Society of Actuaries

Canadian Institute of Actuaries

# Exam MLC
# Models for Life Contingencies

## Tuesday,
## April 24, 2018
## 8:30 a.m. – 12:45 p.m.

# MLC

## INSTRUCTIONS TO CANDIDATES

**General Instructions**

1. Write your candidate number here _____. Your name must not appear.

2. Do not break the seal of this book until the supervisor tells you to do so.

3. Tables and numerical values necessary for solving some of the questions on this examination will be distributed by the Supervisor.

4. This examination has a total of 96 points. It consists of:

    Section A: 20 multiple-choice questions, each worth 2 points for a total of 40 points, and

    Section B: 6 written-answer questions, worth a total of 56 points. The point value for each written-answer question is indicated at the beginning of the question.

    You may divide your time between the two sections of the examination (written-answer, and multiple-choice) as you choose. You should keep in mind the relative weight of the two sections.

    Your written-answer paper will be graded only if your multiple-choice score is at or above a threshold set after the examination is administered.

5. Failure to stop writing or coding after time is called will result in the disqualification of your answers or further disciplinary action.

6. While every attempt is made to avoid defective questions, sometimes they do occur. If you believe a question is defective, the supervisor or proctor cannot give you any guidance beyond the instructions on the exam booklet.

**Multiple-Choice Instructions**

1. A separate answer sheet for the multiple-choice questions is inside the front cover of this book. During the time allotted for this examination, record all your answers on the back of the answer sheet. NO ADDITIONAL TIME WILL BE ALLOWED FOR THIS PURPOSE.

No credit will be given for anything indicated in the examination book but not transferred to the answer sheet.

2. On the front of the answer sheet, space is provided to write and code candidate information. Complete the information requested by printing in the squares and blackening the circles (one in each column) corresponding to the letters or numbers printed. For each empty box blacken the small circle immediately above the "A" circle. Fill out the boxes titled:

    (a) Name
        (include last name, first name and middle initial)

    (b) Candidate Number
        (Candidate/Eligibility Number, use leading zeros if needed to make it a five digit number)

    (c) Test Site Code
        (The supervisor will supply the number.)

    (d) Examination Part
        (Code the examination that you are taking by blackening the circle to the left of "Exam MLC")

    (e) Booklet Number
        (The booklet number can be found in the upper right-hand corner of this examination book. Use leading zeros if needed to make it a four digit number.)

In box titled "Complete this section only if instructed to do so," fill in the circle to indicate if you are using a calculator and write in the make and model number.

In the box titled "Signature and Date" sign your name and write today's date. **If the answer sheet is not signed, it will not be graded**.

Leave the boxes titled "Test Code" and "Form Code" blank.

On the back of the answer sheet fill in the Booklet Number in the space provided.

**CONTINUED ON INSIDE FRONT COVER**

3. Your score will be based on the number of questions which you answer correctly. No credit will be given for omitted answers and no credit will be lost for wrong answers: hence, you should answer all questions even those for which you have to guess.

4. Five answer choices are given with each multiple-choice question, each answer choice being identified by a key letter (A to E). Answer choices for some questions have been rounded. For each question, blacken the circle on the answer sheet which corresponds to the key letter of the answer choice that you select.

5. Use a soft-lead pencil to mark the answer sheet. To facilitate correct mechanical scoring, be sure that, for each question, your pencil mark is dark and completely fills only the intended circle. Make no stray marks on the answer sheet. If you have to erase, do so completely.

6. Do not spend too much time on any one question. If a question seems too difficult, leave it and go on.

7. Clearly indicated answer choices in the test book can be an aid in grading examinations in the unlikely event of a lost answer sheet.

8. After the examination, the supervisor will collect this book and the answer sheet separately. DO NOT ENCLOSE THE ANSWER SHEET IN THE BOOK OR IN THE ESSAY ANSWER ENVELOPE. All books and answer sheets must be returned. THE QUESTIONS ARE CONFIDENTIAL AND MAY NOT BE TAKEN FROM THE EXAMINATION ROOM.

