Determine the infinite limit.

$$\lim_{x \to 4^+} \ln(x^2 - 16)$$

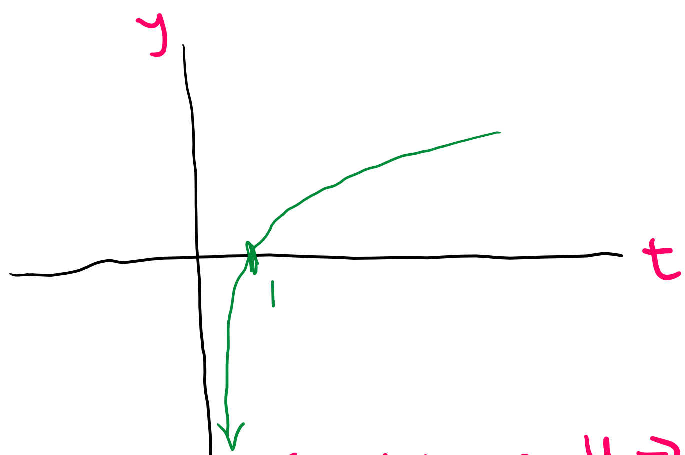
Let $t = x^2 - 16$, as $x \to 4^+$, $x^2 - 16 \to 0$

$\uparrow$

$4^2 - 16$
$= 16 - 16 = 0$

$\Rightarrow \ln(t) \to -\infty$ (see graph below)

Graph of $y = \ln(t)$:



$\leftarrow$ as $t \to 0$, $y \to -\infty$

$$\Rightarrow \lim_{x \to 4^+} \ln(x^2 - 16) = \boxed{-\infty}$$

Guess the value of the limit (if it exists) by evaluating the function at the given numbers.

$$\lim_{x \to -4} \frac{x^2 - 4x}{x^2 - 16}$$

x = −3.5, −3.9, −3.95, −3.99, −3.999, −3.9999, −4.5, −4.1, −4.05, −4.01, −4.001, −4.0001

$$\boxed{f(x) = \frac{x^2 - 4x}{x^2 - 16}}$$

$x \to -4$ from the left

$x = -3.5 \Rightarrow f(-3.5) = \frac{(-3.5)^2 - 4(-3.5)}{(-3.5)^2 - 16} = -7$

$x = -3.9 \Rightarrow f(-3.9) = \frac{(-3.9)^2 - 4(-3.9)}{(-3.9)^2 - 16} = -39$

$x = -3.95 \Rightarrow f(-3.95) = \frac{(-3.95)^2 - 4(-3.95)}{(-3.95)^2 - 16} = -79$

$x = -3.99 \Rightarrow f(-3.99) = \frac{(-3.99)^2 - 4(-3.99)}{(-3.99)^2 - 16} = -399$

$x = -3.999 \Rightarrow f(-3.999) = \frac{(-3.999)^2 - 4(-3.999)}{(-3.999)^2 - 16} = -3{,}999$

$x = -3.9999 \Rightarrow f(-3.9999) = \frac{(-3.9999)^2 - 4(-3.9999)}{(-3.9999)^2 - 16} = -39{,}999$

$f(x) \to -\infty$

Note: as $x \to -4$ from the left, $f(x) \to -\infty$

$$\boxed{\Rightarrow \lim_{x \to -4^-} f(x) = -\infty}$$

$x \to -4$ from the right

$$x=-4.5 \Rightarrow f(-4.5)=\frac{(-4.5)^2-4(-4.5)}{(-4.5)^2-16}=9$$

$$x=-4.1 \Rightarrow f(-4.1)=\frac{(-4.1)^2-4(-4.1)}{(-4.1)^2-16}=41$$

$$x=-4.05 \Rightarrow f(-4.05)=\frac{(-4.05)^2-4(-4.05)}{(-4.05)^2-16}=81$$

$$x=-4.01 \Rightarrow f(-4.01)=\frac{(-4.01)^2-4(-4.01)}{(-4.01)^2-16}=401$$

$$x=-4.001 \Rightarrow f(-4.001)=\frac{(-4.001)^2-4(-4.001)}{(-4.001)^2-16}=4{,}001$$

$$x=-4.0001 \Rightarrow f(-4.0001)=\frac{(-4.0001)^2-4(-4.0001)}{(-4.0001)^2-16}=40{,}001$$

