October 2014 MLC Multiple Choice Solutions

1. Key: C

$$1000\ {}_{5|}q_{\overline{60:70}} = 1000\left[\ {}_5p_{60}\ {}_5p_{70}\ q_{65}\ q_{75} + {}_5q_{60}\ {}_5p_{70}\ q_{75} + {}_5q_{70}\ {}_5p_{60}\ q_{65}\ \right]$$

$$1000[(0.92)(0.88)(0.02132)(0.05169)+(1-0.92)(0.88)(0.05169)$$
$$+(1-0.88)(0.92)(0.02132)]$$
$$=[0.0008922+0.00363898+0.00235373]1000=6.8849$$

2. Key: A

By U.D.D.:

$$0.2 = q_x^{(2)} = q_x'^{(2)}\left(1-\frac{1}{2}q_x'^{(1)}\right)$$
$$= q_x'^{(2)}\left(1-\frac{1}{2}(0.1)\right) = 0.95\ q_x'^{(2)}$$
$$\Rightarrow q_x'^{(2)} = 0.2105$$
$$p_x^{(\tau)} = p_x'^{(1)} p_x'^{(2)} = (0.90)(1-0.2105) = 0.71055$$
$$q_x^{(1)} = q_x^{(\tau)} - q_x^{(2)} = 1 - p_x^{(\tau)} - q^{(2)}$$
$$= 1 - 0.71055 - 0.2 = 0.08945$$

3. Key: C

Path	Probability
S → S → S → S	0.216
S → S → C → S	0.012
S → C → S → S	0.012
S → C → C → S	0.010
Total across all paths:	0.250

4. Key: D

$$A_{35} = A_{35:\overline{15}|}^{\,1} + A_{35:\overline{15}|}^{\;\;\;\,1}\, A_{50}$$

$$0.32 = 0.25 + 0.14\, A_{50}$$

$$A_{50} = \frac{0.07}{0.14} = 0.50$$

5. Key: D

$$P = 30{,}000\ {}_{20|}\ddot{a}_{40} + PA_{40:\overline{20}|}^{\,1}$$

$$\Rightarrow \quad P = 30{,}000\ {}_{20|}\ddot{a}_{40} \,/\left(1 - A_{40:\overline{20}|}^{\,1}\right)$$

$$= 30{,}000(3.0554)\,/\,(1 - 0.0601267) = 97{,}526$$

6. Key: E

$$1{,}000{,}000 = \text{APV(benefits)} = 100{,}000 A_{65} + R\ddot{a}_{65:65} + 0.60R\ddot{a}_{65|65} + 0.70R\ddot{a}_{65|65}$$

$$\left(\text{where } \ddot{a}_{65|65} = \ddot{a}_{65} - \ddot{a}_{65:65}\right)$$

$$= 100{,}000 A_{65} + R\left(1.3\ddot{a}_{65} - 0.3\ddot{a}_{65:65}\right)$$

$$R = \frac{1{,}000{,}000 - 100{,}000 A_{65}}{1.3\ddot{a}_{65} - 0.3\ddot{a}_{65:65}}$$

$$A_{65} = 0.43980$$
$$\ddot{a}_{65} = 9.8969$$
$$\ddot{a}_{65:65} = 7.8552$$

$$R = \frac{956{,}020}{10.50941} = 90{,}968$$

7. **Key: C**

Let π be the annual premium, so that $\pi\ddot{a}_{50} = A_{50} + 0.01\ddot{a}_{50} + 0.19$

$$\Rightarrow \pi = \frac{A_{50} + 0.19}{\ddot{a}_{50}} + 0.01 = \frac{0.24905 + 0.19}{13.2668} + 0.01 = 0.04309389$$

Loss at issue: $L_0 = v^{k+1} - (\pi - 0.01)\ddot{a}_{\overline{k+1}|}\left(1 - v^{k+1}\right)/d + 0.19$

$$\Rightarrow Var\left[L_0\right] = \left(1 + \frac{(\pi - 0.01)}{d}\right)^2 \left({}^2A_{50} - A_{50}^2\right)$$
$$= (2.511143)(0.09476 - 0.24905^2)$$
$$= (2.511143)(0.032734)$$
$$= 0.082$$

