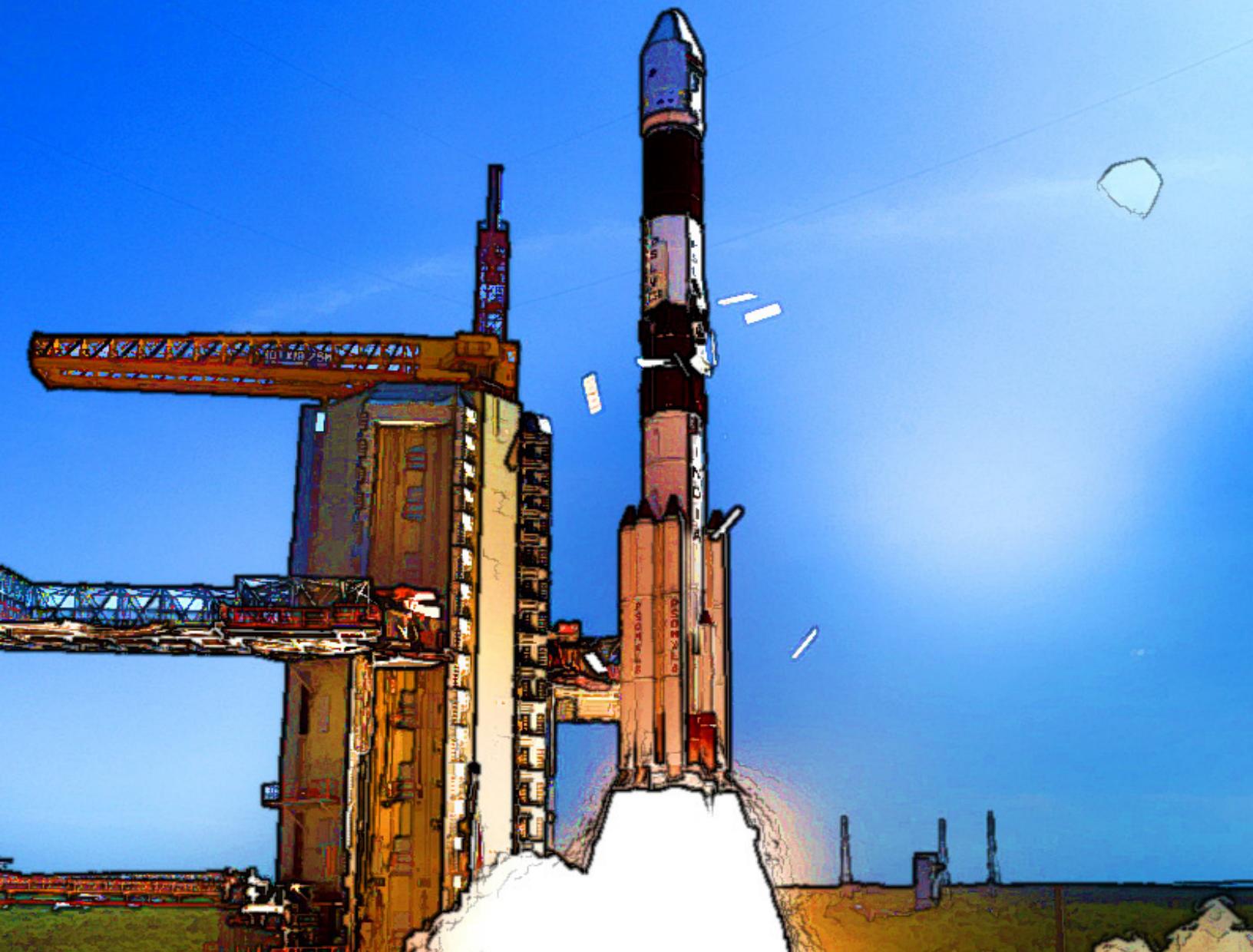


# Cryogenic Engineering: Software Solutions: Part-IV

Cryogenic Instrumentation: Theory + Problems  
(Mathcad + EES)

Dr. M. Thirumaleshwar



**DR. M. THIRUMALESHWAR**

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**CRYOGENIC  
ENGINEERING:  
SOFTWARE SOLUTIONS:  
PART-IV  
CRYOGENIC  
INSTRUMENTATION: THEORY +  
PROBLEMS (MATHCAD + EES)**

Cryogenic Engineering: Software Solutions: Part-IV:  
Cryogenic Instrumentation: Theory + Problems (Mathcad + EES)  
1<sup>st</sup> edition  
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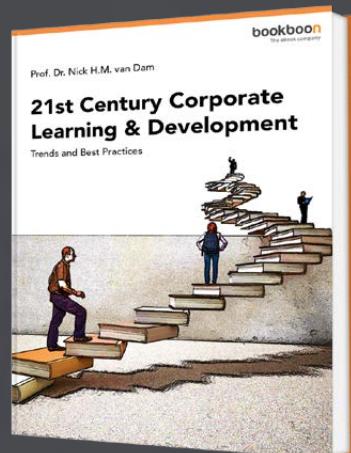
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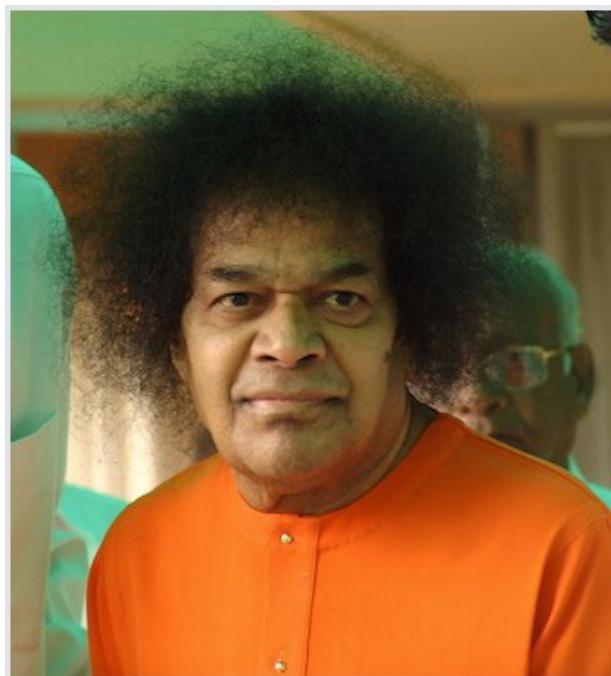
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# DEDICATION

**This work is lovingly dedicated at the lotus feet of**

**Bhagavan Sri Sathya Sai Baba**



“There is only one religion, the religion of Love.  
There is only one caste, the caste of Humanity.  
There is only one language, the language of the Heart.  
There is only one God, He is Omnipresent.”

**“Help Ever, Hurt Never!”**

*...Bhagavan Sri Sathya Sai Baba*

# PREFACE

This book, viz. **Cryogenic Engineering: Software solutions – Part-IV**, is the fourth in the series on **Cryogenic Engineering: Software Solutions**.

This book deals with the topics on **Cryogenic Instrumentation**.

**This part** contains a brief summary of background theory, definitions and formulas and the problems solved with Mathcad and Engineering Equation Solver (EES). As with the Part-I, II and III of the series, the focus is on the solutions of problems in cryogenic engineering using software such as Mathcad and EES. *Only the essential theory and summary of equations required for calculations are given at the beginning of the chapter.*

*Advantages* of using computer software to solve problems are reiterated:

- i) It helps in solving the problems fast and accurately.
- ii) Parametric analysis (what-if analysis) and graphical visualization is done very easily.  
This helps in an in-depth analysis of the problem.
- iii) Once a particular type of problem is solved, it can be used as a *template* and solving similar problems later becomes extremely easy.
- iv) In addition, one can plot the data, curve fit, write functions for various properties or calculations and reuse them.
- v) These possibilities create interest, curiosity and wonder in the minds of students and enthuse them to know more and work more.

**In this book, following topics are dealt with: Temperature measurement, Flow rate and fluid ‘quality’ measurement, and Liquid level measurement.**

*Under temp measurement*, we consider: Temp scales and fixed points, Metallic resistance thermometers, Semiconductor resistance thermometers, Thermocouples, Constant volume gas thermometer, Vapor pressure thermometer and Magnetic thermometers.

*Under Flow rate measurement*, topics considered are: Orifice meter, Venturi meter, Turbine flow meter, and Fluid quality measurement.

*Under Liquid level measurement*, we study: Hydrostatic gauges, Electric resistance gauges, Capacitance probes and Thermodynamic liquid level gauges.

Data on some practical units for cryogenic instrumentation from commercial pamphlets are also presented.

**Several Functions are written in Mathcad and EES to simplify the standard and most required calculations. Students, teachers, researchers and professionals may find them very useful.**

Many numerical problems are solved to illustrate the ease of calculations and graphing using Mathcad and EES software.

S.I. Units are used throughout this book. Wide variety of worked examples presented in the book should be useful for those appearing for University, AMIE and Engineering Services examinations.

**Acknowledgements:** Firstly, I would like to **thank all my students**, who have been an inspiration to me in all my academic efforts.

Sincere thanks are due to **Rev. Fr. Joseph Lobo**, Director, St. Joseph Engineering College, Mangalore, for his kindness, regard and words of encouragement.

I am also thankful to **Dr. Thirumaleshwara Bhat**, Principal, Sri Madhwa Vadiraja Institute of Technology and Management, Bantakal, Udupi, for giving me support in my academic activities.

I gratefully remember my former colleagues at the Cryogenics section of Technical Physics Division, Bhabha Atomic Research Centre (BARC), Bombay and Centre for Advanced Technology, Indore for their sincere cooperation in a true spirit of team-work in all the projects that we undertook.

I particularly salute and admire the vision and foresight of former Heads of Technical Physics Division, BARC viz. late Mr. C. Ambasankaran, Mr. R.Y. Deshpande, Dr. S.R. Gowariker and late Mr. S.S. Ramamurthy in initiating and guiding many of the ‘first of its kind’ projects for Indian Space Research Organization (ISRO), wherein the Cryogenics section was deeply involved in the design and execution of the projects.

**I am especially grateful to Prof. R.G. Scurlock, former Director of Institute of Cryogenics, University of Southampton (UK), for writing a message for the Part-I of this series.**

It was indeed gracious of my former Professor, under whom I studied for M.Sc. in Cryogenics at the University of Southampton, UK during 1970–72, and worked as a Visiting Research Fellow during 1993–94, to honor me by writing this message.

My special thanks to **Bookboon.com** for publishing this *free ebook*. **Ms Karin Jakobsen** and the editorial staff have been most patient and helpful.

Finally, I would like to express my sincere thanks and appreciation to my **wife, Kala**, who, as usual, has given me continuous support, help and encouragement in all my academic activities, making many silent sacrifices.

**M. Thirumaleshwar**

**April, 2017**

**Email:** [tmuliya@rediffmail.com](mailto:tmuliya@rediffmail.com)

**Note: Along with this book, following three ZIP files are also available for free download:**

- i) **Cryogenic Instrumentation (14 files)**
- ii) **Cryogenic Fluid props (4 files), and**
- iii) **Cryogenic material props (12 files)**

**They contain several ‘stand-alone’ EXE files (originally written in EES), which should be of great use for students, teachers and researchers for quick verification of their design calculations.**

# ABOUT THE AUTHOR

Dr. M. Thirumaleshwar graduated in Mechanical Engineering from Karnataka Regional Engineering College, Surathkal, Karnataka, India, in the year 1965. He obtained M.Sc (cryogenics) from University of Southampton, UK and Ph.D. (cryogenics) from Indian Institute of Science, Bangalore, India.

He is a Fellow of Institution of Engineers (India), Life Member, Indian Society for Technical Education, and a Foundation Fellow of Indian Cryogenics Council.

He has worked in India and abroad on large projects in the areas involving heat transfer, fluid flow, vacuum system design, cryo-pumping etc.

He worked as Head of Cryogenics Dept. in Bhabha Atomic Research Centre (BARC), Bombay and Centre for Advanced Technology (CAT), Indore, from 1966 to 1992.

He worked as Guest Collaborator with Superconducting Super Collider Laboratory of Universities Research Association, in Dallas, USA from 1990 to 1993.

He also worked at the Institute of Cryogenics, Southampton, UK as a Visiting Research Fellow from 1993 to 1994.

He was Head of the Dept. of Mechanical Engineering, Fr. Conceicao Rodrigues Institute of Technology, Vashi, Navi Mumbai, India for eight years.

He also worked as Head of Dept. of Mechanical Engineering and Civil Engineering, and then as Principal, Vivekananda College of Engineering and Technology, Puttur (DK), India.

**He was Professor and coordinator of Post-graduate program in the Dept. of Mechanical Engineering in St. Joseph Engineering College, Vamanjoor, Mangalore, India.**

A book entitled “**Fundamentals of Heat and Mass Transfer**” authored by him and published by M/s Pearson Education, India (2006) **has been adopted as a Text book** for third year engineering students by the Visweswaraya Technological University (V.T.U.), Belgaum, India.

He has authored a *free e-book* entitled “**Software Solutions to Problems on Heat Transfer**” wherein problems are solved using 4 software viz. Mathcad, EES, FEHT and EXCEL. This book, containing about 2750 pages, is presented in 9 parts and all the 9 parts can be downloaded for free from [www.bookboon.com](http://www.bookboon.com)

He has also authored *free e-books* on **Thermodynamics** entitled “**Basic Thermodynamics: Software Solutions**” and “**Applied Thermodynamics: Software Solutions**” wherein problems are solved using 3 software viz. Mathcad, EES, and TEST. Each of these titles is presented in 5 parts and all the books can be downloaded for free from [www.bookboon.com](http://www.bookboon.com)

In addition, he has authored following two useful free ebooks:

- i) Applied Thermodynamics: Software Solutions: Vapor compression Refrigeration cycle + Problems (Mathcad)

<http://bookboon.com/en/applied-thermodynamics-software-solutions-vapor-ebook>

- ii) Applied Thermodynamics: Software Solutions: Vapor Power cycles (Rankine cycle) + Problems (Mathcad)

<http://bookboon.com/en/applied-thermodynamics-software-solutions-vapor-po-ebook>

His earlier free ebooks in this **Cryogenic Engineering series**, viz. **Cryogenic Engineering: Software Solutions – Parts-I, II-A and II-B, III-A and III-B** were published by Bookboon recently.

He has also authored **three motivational, free ebooks**, published by [www.bookboon.com](http://www.bookboon.com), entitled as follows:

1. Towards Excellence... How to Study (A Guide book to Students)
2. Towards Excellence... How to teach (A guide book to Teachers)
3. Towards Excellence... Seminars, GD's and Personal Interviews

Dr. M. Thirumaleshwar has attended several National and International conferences and has more than 50 publications to his credit.

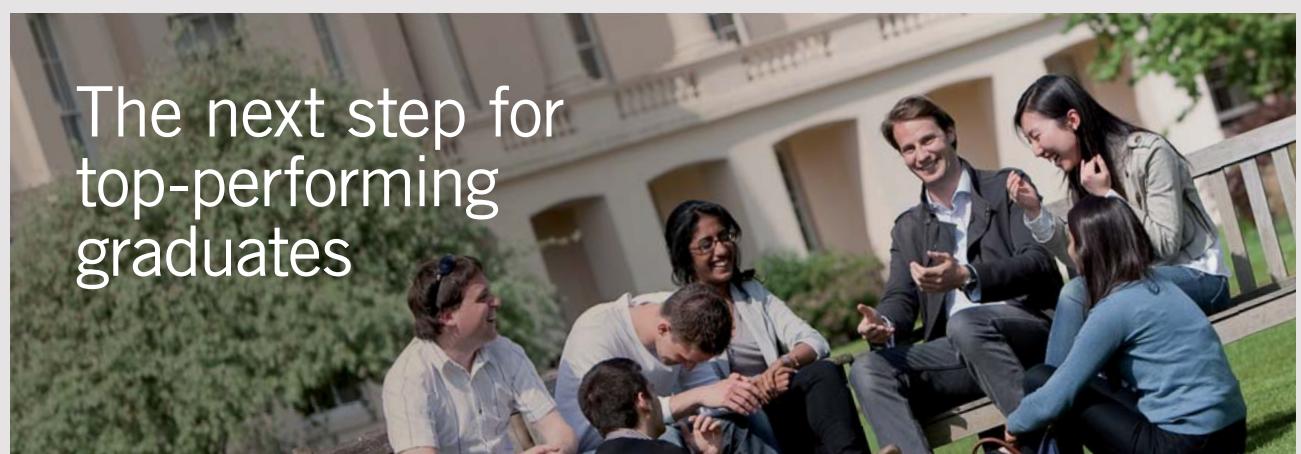
# ABOUT THE SOFTWARE USED

Following software are used while solving problems in this book series:

1. Mathcad 7 and Mathcad 15 (Ref: [www.ptc.com](http://www.ptc.com))
2. Engineering Equation Solver (EES) (Ref: [www.fchart.com](http://www.fchart.com)), and

For a brief introduction to Mathcad, EES and EXCEL see the chapter 1 of the following free ebook by the author:

**“Software Solutions to Problems on Heat Transfer – CONDUCTION-Part-I”:**  
<http://bookboon.com/en/software-solutions-to-problems-on-heat-transfer-ebook>



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# 5 CRYOGENIC INSTRUMENTATION

## Learning objectives:

1. In this chapter, topics on '**Cryogenic Instrumentation**' are dealt with.
  2. First, absolute temp scale and practical temp scales (IPTS-68 and IPTS-90) are explained.
  3. Next, Pt-resistance thermometer and the calculation equations are given. Here, a free software PRTCALC to quickly get the temp value when the resistance value is known (and vice-versa) is also explained. Germanium and carbon resistance thermometers are mentioned and some practical info from commercial catalogues are also given.
  4. Next, various types of Thermocouples used in Cryogenics and their emf-temp calculation formulas and emf-temp tables from NIST are given in detail.
  5. Then, we explain: const. volume gas thermometer, vapor pressure thermometer, and magnetic thermometers.
  6. And, the various types of flow meters explained are: orifice flow meter, venturi flow meter, turbine flow meter, along with some practical data from commercial catalogues. Capacitance-type fluid 'quality' measurement is also mentioned.
  7. Under the topic of liquid level measurement, we deal with: hydrostatic gauges, electric resistance gauges, capacitance probes and thermodynamic level gauges. As examples of commercially available liquid level gauges (one for LN<sub>2</sub> and the other one, for LHe), info from Application notes of M/s American Magnetics are reproduced.
  8. Many numerical problems are solved to illustrate the ease of doing calculations and graphing using Mathcad and EES software.
  9. Several Functions/Procedures are written in Mathcad/EES which greatly simplify the calculations. These should be very much useful to teachers, students and researchers working in this field.
- 

## 5.1 DEFINITIONS, STATEMENTS AND FORMULAS USED [1-9]:

In a cryogenic system, generally, we are interested in measurement/control of temperature, pressure, flow rate, fluid level, fluid quality etc.

We shall summarize data and equations necessary to solve problems on these topics.

### 5.1.1 MEASUREMENT OF TEMPERATURE:

#### Absolute temp scale:

This is defined with respect to a Carnot engine.

We have, for a Carnot engine:

$$\eta_{\text{Carnot}} = \frac{W_{\text{net}}}{Q_a} = \left( \frac{Q_a - Q_r}{Q_a} \right) = \frac{T_h - T_c}{T_h}$$

where:

$Q_a$  is the heat added,  $Q_r$  is the heat rejected,  $T_h$  is the high temp source from which heat is added,  $T_c$  is the low temp sink to which heat is rejected.

$$\text{i.e. } \frac{T_c}{T_h} = \frac{Q_r}{Q_a}$$

Now, Triple Point of water is fixed as 273.16 K. So, in principle, any other temp can be determined by measuring the heat rejected and added in a Carnot engine operating between the Triple point of water and the temp to be measured. Of course, this is only hypothetical since an actual Carnot engine is not practicable, and in practice, we should have some calibrated devices to measure temperature.

#### Practical temp scale:

International Practical Temp Scale of 1968 (IPTS-68) defines the temp scale up to the Triple Point of Hydrogen, i.e. 13.81 K. Following are the Fixed Points as per IPTS-68 [1]:

Temperatures of primary fixed points on the IPTS-68

Fixed Point	Temperature		
	°C	K	°R
NMP <sup>a</sup> of gold (gold point)	1064.43	1337.58	2407.64
NMP of silver	961.93	1235.08	2223.14
NMP of zinc	419.58	692.73	1246.91
NBP <sup>b</sup> of water (steam point)	100.00	373.15	671.67
Standard—triple point of water	0.01	273.16	491.69
NBP of oxygen	-182.962	90.188	162.338
TP <sup>c</sup> of oxygen	-218.789	54.361	97.850
NBP of neon	-246.048	27.102	48.784
NBP of hydrogen	-252.87	20.28	36.50
B.P. of hydrogen at 25 torr	-256.108	17.042	30.676
TP of hydrogen	-259.34	13.81	24.86

<sup>a</sup>NMP = normal melting point or freezing point

<sup>b</sup>NBP = normal boiling point

<sup>c</sup>TP = triple point

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**However, ITS-90 is the new temp scale which replaces the IPTS-68 [5]:**

CCT ITS-90 is designed to represent the thermodynamic (absolute) temperature scale (referencing absolute zero) as closely as possible throughout its range. Many different thermometer designs are required to cover the entire range. These include helium vapor pressure thermometers, helium gas thermometers, standard platinum resistance thermometers (known as SPRTs, PRTs or Platinum RTDs) and monochromatic radiation thermometers.

The table below lists the defining fixed points of ITS-90:

Substance and its state	Defining point (range)			
	K	°C	°R	°F
Triple point of hydrogen	13.8033	-259.3467	24.8459	-434.8241
Triple point of neon	24.5561	-248.5939	44.2010	-415.4690
Triple point of oxygen	54.3584	-218.7916	97.8451	-361.8249
Triple point of argon	83.8058	-189.3442	150.8504	-308.8196
Triple point of mercury	234.3156	-38.8344	421.7681	-37.9019
Triple point of water <sup>[note 1]</sup>	273.16	0.01	491.69	32.02
Melting point <sup>[note 2]</sup> of gallium	302.9146	29.7646	545.2463	85.5763
Freezing point <sup>[note 2]</sup> of indium	429.7485	156.5985	773.5473	313.8773
Freezing point <sup>[note 2]</sup> of tin	505.078	231.928	909.140	449.470
Freezing point <sup>[note 2]</sup> of zinc	692.677	419.527	1,246.819	787.149
Freezing point <sup>[note 2]</sup> of aluminum	933.473	660.323	1,680.251	1,220.581
Freezing point <sup>[note 2]</sup> of silver	1,234.93	961.78	2,222.87	1,763.20
Freezing point <sup>[note 2]</sup> of gold	1,337.33	1,064.18	2,407.19	1,947.52
Freezing point <sup>[note 2]</sup> of copper	1,357.77	1,084.62	2,443.99	1,984.32

1. ^ The triple point of water is frequently approximated by the using the melting point of water at standard conditions for temperature and pressure.

2. ^ a b c d e f g h Melting and freezing points are distinguished by whether heat is entering or leaving the sample when its temperature is measured. See [melting point](#) for more information.

**Standard interpolating thermometers and their ranges:**

Lower (K)	Upper (K)	Variations	Thermometer	Calibration and interpolation strategy
0.65	3.2	1	Helium-3 vapor pressure thermometer	Vapor pressure–temperature relationship fixed by a specified function.
1.25	2.1768	1	Helium-4 vapor pressure thermometer	Vapor pressure–temperature relationship fixed by a specified function.
2.1768	5.0	1	Helium-4 vapor pressure thermometer	Vapor pressure–temperature relationship fixed by a specified function.
3.0	24.5561	1	Helium gas thermometer	Calibrated at three fixed points in this range and interpolated in a specified way.
13.8033	1234.93	11	Platinum resistance thermometer	Resistance calibrated at various fixed points and interpolated in a specified way. Eleven distinct calibration procedures are specified.
1234.93	$\infty$	3	Optical pyrometer	Calibrated at one fixed point, and extrapolated according to Planck's law. May be calibrated at Ag, Au, or Cu freezing point.

**From Ref:[6] :**

- Between 0.65 K and 5.0 K  $T_{90}$  is defined by vapor pressure relations of  ${}^3\text{He}$  and  ${}^4\text{He}$

$$T_{90}(K) = A_0 + \sum_{i=1}^9 A_i \left\{ \frac{\ln[P(Pa)] - B}{C} \right\}^i$$

**Table A.3. Values of the Constants for the Helium Vapor Pressure Eqs. (A.3a) and (A.3b), and the Temperature Range for Which Each Equation, Identified by Its Set of Constants, Is Valid**

	${}^3\text{He}$ 0.65 K to 3.2 K	${}^4\text{He}$ 1.25 K to 2.1768 K	${}^4\text{He}$ 2.1768 K to 5.0 K
$A_0$	1.053447	1.392408	3.146631
$A_1$	0.980106	0.527153	1.357655
$A_2$	0.676380	0.166756	0.413923
$A_3$	0.372692	0.050988	0.091159
$A_4$	0.151656	0.026514	0.016349
$A_5$	-0.002263	0.001975	0.001826
$A_6$	0.006596	-0.017976	-0.004325
$A_7$	0.088966	0.005409	-0.004973
$A_8$	-0.004770	0.013259	0
$A_9$	-0.054943	0	0
$B$	7.3	5.6	10.3
$C$	4.3	2.9	1.9

From Modern Gas-Based Temperature and Pressure Measurements by F. Pavese & G. Molinar  
Plenum Press, NY 1992

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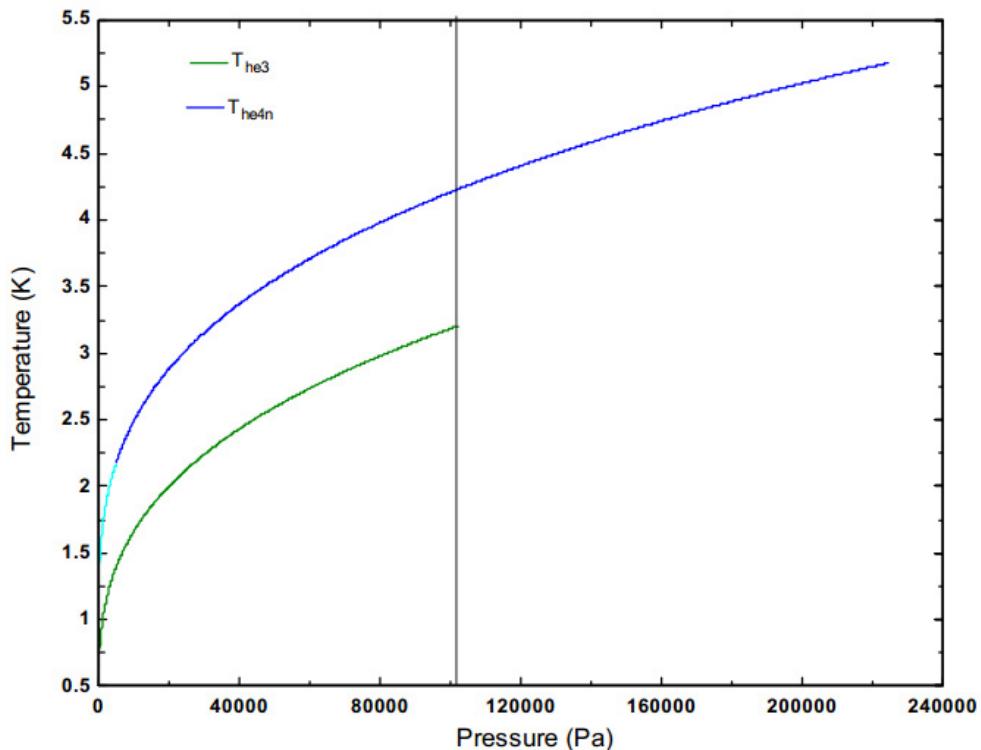
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- Between 3.0 K and the triple point of neon,  $T_{90}$  is defined by means of a helium gas thermometer calibrated at three temperatures:
  - Triple point of neon: 24.5561 K
  - Triple point of equilibrium hydrogen: 13.8033 K
  - A temperature between 3.0 K and 5.0 K determined via a  $^3\text{He}$  or  $^4\text{He}$  vapor pressure thermometer
- Between the triple point of equilibrium hydrogen and the freezing point of silver (961.78 K)  $T_{90}$  is defined by means of platinum resistance thermometers calibrated at specified sets of defining fixed points and using specified interpolations procedures.
- Above the freezing point of silver,  $T_{90}$  is defined in terms of a defining fixed point and the Planck radiation law.

**The standard instrument chosen between 13.81 K and the freezing point of antimony (903.89 K) is a strain-free Platinum resistance thermometer[1]:**

**The resistance-temp function for the temp range: 13.81 K to 273.15 K is:**

$$T(K) = 273.15 + \sum_{n=1}^{20} A_n [\ln(R_e/R_0)]^n$$

where ( $R_e/R_0$ ) is the ratio of resistances at  $T$  and 273.15 K, and  $A_n$  are specified constants.

**For the temp range 273.15 K–903.89 K, we have the modified Callendar equation:**

$$R_e/R_0 = 1 + At' - Bt'(t' - 100)$$

And, the corrected temp:

$$t(^{\circ}\text{C}) = t' + 0.045 \left( \frac{t'}{100} \right) \left( \frac{t'}{100} - 1 \right) \left( \frac{t'}{419.58} - 1 \right) \left( \frac{t'}{630.74} - 1 \right)$$

In the above eqns, A and B are determined from the measured electrical resistances at steam point (100 °C) and the Zinc point (419.58 °C).

**In the range 4.24 K–13.84 K, germanium resistance thermometer scale is used.** Here, temp is related to resistance as follows:

$$\ln T = \sum_{n=0}^8 B_n (\ln R_e)^n$$

Constants  $B_n$  are found by calibration at different temperatures.

**In the range 1.5 K–4.24 K, He<sup>4</sup> vapor pressure scale is used.** Vapor pressure values are given in the following table. Interpolation between two temperatures T<sub>1</sub> and 2 is given by:

$$\frac{1}{T} = \frac{1}{T_1} + \frac{(T_1 - T_2)}{T_1 T_2} \frac{\ln(p/p_1)}{\ln(p_2/p_1)}$$

**In the range 0.8 K–1.5 K, He<sup>3</sup> vapor pressure scale is used.** Vapor pressures for He<sup>3</sup> are also given in the following table, and the interpolation between T<sub>1</sub> and T<sub>2</sub> is also done as per the above eqn.

Vapor pressure of He<sup>4</sup> and He<sup>3</sup>

Pressure (torr)	Temperature, K		Pressure (torr)	Temperature, K	
	He <sup>4</sup>	He <sup>3</sup>		He <sup>4</sup>	He <sup>3</sup>
800	4.2700	3.2395	110	2.6888	1.8343
750	4.2009	3.1779	100	2.6354	1.7881
700	4.1287	3.1134	90	2.5781	1.7387
650	4.0529	3.0455	80	2.5163	1.6856
600	3.9731	2.9739	70	2.4489	1.6279
550	3.8886	2.8981	60	2.3745	1.5646
500	3.7987	2.8175	50	2.2911	1.4939
450	3.7026	2.7313	40	2.1952	1.4131
400	3.5990	2.6285	30	2.0827	1.3176
350	3.4863	2.5376	20	1.9427	1.1978
300	3.3622	2.4269	18	1.9092	1.1692
250	3.2231	2.3033	16	1.8729	1.1384
200	3.0637	2.1623	14	1.8333	1.1049
180	2.9924	2.0994	12	1.7893	1.0679
160	2.9153	2.0318	10	1.7396	1.0266
150	2.8744	1.9959	9	1.7120	1.0038
140	2.8315	1.9585	8	1.6820	0.9791
130	2.7865	1.9192	7	1.6490	0.9523
120	2.7390	1.8779	6	1.6123	0.9227
			5	1.5707	0.8894
			4	1.5221	0.8512



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In the range **0.006 K to 0.8 K**, a magnetic scale based on the **magnetic susceptibility of Cerium Magnesium Nitrate (CMN)** is used.

Magnetic susceptibility  $\chi$  is defined as follows:

$$\chi = \frac{M}{H}$$

And,  $\chi$  is expressed as:

$$\chi = \frac{C}{T - \theta}$$

where, C = Curie constant, and  $\theta$  is a calibration factor.

### Resistance Thermometer Detectors (RTD) [19]:

RTDs generally refer to metallic and semiconductor resistance thermometers. Since Platinum resistance thermometer is most popular and an industry standard, RTDs are synonymous with Pt-resistance thermometers.

**Metals used are: Platinum, Copper, Aluminium, Lead, Indium, Gold and Silver.**

Generally, **Platinum** is used because of its *advantages*:

- i) it has a wide range and non-reactive up to high temperatures of around 850 deg. C,
- ii) it has good accuracy
- iii) it has good interchangeability
- iv) it has long term stability
- v) it can be drawn into wires
- vi) it is not too soft
- vii) it gives reproducible results, and
- viii) it is not affected by corrosion and oxidation

*Disadvantages* of Pt are:

- i) expensive
- ii) influenced by lead wire resistance
- iii) slow thermal response, and
- iv) low resistance to shock and vibration

Pure Pt (99.99%) is used, since impurities in the resistor will affect the electrical resistance. Pt-resistance thermometers are known as PRT. According to the IEC and DIN Standards, temperature coefficient of the resistance wire must be  $0.003850^{\circ}\text{C}^{-1}$  (i.e. mean resistance change referred to the nominal resistance at zero deg.C) and it should have a resistance of 100 ohms at 0 deg.C; so, it is known as PT-100. Its resistance changes approximately 0.4 ohms/degree C.

**For analytical calculations, the resistance-temperature characteristics is expressed as follows:**

A third order polynomial in the temperature range (-200 deg.C to 0 deg.C):

$$R(t)=R_0 \cdot [ (1 + A \cdot t + B \cdot t^2) + C \cdot (t - 100) \cdot t^3 ] \quad \dots(2.10)$$

A second order polynomial for the temperature range: (0 to 850 deg.C):

$$R(t)=R_0 \cdot (1 + A \cdot t + B \cdot t^2) \quad \dots(2.11)$$

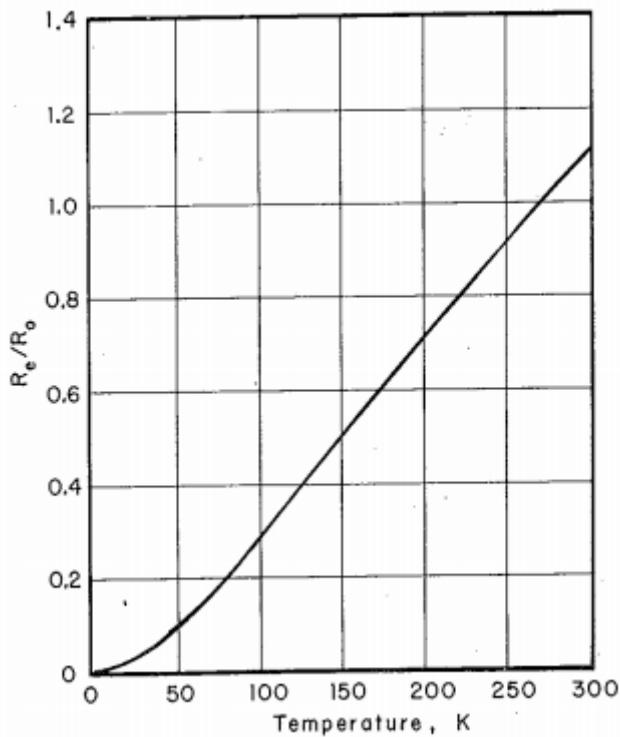
Here, constants A, B and C are given by:

$$A = 3.9083 \times 10^{-3} ^{\circ}\text{C}^{-1}$$

$$B = -5.775 \times 10^{-7} ^{\circ}\text{C}^{-2}$$

$$C = -4.183 \times 10^{-12} ^{\circ}\text{C}^{-4}$$

Pt-resistance thermometer is almost linear for most of the range. Following graph gives  $R_e/R_0$  against temp for Pt [1]:



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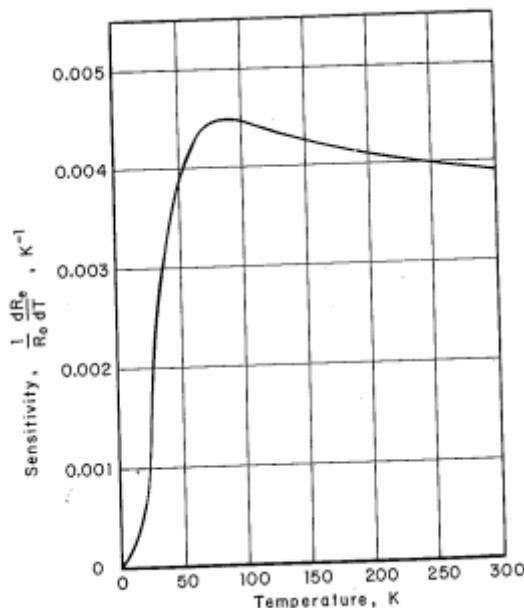
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And, Sensitivity of Pt-resistance thermometer (PRT) is given by:

$$S_0 = \frac{dR_e}{dT} = R_0[A + 2Bt + Ct^2(4t - 300)]$$

Following graph gives Sensitivity of PRT [1]:



***Free software to quickly determine temperature for a given resistance and vice-versa:***

For those who use Pt 100 very often, there is a very **versatile free software** to make calculations, known as PRTC Calc., available for download from <http://www.isotech.co.uk/prtcalc/prtcalc.html>. According to its supplier: "It allows for the accurate conversion of resistance to temperature or temperature to resistance. PRTC Calc is a small utility that can sit on the desk top alongside other applications. The calculated results can easily be imported to spreadsheets or other applications for presentation, storage or whatever. Output data can also be saved as a text file".

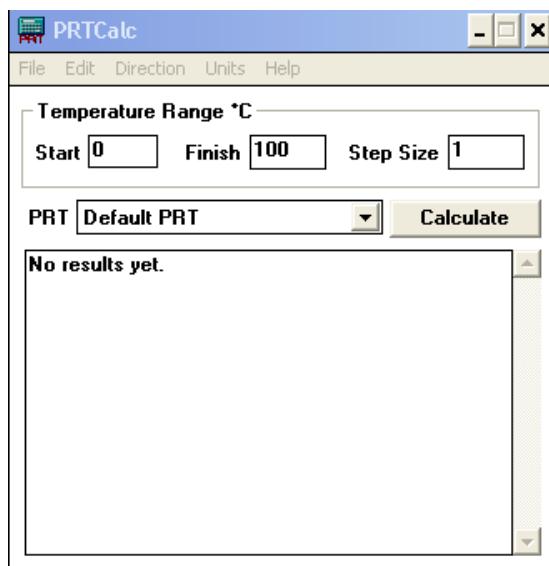
Important features of PRTC Calc are:

1. Can Generate Large Tables or Single Values
2. Convert Temperature to Resistance with high accuracy
3. Convert Resistance to Temperature to approx. 0.01°C
4. Allows very large lists of data
5. Output Can be Saved as a text file
6. Allows marking and copying of result data

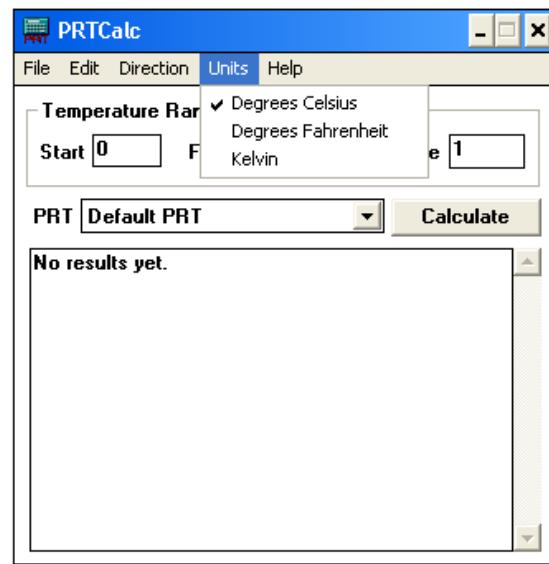
7. Stores coefficients for user entered probes
8. Probe Management.
9. Unit Choice – *Celsius, Fahrenheit or Kelvin*

**Using PRTCalc:**

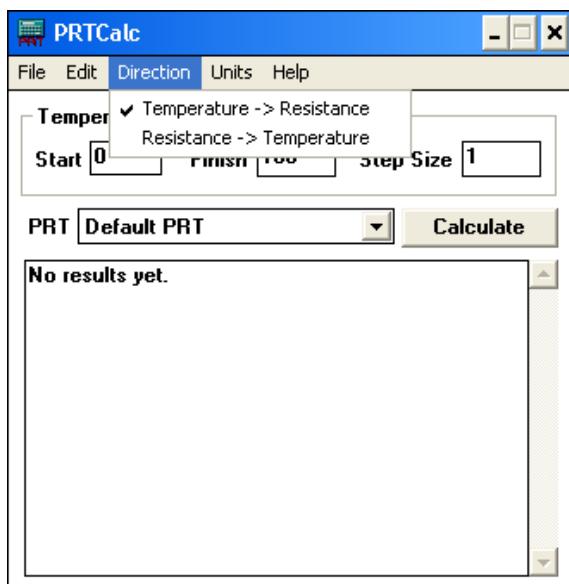
As you start PRTCalc, following screen is presented:



Then, select °C, °F or Kelvin from the **Units** menu:



Choose between calculating from resistance to temperature or temperature to resistance from the **Direction** menu:



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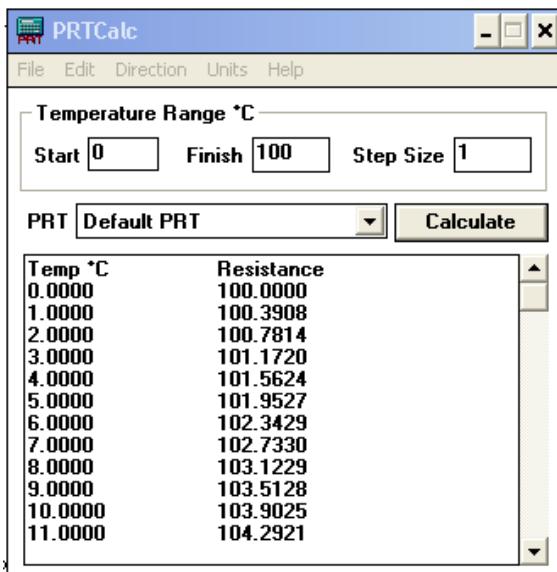


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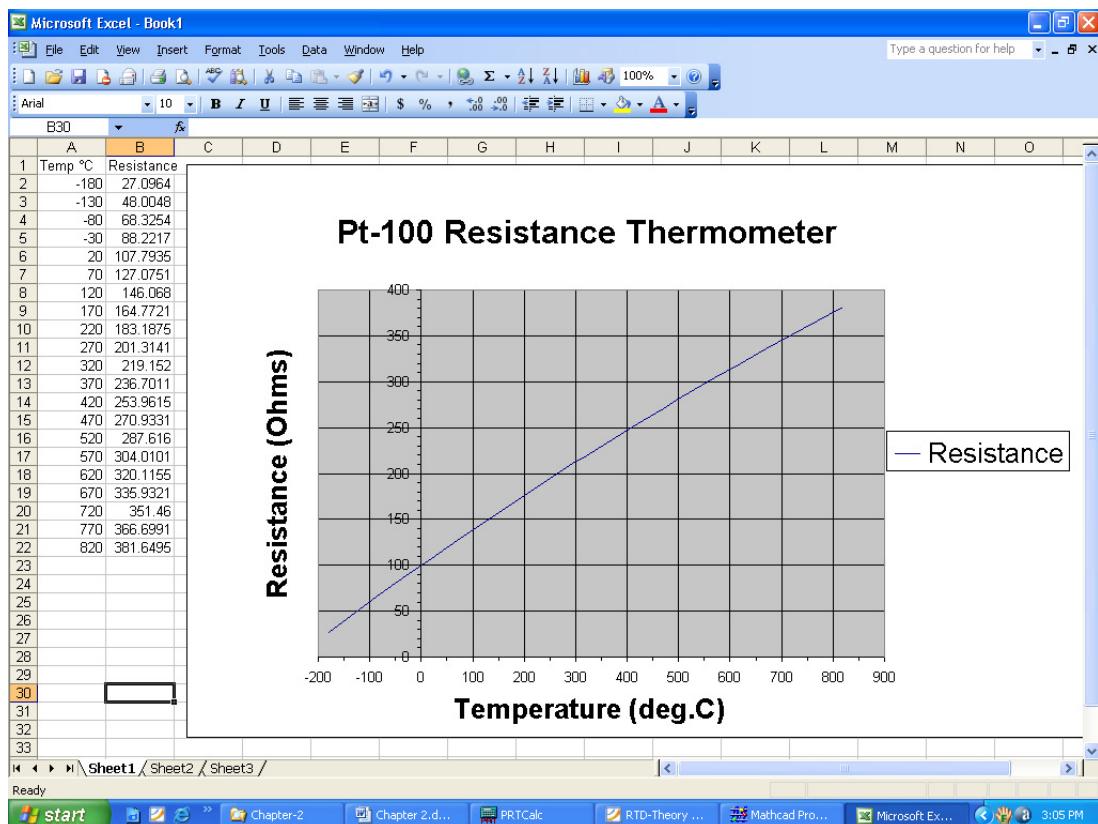
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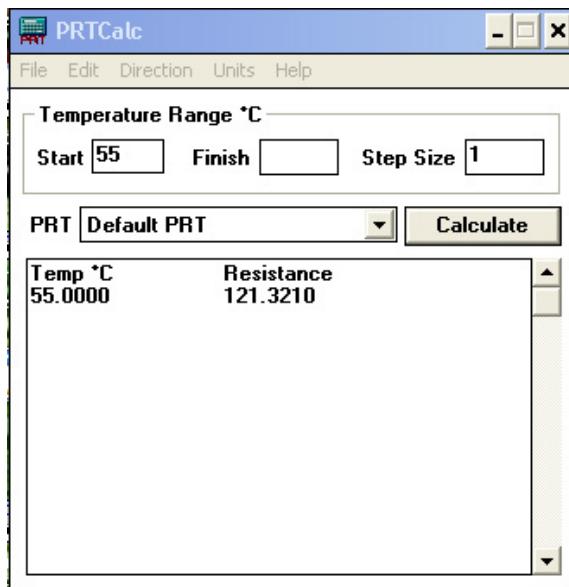
Press '**Calculate**' and the table of Temp vs Resistance appears as shown:



Use the **Edit** menu to copy data to the clipboard; the data can then be pasted into a spreadsheet to produce a graph. As an example, values of resistances are calculated in the temperature range -180 deg.C to 820 deg.C and the table produced is copied to Excel; then, the Temperature vs Resistance chart is produced in Excel, and is shown below:

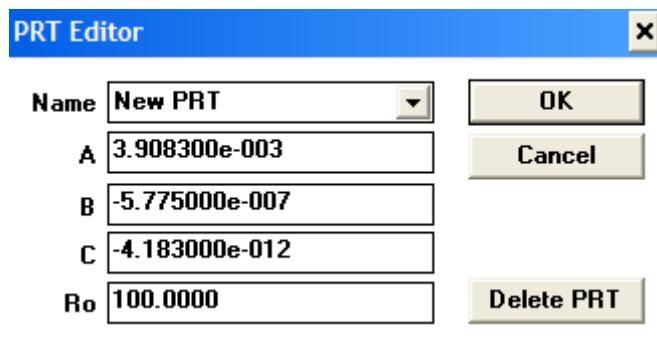


If you wish to calculate a single value only, say the value of resistance at 55 deg.C, then, leave the Finish box empty, and click on ‘Calculate’; you read the resistance as 121.321 ohms.:



*Note:*

1. You can run multiple copies of PRTCALC simultaneously to see the effects of different coefficients or for multiple thermometers.
2. You can also change the values of  $R_0$ , A, B and C by going to File menu and clicking on ‘Open PRT editor’:



### Semiconductor Resistance Thermometers:

**Following information is from the data sheet of Lakeshore Cryotronics**

(Ref: <http://www.lakeshore.com/products/Cryogenic-Temperature-Sensors/Pages/default.aspx>):

**Cernox™ sensors** can be used from 100 mK to 420 K with good sensitivity over the whole range. They have a low magnetoresistance, and are the best choice for applications with magnetic fields up to 30 T (for temperatures greater than 2 K). Cernox™ are resistant to ionizing radiation, and are available in robust mounting packages and probes. Because of their versatility, they are used in a wide variety of cryogenic applications, such as particle accelerators, space satellites, MRI systems, cryogenic systems, and research science.

**Germanium RTDs** have the highest accuracy, reproducibility, and sensitivity from 0.05 K to 100 K. They are resistant to ionizing radiation, but are not recommended for use in magnetic fields. Germanium RTDs are used mostly in research settings when the best accuracy and sensitivity are required. Germanium and Ruthenium Oxide are the *only two* cryogenic temperature sensors that can be used below 100 mK.

**Ruthenium Oxide RTDs** can be used to below 50 mK. Their unique advantage is that they have a low magnetoresistance and follow a standard curve. Their upper temperature range is limited to 40 K, and Cernox™ are better in magnetic fields above 2 K. Ruthenium Oxide sensors are used for applications that require a standard curve in magnetic fields, such as MRI systems. Along with Germanium, they are the only cryogenic temperature sensors that can be used below 100 mK.



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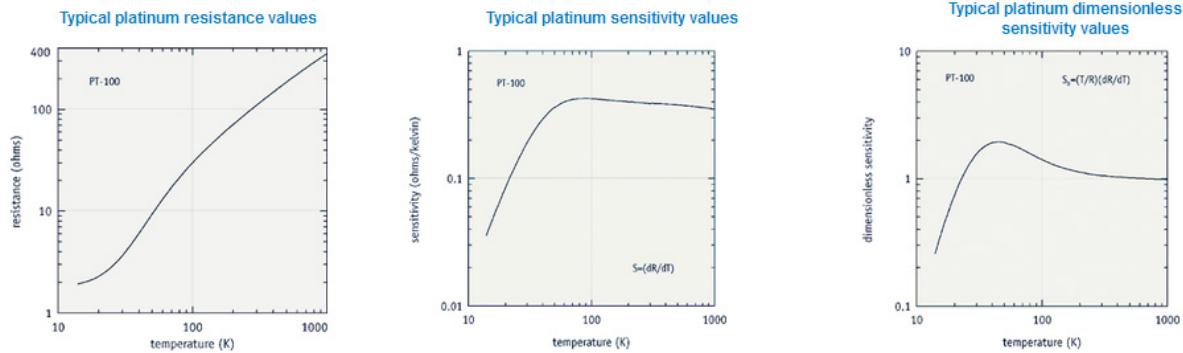


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**Ref:** <http://www.lakeshore.com/products/cryogenic-temperature-sensors/platinum-rtds/models/pages/overview.aspx>

### Platinum Features

- Temperature range: 14 K to 873 K (model dependant)
- Conforms to IEC 751 standards down to 70 K
- High reproducibility:  $\pm 5$  mK at 77 K
- Low magnetic field dependence above 40 K
- Excellent for use in ionizing radiation
- SoftCal™ calibration available



**Germanium resistance thermometers** are calibrated at various temperatures and the resulting data is correlated by an eqn of following type [1]:

$$\ln T = \sum_{n=0}^{8} B_n (\ln R_e)^n$$

**Carbon resistance thermometers** [1]: These are widely used because of their high sensitivity, low cost, small size and relatively simple resistance-temp curve. Following calibration curve is suggested for an ordinary carbon radio resistor (0.1 W size):

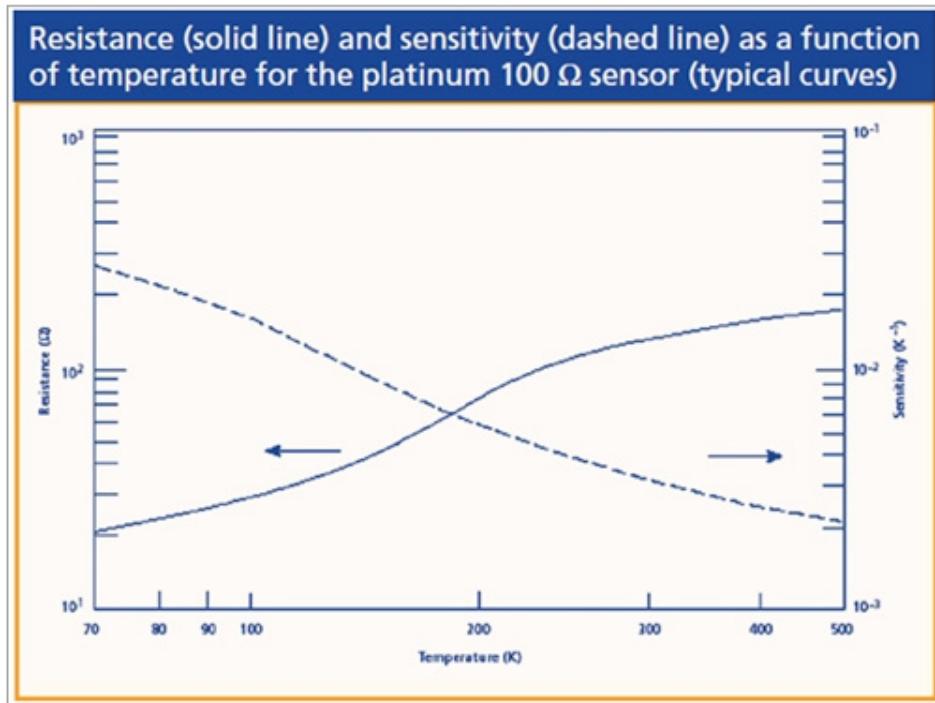
$$T = \frac{B \log_{10} R_e}{(\log_{10} R_e)^2 + A \log_{10} R_e + K}$$

where A, B and K are calibration constants.

**Following information is from the data sheet of Oxford Instruments**

(Ref: <http://www.cryospares.co.uk/Platinum-Resistance-Sensor-70-900K-p/t1-101.htm>)

### Sensor: Platinum Resistance Sensor (70 - 900K)



Following is the information on Platinum RTD's from Cryogenic Control Systems Inc. CA, USA. (Ref: <http://www.cryocon.com/PTsensors/CPGPXP100ds.pdf>)

#### Types of Pt RTD's:

**The CP-100** is a wire-wound ceramic sensor that is fabricated by winding Platinum wire into a coil.

**The GP-100** is fabricated as a platinum band wound onto a glass tube and protected from the environment by a layer of glaze.

**The XP-100 and XP-1K** are thin film sensors. They are manufactured by depositing a thin layer of platinum on a ceramic substrate.

#### Useful Temperature Range:

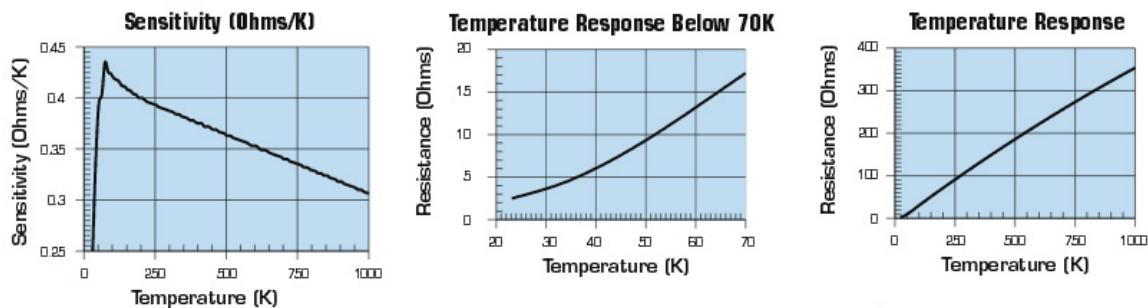
CP-100: 14K to 873K, 100 $\Omega$  @ 0°C

GP-100: 14K to 673K, 100 $\Omega$  @ 0°C

XP-100: 30K to 673K, 100 $\Omega$  @ 0°C

XP-1K: 30K to 673K, 1,000 $\Omega$  @ 0°C

### Typical Performance Charts



### Thermocouples:

Most thermocouples are made by connecting two pieces of dissimilar wire, welded together in a bead. This junction can be made as small as desired by selecting suitably sized wire. It is important that the two metals are in good physical contact.

Thermocouple is a popular device for temperature measurement.

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Its *advantages* are: it is cost effective, easy and convenient to use and is suitable for a wide range of temperatures. Because of low thermal mass of the thermocouple junction, it is quite suitable to measure varying temperatures with fast response.

However, its *disadvantages* are non-linear characteristics of output (i.e. milli-Volts against temperature) and very low values of voltage output, which needs amplification; also, 'cold junction compensation' is required for accuracy.

Voltage developed in thermocouple is a function of temperature difference between junctions. It varies from 1 to 70 microvolts per degree Celsius.

### **Thermocouples popularly used in industry are:**

#### **Thermocouple types:**

Type	Materials	Typical Range (deg.C)	Comments
T	Copper/Constantan	-200 to 400	Non-magnetic; sensitivity of about 43 $\mu\text{V}/^\circ\text{C}$ .
J	Iron/Constantan	-40 to 750	Limited range; sensitivity: approx. 50 $\mu\text{V}/^\circ\text{C}$
K	Chromel/Alumel	-200 to 1300	General purpose; most commonly used. Sensitivity: approx. 41 $\mu\text{V}/^\circ\text{C}$ .
E	Chromel/Constantan	-200 to 900	Suitable for cryogenic use; high sensitivity: 68 $\mu\text{V}/^\circ\text{C}$
S	Pt-10%Rh vs Pt	-50 to 1760	Costly; used for high temp; Low sensitivity: 10 $\mu\text{V}/^\circ\text{C}$
B	(Pt-13%Rh) vs (Pt-6% Rh)	100 to 1800	--do--
R	(Pt-13%Rh) vs Pt	-50 to 1760	--do--
N	(Ni-Cr-Si) vs (Ni-Si-Mg)	-200 to 1300	For high temp. exceeding 1200 $^\circ\text{C}$ . Sensitivity is about 39 $\mu\text{V}/^\circ\text{C}$ at 900 $^\circ\text{C}$ .

**Brief Note on different types of Thermocouples:** Ref:[9]

**Type K** ([chromel-alumel](#)) is the most commonly used general purpose thermocouple. It is inexpensive and, owing to its popularity, available in a wide variety of probes. They are available in the  $-200\text{ }^{\circ}\text{C}$  to  $+1350\text{ }^{\circ}\text{C}$  range. The type K was specified at a time when [metallurgy](#) was less advanced than it is today and, consequently, characteristics vary considerably between examples. Another potential problem arises in some situations since one of the constituent metals, [nickel](#), is magnetic. One characteristic of thermocouples made with magnetic material is that they undergo a step change when the magnetic material reaches its [Curie point](#). This occurs for this thermocouple at  $354\text{ }^{\circ}\text{C}$ . Sensitivity is approximately  $41\text{ }\mu\text{V}/{}^{\circ}\text{C}$ .

**Type E** ([chromel-constantan](#)) has a high output ( $68\text{ }\mu\text{V}/{}^{\circ}\text{C}$ ) which makes it well suited to [cryogenic](#) use. Additionally, it is non-magnetic.

**Type J** ([iron-constantan](#)) is less popular than type K due to its limited range ( $-40$  to  $+750\text{ }^{\circ}\text{C}$ ). The main application is with old equipment that cannot accept modern thermocouples. J types cannot be used above  $760\text{ }^{\circ}\text{C}$  as an abrupt magnetic transformation causes permanent decalibration. The magnetic properties also prevent use in some applications. Type J thermocouples have a sensitivity of about  $50\text{ }\mu\text{V}/{}^{\circ}\text{C}$ .

**Type N** ([nicrosil-nisil](#)) thermocouples are suitable for use at high temperatures, exceeding  $1200\text{ }^{\circ}\text{C}$ , due to their stability and ability to resist high temperature [oxidation](#). Sensitivity is about  $39\text{ }\mu\text{V}/{}^{\circ}\text{C}$  at  $900\text{ }^{\circ}\text{C}$ , slightly lower than type K. Designed to be an improved type K, it is becoming more popular.

**Types B, R, and S** thermocouples use [platinum](#) or a [platinum-rhodium](#) alloy for each conductor. These are among the most stable thermocouples, but have lower sensitivity, approximately  $10\text{ }\mu\text{V}/{}^{\circ}\text{C}$ , than other types. The high cost of these makes them unsuitable for general use. Generally, type B, R, and S thermocouples are used only for high temperature measurements.

