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DISCOVERY OF THE KALMAN FILTER AS A PRACTICAL

TOOL FOR AEROSPACE AND INDUSTRY

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SUMMARY

The Kalman filter in its various forms has become a fundamental tool for analyzing and solving a broad class of estimation problems. The first publicly known application as made at NASA Ames Research Center in the early 1960s during feasibility studies for circumlinear navigation and control of the Apollo space capsule. This paper recounts the fortunate sequence of events, which led the research firs at Ames Research Center to the early discovery of the Kalman filter shortly after its introduction into the literature. The scientific breakthroughs and reformulations that were necessary to transform Kalman's work into a useful tool for a specific aerospace application are described. The resulting extended Kalman filter, as it is now known, is often still referred to simply as the Kalman filter. As the filter's use gained in popularity in the scientific community, the problems of implementation on small spaceborne and airborne computers led to a "square-root" formulation of the filter to overcome numerical difficulties associated with computer word length. The work that led to this new formulation is also discussed, including the first airborne computer implementation and flight test, which was conducted in 1972. Since then the applications of the extended and "square-root" formulations of the Kalman filter have grown rapidly throughout the aerospace industry.

INTRODUCTION

In 1960, Dr. Kalman published his now-famous paper, "A New Approach to Linear Filtering and Prediction Problems" (ref. 1).That paper made a significant contribution to the field of linear filtering by removing the stationary requirements of the Weiner filter and presenting a sequential solution to the time-varying linear filtering problem. Kalman's solution was particularly suited to the dynamical state estimation needs of the space age (ref.2). Commonly known as the Kalman filter, the new formulation had a major effect in related academic and engineering circles. Although the first uses of the Kalman filter were in aerospace applications, the relative simplicity and versatility of the formulation resulted in its rapid adaptation for utilization in many other fields. The Kalman filter in its various forms is clearly established as a fundamental tool for analyzing and solving a broad class of estimation problem.

The events that led to the current widespread use of the Kalman filter are recounted here to the best of our recollections. The description given in (ref. 3) are expanded, giving greater insight into the many problems that had to be overcome, as well as the extremely fortunate sequence of events that resulted in the discovery of the filter and its successful implementation.

The paper describes how the need for a filter such as Kalman's arose from NASA's early work on the manned lunar mission, even before that mission was selected as a national program. Also described is why Kalman's

work was introduced at a "near-perfect" time and why this near-ideal solution to the midcourse navigation problem might have been undiscovered except for a fortunate meeting between Dr. Schmidt (one of the present authors) and Kalman.

Kalman's work was a large step forward, but its usefulness for the Apollo mission was limited by certain features of the formulation. We recount the events that resulted in the scientific breakthrough and the reformulation that transformed Kalman's work into the extremely useful tool that is now known as the "extended Kalman filter."

The reasons for and events that took place during the development of navigation systems for the Apollo and Lockheed C-5A aircraft programs and the way in which the results from those key programs led to a square-root formulation of the Kalman filter, which had features suitable for aircraft applications, are described. A discussion then follows of the development of the first known aircraft flight experiment in which the performance of a square-root formulation coded for an airborne computer was validated.

The paper concludes with a discussion of the tremendous range of problems to which the extended and square-root Kalman filters are being applied in today's highly technical world.

THE FIRST APPLICATION OF THE KALMAN FILTER: APOLLO MISSION

The Need for a New Filter

In the fall of 1959, Dr. Harry J. Goett, then Director of NASA's Goddard Space Flight Center (GSFC}, invited Dr. Schmidt and other members of Dr. Goett's former division at Ames Research Center (ARC/ to meet with the Space Task Group located at Langley Research Center to discuss the future manned spacecraft program. (See (ref. 4) for a history of the Apollo program.) Dr. Goett was eager to get feasibility studies under way at the NASA research centers to refine guidelines for the manned lunar mission.

Of major interest to the present authors were the areas in which the Dynamics Analysis Branch at Ames should concentrate its research efforts. The principal outcome of many meetings of the Space Task Group was the identification two potential areas of research for the Branch. The first was midcourse navigation and guidance for the circumlunar mission, and the second was the autopilot design for large, flexible body liquid-fuel boosters.

The Dynamics Analysis Branch had only eight analytically oriented research persons--too few to carry out two very complex research programs. The most logical choice, considering personnel experience, seemed to be the booster- autopilot- design-problem. However, after many discussions, enthusiasm grew for the midcourse navigation and guidance problem, and there was finally a unanimous decision for the Branch to work in that area. Certainly, it was an ambitious undertaking, one that presented a greater challenge to Branch personnel than had been faced before.

The primary emphasis in the Branch was rapidly brought to bear on the need to develop the concepts and technology for a completely self-contained system. This meant the software for the mission would have to be resident in a reliable, on-board digital computer with considerable memory and relatively high computational speed. The midcourse algorithms would have to be as efficient as possible. The system, with pilot/navigator inputs, would have to navigate and guide the spacecraft from injection into a circumlunar trajectory, around the Moon and

back to Earth, satisfying very restrictive entry corridor requirements on return to Earth.

