Long-Term Actuarial Mathematics
Exam LTAM

Date: Friday, October 23, 2020
Time: 8:30 a.m. – 12:45 p.m.

Recognized by the Canadian Institute of Actuaries.

INSTRUCTIONS TO CANDIDATES

General Instructions

1. Write your candidate number here _____. Your name must not appear.

2. Do not break the seal of this book until the supervisor tells you to do so.

3. Tables for this examination will be distributed by the Supervisor.

4. This examination has a total of 96 points. It consists of:

    Section A: 20 multiple-choice questions, each worth 2 points for a total of 40 points, and

    Section B: 6 written-answer questions, worth a total of 56 points. The point value for each written-answer question is indicated at the beginning of the question.

   You may divide your time between the two sections of the examination (written-answer, and multiple-choice) as you choose. You should keep in mind the relative weight of the two sections.

   Your written-answer paper will be graded only if your multiple-choice score is at or above a threshold set after the examination is administered.

5. Failure to stop writing or coding after time is called will result in the disqualification of your answers or further disciplinary action.

6. While every attempt is made to avoid defective questions, sometimes they do occur. If you believe a question is defective, the supervisor or proctor cannot give you any guidance beyond the instructions on the exam booklet.

Multiple-Choice Instructions

1. A separate answer sheet for the multiple-choice questions is inside the front cover of this book. During the time allotted for this examination, record all your answers on the back of the answer sheet. NO ADDITIONAL TIME WILL BE ALLOWED FOR THIS PURPOSE.

No credit will be given for anything indicated in the examination book but not transferred to the answer sheet.

2. On the front of the answer sheet, space is provided to write and code candidate information. Complete the information requested by printing in the squares and blackening the circles (one in each column) corresponding to the letters or numbers printed. For each empty box blacken the small circle immediately above the "A" circle. Fill out the boxes titled:

   (a) Name
       (include last name, first name and middle initial)

   (b) Candidate Number
       (Candidate/Eligibility Number, use leading zeros if needed to make it a five digit number)

   (c) Test Site Code
       (The supervisor will supply the number.)

   (d) Examination Part
       (Code the examination that you are taking by blackening the circle to the left of "Exam LTAM.")

   (e) Booklet Number
       (The booklet number can be found in the upper right-hand corner of this examination book. Use leading zeros if needed to make it a four digit number.)

In the box titled "Complete this section only if instructed to do so," fill in the circle to indicate if you are using a calculator and write in the make and model number.

In the box titled "Signature and Date" sign your name and write today's date. **If the answer sheet is not signed, it will not be graded**.

Leave the boxes titled "Test Code" and "Form Code" blank.

On the back of the answer sheet fill in the Booklet Number in the space provided.

CONTINUED ON INSIDE FRONT COVER

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Exam LTAM - Front Cover

3. Your score will be based on the number of questions which you answer correctly. No credit will be given for omitted answers and no credit will be lost for wrong answers: hence, you should answer all questions even those for which you have to guess.

4. Five answer choices are given with each multiple-choice question, each answer choice being identified by a key letter (A to E). Answer choices for some questions have been rounded. For each question, blacken the circle on the answer sheet which corresponds to the key letter of the answer choice that you select.

5. Use a soft-lead pencil to mark the answer sheet. To facilitate correct mechanical scoring, be sure that, for each question, your pencil mark is dark and completely fills only the intended circle. Make no stray marks on the answer sheet. If you have to erase, do so completely.

6. Do not spend too much time on any one question. If a question seems too difficult, leave it and go on.

7. Clearly indicated answer choices in the test book can be an aid in grading examinations in the unlikely event of a lost answer sheet.

8. After the examination, the supervisor will collect this book and the answer sheet separately. DO NOT ENCLOSE THE ANSWER SHEET IN THE BOOK OR IN THE ESSAY ANSWER ENVELOPE. All books and answer sheets must be returned. THE QUESTIONS ARE CONFIDENTIAL AND MAY NOT BE TAKEN FROM THE EXAMINATION ROOM.

