Long-Term Actuarial Mathematics

Exam LTAM

Date: Friday, October 25, 2019
Time: 8:30 a.m. – 12:45 p.m.

Recognized by the Canadian Institute of Actuaries.

INSTRUCTIONS TO CANDIDATES

General Instructions

1. Write your candidate number here ____________. Your name must not appear.

2. Do not break the seal of this book until the supervisor tells you to do so.

3. Tables for this examination will be distributed by the Supervisor.

4. This examination has a total of 96 points. It consists of:

   Section A: 20 multiple-choice questions, each worth 2 points for a total of 40 points, and

   Section B: 6 written-answer questions, worth a total of 56 points. The point value for each written-answer question is indicated at the beginning of the question.

   You may divide your time between the two sections of the examination (written-answer, and multiple-choice) as you choose. You should keep in mind the relative weight of the two sections.

   Your written-answer paper will be graded only if your multiple-choice score is at or above a threshold set after the examination is administered.

5. Failure to stop writing or coding after time is called will result in the disqualification of your answers or further disciplinary action.

6. While every attempt is made to avoid defective questions, sometimes they do occur. If you believe a question is defective, the supervisor or proctor cannot give you any guidance beyond the instructions on the exam booklet.

Multiple-Choice Instructions

1. A separate answer sheet for the multiple-choice questions is inside the front cover of this book. During the time allotted for this examination, record all your answers on the back of the answer sheet. NO ADDITIONAL TIME WILL BE ALLOWED FOR THIS PURPOSE.

   No credit will be given for anything indicated in the examination book but not transferred to the answer sheet.

2. On the front of the answer sheet, space is provided to write and code candidate information. Complete the information requested by printing in the squares and blackening the circles (one in each column) corresponding to the letters or numbers printed. For each empty box blacken the small circle immediately above the “A” circle. Fill out the boxes titled:

   (a) Name
   (include last name, first name and middle initial)

   (b) Candidate Number
   (Candidate/Eligibility Number, use leading zeros if needed to make it a five digit number)

   (c) Test Site Code
   (The supervisor will supply the number.)

   (d) Examination Part
   (Code the examination that you are taking by blackening the circle to the left of "Exam LTAM.")

   (e) Booklet Number
   (The booklet number can be found in the upper right-hand corner of this examination book. Use leading zeros if needed to make it a four digit number.)

   In the box titled “Complete this section only if instructed to do so,” fill in the circle to indicate if you are using a calculator and write in the make and model number.

   In the box titled “Signature and Date” sign your name and write today's date. **If the answer sheet is not signed, it will not be graded**.

   Leave the boxes titled “Test Code” and “Form Code” blank.

   On the back of the answer sheet fill in the Booklet Number in the space provided.

CONTINUED ON INSIDE FRONT COVER

3. Your score will be based on the number of questions which you answer correctly. No credit will be given for omitted answers and no credit will be lost for wrong answers: hence, you should answer all questions even those for which you have to guess.

4. Five answer choices are given with each multiple-choice question, each answer choice being identified by a key letter (A to E). Answer choices for some questions have been rounded. For each question, blacken the circle on the answer sheet which corresponds to the key letter of the answer choice that you select.

5. Use a soft-lead pencil to mark the answer sheet. To facilitate correct mechanical scoring, be sure that, for each question, your pencil mark is dark and completely fills only the intended circle. Make no stray marks on the answer sheet. If you have to erase, do so completely.

6. Do not spend too much time on any one question. If a question seems too difficult, leave it and go on.

7. Clearly indicated answer choices in the test book can be an aid in grading examinations in the unlikely event of a lost answer sheet.

8. After the examination, the supervisor will collect this book and the answer sheet separately. DO NOT ENCLOSE THE ANSWER SHEET IN THE BOOK OR IN THE ESSAY ANSWER ENVELOPE. All books and answer sheets must be returned. THE QUESTIONS ARE CONFIDENTIAL AND MAY NOT BE TAKEN FROM THE EXAMINATION ROOM.

Written-Answer Instructions

1. **Write your candidate number at the top of each sheet. Your name must not appear.**

2. Write on only one side of a sheet. Start each question on a fresh sheet. On each sheet, write the number of the question you are answering. Do not answer more than one question on a single sheet.

