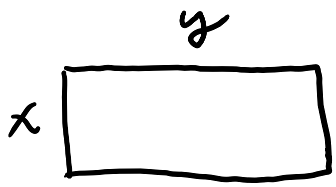
14. Find the dimensions of a rectangle (in m) with area 2,197 m^2 whose perimeter is as small as possible. (Enter the dimensions as a comma separated list.)



$$A = L \cdot W$$

$$\Rightarrow 2197 = y \cdot x$$

$$\Rightarrow \frac{2197}{x} = y$$

$\Rightarrow P = 2W + 2L$ becomes

$$P = 2 \cdot x + 2y$$

$$\Rightarrow P(x) = 2x + 2\left(\frac{2197}{x}\right)$$

$$\Rightarrow P(x) = 2x + \frac{4394}{x}$$

$$\Rightarrow P'(x) = \frac{d}{dx}\left[2x + 4394x^{-1}\right]$$

$$= 2 - 4394x^{-2}$$

$$= 2 - \frac{4394}{x^2}$$

$$= \frac{2x^2 - 4394}{x^2}$$

$\Rightarrow P'(x) = 0$ if $\frac{2x^2}{2} - \frac{4394}{2} = \frac{0}{2}$

$$\Rightarrow x^2 - 2197 = 0$$

$$\Rightarrow x^2 = 2197$$

$$\Rightarrow x = \pm\sqrt{2197}$$

$$= \pm\sqrt{169 \cdot 13}$$

$$\boxed{\begin{array}{c} 2197 \\ 13 \quad 169 \\ \quad\quad 13 \quad 13 \end{array}}$$

$$= \pm\sqrt{169}\cdot\sqrt{13}$$

$$= \pm 13\sqrt{13}$$

Since $x =$ width of a rectangle, $x \neq$ negative

$\Rightarrow x = 13\sqrt{13}$

Here, $P''(x) = \frac{d}{dx}\left[2 - 4394x^{-2}\right]$

$$= 8788x^{-3} = \frac{8788}{x^3}$$

$\Rightarrow P''(13\sqrt{13}) = \dfrac{8788}{(13\sqrt{13})^3} = +$

$\Rightarrow$ by the 2^{nd} derivative test, min P occurs at $x = 13\sqrt{13}$.

$$\Rightarrow y = \frac{2197}{x} = \frac{\overset{169}{\cancel{2197}}}{\cancel{13}\sqrt{13}} = \frac{169\cdot\sqrt{13}}{\sqrt{13}\cdot\sqrt{13}}$$

$$= \frac{169\sqrt{13}}{\sqrt{13^2}} = \frac{\overset{13}{\cancel{169}}\sqrt{13}}{\cancel{13}} = 13\sqrt{13}$$

$\Rightarrow$ Answers: $\boxed{13\sqrt{13},\ 13\sqrt{13}}$

(15)

(e) from (d), $5\cancel{x} + 2y = 750 - 5x$

$\cancel{-5x}$

$$\frac{\cancel{2}y}{\cancel{2}} = \frac{750}{2} - \frac{5x}{2}$$

$$y = 375 - \frac{5}{2}x$$

From (c), $A = y \cdot x$

$$\Rightarrow A(x) = \left(375 - \frac{5}{2}x\right) \cdot x$$

$$\Rightarrow \boxed{A(x) = 375x - \frac{5}{2}x^2}$$

(f) $A'(x) = 375 - \frac{5}{\cancel{2}} \cdot \cancel{2}x$

$$\Rightarrow A'(x) = \cancel{375} - 5x = 0 - 375$$

$\cancel{-375}$

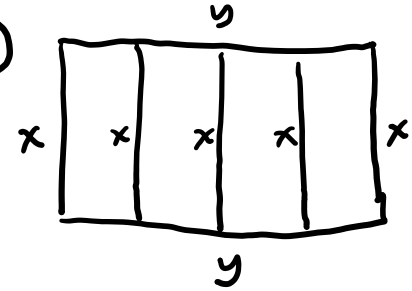
$$\frac{\cancel{-5}x}{\cancel{-5}} = \frac{-375}{-5}$$

$x = 75$ ← critical number.

note: $A''(x) = \frac{d}{dx}(375 - 5x) = 0 - 5 = -5$

$A''(x) = -5 \to$ by 2nd Derivative Test, abs. max is @ $x = 75$

(d)



$5x + 2y = 750$

Given 750 ft of fencing

$\Rightarrow$ Max area is $A(75) = 375(75) - \frac{5}{2}(75)^2$

$= \boxed{14{,}062.5}\ ft^2$

---

(41)

$F(x) = \int f(x)\,dx$

$= \int \left(5^x + 9\sinh(x)\right)dx$

$= \underbrace{\int 5^x dx} + 9\underbrace{\int \sinh(x)\,dx}$

$= \frac{1}{\ln(5)} 5^x + 9 \cdot \cosh(x) + C$

$= \boxed{\frac{5^x}{\ln(5)} + 9\cosh(x) + C} \leftarrow F(x)$

---

(47) Given $f'(t) = \sec(t)\left(\sec(t) + \tan(t)\right)$, $f\left(\frac{\pi}{4}\right) = -4$

find f.

note: $f(t) = \int f'(t)\,dt$

$= \int \sec(t)\left[\sec(t) + \tan(t)\right]dt$

$$= \int \left[\sec^2(t) + \sec(t)\tan(t)\right] dt$$

$$= \underbrace{\int \sec^2(t)\,dt} + \underbrace{\int \sec(t)\tan(t)\,dt}$$

$$f(t) = \tan(t) + \sec(t) + C$$

$$f\left(\tfrac{\pi}{4}\right) = \underbrace{\tan\left(\tfrac{\pi}{4}\right)} + \underbrace{\sec\left(\tfrac{\pi}{4}\right)} + C = -4$$

$$\Rightarrow \cancel{1} + \cancel{\sqrt{2}} + C = -4$$

$$\cancel{-1} \quad - \cancel{\sqrt{2}} \qquad\qquad -1 \quad -\sqrt{2}$$

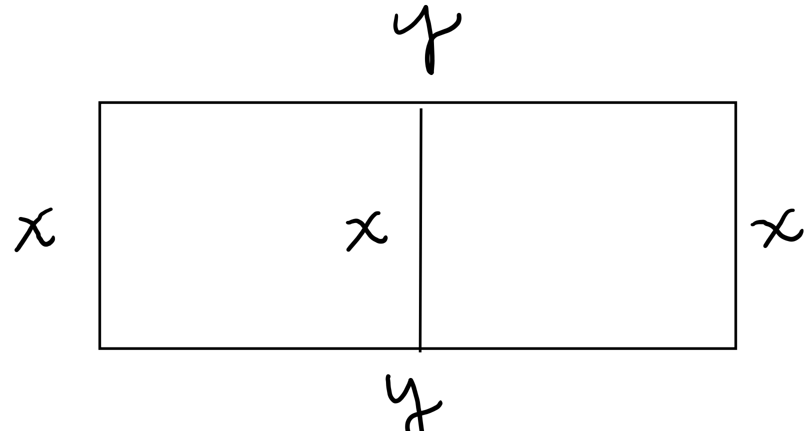
$$\Rightarrow C = -5 - \sqrt{2}$$

$$\Rightarrow f(t) = \tan(t) + \sec(t) + \overbrace{(-5 - \sqrt{2})}^{C}$$

OR

$$\boxed{f(t) = \tan(t) + \sec(t) - 5 - \sqrt{2}}$$

16] A farmer wants to fence an area of 13.5 million square feet in a rectangular field and then divide it in half with a fence parallel to one of the sides of the rectangle. What should the lengths of the sides of the rectangular field be so as to minimize the cost of the fence?



