
$y_i = \beta_0,\ df = n\text{-}1 = 9$

$y_i = \beta_0 + \beta_1 x_i, df = n\text{-}2 = 8$



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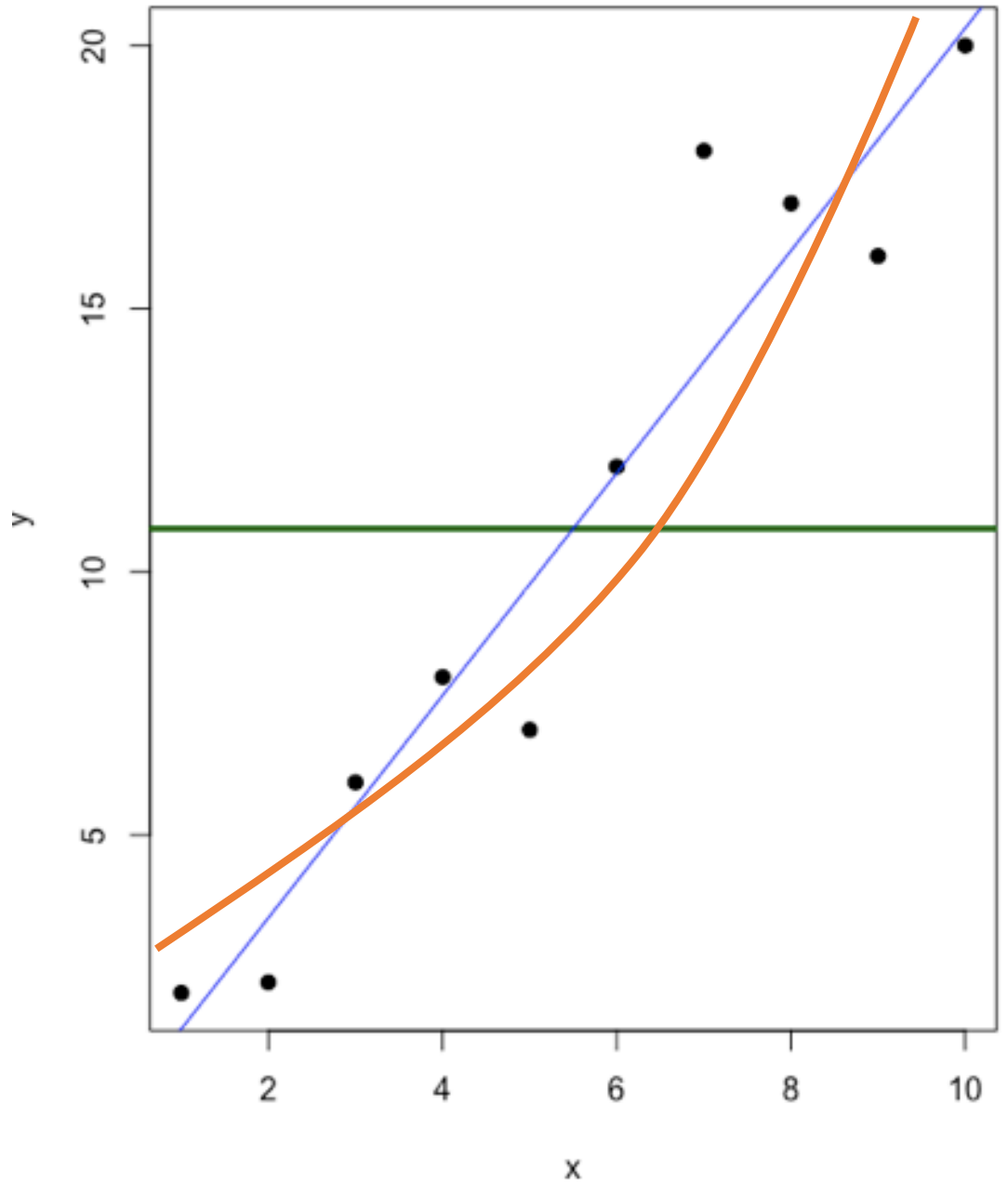

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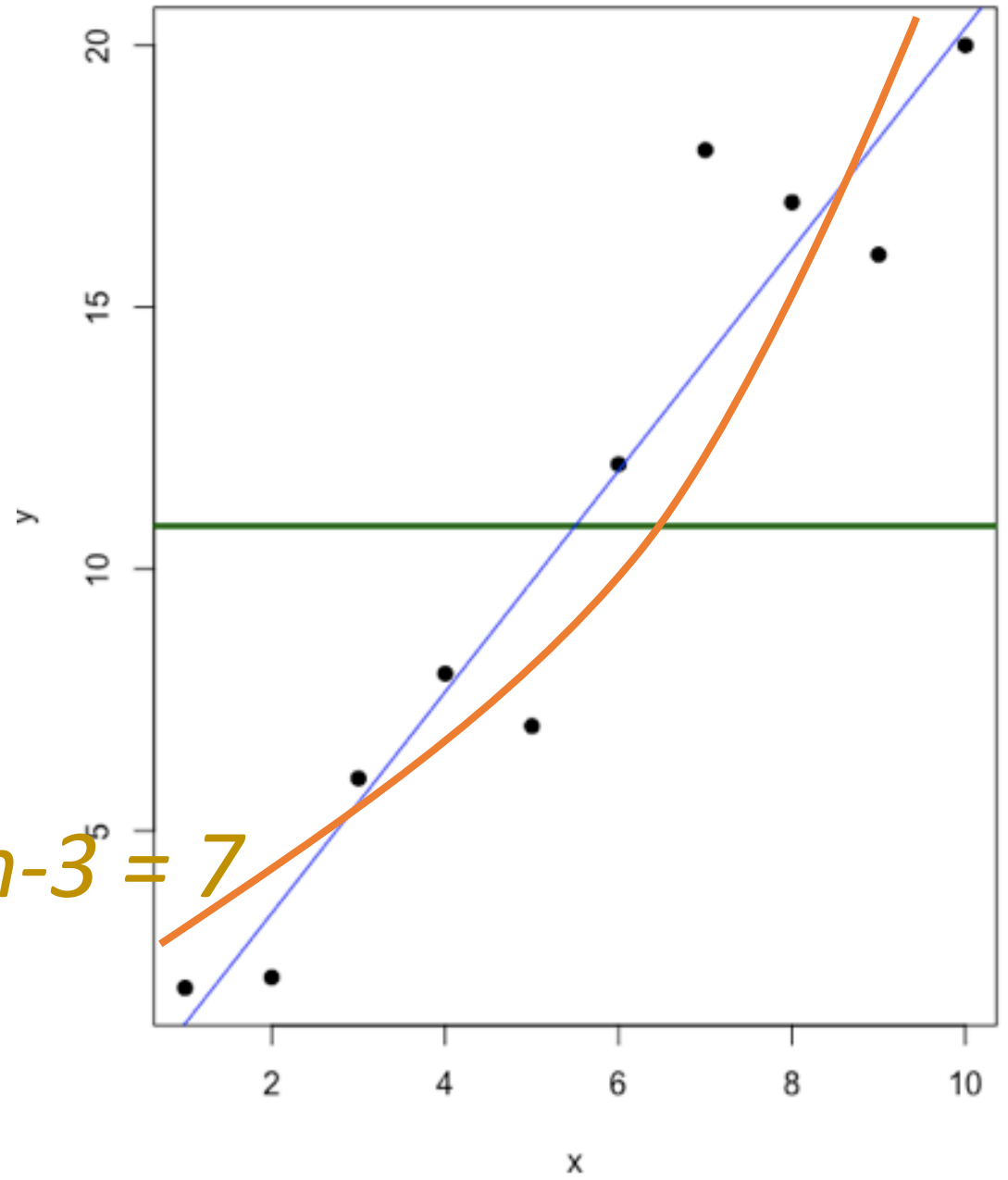

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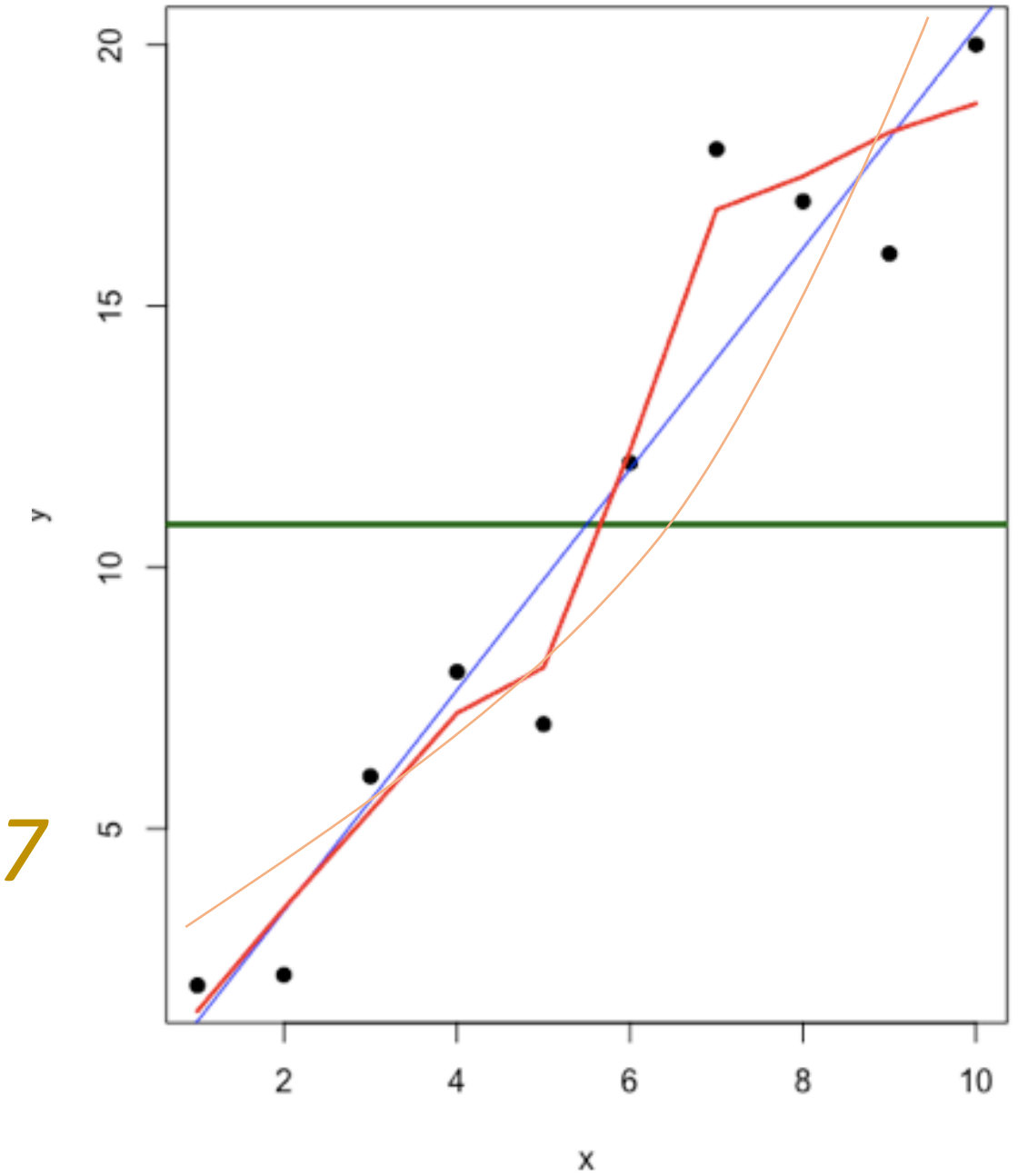
$y_i = \beta_0 + \beta_1 x_i + \beta_1 x_i^2,\ df = 7$