Having selected the problem area and directed our attention to a conceptual design for a solution to the problem, it was clear that we were facing a rather massive effort with a staff that was utterly inexperienced in many of the required tasks (such as lunar trajectory analysis). Perhaps if we had fully comprehended the necessary effort we would have reconsidered our choice of problem area for the Branch research effort. Basically, we were starting with nothing in the way of analytical software tools. Ames had an IBM 704, which would be our simulation computer. Fortunately, FORTRAN was made available to us early in our research efforts, but software for the machine was quite limited. For example, later on, when we were preparing the software to do the Kalman filter matrix operations, we had to write all of our own matrix-handling subroutines. We soon found, however, that the double indexing needed for matrix operations ran so slowly that our matrix handling subroutines had to be rewritten to use single indexing. Otherwise, a circumlunar trajectory with a Kalman filter would have taken hours of computer time.

The first step was to build a trajectory analysis program capable of simulating a trajectory to the Moon and return. Fortunately, Dr. Clarence Gates and others at the Jet Propulsion Laboratory (JPL) gave us invaluable assistance and counseling based on their work in this area. Probably the most timely aid was in the form of an ephemeris tape containing the positions of the Sun and the Moon versus time. By mid-1960, we could calculate free-return trajectories to the Moon and were investigating linear perturbation methods that looked promising for calculating and implementing sm a ll velocity changes to simulate midcourse correction.

Most of the research was going well at that time with the exception of navigation; that is, the use of pilot observations of external bodies to estimate the vehicle state. We had assumed from the beginning that this would be accomplished by the crew operating an inertially referenced optical sensor on the spacecraft to measure the elevation, azimuth, and subtended angles of Earth and Moon. The question was how to process such data in an efficient manner. To this end, we had reviewed the iterative weighted, least-squares estimator in use by JPL and decided that it was not only too complex for state-of-the-art on-board computers but would also put a severe burden on the IBH 704 we were using for simulation. We had also considered taking multiple optical tracking measurements and filtering them with a polynomial. Although this appeared to be a simple enough procedure, the accuracy of the resulting state estimation was not adequate for the level of optical measurement accuracy that could be achieved.

Some of the staff had been working with Weiner filter theory for several years and had made successful applications of that theory to the problems of guidance and navigation for beam-rider and homing missiles. Because the lunar vehicle navigation problem had obvious similarities to missile navigation, we wondered whether the Weiner filter theory could be applied. The difficulties were the nonlinearity of the problem and the requirement, for lunar vehicle navigation, of an irregular series of discrete measurements (whereas missile navigation had assumed continuous measurements). We could not find an approach that would permit applications of the Weiner filter theory without making approximations that would either severely restrict the observation system or destroy the inherent accuracy. Obviously, a new approach was needed for computing the estimated state from on-board measurements--a way that would not overburden our simulation facilities or an on-board computational capability.

Discovery of the Kalman Filter

In retrospect, it seems almost incredible that the next sequence of events should have taken place and that Dr. Kalman's work should be so quickly recognized. The authors recall those events in the following way. Dr. Schmidt, at Ames, and Dr. Kalman, at the Research Institute for Advanced Study (RIAS), had been acquaintances for several years. In the fall of 1960, Dr. Kalman, unaware of the work we were doing, called and arranged to visit Dr. Schmidt to discuss topics of mutual interest. It was during this very fortunate visit that Kalman presented his now famous paper to

members of the staff of the Dynamics Analysis Branch. Because the staff had been thinking of filter theory as a way of modeling the problem, the presentation hit a responsive chord. In particular, the sequential solution features of Dr. Kalman's formulation were of interest because they could certainly relieve some of the computational problems we were facing with the IBM 704. Thus, even though the theory was linear and our application nonlinear, Dr. Schmidt thought the approach might have some merit for our application and assigned key staff members to carefully examine the paper.

That was not an easy task and a great deal of difficulty was experienced in understanding Dr. Kalman's paper because of the relatively new state space approach to control problems used by Dr. Kalman. The notation, as well as the concepts, used by Dr. Kalman were also very difficult for practicing Engineers to grasp. On his next trip to GSFC, Dr. Schmidt arranged to meet with Dr. Kalman at RIAS to discuss the paper further. It was at this meeting that the method of applying Dr. Kalman's theory to a nonlinear system became clear to Dr. Schmidt. He realized that the linear perturbation concepts we had been using in our guidance studies could be used to produce the linear system needed to apply Dr. Kalman's theory. Thus, the combination of Dr. Kalman's linear filter theory with the linear perturbation methods we were already using gave us a potential solution to the nonlinear navigation problem and also to some of the problems posed by the speed and storage limitations of the IBM 704 computer. Clearly, the meeting at RIAS resulted in the breakthrough we needed, and we immediately made plans to produce a digital simulation program to evaluate and validate the Kalman formulation. Whether the Kalman filter would provide the state estimation accuracies necessary for circumlunar navigation remained an open question and was of considerable concern.

