A_0 $r = 0.0185$

48. If \$4,500 is invested at 1.85% interest, find the value (in dollars) of the investment at the end of 6 years if the interest is compounded as follows. (Round your answers to the nearest cent.)

t

note: Compound Interest

$$A = A_0\left(1+\frac{r}{n}\right)^{nt} \Rightarrow A = 4500\left(1+\frac{0.0185}{n}\right)^{\overbrace{n\cdot 6}^{6n}}$$

A = Amt. after time t

A_0 = Initial Amt.

r = Interest rate (changed to a decimal)

n = no. of times interest is compounded per year

(i) annually $\Rightarrow n = 1$

$$A = 4500\left(1+\frac{0.0185}{1}\right)^{6\cdot 1}$$

$$= 4500\,(1.0185)^6$$

$$\approx \boxed{5023.18}$$

(ii) Quarterly $\Rightarrow n = 4$

$$A = 4500\left(1 + \frac{0.0185}{4}\right)^{6 \cdot 4}$$

$$\Rightarrow A = 4500(1.004625)^{24}$$

$$\Rightarrow A \approx 5026.99$$

(iii) $n = 12$

(iv) $n = 52$

(v) $n = 365$

(vi) continuously $\Rightarrow$ use $A = A_0 e^{rt}$

$$\Rightarrow A = 4500\, e^{0.0185 \cdot 6}$$

$$\Rightarrow A \approx 5028.28$$

(b) $$A = 4500\, e^{0.0185t}$$

$$\frac{dA}{dt} = 4500 \cdot \left[\frac{d}{dt}\left(e^{0.0185t}\right)\right]$$

$$= 4500\left[e^{0.0185t} \cdot \underbrace{\frac{d}{dt}(0.0185t)}\right]$$

$$= 4500\left[e^{0.0185t} \cdot 0.0185\right]$$

$$= \underbrace{4500\, e^{0.0185t}} \cdot 0.0185$$

$$= A \cdot 0.0185$$

$$\boxed{\frac{dA}{dt} = 0.0185A}$$

$$t=0 \Rightarrow A(0) = 4500\, e^{0.0185(0)}$$

$$= 4500\, e^{0}$$

$$= 4500 \cdot 1$$

$$= \boxed{4500}$$

① $\frac{d}{dx}\left(x^2 - 10xy + y^2\right) = \frac{d}{dx}(10)$

$$\frac{d}{dx}(x^2) - \underbrace{\frac{d}{dx}(10x\cdot y)}_{\text{use product Rule}} + \frac{d}{dx}(y^2) = 0$$

$$2x - \left[\frac{d}{dx}(10x)\cdot y + 10x\cdot\frac{d}{dx}(y)\right] + 2y\cdot\frac{dy}{dx} = 0$$

$$2x - \left[10\cdot y + 10x\cdot\frac{dy}{dx}\right] + 2y\frac{dy}{dx} = 0$$

$$\cancel{2x} - \cancel{10y} - 10x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$$

$$\cancel{-2x} + \cancel{10y} \qquad\qquad -2x \quad +10y$$

$$-10x\frac{dy}{dx} + 2y\frac{dy}{dx} = 10y - 2x$$

$$\frac{\frac{dy}{dx}\cancel{(-10x+2y)}}{\cancel{(-10x+2y)}} = \frac{10y-2x}{-10x+2y}$$

$$\Rightarrow \frac{dy}{dx} = \frac{10y-2x}{2y-10x} = \frac{\cancel{2}(5y-x)}{\cancel{2}(y-5x)} = \boxed{\frac{5y-x}{y-5x}}$$

10. Find the derivative of the function.

$$y = \arctan\left(\sqrt{\frac{1+x}{1-x}}\right)$$

Note: $\boxed{\arctan = \tan^{-1} \quad \text{and} \quad \frac{d}{dx}\left(\tan^{-1} u\right) = \frac{1}{1+u^2} \cdot \frac{du}{dx}}$

Here, $u = \sqrt{\frac{1+x}{1-x}}$

$$\Rightarrow \frac{du}{dx} = \frac{d}{dx}\left(\sqrt{\frac{1+x}{1-x}}\right)$$

$$= \frac{d}{dx}\left[\left(\frac{1+x}{1-x}\right)^{1/2}\right]$$

$$= \frac{1}{2}\left(\frac{1+x}{1-x}\right)^{1/2-1} \cdot \frac{d}{dx}\left(\frac{1+x}{1-x}\right)$$

$$= \frac{1}{2}\left(\frac{1+x}{1-x}\right)^{-1/2} \cdot \frac{(1-x)\cdot\frac{d}{dx}(1+x) - (1+x)\cdot\frac{d}{dx}(1-x)}{(1-x)^2}$$

$$= \frac{1}{2}\left(\frac{1-x}{1+x}\right)^{1/2} \cdot \frac{(1-x).1 - (1+x).(-1)}{(1-x)^2}$$

$$= \frac{1}{2} \cdot \frac{(1-x)^{1/2}}{(1+x)^{1/2}} \cdot \frac{1-x+1+x}{(1-x)^2}$$

$$= \frac{(1-x)^{1/2}}{2(1+x)^{1/2}} \cdot \frac{2}{(1-x)^2}$$

$$= \frac{\cancel{2}(1-x)^{1/2}}{\cancel{2}(1+x)^{1/2}(1-x)^2}$$

Note: $\frac{(1-x)^{1/2}}{(1-x)^2}$

$= \frac{1}{(1-x)^{2-1/2}}$

$= \frac{1}{(1-x)^{3/2}}$

$$= \underbrace{\frac{1}{(1+x)^{1/2}(1-x)^{3/2}}}$$

$$\Rightarrow y' = \frac{1}{1+u^2} \cdot \frac{du}{dx}$$

$$= \frac{1}{1+\left(\sqrt{\frac{1+x}{1-x}}\right)^2} \cdot \frac{1}{(1+x)^{1/2}(1-x)^{3/2}}$$

Note: from algebra:

$\frac{1}{1+\frac{1+x}{1-x}} \frac{(1-x)}{(1-x)}$

$= \frac{1-x}{1(1-x)+\frac{1+x}{\cancel{1-x}} \cdot \cancel{(1-x)}^{1}}$

$= \frac{1-x}{1-x+1+x}$

$$= \frac{1}{1+\frac{1+x}{1-x}} \cdot \frac{1}{(1+x)^{1/2}(1-x)^{3/2}}$$

$$= \frac{1-x}{1-x+1+x} \cdot \frac{1}{(1+x)^{1/2}(1-x)^{3/2}}$$

$$= \frac{1-x}{2} \cdot \frac{1}{(1+x)^{1/2}(1-x)^{3/2}}$$

$$= \frac{1-x}{2(1+x)^{1/2}(1-x)^{3/2}}$$

Note: $\frac{1-x}{(1-x)^{3/2}} = \frac{(1-x)^1}{(1-x)^{3/2}} = \frac{1}{(1-x)^{3/2-1}} = \frac{1}{(1-x)^{1/2}}$

$$= \frac{1}{2(1+x)^{1/2}(1-x)^{1/2}}$$

$$= \frac{1}{2\sqrt{1+x}\,\sqrt{1-x}}$$

$$= \frac{1}{2\sqrt{(1+x)(1-x)}}$$

$$= \boxed{\frac{1}{2\sqrt{1-x^2}}}$$

Consider the following.

$$\cos(x) + \sqrt{y} = 6$$

(a) Find y' by implicit differentiation.

$$\frac{d}{dx}\left[\cos(x) + \sqrt{y}\right] = \frac{d}{dx}[6]$$

$$\frac{d}{dx}[\cos(x)] + \frac{d}{dx}\left[y^{\frac{1}{2}}\right] = 0$$

$$-\sin(x) + \frac{1}{2}y^{\frac{1}{2}-1} \cdot y' = 0$$

$$-\sin(x) + \frac{1}{2}y^{-1/2} \cdot y' = 0$$

$$-\sin(x) + \frac{1}{2} \cdot \frac{1}{y^{1/2}} \cdot y' = 0$$

$$\cancel{-\sin(x)} + \frac{1}{2\sqrt{y}} \cdot y' = 0 + \sin(x)$$

$$\cancel{+\sin(x)}$$

$$\frac{\cancel{2\sqrt{y}}}{1} \cdot \frac{1}{\cancel{2\sqrt{y}}} y' = \sin(x) \cdot 2\sqrt{y}$$

$$\boxed{y' = 2\sqrt{y} \cdot \sin(x)}$$

(b) Solve the equation explicitly for y and differentiate to get y' in terms of x.

$$\cancel{\cos(x)} + \sqrt{y} = 6 - \cos(x)$$

$$\cancel{-\cos(x)}$$

$$(\sqrt{y})^2 = (6 - \cos(x))^2$$

$$y = (6 - \cos(x))^2$$

using Chain Rule $\Rightarrow y' = 2(6-\cos(x))^1 \frac{d}{dx}[6-\cos(x)]$

$$\Rightarrow y' = 2(6-\cos(x)) \cdot [\underbrace{0-(-\sin(x))}_{\sin(x)}]$$

$$\Rightarrow \boxed{y' = 2(6-\cos(x)) \cdot \sin(x)}$$

(c) Check that your solutions to parts (a) and (b) are consistent by substituting the expression for y into your solution for part (a).

From (a), we have $y' = 2\sqrt{y} \cdot \sin(x)$

From (b), we have $y = (6-\cos(x))^2$

$$\Rightarrow y' = 2\sqrt{(6-\cos(x))^2} \cdot \sin(x)$$

$$= 2|6-\cos(x)| \cdot \sin(x)$$

$$= 2\,{}^*(6-\cos(x)) \cdot \sin(x)$$

$$\Rightarrow \boxed{y' = 2(6-\cos(x))\sin(x)}$$

OR $\boxed{y' = 2\sin(x)(6-\cos(x))}$

Recall: $\boxed{\sqrt{x^2} = |x|}$

*note:
$-1 \le \cos(x) \le 1$
$\Rightarrow 6-\cos(x) > 0$
$\Rightarrow |6-\cos(x)| = 6-\cos(x)$

13] Find the derivative of the function. $G(x) = \sqrt{1-9x^2}\ \arccos(3x)$

note: $G(x) = (1-9x^2)^{1/2} \arccos(3x)$

$$\Rightarrow G'(x) = (1-9x^2)^{1/2} \cdot \frac{d}{dx}\left[\arccos(3x)\right] + \arccos(3x) \cdot \frac{d}{dx}\left[(1-9x^2)^{1/2}\right]$$

$$= (1-9x^2)^{1/2} \cdot \left[-\frac{1}{\sqrt{1-(3x)^2}} \cdot \underbrace{\frac{d}{dx}(3x)}_{3}\right] + \arccos(3x) \cdot \frac{1}{2}(1-9x^2)^{-1/2} \cdot \frac{d}{dx}(1-9x^2)$$

$$= \sqrt{1-9x^2}\left[-\frac{3}{\sqrt{1-9x^2}}\right] + \arccos(3x) \cdot \frac{1}{2\sqrt{1-9x^2}} \cdot (-18x)$$

$$= \frac{-3\sqrt{1-9x^2}}{\sqrt{1-9x^2}} - \frac{18x \arccos(3x)}{2\sqrt{1-9x^2}}$$

$$= -3 - \frac{9x \arccos(3x)}{\sqrt{1-9x^2}}$$

Find the derivative of the function.

$$F(\theta) = \arcsin\left(\sqrt{\sin(3\theta)}\right)$$

Recall:

$$\boxed{\frac{d}{d\theta}\left[\arcsin(u)\right] = \frac{1}{\sqrt{1-u^2}} \cdot \frac{du}{d\theta}}$$

