Society of Actuaries

Canadian Institute of Actuaries

Exam MLC
Models for Life Contingencies

Tuesday, May 3, 2016
8:30 a.m. – 12:45 p.m.

MLC

INSTRUCTIONS TO CANDIDATES

General Instructions

1. Write your candidate number here _____. Your name must not appear.

2. Do not break the seal of this book until the supervisor tells you to do so.

3. Tables and numerical values necessary for solving some of the questions on this examination will be distributed by the Supervisor.

4. This examination has a total of 96 points. It consists of:

   Section A: 20 multiple-choice questions, each worth 2 points for a total of 40 points, and

   Section B: 6 written-answer questions, worth a total of 56 points. The point value for each written-answer question is indicated at the beginning of the question.

   You may divide your time between the two sections of the examination (written-answer, and multiple-choice) as you choose. You should keep in mind the relative weight of the two sections.

   Your written-answer paper will be graded only if your multiple-choice score is at or above a threshold set after the examination is administered.

5. Failure to stop writing after time is called will result in the disqualification of your answers or further disciplinary action.

6. While every attempt is made to avoid defective questions, sometimes they do occur. If you believe a question is defective, the supervisor or proctor cannot give you any guidance beyond the instructions on the exam booklet.

Multiple-Choice Instructions

1. A separate answer sheet for the multiple-choice questions is inside the front cover of this book. During the time allotted for this examination, record all your answers on the back of the answer sheet. NO ADDITIONAL TIME WILL BE ALLOWED FOR THIS PURPOSE.

   No credit will be given for anything indicated in the examination book but not transferred to the answer sheet. Failure to stop writing or coding your answer sheet after time is called will result in the disqualification of your answer sheet or further disciplinary action.

2. On the front of the answer sheet, space is provided to write and code candidate information. Complete the information requested by printing in the squares and blackening the circles (one in each column) corresponding to the letters or numbers printed. For each empty box blacken the small circle immediately above the "A" circle. Fill out the boxes titled:

   (a) Name
   (include last name, first name and middle initial)

   (b) Candidate Number
   (Candidate/Eligibility Number, use leading zeros if needed to make it a five digit number)

   (c) Test Site Code
   (The supervisor will supply the number.)

   (d) Examination Part
   (Code the examination that you are taking by blackening the circle to the left of "Exam MLC")

   (e) Booklet Number
   (The booklet number can be found in the upper right-hand corner of this examination book. Use leading zeros if needed to make it a four digit number.)

   In box titled "Complete this section only if instructed to do so," fill in the circle to indicate if you are using a calculator and write in the make and model number.

   In the box titled "Signature and Date" sign your name and write today's date. **If the answer sheet is not signed, it will not be graded**.

   Leave the boxes titled "Test Code" and "Form Code" blank.

   On the back of the answer sheet fill in the Booklet Number in the space provided.

CONTINUED ON INSIDE FRONT COVER

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Exam MLC - Front Cover

3. Your score will be based on the number of questions which you answer correctly. No credit will be given for omitted answers and no credit will be lost for wrong answers: hence, you should answer all questions even those for which you have to guess.

4. Five answer choices are given with each multiple-choice question, each answer choice being identified by a key letter (A to E). Answer choices for some questions have been rounded. For each question, blacken the circle on the answer sheet which corresponds to the key letter of the answer choice that you select.

5. Use a soft-lead pencil to mark the answer sheet. To facilitate correct mechanical scoring, be sure that, for each question, your pencil mark is dark and completely fills only the intended circle. Make no stray marks on the answer sheet. If you have to erase, do so completely.

6. Do not spend too much time on any one question. If a question seems too difficult, leave it and go on.

7. Clearly indicated answer choices in the test book can be an aid in grading examinations in the unlikely event of a lost answer sheet.

8. Use the blank portions of each page for your scratch work. Extra blank pages are provided at the back of the examination book.

9. After the examination, the supervisor will collect this book and the answer sheet separately. DO NOT ENCLOSE THE ANSWER SHEET IN THE BOOK OR IN THE ESSAY ANSWER ENVELOPE. All books and answer sheets must be returned. THE QUESTIONS ARE CONFIDENTIAL AND MAY NOT BE TAKEN FROM THE EXAMINATION ROOM.