Written-Answer Instructions

1. **Write your candidate number at the top of each sheet. Your name must not appear.**

2. Write on only one side of a sheet. Start each question on a fresh sheet. On each sheet, write the number of the question you are answering. Do not answer more than one question on a single sheet.

3. The answer should be confined to the question as set.

4. When you are asked to calculate, show all your work including any applicable formulas.

5. When you finish, insert your written-answer sheets that you want graded into the Essay Answer Envelope. Be sure to hand in all your answer sheets because they cannot be accepted later. Seal the envelope and write your candidate number in the space provided on the outside of the envelope. Check the appropriate box to indicate Exam LTAM.

6. **Sign your essay answer envelope. If it is not signed, your examination will not be graded.**

7. For all parts of all problems, to maximize the credit earned, candidates should show as much work as possible, considering the time allotted for the question. Answers lacking justification will receive no credit. Answers should be organized so that the methods, logic, and formulas used are readily apparent. Candidates should not round their answers excessively; enough precision should be provided so that their answers can be accurately graded.

   In some cases, candidates are asked to show that a calculation results in a particular number. Typically the answer given will be rounded; candidates should provide a greater level of accuracy than the number given in the question. This structure of question is intended to assist the candidate by giving an indication when the calculation has been done incorrectly, providing an opportunity to explore an alternative approach. It also allows a candidate who cannot obtain the correct answer to use the answer given to proceed with subsequent parts of the problem. (Candidates who are able to solve the problem should use their exact answer for subsequent parts.)

   For questions requiring candidates to derive or write down a formula or equation, the resulting expression should be simplified as far as possible, and where numerical values are provided in the problem, they should be used.

Exam LTAM

SECTION A – Multiple-Choice

1. Which of the following best describes a reversionary annuity?

(A)   An annuity contract in which the policyholder pays a single premium in return for an annuity which commences after a fixed deferred period.

(B)   An annuity issued on two lives which continues while at least one of the lives survives.

(C)   An annuity payable for the balance of the contract term after the death of the policyholder.

(D)   An annuity payable under a structured settlement which may be reduced if the annuitant's health improves.

(E)   An annuity contingent on two lives, with one annuitant and one insured, with the annuity benefits beginning upon the death of the insured provided that the annuitant is still alive.

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2. Which of the following expressions is equal to $E[T_x^2]$?

(A) $\displaystyle\int_0^\infty 2t \, {}_tp_x \, \mu_{x+t} \, dt$

(B) $\displaystyle\int_0^\infty 2t^2 \, {}_tp_x \, \mu_{x+t} \, dt$

(C) $\displaystyle\int_0^\infty 2t \, {}_tp_x \, dt$

(D) $\displaystyle\int_0^\infty 2t^2 \, {}_tp_x \, dt$

(E) $\displaystyle\int_0^\infty 2t^2 \, {}_tp_x^2 \, dt$

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3. A horse trainer purchased 135 stallions, each age 2 at purchase. The animals were all insured, with underwriting, at the time of purchase. The future lifetimes of the stallions are independent.

You are given the following extract from the two-year select and ultimate table for stallion mortality:

x	$l_{[x]}$	$l_{[x]+1}$	l_{x+2}	$x+2$
2	50,000	48,000	45,000	4
3	45,000	43,000	40,000	5
4	40,000	38,000	35,000	6
5	35,000	33,000	30,000	7
6	30,000	28,000	25,000	8

One year after the purchase, 128 stallions are still alive.

