April 2014 MLC Multiple Choice Solutions

1. Key: C

We need to determine $_{3|2.5}q_{90}$.

$$_{3|2.5}q_{90} = \frac{l_{90+3} - l_{90+3+2.5}}{l_{90}} = \frac{l_{93} - l_{95.5}}{l_{90}} = \frac{l_{93} - (l_{95} - 0.5d_{95})}{l_{90}} = \frac{825 - [600 - 0.5(240)]}{1,000} = 0.3450$$

where $l_{90} = 1,000$, $l_{93} = 825$, $l_{97} = \dfrac{d_{97}}{q_{97}} = \dfrac{72}{1} = 72$, $l_{96} = \dfrac{l_{97}}{p_{96}} = \dfrac{72}{0.2} = 360$,

$l_{95} = \dfrac{l_{96}}{p_{95}} = \dfrac{360}{1 - 0.4} = 600$, and $d_{95} = l_{95} - l_{96} = 600 - 360 = 240$.

2. Key: D

$$e_{[61]} = e_{[61]:\overline{3}|} + {_3}p_{[61]}(e_{64})$$

$p_{[61]} = 0.90,$

$_2p_{[61]} = 0.9(0.88) = 0.792,$

$_3p_{[61]} = 0.792(0.86) = 0.68112$

$$e_{[61]:\overline{3}|} = \sum_{k=1}^{3} {_k}p_{[61]} = 0.9 + 0.792 + 0.68112 = 2.37312$$

$$e_{[61]} = 2.37312 + 0.68112 e_{64} = 2.37312 + 0.68112(5.10) = 5.847$$

3. Key: B

Let x be the person's age.
$$
\begin{aligned}
_{15}p_x^{\overline{11}} &= \exp\left[-\int_0^{15} \left(\mu_{x+s}^{10} + \mu_{x+s}^{12} \right) ds \right] \\
&= \exp\left[-\int_0^{15} (0.10 + 0.05) ds \right] \\
&= \exp[-15(0.15)] \\
&= \exp[-2.25] = 0.1054
\end{aligned}
$$

4. Key: B

$$E[Z(1)] = q_x v_{4.5\%} = 0.014354 \Rightarrow q_x = 0.015$$

$$E[Z(2)] = q_x v_{4.5\%} + p_x v_{4.5\%} v_j q_{x+1} = 0.032308 \qquad j \text{ is the forward rate from 1 to 2}$$

$$\Rightarrow 0.014354 + 0.985(v_{4.5\%})(v_j)(0.02) = 0.032308$$

$$\Rightarrow v_j (v_{4.5\%}) = 0.911371$$

$$\Rightarrow (1+i)^2 = 0.911371^{-1} \qquad i \text{ is 2-year spot rate}$$

$$\Rightarrow i = 4.7497\%$$

5. Key: C

Let A_{51}^{ILT} designate A_{51} using the Illustrative Life Table at 6%.

$$
\begin{aligned}
\text{APV (insurance)} &= 1000 \left(\frac{1}{1.05} \right) \left(q_{50} + p_{50} A_{51}^{ILT} \right) \\
&= 1000 \left(\frac{1}{1.05} \right) \left[\frac{5.92}{1000} + \left(1 - \frac{5.92}{1000} \right)(0.25961) \right] \\
&= 251.4220
\end{aligned}
$$

6. Key: D

Let Y_i be the present value random variable of the payment to life i.

$$E[Y_i] = \ddot{a}_x = \frac{1 - A_x}{d} = 11.55 \qquad Var[Y_i] = \frac{{}^2 A_x - \left(A_x \right)^2}{d^2} = \frac{0.22 - 0.45^2}{(0.05/1.05)^2} = 7.7175$$

Then $Y = \sum_{i=1}^{100} Y_i$ is the present value of the aggregate payments.

