Time-consistent innovation policy with creative destruction*

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Abstract

We characterize how strategic behavior among innovating firms affects the time-consistency of patent policy in a Schumpeterian endogenous growth model. Under discretion, a government that inherits a more competitive economy (with smaller leader-laggard technology gaps) chooses stronger patent protection. By fostering competition through the creative destruction process, a government today therefore shifts future governments toward stronger protection, mitigating governments' well-known incentive to expropriate prior investment in innovation. However, in our quantitative model, we find that this influence is quite limited. Correspondingly, patent protection and productivity growth are weak relative to under commitment. Exercises varying the forces of creative destruction, such as the innovativeness of market laggards, illustrate the robustness of our results. Our findings point to the potential benefits of an independent authority setting patent protection mainly focusing on fostering innovation.

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“*Note that the patent system is fragile in the sense that it is dependent on firms’ expectations about future rewards.... A benevolent social planner considering the creation of a new intellectual property rights (IPR) system will face a time-consistency problem.*” Kremer and Williams (2010).

“*Given that resources have been allocated to inventive activity which resulted in a new product or process, the efficient policy is not to permit patent protection.*” Kydland and Prescott (1977).

1. Introduction

Firms’ investment in innovation is a key driver of economic growth. However, without government intervention, competitive markets may insufficiently incentivize innovation (Arrow (1962), Nordhaus (1969)). To stimulate innovation, governments have used patent policy, promising innovators a period of market exclusivity. A well-known challenge is that, ex-post, governments have an incentive to expropriate prior investments in innovation (Kydland and Prescott (1977)). Surprisingly, this time-consistency problem has received no attention in the Schumpeterian endogenous growth literature, which has studied optimal intellectual property rights assuming commitment (Aghion and Howitt (1992) and Acemoglu and Akcigit (2012), among others). Once R&D efforts are sunk, how strong are governments’ incentives to renege on promised protection from market competition? Does optimal time-consistent patent policy differ from optimal policy under commitment and, if so, what are the implications for productivity growth, market power, and welfare? To address these questions, we embed a social planner with discretion *à la* Kydland-Prescott in an endogenous growth model with strategic behavior among innovating firms. This allows to investigate how the time-consistency problem shapes, and is shaped by, the process of creative destruction that propels economic growth.

Our analysis emphasizes that, by fostering competition through the creative destruction process, a planner can shift future planners toward stronger protection, mitigating the well-known incentive to renege. This influence arises because, in a wide range of Schumpeterian frameworks, a planner that inherits a more competitive economy would choose stronger protection.[1] We find that the strength of a planner’s influence over future

[1] As we explain below, in our model and a wide range of Schumpeterian frameworks, a more competitive

planners is limited. In our quantitative model, a planner that inherits a more competitive economy indeed chooses stronger patent protection. However, no matter how competitive the inherited economy is, future planners set the strength of patent protection significantly weaker than would be ex-ante optimal. Therefore, the optimal policy is not time-consistent, with important welfare consequences.

Our framework builds on the strategic competition models of Aghion et al. (2001) and others. There are many industries. In each industry, two firms compete in the goods market and invest in R&D to improve their production technology. R&D increases the probability that a firm improves its technology position relative to its competitor. The productivity gap between firms—the industry's state variable—determines the market power of the leader, and hence markups and profits. A patent policy is time-consistent if firm R&D decisions (conditional on the current expiry rate and expectations of future patent protection) are such that planners in the future would find it optimal to set the expiry rate expected by firms.[2] Under discretion, a planner sets the patent expiry rate taking into account that its choice changes the dynamics of the endogenous state variable (the cross-industry distribution of leader-laggard technology gaps), thereby changing future planners' incentives and choices in a time-consistent equilibrium. We investigate these insights in a model calibrated to match productivity growth, the markup distribution, aggregate R&D intensity, and a measure of the role of reallocation in productivity growth.

Following Kydland and Prescott (1977), Lucas and Stokey (1983), and Debortoli et al. (2021) and others, the main insights can be grasped using a model in which the planner makes decisions at two points in time. A time-0 planner sets an initial expiry rate. At time T, the planner has an opportunity to change the expiry rate.[3] We focus on the patent expiry rate as an (inverse) measure of the strength of patent protection. We use a "dual," global approach to find the optimal time-consistent policy. The first task is to characterize

economy is one with smaller technological gaps between market leaders and laggards.

[2]As discussed in Section 3, a time-consistent policy is a fixed point of the cross-section of time-varying firm R&D decisions (which depend on expected future protection) and future planners' choices (which depend on the time-varying cross-section of technology gaps induced by firm R&D decisions).

[3]Historical evidence indicates that the strength of patent protection is persistent but does experience significant changes (Lerner (2002), Moser (2013)). Our main qualitative results are robust to changing the number of years, T, over which the patent expiry rate is constant (Section 6).

the set of time-consistent policies available to the planner under discretion. We find that, for any initial expiry rate set at date 0, there is a unique time-consistent expiry rate chosen at date T. Comparing across time-consistent policies, a higher initial expiry rate is indeed associated with a lower expiry rate chosen at date T. Thus, even without commitment, the time-0 planner does have some influence over the expiry rate chosen at date T. In particular, starting from a low initial expiry rate (strong initial patent protection), a marginal increase in the initial expiry rate implies that the subsequent planner inherits a notably more competitive economy and sets a significantly stronger degree of patent protection. However, across the set of time-consistent policies, no matter how weak patent protection is set initially, date T protection is much weaker than under commitment.

The second step of our dual approach is to simulate transition dynamics under each time-consistent policy to see which maximizes welfare. In our calibrated model, under the optimal time-consistent policy, the patent protection chosen at date T is much weaker than under commitment. The optimal time-consistent policy also features weaker protection between date 0 and T than under commitment, but this choice does little to address the fact that, from the perspective of time 0, the protection chosen at date T is much too weak. Productivity growth and firm innovation are thus lower under time-consistent policy than under commitment. Although markups are also lower, in our quantitative exercise, households would forgo 0.2% of consumption in perpetuity to switch from time-consistent policy to commitment.

In a number of exercises such as varying externally calibrated parameters, our result that the degree of protection chosen at date T is weaker under discretion is highly robust. However, depending on parameters, optimal time-consistent patent protection can be stronger between dates 0 and T than under commitment (in contrast to the standard intuition that time-consistent policy involves weaker protection). In such parametrizations, the initial planner chooses strong initial protection to partly offset the growth-reducing effects of weak long-run protection. That is, the initial planner chooses to completely forgo weakening protection initially in order to shift the date T choice closer toward the date T choice under commitment.

The optimal policy is not time-consistent because firms' R&D strategies depend on expectations of future patent protection. Stronger future protection increases R&D because the profits from innovation accrue over time (the standard channel). In addition, the process of creative destruction gives rise to novel, strategic channels. Inducing high innovation rates by laggards in competitive industries, strong future protection increases the incentive of leaders in such industries to *escape competition*. However, inducing high innovation by laggards in uncompetitive industries, strong protection reduces the innovation incentives of firms in competitive industries through a *trickle-down effect*. The consistent planner treats past R&D decisions as sunk and therefore ignores these strategic interactions in setting time-consistent patent policy. To investigate the importance of these channels, we decompose the effect of expected future patent protection on productivity growth. The escape-competition, trickle-down, and composition effects on productivity growth from weaker protection chosen at date T are large in magnitude relative to the overall general equilibrium effect and the standard channel.

The differences between optimal policy under discretion and commitment also depend on *how* firm R&D decisions affect welfare. R&D decisions directly affect welfare because innovations by firms at the technology frontier increase the productivity of technologies in use. In our Aghion-Howitt setting, there are also two, more novel ways that firm R&D decisions affect welfare. First, innovation by any firm today affects its industry's technology gap, which in turn affects future R&D decisions through a composition effect (i.e., firms' R&D investments depend on the contemporaneous technology gap). Second, because innovations today impact the evolution of the technology gap distribution, firms' R&D decisions today can affect: future aggregate labor demand (i.e., because high markups depress labor demand); future wages; and ultimately, through wages, future R&D strategies. With sufficiently elastic labor supply, this channel has little effect on the time-consistency problem.[4]

Related literature. Our framework brings together three important areas of the literature. First, our model features strategic behavior among competing firms and an en-

[4]Our main analysis focuses on the connections between the policy choices of the date 0 and date T planners. We also consider how the date 0 expiry rate affects the expiry rates chosen at dates T and $2T$. This analysis, if anything, strengthens our earlier conclusions. Comparing across time-consistent policies, the time 0 planner's choice has almost no effect on the expiry rate set at $2T$.

dogenous distribution of technology gaps and markups, as in Aghion et al. (2001) and Acemoglu and Akcigit (2012). Second we emphasize the dynamic response to changes in innovation policy (in our case, patent policy) as in Atkeson and Burstein (2019). Third, we study the implications of these features for patent policy's time-consistency problem. We thus build on Kydland and Prescott's (1977) original insights on optimal policy under discretion. The endogenous distribution of technology gaps, innovation rates, and markups is crucial for studying patent policy's time-consistency problem, for three reasons. First, it offers a new channel through which a planner can influence future planners' decisions through the persistent gap distribution. Second, the positive and normative consequences of patent policy depend importantly on how such policy affects firms' strategic interactions, for example, through escape-competition and trickle-down incentives. Third, our approach allows us to identify how patent policy affects the evolution of the entire distribution of markups and the associated labor and production distortions. Overall, our results are broadly consistent with Atkeson and Burstein's (2019) in that we find that changes in innovation policy have effects on productivity growth that are modest relative to typical business cycle fluctuations, but nonetheless have important welfare consequences due to the cumulative impact on productivity and output over time. We also introduce time-varying patent policy and discretion into the strategic competition framework of Aghion et al. (2001) in a tractable way, allowing us to uncover novel aspects of the time-consistency problem that arise from the strategic and composition effects of patent policy.

The literature on endogenous growth and creative destruction treats patent policy as exogenous or studies optimal policy only under commitment. Most research has emphasized the static trade-off between incentives for private research and the social costs of market exclusivity for patented goods (for example, Arrow (1962), Nordhaus (1969), Aghion and Howitt (1992), Green and Scotchmer (1995)). Motivated by Nordhaus (1972), one branch of this literature focuses on the optimal length and breadth of patents (Klemperer (1990), Gallini and Scotchmer (2002)). Mostly closely related to our paper, Acemoglu and Akcigit (2012) focus on optimal patent policy under commitment, along the balanced growth path (BGP), in a dynamic framework with sequential innovation. Relatedly, Akcigit and Ates (2022) study the consequences for BGP welfare of changing the exogenous rate of technology diffusion.

It is interesting to look at our paper through the lens of the literature on time-consistent fiscal and monetary policy. In particular, Lucas and Stokey (1983) and others have emphasized that, by choosing the debt maturity structure, a planner can promote adherence of future planners to optimal fiscal policy. In our setting, the planner can influence future planners through the technology gap distribution. Thus, our result that patent policy's time-consistency problem cannot be undone by fostering competition (through patent policy) is somewhat similar in spirit to the limitations to the Lucas-Stokey result uncovered in Debortoli et al. (2021). Our results also point to a potential role for an independent patent authority that places "too large" a weight on fostering innovation and productivity growth, relative to reducing market power, akin to Rogoff (1985)'s "conservative" central banker.

2. Private-sector equilibrium

This section describes the private sector equilibrium in an Aghion-Howitt endogenous growth model, for a given patent policy (i.e., a path for the patent expiry rate). Throughout, we focus on Markov Perfect Equilibria, where firms' and planners' strategies are functions only of the payoff-relevant state variables.

Preferences and final goods. The economy admits a representative household with utility function

$$\int_{t=0}^{\infty} e^{-\rho t} \big[\ln(C(t)) - L(t) \big] dt, \tag{1}$$

where $C(t)$ is consumption of the final good, $L(t)$ is labor, and $\rho > 0$ is the discount rate.[5]

A continuum of intermediate-goods industries is indexed by $j \in [0,1]$. Each industry includes two firms that produce perfect substitutes. The final good is produced using intermediate goods according to the Cobb-Douglas production function, $\ln Y(t) = \int_0^1 \ln y(j,t) dj$, where $y(j,t)$ is the output of the j-th intermediate at time t. The final good is

[5]This preference specification implies that labor supply is infinitely elastic, as in Aghion et al. (2001) and others. Section 3.3 discusses the implications for the time-consistency problem of assuming, as in Acemoglu and Akcigit (2012), inelastic labor supply.

the numeraire and sold in a perfectly competitive market.

Intermediate goods market and productivity. Each industry j includes two firms. A firm i in industry j can produce the intermediate good j using a linear production technology, $y_{i,j} = q_{i,j} l_{i,j}$, where $q_{i,j}$ is the firm's labor productivity and $l_{i,j}$ is the amount of labor hired. The price of intermediate good j is p_j. Two small comments are in order. First, we (only) drop time subscripts to describe the static allocation, or the price and quantity of each intermediate good taking each firm's productivity as given. Second, with a slight abuse of notation, the firm with the highest productivity is referred to using the subscript i and the other firm is referred to using $-i$.

We assume Bertrand competition, in which firms set prices. Each intermediate good is therefore sold at its limit price, $p_j = w/q_{-i,j}$, where w is the wage rate. Only the leader firm produces. Because of the Cobb-Douglas technology for final good production, sales are equalized across industries, with $p_j y_j = Y = C$.

The gross markup is $\psi_j \equiv p_j \frac{q_{i,j}}{w} = \frac{q_{i,j}}{q_{-i,j}}$. Labor demand and profits are functions of the industry markup, with

$$l_j = \psi_j^{-1} \frac{Y}{w} \quad \text{and} \quad \Pi_j = (1 - \psi_j^{-1}) Y. \tag{2}$$

Higher markups depress labor demand. Industry profits and labor demand do not depend on the level of firms' productivity. In a neck-and-neck industry, the two firms have the same productivity, $q_{i,j} = q_{-i,j}$, profits are zero, and the wage is the marginal product of labor.

Productivity ladder. Firms in each industry are ordered on a quality ladder. Each rung represents a proportional productivity improvement of scale $\lambda > 1$. The number of rungs separating leader and laggard at time t is $s(t) \in \{0, 1, ..., \bar{s}\} \equiv S^+$. We assume that the maximum possible technology gap between leader and laggard within an industry is $\bar{s}$. The *technology position* of a firm at time t is denoted by $\sigma(t) \in \{-\bar{s}, \ldots, \bar{s}\} \equiv S$. In an industry with gap $s(t) > 0$, the leader has technology position $\sigma(t) = s(t)$ and the laggard's position is $\sigma(t) = -s(t)$. A firm with position $\sigma(t) = 0$ is tied.

An innovating leader advances one rung and its productivity increases by a factor λ. An innovating laggard can advance incrementally or somewhat radically. With probability $(1-\phi)$, an innovating laggard advances one rung. With probability ϕ, an innovating laggard achieves a radical ("quick catch-up") innovation and closely the gap completely. A firm in position σ at time t innovates at rate $x_\sigma(t)$ by hiring $G(x_\sigma(t); B) = [x_\sigma(t)/B]^{\frac{1}{\gamma}}$ workers as R&D scientists. $B > 0$ is an R&D cost scaling parameter and $\gamma > 0$ captures the convexity of R&D costs in the arrival rate.

Patent expiry. Following recent work by Acemoglu and Akcigit (2012) and Akcigit and Ates (2022), patent expiry allows a laggard to completely catch up with its competitor. The patent expiry rate $\eta(t)$ is time-varying. Between periods 0 and T, the patent expiry rate is $\eta(t) = \eta_1 \geq 0$. From period T onward, the patent expiry rate is $\eta(t) = \eta_2 \geq 0$. (In Section 7, we relax the assumption that the expiry rate is constant from T onward.)

2.1 Private sector allocations

We focus on Markov perfect equilibria. The scaled value function of a firm in position σ at time t is $v_\sigma(t) \equiv \frac{V_\sigma(t)}{Y(t)}$, where $V_\sigma(t)$ is the firm's discounted expected net profits. Scaled operating profits for a firm in position σ at time t are $\pi_\sigma(t)$, from (2).

Household and firm maximization. Under preferences (1), household maximization implies the Euler equation

$$\frac{\dot{C}(t)}{C(t)} = \frac{\dot{Y}(t)}{Y(t)} = g(t) = r(t) - \rho \tag{3}$$

and scaled wage $\omega(t) = 1$.

Denote the innovation strategy of a firm's competitor by $\{x_\sigma^c(t)\}_{\sigma \in S}$, where $x_\sigma^c(t)$ is the innovation rate of a competitor in technology position σ at time t. Each firm chooses its innovation strategy to maximize discounted expected future profits, taking as given its competitor's innovation strategy and the wage rate. The firm trades off R&D costs against the future discounted profits from an innovation. Operating profits are taxed at rate τ.

For a leader (i.e., a firm with $\sigma > 0$ at time t), using the Euler equation, the steady state value function satisfies

$$\rho v_\sigma(t) - \dot{v}_\sigma(t) = \max_{x_\sigma(t)} \underbrace{(1-\tau)\pi_\sigma(t)}_{\text{Operating profits}} - \underbrace{G(x_\sigma(t))\omega(t)}_{\text{R\&D costs}} + \underbrace{x_\sigma(t)\Delta v_\sigma(t)}_{\text{Own innovation}} +$$
$$\underbrace{x^c_{-\sigma}(t)(\phi v_0(t) + (1-\phi)v_{\sigma-1}(t) - v_\sigma(t))}_{\text{Competitor innovation}} + \underbrace{\eta(t)(v_0(t) - v_\sigma(t))}_{\text{Patent expiry}}. \quad (4)$$

The leader value function has three main parts. First, leaders earn after-tax operating profits $(1-\tau)\pi_\sigma(t)$. Second, leaders incur R&D costs and, if successful in R&D, earn the expected capital gain from own innovation, $\Delta v_\sigma(t)$, with $\Delta v_\sigma(t) = v_{\sigma+1}(t) - v_\sigma(t)$ for $\sigma \geq 0$. Third, leaders can experience capital losses from a competitor innovation or patent expiry. Competitor innovation, with arrival rate $x^c_{-\sigma}(t)$, reduces the leader's technology position to zero (with probability ϕ) or by one rung (with probability $1-\phi$). Patent expiry, with arrival rate $\eta(t)$, reduces the leader's technology position to zero.

Similarly, the value function for a laggard (i.e., a firm with $\sigma < 0$ at time t) is

$$\rho v_\sigma(t) - \dot{v}_\sigma(t) = \max_{x_\sigma} - \underbrace{G(x_\sigma(t))\omega(t)}_{\text{R\&D costs}} + \underbrace{x_\sigma(t)\Delta v_\sigma(t)}_{\substack{\text{Own}\\ \text{innovation}}} + \underbrace{x^c_{-\sigma}(t)(v_{\sigma-1}(t) - v_\sigma(t))}_{\text{Competitor innovation}} + \underbrace{\eta(t)(v_0(t) - v_\sigma(t))}_{\text{Patent expiry}}. \quad (5)$$

For a firm in technology position $\sigma < 0$ at time t, the capital gain from a successful innovation is $\Delta v_\sigma(t) = \phi v_0(t) + (1-\phi)v_{\sigma+1}(t) - v_\sigma(t)$. Equation (5) also holds for tied firms if one substitutes $\eta(t) = 0$.

At time t, for a firm in any position σ, profit maximization implies

$$x_\sigma(t) = G'^{-1}\left(\frac{\Delta v_\sigma(t)}{\omega(t)}\right). \quad (6)$$

Because G is twice differentiable and strictly convex, the firm innovation rate $x_\sigma(t)$ is increasing in the capital gain from innovation and decreasing in the cost of R&D measured by the scaled wage $\omega(t)$ (with $\omega(t) = 1$ given preferences (1), as noted above).

