# Spring 2012
# Exam MLC Solutions

**Question #1**
**Key: D**

$$1-\frac{l_{[75]+2}}{l_{[75]+1}}=q_{[75]+1}=0.95q_{75+1}$$

$$l_{[75]+2}=l_{77}=96{,}124$$

$$1-\frac{96{,}124}{l_{[75]+1}}=0.95\times\left(1-\frac{96{,}124}{98{,}153}\right)$$

$$l_{[75]+1}=98{,}050$$

**Question #2**
**Key: A**

$$(x,x+1)\text{ U.D.D.}\Rightarrow {}_{0.25}p_{x+0.75}=1-{}_{0.25}q_{x+0.75}=1-\frac{0.25q_x}{1-0.75q_x}=0.99233$$
$$(x+1,\ x+2)\text{ Constant force }\Rightarrow {}_{0.75}p_{x+1}=(0.95)^{0.75}=0.96226$$

$$p_{x+0.75}={}_{0.25}p_{x+0.75}\times{}_{0.75}p_{x+1}=0.99233\times0.96226=0.95488$$

$$e_{x+0.75}=p_{x+0.75}\left(1+e_{x+1.75}\right)=0.95488\times(1+18.5)=18.62$$

**Question #3**
**Key: B**

Let ${}_t p'_{40}$ denote survival probabilities for this insured.
Let ${}_t p_{40}$ denote survival probabilities using the Illustrative Life Table.

$${}_t p'_{40} = \exp\left[-\int_0^t \left(\mu_{40+s} + 0.02\right) ds\right] = {}_t p_{40}\, e^{-0.02t}$$

So for this insured,

$$\begin{aligned}\ddot{a}_{40:\overline{3}|} &= 1 + v p_{40}\, e^{-0.02} + v^2\, {}_2 p_{40} e^{-0.04} \\ &= 1 + v \times 0.99722 e^{-0.02} + v^2 \times (0.99722)(0.99702) e^{-0.04} \\ &= 1 + 0.92215 + 0.85018 = 2.77233\end{aligned}$$

$$\begin{aligned}A_{40:\overline{3}|} &= 1 - d\ddot{a}_{40:\overline{3}|} \\ &= 1 - \frac{0.06}{1.06} \times 2.77233 = 0.84308\end{aligned}$$

$$\begin{aligned}A_{40:\overset{1}{\overline{3}|}} &= p_{40} p_{41} p_{42} e^{-0.06} v^3 \\ &= (0.99722)(0.99702)(0.99680) e^{-0.06} v^3 = 0.78366\end{aligned}$$

$$A^{1}_{40:\overline{3}|} = 0.84308 - 0.78366 = 0.05942$$

$$P = (10,000)(0.05942) / 2.77233 = 214.33$$

**Question #4**
**Key: C**

$$A^{\ 1}_{[75]:\overline{3}|} = q_{[75]}v + {}_{1|}q_{[75]}v^2 + {}_{2|}q_{[75]}v^3$$
$$= \left[\left(15,930 - 15,668\right)v + \left(15,668 - 15,286\right)v^2 + \left(15,286 - 14,816\right)v^3\right] / 15,930$$
$$= \left[262v + 382v^2 + 470v^3\right] / 15,930 = 0.06163$$

$$\ddot{a}_{[75]:\overline{3}|} = 1 + p_{[75]}v + {}_2p_{[75]}v^2 = 1 + \left(15,668v + 15,286v^2\right) / 15,930 = 2.78190$$
$${}_3E_{[75]} = {}_3p_{[75]}v^3 = \frac{14,816}{15,930}v^3 = 0.78090$$
$$\ddot{a}^{(4)}_{[75]:\overline{3}|} = \ddot{a}_{[75]:\overline{3}|}\,\alpha(4) - \beta(4)\left(1 - {}_3E_{[75]}\right) = 2.78190(1.00027) - 0.38424(1 - 0.78090) = 2.69846$$

The annual premium amount is $1000 \times 0.06163 / 2.69846 = 22.839$

Each quarterly premium $= 22.839 / 4 = 5.71$

**Question #5**
**Key: D**

$$\ddot{a}^{(12)}_{80} \approx \ddot{a}_{80} - (12-1)/24 - (12^2 - 1)/(12 \times 12^2) \times (\mu_{80} + \delta)$$
$$= 6.000 - 0.458333 - 0.082755 \times (0.033 + 0.0488)$$
$$= 5.5349$$

First year commission $= 0.5 \times 900 = 450$
Subsequent commission $= 0.1 \times 900 = 90$

Expected present value of commission expense
$$= (450 - 90) + 90 \times \ddot{a}_{80}$$
$$= 360 + 90 \times 6 = 900$$

