1. ASHRAE: SUPPORT DOCUMENT

1.1. SOLAR RADIATION PROFILE

1.1.1. NOMENCLATURE

$E_0 - Extraterrestrial\ radiant\ flux - \left[\frac{W}{m^2}\right]$

$E_{sc} - Solar\ constant - \left[\frac{W}{m^2}\right]$

$n - Day\ of\ the\ year - [days]$

Γ – auxiliar variable – [°]

$ET - Equation\ of\ time - [minutes]$

$AST - Apparent\ Solar\ time - [hours]$

$LST - Local\ Standard\ time - [hours]$

$LON - Longitude\ of\ site - [°E\ of\ Greenwhich]$

$LSM - Longitude\ of\ local\ standard\ time\ meridian - [°E\ of\ Greenwhich]$

$TZ - Time\ \ Zone - [hours]$

$DST - Daylights\ saving\ time - [hours]$

$\delta - Solar\ declination - [°]$

$H - Hour\ Angle - [°]$

$\beta - Solar\ Altitude\ Angle - [°]$

$L - Latitude - [°N]$

$\phi - Azimuth\ Angle - [°]$

$\gamma - Surface\ solar\ azimuth\ angle - [°]$

$E_b - Beam\ normal\ irradiance - \left[\frac{W}{m^2}\right]$

$E_d - Diffuse\ horizontal\ irradiance - \left[\frac{W}{m^2}\right]$

$m - Relative\ Optical\ Air\ Mass - [-]$

$ab - Empirical\ constant - [-]$

$ad - Empirical\ constant - [-]$

$\theta - angle\ of\ incidence - [°]$

$Y - auxiliar\ variable - [-]$

$E_t - Clear\ sky\ irradiance\ reaching\ the\ receiving\ surface - \left[\frac{W}{m^2}\right]$

$E_{t,b} -\ Clear\ sky\ irradiance\ reaching\ the\ receiving\ surface\ (beam\ component) - \left[\frac{W}{m^2}\right]$

$E_{t,d} -\ Clear\ sky\ irradiance\ reaching\ the\ receiving\ surface\ (diffuse\ component) - \left[\frac{W}{m^2}\right]$

$E_{t,r} -\ Clear\ sky\ irradiance\ reaching\ the\ receiving\ surface\ (ground\ reflected\ component) - \left[\frac{W}{m^2}\right]$

$\tau_b - Beam\ optical\ depth$ - [-]

$\tau_d - Diffuse\ optical\ depth - [-]$

$\rho_g - Ground\ reflectance - [-]$

1.1.2. CALCULATION PROCEDURE

The following procedure is advised by ASHRAE in Ashrae Fundamentals 2017, Chapter 14. The relevant parameters and its calculation is presented bellow.

All trigonometric functions and other expressions which operate with angles are in degrees.

The solar constant (E_{sc}) is defined as the intensity of solar radiation on a surface normal to the sun's rays, just beyond the earth's atmosphere, at the average earth-sun distance.

The extraterrestrial radiant flux E_0 varies throughout the year, its variation is accounted by the equation:

$$E_0 = E_{sc} \cdot \left(1 + 0.033 \cdot \cos\left(360 \cdot \frac{n-3}{365}\right)\right)$$

Example of calculation for each day number:

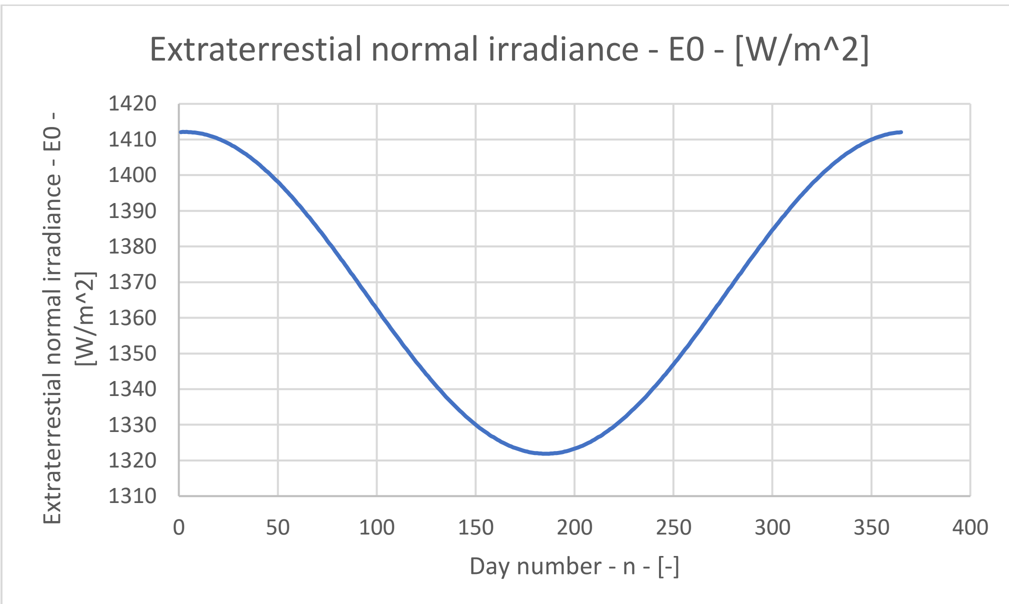


Figure 1

The apparent solar time (AST) is given by:

$$AST = LST + \frac{ET}{60} + \frac{LON + LSM}{15}$$

The longitude of the local time meridian (LSM) can be approximated by:

$$LSM \approx 15 \cdot TZ$$

If Daylight savings time (DST) is implemented during the time of year that is being analyzed, it has to be accounted for:

$$LST = DST - 1$$

The equation of time, which accounts for a difference in minutes in the apparent solar time, caused by the earth's motion relative to the sun, can be calculated by:

$$ET = 2.2918 \cdot [0.0075 + 0.1868 \cdot \cos(\Gamma) - 3.2077 \cdot \sin(\Gamma) - 1.4615 \cdot \cos(2 \cdot \Gamma) - 4.089 \cdot \sin(2 \cdot \Gamma)]$$

$$\Gamma = 360 \cdot \frac{n-1}{365}$$

Example of calculation for each day number:

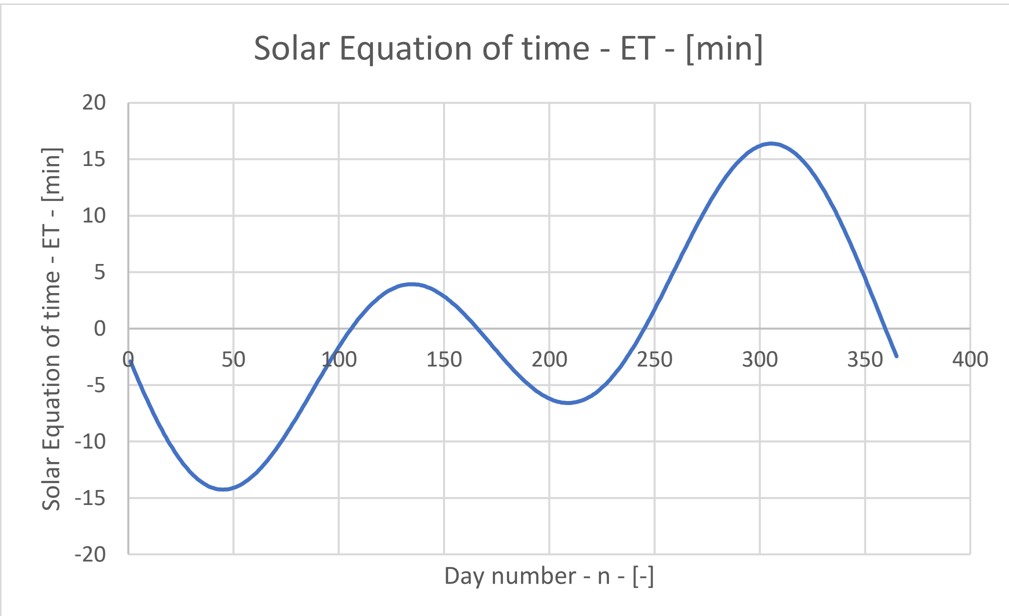


Figure 2

The earth's equatorial plane is tilted at an angle of 23.45° to the orbital plane. The solar declination angle, related with this angle, varies throughout the year. It is responsible for the changing seasons and their unequal periods of daylight and darkness.

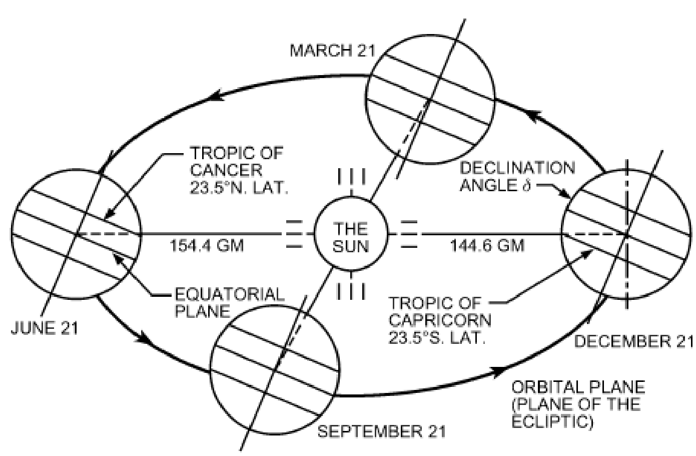


Figure 3 – Solar declination illustration.

The solar declination angle is given by:

$$\delta = 23.45 \cdot \sin\left(360 \cdot \frac{n + 284}{365}\right)$$

Example of calculation for each day number:

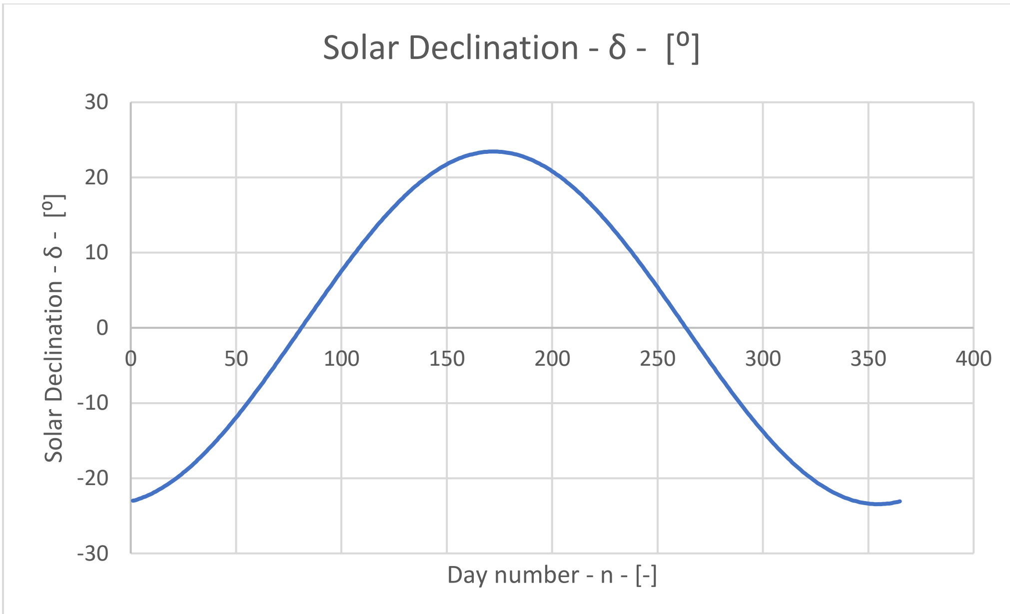


Figure 4

Additional angles of interest are represented in the figure bellow for an example surface.