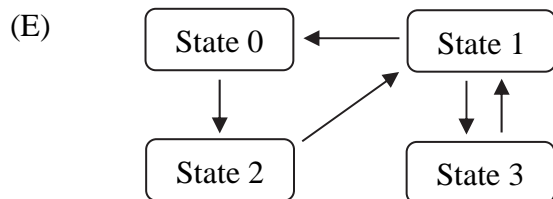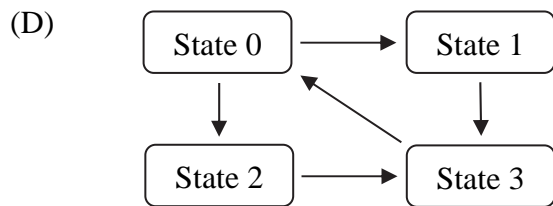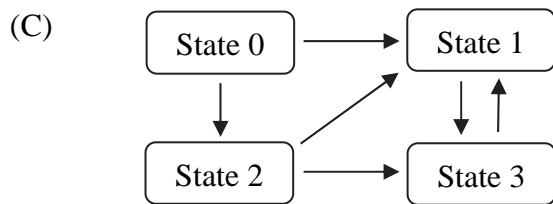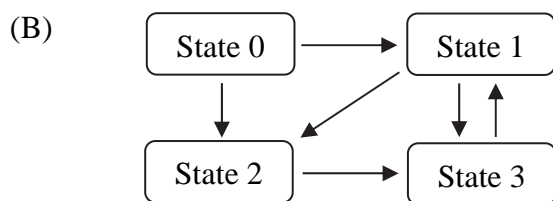Calculate the variance in the number of deaths between age 3 and age 7.
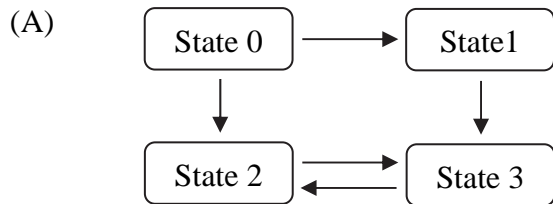
(A)   27

(B)   28

(C)   29

(D)   30

(E)   31

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4. Each of the following diagrams represents a Markov multiple state model. Identify the model for which $_{t}p_{x}^{\overline{22}} = {}_{t}p_{x}^{22}$.

(A)

State 0 → State 1

State 0 → State 2

State 1 → State 3

State 3 → State 2

(B)

State 0 → State 1

State 0 → State 2

State 1 → State 2

State 1 ↔ State 3

State 2 → State 3

(C)

State 0 → State 1

State 0 → State 2

State 2 → State 1

State 1 ↔ State 3

State 2 → State 3

(D)

State 0 → State 1

State 0 → State 2

State 3 → State 0

State 1 → State 3

State 2 → State 3

(E)

State 1 → State 0

State 0 → State 2

State 2 → State 1

State 1 ↔ State 3

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5. For a double decrement table, you are given:

(i) Decrements are uniformly distributed between integer ages in the associated single decrement tables.

(ii) $q_x^{(1)} = 0.138$

(iii) $q_x'^{(2)} = 0.070$

Calculate $1000 q_x^{(2)}$.

(A) 65

(B) 67

(C) 69

(D) 71

(E) 73

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6. In a mortality study of a cohort of 50 lives observed from birth, you are given:

     (i)   There were no censored observations.

     (ii)   No more than one life died at any of the times of death.

Calculate the Nelson-Aalen estimate of the survival function at the time of the 2nd death.

(A)   0.9600

(B)   0.9604

(C)   0.9608

(D)   0.9612

(E)   0.9616

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7. You are given the following excerpt from a select life table with a select period of three years.

x	$l_{[x]}$	$l_{[x-1]+1}$	$l_{[x-2]+2}$	l_x
60	96,415	96,536	96,609	96,634
61	96,062	96,197	96,278	96,306
62	95,670	95,819	95,910	95,941
63	95,233	95,399	95,500	95,534
64	94,747	94,932	95,044	95,083

You are also given

(i) $i = 0.06$

(ii) $\ddot{a}_{63} = 12.886$
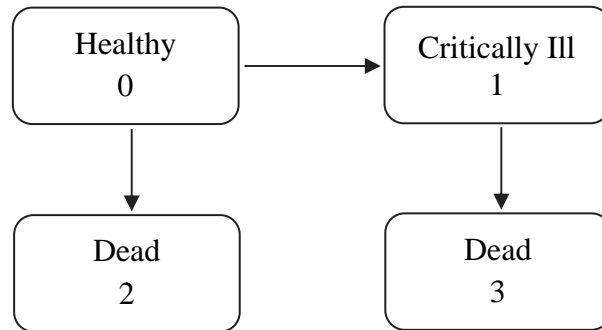
Calculate $\ddot{a}_{[60]+1}$.

