



Society of Actuaries



Canadian Institute of  
Actuaries

# Exam MLC Models for Life Contingencies

**Tuesday,  
April 28, 2015  
8:30 a.m. – 12:45 p.m.**

# MLC

## INSTRUCTIONS TO CANDIDATES

### General Instructions

1. Write your candidate number here \_\_\_\_\_. Your name must not appear.
2. Do not break the seal of this book until the supervisor tells you to do so.
3. Tables and numerical values necessary for solving some of the questions on this examination will be distributed by the Supervisor.
4. This examination has a total of 96 points. It consists of:

Section A: 20 multiple-choice questions, each worth 2 points for a total of 40 points, and

Section B: 7 written-answer questions, worth a total of 56 points. The point value for each written-answer question is indicated at the beginning of the question.

You may divide your time between the two sections of the examination (written-answer, and multiple-choice) as you choose. You should keep in mind the relative weight of the two sections.

Your written-answer paper will be graded only if your multiple-choice score is at or above a threshold set after the examination is administered.

5. Failure to stop writing after time is called will result in the disqualification of your answers or further disciplinary action.
6. While every attempt is made to avoid defective questions, sometimes they do occur. If you believe a question is defective, the supervisor or proctor cannot give you any guidance beyond the instructions on the exam booklet.

### Multiple-Choice Instructions

1. A separate answer sheet for the multiple-choice questions is inside the front cover of this book. During the time allotted for this examination, record all your answers on the back of the answer sheet. **NO ADDITIONAL TIME WILL BE ALLOWED FOR THIS PURPOSE.**

No credit will be given for anything indicated in the examination book but not transferred to the answer sheet. Failure to stop writing or coding your answer sheet after time is called will result in the disqualification of your answer sheet or further disciplinary action.

2. On the front of the answer sheet, space is provided to write and code candidate information. Complete the information requested by printing in the squares and blackening the circles (one in each column) corresponding to the letters or numbers printed. For each empty box blacken the small circle immediately above the "A" circle. Fill out the boxes titled:

- (a) Name  
(include last name, first name and middle initial)
- (b) Candidate Number  
(Candidate/Eligibility Number, use leading zeros if needed to make it a five digit number)
- (c) Test Site Code  
(The supervisor will supply the number.)
- (d) Examination Part  
(Code the examination that you are taking by blackening the circle to the left of "Exam MLC")
- (e) Booklet Number  
(The booklet number can be found in the upper right-hand corner of this examination book. Use leading zeros if needed to make it a four digit number.)

In box titled "Complete this section only if instructed to do so," fill in the circle to indicate if you are using a calculator and write in the make and model number.

In the box titled "Signature and Date" sign your name and write today's date. **If the answer sheet is not signed, it will not be graded.**

Leave the boxes titled "Test Code" and "Form Code" blank.

On the back of the answer sheet fill in the Booklet Number in the space provided.

**CONTINUED ON INSIDE FRONT COVER**

3. Your score will be based on the number of questions which you answer correctly. No credit will be given for omitted answers and no credit will be lost for wrong answers: hence, you should answer all questions even those for which you have to guess.
4. Five answer choices are given with each multiple-choice question, each answer choice being identified by a key letter (A to E). Answer choices for some questions have been rounded. For each question, blacken the circle on the answer sheet which corresponds to the key letter of the answer choice that you select.
5. Use a soft-lead pencil to mark the answer sheet. To facilitate correct mechanical scoring, be sure that, for each question, your pencil mark is dark and completely fills only the intended circle. Make no stray marks on the answer sheet. If you have to erase, do so completely.
6. Do not spend too much time on any one question. If a question seems too difficult, leave it and go on.
7. Clearly indicated answer choices in the test book can be an aid in grading examinations in the unlikely event of a lost answer sheet.
8. Use the blank portions of each page for your scratch work. Extra blank pages are provided at the back of the examination book.
9. After the examination, the supervisor will collect this book and the answer sheet separately. **DO NOT ENCLOSE THE ANSWER SHEET IN THE BOOK OR IN THE ESSAY ANSWER ENVELOPE.** All books and answer sheets must be returned. **THE QUESTIONS ARE CONFIDENTIAL AND MAY NOT BE TAKEN FROM THE EXAMINATION ROOM.**