$A = xy$

$\Rightarrow 13.5 \times 10^6 = xy$

$\Rightarrow \dfrac{13.5 \times 10^6}{x} = y$

Minimize Cost C

$\Rightarrow$ minimize amount of fence used

$\Rightarrow$ minimize $C = 3x + 2y$

Since $y = \dfrac{13.5 \times 10^6}{x}$, $C(x) = 3x + 2\left(\dfrac{13.5 \times 10^6}{x}\right)$

$\Rightarrow C(x) = 3x + \dfrac{27 \times 10^6}{x}$

OR $C(x) = 3x + 27{,}000{,}000\, x^{-1}$

$\Rightarrow C'(x) = 3 - 27{,}000{,}000\, x^{-2}$

$$= 3 - \frac{27{,}000{,}000}{x^2}$$

$\Rightarrow$ We find critical number by setting

$C'(x) = 0$

$\Rightarrow x^2 \cdot \left(3 - \frac{27{,}000{,}000}{x^2}\right) = 0 \cdot x^2$

$\Rightarrow 3x^2 - 27{,}000{,}000 = 0$

$\Rightarrow 3(x^2 - 9{,}000{,}000) = 0$

OR $\frac{\cancel{3}x^2}{\cancel{3}} = \frac{27{,}000{,}000}{3}$

$\sqrt{x^2} = \pm\sqrt{9{,}000{,}000}$

$x = \pm 3000$

$\Rightarrow 3(x + 3000)(x - 3000) = 0$

$\Rightarrow x + 3000 = 0$ OR $x - 3000 = 0$

$\Rightarrow \cancel{x = -3000}$ OR $x = 3000$

Note: x = measurement $\Rightarrow x$ Cannot = neg. #

$\Rightarrow x = 3000$

now $C''(x) = 54{,}000{,}000\, x^{-3} = \dfrac{54{,}000{,}000}{x^3}$

Since $x \neq$ neg. # $\Rightarrow x^3 \neq$ neg. # $\Rightarrow C''(x) > 0$

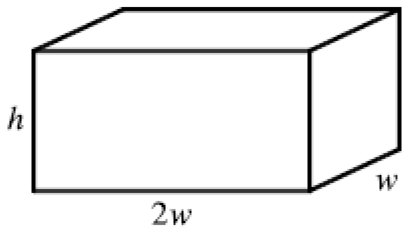
$\Rightarrow$ By the 2nd derivative test, $C(x)$ has a minimum at the critical point

$\Rightarrow C(x)$ is minimum at $x = 3000$

$$\Rightarrow y = \frac{13.5 \times 10^6}{3000} = \frac{13{,}500{,}000}{3000} = 4500$$

$\Rightarrow$ Cost is minimum when the sides are $\boxed{3000 \text{ ft}}$ and $\boxed{4500 \text{ ft}}$.

A rectangular storage container without a lid is to have a volume of 10 m^3. The length of its base is twice the width. Material for the base costs $15 per square meter. Material for the sides costs $9 per square meter. Find the cost (in dollars) of materials for the least expensive such container. (Round your answer to the nearest cent.)



$V = lwh = (2w)\cdot w \cdot h$

$\Rightarrow 10 = 2w^2 h \Rightarrow \frac{10}{2w^2} = h$

$\Rightarrow h = \frac{5}{w^2}$

Material for the base costs $15 per square meter

$\text{base} = 2w \cdot w = 2w^2 \Rightarrow \text{Cost} = \$15 \cdot (2w^2) = 30w^2$

Material for the sides costs $9 per square meter

Left & Right side: one side $= w \cdot h = w \cdot \left(\frac{5}{w^2}\right) = \frac{5}{w} \Rightarrow$ 2 sides $= 2 \cdot \left(\frac{5}{w}\right) = \frac{10}{w}$

front & back side: one side $= 2w \cdot h = 2w\left(\frac{5}{w^2}\right) = \frac{10}{w} \Rightarrow$ 2 sides $= 2\left(\frac{10}{w}\right) = \frac{20}{w}$

$\Rightarrow$ total surface area $= \frac{10}{w} + \frac{20}{w} = \frac{30}{w}$

$\Rightarrow$ Cost for the sides $= \$9 \cdot \left(\frac{30}{w}\right) = \frac{270}{w}$

$\Rightarrow$ Total cost is base + 4 sides

$\Rightarrow C(w) = 30w^2 + \frac{270}{w} = 30w^2 + 270w^{-1}$

$$\Rightarrow C'(w) = 60w - 270w^{-2} = 60w - \frac{270}{w^2}$$

$$\Rightarrow C'(w) = 0 \text{ when } 60w - \frac{270}{w^2} = 0$$

$$\Rightarrow w^2 \cdot \left(60w - \frac{270}{w^2}\right) = (0) \cdot w^2$$

$$\Rightarrow 60w^3 - 270 = 0$$

$$\Rightarrow \frac{60w^3}{60} = \frac{270}{60}$$

$$\Rightarrow w^3 = \frac{9}{2}$$

$$\Rightarrow w = \sqrt[3]{\frac{9}{2}} = \text{critical number}$$

note: $C''(w) = \frac{d}{dw}\left(60w - 270w^{-2}\right)$

$$= 60 + 540w^{-3} = 60 + \frac{540}{w^3} > 0 \text{ since}$$

$w = \text{width} = +$

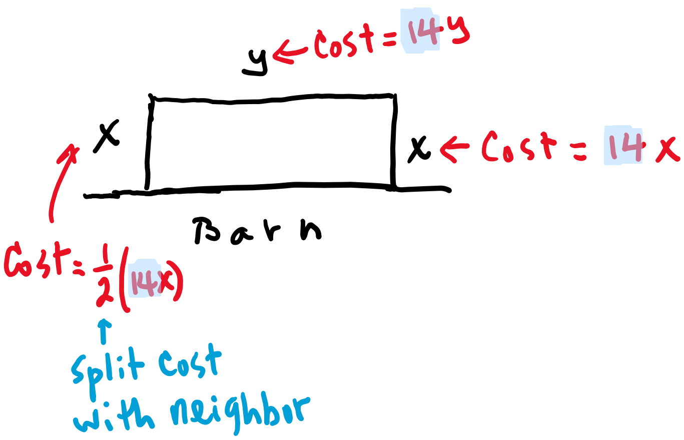
$$\Rightarrow C''\left(\sqrt[3]{\frac{9}{2}}\right) > 0 \Rightarrow \text{by 2nd Derivative test,}$$

Minimum is at $w = \sqrt[3]{\frac{9}{2}}$

$$\Rightarrow \text{Minimum cost is } C\left(\sqrt[3]{\frac{9}{2}}\right) = 30\left(\sqrt[3]{\frac{9}{2}}\right)^2 + \frac{270}{\sqrt[3]{\frac{9}{2}}}$$

$$\approx \boxed{\$245.31}$$

19. A farmer wants to fence in a rectangular plot of land adjacent to the north wall of his barn. No fencing is needed along the barn, and the fencing along the west side of the plot is shared with a neighbor who will split the cost of that portion of the fence. If the fencing costs $14 per linear foot to install and the farmer is not willing to spend more than $7000, find the dimensions for the plot that would enclose the most area. (Enter the dimensions as a comma separated list.)



$$\Rightarrow \text{total cost} = \frac{1}{2}(14x) + 14y + 14x$$

$$7000 = 7x + 14y + 14x$$

$$7000 = 21x + 14y$$

$$\Rightarrow 7000 - 21x = 14y$$

$$\Rightarrow \frac{7000}{14} - \frac{21x}{14} = y$$

$$\Rightarrow 500 - \frac{3}{2}x = y$$

Recall: Area of a rectangle = Length · Width

$$\Rightarrow A = y \cdot x$$

$$\Rightarrow A(x) = \left(500 - \frac{3}{2}x\right) \cdot x$$

$$\Rightarrow A(x) = 500x - \frac{3}{2}x^2$$

$$\Rightarrow A'(x) = 500 - 3x$$

$$\Rightarrow A'(x) = 0 \Rightarrow 500 - 3x = 0$$

$$500 = 3x$$

$$\frac{500}{3} = x$$

Critical number

note: $A''(x) = \frac{d}{dx}(500 - 3x) = -3$

$\Rightarrow A''\left(\frac{500}{3}\right) = -3$

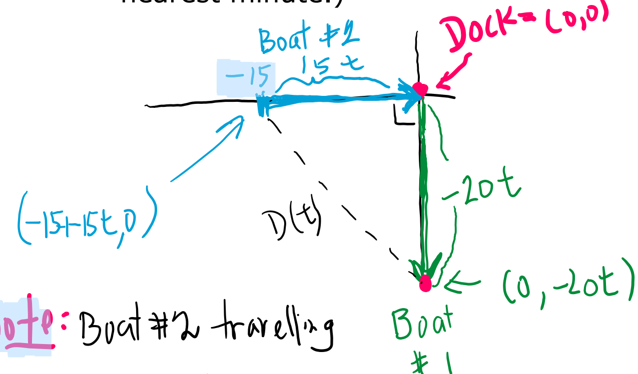
$\Rightarrow$ By the 2^{nd} Derivative test, maximum A occurs at $x = \frac{500}{3}$.