**Type B** thermocouples use a platinum-rhodium alloy for each conductor. One conductor contains 30% rhodium while the other conductor contains 6% rhodium. These thermocouples are suited for use at up to  $1800\text{ }^{\circ}\text{C}$ . Type B thermocouples produce the same output at  $0\text{ }^{\circ}\text{C}$  and  $42\text{ }^{\circ}\text{C}$ , limiting their use below about  $50\text{ }^{\circ}\text{C}$ .

**Type R** thermocouples use a platinum-rhodium alloy containing 13% rhodium for one conductor and pure platinum for the other conductor. Type R thermocouples are used up to  $1600\text{ }^{\circ}\text{C}$ .

**Type S** thermocouples use a platinum–rhodium alloy containing 10% rhodium for one conductor and pure platinum for the other conductor. Like type R, type S thermocouples are used up to 1600 °C. In particular, type S is used as the standard of calibration for the melting point of [gold](#) (1064.43 °C).

**Type T** ([copper-constantan](#)) thermocouples are suited for measurements in the -200 to 350 °C range. Often used as a differential measurement since only copper wire touches the probes. As both conductors are non-magnetic, type T thermocouples are a popular choice for applications such as [electrical generators](#) which contain strong magnetic fields. Type T thermocouples have a sensitivity of about 43 µV/°C.

**Type C** ([tungsten](#) 5% [rhenium](#) – [tungsten](#) 26% [rhenium](#)) thermocouples are suited for measurements in the 0 °C to 2320 °C range. This thermocouple is well-suited for [vacuum furnaces](#) at extremely high temperatures and must never be used in the presence of [oxygen](#) at temperatures above 260 °C.



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**Type M** thermocouples use a nickel alloy for each wire. The positive wire contains 18% molybdenum while the negative wire contains 0.8% cobalt.<sup>[5]</sup> These thermocouples are used in the vacuum furnaces for the same reasons as with type C. Upper temperature is limited to 1400 °C. Though it is a less common type of thermocouple, look-up tables to correlate temperature to EMF (milli-volt output) are available.

In chromel-gold/iron **thermocouples**, the positive wire is chromel and the negative wire is gold with a small fraction (0.03–0.15 atom percent) of iron. It can be used for cryogenic applications (1.2–300 K and even up to 600 K). Both the sensitivity and the temperature range depends on the iron concentration. The sensitivity is typically around 15 µV/K at low temperatures and the lowest usable temperature varies between 1.2 and 4.2 K.

**Standard reference tables** for Thermocouples are normally referenced to 0 deg.C cold junction. NIST reference functions and tables of thermocouple electromotive force (emf) versus temperature have been adopted as standards by the American Society for Testing and Materials (ASTM) and the International Electrotechnical Commission (IEC). Distributed by Standard Reference Data Program of the National Institute of Standards and Technology. NIST look up tables and polynomial coefficients for different Types of thermocouples are available for download from Ref.[10], i.e.: <https://srdata.nist.gov/its90/download/download.html>

This website presents the following screen:

<b>Download Tables:</b>		<b>Definitions and Format</b>
<p>Click the desired type to download a table of thermoelectric voltages and coefficients of that type.</p> <p>Click All Thermocouple Types to download a table of thermoelectric voltages and coefficients of all types (B - T).</p> <p>Click Coefficients of All Thermocouple Types to download a table of ONLY coefficients of all types (B - T).</p>	<ul style="list-style-type: none"><li>• Type B</li><li>• Type E</li><li>• Type J</li><li>• Type K</li><li>• All Thermocouple Types</li></ul>	<ul style="list-style-type: none"><li>• Type N</li><li>• Type R</li><li>• Type S</li><li>• Type T</li><li>• Coefficients of All Thermocouple Types</li></ul>

### Reference Function Coefficients Definitions:

The general form for the emf, E, as a function of  $t_{90}$  for all except type K thermocouples is:

$$E = \sum_{i=0}^n c_i (t_{90})^i$$

where E is in mV and  $t_{90}$  is in °C. For type K thermocouples above 0 °C, there is an additional term to account for a magnetic ordering effect:

$$E = \sum_{i=0}^n c_i (t_{90})^i + a_0 e^{a_1(t_{90}-a_2)^2}$$

where e is the natural logarithm constant, E is in mV, and  $t_{90}$  is in °C. The constants  $a_0$ ,  $a_1$ , and  $a_2$  have the values:

$$\begin{aligned} a_0 &= 0.118597600000 \times 10^0 \\ a_1 &= -0.118343200000 \times 10^{-3} \\ a_2 &= 0.126968600000 \times 10^3 \end{aligned}$$

**As an example, the coefficients for type T (copper-constantan) and type K (Chromel-Alumel) Thermocouples are given below:**

```
* This section contains coefficients for type T thermocouples for
* the two subranges of temperature listed below. The coefficients
* are in units of °C and mV and are listed in the order of constant
* term up to the highest order. The equation is of the form
* E = sum(i=0 to n) c_i t^i.
*
*      Temperature Range (°C)
*      -270.000 to 0.000
*      0.000 °C to 400.000
*****
name: reference function on ITS-90
type: T
temperature units: °C
emf units: mV
```

```
range: -270.000, 0.000, 14
0.000000000000E+00
0.387481063640E-01
0.441944343470E-04
0.118443231050E-06
0.200329735540E-07
0.901380195590E-09
0.226511565930E-10
0.360711542050E-12
0.384939398830E-14
0.282135219250E-16
0.142515947790E-18
0.487686622860E-21
0.107955392700E-23
0.139450270620E-26
0.797951539270E-30
range: 0.000, 400.000, 8
0.000000000000E+00
0.387481063640E-01
0.332922278800E-04
0.206182434040E-06
-0.218822568460E-08
0.109968809280E-10
-0.308157587720E-13
```

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```

0.454791352900E-16
-0.275129016730E-19
*****
* This section contains coefficients of approximate inverse
* functions for type T thermocouples for the subranges of
* temperature and voltage listed below. The range of errors of
* the approximate inverse function for each subrange is also given.
* The coefficients are in units of °C and mV and are listed in
* the order of constant term up to the highest order.
* The equation is of the form t_90 = d_0 + d_1*E + d_2*E^2 + ...
*      + d_n*E^n,
* where E is in mV and t_90 is in °C.
*
*      Temperature          Voltage          Error
*      range                range            range
*      (°C)                 (mV)             (° C)
*      -200. to 0. -5.603 to 0.000 -0.02 to 0.04
*      .0 to 400. 0.000 to 20.872 -0.03 to 0.03
*****

```

Inverse coefficients for type T:

Temperature	-200.	0.
Range:	0.	400.
Voltage	-5.603	0.000
Range:	0.000	20.872
	0.0000000E+00	0.000000E+00
	2.5949192E+01	2.592800E+01
	-2.1316967E-01	-7.602961E-01
	7.9018692E-01	4.637791E-02
	4.2527777E-01	-2.165394E-03
	1.3304473E-01	6.048144E-05
	2.0241446E-02	-7.293422E-07
	1.2668171E-03	0.000000E+00
Error	-0.02	-0.03
Range:	0.04	0.03

---

```

* This section contains coefficients for type K thermocouples for
* the two subranges of temperature listed below. The coefficients
* are in units of °C and mV and are listed in the order of constant
* term up to the highest order. The equation below 0 °C is of the form
* E = sum(i=0 to n) c_i t^i.
*
* The equation above 0 °C is of the form

```

```

* E = sum(i=0 to n) c_i t^i + a0 exp(a1 (t - a2)^2).
*
* Temperature Range (°C)
* -270.000 to 0.000
* 0.000 to 1372.000
*****
name: reference function on ITS-90
type: K
temperature units: °C
emf units: mV
range: -270.000, 0.000, 10
    0.00000000000E+00
    0.394501280250E-01
    0.236223735980E-04
    -0.328589067840E-06
    -0.499048287770E-08
    -0.675090591730E-10
    -0.574103274280E-12
    -0.310888728940E-14
    -0.104516093650E-16
    -0.198892668780E-19
    -0.163226974860E-22
range: 0.000, 1372.000, 9
    -0.176004136860E-01
    0.389212049750E-01
    0.185587700320E-04
    -0.994575928740E-07
    0.318409457190E-09
    -0.560728448890E-12
    0.560750590590E-15
    -0.320207200030E-18
    0.971511471520E-22
    -0.121047212750E-25
exponential:
    a0 = 0.118597600000E+00
    a1 = -0.118343200000E-03
    a2 = 0.126968600000E+03