Modification of the Formulation

The trajectory analysis work that had been progressing in the meantime had given us some insight into the problem of how to schedule the time of the simulated on-board optical measurements and course corrections we planned to use to accomplish the navigation and guidance function. The schedule called for optical measurements to be taken in a short sequence, with the measurements evenly spaced in time. The sequences could be separated by relatively long periods of time. The starting time of the measurement sequences and the timing of the course corrections would be variables for later analysis. These features were permitted by the numerical integration routine used to solve equations of motion for the trajectory. The key features of the routine were its ability to vary the integration step (time) over a wide range and the ability to be stopped and restarted at any arbitrary time. The variable step-size was critical to us because it brought the IBM 704 computation time down to a value that at least let us consider doing the problem.

Dr. Kalman's original formulation would have required an on-board crew to make a continuous sequence of optical measurements equally spaced in time throughout the lunar mission, an impractical scenario. Therefore, to implement our measurement and course-correction schedule, the original formulation had to be revised. The solution to the problem was obtained by decomposing the original formulation into a discrete-time update portion and a discrete-time optical measurement update portion which provided a much more natural and intuitively appealing way of expressing Dr. Kalman's algorithm. Looking back, the decomposition seems almost trivial; at the time, however, it was a major and critical step forward and one in which an unrecognized error could have been disastrous.

The Extended Kalman Filter

The original studies using perturbation methods and the above mentioned decomposition of the filter were based on a linearization about a nominal (reference) trajectory. It soon became apparent that a re linearization about the current estimated state might offer substantial advantages over the technique previously used. We reasoned that "on the average," the estimated state would be closer to the actual. or true, state than to the reference. or nominal, state and thus the linearity of the approximation would be retained better than with a nominal, or reference, state. The correctness of this presumption was borne out in a rather extreme incident. An accidental error in input conditions to a simulation run caused the true trajectory to remain in an orbit around Earth, but the estimated trajectory had the proper starting conditions for a lunar transfer trajectory. The simulated optical measurements used to estimate the position of the space vehicle were intentionally sparse in the early part of the run because the trajectories were quite nonlinear in that region. At first, the Kalman-filter estimator caused some relatively large overshoots in the estimated state owing to the nonlinear effects, but the estimated state soon successfully converged close to the true state. This modification to the implementation has come to be known as the "extended Kalman filter."

Development and Testing of the Filter Simulation

The design of the digital simulation program proceeded fairly rapidly once it was clearly understood how the Kalman filter was to be implemented. The transition matrix for the discrete filter time update remained a problem, however. This matrix was to be used to transform a position and velocity error state at one time into a new error state at a later time. It is a key element in the Kalman-filter formulation.

Another group at Ames specializing in software accepted the task of developing the programs to compute the transition matrix using data from a stored lunar trajectory. The data were not equally spaced in time, and as a result interpolation would be required to retrieve the desired position and velocity at a specified time. A set of variational equations would then be evaluated from the trajectory data to produce the transition matrix. It was thought that this approach would be relatively fast and would reduce the computational time on the IBM 704 over that of the straightforward approach of simply solving the variational equations by integrating an additional set of 18 complicated, second-order differential equations. As it turned out, the software development task was much more difficult than originally thought.

By the time the simulation program implementing the Kalman filter was coded in FORTRAN and the coding extensively verified, the group working on the transition matrix expected their software could be completed in a few additional weeks. Everyone was extremely eager to give the Kalman-filter program a try. When the time came and passed for the transition matrix program to be operational, it was becoming increasingly clear that work should be stopped on this approach because it was producing software that was taking much longer to execute than was originally expected. The only remaining alternative was to integrate the additional 18 second- order differential equations along with those for the true, reference, and estimated trajectories. Although this was a setback in the IBM 704 time budget for the Kalman-filter simulation program, compensating savings were made by the discovery that the transition matrix could be inverted merely by rearranging terms.

At this point, we were about 6 months behind the schedule we had set for our- selves. It was decided to build a test program to debug the modular software for computing the transition matrix. In this way, the modular elements could be used directly by the Kalman-filter simulation program with very few changes. The test program was put together in a few days from coding that was borrowed from other programs we had developed. Very little debugging time was required because most of the complex software was by then already running in other programs and because the problem was very straightforward. This new program produced the transition matrix from injection into the

translunar trajectory to periselene (closest approach to the Moon). It is also used to produce a transition matrix from trans-Earth injection to perigee. The matrix elements were then punched on cards to be read into the simulation program for use with the guidance laws to compute course corrections.

In the Kalman-filter program, the transition matrix for a time-update of the covariance matrix was always available by integrating the 18 second-order differential equations in the simulation program from injection to the time of the first optical measurement and then from that measurement time to the next and so on. The product of all these matrices gave the transition matrix from injection to the time of the latest optical measurement. This matrix was required by the midcourse correction algorithm. The variational equations could be solved about either the reference or the estimated states. Later the program was modified to do the return (inbound) trajectory by reinitializing at periselene with a11other stored transition matrix from periselene to Earth reentry.

