LTAM Fall 2019

Model Solutions
Written Answer Questions

Question 1 Model Solution

Learning Outcomes: 2(b), 2(k), 3(a), 3(c), 4(b), 4(d), 5(a), 5(d)

Chapter References: AMLCR Chapters 3, 4, 5, 6, 7 (example 7.11)

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a)

$$G\ddot{a}_{50} = 100{,}000\,\bar{A}_{50} + 500 + 50\,\ddot{a}_{50} + 0.05G\,\ddot{a}_{50}$$
$$\bar{A}_{50} = \frac{i}{\delta}A_{50} = 1.0248\,(0.18931) = 0.194005 \text{ ; UDD}$$
$$\ddot{a}_{50} = 17.0245$$

$$G = \frac{19{,}400.5 + 500 + 851.225}{16.173275} = \frac{20{,}751.725}{16.173275} = 1283.09$$

Comment: This part was done correctly by almost all candidates.

b) (i) ${}_{0.7+s}q_{80} = {}_{0.7}q_{80} + {}_{0.7}p_{80} \cdot {}_{s}q_{80.7}$

$$(0.7+s)\,q_{80} = (0.7)\,q_{80} + (1-(0.7)\,q_{80}) \cdot {}_{s}q_{80.7}\;;\;\; 0 < s \le 0.3 \quad \text{(UDD)}$$

$${}_{s}q_{80.7} = \frac{s \cdot q_{80}}{1 - 0.7\,q_{80}} = s\left(\frac{0.032658}{0.977139}\right) = 0.033422\,s;\;\; 0 < s \le 0.3$$

Alternatively, ${}_{0.7+s}p_{80} = {}_{0.7}p_{80} \cdot {}_{s}p_{80.7}$

$${}_{s}p_{80.7} = \frac{1-(0.7+s)q_{80}}{1-(0.7)\,q_{80}} = 1 - \frac{s \cdot q_{80}}{1 - 0.7\,q_{80}}$$
$$and\;\; {}_{s}q_{80.7} = 1 - {}_{s}p_{80.7} = \frac{s \cdot q_{80}}{1 - 0.7\,q_{80}} = 0.033422\,s\;;\;\; 0 < s \le 0.3$$

(ii) Acceptable expressions include:

$$\bar{A}_{80.7:\overline{s}|} = \int_0^s v^t\;{}_{t}p_{80.7}\,\mu_{80.7+t}\,dt$$
$$\bar{A}_{80.7:\overline{s}|} = \int_0^s v^t\;\frac{d}{dt}\,{}_{t}q_{80.7}\,dt$$
$$\bar{A}_{80.7:\overline{s}|} = 0.033422\int_0^s v^t\,dt$$

(iii) $\bar{A}_{80.7:\overline{s}|} = 0.033422\int_0^s v^t\,dt = 0.033422\,\bar{a}_{\overline{s}|} = 0.033422\,\frac{1-v^s}{\delta}\;;\;\; 0 < s \le 0.3$

$$\bar{A}_{80.7:\overline{0.3}|} = 0.033422\,\frac{1-v^{0.3}}{\delta} = 0.033422\,\frac{1-0.98547}{ln(1.05)} = 0.009953577$$
$$100{,}000\,\bar{A}_{80.7:\overline{0.3}|} = 995.36$$

Comments:

1. *Part (i) was done correctly by most candidates.*
2. *Most candidates received partial credit for parts (ii) and (iii).*
3. *Only well-prepared candidates received full credit for this part.*

c) ${}_{30.7}V = 100{,}000\,\bar{A}_{80.7:\overline{0.3|}} + {}_{0.3}E_{80.7} \cdot {}_{31}V$

where

$${}_{31}V = 100{,}000\,\bar{A}_{81} + 50\,\ddot{a}_{81} - 0.95G\,\ddot{a}_{81}$$
$$= 100{,}000\,\frac{i}{\delta}\,A_{81} + 50\,\ddot{a}_{81} - 0.95G\,\ddot{a}_{81};\ \text{UDD}$$
$$= 100{,}000(1.0248)(0.60984) + \big(50 - (0.95)(1283.09)\big)(8.1934) = 52{,}918.85$$