(A) 13.29

(B) 13.33

(C) 13.37

(D) 13.41

(E) 13.44

8. You are given the following critical illness model:

You are also given the following information for a 20-year critical illness policy on a healthy life aged 55:

(i) A benefit of 800,000 is paid immediately on death if the insured had a critical illness diagnosis prior to death.

(ii) $i = 0.05$

(iii) The following actuarial functions:

x	$\bar{A}_x^{01}$	$\bar{A}_x^{03}$	$\bar{A}_x^{13}$	$_{20}p_x^{00}$	$_{20}p_x^{01}$
55	0.2833	0.1856	0.4145	0.7335	0.1361
75	0.6494	0.4910	0.7657	--	--

Calculate the actuarial present value of the death benefit.

(A) 8,470

(B) 16,320

(C) 24,180

(D) 32,040

(E) 39,890

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9. You are given the following information for an accidental death benefit, based on a 3-state model. State 0 represents the active state, State 1 represents death by natural causes, and State 2 represents accidental death.

(i) The force of decrement for accidental death is $\mu^{02}_{50+t} = 0.00022$ for all $t > 0$.

(ii) The total force of mortality, $\mu^{01}_{50+t} + \mu^{02}_{50+t} = \mu_{50+t}$ for all $t > 0$, where μ_{50+t} is the force of mortality of the Standard Ultimate Life Table.

(iii) $i = 0.05$

(iv) $\bar{a}_{50} = 16.520$ using the Standard Ultimate Life Table.

Calculate the expected present value of a benefit of 200,000 paid immediately on accidental death, to a life currently age 50.

(A) 719

(B) 721

(C) 723

(D) 725

(E) 727

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10. For a fully discrete 10-year term insurance paying a death benefit of 1,000,000 on the last death of (50) and (60) you are given:

  (i) The future lifetimes of (50) and (60) are independent.

  (ii) Mortality follows the Standard Ultimate Life Table.

  (iii) $i = 0.05$

  (iv) Premiums are paid annually, reducing by ½ on the first death.

Calculate the annual premium payable while both lives are alive.

  (A) 80

  (B) 84

  (C) 88

  (D) 92

  (E) 96

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11. Jacob, (30), is considering purchasing a 35-year term insurance policy, with sum insured 100,000, to provide cover until he retires at age 65. The death benefit is paid at the end of the year of death. The single net premium for the insurance is 12,000.

Jacob has the option to purchase the same insurance, ceasing at age 65, but with a reduced benefit of 10,000 if death occurs during the first 3 months. The single net premium for this contract is 11,865.

The insurer calculates the premium assuming:

> (i) Uniform distribution of deaths between integer ages.
>
> (ii) $i = 0.03$

Calculate q_{30}.

(A) 0.00568

(B) 0.00583

(C) 0.00600

(D) 0.00618

(E) 0.00630

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12. Roger, (65), purchases a special fully discrete 10-year term insurance policy. You are given the following information:

> (i)   The insurance amount is 100,000.
>
> (ii)   Premiums are returned without interest if the policyholder is alive at the end of the 10-year contract.
>
> (iii)   Initial expenses are 2,000 and 50% of the first premium.
>
> (iv)   Renewal expenses are 2.5% of the second and subsequent premiums.
>
> (v)   Expenses of 500 are incurred at the end of the year of death, or on the payment of the return of premium benefit.
>
> (vi)   Gross premiums are calculated using the equivalence principle.
>
> (vii)   Mortality follows the Standard Ultimate Life Table.
>
> (viii) $i = 0.05$

Calculate the gross premium.