### **Written-Answer Instructions**

1. Write your candidate number at the top of each sheet. Your name must not appear.
2. Write on only one side of a sheet. Start each question on a fresh sheet. On each sheet, write the number of the question you are answering. Do not answer more than one question on a single sheet.
3. The answer should be confined to the question as set.
4. When you are asked to calculate, show all your work including any applicable formulas.
5. When you finish, insert all your written-answer sheets into the Essay Answer Envelope. Be sure to hand in all your answer sheets because they cannot be accepted later. Seal the envelope and write your candidate number in the space provided on the outside of the envelope. Check the appropriate box to indicate Exam MLC.
6. Be sure your essay answer envelope is signed because if it is not, your examination will not be graded.
7. For all parts of all problems, to maximize the credit earned, candidates should show as much work as possible, considering the time allotted for the question. Answers lacking justification will receive no credit. Answers should be organized so that the methods, logic, and formulas used are readily apparent. Candidates should not round their answers excessively; enough precision should be provided so that their answers can be accurately graded.

In some cases, candidates are asked to show that a calculation results in a particular number. Typically the answer given will be rounded; candidates should provide a greater level of accuracy than the number given in the question. This structure of question is intended to assist the candidate by giving an indication when the calculation has been done incorrectly, providing an opportunity to explore an alternative approach. It also allows a candidate who cannot obtain the correct answer to use the answer given to proceed with subsequent parts of the problem. (Candidates who are able to solve the problem should use their exact answer for subsequent parts.)

For questions requiring candidates to derive or write down a formula or equation, the resulting expression should be simplified as far as possible, and where numerical values are provided in the problem, they should be used.

## **Exam MLC**

### **SECTION A – Multiple-Choice**

**\*\*BEGINNING OF EXAMINATION\*\***

- 1.** A club is established with 2000 members, 1000 of exact age 35 and 1000 of exact age 45.

You are given:

- (i) Mortality follows the Illustrative Life Table.
- (ii) Future lifetimes are independent.
- (iii)  $N$  is the random variable for the number of members still alive 30 years after the club is established.

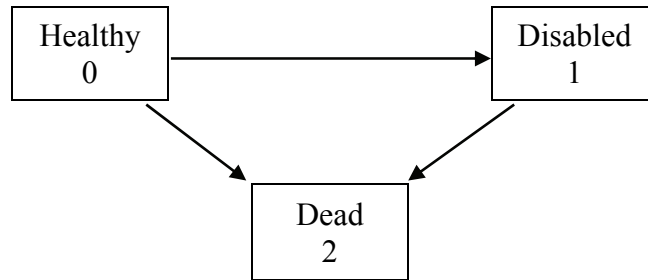
Using the normal approximation, without the continuity correction, calculate the smallest  $n$  such that  $\Pr(N \geq n) \leq 0.05$ .

- (A) 1414
- (B) 1422
- (C) 1428
- (D) 1434
- (E) 1440

**USE THIS PAGE FOR YOUR SCRATCH WORK**

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2. You are pricing a type of disability insurance using the following model:



The insurance will pay a benefit only if, by age 65, the insured had been disabled for a period of at least one year. You are given the following forces of transition:

(i)  $\mu^{01} = 0.02$

(ii)  $\mu^{02} = 0.03$

(iii)  $\mu^{12} = 0.11$

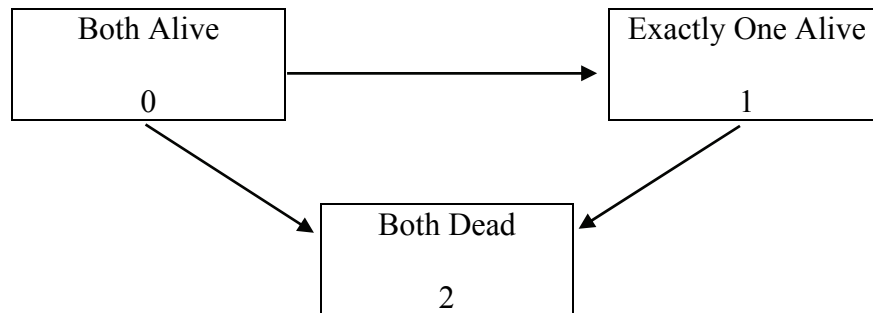
Calculate the probability that a benefit will be paid for a Healthy life aged 50 who purchases this insurance.

