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Homework* 8
Eigenvalues and Eigenvectors

Due Date: *Thursday, June 2, 2016*

Problem 1 (Chap 7.1, Problems 24)

Characteristic Equation, Eigenvalues, and Eigenvectors
find (a) the characteristic equation and (b) the eigenvalues (and corresponding eigenvectors) of the matrix.

24. $\begin{bmatrix} 3 & 2 & -3 \\ -3 & -4 & 9 \\ -1 & -2 & 5 \end{bmatrix}$

Problem 2 (Chap 7.1, Problem 46)

Eigenvalues and Eigenvectors of Linear Transformations
consider the linear transformation $T: R^n \to R^n$ whose matrix A relative to the standard basis is given. Find (a) the eigenvalues of A, (b) a basis for each of the corresponding eigenspaces, and (c) the matrix A' for T relative to the basis B', where B' is made up of the basis vectors found in part (b).

46. $A = \begin{bmatrix} 3 & 1 & 4 \\ 2 & 4 & 0 \\ 5 & 5 & 6 \end{bmatrix}$

* *Collaboration is allowed*

<u>Problem 3</u> <u>(Chap 7.2, Problem 5)</u>

Diagonalizable Matrices and Eigenvalues
(a) verify that A is diagonalizable by computing $P^{-1}AP$, and (b) use the result of part (a) and Theorem 7.4 to find the eigenvalues of A.

5. $A = \begin{bmatrix} -1 & 1 & 0 \\ 0 & 3 & 0 \\ 4 & -2 & 5 \end{bmatrix}$, $P = \begin{bmatrix} 0 & 1 & -3 \\ 0 & 4 & 0 \\ 1 & 2 & 2 \end{bmatrix}$

<u>Problem 4</u> <u>(Chap 7.2, Problem 12)</u>

Diagonalizing a Matrix for each matrix A, find (if possible) a nonsingular matrix P such that $P^{-1}AP$ is diagonal. Verify that $P^{-1}AP$ is a diagonal matrix with the eigenvalues on the main diagonal.

12. $A = \begin{bmatrix} 3 & 2 & -3 \\ -3 & -4 & 9 \\ -1 & -2 & 5 \end{bmatrix}$

<u>Note</u>: this is the matrix from **Problem 1**. Reuse results of that problem.

<u>Problem 5</u> <u>(Chap 7.3, Problem 4)</u>

Determining Whether a Matrix Is Symmetric
determine whether the matrix is symmetric.

4. $\begin{bmatrix} 1 & -5 & 4 \\ -5 & 3 & 6 \\ -4 & 6 & 2 \end{bmatrix}$

<u>MATLAB EXERCISES</u>

<u>Problem 6</u> <u>(Matlab Exercise on *Eigenvalues and Eigenvectors*, **Method 1**)</u>

Study* Matlab functions **`poly()`**† and **`roots()`**. Lets find eigenvalues and eigenvectors of the matrix

$$\begin{bmatrix} 7 & -4 & 0 \\ 8 & -5 & 0 \\ -4 & 4 & 3 \end{bmatrix}$$

* >> **`help poly, help roots`**

† Note that **`poly()`** operates in two *different* modes depending on the type of input

```
>> % Looking for eigenvalues and corresponding eigenvectors for matrix A:
>> A = [7 -4 0; 8 -5 -0; -4 4 3]
>> % The characteristic polynomial of A is given by coefficients
>> % in a vector c
>> c = poly(A)
c =
     1    -5     3     9
>> % The roots (eigenvalues) of the polynomial given by its coeff's in c are
>> lambda = roots(c)
lambda =
    3.0000
    3.0000
   -1.0000
>> % Checking the number of eigenvalues in lambda-vector
>> [n,m] = size(lambda)
n =
     3
m =
     1
>> lambda = roots(c)
lambda =
    3.0000
    3.0000
   -1.0000
>> % Observe what round-off errors may lead to:
>> lambda = unique(lambda) % unique() removes identical components
lambda =
   -1.0000
    3.0000
    3.0000
>> % We need to have distinct eigenvalues. However due to the
>> % numerical round-off errors for analytically identical eigenvalues
>> % we may obtain numerically distinct ones. This is to say,
>> % for purposes of EE20 let's assume that if two numerical values are
>> % identical to 4 digits after the dot then they are same. In Matlab this
>> % can be done using the following procedure (or by using the function
>> % round2()):
>> lambda = unique(round(roots(c)*10000)/10000)
lambda =
    -1
```

```
     3
>> % Checking the number of unique eigenvalues in lambda-vector
>> [nlambda,ndum] = size(lambda)
nlambda =
     2
ndum =
     1
>> % For each distinct lambda(i) we need to find its eigenvector(s)
>> % by solving the homogenous equation M*x = 0 where M = A - lambda(i)*I.
>> % Review Hw 7 how it was done using homsoln() function.
>> M1 = A - lambda(1)*eye(size(A)) % M1 = A - lambda(1)*I
M1 =
     8    -4     0
     8    -4     0
    -4     4     4
>> homsoln(M1,1) % solution of the homogenous equation M1*x = 0

The columns of the following matrix are a basis
for the solution space of homogeneous system
Ax = 0.


    -1
    -2
     1
The general solution is:
r * col(1)
ans =
    -1
    -2
     1
>> % Similarly finding the eigenvectors for the rest of eigenvalues
>> disp(sprintf('\nEigenvectors for eigenvalue:\n lambda = %d\n', lambda(2)));
Eigenvectors for eigenvalue:
 lambda = 3
>> homsoln(A - lambda(2)*eye(size(A)),1)

The columns of the following matrix are a basis
for the solution space of homogeneous system
Ax = 0.
     1     0
```

```
     1     0
     0     1
 The general solution is:
 r * col(1) + s * col(2)
ans =
     1     0
     1     0
     0     1
```

The above solution gives: eigenvector $\mathbf{e_1}$ = [-1 -2 1] for the eigenvalue $\lambda_1 = -1$ and two eigenvectors $\mathbf{e_2}$ = [1 1 0] and e_3 = [0 0 1] for the eigenvalue $\lambda_2 = 3$.

Using the steps provided in the example above[*], write a Matlab function **`eigcomp(A)`** which computes and displays eigenvalues and their corresponding eigenvectors. Verify its correctness against the matrix given in the example above.

Problem 11 (Matlab Exercise on *Eigenvalues and Eigenvectors*, **Method 2**)

Study[†] Matlab function **`eig()`**. Find eigenvalues and eigenvectors of the matrix

$$\begin{bmatrix} 7 & -4 & 0 \\ 8 & -5 & 0 \\ -4 & 4 & 3 \end{bmatrix}$$

Discuss the results.

[*] *Hint*: it is just a matter of writing a **`for`**-loop running through unique eigenvalues in the above procedure.

[†] >> **`help poly, help roots`**

***** END OF HW 8 *****