$$E[Y] = 100 E[Y_i] = 1155 \text{ and } Var[Y] = 100 Var[Y_i] = 771.75$$

$$\Pr[Y \le F] = \Pr \left[Z \le \frac{F - 1155}{\sqrt{771.75}} \right] = 0.95 \Rightarrow \frac{F - 1155}{\sqrt{771.75}} = 1.645$$

$$\Rightarrow F = 1155 + 1.645\sqrt{771.75} = 1200.699$$

7. Key: A

$$\text{APV(insurance)} = 1000 \int_0^\infty e^{-0.05t} \, {}_t p_{xy}^{00} \, \mu^{03} \, dt$$

$$= 1000(0.005) \int_0^\infty e^{-0.05t} \, e^{-0.045t} \, dt$$

$$= 1000 \frac{(0.005)}{0.095} = 52.63158$$

8. Key: B

$$100,000 A_{40} = P[\ddot{a}_{40:\overline{10|}} + 0.5 \, {}_{10|}\ddot{a}_{40:\overline{10|}}]$$

$$P = \frac{100,000 A_{40}}{\ddot{a}_{40:\overline{10|}} + 0.5 \, {}_{10|}\ddot{a}_{40:\overline{10|}}} = \frac{100,000(0.16132)}{7.6967 + 0.5(4.0645)} = \frac{16,132}{9.72895} = 1658.14$$

where
$$\ddot{a}_{40:\overline{10|}} = \ddot{a}_{40} - {}_{10}E_{40} \, \ddot{a}_{50} = 14.8166 - (0.53667)(13.2668) = 7.6967$$

and
$${}_{10|}\ddot{a}_{40:\overline{10|}} = {}_{10}E_{40} \left[\ddot{a}_{50} - {}_{10}E_{50} \, \ddot{a}_{60} \right] = 0.53667[13.2668 - (0.51081)(11.1454)] = 4.0645$$

There are several other ways to write the right hand side of the first equation.

9. Key: B

Woolhouse: $^{W}\ddot{a}_x^{(4)} = 3.4611 - \dfrac{3}{8} = 3.0861$

UDD:
$$^{UDD}\ddot{a}_x^{(4)} = \alpha(4)\ddot{a}_x - \beta(4)$$
$$= 1.00027(3.4611) - 0.38424$$
$$= 3.0778$$

and $A_x = 1 - d\,\ddot{a}_x = 1 - (0.05660)(3.4611) = 0.80410$

$$P^{(W)} = \dfrac{1000(0.80410)}{3.0861} = 260.56$$

$$P^{(UDD)} = \dfrac{1000(0.80410)}{3.0778} = 261.26$$

$$\dfrac{P^{(UDD)}}{P^{(W)}} = \dfrac{261.26}{260.56} = 1.0027$$

10. Key: A

$$P_{30:\overline{20|}} = \dfrac{1}{\ddot{a}_{30:\overline{20|}}} - d \Rightarrow \dfrac{2{,}143}{100{,}000} + 0.05 = \dfrac{1}{\ddot{a}_{30:\overline{20|}}} \Rightarrow \ddot{a}_{30:\overline{20|}} = 14$$

$$A_{30:\overline{20|}} = 1 - d\,\ddot{a}_{30:\overline{20|}} = 1 - 0.05(14) = 0.3$$

$$G\ddot{a}_{30:\overline{20|}} = 100{,}000\,A_{30:\overline{20|}} + (200 + 50\ddot{a}_{30:\overline{20|}}) + (0.33G + 0.06G\,\ddot{a}_{30:\overline{20|}})$$

$$14G = 100{,}000(0.3) + [200 + 50(14)] + (0.33G + 0.84G)$$

$$12.83\,G = 30{,}900$$

$$G = 2408$$

11. Key: D

$$P = \frac{10,000\left(v q_{\overline{30:30}} + v^2 {}_{1|}q_{\overline{30:30}}\right)}{1 + v p_{\overline{30:30}}}$$