The digital simulation program was designed to inte6rate nonlinear differential equations for the true trajectory, the nominal (or reference) trajectory, and the estimated trajectory. The simulated optical measurements were calculated as functions of the true state plus additive biases and noise. The Kalman-filter algorithm accepted these measurements and provided incremental changes in the estimated state. The difference between the estimated state and nominal state was used in the midcourse correction algorithm. This algorithm gave the V vector required to drive the estimated position to the nominal position at periselene at the end of the outbound and perigee at the end of the inbound trajectory. To simulate the mid- course correction, the computed V vector was added to the estimated trajectory 1irectly. Random magnitude and pointing errors were added to the computed V vector to simulate control system errors, and the result was added to the true trajectory. The covariance matrix of the estimated state was also increased in a manner corresponding to the statistics of the simulated control system errors. When the simulation program was finally ready for a test run on the IBM 704, it took a little over an hour of execution time. The first run showed disappointing results in the state estimation scheme. Naturally, we were shocked. What if the Kalman filter did not work? Then, an error was found in the way we were calling a subroutine. On the second run everything worked fine!

By early 1961, the simulation program had been used extensively t0 validate the extended Kalman filter and the guidance laws we had developed. Our very encouraging results indicated that on-board optical measurements combined with the knowledge of the equations of motion could yield adequate accuracy for the circumlunar navigation/guidance problem. This was the breakthrough that we had set out to achieve at the beginning when we had selected the midcourse navigation and guidance for the circumlunar mission as the study area for the Dynamics Analysis Branch. At that time it was clear that we had achieved a potentially significant result for on-board navigation systems. Our studies indicated that the extended Kalman filter would give accuracies comparable to those of weighted least-square estimators, but with a tremendous reduction in requirements for on-board computer memory and computational speed. We had not, however, verified that the extended Kalman filter could be mechanized and operated properly with the flight computers available at this time.

As it turned out, we had been fortunate in the way we had chosen to mechanize the computations for the extended Kalman filter. The order chosen in which to multiply the three Matrices in the measurement update equation, that is, (KH)P rather than K(HP) was the most numerically stable of the two possibilities we could have selected. As a result, we had no apparent computer round-off difficulties. If we had done things differently, however, we might have found that double-precision matrix operations were necessary with a resultant large increase in computer time, which, in turn, would have made the filter appear much less attractive for spacecraft applications.

Dissemination of the Simulation Results

As we recall, two of the first persons (outside of Ames) that we told of our results were Dr. John V. Breakwell and Dr. Charlotte Striebel, both of Lockheed Missiles and Space Company (LMSC). Both Dr. Breakwell and Dr. Striebel were sufficiently impressed with the results to begin further research on their part to explain the equivalences to other trajectory determination methods in use at the time.

At about this same time in early 1961, Dr. Schmidt told Dr. Battin of our results. Dr. Battin was at the Instrumentation Laboratory of the Massachusetts Institute of Technology and was en£3ged 1n studies of the Apollo mission. Dr. Battin had been independently engaged in work along lines similar to those of Dr. Kalman but was unfamiliar with Dr. Kalman's work. In September 1961, Dr. Battin pub1ished an Instrumentation Laboratory Report titled "A Statistical Optimizing Navigation Procedure for Space Flight" (ref. 5). In the introduction to his report, Dr. Battin made the following statement:

The formulation of an optimum linear estimator as a recursion operation in which the current best estimate is combined with newly acquired information to produce a still better estimate was presented by Kalman. The author is indebted to Dr. Stanley F. Schmidt for directing his attentions to Kalman's excellent work. In fact, the original application of Kalman's theory to space navigation was made by Schmidt and his associates. The work described in the following sections of this paper was done without and detailed knowledge of Schmidt's activities ..

Dr. Battin continued to investigate using the filter for application to the Apollo navigation system. Later, Potter (ref. 6}, working with Dr. Battin at MIT, devised the first square-root filter implementation, which was used for the Apollo system. His formulation could not handle random forcing functions, Jut it was useful for implementation in small-word-size on-board computers, such as that of the Apollo system. Still la r, Potter's implementation was included as part of other square- root implementations that could handle random forcing functions. Some of these implementations will be discussed later.

The word of our work began to spread rapidly throughout scientific, academic, and engineering circles. During the summer and fall of 1961, any visitors from all parts of the country had come to discuss the work we had done. In the summer of 1961, two papers (refs. 7 and 8), which were shortened version of two NASA reports (refs. 9 and 10) published in the following year, were presented in San Francisco at an American Astronautical Society (AAS} conference. This was the first formal introduction of our work on navigation and guidance studies for the circumlunar mission before a group of scientists and engineers. Because of the extreme interest in the Apollo program and the potential our work held for that program, extensive informal discussions were held after th2 conference with many representatives of both government and industry.

During the early period, Dr. Schmidt and his staff of researchers at Ames can be credited with the following technical breakthroughs, which led to this first major application of the Kalman filter.

1. Demonstration that Kalman's original theory could be adapted to nonlinear problems.

2. Development of the extended Kalman filter, which linearized abo t the current best estimate of the state to reduce the effects of nonlinearities in the problem.

3. Decomposition and reformulation of Kalman's original algorithm into separate time-update measurement-update portions so that measurements could be processed at any arbitrary time interval.

