

# **GEOGG141/ GEOG3051**

## **Principles & Practice of Remote Sensing (PPRS)**

### **Radiative Transfer Theory at optical wavelengths applied to vegetation canopies: part 2**

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**<http://www2.geog.ucl.ac.uk/~mdisney/teaching/GEOGG141/GEOGG141.html>**

**<http://www2.geog.ucl.ac.uk/~mdisney/teaching/3051/GEOG3051.html>**

Notes adapted from Prof. P. Lewis [plewis@geog.ucl.ac.uk](mailto:plewis@geog.ucl.ac.uk)

# Reading

## Full notes for these lectures

[http://www2.geog.ucl.ac.uk/~mdisney/teaching/GEOGG141/rt\\_theory/rt\\_notes1.pdf](http://www2.geog.ucl.ac.uk/~mdisney/teaching/GEOGG141/rt_theory/rt_notes1.pdf)

[http://www2.geog.ucl.ac.uk/~mdisney/teaching/GEOGG141/rt\\_theory/rt\\_notes2.pdf](http://www2.geog.ucl.ac.uk/~mdisney/teaching/GEOGG141/rt_theory/rt_notes2.pdf)

## Books

Jensen, J. (2007) *Remote Sensing: an Earth Resources Perspective*, 2<sup>nd</sup> ed., Chapter 11 (355-408), 1<sup>st</sup> ed chapter 10.

Liang, S. (2004) *Quantitative Remote Sensing of Land Surfaces*, Wiley, Chapter 3 (76-142).

Monteith, J. L. and Unsworth, M. H. (1990) *Principles of Environmental Physics*, 2<sup>nd</sup> ed., ch 5 & 6.

## Papers

Disney et al. (2000) Monte Carlo ray tracing in optical canopy reflectance modelling, *Remote Sensing Reviews*, 18, 163 – 196.

Feret, J-B. et al. (2008) PROSPECT-4 and 5: Advances in the leaf optical properties model separating photosynthetic pigments, *RSE*, 112, 3030-3043.

Jacquemoud, S. and Baret, F. (1990) PROSPECT: A model of leaf optical properties spectra, *RSE*, 34, 75-91.

Lewis, P. and Disney, M. I. (2007) Spectral invariants and scattering across multiple scale from within-leaf to canopy, *RSE*, 109, 196-206.

Nilson, T. and Kuusk, A. (1989) A canopy reflectance model for the homogeneous plant canopy and its inversion, *RSE*, 27, 157-167.

Price, J. (1990), On the information content of soil reflectance spectra *RSE*, 33, 113-121

Walthall, C. L. et al. (1985) Simple equation to approximate the bidirectional reflectance from vegetative canopies and bare soil surfaces, *Applied Optics*, 24(3), 383-387.

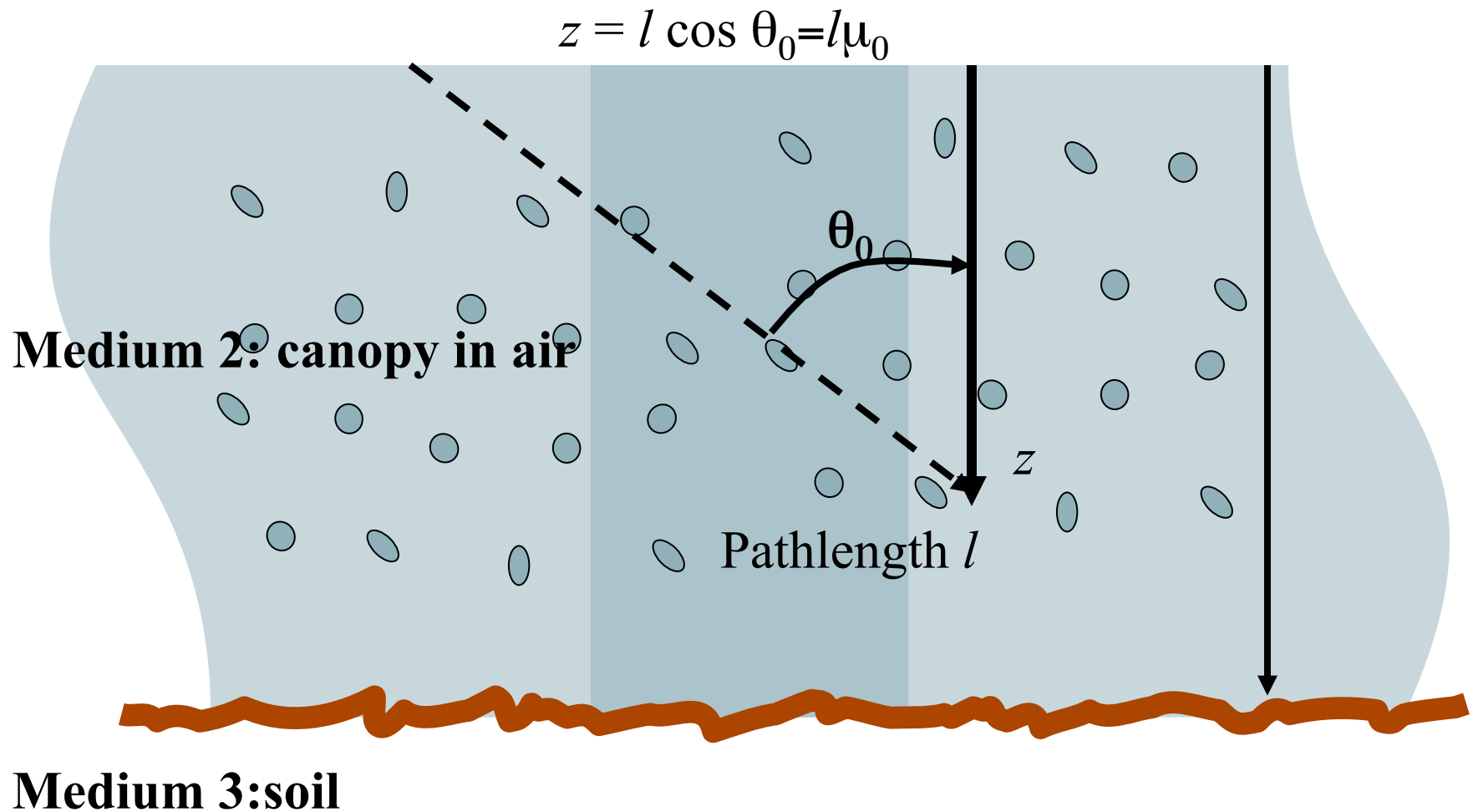
# Radiative Transfer equation

- Describe propagation of radiation through a medium under absorption, emission and scattering processes
- Origins
  - Schuster (1905), Schwarzschild (1906, 1914), Eddington (1916)....
  - Chandrasekhar (1950) – key developments in star formation, showed how to solve under variety of assumptions & cases
  - Applications in nuclear physics (neutron transport), astrophysics, climate, biology, ecology etc. etc.
- Used extensively for (optical) vegetation since 1960s (Ross, 1981)
- Used for microwave vegetation since 1980s