8. **Key: B**

EPV(premiums) = EPV(benefits)

$$P\left(1 + vp_x + v^2\,{}_2p_x\right) = P\left(vq_x + 2v^2 p_x\, q_{x+1}\right) + 10000\left(v^3\,{}_2p_x\, q_{x+2}\right)$$
$$P\left(1 + \frac{0.9}{1.04} + \frac{0.9 \times 0.88}{1.04^2}\right) = P\left(\frac{0.1}{1.04} + \frac{2 \times 0.9 \times 0.12}{1.04^2}\right) + 10000\left(\frac{0.9 \times 0.88 \times 0.15}{1.04^3}\right)$$
$$2.5976P = 0.29588P + 1056.13$$
$$P = 459$$

9. **Key: C**

$$P \times \ddot{a}_{75:\overline{15}|} = 1000\left(A^{\,1}_{75:\overline{15}|} + 15 \times P \times A_{75:\overline{15}|}^{\;\;\;\;\;\;1}\right) \rightarrow P = \frac{1000A^{\,1}_{75:\overline{15}|}}{\ddot{a}_{75:\overline{15}|} - 15 \times A_{75:\overline{15}|}^{\;\;\;\;\;\;1}}$$

$$A^{\,1}_{75:\overline{15}|} = A_{75:\overline{15}|} - A_{75:\overline{15}|}^{\;\;\;\;\;\;1} = 0.7 - 0.11 = 0.59$$

$$\ddot{a}_{75:\overline{15}|} = \frac{1 - A_{75:\overline{15}|}}{d} = (1 - 0.7)/0.04 = 7.5$$

So $P = \dfrac{590}{7.5 - 15(0.11)} = 100.85$

10. Key: C

Let the monthly net premium $= \pi$

$$12\pi = \frac{100{,}000\bar{A}_{45}}{\ddot{a}^{(12)}_{45:\overline{20|}}}$$

$$\alpha(12) = 1.00028$$

$$\beta(12) = 0.46812$$

$$\frac{i}{\delta} = 1.02971$$

$$100{,}000\bar{A}_{45} = 100{,}000\,\frac{i}{\delta}A_{45} = (1.02971)20{,}120 = 20{,}717.7652$$

$$\ddot{a}^{(12)}_{45:\overline{20|}} = \alpha(12)\ddot{a}_{45:\overline{20|}} - \beta(12)\left(1 - {}_{20}E_{45}\right)$$

$$\left(\ddot{a}_{45} - {}_{20}E_{45}\,\ddot{a}_{65}\right)$$

$$= 1.00028\left[14.1121 - 0.25634(9.8969)\right] - 0.46812(1 - 0.25634)$$

$$= 11.230$$

$$12\pi = \frac{20717.7652}{11.230}$$

$$12\pi = 1844.858878$$

$$\pi = 153.7382$$

11. Key: D

$$G\ddot{a}_{x:\overline{30|}} = \text{APV}\left[\text{gross premium}\right] = \text{APV}\left[\text{Benefits + expenses}\right]$$

$$= FA_x + \left(30 + 30\ddot{a}_x\right) + G\left(0.6 + 0.10\ddot{a}_{x:\overline{30|}} + 0.10\ddot{a}_{x:\overline{15|}}\right)$$

$$G = \frac{FA_x + 30 + 30\ddot{a}_x}{\ddot{a}_{x:\overline{30|}} - 0.6 - 0.1\ddot{a}_{x:\overline{30|}} - 0.1\ddot{a}_{x:\overline{15|}}}$$

$$= \frac{FA_x + 30 + 30(15.3926)}{14.0145 - 0.6 - 0.1(14.0145) - 0.1(10.1329)}$$

$$= \frac{FA_x + 491.78}{10.9998}$$

$$= \frac{FA_x}{10.9998} + \frac{491.78}{10.9998} = \frac{FA_x}{10.9998} + 44.71$$

$$\Rightarrow h = 44.71$$

12. Key: D

Intuitively:

(A) A lower interest rate increases premium, but a higher recovery rate decreases premium, because there are lower projected benefits and more policyholders paying premiums.

(B) A lower death rate of healthy lives $\rightarrow$ more pay premium $\rightarrow$ lower premium

(C) A higher death rate of sick lives $\rightarrow$ fewer benefits $\rightarrow$ lower premium

(D) A lower recovery rate $\rightarrow$ higher sickness benefits $\rightarrow$ higher premium
A lower death rate of sick lives $\rightarrow$ higher sickness benefits $\rightarrow$ higher premium

(E) A higher rate of interest $\rightarrow$ lower premium
A lower mortality rate for healthy lives may result in lower premium because more healthy lives are paying premium.

