Long-Term Actuarial Mathematics
Exam LTAM

Date: Tuesday, April 30, 2019
Time: 8:30 a.m. – 12:45 p.m.

Recognized by the Canadian Institute of Actuaries.

INSTRUCTIONS TO CANDIDATES

General Instructions

1. Write your candidate number here _____ . Your name must not appear.

2. Do not break the seal of this book until the supervisor tells you to do so.

3. Tables for this examination will be distributed by the Supervisor.

4. This examination has a total of 96 points. It consists of:

    Section A: 20 multiple-choice questions, each worth 2 points for a total of 40 points, and

    Section B: 6 written-answer questions, worth a total of 56 points. The point value for each written-answer question is indicated at the beginning of the question.

    You may divide your time between the two sections of the examination (written-answer, and multiple-choice) as you choose. You should keep in mind the relative weight of the two sections.

    Your written-answer paper will be graded only if your multiple-choice score is at or above a threshold set after the examination is administered.

5. Failure to stop writing or coding after time is called will result in the disqualification of your answers or further disciplinary action.

6. While every attempt is made to avoid defective questions, sometimes they do occur. If you believe a question is defective, the supervisor or proctor cannot give you any guidance beyond the instructions on the exam booklet.

Multiple-Choice Instructions

1. A separate answer sheet for the multiple-choice questions is inside the front cover of this book. During the time allotted for this examination, record all your answers on the back of the answer sheet. NO ADDITIONAL TIME WILL BE ALLOWED FOR THIS PURPOSE.

No credit will be given for anything indicated in the examination book but not transferred to the answer sheet.

2. On the front of the answer sheet, space is provided to write and code candidate information. Complete the information requested by printing in the squares and blackening the circles (one in each column) corresponding to the letters or numbers printed. For each empty box blacken the small circle immediately above the "A" circle. Fill out the boxes titled:

    (a) Name
         (include last name, first name and middle initial)

    (b) Candidate Number
         (Candidate/Eligibility Number, use leading zeros if needed to make it a five digit number)

    (c) Test Site Code
         (The supervisor will supply the number.)

    (d) Examination Part
         (Code the examination that you are taking by blackening the circle to the left of "Exam LTAM")

    (e) Booklet Number
         (The booklet number can be found in the upper right-hand corner of this examination book. Use leading zeros if needed to make it a four digit number.)

    In the box titled "Complete this section only if instructed to do so," fill in the circle to indicate if you are using a calculator and write in the make and model number.

    In the box titled "Signature and Date" sign your name and write today's date. **If the answer sheet is not signed, it will not be graded**.

    Leave the boxes titled "Test Code" and "Form Code" blank.

    On the back of the answer sheet fill in the Booklet Number in the space provided.

CONTINUED ON INSIDE FRONT COVER

Exam LTAM

SECTION A – Multiple-Choice

1.   You are given the following information about a select and ultimate mortality table:

(i)   The select period is 2 years.

(ii)   The mortality rates in the select period are equal to 75% of the Standard Ultimate Life Table rates.

(iii)   Ultimate mortality follows the Standard Ultimate Life Table.

Calculate $_{12}p_{[68]}$.

(A)   0.816

(B)   0.818

(C)   0.820

(D)   0.822

(E)   0.824

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2. Which of the following characteristics is unique to an *n*-year endowment insurance, when compared to an *n*-year term insurance, an *n*-year pure endowment, and a whole life insurance?

(A)   The sum insured is paid if a death occurs before the end of the nth policy year.

(B)   The sum insured is paid within *n* years, whether the policyholder lives or dies.

(C)   The sum insured is linked to the performance of an investment fund.

(D)   The sum insured is paid out at the end of the nth policy year.

(E)   The sum insured is paid out if a death occurs after the nth policy year.

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3. You are given:

(i) A life table uses a Makeham's mortality model with parameters
$A = 0.00022$, $B = 2.7 \times 10^{-6}$, $c = 1.124$

(ii) $_{10}p_{50} = 0.9803$

Calculate $\dfrac{d}{dt}\,_t q_{50}$ at $t = 10$.