$\Rightarrow y = 500 - \frac{3}{2}x = 500 - \cancel{\frac{3}{2}}^{1}\left(\cancel{\frac{500}{3}}^{250}\right)$

$= 500 - 250 = 250$

$\Rightarrow$ The dimensions that would enclose the most area are $\boxed{\frac{500}{3}, 250}$

A boat leaves a dock at 1:00 PM and travels due south at a speed of 20 km/h. Another boat has been heading due east at 15 km/h and reaches the same dock at 2:00 PM. How many minutes after 1:00 PM were the two boats closest together? (Round your answer to the nearest minute.)



note: Boat #2 travelling at 15mph and reaches the dock in one hour $\Rightarrow$ its distance from the dock was $15 \cdot 1 = 15$ mi west of the dock.

Note: $[D(t)]^2 = (-20t)^2 + (-15+15t)^2$

$$\Rightarrow [D(t)]^2 = 400t^2 + 225 - 450t + 225t^2$$

$$\Rightarrow [D(t)]^2 = 625t^2 - 450t + 450$$

Note: $\boxed{\text{Minimum Value for } D(t) \text{ occurs at the same } t \text{ as } [D(t)]^2 \text{ so we will find } t \text{ where } [D(t)]^2 \text{ is the minimum}}$

Let $f(t) = [D(t)]^2 = 625t^2 - 450t + 450$

$$\Rightarrow f'(t) = 1250t - 450$$

$$\Rightarrow f'(t) = 0 \text{ when } 1250t - 450 = 0$$

$$\Rightarrow 1250t = 450$$

$$\Rightarrow t = \frac{450}{1250} = 0.36 \text{ hr.}$$

note: speed was given as mph $\Rightarrow t = $ hr.

also, $f''(t) = 1250 \Rightarrow f''(0.36) = 1250 > 0 \Rightarrow$ minimum @ $t = 0.36$ hr

Need Answer in minutes $\Rightarrow 0.36 \text{ hr} \times \frac{60 \text{ min}}{1 \text{ hr}} = 21.6 \text{ min} \simeq \boxed{22}$ min

At which points on the curve $y = 1 + 40x^3 - 3x^5$ does the tangent line have the largest slope?

Recall: slope of tangent line $= y'$ $\Rightarrow$ find derivative first.

$$\Rightarrow y' = \frac{d}{dx}\left(1 + 40x^3 - 3x^5\right) = 120x^2 - 15x^4.$$

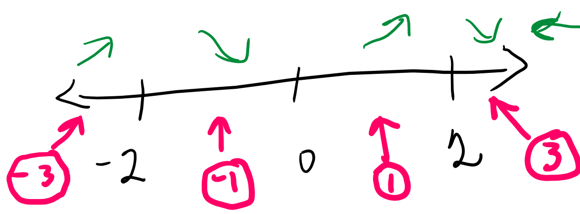
$\Rightarrow$ slope function at $x = a$ is

$$m(a) = 120a^2 - 15a^4 \leftarrow \text{function to maximize}$$

$$\Rightarrow m'(a) = 240a - 60a^3 = 60a(4 - a^2) = 60a(2+a)(2-a)$$

$$\Rightarrow m'(a) = 0 \text{ when } a = 0,\ a = -2,\ a = 2$$

$\Rightarrow$



$\Rightarrow$

Interval	a = Test point	60a	2+a	2-a	m'(a)	Inc/Dec.
$(-\infty, -2)$	-3	$-$	$-$	$+$	$+$	Increasing
$(-2, 0)$	-1	$-$	$+$	$+$	$-$	Decreasing
$(0, 2)$	1	$+$	$+$	$+$	$+$	Increasing
$(2, \infty)$	3	$+$	$+$	$-$	$-$	Decreasing

$\Rightarrow$ max m @ $a = -2$ or $a = 2$

$$\left.\begin{aligned} a = -2 &\Rightarrow m(-2) = 120(-2)^2 - 15(-2)^4 = 480 - 240 = 240 \\ a = 2 &\Rightarrow m(2) = 120(2)^2 - 15(2)^4 = 480 - 240 = 240 \end{aligned}\right\} \text{same} \rightarrow \text{max } m \text{ at both } -2 \text{ and } 2$$

at $x=-2$, $y = 1+40(-2)^3-3(-2)^5$

$= 1+40(-8)-3(-32)$

$= 1-320+96$

$= -223$

$\Rightarrow$ Smaller x-value : $(x,y) = \boxed{(-2,-223)}$

at $x=2$, $y = 1+40(2)^3-3(2)^5$

$= 1+40(8)-3(32)$

$= 1+320-96$

$= 225$

$\Rightarrow$ larger x-value : $(x,y) = \boxed{(2,225)}$

Consider the following.

$$3x^4 - 8x^3 + 9 = 0, \quad [2, 3]$$

(a) Explain how we know that the given equation must have a solution in the given interval.

Let $f(x) = 3x^4 - 8x^3 + 9$.

The polynomial f is continuous on $[2, 3]$, $f(2) = \boxed{-7} < 0$, and $f(3) = \boxed{36} > 0$,

$f(2) = 3(2)^4 - 8(2)^3 + 9$
$= 3(16) - 8(8) + 9$
$= 48 - 64 + 9$
$= -7$

$f(3) = 3(3)^4 - 8(3)^3 + 9$
$= 3(81) - 8(27) + 9$
$= 243 - 216 + 9$
$= 36$

Recall: The Intermediate Value Theorem
Suppose that f is continuous on the closed interval $[a, b]$ and let N be any number between $f(a)$ and $f(b)$, where $f(a) \neq f(b)$. Then there exists a number c in (a, b) such that $f(c) = N$.

so by the Intermediate Value Theorem, there is a number c in $(2, 3)$ such that

$$f(c) = \boxed{0}$$

In other words, the equation $3x^4 - 8x^3 + 9 = 0$ has a solution in $[2, 3]$.

(b)Use Newton's method to approximate the solution correct to six decimal places.