$${}_{0.3}E_{80.7} = v^{0.3}\ {}_{0.3}p_{80.7} = (1.05^{-0.3})\,(1 - 0.033422(0.3)) = 0.9755886$$

$${}_{30.7}V = 995.36 + (0.9755886)(52{,}918.85) = 52{,}622.39$$

Alternatively,

$$({}_{30}V + 0.95G - 50)\ \cdot (1+i)^{0.7}$$
$$= 100{,}000 \cdot (1+i)^{0.7} \cdot \int_0^{0.7} v^t\ {}_tp_{80}\,\mu_{80+t}\,dt + {}_{0.7}p_{80}\ \cdot {}_{30.7}V$$

where ${}_tp_{80}\,\mu_{80+t} = q_{80} = 0.032658;\ \ 0 < t < 1$; UDD
and $\int_0^{0.7} v^t\ dt = 0.688181$;

$${}_{30}V = 100{,}000\,\bar{A}_{80} + 50\,\ddot{a}_{80} - 0.95G\,\ddot{a}_{80}$$
$$= 100{,}000\,\frac{i}{\delta}\,A_{80} + 50\,\ddot{a}_{80} - 0.95G\,\ddot{a}_{80};\ \text{UDD}$$
$$= 100{,}000(1.0248)(0.59293) + \big(50 - (0.95)(1283.09)\big)(8.5484) = 50{,}770.94$$

$${}_{30.7}V = \frac{53{,}744.42 - 2{,}325.55}{0.977139} = 52{,}621.86$$

Comments:

1. *Performance on this part was mixed.*
2. *Candidates who recognized that the result in part b) could be used in a recursion formula to find the reserve at 30.7 did well.*

d) Interpolating between these reserves would ignore the fact that the reserve will increase immediately after the premium is paid. Here, interpolating between the sum of the reserve at time 30 and the premium paid at 30, net of expenses; and the value of the reserve at time 31 would give a good estimate of the reserve at time 30.7.

Comments:

1. *Most candidates did poorly on this part.*
2. *Only well-prepared candidates correctly identified the impact of the premiums received at the beginning of the year.*

Question 2 Model Solution

Learning Outcomes: 2(a), 2(b), 4(b), 4(c), 5(a), 5(b)

Chapter References: AMLCR Chapter 8

General comment: Most candidates did very well on this question.

a) $E[L_0] = EPV(DB) - EPV(\text{premiums net of commissions}) = -0.1G$

$EPV(DB) = 50{,}000\, A_{55}^{(12)\,02} = 24{,}000$
$EPV(\text{premiums net of commissions}) = (0.95)\, G\, \ddot{a}_{55}^{00} = (0.95)(8.832)G = 8.3904\, G$

$24{,}000 - 8.3904\, G = -0.1G$
$G = \frac{24{,}000}{8.2904} = 2894.91$

Comments:

1. *This part was done correctly by virtually all candidates.*
2. *A few candidates incorrectly used the equivalence principle and solved by setting for G using $E[L_0\} = 0$.*

b) ${}_{10}V^{(0)} = 50{,}000\, A_{65}^{(12)\,02} - (0.95)G\, \ddot{a}_{65}^{00}$
$= 50{,}000(0.634) - (0.95)(2894.91)(5.416) = 31{,}700 - 14{,}894.89 = 16{,}805.11$

Comments:

1. *This part was also done correctly by most students.*
2. *The most common error was to forget commissions on the premiums.*

c) (i) $\left({}_{10}V^{(0)} + (0.95)G\right)(1.05)^{1/12} = {}_{\frac{1}{12}}p_{65}^{00} \cdot {}_{10\frac{1}{12}}V^{(0)} + {}_{\frac{1}{12}}p_{65}^{01} \cdot {}_{10\frac{1}{12}}V^{(1)} + {}_{\frac{1}{12}}p_{65}^{02}\,(50{,}000)$