4. Demonstration, by means of a comprehensive digital simulation, using items (1)-(3) above, of the Kalman filter's

potential application to a nonlinear, on-board spacecraft navigation and guidance problem.

5. Dissemination of the results of the simulation work to the M T Instrumentation Laboratory for possible inclusion in the Apollo on-board guidance and control system.

6. Dissemination of information on the Ames Kalman filter work to a large segment of the · scientific and aerospace communities through presentations and formal papers.

It should be noted that at this point in time (1961) no problems had been experienced in the digital simulations owing to truncation or modeling errors. Later, however, many researchers began to report substantial problems of instability with Kalman filtering, problems on which considerable effort and research would be expended.

FURTHER FILTER IMPROVEMENTS

Support for the Apollo Mission

Following Dr. Schmidt's departure from ARC in late 1961 to join Lockheed Missiles and Space Company, the staff at ARC continued to apply the extended Kalman filter to problems of interest to NASA for the Apollo program. From mid-1962 to mid-1964, research at ARC W3S directed to three general areas using essentially the same software with the same Earth-Moon and Moon-Earth trajectories:

1. To study the effect of modeling errors of secondary importance and of off- design conditions on optimal estimation of a space vehicle trajectory.

2. To study the effect of the relatively short word length accommodated by most of the airborne computers of that time

3. To study the effect on midcourse guidance of using ground radar tracking in addition to on-board observation data. It was during the early phases of this work that the divergence" problem was first noticed. Apparently, the problem chosen for the initial studies with the extended Kalman filter was not particularly sensitive to nonlinearities, effects of computer round-off, unmodeled parameters, or a priori statistics. As mentioned earlier, the computer round-off problem was first noticed and occurred when the sequence of multiplying three matrices together was changed. Initially, this led to the assumption that computer round-off was the only problem. Mr. Gerald Smith communicated our discovery to Dr. Schmidt, but little was done at ARC at this time because the problem and the filter formulation we were using seemed quite stable. When undertaken some time later, the first attempts to fix the computer round-off problem involved working with the covariance matrix P to make it symmetric with nonnegative eigenvalues. After computing P in the normal way, some fixes were tried, four of which are mentioned here. First, by forcing P to be symmetric by selecting either the upper or lower off-diagonal elements and using only those diagonal elements to form a symmetric matrix. Second, by averaging the off-diagonal terms to force symmetry. Third, the same as the second, then computing the off-diagonal correlation coefficients. I any of magnitudes were greater than one, the coefficients would be printed and the program stopped. And fourth, by adding a small positive number to all of the diagonal terms of the covariance matrix, P, after measurement and time-update operations. This fix associated numerical truncation errors with process noise, thus allowing an increase in the covariance matrix after numerical operations.

As we recall it, the third and fourth fixes worked best but during the time that work was going on another

problem was being uncovered, both at ARC and elsewhere. The new problem was most apparent after a series of very accurate measurements had been processed by the filter, causing the covariance matrix P to become so "small" that additional measurements would be essentially ignored by the filter. When this happened, only very small corrections to the estimated state would be computed as the result of a measurement, and the estimated state would diverge from the true state. The basic problem here is due to modeling errors. The use of "pseudo noise" in the time-update proved quite effective against this problem.

Other researchers were encountering computational difficulties because of round-off. For example, a Honeywell, Inc. interoffice memorandum by R. C. K. Lee, dated July 8, 1964, recommended that the symmetry problem with P be solved by computing only the diagonal plus the upper triangular elements of the matrix. The memorandum also recommended that the problem of P becoming negative-definite after a measurement update be overcome by using the equivalent but more symmetric expression:

$$P_{t+1} = (I - K_{t+1}*H)*M_{t+1}(I - K_{t-1}*H)^T + K_{t+1}RX_{t+1}^T$$

Where

$$M_{t+1} = \phi P \phi^T + Q$$

$$K_{t+1} = M_{t+1}H^T(HM_{t+1}H + R)^{-1}$$

This equation for Pt+1 is frequently referred to as Joseph's formulation (ref. 11).It reduces to Kalman's measurement equation when the gain Kt+1 is optimal, as shown. It is also the general equation for the covariance matrix after a state change owing to an arbitrary gain.

The results of the filter research in support of the Apollo program (refs. 12-14)made clear the following:

1. How to include astrodynamics constant uncertainties and bias-type errors in the estimation process, and how to compute the performance of a system subjected to unrecognized 0r ignored bias errors.

2. That for the particular circumlunar trajectories being investigated, the simulation computer word mantissa of 2 bits was adequate but that some computations could be carried out with the lesser pre cision. It also verified that when the covariance matrix P is a too-optimistic representation of estimated state errors, external measurements are given too little weight.

3. That ground radar data are generally superior to on-board measurements for estimating the trajectory of the spacecraft, but that use of radar data does not save significant midcourse correction fuel and the control of the trajectory endpoint is not greatly enhanced. These results supported the ultimate decisions to have the primary Apollo navigation conducted from the ground, using ground radar data with a backup system on the spacecraft. That was an important and timely result.

