

## 202A

### Social Planner's Problem

1. Write down all equations from the prompt
2. Check if anything can be simplified easily. Do not simplify if it is not helpful

### Tricks

Additive Consumption, e.g.

### Examples

### Recursive Competitive Equilibrium

A Recursive Competitive Equilibrium consists of:

1. Household's Problem
2. Firm's Problem(s)
3. Household decision rules
4. Firm decision rules
5. Pricing functions
6. Aggregate Perception
7. Pricing functions, Aggregate Perception, and Household decision rules solve Household's Problem
8. Pricing function and Firm decision rules solve Firm's problem
9. Market's clear
10. Perception's are correct

### Log Linearization

We define a variable in log deviation from its steady state as:

$$\hat{x} = \log\left(\frac{x}{\bar{x}}\right)$$

By reverse engineering, we can write  $x$  as:

$$x = \bar{x} \exp(\hat{x})$$

The final step in the process, when the approximation plays a role, is that up to first order (meaning,  $\hat{x}$  is small enough).

$$\exp(\hat{x}) = f(0) + \sum_{i=1}^{\infty} \frac{\partial^i f}{\partial x^i}(0) \frac{(\hat{x} - 0)^i}{i!} \approx 1 + \exp(0)(\hat{x} - 0) = 1 + \hat{x}$$

Then  $x \approx \bar{x}(1 + \hat{x})$

### Tricks

Let  $x, y, z$  be variables, and  $\alpha, \beta, c$  constants.

1.  $\hat{c} = 0$
2. If  $z = x + y$  then  $\bar{z}\hat{z} \approx \bar{x}\hat{x} + \bar{y}\hat{y}$ . And since  $\bar{z} = \bar{x} + \bar{y}$ , then

$$\hat{z} \approx \frac{\bar{x}}{\bar{x} + \bar{y}} \hat{x} + \frac{\bar{y}}{\bar{x} + \bar{y}} \hat{y}$$

3. if  $z = x^\alpha y^\beta$  then

$$\hat{z} = \alpha \hat{x} + \beta \hat{y}$$

### Calibration

### Transversality Condition

### Characterization

Characterization is usually done using FOC, EC, and budget constraint

### Unique Questions

1. Derive expressions that determine how the planner allocates a given amount of capital and labor across the two market sectors. Prove that the same fraction of each input is allocated to a given sector in period  $t$
2. Show that the result obtained in part B can be used to aggregate the resource constraints (\*) and (\*\*) into one resource constraint (derive it). Repeat part A given this result.
3. Suppose an empirical fact about the world is that as durable goods become cheaper, households spend less time in nonmarket activities (female labor supply increases, for example). If you wanted this model to be consistent with this fact, what would this mean for the functional form you would choose for the home production technology,  $F$ ? (Note: This question is not so much asking for a functional form for  $F$ , although that would be fine, but is asking for list of properties that this function should possess.)