Final Exam
Econ 202B
Winter 2021, Atkeson

Question 1: Total 60 points

This question leads you through building a model of how a "savings glut" might drive down the world interest rate. The setup has a world economy with two countries, labeled A and B. The two economies have a unified capital market, so the interest rate is assumed to be the same in both economies.The world economy starts in a steady state in which country A has much higher labor productivity and hence much more effective labor than country B. We also assume that agents in country B are much more patient, and thus have a higher propensity to save, than agents in country A. We then assume that country B experiences a large positive shock to its labor productivity, raising its stock of effective labor relative to than in country A. I will ask you to compute the initial and the new steady-state interest rate as well as the net foreign asset position of country A in terms of the bonds it holds payable by residents of country B (if this goes negative, it means that residents of country B lend to residents of country A).

Consider the following overlapping generations model of the world economy.

There are two countries, labeled A and B. The population of country A is constant at $2N_A$ and that in country B at $2N_B$.

Time is denoted $t = 1, 2, 3, \ldots$. In period $t = 1$, there are N_A agents in country A who we refer to as the "initial old" who consumes only at $t = 1$ with that consumption denoted $c^o_{A,1}$. Likewise, in country B, there are N_B initial old whose consumption at $t = 1$ is denoted by $c^o_{B,1}$.

Corresponding to every $t \geq 1$, in country A, there are N_A agents who we call "the generation born at t" who consume at t and $t + 1$ with consumption denoted $(c^y_{A,t}, c^o_{A,t+1})$ and utility

$$\log(c^y_{A,t}) + \beta_A \log(c^o_{A,t+1})$$

In country B, there are N_B agents who we call "the generation born at t" who consume at t and $t + 1$ with consumption denoted $(c^y_{B,t}, c^o_{B,t+1})$ and utility

$$\log(c^y_{B,t}) + \beta_B \log(c^o_{B,t+1})$$

Assume that the initial old in countries A and B are each endowed with $k_{A,0}$ and $k_{B,0}$ units of capital respectively, and that each member of the generation born at t is endowed with a single unit of labor which they supply inelastically. Aggregate capital in country A is then $K_{A,0} = N_A k_{A,0}$ and in country B, $K_{B,0} = N_B k_{B,0}$. Aggregate labor in each country at each date is given by N_A and N_B, respectively. At each date $t \geq 1$, let $k_{A,t-1}$ denote the capital held by each old person in country A and $k_{B,t-1}$ the capital held by each old person in country B and let

(1) $$K_{A,t-1} = N_A k_{A,t-1} \text{ and } K_{B,t-1} = N_B k_{B,t-1}$$

denote the aggregate capital stocks in each country.

Output in country A is produced according to

(2) $$Y_{A,t} = K^{\alpha}_{A,t-1}(Z_A N_A)^{(1-\alpha)}$$

with Z_A denoting effective labor productivity in country A.

Output in country B is produced according to

(3) $$Y_{B,t} = K^{\alpha}_{B,t-1}(Z_B N_B)^{(1-\alpha)}$$

with Z_B denoting effective labor productivity in country B.

There is free trade in goods and capital across countries, so the world resource constraint is given by

(4) $$N_A(c^o_{A,t} + c^y_{A,t} + k_{A,t}) + N_B(c^o_{B,t} + c^y_{B,t} + k_{B,t}) =$$

$$Y_{A,t} + Y_{B,t} + (1-\delta)(K_{A,t-1} + K_{B,t-1})$$

The old in country A rent their capital and the young in that country sell their labor to a firm that chooses capital and labor inputs K and N to maximize

$$\max_{K,N} K^{\alpha}(Z_A N)^{(1-\alpha)} - R_{AK,t}K - W_{A,t}N$$

and likewise in country B firms maximize profits

$$\max_{K,N} K^{\alpha}(Z_B N)^{(1-\alpha)} - R_{BK,t}K - W_{B,t}N$$

The worldwide interest rate on bonds is denoted by R_t. Each young person in country A faces budget constraints

$$c^y_{A,t} + k_{A,t} + b_{A,t} = W_{A,t}$$

$$c^o_{A,t+1} = (R_{AK,t+1} + (1-\delta))k_{A,t} + R_t b_{A,t}$$

with $k_{A,t} \geq 0$ and each young agent in country B faces constraints

$$c^y_{B,t} + k_{B,t} + b_{B,t} = W_{B,t}$$

$$c^o_{B,t+1} = (R_{BK,t+1} + (1-\delta))k_{B,t} + R_t b_{B,t}$$

with $k_{B,t} \geq 0$.