Expected present value of maintenance expense
$$= 3 \times 12 \times \ddot{a}^{(12)}_{80} \approx 3 \times 12 \times 5.5349 = 199$$

Expected present value of all expenses = 1099

**Question #6**
**Key: B**

Pr(Hannibal Survives 3 Years)

$= {}_3p_{[4]+2}$

$= p_{[4]+2} \times p_7 \times p_8$

$= (1-q_{[4]+2}) \times (1-q_7) \times (1-q_8)$

$= (1-0.06\times 1.1) \times (1-0.07) \times (1-0.08)$

$= 0.7991$

Pr(Jack Survives 3 Years)

$= {}_3p_{[6]}$

$= p_{[6]} \times p_{[6]+1} \times p_{[6]+2}$

$= (1-q_{[6]}) \times (1-q_{[6]+1}) \times (1-q_{[6]+2})$

$= (1-0.06\times 1.5) \times (1-0.07\times 1.3) \times (1-0.08\times 1.1)$

$= 0.7544$

Pr(Exactly one survives)

$= 1 - \Pr(\text{Both Survive}) - \Pr(\text{Both Die})$

$= 1 - {}_3p_{[4]+2} \times {}_3p_{[6]} - {}_3q_{[4]+2} \times {}_3q_{[6]}$

$= 1 - 0.7991\times 0.7544 - (1-0.7991)\times(1-0.7544)$

$= 0.3478$

Expected Present Value of Benefit = Benefit $\times$ Pr(Exactly one survives) $\times$ discount factor

$= 100,000 \times 0.3478 \times (1.06)^{-3}$

$= 29,202$

**Question #7**
**Key: D**

Combining the interest rate and the growth of cost of a new computer gives a net yearly discount factor of $1.02/1.04 = 0.98077 = v$

For cost of computer *C*, net cost of insurance

$$= C[0.01v + 0.01v^2(0.8) + 0.02v^3(0.6) + 0.02v^4(0.4) + 0.02v^5(0.2)]$$
$$= C[0.00981 + 0.00770 + 0.01132 + 0.00740 + 0.00363] = 0.03986 \times C$$

For example, the third term in that formula is (Probability of claim in year 3)$\times$(Amount of claim in year 3)$\times$(interest discount in year 3)

$$= {}_{2|}q_0 \times (\text{replacement cost}) \times (\%\text{ covered}) \times (\text{interest discount in year 3})$$
$$= 0.02 \times \left(1.02^3 C\right) \times 0.6 / 1.04^3$$
$$= C \times 0.02 \times (1.02/1.04)^3 \times 0.6$$
$$= 0.01132$$

Gross cost of insurance $G = 0.03986 \times C + c \times G + 0.05 \times G$
$0.1 \times C = G$ gives $G = 0.3986 \times G + c \times G + 0.05 \times G$
So $c = 1 - 0.3986 - 0.05 = 0.5514$

**Question #8**
**Key: B**

$$\ddot{a}_{81} = \left(\ddot{a}_{80} - 1\right) \times \left(1 + i\right) / p_{80}$$
$$= \left(5.89 - 1\right) \times \left(1.06\right) / \left(1 - 0.077\right) = 5.616$$
$$A_{81} = 1 - d\ \ddot{a}_{81} = 1 - \left(0.06/1.06\right) \times 5.616 = 0.6821$$

Modified renewal net premium

$$= 1000 \times 0.6821 / 5.616 = 121.5$$
$$A_{90} = 1 - d\ \ddot{a}_{90} = 1 - \left(0.06/1.06\right) \times 3.65 = 0.7934$$
$${}_{10}V^{\text{FPT}} = 1000 \times 0.7934 - 3.65 \times 121.5 = 350$$

Note: Other reserve formulas could also be used, including $1000 \times \left(1 - \ddot{a}_{90} / \ddot{a}_{81}\right)$

**Question #9**
**Key: C**

$$\text{EPV of benefits} = 30,000vq_x + 40,000v^2 p_x\, q_{x+1} + 50,000v^3\, {}_2p_x$$
$$= 2,571.43 + 3,961.90 + 34,588.06 = 41,121.39$$

Let P be the premium in the first year.

$$\text{EPV of premiums} = P\left(1 + vp_x(1.1) + v^2\, {}_2p_x(1.21)\right) = 2.8322P$$

$$P = \frac{41,121.39}{2.8322} = 14,519.24$$

$${}_2V = 50,000/1.05 - \text{year 3 premium}$$
$$= 47,619.05 - 14,519.24 \times 1.1^2$$
$$= 30,050.77$$

