Society of Actuaries

Canadian Institute of Actuaries

Exam MLC
Models for Life Contingencies

Tuesday,
April 25, 2017
8:30 a.m. – 12:45 p.m.

MLC

INSTRUCTIONS TO CANDIDATES

General Instructions

1. Write your candidate number here _____. Your name must not appear.

2. Do not break the seal of this book until the supervisor tells you to do so.

3. Tables and numerical values necessary for solving some of the questions on this examination will be distributed by the Supervisor.

4. This examination has a total of 96 points. It consists of:

   Section A: 20 multiple-choice questions, each worth 2 points for a total of 40 points, and

   Section B: 6 written-answer questions, worth a total of 56 points. The point value for each written-answer question is indicated at the beginning of the question.

   You may divide your time between the two sections of the examination (written-answer, and multiple-choice) as you choose. You should keep in mind the relative weight of the two sections.

   Your written-answer paper will be graded only if your multiple-choice score is at or above a threshold set after the examination is administered.

5. Failure to stop writing after time is called will result in the disqualification of your answers or further disciplinary action.

6. While every attempt is made to avoid defective questions, sometimes they do occur. If you believe a question is defective, the supervisor or proctor cannot give you any guidance beyond the instructions on the exam booklet.

Multiple-Choice Instructions

1. A separate answer sheet for the multiple-choice questions is inside the front cover of this book. During the time allotted for this examination, record all your answers on the back of the answer sheet. NO ADDITIONAL TIME WILL BE ALLOWED FOR THIS PURPOSE.

No credit will be given for anything indicated in the examination book but not transferred to the answer sheet. Failure to stop writing or coding your answer sheet after time is called will result in the disqualification of your answer sheet or further disciplinary action.

2. On the front of the answer sheet, space is provided to write and code candidate information. Complete the information requested by printing in the squares and blackening the circles (one in each column) corresponding to the letters or numbers printed. For each empty box blacken the small circle immediately above the "A" circle. Fill out the boxes titled:

   (a) Name
       (include last name, first name and middle initial)

   (b) Candidate Number
       (Candidate/Eligibility Number, use leading zeros if needed to make it a five digit number)

   (c) Test Site Code
       (The supervisor will supply the number.)

   (d) Examination Part
       (Code the examination that you are taking by blackening the circle to the left of "Exam MLC")

   (e) Booklet Number
       (The booklet number can be found in the upper right-hand corner of this examination book. Use leading zeros if needed to make it a four digit number.)

In box titled "Complete this section only if instructed to do so," fill in the circle to indicate if you are using a calculator and write in the make and model number.

In the box titled "Signature and Date" sign your name and write today's date. **If the answer sheet is not signed, it will not be graded**.

Leave the boxes titled "Test Code" and "Form Code" blank.

On the back of the answer sheet fill in the Booklet Number in the space provided.

3. Your score will be based on the number of questions which you answer correctly. No credit will be given for omitted answers and no credit will be lost for wrong answers: hence, you should answer all questions even those for which you have to guess.

4. Five answer choices are given with each multiple-choice question, each answer choice being identified by a key letter (A to E). Answer choices for some questions have been rounded. For each question, blacken the circle on the answer sheet which corresponds to the key letter of the answer choice that you select.

5. Use a soft-lead pencil to mark the answer sheet. To facilitate correct mechanical scoring, be sure that, for each question, your pencil mark is dark and completely fills only the intended circle. Make no stray marks on the answer sheet. If you have to erase, do so completely.

6. Do not spend too much time on any one question. If a question seems too difficult, leave it and go on.

7. Clearly indicated answer choices in the test book can be an aid in grading examinations in the unlikely event of a lost answer sheet.

8. Use the blank portions of each page for your scratch work. Extra blank pages are provided at the back of the examination book.

9. After the examination, the supervisor will collect this book and the answer sheet separately. DO NOT ENCLOSE THE ANSWER SHEET IN THE BOOK OR IN THE ESSAY ANSWER ENVELOPE. All books and answer sheets must be returned. THE QUESTIONS ARE CONFIDENTIAL AND MAY NOT BE TAKEN FROM THE EXAMINATION ROOM.

Written-Answer Instructions

1. Write your candidate number at the top of each sheet. Your name must not appear.

2. Write on only one side of a sheet. Start each question on a fresh sheet. On each sheet, write the number of the question you are answering. Do not answer more than one question on a single sheet.