3. The answer should be confined to the question as set.

4. When you are asked to calculate, show all your work including any applicable formulas.

5. When you finish, insert your written-answer sheets that you want graded into the Essay Answer Envelope. Be sure to hand in all your answer sheets because they cannot be accepted later. Seal the envelope and write your candidate number in the space provided on the outside of the envelope. Check the appropriate box to indicate Exam LTAM.

6. **Sign your essay answer envelope. If it is not signed, your examination will not be graded.**

7. For all parts of all problems, to maximize the credit earned, candidates should show as much work as possible, considering the time allotted for the question. Answers lacking justification will receive no credit. Answers should be organized so that the methods, logic, and formulas used are readily apparent. Candidates should not round their answers excessively; enough precision should be provided so that their answers can be accurately graded.

   In some cases, candidates are asked to show that a calculation results in a particular number. Typically the answer given will be rounded; candidates should provide a greater level of accuracy than the number given in the question. This structure of question is intended to assist the candidate by giving an indication when the calculation has been done incorrectly, providing an opportunity to explore an alternative approach. It also allows a candidate who cannot obtain the correct answer to use the answer given to proceed with subsequent parts of the problem. (Candidates who are able to solve the problem should use their exact answer for subsequent parts.)

   For questions requiring candidates to derive or write down a formula or equation, the resulting expression should be simplified as far as possible, and where numerical values are provided in the problem, they should be used.

Exam LTAM

SECTION A – Multiple-Choice

BEGINNING OF EXAMINATION

1. You are given that ${}_tp_{40} = \exp(-0.06(1.12^t - 1))$ for all $t \geq 0$.

Calculate the probability that a life age 45 survives 10 years.

(A) 0.80

(B) 0.83

(C) 0.86

(D) 0.90

(E) 0.93

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2. For a double decrement model, you are given:

(i) ${}_{0.4}q_{40}^{(1)} = 0.032$

(ii) ${}_{0.6}q_{40}^{(2)} = 0.045$

(iii) Each decrement in the multiple decrement table is uniformly distributed between integer ages.

Calculate $p_{40}^{(\tau)}$.

(A) 0.815

(B) 0.825

(C) 0.835

(D) 0.845

(E) 0.855

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3. You are given the following information about three policyholders:

Age at policy purchase	Age at policy termination	Reason for policy termination
74.0	75.8	Death
75.0	76.5	Death
75.5	75.9	Death

Calculate the estimate for q_{75} obtained from these data, using the exact exposure method.

(A) 0.50

(B) 0.60

(C) 0.70

(D) 0.80

(E) 0.90

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4. You are given the following parameters for the Lee Carter model in year Y:

$$\alpha_{65} = -2.6, \quad \beta_{65} = 0.04, \quad K_Y = -9.0, \quad c = -0.35, \quad \sigma_k = 0.75$$

Calculate the 80th percentile of $\log\left(m(65, Y+1)\right)$.

(A) -2.91

(B) -2.93

(C) -2.95

(D) -2.97

(E) -2.99

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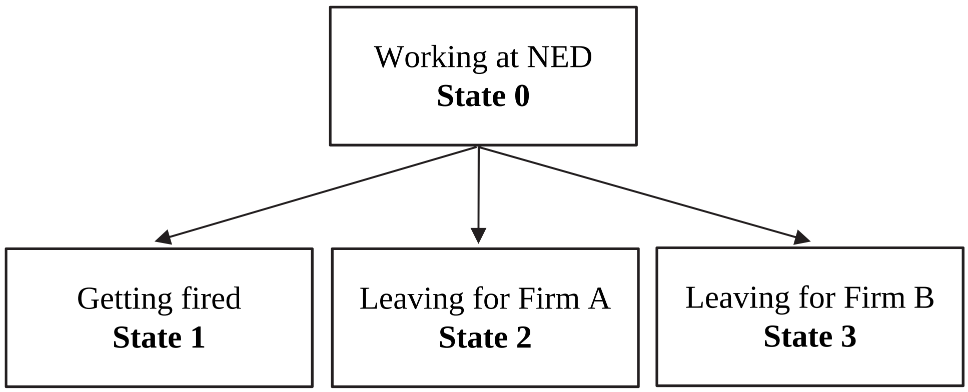
5. Kevin (age x) just started a new job at NED Industries. Kevin intends to work for this firm for the next three years, and will look for new opportunities in the fourth year. Kevin uses the following four-state model.