(A)   4333

(B)   4823

(C)   5347

(D)   5882

(E)   6314

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13. You are given the following information for a whole life policy on (55), who is Healthy.

   (i)   A benefit is payable continuously while the insured is in the Sick state, at a rate of 12,000 per year.

   (ii)   A benefit of 100,000 is payable immediately upon death.

   (iii) Premiums are payable continuously while the insured is in the Healthy state.

   (iv)   Decrements follow the Standard Sickness-Death Model.

   (v) $i = 0.05$

Calculate the annual net premium rate for this policy.

   (A)   5645

   (B)   6128

   (C)   6622

   (D)   7120

   (E)   7622

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14. For a fully-discrete whole life insurance of 250,000 on (35), you are given:

(i) Expenses, which are incurred at the beginning of the policy year, are as follows:

	Percent of Gross Premium	Per policy
First Year	30%	500
Second and subsequent years	8%	100

(ii) The gross premium is calculated using the equivalence principle.

(iii) Mortality follows the Standard Ultimate Life Table.

(iv) $i = 0.05$

Calculate the gross premium reserve at the end of the third year.

(A) 2961

(B) 3063

(C) 3164

(D) 3263

(E) 3364

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15. An insurer is conducting a profit test of a monthly premium whole life insurance issued to (58).

You are given the following information.

      (i)   Premiums are 100 per month.

      (ii)   Renewal expenses are 5 at the start of each month.

      (iii)   The death benefit is 75,000, payable at the end of the month of death.

      (iv) $_{\frac{1}{12}}q_{60} = 0.000283$

      (v)   At the end of each month 0.25% of surviving policyholders lapse their contracts with no cash value.

      (vi)   The policy reserves at times 2 and $2\frac{1}{12}$ are: $_2V = 2430$; $_{2\frac{1}{12}}V = 2520$.

      (vii)   The insurer's interest rate on investments is $i^{(12)} = 0.09$.

Calculate the expected profit emerging in the first month of the third year, per policy in force at the start of the month.

(A)   9.7

(B)   10.4

(C)   11.1

(D)   11.8

(E)   12.5

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16. For a fully discrete 2-year term insurance on (80) you are given:

(i) The annual gross premium is 100.

(ii) Pre-contract cash flows are $Pr_0 = -162$.

(iii) The schedule of profit emergence at t, per policy in force at $t-1$, for $t = 1, 2$, is:

Time in years, t	1	2
Pr_t	60	175

(iv) $p_{80} = 0.850$; $p_{81} = 0.845$

(v) There are no lapses.

(vi) The hurdle rate is r per year.

(vii) The profit margin is 8.7%.

Calculate r.

(A) 8.5%

(B) 9.0%

(C) 9.5%

(D) 10.0%

(E) 10.5%

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17. Tom is a member of a final average salary defined benefit pension plan where the benefit is a whole life annuity-due payable annually.

You are given the following valuation information and assumptions:

   (i)   Tom is age 50 at the valuation date; he entered the plan at age 42.

   (ii)   The accrual rate is 1.8% per year of service.

   (iii)   There are no exits from the plan before age 65 other than death.

   (iv)   All lives in employment at age 65 retire at that time.

   (v)   Tom's projected final average salary at retirement is 108,650.

   (vi)   Mortality follows the Standard Ultimate Life Table.

   (vii) $i = 0.05$

   (viii) The plan is funded using the Projected Unit Credit method.

Calculate the normal contribution for Tom's retirement benefit.

(A)   11,030

(B)   11,330

(C)   11,630

(D)   11,930

(E)   12,230

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18. An insurer issues a fully discrete whole life insurance with sum insured 100,000 to (75). You are given the following information.

(i)   Mortality follows the Standard Ultimate Life Table.