$y_i = LOESS\ f,\ df = n\text{-}5 = 5$



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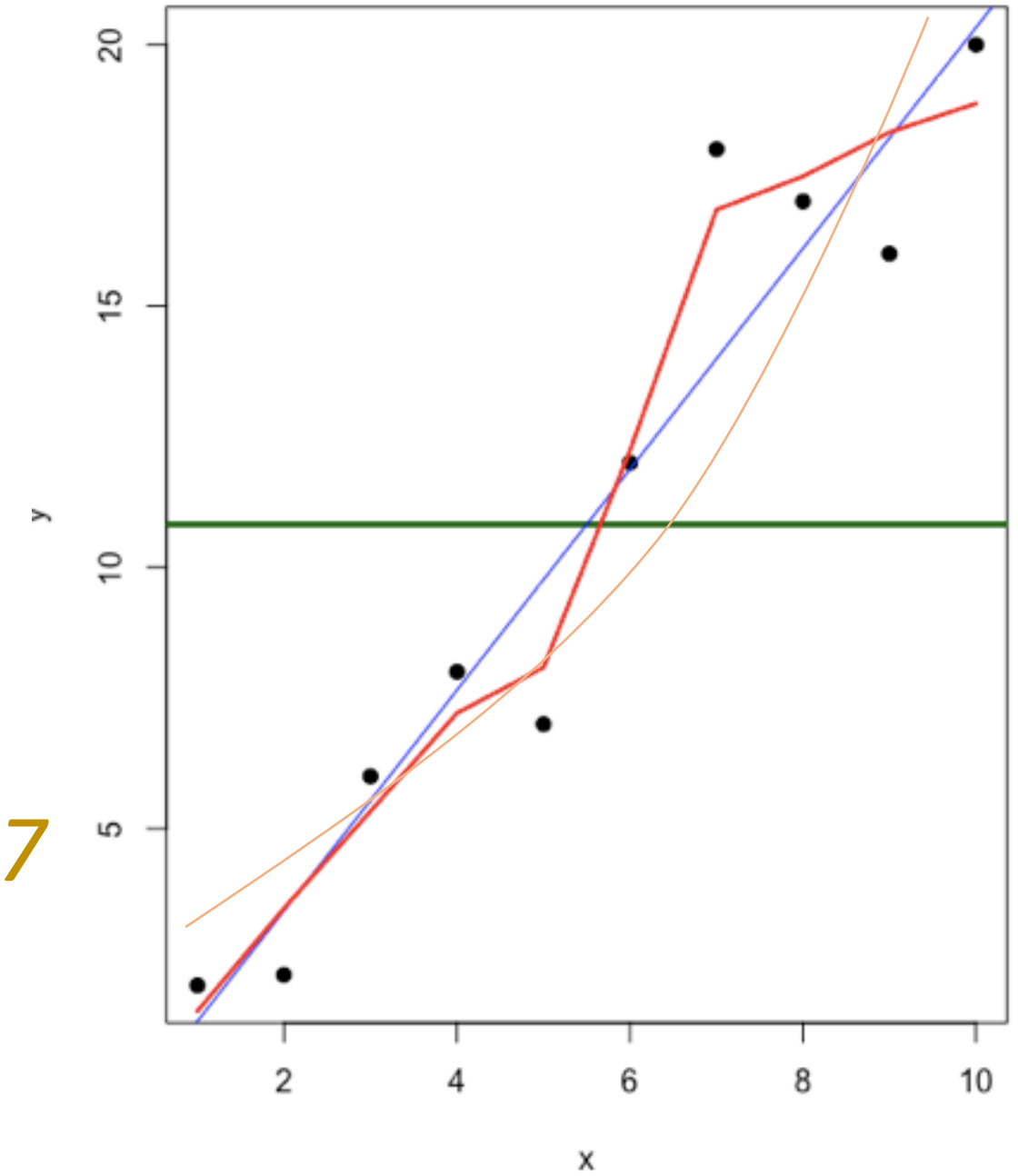
$y_i = LOESS\ f,\ df = n\text{-}5 = 5$

$y_i = y_i,\ df = n\text{-}10 = 0$



Model selection

Top – down: increasing fit

$y_i = \beta_0,\ df = n\text{-}1 = 9$

$y_i = \beta_0 + \beta_1 x_i,\ df = n\text{-}2 = 8$

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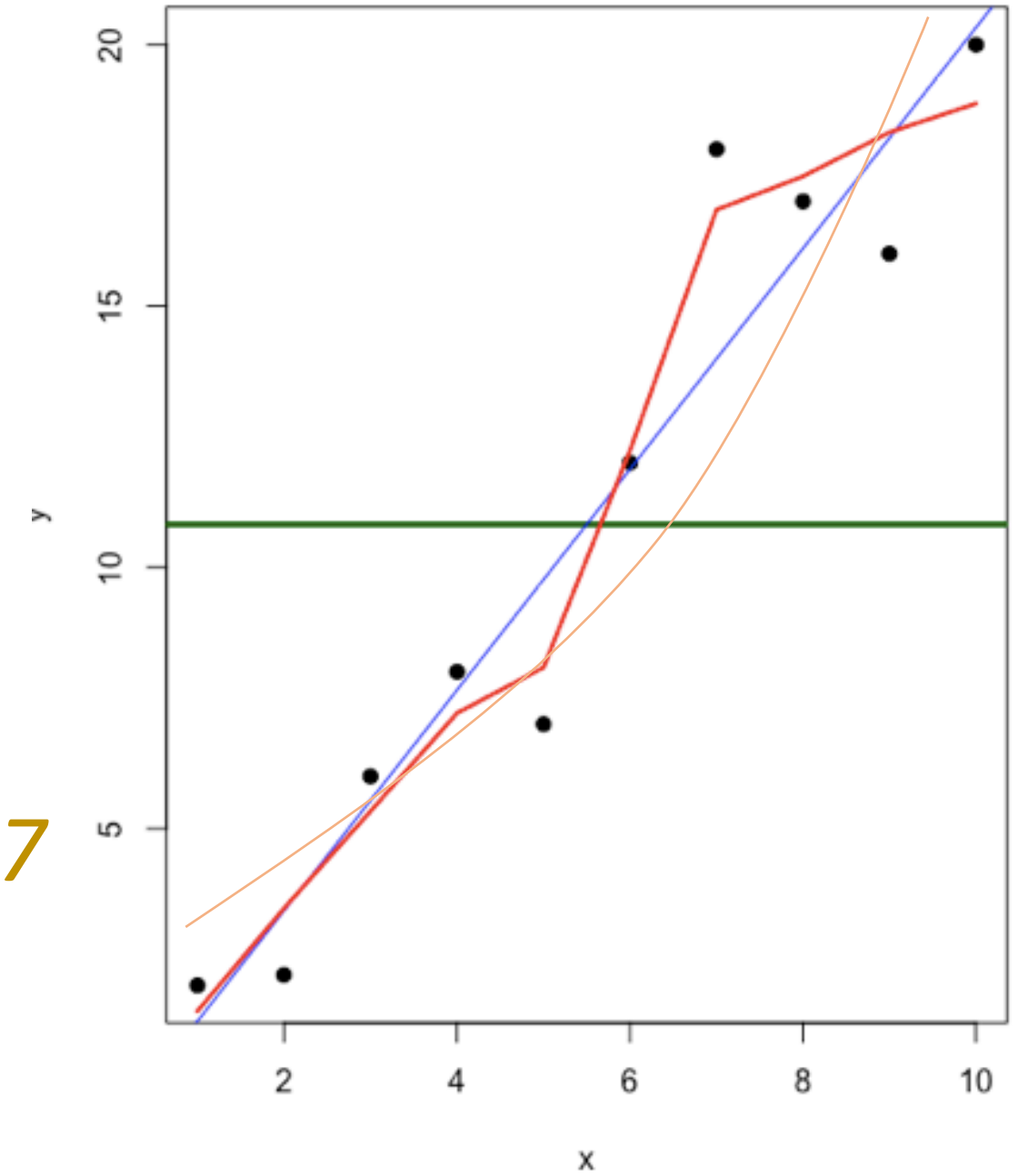

