I. Series más usadas

Sumatorios

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$

Series geométricas

$$\sum_{i=0}^{n} c^i = \frac{c^{n+1}-1}{c-1}, c \neq 1 \qquad \sum_{i=0}^{\infty} c^i = \frac{1}{1-c}, |c| < 1 \qquad \sum_{i=1}^{\infty} c^i = \frac{c}{1-c}, |c| < 1$$

$$\sum_{i=0}^{n} i \cdot c^i = \frac{nc^{n+2}-(n+1)c^{n+1}+c}{(c-1)^2}, c \neq 1 \qquad \sum_{i=0}^{\infty} i c^i = \frac{c}{(1-c)^2}, |c| < 1$$

Números combinatorios

$$\binom{n}{k} = \frac{n!}{(n-k)!k!} \qquad \binom{n}{k} = \binom{n}{n-k} \qquad \binom{n}{m}\binom{m}{k} = \binom{n}{k}\binom{n-k}{m-k}$$

$$\sum_{k=0}^{n} \binom{n}{k}^2 = \binom{2n}{k} \qquad \sum_{k=0}^{n} \binom{n}{k} = 2^n \qquad \sum_{k=0}^{n} \binom{r+k}{k} = \binom{r+n+1}{n}$$

$$\sum_{k=0}^{n} \binom{k}{m} = \binom{n+1}{m+1}$$

$$\sum_{k=0}^{m-1} \binom{m}{k}\binom{n-1}{m-k-1} = \binom{n+m-1}{n} \qquad \sum_{k=0}^{n} \binom{r}{k}\binom{s}{n-k} = \binom{r+s}{n}$$

$$\binom{n}{k} = \sum_{j=1}^{n-k+1} \binom{n-j}{k-1} = \sum_{j=0}^{k} \binom{n-j-1}{k-j}$$

$$\binom{n}{k} = (-1)^k \binom{k-n-1}{k} \qquad \sum_{k=1}^{n} \frac{\binom{n-1}{k-1}}{\binom{n+m}{k}} = \frac{n+m+1}{(m+1)(m+2)}$$

Fórmula Stirling

$$n! \sim \sqrt{2n\pi}n^n e^{-n}$$

en el sentido de que el cociente de ambos tiende hacia 1 cuando n crece hacia infinito. Con más precisión

$$1 < \frac{n!}{\sqrt{2n\pi}n^n e^{-n}} < e^{1/(8n)}$$

Variables Aleatorias

Binomial. Número de aciertos en n pruebas de Bernoulli.

$$X = B(n,p) \; P\{X = x\} = \binom{n}{x} p^x (1-p)^{n-x}, x = 0, 1, ..., n$$

$$E[X] = np \qquad \sigma^2(X) = np(1-p)$$

Geométrica. Número de intentos hasta el primer acierto.

$$X = Geom(p) \; P\{X = x\} = (1-p)^x p, x = 0, 1, 2, ...$$

$$E[X] = \frac{1-p}{p} \qquad \sigma^2(X) = \frac{1-p}{p^2}$$

Binomial Negativa. Número de fallos hasta el n acierto

$$X = BN(n,p) \; P\{X = x\} = \binom{n+x-1}{n-1} (1-p)^x p^n, x = 0, 1, 2, ...$$

$$E[X] = n\frac{1-p}{p} \qquad \sigma^2(X) = n\frac{1-p}{p^2}$$

Poisson

$$X = Poisson(\lambda) \; P_X(x) = \frac{e^{-\lambda}\lambda^x}{x!}, x = 0, 1, 2, ...$$

$$E[X] = \lambda = \sigma^2(X)$$

Normal

$$X = N(\mu, \sigma) \; f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}, -\infty < x < \infty$$

$$E[X] = \mu \qquad \sigma^2(X) = \sigma^2$$

Número e

$$e = 1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{24} + \frac{1}{120} + \cdots$$

$$e^x = \lim_{n\to\infty}\left(1 + \frac{x}{n}\right)^n$$

$$\left(1 + \frac{1}{n}\right)^n < e < \left(1 + \frac{1}{n}\right)^{n+1}$$

Sumatorio de Poisson

$$\sum_{k=0}^{\infty} \frac{\lambda^k}{k!} = e^{\lambda}$$

Función Gamma

$$\Gamma(p) = \int_{0^+}^{+\infty} x^{p-1} e^{-x}\, dx$$

$$\Gamma(p+1) = p \cdot \Gamma(p)$$

$\Gamma(0) = (-1)! = \infty$ $\Gamma(1) = 0!$ $\Gamma(2) = 1!$ $\Gamma(3) = 2!$

$\Gamma(1/2) = \sqrt{\pi}$ $\Gamma(3/2) = 1/2\sqrt{\pi}$ $\Gamma(5/2) = 3/4\sqrt{\pi}$ $\Gamma(7/2) = 15/8\sqrt{\pi}$

Función Beta

$$\beta(p,q) = \int_{0^+}^{1^-} x^{p-1}(1-x)^{q-1}\, dx$$

$$\beta(p,q) = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)},\ (p > 0, q > 0)$$

Teorema de Taylor

$$f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \ldots \frac{f^{(n)}(a)}{n!}(x-a)^n + E_n(x)$$

$$e^x = 1 + x + \frac{1}{2}x^2 + \frac{1}{6} + \cdots = \sum_{i=0}^{\infty} \frac{x^i}{i!}$$

$$ln(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \cdots = \sum_{i=1}^{\infty} (-1)^{i+1}\frac{x^i}{i}$$

$$ln\frac{1}{1-x} = x + \frac{1}{2}x^2 + \frac{1}{3}x^3 + \frac{1}{4}x^4 + \cdots = \sum_{i=1}^{\infty} \frac{x^i}{i}$$

$$(1+x)^n = 1 + nx + \frac{n(n+1)}{2}x^2 + \cdots = \sum_{i=0}^{\infty} \binom{n}{i} x^i$$