Write the following expression as a single definite integral of the form $\int_a^b f(x)\,dx$

$$\int_{-5}^{2} f(x)\,dx + \int_{2}^{5} f(x)\,dx - \int_{-5}^{-3} f(x)\,dx$$

Note: property of integrals

$$\boxed{\int_a^c f(x)dx + \int_c^b f(x)dx = \int_a^b f(x)dx}$$

$$\Rightarrow \int_{-5}^{2} f(x)\,dx + \int_{2}^{5} f(x)\,dx = \int_{-5}^{5} f(x)\,dx$$

and

$$\boxed{\int_b^a f(x)dx = -\int_a^b f(x)dx}$$

$$\Rightarrow \int_{-5}^{-3} f(x)\,dx = -\int_{-3}^{-5} f(x)\,dx$$

$$\Rightarrow \underbrace{\int_{-5}^{2} f(x)\,dx + \int_{2}^{5} f(x)\,dx}_{} - \int_{-5}^{-3} f(x)\,dx$$

$$= \int_{-5}^{5} f(x)\,dx - \left(-\int_{-3}^{-5} f(x)\,dx\right)$$

$$= \int_{-5}^{5} f(x)\,dx + \int_{-3}^{-5} f(x)\,dx = \int_{-3}^{-5} f(x)\,dx + \int_{-5}^{5} f(x)\,dx \qquad \int_{\boxed{-3}}^{\boxed{5}} f(x)\,dx$$

Evaluate the integral.

$$\int_4^6 (x^2+2x-5)\,dx$$

$$= \frac{1}{2+1}x^{2+1} + 2\cdot\frac{1}{1+1}x^{1+1} - 5\cdot x \Big]_4^6$$

$$= \frac{1}{3}x^3 + \cancel{2}^{1}\cdot\frac{1}{\cancel{2}}x^2 - 5x \Big]_4^6$$

$$= \frac{1}{3}x^3 + x^2 - 5x \Big]_4^6$$

$$= \left[\frac{1}{3}(6)^3 + (6)^2 - 5(6)\right] - \left[\frac{1}{3}(4)^3 + (4)^2 - 5(4)\right]$$

$$= \left[\frac{1}{3}(216) + 36 - 30\right] - \left[\frac{1}{3}(64) + 16 - 20\right]$$

$$= \left[\frac{216}{3} + \frac{6\cdot 3}{1\cdot 3}\right] - \left[\frac{64}{3} - \frac{4\cdot 3}{1\cdot 3}\right]$$

$$= \left[\frac{216}{3} + \frac{18}{3}\right] - \left[\frac{64}{3} - \frac{12}{3}\right]$$

$$= \left[\frac{234}{3}\right] - \left[\frac{52}{3}\right]$$

$$= \frac{234-52}{3} = \boxed{\frac{182}{3}} \leftarrow \text{Answer}$$

Evaluate the integral.

$$\int_{-4}^{4} f(x)\,dx \quad \text{where} \quad f(x) = \begin{cases} 4 & \text{if } -4 \le x \le 0 \\ 16 - x^2 & \text{if } 0 \le x \le 4 \end{cases}$$

note: $\displaystyle\int_{-4}^{4} f(x)\,dx = \int_{-4}^{0} f(x)\,dx + \int_{0}^{4} f(x)\,dx$

$$= \int_{-4}^{0} 4\,dx + \int_{0}^{4} (16 - x^2)\,dx$$

$$= 4x\Big]_{-4}^{0} + \left(16x - \frac{1}{3}x^3\right)\Big]_{0}^{4}$$

$$= \left[4(0) - 4(-4)\right] + \left[\left(16(4) - \frac{1}{3}(4)^3\right) - \left(16(0) - \frac{1}{3}(0)^3\right)\right]$$

$$= [0 + 16] + \left[\left(64 - \frac{64}{3}\right) - (0 - 0)\right]$$

$$= 16 + \left[\left(\frac{192}{3} - \frac{64}{3}\right) - (0)\right]$$

$$= \frac{16}{1}\frac{\cdot 3}{\cdot 3} + \frac{128}{3} - 0$$

$$= \frac{48}{3} + \frac{128}{3} = \boxed{\frac{176}{3}}$$

43. Evaluate the definite integral.

$$\int_0^{\frac{\pi}{4}} \frac{6 + 7\cos^2(\theta)}{\cos^2(\theta)}\, d\theta$$

$$= \int_0^{\frac{\pi}{4}} \left[\frac{6}{\cos^2(\theta)} + \frac{7\cancel{\cos^2(\theta)}}{\cancel{\cos^2(\theta)}} \right] d\theta$$