The bond market clearing condition is given by

$$(5) \quad N_A b_{A,t} + N_B b_{B,t} = 0$$

A competitive equilibrium in this economy is an allocation and bondholdings

$$\{c^y_{A,t}, c^o_{A,t}, k_{A,t}, b_{A,t}, c^y_{b,t}, c^o_{B,t}, k_{B,t}, b_{B,t}, Y_{A,t}, Y_{B,t}, K_{A,t}, K_{B,t}\}_{t=1}^{\infty}$$

together with a sequence of interest rates, wage rates, and rental rates on capital,

$$\{R_t, W_{A,t}, W_{B,t}, R_{AK,t}, R_{BK,t}\}_{t=1}^{\infty}$$

such that the allocation and bond holdings maximize each agents utility given their budget constraints, such that the firms in each country maximize profits given local wage rates and rental rates on capital, and such that equations 1, 2, 3, 10, and 5 are satisfied.

Part A: 30/60 points

1. Show that in any equilibrium with $k_{A,t}, k_{B,t} > 0$,

   $$R_t = R_{AK,t+1} + (1-\delta) = R_{AK,t+1} + (1-\delta)$$

   Answer: This argument is done in a homework. If this condition is violated with the left side smaller than the right side, then the young can obtain unbounded consumption while old by borrowing to finance more capital, and if the inequality is reversed, then the young can raise consumption while old by setting capital to zero and investing all savings in bonds.

2. Show that in any equilibrium with $k_{A,t}, k_{B,t} > 0$,

$$R_{KA,t} = \alpha \frac{Y_{A,t}}{K_{A,t-1}}$$

$$R_{KB,t} = \alpha \frac{Y_{B,t}}{K_{B,t-1}}$$

$$W_{A,t} = (1 - \alpha) \frac{Y_{A,t}}{Z_A N_A}$$

$$W_{B,t} = (1 - \alpha) \frac{Y_{B,t}}{Z_B N_A}$$

   Answer: These are the first order conditions of the firms' profit maximization problem.

3. Show that in any equilibrium with $k_{A,t}, k_{B,t} > 0$,

$$\frac{Y_{A,t}}{K_{A,t-1}} = \frac{Y_{B,t}}{K_{B,t-1}}$$

   and hence

$$\frac{k_{A,t-1}}{k_{B,t-1}} = \frac{Z_A}{Z_B}$$

   Answer: Since the interest rate R_t is common to both countries, the result in item 1 above gives that the rental rate on capital is also the same in both countries. These results then follow from the results in item 2 and equation 1.

4. Show that in any equilibrium,

$$c^y_{A,t} = \frac{1}{1 + \beta_A} W_{A,t}$$

$$k_{A,t} + b_{A,t} = \frac{\beta_A}{1 + \beta_A} W_{A,t}$$

$$c^y_{B,t} = \frac{1}{1+\beta_B} W_{B,t}$$

$$k_{B,t} + b_{B,t} = \frac{\beta_B}{1+\beta_B} W_{B,t}$$

Answer: These results were derived in the homework.

5. Show that in any equilibrium

$$K_{A,t} + K_{B,t} = (1-\alpha)\left[\frac{\beta_A}{1+\beta_A} Y_{A,t} + \frac{\beta_B}{1+\beta_B} Y_{B,t}\right]$$

Answer: One obtains this equation by multiplying the decision rules derived in the previous item by N_A and N_B respectively and then using equations 1 and 5 together with the result above that the compensation of labor in both countries is fraction $1-\alpha$ of output.

Part B: 10/60 points Use these results from Part A together with equations 1, 2, 3, 10, and 5 to derive the following first order difference equations for capital and bonds in this world economy for all $t \geq 1$

$$K_{B,t+1}\left(1 + \frac{Z_A N_A}{Z_B N_B}\right) = (1-\alpha)\left[\frac{\beta_A}{1+\beta_A} K^{\alpha}_{A,t} (Z_A N_A)^{1-\alpha} + \frac{\beta_B}{1+\beta_B} K^{\alpha}_{B,t} (Z_B N_B)^{1-\alpha}\right]$$

$$K_{A,t+1} = \frac{Z_A N_A}{Z_B N_B} K_{B,t+1}$$

$$N_A b_{A,t+1} = (1-\alpha)\frac{\beta_A}{1+\beta_A} K^{\alpha}_{A,t} (Z_A N_A)^{1-\alpha} - K_{A,t+1}$$

$$N_B b_{B,t+1} = (1-\alpha)\frac{\beta_B}{1+\beta_B} K^{\alpha}_{B,t} (Z_B N_B)^{1-\alpha} - K_{B,t+1}$$

Answer: These equations can be derived from the results in Part A. The first equation is derived from items 3 and 5 in Part A. The second equation is derived from item 3 in Part A. The last two equations are derived by multiplying the decision rules of young agents in item 4 by N_A and N_B respectively.

