Economics 202A
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Assignment 1: Neoclassical Growth and Dynamic Programming

1. Consider the following problem solved by a representative agent:

$$\max \sum_{t=0}^{\infty} \beta^t \log c_t \quad , 0 < \beta < 1$$

subject to

$$c_t + k_{t+1} \leq k_t^{\theta} \quad , 0 < \theta < 1$$
$$k_0 \text{ given.}$$

The variables c_t and k_t, are time t values of consumption and the capital stock respectively.

A. Write this optimization problem as a dynamic programming problem. Find the optimal law of motion expressing k_{t+1} as a function of the state.

B. Compute the steady state from the optimal law of motion found in part A.

C. Derive the first order condition and envelope condition associated with the dynamic programming problem in part A. Use these to find the steady state capital stock. Is it the same as the one found in part B?

2. Consider the following more complicated problem:

$$\max \sum_{t=0}^{\infty} \beta^t \left(\log c_t - B h_t \right) \quad , 0 < \beta < 1$$

subject to

$$c_t + k_{t+1} \leq A k_t^{\theta} h_t^{1-\theta} \quad , 0 < \theta < 1$$
$$k_0 \text{ given.}$$

In this case h_t is hours worked in period t.

A. Write this optimization problem as a dynamic programming problem. Find the optimal law of motion expressing k_{t+1} as a function of the state and the optimal decision rule expressing h_t as a function of the state.

B. Derive the first order condition and envelope condition associated with the dynamic programming problem in part A. Use these to find the steady state capital stock and hours worked.

3. Consider again problem 1. Set $\beta = .99$ and $\theta = .36$.

a. What is the maximum sustainable capital stock? Explain.

b. Write a program in Matlab (or other language of your choice) that computes the optimal value function and decision rule by value iteration. Use a grid with a minimum capital stock of .01, a maximum that is equal to the maximum sustainable capital stock, and 1000 equally spaced grid points.

c. Plot the decision rule you obtain with k on the horizontal axis and k' on the vertical axis. Plot the true decision rule obtained in question 1 on the same plot.

d. Repeat part b and c, except allow capital to depreciate at the rate $\delta = .02$. In this case, of course, you are comparing your computed decision rule with the true decision rule when $\delta = 1$.