$$q_{\overline{30:30}} = (q_{30})^2 = 0.0016$$

$$p_{\overline{30:30}} = 1 - q_{\overline{30:30}} = 1 - 0.0016 = 0.9984$$

$${}_{1|}q_{\overline{30:30}} = {}_{2}q_{\overline{30:30}} - q_{\overline{30:30}} = ({}_{2}q_{30})^2 - 0.0016 = [0.04 + 0.96(0.06)]^2 - 0.0016 = 0.00793$$

$$\text{APV of Benefits} = 10,000\left(\frac{0.0016}{1.05} + \frac{0.00793}{1.05^2}\right) = 87.17$$

$$P = \frac{87.17}{1 + \frac{0.9984}{1.05}} = 44.68$$

12. Key: B

$$L_A = v^T - 0.10\bar{a}_{\overline{T}|} = \left(1 + \frac{10}{6}\right)v^T - \frac{10}{6}$$

$$Var[L_A] = \left(1 + \frac{10}{6}\right)^2 Var[v^T] = 0.455 \Rightarrow Var[v^T] = 0.06398$$

$$L_B = 2v^T - 0.16\bar{a}_{\overline{T}|} = \left(2 + \frac{16}{6}\right)v^T - \frac{16}{6}$$

$$Var[L_B] = \left(2 + \frac{16}{6}\right)^2 Var[v^T] = \left(2 + \frac{16}{6}\right)^2 (0.06398) = 1.39$$

13. Key: E

In the final year: $({}_{24}V + P)(1 + i) = b_{25}(q_{68}) + 1(p_{68})$

Since $b_{25} = 1$, this reduces to $({}_{24}V + P)(1 + i) = 1 \Rightarrow (0.6 + P)(1.04) = 1 \Rightarrow P = 0.36154$

Looking back to the 12$^{\text{th}}$ year: $({}_{11}V + P)(1 + i) = b_{12}(q_{55}) + {}_{12}V(p_{55})$

$$\Rightarrow (5.36154)(1.04) = 14(0.15) + {}_{12}V(0.85) \Rightarrow {}_{12}V = 4.089$$

14. Key: A

This first solution recognizes that the full preliminary term reserve at the end of year 10 for a 30 year endowment insurance on (40) is the same as the net premium reserve at the end of year 9 for a 29 year endowment insurance on (41). Then, using superscripts of FPT and NLP to distinguish the reserves, we have

$$1000\,_{10}V^{FPT} = 1000\,_9V^{NLP} = 1000(A_{50:\overline{20}|} - P_{41:\overline{29}|}\ddot{a}_{50:\overline{20}|})$$

$$= 1000[0.36084 - 0.01714(11.2918)] = 167$$

or $$= 1000\left(1 - \frac{\ddot{a}_{50:\overline{20}|}}{\ddot{a}_{41:\overline{29}|}}\right) = 1000\left(1 - \frac{11.2918}{13.5597}\right) = 167$$

where

$$\ddot{a}_{50:\overline{20}|} = \ddot{a}_{50} - {}_{20}E_{50}\,\ddot{a}_{70}$$

$$= 13.2668 - (0.23047)(8.5693)$$

$$= 11.2918$$

$$A_{50:\overline{20}|} = 1 - d(11.2918) = 0.36084$$

$$\ddot{a}_{41:\overline{29}|} = \ddot{a}_{41} - {}_{29}E_{41}\,\ddot{a}_{70}$$

$$= 14.6864 - (0.1314764)(8.5693)$$

$$= 13.5597$$

$$A_{41:\overline{29}|} = 1 - d(13.5597) = 0.23247$$

$${}_{29}E_{41} = v^{29}\left(\frac{l_{70}}{l_{41}}\right) = (0.1845567)\frac{6,616,155}{9,287,264} = 0.1314764$$