**Written-Answer Instructions**

1. Write your candidate number at the top of each sheet. Your name must not appear.

2. Write on only one side of a sheet. Start each question on a fresh sheet. On each sheet, write the number of the question you are answering. Do not answer more than one question on a single sheet.

3. The answer should be confined to the question as set.

4. When you are asked to calculate, show all your work including any applicable formulas.

5. When you finish, insert all your written-answer sheets into the Essay Answer Envelope. Be sure to hand in all your answer sheets because they cannot be accepted later. Seal the envelope and write your candidate number in the space provided on the outside of the envelope. Check the appropriate box to indicate Exam MLC.

6. Be sure your essay answer envelope is signed because if it is not, your examination will not be graded.

7. For all parts of all problems, to maximize the credit earned, candidates should show as much work as possible, considering the time allotted for the question. Answers lacking justification will receive no credit. Answers should be organized so that the methods, logic, and formulas used are readily apparent. Candidates should not round their answers excessively; enough precision should be provided so that their answers can be accurately graded.

In some cases, candidates are asked to show that a calculation results in a particular number. Typically the answer given will be rounded; candidates should provide a greater level of accuracy than the number given in the question. This structure of question is intended to assist the candidate by giving an indication when the calculation has been done incorrectly, providing an opportunity to explore an alternative approach. It also allows a candidate who cannot obtain the correct answer to use the answer given to proceed with subsequent parts of the problem. (Candidates who are able to solve the problem should use their exact answer for subsequent parts.)

For questions requiring candidates to derive or write down a formula or equation, the resulting expression should be simplified as far as possible, and where numerical values are provided in the problem, they should be used.

# Exam MLC

# SECTION A – Multiple-Choice

**BEGINNING OF EXAMINATION**

1. A select and ultimate survival model has a 3-year select period, and is specified as follows:

(i) The ultimate part of the model follows the Illustrative Life Table.

(ii) The select part of the model is such that:

$$p_{[x]} = p_{x-5} \qquad p_{[x-1]+1} = p_{x-3} \qquad p_{[x-2]+2} = p_{x-1}$$

Calculate $_4q_{[45]}$.

(A) 0.0125

(B) 0.0135

(C) 0.0145

(D) 0.0155

(E) 0.0165

**2.** You are given:

(i) $_{0.2}q_{50.6} = 0.10$

(ii) Deaths are uniformly distributed over each year of age.

Calculate $q_{50}$.

(A) 0.365

(B) 0.370

(C) 0.375

(D) 0.380

(E) 0.385

**USE THIS PAGE FOR YOUR SCRATCH WORK**

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**3.** A couple, both age 65, have the option to receive one of the following:

- A life annuity of $F$ per year, payable at the beginning of each year while at least one is alive.
- A lump sum of 100,000 if both lives survive 5 years.

You are given:

(i) Mortality follows the Illustrative Life Table.

(ii) $i = 0.06$

(iii) Their future lifetimes are independent.

(iv) The actuarial present values of the payments under the two options are equal.

Calculate $F$.

(A) 3560

(B) 4830

(C) 6460

(D) 7340

(E) 9820

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**4.** You are given:

      (i)     $i = 0.05$

      (ii)    $1000 \, {}_{15}E_{50} = 462$

      (iii)   $1000 A_{50}^{(12)} = 189$

      (iv)   $1000 A_{65}^{(12)} = 354$

Calculate $\ddot{a}_{50:\overline{15}|}^{(12)}$.

(A)     9.6

(B)     9.9

(C)     10.2

(D)     10.5

(E)     10.8

**5.** An insurer issues a 3-year deferred whole life annuity-due of 100 per year to (80).

You are given:

(i) Mortality follows the Illustrative Life Table.

(ii) $f(k,k+1)$ is the $k$-year deferred one-year forward rate.