Part C: 20/60 points Answer the following questions about these difference equations

1. Does this system have a unique steady-state (K_A, K_B, b_A, b_B) with $K_{B,t} > 0$? Show your work in answering this question.

   Answer: Yes, this difference equation has a unique steady state with $K_{B,t} > 0$. One can show this result as follows. After period $t = 1$, we have that

   $$K_{A,t+1} = \frac{Z_A N_A}{Z_B N_B} K_{B,t+1}, \quad \forall t$$

   Thus we can write the first equation as

   $$K_{B,t+1}\left(1 + \frac{Z_A N_A}{Z_B N_B}\right) = (1-\alpha)\left[\frac{\beta_A}{1+\beta_A}\left(\frac{Z_A N_A}{Z_B N_B}\right) + \frac{\beta_B}{1+\beta_B}\right] K_{B,t}^{\alpha}(Z_B N_B)^{1-\alpha}$$

   which we showed had a unique steady-state in the homework. The remaining steady-state quantities are then derived directly from the remaining equations.

2. What is the steady-state interest rate? Derive a formula as a function of the parameters.

   Answer: We compute the steady-state interest rate off of the steady-state rental rate on capital. This is given by

   $$\bar{R} = \bar{R}_{BK} + (1-\delta) = \alpha \frac{\bar{Y}_B}{\bar{K}_B} + (1-\delta)$$

   From the difference equation above, we have

   $$\alpha \frac{\bar{Y}_B}{\bar{K}_B} = \frac{\alpha}{1-\alpha} \frac{\left[1 + \frac{Z_A N_A}{Z_B N_B}\right]}{\left[\frac{\beta_A}{1+\beta_A}\left(\frac{Z_A N_A}{Z_B N_B}\right) + \frac{\beta_B}{1+\beta_B}\right]}$$

3. Compute the steady-state interest rate when $\alpha = 1/3$, $\delta = 1$, $\beta_A = 1/2$, $\beta_B = 3/4$, $N_A = 1$, $N_B = 4$, $Z_A = 1$, $Z_B = 1/10$. What is the relative size of countries A and B? What is the steady state bond position of country A $(N_A b_A)$?

4. Now imagine that country B experiences economic growth through an increase in productivity from $Z_B = 1/10$ to $Z_B = 1/4$. Now what is the steady state interest rate? What is the relative size of countries A and B? What is the steady state bond position of country A $(N_A b_A)$?

Extra Credit: 5 points Explain in words why this increase in productivity in country B results in a decrease in the world interest rate.

Question 2: Total 60 points

In this problem, you will use the code you wrote for homework to assess quantitatively how a decrease in idiosyncratic income uncertainty impacts the steady-state interest rate in a Bewley Model. This was the MATLAB homework assignment in section 7 of the notes idiosyncraticrisk.pdf.

As a baseline, we will use the economy for which you found the steady-state numerically as homework with one alteration — we will make the grid of bondholdings finer.

So, assume that bondholdings are restricted to lie on a finite grid, so $b \in B = \{b^1, b^2, \ldots, b^N\}$ where $b^1 = \bar{b}$. Let us assume that there are only two levels of the endowment shock y so $Y = \{y^1, y^2\}$, with $y^1 = 1$ and $y^2 = 3$. Let the Markov transitions be given by $\varphi_{11} = \varphi_{22} = .8$, and $\varphi_{12} = \varphi_{21} = .2$. With these assumptions, we can write the distribution of bondholdings and endowments across agents as a vector of length $2N$ with typical element $f(b^n, y^i)$ giving the fraction of agents starting the period with bondholdings $b = b^n$ and endowment $y = y^i$. Assume $\beta = .95$, $u(c) = \log(c)$, and a borrowing constraint of $\bar{b} = b^1 = -15$. Choose $b^N = 150$ with a step size of $1/5$ so that $N = 826$. We have to iterate on guesses at q to find the equilibrium bond price. Start with a guess of $q = \beta$.

