Calculus 2 Formula List

$$cos^2(x) = \frac{1}{2}(1 + \cos(2x))$$

$$sin^2(x) = \frac{1}{2}(1 - \cos(2x))$$

$$\sin(2x) = 2\sin(x)\cos(x)$$

$$W = Fd \qquad \text{work} = \text{force} \times \text{distance}$$

The Mean Value Theorem for Integrals

If f is continuous on $[a, b]$, then there exists a number c in $[a, b]$ such that

$$f(c) = f_{\text{avg}} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

that is,

$$\int_a^b f(x)\,dx = f(c)\,(b-a)$$

Table of Trigonometric Substitutions

Expression	Substitution	Identity
$\sqrt{a^2 - x^2}$	$x = a\sin\theta, \quad -\frac{\pi}{2} \leqslant \theta \leqslant \frac{\pi}{2}$	$1 - \sin^2\theta = \cos^2\theta$
$\sqrt{a^2 + x^2}$	$x = a\tan\theta, \quad -\frac{\pi}{2} < \theta < \frac{\pi}{2}$	$1 + \tan^2\theta = \sec^2\theta$
$\sqrt{x^2 - a^2}$	$x = a\sec\theta, \quad 0 \leqslant \theta < \frac{\pi}{2}$ or $\pi \leqslant \theta < \frac{3\pi}{2}$	$\sec^2\theta - 1 = \tan^2\theta$

Strategy for Evaluating $\int \sin^m x \cos^n x \, dx$

(a) If the power of cosine is odd ($n = 2k + 1$), save one cosine factor and use $\cos^2 x = 1 - \sin^2 x$ to express the remaining factors in terms of sine:

$$\begin{aligned}\int \sin^m x \cos^{2k+1} x \, dx &= \int \sin^m x \, (\cos^2 x)^k \cos x \, dx \\ &= \int \sin^m x \, (1 - \sin^2 x)^k \cos x \, dx\end{aligned}$$

Then substitute $u = \sin x$. See Example 1.

(b) If the power of sine is odd ($m = 2k + 1$), save one sine factor and use $\sin^2 x = 1 - \cos^2 x$ to express the remaining factors in terms of cosine:

$$\begin{aligned}\int \sin^{2k+1} x \cos^n x \, dx &= \int (\sin^2 x)^k \cos^n x \sin x \, dx \\ &= \int (1 - \cos^2 x)^k \cos^n x \sin x \, dx\end{aligned}$$

Then substitute $u = \cos x$. See Example 2.

[Note that if the powers of both sine and cosine are odd, either (a) or (b) can be used.]

(c) If the powers of both sine and cosine are even, use the half-angle identities

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x) \qquad \cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

See Examples 3 and 4. It is sometimes helpful to use the identity

$$\sin x \cos x = \frac{1}{2} \sin 2x$$

Strategy for Evaluating $\int \tan^m x \sec^n x \, dx$

(a) If the power of secant is even ($n = 2k$, $k \geqslant 2$), save a factor of $\sec^2 x$ and use $\sec^2 x = 1 + \tan^2 x$ to express the remaining factors in terms of $\tan x$:

$$\begin{aligned}\int \tan^m x \sec^{2k} x \, dx &= \int \tan^m x \, (\sec^2 x)^{k-1} \sec^2 x \, dx \\ &= \int \tan^m x \, (1 + \tan^2 x)^{k-1} \sec^2 x \, dx\end{aligned}$$

Then substitute $u = \tan x$. See Example 5.

(b) If the power of tangent is odd ($m = 2k + 1$), save a factor of $\sec x \tan x$ and use $\tan^2 x = \sec^2 x - 1$ to express the remaining factors in terms of $\sec x$:

$$\begin{aligned}\int \tan^{2k+1} x \sec^n x \, dx &= \int (\tan^2 x)^k \sec^{n-1} x \sec x \tan x \, dx \\ &= \int (\sec^2 x - 1)^k \sec^{n-1} x \sec x \tan x \, dx\end{aligned}$$

Then substitute $u = \sec x$. See Example 6.

Midpoint Rule

$$\int_a^b f(x) \, dx \approx Mn = \Delta x[f(\overline{x}_1) + f(\overline{x}_2) + \cdots + f(\overline{x}_n)]$$

Where

$$\Delta x = \frac{b - a}{n}$$

and

$$\overline{x_i} = \frac{1}{2}(x_{i-1} + x_i) = \text{midpoint of } [x_{i-1,} \, x_i]$$

Trapezoidal Rule

$$\int_a^b f(x)\,dx \approx T_n = \frac{\Delta x}{2}[f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n)]$$

where $\Delta x = (b-a)/n$ and $x_i = a + i\,\Delta x$.

$$E_T = \int_a^b f(x)\,dx - T_n$$

$$E_M = \int_a^b f(x)\,dx - M_n$$

3 Error Bounds

Suppose $|f''(x)| \leqslant K$ for $a \leqslant x \leqslant b$. If E_T and E_M are the errors in the Trapezoidal and Midpoint Rules, then

$$|E_T| \leqslant \frac{K(b-a)^3}{12n^2}$$

and

$$|E_M| \leqslant \frac{K(b-a)^3}{24n^2}$$

Simpson's Rule

$$\int_a^b f(x)\,dx \approx S_n = \frac{\Delta x}{3}[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots$$
$$+ 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)]$$

where n is even and $\Delta x = (b-a)/n$.

4 Error Bound for Simpson's Rule

Suppose that Suppose that $\left|f^{(4)}(x)\right| \leqslant K$ for $a \leqslant x \leqslant b$. If E_s is the error involved in using Simpson's Rule, then

$$|E_S| \leqslant \frac{K(b-a)^5}{180n^4}$$

2 The Arc Length Formula

If f' is continuous on $[a, b]$, then the length of the curve $y = f(x)$, $a \leqslant x \leqslant b$, is

$$L = \int_a^b \sqrt{1 + [f'(x)]^2}\,dx$$

$$S = \int_a^b 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$$

$$S = \int_c^d 2\pi y \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

Both formulas 5 and 6 can be summarized symbolically, using the notation for arc length given in Section 8.1, as

7

$$S = \int 2\pi y \, ds$$

For rotation about the y-axis we can use a similar procedure to obtain the following symbolic formula for surface area:

