

Society of Actuaries Course 3 Exam Fall 2003

****BEGINNING OF EXAMINATION****

- 1.** For two independent lives now age 30 and 34, you are given:

x	q_x
30	0.1
31	0.2
32	0.3
33	0.4
34	0.5
35	0.6
36	0.7
37	0.8

Calculate the probability that the last death of these two lives will occur during the 3rd year from now (i.e. ${}_2|q_{\overline{30:34}}$).

- (A) 0.01
- (B) 0.03
- (C) 0.14
- (D) 0.18
- (E) 0.24

- 2.** For a whole life insurance of 1000 on (x) with benefits payable at the moment of death:

(i) $\delta_t = \begin{cases} 0.04, & 0 < t \leq 10 \\ 0.05, & 10 < t \end{cases}$

(ii) $\mu_x(t) = \begin{cases} 0.06, & 0 < t \leq 10 \\ 0.07, & 10 < t \end{cases}$

Calculate the single benefit premium for this insurance.

- (A) 379
(B) 411
(C) 444
(D) 519
(E) 594

3. A health plan implements an incentive to physicians to control hospitalization under which the physicians will be paid a bonus B equal to c times the amount by which total hospital claims are under 400 ($0 \leq c \leq 1$).

The effect the incentive plan will have on underlying hospital claims is modeled by assuming that the new total hospital claims will follow a two-parameter Pareto distribution with $\alpha = 2$ and $\theta = 300$.

$$E(B) = 100$$

Calculate c .

- (A) 0.44
- (B) 0.48
- (C) 0.52
- (D) 0.56
- (E) 0.60

4. Computer maintenance costs for a department are modeled as follows:

- (i) The distribution of the number of maintenance calls each machine will need in a year is Poisson with mean 3.
- (ii) The cost for a maintenance call has mean 80 and standard deviation 200.
- (iii) The number of maintenance calls and the costs of the maintenance calls are all mutually independent.

The department must buy a maintenance contract to cover repairs if there is at least a 10% probability that aggregate maintenance costs in a given year will exceed 120% of the expected costs.

Using the normal approximation for the distribution of the aggregate maintenance costs, calculate the minimum number of computers needed to avoid purchasing a maintenance contract.

- (A) 80
- (B) 90
- (C) 100
- (D) 110
- (E) 120

5. N is the random variable for the number of accidents in a single year. N follows the distribution:

$$\Pr(N = n) = 0.9(0.1)^{n-1}, \quad n = 1, 2, \dots$$

X_i is the random variable for the claim amount of the i th accident. X_i follows the distribution:

$$g(x_i) = 0.01 e^{-0.01x_i}, \quad x_i > 0, \quad i = 1, 2, \dots$$

Let U and V_1, V_2, \dots be independent random variables following the uniform distribution on $(0, 1)$. You use the inverse transformation method with U to simulate N and V_i to simulate X_i with small values of random numbers corresponding to small values of N and X_i . You are given the following random numbers for the first simulation:

u	v_1	v_2	v_3	v_4
0.05	0.30	0.22	0.52	0.46

Calculate the total amount of claims during the year for the first simulation.

- (A) 0
- (B) 36
- (C) 72
- (D) 108
- (E) 144

6. You are simulating L , the loss-at-issue random variable for a fully continuous whole life insurance of 1 on (x) . The policy has a double indemnity provision which provides that an additional benefit of 1 will be paid if death is by accidental means.
- (i) The contract premium is 0.025.
 - (ii) $\mu_x^{(\tau)}(t) = 0.02, \quad t > 0$
 - (iii) $\mu_x^{(adb)}(t) = 0.005, \quad t > 0$, is the force of decrement due to death by accidental means.
 - (iv) $\delta = 0.05$
 - (v) Your random number for simulating the time of death is 0.350, where low random numbers correspond to long times until death.
 - (vi) Your random number for simulating the cause of death is 0.775, where high random numbers correspond to deaths by accidental means.

Calculate the simulated value of L .

- (A) -0.391
- (B) -0.367
- (C) -0.341
- (D) -0.319
- (E) -0.297

- 7.** A whole life policy provides that upon accidental death as a passenger on an airplane a benefit of 1,000,000 will be paid. If death occurs from other accidental causes, a death benefit of 500,000 will be paid. If death occurs from a cause other than an accident, a death benefit of 250,000 will be paid.