Model selection

Top – down: increasing fit

Bottom – up: loss of *df*s

$y_i = \beta_0,\ df = n\text{-}1 = 9$

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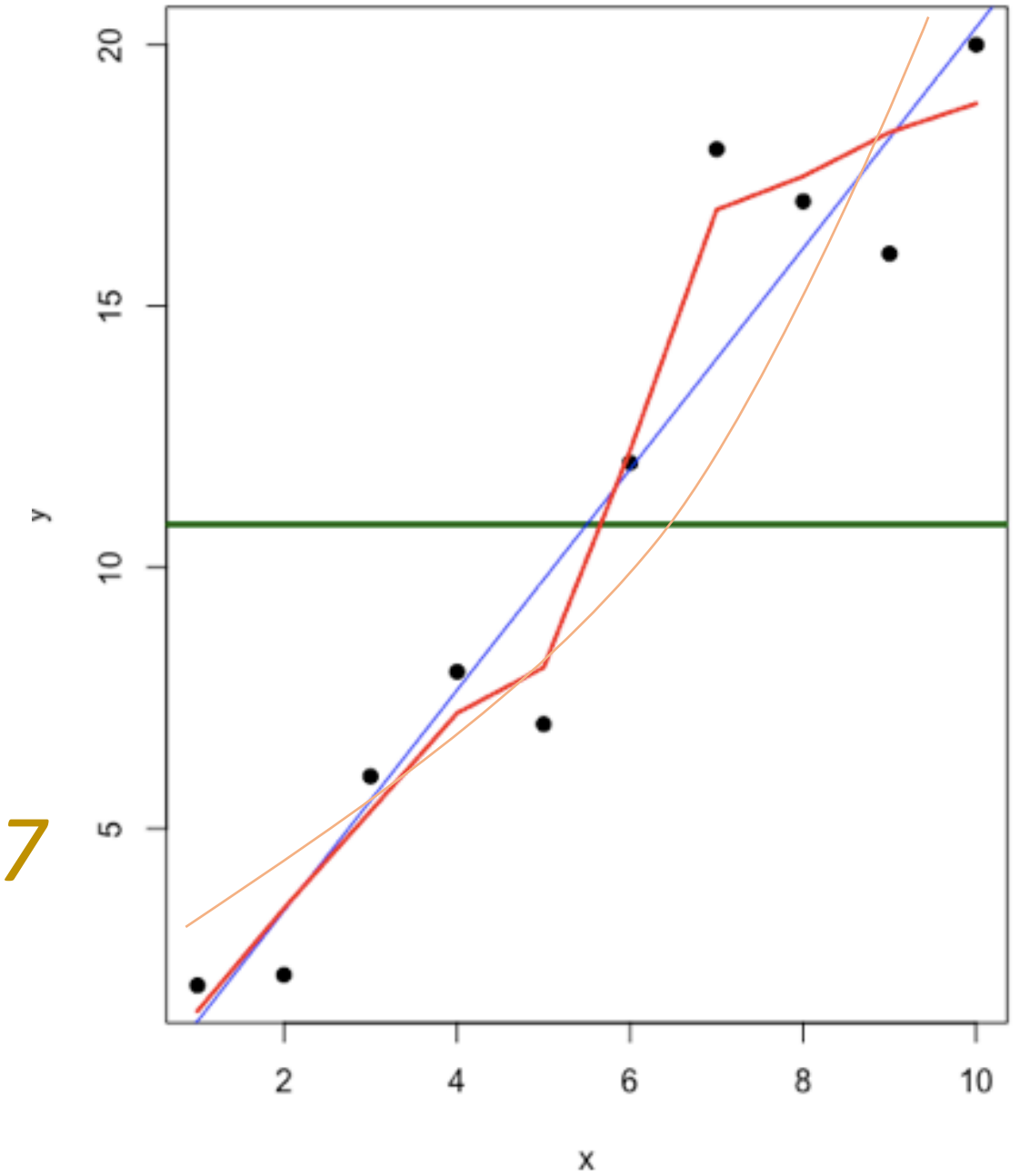

Model selection

Top – down: increasing fit

Bottom – up: loss of *df*s

Trade-off explanatory power with complexity

$y_i = \beta_0,\ df = n\text{-}1 = 9$

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$y_i = \beta_0 + \beta_1 x_i + \beta_1 x_i^2,\ df = 7$

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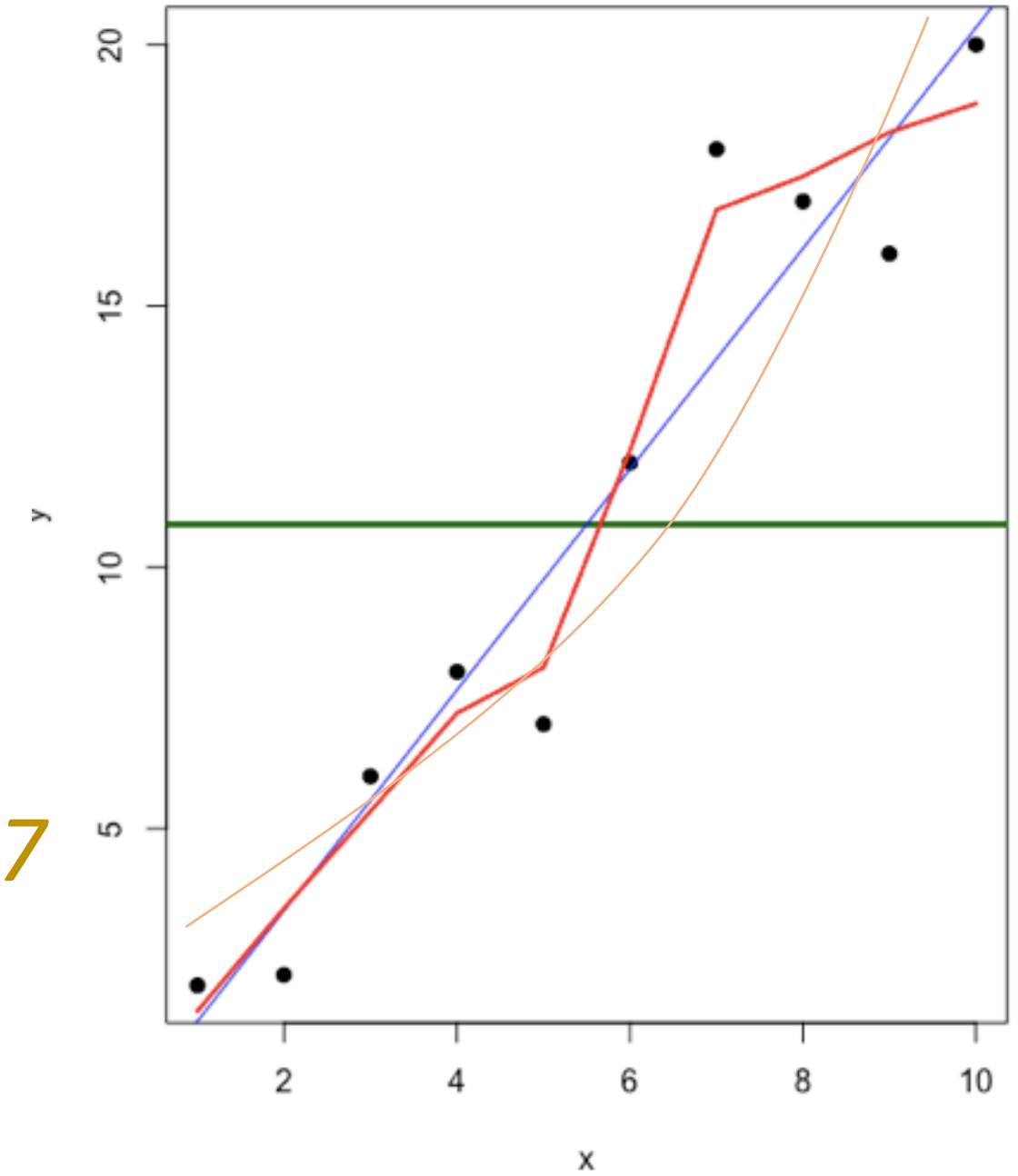

Model selection

Which model to select?

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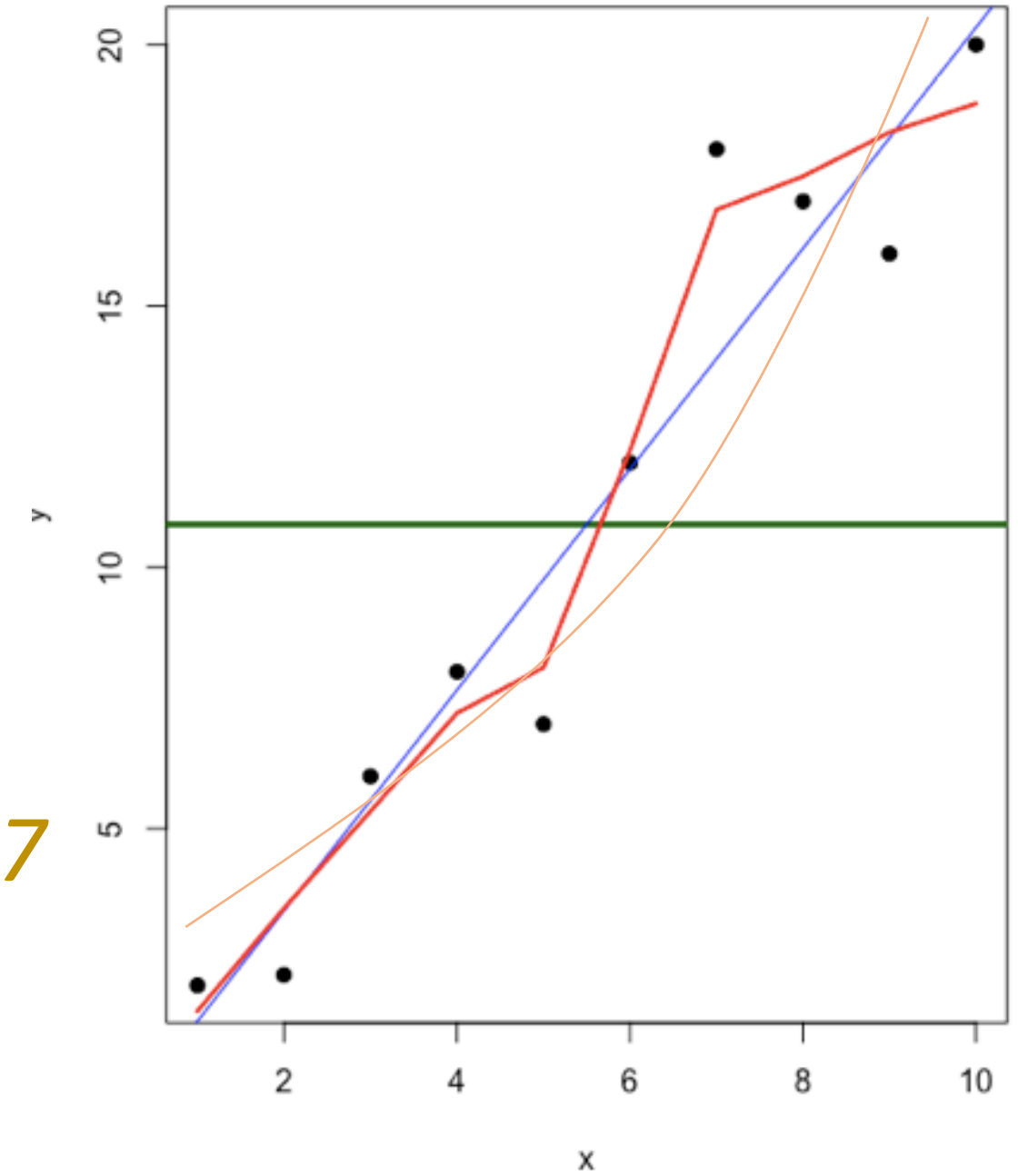