$f(x) \to +\infty$

Note: as $x \to -4$ from the right, $f(x) \to \infty$

$$\Rightarrow \lim_{x \to -4^+} f(x) = +\infty$$

Since $\lim_{x\to -4^-} f(x) = -\infty$ and $\lim_{x \to 4^+} f(x) = +\infty$,

$$\lim_{x\to -4} f(x) = \boxed{DNE}$$ ← Answer

If $3x - 2 \le f(x) \le x^2 - 3x + 7$ for $x \ge 0$, find $\lim_{x\to 3} f(x)$.

Recall: The Squeeze Theorem

If $f(x) \le g(x) \le h(x)$ when x is near a (except possibly at a) and $\lim_{x\to a} f(x) = \lim_{x\to a} h(x) = L$, then

$$\lim_{x\to a} g(x) = L$$

Here, $\lim_{x\to 3} (3x-2) = 3(3) - 2 = 9 - 2 = 7$

and $\lim_{x\to 3} (x^2 - 3x + 7) = (3)^2 - 3(3) + 7 = 9 - 9 + 7 = 7$

so, $\lim_{x\to 3} (3x-2) = \lim_{x\to 3} (x^2 - 3x + 7) = 7$

Therefore, according to the Squeeze theorem,

$$\lim_{x\to 3} f(x) = \boxed{7}$$

Answer

note:

If $\lim\limits_{x\to a^-} f(x) = \lim\limits_{x\to a^+} f(x) = n \leftarrow$ a number

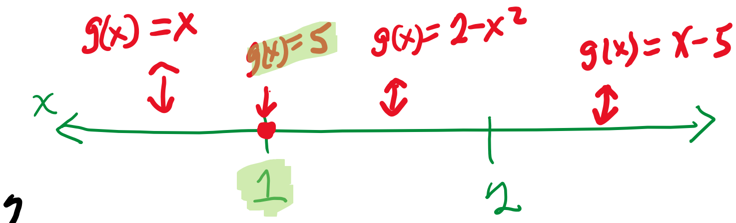
left side right side

Then $\lim\limits_{\underbrace{x\to a}_{\text{2-sided limit}}} f(x) = n$

* If $\lim\limits_{x\to a^-} f(x) \neq \lim\limits_{x\to a^+} f(x)$, Then $\lim\limits_{x\to a} f(x) = DNE$

(46)

$$g(x) = \begin{cases} x & \text{if } x<1 \\ 5 & \text{if } x=1 \\ 2-x^2 & \text{if } 1<x\leq 2 \\ x-5 & \text{if } x>2 \end{cases}$$



(a) (i) $\lim\limits_{x\to 1^-} g(x) = \lim\limits_{x\to 1^-} x = \boxed{1}$

Left side of 1 $\Rightarrow x<1$

(ii) $\lim\limits_{\underbrace{x\to 1}_{\text{2 sided limit.}}} g(x)$ $\Rightarrow$ we need to find $\lim\limits_{x\to 1^+} g(x)$ first.

$\Rightarrow \lim_{x \to 1^+} g(x) = \lim_{x \to 1^+} (2 - x^2) = 2 - (1)^2 = 2 - 1$
$= \boxed{1}$