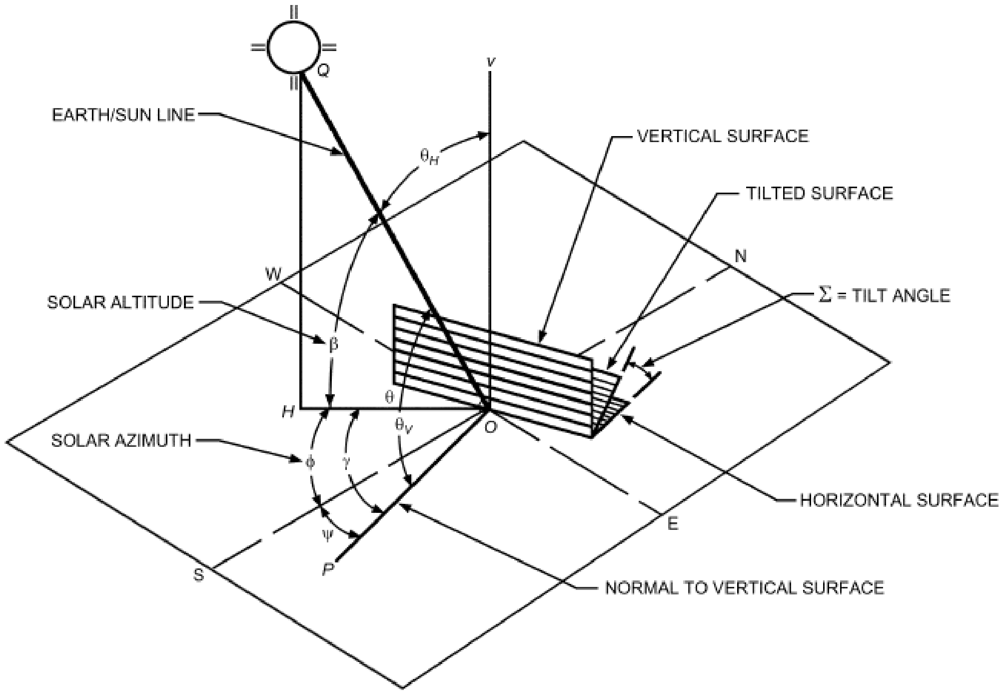


Figure 5

The hour angle, defined as the angular displacement of the sun east or west of the local meridian caused by the rotation of the earth.

$$H = 15 \cdot (AST - 12)$$

The solar altitude angle is defined by the equation bellow:

$$\sin(\beta) = \cos(L) \cdot \cos(\delta) \cdot \cos(H) + \sin(L) \cdot \sin(\delta)$$

The azimuth angle can be obtained by solving the system of equations bellow:

$$\begin{cases} \sin(\phi) = \sin(H) \cdot \left(\dfrac{\cos(\delta)}{\cos(\beta)}\right) \\ \cos(\phi) = \dfrac{\cos(H) \cdot \cos(\delta) \cdot \sin(L) - \sin(\delta) \cdot \cos(L)}{\cos(\beta)} \end{cases}$$

The solar-surface azimuth angle is defined bellow:

$$\gamma = \phi - \psi$$

The surface azimuth is a direct measure of the orientation of the surface:

Table 1

Surface Azimuth - ψ - [º]							
N	NE	E	SE	S	SW	W	NW
180	-135	-90	-45	0	45	90	135

The incidence angle can be calculated through:

$$\cos(\theta) = \cos(\beta) \cdot \cos(\gamma) \cdot \sin(\Sigma) + \sin(\beta) \cdot \cos(\Sigma)$$

Beam and diffuse clear sky irradiance depend on the location specific parameters, such as the optical depths (τ_d and τ_b - measured in weather stations) and the relative optical mass. Optical depts vary with the month of the year and have to be determined for the month of interest.

$$E_b = E_0 \cdot \exp(-\tau_b \cdot m^{ab})$$

$$E_d = E_0 \cdot \exp(-\tau_d \cdot m^{ad})$$

The relative optical mass can be calculated through the following correlation:

$$m = \frac{1}{\sin(\beta) + 0.50572 \cdot (6.07995 + \beta)^{-1.6364}}$$

With the empirical coefficients ab and ad given by:

$$ab = 1.454 - 0.406 \cdot \tau_b - 0.268 \cdot \tau_d + 0.021 \cdot \tau_b \cdot \tau_d$$

$$ad = 0.507 + 0.205 \cdot \tau_b - 0.080 \cdot \tau_d - 0.190 \cdot \tau_b \cdot \tau_d$$

Some outlier values in solar irradiance were detected, after some investigation it was traced back to very high values of m. A literature review on m was conducted and a survey of the value of m when calculated with different correlations was found. The survey indicated that a value of m higher than 16 was extremely unlikely. The formula used was limited to a maximum value of 16 and the outliers where eliminated.

