Chapter 15
Economic Growth

15.1. Introduction

This chapter describes basic nonstochastic models of sustained economic growth. We begin by describing a benchmark exogenous growth model in which sustained growth is driven by exogenous growth in labor productivity. Then we turn our attention to several endogenous growth models in which sustained growth of labor productivity is somehow *chosen* by the households in the economy. We describe several models that differ in whether the equilibrium market economy matches what a benevolent planner would choose. Where the market outcome doesn't match the planner's outcome, there can be room for welfare-improving government interventions. The objective of the chapter is to shed light on the mechanisms at work in different models. We facilitate comparison by using the same production function and simply changing the meaning of one argument.

In the spirit of Arrow's (1962) model of learning by doing, Romer (1986) presents an endogenous growth model in which the accumulation of capital (or knowledge) is associated with a positive externality on the available technology. The aggregate of all agents' holdings of capital is positively related to the level of technology, which in turn interacts with individual agents' savings decisions and thereby determines the economy's growth rate. Thus, the households in this economy are *choosing* how fast the economy is growing, but they do so in an unintentional way. The competitive equilibrium growth rate is less than the socially optimal one.

Another approach assumes that all production factors are reproducible. Following Uzawa (1965), Lucas (1988) formulates a model with accumulation of both physical and human capital. The joint accumulation of all inputs ensures that growth will not come to a halt even though each individual factor in the final-good production function is subject to diminishing returns. In the absence of externalities, the growth rate in the competitive equilibrium coincides with the social optimum.

Romer (1987) constructs a model in which agents can choose to engage in research that produces technological improvements. Each invention represents a technology for producing a new type of intermediate input that can be used in the production of final goods without affecting the marginal product of existing intermediate inputs. The introduction of new inputs enables the economy to experience sustained growth even though each intermediate input taken separately is subject to diminishing returns. In a decentralized equilibrium, private agents will expend resources on research only if they are granted property rights over their inventions. Under the assumption of infinitely lived patents, Romer solves for a monopolistically competitive equilibrium that exhibits the classic tension between static and dynamic efficiency. Patents and the associated market power are necessary for there to be research and new inventions in a decentralized equilibrium, while the efficient production of existing intermediate inputs would require marginal-cost pricing, that is, the abolition of granted patents. The monopolistically competitive equilibrium is characterized by smaller supplies of intermediate inputs and a lower growth rate than is socially optimal.

Finally, we revisit the question of when nonreproducible factors may not pose an obstacle to growth. Rebelo (1991) shows that even if there are nonreproducible factors in fixed supply in a neoclassical growth model, sustained growth is possible if there is a "core" of capital goods that is produced without direct or indirect use of the nonreproducible factors. Because of the ever-increasing relative scarcity of a nonreproducible factor, Rebelo finds that its price increases over time relative to a reproducible factor. Romer (1990) assumes that research requires the input of labor and not only goods as in his earlier model (1987). Now, if labor is in fixed supply and workers' innate productivity is constant, it follows immediately that growth must asymptotically come to an halt. To make sustained growth feasible, we can take a cue from our earlier discussion. One modeling strategy would be to introduce an externality that enhances researchers' productivity, and an alternative approach would be to assume that researchers can accumulate human capital. Romer adopts the first type of assumption, and we find it instructive to focus on its role in overcoming a barrier to growth that nonreproducible labor would otherwise pose.

15.2. The economy

The economy has a constant population of a large number of identical agents who order consumption streams $\{c_t\}_{t=0}^{\infty}$ according to

$$\sum_{t=0}^{\infty} \beta^t u\left(c_t\right), \text{ with } \beta \in (0,1) \text{ and } u\left(c\right) = \frac{c^{1-\sigma}-1}{1-\sigma} \text{ for } \sigma \in [0,\infty), \quad (15.2.1)$$

and $\sigma = 1$ is taken to be logarithmic utility.[1] Lowercase letters for quantities, such as c_t for consumption, are used to denote individual variables, and uppercase letters stand for aggregate quantities.

