



Society of Actuaries



Canadian Institute of  
Actuaries

# Exam MLC Models for Life Contingencies

Friday,  
October 31, 2014  
8:30 a.m. – 12:45 p.m.

# MLC

## INSTRUCTIONS TO CANDIDATES

### General Instructions

1. Write your candidate number here \_\_\_\_\_. Your name must not appear.
2. Do not break the seal of this book until the supervisor tells you to do so.
3. Tables and numerical values necessary for solving some of the questions on this examination will be distributed by the Supervisor.
4. This examination has a total of 96 points. It consists of:

Section A: 20 multiple-choice questions, each worth 2 points for a total of 40 points, and

Section B: 6 written-answer questions, worth a total of 56 points. The point value for each written-answer question is indicated at the beginning of the question.

You may divide your time between the two sections of the examination (written-answer, and multiple-choice) as you choose. You should keep in mind the relative weight of the two sections.

Your written-answer paper will be graded only if your multiple-choice score is at or above a threshold set after the examination is administered.

5. Failure to stop writing after time is called will result in the disqualification of your answers or further disciplinary action.
6. While every attempt is made to avoid defective questions, sometimes they do occur. If you believe a question is defective, the supervisor or proctor cannot give you any guidance beyond the instructions on the exam booklet.

### Multiple-Choice Instructions

1. A separate answer sheet for the multiple-choice questions is inside the front cover of this book. During the time allotted for this examination, record all your answers on the back of the answer sheet. **NO ADDITIONAL TIME WILL BE ALLOWED FOR THIS PURPOSE.**

No credit will be given for anything indicated in the examination book but not transferred to the answer sheet. Failure to stop writing or coding your answer sheet after time is called will result in the disqualification of your answer sheet or further disciplinary action.

2. On the front of the answer sheet, space is provided to write and code candidate information. Complete the information requested by printing in the squares and blackening the circles (one in each column) corresponding to the letters or numbers printed. For each empty box blacken the small circle immediately above the "A" circle. Fill out the boxes titled:

- (a) Name  
(include last name, first name and middle initial)
- (b) Candidate Number  
(Candidate/Eligibility Number, use leading zeros if needed to make it a five digit number)
- (c) Test Site Code  
(The supervisor will supply the number.)
- (d) Examination Part  
(Code the examination that you are taking by blackening the circle to the left of "Exam MLC")
- (e) Booklet Number  
(The booklet number can be found in the upper right-hand corner of this examination book. Use leading zeros if needed to make it a four digit number.)

In box titled "Complete this section only if instructed to do so," fill in the circle to indicate if you are using a calculator and write in the make and model number.

In the box titled "Signature and Date" sign your name and write today's date. **If the answer sheet is not signed, it will not be graded.**

Leave the boxes titled "Test Code" and "Form Code" blank.

On the back of the answer sheet fill in the Booklet Number in the space provided.

CONTINUED ON INSIDE FRONT COVER

3. Your score will be based on the number of questions which you answer correctly. No credit will be given for omitted answers and no credit will be lost for wrong answers: hence, you should answer all questions even those for which you have to guess.
4. Five answer choices are given with each multiple-choice question, each answer choice being identified by a key letter (A to E). Answer choices for some questions have been rounded. For each question, blacken the circle on the answer sheet which corresponds to the key letter of the answer choice that you select.
5. Use a soft-lead pencil to mark the answer sheet. To facilitate correct mechanical scoring, be sure that, for each question, your pencil mark is dark and completely fills only the intended circle. Make no stray marks on the answer sheet. If you have to erase, do so completely.
6. Do not spend too much time on any one question. If a question seems too difficult, leave it and go on.
7. Clearly indicated answer choices in the test book can be an aid in grading examinations in the unlikely event of a lost answer sheet.
8. Use the blank portions of each page for your scratch work. Extra blank pages are provided at the back of the examination book.
9. After the examination, the supervisor will collect this book and the answer sheet separately. **DO NOT ENCLOSE THE ANSWER SHEET IN THE BOOK OR IN THE ESSAY ANSWER ENVELOPE.** All books and answer sheets must be returned. **THE QUESTIONS ARE CONFIDENTIAL AND MAY NOT BE TAKEN FROM THE EXAMINATION ROOM.**

