

****BEGINNING OF EXAMINATION****

- 1.** Which of the following is **not** a traditional life insurance policy?
- (A) A whole life insurance policy.
 - (B) A universal life insurance policy.
 - (C) An endowment insurance policy.
 - (D) A term insurance policy.
 - (E) A joint life term insurance policy.

2. You are given:

$$S_0(x) = e^{-0.001(e^{0.1x}-1)}$$

Calculate μ_{80} .

- (A) 0.20
- (B) 0.22
- (C) 0.24
- (D) 0.28
- (E) 0.30

3. The estimated survival function at time 10 is $\hat{S}(10) = 0.985$.

You are given that $\text{Var}(\hat{S}(10)) \approx 0.01^2$.

Let S_u denote the upper bound of the 95% log-confidence interval for $S(10)$.

Calculate S_u .

(A) 0.994

(B) 0.995

(C) 0.996

(D) 0.997

(E) 0.998

4. A mortality study has nine participants all of whom enter at time 0. There are two deaths: one at time 0.5 and the other at time 0.8. There are no other deaths before the study ends at time 2.0.

Calculate the Nelson-Aalen estimate for the survival function at time 1.1.

- (A) 77.8%
- (B) 78.1%
- (C) 78.4%
- (D) 78.7%
- (E) 79.0%

5. For a double-decrement model you are given:

(i) $q_x^{(1)} = 0.05$

(ii) $q_x^{(\tau)} = 0.15$

(iii) Forces of decrement are constant between integer ages.

Calculate $q_x'^{(2)}$, the independent rate of exit for decrement 2.

(A) 0.101

(B) 0.103

(C) 0.105

(D) 0.107

(E) 0.109

6. For a whole life insurance of 100,000 on (80) you are given:

(i) Benefits are payable at the end of the year of death.

(ii) $i = 0.08$

(iii) The following life table:

x	80	81	82	83	84	85	86	87	88
l_x	100	96	88	78	65	50	34	17	0

Calculate the probability that the present value of the benefit is less than or equal to 75,000.

(A) 0.34

(B) 0.50

(C) 0.65

(D) 0.78

(E) 0.88

7. You are given the following annual transition probabilities for seniors in a continuing care retirement community.

	Healthy	Long Term Care	Dead
Healthy	0.66	0.22	0.12
Long Term Care	0.24	0.42	0.36
Dead	0.00	0.00	1.00

You are also given:

- (i) Costs are the following:
 - Healthy: 20,000 (paid at the beginning of the year)
 - Long Term Care: 100,000 (paid at the beginning of the year)
 - Death: 5,000 (paid at the end of the year of death)
- (ii) Transitions are assumed to occur $\frac{1}{2}$ -way through the year.
- (iii) $i = 0.05$

Calculate the expected present value of the costs over the next 2 years for a senior who is currently Healthy.

- (A) 54,814
- (B) 55,825
- (C) 56,926
- (D) 58,027
- (E) 59,128

8. For two lives, (40) and (50), with independent future lifetimes, you are given:

(i) Mortality follows the Standard Ultimate Life Table.

(ii) $i = 0.05$

Calculate $a_{40:50:\overline{22}|}$

(A) 12.50

(B) 12.63

(C) 12.71

(D) 12.84

(E) 13.09

9. For a fully continuous whole life insurance of 1 on (x) you are given:

- (i) L is the present value of the loss at issue random variable using the premium based on the equivalence principle.
- (ii) The premium based on the equivalence principle is 0.02.
- (iii) L' is the present value of the loss at issue random variable using the premium π .
- (iv) $\delta = 0.06$
- (v) $\text{Var}[L] = 0.16$
- (vi) $\text{Var}[L'] = 0.49$

Calculate π .

- (A) 0.06
- (B) 0.08
- (C) 0.10
- (D) 0.12
- (E) 0.14

- 10.** An insurer issues a whole-life insurance with sum insured 200,000 to (55). Level annual premiums are payable throughout the term of the policy.

