202C HW1

John Friedman

April 7, 2024

Question 1: In this question, we consider a 4-country version of the Armington model. Consider the following parameter values: $\sigma = 3$, $a_{i,j} = 2$ for all $i = j$, $a_{i,j} = 1$ for all $i \neq j$, $L_1 = 2$, $L_2 = L_3 = L_4 = 1$, $A_1 = A_2 = A_3 = A_4 = 0.6$, and trade cost $\tau_{i,j}$ corresponds to the element i, j in the following matrix:

$$T = \begin{pmatrix} 1 & 1.1 & 1.2 & 1.3 \\ 1.3 & 1 & 1.3 & 1.4 \\ 1.2 & 1.2 & 1 & 1.1 \\ 1.1 & 1.1 & 1.1 & 1 \end{pmatrix}$$

1. Solve for equilibrium wages in countries 2, 3 and 4 relative to country 1.

Solution:

1. We will use the good of country 1 as the numeraire ($p_1 = 1$)
2. The excess demand function of the Armington Model is:

$$Z_i(w) = \{\sum_{j \in S} \left(\frac{a_{ij} \tau_{ij}^{1-\sigma} (\frac{w_i}{A_i})^{1-\sigma}}{\sum_{k \in S} a_{kj} \tau_{kj}^{1-\sigma} (\frac{w_k}{A_k})^{1-\sigma}} \right) \frac{w_j L_j}{w_i} \} - L_i$$

3. We solve by guessing a vector of wages, and iterating until the excess demand function is zero.

   Using the code attached in the appendix: $\frac{w_2}{w_1} = 1.1458$, $\frac{w_3}{w_1} = 1.2238$, $\frac{w_4}{w_1} = 1.2614$

2. Solve for bilateral trade shares, λ_{ij} for $i = 1, 2, 3, 4$ and $j = 1, 2, 3, 4$

Solution: Since we have already solved for wages, we can use the formula $\lambda_{ij} = \frac{X_{ij}}{Y_j}$ To solve for bilateral trade shares we use the code attached in the appendix. We get:

$$\lambda = \begin{pmatrix} 1.1649 & 0.2164 & 0.1892 & 0.1682 \\ 0.2625 & 0.3989 & 0.1228 & 0.1105 \\ 0.2701 & 0.1214 & 0.3637 & 0.1568 \\ 0.3025 & 0.136 & 0.1415 & 0.3573 \end{pmatrix}$$

3. Consider (only in this question) that country 2's productivity increases by a factor of 2, from $A_2 = 0.6$ to $A_2' = 1.2$, while the others remain unchanged.

   a) What's the change in welfare for country 2 from the productivity shock?

> **Solution:** The formula for welfare is $W_i = \lambda ii^{\frac{1}{1-\sigma}} a_{ii}^{\frac{1}{\sigma-1}} A_i$ So using our code:
>
> $$W_i^{new}/W_i^{old} = \frac{3.1484}{1.3435} = 2.3434$$

b) What's the change in welfare for country 2 from the productivity shock under autarky ? (hint: you simply need to use an equation from the lecture and no need to solve the model)

> **Solution:** From Sunny's first section, we know that
>
> $$W_i = \lambda_{ii}^{\frac{1}{1-\sigma}} \alpha_{ii}^{\frac{1}{\sigma-1}} A_i$$
>
> The change in welfare is:
>
> $$\frac{W_i^{new}}{W_i^{old}} = \frac{\lambda_{ii}^{\frac{1}{1-\sigma}} \alpha_{ii}^{\frac{1}{\sigma-1}} A_i'}{\lambda_{ii}^{\frac{1}{1-\sigma}} \alpha_{ii}^{\frac{1}{\sigma-1}} A_i} = \frac{A_i'}{A_i} = \frac{1.2}{0.6} = 2$$

c) Provide intuition for the difference in your answers in (a) and (b)

> **Solution:** In part (a) and (b) we have shown that the increase in welfare from a individual country's productivity shock is greater in an open economy than under autarky for that country.
>
> Under this model's parameters, as country 2 becomes more productive, it's income effects dominates its substition effects, and so it's increase in welfare is higher than in autarky. This is due to a strong taste shock preference for its own goods.
>
> Note that if the country preferred other countries goods to its own, the likely result would be for country 2 to trade more with the world, which would reduce its relative price and import share. e.g. terms of trade would become more unfavorable for country 2.

4.

$$T = \begin{pmatrix} 1 & 1 & 1.2 & 1.2 \\ 1 & 1 & 1.2 & 1.2 \\ 1 & 1.2 & 1 & 1.3 \\ 1 & 1.2 & 1.2 & 1 \end{pmatrix}$$

1. Solve for equilibrium wages in countries 2, 3 and 4 relative to country 1.

> **Solution:** $\frac{w_2}{w_1} = 1.2337 \frac{w_3}{w_1} = 1.2356 \frac{w_4}{w_1} = 1.2506$

2. Solve for bilateral trade shares, λ_{ij} for $i = 1, 2, 3, 4$ and $j = 1, 2, 3, 4$

> **Solution:**
>
> $$\lambda = \begin{pmatrix} 1.0123 & 0.2523 & 0.1935 & 0.1971 \\ 0.3325 & 0.3315 & 0.1271 & 0.1295 \\ 0.3316 & 0.1148 & 0.365 & 0.11 \\ 0.3236 & 0.112 & 0.1237 & 0.363 \end{pmatrix}$$

5. Solve for the change in wage in each country (relative to country 1's wage) using the system in changes discussed in Section 5 in the Lecture Notes. Verify that you get the same result as in 4. What is the advantage of solving the system in changes rather than in levels (two times)?

Solution: TODO