Technology gap distribution. Let $\mu_s(t)$ denote the share of industries with technology gap s at time t. For $s > 0$, the net inflow to s is

$$\dot{\mu}_s(t) = \underbrace{\mu_{s+1}(t)(1-\phi)x_{-(s+1)}(t) + \mu_{s-1}(t)x_{s-1}(t)(1 + \mathbb{1}_{s=1})}_{\text{Inflow to gap } s} - \underbrace{\left[x_s(t) + x_{-s}(t) + \eta(t)\right]\mu_s(t)}_{\text{Outflow from gap } s}. \quad (7)$$

For an industry with a positive gap s, inflows occur when there is an incremental laggard innovation in an industry with gap $s+1$ or a leader innovation in an industry with gap $s-1$. (The indicator function for $s=1$, or $\mathbb{1}_{s=1}$, is used to take into account that a tied industry shifts to a one-rung gap when either of the two tied firms innovates.) Outflows occur because of leader or laggard innovation or patent expiry.

The net inflow to neck-and-neck competition is

$$\dot{\mu}_0(t) = \underbrace{\mu_1(t)\big[x_{-1}(t)+\eta(t)\big] + \sum_{\sigma\in(2,\ldots,\bar{s})} \mu_\sigma(t)[\phi x_{-\sigma}(t)+\eta(t)]}_{\text{Inflows}} - \underbrace{2x_0(t)\mu_0(t)}_{\text{Outflows}} \tag{8}$$

Inflows to neck-and-neck competition occur due to patent expiry, any laggard innovation in an industry with a one-rung gap, and quick catch-up innovation in industries with larger gaps. Outflows from neck-and-neck competition occur when either of the two firms innovates. The stationary distribution of gaps, $\{\mu_s\}_{s\in S^+}$, is determined by setting expression (7) equal to 0 for all $s>0$, together with the normalization $\sum_{s\in S^+}\mu_s = 1$.

Output, the quality index, and markups. Denote the highest productivity in industry j at time t by $q_{H,j}(t) = \max\{q_{i,j}(t), q_{-i,j}(t)\}$. If industry j at time t has a positive technology gap $(s>0)$, then the highest productivity is the leader's productivity. If industry j is neck-and-neck, then the highest productivity is the productivity of both firms. Denote the labor demand of the firm with the highest productivity in industry j by $l_{H,j}(t)$. Aggregate output $Y(t)$ satisfies:

$$\ln Y(t) = \int_0^1 \ln\left[q_{H,j}(t) l_{H,j}(t)(1+\mathbb{1}_{s(j,t)=0})\right] d\nu. \tag{9}$$

Define the quality index $Q(t)$ as the aggregate of the highest productivity across the different industries, that is,

$$\ln Q(t) = \int_0^1 \ln q_{H,j}(t) d\nu \tag{10}$$

Aggregate output is increasing in the quality index and decreasing in the average markup:

$$
\begin{aligned}
\ln Y(t) &= \ln Q(t) + \int_0^1 \ln\left[l_H(j,t)(1+\mathbb{1}_{s(j,t)=0})\right] dj \\
&= \ln Q(t) + \sum_{s=0}^{\bar{s}} \mu_s(t) \ln l_s = \underbrace{\ln Q(t)}_{\text{Quality index}} \underbrace{- \sum_{s=0}^{\bar{s}} \mu_s(t) \ln \psi_s}_{\text{Average markup}}
\end{aligned} \tag{11}
$$

where l_s is defined as the labor demand (expression (2)) in an industry with gap s and ψ_s is the gross markup in an industry with gap s. Holding constant the quality index, a lower average log gross markup contributes positively to output, because higher markups depress labor demand. From Eq. (11), the log-difference of output $Y(t)$ and the quality index $Q(t)$ is equal to the (negative) average markup. Thus, if all firms have zero markups, there will be no wedge between actual output and the quality index.

Growth of output, the quality index, and aggregate productivity. From (11), the growth rate of output $Y(t)$ is

$$
\frac{\dot{Y}(t)}{Y(t)} = \underbrace{\frac{\dot{Q}(t)}{Q(t)}}_{\substack{\text{Growth of} \\ \text{quality index}}} \underbrace{- \sum_{s \in S^+} \dot{\mu}_s(t) \ln \psi_s}_{\substack{\text{Change in average} \\ \text{markup}}} . \tag{12}
$$

Productivity growth is increasing in the growth of the quality index and decreasing in the change in the average markup. The quality index grows at rate $g^Q(t)$

$$
g^Q(t) \equiv \frac{\dot{Q}(t)}{Q(t)} = \ln \lambda \sum_{s \in S^+} \mu_s(t) x_s(t)(1+\mathbb{1}_{s=0}), \tag{13}
$$

reflecting that innovations by tied and leader firms increase in the productivity of technologies in use for production.

Aggregate labor is

$$
L(t) = \underbrace{\sum_{s \in S^+} \mu_s(t) \psi_s^{-1}}_{\text{Production labor, } L_p(t)} + \underbrace{\sum_{s \in \{1,\ldots,\bar{s}\}} \Big[\mu_s(t)(G(x_s(t)) + G(x_{-s}(t)))\Big] + 2\mu_0(t) x_0(t)}_{\text{R\&D labor}} \tag{14}
$$

where production labor in an industry with gap S is obtained from (2). Aggregate productivity $Z(t)$ is defined as the ratio of aggregate output to aggregate production labor, or $Z(t) \equiv Y(t)/L_p(t)$. Using (12) and the expression for production labor in (14), aggregate productivity growth $g^{TFP}(t)$ is

$$g^{TFP}(t) = \frac{d \ln \frac{Y(t)}{L_p(t)}}{dt} = \underbrace{\frac{\dot{Q}(t)}{Q(t)}}_{\text{Growth of quality index}} - \underbrace{\sum_{s \in S^+} \dot{\mu}_s(t) \ln \psi_s}_{\text{Change in average markup}} - \underbrace{\sum_{s \in S^+} \dot{\mu}_s(t) \frac{l_s}{L_p(t)}}_{\text{Growth of aggregate production labor}} . \tag{15}$$

Definition. Given a time-0 gap distribution $\mu(0) = \{\mu_s(0)\}_{s \in S^+}$ and a path for patent policy $\{\eta(t)\}_{t \in [0,\infty)}$, an allocation is a sequence

$$\{r(t), w(t), g(t), \{p_j(t), y_j(t), l_j(t)\}_{j \in [0,1]}, \{x_\sigma(t)\}_{\sigma \in S}, \{\mu_s(t)\}_{s \in S^+}, L(t), Y(t), C(t), Q(t)\}_{t \in [0,\infty)}, \tag{16}$$

such that (i) the sequence of prices, quantities, and labor demand $\{p_j(t), y_j(t), l_j(t)\}$ satisfies (2) and maximizes the operating profits of each firm in industry j; (ii) $\forall \sigma$, $x_\sigma(t)$ is a best response to $\{x^c_\sigma(t)\}_{\sigma \in S}$ (i.e., $x_\sigma(t)$ satisfies first order condition (6) where the value function $\{v_\sigma(t)\}_{t \in [0,\infty), \sigma \in S}$ is the solution to (4)-(5)); (iii) $\forall \sigma$, $x^c_\sigma = x_\sigma$ (symmetry); (iv) $Y(t)$, $L(t)$, and $Q(t)$ are given by (9), (10), and (14); (v) $C(t) = Y(t)$; (vi) the distribution of technology gaps $\{\mu_s(t)\}_{s \in S^+}$ satisfies (7)–(8); (vii) interest rate $r(t)$ satisfies the Euler equation (3); (viii) aggregate productivity growth $g(t)$ is given by (15).

A BGP equilibrium is an allocation with constant patent expiry rate η, output and productivity growth g, scaled wage ω, aggregate labor L, innovation rates $\{x_\sigma\}_{\sigma \in S}$, and technology gaps $\{\mu_s\}_{s \in S^+}$.

Computing the equilibrium. To obtain the allocation (16), we first solve the steady-state version of the HJB equations (4)-(5) (i.e., with $\dot{v}_\sigma(t) = 0$ and $\eta(t) = \eta_2$). Internet Appendix A.1 provides a result that $x_\sigma(t)$ and $v_\sigma(t)$ are equal to their BGP values for $t \geq T$. The path for $\{v_\sigma(t)\}_{t \in [0,T)}$ is then obtained as the solution to a system of differential equations (HJB equations (4)-(5)) with the boundary conditions that $v_\sigma(T)$ equals its BGP value $\forall \sigma \in S$. Then, the path for firms' innovation strategy $\{x_\sigma(t)\}_{t \in [0,\infty)}$ is obtained from (6). We obtain $\{\mu_s(t)\}_{t \in [0,\infty)}$ as the solution to the system of differential equations (7) and (8), conditional on $\{x_\sigma(t)\}_{t \in [0,\infty)}$. Finally, we obtain the quality index growth rate from (13) and the productivity growth rate from (15). Internet Appendix A.1 describes this procedure in detail.

3. Time-consistent patent policy

3.1 Preliminaries

Following Kydland and Prescott (1977) (hereafter, KP), Lucas and Stokey (1983), and Debortoli et al. (2021), the main insights about time-consistent patent policy can be grasped using a model in which the planner makes decisions at two points in time. In this section, patent policy is a duple that consists of an initial expiry rate η_1 between time 0 and time T, and a long-run expiry rate η_2 from time T onward. Let x_1 be a path for firms' innovation strategy $\{x_\sigma(t)\}_{\sigma \in S}$, for $t < T$. Similarly, x_2 is a path for firms' innovation strategy for $t \geq T$. From (1) and (9), households' welfare given firms' innovation strategy (x_1, x_2) is

$$\mathbb{W}(x_1, x_2, \eta_1, \eta_2) = \underbrace{\int_{t=0}^{\infty} e^{-\rho t} \ln Q(t) dt}_{\text{Productivity index component}} + \underbrace{\int_{t=0}^{\infty} e^{-\rho t} \Big(- \sum_{s=0}^{\bar{s}} \mu_s(t) \ln \psi_s - L(t) \Big) dt}_{\text{Markup distortions and disutility of labor}}, \tag{17}$$

where $\{Q(t), \mu(t), L(t)\}_{t \in [0,\infty)}$ are the path of the quality index, the gap distribution, and aggregate labor consistent with (x_1, x_2) and the policy duple (η_1, η_2). That is, $\mu(t)$ is determined by the inflow-outflow equations (7) and (8) conditional on the paths of firms' strategy x_1 and x_2 and the patent policy (η_1, η_2).

Let $X_1(\eta_1, \eta_2)$ be a mapping from a patent policy duple (η_1, η_2) into the *equilibrium* path for firms' innovation strategy prior to time T *conditional* on the patent policy (η_1, η_2). Also, define $X_2(x_1, \eta_1, \eta_2)$ as the equilibrium path for firms' innovation strategy beyond T, conditional on firms' innovation strategies in the past (x_1), the patent policy duple (η_1, η_2), and the initial gap distribution $\mu(0)$.[6]

3.2 Planner problems under commitment and discretion

Using the definitions of firms' actions in Section 3.1, this section states the planner problems using KP's general framework for a two-period policy game with policies (η_1, η_2),

[6]An equilibrium path conditional on firms' innovation strategies in the past (x_1), patent policy (η_1, η_2), and the initial gap distribution $\mu(0)$ is an equilibrium as defined in Section 2.1, except that firms' innovation strategies between 0 and T are given by x_1 and do not necessarily satisfy the first order condition (6).

agents' decisions (x_1, x_2), and a non-specific welfare function $\mathbb{W}(x_1, x_2, \eta_1, \eta_2)$. In the next section, we return to our specific setting of endogenous growth with creative destruction to further characterize the equilibrium of the policy game.

Optimal policy with commitment. With commitment, the planner chooses $(\eta_1, \eta_2) \in \mathbb{R}^2_+$ to maximize (17) subject to

$$x_1 = X_1(\eta_1, \eta_2) \tag{18}$$

$$x_2 = X_2(x_1, \eta_1, \eta_2), \tag{19}$$

where $\mathbb{R}_+$ is the set of weakly positive real numbers. We denote the optimal policy under commitment as $\eta^{C,*} = (\eta_1^{C,*}, \eta_2^{C,*})$.

Planner problem at T, without commitment. With a finite horizon, the policy game under discretion can be solved backward. Without commitment, the planner at T chooses the long-run expiry rate to maximize welfare (17), taking as given past (sunk) policy decision η_1, private decisions x_1, and the constraint (19). That is, conditional on x_1 and η_1, the planner at T chooses the long-run expiry rate

$$\tilde{\eta}_2(x_1, \eta_1) = \arg\max_{\eta_2} \mathbb{W}(x_1, X_2(x_1, \eta_1, \eta_2), \eta_1, \eta_2). \tag{20}$$

In a Markov Perfect Equilibrium without commitment, the planner's choice at T depends on x_1 and η_1 only through the payoff-relevant state variable, the time-T gap distribution $\mu(T)$. For a duple (η_1, η_2) to be time-consistent, it must satisfy

$$\eta_2 = \tilde{\eta}_2(X_1(\eta_1, \eta_2), \eta_1) \tag{21}$$

A duple (η_1, η_2) is time-consistent if firm R&D decisions prior to T (conditional on (η_1, η_2)) are such that the long-run expiry rate chosen by a planner with discretion at time T ($\tilde{\eta}_2$) is in fact the long-run expiry rate expected by firms (η_2). The set, η^{TC}, of all time-consistent duples is

$$\eta^{TC} \equiv \{(\eta_1^{TC}, \eta_2^{TC} \in \mathbb{R}^2_+ : \eta_2^{TC} = \tilde{\eta}_2(X_1(\eta_1^{TC}, \eta_2^{TC}), \eta_1^{TC})\}. \tag{22}$$

Optimal time-consistent policy. The optimal policy without commitment maximizes (17) subject to (18), (19), and the requirement that the duple is time-consistent (i.e., $(\eta_1, \eta_2) \in$

η^{TC}), expression (22). Thus, the key implication of discretion is that the planner must choose (η_1, η_2) from the set of time-consistent duples η^{TC}, whereas the planner with commitment can choose any duple of weakly positive patent expiry rates (i.e., $(\eta_1, \eta_2) \in \mathbb{R}_+^2$). We denote the optimal time-consistent policy (duple) as $\eta^{TC,*} = (\eta_1^{TC,*}, \eta_2^{TC,*})$.

3.3 Time consistency in the Aghion-Howitt framework

3.3.1 Consistent planner's problem at T

For given x_1 and η_1, assuming differentiability and an interior solution, the first order condition of the consistent planner's problem at time T is

$$\int_T^\infty \sum_{\sigma \in S} \left(\frac{\partial \mathbb{W}}{\partial x_\sigma(t)} \frac{\partial x_\sigma(t)}{\partial \eta_2} \right) dt + \frac{\partial \mathbb{W}}{\partial \eta_2} = 0. \tag{23}$$

The consistent policy takes into account how the long-run expiry rate η_2 affects welfare through firms' strategy beyond T (the integral in (23)) and through the direct effect of patent expiry on the gap distribution from T onward (the second term, $\frac{\partial \mathbb{W}}{\partial \eta_2}$). The consistent policy ignores the effects of η_2 on firms' strategy prior to T. In contrast, under commitment, at time T, the first order condition for η_2 is

$$\begin{aligned} &\int_T^\infty \sum_{\sigma \in S} \left(\frac{\partial \mathbb{W}}{\partial x_\sigma(t)} \frac{\partial x_\sigma(t)}{\partial \eta_2} \right) dt + \frac{\partial \mathbb{W}}{\partial \eta_2} \\ &+ \int_0^T \sum_{\sigma \in S} \frac{\partial x_\sigma(t)}{\partial \eta_2} \left[\frac{\partial \mathbb{W}}{\partial x_\sigma(t)} + \int_T^\infty \sum_{\sigma' \in S} \frac{\partial x_{\sigma'}(t')}{\partial x_\sigma(t)} \frac{\partial \mathbb{W}}{\partial x_{\sigma'}(t')} d(t') \right] dt = 0. \end{aligned} \tag{24}$$

Firms' strategy prior to T can affect welfare ($\mathbb{W}$) both directly and indirectly through firms' strategy beyond T. If, for all $t < T$ and $\sigma \in S$, the effect of η_2 on $x_\sigma(t)$ is zero (i.e., $\frac{\partial x_\sigma(t)}{\partial \eta_2} = 0$) *or* $x_\sigma(t)$ has no effect on welfare both directly (i.e., $\frac{\partial \mathbb{W}}{\partial x_\sigma(t)} = 0$) and indirectly through firms' strategy beyond T (i.e., $\int_T^\infty \sum_{\sigma' \in S} \frac{\partial x_{\sigma'}(t')}{\partial x_\sigma(t)} \frac{\partial \mathbb{W}}{\partial x_{\sigma'}(t')} d(t') = 0$), then optimal consistent and commitment policies are the same. However, if these conditions fail to hold for some $t < T$ and $\sigma \in S$, then, generically, the optimal consistent policy is not the optimal policy under commitment. We now discuss each of these conditions.

Unpacking $\frac{\partial x_\sigma(t)}{\partial \eta_2}$ for $t < T$. Firms' strategy $x_\sigma(t)$ is determined by the capital gain from innovation Δv_σ (Eq.(6)). The expiry rate beyond T affects this capital gain prior to T through two channels. First, stronger protection beyond T increases the capital gain from an innovation prior to T if such an innovation increases expected profits beyond T. This is the channel—firms' expectations of future protection for their present innovations—emphasized by Kydland and Prescott (1977) and Kremer and Williams (2010).[7]

Second, the long-run expiry rate affects the capital gain from innovation prior to T through a strategic channel. The capital gain from innovation at $t < T$ depends on *competitors*' strategy beyond t (expressions (4)–(5)). Higher innovation beyond t by laggards in competitive industries increases the capital gain from an innovation at t for leaders in competitive industries, because innovations allow such leaders to *escape competition* (Aghion and Howitt (1992)). In contrast, higher innovation beyond t by laggards in uncompetitive industries reduces the capital gain from innovation for a leader in a competitive industry at t: This so-called *trickle-down* effect arises because an innovation at t for a leader in a competitive industry makes this leader more likely to face far-behind laggards beyond t (Acemoglu and Akcigit (2012)). Through this intertemporal strategic channel, the effects of the long-run expiry rate cascade backward in time through firms' expectations about their competitors' future strategy.

Unpacking $\frac{\partial \mathbb{W}}{\partial x_\sigma(t)}$, for $t < T$. Changes in firms' strategy prior to T are expected to affect welfare. In particular, a higher innovation rate for a firm in position σ at time t contributes directly to the growth rate of the quality index at t, if the firm is tied or a leader ($\sigma \geq 0$) and if there is positive mass of industries with gap $|\sigma|$. A faster growth rate, $\dot{Q}(t)/Q(t)$, increases the level of the quality index from t onward, which appears in the welfare expression (17). A higher innovation rate for a firm in position σ at time t also affects welfare through the change at t in the gap distribution (as shown in expressions (7)–(8)), thereby affecting the entire path of the gap distribution from t onward. The gap distribution determines the

[7] Kydland and Prescott (1977) and Kremer and Williams (2010) discuss the time-consistency problem through the lens of the policy tradeoff in Nordhaus (1969), which ignores strategic interactions and other aspects of creative destruction emphasized in the Aghion-Howitt literature. For example, per Kydland and Prescott (1977), "The question would be posed in terms of the optimal patent life (see, e.g., Nordhaus (1969)), which takes into consideration both the incentive for inventive activity provided by patent protection and the loss in consumer surplus that results when someone realizes monopoly rent."

markup components of welfare; it also affects productivity growth (expression (13)).