- (A) 0.14  
(B) 0.16  
(C) 0.18  
(D) 0.20  
(E) 0.22

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3. You are using the following multiple state model for the future lifetimes of (30) and (30):



You are given:

(i)  $\mu_{30+t:30+t}^{01} = 0.014 + 0.0007 \times 1.075^{(30+t)}, \quad t \geq 0$

(ii)  $\mu_{30+t:30+t}^{02} = 0.006, \quad t \geq 0$

Calculate  ${}_{10}p_{30:30}^{00}$ .

- (A) 0.73
- (B) 0.75
- (C) 0.77
- (D) 0.79
- (E) 0.81



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4. The present value random variable for an insurance policy on  $(x)$  is expressed as:

$$Z = \begin{cases} 0, & \text{if } T_x \leq 10 \\ v^{T_x}, & \text{if } 10 < T_x \leq 20 \\ 2v^{T_x}, & \text{if } 20 < T_x \leq 30 \\ 0, & \text{thereafter} \end{cases}$$

Which of the following is a correct expression for  $E[Z]$ ?

- (A)  ${}_{10|}\bar{A}_x + {}_{20|}\bar{A}_x - {}_{30|}\bar{A}_x$
- (B)  $\bar{A}_x + {}_{20}E_x \bar{A}_{x+20} - 2 {}_{30}E_x \bar{A}_{x+30}$
- (C)  ${}_{10}E_x \bar{A}_x + {}_{20}E_x \bar{A}_{x+20} - 2 {}_{30}E_x \bar{A}_{x+30}$
- (D)  ${}_{10}E_x \bar{A}_{x+10} + {}_{20}E_x \bar{A}_{x+20} - 2 {}_{30}E_x \bar{A}_{x+30}$
- (E)  ${}_{10}E_x \left[ \bar{A}_{x+10} + {}_{10}E_{x+10} \bar{A}_{x+20} - {}_{10}E_{x+20} \bar{A}_{x+30} \right]$

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**5.** You are given:

- (i)  $Z_1$  is the present value random variable for an  $n$ -year term insurance of 1000 issued to  $(x)$ .
- (ii)  $Z_2$  is the present value random variable for an  $n$ -year endowment insurance of 1000 issued to  $(x)$ .
- (iii) For both  $Z_1$  and  $Z_2$  the death benefit is payable at the end of the year of death.
- (iv)  $E[Z_1] = 528$
- (v)  $Var(Z_2) = 15,000$
- (vi)  $A_{x:n|}^1 = 0.209$
- (vii)  ${}^2A_{x:n|}^1 = 0.136$

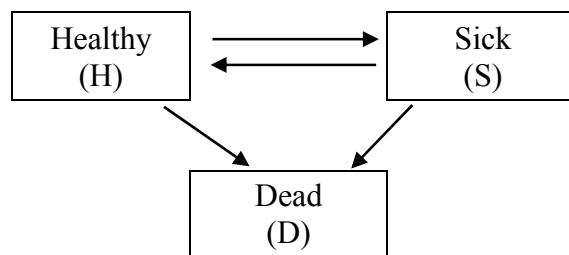
Calculate  $Var(Z_1)$ .

- (A) 143,400
- (B) 177,500
- (C) 211,200
- (D) 245,300
- (E) 279,300

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6. A life insurance company uses the following 3-state model to calculate premiums for a 3-year sickness policy issued to Healthy lives.



The company will pay a benefit of 20,000 at the end of each year if the policyholder is Sick at that time.

