21. Find the limit. (If the limit is infinite, enter '∞' or '-∞', as appropriate. If the limit does not otherwise exist, enter DNE.)

$$\lim_{x \to \infty} \left(\sqrt{81x^2 + x} - 9x \right)$$

note: $\dfrac{\underbrace{(\sqrt{81x^2+x}}_{(A} - \underbrace{9x)}_{B)} \cdot \underbrace{(\sqrt{81x^2+x}}_{(A} + \underbrace{9x)}_{B)}}{(1) \cdot (\sqrt{81x^2+x} + 9x)}$

$$= \frac{\left(\sqrt{81x^2+x}\right)^2 - (9x)^2}{\sqrt{81x^2+x} + 9x}$$

note: $(A-B)(A+B) = A^2 - B^2$

$$= \frac{\cancel{81x^2} + x - \cancel{81x^2}}{\sqrt{81x^2+x} + 9x}$$

$$= \frac{(x)/x}{\left(\sqrt{81x^2+x} + 9x\right)/x}$$

$$= \frac{1}{\frac{\sqrt{81x^2+x}}{x} + \frac{9\cancel{x}}{\cancel{x}}}$$

$x = \sqrt{x^2}$

if $x \geq 0$

$$= \frac{1}{\frac{\sqrt{81x^2+x}}{\sqrt{x^2}} + 9}$$

$$= \frac{1}{\sqrt{\frac{81x^2+x}{x^2}} + 9}$$

$$= \frac{1}{\sqrt{\frac{81\cancel{x^2}}{\cancel{x^2}} + \frac{\cancel{x}^{\,1}}{\cancel{x^2}_{\,x}}} + 9}$$

$$= \boxed{\frac{1}{\sqrt{81+\frac{1}{x}} + 9}}$$

$$\Rightarrow \lim_{x\to\infty}\left(\sqrt{81x^2+9} - 9x\right)$$

$$= \lim_{x\to\infty}\left(\frac{1}{\sqrt{81+\cancel{\frac{1}{x}}^{\,\to 0}} + 9}\right)$$

Note:

$$\lim_{x\to\pm\infty} \frac{1}{x} = 0$$

$$\overset{x\to\infty}{=} \frac{1}{\sqrt{81+0} + 9}$$

$$= \frac{1}{\sqrt{81} + 9}$$

$$= \frac{1}{9+9} = \boxed{\frac{1}{18}}$$

(22) $$\lim_{x\to\infty} \frac{(x^4-7x^2+x)/x^3}{(x^3-x+4)/x^3}$$

$$= \lim_{x\to\infty} \frac{\frac{\cancel{x^4}^{\,x}}{\cancel{x^3}} - \frac{7\cancel{x^2}}{\cancel{x^3}_{\,1}} + \frac{\cancel{x}^{\,1}}{\cancel{x^3}_{\,2}}}{\frac{\cancel{x^3}^{\,1}}{\cancel{x^3}} - \frac{\cancel{x}^{\,1}}{\cancel{x^3}_{\,2}} + \frac{4}{x^3}}$$

$$= \lim_{x\to\infty} \frac{\cancel{x}^{\to\infty} - \cancel{\frac{7}{x}}^{\to 0} + \cancel{\frac{1}{x^2}}^{\to 0}}{1 - \cancel{\frac{1}{x^2}}^{\to 0} + \cancel{\frac{4}{x^3}}^{\to 0}} = \frac{\infty}{1} = \boxed{\infty}$$

note:

(1) $$\lim_{x\to\pm\infty} x = \pm\infty$$

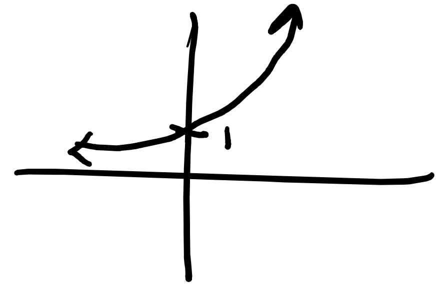
(2) $$\lim_{x\to\pm\infty} \frac{1}{x} = 0$$

(3) $$\lim_{x\to\pm\infty} \frac{c}{x^n} = 0$$

c = constant, n = positive Integer

(4) $$\lim_{x\to\pm\infty} c = c$$

(24) note: $y = e^x \Rightarrow$



$\Rightarrow$ as $x \to \infty$, $e^x \to \infty$

$$\Rightarrow \lim_{x\to\infty} \frac{3 - e^x}{3 + 7e^x} \Rightarrow \boxed{\text{Let } y = e^x}$$