Unpacking $\int_T^\infty \sum_{\sigma' \in S} \frac{\partial x_{\sigma'}(t')}{\partial x_\sigma(t)} \frac{\partial \mathbb{W}}{\partial x_{\sigma'}(t')} d(t')$ **for** $t < T$. This term captures how firms' strategy prior to T affects welfare indirectly through firms' strategy beyond T. A key component of this term is $\frac{\partial x_{\sigma'}(t')}{\partial x_\sigma(t)}$, the effect of firms' strategy prior to T on firms' strategy beyond T. This component could be non-zero because firms' strategy prior to T affects the gap distribution beyond T. From the HJB equations (4)-(5), the gap distribution beyond T can only affect firms' innovation strategy beyond T through the scaled wage. With log preferences for consumption and linear disutility of labor (expression (1)), the scaled wage is constant and equals 1. Hence, in our model, $\frac{\partial x_{\sigma'}(t')}{\partial x_\sigma(t)} = 0$. More generally, however, when labor supply is imperfectly elastic, the scaled wage in Aghion-Howitt models does depend on the gap distribution, because the gap distribution affects labor demand (14).[8]

3.3.2 Consistent planner's problem at time 0

The initial expiry rate η_1 affects the dynamics of the gap distribution at time T, thereby affecting the long-run expiry rate (and expectations of the long-run expiry rate) in a time-consistent equilibrium. The first order condition for the initial expiry rate η_1 for the consistent planner is:[9]

$$\int_0^T \sum_{\sigma \in S} (\frac{\partial \mathbb{W}}{\partial x_\sigma(t)} \frac{\partial x_\sigma(t)}{\partial \eta_1}) dt + \frac{\partial \mathbb{W}}{\partial \eta_1} - \zeta \frac{d\tilde{\eta}_2}{d\eta_1} = 0 \tag{25}$$

where ζ is the Lagrange multiplier for the time-consistency constraint (21). The terms $\int_0^T \sum_{\sigma \in S} (\frac{\partial \mathbb{W}}{\partial x_\sigma(t)} \frac{\partial x_\sigma(t)}{\partial \eta_1}) dt + \frac{\partial \mathbb{W}}{\partial \eta_1}$ are the effect on welfare from increasing the initial expiry rate η_1, for a given long-run expiry rate η_2. The term $\frac{d\tilde{\eta}_2}{d\eta_1}$ is the change in the time-consistent long-run expiry rate from increasing the initial expiry rate. As shown in Section 3.3.1, the constraint (21) is binding under time-consistent policy. The consistent planner at time 0

[8] As in Aghion et al. (2001) and others, we assume perfectly elastic labor supply, making the analysis of transition dynamics tractable. With this assumption, there are no indirect effects of firms' strategy prior to T on firms' strategy beyond T. However, we do highlight that this channel can be present under alternative preferences.

[9] Note that Eq. (25) reflects that, with perfectly elastic labor supply, $\frac{\partial x_\sigma(t)}{\partial \eta_1} = 0$ and $\frac{\partial x_\sigma(t)}{\partial x_{\sigma'(t')}} = 0$, $\forall (\sigma, \sigma') \in S \times S, t \in [T, \infty]$, and $t' \in [0, T]$. That is, firms' innovation strategy beyond T does not depend on the expiry rate prior to T or firms' innovation strategy prior to T. This arises because, with perfectly elastic labor supply, firms' innovation strategy does not depend on the gap distribution.

takes the mapping $\tilde{\eta}_2$ as given but understands how the time-consistent long-run expiry rate changes with the time-T gap distribution and hence with η_1.

The Lagrange multiplier for the time-consistency constraint (21) is:

$$\zeta = -\frac{1}{1 - \int_0^T \frac{\partial \tilde{\eta}_2}{\partial x_\sigma(t)} \frac{\partial x_\sigma(t)}{\partial \eta_2} dt} \times \\ \left(\int_T^\infty \sum_{\sigma \in S} \left(\frac{\partial \mathbb{W}}{\partial x_\sigma(t)} \frac{\partial x_\sigma(t)}{\partial \eta_2} \right) dt + \frac{\partial \mathbb{W}}{\partial \eta_2} + \int_0^T \sum_{\sigma \in S} \frac{\partial x_\sigma(t)}{\partial \eta_2} \left[\frac{\partial \mathbb{W}}{\partial x_\sigma(t)} + \int_T^\infty \sum_{\sigma' \in S} \frac{\partial x_{\sigma'}(t')}{\partial x_\sigma(t)} \frac{\partial \mathbb{W}}{\partial x_{\sigma'}(t')} d(t') dt \right] \right) \tag{26}$$

The time-consistency constraint (21) requires that there is zero gap between the long-run expiry rate (η_2) expected by firms prior to T and the long-run expiry rate ($\tilde{\eta}_2$) chosen by the planner at time T with discretion. This gap is $\eta_2 - \tilde{\eta}_2(\eta_2)$. An increase in the expected long-run expiry rate changes this gap by $1 - \frac{d\tilde{\eta}_2}{d\eta_2}$, giving rise to the multiplier term on the first line of Eq. (26). If the long-run expiry rate expected by firms has no effect on firms' decisions prior to T (and hence no effect the long-run rate chosen at T under discretion), then this multiplier is equal to 1. The remainder of expression (26) is familiar from (24): it is the marginal welfare benefit of increasing the long-run expiry rate, including through firms' innovation strategy over the initial period. With commitment, the time-consistency constraint (21) is not imposed and the marginal welfare benefit of increasing the long-run expiry rate is zero. The first order condition for the initial expiry rate η_1 under commitment is hence the same as (25) but with the Lagrange multiplier ζ equal to 0.

Thus, the consistent planner at time 0 takes into account how the initial expiry rate affects the endogenous technology gap distribution at time T and hence the long-run expiry rate and welfare in a time-consistent equilibrium. We note that the elasticities that determine the marginal benefits of increasing the initial expiry rate ($\frac{\partial x_\sigma(t)}{\partial \eta_1}$, $\frac{\partial \mathbb{W}}{\partial x_\sigma(t)}$, $\frac{\partial \mathbb{W}}{\partial \eta_1}$) reflect strategic and composition effects, similar to the elasticities that determine the marginal benefits of increasing the long-run expiry rate (as discussed in Section 3.3.1).

Relation to Kydland and Prescott (1977). Internet Appendix B shows how the patent policy's time-consistency problem can be mapped exactly into KP's general analysis of time-consistency problems. By analyzing time consistency in an explicit Aghion-Howitt framework, we uncover and characterize new aspects of patent policy's time-consistency problem in the presence of creative destruction—related to firms' strategic interactions, composition effects, and general-equilibrium wage effects. An additional advantage of an explicit Aghion-Howitt framework is that we can analyze the problem quantitatively, as in the remainder of the paper.

3.4 Computing optimal time-consistent policy

Our quantitative analysis takes a dual approach and uses a global solution method. We first characterize the set of time-consistent duples η^{TC} (Eq. (22)). For a given η_1, we solve the fixed-point problem (21) using the solution method for private-sector equilibrium described in Section 2.1. As shown in Section 4.3, in our calibrated model, we find that, for each η_1, Eq. (21) has a unique solution. Repeating these steps for N values of η_1, with N large, we obtain N elements of η^{TC} and find that the set $(\eta_1^{TC}, \eta_2^{TC})$ is very well approximated using a spline. Using this approximation, we maximize welfare (expression (17)) subject to Eqs. (18), (19), and the requirement that the duple is time-consistent (i.e., $(\eta_1, \eta_2) \in \eta^{TC}$), again making use of the private-sector equilibrium solution method for each duple for which we evaluate expression (17).

Further weakening commitment. Thus far, we have introduced a minimal departure from the literature's benchmark of full commitment; that is, the planner makes decisions at times $t = 0$ and $t = T$. In Section 7, we weaken commitment further, studying a setting with planner decisions at $t \in \{0, T, 2T\}$.

4. Results

4.1 Calibration

We set the discount rate to 3% and the corporate tax rate to 20%. We externally calibrate the R&D curvature parameter $\gamma = 0.33$, following Kortum (1993), Acemoglu and Akcigit (2012), and Akcigit and Ates (2022). We calibrate (ϕ, λ, B, η). Section 6 provides robustness.

The targeted moments, listed in Table 1, are: productivity growth, equal to average total factor productivity growth from Fernald et al. (2017); the mean and several upper percentiles of the industry-level markup distribution (to capture the right tail), from Hall (2018); and R&D-to-GDP, from the U.S. Bureau of Economic Analysis. We also target the adjusted "within" moment of Foster et al. (2001) (hereafter, FHK), which captures the share of continuing firms' productivity growth attributable to those firms' productivity improvements holding market shares constant (i.e., absent reallocation).[10]

Our sample period for productivity growth and R&D-to-GDP is 2000-2019, a period of low productivity growth and reduced business dynamism (e.g., Olmstead-Rumsey (2020), Akcigit and Ates (2022)).[11] For the markup moments, our sample period is 2000-2015, matching the end-date of Hall's (2018) analysis. Data for the FHK decomposition is from Foster et al. (2008). We calibrate the parameters (ϕ, λ, B, η) using the simulated method of moments. We compare moments in the model and the data, choosing the parameters that minimize the criterion

$$\min \sum_{m=1}^{8} weight_m \left(\frac{|\text{model}(m) - \text{data}(m)|}{\frac{1}{2}|\text{model}(m)| + \frac{1}{2}|\text{data}(m)|} \right). \tag{27}$$

Because there are five moments characterizing the markup distribution, we give these five moments a weight half as large as the weights given to the other moments.

The model-implied moments fit the data well (Table 1). The calibrated patent expiry rate η is 2.1% per year, indicating strong protection of intellectual property rights. Only $\phi = 7.9\%$ of laggard innovations close the technology gap; all other laggard innovations are incremental. Each rung on the productivity ladder represents a productivity improvement of 4.7%. Next, we study an economy in which the planner at $t = 0$ sets an initial expiry rate knowing that the planner at T chooses the long-run expiry rate. We set $T = 25$ years, consistent with evidence in Lerner (2002) that countries have undertaken major changes to their patent policies at roughly this frequency. Section 6 studies the implications of varying T.

[10]See Internet Appendix C for a full description of the FHK within moment. Continuing firms are firms that do not enter or exit during the sample period.

[11]In Section 6, we show that our main conclusions are robust to varying parameter values and initial conditions.

Table 1: **Parameters and moments.**

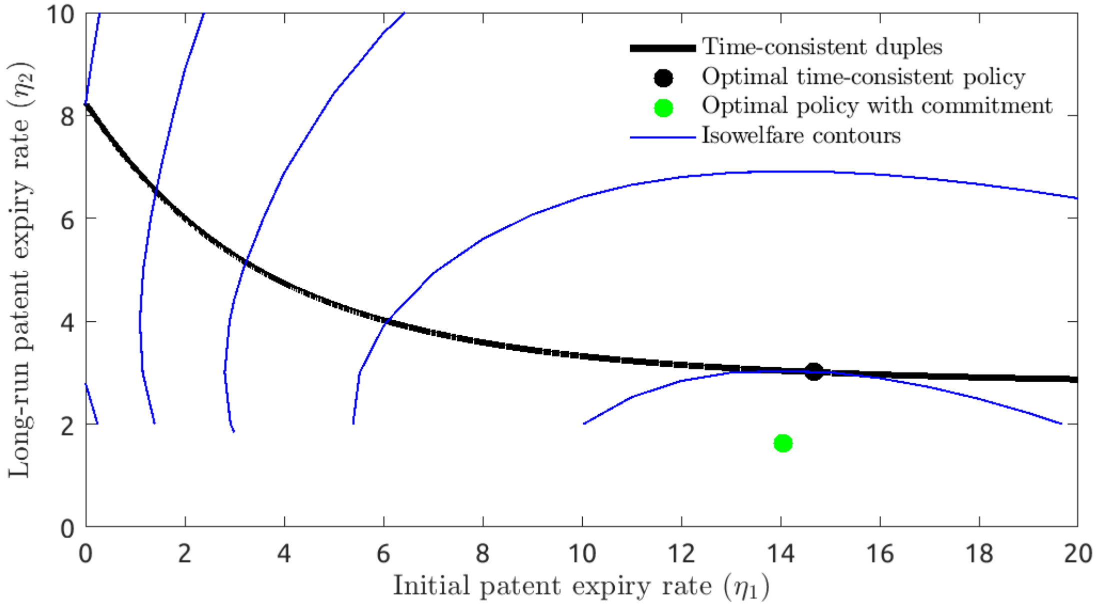
Parameter Estimates		Moments used in estimation		
Parameter	Value	Description	Model	Data
ϕ	0.079	Productivity growth	0.76 %	0.76 %
λ	1.047	R&D to GDP	4.45 %	4.43 %
B	0.480	Markups		
η	0.021	Mean	30.4 %	34.7 %
		75th percentile	38.0 %	45.1 %
		90th percentile	68.4 %	68.4 %
		95th percentile	93.0 %	87.0 %
		99th percentile	156.1 %	135.7 %
		FHK within	88.0 %	90.8 %

4.2 Optimal policy under commitment

Under commitment, optimal patent policy calls for weak patent protection during the initial period (between 0 and T), followed by a strengthening of patent protection, with a long-run expiry rate of 1.63%, below the calibrated patent expiry rate. The optimal duple is shown by the green circle in Figure 1. (The high initial expiry rate reflects the sclerotic nature of the calibrated economy, as discussed further in Section 6.) Each blue iso-welfare line in Figure 1 shows the set of duples that give rise to the same welfare. With commitment, the planner can choose any weakly positive duple.

As shown in Section 3, the optimal policy under commitment is expected to be time-inconsistent. That is, in setting the long-run expiry rate η_2, the time-0 planner under commitment takes into account how η_2 affects firms' innovation strategies prior to T, but the forward-looking planner at time T treats those past decisions as sunk. Consider the problem of the consistent planner at T, if past decisions are those associated with optimal policy under commitment (i.e., $x_1 = X_1(\eta_1^{C,*}, \eta_2^{C,*})$ and $\eta_1 = \eta_1^{C,*})$). Rather than choose the commitment long-run expiry rate of 1.63%, the consistent planner at T would choose

Figure 1: **Time-consistent patent expiry schedules and welfare**



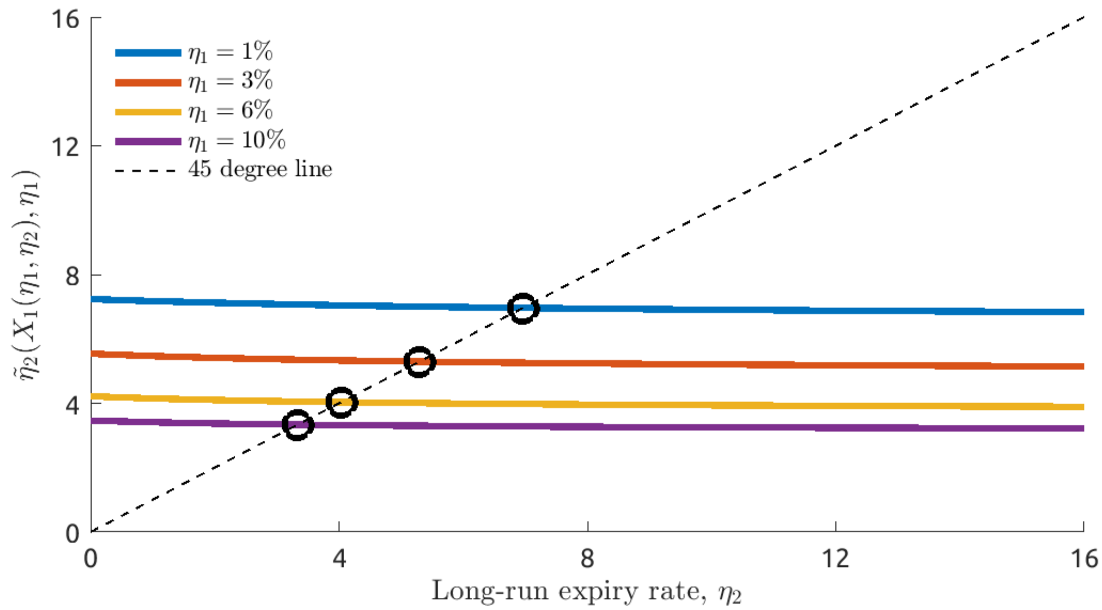
$\tilde{\eta}_2(X_1(\eta_1^{C,*}, \eta_2^{C,*}), \eta_1^{C,*}) = 3.08\%$. Thus, in our calibrated model, the optimal policy under commitment is not time-consistent. (Put differently, the optimal commitment policy is not a fixed point of Eq. (20).)

4.3 Characterization of the set of time-consistent policies

The black solid line in Figure 1 depicts the set of time-consistent expiry rate duples $(\eta_1, \eta_2) \in \eta^{TC}$ (Expression (22)). There are three key properties.

Property 1. Uniqueness. As shown in Figure 1, for each initial expiry rate η_1, there is a unique long-run expiry rate η_2 such that the duple (η_1, η_2) is time-consistent. Figure 2 illustrates the determination of the time-consistent long-run expiry rate (Eq. (21)) for four different values of the initial expiry rate. For each initial expiry rate, the long-run expiry rate $\tilde{\eta}_2$ chosen at time T by the planner with discretion is shown as a function of the long-run expiry rate expected by firms prior to T. The time-consistent long-run expiry rate

Figure 2: **Determination of the set of time-consistent duples**



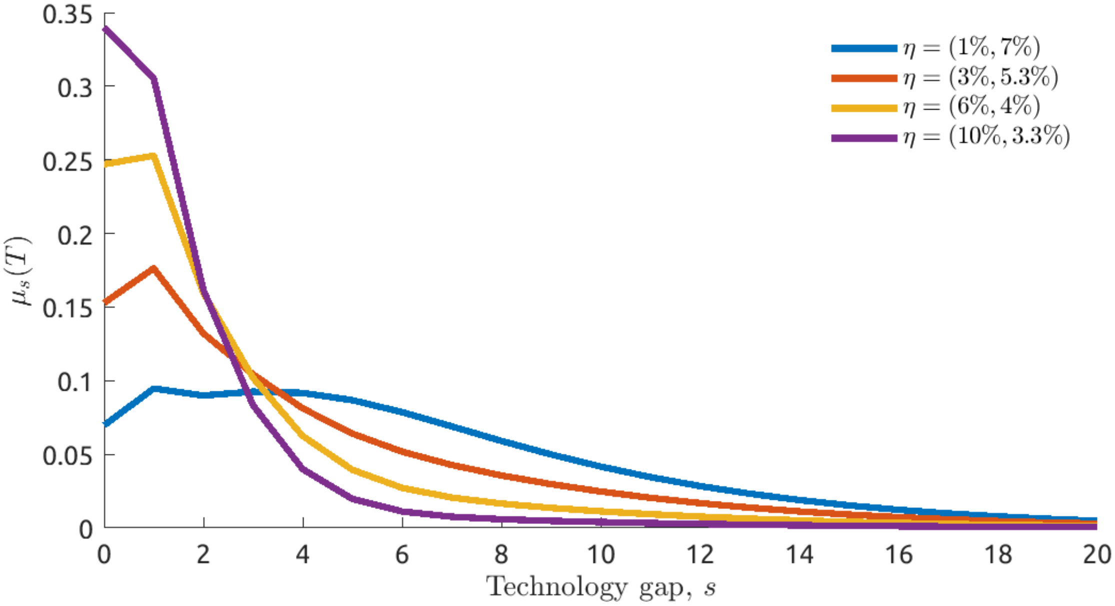
This figure illustrates the fixed point problem (20)–(21). See Section 4.3 for details.

is the unique intersection of $\tilde{\eta}_2$ with the 45-degree line. The long-run rate chosen under discretion at T is weakly decreasing in the expected long-run rate, a sufficient condition for uniqueness. The slope of $\tilde{\eta}_2(\eta_2)$ reflects that *expectations of* weaker long-run protection induce less R&D prior to time T, implying an economy with smaller technology gaps at time T and hence stronger protection actually chosen at time T.