Note that when you construct the transition matrix for the joint distribution of bond holdings and income shocks, you may want to use the ability of MATLAB to handle sparse matrices.

Use your code modified to have a larger number of gridpoints for bondholdings to answer the following questions.

1. (5 points) What is the long run or unconditional average income for an agents in this economy? What is the standard deviation of the logarithm of income in the cross section of agents in this economy? What is the mean and standard deviation of changes in income from t to $t+1$ if that agent currently has high income? What is the mean and standard deviation of changes in income from t to $t+1$ if that agent currently has low income?

2. (5 points) What is the equilibrium interest rate in steady-state in this model economy?

3. (5 points) What is cross section standard deviation of the logarithm of consumption in this model economy?

4. (5 points) What do your decision rules imply is the standard deviation of innovations to the logarithm of consumption between t and $t+1$ for an agent who has high income and the maximum bondholdings b^N at time t? What do your decision rules imply is the standard deviation of innovations to the logarithm of consumption between t and $t+1$ for an agent who has high income and the minimum bondholdings b^1 at time t?

   Now for the next four items to be answered, modify the income process so that $y^1 = 1.5$ and $y^2 = 2.5$, but leave all other parameters unchanged.

5. (5 points) What is the long run or unconditional average income for an agents in this economy? What is the standard deviation of the logarithm of income in the cross section of agents in this economy? What is the mean and standard deviation of changes in income from t to $t+1$ if that agent currently has high income? What is the mean and standard deviation of changes in income from t to $t+1$ if that agent currently has low income?

6. (5 points) What is the equilibrium interest rate in steady-state in this model economy?

7. (5 points) What is cross section standard deviation of the logarithm of consumption in this model economy?

8. (5 points) What do your decision rules imply is the standard deviation of innovations to the logarithm of consumption between t and $t+1$ for an agent who has high income and the maximum bondholdings b^N at time t? What do your decision rules imply is the standard deviation of innovations to the logarithm of consumption between t and $t+1$ for an agent who has high income and the minimum bondholdings b^1 at time t?

   And now, for the next four items to be answered, go back to the assumption that $y^1 = 1$ and $y^2 = 3$ but now set transition probabilities $\varphi_{11} = \varphi_{22} = .95$, and $\varphi_{12} = \varphi_{21} = .05$. Leave all other parameters unchanged.

9. (5 points) What is the long run or unconditional average income for an agents in this economy? What is the standard deviation of the logarithm of income in the cross section of agents in this economy?

What is the mean and standard deviation of changes in income from t to $t+1$ if that agent currently has high income? What is the mean and standard deviation of changes in income from t to $t+1$ if that agent currently has low income?

10. (5 points) What is the equilibrium interest rate in steady-state in this model economy?

11. (5 points) What is cross section standard deviation of the logarithm of consumption in this model economy?

12. (5 points) What do your decision rules imply is the standard deviation of innovations to the logarithm of consumption between t and $t+1$ for an agent who has high income and the maximum bondholdings b^N at time t? What do your decision rules imply is the standard deviation of innovations to the logarithm of consumption between t and $t+1$ for an agent who has high income and the minimum bondholdings b^1 at time t?

Note that in both alternative specifications of the model, one might say that individual income risk is reduced relative to the first baseline case. These alterations should then raise steady-state interest rates. Which alterations has a bigger impact on the steady-state interest rate?

Question 3: Total 60 points In this problem, we examine the impact of taxation on aggregate output in a Lucas span of control model in which agents enjoy leisure.

Consider a static model (only one time period). We assume that there is a population of measure one individuals in this economy and that each of these individuals has some capacity z as a manager and some capacity 1 if employed as a production worker. For convenience, assume that the CDF of manager qualities in the population is given by $M(z)$ with density $m(z)$.

We let $h(z)$ denote the hours worked by a person with talent z if he or she works as a manager and $n(z)$ the hours worked by that person if he or she works as a worker. This individual obtains consumption $c^m(z)$ if he or she works as a manager and consumption $c^w(z)$ if he or she works as a worker. Let W denote the wage available to workers. Let $l(z)$ denote the amount of labor hired by a manager with talent z in his or her firm if he or she choose to manage a firm.