Here, $u = \sqrt{\sin(3\theta)} = \left[\sin(3\theta)\right]^{1/2}$

$$\Rightarrow \frac{du}{d\theta} = \frac{1}{2}\left[\sin(3\theta)\right]^{1/2-1} \cdot \frac{d}{d\theta}\left[\sin(3\theta)\right]$$

$$= \frac{1}{2}\left[\sin(3\theta)\right]^{-1/2} \cdot \cos(3\theta) \cdot \frac{d}{d\theta}(3\theta)$$

$$= \frac{1}{2} \cdot \frac{1}{\left(\sin(3\theta)\right)^{1/2}} \cdot \frac{\cos(3\theta)}{1} \cdot \frac{3}{1}$$

$$= \frac{3\cos(3\theta)}{2\sqrt{\sin(3\theta)}}$$

$$\Rightarrow F'(\theta) = \frac{1}{\sqrt{1-u^2}} \cdot \frac{du}{dx}$$

becomes

$$F'(\theta) = \frac{1}{\sqrt{1-\left(\sqrt{\sin(3\theta)}\right)^2}} \cdot \frac{3\cos(3\theta)}{2\sqrt{\sin(3\theta)}}$$

$$= \frac{3\cos(3\theta)}{\sqrt{1-\sin(3\theta)} \cdot 2\sqrt{\sin(3\theta)}}$$

$$= \boxed{\frac{3\cos(3\theta)}{2\sqrt{1-\sin(3\theta)}\sqrt{\sin(3\theta)}}}$$

Differentiate the function.

$$f(x) = \log_{10}(x^6 + 2)$$

Recall: $\boxed{\frac{d}{dx}(\log_b u) = \frac{1}{u\ln(b)} \cdot \frac{du}{dx}}$

Here, $u = x^6 + 2$, $b = 10$

$$\Rightarrow f'(x) = \frac{1}{(x^6+2)\ln(10)} \cdot \underbrace{\frac{d}{dx}(x^6+2)}$$

$$= \frac{1}{(x^6+2)\ln(10)} \cdot \frac{6x^5}{1}$$

$$= \boxed{\frac{6x^5}{(x^6+2)\ln(10)}}$$

17. Differentiate the function.

$$F(t) = (\ln(t))^2 \cos(t)$$

$$F'(t) = \frac{d}{dt}\left[(\ln(t))^2 \cos(t)\right] \leftarrow \boxed{\text{use Product Rule}}$$

$$= \frac{d}{dt}\left[(\ln(t))^2\right] \cdot \cos(t) + (\ln(t))^2 \cdot \frac{d}{dt}\left[\cos(t)\right]$$

$$= \overbrace{2(\ln(t)) \cdot \frac{1}{t}}^{\text{Chain Rule}} \cdot \cos(t) + (\ln(t))^2 \cdot \left[-\sin(t)\right]$$

$$= \frac{2(\ln(t))\cos(t)}{t} - (\ln(t))^2 \sin(t)$$

__OR:__ $$\boxed{(\ln(t))\left(\frac{2\cos(t)}{t} - \ln(t)\sin(t)\right)}$$

Find y' and y''.

Note:

$$\boxed{\frac{d}{dx}\left[\ln(u)\right] = \frac{1}{u}\cdot\frac{du}{dx}}$$

$$y = \frac{\ln(7x)}{x^7}$$

$$\Rightarrow y' = \frac{x^7\cdot\frac{d}{dx}\left[\ln(7x)\right] - \ln(7x)\cdot\frac{d}{dx}(x^7)}{(x^7)^2}$$

$$= \frac{x^7\cdot\frac{1}{7x}\cdot\overbrace{\frac{d}{dx}(7x)}^{7} - \ln(7x)\cdot 7x^6}{x^{14}}$$

$$= \frac{\frac{\cancel{7}x^{\cancel{7}\,6}}{\cancel{7}\cancel{x}} - 7x^6\ln(7x)}{x^{14}}$$

$$= \frac{x^6 - 7x^6\ln(7x)}{x^{14}} \leftarrow \boxed{GCF = x^6}$$

$$= \frac{\cancel{x^6}\left[1 - 7\ln(7x)\right]}{x^{14-6}}$$

$$\Rightarrow \; y' = \boxed{\frac{1 - 7\ln(7x)}{x^8}}$$

$$y'' = \frac{d}{dx}\left[\frac{1 - 7\ln(7x)}{x^8}\right]$$

$$= \frac{x^8 \cdot \frac{d}{dx}\left[1 - 7\ln(7x)\right] - \left[1 - 7\ln(7x)\right] \cdot \frac{d}{dx}(x^8)}{(x^8)^2}$$

$$= \frac{x^8 \cdot \left[0 - \cancel{7}^{1} \cdot \frac{1}{\cancel{7}_{1} x} \cdot \overbrace{\frac{d}{dx}(7x)}^{7}\right] - \left[1 - 7\ln(7x)\right] \cdot (8x^7)}{x^{16}}$$

$$= \frac{x^{\cancel{8}\,7}\left[-\frac{7}{\cancel{x}}\right] - 8x^7\left[1 - 7\ln(7x)\right]}{x^{16}}$$

$$= \frac{-7x^7 - 8x^7\left[1 - 7\ln(7x)\right]}{x^{16}}$$

$$= \frac{-7x^7 - 8x^7 + 56x^7 \ln(7x)}{x^{16}}$$

$$= \frac{-15x^7 + 56x^7 \ln(7x)}{x^{16}}$$

$$= \frac{\cancel{x^7}\left[-15 + 56\ln(7x)\right]}{x^{16-7}}$$

$$= \frac{-15 + 56\ln(7x)}{x^9}$$

<u>OR</u> $$y'' = \boxed{\frac{56 + \ln(7x) - 15}{x^9}}$$

(2) $\frac{d}{dx}\left(x\cdot e^{y}\right) = \frac{d}{dx}\left(x - y\right)$

$$\underbrace{\frac{d}{dx}(x)}\cdot e^{y} + x\cdot\frac{d}{dx}\left(e^{y}\right) = \frac{d}{dx}(x) - \frac{d}{dx}(y)$$

$$1\cdot e^{y} + x\cdot e^{y}\cdot\underbrace{\frac{d}{dx}(y)}_{\frac{dy}{dx}} = 1 - \frac{dy}{dx}$$

$$e^{y} + xe^{y}\frac{dy}{dx} = 1 - \cancel{\frac{dy}{dx}}$$

$$+\frac{dy}{dx} \qquad \cancel{+\frac{dy}{dx}}$$

$$\cancel{e^{y}} + xe^{y}\frac{dy}{dx} + \frac{dy}{dx} = 1 - e^{y}$$

$$-\cancel{e^{y}}$$

$$xe^{y}\frac{dy}{dx} + 1\cdot\frac{dy}{dx} = 1 - e^{y}$$

$$\frac{\frac{dy}{dx}\left(xe^{y}+1\right)}{\left(xe^{y}+1\right)} = \frac{1-e^{y}}{\left(xe^{y}+1\right)}$$

$$\Rightarrow \quad \frac{dy}{dx} = \boxed{\frac{1-e^{y}}{xe^{y}+1}}$$

Use logarithmic differentiation to find the derivative of the function.

$$y = x^{3\sin(x)}$$

$$\ln(y) = \ln\left(x^{3\sin(x)}\right) \leftarrow$$ Recall: $\boxed{\ln x^n = n\ln(x)}$

$$\Rightarrow \ln(y) = 3\sin(x)\cdot\ln(x)$$

$$\Rightarrow \frac{d}{dx}\Big(\ln(y)\Big) = \frac{d}{dx}\Big(3\sin(x)\cdot\ln(x)\Big) \leftarrow$$ use product Rule

$$\Rightarrow \frac{1}{y}\cdot\frac{dy}{dx} = \underbrace{\frac{d}{dx}\big(3\sin(x)\big)}\cdot\ln(x) + 3\sin(x)\cdot\underbrace{\frac{d}{dx}\big(\ln(x)\big)}$$

$$\Rightarrow \frac{1}{y}\cdot\frac{dy}{dx} = 3\cos(x)\cdot\ln(x) + 3\sin(x)\cdot\frac{1}{x}$$

$$\Rightarrow \frac{1}{y}\cdot\frac{dy}{dx} = 3\cos(x)\cdot\ln(x) + \frac{3\sin(x)}{x}$$

$$\Rightarrow \cancel{y}\cdot\left(\frac{1}{\cancel{y}}\cdot\frac{dy}{dx}\right) = \left(3\cos(x)\ln(x) + \frac{3\sin(x)}{x}\right)\cdot y$$

$$\Rightarrow \frac{dy}{dx} = 3\left(\cos(x)\cdot\ln(x) + \frac{\sin(x)}{x}\right)\cdot x^{3\sin(x)}$$

$$\Rightarrow \frac{dy}{dx} = \boxed{3x^{3\sin(x)}\left(\cos(x)\ln(x) + \frac{\sin(x)}{x}\right)}$$

Use logarithmic differentiation to find the derivative of the function.

$$y = (\sin(2x))^x$$

$$\ln(y) = \ln\left(\sin(2x)\right)^x$$

$$\Rightarrow \ln(y) = x \cdot \ln(\sin(2x))$$

$$\Rightarrow \frac{d}{dx}\left(\ln(y)\right) = \frac{d}{dx}\left(x \cdot \ln(\sin(2x))\right)$$

Recall:

$$\boxed{\frac{d}{dx}\left(\ln(u)\right) = \frac{1}{u} \cdot \frac{du}{dx}}$$

$$\Rightarrow \frac{1}{y} \cdot \frac{dy}{dx} = \underbrace{\frac{d}{dx}(x)} \cdot \ln(\sin(2x)) + \overbrace{x \cdot \frac{d}{dx}\left(\ln(\underbrace{\sin(2x)}_{u})\right)}$$

$$\Rightarrow \frac{1}{y} \cdot \frac{dy}{dx} = 1 \cdot \ln(\sin(2x)) + x \cdot \frac{1}{\sin(2x)} \cdot \underbrace{\frac{d}{dx}\left(\sin(2x)\right)}_{\text{use chain rule}}$$

$$\Rightarrow \frac{1}{y} \cdot \frac{dy}{dx} = \ln(\sin(2x)) + \frac{x}{\sin(2x)} \cdot \cos(2x) \cdot \underbrace{\frac{d}{dx}(2x)}_{2}$$

$$\Rightarrow \frac{1}{y} \cdot \frac{dy}{dx} = \ln(\sin(2x)) + \frac{2x\cos(2x)}{\sin(2x)}$$

$$\Rightarrow \cancel{y} \cdot \left(\frac{1}{\cancel{y}} \cdot \frac{dy}{dx}\right) = \left(\ln(\sin(2x)) + \frac{2x\cos(2x)}{\sin(2x)}\right) \cdot y$$

$$\Rightarrow \frac{dy}{dx} = \left(\ln(\sin(2x)) + \frac{2x\cos(2x)}{\sin(2x)}\right) \cdot (\sin(2x))^x$$

$$\Rightarrow \frac{dy}{dx} = \boxed{(\sin(2x))^x\left(\ln(\sin(2x)) + 2x\cot(2x)\right)}$$

32. A particle moves according to a law of motion $s = f(t)$, $t \geq 0$, where t is measured in seconds and s in feet. (If an answer does not exist, enter DNE.)