$$\Rightarrow \lim_{x\to\infty} \frac{3 - e^x}{3 + 7e^x}$$

$\boxed{\text{as } x \to \infty,\ e^x \to \infty \ \Rightarrow y \to \infty}$

$$= \lim_{y\to\infty} \frac{(3 - y)/y}{(3 + 7y)/y}$$

$$= \lim_{y\to\infty} \frac{\frac{3}{y} - \frac{\cancel{y}}{\cancel{y}}1}{\frac{3}{y} - \frac{7\cancel{y}}{\cancel{y}}}$$

$$= \lim_{y\to\infty} \frac{\cancel{\frac{3}{y}}\to 0 - 1}{\cancel{\frac{3}{y}}\to 0 - 7}$$

$$= \frac{0 - 1}{0 - 7} = \frac{-1}{-7} = \boxed{\frac{1}{7}}$$

Find the limit. (If the limit is infinite, enter '∞' or '-∞', as appropriate. If the limit does not otherwise exist, enter DNE.)

$$\lim_{x\to\infty}\left(e^{-2x}\cos(x)\right)$$

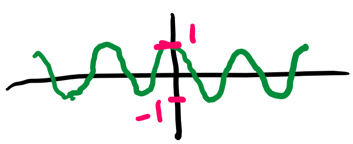
Recall from precalculus:

① Since $e \approx 2.71828\cdots$, $e^{-2x} > 0$

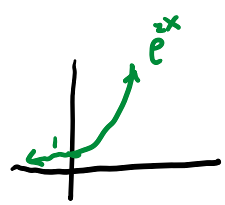
↑ positive number raised to any exponent is positive

② $-1 \le \cos(x) \le 1$

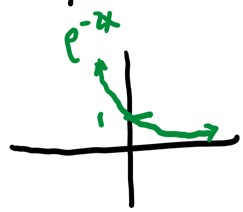
Recall graph of $y=\cos(x)$



③ Graph of $y=e^{2x}$:

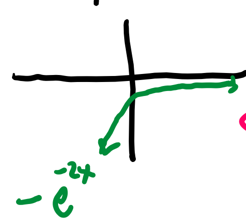


$\Rightarrow$ Graph of $y=e^{-2x}$:



← Reflect e^{2x} across y-axis

$\Rightarrow$ Graph of $y=-e^{-2x}$:



← Reflect e^{-2x} across x-axis

$$\Rightarrow \lim_{x\to\infty}\left(e^{-2x}\right)=0 \quad \text{and} \quad \lim_{x\to\infty}\left(-e^{-2x}\right)=0$$

So, $$-1 \le \cos(x) \le 1$$

$$\Rightarrow \quad e^{-2x} \cdot (-1) \le e^{-2x} \cdot \cos(x) \le e^{-2x} \cdot (1)$$

$$\Rightarrow \quad \underbrace{-e^{-2x}}_{f(x)} \le \underbrace{e^{-2x}\cos(x)}_{g(x)} \le \underbrace{e^{-2x}}_{h(x)}$$

From sec. 2.3:

- **The Squeeze Theorem** If $f(x) \le g(x) \le h(x)$ when x is near a (except possibly at a) and

$$\lim_{x \to a} f(x) = \lim_{x \to a} h(x) = L$$

then

$$\lim_{x \to a} g(x) = L$$

$\Rightarrow$ Since $\lim_{x \to \infty} \left(-e^{-2x}\right) = \lim_{x \to \infty} \left(e^{-2x}\right) = 0$,

$$\lim_{x \to \infty} e^{-2x}\cos(x) = \boxed{0}$$

Answer.

Find the horizontal and vertical asymptotes of the curve. You may want to use a graphing calculator (or computer) to check your work by graphing the curve and estimating the asymptotes. (Enter your answers as comma-separated lists. If an answer does not exist, enter DNE.)

$$y = \frac{x^3 - x}{x^2 - 5x + 4}$$

vertical asymptote:

note:

$$\frac{x^3 - x}{x^2 - 5x + 4} = \frac{x(x^2 - 1)}{(x-4)(x-1)}$$

$$= \frac{x(x+1)\cancel{(x-1)}}{(x-4)\cancel{(x-1)}}$$

$$= \frac{x(x+1)}{x-4} \quad x - 4 \leftrightarrow x - 4 = 0$$

$$\Rightarrow \quad x = 4$$

$\Rightarrow$ vertical asymptote is $\boxed{x = 4}$

Horizontal asymptote:

$$\lim_{x \to \infty} y = \lim_{x \to \infty} \frac{x^3 - x}{x^2 - 5x + 4}$$

$$= \lim_{x \to \infty} \frac{x(x+1)}{x-4} \Big\} \text{ from last page}$$

$$= \lim_{x \to \infty} \left(\frac{x^2 + x}{x - 4} \right)$$

$$= \lim_{x \to \infty} \frac{(x^2 + x) \cdot 1/x}{(x - 4) \cdot 1/x}$$

$$= \lim_{x \to \infty} \left(\frac{\frac{x^2}{x} + \frac{x}{x}}{\frac{x}{x} - \frac{4}{x}} \right)$$

$$= \lim_{x \to \infty} \left(\frac{\cancel{x}^{\to \infty} + 1}{1 - \cancel{\frac{4}{x}}^{\to 0}} \right)$$

$$= \infty \Rightarrow \boxed{DNE} \Leftarrow \text{Answer}$$

Note: same for $x \to -\infty$ $\Rightarrow$ No horizontal asymptote

Find the limit. (If the limit is infinite, enter '∞' or '-∞', as appropriate. If the limit does not otherwise exist, enter DNE.)