(iii)

| $f(0,1)$ | $f(1,2)$ | $f(2,3)$ | $f(3,4)$ | $f(k,k+1),$ $k=4,5,6,...$ |
|---|---|---|---|---|
| 0.04 | 0.045 | 0.05 | 0.055 | 0.06 |

Calculate the expected present value of the annuity.

(A) 341

(B) 343

(C) 345

(D) 347

(E) 349

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**6.** A single premium, 10-year term insurance policy with death benefit of 100,000 is issued to (40). The benefit is payable at the moment of death.

You are given:

     (i)    The single premium is 3144.

     (ii)    Mortality follows the Illustrative Life Table.

     (iii)    Deaths are uniformly distributed over each year of age.

     (iv)    $i = 0.04$

Calculate the probability that the present value of the loss at issue is more than 75,000.

(A)    0.001

(B)    0.006

(C)    0.011

(D)    0.016

(E)    0.021

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**7.** For a 20-year deferred annuity-due issued to (45) that pays 150,000 annually for life, you are given:

   (i)   Premiums of $G$ are paid annually during the deferral period.

   (ii)  First year commissions are 40% of premium.

   (iii) Commissions for years 2 through 10 are 10% of premium.

   (iv)  Commissions for years 11-20 are 5% of premium.

   (v)   $\ddot{a}_{45:\overline{10|}} = 6.25; \quad \ddot{a}_{55:\overline{10|}} = 6.00; \quad \ddot{a}_{65} = 7.40$

   (vi)  $_{10}E_{45} = 0.30; \quad _{10}E_{55} = 0.27$

   (vii) Premiums are calculated using the equivalence principle.

Calculate $G$.

(A)  12,580

(B)  12,680

(C)  12,780

(D)  12,880

(E)  12,980

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**8.** Jenn and Dave are both age 75 with independent future lifetimes. They purchase an insurance policy which pays 100,000 immediately on the death of Jenn provided she dies after Dave. Premiums are payable continuously at a rate $P$ per year while both lives are alive.

You are given:

(i) $\bar{A}_{75} = 0.6091; \quad \bar{A}_{75:75} = 0.7228$

(ii) $i = 6\%$

Calculate $P$.

(A) 4600

(B) 5200

(C) 7600

(D) 8600

(E) 10100

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**9.** A life insurer sells identical fully discrete whole life insurance policies to 1000 independent lives age 45. Premiums are determined using a profit test of the portfolio.

You are given the following profit test assumptions and results:

  (i)   Mortality follows the Illustrative Life Table.

  (ii)  The risk discount rate is 6%.

  (iii) The Net Present Value for the portfolio is 10,000,000.
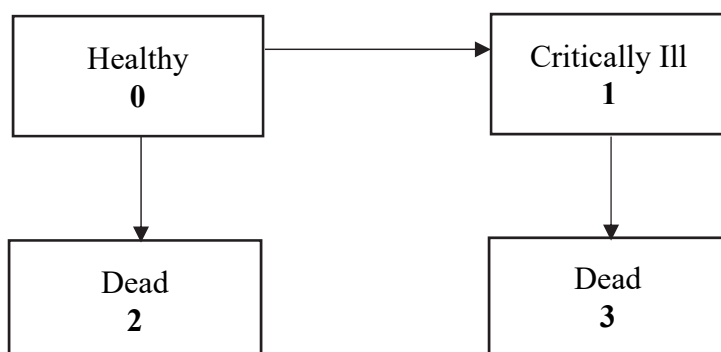
  (iv)  The profit margin is 15%.

Calculate the annual premium for each policy.

(A) 4024

(B) 4724

(C) 5424

(D) 6124

(E) 6824

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**10.** A whole life policy with Critical Illness benefits is issued to a Healthy life age 60. The insurer uses the following multiple state model to value the benefits:



The premium $P$ is payable continuously while the policyholder is Healthy. A benefit of 50,000 is paid immediately on diagnosis of Critical Illness (CI), with another 50,000 paid on death after CI. If the policyholder dies without a CI diagnosis, the full 100,000 is paid immediately on death.

