BEGINNING OF EXAMINATION

1. You are given:

$$m(x) = \begin{cases} 0.04, & 0 < x < 40 \\ 0.05, & x > 40 \end{cases}$$

Calculate $\mathring{e}_{25:\overline{25}|}$.

(A)   14.0

(B)   14.4

(C)   14.8

(D)   15.2

(E)   15.6

2. For a select-and-ultimate mortality table with a 3-year select period:

(i)

x	$q_{[x]}$	$q_{[x]+1}$	$q_{[x]+2}$	q_{x+3}	$x+3$
60	0.09	0.11	0.13	0.15	63
61	0.10	0.12	0.14	0.16	64
62	0.11	0.13	0.15	0.17	65
63	0.12	0.14	0.16	0.18	66
64	0.13	0.15	0.17	0.19	67

(ii)   White was a newly selected life on 01/01/2000.

(iii)   White's age on 01/01/2001 is 61.

(iv) P is the probability on 01/01/2001 that White will be alive on 01/01/2006.

Calculate P.

(A) $0 \le P < 0.43$

(B) $0.43 \le P < 0.45$

(C) $0.45 \le P < 0.47$

(D) $0.47 \le P < 0.49$

(E) $0.49 \le P \le 1.00$

3. For a continuous whole life annuity of 1 on (x):

(i) $T(x)$ is the future lifetime random variable for (x).

(ii) The force of interest and force of mortality are constant and equal.

(iii) $\bar{a}_x = 12.50$

Calculate the standard deviation of $\bar{a}_{\overline{T(x)}|}$.

(A) 1.67

(B) 2.50

(C) 2.89

(D) 6.25

(E) 7.22

4. For a special fully discrete whole life insurance on (x):

(i) The death benefit is 0 in the first year and 5000 thereafter.

(ii) Level benefit premiums are payable for life.

(iii) $q_x = 0.05$

(iv) $v = 0.90$

(v) $\ddot{a}_x = 5.00$

(vi) $_{10}V_x = 0.20$

(vii) $_{10}V$ is the benefit reserve at the end of year 10 for this insurance.

Calculate $_{10}V$.

(A) 795

(B) 1000

(C) 1090

(D) 1180

(E) 1225

5.   For a fully discrete 2-year term insurance of 1 on (x):

(i)   0.95 is the lowest premium such that there is a 0% chance of loss in year 1.

(ii) $p_x = 0.75$

(iii) $p_{x+1} = 0.80$

(iv) Z is the random variable for the present value at issue of future benefits.

Calculate $\text{Var}(Z)$.

(A)   0.15

(B)   0.17

(C)   0.19

(D)   0.21

(E)   0.23

6. A group dental policy has a negative binomial claim count distribution with mean 300 and variance 800.

Ground-up severity is given by the following table:

Severity	Probability
40	0.25
80	0.25
120	0.25
200	0.25

You expect severity to increase 50% with no change in frequency. You decide to impose a per claim deductible of 100.

Calculate the expected total claim payment after these changes.

(A)   Less than 18,000

(B)   At least 18,000, but less than 20,000

(C)   At least 20,000, but less than 22,000

(D)   At least 22,000, but less than 24,000

(E)   At least 24,000

7. You own a fancy light bulb factory. Your workforce is a bit clumsy – they keep dropping boxes of light bulbs. The boxes have varying numbers of light bulbs in them, and when dropped, the entire box is destroyed.

You are given:

Expected number of boxes dropped per month:	50
Variance of the number of boxes dropped per month:	100
Expected value per box:	200
Variance of the value per box:	400

You pay your employees a bonus if the value of light bulbs destroyed in a month is less than 8000.

Assuming independence and using the normal approximation, calculate the probability that you will pay your employees a bonus next month.

(A) 0.16

(B) 0.19

(C) 0.23

(D) 0.27

(E) 0.31

8. Each of 100 independent lives purchase a single premium 5-year deferred whole life insurance of 10 payable at the moment of death. You are given:

(i) $m = 0.04$

(ii) $d = 0.06$

(iii) F is the aggregate amount the insurer receives from the 100 lives.

Using the normal approximation, calculate F such that the probability the insurer has sufficient funds to pay all claims is 0.95.

(A)   280

(B)   390

(C)   500

(D)   610

(E)   720

9. For a select-and-ultimate table with a 2-year select period:

x	$p_{[x]}$	$p_{[x]+1}$	p_{x+2}	$x+2$
48	0.9865	0.9841	0.9713	50
49	0.9858	0.9831	0.9698	51
50	0.9849	0.9819	0.9682	52
51	0.9838	0.9803	0.9664	53

Keith and Clive are independent lives, both age 50. Keith was selected at age 45 and Clive was selected at age 50.

