1. $\overset{0}{e}_{[58]+2} = e_{[58]+2} + 0.5$

$$e_{[58]+2} = p_{[58]+2}(1+e_{61}) = p_{[58]+2}\left[1+\frac{e_{60}}{p_{60}}-1\right]$$

$$= \frac{\ell_{61}}{\ell_{[58]+2}} \times \frac{e_{60}}{p_{60}} = \frac{2210}{3548} \times \frac{1}{(2210/3904)} = \frac{3904}{3549} = 1.100338$$

$$\overset{0}{e}_{[58]+2} = 1.100338 + 0.5 = 1.6$$

ANSWER: B

2. ${}_2q_{53}^{(1)} = q_{53}^{(1)} + p_{53}^{(\tau)} \cdot q_{54}^{(1)}$

$$q_{54}^{(1)} = q_{54}^{(\tau)} - q_{54}^{(2)} = (1-p_{54}^{(\tau)}) - q_{54}^{(2)} = \left(1-\frac{4625}{5000}\right) - 0.040 = 0.035$$

$$p_{53}^{(\tau)} = 1 - q_{53}^{(1)} - q_{53}^{(2)} = 1 - 0.025 - 0.030 = 0.945$$

$${}_2q_{53}^{(1)} = 0.025 + 0.945 \cdot 0.035 = 0.058$$

ANSWER: C

3. The probability of death by year 3:

$0.4 + 0.36 \times 0.20 + 0.24 \times 0.4 = 0.392$

Expected number of deaths = $1000 \times 0.392 = 392$

Variance of the number of deaths = $1000 \times 0.392 \times 0.608 = 238.336$

$$\Pr(X < 375) = \Pr\left(\frac{X-392}{\sqrt{238.336}} < \frac{375-392}{\sqrt{238.336}}\right) = \Pr(Z < -1.10) = \Phi(-1.10) = 0.1357$$

ANSWER: A

4. $p_{45}^{ILT} = \dfrac{9{,}127{,}246}{9{,}164{,}051} = 0.995984$

$$p_{45}^{S} = \exp\left(-\int_0^1 \mu_{45+t}^{S} dt\right) = \exp\left(-\int_0^1 \mu_{45+t}^{ILT} + 0.05dt\right) = \exp\left(-\int_0^1 \mu_{45+t}^{ILT} dt\right) \times \exp(-0.05)$$
$$= p_{45}^{ILT} e^{-0.05} = 0.995984 \times 0.951229 = 0.947409$$
$$\ddot{a}_{45}^{S} = 1 + \left(\frac{1}{1.06}\right) p_{45}^{S} \ddot{a}_{45}^{ILT} = 1 + 0.943396 \times 0.947409 \times 13.9546 = 13.4724$$

$$100\ddot{a}_{45}^{S} = 1347.24$$

ANSWER: A

Note: This solution has been revised. While the answer is unchanged, the above approach is more appropriate.

5.

$$q_{80}^{Ming} = 0.8q_{80}^{ILT} = 0.06424 \Rightarrow p_{80}^{Ming} = 0.93576$$
$$A_{80}^{Ming} = vp_{80}^{Ming} A_{81}^{ILT} + vq_{80}^{Ming} = \frac{1}{1.06} \times 0.93576 \times 0.680 + \frac{1}{1.06} \times 0.06424 = 0.66090$$
$$100{,}000 A_{80}^{Ming} = 66{,}090$$

ANSWER: B

6.

$$Var(Z) = 0.10E[Z] \Rightarrow v^{50}\,{}_{25}p_x(1 - {}_{25}p_x) = 0.10 \cdot v^{25}\,{}_{25}p_x$$
$$\Rightarrow \frac{(1-0.57)}{(1+i)^{50}} = 0.10 \times \frac{1}{(1+i)^{25}}$$
$$\Rightarrow (1+i)^{25} = \frac{0.43}{0.10} = 4.3 \Rightarrow i = 0.06$$

ANSWER: B

7.

$$P\ddot{a}_{55:\overline{10|}} = 0.51213P + v^{10}\,{}_{10}p_{55}\,\ddot{a}_{65}$$
$$\ddot{a}_{55} = \ddot{a}_{55:\overline{10|}} + v^{10}\,{}_{10}p_{55}\,\ddot{a}_{65}$$
$$v^{10}\,{}_{10}p_{55}\,\ddot{a}_{65} = 12.2758 - 7.4575 = 4.8183$$
$$7.4575P = 0.51213P + 4.8183$$
$$\Rightarrow P = 0.693742738$$
$$\Rightarrow 300P = 208.12$$