**Question #10**
**Key: E**

Pr(second premium is paid) $= 0.8$
Pr(third premium is paid) $= 0.8 \times 0.9 = 0.72$
Pr(exactly 1 paid) $= 1 - 0.8 = 0.2$
Pr(exactly 2 paid) $= 0.8 - 0.72 = 0.08$
Pr(all three paid) $= 0.72$

| $Y = PV$ of premiums paid | | | | |
|---|---|---|---|---|
| Premiums paid | Prob ($p$) | $Y$ | $p \times Y$ | $p \times (Y^2)$ |
| Exactly 1 | 0.20 | 100.00 | 20.00 | 2,000.00 |
| Exactly 2 | 0.08 | 148.08 | 11.95 | 1,754.21 |
| All 3 | 0.72 | 194.30 | 139.90 | 27,181.79 |
| | | | | |
| | | Sum | 171.75 | 30,936.00 |
| | | Var | | 1,437.94 |
| | | Std Dev | | 37.92 |

Note: since $\mathrm{Var}(Y) = \mathrm{Var}(Y - k)$ for a constant *k*, and since everyone pays the first premium, you could get the right answer by calculating the standard deviation of the present value of premiums 2 and 3:
$\Pr(0) = 0.20$
$\Pr(48.08) = 0.08$
$\Pr(94.30) = 0.72$

**Question #11**
**Key: C**

Subscripts below denote policy months.

$i^{(12)} = 0.048$ is interest of 0.004 per month
$\mathrm{AV}_{11} =$ Cash Surrender Value + Surrender Charge = 1,000 + 300 = 1,300
$\mathrm{COI}_{12} = 10,000 \times 0.001 / 1.004 = 9.96$
$\mathrm{AV}_{12} = [1,300 + 100 \times (1 - 0.3)) - 5 - 9.96] \times 1.004 = 1,360.46$
$\mathrm{COI}_{13} = 10,000 \times 0.002 / 1.004 = 19.92$
$\mathrm{AV}_{13} = [1,360.46 + 100 \times (1 - 0.1)) - 5 - 19.92] \times 1.004 = 1,431.24$

Cash surrender value = 1,431.24 – 100 = 1,331.24

**Question #12**
**Key: A**

Replacing age by years of service, the probability that John will be an executive in 35 years is ${}_{35}p_0^{01}$.

$$
\begin{aligned}
&{}_{35}p_0^{01}\\
&=\int_0^{35} {}_t p_0^{00}\times\mu_t^{01}\times {}_{35-t}p_t^{11}\,dt\\
&=\int_0^{35} e^{-\int_0^t(\mu_s^{01}+\mu_s^{02})ds}\times\mu_t^{01}\times e^{-\int_t^{35}\mu_s^{12}ds}dt=\int_0^{35} e^{-(\mu_t^{01}+\mu_t^{02})t}\times\mu_t^{01}\times e^{-\mu_t^{12}(35-t)}dt\\
&=\int_0^{35} e^{-(0.01+0.006)t}\times 0.01\times e^{-(0.002)(35-t)}dt\\
&=0.01\times e^{-0.07}\times\int_0^{35} e^{-0.014t}dt\\
&=\frac{0.01}{0.014}\times e^{-0.07}\times\left(1-e^{-0.49}\right)\\
&=0.2580
\end{aligned}
$$

For John to retire as an executive, he will need to become an executive in Company ABC in 35 years and survive to age 65.

So the probability that John will retire from Company ABC as an executive is $0.9\times 0.2580=0.232.$

**Question #13**
**Key: D**

For [0],

$$p_{[0]} = 1 - 0.1 = 0.9$$
$${}_2p_{[0]} = 0.9(1 - 0.167) = 0.7497$$
$${}_3p_{[0]} = 0.7497(1 - 0.333) = 0.5000$$
$${}_4p_{[0]} = 0.5000(1 - 0.5) = 0.2500$$
$${}_5p_{[0]} = 0.2500(1 - 1) = 0$$

For [1],

$$p_{[1]} = 1 - 0.1 = 0.9$$
$${}_2p_{[1]} = 0.9(1 - 0.333) = 0.6003$$
$${}_3p_{[1]} = 0.6003(1 - 0.5) = 0.3002$$
$${}_4p_{[1]} = 0.3002(1 - 1) = 0$$

| | | |
|---|---|---|
| Expected cars bought new | $= 20$ | just purchased |
| | $+20\ p_{[0]}$ | bought 1 year ago |
| | $+20\ {}_2p_{[0]}$ | bought 2 years ago |
| | $+\ldots$ | |
| | $= 20(1 + 0.9 + 0.7497 + 0.50 + 0.250)$ | |
| | $= 67.99$ | |

Expected cars bought 1 year old

$$= 10(1 + 0.9 + 0.6003 + 0.3002) = 28.01$$

Expected cars $= 68 + 28 = 96$

**Question #14**
**Key: A**

Let $X_i$ denote the claim amount for policy *i*.