Property 2. Downward-sloping. The schedule of time-consistent duples in Figure 1 is downward-sloping: a lower initial expiry rate η_1 is associated with a higher long-run expiry rate η_2. As shown by Figure 3, comparing duples along this schedule, the duple with a higher initial expiry rate (weaker initial protection) implies that the planner at T inherits an economy that is more competitive, with lower markups and technology gaps. With a more competitive economy, the planner at T chooses relatively strong long-run protection.[12]

[12]Put differently, $\tilde{\eta}_2(X_1(\eta_1, \eta_2), \eta_1)$ is decreasing in η_1, as shown in Figure 2.

Figure 3: **Distribution of technology gaps at time T**



This figure shows the distribution of technology gaps at time T under different time-consistent duples.

Property 3. Lower bound on the long-run expiry rate. The schedule of time-consistent duples flattens out at high values of the initial expiry rate. No matter how high the planner sets the initial patent expiry rate, the long-run expiry rate is always greater than 2.6%. The optimal long-run expiry rate under commitment, $\eta_2^{C,*} = 1.63\%$, therefore is not part of any time-consistent duple.

In Section 6, we study how these properties are affected by key features of the creative destruction process, such as the speed of laggard catch-up through innovation (parametrized by ϕ), the initial distribution of technology gaps and markups (parametrized by $\mu(0)$), and the R&D production function (parametrized by B and γ). When we vary these parameters, the three key properties continue to obtain—uniqueness; the negative slope of the schedule of time-consistent duples; and the lower bound on the long-run expiry rate. However, varying these parameters does affect the time-consistency problem, shifting the slope and lower bound of the schedule of time-consistent duples.

4.4 Optimal time-consistent policy

The optimal time-consistent duple corresponds to the tangency of the schedule of time-consistent duples with the iso-welfare line in Figure 1, marked by a black solid circle. The main difference relative to commitment is that long-run protection is much weaker under time-consistent policy. This difference corresponds to an expected life of a patent (absent creative destruction through competitors' innovation) of $1/\eta_2^{TC,*} \approx 60$ years under commitment, but $1/\eta_2^{TC,*} \approx 30$ years under discretion. Weak long-run protection is guaranteed by the shape of the schedule of time-consistent duples; no matter the initial patent expiry rate, the time-consistent long-run expiry rate is well above the long-run expiry rate under commitment. This results accords with the basic insight of Kydland and Prescott (1977) that patent policy under commitment offers stronger protection than time-consistent patent policy. Relative to discretion, in our calibrated model the initial patent protection is also stronger under commitment, but this difference (measured in the expected life of a patent) is quantitatively smaller.[13]

What determines the strength of initial protection under optimal time-consistent policy, relative to under commitment? Suppose that, starting from the optimal time-consistent duple (black circle in Figure 1), the consistent planner at time 0 would have moved to the left and upward along the schedule of time-consistent duples (black line). That is, the planner would have considered a time-consistent policy with stronger initial protection (lower initial expiry rate) and weaker long-run protection (higher long-run expiry rate). Such a deviation in policy induces two opposing effects on welfare. First, from Property 3 above it follows that the consistent planner long-run protection is very weak (relative to optimal long-run protection under commitment) everywhere along the set of time-consistent duples. Hence choosing strong initial protection (holding constant the long-run expiry rate) *increases* welfare. This increases arises because the initial patent protection offsets the disincentive for R&D coming expectations from weak long-run protection. Second, again because long-run protection is weak, a higher long-run expiry rate *decreases*

[13]The result that initial protection under time-consistent policy is weaker than under commitment can reverse under alternative parametrizations, depending on parameters governing the creative-destruction process, as shown in Section 6 below.

welfare. At the optimal time-consistent duple, by definition, these two forces are exactly offsetting, for a marginal move along the set of time-consistent duples.[14]

Figure 4 shows the implications of optimal time-consistent and commitment policies for firms' innovation strategy and the gap distribution, at a given point in time ($t = 2T$). Long-run patent protection is stronger under commitment and correspondingly commitment is associated with higher innovation rates, for firms in all technology positions. (For each long-run expiry rate, innovation is highest in the most competitive industries (the "escape competition" effect of Aghion and Howitt (1992)).) The stronger (initial and long-run) patent protection under commitment is also associated with a less competitive long-run gap distribution and thus higher markups. The results in Figure 4 are thus in accord with the tradeoff for patent policy identified by Nordhaus (1969): Stronger protection fosters more innovation and productivity growth, generally at the expense of higher markups and markup-related distortions. Internet Appendix D.1 illustrates the Nordhaus tradeoff in our model, taking account of transition dynamics and the time-consistency problem for patent policy.

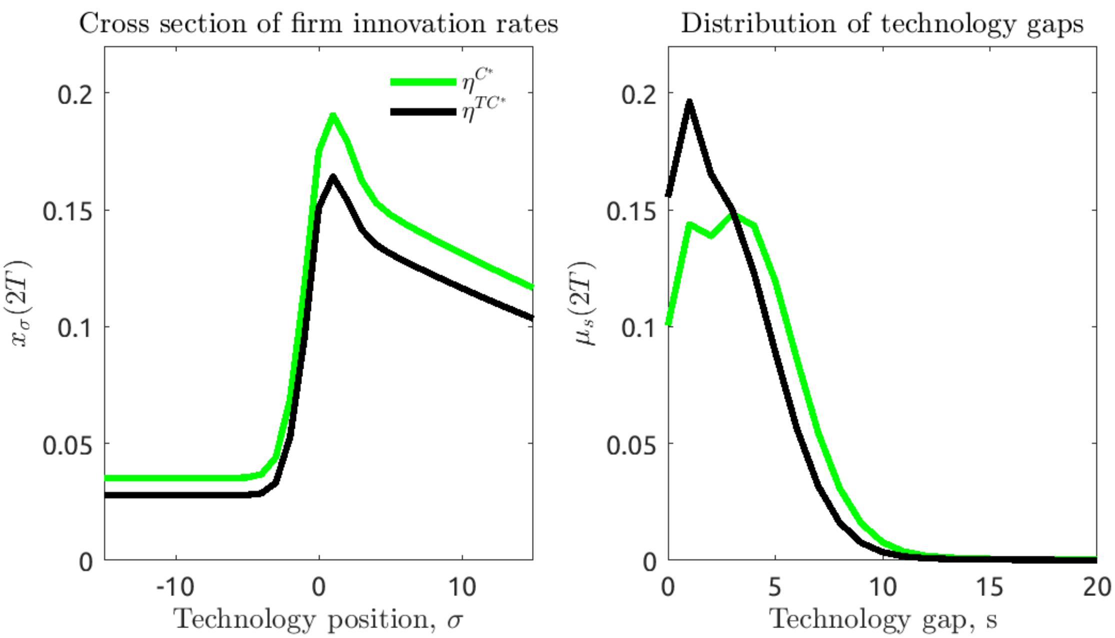
4.5 Dynamics of productivity growth and markups

Figure 5, left panel, shows the evolution of the quality-index and TFP growth rates, under the optimal time-consistent policy. The right panel shows the evolution of the markup distribution. The new policy offers less patent protection in the short-run and long-run, relative to the pre-announcement calibrated economy. Correspondingly, at time $t = 0$, when the policy is announced, firms reduce their R&D (x_σ declines) and growth of TFP and the quality index falls.[15] The announcement also triggers a sharp decline in markups, reflecting the mechanical effects of a patent expiry rate that is temporarily higher than prior to the reform at $t = 0$. The markup decline also reflects an endogenous reduction in

[14]As above noted, consistent with the basic intuition of KP, in the calibrated model, these forces imply that initial patent protection is weaker under optimal time-consistent policy than under commitment. However, this result can *reverse* under alternative parametrizations, depending on parameters governing the creative-destruction process, as shown in Section 6 below.

[15]TFP growth does not fall as much as growth of the quality index due to "composition terms" that create a wedge between the growth rates of TFP and the quality index when the economy is transitioning from one BGP to another (See Eq. (15)).

Figure 4: **Innovation strategy and gap distribution at $t = 2T$, under optimal consistent ($TC*$) and commitment ($C*$) policies**

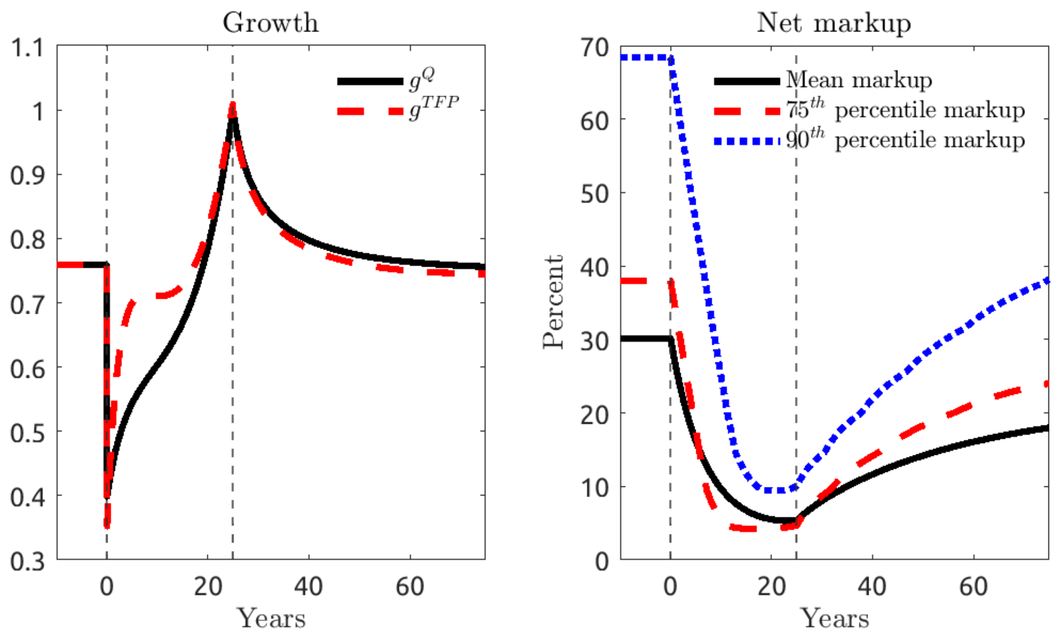


R&D by tied firms and leaders.

As T approaches, strengthening of patent protection (relative to the $[0, T]$ period) becomes nearer. Correspondingly, R&D increases (x_σ rises), TFP growth picks up, and there is slowing of the leftward shift of the markup distribution. At time T, the patent expiry falls, R&D and TFP growth rise, and the markup distribution shifts outward. In the long-run, TFP growth is slightly lower than the previous (calibrated) BGP and the average markup is about 10 percentage points lower.

Comparison with outcomes under commitment. Table 2 shows how the evolution of the patent expiry rate, productivity growth, and the average markup differ under commitment and under time-consistent policy. Our main analysis is a setting, as in Lucas and Stokey (1983) and Debortoli et al. (2021), where a planner acts at two times, 0 and T. This setting is denoted "Optimizing at $\{0, T\}$" in Table 2. For the patent expiry rate, TFP growth, and the average markup, the first column shows the average outcome between times 0 and

Figure 5: **Productivity growth and markups under the optimal time-consistent policy.**



The left panel shows the evolution of the growth rates of the quality index Q and of TFP, if the optimal time-consistent policy is announced at time $t = 0$. The right panel shows the evolution of the markup distribution.

T; the second column, between T and $2T$; the third column, between $2T$ and $3T$; and the final column, along the long-run BGP.

Due to firms' anticipation of stronger long-run protection under commitment, productivity growth is initially slightly higher under commitment. The difference in productivity growth increases once the long-run expiry rate comes into effect, and remains positive in the longer run. The compounding effects of higher long-run productivity growth under commitment imply that time-0 welfare is higher under commitment than under time-consistent policy, even though markups are higher under commitment. We compare welfare under different policies in consumption-equivalent terms (Internet Appendix A.3). At time 0, households would be willing to forgo 0.17% of their consumption, in perpetuity, to switch from optimal time-consistent policy to optimal policy under commitment. Thus, even a minimal deviation from full commitment has important implications for produc-

Table 2: **Comparing outcomes under time-consistent and commitment policies**

	[0,T]	[T,2T]	[2T,3T]	Long-run BGP
Patent expiry rate				
Optimizing at {0,T}				
Time-consistent	14.64	3.02	3.02	3.02
Commitment	14.04	1.63	1.63	1.63
Optimizing at {0,T,2T}				
Time-consistent	15.69	6.51	3.39	3.39
Commitment	14.06	1.30	1.81	1.81
TFP growth				
Optimizing at {0,T}				
Time-consistent	0.74	0.81	0.75	0.75
Commitment	0.78	0.89	0.78	0.76
Optimizing at {0,T,2T}				
Time-consistent	0.68	0.77	0.77	0.74
Commitment	0.79	0.88	0.77	0.76
Average markup				
Optimizing at {0,T}				
Time-consistent	10.89	10.52	16.32	21.20
Commitment	11.38	12.87	22.42	38.72
Optimizing at {0,T,2T}				
Time-consistent	10.14	7.24	12.32	18.81
Commitment	11.38	13.32	22.64	35.23

This table characterizes the evolution of the patent expiry rate, TFP growth, and mean markup, under different assumptions about commitment and about the timing of patent expiry rate changes. Each column shows the average of the indicated outcome over the time span indicated in the column heading.

tivity growth, markups, and welfare.

5. Inspecting the sources of time-consistency problem

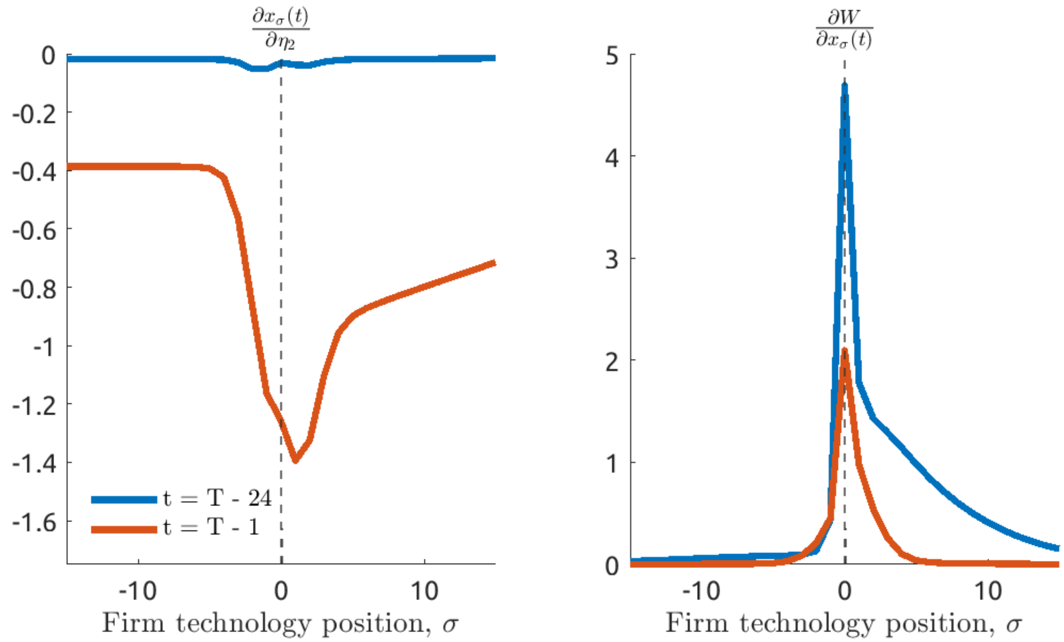
5.1 Effects of patent protection beyond T on welfare through firms' strategy prior to T

As shown in Section 3, the optimal time-consistent policy differs from the commitment policy if the expiry rate beyond T affects firms' strategy prior to T *and* firms' strategy prior to T affects welfare directly or indirectly. That is, if, for all $t < T$ and all $\sigma \in S$, the effect of η_2 on $x_\sigma(t)$ is zero (i.e., $\frac{\partial x_\sigma(t)}{\partial \eta_2} = 0$) or $x_\sigma(t)$ has no effect on welfare directly (i.e., $\frac{\partial \mathbb{W}}{\partial x_\sigma(t)} = 0$), then the optimal consistent policy is the optimal policy under commitment.[16] In our calibrated model, weaker patent protection beyond T reduces firms' innovation strategy $x_\sigma(t)$ for firms in all technology positions $\sigma \in S$ and for all $t < T$, as shown by the left panel in Figure 6. This effect is more powerful as the economy approaches time T (i.e., $\frac{\partial x_\sigma(t)}{\partial \eta_2}$ increases in magnitude as t rises), reflecting that, as T approaches, a greater share of the expected profits from an innovation will be earned beyond T, making patent protection beyond T more relevant for firms' decisions. As shown by the right panel of Figure 6, declines in firms' innovation strategy $x_\sigma(t)$ prior to T also have direct effects on welfare, for all $\sigma \in S$ and for all $t < T$. The channels giving rise to these effects include that higher innovation by tied and leader firms directly increase aggregate productivity.[17] These effects (i.e., $\frac{\partial \mathbb{W}}{\partial x_\sigma(t)}$) are generally larger at $t = 1$ than at $t = 24$ years, partly because increases in aggregate productivity are discounted in welfare (17). The dot product of the terms in the left and right panels of Figure 6 for given t (i.e., $\sum_{\sigma \in S} \frac{\partial x_\sigma(t)}{\partial \eta_2} \frac{\partial \mathbb{W}}{\partial x_\sigma(t)}$) are exactly the effects on welfare of η_2 through firms' innovation strategy at $t < T$, which the commitment planner accounts for but the consistent planner at T ignores.

[16] This conclusion is reached by comparing expressions (23) and (24) and then noting, as discussed in Section 3.3, that there are no indirect effects of $x_\sigma(t)$, for $t < T$, on welfare through firm's strategy beyond T in our model, due to perfectly elastic labor supply.

[17] See Section 3.3 for additional discussion.

Figure 6: **How does the time-consistency problem arise?**



Partial derivatives are evaluated at the equilibrium associated with optimal policy under discretion.

5.2 Strategic and composition effects

The effect of the long-run expiry rate on the growth rate of technological progress at time t is shown by the black line in the top panel of Figure 7. Using Eq. (13), this effect is decomposed into an intensive margin effect (blue) and a composition effect (red) as follows:

$$\frac{dg^Q(t)}{d\eta_2} = \underbrace{\sum_{s\in S^+} \mu_s(t)\frac{dx_s(t)}{d\eta_2}(1+1_{s=0})}_{\text{intensive margin}} + \underbrace{\sum_{s\in S^+} \frac{d\mu_s(t)}{d\eta_2}x_s(t)(1+1_{s=0})}_{\text{composition effect}} \tag{28}$$

A higher long-run expiry rate reduces the growth rate prior to T (an effect ignored by the planner with discretion at T). Firms' expectations of weaker long-run protection beyond T imply a reduction in R&D by firms in each technology position (Figure 6, left panel), giving rise to the intensive margin effect. The decline in the rate of technological progress through the intensive margin is partly offset by a growth-enhancing composition effect.