ANSWER: E

8.

$$G\ddot{a}_{70:\overline{10|}} = v^{10}\,{}_{10}p_{70}\,\ddot{a}_{80} + 0.05G\ddot{a}_{70:\overline{10|}} + 0.7G$$
$$\ddot{a}_{70:\overline{10|}} = \ddot{a}_{70} - v^{10}\,{}_{10}p_{70}\,\ddot{a}_{80} = 8.5693 - 0.33037 \times 5.905 = 6.61846515$$
$$6.61846515G = 0.33037 \times 5.905 + 6.61846515 \times 0.05 \times G + 0.7G$$
$$\Rightarrow G = 0.34914$$
$$\Rightarrow 100{,}000G = 34{,}914$$

ANSWER: D

9. Actuarial present value of insured benefits:

$$100{,}000\left[\frac{0.95 \times 0.02}{1.06^6} + \frac{0.95 \times 0.98 \times 0.03}{1.06^8} + \frac{0.95 \times 0.98 \times 0.97 \times 0.04}{1.06^8}\right] = 5{,}463.32$$

$$\Rightarrow P\left(1 + \frac{0.95}{1.06^5}\right) = 5{,}463.32 \Rightarrow P = 3{,}195.12$$

ANSWER: A

10.

$$G\ddot{a}^{(12)}_{40:\overline{20|}} = 100{,}000\left(\frac{i}{\delta}\right)A_{40} + 200 + 0.04G\ddot{a}^{(12)}_{40:\overline{20|}}$$

$$\ddot{a}_{40:\overline{20|}} = 14.8166 - 0.27414 \times 11.1454 = 11.7612$$

$$\ddot{a}^{(12)}_{40:\overline{20|}} = \alpha(12)\ddot{a}_{40:\overline{20|}} - (1 - {}_{20}E_{40})\beta(12)$$

$$= 1.00028 \cdot 11.7612 - (1 - 0.27414) \cdot 0.46812 = 11.4247$$

$$G = \frac{100{,}000 \times (.06 / 0.05827) \times 11.1454}{0.96 \times 11.4247} = 1532.795$$

$\Rightarrow$ G/12 = 128

ANSWER: C

11. The earlier the death (before year 20), the larger the loss. Since we are looking for the 95th percentile of the present value of benefits random variable, we must find the time at which 5% of the insureds have died. The present value of the death benefit for that insured is what is being asked for.

$$l_{45} = 9{,}164{,}051 \Rightarrow 0.95l_{45} = 8{,}705{,}848$$

$$l_{54} = 8{,}712{,}621$$

$$l_{55} = 8{,}640{,}861$$

So, the time is between ages 54 and 55, i.e. time 9 and time 10.

$$l_{45} - l_{54} = 9{,}164{,}051 - 8{,}712{,}621 = 451{,}430$$

$$0.05l_{45} = 458{,}202.6$$

$$458{,}203 - 451{,}430 = 6{,}773$$

$$l_{54} - l_{55} = 8{,}712{,}621 - 8{,}640{,}861 = 71{,}760$$

$$6{,}773 / 71{,}760 = 0.0944$$

The time just before the last 5% of deaths is expected to occur is: 9 + 0.0944 = 9.0944

The present value of death benefits at this time is:

$$100{,}000e^{-9.0944(0.06)} = 57{,}945$$

ANSWER: C

12.

$$Var(L_0) = \frac{{}^2A_{45} - (A_{45})^2}{(d\ddot{a})^2}$$

$$= \frac{0.06802 - 0.2012^2}{\left(\frac{.06}{1.06} \times 14.1121\right)^2} = \frac{0.02753856}{0.638078425} = 0.04315858$$

$$\sigma(L_0) = 0.207746437$$

$$200{,}000\sigma(L_0) = 41{,}549.29$$

ANSWER: A

13.

$$1{,}000P = 1{,}000\frac{A_{35}}{\ddot{a}_{35}} = \frac{128.72}{15.3926} = 8.36246$$

Benefits paid during July 2018:

$$10{,}000 \times 1{,}000 \times q_{35} = 10{,}000 \times 2.01 = 20{,}100$$

Premiums payable during July 2018:

$$10{,}000 \times (1 - q_{35}) \times 8.36246 = 9{,}979.9 \times 8.36246 = 83{,}456.51$$

Cash flow during July 2018:

$$20{,}100 - 83{,}456.51 = -63{,}356.51$$

ANSWER: A

14.