3. The answer should be confined to the question as set.

4. When you are asked to calculate, show all your work including any applicable formulas.

5. When you finish, insert all your written-answer sheets into the Essay Answer Envelope. Be sure to hand in all your answer sheets because they cannot be accepted later. Seal the envelope and write your candidate number in the space provided on the outside of the envelope. Check the appropriate box to indicate Exam MLC.

6. Be sure your essay answer envelope is signed because if it is not, your examination will not be graded.

7. For all parts of all problems, to maximize the credit earned, candidates should show as much work as possible, considering the time allotted for the question. Answers lacking justification will receive no credit. Answers should be organized so that the methods, logic, and formulas used are readily apparent. Candidates should not round their answers excessively; enough precision should be provided so that their answers can be accurately graded.

   In some cases, candidates are asked to show that a calculation results in a particular number. Typically the answer given will be rounded; candidates should provide a greater level of accuracy than the number given in the question. This structure of question is intended to assist the candidate by giving an indication when the calculation has been done incorrectly, providing an opportunity to explore an alternative approach. It also allows a candidate who cannot obtain the correct answer to use the answer given to proceed with subsequent parts of the problem. (Candidates who are able to solve the problem should use their exact answer for subsequent parts.)

   For questions requiring candidates to derive or write down a formula or equation, the resulting expression should be simplified as far as possible, and where numerical values are provided in the problem, they should be used.

Exam MLC

SECTION A – Multiple-Choice

BEGINNING OF EXAMINATION

1. You are given the following extract from a triple decrement table:

x	$\ell_x^{(\tau)}$	$q_x^{(1)}$	$q_x^{(2)}$	$q_x^{(3)}$
40	15,000	0.01	0.04	0.05
41	-	0.04	0.08	0.10

After the table was prepared, you discover that $q_{40}^{(1)}$ should have been 0.02, and that all other numerical values shown above are correct.

Calculate the resultant change in $d_{41}^{(3)}$.

(A) Decrease by 20.

(B) Decrease by 15.

(C) No Change.

(D) Increase by 15.

(E) Increase by 20.

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2. Barry and Steve are both age 61. Barry has just purchased a whole life insurance policy. Steve purchased a whole life insurance policy one year ago.

Both Barry and Steve are subject to the following 3-year select and ultimate table:

x	$\ell_{[x]}$	$\ell_{[x]+1}$	$\ell_{[x]+2}$	ℓ_{x+3}	$x+3$
60	10,000	9,600	8,640	7,771	63
61	8,654	8,135	6,996	5,737	64
62	7,119	6,549	5,501	4,016	65
63	5,760	4,954	3,765	2,410	66

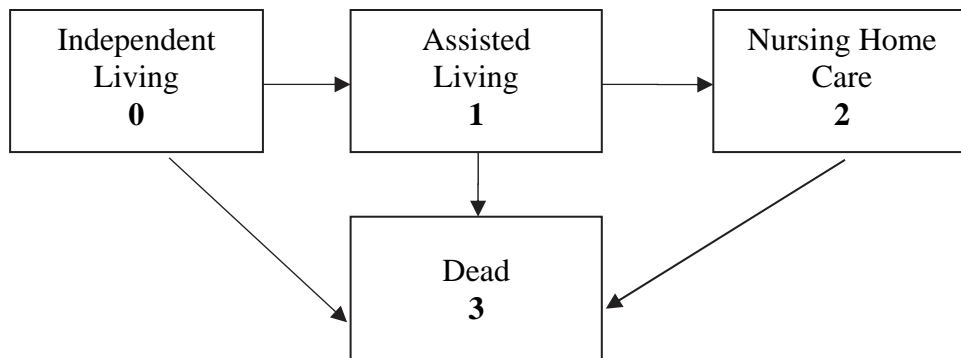The force of mortality is constant over each year of age.

Calculate the difference in the probability of survival to age 64.5 between Barry and Steve.

(A)   0.035

(B)   0.045

(C)   0.055

(D)   0.065

(E)   0.075

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3. You are given the following Markov model:

The forces of transition are the following:

- $\mu^{01} = 0.01$
- $\mu^{03} = 0.02$
- $\mu^{12} = 0.30$
- $\mu^{13} = 0.40$
- $\mu^{23} = 0.70$

Calculate the probability that a person in Independent Living today will be in Assisted Living at the end of 5 years.

