# Random Variates Documentation and Testing

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## 1 RandomVariates

**A library for generating random values from probability distributions**

### 1.1 Introduction

RandomVariates is a library of random variate generation routines. The purpose behind this library was purely for educational purposes as a way to learn how to generate random variates using such methods as inverse transform, convolution, acceptance-rejection and composition methods.

Many of routines in this library are based off the lecture notes taken from Georgia Tech's ISYE664 as well as from the fifth edition of Simulation Modeling and Analysis - Law (2015).

Additionally, this project was an excuse to become familiar with random number generators such as linear congruential generators, Tausworthe Generators and Widynski's "Squares: A Fast Counter-Based RNG".

Finally, instead of implementing the acceptance-rejection algorithms for generating gamma variates described by Law (2015), it was decided to implement Marsaglia and Tsang's transformation-rejection method of using one normal variate and one uniform variate (Marsaglia and Tsang's 2002: https://dl.acm.org/doi/10.1145/358407.358414)

### 1.2 Pseudo Random Number Generators

The following pseudo random number (PRN) generators are contained in this project:

- A basic "desert island" linear congruential generator (implemented in the uniform function)
- taus() and tausunif(): A basic Tausworthe PRN generator and a Tausworthe Uniform PRN generator
- squaresrng(): Widynski's "Squares: A Fast Counter-Based RNG"
  - https://arxiv.org/pdf/2004.06278.pdf

### 1.2.1 Helper functions

The RandomVariates library contains various helper functions to take advantage of the PRN generators. These include:

- **randseed()**: Helper function to grab a "smaller" PRN from the Widynski squares PRN generator. By default the PRNs generated by the squaresrng() PRN generator tend to be extremely large. The randseed() function takes these extremely large PRNs and returns mod(100000000000).

- **generateseed()**: Helper function to generate random seeds if the initial seed has not been set. The generateseed() function calls randseed(), the adds 16807 and return mod(2^31 - 1). If the user sets a seed value, we add 16807, take the mod(2^31 - 1) then raise the result by pi. It was discovered that small seed values tended to create "RANDU"-like behavior.
- **set_seed()** and **get_seed()**: Functions to get and set the seed.
- **reverse()**: Helper function to reverse an integer. The function randseed() generates two system date timestamps in nanoseconds. One timestamp is fed into the squaresrng() function as the key and the second timestamp is reversed via the reverse() function and passed into squaresrng() as the center.

## 1.3 Random Variate Generation Routines

**The following is a summary of the random variate generation routines included in this library**

- **uniform()**: Routine to generate uniform random values between a and b: **Default uniform(a=0, b=1)**
- **norm()**: Method to generate random normals: **Default norm(mu=0, sd=1)**
- **exponential()**: Function to generate exponential random variates: **Default exponential(lam=1)**
- **erlang()**: Routine to generate Erlang_k(lambda) random values: **Default erlang(lam=1, k=1, n=1)**
- **weibull()**: Method to generate weibull random variates: **Default weibull(lam=1, beta=1)**
- **triangular()**: Function to generate triangular random values with **a** lower bound, **b** mode and **c** upper bound: **Default triangular(a=0, b=1, c=2)**
- **bernoulli()**: Function to generate bernoulli random variates: **Default bernoulli(p=0.5)**
- **binomial()**: Routine to generate binomial random values: **Default binomial(t=1, p=0.5)**
- **dicetoss()**: Simple/fun method to generate X-sides dice toss: **Default is a simple 6-sided dicetoss(sides=6)**
- **geometric()**: Method to generate geometric random values: **Default geometric(p=0.5)**
- **negbin()**: Routine to generate discrete random negative binomials: **Default negbin(t=1, p=0.5)**
- **chisq()**: Generate Chi-squared random variates: **Default chisq(df=1)**

- **poisson()**: Method to generate Poisson random variates: **Default poisson(lam=1)**
- **gamma()**: Function to generate gamma random variates with shape parameter k and a scale parameter . Implementation is based on Marsaglia and Tsang's transformation-rejection method of generating gamma random variates (https://dl.acm.org/doi/10.1145/358407.358414): **Default gamma(k=1.0, theta=1)**
- **lognormal()**: Routine to generate lognormal random variates: **Default lognormal(mu=0, sd=1)**
- **beta()**: Routine to generate beta random values: **Default beta(a=1, b=1)**

### 1.3.1 Limitations

- Unlike Numpy's random variate generation routines, these are written in python. Numpy's random routines are written in C hence are much, much faster. Note that Numpy was used in this library to help speed up vector calculations but the Numpy random variate routines were not used.
- Beta and Gamma distributions only accept a, b, k and theta greater than one. Other random variate implementations, such as Numpy can handle values between 0 and 1.
- Setting the seed does not affect the Tausworthe and Tausworthe Uniform PRN generators

### 1.3.2 Distributions not currently implemented

- Pearson Type V
- Pearson Type VI
- Log-Logistic
- Johnson Bounded and Johnson unbounded
- Bézier
- Others …

## 1.4 Installation

### 1.4.1 Requirements:

- **Python 3.x**
- **pip** (https://pip.pypa.io/en/stable/installation/)
- **numpy**: If numpy is not installed, the pip command below will automatically install numpy.

To install the library, simply run the command:

- **pip install randvars**

The pip package can be located here: https://pypi.org/project/randvars/

Source code can be located here: https://github.com/jgoodie/randomvariates

[1]:
```
import numpy as np
from scipy import stats
import statsmodels.api as sm
from collections import defaultdict
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D
```

```
%matplotlib inline
```

## 1.5 Usage

To use the RandomVariates library, you need to import the library into your python script then create an instance of RandomVariates:

[2]:
```
import randomvariates
rv = randomvariates.random.RandomVariates()
```

Alternately, you can import random from randomvariates:

[2]:
```
from randomvariates import random
rv = random.RandomVariates()
```

### 1.5.1 Seeds

By default, when an instance of randomvariates is created, a seed is not set (None). When a seed is set to None, randomvariates will randomly generate values for thevarious random variate routines.

For repeatability, we can set a seed by calling the *set_seed()* method. Once a seed has been set, we can verify by calling the *get_seed()* method.

[3]:
```
rv.set_seed(42)
rv.get_seed()
```

[3]:
```
42
```

### 1.5.2 Pseudo Random Number Generators

**Widynski Squares PRN** To call the Widynski Squares PRN we can call the **squaresrng()** method.

The **squaresrng()** method takes a center and key value. By default, the center and key are set to 1:

**squaresrng(ctr=1, key=1)**

[7]:
```
rv.squaresrng()
```

[7]:
```
433229639706377321950039771238786822355045914855571895475319001318162846036062628
349773053565655627551024270771116047099536343O4
```

To set a different center and key value, simply pass them into the function:

[9]:
```
rv.squaresrng(ctr=42,key=24)
```

[9]:
```
193772339616808025565989058346213049264286127727251181977890903946937070895370548
883125079311556190025146894321870665294602973438363900432540933891418338358617258
905971982336
```

**Tausworthe PRN generators** As of version 0.0.17, the Tausworthe PRN and Tausworthe Uniform PRN generator does not take a seed value (See Limitations above)

To call the Tausworthe generators, simply call **taus()** and **tausunif()**.

By default **taus()** will generate 100 binary PRNs and **tausunif()** will generate a single uniform(0,1):

[10]:
```
# rv.taus(n=100)
rv.taus()
```

[10]:
```
array([0, 0, 0, 1, 0, 0, 0, 1, 0, 1, 1, 1, 0, 0, 1, 0, 0, 1, 1, 0, 0, 0,
       0, 0, 0, 1, 1, 1, 0, 0, 0, 1, 0, 0, 1, 0, 1, 1, 1, 1, 1, 0, 1, 1,
       0, 0, 0, 1, 1, 1, 0, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 0, 1, 0, 0,
       0, 0, 0, 1, 0, 1, 1, 0, 0, 1, 1, 1, 1, 1, 0, 0, 1, 1, 0, 0, 0, 0,
       0, 1, 0, 0, 1, 0, 0, 0, 0, 1, 1, 1])
```

[11]:
```
# rv.tausunif(n=1)
rv.tausunif()
```

[11]:
```
array([0.19396465])
```

To generate more than one tausunif() simply pass in the number of uniforms you wish to generate:

[14]:
```
rv.tausunif(n=25)
```

[14]:
```
array([0.86916423, 0.89400616, 0.04833898, 0.92461759, 0.99822675,
       0.61682276, 0.36837539, 0.25302119, 0.4327886 , 0.18738917,
       0.35105142, 0.77302346, 0.95331382, 0.15204929, 0.38671182,
       0.39694071, 0.98581397, 0.93458211, 0.94700308, 0.02416949,
       0.46230879, 0.49911337, 0.80841138, 0.18418769, 0.62651059])
```

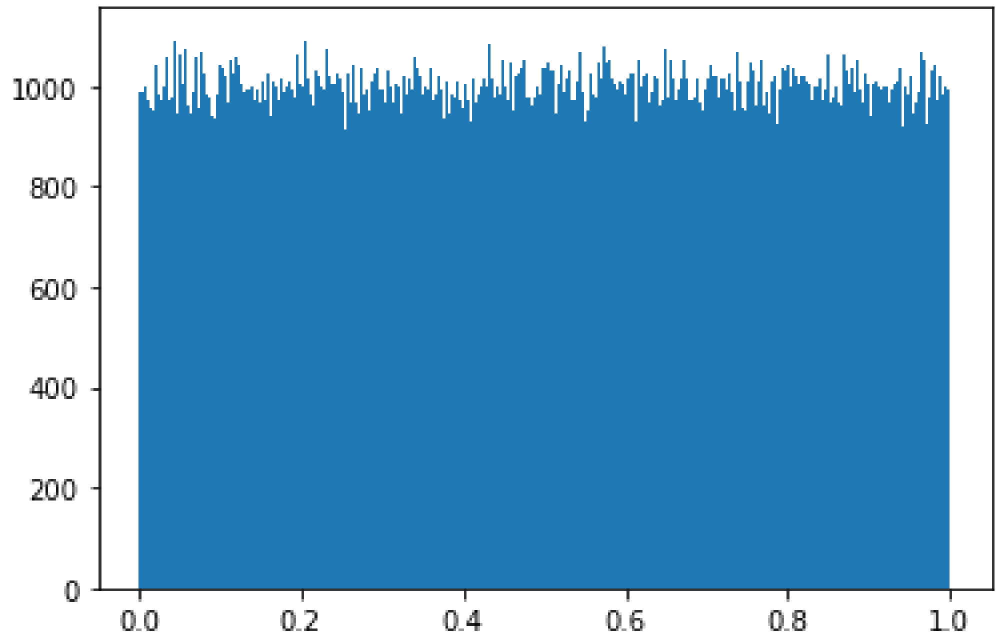
**Let's look at the Tausworthe uniform values to see what we get for a mean and variance:**

[40]:
```
unifs = rv.tausunif(n=1000000)
print(np.mean(unifs))
print(np.var(unifs)) # 1/12 = 0.0833333333
```

```
0.4999426194461891
0.08330764392837407
```

**Plot of the Tausworthe generated uniforms.**

[41]:
```
plt.hist(unifs, bins=1000)
plt.show()
```




**Chi-squared goodness of fit for the Tausworthe generated uniforms**

Note the test statistic of 3.22586 and p-value of 0.91939. As a sanity check, the

Chi-squared with df=9 at an alpha of 0.05 we get 16.919. From the output below we it's obvious that 3.22586 is less than 16.919 which makes sense given our huge p-value of 0.91939. **In this case we fail to reject H0.**

https://people.richland.edu/james/lecture/m170/tbl-chi.html

[42]:
```
n=10
unifs = rv.tausunif(n=n)
exp = np.ones(n)*0.5
stats.chisquare(f_obs=unifs, f_exp=exp, ddof=1, axis=0)
```

[42]:
```
Power_divergenceResult(statistic=3.225862656076932, pvalue=0.9193941504421734)
```

**Linear Congruential Generator (LCG) and Uniform Distribution** The primary Uniform PRN generator for the randomvariates library is based off a "desert island" LCG of the form:

$$X_i = 16807X_{i-1}mod(2^{32} - 1) \tag{1}$$

**To call the uniform PRN generator simply call uniform() method:**

[4]:
```
rv.uniform()
```

[4]: array([0.05257237])

**To generate more than one unif(0,1), call the method with n=X where X is the number of unif(0,1)s to generate:**

[5]:
```python
rv.uniform(n=25)
```

[5]:
```
array([0.05257237, 0.58375326, 0.14097247, 0.32435023, 0.35433263,
       0.26846135, 0.0299879 , 0.00658945, 0.74895567, 0.69787797,
       0.23502476, 0.06106509, 0.32097407, 0.61113629, 0.36767406,
       0.49790078, 0.21841041, 0.82377019, 0.10552924, 0.62995875,
       0.71667283, 0.12018592, 0.96479814, 0.36233659, 0.79098412])
```

**If we want to generate something other than unif(0,1), we can call the function with a=X and b=Y where X and Y are the lower and upper bounds of the uniform distribution:**

[6]:
```python
rv.uniform(a=7, b=11, n=25)
```

[6]:
```
array([ 7.21028946,  9.33501302,  7.56388989,  8.29740092,  8.41733051,
        8.07384542,  7.11995159,  7.02635782,  9.99582266,  9.79151188,
        7.94009902,  7.24426036,  8.28389627,  9.44454517,  8.47069623,
        8.99160312,  7.87364164, 10.29508075,  7.42211697,  9.51983499,
        9.8666913 ,  7.48074369, 10.85919256,  8.44934634, 10.16393646])
```

**Again, as we did with the Tausworthe uniforms, let's check the mean and variance:**

Note: Our "desert island" generator seems to be much better than the Tausworthe version. When testing the Tausworthe and the LCG uniform generators, it was observed that the LCG was more consistent in it's generation of uniform values while the Tausworthe generator didn't seem as consistent. It's also important to note that this was a "gut" feeling and the results from chi-squared goodness of fit tests for the Tausworthe and LCG always tended to return large p-values in favor of the null hypothesis of uniformity.