(ii)   Gross premiums are 7200 per year payable annually.

(iii)   Commissions are 5% of premiums.

(iv)   Deaths are uniformly distributed between integer ages.

(v) $i = 0.05$

(vi) $_9V = 17,138$

Calculate the gross premium reserve at time 9.5.

(A)   22,640

(B)   23,150

(C)   23,660

(D)   24,170

(E)   24,680

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19. You are given:

    (i) The annual premium for retiree healthcare cover, for a life age 64 at the valuation date is $B(64,0) = 6396$.

    (ii) $\dfrac{B(64+t,0)}{B(64,0)} = 1.03^t$ for all $t > 0$.

    (iii) The annual rate of inflation for healthcare premiums is 4%.

    (iv) The valuation interest rate is 6%.

    (v) Retirements follow the Standard Service Table.

    (vi) Retirements before age 65 are assumed to occur exactly ½ -way through the year of age.

    (vii) $\ddot{a}_{64.5|\,i^*} = 27.2753$ and $\ddot{a}_{65|\,i^*} = 26.7083$, where $i^* = \dfrac{1.06}{(1.04)(1.03)} - 1$.

Calculate the actuarial value of the total healthcare benefit for a life age 64 at the valuation date.

(A) 170,420

(B) 171,860

(C) 173,300

(D) 174,740

(E) 176,180

20. For a fully continuous whole life insurance with sum insured 100,000, issued s years ago to (x), you are given the following reserve information:

(i) The annual gross premium rate is $G = 2500$.

(ii) Premium expenses are 5% of premiums, incurred continuously.

(iii) Claim expenses are 1000.

(iv) Initial expenses are 50% of the annual gross premium rate.

(v) The force of interest is 4%.

(vi) The gross premium reserve at time s is $_sV^g = 46,531$.

(vii) $\mu_{x+s} = 0.031313$

Calculate $\dfrac{d}{dt}\, _tV^g$ at time $t = s$.

(A) 2380

(B) 2439

(C) 2483

(D) 2531

(E) 2587

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Exam LTAM

SECTION B – Written-Answer

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1. (*9 points*) Steve, (45), was hit by a truck belonging to XYZ. Steve suffered permanent injuries and, as the result of a lawsuit, was awarded a non-reviewable structured settlement.

The structured settlement will provide Steve with the following payments:

- A death benefit of 100,000 payable at the moment of death if Steve dies before age 65.

- A survival benefit of 200,000 payable on Steve's 65th birthday.

- A monthly life annuity-due of 4,000 per month.

XYZ decides to purchase the structured settlement from the JAB Insurance Company. JAB uses the following assumptions to calculate the present value of the benefits:

(i)   Steve's force of mortality at age $x \geq 45$ is $\mu_x = \mu_x^{SULT} + 0.02$, where μ_x^{SULT} is the force of mortality of the Standard Ultimate Life Table.

(ii) $\delta = 0.045$

(iii)   The two-term Woolhouse formula is used to calculate monthly and continuous annuity values, and to calculate insurance functions from the annuity values.

You are given that, based on the assumptions above, including the extra mortality,

$$\ddot{a}_{45} = 14.4893 \qquad\qquad \ddot{a}_{45:\overline{20|}} = 11.4411$$

(a)   (*5 points*)

(i)   Show that the actuarial present value of the survival benefit is 52,050 to the nearest 10. You should calculate the value to the nearest 0.1.

(ii)   Calculate the actuarial present value of the monthly annuity-due.

(iii)   Calculate the actuarial present value of the death benefit.

(b)   (*2 points*) Determine the reserve that JAB should hold at the end of 20 years, assuming that Steve is alive at that time.

1. Continued

    (c)   (*2 points*) JAB recalculates the actuarial present value of the settlement benefits using the following assumptions:

- Mortality follows the Standard Ultimate Life Table with the addition of 0.01 to the force of mortality.