You are given:

(i) $\mu_{x+t}^{01} = 0.01 \qquad 0 \le t \le 4$

(ii) $\mu_{x+t}^{02} = \begin{cases} 0.00 & 0 \le t \le 3 \\ 0.12 & 3 < t \le 4 \end{cases}$

(iii) $\mu_{x+t}^{03} = \begin{cases} 0.00 & 0 \le t \le 3 \\ 0.08 & 3 < t \le 4 \end{cases}$

Calculate the probability that Kevin will still be working for NED at the end of four years.

(A) 0.533

(B) 0.782

(C) 0.787

(D) 0.811

(E) 0.990

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6. A couple, Franky (55) and Johnny (65) purchase a single premium annuity which pays 10,000 per year continuously while they are both alive, and pays 8,000 per year continuously while exactly one is alive. You are given:

(i) Franky and Johnny have independent future lifetimes.

(ii) $\delta = 0.05$

(iii) $\bar{a}_{65} = 12.8916, \qquad \bar{a}_{75} = 9.7262$

(iv) $\mu_{65} = 0.005605, \qquad \mu_{75} = 0.017552$

(v)
- ${}_tV^{(0)}$ represents the reserve at time *t* given that both Franky and Johnny are alive at time *t*.
- ${}_tV^{(1)}$ represents the reserve at time *t* given that Franky is alive at time *t*, and that Johnny has died before time *t*.
- ${}_tV^{(2)}$ represents the reserve at time *t* given that Johnny is alive at time *t*, and that Franky has died before time *t*.

(vi) ${}_{10}V^{(0)} = 109,650$

Calculate $\frac{d}{dt}\,{}_tV^{(0)}$ at time $t = 10$.

(A) $-4,225$

(B) $-4,425$

(C) $-4,625$

(D) $-4,825$

(E) $-5,025$

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7. An insurance company issues a special policy to (65) with the following benefits:

- A death benefit of 100,000, payable at the end of year of death, provided death occurs before age 85.
- A whole life annuity-due of 45,000 per year starting on the policyholder's 85^{th} birthday.

You are also given:

(i) Annual level premiums of P are payable for 20 years.

(ii) Premiums are determined using the equivalence principle.

(iii) Mortality follows the Standard Ultimate Life Table.

(iv) $i = 0.05$

Calculate P.

(A) 7670

(B) 7870

(C) 8070

(D) 8270

(E) 8470

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8. You are given the following information:

(i) Woolhouse's 3-term formula is used to calculate values for 1/*m*-thly functions.

(ii) Mortality follows the Standard Ultimate Life Table.

(iii) In the Standard Ultimate Life Table:

$\mu_{80} = 0.030162$ and $\mu_{90} = 0.096590$

(iv) $i = 0.05$

Calculate $20,000\,\ddot{a}^{(4)}_{80:\overline{10}|}$.

(A) 130,680

(B) 130,710

(C) 130,740

(D) 130,770

(E) 130,800

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9. Justin, age 50, has been awarded 1,250,000 in compensation for injuries due to an accident during his employment. He will have a structured settlement with the following payments:

- An immediate payment of 250,000.
- A whole life annuity-due of X per year, beginning at age 50.
- A payment of 100,000 payable at the end of the year of death.

You are given:

(i) Justin's post-accident mortality follows the Standard Ultimate Life Table, with a 5-year age rating.

(ii) The structured settlement benefits are determined using the equivalence principle.

(iii) $i = 0.05$

Calculate X.

(A) 59,600

(B) 59,900

(C) 60,200

(D) 60,500

(E) 60,800

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10. A 5-year sickness insurance policy issued to a Healthy life age 60 pays 1000 at the end of each year if the policyholder is Sick at that time. The insurer values the benefit using a three-state model, where State 0 is Healthy, State 1 is Sick, and State 2 is Dead.

You are given:

(i) The following table of probabilities for the sickness model.

t	${}_t p_{60}^{00}$	${}_t p_{60}^{01}$	${}_t p_{60}^{02}$
1	0.98	0.01	0.01
2	0.95	0.03	0.02
3	0.93	0.04	0.03
4	0.91	0.05	0.04
5	0.88	0.07	0.05

(ii) $i = 0.04$

Calculate the expected present value of the sickness benefit for this policy.