The utility of an agent with talent z who chooses to be a manager is

$$\log c^m(z) + \log(1 - h(z))$$

and if he or she chooses to be a worker

$$\log c^w(z) + \log(1 - n(z))$$

Let W be the wage rate and τ the income tax rate. An individual with talent z who chooses to act as a manager chooses consumption $c^m(z)$, hours $h(z)$ of his or her own effort, and labor hours to hire $l(z)$ subject to constraints

$$(6) \quad c^m(z) = (1 - \tau)\left[(zh(z))^{1-\nu} l(z)^\nu - W l(z)\right]$$

Note that the manager chooses three items here: $c^m(z), h(z), l(z)$. An individual with talent z who chooses to act as a worker choose consumption $c^w(z)$ and hours $n(z)$ subject to constraints

$$(7) \quad c^w(z) = (1 - \tau) W n(z)$$

It will be obvious that in equilibrium, high z individuals should be managers and low z individuals should be workers. Let z^* denote the cutoff value of z such that agents with $z > z^*$ choose to be managers and those with $z \leq z^*$ choose to be workers. With this decision rule, we have that aggregate labor is given by

$$(8) \quad L_p = \int_{z \leq z^*} n(z) m(z) dz$$

so labor market clearing is given by

$$L_p = \int_{z>z^*} l(z)m(z)dz$$

Aggregate output is then given by

$$(9) \quad Y = \int_{z>z^*} (zh(z))^{1-\nu} l(z)^{\nu} m(z)dz$$

and the resource constraint for consumption is given by

$$(10) \quad G + \int_{z \le z^*} c^w(z)m(z)dz + \int_{z>z^*} c^m(z)m(z)dz = Y$$

where G is government spending.

The government's budget constraint is

$$(11) \quad G = \tau Y$$

An equilibrium in this economy is an allocation $\{c^w(z), c^m(z), n(z), h(z), l(z), Y, L_p, z^*\}$ and a wage rate W and tax rate τ and government spending G such that the allocation is feasible in that equations 8 and 10 are satisfied, such that the government budget constraint 11 is satisfied, such that $c^w(z), n(z)$ maximize the utility of an agent who chooses to be a worker given budget constraint 7, such that $c^m(z), h(z), l(z)$ maximize the utility of an agent who chooses to be a manager given budget constraint 6, and such that

$$\log c^m(z) + \log(1 - h(z)) > \log c^w(z) + \log(1 - n(z))$$

if and only if $z > z^*$.

Part A: 30/60 points

Derive the following results regarding equilibrium

1. Do the hours worked by a worker $n(z)$ vary with the wage rate W and/or tax rate τ? If not, why not?

   Answer: No. The first order conditions are

   $$\frac{c^w(z)}{1 - n(z)} = (1 - \tau)W$$

   which, together with the budget constraint 7 implies

   $$\frac{n(z)}{1 - n(z)} = 1$$

   or $n(z) = 1/2$. We have that income and substitution effects cancel out with the preferences that we have assumed.

2. If a manager with talent z works $h(z)$ hours, how much labor $l(z)$ does he or she hire given the wage and tax rates? Show that the answer takes the form $l(z) = Kzh(z)$ where K is a factor or proportionality that you must solve for.

   Answer: The first order condition for the choice of labor to hire in equation 6 is

   $$\nu \left(\frac{zh(z)}{l(z)} \right)^{1-\nu} = W$$

   or

   $$l(z) = zh(z) \left(\frac{\nu}{W} \right)^{1/(1-\nu)}$$

   so

   $$K = \left(\frac{\nu}{W} \right)^{1/(1-\nu)}$$

3. If a manager with talent z works $h(z)$ hours and hires the amount of labor you found in the previous item, what are his or her after-tax earnings? Show that these take the form $(1 - \tau)(1 - \nu)K^{\nu}zh(z)$.

   Answer: By plugging in the form for $l(z)$, we get that firm output is given by $y(z) = zh(z)K^{\nu}$. The wage bill is $Wl(z) = \nu y(z)$, so pretax earnings for the manager are $(1 - \nu)y(z)$. After tax earnings are then given as above.

4. Do the hours worked by a manager $h(z)$ vary with the wage rate W and/or tax rate τ?

   Answer: They do not. The first order conditions for the manager after solving out for optimal $l(z)$ are

   $$\frac{c^m(z)}{1 - h(z)} = (1 - \tau)(1 - \nu)K^{\nu}z$$

   The budget constraint for the manager is

   $$c^m(z) = (1 - \tau)(1 - \nu)K^{\nu}zh(z)$$

   This then gives $h(z) = 1/2$.