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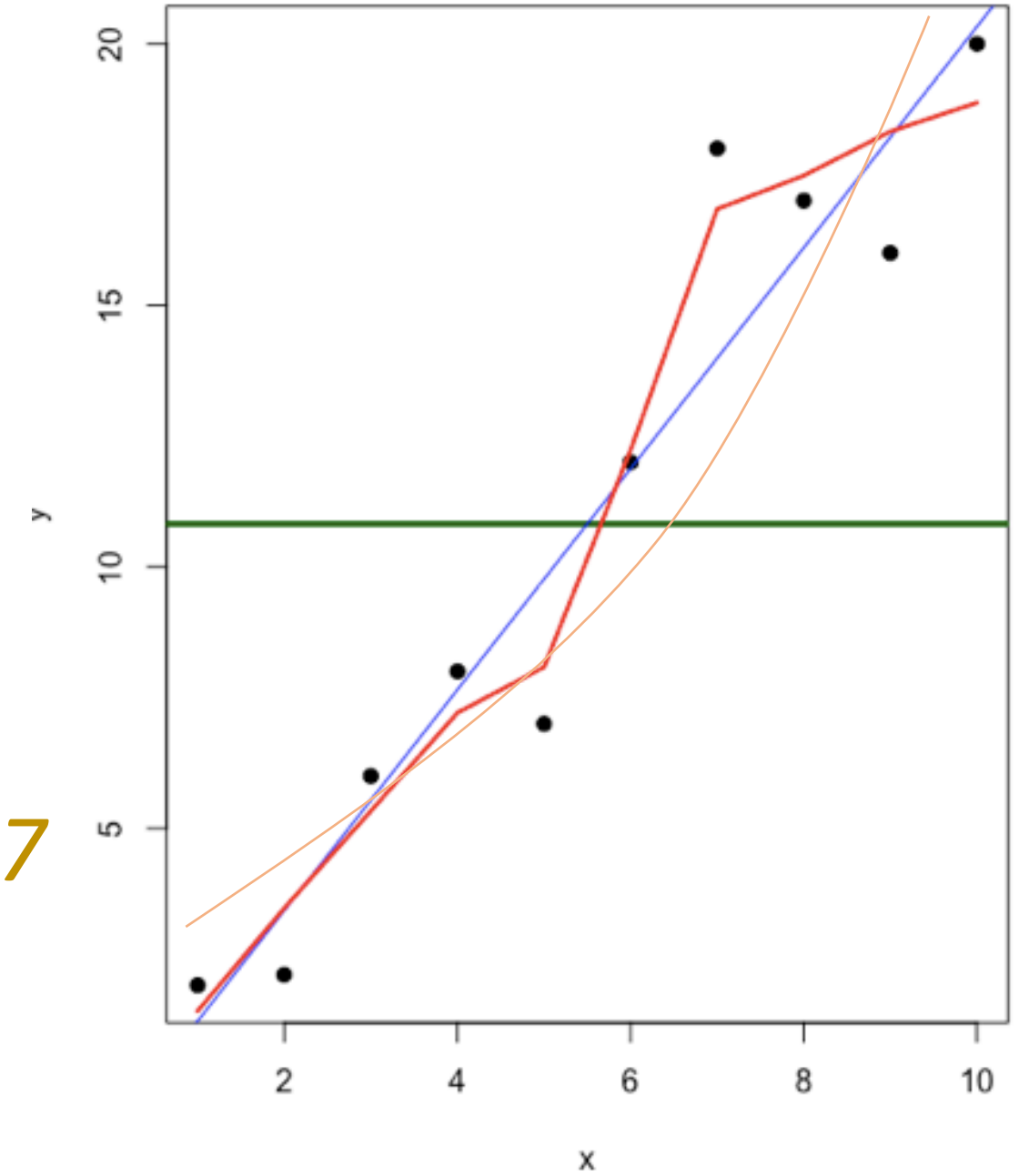

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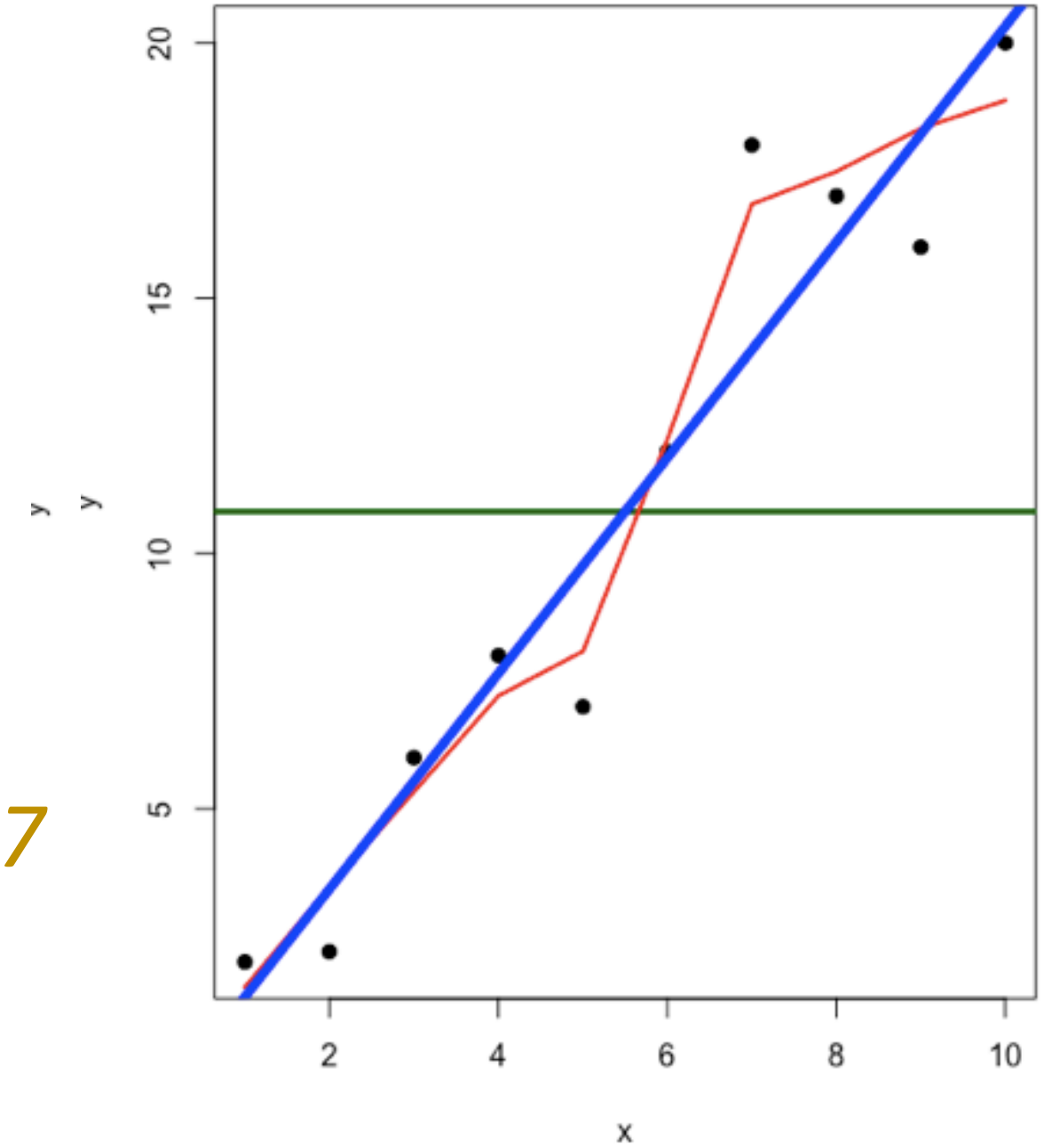

Model selection

Which model to select?

- Sparrow parental care in relationship to offspring number

Model selection

Which model to select?

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- Expect a positive slope

Model selection

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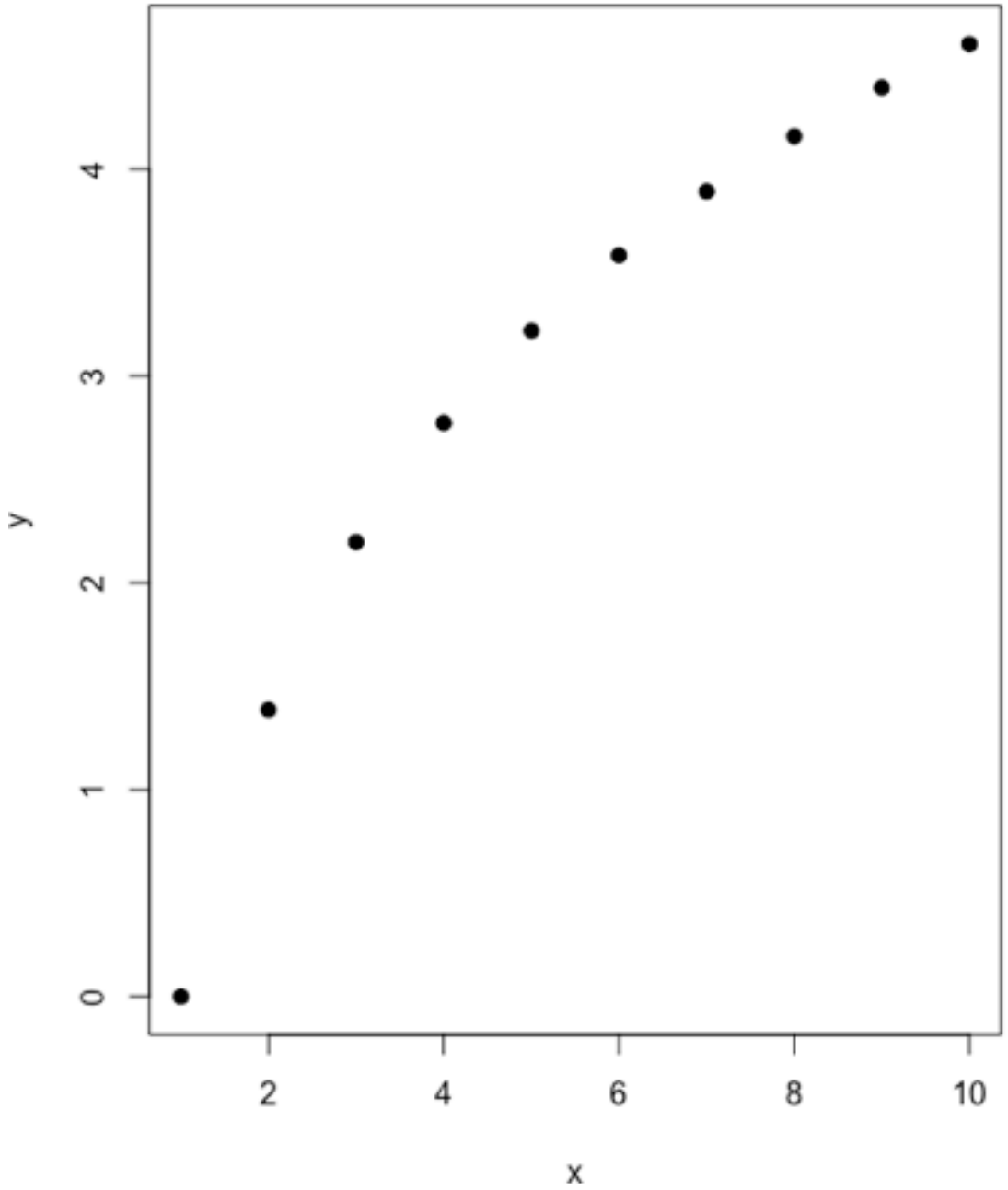

Model selection

Which model to select?