$$= \int_0^{\frac{\pi}{4}} \left[6 \cdot \frac{1}{\cos^2(\theta)} + 7 \right] d\theta$$

$$= \int_0^{\frac{\pi}{4}} \left[6\sec^2(\theta) + 7 \right] d\theta$$

note: $\int \sec^2\theta \, d\theta = \tan\theta + C$

$$= 6\tan(\theta) + 7\theta \Big]_0^{\frac{\pi}{4}}$$

$$= \left[6\underbrace{\tan\left(\tfrac{\pi}{4}\right)}_{1} + 7\left(\tfrac{\pi}{4}\right)\right] - \left[6\tan(0) + 7(0)\right]$$

$$= \left[\underbrace{6 \cdot 1}_{6} + \frac{7\pi}{4}\right] - \left[\underbrace{6 \cdot 0}_{0} + 0\right]$$

$$= 6 + \frac{7\pi}{4} - 0$$

$$= \boxed{6 + \frac{7\pi}{4}}$$

45. Evaluate the definite integral.

$$\int_{-8}^{8} \frac{6e^{x}}{\sinh(x)+cash\,(x)}\,dx$$

Recall:

$$\sinh(x) = \frac{e^{x}-e^{-x}}{2}$$

$$\cosh(x) = \frac{e^{x}+e^{-x}}{2}$$

$$= \int_{-8}^{8} \frac{6e^{x}}{\frac{e^{x}-e^{-x}}{2} + \frac{e^{x}+e^{-x}}{2}}\,dx$$

$$= \int_{-8}^{8} \frac{6e^{x}}{\frac{e^{x}-e^{-x}+e^{x}+e^{-x}}{2}}\,dx$$

$$= \int_{-8}^{8} \frac{6e^{x}}{\frac{\cancel{2}^{1}e^{x}}{\cancel{2}}}\,dx$$

$$= \int_{-8}^{8} \frac{6\cancel{e^{x}}^{1}}{\cancel{e^{x}}}\,dx$$

$$= \int_{-8}^{8} 6\,dx$$

$$= 6x\Big]_{-8}^{8} = 6\,(8-(-8))$$

$$= 6\,(8+8)$$

$$= 6\,(16) = \boxed{96}$$

49. Evaluate the definite integral.

$$\int_0^{3\pi/2} 8|\sin(x)|\,dx$$

note :

$\sin(x) \geq 0$ for $0 \leq x \leq \pi$

and $\sin(x) < 0$ for $\pi < x \leq \frac{3\pi}{2}$

$$\Rightarrow |\sin(x)| = \begin{cases} \sin(x) & \text{if } 0 \leq x \leq \pi \\ -\sin(x) & \text{if } \pi < x \leq \frac{3\pi}{2} \end{cases}$$

$$= 8\int_0^{3\pi/2} |\sin(x)|\,dx$$

$$= 8\left[\int_0^{\pi} \sin(x)\,dx + \int_{\pi}^{3\pi/2} -\sin(x)\,dx\right]$$

$$= 8\left[\left(-\cos(x)\right)\Big|_0^{\pi} + \left(\cos(x)\right)\Big|_{\pi}^{3\pi/2}\right]$$

