Instructor: Roman Chomko

Homework* 7

Linear Transformations: Kernel, Range, One-to-One and Onto Transformations

Due Date: *Tuesday, May 31, 2016*

Problem 1 (Chap 6.2, Problem 7)

Find the kernel of the linear transformation:

7. $T: P_2 \rightarrow P_1$, $T(a_0 + a_1x + a_2x^2) = a_1 + 2a_2x$

Problem 2 (Chap 6.2, Problem 16)

Finding Bases for the Kernel and Range $T(\mathbf{v}) = A\mathbf{v}$ represents the linear transformation T. Find a basis for (a) the kernel of T and (b) the range of T.

16. $A = \begin{bmatrix} 1 & 1 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$

Problem 3 (Chap 6.2, Problem 49)

Determining Whether T Is One-to-One, Onto, or Neither determine whether the linear transformation is one-to-one, onto, or neither.

49. $T: R^2 \rightarrow R^3$, $T(\mathbf{x}) = A\mathbf{x}$, where A is $A = \begin{bmatrix} 5 & -3 \\ 1 & 1 \\ 1 & -1 \end{bmatrix}$

* *Collaboration is allowed*

Problem 4 (Chap 6.3, Problem 2)

The Standard Matrix for a Linear Transformation find the standard matrix for the linear transformation T.

2. $T(x, y) = (2x - 3y, x - y, y - 4x)$

Problem 5 (Chap 6.3, Problems 8)

Finding the Image of a Vector use the standard matrix for the linear transformation T to find the image of the vector $\mathbf{v}$.

8. $T(x, y) = (x + y, x - y, 2x, 2y)$, $\mathbf{v} = (3, -3)$

Problem 6 (Chap 6.3, Problem 34)

Finding the Inverse of a Linear Transformation determine whether the linear transformation is invertible. If it is, find its inverse.

34. $T(x, y) = (x + y, x - y)$

Problem 7 (Chap 6.3, Problem 38)

Finding the Image Two Ways find $T(\mathbf{v})$ by using (a) the standard matrix and (b) the matrix relative to B and B'.

38. $T: R^3 \rightarrow R^2$, $T(x, y, z) = (x - y, y - z)$, $\mathbf{v} = (1, 2, 3)$,
$B = \{(1, 1, 1), (1, 1, 0), (0, 1, 1)\}$, $B' = \{(1, 2), (1, 1)\}$

Problem 8 (Chap 6.4, Problem 9)

Finding a Matrix for a Linear Transformation find the matrix A' for T relative to the basis B'.

9. Let $B = \{(1, 3), (-2, -2)\}$ and $B' = \{(-12, 0), (-4, 4)\}$ be bases for R^2, and let

$$A = \begin{bmatrix} 3 & 2 \\ 0 & 4 \end{bmatrix}$$

be the matrix for $T: R^2 \rightarrow R^2$ relative to B.

(a) Find the transition matrix P from B' to B.

(b) Use the matrices P and A to find $[\mathbf{v}]_B$ and $[T(\mathbf{v})]_B$, where

$[\mathbf{v}]_{B'} = [-1 \quad 2]^T$.

(c) Find P^{-1} and A' (the matrix for T relative to B').

(d) Find $[T(\mathbf{v})]_{B'}$ two ways.

Problem 9 (Chap 6.4, Problem 18)

Similar Matrices **use the matrix P to show that the matrices A and A' are similar.**

18. $P = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}, A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}, A' = \begin{bmatrix} 2 & 0 & 0 \\ -1 & 1 & 0 \\ 2 & 2 & 3 \end{bmatrix}$

MATLAB EXERCISES[*]

Problem 10 (Matlab Exercise)