Written-Answer Instructions

1. Write your candidate number at the top of each sheet. Your name must not appear.

2. Write on only one side of a sheet. Start each question on a fresh sheet. On each sheet, write the number of the question you are answering. Do not answer more than one question on a single sheet.

3. The answer should be confined to the question as set.

4. When you are asked to calculate, show all your work including any applicable formulas.

5. When you finish, insert all your written-answer sheets into the Essay Answer Envelope. Be sure to hand in all your answer sheets because they cannot be accepted later. Seal the envelope and write your candidate number in the space provided on the outside of the envelope. Check the appropriate box to indicate Exam MLC.

6. Be sure your essay answer envelope is signed because if it is not, your examination will not be graded.

7. For all parts of all problems, to maximize the credit earned, candidates should show as much work as possible, considering the time allotted for the question. Answers lacking justification will receive no credit. Answers should be organized so that the methods, logic, and formulas used are readily apparent. Candidates should not round their answers excessively; enough precision should be provided so that their answers can be accurately graded.

   In some cases, candidates are asked to show that a calculation results in a particular number. Typically the answer given will be rounded; candidates should provide a greater level of accuracy than the number given in the question. This structure of question is intended to assist the candidate by giving an indication when the calculation has been done incorrectly, providing an opportunity to explore an alternative approach. It also allows a candidate who cannot obtain the correct answer to use the answer given to proceed with subsequent parts of the problem. (Candidates who are able to solve the problem should use their exact answer for subsequent parts.)

   For questions requiring candidates to derive or write down a formula or equation, the resulting expression should be simplified as far as possible, and where numerical values are provided in the problem, they should be used.

Exam MLC

SECTION A – Multiple-Choice

****BEGINNING OF EXAMINATION****

1. You are using a Markov model for the future repair status of televisions. The three states are:

Fully Functional (F),
Requires Repairs (R), and
Beyond Repair (B).

You are given:

(i) The following annual probability transition matrix:

$$\begin{array}{c} \\ F \\ R \\ B \end{array} \begin{array}{c} \begin{array}{ccc} F & R & B \end{array} \\ \begin{pmatrix} 0.82 & 0.10 & 0.08 \\ 0.60 & 0.05 & 0.35 \\ 0.00 & 0.00 & 1.00 \end{pmatrix} \end{array}$$

(ii) There are now five televisions that are Fully Functional.

(iii) The status of each television is independent of the status of the others.

Calculate the probability that exactly two of these five televisions will be Fully Functional at the end of two years.

(A) 0.10

(B) 0.28

(C) 0.41

(D) 0.54

(E) 0.67

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2. You are given the survival function:

$$S_0(x) = \left(1 - \frac{x}{60}\right)^{\frac{1}{3}}, \quad 0 \le x \le 60.$$

Calculate $1000\mu_{35}$.

(A) 5.6

(B) 6.7

(C) 13.3

(D) 16.7

(E) 20.1

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3. A fund is established for the benefit of 400 workers all age 25 with independent future lifetimes. When they reach age 65, the fund will be dissolved and distributed to the survivors.

The fund will earn interest at a rate of 6% per year.

The initial fund balance, F, is determined so that the probability that the fund will pay at least 15,200 to each survivor is 86%, using the normal approximation.

Mortality follows the Illustrative Life Table.

Calculate F.

(A) 440,000

(B) 450,000

(C) 460,000

(D) 470,000

(E) 480,000

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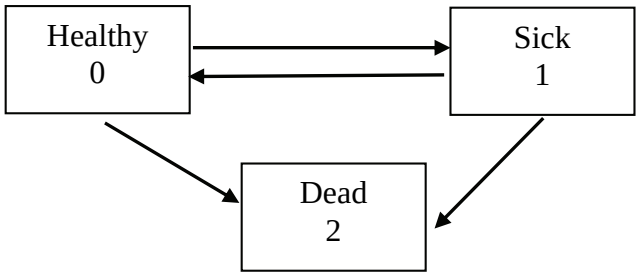
4. A 5-year sickness insurance policy is based on the following Markov model:



You are given the following constant forces of transition:

(i) $\mu^{01} = 0.05$

(ii) $\mu^{10} = 0.02$

(iii) $\mu^{02} = 0.01$

(iv) $\mu^{12} = 0.06$

Calculate the probability that a Healthy life will become Sick exactly once during the 5 years and remain continuously Sick from that point until the end of the 5 years.

