National University of Singapore
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Exercise 1. – *to be handed in by 17 February 2023*

Give a detailed proof of the strictness theorem (Theorem 3.3 in the lecture notes).

In more detail, prove that any monoidal category $(\mathcal{C}, \mathbf{1}, \otimes, \alpha, \lambda, \rho)$ is monoidally equivalent to the strict monoidal category $\mathbf{End}_{\mathrm{mod}-\mathcal{C}}(\mathcal{C})$ of right $\mathcal{C}$-module endofuctors of $\mathcal{C}$ (where we consider $\mathcal{C}$ as a right module category over $\mathcal{C}$, as in Example 2.3(1) in the lecture notes).

Remark: In view of Remark 1.15(3), it suffices to construct a monoidal functor

$$(F, \varphi, \varepsilon) \colon \mathcal{C} \longrightarrow \mathbf{End}_{\mathrm{mod}-\mathcal{C}}(\mathcal{C})$$

and a functor $G \colon \mathbf{End}_{\mathrm{mod}-\mathcal{C}}(\mathcal{C}) \to \mathcal{C}$ such that $G \circ F$ is naturally isomorpic to the identity functor on $\mathcal{C}$ and $F \circ G$ is naturally isomorphic to the identity functor on $\mathbf{End}_{\mathrm{mod}-\mathcal{C}}(\mathcal{C})$. You do not need to endow G with the structure of a monoidal functor or check that the natural isomorphisms are monoidal. (But you are still encouraged to do this for yourself.)