# Radiative Transfer equation

- Consider energy balance across elemental volume
- Generally use scalar form (SRT) in optical
- Generally use vector form (VRT) for microwave

# Medium 1: air



Path of radiation - - - - -

# Scalar Radiative Transfer Equation

- 1-D scalar radiative transfer (SRT) equation
  - for a plane parallel medium (air) embedded with a low density of small scatterers
  - change in specific Intensity (Radiance)  $I(z, \underline{\Omega})$  at depth  $z$  in direction  $\underline{\Omega}$  wrt  $z$ :

$$\mu \frac{\partial I(z, \underline{\Omega})}{\partial z} = -K_e I(\underline{\Omega}, z) + J_s(\underline{\Omega}, z)$$

- Crucially, an integro-differential equation (i.e. hard to solve)

$$\mu \frac{\partial I(z, \underline{\Omega})}{\partial z} = -\kappa_e I(\underline{\Omega}, z) + J_s(\underline{\Omega}, z)$$

- Source Function:

$$J_s(\underline{\Omega}, z) = \int_{4\pi} P(z, \underline{\Omega}' \rightarrow \underline{\Omega};) I(\underline{\Omega}, \underline{\Omega}') d\underline{\Omega}'$$

- $\mu$  - cosine of the direction vector ( $\underline{\Omega}$ ) with the local normal  
– accounts for path length through the canopy
- $\kappa_e$  - volume extinction coefficient
- $P(z, \underline{\Omega}' \rightarrow \underline{\Omega})$  is the volume scattering phase function i.e. prob. of photon at depth  $z$  being scattered from illum direction  $\underline{\Omega}$  to view direction  $\underline{\Omega}'$

# Extinction Coefficient and Beer's Law

$$K_e$$

- Volume extinction coefficient:
  - ‘total interaction cross section’
  - ‘extinction loss’
  - ‘number of interactions’ per unit length
- a measure of attenuation of radiation in a canopy (or other medium).

$$I(l) = I_0 e^{-K_e l}$$

Beer's Law



# Extinction Coefficient and Beers Law

$$I(l) = I_0 e^{-\kappa_e l}$$

$$\begin{aligned} \frac{dI}{dl} &= -\kappa_e I_0 e^{-\kappa_e l} \\ &= -\kappa_e I \end{aligned}$$

No source version of SRT eqn  $\mu \frac{\partial I(z, \underline{\Omega})}{\partial z} = -\kappa_e I(\underline{\Omega}, z) + J_s(\underline{\Omega}, z)$

# Optical Extinction Coefficient for Oriented Leaves

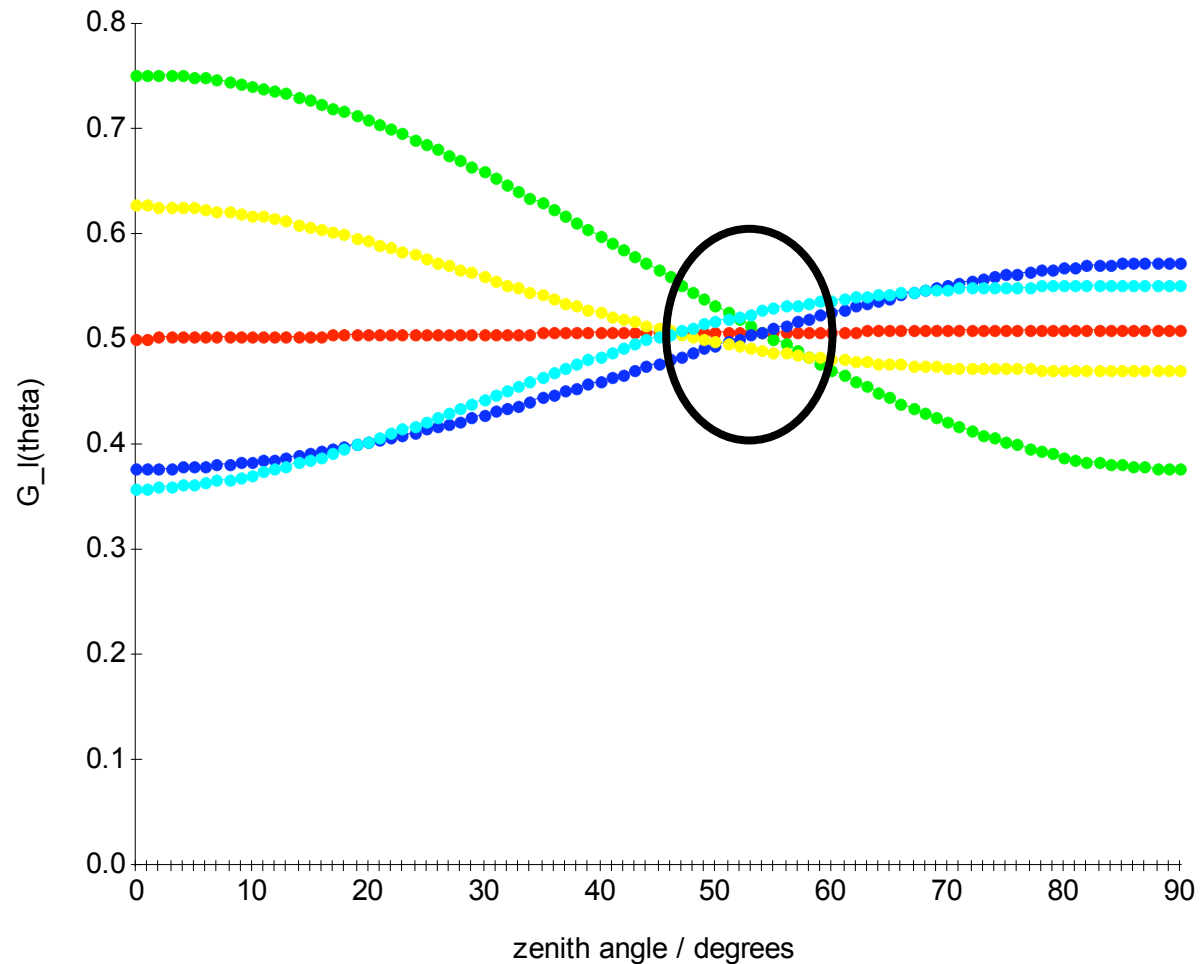
- Volume extinction coefficient:

$$\kappa_e(\underline{\Omega}) = u_l G_l(\underline{\Omega})$$

- $u_l$  : leaf area density
  - Area of leaves per unit volume
- $G_l$  : (Ross) projection function

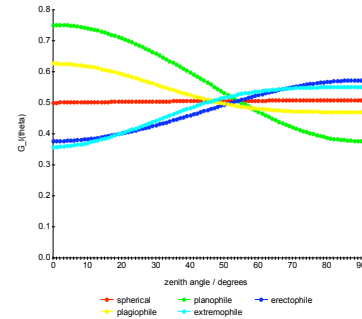
$$G_l(\underline{\Omega}) = \frac{1}{2\pi} \int_{2\pi+} g_l(\underline{\Omega}_l) |\underline{\Omega} \cdot \underline{\Omega}_l| d\underline{\Omega}_l$$

# Optical Extinction Coefficient for Oriented Leaves



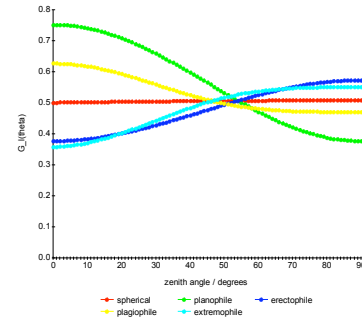
$$G_l(\underline{\Omega}) = \frac{1}{2\pi} \int_{2\pi+} g_l(\underline{\Omega}_l) |\underline{\Omega} \cdot \underline{\Omega}_l| d\underline{\Omega}_l$$

- spherical
- planophile
- erectophile
- plagiophile
- extremophile



- range of G-functions small (0.3-0.8) and smoother than leaf inclination distributions;
- planophile canopies, G-function is high ( $>0.5$ ) for low zenith and low ( $<0.5$ ) for high zenith;
- converse true for erectophile canopies;
- G-function always close to 0.5 between  $50^\circ$  and  $60^\circ$
- essentially invariant at 0.5 over different leaf angle distributions at  $57.5^\circ$ .

$$\int_{z=0}^{z=-H} -\frac{u_l G(\underline{\Omega})}{\mu} dz = \frac{LG(\underline{\Omega})}{\mu}$$



- so, radiation at bottom of canopy for spherical:

$$I_0 e^{-\frac{LG(\underline{\Omega})}{\mu}} = I_0 e^{-\frac{0.5L}{\mu}}$$

- for horizontal:

$$= I_0 e^{-L}$$

# A Scalar Radiative Transfer Solution

- Attempt similar first Order Scattering solution
  - in optical, consider total number of interactions
    - with leaves + soil
- Already have extinction coefficient:

$$\kappa_e(\underline{\Omega}) = u_l G(\underline{\Omega})$$

# SRT

- Phase function: 
$$P(\underline{\Omega}' \rightarrow \underline{\Omega}) = \frac{1}{\mu'} u_l \Gamma(\underline{\Omega}' \rightarrow \underline{\Omega})$$
- Probability of photon being scattered from incident ( $\underline{\Omega}'$ ) to view ( $\underline{\Omega}$ )
- $u_l$  - leaf area density;
- $\mu'$  - cosine of the incident zenith angle
- $\Gamma$  - area scattering phase function.

# SRT

- Area scattering phase function:

$$\Gamma(\underline{\Omega}' \rightarrow \underline{\Omega}) = \frac{1}{4\pi} \int_{4\pi} \omega_l g_l(\underline{\Omega}_l) |\underline{\Omega} \cdot \underline{\Omega}_l| |\underline{\Omega}' \cdot \underline{\Omega}_l| d\underline{\Omega}_l$$

- double projection, modulated by spectral terms
- $\omega_l$  : leaf single scattering albedo
  - Probability of radiation being scattered rather than absorbed at leaf level
  - Function of wavelength – low transmission, low fwd. scattering and vice versa