(A) 0.06

(B) 0.09

(C) 0.12

(D) 0.15

(E) 0.18

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5. For a select and ultimate mortality model with a one-year select period, you are given:

(i) $p_{[x]} = (1+k)p_x$, for some constant k

(ii) $\ddot{a}_{x:\overline{n}|} = 21.854$

(iii) $\ddot{a}_{[x]:\overline{n}|} = 22.167$

Calculate k.

(A) 0.005

(B) 0.010

(C) 0.015

(D) 0.020

(E) 0.025

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6. For a 10-year term insurance paying a death benefit of 100,000 at the end of the year of the last death of (50) and (60), provided the last death occurs within the 10 years, you are given:

(i) Mortality follows the Illustrative Life Table.

(ii) Future lifetimes are independent.

(iii) $i = 0.06$

Calculate the single net premium.

(A) 1050

(B) 1260

(C) 1460

(D) 1660

(E) 1870

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7. For a fully discrete whole life insurance policy of 100,000 on (35), you are given:

(i) First year commissions are 19% of the annual gross premium.

(ii) Renewal year commissions are 4% of the annual gross premium.

(iii) Mortality follows the Illustrative Life Table.

(iv) $i = 0.06$

Calculate the annual gross premium for this policy using the equivalence principle.

(A) 640

(B) 720

(C) 800

(D) 880

(E) 960

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8. For a fully continuous 20-year term insurance policy of 100,000 on (50), you are given:

(i) Gross premiums, calculated using the equivalence principle, are payable at an annual rate of 4500.

(ii) Expenses at an annual rate of R are payable continuously throughout the life of the policy.

(iii) $\mu_{50+t} = 0.04$, for $t > 0$

(iv) $\delta = 0.08$

Calculate R.

(A) 400

(B) 500

(C) 600

(D) 700

(E) 800

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9. For a fully discrete whole life insurance policy of 50,000 on (35), with premiums payable for a maximum of 10 years, you are given:

(i) Expenses of 100 are payable at the end of each year including the year of death.

(ii) Mortality follows the Illustrative Life Table.

(iii) $i = 0.06$

Calculate the annual gross premium using the equivalence principle.

(A) 990

(B) 1000

(C) 1010

(D) 1020

(E) 1030

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10. For an n-year endowment insurance of 1000 on (x), you are given:

(i) Death benefits are payable at the moment of death.

(ii) Premiums are payable annually at the beginning of each year.

(iii) Deaths are uniformly distributed over each year of age.

(iv) $i = 0.05$

(v) ${}_nE_x = 0.172$

(vi) $\overline{A}_{x:\overline{n}|} = 0.192$

Calculate the annual net premium for this insurance.

(A) 10.1

(B) 11.3

(C) 12.5

(D) 13.7

(E) 14.9

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11. XYZ Insurance writes 10,000 fully discrete whole life insurance policies of 1000 on lives age 20 and an additional 10,000 fully discrete whole life policies of 1000 on lives age 50.

XYZ used the following assumptions to determine the net premiums for these policies:

(i) Mortality follows the Illustrative Life Table.

(ii) $i = 0.06$

During the first ten years, mortality did follow the Illustrative Life Table.

Calculate the average net premium per policy in force received at the beginning of the eleventh year.

(A) 11.08

(B) 11.17

(C) 11.26

(D) 11.36

(E) 11.45

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12. For two fully discrete whole life insurance policies on (x), you are given:

(i)

	Death Benefit	Annual Net Premium	Variance of Loss at Issue
Policy 1	8	1.250	20.55
Policy 2	12	1.875	W

(ii) $i = 0.06$

(iii) The two policies are priced using the same mortality table.