8

$$S = \int 2\pi x \, ds$$

where, as before (see Equations 8.1.7 and 8.1.9), we can use either

$$ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

or

$$ds = \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} \qquad \text{if} \quad \frac{dx}{dt} \neq 0$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}$$

$$A = \int_a^b y\,dx = \int_\alpha^\beta g(t)f'(t)\,dt \qquad \left[\text{or} \quad \int_\beta^\alpha g(t)f'(t)\,dt\right]$$

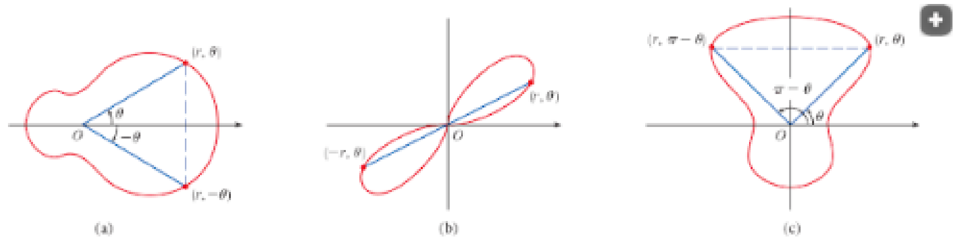
$$L = \int_\alpha^\beta \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$$

$$S = \int_\alpha^\beta 2\pi y\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$$

$$x = r\cos\theta \qquad y = r\sin\theta$$

$$r^2 = x^2 + y^2 \qquad \tan\theta = \frac{y}{x}$$

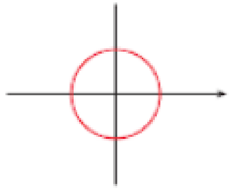
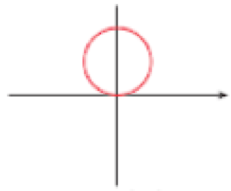
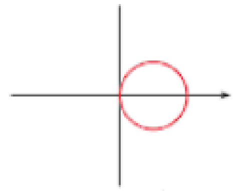
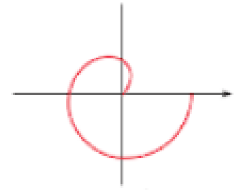
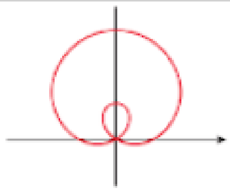
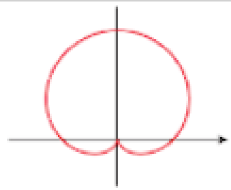
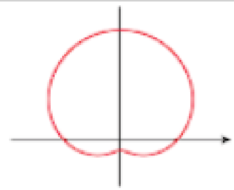
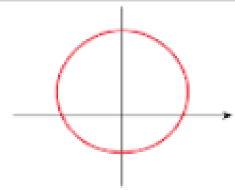
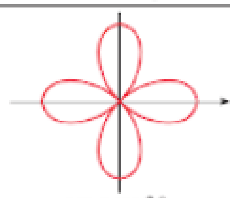
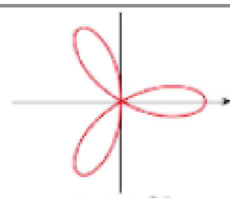
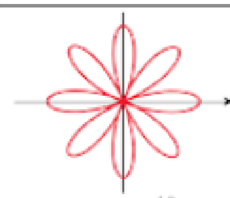
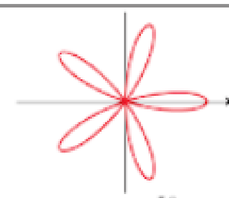
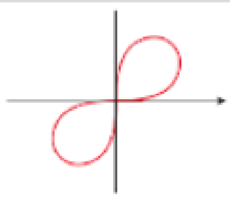
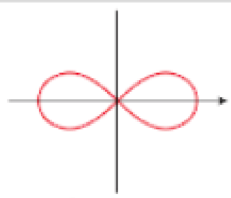
Figure 14



(a) If a polar equation is unchanged when θ is replaced by $-\theta$, the curve is symmetric about the polar axis.

(b) If the equation is unchanged when r is replaced by $-r$, or when θ is replaced by $\theta + \pi$, the curve is symmetric about the pole. (This means that the curve remains unchanged if we rotate it through 180° about the origin.)

(c) If the equation is unchanged when θ is replaced by $\pi - \theta$, the curve is symmetric about the vertical line $\theta = \pi/2$.

Circles and Spiral	 $r = a$ circle	 $r = a \sin \theta$ circle	 $r = a \cos \theta$ circle	 $r = a\theta$ spiral
Limaçons $r = a \pm b \sin \theta$ $r = a \pm b \cos \theta$ $(a > 0, b > 0)$ Orientation depends on the trigonometric function (sine or cosine) and the sign of b	 $a < b$ limaçon with inner loop	 $a = b$ cardioid	 $a > b$ dimpled limaçon	 $a \geq 2b$ convex limaçon
Roses $r = a \sin n\theta$ $r = a \cos n\theta$ n-leaved if n is odd $2n$-leaved if n is even	 $r = a \cos 2\theta$ four-leaved rose	 $r = a \cos 3\theta$ three-leaved rose	 $r = a \cos 4\theta$ eight-leaved rose	 $r = a \cos 5\theta$ five-leaved rose
Lemniscates Figure-eight-shaped curves	 $r^2 = a^2 \sin 2\theta$ lemniscate	 $r^2 = a^2 \cos 2\theta$ lemniscate		

$$A = \int_a^b \frac{1}{2} r^2 \, d\theta$$

$$L = \int_a^b \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} \, d\theta$$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{\frac{dr}{d\theta} \sin \theta + r \cos \theta}{\frac{dr}{d\theta} \cos \theta - r \sin \theta}$$

4 The geometric series

$$\sum_{n=1}^{\infty} ar^{n-1} = a + ar + ar^2 + \cdots$$

is convergent if $|r| < 1$ and its sum is

$$\sum_{n=1}^{\infty} ar^{n-1} = \frac{a}{1-r} \qquad |r| < 1$$

If $|r| \geqslant 1$, the geometric series is divergent.

In words: the sum of a convergent geometric series is

$$\frac{\text{first term}}{1 - \text{common ratio}}$$

Standard Series for Use with the Comparison Tests

In using the Direct Comparison Test we must, of course, have some known series $\sum b_n$ for the purpose of comparison. Most of the time we use one of these series:

- A p-series [$\sum 1/n^p$ converges if $p > 1$ and diverges if $p \leqslant 1$; see (11.3.1)]
- A geometric series [$\sum ar^{n-1}$ converges if $|r| < 1$ and diverges if $|r| \geqslant 1$; see (11.2.4)]

The Direct Comparison Test

Suppose that $\sum a_n$ and $\sum b_n$ are series with positive terms.