*****
* This section contains coefficients of approximate inverse
* functions for type K thermocouples for the subranges of
* temperature and voltage listed below. The range of errors of
* the approximate inverse function for each subrange is also given.
* The coefficients are in units of °C and mV and are listed in
* the order of constant term up to the highest order.
* The equation is of the form t_90 = d_0 + d_1*E + d_2*E^2 + ...
*      + d_n*E^n,
* where E is in mV and t_90 is in °C.

```

Temperature range (°C)	Voltage range (mV)	Error range (° C)
-200. to 0.	-5.891 to 0.000	-0.02 to 0.04
0. to 500.	0.000 to 20.644	-0.05 to 0.04
500. to 1372.	20.644 to 54.886	-0.05 to 0.06

Inverse coefficients for type K:

Temperature	-200.	0.	500.
Range:	0.	500.	1372.

Voltage	-5.891	0.000	20.644
Range:	0.000	20.644	54.886

0.0000000E+00	0.000000E+00	-1.318058E+02
2.5173462E+01	2.508355E+01	4.830222E+01
-1.1662878E+00	7.860106E-02	-1.646031E+00
-1.0833638E+00	-2.503131E-01	5.464731E-02
-8.9773540E-01	8.315270E-02	-9.650715E-04
-3.7342377E-01	-1.228034E-02	8.802193E-06
-8.6632643E-02	9.804036E-04	-3.110810E-08
-1.0450598E-02	-4.413030E-05	0.000000E+00

The advertisement features several circular icons: a green one with 'Reduce reuse Recycle' text, a yellow one with 'WORK WITH US', a red one with a bird, a pink one with 'togetherness', a blue one with 'Save water. shower together.', a white one with a lamp, and a blue one with 'everyone deserves good design'. The central text reads: 'It's only an opportunity if you act on it' and 'IKEA.SE/STUDENT'.

-5.1920577E-04	1.057734E-06	0.000000E+00
0.0000000E+00	-1.052755E-08	0.000000E+00
Error	-0.02	-0.05
Range:	0.04	0.04
		-0.05
		0.06

**Now, temp-emf tables are given below for ready reference:**

**Typical Table of Thermoelectric voltages for type T Thermocouple:**

ITS-90 Table for type T thermocouple											
°C	0	-1	-2	-3	-4	-5	-6	-7	-8	-9	-10
Thermoelectric Voltage in mV											
-270	-6.258										
-260	-6.232	-6.236	-6.239	-6.242	-6.245	-6.248	-6.251	-6.253	-6.255	-6.256	-6.258
-250	-6.180	-6.187	-6.193	-6.198	-6.204	-6.209	-6.214	-6.219	-6.223	-6.228	-6.232
-240	-6.105	-6.114	-6.122	-6.130	-6.138	-6.146	-6.153	-6.160	-6.167	-6.174	-6.180
-230	-6.007	-6.017	-6.028	-6.038	-6.049	-6.059	-6.068	-6.078	-6.087	-6.096	-6.105
-220	-5.888	-5.901	-5.914	-5.926	-5.938	-5.950	-5.962	-5.973	-5.985	-5.996	-6.007
-210	-5.753	-5.767	-5.782	-5.795	-5.809	-5.823	-5.836	-5.850	-5.863	-5.876	-5.888
-200	-5.603	-5.619	-5.634	-5.650	-5.665	-5.680	-5.695	-5.710	-5.724	-5.739	-5.753
-190	-5.439	-5.456	-5.473	-5.489	-5.506	-5.523	-5.539	-5.555	-5.571	-5.587	-5.603
-180	-5.261	-5.279	-5.297	-5.316	-5.334	-5.351	-5.369	-5.387	-5.404	-5.421	-5.439
-170	-5.070	-5.089	-5.109	-5.128	-5.148	-5.167	-5.186	-5.205	-5.224	-5.242	-5.261
-160	-4.865	-4.886	-4.907	-4.928	-4.949	-4.969	-4.989	-5.010	-5.030	-5.050	-5.070
-150	-4.648	-4.671	-4.693	-4.715	-4.737	-4.759	-4.780	-4.802	-4.823	-4.844	-4.865
-140	-4.419	-4.443	-4.466	-4.489	-4.512	-4.535	-4.558	-4.581	-4.604	-4.626	-4.648
-130	-4.177	-4.202	-4.226	-4.251	-4.275	-4.300	-4.324	-4.348	-4.372	-4.395	-4.419
-120	-3.923	-3.949	-3.975	-4.000	-4.026	-4.052	-4.077	-4.102	-4.127	-4.152	-4.177
-110	-3.657	-3.684	-3.711	-3.738	-3.765	-3.791	-3.818	-3.844	-3.871	-3.897	-3.923
-100	-3.379	-3.407	-3.435	-3.463	-3.491	-3.519	-3.547	-3.574	-3.602	-3.629	-3.657
-90	-3.089	-3.118	-3.148	-3.177	-3.206	-3.235	-3.264	-3.293	-3.322	-3.350	-3.379
-80	-2.788	-2.818	-2.849	-2.879	-2.910	-2.940	-2.970	-3.000	-3.030	-3.059	-3.089
-70	-2.476	-2.507	-2.539	-2.571	-2.602	-2.633	-2.664	-2.695	-2.726	-2.757	-2.788
-60	-2.153	-2.186	-2.218	-2.251	-2.283	-2.316	-2.348	-2.380	-2.412	-2.444	-2.476
-50	-1.819	-1.853	-1.887	-1.920	-1.954	-1.987	-2.021	-2.054	-2.087	-2.120	-2.153
-40	-1.475	-1.510	-1.545	-1.579	-1.614	-1.648	-1.683	-1.717	-1.751	-1.785	-1.819
-30	-1.121	-1.157	-1.192	-1.228	-1.264	-1.299	-1.335	-1.370	-1.405	-1.440	-1.475
-20	-0.757	-0.794	-0.830	-0.867	-0.904	-0.940	-0.976	-1.013	-1.049	-1.085	-1.121
-10	-0.383	-0.421	-0.459	-0.496	-0.534	-0.571	-0.608	-0.646	-0.683	-0.720	-0.757
0	0.000	-0.039	-0.077	-0.116	-0.154	-0.193	-0.231	-0.269	-0.307	-0.345	-0.383
°C	0	-1	-2	-3	-4	-5	-6	-7	-8	-9	-10

ITS-90 Table for type T thermocouple

°C	0	1	2	3	4	5	6	7	8	9	10
Thermoelectric Voltage in mV											
0	0.000	0.039	0.078	0.117	0.156	0.195	0.234	0.273	0.312	0.352	0.391
10	0.391	0.431	0.470	0.510	0.549	0.589	0.629	0.669	0.709	0.749	0.790
20	0.790	0.830	0.870	0.911	0.951	0.992	1.033	1.074	1.114	1.155	1.196
30	1.196	1.238	1.279	1.320	1.362	1.403	1.445	1.486	1.528	1.570	1.612
40	1.612	1.654	1.696	1.738	1.780	1.823	1.865	1.908	1.950	1.993	2.036
50	2.036	2.079	2.122	2.165	2.208	2.251	2.294	2.338	2.381	2.425	2.468
60	2.468	2.512	2.556	2.600	2.643	2.687	2.732	2.776	2.820	2.864	2.909
70	2.909	2.953	2.998	3.043	3.087	3.132	3.177	3.222	3.267	3.312	3.358
80	3.358	3.403	3.448	3.494	3.539	3.585	3.631	3.677	3.722	3.768	3.814
90	3.814	3.860	3.907	3.953	3.999	4.046	4.092	4.138	4.185	4.232	4.279
100	4.279	4.325	4.372	4.419	4.466	4.513	4.561	4.608	4.655	4.702	4.750
110	4.750	4.798	4.845	4.893	4.941	4.988	5.036	5.084	5.132	5.180	5.228
120	5.228	5.277	5.325	5.373	5.422	5.470	5.519	5.567	5.616	5.665	5.714
130	5.714	5.763	5.812	5.861	5.910	5.959	6.008	6.057	6.107	6.156	6.206
140	6.206	6.255	6.305	6.355	6.404	6.454	6.504	6.554	6.604	6.654	6.704
150	6.704	6.754	6.805	6.855	6.905	6.956	7.006	7.057	7.107	7.158	7.209
160	7.209	7.260	7.310	7.361	7.412	7.463	7.515	7.566	7.617	7.668	7.720
170	7.720	7.771	7.823	7.874	7.926	7.977	8.029	8.081	8.133	8.185	8.237
180	8.237	8.289	8.341	8.393	8.445	8.497	8.550	8.602	8.654	8.707	8.759
190	8.759	8.812	8.865	8.917	8.970	9.023	9.076	9.129	9.182	9.235	9.288
200	9.288	9.341	9.395	9.448	9.501	9.555	9.608	9.662	9.715	9.769	9.822
210	9.822	9.876	9.930	9.984	10.038	10.092	10.146	10.200	10.254	10.308	10.362
220	10.362	10.417	10.471	10.525	10.580	10.634	10.689	10.743	10.798	10.853	10.907
230	10.907	10.962	11.017	11.072	11.127	11.182	11.237	11.292	11.347	11.403	11.458
240	11.458	11.513	11.569	11.624	11.680	11.735	11.791	11.846	11.902	11.958	12.013
250	12.013	12.069	12.125	12.181	12.237	12.293	12.349	12.405	12.461	12.518	12.574
260	12.574	12.630	12.687	12.743	12.799	12.856	12.912	12.969	13.026	13.082	13.139
270	13.139	13.196	13.253	13.310	13.366	13.423	13.480	13.537	13.595	13.652	13.709
280	13.709	13.766	13.823	13.881	13.938	13.995	14.053	14.110	14.168	14.226	14.283
290	14.283	14.341	14.399	14.456	14.514	14.572	14.630	14.688	14.746	14.804	14.862
300	14.862	14.920	14.978	15.036	15.095	15.153	15.211	15.270	15.328	15.386	15.445
310	15.445	15.503	15.562	15.621	15.679	15.738	15.797	15.856	15.914	15.973	16.032
320	16.032	16.091	16.150	16.209	16.268	16.327	16.387	16.446	16.505	16.564	16.624
330	16.624	16.683	16.742	16.802	16.861	16.921	16.980	17.040	17.100	17.159	17.219
340	17.219	17.279	17.339	17.399	17.458	17.518	17.578	17.638	17.698	17.759	17.819
350	17.819	17.879	17.939	17.999	18.060	18.120	18.180	18.241	18.301	18.362	18.422
360	18.422	18.483	18.543	18.604	18.665	18.725	18.786	18.847	18.908	18.969	19.030
370	19.030	19.091	19.152	19.213	19.274	19.335	19.396	19.457	19.518	19.579	19.641
380	19.641	19.702	19.763	19.825	19.886	19.947	20.009	20.070	20.132	20.193	20.255
390	20.255	20.317	20.378	20.440	20.502	20.563	20.625	20.687	20.748	20.810	20.872
400	20.872										
°C	0	1	2	3	4	5	6	7	8	9	10

### Typical Table of Thermoelectric voltages for type K Thermocouple (Chromel-Alumel):

ITS-90 Table for type K thermocouple	0	-1	-2	-3	-4	-5	-6	-7	-8	-9	-10
	°C										
Thermoelectric Voltage in mV											
-270	-6.458										
-260	-6.441	-6.444	-6.446	-6.448	-6.450	-6.452	-6.453	-6.455	-6.456	-6.457	-6.458
-250	-6.404	-6.408	-6.413	-6.417	-6.421	-6.425	-6.429	-6.432	-6.435	-6.438	-6.441
-240	-6.344	-6.351	-6.358	-6.364	-6.370	-6.377	-6.382	-6.388	-6.393	-6.399	-6.404
-230	-6.262	-6.271	-6.280	-6.289	-6.297	-6.306	-6.314	-6.322	-6.329	-6.337	-6.344
-220	-6.158	-6.170	-6.181	-6.192	-6.202	-6.213	-6.223	-6.233	-6.243	-6.252	-6.262
-210	-6.035	-6.048	-6.061	-6.074	-6.087	-6.099	-6.111	-6.123	-6.135	-6.147	-6.158
-200	-5.891	-5.907	-5.922	-5.936	-5.951	-5.965	-5.980	-5.994	-6.007	-6.021	-6.035
-190	-5.730	-5.747	-5.763	-5.780	-5.797	-5.813	-5.829	-5.845	-5.861	-5.876	-5.891
-180	-5.550	-5.569	-5.588	-5.606	-5.624	-5.642	-5.660	-5.678	-5.695	-5.713	-5.730
-170	-5.354	-5.374	-5.395	-5.415	-5.435	-5.454	-5.474	-5.493	-5.512	-5.531	-5.550
-160	-5.141	-5.163	-5.185	-5.207	-5.228	-5.250	-5.271	-5.292	-5.313	-5.333	-5.354
-150	-4.913	-4.936	-4.960	-4.983	-5.006	-5.029	-5.052	-5.074	-5.097	-5.119	-5.141
-140	-4.669	-4.694	-4.719	-4.744	-4.768	-4.793	-4.817	-4.841	-4.865	-4.889	-4.913
-130	-4.411	-4.437	-4.463	-4.490	-4.516	-4.542	-4.567	-4.593	-4.618	-4.644	-4.669
-120	-4.138	-4.166	-4.194	-4.221	-4.249	-4.276	-4.303	-4.330	-4.357	-4.384	-4.411
-110	-3.852	-3.882	-3.911	-3.939	-3.968	-3.997	-4.025	-4.054	-4.082	-4.110	-4.138
-100	-3.554	-3.584	-3.614	-3.645	-3.675	-3.705	-3.734	-3.764	-3.794	-3.823	-3.852

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-90	-3.243	-3.274	-3.306	-3.337	-3.368	-3.400	-3.431	-3.462	-3.492	-3.523	-3.554
-80	-2.920	-2.953	-2.986	-3.018	-3.050	-3.083	-3.115	-3.147	-3.179	-3.211	-3.243
-70	-2.587	-2.620	-2.654	-2.688	-2.721	-2.755	-2.788	-2.821	-2.854	-2.887	-2.920
-60	-2.243	-2.278	-2.312	-2.347	-2.382	-2.416	-2.450	-2.485	-2.519	-2.553	-2.587
-50	-1.889	-1.925	-1.961	-1.996	-2.032	-2.067	-2.103	-2.138	-2.173	-2.208	-2.243
-40	-1.527	-1.564	-1.600	-1.637	-1.673	-1.709	-1.745	-1.782	-1.818	-1.854	-1.889
-30	-1.156	-1.194	-1.231	-1.268	-1.305	-1.343	-1.380	-1.417	-1.453	-1.490	-1.527
-20	-0.778	-0.816	-0.854	-0.892	-0.930	-0.968	-1.006	-1.043	-1.081	-1.119	-1.156
-10	-0.392	-0.431	-0.470	-0.508	-0.547	-0.586	-0.624	-0.663	-0.701	-0.739	-0.778
0	0.000	-0.039	-0.079	-0.118	-0.157	-0.197	-0.236	-0.275	-0.314	-0.353	-0.392

°C      0      -1      -2      -3      -4      -5      -6      -7      -8      -9      -10

ITS-90 Table for type K thermocouple

°C	0	1	2	3	4	5	6	7	8	9	10
Thermoelectric Voltage in mV											
0	0.000	0.039	0.079	0.119	0.158	0.198	0.238	0.277	0.317	0.357	0.397
10	0.397	0.437	0.477	0.517	0.557	0.597	0.637	0.677	0.718	0.758	0.798
20	0.798	0.838	0.879	0.919	0.960	1.000	1.041	1.081	1.122	1.163	1.203
30	1.203	1.244	1.285	1.326	1.366	1.407	1.448	1.489	1.530	1.571	1.612
40	1.612	1.653	1.694	1.735	1.776	1.817	1.858	1.899	1.941	1.982	2.023
50	2.023	2.064	2.106	2.147	2.188	2.230	2.271	2.312	2.354	2.395	2.436
60	2.436	2.478	2.519	2.561	2.602	2.644	2.685	2.727	2.768	2.810	2.851
70	2.851	2.893	2.934	2.976	3.017	3.059	3.100	3.142	3.184	3.225	3.267
80	3.267	3.308	3.350	3.391	3.433	3.474	3.516	3.557	3.599	3.640	3.682
90	3.682	3.723	3.765	3.806	3.848	3.889	3.931	3.972	4.013	4.055	4.096
100	4.096	4.138	4.179	4.220	4.262	4.303	4.344	4.385	4.427	4.468	4.509
110	4.509	4.550	4.591	4.633	4.674	4.715	4.756	4.797	4.838	4.879	4.920
120	4.920	4.961	5.002	5.043	5.084	5.124	5.165	5.206	5.247	5.288	5.328
130	5.328	5.369	5.410	5.450	5.491	5.532	5.572	5.613	5.653	5.694	5.735
140	5.735	5.775	5.815	5.856	5.896	5.937	5.977	6.017	6.058	6.098	6.138
150	6.138	6.179	6.219	6.259	6.299	6.339	6.380	6.420	6.460	6.500	6.540
160	6.540	6.580	6.620	6.660	6.701	6.741	6.781	6.821	6.861	6.901	6.941
170	6.941	6.981	7.021	7.060	7.100	7.140	7.180	7.220	7.260	7.300	7.340
180	7.340	7.380	7.420	7.460	7.500	7.540	7.579	7.619	7.659	7.699	7.739
190	7.739	7.779	7.819	7.859	7.899	7.939	7.979	8.019	8.059	8.099	8.138

200	8.138	8.178	8.218	8.258	8.298	8.338	8.378	8.418	8.458	8.499	8.539
210	8.539	8.579	8.619	8.659	8.699	8.739	8.779	8.819	8.860	8.900	8.940
220	8.940	8.980	9.020	9.061	9.101	9.141	9.181	9.222	9.262	9.302	9.343
230	9.343	9.383	9.423	9.464	9.504	9.545	9.585	9.626	9.666	9.707	9.747
240	9.747	9.788	9.828	9.869	9.909	9.950	9.991	10.031	10.072	10.113	10.153
250	10.153	10.194	10.235	10.276	10.316	10.357	10.398	10.439	10.480	10.520	10.561
260	10.561	10.602	10.643	10.684	10.725	10.766	10.807	10.848	10.889	10.930	10.971
270	10.971	11.012	11.053	11.094	11.135	11.176	11.217	11.259	11.300	11.341	11.382
280	11.382	11.423	11.465	11.506	11.547	11.588	11.630	11.671	11.712	11.753	11.795
290	11.795	11.836	11.877	11.919	11.960	12.001	12.043	12.084	12.126	12.167	12.209
300	12.209	12.250	12.291	12.333	12.374	12.416	12.457	12.499	12.540	12.582	12.624
310	12.624	12.665	12.707	12.748	12.790	12.831	12.873	12.915	12.956	12.998	13.040
320	13.040	13.081	13.123	13.165	13.206	13.248	13.290	13.331	13.373	13.415	13.457
330	13.457	13.498	13.540	13.582	13.624	13.665	13.707	13.749	13.791	13.833	13.874
340	13.874	13.916	13.958	14.000	14.042	14.084	14.126	14.167	14.209	14.251	14.293
350	14.293	14.335	14.377	14.419	14.461	14.503	14.545	14.587	14.629	14.671	14.713
360	14.713	14.755	14.797	14.839	14.881	14.923	14.965	15.007	15.049	15.091	15.133
370	15.133	15.175	15.217	15.259	15.301	15.343	15.385	15.427	15.469	15.511	15.554
380	15.554	15.596	15.638	15.680	15.722	15.764	15.806	15.849	15.891	15.933	15.975
390	15.975	16.017	16.059	16.102	16.144	16.186	16.228	16.270	16.313	16.355	16.397
400	16.397	16.439	16.482	16.524	16.566	16.608	16.651	16.693	16.735	16.778	16.820
410	16.820	16.862	16.904	16.947	16.989	17.031	17.074	17.116	17.158	17.201	17.243
420	17.243	17.285	17.328	17.370	17.413	17.455	17.497	17.540	17.582	17.624	17.667
430	17.667	17.709	17.752	17.794	17.837	17.879	17.921	17.964	18.006	18.049	18.091
440	18.091	18.134	18.176	18.218	18.261	18.303	18.346	18.388	18.431	18.473	18.516
450	18.516	18.558	18.601	18.643	18.686	18.728	18.771	18.813	18.856	18.898	18.941
460	18.941	18.983	19.026	19.068	19.111	19.154	19.196	19.239	19.281	19.324	19.366
470	19.366	19.409	19.451	19.494	19.537	19.579	19.622	19.664	19.707	19.750	19.792
480	19.792	19.835	19.877	19.920	19.962	20.005	20.048	20.090	20.133	20.175	20.218
490	20.218	20.261	20.303	20.346	20.389	20.431	20.474	20.516	20.559	20.602	20.644

°C	0	1	2	3	4	5	6	7	8	9	10
ITS-90 Table for type K thermocouple											
°C	0	1	2	3	4	5	6	7	8	9	10
Thermoelectric Voltage in mV											
500	20.644	20.687	20.730	20.772	20.815	20.857	20.900	20.943	20.985	21.028	21.071
510	21.071	21.113	21.156	21.199	21.241	21.284	21.326	21.369	21.412	21.454	21.497
520	21.497	21.540	21.582	21.625	21.668	21.710	21.753	21.796	21.838	21.881	21.924
530	21.924	21.966	22.009	22.052	22.094	22.137	22.179	22.222	22.265	22.307	22.350
540	22.350	22.393	22.435	22.478	22.521	22.563	22.606	22.649	22.691	22.734	22.776
550	22.776	22.819	22.862	22.904	22.947	22.990	23.032	23.075	23.117	23.160	23.203
560	23.203	23.245	23.288	23.331	23.373	23.416	23.458	23.501	23.544	23.586	23.629
570	23.629	23.671	23.714	23.757	23.799	23.842	23.884	23.927	23.970	24.012	24.055
580	24.055	24.097	24.140	24.182	24.225	24.267	24.310	24.353	24.395	24.438	24.480
590	24.480	24.523	24.565	24.608	24.650	24.693	24.735	24.778	24.820	24.863	24.905
600	24.905	24.948	24.990	25.033	25.075	25.118	25.160	25.203	25.245	25.288	25.330
610	25.330	25.373	25.415	25.458	25.500	25.543	25.585	25.627	25.670	25.712	25.755
620	25.755	25.797	25.840	25.882	25.924	25.967	26.009	26.052	26.094	26.136	26.179
630	26.179	26.221	26.263	26.306	26.348	26.390	26.433	26.475	26.517	26.560	26.602
640	26.602	26.644	26.687	26.729	26.771	26.814	26.856	26.898	26.940	26.983	27.025

650 27.025 27.067 27.109 27.152 27.194 27.236 27.278 27.320 27.363 27.405 27.447  
660 27.447 27.489 27.531 27.574 27.616 27.658 27.700 27.742 27.784 27.826 27.869  
670 27.869 27.911 27.953 27.995 28.037 28.079 28.121 28.163 28.205 28.247 28.289  
680 28.289 28.332 28.374 28.416 28.458 28.500 28.542 28.584 28.626 28.668 28.710  
690 28.710 28.752 28.794 28.835 28.877 28.919 28.961 29.003 29.045 29.087 29.129  
  
700 29.129 29.171 29.213 29.255 29.297 29.338 29.380 29.422 29.464 29.506 29.548  
710 29.548 29.589 29.631 29.673 29.715 29.757 29.798 29.840 29.882 29.924 29.965  
720 29.965 30.007 30.049 30.090 30.132 30.174 30.216 30.257 30.299 30.341 30.382  
730 30.382 30.424 30.466 30.507 30.549 30.590 30.632 30.674 30.715 30.757 30.798  
740 30.798 30.840 30.881 30.923 30.964 31.006 31.047 31.089 31.130 31.172 31.213  
  
750 31.213 31.255 31.296 31.338 31.379 31.421 31.462 31.504 31.545 31.586 31.628  
760 31.628 31.669 31.710 31.752 31.793 31.834 31.876 31.917 31.958 32.000 32.041  
770 32.041 32.082 32.124 32.165 32.206 32.247 32.289 32.330 32.371 32.412 32.453  
780 32.453 32.495 32.536 32.577 32.618 32.659 32.700 32.742 32.783 32.824 32.865  
790 32.865 32.906 32.947 32.988 33.029 33.070 33.111 33.152 33.193 33.234 33.275

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810	33.685	33.726	33.767	33.808	33.848	33.889	33.930	33.971	34.012	34.053	34.093
820	34.093	34.134	34.175	34.216	34.257	34.297	34.338	34.379	34.420	34.460	34.501
830	34.501	34.542	34.582	34.623	34.664	34.704	34.745	34.786	34.826	34.867	34.908
840	34.908	34.948	34.989	35.029	35.070	35.110	35.151	35.192	35.232	35.273	35.313
850	35.313	35.354	35.394	35.435	35.475	35.516	35.556	35.596	35.637	35.677	35.718
860	35.718	35.758	35.798	35.839	35.879	35.920	35.960	36.000	36.041	36.081	36.121
870	36.121	36.162	36.202	36.242	36.282	36.323	36.363	36.403	36.443	36.484	36.524
880	36.524	36.564	36.604	36.644	36.685	36.725	36.765	36.805	36.845	36.885	36.925
890	36.925	36.965	37.006	37.046	37.086	37.126	37.166	37.206	37.246	37.286	37.326
900	37.326	37.366	37.406	37.446	37.486	37.526	37.566	37.606	37.646	37.686	37.725
910	37.725	37.765	37.805	37.845	37.885	37.925	37.965	38.005	38.044	38.084	38.124
920	38.124	38.164	38.204	38.243	38.283	38.323	38.363	38.402	38.442	38.482	38.522
930	38.522	38.561	38.601	38.641	38.680	38.720	38.760	38.799	38.839	38.878	38.918
940	38.918	38.958	38.997	39.037	39.076	39.116	39.155	39.195	39.235	39.274	39.314
950	39.314	39.353	39.393	39.432	39.471	39.511	39.550	39.590	39.629	39.669	39.708
960	39.708	39.747	39.787	39.826	39.866	39.905	39.944	39.984	40.023	40.062	40.101
970	40.101	40.141	40.180	40.219	40.259	40.298	40.337	40.376	40.415	40.455	40.494
980	40.494	40.533	40.572	40.611	40.651	40.690	40.729	40.768	40.807	40.846	40.885
990	40.885	40.924	40.963	41.002	41.042	41.081	41.120	41.159	41.198	41.237	41.276

°C      0      1      2      3      4      5      6      7      8      9      10

ITS-90 Table for type K thermocouple

°C	0	1	2	3	4	5	6	7	8	9	10
	Thermoelectric Voltage in mV										
1000	41.276	41.315	41.354	41.393	41.431	41.470	41.509	41.548	41.587	41.626	41.665
1010	41.665	41.704	41.743	41.781	41.820	41.859	41.898	41.937	41.976	42.014	42.053
1020	42.053	42.092	42.131	42.169	42.208	42.247	42.286	42.324	42.363	42.402	42.440
1030	42.440	42.479	42.518	42.556	42.595	42.633	42.672	42.711	42.749	42.788	42.826
1040	42.826	42.865	42.903	42.942	42.980	43.019	43.057	43.096	43.134	43.173	43.211
1050	43.211	43.250	43.288	43.327	43.365	43.403	43.442	43.480	43.518	43.557	43.595
1060	43.595	43.633	43.672	43.710	43.748	43.787	43.825	43.863	43.901	43.940	43.978
1070	43.978	44.016	44.054	44.092	44.130	44.169	44.207	44.245	44.283	44.321	44.359
1080	44.359	44.397	44.435	44.473	44.512	44.550	44.588	44.626	44.664	44.702	44.740
1090	44.740	44.778	44.816	44.853	44.891	44.929	44.967	45.005	45.043	45.081	45.119
1100	45.119	45.157	45.194	45.232	45.270	45.308	45.346	45.383	45.421	45.459	45.497
1110	45.497	45.534	45.572	45.610	45.647	45.685	45.723	45.760	45.798	45.836	45.873
1120	45.873	45.911	45.948	45.986	46.024	46.061	46.099	46.136	46.174	46.211	46.249
1130	46.249	46.286	46.324	46.361	46.398	46.436	46.473	46.511	46.548	46.585	46.623
1140	46.623	46.660	46.697	46.735	46.772	46.809	46.847	46.884	46.921	46.958	46.995
1150	46.995	47.033	47.070	47.107	47.144	47.181	47.218	47.256	47.293	47.330	47.367
1160	47.367	47.404	47.441	47.478	47.515	47.552	47.589	47.626	47.663	47.700	47.737
1170	47.737	47.774	47.811	47.848	47.884	47.921	47.958	47.995	48.032	48.069	48.105
1180	48.105	48.142	48.179	48.216	48.252	48.289	48.326	48.363	48.399	48.436	48.473
1190	48.473	48.509	48.546	48.582	48.619	48.656	48.692	48.729	48.765	48.802	48.838

1200	48.838	48.875	48.911	48.948	48.984	49.021	49.057	49.093	49.130	49.166	49.202
1210	49.202	49.239	49.275	49.311	49.348	49.384	49.420	49.456	49.493	49.529	49.565
1220	49.565	49.601	49.637	49.674	49.710	49.746	49.782	49.818	49.854	49.890	49.926
1230	49.926	49.962	49.998	50.034	50.070	50.106	50.142	50.178	50.214	50.250	50.286
1240	50.286	50.322	50.358	50.393	50.429	50.465	50.501	50.537	50.572	50.608	50.644
1250	50.644	50.680	50.715	50.751	50.787	50.822	50.858	50.894	50.929	50.965	51.000
1260	51.000	51.036	51.071	51.107	51.142	51.178	51.213	51.249	51.284	51.320	51.355
1270	51.355	51.391	51.426	51.461	51.497	51.532	51.567	51.603	51.638	51.673	51.708
1280	51.708	51.744	51.779	51.814	51.849	51.885	51.920	51.955	51.990	52.025	52.060
1290	52.060	52.095	52.130	52.165	52.200	52.235	52.270	52.305	52.340	52.375	52.410
1300	52.410	52.445	52.480	52.515	52.550	52.585	52.620	52.654	52.689	52.724	52.759
1310	52.759	52.794	52.828	52.863	52.898	52.932	52.967	53.002	53.037	53.071	53.106
1320	53.106	53.140	53.175	53.210	53.244	53.279	53.313	53.348	53.382	53.417	53.451
1330	53.451	53.486	53.520	53.555	53.589	53.623	53.658	53.692	53.727	53.761	53.795
1340	53.795	53.830	53.864	53.898	53.932	53.967	54.001	54.035	54.069	54.104	54.138
1350	54.138	54.172	54.206	54.240	54.274	54.308	54.343	54.377	54.411	54.445	54.479
1360	54.479	54.513	54.547	54.581	54.615	54.649	54.683	54.717	54.751	54.785	54.819
1370	54.819	54.852	54.886								

°C      0      1      2      3      4      5      6      7      8      9      10

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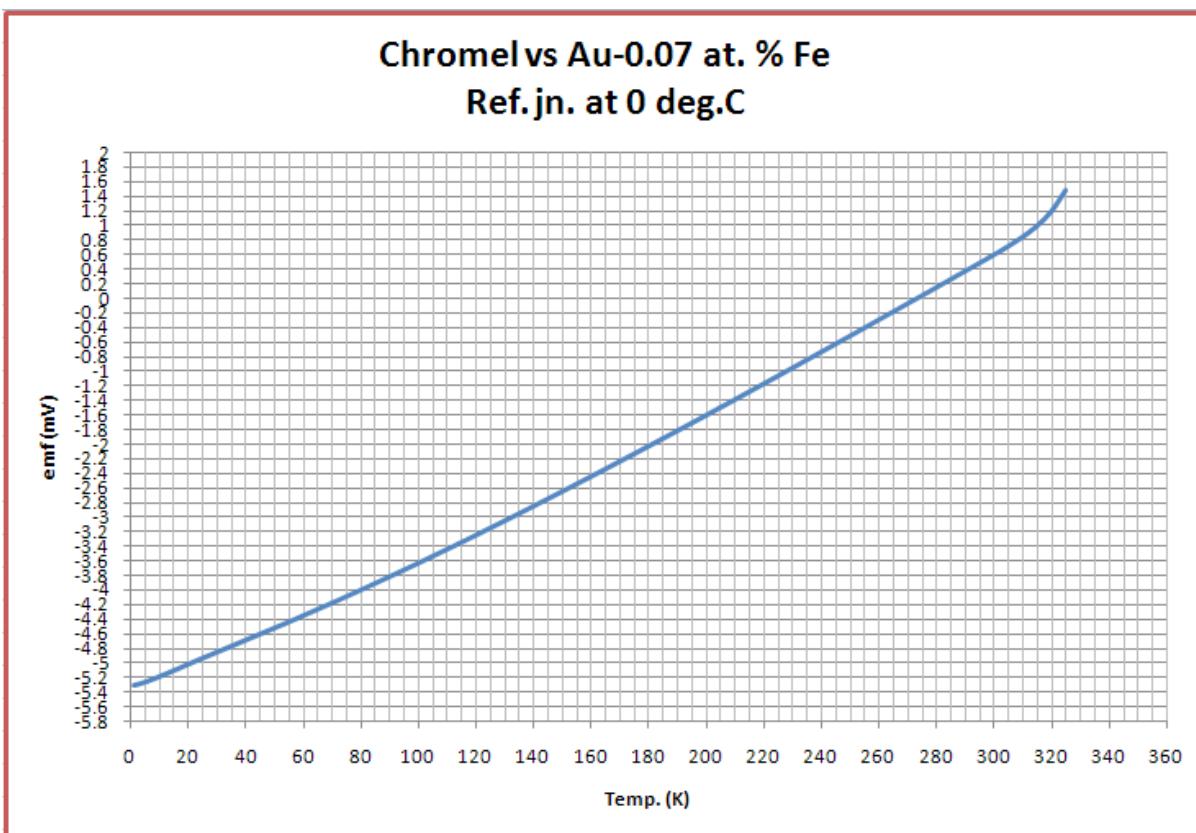
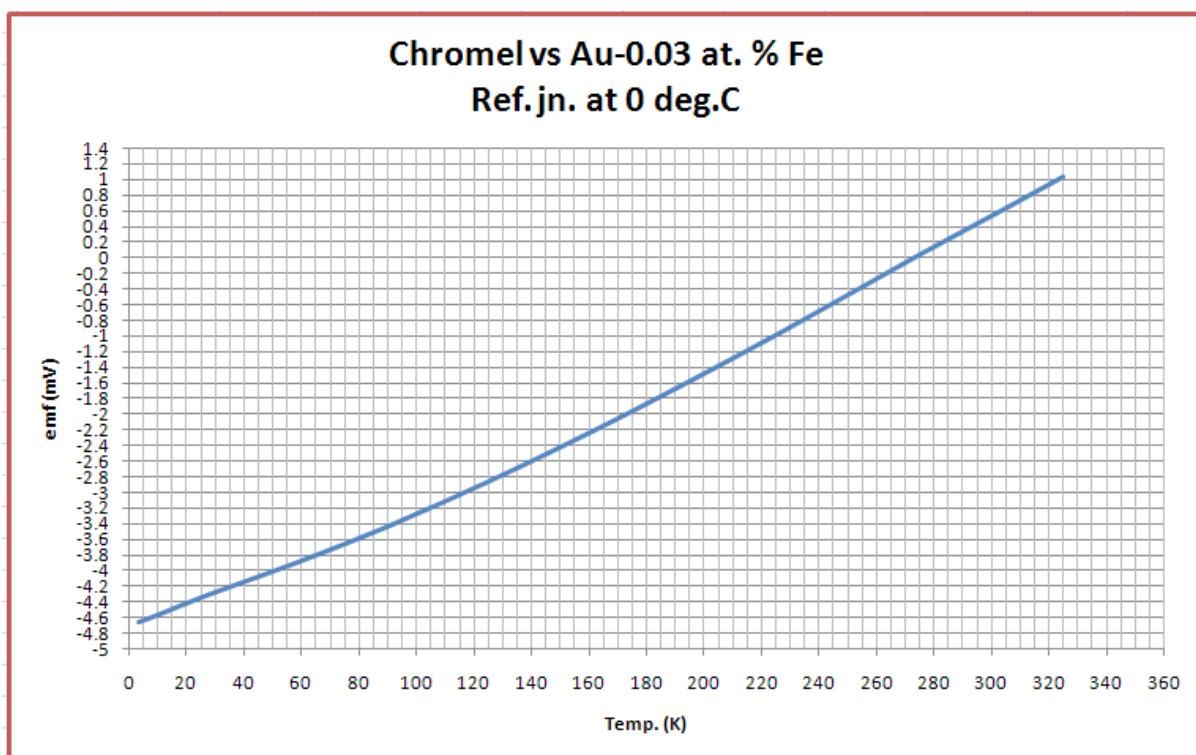


**Reference Tables for Chromel vs Gold/Iron thermocouples, used in cryogenic applications, are given in Ref. [11]:**

**Thermocouple Curves – Chromel versus Gold/Iron**

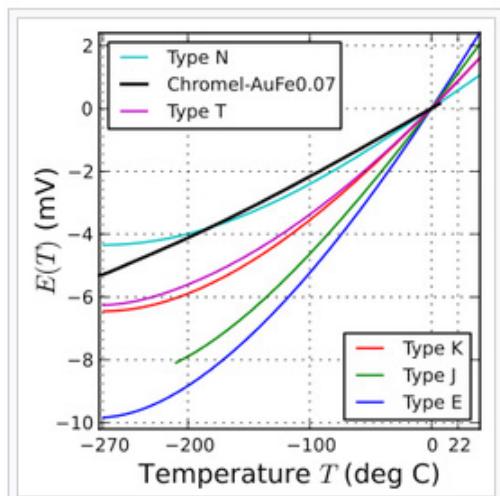
Chromel vs. Au-0.03 at.% Fe		Chromel vs. Au-0.07 at.% Fe	
Temp (K)	V <sub>TC</sub> (mV)	Temp (K)	V <sub>TC</sub> (mV)
3.5	-4.6676	1.4	-5.2982
8.0	-4.6067	3.0	-5.2815
13.5	-4.5259	4.8	-5.2594
18.0	-4.4571	7.0	-5.2285
24.0	-4.3703	10.5	-5.1742
30.0	-4.2869	19.0	-5.0315
52.0	-3.9928	26.0	-4.9126
60.0	-3.8830	48.0	-4.5494
65.0	-3.8126	58.0	-4.3810
70.0	-3.7411	70.0	-4.1733
80.0	-3.5948	80.0	-3.9952
90.0	-3.4436	90.0	-3.8132
105.0	-3.2026	100.0	-3.6270
115.0	-3.0374	110.0	-3.4370
125.0	-2.8689	120.0	-3.2435
135.0	-2.6957	135.0	-2.9477
145.0	-2.5184	150.0	-2.6452
160.0	-2.2468	165.0	-2.3372
170.0	-2.0615	180.0	-2.0242
180.0	-1.8725	200.0	-1.6004
195.0	-1.5839	220.0	-1.1693
210.0	-1.2905	245.0	-0.6232
225.0	-0.9912	270.0	-0.0705
240.0	-0.6847	300.0	+0.5986
265.0	-0.1670	305.0	+0.7158
275.0	+0.0378	310.0	+0.8431
285.0	+0.2387	315.0	+0.9944
305.0	+0.6350	320.0	+1.1940
325.0	+1.0387	325.0	+1.4841

Graphs of above tables, drawn in Excel, are given below:



### Comparison of Thermocouples used for low temps:

(Ref: <https://en.wikipedia.org/wiki/Thermocouple>)



**Note:** The AuFe-based thermocouple shows a steady sensitivity down to low temperatures, whereas conventional types soon flatten out and lose sensitivity at low temperature.

**Correct method** of connecting a thermocouple is shown below:

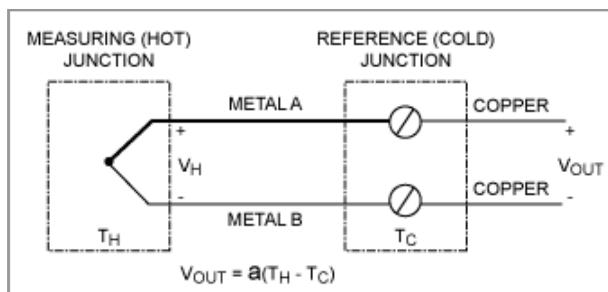


Fig. Using Reference junction for thermocouples

Reference junction is normally at 0 deg. C. However, if it is difficult to maintain 0 deg. C reference in practice, by suitable electronic circuitry, 'cold junction compensation' is provided.

**Barron [1] gives the following information on calibration coefficients for Thermocouples commonly used in cryogenics:**

**Polynomials for emf and temp:**

$$e = a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 \quad (6.12)$$

$$t = b_1 e + b_2 e^2 + b_3 e^3 + b_4 e^4 \quad (6.13)$$

Table 6.3. Coefficients in the calibration equations for cryogenic thermocouples. The reference junction for Type K and Type T thermocouples is 0°C (32°F); for the chromel-Au/0.03 Fe thermocouple, the reference junction is 77.3 K (139.1°R).  $t$  = temperature minus reference temperature;  $e$  = absolute value of emf.  $t$  is in °C or K;  $e$  is in mV. Refer to eqns. (6.12) and (6.13).

Coefficient	Copper-Constantan Type T	Chromel-Alumel Type K	Chromel-Au/0.03 Fe
$a_1$	$-3.87706 \times 10^{-2}$	$-3.94841 \times 10^{-2}$	$-1.53129 \times 10^{-2}$
$a_2$	$-4.56877 \times 10^{-5}$	$-2.83938 \times 10^{-5}$	$-7.87084 \times 10^{-5}$
$a_3$	$4.35205 \times 10^{-8}$	$1.13868 \times 10^{-7}$	$-9.79295 \times 10^{-7}$
$a_4$	$1.51931 \times 10^{-11}$	$2.57457 \times 10^{-11}$	$-1.59091 \times 10^{-9}$
$b_1$	-25.47763	-24.90286	-64.79915
$b_2$	-1.45756	-1.33496	-28.39826
$b_3$	0.29713	0.37298	24.75019
$b_4$	-0.06419	-0.06882	-2.34307

In the above Table, type T and K thermocouples are used in the range -200 C to ambient (and above).

Chromel-Au/0.03 Fe thermocouple is used in the range 10 K to 80 K.

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### Constant volume gas thermometer [1]:

Here, temperature is correlated to the pressure of the thermometer gas in a constant volume bulb.

The thermometer is calibrated by measuring the gas pressure  $p_s$  at a standard temp  $T_s$  (ex: triple point of water). Then, assuming that the gas behaves as an ideal gas, and neglecting the dead volume gas which is not at the bulb temperature, temp  $T$  corresponding to a bulb volume pressure  $p$  is given by:

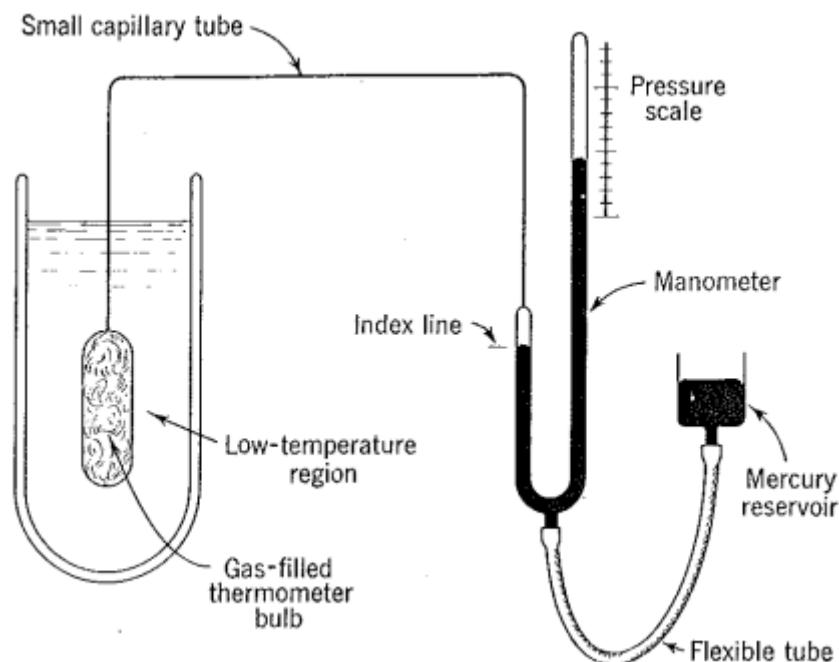
$$T = p \left( \frac{T_s}{p_s} \right)$$

For accurate work, we should include a correction to take in to account the fact that the dead volume  $V_0$  is not at the same temp as the bulb temp. Then, the temperature corresponding to a bulb pressure 'p' is given by:

$$T = K_1 \cdot p \cdot (T_s / p_s), \text{ where}$$

$K_1$  is the dead volume correction factor, given by:

$$K_1 = \left[ 1 + \left( 1 - \frac{p}{p_s} \right) \cdot \frac{V_0}{V} \cdot \frac{T_s}{T_0} \right]^{-1}$$



We should also take in to consideration the fact that the gas may not follow the ideal gas equation exactly. Then, for a real gas, equation of state in a virial expansion is:

$$pv = RT[1 + B(T)p + C(T)p^2 + D(T)p^3 + \dots]$$

where  $B(T)$ ,  $C(T)$  ... etc are virial coefficients.

For low pressures, the deviation from the ideal gas eqn is small, and we may use the first two terms of the virial eqn of state. Then, the density i.e. reciprocal of specific volume,  $v$ ) can be written as:

$$\rho = p/RT[1 + B(T)p]$$

And, the corrected temp is given by:

$$T = K_1(1 + K_2)(T_s/p_s)p$$

where  $K_2$  is the gas-imperfection correction factor, given by:

$$K_2 = \frac{B(T_s)p_s - B(T')p}{1 + B(T')p}$$

And,

$$T' = K_1(T_s/p_s)p$$

For Helium gas, between 4 K and 400 K, the second virial coeff is determined from:

$$B(T) = 0.0072929 T^{-5/4}(1 - C_1 T^{-1/2} - C_2 T^{-1} + C_3 T^{-3/2})$$

where

$$\begin{aligned}C_1 &= 3.04135 \\C_2 &= 14.14250 \\C_3 &= 17.35052\end{aligned}$$

And,  $B(T')$  is in  $\text{kPa}^{-1}$  and  $T$  in kelvins.

### Sensitivity of constant volume gas thermometer:

If the dead volume and gas imperfection corrections can be neglected, the sensitivity of a constant volume gas thermometer becomes a constant:

$$S_0 = \frac{dp}{dT} = \frac{p_s}{T_s}$$

If the dead volume effect cannot be neglected, we get the Sensitivity as:

$$S_0 = \frac{\frac{p_s}{T_s}}{K_1 \left( 1 + \frac{T}{T_0} \cdot V_0 \text{byV} \right)} = \frac{\frac{p_s}{T_s}}{\left( 1 + \frac{T}{T_0} \cdot V_0 \text{byV} \right) \cdot \left[ 1 + \left( 1 - \frac{p}{p_s} \right) \cdot V_0 \text{byV} \cdot \frac{T_s}{T_0} \right]^{-1}}$$

### Vapor pressure Thermometer [1]:

Since vapor pressure is a function of temp of liquid in contact with the vapor, this property is used to measure temperature.



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Following is the schematic fig:

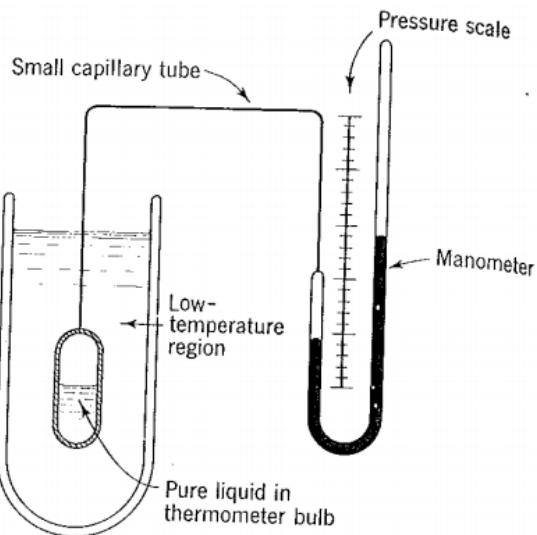


Fig. Vapor-pressure thermometer.

**Advantages:** Highly sensitive in the range of use, and, there is no need to have corrections for 'dead volume' and 'gas imperfections', as in the case of a gas thermometer.

**Disadvantage:** Limited range of use. Generally used in the pressure range of 5 kPa to 250 kPa. Also, thermometer fluid should be as pure as possible, since even slight impurities can cause deviations in vapor pressure-temp relations.

For a pure substance, vapor pressure is related to sat. temp. as follows:

$$\ln\left(\frac{P}{P_0}\right) = C_1 - \frac{C_2}{T} - C_3 \ln\left(\frac{T}{T_0}\right) - C_4 \cdot T + C_5 \cdot T^2$$

where,  $P_0 = 101.325$  kPa, and  $T_0$  = normal boiling point temp.

Constants  $C_1, C_2\dots$  are given in following table, for a few gases:

Table 6.4. Constants in the vapor-pressure relationship,  $\ln(p/p_0) = C_1 - C_2/T - C_3 \ln(T/T_0) - C_4 T + C_5 T^2$  where  $p_0 = 101.325 \text{ kPa}$ ,  $T_0 = 77.344 \text{ K}$  for N<sub>2</sub> and 90.188 K for O<sub>2</sub>, and the temperature  $T$  is in kelvins. Log base  $e$  is used.

	Hydrogen	Neon	Nitrogen	Oxygen
$C_1$	3.940796	10.618417	13.569758	13.726967
$C_2$	101.33783	244.96075	930.15333	1076.35667
$C_3$	0	0	2.3668	1.664512
$C_4$	-0.0543201	0.0848111	0.0328844	0.0304241
$C_5$	$-1.10563 \times 10^{-4}$	$9.78350 \times 10^{-4}$	$1.67138 \times 10^{-4}$	$1.16981 \times 10^{-4}$

### Sensitivity:

Sensitivity of vapor pressure thermometer is given by:

$S_0 = dp/dT$ , and we get:

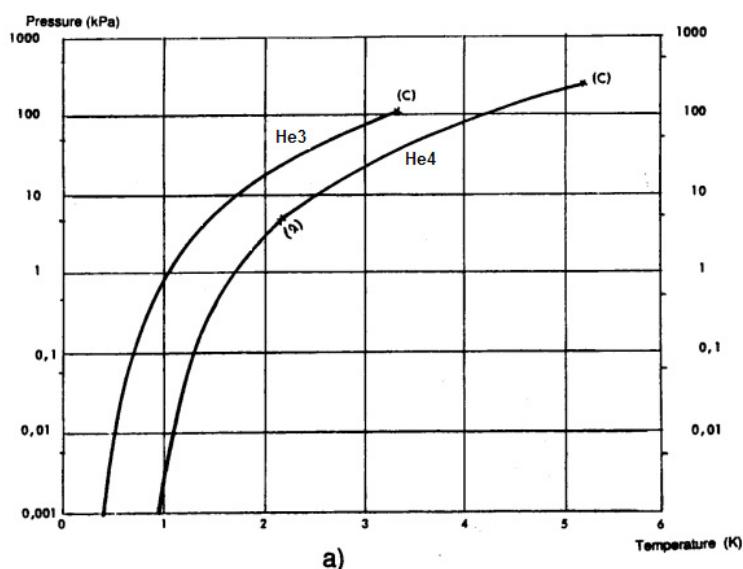
$$S_0 := p \left( \frac{C_2}{T^2} - \frac{C_3}{T} - C_4 + 2 \cdot C_5 \cdot T \right)$$

Precautions must be taken to ensure that the 'dead volume' is minimum such that the sensing element is not completely filled with liquid at the minimum temp. Also, no part of the connecting tube must be at a temp lower than that of the sensing element.

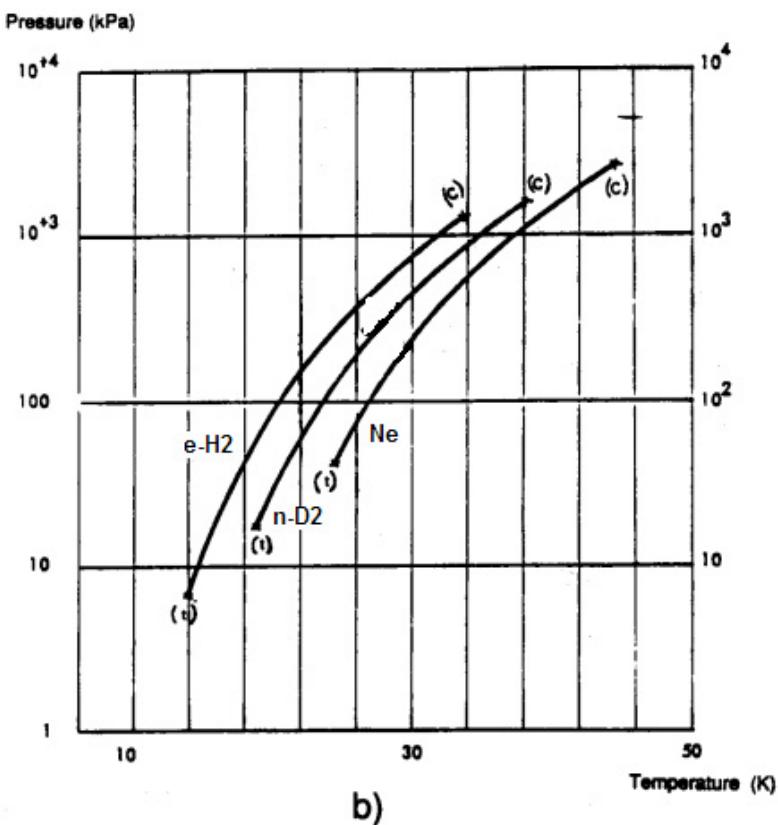
### Vapor pressure curves for a few substances:

Graphs (a) and (b) below are from Ref. [12]:

#### Vapor pressure curves for He3 and He4:



Vapor pressure curves for normal H<sub>2</sub>, D<sub>2</sub> and Ne:

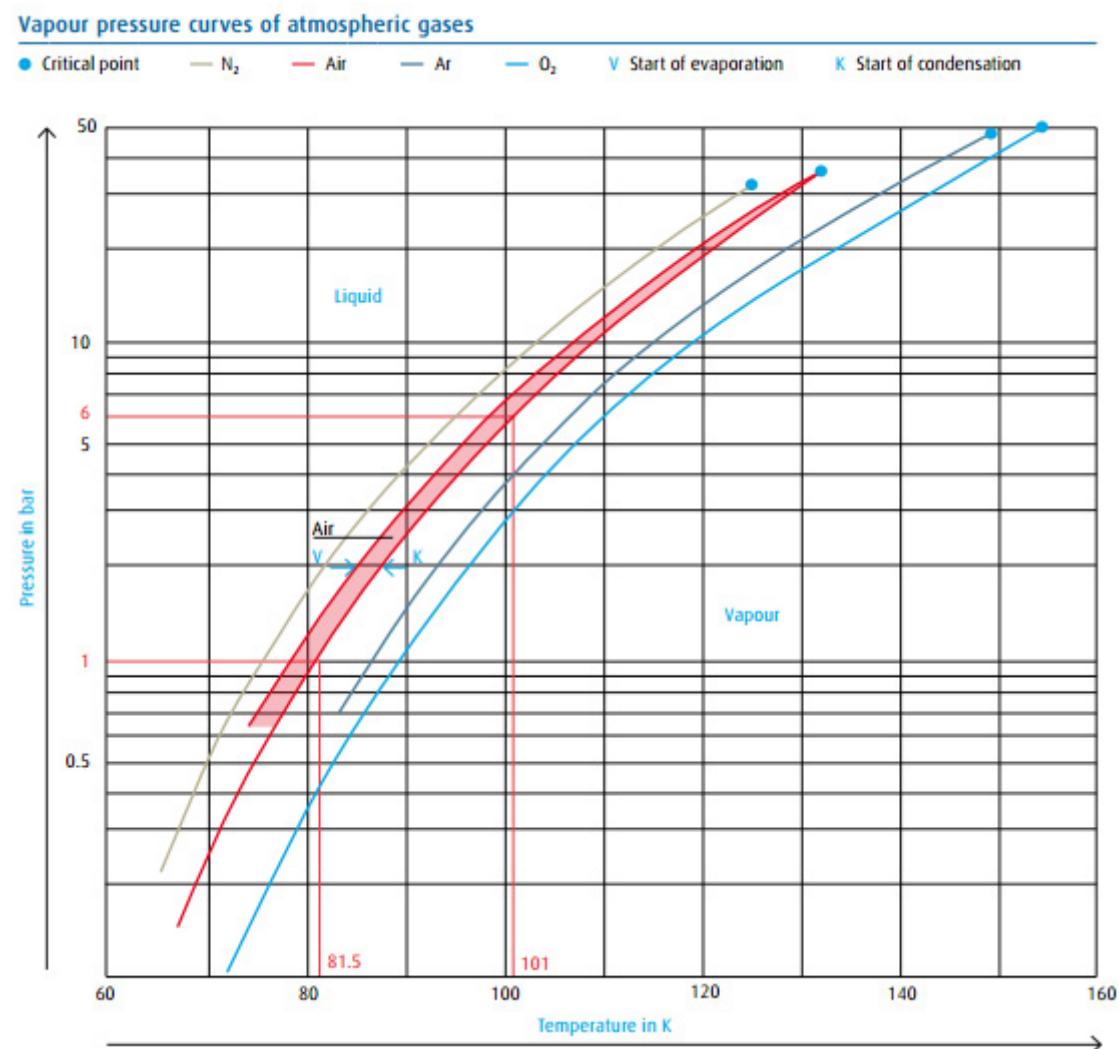


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**Note:** In the above graphs, points marked 'c' for critical point and those marked 't' for triple point.

**Vapor pressure curves for N<sub>2</sub>, O<sub>2</sub>, Air and A are given below:**

(Ref: Air separation – Linde Engineering)



### Magnetic Thermometer [1]:

This is used for temperatures below about 1 K.

Here, the principle is to measure the magnetic susceptibility of a paramagnetic material and relate it to the temp.

Magnetic susceptibility is:  $\chi = I/H$  where I is the magnetic moment per unit mass and H is the applied magnetic field intensity.

For paramagnetic materials which obey the Curie law, we have:

$$X = I/H = C/T \text{ where } C \text{ is the Curie constant.}$$

However, deviations from Curie law do occur at very low temp for all paramagnetic materials. Then, we define a ‘magnetic temp.  $T_{\text{star}}$ ’ by the Curie law:

$$T_{\text{star}} = C / X$$

When the Curie law is valid,  $T_{\text{star}} = T$ ; otherwise, the two temps are different.

Barron [1] describes the procedure of drawing the graph of absolute temp,  $T$  vs magnetic temp  $T_{\text{star}}$  for a paramagnetic material.

Ref.[1] gives the relation between the absolute temp and magnetic temp for some paramagnetic materials:

**For Cerium magnesium nitrate, for  $T \geq 0.004$  K:**

$$T^* = T + (0.236 + 0.004137/T)(10^{-3})$$

**For Chromium methylammonium alum, for  $T \geq 0.07$  K:**

$$T^* = T + 0.00250 + 0.002422/T$$

**For Chromium potassium alum, for  $T \geq 0.1$  K:**

$$T^* = T + 0.000862 + 0.002057/T$$

**Note:** In the above equations, temps are in kelvins.

### 5.1.2 MEASUREMENT OF VOLUMETRIC FLOW RATE AND MASS FLOW RATE [1]:

This is important in cryogenic fluid transfer systems.

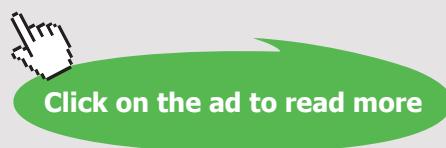
A List of Cryogenic flow meters and their operating principles is given in **IGC Doc 07/03/E of European Industrial Gases Association (EIGA)**:

#### **Appendix 2: List of cryogenic meters and their operating principles**

The meters listed in the following table represent a selection of those which have been used or thought to be suitable for cryogenic service. They are typical only, are not intended as a recommendation, and do not necessarily fall within the scope of the document.

Item	Type	Classification	Operating principles	Description of operation
1	Turbine (10,11)	Volumetric (indirect) velocity	A freely rotating element in the fluid stream measures mean velocity, volume being proportional to rotor revolutions.	A rotor with blades inclined at an angle to the axis of flow of the fluid is mounted on bearings inside a chamber. The number of revolutions of the impeller due to the passage of the liquid into meter is detected by means of a coil mounted on the chamber.
2	Turbine with density compensation *) (11,12)	Mass (indirect)	As item 1 with the addition of liquid temperature measurement to give an inferred density. The volumetric output from the turbine is modified according to variations in the liquid density to give flow in mass units.	The meter normally consists of a turbine as item 1, a temperature sensor and an electronic package. Signals from turbine pick -up and temperature sensor are converted by electronic circuits to a mass reading.

\*)The principle of using density correction to give a reading of flow in mass units may equally be applied to volumetric primary elements other than the turbine.



Item	Type	Classification	Operating principles	Description of operation
3	Coriolis force	Mass	Measure of mass, in liquid or vapour form, no influence of temperature, no need for straight piping upstream and downstream	A mass flow dependent Coriolis force occurs when a moving mass is subjected to an oscillation perpendicular to the flow direction. The measuring system accurately determines and evaluates the resulting effects on the measuring tubes.
4	Vortex shedding (13)	Volumetric (indirect) Velocity or Mass	An obstruction in the fluid stream sheds vortices at a frequency which is a measure of the velocity	The sensing element is in a stationary bluff body located across the flow stream. The sensor detects vortices shed alternately on either side in a periodic fashion. The signal produced is directly proportional to volumetric flow rate.
5	Orifice or nozzle or venturi (10,14)	Volumetric (indirect) Velocity or Mass	Change in velocity of the fluid passing through the element causes a change in kinetic energy. Differential pressure generated is a measure of fluid velocity.	An orifice plate or venturi device of known dimensions is placed in a length of pipe. Differential pressure created by flow of liquid through the restriction is measured using tappings located upstream and downstream.
6	Ultrasonic	Volumetric (indirect) Velocity or Mass ( indirect)	Determination of fluid velocity by observing the effect of fluid flow on the apparent velocity of propagation of sound waves in the fluid.	A number of alternative forms using different transducer and electronic arrangements are used.

### Orifice meters[1]:

Simple sharp edged orifice meters are used effectively to measure flow rates of cryogenic fluids. Here, upstream of the orifice, a straight length of at least 10 times the tube diameter should be provided to ensure symmetrical velocity distribution, and a straight length of about 5 times the tube dia should be provided on the downstream side.

**Calibration curve obtained with water as the flowing fluid can be applied for flow with liquid nitrogen, liquid oxygen and liquid hydrogen too as far as the flow upstream of the orifice is in single phase.**

A schematic fig. of the orifice meter is shown below.

Here note that pressure taps are at 'D and D/2', which means that on the upstream side, pressure tap is at a distance of D (i.e. pipe dia) from the orifice, and on the downstream side, the pressure tap is at a distance of D/2 from the orifice.

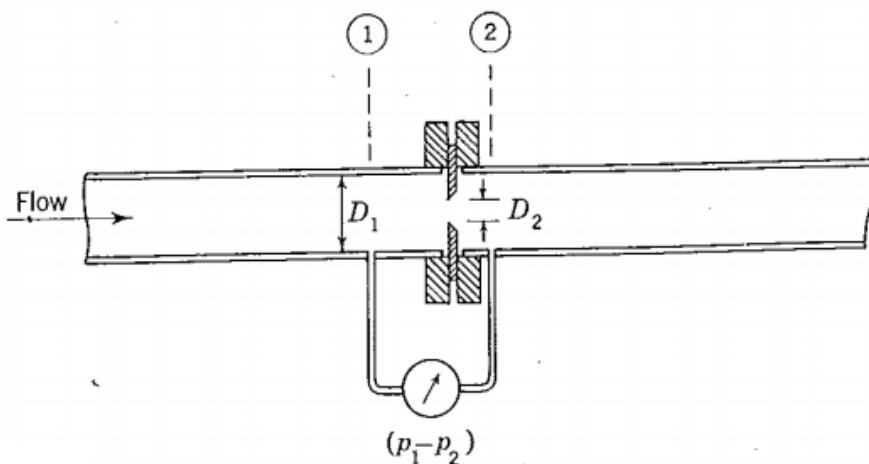


Fig. Orifice flowmeter.

Orifice meter is a simple and popular instrument to measure flow. However, compared to other instruments, it produces a larger pressure drop, which may not be desirable in some cases.

For liquid mass flow rate measurements, the governing equation for orifice meter is:

$$\dot{m} = C_d \cdot C_a \cdot A_0 \cdot (2 \cdot \rho \cdot \Delta P)^{\frac{1}{2}}$$

where

$C_d$  = discharge coeff.

$C_a$  = velocity of approach coeff.

$A_0$  = area of orifice

$\rho$  = fluid density

$\Delta P$  = pressure drop across the orifice

For a meter with 'D and D/2 pressure taps', coeff.  $C_d$  is given as:

$$C_d = 0.5959 + 0.0312 \cdot \beta^{2.1} - 0.184 \cdot \beta^8 + \frac{0.039 \cdot \beta^4}{1 - \beta^4} - 0.01584 \cdot \beta^3 + 91.71 \cdot \beta^{2.5} \cdot N_{Re}^{-0.75}$$

where

$$\beta = D_0/D_p$$

$D_0$  = orifice dia

$D_p$  = pipe inside dia

$$N_{Re} = \text{pipe Reynolds No.} = D_p * G / \mu$$

$G = m / A_p$  where  $m$  = mass flow rate, and  $A_p$  = pipe cross-sectional area



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Velocity of approach factor,  $C_a$  is given by:

$$C_a = \left(1 - \beta^4\right)^{\frac{-1}{2}}$$

**As an example of commercial Orifice meter for Cryogenic fluids**, see following info from the catalogue of Emerson Process.com/ Rosemount, Technical Note No. 00840-0200-4810, Rev AA, April 2016:[Ref. 13]

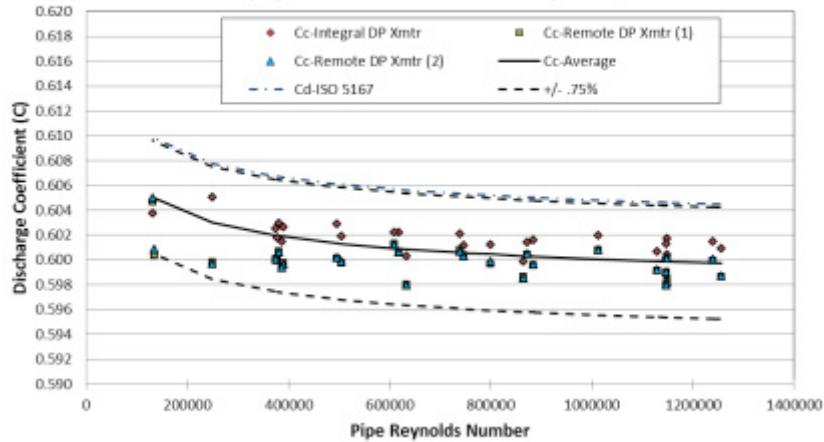
"Industrial gases are often moved or stored in a cryogenic liquid state, that is, at cold temperatures and pressures slightly above vapor pressure. In addition to industrial gases, natural gas is now often liquefied (referred to as LNG) and transported great distances to market. Flow metering in these conditions presents special challenges, the solutions to which are the subject of this paper.

At cryogenic conditions, these liquids provide little lubrication for moving parts and therefore create major challenges for traditional turbine and positive displacement flow meters. Additionally, the large temperature gradient presents concerns when O-rings, glands, welds, or dissimilar metals are present in the flow stream. As a result, reliable measurement of cryogenic liquids is difficult with traditional flow metering technologies."



Rosemount 3051SFC Compact  
Conditioning Differential Pressure  
Flowmeter

**Figure 1-2. Discharge Coefficient vs. Reynolds Number**  
**NIST Cryogenic Lab Test Results - July 29th, 2015**



### Venturi flow meters [1]:

One *disadvantage* of Orifice meter is the large frictional pressure drop. To avoid this, Venturi meters are used.

A schematic fig. of Venturi meter is shown below:

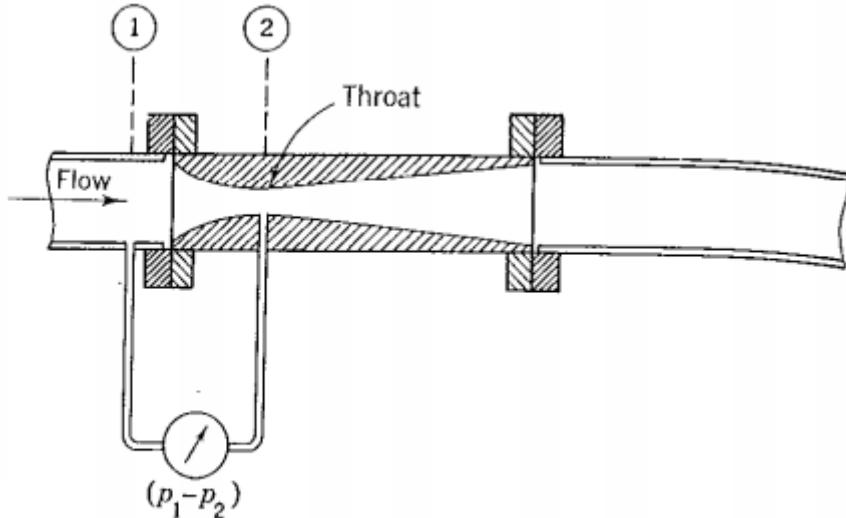


Fig. Venturi flowmeter.

Venturi meter consists of a conical reducer section, straight throat section, and a gradual enlargement to the original pipe diameter. Inlet cone angle is usually 20 to 22 degrees, and exit cone angle is about 5 to 7 degrees. Throat dia is about one-half the pipe dia. Pressure taps are placed at one-half pipe dia at the upstream of venture entrance and at the middle of the throat section.

Mass flow rate is given by the same expression as for an Orifice meter:

$$\dot{m} = C_d \cdot C_a \cdot A_0 \cdot (2 \cdot \rho \cdot \Delta P)^{\frac{1}{2}}$$

However, discharge coeff.  $C_d$  is nearly unity for Venturi meter.

For design purposes, following equations are used to determine  $C_d$ :

1. For  $3000 \leq N_{Re} \leq 2 \times 10^5$ ,

$$C_d = \frac{\log_{10} N_{Re}}{0.60 + 0.90 \log_{10} N_{Re}}$$

2. For  $N_{Re} > 2 \times 10^5$ ,

$$C_d = 0.988$$



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And, Reynolds No. is determined from:

$$N_{Re} = D_{\text{pipe}} \cdot \beta^2 \cdot \frac{(2 \cdot \rho \cdot \Delta P)^{\frac{1}{2}}}{\mu}$$

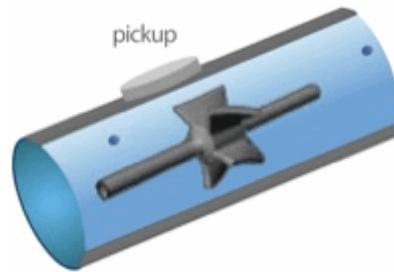
In the above,  $\beta$  is the ratio of throat dia to the pipe dia.

**Note:** To avoid cavitation problem, sufficiently high upstream pressure should be maintained so that vapor pressure is not reached at the throat.

### Turbine Flow meters [1]:

Following info is from the Ref. [14], i.e. <http://www.flowmeters.com/turbine-technology#catdesc>

#### Turbine Flow meter Technology:



“Turbine flow meters use the mechanical energy of the fluid to rotate a “pinwheel” (rotor) in the flow stream. Blades on the rotor are angled to transform energy from the flow stream into rotational energy. The rotor shaft spins on bearings. When the fluid moves faster, the rotor spins proportionally faster.

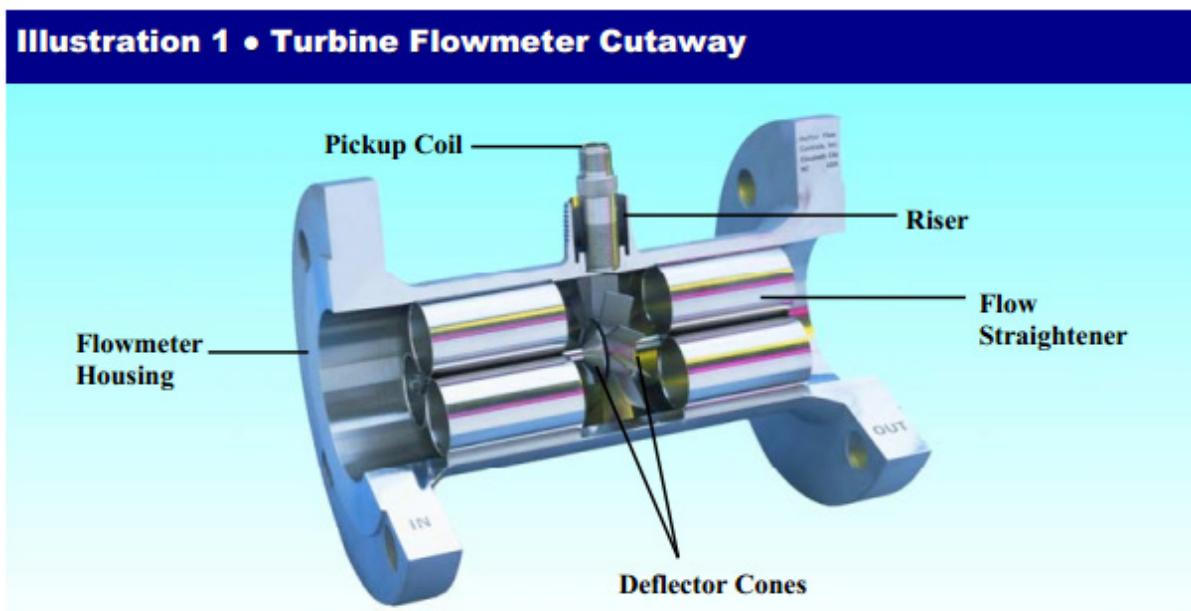
Shaft rotation can be sensed mechanically or by detecting the movement of the blades. Blade movement is often detected magnetically, with each blade or embedded piece of metal generating a pulse. Turbine flow meter sensors are typically located external to the flowing stream to avoid material of construction constraints that would result if wetted sensors were used. When the fluid moves faster, more pulses are generated. The transmitter processes the pulse signal to determine the flow of the fluid. Transmitters and sensing systems are available to sense flow in both the forward and reverse flow directions.

Turbine flow meters measure the velocity of liquids, gases and vapors in pipes, such as hydrocarbons, chemicals, water, cryogenic liquids, air, and industrial gases. High accuracy turbine flow meters are available for custody transfer of hydrocarbons and natural gas. These flow meters often incorporate the functionality of a flow computer to correct for pressure, temperature and fluid properties in order to achieve the desired accuracy for the application.

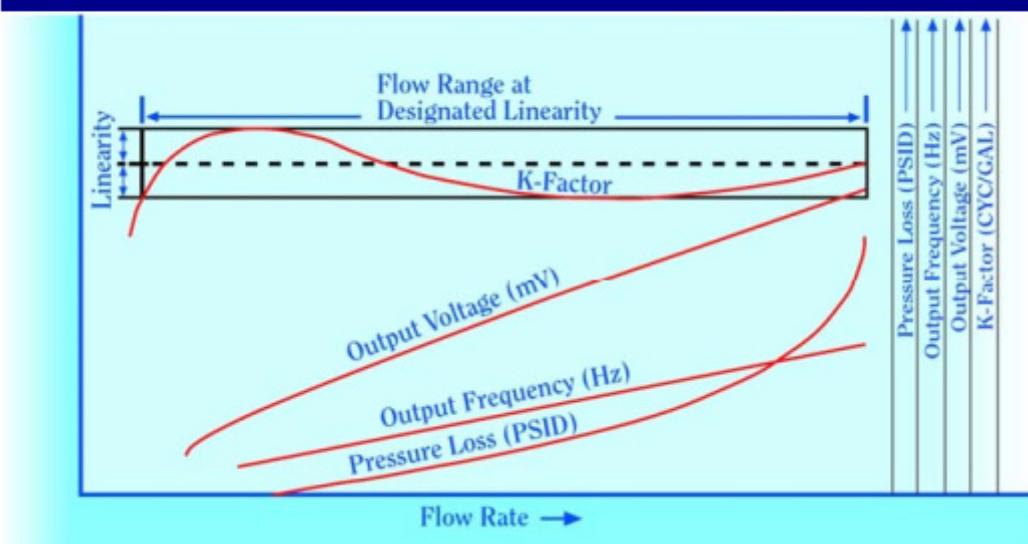
**Caution:** Abrupt transitions from gas flow to liquid flow should be avoided because they can mechanically stress the flow meter, degrade accuracy, and/or damage the flow meter. These conditions generally occur when filling the pipe and under slug flow conditions. Two-phase flow conditions can also cause turbine flow meters to measure inaccurately.”

---

A typical turbine flow meter of M/s Hoffer Flow Controls Inc. ([www.hofferflow.com](http://www.hofferflow.com)) and the performance characteristics are shown below Ref. [15]:



**Figure 1 • Turbine Flowmeter Performance Characteristics**



**Note:** The K-factor of the flow meter relates the output frequency of the coil and the flow rate (pulses per gallon or pulses per liter) and is established in the factory at the time of manufacture.



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**Barron [1] gives the following calculation equations for Turbine flow meters:**

At high Reynolds Numbers (about 6000, based on rotor tip velocity and meter bore dia), volumetric flow rate,  $V_{\text{dot}}$  ( $\text{m}^3/\text{s}$ ) is related to the rotational speed of the turbine 'n' (rps) by:

$$V_{\text{dot}} = \frac{\pi \cdot D_b \cdot A_{\text{ff}} \cdot n}{\tan(\theta_b)}$$

where

$D_b$  = rotor blade tip dia (m)

$\theta_b$  = angle between the blade and meter centre line (deg.)

$A_{\text{ff}}$  = free flow area through the turbine ( $\text{m}^2$ ),

And,

$$A_{\text{ff}} = \frac{\pi \cdot (D^2 - D_h^2)}{4} \cdot \left[ 1 - 2 \cdot m \cdot \frac{t}{\pi} \cdot (D + D_h) \cdot \cos(\theta_b) \right]$$

where

$D_h$  = rotor hub dia (m)

$D$  = meter bore dia (m)

$m$  = no. of blades

$t$  = blade thickness (m), and

$n$  = rotational speed of the turbine (rotations/s)

### Fluid 'quality' measurement [1]:

In flow rate measurement, we need the density of flowing fluid. When the flowing fluid is in single phase (liquid or gas), density of fluid is calculated by knowing the pressure and temp. But, if the fluid is in two-phase, we need to know the 'quality' (i.e. fraction of total mass flowing that is in vapor phase) to calculate the flow rate.

If the quality is known, density of fluid mixture is calculated as follows:

$$\frac{1}{\rho} = \frac{1-x}{\rho_f} + \frac{x}{\rho_g} = \frac{1}{\rho_f} + x \cdot \left( \frac{1}{\rho_g} - \frac{1}{\rho_f} \right)$$

where  $\rho_f$  = density of sat. liquid, and  $\rho_g$  = density of sat. vapor.

**To measure 'quality', a capacitance probe is used.**

Here, the principle used is: capacitance of fluid mixture is related to fluid quality.

A schematic of such a probe is shown below:

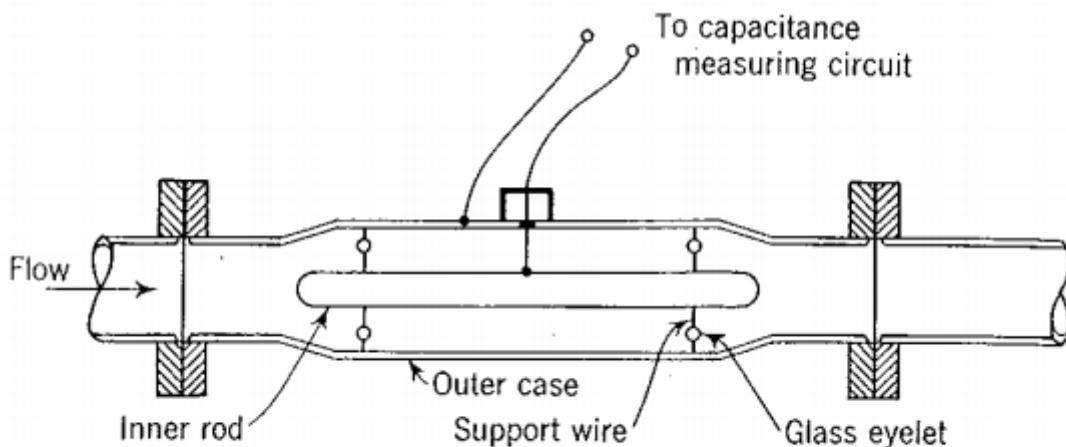


Fig. Capacitance quality meter (Killian and Simpson 1960).

The probe consists of an inner rod concentric to an outer cylindrical case, such that the cross-sectional area of the annulus is equal to the cross-sectional area of the line where the probe is placed. The inner rod is electrically insulated from the outer casing by supporting the inner rod by wires passing through glass eyelets.

Probe capacitance is calibrated against fluid quality in the factory.

**Calculation formulas are given below:**

Capacitance of a concentric-cylinder capacitor, long enough to neglect the end effects, is given by:

$$C = \frac{2\pi L \cdot \epsilon \cdot \epsilon_0}{\ln\left(\frac{D_o}{D_i}\right)}$$

where  $L$  = Length of cylinder,  $\epsilon$  = dielectric const of material in the annulus, and

$$\epsilon_0 := 8.8542 \cdot 10^{-12} \quad \text{F/m} = \text{permittivity of free space}$$

Dielectric const is linearly related to the volumetric fraction of vapor and liquid.

If  $y$  = liquid volume ratio, then the vapor volume fraction is given by:

$$1 - y = \frac{\epsilon_f - \epsilon}{\epsilon_f - \epsilon_g}$$

where  $\epsilon_f$  and  $\epsilon_g$  are the dielectric constants of sat. liquid and sat. vapor.

Density of two-phase mixture is written as follows:

$$\rho = y \cdot \rho_f + (1 - y) \cdot \rho_g = \rho_f - (1 - y) \cdot (\rho_f - \rho_g)$$

Then, the volume ratio and the quality are related by:

$$x = \frac{m_g}{m} = \frac{V_g \cdot \rho_g}{(V_f + V_g) \cdot \rho} = (1 - y) \cdot \frac{\rho_g}{\rho}$$

**Note:** Dielectric constants for sat. liquid and sat. vapor vary with pressure; so, calibration must be made at the pressure at which measurements are to be taken.



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**Some data on dielectric constants for a few fluids are given below, from Ref.[2]:**

**For fluid: H<sub>2</sub>:**

**f. Dielectric Constant.** The dielectric constant for fluid parahydrogen may be calculated from

$$\frac{\varepsilon - 1}{\varepsilon + 2} = A\rho + B\rho^2 + C\rho^3 \quad (3.39)$$

where  $A = 0.99575$ ,  $B = -0.09069$ ,  $C = 1.1227$ , and  $\varepsilon$  is the dielectric constant. The density,  $\rho$ , must be in units of grams per cubic centimeter. The equation is valid over the range of the tables and may be extrapolated in the fluid phase with reasonable results.

Range of values (dimensionless) :

Triple point			Boiling point			
Liquid	Vapor	Solid	Liquid	Vapor	Critical point	300 K, 1 atm
1.252	1.0004	1.285	1.229	1.004	1.0980	1.00025

The uncertainty of  $\varepsilon - 1$  is estimated to be no greater than 0.1% for the fluid phase and 0.2% for the solid phase. Values of the dielectric constant are listed in [Table 3.56](#).

**Table 3.56** Dielectric Constant for Parahydrogen

Density (g/cm <sup>3</sup> )	Dielectric constant, $\varepsilon$
0.005	1.01515
0.010	1.03046
0.015	1.04594
0.020	1.06158
0.025	1.07739
0.030	1.09336
0.035	1.10950
0.040	1.12580
0.045	1.14226
0.050	1.15889
0.055	1.17569
0.060	1.19265
0.065	1.20977
0.070	1.22705
0.075	1.24449
0.080	1.26210

**For fluid: Argon:**

**c. Dielectric Constant.** The Clausius–Mosotti function

$$\frac{\varepsilon - 1}{\varepsilon + 2} \left( \frac{1}{\rho} \right) \quad (3.19)$$

should be a constant and independent of temperature provided the molecules of the liquid studied have no permanent dipole moment. For liquid argon this quantity

is constant within the accuracy of the data.  $\varepsilon$  is the dielectric constant and  $\rho$  is the density.

For liquid argon, values for the dielectric constant are given in [Table 3.33](#)

For gaseous argon at STP, according to Jelatis (1948),

$$\varepsilon = 1.000554$$

**Table 3.33** Dielectric Constant for Argon—Selected Values

Temp. (K)	Density (g/cm <sup>3</sup> )	Dielectric constant (ref. to vacuum)
88.8	1.393	1.516
88.5	1.395	1.518
87.1	1.404	1.520
85.8	1.414	1.525
84.3	1.422	1.530
82.4 <sup>a</sup>	1.434	1.537

<sup>a</sup> Supercooled liquid.

Source: McLennan et al. (1930).

**For fluid: N2:**

**c. Dielectric Constant.** The dielectric constant of nitrogen may be calculated from the Clausius–Mossotti equation,

$$\frac{\varepsilon - 1}{\varepsilon + 2} \left( \frac{1}{\rho} \right) = p \quad (3.8)$$

where  $\varepsilon$  is the dielectric constant,  $\rho$  is the density, and  $p$  is the specific polarization, a property of the substance having dimensions of specific volume:

$$p = A_n + B_n \rho + C_n \rho^2 \quad (3.9)$$

where  $p$  is the specific polarization in cm<sup>3</sup>/mol and  $\rho$  is in units of mol/cm<sup>3</sup>.

The parameters  $A_n$ ,  $B_n$ , and  $C_n$  are

$$A_n = 4.389 \text{ cm}^3/\text{mol}, \quad B_n = 2.2(\text{cm}^3/\text{mol})^2, \quad C_n = -114.0(\text{cm}^3/\text{mol})^3$$

### For fluid: O<sub>2</sub>:

**g. Dielectric Constant.** The dielectric constant can be calculated from an extension of the Clausius–Mossotti relationship

$$\frac{\varepsilon - 1}{\varepsilon + 2} = A\rho + B\rho^2 + C\rho^3 \quad (3.3)$$

where  $A = 0.12361$ ,  $B = 3.2 \times 10^{-4}$ ,  $C = -1.21 \times 10^{-3}$ , and  $\rho$  is in g/cm<sup>3</sup>. The equation is valid over the range 54.35–340 K, 0.2–340 atm, and will yield reasonable values upon extrapolation.

The uncertainty in the term  $\varepsilon - 1$  varies from 0.15% at low densities to less than 0.05% at high densities.

### For fluid: He4:

**d. Dielectric Constant.** The dielectric constant of a fluid may be calculated from the Clausius–Mossotti equation

$$\frac{\varepsilon - 1}{\varepsilon + 2} \left( \frac{3M}{4\pi\rho} \right) = P \quad (3.41)$$

where  $\varepsilon$  is the dielectric constant,  $\rho$  is the density, and  $P$  is the specific polarizability, a property of the substance having dimensions of specific volume. Measurements of the dielectric constant indicated that for helium-4 the specific polarizability is a weak function of density and that the first density correction is negative. The equation



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$$p = 0.123396 - 0.0014\rho \quad (3.42)$$

can be used, where  $p$  is the specific polarizability in  $\text{cm}^3/\text{g}$  and  $\rho$  is the density in  $\text{g}/\text{cm}^3$ . The uncertainty of tabulated values of dielectric constant shown in Table 3.61 is estimated to be 0.01%.

**Table 3.61** Thermodynamic Properties of Coexisting Liquid and Gaseous Helium

Temp. (K)	Pressure (atm)	Density <sup>a</sup> ( $\text{g}/\text{cm}^3 \times 100$ )	Enthalpy <sup>a</sup> (J/g)	Entropy <sup>a</sup> (J/(g.K))	Spec. heat (J/(g K))		Thermal conductivity (mW/cm. K)	Viscosity <sup>a</sup> ( $\text{g}/(\text{cm s}) \times 10^6$ )	Dielectric constant $\epsilon$	Prandtl number $N_{pr}$
					$C_V$	$C_P$				
2.177 <sup>b</sup>	0.04969	14.62 0.1177	25.41 3.276	11.92 1.671	3.20	5.61	0.0447	5.38	1.00044	0.675
2.20	0.05256	14.61 0.1235	25.51 3.276	11.85 1.671	3.10	5.16	0.148	36.1	1.05500	0.778
2.30	0.06629	14.58 0.1503	3.567 25.91	1.796 11.57	2.46	2.56	0.0453	5.45	1.00046	0.677
2.40	0.08228	14.53 0.1805	3.816 26.30	1.898 11.32	2.10	2.25	0.0480	36.7	1.05486	0.605
2.50	0.1008	14.48 0.2144	4.045 26.88	1.986 11.09	1.93	2.13	0.165	6.07	1.00067	0.690
2.60	0.1219	14.42 0.2521	4.269 27.04	2.068 10.87	1.85	2.10	0.168	37.1	1.05468	0.521
2.70	0.1460	14.35 0.2939	4.496 27.38	2.147 10.66	1.84	2.14	0.171	37.3	1.05448	0.482
2.80	0.1730	14.28 0.3401	4.731 27.71	2.225 10.47	1.85	2.22	0.174	7.01	1.00109	0.717
2.90	0.2033	14.20 0.3908	4.975 28.03	2.304 10.28	1.88	2.31	0.176	36.8	1.00126	0.728
3.00	0.2371	14.11 0.4463	5.231 28.33	2.382 10.11	1.92	2.42	0.178	7.96	1.00145	0.741
3.10	0.2744	14.02 0.5070	5.501 28.61	2.462 9.937	1.96	2.54	0.180	36.5	1.00165	0.755
3.20	0.3156	13.93 0.5731	5.783 28.87	2.542 9.774	2.00	2.67	0.182	8.29	1.00188	0.770
3.30	0.3607	13.83 0.6452	6.081 29.11	2.624 9.616	2.04	2.80	0.184	35.8	1.00212	0.787

(Continued)

**Table 3.61 (Continued)**

Temp. (K)	Pressure (atm)	Density <sup>a</sup> ( $\text{g}/\text{cm}^3 \times 100$ )	Enthalpy <sup>a</sup> (J/g)	Entropy <sup>a</sup> (J/(g.K))	Spec. heat (J/(g K))		Thermal conductivity (mW/cm. K)	Viscosity <sup>a</sup> ( $\text{g}/(\text{cm s}) \times 10^6$ )	Dielectric constant $\epsilon$	Prandtl number $N_{pr}$
					$C_V$	$C_P$				
3.40	0.4100	13.72 0.7235	6.393 29.32	2.706 9.463	2.07	2.95	0.186	35.5	1.05157	0.562
3.50	0.4637	13.60 0.8085	6.722 29.52	2.790 9.314	2.11	3.10	0.188	9.30	1.00268	0.828
3.60	0.5220	13.48 0.9008	7.068 29.69	2.875 9.169	2.14	3.28	0.0813	35.1	1.05113	0.580
3.70	0.5849	13.35 1.001	7.432 29.84	2.962 9.025	2.18	3.47	0.0843	9.65	1.00300	0.852
3.80	0.6528	13.21 1.109	7.816 29.96	3.050 8.884	2.21	3.68	0.190	34.7	1.00334	0.599
3.90	0.7257	13.06 1.228	8.221 30.06	3.141 8.743	2.25	3.92	0.194	10.0	1.00455	0.878
4.00	0.8040	12.90 1.356	8.650 30.12	3.234 8.603	2.28	4.19	0.195	32.8	1.004845	0.707
4.10	0.8878	12.73 1.496	9.105 30.15	3.330 8.463	2.32	4.51	0.196	11.5	1.00503	1.02
4.20	0.9772	12.54 1.649	9.508 30.14	3.429 8.322	2.36	4.88	0.196	32.3	1.004780	0.745
4.224	1.000	12.50 1.689	9.711 30.13	3.454 8.287	2.37	4.98	0.196	11.9	1.00555	1.07
4.30	1.073	12.35 1.818	10.10 30.08	3.532 8.177	2.40	5.32	0.197	31.3	1.04635	0.847
4.40	1.174	12.13 2.005	10.66 29.98	3.640 8.029	2.44	5.86	0.197	12.8	1.00674	1.20

4.50	1.282	11.89 2.213	11.25 29.81	3.753 7.876	2.49 3.37	6.55 12.1	0.199 0.128	30.1 13.8	1.04463 1.00821	0.994 1.39
4.60	1.397	11.63	11.90	3.873	2.54	7.44	0.200	29.5	1.04364	1.10
		2.449	29.58	7.714	3.37	13.5	0.127	14.3	1.00909	1.52
4.70	1.519	11.34	12.61	4.002	2.59	8.68	0.202	28.9	1.04252	1.24
		2.179	29.25	7.540	3.38	15.5	0.135	14.8	1.01010	1.70
4.80	1.648	11.01	13.40	4.144	2.66	10.5	0.205	28.2	1.04125	1.45
		3.037	28.80	7.350	3.38	18.5	0.147	15.5	1.01128	1.95
4.90	1.784	10.61	14.30	4.304	2.73	13.6	0.210	27.4	1.03974	1.78
		3.425	28.18	7.133	3.39	23.6	0.164	16.2	1.01273	2.32
5.00	1.929	10.11	15.39	4.495	2.81	19.9	0.222	26.4	1.03785	2.37
		3.930	27.28	6.871	3.39	34.6	0.198	17.0	1.01461	2.97
5.10	2.082	9.489	16.82	4.747	2.92	38.5	0.263	25.1	1.03520	3.68
		4.680	25.83	6.513	3.37	71.5	0.301	18.1	1.01742	4.30
5.201 <sup>b</sup>	2.245	6.964	21.36	5.589						
		6.964	21.36	5.589						

<sup>a</sup> Top line of data = saturated liquid properties; bottom line data = saturated vapor properties.

<sup>b</sup> Lambda point and critical point

Source: McCarty (1972).

### Liquid level measurement:

In general, various types of liquid level indicators are: [16]

- Sight glass type
- Simple float type
- Hydrostatic device...principle is to measure hydrostatic pressure
- Load cells...essentially strain gauge device.... Non-contact type
- Magnetic level gauges...similar to float devices, but the float carries a set of strong permanent magnets
- Capacitance transmitters...principle is that liquids and vapors of a fluid have different dielectric constants; parallel-plate or concentric-tube capacitors are used.
- Magneto-restrictive level transmitters, Ultrasonic level transmitters, Laser level transmitters, and Guided-wave radar level transmitters are the other types of level meters which can be classified as belonging to the 'emerging technologies'.

Barron [1] explains various types of level meters used in cryogenics, and gives the calculation formulas:

#### Hydrostatic gauges [1]:

This is the simplest type of level gauge and a schematic is shown below.

Here, differential pressure is measured using a Bourdon gauge, U-tube manometer, or a diaphragm transducer.

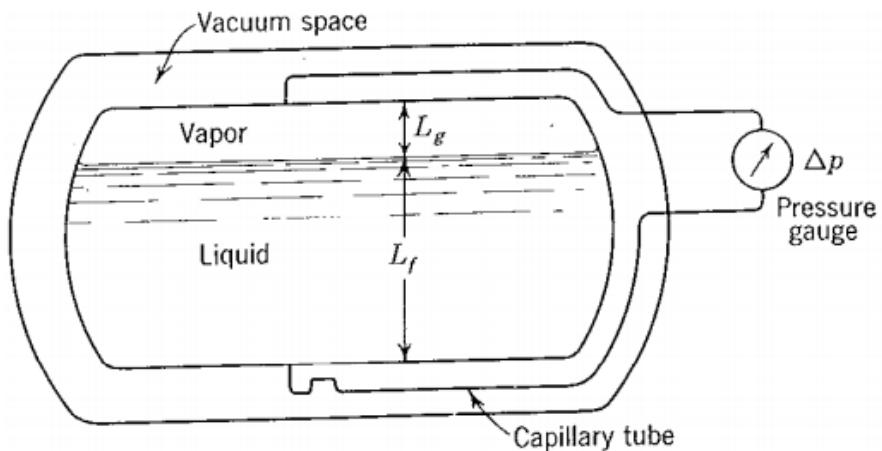


Fig. Hydrostatic liquid-level gauge.

Hydrostatic pressure is related to the liquid level  $L_f$  by:

$$\Delta P = \rho_f L_f g + \rho_g L_g g$$

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where

$L_f$  = height of liquid column

$L_g = L - L_f$  = height of vapor column

$L$  = inside dia of vessel

$\rho_f$  = sat. liquid density

$\rho_g$  = sat. vapor density

$g$  = accn. due to gravity

Eliminating  $L_g$ , we get:

$$\Delta P = (\rho_f - \rho_g) \cdot L_f \cdot g + \rho_g \cdot L \cdot g$$

For fluids other than H<sub>2</sub> and He, vapor density << liquid density, and we can write:

$$\Delta P = \rho_f L_f g$$

*Disadvantage:* low sensitivity for LH<sub>2</sub> and LHe.

### Sensitivity:

Sensitivity of hydrostatic liquid level gauge is determined from:

$$S_0 = \frac{d(\Delta P)}{dL_f} = (\rho_f - \rho_g) \cdot g \quad \text{Pa/m}$$

i.e. Sensitivity is directly proportional to the difference between the liquid and vapor densities.

### Electric resistance gauges [1]:

Here, a small wire is electrically heated and moved upwards through the liquid, and since the heat transfer coeff in liquid is two or three orders of magnitude higher in liquid than in vapor, temp of wire increases substantially when the heated wire is exposed to vapor. This results in change in electric resistance of the wire and it is measured. Following is the schematic:

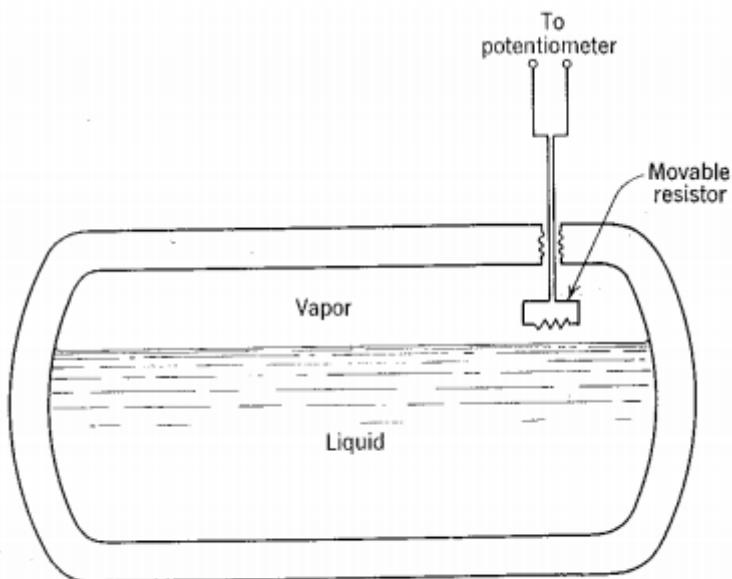


Fig. Movable electric resistance liquid-level gauge.

Heat transfer coeff for a small wire in free convection is given by:

$$h_c = \frac{\left( \frac{2k_t}{D} \right)}{\ln \left( 1 + \phi \cdot N_{Gr}^{\frac{1}{4}} \right)}$$

where

$$\phi = 3.70 \cdot (N_{Pr} + 0.952)^{\frac{1}{4}} \cdot N_{Pr}^{-\frac{1}{2}}$$

$$N_{Gr} = g \cdot \beta \cdot \Delta T \cdot \frac{\rho^2}{\mu^2} \cdot D^3 = \text{Grashof Number} \quad N_{Pr} = \text{Prandtl Number}$$

$$k_t = \text{thermal\_cond} \quad D = \text{wire\_dia} \quad \beta = \text{thermal\_expn\_coeff}$$

$$\rho = \text{density} \quad \mu = \text{viscosity} \quad \Delta T = \text{temp\_diff\_between\_wire\_and\_fluid}$$

Another variation of electric resistance type of level meter uses vertical wire or strips and the length of wire (or strip) immersed in liquid gives an indication of level of liquid in the vessel. It has the advantage of continuous level indication, but has a disadvantage that more energy is dissipated in the vessel. Fig. of such a level meter is shown below:

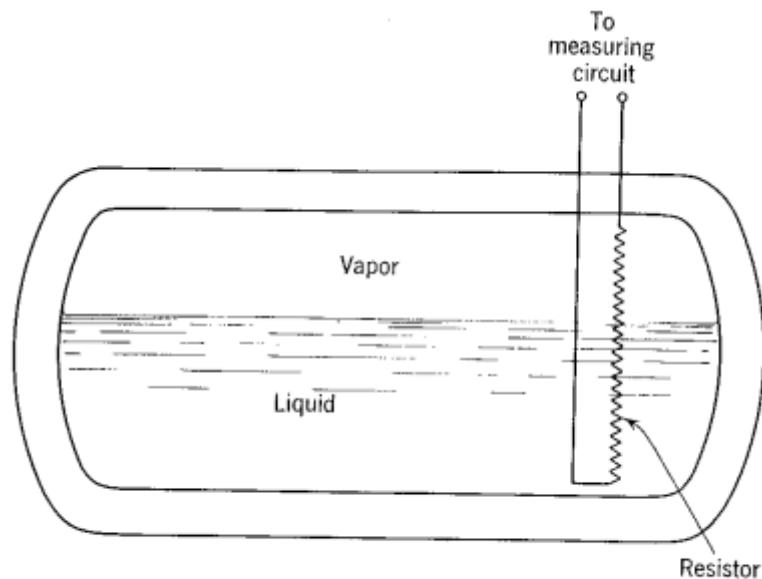
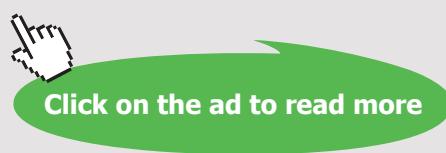


Fig. Fixed electric resistance liquid-level gauge.



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### Capacitance liquid level probes [1]:

Here, two concentric electrodes are placed vertically in the vessel and the principle is: dielectric constants of liquid and vapor phases are different.

Schematic fig. is shown below:

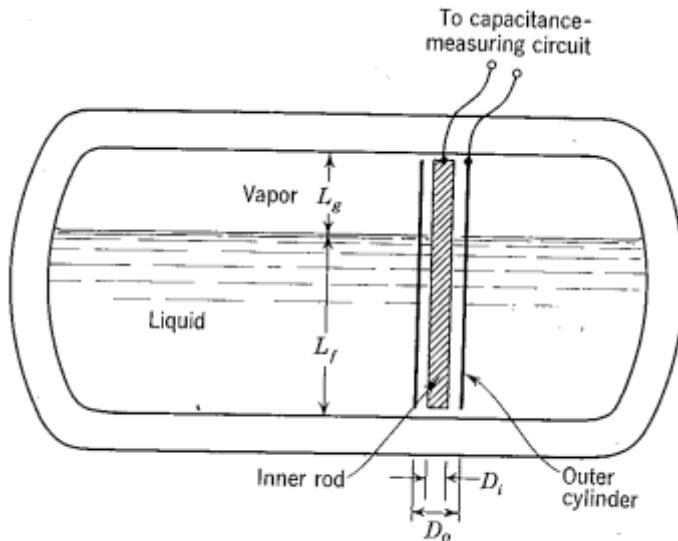


Fig. Capacitance liquid-level gauge.

Capacitance of a concentric-tube capacitor, long enough that the end effects can be neglected, is given by:

$$C = \frac{2 \cdot \pi \cdot L \cdot \epsilon \cdot \epsilon_0}{\ln\left(\frac{D_o}{D_i}\right)}$$

where L= Length of cylinder,  $\epsilon$  = dielectric const of material in the annulus, and

$$\epsilon_0 := 8.8542 \cdot 10^{-12} \text{ F/m} = \text{permittivity of free space}$$

Total capacitance of the gauge is given by:

$$C = (C_f + C_g) = \frac{2 \cdot \pi \cdot (L_f \cdot \epsilon_f + L_g \cdot \epsilon_g) \cdot \epsilon_0}{\ln\left(\frac{D_o}{D_i}\right)}$$

where

$L_f$  = Length of gauge immersed in liquid

$L_g = L - L_f$  = Length of gauge immersed in vapor

Then, the total capacitance can be written as:

$$C = \frac{2\pi L \cdot \epsilon_0}{\ln\left(\frac{D_0}{D_i}\right)} \left[ \epsilon_g + (\epsilon_f - \epsilon_g) \cdot \frac{L_f}{L} \right]$$

Solving for liquid level, we get:

$$L_f = \frac{C \cdot \ln\left(\frac{D_0}{D_i}\right)}{2\pi \cdot (\epsilon_f - \epsilon_g) \cdot \epsilon_0} - \frac{\epsilon_g \cdot L}{(\epsilon_f - \epsilon_g)}$$

From the above, Sensitivity of capacitance liquid level gauge is:

$$S_0 = \frac{dC}{dL_f} = \frac{2\pi \cdot (\epsilon_f - \epsilon_g) \cdot \epsilon_0}{\ln\left(\frac{D_0}{D_i}\right)}$$

Note that the Sensitivity is *independent* of the length of the probe or liquid level. And, Sensitivity can be increased by making the diameter ratio as small as possible, without causing arcing across the capacitor surfaces.

### Thermodynamic liquid level gauge [1]:

These probes work on the principle that a liquid undergoes a large change in volume when it evaporates.

Schematic of such a liquid-level probe is shown below.

A capillary tube, slightly heated by an electric current, is connected to a dead volume that is at ambient temp. The gauge is initially charged with a known amount of gas of the same species that is in the tank. As the capillary is immersed in the cryogenic fluid, the gas in the immersed portion of the tube is condensed and pressure of gas in the capillary and dead volume reduces. The pressure in the dead volume is used as an indication of the level.

Using the fact that total mass,  $m$  is a constant, and is equal to sum of mass of liquid in the capillary tube,  $m_p$ , mass of vapor in the capillary,  $m_g$ , and mass of gas in the dead volume,  $m_0$ , we write:

$$m = m_f + m_g + m_0 = \rho_f V_f + \rho_g V_g + \rho_0 V_0$$

where  $\rho$  = density, and  $V$  = volume



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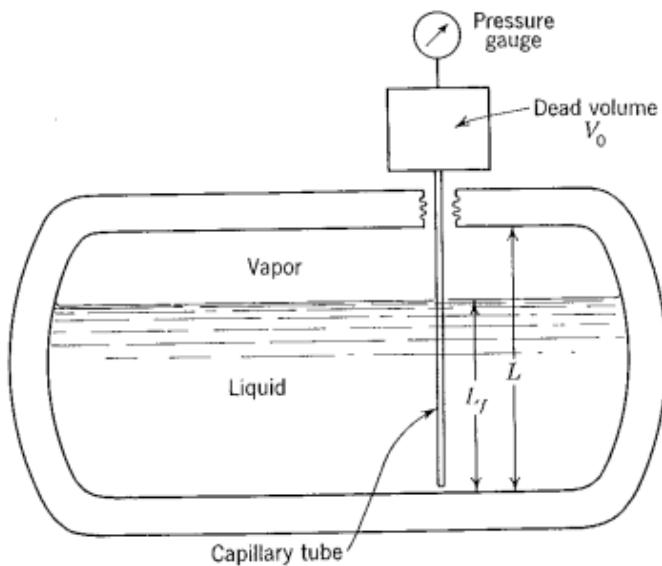


Fig. Thermodynamic liquid-level gauge.

Capillary tube has a constant cross-sectional area,  $A_c$ . So, we have:

$$m = \rho_f A_c L_f + \rho_g A_c L_g + \rho_0 V_0$$

$$\text{Or: } m = A_c [(\rho_f - \rho_g) L_f + \rho_g L] + \rho_0 V_0$$

where  $L = L_f + L_g$  = total length of capillary tube, and  $L_f$  = liquid level.

Solving for liquid level, we get:

$$\frac{L_f}{L} = \frac{\left( \frac{(m - \rho_0 V_0)}{A_c L} \right) - \rho_g}{\rho_f - \rho_g}$$

Note that densities in the above eqn are determined from the pressure  $p$  indicated by the gauge, and temp of dead volume,  $T_0$ , liquid temp,  $T_f$  and vapor temp,  $T_g$  in the capillary.

Equation of state for both the liquid and vapor phase is expressed as:

$$p = Z \cdot \rho \cdot R \cdot T$$

where  $Z$  is the “supercompressibility factor” which is a function of pressure and temp of fluid.

Values of  $Z$  for LH<sub>2</sub> and LN<sub>2</sub> are presented in Tables 6.6 and 6.7, and the generalized supercompressibility factor for gases is presented in Table 6.8.

Table 6.6. Supercompressibility factor  $Z$  for liquid hydrogen  $Z = pv/RT$

Temperature (K)	Saturated Liquid	Pressure (atm)						
		10	20	30	40	60	80	100
16	0.004107	0.2022	0.3999	0.5941	0.7851	1.159		
17	0.006332	0.1923	0.3798	0.5637	0.7444	1.097	1.444	1.787
18	0.008451	0.1835	0.3624	0.5370	0.7087	1.044	1.371	1.695
19	0.012056	0.1760	0.3471	0.5140	0.6773	0.9960	1.306	1.612
20	0.01529	0.1693	0.3337	0.4936	0.6498	0.9542	1.250	1.539
21	0.02032	0.1636	0.3218	0.4756	0.6256	0.9168	1.199	1.474
22	0.02533	0.1587	0.3113	0.4595	0.6036	0.8838	1.154	1.416
23	0.03100	0.1545	0.3023	0.4456	0.5850	0.8552	1.114	1.366
24	0.03936	0.1510	0.2944	0.4332	0.5685	0.8290	1.079	1.321
25	0.04918	0.1480	0.2880	0.4229	0.5538	0.8052	1.046	1.280
26	0.05847	0.1457	0.2828	0.4137	0.5403	0.7841	1.017	1.243
27	0.07101	0.1440	0.2782	0.4057	0.5286	0.7645	0.9908	1.210
28	0.8442	0.1431	0.2744	0.3988	0.5185	0.7471	0.9666	1.179
29	0.1042	0.1437	0.2718	0.3932	0.5095	0.7317	0.9450	1.151
30	0.1208	0.1456	0.2708	0.3890	0.5026	0.7185	0.9255	1.124
31	0.1407	0.1498	0.2712	0.3864	0.4969	0.7069	0.9076	1.101
32	0.1799	...	0.2732	0.3848	0.4919	0.6965	0.8912	1.079
33	...	...	0.2781	0.3845	0.4876	0.6869	0.8757	1.058
34	...	...	0.2929	0.3897	0.4890	0.6803	0.8640	1.0413
35	...	...	0.3157	0.3954	0.4891	0.6746	0.8522	1.0256
36	...	...	0.3627	0.4052	0.4915	0.6703	0.8425	1.0103

Table 6.7. Supercompressibility factor  $Z$  for liquid nitrogen,  $Z = pv/RT$

Temperature (K)	Saturated Liquid	Pressure (atm)				
		5	10	20	40	60
70	0.002206	0.02900	0.05793	0.11562	0.23027	0.34403
75	0.004173	0.02779	0.05551	0.11073	0.22035	0.32894
80	0.007241	0.02681	0.05352	0.10671	0.21214	0.31639
85	0.011743	0.02602	0.05193	0.10346	0.20541	0.30600
90	0.018066	0.02542	0.05071	0.10092	0.20001	0.29751
95	0.026640	...	0.04985	0.09908	0.19588	0.29080
100	0.038003	...	0.04941	0.09798	0.19304	0.28579
105	0.052869	...	...	0.09777	0.19159	0.28254
110	0.072282	...	...	0.09883	0.19184	0.28124
115	0.098078	...	...	0.10219	0.19448	0.28232
120	0.134410	...	...	...	0.20122	0.28659

Table 6.8. Supercompressibility factor  $Z = pv/RT$  for gases (Lydersen et al. 1955).  $p_r = p/p_c$  and  $T_r = T/T_c$ , where  $p_c$  = critical pressure,  $T_c$  = critical temperature

$T_r$	$p_r$											
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.5	2.0
Sat.Vap.	0.898	0.833	0.783	0.738	0.693	0.641	0.583	0.519	0.443	0.270	...	...
0.80	0.921	...	...	...	...	...	...	...	...	0.145	0.215	0.284
0.85	0.933	0.861	0.789	...	...	...	...	...	...	0.146	0.216	0.283
0.90	0.947	0.890	0.826	0.764	...	...	...	...	...	0.148	0.217	0.283
0.92	0.951	0.901	0.842	0.783	0.710	...	...	...	...	0.151	0.219	0.284
0.94	0.955	0.909	0.856	0.798	0.735	0.660	...	...	...	0.155	0.223	0.287
0.96	0.958	0.915	0.868	0.817	0.761	0.700	0.613	...	...	0.161	0.230	0.291
0.98	0.962	0.922	0.879	0.832	0.782	0.731	0.665	0.580	...	0.174	0.241	0.298
1.00	0.965	0.927	0.889	0.846	0.801	0.755	0.704	0.640	0.520	0.270	0.247	0.306
1.05	0.971	0.940	0.908	0.873	0.838	0.802	0.766	0.723	0.670	0.611	0.332	0.341
1.10	0.976	0.950	0.923	0.894	0.866	0.837	0.805	0.733	0.738	0.700	0.496	0.400
1.20	0.983	0.965	0.946	0.924	0.915	0.905	0.862	0.840	0.818	0.795	0.682	0.573
1.40	0.990	0.982	0.972	0.959	0.949	0.937	0.928	0.921	0.912	0.899	0.846	0.801
1.60	0.992	0.988	0.985	0.978	0.973	0.965	0.964	0.960	0.955	0.948	0.919	0.888
1.80	0.993	0.991	0.990	0.987	0.985	0.982	0.981	0.980	0.978	0.974	0.956	0.935
2.0	0.994	0.994	0.994	0.994	0.993	0.992	0.992	0.990	0.990	0.988	0.976	0.966
3.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	0.986
4.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.990
6.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.995
8.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.008	0.998
10.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.010	1.000
15.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.020	1.020



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Using the value of  $\rho$  from the above equation in the equation for liquid level, we get:

$$\frac{L_f}{L} = \frac{\frac{m \cdot R \cdot T_f}{A_c \cdot L_p} - \frac{V_0 \cdot T_f}{Z_0 \cdot T_0 \cdot A_c \cdot L}}{\frac{1}{Z_f} - \frac{T_f}{Z_g \cdot T_g}}$$

Once the pressure  $p$  is read by the gauge, supercompressibility factors are determined and the liquid level is calculated.

Knowing the charging pressure,  $p_0$  when the capillary and dead volume are at temp.  $T_0$ , total mass within the gauge is determined as follows:

$$m = \left( \frac{p_0 \cdot V_0}{Z_0 \cdot R \cdot T_0} \right) \cdot \left( 1 + \frac{A_c \cdot L}{V_0} \right)$$

**Note:** Initial charging pressure must be such that pressure within the gauge during operation must never fall below the sat. pressure of boiling liquid in the storage tank.

### Sensitivity:

Since the dependence of supercompressibility factor on the gauge pressure can not be expressed by a simple analytical relation, it is difficult to write an expression for the sensitivity of the thermodynamic liquid level probe. However, sensitivity can be improved by reducing the ratio of the dead volume to capillary volume.

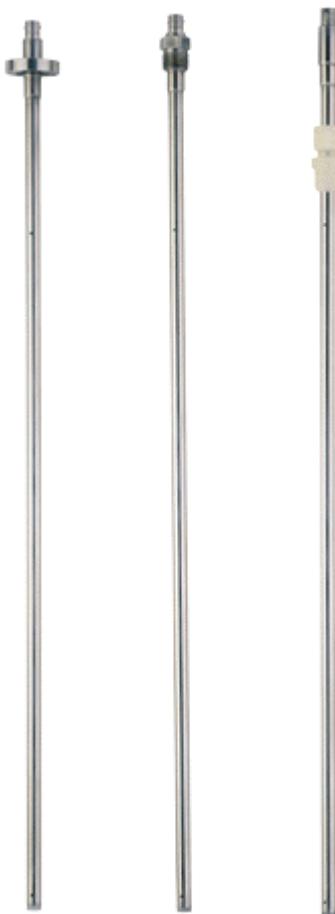
### Practical examples of liquid level gauges:

*Two typical examples of commercially available liquid level gauges (one for LN2 and the other one, for LHe) from Application notes of M/s American Magnetics are reproduced below:*

#### Liquid Nitrogen Level Measurement and Control [17]:

For magnet systems that use nitrogen shielded dewars, an [AMI liquid level controller](#) can automate the process of transferring nitrogen to the dewar. [AMI's capacitance-based sensor](#) utilizes a very reliable liquid nitrogen level measurement method. The sensing element is a linear capacitor consisting of two concentric stainless steel tubes in which the cryogenic liquid becomes the dielectric. The sensor capacitance is directly related to the liquid level in the sensor. Level is typically displayed from 0% to 100% by a liquid level instrument.

**Capacitance-based liquid level sensors**, are typically manufactured of 3/8 inch OD stainless steel tubing and will operate with virtually all common laboratory cryogenic liquids including nitrogen, liquid neon, and liquid xenon. Sensors for common Industrial Use Cryogenics are also available for materials such as liquid oxygen (LOX), liquid hydrogen LH<sub>2</sub>), liquid carbon dioxide (liquid CO<sub>2</sub>), liquefied or liquefied natural gas (LNG), liquid argon, liquid propane, and liquid butane. Upon request, special assembly techniques can be applied for sensors required for liquid oxygen or hydrogen measurement. Sensors can be supplied in single-section overall lengths of up to 50 feet. Alternatively segmented sensors are available that can be bolted together during installation. Three standard sensor mounting configurations are available. The typical configuration includes a hermetically sealed BNC connector with an adjustable 3/8 NPT nylon feed-through. For higher pressure or vacuum applications, a welded stainless steel 3/8 NPT BNC connector fitting or conflat flange BNC connector fitting, respectively, is available. A connecting coaxial cable and in-line oscillator/transmitter are required to interface the sensor to the non-industrial instrument.



#### Helium Level Measurement and Control [18]:



Liquid helium level measurement is nearly indispensable in that it gives an instantaneous and reliable readout of the liquid helium level in the dewar, a must for avoiding operation of the magnet with too little helium.

Unlike single point sensor methods of determining the liquid helium level in dewars, [AMI's liquid helium level](#) instruments provide a *continuous readout* of the liquid helium level in a reliable and convenient manner.

The [liquid helium level sensor](#) is comprised of a fine superconducting wire in a  $\frac{1}{4}$ -inch outside diameter, non-conductive tube. During operation, a small current is conducted through the wire, causing it to be resistive in the helium gas and superconducting in the liquid. This technique results in a voltage being established along the sensor that is proportional to the length of the wire above the liquid helium, and provides a continuous measure of the helium depth. Measurements of this voltage are made using a four wire method to avoid errors caused by variations in the length of the leads to the sensor. The small amount of heat generated in the probe is dissipated primarily in the helium gas rather than in the liquid helium. The sensor is rugged, reliable, and functions in magnetic fields up to 10 Tesla.

A host of helium level monitors are available from AMI, from a simple 0% to 100% analog meter display to the more sophisticated, microprocessor-based, sample-and-hold design. Helium level monitors which incorporate AMI's patented sample-and-hold design are ideal for operation in systems where it is important to monitor the helium level and minimize the liquid helium losses.



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Standard AMI liquid helium sensors are constructed using 3/16", 1/4", and 3/8" phenolic (normally G-10) tubing. Flexible sensors are available in 3/32" and 5/16" outer diameters. Immediate shipment is available for 1/4" diameter sensors having active lengths of 1" to 36" inches (1 inch increments) and 6" Teflon-insulated leads. Other active lengths can be made to customer specifications up to 80", and can incorporate longer leads or, for 1/4" and 3/16" sensors, stainless steel sleeved Lemo connectors. Overall sensor length typically exceeds the active length by at least one inch for phenolic tubing sensors, and two inches for flexible sensors.



### How They Work

The liquid helium level sensor operates by measuring the resistance of a superconductive filament contained within a protective tube. The current through the sensor maintains the portion of the filament in helium gas in the normal (resistive) state, while the portion in liquid remains in the superconducting state (zero resistance). The resulting voltage along the sensor is proportional to the length of filament above the liquid helium and provides a *continuous* measure of the helium depth.

Voltage measurements are made using a four wire technique to eliminate errors resulting from variations in the length of the leads. The small amount of heat generated in the probe is dissipated primarily in the helium gas rather than in the liquid helium. A standard sensor will not function properly through the lambda point (see [Model 135-2K](#)).



<b><u>Specifications</u></b>	
Sensor Diameter:	Rigid: 3/16", 1/4", and 3/8" Flexible: 3/32" and 5/16"
Metal Sleeve Diameter (for Lemo Connector):	1/2", 3/8", 12mm, 10mm or 8mm
Active length:	Customer specified
Overall length:	Rigid: Active length plus 1 in. Flexible: Active length plus 2 in.
Std. sensor current:	75 mA
Std. sensor resistance:	11.6 ohms/in. at 20 K 13.7 ohms/in. at 300 K
Magnetic field operation:	Unaffected to at least 10 Tesla

## 5.2 PROBLEMS SOLVED WITH MATHCAD:

**Prob 5.2.1** A Pt-100 RTD shows a resistance of 38.6 ohms. Find out the temperature.

### Solution:

Since the resistance is less than 100 ohms, temperature is less than 0 deg.C, and we use the eqn:

$$R(t) = R_0 \left[ (1 + A \cdot t + B \cdot t^2) + C \cdot (t - 100) \cdot t^3 \right]$$

where  $R_0 := 100$  ohms

$$A := 3.9083 \cdot 10^{-3} \text{ deg.C/C}$$

$$B := -5.775 \cdot 10^{-7} \text{ deg.C/C}^2$$

$$C := -4.183 \cdot 10^{-12} \text{ deg.C/C}^4$$

We use the ‘Solve block’ to solve this. First, assume a trial value for temperature, say -10 deg.C. Then, type Given, and under ‘Given’ write the constraint eqn. i.e.  $R(t)$  is equal to 38.6 ohms. Now, simply type

‘ $t =$ ’ and the answer appears immediately as -152.694 deg.C.



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$t := -10 \text{ deg.C} \dots \text{trial value}$

Given

$$38.6 = R_0 [ (1 + A \cdot t + B \cdot t^2) + C \cdot (t - 100) \cdot t^3 ]$$

$$\text{Find}(t) = -152.694$$

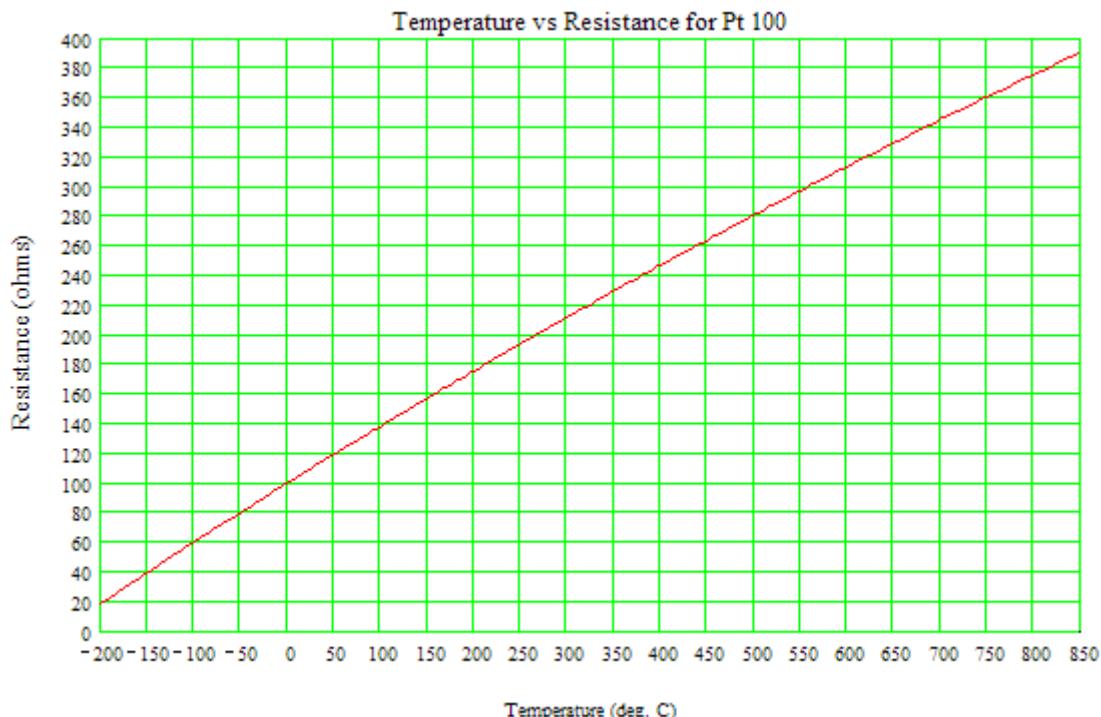
i.e. the temperature is **-152.694 deg. C.....Ans.**

To draw Temp vs Resistance graph for Pt 100:

$$\text{Resist}(t) := \begin{cases} R_0 [ (1 + A \cdot t + B \cdot t^2) + C \cdot (t - 100) \cdot t^3 ] & \text{if } t < 0 \\ R_0 [ (1 + A \cdot t + B \cdot t^2) ] & \text{otherwise} \end{cases} \quad \dots \text{Define Resistance as function of temp in the two ranges i.e. below 0 deg.C and above 0 deg.C}$$

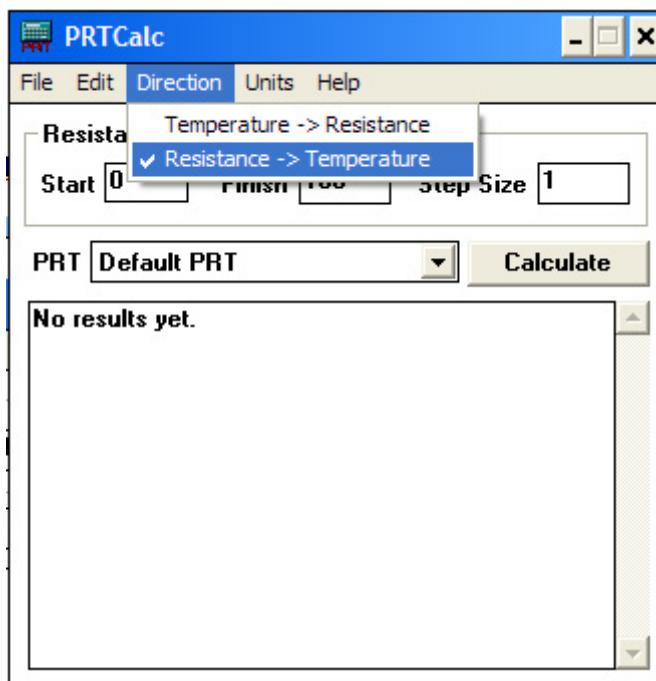
For example:  $\text{Resist}(-152.7) = 38.597 \text{ ohms}$   
 $\text{Resist}(100) = 138.505 \text{ ohms}$

$t := -200, -195..850 \quad \dots \text{define a range variable } t \text{ from -200 C to 850 C with an increment of 5 deg.C}$

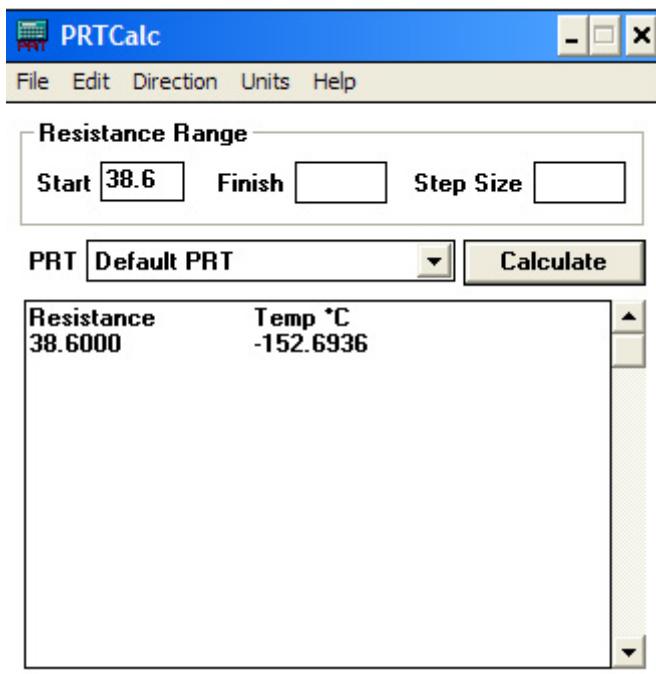


### Using the Software PRTCALC:

When PRTCALC is started, choose Resistance-temp conversion:



Enter the value of Resistance as 38.6 Ohms, and leave the 'Finish' blank, and press 'Calculate'. We get:



Note that temp is -152.6936 C...matches with the result obtained with Mathcad.

**Prob. 5.2.2** The electric resistance of a platinum resistance thermometer is found 50.000 ohms at the ice point, 69.625 ohms at the steam point, and 128.275 at the zinc point. Determine the constants  $R_0$ , A, and B in the resistance function given in eqn. (6.4). Using these values, determine the temperature (including the correction given by eqn. 6.5) when the electric resistance of the thermometer is 88.650 ohms.[1]

**Solution:**

$$R_e = R_0(1 + A \cdot t' - B \cdot t' \cdot (t' - 100)) \quad \dots \text{Temp-resistance relation given in eqn.(6.4)}$$

Use the 'Solve Block' of Mathcad:

$$A := 1 \quad B := 1 \quad C := 1 \quad \dots \text{guess values}$$

$$R_0 := 50 \quad \dots \text{by data...resist.at Ice point, } 0 \text{ C}$$



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Given

$$69.625 = 50 \cdot (1 + A \cdot 100 - B \cdot 100 \cdot (100 - 100)) \quad \dots \text{at Steam point, } 100 \text{ C}$$

$$128.275 = 50 \cdot (1 + A \cdot 419.58 - B \cdot 419.58 \cdot (419.58 - 100)) \quad \dots \text{at Zinc point, } 419.58 \text{ C}$$

$$\text{Find}(A, B) = \begin{bmatrix} 3.925 \cdot 10^{-3} \\ 6.06696 \cdot 10^{-7} \end{bmatrix}$$

$$\text{i.e. } A := 3.925 \cdot 10^{-3} \quad \dots \text{Ans.}$$

$$B := 6.06696 \cdot 10^{-7} \quad \dots \text{Ans.}$$

Find the temp when  $R_e = 88.65$  ohms:

$$T := 150 \quad \text{C} \dots \text{guess value}$$

Given

$$88.65 = R_0 \cdot (1 + A \cdot T - B \cdot T \cdot (T - 100))$$

$$\text{Find}(T) = 200.036$$

$$\text{i.e. } T := 200.036 \quad \text{C} \dots \text{Ans.}$$

Temp. correction:

$$T_C := T + 0.045 \cdot \left( \frac{T}{100} \right) \cdot \left( \frac{T}{100} - 1 \right) \cdot \left( \frac{T}{419.58} - 1 \right) \cdot \left( \frac{T}{630.74} - 1 \right)$$

$$\text{i.e. } T_C = 200.068 \quad \text{C.... corrected value of } T \dots \text{Ans.}$$

=====

**Prob.5.2.3** A platinum resistance thermometer is calibrated according to eqn,(6.9) with an electric resistance at 0 deg.C of 100.00 ohms and the constants A, B, and C are the same as those given after eqn. (6.9). If the electric resistance indication of the thermometer is 50.00 ohms, determine the temperature indication. Determine the sensitivity of the thermometer at this temperature. Ref:[1]

**Solution:**

$$R_e = R_0 [1 + A \cdot t + B \cdot t^2 + C \cdot t^3 \cdot (t - 100)] \quad \dots \text{Temp-resistance relation, } t \text{ in deg.C}$$

where:

$$A = 3.946 \cdot 10^{-3} \text{ C}^{-1} \quad B = -1.108 \cdot 10^{-6} \text{ C}^{-2} \quad C = 3.33 \cdot 10^{-12} \text{ C}^{-4}$$

**Sensitivity:**

$$S_0 = \frac{d}{dT} R_e = R_0 [A + 2B \cdot t + C \cdot t^2 \cdot (4t - 300)] \quad \text{Ohms/deg.C}$$


---

**Data:**

$$R_0 := 100 \text{ Ohms}$$

$$A := 3.946 \cdot 10^{-3} \text{ C}^{-1} \quad B := -1.108 \cdot 10^{-6} \text{ C}^{-2} \quad C := 3.33 \cdot 10^{-12} \text{ C}^{-4}$$

$$\text{Now: } R_e := 50 \text{ Ohms}$$

**Then, to find Temp, use the 'Solve block' of Mathcad:**

$$t := 100 \text{ C.... guess value}$$

Given

$$R_e = R_0 [1 + A \cdot t + B \cdot t^2 + C \cdot t^3 \cdot (t - 100)]$$

$$\text{Find}(t) = -122.82313$$

$$\text{i.e. } t := -122.82313 \text{ C .... Ans.}$$

**Sensitivity at this temp:**

$$S_0 := R_0 [A + 2B \cdot t + C \cdot t^2 \cdot (4t - 300)]$$

$$\text{i.e. } S_0 = 0.418 \text{ Ohms/deg.C .... Ans.}$$


---

**Prob.5.2.4** A germanium semiconductor thermometer has the following resistance values: 2000 ohms at 10 K; 200 ohms at 20 K; 30 ohms at 40 K; and 6 ohms at 100 K. Determine the values of the constants in the calibration equation:  $\ln T = B_0 + B_1 \cdot \ln R_e + B_2 \cdot (\ln R_e)^2 + B_3 \cdot (\ln R_e)^3$ .

Using these values, determine the temperature indication of the thermometer when the electric resistance is 100 ohm. Determine the sensitivity of the thermometer at this temperature.  
Ref:[1]

**Solution:**

Use the 'Solve Block' of Mathcad to find  $B_0$ ,  $B_1$ ,  $B_2$ ,  $B_3$  and  $B_4$ :



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$$B_0 := 100 \quad B_1 := 100 \quad B_2 := 100 \quad B_3 := 100 \quad \dots \text{guess values}$$

Given

$$\ln(10) = B_0 + B_1 \cdot \ln(2000) + B_2 \cdot (\ln(2000))^2 + B_3 \cdot (\ln(2000))^3 \quad \dots \text{at } 10 \text{ K}$$

$$\ln(20) = B_0 + B_1 \cdot \ln(200) + B_2 \cdot (\ln(200))^2 + B_3 \cdot (\ln(200))^3 \quad \dots \text{at } 20 \text{ K}$$

$$\ln(40) = B_0 + B_1 \cdot \ln(30) + B_2 \cdot (\ln(30))^2 + B_3 \cdot (\ln(30))^3 \quad \dots \text{at } 40 \text{ K}$$

$$\ln(100) = B_0 + B_1 \cdot \ln(6) + B_2 \cdot (\ln(6))^2 + B_3 \cdot (\ln(6))^3 \quad \dots \text{at } 100 \text{ K}$$

$$\text{Find} \{B_0, B_1, B_2, B_3\} = \begin{bmatrix} 6.21786 \\ -1.11924 \\ 0.13554 \\ -7.37532 \cdot 10^{-3} \end{bmatrix}$$

i.e. We get:

$$B_0 := 6.21786 \quad B_1 := -1.11924 \quad B_2 := 0.13554 \quad B_3 := -7.37532 \cdot 10^{-3} \quad \dots \text{Ans.}$$

**Then, find temp when the resistance is 100 Ohms:**

$$R_e := 100 \quad \text{Ohms}$$

Temp, T is given by:

$$T := \exp[B_0 + B_1 \cdot \ln(R_e) + B_2 \cdot (\ln(R_e))^2 + B_3 \cdot (\ln(R_e))^3]$$

$$\text{i.e. } T = 24.972 \quad \text{K ..... Ans.}$$

**Sensitivity at this temp of 24.972 K:**

We have:

$$T = \exp[B_0 + B_1 \cdot \ln(R_e) + B_2 \cdot (\ln(R_e))^2 + B_3 \cdot (\ln(R_e))^3]$$

Then:

$$dT/dR_e =$$

$$\left( \frac{B_1}{R_e} + 2 \cdot B_2 \cdot \frac{\ln(R_e)}{R_e} + 3 \cdot B_3 \cdot \frac{\ln(R_e)^2}{R_e} \right) \cdot \exp(B_0 + B_1 \ln(R_e) + B_2 \ln(R_e)^2 + B_3 \ln(R_e)^3) = -0.08493$$

K/Ohms

OR:

$$\frac{dR_e}{dT} = \frac{-1}{0.08493} = -11.774 \text{ Ohms/K} \dots \text{Ans.}$$

=====

**Prob.5.2.5** A carbon resistance thermometer has the following electric resistance values: 1460 ohms at 4.2 K; 133 ohms at 77.3 K; and 100 ohms at 300 K. Determine the constants in the calibration relationship, eqn. (6.11). Using these values, determine the temperature indication when the thermometer resistance is 250 ohms. Ref:[1]

**Solution:**

We have:

$$T = \frac{B \cdot \log(R_e)}{\log(R_e)^2 + A \cdot \log(R_e) + K} \dots \text{expression for temp (K) ... eqn. (6.11) in Ref.[1]}$$

**Use the 'Solve block' of Mathcad to determine the constants:**

$$A := 100 \quad B := 100 \quad K := 100 \quad \dots \text{guess values}$$

Given

$$4.2 = \frac{B \cdot \log(1460)}{\log(1460)^2 + A \cdot \log(1460) + K} \dots \text{at 4.2 K}$$

$$77.3 = \frac{B \cdot \log(133)}{\log(133)^2 + A \cdot \log(133) + K} \dots \text{at 77.3 K}$$

$$300 = \frac{B \cdot \log(100)}{\log(100)^2 + A \cdot \log(100) + K} \quad \dots \text{at } 300 \text{ K}$$

$$\text{Find}(A, B, K) = \begin{bmatrix} -3.75451 \\ 2.19826 \\ 3.52368 \end{bmatrix}$$

i.e. We get:

$$A := -3.75451 \quad B := 2.19826 \quad K := 3.52368 \quad \dots \text{Ans.}$$



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**When resistance is 250 Ohms, the temp is:**

$$R_e := 250 \text{ Ohms}$$

$$T := \frac{B \cdot \log(R_e)}{\log(R_e)^2 + A \cdot \log(R_e) + K}$$

$$\text{i.e. } T = 19.472 \text{ K .... Ans.}$$

=====

**Prob.5.2.6.** Write a Mathcad Function to find the temp when emf value is known, for a type T Thermocouple (i.e. copper-constantan). The cold junction is at 0 C.

**Solution:**

**Take the coefficient values for type T Thermocouple from NIST Thermocouple data:**

We have from NIST data:

**To find emf (mV) when temp (deg.C) values are given, in the temp range: -270 to 0 C:**

$$C1_0 := 0 \quad C1_1 := 0.38748106364 \cdot 10^{-1} \quad C1_2 := 0.44194434347 \cdot 10^{-4}$$

$$C1_3 := 0.11844323105 \cdot 10^{-6} \quad C1_4 := 0.20032973554 \cdot 10^{-7} \quad C1_5 := 0.90138019559 \cdot 10^{-9}$$

$$C1_6 := 0.22651156593 \cdot 10^{-10} \quad C1_7 := 0.36071154205 \cdot 10^{-12} \quad C1_8 := 0.38493939883 \cdot 10^{-14}$$

$$C1_9 := 0.28213521925 \cdot 10^{-16} \quad C1_{10} := 0.14251594779 \cdot 10^{-18} \quad C1_{11} := 0.48768662286 \cdot 10^{-21}$$

$$C1_{12} := 0.1079553927 \cdot 10^{-23} \quad C1_{13} := 0.13945027062 \cdot 10^{-26} \quad C1_{14} := 0.79795153927 \cdot 10^{-30}$$

**To find emf (mV) when temp (deg.C) values are given, in the temp range: 0 to 400 C:**

$$C2_0 := 0 \quad C2_1 := 0.38748106364 \cdot 10^{-1} \quad C2_2 := 0.3329222788 \cdot 10^{-4}$$

$$C2_3 := 0.20618243404 \cdot 10^{-6} \quad C2_4 := -0.21882256846 \cdot 10^{-8} \quad C2_5 := 0.10996880928 \cdot 10^{-10}$$

$$C2_6 := -0.30815758772 \cdot 10^{-13} \quad C2_7 := 0.4547913529 \cdot 10^{-16} \quad C2_8 := -0.27512901673 \cdot 10^{-19}$$

### In the following Function:

**t is in deg.C, emf is in mV , reference junction at zero deg.C**