- Sparrow parental care in relationship to offspring number
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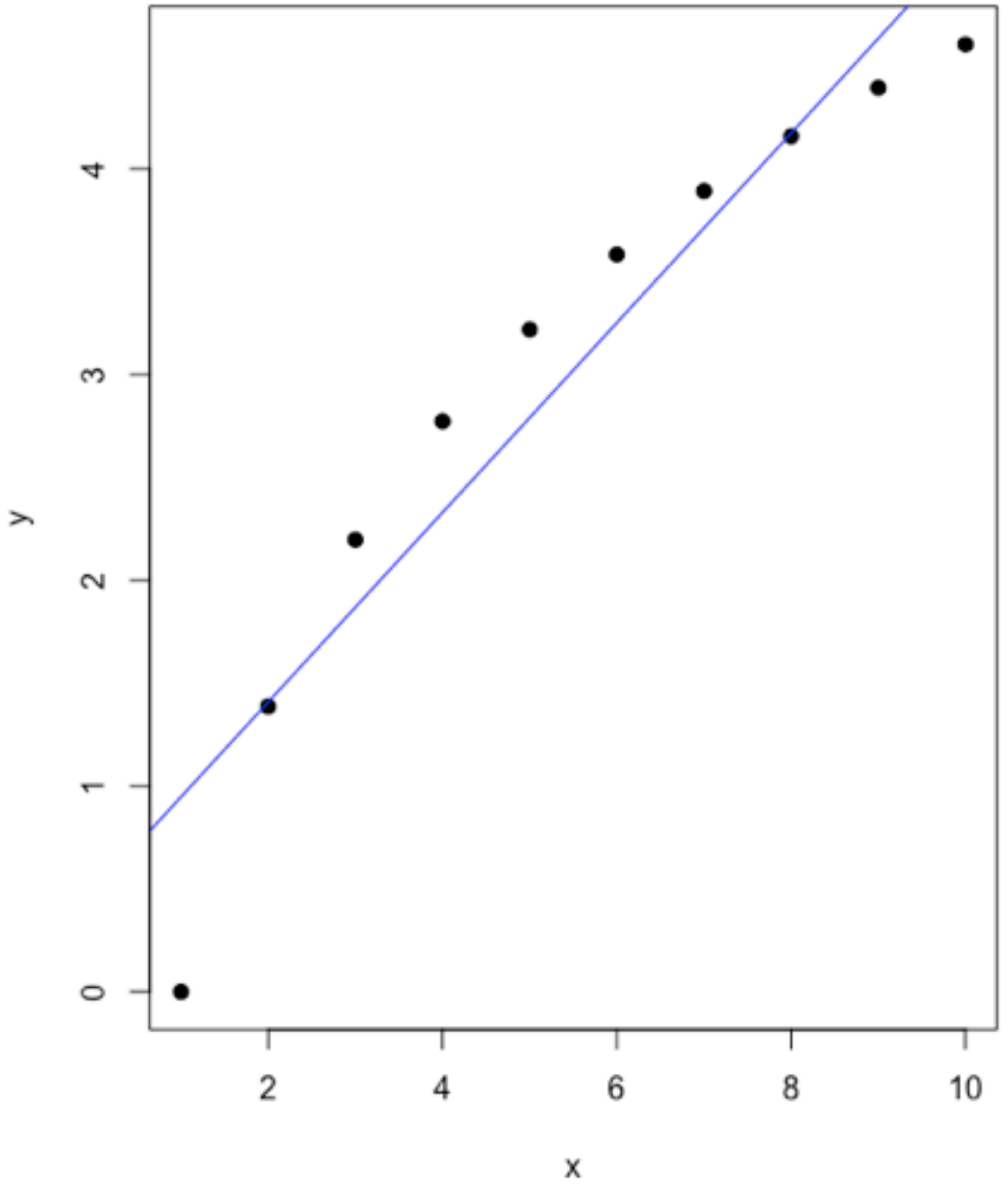
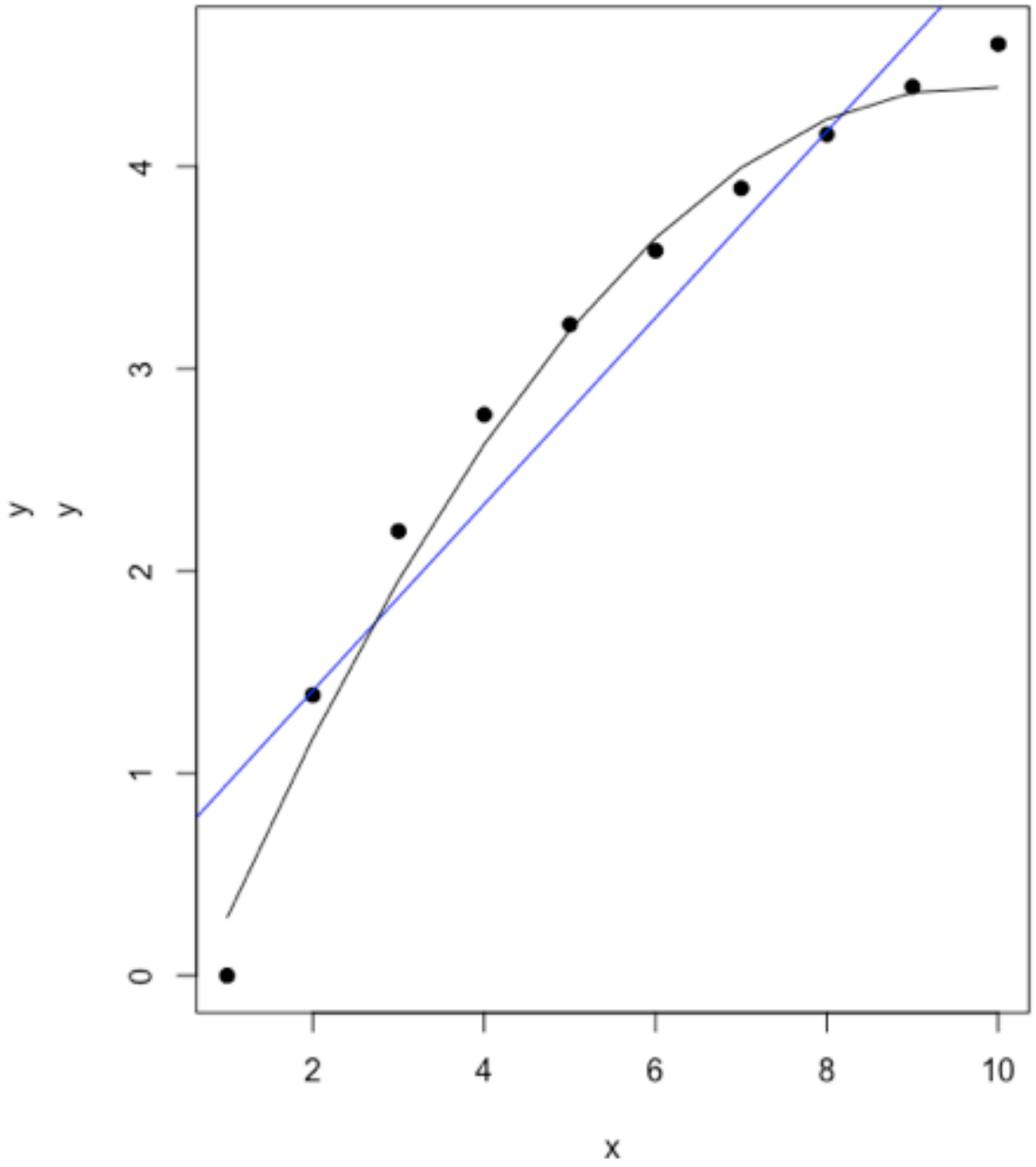


Model selection

Which model to select?

