Instructor: Roman Chomko

Homework* 6

Gram-Schmidt Orthonormalization, Fourier Transform, Least Square Approximation and Linear Transformations

Due Date: *Tuesday, May 24, 2016*

Note: Matlab solutions are not required in this homework

Problem 1 (Chap 5.2, Problem 78)

find the orthogonal projection of f onto g. Use the inner product in $C[a, b]$

$$\langle f, g\rangle = \int_a^b f(x)g(x)\,dx.$$

78. $C[-1, 1]$, $f(x) = x^3 - x$, $g(x) = 2x - 1$

Problem 2 (Chap 5.3, Problem 34)

Applying the Gram-Schmidt Process In Exercise 34 apply the Gram-Schmidt orthonormalization process to transform the given basis for R^n into an orthonormal basis. Use the Euclidean inner product on R^n and use the vectors in the order in which they are given.

34. $B = \{(0, 1, 2), (2, 0, 0), (1, 1, 1)\}$

Problems continued in next page …

* *Collaboration is allowed*

Problem 3 (Chap 5.3, Problem 64)

Orthonormal Sets in P_2 In Exercise 64, let $p(x) = a_0 + a_1x + a_2x^2$ and $q(x) = b_0 + b_1x + b_2x^2$ be vectors in P_2 with $\langle p, q \rangle = a_0b_0 + a_1b_1 + a_2b_2$. Determine whether the given second-degree polynomials form an orthonormal set, and if not, then apply the Gram-Schmidt orthonormalization process to form an orthonormal set.

64. $\{x^2 - 1, x - 1\}$

Problem 4 (Chap 5.5, Problem 72)

Finding a Least Squares Approximation In Exercise 72, find the least squares approximation $g(x) = a_0 + a_1x + a_2x^2$ of the function f

72. $f(x) = \cos x, \quad -\pi/2 \le x \le \pi/2$

Problem 5 (Chap 5.5, Problem 80)

Finding a Fourier Approximation In Exercise 80 find the Fourier approximation with the specified order of the function on the interval $[0, 2\pi]$.

80. $f(x) = e^{-2x}$, second order

Problem 6 (Chap 6.1, Problems 3, 5)

Use the function to find **(a)** the image of and **(b)** the preimage of **w**:

3. $T(v_1, v_2, v_3) = (v_2 - v_1, v_1 + v_2, 2v_1)$, $\mathbf{v} = (2, 3, 0)$, $\mathbf{w} = (-11, -1, 10)$

5. $T(v_1, v_2, v_3) = (4v_2 - v_1, 4v_1 + 5v_2)$, $\mathbf{v} = (2, -3, -1)$, $\mathbf{w} = (3, 9)$

Problem 7 (Chap 6.1, Problems 12, 17)

Linear Transformations determine whether the function is a linear transformation.

12. $T: R^3 \to R^3, T(x, y, z) = (x + 1, y + 1, z + 1)$

17. $T: M_{2,2} \to R, T(A) = a + b - c + d$, where

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Problem 8 (Chap 6.1, Problems 34, 40)

Linear Transformation Given by a Matrix

define the linear transformation $T: R^n \rightarrow R^m$ by $T(\mathbf{v}) = A\mathbf{v}$. Find the dimensions of R^n and R^m.

34. $A = \begin{bmatrix} 1 & 2 \\ -2 & 4 \\ -2 & 2 \end{bmatrix}$

40. For the linear transformation from Exercise 3-34 above, find **(a)** $T(2, 4)$ and **(b)** the preimage of (-1, 2, 2). **(c)** Then explain why the vector (1, 1, 1) has no preimage under this transformation.

MATLAB EXERCISES*

Study the usage of Matlab's functions dot(), cross(), norm(), abs() which depending on the taste of the programmer may or may not be convenient.

```
>> % Dot products
>> u = [ 1 2 3], v = [1 1 0]
u =
     1     2     3
v =
     1     1     0
>> % Dot product between u and v
>> u*v'
ans =
     3
>> % or equivalently
>> dot(u,v)
ans =
     3
>> % Norm (length) of vectors
>> length = sqrt(dot(u,u))
length =
    3.7417
>> % or equivalently
```

* Print out solutions for Matlab problems and submit them along with the rest of the homework.

```
>> length = norm(u)
length =
    3.7417
>> % Distance between u and v
>> d = norm(u-v)
d =
    3.1623
>> % Angle between u and v
>> % Cos of angle between u and v
>> cosuv = dot(u,v)/(norm(u)*norm(v))
cosuv =
    0.5669
>> thetaRad = acos(cosuv) % angle in radians
thetaRad =
    0.9680
>> thetaDeg = (thetaRad/pi)*180 % angle in degrees
thetaDeg =
   55.4624
>> % Cross products
>> cross(u,v)
ans =
    -3     3    -1
>> % Absolute values
>> u = -1; % a scalar
>> abs(u)
ans =
     1
>> u1 = [-1]; % actually Matlab sees it as a vector
>> abs(u1)
ans =
     1
>> u2 = [-1 -2 3] % check this out
>> abs(u2)
ans =
     1     2     3
```

Problem 9 (Matlab Exercise)

Write a Matlab function **`projectUV()`**, that is,

`function [w] = projectUV(u,v)`

which computes a projection vector of **u** on **v** thus performing the operation

$$\mathrm{proj}_{\mathbf{v}}\mathbf{u} = \frac{\mathbf{u} \circ \mathbf{v}}{\|\mathbf{v}\|}\mathbf{v}$$

Test the function by computing the projection of vector **u** = (1, 2, 3) onto **v** = (1, 1, 0).

Problem 10 (Matlab Exercise)

Study and copy/paste the function into a new Matlab m-file* **`gschmidt.m`**

```
function W = gschmidt(A,d)            %last updated 1/19/96
%GSCHMIDT  The Gram-Schmidt process on the columns in matrix
%          A. The orthonormal basis appears in the columns of W
%          unless there is a second argument in which case W
%          contains only an orthogonal basis. The second argument
%          can have any dummy value d.
%
%          Use in the form  ==> W = gschmidt(A)    <==
%                       or  ==> W = gschmidt(A,d)  <==
%
%  By: David R. Hill, MATH Department, Temple University
%      Philadelphia, Pa., 19122        Email: hill@math.temple.edu
%
[m,n]=size(A);
W=A(:,1);
for k = 2:n  % orthogonalization process
   z=0;
   for j=1:k-1
      p=W(:,j);
      z=z+((p'*A(:,k))/(p'*p))*p;
   end
   W=[W A(:,k)-z];
end
if nargin == 1
   for j=1:n, W(:,j) = W(:,j)/norm(W(:,j));end % normalizing
end
```

Solve **Problem 2** using Matlab and this function **`gschmidt()`** and thus verify your solution of **Problem 2**.

* as a text file using for example either Notepad or WordPad. **Don't** use Word for creating m-files.

***** END OF HW 6 *****