The insurance company uses the following transition probabilities, applicable in each of the three years:

	H	S	D
H	0.950	0.025	0.025
S	0.300	0.600	0.100
D	0.000	0.000	1.000

Calculate the expected present value at issue of sickness benefit payments using an interest rate of 6%.

- (A) 1805
- (B) 1870
- (C) 1935
- (D) 2000
- (E) 2065

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7. You are given:

(i)  $A_{35} = 0.188$

(ii)  $A_{65} = 0.498$

(iii)  ${}_{30}p_{35} = 0.883$

(iv)  $i = 0.04$

Calculate  $1000 \ddot{a}_{35:\overline{30}|}^{(2)}$  using the two-term Woolhouse approximation.

(A) 17,060

(B) 17,310

(C) 17,380

(D) 17,490

(E) 17,530



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**8.** For a fully continuous whole life insurance of 1 on  $(x)$ , you are given:

- (i)  $L$  is the present value of the loss at issue random variable if the premium rate is determined by the equivalence principle.
- (ii)  $L^*$  is the present value of the loss at issue random variable if the premium rate is 0.06.
- (iii)  $\delta = 0.07$
- (iv)  $\bar{A}_x = 0.30$
- (v)  $Var(L) = 0.18$

Calculate  $Var(L^*)$ .

- (A) 0.18
- (B) 0.21
- (C) 0.24
- (D) 0.27
- (E) 0.30

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9. For a fully discrete 10-year deferred whole life annuity-due of 1000 per month on (55), you are given:

- (i) The premium,  $G$ , will be paid annually at the beginning of each year during the deferral period.
- (ii) Expenses are expected to be 300 per year for all years, payable at the beginning of the year.
- (iii) Mortality follows the Illustrative Life Table.
- (iv)  $i = 0.06$

Using the two-term Woolhouse approximation, the expected loss at issue is  $-800$ .

Calculate  $G$ .

- (A) 7800
- (B) 7900
- (C) 8000
- (D) 8100
- (E) 8200

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**10.** For a special fully discrete whole life insurance policy of 1000 on (90), you are given:

- (i) The first year premium is 0.
- (ii)  $P$  is the renewal premium.
- (iii) Mortality follows the Illustrative Life Table.
- (iv)  $i = 0.06$
- (v) Premiums are calculated using the equivalence principle.

Calculate  $P$ .

- (A) 230
- (B) 250
- (C) 275
- (D) 300
- (E) 330

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**11.** For a special fully continuous whole life insurance on  $(x)$ , you are given:

(i) Premiums and benefits:

	First 20 years	After 20 years
Premium Rate	$3P$	$P$
Benefit	1,000,000	500,000

(ii)  $\mu_{x+t} = 0.03, t \geq 0$

(iii)  $\delta = 0.06$

Calculate  $P$  using the equivalence principle.

(A) 10,130

(B) 10,190

(C) 10,250

(D) 10,310

(E) 10,370



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**12.** For a fully discrete 5-payment whole life insurance of 1000 on (40), you are given:

- (i) Expenses incurred at the beginning of the first five policy years are as follows:

	Year 1		Years 2-5	
	Percent of premium	Per policy	Percent of premium	Per policy
Sales Commissions	20%	0	5%	0
Policy Maintenance	0%	10	0%	5

- (ii) No expenses are incurred after Year 5.
- (iii) Mortality follows the Illustrative Life Table.
- (iv)  $i = 0.06$

Calculate the gross premium using the equivalence principle.

- (A) 41
- (B) 46
- (C) 51
- (D) 56
- (E) 61

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- 13.** Bill and Laura, each age 45, with independent future lifetimes, purchase a special life insurance policy with the following provisions:
- (i) Premiums are payable annually, at the beginning of the year, for as long as Bill and Laura are both alive.
  - (ii) The policy pays, at the beginning of the year, 60,000 per year while only Laura is alive.
  - (iii) The policy pays, at the beginning of the year, 3 times the net premium per year while only Bill is alive.
  - (iv) Mortality follows the Illustrative Life Table.
  - (v)  $i = 0.06$

Calculate the net premium for this special life insurance policy.