[7]:
```python
unifs = rv.uniform(n=1000000)
print(np.mean(unifs)) # 0.5
print(np.var(unifs)) # 1/12 = 0.0833333333
```

```
0.49926189331773324
0.08334189169747183
```

**Check for uniformity by running a goodness of fit test**

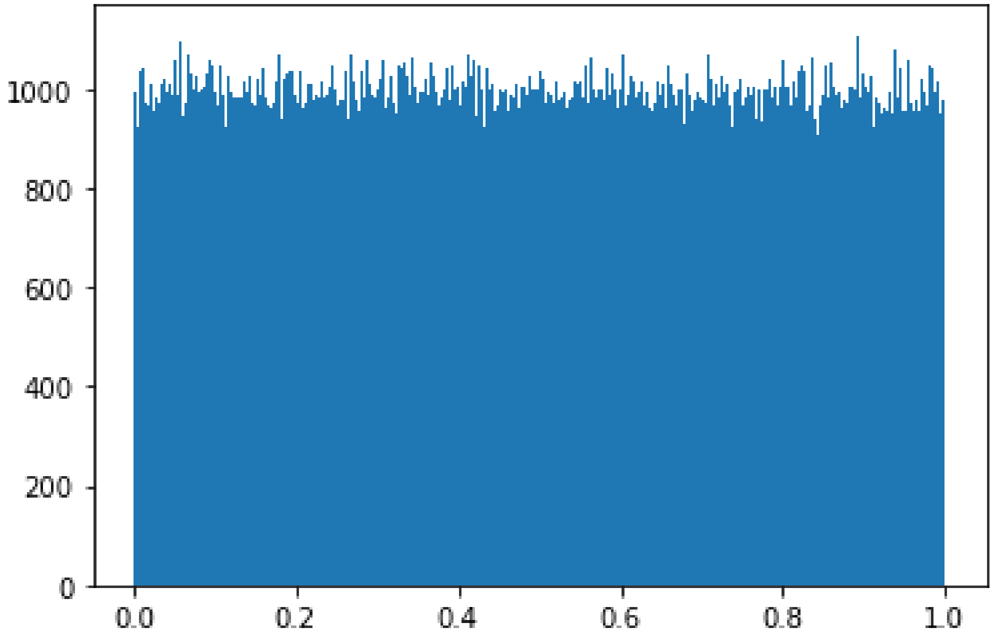
Note the large p-value. We fail to reject H0, there's not enough evidence to say these aren't uniform

[8]:
```python
n=10
unifs = rv.uniform(n=n)
exp = np.ones(n)*0.5
stats.chisquare(f_obs=unifs, f_exp=exp, ddof=1, axis=0)
```

[8]: Power_divergenceResult(statistic=2.0145775757113675, pvalue=0.9805616826416578)

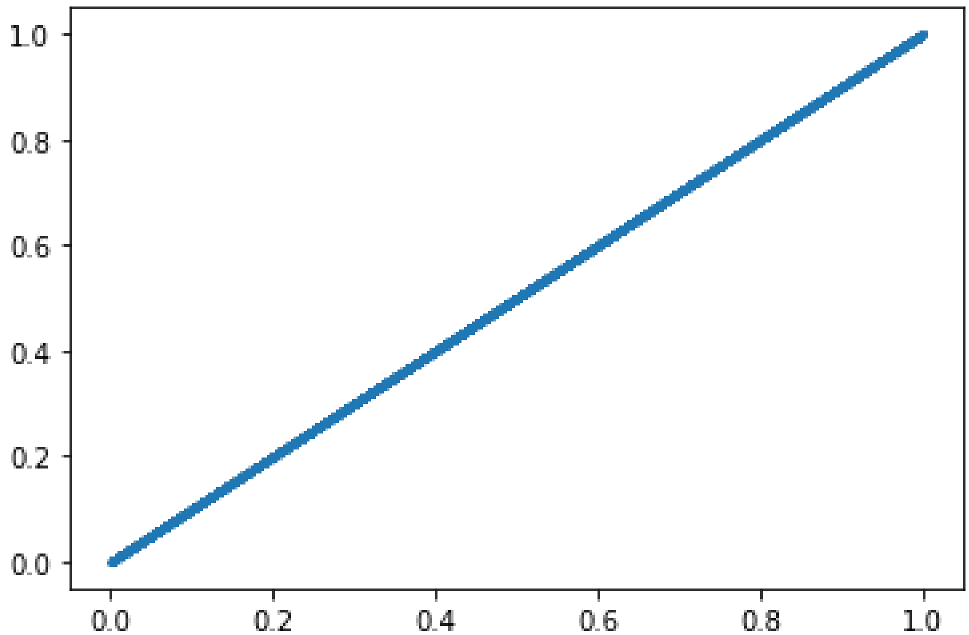
**Again, let's plot the uniforms to see what they look like**

[9]:
```
unifs = rv.uniform(n=1000000)
plt.hist(unifs, bins=1000)
plt.show()
```




**Note when we generate two sets of uniforms with the same seed and plot them against each other, we get a straight line**

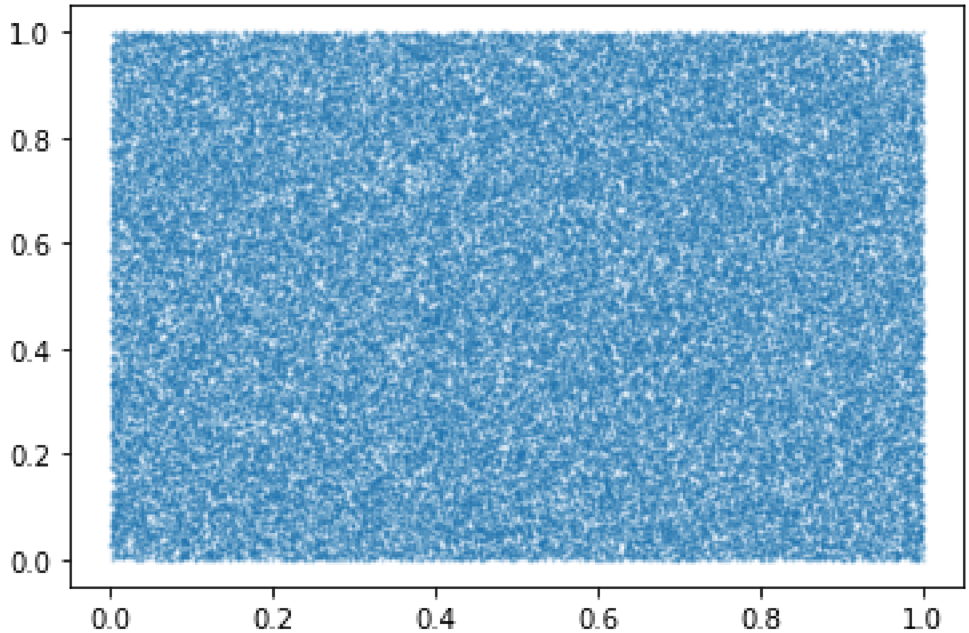
[10]:
```
x = rv.uniform(n=100000)
y = rv.uniform(n=100000)

plt.scatter(x, y, s=0.8, alpha=0.2)
plt.show()
```




**Let's try again with each uniform having a different seed**

Note how the image below looks like TV static.

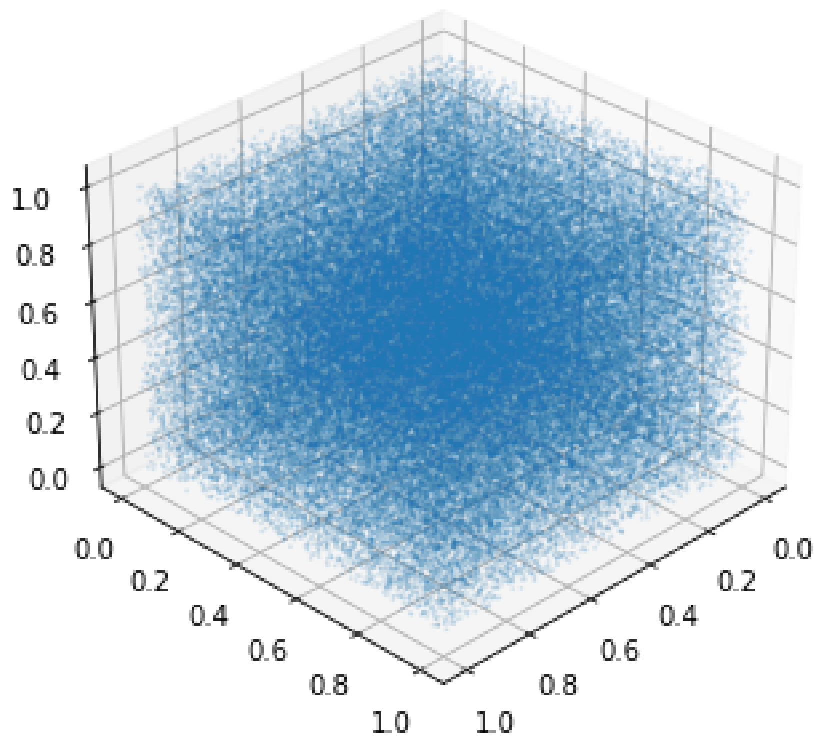
[11]:
```
rv.set_seed(0)
x = rv.uniform(n=100000)
rv.set_seed(1)
y = rv.uniform(n=100000)

plt.scatter(x, y, s=0.8, alpha=0.2)
plt.show()
```




**To make sure we don't get any "RANDU" effects, let's create a 3-D plot**

Notice how the image below looks like a big static square.

[12]:
```
x = random.RandomVariates()
x.set_seed(1*3.141)

y = random.RandomVariates()
y.set_seed(2*3.141)

z = random.RandomVariates()
z.set_seed(3*3.141)

fig = plt.figure()
ax = Axes3D(fig, auto_add_to_figure=False, azim=45)
fig.add_axes(ax)

sequence_containing_x_vals = x.uniform(n=100000)
sequence_containing_y_vals = y.uniform(n=100000)
sequence_containing_z_vals = z.uniform(n=100000)

ax.scatter(sequence_containing_x_vals, sequence_containing_y_vals,␣
 ↪sequence_containing_z_vals, s=0.8, alpha=0.1)
plt.show()
```




### 1.5.3 Normal Distribution

To generate random normal random variates, call the **norm()** function. By default, the **norm()** function will generate values with mean = 0 and standard deviation = 1.

[13]:
```
rv.set_seed(42) # Set our seed back to 42
rv.norm(n=25)
```

[13]:
```
array([-1.41895603,  1.03055739,  0.64797237, -0.54395554,  1.41565353,
       -1.58770622, -2.02602564, -0.29023875,  0.71303646, -0.09758493,
        0.67348912, -1.57582437,  0.2270985 , -0.10001884,  0.41797698,
       -0.99798613, -1.73261847,  0.53073999,  1.78233654,  0.94442933,
       -0.38071356, -0.51034473,  0.22164846, -0.66787692, -0.32286354])
```

**To generate normals with other means and standard deviations, simply specify them when calling the function:**

[14]:
```
rv.norm(mu=42, sd=21, n=25)
```

[14]:
```
array([12.20192344, 63.64170513, 55.60741974, 30.57693373, 71.72872421,
        8.65816942, -0.54653844, 35.90498633, 56.97376558, 39.95071654,
       56.14327153,  8.90768817, 46.76906844, 39.89960435, 50.77751655,
```

```
       21.04229136,  5.61501208, 53.1455398 , 79.42906729, 61.83301601,
       34.00501532, 31.28276068, 46.65461775, 27.97458477, 35.21986557])
```

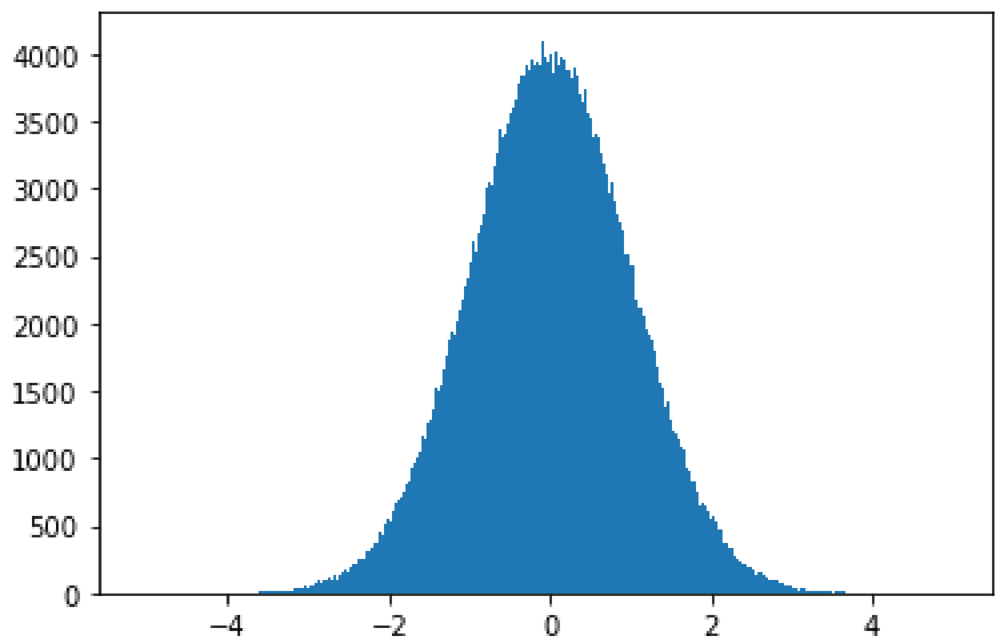
**To check that our Nor(0,1) are generating a mean of 0 and variance of 1**

