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Homework* 5

Change of Basis and Inner Products

Due Date: *Thursday, May 12, 2016*

Problem 1 (Chap 4.7, Problem 14)

Find the coordinate matrix of \mathbf{x} in R^n relative to the basis B' :

14. $B' = \{(3/2, 4, 1), (3/4, 5/2, 0), (1, 1/2, 2)\}$, $\mathbf{x} = (3, -1/2, 8)$

Problem 2 (Chap 4.7, Problem 38)

(a) find the transition matrix from B to B' , (b) find the transition matrix from B' to B ,
(c) verify that the two transition matrices are inverses of each other, and (d) find the coordinate matrix $[\mathbf{x}]_B$, given the coordinate matrix $[\mathbf{x}]_{B'}$.

38. $B = \{(1, 1, 1), (1, -1, 1), (0, 0, 1)\}$,

$B' = \{(2, 2, 0), (0, 1, 1), (1, 0, 1)\}$,

$$[\mathbf{x}]_{B'} = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$$

Problem 3 (Chap 4.7, Problem 44)

Find the coordinate matrix of p relative to the standard basis for P_3

44. $p = 3x^2 + 114x + 13$

Problem 4 (Chap 5.1, Problem 26)

Finding Dot Products

find (a) $\mathbf{u} \cdot \mathbf{v}$, (b) $\mathbf{v} \cdot \mathbf{v}$, (c) $\|\mathbf{u}\|^2$, (d) $(\mathbf{u} \cdot \mathbf{v})\mathbf{v}$, and (e) $\mathbf{u} \cdot (5\mathbf{v})$.

26. $\mathbf{u} = (4, 0, -3, 5)$, $\mathbf{v} = (0, 2, 5, 4)$

* Collaboration is allowed

Problem 5 (Chap 5.1, Problem 44)

Finding the Angle Between Two Vectors

find the angle θ between the vectors.

44. $\mathbf{u} = (1, -1, 0, 1), \quad \mathbf{v} = (-1, 2, -1, 0)$

Problem 6 (Chap 5.2, Problem 6)

Showing That a Function Is an Inner Product

In Exercise 6., show that the function defines an inner product on R^3 , where $\mathbf{u} = (u_1, u_2, u_3)$ and $\mathbf{v} = (v_1, v_2, v_3)$.

6. $\langle \mathbf{u}, \mathbf{v} \rangle = u_1v_1 + 2u_2v_2 + u_3v_3$

Problem 7 (Chap 5.2, Problem 24)

Finding Inner Product, Length, and Distance In

Exercises 17–26, find (a) $\langle \mathbf{u}, \mathbf{v} \rangle$, (b) $\|\mathbf{u}\|$, (c) $\|\mathbf{v}\|$, and (d) $d(\mathbf{u}, \mathbf{v})$ for the given inner product defined on R^n .

24. $\mathbf{u} = (1, 1, 1), \quad \mathbf{v} = (2, 5, 2),$
 $\langle \mathbf{u}, \mathbf{v} \rangle = u_1v_1 + 2u_2v_2 + u_3v_3$

Problem 8 (Chap 5.2, Problem 28)

Showing That a Function Is an Inner Product

let

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

be matrices in the vector space $M_{2,2}$. Show that the function defines an inner product on $M_{2,2}$.

28. $\langle A, B \rangle = 2a_{11}b_{11} + a_{21}b_{21} + a_{12}b_{12} + 2a_{22}b_{22}$

Problem 9 (Chap 5.1, Problem 40)

In Exercise 40, use the functions f and g in $C[-1, 1]$ to find (a) $\langle f, g \rangle$, (b) $\|f\|$, (c) $\|g\|$, and (d) $d(f, g)$ for the inner product

$$\langle f, g \rangle = \int_{-1}^1 f(x)g(x) dx.$$

40. $f(x) = -x, \quad g(x) = x^2 - x + 2$

Problem 10 (Chap 5.1, Problems 46, 48, 50, 51)

Finding the Angle Between Two Vectors

find the angle θ between the vectors.

50. $p(x) = 1 + x^2$, $q(x) = x - x^2$,
 $\langle p, q \rangle = a_0b_0 + 2a_1b_1 + a_2b_2$

51. $f(x) = x$, $g(x) = x^2$,
 $\langle f, g \rangle = \int_{-1}^1 f(x)g(x) dx$

*** END OF HW 5 ***