Review of Precalculus

A. Expanding Binomials:

1. $(x+y)^2$
2. $3x(2x+1)^2$
3. $(a+2b)^3$
4. $(u-v)^4$

B. Factoring:

1. $2x^3-2xy^2$
2. $x^2+xy-6y^2$
3. $3ax+2x-3ay-2y$
4. $4a(x+y)^2-2a(x+y)$
5. $2x^{3/2}+4x^{1/2}$

C. Converting Fractional Exponents to Radicals:

1. $x^{2/3}$
2. $3x^{4/5}$
3. $7(x+1)^{1/2}$
4. $5w^{-2/3}$

D. Converting Radicals to Fractional Exponents:

1. $\sqrt[3]{x^4}$
2. $3\sqrt[5]{x^2}$
3. $\sqrt[4]{(x+1)^3}$
4. $1/\sqrt[3]{x^2}$

E. Finding the Equation of a Line in the xy-Plane

1. slope $= -\frac{2}{3}$ and passes through $(6,4)$
2. passes through $(3,1)$ and $(2,7)$
3. vertical and passes through $(2,3)$
4. horizontal through $(4,-8)$

F. Solving Linear Equations

1. Solve $\frac{1}{2}x+\frac{1}{3}(\frac{1}{2}x-1)=4$ for x
2. Solve $Ax+B=Cx+D$ for x
3. Solve $\frac{1}{C_1}+\frac{1}{C_2}=\frac{1}{C}$ for C_1
4. Solve $xyy'+x=x^2y'+y^3$ for y'.

G. Rationalizing Denominators

1. $\dfrac{1}{\sqrt{3}-\sqrt{2}}$
2. $\dfrac{3}{\sqrt{x}-1}$
3. $\dfrac{4x}{\sqrt{4+x}-2}$

H. Function Notation

1. $f(x)=3x^2+3x+1$

   $f(1)=$

   $f(7)=$

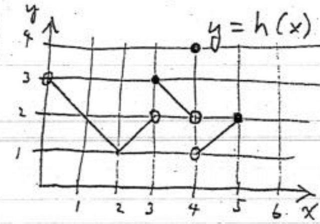
   $f(x+h)=$

2. $g(x)=\begin{cases} x+1, & x\ge 1 \\ x^3, & x<1 \end{cases}$

   $g(0)=$

   $g(1)=$

   $g(2)=$

3. 

   $h(0)=$ $h(3)=$

   $h(1)=$ $h(4)=$

   $h(2)=$ $h(5)=$

I. Composition

$\begin{cases} f(x)=x^2+4 \\ g(x)=2x+3 \end{cases}$

1. $f(g(2))=$
2. $g(f(3))=$
3. $f(g(x))=$
4. $g(f(x))=$

J. Reverse Composition

Find two functions whose composition is equal to the following:

1. $y=\sqrt{x^2+4}$
2. $y=3(x+7)^4$
3. $y=2\sin^3 x$
4. $y=\sin(7x+4)$

K. Solving Log and Exponential Equations

1. $\log_2(3x+1)=4$
2. $\ln(4x+8)-2\ln 2=3$
3. $2^{4x-1}=8$
4. $3e^{2x+1}=6$

L. Simplifying Trig Expressions Using Identities:

1. $3+2\sin^2 x+2\cos^2 x$
2. $\sin x\cot x+\cos x$
3. $4+4\tan^2 x+3/\cos^2 x$
4. $4\sin^2 x\cos^2 x$

Review of Precalculus

M. Solving Trig Equations

① $2\sin x - 1 = 0$ on $[0, 2\pi)$

② $\sqrt{3}\tan x + 1 = 0$ on $[0, 2\pi)$

③ $2\cos x + \sqrt{3} = 0$ on $[0, 2\pi)$

④ $2\cos^2 x - 1 = 0$ on $[0, 2\pi)$

N. Evaluating Trig Expressions

① $\sin \frac{2\pi}{3}$ ⑤ $\cot \frac{\pi}{4}$ ⑨ $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

② $\cos \frac{7\pi}{6}$ ⑥ $\csc \frac{7\pi}{4}$ ⑩ $\cos^{-1}\left(-\frac{1}{2}\right)$

③ $\tan 3\pi$ ⑦ $\tan^{-1}(\sqrt{3})$ ⑪ $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

④ $\sec \frac{2\pi}{3}$ ⑧ $\sin^{-1}\left(\frac{1}{2}\right)$ ⑫ $\sin^{-1}(1)$

O. Sketching Graphs of Basic Functions

Sketch a careful graph of each of the following functions labeling all intercepts, asymptotes, and any other important parts of the graph.