(A) 0.00292

(B) 0.00298

(C) 0.00304

(D) 0.00310

(E) 0.00316

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4. The following probabilities have been calculated using a multiple state model with 3 states: Healthy (0), Disabled (1), and Dead (2).

x	p_x^{00}	p_x^{01}	p_x^{02}	p_x^{10}	p_x^{12}
70	0.60	0.30	0.10	0.10	0.15
71	0.50	0.30	0.20	0.10	0.25

You are given the following additional information:

(i)   There are 100 Healthy lives, all age 70.

(ii)   The future states of the 100 lives are independent.

(iii) N^d is the random variable representing the number of deaths within the first two years.

Calculate the standard deviation of N^d.

(A)   4.6

(B)   5.6

(C)   6.6

(D)   7.6

(E)   8.6

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5. A pension plan uses the following multiple decrement model:
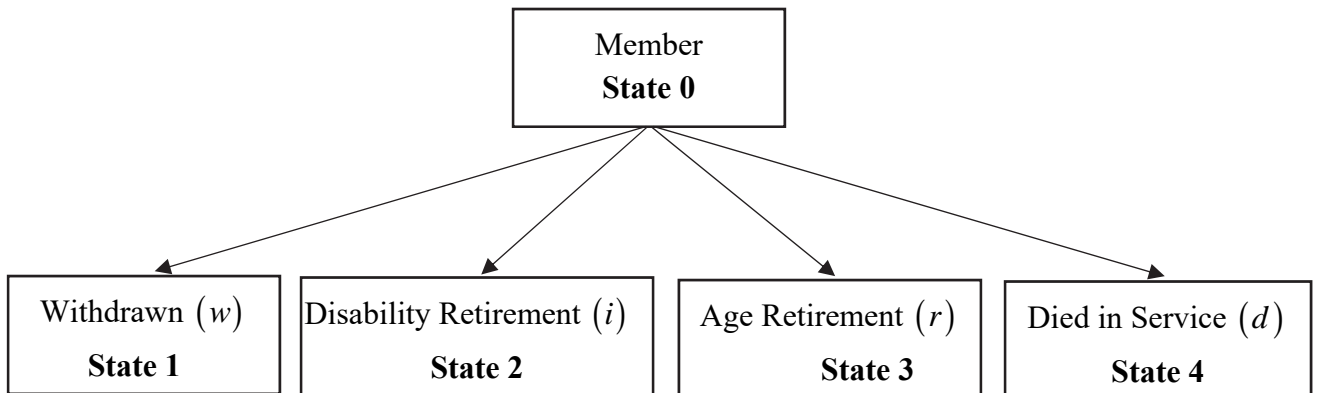
You are given the following information:

(i) All transitions are modeled assuming constant forces of transition between integer ages.

(ii) The following excerpt from the multiple decrement table:

x	l_x	w_x	i_x	r_x	d_x
61	58,622	3,201	812	28,460	413
62	25,736	--	--	--	--

Calculate $_{0.6}p_{61}^{01}$.

(A) 0.038

(B) 0.041

(C) 0.048

(D) 0.055

(E) 0.059

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6. You are given the following information from a mortality study:

      (i)   The first death occured at time 2.1, and the second and third deaths occurred at time 3.2.

      (ii)   The Nelson-Åalen estimators at those times are $\hat{H}(2.1) = \dfrac{1}{30}$ and $\hat{H}(3.2) = \dfrac{1}{6}$.

Calculate the number of lives in the risk set at time 3.2.

(A)   12

(B)   15

(C)   18

(D)   21

(E)   24

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7. Improving maintenance protocols extends the lifetimes of a type of industrial robot. The robots' initial mortality rates and annual improvement factors are given below:

x	$q(x,0)$	φ_x
0	0.4	0.10
1	0.5	0.15
2	0.6	0.20

Calculate the probability that a new robot placed into service at time 0 is still functioning at the end of three years.