You are given the following assumptions:

- (i) The death benefit is paid 4 months after death.
- (ii) Initial expenses are 1935 plus 30% of the gross premium.
- (iii) Renewal expenses are 5% of each gross premium starting in the second year.
- (iv) Mortality follows the Standard Ultimate Life Table.
- (v) Deaths are uniformly distributed between integer ages.
- (vi) $i = 0.05$
- (vii) The gross premium is calculated using the equivalence principle.

Calculate the gross annual premium.

- (A) 3,250
- (B) 3,260
- (C) 3,270
- (D) 3,280
- (E) 3,290

11. You are given the following extract from a select life table, with a 4-year select period.

x	$l_{[x]}$	$l_{[x]+1}$	$l_{[x]+2}$	$l_{[x]+3}$	l_{x+4}	$x+4$
40	101,316	101,114	100,841	100,476	100,000	44
41	100,841	100,625	100,325	99,920	99,389	45
42	100,315	100,079	99,751	99,300	98,708	46

A select life age 41 purchased a fully discrete 3-year term insurance with a benefit of S .

You are given:

- (i) Mortality follows the select table above.
- (ii) $i = 0.05$
- (iii) Expenses are 8% of each premium plus an initial cost of 35.
- (iv) Claim expenses of 1080 are incurred at the time of payment of death benefit.
- (v) The gross annual premium is 170.
- (vi) Premiums are calculated using the equivalence principle.

Calculate S .

- (A) 48,300
- (B) 49,000
- (C) 49,700
- (D) 50,400
- (E) 51,100

- 12.** An insurer offers a death-in-service benefit for employees who die in service before age 60. The sum insured is 100,000. Level premiums are paid annually while in employment.

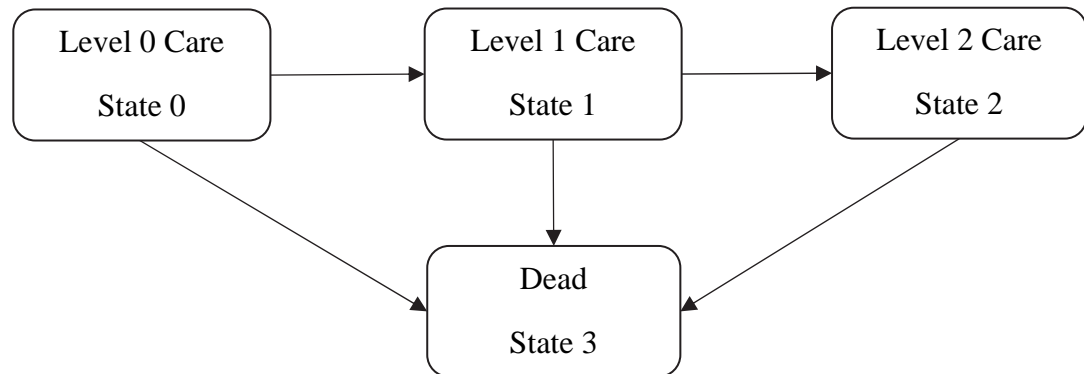
You are given:

- (i) Decrements before age 60 follow the Standard Service Table. All lives are assumed to retire at age 60.
- (ii) Deaths are assumed to occur $\frac{1}{2}$ -way through the year of death.
- (iii) The benefit is paid immediately on death.
- (iv) $i = 0.05$

Calculate the annual net premium for a new employee age 57.