(A)   0.008

(B)   0.010

(C)   0.012

(D)   0.023

(E)   0.034

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4. You are given the following Markov model:

(i) Annual transition probabilities are as follows:

	Healthy	Sick	Terminated
Healthy	0.90	0.05	0.05
Sick	0.30	0.60	0.10
Terminated	0.00	0.00	1.00

(ii) The annual health care costs each year, paid at the middle of the year, for each state, are:

Healthy: 500   Sick: 5000   Terminated: 0

(iii) Transitions occur at the end of the year.

(iv) $\delta = 0.04$

Calculate the actuarial present value of health care costs over the next three years for an individual who is currently Healthy.

(A) 1710

(B) 1760

(C) 1810

(D) 1860

(E) 1910

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5. You are given that T, the time to first failure of an industrial robot, has a density $f(t)$ given by

$$f(t) = \begin{cases} 0.1, & 0 \le t < 2 \\ 0.4t^{-2}, & 2 \le t < 10 \end{cases}$$

with $f(t)$ undetermined on $[10, \infty)$.

Consider a supplemental warranty on this robot which pays 100,000 at the time T of its first failure if $2 \le T \le 10$, with no benefits payable otherwise.

You are also given that $\delta = 5\%$.

Calculate the 90th percentile of the present value of the future benefits under this warranty.

(A)   82,000

(B)   84,000

(C)   87,000

(D)   91,000

(E)   95,000

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6. For an annual life annuity-due of 1 with a 5-year certain period on (55), you are given:

  (i)   Mortality follows the Illustrative Life Table

  (ii) $i = 0.06$

Calculate the probability that the sum of the undiscounted payments actually made under this annuity will exceed the expected present value, at issue, of the annuity.

(A)   0.79

(B)   0.81

(C)   0.83

(D)   0.85

(E)   0.87

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7. For a special fully discrete 10-year deferred whole life insurance of 100 on (50), you are given:

(i) Premiums are payable annually, at the beginning of each year, only during the deferral period.

(ii) For deaths during the deferral period, the benefit is equal to the return of all premiums paid, without interest.

(iii) $i = 0.05$

(iv) $\ddot{a}_{50} = 17.0$

(v) $\ddot{a}_{60} = 15.0$

(vi) $_{10}E_{50} = 0.60$

(vii) $(IA)^1_{50:\overline{10}|} = 0.15$

Calculate the annual net premium for this insurance.

(A) 1.3

(B) 1.6

(C) 1.9

(D) 2.2

(E) 2.5

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8. For a fully continuous whole life insurance of 100,000 on (35), you are given:

   (i)   The annual rate of premium is 926.

   (ii)   Mortality follows the Illustrative Life Table.

   (iii)   Deaths are uniformly distributed over each year of age.

   (iv) $i = 0.06$

Calculate the 75th percentile of the loss at issue random variable for this policy.

   (A)   900

   (B)   1000

   (C)   1100

   (D)   1200

   (E)   1300

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9. Pat and Mel are both age 40 with independent future lifetimes. They purchase a deferred annuity-due that pays 1,000,000 per year while both are alive, starting at age 85.

You are given:

(i) The net annual premium, P, is payable for ten years, but only while both Pat and Mel are alive.

(ii) Mortality follows the Illustrative Life Table.

(iii) $i = 0.06$

Calculate P.

(A) 1900

(B) 1950

(C) 2000

(D) 2050

(E) 2100

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10. For a fully continuous whole life insurance of 10,000 issued to (40) you are given the following information:

(i) Premiums are paid at a rate of 100 per year.

(ii) $\delta = 0.05$

(iii) $\mu_{70.5} = 0.038$

(iv) For $t = 30.5$, $\dfrac{d}{dt}{}_tV = 292$

Calculate ${}_{30.5}V$.

(A) 5000

(B) 5500

(C) 6000

(D) 6500

(E) 7000

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11. For a fully discrete 10-payment whole life insurance of H on (45), you are given:

(i) Expenses payable at the beginning of each year are as follows:

Expense Type	First Year	Years 2-10	Years 11+
Per policy	100	20	10
% of Premium	105%	5%	0%

(ii) Mortality follows the Illustrative Life Table.

(iii) $i = 0.06$

(iv) $\ddot{a}_{45:\overline{10}|} = 7.649$

(v) The gross annual premium, calculated using the equivalence principle, is of the form:

$$G = gH + f$$

where g is the premium rate per 1 of insurance and f is the per policy fee.