```
TC_Cu_Constantan_emf(t) := | return "temp should be between -270 C and 400 C!"" if t<-270
                             | return "temp should be between -270 C and 400 C!"" if t>400
                             | sum← 0
                             | if t≤0
                             |   for i∈ 0.. 14
                             |     sum← sum + C1i·ti
                             |   e← sum
                             | if t>0
                             |   for i∈ 0.. 8
                             |     sum← sum + C2i·ti
                             |   e← sum
```

Ex:

t := -10 C      TC\_Cu\_Constantan\_emf(t) = -0.383 mV

t := 400 C      TC\_Cu\_Constantan\_emf(t) = 20.872 mV

---

**To find temp (deg.C) when emf (mV) values are given, in the emf range (-5.603 to 0 mV), i.e. temp range: -200 to 0 C:**

**NIST coefficients, for t = -200 C to 0 C (i.e. emf = -5.603 to 0 mV):**

$$D1_0 := 0 \quad D1_1 := 2.5949192 \cdot 10^1 \quad D1_2 := -2.1316967 \cdot 10^{-1}$$

$$D1_3 := 7.9018692 \cdot 10^{-1} \quad D1_4 := 4.2527777 \cdot 10^{-1} \quad D1_5 := 1.3304473 \cdot 10^{-1}$$

$$D1_6 := 2.0241446 \cdot 10^{-2} \quad D1_7 := 1.2668171 \cdot 10^{-3}$$

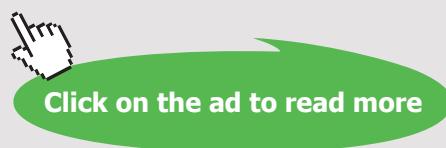
**NIST coefficients, for t = 0 C to 400 C (i.e. emf = 0 to 20.872 mV):**

$$D2_0 := 0 \quad D2_1 := 2.5928 \cdot 10^1 \quad D2_2 := -7.602961 \cdot 10^{-1}$$

$$D2_3 := 4.637791 \cdot 10^{-2} \quad D2_4 := -2.165394 \cdot 10^{-3} \quad D2_5 := 6.048144 \cdot 10^{-5}$$

$$D2_6 := -7.293422 \cdot 10^{-7} \quad D2_7 := 0$$

```
TC_Cu_Constantan_temp(E) := | return "emf should be between -5.603 mV and 20.872 mV!"" if E<-5.603
                             | return "emf should be between -5.603 mV and 20.872 mV!"" if E>20.872
                             | sum← 0
                             | if E≤0
                             |   for i∈ 0..7
                             |     sum← sum + D1i·Ei
                             |     t← sum
                             |   if E>0
                             |     for i∈ 0..7
                             |       sum← sum + D2i·Ei
                             |       t← sum
```



Ex:

$$E := -5.603 \text{ mV} \quad TC_{Cu\text{-Constantan}}(E) = -199.964 \text{ C}$$

$$E := 20.872 \text{ mV} \quad TC_{Cu\text{-Constantan}}(E) = 399.975 \text{ C}$$


---

**Prob.5.2.7.** Write a Mathcad Function to find the temp when emf value is known, for a type K Thermocouple (i.e. Chromel-Alumel). The cold junction is at 0 C.

**Solution:**

**Take the coefficient values for type K Thermocouple from NIST Thermocouple data:**

We have from NISTdata:

**To find emf (mV) when temp (deg.C) values are given, in the temp range: -270 to 0 C:**

$$CC_0 := 0 \quad CC_1 := 0.39450128025 \cdot 10^{-1} \quad CC_2 := 0.23622373598 \cdot 10^{-4}$$

$$CC_3 := -0.32858906784 \cdot 10^{-6} \quad CC_4 := -0.49904828777 \cdot 10^{-8} \quad CC_5 := -0.67509059173 \cdot 10^{-10}$$

$$CC_6 := -0.57410327428 \cdot 10^{-12} \quad CC_7 := -0.31088872894 \cdot 10^{-14} \quad CC_8 := -0.10451609365 \cdot 10^{-16}$$

$$CC_9 := -0.19889266878 \cdot 10^{-19} \quad CC_{10} := -0.16322697486 \cdot 10^{-22}$$

**To find emf (mV) when temp (deg.C) values are given, in the temp range: 0 to 1372 C:**

$$DD_0 := -0.17600413686 \cdot 10^{-1} \quad DD_1 := 0.38921204975 \cdot 10^{-1} \quad DD_2 := 0.18558770032 \cdot 10^{-4}$$

$$DD_3 := -0.99457592874 \cdot 10^{-7} \quad DD_4 := 0.31840945719 \cdot 10^{-9} \quad DD_5 := -0.56072844889 \cdot 10^{-12}$$

$$DD_6 := 0.56075059059 \cdot 10^{-15} \quad DD_7 := -0.32020720003 \cdot 10^{-18} \quad DD_8 := 0.97151147152 \cdot 10^{-22}$$

$$DD_9 := -0.12104721275 \cdot 10^{-25}$$

**Exponentials coeffs.:**

$$a0 := 0.1185976 \quad a1 := -0.1183432 \cdot 10^{-3} \quad a2 := 0.1269686 \cdot 10^3$$

### In the following Function:

t is in deg.C, emf is in mV , reference junction at zero deg.C

```
TC_Chromel_Alumel_emf(t) := | return "temp should be between -270 C and 1372 C!"" if t<- 270
                                | return "temp should be between -270 C and 1372 C!"" if t>1372
                                | sum← 0
                                | if t≤0
                                |   | for i∈ 0.. 10
                                |   |   | sum← sum + CCi·ti
                                |   |   | e← sum
                                |   | if t>0
                                |   |   | for i∈ 0.. 9
                                |   |   |   | sum← sum + DDi·ti
                                |   |   |   | e← sum + a0·exp[a1·(t - a2)2]
```

---

To find temp (deg.C) when emf (mV) values are given, in the emf range (-5.891 to 0 mV), i.e. temp range: -200 to 0 C:

NIST coefficients, for t = -200 C to 0 C (i.e. emf = -5.891 to 0 mV):

$$DD1_0 := 0 \quad DD1_1 := 2.5173462 \cdot 10^1 \quad DD1_2 := -1.1662878$$

$$DD1_3 := -1.0833638 \quad DD1_4 := -8.977354 \cdot 10^{-1} \quad DD1_5 := -3.7342377 \cdot 10^{-1}$$

$$DD1_6 := -8.6632643 \cdot 10^{-2} \quad DD1_7 := -1.0450598 \cdot 10^{-2} \quad DD1_8 := -5.1920577 \cdot 10^{-4}$$

$$DD1_9 := 0$$

NIST coefficients, for t = 0 C to 500 C (i.e. emf = 0 to 20.644 mV):

$$DD2_0 := 0 \quad DD2_1 := 2.508355 \cdot 10^1 \quad DD2_2 := 7.860106 \cdot 10^{-2}$$

$$DD2_3 := -2.503131 \cdot 10^{-1} \quad DD2_4 := 8.31527 \cdot 10^{-2} \quad DD2_5 := -1.228034 \cdot 10^{-2}$$

$$DD2_6 := 9.804036 \cdot 10^{-4} \quad DD2_7 := -4.41303 \cdot 10^{-5} \quad DD2_8 := 1.057734 \cdot 10^{-6}$$

$$DD2_9 := -1.052755 \cdot 10^{-8}$$

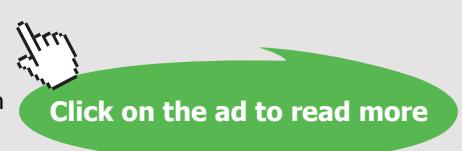
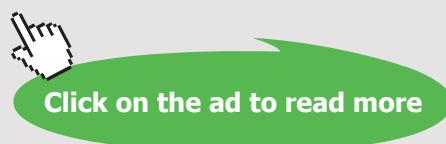
**NIST coefficients, for t = 500 C to 1372 C (i.e. emf = 20.644 to 54.886 mV):**

$$DD3_0 := -1.318058 \cdot 10^2 \quad DD3_1 := 4.830222 \cdot 10^1 \quad DD3_2 := -1.646031$$

$$DD3_3 := 5.464731 \cdot 10^{-2} \quad DD3_4 := -9.650715 \cdot 10^{-4} \quad DD3_5 := 8.802193 \cdot 10^{-6}$$

$$DD3_6 := -3.11081 \cdot 10^{-8} \quad DD3_7 := 0 \quad DD3_8 := 0 \quad DD3_9 := 0$$

TC\_Chromel\_Alumel\_temp(E) :=



```

return "emf should be between -5.891 mV and 54.886 mV!"" if E<-5.891
return "emf should be between -5.891 mV and 54.886 mV!"" if E>54.886
sum← 0
if E≤0
    for i∈ 0..9
        sum← sum + DD1i·Ei
    t← sum
    return t
if E≤20.644
    for i∈ 0..9
        sum← sum + DD2i·Ei
    t← sum
    return t
if E≤54.886
    for i∈ 0..9
        sum← sum + DD3i·Ei
    t← sum
    return t

```

Ex:

E := -3.243 mV      TC\_Chromel\_Alumel\_temp(E) = -90.007 C

E := 54.886 mV      TC\_Chromel\_Alumel\_temp(E) = 1.372·10<sup>3</sup> C

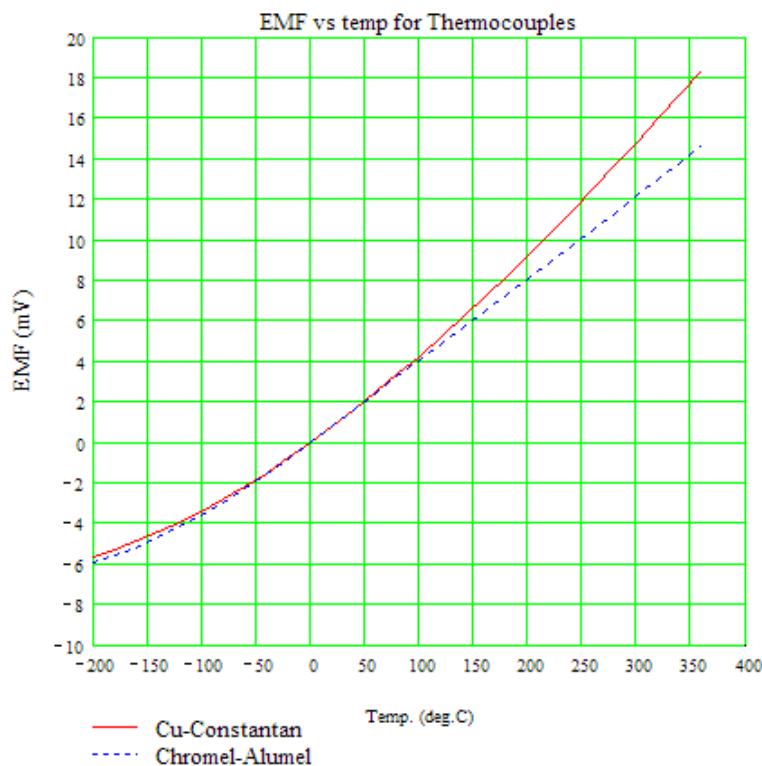
E := 31.213 mV      TC\_Chromel\_Alumel\_temp(E) = 749.988 C

**Prob. 5.2.8** Plot Temp vs emf for type T and type K thermocouples, for the temp range: -200 C to 350 C:

**Solution:**

t := -200, -180..360 C .... define a range variable

t	TC_Cu_Constantan_emf(t)	TC_Chromel_Alumel_emf(t)
-200	-5.603	-5.891
-180	-5.261	-5.55
-160	-4.865	-5.141
-140	-4.419	-4.669
-120	-3.923	-4.138
-100	-3.379	-3.554
-80	-2.788	-2.92
-60	-2.153	-2.243
-40	-1.475	-1.527
-20	-0.757	-0.778
0	0	0
20	0.79	0.798
40	1.612	1.612
60	2.468	2.436
80	3.358	3.267
100	4.279	4.096
120	5.228	4.92
140	6.206	5.735
160	7.209	6.54
180	8.237	7.34
200	9.288	8.138
220	10.362	8.94
240	11.458	9.747
260	12.574	10.561
280	13.709	11.382
300	14.862	12.209
320	16.032	13.04
340	17.219	13.874
360	18.422	14.713



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**Prob.5.2.9.** Write a Mathcad Function to find the temp when emf value is known and vice-versa, for a Chromel vs Au-0.03 at.%Fe Thermocouple. The cold junction is at 0 C (i.e. 273 K).

**Solution:**

From the data given by Lakeshore (Ref: <http://www.lakeshore.com/Documents/F036-00-00.pdf>), we have:

Temp (K) E\_TC (mV)

3.5	-4.6676
8	-4.6067
13.5	-4.5259
18	-4.4571
24	-4.3703
30	-4.2869
52	-3.9928
60	-3.883
65	-3.8126
70	-3.7411
80	-3.5948
90	-3.4436
105	-3.2026
115	-3.0374
-----	
MM1 :=	125 -2.8689
	135 -2.6957
	145 -2.5184
	160 -2.2468
	170 -2.0615
	180 -1.8725
	195 -1.5839
	210 -1.2905
	225 -0.9912
	240 -0.6847
	265 -0.167
	275 0.0378
	285 0.2387
	305 0.635
	325 1.0387

Let: temp := MM1<sup><0></sup>      EMF := MM1<sup><1></sup>

**Now, write the Function to find emf (mV) when temp (K) is known:**

```
TC_Chromel_Au_03Fe_emf(t) := | return "temp should be between 3.5 K and 325 K!"" if t<3.5
                                | return "temp should be between 3.5 K and 325 K!"" if t>325
                                | emf← linterp(temp,EMF,t)
```

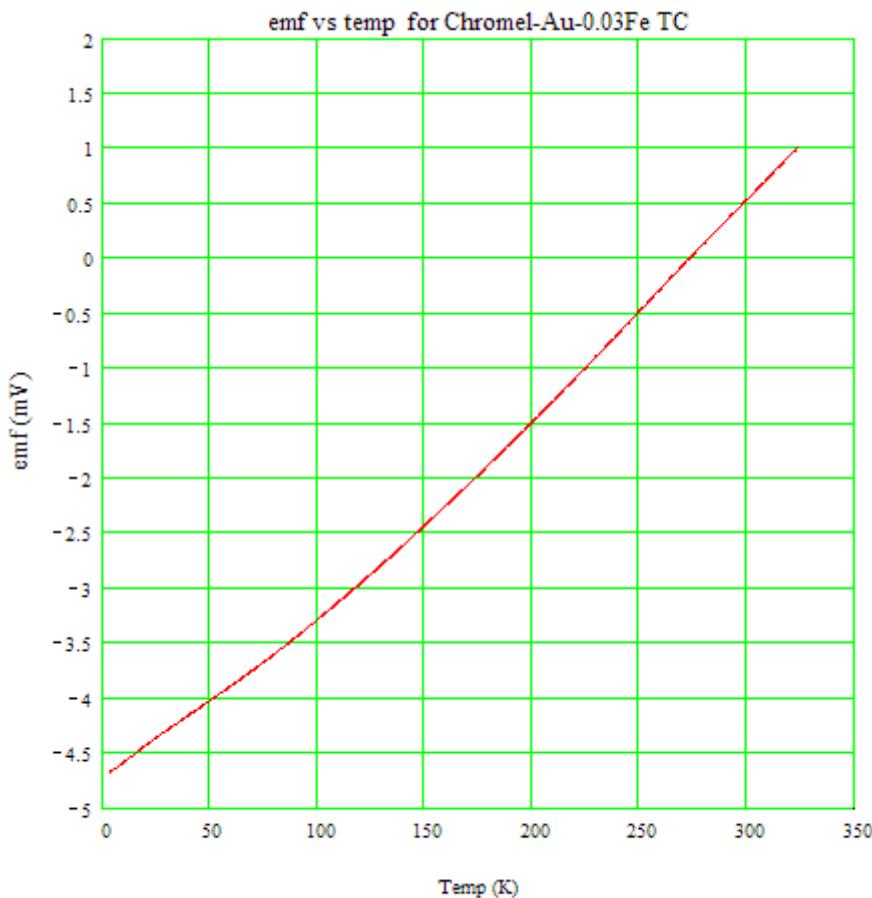
Ex:       $t := 5 \text{ K}$

$\text{TC_Chromel\_Au\_03Fe\_emf}(t) = -4.6473 \text{ mV}$

---

**Plot emf vs temp for Chromel – Au-0.03 at. % Fe thermocouple:**

$t := 3.5, 4..325 \text{ K}$ ...define a range variable



**Function to find temp (K) when emf (mV) is known:**

TC\_Chromel\_Au\_03Fe\_temp(emf) :=

```
return "emf should be between -4.6676 mV and 1.0387 mV!"" if emf<-4.6676  
return "emf should be between -4.6676 mV and 1.0387 mV!"" if emf>1.0387  
temp←linterp(EMF,temp,emf)
```

Ex:      emf := -4            mV

TC\_Chromel\_Au\_03Fe\_temp(emf) = 51.46141      K

---

**Prob.5.2.10.** Write a Mathcad Function to find the temp when emf value is known and vice-versa, for a Chromel vs Au-0.07 at.%Fe Thermocouple. The cold junction is at 0 C (i.e. 273 K).

**Solution:**

From the data given by Lakeshore (Ref: <http://www.lakeshore.com/Documents/F036-00-00.pdf>),



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we have:

Temp (K)	E_TC (mV)
1.4	- 5.2982
3	- 5.2815
4.8	- 5.2594
7	- 5.2285
10.5	- 5.1742
19	- 5.0315
26	- 4.9126
48	- 4.5494
58	- 4.381
70	- 4.1733
80	- 3.9952
90	- 3.8132
100	- 3.627
110	- 3.437
MM2 :=	120 - 3.2435
135	- 2.9477
150	- 2.6452
165	- 2.3372
180	- 2.0242
200	- 1.6004
220	- 1.1693
245	- 0.6232
270	- 0.0705
300	0.5986
305	0.7158
310	0.8431
315	0.9944
320	1.194
325	1.4841

Let:  $\text{temp2} := \text{MM2}^{<0>} \quad \text{EMF2} := \text{MM2}^{<1>}$

**Now, write the Function:**

```
TC_Chromel_Au_07Fe_emf(t) := | return "temp should be between 1.4 K and 325 K!"" if t<1.4
                                | return "temp should be between 1.4 K and 325 K!"" if t>325
                                | emf← linterp(temp2,EMF2,t)
```

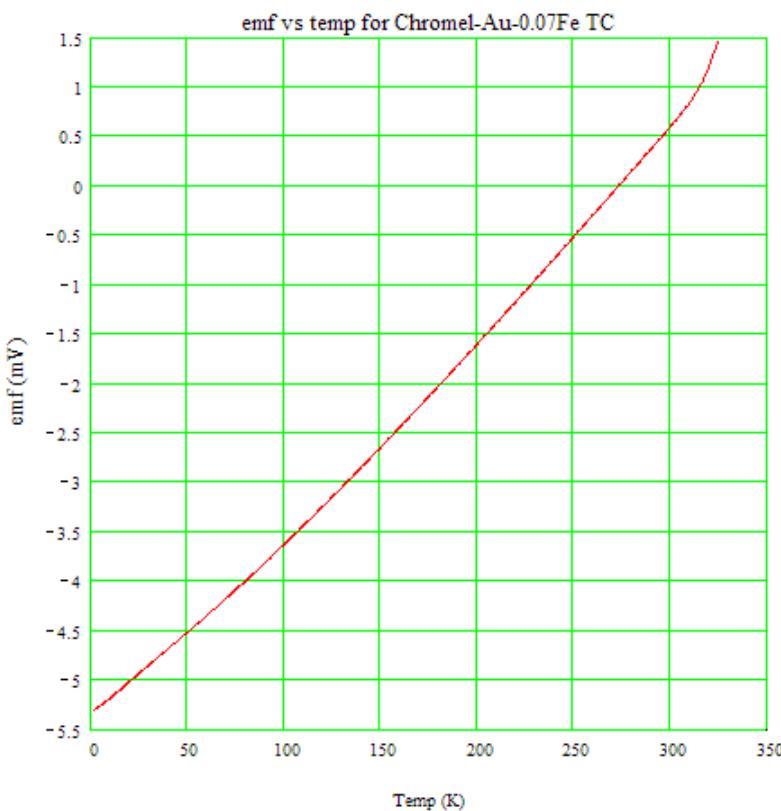
Ex:  $t := 3 \text{ K}$

$\text{TC_Chromel_Au_07Fe_emf}(t) = -5.2815 \text{ mV}$

---

### Plot emf vs temp for Chromel – Au-0.07 at % Fe thermocouple:

$t := 1.4, 1.9..325 \text{ K}$ ...define a range variable



### Function to find temp (K) when emf (mV) is known:

```
TC_Chromel_Au_07Fe_temp(emf) := 
  | return "emf should be between -5.2982 mV and 1.4841 mV!"" if emf < -5.2982
  | return "emf should be between -5.2982 mV and 1.4841 mV!"" if emf > 1.4841
  | temp ← linterp(EMF2, temp2, emf)
```

---

Ex:  $\text{emf} := -4 \text{ mV}$

$\text{TC_Chromel_Au_07Fe_temp(emf)} = 79.73049 \text{ K}$

---

**Prob. 5.2.11** Determine the sensitivity,  $S_0$  for a copper-constantan thermocouple (reference junction at the ice point) at a temperature of -100 C. Ref:[1]

We have, from NIST data on Thermocouples, given earlier, for copper-constantan Thermocouple (see Prob.5.2.6):

$t := -100 \text{ } ^\circ\text{C}$

$$e = \sum_{i=0}^{14} C_{1i} t^i$$

Then:  $(de / dt) =$

$$C_{11} + 2 \cdot C_{12} \cdot t + 3 \cdot C_{13} \cdot t^2 + 4 \cdot C_{14} \cdot t^3 + 5 \cdot C_{15} \cdot t^4 + 6 \cdot C_{16} \cdot t^5 + 7 \cdot C_{17} \cdot t^6 + 8 \cdot C_{18} \cdot t^7 + 9 \cdot C_{19} \cdot t^8 + 10 \cdot C_{110} \cdot t^9 + 11 \cdot C_{111} \cdot t^{10} + 12 \cdot C_{112} \cdot t^{11} + 13 \cdot C_{113} \cdot t^{12} + 14 \cdot C_{114} \cdot t^{13}$$

i.e. we get:  $(de / dt)$  at  $-100 \text{ } ^\circ\text{C} = 0.028 \text{ mV/C...Ans.}$



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**Alternatively:**

**Let us use the Mathcad Function written earlier in Prob. 5.2.6:**

At -100 C:

$$t := -100 \text{ C} \quad TC\_Cu\_Constantan\_emf(t) = -3.379 \text{ mV}$$

At - 101 C:

$$t := -101 \text{ C} \quad TC\_Cu\_Constantan\_emf(t) = -3.407 \text{ mV}$$

Therefore:

$$\frac{de}{dt} = \frac{-3.407 - (-3.379)}{-101 - (-100)} = 0.028 \text{ mV/C ... Ans.}$$

i.e.  $(de / dt)$  at -100 C = 0.028 mV/C.... Ans.... verified.

---

**Prob. 5.2.12** A constant-volume gas thermometer using helium gas as the working medium is calibrated at the ice point, 273.15 K or 491.67°R. At this point, the pressure indication of the thermometer is 75 kPa (10.88 psia). The ratio of the dead-volume to sensing element volume is 0.80. Determine the temperature indication when the pressure indication is 25 kPa (3.63 psia) and the dead-volume gas temperature is 300 K. Neglect gas imperfections. Ref:[1]

**Solution:**

We have:

$$T_s := 273.15 \text{ K} \quad p_s := 75 \text{ kPa} \quad p := 25 \text{ kPa}$$

$$T_0 := 300 \text{ K} \quad V0byV := 0.8$$

Then, the dead-volume correction is:

$$K_1 := \left[ 1 + \left( 1 - \frac{p}{p_s} \right) \cdot V0byV \cdot \left( \frac{T_s}{T_0} \right) \right]^{-1}$$

$$\text{i.e. } K_1 = 0.673$$

Then, temp indicated is:

$$T := K_1 \cdot p \cdot \left( \frac{T_s}{p_s} \right)$$

$$\text{i.e. } T = 61.288 \quad \text{K .... Ans.}$$

And, the Sensitivity is:

$$S_0 := \frac{\frac{p_s}{T_s}}{K_1 \cdot \left( 1 + \frac{T}{T_0} \cdot V_0 \text{byV} \right)}$$

$$\text{i.e. } S_0 = 0.351 \quad \text{kPa/K ... Ans.}$$


---

If the gas imperfections are not neglected:

The indicated temp calculated above is now termed as  $T_{\text{prime}}$ :

$$\text{i.e. } T_{\text{prime}} := 61.288 \quad \text{K}$$

At this temp. second virial coeff. is calculated as follows:

$$B(T_{\text{prime}}) := 0.0072929 \cdot T_{\text{prime}}^{-\frac{5}{4}} \cdot \left( 1 - 3.04135 \cdot T_{\text{prime}}^{-\frac{1}{2}} - 14.1425 \cdot T_{\text{prime}}^{-1} + 17.35052 \cdot T_{\text{prime}}^{\frac{-3}{2}} \right)$$

$$\text{i.e. } B(T_{\text{prime}}) = 1.773 \cdot 10^{-5} \quad \text{kPa}^{-1}$$

Similarly, at  $T_s = 273.15 \text{ K}$ :

$$B(T_s) = 5.044 \cdot 10^{-6} \quad \text{kPa}^{-1}$$

Therefore, gas imperfection correction factor is:

$$K_2 := \frac{B(T_s) \cdot p_s - B(T_{\text{prime}}) \cdot p}{1 + B(T_{\text{prime}}) \cdot p}$$

$$\text{i.e. } K_2 = -6.493 \cdot 10^{-5}$$

Therefore, temp error due to gas imperfections is:

$$\Delta T := K_2 \cdot T_{\text{prime}} \quad \text{i.e.} \quad \Delta T = -3.98 \cdot 10^{-3} \quad \text{K}$$

Then, the temp when gas imperfection effect is included is:

$$T_c := T_{\text{prime}} + \Delta T$$

$$\text{i.e.} \quad T_c = 61.284 \quad \text{K} \dots \text{corrected temp.... Ans.}$$

---

Plot the reduced Sensitivity,  $[(T_s/p_s) \cdot (dp/dT)]$  against temp  $T$  (K) for a constant volume gas thermometer with  $T_s = 273$  K and  $p_s = 101.325$  kPa, neglecting the gas imperfections:

We have, reduced sensitivity:

$$S_0_{\text{reduced}} = \frac{T_s}{p_s} \cdot \frac{dp}{dT}$$



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$$\text{But, } \frac{dp}{dT} = S_0 = \frac{\frac{p_s}{T_s}}{K_1 \left( 1 + \frac{T}{T_0} \cdot V_0 \text{byV} \right)}$$

Therefore,

$$S_0_{\text{reduced}} = \frac{T_s \cdot \frac{p_s}{T_s}}{p_s K_1 \left( 1 + \frac{T}{T_0} \cdot V_0 \text{byV} \right)} = \frac{1}{\left[ K_1 \left( 1 + \frac{T}{T_0} \cdot V_0 \text{byV} \right) \right]}$$

$$\text{And, } K_1 = \left[ 1 + \left( 1 - \frac{p}{p_s} \right) \cdot V_0 \text{byV} \cdot \frac{T_s}{T_0} \right]^{-1}$$

So, writing Reduced sensitivity as a function of T and V0byV. we get:

$$S_0_{\text{reduced}}(T, V_0 \text{byV}) := \frac{1}{\left[ \left[ 1 + \left( 1 - \frac{p}{p_s} \right) \cdot V_0 \text{byV} \cdot \frac{T_s}{T_0} \right]^{-1} \cdot \left( 1 + \frac{T}{T_0} \cdot V_0 \text{byV} \right) \right]}$$

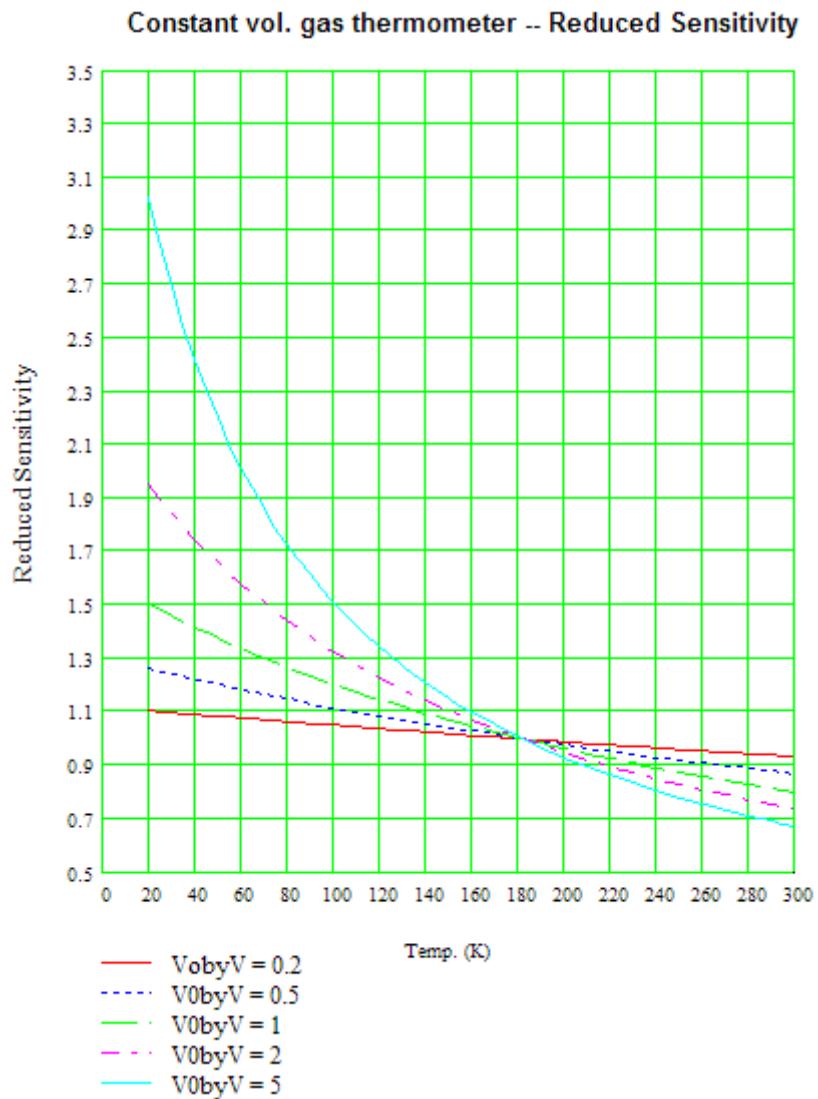
For the above case we have the data:

$$T_s := 273.15 \quad \text{K} \quad p_s := 75 \quad \text{kPa} \quad p := 25 \quad \text{kPa}$$

$$T_0 := 300 \quad \text{K}$$

Now, to plot the graph:

$$T := 20, 25..300 \quad \text{K} \dots \text{define a range variable}$$



**Prob. 5.2.13** A constant-volume gas thermometer is calibrated at the triple point of water, at which point the pressure indication is 150 kPa. Determine the temperature indication and sensitivity of the thermometer when the pressure reading is 20 kPa (2.90 psia). The ratio of dead volume to sensing element volume is 2.00, and the dead-volume gas is at 300 K (540°R). Neglect gas imperfections.[1]

**Solution:**

**Data:**

$$T_s := 273.16 \quad K \quad p_s := 150 \quad \text{kPa} \quad p := 20 \quad \text{kPa}$$

$$T_0 := 300 \quad K \quad V_0/V := 2$$

Then, the dead-volume correction is:

$$K_1 := \left[ 1 + \left( 1 - \frac{p}{p_s} \right) \cdot \frac{V_0}{V} \cdot \frac{T_s}{T_0} \right]^{-1}$$

i.e.  $K_1 = 0.388$

Then, temp indicated is:

$$T := K_1 p \cdot \frac{T_s}{p_s}$$

i.e.  $T = 14.126 \text{ K .... Ans.}$



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And, the Sensitivity is:

$$S_0 := \frac{\frac{p_s}{T_s}}{K_1 \left( 1 + \frac{T}{T_0} \cdot V_{0\text{by}V} \right)}$$

i.e.  $S_0 = 1.294 \text{ kPa/K ... Ans.}$

---

**Prob. 5.2.14** Determine the rate of change of pressure indication with change in dead volume temperature,  $dp/dT_0$ , for a constant-volume gas thermometer. Neglect gas imperfections, and assume that the temperature of the gas in the sensing element remains constant. Calculate a numerical value for the change in pressure indication per unit change in dead-volume temperature for  $T_s = 273.16 \text{ K}$  ( $492.69^\circ\text{R}$ ),  $p_s = 120 \text{ kPa}$  ( $17.40 \text{ psia}$ ),  $p = 40 \text{ kPa}$  ( $5.80 \text{ psia}$ ),  $T_0 = 300 \text{ K}$  ( $540^\circ\text{R}$ ), and  $V_{0/V} = 1$ .[1]

**Solution:**

**By data:**

$$p_s := 120 \text{ kPa} \quad T_s := 273.16 \text{ K} \quad p := 40 \text{ kPa} \quad T_0 := 300 \text{ K}$$

$$V_{0\text{by}V} := 1$$

We have:

$$T = \frac{p \cdot \left( \frac{T_s}{p_s} \right)}{1 + \left( 1 - \frac{p}{p_s} \right) \cdot (V_{0\text{by}V}) \cdot \left( \frac{T_s}{T_0} \right)}$$

i.e.  $T = 56.66 \text{ K}$

Then,

**Solving for p:**

$$p = \frac{- \left( T \cdot p_s \cdot T_0 + T \cdot V_{0\text{by}V} \cdot T_s \cdot p_s \right)}{\left( -T \cdot V_{0\text{by}V} \cdot T_s - T_s \cdot T_0 \right)}$$

Then, we get, by differentiating in Mathcad:

$$\frac{dp}{dT_0} = T \cdot \frac{p_s}{\left( -T \cdot V0byV \cdot T_s - T_s \cdot T_0 \right)} + \frac{\left( -T \cdot p_s \cdot T_0 - T \cdot V0byV \cdot T_s \cdot p_s \right)}{\left( -T \cdot V0byV \cdot T_s - T_s \cdot T_0 \right)^2} \cdot T_s \quad ...Ans.$$

Now, numerical values given are:

$$p_s := 120 \text{ kPa} \quad T_s := 273.16 \text{ K} \quad p := 40 \text{ kPa} \quad T_0 := 300 \text{ K}$$

$$V0byV := 1$$

Then, using the above eqn for  $dp/dT_0$ , we get:

$$dp/dT_0 =$$

$$\left[ -T \cdot \frac{p_s}{\left( -T \cdot V0byV \cdot T_s - T_s \cdot T_0 \right)} + \frac{\left( -T \cdot p_s \cdot T_0 - T \cdot V0byV \cdot T_s \cdot p_s \right)}{\left( -T \cdot V0byV \cdot T_s - T_s \cdot T_0 \right)^2} \cdot T_s \right] = -0.042 \text{ kPa/K} ...Ans.$$

=====

**Prob. 5.2.15** Determine the rate of change in sensitivity with change in dead-volume ratio,  $dS_0/d(V_0/V)$ , for a constant-volume gas thermometer. Neglect gas imperfections, and assume that the pressure  $p$  and temperature  $T$  are constant. Calculate a numerical value for the change in sensitivity with respect to change in dead volume ratio for  $T_s = 273.16 \text{ K}$  ( $492.69^\circ\text{R}$ ),  $p_s = 120 \text{ kPa}$  ( $17.40 \text{ psia}$ ),  $p = 40 \text{ kPa}$  ( $5.80 \text{ psia}$ ),  $T_0 = 300 \text{ K}$  ( $540^\circ\text{R}$ ), and  $V_0/V = 1.00$ . [1]

**Data:**

$$p_s := 120 \text{ kPa} \quad T_s := 273.16 \text{ K} \quad p := 40 \text{ kPa} \quad T_0 := 300 \text{ K}$$

$$V0byV := 1$$

Then, we have:

$$T := \frac{p \cdot \left( \frac{T_s}{p_s} \right)}{1 + \left( 1 - \frac{p}{p_s} \right) \cdot (V0byV) \cdot \left( \frac{T_s}{T_0} \right)} \quad \text{i.e. } T = 56.66 \text{ K}$$

We have, for Sensitivity:

$$S_0 = \frac{\frac{p_s}{T_s}}{\left(1 + \frac{T}{T_0} \cdot V_0 \text{byV}\right) \cdot \left[1 + \left(1 - \frac{p}{p_s}\right) \cdot V_0 \text{byV} \cdot \frac{T_s}{T_0}\right]^{-1}}$$

Then, by differentiating in Mathcad:

$$dS_0/d(V_0 \text{byV}) =$$

$$\frac{-p_s}{\left[\frac{T_s}{T_0} \cdot \left(1 + \frac{T}{T_0} \cdot V_0 \text{byV}\right)^2\right]} \cdot \left[1 + \left(1 - \frac{p}{p_s}\right) \cdot V_0 \text{byV} \cdot \frac{T_s}{T_0}\right] \frac{T}{T_0} + \frac{p_s}{\left(1 + \frac{T}{T_0} \cdot V_0 \text{byV}\right)} \cdot \frac{\left(1 - \frac{p}{p_s}\right)}{T_0}$$



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And, we get:

$$dS_0/d(V_0 \text{by} V) =$$

$$\frac{-p_s}{\left[ T_s \left( 1 + \frac{T}{T_0} \cdot V_0 \text{by} V \right)^2 \right]} \left[ 1 + \left( 1 - \frac{p}{p_s} \right) \cdot V_0 \text{by} V \cdot \frac{T_s}{T_0} \right] \cdot \frac{T}{T_0} + \frac{p_s}{\left( 1 + \frac{T}{T_0} \cdot V_0 \text{by} V \right)} \cdot \frac{\left( 1 - \frac{p}{p_s} \right)}{T_0} = 0.13 \quad \dots \text{Ans.}$$


---

**Prob. 5.2.16** Determine the temp indication for a liquid hydrogen vapor pressure thermometer when the pressure reading is 14 kPa. [1]

**Solution:**

Normal boiling point for LH<sub>2</sub> at 1 atm = T<sub>0</sub> = 36.7 R = 20.3889 K

**Data:**

$$p_0 := 101.325 \text{ kPa} \quad T_0 := 20.3889 \text{ K} \quad p := 14 \text{ kPa}$$

**Vapor pressure-temp relation is:**

$$\ln\left(\frac{p}{p_0}\right) = C1 - \frac{C2}{T} - C3 \cdot \ln\left(\frac{T}{T_0}\right) - C4 \cdot T + C5 \cdot T^2$$

where, the constants C<sub>1</sub>,...,C<sub>5</sub> for H<sub>2</sub> are given below [1]:

$$C1 := 3.940796 \quad C2 := 101.33783 \quad C3 := 0 \quad C4 := -0.0543201 \quad C5 := -1.10563 \cdot 10^{-4}$$

**To find T corresponding to a pressure p = 14 kPa:**

Use the 'Solve block' of Mathcad:

$$T := 20 \text{ K} \dots \text{guess value}$$

Given

$$\ln\left(\frac{p}{p_0}\right) = C1 - \frac{C2}{T} - C3 \cdot \ln\left(\frac{T}{T_0}\right) - C4 \cdot T + C5 \cdot T^2$$

$$T := \text{Find}(T)$$

$T := \text{Find}(T)$

i.e.  $T = 15.092 \text{ K}$  .... Ans.

=====

**Prob. 5.2.17** Determine the temp indication for a liquid neon vapor pressure thermometer when the pressure reading is 225 kPa. Also, determine the sensitivity at this point. [1]

**Solution:**

Normal boiling point for Liquid Neon at 1 atm =  $T_0 = 48.8 \text{ R} = 27.1111 \text{ K}$

**Data:**

$$p_0 := 101.325 \text{ kPa} \quad T_0 := 27.1111 \text{ K} \quad p := 225 \text{ kPa}$$

Vapor pressure-temp relation is:

$$\ln\left(\frac{p}{p_0}\right) = C1 - \frac{C2}{T} - C3 \cdot \ln\left(\frac{T}{T_0}\right) - C4 \cdot T + C5 \cdot T^2$$

where, the constants  $C1, \dots, C5$  for Neon are given below [1]:

$$C1 := 10.618417 \quad C2 := 244.96075 \quad C3 := 0 \quad C4 := 0.0848111 \quad C5 := 9.78350 \cdot 10^{-4}$$

**To find T corresponding to a pressure  $p = 225 \text{ kPa}$ :**

Use the 'Solve block' of Mathcad:

$T := 25 \text{ K}$  .... guess value

Given

$$\ln\left(\frac{p}{p_0}\right) = C1 - \frac{C2}{T} - C3 \cdot \ln\left(\frac{T}{T_0}\right) - C4 \cdot T + C5 \cdot T^2$$

$T := \text{Find}(T)$

i.e.  $T = 30.035 \text{ K}$  .... Ans.

### Sensitivity at this temp. T:

From the vapor pressure eqn. we have:

$$p = p_0 \cdot \exp\left(C1 - \frac{C2}{T} - C3 \cdot \ln\left(\frac{T}{T_0}\right) - C4 \cdot T + C5 \cdot T^2\right)$$

Differentiating p w.r.t. T in Mathcad, we get for Sensitivity:

$$\frac{dp}{dT} =$$

$$p_0 \cdot \left(\frac{C2}{T^2} - \frac{C3}{T} - C4 + 2 \cdot C5 \cdot T\right) \cdot \exp\left(C1 - \frac{C2}{T} - C3 \cdot \ln\left(\frac{T}{T_0}\right) - C4 \cdot T + C5 \cdot T^2\right) = 55.239 \text{ kPa/K .... Ans.}$$

Check:

From Ref.[1], i.e. Barron, we have, for Sensitivity:

$$S_0 := p \cdot \left(\frac{C2}{T^2} - \frac{C3}{T} - C4 + 2 \cdot C5 \cdot T\right)$$

i.e.  $S_0 = 55.239 \text{ kPa/K .... verified.}$

=====



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**Prob. 5.2.18** Chromium potassium alum is used as a magnetic thermometer. The Curie constant for this material is  $45.8 \times 10^{-6}$  K-m<sup>3</sup>/kg. Determine the absolute temperature indication of the thermometer when the measured magnetic susceptibility is  $1.167 \times 10^{-4}$  m<sup>3</sup>/kg. At this point, determine the sensitivity of the thermometer, where  $S_0 = dX/dT$ . [1]

**Solution:**

**Data:**

Material: Chromium potassium alum

$$C := 45.8 \cdot 10^{-6} \text{ K-m}^3/\text{kg} \dots \text{Curie constant}$$

$$X := 1.167 \cdot 10^{-4} \text{ m}^3/\text{kg} \dots \text{magnetic susceptibility}$$

For Chromium potassium alum, we have the relation between absolute temp and magnetic temp:

$$T_{\text{star}} = T + 0.000862 + \frac{0.002057}{T} \dots \text{eqn. A}$$

Now,  $T_{\text{star}}$  is given by Curie law:

$$T_{\text{star}} := \frac{C}{X}$$

$$\text{i.e. } T_{\text{star}} = 0.392 \text{ K} \dots \text{magnetic temp.}$$

Now, using the above eqn. A, and the 'Solve block' of Mathcad:

$$T := 0.1 \text{ K} \dots \text{guess value}$$

Given

$$T_{\text{star}} = T + 0.000862 + \frac{0.002057}{T}$$

$$T := \text{Find}(T)$$

$$\text{i.e. } T = 0.386 \text{ K} \dots \text{absolute temp..... Ans.}$$

**Sensitivity:**

$$S_0 = \frac{dX}{dT}$$

$$\text{Now, we have: } T_{\text{star}} = \frac{C}{X}$$

and,

$$T_{\text{star}} = T + 0.000862 + \frac{0.002057}{T}$$

Therefore:

$$\frac{C}{X} = T + 0.000862 + \frac{0.002057}{T}$$

OR:  $X = \frac{C}{\left( T + 0.000862 + \frac{0.002057}{T} \right)}$

And, differentiating X w.r.t T in Mathcad, we get, Sensitivity =

$$dX/dT =$$

$$\frac{-C}{\left( T + 8.62 \cdot 10^{-4} + \frac{0.002057}{T} \right)^2} \cdot \left( 1 - \frac{0.002057}{T^2} \right) = -2.933 \cdot 10^{-4}$$

i.e. Sensitivity =  $-2.933 \times 10^{-4} \text{ m}^3/\text{kg.K}$  .... Ans.

=====

**Prob. 5.2.19** A cerium magnesium nitrate magnetic thermometer (Curie constant =  $5.22 \times 10^{-6} \text{ K-m}^3/\text{kg}$ ) indicates a magnetic susceptibility of  $1.044 \times 10^{-4} \text{ m}^3/\text{kg}$ . Determine the temperature indication of the thermometer.[1]

**Data:**

Material: Cerium magnesium nitrate

$$C := 5.22 \cdot 10^{-6} \text{ K-m}^3/\text{kg} \text{ .... Curie constant}$$

$$X := 1.044 \cdot 10^{-4} \text{ m}^3/\text{kg} \text{ .... magnetic susceptibility}$$

Now,  $T_{\text{star}}$ , the magnetic temp. is given by Curie law:

$$T_{\text{star}} := \frac{C}{X}$$

$$\text{i.e. } T_{\text{star}} = 0.05 \text{ K .... magnetic temp.}$$

For Cerium magnesium nitrate, we have the relation between absolute temp and magnetic temp:

$$T_{\text{star}} = T + \left(0.236 + \frac{0.004137}{T}\right) \cdot 10^{-3} \quad \dots \text{eqn. B}$$

Now, using the above eqn. B, and the 'Solve block' of Mathcad:

$T := 0.1 \text{ K}$  .... guess value

Given

$$T_{\text{star}} = T + \left(0.236 + \frac{0.004137}{T}\right) \cdot 10^{-3}$$

$T := \text{Find}(T)$

i.e.  $T = 0.04968 \text{ K}$  .... Absolute temp..... Ans.



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**Sensitivity at this temp:**

We have:

$$\frac{C}{X} = T + \left( 0.236 + \frac{0.004137}{T} \right) \cdot 10^{-3}$$

OR:  $X = \frac{C}{\left[ T + \left( 0.236 + \frac{0.004137}{T} \right) \cdot 10^{-3} \right]}$

And, differentiating X w.r.t T in Mathcad, we get, Sensitivity =

$$dX/dT =$$

$$\frac{-C}{\left( T + 2.360000000000000000000000 \cdot 10^{-4} + \frac{4.137000000000000000000000 \cdot 10^{-6}}{T} \right)^2} \cdot \left\{ 1 - \frac{4.137000000000000000000000 \cdot 10^{-6}}{T^2} \right\} = -2.085 \cdot 10^{-3}$$

i.e. Sensitivity =  $-2.085 \times 10^{-3} \text{ m}^3/\text{kg.K}$  .... Ans.

=====

**Prob. 5.2.20** Write a Mathcad Function to determine the coeff. of discharge, Cd for an orifice meter with 'D and D/2 pressure taps', using the ISO-ASME expression for Cd.

Then, write a Function to determine the mass flow rate and volume flow rate for such an orifice meter when the pipe dia, orifice dia, fluid density, viscosity and pressure drop are given.

**Function to determine Cd of orifice:**

$$C_d(\beta, N_{Re}) := 0.5959 + 0.0312 \cdot \beta^{2.1} - 0.184 \cdot \beta^8 + \frac{0.039 \cdot \beta^4}{1 - \beta^4} - 0.01584 \cdot \beta^3 + 91.71 \cdot \beta^{2.5} \cdot N_{Re}^{-0.75}$$

Ex:  $\beta := 0.5 \quad N_{Re} := 2.639 \cdot 10^5$

$$C_d(\beta, N_{Re}) = 0.604$$

=====

### Function to determine the Cd, mass flow rate and volume flow rate of an orifice meter:

#### Inputs:

$D_p$  = pipe dia (m),  $D_0$  = orifice dia (m),  $\rho$  = density ( $\text{kg}/\text{m}^3$ ),  $\mu$  = viscosity ( $\text{Pa}\cdot\text{s}$ ),  $\Delta P$  = pressure drop (Pa)

#### Outputs:

$\beta = D_0/D_p$ ,  $C_a$  = Vel. of approach factor,  $C_d$  = coeff of discharge,  $Re$  = Reynolds No.,  
 $m_{dot}$  = mass flow rate ( $\text{kg}/\text{s}$ ),  $V_{dot}$  = vol. flow rate ( $\text{m}^3/\text{s}$ )

```

Orifice_meter(D_p,D_0,rho,mu,dP) := 
    β ←  $\frac{D_0}{D_p}$ 
     $C_a \leftarrow \left(1 - \beta^4\right)^{-\frac{1}{2}}$ 
     $A_0 \leftarrow \frac{\pi \cdot D_0^2}{4}$ 
     $A_p \leftarrow \frac{\pi \cdot D_p^2}{4}$ 
     $Cd_{guess} \leftarrow 0.6$ 
     $m_{dot1} \leftarrow Cd_{guess} \cdot C_a \cdot A_0 \cdot (2 \cdot \rho \cdot \Delta P)^{\frac{1}{2}}$ 
     $N_{Re1} \leftarrow \frac{m_{dot1}}{A_p} \cdot \frac{D_p}{\mu}$ 
     $Cd_2 \leftarrow C_d(\beta, N_{Re1})$ 
    while  $|Cd_2 - Cd_{guess}| > 0.0001$ 
         $Cd_{guess} \leftarrow Cd_2$ 
         $m_{dot1} \leftarrow Cd_{guess} \cdot C_a \cdot A_0 \cdot (2 \cdot \rho \cdot \Delta P)^{\frac{1}{2}}$ 
         $N_{Re1} \leftarrow \frac{m_{dot1}}{A_p} \cdot \frac{D_p}{\mu}$ 
         $Cd_2 \leftarrow C_d(\beta, N_{Re1})$ 
     $V_{dot1} \leftarrow \frac{m_{dot1}}{\rho}$ 
    [ "beta=D0/Dp" "Ca" "Cd" "Re" "m_dot(kg/s)" "V_dot(m^3/s)" ]
    [ β C_a Cd_guess N_Re1 m_dot1 V_dot1 ]

```

**Prob. 5.2.21** An Orifice meter is used to measure the flow of LN2 through a tube of ID = 102 mm. Dia of orifice is 51 mm. And, measured pressure drop = 0.150 kPa. Temp of LN2 is 85 K. Determine the mass flow rate of LN2. [1]

At 85 K density and viscosity of LN2 are:  $\rho = 771 \text{ kg/m}^3$ , and  $\mu = 119 \times 10^{-6} \text{ Pa-s}$ . [1]

**Solution:**

**Data:**

$$D_p := 0.102 \text{ m} \quad D_0 := 0.051 \text{ m} \quad \Delta P := 150 \text{ Pa} \quad \rho := 771 \text{ kg/m}^3$$

$$\mu := 119 \cdot 10^{-6} \text{ Pa-s}$$

**Using the Mathcad Function for an orifice meter, written above:**

Orifice\_meter( $D_p, D_0, \rho, \mu, \Delta P$ ) =

$$\begin{bmatrix} "beta=D0/Dp" & "Ca" & "Cd" & "Re" & "m_dot(kg/s)" & "V_dot(m^3/s)" \\ 0.5 & 1.0328 & 0.60711 & 6.46199 \cdot 10^4 & 0.61603 & 7.99003 \cdot 10^{-4} \end{bmatrix}$$



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**Thus:**

$$\text{Mass flow rate} = m_{\text{dot}} = 0.61603 \text{ kg/s} \dots \text{Ans.}$$

$$\text{Vol. flow rate} = V_{\text{dot}} = 7.99003 \times 10^{-4} \text{ m}^3/\text{s} = 0.799003 \text{ lit/s} \dots \text{Ans.}$$

$$Cd \text{ for the orifice meter} = 0.60711 \dots \text{Ans.}$$

=====

**Prob. 5.2.22** An orifice meter is used to measure the flow of liquid oxygen through a 50 mm (1.969-in.) diameter tube. The orifice diameter is 25 mm (0.984 in.), and the measured pressure drop across the orifice (with D and D/2 pressure taps) is 250 Pa (0.0363 psi). If the temperature of the liquid oxygen is 100 K (180°R), determine the volumetric flow rate and the mass flow rate of the liquid oxygen.[1]

**Solution:**

**Data:**

$$D_p := 0.050 \text{ m} \quad D_0 := 0.025 \text{ m} \quad \Delta P := 250 \text{ Pa}$$

$$\rho := 1090.9 \text{ kg/m}^3 \quad \mu := 0.00015243 \text{ Pa-s}$$

**Using the Mathcad Function for an orifice meter, written above:**

$$\text{Orifice\_meter}(D_p, D_0, \rho, \mu, \Delta P) =$$

$$\begin{bmatrix} "beta=D0/Dp" & "Ca" & "Cd" & "Re" & "m_dot(kg/s)" & "V_dot(m^3/s)" \\ 0.5 & 1.0328 & 0.60909 & 3.8099 \times 10^4 & 0.22806 & 2.09054 \times 10^{-4} \end{bmatrix}$$

**Thus:**

$$\text{Mass flow rate} = m_{\text{dot}} = 0.22806 \text{ kg/s} \dots \text{Ans.}$$

$$\text{Vol. flow rate} = V_{\text{dot}} = 2.0905 \times 10^{-4} \text{ m}^3/\text{s} = 0.20905 \text{ lit/s} \dots \text{Ans.}$$

$$Cd \text{ for the orifice meter} = 0.60909 \dots \text{Ans.}$$

=====

**Plot the liquid flow rate as the pressure drop,  $\Delta P$  varies from 50 Pa to 400 Pa:**

We re-write the above results as functions of  $\Delta P$ :

$$Cd(\Delta P) := \text{Orifice\_meter}(D_p, D_0, \rho, \mu, \Delta P)_{1,2} \quad \text{i.e. } Cd(\Delta P) = 0.609$$

$$Re(\Delta P) := \text{Orifice\_meter}(D_p, D_0, \rho, \mu, \Delta P)_{1,3} \quad \text{i.e. } Re(\Delta P) = 3.8099 \cdot 10^4$$

$$m_{\text{dot}}(\Delta P) := \text{Orifice\_meter}(D_p, D_0, \rho, \mu, \Delta P)_{1,4} \quad \text{i.e. } m_{\text{dot}}(\Delta P) = 0.2281 \text{ kg/s}$$

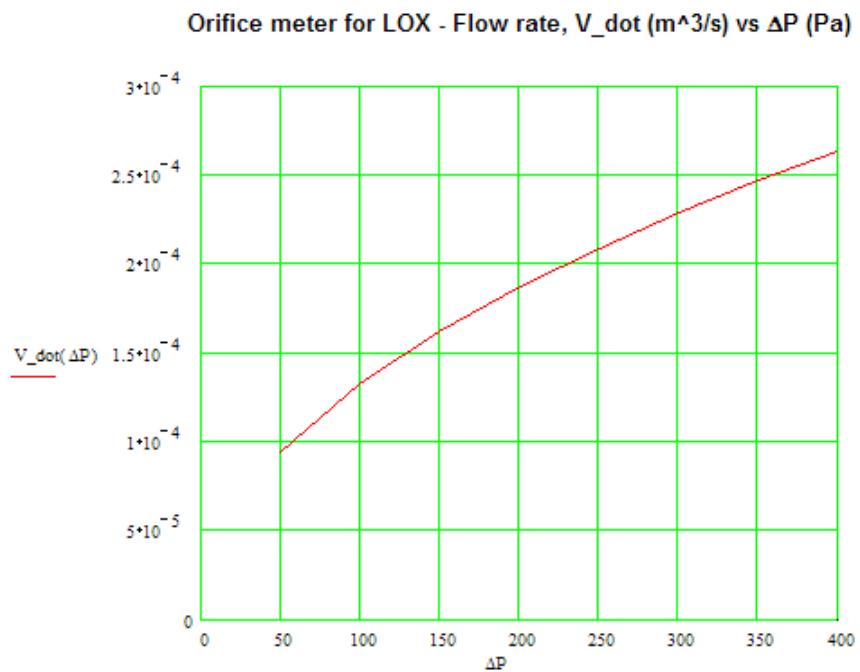
$$V_{\text{dot}}(\Delta P) := \text{Orifice\_meter}(D_p, D_0, \rho, \mu, \Delta P)_{1,5} \quad \text{i.e. } V_{\text{dot}}(\Delta P) = 2.091 \cdot 10^{-4} \text{ m}^3/\text{s}$$

**Compute the Parametric table, as  $\Delta P$  varies from 50 Pa to 400 Pa:**

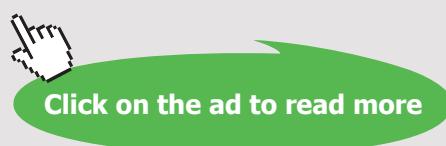
$\Delta P := 50, 100..400 \text{ Pa}$  ... define a range variable

$\Delta P$	$Cd(\Delta P)$	$Re(\Delta P)$	$m_{\text{dot}}(\Delta P)$	$V_{\text{dot}}(\Delta P)$
50	0.614	17172.46	0.10279	0.000094
100	0.611	24188.703	0.14479	0.000133
150	0.61	29572.885	0.17702	0.000162
200	0.61	34106.164	0.20416	0.000187
250	0.609	38099.025	0.22806	0.000209
300	0.609	41708.165	0.24966	0.000229
350	0.608	45026.625	0.26953	0.000247
400	0.608	48115.007	0.28801	0.000264

Now, plot  $V_{dot}$  vs  $\Delta P$ :



=====



**Prob. 5.2.23** Write a Mathcad Function to determine: Reynolds No. (N\_Re), the coefficients Cd) and Ca, mass flow rate (m\_dot, kg/s) and volume flow rate (V\_dot, m^3/s) for a Venturi flow meter when the pipe dia, throat dia, fluid density, viscosity and pressure drop are given.

**Solution:**

**Function to determine Cd etc for a Venturi meter:**

We have:

$$\beta = \frac{D_{\text{throat}}}{D_{\text{pipe}}} \quad N_{\text{Re}} = D_{\text{pipe}} \cdot \beta^2 \cdot \frac{(2 \cdot \rho \cdot \Delta P)^{\frac{1}{2}}}{\mu}$$

**Inputs:**

D\_pipe = pipe dia (m), D\_throat = throat dia (m), ρ = density (kg/m^3), μ = viscosity (Pa-s), DP = pressure drop (Pa)

**Outputs:**

$\beta = D_{\text{throat}}/D_{\text{pipe}}$ , Ca = Vel. of approach factor, Cd = coeff of discharge, N\_Re = Reynlds No., m\_dot = mass flow rate (kg/s), V\_dot = vol. flow rate (m^3/s)

$$\begin{aligned} \text{Venturi\_meter}\left(D_{\text{throat}}, D_{\text{pipe}}, \rho, \mu, \Delta P\right) := & \left| \begin{array}{l} \beta \leftarrow \frac{D_{\text{throat}}}{D_{\text{pipe}}} \\ A_0 \leftarrow \frac{\pi}{4} \cdot D_{\text{throat}}^2 \\ N_{\text{Re}} \leftarrow D_{\text{pipe}} \cdot \beta^2 \cdot \frac{(2 \cdot \rho \cdot \Delta P)^{\frac{1}{2}}}{\mu} \\ \text{if } (N_{\text{Re}} \geq 3000) \cdot (N_{\text{Re}} \leq 2 \cdot 10^5) \\ \quad \text{Cd} \leftarrow \frac{\log(N_{\text{Re}})}{0.6 + 0.9 \cdot \log(N_{\text{Re}})} \\ \quad \text{Cd} \leftarrow 0.988 \text{ if } N_{\text{Re}} > 2 \cdot 10^5 \\ \quad \text{Ca} \leftarrow \left(1 - \beta^4\right)^{\frac{-1}{2}} \\ \quad m_{\text{dot}} \leftarrow \text{Cd} \cdot \text{Ca} \cdot A_0 \cdot (2 \cdot \rho \cdot \Delta P)^{\frac{1}{2}} \\ \quad V_{\text{dot}} \leftarrow \frac{m_{\text{dot}}}{\rho} \\ \end{array} \right. \\ & \left[ \begin{array}{c} \text{"beta=Dt/Dp"} \quad \text{"Ca"} \quad \text{"Re"} \quad \text{"Cd"} \quad \text{"m_dot(kg/s)"} \quad \text{"V_dot(m}^3/\text{s)"} \\ \beta \quad \text{Ca} \quad N_{\text{Re}} \quad \text{Cd} \quad m_{\text{dot}} \quad V_{\text{dot}} \end{array} \right] \end{aligned}$$

**Prob. 5.2.24** Liquid hydrogen at 22 K (39.6°R) flows through a pipe having an inside diameter of 42.8 mm (1.685 in.). A venturi meter having a throat diameter of 21.4 mm (0.843 in.) is placed in the line to measure the flow rate of the fluid. Determine the volumetric flow rate and mass flow rate of the liquid hydrogen if the pressure difference for the meter is 1.35 kPa (0.196 psi).[1]

**Solution:**

**Data:**

$$D_{\text{pipe}} := 0.0428 \text{ m} \quad D_{\text{throat}} := 0.0214 \text{ m} \quad \Delta P := 1350 \text{ Pa}$$

$$\rho := 68.72 \text{ kg/m}^3 \text{ ... for LH2 at 22 K}$$

$$\mu := 11.87 \cdot 10^{-6} \text{ Pa-s ... for LH2 at 22 K}$$

**Using the Mathcad Function written above:**

$$\text{Venturi\_meter}(D_{\text{throat}}, D_{\text{pipe}}, \rho, \mu, \Delta P) =$$

$$\begin{bmatrix} \text{"beta=Dt/Dp"} & \text{"Ca"} & \text{"Re"} & \text{"Cd"} & \text{"m_dot(kg/s)" } & \text{"V_dot(m}^3/\text{s)"} \\ 0.5 & 1.033 & 3.883 \cdot 10^5 & 0.988 & 0.158 & 2.301 \cdot 10^{-3} \end{bmatrix}$$

**Thus:**

$$\text{Reynolds No} = \text{Re} = 3.883 \times 10^5 \text{ ... Ans.}$$

$$\text{Mass flow rate} = m_{\text{dot}} = 0.158 \text{ kg/s ... Ans.}$$

$$\text{Vol. flow rate} = V_{\text{dot}} = 2.301 \cdot 10^{-3} \text{ m}^3/\text{s} = 2.301 \text{ lit/s ... Ans.}$$

$$\text{Cd for the orifice meter} = 0.988 \text{ ... Ans.}$$


---

**Plot the liquid flow rate as the pressure drop,  $\Delta P$  varies from 100 Pa to 1500 Pa:**

We re-write the above results as functions of  $\Delta P$ :

$$\text{Re}(\Delta P) := \text{Venturi\_meter}(D_{\text{throat}}, D_{\text{pipe}}, \rho, \mu, \Delta P)_{1,2} \quad \text{i.e.} \quad \text{Re}(\Delta P) = 3.883 \cdot 10^5$$

$$\text{Cd}(\Delta P) := \text{Venturi\_meter}(D_{\text{throat}}, D_{\text{pipe}}, \rho, \mu, \Delta P)_{1,3} \quad \text{i.e.} \quad \text{Cd}(\Delta P) = 0.988$$

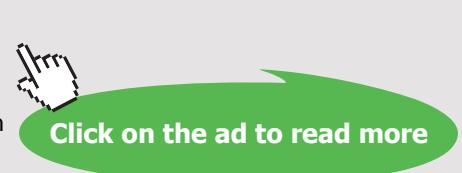
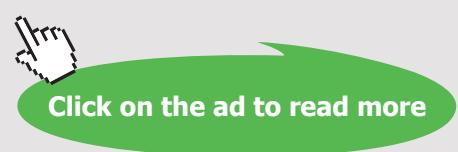
$$m_{\text{dot}}(\Delta P) := \text{Venturi\_meter}(D_{\text{throat}}, D_{\text{pipe}}, \rho, \mu, \Delta P)_{1,4} \quad \text{i.e.} \quad m_{\text{dot}}(\Delta P) = 0.158 \text{ kg/s}$$

$$V_{\text{dot}}(\Delta P) := \text{Venturi\_meter}(D_{\text{throat}}, D_{\text{pipe}}, \rho, \mu, \Delta P)_{1,5} \quad \text{i.e.} \quad V_{\text{dot}}(\Delta P) = 2.301 \cdot 10^{-3} \text{ m}^3/\text{s}$$

**Compute the Parametric table, as  $\Delta P$  varies from 100 Pa to 1500 Pa:**

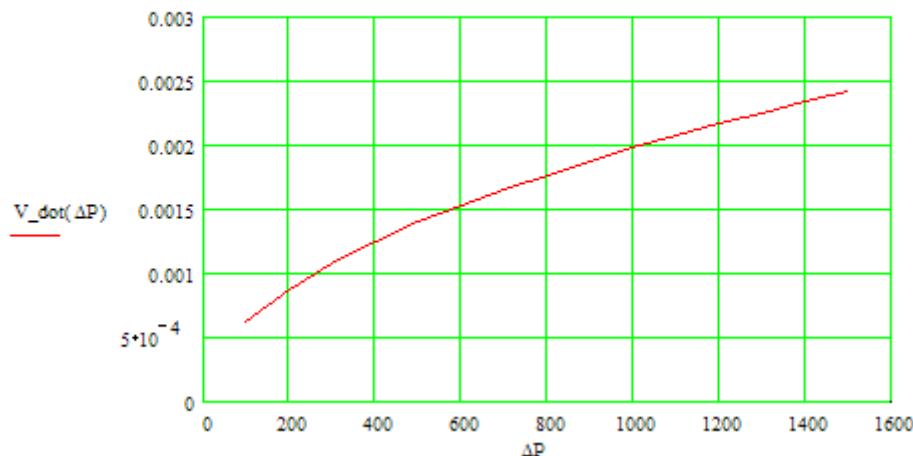
$\Delta P := 100, 200.. 1500 \text{ Pa}$ ....define a range variable

$\Delta P$	$Re(\Delta P)$	$Cd(\Delta P)$	$m_{dot}(\Delta P)$	$V_{dot}(\Delta P)$
100	105679.228	0.981	0.043	0.001
200	149452.997	0.984	0.061	0.001
300	183041.792	0.986	0.074	0.001
400	211358.456	0.988	0.086	0.001
500	236305.938	0.988	0.096	0.001
600	258860.185	0.988	0.105	0.002
700	279600.956	0.988	0.114	0.002
800	298905.995	0.988	0.122	0.002
900	317037.684	0.988	0.129	0.002
1000	334187.062	0.988	0.136	0.002
1100	350498.347	0.988	0.143	0.002
1200	366083.584	0.988	0.149	0.002
1300	381031.875	0.988	0.155	0.002
1400	395415.464	0.988	0.161	0.002
1500	409293.89	0.988	0.167	0.002



Now, plot  $V_{dot}$  vs  $\Delta P$ :

Venturi meter for LH<sub>2</sub> - Flow rate,  $V_{dot}$  (m<sup>3</sup>/s) vs  $\Delta P$  (Pa)



**Prob. 5.2.25.** A turbine flow meter has six blades with a thickness of 2.5 mm (0.098 in.) and a blade angle of 45°. The meter bore diameter is 75 mm (2.953 in.), the rotor blade-tip diameter is 70 mm (2.756 in.), and the rotor hub diameter is 35 mm (1.378 in.). If the rotational speed of the rotor is 1.50 rev./second, determine the volumetric flow rate and mass flow rate for liquid nitrogen flowing at 80 K (144°R).[1]

### Solution:

For a turbine flow meter, volumetric flow rate  $V_{dot}$  is given by:

$$V_{dot} = \frac{\pi \cdot D_b \cdot A_{ff} \cdot n}{\tan(\theta_b)}$$

where

$D_b$  = rotor blade tip dia

$\theta_b$  = angle between the blade and meter centre line

$A_{ff}$  = free flow area through the turbine, where

$$A_{ff} = \frac{\pi \cdot (D^2 - D_h^2)}{4} \cdot \left[ 1 - 2 \cdot m \cdot \frac{t}{\pi} \cdot (D + D_h) \cdot \cos(\theta_b) \right]$$

$D_h$  = rotor hub dia

$D$  = meter bore dia

$m$  = no. of blades

$t$  = blade thickness, and

$n$  = rotational speed of the turbine

**Data:**

$t := 0.0025 \text{ m}$  ... blade thickness  
 $\theta_b := 45 \text{ deg.}$   
 $D := 0.075 \text{ m}$  ... meter bore dia  
 $D_b := 0.07 \text{ m}$  ... rotor blade tip dia  
 $D_h := 0.035 \text{ m}$  ... rotor hub dia  
 $n := 1.5 \text{ rps}$  .... rotational speed of rotor  
 $m := 6$  .... no. of blades  
 $\rho := 795.1 \text{ kg/m}^3$  ... density of LN2 at 80 K

Therefore:

$$A_{ff} := \frac{\pi \cdot (D^2 - D_h^2)}{4} \left[ 1 - 2 \cdot m \cdot \frac{t}{\pi} \cdot (D + D_h) \cdot \cos\left(\theta_b \cdot \frac{\pi}{180}\right) \right] \quad \dots \text{Note that angle is converted to radians}$$

i.e.  $A_{ff} = 3.453 \cdot 10^{-3} \text{ m}^2$  .... free flow area

Then, volumetric flow rate is given by:

$$V_{dot} := \frac{\pi \cdot D_b \cdot A_{ff} \cdot n}{\tan\left[\theta_b \cdot \left(\frac{\pi}{180}\right)\right]}$$

i.e.  $V_{dot} = 1.139 \cdot 10^{-3} \text{ m}^3/\text{s} = 1.139 \text{ lit/s}$  .... Ans.

And,

$m_{dot} := V_{dot} \cdot \rho \text{ kg/s}$  ... mass flow rate

i.e.  $m_{dot} = 0.906 \text{ kg/s}$  ... mass flow rate .... Ans.

=====

**Plot  $m_{dot}$  (kg/s) as rotational speed,  $n$  varies from 1 rev/s to 3 rev/s:**

First, write  $V_{dot}$  and  $m_{dot}$  as functions of  $n$ :

$$V_{dot}(n) := \frac{\pi \cdot D_b \cdot A_{ff} \cdot n}{\tan\left[\theta_b \cdot \left(\frac{\pi}{180}\right)\right]}$$

$m_{dot}(n) := V_{dot}(n) \cdot \rho$

**Now:**

`n := 1, 1.2.. 3 rps... define a range variable`

**We get:**

n	m_dot(n)
1	0.604
1.2	0.725
1.4	0.845
1.6	0.966
1.8	1.087
2	1.208
2.2	1.328
2.4	1.449
2.6	1.57
2.8	1.691
3	1.811

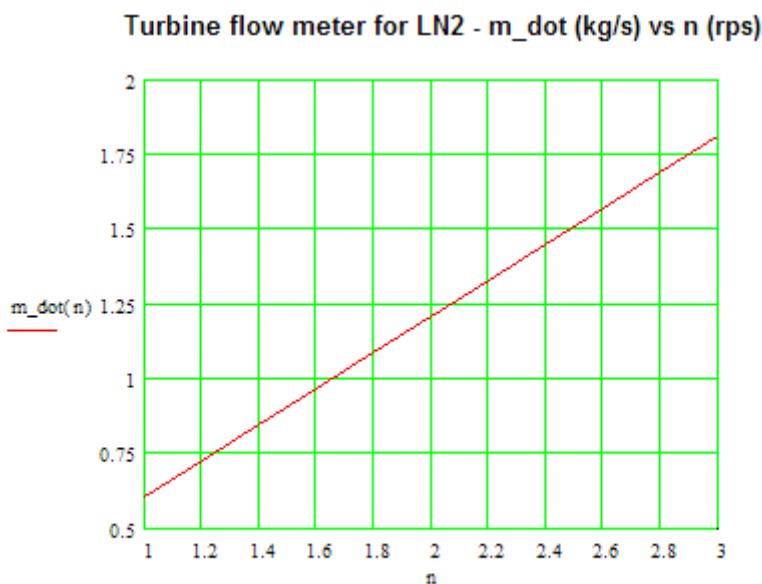


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**And, the plot:**



=====

**Prob. 5.2.26** Liquid hydrogen flows in a tube at 25 K and 328.8 kPa. A quality probe having a length of 600 mm, an inner rod dia of 38.1 mm and an outer cylinder ID is 66.9 mm. If the capacitance indication of probe is 66 pF, determine the quality of flowing fluid. [1]

**Solution:**

At 25 K, for sat. liq. and vapor hydrogen, we get from NIST website:

$$\rho_f := 0.064703 \text{ g/cm}^3 \quad \rho_g := 0.0038939 \text{ g/cm}^3$$

And:

$$D_i := 0.0381 \text{ m} \quad D_o := 0.0669 \text{ m}$$

$$L := 0.6 \text{ m} \quad CC := 66 \cdot 10^{-12} \text{ F}$$

**Dielectric constants for sat. liq. and sat. vap. hydrogen:**

For H<sub>2</sub>: From Cryog. Engg. by Thomas Flynn:

Dielectric const. is given by:

$$\frac{\varepsilon - 1}{\varepsilon + 2} = A \cdot \rho + B \cdot \rho^2 + C \cdot \rho^3 \quad \text{where}$$

$$A := 0.99575 \quad B := -0.09069 \quad C := 1.1227$$

**Then, dielectric constant for sat. LH2 and vapor H2:**

**For sat. liq H2:**

$$\frac{\varepsilon_f - 1}{\varepsilon_f + 2} = A \cdot \rho_f + B \cdot \rho_f^2 + C \cdot \rho_f^3$$

$$A := 0.99575 \quad B := -0.09069 \quad C := 1.1227$$

Solving for  $\varepsilon_f$  with Mathcad, we get:

$$\varepsilon_f =$$

$$\frac{-\left(1 - 2 \cdot A \cdot \rho_f - 2 \cdot B \cdot \rho_f^2 - 2 \cdot C \cdot \rho_f^3\right)}{\left(1 - A \cdot \rho_f - B \cdot \rho_f^2 - C \cdot \rho_f^3\right)} = 1.206336 \quad \text{....dielectric const for sat. LH2}$$

**Similarly, for sat. vapor H2, we have:**

$$\varepsilon_g =$$

$$\varepsilon_g =$$

$$\frac{-\left(1 - 2 \cdot A \cdot \rho_g - 2 \cdot B \cdot \rho_g^2 - 2 \cdot C \cdot \rho_g^3\right)}{\left(1 - A \cdot \rho_g - B \cdot \rho_g^2 - C \cdot \rho_g^3\right)} = 1.011673 \quad \text{....dielectric const for sat. GH2}$$

i.e.

$$\varepsilon_f := 1.206336 \quad \text{and,}$$

$$\varepsilon_g := 1.011673$$

Then, we calculate the dielectric const for two phase mixture, from the following eqn:

$$CC = \frac{2 \cdot \pi \cdot L \cdot \varepsilon \cdot \varepsilon_0}{\ln\left(\frac{D_o}{D_i}\right)}$$

where L= Length of cylinder,  $\varepsilon$  = dielectric const of material in the annulus, and

$$\varepsilon_0 := 8.8542 \cdot 10^{-12} \quad \text{F/m = permittivity of free space}$$

Therefore:

$$\varepsilon := \frac{CC \cdot \ln\left(\frac{D_0}{D_i}\right)}{2 \cdot \pi \cdot L \cdot \varepsilon_0}$$

i.e.  $\varepsilon = 1.11317$  ... dielectric const. of two phase mixture

Vapor volume fraction is determined from:

$$1 - y = \frac{\varepsilon_f - \varepsilon}{\varepsilon_f - \varepsilon_g}$$

i.e.  $1 - y = 0.47862$

Then, density of two phase mixture is given by:

$$\rho = \rho_f - (1 - y) \cdot (\rho_f - \rho_g)$$

i.e.  $\rho = \rho_f - (0.47862) \cdot (\rho_f - \rho_g) = 0.035599$  g/cm<sup>3</sup> .... density of two phase mixture.



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Then, quality is calculated as follows:

$$x = \frac{m_g}{m} = \frac{V_g \cdot \rho_g}{(V_f + V_g) \cdot \rho} = (1 - y) \cdot \left( \frac{\rho_g}{\rho} \right) = \frac{0.47862 \cdot 0.0038939}{0.035599}$$

i.e.  $x = 0.05235$  ....quality .... Ans.

=====

**Prob.5.2.27** A two-phase mixture of liquid and vapor argon flows in a tube at 90 K (162°R). A capacitance quality probe having a length of 1.060 m (41.7 in.), an inner diameter of 48 mm (1.890 in.), and an outer diameter of 57 mm (2.244 in.) is used to measure the quality. The capacitance indication of the gauge is 415 pF. What is the quality of the flowing argon? [1]

### Solution:

At 90 K, for sat. liq. and vapor Argon, we get from NIST website:

$$\rho_f := 1.3786 \text{ g/cm}^3 \quad \rho_g := 0.0074362 \text{ g/cm}^3$$

And:

$$D_i := 0.048 \text{ m} \quad D_o := 0.057 \text{ m}$$

$$L := 1.06 \text{ m} \quad CC := 415 \cdot 10^{-12} \text{ F}$$

### Dielectric constants for sat. liq. and sat. vap. Argon:

For Argon: From Cryog. Engg. by Thomas Flynn:

**Table 3.33** Dielectric Constant for Argon—Selected Values

Temp. (K)	Density (g/cm <sup>3</sup> )	Dielectric constant (ref. to vacuum)
88.8	1.393	1.516
88.5	1.395	1.518
87.1	1.404	1.520
85.8	1.414	1.525
84.3	1.422	1.530
82.4 <sup>a</sup>	1.434	1.537

<sup>a</sup> Supercooled liquid.

Source: McLennan et al. (1930).

i.e. We have: at 88.8 K,  $\epsilon_f = 1.516$ , and at 88.5 K,  $\epsilon_f = 1.518$

Therefore, extrapolating, we get the value of  $\epsilon_f$  at 90 K as:

$$1.516 - \frac{0.002}{0.3} \cdot 1.2 = 1.508 \quad \dots \epsilon_f \text{ at } 90 \text{ K}$$

**And, for sat. vapor Argon at 90 K, the value of  $\epsilon_g$  is a constant, viz.**

$$\epsilon_g = 1.000554$$

$$\text{i.e. } \epsilon_f := 1.508 \quad \epsilon_g := 1.000554$$

Then, we calculate the dielectric const for two phase mixture, from the following eqn:

$$CC = \frac{2 \cdot \pi \cdot L \cdot \epsilon \cdot \epsilon_0}{\ln \left( \frac{D_0}{D_i} \right)}$$

where  $L$  = Length of cylinder,  $\epsilon$  = dielectric const of material in the annulus, and

$$\epsilon_0 := 8.8542 \cdot 10^{-12} \quad \text{F/m} = \text{permittivity of free space}$$

Therefore:

$$\epsilon := \frac{CC \cdot \ln \left( \frac{D_0}{D_i} \right)}{2 \cdot \pi \cdot L \cdot \epsilon_0}$$

$$\text{i.e. } \epsilon = 1.20938 \quad \dots \text{dielectric const. of two phase mixture}$$

Vapor volume fraction is determined from:

$$1 - y = \frac{\epsilon_f - \epsilon}{\epsilon_f - \epsilon_g}$$

$$\text{i.e. } 1 - y = 0.58847$$

Then, density of two phase mixture is given by:

$$\rho = \rho_f - (1 - y) \cdot (\rho_f - \rho_g)$$

$$\text{i.e. } \rho = \rho_f - (0.58847) \cdot (\rho_f - \rho_g) = 0.572 \quad \text{g/cm}^3$$

$$\text{i.e. } \rho = 0.572 \quad \text{g/cm}^3 \dots \text{density of two phase mixture.}$$

Then, quality is calculated as follows:

$$x = \frac{m_g}{m} = \frac{V_g \cdot \rho_g}{(V_f + V_g) \cdot \rho} = (1 - y) \cdot \left( \frac{\rho_g}{\rho} \right) = \frac{0.58847 \cdot 0.0074362}{0.572}$$

i.e.  $x = 0.00765$  ....quality .... Ans.

=====

**Prob.5.2.28** For an 2.440-m inside-diameter liquid-oxygen storage vessel (horizontal cylindrical vessel), the hydrostatic pressure indication on a hydrostatic liquid-level gauge is 20 kPa (2.90 psi). If the fluid in the vessel is under a pressure of 101.3 kPa (1 atm) determine the liquid level and the sensitivity of the liquid level gauge.[1]

**Solution:**

We have for Hydrostatic level gauges:

$$\Delta P = (\rho_f - \rho_g) \cdot L_f g + \rho_g \cdot L \cdot g$$



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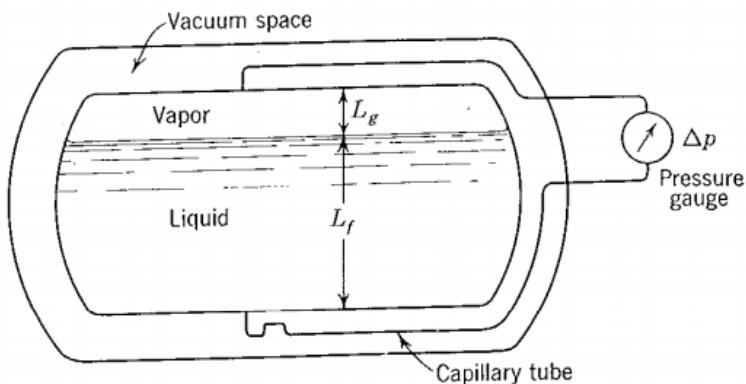


Fig. Hydrostatic liquid-level gauge.

Since for fluids other than hydrogen and helium, vapor density can be neglected compared to the liquid density, we write:

$$\Delta P = \rho_f L_f g$$

where

$$\Delta P := 20 \cdot 10^3 \text{ Pa} \quad g := 9.81 \text{ m/s}^2 \quad L := 2.44 \text{ m} \dots \text{dia of tank}$$

$$\rho_f := 1141 \text{ kg/m}^3 \text{ at } 101.3 \text{ kPa, } 90.18 \text{ K} \quad \rho_g := 4.47 \text{ kg/m}^3 \text{ at } 101.3 \text{ kPa, } 90.18 \text{ K}$$

Therefore:

$$L_f := \frac{\Delta P}{\rho_f g}$$

$$\text{i.e. } L_f = 1.787 \text{ m} \dots \text{height of liquid from bottom of tank} \dots \text{Ans.}$$

$$\text{i.e. the horizontal tank is } 73.24\% \text{ full} \dots \text{Ans.}$$

Sensitivity,  $S_0$ , is given by:

$$S_0 = \frac{d(\Delta P)}{dL_f} = (\rho_f - \rho_g) \cdot g \quad \text{Pa/m}$$

Therefore:

$$S_0 := (\rho_f - \rho_g) \cdot g$$

$$\text{i.e. } S_0 = 1.115 \cdot 10^4 \text{ Pa/m} \dots \text{Ans.}$$

**Prob.5.2.29** For a 1.83 m ID LH<sub>2</sub> storage vessel, hydrostatic pressure indication is 0.86 kPa. If the fluid pressure in the ullage space is 101.3 kPa, determine the liquid level. Compare the sensitivity of LH<sub>2</sub> level indicator with a hydrostatic gauge for LN<sub>2</sub> level indication.[1]

**Solution:**

We have for Hydrostatic level gauges:

$$\Delta P = (\rho_f - \rho_g) \cdot L_f \cdot g + \rho_g \cdot L \cdot g$$

Then, liquid level is given by:

$$L_f = \frac{\Delta P - \rho_g \cdot L \cdot g}{(\rho_f - \rho_g) \cdot g}$$

Now, by data:

$$\Delta P := 0.86 \cdot 10^3 \text{ Pa} \quad g := 9.81 \text{ m/s}^2 \quad L := 1.83 \text{ m}$$

$$\rho_f := 70.79 \text{ kg/m}^3 \text{ at } 101.3 \text{ kPa, } 20.3 \text{ K} \quad \rho_g := 1.331 \text{ kg/m}^3 \text{ at } 101.3 \text{ kPa, } 20.39 \text{ K}$$

Therefore:

$$L_f := \frac{\Delta P - \rho_g \cdot L \cdot g}{(\rho_f - \rho_g) \cdot g}$$

$$\text{i.e. } L_f = 1.227 \text{ m .... height of liquid from bottom of tank .... Ans.}$$

i.e. the horizontal tank is 67.05 % full .... Ans.

If we neglect the density of vapor, the liquid level will be:

$$L_f := \frac{\Delta P}{\rho_f g} \quad \text{i.e. } L_f = 1.238 \text{ m .... Ans.}$$

Sensitivity,  $S_0$ , is given by:

$$S_0 = \frac{d(\Delta P)}{dL_f} = (\rho_f - \rho_g) \cdot g \quad \text{Pa/m}$$

Therefore:

$$S_0 := (\rho_f - \rho_g) \cdot g$$

i.e.  $S_0 = 681.393 \text{ Pa/m} \dots \text{Ans.}$

Sensitivity for LN2:

For LN2 at 101.3 kPa, first, we get densities for sat. liquid and sat.vapor:

$$\rho_f := 807.3 \text{ kg/m}^3 \text{ at } 101.3 \text{ kPa, } 77.36 \text{ K} \quad \rho_g := 4.604 \text{ kg/m}^3 \text{ at } 101.3 \text{ kPa, } 77.36 \text{ K}$$

Therefore:

$$S_0 := (\rho_f - \rho_g) \cdot g$$

i.e.  $S_0 = 7.874 \cdot 10^3 \text{ Pa/K} \dots \text{Ans.}$

Note that the gauge for LN2 is 11.56 times more sensitive.

---



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**Plot for LH<sub>2</sub>, liquid level L<sub>f</sub> against ΔP, as ΔP varies from 100 Pa to 900 Pa:**

For Hydrogen, we have:

$$g := 9.81 \text{ m/s}^2 \quad L := 1.83 \text{ m}$$

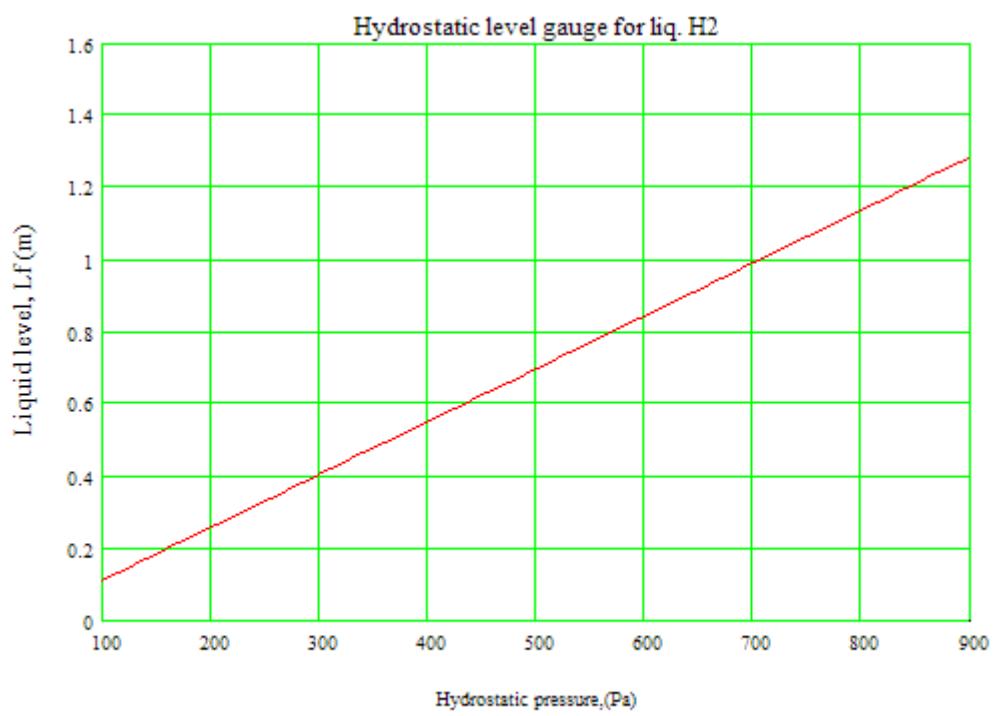
$$\rho_f := 70.79 \text{ kg/m}^3 \text{ at } 101.3 \text{ kPa, } 20.3 \text{ K} \quad \rho_g := 1.331 \text{ kg/m}^3 \text{ at } 101.3 \text{ kPa, } 20.39 \text{ K}$$

First, let us write L<sub>f</sub> as a function of ΔP:

$$L_f(\Delta P) := \frac{\Delta P - \rho_g \cdot L \cdot g}{(\rho_f - \rho_g) \cdot g}$$

Now, to plot:

$$\Delta P := 100, 110..900 \text{ Pa} \dots \text{ define a range variable}$$



**Prob.5.2.30** A liquid-level gauge consists of a vertical wire immersed in liquid nitrogen at 78 K (1404). When the entire wire is immersed in liquid nitrogen, electric resistance of the wire is 300 ohms. The total length of the wire is 2.00 m. Determine the electric resistance of the wire when 1.00 m is immersed in liquid and 1.00 m is exposed to nitrogen vapor at 78 K. Assume that the temperature of the portion of wire in the liquid is at a temperature of 78 K, and the average temperature of the portion of the wire in the vapor is given by:

$$T - T_f = \frac{(T_0 - T_f) \cdot \cosh(M \cdot L)}{\cosh(M \cdot L) - 1} \cdot \left( 1 - \frac{\tanh(M \cdot L)}{M \cdot L} \right)$$

where  $T_f$  = liquid temperature = 78 K;  $T_0$  = 100 K;  $M = 4h_c / k_t D = 6.56 \text{ m}^{-1}$ ;

$h_c$  = heat-transfer coefficient for the wire,  $k_t$  = wire thermal conductivity;  $D$  = wire diameter; and  $L$  = length of wire exposed to the vapor = 1.00 m. The electric resistance of the wire varies with the average temperature of the wire as follows:

$$R = R_0 [1 + a(T - T_f)]$$

where  $R_0 = 150$  ohms for 1.00 m of wire = wire electric resistance at  $T_f$ ;  $a = 4.0 \times 10^{-3} \text{ K}^{-1}$ ; and  $T$  = average temperature of the part of the wire exposed to the vapor.[1]

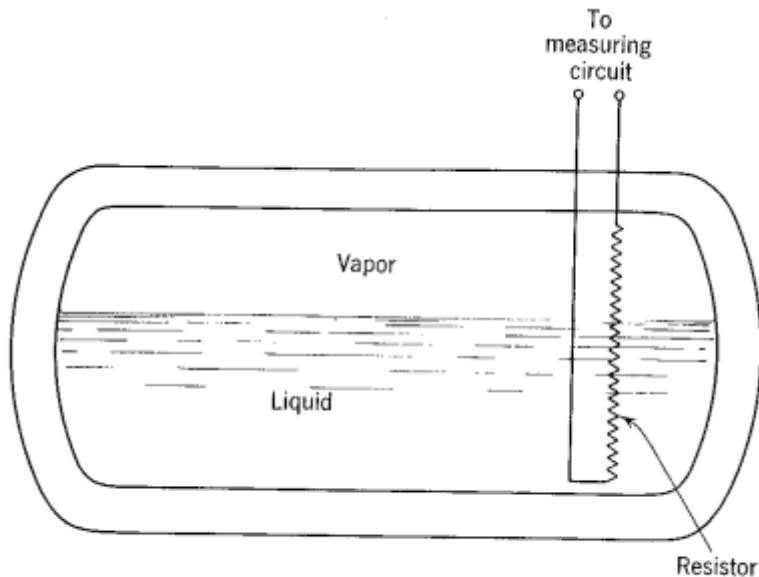


Fig. Fixed electric resistance liquid-level gauge.

**Solution:**

We have:

$$T_f := 78 \quad K \quad T_0 := 100 \quad K \quad M := 6.56 \quad m^{-1}$$

$L := 1 \quad m$  ... length of wire exposed to vapor

$L_{tot} := 2 \quad m$  ... total length of wire

$R_{tot} := 300 \quad \text{Ohms}$  ... resistance when the entire wire (2.0 m) is immersed in LN2

$R_0 := 150 \quad \text{Ohms}$  ... resistance when 1.0 m wire is immersed in LN2

$$a := 4 \cdot 10^{-3} \quad K^{-1}$$

$T$  = average temp of wire exposed to vapor

Now, avg temp.  $T$  is given by:

$$T := T_f + \frac{(T_0 - T_f) \cdot \cosh(M \cdot L)}{\cosh(M \cdot L) - 1} \cdot \left( 1 - \frac{\tanh(M \cdot L)}{M \cdot L} \right)$$

i.e.  $T = 96.699 \quad K$



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Therefore, electric resist of wire exposed to vapor:

$$R_{vap} := R_0 [1 + a(T - T_f)]$$

i.e.  $R_{vap} = 161.22$  Ohms ... resist of 1 m wire exposed to vapor

And, total resist of wire when 1 m is immersed in liquid and 1 m is exposed to vapor:

$$R := R_0 + R_{vap}$$

i.e.  $R = 311.22$  Ohms .... Ans.

=====

**Prob. 5.2.31** A capacitance liquid-level probe is used to measure liquid level in a liquid-nitrogen storage vessel. The total length of the probe is 1.220 m (48.0 in.), and the inner and outer diameters of the annular space of the probe are 75 mm (2.953 in.) and 100 mm (3.937 in.), respectively. If the capacitance reading of the gauge is 312 pF, determine the liquid level and the sensitivity of the gauge. The pressure within the storage vessel is 101,3 kPa (1 atm), and the dielectric constants are  $\epsilon_f = 1.4319$  for the liquid, and  $\epsilon_g = 1.0020$  for the vapor.[1]

**Solution:**

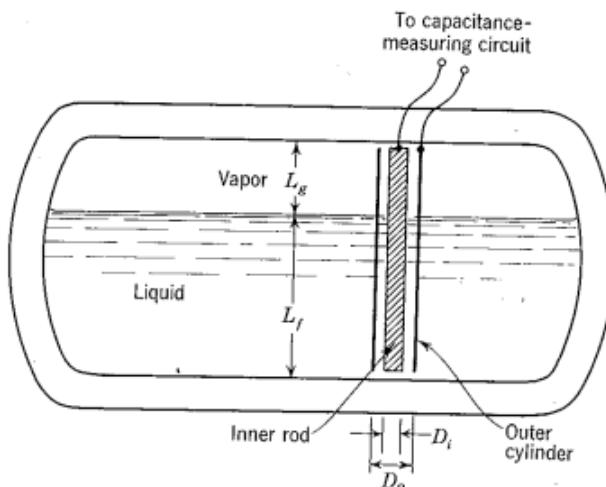


Fig. Capacitance liquid-level gauge.

$L := 1.22 \text{ m}$  ... length of probe

$D_i := 0.075 \text{ m}$  ... ID of probe annular space

$D_0 := 0.1 \text{ m}$  ... OD of probe annular space

$C := 312 \cdot 10^{-12} \text{ F}$  ... capacitance reading

$\epsilon_f := 1.4319$  ... dielectric const of liq.

$\epsilon_g := 1.002$  ... dielectric const of vapor

$\epsilon_0 := 8.8542 \cdot 10^{-12} \text{ F/m}$  ... permittivity of free space

Then, we have the liquid level,  $L_f$

$$L_f := \left[ \frac{\frac{C \cdot \ln\left(\frac{D_0}{D_i}\right)}{2 \cdot \pi \cdot (\epsilon_f - \epsilon_g) \cdot \epsilon_0}}{\frac{1}{(\epsilon_f - \epsilon_g)}} - \frac{\epsilon_g \cdot L}{(\epsilon_f - \epsilon_g)} \right]$$

i.e.  $L_f = 0.909 \text{ m}$  ... liquid level from bottom of tank ... Ans.

Sensitivity is given by:

$$S_0 := \frac{\frac{2 \cdot \pi \cdot (\epsilon_f - \epsilon_g) \cdot \epsilon_0}{\ln\left(\frac{D_0}{D_i}\right)}}{1}$$

i.e.  $S_0 = 8.313 \cdot 10^{-11} \text{ F/m}$  .... Ans.

i.e.  $S_0 = 83.13 \text{ pF/m}$  .... Ans.

---

**Prob.5.2.32** While solving problems on Thermodynamic liquid gauges, we need the values of values of 'Supercompressibility factors' for liquid hydrogen, liquid nitrogen, and also for gases.

In Ref.[1], these are given in Tabular form, i.e. in Table 6.6 for LH2, Table 6.7 for LN2 and Table 6.8 for Gases.

**Write a Mathcad Function to find the Supercompressibility for liquid hydrogen, when P (atm) and T (K) are given, and also for sat. liquid.**

**Solution:**

First copy the Table 6.6 from Ref.[1] in to Mathcad as a Matrix:

**Supercompressibility factors for LH2: Ref: Table 6.6 of Barron:**

T(K)	Pressure (atm.)								
	10	20	30	40	60	80	100	120	
16	0.2022	0.3999	0.5941	0.7851	1.159	0	0	0	
17	0.1923	0.3798	0.5637	0.7444	1.097	1.444	1.787	0	
18	0.1835	0.3624	0.537	0.7087	1.044	1.371	1.695	2.002	
19	0.176	0.3471	0.514	0.6773	0.996	1.306	1.612	1.906	
20	0.1693	0.3337	0.4936	0.6498	0.9542	1.25	1.539	1.821	
21	0.1636	0.3218	0.4756	0.6256	0.9168	1.199	1.474	1.746	
22	0.1587	0.3113	0.4595	0.6036	0.8838	1.154	1.416	1.676	
23	0.1545	0.3023	0.4456	0.585	0.8552	1.114	1.366	1.615	
24	0.151	0.2944	0.4332	0.5685	0.829	1.079	1.321	1.559	
25	0.148	0.288	0.4229	0.5538	0.8052	1.046	1.28	1.509	
M_LH2 :=	26	0.1457	0.2828	0.4137	0.5403	0.7841	1.017	1.243	1.464
	27	0.144	0.2782	0.4057	0.5286	0.7645	0.9908	1.21	1.421
	28	0.1431	0.2744	0.3988	0.5185	0.7471	0.9666	1.179	1.382
	29	0.1437	0.2718	0.3932	0.5095	0.7317	0.945	1.151	1.347
	30	0.1456	0.2708	0.389	0.5026	0.7185	0.9255	1.124	1.316
	31	0.1498	0.2712	0.3864	0.4969	0.7069	0.9076	1.101	1.287
	32	0	0.2732	0.3848	0.4919	0.6965	0.8912	1.079	1.261
	33	0	0.2781	0.3845	0.4876	0.6869	0.8757	1.058	1.235
	34	0	0.2929	0.3897	0.489	0.6803	0.864	1.0413	1.215
	35	0	0.3157	0.3954	0.4891	0.6746	0.8522	1.0256	1.195
	36	0	0.3627	0.4052	0.4915	0.6703	0.8425	1.0103	1.177

Now, write the pressure range in to a Vector:

```
Pressures_LH2 :=(10 20 30 40 60 80 100 120) ...pressure range in a vector
```

**Now, write the Function:**

In the Function below, we have:

**Inputs:** Pressure, P (atm.), Temp. T (K)

**Outputs:** In a Matrix.. P, T and Z for LH2

**Note:** In the Matrix above, '0' indicates no value for that entry. So, input the values judiciously.