- (A) 264
- (B) 272
- (C) 286
- (D) 290
- (E) 294

13. A CCRC uses the following multiple state model:



You are given:

- (i) The monthly costs, per resident, incurred to cover maintenance of the facility, medical care and services, and other related expenses, depend on the level of care as follows:
 - Level 0 Care: 3,500
 - Level 1 Care: 8,000
 - Level 2 Care: 20,000
 Costs are assumed to be incurred at the start of each month.
- (ii) Entrants into level 0 pay a fee of 50,000 on entry, plus a level monthly fee, which does not change on movements between care levels. The fee is payable at the start of each month.
- (iii) The entry fee is refunded immediately on death.
- (iv) Fees are calculated using the equivalence principle.
- (v) $i = 0.05$
- (vi) $\ddot{a}_{65}^{(12)00} = 12.495$; $\ddot{a}_{65}^{(12)01} = 0.789$; $\ddot{a}_{65}^{(12)02} = 0.252$; $\bar{A}_{65}^{03} = 0.127$

Calculate the monthly fee for a new resident, age 65, entering Level 0 Care.

- (A) 3,600
- (B) 3,700
- (C) 3,800
- (D) 3,900
- (E) 4,000

14. For a fully discrete 20-payment whole life of 1000 on (35), you are given:

- (i) Mortality follows the Standard Ultimate Life Table.
- (ii) $i = 0.05$

Calculate the Full Preliminary Term reserve at time 10.

- (A) 75
- (B) 79
- (C) 82
- (D) 87
- (E) 92

15. For a fully continuous whole life insurance, with sum insured 100,000, issued to (50), you are given:

- (i) Mortality follows Gompertz' law, with $B = 0.0003$ and $c = 1.075$.
- (ii) Expenses of 5% of the premium plus 100 per year are payable continuously.
- (iii) The gross premium rate is 3,400.
- (iv) The gross premium reserve at time t is denoted by ${}_tV$.
- (v) ${}_{20}V = 52,225$
- (vi) $\delta = 0.05$

Calculate $\frac{d}{dt}{}_tV$ at $t = 20$.

- (A) 3,255
- (B) 3,366
- (C) 3,477
- (D) 3,588
- (E) 3,699

16. A whole life sickness-death benefit policy was purchased by (60).

You are given:

- (i) The policy pays 1,000 per year, continuously, while the policyholder is Sick.
- (ii) The policy pays 10,000 at the moment of the policyholder's death.
- (iii) The policyholder pays a premium of 850 continuously while the policyholder is Healthy.
- (iv) Transitions follow the Standard Sickness-Death Model.
- (v) $i = 0.05$

Calculate the gross premium reserve at time 5, given that the policyholder is Sick at that time.

- (A) 14,280
- (B) 14,340
- (C) 14,400
- (D) 14,460
- (E) 14,520

- 17.** A couple, who are both aged 50, purchase a deferred annuity, with a 10-year deferred period.

Level premiums are payable annually during the deferred period, conditional on both lives surviving.

The policy benefits are as follows:

- If either life dies during the deferred period, a death benefit of 100,000 is paid at the end of the year of the first death, and the policy expires.
- If both lives survive the deferred period, an annuity of 10,000 per year is paid annually in advance until the second death. There are no further benefits.

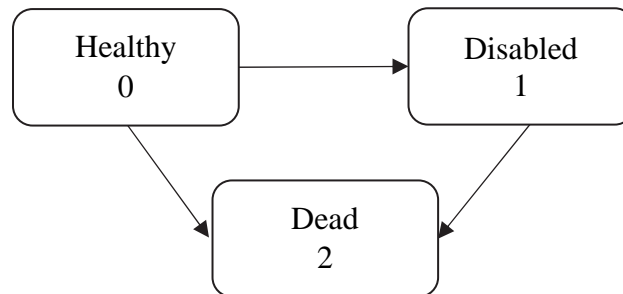
You are given the following assumptions:

- (i) The couple have independent future lifetimes.
- (ii) The mortality of each life follows the Standard Ultimate Life Table.
- (iii) $i = 0.05$
- (iv) The net premium is 12,570 per year.

Calculate the net premium reserve at time 9, assuming both lives survive.