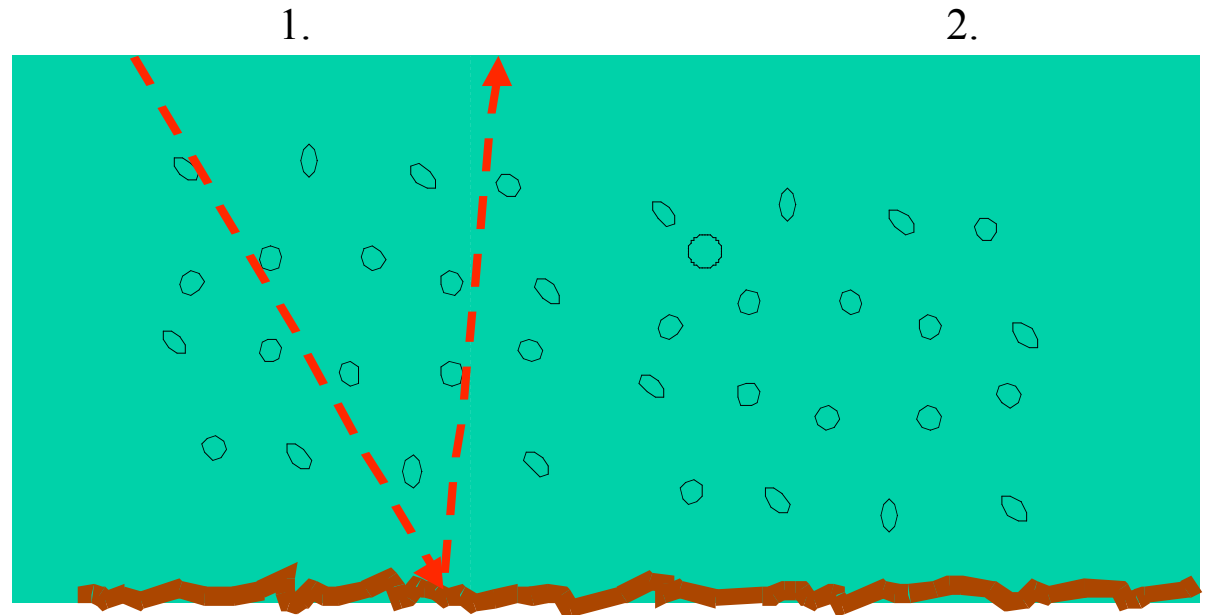
# SRT

$$\begin{aligned}
 \underline{I}^+(\underline{\Omega}_s, z) = & e^{-L \left( \frac{G(\underline{\Omega}_s)\mu_0 + G(\underline{\Omega}_0)\mu_s}{\mu_s\mu_0} \right)} \rho_{soil}(\underline{\Omega}_s, \underline{\Omega}_0) I_0 \delta(\underline{\Omega}_s - \underline{\Omega}_0) + \\
 & \frac{I_0 \Gamma(\underline{\Omega}' \rightarrow \underline{\Omega})}{G(\underline{\Omega}_s)\mu_0 + G(\underline{\Omega}_0)\mu_s} \left( 1 - e^{-L \left( \frac{G(\underline{\Omega}_s)\mu_0 + G(\underline{\Omega}_0)\mu_s}{\mu_s\mu_0} \right)} \right)
 \end{aligned}$$

$$e^{-L \left( \frac{G(\underline{\Omega}_s)\mu_0 + G(\underline{\Omega}_0)\mu_s}{\mu_s\mu_0} \right)} \rho_{soil}(\underline{\Omega}_s, \underline{\Omega}_0)$$

$$I^+(\underline{\Omega}_s, z) = \frac{I_0 \Gamma(\underline{\Omega}' \rightarrow \underline{\Omega})}{G(\underline{\Omega}_s)\mu_0 + G(\underline{\Omega}_0)\mu_s} \left( 1 - e^{-L \left( \frac{G(\underline{\Omega}_s)\mu_0 + G(\underline{\Omega}_0)\mu_s}{\mu_s\mu_0} \right)} \right)$$

- through canopy, reflected from soil & back through canopy

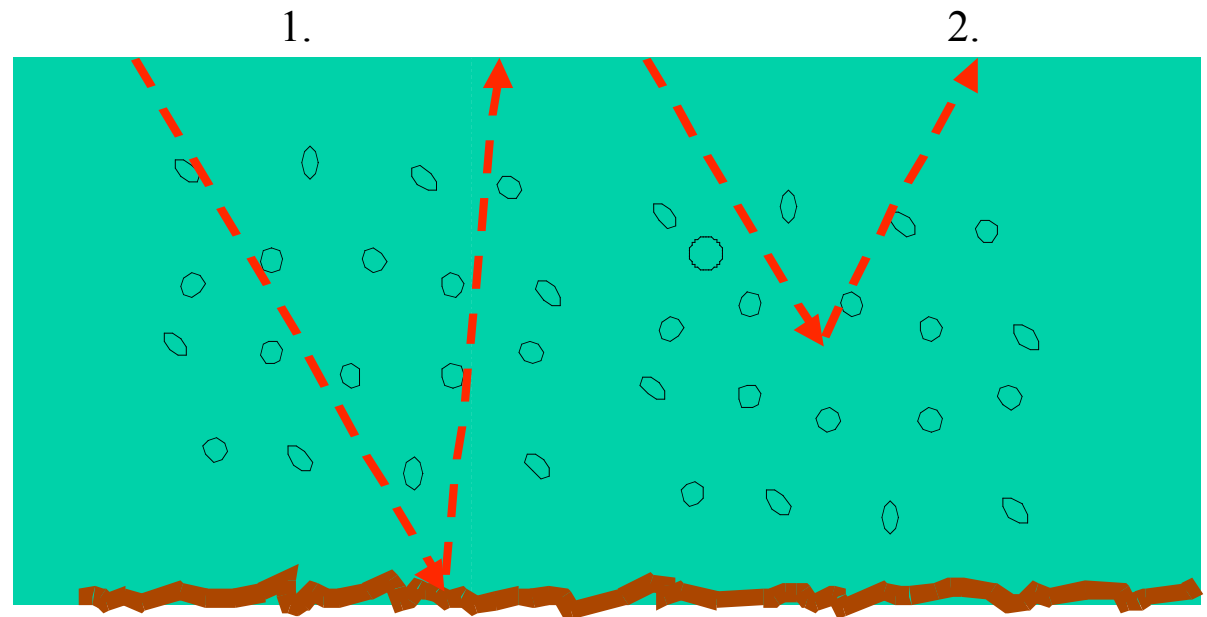


$$\frac{\Gamma(\underline{\Omega}' \rightarrow \underline{\Omega})}{G(\underline{\Omega}_s)\mu_0 + G(\underline{\Omega}_0)\mu_s} \left( 1 - e^{-L \left( \frac{G(\underline{\Omega}_s)\mu_0 + G(\underline{\Omega}_0)\mu_s}{\mu_s\mu_0} \right)} \right) \quad I^+(\underline{\Omega}_s, z) = e^{-L \left( \frac{G(\underline{\Omega}_s)\mu_0 + G(\underline{\Omega}_0)\mu_s}{\mu_s\mu_0} \right)} \rho_{soil}(\underline{\Omega}_s, \underline{\Omega}_0) I_0 \delta(\underline{\Omega}_s - \underline{\Omega}_0) + \frac{I_0 \Gamma(\underline{\Omega}' \rightarrow \underline{\Omega})}{G(\underline{\Omega}_s)\mu_0 + G(\underline{\Omega}_0)\mu_s} \left( 1 - e^{-L \left( \frac{G(\underline{\Omega}_s)\mu_0 + G(\underline{\Omega}_0)\mu_s}{\mu_s\mu_0} \right)} \right)$$