Aggregate claims $S = \sum_{i=1}^{n} X_i$

$E(X_i) = 5(0.10) + 10(0.01) = 0.6$

$E\left[(X_i)^2\right] = 5^2 \times 0.10 + 10^2 \times 0.01 = 3.5$

$Var\left(X_i\right) = 3.5 - 0.6^2 = 3.14$

So for *n* policies sold, $E[S] = 0.6n$ and $Var[S] = 3.14n$

95th percentile of total claims $= E(S) + 1.645\,\text{Std Dev}(S)$

$= 0.6n + 1.645 \times \sqrt{3.14n}$

$= 0.6n + 2.915\sqrt{n}$

Benefit premiums from *n* policies = 0.6*n*

Gross premiums from *n* policies $= (1.25 \times 0.6)n = 0.75n$

$0.75n \geq 0.6n + 2.915\sqrt{n}$

$\sqrt{n} = \dfrac{2.915}{0.15} = 19.433$

$n \geq 377.64$

**Question # 15**
**Key: C**

Want $\Pr\left(Y < 1000\right) = \Pr\left(100\ddot{a}_{\overline{K_{65}+1|}} < 1000\right) = \Pr\left(K_{65} \leq 12\right)$

since $\ddot{a}_{\overline{13|}} = 9.86$ and $\ddot{a}_{\overline{14|}} = 10.4$

Then, $\Pr\left(K_{65} \leq 12\right) = \Pr\left(T_{65} < 13\right)$ and

${}_{13}p_{65} = e^{-\int_{65}^{78} \mu_x d_x} = e^{-\int_{65}^{78} 0.001 \times dx}$

$= e^{-0.0005x^2\Big|_{65}^{78}} = 0.3948$

And so,

$P\left(T_{65} < 13\right) = {}_{13}q_{65} = 1 - 0.3948 = 0.61$

**Question #16**
**Key: B**

$$
\begin{aligned}
&\text{APV} \\
&= 25{,}000 \times \bar{a}_{xy} + 15{,}000 \times \left(\bar{a}_{\overline{xy}} - \bar{a}_{xy}\right) + 30{,}000 \times \bar{A}_{xy} \\
&= 25{,}000 \times \bar{a}_{xy} + 15{,}000 \times \left(\bar{a}_{\overline{xy}} - \bar{a}_{xy}\right) + 30{,}000 \times \left(1 - \bar{a}_{xy} \times \delta\right) \\
&= 25{,}000 \times 8 + 15{,}000 \times (10 - 8) + 30{,}000 \times (1 - 8 \times \log 1.06) \\
&= 246{,}015
\end{aligned}
$$

**Question # 17**
**Key: B**

Let $G^M$ and $G^E$ denote the gain from mortality and gain from expenses respectively

$$P = \frac{50{,}000 A_{40}}{\ddot{a}_{40}} = \frac{50(161.32)}{14.8166} = 544.39, \text{ so } G = 680.49$$

$${}_{11}V = (50{,}000 + 300) A_{51} - (1 - 0.05) G\, \ddot{a}_{51} = (50{,}300)(0.25961) - (0.95)(680.49)(13.0803) = 4{,}602.42$$

per person.

$$
\begin{aligned}
G^M &= (\text{Expected } q - \text{Actual } q)(50{,}000 - {}_{11}V + 300)(1{,}000) \\
&= (0.00592 - 0.005)(50{,}300 - 4{,}602.42)(1{,}000) = 42{,}042
\end{aligned}
$$

$G^E$ = Actual deaths $\times$ (Expected claims expense – Actual claims expense) + 1,000$G$(Expected premium expense – Actual premium expense)$\times(1+i)$

$$
\begin{aligned}
&= 5 \times (300 - 100) + (1{,}000)(680.49)(0.05 - 0.06)(1.06) \\
&= -6{,}213
\end{aligned}
$$

Note: You do not need to calculate the numerical value of $G^M$ to solve for $G^E$ here, but do need to know the order of calculation. The numerical result for $G^E$ would be different if the order were "Calculate $G^E$, then calculate $G^M$."