### **Written-Answer Instructions**

1. Write your candidate number at the top of each sheet. Your name must not appear.
2. Write on only one side of a sheet. Start each question on a fresh sheet. On each sheet, write the number of the question you are answering. Do not answer more than one question on a single sheet.
3. The answer should be confined to the question as set.
4. When you are asked to calculate, show all your work including any applicable formulas.
5. When you finish, insert all your written-answer sheets into the Essay Answer Envelope. Be sure to hand in all your answer sheets because they cannot be accepted later. Seal the envelope and write your candidate number in the space provided on the outside of the envelope. Check the appropriate box to indicate Exam MLC.
6. Be sure your essay answer envelope is signed because if it is not, your examination will not be graded.
7. For all parts of all problems, to maximize the credit earned, candidates should show as much work as possible, considering the time allotted for the question. Answers lacking justification will receive no credit. Answers should be organized so that the methods, logic, and formulas used are readily apparent. Candidates should not round their answers excessively; enough precision should be provided so that their answers can be accurately graded.

In some cases, candidates are asked to show that a calculation results in a particular number. Typically the answer given will be rounded; candidates should provide a greater level of accuracy than the number given in the question. This structure of question is intended to assist the candidate by giving an indication when the calculation has been done incorrectly, providing an opportunity to explore an alternative approach. It also allows a candidate who cannot obtain the correct answer to use the answer given to proceed with subsequent parts of the problem. (Candidates who are able to solve the problem should use their exact answer for subsequent parts.)

For questions requiring candidates to derive or write down a formula or equation, the resulting expression should be simplified as far as possible, and where numerical values are provided in the problem, they should be used.

## **Exam MLC**

### **SECTION A – Multiple-Choice**

**\*\*BEGINNING OF EXAMINATION\*\***

- 1.** For two lives, (60) and (70), with independent future lifetimes, you are given:

(i)  ${}_5p_{60} = 0.92$

(ii)  ${}_5p_{70} = 0.88$

(iii)  $1000q_{65} = 21.32$

(iv)  $1000q_{75} = 51.69$

Calculate  $1000 {}_5|q_{\overline{60:70}}$ .

- (A) 6.3  
(B) 6.6  
(C) 6.9  
(D) 7.2  
(E) 7.5

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**2.** For a double decrement table, you are given:

(i)  $q'_x^{(1)} = 0.1$

(ii)  $q_x^{(2)} = 0.2$

(iii) Each decrement is uniformly distributed over each year of age in its associated single decrement table.

Calculate  $q_x^{(1)}$ .

(A) 0.0895

(B) 0.0915

(C) 0.0935

(D) 0.0955

(E) 0.0975

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3. Patients are classified as Sick (S), Critical (C), or Discharged (D). Transitions occur daily according to the following transition matrix:

$$\begin{array}{c} \begin{array}{ccc} & \text{S} & \text{C} & \text{D} \\ \text{S} & \begin{pmatrix} 0.60 & 0.20 & 0.20 \end{pmatrix} \\ \text{C} & \begin{pmatrix} 0.10 & 0.50 & 0.40 \end{pmatrix} \\ \text{D} & \begin{pmatrix} 0.00 & 0.00 & 1.00 \end{pmatrix} \end{array} \end{array}$$

Calculate the probability that a patient who is classified as Sick today will be classified as Sick three days later.

- (A) 0.216
- (B) 0.234
- (C) 0.250
- (D) 0.267
- (E) 0.284



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**4.** You are given:

(i)  $A_{35:\overline{15}|} = 0.39$

(ii)  $A_{35:\overline{15}|}^1 = 0.25$

(iii)  $A_{35} = 0.32$

Calculate  $A_{50}$ .

(A) 0.35

(B) 0.40

(C) 0.45

(D) 0.50

(E) 0.55

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- 5.** For a 20-year deferred whole life annuity-due with annual payments of 30,000 on (40), you are given:
- (i) The single net premium is refunded without interest at the end of the year of death if death occurs during the deferral period.
  - (ii) Mortality follows the Illustrative Life Table.
  - (iii)  $i = 0.06$

Calculate the single net premium for this annuity.