For most part of our discussion of economic growth, the production function takes the form

$$F\left(K_t, X_t\right) = X_t f\left(\hat{K}_t\right), \quad \text{where } \hat{K}_t \equiv \frac{K_t}{X_t}. \quad (15.2.2)$$

That is, the production function $F(K,X)$ exhibits constant returns to scale in its two arguments, which via Euler's theorem on linearly homogeneous functions implies

$$F\left(K, X\right) = F_1\left(K, X\right) K + F_2\left(K, X\right) X, \quad (15.2.3)$$

where $F_i(K,X)$ is the derivative with respect to the ith argument (and $F_{ii}(K,X)$ will be used to denote the second derivative with respect to the ith argument). The input K_t is physical capital with a rate of depreciation equal to δ. New capital can be created by transforming one unit of output into one unit of capital. Past investments are reversible. It follows that the relative price of capital in terms of the consumption good must always be equal to 1. The second argument X_t captures the contribution of labor. Its precise meaning will differ among the various setups that we will examine.

We assume that the production function satisfies standard assumptions of positive but diminishing marginal products,

$$F_i\left(K, X\right) > 0, \quad F_{ii}\left(K, X\right) < 0, \quad \text{for } i = 1, 2;$$

and the Inada conditions,

$$\lim_{K \to 0} F_1\left(K, X\right) = \lim_{X \to 0} F_2\left(K, X\right) = \infty,$$
$$\lim_{K \to \infty} F_1\left(K, X\right) = \lim_{X \to \infty} F_2\left(K, X\right) = 0,$$

[1] By virtue of L'Hôpital's rule, the limit of $(c^{1-\sigma}-1)/(1-\sigma)$ is $\log(c)$ as σ goes to 1.

which imply

$$\lim_{\hat{K}\to 0} f'\left(\hat{K}\right) = \infty, \quad \lim_{\hat{K}\to\infty} f'\left(\hat{K}\right) = 0. \tag{15.2.4}$$

We will also make use of the mathematical fact that a linearly homogeneous function $F(K, X)$ has first derivatives $F_i(K, X)$ homogeneous of degree 0; thus, the first derivatives are only functions of the ratio $\hat{K}$. In particular, we have

$$F_1(K, X) = \frac{\partial X f(K/X)}{\partial K} = f'\left(\hat{K}\right), \tag{15.2.5a}$$

$$F_2(K, X) = \frac{\partial X f(K/X)}{\partial X} = f\left(\hat{K}\right) - f'\left(\hat{K}\right)\hat{K}. \tag{15.2.5b}$$

15.2.1. Balanced growth path

We seek additional technological assumptions to generate market outcomes with steady-state growth of consumption at a constant rate $1 + \mu = c_{t+1}/c_t$. The literature uses the term "balanced growth path" to denote a situation where all endogenous variables grow at constant (but possibly different) rates. Along such a steady-state growth path (and during any transition toward the steady state), the return to physical capital must be such that households are willing to hold the economy's capital stock.

In a competitive equilibrium where firms rent capital from the agents, the rental payment r_t is equal to the marginal product of capital,

$$r_t = F_1(K_t, X_t) = f'\left(\hat{K}_t\right). \tag{15.2.6}$$

Households maximize utility given by equation (15.2.1) subject to the sequence of budget constraints

$$c_t + k_{t+1} = r_t k_t + (1-\delta) k_t + \chi_t, \tag{15.2.7}$$

where χ_t stands for labor-related budget terms. The first-order condition with respect to k_{t+1} is

$$u'(c_t) = \beta u'(c_{t+1})(r_{t+1} + 1 - \delta). \tag{15.2.8}$$

After using equations (15.2.1) and (15.2.6) in equation (15.2.8), we arrive at the following equilibrium condition:

$$\left(\frac{c_{t+1}}{c_t}\right)^{\sigma} = \beta\left[f'\left(\hat{K}_{t+1}\right) + 1 - \delta\right]. \tag{15.2.9}$$