(A)   0.2025

(B)   0.2075

(C)   0.2125

(D)   0.2175

(E)   0.2225

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8. For a fully discrete, three-year term insurance issued to a select life age 60, you are given the following information:

(i) The death benefit is 50,000.

(ii) An excerpt from the two-year select and ultimate life table applicable to this insurance:

x	$l_{[x]}$	$l_{[x]+1}$	l_{x+2}	$x+2$
60	100,000	99,000	97,000	62
61	98,000	96,000	93,000	63
62	95,000	92,000	89,000	64
63	91,000	88,000	85,000	65

(iii) $i = 0.05$

Calculate the probability that the present value of the death benefit is less than 45,000.

(A) 0.91

(B) 0.93

(C) 0.95

(D) 0.97

(E) 0.99

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9. A two-year term disability income insurance, issued to a Healthy life age x, offers a benefit of 25,000 at the end of each year if the policyholder is Disabled at that time.
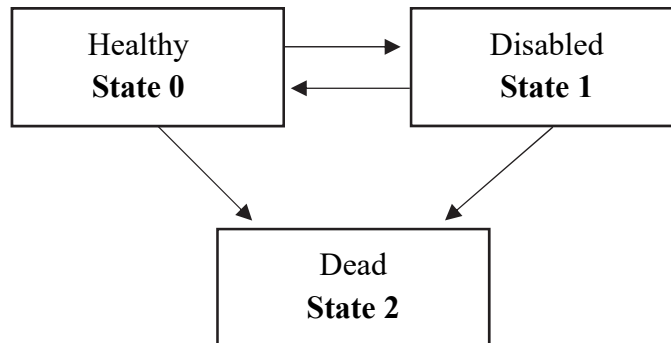
You are given that:

(i) $p^{00}_{x+t} = 0.92$, $\quad p^{01}_{x+t} = 0.06$, $\quad p^{11}_{x+t} = 0.40$, $\quad$ for $t = 0,1$

(ii) $i > 0$

(iii) The expected present value of the benefits is 3,000.

Calculate i.

(A) 0.08

(B) 0.09

(C) 0.10

(D) 0.11

(E) 0.12

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10. You are given the following present value random variable for an annuity on (x):

$$Y = \begin{cases} \overline{a}_{\overline{10}|} & \text{if } T_x \leq 10 \\ \overline{a}_{\overline{T_x}|} & \text{if } T_x > 10 \end{cases}$$

For which of the following types of annuity is Y the present value?

(A)   A 10-year term annuity

(B)   A 10-year guaranteed annuity

(C)   A 10-year deferred annuity

(D)   A 10-year reversionary annuity

(E)   A 10-year family income annuity

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11. An insurer sells a single premium, 22-year term policy with a face amount of 500,000 to (38). You are given:

  (i) The death benefit is payable at the end of the year of death.

  (ii) The premium is calculated assuming that mortality follows the Standard Ultimate Life Table (SULT).

  (iii) There are no expenses.

  (iv) $i = 0.05$

  (v) The single premium is 7,245.

Immediately after issue, it is discovered that the mortality rates should have been 300% of the SULT during the first 2 policy years, but reverting to the SULT mortality rates for the remaining 20 years. The premium is unchanged.

Calculate the expected loss at issue on this policy.

  (A) 580

  (B) 610

  (C) 640

  (D) 670

  (E) 700

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12. For two lives, both age 50, you are given:

      (i)   Mortality follows the Standard Ultimate Life Table.

      (ii)   The future lifetimes are independent.

      (iii) $i = 0.05$

Calculate $\ddot{a}_{\overline{50:50:\overline{20}|}}$.

(A)   12.60

(B)   12.84

(C)   13.08

(D)   13.32

(E)   13.56

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13. QMX Life issues a fully discrete 20-year deferred life annuity-due to (45). You are given:

(i) Premiums are payable at the beginning of each year during the deferral period.