The declines in R&D for tied firms and leaders are large relative to those for laggards. A higher long-run expiry rate therefore contributes to a more competitive economy, boosting growth because leaders innovate more in more competitive industries. The intensive margin effect is further decomposed in the bottom panel of Figure 7. Weaker long-run protection reduces growth through firms' expectations of reduced protection for their own innovations, as shown by the yellow line, which shows the effect of weaker long-run protection on growth without including any strategic effects (i.e., holding constant firms' beliefs about their competitors' strategy $\{x^c_\sigma\}_{\sigma\in S}$). This own-innovation effect is amplified by a growth-reducing escape-competition effect, but dampened by a growth-enhancing trickle-down effect. The escape competition and trickle down effects were discussed in Section 3.3.1. Internet Appendix D.2 presents this decomposition in greater detail.

5.3 Creative destruction and the time pattern of profits from innovation

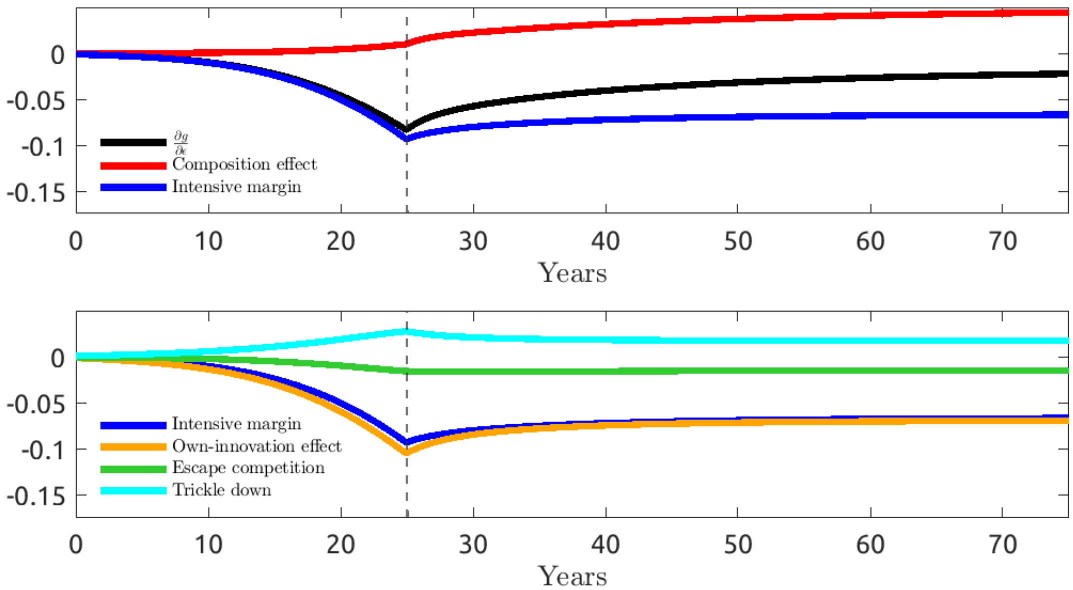
The time pattern of expected profits from innovation shapes the time consistency problem and is closely connected to the creative destruction process. A necessary condition for a time-consistency problem is that firms' strategy prior to T depends on patent protection beyond T. For a firm in a given technology position at $t < T$, this dependence can arise when the nature of creative destruction—through patent expiry, and competitor innovation—is such that an innovation for this firm generates a long-lasting stream of expected profits. In contrast, if the profits from an innovation at $t < T$ do not extend beyond T, then the expiry rate beyond T can only affect the firm's strategy through the response to changes in its competitor's strategy.

For a firm in position σ at time t, let $\mathbb{D}_{\sigma,t,z}$ denote the discounted expected profits at $t+z$ from an innovation at t. For a leader or tied firm ($\sigma \geq 0$),

$$\mathbb{D}_{\sigma,t,z} = e^{-\rho z}\big(\mathbb{E}_{\sigma+1,t} - \mathbb{E}_{\sigma,t}\big)\Pi^N_{\varsigma(t+z)}, \tag{29}$$

where $\mathbb{E}_{\sigma,t}$ is the expectation operator over a firm's position $\varsigma(t+z)$ at time $t+z$ conditional on having position σ at time t. Net operating profits—operating profits less R&D costs—

Figure 7: **The effect of long-run patent protection on technological progress**



The black line in the top panel shows the effect of a higher long-run expiry rate on the growth rate of the quality index, or $dg^Q(t)/d\eta_2$. The top panel also shows a decomposition of $dg^Q(t)/d\eta_2$ into intensive margin and composition components (Eq. (28)). The bottom panel further decomposes the intensive margin effect into a component driven by firms' expectation of weaker protection for their own innovations (i.e., holding competitors' strategy constant) and two strategic components.

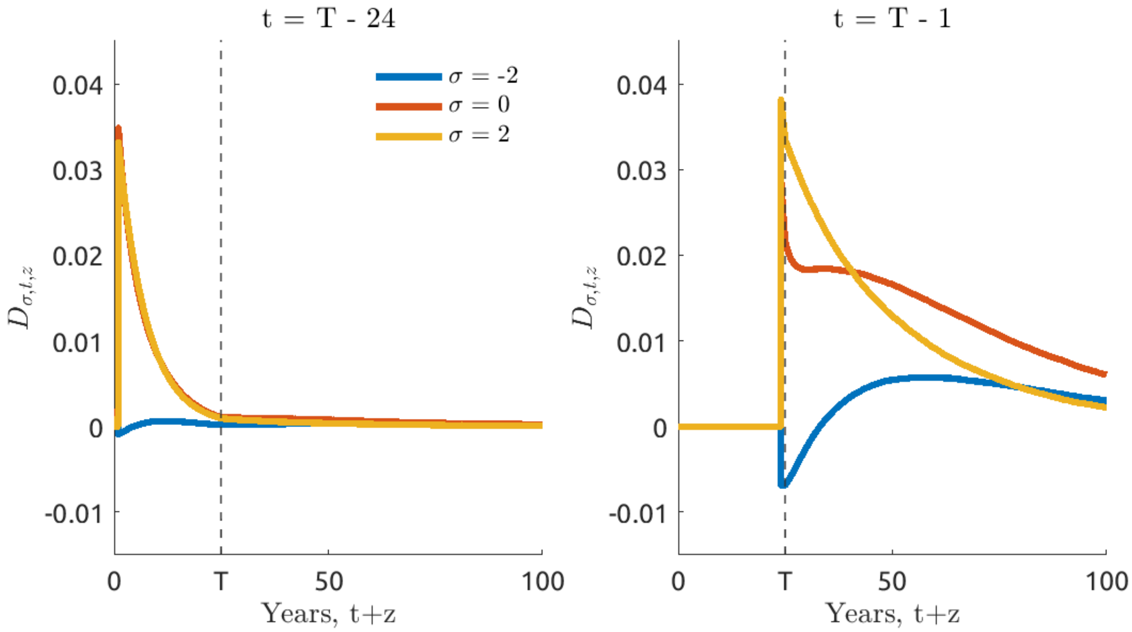
conditional on having position $\varsigma(z)$ are given by $\Pi^N_{\varsigma(z)}$. For a laggard ($\sigma < 0$),

$$\mathbb{D}_{\sigma,t,z} = \Big(\phi\mathbb{E}_{0,t} + (1-\phi)\mathbb{E}_{\sigma+1,t} - \mathbb{E}_{\sigma,t}\Big)\Pi^N_{\varsigma(t+z)}. \tag{30}$$

The capital gain from innovation at t is thus $\Delta v_\sigma(t) = \int_0^\infty \mathbb{D}_{\sigma,t,z} dz$.

The left panel of Figure 8 shows the time pattern of discounted expected profits from innovation at time $t = 1$ year, for firms in different positions σ at time t. Discounted expected profits are shown conditional on optimal policy under commitment. For tied and leader firms, an innovation at time $t = 1$ year immediately increases gross operating profits (by increasing the technology gap by one rung) and decreases R&D costs (because leaders conduct more R&D in more competitive industries). The corresponding initial increase

Figure 8: **Time pattern of expected profits from innovation**



$\mathbb{D}_{t,z,\sigma}$, or discounted expected profits at time $t+z$ from an innovation at time t, is shown for firms in different technology positions at time t.

in net operating profits then erodes over time as patent expiries and quick-catch-up laggard innovations reset the technology gap to zero. Under optimal time-consistent policy, this erosion occurs quickly because the initial expiry rate is high. Very little of the capital gain from innovation at $t = 1$ year is accounted for by discounted expected profits beyond T, when the second planner changes patent policy. For the laggard, the expected profits from innovation are very small (albeit relatively long-lasting), because an innovation at time t increases R&D costs (as the industry becomes more competitive) without any initial increase in operating profits. The small discounted expected profits that accrue beyond T from an innovation at $t = 1$ year explain why firms' innovation strategy at $t = 1$ year is fairly insensitive to η_2, as shown in the left panel of Figure 6. This insensitivity reflects that an innovation at $t = 1$ occurs 24 years prior to the change in the expiry rate to η_2, and that the initial expiry rate $\eta_1^{C,*}$ is high (e.g., implying patent protection for leader innovations expires between t and T at a high rate). In contrast, for an innovation at $t = 24$, almost all of the expected profits accrue beyond the change in the expiry rate (Figure 8, right panel). Correspondingly, firms' innovation strategy at $t = 24$ years is highly sensitive to $\eta_2,$ as shown in the left panel of Figure 6. The increasing sensitivity of firms' innovation strategy as the change in patent protection draws nearer (i.e., as t increases toward T) explains the increasing magnitude of the intensive margin effect of the long-run expiry rate on the quality-index growth rate over the same period, as shown in Figure 7.

6. Creative-destruction parameters and optimal policy

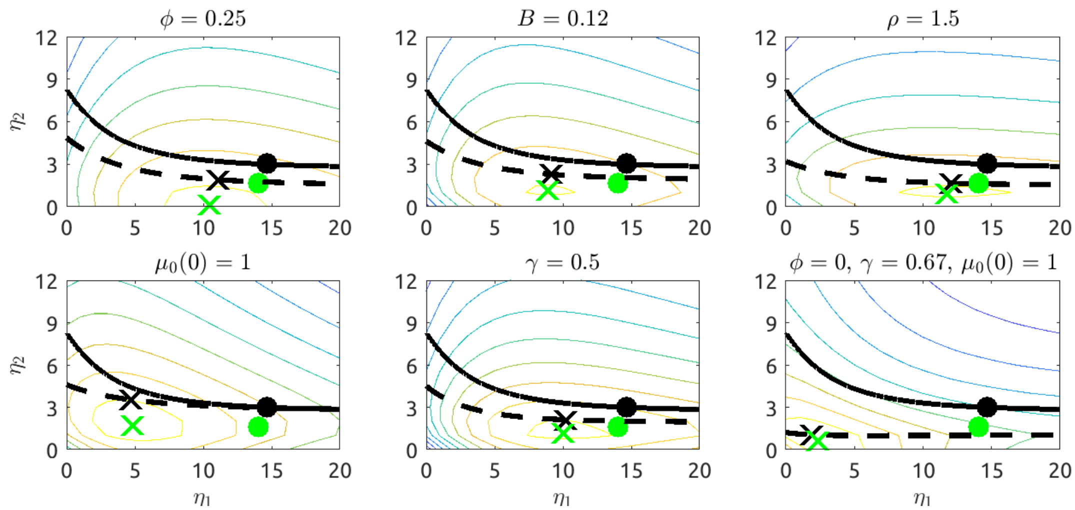
In this section, we investigate how the time-consistency problem is affected by parameters characterizing the creative destruction process. Each panel in Figure 9 shows the schedule of time-consistent duples (dashed black line) and the optimal consistent (green X) and commitment (black X) duples, when the parameter indicated in the panel title is altered. For reference, each panel in Figure 9 also shows—from Figure 1— the schedule of time-consistent duples (solid black line) and the optimal consistent (green circle) and commitment (black circle) duples in the calibrated model.[18]

The left panels (the first column) correspond to economies that are more competitive either because of a higher probability of quick catch-up for innovating laggards ($\phi = 0.25$) or a perfectly competitive initial distribution of technology gaps ($\mu_0(0) = 1$). The middle panels correspond to economies in which the R&D cost function is altered, with uniformly higher R&D costs ($B = 0.12$) or a less convex R&D cost function ($\gamma = 0.5$). The top right panel corresponds to a lower discount rate ($\rho = 0.015$) and the bottom right panel corresponds to a combination of parameter changes.

Across all of these exercises, the schedule of time-consistent duples retains the three properties emphasized in Section 4: a unique time-consistent long-run expiry rate associated with each initial expiry rate; a negative relation between the initial and long-run expiry rates, when comparing across time-consistent duples; and a lower bound on the time-consistent long-run expiry rate. In addition, the optimal time-consistent long-run

[18]As an additional robustness exercise, Internet Appendix D.3 presents results from a calibration with an alternative sample period.

Figure 9: **Parameter values and the time-consistency problem**



Each panel shows the schedule of time-consistent duples (dashed black line) and the optimal consistent (green X) and commitment (black X) duples, when the indicated parameter is altered. Also shown are results for calibrated model. For the calibrated model, the schedule of time-consistent duples is the solid black line and the optimal consistent and commitment duples are the green and black circles, respectively.

expiry rate is markedly above optimal long-run expiry rate under commitment, in each exercise, consistent with the basic intuition of Kydland and Prescott (1977) that policymakers have an incentive to renege on promised patent protection. However, depending on parameters governing the creative destruction process, the reverse can obtain with respect to the *initial* strength of patent protection. That is, initial protection can be *stronger* under optimal time-consistent policy, relative to under commitment (as shown by the bottom left and bottom right panels, for example). In these economies, under discretion, the initial planner chooses strong initial protection to partly offset the growth-reducing effects of weak long-run protection.

When the economy is less sclerotic than the benchmark economy (left panels of Figure 9), initial patent protection under commitment is stronger than in the benchmark economy. Initial protection under discretion is also stronger than in the benchmark economy. Thus, weak initial patent protection in our benchmark model reflects limitations on creative destruction through laggard innovation (low ϕ) and the sclerotic initial gap distribution.[19] A less sclerotic economy (again, higher ϕ or a competitive initial gap distribution) also makes stronger long-run protection time-consistent, partly because, for a given initial expiry rate, the planner at T inherits a more competitive economy.

Lower B raises the cost of achieving a given innovation rate, reducing productivity growth and markups if the patent policy is held constant. Thus, lower B increases the welfare benefits of patent protection, because patent protection generally increases productivity growth at the expense of higher markup distortions. Correspondingly, lower B makes stronger long-run patent protection time-consistent, and implies stronger initial and long-run patent protection. Higher γ reduces the convexity of the R&D cost function $G(x)$. In the neighborhood of the equilibrium innovation rates in the benchmark economy, higher γ also has the effect of increasing the cost of achieving a given innovation rate. Thus, the effects of higher γ on optimal policy are similar qualitatively to the effects of lower B. A lower discount rate (top right panel) implies that the planner puts more weight on productivity growth, relative to markup distortions.[20] Correspondingly, a lower discount rate shifts downward the schedule of time-consistent duples and implies stronger patent protection, especially in the long run.

Figure 10 illustrates the consequences of varying T. For values of T well above and below the benchmark value, long-run patent protection is stronger under commitment, relative to under time-consistent policy (left panel). Regardless of the value of T, relative to the consistent policy (middle and right panels), longer-run productivity growth and markups are systematically higher under commitment.[21]

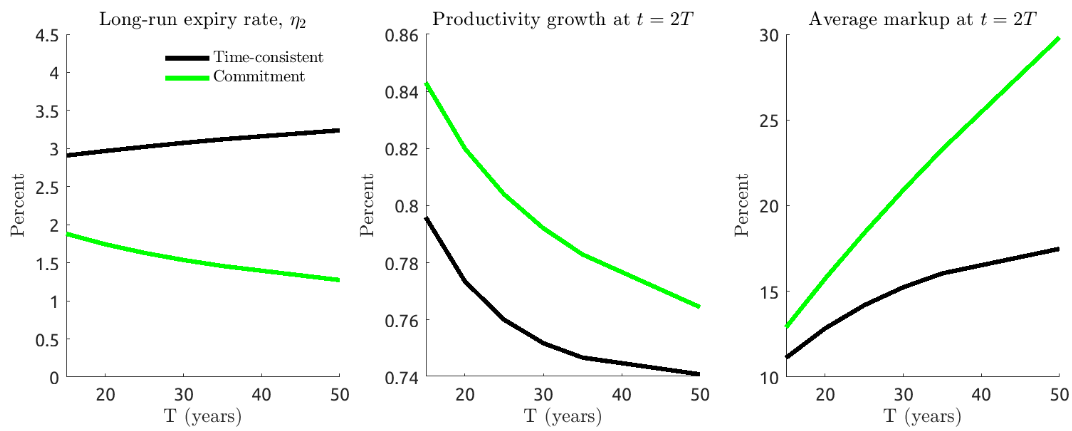
7. Extensions

[19]The calibrated value of ϕ and the initial gap distribution are disciplined by the markup distribution targets.

[20]A way to grasp the role of the discount rate is to look at the expression for welfare for an economy in a BGP at time 0, $\mathbb{W} = \frac{\ln(Y(0))-L}{\rho} + \frac{g}{\rho^2}$. For given quality index $Q(0)$, the term $\ln(Y(0)) - L$ is determined by the distribution of markups, which depress labor demand. The weight on the productivity growth term, relative to the weight on the markup term, is increasing in ρ.

[21]Internet Appendix D.5 presents the schedule of time-consistent duples and isowelfare curves for $T = 50$.

Figure 10: **Implications of varying initial expiry rate period T**



7.1 Further weakening commitment

To gauge how further departing from full commitment affects productivity growth, market competition, and welfare, we relax commitment further by allowing changes to the patent expiry rate at times T and $2T$.

7.1.1 Formal characterization

The expiry rate and path for firms' strategy between 0 and T continue to be denoted by η_1 and x_1, respectively. Let η_2 and x_2 denote the expiry rate and path for firms' strategy between times T and $2T$; and η_3 and x_3, from $2T$ onward. Thus, patent policy is now the triple $\eta = (\eta_1, \eta_2, \eta_3)$ and firms' strategy is $x = (x_1, x_2, x_3)$. Similarly, $X_1(\eta)$ maps patent policy to the equilibrium path for firms' strategy between 0 and T; $X_2(x_1, \eta)$ and $X_3(x_1, x_2, \eta)$ are the equilibrium paths for firms' strategy (between T and $2T$ and from time $2T$ onward, respectively) conditional on past strategy and patent policy. For a given path of firms' strategy and patent policy, welfare is $\mathbb{W}(x, \eta)$. Under commitment, the planner selects η to maximize welfare subject to $x_1 = X_1(\eta)$, $x_2 = X_2(x_1, \eta)$, and $x_3 = X_3(x_1, x_2, \eta)$.

Taking as given past decisions (x_1, x_2) and (η_1, η_2), the consistent planner at $2T$ chooses η_3 to maximize $\mathbb{W}(x, \eta)$ subject to $x_3 = X_3(x_1, x_2, \eta)$. We denote the solution as $\tilde{\eta}_3(x_1, x_2, \eta_1, \eta_2)$.

Taking as given past decisions x_1 and η_1, the consistent planner at T chooses η_2 and η_3 to maximize $\mathbb{W}(x, \eta)$ subject to the constraints $x_2 = X_2(x_1, \eta)$, $x_3 = X_3(x_1, x_2, \eta)$, and

$$\eta_3 = \tilde{\eta}_3(x_1, x_2, \eta_1, \eta_2). \tag{31}$$

The constraint (31) implies that the planner with discretion at T cannot choose any weakly positive duple (η_2, η_3), but instead must choose from the set of weakly positive duples such that, if the planner at T chooses η_2, then the planner at $2T$ will choose η_3. A triple η (with each element weakly positive) is time-consistent if: (i) conditional on $x_1 = X_1(\eta)$ and η_1, the planner at T chooses (η_2, η_3); and (ii) conditional on $x_1 = X_1(\eta)$, $x_2 = X_2(x_1, \eta)$, η_1, and η_2, the planner at $2T$ chooses η_3.