Calculate the probability that exactly one will be alive at the end of three years.

(A)   Less than 0.115

(B)   At least 0.115, but less than 0.125

(C)   At least 0.125, but less than 0.135

(D)   At least 0.135, but less than 0.145

(E)   At least 0.145

10-11. *Use the following information for questions* 10 *and* 11.

For a tyrannosaur with 10,000 calories stored:

(i)   The tyrannosaur uses calories uniformly at a rate of 10,000 per day. If his stored calories reach 0, he dies.

(ii)   The tyrannosaur eats scientists (10,000 calories each) at a Poisson rate of 1 per day.

(iii)   The tyrannosaur eats only scientists.

(iv)   The tyrannosaur can store calories without limit until needed.

10. Calculate the probability that the tyrannosaur dies within the next 2.5 days.

(A)   0.30

(B)   0.40

(C)   0.50

(D)   0.60

(E)   0.70

10-11. *(Repeated for convenience) Use the following information for questions* 10 *and* 11.

For a tyrannosaur with 10,000 calories stored:

(i)   The tyrannosaur uses calories uniformly at a rate of 10,000 per day. If his stored calories reach 0, he dies.

(ii)   The tyrannosaur eats scientists (10,000 calories each) at a Poisson rate of 1 per day.

(iii)   The tyrannosaur eats only scientists.

(iv)   The tyrannosaur can store calories without limit until needed.

11.   Calculate the expected calories eaten in the next 2.5 days.

     (A)   17,800

     (B)   18,800

     (C)   19,800

     (D)   20,800

     (E)   21,800

12-13. *Use the following information for questions* 12 *and* 13.

A fund is established by collecting an amount P from each of 100 independent lives age 70. The fund will pay the following benefits:

- 10, payable at the end of the year of death, for those who die before age 72, or

- P, payable at age 72, to those who survive.

You are given:

(i)   Mortality follows the Illustrative Life Table.

(ii) $i = 0.08$

12. Calculate P, using the equivalence principle.

(A)   2.33

(B)   2.38

(C)   3.02

(D)   3.07

(E)   3.55

12-13. *(Repeated for convenience) Use the following information for questions* 12 *and* 13.

A fund is established by collecting an amount P from each of 100 independent lives age 70. The fund will pay the following benefits:

- 10, payable at the end of the year of death, for those who die before age 72, or

- P, payable at age 72, to those who survive.

You are given:

(i)   Mortality follows the Illustrative Life Table.

(ii) $i = 0.08$

13.   For this question only, you are also given:

The number of claims in the first year is simulated from the binomial distribution using the inverse transform method (where smaller random numbers correspond to fewer deaths). The random number for the first trial, generated using the uniform distribution on [0, 1], is 0.18.

Calculate the simulated claim amount.

(A)   0

(B)   10

(C)   20

(D)   30

(E)   40

14. You are simulating a continuous surplus process, where claims occur according to a Poisson process with frequency 2, and severity is given by a Pareto distribution with parameters $a = 2$ and $q = 1000$. The initial surplus is 2000, and the relative security loading is 0.1. Premium is collected continuously, and the process terminates if surplus is ever negative.

You simulate the time between claims using the inverse transform method (where small numbers correspond to small times between claims) using the following values from the uniform distribution on [0,1]: 0.83, 0.54, 0.48, 0.14.

You simulate the severities of the claims using the inverse transform method (where small numbers correspond to small claim sizes) using the following values from the uniform distribution on [0,1]: 0.89, 0.36, 0.70, 0.61.

Calculate the simulated surplus at time 1.

(A)   1109

(B)   1935

(C)   2185

(D)   4200

(E)   Surplus becomes negative at some time in [0,1].

15. You are given:

(i) $P_x = 0.090$

(ii) $_nV_x = 0.563$

(iii) $P_{x:\overline{n}|}^{1} = 0.00864$

Calculate $P_{x:\overline{n}|}^1$.

(A)   0.008

(B)   0.024

(C)   0.040

(D)   0.065

(E)   0.085

16. You are given:

(i)   Mortality follows De Moivre's law with $w = 100$.

(ii) $i = 0.05$

(iii)   The following annuity-certain values:

$$\bar{a}_{\overline{40|}} = 17.58$$
$$\bar{a}_{\overline{50|}} = 18.71$$
$$\bar{a}_{\overline{60|}} = 19.40$$

Calculate $_{10}\bar{V}\left(\bar{A}_{40}\right)$.