Right side of 1 $\Rightarrow x > 1 \Rightarrow g(x) = 2 - x^2$

$\Rightarrow$ Since $\lim_{x \to 1^-} g(x) = \lim_{x \to 1^+} g(x) = 1$

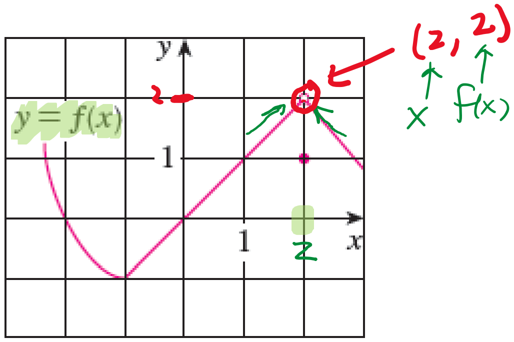
$\Rightarrow \lim_{x \to 1} g(x) = \boxed{1}$ ← answer for (ii)

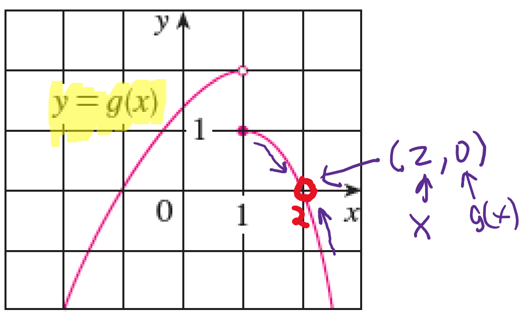
(iii) $g(1) = \boxed{5}$

$y = g(x) \Rightarrow x = 1$

note: parts (iv), (v), and (vi) are worked the same way but now $x = 2$

50. The graphs of *f* and *g* are given. Use them to evaluate each limit, if it exists. (If an answer does not exist, enter DNE.)





ⓐ $\lim_{x \to 2} [f(x) + g(x)]$

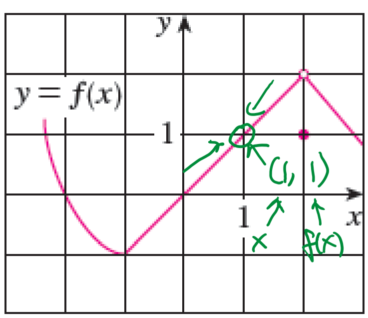
$= \underbrace{\lim_{x \to 2} f(x)}_{} + \underbrace{\lim_{x \to 2} g(x)}_{}$

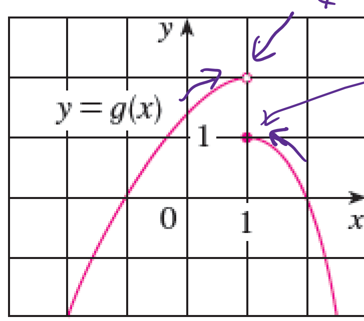
$= \quad 2 \quad + \quad 0$

$= \boxed{2}$ ← Answer

note:

ⓑ $\lim_{x \to 1} [f(x) + g(x)]$





$\lim_{x \to 1^-} g(x) = 2$

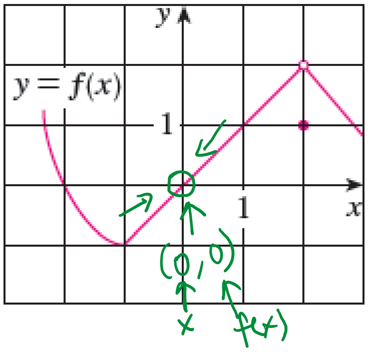
$\lim_{x \to 1^+} g(x) = 1$

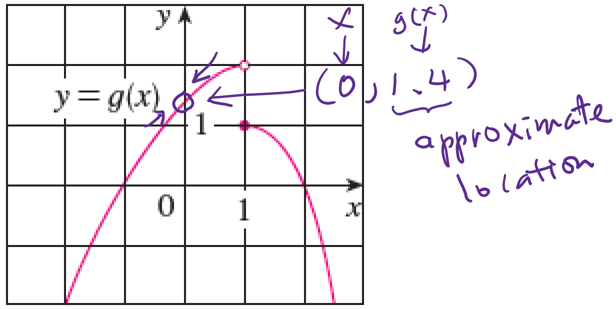
$\lim_{x \to 1^-} g(x) \neq \lim_{x \to 1^+} g(x)$