- (A) 144,570
- (B) 144,660
- (C) 144,750
- (D) 144,840
- (E) 144,930

- 18.** You are conducting a profit test on a permanent disability insurance policy issued to (x) based on the following model:



You are given:

- (i) $p_{x+4}^{01} = 0.02$ $p_{x+4}^{02} = 0.01$ $p_{x+4}^{12} = 0.04$
- (ii) The policy pays 500 at the end of the year of becoming Disabled, provided that the policyholder is still alive at the year end.
- (iii) The policy pays 2000 at the end of the year of death.
- (iv) Annual premiums of 100 are payable at the beginning of the year, whenever Healthy.
- (v) Expenses are 5 per year payable at the beginning of each year.
- (vi) Interest of 5% is earned on the insurer's funds each year
- (vii) The reserves at end of years 4 and 5 for each state are:

Year	Healthy	Disabled
4	218	345
5	228	370

Calculate $\text{Pr}_5^{(0)}$

- (A) 64
- (B) 66
- (C) 68
- (D) 70
- (E) 72

19. For a defined benefit pension plan, you are given:

- (i) Sally, who is age 55 at the valuation date, has 15 years of service.
- (ii) Sally's salary in the year following valuation is 90,000.
- (iii) The retirement benefit is a monthly annuity due payable starting at age 65 based on the final one-year salary.
- (iv) The accrual rate is 1.5% per year of service.
- (v) Salaries increase at 2.5% per year.
- (vi) There are no exits from the plan other than death and retirement.
- (vii) Mortality follows the Standard Ultimate Life Table.
- (viii) $i = 0.05$
- (ix) $\ddot{a}_{65}^{(12)} = 13.0870$
- (x) The pension plan is funded using the Projected Unit Credit method.

Calculate the normal contribution for Sally's retirement benefit.

- (A) 13,093
- (B) 13,214
- (C) 13,335
- (D) 13,552
- (E) 13,769

20. You are given the following information about a post-retirement health benefit plan:

- (i) $B(x, t)$ denotes the annual supplementary health insurance premium for a life age x at time t .
- (ii) For $k > 0$, $B(x + k, t) = (1.04)^k B(x, t)$.
- (iii) Premium inflation at all ages is 6% per year.
- (iv) Mortality follows the Standard Ultimate Life Table.
- (v) $i = 0.07$
- (vi) $\ddot{a}_{67,0}^B = 31.75$

Calculate $\ddot{a}_{69,2}^B$.

- (A) 27.75
- (B) 28.45
- (C) 29.75
- (D) 30.45
- (E) 31.75

1.

(10 points) A whole life insurance policy on (50) has an initial sum insured of 200,000 payable at the end of the year of death. At the beginning of each policy year after the first, the sum insured will increase by 5,000. The insured will pay level annual premiums payable for at most 20 years.

You are given:

- (i) Mortality follows the Standard Ultimate Life Table
- (ii) $i = 0.05$

(a) (2 points)

- (i) Show that $(IA)_x = A_x + v p_x (IA)_{x+1}$
- (ii) Use this relationship to show that if $(IA)_{50} = 5.8255$, then $(IA)_{51} = 5.9$ rounded to the nearest 0.1. You should calculate the value to the nearest 0.001.

Annual gross premiums are calculated using the equivalence principle based on expenses of:

- 1000 on the payment of the death benefit.
 - 15% of the first premium.
 - 5% of all other premiums.
- (b) (2 points) Show that the annual gross premium is 5,500 to the nearest 100. You should calculate the value to the nearest 1.
- (c) (2 points) Show that the gross premium reserve at the end of the second policy year is 10,080 to the nearest 10. You should calculate the value to the nearest 1.

1. Continued

(d) (4 points) You are given:

- The gross premium reserves at times 10 and 11 are $_{10}V^g = 63,208$ and $_{11}V^g = 71,217$.
- At the start of the 11th policy year, at time $t=10$, there were 1000 identical, independent policies in force.
- During the 11th year, the experience was as follows:
 - The interest rate earned was 6%.
 - There were six deaths.
 - Expenses of 6% of premiums, plus 1100 for each death benefit, were incurred.

