## Instructions and hints:

- Read both texts carefully before proceeding with their summary or translation.
- Write with medium sized clear calligraphy. It is recommended that you produce a first draft on rough paper.
- **No dictionaries are allowed.** If you do not know the meaning of a word or expression, leave it untranslated and underlined. We are interested in what you know by yourself, not in the quality of your dictionary.
- Your summary should be $150 \pm 50$ words in length. Deviating from these limits will be penalized.
- To prevent grammatical errors use simple and short sentences. If you end up with a very long sentence try to split it up.
- Avoid literal translations (word for word). Try to understand the text and explain it in your own words.
- Notice that the mark obtained here represents 60% of your final mark.

1. **(5 points) Summarize the following text in approximately 150 words.**

   [**Note**: In your summary, do not focus on definitions (you may skip them completely) but on the meaning of the problem and the solution proposed.]

(W.H. Gates and C.H. Papadimitriu (1979). Bounds for sorting by prefix reversal. Discrete Mathematics, 27:47-57)

### 1. Introduction

We introduce our problem by the following quotation from [1]:

> *The chef in our place is sloppy, and when he prepares a stack of pancakes they come out all different sizes. Therefore, when I deliver them to a customer, on the way to the table I rearrange them (so that the smallest winds up on top, and so on, down to the largest at the bottom) by grabbing several from the top and flipping them over, repeating this (varying the number I flip) as many times as necessary. If there are $n$ pancakes, what is the maximum number of flips (as a function $f(n)$ of $n$) that I will ever have to use to rearrange them?*

In this paper we derive upper and lower bounds for $f(n)$. Certain bounds were already known. For example, consider any stack of pancakes. An adjacency in this stack is a pair of pancakes that are adjacent in the stack and such that no other pancake has size intermediate between the two.If the largest pancake is on the bottom, this also counts as one extra adjacency. Now, for $n \geq 4$ there are stacks of $n$ pancakes that have no adjacencies whatsoever. On the other hand, a sorted stack must have all $n$ adjacencies and each move (flip) can create at most one adjacency. Consequently, for $n \geq 4$, $f(n) \geq n$. [...] For upper bounds —algorithms, that is— it was known that $f(x) \geq 2n$. This can be seen as follows. Given any stack we may start by bringing the largest pancake on top and and then flip the whole stack: the largest pancake is now at the bottom, after two moves. Inductively, bring to the top the largest pancake that has not been sorted yet, and then flip it to the bottom of the unsorted stack. By $2n$ moves we will have thus sorted the whole thing. In fact, $2n$ can be improved to $2n - c$, constant $c$, by sorting the few last pancakes by a more clever method. [...]

### 2. An algorithm

We will represent permutations in $S_n$ as strings in $\Sigma_n^*$, where $\Sigma_n = \{1, 2, \ldots, n\}$. We will define a binary relation "$\rightarrow$" in $S_n$ by writing $\pi \rightarrow \sigma$ whenever $\pi = xy$, $\sigma = x^R y$, where $x, y \in \Sigma_n^*$ and $x^R$ is the string $x$ reversed (read backwards). If $\pi$ is a permutation, $f(\pi)$ is the smallest $k$ such that there exists a sequence of permutations $\pi = \pi_0 \to \pi_1 \to \cdots \to \pi_k \equiv e_n$, where $e_n = 123\ldots n$ is the identity permutation. $f(n)$ is the largest $f(\pi)$ over all $\pi \in S_n$. [...] Our algorithm will sort the permutation $\pi$ so as to create a total of $n-1$ adjacencies, that is, a block $b$ of size $n$ [...] These permutations can be transformed to $e_n$ via at most four flippings. [...]

### 3. A lower bound

Let $\tau = 17536428$. For $k$ a positive integer, $\tau_k$ denotes $\tau$ with each of the integers increased by $8 \cdot (k-1)$. In other words, $\tau_k$ is the sequence $1_k 7_k 5_k 3_k 6_k 4_k 2_k 8_k$, where $m_k = m + (k-1) \cdot 8$. Consider the permutation $\chi = \tau_1 \tau_2^R \tau_3 \tau_4^R \cdots \tau_{m-1} \tau_m^R$, where $m$ is an even integer and $n = |\chi| = 8 \cdot m$.

**Theorem 2.** $19n/16 \leq f(\chi) \leq 17n/16$.

[...]

### 5. Discussion

We presented an algorithm sorting any permutation of length $n$ in about $5n/3$ prefix reversals; improving the multiplicative constant seems to be quite challenging. We also described a technique for deriving lower bounds for $f(n)$, and showed how it can be used to establish that $f(n) \geq 17n/16$. Improving on this particular lower bound does not appear too hard; in fact, we conjecture that for our "hard" permutation $\chi$, $f(\chi) = 19n/16$. Also, slightly better lower bounds may be conceivably proved by using different $\tau$'s —of length $7$, say. However, we do not know how the upper and lower bounds can be narrowed significantly. Naturally, it is not clear at all that $f(n)/n$ converges, and hence it may be that no better bounds are attainable.

[1] Amer. Math. Monthly 81(1) (1975) 1010

2. **(5 points) Translate the following text into Spanish.**

(A. Boole Stott (1900). On certain series of sections of the regular four-dimensional hypersolids. Verhandelingen der Koninklijke Akademie van Wetenschappen te Amsterdam (1e sectie), 7:1-24)

In making series of sections of the regular four-dimensional figures by the method given in this paper it is only necessary to know the number of solids meeting at each vertex. The total number in each figure can be found by counting the number of solids cut in the sections.

Taking the figures bounded by tetrahedra it is evident that a section by a space cutting the edges meeting in a vertex at equal distances from that vertex will give an equilateral triangular section of each tetrahedron. Hence the complete section will be a three-dimensional regular figure bounded by equilateral triangles.

There are only three such figures, the tetrahedron, the octahedron and the icosahedron; so there will be no other four-dimensional figure bounded by tetrahedra except those which have 4, 8 or 20 at each vertex. If groups of tetrahedra arranged so that there are 4, 8 or 20 round a point he cut by parallel spaces close enough together to pass at least once through each edge, then the number of tetra-hedra cut in the three groups respectively will be 5, 16 or 600.

Next taking cubes. The section of a cube by a space cutting the edges meeting at a vertex at equal distances from that vertex is an equilateral triangle. So there will be no other figure bounded by cubes except those having 4, 8 or 20 at each vertex. But 8 cubes exactly fill three-dimensional space and cannot therefore form a four-dimensional angle. Hence there cannot be a figure whose angles are formed by eight cubes. Still less can there be one whose angles are made by 20.

[...] In this manner we meet successively all the regular cells of four-dimensional space.