LTAM MC Solutions, Fall 2019

1. Answer A

   ${}_{15}p_x = {}_5p_x \cdot {}_{10}p_{x+5}$

   ${}_{15}p_x = 0.764591 \qquad {}_5p_x = 0.955290 \quad \Rightarrow \quad {}_{10}p_{x+5} = 0.800$

2. Answer D

   $0.4q_{40}^{(1)} = 0.032 \Rightarrow q_{40}^{(1)} = \dfrac{0.032}{0.4} = 0.08$ and $0.6q_{40}^{(2)} = 0.045 \Rightarrow q_{40}^{(2)} = \dfrac{0.045}{0.6} = 0.075$

   $\Rightarrow p_{40}^{(\tau)} = 1 - q_{40}^{(1)} - q_{40}^{(2)} = 0.845$

3. Answer B

   The central exposed-to-risk is $E_{75}^c = 0.8 + 1 + 0.4 = 2.2$

   $d_x = 2 \Rightarrow \hat{p}_x = e^{-2/2.2} = 0.403$

   $\hat{q}_x = 1 - e^{-2/2.2} = 0.597$

4. Answer C

   $\log(m(65,2018)) = \alpha_{65} + \beta_{65} K_{2018}$

   $K_{2018} \sim N(\mu, \sigma^2)$ where $\mu = -9 - 2(0.35) = -9.7$ and $\sigma = 0.75(\sqrt{2}) = 1.06066$

   $\Rightarrow 80\%ile$ of $K_{2018} = \mu + 0.842\sigma = -8.8069$

   $\Rightarrow 80\%ile$ of $\log(m(65,2018)) = -2.6 + 0.04(-8.8069) = -2.95$

5. Answer C

   ${}_3p_x^{00} = e^{-3(0.01)} = 0.970446$

   ${}_1p_{x+3}^{00} = e^{-(0.01+0.12+0.08)} = 0.810584$

   $\Rightarrow {}_4p_x^{00} = 0.970446 \times 0.810584 = 0.78663$

6. Answer A

   $$\frac{d}{dt}{}_tV^{(0)} = \delta\, {}_tV^{(0)} - 10{,}000 - \mu_{65+t}\left({}_tV^{(1)} - {}_tV^{(0)}\right) - \mu_{55+t}\left({}_tV^{(2)} - {}_tV^{(0)}\right)$$

   At $t = 10$:

   ${}_{10}V^{(1)} = 8{,}000 \times \bar{a}_{65} = 103{,}133; \quad {}_{10}V^{(2)} = 8{,}000 \times \bar{a}_{75} = 77{,}810$

   $$\begin{aligned}\Rightarrow \frac{d}{dt}{}_tV^{(0)} &= 0.05(109{,}650) - 10{,}000 - 0.017552(103{,}133 - 109{,}650) \\ &\quad - 0.005605(77{,}810 - 109{,}650) \\ &= -4224.65\end{aligned}$$

7. Answer B

EPV Death Benefit: $100,000\times A^{\ 1}_{65:\overline{20|}} = 100,000(A_{65:\overline{20|}} - {}_{20}E_{65})$

$= 100,000(0.43371 - 0.24381) = 18,990$

EPV Annuity Benefit: $45,000\times {}_{20}E_{65}\times \ddot{a}_{85} = 45,000(0.24381)(6.7993) = 74,598$

EPV Premiums: $P\ddot{a}_{65:\overline{20|}} = 11.8920P$

$\Rightarrow P = 7,870$

8. Answer D

$\ddot{a}_{80}^{(4)} = \ddot{a}_{80} - \frac{3}{8} - \frac{15}{12\times16}\left(\mu_{80} + \delta\right) = 8.5484 - \frac{3}{8} - \frac{15}{12\times16}\left(0.030162 + \ln(1.05)\right) = 8.1672$