(ii) Annuity payments are 75,000 annually.

(iii) Expenses are as follows:

Expense Type	Year 1	Years 2-20	Years 21+
Percent of Premium	35%	5%	0%
Per Policy	2,000	50	150

(iv) Mortality follows the Standard Ultimate Life Table.

(v) $i = 0.05$

Calculate the gross premium for this policy using the equivalence principle.

(A) 26,200

(B) 27,400

(C) 29,700

(D) 30,800

(E) 32,600

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14. For a fully discrete 10-year endowment insurance of 300,000 on (50), you are given:

(i) The gross annual premium is 26,470.

(ii) Initial non-commission expenses are 17,500.

(iii) Commissions are 5% of all premiums.

(iv) Mortality follows the Standard Ultimate Life Table.

(v) $i = 0.05$

(vi) L_0 denotes the loss-at-issue random variable.

Calculate $\Pr[L_0 < 0]$.

(A) 0.967

(B) 0.971

(C) 0.975

(D) 0.979

(E) 0.983

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15. A life insurer issues a whole life policy of 100,000 to a healthy life age 50, with a partial waiver of premiums on sickness. You are given:

- (i) Net premiums of P per year are payable continuously while the insured is healthy.

- (ii) Net premiums of $0.5P$ per year are payable continuously while the life is sick.

- (iii) The death benefit is paid immediately on death.

- (iv) Mortality and morbidity follow the Standard Sickness-Death Model.

- (v) $i = 0.05$

Calculate P.

- (A) 2500
- (B) 2600
- (C) 2700
- (D) 2800
- (E) 2900

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16. For a fully discrete 20-year endowment insurance of 2,000,000, issued to (50), you are given:

    (i)   Expenses are as follows:

Expense Type	First Year	Renewal
Percent of Premium	35%	5%
Per Policy	2,000	100

    (ii)   The annual gross premium is 60,000.

    (iii)   Mortality follows the Standard Ultimate Life Table.

    (iv) $i = 0.05$

Calculate the gross premium reserve at the end of the 10th policy year.

(A)   700,000

(B)   730,000

(C)   760,000

(D)   790,000

(E)   820,000

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17. For a fully continuous whole life insurance of 100,000, issued to a life age 55, you are given:

(i)   The premium rate is 1,500 per year.

(ii)   There are no expenses.

(iii) Mortality follows the Standard Ultimate Life Table.

(iv)   Deaths are uniformly distributed between integer ages.

(v) $i = 0.05$

Calculate the standard deviation of L_5, the present value of future loss random variable at time 5.

(A)   20,000

(B)   20,200

(C)   20,400

(D)   20,600

(E)   20,800

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18. For a fully discrete 5-year endowment insurance of 5000, you are given:

(i) Expenses incurred at the beginning of each year are:

	First year	Renewal
Percent of Premium	15%	5%
Per Policy	20	8

(ii) The expense reserve 4 years after issue is $_4V^e = -15.78$.

(iii) The annual gross premium, calculated using the equivalence principle, is 960.45.

Calculate the annual net premium.

(A) 740

(B) 790

(C) 840

(D) 890

(E) 940

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19. Abby, who is age 45 on 1/1/2019, is a member of a defined benefit pension plan. The retirement benefit, payable annually at the start of the year, is 1.5% of her three-year final average salary for each year of service.

You are given the following assumptions:

(i)   Abby's salary in 2019 is 65,000, and she has 15 years of service as of 1/1/2019.

(ii)   Her salary is expected to increase by 2.5% in each year on January 1.

(iii)   Retirement occurs only at age 65.

(iv)   No benefits are payable to lives who exit the plan before age 65.

(v) $_{20}p_{45}^{(\tau)} = 0.29$

(vi) $i = 0.06$

(vii) $\ddot{a}_{65} = 9.897$

Calculate the normal contribution for Abby for the year beginning 1/1/2019, using the Projected Unit Credit method.