- $\delta = 0.055$

        (i)   Without further calculation, state whether the actuarial present value of the monthly annuity-due would increase, decrease, or stay the same. Justify your answer.

        (ii)   Without further calculation, state whether the actuarial present value of the death benefit would increase, decrease, or stay the same. Justify your answer.

2. (*9 points*) Consider the following expression for the force of mortality:

$$\mu_{x+s} = \alpha e^{\lambda(x+s)}$$

(a) (*3 points*)

(i) Show that for $\alpha > 0, \lambda > 0,$

$$_t p_x = \exp\left\{-\alpha e^{\lambda x}\left(\frac{e^{\lambda t} - 1}{\lambda}\right)\right\}.$$

(ii) Derive an expression for $_t p_x$ when $\lambda = 0.$

(b) (*3 points*)

(i) State the three conditions that a survival function must satisfy in order to be valid.

(ii) Given that $\alpha > 0$ and $\lambda > 0$, show that $\mu_{x+s} = \alpha e^{\lambda(x+s)}$ defines a valid survival model.

(iii) Determine whether the three conditions are satisfied when $\lambda = 0.$

(c) (*3 points*)

(i) Roughly sketch the pattern of the force of mortality for males under age 30, as seen in typical population mortality studies.

(ii) Explain why the model above does not provide a good fit to mortality in this age range.

3. (*9 points*) You are using the following data extract of a mortality study of 30 lives, each age 75 at the start of the study, to estimate the survival function, $S(t) = {}_t p_{75}$.

i	y_i	s_i	b_i
1	1.5	1	1
2	3.0	1	0
3	4.2	2	2
4	8.9	1	2
5	9.3	1	0
$\vdots$	$\vdots$	$\vdots$	$\vdots$

(a) (*2 points*) Show that the Kaplan Meier estimator for $S(5)$ is 0.86 to the nearest 0.01. You should calculate the value to the nearest 0.001.

Kathy, age 75, purchases a single premium 100,000 5-year pure endowment policy.

You are given the following information:

  (i) The premium and reserves are calculated using the estimated survival functions from part (a).

  (ii) $i = 0.06$

(b) (*1 point*) Calculate the net single premium for the pure endowment product.

(c) (*2 points*) Calculate the net premium reserve at time 2 for this policy.

(d) (*2 points*) Using Greenwood's formula, show that the approximate standard deviation of the Kaplan Meier estimator of $S(5)$ is 0.064 to the nearest 0.001. You should calculate the value to the nearest 0.0001.

(e) (*2 points*)

  (i) Calculate the 95% linear confidence interval for $S(5)$.

  (ii) Assume a single premium of 65,500, net of commission. There are no other expenses. Calculate the 95% confidence interval for the expected loss at issue, using the confidence interval from (e)(i) above.

4. (*10 points*) Dana and Mark are each aged 50, and have independent future lifetimes. They purchase a fully discrete last survivor whole life insurance of 100,000 . You are given:

      (i)   Mortality for each life follows the Standard Ultimate Life Table (SULT).

      (ii) $i = 0.05$

      (iii)   Premiums of *G* per year are payable until the second death.

      (iv)   Premiums are payable semi-annually for the first 20 years, then annually thereafter.

      (v)   *G* is calculated using the equivalence principle.

      (vi)   Commissions are 10% of each premium.

      (vii)   Other acquisition expenses, payable at issue, are 70% of *G*.

      (viii) The two-term Woolhouse formula is used to value single and joint life annuities.

(a)   (*1 point*) Show that the probability that exactly one of Dana and Mark survives 20 years, is 0.14 to the nearest 0.01. You should calculate the probability to the nearest 0.0001.

(b)   (*2 points*) Show that $\ddot{a}^{(2)}_{50:50:\overline{20|}}$ is 12.92 to the nearest 0.01. You should calculate the value to the nearest 0.0001.

(c)   (*2 points*) Show that *G* is 850 to the nearest 10. You should calculate the value to the nearest 0.1.