Table 2

Values of (i) Relative Optical Air Mass of the Atmosphere Calculated as $\mu = \sec \theta$, (ii) Relative Optical Air Mass m_K Calculated by *Kasten* [1966] for the ARDC (Air Research and Development Command Model Atmosphere, 1959) Midlatitude Atmosphere Model; (iii) Relative Optical Air Mass m_{KY} of the Midlatitude Atmosphere Calculated by *Kasten and Young* [1989] for the ISO (International Standards Organization) Standard Atmosphere 1972 Model; (iv) Relative Optical Air Mass m_{75N} Calculated by *Tomasi et al.* [1998] for the July-75°N Atmospheric Model Defined Over the 0–100 km Altitude Range; and (v) Relative Optical Air Mass m_{75S} Calculated by *Tomasi et al.* [1998] for the January-75°S Atmospheric Model Defined Over the 0–100 km Altitude Range, Given for the 30 Selected Values of Apparent Solar Zenith Angle θ (deg) From 0° to 87°

θ (deg)	$\mu = \sec \theta$	m_K [*Kasten*, 1966]	m_{KY} [*Kasten and Young*, 1989]	m_{75N} [*Tomasi et al.*, 1998]	m_{75S} [*Tomasi et al.*, 1998]
0	1.0000	1.0000	1.0000	1.0000	1.0000
10	1.0154	1.0148	1.0154	1.0154	1.0154
15	1.0353	1.0346	1.0352	1.0352	1.0352
20	1.0642	1.0634	1.0640	1.0640	1.0640
25	1.1034	1.1025	1.1031	1.1032	1.1032
30	1.1547	1.1536	1.1543	1.1543	1.1543
35	1.2208	1.2194	1.2202	1.2201	1.2201
40	1.3054	1.3037	1.3045	1.3045	1.3045
45	1.4142	1.4119	1.4128	1.4127	1.4127
50	1.5557	1.5526	1.5535	1.5534	1.5534
55	1.7434	1.7388	1.7398	1.7395	1.7398
60	2.0000	1.9928	1.9939	1.9938	1.9940
65	2.3662 ([a])	2.3539	2.3552	2.3551	2.3555
68	2.6695 ([a])	2.6515	2.6529	2.6528	2.6536
70	2.9238 ([a])	2.8999	2.9016	2.9013	2.9022
72	3.2361 ([a])	3.2035	3.2054	3.2054	3.2067
74	3.6280 ([a])	3.5819	3.5841	3.5840	3.5858
75	3.8637 ([a])	3.8081	3.8105	3.8108	3.8127
76	4.1336 ([b])	4.0656	4.0682	4.0689	4.0713
77	4.4454 ([b])	4.3612	4.3640	4.3647	4.3673
78	4.8097 ([b])	4.7036	4.7067	4.7078	4.7111
79	5.2408 ([b])	5.1047	5.1081	5.1093	5.1139
80	5.7588 ([b])	5.5803	5.5841	5.5859	5.5915
81	6.3925 ([b])	6.1526	6.1565	6.1594	6.1668
82	7.1853 ([b])	6.8531	6.8568	6.8606	6.8709
83	8.2055 ([b])	7.7279	7.7307	7.7364	7.7519
84	9.5668 ([b])	8.8474	8.8475	8.8562	8.8784
85	11.474 ([b])	10.323	10.316	10.331	10.364
86	14.336 ([b])	12.330	12.317	12.335	12.388
87	19.107 ([b])	15.219	15.163	15.216	15.307

[a]Of poorly reliable use.
[b]Of erroneous use.

On the actual surface, the total irradiance is given by:

$$E_t = E_{t,b} + E_{t,d} + E_{t,r}$$

With the beam irradiance given by:

$$E_{t,b} = E_b \cdot \cos(\theta)$$

The diffuse irradiance can be calculated through:

If $\Sigma \leq 90°$:

$$E_{t,d} = E_d \cdot (Y \cdot \sin(\Sigma) + \cos(\Sigma))$$

If $\Sigma > 90°$:

$$E_{t,d} = E_d \cdot (Y \cdot \sin(\Sigma))$$

With:

$$Y = \max\big(0.45\,;0.55 + 0.437 \cdot cos(\theta) + 0.313 \cdot cos^2(\theta)\big)$$

Additionally, the ground-reflected irradiance can be calculated through:

$$E_{t,r} = (E_b \cdot \sin(\beta) + E_d) \cdot \rho_g \cdot \frac{1 + \cos(\beta)}{2}$$

The value for ground reflectance, ρ_g, is a location-specific and can vary greatly. ASHRAE recommends 0.2 as a reasonable approximation.

