1. $${}_{2.5}q_{50} = {}_{2}q_{50} + {}_{2}p_{50}\ {}_{0.5}q_{52}$$
$$= 0.02 + (0.98)\left(\frac{0.5}{2}\right)(0.04)$$
$$= 0.0298$$

ANSWER: B

2. $${}_{20}q_{30} = \frac{S_0(30) - S_0(50)}{S_0(30)} = \frac{\frac{220}{250} - \frac{3}{4}}{\frac{220}{250}} = \frac{440 - 375}{440} = \frac{65}{440} = \frac{13}{88} = 0.1477$$

ANSWER: B

3. The 20-year female survival probability = $e^{-20\mu}$
The 20-year male survival probability = $e^{-30\mu}$

We want 1-year female survival = $e^{-\mu}$

Suppose that there were *M* males and 3*M* females initially. After 20 years, there are expected to be $Me^{-30\mu}$ and $3Me^{-20\mu}$ survivors, respectively. At that time we have:

$$\frac{3Me^{-20\mu}}{Me^{-30\mu}} = \frac{85}{15} \Rightarrow e^{10\mu} = \frac{85}{45} = \frac{17}{9} \Rightarrow e^{-\mu} = \left(\frac{9}{17}\right)^{1/10} = 0.938$$

ANSWER: C

4. $E[Z]=\int_0^\infty 0.04e^{-0.04t}\left(e^{0.02t}\right)e^{-0.06t}dt$

$$=0.04\int_0^\infty e^{-0.08t}dt=\frac{0.04}{0.08}=\frac{1}{2}$$

$$E\left[Z^2\right]=\int_0^\infty\left(0.04e^{-0.04}\right)\left(e^{0.04t}\right)\left(e^{-0.12t}\right)dt=\frac{0.04}{0.12}=\frac{1}{3}$$

$$Var[Z]=\frac{1}{3}-\left(\frac{1}{2}\right)^2=\frac{1}{12}=0.0833$$

ANSWER: E

5. $A^{\;1}_{[50]:\overline{3}|}=v\left(q_{[50]}+p_{[50]}v\left(q_{[50]+1}+p_{[50]+1}vq_{52}\right)\right)$

where: $v=\dfrac{1}{1.04}$

$q_{[50]}=0.7(0.045)=0.0315$
$p_{[50]}=1-q_{[50]}=0.9685$
$q_{[50]+1}=0.8(0.050)=0.040$
$p_{[50]+1}=1-q_{[50]+1}=0.960$
$q_{52}=0.055$

So: $A^{\;1}_{[50]:\overline{3}|}=0.1116$

ANSWER: D

6. The median of K_{48} is the integer m for which

$P(K_{48} < m) \leq 0.5$ and $P(K_{48} > m) \leq 0.5.$

This is equivalent to finding m for which

$$\frac{l_{48+m}}{l_{48}} \geq 0.5 \text{ and } \frac{l_{48+m+1}}{l_{48}} \leq 0.5.$$

Based on the ILT, we have $m = 30$ since

$$l_{78} \geq 4,522,840 \text{ and } l_{79} \leq 4,522,840.$$

So: $APV = 5000 A_{48} + 5000 {}_{30}E_{48} A_{78}$
$= 5000(0.22892) + 5000(0.24193)(0.36044)$
$= 1422.14$

ANSWER: A

7. The minimum premium to prevent lapse will be the premium such that $AV_5 = 0$.

Let P be this premium.

$$AV_5 = 0 = \left(AV_4 + 0.95P - 500 - \frac{20{,}000}{1.045} \right)(1.045) \Rightarrow P = 20{,}146.06$$

ANSWER: A

8. $$EPV\,(\text{NET PREMIUM}) + EPV\,\underbrace{(\text{EXPENSE LOADING})}_{\uparrow p^e} = \text{GROSS PREMIUM}$$

So: $1{,}000{,}000\dfrac{A_{50}}{\ddot{a}_{50}} + p^e = 19{,}526$

So: $p^e = 753.76$

ANSWER: C

9. Let P be the net premium for year 1.

Then:

$$P\left[1 + \frac{1.01}{1.05}0.99\right] = \frac{10^5}{1.05}\left(0.01 + \frac{0.99}{1.05}(1.01)(0.02)\right) \Rightarrow P = 1416.93$$

ANSWER: B

10. The policy is fully discrete, so all cash flows occur at the start or end of a year. There is a loss if death occurs in year 1 or year 2, otherwise the policy was profitable.

$$\Pr\left(\text{death in year 1 or 2}\right) = 1 - e^{-2\mu} = 0.113$$

ANSWER: D

11.
$$\begin{aligned} APV\left(\text{expenses}\right) &= 0.35G + 8 + 0.15G a_{30:\overline{4}|} + 4a_{30:\overline{9}|} \\ &= 0.20G + 4 + 0.15G\,\ddot{a}_{30:\overline{5}|} + 4\ddot{a}_{30:\overline{10}|} \end{aligned}$$

$$G\,\ddot{a}_{30:\overline{5}|} = 0.20G + 4 + 0.15G\,\ddot{a}_{30:\overline{5}|} + 4\ddot{a}_{30:\overline{10}|} + 200{,}000 A^{1}_{30:\overline{10}|}$$

$$G = \frac{200{,}000 A^{1}_{30:\overline{10}|} + 4 + 4\ddot{a}_{30:\overline{10}|}}{0.85\ddot{a}_{30:\overline{5}|} - 0.20}$$

$$\begin{aligned} 200{,}000\, A^{1}_{30:\overline{10}|} &= 200{,}000\left[A_{30} - {}_{10}E_{30}A_{40}\right] \\ &= 200(102.48) - (200)(0.54733)\left(161.32\right) \\ &= 2836.94 \end{aligned}$$

$$G = \frac{2836.94 + 4 + 4\left(7.7465\right)}{0.85\left(4.4516\right) - 0.20} = \frac{2871.926}{3.58386} = 801.35$$

