# 202C HW1

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**Question 1:** In this question, we consider a 4-country version of the Armington model. Consider the following parameter values: $\sigma = 3$, $a_{i,j} = 2$ for all $i = j$, $a_{i,j} = 1$ for all $i \neq j$, $L_1 = 2$, $L_2 = L_3 = L_4 = 1$, $A_1 = A_2 = A_3 = A_4 = 0.6$, and trade cost $\tau_{i,j}$ corresponds to the element $i, j$ in the following matrix:

$$T = \begin{pmatrix} 1 & 1.1 & 1.2 & 1.3 \\ 1.3 & 1 & 1.3 & 1.4 \\ 1.2 & 1.2 & 1 & 1.1 \\ 1.1 & 1.1 & 1.1 & 1 \end{pmatrix}$$

1. Solve for equilibrium wages in countries 2, 3 and 4 relative to country 1.

**Solution:**

1. We will use the good of country 1 as the numeraire ($p_1 = 1$) Since $p_i = \frac{w_i}{A_i} \implies 1 = \frac{w_1}{0.6} \implies w_1 = 0.6$
2. The excess demand function of the Armington Model is:

$$Z_i(w) = \{\sum_{j \in S} (\frac{a_{ij}\tau_{ij}^{1-\sigma}(\frac{w_i}{A_i})^{1-\sigma}}{\sum_{k \in S} a_{kj}\tau_{kj}^{1-\sigma}(\frac{w_k}{A_k})^{1-\sigma}}) \frac{w_j L_j}{w_i}\} - L_i$$

   Note that I have moved $\frac{1}{w_i}$ outside the sum for simplicity.
3. We solve by guessing a vector of wages, and iterating until the excess demand function is zero.

   TODO

2. Solve for bilateral trade shares, $\lambda_{ij}$ for $i = 1, 2, 3, 4$ and $j = 1, 2, 3, 4$
3. Consider (only in this question) that country 2's productivity increases by a factor of 2, from $A_2 = 0.6$ to $A_2' = 1.2$, while the others remain unchanged.
   a) What's the change in welfare for country 2 from the productivity shock?
   b) What's the change in welfare for country 2 from the productivity shock under autarky ? (hint: you simply need to use an equation from the lecture and no need to solve the model)

**Solution:** From Sunny's first section, we know that

$$W_i = \lambda_{ii}^{\frac{1}{1-\sigma}} \alpha_{ii}^{\frac{1}{\sigma-1}} A_i$$

The change in welfare is:

$$\frac{W_i^{new}}{W_i^{old}} = \frac{\lambda_{ii}^{\frac{1}{1-\sigma}} \alpha_{ii}^{\frac{1}{\sigma-1}} A_i'}{\lambda_{ii}^{\frac{1}{1-\sigma}} \alpha_{ii}^{\frac{1}{\sigma-1}} A_i} = \frac{A_i'}{A_i} = \frac{1.2}{0.6} = 2$$

c) Provide intuition for the difference in your answers in (a) and (b)

4.
$$T = \begin{pmatrix} 1 & 1 & 1.2 & 1.2 \\ 1 & 1 & 1.2 & 1.2 \\ 1 & 1.2 & 1 & 1.3 \\ 1 & 1.2 & 1.2 & 1 \end{pmatrix}$$

1. Solve for equilibrium wages in countries 2, 3 and 4 relative to country 1.
2. Solve for bilateral trade shares, $\lambda_{ij}$ for $i = 1, 2, 3, 4$ and $j = 1, 2, 3, 4$

5. Solve for the change in wage in each country (relative to country 1's wage) using the system in changes discussed in Section 5 in the Lecture Notes. Verify that you get the same result as in 4. What is the advantage of solving the system in changes rather than in levels (two times)?

**Solution:**