$\ddot{a}_{90}^{(4)} = \ddot{a}_{90} - \frac{3}{8} - \frac{15}{12\times16}\left(\mu_{90} + \delta\right) = 5.1835 - \frac{3}{8} - \frac{15}{12\times16}\left(0.096590 + \ln(1.05)\right) = 4.7971$

$\ddot{a}_{80:\overline{10|}}^{(4)} = 8.1672 - {}_{10}E_{80}\ 4.7971 = 6.5385$

$\Rightarrow 20,000\,\ddot{a}_{80:\overline{10|}}^{(4)} = 130,770$

9. Answer E

$1,250,000 = 250,000 + X\ddot{a}_{55} + 100,000A_{55} \Rightarrow X = 60,802$

10. Answer D

$EPV = 1000\left(p_{60}^{01}v + {}_2p_{60}^{01}v^2 + {}_3p_{60}^{01}v^3 + {}_4p_{60}^{01}v^4 + {}_5p_{60}^{01}v^5\right)$

$= 1000\left(0.01v + 0.03v^2 + 0.04v^3 + 0.05v^4 + 0.07v^5\right)$

$= 173$

11. Answer D

$12P\left(0.95\times \ddot{a}_{50:\overline{10|}}^{(12)} - 0.6\right) = 1,000,000A^{(12)\,1}_{50:\overline{10|}}$

$\ddot{a}_{50:\overline{10|}}^{(12)} = \alpha(12)\ddot{a}_{50:\overline{10|}} - \beta(12)\left(1 - {}_{10}E_{50}\right) = 1.0002\times 8.0550 - 0.46651(1 - 0.60182) = 7.87086$

$A^{(12)\,1}_{50:\overline{10|}} = \dfrac{i}{i^{(12)}}\left(A_{50:\overline{10|}} - {}_{10}E_{50}\right) = 1.02271\left(0.61643 - 0.60182\right) = 0.014942$

$\Rightarrow P = 181.05$

12. Answer A

$P\bar{a}_{50}^{00} = 30,000\bar{A}_{50}^{01} + 50,000\bar{A}_{50}^{02} + 20,000\bar{A}_{50}^{03}$

$\Rightarrow P = 1193.7$

13. Answer A

$$P^{n} = 1000A_{60} / \ddot{a}_{60} = 33.115; \quad P^{FPT} = 1000A_{61} / \ddot{a}_{61} = 35.110$$

$$\begin{aligned} {}_{10}V^{n} - {}_{10}V^{FPT} &= \left(1000A_{70} - P^{n}\ddot{a}_{70}\right) - \left(1000A_{70} - P^{FPT}\ddot{a}_{70}\right) \\ &= \left(P^{FPT} - P^{n}\right)\ddot{a}_{70} = 17.09 \end{aligned}$$

Or

$${}_{10}V^{n} = 1000\left(1 - \frac{\ddot{a}_{70}}{\ddot{a}_{60}}\right) \quad \text{and} \quad {}_{10}V^{FPT} = 1000\left(1 - \frac{\ddot{a}_{70}}{\ddot{a}_{61}}\right)$$

$${}_{10}V^{n} - {}_{10}V^{FPT} = 1000\left(1 - \frac{\ddot{a}_{70}}{\ddot{a}_{60}}\right) - 1000\left(1 - \frac{\ddot{a}_{70}}{\ddot{a}_{61}}\right) = 1000\left[\frac{\ddot{a}_{70}}{\ddot{a}_{61}} - \frac{\ddot{a}_{70}}{\ddot{a}_{60}}\right] = 17.13$$