Computing equilibrium. To obtain the optimal time-consistent policy, we retain a dual approach where we first characterize globally the set of time-consistent patent policies and then determine which element of this set maximizes welfare.

The planner with discretion at T, taking x_1 and η_1 as given, chooses

$$\tilde{\eta}_2(x_1, \eta_1) = \arg\max_{\eta_2} \mathbb{W}(x, \eta_1, \eta_2, \eta_3) \tag{32}$$

subject to

$$x_2 = X_2(x_1, \eta_1, \eta_2, \eta_3) \tag{33}$$

$$x_3 = X_3(x_1, x_2, \eta_1, \eta_2, \eta_3) \tag{34}$$

$$\eta_3 = \tilde{\eta}_3(x_1, x_2, \eta_1, \eta_2) \tag{35}$$

A triple (η_1, η_2, η_3) is time consistent if $\eta_2 = \tilde{\eta}_2(x_1, \eta_1)$ and $\eta_3 = \tilde{\eta}_3(x_1, x_2, \eta_1, \eta_2)$ where $x_1 = X_1(\eta)$ and $x_2 = X_2(x_1, \eta)$. That is, a triple is time consistent if

$$\eta_2 = \tilde{\eta}_2(X_1(\eta_1, \eta_2, \eta_3), \eta_1) \tag{36}$$

with

$$\eta_3 = \tilde{\eta}_3(X_1(\eta_1, \eta_2, \eta_3), X_2(X_1(\eta_1, \eta_2, \eta_3), \eta_1), \eta_1, \eta_2) \tag{37}$$

For given η_1, Eqs. (36) and (37) are a system of two equations in two unknowns (η_2, η_3). Combining η_1 and a solution to Eqs. (36) and (37) one obtains a time-consistent triple

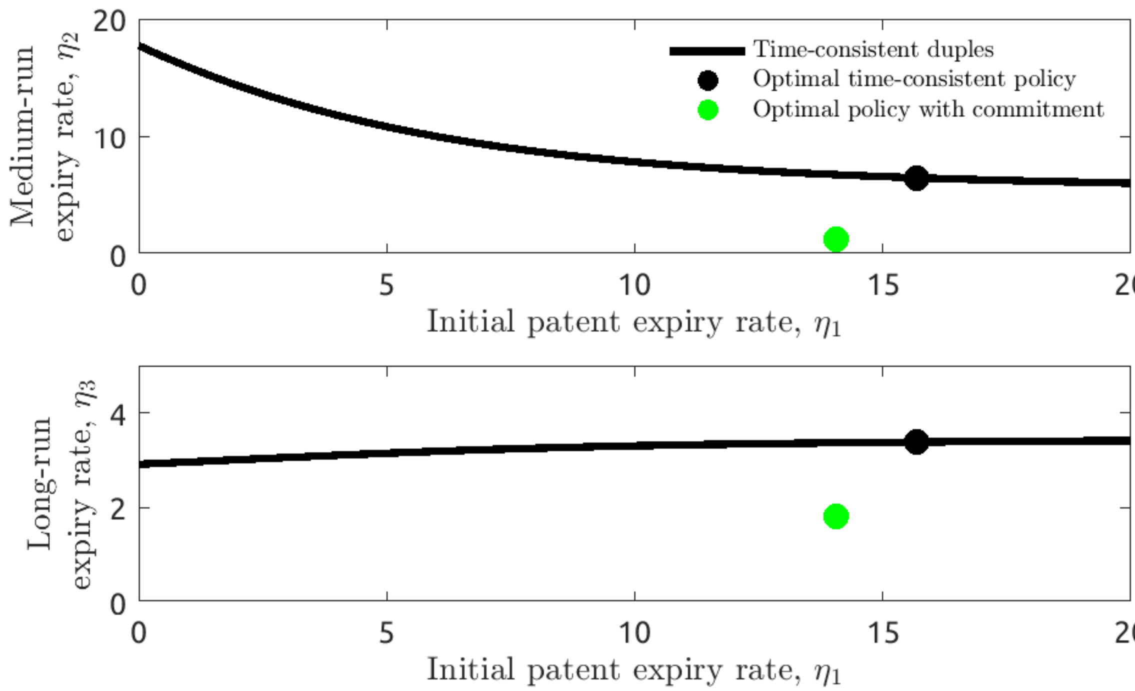
(η_1, η_2, η_3). In our quantitative model, for given η_1, we find there is a unique solution to this system of equations. After repeating this procedure for many η_1 values, we find that, across the set of time-consistent triples, η_2 and η_3 are well-approximated as a spline function of η_1. Using this spline approximation to the set of time-consistent triples, we choose η to maximize welfare subject to $x_1 = X_1(\eta)$, $x_2 = X_2(x_1, \eta)$, and $x_3 = X_3(x_1, x_2, \eta)$, and the constraint that the triple is time-consistent. Additional detail on the solution method is presented in Internet Appendix A.2.

7.1.2 Quantitative implications

In Figure 11, green circles mark the optimal commitment policy and black circles, the optimal consistent policy. Relaxing commitment as in Section 7.1.1 (i.e., with the planner able to alter the patent expiry rate at $\{0, T, 2T\}$, instead of only at $\{0, T\}$) increases the gap between the strength of patent protection under optimal consistent policy and under commitment, especially in the medium run. The expected patent life (ignoring competitor innovation) associated with the commitment medium-run expiry rate is $1/\eta_2^{C,*}$, approximately 75 years, much higher than that associated with consistent policy, $1/\eta_2^{TC,*}$ or 15 years. The thick black lines in Figure 11 show the schedule of time-consistent triples. Analogous to the results with expiry rate choices at $\{0, T\}$, the planner at time 0 without commitment can achieve stronger medium-run patent protection by weakening initial patent protection; but, no matter how weak initial patent protection is set, medium-run protection under consistent policy is always much weaker than under commitment. The planner at time 0 without commitment has effectively no influence over the long-run expiry rate: Comparing across time-consistent triples, η_3^{TC} changes little with η_1^{TC}.

Table 2 shows the implications for the evolution of TFP growth and the average markup, in the rows corresponding to "Optimizing at $\{0, T, 2T\}$" in the table. Annual TFP growth is, on average, 11 basis points lower for the first 50 years under optimal-time consistent policy, relative to under commitment. There are also important differences in the average markup, which increase in magnitude over time. Over the first 25 years, the average markup is approximately the same under optimal consistent policy and under commitment. But, over the subsequent 25 years, average markup is 6 percentage points higher

Figure 11: **Time-consistent patent policy, when further weakening commitment**



This figure compares consistent and commitment policy with the planner can alter the expiry rate at $\{0, T, 2T\}$.

under commitment policy. Overall, the consumption-equivalent loss at time 0 from time-consistent policy (relative to optimal commitment policy) is 0.61%, three times higher than with patent expiry rate choices only at $\{0, T\}$.

7.2 State-dependent patent policy

Acemoglu and Akcigit (2012) studies "state-dependent" patent policy, in which the patent expiry rate in less competitive industries can differ from the expiry rate in more competitive industries (i.e., the expiry rate in an industry with gap s is η_s). Their paper shows that, contrary to a conjecture based on static models, optimal patent policy provides *greater* protection to technologically more advanced leaders. This result arises due to the trickle-down of incentives: stronger protection for advanced leaders directly enhances the inno-

Table 3: **Optimal state-dependent policy**

Parameter	Commitment	Discretion
$\eta_{1,c}$	19.66	19.07
$\eta_{1,u}$	12.62	13.40
η_2	1.60	3.05

vation incentives for such leaders, but also encourages R&D by those with limited leads because of the prospect of reaching technology gaps associated with greater protection. Here, we investigate how state-dependent patent policy interacts with the time-consistency problem. We allow the patent expiry rate over the initial period (i.e., prior to T) to differ between industries with small gaps (one rung) and industries with larger gaps. This "threshold" approach follows Acemoglu and Akcigit (2012). Patent policy then becomes a triple $(\eta_{1,c}, \eta_{1,u}, \eta_2)$, with $\eta_{1,c}$ and $\eta_{1,u}$ the initial expiry rates in competitive and uncompetitive industries. As shown in Table 3, under commitment, consistent with their intuition, initial patent protection in less competitive industries is stronger than initial patent protection in more competitive industries. This result remains in an environment with discretion. However, the optimal difference between protection in more and less competitive industries is attenuated. Under discretion, policies that contribute to a more competitive economy at time T ameliorate an important cost of lack of commitment (weak long-run protection) by making stronger long-run protection time-consistent.

8. Conclusion

The optimal design of patent policies is subject to commitment and credibility issues. Our paper develops a framework for understanding how policymakers' discretion affects patent policy and thereby productivity growth, market competition, and business dynamism. We study time-consistent patent policy in a Schumpeterian endogenous growth model with strategic behavior among competing firms. In our framework, a planner with discretion that inherits a more competitive economy, with smaller leader-laggard technology gaps, chooses stronger patent protection. Thus, by fostering competition through the creative destruction process, a planner today can shift future planners' decisions toward stronger protection, mitigating the well-known incentive of planners to expropriate the fruits of past R&D. We incorporate these insights into a quantitative model that matches aggregate productivity growth, R&D intensity, the distribution of markups, and a measure of the role of reallocation in driving productivity growth. We first characterize the set of all time-consistent policies. Then, we simulate transition dynamics and compute welfare under these alternative policies to determine the optimal time-consistent policy.

We find that the ability of a planner to induce future planners to adhere to the ex-ante optimal policy is quite limited. Thus, optimal time-consistent patent protection is weaker than under commitment; consistent patent policy implies lower productivity growth (but lower markups). Interestingly, and contrary to the standard intuition that the time-consistency problem implies weaker protection (with the planner having an incentive to renege on past promises of protection), patent protection can temporarily be stronger under time-consistent policy, relative to commitment, because the initial planner seeks to partly offset the effects of weak long-run protection. Overall, optimal time-consistent policy is associated with a significant consumption-equivalent welfare loss relative to optimal policy under commitment. These consequences of the time-consistency problem are more severe when we further weaken commitment. We uncover new channels underlying the time consistency problem, including: the effects of future patent policy on past R&D strategies through strategic interactions; and the effects of past R&D strategies on future R&D strategies through a composition effect and the labor market. The planner with discretion ignores these effects.

The findings of our paper also open new directions for future research and policy design. Patent policy's time-consistency problem might be ameliorated if governments can build reputations over time. The time-consistency problem might also be mitigated with an independent authority setting patent protection mainly focusing on fostering innovation. Introducing other frictions in the innovation process—such as asymmetric information (Akcigit et al. (2022a)) or financial frictions (Aghion et al. (2019), Akcigit et al. (2022b), Chikis et al. (2021))—may amplify or dampen patent policy's time-consistency problem.

We have also shown that patent policy's time-consistency problem creates a new welfare benefit for other policies that foster competition, because a more competitive economy ameliorates the time-consistency problem. Relatedly, further research is also warranted to understand how the time-consistency problem for patent policy differs for other types of policies that shape innovation, such as tax incentives for R&D, corporate taxation, government research grants, and competition policy, and how discretion shapes the bundle of such policies.

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Internet Appendix for

“Time-consistent patent policy in an era of sclerosis”

Jonathan Goldberg, David López-Salido, and Nicholas von Turkovich

A. Computing equilibrium

A.1 Private-sector equilibrium (Transition dynamics)

This appendix describes how we solve for an allocation (16), conditional on a time-0 gap distribution $\mu(0) = \{\mu_s(0)\}_{s\in S^+}$ and a time-varying, non-stochastic path for the patent expiry rate $\{\eta(t)\}_{t\in[0,\infty)}$. We assume that $\eta(t)$ reaches a constant terminal value $\eta_{\hat{T}}$ at time $\hat{T} < \infty$, consistent with the exercises in the main text. In this appendix, we denote the firm innovation strategy and value function associated with $\eta_{\hat{T}}$ by $x_{\sigma,BGP,\eta_{\hat{T}}}$ and $v_{\sigma,BGP,\eta_{\hat{T}}}$, respectively. The following result is useful for the development of the solution method.

Lemma. Firms' innovation strategy from $\hat{T}$ onward is constant and equal to the firms' strategy in the BGP associated with $\eta_{\hat{T}}$. That is, $x_\sigma(t) = x_{\sigma,BGP,\eta_{\hat{T}}}, \forall t \geq \hat{T}, \sigma \in S$.

Proof. With preferences (1), the scaled wage satisfies $\omega(t) = 1$ for all $t \in [0,\infty)$. It is then immediate that $v_\sigma(t) = v_{\sigma,BGP,\eta_{\hat{T}}}, \forall t \geq \hat{T}$ solves the HJB equations (4)–(5) for $t \geq \hat{T}$. Then, from (6), setting the scaled wage equal to 1, we have $x_\sigma(t) = x_{\sigma,BGP,\eta_{\hat{T}}}, \forall t \geq \hat{T}, \sigma \in S$.

This result follows from assuming a perfectly elastic labor supply. It implies that if the expiry rate is constant from time $\hat{T}$ onward, then so are firms' value function and investment strategy, even if the economy is not in a BGP from $\hat{T}$ onward (i.e., even if there is an $s \in S^+$ and $t > \hat{T}$ such that $\dot{\mu}_s(t) \neq 0$). Put differently, with perfectly elastic labor supply, the HJB equations do not depend on the gap distribution $\mu(t)$. As a result, even without knowing the gap distribution beyond $\hat{T}$, we can determine the path of the value function and investment strategy beyond $\hat{T}$.

Note that the assumption that the patent expiry rate reaches a constant value ($\eta(t) = \eta_{\hat{T}}$ for $t \geq \hat{T}$)) is consistent with the analyses in the main text, in which patent policy has the form of an $n-$tuple $(\eta_1, ..., \eta_n)$ with $n \in \{2, 3\}$. For given T, where T denotes the length of each time interval of a particular patent expiry rate, the expiry rate path is:

$$\eta(t) = \begin{cases} \eta_1 & t \in [0, T) \\ \eta_2 & t \in [T, 2T) \\ \cdots & \cdots \\ \eta_n & t \in [(n-1)T, \infty). \end{cases} \tag{IA.1}$$

Thus, for such a patent policy, $\hat{T} = (n-1)T$.

Our solution method produces a discrete-time approximation to the path of the value function, innovation strategy, and gap distribution, $\{\{v_\sigma(t), x_\sigma(t)\}_{\sigma \in S}, \{\mu_s(t)\}_{s \in S^+}\}_{t \in \{0, \Delta t, \ldots, \overline{T} - \Delta t, \overline{T}\}}$ for $\overline{T} \geq \hat{T}$. Note that the economy may not reach its new BGP by time $\hat{T}$ as the gap distribution may continue to evolve though the value function and innovation strategy are equal to the terminal BGP values. Hence one may wish to extend the horizon of the analysis $\bar{T}$ to a time beyond $\hat{T}$.

Step 1. Solve for the BGP associated with the terminal patent expiry rate. Following Acemoglu and Akcigit (2012), solve for the BGP value function and innovation strategy using value function iteration and uniformization.

Step 2. Seed the value function and innovation strategy paths between $\hat{T}$ and $\overline{T}$ using BGP values, following the Lemma. In accordance with the above Lemma, set the value function and innovation strategy for $t \geq \hat{T}$ equal to their BGP values. That is, for $t \in \{\hat{T}, \hat{T} + \Delta t, ..., \overline{T}\}$, $v_{\sigma,t} = v_{\sigma,BGP,\eta_{\hat{T}}}$ and $x_{\sigma,t} = x_{\sigma,BGP,\eta_{\hat{T}}} \forall \sigma \in S$.

Step 3. Shoot backward from $\hat{T}$ to obtain the paths for the value function and innovation strategy. Obtain $\{v_{\sigma,t}, x_{\sigma,t}\}_{\sigma \in S, t \in \{0, \Delta t, \ldots, \hat{T} - \Delta t\}}$ by shooting backward, using the discrete-approximation to the HJBs (4)-(5). The discrete-time approximation of the value function for a leader in position $\sigma > 0$ is:

$$\begin{aligned} \rho v_{\sigma,t} - \frac{v_{\sigma,t+\Delta t} - v_{\sigma,t}}{\Delta t} = \max_{x_{\sigma,t}} (1-\tau)\pi_{\sigma,t} - G(x_{\sigma,t})\omega_t + x_{\sigma,t} \Delta v_{\sigma,t+\Delta t} + \\ x^c_{-\sigma,t} \left(\phi v_{0,t+\Delta t} + (1-\phi) v_{\sigma-1,t+\Delta t} - v_{\sigma,t+\Delta t}\right) + \eta_t (v_{0,t+\Delta t} - v_{\sigma,t+\Delta t}). \end{aligned} \tag{IA.2}$$

The discrete-time approximation for the laggard and tied firms HJB equations can be obtained similarly. For all $\sigma \in S$, profit maximization implies:

$$x_{\sigma,t} = G'^{-1}\left(\frac{\Delta v_{\sigma,t+\Delta t}}{\omega_t}\right), \tag{IA.3}$$

where, for $\sigma \geq 0$, $\Delta v_{\sigma,t+\Delta t} = v_{\sigma+1,t+\Delta t} - v_{\sigma,t+\Delta t}$ and for $\sigma < 0$, $\Delta v_{\sigma,t+\Delta t} = \phi v_{0,t+\Delta t} + (1-\phi)v_{\sigma+1,t+\Delta t} - v_{\sigma,t+\Delta t}$. Setting $\omega_t = 1$ (perfectly elastic labor supply) and $x^c_{\sigma,t} = x_{\sigma,t}$ (symmetry) and substituting from (IA.3) into (IA.2), one obtains, for $\sigma > 0$,

$$\begin{aligned}(1+\Delta t\rho)v_{\sigma,t} = \Delta t\Big[&(1-\tau)\pi_{\sigma,t} - G(G'^{-1}(\Delta v_{\sigma,t+\Delta t})) + G'^{-1}(\Delta v_{\sigma,t+\Delta t})\Delta v_{\sigma,t+\Delta t} + \\ &G'^{-1}(\Delta v_{-\sigma,t+\Delta t})(\phi v_{0,t+\Delta t} + (1-\phi)v_{\sigma-1,t+\Delta t} - v_{\sigma,t+\Delta t}) + \eta_t(v_{0,t+\Delta t} - v_{\sigma,t+\Delta t})\Big] + v_{\sigma,t+\Delta t}.\end{aligned} \tag{IA.4}$$

For $t \in \{\hat{T}-\Delta t, \hat{T}-2\Delta t, ..., \Delta t, 0\}$, we iteratively use (IA.4) and an analogous equation for $\sigma \leq 0$ to compute the paths for the value function and iteration strategy prior to $\hat{T}$.

Step 4. Shoot forward from 0 to $\overline{T}$ to obtain the path for the gap distribution. The discrete-time approximation to the differential equation (7) for the share of industries with gap $s > 0$ is

$$\frac{\mu_{s,t+\Delta t} - \mu_{s,t}}{\Delta t} = \mu_{s+1,t}\big[(1-\phi)x_{-(s+1),t}\big] + \mu_{s-1,t}x_{s-1,t}(1+\mathbb{1}_{s=1}) - \big[x_{s,t} + x_{-s,t} + \eta_t\big]\mu_{s,t}. \tag{IA.5}$$

Using the innovation strategy path from Steps 2 and 3, we use (IA.5) (together with the analogous equation for $s = 0$ or, equivalently, the normalization that $\sum_{s\in S^+}\mu_{s,t} = 1$) to obtain the path for the gap distribution, $\{\mu_{s,t}\}_{s\in S^+, t\in\{0,\Delta t,...,\overline{T}-\Delta t,\overline{T}\}}$.