Application of the Filter to the Agena Program

Meanwhile, at the Lockheed Missiles and Space Company, in 1961, Dr. Schmidt had his first opportunity to process actual measurement data with the extended Kalman filter developed at ARC. The purpose of the effort was to validate the performance of the Agena upper stage, using Earth-based tracking data from the downrange stations and telemetry data from the vehicle. A general-purpose post flight analysis program was developed which combined

tracking data from several station with the model of the equations of motion for the vehicle. The error state included tracking-measurement biases and location errors, as well as coefficients of a propulsion model for the thrust of the Agena upper stage.

The procedure developed for the operation was to use the tracking data during coast phases to estimate position and velocity and covariance matrix of errors at the initiation and termination of the thrust phase. The thrust phase was then handled by starting the program with the initial state and covariance matrix (from coast-phase data), with the error state expanded to include coefficients of the thrust model. Tracking data during the thrust phase was processed in the normal manner. At the termination of the thrust, the state estimate from the post burn tracking data was used as a measurement of the six-component state vector, with its covariance matrix used to characterize the accuracy.

This development added the following techniques to the applications technology of the Kalman filter:

1. A bad-data rejection technique was developed which compared the measurement residual magnitude to its standard deviation as computed from the Kalman-filter measurement update algorithm. If the residual magnitude exceeded n times the standard deviation, the measurement was rejected. The value of n used was 3, corresponding to a 3-sigma residual magnitude test.

2. The Kalman filter was used a3 data compression algorithm to form an equivalent measurement and covariance matrix from the multitude of measurements taken during the coast phases of the vehicle.

3. An iterative approach using backward integration and forward filtering was developed to remove effects of nonlinearities from the estimate. This was not data-smoothing, but simply an equivalent procedure to the weighted, least-squares estimators in use at that time.

4. The Kalman filter was used to estimate parameters in the measurement and equation-of-motion models.

Development of the Kalman-Schmidt Filter

After moving to Philco-WDL (now Ford Aerospace and Communications Corp.) in 1962, Dr. Schmidt began work on the development of a general-purpose error-analysis program for GSFC. During this effort, the so-called Kalman-Schmidt filter (ref. 15) was developed, largely as a result of the encouragement of Dr. F. 0. Von Bun at GSFC. Earlier, the extended Kalman filter had been referred to by some authors (ref. 16) as the Kalman-Schmidt filter, probably because of a desire to give credit for the application technology, which resulted in the extended Kalman, filter. The "Kalman-Schmidt" filter as referred to here, includes the effects of (but does not solve for) selected error states. As a result, a means of optimally compensating for modeling errors is provided, when it is known which model errors in the filter equations are significant. This filter was provided as one option in the general-purpose error-analysis program delivered to GSFC (ref. 17).

In addition to the Kalman-Schmidt filter, the developments at Philco-WDL added the following techniques to recursive filtering technology:

1. General computational techniques for saving machine time by taking account of the sparsity of the transition matrix and symmetry of the covariance matrix.

2. Mathematical formulation for combining on-board inertial sensor measurements with ground-tracking data.

3. Ad hoc technique fur adding pseudorandom forcing functions to minimize the effects of numerical errors.

DEVELOPMENT OF THE SQUARE-ROOT FORMULATION

The C-5A Aircraft Navigation System

By 1966, the advantages of the extended Kalman filter were widely recognized. When Lockheed became the prime contractor for the C-5A aircraft, the Kalman filter was specified for the navigation system. The contract for the C-5A navigation system was won by Northrop Corp., which, in turn, hired Dr. Schmidt as a consultant for the Kalman-filter development. The filter combined inertial data and data from various navigational aids to produce the state estimate of the aircraft. This was the first opportunity either of the authors had been afforded to participate in the development of a real-time system on boa u an aircraft, even though they had hoped for a NASA program as early as 1961. The C-5A system, to the best of the authors' knowledge, was the first real-time application of a Kalman filter on board an aircraft.

During the development of the C-5A navigation system, the real-world problems of selecting appropriate error states, adding error estimates to the system outputs, working with a limited word size, and making the whole computational burden work within the time-frames of small computers had to be confronted. Many of the model and numerical compensation techniques (refs. 18 and 19) were put to a demanding test by this developmental work. Also, data compression methods that use measurement averaging for saving computational time at a small expense in accuracy were perfected by John Weinberg while working with Schmidt and Lukesh at Northrop (ref. 20). This development definitely pointed out the need for a general square-root filter formulation that could include random forcing functions and work properly with the fixed-point arithmetic of the available on-board computers. That is, a method was needed wherein the filter's error covariance would be computed and propagated in square-root form and would therefore require less computational precision to maintain filter stability. However, because such a formulation was not available for this system, the standard extended Kalman algorithm, with the epsilon technique and other ad hoc procedures, was used for controlling numerical problems caused by round-off (refs. 18 and 20).