You are given the following information:

| $x$ | $\bar{a}_x^{00}$ | $\bar{A}_x^{01}$ | $\bar{A}_x^{02}$ | $\bar{A}_x^{03}$ | $\bar{A}_x^{13}$ |
|-----|------|-------|-------|-------|-------|
| 60 | 10.989 | 0.390 | 0.181 | 0.280 | 0.546 |

Calculate $P$.

(A)    4300

(B)    4700

(C)    5100

(D)    5500

(E)    5900

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**11.** For a fully discrete whole life insurance of 250,000 on (55), you are given:

   (i)     The annual premium is 5,000.

   (ii)    Mortality follows the Illustrative Life Table.

   (iii)    $i = 0.06$

Calculate the probability that the insurance company will make a profit on this policy.

(A)    0.524

(B)    0.563

(C)    0.607

(D)    0.648

(E)    0.683

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**12.** For a 20-year endowment insurance of 1,000,000 on (40), you are given:

(i)    The death benefit is paid at the end of the year of death.

(ii)    Premiums are payable monthly.

(iii)    Deaths are uniformly distributed over each year of age.

(iv)    Mortality follows the Illustrative Life Table.

(v)    $i = 0.06$

Calculate the monthly net premium.

(A)    2413

(B)    2426

(C)    2438

(D)    2451

(E)    2463

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**13.** For a Type A universal life insurance policy on (55), with a death benefit of 500,000, you are given:

    (i)     The account value at time 10 is $AV_{10} = 66,600$.

    (ii)    Annual premiums of 10,000 are paid at the beginning of each policy year.

    (iii)   Expense charges, payable at the beginning of each year, are 120 plus 5% of premium.

    (iv)   The cost of insurance rate in year 11 is 15 per 1000.

    (v)    $i^c = i^q = 0.04$

Calculate the Additional Death Benefit for this policy at the end of policy year 11.

(A)   427,390

(B)   429,490

(C)   432,390

(D)   433,390

(E)   434,990

**14.** For a fully discrete 20-year term insurance issued to (35) with face amount 1,000,000, you are given:

      (i)      The annual net premium for this insurance is 3489.

      (ii)     Mortality follows the Illustrative Life Table.

      (iii)    $i = 0.06$

      (iv)    The force of mortality is constant between integer ages.

Calculate the net premium reserve at time 10.4.

(A)    15,630

(B)    15,780

(C)    15,930

(D)    16,080

(E)    16,230

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**15.** For a fully continuous whole life insurance of 100,000 on (50), you are given:

    (i)    $\mu_{50+t}$ is a continuous function of $t$.

    (ii)    $\mu_{60} = 0.013$

    (iii)    $\delta = 0.04$

    (iv)    $_tV$ is the gross premium reserve at time $t$.

    (v)    $_{10}V = 18,700$

    (vi)    Maintenance expenses are 5% of the premium, payable continuously.

    (vii)    Claims expenses are 2.5% of the death benefit.

    (viii)    The gross premium rate is 2700 per year.

Calculate $\dfrac{d}{dt}\,_tV$ at $t = 10$.

    (A)    2148

    (B)    2224

    (C)    2256

    (D)    2314

    (E)    2391

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**16.** For a fully discrete whole life insurance of 1000 on (40), you are given:

(i)     $1000\,q_{40} = 4.23$

(ii)     $i = 0.06$

(iii)     $\ddot{a}_{40} = 13.25; \quad \ddot{a}_{41} = 13.04; \quad \ddot{a}_{60} = 9.72$

(iv)     $A_{40} = 0.25; \quad A_{60} = 0.45$

(v)     $_{20}V$ is the net premium reserve at the end of year 20.

(vi)     $_{20}V^{FPT}$ is the Full Preliminary Term (FPT) reserve at the end of year 20.

Calculate $_{20}V - {_{20}V}^{FPT}$.

(A)     12

(B)     13

(C)     14

(D)     15

(E)     16

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**17.** For a whole life insurance of 100,000 on (45), you are given:

     (i)     Gross premiums of 900 are paid every ½ year.