(A)   0.075

(B)   0.077

(C)   0.079

(D)   0.081

(E)   0.083

17. For a group of individuals all age x, you are given:

(i)   30% are smokers and 70% are non-smokers.

(ii) The constant force of mortality for smokers is 0.06.

(iii) The constant force of mortality for non-smokers is 0.03.

(iv) $d = 0.08$

Calculate $\text{Var}\left(\bar{a}_{\overline{T(x)|}}\right)$ for an individual chosen at random from this group.

(A)   13.0

(B)   13.3

(C)   13.8

(D)   14.1

(E)   14.6

18. For a certain company, losses follow a Poisson frequency distribution with mean 2 per year, and the amount of a loss is 1, 2, or 3, each with probability 1/3. Loss amounts are independent of the number of losses, and of each other.

An insurance policy covers all losses in a year, subject to an annual aggregate deductible of 2.

Calculate the expected claim payments for this insurance policy.

(A)   2.00

(B)   2.36

(C)   2.45

(D)   2.81

(E)   2.96

19. A Poisson claims process has two types of claims, Type I and Type II.

(i)   The expected number of claims is 3000.

(ii)   The probability that a claim is Type I is 1/3.

(iii)   Type I claim amounts are exactly 10 each.

(iv)   The variance of aggregate claims is 2,100,000.

Calculate the variance of aggregate claims with Type I claims excluded.

(A)   1,700,000

(B)   1,800,000

(C)   1,900,000

(D)   2,000,000

(E)   2,100,000

20. Don, age 50, is an actuarial science professor. His career is subject to two decrements:

(i)   Decrement 1 is mortality. The associated single decrement table follows De Moivre's law with $w = 100$.

(ii)   Decrement 2 is leaving academic employment, with

$$\mu_{50}^{(2)}(t) = 0.05, \quad t \geq 0$$

Calculate the probability that Don remains an actuarial science professor for at least five but less than ten years.

(A)   0.22

(B)   0.25

(C)   0.28

(D)   0.31

(E)   0.34

21. For a double decrement model:

(i) In the single decrement table associated with cause (1), $q'^{(1)}_{40} = 0.100$ and decrements are uniformly distributed over the year.

(ii) In the single decrement table associated with cause (2), $q'^{(2)}_{40} = 0.125$ and all decrements occur at time 0.7.

Calculate $q^{(2)}_{40}$.

(A) 0.114

(B) 0.115

(C) 0.116

(D) 0.117

(E) 0.118

22. A taxi driver provides service in city R and city S only.

If the taxi driver is in city R, the probability that he has to drive passengers to city S is 0.8. If he is in city S, the probability that he has to drive passengers to city R is 0.3.

The expected profit for each trip is as follows:

a trip within city R:	1.00
a trip within city S:	1.20
a trip between city R and city S:	2.00

Calculate the long-run expected profit per trip for this taxi driver.

(A)   1.44

(B)   1.54

(C)   1.58

(D)   1.70

(E)   1.80

23. The Simple Insurance Company starts at time 0 with a surplus of 3. At the beginning of every year, it collects a premium of 2. Every year, it pays a random claim amount as shown:

Claim Amount	Probability of Claim Amount
0	0.15
1	0.25
2	0.40
4	0.20

If, at the end of the year, Simple's surplus is more than 3, it pays a dividend equal to the amount of surplus in excess of 3. If Simple is unable to pay its claims, or if its surplus drops to 0, it goes out of business. Simple has no administration expenses and interest is equal to 0.

Calculate the probability that Simple will be in business at the end of three years.

(A)   0.00

(B)   0.49

(C)   0.59

(D)   0.90

(E)   1.00

24. For a special 2-payment whole life insurance on (80):

(i) Premiums of p are paid at the beginning of years 1 and 3.

(ii) The death benefit is paid at the end of the year of death.

(iii) There is a partial refund of premium feature:

If (80) dies in either year 1 or year 3, the death benefit is $1000 + \dfrac{p}{2}$.

Otherwise, the death benefit is 1000.

(iv) Mortality follows the Illustrative Life Table.

(v) $i = 0.06$

Calculate p, using the equivalence principle.

(A) 369

(B) 381

(C) 397

(D) 409

(E) 425

25. For a special fully continuous whole life insurance on (65):

(i)   The death benefit at time t is $b_t = 1000\,e^{0.04t}, \quad t \geq 0$.

(ii)   Level benefit premiums are payable for life.