$$\underbrace{\lim_{x \to -\infty}}_{x<0} \left(\sqrt{25x^2 + 7x} + 5x\right) = \boxed{-\frac{7}{10}} \leftarrow \text{Answer}$$

note: $\dfrac{\left(\sqrt{25x^2+7x} + 5x\right)}{(1)} \cdot \dfrac{\left(\sqrt{25x^2+7x} - 5x\right)}{\left(\sqrt{25x^2+7x} - 5x\right)}$

Recall:
$(A+B)(A-B)$
$= A^2 - B^2$
Here, $A = \sqrt{25x^2+7x}$
$B = 5x$

$$= \frac{\left(\sqrt{25x^2+7x}\right)^2 - (5x)^2}{\sqrt{25x^2+7x} - 5x}$$

$$= \frac{\cancel{25x^2} + 7x - \cancel{25x^2}}{\sqrt{25x^2+7x} - 5x}$$

$$= \frac{(7x) \cdot \frac{1}{x}}{\left(\sqrt{25x^2+7x} - 5x\right) \cdot \frac{1}{x}}$$

$$= \frac{\frac{7\cancel{x}}{\cancel{x}}}{\frac{\sqrt{25x^2+7x}}{\sqrt{x^2}} - \frac{5\cancel{x}}{\cancel{x}}}$$

note: $\sqrt{x^2} = |x|$
$\Rightarrow x \geq 0, \sqrt{x^2} = x$
$x < 0, \sqrt{x^2} = -x$

$$= \frac{7}{-\sqrt{\frac{25x^2+7x}{x^2}} - 5}$$

$$= \frac{7}{-\sqrt{\frac{25\cancel{x^2}}{\cancel{x^2}} + \frac{7\cancel{x}}{\cancel{x^2}x}} - 5}$$

$$= \frac{7}{-\sqrt{25+\frac{7}{x}} - 5}$$

$$\Rightarrow \lim_{x\to-\infty} \left(\sqrt{25x^2+7x} - 5x\right)$$

$$= \lim_{x\to-\infty} \frac{7}{-\sqrt{25+\cancel{\frac{7}{x}}\to 0} - 5}$$

$$= \frac{7}{-\sqrt{25+0} - 5} = \frac{7}{-\sqrt{25}-5} = \frac{7}{-5-5} = \frac{7}{-10} = \boxed{-\frac{7}{10}}$$

$$\lim_{x \to -\infty} \left(\sqrt{9x^2+7x} + 3x \right)$$

Note: $$\frac{\left(\sqrt{9x^2+7x} + 3x\right)}{(1)} \cdot \frac{\left(\sqrt{9x^2+7x} - 3x\right)}{\left(\sqrt{9x^2+7x} - 3x\right)}$$

$$= \frac{\left(\sqrt{9x^2+7x}\right)^2 - (3x)^2}{\sqrt{9x^2+7x} - 3x}$$

$$= \frac{9x^2+7x-9x^2}{\sqrt{9x^2+7x} - 3x}$$

$$= \frac{7x}{\sqrt{9x^2+7x} - 3x}$$

$$\Rightarrow \lim_{x \to -\infty} \left(\sqrt{9x^2+7x} + 3x \right)$$

$$= \lim_{x \to -\infty} \left(\frac{7x}{\sqrt{9x^2+7x} - 3x} \right)$$

$$= \lim_{x \to -\infty} \frac{(7x) \cdot 1/x}{\left(\sqrt{9x^2+7x} - 3x\right) \cdot 1/x}$$

$$= \lim_{x \to -\infty} \left(\frac{\frac{7x}{x}}{\frac{\sqrt{9x^2+7x}}{x} - \frac{3x}{x}} \right)$$

$$= \lim_{x \to -\infty} \left(\frac{7}{\frac{\sqrt{9x^2+7x}}{-\sqrt{x^2}} - 3} \right)$$