Using $\{\{v_{\sigma,t}, x_{\sigma,t}\}_{\sigma\in S}, \{\mu_{s,t}\}_{s\in S^+}\}_{t\in\{0,\Delta t,...,\overline{T}-\Delta t,\overline{T}\}}$, the path of the remaining objects in the allocation (e.g., the interest rate r_t and output Y_t) can be obtained as described in Section 2.1.

A.2 Optimal consistent policy

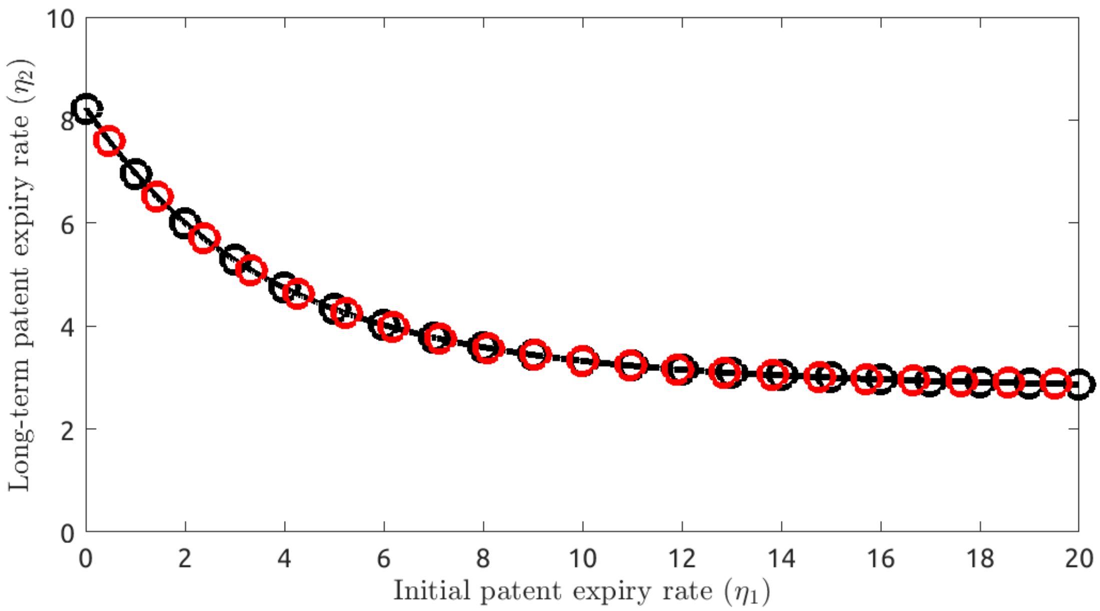
To obtain the welfare-maximizing time-consistent policy, we use a "dual" and global approach as explained in Section 3.4 of the main text. First, we characterize the set of time-consistent policies η^{TC} (defined in Eq. (22)). We find that, for each initial expiry rate η_1, there is a unique long-run expiry rate η_2 such that (η_1, η_2) is time-consistent (i.e., satisfies Eq. (21)). For a given η_1, to solve for the time-consistent η_2, we use a bisection method. We choose an initial search space with lower bound η_2^{lb} and upper bound η_2^{ub}. We make an initial guess for the time-consistent η_2 from within this search space. For the patent policy (η_1, η_2), we simulate the path for the economy, as described in Section 2.1 of the main text and Internet Appendix A.1, and extract the gap distribution at time T, $\mu(T)$. Using this gap distribution as the initial condition of the problem of the planner with discretion at time T, we solve for $\tilde{\eta_2}$ (Eq. (20)). If $\eta_2 > \tilde{\eta_2}(X_1(\eta_1, \eta_2), \eta_1)$, we set $\eta_2^{ub} = \eta_2$ and repeat the process with a new guess for the long-run expiry rate. Likewise, if $\eta_2 < \tilde{\eta_2}(X_1(\eta_1, \eta_2), \eta_1)$, we set $\eta_2^{lb} = \eta_2$. We repeat this process until $|\eta_2 - \tilde{\eta_2}| < \epsilon^{tol}$, where ϵ^{tol} is an extremely low tolerance parameter. Figure 2 illustrates this fixed-point problem. Given an appropriately wide initial search space (η_1^{lb}, η^{ub}), the downward slope of $\tilde{\eta_2}(\eta_2)$ is a sufficient condition for convergence and uniqueness.

Using this procedure and a grid of values for η_1, we calculate the time-consistent η_2 for each η_1 in the grid. We then approximate the complete set of time-consistent policies using a spline interpolation for a much finer grid of η_1 values. As shown in Figure IA.1, a spline allows a very high quality approximation. We then simulate the path of the economy under each duple formed by an η_1 value from the finer grid and the associated time-consistent η_2 value. We compute time-0 welfare under each duple using Eq. (17); see Internet Appendix A.3 for details. We then identify η^{TC^*} as the duple that induces the highest time-0 welfare.

Further weakening commitment. As described in Section 7, we relax commitment further by allowing for changes to the patent expiry rate at two points in time, T and $2T$. A triple (η_1, η_2, η_3) is time-consistent if it satisfies Eqs. (36)–(37). For a candidate triple (η_1, η_2, η_3), we define a loss function equal to the sum of the magnitude of the residual of each of these equations:

$$loss(\eta_1, \eta_2, \eta_3) = |\eta_2 - \tilde{\eta_2}| + |\eta_3 - \tilde{\eta_3}| \tag{IA.6}$$

Figure IA.1: **Spline interpolation of time-consistent duples**



This figure shows the spline approximation used in the construction of the set of time-consistent policies η^{TC}. Each black circle corresponds to a time-consistent duple that is solved for directly (i.e., satisfying Eq. (21)). The black line is the spline interpolation for a finer grid of η_1 values. As a test of the quality of the approximation, we directly solve for an additional time-consistent duples with initial expiry rates not in the original set of η_1 values. Each such duple is shown by a red circle. The red circles correspond extremely closely to the spline approximation (black line) of the original set of duples.

For a given η_1, we search for the pairs (η_2, η_3) such that this loss function is zero. (As discussed next, we find there is a unique such pair for each η_1.) To obtain the loss function for a pair (η_2, η_3), we simulate the path of the economy under the candidate triple (η_1, η_2, η_3) and extract $\mu(T)$ and $\mu(2T)$, the gap distribution at two times at which the patent expiry rate can change. Taking these gap distributions as the initial condition of the planner's problem at periods T and $2T$, respectively, we calculate the planner's choices under discretion ($\tilde{\eta}_2$ and $\tilde{\eta}_3$), allowing us to obtain the loss function Eq. (IA.6).

Figure IA.2 shows the loss function as a function of η_2 and η_3, for four different values of η_1. (This figure is roughly analogous to Figure 2 of the main text, which pertained to the case of one change in the expiry rate.) For each initial expiry rate, there is a unique triple (η_1, η_2, η_3) that is time-consistent (i.e., such that the loss function is zero). Moreover, the loss function is well behaved (e.g., convex in a large neighborhood around the pair (η_2, η_3) that gives a zero loss). With this motivation, we define a grid of η_1 values and for each, we solve for the time-consistent triple (η_1, η_2, η_3) by using a local minimization method for the loss function and then verifying that the loss function for the time-consistent triple is below an extremely low threshold.

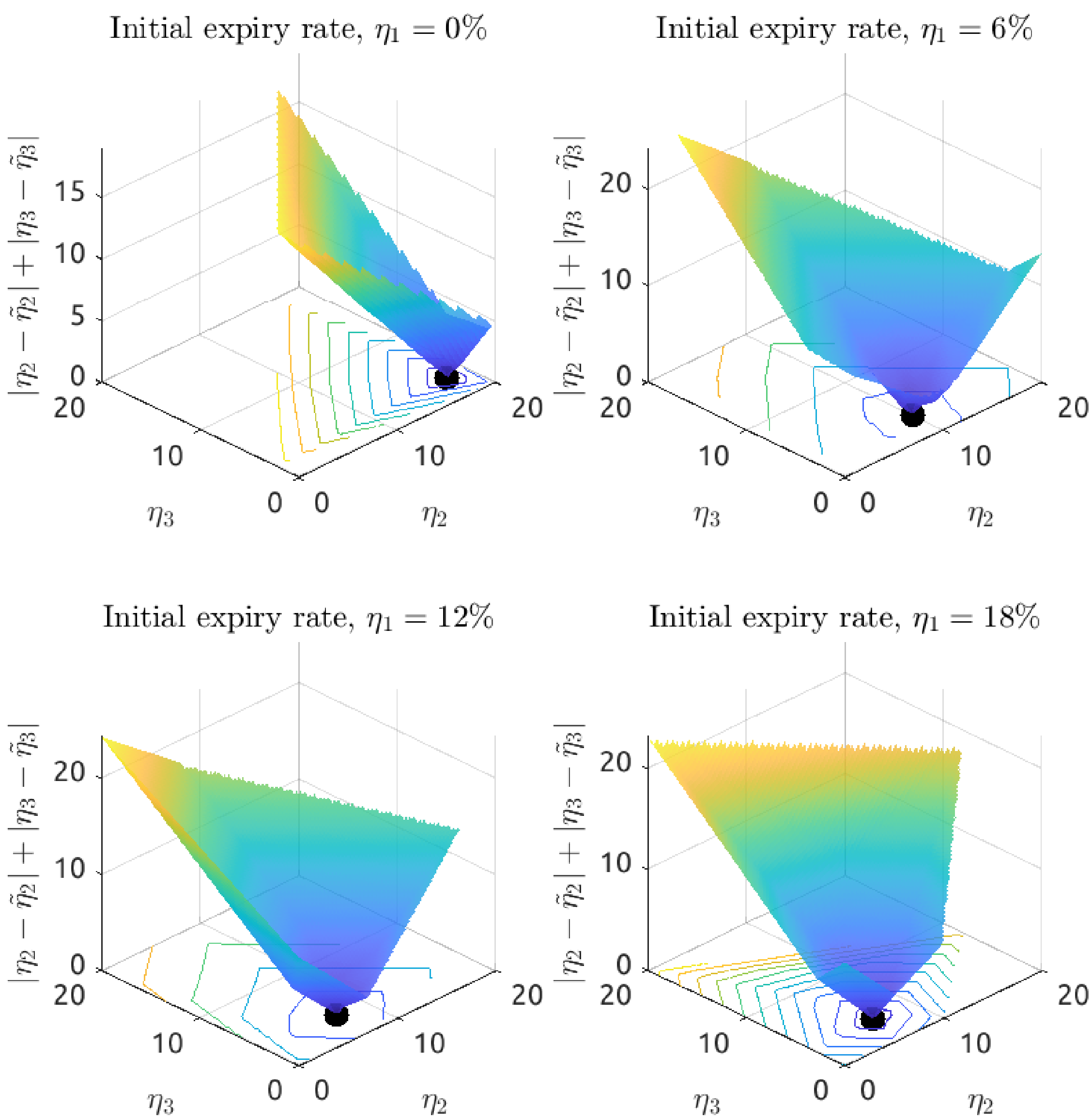
This approach is computationally expensive: for each η_1, it requires the simulation of many transition paths (corresponding to many (η_2, η_3) candidates) and then, for *each* $\mu(T)$ and $\mu(2T)$ for a given candidate, simulating many additional transition paths to find the optimal choices $\tilde{\eta}_2$ and $\tilde{\eta}_3$ of planners with discretion at T and $2T$. We increase the efficiency of search process by "sharing" a variety of information from residual calculations for nearby (η_2, η_3) pairs (for given η_1) and from searches already completed for nearby η_1 values. Such sharing increases efficiency but the time-consistency of the final triple is verified directly. We then approximate the set of time-consistent triples (using a finer grid of η_1) and obtain time-0 welfare by simulating the transition path for each triple; the optimal time-consistent triple that is the time-consistent triple that maximizes welfare.

A.3 Welfare and consumption-equivalent gains

For a given initial distribution of technology gaps $\{\mu_s(0)\}_{s\in S^+}$ and initial quality index $Q(0)$, we calculate welfare $\mathbb{W}(x_1, x_2, \eta_1, \eta_2)$ (Eq. (17)) as follows. First, guess—and, later, verify—a value for $\overline{T}$ such that the economy is growing along a BGP for $t > \overline{T}$.[22] Second, following step 1 of Internet Appendix A.1, obtain the BGP growth rate and gap distribution associated with x_2. Third, following Step 4 of Internet Appendix A.1, "shoot forward" to obtain the path $\{\mu_t\}_{t\in\{0,\Delta t,\ldots,\overline{T}-\Delta t,\overline{T}\}}$ conditional on the initial gap distribution, μ_0, the patent policy (η_1, η_2), and the innovation strategy path given by x_1 and x_2. Fourth, confirm that the distance between $\mu_{\overline{T}}$ and the BGP gap distribution is zero (in practice, below an extremely low threshold). If this condition does not hold, guess a larger value for $\hat{T}$ and restart the

[22]Consider the setting in which patent policy can be altered at times 0 and T, as in Section 3. In such a setting, per the Lemma in Appendix A.1, the innovation strategy $x_\sigma(t)$ is constant for $t > T$.

Figure IA.2: **Determination of the set of time-consistent triples**



This figure shows the loss function (Eq. (IA.6)) as a function of (η_2, η_3), for four different values of η_1. For each η_1, the time-consistent triple corresponds to the (η_2, η_3) pair such that the loss is zero, implying that the triple satisfies Eqs. (36) and (37).

welfare calculation. Fifth, obtain $\{Q_t, C_t, L_t\}_{t\in\{0,\Delta t,\ldots,\overline{T}\}}$ as described in Section 3. Sixth, calculate

$$\mathbb{W}(x_1, x_2, \eta_1, \eta_2) = \sum_{t\in\{0,\Delta t,\ldots,\overline{T}-\Delta t\}} \exp(-\rho t)\Delta t\big(\ln(C(t)) - L(t)\big) + \exp(-\rho\overline{T})\mathbb{W}(\overline{T}), \tag{IA.7}$$

where $\mathbb{W}(\overline{T})$ is the BGP welfare

$$\mathbb{W}(\overline{T}) = \frac{\ln(Y_{\overline{T}}) - L_{\overline{T}}}{\rho} + \frac{g}{\rho^2} \tag{IA.8}$$

with g equal to the BGP growth rate.

Conditioning on Q(0). An assumption of an alternative initial quality index $\tilde{Q}(0) \neq Q(0)$ implies an alternative time-0 welfare $\mathbb{W}(x_1, x_2, \eta_1, \eta_2; \tilde{Q}(0)) = \frac{1}{\rho}(\ln(\tilde{Q}(0)) - \ln(Q(0))) + \mathbb{W}(x_1, x_2, \eta_1, \eta_2; Q(0))$. Therefore, for comparisons of welfare across economies with different policies, it is sufficient to compare two economies with the same level of $Q(0)$ and the same initial gap distribution μ_0, but different policies.

Consumption-equivalent welfare (χ). Consider two alternative policies characterized by the duples (η_1^A, η_2^A) and (η_1^B, η_2^B). Denote welfare under policy A by W_A:

$$W_A = \mathbb{W}(X_1(\eta_1^A, \eta_2^A), X_2(X_1(\eta_1^A, \eta_2^A), \eta_1^A, \eta_2^A), \eta_1^A, \eta_2^A), \eta_1^A, \eta_2^A), \tag{IA.9}$$

and similarly denote welfare under policy B by W_B. Under preferences (1), the consumption-equivalent welfare gain of switching from policies B to policies A is denoted by χ, where $\chi = \exp(\rho(\mathbb{W}_A - \mathbb{W}_B)) - 1$.

B. Relation to Kydland and Prescott (1977)

KP described a general two-period problem with policies (η_1, η_2) for period 1 and 2, agents' decisions (x_1, x_2) and a non-specific welfare function $\mathbb{W}(x_1, x_2, \eta_1, \eta_2)$. Agents' decisions in period t depend upon both policy decisions and agents' past decisions (i.e., $x_1 = X_1(\eta_1, \eta_2)$ and $x_2 = X_2(x_1, \eta_1, \eta_2)$). In their setting, under commitment, the first order condition is

$$\frac{\partial \mathbb{W}}{\partial x_2}\frac{\partial X_2}{\partial \eta_2} + \frac{\partial \mathbb{W}}{\partial \eta_2} + \frac{\partial X_1}{\partial \eta_2}\left[\frac{\partial \mathbb{W}}{\partial x_1} + \frac{\partial \mathbb{W}}{\partial x_2}\frac{\partial X_2}{\partial x_1}\right] = 0. \tag{IA.10}$$

The time-consistency problem in our setting can be seen through the lens of KP. The terms on the first line of (24) correspond to $\frac{\partial \mathbb{W}}{\partial x_2}\frac{\partial X_2}{\partial \eta_2} + \frac{\partial \mathbb{W}}{\partial \eta_2}$ in Eq. (IA.10). These terms are taken into account by the consistent planner at T and under commitment. The terms on the second line of (24) correspond to $\frac{\partial X_1}{\partial \eta_2}[\frac{\partial \mathbb{W}}{\partial x_1} + \frac{\partial \mathbb{W}}{\partial x_2}\frac{\partial X_2}{\partial x_1}]$ in Eq. (IA.10). In the Aghion-Howitt model, these terms correspond to the effect of η_2 on welfare through x_1 (the path for firms' innovation strategy *prior to* T), directly and through x_2 (firms' innovation strategy beyond T). As in KP, the first order condition of the consistent planner ignores these terms.

By analyzing time consistency in an explicit Aghion-Howitt framework, we uncover and characterize new aspects of patent policy's time-consistency problem in the presence of creative destruction. First, the effect of patent protection beyond T on firms' strategy prior to T reflects both long-lasting expected profits from innovation as well as a new strategic channel that can propagate such effects backward in time through the dependence of firms' decisions on their competitors' future strategy. Second, firms' strategy prior to T affects welfare both through a direct channel (i.e., through the productivity of technologies in use and the evolution of the gap distribution) and through an indirect channel (i.e., through composition changes that affect aggregate labor demand and hence can affect firm's strategies beyond T). We also show that this second channel depends crucially on the labor market and that, with sufficiently elastic labor supply, has little effect on the time-consistency problem. An additional advantage of an explicit Aghion-Howitt framework is that we can analyze the problem quantitatively, as in the remainder of the paper.

C. Calibration details

This appendix provides additional information about moments targeted in the calibration (Section 4.1 of the main text).

Markup distribution. We obtain data values from Hall (2018), who estimates the distribution of Lerner indexes. The Lerner index is defined as the ratio of price minus marginal cost to price. To obtain markup moments, we make many draws from the distribution of

Lerner indexes in Hall (2018) and convert each Lerner index draw to a markup.

R&D to GDP. In the numerator, we use private fixed investment in intellectual property products (excluding "Entertainment, literary, and artistic originals").