ANSWER: C

12.
$$\begin{aligned} {}_{3}V^{FPT} &= 100{,}000\, A_{[55]+3} - 100{,}000\, P_{[55]+1}\,\ddot{a}_{[55]+3} \\ &= 100{,}000\, A_{58} - 100{,}000 \frac{A_{[55]+1}}{\ddot{a}_{[55]+1}}\ddot{a}_{58} \\ &= 100{,}000\left(0.27 - \frac{0.24}{\frac{1-0.24}{d}} \cdot \frac{1-0.27}{d} \right) \\ &= 3947.37 \end{aligned}$$

ANSWER: B

13. $${}_1V = \left({}_0V + P\right)(1+i) - \left(25{,}000 + {}_1V - {}_1V\right)q_x$$
$${}_2V = \left({}_1V + P\right)(1+i) - \left(50{,}000 + {}_2V - {}_2V\right)q_{x+1} = 50{,}000$$
$$\left(\left(P(1+i) - 25{,}000q_x\right) + P\right)(1+i) - 50{,}000q_{x+1} = 50{,}000$$
$$\left(\left(P(1.05) - 25{,}000(0.15)\right) + P\right)(1.05) - 50{,}000(0.15) = 50{,}000$$

Solving for *P*, we get

$$P = \frac{61{,}437.50}{2.1525} = 28{,}542.39$$

ANSWER: D

14. $$AV_2 = \left(AV_1 + 3000(1-0.07) - 10\right)(1.05) - \frac{3.0}{1000}\left(50{,}000 - AV_2\right)$$
$$= 5113.211 + 0.003AV_2$$
$$\Rightarrow AV_2 = \frac{5113.211}{0.997} = 5128.60$$

ANSWER: D

15. $$DPP = \min\left\{t : NPV(t) \geq 0\right\}$$
$$NPV(0) = \pi_0 = -550$$
$$NPV(1) = \pi_0 + \pi_1 v = -550 + \frac{300}{1.12} = -282$$
$$NPV(2) = NPV(1) + \pi_2 v = -282 + \frac{275}{1.12^2} = -62.91$$
$$NPV(3) = NPV(2) + \pi_3 v^3 = -62.91 + \frac{75}{1.12^3} = -9.53$$
$$NPV(4) = NPV(3) + \pi_4 v^4 = -9.53 + \frac{150}{1.12^4} = 85.80$$
$$NPV(4) \geq 0 \Rightarrow DPP = 4$$

ANSWER: D

16. $${}_4V = \frac{(505+220-30)(1.05)-10{,}000q_{53}}{1-q_{53}} = 666.2807$$

The profit for policy year 4 is

$$4885\ [(505+220-30)(0.01)+(30-34)(1.06)+(10{,}000-666.2807)(0.0068-\frac{42}{4885})]$$

$$=-68{,}730.37$$

ANSWER: C

17. $${}_1V^e = \frac{\left(\overbrace{G-187}^{p^e}-0.25G-10\right)(1.03)}{0.992} = -38.7$$

$$\Rightarrow 0.75G = \frac{-38.7(0.992)}{1.03}+187+10 = 159.72$$

$$\Rightarrow G = 212.97$$

ANSWER: B

18. At 66, the total retirement fund is $1{,}500{,}000(1.08)$ and is to be used to purchase a quarterly annuity equal to $X\ddot{a}_{66}^{(4)}$ so that:

$$X = \frac{1{,}500{,}000(1.08)}{\ddot{a}_{66}-\frac{3}{8}} = \frac{1{,}620{,}000}{9.6362-\frac{3}{8}} = 174{,}923.30$$

$$\text{Replacement Ratio} = \frac{X}{250{,}000} = \frac{174{,}923.30}{250{,}000} = 0.70$$

ANSWER: E

19. No projected salary for traditional unit credit method:

$${}_0V = (0.02)(10)(150{,}000)v^{20}\ {}_{20}p_{45}\ddot{a}_{65}^{(12)} = 127{,}157.50$$

$${}_1V = (0.02)(11)(150{,}000)v^{19}\ {}_{19}p_{46}\ddot{a}_{65}^{(12)}$$

Let C = normal contribution

$${}_0V + C = \underbrace{vp_{45}\ {}_1V}_{\frac{11}{10}\bullet {}_0V} \Rightarrow C = \frac{1}{10}\ {}_0V = 12{,}715.75$$

ANSWER: A

20. Kaitlyn's annual retirement benefit is

$$\frac{50{,}000}{5}\left(1.025^{26}+1.025^{27}+1.025^{28}+1.025^{29}+1.025^{30}\right)\times 31\times\left(1-0.07\left(3\right)\right)\times\left(0.02\right)$$
$$=10{,}000\left(31\right)\left(0.02\right)\left(0.79\right)\left(9.988563\right)=48{,}923.98$$

ANSWER: D