- (A) 7,500
- (B) 10,000
- (C) 12,500
- (D) 15,000
- (E) 17,500

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**14.** For a fully discrete whole life insurance of 1 on  $(x)$ , you are given:

- (i) The net premium reserve at the end of the first year is 0.012.
- (ii)  $q_x = 0.009$
- (iii)  $i = 0.04$

Calculate  $\ddot{a}_x$ .

- (A) 17.1
- (B) 17.6
- (C) 18.1
- (D) 18.6
- (E) 19.1

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**15.** For a fully discrete 2-year term life insurance on (50), you are given:

- (i) Cash flows are annual.
- (ii) The annual gross premium is 250.
- (iii) The annual hurdle rate used for profit calculations is 10%.
- (iv) The profit vector is  $(-165, 100, 125)$ .
- (v) The profit margin for this insurance is 6%.

Calculate the probability that (50) will survive one year.

- (A) 0.95
- (B) 0.96
- (C) 0.97
- (D) 0.98
- (E) 0.99



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**16.** For a Type A universal life insurance on (50), you are given:

- (i) The face amount of the policy is 30,000.
- (ii) The account value at the end of year 5 is 20,000.
- (iii) A premium of 1500 is paid at the beginning of year 6.
- (iv) Expense charges of 145 are deducted at the start of each year.
- (v) The death benefit is paid at the end of the year of death.
- (vi) Interest is credited at 6% per year.
- (vii) The cost of insurance is based on 125% of the mortality rates from the Illustrative Life Table, discounted at an interest rate of 4%.
- (viii) The corridor factor is 1.8.
- (ix)  $AV_6^{cf}$  represents the account value at the end of year 6 assuming the death benefit is based on the corridor factor.

Calculate  $AV_6^{cf}$ .

- (A) 21,970
- (B) 22,190
- (C) 22,310
- (D) 22,430
- (E) 22,550

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**17.** For a fully discrete whole life insurance policy on (61), you are given:

- (i) The annual gross premium using the equivalence principle is 500.
- (ii) Initial expenses, incurred at policy issue, are 15% of the premium.
- (iii) Renewal expenses, incurred at the beginning of each year after the first, are 3% of the premium.
- (iv) Mortality follows the Illustrative Life Table.
- (v)  $i = 0.06$

Calculate the amount of the death benefit.

- (A) 13,660
- (B) 13,700
- (C) 13,740
- (D) 13,780
- (E) 13,820

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**18.** For a fully discrete whole life insurance of 100,000 on (40) you are given:

- (i) Expenses incurred at the beginning of the first year are 300 plus 50% of the first year premium.
- (ii) Renewal expenses, incurred at the beginning of the year, are 10% of each of the renewal premiums.
- (iii) Mortality follows the Illustrative Life Table.
- (iv)  $i = 0.06$
- (v) Gross premiums are calculated using the equivalence principle.

Calculate the gross premium reserve for this insurance immediately after the second premium and associated renewal expenses are paid.

- (A) 200
- (B) 540
- (C) 880
- (D) 1220
- (E) 1560

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**19.** You are given the following information for John, exact age 30, who just joined a defined benefit pension plan:

- (i) The plan provides a retirement pension of 1.6% of final average salary for each year of service. The final average salary is defined as the average salary in the three years before retirement.
- (ii) John's salary is currently 40,000 and is expected to increase 3.5% annually on his birthdays.
- (iii) John will retire at exact age 65.

Calculate the replacement ratio provided by his pension.

- (A) 46%
- (B) 48%
- (C) 50%
- (D) 52%
- (E) 54%



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**20.** Fred and Glenn are identical twins currently age 90.

- Both started working for the same company at age 25.
- Both were paid 120,000 per year at age 59.
- The company gives salary increases of 4800 per year after age 59.