① $y = x$ ⑨ $y = e^x$ ⑯ $y = \csc x$

② $y = x^2$ ⑩ $y = e^{-x}$ ⑰ $y = \sec x$

③ $y = x^3$ ⑪ $y = e^{-x^2}$ ⑱ $y = \cot x$

④ $y = \frac{1}{x}$ ⑫ $y = \ln x$ ⑲ $y = \sin^{-1} x$

⑤ $y = \frac{1}{x^2}$ ⑬ $y = \sin x$ ⑳ $y = \cos^{-1} x$

⑥ $y = \sqrt{x}$ ⑭ $y = \cos x$ ㉑ $y = \tan^{-1} x$

⑦ $y = \sqrt[3]{x}$ ⑮ $y = \tan x$

⑧ $y = |x|$

P. Expanding a Log Expression

① $\ln \frac{x^2 y^3}{z^4}$

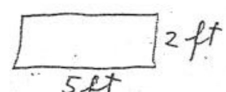
② $\ln \frac{(x+1)^4 x^5}{(x+y)^6}$

③ $\ln \frac{\sqrt[3]{(x+1)^2}\, x^4}{x^3 \sqrt[4]{x}}$

Q. Finding Areas and Perimeters

Find the area and perimeter of each figure.

①



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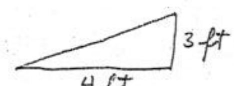

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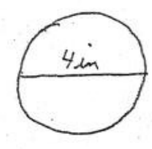
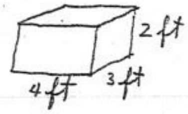


R. Finding Volumes and Surface Area

Find the volume and surface area of each figure.

①



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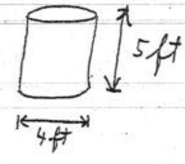

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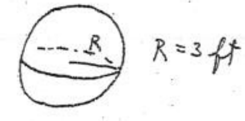


Review of Precalculus Solutions

A. ① $(x+y)^2$
$= (x+y)(x+y)$
$= \boxed{x^2+2xy+y^2}$

② $3x(2x+1)^2$
$= 3x(2x+1)(2x+1)$
$= 3x(4x^2+4x+1)$
$= \boxed{12x^3+12x^2+3x}$

③ $(a+2b)^3$
$= (a+2b)(a+2b)(a+2b)$
$= (a^2+4ab+4b^2)(a+2b)$
$= a^3+4a^2b+4ab^2+2a^2b+8ab^2+8b^3$
$= \boxed{a^3+6a^2b+12ab^2+8b^3}$

④ $(u-v)^4$
$= (u-v)(u-v)(u-v)(u-v)$
$= (u^2-2uv+v^2)(u^2-2uv+v^2)$
$= \boxed{u^4-4u^3v+6u^2v^2-4uv^3+v^4}$

B. ① $2x^3-2xy^2$
$= 2x(x^2-y^2)$
$= \boxed{2x(x-y)(x+y)}$

② $x^2+xy-6y^2$
$= \boxed{(x+3y)(x-2y)}$

③ $3ax+2x-3ay-2y$
$= x(3a+2)-y(3a+2)$
$= \boxed{(3a+2)(x-y)}$

④ $4a(x+y)^2-2a(x+y)$
$= 2a(x+y)[2(x+y)-1]$
$= \boxed{2a(x+y)[2x+2y-1]}$

B. ⑤ $2x^{3/2}+4x^{1/2}$
$= 2x^{1/2}(x^{2/2}+2)$
$= \boxed{2x^{1/2}(x+2)}$

C. ① $x^{2/3} = \boxed{\sqrt[3]{x^2}}$

② $3x^{4/5} = \boxed{3\sqrt[5]{x^4}}$

③ $7(x+1)^{1/2} = \boxed{7\sqrt{x+1}}$

④ $5w^{-2/3} = \dfrac{5}{w^{2/3}} = \boxed{\dfrac{5}{\sqrt[3]{w^2}}}$

D. ① $\sqrt[3]{x^4} = \boxed{x^{4/3}}$

② $3\sqrt[5]{x^2} = \boxed{3x^{2/5}}$

③ $\sqrt[4]{(x+1)^3} = \boxed{(x+1)^{3/4}}$

④ $1/\sqrt[3]{x^2} = \dfrac{1}{x^{2/3}} = \boxed{x^{-2/3}}$

E. ① $m=-\frac{2}{3}$ and $(x_1,y_1)=(6,4)$
$y-y_1 = m(x-x_1)$
$y-4 = -\frac{2}{3}(x-6)$
$y-4 = -\frac{2}{3}x+4$
$y = -\frac{2}{3}x+4+4$
$\boxed{y = -\frac{2}{3}x+8}$