```

Z_LH2(P,T) := | return "T should be between 16 and 36 K" if (T<16) + (T>36)
                | return "P should be between 10 and 120 atm." if (P<10) + (P>120)
                | for k ∈ 0..7
                |   if (P=Pressures_LH20,k)
                |     Z←linterp(M_LH2<0,M_LH2<k+1,T)
                |     return [ "P(atm)" "T(K)" "Z for LH2" ]
                |   if (P>Pressures_LH20,k) · (P<Pressures_LH20,k+1)
                |     PL←Pressures_LH20,k
                |     PH←Pressures_LH20,k+1
                |     A←linterp(M_LH2<0,M_LH2<k+1,T)
                |     B←linterp(M_LH2<0,M_LH2<k+2,T)
                |     Z←A + (P - PL) · (B - A)
                |     return [ "P(atm)" "T(K)" "Z for LH2" ]

```



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Ex:  $P := 120 \text{ atm}$        $T := 29.5 \text{ K}$

$$Z_{\text{LH2}}(P, T) = \begin{bmatrix} "P(\text{atm})" & "T(\text{K})" & "Z \text{ for LH2}" \\ 120 & 29.5 & 1.3315 \end{bmatrix}$$

### Z for LH2 when the LH2 is saturated:

From Table 6.6, we have:

Tsat(K)	Z
16	0.004107
17	0.006332
18	0.008451
19	0.012056
20	0.01529
21	0.02032
22	0.02533
23	0.031
24	0.03936
25	0.04918
M_LH2_SAT := 26	0.05847
27	0.07101
28	0.08442
29	0.1042
30	0.1208
31	0.1407
32	0.1799
33	0
34	0
35	0
36	0

$$Z_{\text{LH2\_SAT}}(T_{\text{sat}}) := \begin{cases} \text{return "T should be between 16 and 32 K" if } (T_{\text{sat}} < 16) + (T_{\text{sat}} > 32) \\ Z \leftarrow \text{linterp}(M_{\text{LH2\_SAT}}^{<0>} , M_{\text{LH2\_SAT}}^{<1>} , T_{\text{sat}}) \\ \begin{bmatrix} "T_{\text{sat}}(\text{K})" & "Z \text{ for sat.LH2}" \\ T_{\text{sat}} & Z \end{bmatrix} \end{cases}$$

Ex:  $T_{sat} := 31.5 \text{ K}$

$$Z_{LH2\_SAT}(T_{sat}) = \begin{bmatrix} "T_{sat}(K)" & "Z for sat.LH2" \\ 31.5 & 0.1603 \end{bmatrix}$$

---

**Prob.5.2.33** While solving problems on Thermodynamic liquid gauges, we need the values of values of ‘Supercompressibility factors’ for liquid hydrogen, liquid nitrogen, and also for gases.

In Ref.[1], these are given in Tabular form, in Table 6.6 for LH2, Table 6.7 for LN2 and Table 6.8 for Gases.

**Write a Mathcad Function to find the Supercompressibility for liquid nitrogen, when P (atm) and T (K) are given, and also for sat. liquid.**

**Solution:**

First copy the Table 6.7 from Ref.[1] in to Mathcad as a Matrix:



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### Supercompressibility factors for LN2: Ref: Table 6.7 of Barron

T(K)	Pressure (atm.)					
	5	10	20	40	60	
M_LIN :=	70	0.029	0.05793	0.11562	0.23027	0.34403
	75	0.02779	0.05551	0.11073	0.22035	0.32894
	80	0.02681	0.05352	0.10671	0.21214	0.31639
	85	0.02602	0.05193	0.10346	0.20541	0.306
	90	0.02542	0.05071	0.10092	0.20001	0.29751
	95	0	0.04985	0.09908	0.19588	0.2908
	100	0	0.04941	0.09798	0.19304	0.28579
	105	0	0	0.09777	0.19159	0.28254
	110	0	0	0.09883	0.19184	0.28124
	115	0	0	0.10219	0.19448	0.28232
	120	0	0	0	0.20122	0.28659

Now, write the pressure range in to a Vector:

Pressures\_LIN :=(5 10 20 40 60) ... pressure range in to a Vector:

Now, write the Function:

In the Function below, we have:

**Inputs:** Pressure, P (atm.), Temp. T (K)

**Outputs:** In a Matrix.. P, T and Z for LN2

**Note:** In the Matrix above, '0' indicates no value for that entry. So, input the values judiciously.

```

Z_LIN(P, T) := | return "T should be between 70 and 120 K" if (T<70) + (T>120)
                | return "P should be between 5 and 60 atm." if (P<5) + (P>60)
                | for k ∈ 0..4
                |   if (P=Pressures_LIN0,k)
                |     Z←linterp(M_LIN<0, M_LIN<k+1, T)
                |     return ["P(atm)" "T(K)" "Z for LN2"]
                |   if (P>Pressures_LIN0,k) · (P<Pressures_LIN0,k+1)
                |     PL←Pressures_LIN0,k
                |     PH←Pressures_LIN0,k+1
                |     A←linterp(M_LIN<0, M_LIN<k+1, T)
                |     B←linterp(M_LIN<0, M_LIN<k+2, T)
                |     Z←A + (P - PL) · (B - A)
                |     return ["P(atm)" "T(K)" "Z for LN2"]
                |

```

Ex: P := 60 atm      T := 120 K

$$Z_{\text{LIN}}(P, T) = \begin{bmatrix} "P(\text{atm})" & "T(\text{K})" & "Z \text{ for LN2}" \\ 60 & 120 & 0.28659 \end{bmatrix}$$


---

### Z for LN2 when the LN2 is saturated:

From Table 6.7, we have:

T <sub>sat</sub> (K)	Z
70	0.002206
75	0.004173
80	0.007241
85	0.011743
90	0.018066
M_LIN_SAT := 95	0.02664
100	0.038003
105	0.052869
110	0.072282
115	0.098078
120	0.13441

$Z_{LIN\_SAT}(T_{sat}) := \begin{cases} \text{return "T}_{sat}\text{ should be between 70 and 120 K" if } (T_{sat}<70) + (T_{sat}>120) \\ Z \leftarrow \text{linterp}\{M_{LIN\_SAT}^{<0>} , M_{LIN\_SAT}^{<1>} , T_{sat}\} \\ \begin{bmatrix} "T_{sat}(K)" & "Z for sat.LN2" \\ T_{sat} & Z \end{bmatrix} \end{cases}$

Ex:  $T_{sat} := 98 \text{ K}$

$$Z_{LIN\_SAT}(T_{sat}) = \begin{bmatrix} "T_{sat}(K)" & "Z for sat.LN2" \\ 98 & 0.033458 \end{bmatrix}$$

**Prob.5.2.34** While solving problems on Thermodynamic liquid gauges, we need the values of values of ‘Supercompressibility factors’ for liquid hydrogen, liquid nitrogen, and also for gases.

In Ref.[1], these are given in Tabular form, in Table 6.6 for LH2, Table 6.7 for LN2 and Table 6.8 for Gases.



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**Write a Mathcad Function to find the Supercompressibility for gases, when reduced pressure  $Pr (= P/P_c)$  and reduced temp  $Tr (= T/T_c)$  are given, and also for sat. gas.**

**Solution:**

First copy the Table 6.8 from Ref.[1] in to Mathcad as a Matrix:

**Supercompressibility factors for Gases: Ref: Table 6.8 of Barron**

		Pr = P/Pc												
Tr=	T/Tc	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	1.5	2	
0.8	0.921	0	0	0	0	0	0	0	0	0	0.145	0.215	0.284	
0.85	0.933	0.861	0.789	0	0	0	0	0	0	0	0.146	0.216	0.283	
0.9	0.947	0.89	0.826	0.764	0	0	0	0	0	0	0.148	0.217	0.283	
0.92	0.951	0.901	0.842	0.783	0.71	0	0	0	0	0	0.151	0.219	0.284	
0.94	0.955	0.909	0.856	0.798	0.735	0.66	0	0	0	0	0.155	0.223	0.287	
0.96	0.958	0.915	0.868	0.817	0.761	0.7	0.613	0	0	0	0.161	0.23	0.291	
0.98	0.962	0.922	0.879	0.832	0.782	0.731	0.665	0.58	0	0	0.174	0.241	0.298	
1	0.965	0.927	0.889	0.846	0.801	0.755	0.704	0.64	0.52	0.27	0.247	0.306		
1.05	0.971	0.94	0.908	0.873	0.838	0.802	0.766	0.723	0.67	0.611	0.332	0.341		
1.1	0.976	0.95	0.923	0.894	0.866	0.837	0.805	0.733	0.738	0.7	0.496	0.4		
M_GASES :=	1.2	0.983	0.965	0.946	0.924	0.915	0.905	0.862	0.84	0.818	0.795	0.682	0.573	
	1.4	0.99	0.982	0.972	0.959	0.949	0.937	0.928	0.921	0.912	0.899	0.846	0.801	
	1.6	0.992	0.988	0.985	0.978	0.973	0.965	0.964	0.96	0.955	0.948	0.919	0.888	
	1.8	0.993	0.991	0.99	0.987	0.985	0.982	0.981	0.98	0.978	0.974	0.956	0.935	
	2	0.994	0.994	0.994	0.994	0.993	0.992	0.992	0.99	0.99	0.988	0.976	0.966	
	3	1	1	1	1	1	1	1	1	1	1	0.999	0.986	
	4	1	1	1	1	1	1	1	1	1	1	1	0.99	
	6	1	1	1	1	1	1	1	1	1	1	1	1.004	0.995
	8	1	1	1	1	1	1	1	1	1	1	1	1.008	0.998
	10	1	1	1	1	1	1	1	1	1	1	1	1.01	1
	15	1	1	1	1	1	1	1	1	1	1	1	1.02	1.02

Now, write the reduced pressure range in to a Vector:

Red\_Pressures\_Gases :=(0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1 1.5 2)

.... reduced pressure range in a Vector.

In the Function below, we have:

**Inputs:** Pr, Tr

**Outputs:** In a Matrix.. Pr, Tr and Z for Gas

**Note:** In the Matrix above, '0' indicates no value for that entry. So, input the values judiciously.

```
Z_GASES(Pr, Tr) := | return "Tr should be between 0.8 and 15" if (Tr<0.8) + (Tr>15)
                    | return "Pr should be between 0.1 and 2" if (Pr<0.1) + (Pr>2)
                    | for k ∈ 0..11
                    |   if (Pr=Red_Pressures_Gases0,k)
                    |     Z←linterp(M_GASES<0>, M_GASES<k+1>, Tr)
                    |     return [ "Pr" "Tr" "Z for Gas" ]
                    |   if (Pr>Red_Pressures_Gases0,k) · (Pr<Red_Pressures_Gases0,k+1)
                    |     PL← Red_Pressures_Gases0,k
                    |     PH← Red_Pressures_Gases0,k+1
                    |     A←linterp(M_GASES<0>, M_GASES<k+1>, Tr)
                    |     B←linterp(M_GASES<0>, M_GASES<k+2>, Tr)
                    |     Z← A + (Pr - PL) · (B - A)
                    |     return [ "Pr" "Tr" "Z for Gas" ]
                    |   
```

Ex:      Tr := 2.377                  Pr := 0.149

For these values, get Z:

$$Z_GASES(Pr, Tr) = \begin{bmatrix} "Pr" & "Tr" & "Z for Gas" \\ 0.149 & 2.377 & 0.99626 \end{bmatrix}$$

=====

**Z\_gas for Sat. vapor. for various values of Pr = P/Pc:**

**From Table 6.8, we have:**

Pr	Z
0.1	0.898
0.2	0.833
0.3	0.783
0.4	0.738
0.5	0.693
0.6	0.641
0.7	0.583
0.8	0.519
0.9	0.443
1	0.27

**Z\_GASES\_SAT(Pr) :=** 
$$\begin{cases} \text{return "Pr should be between 0.1 and 1" if } (\text{Pr}<0.1) + (\text{Pr}>1) \\ Z \leftarrow \text{linterp}\langle M\_GASES\_SAT^{<0>} , M\_GASES\_SAT^{<1>} , \text{Pr} \rangle \\ \begin{bmatrix} \text{"Pr"} & \text{"Z for sat.Gas"} \\ \text{Pr} & Z \end{bmatrix} \end{cases}$$



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Ex:  $\text{Pr} := 0.35$

$$\text{Z\_GASES\_SAT}(\text{Pr}) = \begin{bmatrix} \text{"Pr"} & \text{"Z for sat.Gas"} \\ 0.35 & 0.7605 \end{bmatrix}$$

=====:

**Prob. 5.2.35** A thermodynamic liquid-level gauge for liquid hydrogen has a sensing element length of 1.520 m (59.8 in.), a sensing element cross-sectional area of 30 mm<sup>2</sup> (0.0465 in<sup>2</sup>), and a dead volume of 9.50 dm<sup>3</sup> (580 in<sup>3</sup>). The temperature of the liquid to be gauged is 20 K (36°R), and the gas within the sensing element is at a mean temperature of 32 K (57.6°R). The temperature of the dead-volume gas is 300 K (540°R). If the element is initially charged with 10.5 g (0.0231 lbm) of hydrogen gas, determine the liquid level when the pressure indicated by the gauge is 1.013 MPa (10 atm). [1]

**Solution:**

For H<sub>2</sub>, critical pressure and temp are:

$$P_c := 1.315 \cdot 10^6 \quad \text{Pa} \quad T_c := 33.2 \quad \text{K}$$

Other data:

$$R := 4124.2 \quad \text{J/kg.K} \quad \dots \text{Gas constant for H}_2$$

$$p := 1.013 \cdot 10^6 \quad \text{Pa} \quad \dots \text{pressure indicated by gauge}$$

$$m := 10.5 \cdot 10^{-3} \quad \text{kg} \quad \dots \text{initial charging mass}$$

$$T_f := 20 \quad \text{K}$$

$$L := 1.52 \quad \text{m} \quad \dots \text{length of sensing element}$$

$$A_c := 30 \cdot 10^{-6} \quad \text{m}^2 \quad \dots \text{crosssectional area of sensing element}$$

$$V_0 := 9.5 \cdot 10^{-3} \quad \text{m}^3 \quad \dots \text{dead volume}$$

$$T_0 := 300 \quad \text{K} \quad \dots \text{temp of dead volume}$$

$$T_g := 32 \quad \text{K} \quad \dots \text{temp of gas in the sensing element}$$

### Calculations:

Eqn. for liquid level is:

$$\frac{L_f}{L} = \frac{\frac{m \cdot R \cdot T_f}{A_c \cdot L \cdot p} - \frac{V_0 \cdot T_f}{Z_0 \cdot T_0 \cdot A_c \cdot L}}{\frac{1}{Z_f} - \frac{T_f}{Z_g \cdot T_g}}$$

For dead volume conditions:

$$\frac{T_0}{T_c} = 9.036 \quad \frac{p}{p_c} = 0.77$$

For these values, get  $Z_0$  from Table 6.8, or the Mathcad Function written earlier:

$$Z_0 := Z\_GASES(0.77, 9.036)$$

i.e.  $Z_0 = \begin{bmatrix} "Pr" & "Tr" & "Z for Gas" \\ 0.77 & 9.036 & 1 \end{bmatrix}$

i.e.  $Z_0 := 1$

Now, from NIST website, we have, for LH2:

### Liquid Phase Data

#### Data on Saturation Curve

Temperature (K)	Pressure (atm)	Density (kg/m <sup>3</sup> )	Volume (m <sup>3</sup> /kg)
30.000	7.9381	54.540	0.018335
31.000	9.2934	51.382	0.019462
32.000	10.813	47.086	0.021238
33.000	12.527	38.080	0.026260

So, we get:

For the capillary tube:

At 32 K, the sat. pressure is 10.813 atm. But, the pressure gauge reads 10 atm.

So, we have sat. vapor in capillary. And, we use the Mathcad Function for gases, with saturation condition to get  $Z_g$ :

$$\frac{P}{P_c} = 0.77$$

For these values, get Zg from Table 6.8, or from the Mathcad Function:

$$Z_g := Z\_GASES\_SAT(0.77)$$

i.e.  $Z_g = \begin{bmatrix} "Pr" & "Z for sat.Gas" \\ 0.77 & 0.5382 \end{bmatrix}$

i.e.  $Z_g := 0.5382$

For the liquid conditions:

$$T_f = 20 \quad K \quad p = 1.013 \cdot 10^6 \quad = 10 \text{ atm.}$$

For these values, get Zf from Table 6.6, or from the Mathcad Function written earlier:



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$$P := 10 \text{ atm} \quad T_f = 20 \text{ K}$$

$$Z_f := Z_{\text{LH2}}(P, T_f)$$

$$\text{i.e. } Z_f = \begin{bmatrix} "P(\text{atm})" & "T(K)" & "Z \text{ for LH2}" \\ 10 & 20 & 0.1693 \end{bmatrix}$$

$$\text{i.e. } Z_f := 0.1693$$

Therefore, liquid level is given by:

$$\frac{L_f}{L} = \frac{\frac{m \cdot R \cdot T_f}{A_c \cdot L \cdot p} - \frac{V_0 \cdot T_f}{Z_0 \cdot T_0 \cdot A_c \cdot L} - \frac{T_f}{Z_g \cdot T_g}}{\frac{1}{Z_f} - \frac{T_f}{Z_g \cdot T_g}}$$

$$\text{i.e. } \frac{L_f}{L} = 0.77952$$

Then, liquid level  $L_f$  is equal to:

$$L_f := 0.77952 \cdot L$$

$$\text{i.e. } L_f = 1.185 \text{ m .... Ans.}$$


---

**Prob. 5.2.36** A thermodynamic liquid-level gauge for liquid nitrogen has a ratio of dead volume to capillary tube volume of  $V_0/A_c \cdot L = 200$ . The temperature of the liquid to be gauged is 77.4 K (139.3°R), and the gas within the sensing element is at an average temperature of 95 K (171°R). The temperature of the dead volume is 300 K (540°R). The gauge is initially charged with nitrogen gas so that the ratio of charging mass to dead volume is 7.55 kg/m<sup>3</sup> (0.471 lbm/ft<sup>3</sup>), and the sensing element length is 1.220 m (48.0 in.). Determine the liquid-level indication when the pressure indicated by the gauge is 506.6 kPa (5 atm).[1]

**Solution:**

For N<sub>2</sub>, critical pressure and temp are:

$$p_c := 3.3958 \cdot 10^6 \text{ Pa} \quad T_c := 126.192 \text{ K}$$

Other data:

$$R := 296.8 \text{ J/kg.K} \dots \text{Gas constant for N}_2$$

$$p := 506.6 \cdot 10^3 \text{ Pa} \dots \text{pressure indicated by gauge}$$

$$T_f := 77.4 \text{ K}$$

$$L := 1.22 \text{ m} \dots \text{length of sensing element}$$

$$T_0 := 300 \text{ K} \dots \text{temp of dead volume}$$

$$T_g := 95 \text{ K} \dots \text{temp of gas in the sensing element}$$

$$\frac{V_0}{A_c L} = 200$$

$$\frac{m}{V_0} = 7.55 \text{ kg/m}^3$$

Calculations:

Eqn. for liquid level is:

$$\frac{L_f}{L} = \frac{\frac{m \cdot R \cdot T_f}{A_c \cdot L \cdot p} - \frac{V_0 \cdot T_f}{Z_0 \cdot T_0 \cdot A_c \cdot L} - \frac{T_f}{Z_g \cdot T_g}}{\frac{1}{Z_f} - \frac{T_f}{Z_g \cdot T_g}}$$

For dead volume conditions:

$$\frac{p}{p_c} = 0.149 \quad \frac{T_0}{T_c} = 2.377$$

For these values, get  $Z_0$  from Table 6.8, or the Mathcad Function written earlier:

$$Z_0 := Z\_GASES(0.149, 2.377)$$

$$\text{i.e. } Z_0 = \begin{bmatrix} "Pr" & "Tr" & "Z for Gas" \\ 0.149 & 2.377 & 0.996 \end{bmatrix}$$

$$\text{i.e. } Z_0 := 0.996$$

Now, from NIST website, we have, for LN2:

### Liquid Phase Data

#### Data on Saturation Curve

Temperature (K)	Pressure (atm)	Density (kg/m <sup>3</sup> )	Volume (m <sup>3</sup> /kg)
93.000	4.5606	729.19	0.0013714
94.000	4.9365	723.77	0.0013817
95.000	5.3346	718.26	0.0013922
96.000	5.7553	712.67	0.0014032

So, we get:

For the capillary tube:

At 95 K, the sat. pressure of N<sub>2</sub> is 5.3346 atm. But, the pressure gauge reads 5 atm.

So, we have sat. vapor in capillary. And, we use the Mathcad Function for gases, with saturation condition to get Z<sub>g</sub>:



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We have:  $\frac{p}{p_c} = 0.149$

For this value, get  $Z_g$  from Table 6.8, or from the Mathcad Function:

$$Z_g := Z\_GASES\_SAT(0.149)$$

i.e.  $Z_g = \begin{bmatrix} "Pr" & "Z for sat.Gas" \\ 0.149 & 0.86615 \end{bmatrix}$

i.e.  $Z_g := 0.86615$

For the liquid conditions:

$$T_f = 77.4 \text{ K} \quad p = 5.066 \cdot 10^5 \text{ Pa} = 5.00 \text{ atm.}$$

For these values, get  $Z_f$  from Table 6.7, or from the Mathcad Function:

$$P := 5 \text{ atm} \quad T_f = 77.4 \text{ K}$$

$$Z\_LIN(P, T_f) = \begin{bmatrix} "P(atm)" & "T(K)" & "Z for LN2" \\ 5 & 77.4 & 0.02732 \end{bmatrix}$$

i.e.  $Z_f := 0.02732$

Therefore, liquid level is given by:

$$\frac{L_f}{L} = \frac{\frac{m \cdot R \cdot T_f}{p \cdot A_c \cdot L} - \frac{V_0 \cdot T_f}{Z_0 \cdot T_0 \cdot A_c \cdot L} - \frac{T_f}{Z_g \cdot T_g}}{\frac{1}{Z_f} - \frac{T_f}{Z_g \cdot T_g}}$$

Now, we see that:

$$\frac{m}{A_c \cdot L} = \frac{m}{V_0 \cdot A_c \cdot L} = \frac{V_0}{A_c \cdot L} = 7.55 \cdot 200 = 1510$$

Substituting in the above eqn. for  $L_f/L$ , we get:

i.e.  $\frac{L_f}{L} = 0.441$

Therefore, liquid level is:

$$L_f := 0.441 \cdot L$$

$$\text{i.e. } L_f = 0.538 \text{ m ... Ans.}$$

=====

**Plot  $L_f$ \_by\_ $L$  against gauge pressure  $p$  as  $p$  varies from  $5.5 \cdot 10^5$  Pa**

**to  $6.5 \cdot 10^5$  Pa:**

First, write the various parameters as functions of  $p$ :

Here, note that we have chosen the gauge pressures in such a way that in the capillary we have only superheated vapor.

Let:

$$p := 5.5 \cdot 10^5 \text{ Pa} \quad p_c = 3.396 \cdot 10^6 \text{ Pa}$$

$$\frac{p}{p_c} = 0.162 \quad \frac{T}{T_c} = 0.951 \quad \frac{T_0}{T_c} = 2.377$$

$$T_f = 77.4 \text{ K} \quad T_0 = 300 \text{ K}$$

Then, we get:

$$Z_g(p) := Z_{\text{GASES\_SAT}}\left(\frac{p}{p_c}\right)_{1,1} \quad \text{i.e. } Z_g(p) = 0.858$$

$$Z_0(p) := Z_{\text{GASES}}\left(\frac{p}{p_c}, \frac{T_0}{T_c}\right)_{1,2} \quad \text{i.e. } Z_0(p) = 0.996$$

$$Z_f(p) := Z_{\text{LIN}}\left(\frac{p}{101300}, T_f\right)_{1,2} \quad \text{i.e. } Z_f(p) = 0.03$$

Then, liquid level is given by:

$$L_{f\_by\_L}(p) := \frac{\frac{1510 \cdot R \cdot T_f}{p} - \frac{200 \cdot T_f}{Z_0(p) \cdot T_0} - \frac{T_f}{Z_g(p) \cdot T_g}}{\frac{1}{Z_f(p)} - \frac{T_f}{Z_g(p) \cdot T_g}}$$

i.e.  $L_{f\_by\_L}(p) = 0.315$

**Now, to plot:**

$p := 550000, 555000.. 650000$  Pa....define a range variable



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We get:

p	Lf_by_L(p)
550000	0.315
555000	0.301
560000	0.286
565000	0.271
570000	0.257
575000	0.242
580000	0.228
585000	0.213
590000	0.198
595000	0.184
600000	0.169
605000	0.154
610000	0.14
615000	0.125
620000	0.11
625000	0.095
630000	0.081
635000	0.066
640000	0.051
645000	0.037
650000	0.022

**Plot:**

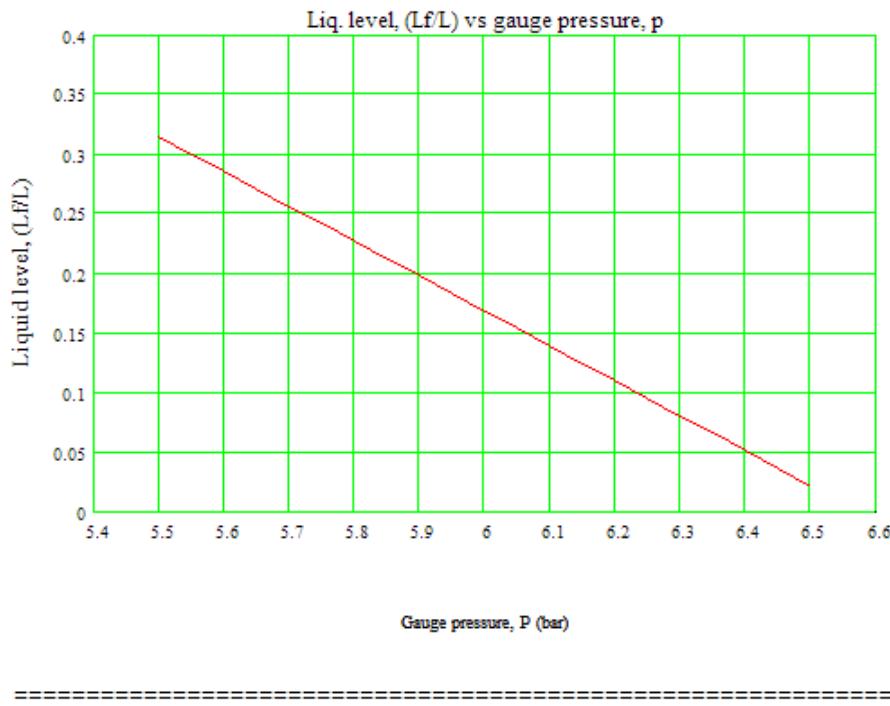
**For:**

**Nitrogen**

$$V_0/(Ac.L) = 200$$

$$m/V_0 = 7.55 \text{ kg/m}^3$$

$$T_0 = 300 \text{ K}, T_f = 77.4 \text{ K}$$



### 5.3 PROBLEMS SOLVED WITH EES:

**Prob 5.3.1** Write an EES Function to determine the resistance of a Pt-100 resist. thermometer when the temp. is known.

Use that Function to find the temp when a Pt-100 RTD shows a resistance of 38.6 ohms.

Also, plot the Resistance vs temp graph for the temp range: -200 C to 400 C.

**Solution:**

**First, write the EES Function:**

FUNCTION Pt\_Resist(temp)

{Finds the resistance (ohms) of Pt-resist thermometer for given temp (deg.C)}

A := 3.9083 \* 10<sup>-3</sup> "deg.C/C"

B := -5.775 \* 10<sup>-7</sup> "deg.C / C<sup>2</sup>"

C := -4.183 \* 10<sup>-12</sup> "deg.C / C<sup>4</sup>"

R\_0 := 100 "ohms"

IF (temp < 0) THEN

Pt\_Resist := R\_0 \* (1 + A \* temp + B \* temp^2 + C \* (temp - 100) \* temp^3)

ELSE

Pt\_Resist := R\_0 \* (1 + A \* temp + B \* temp^2)

ENDIF

END

“=====”

**Now, solve the problem:**

R = 38.6 “ohm...by data”

R = Pt\_Resist(temp)



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**Note:** Great advantage of EES is that the unknown need not be on the LHS of the eqn.

Now, press F2 to get the solution:

Unit Settings: SI C kPa kJ mass deg

R = 38.6 [Ω]

temp = -152.694 [C]

Thus:

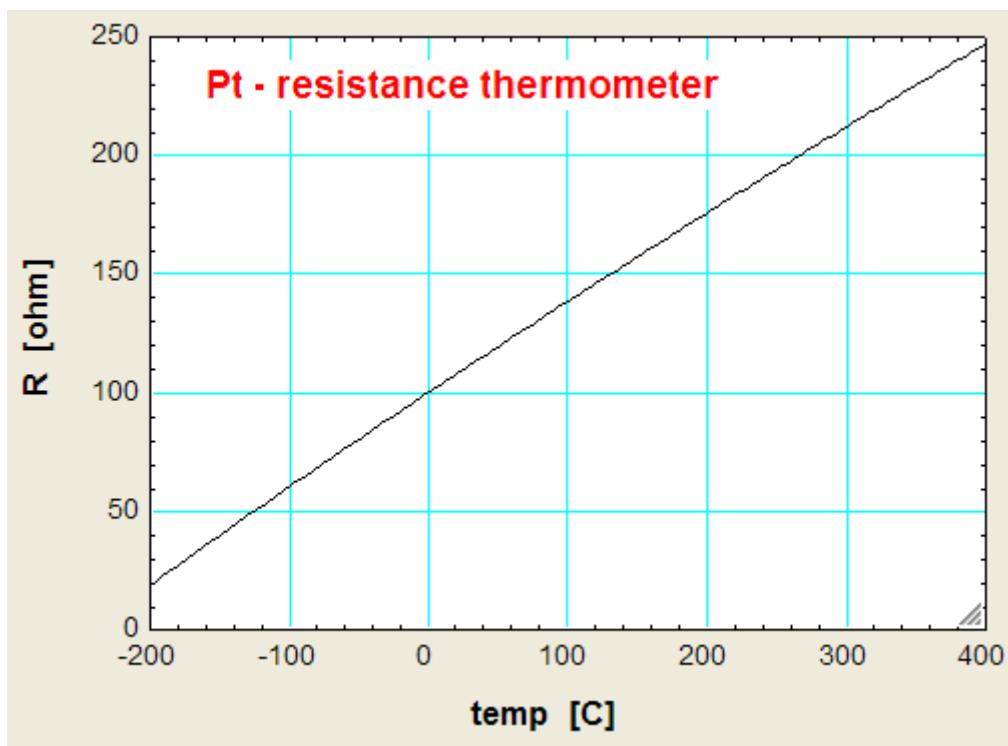
Temp = -152.694 C when the Pt-resist is 38.6 ohms...Ans.

Now, to plot the Temp-Resist graph:

First compute the Parametric Table:

	temp [C]	R [Ω]		temp [C]	R [Ω]
Run 1	-200.000	18.52	Run 16	100.000	138.5
Run 2	-180.000	27.1	Run 17	120.000	146.1
Run 3	-160.000	35.54	Run 18	140.000	153.6
Run 4	-140.000	43.88	Run 19	160.000	161.1
Run 5	-120.000	52.11	Run 20	180.000	168.5
Run 6	-100.000	60.26	Run 21	200.000	175.9
Run 7	-80.000	68.33	Run 22	220.000	183.2
Run 8	-60.000	76.33	Run 23	240.000	190.5
Run 9	-40.000	84.27	Run 24	260.000	197.7
Run 10	-20.000	92.16	Run 25	280.000	204.9
Run 11	0.000	100	Run 26	300.000	212.1
Run 12	20.000	107.8	Run 27	320.000	219.2
Run 13	40.000	115.5	Run 28	340.000	226.2
Run 14	60.000	123.2	Run 29	360.000	233.2
Run 15	80.000	130.9	Run 30	380.000	240.2
			Run 31	400.000	247.1

**Now, plot:**



**Prob. 5.3.2** The electric resistance of a platinum resistance thermometer is found 50.000 ohms at the ice point, 69.625 ohms at the steam point, and 128.275 at the zinc point. Determine the constants  $R_0$ , A, and B in the resistance function given in eqn. (6.4). Using these values, determine the temperature (including the correction given by eqn. 6.5) when the electric resistance of the thermometer is 88.650 ohms.[1]

**“Solution:”**

“We have:”

{ $R_e = R_0 * (1 + A * t_{\text{prime}} - B * t_{\text{prime}} * (t_{\text{prime}} - 100))$  “...Temp-resistance relation given in eqn.(6.4)”}

$R_0 = 50$  “ohms...resist at ice point, i.e. at  $t_{\text{prime}} = 0$  deg.C”

$69.625 = R_0 * (1 + A * 100 - B * 100 * (100 - 100))$  “...Resist at steam point, i.e.  $t_{\text{prime}} = 100$  C”

$128.275 = R_0 * (1 + A * 419.58 - B * 419.58 * (419.58 - 100))$  “...Resist at zinc point, i.e.  $t_{\text{prime}} = 419.58$  C”

“Solving the above 2 eqns we get A and B.”

“Now, when the resistance,  $R_e = 88.65$  ohms, the temp,  $t_{\text{prime}}$  is given by:”

$R_e = 88.65$  “ohms”

$$R_e = R_0 * (1 + A * t_{\text{prime}} - B * t_{\text{prime}} * (t_{\text{prime}} - 100))$$

“Temp. correction given in eqn. 6.5:”

$$T_{\text{corrected}} = t_{\text{prime}} + 0.045 * (t_{\text{prime}}/100) * (t_{\text{prime}}/100 - 1) * (t_{\text{prime}}/419.58 - 1) * (t_{\text{prime}}/630.74 - 1)$$

### Results:

**Unit Settings: SI C kPa kJ mass deg**

$$A = 0.003925$$

$$B = 6.067E-07$$

$$R_0 = 50 \text{ } [\Omega]$$

$$R_e = 88.65 \text{ } [\Omega]$$

$$T_{\text{corrected}} = 200.068 \text{ } [C]$$

$$t' = 200.036 \text{ } [C]$$



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Thus:

Temp. when the resist. is 88.65 ohms =  $t_{\text{prime}} = 200.036 \text{ C}$ ...Ans.

Corrected temp =  $T_{\text{corrected}} = 200.068 \text{ C}$ ...Ans.

---

**Prob.5.3.3** A platinum resistance thermometer is calibrated according to eqn. (6.9) with an electric resistance at  $^{\circ}\text{C}$  of 100.00 ohms and the constants A, B, and C are the same as those given after eqn. (6.9). If the electric resistance indication of the thermometer is 50.00 ohms, determine the temperature indication. Determine the sensitivity of the thermometer at this temperature.[1]

**“Solution:”**

“According to eqn. 6.9, the temp-resistance relation is:”

$$R_e = R_0 * (1 + A * t + B * t^2 + C * T^3 * (t - 100)) \text{ “where”}$$

$$A = 3.946E-03 \text{ “C}^{-1}$$

$$B = -1.108E-06 \text{ “C}^{-2}$$

$$C = 3.33E-12 \text{ “C}^{-4}$$

**“Sensitivity:  $S_0 = d R_e / dt:$ ”**

$$S_0 = R_0 * (A + 2 * B * t + C * t^2 * (4 * t - 300)) \text{ “ohms/ deg.C”}$$

“By data, we have:”

$$R_0 = 100 \text{ “ohms”}$$

$$R_e = 50 \text{ “ohms”}$$

### Results:

**Unit Settings: SI C kPa kJ mass deg**

$$A = 0.003946$$

$$B = -0.000001108$$

$$C = 3.330E-12$$

$$R_0 = 100 \text{ } [\Omega]$$

$$R_e = 50 \text{ } [\Omega]$$

$$S_0 = 0.4178 \text{ } [\Omega/C]$$

$$t = -122.823 \text{ } [C]$$

Thus:

Temp. when resistance is 50 ohms =  $t = -122.823 \text{ C...Ans.}$

Sensitivity =  $S_0 = 0.4178 \text{ ohms/C...Ans.}$

---

**Prob.5.3.4** A germanium semiconductor thermometer has the following resistance values: 2000 ohms at 10 K; 200 ohms at 20 K; 30 ohms at 40 K; and 6 ohms at 100 K. Determine the values of the constants in the calibration equation,  $\ln T = B_0 + B_1 \cdot \ln R_e + B_2 \cdot (\ln R_e)^2 + B_3 \cdot (\ln R_e)^3$

Using these values, determine the temperature indication of the thermometer when the electric resistance is 100 ohm. Determine the sensitivity of the thermometer at this temperature.[1]

**“Solution:”**

**“Calibration eqn. is:”**

$$\ln(T) = (B_0 + B_1 * \ln(R_e) + B_2 * (\ln(R_e))^2 + B_3 * (\ln(R_e))^3) \text{ “where:”}$$

$$\ln(10) = (B_0 + B_1 * \ln(2000) + B_2 * (\ln(2000))^2 + B_3 * (\ln(2000))^3) \text{ “...at 10 K, resist is 2000 ohms”}$$

$$\ln(20) = (B_0 + B_1 * \ln(200) + B_2 * (\ln(200))^2 + B_3 * (\ln(200))^3) \text{ “...at 20 K, resist is 200 ohms”}$$

$$\ln(40) = (B_0 + B_1 * \ln(30) + B_2 * (\ln(30))^2 + B_3 * (\ln(30))^3) \text{ “...at 40 K, resist is 30 ohms”}$$

$$\ln(100) = (B_0 + B_1 * \ln(6) + B_2 * (\ln(6))^2 + B_3 * (\ln(6))^3) \text{ “...at 100 K, resist is 6 ohms”}$$

**“Solving the above 4 eqns, we get B\_0, B\_1, B\_2 and B\_3”**

**“When the thermometer resist is 100 ohms:”**

$$R_e = 100 \text{ “ohms”}$$

**“Sensitivity,  $S_0 = dt / dR_e$ ”**

“We shall use the EES built-in function: DIFFERENTIATE.

However, to use this function, we need to have the results of the function in a Tabular form in a ‘Lookup Table’. And, we generate a Parametric Table of Temp values for various values of resistance,  $R_e$  and copy it into a Lookup Table and name it as ‘GermaniumThermo’.

Regarding the use of DIFFERENTIATE Function, following extract is from EES help file:

The DIFFERENTIATE function returns the derivative determined from two columns of tabular data in a Lookup Table or a Lookup file.

**DIFFERENTIATE(‘Table Name’,‘ColName1’,‘ColName2’,ColName2=Value)**

‘Table Name’ is a string constant or string variable that provides the name of the Lookup table in the Lookup Window.

ColName1 and ColName2 are the column header names.



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The final parameter is of the form ColName2=Value where the text to the left of the equal sign can be either of the column header names (ColName1 or ColName2) specified with the two previous parameters. Note that if a string variable was provided for the column name, then the same string variable must be used for this parameter. Value is a numerical value or expression.

EES will return an estimate of the derivative  $d(\text{ColName1})/ d(\text{ColName2})$  at a point fixed by the specified value of either ColName1 or ColName2.”

### Following is the Lookup Table:

GermaniumThermo		
	1 Resist [ $\Omega$ ]	2 Temp [K]
Row 1	50	32.21
Row 2	60	30
Row 3	70	28.32
Row 4	80	26.99
Row 5	90	25.89
Row 6	100	24.97
Row 7	110	24.18
Row 8	120	23.5
Row 9	130	22.89
Row 10	140	22.35
Row 11	150	21.86

“So, we have:”

$S_0 = \text{DIFFERENTIATE}(\text{'GermaniumThermo'}, \text{'Resist'}, \text{'Temp'}, \text{Resist} = 100)$

### Results:

**Unit Settings: SI C kPa kJ mass deg**

$$B_0 = 6.2179$$

$$B_1 = -1.1192$$

$$B_2 = 0.1355$$

$$B_3 = -0.007375$$

$$R_e = 100 \text{ } [\Omega]$$

$$S_0 = -11.765 \text{ } [\Omega/K]$$

$$T = 24.972 \text{ [K]}$$

Thus:

Temp when the resistance is 100 ohms = 24.972 K...Ans.

Sensitivity at this temp.,  $S_0 = -11.765 \text{ Ohm/K...Ans.}$

---

**Prob.5.3.5** A carbon resistance thermometer has the following electric resistance values: 1460 ohms at 4.2 K; 133 ohms at 77.3 K; and 100 ohms at 300 K. Determine the constants in the calibration relationship, eqn. (6.11). Using these values, determine the temperature indication when the thermometer resistance is 250 ohms.[1]

“Solution:”

“Expression for temp, eqn. 6.11 of Ref. [1] is:”

$$T = B * \log_{10}(R_e) / (\log_{10}(R_e)^2 + A * \log_{10}(R_e) + K)$$

“Applying the above eqn. for  $T = 4.2 \text{ K}, 77.3 \text{ K} \text{ and } 300 \text{ K}$ , we get A, B and K:”

“And, we write the above eqn. for the three temp values as follows:”

$$B * \log_{10}(1460) = 4.2 * (\log_{10}(1460)^2 + A * \log_{10}(1460) + K)$$

$$B * \log_{10}(133) = 77.3 * (\log_{10}(133)^2 + A * \log_{10}(133) + K)$$

$$B * \log_{10}(100) = 300 * (\log_{10}(100)^2 + A * \log_{10}(100) + K)$$

$R_e = 250 \text{ "ohms"}$

**Results:**

**Unit Settings: SI C kPa kJ mass deg**

$$A = -3.75451 \quad B = 2.19826 \quad K = 3.52368 \quad R_e = 250 \text{ } [\Omega]$$

$$\boxed{T = 19.473 \text{ [K]}}$$

**Thus:**

**The constants in the eqn are:**

**A = -3.75451, B = 2.19826, K = 3.52368...Ans.**

**Temp when R<sub>e</sub> is 250 ohms = 18.743 K...Ans.**

---

**Prob.5.3.6.** Write an EES Function to find the emf when temp. value is known, for a type T Thermocouple (i.e. copper-constantan). The cold junction is at 0 C.

**Solution:**

**“From NIST data for Type T Thermocouple, i.e. cu-const. TC:”**

FUNCTION TC\_Cu\_Const\_emf(T)

{Finds the emf value (mV) when temp(deg.C) is known, for a type T Thermocouple (i.e. copper-constantan). The cold junction is at 0 C.}



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IF ((T < -270) OR (T > 400)) THEN

CALL ERROR ('Temp should be between -270 C and 400 C !!')

ENDIF

**"Coeffs. for temp range: -270 to 0 deg.C:"**

```
C1[0]:=0.000000000000E+00;C1[1]:=0.387481063640E-01;C1[2]:=0.441944343470E-04  
C1[3]:=0.118443231050E-06;C1[4]:=0.200329735540E-07;C1[5]:=0.901380195590E-09  
C1[6]:=0.226511565930E-10;C1[7]:=0.360711542050E-12;C1[8]:=0.384939398830E-14  
C1[9]:=0.282135219250E-16;C1[10]:=0.142515947790E-18;C1[11]:=0.487686622860E-21  
C1[12]:=0.107955392700E-23;C1[13]:=0.139450270620E-26;C1[14]:=0.797951539270E-30
```

**"Coeffs. for temp range: 0 to 400 deg.C:"**

```
C2[0]:=0.000000000000E+00;C2[1]:=0.387481063640E-01;C2[2]:=0.332922278800E-04  
C2[3]:=0.206182434040E-06;C2[4]:=-0.218822568460E-08;C2[5]:=0.109968809280E-10  
C2[6]:=-0.308157587720E-13;C2[7]:=0.454791352900E-16;C2[8]:=-0.275129016730E-19  
sum := 0
```

i := 0

IF (T <= 0) THEN

REPEAT

i := i + 1

sum:= sum + C1[i] \* T^i

UNTIL (i >= 14)

TC\_Cu\_Const\_emf = sum

ENDIF

IF (T > 0) THEN

REPEAT

sum:= sum + C2[i] \* T^i

i := i + 1

UNTIL (i > 8)

TC\_Cu\_Const\_emf = sum

ENDIF

END

**"=====**

“Ex:”

T = -190 “C”

EMF = TC\_Cu\_Const\_emf(T)

**Results:**

**Unit Settings: SI C kPa kJ mass deg**

EMF = -5.439 [mV]

**T = -190.000 [C]**

---

**Prob.5.3.7.** Write an EES Function to find the temp when emf value is known, for a type T Thermocouple (i.e. copper-constantan). The cold junction is at 0 C.

**“From NIST data for Type T Thermocouple, i.e. cu-const. TC:”**

FUNCTION TC\_Cu\_Const\_temp(E)

{ Finds the temp(deg.C) when emf value (mV) is known, for a type T Thermocouple (i.e. copper-constantan). The cold junction is at 0 C.}

IF ((E < -5.603) OR (E > 20.872)) THEN

CALL ERROR ('emf (E) should be between -5.603 mV and 20.872 mV !!')

ENDIF

**“Coeffs. for emf range: -5.603 to 0 mV:”**

```
D1[0] := 0.000000000000E+00 ; D1[1] := 2.5949192E+01; D1[2] := -2.1316967E-01  
D1[3] := 7.9018692E-01 ; D1[4] := 4.2527777E-01; D1[5] := 1.3304473E-01  
D1[6] := 2.0241446E-02; D1[7] := 1.2668171E-03
```

**“Coeffs. for emf range: 0 to 20.872 mV:”**

```
D2[0] := 0.000000000000E+00; D2[1] := 2.592800E+01; D2[2] := -7.602961E-01  
D2[3] := 4.637791E-02; D2[4] := -2.165394E-03; D2[5] := 6.048144E-05  
D2[6] := -7.293422E-07; D2[7] := 0.000000E+00
```

sum := 0

i := 0

IF (E <= 0) THEN

REPEAT

i := i + 1

sum:= sum + D1[i] \* E<sup>i</sup>

UNTIL (i >= 7)

TC\_Cu\_Const\_temp = sum

ENDIF

IF (E > 0) THEN



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REPEAT

```
sum:= sum + D2[i] * E^i
```

```
i := i + 1
```

```
UNTIL (i > 7)
```

```
TC_Cu_Const_temp := sum
```

```
ENDIF
```

```
END
```

“=====”

“Ex:”

E = 20 “mV”

T = TC\_Cu\_Const\_temp(E)

**Results:**

**Unit Settings: SI C kPa kJ mass deg**

E = 20 [mV]

T = 385.865 [C]

---

**Prob.5.3.8.** Write an EES Function to find the temp when emf value is known, for a type K Thermocouple (i.e. Chromel-Alumel). The cold junction is at 0 C. Take the coefficient values for type K Thermocouple from NIST Thermocouple data.

**Solution:**

FUNCTION TC\_Chromel\_Alumel\_emf(T)

{ Finds the emf value (mV) when temp(deg.C) is known, for a type K Thermocouple (i.e. chromel-alumel). The cold junction is at 0 C.}

IF ((T < -270) OR (T > 1372)) THEN

CALL ERROR ('Temp should be between -270 C and 1372 C !!')

ENDIF

**“Coeffs. for temp range: -270 to 0 deg.C:”**

```
CC[0]:=0.000000000000E+00;CC[1]:=0.394501280250E-01;CC[2]:=0.236223735980E-04  
CC[3]:=-0.328589067840E-06;CC[4]:=-0.499048287770E-08;CC[5]:=-0.675090591730E-10  
CC[6]:=-0.574103274280E-12;CC[7]:=-0.310888728940E-14;CC[8]:=-0.104516093650E-16  
CC[9]:=-0.198892668780E-19;CC[10]:=-0.163226974860E-22
```

**“Coeffs. for temp range: 0 to 1372 deg.C:”**

```
DD[0]:=-0.176004136860E-01;DD[1]:=0.389212049750E-01;DD[2]:=0.185587700320E-04  
DD[3]:=-0.994575928740E-07;DD[4]:=0.318409457190E-09;DD[5]:=-0.560728448890E-12  
DD[6]:=0.560750590590E-15;DD[7]:=-0.320207200030E-18;DD[8]:=0.971511471520E-22  
DD[9]:=-0.121047212750E-25
```

**“Exponential coeffs.:”**

a0:=0.118597600000E+00; a1:=-0.118343200000E-03; a2:=0.126968600000E+03

sum := 0

i := 0

IF (T <= 0) THEN

REPEAT

i := i + 1

sum:= sum + CC[i] \* T^i

UNTIL (i >= 10)

TC\_Chromel\_Alumel\_emf := sum

ENDIF

IF ( $T > 0$ ) THEN

REPEAT

sum:= sum + DD[i] \*  $T^i$

$i := i + 1$

UNTIL ( $i > 9$ )

TC\_Chromel\_Alumel\_emf := sum +  $a_0 * \exp(a_1 * (T - a_2)^2)$

ENDIF

END

“=====”



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“Ex:”

T = 1370 “C”

EMF = TC\_Chromel\_Alumel\_emf(T)

**Unit Settings: SI C kPa kJ mass deg**

EMF = 54.8186 [mV]

T = 1,370.000 [C]

---

**Prob.5.3.9.** Write an EES Function to find the temp when emf value is known, for a type K Thermocouple (i.e. chromel-alumel). The cold junction is at 0 C.

**Solution:**

“From NIST data for Type K Thermocouple, i.e. chromel-alumel. TC:”

FUNCTION TC\_Chromel\_Alumel\_temp(E)

{ Finds the temp(deg.C) when emf value (mV) is known, for a type K Thermocouple (i.e. chromel-alumel). The cold junction is at 0 C.}

IF ((E < -5.891) OR (E > 54.886)) THEN

CALL ERROR ('emf (E) should be between -5.891 mV and 54.886 mV !!')

ENDIF

“Coeffs. for emf range: -5.891 to 0 mV:”

```
DD1[0]:=0.000000000000E+00 ; DD1[1]:=2.5173462E+01 ; DD1[2]:=-1.1662878E+00  
DD1[3]:=-1.0833638E+00 ; DD1[4]:=-8.9773540E-01 ; DD1[5]:=-3.7342377E-01  
DD1[6]:=-8.6632643E-02 ; DD1[7]:=-1.0450598E-02 ; DD1[8]:=-5.1920577E-04  
DD1[9]:=0.0000000E+00
```

**“Coeffs. for emf range: 0 to 20.644 mV:”**

```
DD2[0] := 0.000000000000E+00; DD2[1] := 2.508355E+01 ; DD2[2] := 7.860106E-02  
DD2[3] := -2.503131E-01 ; DD2[4] :=8.315270E-02; DD2[5] :=-1.228034E-02  
DD2[6] := 9.804036E-04 ; DD2[7] :=-4.413030E-05 ; DD2[8] :=1.057734E-06  
DD2[9] :=-1.052755E-08
```

**“Coeffs. for emf range: 20.644 to 54.886 mV:”**