- (i) Show that the surplus earned on the portfolio in the 11th policy year, per policy in force at the start of the year, was 160 to the nearest 10. You should calculate the value to the nearest 1.
- (ii) Calculate the gain by source, in the order: (1) Interest; (2) Mortality; (3) Expenses.

2.

(10 points) XYZ Insurance Company sells three-year term policies with a death benefit of 1,000,000 payable at the end of the year of death, to lives age x . Premiums are payable annually.

The insurer holds net premium reserves for the policy, based on an interest rate $i = 0.05$ and mortality of $q_{x+t} = 0.10 + 0.10t$ for $t = 0, 1, 2$.

The net premium for this policy is 177,313.20. The net premium reserve at the end of the first year, ${}_1V^n$, is 95,754.

The assumptions used for profit testing this policy are:

- (i) The pre-contract underwriting expense is 4,000 per policy.
 - (ii) Commissions are 20% of premium in year 1 and 5% of premium each year thereafter. The extra 15% of premium in commissions in the first year is treated as a pre-contract cash flow for profit testing purposes.
 - (iii) Maintenance expense is 100 per policy at the beginning of every year including the first year.
 - (iv) The earned interest rate is 7%.
 - (v) Mortality is equal to the reserve mortality.
 - (vi) At the end of each of the first two years, 10% of the surviving policyholders withdraw.
 - (vii) There are no cash values.
-
- (a) (1 point) List four ways that profit tests are applied in practice.
 - (b) (1 point) Show that the net premium reserve at the end of the second year is 108,400 to the nearest 10. You should calculate the value to the nearest 1.

2. Continued

XYZ uses a gross premium of 210,000 in its first profit test for this three-year term policy. Using this premium, $Pr_2 = 37,766$ and $Pr_3 = 29,347$.

(c) (2 points)

(i) Determine Pr_0 .

(ii) Determine Pr_1 .

(d) (3 points)

(i) Determine the profit signature for this policy.

(ii) Calculate the NPV for this policy using a risk discount rate of 12%.

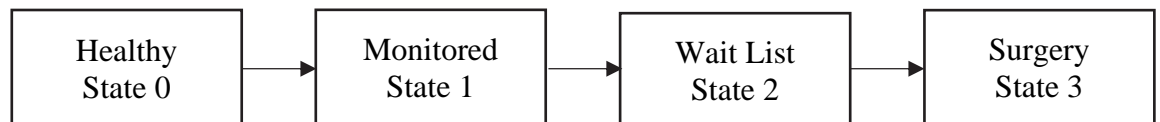
(iii) Calculate the profit margin for this policy.

(e) (3 points) XYZ wants to achieve a profit margin of 10% on this policy. Calculate the gross premium that XYZ should charge to meet that objective.

3.

(9 points) You are analyzing waiting times and expenses associated with optometric procedures. The analysis is based on the multiple state model illustrated below.

An individual who is diagnosed with a specific eye condition moves into State 1, where they are Monitored. When the condition progresses, they move onto a Wait List for surgery (State 2). Surgery (State 3) is performed once their name reaches the top of the wait list. The time spent on the wait list is random, depending on the progression of the disease and the availability of surgical resources.



You are given the following information:

- (i) $\mu_{x+t}^{01} = a + be^{c(x+t)}$ where $a = 0.004$, $b = 0.015$, and $c = 0.005$;
 - (ii) $\mu_{x+t}^{12} = 0.2$; $\mu_{x+t}^{23} = 0.4$.
 - (iii) $\delta = 0.05$
-
- (a) (2 points) Calculate the probability that a Healthy life age 60 will be diagnosed with the eye condition before age 65.
 - (b) (1 point) Show that ${}_t p_x^{12} = (e^{-0.2t} - e^{-0.4t})$.
 - (c) (1 point) Calculate the probability that a life currently being Monitored (State 1) will have Surgery (State 3) within 5 years.