- Sparrow parental care in relationship to offspring number
- Expect a positive slope
- But diminishing

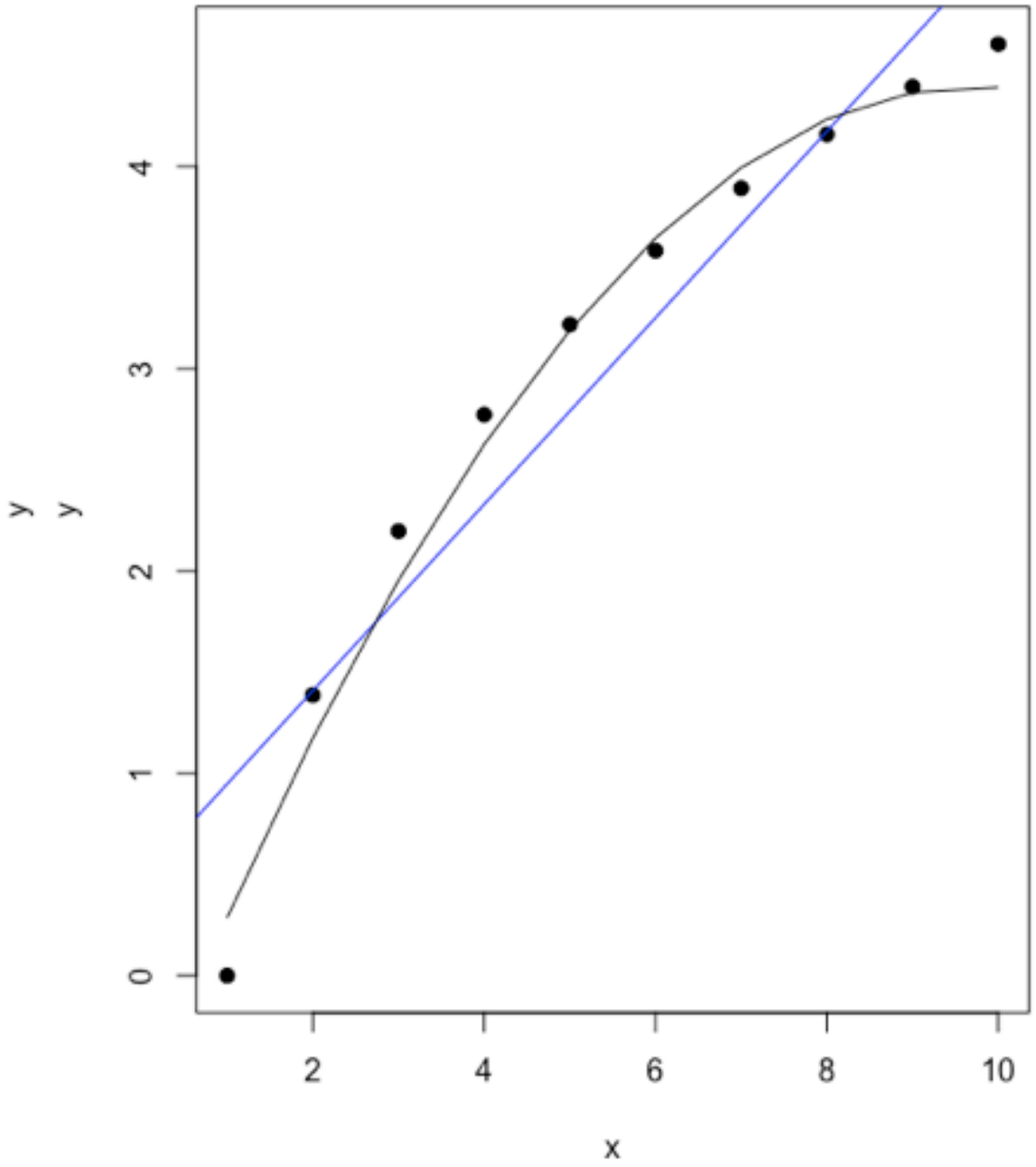


Model selection

Which model to select?

- Sparrow parental care in relationship to offspring number
- Expect a positive slope
- But diminishing

- Biology (and common sense) dictates a squared term



Model selection

- Step-wise deletion

Model selection

- Step-wise deletion
- Used to be go-to method but now not any longer suggested

Model selection

- Step-wise deletion
- Used to be go-to method but now not any longer suggested
- Make full model with all parameters and interactions

Model selection

- Step-wise deletion
- Used to be go-to method but now not any longer suggested
- Make full model with all variable and interactions
- Delete least significant variable – interactions first!

Model selection

- Step-wise deletion
- Used to be go-to method but now not any longer suggested
- Make full model with all variable and interactions
- Delete least significant variable – interactions first!
- **NEVER have interaction in but not the main effects!!!!!**

Model selection

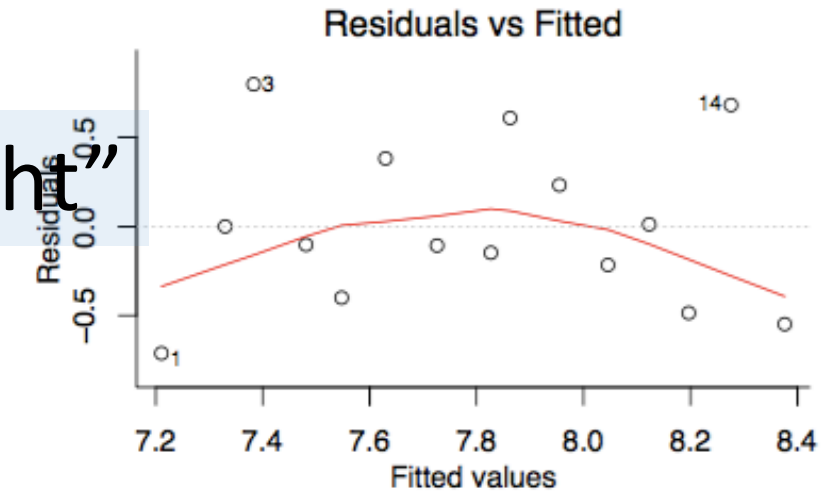
- Step-wise deletion
- Used to be go-to method but now not any longer suggested
- Make full model with all variable and interactions
- Delete least significant variable – interactions first!
- **NEVER have interaction in but not the main effects**
- Continue until all effects are significant, or needed otherwise

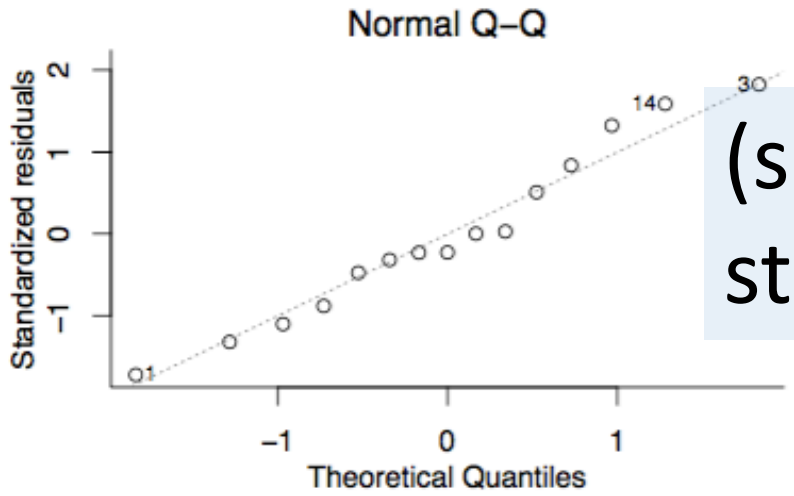
Process – common problems

1. Outliers?
2. Homogeneity of variances?
3. Normal distributed?
4. Zero-inflation?
5. Collinearity among covariates?
6. Plot data
7. Which covariates, fixed factors, and interactions?
8. Maximal model
9. Model selection
10. Make a decision
11. Model validation
12. Interpretation

Diagnostic plots

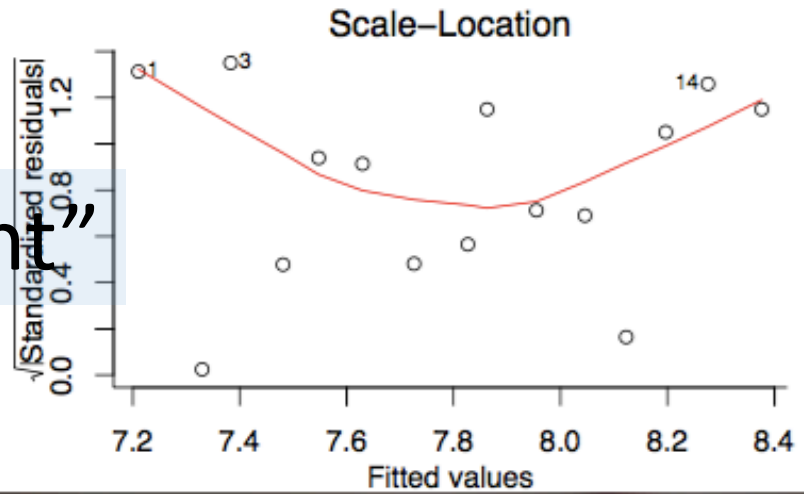
```
> mod <- lm(y ~ x, data=myData)
> plot(mod)
```

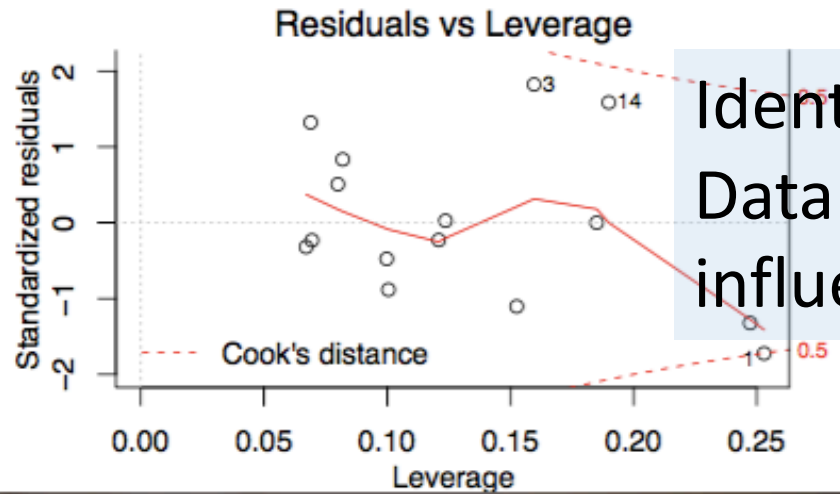




“starry night”

(somewhat) straight line





“starry night”