Canopy only scattering

Direct function of  $\omega$

Function of  $g$ ,  $L$ , and



# 1st O SRT

- Special case of spherical leaf angle:

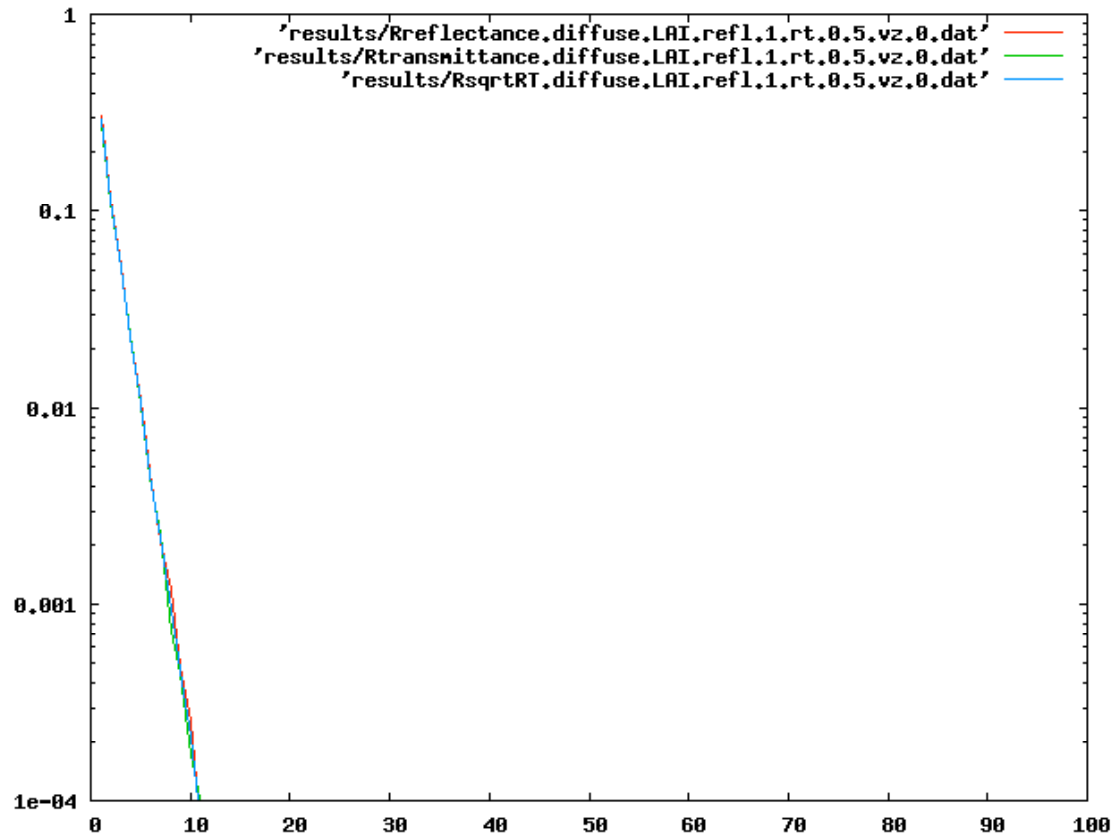
$$G(\underline{\underline{\Omega}}) = 0.5$$

$$\Gamma(\underline{\underline{\Omega}}' \rightarrow \underline{\underline{\Omega}}) = \frac{\rho_l + \tau_l}{3\pi} (\sin \gamma - \gamma \cos \gamma) + \frac{\rho_l}{3} \cos \gamma$$

$$\cos \gamma = |\underline{\underline{\Omega}} \cdot \underline{\underline{\Omega}}'|$$