Recall: Newton's Method

If the nth approximation is x_n and $f'(x_n) \neq 0$, then the next approximation is given by

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

If the numbers x_n become closer and closer to r as n becomes large, then we say that the sequence *converges* to r and we write

$$\lim_{n\to\infty} x_n = r$$

$f(x) = 3x^4 - 8x^3 + 9 \Rightarrow f'(x) = 12x^3 - 24x$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$\Rightarrow x_{n+1} = x_n - \frac{3x_n^4 - 8x_n^3 + 9}{12x_n^3 - 24x_n}$$

Let $x_1 = 2.5 \Rightarrow x_2 = 2.5 - \dfrac{3(2.5)^4 - 8(2.5)^3 + 9}{12(2.5)^3 - 24(2.5)} \approx 2.490686$

$$x_3 = 2.490686 - \frac{3(2.490686)^4 - 8(2.490686)^3 + 9}{12(2.490686)^3 - 24(2.490686)} \approx 2.483978$$

$$x_4 = 2.483978 - \frac{3(2.483978)^4 - 8(2.483978)^3 + 9}{12(2.483978)^3 - 24(2.483978)} \approx 2.479151$$

$$x_5 = 2.479151 - \frac{3(2.479151)^4 - 8(2.479151)^3 + 9}{12(2.479151)^3 - 24(2.479151)} \approx 2.475679$$

$$x_6 = 2.475679 - \frac{3(2.475679)^4 - 8(2.475679)^3 + 9}{12(2.475679)^3 - 24(2.475679)} \approx 2.473183$$

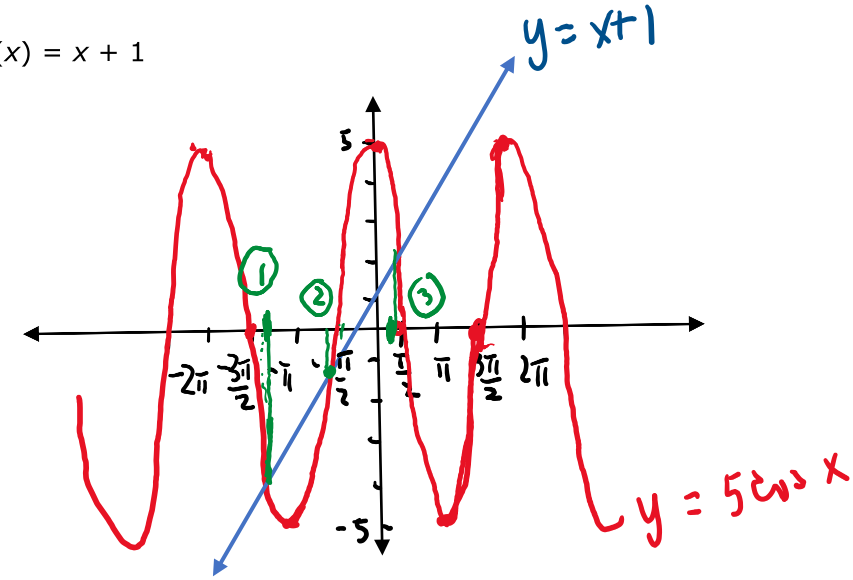
$$\vdots$$

$$\left.\begin{array}{l} x_{31} \approx 2.466813 \\ x_{32} \approx 2.466813 \end{array}\right\} \text{same} \Rightarrow x \approx \boxed{2.466813}$$

33] Use Newton's method to find all solutions of the equation correct to six decimal places. (Enter your answers as a comma-separated list.)

$5\cos(x) = x + 1$

note:



Let $f(x) = 5\cos(x) - x - 1$

$$\Rightarrow f'(x) = -5\sin(x) - 1$$

$$\Rightarrow x_{n+1} = x_n - \frac{5\cos(x_n) - x_n - 1}{-5\sin(x_n) - 1}$$

$$\Rightarrow \boxed{x_{n+1} = x_n + \frac{5\cos(x_n) - x_n - 1}{5\sin(x_n) + 1}}$$

1 Try $x_1 = -4$ radians

$$\Rightarrow x_2 = -4 + \frac{5\cos(-4) - (-4) - 1}{5\sin(-4) + 1}$$

$$= -4.0560655\ldots \approx \boxed{-4.056066}$$

$$x_3 = x_2 + \frac{5\cos(x_2) - (x_2) - 1}{5\sin(x_2) + 1}$$

$$= -4.0550528\ldots \approx \boxed{-4.055053}$$

$$x_4 = x_3 + \frac{5\cos(x_3) - (x_3) - 1}{5\sin(x_3) + 1}$$

$$= -4.0550525\ldots \approx \boxed{-4.055053}$$

Note: since $x_4 = x_3$, $x = -4.055053$

---

2 Try $x_1 = -2$

$$\Rightarrow x_2 = -2 + \frac{5\cos(-2) - (-2) - 1}{5\sin(-2) + 1}$$

$$= -1.6952662\ldots \approx \boxed{-1.695266}$$

$$x_3 = x_2 + \frac{5\cos(x_2) - (x_2) - 1}{5\sin(x_2) + 1}$$

$$= -1.7140787\cdots \approx \boxed{-1.714079}$$

$$x_4 = x_3 + \frac{5\cos(x_3) - (x_3) - 1}{5\sin(x_3) + 1}$$

$$= -1.71410\,79\cdots \approx \boxed{-1.714108}$$

$$x_5 = x_4 + \frac{5\cos(x_4) - (x_4) - 1}{5\sin(x_4) + 1}$$

$$= -1.7141079\cdots \approx \boxed{-1.714108}$$

NOTE:

$\Rightarrow$ Since $x_5 = x_4$, $x = -1.714108$

③ Try $x_1 = 1$

$$\Rightarrow x_2 = 1 + \frac{5\cos(1) - (1) - 1}{5\sin(1) + 1}$$

$$= 1.1347155\cdots \approx \boxed{1.134716}$$

$$x_3 = x_2 + \frac{5\cos(x_2) - (x_2) - 1}{5\sin(x_2) + 1}$$

$$= 1.13066006\cdots \approx \boxed{1.130601}$$

$$x_4 = x_3 + \frac{5\cos(x_3) - (x_3) - 1}{5\sin(x_3) + 1}$$

$$= 1.1305973\cdots \approx \boxed{1.130597}$$

$$x_5 = x_4 + \frac{5\cos(x_4) - (x_4) - 1}{5\sin(x_4) + 1}$$

$$= 1.1305973\cdots \approx \boxed{1.130597}$$

NOTE: Since $x_5 = x_4$, $x = 1.130597$

$$\Rightarrow \boxed{x = -4.055053,\ -1.714108,\ 1.130597}$$

Answers

Use the guidelines of this section to sketch the curve.

$$f(x) = \frac{x^2}{x^2 + 27}$$

Guidelines:

A. Domain

Note: f is a rational function $\Rightarrow$ only undefined if denominator $=0$

Here denominator $= x^2+27$ which will never $=0$

$\Rightarrow$ Domain $=$ dom $f = (-\infty, \infty) =$ all real numbers

B. x-intercept(s) & y-intercept

x-intercept(s): Let $f(x)=0 \Rightarrow 0 = \frac{x^2}{x^2+27}$

$\Rightarrow 0 = x^2$

$\Rightarrow 0 = x \Rightarrow$ x-intercept is $(0,0)$

y-intercept: Let $x=0 \Rightarrow f(0) = \frac{0^2}{0^2+27} = \frac{0}{27} = 0 \Rightarrow$ y-intercept is also $(0,0)$

C. Symmetry

Recall: $\boxed{\begin{array}{l} f(-x) = f(x) \Leftrightarrow f \text{ is even} \Leftrightarrow \text{Symmetry with respect to y-axis} \\ f(-x) = -f(x) \Leftrightarrow f \text{ is odd} \Leftrightarrow \text{Symmetry with respect to the origin} \end{array}}$

Here, $f(-x) = \frac{(-x)^2}{(-x)^2+27} = \frac{x^2}{x^2+27} = f(x) \Rightarrow f$ is even

$\Rightarrow$ we have Symmetry with respect to the y-axis

D. Asymptotes

Vertical Since dom $f = (-\infty, \infty)$, we have no vertical asymptote.