$$V_1 = 1 - \frac{\ddot{a}_{41}}{\ddot{a}_{40}} = 1 - \frac{14.6864}{14.8166} = 0.008787441$$

$$V_1 = B \times A_{41}$$

$$\Rightarrow 0.008787441 = B \times 0.16869$$

$$\Rightarrow B = 0.052092247$$

$$\Rightarrow 1{,}000{,}000B = 52{,}092.25$$

ANSWER: D

15.

$$A_{65} = (P+W)\times \ddot{a}_{65}$$
$$A_{45} = P\ddot{a}_{45} + W\ {}_{20}E_{45}\times \ddot{a}_{65}$$

$$0.4398 = (P+W)(9.8969)$$
$$0.2012 = 14.1121P + W(0.25634)(9.8969)$$
$$P+W = 0.044438157$$
$$\Rightarrow P = 0.044438157 - W$$

$$0.2012 = 14.1121(0.044438157 - W) + W(0.25634)(9.8969)$$
$$\Rightarrow W = 0.03679576$$
$$\Rightarrow 1{,}000W = 36.79576$$

ANSWER: D

16.

$$V_{10} = 2{,}290 = B\left(1-\frac{\ddot{a}_{x+10}}{\ddot{a}_x}\right) = B\left(1-\frac{11.4}{14.8}\right) \Rightarrow B = 9{,}969.53$$
$$G\ddot{a}_x = 25 + 5\ddot{a}_x + B\times A_x$$
$$A_x = 1 - d\ddot{a}_x = 1-\left(\frac{0.04}{1.04}\times 14.8\right) = 0.430769231$$
$$G\times 14.8 = 25 + 5\times 14.8 + 9{,}969.53\times 0.430769231$$
$$\Rightarrow G = 296.86$$

$${}_{10}V^g = 9{,}969.53A_{x+10} + 5\ddot{a}_{x+10} - 296.86\ddot{a}_{x+10}$$
$$A_{x+10} = 1 - d\ddot{a}_{x+10} = 1-\left(\frac{0.04}{1.04}\times 11.4\right) = 0.561538462$$
$${}_{10}V^g = 9{,}969.53\times 0.561538462 + 5\times 11.4 - 296.86\times 11.4$$
$$\Rightarrow {}_{10}V^g = 2{,}271.07$$

ANSWER: E

17.

$$AV_6 = [200{,}000 + 25{,}000(1-0.02) - COI] \times 1.05$$

$$COI = \left(\frac{AV_6 \times 2.5 - AV_6}{1.05} \right) \times \frac{30}{1{,}000}$$

$$AV_6 = 235{,}725 - 1.05COI = 235{,}725 - \frac{30}{1{,}000} \times \left(2.5AV_6 - AV_6 \right)$$

$$1.045AV_6 = 235{,}725 \Rightarrow AV_6 = 225{,}574.16$$

$$\Rightarrow ADB = AV_6 \times 2.5 - AV_6 = 338{,}361.24$$

ANSWER: E

18.

$$L_{10} = 10{,}000A_{35} = 1{,}287.20$$

$$L_{10}^{*} = 10{,}000$$

$$L_{10}^{*} - L_{10} = 10{,}000 - 1{,}287.20 = 8{,}712.80$$

ANSWER: E

19.

$$\left(\bar{I}\bar{a} \right)_{40:\overline{t|}} = \int_0^t s\, {}_s p_{40} v^s \, ds$$

$$\frac{d\left(\bar{I}\bar{a} \right)_{40:\overline{t|}}}{dt} = t\, {}_t p_{40} v^t$$

At $t = 10.5$,

$$10.5\, {}_{10.5}E_{40} = 10.5\, {}_{10}p_{40}\, {}_{0.5}p_{50} v^{10.5}$$

$$= 10.5\, {}_{10}E_{40}\, {}_{0.5}p_{50} v^{0.5}$$

$$= 10.5 \times 0.53667 \times (1 - 0.5 \times 0.00592)(0.97128586)$$

$$= 5.45703$$

ANSWER: C

20. Years of service at age 65: $15+(65-45)=35$

Final one-year salary: $(120{,}000)(1.04^{20})=262{,}935$

Projected pension: $(262{,}935)(35)(0.015)=138{,}041$

Actuarial present value of projected pension:

$$\frac{(138{,}041)(0.552)(10.60)}{1.05^{20}}=304{,}415.7$$

Actuarial liability: $\left(\dfrac{15}{35}\right)\left(304{,}415.7\right)=130{,}464$

Normal cost under projected unit credit with no benefits paid on next year's terminations is:

$$\frac{130{,}464}{15}=8{,}697.6$$

ANSWER: E