Calculate f.

(A) 37.20

(B) 38.90

(C) 40.90

(D) 44.80

(E) 47.50

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12. A warranty pays 2000 at the end of the year of the first failure if a washing machine fails within three years of purchase. The warranty is purchased with a single premium, G, paid at the time of purchase of the washing machine.

You are given:

(i) 10% of the washing machines that are working at the start of each year fail by the end of that year.

(ii) $i = 0.08$

(iii) The sales commission is 35% of G.

(iv) G is calculated using the equivalence principle.

Calculate G.

(A) 630

(B) 660

(C) 690

(D) 720

(E) 750

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13. Tom and Jo are both age 55 with independent future lifetimes. They purchase a special, fully discrete, 10-year deferred joint and last survivor annuity policy. You are given:

(i)   The annuity is payable at the beginning of the year, starting at age 65, as long as at least one individual is alive.

(ii)   The benefit is 10,000 per year while both individuals are alive, and is 6,000 per year if only one individual is alive.

(iii)   The premium of P per year is paid annually through the deferred period conditional on both individuals surviving.

(iv)   Mortality follows the Illustrative Life Table.

(v) $i = 0.06$

Calculate P.

(A)   7050

(B)   7170

(C)   7290

(D)   7410

(E)   7530

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14. For a Type B universal life policy, you are given:

(i) $AV_5 = 100,000$

(ii) The additional death benefit is 1,000,000.

(iii) Expense charges are 3.5% of premium plus 100 per year, paid at the beginning of the year.

(iv) The cost of insurance charge, COI, is 50 per 1000 of insurance in policy year 6.

(v) i^c, the credited interest rate per year, is 0.04.

(vi) i^q, the interest rate per year for discounting the net amount at risk in the COI, is 0.06.

(vii) A surrender charge of 6% of account value is collected upon surrender.

(viii) A premium of 25,000 is paid at the beginning of policy year 6.

Calculate the cash surrender value at the end of policy year 6.

(A) 69,500

(B) 70,900

(C) 72,300

(D) 73,700

(E) 75,100

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15. For a portfolio of fully discrete whole life insurance policies of 100,000 on lives age 35, you are given:

  (i) The following expenses are payable at the beginning of the 11th year:

Per policy	50
Percent of Premium	15%

  (ii) The gross premium equals 1100 per policy.

  (iii) The asset share per policy at the end of the 10th year is 10,000.

  (iv) During year 11 the realized investment rate is 8%.

  (v) During year 11 the realized mortality rate is 0.005.

Calculate the asset share per policy at the end of the 11th year.

(A) 10,900

(B) 11,000

(C) 11,100

(D) 11,200

(E) 11,300

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16. For a fully discrete whole life insurance of 1000 on (x), you are given:

    (i)   For calculating gross premium reserves in year 8, the following assumptions are made:
- $q_{x+7} = 0.03$
- Annual expenses of 100, payable at the beginning of the year
- $i = 0.07$

    (ii)   Actual experience during year 8 for this policy is:
- The policy is in force at the end of year 8.
- The annual expenses are 75, paid at the beginning of the year.
- The interest earned is 3%.

    (iii)   Gain by source for year 8 is analyzed in the following order: mortality, expense, interest.

Calculate the gain from expense in policy year 8.

(A)   25.00

(B)   25.75

(C)   26.75

(D)   27.50

(E)   28.50

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17. You are conducting a profit test on a fully discrete 3-year term insurance policy of 1000 issued to (55). Some details of your profit test calculations are summarized below.

Policy Year k	Starting Reserve	Gross Premium	Expenses	Investment Earnings	Expected Death Benefit	Expected Cost of Reserve	q_{55+k-1}
1	0	75	20	2.80	10.00	64.35	0.010
2	65	75	20	6.00	15.00	123.13	0.015
3	125	75	20	9.00	21.00	0.00	0.021

You are also given:

(i) The pre-contract expenses are 100.

(ii) There are no lapses.

(iii) The hurdle rate is 10%.

Calculate the net present value of this policy.

(A) -2.5

(B) -1.9

(C) 0

(D) 1.9

(E) 2.5

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18. Tom, who is age 50 today, is a participant in a pension plan with the following supplemental benefits payable at retirement:

Age at Retirement	Lump Sum Supplemental Benefit
63	20,000
64	25,000
65	30,000

Supplemental benefits are not provided at any other ages.