$\Rightarrow \lim_{x \to 1} g(x) = DNE$

$= \lim_{x \to 1} f(x) + \lim_{x \to 1} g(x)$

$= \underbrace{\quad 1 \quad}_{} + \underbrace{\quad DNE \quad}_{} = \boxed{DNE}$

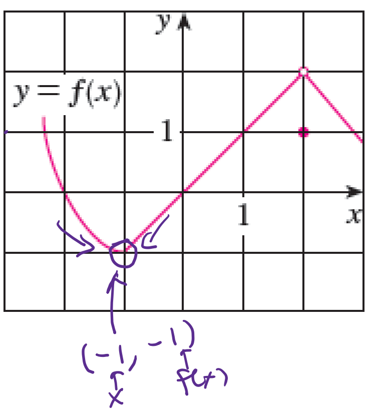


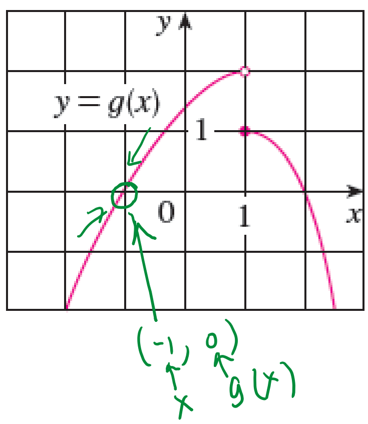


ⓒ $\lim\limits_{x\to 0} [f(x)\,g(x)]$

$$= \underbrace{\lim_{x\to 0} f(x)} \cdot \underbrace{\lim_{x\to 0} g(x)}$$

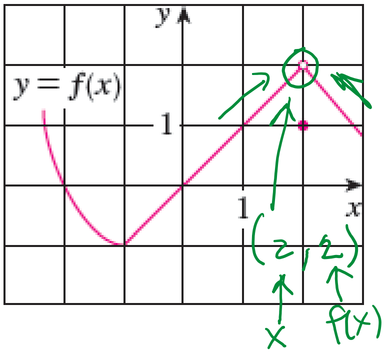
$$\approx 0 \cdot 1.4 = \boxed{0}$$





ⓓ $\lim\limits_{x\to -1} \dfrac{f(x)}{g(x)}$

$$= \frac{\lim\limits_{x\to -1} f(x)}{\lim\limits_{x\to -1} g(x)} = \frac{-1}{0} = \text{undefined} = \boxed{\text{DNE}}$$

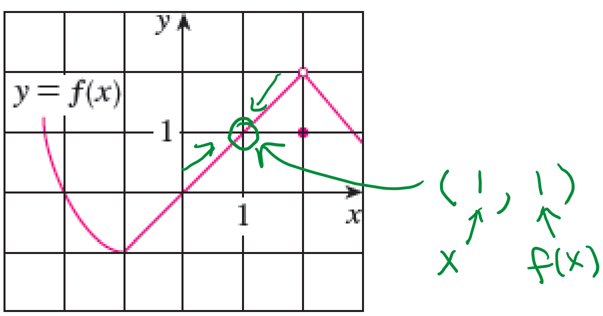


ⓔ $\lim_{x\to 2}\left[x^3 f(x)\right]$

$$= \lim_{x\to 2} x^3 \cdot \underbrace{\lim_{x\to 2} f(x)}$$

$$= (2)^3 \cdot 2$$

$$= 8 \cdot 2 = \boxed{16}$$



ⓕ $\lim_{x\to 1}\sqrt{3+f(x)}$

$$= \sqrt{3 + \lim_{x\to 1} f(x)}$$

$$= \sqrt{3+1} = \sqrt{4} = \boxed{2}$$

If $\displaystyle\lim_{x\to 1} \frac{f(x)-9}{x-1} = 8$, find $\displaystyle\lim_{x\to 1} f(x)$.