The company has a pension plan that pays a continuous pension benefit as follows:

- The pension benefit is 2% of final one-year salary for each year of service.
- The normal retirement age is 65.
- For retirement before age 65, the pension reduction factor is 4% per year.

Fred retired at exact age 60 and Glenn at exact age 65.

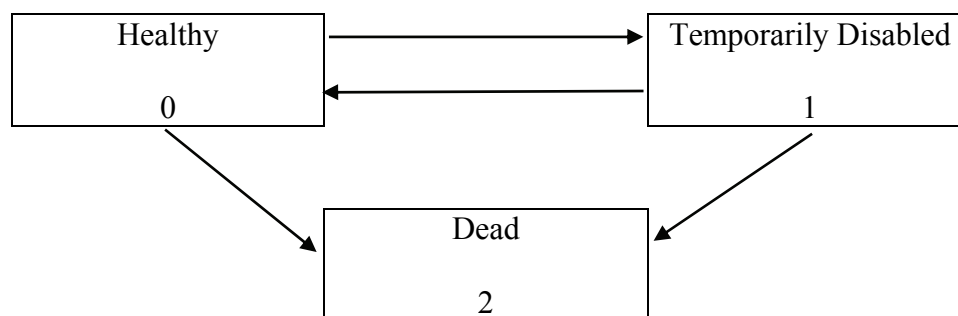
Calculate the age by which they had both received the same total benefits (discounted at 0%).

- (A) 71.0
- (B) 71.5
- (C) 72.0
- (D) 72.5
- (E) 73.0

## **Exam MLC**

### **SECTION B – Written-Answer**

1. (9 points) You are using the following 3-state Markov model to price a 10-year disability insurance product.



- (a) (2 points) Show that for this model,  $\sum_{j=0}^2 \bar{a}_{x:\overline{10}|}^{0j} = \bar{a}_{\overline{10}|}$ .

The product has the following features:

- The product is issued to individuals age  $x$  who are in the Healthy state.
- The product pays a continuous disability benefit at a rate of 1000 per year while the insured is in the Temporarily Disabled state.
- The product pays a death benefit of 10,000 at the moment of death.
- Net premiums are payable continuously while the insured is in the Healthy state.

You are also given the following information:

$$\begin{array}{llll} \delta = 0.1 & \bar{a}_{x:\overline{10}|}^{00} = 4.49 & \bar{a}_{x:\overline{10}|}^{02} = 1.36 & \bar{A}_{x:\overline{10}|}^{02} = 0.3871 \\ \mu_{x+t}^{01} = 0.04 & \mu_{x+t}^{02} = 0.02t & \mu_{x+t}^{10} = 0.05 & \mu_{x+t}^{12} = 0.04t \end{array}$$

- (b) (2 points) Show that the net premium rate for this policy is 970 per year to the nearest 10. You should calculate the rate to the nearest 1.

Let  ${}_tV^{(i)}$  denote the net premium reserve for a policy in state  $i$  at time  $t$ . You are also given:

$${}_3V^{(0)} = 1304.54 \qquad {}_3V^{(1)} = 7530.09$$

- (c) (2 points) Calculate  $\frac{d}{dt} {}_tV^{(0)}$  at  $t = 3$ .

**1. Continued**

- (d) (3 points) Your company is considering adding an additional feature to this product. Under this additional feature, the insurer would return the sum of the premiums paid at the end of 10 years without interest if no benefits were paid during the life of the policy.

Calculate the increase in the net premium rate payable continuously for the product as a result of including this feature.

2. (6 points) You are given the following excerpt from a triple decrement table:

$x$	$l_x^{(\tau)}$	$d_x^{(1)}$	$d_x^{(2)}$	$d_x^{(3)}$
60	1000	$d_{60}^{(1)}$	60	45
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

You are also given the following information about the decrements:

- $\mu_{60+t}^{(1)} = 1.2t$  for  $0 \leq t \leq 1$
  - Decrement 2 happens exactly halfway through the year.
  - Decrement 3 happens at the end of the year.
- (a) (2 points) Calculate  $q_{60}'^{(2)}$ .
- (b) (2 points) Calculate  $d_{60}^{(1)}$ .
- Now suppose instead that Decrement 2 occurs at the start of the year, and that each  $q_{60}'^{(i)}$  remains unchanged.
- (c) (2 points) State with reasons the effect (increase, decrease, no change, cannot be determined) that this change would have on the following probabilities:
- (i)  $q_{60}^{(1)}$
  - (ii)  $q_{60}^{(2)}$
  - (iii)  $q_{60}^{(3)}$

3. (8 points) A life insurance company issues a whole life annuity immediate with annual payments. The annuity is issued to (65). The annuity pays 50,000 at the end of each year for the life of the annuitant with the first payment being made at age 66.

A single premium is paid to purchase this annuity.

You are given:

- (i) Mortality follows the Illustrative Life Table.
  - (ii)  $i = 0.06$
  - (iii) An expense of 100 is incurred to make each annuity payment.
  - (iv) Issue expense incurred at issue is 3000 per policy.
  - (v) Commissions are 10% of the single premium.
- (a) (2 points) Assume that the single gross premium,  $G$ , is 110% of the expected present value at issue of benefits and expenses. Show that  $G$  is 555,000 to the nearest 5,000. You should calculate  $G$  to the nearest 100.
- (b) (2 points) Using  $G$  from Part (a), calculate the probability that this policy will generate a profit.

Another life insurance company issued 8000 independent annuities identical to that described above. This company determines the single gross premium per policy,  $G^P$ , using the portfolio percentile premium principle such that the probability that the present value of the loss at issue on the portfolio is negative is 90%.

- (c) (3 points) Show that  $G^P$  is 500,000 to the nearest 10,000. You should calculate  $G^P$  to the nearest 100.
- (d) (1 point)  $G^P$  is calculated so that the portfolio of annuities has a 90% chance of generating a profit.  $G$  is such that the probability of an individual policy having a profit is less than 90%.

Explain why  $G^P$  is less than  $G$ .

4. (9 points) For a three-year special decreasing term insurance policy on  $(x)$ , the death benefit is paid at the end of the year of death. The level annual net premium is  $P$ .

You are given:

- (i)  $d = 0.10$
  - (ii)  $q_{x+k} = 0.1(2^k)$ ,  $k = 0, 1, 2$
  - (iii)  ${}_1V^n = 8147.08$  and  ${}_2V^n = 12,480.86$ , are the net premium reserves at the end of year 1 and year 2, respectively.
  - (iv) The death benefit during the first year is 180,000.
- (a) (2 points) Show that  $P$  is 23,000 to the nearest 500. You should calculate  $P$  to the nearest 1.
  - (b) (2 points) Show that the death benefits, to the nearest 5000, during the second and third year, are 120,000 and 100,000, respectively. You should calculate the death benefits to the nearest 1000.

You calculate reserves for this policy using the Full Preliminary Term (FPT) reserve method.

- (c) (3 points)
  - (i) Calculate the net premium for the first year using the FPT reserve method.
  - (ii) Calculate the net premium for years 2 and 3 using the FPT reserve method.
- (d) (1 point) Calculate the reserve at the end of the second year using the FPT reserve method.
- (e) (1 point) FPT is a modified net premium reserve method. Explain the purpose of modified net premium reserve methods.



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5. (9 points) A fully discrete whole life insurance of 10,000 was issued 20 years ago to a policyholder then age 40.

You are given:

- (i) The original gross premium is 120.
  - (ii) Commissions are 5% of premium before and after any change.
  - (iii) Mortality follows the Illustrative Life Table.
  - (iv)  $i = 0.06$
  - (v) At the end of 20 years, the cash surrender value is 85% of the gross premium reserve.
  - (vi) For evaluating policy changes, the insurer uses the equivalence principle and:
    - The cash surrender value
    - A charge of 80 for the change
    - The premiums, commissions and benefits after the change
- (a) (2 points) Show that the cash surrender value at the end of year 20 is 2060 to the nearest 10. You should calculate the value to the nearest 1.
- (b) (5 points)
- (i) The policyholder is considering changing his policy to a level term insurance expiring at age 70. He would continue to pay premiums of 120 until age 70.  
  