**Question #18**
**Key: C**

$$100,000P_{50} = 24,905/13.2668 = 1,877.2 \text{ so } G = 1.2P_{50} = 2,252.6$$

$${}_{10}V = 100,000\left(A_{60} - P_{50}\,\ddot{a}_{60}\right) = 36,913 - 1,877.2(11.1454) = 15,990.9$$

$${}_{11}V = 100,000\left(A_{61} - P_{50}\,\ddot{a}_{61}\right) = 38,279 - 1,877.2(10.9041) = 17,809.8$$

$$\text{EOY Gain } = [{}_{10}V + G - 0.05G - 40]1.07 - 0.7q_{60}\,100,000 - (1 - 0.7q_{60})\,{}_{11}V$$

$$= [15,991 + 2,253 - 113 - 40]1.07 - 9.632(100) - 0.99037(17,810)$$

$$= 19,357 - 963 - 17,638 = 756$$

**Question #19**
**Key: D**

$$\text{Pr(Transition from 1 to 0 at time } t\text{, then not remain continuously in 0 until 5)} = \int_0^5 {}_t p_x^{\overline{11}}\,\mu_{x+t}^{10}\left(1 - {}_{5-t}p_{x+t}^{\overline{00}}\right)dt$$

$$= \int_0^5 \exp\left[-\int_0^t \left(\mu^{10} + \mu^{12}\right)ds\right]\mu^{10}\left[1 - \exp\left[-\int_t^5 \left(\mu^{01} + \mu^{02}\right)ds\right]\right]dt$$

$$= 0.03\int_0^5 \exp\left[-\int_0^t 0.07\,ds\right]\left[1 - \exp\left[-\int_t^5 0.07\,ds\right]\right]dt$$

$$= 0.03\int_0^5 e^{-0.07t}\left[1 - e^{-0.07(5-t)}\right]dt$$

$$= 0.03\int_0^5 \left(e^{-0.07t} - e^{-0.35}\right)dt$$

$$= 0.03\left[\frac{1 - e^{-0.35}}{0.07} - (5)\left(e^{-0.35}\right)\right] = (0.03)(4.2187 - 3.5234) = 0.0209$$

**Question # 20**
**Key: E**

Retirement Benefit

| Age at Retirement | Salary | Interest discount | Prob | $20\% \times$ Years of Service | Reduction | Total APV |
|---|---|---|---|---|---|---|
| 63 | 253,094 | $e^{-0.05(3)}$ | 0.06 | 0.6 | 0.40 | 4,705 |
| 64 | 263,218 | $e^{-0.05(4)}$ | 0.05 | 0.8 | 0.20 | 6,896 |
| 65 | 273,747 | $e^{-0.05(5)}$ | 0.43 | 1.0 | 0.0 | 91,674 |
| | | | | | Sum | 103,275 |

APV above = Salary $\times$ Interest discount $\times$ prob $\times$ (20% $\times$ Years of service) $\times$ $(1-\text{reduction})$

Death Benefit

| Age at Death | Salary | Interest discount | prob | total |
|---|---|---|---|---|
| 60 | 225,000 | $e^{-0.05}$ | 0.01 | 2,140 |
| 61 | 234,000 | $e^{-0.05(2)}$ | 0.01 | 2,117 |
| 62 | 243,360 | $e^{-0.05(3)}$ | 0.01 | 2,095 |
| 63 | 253,094 | $e^{-0.05(4)}$ | 0.01 | 2,072 |
| 64 | 263,218 | $e^{-0.05(5)}$ | 0.01 | 2,050 |
| | | | sum | 10,474 |

Total APV of both benefits = 103,275 + 10,474 = 113,749.

**Question #21**
**Key: A**

$$\frac{d\left({}_tV\right)}{dt} = \delta \times {}_tV + P - \left(S - {}_tV\right) \times \mu_{x+t}$$

$$20.5 = 0.05 \times {}_tV + 10 - \left(1000 - {}_tV\right) \times 0.026$$

$$20.5 - 10 + 1000 \times 0.026 = \left(0.05 + 0.026\right) \times {}_tV$$

$${}_tV = 36.5 / 0.076 = 480.26$$

**Question #22**
**Key: C**

$$P = \frac{100000\ {}_{10|}A_{30}}{\ddot{a}_{30:\overline{10|}}} = \frac{100(0.54733)(161.32)}{15.8561 - 14.8166(0.54733)}$$

$$= \frac{100(88.2953)}{7.7465} = 1139.81, \text{ so } G = 1.2(1139.81) = 1367.77$$

$$E\left[L_0\right] = -G\ddot{a}_{30:\overline{10|}} + 100000\ {}_{10|}A_{30} + 0.07G + 0.03G\ddot{a}_{30:\overline{10|}} + 60 + 40\ddot{a}_{30}$$

$$= -10,595.43 + 8,829.53 + 95.74 + 317.86 + 60 + 634.24 = -658.06$$

**Question #23**
**Key: E**

Let $L_i$ be the present value of future losses random variable for policyholder $i$.