(A)   1360

(B)   1460

(C)   1560

(D)   1660

(E)   1760

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20. For a retiree health care coverage for a life aged 60 with 30 years of service at the valuation date, you are given the following information:

(i) The health care benefit premium at age 65, $B(65,0)$, is 3000.

(ii) The benefit premium increases each year with a rate of health inflation of $j = 0.04$.

(iii) The benefit premium increases for each year of age by a factor of $c = 1.03$.

(iv) Mortality is the only decrement, and is assumed to follow the Standard Ultimate Life Table.

(v) Retirement occurs only at age 65.

(vi) $\ddot{a}_B(65,0) = 30.5$

(vii) Assume linear accrual of the health care benefit.

(viii) $i = 0.05$

Calculate the normal contribution for this life at the valuation date.

(A) 2000

(B) 2230

(C) 2440

(D) 2560

(E) 2680

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Exam LTAM

SECTION B – Written-Answer

1. (*9 points*) ABC Insurance Company issues a fully discrete whole life insurance of 100,000 to (50). You are given:

(i) The commission is 50% of the first premium and 10% of each subsequent premium.

(ii) Mortality follows the Standard Ultimate Life Table.

(iii) $i = 0.05$

(a) (*3 points*)

(i) Write down an expression for L_0^g, the loss-at-issue random variable for a single policy, in terms of K_{50}, the gross premium G, and interest rate functions.

(ii) Show that the gross premium calculated using the equivalence principle is 1270 to the nearest 10. You should calculate the gross premium to the nearest 1.

The insurance company realizes that using the equivalence principle premium would create an unacceptably high probability of losing money. It decides to set the premium, G^*, using the portfolio percentile premium principle, and a normal approximation, such that the probability of a loss greater than zero, on a portfolio of 100 identical policies issued to independent lives age 50, is 5%.

(b) (*5 points*)

(i) Write an expression for $\mathrm{Var}[L_0^g]$ as a function of G^*. You should simplify your expression as far as possible using standard actuarial functions.

(ii) Show that G^* is 1440 to the nearest 10. You should calculate G^* to the nearest 1.

(c) (*1 point*) Without further calculation, state whether the probability of loss greater than zero on the portfolio would increase, decrease or stay the same if the insurer sells 1000 policies instead of 100, each with a premium of G^*. Justify your answer.

2. (*10 points*) Gary, who is age 45, has recently suffered a disabling injury on the job. His prognosis is uncertain. His annual salary before the accident was 100,000.

Gary receives a structured settlement from MRH Insurance. The settlement is a life annuity, starting immediately, and payable continuously at a rate of 90,000 per year while Gary is disabled. An additional annuity of 20,000 per year is payable continuously while Gary's prognosis is uncertain for up to two years, to offset medical expenses.

(a) (*1 point*) State two reasons why structured settlements often use an annuity format rather than a lump sum.

(b) (*1 point*) State two possible reasons why the long term annuity would replace less than 100% of Gary's pre-injury earnings.

MRH uses the following multiple state model:
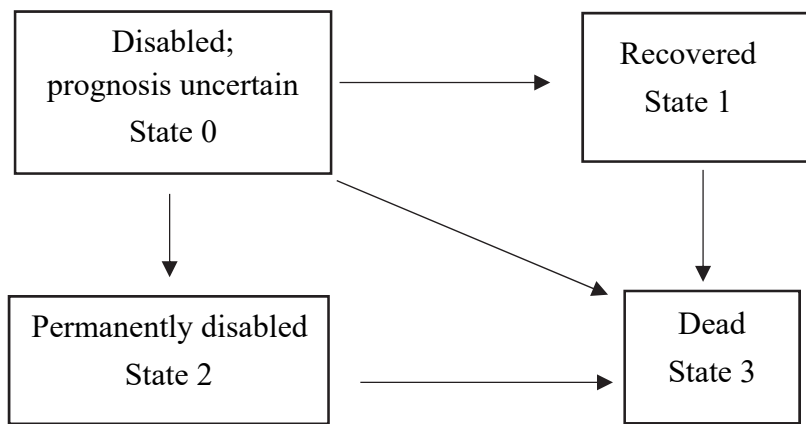
You are given:

(i) MRH holds reserves equal to the expected present value of future benefits.