You are given:

- (i) Death benefits are payable at the moment of death.
- (ii) $\mu^{(1)} = 1/2,000,000$ where (1) indicates accidental death as a passenger on an airplane.
- (iii) $\mu^{(2)} = 1/250,000$ where (2) indicates death from other accidental causes.
- (iv) $\mu^{(3)} = 1/10,000$ where (3) indicates non-accidental death.
- (v) $\delta = 0.06$

Calculate the single benefit premium for this insurance.

- (A) 450
- (B) 460
- (C) 470
- (D) 480
- (E) 490

8. For a special fully discrete whole life insurance of 1000 on (40):

- (i) The level benefit premium for each of the first 20 years is π .
- (ii) The benefit premium payable thereafter at age x is $1000 {}_v q_x$, $x = 60, 61, 62, \dots$
- (iii) Mortality follows the Illustrative Life Table.
- (iv) $i = 0.06$

Calculate π .

- (A) 4.79
- (B) 5.11
- (C) 5.34
- (D) 5.75
- (E) 6.07

9. For an annuity payable semiannually, you are given:

- (i) Deaths are uniformly distributed over each year of age.
- (ii) $q_{69} = 0.03$
- (iii) $i = 0.06$
- (iv) $1000\bar{A}_{70} = 530$

Calculate $\ddot{a}_{69}^{(2)}$.

- (A) 8.35
- (B) 8.47
- (C) 8.59
- (D) 8.72
- (E) 8.85

- 10.** For a sequence, $u(k)$ is defined by the following recursion formula

$$u(k) = \alpha(k) + \beta(k) \times u(k-1) \text{ for } k = 1, 2, 3, \dots$$

(i) $\alpha(k) = -\left(\frac{q_{k-1}}{p_{k-1}}\right)$

(ii) $\beta(k) = \frac{1+i}{p_{k-1}}$

(iii) $u(70) = 1.0$

Which of the following is equal to $u(40)$?

(A) A_{30}

(B) A_{40}

(C) $A_{40:\overline{30}|}$

(D) $A_{40:\overline{30}|}^1$

(E) $A_{40:\overline{30}|}^{\frac{1}{2}}$

- 11.** Subway trains arrive at a station at a Poisson rate of 20 per hour. 25% of the trains are express and 75% are local. The type of each train is independent of the types of preceding trains. An express gets you to the stop for work in 16 minutes and a local gets you there in 28 minutes. You always take the first train to arrive. Your co-worker always takes the first express. You both are waiting at the same station.

Calculate the probability that the train you take will arrive at the stop for work before the train your co-worker takes.

- (A) 0.28
- (B) 0.37
- (C) 0.50
- (D) 0.56
- (E) 0.75

12. A new disease has the following characteristics:

- (i) Once an individual contracts the disease, each year they are in only one of the following states with annual treatment costs as shown:

State	Annual Treatment Costs
Acutely ill	10
In remission	1
Cured or dead	0

Annual treatment costs are assumed not to change in the future.

- (ii) Changes in state occur only at the end of the year.
- (iii) 30% of those who are acutely ill in a given year are in remission in the following year and 10% are cured or dead.
- (iv) 20% of those who are in remission in a given year become acutely ill in the following year and 30% are cured or dead.
- (v) Those who are cured do not become acutely ill or in remission again.

Recall:

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}^{-1} = \begin{pmatrix} a_{22}/d & -a_{12}/d \\ -a_{21}/d & a_{11}/d \end{pmatrix} \text{ where } d = a_{11}a_{22} - a_{12}a_{21}$$

Calculate the expected total treatment costs, for the current and future years, for an individual currently acutely ill.

- (A) 18
- (B) 23
- (C) 28
- (D) 33
- (E) 38

13. A continuous-time surplus process has a compound Poisson claims process with $\lambda = 0.8$.

- (i) All claims are size 1000.
- (ii) Premiums are collected continuously with a relative security loading of 0.25.
- (iii) Initial surplus is 1000.
- (iv) Ruin occurs if surplus drops below 0.