${}_{\frac{1}{12}}p_{65}^{00} = 1 - {}_{\frac{1}{12}}p_{65}^{01} - {}_{\frac{1}{12}}p_{65}^{02} = 1 - 0.00461 - 0.00293 = .99246$

$\left(16{,}805.11 + (0.95)(2894.91)\right)(1.05)^{1/12} = (0.99246)\,{}_{10\frac{1}{12}}V^{(0)} + (0.00461)(34{,}110)$
$+(0.00293)(50{,}000)$

$$\Rightarrow {}_{10\frac{1}{12}}V^{(0)} = \frac{19{,}634.945 - 157.247 - 146.5}{0.99246} = 19{,}478.06$$

(ii) ${}_{10\frac{1}{12}}V^{(0)}\,(1.05)^{1/12} = {}_{\frac{1}{12}}p^{00}_{65\frac{1}{12}} \cdot {}_{10\frac{2}{12}}V^{(0)} + {}_{\frac{1}{12}}p^{01}_{65\frac{1}{12}} \cdot {}_{10\frac{2}{12}}V^{(1)} + {}_{\frac{1}{12}}p^{02}_{65\frac{1}{12}}\,(50{,}000)$

$${}_{\frac{1}{12}}p^{00}_{65\frac{1}{12}} = 1 - {}_{\frac{1}{12}}p^{01}_{65\frac{1}{12}} - {}_{\frac{1}{12}}p^{02}_{65\frac{1}{12}} = 1 - 0.00467 - 0.00295 = 0.99238$$

$$19{,}557.42 = (0.99238)\,{}_{10\frac{2}{12}}V^{(0)} + (0.00467)(34{,}170) + (0.00295)(50{,}000)$$

$$\Rightarrow {}_{10\frac{2}{12}}V^{(0)} = \frac{19{,}557.42 - 159.57 - 147.50}{0.99238} = \frac{19{,}250.35}{0.99238} = 19{,}398.16$$

Comments:

1. *Once again, the majority of candidates got part (i) correct.*
2. *Fewer candidates got part (ii) correct with the most common error being to include the premium.*

d) (i) ${}_{10\frac{2}{12}}V^{(0)} = EPV(\textit{benefits}) - EPV(\textit{future premiums net of commissions})$

$EPV(\textit{benefits})$ is not affected.
Since $EPV(\textit{future premiums net of commissions})$ will decrease, ${}_{10\frac{2}{12}}V^{(0)}$ will increase.

(ii) ${}_{10\frac{2}{12}}V^{(1)} = EPV(\textit{benefits}) - EPV(\textit{future premiums net of commissions})$

$EPV(\textit{benefits})$ is not affected.
Since premiums are waived while in State 1, no commissions are paid;
<u>and</u> since there are no transitions back to State 0 after reaching State 1,
the higher commissions will have no impact on ${}_{10\frac{2}{12}}V^{(1)}$.

Comments:

1. *Candidates did not do well on this part.*
2. *Many candidates assumed that the reserve in part (i) would decrease arguing that since less premium (net of commissions) is being collected that the reserve has to be less.*
3. *Most candidates correctly stated that the reserve would not change. However, not many students were able to completely explain why.*

Question 3 Model Solution

Learning Outcomes: 2(a), 2(b), 2(j), 3(a), 3(b), 4(b), 4(c)

Chapter References: AMLCR Chapters 8, 9

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a) Common shock:
Simultaneous deaths represented by transitions from State 0 to State 4.
Broken heart syndrome:
Mortality of surviving life is higher than individual mortality when both are alive, $\mu^{13}>\mu^{02}$ and $\mu^{23}>\mu^{01}$.