     (ii)    The death benefit is paid at the end of the ½-year of death.

     (iii)   Mortality follows the Illustrative Life Table.

     (iv)   $i = 0.05$

     (v)    Renewal expenses are 10% of the gross premium.

     (vi)   Deaths are uniformly distributed between integer ages.

     (vii)  The gross premium reserve at duration 10.5, immediately before the premium payment, is $_{10.5}V^g = 16,074$.
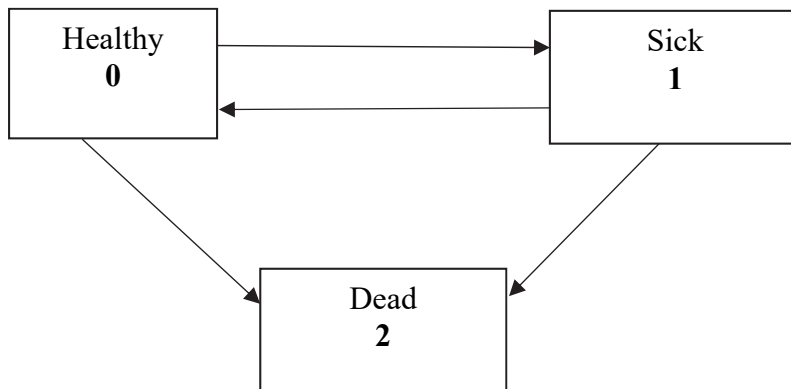
Calculate the gross premium reserve at the end of the 11$^{\text{th}}$ policy year, $_{11}V^g$.

(A)    16,930

(B)    16,980

(C)    17,010

(D)    17,220

(E)    17,320

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**18.** You are given the following health-sickness model:



For a certain age $x$, you are given:

(i) $\mu^{02}_{x+5} = \mu^{12}_{x+5} = 0.0020$

(ii) At $t = 5$, $\dfrac{d}{dt}\,{}_{t}p^{00}_{x} = -0.0030$

(iii) At $t = 5$, $\dfrac{d}{dt}\,{}_{t}p^{01}_{x} = 0.0012$

Calculate ${}_{5}p^{02}_{x}$

(A)   0.05

(B)   0.10

(C)   0.15

(D)   0.20

(E)   0.25

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**19.** Mark is covered under a defined benefit pension plan. You are given:

(i) The annual benefit payable as a life annuity-due is 2% of the 3-year final average salary per year of service.

(ii) Mark retires at age 65 with 30 years of service.

(iii) Mark's salary in his final year of employment was 100,000.

(iv) At the start of each of the last three years of employment, Mark's salary increased by 3%.

(v) Mortality follows the Illustrative Life Table.

(vi) $i = 0.06$

(vii) Mark chooses to take his benefit as a 10-year guaranteed whole life annuity-due.

Calculate the annual payment he will receive using the equivalence principle.

(A) 52,400

(B) 52,800

(C) 53,200

(D) 53,600

(E) 54,000

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**20.** XYZ offers a pension plan with the following lump sum death-in-service benefits, payable immediately on death:

- 10,000 for each full year of service on death in service between ages 64 and 65.
- 15,000 for each full year of service on death in service between ages 65 and 66.

You are given:

(i)    Death is assumed to occur half-way through the year of age.

(ii)   Decrements for this pension plan follow the Illustrative Service Table.

(iii)   $i = 0.05$

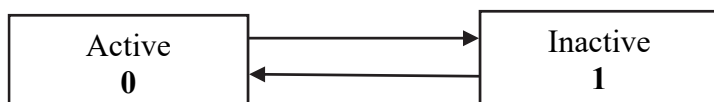(iv)   XYZ uses the Traditional Unit Credit funding method.

Calculate the normal cost for this benefit for a new employee who is age 50.