Calculate the probability of ruin by time 2.

- (A) 0.19
- (B) 0.22
- (C) 0.28
- (D) 0.33
- (E) 0.38

- 14.** For a fully discrete whole life insurance of 1000 on (40), the contract premium is the level annual benefit premium based on the mortality assumption at issue. At time 10, the actuary decides to increase the mortality rates for ages 50 and higher.

You are given:

- (i) $d = 0.05$
- (ii) Mortality assumptions:

At issue	${}_k q_{40} = 0.02, k = 0, 1, 2, \dots, 49$
Revised prospectively at time 10	${}_k q_{50} = 0.04, k = 0, 1, 2, \dots, 24$

- (iii) ${}_{10}L$ is the prospective loss random variable at time 10 using the contract premium.

Calculate $E[{}_{10}L | K(40) \geq 10]$ using the revised mortality assumption.

- (A) Less than 225
- (B) At least 225, but less than 250
- (C) At least 250, but less than 275
- (D) At least 275, but less than 300
- (E) At least 300

15. For a group of individuals all age x , of which 30% are smokers and 70% are non-smokers, you are given:

(i) $\delta = 0.10$

(ii) $\bar{A}_x^{\text{smoker}} = 0.444$

(iii) $\bar{A}_x^{\text{non-smoker}} = 0.286$

(iv) T is the future lifetime of (x) .

(v) $\text{Var}[\bar{a}_{T|}^{\text{smoker}}] = 8.818$

(vi) $\text{Var}[\bar{a}_{T|}^{\text{non-smoker}}] = 8.503$

Calculate $\text{Var}[\bar{a}_{T|}]$ for an individual chosen at random from this group.

(A) 8.5

(B) 8.6

(C) 8.8

(D) 9.0

(E) 9.1

16. For a space probe to Mars:

- (i) The probe has three radios, whose future lifetimes are independent, each with mortality following

$${}_k|q_0 = 0.1(k+1), \quad k = 0, 1, 2, 3$$

where time 0 is the moment the probe lands on Mars.

- (ii) The failure time of each radio follows the hyperbolic assumption within each year.
(iii) The probe will transmit until all three radios have failed.

Calculate the probability that the probe is no longer transmitting 2.25 years after landing on Mars.

- (A) 0.053
(B) 0.059
(C) 0.063
(D) 0.067
(E) 0.069

17. T , the future lifetime of (0) , has a spliced distribution.

- (i) $f_1(t)$ follows the Illustrative Life Table.
- (ii) $f_2(t)$ follows DeMoivre's law with $\omega = 100$.
- (iii) $f_T(t) = \begin{cases} k f_1(t), & 0 \leq t \leq 50 \\ 1.2 f_2(t), & 50 < t \end{cases}$

Calculate ${}_{10}P_{40}$.

- (A) 0.81
- (B) 0.85
- (C) 0.88
- (D) 0.92
- (E) 0.96

- 18.** A population has 30% who are smokers with a constant force of mortality 0.2 and 70% who are non-smokers with a constant force of mortality 0.1.

Calculate the 75th percentile of the distribution of the future lifetime of an individual selected at random from this population.

- (A) 10.7
- (B) 11.0
- (C) 11.2
- (D) 11.6
- (E) 11.8

19. Aggregate losses for a portfolio of policies are modeled as follows:

- (i) The number of losses before any coverage modifications follows a Poisson distribution with mean λ .
- (ii) The severity of each loss before any coverage modifications is uniformly distributed between 0 and b .

The insurer would like to model the impact of imposing an ordinary deductible, d ($0 < d < b$), on each loss and reimbursing only a percentage, c ($0 < c \leq 1$), of each loss in excess of the deductible.

It is assumed that the coverage modifications will not affect the loss distribution. The insurer models its claims with modified frequency and severity distributions. The modified claim amount is uniformly distributed on the interval $[0, c(b-d)]$.

Determine the mean of the modified frequency distribution.