(i) If $\sum b_n$ is convergent and $a_n \leqslant b_n$ for all n, then $\sum a_n$ is also convergent.

(ii) If $\sum b_n$ is divergent and $a_n \geqslant b_n$ for all n, then $\sum a_n$ is also divergent.

The Integral Test

Suppose f is a continuous, positive, decreasing function on $[1, \infty)$ and let $a_n = f(n)$. Then the series $\sum_{n=1}^{\infty} a_n$ is convergent if and only if the improper integral $\int_1^{\infty} f(x)\, dx$ is convergent. In other words:

(i) If $\int_1^{\infty} f(x)\, dx$ is convergent, then $\sum_{n=1}^{\infty} a_n$ is convergent.

(ii) If $\int_1^{\infty} f(x)\, dx$ is divergent, then $\sum_{n=1}^{\infty} a_n$ is divergent.

The Limit Comparison Test

Suppose that $\sum a_n$ and $\sum b_n$ are series with positive terms. If

$$\lim_{n\to\infty} \frac{a_n}{b_n} = c$$

where c is a finite number and $c > 0$, then either both series converge or both diverge.

Alternating Series Test

If the alternating series

$$\sum_{n=1}^{\infty} (-1)^{n-1} b_n = b_1 - b_2 + b_3 - b_4 + b_5 - b_6 + \cdots \qquad (b_n > 0)$$

satisfies the conditions

(i) $b_{n+1} \leqslant b_n$ for all n

(ii) $\lim_{n \to \infty} b_n = 0$

then the series is convergent.

Alternating Series Estimation Theorem

If $s = \Sigma(-1)^{n-1} b_n$, where $b_n > 0$, is the sum of an alternating series that satisfies

(i) $b_{n+1} \leqslant b_n$ and

(ii) $\lim_{n \to \infty} b_n = 0$

then

$$|R_n| = |s - s_n| \leqslant b_{n+1}$$

The Ratio Test

(i) If $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = L < 1$, then the series $\sum_{n=1}^{\infty} a_n$ is absolutely convergent (and therefore convergent).

(ii) If $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = L > 1$ or $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \infty$, then the series $\sum_{n=1}^{\infty} a_n$ is divergent.

(iii) If $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = 1$, the Ratio Test is inconclusive; that is, no conclusion can be drawn about the convergence or divergence of $\sum a_n$.

The Root Test

(i) If $\lim_{n\to\infty} \sqrt[n]{|a_n|} = L < 1$, then the series $\sum_{n=1}^{\infty} a_n$ is absolutely convergent (and therefore convergent).

(ii) If $\lim_{n\to\infty} \sqrt[n]{|a_n|} = L > 1$ or $\lim_{n\to\infty} \sqrt[n]{|a_n|} = \infty$, then the series $\sum_{n=1}^{\infty} a_n$ is divergent.

(iii) If $\lim_{n\to\infty} \sqrt[n]{|a_n|} = 1$, the Root Test is inconclusive.

1. **Test for Divergence** If you can see that $\lim_{n\to\infty} a_n$ may be different from 0, then apply the Test for Divergence.

2. **p-Series** If the series is of the form $\sum 1/n^p$, then it is a p-series, which we know to be convergent if $p > 1$ and divergent if $p \leqslant 1$.

3. **Geometric Series** If the series has the form $\sum ar^{n-1}$ or $\sum ar^n$, then it is a geometric series, which converges if $|r| < 1$ and diverges if $|r| \geqslant 1$. Some preliminary algebraic manipulation may be required to bring the series into this form.

4. **Comparison Tests** If the series has a form that is similar to a p-series or a geometric series, then one of the comparison tests should be considered. In particular, if a_n is a rational function or an algebraic function of n (involving roots of polynomials), then the series should be compared with a p-series. Notice that most of the series in Exercises 11.4 have this form. (The value of p should be chosen as in Section 11.4 by keeping only the highest powers of n in the numerator and denominator.) The comparison tests apply only to series with positive terms, but if $\sum a_n$ has some negative terms, then we can apply a comparison test to $\sum |a_n|$ and test for absolute convergence.

5. **Alternating Series Test** If the series is of the form $\sum (-1)^{n-1} b_n$ or $\sum (-1)^n b_n$, then the Alternating Series Test is an obvious possibility. Note that if $\sum b_n$ converges, then the given series is absolutely convergent and therefore convergent.

6. **Ratio Test** Series that involve factorials or other products (including a constant raised to the n th power) are often conveniently tested using the Ratio Test. Bear in mind that $|a_{n+1}/a_n| \to 1$ as $n \to \infty$ for all p-series and therefore all rational or algebraic functions of n. Thus the Ratio Test should not be used for such series.

7. **Root Test** If a_n is of the form $(b_n)^n$, then the Root Test may be useful.

8. **Integral Test** If $a_n = f(n)$, where $\int_1^\infty f(x)\,dx$ is easily evaluated, then the Integral Test is effective (assuming the hypotheses of this test are satisfied).