Then, $S = \sum_{i=1}^{10} L_i$ is the present value of future losses for the portfolio

$$L_i = \begin{cases} -90 & \text{with probability } 1-Q \\ 1000v - 90 = 871.54 & \text{with probability } Q \end{cases}$$

$$Q = \begin{cases} 0 & \text{with probability } 0.3 \\ 0.2 & \text{with probability } 0.7 \end{cases}$$

Then, $Var(S) = E\left[Var(S|Q)\right] + Var\left[E(S|Q)\right]$

$$E\left[S|Q=0\right] = 10E\left[L|Q=0\right] = 10(-90) = -900$$

$$E\left[S|Q=0.2\right] = 10E\left[L|Q=0.2\right] = 10\left[-90(0.8) + 871.54(0.2)\right] = 1023.08$$

$$E[S] = E[E(S|Q)] = -900(0.3) + 1023.08(0.7) = 446.156$$

$$Var(S|Q=0) = 10Var(L|Q=0) = 0$$

$$Var\left(S|Q=0.2\right) = 10Var\left(L|Q=0.2\right) = 10\left[\left(-90\right)^2\left(0.8\right) + 871.54^2(0.2) - 102.308^2\right] = 1,479,295$$

$$E\left[Var\left(S|Q\right)\right] = 0(0.3) + 1,479,295(0.7) = 1,035,507$$

$$Var\left[E\left(S|Q\right)\right] = (-900)^2(0.3) + (1023.08)^2(0.7) - 446.156^2 = 776,630$$

$$Var(S) = 1,035,507 + 776,630 = 1,812,137$$

Alternatively, Let $N$ = number of deaths.
Then $S = 1000vN - 900$ and given $Q$, $N$~binomial (10,$Q$)

$$Var(S) = (1000v)^2 Var(N)$$

$$\begin{aligned}
Var(N) &= E\left[Var(N|Q)\right] + Var\left[E(N|Q)\right] \\
&= E\left[10Q(1-Q)\right] + Var\left[10Q\right] \\
&= 10E[Q] - 10E\left[Q^2\right] + 100\left(E\left[Q^2\right] - E\left[Q\right]^2\right) \\
&= 90E\left[Q^2\right] + 10E\left[Q\right] - 100E\left[Q\right]^2 \\
&= 90\left[0^2(0.3) + 0.2^2(0.7)\right] + 10\left[0(0.3) + 0.2(0.7)\right] - 100\left[0(0.3) + 0.2(0.7)\right]^2 \\
&= 1.96
\end{aligned}$$

$$Var(S) = \left(1000v\right)^2 (1.96) = 1{,}812{,}130$$

It is possible to obtain the answer without knowing the conditional variance formula.

$$E\left(N|Q=0.0\right) = Var\left(N|Q=0.0\right) = E\left(N^2|Q=0.0\right) = 0$$

$$E\left[N|Q=0.2\right] = 10(0.2) = 2$$

$$Var\left(N|Q=0.2\right) = 10(0.2)(0.8) = 1.6$$

$$E\left[N^2|Q=0.2\right] = 1.6 + 2^2 = 5.6$$

$$E(N) = E\left(N|Q=0.0\right)\Pr\left(Q=0.0\right) + E\left(N|Q=0.2\right)\Pr\left(Q=0.2\right) = 0(0.3) + 2(0.7) = 1.4$$

$$E\left(N^2\right) = E\left(N^2|Q=0.0\right)\Pr\left(Q=0.0\right) + E\left(N^2|Q=0.2\right)\Pr\left(Q=0.2\right) = 0(0.3) + 5.6(0.7) = 3.92$$

$$Var(N) = E\left(N^2\right) - E(N)^2 = 3.92 - 1.4^2 = 1.96$$

$$Var(S) = (1000v)^2(1.96) = 1{,}812{,}130$$

Note: While this question illustrates the additional variance when $q$ is uncertain, the premium of 90 would typically not be used as the expected loss is positive.

**Question #24**
**Key: B**

Notation for forward rates and yield rates varies by text. For this solution,

- $y_k$ denotes the annual effective rate for a zero-coupon bond purchased at time 0 maturing in *k* years
- $f(j,k)$ denotes the annual forward rate from start time *j* to end time *k*.