Note: we need to build $\dfrac{f(x)-9}{x-1}$ from $f(x)$ to use the limit value given in the problem.

$$\Rightarrow \quad f(x) = f(x) - 9 + 9$$

$$= \frac{f(x)-9}{1} + 9$$

$$= \frac{[f(x)-9]\cdot(x-1)}{[1]\cdot(x-1)} + 9$$

$$= \frac{f(x)-9}{x-1}\cdot(x-1) + 9$$

So, $$\lim_{x\to 1} f(x) = \lim_{x-1}\left[\frac{f(x)-9}{x-1}\cdot(x-1) + 9\right]$$

$$= \underbrace{\lim_{x-1}\left[\frac{f(x)-9}{x-1}\right]} \cdot \underbrace{\lim_{x\to 1}(x-1)} + \underbrace{\lim_{x\to 1} 9}$$

$$= 8 \cdot 0 + 9$$

$$= 0 + 9 = \boxed{9}$$

Answer

If $\lim_{x\to 1} \dfrac{f(x)-2}{x-1} = 10$, find $\lim_{x\to 1} f(x)$.

Note: $\lim_{x\to 1} [f(x)-2]$

$$= \lim_{x\to 1} \frac{[f(x)-2]\cdot(x-1)}{(1)\cdot(x-1)}$$

$$= \lim_{x\to 1} \left[\frac{f(x)-2}{x-1}\cdot(x-1)\right]$$

$$= \underbrace{\lim_{x\to 1}\left[\frac{f(x)-2}{x-1}\right]} \cdot \underbrace{\lim_{x\to 1}(x-1)}$$

$$= \boxed{10} \cdot (1-1)$$

Given in the problem

$$= 10 \cdot 0 = 0$$

So, $\lim\limits_{x \to 1} f(x) = \lim\limits_{x \to 1} \left[f(x) + 0 \right]$

$$= \lim_{x \to 1} \left[f(x) - 2 + 2 \right]$$

$$= \lim_{x \to 1} \left(\left[f(x) - 2 \right] + 2 \right)$$

$$= \lim_{x \to 1} \left[f(x) - 2 \right] + \lim_{x \to 1} 2$$

$$= 0 + 2 = \boxed{2}$$

from last page

Answer

If $\lim_{x\to 0} \frac{f(x)}{x^2} = 10$, find each of the following limits.

ⓐ $\lim_{x\to 0} f(x) = \lim_{x\to 0}\left[\frac{f(x)\cdot x^2}{1\cdot x^2}\right]$

$= \lim_{x\to 0}\left[\frac{f(x)}{x^2}\cdot x^2\right]$

$= \underbrace{\left[\lim_{x\to 0}\frac{f(x)}{x^2}\right]}\cdot\underbrace{\left[\lim_{x\to 0} x^2\right]}$

$= [10]\cdot[0] = \boxed{0}$

Answer

Note: we need to build $\frac{f(x)}{x^2}$ from $f(x)$.

ⓑ $\lim_{x\to 0}\frac{f(x)}{x} = \lim_{x\to 0}\left[\frac{f(x)\cdot x}{x\cdot x}\right]$

$= \lim_{x\to 0}\left[\frac{f(x)}{x^2}\cdot x\right]$

$= \underbrace{\left[\lim_{x\to 0}\frac{f(x)}{x^2}\right]}\cdot\left[\lim_{x\to 0} x\right]$

$= [10]\cdot[0] = \boxed{0}$

Answer

Note: we need to build $\frac{f(x)}{x^2}$ from $\frac{f(x)}{x}$

49] Find the number *a* such that the limit exists.