(iii) $\mu_{65}(t) = 0.02, \quad t \geq 0$

(iv) $\delta = 0.04$

Calculate $_2\bar{V}$, the benefit reserve at the end of year 2.

(A)   0

(B)   29

(C)   37

(D)   61

(E)   83

26. You are given:

(i) $A_x = 0.28$

(ii) $A_{x+20} = 0.40$

(iii) $A_{x:\overline{20}|}^{\ 1} = 0.25$

(iv) $i = 0.05$

Calculate $a_{x:\overline{20}|}$.

(A) 11.0

(B) 11.2

(C) 11.7

(D) 12.0

(E) 12.3

27. On his walk to work, Lucky Tom finds coins on the ground at a Poisson rate. The Poisson rate, expressed in coins per minute, is constant during any one day, but varies from day to day according to a gamma distribution with mean 2 and variance 4.

Calculate the probability that Lucky Tom finds exactly one coin during the sixth minute of today's walk.

(A) 0.22

(B) 0.24

(C) 0.26

(D) 0.28

(E) 0.30

28. The unlimited severity distribution for claim amounts under an auto liability insurance policy is given by the cumulative distribution:

$$F(x) = 1 - 0.8e^{-0.02x} - 0.2e^{-0.001x}, \qquad x \geq 0$$

The insurance policy pays amounts up to a limit of 1000 per claim.

Calculate the expected payment under this policy for one claim.

(A)   57

(B)   108

(C)   166

(D)   205

(E)   240

29. A machine is in one of four states (F, G, H, I) and migrates annually among them according to a Markov process with transition matrix:

	F	G	H	I
F	0.20	0.80	0.00	0.00
G	0.50	0.00	0.50	0.00
H	0.75	0.00	0.00	0.25
I	1.00	0.00	0.00	0.00

At time 0, the machine is in State F. A salvage company will pay 500 at the end of 3 years if the machine is in State F.

Assuming $v = 0.90$, calculate the actuarial present value at time 0 of this payment.

(A)   150

(B)   155

(C)   160

(D)   165

(E)   170

30. The claims department of an insurance company receives envelopes with claims for insurance coverage at a Poisson rate of $\lambda = 50$ envelopes per week. For any period of time, the number of envelopes and the numbers of claims in the envelopes are independent. The numbers of claims in the envelopes have the following distribution:

Number of Claims	Probability
1	0.20
2	0.25
3	0.40
4	0.15

Using the normal approximation, calculate the 90th percentile of the number of claims received in 13 weeks.

(A)   1690

(B)   1710

(C)   1730

(D)   1750

(E)   1770

31. An insurer's losses are given by a Poisson process with mean 12 per year. Severity is constant at 1. Annual premiums of 18 are collected. Initial surplus is 6.

Which of the following best describes the insurer's future?

(A)   Ruin is certain within 3 years.

(B)   Ruin is certain, but not necessarily within 3 years.

(C)   Ruin is possible, but not certain.

(D)   Ruin is possible, but can be avoided with certainty by increasing the initial surplus.

(E)   The current surplus is adequate so that ruin will not occur.

32. An actuary is evaluating two methods for simulating $T(xy)$, the future lifetime of the joint-life status of independent lives (x) and (y):

(i)   Mortality for (x) and (y) follows De Moivre's law with $w = 100$.

(ii) $0 < x \leq y < 100$

(iii) Both methods select random numbers R_1 and R_2 independently from the uniform distribution on [0, 1].

(iv) Method 1 sets:

   (a) $T(x) = (100 - x)R_1$

   (b) $T(y) = (100 - y)R_2$

   (c) $T(xy) = $ smaller of $T(x)$ and $T(y)$

(v)   Method 2 first determines which lifetime is shorter:

   (a)   If $R_1 \leq 0.50$, it chooses that (x) is the first to die, and sets
         $T(xy) = T(x) = (100 - x)R_2$.

   (b)   If $R_1 > 0.50$, it chooses that (y) is the first to die, and sets
         $T(xy) = T(y) = (100 - y)R_2$.

Which of the following is correct?

(A)   Method 1 is valid for $x = y$ but not for $x < y$; Method 2 is never valid.

(B)   Method 1 is valid for $x = y$ but not for $x < y$; Method 2 is valid for $x = y$
      but not for $x < y$.

(C)   Method 1 is valid for $x = y$ but not for $x < y$; Method 2 is valid for all x and y.

(D)   Method 1 is valid for all x and y; Method 2 is never valid.

(E)   Method 1 is valid for all x and y; Method 2 is valid for $x = y$ but not for $x < y$.