$f(t) = t^3 - 9t^2 + 24t$

(a) Find the velocity (in ft/s) at time t.

note: velocity $= v = s' = f'$

$$\Rightarrow f'(t) = \frac{d}{dt}\left(t^3 - 9t^2 + 24t\right)$$

$$= 3t^2 - 18t + 24$$

$$\Rightarrow \boxed{v(t) = 3t^2 - 18t + 24}$$

(b) What is the velocity (in ft/s) after $\underbrace{\text{1 second}}_{t=1}$?

$$\Rightarrow v(1) = 3(1)^2 - 18(1) + 24$$

$$= 3(1) - 18 + 24$$

$$= 3 - 18 + 24 = \boxed{9} \text{ ft/s}$$

(c) When is the particle at rest? (Enter your answers as a comma-separated list.)

note: "at rest" $\Rightarrow v(t) = 0$

$$\Rightarrow 3t^2 - 18t + 24 = 0$$

$$\Rightarrow 3(t^2 - 6t + 8) = 0$$

$$\Rightarrow 3(t-2)(t-4) = 0$$

$$\Rightarrow t - 2 = 0 \quad \text{OR} \quad t - 4 = 0$$

$$\Rightarrow t = 2 \quad \text{OR} \quad t = 4$$

$$\Rightarrow t = \boxed{2, 4}$$

(d) When is the particle moving in the positive direction? (Enter your answer using interval notation.)

Note: Particle is moving in the positive direction when $v(t) > 0$

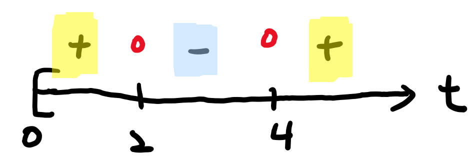
$$\Rightarrow 3t^2 - 18t + 24 > 0$$

$$\Rightarrow 3(t^2 - 6t + 8) > 0$$

$$\Rightarrow 3(t-2)(t-4) > 0$$

$$\Rightarrow (t-2)(t-4) > 0$$

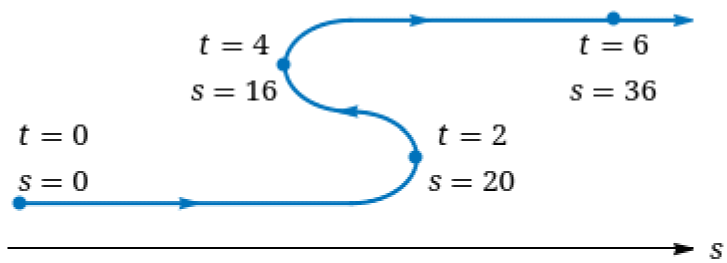
From precalculus:



Interval	Test Point	$(t-2)$	$(t-4)$	$(t-2)(t-4)$
$[0, 2)$	1	$1-2= -$	$1-4= -$	$- \; - = +$
$(2, 4)$	3	$3-2= +$	$3-4= -$	$+ \; - = -$
$(4, \infty)$	5	$5-2= +$	$5-4= +$	$+ \; + = +$

$$\Rightarrow v(t) > 0 \text{ on } \boxed{[0, 2) \cup (4, \infty)}$$

(e) Draw a diagram to illustrate the motion of the particle and use it to find the total distance (in ft) traveled during the first 6 seconds.



from $t=0$ to $t=2$, distance traveled $= |f(2) - f(0)|$

$$= |20 - 0|$$

$$= |20| = 20 \text{ ft}$$

from $t=2$ to $t=4$, distance traveled $= |f(4) - f(2)|$

NOTE: partical changed direction since $v(t)<0$ on $(2, 4)$

$$= |16 - 20|$$

$$= |-4| = 4 \text{ ft}$$

from $t=4$ to $t=6$, distance traveled $= |f(6) - f(4)|$

$$= |36 - 16|$$

$$= |20| = 20 \text{ ft}$$

$\Rightarrow$ total distance traveled $= 20 \text{ ft} + 4 \text{ ft} + 20 \text{ ft}$

$$= \boxed{44} \text{ ft}$$

(f) Find the acceleration (in ft/s^2) at time t.

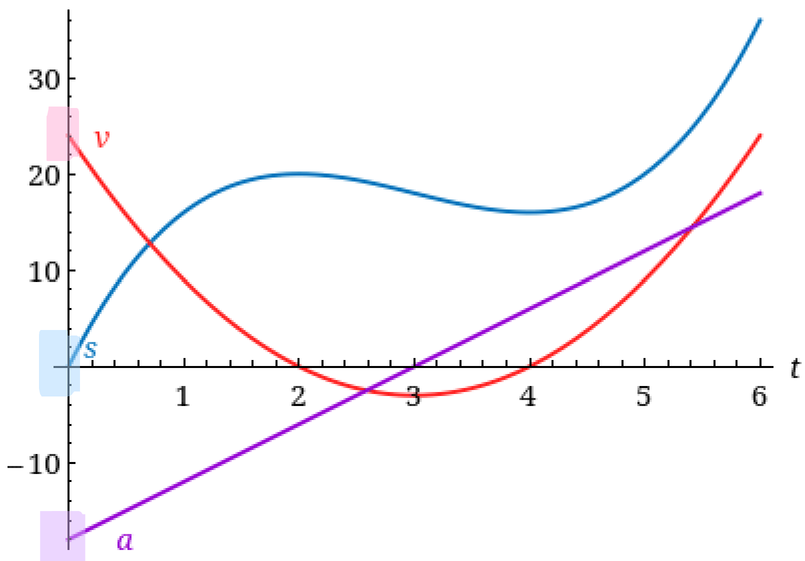
$$\text{acceleration} = a(t) = v'(t) = \frac{d}{dt}\left(3t^2 - 18t + 24\right)$$

$$\Rightarrow a(t) = 6t - 18$$

Find the acceleration (in ft/s^2) after 1 second.

$$a(1) = 6(1) - 18 = 6 - 18 = \boxed{-12} \text{ ft/s}^2$$

(g)



Note: This is the only choice where $s(0) = 0$, $v(0) = 24$, and $a(0) = -18$

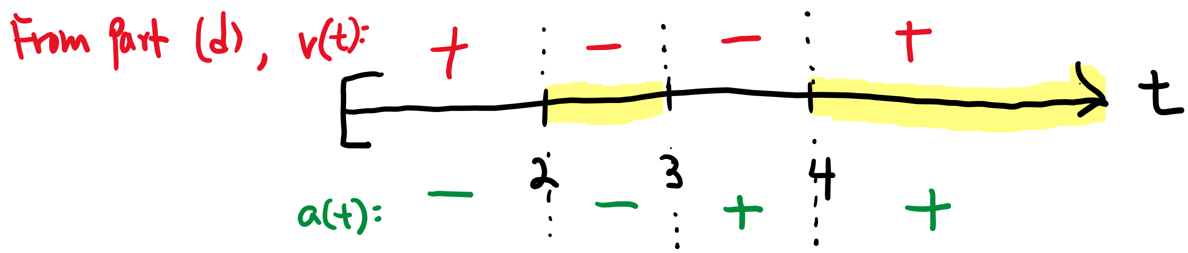
(h) When is the particle speeding up? (Enter your answer using interval notation.)

Note: partical is speeding up if $v(t)$ and $a(t)$ have the same sign.

Here, $a(t) > 0 \Rightarrow 6t - 18 > 0$

$\Rightarrow 6t > 18$

$\Rightarrow t > 3 \Rightarrow a(t) < 0$ when $0 \le t < 3$



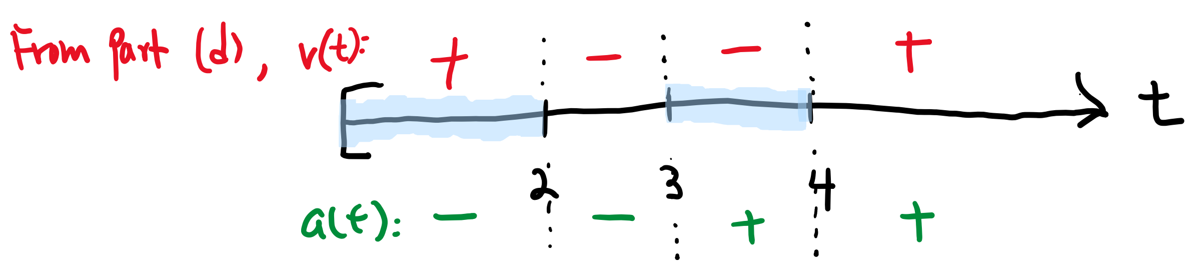
$\Rightarrow$ Both $v(t)$ and $a(t)$ are negative on $(2, 3)$

Both $v(t)$ and $a(t)$ are positive on $(4, \infty)$

$\Rightarrow$ particle is speeding up on $\boxed{(2, 3), (4, \infty)}$

When is it slowing down? (Enter your answer using interval notation.)

Note: partical is slowing down when $a(t)$ and $v(t)$ have opposite signs.

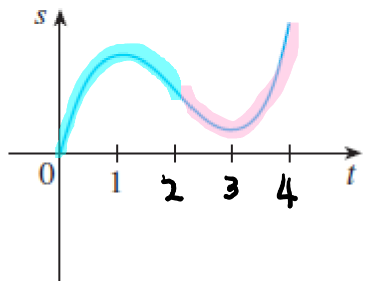


$\Rightarrow$ partical is slowing down on $\boxed{[0, 2), (3, 4)}$

$\leftarrow s(t) \Rightarrow v(t) = s'(t) = m_{tan}$

34] Graphs of the *position* functions of two particles are shown, where *t* is measured in seconds.

(a)



$v(t) > 0$ on $(0,1) \cup (3,4)$

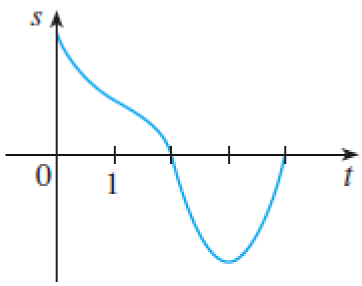
$v(t) < 0$ on $(1,3)$

note: $a(t) = v'(t) = s''(t)$

$a(t) > 0$ on $(2,4)$

$a(t) < 0$ on $(0,2)$

(b)

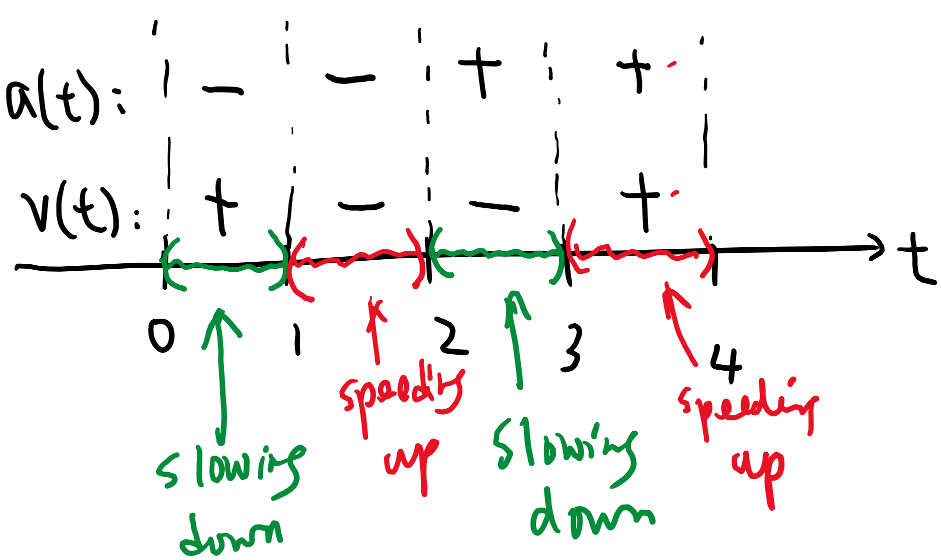


When is the particle in figure (a) speeding up? (Enter your answer using interval notation.)

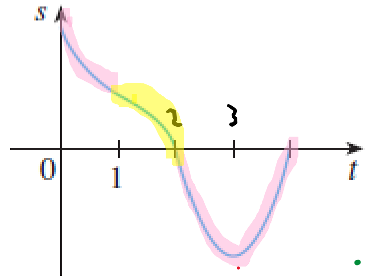
$\boxed{(1,2) \cup (3,4)}$

When is the particle in figure (a) slowing down? (Enter your answer using interval notation.)