Horizontal

Note: If $\lim_{x\to\pm\infty} f(x) = L$ Then $y = L$ is the horizontal asymptote

$\Rightarrow \lim_{x\to\pm\infty} f(x) = \lim_{x\to\pm\infty}\left(\frac{x^2}{x^2+27}\right) = \lim_{x\to\pm\infty}\left(\frac{\frac{x^2}{x^2}}{\frac{x^2}{x^2}+\frac{27}{x^2}}\right)$

$= \lim_{x\to\pm\infty} \frac{1}{1+\frac{27}{x^2}} = \frac{1}{1+0} = \frac{1}{1} = 1 \Rightarrow$ $\boxed{\text{Horizontal asymptote is } y=1}$

E. Intervals of increase & decrease → Need to find f'.

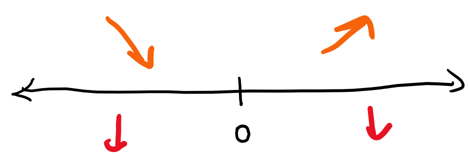
$$f'(x) = \frac{d}{dx}\left[\frac{x^2}{x^2+27}\right]$$

$$= \underbrace{\frac{(x^2+27)\cdot\frac{d}{dx}(x^2) - x^2\cdot\frac{d}{dx}(x^2+27)}{(x^2+27)^2}}_{\text{Use Quotient Rule}}$$

$$= \frac{(x^2+27)\cdot(2x) - x^2\cdot(2x)}{(x^2+27)^2}$$

$$= \frac{2x^3+54x-2x^3}{(x^2+27)^2} = \frac{54x}{(x^2+27)^2} \Rightarrow f'(x)=0 \text{ at } x=0$$

$\Rightarrow$



$x=-1$

$$f'(-1) = \frac{54(-1)}{[(-1)^2+27]^2} = \frac{-}{+} = -$$

$x=1$

$$f'(1) = \frac{54(1)}{(1^2+27)^2} = \frac{+}{+} = +$$

$\Rightarrow$ $\boxed{\text{Decrasing on } (-\infty, 0) \text{ and Increasing on } (0, \infty)}$

F. Local extrema (max & min)

Since ↘ ↗ across $x=0$, we have a local minimum at $x=0$

$$\Rightarrow \text{Local minimum is } f(0) = \frac{0^2}{0^2+27} = \frac{0}{27} = 0.$$

G. Intervals of concavity → Need f''.

$$f''(x) = \frac{d}{dx}\left[f'(x)\right] = \frac{d}{dx}\left[\frac{54x}{(x^2+27)^2}\right]$$

$$= \frac{(x^2+27)^2 \cdot \frac{d}{dx}(54x) - 54x \cdot \frac{d}{dx}\left[(x^2+27)^2\right]}{\left[(x^2+27)^2\right]^2}$$

$$= \frac{(x^2+27)^2 \cdot (54) - 54x \cdot \left[2(x^2+27)\cdot 2x\right]}{(x^2+27)^4}$$

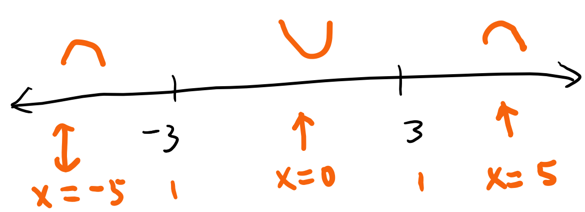
$$= \frac{54(x^2+27)^2 - 216x^2(x^2+27)}{(x^2+27)^4}$$

$$= \frac{54\cancel{(x^2+27)}\left[(x^2+27) - 4x^2\right]}{(x^2+27)^{\cancel{4}\,3}}$$

$$= \frac{54(x^2+27-4x^2)}{(x^2+27)^3} = \frac{54\underbrace{(27-3x^2)}_{3(9-x^2)}}{(x^2+27)^3} = \frac{162(3+x)(3-x)}{(x^2+27)^3}$$

$\Rightarrow f''(x) = 0$ at $(3+x)(3-x) = 0$

$\Rightarrow x = -3$ or $x = 3$



$$f''(-5) = \frac{162(3+(-5))(3-(-5))}{(x^2+27)^3} = \frac{+(-)(+)}{+} = -$$

$$f''(0) = \frac{162(3+0)(3-0)}{(0^2+27)^3} = \frac{+(+)(+)}{+} = +$$

$$f''(5) = \frac{162(3+5)(3-5)}{(5^2+27)^3} = \frac{+(+)(-)}{+} = -$$

⇒ Concave down on $(-\infty, -3) \cup (3, \infty)$ and Concave up on $(-3, 3)$

Inflection points @ $x = \pm 3$.

$$\Rightarrow f(\pm 3) = \frac{(\pm 3)^2}{(\pm 3)^2 + 27} = \frac{9}{9+27} = \frac{9}{36} = \frac{1}{4}$$

⇒ Inflection points are $(-3, \frac{1}{4})$ and $(3, \frac{1}{4})$

H. Graph

Summary:

A. dom $f = (-\infty, \infty)$

B. x-intercept = y-intercept = $(0, 0)$

C. Symmetry w.r.t. y-axis.

D. No vertical asymptote

Horizontal asymptote is $\boxed{y = 1}$

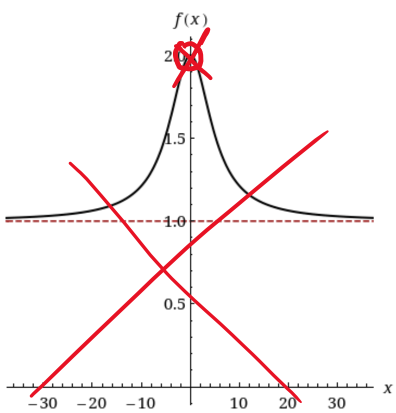
E. Decreasing on $(-\infty, 0)$

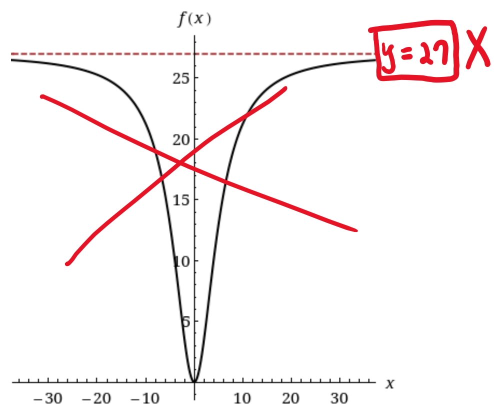
Increasing on $(0, \infty)$

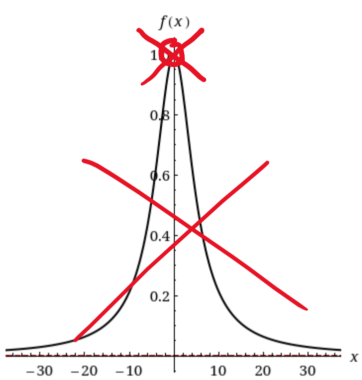
F. Local min of $f(0) = 0$.

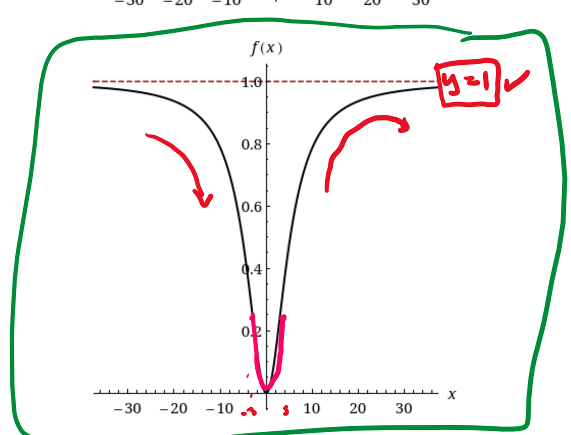
Inflection pts $(-3, \frac{1}{4}), (3, \frac{1}{4})$ → G. Concave upward on $(-3, 3)$