[15]:
```python
z = rv.norm(n=1000000)
mean = np.mean(z)
var = np.var(z)
print(f"mean: {mean}")
print(f"var: {var}")
```

```
mean: -0.0010792786142373445
var: 1.0020209597387706
```

**Do our normal random values actually look normal?**

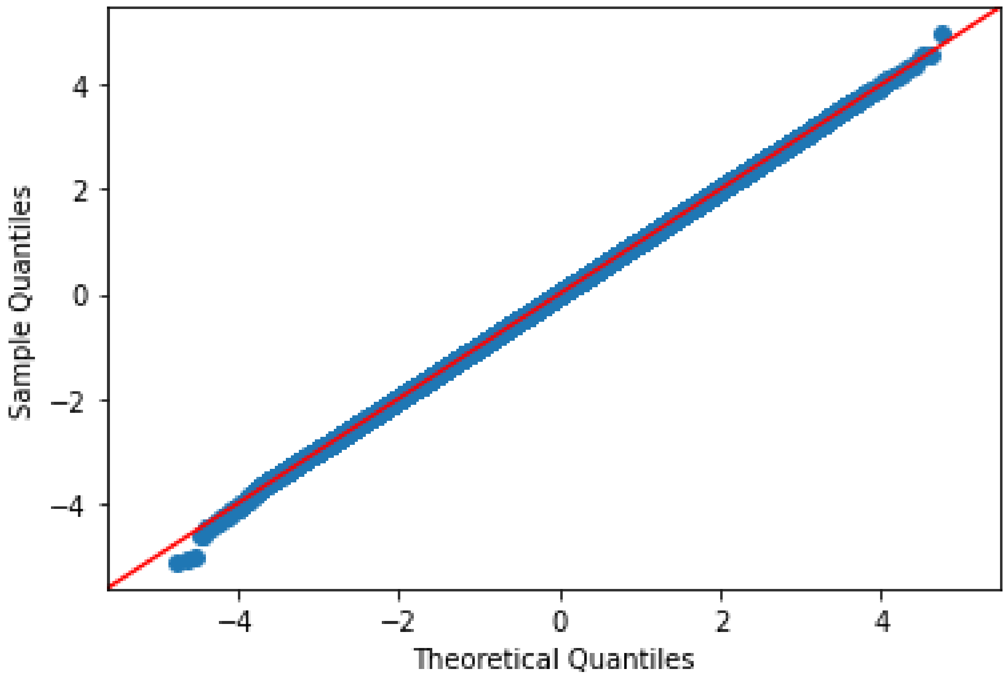
[16]:
```python
z = rv.norm(mu=0, sd=1, n=1000000)
plt.hist(z, bins=1000)
plt.show()
```




**Let's see what the Q-Q plot looks like:**

Note that our random normals fall nicely on the 45 degree line.

[17]:
```python
z = rv.norm(mu=0, sd=1, n=1000000)
sm.qqplot(z, line='45');
```




**Finally let's run a Shapiro Wilk test for normality.**

Note the test statistic and p-value. Since the p-value is much greater that 0.05, we fail to reject the null hypothesis.

[18]:
```
z = rv.norm(mu=0, sd=1, n=25)
stats.shapiro(z)
```

[18]:
```
ShapiroResult(statistic=0.9745835065841675, pvalue=0.7614504098892212)
```

### 1.5.4 Exponential Random Variates

To generate exponential random values, we can call the the **exponential()** function.

By default, the **exponential()** function will generate a single, lambda=1 random variate.

[19]:
```
rv.exponential()
```

[19]:
```
array([0.05400472])
```

To generate exponentials with different rates (lambda), call the exponential function with lam=X:

[20]:
```
rv.exponential(lam=3, n=25)
```

[20]:
```
array([0.01800157, 0.29215902, 0.05065144, 0.13069348, 0.1458236 ,
       0.10420174, 0.01014891, 0.00220375, 0.46070858, 0.39897476,
       0.08930394, 0.02100304, 0.12903199, 0.31484212, 0.15278343,
```

```
0.22965251, 0.08214183, 0.57865546, 0.03717436, 0.33138026,
0.42038432, 0.04268156, 1.11555212, 0.14998157, 0.52178168])
```

**Check the mean and variance of our exponential random variates.**

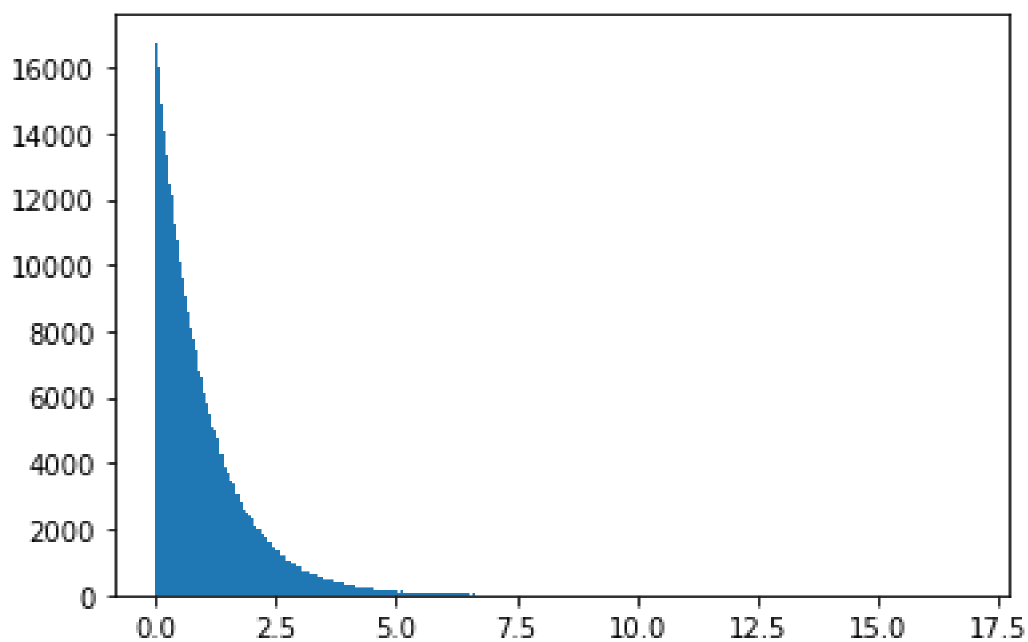
If lambda is 2, then we should see a mean of 1/2 and variance of 1/4

[21]:
```python
# expo mean = 1/lam
# expo var = 1/lam^2
e = rv.exponential(lam=2, n=1000000)
mean = np.mean(e)
var = np.var(e)
print(f'mean: {mean}') # 1/2
print(f'var: {var}') # 1/4
```

```
mean: 0.499091656691787
var: 0.25017014791880715
```

**Plot of Exp(1)**

[23]:
```python
e = rv.exponential(lam=1, n=1000000)
plt.hist(e, bins=1000)
plt.show()
```




**Chi-squared goodness-of-fit for Exponentials**

Note the mean of 9.00471853961017 for Exp(1/9)

[25]:
```
rv.set_seed(55)
e_obs = rv.exponential(lam=1/9, n=100)
mean = np.mean(e_obs)
var = np.var(e_obs)
print(f'mean: {mean}') # 9.00
```

```
mean: 9.00471853961017
```

Find the intervals for the chi-squared test

[27]:
```
# 0.2 = 1/5 or 5 intervals
for i in range(5):
    print(-9*np.log(1-0.2*i))
```

```
-0.0
2.0082919618278874
4.597430613893916
8.246616586867399
14.484941211906904
```

[28]:
```
intervals = defaultdict(int)
for e in e_obs:
    if 0.0 <= e < 2.01:
        intervals[1] += 1
    elif 2.01 <= e < 4.60:
        intervals[2] += 1
    elif 4.60 <= e < 8.25:
        intervals[3] += 1
    elif 8.25 <= e < 14.48:
        intervals[4] += 1
    else:
        intervals[5] += 1
```

$$X^2_{0.05,3} = 7.81 \tag{2}$$

Since 0 (1.9) is less than 7.81 we fail to reject the null hypothesis

[29]:
```
Oi = np.array(list(intervals.values()))
Ei = np.ones(5)*(100/5)
x0 = np.sum(((Oi - Ei)**2)/Ei)
x0
```

[29]: 1.9

### 1.5.5 Erlang Random Variates

Random Erlang variates can be generated by calling the **erlang()** function.

By default, the erlang function will generate variates with lambda = 1 and shape (k) = 1:

[30]:
```
rv.erlang()
```

[30]:
```
array([2.90653443])
```

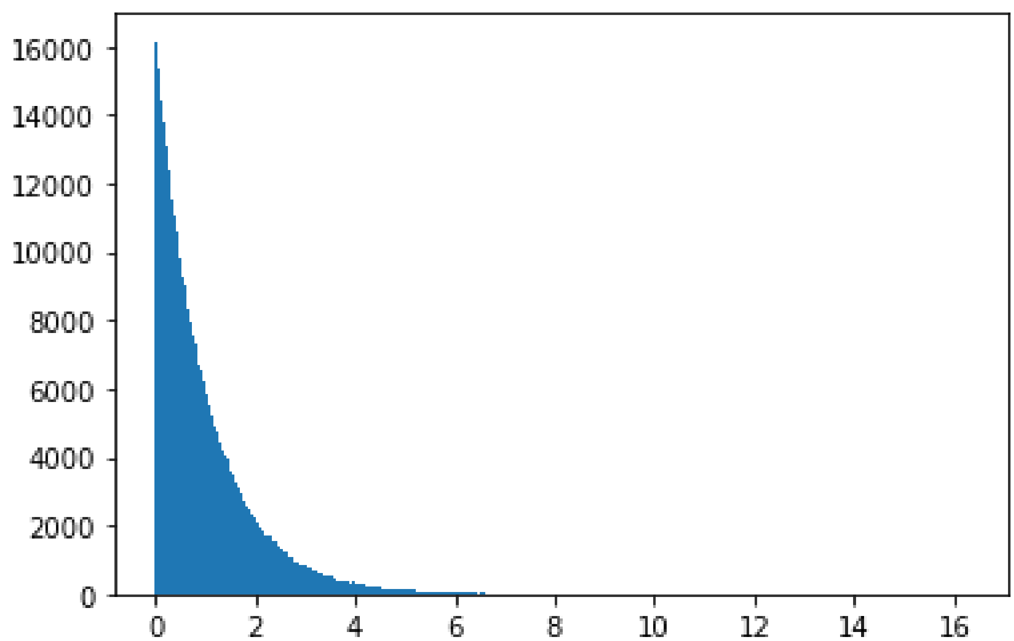
To generate erlangs with different rate and shape parameters, set lam=X and k = Y, where X is the lambda rate and Y is the shape:

[31]:
```
rv.erlang(lam=5, k=5, n=25)
```

[31]:
```
array([1.80165983, 1.58274981, 0.92871473, 0.52756431, 0.53273064,
       1.67822669, 1.45200378, 0.75477015, 1.05308769, 0.39410616,
       0.98431467, 1.02531066, 0.56406195, 0.64748447, 1.35281805,
       1.18196551, 1.29296929, 0.55254751, 0.46277568, 0.79741588,
       1.88896114, 1.33347742, 1.04105106, 1.36739384, 1.15008453])
```

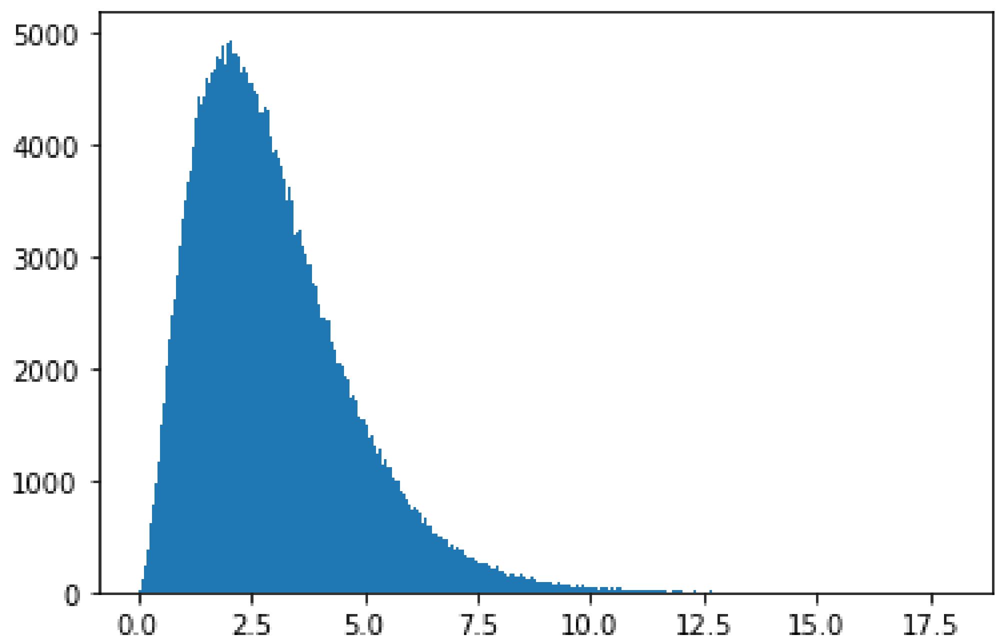
**Let's create some plots of our erlang random values**

[32]:
```
erls = rv.erlang(lam=1, k=1, n=1000000)
plt.hist(erls, bins=1000)
plt.show()
```




**Note the shape when we specify k = 3**

[33]:
```
erls = rv.erlang(lam=1, k=3, n=1000000)
plt.hist(erls, bins=1000)
plt.show()
```




### 1.5.6 Weibull Random Variates

To generate values from the Weibull distribution, call the **weibull()** method with lam and beta.