5. Does the choice of whether to be a manager or a worker vary with the tax rate τ?

   Answer: It does not. Because hours worked are the same for managers and workers, one will choose to be a manager if that offers a higher after tax earnings. That occurs when

   $$(1-\tau)(1-\nu)\left(\frac{\nu}{W}\right)^{\nu/(1-\nu)} z\frac{1}{2} > (1-\tau)W\frac{1}{2}$$

   and we see that $1-\tau$ cancels from both sides. To complete the proof, we do have to show that the market clearing wage does not vary with τ. Note that holding z^* fixed, we have $L_p = \frac{1}{2}\int_{z\leq z^*} m(z)dz$ and that since

   $$l(z) = \frac{z}{2}\left(\frac{\nu}{W}\right)^{1/(1-\nu)}$$

   we have from equation 8 that the market clearing wage W and z^* do not vary with the tax rate.

6. In the cross section of the population in this model, do high-earning managers work more or fewer hours than workers?

Note that in this specification of our model, we have obtained the result that the tax rate does not impact aggregate output even if agents enjoy leisure. We now show that this result is connected to the result that in our model, in the cross section, high earning managers work the same number of hours as workers. In this way, we have connected data on Engel curves for consumption and leisure with implications for the distortionary impact of income taxation.

Part B: 30/60 points Now consider a variant of our model in which agents have preferences of the following form.

Let the utility of an agent with talent z who chooses to be a manager be

$$\frac{c^m(z)^{1-\sigma}}{1-\sigma} - h(z)$$

and if he or she chooses to be a worker

$$\frac{c^w(z)^{1-\sigma}}{1-\sigma} - n(z)$$

Assume $\sigma > 0$.

1. Do the hours worked by a worker $n(z)$ vary with the wage rate W and/or tax rate τ? Do they rise or fall with the tax rate? How does that depend on parameters?

   Answer: Yes. The first order conditions are

   $$c^w(z)^\sigma = (1-\tau)W$$

   which, together with the budget constraint 7 implies

   $$n(z) = [(1-\tau)W]^{(1-\sigma)/\sigma}$$

   so the sign of the change in labor with tax rates depends on whether $\sigma > 1$ or $\sigma < 1$.

2. If a manager with talent z works $h(z)$ hours, how much labor $l(z)$ does he or she hire given the wage and tax rates? Show that the answer takes the form $l(z) = Kzh(z)$ where K is a factor or proportionality that you must solve for.

   Answer: This is the same as in part A.

3. If a manager with talent z works $h(z)$ hours and hires the amount of labor you found in the previous item, what are his or her after-tax earnings? Show that these take the form $(1-\tau)(1-\nu)K^\nu zh(z)$.

   Answer: This is the same as in part A.

4. Do the hours worked by a manager $h(z)$ vary with the wage rate W and/or tax rate τ? Do they rise or fall with the tax rate? How does that depend on parameters?

   Answer: They do. The first order conditions for the manager after solving out for optimal $l(z)$ are

   $$c^m(z)^\sigma = (1-\tau)(1-\nu)K^\nu z$$

   The budget constraint for the manager is

   $$c^m(z) = (1-\tau)(1-\nu)K^\nu zh(z)$$

This then gives

$$h(z) = [(1 - \tau)(1 - \nu)K^{\nu} z]^{(1-\sigma)/\sigma}$$

Again, the sign of the change in labor with tax rates depends on whether $\sigma > 1$ or $\sigma < 1$.

5. Do the hours worked by a manager $h(z)$ vary with z? Do they rise with z or fall with z? How does that depend on parameters? Could we use information about how manager's hours vary in the cross section of manager earnings to estimate how manager and worker hours would respond to changes in taxes?

   Answer: We can. From the previous item, we have

   $$h(z) = [(1 - \tau)(1 - \nu)K^{\nu} z]^{(1-\sigma)/\sigma}$$

   so the responsiveness of manager hours to z in the cross section correspond to the reponsiveness of manager hours to changes in tax rates $(1 - \tau)$. Note from items above that the after tax earnings per hour of a manager in the cross section are linear in z, so a change in z in the cross section corresponds directly to a change in after-tax earnings per hour.

6. What would you guess the cross section data on hours worked by managers at different levels of earnings would show? How would that data shape your views on the impact of income taxes on hours worked by managers?