

### HW3

1.) A.)

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minMax(A)
j=0
initialize minarray and maxarray
for i=0 to A.length
    x=A[i]
    y=A[(i+1)%n]
    i=i+2
    if(min<max)
        minarray[j]=x
        maxarray[j]=y
        j++
    else
        minarray[j]=y
        maxarray[j]=x
        j++

min=minarray[0]
max=maxarray[0]
for i=1 to minarray.length
    if minarray[i]<min
        min=minarray[i];
    i++
for i=1 to maxarray.length
    if minarray[i]>max
        max=maxarray[i]
    i++

```

B.) In the beginning of the algorithm, comparing the values to separate the array into minarray and maxarray is  $n/2$  comparisons. Next, comparing the values within the array minarray takes  $n/2$  comparisons. Finally, comparing in the max array takes  $n/2$  comparisons. Therefore,  $n/2+n/2+n/2=3n/2$

2.)

a.) Dividing into 7 would have 4 elements greater than x. Therefore leading to

$$4 \left( \frac{1}{2} \left\lceil \frac{n}{7} \right\rceil - 2 \right) \geq \frac{2n}{7} - 8$$

Looking at step 5 has a size  $\frac{5n}{7} + 8$

The Recurrence would then be  $T(n) = T\left(\frac{n}{7}\right) + T\left(\frac{5n}{7} + 8\right) + n$  assume  $T(n) \leq cn$

$$\begin{aligned}
&\leq c \left\lceil \frac{n}{7} \right\rceil + c \left( \frac{5n}{7} + 8 \right) + an \\
&\leq \frac{cn}{7} + c + \frac{5cn}{7} + 8c + an \\
&= \frac{6cn}{7} + 9c + an \\
&= cn + \left( -\frac{cn}{7} + 9c + an \right) \\
&\leq cn \\
&= O(n) \\
-\frac{cn}{7} + 9c + an &\leq 0 \\
c \left( \frac{n}{7} - 9 \right) &\geq -an \\
\frac{c(n-63)}{7} &\geq -an \\
c &\geq \frac{7an}{n-63}
\end{aligned}$$

$n > 83$  we can find a constant that satisfy this equation

let  $n=126$

$$14a \leq c$$

Then find a constant  $a$  and  $c$  such that  $n \geq 140$  then  $T(n) = O(n)$

b) Select in groups of 3

Elements greater than  $x^2 \left( \left\lceil \frac{1}{2} \left\lceil \frac{n}{3} \right\rceil \right\rceil - 2 \right)$

$$\text{Step 5 } n - \left( \frac{n}{3} - 4 \right) = \frac{2n}{3} + 4$$

$$\text{Recurrence } T(n) \leq T \left( \left\lceil \frac{n}{3} \right\rceil \right) + T \left( \frac{2n}{3} + 4 \right) + n$$

$$\begin{aligned}
T(n) &> c \left\lceil \frac{n}{3} \right\rceil + c \left( \frac{2n}{3} + 2 \right) + an \\
&> \frac{cn}{3} + c + \frac{2cn}{3} + 2c + an \\
&= cn + 3c + an \\
&> cn \\
\Omega(n) \quad c > 0
\end{aligned}$$

2.) A.)

QUICKSORT(A,p,r)

1. If (p<r)
2.     n=h-p+1 //size of subarray
3.     m=med(A,p,r,n/2) //median of array
4.     q=partition(A,p,r,med)
5.     QUICKSORT(A,p,q-1)
6.     QUICKSORT(A,q+1,r)

b.)  $T(n) = 2T(n/2) + O(n)$

c) Calling the partition in quicksort takes in the median of the input array. This is the pivot element. The worst-case of SELECT is  $O(n)$ . Partition is now split into two which guarantees best case partitioning. Thus giving us the recurrence shown at (b)