② $(x_1,y_1)=(3,1)$ and $(x_2,y_2)=(2,7)$
$m = \dfrac{y_2-y_1}{x_2-x_1} = \dfrac{7-1}{2-3} = \dfrac{6}{-1} = -6$
$y-y_1 = m(x-x_1)$
$y-1 = -6(x-3)$
$y-1 = -6x+18$
$y = -6x+18+1$
$\boxed{y = -6x+19}$

E. ③ m is undefined and line passes through $(2,3)$
Line: $\boxed{x=2}$

④ $m=0$ and line passes through $(4,-8)$
Line: $\boxed{y=-8}$

F. ① $\frac{1}{2}x+\frac{1}{3}(\frac{1}{2}x-1)=4$
$\frac{1}{2}x+\frac{1}{6}x-\frac{1}{3}=4$
$6\cdot\frac{1}{2}x+6\cdot\frac{1}{6}x-6\cdot\frac{1}{3}=6\cdot 4$
$3x+x-2=24$
$4x=24+2$
$4x=26$
$x=\frac{26}{4}$ → $\boxed{x=\frac{13}{2}}$

② $Ax+B = Cx+D$
$Ax-Cx = D-B$
$(A-C)x = D-B$
$\boxed{x = \dfrac{D-B}{A-C}}$

③ $\dfrac{1}{C_1}+\dfrac{1}{C_2}=\dfrac{1}{C}$
$C_1C_2C\cdot\frac{1}{C_1}+C_1C_2C\cdot\frac{1}{C_2}=C_1C_2C\cdot\frac{1}{C}$
$C_2C+C_1C = C_1C_2$
$C_2C = C_1C_2-C_1C$
$C_2C = (C_2-C)C_1$
$\boxed{\dfrac{C_2C}{C_2-C} = C_1}$

④ $xyy'+x = x^2y'+y^3$
$xyy'-x^2y' = y^3-x$
$(xy-x^2)y' = y^3-x$
$\boxed{y' = \dfrac{y^3-x}{xy-x^2}}$

Review of Precalculus Solutions

G. ① $\dfrac{1}{\sqrt{3}-\sqrt{2}} =$

$= \dfrac{1}{(\sqrt{3}-\sqrt{2})} \cdot \dfrac{(\sqrt{3}+\sqrt{2})}{(\sqrt{3}+\sqrt{2})}$

$= \dfrac{\sqrt{3}+\sqrt{2}}{(\sqrt{3})^2-(\sqrt{2})^2}$

$= \dfrac{\sqrt{3}+\sqrt{2}}{3-2}$

$= \boxed{\sqrt{3}+\sqrt{2}}$

② $\dfrac{3}{\sqrt{x}-1} =$

$= \dfrac{3}{(\sqrt{x}-1)} \cdot \dfrac{(\sqrt{x}+1)}{(\sqrt{x}+1)}$

$= \dfrac{3(\sqrt{x}+1)}{(\sqrt{x})^2-(1)^2}$

$= \boxed{\dfrac{3(\sqrt{x}+1)}{x-1}}$

③ $\dfrac{4x}{\sqrt{4+x}-2}$

$= \dfrac{4x}{(\sqrt{4+x}-2)} \cdot \dfrac{(\sqrt{4+x}+2)}{(\sqrt{4+x}+2)}$

$= \dfrac{4x(\sqrt{4+x}+2)}{(\sqrt{4+x})^2-(2)^2}$

$= \dfrac{4x(\sqrt{4+x}+2)}{4+x-4}$

$= \dfrac{4x(\sqrt{4+x}+2)}{x}$

$= \boxed{4(\sqrt{4+x}+2)}$

H. ① $f(1) = 3(1)^2+3(1)+1$

$= 3\cdot 1+3+1$

$= \boxed{7}$

$f(7) = 3(7)^2+3(7)+1$

$= 3(49)+21+1$

$= \boxed{169}$

$f(x+h) = 3(x+h)^2+3(x+h)+1$

$= 3(x^2+2xh+h^2)+3x+3h+1$

$= \boxed{3x^2+6xh+3h^2+3x+3h+1}$

② $g(0) = (0)^3 = \boxed{0}$ since $x=0<1$

$g(1) = (1)+1 = \boxed{2}$ since $x=1\geq 1$

$g(2) = (2)+1 = \boxed{3}$ since $x=2\geq 1$

③ $h(0)$ is undefined

$h(1) = 2$ $\quad h(4) = 4$

$h(2) = 1$ $\quad h(5) = 2$

$h(3) = 3$ $\quad h(6)$ is undefined.