$\boxed{(0,1) \cup (2,3)}$



(b)



$V(t) = s'(t)$

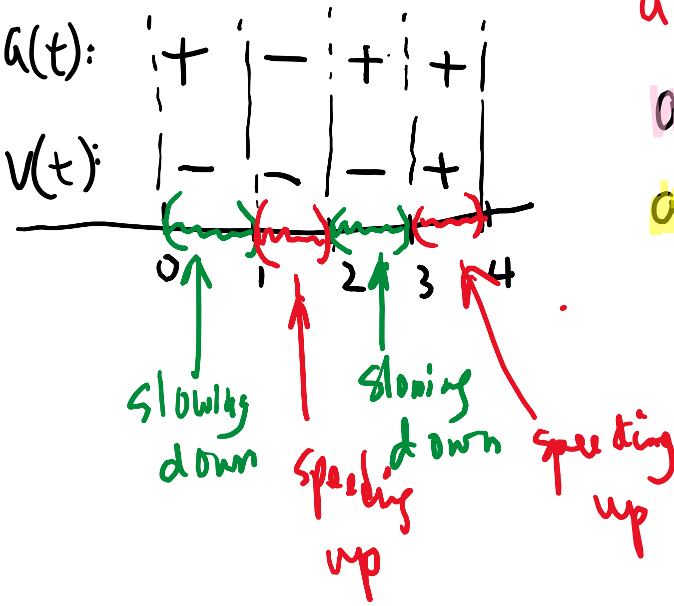
$v(t) < 0$ on $(0, 3)$

$v(t) > 0$ on $(3, 4)$

$a(t) = v'(t) = s''(t)$

$a(t) > 0$ on $(0,1) \cup (2,4)$

$a(t) < 0$ on $(1,2)$



When is the particle in figure (b) speeding up? (Enter your answer using interval notation.)

$\boxed{(1,2) \cup (3,4)}$

When is the particle in figure (b) slowing down? (Enter your answer using interval notation.)

$\boxed{(0,1) \cup (2,3)}$

The height (in meters) of a projectile shot vertically upward from a point 3 m above ground level with an initial velocity of 23.5 m/s is $h = 3 + 23.5t - 4.9t^2$ after t seconds.

(a) Find the velocity (in m/s) after 2 seconds and after 4 seconds.

Note: $\boxed{\text{velocity} = v(t) = h'(t)}$

$$\Rightarrow h'(t) = \frac{d}{dt}\left(3 + 23.5t - 4.9t^2\right)$$

$$= 0 + 23.5 - 4.9 \cdot 2t$$

$$= 23.5 - 9.8t$$

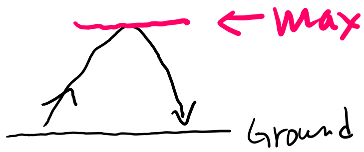
$$\Rightarrow v(t) = 23.5 - 9.8t$$

$t = 2 \Rightarrow v(2) = 23.5 - 9.8(2) = 23.5 - 19.6 = \boxed{3.9}$ m/s

$t = 4 \Rightarrow v(4) = 23.5 - 9.8(4) = 23.5 - 39.2 = \boxed{-15.7}$ m/s

(b) When does the projectile reach its maximum height? (Round your answer to two decimal places.)

Note:



← max height when $m_{tan} = 0$

$\Rightarrow h'(t) = 0$

$\Rightarrow v(t) = 0$

Here, $v(t) = 23.5 - 9.8t$

$\Rightarrow$ Max height is reached when $23.5 - 9.8t = 0$

$+9.8t \qquad +9.8t$

$$\Rightarrow \frac{23.5}{9.8} = \frac{9.8t}{9.8}$$

$$\Rightarrow \boxed{2.40}\ \text{s} = t$$

(c) What is the maximum height? (Round your answer to two decimal places.)

From part (b), maximum height is reached when

$t \approx 2.40s \Rightarrow \text{height} = h(2.40) = 3 + 23.5(2.40) - 4.9(2.40)^2$

$= 31.176 \approx \boxed{31.18}$ m

(d) When does it hit the ground? (Round your answer to two decimal places.)

Note: on the ground $\Rightarrow h = 0$

$\Rightarrow 3 + 23.5t - 4.9t^2 = 0$ $+4.9t^2 - 23.5t - 3$

$-3 \quad -23.5t \quad +4.9t^2$

$\Rightarrow 0 = 4.9t^2 - 23.5t - 3$

Quadratic formula $\Rightarrow a = 4.9,\ b = -23.5,\ c = -3$

$$\Rightarrow t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-23.5) \pm \sqrt{(-23.5)^2 - 4(4.9)(-3)}}{2(4.9)}$$

$$= \frac{23.5 \pm \sqrt{611.05}}{9.8}$$

$\nearrow t = \frac{23.5 + \sqrt{611.05}}{9.8} \approx \boxed{4.92}$ s

Answer

$\searrow t = \frac{23.5 - \sqrt{611.05}}{9.8} \approx -\cancel{0.12}$

time is not negative

(e)With what velocity (in m/s) does it hit the ground? (Round your answer to two decimal places.)

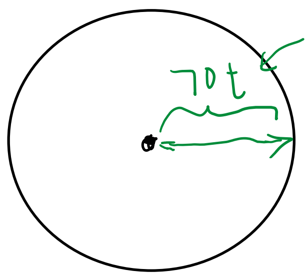
Note: From part (d), the projectile hits the ground when $t \simeq 4.92$ s

$\Rightarrow$ velocity when it hits the ground is

$$v(4.92) = 23.5 - 9.8(4.92)$$
$$= 23.5 - 48.216$$
$$= -24.716 \approx \boxed{-24.72} \text{ m/s}$$

A stone is dropped into a lake, creating a circular ripple that travels outward at a speed of 70 cm/s. Find the rate (in cm²/s) at which the area within the circle is increasing after each of the following.

Visual aid:



radius = distance from center to edge of circle

= Rate · time

= 70 · t = 70t

Note: Area of the circle formula is

$$A = \pi r^2, \quad r = \text{radius}$$

$$\Rightarrow A(t) = \pi(70t)^2$$

$$\Rightarrow A(t) = \pi(4900t^2)$$

$$\Rightarrow A(t) = 4900\pi t^2$$

$\Rightarrow$ rate of change for area of the circle $= A'(t) = 4900\pi \cdot \frac{d}{dt}(t^2)$

$$= 4900\pi \cdot 2t$$

$$\Rightarrow A'(t) = 9800\pi t$$

(a) after 2 s $\Rightarrow A'(2) = 9800\pi(2) = \boxed{19600\pi}$* cm²/s

(b) after 4 s $\Rightarrow A'(4) = 9800\pi(4) = \boxed{39200\pi}$* cm²/s

(c) after 7 s $\Rightarrow A'(7) = 9800\pi(7) = \boxed{68600\pi}$* cm²/s

* Be sure to use π in your answer for exact numbers.

4. Use implicit differentiation to find an equation of the tangent line to the curve at the given point.

↳ need $m = \frac{dy}{dx}$

$$y\sin(16x) = x\cos(2y), \quad (\pi/2, \pi/4)$$

$$\frac{d}{dx}\left(y\sin(16x)\right) = \frac{d}{dx}\left(x\cdot\cos(2y)\right)$$

$$\frac{d}{dx}(y)\cdot\sin(16x) + y\cdot\frac{d}{dx}\left[\sin(16x)\right] = \frac{d}{dx}(x)\cdot\cos(2y) + x\cdot\frac{d}{dx}\left[\cos(2y)\right]$$

$$\frac{dy}{dx}\cdot\sin(16x) + y\cdot\cos(16x)\cdot\underbrace{\frac{d}{dx}(16x)}_{16} = 1\cdot\cos(2y) + x\cdot\left[-\sin(2y)\right]\cdot\frac{d}{dx}(2y)$$

$$\frac{dy}{dx}\cdot\sin(16x) + y\cdot\cos(16x)\cdot 16 = \cos(2y) - x\cdot\sin(2y)\cdot 2\cdot\frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx}\cdot\sin(16x) + 16y\cos(16x) = \cos(2y) - 2x\sin(2y)\frac{dy}{dx}$$

$\left(\frac{\pi}{2}, \frac{\pi}{4}\right)$ ↑x ↑y

$$\Rightarrow \frac{dy}{dx}\cdot\sin\left(16\cdot\frac{\pi}{2}\right) + 16\cdot\frac{\pi}{4}\cdot\cos\left(16\cdot\frac{\pi}{2}\right) = \cos\left(2\cdot\frac{\pi}{4}\right) - 2\cdot\frac{\pi}{2}\sin\left(2\cdot\frac{\pi}{4}\right)\cdot\frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx}\cdot\underbrace{\sin(8\pi)} + 4\pi\cdot\underbrace{\cos(8\pi)} = \underbrace{\cos\left(\frac{\pi}{2}\right)} - \pi\cdot\underbrace{\sin\left(\frac{\pi}{2}\right)}\cdot\frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx}\cdot 0 + 4\pi\cdot 1 = 0 - \pi\cdot 1\cdot\frac{dy}{dx}$$

$$\frac{4\pi}{-\pi} = \frac{-\pi\frac{dy}{dx}}{-\pi} \Rightarrow \frac{dy}{dx} = -4$$

$$\Rightarrow m_{tan} = -4 \Rightarrow \text{Equation is}$$

$$y - \frac{\pi}{4} = -4\left(x - \frac{\pi}{2}\right)$$

$$\Rightarrow y - \frac{\pi}{4} = -4x + 2\pi \Rightarrow \boxed{y = -4x + \frac{9\pi}{4}}$$

$$+\frac{\pi}{4} \qquad\qquad +\frac{\pi}{4}$$

Answer

A particle moves according to a law of motion $s = f(t)$, $t \geq 0$, where t is measured in seconds and s in feet. (If an answer does not exist, enter DNE.)

$$f(t) = \sin\left(\frac{\pi t}{2}\right)$$

(a) Find the velocity (in ft/s) at time t.

$$V(t) = s' = f'(t) = \frac{d}{dt}\left[\sin\left(\frac{\pi t}{2}\right)\right]$$
$$= \cos\left(\frac{\pi t}{2}\right) \cdot \frac{d}{dt}\left(\frac{\pi t}{2}\right)$$
$$= \cos\left(\frac{\pi t}{2}\right) \cdot \frac{\pi}{2}$$
$$\Rightarrow \boxed{V(t) = \frac{\pi}{2}\cos\left(\frac{\pi t}{2}\right)}$$

(b) What is the velocity (in ft/s) after 1 second?

$$V(1) = \frac{\pi}{2}\cos\left(\frac{\pi \cdot 1}{2}\right) = \frac{\pi}{2}\cos\left(\frac{\pi}{2}\right) = \frac{\pi}{2} \cdot 0 = \boxed{0} \text{ ft/s}$$

(c) When is the particle at rest? (Use the parameter n as necessary to represent any integer.)

note: At rest $\Rightarrow v(t) = 0$

$$\Rightarrow \cancel{\frac{\pi}{2}}\cos\left(\frac{\pi t}{2}\right) = 0 \,/\, \frac{\pi}{2}$$

$\pi/2$

$$\Rightarrow \cos\left(\frac{\pi t}{2}\right) = 0$$

$$\Rightarrow \frac{2}{\pi}\left(\frac{\pi t}{2}\right) = \frac{2}{\pi}\left(\frac{\pi}{2} + n\pi\right)$$

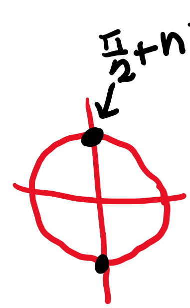
$$\Rightarrow t = \frac{2}{\pi}\left(\frac{\pi}{2} + n\pi\right) = \frac{\cancel{2}}{\cancel{\pi}} \cdot \frac{\cancel{\pi}}{\cancel{2}} + \frac{2}{\cancel{\pi}} \cdot n\cancel{\pi} = \boxed{1 + 2n}$$

$\theta = \frac{\pi t}{2}$

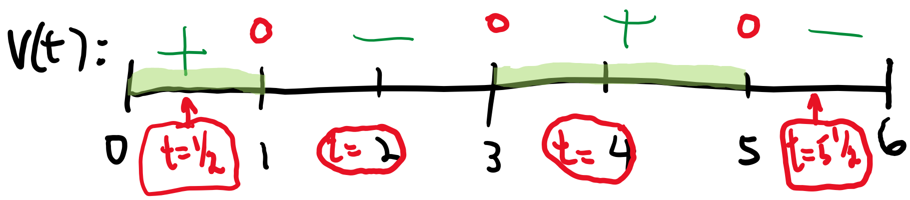
$\downarrow$

$\cos\theta = 0$

find θ



(d) When is the particle moving in the positive direction for $0 \le t \le 6$? (Enter your answer using interval notation.)