Study and copy/paste the following two functions into separate m-files[†]: 1) to compute the *range* of a matrix of a linear transformation **lisub.m**, and 2) to compute the *kernel* of a matrix of a linear transformation **homsoln.m**

```
function S = lisub(A,code)              %last updated 10/3/93
%LISUB  Find a linearly independent subset of vectors.
%       If code = 'r' the vectors are the rows of A.
%       If code = 'c' the vectors are the columns of A.
%       Any other value in code causes an error.
%       The routine returns a subset of linearly independent vectors
%       from the original set.
%
%       Use in the form  -->  S = lisub(A,'r')  or  lisub(A,'c')  <--
%
% By: David R. Hill, Math., Dept.,
%     Temple University, Philadelphia, Pa. 19122

err='ERROR: Improper code; second argument must be ''r'' or ''c''';
if code=='r'
   B=A';
elseif code == 'c'
   B=A;
else
   disp(err)
   return
end
[m,n]=size(B);
B=rref(B);
l1=[];
for ki=1:m
   z=find(B(ki,:)>0);
   if isempty(z)==0,
      l1=[l1 z(1)];
   end
end %now l1 contains the column #s with leading 1's
if length(l1)~=0
   if code=='r'
```

[*] Solutions for Matlab problems are to be submitted via iLearn's assignment
[†] as a text file using for example either Notepad or WordPad. **Don't** use Word for creating m-files.

```
      S=A(l1,:); %rows of S are a L.I. subset
   else
      S=A(:,l1); %columns of S are a L.I. subset
   end
else
   S=[];  %empty matrix returned only if A is the zero matrix
end
```

Listing 1. M-file: **`lisub.m`**

```
function NS = homsoln(A,code)              % last updated 8/15/91
%HOMSOLN  Find the general solution of a homogeneous system of
%         equations.The routine returns a set of basis vectors for
%         the null space of Ax = 0. Use in the form
%
%                     -->  ns = homsoln(A)  <--
%
%         If there is a second argument the general solution is
%         displayed. Use in the form
%
%                     -->  homsoln(A,1)  <--
%
%         This option assumes that the general solution has at
%         most 10 arbitrary constants.
%
% By: David R. Hill, Math., Dept.,
%     Temple University, Philadelphia, Pa. 19122
s0=' ';
s1=['The columns of the following matrix are a basis';
    'for the solution space of homogeneous system   ';
    'Ax = 0.                                        ';];
s2='The general solution is:';
s3='The only solution is the zero vector.';
var='rstuvwabcd'; %variable names for general solution
[m,n]=size(A);
A=rref(A);
l1=[];
for ki=1:m
   z=find(A(ki,:)>0);
   if isempty(z)==0,
      l1=[l1 z(1)];
   end
end %now l1 contains the column #s with leading 1's
la=1:n;  %all numbers from 1 to n
la(l1)=zeros(1,length(l1));
lv=find(la>0); %lv contains the numbers of the arbitrary variables
if isempty(lv)==1
   NS=zeros(n,1);
else
   %build the columns that span the null space
   I=eye(n);
   NS=[];
   r=rank(A);
   for ki=1:length(lv)
      c=I(:,lv(ki));
      for kj = 1:r
```

```
         c(l1(kj)) = -A(kj,lv(ki));
      end
      NS = [NS c];
   end
end
if nargin > 1   %display general solution
%   clc % 'clear screen' command, uncomment this line if needed
   s=[];
   for ki = 1:length(lv)
     s=[s var(ki) ' * col(' num2str(ki) ') '];
     if ki~=length(lv)
        s=[s '+ '];
     end
   end
   if isempty(s)==1
      disp(s3),disp(s0)
   else
      disp(s1),disp(s0),disp(NS)
      disp(s0),disp(s2),disp(s0),disp(s)
   end
end
```

Listing 2. M-file: **`homsoln.m`**

Use Matlab and the two functions above **`lisub()`** and **`homsoln()`** to solve:

Finding Bases for the Kernel and Range

$T(\mathbf{v}) = A\mathbf{v}$ represents the linear transformation T. Find a basis for (a) the kernel of T and (b) the range of T.

16. $A = \begin{bmatrix} 1 & 1 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$

and thus verify your solution of **Problem 2**.

***** END OF HW 7 *****