Instructor: Roman Chomko

Homework* 3
LU Factorization, Determinants, Cramer's Rule

Due Date: *Thursday, April 21, 2016*

Note: unless specified otherwise, all results must be confirmed with Matlab.

CRITICAL NOTE !!! The homework is to be turned in in **TWO parts**:

1. **Theoretical**: hand calculations (not necessarily in hand-writing)
2. **Matlab**: session log files which contain solutions to all Matlab related problems and are to be printed out and turned in along with the rest of the homework.

Matlab session log files must contain **your name** and the **hw's number** in the first few comment lines on top.

Problem 1 (Chap 2.4, Problems 42, 44)

Find an *LU*-factorization of the following matrices:

$$\textbf{42.}\begin{bmatrix}-2 & 1\\ -6 & 4\end{bmatrix} \qquad \textbf{44.}\begin{bmatrix}2 & 0 & 0\\ 0 & -3 & 1\\ 10 & 12 & 3\end{bmatrix}$$

Problem 2 (Chap 2.4, Problem 46)

Solving a Linear System Using *LU*-Factorization
In Exercise 46, solve the linear system $A\mathbf{x} = \mathbf{b}$ by
(a) finding an *LU*-factorization of the coefficient matrix A,
(b) solving the lower triangular system $L\mathbf{y} = \mathbf{b}$, and
(c) solving the upper triangular system $U\mathbf{x} = \mathbf{y}$.

46.
$$\begin{aligned} 2x_1 &= 4\\ -2x_1 - x_3 + x_2 &= -4\\ 6x_1 + 2x_2 + x_3 &= 15\\ -x_4 &= -1 \end{aligned}$$

!!! observe the order of variables

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* *Collaboration is allowed*

Problem 3 (Chap 3.1, Problem 16)

Finding the Minors and Cofactors of a Matrix

find all (a) minors and (b) cofactors of the matrix.

$$\textbf{16.}\begin{bmatrix} -3 & 4 & 2 \\ 6 & 3 & 1 \\ 4 & -7 & -8 \end{bmatrix}$$

Problem 4 (Chap 3.1, Problem 22)

Use expansion by cofactors to find the determinant of the matrix:

$$\textbf{22.}\begin{bmatrix} -3 & 0 & 0 \\ 7 & 11 & 0 \\ 1 & 2 & 2 \end{bmatrix}$$

Problem 5 (Chap 3.1, Problem 40)

Find the determinant of a triangular matrix

$$\textbf{40.}\begin{bmatrix} 5 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & -3 \end{bmatrix}$$

Problem 6 (Chap 3.2, Problem 28)

Use elementary row or column operations to find the determinant of the following matrix:

$$\textbf{28.}\begin{vmatrix} 3 & 0 & 6 \\ 2 & -3 & 4 \\ 1 & -2 & 2 \end{vmatrix}$$

Problem 7 (Chap 3.3, Problem 20)

Using a determinant to find out whether a matrix is singular or non-singular

$$\textbf{20.}\begin{bmatrix} 1 & 0 & 4 \\ 0 & 6 & 3 \\ 2 & -1 & 4 \end{bmatrix}$$

Problem 8 (Chap 3.3, Problem 28)

The Determinant of the Inverse of a Matrix

In Exercises 25–30, find $|A^{-1}|$. Begin by finding A^{-1}, and then evaluate its determinant. Verify your result by finding $|A|$ and then applying the formula from Theorem 3.8, $|A^{-1}| = \dfrac{1}{|A|}$.

$$\textbf{28. } A = \begin{bmatrix} 1 & 0 & 1 \\ 2 & -1 & 2 \\ 1 & -2 & 3 \end{bmatrix}$$

Problem 9 (Chap 3.3, Problem 38)

Finding Determinants **In Exercises 37–44, find (a) $|A^T|$, (b) $|A^2|$, (c) $|AA^T|$, (d) $|2A|$, and (e) $|A^{-1}|$.**

38. $A = \begin{bmatrix} -4 & 10 \\ 5 & 6 \end{bmatrix}$

Problem 10 (Chap 3.4, Problem 4)

Find the adjoint and inverse of a matrix A

4. $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & -1 \\ 2 & 2 & 2 \end{bmatrix}$

Problem 11 (Chap 3.4, Problem 28)

Use Cramer's Rule to solve the system of linear equations

28.
$$\begin{aligned} 14x_1 - 21x_2 - 7x_3 &= -21 \\ -4x_1 + 2x_2 - 2x_3 &= 2 \\ 56x_1 - 21x_2 + 7x_3 &= 7 \end{aligned}$$