# Multiple Scattering

Contributions to reflectance and transmittance

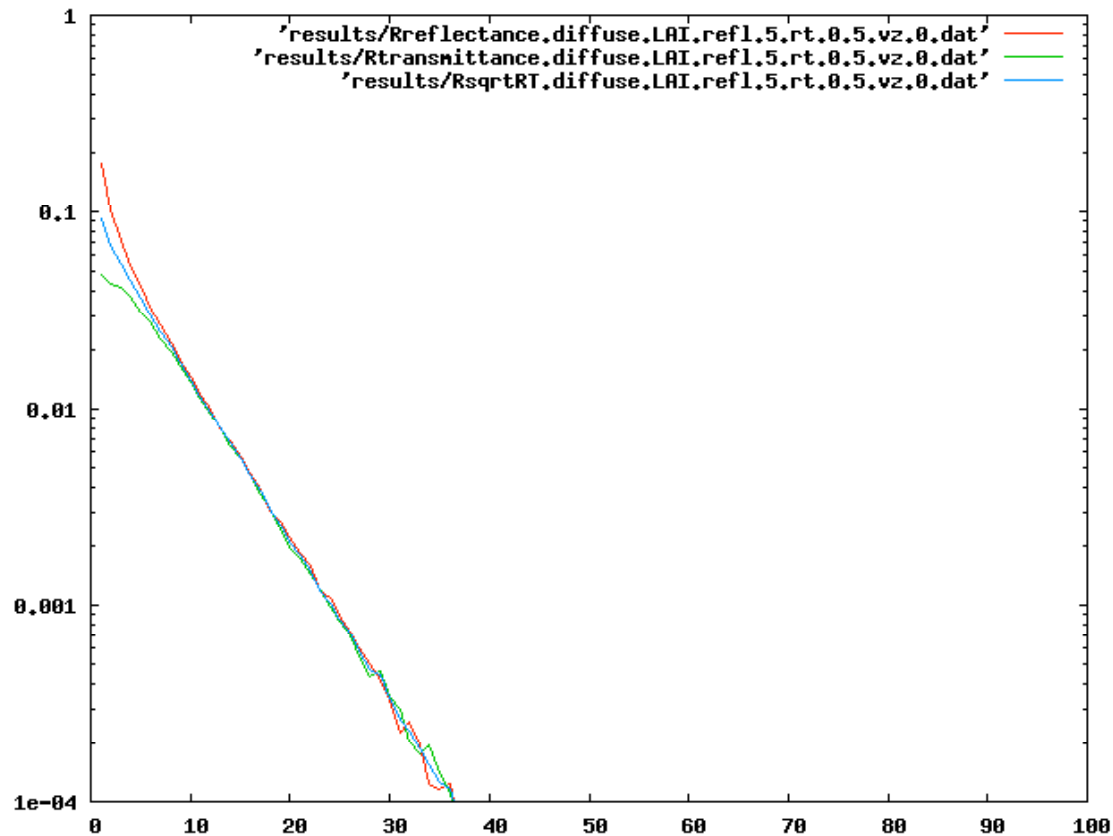


Scattering order

LAI 1

# Multiple Scattering

## Contributions to reflectance and transmittance

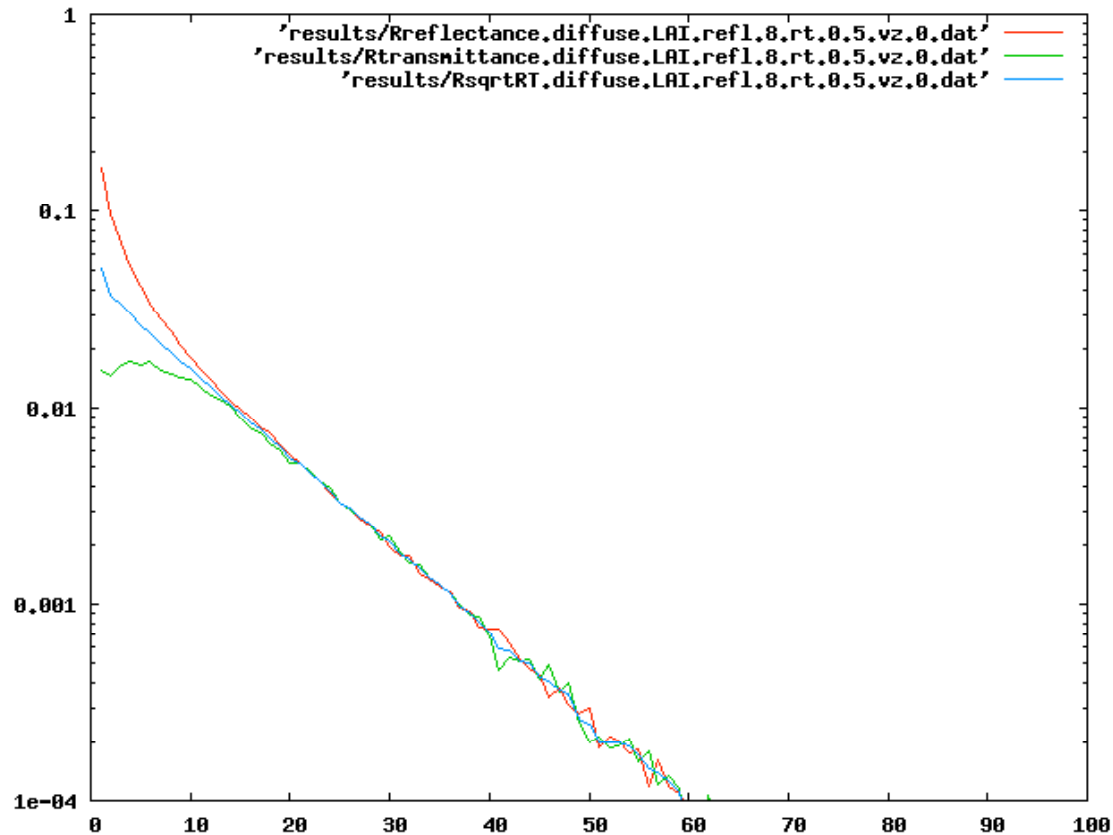


Scattering order

LAI 5

# Multiple Scattering

Contributions to reflectance and transmittance



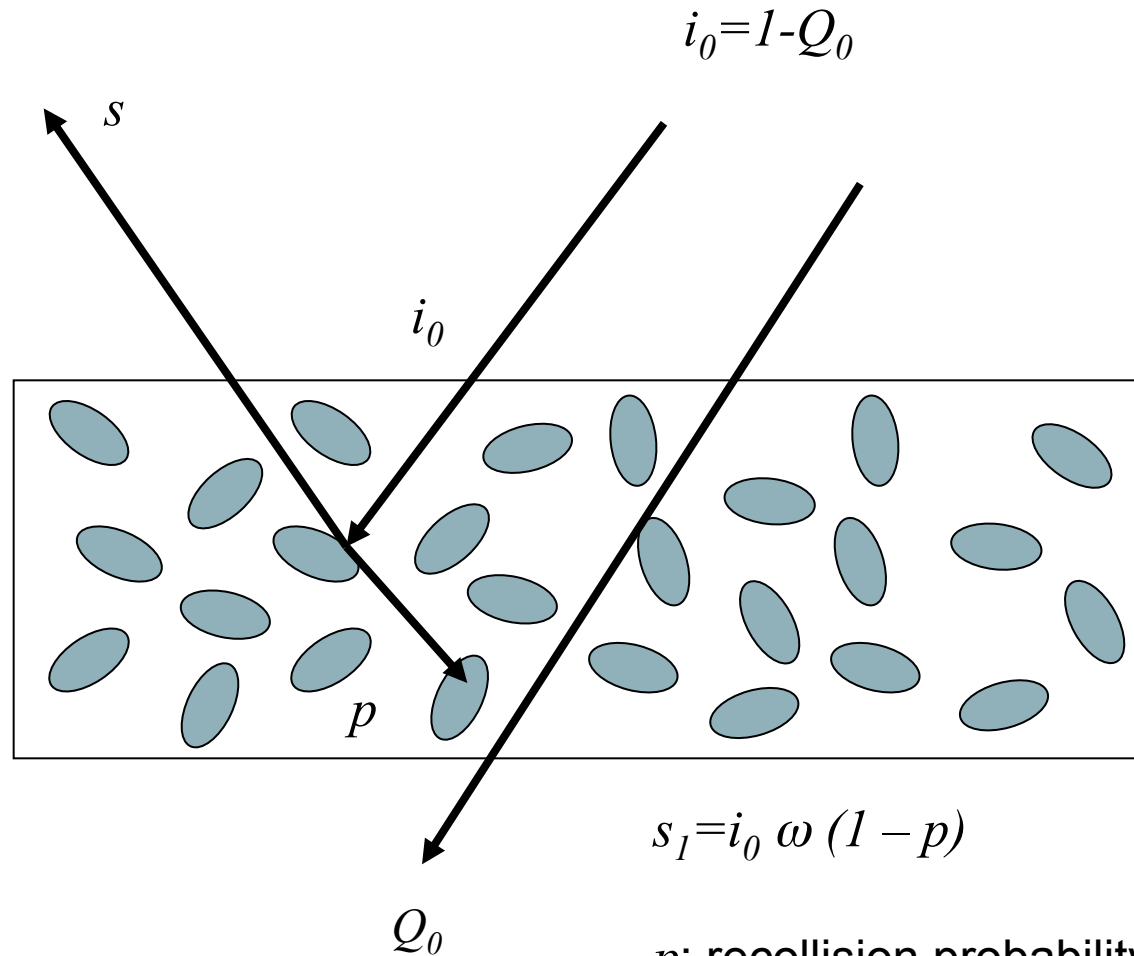
Scattering order

LAI 8

# Multiple Scattering

- range of approximate solutions available
  - Successive orders of scattering (SOSA)
  - 2 & 4 stream approaches etc. etc.
  - Monte Carlo ray tracing (MCRT)
- Recent advances using concept of recollision probability,  $p$ 
  - Huang et al. 2007



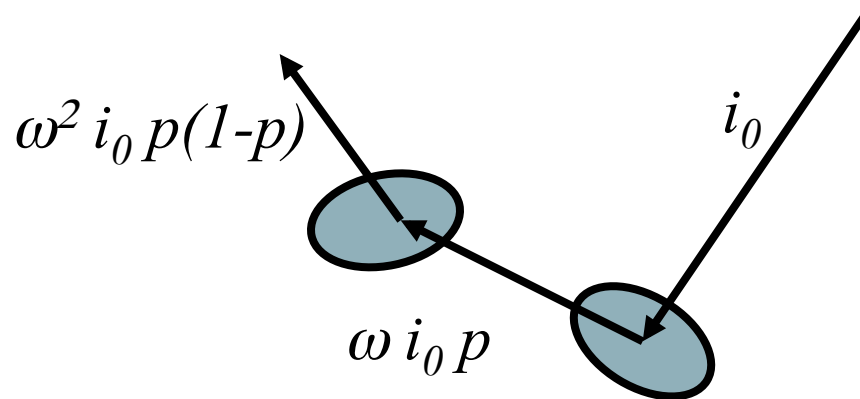


$i_0$  = intercepted (incoming)  
 $Q_0$  = transmitted (uncollided)

$p$ : recollision probability  
 $\omega$ : single scattering albedo of leaf