Show that the revised death benefit would be 20,500 to the nearest 100. You should calculate the value to the nearest 1.
  - (ii) The policyholder is considering changing his policy to an endowment insurance of 10,000 maturing at age 70. Calculate the revised premium.
  - (iii) The policyholder is considering changing his policy to whole life insurance of 9,000. Calculate the revised premium.
- (c) (1 point) Based on the revised premiums and benefits in (b), state which of those three possible changes the policyholder would be least likely to make. Justify your answer.

**5. Continued**

- (d) *(1 point)* Many companies would do underwriting for some policy changes or would not use the same mortality assumption for all possible changes. However, this insurer does no additional underwriting with respect to policy changes.

State which of those three possible changes would be the most likely to be evaluated using a higher mortality assumption. Justify your answer.

6. (9 points) For universal life insurance policies with a death benefit of 100,000 plus the account value issued to independent lives age 50, you are given:

(i)

Policy Year	Annual Premium	Percent of Premium Charge	Annual Cost of Insurance Rate per 1000	Annual Expense Charge	Surrender Charge
1	2500	5%	5.00	30	$0.8 * AV_1$

(ii) The credited interest rate is  $i = 0.06$ .

- (a) (2 points) Show that the cash surrender value at the end of policy year 1 is 400 to the nearest 10. You should calculate the value to the nearest 1.

You are also given the following information for profit-testing these policies:

(i) The expenses for each policy are:

- Pre-contract expenses of 200
- Annual maintenance expenses of 120 at the beginning of each year including the first year
- Surrender benefit expenses of 100
- Death benefit expenses of 200

(ii)  $q_{50} = 0.004$

(iii) Death benefits are payable at the end of the year of death.

(iv) At the end of the first year, 10% of the policyholders in force are expected to surrender.

(v) The earned rate is 11%.

(vi) The hurdle rate is 14%.

- (b) (3 points) Show that the expected profit emerging at the end of year 1 is 400 to the nearest 10. You should calculate the value to the nearest 1.

- (c) (1 point) Calculate NPV(1), the actuarial present value of expected profits through year 1.

**6. Continued**

You have calculated that the Net Present Value (NPV) for the policy is 2000. Your supervisor suggests that the 10% surrender assumption is too low. He recommends assuming 20% of the policies in force surrender at the end of year 1.

- (d) (3 points) Calculate the revised NPV using the new surrender assumption.

7. (6 points) Susie begins work at age 40 at ABC Life on January 1, 2014, with a starting salary of 30,000. She will switch jobs to XYZ Re at some time before age 55 at her then-current salary and will remain at XYZ Re until retirement.

- Both companies offer 2% annual salary raises on January 1 of each year.
- The annual retirement benefit at ABC Life is 900 per year of service.
- The annual retirement benefit at XYZ Re is 3% of the final 3-year average salary for each year of service.

Susie will retire on her 65<sup>th</sup> birthday. She will receive retirement benefits from both companies.

- (a) (2 points) Assume Susie stays at ABC Life for 9.5 years and then switches to XYZ Re. Show that the replacement ratio would be 63% to the nearest 1%. You should calculate the replacement ratio to the nearest 0.1%.
- (b) (1 point) Calculate the maximum length of time that Susie can remain at ABC Life and still attain a replacement ratio of at least 65%.

Susie switches to XYZ Re after seven years on January 1, 2021 (and gets the annual raise). Later, on January 1, 2029, XYZ Re decides to stop all benefit accruals. No further benefits accrue after this date.

On January 1, 2029, Susie purchases a 10-year deferred whole life annuity due, with premiums payable annually during the deferred period. The annuity payments, combined with her accrued retirement benefits, will give her a replacement ratio of 65%.

You are given:

- Susie's mortality follows the Illustrative Life Table
  - $i = 6\%$
- (c) (3 points) Calculate the annual net premium for the annuity.

**\*\*END OF EXAMINATION\*\***

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