Martingale Report

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Question 1: **Experiment 1 Win Probability**

The estimated probability of winning $80 in Experiment 1 is 100%. Programmatically, I solved this by tracking the number of episodes which reached $80; this was shown to be every episode. Theoretically speaking, this is possible because in Experiment 1 we have an "unlimited bankroll" and double our bets after losing. Further, there is no state in which we can lose enough money that we will have to stop playing, and we are not risk averse to doubling our bet to cover our losses at any amount. Because of these principles, in the span of 1000 turns there was always a time where we hit $80. We can see this is consistent with Figure 1, where the green average line converges at $80 as the number of sequential bets grows.

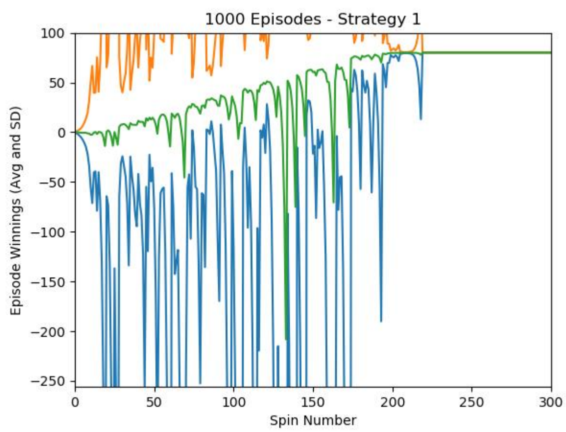

Figure 1— The plot above shows the average state of the episode winnings at each turn in green, the upper boundary of standard deviation in orange, and the lower boundary of standard deviation in blue for Experiment 1.

Question 2: **Experiment 1 Expected Value**

The estimated value of winnings after 1000 turns was $80. Since we saw earlier that we hit our $80 cap on 100% of the episodes, and we had an upper limit of $80 that is our expected value. In this case, since there was no variance in final outcome our EV is simply $80.

Question 3: **Experiment 1 Standard Deviation**

The upper and lower standard deviation lines do not reach a maximum/minimum value and stabilize in experiment one. They also do not converge over sequential bets. The instability of the lines is due to the unlimited bankroll principle, enabling there to be large bets across each sample. Because we will double our bet with a loss, as sequential bets accumulate we have a theoretical range of ($-2^{\text{\#of iterations-1}}$, $2^{\text{\#of iterations-1}}$), where each sequential bet has a higher possible maximum bet as it is possible to have the most consecutive losses (i.e. bet 3 has a possible 2 consecutive losses, whereas bet 100 has a possible 999 consecutive losses preceding it). The range in magnitude of bets, combined with a 48.7% win probability leads to high instability in variance. Because the bets have a higher theoretical maximum as the sequence continues the lines do not converge over the sequence. In Figure 2, we can see that the variance regularly fluctuates at magnitudes that are well beyond our boundaries of $100 and -$256.

Figure 2— The plot above shows the median value of the episode winnings at each turn in green, the upper boundary of standard deviation in orange, and the lower boundary of standard deviation in blue for Experiment 1.

Question 4: **Experiment 2 Winning Probability**

The estimated probability of winning $80 in experiment 2 is 57.1%. I derived this programmatically by taking the proportion of the 1000 episodes that reached $80. In contrast to experiment 1, we have a maximum loss value of -$256 in Experiment 2 creating scenarios where we have to stop betting and cannot reach $80. The finite bankroll combined with the risk of losing large sums when doubling our bets creates the difference in probability between experiments 1 and 2.

Question 5: **Experiment 2 Expected Value**

The expected value of winnings in Experiment 2 is -$63.83. This is represented by the average of the end winnings of each episode. Because there are many scenarios where we lose our entire bankroll ($256), which is over 3x in magnitude of $80 upper winnings limit, we end up with a negative expected value. While doubling our bet in theory could get us back to even or into profitability, the slightly negative odds of winning (48.7%) make it a risky strategy and create a losing expected value over time. We see in Figure 3 that the mean value becomes slightly negative over sequential bets.