3. Continued

(d) (2 points)

- (i) Show that $\bar{a}_{x:\overline{5}|}^{11} = 2.85$ to the nearest 0.01. You should calculate the value to the nearest 0.001.
- (ii) Show that $\bar{a}_{x:\overline{5}|}^{12} = 0.87$ to the nearest 0.01. You should calculate the value to the nearest 0.001.
- (iii) Show algebraically that $\bar{a}_{x:\overline{5}|}^{11} + \bar{a}_{x:\overline{5}|}^{12} + \bar{a}_{x:\overline{5}|}^{13} = \bar{a}_{\overline{5}|}$

(e) (3 points) The health authority is considering two alternatives for improving care for those diagnosed with the eye condition. One criterion for deciding which course of action to choose is based on a quality of life (QL) index, as follows:

- In State 1, $QL = 0.5$ per year.
- In State 2, $QL = 0.2$ per year
- In State 3, $QL = 1.0$ per year.

The authority will choose one of the following options:

Option A: Open a new surgical center. This would increase μ_x^{23} to 1.0 without changing any other transition intensities.

Option B: Improve care for those in States 1 and 2. This would improve the QL index to 0.55 in State 1, and to 0.3 in State 2.

Calculate whether the health authority should select Option A or Option B, based on the increase in the expected present value of the 5-year QL of lives currently in State 1, using a force of interest of $\delta = 0.05$.

4.

(10 points) Single premium 10-year deferred whole-life annuity contracts are issued to 500 independent lives aged 50. In case of death during the deferred period, the single premium is refunded at the end of the year of death.

The 500 lives have been classified into three groups, based on their mortality rates, as follows:

Group A:

- 250 lives have mortality that follows the Standard Ultimate Life Table (SULT).
- The force of mortality for this group is denoted by μ_x^A
- The single premium for Group A contracts is 100,000.

Group B:

- 150 lives have force of mortality $\mu_x^B = 0.8 \mu_x^A$.
- The single premium for Group B contracts is 120,000.

Group C:

- 100 lives have force of mortality $\mu_x^C = \mu_x^A + 0.01$.
- The single premium for Group C contracts is 90,000.

(a) (3 points)

- (i) Calculate the 10-year survival probability for each group.
- (ii) Given that a contract has had its premium refunded, show that the probability that it is a Group B contract is 0.13 to the nearest 0.01. You should calculate the value to the nearest 0.001.

(b) (3 points) Let N be the total number of refunded premiums from all groups. Calculate the standard deviation of N .

4. Continued

- (c) (3 points) A single-factor age-based mortality improvement scale is used to model future mortality rates of the Standard lives.

You are given:

- The probability that a Group A life aged x at time t dies within a year is $q^A(x, t) = q^A(x, 0) (0.98)^t$ $t = 1, 2, 3, \dots$
- The base mortality for Group A, $q^A(x, 0)$, follows the SULT.
- The force of mortality for each of the other two groups satisfies the same relationship as above, that is:

$$\mu^B(x, t) = 0.8\mu^A(x, t) \text{ and } \mu^C(x, t) = \mu^A(x, t) + 0.01 \text{ for all } t \geq 0;$$

where $\mu(x, t)$ is the force of mortality under the mortality improvement model.

Under the mortality improvement model, the probability that a Group A life aged 50 at $t=0$ survives 8 years is ${}_8p^A(50, 0) = 0.987068$.

- (i) Calculate ${}_{10}p^A(50, 0)$.
- (ii) Given that a contract has had its premium refunded, calculate the probability that it is a Group B contract.
- (d) (1 point) Write down two reasons why single-factor mortality improvement scales, depending only on age, have proven to be too simplistic.