<u>Note</u>: $x \to -\infty \Rightarrow \sqrt{x^2} = -x \Rightarrow x = -\sqrt{x^2}$

$$= \lim_{x \to -\infty} \frac{7}{-\sqrt{\frac{9x^2}{x^2} + \frac{7x}{x^2}} - 3}$$

$$= \lim_{x \to -\infty} \frac{7}{-\sqrt{9 + \cancel{\frac{7}{x}}^{\to 0}} - 3}$$

$$= \frac{7}{-\sqrt{9+0} - 3}$$

$$= \frac{7}{-\sqrt{9} - 3}$$

$$= \frac{7}{-3-3} = \frac{7}{-6} = \boxed{-\frac{7}{6}}$$

32. Find the limit. (If the limit is infinite, enter '∞' or '-∞', as appropriate. If the limit does not otherwise exist, enter DNE.)

$$\lim_{x\to\infty} \frac{4u^4+4}{(u^2-9)(4u^2-1)}$$

$$= \lim_{x\to\infty} \frac{(4u^4+4)\cdot\frac{1}{u^4}}{(u^4-37u^2+9)\cdot\frac{1}{u^4}}$$

$$= \lim_{x\to\infty} \frac{\frac{4\cancel{u^4}}{\cancel{u^4}}+\frac{4}{u^4}}{1\frac{\cancel{u^4}}{\cancel{u^4}}-\frac{37\cancel{u^2}}{u^{\cancel{4}2}}+\frac{9}{u^4}}$$

$$= \lim_{x\to\infty} \frac{4+\cancel{\frac{4}{u^4}}\to 0}{1-\cancel{\frac{37}{u^2}}\to 0+\cancel{\frac{9}{u^4}}\to 0}$$

$$= \frac{4+0}{1-0+0} = \frac{4}{1} = \boxed{4}$$

note:

$$(u^2-9)(4u^2-1)$$

$$= u^4 - u^2 - 36u^2 + 9$$

$$= u^4 - 37u^2 + 9$$

Recall:

$$\lim_{x\to\pm\infty} \frac{c}{x^n} = 0$$

$n>0$, c = constant.

33. Find the limit. (If the limit is infinite, enter '∞' or '-∞', as appropriate. If the limit does not otherwise exist, enter DNE.)

$$\lim_{x\to\infty} \frac{x+2}{\sqrt{49x^2+1}}$$

$x>0$

note: $\sqrt{x^2} = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x<0 \end{cases}$

$$= \lim_{x\to\infty} \frac{(x+2)\cdot 1/x}{\left(\sqrt{49x^2+1}\right)\cdot 1/\sqrt{x^2}}$$

$$\frac{\sqrt{49x^2+1}}{1}\cdot\frac{1}{\sqrt{x^2}} = \frac{\sqrt{49x^2+1}}{\sqrt{x^2}} = \sqrt{\frac{49x^2+1}{x^2}}$$

$$= \lim_{x\to\infty} \frac{\frac{\cancel{x}}{\cancel{x}}+\frac{2}{x}}{\sqrt{\frac{49x^2+1}{x^2}}}$$

$$= \lim_{x\to\infty} \frac{1+\frac{2}{x}}{\sqrt{\frac{49\cancel{x^2}}{\cancel{x^2}}+\frac{1}{x^2}}}$$

$$= \lim_{x\to\infty} \frac{1+\cancel{\frac{2}{x}}\to 0}{\sqrt{49+\cancel{\frac{1}{x^2}}\to 0}} = \frac{1+0}{\sqrt{49+0}} = \frac{1}{\sqrt{49}} = \boxed{\frac{1}{7}}$$

Answer

34] Find the limit. (If the limit is infinite, enter '∞' or '-∞', as appropriate. If the limit does not otherwise exist, enter DNE.)

$$\lim_{x \to \infty} \frac{(x + x^3 + x^5) \cdot 1/x^4}{(5 - x^2 + x^4) \cdot 1/x^4}$$

$$= \lim_{x \to \infty} \frac{\frac{x}{x^4} + \frac{x^3}{x^4} + \frac{x^5}{x^4}}{\frac{5}{x^4} - \frac{x^2}{x^4} + \frac{x^4}{x^4}}$$

$$= \lim_{x \to \infty} \frac{\frac{1}{x^3} \to 0 + \frac{1}{x} \to 0 + x \to \infty}{\frac{5}{x^4} \to 0 - \frac{1}{x^2} \to 0 + 1}$$

$$\to \frac{0 + 0 + \infty}{0 - 0 + 1} \to \frac{\infty}{1} \to \boxed{\infty}$$

40. Find an equation of the tangent line to the graph of *f* at the given point.

$$y = x^3 - 2x + 2, \quad (3, 23)$$

$$m = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} (3x^2 + \cancel{3xh}^{0} + \cancel{h^2}^{0} - 2)$$

$$\Rightarrow \boxed{m = 3x^2 - 2}$$

note: $f(x) = x^3 - 2x + 2$

Step1: $f(x+h) = (x+h)^3 - 2(x+h) + 2$

$$= \underbrace{x^3 + 3x^2h + 3xh^2 + h^3}_{(x+h)^3} - 2x - 2h + 2$$

Step2: $f(x+h) - f(x)$

$$= \underbrace{(x^3 + 3x^2h + 3xh^2 + h^3 - 2x - 2h + 2)}_{f(x+h)} - \underbrace{(x^3 - 2x + 2)}_{f(x)}$$