$t = \frac{1}{2}, \quad v\left(\frac{1}{2}\right) = \frac{\pi}{2}\cos\left(\frac{\pi \cdot \frac{1}{2}}{2}\right) = \frac{\pi}{2}\cos\left(\frac{\pi}{4}\right) = \frac{\pi}{2}\cdot\frac{\sqrt{2}}{2} = \frac{\sqrt{2}\,\pi}{4} = +$

$t = 2, \quad v(2) = \frac{\pi}{2}\cos\left(\frac{\pi \cdot 2}{2}\right) = \frac{\pi}{2}\cos(\pi) = \frac{\pi}{2}(-1) = -\frac{\pi}{2} = -$

$t = 4, \quad v(4) = \frac{\pi}{2}\cos\left(\frac{\pi \cdot 4}{2}\right) = \frac{\pi}{2}\cos(2\pi) = \frac{\pi}{2}(1) = \frac{\pi}{2} = +$

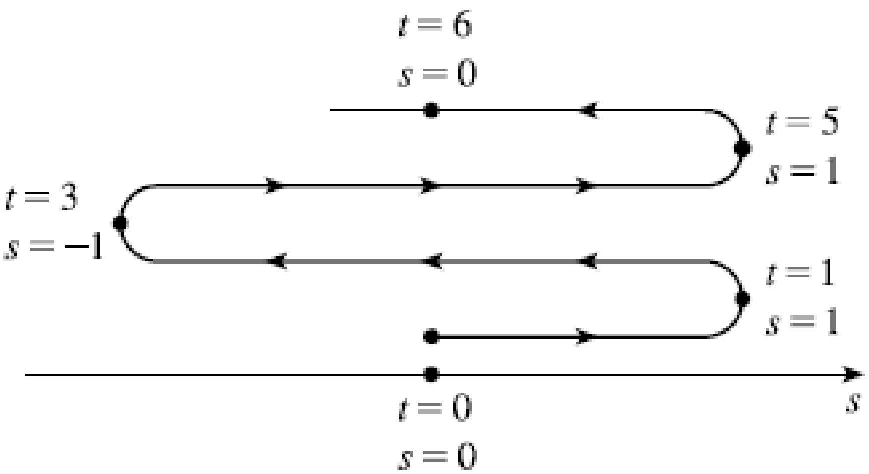
$t = 5\frac{1}{2}, \quad v\left(5\frac{1}{2}\right) = \frac{\pi}{2}\cos\left(\frac{\pi \cdot 5\frac{1}{2}}{2}\right) = \frac{\pi}{2}\cos\left(\frac{11\pi}{4}\right) = \frac{\pi}{2}\left(-\frac{\sqrt{2}}{2}\right) = -$

$$\boxed{\frac{5\frac{1}{2}}{2} = \frac{\frac{11}{2}}{2} = \frac{11}{2}\cdot\frac{1}{2} = \frac{11}{4}}$$

note: Particle is moving in the positive direction when $v(t) > 0 \Rightarrow$ answer is $(0,1) \cup (3,5)$

(e) Draw a diagram to illustrate the motion of the particle and use it to find the total distance (in ft) traveled during the first 6 seconds.

The following figure illustrates the motion of the particle. (Figure not drawn to scale.)



$$\Rightarrow \text{Total distance} = |f(1) - f(0)| + |f(3) - f(1)| + |f(5) - f(3)| + |f(6) - f(5)|$$
$$= |1 - 0| + |-1 - 1| + |1 - (-1)| + |0 - 1|$$
$$= |1| + |-2| + |2| + |-1|$$
$$= 1 + 2 + 2 + 1 = \boxed{6 \text{ ft}}$$

(f) Find the acceleration (in ft/s^2) at time t.

note: acceleration $= a(t) = v'(t)$

$$\Rightarrow a(t) = \frac{d}{dt}\left[\frac{\pi}{2}\cos\left(\frac{\pi t}{2}\right)\right]$$
$$= \frac{\pi}{2} \cdot \frac{d}{dt}\left[\cos\left(\frac{\pi t}{2}\right)\right]$$

$$= \frac{\pi}{2} \cdot \left[-\sin\left(\frac{\pi t}{2}\right)\right] \cdot \frac{d}{dt}\left(\frac{\pi t}{2}\right)$$

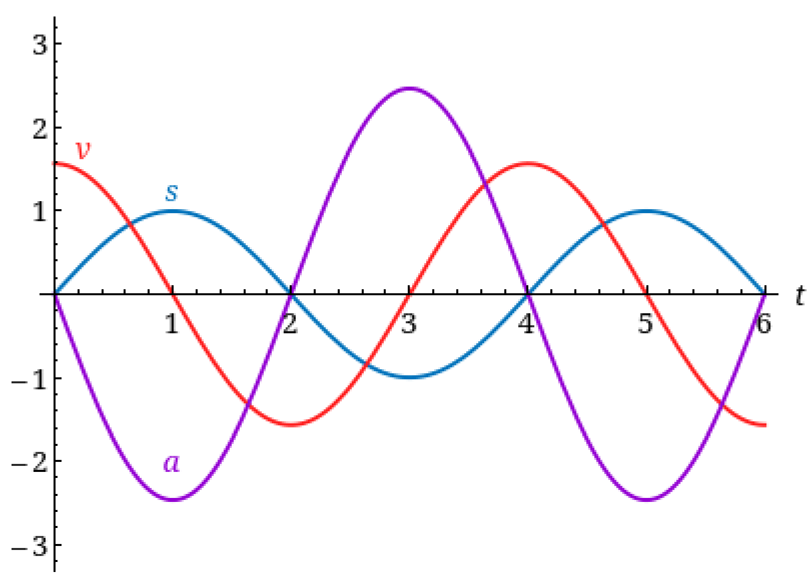
$$= -\frac{\pi}{2} \sin\left(\frac{\pi t}{2}\right) \cdot \frac{\pi}{2} = -\frac{\pi^2}{4} \sin\left(\frac{\pi t}{2}\right)$$

$$\Rightarrow \boxed{a(t) = -\frac{\pi^2}{4} \sin\left(\frac{\pi t}{2}\right)}$$

Find the acceleration (in ft/s^2) after 1 second.

$$a(1) = -\frac{\pi^2}{4} \sin\left(\frac{\pi \cdot 1}{2}\right) = -\frac{\pi^2}{4} \underbrace{\sin\left(\frac{\pi}{2}\right)}_{1} = \boxed{-\frac{\pi^2}{4}}$$

(g) Graph the position, velocity, and acceleration functions for $0 \le t \le 6$.



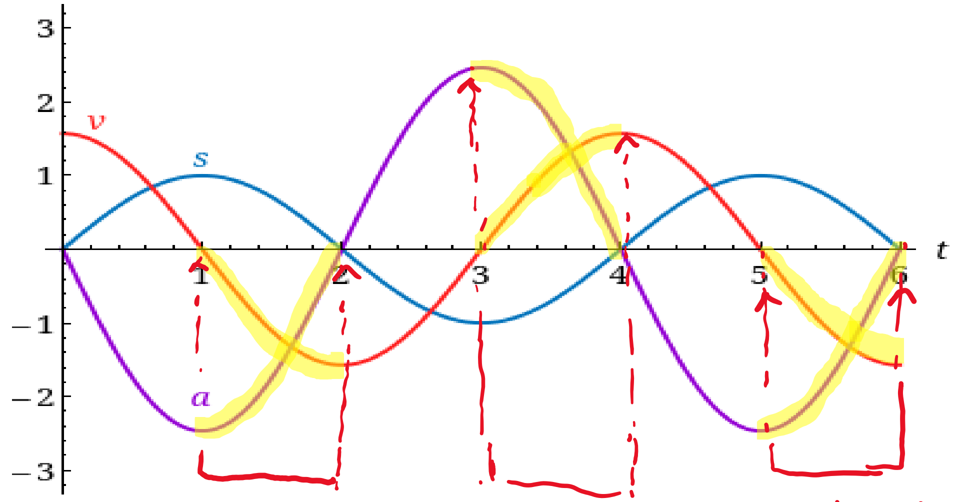
$$S = \sin\left(\frac{\pi t}{2}\right)$$

$$V(t) = \frac{\pi}{2} \cos\left(\frac{\pi t}{2}\right)$$

$$a(t) = -\frac{\pi^2}{4} \sin\left(\frac{\pi t}{2}\right)$$

(h) When is the particle speeding up? (Enter your answer using interval notation.)

note: speeding up $\Rightarrow$ v and a have same sign

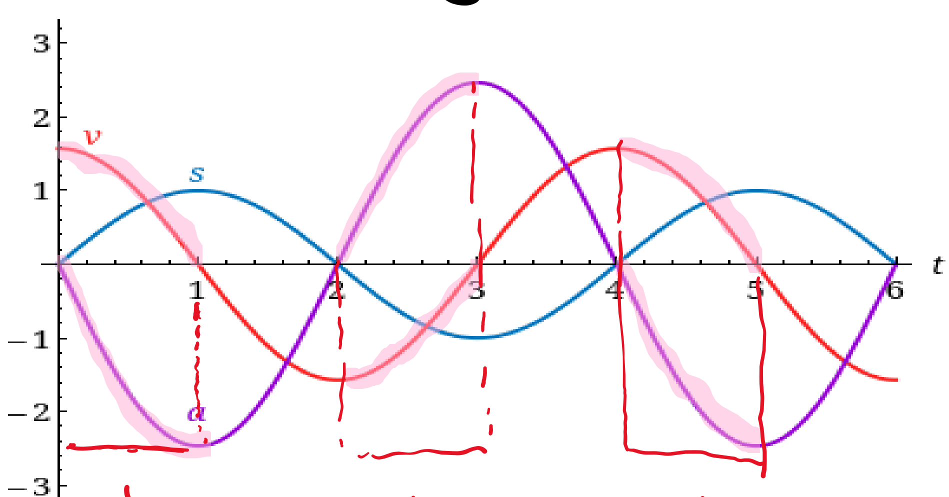


$1 < t < 2$ $\quad$ $3 < t < 4$ $\quad$ $5 < t < 6$

$\Rightarrow (1, 2) \cup (3, 4) \cup (5, 6)$

When is it slowing down? (Enter your answer using interval notation.)

note: slowing down $\Rightarrow$ v and a have opposite signs.