Comments:

1. *Performance on this part was mixed.*
2. *Many candidates identified only one way that dependency is incorporated in the model.*
3. *A common error was to compare incorrectly forces of mortality, e.g. compare μ^{13} and μ^{23}.*

b) $(i)\ {}_{10}p_{50}^{23} = 1 - e^{-\int_0^{10} 1.05\, \mu_{50+t}^{*}\, dt}$

$$= 1 - \left(e^{-\int_0^{10} \mu_{50+t}^{*}\, dt}\right)^{1.05} = 1 - \left({}_{10}p_{50}^{SULT}\right)^{1.05} = 1 - \left(\frac{96{,}634.1}{98{,}576.4}\right)^{1.05} = 0.0206784$$

$(ii)\ {}_{10}p_{40:50}^{00} = e^{-\int_0^{10} \mu_{40+t:50+t}^{01} + \mu_{40+t:50+t}^{02} + \mu_{40+t:50+t}^{04}\, dt}$

$$= e^{-\int_0^{10} (\mu_{50+t}^{*} - 0.0005) + (\mu_{40+t}^{*} - 0.0005) + 0.0005 dt}$$

$$= e^{(0.0005)(10)} \cdot {}_{10}p_{40}^{SULT} \cdot {}_{10}p_{50}^{SULT} = e^{0.005} \cdot {}_{20}p_{40}^{SULT}$$

$$= e^{0.005}\left(\frac{96{,}634.1}{99{,}338.3}\right) = 0.9776539$$

Comments:

1. *Only the most well-prepared candidates achieved full or nearly full credit on this part.*
2. *Partial credit was awarded to candidates who showed some understanding of the question by writing down some formulas relevant to the calculation of these probabilities of transition.*
3. *A common error in part (ii) was to integrate an exponential function of the transition intensities, e.g. $\int_0^{10} exp(\mu_{40+t:50+t}^{01} + \mu_{40+t:50+t}^{02} + \mu_{40+t:50+t}^{04})\, dt$.*

c) Note that we must use the givens of $\bar{a}^{00}_{40:50:\overline{10|}} = 7.8487$ and $\bar{A}^{03}_{40:50:\overline{10|}} = 0.00789$.

$$P\,\bar{a}^{00}_{40:50:\overline{10|}} = 100{,}000\,\bar{A}^{03}_{40:50:\overline{10|}} + 300{,}000\,\bar{A}^{04}_{40:50:\overline{10|}}$$

$$\bar{A}^{04}_{40:50:\overline{10|}} = \int_0^{10} e^{-\delta t}\ {}_tp^{00}_{40:50}\ \mu^{04}_{40+t:50+t}\,dt = 0.0005\int_0^{10} e^{-\delta t}\ {}_tp^{00}_{40:50}\ dt$$

$$= 0.0005\,\bar{a}^{00}_{40:50:\overline{10|}} = 0.0005\,(7.8487) = 0.00392435$$

$$7.8487\,P = 789.0 + 1177.305$$
$$P = 250.5262$$

Comments:

1. *Most candidates were able to write down the formula needed to calculate the premium.*
2. *Some candidates were also able to write down a correct expression for $\bar{A}^{04}_{40:50:\overline{10|}}$ but only well-prepared candidates recognized that it could be calculated from the given value for $\bar{a}^{00}_{40:50:\overline{10|}}$.*

d) (i) $\bar{a}^{00}_{x:y:\overline{10|}}$: Stays the **same**.

The transition intensities leaving State 0 do not change and it is impossible to return to State 0 after leaving it.

(ii) $\bar{A}^{03}_{x:y:\overline{10|}}$: Would be **higher**.

If (y) dies first, no impact. If (x) dies first, (y) is likely to die sooner which will increase the EPV of the insurance benefit.

(iii) $\bar{a}_{x|y}$: This corresponds to $\bar{a}^{02}_{x:y}$ in this model. Would be **lower**.

This annuity is payable while (y) is alive after the death of (x). Since in this case, (y) is likely to die sooner, the EPV of the annuity payments will decrease.

(iv) Premium: Would be **higher**.