33. You are given:

(i) The survival function for males is $s(x) = 1 - \dfrac{x}{75}, \quad 0 < x < 75$.

(ii) Female mortality follows De Moivre's law.

(iii) At age 60, the female force of mortality is 60% of the male force of mortality.

For two independent lives, a male age 65 and a female age 60, calculate the expected time until the second death.

(A) 4.33

(B) 5.63

(C) 7.23

(D) 11.88

(E) 13.17

34. For a fully continuous whole life insurance of 1:

(i) $m = 0.04$

(ii) $d = 0.08$

(iii) L is the loss-at-issue random variable based on the benefit premium.

Calculate Var (L).

(A) $\dfrac{1}{10}$

(B) $\dfrac{1}{5}$

(C) $\dfrac{1}{4}$

(D) $\dfrac{1}{3}$

(E) $\dfrac{1}{2}$

35. The random variable for a loss, X, has the following characteristics:

x	$F(x)$	$E(X \wedge x)$
0	0.0	0
100	0.2	91
200	0.6	153
1000	1.0	331

Calculate the mean excess loss for a deductible of 100.

(A)   250

(B)   300

(C)   350

(D)   400

(E)   450

36. WidgetsRUs owns two factories. It buys insurance to protect itself against major repair costs. Profit equals revenues, less the sum of insurance premiums, retained major repair costs, and all other expenses. WidgetsRUs will pay a dividend equal to the profit, if it is positive.

You are given:

(i)   Combined revenue for the two factories is 3.

(ii)   Major repair costs at the factories are independent.

(iii)   The distribution of major repair costs for each factory is

k	Prob (k)
0	0.4
1	0.3
2	0.2
3	0.1

(iv)   At each factory, the insurance policy pays the major repair costs in excess of that factory's ordinary deductible of 1. The insurance premium is 110% of the expected claims.

(v)   All other expenses are 15% of revenues.

Calculate the expected dividend.

(A)   0.43

(B)   0.47

(C)   0.51

(D)   0.55

(E)   0.59

37. For watches produced by a certain manufacturer:

(i) Lifetimes follow a single-parameter Pareto distribution with $a > 1$ and $q = 4$.

(ii) The expected lifetime of a watch is 8 years.

Calculate the probability that the lifetime of a watch is at least 6 years.

(A) 0.44

(B) 0.50

(C) 0.56

(D) 0.61

(E) 0.67

38. For a triple decrement model:

(i)

x	$q_x^{\prime(1)}$	$q_x^{\prime(2)}$	$q_x^{\prime(3)}$
63	0.02000	0.03000	0.25000
64	0.02500	0.03500	0.20000
65	0.03000	0.04000	0.15000

(ii) $q_{65}^{(1)} = 0.02716$

(iii)   Each decrement has a constant force over each year of age.

Calculate $_2q_{64}^{(1)}$.

(A)   0.04182

(B)   0.04248

(C)   0.04314

(D)   0.04480

(E)   0.04546

39. For a special 3-year deferred whole life annuity-due on (x):

(i) $i = 0.04$

(ii)   The first annual payment is 1000.

(iii)   Payments in the following years increase by 4% per year.

(iv)   There is no death benefit during the three year deferral period.

(v)   Level benefit premiums are payable at the beginning of each of the first three years.

(vi) $e_x = 11.05$ is the curtate expectation of life for (x).

(vii)

k	1	2	3
$_k p_x$	0.99	0.98	0.97

Calculate the annual benefit premium.

(A)   2625

(B)   2825

(C)   3025

(D)   3225

(E)   3425

40. For a special fully discrete 10-payment whole life insurance on (30) with level annual benefit premium p :

(i) The death benefit is equal to 1000 plus the refund, without interest, of the benefit premiums paid.

(ii) $A_{30} = 0.102$

(iii) $_{10|}A_{30} = 0.088$

(iv) $(IA)^1_{30:\overline{10|}} = 0.078$

(v) $\ddot{a}_{30:\overline{10|}} = 7.747$

Calculate p .

(A) 14.9

(B) 15.0

(C) 15.1

(D) 15.2

(E) 15.3

END OF EXAMINATION

Final November 2001 Course 3 Exam Answer Key

Exam

Test	Key
1	E
2	C
3	E
4	D
5	D
6	D
7	A
8	A
9	D
10	C
11	B
12	C
13	C
14	C
15	E
16	A
17	D
18	B
19	D
20	A
21	C
22	B
23	D
24	C
25	E
26	B
27	A
28	C
29	E
30	B
31	C
32	D
33	E
34	B
35	B
36	E
37	A
38	B
39	B
40	B