$0 < t < 1$ $\quad$ $2 < t < 3$ $\quad$ $4 < t < 5$

$\Rightarrow (0, 1) \cup (2, 3) \cup (4, 5)$

A particle moves with position function $s = t^4 - 4t^3 - 20t^2 + 25t, \quad t \geq 0$

(a) At what time does the particle have a velocity of 25 m/s? (Enter your answers as a comma-separated list.)

note: $v(t) = s'(t)$

$$\Rightarrow v(t) = \frac{d}{dt}\left(t^4 - 4t^3 - 20t^2 + 25t\right)$$

$$= 4t^3 - 12t^2 - 40t + 25$$

$$v(t) = 25 \Rightarrow 4t^3 - 12t^2 - 40t + 25 = 25$$

$$\underline{-25} \quad \underline{-25}$$

$$\Rightarrow 4t^3 - 12t^2 - 40t = 0$$

$$\Rightarrow 4t(t^2 - 3t - 10) = 0$$

$$\Rightarrow 4t(t-5)(t+2) = 0$$

$$\Rightarrow 4t = 0 \quad \text{OR} \quad t - 5 = 0 \quad \text{OR} \quad t + 2 = 0$$

$$\Rightarrow t = 0 \quad \text{OR} \quad t = 5 \quad \text{OR} \quad t = -2$$

Since $t \geq 0$, $\boxed{t = 0, 5}$

(b) At what time is the acceleration 0? (Round your answer to two decimal places.)

note: $a(t) = v'(t)$

$$\Rightarrow a(t) = \frac{d}{dt}\left[4t^3 - 12t^2 - 40t + 25\right]$$

$$= 12t^2 - 24t - 40$$

$$\frac{12t^2}{4} - \frac{24t}{4} - \frac{40}{4} = \frac{0}{4}$$

$$\Rightarrow 3t^2 - 6t - 10 = 0$$

$a = 3$, $b = -6$, $c = -10$

$$\Rightarrow t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-6) \pm \sqrt{(-6)^2 - 4(3)(-10)}}{2(3)}$$

$$= \frac{6 \pm \sqrt{36 + 120}}{6}$$

$$= \frac{6 \pm \sqrt{156}}{6} \Rightarrow t = \frac{6 + \sqrt{156}}{6} \approx 3.08$$

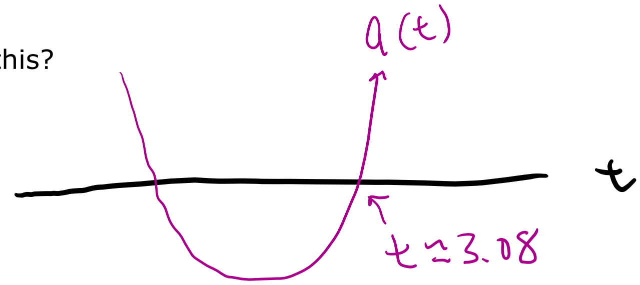
OR

$$t = \frac{6 - \sqrt{156}}{6} \approx -1.08$$

Since $t \geq 0$, $t = \boxed{3.08}$ sec.

What is the significance of this?

note :



◉ At this time, the acceleration changes from negative to positive and the velocity attains its minimum value.

47. When a cold drink is taken from a refrigerator, its temperature is 5°C. After 25 minutes in a 20°C room its temperature has increased to 10°C. (Round your answers to two decimal places.)

(a) What is the temperature of the drink (in °C) after 55 minutes?

Recall: Newton's Law of Cooling

$$T(t) = T_s + (T_0 - T_s)e^{kt}$$

T(t) = Temperature after time t

T_s = Surrounding temperature

T_0 = Initial temperature

k = Cooling constant

note: 5°C when drink is taken from refrigerator $\Rightarrow T_0 = 5$

20°C Room $\Rightarrow T_s = 20$

Temperature increased to 10°C after 25 minutes $\Rightarrow T(25) = 10$

Note: Formula was $T(t) = T_s + (T_s - T_0)e^{kt}$

But here, the temperature increased NOT decreased

$\Rightarrow$ we use $(T_0 - T_s)$ instead.

$\Rightarrow$ Formula becomes

$$T(t) = T_s + (T_0 - T_s)e^{kt}$$

$\Rightarrow$ here, we have $T(t) = 20 + (5 - 20)e^{kt}$

$$\Rightarrow T(t) = 20 - 15e^{kt}$$

$\Rightarrow$ we need to find K

$\Rightarrow$ use $T(25) = 10$

$$\Rightarrow \quad 20 - 15e^{k(25)} = 10$$
$$\underline{-20} \qquad\qquad \underline{-20}$$

$$\Rightarrow \quad \frac{-15e^{25k}}{-15} = \frac{-10}{-15}$$

$$\Rightarrow \quad e^{25k} = \frac{2}{3}$$

$$\Rightarrow \quad \ln(e^{25k}) = \ln\left(\frac{2}{3}\right)$$

$$\Rightarrow \quad 25k = \ln\left(\frac{2}{3}\right)$$

$$\Rightarrow \quad k = \frac{1}{25}\ln\left(\frac{2}{3}\right)$$

$\Rightarrow$ Formula becomes

$$T(t) = 20 - 15e^{\frac{1}{25}\ln\left(\frac{2}{3}\right)t}$$

$t = 55$ minutes $\Rightarrow$ $T(55) = 20 - 15e^{\frac{1}{25}\ln\left(\frac{2}{3}\right)\cdot(55)}$

$$= 20 - 15e^{\frac{55}{25}\ln\left(\frac{2}{3}\right)}$$

$$= 20 - 15e^{\frac{11}{5}\ln\left(\frac{2}{3}\right)}$$

$$= 20 - 15e^{\ln\left(\frac{2}{3}\right)^{11/5}}$$

$$= 20 - 15\left(\frac{2}{3}\right)^{11/5}$$

$$\approx \boxed{13.85}\ ^\circ C$$

(b) After how many minutes will its temperature reach 12°C?

$$T(t) = 20 - 15e^{\frac{1}{25}\ln\left(\frac{2}{3}\right)t}$$

$$\Rightarrow 12 = 20 - 15e^{\frac{1}{25}\ln\left(\frac{2}{3}\right)t}$$

$$\underline{-20} \qquad \underline{-20}$$

$$\Rightarrow \frac{-8}{-15} = \frac{-15e^{\frac{1}{25}\ln\left(\frac{2}{3}\right)t}}{-15}$$

$$\Rightarrow \frac{8}{15} = e^{\ln\left(\frac{2}{3}\right)^{\frac{1}{25}}\cdot t}$$

$$\Rightarrow \frac{8}{15} = \left[\left(\frac{2}{3}\right)^{\frac{1}{25}}\right]^{t}$$

$$\Rightarrow \frac{8}{15} = \left(\frac{2}{3}\right)^{\frac{t}{25}}$$

$$\Rightarrow \ln\left(\frac{8}{15}\right) = \ln\left(\frac{2}{3}\right)^{\frac{t}{25}}$$

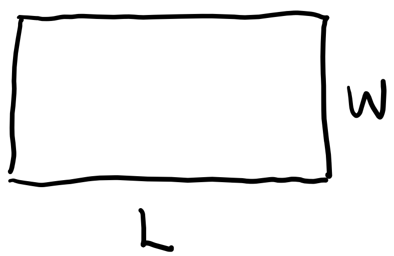
$$\Rightarrow \ln\left(\frac{8}{15}\right) = \frac{t}{25}\ln\left(\frac{2}{3}\right)$$

$$\Rightarrow 25\ln\left(\frac{8}{15}\right) = t\ln\left(\frac{2}{3}\right)$$

$$\Rightarrow \frac{25\ln\left(\frac{8}{15}\right)}{\ln\left(\frac{2}{3}\right)} = t \qquad \Rightarrow t \approx \boxed{38.76}\text{ min.}$$

(50)

The length of a rectangle is increasing at a rate of 4 cm/s and its width is increasing at a rate of 8 cm/s. When the length is 9 cm and the width is 4 cm, how fast is the area of the rectangle increasing?



$$\frac{dL}{dt} = 4 \quad , L = 9$$

$$\frac{dW}{dt} = 8 \quad , W = 4$$

$$A = L \cdot W$$

$$\Rightarrow \frac{d}{dt}(A) = \frac{d}{dt}(LW)$$

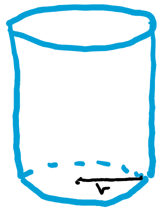
$$\Rightarrow \frac{dA}{dt} = \frac{dL}{dt} \cdot W + L \cdot \frac{dW}{dt}$$ ← use product Rule

$$= (4)(4) + (9)(8)$$

$$= 16 + 72$$

$$= \boxed{88}$$

51



Recall: Volume of a Cylinder

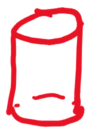
$V = \underbrace{\text{Base}}_{\substack{\text{Area} \\ \text{of} \\ \text{a Circle}}} \times \text{height}$

$V = \pi r^2 \cdot h \Rightarrow V = \pi r^2 h$

$r = 5 \Rightarrow V = \pi (5)^2 h$

$V = 25\pi h$

$\Rightarrow V(h) = 25\pi h$



$\leftarrow V = \pi r^2 h$

$r = 5$

Got: $\boxed{\frac{dV}{dt} = 2}$

A cylindrical tank with radius 5 m is being filled with water at a rate of 2 m^3/min. How fast is the height of the water increasing (in m/min)?

Find $\boxed{\frac{dh}{dt}} = ?$

$V = \pi(25)h$

$\Rightarrow V = 25\pi h$

$\Rightarrow \frac{d}{dt}(V) = \frac{d}{dt}(25\pi h)$

$\Rightarrow \frac{dV}{dt} = 25\pi \cdot \frac{d}{dt}(h)$

$\Rightarrow \frac{dV}{dt} = 25\pi \frac{dh}{dt}.$

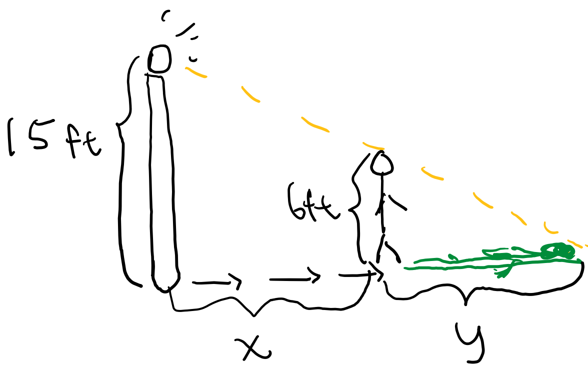
$\Rightarrow \frac{2}{25\pi} = \frac{\cancel{25\pi}}{\cancel{25\pi}} \frac{dh}{dt}$

$\boxed{\frac{2}{25\pi}} = \frac{dh}{dt}$

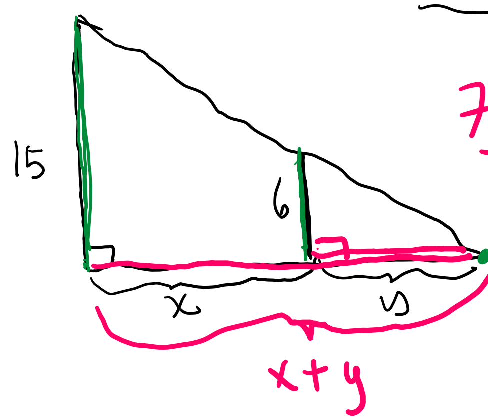
Answer

A streetlight is mounted at the top of a 15-ft-tall pole. A man 6 feet tall walks away from the pole with a speed of 5 ft/s along a straight path. How fast (in ft/s) is the tip of his shadow moving when he is 40 feet from the pole?