$$\lim_{x \to -2} \frac{2x^2 + ax + a + 6}{x^2 + x - 2}$$

note: $$f(-2) = \frac{2(-2)^2 + a(-2) + a + 6}{(-2)^2 + (-2) - 2}$$

$$= \frac{8 - 2a + a + 6}{4 - 2 - 2}$$

$$= \frac{14 - a}{0}$$

$\Rightarrow$ limit only exists if $\frac{0}{0}$

$\Rightarrow 14 - a = 0 \Rightarrow \boxed{a = 14}$

So, the limit in this problem becomes

$$\lim_{x \to -2} \frac{2x^2 + \overset{\overset{a}{\downarrow}}{14x} + \overset{\overset{a}{\downarrow}}{14} + 6}{x^2 + x - 2}$$

$$= \lim_{x \to -2} \frac{2x^2 + 14x + 20}{x^2 + x - 2}$$

$$= \lim_{x \to -2} \frac{2(x^2 + 7x + 10)}{(x+2)(x-1)}$$

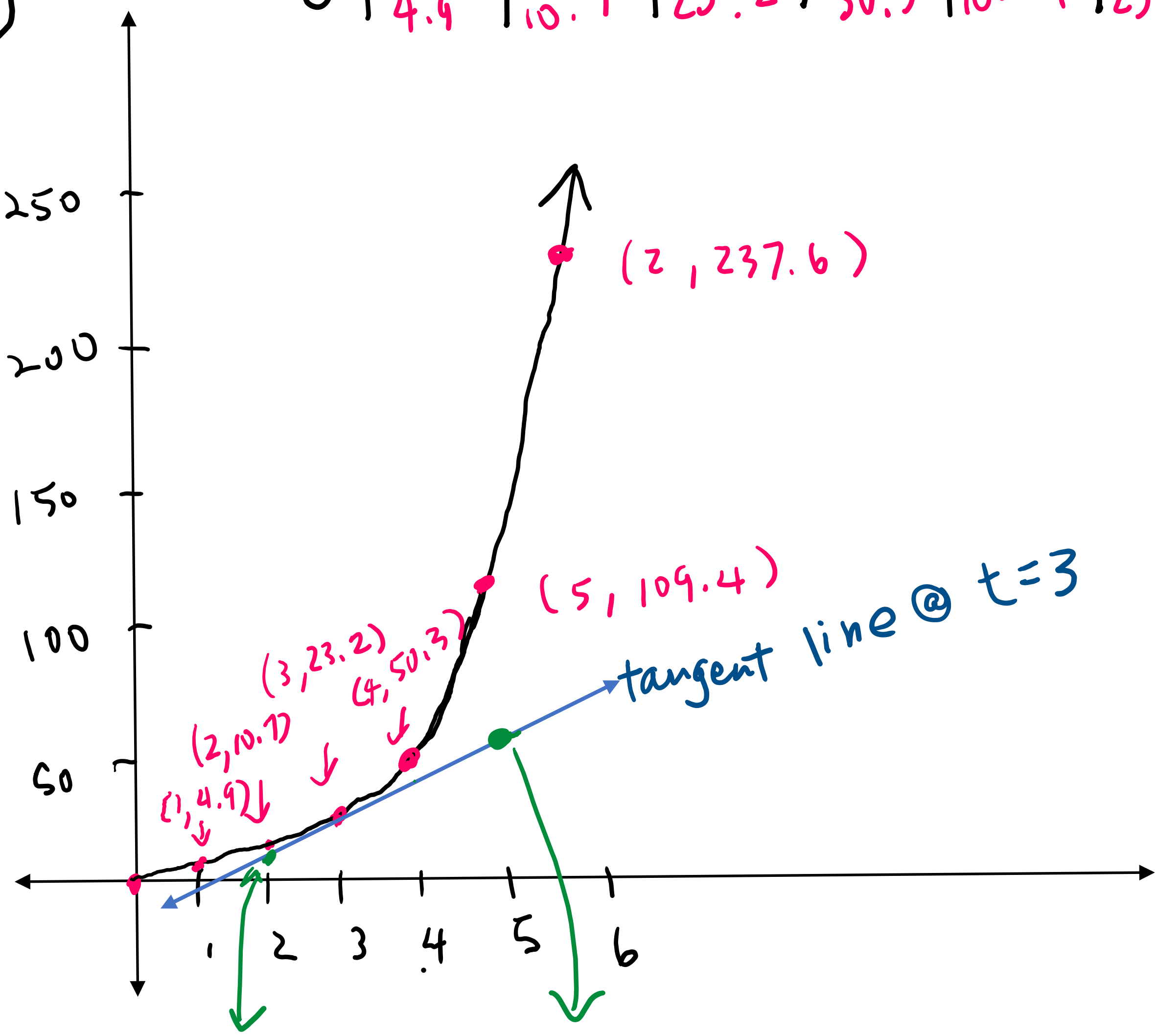
$$= \lim_{x \to -2} \frac{2(x+2)(x+5)}{(x+2)(x-1)}$$

