CSE 310 Hw1 Joshua Godinez

2.)

| COST | # of times | |
|---|---|---|
| $C_1$ | n | |
| $C_2$ | n-1 | |
| $C_3$ | $\frac{n(n-1)}{2}-1$ | (arithmetic series) |
| $C_4$ | $\frac{n(n-1)}{2}-1$ | Inside second for loop |
| $C_5$ | $\frac{n(n-1)}{2}-1$ | |
| $C_6$ | $\frac{n(n-1)}{2}-1$ | |
| $C_7$ | $\frac{n(n-1)}{2}-1$ | |
| $C_8$ | $\frac{n(n-1)}{2}-1$ | |
| $C_9$ | $\frac{n(n-1)}{2}-1$ | |

T(n)=O($n^2$) Worst Case
T(n)=O(n) Best Case (inner for loop will not run)

3.)
a. $f(n) = 0.02n^2 + 20n \in f(n) = \Theta(n^2)$

$$f(n) \geq c * g(n) \geq 0$$
$$0.02n^2 + 20n \geq c * n^2 \text{ (divide both sides by } n^2\text{)}$$
$$.02 + \frac{20}{n} \geq c \quad \text{(let n=20)}$$
$$1.02 \geq c$$
let c=1 and n=20
$$f(n) = \Theta(n^2)$$

b. $f(n) = \Theta(n^2)$ and $g(n) = 0(n^2) \in f(n) + g(n) = \theta(n^2)$

$$f(n) \text{ in this case means } c_1 * n^2 \leq f(n) \leq c_2 * n^2$$
$$g(n) \text{in this case means } g(n) \leq c_3 * n^2$$
$$c_1 * n^2 \leq f(n) + g(n) \leq c_2 * n^2 + c_3 * n^2$$
$$c_1 * n^2 \leq f(n) + g(n) \leq (c_2 + c_3) * n^2 \quad \text{(let } c_{1,}c_{2,}c_{3,} = 1 \text{ and } n = 1\text{)}$$
$$f(n) + g(n) = \theta(n^2)$$

c. $f(n) = 6n^2 + 7n + 5 => O(n^2)$

$$0 \leq f(n) \leq c * g(n)$$
$$6n^2 + 7n + 5 \leq c * n^2$$

$$6 + \frac{7}{n} + \frac{5}{n^2} \leq c \quad \text{(Let n=1)}$$

$$18 \leq c$$

let n=1 and c=19

f(n)=O($n^2$)

$f(n) = 6n\text{^}2 + 7n + 5 => \Omega(n^2)$

$$0 \leq c * g(n) \leq f(n)$$

$$c * n^2 \leq 6n^2 + 7n + 5$$

$$c \leq 6 + \frac{7}{n} + \frac{5}{n^2} \quad \text{(let n=1)}$$

$$c \leq 18$$

let n=1 and c=17

$$f(n) = \Omega(n^2)$$

$f(n) = 6n\text{^}2 + 7n + 5 => \theta(n\text{^}2)$

$$c_1 * g(n) \leq f(n) \leq c_2 * g(n)$$

LHS: $c_1 * n^2 \leq 6n^2 + 7n + 5$

$$c_1 \leq 6 + \frac{7}{n} + \frac{5}{n^2} \quad \text{(let n=1)}$$

$$c \leq 18$$

$$let\ c_1 and\ c_2 = 18\ and\ n = 1$$

f(n) = $\theta(n\text{^}2)$

d.

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

Base case: $1^2 + 2^2 + 3^2 + \ldots. + n^2 = \frac{n(n+1)(2n+1)}{6}$

Let n=1

$$1^2 = \frac{1(1+1)(2+1)}{6} => \frac{6}{6} => 1$$

n=k

Induction:

Assume n=k

$$1^2 + 2^2 + 3^2 + \cdots + k^2 = \frac{k(k+1)(2k+1)}{6}$$

therefore n=k+1 must be true

LHS: $(1^2 + 2^2 + 3^2 + \ldots\ldots + k^2) + (k+1)^2$

$$RHS: \quad = \frac{k(k+1)(2k+1)}{6} + (k+1)^2$$

$$= \frac{k(k+1)(2k+1) + 6(k+1)^2}{6}$$

$$= \frac{k(k+1)(2k^2+7k+6)}{6}$$

$$= \frac{(k+1)(k+2)(2k+3)}{6}$$

Therefore the LHS=RHS

By induction, $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$ for n>0

4.) Power(x, n){
if(n==0)
return 1
int half=Power(x, n/2)
if(n%2==0)
return half*half
else
return x*half*half
}
O(logn)- Recursion called once. Using n/2 at each iteration step

5.) Closest(p)

distance=0; O(1)
n<- p.length n
for i=1 to n n
for j=i+1 to n $n^2$
if GET_DISTANCE(p(i),p(j))<distance
distance=Get_Distance(p(i),p(j))

int s=p(i)
int d=p(j)

$O(n^2)$