- (A) λ
- (B) λc
- (C) $\lambda \frac{d}{b}$
- (D) $\lambda \frac{b-d}{b}$
- (E) $\lambda c \frac{b-d}{b}$

20. The RIP Life Insurance Company specializes in selling a fully discrete whole life insurance of 10,000 to 65 year olds by telephone. For each policy:

- (i) The annual contract premium is 500.
- (ii) Mortality follows the Illustrative Life Table.
- (iii) $i = 0.06$

The number of telephone inquiries RIP receives follows a Poisson process with mean 50 per day. 20% of the inquiries result in the sale of a policy.

The number of inquiries and the future lifetimes of all the insureds who purchase policies on a particular day are independent.

Using the normal approximation, calculate the probability that S , the total prospective loss at issue for all the policies sold on a particular day, will be less than zero.

- (A) 0.33
- (B) 0.50
- (C) 0.67
- (D) 0.84
- (E) 0.99

21. For a special fully discrete whole life insurance on (40):

- (i) The death benefit is 1000 for the first 20 years; 5000 for the next 5 years; 1000 thereafter.
- (ii) The annual benefit premium is $1000P_{40}$ for the first 20 years; $5000P_{40}$ for the next 5 years; π thereafter.
- (iii) Mortality follows the Illustrative Life Table.
- (iv) $i = 0.06$

Calculate ${}_{21}V$, the benefit reserve at the end of year 21 for this insurance.

- (A) 255
- (B) 259
- (C) 263
- (D) 267
- (E) 271

22. For a whole life insurance of 1 on (41) with death benefit payable at the end of year of death, you are given:

- (i) $i = 0.05$
- (ii) $p_{40} = 0.9972$
- (iii) $A_{41} - A_{40} = 0.00822$
- (iv) ${}^2A_{41} - {}^2A_{40} = 0.00433$
- (v) Z is the present-value random variable for this insurance.

Calculate $\text{Var}(Z)$.

- (A) 0.023
- (B) 0.024
- (C) 0.025
- (D) 0.026
- (E) 0.027

- 23.** XYZ insurance company's claims follow a compound Poisson surplus process where individual claims have a uniform distribution over $(0, 10)$.

The relative security loading is positive.

The current surplus of XYZ is 40.

Given that the company's surplus falls below 40, calculate the probability that it will fall below 35 the first time that it falls below 40.

- (A) 0.25
- (B) 0.30
- (C) 0.35
- (D) 0.40
- (E) 0.45

24. For a perpetuity-immediate with annual payments of 1:

- (i) The sequence of annual discount factors follows a Markov chain with the following three states:

State number	0	1	2
Annual discount factor, v	0.95	0.94	0.93

- (ii) The transition matrix for the annual discount factors is:

$$\begin{bmatrix} 0.0 & 1.0 & 0.0 \\ 0.9 & 0.0 & 0.1 \\ 0.0 & 1.0 & 0.0 \end{bmatrix}$$

Y is the present value of the perpetuity payments when the initial state is 1.

Calculate $E(Y)$.

- (A) 15.67
(B) 15.71
(C) 15.75
(D) 16.82
(E) 16.86

- 25.** John has an option to buy stock A and another option to buy stock B. Each option gives him the right to buy the stock at a specific price, the strike price, at the end of 1000 days.

On day n ($n = 2, 3, 4, \dots, 1000$), for each stock, $\Pr(\text{market price} > \text{strike price})$ depends only on the relationships on day $n - 1$ as follows:

Relationship of stocks on day $n - 1$	$\Pr(\text{market price} > \text{strike price})$ on day n
Market price $>$ strike price for neither stock	$\frac{1}{4}$
Market price $>$ strike price for exactly one stock	$\frac{1}{2}$
Market price $>$ strike price for both stocks	$\frac{3}{4}$

Calculate the probability that both stocks will have a market price in excess of their strike price at the end of 1000 days.

- (A) $\frac{2}{7}$
- (B) $\frac{3}{7}$
- (C) $\frac{3}{8}$
- (D) $\frac{3}{16}$
- (E) $\frac{9}{16}$

- 26.** A member of a high school math team is practicing for a contest. Her advisor has given her three practice problems: #1, #2, and #3.

She randomly chooses one of the problems, and works on it until she solves it. Then she randomly chooses one of the remaining unsolved problems, and works on it until solved. Then she works on the last unsolved problem.

She solves problems at a Poisson rate of 1 problem per 5 minutes.