Calculate W.

(A) 30.8

(B) 38.5

(C) 46.2

(D) 53.9

(E) 61.6

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13. For a fully discrete 30-year endowment insurance of 2000 on (35), you are given:

(i) At the end of 15 years, the policy is converted to a reduced paid-up endowment insurance of 800.

(ii) At the time of conversion, the actuarial present value of the reduced paid-up endowment insurance equals the 15^{th} year net premium reserve on the original policy less the amount of the surrender charge (*SC*).

(iii) $i = 5\%$

(iv) $A_{35:\overline{30}|} = 0.255$

(v) $A_{50:\overline{15}|} = 0.506$

Calculate *SC*.

(A) 140

(B) 270

(C) 320

(D) 380

(E) 500

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14. Greg, age 36, buys a Type B universal life policy. The initial premium is 4450 and the additional death benefit is 200,000.

The policy charges are:

- Cost of insurance rate: 120% of the mortality from the Illustrative Life Table
- Expense charges payable at the start of each year: 56 plus 2% of premium.

You are given:

(i) $i^c = i^q = 0.06$

(ii) All cash flows occur at policy anniversaries.

(iii) Account values are calculated annually.

Calculate the account value at the end of the first year.

(A) 4050

(B) 4060

(C) 4070

(D) 4080

(E) 4090

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15. For a 40-year endowment insurance of 10,000 issued to (25), you are given:

(i) $i = 0.04$

(ii) $p_{25} = 0.995$

(iii) $\ddot{a}_{25:\overline{20}|} = 11.087$

(iv) $\ddot{a}_{25:\overline{40}|} = 16.645$

(v) The annual level net premium is 216.

A modified premium reserving method is used for this policy, where the modified premiums are:

I. A first year premium equal to the first year net cost of insurance,

II. Level premiums of β for years 2 through 20, and

III. Level premiums of 216 thereafter.

Calculate β.

(A) 140

(B) 170

(C) 200

(D) 230

(E) 260

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16. For a Type A universal life insurance policy issued to (50), you are given:

(i) The total death benefit is 500,000.

(ii) Level annual premiums are 5000 payable at the beginning of each year.

(iii) Death benefits are paid at the end of the year of death.

(iv) The cost of insurance rates are 120% of the mortality from the Illustrative Life Table.

(v) $i^q = 0.03$ and $i^c = 0.045$ for all years.

(vi) Expense charges payable at the start of each year: 75 plus 3.5% of each annual premium.

(vii) Account values are calculated annually.

(viii) The account value at the end of the first year is 1369.90.

Calculate the account value at the end of the second year.

(A) 2500

(B) 3000

(C) 3500

(D) 4000

(E) 4500

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17. For a special fully discrete whole life insurance, you are given:

(i) The death benefit is $1000(1.03)^k$ for death in policy year k, for $k = 1, 2, 3...$

(ii) $q_x = 0.05$

(iii) $i = 0.06$

(iv) $\ddot{a}_{x+1} = 7.00$

(v) The annual net premium for this insurance at issue age x is 110.

Calculate the annual net premium for this insurance at issue age $x+1$.

(A) 110

(B) 112

(C) 116

(D) 120

(E) 122

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18. You are given the following information about a 60 year old member of a defined benefit pension plan with 35 years of past service:

(i) S_x denotes the salary earned from age x to $x+1$.

(ii) Death is the only decrement other than retirement and occurs mid-year.

(iii) There are no benefits paid upon death.

(iv) ${}_tV$ represents the actuarial liability at age $25+t$.

(v) Funding is based on the projected unit credit method.

Which of the following is a correct expression for the normal contribution for the retirement benefit at the start of the current year?

(A) $\frac{1}{60}\,{}_{35}V$

(B) $\left(1-\frac{25S_{59}}{26S_{60}}\right){}_{35}V$

(C) $\frac{1}{35}\,{}_{35}V$

(D) $\left(\frac{26S_{60}}{25S_{59}}-1\right){}_{35}V$

(E) $\frac{1}{25}\,{}_{35}V$

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19. A pension plan provides an annual retirement benefit of 2% of final year's salary for each year of service, payable at the start of each month, upon retirement at age 65. The annual retirement benefit cannot exceed 60% of final year's salary.