$$= \lim_{x \to -2} \frac{2(x+5)}{x-1} = \frac{2(-2+5)}{-2-1} = \frac{2(3)}{-3} = \boxed{-2}$$

Answer

(b)

t	1	2	3	4	5	6
s	4.9	10.7	23.2	50.3	109.4	237.6



(2, 4) (5, 52)

← Best estimate →

$$\Rightarrow m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{52 - 4}{5 - 2} = \frac{48}{3} = 18$$

⇒ Best estimate for instantaneous velocity is $\boxed{18.0}$ ft/s

Evaluate the limit, if it exists. (If an answer does not exist, enter DNE.)

$$\lim_{x \to 8} \frac{\frac{1}{x} - \frac{1}{8}}{x-8}$$

Note: $x = 8 \Rightarrow \dfrac{\frac{1}{8} - \frac{1}{8}}{8-8} = \dfrac{0}{0} \Rightarrow$ Need to reduce/Simplify the fraction First.

$\boxed{LCD = 8x} \Rightarrow \dfrac{\left(\frac{1}{x} - \frac{1}{8}\right) \cdot 8x}{(x-8) \cdot 8x} = \dfrac{\frac{1}{\cancel{x}} \cdot 8\cancel{x} - \frac{1}{\cancel{8}} \cdot \cancel{8}x}{(x-8) \cdot 8x}$ ← Leave in Factored form to reduce

$$= \frac{8 - x}{(x-8) \cdot 8x}$$

Note: $8 - x \neq x - 8$ $\Rightarrow$ need to rewrite $8-x$ to have x first

$$= \frac{-x + 8}{(x-8) \cdot 8x}$$

$$= \frac{-1\cancel{(x-8)}}{\cancel{(x-8)} \cdot 8x} = \frac{-1}{8x}$$

$$\Rightarrow \lim_{x \to 8} \frac{\frac{1}{x} - \frac{1}{8}}{x-8} = \lim_{x \to 8} \frac{-1}{8x}$$

$$= \frac{-1}{8(8)}$$

$$= \frac{-1}{64} = \boxed{-\frac{1}{64}}$$ ← Answer

Evaluate the limit, if it exists. (If an answer does not exist, enter DNE.)

$$\lim_{x \to 4} \frac{\frac{1}{x} - \frac{1}{4}}{x-4}$$

note: when $x=4$, we get $\dfrac{\frac{1}{4} - \frac{1}{4}}{4-4} = \dfrac{0}{0}$

$\Rightarrow$ Need to Simplify the fraction.

$$\Rightarrow \frac{\frac{1}{x} - \frac{1}{4}}{x-4} = \frac{\frac{4}{4x} - \frac{x}{4x}}{x-4}$$

$$= \frac{\frac{4-x}{4x}}{x-4}$$

$$= \frac{4-x}{4x} \div \frac{(x-4)}{1}$$

$$= \frac{4-x}{4x} \cdot \frac{1}{x-4}$$

$$= \frac{-1(\cancelto{1}{x-4})}{4x} \cdot \frac{1}{\cancelto{1}{x-4}}$$

$$= \frac{-1}{4x}$$

$$\Rightarrow \lim_{x \to 4} \frac{\frac{1}{x} - \frac{1}{4}}{x-4} = \lim_{x \to 4} \frac{-1}{4x}$$

$$= \frac{-1}{4(4)}$$

$$= \frac{-1}{16} \text{ or } \boxed{-\frac{1}{16}}$$

↑

Answer