Identify outliers
Data with lots of influence

Process – common problems

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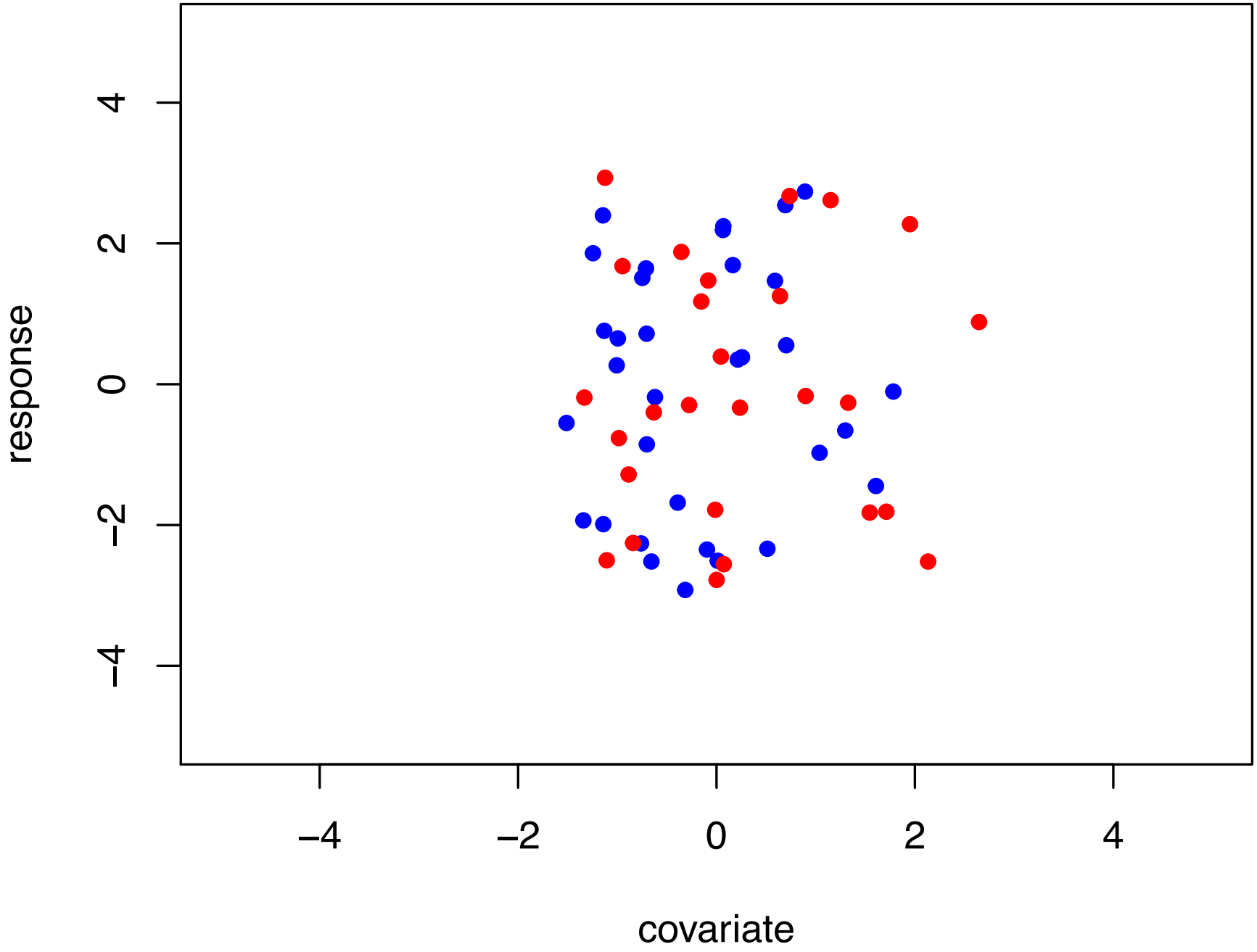
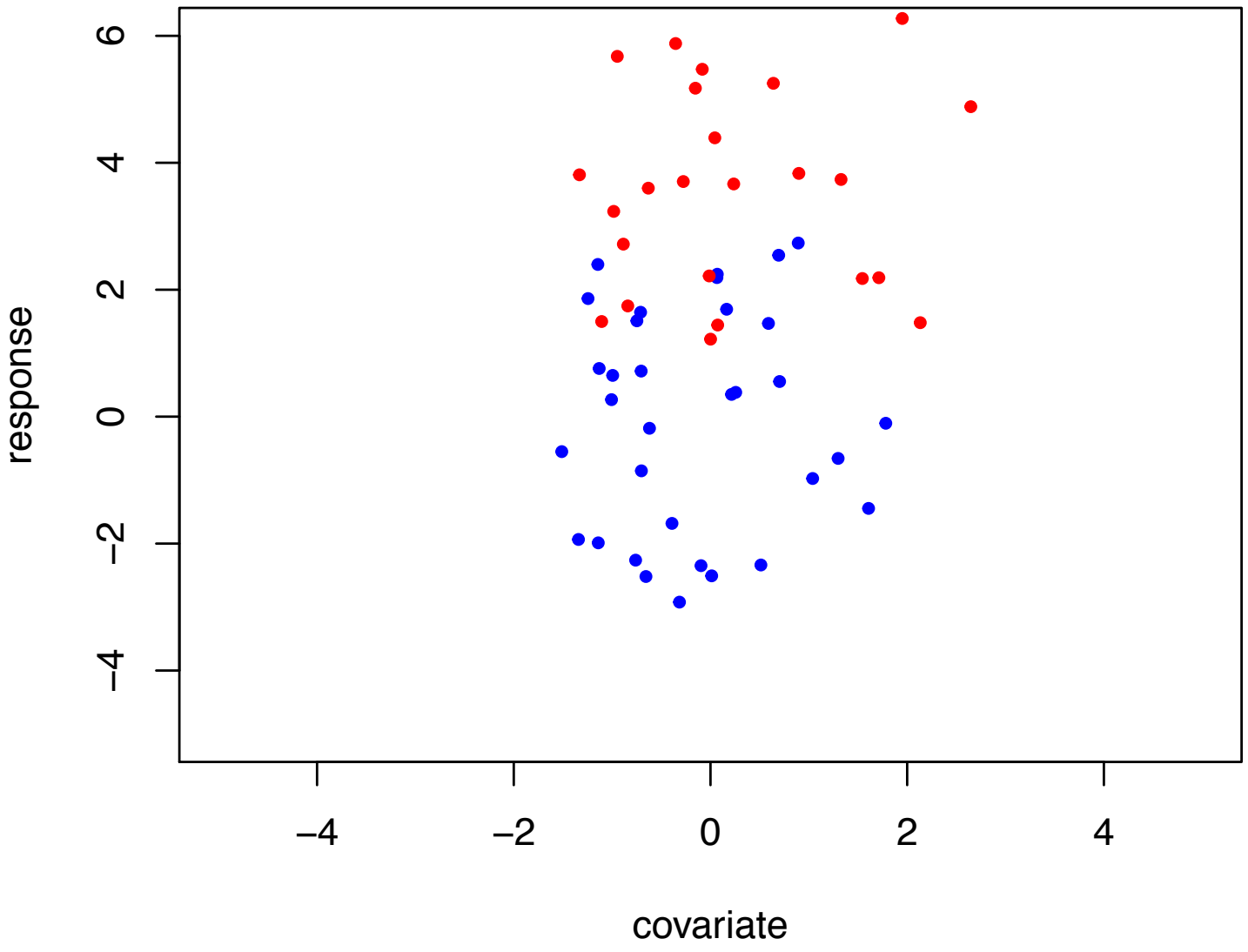
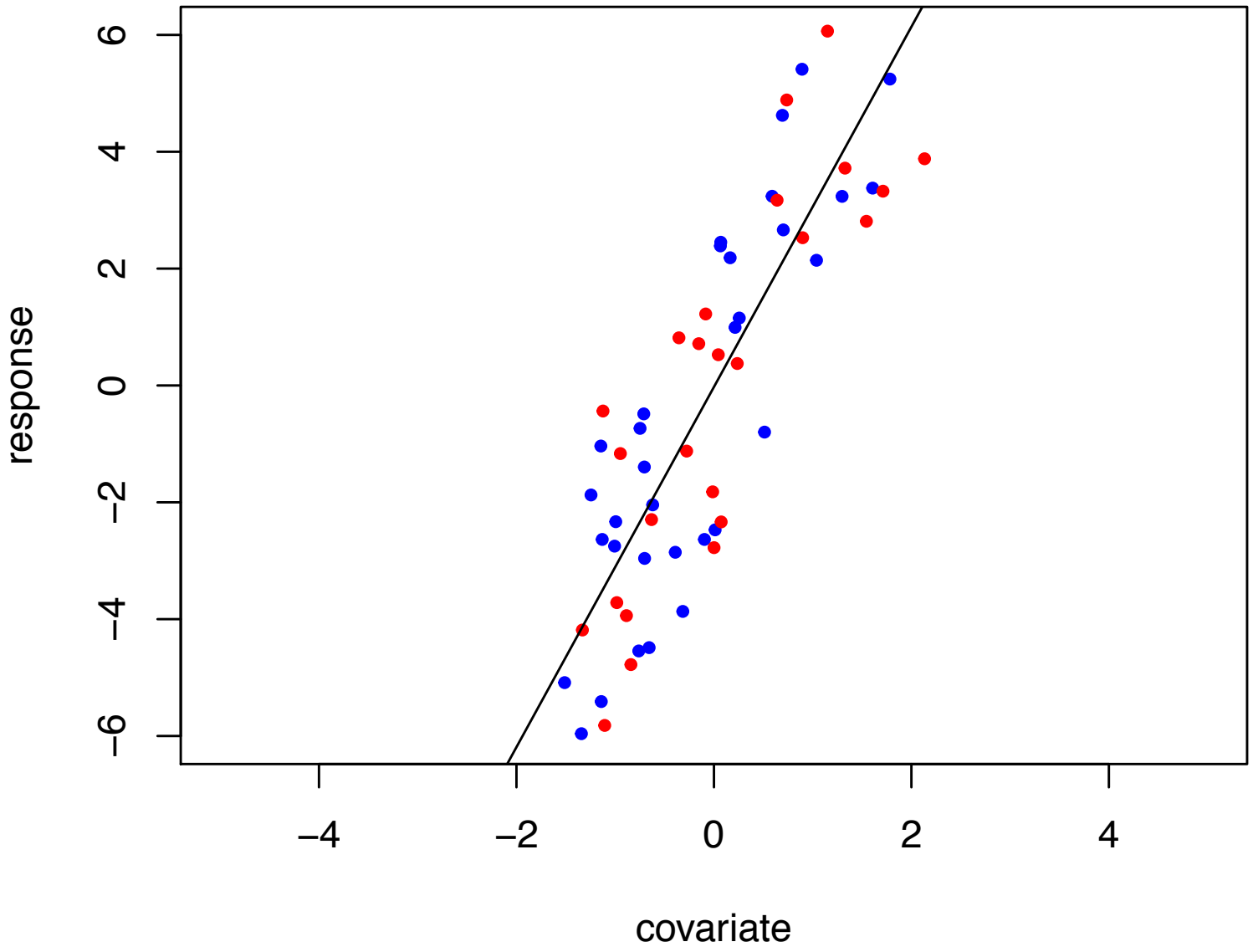