- (A) 91,700
- (B) 93,600
- (C) 95,600
- (D) 97,500
- (E) 99,500

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6. Mr. and Mrs. Peters, both age 65, purchase a contract providing the following benefits:

- (i) An annuity-due of  $R$  per year payable while both are alive, reducing to  $0.6R$  per year after the death of Mr. Peters as long as Mrs. Peters is alive, and reducing to  $0.7R$  per year after the death of Mrs. Peters as long as Mr. Peters is alive.
- (ii) A life insurance benefit of 100,000 payable at the end of the year of death of Mr. Peters, whether or not Mrs. Peters is alive.

You are given:

- (i) Future lifetimes are independent.
- (ii) The mortality of each life follows the Illustrative Life Table.
- (iii)  $i = 0.06$
- (iv) The single net premium for this contract is 1,000,000.

Calculate  $R$ .

- (A) 53,000
- (B) 66,500
- (C) 73,000
- (D) 82,000
- (E) 91,000

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7. For a fully discrete whole life insurance of 1 on (50), you are given:

- (i) Expenses of 0.20 at the start of the first year and 0.01 at the start of each renewal year are incurred.
- (ii) Mortality follows the Illustrative Life Table.
- (iii)  $i = 0.06$

Gross premiums are determined using the equivalence principle.

Calculate the variance of  $L_0$ , the gross loss-at-issue random variable.

- (A) 0.044
- (B) 0.058
- (C) 0.082
- (D) 0.093
- (E) 0.102



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**8.** For a special fully discrete 3-year term insurance on (75), you are given:

(i) The death benefit during the first two years is the sum of the net premiums paid without interest.

(ii) The death benefit in the third year is 10,000.

(iii)

$x$	$p_x$
75	0.90
76	0.88
77	0.85

(iv)  $i = 0.04$

Calculate the annual net premium.

(A) 449

(B) 459

(C) 469

(D) 479

(E) 489

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9. For a special fully discrete 15-year endowment insurance on (75), you are given:

- (i) The death benefit is 1000.
- (ii) The endowment benefit is the sum of the net premiums paid without interest.
- (iii)  $d = 0.04$
- (iv)  $A_{75:\overline{15}|} = 0.70$
- (v)  $A_{75:\overline{15}|}^{\frac{1}{d}} = 0.11$

Calculate the annual net premium.

- (A) 80
- (B) 90
- (C) 100
- (D) 110
- (E) 120

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**10.** For a whole life insurance of 100,000 on (45) with premiums payable monthly for a period of 20 years, you are given:

- (i) The death benefit is paid immediately upon death.
- (ii) Mortality follows the Illustrative Life Table.
- (iii) Deaths are uniformly distributed over each year of age.
- (iv)  $i = 0.06$

Calculate the monthly net premium.

- (A) 150
- (B) 152
- (C) 154
- (D) 156
- (E) 158

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**11.** For fully discrete 30-payment whole life insurance policies on  $(x)$ , you are given:

- (i) The following expenses payable at the beginning of the year:

	1 <sup>st</sup> Year	Years 2 – 15	Years 16 – 30	Years 31 and after
Per policy	60	30	30	30
Percent of premium	80%	20%	10%	0%

- (ii)  $\ddot{a}_x = 15.3926$
- (iii)  $\ddot{a}_{x:\overline{15}|} = 10.1329$
- (iv)  $\ddot{a}_{x:\overline{30}|} = 14.0145$
- (v) Annual gross premiums are calculated using the equivalence principle.
- (vi) The annual gross premium is expressed as  $kF + h$ , where  $F$  is the death benefit and  $k$  and  $h$  are constants for all  $F$ .

Calculate  $h$ .