$$= \cancel{x^3} + 3x^2h + 3xh^2 + h^3 \cancel{- 2x} - 2h \cancel{+ 2} \cancel{- x^3} \cancel{+ 2x} \cancel{- 2}$$

$$= 3x^2h + 3xh^2 + h^3 - 2h$$

Step3: $\frac{f(x+h) - f(x)}{h} = \frac{3x^2h + 3xh^2 + h^3 - 2h}{h}$

$$= \frac{\cancel{h}^{1}(3x^2 + 3xh + h^2 - 2)}{\cancel{h}} = 3x^2 + 3xh + h^2 - 2$$

$\Rightarrow$ at $(3, 23)$, $x = 3$

$\Rightarrow m = 3x^2 - 2$ becomes $m = 3(3)^2 - 2$

$= 3(9) - 2$

$= 27 - 2 = 25$

<u>Recall</u>: point-slope form for equation of a line

$$y - y_1 = m(x - x_1)$$

Here, $x_1 = 3$, $y_1 = 23$, $m = 25$

$$\Rightarrow \quad y - 23 = 25(x - 3)$$

$$y - \cancel{23} = 25x - 75 \Rightarrow \boxed{y = 25x - 52}$$

$$+\cancel{23} \qquad\qquad +23$$

41. (a) Find the slope, m, of the tangent to the curve $y = 5 + 5x^2 - 2x^3$ at the point where $x = a$.

note: $\boxed{m_{tan} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$

$x = a \Rightarrow m_{tan} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$

$\boxed{f(x) = 5 + 5x^2 - 2x^3}$

Step 1: $f(a+h) = 5 + 5(a+h)^2 - 2(a+h)^3$

$= 5 + 5(a^2 + 2ah + h^2) - 2(a^3 + 3a^2h + 3ah^2 + h^3)$

$= 5 + 5a^2 + 10ah + 5h^2 - 2a^3 - 6a^2h - 6ah^2 - 2h^3$

Step 2: $f(a+h) - f(a)$

$= (5 + 5a^2 + 10ah + 5h^2 - 2a^3 - 6a^2h - 6ah^2 - 2h^3) - (5 + 5a^2 - 2a^3)$

$= \cancel{5} + \cancel{5a^2} + 10ah + 5h^2 \cancel{-2a^3} - 6a^2h - 6ah^2 - 2h^3 \cancel{-5} \cancel{-5a^2} \cancel{+2a^3}$

$= 10ah + 5h^2 - 6a^2h - 6ah^2 - 2h^3$

Step3 $\dfrac{f(a+h)-f(a)}{h}$

$$= \frac{10ah+5h^2-6a^2h-6ah^2-2h^3}{h}$$

$$= \frac{10a\cancel{h}}{\cancel{h}} + \frac{5\cancel{h^2}^{\,h}}{\cancel{h}} - \frac{6a^2\cancel{h}}{\cancel{h}} - \frac{6a\cancel{h^2}^{\,h}}{\cancel{h}} - \frac{2\cancel{h^3}^{\,h^2}}{\cancel{h}}$$

$$= 10a+5h-6a^2-6ah-2h^2$$

Step4:

$$m_{tan}\Big|_{x=a} = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$$

$$= \lim_{h\to 0} 10a+5\cancel{h}^{\to 0}-6a^2-6\cancel{ah}^{\to 0}-2\cancel{h^2}^{\to 0}$$

$$= \boxed{10a-6a^2}$$

$=$

(b)Find equations of the tangent lines at the following points.

$(1, 8)$ $\Rightarrow$ @ $x=1 \Rightarrow a=1 \Rightarrow m_{tan} = 10(1) - 6(1)^2$

x_1 $f(x) = y_1$

$= 10 - 6 \cdot 1$

$= 10 - 6 = \boxed{4}$

Recall: point-slope form for equation of a line:

$$y - y_1 = m(x - x_1)$$

Here, $x_1 = 1$, $y_1 = 8$, $m = 4$

$$\Rightarrow \quad y - 8 = 4(x - 1)$$

$$y - \cancel{8} = 4x - 4$$

$$\cancel{+8} \qquad +8$$

$$\boxed{y = 4x + 4}$$

@ $(2, 9)$ $\Rightarrow a = 2$, $m = 10(2) - 6(2)^2$

x_1 y_1

$= 20 - 6(4)$

$= 20 - 24 = \boxed{-4}$

$$\Rightarrow \quad y - 9 = -4(x-2)$$

$$y - \cancel{9} = -4x + 8$$

$$\cancel{+9} \qquad\qquad +9$$

$$\boxed{y = -4x + 17}$$

4b

$$\frac{f(a+h) - f(a)}{h}$$

$$= \frac{\dfrac{2(a+h)+2}{(a+h)+5} - \dfrac{2a+2}{a+5}}{h} \quad \leftarrow \text{LCD} = (a+h+5)(a+5)$$