(A) 71

(B) 94

(C) 116

(D) 173

(E) 186

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11. An insurer issues a 10-year term insurance policy with a sum insured of 1,000,000 to (50).

You are given the following information:

(i) The sum insured is paid at the end of the month of death.

(ii) Premiums are payable monthly.

(iii) Initial expenses, payable at the start of the contract, are 60% of the annualized premium.

(iv) Maintenance expenses are 5% of premiums including the first.

(v) Mortality follows the Standard Ultimate Life Table.

(vi) Deaths are uniformly distributed between integer ages.

(vii) $i = 0.05$

(viii) Premiums are determined using the equivalence principle.

Calculate the monthly premium for this policy.

(A) 150

(B) 160

(C) 170

(D) 180

(E) 190

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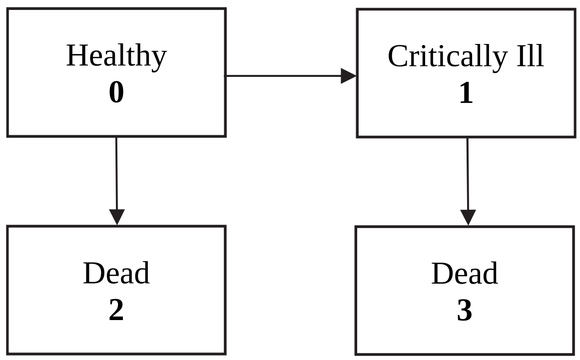
12. The following model is used for pricing a fully continuous whole life critical illness insurance policy issued to a Healthy life age 50:



You are given:

(i) The policy has the following benefits:

- A partly accelerated benefit of 30,000 upon a Critical Illness (CI) diagnosis
- A death benefit of 50,000 if the policyholder dies before a CI diagnosis
- A death benefit of 20,000 if the policyholder dies after a CI diagnosis

(ii) Premiums are payable continuously while in State 0.

(iii) The following extract of table functions:

x	$\bar{a}_x^{00}$	$\bar{A}_x^{01}$	$\bar{A}_x^{02}$	$\bar{A}_x^{03}$	$\bar{A}_x^{13}$
50	13.30678	0.22409	0.12667	0.14176	0.34988

Calculate the annual rate of net premium for this policy.

(A) 1200

(B) 1300

(C) 1400

(D) 1500

(E) 1600

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13. For a fully discrete whole life insurance of 1000 on (60), you are given:

(i) $\ddot{a}_{60} = 11.146, \quad \ddot{a}_{61} = 10.903, \quad \ddot{a}_{70} = 8.569$

(ii) $A_{60} = 0.3691, \quad A_{61} = 0.3828, \quad A_{70} = 0.5150$

Calculate the difference between the net premium reserve and the full preliminary term reserve at the end of the 10^{th} year.

(A) 17

(B) 18

(C) 19

(D) 20

(E) 21

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14. For a special fully discrete whole life insurance issued to (61), you are given the following:

(i) Mortality follows the Standard Ultimate Life Table.

(ii) $i = 0.05$

The death benefit is 50,000 in the first 20 years and 100,000 afterwards. Level premiums are payable annually.

Calculate the net premium reserve at the end of the 20^{th} year.

(A) 46,640

(B) 47,640

(C) 48,640

(D) 49,640

(E) 50,640

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15. An insurer issues a special semi-continuous 5-year term insurance to (60).

The policy benefits are:

- 100,000 immediately on death from natural causes (Decrement 1).
- 200,000 immediately on death from other causes (Decrement 2).

Premiums of 7000 per year are payable annually.

You are given:

(i) $\mu_{61+t}^{(1)} = 0.05 \quad 0 \le t \le 4$

(ii) $\mu_{61+t}^{(2)} = 0.01 \quad 0 \le t \le 4$

(iii) $\delta = 0.02$

Calculate the reserve at the end of the first year.

(A) –170

(B) –370

(C) –570

(D) –770

(E) –970

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16. For a profit test of a fully discrete 10-year term insurance issued to (60), you are given:

(i) The annual premium is 700.

(ii) The death benefit is 100,000.

(iii) Mortality follows the Standard Ultimate Life Table.