13. Key: A

$$\left({}_5V + 0.96G - 50\right)(1.05) = q_{50}(100,200) + p_{50}\ {}_6V$$
$$\left(5500 + 0.96G - 50\right)(1.05) = (0.009)(100,200) + (1-0.009)(7100)$$
$$(1.05)(0.96)G + 5722.5 = 7937.9$$
$$(1.05)(0.96)G = 2215.4$$
$$G = 2197.8$$

14. Key: E

$${}_{15.6}V\left(1+i\right)^{0.4} = {}_{0.4}p_{x+15.6}\ \ {}_{16}V + {}_{0.4}q_{x+15.6}\ \ 100$$
$${}_{15.6}V(1.05)^{0.4} = 0.957447(49.78) + 0.042553(100)$$
$${}_{15.6}V = 50.91$$

15. Key: D

APV future benefits

$$= 1000\left[0.04v + 0.05\times 0.96v^2 + 0.96\times 0.95\times(0.06 + 0.94\times 0.683)v^3\right] = 630.25$$

APV future premiums $= 130(1 + 0.96v) = 248.56$

$$E[{}_3L] = 630.25 - 248.56 = 381.69$$

16. Key: D

$$\frac{V\left[{}_\tau L\right]}{V\left[{}_{\tau+1}L\right]} = \frac{\left(1+\frac{p}{d}\right)^2\left({}^2A_{x+\tau} - A_{x+\tau}^2\right)}{\left(1+\frac{p}{d}\right)^2\left({}^2A_{x+\tau+1} - A_{x+\tau+1}^2\right)}$$

$$A_{x+t} = vq_{x+\tau} + vp_{x+\tau}\ A_{x+\tau+1}$$

$$= (0.90703)^{1/2}(0.02067) + (0.90703)^{1/2}(1 - 0.02067)(0.52536) = 0.50969$$

$${}^2A_{x+\tau} = v^2 q_{x+\tau} + v^2 p_{x+\tau}\ {}^2A_{x+\tau+1}$$

$$= (0.90703)(0.02067) + (0.90703)(1 - 0.02067)(0.30783) = 0.29219$$

$$\Rightarrow \frac{\mathrm{Var}\left({}_k L\right)}{\mathrm{Var}\left({}_{k+1} L\right)} = \frac{(0.29219) - (0.50969)^2}{(0.30783) - (0.52536)^2} = \frac{0.03241}{0.03183} = 1.018$$

17. Key: B

$$\mathrm{Pr}_2 = {}_1V + P - E + I - EDB - E_2V$$

$$= (400 + 1500 - 100)1.072 - 1200 - 0.988\times 700$$

$$= 38.00$$

18. Key: A

x	t	Premium	Expense charges	COI rate	Amount at Risk	COI	Credited Interest	End of Year Account Value
40	0	2500	75.00	0.0028	100,000	266.67	97.12	2255.45
41	1	3000	80.00	0.0030	95,000	271.43	255.00	5159.03

The COI calculation reflects discounting the amount at risk at 5%, (e.g., in year 1, $(100{,}000/1.05)\times 0.0028 = 266.67$)
Credited interest is 4.5% in year 1 and 5.2% in year 2.

19. Key: E

Annual Retirement Benefit

$$(0.0175)\left[525{,}000+\sum_{K=0}^{9}(50{,}000)(1.03)^{K}\right]=19{,}218.39$$

APV at age 65

$$(19{,}218.39)\ {}_{10|}\ddot{a}_{55}^{(12)}=19{,}218.39(1.04)^{-10}\left({}_{10}p_{55}\right)\ddot{a}_{65}^{(12)}$$
$$=19{,}218.39(0.6756)(0.925)(12.60)=151{,}328$$

20. Key: B

$$\begin{aligned}
{}_{2.5}q_{[50]+0.4} &= 1-{}_{2.5}p_{[50]+0.4}=1-{}_{2.9}p_{[50]}/\left(p_{[50]}\right)^{0.4}\\
&=1-\left\{p_{[50]}\,p_{[50]+1}\left(p_{52}\right)^{0.9}\right\}/\left(1-q_{[50]}\right)^{0.4}\\
&=1-\left\{\left(1-q_{[50]}\right)\left(1-q_{[50]+1}\right)\left(1-q_{52}\right)^{0.9}\right\}/\left(1-q_{[50]}\right)^{0.4}\\
&=1-\left\{(1-0.0050)(1-0.0063)(1-0.0080)^{0.9}\right\}/(1-0.0050)^{0.4}\\
&=0.01642
\end{aligned}$$

$$1000\ {}_{2.5}q_{[50]+0.4}=16.42$$