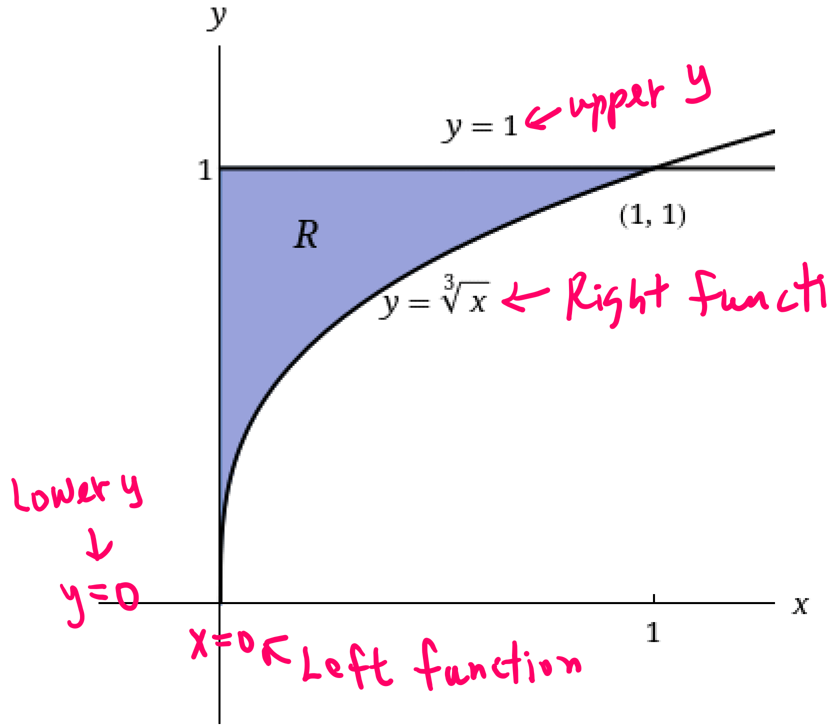
$$= 8\left[-\left(\cos(\pi) - \cos(0)\right) + \left(\cos\left(\frac{3\pi}{2}\right) - \cos(\pi)\right)\right]$$

$$= 8\left[-\left(-1 - 1\right) + \left(0 - (-1)\right)\right]$$

$$= 8\left[-(-2) + (1)\right]$$

$$= 8\left[2 + 1\right] = 8(3) = \boxed{24}$$

The boundaries of the shaded region are the y-axis, the line $y = 1$, and the curve $y = \sqrt[3]{x}$.



$\Rightarrow (y)^3 = (\sqrt[3]{x})^3 \Rightarrow y^3 = x$

$\Rightarrow x = y^3$

Find the area of the region R by writing x as a function of y and integrating with respect to y.

Note: Integrate with respect to $y \Rightarrow$ use the following:

$$A = \int_{\text{lower } y}^{\text{upper } y} \left(\text{Right Function} - \text{Left Function} \right) dy.$$

$$= \int_0^1 (y^3 - 0)\, dy$$

$$= \int_0^1 y^3\, dy$$

$$= \frac{1}{4} y^4 \Big]_0^1 = \frac{1}{4}\left[(1)^3 - (0)^3\right] = \frac{1}{4}[1 - 0] = \frac{1}{4}(1) = \boxed{\frac{1}{4}}$$

69. Evaluate the indefinite integral. (Use C for the constant of integration.)

$$\int \frac{x^7}{1+x^{16}}\,dx$$

$$= \int \frac{1}{1+(x^8)^2} \cdot x^7\,dx$$

$$= \int \frac{1}{1+u^2} \cdot \frac{1}{8}\,du$$

$$= \frac{1}{8} \int \frac{1}{1+u^2}\,du$$

$$= \frac{1}{8} \tan^{-1}(u) + C$$

$$= \boxed{\frac{1}{8} \tan^{-1}(x^8) + C}$$

$$\boxed{\begin{aligned} &\text{Let } u = x^8 \\ &\Rightarrow \frac{du}{dx} = 8x^7 \\ &\Rightarrow du = 8x^7\,dx \\ &\Rightarrow \frac{1}{8}\,du = x^7\,dx \end{aligned}}$$

70. Evaluate the indefinite integral. (Use C for the constant of integration.)

$$\int \frac{7+5x}{1+x^2}\,dx$$

$$= \int \left(\frac{7}{1+x^2} + \frac{5x}{1+x^2}\right) dx$$

$$= \int \frac{7}{1+x^2}\,dx + \int \frac{5x}{1+x^2}\,dx$$

$$= 7\int \frac{1}{1+x^2}\,dx + 5\int \frac{x}{1+x^2}\,dx$$

$$= 7\cdot \tan^{-1}(x) + 5\cdot \frac{1}{2}\ln(1+x^2) + C$$

$$= \boxed{7\tan^{-1}(x) + \frac{5}{2}\ln(1+x^2) + C}$$

Note:

$$\int \frac{x}{1+x^2}\,dx$$

$\Rightarrow u = 1+x^2$

$\Rightarrow \frac{du}{dx} = 2x$

$\Rightarrow du = 2x\,dx$

$\Rightarrow \frac{1}{2}du = x\,dx$

$$\Rightarrow \int \frac{x}{1+x^2}\,dx$$

$$= \int \frac{1}{1+x^2}\cdot x\,dx$$

$$= \int \frac{1}{u}\cdot \frac{1}{2}\,du$$

$$= \frac{1}{2}\int \frac{1}{u}\,du$$

$$= \frac{1}{2}\ln|u| + C$$

$$= \frac{1}{2}\ln|1+x^2| + C$$

$$= \frac{1}{2}\ln(1+x^2) + C$$

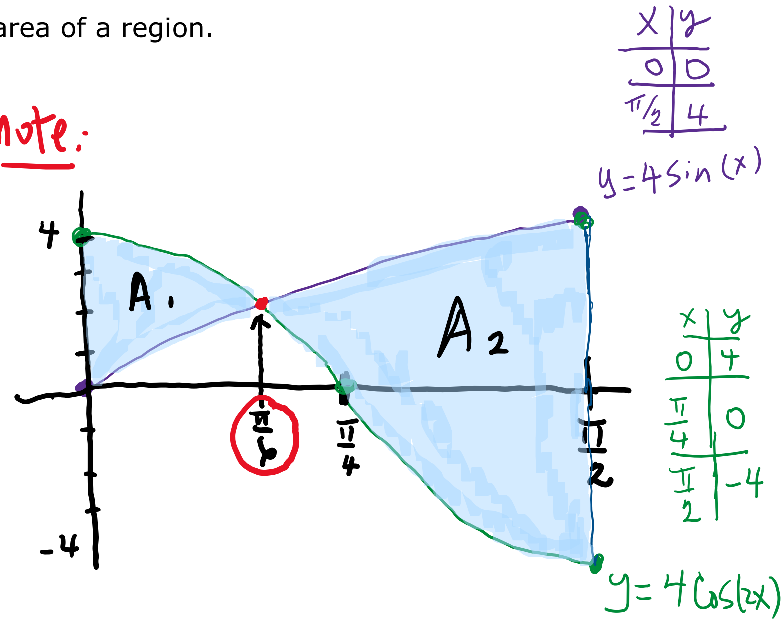
note: $1+x^2 > 0$ for all x

$\Rightarrow |1+x^2| = 1+x^2$

79. Evaluate the integral and interpret it as the area of a region.

$$\int_0^{\pi/2} |4\sin(x) - 4\cos(2x)|\, dx$$

note:



point of intersection:

$4\sin(x) = 4\cos(2x)$

$4\sin(x) = 4(1-2\sin^2(x))$

$4\sin(x) = 4 - 8\sin^2(x)$

$4\sin(x) - 4 + 8\sin^2(x) = 0$

$8\sin^2(x) + 4\sin(x) - 4 = 0$

$2\sin^2(x) + \sin(x) - 1 = 0$ ←

If $y = \sin(x)$
Then we have
$2y^2 + y - 1 = 0$
$(2y-1)(y+1) = 0$

$(2\sin(x) - 1)(\sin(x) + 1) = 0$

$\Rightarrow 2\sin(x) - 1 = 0$ or $\sin(x) + 1 = 0$

$2\sin(x) = 1$	$\sin(x) = -1$
$\sin(x) = \frac{1}{2}$	$x = \frac{3\pi}{2} + 2k\pi$
$x \in [0, \frac{\pi}{2}] \Rightarrow \boxed{x = \frac{\pi}{6}}$ or	not in $[0, \frac{\pi}{2}]$
	$\Rightarrow \emptyset$

$$A = A_1 + A_2$$

$$= \underbrace{\int_0^{\frac{\pi}{6}} \left[4\cos(2x) - 4\sin(x)\right]dx}_{A_1} + \underbrace{\int_{\frac{\pi}{6}}^{\pi/2} \left[4\sin(x) - 4\cos(2x)\right]dx}_{A_2}$$

$$= 4\int_0^{\frac{\pi}{6}} \left[\underbrace{\cos(2x)}_{\frac{1}{2}\sin(2x)} - \underbrace{\sin(x)}_{\int \sin(x)dx = -\cos(x)}\right]dx + 4\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \left[\underbrace{\sin(x)}_{-\cos(x)} - \underbrace{\cos(2x)}_{\frac{1}{2}\sin(2x)}\right]dx$$