By default, lam and beta are set to 1:

weibull(lam=1, beta=1).

[4]:
```
rv.weibull()
```

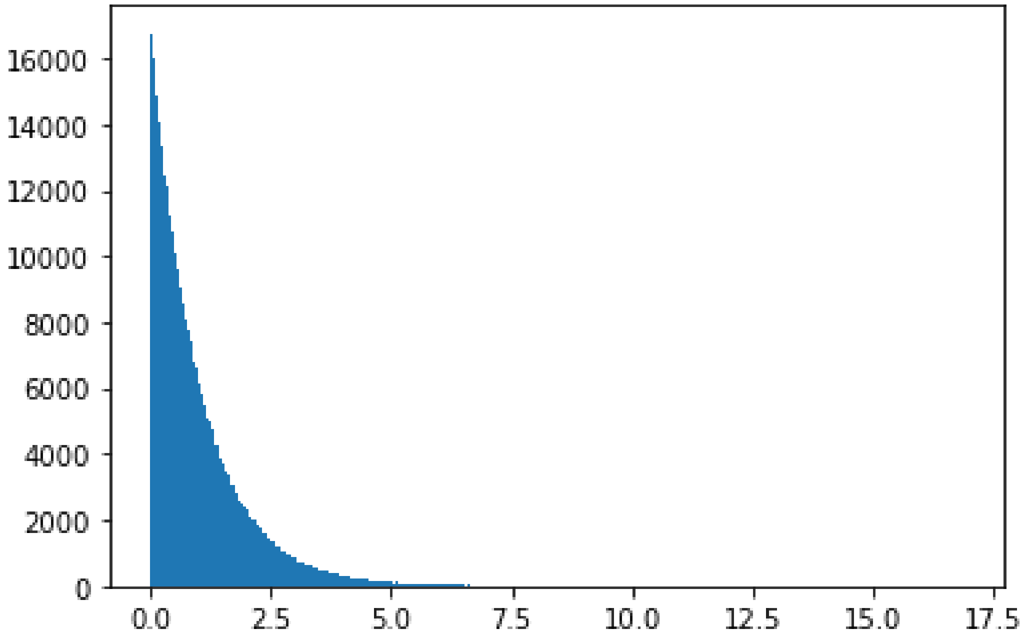
[4]:
```
array([0.05400472])
```

To generate weibull values with different lam (shape) and beta (scale), set lam and beta as such:

[5]:
```
rv.weibull(lam=3, beta=5, n=25)
```

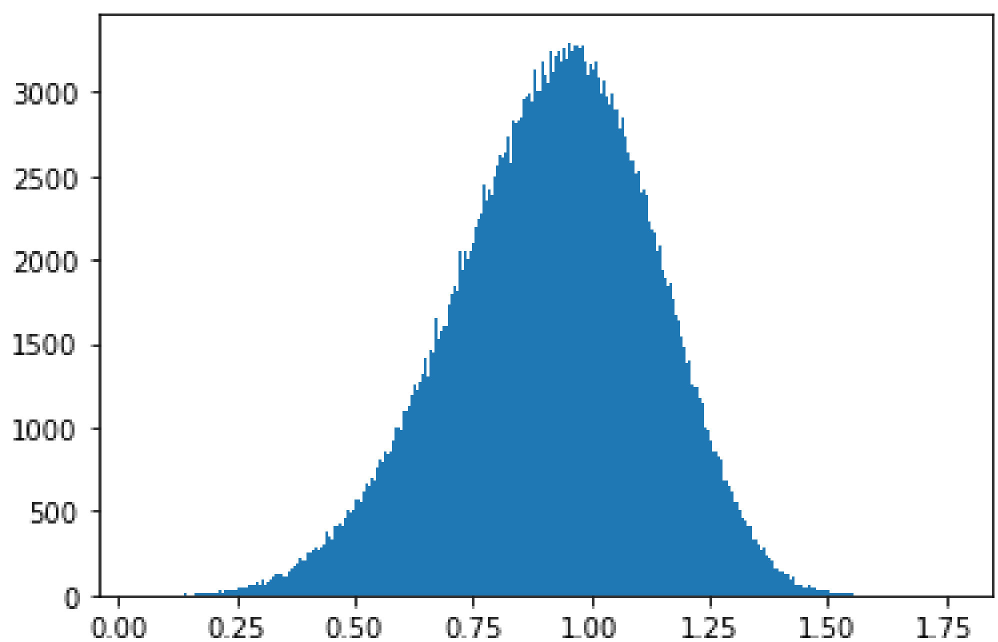
[5]:
```
array([0.18593669, 0.32465856, 0.2286764 , 0.27641002, 0.2825326 ,
       0.26416665, 0.1658009 , 0.12216185, 0.3556217 , 0.34553503,
       0.25613983, 0.19176062, 0.27570362, 0.32955019, 0.28517946,
       0.30939781, 0.25189287, 0.37220912, 0.21495705, 0.3329418 ,
       0.34916628, 0.22097902, 0.42442501, 0.28412574, 0.36458665])
```

[6]:
```
w = rv.weibull(lam=1,beta=1,n=1000000)
plt.hist(w, bins=1000)
plt.show()
```



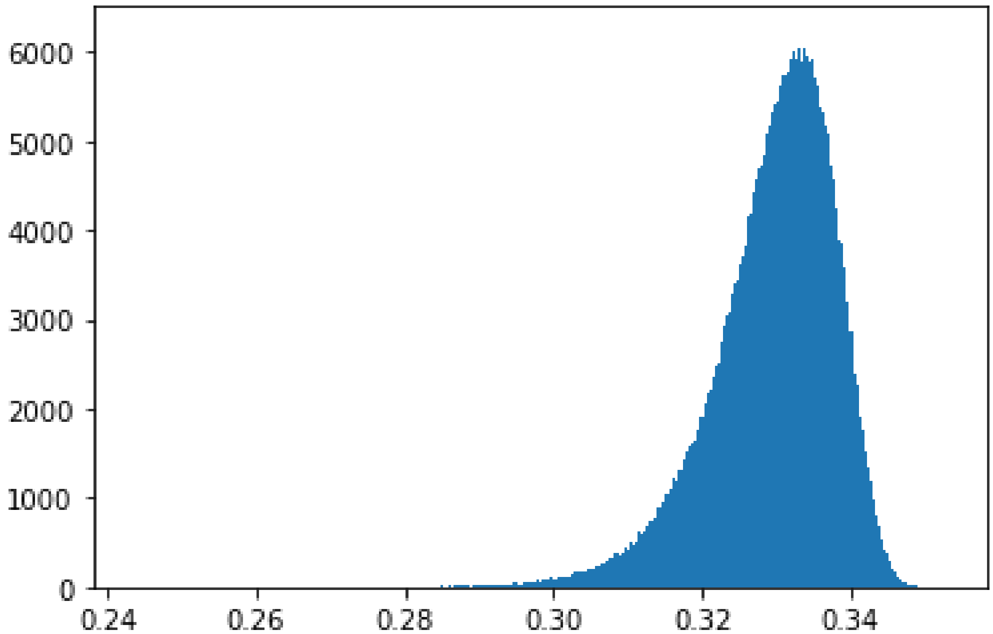

**Weibull with beta shape parameter set to 5**

[7]:
```
w = rv.weibull(lam=1,beta=5,n=1000000)
plt.hist(w, bins=1000)
plt.show()
```



**Weibull with lambda = 3 and beta set to 50**

[8]:
```
w = rv.weibull(lam=3,beta=50,n=1000000)
plt.hist(w, bins=1000)
plt.show()
```




### 1.5.7 Triangular Random Variates

By default, the randomvariates library will generate Triangular(0,1,2) values from a triangular distribution:

[10]:
```
rv.triangular()
```

[10]:
```
array([0.32426028])
```

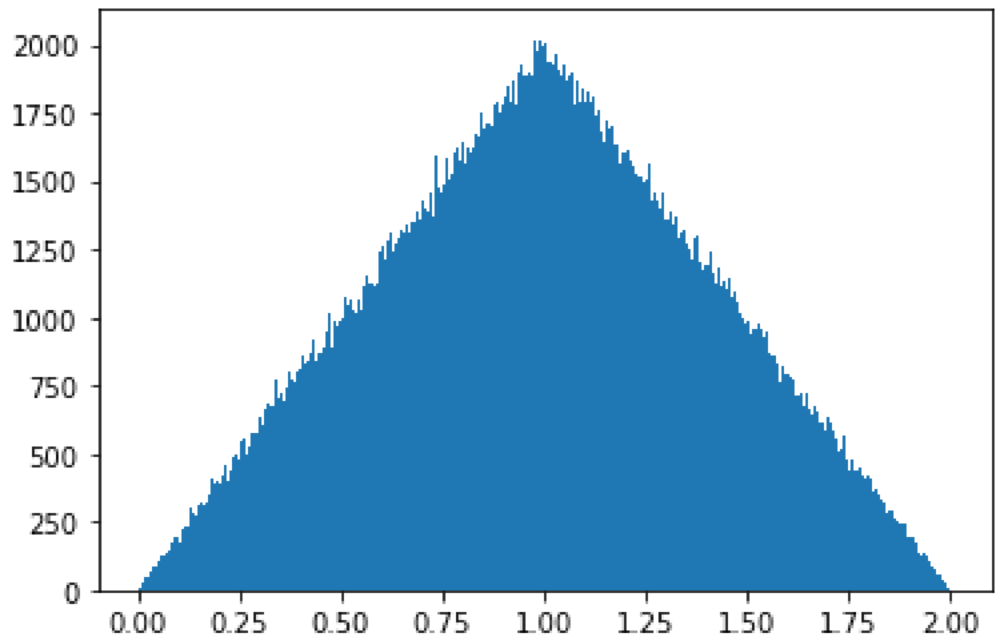
To generate values from a Triangular distribution with lower bound a, mode b and upper bound c, call the triangular() function with a, b, and c set:

[11]:
```
rv.triangular(a=-5, b=0, c=5, n=25)
```

[11]:
```
array([-3.37869858,  0.43794594, -2.34507559, -0.97290284, -0.79088711,
       -1.3362495 , -3.7755022 , -4.42600287,  1.45708923,  1.11334314,
       -1.57199216, -3.25264356, -0.99391671,  0.59055725, -0.71237794,
       -0.01050714, -1.69537891,  2.03158449, -2.70294491,  0.6985976 ,
        1.23617765, -2.54861343,  3.67331503, -0.74361312,  1.76723119])
```

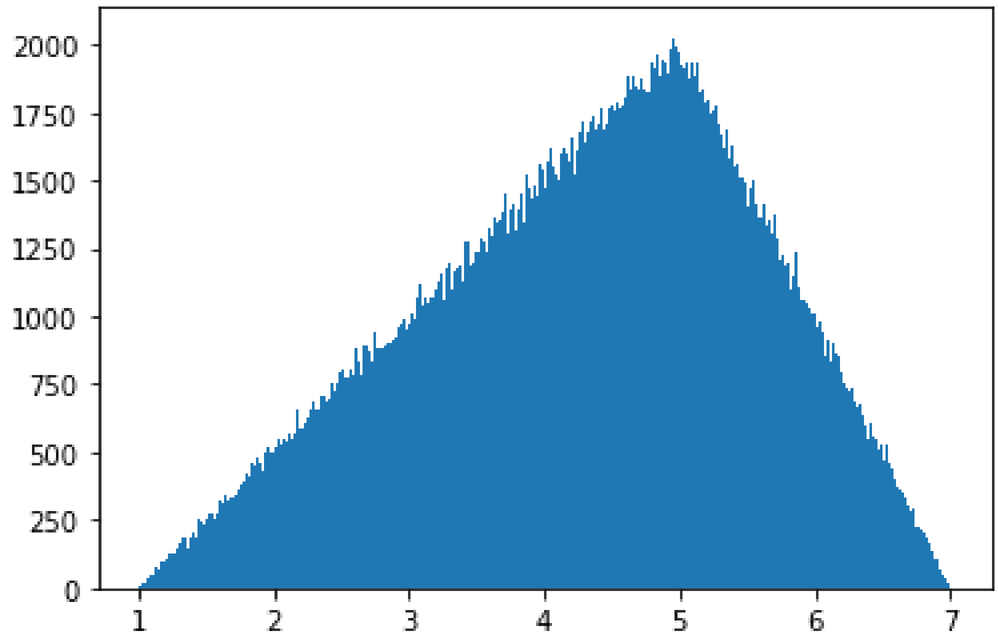
**Plot of Tri(0,1,2)**

[12]:
```
t = rv.triangular(a=0, b=1, c=2, n=1000000)
plt.hist(t, bins=1000)
plt.show()
```




**Plot of Tri(1,5,7)**

[13]:
```
t = rv.triangular(a=1, b=5, c=7, n=1000000)
plt.hist(t, bins=1000)
plt.show()
```




### 1.5.8 Bernoulli Random Values

To generate bernoulli(p) random values, call the **bernoulli()** method with probability, p. By default, the **bernoulli()** method generates bernoulli(0.5) random values.