The given forward rates imply the following yield rates:

$$(1+y_3)^3 = (1+f(0,3))^3, \text{ so } y_3 = \left((1.05)^3\right)^{1/3} - 1 = 0.05$$

$$(1+y_2)^2(1+f(2,3)) = (1+f(0,3))^3, \text{ so } y_2 = \left(1.05^3/1.091\right)^{1/2} - 1 = 0.03$$

$$(1+y_1)(1+f(1,3))^2 = (1+f(0,3))^3, \text{ so } y_1 = \left(1.05^3\right)/\left(1.07^2\right) - 1 = 0.011$$

So $A^{1}_{65:\overline{3}|} = 10{,}000[0.00355/1.011 + (1-0.00355)(0.00397)/1.03^2$

$+ (1-0.00355)(1-0.00397)(0.00444)/1.05^3]$

$= 10{,}000\left[0.00351 + 0.00373 + 0.00381\right] = 110.5$

**Question #25**
**Key: A**

Let $L_{0,i}$ denote the loss at issue for policy i. (The curtate future lifetime random variable, $K_{40}$, will take on different values for different policies, but this solution will not denote that explicitly.)

$$L_{0,i} = 100,000 v^{K_{40}+1} + (0.2 - 0.06)G - (1 - 0.06)G\ddot{a}_{\overline{K_{40}+1}|}$$

$$\begin{aligned}E(L_{0,i}) &= 100,000 \times A_{40} + (0.2 - 0.06)G - (1 - 0.06)\ddot{a}_{40}\, G \\ &= 100,000 \times 0.161 + (0.2 - 0.06)G - (1 - 0.06) \times 14.822G \\ &= 16,100 - 13.7927G\end{aligned}$$

$$Var(L_{0,i}) = \left(100,000 + \frac{0.94G}{d}\right)^2 \times \left({}^2A_{40} - A_{40}^2\right) = (100,000 + 16.607G)^2 \times 0.02208$$

Let $L$ denote the loss on the portfolio, $L = \sum_{i=1}^{40,000} L_{0,i}$

$$E(L) = 40,000 \times E(L_{0,i}) = 40,000 \times (16,100 - 13.7927G)$$

$$Var(L) = 40,000 \times Var(L_{0,i}) = 40,000 \times (100,000 + 16.607G)^2 \times 0.02208$$

$$\Pr\left[L < 0\right] = \Phi\left(\frac{-E(L)}{\sqrt{Var(L)}}\right)$$

$$= \Phi\left(\frac{-40,000 \times (16,100 - 13.7927G)}{200 \times (100,000 + 16.607G) \times \sqrt{0.02208}}\right)$$

$$= \Phi\left(\frac{200 \times (13.7927G - 16,100)}{(100,000 + 16.607G) \times 0.14859}\right)$$

$$= 0.95$$

Therefore, $\dfrac{200 \times (13.7927G - 16,100)}{(100,000 + 16.607G) \times 0.14859} = 1.645$ which gives $G = 1178.$

So the annual premium per policy for a portfolio of 40,000 identical and independent lives is $1,215 - 1,178 = 37$ less than that for a portfolio of 2,000 lives.

**Question #26**
**Key: E**

Preliminary calculations from the Illustrative Life Table at 6%:

$${}_{30}E_{35} = {}_{20}E_{35} \times {}_{10}E_{55} = 0.28600 \times 0.48686 = 0.13924$$

$$\ddot{a}_{35:\overline{30}|} = \ddot{a}_{35} - {}_{30}E_{35}\,\ddot{a}_{65} = 15.3926 - 0.13924 \times 9.8969 = 14.0146$$

$$A^{1}_{35:\overline{30}|} = A_{35} - {}_{30}E_{35}\,A_{65} = 0.12872 - 0.13924 \times 0.43980 = 0.06748$$

EPV Premium inflow = EPV Benefit outflow + EPV Expense outflow.
Let $G$ denote the initial gross premium. Then:

EPV Premium inflow $= \left(\dfrac{2 \times \ddot{a}_{35:\overline{30}|} + 1 \times \ddot{a}_{35}}{3}\right) G;$

$= 14.4734G$

EPV Benefit outflow $= 60{,}000 \times \left(\dfrac{3 \times A^{1}_{35:\overline{30}|} + 1 \times A_{35}}{4}\right);$

4,967.4

EPV Expense outflow $= \left(0.35 + 0.05\dfrac{2\ddot{a}_{35:\overline{30}|} + 1\ddot{a}_{35}}{3}\right) \times G.$

$1.0737G$

$$14.4734G = 4{,}967.4 + 1.0737G$$

$$G = \frac{4{,}967.4}{14.4734 - 1.0737} = 370.71$$

Note: other formulas for the nonlevel benefits and premiums would lead to the same EPVs and the same $G$.