You are given the following assumptions:

    (i)   Retirement takes place in the middle of each year of age.

    (ii)   Decrements for this pension plan follow the Illustrative Service Table.

    (iii) $i = 0.05$

Calculate the actuarial present value of Tom's supplemental benefits.

(A)   3390

(B)   3490

(C)   3590

(D)   3690

(E)   3790

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19. A defined benefit pension plan provides its members, upon retirement at age 65, an annual pension equal to 2% of final salary per year of service. Final salary is defined to be the salary earned in the calendar year immediately preceding retirement. The pension benefit will be paid in the form of a life annuity payable at the beginning of each month.

Justin joined the pension plan at exact age 30 on January 1, 2002 with a salary of 63,000. He expected this salary to increase by 5% each year on his birthday. As of January 1, 2017, Justin's annual salary has always increased at that rate.

You are given: $\ddot{a}_{65}^{(12)} = 11$ and $_{20}E_{45} = 0.15$

Calculate the actuarial accrued liability on January 1, 2017, of Justin's retirement benefit using the Traditional Unit Credit method.

(A)   62,000

(B)   65,000

(C)   156,000

(D)   164,000

(E)   172,000

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20. On January 1, 2017, Louis is a 51 year old member of a defined benefit pension plan. Louis has 10 years of past service and his salary for 2017 is 70,400. You are given:

(i) The annual retirement benefit is 2% of the final year's salary for each year of service. There are no death benefits.

(ii) His salary for 2016 was 68,700.

(iii) The pension is payable as a monthly single life annuity-due upon retirement.

(iv) His normal retirement date is December 31, 2030 at age 65. There are no decrements other than death prior to retirement.

(v) Mortality follows the Illustrative Life Table.
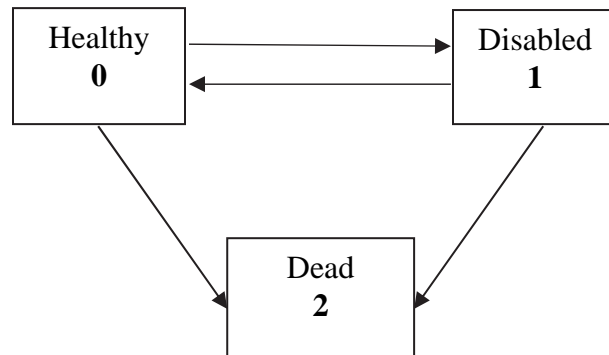
(vi) $i = 0.06$

(vii) $\ddot{a}_{65}^{(12)} = 9.4316$

Calculate the normal contribution at January 1, 2017 under the Traditional Unit Credit cost method.

(A) 6,200

(B) 7,200

(C) 8,200

(D) 9,200

(E) 10,200

Exam MLC

SECTION B – Written-Answer

1. (*10 points*) You are using the following 3-state Markov model to price an insurance product:

$$\mu_{40+t}^{01} = 0.03 \quad \mu_{40+t}^{10} = 0.01 \quad \mu_{40+t}^{02} = 0.01 + 0.01t \quad \mu_{40+t}^{12} = 0.01 + 0.02t \ , \ \text{for } t \geq 0$$

You are also given:

(i)   The product is issued to 100 Healthy individuals age 40 with independent future lifetimes.

(ii)   The premiums are payable continuously while the insured is Healthy.

(iii)   The product pays a continuous disability benefit at a rate of 5,000 per year while the insured is Disabled, and pays a death benefit of 50,000 at the moment of death.

(iv)   The product has a 5-year term. No benefits are paid after 5 years.

(a)   (*1 point*) For a single policy, show that the probability that no benefits are paid during the course of the policy is 0.7 to the nearest 0.1. You should calculate the value to the nearest 0.01.

(b)   (*2 points*) Using the normal approximation with continuity correction, calculate the probability that no benefits are paid for at least 65 of the 100 policies issued.

1. **Continued**

    (c)   (*1 point*) Write down Kolmogorov's forward differential equations with the associated boundary conditions for this model for:

        (i) $_t p_{40}^{00}$

        (ii) $_t p_{40}^{01}$

        (iii) $_t p_{40}^{02}$

    (d)   (*2 points*) Use Euler's forward method with a step size of $h = 0.1$ to calculate the expected number of insureds in the Disabled state at time 0.2.

