

Econ 202A Notes: Macroeconomic Theory I

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1 Basic Neoclassical Growth Model (Cass-Koopmans)

Economy consists of many identical infinitely lived households – all have the same preferences and endowments.

Several Interpretations

1. Representative Agent (Robinson Crusoe)
2. Social Planner
3. Infinitely Lived Family (dynasty)

One production sector – output produced from capital and labor. Can be consumed or invested. Investment becomes productive capital after one period.

$$\begin{aligned} \max \quad & \sum_{t=0}^{\infty} \beta^t u(c_t) \\ \text{s.t.} \quad & \beta \in (0, 1) \\ & c_t + i_t \leq y_t = f(k_t, n_t) \\ & k_{t+1} \leq (1 - \delta)k_t + i_t \\ & 0 \leq n_t \leq 1 \\ & k_0, k_1 > 0 \end{aligned}$$

Note that no prices are given here!

1.1 Production Function

$$F : \mathbb{R}_+^2 \rightarrow \mathbb{R}_+$$

1. continuously differentiable
2. homogenous of degree 1 \Rightarrow constant returns to scale
3. strictly quasi-concave

4. $F(0,n) = 0$
 - (a) implies capital is essential
 - (b) $F_k =$ marginal product of capital > 0
 - (c) $F_n =$ marginal product of labor > 0
5. $\lim_{k \rightarrow 0} F(k, 1) = \infty$
6. $\lim_{k \rightarrow \infty} F(k, 1) = 0$

1.2 Utility Function

$u : \mathbb{R}_+ \rightarrow \mathbb{R}$

1. bounded - needed for dynamic programming
2. continuously differentiable
3. strictly increasing
4. strictly concave
5. $\lim_{c \rightarrow 0} u'(c) = \infty$

Note: We will use functional forms for F and N for most everything we do in this class.

1.3 Simplifying the Planner's Problem

1. $F_N > 0$ *work from here*