[14]:
```
rv.bernoulli()
```

[14]:
```
array([0])
```

[15]:
```
rv.bernoulli(n=25)
```

[15]:
```
array([0, 1, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 1, 1, 0,
       1, 0, 1])
```

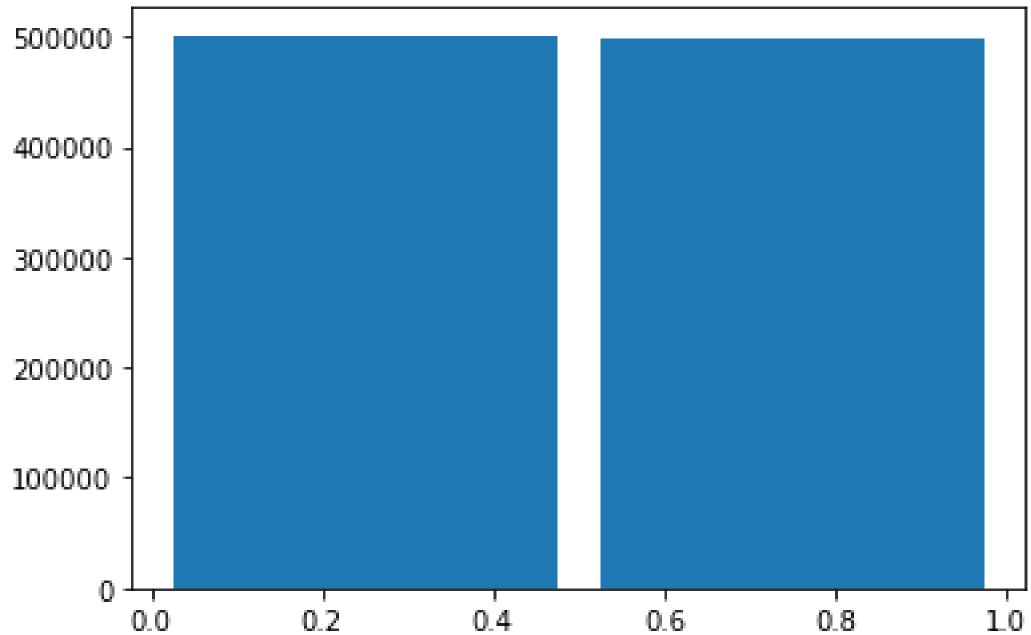
**To generate bernoulli(0.8) random values, set p=0.8:**

[16]:
```
rv.bernoulli(p=0.8, n=25)
```

[16]:
```
array([0, 1, 0, 1, 1, 1, 0, 0, 1, 1, 1, 0, 1, 1, 1, 1, 1, 1, 0, 1, 1, 0,
       1, 1, 1])
```

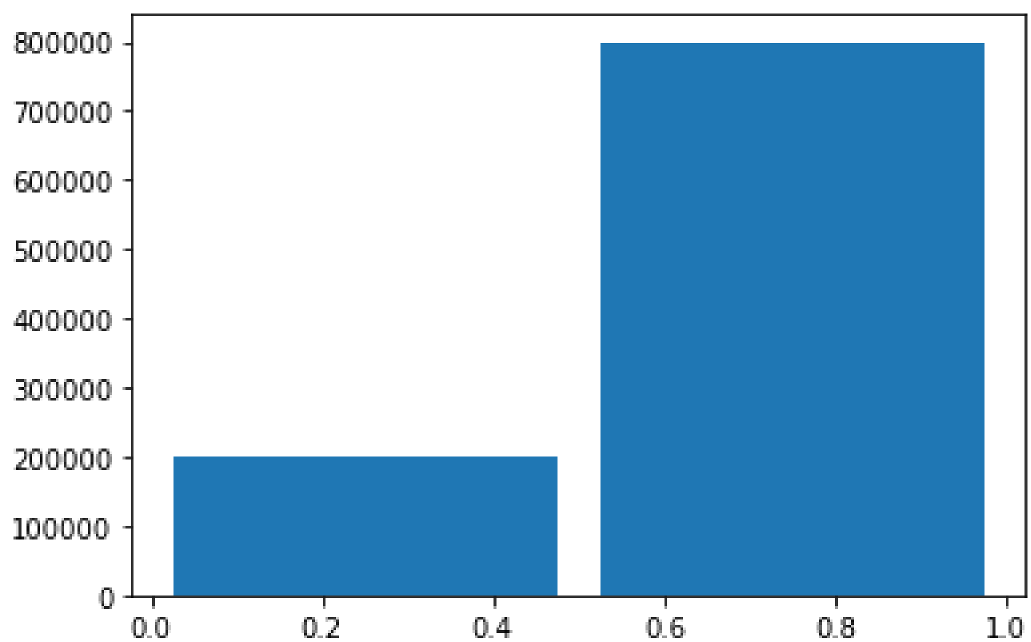
**Plot of bernoulli p=0.5**

[17]:
```
b = rv.bernoulli(p=0.5, n=1000000)
plt.hist(b, bins=2, rwidth=0.9)
plt.show()
```




**Plot of bernoulli p=0.8**

[18]:
```
b = rv.bernoulli(p=0.8, n=1000000)
plt.hist(b, bins=2, rwidth=0.9)
plt.show()
```



### 1.5.9 Binomial Random Variatates

Binomial(n,p) random values can be generated with the **binomial()** function. By default, the **binomial()** function generates 1 trial at p=0.5:

[19]:
```
rv.binomial()
```

[19]:
```
array([0])
```

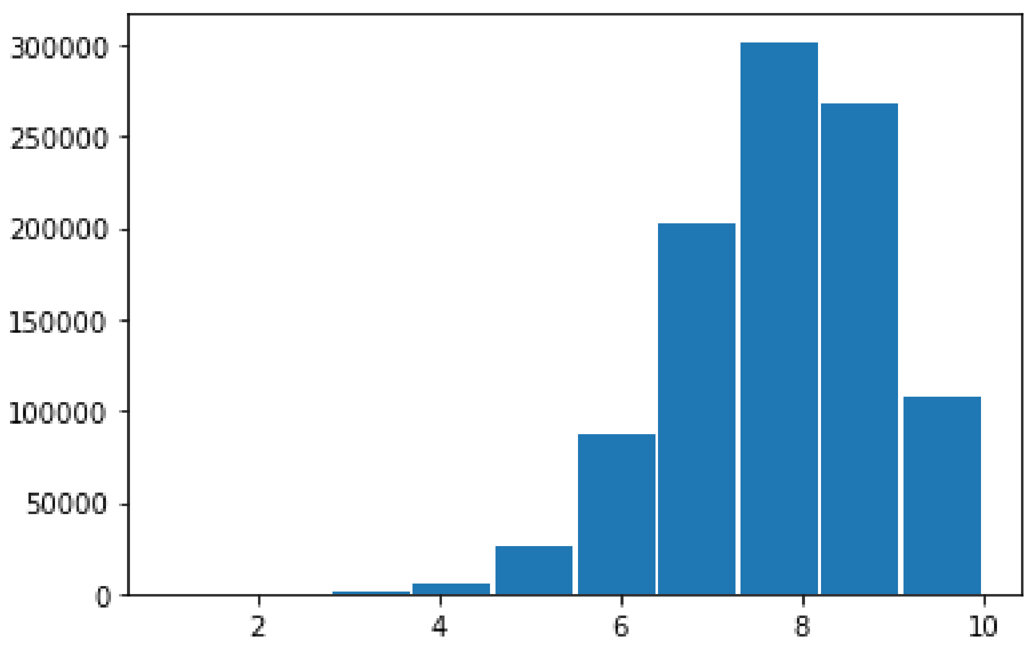
**Note: Don't confuse t=trials and n=number of values to generate. To generate 25 binomials with 10 trials, and probability 0.5, we would specify binomial(t=10, p=0.5, n=25):**

[20]:
```
rv.binomial(t=10, p=0.5, n=25)
```

[20]:
```
array([3, 7, 6, 8, 7, 5, 4, 5, 6, 6, 2, 6, 4, 5, 3, 3, 4, 7, 6, 3, 5, 2,
       6, 4, 6])
```

**Plot of binomial(10, 0.8)**

[21]:
```
b = rv.binomial(t=10, p=0.8, n=1000000)
plt.hist(b, bins=10, rwidth=0.95)
plt.show()
```



#### 1.5.10 Random X-sided Dice Toss

For the D&D fans, the **dicetoss()** function allows you to generate an X-sided die toss.

For example, to generate 10, 20-sided dice tosses, simply call the **dicetoss()** function.

By default, **dicetoss()** defaults to a 6-sided die:

[22]:
```
rv.dicetoss(n=10)
```

[22]:
```
array([1., 4., 1., 2., 3., 2., 1., 1., 5., 5.])
```

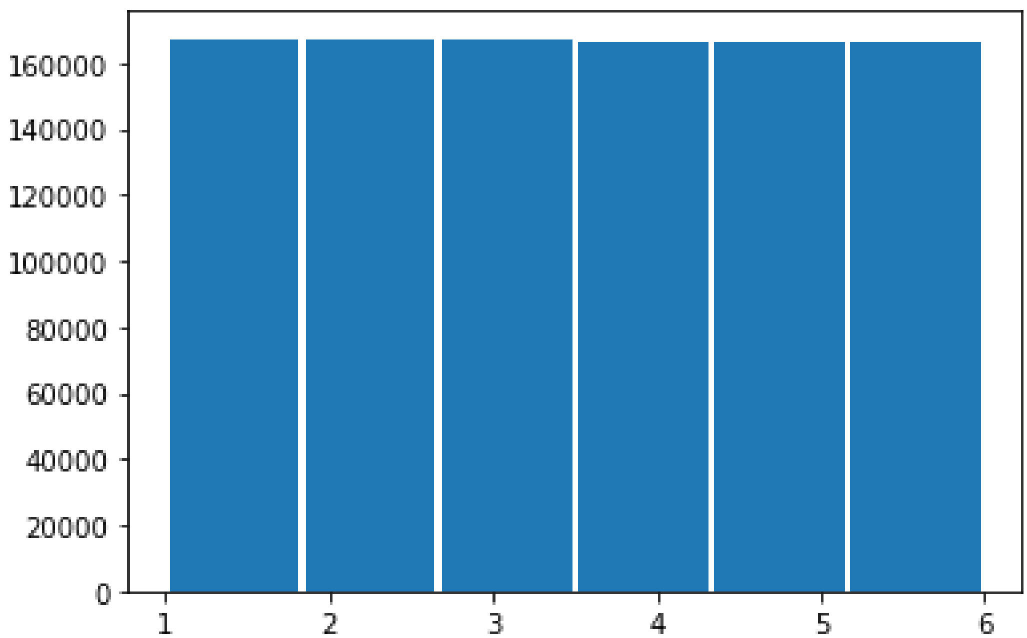
**To generate 10, 20-sided dice toss, set the side variable to 20:**

[23]:
```
rv.dicetoss(sides=20, n=10)
```

[23]:
```
array([ 2., 12.,  3.,  7.,  8.,  6.,  1.,  1., 15., 14.])
```

**Six sided dice toss. Note how all sides have equal probability.**

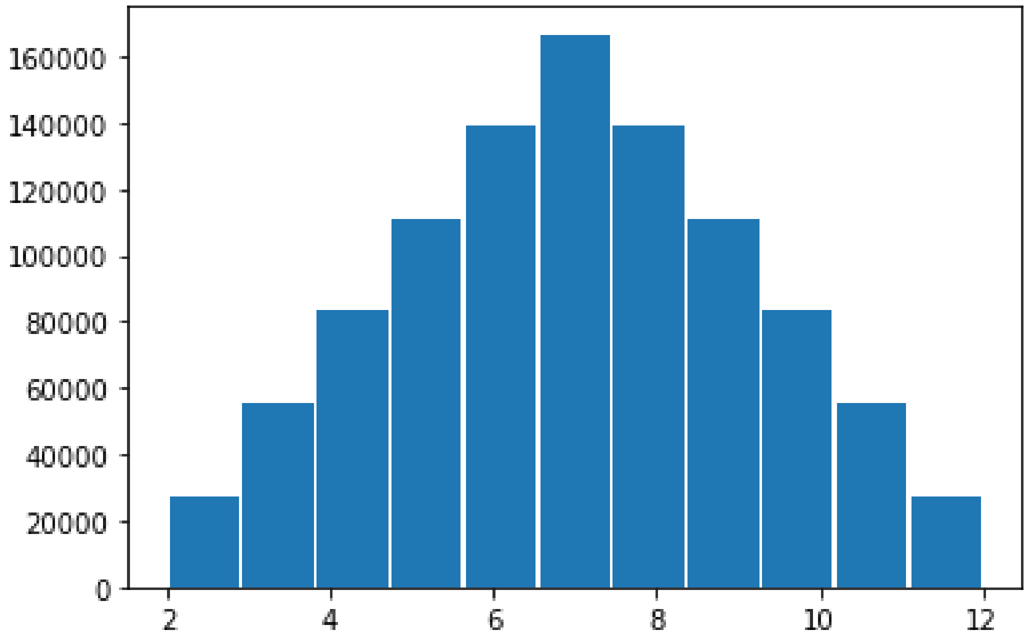
[24]:
```
b = rv.dicetoss(n=1000000)
plt.hist(b, bins=6, rwidth=0.95)
plt.show()
```




**Sum of two 6-sided dice tosses**

[25]:
```
rv1 = random.RandomVariates()
rv2 = random.RandomVariates()
a = rv1.dicetoss(n=1000000)
```

```
b = rv2.dicetoss(n=1000000)
plt.hist(a+b, bins=11, rwidth=0.95)
plt.show()
```




#### 1.5.11 Geometric Random Variates

To generate geometric random values, use the **geometric()** function. By default, the geometric function is set to a probability of 0.5:

[26]:
```
rv.geometric()
```

[26]:
```
array([1.])
```

To generate geometric values with a different probability, set p equal to the new probability:

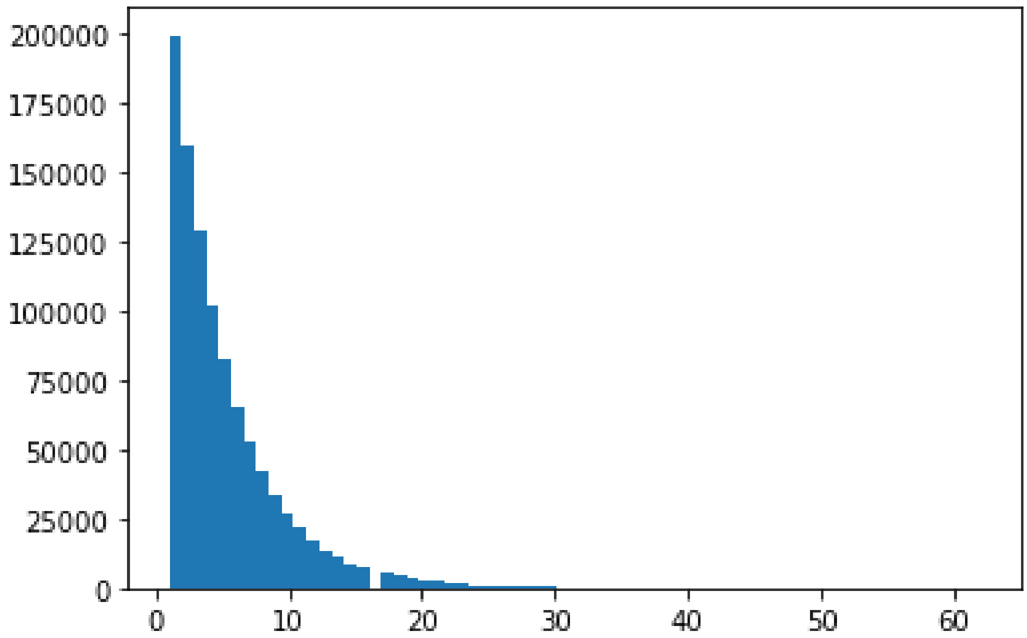
[27]:
```
rv.geometric(p=0.42, n=25)
```

[27]:
```
array([1., 2., 1., 1., 1., 1., 1., 1., 3., 3., 1., 1., 1., 2., 1., 2., 1.,
       4., 1., 2., 3., 1., 7., 1., 3.])
```

**Let's plot some geometric random values to see what things look like:**

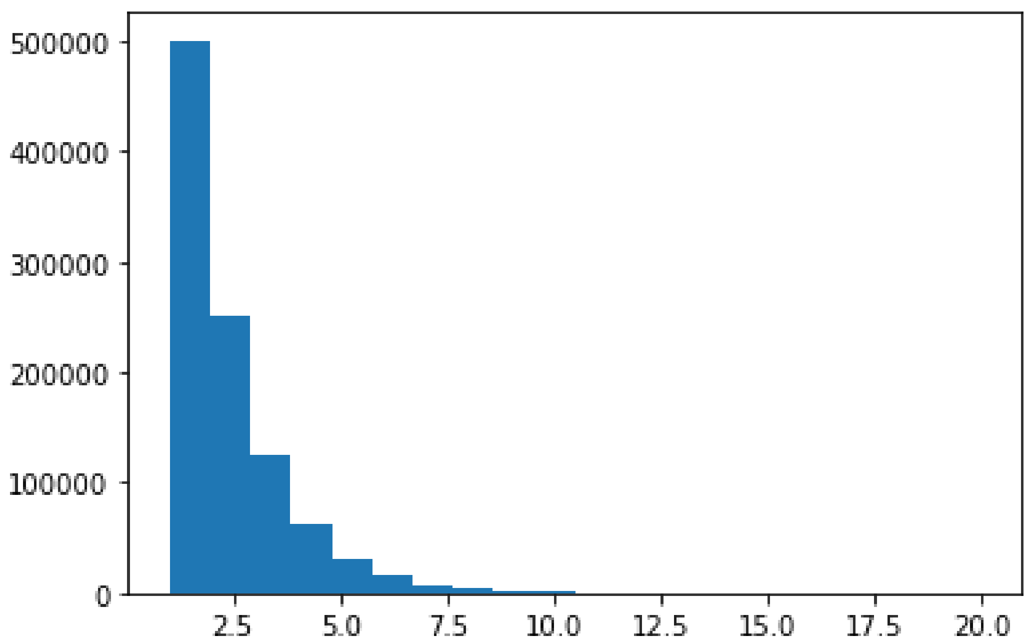
**Geometric with p = 0.2**

[28]:
```
rv.set_seed(101) # Set seed to 101 since 42 doesn't look as nice
g = rv.geometric(p=0.2, n=1000000)
plt.hist(g, bins=65)
plt.show()
```



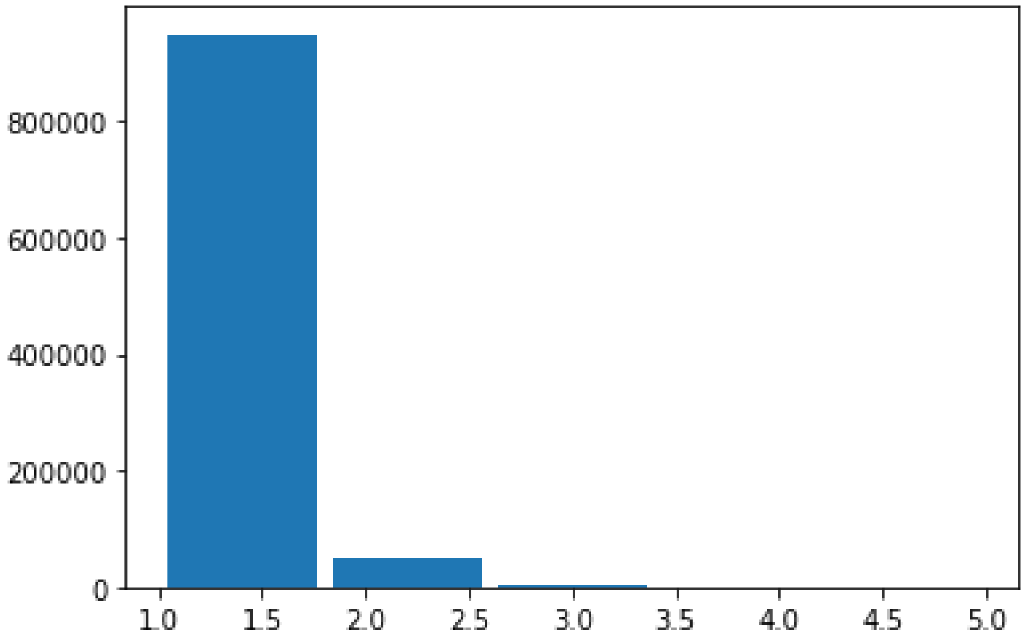

**Geometric with p = 0.5**

[29]:
```
rv.set_seed(123454321) # Set seed to 123454321 for a nice graph
g = rv.geometric(p=0.5, n=1000000)
plt.hist(g, bins=20)
plt.show()
```



**Geometric with p = 0.95**

[30]:
```
rv.set_seed(101)
g = rv.geometric(p=0.95, n=1000000)
plt.hist(g, bins=5, rwidth=0.9)
plt.show()
```




### 1.5.12 Negative Binomial Random Variates

To generate negative binomial random variates, call the **negbin()** funtion.

By default **negbin()** will generate values with a probability of 0.5 and 1 trial:

[31]:
```
rv.set_seed(42)
rv.negbin()
```

[31]:
```
array([1.])
```

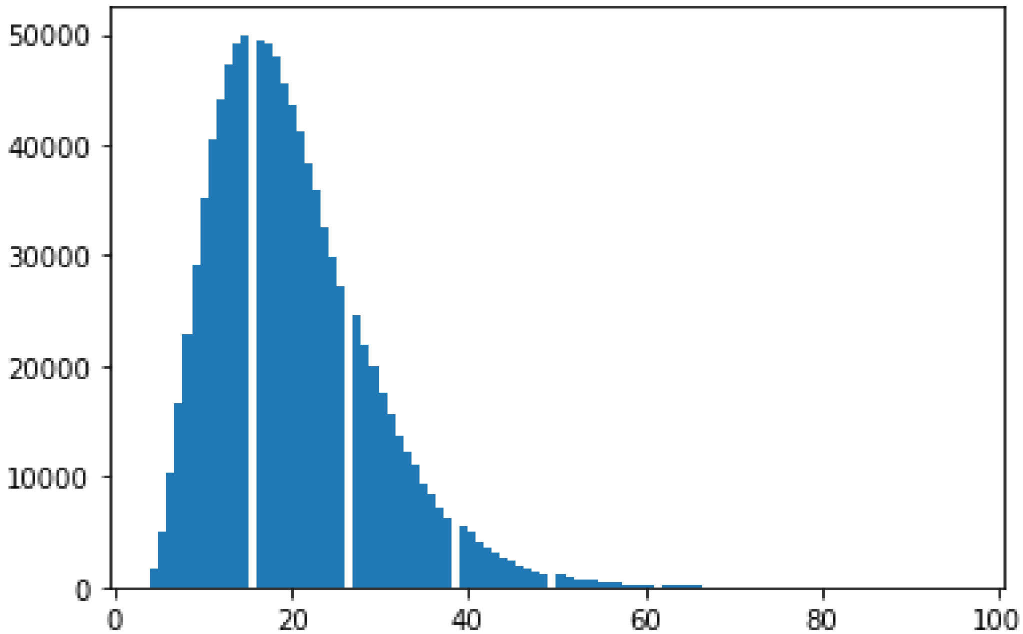
To generate 25 negbin values with a probability of 0.42 and 10 trials:

[32]:
```
rv.negbin(t=10, p=0.42, n=25)
```

[32]:
```
array([15., 28., 35., 35., 26., 17., 21., 41., 31., 27., 16., 19., 23.,
       26., 18., 24., 20., 27., 32., 17., 18., 16., 31., 21., 33.])
```

**Nice looking negbin distribution with t = 4 and. p = 0.2**

[33]:
```
nb = rv.negbin(t=4, p=0.2, n=1000000)
plt.hist(nb, bins=100)
plt.show()
```




#### 1.5.13 Chi-Squared Random Variates

Chi-Squared random values can be generated by calling the **chisq()** method.

By default, **chisq()** generates values with df=1:

[34]:
```
rv.chisq()
```

[34]:
```
array([2.01343621])
```

To generate chi-squared values with different degrees of freedom, set df=X where X is the degrees of freedom:

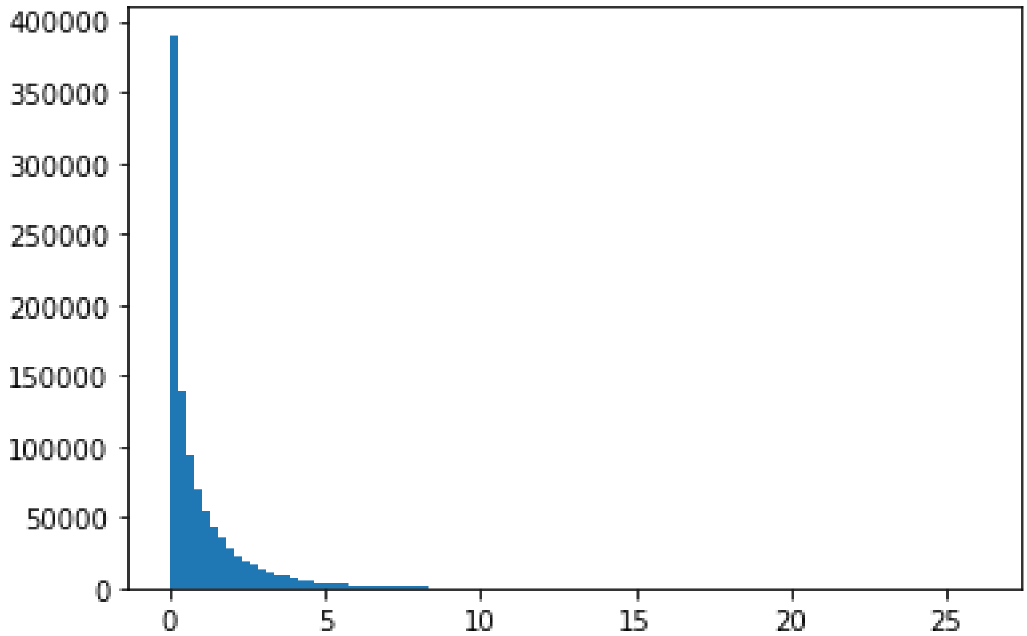
[35]:
```
rv.chisq(df=3, n=25)
```

[35]:
```
array([4.03013192, 2.1255032 , 1.41496674, 2.49301795, 4.34632967,
       7.07483573, 8.80603908, 0.40890643, 1.02559277, 0.3263966 ,
       1.16851057, 9.41171507, 0.10331964, 0.4620984 , 1.30332824,
       2.86123596, 6.30155659, 2.34574672, 6.51270442, 1.8040176 ,
       2.73061465, 2.18939106, 0.17322089, 1.95769521, 1.34417982])
```

**Plotting out some chi-squared random values**

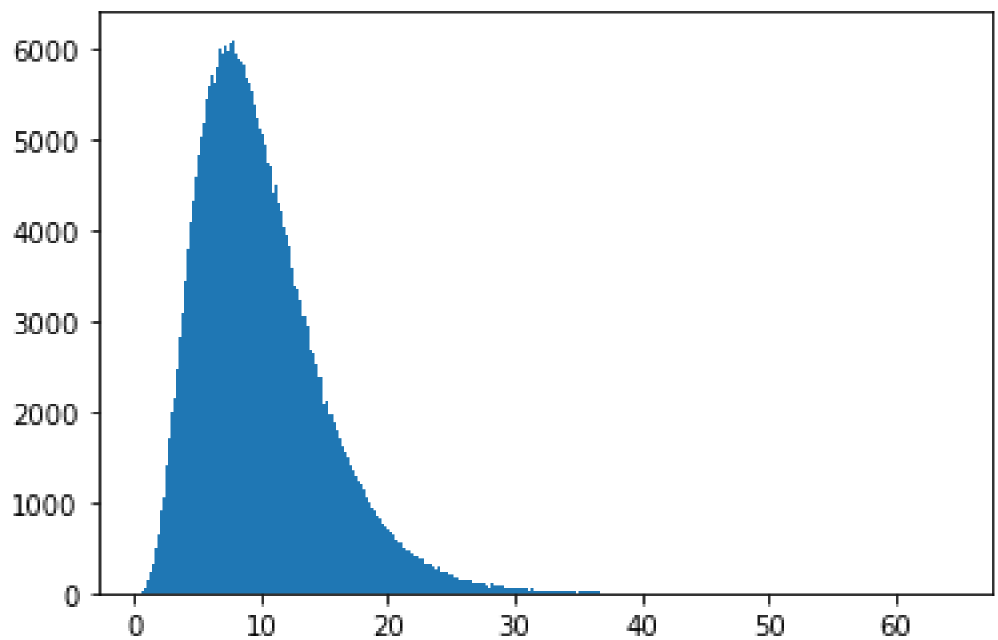
Chi-squared with df = 1

[36]:
```
cs = rv.chisq(df=1, n=1000000)
plt.hist(cs, bins=100)
plt.show()
```




**Chi-squared with df = 10**

[37]:
```
cs = rv.chisq(df=10, n=1000000)
plt.hist(cs, bins=1000)
plt.show()
```




### 1.5.14 Poisson Random Variates

By default, the **poisson()** method will generate poission random values with lam=1:

[38]:
```
rv.poisson()
```

[38]:
```
array([3])
```

To generate possion random variates for different lambda values, set lam=X, where X is the new labmda value:

[39]:
```
rv.poisson(lam=3, n=25)
```

[39]:
```
array([4, 7, 3, 2, 3, 3, 0, 3, 5, 1, 3, 2, 9, 3, 1, 3, 2, 3, 4, 5, 2, 4,
       5, 7, 2])
```

**Check the poisson mean and variance. They should be equal to lambda!**

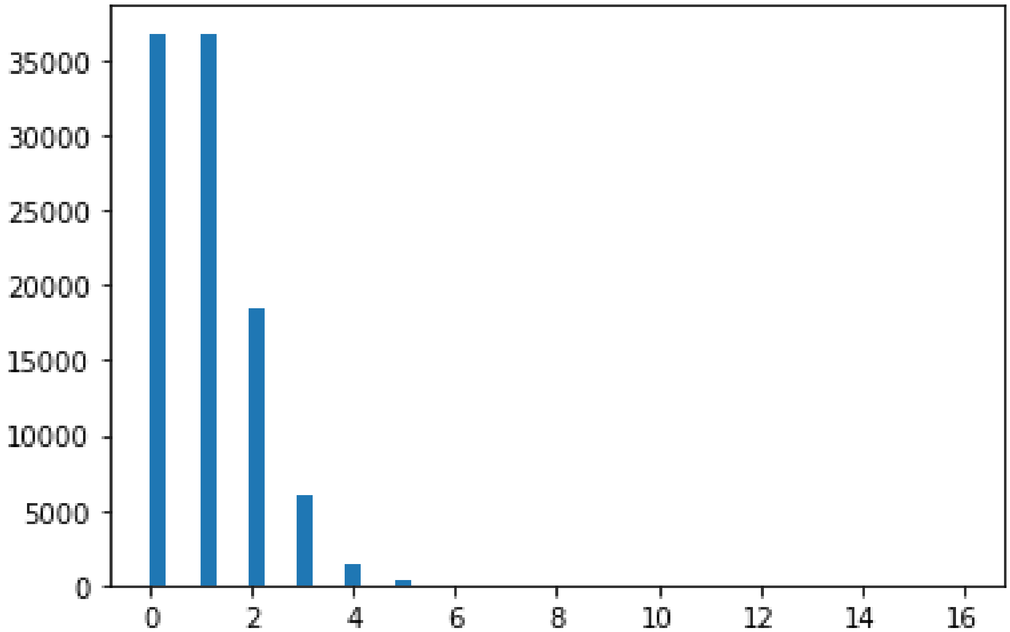
[40]:
```
rv.set_seed(42)
p = rv.poisson(lam=3, n=1000000)
print(f"mean: {np.mean(p)}")
print(f"var: {np.var(p)}")
```

```
mean: 2.99949
var: 3.038947739900001
```

**Create some plots of our poisson random values while also checking mean and variance**

**Poisson with lambda = 1**

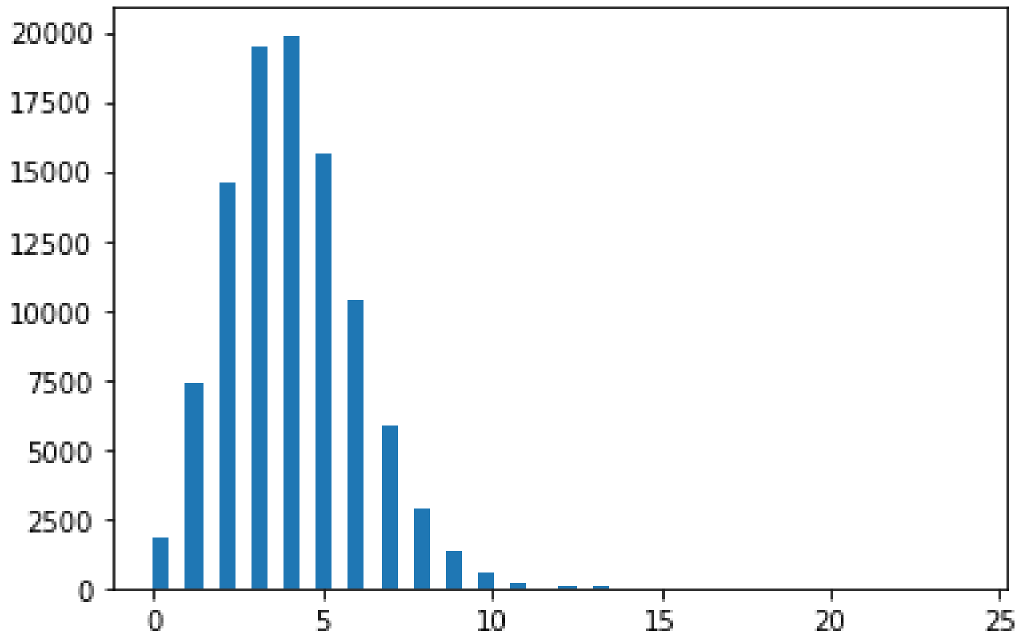
[41]:
```
rv.set_seed(876)
p = rv.poisson(lam=1, n=100000)
plt.hist(p, bins=50)
plt.show()
print(f"mean: {np.mean(p)}")
print(f"var: {np.var(p)}")
```




```
mean: 0.99825
var: 1.0007269375
```

**Poisson with lambda = 4**

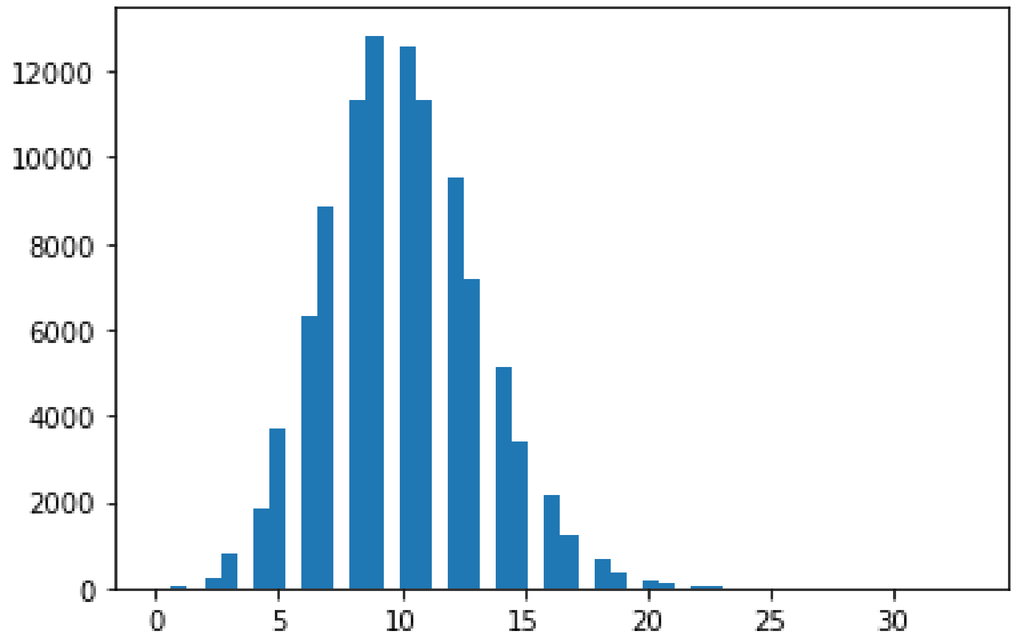
[42]:
```
p = rv.poisson(lam=4, n=100000)
plt.hist(p, bins=50)
plt.show()
print(f"mean: {np.mean(p)}")
print(f"var: {np.var(p)}")
```




```
mean: 3.99773
var: 4.016224847099999
```

**Poisson with lambda = 10**

[43]:
```
p = rv.poisson(lam=10, n=100000)
plt.hist(p, bins=50)
plt.show()
print(f"mean: {np.mean(p)}")
print(f"var: {np.var(p)}")
```




```
mean: 9.99379
var: 10.021511435899999
```

#### 1.5.15 Gamma Random Variates

Gamma random values can be generated by calling the **gamma()** function.

By default, **gamma()** generates values with a shape parameter (k) and scale parameter (theta) equal to one:

[44]:
```
rv.set_seed(42)
rv.gamma()
```

[44]:
```
array([0.0496442])
```

To generate gamma values with different shape and scale parameters set k = shape and theta = scale. i.e.) k=3, theta=3

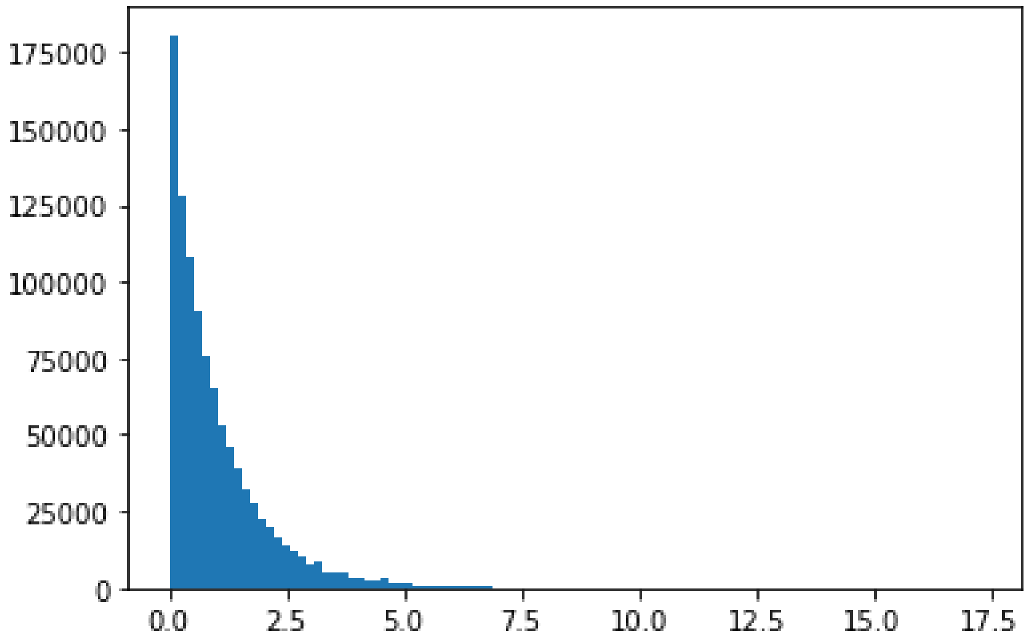
[45]:
```
rv.gamma(k=3, theta=3, n=25)
```

[45]:
```
array([ 2.86760705,  8.28296535, 15.61018946, 13.89795502, 28.71023072,
        0.98039742, 14.78770565,  8.31682721,  7.12832689, 11.31451427,
       14.12970636,  9.03501294, 18.95392932,  8.94168958,  2.80143093,
        7.09702805,  1.98142127,  6.69417433,  7.64163982, 12.51436153,
        9.84781027,  7.80807741,  6.79817083,  7.22277182, 13.64361073])
```

**Gererating some histograms of our gamma random variables**

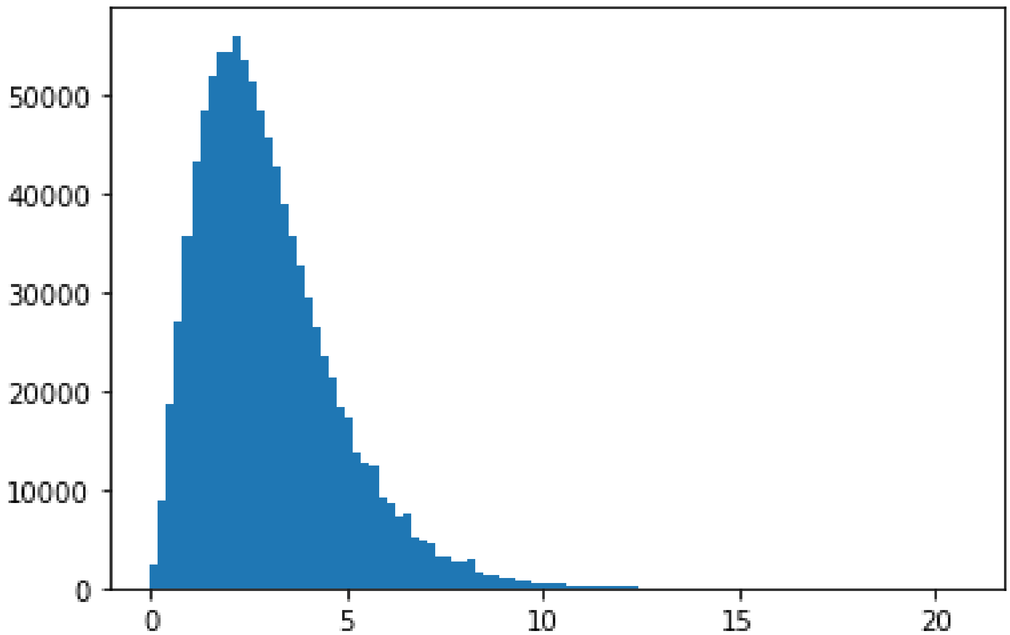
Gamma with k = 1 and theta = 1

[46]:
```
g = rv.gamma(k=1, theta=1, n=1000000)
plt.hist(g, bins=100)
plt.show()
```



