COURSE 3
MAY 2001

MULTIPLE-CHOICE ANSWER KEY

1	E
2	A
3	C
4	E
5	D
6	B
7	C
8	D
9	C
10	A
11	B
12	A
13	E
14	B
15	A
16	B
17	A
18	E
19	C
20	C
21	E
22	D
23	A
24	B
25	C

26	B
27	D
28	D
29	E
30	D
31	D
32	A
33	D
34	A
35	B
36	E
37	C
38	A
39	E
40	B

SOLUTIONS FOR MAY 2001 COURSE 3 EXAM

Test Question: 1 **Key: E**

For de Moivre's law,

$$\overset{\circ}{e}_{30} = \int_0^{w-30}\left(1-\frac{t}{w-30}\right)dt$$

$$= \left[t-\frac{t^2}{2(w-30)}\right]_0^{w-30}$$

$$= \frac{w-30}{2}$$

Prior to medical breakthrough $w = 100 \Rightarrow \overset{\circ}{e}_{30} = \dfrac{100-30}{2} = 35$

After medical breakthrough $\overset{\circ}{e}'_{30} = \overset{\circ}{e}_{30} + 4 = 39$

so $\overset{\circ}{e}'_{30} = 39 = \dfrac{w'-30}{2} \Rightarrow w' = 108$

Test Question: 2 **Key: A**

$${}_0L = 100{,}000v^{2.5} - 4000\ddot{a}_{\overline{3}|} \quad @5\%$$

$$= 77{,}079$$

Test Question: 3 **Key: C**

$$E[N] = E_\Lambda\left[E[N|\Lambda]\right] = E_\Lambda[\Lambda] = 2$$
$$Var[N] = E_\Lambda\left[Var[N|\Lambda]\right] + Var_\Lambda\left[E[N|\Lambda]\right]$$
$$= E_\Lambda[\Lambda] + Var_\Lambda[\Lambda] = 2 + 2 = 4$$

Distribution is negative binomial (<u>Loss Models</u>, 3.3.2)

Per supplied tables

$$mean = r\beta = 2$$
$$Var = r\beta(1+\beta) = 4$$
$$(1+\beta) = 2$$
$$\beta = 1$$
$$r\beta = 2$$
$$r = 2$$

From tables

$$p_3 = \frac{r(r+1)(r+2)\beta^3}{3!(1+\beta)^{r+3}} = \frac{(2)(3)(4)1^3}{3!\,2^5} = \frac{4}{32} = 0.125$$

$1000\ p_3 = 125$

Test Question: 4 Key: E

$\mathrm{E}\left[N\right] = Var\left[N\right] = (60)(0.5) = 30$
$\mathrm{E}\left[X\right] = (0.6)(1) + (0.2)(5) + (0.2)(10) = 3.6$
$\mathrm{E}\left[X^2\right] = (0.6)(1) + (0.2)(25) + (0.2)(100) = 25.6$
$Var\left[X\right] = 25.6 - 3.6^2 = 12.64$

For any compound distribution, per <u>Loss Models</u>
$Var\left[S\right] = E\left[N\right]Var\left[X\right] + Var\left[N\right]\left(E\left[X\right]\right)^2$
$= (30)\ (12.64) + (30)\ \left(3.6^2\right)$
$= 768$

For specifically Compound Poisson, per <u>Probability Models</u>
$Var\left[S\right] = l\,t\,E\left[X^2\right] = (60)\ (0.5)\ (25.6) = 768$

Alternatively, consider this as 3 Compound Poisson processes (coins worth 1; worth 5; worth 10), where for each $Var(X) = 0$, thus for each $Var(S) = Var(N)E\left[X\right]^2$.
Processes are independent, so total Var is

$$Var = (60)(0.5)(0.6)1^2 + (60)(0.5)(0.2)5^2 + (60)(0.5)(0.2)(10)^2$$
$$= 768$$

Test Question: 5 **Key: D**

$$1000\,{}_{20}^{20}V_x = 1000\,A_{x+20} = \frac{1000\left({}_{19}^{20}V_x + {}_{20}P_x\right)(1.06) - q_{x+19}(1000)}{p_{x+19}}$$

$$= \frac{(342.03 + 13.72)(1.06) - 0.01254(1000)}{0.98746} = 369.18$$

$$\ddot{a}_{x+20} = \frac{1 - 0.36918}{(0.06/1.06)} = 11.1445$$

$$so \quad 1000P_{x+20} = 1000\frac{A_{x+20}}{\ddot{a}_{x+20}} = \frac{369.18}{11.1445} = 33.1$$