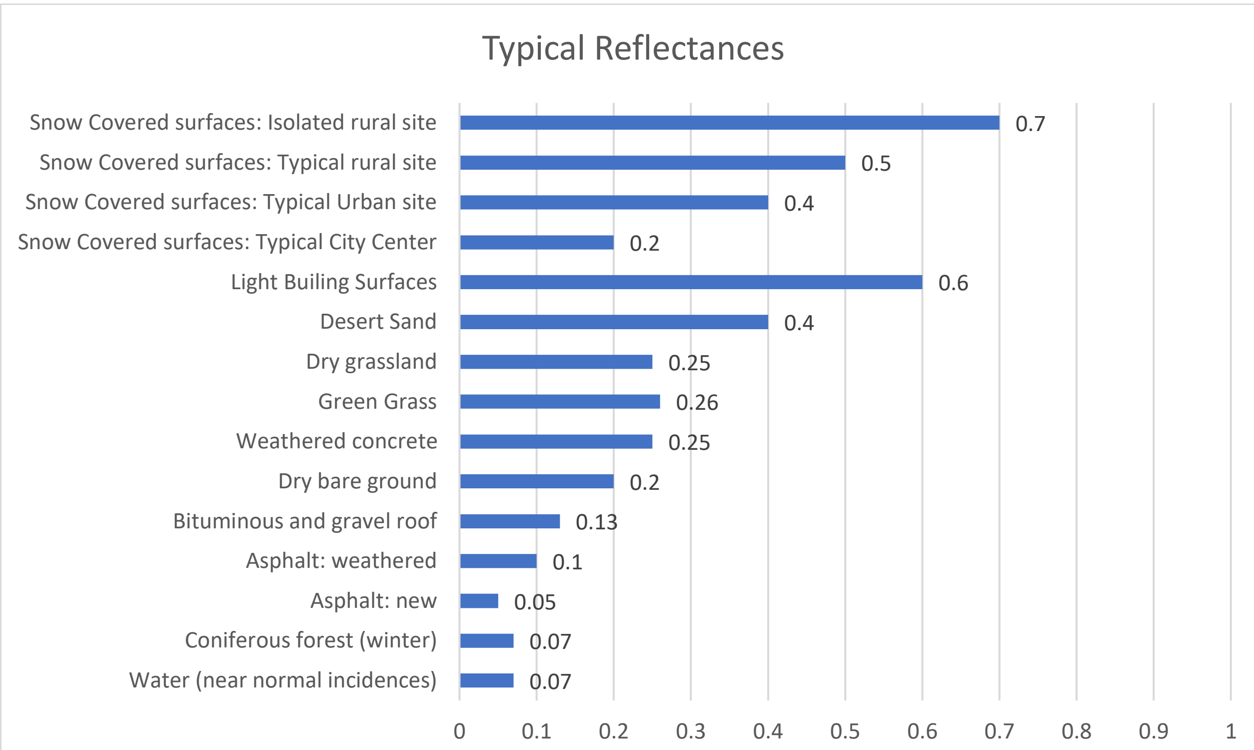
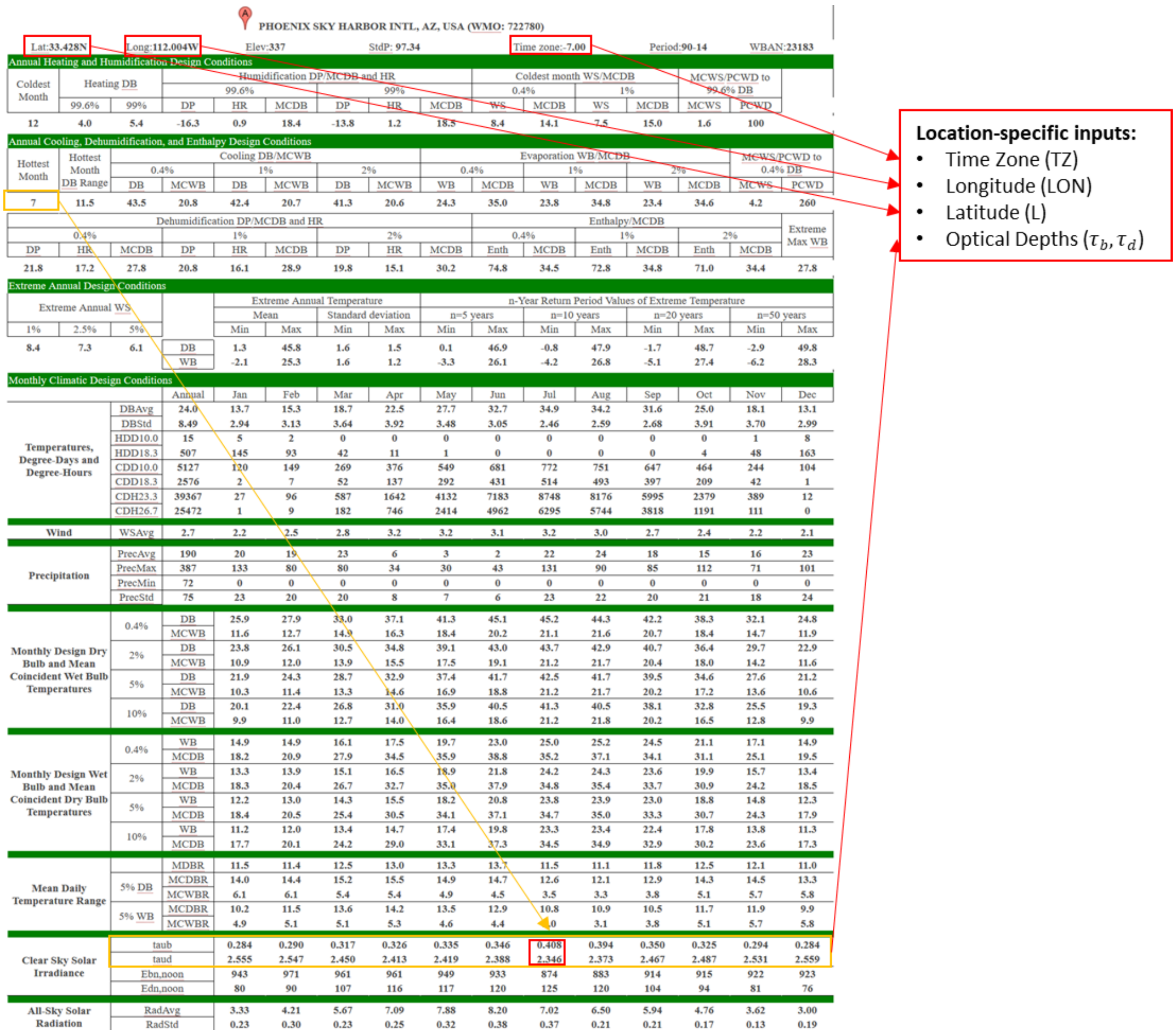


Figure 6

With the parameters described before, the solar irradiance hour profile can be determined for every location if it is known:

- Location specific parameters: Latitude, Longitude, Time Zone and Optical Depths (available from Weather Stations data).
- Date of interest: If the goal is to determine the maximum solar load, the 21st of each month is usually the day where irradiance is at its peak. The month of interest for cooling is usually the result of climate data analysis and corresponds to the hottest month – also location-specific.
- Surface specific parameters: surface azimuth, tilt angle.