Concave downward on $(-\infty, -3) \cup (3, \infty)$









Answer

50. Find a function f such that $f'(x) = 3x^3$ and the line $81x + y = 0$ is tangent to the graph of f.

Solution,

note: $f'(x) = 3x^3 \Rightarrow f(x) = \frac{3}{4}x^4 + C$

Given line: $81x + y = 0 \Rightarrow y = -81x \Rightarrow m = -81$

Since line is tangent to the graph,

$m = f'(x)$

$\Rightarrow \frac{-81}{3} = \frac{3x^3}{3} \Rightarrow -27 = x^3$

$\Rightarrow \sqrt[3]{-27} = \sqrt[3]{x^3}$

$\Rightarrow -3 = x$

$\Rightarrow y = -81x = -81(-3) = 243$

$\Rightarrow$ The point $(-3, 243)$ is on the graph

$\Rightarrow f(-3) = 243$

$\Rightarrow \frac{3}{4}(-3)^4 + C = 243$

$\Rightarrow \frac{3}{4}(81) + C = 243$

$\Rightarrow \frac{243}{4} + C = \underset{1\cdot 4}{243}\cdot 4 - \frac{243}{4} = \frac{972}{4} - \frac{243}{4} = \frac{729}{4}$

$-\frac{243}{\cancel{4}}$

$\Rightarrow C = \frac{729}{4} \Rightarrow \boxed{f(x) = \frac{3}{4}x^4 + \frac{729}{4}}$

55] A car is traveling at 106 km/h when the driver sees an accident 65 m ahead and slams on the brakes. What minimum constant deceleration is required to stop the car in time to avoid a pileup? (Round your answer to two decimal places.)

Let $a(t) = K$ (km/h^2) where $K < 0$

at $t = 0$, $s(0) = 0$ and $V(0) = 106$

note ①: $V(t) = \int a(t)dt = \int K\,dt = Kt + C_1$

$$v(0) = 106 \Rightarrow 106 = K(0) + C_1$$

$$\Rightarrow 106 = 0 + C_1$$

$$\Rightarrow 106 = C_1$$

$$\Rightarrow v(t) = Kt + 106$$

② $s(t) = \int v(t)dt$

$$= \int (Kt + 106)dt$$

$$= \int Kt\,dt + \int 106\,dt$$

$$= K\int t\,dt + 106\int dt$$

$$= K \cdot \frac{t^2}{2} + 106t + C_2$$

$$\Rightarrow S(t) = \frac{Kt^2}{2} + 106t + C_2$$

Given $S(0) = 0 \Rightarrow \frac{K(0)^2}{2} + 106(0) + C_2 = 0$

$$\Rightarrow 0 + 0 + C_2 = 0 \Rightarrow C_2 = 0$$

$$\Rightarrow S(t) = \frac{Kt^2}{2} + 106t$$

<u>note</u>: when the car stopped $V(t) = 0$

$$\Rightarrow Kt + 106 = 0 \Rightarrow t = \frac{-106}{K}$$

and $S(t) < 65$ meters to not hit cars in the accident

or, $S(t) < 0.065\,km$

$$\Rightarrow 2\left(\frac{Kt^2}{\cancel{2}}\right) + 2\cdot(106t) < 2(0.065)$$

$$\Rightarrow Kt^2 + 212t < 0.13$$

$$\Rightarrow K\left(\frac{-106}{K}\right)^2 + 212\left(-\frac{106}{K}\right) < 0.13$$

$$\Rightarrow \cancel{K}\left(\frac{11,236}{\cancel{K^2}\,K}\right) - \frac{22,472}{K} < 0.13$$

$$\Rightarrow \frac{11,236}{K} - \frac{22,472}{K} < 0.13$$

$$\Rightarrow \cancel{K}\left(-\frac{11236}{\cancel{K}}\right) <^{K} (0.13)$$

switch to >

$$\boxed{K<0} \Rightarrow \frac{-11236}{0.13} > \frac{0.13\,K}{0.13}$$

$$\Rightarrow -86430.76923\ldots > K$$

km/hr²

$$\Rightarrow -86430.76923\, \frac{\cancel{km}}{\cancel{hr^2}} \times \frac{1000\, m}{1\, \cancel{km}} \times \frac{1\, \cancel{hr}}{3600\, sec} \times \frac{1\, \cancel{hr}}{3600\, sec}$$

Decelerate ⟷ (−)

$$= \boxed{6.67\ m/sec^2}$$

55. A car is traveling at 88 km/h when the driver sees an accident 50 m ahead and slams on the brakes. What minimum constant deceleration (in m/s^2) is required to stop the car in time to avoid a multi-car pileup? (Round your answer to two decimal places.)

Initial Velocity of the car = 88 Km/h $\Rightarrow V(0) = 88$

Now Let $S(t)$ = position function $\Rightarrow S(0) = 0$

Because The accident is 50 m ahead $\Rightarrow$ 0.05 Km ahead

← Need to change m to km since $V(t)$ is in km/h

$\Rightarrow$ To avoid hitting the accident, we need

$$\boxed{s(t) < 0.05}$$ ← Need to find $S(t)$.

Let acceleration $= a(t) = K$, where K = a Constant

$$\Rightarrow \quad v'(t) = K \quad \text{since } v'(t) = a(t)$$

$$\Rightarrow \quad v(t) = Kt + C$$

but $V(0) = 88 \Rightarrow K(0) + C = 88$

$$\Rightarrow 0 + C = 88 \Rightarrow C = 88$$

$$\Rightarrow \boxed{v(t) = Kt + 88}$$

Since $v(t) = s'(t)$, we have $s(t) = \frac{K}{2}t^2 + 88t + D$

↑ Constant

Given $S(0) = 0 \Rightarrow \frac{K}{2}(0)^2 + 88(0) + D = 0$

$$0 + 0 + D = 0$$

$$\Rightarrow D = 0$$

$$\Rightarrow \boxed{S(t) = \frac{K}{2}t^2 + 88t}$$

Note: when the car stops, $V(t) = 0$

$$\Rightarrow \quad Kt + 88 = 0$$

$$\Rightarrow \quad Kt = -88 \Rightarrow t = \frac{-88}{K}$$ is when the car stops

$\Rightarrow$ The position where the car stops is

$$s\left(-\frac{88}{k}\right) = \frac{k}{2}\left(-\frac{88}{k}\right)^2 + 88\left(-\frac{88}{k}\right)$$

$$= \frac{\cancel{k}^{1}}{\cancel{2}_{1}}\left(\frac{\cancel{7744}^{3872}}{\cancel{k^2}_{k}}\right) - \frac{7744}{k}$$

$$= \frac{3872}{k} - \frac{7744}{k}$$

$$= -\frac{3872}{k}$$

To avoid the accident, we need

$$s\left(-\frac{88}{k}\right) < 0.05$$

$$\Rightarrow k\cdot\left(-\frac{3872}{k}\right) < (0.05)\cdot k$$

Note:

Since we are decelerating, $k < 0 \Rightarrow$ need to switch direction of inequality because we are multiplying a negative number on both sides.

$$\Rightarrow -3872 > 0.05k$$

$$\Rightarrow -\frac{3872}{0.05} > k$$

$$\Rightarrow -77{,}440 > k$$

<u>OR</u> $k < -77{,}440 \ km/h^2$

<u>But:</u> we want answer in m/s^2, we need to convert km/h^2

$$\Rightarrow \frac{77{,}440 \ km}{h^2} = \frac{77440 \ \cancel{km}}{1 \ \cancel{h^2}} \times \frac{1000 \ m}{1 \ \cancel{km}} \times \frac{1 \ \cancel{h}}{3600 \ s} \times \frac{1 \ \cancel{h}}{3600 \ s} \approx \boxed{5.98 \ m/s^2}$$

57. A particular bullet train accelerates and decelerates at the rate of 4 ft/s^2. Its maximum cruising speed is 60 mi/h. (Round your answers to three decimal places.)