    (e)   (*4 points*) You are also given:

        • $\delta = 0.05$

        •   The following EPVs:

k	$\bar{A}^{01}_{40+k:\overline{5-k}}$	$\bar{A}^{02}_{40+k:\overline{5-k}}$	$\bar{A}^{12}_{40+k:\overline{5-k}}$	$\bar{a}^{00}_{40+k:\overline{5-k}}$	$\bar{a}^{01}_{40+k:\overline{5-k}}$	$\bar{a}^{02}_{40+k:\overline{5-k}}$	$\bar{a}^{10}_{40+k:\overline{5-k}}$	$\bar{a}^{11}_{40+k:\overline{5-k}}$	$\bar{a}^{12}_{40+k:\overline{5-k}}$
0	0.117	0.146	0.220	3.887	0.262	0.275	0.089	3.924	0.411
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
3	0.053	0.092	0.156	1.767	0.050	0.086	0.017	1.740	0.147

        (i)   Show that the annual net premium rate per policy is 2200 to the nearest 100. You should calculate the premium rate to the nearest 1.

        (ii)   At time 3, 80 insureds are Healthy, 11 are Disabled, and 9 are Dead. Calculate the total net premium reserve for this block of business at time 3.

2. (*11 points*) Consider a special 10-year deferred, 25-year endowment insurance policy sold to Zack, age 40. The policy pays no benefit in case of death before age 50. A death benefit of 100,000 is payable at the time of death, in case of death between ages 50 and 75. In case of survival to age 75, Zack will receive a benefit of 50,000.

You are given:

(i)   Mortality follows the Illustrative Life Table.

(ii) $i = 0.06$

(iii)   Deaths are uniformly distributed over each year of age

(a)   (*1 point*) Write down an expression for the present value random variable Z for this insurance in terms of T_{40} .

(b)   (*3 points*) Sketch a graph of the present value of the benefit as a function of the time of death. Clearly label the axes and show all key values.

(c)   (*2 points*) Calculate the probability that a single premium of 15,000 will be sufficient to cover the benefit payable under this policy.

(d)   (*2 points*) Calculate the smallest single premium that will be sufficient to cover the benefit payable under this policy with a probability of at least 60%.

(e)   (*3 points*) Calculate the EPV of the benefit for this policy.

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3. (*8 points*) Sarah purchases a Type A universal life policy with a face amount of 100,000 and pays an annual premium of 50,000.

You are given:

Policy Year	Percent of Premium Charge	Annual Expense Charge	COI Rate per 1 of Insurance	Annual Discount Rate for COI	Annual Credited Interest Rate	Corridor Factor
1	20%	75	0.025	4.5%	6.50%	1.5
2	8%	75	0.030	4.5%	5.75%	1.4

(a) (*3 points*) Show that the account value at the end of two years is 91,000 to the nearest 1000. You should calculate the account value to the nearest 1.

Sarah surrenders the policy at the end of the second year, when she is age 60. The policy does not have a surrender charge. Sarah uses the surrender value as a net single premium to purchase a special last-survivor life annuity-due with her husband, Grant, who is 10 years older than Sarah.

This annuity provides the following payments at the beginning of each quarter:

- For the first ten years, a guaranteed payment of Q
- After the first ten years, a payment of Q if Sarah is alive
- After the first ten years, a payment of $0.6Q$ if Grant is alive and Sarah is dead

You are given that the net single premium and reserves for this annuity are calculated based on the following:

(i) The future lifetimes of Sarah and Grant are independent.

(ii) Mortality follows the Illustrative Life Table.

(iii) $i = 0.06$

(iv) $\ddot{a}^{(4)}_{\overline{10|}} = 7.634$

(v) The two-term Woolhouse formula

(b) (*3 points*) Show that Q is 1950 to the nearest 50. You should calculate Q to the nearest 1.

3. **Continued**

Grant dies during the tenth year of this annuity.

(c)   (*1 point*) Calculate the net premium reserve immediately prior to the payment of
       Q at the start of the eleventh year.

In the eleventh year, Sarah decides that she no longer needs the annuity and asks the
company to pay her the reserve. The insurance company refuses to pay the full reserve.

(d)   (*1 point*) Explain why the insurance company would not agree to Sarah's request.