$$= \frac{\dfrac{(2a+2h+2)\cdot(a+5)}{(a+h+5)\cdot(a+5)} - \dfrac{(2a+2)\cdot(a+h+5)}{(a+5)\cdot(a+h+5)}}{h}$$

$$(2a^2+10a+2ah+10h+2a+10) \qquad (2a^2+2ah+10a+2a+2h+10)$$

$$= \frac{\dfrac{(2a+h+2)(a+5) - (2a+2)(a+h+5)}{(a+h+5)(a+5)}}{\dfrac{h}{1}}$$

$$= \frac{8\cancel{h}^{1}}{(a+h+5)(a+5)} \cdot \frac{1}{\cancel{h}_{1}}$$

$$= \frac{8}{(a+h+5)(a+5)}$$

$$\Rightarrow f'(a) = \lim_{h \to 0} \frac{8}{(a+\cancel{h}^{\to 0}+5)(a+5)}$$

$$= \frac{8}{(a+0+5)(a+5)} = \frac{8}{(a+5)(a+5)} = \boxed{\frac{8}{(a+5)^2}}$$

43] If a ball is thrown into the air with a velocity of 37 ft/s, its height (in feet) after t seconds is given by $y = 37t - 16t^2$. Find the velocity when $t = 1$.

Note:

velocity at time $t = \lim_{h \to 0} \dfrac{f(t+h) - f(t)}{h}$

Here, $t = 1$, $y = f(t) = 37t - 16t^2$

So, velocity at time $t = 1$ is

$$\lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$$

*we will find this limit in steps to make it easier.

step 1: find $f(1+h)$

Recall: $f(t) = 37t - 16t^2$

So $f(1+h) = 37(1+h) - 16(1+h)^2$

$= 37(1+h) - 16\underbrace{(1+h)(1+h)}_{\text{FOIL}}$

$= 37 + 37h - 16(1 + \underbrace{h + h} + h^2)$

$= 37 + 37h - 16(1 + 2h + h^2)$

$= 37 + 37h - 16 - 32h - 16h^2$

$= 21 + 5h - 16h^2$

$\boxed{37 - 16}$ $\boxed{37h - 32h}$

Step 2: find $f(1)$

$f(1) = 37(1) - 16(1)^2$

$= 37 - 16 = 21$

Step 3: Find $\dfrac{f(1+h) - f(1)}{h}$

from step 1, $f(1+h) = 21 + 5h - 16h^2$

from Step 2, $f(1) = 21$

so $\dfrac{f(1+h) - f(1)}{h}$

$$= \frac{\overbrace{(21 + 5h - 16h^2)}^{f(1+h)} - \overbrace{21}^{f(1)}}{h}$$

$$= \frac{\cancel{21} + 5h - 16h^2 - \cancel{21}}{h}$$

$$= \frac{5h - 16h^2}{h} = \frac{\cancel{h}(5 - 16h)}{\cancel{h}}$$

$$= 5 - 16h$$

Step 4: Find the limit.

$$\lim_{h\to 0} \frac{f(1+h)-f(1)}{h}$$

$$= \lim_{h\to 0} \left(\underbrace{5-16h}_{\text{from Step 3}}\right)$$

$$= 5 - 16(0)$$

$$= 5 - 0 = \boxed{5}$$

So, the velocity at $t = 1$ is

$$\boxed{5 \text{ ft/sec}}$$

Answer

(51) Given $f(x) = |x-5|$

note: $|x-5| = \begin{cases} x-5 & \text{if } x-5 \geq 0 \\ \underbrace{-(x-5)}_{-x+5} & \text{if } x-5 < 0 \end{cases} = \begin{cases} x-5 & \text{if } x \geq 5 \\ 5-x & \text{if } x<5 \end{cases}$

$$\Rightarrow f(x) = \begin{cases} x-5 & \text{if } x \geq 5 \\ 5-x & \text{if } x<5 \end{cases}$$

$\boxed{f(5) = |5-5| = |0| = 0}$

$$\Rightarrow \lim_{x \to 5^+} \frac{f(x) - f(5)}{x-5} = \lim_{x\to 5^+} \frac{\overbrace{x-5}^{f(x)} - 0}{x-5}$$