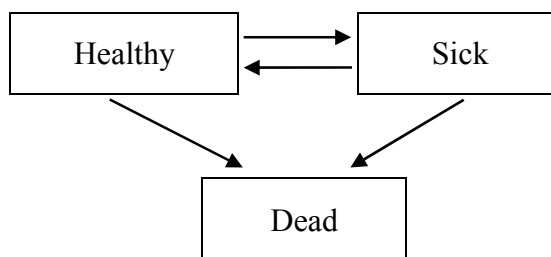
- (A) 30.3
- (B) 35.1
- (C) 39.9
- (D) 44.7
- (E) 49.5



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12. You are analyzing the sensitivity of some of the assumptions used in setting the premium rate for a sickness policy. You are basing your calculations on a multiple state model as diagrammed below:



You are given:

- (i) Level premiums are paid continuously by Healthy policyholders.
- (ii) Level sickness benefits are paid continuously to Sick policyholders.
- (iii) There is no death benefit.

Which one of the following changes to the assumptions will be certain to increase the premium rate?

- (A) A lower rate of interest and a higher recovery rate from the Sick state.
- (B) A lower mortality rate for those in the Healthy state and a lower mortality rate for those in the Sick state.
- (C) A higher mortality rate for those in the Healthy state and a higher mortality rate for those in the Sick state.
- (D) A lower recovery rate from the Sick state and a lower mortality rate for those in the Sick state.
- (E) A higher rate of interest and a lower mortality rate for those in the Healthy state.

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**13.** For a fully discrete whole life insurance of 100,000 on (45), you are given:

- (i) The gross premium reserve at duration 5 is 5500 and at duration 6 is 7100.
- (ii)  $q_{50} = 0.009$
- (iii)  $i = 0.05$
- (iv) Renewal expenses at the start of each year are 50 plus 4% of the gross premium.
- (v) Claim expenses are 200.

Calculate the gross premium.

- (A) 2200
- (B) 2250
- (C) 2300
- (D) 2350
- (E) 2400

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**14.** For a fully discrete whole life insurance of 100 on  $(x)$ , you are given:

- (i)  $q_{x+15} = 0.10$
- (ii) Deaths are uniformly distributed over each year of age.
- (iii)  $i = 0.05$
- (iv)  ${}_tV$  denotes the benefit reserve at time  $t$ .
- (v)  ${}_{16}V = 49.78$

Calculate  ${}_{15.6}V$ .

- (A) 49.7
- (B) 50.0
- (C) 50.3
- (D) 50.6
- (E) 50.9

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**15.** For a fully discrete 5-payment whole life insurance of 1000 on (80), you are given:

- (i) The gross premium is 130.
- (ii)  $q_{80+k} = 0.01(k+1), \quad k = 0, 1, 2, \dots, 5$
- (iii)  $v = 0.95$
- (iv)  $1000A_{86} = 683$
- (v)  ${}_3L$  is the prospective loss random variable at time 3, based on the gross premium.

Calculate  $E[{}_3L]$ .

- (A) 330
- (B) 350
- (C) 360
- (D) 380
- (E) 390



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**16.** For a fully discrete whole life insurance of 1 on  $(x)$ , you are given:

(i)  $q_{x+10} = 0.02067$

(ii)  $v^2 = 0.90703$

(iii)  $A_{x+11} = 0.52536$

(iv)  ${}^2A_{x+11} = 0.30783$

(v)  ${}_kL$  is the prospective loss random variable at time  $k$ .

Calculate  $\frac{\text{Var}({}_{10}L)}{\text{Var}({}_{11}L)}$ .

(A) 1.006

(B) 1.010

(C) 1.014

(D) 1.018

(E) 1.022

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- 17.** An insurer issues identical fully discrete 10-year term insurance policies of 100,000 to independent lives, all age 60. The annual premium is 1500 for each policy.

You are given the following profit test assumptions:

- For each policy, expenses of 100 are incurred at the beginning of each of the first 2 years.
- $q_{60} = 0.010$ ,  $q_{61} = 0.012$
- The reserve for a policy in force at the end of the first year is 400, and for a policy in force at the end of the second year is 700.
- $i = 0.072$

Calculate the expected profit emerging at the end of the second year, per policy in force at the start of the second year.

