Instructor: Roman Chomko

Homework* 4
Vector Spaces

Due Date: *Thursday, May 5, 2016*

Note: this hw doesn't contain Matlab exercises and its usage is not required.

Problem 1 (Chap 4.1, Problems 45, 47)

Write **v** as a linear combination of $\mathbf{u_1}$, $\mathbf{u_2}$ and $\mathbf{u_3}$ and if possible:

45. $\mathbf{v} = (10, 1, 4)$, $\mathbf{u}_1 = (2, 3, 5)$, $\mathbf{u}_2 = (1, 2, 4)$,
$\mathbf{u}_3 = (-2, 2, 3)$

47. $\mathbf{v} = (0, 5, 3, 0)$, $\mathbf{u}_1 = (1, 1, 2, 2)$, $\mathbf{u}_2 = (2, 3, 5, 6)$,
$\mathbf{u}_3 = (-3, 1, -4, 2)$

Problem 2 (Chap 4.1, Problem 56)

Illustrate properties 1-10 of Theorem 4.2 of Lecture 11, Slide 8 for

$$\mathbf{u} = (2, -1, 3),\ \mathbf{v} = (3, 4, 0),\ \mathbf{w} = (7, 8, -4),\ c = 2 \text{ and } d = -1$$

Problem 3 (Chap 4.2, Problems 19, 22, 26, 33)

Determine whether the set, together with the indicated operations, is a vector space. If it is not, then identify at least one of the vector space axioms that fails.

19. The set of all polynomials of degree four or less with the standard operations

22. The set $\{(x, y):\ x \geq 0,\ y \geq 0\}$ with the standard operations in R^2

26. The set of all 2 x 2 matrices of the form

$$\begin{bmatrix} a & b \\ c & 1 \end{bmatrix}$$

33. $C[0,1]$, the set of all continuous functions defined on the interval [0, 1], with the standard operations.

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* *Collaboration is allowed*

Problem 4 (Chap 4.3, Problems 2, 3)

Verify the W is a subspace of V. In each case, assume that V has the standard operations.

2. $W = \{(x, y, 2x - 3y)$: x and y are real numbers$\}$, $V = R^3$

3. W is a set of all 2 x 2 matrices of the form $\begin{bmatrix} 0 & a \\ b & 0 \end{bmatrix}$, $V = M_{2,2}$.

Problem 5 (Chap 4.3, Problem 8)

Show that W is not a subspace of a vector space where W is a set of all vectors in R^2 whose second component is 1. Verify this by giving a specific example that violates the test for a vector subspace.

Problem 6 (Chap 4.3, Problem 29)

Determining Subspaces **determine whether the subset of $M_{n,n}$ is a subspace of $M_{n,n}$ with the standard operations of matrix addition and scalar multiplication.**

29. The set of all $n \times n$ upper triangular matrices

Problem 7 (Chap 4.3, Problems 37, 38, 40)

Determining Subspaces **determine whether the set W is a subspace of R^3 with the standard operations. Justify your answer.**

37. $W = \{(x_1, x_2, 0)$: x_1 and x_2 are real numbers$\}$

38. $W = \{(x_1, x_2, 4)$: x_1 and x_2 are real numbers$\}$

40. $W = \{(s, s - t, t)$: s and t are real numbers$\}$

Problem 8 (Chap 4.3, Problem 48)

Determine whether the set

$$S = \left\{f \in C[0, 1]: \int_0^1 f(x)\, dx = 0\right\}$$

is a subspace of $C[0, 1]$. Prove your answer.

Problem 9 (Chap 4.4, Problems 9, 15)

Spanning Sets **In Exercises 9–20, determine whether the set S spans R^2. If the set does not span R^2, then give a geometric description of the subspace that it does span.**

9. $S = \{(2, 1), (-1, 2)\}$

15. $S = \{(1, 3), (-2, -6), (4, 12)\}$

Problem 10 (Chap 4.4, Problem 46)

Testing for Linear Independence
determine whether the set of vectors in $M_{2,2}$ is linearly independent or linearly dependent.

46. $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, C = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$

Problem 11 (Chap 4.5, Problems 22, 26)

Explain why S is not a basis for P_2.

22. $S = \{2, x, x + 3, 3x^2\}$

Explain why S is not a basis for $M_{2,2}$.

26. $S = \left\{ \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \right\}$

Problem 12 (Chap 4.6, Problems 8, 9)

Finding a Basis for a Row Space and Rank **In Exercises 5–10, find (a) a basis for the row space and (b) the rank of the matrix.**

8. $\begin{bmatrix} 2 & -3 & 1 \\ 5 & 10 & 6 \\ 8 & -7 & 5 \end{bmatrix}$

9. $\begin{bmatrix} -2 & -4 & 4 & 5 \\ 3 & 6 & -6 & -4 \\ -2 & -4 & 4 & 9 \end{bmatrix}$

Problem 13 (Chap 4.6, Problems 25, 28, 30)

Finding the Nullspace of a Matrix **In Exercises 25–36, find the nullspace of the matrix.**

25. $A = \begin{bmatrix} 2 & -1 \\ -6 & 3 \end{bmatrix}$

28. $A = \begin{bmatrix} 1 & 4 & 2 \end{bmatrix}$

30. $A = \begin{bmatrix} 1 & 4 & 2 \\ 0 & 0 & 1 \end{bmatrix}$

Problem 14 (Chap 4.6, Problems 38, 39)

Finding a Basis and Dimension In Exercises 37–46, find (a) a basis for and (b) the dimension of the solution space of the homogeneous system of linear equations.

38.
$$\begin{aligned} x - y &= 0 \\ -x + y &= 0 \end{aligned}$$

39.
$$\begin{aligned} -x + y + z &= 0 \\ 3x - y &= 0 \\ 2x - 4y - 5z &= 0 \end{aligned}$$

Problem 15 (Chap 4.6, Problem 50)

Nonhomogeneous System In Exercises 49–54, (a) determine whether the nonhomogeneous system $A\mathbf{x} = \mathbf{b}$ is consistent, and (b) if the system is consistent, then write the solution in the form $\mathbf{x} = \mathbf{x}_p + \mathbf{x}_h$, where $\mathbf{x}_p$ is a particular solution of $A\mathbf{x} = \mathbf{b}$ and $\mathbf{x}_h$ is a solution of $A\mathbf{x} = \mathbf{0}$.

50.
$$\begin{aligned} 2x - 4y + 5z &= 8 \\ -7x + 14y + 4z &= -28 \\ 3x - 6y + z &= 12 \end{aligned}$$

***** END OF HW 4 *****