I. ① $g(2) = 2(2)+3 = 7$

$f(g(2)) = f(7) = (7)^2+4 = \boxed{53}$

② $f(3) = (3)^2+4 = 13$

$g(f(3)) = g(13) = 2(13)+3 = \boxed{29}$

③ $f(g(x)) = [2x+3]^2+4$

$= 4x^2+12x+9+4$

$= \boxed{4x^2+12x+13}$

④ $g(f(x)) = 2[x^2+4]+3$

$= 2x^2+8+3$

$= \boxed{2x^2+11}$

J. ① $y = \sqrt{x^2+4}$

$\boxed{\text{Set } u = x^2+4, \quad y = \sqrt{u}}$

② $y = 3(x+7)^4$

$\boxed{u = x+7, \quad y = 3u^4}$

③ $y = 2\sin^3 x$

$y = 2(\sin x)^3$

$\boxed{u = \sin x, \quad y = 2u^3}$

④ $y = \sin(7x+4)$

$\boxed{u = 7x+4, \quad y = \sin u}$

K. ① $\log_2(3x+1) = 4$

$3x+1 = 2^4$

$3x+1 = 16$

$3x = 15$

$\boxed{x = 5}$

② $\ln(4x+8) - 2\ln 2 = 3$

$\ln(4x+8) - \ln 2^2 = 3$

$\ln\dfrac{(4x+8)}{2^2} = 3$

$\ln(x+2) = 3$

$x+2 = e^3$

$\boxed{x = e^3-2}$

③ $2^{4x-1} = 8$

$4x-1 = \log_2 8$

$4x-1 = 3$

$4x = 3+1$

$4x = 4$

$\boxed{x=1}$

④ $3e^{2x+1} = 6$

$e^{2x+1} = 2$

$2x+1 = \ln 2$

$2x = -1+\ln 2$

$\boxed{x = \dfrac{-1+\ln 2}{2}}$

Review of Precalculus Solutions

L. ① $3 + 2\sin^2 x + 2\cos^2 x$

$= 3 + 2(\sin^2 x + \cos^2 x)$

$= 3 + 2 \cdot 1$

$= \boxed{5}$

② $\sin x \cot x + \cos x$

$= \sin x \cdot \dfrac{\cos x}{\sin x} + \cos x$

$= \cos x + \cos x$

$= \boxed{2\cos x}$

③ $4 + 4\tan^2 x + \dfrac{3}{\cos^2 x}$

$= 4(1 + \tan^2 x) + 3\sec^2 x$

$= 4\sec^2 x + 3\sec^2 x$

$= \boxed{7\sec^2 x}$

④ $4\sin^2 x \cos^2 x$

$= (2\sin x \cos x)^2$

$= (\sin 2x)^2$

$= \boxed{\sin^2 2x}$

M. ① $2\sin x - 1 = 0$

$\sin x = \frac{1}{2}$

$\boxed{x = \frac{\pi}{6} \text{ or } x = \frac{5\pi}{6}}$

② $\sqrt{3}\tan x + 1 = 0$

$\tan x = -\frac{1}{\sqrt{3}}$

$\boxed{x = \frac{5\pi}{6} \text{ or } x = \frac{11\pi}{6}}$

③ $2\cos x + \sqrt{3} = 0$

$\cos x = -\frac{\sqrt{3}}{2}$

$\boxed{x = \frac{5\pi}{6} \text{ or } x = \frac{7\pi}{6}}$

④ $2\cos^2 x - 1 = 0$

$\cos^2 x = \frac{1}{2}$

$\cos x = \pm\frac{1}{\sqrt{2}}$

$\cos x = \pm\frac{\sqrt{2}}{2}$

$\boxed{x = \frac{\pi}{4} \text{ or } x = \frac{3\pi}{4} \text{ or } x = \frac{5\pi}{4} \text{ or } x = \frac{7\pi}{4}}$

N.