- (A) 30
- (B) 38
- (C) 46
- (D) 54
- (E) 62

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- 18.** For a Type B universal life insurance policy, you are given:

Policy Year	Annual Premium	Percent of Premium Charge	Annual Expense Charge	Additional Death Benefit	Annual Cost of Insurance (COI) Rate	Annual Discount Rate for COI	Annual Credited Interest Rate
1	2500	1%	50	100,000	0.0028	5.0%	4.5%
2	3000	1%	50	95,000	0.0030	5.0%	5.2%

Calculate the account value at the end of year 2.

- (A) 5160
- (B) 5180
- (C) 5200
- (D) 5220
- (E) 5240

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**19.** For a career average pension plan with one participant currently exact age 55, you are given:

- (i) The annual retirement benefit is 1.75% of total earnings during the participant's years of service. Retirement occurs at age 65. The retirement benefit is payable monthly with the first payment at age 65.
- (ii) Current salary after the increase at age 55 is 50,000. Future salary increases will be 3% and will occur on each birthday.
- (iii) Total past earnings are 525,000.
- (iv)  $i = 0.04$
- (v)  $\ddot{a}_{65}^{(12)} = 12.60$
- (vi) Death is the only decrement before age 65.
- (vii)  $_{10}p_{55} = 0.925$

Calculate the actuarial present value at the participant's current age of the annual retirement benefit.

- (A) 129,300
- (B) 134,800
- (C) 140,300
- (D) 145,800
- (E) 151,300



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**20.** For a mortality table with a select period of two years, you are given:

(i)

$x$	$q_{[x]}$	$q_{[x]+1}$	$q_{x+2}$	$x+2$
50	0.0050	0.0063	0.0080	52
51	0.0060	0.0073	0.0090	53
52	0.0070	0.0083	0.0100	54
53	0.0080	0.0093	0.0110	55

(ii) The force of mortality is constant between integral ages.

Calculate  $1000 {}_{2.5}q_{[50]+0.4}$ .

- (A) 15.2
- (B) 16.4
- (C) 17.7
- (D) 19.0
- (E) 20.2

## **Exam MLC**

### **SECTION B – Written-Answer**

1. (12 points) For a fully discrete whole life insurance of 100,000 on (35), you are given:

(i)

Policy Year	Commission Rate	Per Policy Expenses
1	100%	100
2-10	10%	8
11 and later	0	8

(ii) Per policy expenses are incurred at the beginning of each policy year.

(iii) Mortality follows the Illustrative Life Table.

(iv)  $i = 0.06$

(v)  $G$  is the gross premium.

(vi)  $G$  is calculated such that the actuarial present value of gross premiums is 110% of the actuarial present value of benefits, commissions, and per policy expenses.

(vii)  $C$  is the random variable for the present value at issue of commissions.

(a) (2 points)  $E[C]$  can be written in the form  $kG$ . Calculate  $k$ .

(b) (1 point) Calculate the expected present value at issue of per policy expenses.

(c) (2 points) Show that  $G$  is 1060, rounded to the nearest 10. You should calculate  $G$  to the nearest 1.

(d) (3 points) Calculate  $\text{Var}(C)$ .

The company charges a premium of 95 per month for a policyholder who wishes to pay monthly. Commission rates during renewal policy years are as given above for annual premium policies.

$C^*$  is the present value at issue of commissions for a policy paying premiums monthly.

**1. Continued**

- (e) (3 points) Using the 2-term Woolhouse formula, calculate the commission rate during the first policy year for a policy paying premiums monthly, so that  $E[C^*] = E[C]$ .

Your company also sells a Type A universal life policy of 100,000 on (35) with commission rates the same as those on the whole life policy.

- (f) (1 point) Identify a characteristic of universal life which would suggest that commissions paid on the universal life policy would be more variable than commissions paid on the whole life policy. Support your answer.

2. (9 points) XYZ Company sponsors a defined benefit pension plan. Key attributes of the pension plan are:

- Benefits are payable as a single life annuity, paid at the beginning of each month.
- The annual benefit payable at age 65 is calculated as 2% of the final 3-year average salary up to 100,000 multiplied by years of service, plus 3% of the final 3-year average salary over 100,000 multiplied by years of service. The monthly benefit is the annual benefit divided by 12.
- You are given:
  - (i)  $i = 5\%$
  - (ii)  $A_{65}^{(12)} = 0.470$

Tom joined XYZ Company on January 1, 2009 at exact age 40. He is planning to retire on January 1, 2034 at age 65. Tom's salary for 2014 is 80,000 and he receives annual raises of 4% each January 1.