We see that a constant consumption growth rate on the left side is sustained in an equilibrium by a constant rate of return on the right side. It was also for this reason that we chose the class of utility functions in equation (15.2.1) that exhibits a constant intertemporal elasticity of substitution. These preferences allow for balanced growth paths.[2]

Equation (15.2.9) makes clear that capital accumulation alone cannot sustain steady-state consumption growth when the labor input X_t is constant over time, $X_t = L$. Given the second Inada condition in equations (15.2.4), the limit of the right side of equation (15.2.9) is $\beta(1-\delta)$ when $\hat{K}$ approaches infinity. The steady state with a constant labor input must therefore be a constant consumption level and a capital-labor ratio $\hat{K}^\star$ given by

$$f'\left(\hat{K}^\star\right) = \beta^{-1} - (1-\delta). \tag{15.2.10}$$

In chapter 5 we derived a closed-form solution for the transition dynamics toward such a steady state in the case of logarithmic utility, a Cobb-Douglas production function, and $\delta = 1$.

15.3. Exogenous growth

As in Solow's (1956) classic article, the simplest way to ensure steady-state consumption growth is to postulate exogenous labor-augmenting technological change at the constant rate $1 + \mu \geq 1$,

$$X_t = A_t L, \qquad \text{with } A_t = (1+\mu) A_{t-1},$$

where L is a fixed stock of labor. Our conjecture is then that both consumption and physical capital will grow at that same rate $1+\mu$ along a balanced growth path. The same growth rate of K_t and A_t implies that the ratio $\hat{K}$ and therefore the marginal product of capital remain constant in the steady state. A time-invariant rate of return is in turn consistent with households choosing a constant growth rate of consumption, given the assumption of isoelastic preferences.

[2] To ensure well-defined maximization problems, a maintained assumption throughout the chapter is that parameters are such that any derived consumption growth rate $1+\mu$ yields finite lifetime utility; i.e., the implicit restriction on parameter values is that $\beta(1+\mu)^{1-\sigma} < 1$. To see that this condition is needed, substitute the consumption sequence $\{c_t\}_{t=0}^\infty = \{(1+\mu)^t c_0\}_{t=0}^\infty$ into equation (15.2.1).

Evaluating equation (15.2.9) at a steady state, the optimal ratio $\hat{K}^\star$ is given by

$$(1+\mu)^\sigma \;=\; \beta\left[f'\left(\hat{K}^\star\right)+1-\delta\right]. \tag{15.3.1}$$

While the steady-state consumption growth rate is exogenously given by $1+\mu$, the endogenous steady-state ratio $\hat{K}^\star$ is such that the implied rate of return on capital induces the agents to choose a consumption growth rate of $1+\mu$. As can be seen, a higher degree of patience (a larger β), a higher willingness intertemporally to substitute (a lower σ), and a more durable capital stock (a lower δ) each yield a higher ratio $\hat{K}^\star$, and therefore more output (and consumption) at a point in time, but the growth rate remains fixed at the rate of exogenous labor-augmenting technological change. It is straightforward to verify that the competitive equilibrium outcome is Pareto optimal, since the private return to capital coincides with the social return.

Physical capital is compensated according to equation (15.2.6), and labor is also paid its marginal product in a competitive equilibrium,

$$w_t = F_2\left(K_t, X_t\right)\frac{\mathrm{d}\,X_t}{\mathrm{d}\,L} = F_2\left(K_t, X_t\right)\,A_t. \tag{15.3.2}$$

So, by equation (15.2.3), we have

$$r_t K_t + w_t L = F\left(K_t, A_t L\right).$$

Factor payments are equal to total production, which is the standard result of a competitive equilibrium with constant-returns-to-scale technologies. However, it is interesting to note that if A_t were a separate production factor, there could not exist a competitive equilibrium, since factor payments based on marginal products would exceed total production. In other words, the dilemma would then be that the production function $F(K_t, A_t L)$ exhibits increasing returns to scale in the three "inputs" K_t, A_t, and L, which is not compatible with the existence of a competitive equilibrium. This problem is to be kept in mind as we now turn to one way to endogenize economic growth.