(a) What is the maximum distance (in mi) the train can travel if it accelerates from rest until it reaches its cruising speed and then runs at that speed for 20 minutes?

note: $a(t) = 4 \Rightarrow v(t) = \int a(t)\,dt = \int 4\,dt = 4t + C$

but $v(0) = 0 \Rightarrow 4(0) + C = 0$

$\Rightarrow 0 + C = 0 \Rightarrow C = 0$

$\Rightarrow v(t) = 4t$

Since acceleration is in ft/s, we need to convert speed from mi/h to ft/s.

$$\Rightarrow v = 60 \text{ mi/h} = \frac{60\,\cancel{\text{mi}}}{1\,\cancel{\text{hr}}} \cdot \frac{1\,\cancel{\text{hr}}}{3600\,\text{s}} \cdot \frac{5280\,\text{ft}}{1\,\cancel{\text{mi}}} = 88 \text{ ft/s}$$

$\Rightarrow v(t) = 88 \Rightarrow 4t = 88 \Rightarrow t = 22$

$\Rightarrow$ It takes 22 seconds to reach 88 ft/s

Since $v(t) = s'(t)$, we get $s(t) = \int v(t)\,dt$

$= \int 4t\,dt$

$= 2t^2 + C$

$s(0) = 0 \Rightarrow 2(0)^2 + C = 0 \Rightarrow 0 + C = 0 \Rightarrow C = 0$

$\Rightarrow$ The position function is $s(t) = 2t^2$

$\Rightarrow$ The position at $t = 22$ sec is $s(22) = 2(22)^2 = 968$ ft

Since the cruising speed is $v = 88$ ft/s and 20 min = 1200 sec,

the train went another $88 \cdot 1200 = 105{,}600$ ft

$\Rightarrow$ the train traveled $968\text{ ft} + 105{,}600\text{ ft} = 106{,}568\text{ ft}$

Converting to miles, we get

$$106{,}568\text{ ft} \cdot \frac{1\text{ mi}}{5280\text{ ft}} = \frac{106{,}568}{5280}\text{ mi} \approx \boxed{20.183}\text{ mi}$$

(b) Suppose that the train starts from rest and must come to a complete stop in 20 minutes. What is the maximum distance (in mi) it can travel under these conditions?

From part (a), train takes 22 seconds to reach 88 ft/s and travels 968 ft. Therefore, it needs to decelerate for another 22 seconds and travels another 968 ft at the end of its trip $\Rightarrow 22 + 22 = 44$ seconds to accelerate and decelerate.

$\Rightarrow$ During the 20 minutes = 1200 seconds trip,

the train is cruising at 88 ft/s for

$$1200\text{ sec} - 44\text{ sec} = 1156\text{ seconds}$$

$\Rightarrow$ distance traveled during that time is

$$88 \cdot 1156 = 101{,}728\text{ ft}$$

$\Rightarrow$ Total distance traveled is

$$\underbrace{968}_{\substack{\text{Distance}\\\text{traveled}\\\text{while}\\\text{accelerating}}} + \underbrace{101,728}_{\substack{\text{Distance}\\\text{traveled}\\\text{while}\\\text{Cruising}}} + \underbrace{968}_{\substack{\text{Distance}\\\text{traveled}\\\text{while}\\\text{decelerating}}} = 103,664 \text{ ft}$$

Converting to miles, we get $\dfrac{103,664}{5280} \approx \boxed{19.633} \text{ mi}$

$\boxed{5280 \text{ ft} = 1 \text{ mi}}$

(c) Find the minimum time (in min) that the train takes to travel between two consecutive stations that are 30 miles apart.

note: 30 miles = 30(5280) = 158,400 ft

$$\Rightarrow \begin{array}{c}\text{Distance}\\\text{traveled}\\\text{at}\\\text{Cruising}\\\text{speed}\end{array} = 158,400 \text{ ft} - \underbrace{968 \text{ ft}}_{\substack{\text{Distance}\\\text{traveled}\\\text{during}\\\text{acceleration}}} - \underbrace{968 \text{ ft}}_{\substack{\text{Distance}\\\text{traveled}\\\text{during}\\\text{deceleration}}}$$

$$= 156,464 \text{ ft}$$

Since $D = R \cdot T \Rightarrow T = \dfrac{D}{R}$, we get time spent traveling 156,464 ft at cruising speed of 88 ft/s as $T = \dfrac{156,464}{88} = 1778$ seconds

$\Rightarrow$ Total time needed to travel 60 mi $= \underset{\substack{\uparrow\\ \text{time spent}\\ \text{accelerating}}}{22} + \underset{\substack{\uparrow\\ \text{time spent}\\ \text{Cruising}}}{1778} + \underset{\substack{\uparrow\\ \text{time spent}\\ \text{decelerating}}}{22} = 1822 \text{ sec}$

Since there are 60 seconds in 1 minute,

we get $\text{time} = \frac{1822}{60} \approx \boxed{30.367}$ minutes

(d) The trip from one station to the next takes at minimum 37.5 minutes. How far (in mi) apart are the stations?

note: 37.5 minutes $= 37.5(60) = 2250$ seconds

$\Rightarrow$ 2250 seconds $=$ acceleration time $+$ Cruising time $+$ deceleration time

$\Rightarrow 2250 = 22 + \text{Cruising time} + 22$

$\Rightarrow$ Cruising time $= 2250 - 22 - 22 = 2206$ sec.

$D = R \cdot T$

$\Rightarrow$ distance traveled while Cruising at 88 ft/s $= 88 \cdot 2206 = 194{,}128$ ft

$\Rightarrow$ Total Distance $= \underbrace{968}_{\substack{\text{Distance traveled}\\ \text{while accelerating}}} + \underbrace{194{,}128}_{\substack{\text{Distance traveled}\\ \text{while at Crusing speed}}} + \underbrace{968}_{\substack{\text{Distance traveled}\\ \text{while decelerating}}} = 196{,}064$ ft

Converting answer to miles using

1 mi = 5280 ft, we get

$$\text{Distance} = \frac{196{,}064}{5280} \approx \boxed{37.133}\ \text{mi}$$

Use the guidelines of this section to sketch the curve. (In guideline D find an equation of the slant asymptote.)

$$f(x) = \frac{x^2}{x-4}$$

Guidelines:

A: Domain

note: f is a fraction $\Rightarrow$ denominator $\neq 0$

$\Rightarrow x-4 \neq 0$

$\Rightarrow x \neq 4$

$\Rightarrow$ domain $= (-\infty, 4) \cup (4, \infty)$

B: x-intercept & y-intercept

x-intercept: Let $f(x)=0 \Rightarrow \frac{x^2}{x-4} = 0$

$\Rightarrow x^2 = 0 \Rightarrow x = 0$

$\Rightarrow (0,0)$

y-intercept: Let $x=0 \Rightarrow f(0) = \frac{0^2}{0-4} = \frac{0}{-4} = 0 \Rightarrow (0,0)$

C: Symmetry

Recall:

$$\boxed{\begin{array}{l} f(-x) = -f(x) \Leftrightarrow f \text{ is odd} \Leftrightarrow \text{Symmetric with respect to origin} \\ f(-x) = f(x) \Leftrightarrow f \text{ is even} \Leftrightarrow \text{symmetric with respect to y-axis} \end{array}}$$

Here, $f(-x) = \frac{(-x)^2}{-x-4} = \frac{x^2}{-x-4} \left.\begin{array}{l} \neq -f(x) \\ \neq f(x) \end{array}\right\} \Rightarrow$ no symmetry

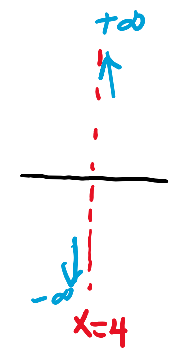
D: Asymptotes:

Vertical

* since $x \neq 4$, $x=4$ is possible vertical asymptote.

verify: $\lim\limits_{x \to 4^-} f(x) = \lim\limits_{x \to 4^-} \dfrac{x^2}{x-4} \begin{matrix} \to + \\ \to - \end{matrix} \Big\} \to -\infty$

$\lim\limits_{x \to 4^+} f(x) = \lim\limits_{x \to 4^+} \dfrac{x^2}{x-4} \begin{matrix} \to + \\ \to + \end{matrix} \Big\} \to +\infty$