Calculate the probability that she has solved problem #3 within 10 minutes of starting the problems.

- (A) 0.18
- (B) 0.34
- (C) 0.45
- (D) 0.51
- (E) 0.59

27. For a double decrement table, you are given:

(i) $\mu_x^{(1)}(t) = 0.2 \mu_x^{(\tau)}(t), \quad t > 0$

(ii) $\mu_x^{(\tau)}(t) = k t^2, \quad t > 0$

(iii) $q_x^{(1)} = 0.04$

Calculate ${}_2q_x^{(2)}$.

(A) 0.45

(B) 0.53

(C) 0.58

(D) 0.64

(E) 0.73

28. For (x) :

(i) K is the curtate future lifetime random variable.

(ii) $q_{x+k} = 0.1(k+1)$, $k = 0, 1, 2, \dots, 9$

Calculate $\text{Var}(K \wedge 3)$.

(A) 1.1

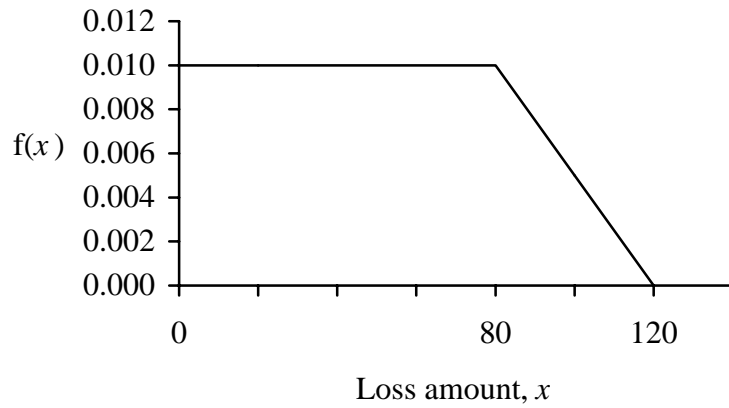
(B) 1.2

(C) 1.3

(D) 1.4

(E) 1.5

29. The graph of the density function for losses is:



Calculate the loss elimination ratio for an ordinary deductible of 20.

- (A) 0.20
- (B) 0.24
- (C) 0.28
- (D) 0.32
- (E) 0.36

30. Michel, age 45, is expected to experience higher than standard mortality only at age 64. For a special fully discrete whole life insurance of 1 on Michel, you are given:

- (i) The benefit premiums are not level.
- (ii) The benefit premium for year 20, π_{19} , exceeds P_{45} for a standard risk by 0.010.
- (iii) Benefit reserves on his insurance are the same as benefit reserves for a fully discrete whole life insurance of 1 on (45) with standard mortality and level benefit premiums.
- (iv) $i = 0.03$
- (v) ${}_{20}V_{45} = 0.427$

Calculate the excess of q_{64} for Michel over the standard q_{64} .

- (A) 0.012
- (B) 0.014
- (C) 0.016
- (D) 0.018
- (E) 0.020

31. For a block of fully discrete whole life insurances of 1 on independent lives age x , you are given:

- (i) $i = 0.06$
- (ii) $A_x = 0.24905$
- (iii) ${}^2A_x = 0.09476$
- (iv) $\pi = 0.025$, where π is the contract premium for each policy.
- (v) Losses are based on the contract premium.

Using the normal approximation, calculate the minimum number of policies the insurer must issue so that the probability of a positive total loss on the policies issued is less than or equal to 0.05.

- (A) 25
- (B) 27
- (C) 29
- (D) 31
- (E) 33

- 32.** Your company currently offers a whole life annuity product that pays the annuitant 12,000 at the beginning of each year. A member of your product development team suggests enhancing the product by adding a death benefit that will be paid at the end of the year of death.

Using a discount rate, d , of 8%, calculate the death benefit that minimizes the variance of the present value random variable of the new product.

- (A) 0
- (B) 50,000
- (C) 100,000
- (D) 150,000
- (E) 200,000

- 33.** A towing company provides all towing services to members of the City Automobile Club. You are given:

(i)

Towing Distance	Towing Cost	Frequency
0-9.99 miles	80	50%
10-29.99 miles	100	40%
30+ miles	160	10%

- (ii) The automobile owner must pay 10% of the cost and the remainder is paid by the City Automobile Club.
- (iii) The number of towings has a Poisson distribution with mean of 1000 per year.
- (iv) The number of towings and the costs of individual towings are all mutually independent.