Test Question: 6 **Key: B**

$$ {}_k p_x^{(t)} = e^{-\int_0^k m_x^{(t)}(t)dt} = e^{-\int_0^k 2\, m_x^{(1)}(t)dt} $$

$$ = \left(e^{-\int_0^k m_x^{(1)}(t)dt} \right)^2 $$

$= \left({}_k p_x \right)^2$ where ${}_k p_x$ is from Illustrative Life Table, since $m^{(1)}$ follows I.L.T.

$$ {}_{10} p_{60} = \frac{6{,}616{,}155}{8{,}188{,}074} = 0.80802 $$

$$ {}_{11} p_{60} = \frac{6{,}396{,}609}{8{,}188{,}074} = 0.78121 $$

$$ {}_{10|}q_{60}^{(t)} = {}_{10}p_{60}^{(t)} - {}_{11}p_{60}^{(t)} $$

$= \left({}_{10} p_{60} \right)^2 - \left({}_{11} p_{60} \right)^2$ from I.L.T.

$$ = 0.80802^2 - 0.78121^2 = 0.0426 $$

Test Question: 7 **Key: C**

State 1: light Training
State 2: heavy Training

$$P_{11} = 0.4\times 0.5+0.6\times 0 = 0.2$$
$$P_{12} = 0.4\times 0.5+0.6\times 1 = 0.8$$
$$P_{21} = 0.8\times 0.5+0.2\times 0 = 0.4$$
$$P_{22} = 0.8\times 0.5+0.2\times 1 = 0.6$$
$$P = \begin{bmatrix} 0.2 & 0.8 \\ 0.4 & 0.6 \end{bmatrix}$$
$$p_1 = 0.2p_1 + 0.4p_2$$
$$p_2 = 0.8p_1 + 0.6p_2$$
$$p_1 + p_2 = 1$$
$$\Rightarrow 1-p_2 = 0.2(1-p_2) + 0.4.\times p_2 = 0.2+0.2p_2 \Rightarrow 1.2p_2 = 0.8 \qquad p_2 = \frac{8}{12} = \frac{2}{3}$$

Note: the notation in <u>Probability Models</u> would label the states 0 and 1, and would label the top row and left column of the matrix *P* with subscript 0. The underlying calculations are the same. The matrix *P* would look different, but the result would be the same, if you chose to make "heavy" the lower-numbered state.

Test Question: 8 **Key: D**

$s = \sqrt{0.0004} = 0.02$

$Y(1)$ is normal $(0.01, \quad 0.0004)$

0.1587 corresponds to -1 standard deviation $\Rightarrow$

$Y(1) = 0.01 - (1)(0.02) = -0.01$

$Y(2) - Y(1)$ is normal $(0.01, \quad 0.0004)$

0.9332 corresponds to +1.5 standard deviation $\Rightarrow$

$$\begin{aligned} Y(2) &= Y(1) + 0.01 + (1.5)(0.02) \\ &= -0.01 + 0.01 + 0.03 \\ &= 0.03 \end{aligned}$$

$F = 100e^{Y(1)} = 100e^{-0.01} = 99.00$

$G = 100e^{Y(2)} = 100e^{0.03} = 103.05$

$G - F = 4.05$

Test Question: 9 **Key: C**

$P_s = \frac{1}{\mathring{e}_s} - d$, where s can stand for any of the statuses under consideration.

$$\mathring{e}_s = \frac{1}{P_s + d}$$

$$\mathring{e}_x = \mathring{e}_y = \frac{1}{0.1 + 0.06} = 6.25$$

$$\mathring{e}_{\overline{xy}} = \frac{1}{0.06 + 0.06} = 8.333$$

$$\mathring{e}_{\overline{xy}} + \mathring{e}_{xy} = \mathring{e}_x + \mathring{e}_y$$

$$\mathring{e}_{xy} = 6.25 + 6.25 - 8.333 = 4.167$$

$$P_{xy} = \frac{1}{4.167} - 0.06 = 0.18$$

Test Question: 10 Key: A

$$d_0^{(t)} = 1000\int_0^1 e^{-(m+0.04)t}(m+0.04)dt$$

$$= 1000\left(1 - e^{-(m+0.04)}\right) = 48$$

$$e^{-(m+0.04)} = 0.952$$

$$m + 0.04 = -\ln(0.952)$$

$$= 0.049$$

$$m = 0.009$$

$$d_3^{(1)} = 1000\int_3^4 e^{-0.049t}(0.009)dt$$

$$= 1000\frac{0.009}{0.049}\left(e^{-(0.049)(3)} - e^{-(0.049)(4)}\right) = 7.6$$