4. (*9 points*) You are designing and pricing a single premium whole life annuity with benefits of 10,000 payable annually on Kevin, age 35. You are given:

- (i) The first annuity payment is at age 65.

- (ii) Initial expenses are 4% of premium.

- (iii) Maintenance expenses of 50 per year are payable at the beginning of each year, including the first.

- (iv) Additional administrative expenses of 50 are incurred with each benefit payment.

- (v) Mortality follows the Illustrative Life Table.

- (vi) $i = 0.06$

- (vii) $_0L$ denotes the random variable for the present value of the loss at issue.

- (a) (*2 points*) Calculate the single gross premium using the equivalence principle.

- (b) (*3 points*) Show that the smallest single gross premium such that $\Pr(_0L > 5000) < 0.10$ is 21,000 to the nearest 1000. You should calculate the premium to the nearest 10.

Your supervisor suggests that you change the design of the annuity to refund the single gross premium with interest at 6% per year if Kevin dies before age 65. The refund would be paid at the end of the year of death.

- (c) (*2 points*) Calculate the revised single gross premium using the equivalence principle.

- (d) (*2 points*)

  - (i) Show that the revised smallest single gross premium such that $\Pr(_0L > 5000) < 0.10$ is the same as the premium calculated in (b).

  - (ii) Explain why in this case the premium does not change.

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5. (*9 points*) You have conducted a profit test for a fully discrete 100,000 5-year term life insurance policy issued to (50). For this profit test, you are given:

- Mortality follows the Illustrative Life Table.
- There are no withdrawals.
- The annual gross premium is 770.
- The pre-contract expense is 200.
- Maintenance expenses, payable at the beginning of the year, are 20 per year, including the first year.
- The reserves used for the profit test:

k	$_kV$
0	0
1	100
2	160
3	170
4	120
5	0

- The earned asset rate is 4%.
- The hurdle rate is 6%.
- The NPV for this policy is 144.56.

(a) (*3 points*) Show that NPV(2) for this policy is –43 to the nearest 1. You should calculate the value to the nearest 0.01.

(b) (*1 point*) Calculate the discounted payback period (DPP) for this policy.

Regulators now allow the insurer to hold zero reserves at time 1. You redo the profit test, setting $_1V = 0$ and leaving the rest of the reserves and profit test basis unchanged.

(c) (*3 points*)

    (i) Show that the revised Pr₂ is –21 to the nearest 1. You should calculate the value to the nearest 1.

    (ii) Recalculate the NPV.

5. Continued

(d) (*2 points*) Your manager suggests that the negative emerging profit in year 2 should be eliminated.

(i) Calculate the minimum $_1V$ such that the emerging profit in year 2 is zero.

(ii) Explain why the negative emerging profit in year 2 is undesirable from the perspective of the insurer.

6. (*9 points*) Kira, age 63, is a participant in XYZ's defined benefit pension plan. She was hired at age 35 at a salary of 40,000. XYZ provides salary increases of 3% at the end of each year of age. You are given:

(i) The normal retirement benefit, payable from age 65, is 2% of the final twelve months' salary per year of service.

(ii) The retirement benefit is payable as a life annuity-due, with semiannual payments.

(iii) For early retirements, the pension benefit is reduced by 0.5% for each month that retirement precedes age 65.

(iv) Decrements from active employment before age 65 follow the Illustrative Service Table, with decrements occurring in the middle of the year.

(v) All employees who reach age 65 retire immediately.

(vi) There are no death benefits.

(vii) Mortality after retirement follows the Illustrative Life Table, with deaths uniformly distributed over each year of age.

(viii) $i = 0.06$

(ix) For a retired employee, $\ddot{a}^{(2)}_{63.5} = 10.0282$.

(a) (*2 points*) Show that Kira's replacement ratio, if she retires at age 64.5, is 57% to the nearest 1%. You should calculate the replacement ratio to the nearest 0.1%.

(b) (*2 points*) Show that for a retired employee, $\ddot{a}^{(2)}_{64.5} = 9.77$ to the nearest 0.01. You should calculate the value to the nearest 0.0001.

(c) (*5 points*)

(i) Show that the actuarial accrued liability at age 63 for Kira's retirement benefits, using the projected unit credit cost method, is 436,500 to the nearest 100. You should calculate the value to the nearest 10.

(ii) Calculate the normal contribution for the year starting at age 63 for Kira's retirement benefits, using the projected unit credit cost method.

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