202C HW1

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Question 1: In this question, we consider a 4-country version of the Armington model. Consider the following parameter values: $\sigma = 3$, $a_{i,j} = 2$ for all $i = j$, $a_{i,j} = 1$ for all $i \neq j$, $L_1 = 2$, $L_2 = L_3 = L_4 = 1$, $A_1 = A_2 = A_3 = A_4 = 0.6$, and trade cost $\tau_{i,j}$ corresponds to the element i, j in the following matrix:

$$T = \begin{pmatrix} 1 & 1.1 & 1.2 & 1.3 \\ 1.3 & 1 & 1.3 & 1.4 \\ 1.2 & 1.2 & 1 & 1.1 \\ 1.1 & 1.1 & 1.1 & 1 \end{pmatrix}$$

1. Solve for equilibrium wages in countries 2, 3 and 4 relative to country 1.

Solution:

1. We will use the good of country 1 as the numeraire ($p_1 = 1$)
2. The excess demand function of the Armington Model is:

$$Z_i(w) = \{\sum_{j \in S} (\frac{a_{ij}\tau_{ij}^{1-\sigma}(\frac{w_i}{A_i})^{1-\sigma}}{\sum_{k \in S} a_{kj}\tau_{kj}^{1-\sigma}(\frac{w_k}{A_k})^{1-\sigma}})\frac{w_j L_j}{w_i}\} - L_i$$

3. We solve by guessing a vector of wages, and iterating until the excess demand function is zero.

   Using the code attached in the appendix: $\frac{w_2}{w_1} = 1.1458$, $\frac{w_3}{w_1} = 1.2238$, $\frac{w_4}{w_1} = 1.2614$

2. Solve for bilateral trade shares, λ_{ij} for $i = 1, 2, 3, 4$ and $j = 1, 2, 3, 4$

Solution: Since we have already solved for wages, we can use the formula $\lambda_{ij} = \frac{X_{ij}}{Y_j}$ To solve for bilateral trade shares we use the code attached in the appendix. We get:

$$\lambda = \begin{pmatrix} 0.5825 & 0.248 & 0.2315 & 0.2122 \\ 0.1313 & 0.4571 & 0.1502 & 0.1393 \\ 0.135 & 0.1391 & 0.4451 & 0.1978 \\ 0.1513 & 0.1558 & 0.1731 & 0.4507 \end{pmatrix}$$

3. Consider (only in this question) that country 2's productivity increases by a factor of 2, from $A_2 = 0.6$ to $A_2' = 1.2$, while the others remain unchanged.

   a) What's the change in welfare for country 2 from the productivity shock?

Solution: The formula for welfare is $W_i = \lambda ii^{\frac{1}{1-\sigma}} a_{ii}^{\frac{1}{\sigma-1}} A_i$ So using our code:

$$W_i^{new}/W_i^{old} = \frac{2.2863}{1.2551} = 1.82$$

b) What's the change in welfare for country 2 from the productivity shock under autarky ? (hint: you simply need to use an equation from the lecture and no need to solve the model)

Solution: From Sunny's first section, we know that

$$W_i = \lambda_{ii}^{\frac{1}{1-\sigma}} \alpha_{ii}^{\frac{1}{\sigma-1}} A_i$$

The change in welfare is:

$$\frac{W_i^{new}}{W_i^{old}} = \frac{\lambda_{ii}^{\frac{1}{1-\sigma}} \alpha_{ii}^{\frac{1}{\sigma-1}} A_i'}{\lambda_{ii}^{\frac{1}{1-\sigma}} \alpha_{ii}^{\frac{1}{\sigma-1}} A_i} = \frac{A_i'}{A_i} = \frac{1.2}{0.6} = 2$$

c) Provide intuition for the difference in your answers in (a) and (b)

Solution: In part (a) and (b), we showed that the increase in welfare from a home productivity shock is bigger under autarky than in an open economy for the affected country. The intuition for this is that in an open economy, as country 2 becomes more productive it increases the supply of goods to the rest of the world, which pushes down its relative price and import share. e.g. Country 2's terms of trade decrease. In autarky, the country fully absorbs the benefits of its productivity shock, as it is not trading with the rest of the world.

4.

$$T = \begin{pmatrix} 1 & 1 & 1.2 & 1.2 \\ 1 & 1 & 1.2 & 1.2 \\ 1 & 1.2 & 1 & 1.3 \\ 1 & 1.2 & 1.2 & 1 \end{pmatrix}$$

1. Solve for equilibrium wages in countries 2, 3 and 4 relative to country 1.

Solution: $\frac{w_2}{w_1} = 1.2337 \frac{w_3}{w_1} = 1.2356 \frac{w_4}{w_1} = 1.2506$

2. Solve for bilateral trade shares, λ_{ij} for $i = 1, 2, 3, 4$ and $j = 1, 2, 3, 4$

Solution:

$$\lambda = \left(\begin{pmatrix} 0.5062 & 0.3112 & 0.2391 & 0.2465 \\ 0.1663 & 0.409 & 0.1571 & 0.162 \\ 0.1658 & 0.1416 & 0.451 & 0.1376 \\ 0.1618 & 0.1382 & 0.1528 & 0.4539 \end{pmatrix} \right)$$

5. Solve for the change in wage in each country (relative to country 1's wage) using the system in changes discussed in Section 5 in the Lecture Notes. Verify that you get the same result as in 4. What is the advantage of solving the system in changes rather than in levels (two times)?

Solution: TODO