```
DD3[0] := -1.318058E+02; DD3[1] := 4.830222E+01 ; DD3[2] := -1.646031E+00  
DD3[3] := 5.464731E-02 ; DD3[4] :=-9.650715E-04; DD3[5] :=8.802193E-06  
DD3[6] := -3.110810E-08 ; DD3[7] :=0 ; DD3[8] :=0; DD3[9] :=0
```

sum := 0

i := 0

IF (E < 0) THEN

REPEAT

i := i + 1

sum:= sum + DD1[i] \* E^i

UNTIL (i >= 9)

TC\_Chromel\_Alumel\_temp := sum

ENDIF

IF ((E > 0) AND (E <= 20.644)) THEN

REPEAT

sum:= sum + DD2[i] \* E^i

i := i + 1

UNTIL (i > 9)

TC\_Chromel\_Alumel\_temp := sum

ENDIF

IF ((E > 20.644) AND (E <= 54.886)) THEN

REPEAT

sum:= sum + DD3[i] \* E^i

i := i + 1

UNTIL (i > 9)

TC\_Chromel\_Alumel\_temp := sum

ENDIF

END

“=====”



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“Ex:”

$$E = 31.213 \text{ "mV"}$$

$$T = \text{TC_Chromel_Alumel_temp}(E)$$

**Results:**

**Unit Settings: SI C kPa kJ mass deg**

E = 31.2130 [mV]

T = 749.988 [C]

---

**Prob. 5.3.10** Write an EES Function to find the temp when emf value is known, for Chromel-Au/0.03 atomic % Fe Thermocouple, and Chromel-Au/0.07 atomic % Fe Thermocouple. The cold junction is at 0 C.

Also, plot Temp vs emf for type T and type K thermocouples, for the temp range: -200 C to 350 C.

Find the EMF's at a temp of -100 C for both these TC's and also find the sensitivities at that temp.

“Solution:”

**First, copy the two Temp vs EMF tables in to two Lookup Tables, and name them as ‘Chromel-Au03Fe\_TC’ and ‘Chromel-Au07Fe\_TC’ respectively. EMF values at intermediate values of Temps are obtained by using the Interpolate Function of EES.**

**Lookup Tables are shown below:**

Chromel-Au03Fe_TC		
	Temp [K]	EMF [mV]
Row 1	3.5	-4.668
Row 2	8	-4.607
Row 3	13.5	-4.526
Row 4	18	-4.457
Row 5	24	-4.37
Row 6	30	-4.287
Row 7	52	-3.993
Row 8	60	-3.883
Row 9	65	-3.813
Row 10	70	-3.741
Row 11	80	-3.595
Row 12	90	-3.444
Row 13	105	-3.203
Row 14	115	-3.037
Row 15	125	-2.869
Row 16	135	-2.696
Row 17	145	-2.518
Row 18	160	-2.247
Row 19	170	-2.062
Row 20	180	-1.873
Row 21	195	-1.584
Row 22	210	-1.291
Row 23	225	-0.9912
Row 24	240	-0.6847
Row 25	265	-0.167
Row 26	275	0.0378
Row 27	285	0.2387
Row 28	305	0.635
Row 29	325	1.039

Chromel-Au07Fe_TC		
	Temp [K]	EMF [mV]
Row 1	1.4	-5.298
Row 2	3	-5.282
Row 3	4.8	-5.259
Row 4	7	-5.229
Row 5	10.5	-5.174
Row 6	19	-5.032
Row 7	26	-4.913
Row 8	48	-4.549
Row 9	58	-4.381
Row 10	70	-4.173
Row 11	80	-3.995
Row 12	90	-3.813
Row 13	100	-3.627
Row 14	110	-3.437
Row 15	120	-3.244
Row 16	135	-2.948
Row 17	150	-2.645
Row 18	165	-2.337
Row 19	180	-2.024
Row 20	200	-1.6
Row 21	220	-1.169
Row 22	245	-0.6232
Row 23	270	-0.0705
Row 24	300	0.5986
Row 25	305	0.7158
Row 26	310	0.8431
Row 27	315	0.9944
Row 28	320	1.194
Row 29	325	1.484

**Now, write the EES Functions for Chromel-Au03Fe Thermocouple:**

“Chromel-Au\_0.03 at.%Fe Thermocouple:”

FUNCTION Chromel\_Au\_0.03Fe\_emf(temp)

“Gives emf of TC when temp is given, Ref. junction at 0 C (i.e. 273 K)”

“Input: temp in Kelvin; Output: emf in mV”

“Linear interpolation from the Lookup Table”

Chromel\_Au\_0.03Fe\_emf:=INTERPOLATE1(Chromel-Au03Fe\_TC,Temp,EMF,Temp=temp)

END

=====

**And, now, write the EES Functions for Chromel-Au07Fe Thermocouple:**

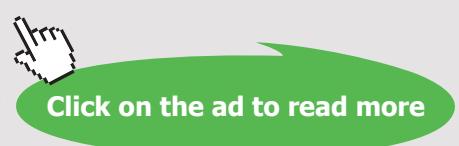
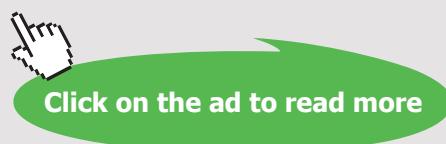
“Chromel-Au\_0.07 at.%Fe Thermocouple:”

FUNCTION Chromel\_Au\_0.07Fe\_emf(temp)

“Gives emf of TC when temp is given, Ref. junction at 0 C (i.e. 273 K)”

“Input: temp in Kelvin; Output: emf in mV”

“Linear interpolation from the Lookup Table”



Chromel\_Au\_0.07Fe\_emf:=INTERPOLATE1(Chromel-Au07Fe\_TC,Temp,EMF,Temp=temp)

END

“=====”

**Also, plot Temp vs emf for type T and type K thermocouples, for the temp range: -200 C to 350 C.**

**Find the EMF's at a temp of -100 C for both these TC's and also find the sensitivities at that temp:**

**“We shall use the EES Functions written above.”**

{T = -100 “C”}

**“For Type T (cu – constantan) TC:”**

E\_T = TC\_Cu\_Const\_emf(T)

**“For Type K (chromel – alumel) TC:”**

E\_K = TC\_Chromel\_Alumel\_emf(T)

**Results:**

**Unit Settings: SI C kPa kJ mass deg**

E\_K = -3.554 [mV]

E\_T = -3.379 [mV]

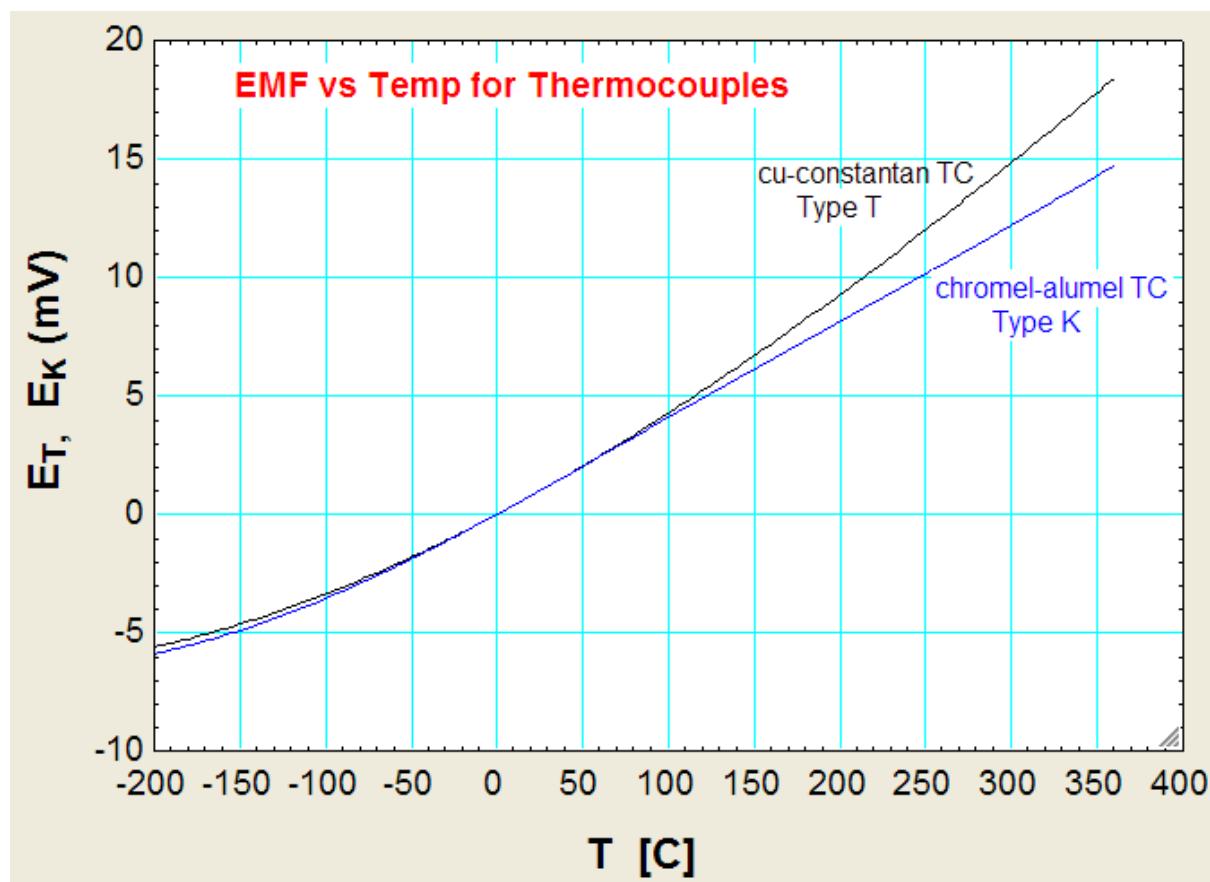
T = -100.000 [C]

**Now, to draw the plot:**

**First, compute the Parametric Table:**

1..29	1 T [C]	2 E <sub>T</sub> [mV]	3 E <sub>K</sub> [mV]	1..29	1 T [C]	2 E <sub>T</sub> [mV]	3 E <sub>K</sub> [mV]
Run 1	-200.000	-5.603	-5.891	Run 16	100.000	4.279	4.096
Run 2	-180.000	-5.261	-5.55	Run 17	120.000	5.228	4.92
Run 3	-160.000	-4.865	-5.141	Run 18	140.000	6.206	5.735
Run 4	-140.000	-4.419	-4.669	Run 19	160.000	7.209	6.54
Run 5	-120.000	-3.923	-4.138	Run 20	180.000	8.237	7.34
Run 6	-100.000	-3.379	-3.554	Run 21	200.000	9.288	8.138
Run 7	-80.000	-2.788	-2.92	Run 22	220.000	10.36	8.94
Run 8	-60.000	-2.153	-2.243	Run 23	240.000	11.46	9.747
Run 9	-40.000	-1.475	-1.527	Run 24	260.000	12.57	10.56
Run 10	-20.000	-0.7568	-0.7775	Run 25	280.000	13.71	11.38
Run 11	0.000	0	0	Run 26	300.000	14.86	12.21
Run 12	20.000	0.7896	0.7981	Run 27	320.000	16.03	13.04
Run 13	40.000	1.612	1.612	Run 28	340.000	17.22	13.87
Run 14	60.000	2.468	2.436	Run 29	360.000	18.42	14.71
Run 15	80.000	3.358	3.267				

**Now, plot:**



**Find the EMF's at a temp of -100 C for both these TC's and also find the sensitivities at that temp.:**

**From the Parametric Table computed using the EES Functions, shown above, we see that:**

**EMF at -100 C:**

**Unit Settings: SI C kPa kJ mass deg**

**E<sub>K</sub> = -3.554 [mV]**

**E<sub>T</sub> = -3.379 [mV]**

**T = -100.000 [C]**

**Sensitivities: (S\_0 = dE/dT .... mV/C)**

We need to first copy the Parametric Table in to a Lookup Table and then use the built-in DIFFERENTIATE function of EES.

Following is the format of DIFFERENTIATE function:

**DIFFERENTIATE('Table Name','ColName1','ColName2',ColName2=Value)**

**See Prob. 5.3.4 for Notes on the use of DIFFERENTIATE Function.**



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**Now, find the derivative dE/dT for Type T and Type K Thermocouples at -100 C:**

“To find the sensitivities for both the TC’s at -100 C:”

“Copy the Parametric Table in to a Lookup Table and name it as TC\_EMFvsT.

Now, find the derivative dE/dT for Type T and Type K Thermocouples at -100 C.”

“Format of this built-in function is: DIFFERENTIATE(‘Table Name’,‘ColName1’,‘ColName2’,ColName2=Value)

EES will return an estimate of the derivative  $d(\text{ColName1})/ d(\text{ColName2})$  at a point fixed by the specified value of either ColName1 or ColName2.”

$dTbydE_{\text{TypeT}} = \text{DIFFERENTIATE}(\text{'TC\_EMFvsT'}, \text{'Temp'}, \text{'EforTypeT'}, \text{Temp} = -100)$   
“C/mV”

$dTbydE_{\text{TypeK}} = \text{DIFFERENTIATE}(\text{'TC\_EMFvsT'}, \text{'Temp'}, \text{'EforTypeK'}, \text{Temp} = -100)$   
“C/mV”

“**And Sensitivities are:**”

$S_{0,\text{TypeT}} = 1 / dTbydE_{\text{TypeT}}$  “mV/C”

$S_{0,\text{TypeK}} = 1 / dTbydE_{\text{TypeK}}$  “mV/C”

**Results:**

**Unit Settings: SI C kPa kJ mass deg**

$dTbydE_{\text{TypeK}} = 32.78 \text{ [C/mV]}$	$dTbydE_{\text{TypeT}} = 35.21 \text{ [C/mV]}$	$E_K = -3.554 \text{ [mV]}$
$E_T = -3.379 \text{ [mV]}$	$S_{0,\text{TypeK}} = 0.031 \text{ [mV/C]}$	$S_{0,\text{TypeT}} = 0.028 \text{ [mV/C]}$
$T = -100.000 \text{ [C]}$		

**Thus:**

**Sensitivity of Type T Thermocouple at -100 C = 0.028 mV/C...Ans.**

**Sensitivity of Type K Thermocouple at -100 C = 0.031 mV/C...Ans.**

**Prob. 5.3.11** A constant-volume gas thermometer using helium gas as the working medium is calibrated at the ice point, 273.15 K or 491.67°R. At this point, the pressure indication of the thermometer is 75 kPa (10.88 psia). The ratio of the dead-volume to sensing element volume is 0.80. Determine the temperature indication when the pressure indication is 25 kPa (3.63 psia) and the dead-volume gas temperature is 300 K. Neglect gas imperfections.

**Solution:**

**“Data:”**

$$T_s = 273.15 \text{ [K]}$$

$$p_s = 75 \text{ [kPa]}$$

$$p = 25 \text{ [kPa]}$$

$$T_0 = 300 \text{ [K]}$$

$$V_0/\text{V} = 0.8$$

**“Then, dead volume correction is:”**

$$K_1 = (1 + (1 - p/p_s) * V_0/\text{V} * (T_s / T_0))^{-1}$$

**“Then, temp indicated is:”**

$$T = K_1 * p * (T_s / p_s)$$

**“And, Sensitivity is:”**

$$S_0 = (p_s / T_s) / (K_1 * (1 + T/T_0 * V_0/\text{V}))$$

**“When gas imperfections are not neglected:”**

“The indicated temp calculated above is now termed as  $T_{\text{prime}}$ :”

$$T_{\text{prime}} = T$$

**"At this temp. second virial coeff. is calculated as follows:"**

$$BT_{\text{prime}} = 0.0072929 * T_{\text{prime}}^{-(-5/4)} * (1 - 3.04135 * T_{\text{prime}}^{-(-1/2)} - 14.1425 * T_{\text{prime}}^{(-1)} + 17.35052 * T_{\text{prime}}^{(-3/2)}) \text{ "kPa}^{-1}$$

**"Similarly, at  $T_s = 273.15 \text{ K}$ :**

$$BT_s = 0.0072929 * T_s^{-(-5/4)} * (1 - 3.04135 * T_s^{-(-1/2)} - 14.1425 * T_s^{(-1)} + 17.35052 * T_s^{(-3/2)}) \text{ "kPa}^{-1}$$

**"Therefore, gas imperfection correction factor is:"**

$$K_2 = (BT_s * p_s - BT_{\text{prime}} * p) / (1 + BT_{\text{prime}} * p)$$

**"Therefore, temp error due to gas imperfections is:"**

$$\Delta T = K_2 * T_{\text{prime}} \text{ "K"}$$



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**“Then, the temp when gas imperfection effect is included, is:”**

$$T_c = T_{\text{prime}} + \Delta T \text{ "K...corrected temp."}$$

**Results:**

**Unit Settings: SI C kPa kJ mass deg**

BTprime = 0.00001773	BTs = 0.000005044	$\Delta T = -0.00398$
K1 = 0.6731	K2 = -0.00006493	$p = 25 \text{ [kPa]}$
$p_s = 75 \text{ [kPa]}$	$S_0 = 0.351 \text{ [kPa/K]}$	$T = 61.288 \text{ [K]}$
$T_0 = 300 \text{ [K]}$	$T_c = 61.284 \text{ [K]}$	$T' = 61.288 \text{ [K]}$
$T_s = 273.1 \text{ [K]}$	$V_0/V = 0.8$	

**Thus:**

**Temp when p is 25 kPa, neglecting gas imperfections =  $T = 61.288 \text{ K...Ans.}$**

**Temp when p is 25 kPa, including effect of gas imperfections =  $T_c = 61.284 \text{ K...Ans.}$**

---

**Prob.5.3.12** For a const. vol. gas thermometer, plot reduced sensitivity,  $S_0\text{-reduced}$ , vs Temp (K) when  $T_s = 273.15 \text{ K}$  and  $p_s = 75 \text{ kPa}$ :

**“Solution:”**

**“Data:”**

$$T_s = 273.15 \text{ [K]}; p_s = 75 \text{ [kPa]}; p = 25 \text{ [kPa]}; T_0 = 300 \text{ [K]}$$

$$\{T = 20 \text{ [K]}\}$$

$$V_0/V = 5$$

**“Then, we have:”**

$$S_0\text{-reduced} = (T_s / p_s) * (dp/dT)$$

$$\text{But, } dp/dT = S_0 = (p_s / T_s) / (K1 * (1 + T/T_0 * V_0/V))$$

**“Therefore:”**

$$S_{0\_reduced} = (T_s / p_s) * (dp/dT) = (K_1 * (1 + T/T_0 * V_0 by V))$$

“i.e.”

$$S_{0\_reduced} = 1 / (K_1 * (1 + T/T_0 * V_0 by V))$$

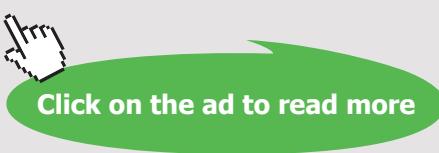
$$K_1 = (1 + (1 - p/p_s) * V_0 by V * (T_s / T_0))^{-1}$$

### Results:

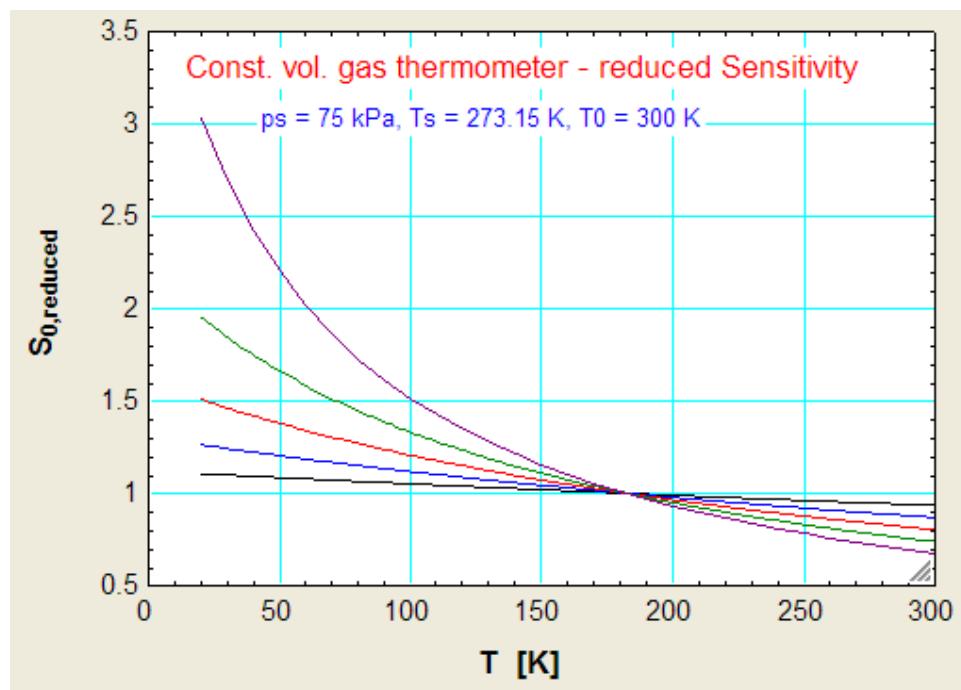
**First, compute the Parametric Table:**

Run	T [K]	V0by V =				
		0.2	0.5	1.0	2.0	5.0
Run 1	20.000	1.107	1.261	1.507	1.954	3.026
Run 2	30.000	1.099	1.241	1.461	1.845	2.69
Run 3	40.000	1.092	1.222	1.418	1.748	2.421
Run 4	50.000	1.085	1.203	1.377	1.661	2.201
Run 5	60.000	1.078	1.185	1.339	1.581	2.017
Run 6	70.000	1.071	1.167	1.303	1.51	1.862
Run 7	80.000	1.065	1.15	1.269	1.444	1.729
Run 8	90.000	1.058	1.133	1.236	1.384	1.614
Run 9	100.000	1.051	1.117	1.205	1.328	1.513
Run 10	110.000	1.045	1.102	1.176	1.277	1.424
Run 11	120.000	1.038	1.086	1.148	1.23	1.345
Run 12	130.000	1.032	1.071	1.121	1.186	1.274
Run 13	140.000	1.026	1.057	1.096	1.145	1.21
Run 14	150.000	1.019	1.043	1.071	1.107	1.153
Run 15	160.000	1.013	1.029	1.048	1.071	1.1

	V0by V =	0.2	0.5	1.0	2.0	5.0
	T [K]	S <sub>0,reduced</sub>				
Run 16	170.000	1.007	1.016	1.026	1.038	1.053
Run 17	180.000	1.001	1.003	1.004	1.006	1.009
Run 18	190.000	0.9953	0.99	0.9839	0.9768	0.9684
Run 19	200.000	0.9895	0.9776	0.9642	0.9489	0.9312
Run 20	210.000	0.9837	0.9656	0.9453	0.9225	0.8967
Run 21	220.000	0.978	0.9538	0.9271	0.8976	0.8646
Run 22	230.000	0.9723	0.9423	0.9096	0.8739	0.8348
Run 23	240.000	0.9667	0.9311	0.8928	0.8515	0.807
Run 24	250.000	0.9612	0.9201	0.8765	0.8303	0.781
Run 25	260.000	0.9557	0.9094	0.8609	0.81	0.7566
Run 26	270.000	0.9503	0.899	0.8458	0.7907	0.7336
Run 27	280.000	0.945	0.8888	0.8312	0.7723	0.7121
Run 28	290.000	0.9397	0.8788	0.8171	0.7548	0.6917
Run 29	300.000	0.9345	0.869	0.8035	0.738	0.6725



Now, plot the results:



**Prob. 5.3.13** A constant-volume gas thermometer is calibrated at the triple point of water, at which point the pressure indication is 150 kPa. Determine the temperature indication and sensitivity of the thermometer when the pressure reading is 20 kPa (2.90 psia). The ratio of dead volume to sensing element volume is 2.00, and the dead-volume gas is at 300 K (540°R). Neglect gas imperfections.[1]

**Solution:**

**“Data:”**

$$T_s = 273.16 \text{ [K]}$$

$$p_s = 150 \text{ [kPa]}$$

$$p = 20 \text{ [kPa]}$$

$$T_0 = 300 \text{ [K]}$$

$$V_0/\text{by}V = 2$$

**“Then, dead volume correction is:”**

$$K_1 = (1 + (1 - p/p_s) * V_0 \text{by} V * (T_s / T_0))^{-1}$$

**“Then, temp indicated is:”**

$$T = K_1 * p * (T_s / p_s)$$

**“And, Sensitivity is:”**

$$S_0 = (p_s / T_s) / (K_1 * (1 + T/T_0 * V_0 \text{by} V))$$


---

### Results:

**Unit Settings: SI C kPa kJ mass deg**

$$K_1 = 0.3879$$

$$p = 20 \text{ [kPa]}$$

$$p_s = 150 \text{ [kPa]}$$

$$S_0 = 1.2939380 \text{ [kPa/K]}$$

$$T = 14.126335 \text{ [K]}$$

$$T_0 = 300 \text{ [K]}$$

$$T_s = 273.2 \text{ [K]}$$

$$V_0 \text{by} V = 2$$

### Thus:

When  $p = 20 \text{ kPa}$ , temp.  $T = 14.126 \text{ K}$  .... Ans.

Sensitivity,  $S_0 = 1.2930 \text{ kPa/K}$  .... Ans.

---

**Prob. 5.3.14** Determine the rate of change of pressure indication with change in dead volume temperature,  $dp/dT_0$ , for a constant-volume gas thermometer. Neglect gas imperfections, and assume that the temperature of the gas in the sensing element remains constant. Calculate a numerical value for the change in pressure indication per unit change in dead-volume temperature for  $T_s = 273.16 \text{ K}$  ( $492.69^\circ\text{R}$ ),  $p_s = 120 \text{ kPa}$  ( $17.40 \text{ psia}$ ),  $p = 40 \text{ kPa}$  ( $5.80 \text{ psia}$ ),  $T_0 = 300 \text{ K}$  ( $540^\circ\text{R}$ ), and  $V_0/V = 1$ . [1]

**Solution:**

**First, find out temp of gas (T) in the sensing element:**

**“Data:”**

$$T_s = 273.16[\text{K}]$$

$$p_s = 120[\text{kPa}]$$

$$p = 40 \text{ [kPa]}$$

$$T_0 = 300 \text{ [K]}$$

$$V_0\text{by}V = 1$$

**“We have:**

$$K_1 = (1 + (1 - p/p_s) * V_0\text{by}V * (T_s / T_0))^{-1}$$



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“Then, temp indicated is:

$$T = K_1 * p * (T_s / p_s)$$

“i.e.”

$$T = p * (T_s / p_s) / (1 + (1 - p/p_s) * V_0 \text{by} V * (T_s / T_0))$$

We get:

Unit Settings: [kJ]/[C]/[kPa]/[kg]/[degrees]

$$\begin{array}{lll} p = 40 \text{ [kPa]} & p_s = 120 \text{ [kPa]} & T = 56.660 \text{ [K]} \\ T_0 = 300 \text{ [K]} & V_0 \text{by} V = 1 & T_s = 273.2 \text{ [K]} \end{array}$$

Thus:

**Temp of sensing element,  $T = 56.66 \text{ K}$ ...Ans.**

Now, keep this temp constant, and find the variation of  $p$  with  $T_0$ , in a parametric table.

To find  $dp/dT_0$ , copy this parametric table in to a Lookup Table, and use the EES built-in function DIFFERENTIATE to get  $dT_0/dp$  first, and then get  $dp/dT_0$ :

“Then, we get:”

$$p = - (T * p_s * T_0 + (T * V_0 \text{by} V * T_s * p_s)) / (-T * V_0 \text{by} V * T_s - T_s * T_0)$$

“First, get variation of  $p$  with  $T_0$  in a parametric table, then copy it in to a Lookup Table, and name it as ConstVolGasThermo1, and use the EES built-in function DIFFERENTIATE to get the derivative  $dT_0/dp$ :”

$$dT_0 \text{by} dP = \text{DIFFERENTIATE}(\text{ConstVolGasThermo1}, \text{'Temp'}, \text{'Pressure'}, \text{Temp} = 300)$$

“Now, get  $dp/dT_0$ :”

$$dP \text{by} dT_0 = 1 / dT_0 \text{by} dP$$

We get:

TC_EMFvsT   ConstVolGasThermo1		<input type="button" value="&lt;"/>	<input type="button" value="&gt;"/>
	1 Temp [K]	2 Pressure [kPa]	
Row 1	260	41.91	
Row 2	270	41.39	
Row 3	280	40.9	
Row 4	290	40.44	
Row 5	300	40	
Row 6	310	39.59	
Row 7	320	39.2	
Row 8	330	38.83	
Row 9	340	38.48	
Row 10	350	38.14	

### Results:

Unit Settings: [kJ]/[C]/[kPa]/[kg]/[degrees]

$$dP/dT_0 = -0.04233 \text{ [kPa/K]}$$

$$dT_0/dP = -23.62 \text{ [K/kPa]}$$

Thus:

$$dP/dT_0 = -0.04233 \text{ kPa/K...Ans.}$$


---

**Prob. 5.2.15** Determine the rate of change in sensitivity with change in dead-volume ratio,  $dS_0/d(V_0/V)$ , for a constant-volume gas thermometer. Neglect gas imperfections, and assume that the pressure  $p$  and temperature  $T$  are constant. Calculate a numerical value for the change in sensitivity with respect to change in dead volume ratio for  $T_s = 273.16 \text{ K}$  ( $492.69^\circ\text{R}$ ),  $p_s = 120 \text{ kPa}$  (17.40 psia),  $p = 40 \text{ kPa}$  (5.80 psia),  $T_0 = 300 \text{ K}$  ( $540^\circ\text{R}$ ), and  $V_0/V = 1.00$ . [1]

### Solution:

“Data:”

$$T_s = 273.16 \text{ [K]}$$

$$p_s = 120 \text{ [kPa]}$$

$$p = 40 \text{ [kPa]}$$

$$T_0 = 300 \text{ [K]}$$

$$V_{0byV} = 1$$

“Then, dead volume correction is:”

$$K1 = (1 + (1 - p/p_s) * V_{0byV} * (T_s / T_0))^{-1}$$

“Then, temp indicated is:”

$$T = K1 * p * (T_s / p_s)$$

We get:

Unit Settings: [kJ]/[C]/[kPa]/[kg]/[degrees]

$K1 = 0.6223$	$p = 40 \text{ [kPa]}$	$p_s = 120 \text{ [kPa]}$	$T = 56.660 \text{ [K]}$
$T_0 = 300 \text{ [K]}$	$T_s = 273.2 \text{ [K]}$	$V_{0byV} = 1$	

i.e.  $T = 56.66 \text{ K.}$



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“Now, Sensitivity,  $S_0$  is given by:”

$$S_0 = (p_s / T_s) / (K_1 * (1 + T/T_0 * V_0/V))$$

“Substituting for  $K_1$ , we get:”

$$S_0 = (p_s / T_s) / (((1+(1-p/p_s) * V_0/V * (T_s / T_0))^{(-1)}) * (1 + T/T_0 * V_0/V))$$

**And, we get:**

Unit Settings: [kJ]/[C]/[kPa]/[kg]/[degrees]

$$K_1 = 0.6223$$

$$p = 40 \text{ [kPa]}$$

$$p_s = 120 \text{ [kPa]}$$

$$S_0 = 0.594 \text{ [kPa/K]}$$

$$T = 56.660 \text{ [K]}$$

$$T_0 = 300 \text{ [K]}$$

$$T_s = 273.2 \text{ [K]}$$

$$V_0/V = 1$$

i.e. Sensitivity,  $S_0 = 0.594 \text{ kPa/K}$

**To find  $dS_0/d(V_0/V)$ :**

“First, get variation of  $S_0$  with  $V_0/V$  in a parametric table, then copy it in to a Lookup Table, and name it as ConstVolGasThermo2, and use the EES built-in function DIFFERENTIATE to get the derivative  $dV_0/V/dS_0$ :”

$$T_s = 273.16[\text{K}]; p_s = 120[\text{kPa}]; p = 40 \text{ [kPa]}; T_0 = 300 \text{ [K]}; V_0/V = 1; T = 56.66 \text{ "K"}$$

$$S_0 = (p_s / T_s) / (((1+(1-p/p_s) * V_0/V * (T_s / T_0))^{(-1)}) * (1 + T/T_0 * V_0/V))$$

	$V_0 \text{by} V$	$S_0$
Row 1	0.2	0.475
Row 2	0.3	0.491
Row 3	0.4	0.508
Row 4	0.5	0.523
Row 5	0.6	0.538
Row 6	0.7	0.553
Row 7	0.8	0.567
Row 8	0.9	0.581
Row 9	1	0.594
Row 10	1.1	0.607
Row 11	1.2	0.619
Row 12	1.3	0.631
Row 13	1.4	0.643
Row 14	1.5	0.654
Row 15	1.6	0.665
Row 16	1.7	0.676
Row 17	1.8	0.686
Row 18	1.9	0.696
Row 19	2	0.706

Get the derivative from the Lookup Table as follows: (Also, see Prob. 5.3.4 and 5.3.14)

$$dV_0 \text{by} V \text{by} dS_0 = \text{DIFFERENTIATE}(\text{'ConstVolGasThermo2'}, \text{'V0byV'}, \text{'S0'}, S_0 = 0.594)$$

“Now, get  $dS_0/dV_0 \text{by} V$ :”

$$dS_0 \text{by} dV_0 \text{by} V = 1 / dV_0 \text{by} V \text{by} dS_0$$

### Results:

Unit Settings: [kJ]/[C]/[kPa]/[kg]/[degrees]

$$dS_0 \text{by} dV_0 \text{by} V = 0.132$$

$$p_s = 120 \text{ [kPa]}$$

$$T_0 = 300 \text{ [K]}$$

$$dV_0 \text{by} V \text{by} dS_0 = 7.578$$

$$S_0 = 0.594 \text{ [kPa/K]}$$

$$T_s = 273.2 \text{ [K]}$$

$$p = 40 \text{ [kPa]}$$

$$T = 56.660 \text{ [K]}$$

$$V_0 \text{by} V = 1$$

Thus:

$$d(S_0) / d(V_0 \text{by} V) = 0.132 \dots \text{Ans.}$$

**Prob. 5.3.16** Determine the temp indication for a liquid hydrogen vapor pressure thermometer when the pressure reading is 14 kPa. [1]

**“Solution:”**

**“Normal boiling point for LH<sub>2</sub> at 1 atm = T<sub>0</sub> = 36.7 R = 20.3889 K”**

$$T_0 = 20.3889 \text{ "K"}$$

$$p_0 = 101.325 \text{ "kPa"}$$

$$p = 14 \text{ "kPa"}$$

**“Vapor pressure-temp relation for hydrogen is:”**

$$\ln(p / p_0) = C1 - C2/T - C3 * \ln(T/T_0) - C4 * T + C5 * T^2 \text{ ..finds } T \text{ (K)}$$

“where”



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C1 = 3.940796

C2 = 101.33783

C3 = 0

C4 = -0.0543201

C5 = -1.10563E-04

**Results:**

Unit Settings: [kJ]/[C]/[kPa]/[kg]/[degrees]

C1 = 3.941      C2 = 101.3      C3 = 0      C4 = -0.05432

C5 = -0.0001106      p = 14 [kPa]       $p_0 = 101.3$  [kPa]      T = 15.092 [K]

$T_0 = 20.39$  [K]

**Thus:**

**Temp corresponding to a vapor pressure of 14 kPa = 15.092 K...Ans.**

---

**Prob. 5.3.17** Instead of using the Temp-vapor pressure equations, it is more convenient to use a Temp-pressure table for a Cryogenic fluid, if it is available.

Build such a vapor pressure tables with the data from NIST web site, for Helium, Hydrogen, Oxygen, Nitrogen, and Neon, and write an EES Procedure to find the Temp (K) and Sensitivity ( $= dP/dT$ , bar/K) when the pressure (bar) is input.

(b) Determine the temp indication for a liquid neon vapor pressure thermometer when the pressure reading is 225 kPa (=2.25 bar). Also, determine the sensitivity at this point.

**“Solution:”**

**First, get the Temp-pressure data from the NIST web site:**

**Ref:** <http://webbook.nist.gov/chemistry/fluid/>

For Nitrogen:		For Oxygen:		For Hydrogen:	
Temp.(K)	Pressure (bar)	Tempera ture (K)	Pressure (bar)	Tempera ture (K)	Pressure (bar)
64	0.14602	55	0.0017857	14	0.075414
66	0.20623	57	0.0032314	14.5	0.099458
68	0.28481	59	0.0055977	15	0.12898
70	0.38545	61	0.0093243	15.5	0.16475
72	0.51213	63	0.014993	16	0.20755
74	0.66914	65	0.023349	16.5	0.25821
76	0.86102	67	0.035326	17	0.31759
78	1.0926	69	0.052059	17.5	0.38656
80	1.3687	71	0.074901	18	0.46602
82	1.6947	73	0.10543	18.5	0.55688
84	2.0757	75	0.14547	19	0.66006
86	2.5174	77	0.19708	19.5	0.77651
88	3.0251	79	0.26253	20	0.90717
90	3.6046	81	0.34436	20.5	1.053
92	4.2616	83	0.4453	21	1.215
94	5.002	85	0.56831	21.5	1.3941
96	5.8316	87	0.71653	22	1.5913
98	6.7565	89	0.89331	22.5	1.8075
100	7.7827	91	1.1022	23	2.0438
102	8.9166	93	1.3467	23.5	2.3013
104	10.164	95	1.6308	24	2.5807
106	11.533	97	1.9584	24.5	2.8833
108	13.028	99	2.3334	25	3.21
110	14.658	101	2.7601	25.5	3.5618
112	16.43	103	3.2426	26	3.9399
114	18.351	105	3.7853	26.5	4.3452
116	20.431	107	4.3925	27	4.7789
118	22.678	109	5.0687	27.5	5.2421
120	25.106	111	5.8183	28	5.7359
122	27.727	113	6.6458	28.5	6.2616
124	30.562	115	7.5559	29	6.8205
126	33.645	117	8.5532	29.5	7.4139
		119	9.6423	30	8.0432
		121	10.828	30.5	8.7102
		123	12.115	31	9.4165
		125	13.509	31.5	10.164
		127	15.014	32	10.957
		129	16.635	32.5	11.797
		131	18.378	33	12.693

	133	20.248
	135	22.25
	137	24.392
	139	26.678
	141	29.116
	143	31.713
	145	34.477
	147	37.418
	149	40.547
	151	43.878
	153	47.434

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For Helium:		For Neon:	
Temperatur e (K)	Pressure (bar)	Tempera ture (K)	Pressure (bar)
2.18	0.04896	25	0.5092
2.23	0.055406	26	0.71611
2.28	0.06238	27	0.98173
2.33	0.069907	28	1.3159
2.38	0.078015	29	1.7287
2.43	0.086732	30	2.2307
2.48	0.096083	31	2.8324
2.53	0.1061	32	3.5446
2.58	0.1168	33	4.3782
2.63	0.12821	34	5.3443
2.68	0.14036	35	6.4543
2.73	0.15328	36	7.7202
2.78	0.16699	37	9.1543
2.83	0.18151	38	10.77
2.88	0.19687	39	12.581
2.93	0.21309	40	14.603
2.98	0.2302	41	16.855
3.03	0.24822	42	19.355
3.08	0.26717	43	22.121
3.13	0.28708	44	25.168
3.18	0.30797		
3.23	0.32986		
3.28	0.35278		
3.33	0.37674		
3.38	0.40177		
3.43	0.4279		
3.48	0.45514		
3.53	0.48351		
3.58	0.51304		
3.63	0.54376		
3.68	0.57568		
3.73	0.60882		
3.78	0.64322		

3.83	0.67888
3.88	0.71584
3.93	0.75412
3.98	0.79375
4.03	0.83474
4.08	0.87713
4.13	0.92093
4.18	0.96618
4.23	1.0129
4.28	1.0611
4.33	1.1109
4.38	1.1622
4.43	1.2151
4.48	1.2697
4.53	1.3259
4.58	1.3838
4.63	1.4435
4.68	1.505
4.73	1.5683
4.78	1.6335
4.83	1.7008
4.88	1.77
4.93	1.8415
4.98	1.9152
5.03	1.9913
5.08	2.07
5.13	2.1517
5.18	2.2368

Now, copy these tables as Lookup Tables in EES and name them, respectively, as:

‘N2-Vapor-pressure’, ‘O2-Vapor-pressure’, ‘H2-Vapor-pressure’, ‘He4-Vapor-pressure’, and  
‘Neon-Vapor-pressure’.

Now, write the EES Procedure:

\$UnitSystem SI kPa K kJ deg

“Write an EES Procedure to find the temp and sensitivity of a vapor pressure thermometer when pressure is given, for different cryogenic fluids”

PROCEDURE Cryog\_Vapor\_Pressure\_Thermo(Fluid\$, p : T, S\_0)

“Gives Temp (K) and Sensitivity ( $S_0 = dP/dT$ ) when Pressure (bar) is given, by interpolation from the Lookup Tables”

IF (Fluid\$ = ‘Hydrogen’) THEN

IF ((p < 0.075414) OR (p > 12.693)) THEN

CALL ERROR(‘Pressure should be between 0.075414 bar and 12.693 bar !!’)

ENDIF

T:=INTERPOLATE1(H2-Vapor-pressure,Temp,Pressure,Pressure=p)

“Format of the built-in function is: DIFFERENTIATE(‘Table Name’,‘ColName1’,‘ColName2’,ColName2=Value)

EES will return an estimate of the derivative  $d(\text{ColName1})/d(\text{ColName2})$  at a point fixed by the specified value of either ColName1 or ColName2.”



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```
dTbydP = DIFFERENTIATE('H2-Vapor-pressure', 'Temp', 'Pressure', Pressure= p) "K/bar"
S_0 :=1/dTbydP "bar/K"
ENDIF

IF (Fluid$ = 'Neon') THEN
    IF ((p < 0.5092) OR (p > 25.168)) THEN
        CALL ERROR('Pressure should be between 0.5092 bar and 25.168 bar !!')
    ENDIF
    T:=INTERPOLATE1(Neon-Vapor-pressure,Temp,Pressure,Pressure=p)
    dTbydP = DIFFERENTIATE('Neon-Vapor-pressure', 'Temp', 'Pressure', Pressure= p) "K/bar"
    S_0 :=1/dTbydP "bar/K"
ENDIF

IF (Fluid$ = 'Helium') THEN
    IF ((p < 0.04896) OR (p > 2.2368)) THEN
        CALL ERROR('Pressure should be between 0.04896 bar and 2.2368 bar !!')
    ENDIF
    T:=INTERPOLATE1(He4-Vapor-pressure,Temp,Pressure,Pressure=p)
    dTbydP = DIFFERENTIATE('He4-Vapor-pressure', 'Temp', 'Pressure', Pressure= p) "K/bar"
    S_0 :=1/dTbydP "bar/K"
ENDIF

IF (Fluid$ = 'Oxygen') THEN
```

IF ((p < 0.0017857) OR (p > 47.434)) THEN

CALL ERROR('Pressure should be between 0.0017857 bar and 47.434 bar !!')

ENDIF

T:=INTERPOLATE1(O2-Vapor-pressure,Temp,Pressure,Pressure=p)

dTbydP = DIFFERENTIATE('O2-Vapor-pressure', 'Temp', 'Pressure', Pressure= p) "K/bar"

S\_0 :=1/dTbydP "bar/K"

ENDIF

IF (Fluid\$ = 'Nitrogen') THEN

IF ((p < 0.14602) OR (p > 33.645)) THEN

CALL ERROR('Pressure should be between 0.14602 bar and 33.645 bar !!')

ENDIF

T:=INTERPOLATE1(N2-Vapor-pressure,Temp,Pressure,Pressure=p)

dTbydP = DIFFERENTIATE('N2-Vapor-pressure', 'Temp', 'Pressure', Pressure= p) "K/bar"

S\_0 :=1/dTbydP "bar/K"

ENDIF

END

"===== "

**(b) Now, solve the problem:**

Fluid\$ = 'Neon'

p = 2.25 "bar"

CALL Cryog\_Vapor\_Pressure\_Thermo(Fluid\$, p : T, S\_0)

**Results:**

**Unit Settings: SI K kPa kJ mass deg**

Fluid\$ = 'Neon'

p = 2.25 [bar]

S<sub>0</sub> = 0.5515 [bar/K]

T = 30.032 [K]

Thus:

Temp corresponding to a vapor pressure of 2.25 bar = 30.032 K...Ans.

Sensitivity = S\_0 = 0.5515 bar/K...Ans.

---

**Prob. 5.3.18** Chromium potassium alum is used as a magnetic thermometer. The Curie constant for this material is  $45.8 \times 10^{-6}$  K-m<sup>3</sup>/kg. Determine the absolute temperature indication of the thermometer when the measured magnetic susceptibility is  $1.167 \times 10^{-4}$  m<sup>3</sup>/kg. At this point, determine the sensitivity of the thermometer, where  $S_0 = dX/dT$ . [1]



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**Solution:**

**“Data:”**

$C = 45.8E-06 \text{ [K-m}^3/\text{kg]}$  “...Curie const.”

$X = 1.167E-04 \text{ [m}^3/\text{kg}]$  “...Magn. susceptibility of Chromium potassium alum”

**“Now,  $T_{\text{star}}$  is given by Curie law:”**

$$T_{\text{star}} = C/X$$

**“For Chromium potassium alum, we have the relation between absolute temp and magnetic temp”**

$$T_{\text{star}} = T + 0.000862 + 0.002057/T \text{ “...eqn. A...finds abs. temp. } T \text{ “}$$

**“Sensitivity:  $S_0 = dX/dT$ ”**

**“We have:”**

$$X = C/T_{\text{star}} \text{ i.e.}$$

$$X = C / (T + 0.000862 + 0.002057/T) \text{ ”}$$

**“Then, we get :  $S_0 = dX/dT$ : .... Also, see Prob. 5.2.18 to get expression for  $S_0$ ”**

$$S_0 = -C * (1 - 0.002057 / T^2) / (T + 8.62E-04 + 0.002057/T)^2$$

**Results:**

Unit Settings: [kJ]/[C]/[kPa]/[kg]/[degrees]

$$C = 0.0000458 \text{ [K-m}^3/\text{kg}]$$

$$S_0 = -0.0002933 \text{ [m}^3/\text{kg-K}]$$

$$T = 0.386 \text{ [K]}$$

$$T_{\text{star}} = 0.3925 \text{ [K]}$$

$$X = 0.0001167 \text{ [m}^3/\text{kg}]$$

**Thus:**

**Abs. temp =  $T = 0.386 \text{ K}$ ...Ans.**

**Sensitivity at that temp =  $S_0 = -0.0002933 \text{ m}^3/\text{kg-K}$ ...Ans.**

**Prob. 5.319** A cerium magnesium nitrate magnetic thermometer (Curie constant =  $5.22 \times 10^{-6}$  K-m<sup>3</sup>/kg) indicates a magnetic susceptibility of  $1.044 \times 10^{-4}$  m<sup>3</sup>/kg. Determine the temperature indication of the thermometer.[1]

**Solution:**

**“Data:”**

$$C = 5.22E-06 \text{ [K-m}^3\text{/kg]} \text{ “...Curie const.”}$$

$$X = 1.044E-04 \text{ [m}^3\text{/kg]} \text{ “...Magn. susceptibility of CMN”}$$

**“Now, T\_star is given by Curie law:”**

$$T_{\text{star}} = C/X$$

**“For Chromium magnesium nitrate (CMN), we have the relation between absolute temp and magnetic temp:”**

$$T_{\text{star}} = T + (0.236 + 0.004137/T) * 10^{-3} \text{ “...eqn. B .... finds abs. temp. T”}$$

**“Sensitivity: S\_0 = dX/dT”**

**“We have:”**

$$X = C/T_{\text{star}} \text{ i.e.}$$

$$X = C / (T + (0.236 + 0.004137/T) * 10^{-3})$$

**“Then, we get : S\_0 = dX/dT:”**

$$S_0 = -C * (1 - 0.000004137/T^2) / (T + 0.000236 + 4.137E-06/T)^2$$

**Results:**

Unit Settings: [kJ]/[C]/[kPa]/[kg]/[degrees]

$$C = 0.00000522 \text{ [K-m}^3\text{/kg]}$$

$$S_0 = -0.0020845 \text{ [m}^3\text{/kg-K]}$$

$$T = 0.049681 \text{ [K]}$$

$$T_{\text{star}} = 0.05000 \text{ [K]}$$

$$X = 0.0001044 \text{ [m}^3\text{/kg]}$$

**Thus:**

**Abs. temp = T = 0.049681 K...Ans.**

**Sensitivity at that temp = S\_0 = -0.0020845 m<sup>3</sup>/kg-K...Ans.**

---

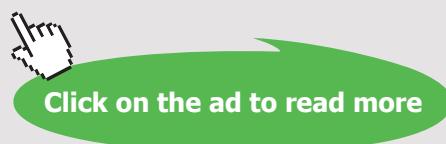
**Prob. 5.3.20** Write an EES Function to determine the coeff. of discharge, Cd for an orifice meter with 'D and D/2 pressure taps', using the ISO-ASME expression for Cd.

Then, write an EES PROCEDURE to determine the mass flow rate and volume flow rate for such an orifice meter when the pipe dia, orifice dia, fluid density, viscosity and pressure drop are given.

**Solution:**

\$UnitSystem SI kJ K kPa deg

FUNCTION Cd\_orifice(beta, Re)



{ Finds the Cd of orifice when beta( = dia of orifice / dia of pipe =  $D_o/D_p$ ) and pipe-Reynolds no. Re are known.}

$$Cd_{orifice} := 0.5959 + 0.0312 * \beta^{2.1} - 0.184 * \beta^8 + 0.039 * \beta^4 / (1 - \beta^4) - 0.01584 * \beta^3 + 91.71 * \beta^{2.5} * Re^{-0.75}$$

END

“=====”

“PROCEDURE to determine the Cd, mass flow rate, volume flow rate...etc. of an orifice meter:”

PROCEDURE Orifice\_meter(Fluid\$, D\_p, D\_0, T\_s, DELTAP : beta, Re, C\_a, C\_d, rho, mu, m\_dot, V\_dot)

{

Inputs:

Fluid\$ = Fluid flowing in the pipe, D\_p = pipe dia (m), D\_0 = orifice dia (m), T\_s = sat. temp of fluid (K), DELTAP = pressure drop (Pa)

Outputs:

$\beta = D_0/D_p$ ,  $C_a$  = Vel. of approach factor,  $C_d$  = coeff of discharge,  $Re$  = Reynolds No.,  $m_{dot}$  = mass flow rate (kg/s),  $V_{dot}$  = vol. flow rate ( $m^3/s$ )

}

$$\beta := D_0/D_p$$

$$C_a := (1 - \beta^4)^{-1/2}$$

$$A_0 := \pi * D_0^2 / 4$$

$$A_p := \pi * D_p^2 / 4$$

$$Cd_{guess} := 0.6$$

P\_s := P\_SAT(Fluid\$, T=T\_s)"kPa ... sat. pressure for T\_s (K)"

rho := DENSITY(Fluid\$, T=T\_s, P=P\_s \* 1.001) "kg/m^3...multiplied to make sure that the pressure is 0.1% higher than the saturation pressure to ensure that liquid properties are returned"

mu := VISCOSITY(Fluid\$, T=T\_s, P=P\_s \* 1.001)" Pa-s...multiplied to make sure that the pressure is 0.1% higher than the saturation pressure to ensure that liquid properties are returned"

mdot1 := Cd\_guess \* C\_a \* A\_0 \* (2 \* rho \* DELTAP)^(1/2)

Re1 := (mdot1 / A\_p) \* (D\_p / mu) "Reynolds No."

{C\_d := 0.5959 + 0.0312 \* beta^2.1 - 0.184 \* beta^8 + 0.039 \* beta^4 / (1 - beta^4) - 0.01584 \* beta^3 + 91.71 \* beta^2.5 \* Re1^(-0.75)}

C\_d := Cd\_orifice(beta, Re1)

IF (ABS(C\_d - Cd\_guess) > 0.001) THEN

Cd\_guess := C\_d

m\_dot := Cd\_guess \* C\_a \* A\_0 \* (2 \* rho \* DELTAP)^(1/2)

Re := (m\_dot / A\_p) \* (D\_p / mu)

{C\_d := 0.5959 + 0.0312 \* beta^2.1 - 0.184 \* beta^8 + 0.039 \* beta^4 / (1 - beta^4) - 0.01584 \* beta^3 + 91.71 \* beta^2.5 \* Re^(-0.75)}

C\_d := Cd\_orifice(beta, Re)

ELSE

C\_d := Cd\_guess

Re := Re1

m\_dot := m\_dot1

ENDIF

V\_dot := m\_dot / rho

END

“=====”

**Prob. 5.3.21** An Orifice meter is used to measure the flow of LN2 through a tube of ID = 102 mm. Dia of orifice is 51 mm. And, measured pressure drop = 0.150 kPa. Temp of LN2 is 85 K. Determine the mass flow rate and volume flow rate of LN2.[1]



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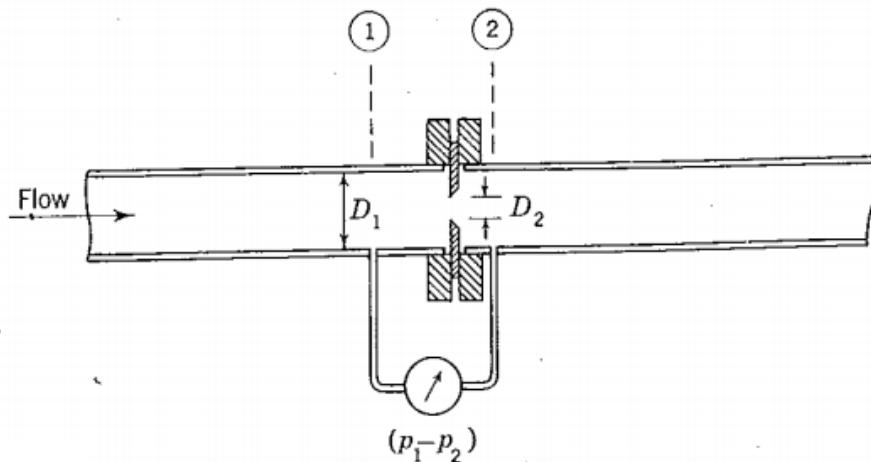


Fig. Orifice flowmeter.

**“Data:”**

$$T_s = 85 \text{ "K"}$$

Fluid\$ = 'Nitrogen'

$$D_p = 0.102 \text{ "m ... dia of pipe"}$$

$$D_0 = 0.051 \text{ "m ... dia of orifice"}$$

$$\Delta P = 150 \text{ "Pa"}$$

**“Now, Call the EES Procedure written above:”**

```
CALL Orifice_meter(Fluid$, D_p, D_0, T_s, DELTAP : beta, Re, C_a, C_d, rho, mu,
m_dot, V_dot)
```

**Results:**

Unit Settings: [kJ]/[K]/[kPa]/[kg]/[degrees]

$$\beta = 0.5$$

$$\Delta P = 150 \text{ [Pa]}$$

Fluid\$ = 'Nitrogen'

$$Re = 60911$$

$$V = 0.0007997 \text{ [m}^3/\text{s] }$$

$$C_a = 1.0328$$

$$D_0 = 0.051 \text{ [m]}$$

$$\mu = 0.0001262 \text{ [Pa-s]}$$

$$\rho = 770 \text{ [kg/m}^3\text{]}$$

$$C_d = 0.6073$$

$$D_p = 0.102 \text{ [m]}$$

$$m = 0.6158 \text{ [kg/s]}$$

$$T_s = 85 \text{ [K]}$$

**Thus:**

**Cd = 0.6073...Ans.**

**Mass flow rate = m\_dot = 0.6158 kg/s...Ans.**

**Volume flow rate = V\_dot = 0.0007997 m^3/s...Ans.**

---

**Prob. 5.3.22** An orifice meter is used to measure the flow of liquid oxygen through a 50 mm (1.969-in.) diameter tube. The orifice diameter is 25 mm (0.984 in.), and the measured pressure drop across the orifice (with D and D/2 pressure taps) is 250 Pa (0.0363 psi). If the temperature of the liquid oxygen is 100 K (180°R), determine the volumetric flow rate and the mass flow rate of the liquid oxygen.[1]

Also, plot the liquid flow rate as the pressure drop,  $\Delta P$  varies from 50 Pa to 400 Pa:

**Solution:**

**“Data:”**

T\_s = 100 “K”

Fluid\$ = ‘Oxygen’

D\_p = 0.050 “m”

D\_0 = 0.025 “m”

{DELTAP = 250 “Pa”}

CALL Orifice\_meter(Fluid\$, D\_p, D\_0, T\_s, DELTAP : beta, Re, C\_a, C\_d, rho, mu, m\_dot, V\_dot)

**Results:**

**Unit Settings: SI K kPa kJ mass deg**

$$\beta = 0.5$$

$$C_a = 1.0328$$

$$C_d = 0.60904$$

$$\Delta P = 250 \text{ [Pa]}$$

$$D_0 = 0.025 \text{ [m]}$$

$$D_p = 0.05 \text{ [m]}$$

Fluid\$ = 'Oxygen'

$$\mu = 0.000153 \text{ [Pa-s]}$$

$$\dot{m} = 0.22813 \text{ [kg/s]}$$

$$Re = 37,978$$

$$\rho = 1.092 \text{ [kg/m}^3\text{]}$$

$$T_s = 100 \text{ [K]}$$

$$\dot{V} = 0.000209 \text{ [m}^3/\text{s}]$$

**Thus:**

**Cd = 0.60904...Ans.**

**Mass flow rate = m\_dot = 0.22813 kg/s...Ans.**

**Volume flow rate = V\_dot = 0.000209 m^3/s...Ans.**

**Also, plot the liquid flow rate as the pressure drop, DP varies from 50 Pa to 400 Pa:**

**First, compute the Parametric Table:**



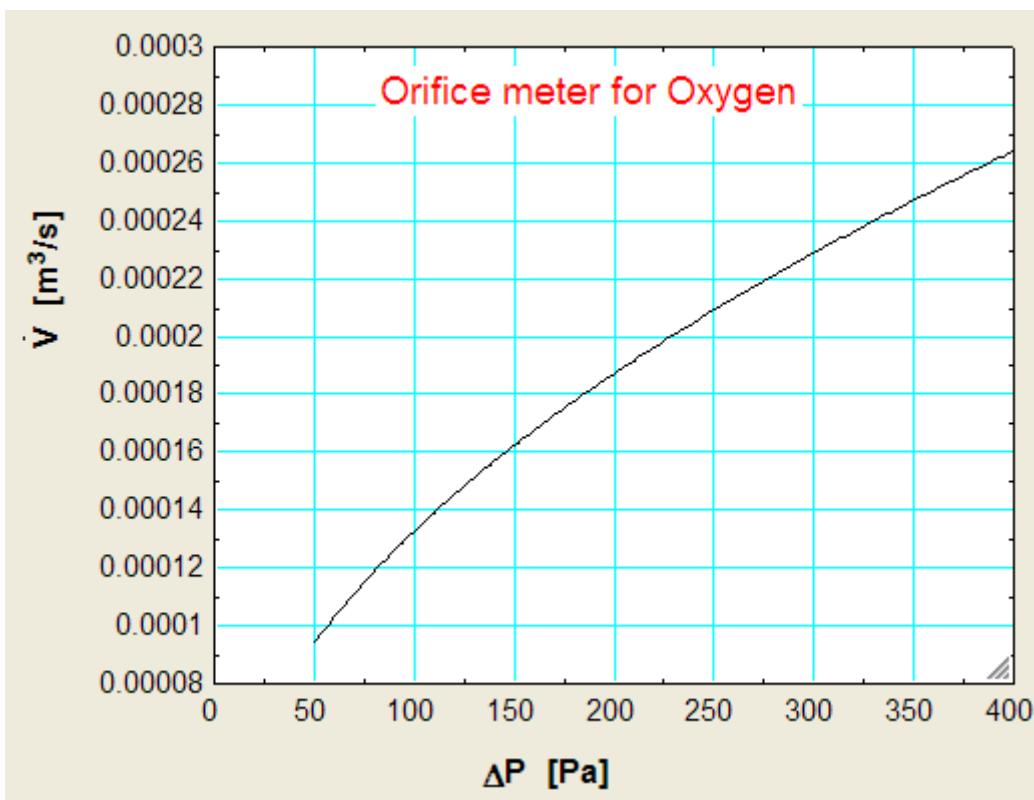
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 1..8	1 $\Delta P$ [Pa]	2 $C_d$	3 $Re$	4 $m$ [kg/s]	5 $\dot{V}$ [ $m^3/s$ ]
Run 1	50	0.61391	17,124	0.10286	0.000094
Run 2	100	0.61146	24,117	0.14487	0.000133
Run 3	150	0.61029	29,479	0.17708	0.000162
Run 4	200	0.60955	33,998	0.20422	0.000187
Run 5	250	0.60904	37,979	0.22813	0.000209
Run 6	300	0.60865	41,576	0.24974	0.000229
Run 7	350	0.60834	44,884	0.26961	0.000247
Run 8	400	0.60808	47,963	0.28810	0.000264

Now, plot:



**Prob. 5.3.23** Write an EES Procedure to determine: Reynolds No. ( $N_{Re}$ ), the coefficients  $C_d$  and  $C_a$ , mass flow rate ( $m_{dot}$ , kg/s) and volume flow rate ( $V_{dot}$ ,  $m^3/s$ ) for a Venturi flow meter when the pipe dia, throat dia, fluid density, viscosity and pressure drop are given.