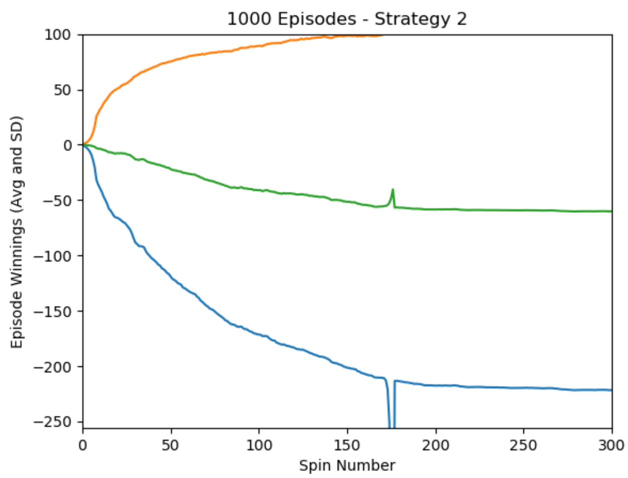

Figure 3— The plot above shows the mean value of the episode winnings at each turn in green, the upper boundary of standard deviation in orange, and the lower boundary of standard deviation in blue for Experiment 2.

Question 6: **Experiment 2 Standard Deviations**

The upper and lower standard deviation lines do stabilize over time in Experiment 2, but they do not converge. The stabilization occurs because we cannot bet an amount greater than our $256 bankroll. This creates a range of (-256, 256) which the variance approaches over the run of sequential bets. The variance still does not converge because the magnitude of variance is still increasing with each iteration, not decreasing. This is similar to Experiment 1, except now we have a smaller minimum/maximum with the finite bankroll to consider. In Figure 4, we can see the lower standard deviation line stabilize at it approaches the -$256 minimum.

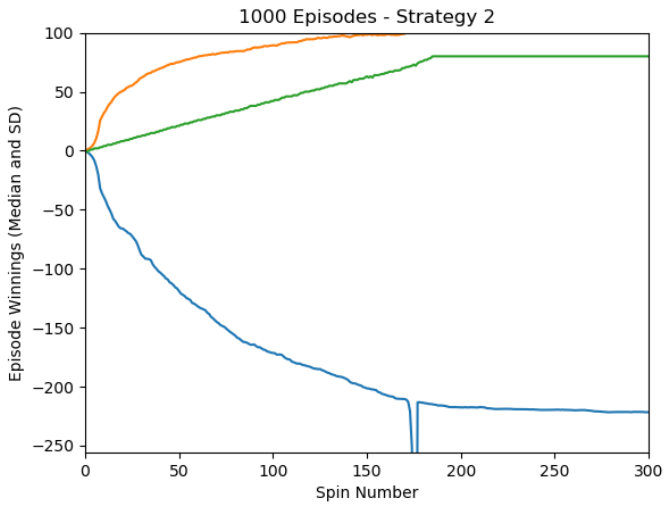

Figure 4— The plot above shows the median value of the episode winnings at each turn in green, the upper boundary of standard deviation in orange, and the lower boundary of standard deviation in blue for Experiment 2.

Question 7: Benefits of Expected Values

Expected values give us a better estimation of the anticipated outcome and trends of a system because collections of samples account for variance. Especially when randomness is at play, represented by the Roulette roll in our experiments, a single value gives us a very incomplete picture of the central tendencies and variance of a system. When we take a large sample of a system, we can begin to describe the most-likely outcomes and the magnitude of variance associated with the system. When randomness is at play, there is always some level of risk associated with a given system. Encapsulating the magnitude of variance or "risk" is key when evaluating a system and describing the nature of variables. This allows us to make informed decisions and most-accurately depict data.