Test Question: 11 **Key: B**

$${}_2p_{70}^{(t)} = 1 - 0.1 - 0.1 - 0.1 - 0.5 = 0.2$$

$$F(0) = {}_2p_{70}^{(t)} = 0.20$$

$$F(1000v) = F(943) = F(0) + {}_{0|}q_{70}^{(1)} = 0.30$$

$$F\left(1100v^2\right) = F(979) = F(943) + {}_{1|}q_{70}^{(1)} = 0.40$$

$$F(1100v) = F(1038) = F(979) + {}_{0|}q_{70}^{(2)} = 0.50$$

$$F\left(1200v^2\right) = F(1068) = F(1038) + {}_{1|}q_{70}^{(2)} = 1.00 \ \left[\text{good; must have } F(\text{maximum possible}) = 1\right]$$

$F(943)$ < random number < $F(979)$, so choose 979

Test Question: 12 **Key: A**

Let Z_i be random variable indicating death; W_i be random variable indicating lapse for policy. Let U denote the random number used.

policy # 1: $q_{100} = 0.40812$ from Illustrative Life Table

$U = 0.3 < 0.40812$ $\quad Z_1 = 1$ $\quad W_1 = 0$

policy # 2: $q_{91} = 0.20493$ from Illustrative Life Table

$U = 0.5 > 0.20493$ $\quad Z_2 = 0$

next checking lapse $U = 0.1 < 0.15$ (surrender rate) $\Rightarrow W_2 = 1$

policy # 3 $q_{96} = 0.30445$

$U = 0.4 > 0.30445$ $\quad Z_3 = 0$

next checking lapse $U = 0.8 > 0.15$ $\Rightarrow W_3 = 0$

$\Rightarrow$ total Death and Surrender Benefits = 10+20+0 = 30

Test Question: 13 **Key: E**

$${}_2p_x = 1 - 0.1 - 0.2 = 0.7$$

$${}_3p_x = 0.7 - 0.3 = 0.4$$

Use $l_x = 1$ (arbitrary, doesn't affect solution)

so $l_{x+2} = 0.7$ $l_{x+3} = 0.4$

By hyperbolic $\dfrac{1}{l_{x+2.5}} = .5\dfrac{1}{l_{x+2}} + .5\dfrac{1}{l_{x+3}}$

$$= \frac{.5}{.7} + \frac{.5}{.4} = 1.9643$$

$$l_{x+2.5} = 0.5091 = {}_{2.5}p_x$$

$${}_{2.5}q_x = 1 - 0.5091 = 0.4909$$

Prob (all 3 failed) $= (0.4909)^3 = 0.118$

Test Question: 14 **Key: B**

This is a graph of $l_x m(x)$.

$m(x)$ would be increasing in the interval $(80,100)$.

The graphs of $l_x p_x$, l_x and l_x^2 would be decreasing everywhere.

The graph shown is comparable to Figure 3.3.2 on page 65 of Actuarial Mathematics

Test Question: 15 **Key: A**

Using the conditional mean and variance formulas:

$$E[N] = E_\Lambda(N|\Lambda)$$

$$Var[N] = Var_\Lambda(E(N|\Lambda)) + E_\Lambda(Var(N|\Lambda))$$

Since N, given lambda, is just a Poisson distribution, this simplifies to:

$$E[N] = E_\Lambda(\Lambda)$$
$$Var[N] = Var_\Lambda(\Lambda) + E_\Lambda(\Lambda)$$

We are given that $E[N] = 0.2$ and $Var[N] = 0.4$, subtraction gives $Var\ (\Lambda) = 0.2$