(A)    60

(B)    70

(C)    80

(D)    90

(E)    100

# Exam MLC

# SECTION B – Written-Answer

**1.** (*9 points*) TCFA is a firm with two categories of employment, active and inactive. You are using the following Markov model to project employment at TCFA:

| Active 0 | → ← | Inactive 1 |
|---|---|---|

You are given the following additional information:

(i) Transition forces are $\mu^{01} = 0.2$ and $\mu^{10} = 0.1$

(ii) All new hires are in the Active state.

(iii) TCFA pays a rehiring bonus of 10,000 at the moment of rehiring when an Inactive employee becomes Active. Only one bonus is paid in each year, at the time of the first transition back to the Active state.

(iv) $\delta = 0.05$

(a) (*1 point*) Show that the probability that an Active employee becomes Inactive at least once in the first 7 months is 0.11 to the nearest 0.01. You should calculate the probability to the nearest 0.001.

(b) (*3 points*)

(i) Show that the probability that a newly hired employee has made one transition to the Inactive state, and has not moved back to the Active state by time *t*, is $2\left(e^{-0.1t} - e^{-0.2t}\right)$.

(ii) Calculate the expected present value of the rehiring bonus for a single employee during the first year of employment.

# 1. Continued

TCFA has now decided to change its employment practices as follows:

- An inactive employee who has transitioned only once is eligible to return to active employment and has a force of transition for such return of 0.10.
- The force of transition from active to inactive is 0.20, for both the first and second transitions to inactive.
- Once employees have become inactive for the second time, they are no longer eligible to return to active employment.

(c)  (*2 points*)  Draw a diagram of a 4-state Markov model that could be used to model employee transitions following the change in hiring practices. You should define the states and write down the values of all non-zero transition intensities.

(d)  (*3 points*)  Use the revised model to calculate the probability that a newly hired employee is actively employed at the end of 5 years.

**2.**   (*10 points*)  Marcel, age $x$, and Jeanne, age $y$, have just retired.  They have 200,000 to purchase a single premium, joint and last survivor annuity.

You are given:

   (i)    Pricing uses the equivalence principle.

   (ii)   The future lifetimes are independent.

   (iii)  $\delta = 0.06$

   (iv)   $\bar{A}_x = 0.23, \quad \bar{A}_y = 0.19, \quad \bar{A}_{xy} = 0.35$

   (v)    $^2\bar{A}_x = 0.13, \quad ^2\bar{A}_y = 0.10, \quad ^2\bar{A}_{xy} = 0.21$

(a)   (*2 points*)  Show that the 200,000 single premium would buy a continuous whole life last survivor annuity with an annual payment rate of 12,900 to the nearest 100. You should calculate your answer to the nearest 1.

(b)   (*2 points*)  Calculate the standard deviation of the present value of the payments for this last survivor annuity.

Instead of the level last survivor annuity, the couple decides to use the 200,000 to buy a special annuity that pays $X$ per year continuously if both Marcel and Jeanne are alive, and pays $0.75X$ per year continuously if only one of them is alive.

(c)   (*1 point*)  Calculate $X$.

(d)   (*5 points*)

   (i)    Show that the covariance between $v^{T_{xy}}$ and $v^{T_{\overline{xy}}}$ is
   $$Cov\left(v^{T_{xy}}, v^{T_{\overline{xy}}}\right) = \left(\bar{A}_x - \bar{A}_{xy}\right)\left(\bar{A}_y - \bar{A}_{xy}\right)$$

   (ii)   Hence, show that the covariance between $\bar{a}_{\overline{T_{xy}|}}$ and $\bar{a}_{\overline{T_{\overline{xy}}|}}$ is
   $$Cov\left(\bar{a}_{\overline{T_{xy}|}}, \bar{a}_{\overline{T_{\overline{xy}}|}}\right) = \left(\bar{a}_x - \bar{a}_{xy}\right)\left(\bar{a}_y - \bar{a}_{xy}\right)$$

   (iii)  Calculate the standard deviation of the present value of the annuity payments for this special survivorship annuity.