Since $\bar{a}^{00}_{x:y:\overline{10|}}$ and $\bar{A}^{04}_{x:y:\overline{10|}}$ would stay the same but $\bar{A}^{03}_{40:50:\overline{10|}}$ would increase, the premium in c) would increase.

Comments:

1. *Most candidates received partial credit for correctly justifying the impact of the change on some of the four given actuarial functions.*
2. *Only well-prepared candidates adequately justified the impact of the change on all four functions.*

Question 4 Model Solution

Learning Outcomes: 1(b), 3(a), 4(a), 4(b)

Chapter References: AMLCR Chapters 1, 4, 5, 6

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a) Mortality:
Direct marketed policies generally offer relatively low benefits with little or no medical evidence except for a standard questionnaire. Because of the potential for adverse selection, insurers assume higher mortality for these policies.

Expenses:
Because there is less underwriting done on these policies and typically lower commissions, the expenses for writing new business are lower.

Comments:

1. *Performance on this part was mixed. Most candidates either achieved full or near full credit or received little or no credit.*
2. *A common reason for achieving only partial credit was to state the differences in mortality and expenses assumptions without providing a reason why these differences exist.*

b) Let P be the annual gross premium.

$$L^g = \begin{cases} 50{,}000\, v^{K^*+1} - (0.97)\, P\, \ddot{a}_{\overline{K^*+1}|} & K^* = 0, 1, \ldots, 9 \\ -(0.97)\, P\, \ddot{a}_{\overline{10}|} & K^* = 10, 11, \ldots \end{cases}$$

where $K^* = K_{40+5}$ (from SULT)

Comments:

1. *Most candidates received little or no credit for this part for failing to define a proper random variable. A common error was to define L^g as the difference between EPVs.*
2. *Only well-prepared candidates provided a complete and correct definition of the loss-at-issue random variable.*

c) $E[L^g] = 0$ => $50{,}000\, A^{\,1}_{45:\overline{10}|} = (0.97)\, P\, \ddot{a}_{45:\overline{10}|}$;

$$P = \frac{50{,}000(A_{45} - {}_{10}E_{45}A_{55})}{(0.97)\ddot{a}_{45:\overline{10}|}} = \frac{50{,}000\big(0.15161 - (0.60655)(0.23524)\big)}{(0.97)(8.0751)} = 56.97$$

Alternatively,

$$P = \frac{50{,}000(A_{45:\overline{10}|} - {}_{10}E_{45})}{(0.97)\ddot{a}_{45:\overline{10}|}} = \frac{50{,}000(0.61547 - 0.60655)}{(0.97)(8.0751)} = 56.94$$

Comments:

1. *Almost all candidates achieved full or nearly full credit for this part.*
2. *For those who did not receive full credit, common errors included using age 40 instead of 45 in the SULT and using an incorrect EPV for the death benefit and/or premiums.*

d)

(i) Let $Z_1 = \begin{cases} v^{K^*+1} & K^* = 0, 1, \ldots, 9 \\ 0 & K^* = 10, 11, \ldots \end{cases}$

Then,

$$V[Z_1] = {}^2A_{\overset{1}{45}:\overline{10}|} - \left(A_{\overset{1}{45}:\overline{10}|}\right)^2$$

${}^2A_{\overset{1}{45}:\overline{10}|} = ({}^2A_{45} - v^{10}\,{}_{10}E_{45} \cdot {}^2A_{55})$ and $A_{\overset{1}{45}:\overline{10}|} = (A_{45:\overline{10}|} - {}_{10}E_{45})$

$$V[Z_1] = [0.03463 - (1.05^{-10})(0.60655)(0.07483)] - (0.61547 - 0.60655)^2 = 0.006686055$$

Variance of pv of benefit: $50{,}000^2(0.006686055) = 16{,}715{,}137.62$
And SD = 4088.42

(ii) Since $L^g = 50{,}000\, Z_1 - (0.97)\, P\left(\frac{1-Z_2}{d}\right)$, the total variance will increase by the variance of the EPV of net premiums and twice the (negative) covariance between the EPV of benefits and the EPV of net premiums.
Since the premiums are approximately 0.1% of the benefit, the total variance will increase by a very small amount.
In the case of survival for 10 years, there will be no benefit and 10 premiums paid. In case of death, the 50,000 benefit is paid with a relatively small additional uncertainty about the EPV of premiums received. So the variance is almost entirely due to the uncertainty about the 50,000 benefit.