Test Question: 16 **Key: B**

N = number of salmon
X = eggs from one salmon
S = total eggs.
$E(N) = 100t$
$Var(N) = 900t$

$$E(S) = E(N)E(X) = 500t$$
$$Var(S) = E(N)Var(X) + E^2(X)Var(N) = 100t \cdot 5 + 25 \cdot 900t = 23{,}000t$$
$$P(S > 10{,}000) = P\left(\frac{S - 500t}{\sqrt{23{,}000t}} > \frac{10{,}000 - 500t}{\sqrt{23{,}000t}}\right) = .95 \Rightarrow$$
$$10{,}000 - 500t = -1.645 \cdot \sqrt{23000}\sqrt{t} = -250\sqrt{t}$$
$$40 - 2t = -\sqrt{t}$$
$$2\left(\sqrt{t}\right)^2 - \sqrt{t} - 40 = 0$$
$$\sqrt{t} = \frac{1 \pm \sqrt{1 + 320}}{4} = 4.73$$
$$t = 22.4$$
round up to 23

Test Question: 17 Key: A

$$A\ P\ V\,(x\text{'s benefits}) = \sum_{k=0}^{2} v^{k+1} b_{k+1}\ {}_k p_x\, q_{x+k}$$

$$= 1000\left[300v(0.02) + 350v^2(0.98)(0.04) + 400v^3(0.98)(0.96)(0.06)\right]$$

$$= 36{,}829$$

Test Question: 18 Key: E

p denotes benefit premium

${}_{19}V = APV$ future benefits - APV future premiums

$$0.6 = \frac{1}{1.08} - p \Rightarrow p = 0.326$$

$$\begin{aligned} {}_{11}V &= \frac{\left({}_{10}V + p\right)(1.08) - \left(q_{65}\right)(10)}{p_{65}} \\ &= \frac{(5.0 + 0.326)(1.08) - (0.10)(10)}{1 - 0.10} \\ &= 5.28 \end{aligned}$$

Test Question: 19 Key: C

X = losses on one life

$$E[X] = (0.3)(1) + (0.2)(2) + (0.1)(3)$$
$$= 1$$

S = total losses

$$E[S] = 3E[X] = 3$$

$$E[(S-1)_+] = E[S] - 1(1 - F_s(0))$$
$$= E[S] - (1)(1 - f_s(0))$$
$$= 3 - (1)(1 - 0.4^3)$$
$$= 3 - 0.936$$
$$= 2.064$$

Test Question: 20 **Key: C**

$$M_x(r) = E\left[e^{rx}\right]$$
$$= \frac{e^r + e^{2r} + e^{3r}}{3}$$
$$M_x(0.5) = \frac{e^{0.5} + e + e^{1.5}}{3} = 2.95$$
$$p_1 = E[X] = \frac{1+2+3}{3} = 2$$

$$l\left[M_x(r) - 1\right] = cr$$

Since $l = 2$ and $r = 0.5$,

$$2\left[M_x(0.5) - 1\right] = 0.5c$$
$$2(2.95 - 1) = 0.5c$$
$$3.9 = 0.5c$$
$$c = 7.8 = \text{premium rate per period}$$

Test Question: 21 Key: E

Simple's surplus at the end of each year follows a Markov process with four states:

State 0: out of business
State 1: ending surplus 1
State 2: ending surplus 2
State 3: ending surplus 3 (after dividend, if any)

State 0 is absorbing (recurrent). All the other states are transient states.
Thus eventually Simple must reach state 0.

Test Question: 22 Key: D

(See solution to problem 21 for definition of states).

$t=0$

$$[0 \quad 0 \quad 0 \quad 1]\begin{bmatrix} 1.0 & 0.0 & 0.00 & 0.00 \\ 0.1 & 0.5 & 0.25 & 0.15 \\ 0.1 & 0.0 & 0.50 & 0.40 \\ 0.0 & 0.1 & 0.00 & 0.90 \end{bmatrix} = [0.0 \quad 0.1 \quad 0.0 \quad 0.9] \text{ at } t=1$$

$t=1$

$$[0.0 \quad 0.1 \quad 0.0 \quad 0.9]\begin{bmatrix} 1.0 & 0.0 & 0.00 & 0.00 \\ 0.1 & 0.5 & 0.25 & 0.15 \\ 0.1 & 0.0 & 0.50 & 0.40 \\ 0.0 & 0.1 & 0.00 & 0.90 \end{bmatrix} = [0.01 \quad 0.14 \quad 0.025 \quad 0.825] \text{ at } t=2$$

Expected dividend at the end of the third year =

$$\sum_{k=0}^{3} \text{(probability in state } k \text{ at } t = 2) \times \text{(expected dividend if in state } k)$$

