INSTRUCTIONS AND HINTS * Read the text carefully before proceeding with your summary * Dictionaries are allowed. * Deviating from the answer length limits will be penalized. * Avoid literal translations (word for word). Try to understand the text and explain it in your own words. The use of literal sentences from the original text will be penalized. * To prevent grammatical errors use simple and short sentences. If you end up with a very long sentence try to split it up. * Notice that the mark obtained here represents 60% of your final mark.

A continuación se muestra el examen

Pregunta 1

(J.M Heffernan, R.J Smith, and L.M Wahl (2005). Perspectives on the basic reproductive ratio. J.R.Soc. Interface, 2(4):281-293)

The basic reproductive ratio, R_0, is a key concept in epidemiology, and is inarguably `one of the foremost and most valuable ideas that mathematical thinking has brought to epidemic theory' (Heesterbeek & Dietz 1996).

[...]

As a general definition, R_0 is the expected number of secondary individuals produced by an individual in its lifetime. The interpretation of `secondary', however, depends on context. In demographics and ecology, R_0 is taken to mean the lifetime reproductive success of a typical member of the species. In epidemiology, we take R_0 to mean the number of individuals infected by a single infected individual during his or her entire infectious period, in a population which is entirely susceptible. For in-host dynamics, R_0 gives the number of newly infected cells produced by one infected cell during its lifetime, assuming all other cells are susceptible.

[...]

2. Derivations of R_0 from a deterministic model

The derivation of R_0 from a non-spatial, deterministic model is fairly straightforward from first principles. The survival function method (§2.1) gives the `gold standard' determination of R_0, and is applicable even when non-constant transmission probabilities between classes (i.e. non-exponential lifetime distributions) are assumed.

[...]

2.1 Survival function

The method we describe as the `survival function' approach is, in essence, a first-principles definition of R_0, and thus has a rich history of use. The approach is described in detail in Heesterbeek & Dietz (1996), who also give an interesting historical overview.

Consider a large population and let $F(a)$ be the probability that a newly infected individual remains infectious for at least time a. This is called the survival probability. Also, let $b(a)$ denote the average number of newly infected individuals that an infectious individual will produce per unit time when infected for total time a. Then, R_0 is given by:

$$R_0 = \int_0^\infty b(a)F(a)\ da$$

As this expression yields R_0 by definition, this approach will be appropriate for any model in which closed-form expressions can be given for the underlying survival probability, $F(a)$, and the infectivity as a function of time, $b(a)$. In particular, it is straightforward to handle situations in which infectivity depends on time, since infection, or other transmission probabilities between states, vary with time. Thus, this derivation of R_0 is not restricted to systems described by ordinary differential equations (ODEs).

This method can also be naturally extended to describe models in which a series of states are involved in the `reproduction' of an infected individual. As an example of the latter technique, consider epidemic modelling of malaria. An infected human may pass the infection to a mosquito, which may in turn infect more humans. This complete cycle must be taken into account in our derivation of R_0, which we might expect to yield the total number of infected humans produced by one infected human. In general, if only two distinct infectious states are involved in such an infection cycle, $F(a)$ can be defined as the probability that an individual in state 1 at time zero produces an individual who is in state 2 until at least time a. Similarly, $b(a)$ is the average number of new individuals in state 1 produced by an individual who has been in state 2 for time a. In modelling malaria, $F(a)$ could be the probability that a human infected at time zero produces an infected mosquito which remains alive until at least time a. In more concrete terms, $F(a)$ would be the integral of the following product:

$$\begin{aligned} F(a) = \int_0^a \quad & \text{prob[human infected at time 0 exists at time t]} \\ & \times \text{prob[human infected for tot. time t infects mosquito]} \\ & \times \text{prob[infected mosquito lives to be age } (a-t)]\ dt \end{aligned}$$

while $b(a)$ would simply be the average number of humans newly infected by a mosquito which has been infected for time a. (Note that we could also take the infected mosquito as state 1, deriving an analogous expression which would yield the same value of R_0)

Unfortunately, derivations such as the equation above become increasingly cumbersome as this method is extended to infection cycles involving three or more states (Hethcote & Tudor 1980; Lloyd 2001b; Huang et al. 2003).

(Este contenido está relacionado con las preguntas enunciadas a continuación y no requiere respuesta por parte del estudiante)

Pregunta 2

Q1 (5 points). Write a summary in English of the text above in scientific style. Your essay should be at least 200 ± 50 words (i.e. 1200 ± 300 characters).

In epidemiology, the basic reproductive ratio, R_0, is the expected number of newly infected individuals for each individual during its infectious period. Deriving R_0 from a model has been an important mathematical problem, and the 'survival function method' is one of its grandest answers.