An example of climatic data and the location-specific inputs consulted is presented bellow:



PHOENIX SKY HARBOR INTL, AZ, USA (WMO: 722780)

Lat:33.428N | Long:112.004W | Elev:337 | StdP: 97.34 | Time zone:-7.00 | Period:90-14 | WBAN:23183

Annual Heating and Humidification Design Conditions

Coldest Month	Heating DB		Humidification DP/MCDB and HR						Coldest month WS/MCDB				MCWS/PCWD to 99.6% DB	
			99.6%			99%			0.4%		1%			
	99.6%	99%	DP	HR	MCDB	DP	HR	MCDB	WS	MCDB	WS	MCDB	MCWS	PCWD
12	4.0	5.4	-16.3	0.9	18.4	-13.8	1.2	18.5	8.4	14.1	7.5	15.0	1.6	100

Annual Cooling, Dehumidification, and Enthalpy Design Conditions

Hottest Month	Hottest Month DB Range	Cooling DB/MCWB						Evaporation WB/MCDB						MCWS/PCWD to 0.4% DB	
		0.4%		1%		2%		0.4%		1%		2%			
		DB	MCWB	DB	MCWB	DB	MCWB	WB	MCDB	WB	MCDB	WB	MCDB	MCWS	PCWD
7	11.5	43.5	20.8	42.4	20.7	41.3	20.6	24.3	35.0	23.8	34.8	23.4	34.6	4.2	260

Dehumidification DP/MCDB and HR									Enthalpy/MCDB						Extreme Max WB
0.4%			1%			2%			0.4%		1%		2%		
DP	HR	MCDB	DP	HR	MCDB	DP	HR	MCDB	Enth	MCDB	Enth	MCDB	Enth	MCDB	
21.8	17.2	27.8	20.8	16.1	28.9	19.8	15.1	30.2	74.8	34.5	72.8	34.8	71.0	34.4	27.8

Extreme Annual Design Conditions

Extreme Annual WS				Extreme Annual Temperature				n-Year Return Period Values of Extreme Temperature							
				Mean		Standard deviation		n=5 years		n=10 years		n=20 years		n=50 years	
1%	2.5%	5%		Min	Max	Min	Max	Min	Max	Min	Max	Min	Max	Min	Max
8.4	7.3	6.1	DB	1.3	45.8	1.6	1.5	0.1	46.9	-0.8	47.9	-1.7	48.7	-2.9	49.8
			WB	-2.1	25.3	1.6	1.2	-3.3	26.1	-4.2	26.8	-5.1	27.4	-6.2	28.3

Monthly Climatic Design Conditions

			Annual	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Temperatures, Degree-Days and Degree-Hours		DBAvg	24.0	13.7	15.3	18.7	22.5	27.7	32.7	34.9	34.2	31.6	25.0	18.1	13.1
		DBStd	8.49	2.94	3.13	3.64	3.92	3.48	3.05	2.46	2.59	2.68	3.91	3.70	2.99
		HDD10.0	15	5	2	0	0	0	0	0	0	0	0	1	8
		HDD18.3	507	145	93	42	11	1	0	0	0	0	4	48	163
		CDD10.0	5127	120	149	269	376	549	681	772	751	647	464	244	104
		CDD18.3	2576	2	7	52	137	292	431	514	493	397	209	42	1
		CDH23.3	39367	27	96	587	1642	4132	7183	8748	8176	5995	2379	389	12
		CDH26.7	25472	1	9	182	746	2414	4962	6295	5744	3818	1191	111	0
Wind		WSAvg	2.7	2.2	2.5	2.8	3.2	3.2	3.1	3.2	3.0	2.7	2.4	2.2	2.1
Precipitation		PrecAvg	190	20	19	23	6	3	2	22	24	18	15	16	23
		PrecMax	387	133	80	80	34	30	43	131	90	85	112	71	101
		PrecMin	72	0	0	0	0	0	0	0	0	0	0	0	0
		PrecStd	75	23	20	20	8	7	6	23	22	20	21	18	24

			Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Monthly Design Dry Bulb and Mean Coincident Wet Bulb Temperatures	0.4%	DB	25.9	27.9	33.0	37.1	41.3	45.1	45.2	44.3	42.2	38.3	32.1	24.8
		MCWB	11.6	12.7	14.9	16.3	18.4	20.2	21.1	21.6	20.7	18.4	14.7	11.9
	2%	DB	23.8	26.1	30.5	34.8	39.1	43.0	43.7	42.9	40.7	36.4	29.7	22.9
		MCWB	10.9	12.0	13.9	15.5	17.5	19.1	21.2	21.7	20.4	18.0	14.2	11.6
	5%	DB	21.9	24.3	28.7	32.9	37.4	41.7	42.5	41.7	39.5	34.6	27.6	21.2
		MCWB	10.3	11.4	13.3	14.6	16.9	18.8	21.2	21.7	20.2	17.2	13.6	10.6
	10%	DB	20.1	22.4	26.8	31.0	35.9	40.5	41.3	40.5	38.1	32.8	25.5	19.3
		MCWB	9.9	11.0	12.7	14.0	16.4	18.6	21.2	21.8	20.2	16.5	12.8	9.9
Monthly Design Wet Bulb and Mean Coincident Dry Bulb Temperatures	0.4%	WB	14.9	14.9	16.1	17.5	19.7	23.0	25.0	25.2	24.5	21.1	17.1	14.9
		MCDB	18.2	20.9	27.9	34.5	35.9	38.8	35.2	37.1	34.1	31.1	25.1	19.5
	2%	WB	13.3	13.9	15.1	16.5	18.9	21.8	24.2	24.3	23.6	19.9	15.7	13.4
		MCDB	18.3	20.4	26.7	32.7	35.0	37.9	34.8	35.4	33.7	30.9	24.2	18.5
	5%	WB	12.2	13.0	14.3	15.5	18.2	20.8	23.8	23.9	23.0	18.8	14.8	12.3
		MCDB	18.4	20.5	25.4	30.5	34.1	37.1	34.7	35.0	33.3	30.7	24.3	17.9
	10%	WB	11.2	12.0	13.4	14.7	17.4	19.8	23.3	23.4	22.4	17.8	13.8	11.3
		MCDB	17.7	20.1	24.2	29.0	33.1	37.3	34.5	34.9	32.9	30.2	23.6	17.3
Mean Daily Temperature Range		MDBR	11.5	11.4	12.5	13.0	13.3	13.7	11.5	11.1	11.8	12.5	12.1	11.0
	5% DB	MCDBR	14.0	14.4	15.2	15.5	14.9	14.7	12.6	12.1	12.9	14.3	14.5	13.3
		MCWBR	6.1	6.1	5.4	5.4	4.9	4.5	3.5	3.3	3.8	5.1	5.7	5.8
	5% WB	MCDBR	10.2	11.5	13.6	14.2	13.5	12.9	10.8	10.9	10.5	11.7	11.9	9.9
		MCWBR	4.9	5.1	5.1	5.3	4.6	4.4	[illegible]	3.1	3.8	5.1	5.7	5.8
Clear Sky Solar Irradiance		taub	0.284	0.290	0.317	0.326	0.335	0.346	0.408	0.394	0.350	0.325	0.294	0.284
		taud	2.555	2.547	2.450	2.413	2.419	2.388	2.346	2.373	2.467	2.487	2.531	2.559
		Ebn,noon	943	971	961	961	949	933	874	883	914	915	922	923
		Edn,noon	80	90	107	116	117	120	125	120	104	94	81	76
All-Sky Solar Radiation		RadAvg	3.33	4.21	5.67	7.09	7.88	8.20	7.02	6.50	5.94	4.76	3.62	3.00
		RadStd	0.23	0.30	0.23	0.25	0.32	0.38	0.37	0.21	0.21	0.17	0.13	0.19