(iv) The insurer earned rate is 6% per year.

(v) Renewal expenses are 4% of each premium after the first.

(vi) The insurer holds the following reserves:

t	${}_tV$
0	0
1	325
2	600
3	822
4	982

Calculate the emerging surplus at the end of the second policy year per policy in force at the start of that year.

(A) 50

(B) 60

(C) 70

(D) 80

(E) 90

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17. John is planning to retire in exactly three years. John's Final Average Salary, used to calculate his pension, is defined as the average salary in the final two years of his employment.

John's firm awards salary increases at the end of each year. The latest awards have just been announced and John did not get a raise.

John models his salary increases using a two-state Markov model with annual transitions.

You are given:

(i) In State 0, his salary increase will be 0%.

(ii) In State 1, his salary increase will be 5%.

(iii) John is currently in State 0.

(iv) John's salary in the next year will be 100,000.

(v) $p^{00} = 0.70, \quad p^{01} = 0.30, \quad p^{10} = 0.60, \quad p^{11} = 0.40$

Calculate the expected value of John's Final Average Salary.

(A) 100,000

(B) 101,500

(C) 102,340

(D) 103,780

(E) 104,840

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18. A defined benefit pension plan provides a retirement benefit with an accrual rate of 1.3% of final one-year salary up to 55,000 per year of service, and 2% of final one-year salary above 55,000 per year of service.

You are given:

(i) The pension is paid as a monthly annuity-due from age 65.

(ii) There are no early retirement or other benefits.

(iii) There are no exits from the plan before retirement except for death.

(iv) Mortality follows the Standard Ultimate Life Table.

(v) Salaries increase at 2.5% at the end of each year.

(vi) $i = 0.05$

(vii) $\ddot{a}_{65}^{(12)} = 13.086$

Elizabeth, who is 50 at the valuation date, has 20 years of service in the plan. Her salary in the year prior to the valuation was 88,000.

Calculate the normal contribution for Elizabeth using the Traditional Unit Credit method.

(A) 13,285

(B) 13,585

(C) 13,885

(D) 14,185

(E) 14,485

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19. A defined benefit plan provides for a retirement benefit of 500 per month for each year of service at the normal retirement age of 65.

You are given:

(i) The only decrement before retirement is death.

(ii) Mortality follows the Standard Ultimate Life Table.

(iii) Deaths are uniformly distributed over each year of age.

(iv) On retirement before age 65, the benefit is adjusted such that the reduced benefit taken immediately on early retirement has the same actuarial present value as the accrued benefit deferred to age 65.

(v) $i = 0.05$

(vi) $\ddot{a}_{65}^{(12)} = 13.086$

Calculate the percentage reduction factor that should be applied to the accrued benefit for an employee who chooses to take their pension at age 64 instead of age 65.

(A) 3.5%

(B) 4.4%

(C) 5.7%

(D) 6.7%

(E) 7.3%

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20. You are given the following information about a post-retirement health benefit plan:

(i) $B(x,t)$ denotes the annual supplementary health insurance premium for a life age x at time t.

(ii) For $k > 0$, $B(x+k,t) = (1.03)^k \; B(x,t)$.

(iii) For all ages, and for $t = 0,1,2,...,$ $B(x,t+1) = (1.04)B(x,t)$.

(iv) Mortality follows the Standard Ultimate Life Table.

(v) $i = 6\%$

(vi) $\ddot{a}_B(65,2) = 26.708$

Calculate $\ddot{a}_B(63,0)$.

(A) 25.4

(B) 26.7

(C) 27.6

(D) 28.7

(E) 29.0

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SECTION B – Written-Answer

1. (*9 points*) Tim, age 50, purchases a semi-continuous whole life insurance policy with a death benefit of 100,000 payable at the moment of death. Premiums are payable annually.

You are given:

(i) Initial expenses are 500.

(ii) Maintenance expenses, payable at the start of each year including the first, are 50 per year plus 5% of the gross premium.

(iii) Mortality follows the Standard Ultimate Life Table.

(iv) Deaths are uniformly distributed between integer ages.

(v) $i = 5\%$

(a) (*2 points*) Show that the gross annual premium, calculated using the equivalence principle is 1280 to the nearest 10. You should calculate the value to the nearest 0.1.