14. Answer B

$$P = \frac{100,000(A_{61} - 0.5A^{\;1}_{61:\overline{20}|})}{\ddot{a}_{61}} = \frac{100,000\left(0.30243 - 0.5\left(0.41417 - 0.28641\right)\right)}{14.6491} = 1628.43$$

or

$$P = \frac{50,000(A_{61} - {}_{20}E_{61} \cdot A_{81})}{\ddot{a}_{61}} = \frac{50,000\left(0.30243 - 0.5\left(0.41417 - (0.28641)(0.60984)\right)\right)}{14.6491} = 1628.41$$

$${}_{20}V^{n} = 100,000A_{81} - P\ddot{a}_{81} = 47,641$$

15. Answer E

$${}_{1}V = 100,000\bar{A}^{01}_{61:\overline{4}|} + 200,000\bar{A}^{02}_{61:\overline{4}|} - 7000\ddot{a}_{61:\overline{4}|}$$

$$\bar{A}^{01}_{61:\overline{4}|} = \int_0^4 v^t\,{}_tp^{00}_{61}\mu^{01}_{61+t}\,dt = \int_0^4 e^{-0.02t}e^{-0.06t}\,0.05\,dt = \frac{0.05}{0.08}\left(1 - e^{-4(0.08)}\right) = 0.17116$$

$$\bar{A}^{02}_{61:\overline{4}|} = \int_0^4 v^t\,{}_tp^{00}_{61}\mu^{02}_{61+t}\,dt = \int_0^4 e^{-0.02t}e^{-0.06t}\,0.01\,dt = \frac{0.01}{0.08}\left(1 - e^{-4(0.08)}\right) = 0.03423$$

$$\ddot{a}^{00}_{61:\overline{4}|} = 1 + v_1p^{00}_{61} + v^2{}_2p^{00}_{61} + v^3{}_3p^{00}_{61} = 1 + e^{-0.08} + e^{-0.16} + e^{-0.24} = 3.5619$$

$$\Rightarrow {}_{1}V = -971$$

16. Answer D

$$\Pr_2 = \left({}_1V + P - E\right)(1+i) - q_{61} \times 100{,}000 - p_{61} \times {}_2V$$

$$= (325 + 700 - (0.04)(700))(1.06) - 100{,}000(0.003792) - (1 - 0.003792)(600)$$

$$= 79.9$$

17. Answer C

States at *t* =1,2	**Probability**	**Final Average Salary**
0, 0	$0.7^2 = 0.49$	(100,000+100,000)/2 = 100,000
0, 1	$0.7 \times 0.3 = 0.21$	(100,000+105,000)/2=102,500
1, 0	$0.3 \times 0.6 = 0.18$	(105,000+105,000)/2=105,000
1, 1	$0.3 \times 0.4 = 0.12$	(105,000+110,250)/2=107,625

So the expected final salary is

$$0.49 \times 100{,}000 + 0.21 \times 102{,}500 + 0.18 \times 105{,}000 + 0.12 \times 110{,}250 = 102{,}340$$

18. Answer C

$$AL_0 = 20\left((55{,}000)(0.013) + (33{,}000)(0.02)\right) {}_{15}p_{50}\, v^{15}(13.086) = 166{,}083$$

$$EPV_0 \text{ of } AL_1 = 21\left((55{,}000)(0.013) + (35{,}200)(0.02)\right) {}_{15}p_{50}\, v^{15}(13.086) = 179{,}968$$

$$NC = 179{,}968 - 166{,}083 = 13{,}885$$

19. Answer E

$$(1-k)B\ddot{a}_{64}^{(12)} = Bvp_{64}\ddot{a}_{65}^{(12)}$$

$$\ddot{a}_{64}^{(12)} = \alpha(12)\ddot{a}_{64} - \beta(12) = (1.0002)(13.8363) - 0.46651 = 13.3726$$

$$\Rightarrow 1-k = \frac{0.994712v(13.086)}{13.3726} = 0.927$$

$$\Rightarrow k = 0.073$$

20. Answer E

$$a_B(63,0) = 1 + (1+j)cvp_{63} + (1+j)^2 c^2 v^2 {}_2p_{63}(a_B(65,2))$$

$$= 1 + (1.04)(1.03)(1.06)^{-1}\left(\frac{95{,}082.5}{95{,}534.4}\right) + (1.04)^2(1.03)^2(1.06)^{-2}\left(\frac{94{,}579.7}{95{,}534.4}\right)(26.708)$$

$$= 29.0$$