$x>5$

$\Rightarrow f(x) = x-5$

$$= \lim_{x\to 5^+} \frac{\cancel{x-5}}{\cancel{x-5}}$$

$$= \lim_{x\to 5^+} 1 = \boxed{1}$$

and:

$$\lim_{x\to 5^-} \frac{f(x)-f(5)}{x-5} = \lim_{x\to 5^-} \frac{\overbrace{5-x}^{f(x)} - \underbrace{0}_{f(5)=|5-5|=|0|=0}}{x-5}$$

$x<5$

$\Rightarrow f(x)=5-x$

$$= \lim_{x\to 5^-} \frac{-x+5}{x-5}$$

$$= \lim_{x\to 5^-} \frac{-1\cancel{(x-5)}}{\cancel{x-5}}$$

$$= \lim_{x\to 5^-} -1 = \boxed{-1}$$

note: since $\displaystyle\lim_{x\to 5^+} \frac{f(x)-f(5)}{x-5} \neq \lim_{x\to 5^-} \frac{f(x)-f(5)}{x-5}$

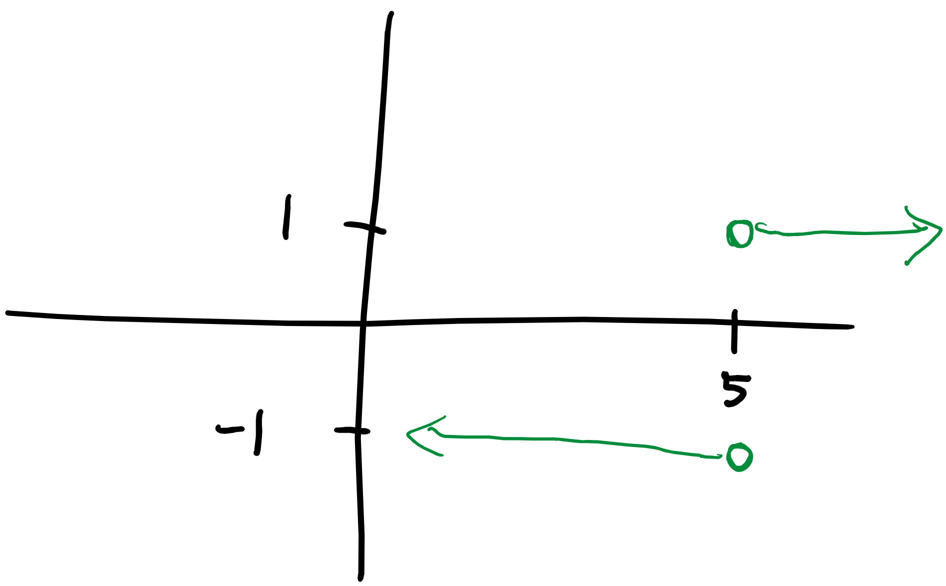
$\Rightarrow f'(5) = \displaystyle\lim_{x\to 5} \frac{f(x)-f(5)}{x-5}$ does not exist

$\Rightarrow f$ is NOT differentiable at $x=5$

However, $f'(x) = 1$ if $x > 5$

and $f'(x) = -1$ if $x < 5$

$\Rightarrow$ Graph of $f'(x)$ is



Show that *f* is continuous on $(-\infty, \infty)$.

$$f(x) = \begin{cases} 1 - x^2 & if\ x \le 1 \\ \ln(x) & if\ x > 1 \end{cases}$$

Notes:

(1)Theorem

The following types of functions are continuous at every number in their domains.

- Polynomials
- Rational functions
- Root functions
- Inverse trigonometric functions
- Trigonometric functions
- Exponential functions
- Logarithmic functions

(2) A function *f* is continuous at a number *a* if

$$\lim_{x \to a} f(x) = f(a)$$

That is, the following must be true:

- ✓ $f(a)$ is defined (that is, *a* is in the domain of *f*) ← $f(1) = 1 - (1)^2 = 1 - 1 = \boxed{0}$ ← $f(a)$
- ✓ $\lim_{x \to a} f(x)$ exists
- ✓ $\lim_{x \to a} f(x) = f(a)$

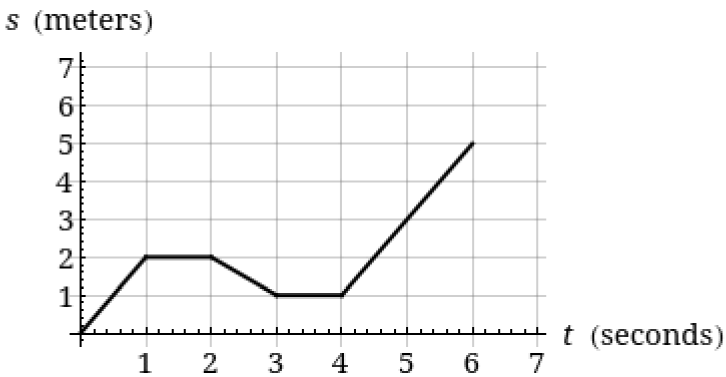
@ $\boxed{x = 1}$ ↑ $a = 1$

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (1 - x^2) = 1 - (1)^2 = 1 - 1 = \boxed{0}$$