Series Convergence Tests

Test	Series	Converges	Diverges	Remarks
For Divergence (TFD)	$\sum_{n=1}^{\infty} a_n$	CANNOT show convergence	$\lim_{n\to\infty} a_n \neq 0$	always check first!
Geometric	$\sum_{n=1}^{\infty} ar^{n-1}$	$\lvert r\rvert < 1$	$\lvert r\rvert \geq 1$	$\text{sum} = \frac{\text{first term}}{1-r}$
Telescoping	$\sum_{n=1}^{\infty} (b_n - b_{n+k})$	$\lim_{n\to\infty} b_{n+k} = L$ L has to be finite	$\lim_{n\to\infty} b_{n+k}$ D.N.E. or inf	write out serval terms then cancel stuff to find paritial sum
P-Series	$\sum_{n=1}^{\infty} \frac{1}{n^p}$	$p > 1$	$p \leq 1$	famous sum $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$
Integral	$\sum_{n=1}^{\infty} a_n$ $a_n = f(n) \geq 0$	$\int_1^{\infty} f(x)\,dx$ Converges	$\int_1^{\infty} f(x)\,dx$ Diverges	$f(x)$ has to be positive, continuous & decreasing for $x \geq 1$
Direct Comparison (DCT)	$\sum_{n=1}^{\infty} a_n$ $a_n > 0$	$\sum_{n=1}^{\infty} a_n \leq$ a known convergent	$\sum_{n=1}^{\infty} a_n \geq$ a known divergent	try to use p-series or geometric series to compare
Limit Comparison (LCT)	$\sum_{n=1}^{\infty} a_n$ $a_n > 0$	$\lim_{n\to\infty} \frac{a_n}{b_n} = L > 0$ & $\sum_{n=1}^{\infty} b_n$ is known to be convergent	$\lim_{n\to\infty} \frac{a_n}{b_n} = L > 0$ & $\sum_{n=1}^{\infty} b_n$ is known to be divergent	this version of LCT is inconclusive if $L = 0$ or $L = \infty$
Alternating (AST)	$\sum_{n=1}^{\infty} (-1)^{n-1} b_n$ $b_n \geq 0$	(1.) $\lim_{n\to\infty} b_n = 0$ (2.) $b_{n+1} \leq b_n$	use TFD $\lim_{n\to\infty} (-1)^{n-1} b_n \neq 0$	$(-1)^{n-1}$ $= \cos((n-1)\pi)$
Ratio	$\sum_{n=1}^{\infty} a_n$	$\lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert = L < 1$	$\lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert = L > 1$	inconclusive if $L = 1$ great for ! and $(\ )^n$
Root	$\sum_{n=1}^{\infty} a_n$	$\lim_{n\to\infty} \sqrt[n]{\lvert a_n\rvert} = L < 1$	$\lim_{n\to\infty} \sqrt[n]{\lvert a_n\rvert} = L > 1$	inconclusive if $L = 1$ great for $(\ )^n$

If $\sum_{n=1}^{\infty} \lvert a_n\rvert$ converges, then $\sum_{n=1}^{\infty} a_n$ is **absolute convergent** (which implies $\sum_{n=1}^{\infty} a_n$ also converges)

If $\sum_{n=1}^{\infty} a_n$ converges but $\sum_{n=1}^{\infty} \lvert a_n\rvert$ diverges, then $\sum_{n=1}^{\infty} a_n$ is **conditional convergent**

4 Theorem

For a power series $\sum_{n=0}^{\infty} c_n(x-a)^n$, there are only three possibilities:

(i) The series converges only when $x = a$.

(ii) The series converges for all x.

(iii) There is a positive number R such that the series converges if $|x-a| < R$ and diverges if $|x-a| > R$.

5 Theorem

If f has a power series representation (expansion) at a, that is, if

$$f(x) = \sum_{n=0}^{\infty} c_n (x-a)^n \qquad |x-a| < R$$

then its coefficients are given by the formula

$$c_n = \frac{f^{(n)}(a)}{n!}$$

2 Theorem

If the power series $\sum c_n (x-a)^n$ has radius of convergence $R > 0$, then the function f defined by

$$f(x) = c_0 + c_1(x-a) + c_2(x-a)^2 + \cdots = \sum_{n=0}^{\infty} c_n (x-a)^n$$

is differentiable (and therefore continuous) on the interval $(a-R,\ a+R)$ and

(i) $\quad f'(x) = c_1 + 2c_2(x-a) + 3c_3(x-a)^2 + \cdots = \sum_{n=1}^{\infty} nc_n (x-a)^{n-1}$

(ii) $\quad \int f(x)\,dx = C + c_0(x-a) + c_1\frac{(x-a)^2}{2} + c_2\frac{(x-a)^3}{3} + \cdots$

$$= C + \textstyle\sum_{n=0}^{\infty} c_n \dfrac{(x-a)^{n+1}}{n+1}$$

The radii of convergence of the power series in Equations (i) and (ii) are both R.

$\boxed{1}$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots = \sum_{n=0}^{\infty} x^n \qquad |x| < 1$$

The series in Equation 6 is called the **Taylor series of the function f at a** (or **about a** or **centered at a**). For the special case $a = 0$ the Taylor series becomes

$\boxed{7}$

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n = f(0) + \frac{f'(0)}{1!} x + \frac{f''(0)}{2!} x^2 + \cdots$$

This case arises frequently enough that it is given the special name **Maclaurin series**.

$\boxed{17}$ The Binomial Series

If k is any real number and $|x| < 1$, then

$$(1+x)^k = \sum_{n=0}^{\infty} \binom{k}{n} x^n = 1 + kx + \frac{k(k-1)}{2!} x^2 + \frac{k(k-1)(k-2)}{3!} x^3 + \cdots$$

Important Maclaurin series and their radii of convergence.

$\dfrac{1}{1-x} = \displaystyle\sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \cdots$	$R = 1$
$e^x = \displaystyle\sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$	$R = \infty$
$\sin x = \displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$	$R = \infty$
$\cos x = \displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$	$R = \infty$
$\tan^{-1} x = \displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots$	$R = 1$
$\ln(1+x) = \displaystyle\sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n} = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$	$R = 1$
$(1+x)^k = \displaystyle\sum_{n=0}^{\infty} \binom{k}{n} x^n = 1 + kx + \frac{k(k-1)}{2!} x^2 + \frac{k(k-1)(k-2)}{3!} x^3 + \cdots$	$R = 1$

The length of the two-dimensional vector $\mathbf{a} = \langle a_1, a_2 \rangle$ is

$$|\mathbf{a}| = \sqrt{a_1^2 + a_2^2}$$

The length of the three-dimensional vector $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ is

$$|\mathbf{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$$

A **unit vector** is a vector whose length is 1. For instance, $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ are all unit vectors. In general, if $\mathbf{a} \neq \mathbf{0}$, then the unit vector that has the same direction as $\mathbf{a}$ is

4

$$\mathbf{u} = \frac{1}{|\mathbf{a}|}\mathbf{a} = \frac{\mathbf{a}}{|\mathbf{a}|}$$

1 Definition of the Dot Product

If $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$, then the **dot product** of $\mathbf{a}$ and $\mathbf{b}$ is the number $\mathbf{a} \cdot \mathbf{b}$ given by

$$\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$$

3 Theorem

If θ is the angle between the vectors $\mathbf{a}$ and $\mathbf{b}$, then

$$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}|\,|\mathbf{b}| \cos \theta$$

6 Corollary

If θ is the angle between the nonzero vectors $\mathbf{a}$ and $\mathbf{b}$, then

$$\cos \theta = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|\,|\mathbf{b}|}$$

7 Two vectors $\mathbf{a}$ and $\mathbf{b}$ are orthogonal if and only if $\mathbf{a} \cdot \mathbf{b} = 0$.

8

$$\cos\,\alpha = \frac{\mathbf{a} \cdot \mathbf{i}}{|\mathbf{a}|\,|\mathbf{i}|} = \frac{a_1}{|\mathbf{a}|}$$

(This can also be seen directly from Figure 3.)