- 2<sup>nd</sup> Order scattering:

$$\frac{s_1}{i_0} = \omega(1 - p)$$



$$\frac{s_2}{i_0} = \frac{s_1}{i_0} \omega p$$

$$\frac{s}{i_0} = \omega(1 - p) + \omega^2(1 - p)p + \omega^3(1 - p)p^2 + \dots$$

$$\frac{s}{i_0} = \omega(1 - p) + \omega^2(1 - p)p + \omega^3(1 - p)p^2 + \dots$$

$$\frac{s}{i_0} = \omega(1 - p) [1 + \omega p + \omega^2 p^2 + \dots]$$

$$\frac{s}{i_0} = \frac{\omega(1 - p)}{1 - p\omega}$$

‘single scattering albedo’ of canopy

$$\frac{s}{i_0} = \frac{\omega(1-p)}{1-p\omega}$$

$p$ : recollision probability

$$n_s(\lambda) = \frac{1}{1-p\omega(\lambda)}$$

**Average number of photon interactions:**  
The degree of multiple scattering

$$\alpha(\lambda) = \frac{1-\omega(\lambda)}{1-p\omega(\lambda)}$$

**Absorptance**

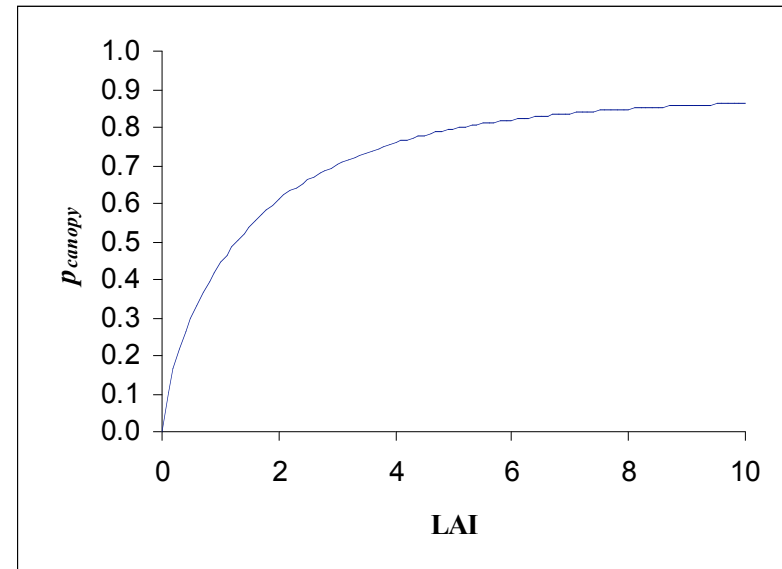
Knyazikhin et al. (1998):  $p$  is eigenvalue of RT equation  
Depends on **structure** only