**Question #27**
**Key: D**

Account Value $= 2029 + 200 - 0.10 \times 200 - 40 - 400 = 1769$
EPV of Term to $49 = 100 \times (1000A_{44} - {}_{5}E_{44} \times 1000A_{49})$
$= 100 \times (192.61 - 0.73117 \times 238.82$
$= 1799$
No lapse reserve $= 1799 - 1769 = 30$

**Question #28**
**Key: A**

With all $\mu$ values constant,

$$
\begin{aligned}
{}_{t+h}p_x^{00} &= {}_tp_x^{00} - 0.1\times {}_tp_x^{00}(\mu^{01}) + 0.1\times {}_tp_x^{01}(\mu^{10}) \\
&= {}_tp_x^{00} - 0.1\times {}_tp_x^{00}(0.3) + 0.1\times {}_tp_x^{01}(0.1) \\
{}_{t+h}p_x^{01} &= {}_tp_x^{01} - 0.1\times {}_tp_x^{01}(\mu^{10}+\mu^{12}) + 0.1\times {}_tp_x^{00}(\mu^{01}) \\
&= {}_tp_x^{01} - 0.1\times {}_tp_x^{01}(0.1+0.1) + 0.1\times {}_tp_x^{00}(0.3) \\
{}_{t+h}p_x^{02} &= {}_tp_x^{02} + 0.1\times {}_tp_x^{01}(\mu^{12}) \text{ [not needed here]} \\
&= {}_tp_x^{02} + 0.1\times {}_tp_x^{01}(0.1) \text{ [not needed here]} \\
{}_{0.7}p_x^{00} &= 0.8370 - (0.1)(0.8370)(0.3) + (0.1)(0.1588)(0.1) = 0.8135 \\
{}_{0.7}p_x^{01} &= 0.1588 - (0.1)(0.1588)(0.1+0.1) + (0.1)(0.8370)(0.3) = 0.1807 \\
{}_{0.8}p_x^{01} &= 0.1807 - (0.1)(0.1807)(0.2) + (0.1)(0.8135)(0.3) = 0.2015
\end{aligned}
$$

**Question #29**
**Key: E**

Derive independent $p_{60}'^{(d)} = p_{60}^{00\left(p_{60}^{0d}/p_{60}^{0*}\right)}$

where $p_{60}^{0*} = p_{60}^{0d} + p_{60}^{0w} = 1 - \dfrac{l_{61}^{(\tau)}}{l_{60}^{(\tau)}}$

$$
\begin{aligned}
&= \left(l_{61}^{(\tau)} / l_{60}^{(\tau)}\right)^{(2{,}580/950{,}000)/\left(1-\ell_{61}^{(\tau)}/\ell_{60}^{(\tau)}\right)} \\
&= 0.897556^{(0.002716/0.102444)} = 0.997139
\end{aligned}
$$

So $\quad p_{60}^{0w} = \log p_{60}'^{(w)} / \log p_{60}^{00} \times \left(1 - p_{60}^{00}\right)$

$= \log 0.95 / \log\left[(0.997139)(0.95)\right] \times (1 - (0.997139)(0.95)) = 0.049929$

So $\quad d_{60}^{(w)} = 950{,}000 \times 0.049929 = 47{,}433$

**Question # 30**
**Key: B**

From the Illustrative Life Table at 6%:

$\alpha(12) = 1.00028 \quad {}_{10}E_{50} = 0.51081$

$\beta(12) = 0.46812 \quad {}_{20}E_{50} = 0.23047$

$\ddot{a}_{50} = 13.2668$

$\ddot{a}_{60} = 11.1454$

$\ddot{a}_{70} = 8.5693$

Expected Present Value $= 1200 \times \left( \ddot{a}_{50}^{(12)} + 3 \times {}_{10|}\ddot{a}_{50}^{(12)} - 4 \times {}_{20|}\ddot{a}_{50}^{(12)} \right)$

$$= 1200 \times \left[ 12.8024 + 3(5.4557) - 4(1.8676) \right] = 26,039$$

$$\ddot{a}_{50}^{(12)} = \alpha(12)\ddot{a}_{50} - \beta(12) = 1.00028(13.2668) - 0.46812 = 12.8024$$

$$\begin{aligned} {}_{10|}\ddot{a}_{50}^{(12)} = \alpha(12)\,{}_{10}E_{50}\ddot{a}_{60} - \beta(12)\,{}_{10}E_{50} &= 1.00028(0.51081)(11.1454) - 0.46812(0.51081) \\ &= 5.4557 \end{aligned}$$

$$\begin{aligned} {}_{20|}\ddot{a}_{50}^{(12)} = \alpha(12)\,{}_{20}E_{50}\,\ddot{a}_{70} - \beta(12)\,{}_{20}E_{50} &= 1.00028(0.23047)8.5693 - 0.46812(0.23047) \\ &= 1.8676 \end{aligned}$$

Note: other approaches for the nonlevel annuity benefits will also work.