(b) (*4 points*)

(i) Show that

$${}_{s}q_{80.7} = 0.033422\,s \quad \text{for } 0 < s \le 0.3.$$

(ii) Hence, write down an integral expression for

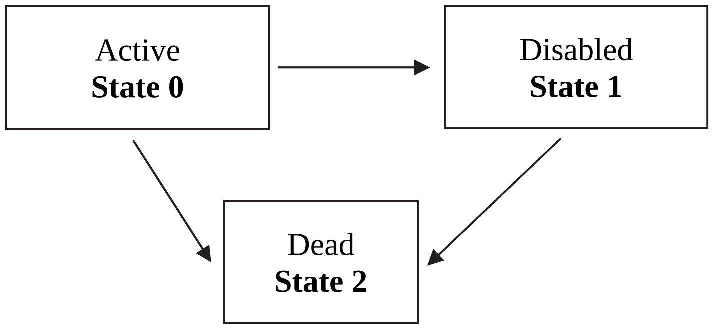
$$\bar{A}^{\,1}_{80.7:\overline{s}|}\,, \text{ for } 0 < s \le 0.3.$$

(iii) Show that $100{,}000\bar{A}^{\,1}_{80.7:\overline{0.3}|} = 995$ to the nearest 1. You should calculate the value to the nearest 0.01.

(c) (*2 points*) Show that the gross premium reserve at time 30.7 for this policy is 52,620 to the nearest 10. You should calculate the value to the nearest 0.1.

(d) (*1 point*) Explain why applying linear interpolation to the gross premium reserves at time 30 and at time 31 would <u>not</u> give a good estimate of the gross premium reserve at time 30.7 in this case.

2. (*9 points*) An insurer issues a whole life insurance of 50,000 to (55), with a waiver of premium on disability. You are evaluating this policy using the following model:



You are given:

(i) The gross premium G is paid at the start of each year if the insured is Active.

(ii) The death benefit is payable at the end of the month of death.

(iii) Commissions are 5% of each premium paid. There are no other expenses.

(iv) $i = 0.05$

(v)

x	$A_x^{(12)\,02}$	$A_x^{(12)\,12}$	$\ddot{a}_x^{00}$
55	0.480	0.531	8.832
65	0.634	0.681	5.416

(vi) ${}_0L^g$ denotes the present value of the gross loss-at-issue random variable.

(vii) ${}_tV^{(j)}$ denotes the gross premium reserve at time t, for a life then in State j.

(a) (*2 points*) The gross premium is determined such that $\mathrm{E}[{}_0L^g] = -0.1G$. Show that $G = 2900$ to the nearest 100. You should calculate G to the nearest 1.

(b) (*2 points*) Show that ${}_{10}V^{(0)}$ is 16,800 to the nearest 100. You should calculate the reserve to the nearest 1.

2. **Continued**

You are also given:

(i) ${}_{10\frac{1}{12}}V^{(1)} = 34{,}110$

(ii) ${}_{10\frac{2}{12}}V^{(1)} = 34{,}170$

(iii) The following probabilities:

x	${}_{\frac{1}{12}}p_x^{01}$	${}_{\frac{1}{12}}p_x^{02}$	${}_{\frac{1}{12}}p_x^{12}$
65	0.00461	0.00293	0.00493
$65\frac{1}{12}$	0.00467	0.00295	0.00497

(c) (*3 points*)

(i) Show that ${}_{10\frac{1}{12}}V^{(0)} = 19{,}500$ to the nearest 100. You should calculate the value to the nearest 1.

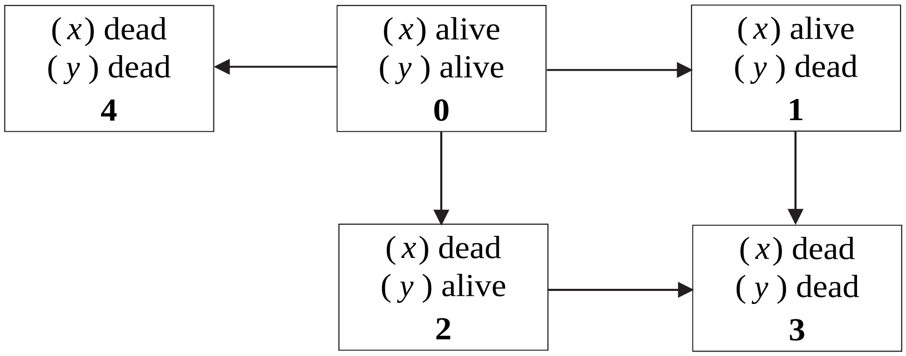
(ii) Calculate ${}_{10\frac{2}{12}}V^{(0)}$.