Using the normal approximation for the distribution of aggregate towing costs, calculate the probability that the City Automobile Club pays more than 90,000 in any given year.

- (A) 3%
- (B) 10%
- (C) 50%
- (D) 90%
- (E) 97%

34. You are given:

- (i) Losses follow an exponential distribution with the same mean in all years.
- (ii) The loss elimination ratio this year is 70%.
- (iii) The ordinary deductible for the coming year is $\frac{4}{3}$ of the current deductible.

Compute the loss elimination ratio for the coming year.

- (A) 70%
- (B) 75%
- (C) 80%
- (D) 85%
- (E) 90%

35. For T , the future lifetime random variable for (0):

(i) $\omega > 70$

(ii) ${}_{40}p_0 = 0.6$

(iii) $E(T) = 62$

(iv) $E(T \wedge t) = t - 0.005t^2, \quad 0 < t < 60$

Calculate the complete expectation of life at 40.

(A) 30

(B) 35

(C) 40

(D) 45

(E) 50

- 36.** Consider the 1-year temporary complete life expectancy of (90), i.e. ${}^{\circ}e_{90:\overline{1}|}$, as based on the Illustrative Life Table. Let C , H , and U be its values under the *constant force*, the *hyperbolic*, and the *uniform* distribution assumptions respectively.

Which of the following is true?

- (A) $C > H > U$
- (B) $C > U > H$
- (C) $U > C > H$
- (D) $U > H > C$
- (E) $H = C = U$

- 37.** Two actuaries use the same mortality table to price a fully discrete 2-year endowment insurance of 1000 on (x) .
- (i) Kevin calculates non-level benefit premiums of 608 for the first year and 350 for the second year.
 - (ii) Kira calculates level annual benefit premiums of π .
 - (iii) $d = 0.05$

Calculate π .

- (A) 482
- (B) 489
- (C) 497
- (D) 508
- (E) 517

38. For a fully discrete 10-payment whole life insurance of 100,000 on (x) , you are given:

- (i) $i = 0.05$
- (ii) $q_{x+9} = 0.011$
- (iii) $q_{x+10} = 0.012$
- (iv) $q_{x+11} = 0.014$
- (v) The level annual benefit premium is 2078.
- (vi) The benefit reserve at the end of year 9 is 32,535.

Calculate $100,000A_{x+11}$.

- (A) 34,100
- (B) 34,300
- (C) 35,500
- (D) 36,500
- (E) 36,700

39. You are given:

- (i) Mortality follows DeMoivre's law with $\omega = 105$.
- (ii) (45) and (65) have independent future lifetimes.

Calculate ${}^{\circ}e_{\overline{45:65}}$.

- (A) 33
- (B) 34
- (C) 35
- (D) 36
- (E) 37

40. You are the consulting actuary to a group of venture capitalists financing a search for pirate gold.

It's a risky undertaking: with probability 0.80, no treasure will be found, and thus the outcome is 0.

The rewards are high: with probability 0.20 treasure will be found. The outcome, if treasure is found, is uniformly distributed on [1000, 5000].

You use the inverse transformation method to simulate the outcome, where large random numbers from the uniform distribution on [0, 1] correspond to large outcomes.

Your random numbers for the first two trials are 0.75 and 0.85.

Calculate the average of the outcomes of these first two trials.

- (A) 0
- (B) 1000
- (C) 2000
- (D) 3000
- (E) 4000

****END OF EXAMINATION****

COURSE 3 Fall 2003
Final Answer Key

<i>Question #</i>	<i>Answer</i>		<i>Question #</i>	<i>Answer</i>
1	E		21	A
2	E		22	C
3	A		23	A
4	C		24	D
5	B		25	A
6	D		26	E
7	B		27	D
8	B		28	A
9	C		29	E
10	C		30	D
11	A		31	B
12	E		32	D
13	C		33	B
14	E		34	C
15	E		35	E
16	E		36	C
17	A		37	B
18	D		38	E
19	D		39	B
20	C		40	B