A member, now age 45, joined the plan at age 30. Her current salary is 50,000 and will increase at the rate of 3% per year at the start of each year in the future.

You are given:

(i) $l_{45}^{(\tau)} = 5000$ and $l_{65}^{(\tau)} = 3000$

(ii) $i = 0.05$

(iii) $\ddot{a}_{65}^{(12)} = 7.80$

Calculate the expected present value of this member's retirement benefit.

(A) 93,000

(B) 101,000

(C) 109,000

(D) 260,000

(E) 411,000

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20. Phillip, who is age 40, joins XYZ company which offers him a choice of two pension plans:

- Plan 1 pays an annual pension of 1250 for each year of service.
- Plan 2 pays an annual pension of 2% of his career average salary for each year of service.

You are given:

(i) His starting salary is S_0 and he will receive a 4% salary increase at the beginning of each year starting at age 41.

(ii) He will retire at age 65.

(iii) Plan 1 and Plan 2 both pay benefits at the beginning of each year.

(iv) Plan 1 and Plan 2 yield the same replacement ratio for him.

Calculate S_0

(A) 27,500

(B) 30,000

(C) 32,500

(D) 35,000

(E) 37,500

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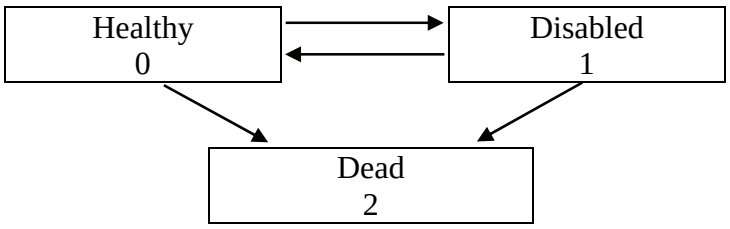
SECTION B – Written-Answer

1. (*12 points*) You are using the following 3-state Markov model to price a special three-year term insurance for Jane, who is exact age 90:



At issue, Jane is Healthy.

The policy provides the following benefits:

- A death benefit of 100,000 payable at the end of the year of death.
- A payment of 25,000 at the end of any year if Jane provides evidence that she is Disabled at that time.

The policy has annual level net premiums payable when Jane is Healthy.

You are given:

(i) $p_{90+t}^{01} = 0.05 + 0.07t$ for $t = 0, 1, 2$

(ii) $p_{90+t}^{02} = 0.10 + 0.10t$ for $t = 0, 1, 2$

(iii) $p_{90+t}^{10} = 0.20$ for $t = 0,1,2$

(iv) $p_{90+t}^{12} = 0.15 + 0.20t$ for $t = 0, 1, 2$

(v) $i = 0.08$

1. Continued

Using the above probability values, the probability that Jane is in State j at time t is as follows:

	Time Since Issue (t)			
State (j)	0	1	2	3
0	1	0.85	0.5880	*A*
1	0	0.05	0.1245	*B*
2	0	0.10	0.2875	*C*

(a) (*3 points*) Calculate *A*, *B*, and *C* in the table above (i.e. calculate the probability of being in State j, 3 years after issue, for $j = 0,1,2$).

(b) (*1 point*) Show that the expected present value of the Disability benefit is 6700 to the nearest 100. You should calculate the expected present value to the nearest 1.

(c) (*1 point*) Show that the expected present value of the Death benefit is 45,000 to the nearest 1000. You should calculate the expected present value to the nearest 1.

(d) (*1 point*) Calculate the annual net premium.

(e) (*3 points*) Calculate the net premium reserves at the end of the second year, ${}_2V^{(0)}$ and ${}_2V^{(1)}$, for this policy.

The policy offers an optional feature that provides a refund of all premiums paid (including the premium for this feature) without interest at the end of three years if the policyholder has not received any other benefits prior to the expiration of the policy.

(f) (*2 points*) Calculate the increase in the annual net premium if this optional feature is added to the policy.