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} \ln(x) = \ln(1) = \boxed{0}$$

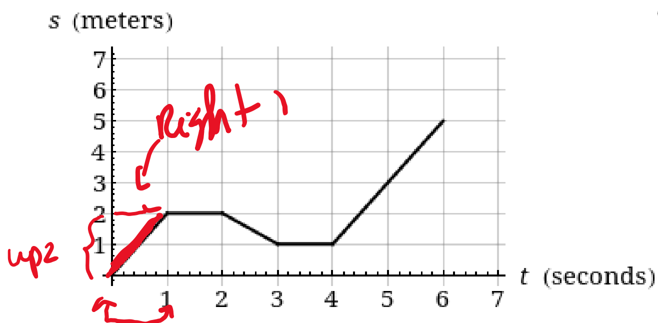
$$\Rightarrow \lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = \lim_{x \to 1} f(x) = \boxed{0}$$

42. A particle starts by moving to the right along a horizontal line; the graph of its position function is shown in the figure.

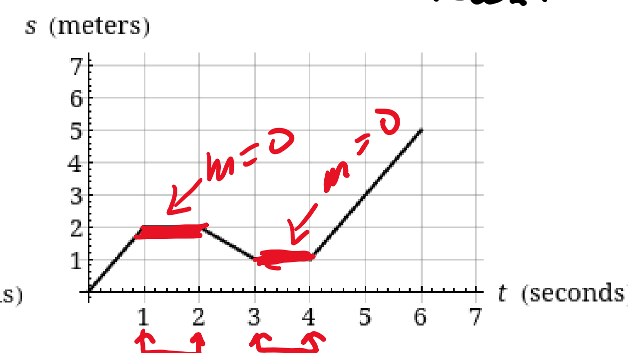


(b) Draw a graph of the velocity function.

Recall: $m = \frac{\text{Rise}}{\text{Run}}$.

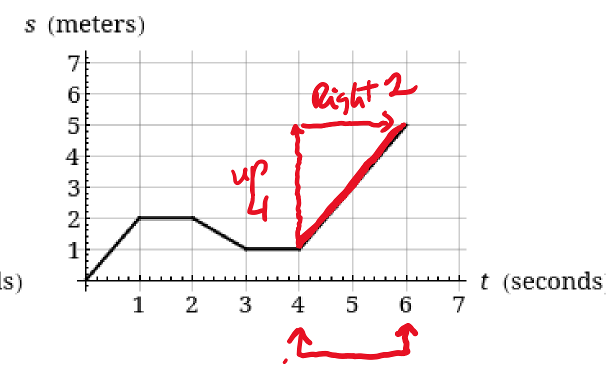


$\boxed{0 < t < 1}$, $m_{tan} = \frac{2}{1} = \boxed{2 = v}$



$\boxed{1 < t < 2}$, $m_{tan} = \boxed{v = 0}$

$\boxed{3 < t < 4}$, $m_{tan} = \boxed{v = 0}$

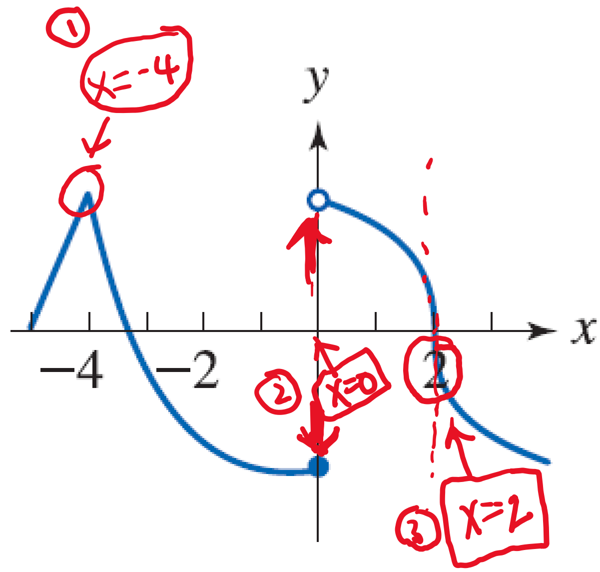


$\boxed{2 < t < 3}$, $m_{tan} = \frac{-1}{1} = \boxed{-1 = v}$

$\boxed{4 < t < 6}$, $m_{tan} = \frac{4}{2} = \boxed{2 = v}$

looking for x-coordinates..

50. The graph of f is given. State the numbers at which f is not differentiable. (Enter your answers as a comma-separated list.)



$\Rightarrow \boxed{x = -4, 0, 2}$

Answers

Recall: Functions are not differentiable at **corners**, **discontinuities**, or **vertical tangents**.

1 2