(ii) $i = 0.04$

You have calculated the following table of annuity values and transition probabilities:

x	$\bar{a}_x^{00}$	$\bar{a}_x^{01}$	$\bar{a}_x^{02}$	$\bar{a}_x^{11}$	$\bar{a}_x^{22}$	$_2p_x^{00}$	$_2p_x^{01}$	$_2p_x^{02}$
45	0.559	5.540	7.161	19.948	10.703	0.0301	0.2766	0.6164
47	0.559	5.420	7.104	19.528	10.623	0.0301	0.2766	0.6162

2. **Continued**

(c)   (*2 points*) Show that the expected present value of future benefits at $t=0$, the start date for the annuity payments, is 705,700 to the nearest 100. You should calculate the value to the nearest 1.

(d)   (*4 points*)

  (i)   Show that $_2V^{(0)}$, the reserve at $t=2$ if Gary is in State 0 at that time, is 689,700 to the nearest 100. You should calculate the reserve to the nearest 1.

  (ii)   Calculate $_2V^{(2)}$, the reserve at $t=2$ if Gary is in State 2 at that time.

  (iii)   Show that the expected present value at $t=0$ of the reserve at $t=2$ is 564,000 to the nearest 1,000. You should calculate the value to the nearest 10.

  (iv)   Calculate the expected present value at $t=0$ of the payments during the first two years.

The chief actuary of MRH reviews your assumptions. She asks you to redo your calculations, increasing μ^{01}_{45+t} by 0.01 for all t, with no other changes.

(e)   (*2 points*)

  (i)   State with reasons whether the value of $_2V^{(2)}$ will increase, decrease, or stay the same as a result of this change.

  (ii)   State with reasons whether the expected present value, at age 45, of the reserve MRH will hold at the end of two years will increase, decrease or stay the same as a result of this change.

3. (*8 points*) Diana is a member of a pension plan that offers an annual retirement pension of 2% of the member's career average earnings for each year of service.

At the valuation date, Diana is exactly 63 years old, has 30 years of service, and her total past earnings are 2,500,000. Her salary in the year following the valuation date will be 160,000 if she works for the full year, and is paid in level monthly payments.

You are given:

(i)   The pension is paid as a monthly life annuity-due.

(ii)   Members can retire at any time after age 60 without actuarial reduction of benefits.

(iii) There are no benefits other than the age retirement pension.

(iv)   The Traditional Unit Credit approach is used for valuation and funding.

(v)   The valuation uses the Standard Service Table.

(vi)   All lives retiring before age 65 are assumed to exit exactly half-way through the year of age.

(vii) Mortality after retirement follows the Standard Ultimate Life Table with Uniform Distribution of Deaths.

(viii) $i = 0.05$

(ix) $\ddot{a}_{63.5}^{(12)} = 13.514, \quad \ddot{a}_{64.5}^{(12)} = 13.231, \quad \ddot{a}_{65}^{(12)} = 13.086$

(a)   (*2 points*) Show that the Actuarial Liability for Diana is 595,000 to the nearest 1000. You should calculate the value to the nearest 1.

(b)   (*3 points*) Show that the Normal Contribution for Diana is 36,000 to the nearest 1000. You should calculate the value to the nearest 10.

3.   Continued

(c)   (*3 points*) Diana is eligible for post-retirement health benefits. The benefits are provided through a group insurance policy with premiums paid annually, starting immediately on retirement. You are given the following information:

- Annual premiums are based on the retiree's age last birthday. The first premium is paid immediately on retirement.

- The premium for a new retiree age 60 last birthday is 2000 at the valuation date, $t = 0$.