Similarly, we also have

9

$$\cos\,\beta = \frac{a_2}{|\mathbf{a}|} \qquad \cos\,\gamma = \frac{a_3}{|\mathbf{a}|}$$

10

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

Scalar projection of $\mathbf{b}$ onto $\mathbf{a}$: $\quad \text{comp}_{\mathbf{a}}\,\mathbf{b} = \dfrac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|}$

Vector projection of $\mathbf{b}$ onto $\mathbf{a}$: $\quad \text{proj}_{\mathbf{a}}\,\mathbf{b} = \left(\dfrac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|}\right)\dfrac{\mathbf{a}}{|\mathbf{a}|} = \dfrac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|^2}\,\mathbf{a}$

12

$$W = |\mathbf{F}|\,|\mathbf{D}|\,\cos\,\theta = \mathbf{F} \cdot \mathbf{D}$$

6

$$\mathbf{a}\times\mathbf{b}=\begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix}\mathbf{i}-\begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix}\mathbf{j}+\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}\mathbf{k}$$

In view of the similarity between Equations 5 and 6, we often write

7

$$\mathbf{a}\times\mathbf{b}=\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

8 Theorem

The vector $\mathbf{a}\times\mathbf{b}$ is orthogonal to both $\mathbf{a}$ and $\mathbf{b}$.

9 Theorem

If θ is the angle between $\mathbf{a}$ and $\mathbf{b}$ (so $0\leqslant\theta\leqslant\pi$), then the length of the cross product $\mathbf{a}\times\mathbf{b}$ is given by

$$|\mathbf{a}\times\mathbf{b}|=|\mathbf{a}|\,|\mathbf{b}|\sin\theta$$

10 Corollary

Two nonzero vectors $\mathbf{a}$ and $\mathbf{b}$ are parallel if and only if

$$\mathbf{a} \times \mathbf{b} = \mathbf{0}$$

14 The volume of the parallelepiped determined by the vectors $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ is the magnitude of their scalar triple product:

$$V = |\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})|$$

2 Parametric equations for a line through the point (x_0, y_0, z_0) and parallel to the direction vector $\langle a, b, c \rangle$ are

$$x = x_0 + at \qquad y = y_0 + bt \qquad z = z_0 + ct$$

3

$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$$

4 The line segment from $\mathbf{r}_0$ to $\mathbf{r}_1$ is given by the vector equation

$$\mathbf{r}(t) = (1 - t)\mathbf{r}_0 + t\mathbf{r}_1 \qquad 0 \leqslant t \leqslant 1$$

7 A **scalar equation of the plane** through point $P_0(x_0, y_0, z_0)$ with normal vector $\mathbf{n} = \langle a, b, c \rangle$ is

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

9 The distance D from the point $P_1(x_1, y_1, z_1)$ to the plane $ax + by + cz + d = 0$ is

$$D = \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$$

Differentiation Rules

General Formulas

1. $\frac{d}{dx}(c) = 0$
2. $\frac{d}{dx}[cf(x)] = cf'(x)$
3. $\frac{d}{dx}[f(x) + g(x)] = f'(x) + g'(x)$
4. $\frac{d}{dx}[f(x) - g(x)] = f'(x) - g'(x)$
5. $\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$ (Product Rule)
6. $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$ (Quotient Rule)
7. $\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$ (Chain Rule)
8. $\frac{d}{dx}(x^n) = nx^{n-1}$ (Power Rule)

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Exponential and Logarithmic Functions

9. $\frac{d}{dx}(e^x) = e^x$

10. $\frac{d}{dx}(b^x) = b^x \ln b$

11. $\frac{d}{dx} \ln |x| = \frac{1}{x}$

12. $\frac{d}{dx}(\log_b x) = \frac{1}{x \ln b}$

Trigonometric Functions

13. $\frac{d}{dx}(\sin x) = \cos x$

14. $\frac{d}{dx}(\cos x) = -\sin x$

15. $\frac{d}{dx}(\tan x) = \sec^2 x$

16. $\frac{d}{dx}(\csc x) = -\csc x \cot x$

17. $\frac{d}{dx}(\sec x) = \sec x \tan x$

18. $\frac{d}{dx}(\cot x) = -\csc^2 x$

Inverse Trigonometric Functions

19. $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$

20. $\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$

21. $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$

22. $\frac{d}{dx}(\csc^{-1} x) = -\frac{1}{x\sqrt{x^2-1}}$

23. $\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$

24. $\frac{d}{dx}(\cot^{-1} x) = -\frac{1}{1+x^2}$

Hyperbolic Functions

25. $\frac{d}{dx}(\sinh x) = \cosh x$

26. $\frac{d}{dx}(\cosh x) = \sinh x$

27. $\frac{d}{dx}(\tanh x) = \operatorname{sech}^2 x$

28. $\frac{d}{dx}(\operatorname{csch} x) = -\operatorname{csch} x \coth x$

29. $\frac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x \tanh x$

30. $\frac{d}{dx}(\coth x) = -\operatorname{csch}^2 x$

Inverse Hyperbolic Functions

31. $\frac{d}{dx}(\sinh^{-1} x) = \frac{1}{\sqrt{1+x^2}}$

32. $\frac{d}{dx}(\cosh^{-1} x) = \frac{1}{\sqrt{x^2-1}}$

33. $\frac{d}{dx}(\tanh^{-1} x) = \frac{1}{1-x^2}$

34. $\frac{d}{dx}(\operatorname{csch}^{-1} x) = -\frac{1}{|x|\sqrt{x^2+1}}$

35. $\frac{d}{dx}(\operatorname{sech}^{-1} x) = -\frac{1}{x\sqrt{1-x^2}}$

36. $\frac{d}{dx}(\coth^{-1} x) = \frac{1}{1-x^2}$

Table of Integrals

Basic Forms

1. $\int u\,dv = uv - \int v\,du$
2. $\int u^n\,du = \frac{u^{n+1}}{n+1} + C, \quad n \neq -1$
3. $\int \frac{du}{u} = \ln|u| + C$
4. $\int e^u\,du = e^u + C$
5. $\int b^u\,du = \frac{b^u}{\ln b} + C$
6. $\int \sin u\,du = -\cos u + C$
7. $\int \cos u\,du = \sin u + C$
8. $\int \sec^2 u\,du = \tan u + C$
9. $\int \csc^2 u\,du = -\cot u + C$
10. $\int \sec u \tan u\,du = \sec u + C$