$$= 4\left[\frac{1}{2}\sin(2x) + \cos(x)\right]_0^{\frac{\pi}{6}} \overset{-}{+} 4\left[\overset{+}{-}\cos(x) \overset{+}{-} \frac{1}{2}\sin(2x)\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$$

$$= 4\left[\left(\underbrace{\frac{1}{2}\sin\left(\overset{\pi/3}{2\cdot\frac{\pi}{6}}\right)}_{\sqrt{3}/4} + \underbrace{\cos\left(\frac{\pi}{6}\right)}_{\sqrt{3}/2}\right) - \left(\frac{1}{2}\underbrace{\sin\left(\overset{0}{2\cdot 0}\right)}_{0} + \underbrace{\cos(0)}_{1}\right)\right]$$

$$-4\left[\left(\underbrace{\cos\left(\frac{\pi}{2}\right)}_{0} + \frac{1}{2}\underbrace{\sin\left(\overset{\pi}{2\cdot\frac{\pi}{2}}\right)}_{0}\right) - \left(\underbrace{\cos\left(\frac{\pi}{6}\right)}_{\sqrt{3}/2} + \frac{1}{2}\underbrace{\sin\left(\overset{\pi/3}{2\cdot\frac{\pi}{6}}\right)}_{\frac{\sqrt{3}}{2}}\right)\right] \quad \frac{\sqrt{3}}{4}$$

$$= 4\left[\left(\frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{2}\right) - \left(0 + \overset{1}{1}\right)\right] - 4\left[\left(0 + \overset{0}{0}\right) - \left(\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{4}\right)\right]$$

$$= 4\left[\frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{2} - 1\right] - 4\left[-\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{4}\right]$$

$$= \sqrt{3} + 2\sqrt{3} - 4 + 2\sqrt{3} + \sqrt{3}$$

$$= \boxed{6\sqrt{3} - 4}$$

note:

Let $u = 2x$

$\Rightarrow du = 2dx$

$\Rightarrow \frac{1}{2}du = dx$

$\Rightarrow \int \cos(2x)dx$

$= \int \cos u \cdot \frac{1}{2}du$

$= \frac{1}{2}\int \cos u \, du$

$= \frac{1}{2}\sin u + C$

$= \frac{1}{2}\sin(2x) + C$

Consider the following theorem.

Theorem

If f is integrable on $[a, b]$, then

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x$$

where $\Delta x = \frac{b-a}{n}$ and $x_i = a + i\Delta x$

Use the given theorem to evaluate the definite integral.

$$b \to \int_{a \to -2}^{0} (7x^2 + 7x)\,dx$$

Note: $\Delta x = \frac{0-(-2)}{n} = \frac{2}{n}$ and $x_i = -2 + i\cdot\frac{2}{n} = -2 + \frac{2i}{n}$

$$\Rightarrow \int_{-2}^{0} (7x^2 + 7x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} f\left(-2 + \frac{2i}{n}\right)\cdot\frac{2}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left[7\left(-2 + \frac{2i}{n}\right)^2 + 7\left(-2 + \frac{2i}{n}\right)\right]\cdot\frac{2}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} 7\left[\left(-2 + \frac{2i}{n}\right)^2 + \left(-2 + \frac{2i}{n}\right)\right]\cdot\frac{2}{n}$$

$$= 7\lim_{n\to\infty} \frac{2}{n} \sum_{i=1}^{n} \left[\left(-2 + \frac{2i}{n}\right)^2 + \left(-2 + \frac{2i}{n}\right)\right]$$

$$= 7\lim_{n\to\infty} \frac{2}{n} \sum_{i=1}^{n} \left[\underbrace{4 - \frac{8i}{n} + \frac{4i^2}{n^2}} - 2 + \frac{2i}{n}\right]$$

$$\left(-2 + \frac{2i}{n}\right)^2 = \left(-2 + \frac{2i}{n}\right)\left(-2 + \frac{2i}{n}\right) = \underset{F}{4} \underset{O}{- \frac{4i}{n}} \underset{I}{- \frac{4i}{n}} \underset{L}{+ \frac{4i^2}{n^2}}$$

$$= 7 \lim_{n\to\infty} \frac{2}{n}\sum_{i=1}^{n}\left[\frac{4i^2}{n^2} - \frac{6i}{n} + 2\right]$$