0.01*0 + 0.14*0 + 0.025(0*0.85 + 1*0.15) + 0.825*(0*0.6 +1*0.25 + 2*0.15) = 0.4575

Test Question: 23 **Key: A**

$$1180 = 70\bar{a}_{30} + 50\bar{a}_{40} - 20\bar{a}_{30:40}$$
$$1180 = (70)(12) + (50)(10) - 20\bar{a}_{30:40}$$
$$\bar{a}_{30:40} = 8$$
$$\bar{a}_{\overline{30:40}} = \bar{a}_{30} + \bar{a}_{40} - \bar{a}_{30:40} = 12 + 10 - 8 = 14$$
$$100\bar{a}_{\overline{30:40}} = 1400$$

Test Question: 24 **Key: B**

$$\overline{a} = \int_0^{\infty} \overline{a}_{\overline{t}|}\, f(t)dt \;\; = \int_0^{\infty} \frac{1-e^{-0.05t}}{0.05} \frac{1}{\Gamma(2)} te^{-t} dt$$

$$= \frac{1}{0.05} \int_0^{\infty} \left(te^{-t} - te^{-1.05t} \right) dt$$

$$= \frac{1}{0.05} \left[-(t+1)e^{-t} + \left(\frac{t}{1.05} + \frac{1}{1.05^2} \right) e^{-1.05t} \right] \Bigg|_0^{\infty}$$

$$= \frac{1}{0.05} \left[1 - \left(\frac{1}{1.05} \right)^2 \right] = 1.85941$$

$20{,}000 \times 1.85941 = 37{,}188$

Test Question: 25 **Key: C**

$$
\begin{aligned}
p(k) &= \frac{2}{k} p(k-1) \\
&= \left[0 + \frac{2}{k}\right] p(k-1)
\end{aligned}
$$

Thus an $(a, b, 0)$ distribution with $a = 0$, $b = 2$.

Thus Poisson with $l = 2$.

$$
\begin{aligned}
p(4) &= \frac{e^{-2} 2^4}{4!} \\
&= 0.09
\end{aligned}
$$

Test Question: 26 Key: B

By the memoryless property, the distribution of amounts paid in excess of 100 is still exponential with mean 200.

With the deductible, the probability that the amount paid is 0 is $F(100) = 1 - e^{-100/200} = 0.393.$

Thus the average amount paid per loss is (0.393) (0) + (0.607) (200) = 121.4

The expected number of losses is (20) (0.8) = 16.

The expected amount paid is (16) (121.4) = 1942.

Test Question: 27 **Key: D**

From UDD $l_{96.5} = \dfrac{l_{96} + l_{97}}{2}$

$$480 = \frac{600 + l_{97}}{2} \Rightarrow l_{97} = 360$$

Likewise, from $l_{97} = 360$ and $l_{97.5} = 288$, we get $l_{98} = 216$

For constant force,

$$e^{-m} = \frac{l_{98}}{l_{97}} = \frac{216}{360} = 0.6$$

$$_{0.5}p_{97} = e^{-.5m} = (0.6)^{1/2} = 0.7746$$

$$l_{97.5} = (0.7746)l_{97} = (0.7746)(360) = 278.86$$

Test Question: 28 **Key: D**

Let M = the force of mortality of an individual drawn at random; and T = future lifetime of the individual.

$$
\begin{aligned}
\Pr[T \le 1] &= E\{\Pr[T \le 1 | M]\} \\
&= \int_0^\infty \Pr[T \le 1 | M = m] f_M(m) dm \\
&= \int_0^2 \int_0^1 m e^{-mt} dt \frac{1}{2} dm \\
&= \int_0^2 \left(1 - e^{-m}\right) \frac{1}{2} du = \frac{1}{2}\left(2 + e^{-2} - 1\right) = \frac{1}{2}\left(1 + e^{-2}\right) \\
&= 0.56767
\end{aligned}
$$

Test Question: 29 **Key: E**

$$E[N] = (0.8)(1) + (0.2)(2) = 1.2$$
$$E[N^2] = (0.8)1 + (0.2)(4) = 1.6$$
$$\text{Var(N)} = 1.6 - 1.2^2 = 0.16$$
$$E[X] = 70 + 100 = 170$$
$$\text{Var(X)} = E[X^2] - E[X]^2 = (7000 + 100{,}000) - 170^2 = 78{,}100$$
$$E[S] = E[N]E[X] = 1.2(170) = 204$$
$$\text{Var(S)} = E[N]\text{Var}(\text{X}) + E[X]^2\,\text{Var}(\text{N}) = 1.2(78{,}100) + 170^2(0.16) = 98{,}344$$