(d) (*2 points*) To increase sales, the insurance company raises its commission rate to 15% of the first premium and 8% of each renewal premium paid. The gross premium is not changed.

(i) Without further calculation, state with reasons whether the value of ${}_{10\frac{2}{12}}V^{(0)}$ will increase, decrease or stay the same as a result of these changes.

(ii) Without further calculation, state with reasons whether the value of ${}_{10\frac{2}{12}}V^{(1)}$ will increase, decrease or stay the same as a result of these changes.

3. (*10 points*) An insurer uses the following model for joint life policies.



You are given the following additional information

(i) μ_w^* is the force of mortality for a life age w under the Standard Ultimate Life Table.

(ii) $\mu_{x+t:y+t}^{01} = \mu_{y+t}^* - 0.0005;\quad \mu_{x+t:y+t}^{02} = \mu_{x+t}^* - 0.0005;\quad \mu_{x+t:y+t}^{04} = 0.0005$

(iii) $\mu_{x+t:y+t}^{13} = 1.2\mu_{x+t}^*;\quad \mu_{x+t:y+t}^{23} = 1.05\mu_{y+t}^*$

(iv) $i = 0.05$

(v) For $x = 40$ and $y = 50$, $\bar{a}_{x:y:\overline{10}|}^{00} = 7.8487$ and $\bar{A}_{x:y:\overline{10}|}^{03} = 0.00789$

(a) (*2 points*) Describe two ways that dependency is incorporated in this model.

(b) (*3 points*)

(i) Calculate ${}_{10}p_{50}^{23}$.

(ii) Calculate ${}_{10}p_{x:y}^{00}$ for $x = 40$ and $y = 50$.

3. **Continued**

(c) (*3 points*) A couple, (x) who is age 40 and (y) who is age 50, buys a special 10-year joint life insurance policy with the following features:

- Level premiums are payable continuously while in State 0.
- A death benefit of 100,000 is payable immediately on the second death.
- If the deaths are simultaneous, the death benefit is increased to 300,000.

Calculate the annual net premium rate.

(d) (*2 points*) State with reasons whether each of the following would be higher, lower or stay the same if $\mu^{23}_{x+t:y+t}$ is increased to $1.15\mu^{*}_{y+t}$, assuming all other transition intensities are as given above.

(i) $\bar{a}^{00}_{x:y:\overline{10|}}$

(ii) $\bar{A}^{03}_{x:y:\overline{10|}}$

(iii) $\bar{a}_{x|y}$

(iv) The annual net premium for the contract in (c).

4. (*10 points*) CTI is a life insurance company that plans to issue a new, high volume, low sum insured term insurance policy through direct marketing. Its current business is predominantly traditionally-brokered term insurance.

CTI assumes the following for calculating premiums and reserves.

(i) For brokered policies, mortality follows the Standard Ultimate Life Table, and expenses are 8% of each premium.

(ii) For direct-marketed policies, mortality follows the Standard Ultimate Life Table with a 5-year addition to the policyholder's age. Expenses are 3% of each premium.

(iii) $i = 0.05$

(iv) Premiums are calculated using the equivalence principle.

(a) (*2 points*) Explain why the mortality and expense assumptions are different for the direct-marketed policies compared to the brokered policies.

Consider a fully discrete direct-marketed 10-year term insurance policy of 50,000 issued to (40), with premium P.

(b) (*1 point*) Let K^* denote the curtate future lifetime of (40). Write down the gross loss-at-issue random variable in terms of K^*, P, and interest rate functions.

(c) (*1 point*) Show that $P = 57$ to the nearest 1. You should calculate your answer to the nearest 0.01

(d) (*3 points*)

(i) Show that the standard deviation of the present value of the benefit is 4088 to the nearest 1. You should calculate the value to the nearest 0.01.

(ii) The standard deviation of the gross loss-at-issue is 4088.5. Without further calculation, explain why the standard deviation of the gross loss-at-issue is very close to the standard deviation of the benefit.