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**3.** (*10 points*) SOA Life Insurance Company issues a special fully discrete whole life policy to (75). The death benefit is 1000 during the first year, and increases by 1000 in each subsequent year. Level annual premiums are payable throughout the term of the policy.

You are given the following information:

> (i) A termination expense of 500 is paid at the end of the year of death.
>
> (ii) Commissions are 40% of the first premium and 8% of all subsequent premiums.
>
> (iii) Issue expenses are 400 per policy.
>
> (iv) Maintenance expenses are 45 per policy at the beginning of each year including the first.
>
> (v) Mortality follows the Illustrative Life Table.
>
> (vi) $i = 0.06$
>
> (vii) The annual net premium is 662.46
>
> (viii) Gross premiums are calculated using the equivalence principle.

(a) (*2 points*) Show that the gross premium is 920 to the nearest 10. You should calculate the value to the nearest 1.

(b) (*2 points*) Calculate the gross premium reserve at the end of each of the first two policy years.

A change in regulation now requires SOA Life to use a net premium reserve method. SOA Life decides to use the Full Preliminary Term (FPT) modified net reserve method.

(c) (*1 point*) Calculate the first year modified net premium under the FPT method.

(d) (*2 points*) Show that the modified net premium for the second and subsequent years under the FPT method is 760 to the nearest 10. You should calculate the value to the nearest 1.

(e) (*2 points*) Calculate the FPT reserve at the end of each of the first two policy years.

**3.** **Continued**

    (f)    (*1 point*)  Explain why the company would use the FPT method, which assumes non-level net premiums, instead of an unmodified net premium reserve with level net premiums.

**4.** (*9 points*)  XYZ Insurance is conducting a profit test of a fully discrete participating whole life policy with a death benefit of 100,000 issued to a life age 80.

XYZ holds net premium reserves for this policy. You are given the following information about the reserves:

- The reserve basis is the Illustrative Life Table with interest at 6%.

- $_kV$ denotes the net premium reserve at time $k$.

- $_1V = 4263$

(a)  (*1 point*)  Show that $_2V = 8450$ to the nearest 50.  You should calculate the value to the nearest 1.

For the profit test you are given the following additional information:

(i)  The gross premium is 13,000.

(ii)  Pre-contract expenses are 1000.

(iii)  Percent of premium expenses are 20% in the first year and 5% thereafter.

(iv)  Maintenance expenses are 75 at the beginning of each year including the first.

(v)  The interest earned on insurer's funds is 7% per year.

(vi)  Mortality rates are 85% of the Illustrative Life Table rates.

(vii)  Policy lapses will be 20% of policies in force at the end of the first year and 5% of policies in force at the end of all subsequent years.

(viii)  The cash surrender values are 20% of the reserves.

(ix)  Cash dividends equal to 70% of the pre-dividend profits in any year are distributed at the year end.  If the policy generates a loss in a policy year, no dividends are paid.

(x)  Dividends are payable with respect to all policies in force at the beginning of the year.

(xi)  $\Pr_t$ denotes the profit emerging at the end of the $t^{th}$ year <u>after</u> distribution of dividends, given that policy was in effect at the beginning of the $t^{th}$ year.

(xii)  $\Pr_1 = 265.72$ and $\Pr_3 = 894.99$

(xiii)  The risk discount rate is 10% per year.

# 4. Continued

(b) (*2 points*) Show that $\text{Pr}_2 = 820$ to the nearest 10. You should calculate the value to the nearest 1.

(c) (*2 points*) Calculate the partial Net Present Value up to time 3, NPV(3), for the policy.

(d) (*3 points*) Now assume that, at each year end, for policies remaining in force the dividend is distributed as a reversionary bonus. The additional death benefit is calculated using the reserve basis and the equivalence principle.

    (i) Calculate the additional death benefit purchased for a continuing policyholder at the end of the first year.

    (ii) Show that the revised value of $\text{Pr}_2$ is 830 to the nearest 10. You should calculate the value to the nearest 1.

(e) (*1 point*) Explain why $\text{Pr}_2$ increases when the dividend is converted to an additional death benefit.