Std dev $(S) = \sqrt{98{,}344} = 313.6$

So B = 204 + 314 = 518

Test Question: 30 Key: D

$$f_s(1000) = (0.8)(0.1) + (0.2)(2)(0.2)(0.1) = 0.088$$
$$f_s(1100) = (0.2)(2)(0.7)(0.1) = 0.028$$
$$f_s(2000) = (0.2)(0.1)^2 = 0.002$$
$$E\big[(S-200)_+\big] = (0.088)(800) + (0.028)(900) + (0.002)(1800)$$
$$= 99.2$$

With 175% relative security loading, cost = (2.75) (99.2) = 272.8

Alternatively,

$$f_s(0) = F_s(0) = (0.8)(0.2) + (0.2)(0.2)^2 = 0.168$$
$$f_s(100) = (0.8)(0.7) + (0.2)(2)(0.2)(0.7) = 0.616$$
$$F_s(100) = 0.168 + 0.616 = 0.784$$
$E\big[S\big] = 204$ [from problem 29]
$$E\big[(S-200)_+\big] = E\big[(S-100)_+\big] - (100)(1 - F_s(100))$$
$$= E\big[S\big] - (100)(1 - F_s(0)) - (100)(1 - F_s(100))$$
$$= 204 - (100)(1 - 0.168) - (100)(1 - 0.784)$$
$$= 99.2$$

cost = (2.75) (99.2) = 272.8

Test Question: 31 **Key: D**

Let $p =$ benefit premium

Actuarial present value of benefits =

$$= (0.03)(200{,}000)v + (0.97)(0.06)(150{,}000)v^2 + (0.97)(0.94)(0.09)(100{,}000)v^3$$
$$= 5660.38 + 7769.67 + 6890.08$$
$$= 20{,}320.13$$

Actuarial present value of benefit premiums

$$= \ddot{a}_{x:\overline{3}|}\, p$$
$$= \left[1 + 0.97v + (0.97)(0.94)v^2\right] p$$
$$= 2.7266p$$
$$p = \frac{20{,}320.13}{2.7266} = 7452.55$$

$${}_1V = \frac{(7452.55)(1.06) - (200{,}000)(0.03)}{1 - 0.03}$$
$$= 1958.46$$

Initial reserve, year 2 = ${}_1V + p$

= 1958.56 + 7452.55

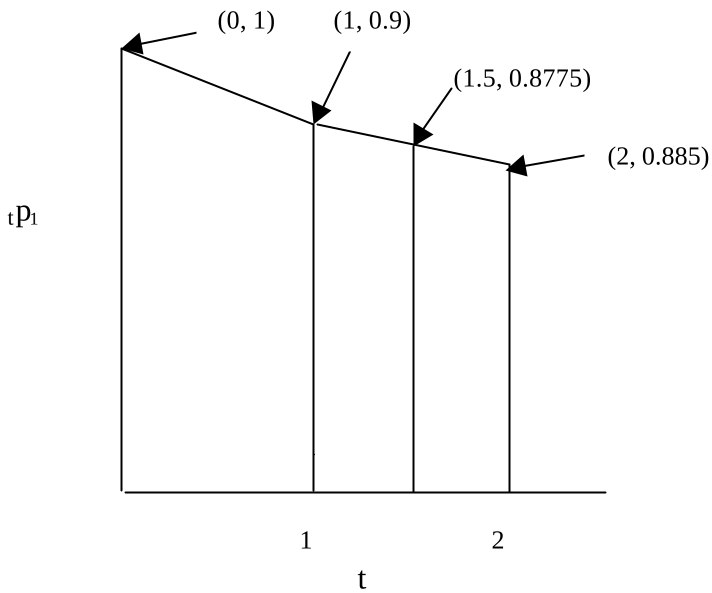
= 9411.01

Test Question: 32 **Key: A**

Let p denote the premium.