(g) (*1 point*) Features such as return of premium options may affect the other benefits claimed on a policy. Explain why that might be true under this policy.

2. (*9 points*) Sam, age 35, purchases two insurance policies with present value random variables Z_1 and Z_2 given by:

$$Z_1 = \begin{cases} 10v^{T_{35}}, & 0 < T_{35} \le 25 \\ 10v^{25}, & 25 < T_{35} \end{cases}$$

$$Z_2 = \begin{cases} 100v^{T_{35}}, & 0 < T_{35} \le 25 \\ 200v^{T_{35}}, & 25 < T_{35} \le 45 \\ 0, & 45 < T_{35} \end{cases}$$

(a) (*1 point*) Write down expressions for the expected values of Z_1 and Z_2 in terms of standard actuarial notation for insurance and endowment benefits.

(b) (*2 points*) Derive expressions for the variances of Z_1 and Z_2 in terms of standard actuarial notation for insurance and endowment benefits.

(c) *(3 points*) Write down an expression for the covariance of Z_1 and Z_2 in terms of standard actuarial notation for insurance and endowment benefits.

You are given:

(i) Mortality follows the Illustrative Life Table.

(ii) Deaths are uniformly distributed over each year of age.

(iii) $i = 0.06$

(d) (3 *points*) Calculate the standard deviation of Z_1.

3. (*8 points*) Your company issues whole life insurance policies to 10,000 lives with independent future lifetimes, all age 40. You are given:

(i) The death benefit for each policy is 50,000 and is payable at the end of the month of death.

(ii) Premiums of P are payable each month for life.

(iii) Initial expenses other than commissions, payable at the start of the first year, are 20% of the total first year annualized premium.

(iv) Commissions are 5% of each premium including the first.

(v) Mortality follows the Illustrative Life Table, and deaths are uniformly distributed over each year of age.

(vi) $i = 0.06$

(vii) ${}_0L$ is the present value loss at issue random variable for a policy.

(a) (*1 point*) Write down an expression for ${}_0L$ as a function of $K_{40}^{(12)}$ and P.

(b) (*2 points*) Write down an expression for $Var({}_0L)$ as a function of P.

(c) (*2 points*) Show that the monthly premium calculated using the equivalence principle would be 50 to the nearest 10. You should calculate your answer to the nearest 1.

You have now been instructed to calculate monthly premiums using the portfolio percentile premium principle, using the Normal approximation without continuity correction.

(d) (*2 points*) Calculate the monthly premium such that the probability of a loss on the portfolio is 10%.

(e) (*1 point*) State what happens to the monthly premium calculated by the portfolio percentile premium principle as the number of policies approaches infinity. Justify your answer. A numerical demonstration is not required.

4. (*9 points*) For fully discrete whole life insurances of 100,000 on (40), you are given:

(i) The gross premium is 2000.

(ii) Commissions are 120% of the first year premium and 10% of each renewal premium.

(iii) Commissions and all other expenses are payable at the start of each year.

(iv) For reserves:

- The Full Preliminary Term (FPT) reserve method is used.
- Mortality follows the Illustrative Life Table.
- $i = 0.06$

(a) (*2 points*) Show that ${}_2V^{FPT}$, the FPT reserve at the end of year 2, is 920 to the nearest 10. You should calculate your answer to the nearest 1.

The actual policy experience for years 1, 2, and 10 for a portfolio of 1000 such policies is:

Policy Year	Policies in Force at the Start of Year	Number of Deaths During Year	Number of Policies Lapsing During Year	Expenses Other than Commissions	Earned Interest Rate
1	1000	10	0	700,000	0.08
2	990	6	0	200,000	0.08
…	…	…	…	…	…
10	800	6	0	21,500	0.08

(b) (*2 points*) Show that the insurer's actual profit in year 2 was 200,000 to the nearest 10,000. You should calculate the profit to the nearest 100.

4. Continued

The policy is participating. Beginning in year 5, the insurer distributes 60% of its profit before distributions of surplus each year.