**5.** (*9 points*) An insurer issues identical fully continuous whole life policies to a number of independent lives, each age 80. The death benefit is 10,000.

You are given:

(i) $N$ policyholders will purchase the insurance.

(ii) There are no expenses.

(iii) The annual premium rate is $P$.

(iv) $L_j$ denotes the present value of the loss at issue on the $j^{\text{th}}$ policy in the portfolio and $L = \sum_{j=1}^{N} L_j$ denotes the total loss at issue for the portfolio.

(v) $\delta = 0.05$

(vi)

| $\delta$ | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 | 0.10 | 0.11 | 0.12 |
|---|---|---|---|---|---|---|---|---|---|
| $\bar{a}_{80}$ | 5.855 | 5.595 | 5.353 | 5.129 | 4.920 | 4.725 | 4.544 | 4.374 | 4.215 |

(a) (*3 points*)

(i) The standard deviation of $L$ can be written in the form $\sqrt{N}(a + bP)$. Calculate $a$ and $b$.

(ii) Show that the risk within the portfolio, as measured by the total loss $L$, is diversifiable.

## 5. Continued

The insurer sets the premium rate at $P = 1300$. Immediately after issuing the policies it is announced that a new drug has been developed which may dramatically reduce mortality of the policyholders. There is a 60% chance that the drug will not work, and the force of mortality will be unchanged, and a 40% chance that the force of mortality will be reduced by 0.01 for all lives in the group, at all ages.

(b)  (*4 points*)

    (i)    Show that the standard deviation of $L_j$ assuming that the drug does not work is 5900 to the nearest 50. You should calculate the value to the nearest 1.

    (ii)    Show that the standard deviation of $L_j$ assuming that the drug does work is 5950 to the nearest 50. You should calculate the value to the nearest 1.

    (iii)    The insurer sells 400 policies. Calculate the standard deviation of the total loss at issue, $L$.

(c)  (*1 point*) Show that the risk within the portfolio, as measured by the total loss $L$, is no longer diversifiable.

(d)  (*1 point*) Explain why the introduction of the new drug has created non-diversifiable risk.

**6.** (*9 points*) A defined benefit pension plan with two members, Finn and Oscar, provides for a pension benefit paid as a monthly whole life annuity-due. The annual pension benefit is 1.7% of the final one-year's salary for each year of service.

You are given:

     (i)      Mortality follows the Illustrative Life Table, assuming deaths are uniformly distributed between integer ages.

     (ii)     Participants reaching age 64.5 retire at that time with probability 50%. All participants reaching age 65 in service retire immediately. There are no other retirements.

     (iii)    There are no withdrawals from the plan other than by death or retirement.

     (iv)    $i = 0.06$

     (v)     $\ddot{a}_{64.5}^{(12)} = 9.5613$

     (vi)    Salaries increase every year on January 1. Future salary increases are assumed to be 2% per year.

     (vii)   On January 1, 2018, Finn is 25 years old. He is a new employee with no past service. His salary in 2018 is 60,000.

     (viii)  On January 1, 2018, Oscar is 64 years old and has 29 years of service. His salary in 2017 was 95,000 and in 2018 is 100,000.

(a)    (*2 points*) Calculate the projected replacement ratios for both Finn and Oscar assuming that they each retire at exact age 65.

(b)    (*2 points*) Calculate the total accrued liability for the plan on January 1, 2018, under the Traditional Unit Credit (TUC) method.

# 6.    Continued

(c)    (*3 points*)

   (i)    Calculate the Normal Cost under the TUC method for Finn.

   (ii)    Calculate the Normal Cost under the TUC method for Oscar.

(d)    (*2 points*)

   (i)    Without further calculation, state with reasons whether the Normal Cost under the Projected Unit Credit (PUC) method will be greater or less than the TUC for Finn.

   (ii)    Without further calculation, state with reasons whether the Normal Cost under the PUC will be greater or less than the TUC for Oscar.

**\*\*END OF EXAMINATION\*\***

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