(d)   (*2 points*)

      (i)   Show that the reserve at time 20, if both lives are alive, is 22,400 to the nearest 100. You should calculate the value to the nearest 1.

      (ii)   Calculate the reserve at time 20 if only one of the lives is alive at that time.

(e)   (*3 points*) Calculate the reserve at time 19.5 if both Dana and Mark are alive at that time. Assume a constant force of mortality between ages 69 and 70 for the individual lives.

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5. (*11 points*) For a fully discrete whole life insurance of 100,000 on (40), you are given:

(i) Expenses are 1000 at issue and 50 at the beginning of each subsequent year.

(ii) Commissions are 50% of the first year's premium and 10% of each subsequent year's premium.

(iii) G denotes the level annual gross premium.

(iv) L_0^G denotes the gross loss-at-issue random variable.

(v) Mortality follows the Standard Ultimate Life Table.

(vi) $i = 0.05$

(a) (*4 points*)

(i) Write down an expression for L_0^G in terms of G, $K_{40,}$ and interest rate functions.

(ii) Show that the premium calculated using the equivalence principle is 860 to the nearest 10. You should calculate the premium to the nearest 0.1.

(b) (*2 points*) Calculate L_0^G, for an individual policy, using the premium from part (a) (ii), and assuming that T_{40} lies at its median value.

The company expects to sell 100 identical such policies to individuals with independent future lifetimes. Let L denote the aggregate loss at issue on the portfolio of 100 policies.

(c) (*1 point*) Explain why, if the company charges the equivalence principle premium, $\Pr[L > 0]$ is approximately 0.5. Detailed calculations are not necessary.

5. **Continued**

The Chief Actuary believes that if the company charges the equivalence principle premium, there is too much risk that the portfolio will not be profitable. She proposes setting the gross premium at $G = 900$.

(d)   (*3 points*) Calculate the probability that the aggregate loss on the portfolio is greater than zero, that is, $\Pr[L > 0]$, assuming a premium of 900 per policy, and using the Normal approximation to the distribution of *L*.

(e)   (*1 point*) Your company will hold full preliminary term reserves for this insurance.

State whether the full preliminary term reserve at duration 5 will be higher, lower, or the same if the gross premium is 900 instead of the equivalence principle premium. Justify your answer. You do not need to calculate the reserve.

6. (*8 points*) You act as the valuation actuary for The ACME Corporation, which sponsors a final average salary defined benefit pension plan for its employees. The age retirement benefit provisions and valuation assumptions for ACME's plan are described below.

- The accrual rate is 2% per year of service.

- The final average salary is defined as the salary over the final year of employment.

- The Normal Form of pension is a life annuity with no guarantee, paid monthly in advance.

- The normal retirement age is 65.

- Salaries increase each year on 1 January at a rate of 2.5% per year.

- $i = 0.05$

- Mortality of active members and retirees follows the Standard Ultimate Life Table.

- There are no exits prior to retirement at age 65, other than death.

- The two-term Woolhouse formula is used for annuities paid more frequently than annually.

- The plan is funded using the Traditional Unit Credit method.

You are also given the following summary membership data, as of the valuation date, 1 January 2020.

Age	Number of members	Status	Pension in payment	Salary per member in 2019	Years of service per member
35	20	Active	-	45,000	8
60	5	Active	-	62,000	25
70	1	Retired	32,000	--	30

(a) (*2 points*) Calculate the total actuarial liability for ACME's pension plan as of the valuation date.

(b) (*3 points*) Calculate the normal cost for 2020, expressed as a percentage of the total payroll at the valuation date.

6. Continued

    (c)   (*1 point*) ACME terminated the employment of all the 35-year-old members on the valuation date.

        Calculate the revised normal contribution rate for ACME, expressed as a percentage of the total payroll of the remaining plan members.

    (d)   (*2 points*) Without further calculation, state whether the change in the normal contribution rate would be greater or smaller under Projected Unit Credit funding. Justify your answer.

END OF EXAMINATION

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