$$P_{41:\overline{29}|} = \frac{0.23247}{13.5597} = 0.01714$$

Alternatively, working from the definition of full preliminary term reserves as having $_1V^{FPT} = 0$ and the discussion of modified reserves in the Notation and Terminology Study Note, let α be the valuation premium in year 1 and β be the valuation premium thereafter. Then (with some of the values taken from above),

$$\alpha = 1000vq_{40} = 2.6226$$

APV (valuation premiums) = APV (benefits)

$$\alpha + {}_1E_{40}(\ddot{a}_{41:\overline{29}|})\beta = 1000A_{40:\overline{30}|}$$

$$2.6226 + 0.94077(13.5597)\beta = 221.32$$

$$\beta = \frac{221.32 - 2.6226}{12.7566} = 17.14$$

14. **Continued**

where
$$_1E_{40} = (1 - 0.00278)v = 0.94077$$
$$A_{40:\overline{30}|} = A_{40} + {}_{20}E_{40}({}_{10}E_{60})(1 - A_{70})$$
$$= 0.16132 + 0.27414(0.45120)(1 - 0.51495) = 0.22132$$
$$_{10}V^{FPT} = 1000A_{50:\overline{20}|} - \beta\ddot{a}_{50:\overline{10}|} = 1000(0.36084) - 17.14(11.2918) = 167$$

15. Key: E

No cash flow beginning of year, the one item earning interest is the reserve at the end of the previous year

Gain due to interest = (reserves at the beginning of year)(actual interest – anticipated interest) $= 1000(8929.18)(0.04 - 0.03) = 89292$.

16. Key: E

Future expenses at $x + 2 = 0.08G + 5$

Expense load at $x + 2 = P^e$

$-23.64 = (0.08G + 5) - P^e$

$\Rightarrow P^e = 58.08$

$1000P_{x:\overline{3}|} = 368.05 - 58.08 = 309.97$

17. Key: C

Because the death benefit is F plus account value, the net amount at risk does not depend on the account value. Therefore the cost of insurance charges are the same for both policies. Therefore the only difference in ending account value is due to the premiums, less expense charges, accumulated with interest.

Anne pays 500 more premium in year 1, or $500(1-0.40)=300$ more after expense charges.

Julie pays 500 more premium in each subsequent year, or $500(1-0.10)=450$ more after expense charges.

Julie's extra account value is
$$-300(1.06^{20})+450\ddot{s}_{\overline{19}|}=-300(3.2071)+450(35.7856)=15,141.39$$

Julie's ending account value is
$$10,660+15,141=25,801$$

18. Key: E

Defined benefit plan projected benefit

$$=50,000\left(\frac{1.02^{22}+1.02^{23}+1.02^{24}}{3}\right)(30)(0.005)$$
$$=50,000(1.5771054)(30)(0.005)$$
$$=11,828$$

The additional income desired $= 42,000 - 11,828 = 30,172$.
The necessary defined contribution accumulation at age 65 is
$30,172\ddot{a}_{65} = 30,172(9.9) = 298,702$.

19. Key: C

Annual pension at age 65 $\quad = 45,000\left[\dfrac{1+(1.04)+\cdots+(1.04)^{29}}{30}\right](0.02)(30)$

$\qquad\qquad\qquad\qquad\qquad = 45,000(0.02)\dfrac{(1.04)^{30}-1}{0.04}=50,476.44$

20. Key: D

Let P be the annual premium

$$\text{APV(premium)} = P\left(1 + v\frac{945}{1000} + v^2\frac{895}{1000}\right) = 2.711791P$$

$$\text{APV(benefits)} = 100\left(v\frac{20}{1000} + v^2\frac{25}{1000} + v^3\frac{895}{1000}\right) = 81.4858$$

$$P = \frac{81.4858}{2.711791} = 30.05$$

Note: The term $v^3\frac{895}{1000}$ is the sum of $v^3\frac{30}{1000}$ for death benefits and $v^3\frac{895-30}{1000}$ for maturity benefits.