- For a new retiree age y last birthday, retiring t years following the valuation date, the initial annual premium is

$$B(y,t) = 2000(1.035)^{y-60}(1.03)^t \quad \text{for} \ \ y = 60, 61, \cdots, 65, \ \text{and} \ t \geq 0.$$

- $\ddot{a}_B(63.5,0) = 30.420, \quad \ddot{a}_B(64.5,0) = 29.124, \quad \ddot{a}_B(65,0) = 28.487$

Calculate the expected present value of Diana's supplementary health insurance benefit.

4. (*11 points*) Pat and Robin, each age 40, buy a fully discrete, last survivor insurance with a sum insured of 100,000.

You are given:

- (i) Premiums are payable while at least one life is alive, for a maximum of 20 years.

- (ii) Mortality of each follows the Standard Ultimate Life Table (SULT).

- (iii) $i = 0.05$

- (iv) With independent future lifetimes, $\ddot{a}_{40:40:\overline{20}|} = 12.9028$.

(a) (*2 points*) Show that the annual net premium assuming that the future lifetimes are independent is 620 to the nearest 10. You should calculate the value to the nearest 1.

(b) (*1 point*) State two reasons why couples may have dependent future lifetimes.

The insurer decides that premiums and reserves for this policy will be determined using a mortality model incorporating dependency.

You are given the following information about this model:

- (i) The future lifetimes for the first 20 years are not independent.

- (ii) If both lives survive 20 years, it is assumed that the future lifetimes from that time will be independent, and will follow the Standard Ultimate Life Table.

- (iii) The mortality of each of Pat and Robin, individually, follows the Standard Ultimate Life Table, whether the other is alive or dead.

- (iv) $\ddot{a}_{40:40:\overline{10}|} = 8.0703$, $\quad \ddot{a}_{40:40:\overline{20}|} = 12.9254$, $\quad _{20}E_{40:40} = 0.35912$, $\quad _{10}E_{50:50} = 0.59290$

- (v) $A_{\overline{50:50}} = 0.13441$

- (vi) $_{10}P_{40:40} = 0.9866$, $\quad _{10}P_{\overline{40:40}} = 0.9980$

4. **Continued**

Use the dependent mortality model for the rest of this question.

(c)   (*3 points*)

(i)   Show that $A_{40:40} = 0.158$ to the nearest 0.001. You should calculate the value to the nearest 0.0001.

(ii)   Show that $_{10}E_{40:40} = 0.606$ to the nearest 0.01. You should calculate the value to the nearest 0.0001.

(iii) Show that $\ddot{a}_{50:50:\overline{10|}} = 8.02$ to the nearest 0.01. You should calculate the value to the nearest 0.0001.

(d)   (*1 point*) Show that the annual net premium is 645 to the nearest 5. You should calculate the value to the nearest 0.1.

(e)   (*3 points*) Let $_{k}L$ denote the net future loss random variable at time k for the insurance.

(i)   Calculate $E\begin{bmatrix} _{10}L \end{bmatrix}$ given that only Pat is alive at time 10.

(ii)   Calculate $E\begin{bmatrix} _{10}L \end{bmatrix}$ given that both Pat and Robin are alive at time 10.

(iii) Calculate $E\begin{bmatrix} _{10}L \end{bmatrix}$ given that at least one of Pat and Robin is alive at time 10.

(f)   (*1 point*) Because the insurer is not informed of the first death for these policies, the actuary decides that they should hold the same reserve for all policies in force. Explain which, if any, of the reserves in part (e) would be a suitable value for the time 10 reserve for each in-force policy.