FHK "within" moment. Foster et al. (2001) decompose productivity growth into five components using the identity

$$\Delta\Theta_t = \underbrace{\sum_{i\in\mathbb{C}_t}\xi_{it-1}\Delta\theta_{it}}_{\text{within}} + \underbrace{\sum_{i\in\mathbb{C}_t}(\theta_{it-1}-\Theta_{t-1})\Delta\xi_{it}}_{\text{between}} + \underbrace{\sum_{i\in\mathbb{C}_t}\Delta\theta_{it}\Delta\xi_{it}}_{\text{cross}} + \underbrace{\sum_{i\in\mathbb{N}_t}\xi_{it}(\theta_{it}-\Theta_{t-1})}_{\text{entry}} + \underbrace{\sum_{i\in\mathbb{X}_t}\xi_{it-1}(\Theta_{t-1}-\theta_{it-1})}_{\text{exit}}, \quad \text{(IA.11)}$$

where $\mathbb{C}_t$ is the set of continuing firms, $\mathbb{N}_t$ of entering firms, and $\mathbb{X}_t$ of exiting firms in industry j between $t-1$ and t. In addition, $\theta_{it} = \ln(\frac{y_{it}}{l_{it}})$ is the log productivity of firm i at time t, $\xi_{it} = \frac{p_{it}y_{it}}{\sum_i p_{it}y_{it}}$ is the revenue share of firm i at time t, and $\Theta_{it} = \sum_i \xi_{it}\theta_{it}$. We also define the *adjusted* within share as the share of *continuing firms*' productivity growth accounted for by those firms' productivity improvements holding market shares constant (i.e., absent reallocation). The adjusted within share is calculated as within divided by the sum of within, between, and cross. We use the adjusted within share as a moment target in order to focus on continuing firms (our model does not include entry). We calculate the FHK decomposition in the model using the industry-level simulation, focusing, as in the data, on observations five years apart.

D. Additional results

D.1 The dynamic Nordhaus tradeoff with time-consistent patent policy

Nordhaus (1969) describes a core tradeoff for patent policy: Stronger protection fosters more innovation and productivity growth, generally at the expense of higher markups and markup-related distortions. Figure IA.3 illustrates the determination of optimal time-consistent policy in our calibrated model. The black line in Figure IA.3 shows the productivity growth and markup components of welfare associated with different duples $(\eta_1^{TC}, \eta_2^{TC}) \in \eta^{TC}$ (see Eq. (17) for the welfare decomposition). As indicated by the curved arrow, weaker initial patent protection increases welfare by reducing (appropriately discounted) markup distortions but decreases welfare by also reducing (appropriately discounted) growth of the quality index. This downward slope of the black line (the negative relation of the productivity and markup components of welfare, as one moves across the set of time-consistent patent policies) captures tradeoff identified by Nordhaus, taking account of transition dynamics and the time-consistency problem for patent policy.

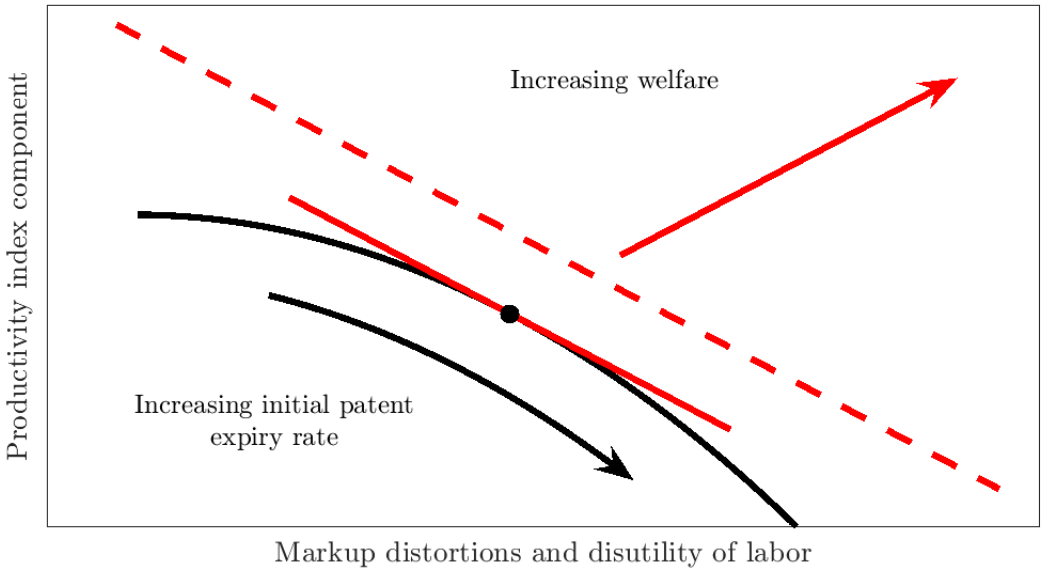
Each red line in Figure IA.3 is an iso-welfare line (with a slope of -45 degrees), showing combinations of productivity and markup components that deliver the same total welfare. The tangency of the black line and the solid red iso-welfare line shows welfare components associated with the optimal time-consistent policy $(\eta_1^{TC,*}, \eta_2^{TC,*})$. Under commitment, the planner can achieve time-0 welfare shown by the dashed red line—that is, under commitment, time-0 welfare is higher.

D.2 Growth decomposition and strategic interactions

The effect of the long-run expiry rate on the growth rate of the quality index is decomposed into an intensive margin component and a composition effect in Eq. (28) in Section 5.2 of the main text (shown in Figure 7, top panel). The intensive margin component is further decomposed in the bottom panel of Figure 7 into an "own innovation" effect (reflecting reduced protection for a firm's own innovations, holding constant each firm's beliefs about its competitor's strategy), a trickle down effect, and an escape competition effect.

$$\frac{dg(t)}{d\eta_2} = \underbrace{\sum_{s\in S^+} \frac{d\mu_s(t)}{d\eta_2} x_s(t)(1+1_{s=0})}_{\text{composition effect}} + \underbrace{\sum_{s\in S^+} \mu_s(t)\frac{dx_{s,own}(t)}{d\eta_2}(1+1_{s=0})}_{\text{own innovation effect}} +$$
$$\underbrace{\sum_{\sigma\geq 0}\mu_\sigma(t)(\sum_{t'>t}\sum_{\sigma'<-\sigma}\frac{\partial x_\sigma(t)}{\partial x^c_{\sigma'}(t')}\frac{dx^c_{\sigma'}(t')}{d\eta_2})(1+1_{\sigma=0})}_{\text{trickle down effect}} + \underbrace{\sum_{\sigma\geq 0}\mu_\sigma(t)(\sum_{t'>t}\sum_{\sigma'\geq-\sigma}\frac{\partial x_\sigma(t)}{\partial x^c_{\sigma'}(t')}\frac{dx^c_{\sigma'}(t')}{d\eta_2})(1+1_{\sigma=0})}_{\text{escape competition effect}} \tag{IA.12}$$

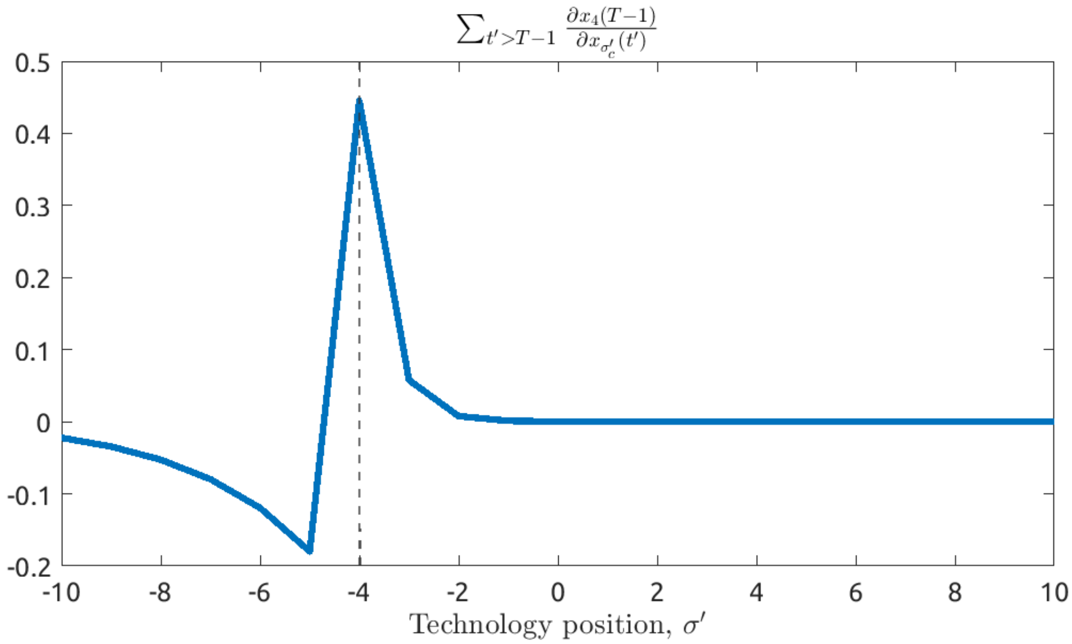
Figure IA.3: **Optimal time-consistent policy**



The black line shows the productivity growth and markup components of welfare associated with different time-consistent duples $(\eta_1^{TC}, \eta_2^{TC}) \in \eta^{TC}$. The tangency of the red solid isowelfare line with the black line corresponds to the welfare components under optimal time-consistent policy. The red dashed line is the isowelfare line associated with optimal policy under commitment.

Each firm's innovation strategy today $\{x_\sigma(t)\}_{\sigma \in S}$ depends on its competitors' innovation strategy today and in the future $\{x^c_{\sigma'}(t)\}_{\sigma' \in S}, t' \geq t$. This dependence can be seen in the Hamilton-Jacobi-Bellman equations (4)-(5). For example, a leader that is currently 4 steps ahead has less to gain from innovation, the greater the innovation rate of its competitor when the competitor is 5 or more steps behind (e.g., the greater is x^c_{-5}), because the leader would face a competitor 5 or more steps behind if it were to innovate one or more times. Acemoglu and Akcigit (2012) refers to such strategic interactions as "trickle down" interactions. In addition, a leader that is currently 4 steps ahead has more to gain from innovation, the greater the innovation rate of its competitor when the competitor is 4 or fewer steps behind (e.g., the greater is x^c_{-4}), because such a leader can avoid a competitor 4 or fewer steps behind if it were to innovate one or more times. Following Aghion and Howitt

Figure IA.4: **Impact of changes in competitor innovation strategy on own innovation strategy**



(1992), we refer to such strategic interactions as "escape competition" interactions. More generally, as in (IA.12), the change in the innovation rate at time t of a firm in position σ at time t due to a change in the innovation rate of its competitor when the competitor is in position σ' at time $t' \geq t$, or $\frac{\partial x_\sigma(t)}{\partial x^c_{\sigma'}(t')}$, is labeled as an escape competition effect if $\sigma \geq -\sigma$ and as a trickle-down effect if $\sigma < -\sigma$. Figure IA.4 illustrates these escape competition and trickle down effects, by showing the effect on the innovation rate at time t of a firm in position $\sigma = 4$ at time t from an increase in its competitor's innovation rate at any time $t' > t$ when the competitor is in position σ'.

D.3 Alternative sample period

This appendix presents results from a calibration targeting data values based on an alternative sample period, 1970–1999. The calibration targets and parameters are shown in

Table IA.I: **Parameters and moments: Alternative sample period.**

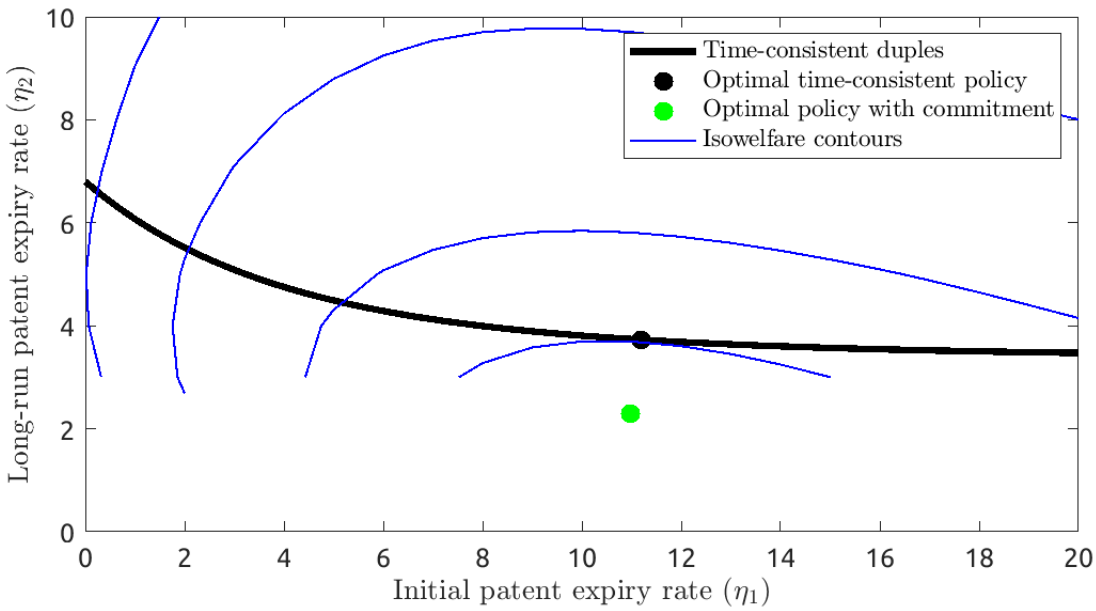
Parameter Estimates		Moments used in estimation		
Parameter	Value	Description	Model	Data
ϕ	0.033	Productivity growth	0.89 %	0.89 %
λ	1.029	R&D to GDP	4.36 %	2.56 %
B	0.906	Markups		
η	0.041	Mean	19.2 %	19.8 %
		75th percentile	24.6 %	26.2 %
		90th percentile	43.3 %	43.3 %
		95th percentile	58.3 %	57.4 %
		99th percentile	95.9 %	94.3 %
		FHK within	87.6 %	90.8 %

Table IA.I. Over this period, the economy exhibited less sclerosis, relative to the sample period used in the calibration in the main text. That is, markups were lower and productivity growth was higher.[23]

For this alternative calibration, Figure IA.5 shows the schedule of time-consistent duples (thick black line), the optimal consistent duple (black circle) and the commitment (green circle) duple. The results are qualitatively similar to those from the benchmark calibration in the main text (compare with Figure 1). Optimal consistent policy implies weaker protection than commitment policy, especially in the long run. The initial expiry rates under discretion and commitment are lower than those for the benchmark calibration, reflecting that this alternative sample period features a less sclerotic economy.

[23] In this calibration, the numerator of the R&D to GDP ratio is private fixed investment in intellectual property products (excluding "Entertainment, literary, and artistic originals" and "Software"). Markup data is for 1988-1999, matching the start of the sample period in Hall (2018).

Figure IA.5: **Time-consistent patent policy for the calibration with alternative sample period**



D.4 State-dependent policy

Section 7.2 of the main text discusses state-dependent patent policy. Figure IA.6 shows the schedule of time-consistent state-dependent patent expiry rate triples $(\eta_1^c, \eta_1^u, \eta_2)$. Similarly to the standard case of uniform expiry rates, there is a unique long-run expiry rate for each pair of initial expiry rates (η_1^c, η_1^u), the long-run expiry rate is decreasing in η_1^u and (though less visually apparent) in η_1^c, and there is a lower bound on the time-consistent long-run expiry rate.

D.5 Time-consistency problem with $T = 50$ years

Figure IA.7 shows that our main qualitative results regarding time-consistent policy are robust to doubling the length of the initial expiry-rate period T, relative to the benchmark calibration. See Section 6 of the main text for additional discussion of the value for T.

Figure IA.6: **State-dependent, time-consistent policy**

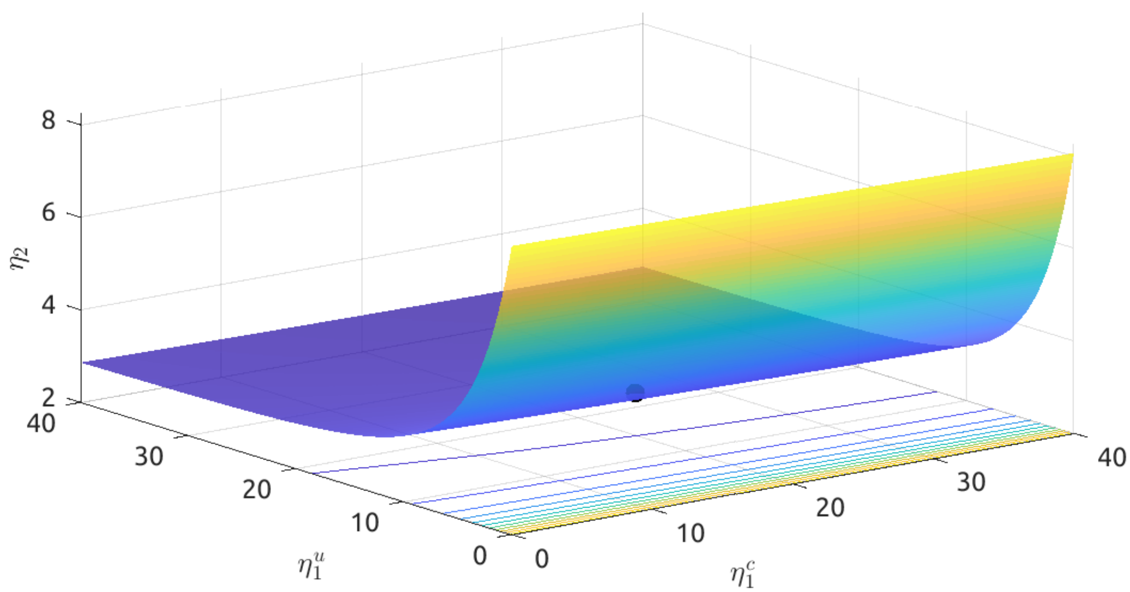
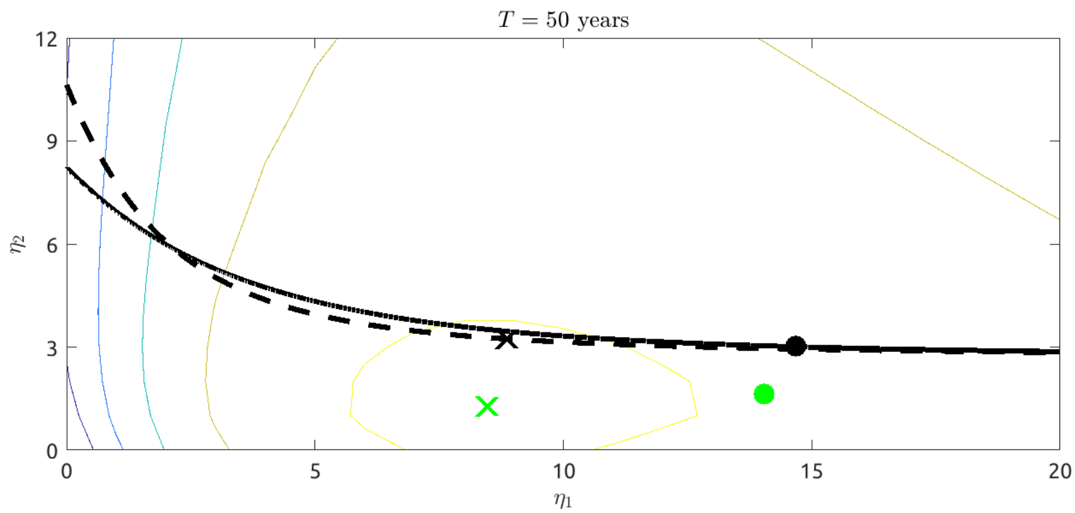


Figure IA.7: **Time-consistency problem when adjusting T**



This figure shows, for an initial expiry-rate period of $T = 50$, the schedule of time-consistent duples (dashed black line) and the optimal consistent (green X) and commitment (black X) duples. Also shown are results for calibrated model, with $T = 25$. For the calibrated model, the schedule of time-consistent duples is the solid black line and the optimal consistent and commitment duples are the green and black circles, respectively.