matrix_algebra_worksheet_ANSWER

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Linear Algebra - Worksheet ANSWER

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 7 & 4 \end{bmatrix},\ B = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix},\ C = \begin{bmatrix} 5 & -1 \\ 9 & 1 \\ 6 & 0 \end{bmatrix},\ D = \begin{bmatrix} 3 & -2 & -1 \\ 1 & 2 & 3 \end{bmatrix}$$

$$u = \begin{bmatrix} 6 & 2 & -3 & 5 \end{bmatrix},\ v = \begin{bmatrix} 3 & 5 & -1 & 4 \end{bmatrix},\ w = \begin{bmatrix} 1 \\ 8 \\ 0 \\ 5 \end{bmatrix}$$

1. Matrix Dimensions

Write the dimansions of each matrix.

1.1) A: $2x3$
1.2) B: $2x2$
1.3) C: $3x2$
1.4) D: $2x3$
1.5) u: $1x4$
1.6) w: $4x1$

2. Vector Operations

Perform the following operations. Assume $\alpha = 6$.

2.1)
$\vec{u} + \vec{v} = \begin{bmatrix} 6 & 2 & -3 & 5 \end{bmatrix} + \begin{bmatrix} 3 & 5 & -1 & 4 \end{bmatrix} = \begin{bmatrix} (6+3) & (2+5) & (-3-1) & (5+4) \end{bmatrix}$
$\Rightarrow \vec{u} + \vec{v} = \begin{bmatrix} 9 & 7 & -4 & 9 \end{bmatrix}$

2.2)
$\vec{u} - \vec{v} = \begin{bmatrix} 6 & 2 & -3 & 5 \end{bmatrix} - \begin{bmatrix} 3 & 5 & -1 & 4 \end{bmatrix} = \begin{bmatrix} (6-3) & (2-5) & (-3+1) & (5-4) \end{bmatrix}$
$\Rightarrow \vec{u} - \vec{v} = \begin{bmatrix} 3 & -3 & -2 & 1 \end{bmatrix}$

2.3)
$\alpha\vec{u} = 6 \cdot \vec{u} = 6 \cdot \begin{bmatrix} 6 & 2 & -3 & 5 \end{bmatrix} = \begin{bmatrix} (6 \cdot 6) & (6 \cdot 2) & (6 \cdot -3) & (6 \cdot 5) \end{bmatrix}$
$\Rightarrow \alpha\vec{v} = \begin{bmatrix} 36 & 12 & -18 & 30 \end{bmatrix}$

2.4)
$\vec{u} \cdot \vec{v} = \begin{bmatrix} 6 & 2 & -3 & 5 \end{bmatrix} \cdot \begin{bmatrix} 3 & 5 & -1 & 4 \end{bmatrix} = (6 \cdot 3) + (2 \cdot 5) + (-3 \cdot (-1)) + (5 \cdot 4)$
$= 18 + 10 + 3 + 20 = 41$
$\Rightarrow \vec{u} \cdot \vec{v} = 41$

2.5)
$\|\vec{u}\| = \sqrt{\vec{u} \cdot \vec{u}} = \sqrt{\begin{bmatrix} 6 & 2 & -3 & 5 \end{bmatrix} \cdot \begin{bmatrix} 6 & 2 & -3 & 5 \end{bmatrix}} = \sqrt{(6 \cdot 6) + (2 \cdot 2) + (-3 \cdot (-3)) + (5 \cdot 5)} =$
$= \sqrt{36 + 4 + 9 + 25}$
$= \sqrt{74}$
$\Rightarrow \|\vec{u}\| = \sqrt{2}\sqrt{37} \approx 9$

3. Matrix Operations

Evaluate each of the foolowing expressionas, if it is defined; else fill in with "not defined". Do your work by hand on scratch paper.

3.1)
"$A + C$ = Not defined": the dimensions of A and C do not match.

3.2)

$$A + C^T = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 7 & 4 \end{bmatrix} \begin{bmatrix} 5 & -1 \\ 9 & 1 \\ 6 & 0 \end{bmatrix}^T = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 7 & 4 \end{bmatrix} \begin{bmatrix} 5 & 9 & 6 \\ -1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1+5 & 2+9 & 3+6 \\ 2-1 & 7+1 & 4+0 \end{bmatrix}$$

$$=> A + C^T = \begin{bmatrix} 6 & 11 & 9 \\ 1 & 8 & 4 \end{bmatrix}$$

3.3)

$$C^T + 3D = \begin{bmatrix} 5 & 9 & 6 \\ -1 & 1 & 0 \end{bmatrix} + 3 \cdot \begin{bmatrix} 3 & -2 & -1 \\ 1 & 2 & 3 \end{bmatrix}$$

$$=> C^T + 3D = \begin{bmatrix} 14 & 3 & 3 \\ 2 & 7 & 9 \end{bmatrix}$$

3.4)

$$BA = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 7 & 4 \end{bmatrix} = \begin{bmatrix} (1-2) & (2-7) & (3-4) \\ (0+2) & (0+7) & (0+4) \end{bmatrix}$$

$$=> BA = \begin{bmatrix} -1 & -5 & -1 \\ 2 & 7 & 4 \end{bmatrix}$$

3.5)
"BA = Not defined"