Figure 7

Example of irradiance day profiles for different surface azimuths:

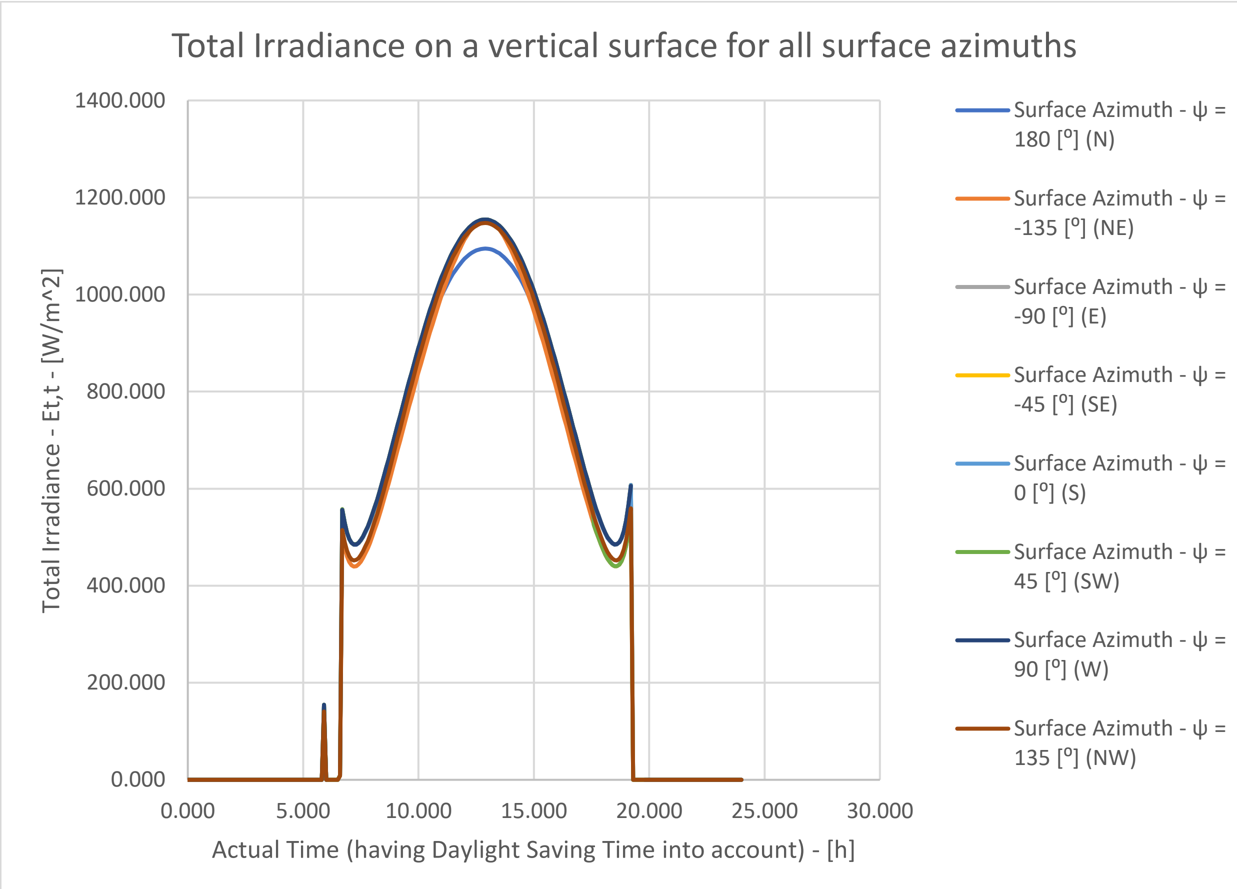

Total Irradiance on a vertical surface for all surface azimuths
Total Irradiance - Et,t - [W/m^2]
1400.000
1200.000
1000.000
800.000
600.000
400.000
200.000
0.000
0.000
5.000
10.000
15.000
20.000
25.000
30.000
Actual Time (having Daylight Saving Time into account) - [h]
Surface Azimuth - ψ = 180 [º] (N)
Surface Azimuth - ψ = -135 [º] (NE)
Surface Azimuth - ψ = -90 [º] (E)
Surface Azimuth - ψ = -45 [º] (SE)
Surface Azimuth - ψ = 0 [º] (S)
Surface Azimuth - ψ = 45 [º] (SW)
Surface Azimuth - ψ = 90 [º] (W)
Surface Azimuth - ψ = 135 [º] (NW)

Figure 8

1.2. DRY-BULB TEMPERATURE PROFILE

To determine the day dry-bulb temperature profile, ASHRAE provides a normalized profile which has been shown to be representative of both dry-bulb and wet-bulb temperature daily profiles in design-days.