5. (*8 points*) A 5-year term life insurance policy with a partially accelerated Critical Illness (CI) rider is sold to a Healthy life age x. The policy is to be profit-tested using the multiple state model:
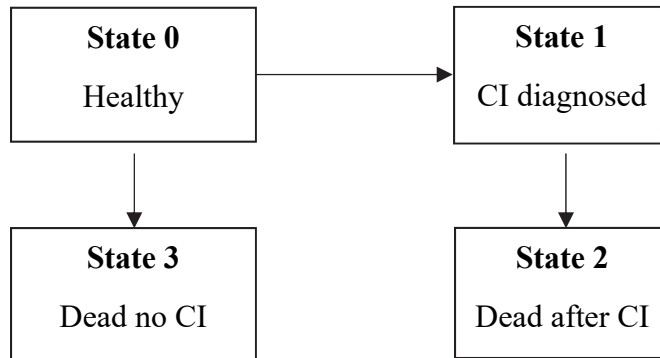
The policy pays 600,000 at the end of the year of CI diagnosis, if the policyholder is still alive at the end of that year; 1,000,000 at the end of the year of death, if the policyholder was in State 0 at the start of that year; and 500,000 at the end of the year of death if the policyholder was in State 1 at the start of that year.

Annual premiums of 30,000 are payable while the policyholder is in State 0.

You are given the following profit test assumptions:

(i) One-year transition probabilities at age $x+k$, for $k=0,1, 2$, are:

$$p^{01}_{x+k} = 0.01+0.002k, \quad p^{02}_{x+k} = 0.008+0.003k, \quad p^{03}_{x+k} = 0.004, \quad p^{12}_{x+k} = 0.25$$

(ii) The state-dependent reserves are:

t	2	3	4
$_tV^{(0)}$	12,000	9,000	5,000
$_tV^{(1)}$	280,000	210,000	120,000

(iii) Pre-contract expenses are 500.

(iv) Commissions are 5% of each premium including the first.

(v) Maintenance expenses in State 1 are 100 at the beginning of the year.

(vi) The insurer earns interest on investments of 6% per year.

(vii) The hurdle rate is 10% per year.

5. **Continued**

(a) (*3 points*) Show that the emerging profit in year 3, conditional on being in State 0 at the start of the year, is 4,900 to the nearest 100. You should calculate your answer to the nearest 1.

(b) (*2 points*) Show that the emerging profit in year 3, conditional on being in State 1 at the start of the year, is 14,200 to the nearest 100. You should calculate your answer to the nearest 1.

(c) (*2 points*) Calculate the profit signature value for year 3, Π_3.

(d) (*1 point*) You are given the first three values of the profit signature:

$$\Pi_0 = -500 \quad \Pi_1 = -770 \quad \Pi_2 = 3536$$

Calculate the partial net present value for the first 3 years, NPV(3).

6. (*10 points*) In a mortality study of a cohort of twelve 80-year-olds, you are given the following observed exit times:

$$1^+, \ 2, \ 2, \ 2^+, \ 4, \ 4^+, \ 5, \ 5^+, \ 7, \ 8, \ 9^+, \ 9^+$$

where "+" indicates censoring; all other exits are deaths.

Let $\hat{S}(t)$ denote the Kaplan-Meier estimate of the time t survival probability, $S(t)$, for a life age 80.

(a) (*3 points*) Specify $\hat{S}(t)$ for all values of $t, 0 < t \leq 9$, and hence show that $\hat{S}(6) = 0.60$ to the nearest 0.01. You should calculate $\hat{S}(6)$ to the nearest 0.001.

(b) (*2 points*) Show that the estimated standard deviation of $\hat{S}(6)$ using Greenwood's formula is 0.16 to the nearest 0.01. You should calculate your value to the nearest 0.001.

(c) (*3 points*)

   (i) Calculate an approximate 95% linear confidence interval for $S(6)$.

   (ii) Calculate an approximate 95% log-transformed confidence interval for $S(6)$.

(d) (*1 point*) Explain why the log-transformed confidence interval is preferred when estimating $S(t)$.

(e) (*1 point*) Suppose that you have a thirteenth observation truncated at time 3 and right-censored before time 4. Without further calculation explain whether this would increase, decrease or have no effect on your estimate of $S(6)$.

END OF EXAMINATION