11. $\displaystyle\int \csc u \cot u \, du = -\csc u + C$

12. $\displaystyle\int \tan u \, du = \ln\ |\sec u| + C$

13. $\displaystyle\int \cot u \, du = \ln\ |\sin u| + C$

14. $\displaystyle\int \sec u \, du = \ln\ |\sec u + \tan u| + C$

15. $\displaystyle\int \csc u \, du = \ln\ |\csc u - \cot u| + C$

16. $\displaystyle\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1}\frac{u}{a} + C, \quad a > 0$

17. $\displaystyle\int \frac{du}{a^2 + u^2} = \frac{1}{a}\tan^{-1}\frac{u}{a} + C$

18. $\displaystyle\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a}\sec^{-1}\frac{u}{a} + C$

19. $\displaystyle\int \frac{du}{a^2 - u^2} = \frac{1}{2a}\ln\left|\frac{u+a}{u-a}\right| + C$

20. $\displaystyle\int \frac{du}{u^2 - a^2} = \frac{1}{2a}\ln\left|\frac{u-a}{u+a}\right| + C$

Forms Involving $\sqrt{a^2+u^2}$, $a>0$

21. $\int \sqrt{a^2+u^2}du = \frac{u}{2}\sqrt{a^2+u^2} + \frac{a^2}{2}\ln\left(u+\sqrt{a^2+u^2}\right)+C$

22. $\int u^2\sqrt{a^2+u^2}\,du = \frac{u}{8}(a^2+2u^2)\,\sqrt{a^2+u^2} - \frac{a^4}{8}\ln\left(u+\sqrt{a^2+u^2}\right)+C$

23. $\int \frac{\sqrt{a^2+u^2}}{u}du = \sqrt{a^2+u^2} - a\ln\left|\frac{a+\sqrt{a^2+u^2}}{u}\right|+C$

24. $\int \frac{\sqrt{a^2+u^2}}{u^2}\,du = -\frac{\sqrt{a^2+u^2}}{u} + \ln\left(u+\sqrt{a^2+u^2}\right)+C$

25. $\int \frac{du}{\sqrt{a^2+u^2}} = \ln\left(u+\sqrt{a^2+u^2}\right)+C$

26. $\int \frac{u^2du}{\sqrt{a^2+u^2}} = \frac{u}{2}\sqrt{a^2+u^2} - \frac{a^2}{2}\ln\left(u+\sqrt{a^2+u^2}\right)+C$

27. $\int \frac{du}{u\sqrt{a^2+u^2}} = -\frac{1}{a}\ln\left|\frac{\sqrt{a^2+u^2}+a}{u}\right|+C$

28. $\int \frac{du}{u^2\sqrt{a^2+u^2}} = -\frac{\sqrt{a^2+u^2}}{a^2u}+C$

29. $\int \frac{du}{(a^2+u^2)^{3/2}} = \frac{u}{a^2\sqrt{a^2+u^2}}+C$

Forms Involving $\sqrt{a^2 - u^2}$, $a > 0$

30. $$\int \sqrt{a^2 - u^2}\, du = \frac{u}{2}\sqrt{a^2 - u^2} + \frac{a^2}{2}\sin^{-1}\frac{u}{a} + C$$

31. $$\int u^2\sqrt{a^2 - u^2}\, du = \frac{u}{8}\left(2u^2 - a^2\right)\sqrt{a^2 - u^2} + \frac{a^4}{8}\sin^{-1}\frac{u}{a} + C$$

32. $$\int \frac{\sqrt{a^2 - u^2}}{u} du = \sqrt{a^2 - u^2} - a\ln\left|\frac{a + \sqrt{a^2 - u^2}}{u}\right| + C$$

33. $$\int \frac{\sqrt{a^2 - u^2}}{u^2} du = -\frac{1}{u}\sqrt{a^2 - u^2} - \sin^{-1}\frac{u}{a} + C$$

34. $$\int \frac{u^2\, du}{\sqrt{a^2 - u^2}} = -\frac{u}{2}\sqrt{a^2 - u^2} + \frac{a^2}{2}\sin^{-1}\frac{u}{a} + C$$

35. $$\int \frac{du}{u\sqrt{a^2 - u^2}} = -\frac{1}{a}\ln\left|\frac{a + \sqrt{a^2 - u^2}}{u}\right| + C$$

36. $$\int \frac{du}{u^2\sqrt{a^2 - u^2}} = -\frac{1}{a^2 u}\sqrt{a^2 - u^2} + C$$

37. $$\int \left(a^2 - u^2\right)^{3/2} du = -\frac{u}{8}\left(2u^2 - 5a^2\right)\sqrt{a^2 - u^2} + \frac{3a^4}{8}\sin^{-1}\frac{u}{a} + C$$

38. $$\int \frac{du}{\left(a^2 - u^2\right)^{3/2}} = \frac{u}{a^2\sqrt{a^2 - u^2}} + C$$

Forms Involving $\sqrt{u^2 - a^2}$, $a > 0$

39. $$\int \sqrt{u^2 - a^2}\, du = \frac{u}{2}\sqrt{u^2 - a^2} - \frac{a^2}{2}\ln\left|u + \sqrt{u^2 - a^2}\right| + C$$

40. $$\int u^2\sqrt{u^2 - a^2}\, du = \frac{u}{8}(2u^2 - a^2)\sqrt{u^2 - a^2} - \frac{a^4}{8}\ln\left|u + \sqrt{u^2 - a^2}\right| + C$$

41. $$\int \frac{\sqrt{u^2 - a^2}}{u} du = \sqrt{u^2 - a^2} - a\cos^{-1}\frac{a}{|u|} + C$$

42. $$\int \frac{\sqrt{u^2 - a^2}}{u^2} du = -\frac{\sqrt{u^2 - a^2}}{u} + \ln\left|u + \sqrt{u^2 - a^2}\right| + C$$