The normalization is as follows:

$$T_{db,i} = T_{db,max} - f_i \cdot T_{db,variation}$$

With f being the normalized factor that account for hour variations in temperatures. f is presented in Figure 9.

The maximum dry-bulb temperature, maximum wet-bulb temperature and respective temperature variations are provided by ASHRAE for the design-day desired (either for cooling, heating or dehumidifying).

Time - [h]	Fraction
1	0.88
2	0.92
3	0.95
4	0.98
5	1
6	0.98
7	0.91
8	0.74
9	0.55
10	0.38
11	0.23
12	0.13
13	0.05
14	0
15	0
16	0.06
17	0.14
18	0.24
19	0.39
20	0.5
21	0.59
22	0.68
23	0.75
24	0.82

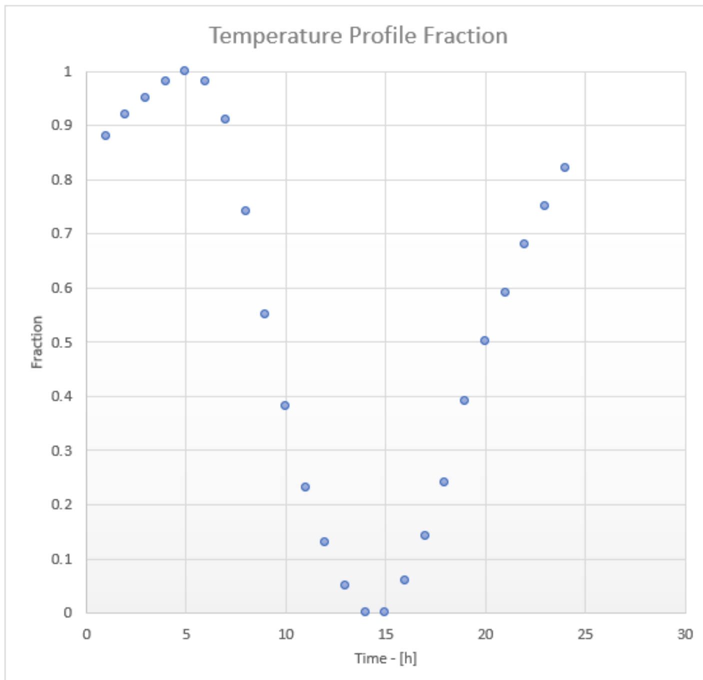
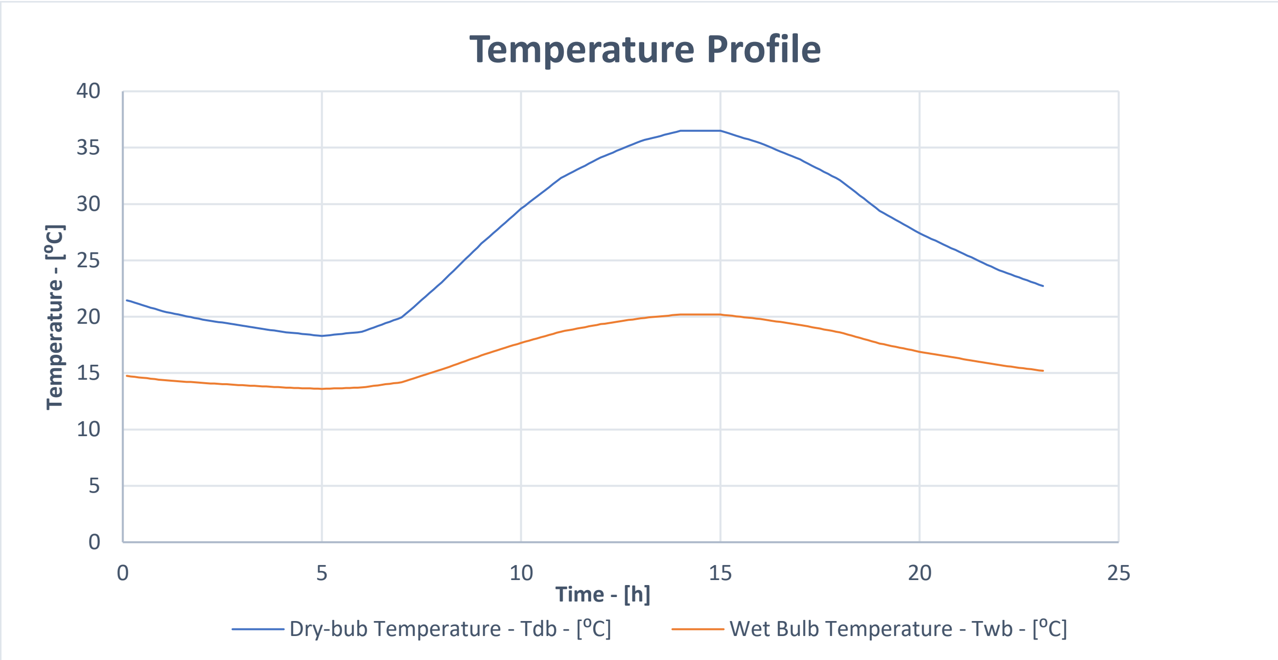


Figure 9

An example of both dry and wet-bulb temperature profile is shown bellow for Riverside, California.


Temperature Profile
Temperature - [ºC]
40
35
30
25
20
15
10
5
0
0
5
10
15
20
25
Time - [h]
Dry-bub Temperature - Tdb - [ºC]
Wet Bulb Temperature - Twb - [ºC]