Pi-D Calculation

Pi-D

$$\Pi_{ij} = P_{ij} - p\delta_{ij}; \quad p = (p_{xx} + p_{yy} + p_{zz})/3; \quad \theta = \nabla \cdot u$$

$$\Pi = \begin{bmatrix} p_{xx} - p & p_{xy} & p_{xz} \\ p_{yx} & p_{yy} - p & p_{yz} \\ p_{zx} & p_{zy} & p_{zz} - p \end{bmatrix}$$

$$D_{ij} = \frac{1}{2}(\partial_i u_j + \partial_j u_i) - \frac{\theta}{3}\delta_{ij}$$

$$D = \begin{bmatrix} \partial_x u_x - \frac{\theta}{3} & \frac{1}{2}(\partial_x u_y + \partial_y u_x) & \frac{1}{2}\partial_x u_z \\ \frac{1}{2}(\partial_y u_x + \partial_x u_y) & \partial_y u_y - \frac{\theta}{3} & \frac{1}{2}\partial_y u_z \\ \frac{1}{2}\partial_x u_z & \frac{1}{2}\partial_y u_z & -\frac{\theta}{3} \end{bmatrix}$$

$$\Pi_{ij}D_{ij} = \Pi_{xx}D_{xx} + \Pi_{yy}D_{yy} + \Pi_{zz}D_{zz} + \Pi_{xy}D_{xy} + \Pi_{yx}D_{yx} + \Pi_{xz}D_{xz} + \Pi_{zx}D_{zx} + \Pi_{yz}D_{yz} + \Pi_{zy}D_{zy}$$

$$\Pi_{ij}D_{ij} = \Pi_{xx}D_{xx} + \Pi_{yy}D_{yy} + \Pi_{zz}D_{zz} + 2(\Pi_{xy}D_{xy} + \Pi_{xz}D_{xz} + \Pi_{yz}D_{yz})$$

$$e^{i\pi} = -1$$