$$L = b_T v^T - p\,\bar{a}_{\overline{T}|} = (1+i)^T \times v^T - p\,\bar{a}_{\overline{T}|}$$
$$= 1 - p\,\bar{a}_{\overline{T}|}$$
$$E[L] = 1 - p\,\bar{a}_x = 0 \quad \Rightarrow p = 1/\bar{a}_x$$
$$\Rightarrow L = 1 - p\,\bar{a}_{\overline{T}|} = 1 - \frac{\bar{a}_{\overline{T}|}}{\bar{a}_x} = \frac{d\,\bar{a}_x - (1 - v^T)}{d\,\bar{a}_x}$$
$$= \frac{v^T - (1 - d\,\bar{a}_x)}{d\,\bar{a}_x} = \frac{v^T - \bar{A}_x}{1 - \bar{A}_x}$$

Test Question: 33 **Key: D**



$${}_1p_1 = (1-0.1) = 0.9$$
$${}_2p_1 = (0.9)(1-0.05) = 0.855$$

since uniform, ${}_{1.5}p_1 = (0.9+0.855)/2$
$= 0.8775$

$$\overset{0}{e}_{1:\overline{1.5}|} = \text{ Area between } t=0 \text{ and } t=1.5$$
$$= \left(\frac{1+0.9}{2}\right)(1) + \left(\frac{0.9+0.8775}{2}\right)(0.5)$$
$$= 0.95 + 0.444$$
$$= 1.394$$

Alternatively,

$$\overset{0}{e}_{1:\overline{1.5}|} = \int_0^{1.5} {}_tp_1 dt$$
$$= \int_0^1 {}_tp_1 dt + {}_1p_1 \int_0^{0.5} {}_xp_2 dx$$
$$= \int_0^1 (1-0.1t)dt + 0.9\int_0^{0.5}(1-0.05x)dx$$
$$= \left[t - \tfrac{0.1t^2}{2}\right]_0^1 + 0.9\left[x - \tfrac{0.05x^2}{2}\right]_0^{0.5}$$
$$= 0.95 + 0.444 = 1.394$$

Test Question: 34 Key: A

$$10{,}000\,A_{63}(1.12) = 5233$$

$$A_{63} = 0.4672$$

$$A_{x+1} = \frac{A_x(1+i) - q_x}{p_x}$$

$$A_{64} = \frac{(0.4672)(1.05) - 0.01788}{1 - 0.01788}$$

$$= 0.4813$$

$$A_{65} = \frac{(0.4813)(1.05) - 0.01952}{1 - 0.01952}$$

$$= 0.4955$$

Single contract premium at 65 = (1.12) (10,000) (0.4955)
= 5550

$$(1+i)^2 = \frac{5550}{5233} \qquad i = \sqrt{\frac{5550}{5233}} - 1 = 0.02984$$

Test Question: 35 Key: B

Original Calculation (assuming independence):

$$m_x = 0.06$$
$$m_y = 0.06$$
$$m_{xy} = 0.06 + 0.06 = 0.12$$
$$\overline{A}_x = \frac{m_x}{m_x + d} = \frac{0.06}{0.06 + 0.05} = 0.54545$$
$$\overline{A}_y = \frac{m_y}{m_y + d} = \frac{0.06}{0.06 + 0.05} = 0.54545$$
$$\overline{A}_{xy} = \frac{m_{xy}}{m_{xy} + d} = \frac{0.12}{0.12 + 0.05} = 0.70588$$
$$\overline{A}_{\overline{xy}} = \overline{A}_x + \overline{A}_y - \overline{A}_{xy} = 0.54545 + 0.54545 - 0.70588 = 0.38502$$

Revised Calculation (common shock model):

$$m_x = 0.06,\ m_x^{T^*(x)} = 0.04$$
$$m_y = 0.06,\ m_y^{T^*(y)} = 0.04$$
$$m_{xy} = m_x^{T^*(x)} + m_y^{T^*(y)} + m^Z + 0.04 + 0.04 + 0.02 = 0.10$$
$$\overline{A}_x = \frac{m_x}{m_x + d} = \frac{0.06}{0.06 + 0.05} = 0.54545$$
$$\overline{A}_y = \frac{m_y}{m_{y+d}} = \frac{0.06}{0.06 + 0.05} = 0.54545$$
$$\overline{A}_{xy} = \frac{m_{xy}}{m_{xy} + d} = \frac{0.10}{0.10 + 0.05} = 0.66667$$
$$\overline{A}_{\overline{xy}} = \overline{A}_x + \overline{A}_y - \overline{A}_{xy} = 0.54545 + 0.54545 - 0.66667 = 0.42423$$