$$= 7 \lim_{n\to\infty} \frac{2}{n}\left[\sum_{i=1}^{n}\frac{4i^2}{n^2} - \sum_{i=1}^{n}\frac{6i}{n} + \sum_{i=1}^{n} 2\right]$$

$$= 7 \lim_{n\to\infty} \frac{2}{n}\left[\frac{4}{n^2}\sum_{i=1}^{n} i^2 - \frac{6}{n}\sum_{i=1}^{n} i + \sum_{i=1}^{n} 2\right]$$

$$= 7 \lim_{n\to\infty} \frac{2}{n}\left[\frac{4}{n^2}\cdot\frac{n(n+1)(2n+1)}{6} - \frac{6}{n}\cdot\frac{n(n+1)}{2} + n\cdot 2\right]$$

$$= 7 \lim_{n\to\infty}\left[\frac{\cancel{8}^{\,4}}{n^{\cancel{3}\,2}}\cdot\frac{\cancel{n}(n+1)(2n+1)}{\cancel{6}\;3} - \frac{\cancel{12}^{\,6}}{n^{\cancel{2}\,1}}\cdot\frac{\cancel{n}(n+1)}{\cancel{2}} + \frac{2}{\cancel{n}}\cdot 2\cancel{n}\right]$$

$$= 7 \lim_{n\to\infty}\left[\frac{4(n+1)(2n+1)}{3n^2} - \frac{6(n+1)}{n} + 4\right]$$

$$= 7 \lim_{n\to\infty}\left[\frac{4(2n^2+3n+1)}{3n^2} - \frac{6n+6}{n} + 4\right]$$

$$= 7 \lim_{n\to\infty}\left[\frac{4}{3}\left(\frac{2n^2}{n^2} + \frac{3n}{n^2} + \frac{1}{n^2}\right) - \frac{6n}{n} - \frac{6}{n} + 4\right]$$

$$= 7 \lim_{n\to\infty}\left[\frac{4}{3}\left(2 + \cancel{\frac{3}{n}}^{\to 0} + \cancel{\frac{1}{n^2}}^{\to 0}\right) - 6 - \cancel{\frac{6}{n}}^{\to 0} + 4\right]$$

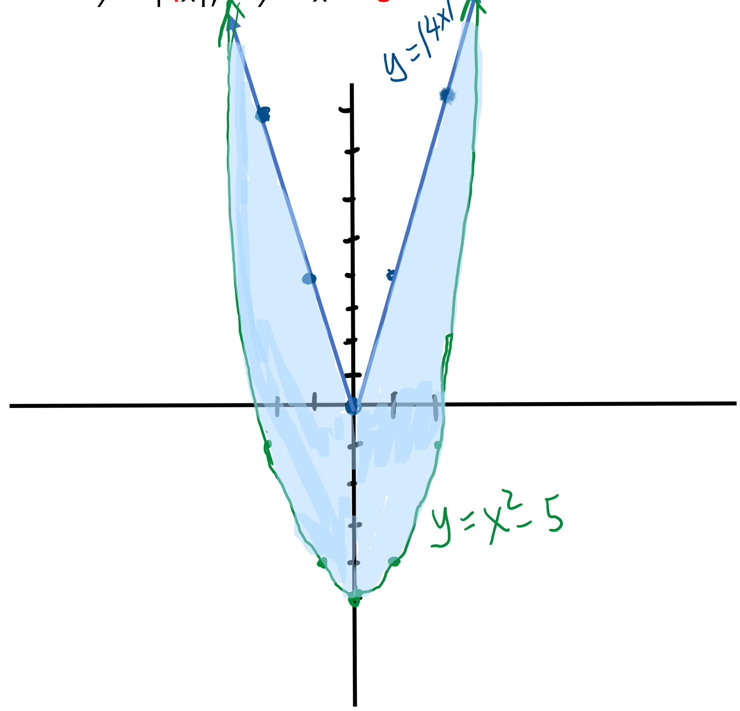
$$= 7\cdot\left[\frac{4}{3}(2 + 0 + 0) - 6 - 0 + 4\right]$$

$$= 7\cdot\left[\frac{4}{3}(2) - 2\right]$$

$$= 7\cdot\left[\frac{8}{3} - 2\right] = 7\left[\frac{8}{3} - \frac{6}{3}\right] = 7\left[\frac{2}{3}\right] = \boxed{\frac{14}{3}}$$