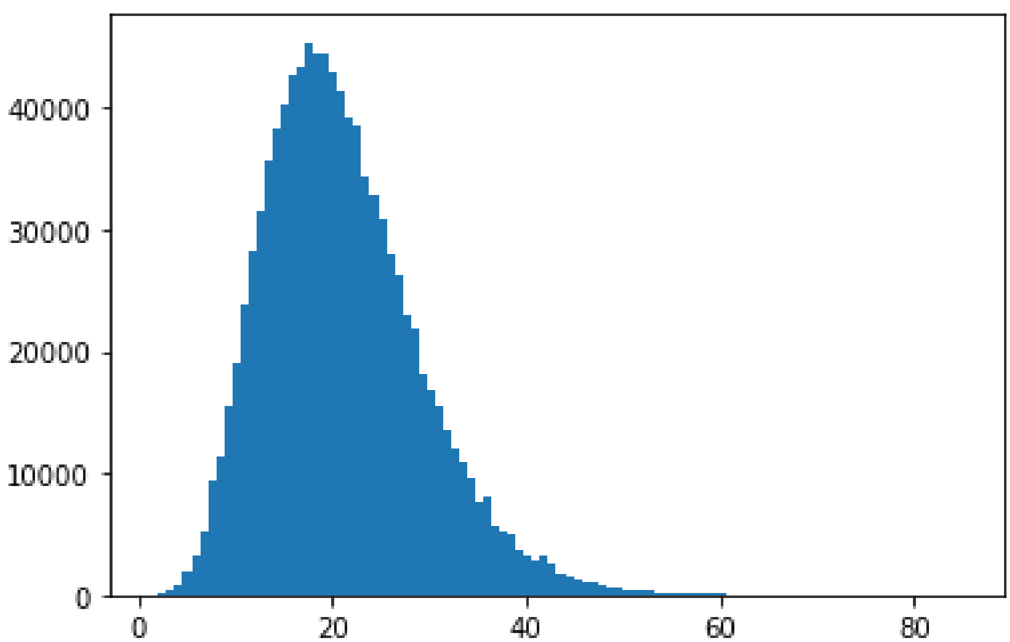

**Gamma with k = 3 and theta = 1**

[47]:
```
g = rv.gamma(k=3, theta=1, n=1000000)
plt.hist(g, bins=100)
plt.show()
```




**Gamma with k = 7 and theta = 3**

[48]:
```
g = rv.gamma(k=7, theta=3, n=1000000)
plt.hist(g, bins=100)
plt.show()
```



### 1.5.16 Lognormal Random Variate

Lognormal values can be generated with the **lognormal()** function:

[49]:
```
rv.set_seed(42)
rv.lognormal()
```

[49]:
```
array([0.24196649])
```

To generate lognormal values with different mean and standard deviation, specify the mu=X and sd=Y parameters where mu=X is the mean and sd=Y is the standard deviation:

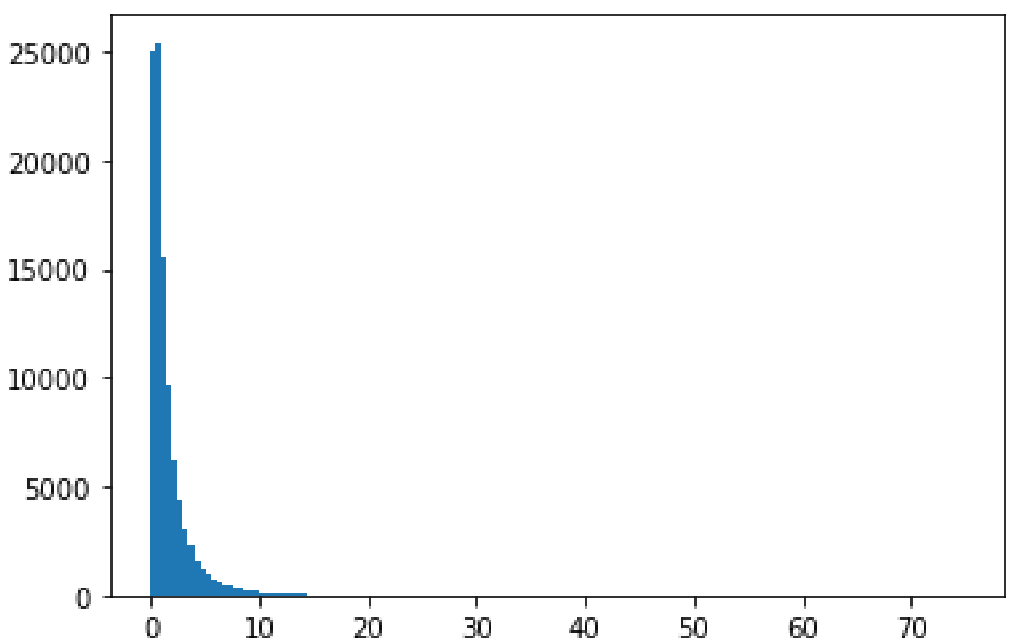
[51]:
```
rv.lognormal(mu=5, sd=2, n=10)
```

[51]:
```
array([   8.68926144, 1165.743983  ,  542.36799958,   50.00329616,
       2518.21870637,    6.20023729,    2.58041108,   83.05661697,
        617.74324652,  122.09875036])
```

**Let's check to see what our lognormal random values look like**

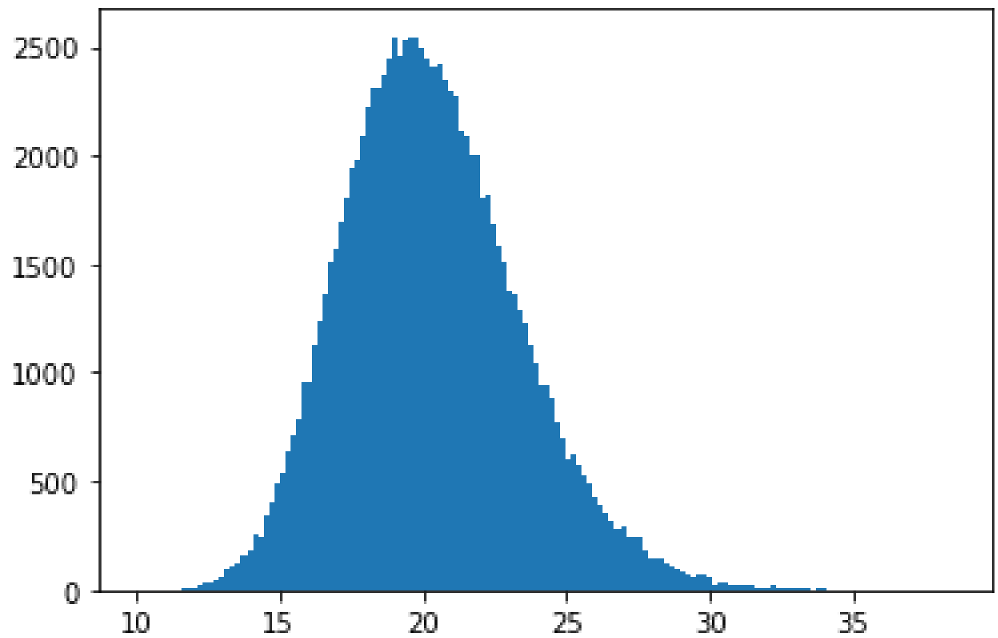
Lognormal with mu = 0 and sd = 1

[52]:
```
ln = rv.lognormal(mu=0, sd=1, n=100000)
plt.hist(ln, bins=150)
plt.show()
```




**Lognormal with mu = 3 and sd = 0.15**

[53]:
```
ln = rv.lognormal(mu=3, sd=0.15, n=100000)
plt.hist(ln, bins=150)
plt.show()
```




### 1.5.17 Beta Random Variates

Beta random values can be generated via the **beta()** method.

The **beta()** method takes two shape parameters - a and b. By default, the a and b parameters are set to 1:

[54]:
```
rv.beta()
```

[54]:
```
array([0.01995716])
```

To generate beta values with different shape parameters, specify different shape values as such:

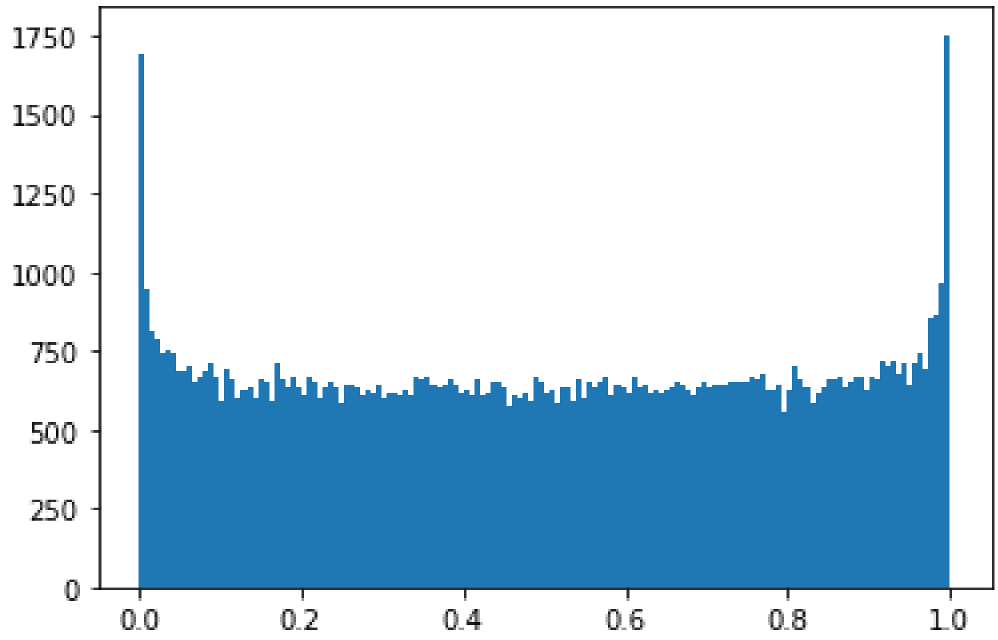
[55]:
```
rv.beta(a = 2, b = 4, n = 25)
```

[55]:
```
array([0.05843224, 0.17390187, 0.50358743, 0.50885326, 0.71498465,
       0.01966699, 0.53999924, 0.18878303, 0.30052695, 0.66454575,
       0.47430718, 0.23385242, 0.69526851, 0.48014572, 0.07415403,
       0.22019374, 0.07180719, 0.17131799, 0.19680721, 0.23783813,
       0.32510779, 0.29333343, 0.20013351, 0.27709332, 0.51118395])
```

**Check some plots of our beta random values**

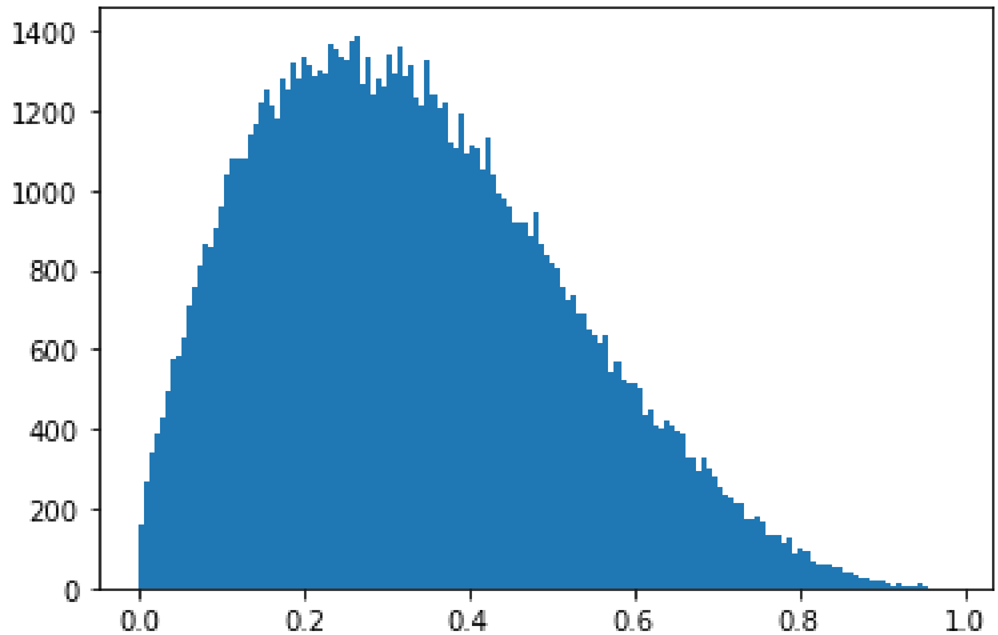
Beta with a = 1 and b = 1

[56]:
```
b = rv.beta(a = 1, b = 1, n = 100000)
plt.hist(b, bins=150)
plt.show()
```




**Beta with a = 2 and b = 4**

[57]:
```
b = rv.beta(a = 2, b = 4, n = 100000)
plt.hist(b, bins=150)
plt.show()
```




**Beta with a = 7 and b = 15**

[58]:
```
b = rv.beta(a = 7, b = 5, n = 100000)
plt.hist(b, bins=150)
plt.show()
```