Difference $= 0.42423 - 0.38502 = 0.03921$

Test Question: 36 Key: E

Treat as three independent Poisson variables, corresponding to 1, 2 or 3 claimants.

$$\text{rate}_1 = 6 \quad \left[= \frac{1}{2}\times 12\right]$$
$$\text{rate}_2 = 4$$
$$\text{rate}_3 = 2$$

$$\text{Var}_1 = 6$$
$$\text{Var}_2 = 16 \quad \left[= 4\times 2^2\right]$$
$$\text{Var}_3 = 18$$

total Var = $6+16+18=40$, since independent.

Alternatively,

$$E\left(X^2\right)=\frac{1^2}{2}+\frac{2^2}{3}+\frac{3^2}{6}=\frac{10}{3}$$

For compound Poisson, $\text{Var}[S] = E[N]E[X^2]$
$$= (12)\left(\frac{10}{3}\right) = 40$$

Test Question: 37 Key: C

$\int_0^3 l(t)dt = 6$ so $N(3)$ is Poisson with $l = 6$.

P is Poisson with mean 3 (with mean 3 since $\text{Prob}(y_i < 500) = 0.5$)

P and Q are independent, so the mean of P is 3, no matter what the value of Q is.

Test Question: 38 Key: A

At age x:

Actuarial Present value (APV) of future benefits = $\left(\frac{1}{5}A_x\right)1000$

APV of future premiums = $\left(\frac{4}{5}\ddot{a}_x\right)p$

$\frac{1000}{5}A_{25} = \frac{4}{5}p\,\ddot{a}_{25}$ by equivalence principle

$\frac{1000}{4}\frac{A_{25}}{\ddot{a}_{25}} = p \Rightarrow p = \frac{1}{4} \times \frac{81.65}{16.2242} = 1.258$

${}_{10}V$ = APV (Future benefits) – APV (Future benefit premiums)

$= \frac{1000}{5}A_{35} - \frac{4}{5}p\,\ddot{a}_{35}$

$= \frac{1}{5}(128.72) - \frac{4}{5}(1.258)(15.3926)$

$= 10.25$

Test Question: 39 Key: E

Let Y = present value random variable for payments on one life

$S = \sum Y =$ present value random variable for all payments

$E\left[Y\right] = 10\ddot{a}_{40} = 148.166$

$$\mathrm{Var}\left[Y\right] = 10^2 \frac{\left({}^2A_{40} - A_{40}^2\right)}{d^2}$$

$$= 100\left(0.04863 - 0.16132^2\right)\left(1.06 / 0.06\right)^2$$

$$= 705.55$$

$E\left[S\right] = 100E\left[Y\right] = 14{,}816.6$

$\mathrm{Var}\left[S\right] = 100\,\mathrm{Var}\left[Y\right] = 70{,}555$

Standard deviation $\left[S\right] = \sqrt{70{,}555} = 265.62$

By normal approximation, need

$E\,[S]$ + 1.645 Standard deviations = 14,816.6 + (1.645) (265.62)

= 15,254

Test Question: 40 Key: B

Initial Benefit Prem

$$= \frac{5A_{30} - 4\left(A^{1}_{30:\overline{20}|}\right)}{5\ddot{a}_{30:\overline{35}|} - 4\ddot{a}_{30:\overline{20}|}}$$

$$= \frac{5(0.10248) - 4(0.02933)}{5(14.835) - 4(11.959)}$$

$$= \frac{0.5124 - 0.11732}{74.175 - 47.836} = \frac{0.39508}{26.339} = 0.015$$

Where

$$A^{1}_{30:\overline{20}|} = \left(A_{30:\overline{20}|} - A_{30:\overline{20}|}^{\;\;\;\;\;\;1}\right) = 0.32307 - 0.29374 = 0.02933$$

and

$$\ddot{a}_{30:\overline{20}|} = \frac{1 - A_{30:\overline{20}|}}{d} = \frac{1 - 0.32307}{\left(\frac{0.06}{1.06}\right)} = 11.959$$

Comment: the numerator could equally well have been calculated as $A_{30} + 4\ {}_{20}E_{30}\ A_{50}$

= 0.10248 + (4) (0.29374) (0.24905)

= 0.39510