43. $$\int \frac{du}{\sqrt{u^2 - a^2}} = \ln\left|u + \sqrt{u^2 - a^2}\right| + C$$

44. $$\int \frac{u^2\, du}{\sqrt{u^2 - a^2}} = \frac{u}{2}\sqrt{u^2 - a^2} + \frac{a^2}{2}\ln\left|u + \sqrt{u^2 - a^2}\right| + C$$

45. $$\int \frac{du}{u^2\sqrt{u^2 - a^2}} = \frac{\sqrt{u^2 - a^2}}{a^2 u} + C$$

46. $$\int \frac{du}{(u^2 - a^2)^{3/2}} = -\frac{u}{a^2\sqrt{u^2 - a^2}} + C$$

Forms Involving $a + bu$

47. $\int \frac{u\,du}{a+bu} = \frac{1}{b^2}(a + bu - a\ln|a+bu|) + C$

48. $\int \frac{u^2\,du}{a+bu} = \frac{1}{2b^3}\left[(a+bu)^2 - 4a(a+bu) + 2a^2\ln|a+bu|\right] + C$

49. $\int \frac{du}{u(a+bu)} = \frac{1}{a}\ln\left|\frac{u}{a+bu}\right| + C$

50. $\int \frac{du}{u^2(a+bu)} = -\frac{1}{au} + \frac{b}{a^2}\ln\left|\frac{a+bu}{u}\right| + C$

51. $\int \frac{u\,du}{(a+bu)^2} = \frac{a}{b^2(a+bu)} + \frac{1}{b^2}\ln|a+bu| + C$

52. $\int \frac{du}{u(a+bu)^2} = \frac{1}{a(a+bu)} - \frac{1}{a^2}\ln\left|\frac{a+bu}{u}\right| + C$

53. $\int \frac{u^2\,du}{(a+bu)^2} = \frac{1}{b^3}\left(a + bu - \frac{a^2}{a+bu} - 2a\ln|a+bu|\right) + C$

54. $\int u\sqrt{a+bu}\,du = \frac{2}{15b^2}(3bu - 2a)(a+bu)^{3/2} + C$

55. $\int \frac{u\,du}{\sqrt{a+bu}} = \frac{2}{3b^2}(bu - 2a)\sqrt{a+bu} + C$

56. $\int \frac{u^2\,du}{\sqrt{a+bu}} = \frac{2}{15b^3}(8a^2 + 3b^2u^2 - 4abu)\sqrt{a+bu} + C$

57. $$\int \frac{du}{u\sqrt{a+bu}} = \frac{1}{\sqrt{a}}\ln\left|\frac{\sqrt{a+bu}-\sqrt{a}}{\sqrt{a+bu}+\sqrt{a}}\right| + C, \quad \text{if } a > 0$$
$$= \frac{2}{\sqrt{-a}}\tan^{-1}\sqrt{\frac{a+bu}{-a}} + C, \quad \text{if } a < 0$$

58. $$\int \frac{\sqrt{a+bu}}{u}du = 2\sqrt{a+bu} + a\int \frac{du}{u\sqrt{a+bu}}$$

59. $$\int \frac{\sqrt{a+bu}}{u^2}du = -\frac{\sqrt{a+bu}}{u} + \frac{b}{2}\int \frac{du}{u\sqrt{a+bu}}$$

60. $$\int u^n\sqrt{a+bu}\,du = \frac{2}{b\,(2n+3)}\left[u^n(a+bu)^{3/2} - na\int u^{n-1}\sqrt{a+bu}\,du\right]$$

61. $$\int \frac{u^n\,du}{\sqrt{a+bu}} = \frac{2\,u^n\sqrt{a+bu}}{b\,(2n+1)} - \frac{2na}{b\,(2n+1)}\int \frac{u^{n-1}\,du}{\sqrt{a+bu}}$$

62. $$\int \frac{du}{u^n\sqrt{a+bu}} = -\frac{\sqrt{a+bu}}{a\,(n-1)\,u^{n-1}} - \frac{b\,(2n-3)}{2a\,(n-1)}\int \frac{du}{u^{n-1}\sqrt{a+bu}}$$

Trigonometric Forms

63. $\int \sin^2 u \, du = \frac{1}{2}u - \frac{1}{4}\sin 2u + C$

64. $\int \cos^2 u \, du = \frac{1}{2}u + \frac{1}{4}\sin 2u + C$

65. $\int \tan^2 u \, du = \tan u - u + C$

66. $\int \cot^2 u \, du = -\cot u - u + C$

67. $\int \sin^3 u \, du = -\frac{1}{3}(2 + \sin^2 u) \cos u + C$

68. $\int \cos^3 u \, du = \frac{1}{3}(2 + \cos^2 u) \sin u + C$

69. $\int \tan^3 u \, du = \frac{1}{2}\tan^2 u + \ln |\cos u| + C$

70. $\int \cot^3 u \, du = -\frac{1}{2}\cot^2 u - \ln |\sin u| + C$

71. $\int \sec^3 u \, du = \frac{1}{2}\sec u \tan u + \frac{1}{2}\ln |\sec u + \tan u| + C$

72. $\int \csc^3 u \, du = -\frac{1}{2}\csc u \cot u + \frac{1}{2}\ln |\csc u - \cot u| + C$

73. $\displaystyle\int \sin^n u\, du = -\frac{1}{n}\sin^{n-1} u \cos u + \frac{n-1}{n}\int \sin^{n-2} u\, du$

74. $\displaystyle\int \cos^n u\, du = \frac{1}{n}\cos^{n-1} u \sin u + \frac{n-1}{n}\int \cos^{n-2} u\, du$

75. $\displaystyle\int \tan^n u\, du = \frac{1}{n-1}\tan^{n-1} u - \int \tan^{n-2} u\, du$

76. $\displaystyle\int \cot^n u\, du = \frac{-1}{n-1}\cot^{n-1} u - \int \cot^{n-2} u\, du$

77. $\displaystyle\int \sec^n u\, du = \frac{1}{n-1}\tan u \sec^{n-2} u + \frac{n-2}{n-1}\int \sec^{n-2} u\, du$

78. $\displaystyle\int \csc^n u\, du = \frac{-1}{n-1}\cot u \csc^{n-2} u + \frac{n-2}{n-1}\int \csc^{n-2} u\, du$