Comments:

1. *Performance on this part was poor with most candidates achieving little or no credit.*
2. *Candidates who did well on part (b) did very well on part (d) (i).*
3. *Even for those who correctly calculated the SD, providing a good explanation for part (ii) was a challenge.*

e)

Let G be the premium paid during the first 5 years. $E[L^g] = 0$.

$$50{,}000A_{\overset{1}{45}:\overline{10}|} = (0.97)G(2\ddot{a}_{45:\overline{10}|} - \ddot{a}_{45:\overline{5}|})$$

Alternatively,

$$50{,}000\, A_{\overset{1}{45}:\overline{10}|} = (0.97)\, G\, (\ddot{a}_{45:\overline{10}|} + {}_5E_{45}\, \ddot{a}_{50:\overline{5}|})$$

$$G = \frac{50{,}000 A^{\;1}_{45:\overline{10}|}}{(0.97)(2\,\ddot{a}_{45:\overline{10}|} - \ddot{a}_{45:\overline{5}|})} = \frac{50{,}000 A^{\;1}_{45:\overline{10}|}}{(0.97)(\ddot{a}_{45:\overline{10}|} + {}_5E_{45}\,\ddot{a}_{50:\overline{5}|})}$$

Using either

$\ddot{a}_{45:\overline{5}|} = \ddot{a}_{45} - {}_5E_{45}\,\ddot{a}_{50} = 17.8162 - (0.77991)(17.0245) = 4.538622$ or

$\ddot{a}_{50:\overline{5}|} = \ddot{a}_{50} - {}_5E_{50}\,\ddot{a}_{55} = 17.0245 - (0.77772)(16.0599) = 4.534395;$

$$G = \frac{446}{(0.97)(2\,(8.0751) - 4.538622)} = 39.5979 \quad or$$

$$G = \frac{446}{(0.97)(8.0751 + (0.77991)(4.534395))} = 39.5981$$

Comments:

1. *Performance on this part was mixed.*
2. *Many candidates failed to correctly reflect the fact that premiums doubled after 5 years.*

f) There is a risk of anti-selection at time 5, i.e. a lapse and re-entry risk.
With the 10-year policy, policyholders have a free option to continue or renew at time 5 and pay a lower premium until then. A policyholder aged 40 could buy the 10-year term with an initial premium which is less than the premium for the 5-year term. At time 5, he/she could lapse the policy if coverage is no longer needed. Continuing coverage is also an option, if he/she can qualify again, buying a new 10-year term policy would be cheaper (for most ages) than keeping the original policy in force. If he/she cannot qualify, maintaining the original 10-year policy by paying the increased premium is an option, one that is not available with the 5-year policy.

Comments:

1. *Only well-prepared candidates correctly identified the anti-selection risk at time 5.*
2. *The candidates who discussed the popularity of the two products on the basis of the health condition of the policyholder (sick versus healthy) received no credit.*

Question 5 Model Solution

Learning Outcomes: 2(a), 2(b), 2(g), 3(a)

Chapter References: AMLCR Chapter 5, SN LTAM 21-18 Revised (Section 4)

General comment: Many candidates omitted this question entirely or only answered parts (a) and (b).

a)

k	*q(65+k, k)*	*P(K_{65} = k)*
0	q_{65}^{SULT}= 0.005915	0.005915
1	0.006619(1-0.033)^{1} =0.00640057	(1-0.005915)(0.00640057)=0.006363
2	0.007409(1-0.031)^{2} =0.00695676	(1-0.005917)(1-0.00640057)(0.00695676) =0.006871