① $\frac{\sqrt{3}}{2}$ ② $-\sqrt{3}/2$ ③ 0 ④ -2

⑤ 1 ⑥ $-\sqrt{2}$ ⑦ $\frac{\pi}{3}$ ⑧ $\frac{\pi}{6}$

⑨ $\frac{\pi}{6}$ ⑩ $\frac{2\pi}{3}$ ⑪ $-\frac{\pi}{3}$ ⑫ $\frac{\pi}{2}$

O. See separate sheet.

P. ① $\ln \dfrac{x^2 y^3}{z^4}$

$= \ln x^2 + \ln y^3 - \ln z^4$

$= \boxed{2\ln x + 3\ln y - 4\ln z}$

② $\ln \dfrac{(x+1)^4 x^5}{(x+y)^6}$

$= \ln(x+1)^4 + \ln x^5 - \ln(x+y)^6$

$= \boxed{4\ln(x+1) + 5\ln x - 6\ln(x+y)}$

③ $\ln \dfrac{\sqrt[3]{(x+1)^2}\, x^4}{x^3 \sqrt[4]{x}}$

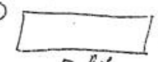
$= \ln \dfrac{(x+1)^{2/3} x^4}{x^3 x^{1/4}}$

$= \ln(x+1)^{2/3} + \ln x^4 - \ln x^3 - \ln x^{1/4}$

$= \frac{2}{3}\ln(x+1) + 4\ln x - 3\ln x - \frac{1}{4}\ln x$

$= \boxed{\frac{2}{3}\ln(x+1) + \frac{3}{4}\ln x}$

Q. ①

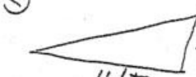


$A = LW = (5)(2) = \boxed{10\text{ ft}^2}$

$P = 2L + 2W$

$= 2(5) + 2(2) = \boxed{14\text{ ft}}$

②



$A = \frac{1}{2}bh = \frac{1}{2}(4)(3) = \boxed{6\text{ ft}^2}$

$c^2 = a^2 + b^2$

$c^2 = 3^2 + 4^2$

$c^2 = 25$

$c = 5$

$P = a + b + c$

$P = 5 + 3 + 4 = \boxed{12\text{ ft}}$

③ $A = \pi R^2 = \pi(2)^2 = \boxed{4\pi \text{ in}^2}$

$C = 2\pi R = 2\pi(2) = \boxed{4\pi \text{ in}}$

R. ① $V = LWH$

$= (4)(3)(2) = \boxed{24\text{ ft}^3}$

$S = 2LW + 2HW + 2LH$