The additional complexity of making an extended Kalman filter work with the C-5A navigation system had thoroughly convinced Dr. Schmidt of the need for a square-root formulation including random forcing functions for airborne applications. Being aware of Potter's square-root measurement update formulation for the Apollo mission, he personally made several attempts to find an efficient square-root filter method that would allow use of random forcing functions in the time-update algorithm. In early 1968, he successfully developed a method (refs. 21 and 22) that looked promising for application on a small, fixed-point on-board computer. (The reader should consult reference 23 for a good survey of the history of the square-root filter development during this time.)

Flight Test of RAINPAL with the First Airborne Square-Root Filter

In 1969, Mr. L. A. McGee (one of the present authors) proposed a flight-test program to support the Shuttle developmer.t work by testing the Kalman filter in an on-boa d configuration to validate its performance with a highly

accurate ranging device manufactured by Cubic Corporation of San Diego, California. Studies had indicated that the potential navigational accuracy of such a system was so great that a highly accurate and independent assessment of its navigational performance would be essential, if the results were to be accepted b: the scientific community. The test site meeting these requirements was the Army's White Sands Miss1le Range (WSMR) in New Mexico. At the time, the cinetheodolite system in operation on the WSMR test range was believed to be the most accurate system in the nation for determining the actual position and velocity of an aircraft during approach, landing, and rollout. The flight-test aircraft was to be a Convair CV-340 with a gross takeoff weight of 44,000 lb.

The on-board computer would be a ruggedized XDS-920 with a 24-bit-word length and a 32K memory. Based on previous studies of the effect of computer word length on Kalman-filter performance and the limited amount of memory available, as well as the need to avoid as many numerical difficulties with the Kalman filter as possible, the decision was made to include in this project an evaluation of a square-root formulation of the Kalman filter that would include random forcing functions.

In 1970, McGee, G. L. Smith, and others at ARC issued a request for proposal for the development of software to implement and test (in flight) a square-root Kalman filter with random forcing functions on a 24-bit, fixed-point XDS-920 computer along with such other software as would be required in order to develop a complete airborne navigation system. This system, later called RAINPAL, was to be a precision approach and landing navigation system.

The competition for the software development work on the RAINPAL system was won by Analytical Mechanics Associates, Inc. (AMA), based on the square-root formulation capable of handling random forcing functions which had recently been developed by Dr. Schmidt, who would be leading the AHA team. The original formulation used Potter's algorithm for a measurement update and a modified Gram-Schmidt algorithm for the time update in order to include the random forcing functions. The Gram-Schmidt triangularization algorithm was used to reduce the non-square augmented square-root covariance matrix to a square form required by the square-root filter formulation. Mr. Paul Kaminski, a doctoral candidate at Stanford University (ref. 23), demonstrated to Dr. Schmidt that another triangularization algorithm by Householder could be used for the time update and save computational operations over the modified Gram-Schmidt algorithm. At this time, it made little difference which algorithm was implemented, so the decision was made to proceed using the faster Householder algorithm. Although Potter's algorithm was used in the Apollo system. The RAINPAL system is believed to be the first application of the complete square-root filter technology, including process noise, on-board an aircraft.

The RAINPAL system was initiated at ARC but, before the flight tests were started, it became a joint program with NASA's Manned Spacecraft Center (MSC) and the Army's Instrumentation Directorate at WSHR (WSMR-ID). Ames' interest was that of flight-testing and validating a square-root Kalman filter with inertial aiding; MSC's interest was to investigate new concepts, which might be suitable for navigation of the Space Shuttle Vehicle (SSV); and the WSMR-ID interest was a desi e to investigate new concepts offering promise for a future instrumentation system at WSHR.

Overall, the square-root Kalman filter was under considerable scrutiny. Failure caused by divergence, computer round-off, or many other potential causes would have been a severe blow to t e proponents of the square-root-filter technology. The square-root filter however, performed flawlessly once it had been debugged early in the software development. In fact, it soon became possible to place the square-root Kalman filter above all suspicion when software problems occurred. With the standard formulation, the filter would always have been suspect because of its propensity to diverge or develop negative eigenvalues, which could cause very peculiar transients in the-

navigation estimate.

It is rare in the development of a system such as RAINPAL that the opportunity for early validation is possible. The RAINPAL validation (refs. 24-26) was provided by the theodolite tracking system at WSHR, which is nationally recognized for its accuracy. The flight-test results showed that with only three range measurements versus seven theodolites, on the final approach to touchdown the RAINPAL system gave position and velocity estimates that were smoother and had accuracies of the same order as those of the WSHR system. Obviously, the RAINPAL system could be operated as an independent reference system against which other systems could be tested or evaluated.

Since the flight test of the RAINPAL system in January 1972, Dr. Schmidt has designed several other square-root-filter applications for aircraft system (ref. 27), including systems for ARC, which were flight tested on TOL aircraft and on helicopters. The same basic square-root approach developed for the RAINPAL system was used in these filters. The early versions of these filters used an experimental microwave landing system (HLS) called MODILS as a primary landing aid (refs. 28-31). The later versions used a prototype MLS and were flight tested on a helicopter (refs. 32 and 33).