- Policies which terminate by death receive their share of current year profits as dividends.
- Policies in force at year end receive their share of current year profits as compound reversionary bonuses.
- Bonuses are valued using the same mortality and interest assumptions that are used for reserves.
- At the end of year 9, the total death benefit, including credited bonuses through year 9, is 110,000 for each policy in force.

For the base benefit of 100,000, the reserves per policy inforce at the end of years 9 and 10 are 8436 and 9666, respectively.

(c) (*1 point*) Show that the reserve at the end of year 9, including the portion for the reversionary bonus, is 10,800 is to the nearest 100. You should calculate your answer to the nearest 1.

(d) (*2 points*) Show that the insurer's profit before distribution of surplus in policy year 10 is 570,000 to the nearest 10,000. You should calculate your answer to the nearest 100.

(e) (*2 points*) Calculate the insurer's compound reversionary bonus rate in year 10.

5. (*9 points*) An insurance company issues 300 fully discrete whole life insurance policies of 100,000 to individuals age 55 with independent future lifetimes.

You are given the following reserve assumptions:

(i) Death is the only decrement assumed and $q_{65} = 0.04$

(ii) $i = 0.05$

(iii) Maintenance expenses are 100 per policy, payable at the beginning of each year.

(iv) Death benefit claim expenses are 300 per death, paid at the end of the year of death.

You are also given the following:

(v) The annual gross premium per policy is 5500.

(vi) ${}_tV^g$ denotes the gross premium reserve per policy at time *t*.

(vii) ${}_{10}V^g = 29,250$

(a) (*2 points*) Show that ${}_{11}V^g$ is 33,720 to the nearest 10. You should calculate the value to the nearest 1.

5. Continued

You are also given:

- The cash surrender benefit in year t is $0.9\times {}_tV^g$. Surrenders occur at the end of the year.
- The following experience for these policies during year 11:

Year	Policies Inforce at Start of Year	Number of Deaths	Number of Surrenders	Maintenance Expenses Incurred	Claim Expenses Incurred	Interest Earned
11	160	9	7	17,700	2500	0.06

(b) (*4 points*)

(i) Calculate the total gain or loss emerging at the end of the eleventh year on the portfolio.

(ii) Calculate the gain or loss by source emerging at the end of the eleventh year in the following order: interest, mortality, expenses, surrenders.

(c) (*2 points*) You are also given that $AS_{10} = 40{,}100$.
Calculate AS_{11}.

(d) (*1 point*) Compare and contrast asset shares and gross premium reserves.

6. (*9 points*) Lucille, who is currently age 45, entered a defined benefit pension plan at age 40 on January 1, 2011. You are given:

(i) The annual retirement benefit is 1.5% of the final three-year average salary for each year of service.

(ii) Lucille's normal retirement age is 65. There are no decrements prior to retirement other than by death.

(iii) Mortality follows the Illustrative Life Table and is subject to a uniform distribution of deaths over each year of age.

(iv) Prior to retirement, $i = 0.05$, and after retirement, $i = 0.06$.

(v) Lucille's salary in calendar year 2015 is 45,000.

(vi) Lucille's salary increases by 3% on January 1 each year.

(vii) There is no death benefit.

(viii) The pension benefit is payable at the start of each month beginning at age 65.

(a) (*2 points*)

(i) Calculate the value of Lucille's accrued pension benefit as of December 31, 2015, using the traditional unit credit approach.

(ii) Calculate the value of Lucille's accrued pension benefit as of December 31, 2015, using the projected unit credit approach.

(b) (*3 points*)

(i) Calculate the normal contribution rate, as a percentage of salary, due at January 1, 2016, under the traditional unit credit approach.

(ii) Calculate the normal contribution rate, as a percentage of salary, due at January 1, 2016, under the projected unit credit approach.

6. Continued

(c) (*4 points*)

(i) On a single set of axes, sketch the curves of the normal contribution rate as a percentage of salary for Lucille under both the traditional unit credit and the projected unit credit approach from entry age to retirement age.

(ii) On a single set of axes, sketch the curves of the accrued benefit values for Lucille under both the traditional unit credit and the projected unit credit approach from entry age to retirement age.

****END OF EXAMINATION****

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