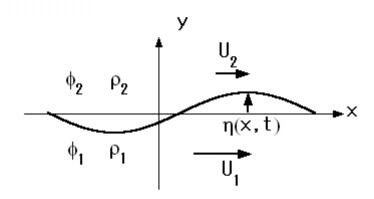
Kelvin Helmholtz Instability



$$egin{aligned} \mathbf{n} \cdot \mathbf{u_{int}} &= \mathbf{n} \cdot \mathbf{u_1} = \mathbf{n} \cdot \mathbf{u_2} \ \mathbf{n} &= rac{1}{\sqrt{1 + \left(rac{\partial \eta}{\partial t}
ight)}} igg(-rac{\partial \eta}{\partial t}, 1igg) \end{aligned}$$

Bernoulli Equation:

$$P_1 + rac{1}{2}
ho v_1^2 = P_2 + rac{1}{2}
ho v_2^2 \ q_1 =
abla \Phi_1 \quad
abla^2 \Phi = 0; \qquad q_2 =
abla \Phi_2 \quad
abla^2 \Phi_2 = 0 \ rac{\partial \Phi_1}{\partial t} + rac{(
abla \Phi_1)^2}{2} + rac{p_1}{
ho_1} + C_1 \qquad rac{\partial \Phi_2}{\partial t} + rac{(
abla \Phi_2)^2}{2} + rac{p_1}{
ho_1} + C_2$$

Kinematic Equations at boundary:

$$egin{aligned} rac{\partial \eta}{\partial t} + rac{\partial \Phi_1}{\partial x} rac{\partial \eta}{\partial x} &= rac{\partial \Phi_1}{\partial t} \ \\ rac{\partial \eta}{\partial t} + rac{\partial \Phi_2}{\partial x} rac{\partial \eta}{\partial x} &= rac{\partial \Phi_2}{\partial t} \end{aligned}$$

If $\Phi_i=U_ix+\phi_i$ where $\phi_i\ll\Phi_i$, then the kinematic and dynamic condition at the boundary y=0 become:

$$egin{aligned} rac{\partial \eta}{\partial t} + U_i rac{\partial \eta}{\partial x} &= rac{\partial \phi_i}{\partial y} \ &
ho_1 \left(rac{\partial \phi_1}{\partial t} + U_1 rac{\partial \phi_1}{\partial x}
ight) &=
ho_2 \left(rac{\partial \phi_2}{\partial t} + U_2 rac{\partial \phi_2}{\partial x}
ight) \end{aligned}$$

Now, we can assume that η goes as a sinusoidal along the boundary

$$\eta = A e^{i(kx-\omega t)}$$
 $\phi_i = ar{\phi}_i e^{\pm ky} e^{i(kx-\omega t)}$

Substituting the sinusoidal into the kinematic and dynamic condition at the boundary, we get

$$(-i\omega+ikU_i)A=\pm kar{\phi}_i$$
 $ho_1(-i\omega+ikU_1)\phi_1=
ho_2(-i\omega+ikU_2)\phi_2$

Writing this system of equations in matrix form, we can find solve the eigenvalue problem to find the dispersion relation:

$$\omega = krac{
ho_{1}U_{1} +
ho_{2}U_{2}}{
ho_{1} +
ho_{2}} \pm ikrac{\sqrt{
ho_{1}
ho_{2}}\left|U_{1} - U_{2}
ight|}{
ho_{1} +
ho_{2}}$$

Note: 1 denotes upper boundary and 2 denotes lower boundary