Comments:

1. *Most candidates who answered this part did very well.*
2. *Common errors for those who achieved partial credit included using incorrect improvement factors and not squaring the improvement factor when calculating q(67, 2).*

b)

k	$y = \ddot{a}_{\overline{min(3,k+1)}|}$	$P(Y = y)$ =(K_{65}= k)	$y \cdot P(Y = y)$	$y^2 \cdot P(Y = y)$
0	1	0.005915	0.005915	0.005915
1	1.952381	0.006363	0.012422	0.024253
2+	2.85941	1-0.005915-0.006363 =0.987722	2.824303	8.075843
Total			$E[Y]$ = 2.842641	$E[Y^2]$ =8.106011

$V(Y) = 8.106011 - (2.842641)^2 = 0.025404$

Alternatively,

$$V(Y) = \frac{{}^2A_{x:\overline{3}|} - (A_{x:\overline{3}|})^2}{d^2} = 0.025433$$

$A_{x:\overline{3}|} = 0.005915 \text{ v } + 0.006363 \text{ v}^2 + \ 0.987722 \text{ v}^3 = 0.86464$

${}^2A_{x:\overline{3}|} = 0.005915 \text{ v}^2 \ + 0.006363 \text{ v}^4 + \ 0.987722 \text{ v}^6 = 0.74766$

And $SD(Y) = \ 0.1594$

Comments:

1. *Candidates did poorly on this part.*
2. *Even candidates who correctly calculated the probabilities of dying in each of the first three years, working with the three-point distribution for Y or calculating the EPVs in the alternative solution proved to be challenging.*

c) (i) $p(x,t) = p(x,0)(1+\Psi_{x,t})$

$$= p(x,0)\left[1+\frac{q(x,0)}{p(x,0)}(1-(1-\varphi_x)^t)\right]$$

$$= \frac{p(x,0)}{p(x,0)}[p(x,0)+q(x,0)-q(x,0)(1-\varphi_x)^t]$$

$$= 1-q(x,0)(1-\varphi_x)^t = 1-q(x,t)$$

(ii) $\Psi_{70,10} = \frac{q(70,0)}{p(70,0)}(1-(1-\varphi_{70})^{10}) = \frac{0.010413}{0.989587}(1-0.97^{10}) = 0.00276297$

(iii) $p(70,t) = p(70{,}0)\left[1+\frac{0.010413}{0.989587}(1-0.97^t)\right] \geq .995$; t ≥24.085, so 25 years.

Comments:

1. *Most candidates omitted this part or received little or no credit for it.*
2. *Only very well-prepared candidates achieved full or nearly credit for this part.*

d) Use improvement factors that are a function of both age and calendar year (cohort effect).
Alternatively,
Use a stochastic model, e.g. Lee-Carter model or CBD model.

Comment:

Performance on this part was mixed with most candidates either achieving full credit or receiving no credit.

Question 6 Model Solution

Learning Outcomes: 4(b), 5(a), 5(c)

Chapter References: AMLCR Chapter 12

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a) $EPV(Premiums) = EPV(return\ of\ premiums) + EPV(payments) + EPV(expenses)$

$P \cdot (0.95\ \ddot{a}_{65:\overline{10|}} - 0.25\ ) = (10\ P)\ {}_{10}E_{65}\ A^{\ 1}_{75:\overline{10|}} + (36{,}000)\ {}_{20}E_{65}\ \ddot{a}_{85} + 900 + 100\ \ \ddot{a}_{65}$

$$P = \frac{36{,}000(0.24381)(6.7993) + 900 + 100(13.5498)}{(0.95)(7.8435) - 0.25 - (10)(0.55305)(0.65142 - 0.44085)} = 10{,}259.385$$

Comments:

1. *Most candidates did very well on this part.*
2. *For those who did not receive full credit, a common error was to incorrectly value the return of premium feature.*

b) $Pr_0 = -E_0 - {}_0V = -7000 - 500 = -7500$

$Pr_t = {}_{t-1}V + CF_t - E_t + I_t - EDB_t - E\,{}_tV; \qquad t \geq 1$

where CF_t is the net cash flow received by the insurer at time t.

$$CF_t = \begin{cases} P = 10{,}259.385 & t = 0,1,\ldots,9 \\ 0 & t = 10,11,\ldots,19 \\ -36{,}000 & t = 20,21,\ldots \end{cases}$$

$Pr_1 = ({}_0V + P - (0.05P + 70))(1 + i) - 0 - (1 - 0.03)(1 - 0.9\ q_{65})\ {}_1V$

$Pr_1 = (500 + (0.95)(10{,}259.385) - 70)(1.07) - (0.9648362)(10{,}150) = 1{,}095.68$

$Pr_{12} = ({}_{11}V - 70(1.02)^{11})(1 + i) - 0.9q_{76}(10)(10{,}259.385) - (1 - 0)(1 - 0.9\ q_{76})\ {}_{12}V$

$Pr_{12} = (143{,}035 - 70(1.02)^{11})(1.07) - 0.0186012(102{,}593.85) - (0.9813988)(151{,}210)$
$= \ 2{,}648.64$

$Pr_{30} = ({}_{29}V - 36{,}000 - 70(1.02)^{29})(1 + i) - 0 - (1 - 0)(1 - 0.9\ q_{94})\ {}_{30}V$

$Pr_{30} = (155{,}745 - 36{,}000 - 70(1.02)^{29})(1.07)\ - (0.8595532)(146{,}275) = 2{,}262.995$

Comments:

1. *Many candidates omitted this part.*
2. *Most candidates who answered did part only achieved partial credit for it.*
3. *The most common error was to use incorrect cash flows when calculating the emerging profits, Pr_t, for t=1, 12, 30.*

c)

Let NPV_1 be the EPV at the start of year 2 (time 1) of future emerging profits per policy in force.

$$NPV_1 = Pr_2\, v_{0.1}^1 + {}_1p_{x+1}^{00}\, Pr_3\, v_{0.1}^2 + {}_2p_{x+1}^{00}\, Pr_4\, v_{0.1}^3 + \ldots$$

$$NPV = \sum_{t=0}^{n} \pi_t\ v_{0.1}^t = 8860$$

where $\pi_0 = Pr_0$ and $\pi_t = {}_{t-1}p_x^{00}\, Pr_t,\ \ t \geq 1.$

$$\begin{aligned} NPV &= Pr_0 + Pr_1\, v_{0.1} + \left({}_1p_x^{00}\, Pr_2\, v_{0.1}^2 + {}_2p_x^{00}\, Pr_3\, v_{0.1}^3 + \ldots\right) \\ &= Pr_0 + Pr_1 v_{0.1} + (\text{EPV at time 0 of profits emerging in years 2, 3, \ldots}) \\ &= Pr_0 + Pr_1\, v_{0.1} + {}_1p_x^{00}\, v_{0.1}\left(Pr_2\, v_{0.1} + {}_1p_{x+1}^{00}\, Pr_3\, v_{0.1}^2 + \ldots\right) \\ &= Pr_0 + Pr_1\, v_{0.1} + {}_1p_x^{00}\, v_{0.1} NPV_1 \end{aligned}$$

$$8860 = -7500 + \frac{1095.68}{1.1} + (1 - 0.03)(1 - 0.9q_{65})\ \frac{1}{1.1}\ \ NPV_1$$

$$NPV_1 = \frac{8860 + 7500 - 1095.68/1.1}{(0.9648362)/1.1} = \frac{15{,}363.9273}{0.877124} = 17{,}516.25$$

Comments:

1. *Most candidates omitted this part or received no credit for it.*
2. *Only very well-prepared candidates correctly derived the relationship between NPV and NPV_1 that was needed to answer this question.*