79. $\displaystyle\int \sin au \sin bu\, du = \frac{\sin(a-b)u}{2(a-b)} - \frac{\sin(a+b)u}{2(a+b)} + C$

80. $\displaystyle\int \cos au \cos bu\, du = \frac{\sin(a-b)u}{2(a-b)} + \frac{\sin(a+b)u}{2(a+b)} + C$

81. $\displaystyle\int \sin au \cos bu\, du = -\frac{\cos(a-b)u}{2(a-b)} - \frac{\cos(a+b)u}{2(a+b)} + C$

82. $\displaystyle\int u \sin u\, du = \sin u - u \cos u + C$

83. $\displaystyle\int u \cos u\, du = \cos u + u \sin u + C$

83. $\displaystyle\int u \cos u\, du = \cos u + u \sin u + C$

84. $\displaystyle\int u^n \sin u\, du = -u^n \cos u + n\int u^{n-1} \cos u\, du$

85. $\displaystyle\int u^n \cos u\, du = u^n \sin u - n\int u^{n-1} \sin u\, du$

86. $\displaystyle\int \sin^n u \cos^m u\, du = -\tfrac{\sin^{n-1} u \cos^{m+1} u}{n+m} + \tfrac{n-1}{n+m}\int \sin^{n-2} u \cos^m u\, du$

$$= \tfrac{\sin^{n+1} u \cos^{m-1} u}{n+m} + \tfrac{m-1}{n+m}\int \sin^n u \cos^{m-2} u\, du$$

Inverse Trigonometric Forms

87. $$\int \sin^{-1}u\,du = u\sin^{-1}u + \sqrt{1-u^2} + C$$

88. $$\int \cos^{-1}u\,du = u\cos^{-1}u - \sqrt{1-u^2} + C$$

89. $$\int \tan^{-1}u\,du = u\tan^{-1}u - \frac{1}{2}\ln\left(1+u^2\right) + C$$

90. $$\int u\sin^{-1}u\,du = \frac{2u^2-1}{4}\sin^{-1}u + \frac{u\sqrt{1-u^2}}{4} + C$$

91. $$\int u\cos^{-1}u\,du = \frac{2u^2-1}{4}\cos^{-1}u - \frac{u\sqrt{1-u^2}}{4} + C$$

92. $$\int u\tan^{-1}u\,du = \frac{u^2+1}{2}\tan^{-1}u - \frac{u}{2} + C$$

93. $$\int u^n\sin^{-1}u\,du = \frac{1}{n+1}\left[u^{n+1}\sin^{-1}u - \int \frac{u^{n+1}du}{\sqrt{1-u^2}}\right],\ n \neq -1$$

94. $$\int u^n\cos^{-1}u\,du = \frac{1}{n+1}\left[u^{n+1}\cos^{-1}u + \int \frac{u^{n+1}\,du}{\sqrt{1-u^2}}\right],\ n \neq -1$$

95. $$\int u^n\tan^{-1}u\,du = \frac{1}{n+1}\left[u^{n+1}\tan^{-1}u - \int \frac{u^{n+1}du}{1+u^2}\right],\ n \neq -1$$

Exponential and Logarithmic Forms

96. $\int ue^{au}\,du = \frac{1}{a^2}(au-1)\,e^{au} + C$

97. $\int u^n e^{au}\,du = \frac{1}{a}u^n e^{au} - \frac{n}{a}\int u^{n-1}e^{au}\,du$

98. $\int e^{au}\sin bu\,du = \frac{e^{au}}{a^2+b^2}(a\sin bu - b\cos bu) + C$

99. $\int e^{au}\cos bu\,du = \frac{e^{au}}{a^2+b^2}(a\cos bu + b\sin bu) + C$

100. $\int \ln u\,du = u\ln u - u + C$

101. $\int u^n \ln u\,du = \frac{u^{n+1}}{(n+1)^2}[(n+1)\ln u - 1] + C$

102. $\int \frac{1}{u\ln u}du = \ln|\ \ln u\,| + c$

Hyperbolic Forms

103. $\int \sinh u\, du = \cosh u + C$

104. $\int \cosh u\, du = \sinh u + C$

105. $\int \tanh u\, du = \ln \cosh u + C$

106. $\int \coth u\, du = \ln\ |\sinh u| + C$

107. $\int \operatorname{sech} u\, du = \tan^{-1} |\sinh u| + C$

108. $\int \operatorname{csch} u\, du = \ln\ \left|\tanh \frac{1}{2}u\right| + C$

109. $\int \operatorname{sech}^2 u\, du = \tanh u + C$

110. $\int \operatorname{csch}^2 u\, du = -\coth u + C$

111. $\int \operatorname{sech} u \tanh u\, du = -\operatorname{sech} u + C$

112. $\int \operatorname{csch} u \coth u\, du = -\operatorname{csch} u + C$

Forms Involving $\sqrt{2au-u^2}$, $a>0$

113. $\int \sqrt{2au-u^2}\,du = \frac{u-a}{2}\sqrt{2au-u^2} + \frac{a^2}{2}\cos^{-1}\left(\frac{a-u}{a}\right) + C$

114. $\int u\sqrt{2au-u^2}\,du = \frac{2u^2-au-3a^2}{6}\sqrt{2au-u^2} + \frac{a^3}{2}\cos^{-1}\left(\frac{a-u}{a}\right) + C$

115. $\int \frac{\sqrt{2au-u^2}}{u}\,du = \sqrt{2au-u^2} + a\cos^{-1}\left(\frac{a-u}{a}\right) + C$

116. $\int \frac{\sqrt{2au-u^2}}{u^2}\,du = -\frac{2\sqrt{2au-u^2}}{u} - \cos^{-1}\left(\frac{a-u}{a}\right) + C$

117. $\int \frac{du}{\sqrt{2au-u^2}} = \cos^{-1}\left(\frac{a-u}{a}\right) + C$

118. $\int \frac{u\,du}{\sqrt{2au-u^2}} = -\sqrt{2au-u^2} + a\cos^{-1}\left(\frac{a-u}{a}\right) + C$

119. $\int \frac{u^2\,du}{\sqrt{2au-u^2}} = -\frac{(u+3a)}{2}\sqrt{2au-u^2} + \frac{3a^2}{2}\cos^{-1}\left(\frac{a-u}{a}\right) + C$

120. $\int \frac{du}{u\sqrt{2au-u^2}} = -\frac{\sqrt{2au-u^2}}{au} + C$