5.

(8 points) NED Life issued a fully discrete whole life insurance of 100,000 to (50). You are given the following assumptions:

- (i) Commissions are 80% of premium in year 1, 12% of premium in years 2-10, and 5% of premium thereafter.
 - (ii) Other expenses, payable at the start of the year, are 1,000 in year 1 and 100 per year thereafter.
 - (iii) Mortality follows the Standard Ultimate Life Table.
 - (iv) $i = 0.05$
 - (v) ${}_0L^g$ is the gross loss at issue random variable.
 - (vi) The gross premium G is calculated such that $E[{}_0L^g] = -0.5G$
 - (vii) ${}_kV^g$ denotes the gross premium reserve at time k .
- (a) (2 points) Show that G is 1,490 to the nearest 10. You should calculate G to the nearest 0.1.
- (b) (2 points) Show that ${}_{10}V^g$ is 9,390 to the nearest 10. You should calculate ${}_{10}V^g$ to the nearest 1.

NED Life holds modified net premium reserves. The modified net premium reserve at time k is denoted by ${}_kV^{\text{mod}}$. You are given:

- (i) The modified net premium is π_1 for each of the first 10 years, and π_2 for each subsequent year.
- (ii) ${}_{10}V^{\text{mod}} = {}_{10}V^g$

5. Continued

(c) (3 points)

(i) Show that π_2 is 1,320 to the nearest 10. You should calculate the value to the nearest 1.

(ii) Calculate π_1 .

(d) (1 point) Explain why ${}_3V^{\text{mod}}$ is greater than ${}_3V^g$.

6.

(9 points) Max enrolled in a defined contribution (DC) pension plan at age 35. Max and their employer each contribute 6% of Max's salary to the pension fund.

Max plans for his retirement based on the following assumptions:

- Max's initial salary rate is 50,000 per year.
- Salaries increase continuously at an annual rate of 2%, compounded continuously.
- Contributions are invested continuously to a fund earning a 6% per year force of interest.
- Max intends to retire on his 60th birthday.

(a) (2 points) Show that Max's projected DC account balance as of his 60th birthday is 425,000 to the nearest 1000. You should calculate the value to the nearest 1.

(b) (2 points) Assume first that, on retirement, Max elects to receive a monthly life annuity with a 10-year guarantee. Annuities are priced using the equivalence principle, based on the following assumptions:

- Mortality follows the Standard Ultimate Life Table.
- The annual effective interest rate is $i = 0.05$.
- Woolhouse's two term formula.

(i) Calculate Max's projected monthly income in retirement.

(ii) Estimate Max's projected Replacement Ratio.

Max's spouse, Charlie, is also 60. Max and Charlie have independent future lifetimes. When they reach age 60, Max's DC account has accumulated to 750,000, after several years of high investment returns.

(c) (2 points) Assume that Max elects to receive a monthly pension payable while both Max and Charlie survive, with a 2/3 survivor's benefit payable after the first death, with no guarantee period. Show that the monthly benefit is 4,200 rounded to the nearest 100. You should show your value to the nearest 1.

6. Continued

- (d) *(1 point)* Max is also eligible for a Government Pension Plan (GPP) which will provide a benefit of 1000 per month starting at age 65, payable during Max's lifetime and with a 50% benefit payable to Charlie after Max's death.

Calculate the monthly benefit payable from Max's entitlements from both the DC pension plan and GPP eight year after Max's retirement under each of the following states:

- (i) Max and Charlie survive.
 - (ii) Only Max survives.
 - (iii) Only Charlie survives.
- (e) *(2 points)* Max elects to receive an integrated pension which will start at X per month at age 60 and will reduce to $X-1000$ per month at age 65, so as to provide a level benefit before and after the start of Max's GPP benefit, assuming Max survives to age 65.

Calculate X , assuming the $2/3$ survivor pension option.

****END OF EXAMINATION****