$= 2(4)(3) + 2(2)(3) + 2(4)(2)$

$= \boxed{52\text{ ft}^2}$

② $V = \pi R^2 H$

$= \pi(2)^2(5) = \boxed{20\pi\text{ ft}^3}$

$S = 2\pi RH + 2\pi R^2$

$= 2\pi(2)(5) + 2\pi(2)^2$

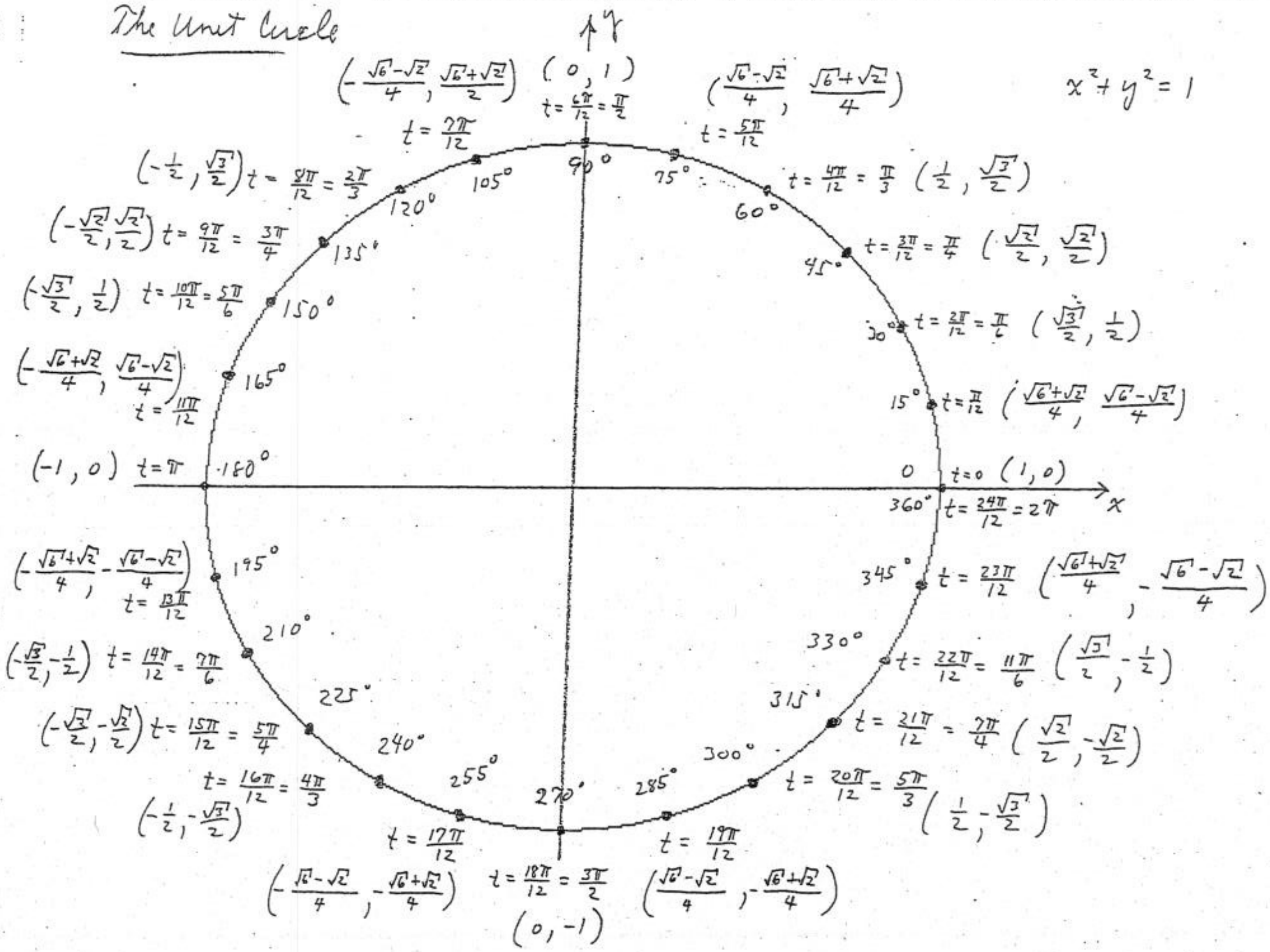
$= \boxed{28\pi\text{ ft}^2}$

③ $V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi(3)^3 = \boxed{36\pi\text{ ft}^3}$

$S = 4\pi R^2 = 4\pi(3)^2 = \boxed{36\pi\text{ ft}^2}$

The Unit Circle

$x^2 + y^2 = 1$



Special Values for Inverse Trig Functions

x	$\sin x$
$-\frac{\pi}{2}$	-1
$-\frac{\pi}{3}$	$-\sqrt{3}/2$
$-\frac{\pi}{4}$	$-\sqrt{2}/2$
$-\frac{\pi}{6}$	$-1/2$
0	0
$\frac{\pi}{6}$	$1/2$
$\frac{\pi}{4}$	$\sqrt{2}/2$
$\frac{\pi}{3}$	$\sqrt{3}/2$
$\frac{\pi}{2}$	1

domain of Sin x

$[-\frac{\pi}{2}, \frac{\pi}{2}]$

x	$\cos x$
0	1
$\frac{\pi}{6}$	$\sqrt{3}/2$
$\frac{\pi}{4}$	$\sqrt{2}/2$
$\frac{\pi}{3}$	$1/2$
$\frac{\pi}{2}$	0
$\frac{2\pi}{3}$	$-1/2$
$\frac{3\pi}{4}$	$-\sqrt{2}/2$
$\frac{5\pi}{6}$	$-\sqrt{3}/2$
π	-1

domain of Cos x

$[0, \pi]$

x	$\tan x$
$-\frac{\pi}{2}$	u.D. $\to -\infty$
$-\frac{\pi}{3}$	$-\sqrt{3}$
$-\frac{\pi}{4}$	-1
$-\frac{\pi}{6}$	$-\sqrt{3}/3$
0	0
$\frac{\pi}{6}$	$\sqrt{3}/3$
$\frac{\pi}{4}$	1
$\frac{\pi}{3}$	$\sqrt{3}$
$\frac{\pi}{2}$	u.D. $\to +\infty$

domain of Tan x

$(-\frac{\pi}{2}, \frac{\pi}{2})$