Other Airborne Applications of a Square-Root Filter

During the time the RAINPAL system was being readied for flight test at WSMR, other researchers we e also developing square-root Kalman filters for airborne applications. One of the best examples of this work was done by Intermetrics, Inc. for the Completely Integrated Reference Instrumentation System (CIRIS) under development at Hollon Air Force Base. This system was very similar to the RAINPAL system in that it employed inertial aiding from an inertial navigation system and also employed precision measurements to transponders many miles apart on the ground. The precision ranging system was a Cubic CR-100 system, which produced both range and range rate from the aircraft to the ground transponder. This was a later and much improved version of the Cubic system used by the RAINPAL system, which provided only precision range from the aircraft to the ground transponder system. Details of this work were published in 1973 by Widnall (ref. 34), who described simulation results for a new square-root algorithm devised by Carlson (ref, 35). Carlson's algorithm maintained the covariance square-root matrix in triangular form, but, more importantly, reduced the computation time from that of the Householder/Potter method. This was a significant step forward, because for a moderate number of states, Carlson's method would approach the speed of the standard extended Kalman-filter formulation. This improvement in speed substantially overcame one of the most serious complaints leveled against the square-root formulation.

The Square-Root Filter Successfully Tested in Space

In 1975 Bierman (ref. 36) introduced his U-D factorization of the Kalman filter which appears to be the most efficient square-root method to be developed up-to-date; it uses only slightly more computer time than the standard extended Kalman filter. (It should be noted that the U-D factorizatio1; is not actually a square-root method in that three matrix products are required to define the covariance matrix. With this algorithm, there is no requirement for square-root operations.) This work, carried out at JPL draws on the work of Dyer and McReynolds in 1969 (ref. 37) whose filter algorithms were successfully used for the navigational computations on the Mariner 10 Venus-Mercury space mission in 1973. This success further established the reliability of the sequential square-root filter concept for real-time operations. Thus, except for the intuitive familiarity provided by the standard extended Kalman filter; there seem to be ample reasons for using the square-root formulation in all future applications.

Two different types of square-root filters have been developed. The first may be regarded as a factorization of the standard extended Kalman filter algorithm; it basically leads to the square root of the error covariance matrix. The

second involves the square root of the information matrix, which is defined as the inverse of the error covariance matrix. Each of these algorithms has attributes to recommend it for certain applications.

CONCLUDING REMARKS

It is clear that the relatively simple and straightforward sequential, extended Kalman filter used by the authors in the early 1960's can be adequate as a fundamental analytical tool for solving some estimation problems. Certainly, persons attempting to construct a Kalman filter without experience would be tempted to take this route because of the intuitive statistical familiarity. However, success or failure with this approach may be dependent on many factors, such as computer round- off errors, inadequate statistical models and nonlinearities in the problem, any or all of which may trigger the filter's potential instability or inaccuracy. Some rather pragmatic yet effective solutions to some of these problems have been devised, as we have seen in the foregoing history. Which of the pragmatic solutions to use often depends on correctly identifying the problem and applying the proper amount of ad hoc stabilization or "fix". If the filter is being run on a large machine, extra precision arithmetic may be a reliable solution to the problem of numerical instability, but stability problems may still remain because of mismodeling. In the case of an airborne application, extra precision is often not a practical alternative.

It appears that those experienced in applying Kalman filters to real-world problems are abandoning the ad hoc stabilization techniques and the standard filter formulation in favor of algorithms that are numerically better conditioned. The square-root filter, by its nature, is inherently more stable and better conditioned than the standard extended Kalman-filter formulation.

Square-root algorithms have gained acceptance rather slowly despite their superior performance as reported, for example L (refs. 24, 25, 32-34). Some of the reluctance of potential users probably stems from the fact that the early algorithms ran slower than the standard Kalman formulation, and the factorization techniques appeared too complicated and used too much computer storage as well as computer time.

As mentioned earlier, the Kalman filter has been used in a variety of fields. Recently, a special issue of the IEEE Transactions on Automatic Control (ref. 38) was devoted to papers on applications that were as wide ranging as possible in their subject matter. The papers cover such diverse subjects as spacecraft orbit determination, prediction of cattle populations in France, radar tracking, navigation, ship motion, natural gamma ray spectroscopy in oil- and gas-·well exploration, measurement of instantaneous flow rates, and estimation and prediction of unmeasurable variables in industrial processes, on-line failure detection in nuclear plant instrumentation, and power station control systems. In many cases, the solutions in these papers were implemented and were operationally successful. Indeed, the broad application of the filter to seemingly unlikely problems suggests that we have only scratched the surface when it comes to possible applications, and that we will likely be amazed at the applications to which this filter will be put in the years to come.

This history of Kalman filtering has naturally dwelt on the events that took place at RC, LMSC, Philco-WDL, and Analytical Mechanics Associates, Inc. The recording of those events is almost totally dependent on the recollections of and personal contacts made by the authors. Omissions in this work, of which there are undoubtedly some, should be attributed to our ignorance of some pertinent events, failures of memory, the inevitable limitations of time, and to the mushrooming publication of papers about the Kalman filter which has taken place since the early 1960's. For those interested in further details, the authors counsel a perusal of the various references cited herein and of the many other papers they in turn are certain to suggest.

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