Chapter 1: Linear Equations in Linear Algebra
Section 1.1: Systems of Linear Equations

BGSU Math 3320

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Sometimes also called "systems of *simultaneous* equations"

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It is important that you clearly understand the definition of equivalence of two linear systems.

A system with many solutions

Example: For this linear system

$$\begin{aligned} x_1 - x_2 - 2x_3 &= 1 \\ 3x_1 - 3x_2 - 5x_3 &= 3 \\ 2x_1 - 2x_2 - 6x_3 &= 2, \end{aligned}$$

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these are examples of <u>"solutions"</u>:

$$\begin{pmatrix} x_1 = 1 \\ x_2 = 0 \\ x_3 = 0 \end{pmatrix}, \qquad \begin{pmatrix} x_1 = 2 \\ x_2 = 1 \\ x_3 = 0 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} x_1 = -1 \\ x_2 = -2 \\ x_3 = 0 \end{pmatrix}$$

Each is <u>in the solution set.</u>

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If a system has at least one solution
then it is **"consistent."** *Otherwise,* **"inconsistent."**

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(we multiplied both sides of 1^{st} eq'n with 7.)

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3. We can **replace** one of the equations with a scalar multiple of another:

None of these three operations alters the solution set.

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Here, we *added the same number to both sides* of the 1^{st} equation, since

$$4 \times (\text{left side of eqn } 3) = 4 \times (\text{right side of eqn } 3)$$

None of these three operations alters the solution set.

Example

Find the solution set of

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$\boxed{\therefore \text{ Solution Set: } \{(1+x_2, x_2, 0) \,|\, x_2 \in \mathbb{R}\}}$ "x_2 is a free variable"

Matrix Notation for linear systems.

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$$\overbrace{\left[\begin{array}{ccc|c} 1 & -1 & -2 & 1 \\ 3 & -3 & -5 & 3 \\ 2 & -2 & -6 & 2 \end{array}\right]}^{\text{Augmented matrix for the system}}$$

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- Note well the difference between **augmented** matrix and the **coefficient** matrix.
- Convention about matrices: "Row,Column" or "$m \times n$".

The **three types of “elementary row operations”**

Things we can do to a **system of equations** without changing the solution set	Things we can do to an **augmented matrix** without changing the solution set of the associated linear system
We can swap the order of the equations	**1. (“swap”) We can interchange any two rows**
We can scale both sides of one of the equations.	**2. (“scale”) We can multiply a row a scalar**
We can add number to both sides of an equation	**3. (“Replacement”) We two can replace a row with the sum of itself and any scalar multiple of another row**

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Two Fundamental Questions about a Linear System:

1. Is the system consistent; that is, does at least one solution exist?
2. If a solution exists, is it the only one; that is, is the solution unique?

Elementary Row Operations

Example

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We can now read-off the solution set, as before.

Intuitively, we seek "Triangular" matrices

Let's take a look at Example 1.1.1 (p. 5) of the text.

$$\begin{aligned} x_1 - 2x_2 + x_3 &= 0 \\ 2x_2 - 8x_3 &= 8 \\ 5x_1 - 5x_3 &= 10 \end{aligned}$$

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So do you see the inituitive meaning of "upper triangular"?
Do you see the advantage of putting a matrix into a triangular form?

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So we're going to be using row operations to put matrices into upper triangular form.

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3. Two equivalent linear systems can have different solution sets.

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Problem 1.1.2

Solve the linear system $\left\{ \begin{array}{rcr} 2x_1 + 4x_2 & = & -4 \\ 5x_1 + 7x_2 & = & 11 \end{array} \right\}$ by using elementary row operations on the augmented matrix. Follow the systematic elimination procedure described in this section.

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We can also present the solution as $(x_1, x_2) = (12, -7)$.

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Why?

To see why, interpret the last row of the triangular matrix as an equation ...