- For canopy:

$$p_{canopy} = p_{max} \left( 1 - \exp(-kLAI^b) \right)$$

Spherical leaf angle distribution

$$p_{max}=0.88, k=0.7, b=0.75$$

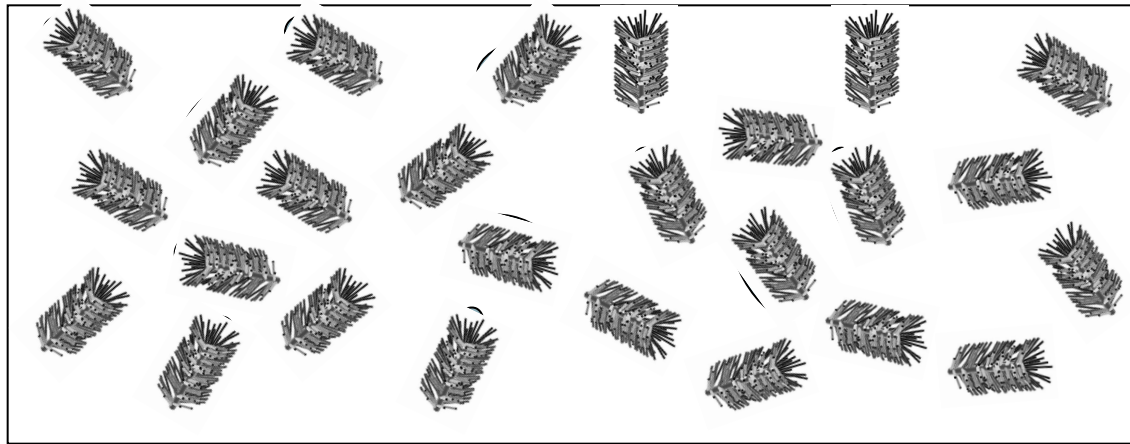


Smolander & Stenberg RSE 2005

# Clumping: aggregation across scales?

Canopy with 'shoots' as fundamental scattering objects:

$$\left( \frac{s}{i_0} \right)_{canopy} = \omega_{canopy} = \frac{(1 - p_{canopy}) \omega_{shoot}}{1 - p_{canopy} \omega_{shoot}}$$



Canopy with 'shoots' as fundamental scattering objects:

$$\left(\frac{s}{i_0}\right)_{canopy} = \omega_{canopy} = \frac{(1 - p_{canopy})\omega_{shoot}}{1 - p_{canopy}\omega_{shoot}}$$

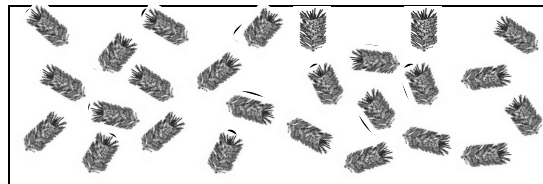
$$\left(\frac{s}{i_0}\right)_{shoot} = \omega_{shoot} = \frac{(1 - p_{shoot})\omega_{needle}}{1 - p_{shoot}\omega_{needle}}$$

$$\left(\frac{s}{i_0}\right)_{canopy} = \omega_{canopy} = \frac{(1 - p_2)\omega_{needle}}{1 - p_2\omega_{needle}}$$

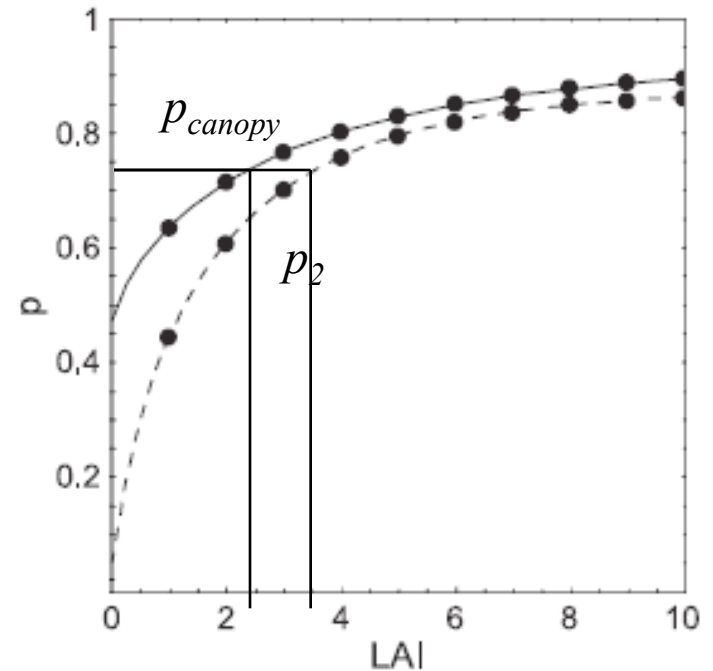
$$p_2 = p_{canopy} + (1 - p_{canopy})p_{shoot}$$

i.e. can use  
approach across  
nested scales

Lewis and Disney, 2007



- $p_{shoot}=0.47$  (scots pine)
- $p_2 < p_{canopy}$
- Shoot-scale clumping reduces apparent LAI



Smolander & Stenberg RSE 2005



## Other RT Modifications

- Hot Spot
  - joint gap probability:  $Q$
  - For far-field objects, treat incident & exitant gap probabilities independently

$$Q(\underline{\Omega}' \rightarrow \underline{\Omega}) = e^{-L \frac{G(\underline{\Omega}) \mu' + G(\underline{\Omega}') \mu}{\mu \mu'}}$$

- product of two Beer's Law terms

## RT Modifications

- Consider retro-reflection direction:
  - assuming independent:

$$Q(\underline{\Omega} \rightarrow \underline{\Omega}) = e^{-\frac{2 L G(\Omega)}{\mu}}$$

- But *should* be:

$$Q(\underline{\Omega} \rightarrow \underline{\Omega}) = e^{-\frac{L G(\Omega)}{\mu}}$$

# RT Modifications

- Consider retro-reflection direction:

- But *should* be:

$$Q(\underline{\Omega} \rightarrow \underline{\Omega}) = e^{-\frac{L G(\underline{\Omega})}{\mu}}$$

- as ‘have already travelled path’
- so need to apply corrections for Q in RT

- e.g.  $Q(\underline{\Omega}' \rightarrow \underline{\Omega}) = P(\underline{\Omega}') P(\underline{\Omega}) C(\underline{\Omega}', \underline{\Omega})$

# RT Modifications

- As result of finite object size, hot spot has angular width
  - depends on ‘roughness’
    - leaf size / canopy height (Kuusk)
    - similar for soils
- Also consider shadowing/shadow hiding

# Summary

- SRT formulation
  - extinction
  - scattering (source function)
- Beer's Law
  - exponential attenuation
  - rate - extinction coefficient
    - $\text{LAI} \times \text{G-function}$  for optical

# Summary

- SRT 1st O solution
  - use area scattering phase function
  - simple solution for spherical leaf angle
  - 2 scattering mechanisms
- Multiple scattering
  - Recollison probability
- Modification to SRT:
  - hot spot at optical