Given: $\frac{dx}{dt} = 5$



$\Rightarrow$



Find: $\frac{d}{dt}(x+y)$ if $x=40$

$$\Rightarrow \frac{15}{6} = \frac{x+y}{y}$$

$$\Rightarrow 15 \cdot y = 6(x+y)$$

$$\Rightarrow \underset{-6y}{15y} = 6x + \cancel{6y} \underset{-\cancel{6y}}{}$$

$$\Rightarrow \frac{\cancel{9}y}{\cancel{9}} = \frac{\overset{2}{\cancel{6}}x}{\cancel{9}_3} \Rightarrow y = \frac{2}{3}x$$

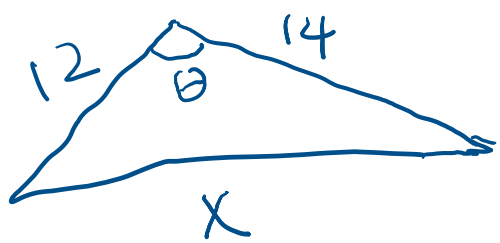
$$\Rightarrow \frac{d}{dt}(x+y) = \frac{d}{dt}\left(1x + \frac{2}{3}x\right) = \frac{d}{dt}\left(\frac{5}{3}x\right) = \frac{5}{3} \cdot \frac{dx}{dt}$$

$$1 + \frac{2}{3} = \frac{3}{3} + \frac{2}{3} = \frac{5}{3}$$

$$= \frac{5}{3} \cdot \left(\frac{5}{1}\right)$$

$$= \boxed{\frac{25}{3}} \text{ ft/s}$$

58] Two sides of a triangle have lengths 12 m and 14 m. The angle between them is increasing at a rate of 2°/min. How fast is the length of the third side increasing when the angle between the sides of fixed length is 60°? (Round your answer to three decimal places.)



$$\boxed{\begin{array}{l} \dfrac{d\theta}{dt} = 2^\circ \times \dfrac{\pi}{180} = \dfrac{\pi}{90} \text{ rad/min} \\[2ex] \text{find } \dfrac{dx}{dt} \text{ if } \theta = 60^\circ \end{array}}$$

using law of cosine, we get

$$x^2 = 12^2 + 14^2 - 2(12)(14)\cos\theta$$

$$x^2 = 144 + 196 - 336\cos\theta$$

$$x^2 = 340 - 336\cos\theta$$

$$\Rightarrow \frac{d}{dt}(x^2) = \frac{d}{dt}(340 - 336\cos\theta)$$

$$\Rightarrow 2x\frac{dx}{dt} = 0 - 336\cdot(-\sin\theta)\cdot\frac{d\theta}{dt}$$

$$\Rightarrow 2x\frac{dx}{dt} = 336\sin\theta\cdot\frac{d\theta}{dt}$$

$$\Rightarrow \frac{dx}{dt} = \frac{\cancel{336}^{168} \sin\theta}{\cancel{2}_{1} x} \cdot \frac{d\theta}{dt}$$

$$\Rightarrow \frac{dx}{dt} = \frac{168 \sin\theta}{x} \frac{d\theta}{dt}$$

$\underline{\text{note}}$: @ $\theta = 60^\circ$, $x^2 = 340 - 336\cos 60^\circ$

$$\Rightarrow x^2 = 340 - 336\left(\tfrac{1}{2}\right)$$

$$\Rightarrow x^2 = 340 - 168$$

$$\Rightarrow x^2 = 172 \Rightarrow x = \pm\sqrt{172}$$

x = side of a $\triangle \Rightarrow x > 0 \Rightarrow x = \sqrt{4 \cdot 43}$

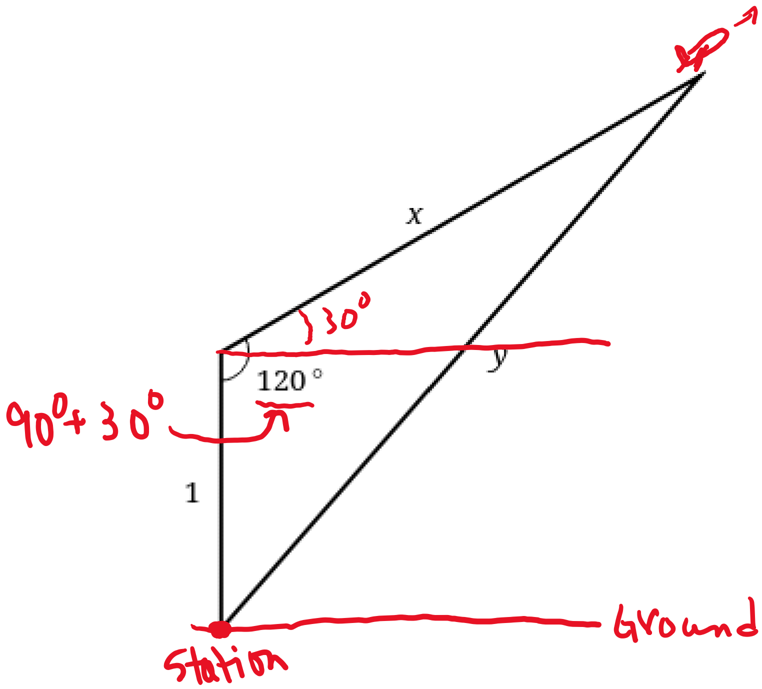
$$\Rightarrow x = 2\sqrt{43}$$

$$\Rightarrow \frac{dx}{dt} = \frac{\cancel{168}^{84} \cdot \sin 60^\circ}{\cancel{2}_{1}\sqrt{43}} \cdot \frac{\pi}{90} = \frac{\cancel{84}^{42} \cdot \frac{\sqrt{3}}{\cancel{2}_{1}} \cdot \pi}{90\sqrt{43}}$$

$$= \frac{{}^{7}\cancel{42}\sqrt{3}\,\pi}{{}_{15}\cancel{90}\sqrt{43}} \approx \boxed{0.387}$$

$\underline{\text{Answer}}$

59. A plane flying with a constant speed of 240 km/h passes over a ground radar station at an altitude of 1 km and climbs at an angle of 30°. At what rate (in km/h) is the distance from the plane to the radar station increasing a minute later? (Round your answer to the nearest whole number.)



$$\frac{dx}{dt} = 240 \text{ km/hr}$$

Find $\frac{dy}{dt}$ when $\underbrace{t = 1 \text{ min}}$

$$\boxed{t = \frac{1}{60} \text{ hr.}}$$

By Law of cosines, we get

$$y^2 = x^2 + 1^2 - 2(x)(1)\cos(120^\circ)$$

$$\Rightarrow y^2 = x^2 + 1 - 2x\left(-\frac{1}{2}\right)$$

$$\Rightarrow y^2 = x^2 + 1 + x$$

$$\Rightarrow \frac{d}{dt}\left(y^2\right) = \frac{d}{dt}\left(x^2 + x + 1\right)$$

$$\Rightarrow 2y\frac{dy}{dt} = 2x\frac{dx}{dt} + 1\cdot\frac{dx}{dt} + 0$$

$$\Rightarrow 2y\frac{dy}{dt} = 2x\frac{dx}{dt} + \frac{dx}{dt}$$

$$\Rightarrow \frac{\cancel{2y}\,\frac{dy}{dt}}{\cancel{2y}} = \frac{(2x+1)\frac{dx}{dt}}{2y}$$

$$\Rightarrow \frac{dy}{dt} = \frac{2x+1}{2y}\frac{dx}{dt}$$

note. Since speed of plane $= 240$ km/hr

$\Rightarrow$ distance $=$ Rate . time

$$= \frac{240\text{ Km}}{1\text{ hr}} \cdot \frac{1}{60}\text{hr.}$$

$$\boxed{t = 1\text{ min} = \frac{1}{60}\text{ hr.}}$$

$$= \frac{240}{60}\text{ Km}$$

$$= \boxed{4\text{ Km} = x}$$

$$y^2 = x^2 + x + 1 \Rightarrow y^2 = 4^2 + 4 + 1$$

$$\Rightarrow y^2 = 21 \Rightarrow y = \pm\sqrt{21}$$

Here, $y =$ distance $\Rightarrow y \geq 0 \Rightarrow y = \sqrt{21}$

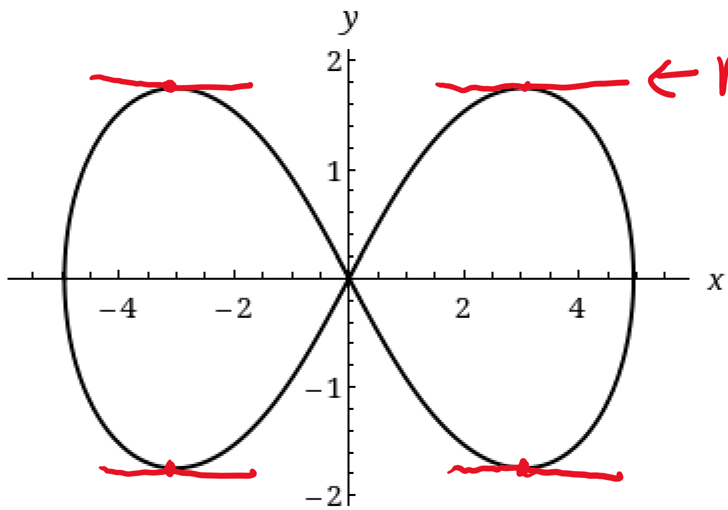
$$\Rightarrow \quad \frac{dy}{dt} = \frac{2x+1}{2y}\frac{dx}{dt} \qquad \text{becomes}$$

$$\frac{dy}{dt} = \frac{\overbrace{2(4)+1}^{9}}{2\sqrt{21}} \cdot 240 \approx \boxed{236} \text{ km/hr.}$$

Answer

Find the points on the lemniscate where the tangent is horizontal. (Order your answers from smallest to largest x, then from smallest to largest y.)

$$2(x^2 + y^2)^2 = 49(x^2 - y^2)$$



$\leftarrow m_{tan} = 0 \Rightarrow y' = 0$

$$\frac{d}{dx}\left[2(x^2+y^2)^2\right] = \frac{d}{dx}\left[49(x^2-y^2)\right]$$

$$4(x^2+y^2)^1 \frac{d}{dx}\left[(x^2+y^2)\right] = 49 \cdot \frac{d}{dx}\left[(x^2-y^2)\right]$$

$$4(x^2+y^2)\cdot\left[2x + 2y\cdot y'\right] = 49\cdot\left[2x - 2y\cdot y'\right]$$

$$8x(x^2+y^2) + 8y(x^2+y^2)y' = 98x - \cancel{98y\cdot y'}$$

$$+98yy' \qquad \cancel{+98yy'}$$

$$\cancel{8x(x^2+y^2)} + 8y(x^2+y^2)y' + 98y\,y' = 98x - 8x(x^2+y^2)$$

$$-\cancel{8x(x^2+y^2)}$$

$$8y(x^2+y^2)y' + 98y\,y' = 98x - 8x(x^2+y^2)$$

$$\frac{y'\left[\cancel{8y(x^2+y^2)+98y}\right]}{\cancel{8y(x^2+y^2)+98y}} = \frac{98x - 8x(x^2+y^2)}{8y(x^2+y^2)+98y}$$

$$\Rightarrow \quad y' = \frac{98x - 8x(x^2+y^2)}{8y(x^2+y^2)+98y}$$

note: Horizontal tangent $\Rightarrow \underbrace{m_{tan}}_{y'} = 0$

$$\Rightarrow \quad y' = 0$$

$$\Rightarrow \quad \frac{98x - 8x(x^2+y^2)}{8y(x^2+y^2)+98y} = 0$$

$$\Rightarrow 98x - 8x(x^2+y^2) = 0$$

$$\Rightarrow 2x\left[49 - 4(x^2+y^2)\right] = 0$$

$$\Rightarrow \frac{\cancel{2}x}{\cancel{2}} = \frac{0}{2} \quad \text{OR} \quad \cancel{49} - 4(x^2+y^2) = 0$$