“PROCEDURE to determine the Cd, mass flow rate, volume flow rate... etc. of a Venturi meter:”

PROCEDURE Venturi\_meter(Fluid\$, D\_pipe, D\_throat, T\_s, DELTAP : beta, Re, C\_a, C\_d, rho, mu, m\_dot, V\_dot)

{

Inputs:

Fluid\$ = Fluid flowing in the pipe, D\_pipe = pipe dia (m), D\_throat = throat dia (m), T\_s = sat. temp of fluid (K), DELTAP = pressure drop (Pa)

Outputs:

beta = D\_throat/D\_pipe, C\_a = Vel. of approach factor, C\_d = coeff of discharge, Re = Reynolds No., m\_dot = mass flow rate (kg/s), V\_dot = vol. flow rate (m^3/s)  
}

beta:= D\_throat/D\_pipe

C\_a := (1 - beta^4)^(-1/2)

A\_0 := pi \* D\_throat^2 / 4

P\_s := P\_SAT(Fluid\$, T=T\_s) "kPa ... sat. pressure for T\_s (K)"

rho := DENSITY(Fluid\$, T=T\_s, P=P\_s \* 1.001) "kg/m^3 ... P\_s is multiplied by 1.001 to make sure that the pressure is 0.1% higher than the saturation pressure to ensure that liquid properties are returned"

mu := VISCOSITY(Fluid\$, T=T\_s, P=P\_s \* 1.001) "Pa-s ... P\_s is multiplied by 1.001 to make sure that the pressure is 0.1% higher than the saturation pressure to ensure that liquid properties are returned"

Re := D\_pipe \* (beta^2 / mu) \* (2 \* rho \* DELTAP)^(1/2)

IF ((Re >= 3000) AND (Re <= 2 \* 10^5)) THEN

C\_d := log10(Re) / (0.6 + 0.9 \* log10(Re))

ENDIF

IF ( $Re > 2 * 10^5$ ) THEN

$C_d := 0.988$

ENDIF

$m_{dot} := C_d * C_a * A_0 * (2 * \rho * \Delta P)^{1/2}$

$V_{dot} := m_{dot} / \rho$

END

“=====”



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**Prob. 5.3.24** Liquid hydrogen at 22 K (39.6°R) flows through a pipe having an inside diameter of 42.8 mm (1.685 in.). A venturi meter having a throat diameter of 21.4 mm (0.843 in.) is placed in the line to measure the flow rate of the fluid. Determine the volumetric flow rate and mass flow rate of the liquid hydrogen if the pressure difference for the meter is 1.35 kPa (0.196 psi).[1]

Plot the liquid flow rate as the pressure drop,  $\Delta P$  varies from 100 Pa to 1500 Pa.

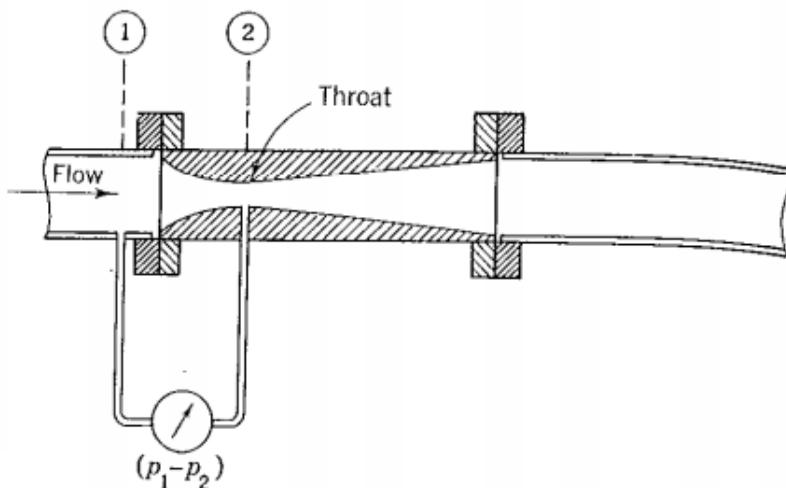


Fig. Venturi flowmeter.

**Solution:**

**“Data:”**

$$T_s = 22 \text{ "K"}$$

$$\text{Fluid\$} = \text{'Hydrogen'}$$

$$D_{\text{pipe}} = 0.0428 \text{ "m"}$$

$$D_{\text{throat}} = 0.0214 \text{ "m"}$$

$$\Delta P = 1350 \text{ "Pa"}$$

**Now, call the EES Procedure written above:**

```
CALL Venturi_meter(Fluid$, D_pipe, D_throat, T_s, DELTAP : beta, Re, C_a, C_d, rho,
mu, m_dot, V_dot)
```

### Results:

**Unit Settings: SI K kPa kJ mass deg**

$$\beta = 0.5$$

$$C_a = 1.0328$$

$$C_d = 0.98800$$

$$\Delta P = 1,350 \text{ [Pa]}$$

$$D_{\text{pipe}} = 0.0428$$

$$D_{\text{throat}} = 0.0214$$

$$\text{Fluid\$} = \text{'Hydrogen'}$$

$$\mu = 0.00001102 \text{ [Pa-s]}$$

$$\dot{m} = 0.15830 \text{ [kg/s]}$$

$$Re = 418,702$$

$$\rho = 68.9 \text{ [kg/m}^3]$$

$$T_s = 22 \text{ [K]}$$

$$\dot{V} = 0.002298 \text{ [m}^3/\text{s}]$$

Thus:

**Cd = 0.988...Ans.**

**Mass flow rate = m\_dot = 0.15830 kg/s...Ans.**

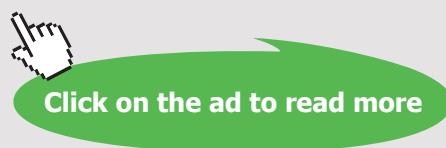
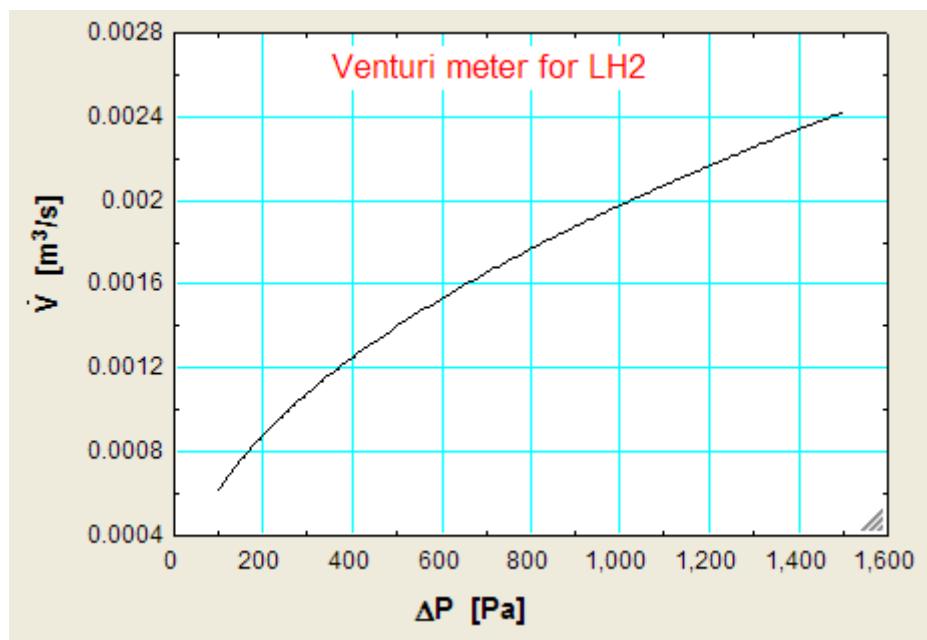
**Volume flow rate = V\_dot = 0.002298 m^3/s...Ans.**

**Also, plot the liquid flow rate as the pressure drop, DP varies from 100 Pa to 1500 Pa:**

**First, compute the Parametric Table:**

1..15	1 ΔP [Pa]	2 Re	3 C <sub>d</sub>	4 m [kg/s]	5 V [m <sup>3</sup> /s]
Run 1	100	113,956	0.98169	0.04281	0.000621
Run 2	200	161,158	0.98500	0.06074	0.000882
Run 3	300	197,378	0.98687	0.07454	0.001082
Run 4	400	227,912	0.98800	0.08617	0.001251
Run 5	500	254,814	0.98800	0.09634	0.001398
Run 6	600	279,135	0.98800	0.10553	0.001532
Run 7	700	301,500	0.98800	0.11399	0.001654
Run 8	800	322,317	0.98800	0.12186	0.001769
Run 9	900	341,869	0.98800	0.12925	0.001876
Run 10	1,000	360,361	0.98800	0.13624	0.001977
Run 11	1,100	377,950	0.98800	0.14289	0.002074
Run 12	1,200	394,756	0.98800	0.14924	0.002166
Run 13	1,300	410,875	0.98800	0.15534	0.002255
Run 14	1,400	426,385	0.98800	0.16120	0.002340
Run 15	1,500	441,350	0.98800	0.16686	0.002422

Now, plot:



**Prob. 5.3.25.** A turbine flow meter has six blades with a thickness of 2.5 mm (0.098in.) and a blade angle of 45°. The meter bore diameter is 75 mm (2.953 in.), the rotor blade-tip diameter is 70 mm (2.756 in.), and the rotor hub diameter is 35 mm (1.378 in.). If the rotational speed of the rotor is 1.50 rev./second, determine the volumetric flow rate and mass flow rate for liquid nitrogen flowing at 80 K (144°R).[1]

**Solution:**

**“Data:”**

Fluid\$ = ‘Nitrogen’

t = 0.0025 “m ...blade thickness”

theta\_b = 45 “deg.”

D = 0.075 “m ... meter bore dia”

D\_b = 0.07 “m ... rotor blade tip dia”

D\_h = 0.035 “m ... rotor hub dia”

n = 1.5 “rps ... rotational speed of rotor”

m = 6 “... no. of blades”

T\_s = 80 “K ... sat. temp of LN2 flowing”

**“Calculations:”**

P\_sat = P\_sat(Fluid\$,T=T\_s) “kPa .... sat. pressure”

rho = Density(Fluid\$,T=T\_s,P=P\_sat \* 1.001) “kg/m^3 ... note that P\_sat is increased by 0.1% to ensure that liquid properties are returned”

**“Then, free flow area through the turbine is:”**

A\_ff = (pi / 4) \* (D^2 - D\_h^2) \* ( 1 - m \*( t /pi) \* (D + D\_h) \* cos(theta\_b)) “m^2 ... free flow area”

**“Then, vol. flow rate is given by:”**

$$V_{\text{dot}} = \pi * D_b * A_{\text{ff}} * n / \tan(\theta_b) \text{ "m}^3 / \text{s"}$$

**“And, mass flow rate is:”**

$$m_{\text{dot}} = V_{\text{dot}} * \rho \text{ "kg/s"}$$

**Results:**

**Unit Settings: SI K kPa kJ mass deg**

$A_{\text{ff}} = 0.003454 \text{ [m}^2]$	$D = 0.075 \text{ [m]}$	$D_b = 0.07 \text{ [m]}$
$D_h = 0.035 \text{ [m]}$	Fluid\$ = 'Nitrogen'	$m = 6$
$m = 0.90470 \text{ [kg/s]}$	$n = 1.5 \text{ [rps]}$	$P_{\text{sat}} = 136.9 \text{ [kPa]}$
$\rho = 793.9 \text{ [kg/m}^3]$	$t = 0.0025 \text{ [m]}$	$\theta_b = 45 \text{ [deg]}$
$T_s = 80 \text{ [K]}$	$V_{\text{dot}} = 0.001140 \text{ [m}^3/\text{s}]$	

Thus:

**Vol. flow rate of LN2 =  $V_{\text{dot}} = 0.001140 \text{ m}^3/\text{s}$ ...Ans.**

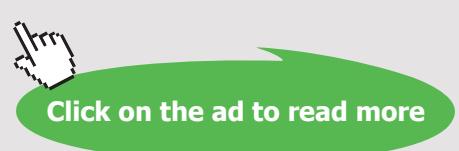
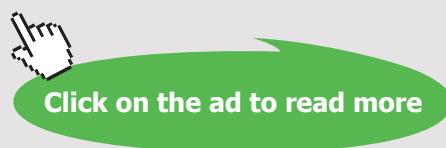
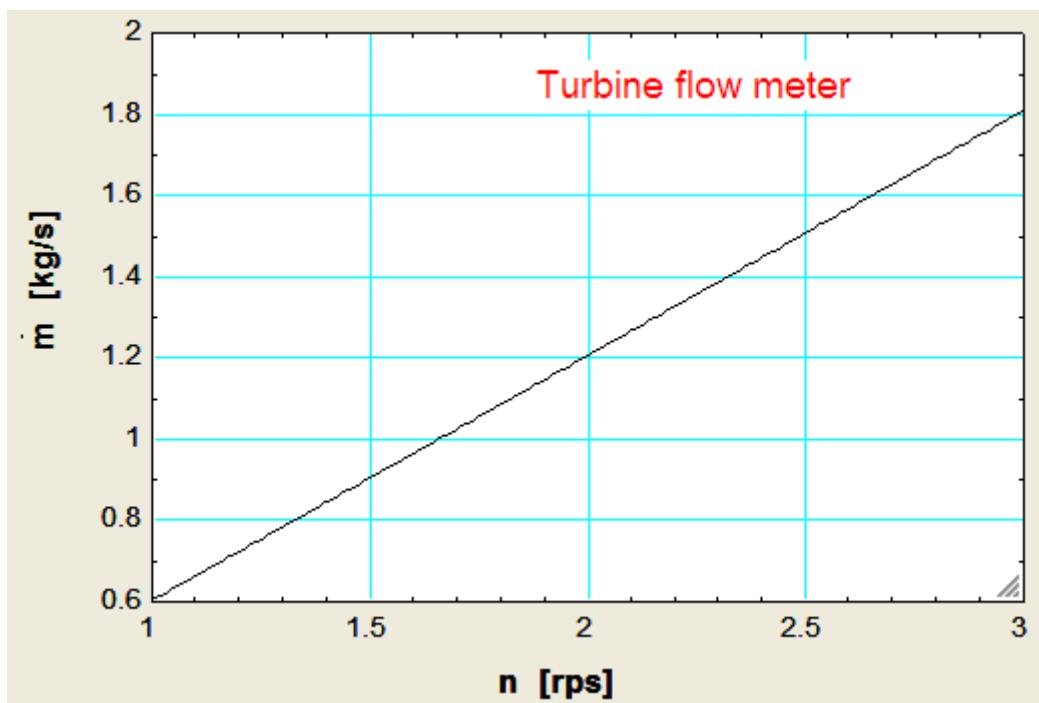
**Mass flow rate of LN2 =  $m_{\text{dot}} = 0.90470 \text{ kg/s}$ ...Ans.**

**Plot  $m_{\text{dot}}$  (kg/s) as rotational speed,  $n$  varies from 1 rev/s to 3 rev/s:**

First, compute the Parametric Table:

1..11	1 n [rps]	2 m [kg/s]
Run 1	1	0.60313
Run 2	1.2	0.72376
Run 3	1.4	0.84439
Run 4	1.6	0.96502
Run 5	1.8	1.08564
Run 6	2	1.20627
Run 7	2.2	1.32690
Run 8	2.4	1.44752
Run 9	2.6	1.56815
Run 10	2.8	1.68878
Run 11	3	1.80940

Now, plot:



**Prob. 5.3.26** Liquid hydrogen flows in a tube at 25 K and 328.8 kPa. A quality probe having a length of 600 mm, an inner rod dia of 38.1 mm and an outer cylinder ID is 66.9 mm. If the capacitance indication of probe is 66 pF, determine the quality of flowing fluid. [1]

**Solution:**

**“Data:”**

Fluid\$ = ‘Hydrogen’

T\_s = 25 “K”

D\_i = 0.0381 “m”

D\_0 = 0.0669 “m”

L = 0.6 “m”

CC = 66E-12 “F... capacitance”

epsilon\_0 = 8.8542E-12 “F/m ... permittivity of free space”

**“Dielectric constants for sat. liq. and sat. vap. hydrogen:”**

**“First, we need densities of sat. liq. and sat. vapor of Hydrogen at 25 K:”**

rho\_f = Density(Fluid\$, T=T\_s, x=0) \* convert (kg/m^3, g/cm^3) “... get density in g/cm3”

rho\_g = Density(Fluid\$, T=T\_s, x=1) \* convert (kg/m^3, g/cm^3) “... get density in g/cm3”

**“Then, for sat. Liq. hydrogen, dielectric const is given by (Ref: Cryog. Engg by Thomas Flynn):”**

$(\text{epsilon}_f - 1) / (\text{epsilon}_f + 2) = 0.99575 * \text{rho}_f - 0.09069 * \text{rho}_f^2 + 1.1227 * \text{rho}_f^3$

**“And, for sat. vapor hydrogen, dielectric const is given by (Ref: Cryog. Engg by Thomas Flynn):”**

$$(\epsilon_g - 1) / (\epsilon_g + 2) = 0.99575 * \rho_g - 0.09069 * \rho_g^2 + 1.1227 * \rho_g^3$$

**“And, dielectric const for two phase mixture ( $\epsilon$ ) is given by:”**

$$\epsilon = CC * \ln(D_0/D_i) / (2 * \pi * L * \epsilon_0)$$

**“Vapor vol. fraction is determined from:”**

$$(1 - y) = (\epsilon_f - \epsilon) / (\epsilon_f - \epsilon_g)$$

**“Then, density of two phase mixture is:”**

$$\rho = \rho_f - (1 - y) * (\rho_f - \rho_g) \text{ “g/cm}^3\text{”}$$

**“And, quality is calculated as follows:”**

$$x = m_g / m = (V_g * \rho_g) / ((V_f + V_g) * \rho) = (1 - y) * (\rho_g / \rho)$$

“i.e.”

$$x = (1 - y) * (\rho_g / \rho) \text{ “...quality of two phase mixture”}$$

### Results:

#### Unit Settings: SI K kPa kJ mass deg

$$CC = 6.600E-11 \text{ [F]}$$

$$D_0 = 0.0669 \text{ [m]}$$

$$D_i = 0.0381 \text{ [m]}$$

$$\epsilon = 1.11317$$

$$\epsilon_0 = 8.854E-12 \text{ [F/m]}$$

$$\epsilon_f = 1.206$$

$$\epsilon_g = 1.012$$

$$\text{Fluid\$} = \text{'Hydrogen'}$$

$$L = 0.6 \text{ [m]}$$

$$\rho = 0.0356 \text{ [g/cm}^3\text{]}$$

$$\rho_f = 0.0647 \text{ [g/cm}^3\text{]}$$

$$\rho_g = 0.003892 \text{ [g/cm}^3\text{]}$$

$$T_s = 25 \text{ [K]}$$

$$x = 0.05233$$

$$y = 0.5214$$

Thus:

**Quality of flowing fluid (i.e. hydrogen) =  $x = 0.05233$ ...Ans.**

**Prob.5.3.27** A two-phase mixture of liquid and vapor argon flows in a tube at 90 K (162°R). A capacitance quality probe having a length of 1.060 m (41.7 in.), an inner diameter of 48 mm (1.890 in.), and an outer diameter of 57 mm (2.244 in.) is used to measure the quality. The capacitance indication of the gauge is 415 pF. What is the quality of the flowing argon? [1]

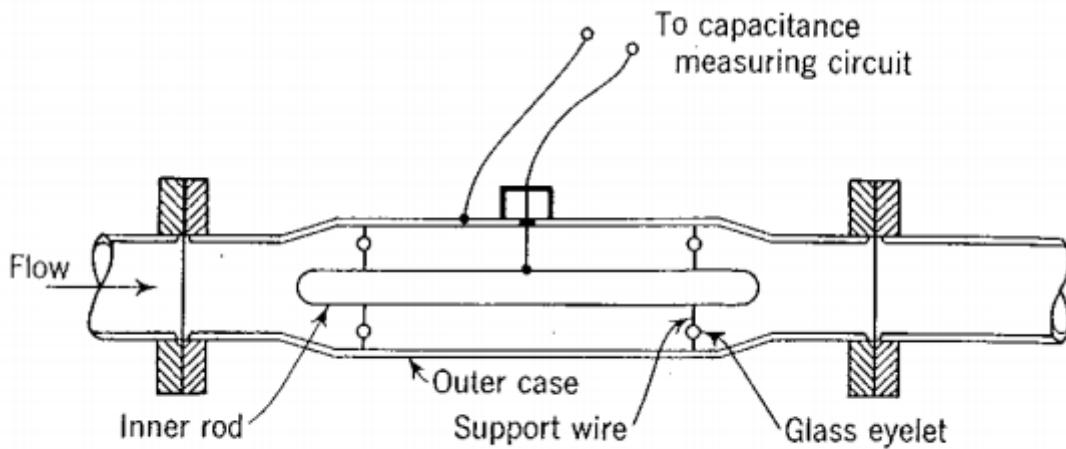


Fig. Capacitance quality meter (Killian and Simpson 1960).

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**Solution:**

**“Data:”**

Fluid\$ = ‘Argon’

T\_s = 90 “K”

D\_i = 0.048 “m”

D\_0 = 0.057 “m”

L = 1.06 “m”

CC = 415E-12 “F... capacitance”

epsilon\_0 = 8.8542E-12 “F/m ... permittivity of free space”

**“Dielectric constants for sat. liq. and sat. vap. Argon:”**

**“First, we need densities of sat. liq. and sat. vapor of Argon at 90 K:”**

rho\_f = Density(Fluid\$,T=T\_s,x=0) \* convert (kg/m^3, g/cm^3) “... get density in g/cm3”

rho\_g = Density(Fluid\$,T=T\_s,x=1)\* convert (kg/m^3, g/cm^3) “... get density in g/cm3”

**“For sat. liq Argon:”**

epsilon\_f = 1.508 “...from Cryog. Engg. by Thomas Flynn”

**“And, for sat. vapor Argon at 90 K, the value of epsilon\_g is a constant, viz.”**

epsilon\_g = 1.000554

**“Then, we calculate the dielectric const for two phase mixture, from the following eqn:”**

epsilon = CC \* ln(D\_0/D\_i) / (2 \* pi\* L \* epsilon\_0)

**“Vapor vol. fraction is determined from:”**

$$(1 - y) = (\epsilon_f - \epsilon) / (\epsilon_f - \epsilon_g)$$

**“Then, density of two phase mixture is:”**

$$\rho = \rho_f - (1 - y) * (\rho_f - \rho_g) \text{ "g/cm}^3\text{"}$$

**“And, quality is calculated as follows:”**

$$x = m_g / m = (V_g * \rho_g) / ((V_f + V_g) * \rho) = (1 - y) * (\rho_g / \rho)$$

“i.e.”

$$x = (1 - y) * (\rho_g / \rho) \text{ "... quality of two phase mixture"}$$

### Results:

#### Unit Settings: SI K kPa kJ mass deg

CC = 4.150E-10 [F]	D <sub>0</sub> = 0.057 [m]	D <sub>j</sub> = 0.048 [m]
<b><math>\epsilon = 1.20938</math></b>	$\epsilon_0 = 8.854E-12 [F/m]$	$\epsilon_f = 1.508$
$\epsilon_g = 1.001$	Fluid\$ = 'Argon'	L = 1.06 [m]
$\rho = 0.5717 \text{ [g/cm}^3\text{]}$	$\rho_f = 1.37868 \text{ [g/cm}^3\text{]}$	$\rho_g = 0.007436 \text{ [g/cm}^3\text{]}$
T <sub>s</sub> = 90 [K]	<b>x = 0.007653</b>	y = 0.4115

Thus:

**Quality of flowing fluid (i.e. Argon) = x = 0.007653...Ans.**

---

**Prob.5.3.28** For an 2.440-m inside-diameter liquid-oxygen storage vessel (horizontal cylindrical vessel), the hydrostatic pressure indication on a hydrostatic liquid-level gauge is 20 kPa (2.90 psi). If the fluid in the vessel is under a pressure of 101.3 kPa (1 atm) determine the liquid level and the sensitivity of the liquid level gauge.[1]

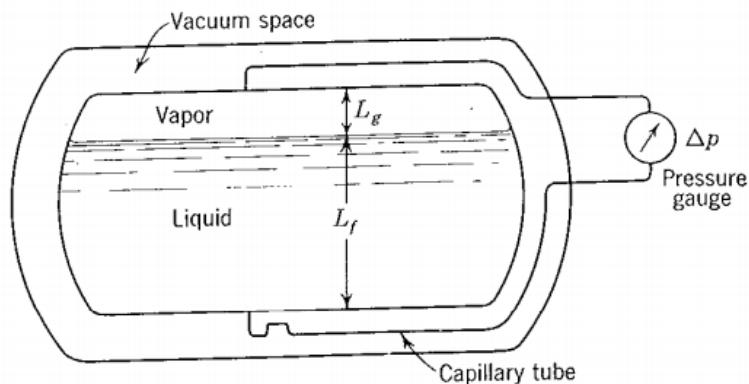


Fig. Hydrostatic liquid-level gauge.

**“Solution:”**

“We have for Hydrostatic level gauges:

$$\text{DELTAP} = \rho_f * L_f * g + \rho_g * L_g * g$$



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**Since for fluids other than hydrogen and helium, vapor density can be neglected compared to the liquid density, we write:**

$$\Delta P = \rho_f * L_f * g$$

**“Data:”**

Fluid\$ = ‘Oxygen’

P\_s = 101.3 “kPa”

DELTA P = 20000 “Pa”

g = 9.81 “m/s^2”

L = 2.44 “m ... dia of tank”

$\rho_f$  = Density(Fluid\$, P=P\_s, x=0) “kg/m^3 .... density of sat. liq.”

$\rho_g$  = Density(Fluid\$, P=P\_s, x=1) “kg/m^3 .... density of sat. vap.”

**“Therefore:”**

$L_f = \Delta P / (\rho_f * g)$  “m ..... height of liquid from bottom of tank”

**“Sensitivity, S\_0, is given by:”**

$$S_0 = d(\Delta P) / d(L_f) = (\rho_f - \rho_g) * g \text{ .... Pa/m}^2$$

$$S_0 = (\rho_f - \rho_g) * g \text{ .... Pa/m}^2$$

**Results:**

**Unit Settings: SI K kPa kJ mass deg**

$$\Delta P = 20,000 \text{ [Pa]}$$

Fluid\$ = ‘Oxygen’

$$g = 9.81 \text{ [m/s}^2]$$

$$L = 2.44 \text{ [m]}$$

$$L_f = 1.785 \text{ [m]}$$

$$P_s = 101.3 \text{ [kPa]}$$

$$\rho_f = 1,142.00192 \text{ [kg/m}^3]$$

$$\rho_g = 4.463 \text{ [kg/m}^3]$$

$$S_0 = 11,159.255 \text{ [Pa/m}^2]$$

Thus:

Liquid level =  $L_f = 1.785 \text{ m}$  from bottom of the horizontal cylindrical tank...Ans.

Sensitivity =  $S_0 = 11159.255 \text{ Pa/m}$ ...Ans.

---

**Prob.5.3.29** For a 1.83 m ID LH2 storage vessel, hydrostatic pressure indication is 0.86 kPa. If the fluid pressure in the ullage space is 101.3 kPa, determine the liquid level. Compare the sensitivity of LH2 level indicator with a hydrostatic gauge for LN2 level indication.[1]

**Solution:**

“We have for Hydrostatic level gauges:

$$\Delta P = \rho_f * L_f * g + \rho_g * L_g * g$$

Then, liquid level is given by:

$$L_f = (\Delta P - \rho_g * L * g) / ((\rho_f - \rho_g) * g)$$

“Data:”

Fluid\$ = ‘Hydrogen’

$P_s = 101.3 \text{ kPa}$

$\Delta P = 860 \text{ Pa}$

$g = 9.81 \text{ m/s}^2$

$L = 1.83 \text{ m ... dia of tank}$

“Find densities of sat. liq. and sat. vapor using the EES built-in Function:”

$\rho_f = \text{Density(Fluid$,P=P_s,x=0)} \text{ kg/m}^3 \text{ .... density of sat. liq.}$

$\rho_g = \text{Density(Fluid$,P=P_s,x=1)} \text{ kg/m}^3 \text{ .... density of sat. vap.}$

**“Therefore:”**

$L_f = (\Delta P - \rho_g * L * g) / ((\rho_f - \rho_g) * g)$  “m ..... height of liquid from bottom of tank”

**“Sensitivity, S<sub>0</sub>, is given by:”**

$S_0 = d(\Delta P) / d(L_f) = (\rho_f - \rho_g) * g$  .... Pa/m “

$S_0 = (\rho_f - \rho_g) * g$  .... Pa/m “

“-----”

**“Sensitivity for LN2:”**

“For LN2 at 101.3 kPa, first, we get densities for sat. liquid and sat.vapor:”

$\rho_f_{N2} = \text{Density}(\text{Nitrogen}, P=P_s, x=0)$  “kg/m<sup>3</sup> .... density of sat. liq.”



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$\rho_{g,N2}$  = Density(Nitrogen,P=P<sub>s</sub>,x=1) "kg/m<sup>3</sup> .... density of sat. vap."

"**Sensitivity, S<sub>0\_N2</sub>, is given by:**"

$S_{0_N2} = (\rho_f - \rho_{g,N2}) * g$  ".... Pa/m "

"-----"

### Results:

**Unit Settings: SI K kPa kJ mass deg**

$\Delta P = 860$  [Pa]

Fluid\$ = 'Hydrogen'

$g = 9.81$  [m/s<sup>2</sup>]

$L = 1.83$  [m]

$L_f = 1.22598$  [m]

$P_s = 101.3$  [kPa]

$\rho_f = 70.8504$  [kg/m<sup>3</sup>]

$\rho_{f,N2} = 806.1$  [kg/m<sup>3</sup>]

$\rho_g = 1.3321$  [kg/m<sup>3</sup>]

$\rho_{g,N2} = 4.611$  [kg/m<sup>3</sup>]

$S_0 = 681.974$  [Pa/m]

$S_{0,N2} = 7,863$  [Pa/m]

Thus:

LH<sub>2</sub> liquid level =  $L_f = 1.22598$  m from bottom of horizontal tank...Ans.

Sensitivity for LH<sub>2</sub> =  $S_0 = 681.974$  Pa/m...Ans.

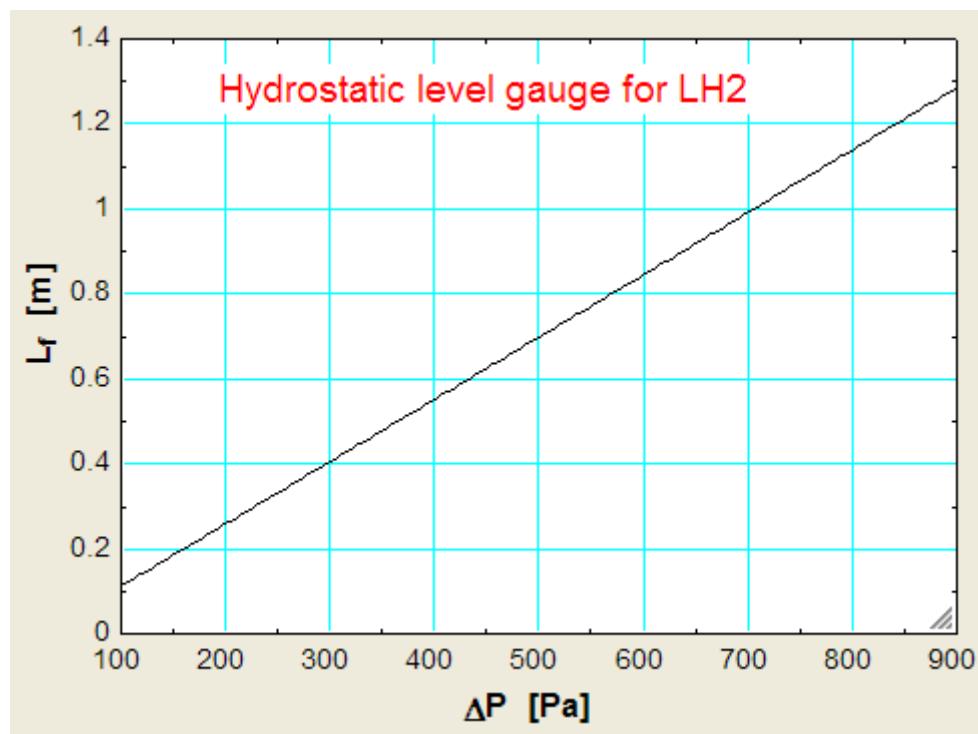
Sensitivity for LN<sub>2</sub> =  $S_{0,N2} = 7863$  Pa/m...Ans.

**Plot for LH<sub>2</sub>, liquid level L<sub>f</sub> against DP, as DP varies from 100 Pa to 900 Pa:**

First, compute the Parametric Table:

1..17	1 $\Delta P$ [Pa]	2 $L_f$ [m]
Run 1	100	0.11157
Run 2	150	0.18488
Run 3	200	0.25820
Run 4	250	0.33152
Run 5	300	0.40483
Run 6	350	0.47815
Run 7	400	0.55147
Run 8	450	0.62478
Run 9	500	0.69810
Run 10	550	0.77142
Run 11	600	0.84473
Run 12	650	0.91805
Run 13	700	0.99137
Run 14	750	1.06468
Run 15	800	1.13800
Run 16	850	1.21132
Run 17	900	1.28463

Now, plot:



**Prob.5.3.30** A liquid-level gauge consists of a vertical wire immersed in liquid nitrogen at 78 K (1404). When the entire wire is immersed in liquid nitrogen, electric resistance of the wire is 300 ohms. The total length of the wire is 2.00 m. Determine the electric resistance of the wire when 1.00 m is immersed in liquid and 1.00 m is exposed to nitrogen vapor at 78 K. Assume that the temperature of the portion of wire in the liquid is at a temperature of 78 K, and the average temperature of the portion of the wire in the vapor is given by:

$$T - T_f = \frac{(T_0 - T_f) \cdot \cosh(M \cdot L)}{\cosh(M \cdot L) - 1} \cdot \left( 1 - \frac{\tanh(M \cdot L)}{M \cdot L} \right)$$

where  $T_f$  = liquid temperature = 78 K;  $T_0$  = 100 K;  $M = 4hc / kt \cdot D = 6.56 \text{ m}^{-1}$ ;  $hc$  = heat-transfer coefficient for the wire,  $kt$  = wire thermal conductivity;  $D$  = wire diameter; and  $L$  = length of wire exposed to the vapor = 1.00 m. The electric resistance of the wire varies with the average temperature of the wire as follows:

$$R = R_0 [1 + a(T - T_f)]$$

where  $R_0 = 150$  ohms for 1.00 m of wire = wire electric resistance at  $T_f$ ;  $a = 4.0 \times 10^{-3} \text{ K}^{-1}$ ; and  $T$  = average temperature of the part of the wire exposed to the vapor.[1]"

### “Solution:”

#### “Data:”

$$T_f = 78 \text{ [K]}$$

$$T_0 = 100 \text{ [K]}$$

$$M = 6.56 \text{ "m}^{-1}$$

$$L = 1 \text{ [m] "length of wire exposed to vapor"}$$

$$L_{\text{tot}} = 2 \text{ [m] "...total length of wire"}$$

$$R_{\text{tot}} = 300 \text{ "Ohms ... resistance when the entire wire (2.0 m) is immersed in LN2"}$$

$$R_0 = 150 \text{ "Ohms ... resistance when 1.0 m wire is immersed in LN2"}$$

$$a = 4E-03 \text{ "K}^{-1}$$

“T = average temp of wire exposed to vapor. And, T is given by:”

$$(T - T_f) = ((T_0 - T_f) * \cosh(M*L) / (\cosh(M*L) - 1)) * (1 - \tanh(M*L)/(M*L))$$

“Therefore, resist. of wire exposed to vapor:”

$$R_{vap} = R_0 * (1 + a * (T - T_f)) \text{ “ohms --- resist of wire exposed to vapor”}$$

“And, total resist of wire when 1 m is immersed in liquid and 1 m is exposed to vapor:”

$$R = R_0 + R_{vap} \text{ “ohms”}$$

### Results:

**Unit Settings: SI K kPa kJ mass deg**

$a = 0.004 \text{ [K}^{-1}]$	$L = 1 \text{ [m]}$	$L_{tot} = 2 \text{ [m]}$	$M = 6.56 \text{ [m}^{-1}]$
$R = 311.220 \text{ [\Omega]}$	$R_0 = 150 \text{ [\Omega]}$	$R_{tot} = 300 \text{ [\Omega]}$	$R_{vap} = 161.220 \text{ [\Omega]}$
$T = 96.699 \text{ [K]}$	$T_0 = 100 \text{ [K]}$	$T_f = 78 \text{ [K]}$	

Thus:

**Electric resistance of the wire when 1 .00 m is immersed in liquid and 1.00 m is exposed to nitrogen vapor at 78 K = R = 311.22 Ohms...Ans.**

---

**Prob. 5.3.31** A capacitance liquid-level probe is used to measure liquid level in a liquid-nitrogen storage vessel. The total length of the probe is 1.220 m (48.0 in.), and the inner and outer diameters of the annular space of the probe are 75 mm (2.953 in.) and 100 mm (3.937 in.), respectively. If the capacitance reading of the gauge is 312 pF, determine the liquid level and the sensitivity of the gauge. The pressure within the storage vessel is 101,3 kPa (1 atm), and the dielectric constants are  $\epsilon_f = 1.4319$  for the liquid, and  $\epsilon_g = 1.0020$  for the vapor.[1]

**“Solution:”**

**“Data:”**

$$L = 1.22[\text{m}] \text{ “length of probe”}$$

$$D_i = 0.075[\text{m}] \text{ “...ID of probe annular space”}$$

$D_0 = 0.1[m]$  "...OD of probe annular space"

$C = 312E-12$  "F ... capacitance reading"

$\epsilon_f = 1.4319$  "... dielectric const of liq."

$\epsilon_g = 1.002$  "... dielectric const of vapor"

$\epsilon_0 = 8.8542E-12$  "F/m ... permittivity of free space"

**"Then, we have the liquid level, Lf:"**

$$L_f = C * \ln(D_0/D_i) / (2 * \pi * (\epsilon_f - \epsilon_g) * \epsilon_0) - \epsilon_g * L / (\epsilon_f - \epsilon_g)$$

**"Sensitivity: is given by:"**

$$S_0 = 2 * \pi * (\epsilon_f - \epsilon_g) * \epsilon_0 / \ln(D_0 / D_i)$$



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### Results:

**Unit Settings: SI K kPa kJ mass deg**

$$C = 3.120E-10 \text{ [F]}$$

$$D_0 = 0.1 \text{ [m]}$$

$$D_l = 0.075 \text{ [m]}$$

$$\epsilon_0 = 8.854E-12 \text{ [F/m]}$$

$$\epsilon_f = 1.4319$$

$$\epsilon_g = 1.002$$

$$L = 1.22 \text{ [m]}$$

$$L_f = 0.90939 \text{ [m]}$$

$$S_0 = 8.313E-11 \text{ [F/m]}$$

Thus:

Liquid level =  $L_f = 0.90930 \text{ m}$  from bottom of horizontal tank...Ans.

Sensitivity =  $S_0 = 8.313E-12 \text{ F/m}$ ...Ans.

**Prob.5.3.32** While solving problems on Thermodynamic liquid gauges, we need the values of values of 'Supercompressibility factors' for liquid hydrogen, liquid nitrogen, and also for gases.

In Ref.[1], these are given in Tabular form, i.e. in Table 6.6 for LH2, Table 6.7 for LN2 and Table 6.8 for Gases.

**Write an EES Function to find the Supercompressibility for liquid hydrogen, when P (atm) and T (K) are given, and also for sat. liquid.**

First, copy Table 6.6 in to EES as a Lookup Table, and name it as **ZforLH2**. Then, write the desired Function as follows:

FUNCTION Z\_for\_LH2(P\_atm, Temp\_K)

{

Inputs:

P\_atm = Pressure (atm), Temp\_K = Temp (K)

Outputs:

Supercompressibility factor for LH2, Z

}

```
IF ((P_atm < 10) OR (P_atm > 120)) THEN  
    CALL ERROR ('P_atm must be between 10 atm and 120 atm !!')  
ENDIF  
  
IF ((Temp_K < 16) OR (Temp_K > 36)) THEN  
    CALL ERROR ('Temp_K must be between 16 K and 36 K !!')  
ENDIF  
  
Z_for_LH2 :=INTERPOLATE2DM('ZforLH2', P_atm, Temp_K)  
  
END
```

“=====”

“Ex:”

P\_atm = 15

Temp\_K= 17 “K”

Z = Z\_for\_LH2(P\_atm, Temp\_K)

We get:

Unit Settings: SI K kPa kJ mass deg

P\_atm = 15

TempK = 17 [K]

Z = 0.286050

---

### Function for Z of Sat. LH2:

First, copy Z for Sat. LH2 from Table 6.6 in to EES as a Lookup Table, and name it as **ZforSatLH2**. Then, write the desired Function as follows:

FUNCTION Z\_for\_Sat\_LH2( Temp\_K)

{

Inputs:

Temp\_K = Temp (K)

Outputs:

Supercompressibility factor for Sat. LH<sub>2</sub>, Z

}

IF ((Temp\_K < 16) OR (Temp\_K > 32)) THEN

CALL ERROR ("Temp\_K must be between 16 K and 32 K !!")

ENDIF



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Z\_for\_Sat\_LH2 := INTERPOLATE1('ZforSatLH2','Temp','ZforSatLH2',Temp=Temp\_K)

END

“=====”

“Ex:”

Temp\_K= 26.5 “K”

Z = Z\_for\_Sat\_LH2( Temp\_K)

We get:

**Unit Settings: SI K kPa kJ mass deg**

TempK = 26.5 [K]

Z = 0.064740

---

**Prob.5.3.33** While solving problems on Thermodynamic liquid gauges, we need the values of values of ‘Supercompressibility factors’ for liquid hydrogen, liquid nitrogen, and also for gases.

In Ref.[1], these are given in Tabular form, in Table 6.6 for LH2, Table 6.7 for LN2 and Table 6.8 for Gases.

Write an EES Function to find the Supercompressibility for liquid nitrogen, when P (atm) and T (K) are given, and also for sat. liquid.

First, copy Table 6.7 in to EES as a Lookup Table, and name it as **ZforLN2**. Then, write the desired Function as follows:

FUNCTION Z\_for\_LN2(P\_atm, Temp\_K)

{

Inputs:

P\_atm = Pressure (atm), Temp\_K = Temp (K)

**Outputs:**

Supercompressibility factor for LN2, Z  
}  
  
IF ((P\_atm < 5) OR (P\_atm > 60)) THEN  
  
CALL ERROR ('P\_atm must be between 5 atm and 60 atm !!')  
  
ENDIF  
  
IF ((Temp\_K < 70) OR (Temp\_K > 120)) THEN  
  
CALL ERROR ('Temp\_K must be between 70 K and 120 K !!')  
  
ENDIF  
  
Z\_for\_LN2 :=INTERPOLATE2DM('ZforLN2', P\_atm, Temp\_K)  
  
END

“=====”

“Ex:”

P\_atm = 7

Temp\_K= 72 “K”

Z = Z\_for\_LN2(P\_atm, Temp\_K)

We get:

**Unit Settings: SI K kPa kJ mass deg**

P<sub>atm</sub> = 7

TempK = 72 [K]

Z = 0.039894

### Function for Z of Sat. LN2:

First, copy Z for Sat. LN2 from Table 6.7 in to EES as a Lookup Table, and name it as **ZforSatLN2**. Then, write the desired Function as follows:

FUNCTION Z\_for\_Sat\_LN2( Temp\_K)

{

Inputs:

Temp\_K = Temp (K)

Outputs:

Supercompressibility factor for Sat. LN2, Z

}



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```
IF ((Temp_K < 70) OR (Temp_K > 120)) THEN  
    CALL ERROR ("Temp_K must be between 70 K and 120 K !!")  
ENDIF  
  
Z_for_Sat_LN2 := INTERPOLATE1('ZforSatLN2','Temp','ZforSatLN2',Temp=Temp_K)  
  
END
```

“=====”

“Ex:”

Temp\_K= 72 “K”

Z = Z\_for\_Sat\_LN2( Temp\_K)

We get:

**Unit Settings: SI K kPa kJ mass deg**

TempK = 72 [K]

Z = 0.002993

---

**Prob.5.3.34** While solving problems on Thermodynamic liquid gauges, we need the values of values of ‘Supercompressibility factors’ for liquid hydrogen, liquid nitrogen, and also for gases.

In Ref.[1], these are given in Tabular form, in Table 6.6 for LH2, Table 6.7 for LN2 and Table 6.8 for Gases.

Write an EES Function to find the Supercompressibility for gases, when reduced pressure  $Pr (= P/P_c)$  and reduced temp  $Tr (= T/T_c)$  are given, and also for sat. vapor.

First, copy Table 6.8 in to EES as a Lookup Table, and name it as **ZforGases**. Then, write the desired Function as follows:

FUNCTION Z\_for\_Gases(P\_reduced, T\_reduced)

{

Inputs:

P\_reduced = P/P\_crit, T\_reduced = T/T\_crit

Outputs:

Supercompressibility factor for Gas, Z

}

IF ((P\_reduced < 0.1) OR (P\_reduced > 2)) THEN

CALL ERROR ('P\_reduced must be between 0.1 and 2 !!')

ENDIF

IF ((T\_reduced < 0.8) OR (T\_reduced > 15)) THEN

CALL ERROR ('T\_reduced must be between 0.8 and 15 !!')

ENDIF

Z\_for\_Gases :=INTERPOLATE2DM('ZforGases', P\_reduced, T\_reduced)

END

“=====”

“Ex:”

P\_reduced = 0.15,

T\_reduced = 0.875

$Z = Z_{\text{for\_Gases}}(P_{\text{reduced}}, T_{\text{reduced}})$

We get:

**Unit Settings: SI K kPa kJ mass deg**

$P_{\text{reduced}} = 0.15$

$T_{\text{reduced}} = 0.875$

$Z = 0.907750$

---

### Function for Z of Sat. Vapors:

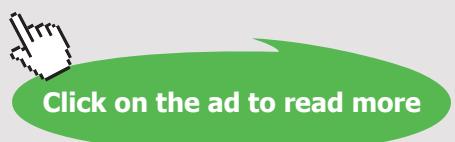
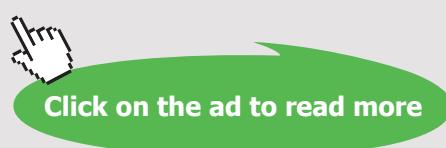
First, copy Z for Sat. Vapor from Table 6.8 in to EES as a Lookup Table, and name it as **ZforSatVapor**. Then, write the desired Function as follows:

FUNCTION  $Z_{\text{for\_Sat\_Vapor}}( P_{\text{reduced}} )$

{

Inputs:

$P_{\text{reduced}} = P / P_{\text{crit}}$



**Outputs:**

Supercompressibility factor for Sat. Vapor, Z

}

IF ((P\_reduced < 0.1) OR (P\_reduced > 1)) THEN

CALL ERROR ('P\_reduced must be between 0.1 and 1 !!')

ENDIF

Z\_for\_Sat\_Vapor := INTERPOLATE1('ZforSatVapor','P\_reduced','ZforSatVapor',P\_reduced=P\_reduced)

END

“=====”

“Ex:”

P\_reduced = 0.15

Z = Z\_for\_Sat\_Vapor(P\_reduced)

We get:

**Unit Settings: SI K kPa kJ mass deg**

P\_reduced = 0.15

Z = 0.865500

“=====”

**Prob. 5.3.35** A thermodynamic liquid-level gauge for liquid hydrogen has a sensing element length of 1.520 m (59.8 in.), a sensing element cross-sectional area of 30 mm<sup>2</sup> (0.0465 in<sup>2</sup>), and a dead volume of 9.50 dm<sup>3</sup> (580 in<sup>3</sup>). The temperature of the liquid to be gauged is 20 K (36°R), and the gas within the sensing element is at a mean temperature of 32 K (57.6°R). The temperature of the dead-volume gas is 300 K (540°R). If the element is initially charged with 10.5 g (0.0231 lbm) of hydrogen gas, determine the liquid level when the pressure indicated by the gauge is 1.013 MPa (10 atm). [1]

**Solution:**

**“For H<sub>2</sub>, critical pressure and temp are:”**

$$p_c = 1.315E06 \text{ [Pa]}$$

$$T_c = 33.2 \text{ [K]}$$

**“And, other data:”**

$$R = 4124.2 \text{ [J/kg.K]} \text{ “...Gas constant for H}_2\text{”}$$

$$p = 1.013E06 \text{ [Pa]} \text{ “...pressure indicated by gauge”}$$

$$m = 10.5E-03 \text{ [kg]} \text{ “...initial charging mass”}$$

$$T_f = 20 \text{ [K]}$$

$$L = 1.52 \text{ [m]} \text{ “...length of sensing element”}$$

$$A_c = 30E-06 \text{ [m}^2\text{]} \text{ “...cross-sectional area of sensing element”}$$

$$V_0 = 9.5E-03 \text{ [m}^3\text{]} \text{ “...dead volume”}$$

$$T_0 = 300 \text{ [K]}$$

$$T_g = 32 \text{ [K]} \text{ “...temp of gas in the sensing element”}$$

**“Calculations:”**

**“For dead volume conditions:”**

$$P_{reduced} = p/p_c$$

$$T_{reduced} = T_0 / T_c$$

**“For the values of  $T_{reduced} = (T_0/T_c)$  and  $P_{reduced} = (p/p_c)$ , get  $Z_0$  from Table 6.8, or the EES Function written earlier:”**

$Z_0 = Z_{\text{for Gases}}(P_{\text{reduced}}, T_{\text{reduced}})$

Now, from NIST website, we have, for LH2:

### Liquid Phase Data

#### Data on Saturation Curve

Temperature (K)	Pressure (atm)	Density (kg/m <sup>3</sup> )	Volume (m <sup>3</sup> /kg)
30.000	7.9381	54.540	0.018335
31.000	9.2934	51.382	0.019462
32.000	10.813	47.086	0.021238
33.000	12.527	38.080	0.026260

So, we get:

“For the capillary tube:”



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“At 32 K, the sat. pressure is 10.813 atm. But, the pressure gauge reads 10 atm.

So, we have sat. vapor in capillary. And, we use the EES Function for gases, with saturation condition to get  $Z_g$ .”

$$Z_g = Z_{\text{for Sat Vapor}}(P_{\text{reduced}})$$

**“For the liquid conditions:”**

“For the values of  $T_f = 20$  K, and  $P = 10$  atm. , get  $Z_f$  from Table 6.6, or from the EES Function written earlier:”

$$P_{\text{atm}} = 10$$

$$\text{Temp}_K = T_f$$

$$Z_f = Z_{\text{for LH2}}(P_{\text{atm}}, \text{Temp}_K)$$

**“Therefore, liquid level is given by:”**

**Eqn. for liquid level is:**

$$\frac{L_f}{L} = \frac{\frac{m \cdot R \cdot T_f}{A_c \cdot L \cdot p} - \frac{V_0 \cdot T_f}{Z_0 \cdot T_0 \cdot A_c \cdot L} - \frac{T_f}{Z_g \cdot T_g}}{\frac{1}{Z_f} - \frac{T_f}{Z_g \cdot T_g}}$$

**Now, let:**

$$A = m * R * T_f / (A_c * L * p)$$

$$B = V_0 * T_f / (Z_0 * T_0 * A_c * L)$$

$$C = T_f / (Z_g * T_g)$$

$$D = (1 / Z_f) - C$$

**“Then:”**

$$L_f / L = (A - B - C) / D \quad \text{.... finds liquid level, } L_f \text{ (m)}$$

## Results:

**Unit Settings: SI K kPa kJ mass deg**

$$A = 18.75$$

$$A_c = 0.00003 \text{ [m}^2\text{]}$$

$$B = 13.89$$

$$C = 1.162$$

$$D = 4.745$$

$$L = 1.52 \text{ [m]}$$

$$L_f = 1.18483 \text{ [m]}$$

$$m = 0.0105 \text{ [kg]}$$

$$p = 1.013E+06 \text{ [Pa]}$$

$$P_{atm} = 10$$

$$p_c = 1.315E+06 \text{ [Pa]}$$

$$P_{reduced} = 0.7703$$

$$R = 4,124.200 \text{ [J/kg.K]}$$

$$Temp_K = 20 \text{ [K]}$$

$$T_0 = 300 \text{ [K]}$$

$$T_c = 33.2 \text{ [K]}$$

$$T_f = 20 \text{ [K]}$$

$$T_g = 32 \text{ [K]}$$

$$T_{reduced} = 9.036$$

$$V_0 = 0.0095 \text{ [m}^3\text{]}$$

$$Z_0 = 1$$

$$Z_f = 0.1693$$

$$Z_g = 0.53798$$

Thus:

Liquid level =  $L_f = 1.18483 \text{ m...from bottom of horizontal tank...Ans.}$

---

**Prob. 5.3.36** A thermodynamic liquid-level gauge for liquid nitrogen has a ratio of dead volume to capillary tube volume of  $V_0/A_c L = 200$ . The temperature of the liquid to be gauged is 77.4 K (139.3°R), and the gas within the sensing element is at an average temperature of 95 K (171°R). The temperature of the dead volume is 300 K (540°R). The gauge is initially charged with nitrogen gas so that the ratio of charging mass to dead volume is 7.55 kg/m<sup>3</sup> (0.471 lbm/ft<sup>3</sup>), and the sensing element length is 1.220 m (48.0 in.). Determine the liquid-level indication when the pressure indicated by the gauge is 506.6 kPa (5 atm).[1]

**Solution:**

“For N<sub>2</sub>, critical pressure and temp are:”

$$p_c = 3.3958E06 \text{ [Pa]}$$

$$T_c = 126.192 \text{ [K]}$$

“And, other data:”

$$R = 296.8 \text{ [J/kg.K]} \text{ “...Gas constant for N}_2\text{”}$$

$$p = 506.6E03 \text{ [Pa]} \text{ “...pressure indicated by gauge”}$$

$$T_f = 77.4 \text{ [K]}$$

$L = 1.22 \text{ [m]}$  "...length of sensing element"

$T_0 = 300 \text{ [K]}$

$T_g = 95 \text{ [K]}$  "...temp of gas in the sensing element"

{

By data:

$V_0 / (A_c * L) = 200$

$m/V_0 = 7.55 \text{ "kg/m}^3\text{"}$

}

"So, we write:"

$V_0/A_c/L = 200$



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$m_{byV0} = 7.55$

$m_{byAcL} = m_{byV0} * V_{0byAcL}$  “ $\text{kg/m}^3$ ”

“Calculations:”

“For dead volume conditions:”

$$P_{reduced} = p/p_c$$

$$T_{reduced} = T_0 / T_c$$

“For the values of  $(T_0/T_c)$  and  $(p/p_c)$ , get  $Z_0$  from Table 6.8, or the EES Function written earlier:”

$$Z_0 = Z_{\text{for Gases}}(P_{reduced}, T_{reduced})$$

Now, from NIST website, we have, for LH2:

### Liquid Phase Data

#### Data on Saturation Curve

Temperature (K)	Pressure (atm)	Density ( $\text{kg/m}^3$ )	Volume ( $\text{m}^3/\text{kg}$ )
93.000	4.5606	729.19	0.0013714
94.000	4.9365	723.77	0.0013817
95.000	5.3346	718.26	0.0013922
96.000	5.7553	712.67	0.0014032

So, we get:

“For the capillary tube:”

“At 95 K, the sat. pressure of N<sub>2</sub> is 5.3346 atm. But, the pressure gauge reads 5 atm.

So, we have sat. vapor in capillary. And, we use the EES Function for gases, with saturation condition to get  $Z_g$ :”

$$Z_g = Z_{\text{for Sat Vapor}}(P_{\text{reduced}})$$

“For the liquid conditions:”

“For the values of  $T_f = 77.4$  K, and  $P = 5$  atm. , get  $Z_f$  from Table 6.7, or from the EES Function written earlier:”

$$P_{\text{atm}} = p / (101.32E03) \text{ “atm”}$$

$$\text{Temp\_K} = T_f$$

$$Z_f = Z_{\text{for LN2}}(P_{\text{atm}}, \text{Temp\_K})$$

“Therefore, liquid level is given by:”

$$\frac{L_f}{L} = \frac{\frac{m \cdot R \cdot T_f}{p \cdot A_c \cdot L} - \frac{V_0 \cdot T_f}{Z_0 \cdot T_0 \cdot A_c \cdot L} - \frac{T_f}{Z_g \cdot T_g}}{\frac{1}{Z_f} - \frac{T_f}{Z_g \cdot T_g}}$$

Now, let:

$$A = m * R * T_f / (A_c * L * p)$$

“i.e.”

$$A = m \text{byAcL} * (R * T_f) / p$$

$$B = V_0 * T_f / (Z_0 * T_0 * A_c * L)$$

“i.e.”

$$B = (V0 \text{byAcL}) * T_f / (Z_0 * T_0)$$

$$C = T_f / (Z_g * T_g)$$

$$D = (1 / Z_f) - C$$

“Then:”

$$L_{fb} = (A - B - C) / D \text{ “...finds } (L_f / L) \text{ “}$$

$$L_f = L_{fb} * L \text{ “...finds liquid level, } L_f \text{ (m) “}$$

“-----”

### Results:

#### Unit Settings: SI K kPa kJ mass deg

A = 68.47	B = 51.79	C = 0.9408	D = 35.66
L = 1.22 [m]	$L_{fb} = 0.4413$	$L_f = 0.53839 \text{ [m]}$	$m_{byAcL} = 1.510 \text{ [kg/m}^3]$
$m_{byV0} = 7.55 \text{ [kg/m}^3]$	$p = 506,600 \text{ [Pa]}$	$P_{atm} = 5$	$p_c = 3.396E+06 \text{ [Pa]}$
$P_{reduced} = 0.1492$	$R = 296,800 \text{ [J/kg.K]}$	$T_{TempK} = 77.4 \text{ [K]}$	$T_0 = 300 \text{ [K]}$
$T_c = 126.2 \text{ [K]}$	$T_f = 77.4 \text{ [K]}$	$T_g = 95 \text{ [K]}$	$T_{reduced} = 2.377$
$V_{0byAcL} = 200$	$Z_0 = 0.9963$	$Z_f = 0.02732$	$Z_g = 0.86603$

Thus:

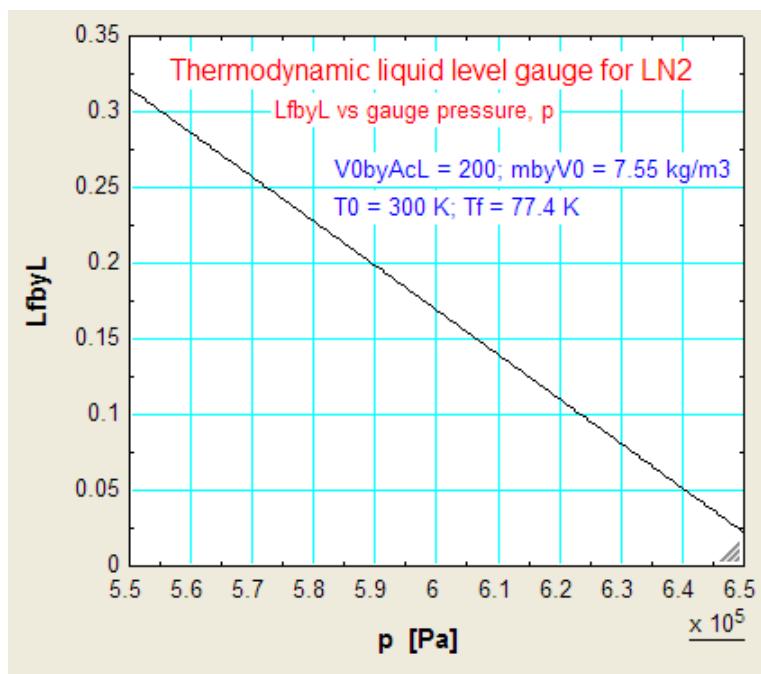
Liquid level =  $L_f = 0.53839 \text{ m...from bottom of horizontal tank...Ans.}$

Plot  $L_f$  against gauge pressure  $p$  as  $p$  varies from  $5.5 \times 10^5 \text{ Pa}$  to  $6.5 \times 10^5 \text{ Pa}$ :

First, compute the Parametric Table:

1..21	1 p [Pa]	2 LfbyL	1..21	1 p [Pa]	2 LfbyL
Run 1	550,000	0.3151	Run 12	605,000	0.1542
Run 2	555,000	0.3005	Run 13	610,000	0.1395
Run 3	560,000	0.2859	Run 14	615,000	0.1249
Run 4	565,000	0.2713	Run 15	620,000	0.1102
Run 5	570,000	0.2567	Run 16	625,000	0.09546
Run 6	575,000	0.2421	Run 17	630,000	0.08074
Run 7	580,000	0.2275	Run 18	635,000	0.06602
Run 8	585,000	0.2128	Run 19	640,000	0.05129
Run 9	590,000	0.1982	Run 20	645,000	0.03656
Run 10	595,000	0.1835	Run 21	650,000	0.02181
Run 11	600,000	0.1689			

**Now. Plot:**



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