Time affects infectivity, and some other techniques are troubled by this. However, the survival function approach solves this in the following manner:
Let us consider b(a) to be the average number of infected individuals by a single infected individual per unit time, and F(a) the probability that an infected individual remains infected for at least a time a. Then the integral between 0 and infinite of b(a)*F(a) is the number of individuals that will get infected by an individual in all of its infectious life, R_0.

Different methods are limited when considering different states of infection. Even though the problem grows more complicated when 3+ states are considered, it is still pretty straightforward for two states. For example, when modeling the epidemic evolution of malaria, humans do not infect each other directly. Instead, there are vectors of disease, a human infects a mosquito, which may in turn infect other humans.

We will consider state 1 to be an infected human and state 2 to be an infected mosquito. Then, we can define F(a) as the probability that a state 1 individual (human) produces a state 2 infected individual (mosquito) that will remain infected at least until time a. On the other hand, b(a) is the average number of infected humans produced by a mosquito.

Here, F(a) would be calculated as the area between 0 and a of the product of three probabilities: The probability that an infected human lasts until some determined time, the probability that that human infects a mosquito, and the probability that that mosquito lives until time a.

Pregunta 3

Q2 (5 points). Elaborate on the following controversial statement by the russian mathematician Vladimir Igorevich Arnold in his address at the discussion on teaching of mathematics in Palais de Découverte in Paris on 7 March 1997. Your discussion should be in English and at least 150 words long.

"Mathematics is a part of physics. Physics is an experimental science, a part of natural science. Mathematics is the part of physics where experiments are cheap.

The Jacobi identity (which forces the heights of a triangle to cross at one point) is an experimental fact in the same way as that the Earth is round (that is, homeomorphic to a ball). But it can be discovered with less expense.

In the middle of the twentieth century it was attempted to divide physics and mathematics. The consequences turned out to be catastrophic. Whole generations of mathematicians grew up without knowing half of their science and, of course, in total ignorance of any other sciences. They first began teaching their ugly scholastic pseudo-mathematics to their students, then to schoolchildren (forgetting Hardy's warning that ugly mathematics has no permanent place under the Sun).

Since scholastic mathematics that is cut off from physics is fit neither for teaching nor for application in any other science, the result was the universal hate towards mathematicians - both on the part of the poor schoolchildren (some of whom in the meantime became ministers) and of the users."

As the content at the core of the statement, this is a diving topic. Is mathematics a science per se, or is it just the pillars other sciences need to support themselves? Probably, a case could be made for each opinion.

In one's point of view, mathematics need no objective relation to the real world to be an object of beauty and study. In the same way the arts serve no further purpose than the enlightenment of the soul, mathematics can and should be studied even if no applications can be derived from it. An object should not need a practical utility to be admired. Human individual lives have proven, time and time again, to be a small cog in great machinery, with the only purpose of advancing. Let's produce more to consume more. Seldom do we reflect upon ourselves, but pure mathematics are an exercise of precisely that.

On the other hand, biological success, and more recently, financial stability is what enables the possibility of living entirely on theoretics. For example, most laboratories would reject a grant proposal with the objective of meditation. And thus, the application of mathematics to other fields allows them to grow in all dimensions. But as the Russian mathematician probably thought, this is not a one-way street. Mathematics have succeeded largely because of physics. If we allow ourselves to forget Leibniz just for a second, Newton created calculus to develop his theories in physics further. That is not the sole occurrence. All along the development of sciences, problems demand new solutions, and only time can determine their future applications.

Physics students should not be afraid of the conceptual, for it is the abstract world that lets them become what they are. And mathematicians should not dismiss the applications of their science because experimental advances physically let them study the abstract. One could even argue that all sciences are just branches of the same tree, growing in different directions, effecting a new whole science when a question asks loudly enough (biostatistics, for example).

Furthermore, when possible, people should be encouraged to explore their passions and interests. Hopefully, they are most likely to excel at them. Whereas some person could feel entranced discovering why that pendulum did eventually stop, some other individual could love discovering the different geometries that arise when one simple postulate is removed. A third one may love both topics and argue that they could be related in some situations. But none of them should be discouraged because probably all three will eventually benefit from the work of the others.

To end the discussion, Plato argued almost 25 centuries ago that our human world was but a shadow of the world of ideals, and that's how mathematics can and should be contemplated. They belong to the world of ideals, unhindered by our local prose and mundane lives. And, even though it may be an even more aggressive opinion the one of Vladimir, I think that the relation between physics and mathematics is defined by what is said above.
Physics is the shadow that mathematics project when shined upon the light of our universe.

Observaciones del docente:

Question 2: 4/5
- Good start with a definition and a need to be covered by the survival function method.
- Very good explanation of the one and two state infection models
- But there is no conclusion (or closing sentence)!

Question 3: 5/5
- Very good start, stating the question to be discussed.
- Please, when refering to authors, use their names (Arnold) and perhaps preceded by their title (Prof. Arnold)
- Good conclusion, maybe too idealistic (too Platonic), but summarizing well the previous discussion.