response
4
2
0
−2
−4
−4
−2
0
2
4
covariate


response
6
4
2
0
−2
−4
−4
−2
0
2
4
covariate


response
6
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2
0
−2
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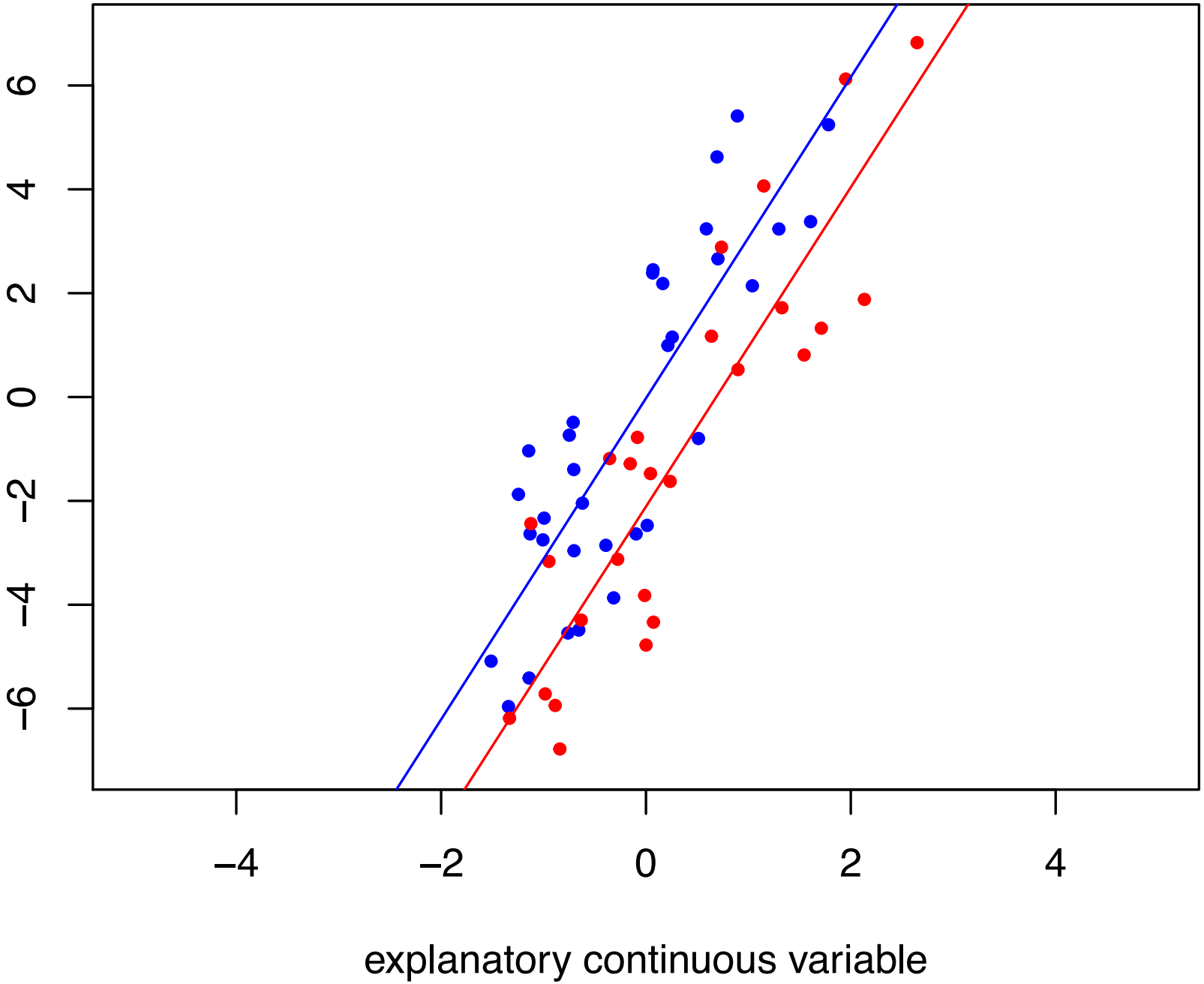
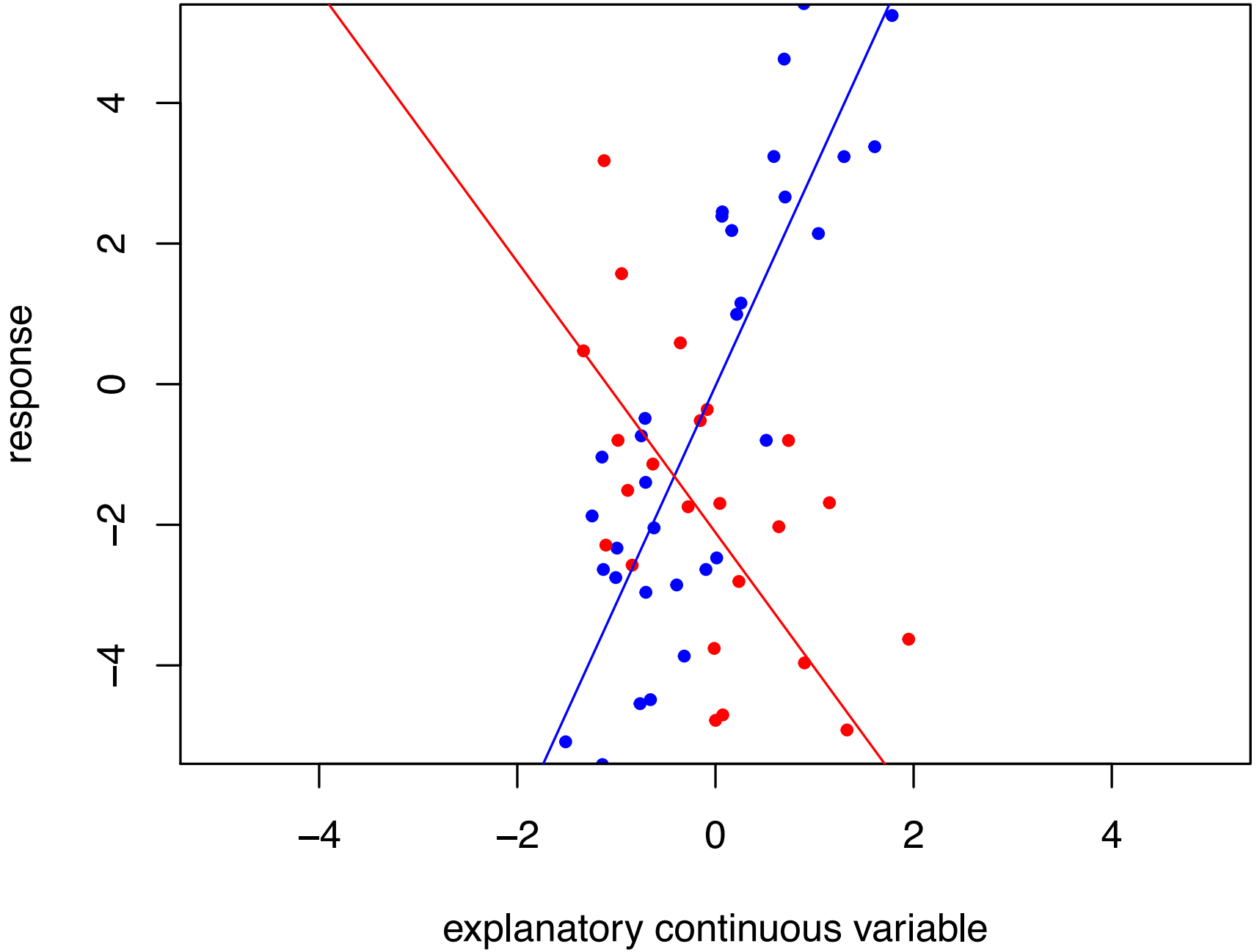
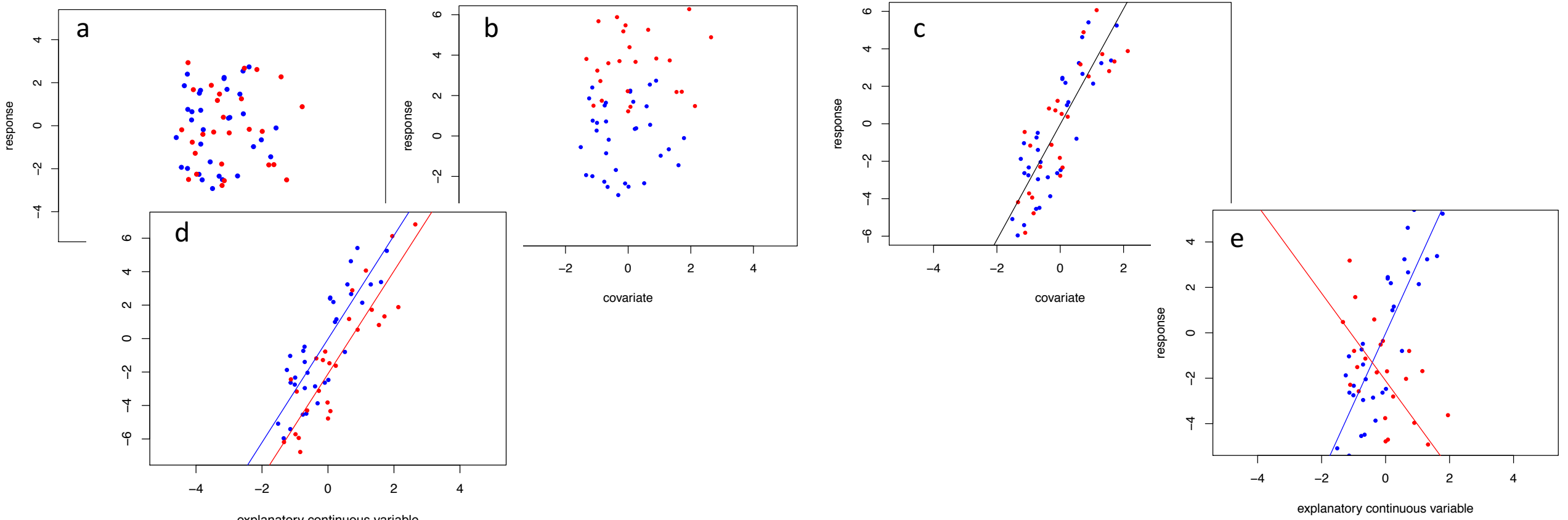
−6
−4
−2
0
2
4
covariate


6
4
2
0
−2
−4
−6
−4
−2
0
2
4
explanatory continuous variable


response
4
2
0
−2
−4
−4
−2
0
2
4
explanatory continuous variable

$$y_i = b_0 + b_1 x_{i0} + b_2 x_{i1} + b_3 x_{i0} x_{i1} + \varepsilon_i$$

plot	a	b	c	d	e
Intercept					
b_1 (sex)	0	+	0	+	+
b_2 (tarsus)	0	0	+	+	+
b_3 (tarsus x sex)	0	0	0	0	-



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- Remember units!
- Effect sizes!

Sparrows!

Unicorns 1.0!

Grasslands!

Unicorns 2.0