$$\cancel{-49} \qquad\qquad -49$$

$$\Rightarrow \cancel{x=0} \quad \text{OR} \quad \frac{\cancel{-4}(x^2+y^2)}{\cancel{-4}} = \frac{\cancel{-}49}{\cancel{-}4}$$

note: no horizontal tangent line @ $x=0$

$$\Rightarrow x^2+y^2 = \frac{49}{4}$$

Recall: $2(x^2+y^2)^2 = 49(x^2-y^2)$

$$\Rightarrow 2\left(\frac{49}{4}\right)^2 = 49(x^2-y^2)$$

$$\Rightarrow \overset{1}{\cancel{2}} \cdot \frac{49 \cdot 49}{\cancel{16}\ 8} = 49(x^2-y^2)$$

$$\Rightarrow \frac{49 \cdot \cancel{49}}{8 \cdot \cancel{49}} = \frac{\cancel{49}(x^2-y^2)}{\cancel{49}}$$

$$\Rightarrow \frac{49}{8} = x^2-y^2$$

$$\Rightarrow \quad x^2 + \cancel{y^2} = \frac{49 \cdot 2}{4 \cdot 2} = \frac{98}{8}$$

$$(+) \quad x^2 - \cancel{y^2} = \frac{49}{8}$$

$$\frac{\cancel{2}x^2}{\cancel{2}} = \frac{147}{8} \cdot \frac{1}{2}$$

$$\Rightarrow \sqrt{x^2} = \pm\sqrt{\frac{147}{16}}$$

$$\Rightarrow \quad x = \pm\sqrt{\frac{147}{16}} = \frac{\pm\sqrt{49 \cdot 3}}{\sqrt{16}} = \frac{\pm 7\sqrt{3}}{4}$$

$$x = \pm\frac{7\sqrt{3}}{4} \Rightarrow \left(\pm\frac{7\sqrt{3}}{4}\right)^2 + y^2 = \frac{49}{4}$$

$$\Rightarrow \cancel{\frac{147}{16}} + y^2 = \frac{\overbrace{49 \cdot 4}^{196}}{\underbrace{4 \cdot 4}_{16}} - \frac{147}{16}$$

$$-\cancel{\frac{147}{16}}$$

$$\Rightarrow \sqrt{y^2} = \frac{196 - 147}{16} = \pm\sqrt{\frac{49}{16}}$$

$$\Rightarrow y = \pm\sqrt{\frac{49}{16}} = \pm\frac{7}{4}$$

$\Rightarrow$ Smaller $x = -\frac{7\sqrt{3}}{4} \Rightarrow y = -\frac{7}{4}, \frac{7}{4}$

$\Rightarrow$ points are $\left(-\frac{7\sqrt{3}}{4}, -\frac{7}{4}\right), \left(-\frac{7\sqrt{3}}{4}, \frac{7}{4}\right)$

and larger $x = \frac{7\sqrt{3}}{4} \Rightarrow y = -\frac{7}{4}, \frac{7}{4}$

$\Rightarrow$ points are $\left(\frac{7\sqrt{3}}{4}, -\frac{7}{4}\right), \left(\frac{7\sqrt{3}}{4}, \frac{7}{4}\right),$

NOTE: Order your answers from smallest to largest *x*, then from smallest to largest *y*.

Answers are

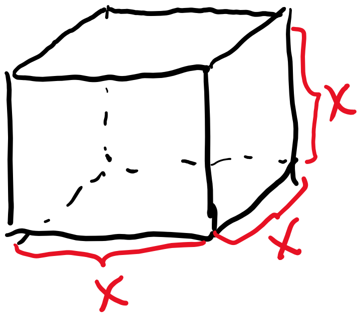
$$\left(-\frac{7\sqrt{3}}{4}, -\frac{7}{4}\right)$$

$$\left(-\frac{7\sqrt{3}}{4}, \frac{7}{4}\right)$$

$$\left(\frac{7\sqrt{3}}{4}, -\frac{7}{4}\right)$$

$$\left(\frac{7\sqrt{3}}{4}, \frac{7}{4}\right)$$

63



when $x = 15$ cm
$dx = 0.2$ cm

(a) $V(x) = x^3$

$$\Rightarrow V'(x) = 3x^2$$

$$\Rightarrow \frac{dV}{dx} = 3x^2$$

$$\Rightarrow dV = 3x^2\,dx$$

$$\Rightarrow dV = 3(15)^2(0.2)$$

$$\Rightarrow dV = \boxed{135 \text{ cm}^3}$$

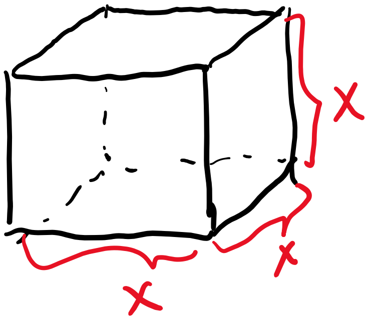
note: Relative error $= \dfrac{dV}{V} \approx \dfrac{3\cancel{x^2}\,dx}{\cancel{x^3}\,x}$

$$= \frac{3\,dx}{x} = \frac{3(0.2)}{15}$$

$$\boxed{\text{Relative error} = 0.0400}$$

$$\Rightarrow \boxed{\text{percentage error} = 4.00\,\%}$$

Recall: $x = 15$ cm
dx



note: each side has
area $= x^2$
and there are 6 sides

(b) Surface area $= S = 6x^2$

$\Rightarrow S(x) = 6x^2$

$\Rightarrow S'(x) = 12x$

$\Rightarrow \dfrac{dS}{dx} = 12x$

$\Rightarrow dS = 12x\,dx$

$= 12(15)(0.2)$

$= \boxed{36 \text{ cm}^2}$

$$\text{Relative error} = \frac{dS}{S}$$

$$= \frac{\overset{2}{\cancel{12}}\cancel{x}\,dx}{\cancel{6}\cancel{x^2}_{\,x}} = \frac{2\,dx}{x} = \frac{2(0.2)}{15}$$

$$\approx \boxed{0.0267}$$

$\Rightarrow$ percentage error $= \boxed{2.67\,\%}$

71 (j) $\lim_{x \to \infty} \frac{\sinh(x)}{e^x}$

$$= \lim_{x \to \infty} \frac{\frac{e^x - e^{-x}}{2}}{e^x}$$

$$= \lim_{x \to \infty} \left[\frac{e^x - e^{-x}}{2} \cdot \frac{1}{e^x} \right]$$

$$= \lim_{x \to \infty} \left[\frac{1}{2} \cdot \frac{e^x - e^{-x}}{e^x} \right]$$

$$= \frac{1}{2} \lim_{x \to \infty} \left[\frac{e^x}{e^x} - \frac{e^{-x}}{e^x} \right]$$

$$= \frac{1}{2} \lim_{x \to \infty} \left[1 - \frac{1}{e^{2x}} \right]$$

$$= \frac{1}{2} \left[\lim_{x \to \infty} 1 - \lim_{x \to \infty} \frac{1}{e^{2x}} \right]$$

$$= \frac{1}{2} \left[1 - 0 \right] = \frac{1}{2} [1] = \frac{1}{2}$$

5. If $xy + 6e^y = 6e$, find the value of y'' at the point where $x = 0$.

note: $\boxed{x=0} \Rightarrow 0 \cdot y + 6e^y = 6e \Rightarrow 6e^y = 6e \Rightarrow e^y = e \Rightarrow \ln e^y = \ln e$

$\Rightarrow \boxed{y = 1}$

now, find y':

$$\frac{d}{dx}\left[xy + 6e^y\right] = \frac{d}{dx}\left[6e\right] \leftarrow \text{use Implicit Differentiation}$$

$$\Rightarrow \frac{d}{dx}[xy] + 6\frac{d}{dx}[e^y] = 0$$

product Rule   chain Rule

$$\Rightarrow \underbrace{\frac{d}{dx}[x] \cdot y + x \cdot \frac{d}{dx}[y]}_{\text{product Rule}} + 6 \cdot e^y \cdot \frac{d}{dx}(y) = 0$$

$$\Rightarrow 1 \cdot y + x \cdot \frac{dy}{dx} + 6 \cdot e^y \cdot \frac{dy}{dx} = 0$$

$$\Rightarrow y + x\frac{dy}{dx} + 6e^y\frac{dy}{dx} = 0 \quad -y$$

$$-y$$

$$\Rightarrow x\frac{dy}{dx} + 6e^y\frac{dy}{dx} = -y$$

$$\Rightarrow \frac{dy}{dx}(x + 6e^y) = -y$$

$$\Rightarrow \frac{dy}{dx} = \frac{-y}{x+6e^y}$$

$$\Rightarrow y' = \frac{-y}{x+6e^y} \Rightarrow y' = \frac{-1}{0+6e^1} = -\frac{1}{6e}$$

$\boxed{x=0,\ y=1}$

$$\Rightarrow y'' = \frac{d}{dx}\left[\frac{-y}{x+6e^y}\right] \leftarrow \text{use Quotient Rule}$$

$$= \frac{(x+6e^y)\cdot\frac{d}{dx}[-y] - (-y)\frac{d}{dx}(x+6e^y)}{(x+6e^y)^2}$$

$$= \frac{(x+6e^y)(-y') + y(1+6e^y\cdot y')}{(x+6e^y)^2}$$

Recall: $x=0$, $y=1$, $y'=-\frac{1}{6e}$ $\Rightarrow$

$$y'' = \frac{(0+6e^1)(-(-\tfrac{1}{6e})) + 1(1+\overbrace{6e^1\cdot(-\tfrac{1}{6e})}^{-\frac{6e}{6e}=-1})}{(0+6e^1)^2}$$

$$= \frac{(6e)(\frac{1}{6e}) + (1-1)}{(6e)^2}$$

$$= \frac{\frac{6e}{6e} + 0}{36e^2} = \frac{1+0}{36e^2}$$

$$= \boxed{\frac{1}{36e^2}}$$

Assignment 4, problem 65

Find the differential of each function.

(a) $y = x^2 \sin(4x)$

(b) $y = \ln(\sqrt{5 + t^2})$

$\boxed{\text{Recall: } dy = y'dx}$

(a)

$y' = \frac{d}{dx}\left(x^2 \sin(4x)\right) \Leftrightarrow$ use product Rule

$= \frac{d}{dx}(x^2) \cdot \sin(4x) + x^2 \cdot \frac{d}{dx}\left[\sin(4x)\right]$

$= 2x \cdot \sin(4x) + x^2 \cdot \underbrace{\cos(4x) \cdot 4}_{\text{Chain Rule}}$

$= 2x \sin(4x) + 4x^2 \cos(4x)$

$\Rightarrow dy = y'dx = \boxed{\left(2x \sin(4x) + 4x^2 \cos(4x)\right)dx}$

Answer

(b) $y' = \frac{d}{dt}\left(\ln\left(\sqrt{5+t^2}\right)\right)$

$$\boxed{\text{Recall: } \frac{d}{dt}(\ln u) = \frac{1}{u}\cdot\frac{du}{dt}}$$

Here $u = \sqrt{5+t^2}$

$$\Rightarrow \frac{du}{dt} = \frac{1}{2\sqrt{5+t^2}}\cdot\frac{d}{dt}(5+t^2)$$

$$= \frac{1}{\cancel{2}\sqrt{5+t^2}}\cdot(\cancel{2}t)$$

$$= \frac{t}{\sqrt{5+t^2}}$$

$$\Rightarrow y' = \frac{1}{u}\cdot\frac{du}{dt} = \frac{1}{\sqrt{5+t^2}}\cdot\frac{t}{\sqrt{5+t^2}}$$

$$\Rightarrow \quad = \frac{t}{\left(\sqrt{5+t^2}\right)^2} = \frac{t}{5+t^2}$$

$$\Rightarrow dy = y'dt = \boxed{\frac{t}{5+t^2}\,dt}$$ Answer