$\Rightarrow$ vertical asymptote is $x=4$

Horizontal

Recall: $\boxed{\text{If } \lim\limits_{x \to \pm\infty} f(x) = L \text{, then } y=L \text{ is the horizontal asymptote.}}$

Here, $f(x) = \dfrac{x^2}{x-4} = x+4+\dfrac{16}{x-4}$

$$\boxed{\begin{array}{r|rrr} 4 & 1 & 0 & 0 \\ & & 4 & 16 \\ \hline & 1 & 4 & 16 \end{array}}$$

$\Rightarrow \lim\limits_{x \to \pm\infty} f(x) = \lim\limits_{x \to \pm\infty} \dfrac{x^2}{x-4} = \lim\limits_{x \to \pm\infty} x+4+\dfrac{16}{x-4}$

$\Rightarrow \lim\limits_{x \to -\infty} f(x) = \lim\limits_{x \to -\infty} x+4+\dfrac{16}{x-4} = -\infty$

and $\lim\limits_{x \to +\infty} f(x) = \lim\limits_{x \to +\infty} x+4+\dfrac{16}{x-4} = +\infty$

$\Big\} \neq L$

$\Rightarrow$ no horizontal asymptote.

Recall:

If $\lim_{x \to \infty} [f(x) - (mx+b)] = 0$

Then $y = mx+b$ is the slant asymptote

Since $f(x) = x + 4 + \frac{16}{x-4}$, $mx+b = x+4$

Verify: $\lim_{x \to \infty} [f(x) - (mx+b)]$

$$= \lim_{x \to \infty} \left[\left(x+4+\frac{16}{x-4}\right) - (x+4)\right]$$

$$= \lim_{x \to \infty} \left[x+4+\frac{16}{x-4} - x - 4\right]$$

$$= \lim_{x \to \infty} \left[\frac{16}{x-4}\right] = 0$$

$\Rightarrow$ $y = x+4$ is the slant asymptote.

E: Intervals of increase & decrease

$$f'(x) = \frac{d}{dx}\left[\frac{x^2}{x-4}\right]$$

$$= \frac{\frac{d}{dx}(x^2)\cdot(x-4) - x^2 \cdot \frac{d}{dx}(x-4)}{(x-4)^2}$$

$$= \frac{2x(x-4) - x^2 \cdot (1)}{(x-4)^2}$$

$$= \frac{2x^2 - 8x - x^2}{(x-4)^2}$$

$$= \frac{x^2 - 8x}{(x-4)^2} = \frac{x(x-8)}{(x-4)^2}$$

$\leftarrow f'(x) = 0$ @ $x=0$, $x=8$

$\leftarrow f'(x) =$ undefined @ $x=4$

$\Rightarrow$ critical numbers are $x = 0, 4, 8$.

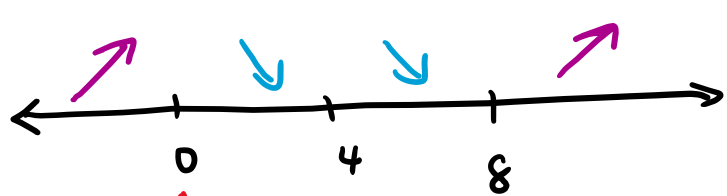
Interval	Test point	$f'(x) = \frac{x(x-8)}{(x-4)^2}$	Increasing ($\nearrow$) Decreasing ($\searrow$)
$(-\infty, 0)$	-1	$f'(-1) = \frac{-1(-1-8)}{(-1-4)^2} = \frac{- \; -}{+} = +$	$\nearrow$
$(0, 4)$	2	$f'(2) = \frac{2(2-8)}{(2-4)^2} = \frac{+ \; -}{+} = -$	$\searrow$
$(4, 8)$	5	$f'(5) = \frac{5(5-8)}{(5-4)^2} = \frac{+ \; -}{+} = -$	$\searrow$
$(8, \infty)$	10	$f'(10) = \frac{10(10-8)}{(10-4)^2} = \frac{+ \; +}{+} = +$	$\nearrow$

$\Rightarrow$ Increasing on $(-\infty, 0), (8, \infty)$

Decreasing on $(0, 4), (4, 8)$

F: Local extrema (max & min)

Visual aid:



Local max @ $x=0$

$\Rightarrow f(0) = \frac{0^2}{0-4} = 0$

max

Local min @ $x=8$

$\Rightarrow f(8) = \frac{8^2}{8-4} = \frac{64}{4} = 16$

min

G: Intervals of concavity

$$f''(x) = \frac{d}{dx}\left[\frac{x^2-8x}{(x-4)^2}\right]$$

$$= \frac{(x-4)^2 \cdot \frac{d}{dx}(x^2-8x) - (x^2-8x)\cdot\frac{d}{dx}\left[(x-4)^2\right]}{\left[(x-4)^2\right]^2}$$

$$= \frac{(x-4)^2 \cdot (2x-8) - (x^2-8x)\cdot\left[2(x-4)\cdot 1\right]}{(x-4)^4}$$

$$= \frac{(x-4)^2 \cdot 2(x-4) - 2(x-4)(x^2-8x)}{(x-4)^4}$$

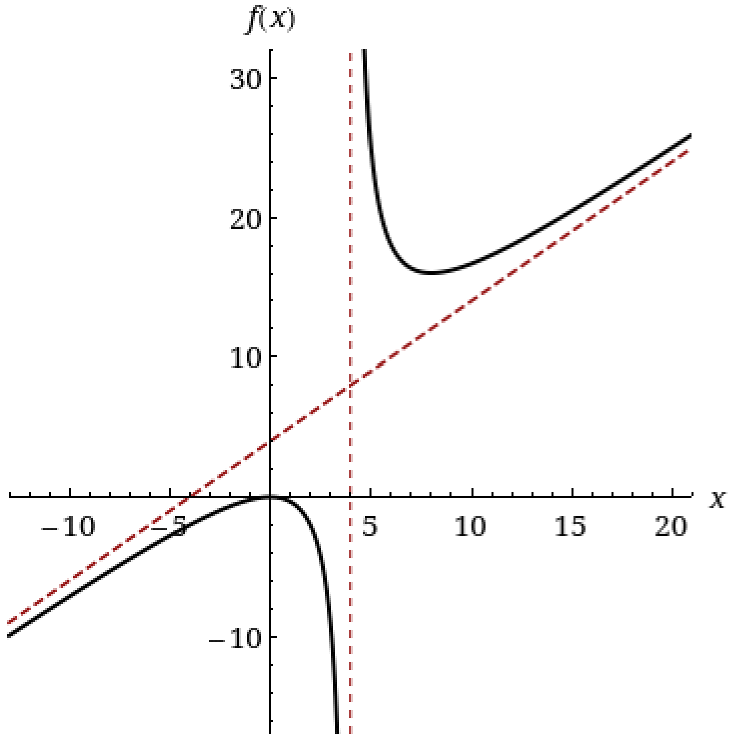
$$= \frac{2\cancel{(x-4)}\left[(x-4)^2 - (x^2-8x)\right]}{(x-4)^{\cancel{4}\,3}}$$

$$= \frac{2\left[\cancel{x^2}\cancel{-8x}+16\cancel{-x^2}\cancel{+8x}\right]}{(x-4)^3}$$

$$= \frac{2(16)}{(x-4)^3} = \frac{32}{(x-4)^3} = f''(x)$$

$\Rightarrow f''(x) \neq 0$ for any x $\Rightarrow$ no inflection point

H: Graph



Summary:

A: dom f = $(-\infty,4) \cup (4,\infty)$

B: x- and y-intercepts = $(0,0)$

C: No symmetry

D: vertical asymptote is $x = 4$

slant asymptote is $y = x+4$

E: Inc. on $(-\infty,0)$, $(8,\infty)$

Dec. on $(0,4)$, $(4,8)$

F: Local max @ $(0,0)$

Local min @ $(8,16)$

G: No inflection point