(e) (*2 points*) CTI would like to offer a revised, direct-marketed 10-year term policy, where the annual premium in the first five years is ½ of the annual premium in the last five years.

Show that the initial annual premium for a 50,000 policy issued to (40) is 40 to the nearest 10. You should calculate the value to the nearest 0.01.

4. Continued

(f) (*1 point*) CTI issues 5-year term insurance policies of 50,000 by direct marketing. At most ages the premium for this policy is greater than the initial premium for the policy described in part (e). Therefore, the new policy is expected to be very popular.

Explain why CTI might not launch this product despite its anticipated popularity.

5. (*9 points*) For the single factor longevity model expressed as $q(x,t) = q(x,0)(1-\varphi_x)^t$, you are given:

(i) Base mortality rates are taken from the Standard Ultimate Life Table.

(ii) The following selected improvement factors:

x	65	66	67	≥ 68
φ_x	0.034	0.033	0.031	0.030

(iii) $i = 0.05$

(iv) K_{65} denotes the curtate future lifetime of a life who is age 65 at time 0.

(a) (*2 points*) Calculate $\Pr[K_{65} = k]$ for $k = 0,1,2$.

(b) (*2 points*) Let *Y* denote the present value of a 3-year temporary life annuity-due of 1 per year issued to a life who is age 65 at time 0.

(i) Calculate the expected value of *Y*.

(ii) Calculate the standard deviation of *Y*.

(c) (*4 points*)

(i) Show that this longevity model can be equivalently expressed as

$$p(x,t) = p(x,0)(1+\psi_{x,t}),$$

where $\psi_{x,t} = \dfrac{q(x,0)}{p(x,0)}\left(1-(1-\varphi_x)^t\right), \quad t = 1,2,\ldots$.

(ii) Calculate $\psi_{70,10}$.

(iii) Calculate the number of years from time 0 before a life then age 70 will have a 1-year survival probability of at least 0.995.

(d) (*1 point*) The single factor longevity model has been considered to be too simplistic. Suggest one way to improve the model.

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6. (*9 points*) An insurer issues a 20-year deferred annuity contract to (65). Level annual premiums will be paid for a maximum of 10 years. On survival to age 85, an annuity of 36,000 per year is paid annually in advance. If the life dies after 10 years, but before the first annuity payment, the premiums paid are returned without interest, at the end of the year of death. There is no payment on death in the first 10 years.

You are given the following premium assumptions:

(i) Mortality follows the Standard Ultimate Life Table.

(ii) Initial expenses are 1000 plus 30% of the premium.

(iii) Expenses of 5% of the premium are payable with the second and subsequent premiums.

(iv) Renewal expenses of 100 per year are payable annually in advance throughout the term of the contract, starting in the second year.

(v) $i = 0.05$

(vi) Premiums are calculated using the equivalence principle.

(a) (*2 points*) Show that the premium is 10,260 to the nearest 10. You should calculate the premium to the nearest 0.1.

The insurer profit tests the policy. You are given the following assumptions for the profit test:

(i) Mortality rates are 90% of the mortality rates in the Standard Ultimate Life Table.

(ii) For the first five years of the contract, 3% of policyholders who survive to each year end lapse at that time. There are no lapses after the first five years.

(iii) Pre-contract expenses are 7000.

(iv) Maintenance expenses start at 70 at the start of the first year, and increase at a rate of 2% per year.

(v) Premium expenses are 5% of each premium, including the first.

(vi) The insurer's funds earn 7% interest each year.

6. **Continued**

(vii) The hurdle rate is 10%.

(viii) Reserves at times 0, 1, 11, 12, 29 and 30 are

$$ {}_{0}V = 500 \qquad {}_{1}V = 10{,}150 $$
$$ {}_{11}V = 143{,}035 \qquad {}_{12}V = 151{,}210 $$
$$ {}_{29}V = 155{,}745 \qquad {}_{30}V = 146{,}275 $$

(b) (*4 points*) Calculate Pr_t for t = 0, 1, 12 and 30.

(c) (*3 points*) The insurer calculates the NPV for each policy issued as 8860. Calculate the NPV at the start of the second year of future cashflows for each policy in force at that time.

****END OF EXAMINATION****

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