- (a) (2 points) Assuming Tom works until age 65, show that the monthly accrued benefit at age 65 is 8050, to the nearest 10. You should calculate the benefit to the nearest 1.
- (b) (1 point) Calculate the replacement ratio at age 65.
- (c) (2 points) Calculate the expected present value of the benefit at age 65.

In addition to accruing a benefit in the defined benefit plan, Tom will contribute a constant percentage of his annual salary to a defined contribution plan.

You are also given:

- (iii) Contributions are made at the start of each year, beginning on January 1, 2014.
- (iv) Contributions earn an investment return of 7% per year.
- (v) The contribution balance is converted to a monthly life annuity using assumptions (i) and (ii) above.
- (d) (4 points) Calculate the percentage of each year's salary that Tom needs to contribute so that his total replacement ratio is 80% at age 65.

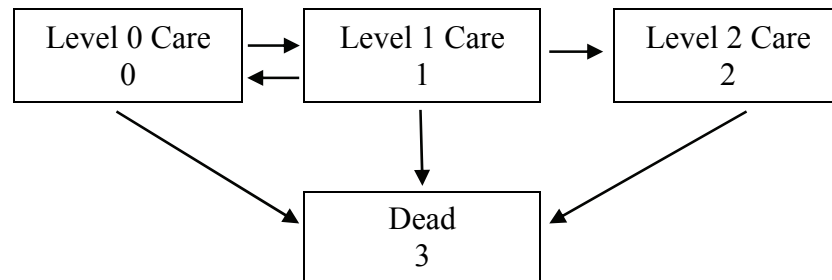
3. (6 points) For  $(x)$  and  $(y)$  with independent future lifetimes, you are given that  $q_x = 0.2$  and  $q_y = 0.1$ .

- (a) (1 point) Explain in words the meaning of the probability described by the symbol  $q_{xy}$ .

You are also given that mortality within integral ages follows a uniform distribution of deaths assumption for each of  $(x)$  and  $(y)$  individually.

- (b) (2 points) Sketch the graph of  ${}_s p_x$  as a function of  $s$  for  $0 \leq s \leq 1$ . You should mark numerical values on each axis.
- (c) (3 points) Show that  ${}_s q_{xy} = s q_{xy} + g(s) q_{xy}^-$  for  $0 \leq s \leq 1$ , where  $g(s)$  is a function of  $s$  that you should specify.

4. (8 points) A long-term care provider offers three care levels. Transitions are modeled as a Markov multiple-state model. Transitions and states are shown in the following diagram:



- (a) (1 point) Write down Kolmogorov's forward differential equations with the associated boundary conditions for this model for:
- (i)  ${}_t p_x^{10}$
- (ii)  ${}_t p_x^{11}$
- (b) (3 points) Estimate  ${}_1 p_{80}^{10}$  using Euler's forward method, a step size of  $h = \frac{1}{3}$ , and the transition intensities and probabilities in the table below:

$t$	${}_t p_{80}^{11}$	$\mu_{80+t}^{01}$	$\mu_{80+t}^{03}$	$\mu_{80+t}^{10}$	$\mu_{80+t}^{12}$	$\mu_{80+t}^{13}$	$\mu_{80+t}^{23}$
0	1.00000	0.10000	0.02981	0.08000	0.15000	0.05962	0.11924
$\frac{1}{3}$	0.90346	0.10000	0.03082	0.08000	0.15000	0.06164	0.12328
$\frac{2}{3}$	0.81652	0.10000	0.03186	0.08000	0.15000	0.06373	0.12746
1	--	0.10000	0.03294	0.08000	0.15000	0.06589	0.13178

- (c) (4 points) You are given the following annuity values at 5%:

$x$	$\bar{a}_x^{00}$	$\bar{a}_x^{01}$	$\bar{a}_x^{02}$	$\bar{a}_x^{11}$	$\bar{a}_x^{10}$	$\bar{a}_x^{12}$	$\bar{a}_x^{22}$
80	5.5793	1.3813	0.6109	3.0936	1.6719	1.7206	4.4712
85	4.8066	1.0396	0.3403	2.6723	0.8834	1.0883	3.2367



#### 4. Continued

You are also given:

- ${}_5p_{80}^{00} = 0.53880$        ${}_5p_{80}^{01} = 0.17327$        ${}_5p_{80}^{02} = 0.06956$
- $i = 0.05$
- Residents pay a service fee of 8000 per year continuously while in Level 0 or Level 1 Care.
- Level 2 Care costs are paid at a continuous rate of 30,000 per year for lives age 80 to 85, and 40,000 per year for lives older than 85.

Ada, who is age 80, is currently in Level 0 Care.

- (i) Calculate the expected present value of Ada's future Level 0 and Level 1 service fees.
- (ii) Calculate the expected present value of Ada's future Level 2 Care costs.

- 5.** (11 points) For a special 3-year term life insurance issued to (50) with a premium refund feature, you are given:

- (i) The death benefit is 100,000.
- (ii) The premium refund feature refunds the last premium payment, without interest, at the end of the 3-year term if the insured is still alive.
- (iii) Mortality follows the Illustrative Life Table.
- (iv) Pre-contract expenses are 155.
- (v) Commissions are 5% of each premium.
- (vi) The hurdle rate is 14%.
- (vii) The reserves of this policy have been set to:

$t$	${}_tV$
0	0
1	400
2	800

- (viii) The annual premium for this policy is 1100.
- (ix) The earned interest rates are:

Year 1	Year 2	Year 3
0.01	0.02	0.03

- (a) (1 point) Show that the expected profit in policy year 2 for a policy in force at the start of year 2 is 37 to the nearest 1. You should calculate your answer to the nearest 0.01.
- (b) (4 points) Calculate the profit vector of this policy.
- (c) (3 points) Calculate the profit signature and Net Present Value (NPV) of this policy.

## 5. Continued

- (d) (3 points) Rank from low to high the Internal Rate of Return (IRR) of the following products, explaining your order.

Product A: The special 3-year term life insurance described above.

Product B: A 3-year term life insurance policy with the following profit signature:  $[-155, 0, 0, 210]$

Product C: The same special 3-year term life insurance as Product A, except that the reserves of the product have been set to:

$t$	${}_tV$
0	0
1	300
2	800

- 6.** (10 points) For a Type A universal life policy issued to (50), you are given:
- (i) The face amount is 100,000.
  - (ii) All cash flows occur at policy anniversaries.
  - (iii) The policyholder pays an initial premium of 15,000.
  - (iv) The cost of insurance (COI) is calculated based on 120% of the mortality in the Illustrative Life Table. The interest rate for discounting the net amount at risk,  $i^q$ , is 0.04.
  - (v) The expense charge is 1% of premium.
  - (vi) The credited interest rate for policy year 1 is 5%.
  - (vii) The corridor factor in year 1 is 2.2.
  - (viii) The surrender charge in policy year 1 is 5% of the premium paid.
- (a) (1 point) Explain why the corridor factor requirement exists for universal life insurance.
- (b) (5 points)
- (i) Calculate the COI in policy year 1 assuming there is no corridor factor requirement.
  - (ii) Calculate the COI in policy year 1 based only on the corridor factor (as if the face amount were 0).
  - (iii) Determine the COI in policy year 1.
- (c) (2 points) Calculate the account value, additional death benefit and cash surrender value at the end of policy year 1.

**6. Continued**

- (d) (2 points) The policy contains a no-lapse guarantee providing term insurance of 100,000 until age 70.

If the expected present value of the guaranteed insurance coverage is greater than the account value, the company holds a reserve for the no-lapse guarantee equal to the difference. The expected present value is based on the Illustrative Life Table at 6% interest and no expenses.

The account value at the end of policy year 10 is 10,000.

Calculate the reserve for the no-lapse guarantee at the end of year 10.

**\*\*END OF EXAMINATION\*\***

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