80. Sketch the region enclosed by the given curves.

$y = |4x|, \quad y = x^2 - 5$



$y = |4x|$

x	y
0	0
1	4
2	8

$y = x^2 - 5$

x	y
0	-5
1	-4
2	-1

Find the area of the region.

note: points of intersection:

For $x \geq 0$, $y = |4x| = 4x$

$$\Rightarrow \quad x^2 - 5 = 4x \Rightarrow x^2 - 4x - 5 = 0$$

$$\Rightarrow (x-5)(x+1) = 0$$

$$\Rightarrow x - 5 = 0 \text{ OR } x + 1 = 0$$

$$\Rightarrow x = 5 \text{ OR } x = -1$$

$$x \geq 0 \Rightarrow \boxed{x = 5}$$

$$\Rightarrow \text{For } x \geq 0, \; A = \int_0^5 \left[4x - (x^2 - 5)\right] dx$$

$$\Rightarrow A = \int_0^5 (4x - x^2 + 5)\,dx$$

$$= 4\cdot\frac{1}{2}x^2 - \frac{1}{3}x^3 + 5x\Big]_0^5$$

$$= 2x^2 - \frac{1}{3}x^3 + 5x\Big]_0^5$$

$$= \left[2\cdot(5)^2 - \frac{1}{3}(5)^3 + 5(5)\right] - \left[2(0)^2 - \frac{1}{3}(0)^3 + 5(0)\right]$$

$$= \left[2\cdot 25 - \frac{1}{3}\cdot 125 + 25\right] - \underbrace{\left[2\cdot 0 - \frac{1}{3}\cdot 0 + 0\right]}_{0}$$

$$= 50 - \frac{125}{3} + 25$$

$$= \frac{75\cdot 3}{1\cdot 3} - \frac{125}{3} = \frac{225}{3} - \frac{125}{3} = \frac{100}{3}$$

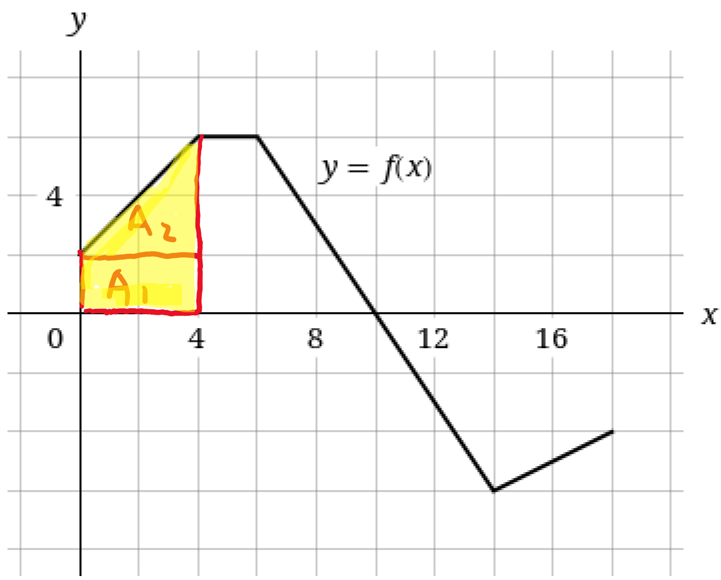
note: Graph is symmetric with respect to y-axis

$\Rightarrow$ Total area between $y = |4x|$ and $y = x^2 - 5$

is $2A = 2\left(\frac{100}{3}\right) = \boxed{\frac{200}{3}}$

Answer

9. The graph of f is shown.



(f) $\int_4^0 f(x) = -\int_0^4 f(x)$

$$= -(A_1 + A_2)$$

$$= -(\overset{L\cdot W}{4\cdot 2} + \overset{\frac{1}{2}\cdot B\cdot H}{\tfrac{1}{2}\cdot 4\cdot 4})$$

$$= -(8 + 8)$$

$$= \boxed{-16} \leftarrow \text{Answer}$$

note: A_1 = Rectangle $= L \cdot W$

with Length = 4

Width = 2

A_2 = Triangle $= \frac{1}{2} \cdot B \cdot H$

with Base = 4

Height = 4