

Week 13, April 20th, 2015

9:30 – 10:20

Review of Homework 7 and Homework 8

- Solutions posted on course website
- We will revisit the DailyTemp (HW 7) and address the uncertainty of temperature on Wednesday and next week

10:20 – 10:45

Lecture

Expected value does not always lead to conclusions.

- How can we trust it in more complicated problems?
- What should actually guide our decisions?

Example 1:

- Square A \rightarrow take \$20 mill or \$10 million
- Obvious choice is to take \$20 mill

Example 1:

- Square A \rightarrow 0.5 for 40M and 0.5 of \$0
- Or \rightarrow choice to take a SURE 10 mill
- $= E(B) = \$20$

Example 3: Marker flipping game vs. \$10,000,000 guaranteed

- Calculate the expected value of flipping game
 - $E(X) = \sum \omega x P(X=x)$
 - Decision tree

St. Petersburg paradox

- Consideration of a case where theoretical value was infinite
- Conclusion: expected value is not a reasonable guide for decisions
- Wednesday's class: expected utility
 - Satisfaction that life brings you – hard to quantify/monetize value

HW 8 Decision Tree Write-up

1.

A)

T: positive test

S: has disease

$$P(S | T) = [P(S) * P(T | S)] / P(T)$$

$$P(S) = 1/1000$$

$$P(T | S) = 0.95 \text{ //sensitive of test}$$

$$P(T | \sim S) = 1 - P(\sim T | S)$$

$$P(T) = P(T, S) + P(T, \sim S) \leftarrow \text{Kolmogorov's first axiom}$$

$$P(T) = P(S) * P(T | S) + P(\sim S) * P(T | \sim S)$$

$$P(T) = 0.094$$

$$P(S | T) = P(S)P(T | S) / P(T) = 0.001 * 0.95 / 0.01094 = 0.086 \text{ or } 8.7\%$$

B)

If you test everyone,

Costs

$$\$10 + 10 \text{ m people} = \$100 \text{ m}$$

$$\$50 \times 10 \text{ m} \times P(T, \sim S) = \$50 \text{ m}$$

Benefits

$$\$10,000 * 10 \text{ m} \times P(T, S) = \$95 \text{ m}$$

Therefore, testing everyone was not a good option after doing cost-benefit analysis.

2.

A)

$$E(0.5X + 0.5Y) = 0.5E(X) + 0.5E(Y) = 15$$

B)

$$\text{var}(0.5X + 0.5Y) = (0.5^2) * \text{var}(X) + (0.5^2) * \text{var}(Y) + 2 * 0.5 * 0.5 * \text{cov}(X, Y)$$

$$\text{var}(0.5X + 0.5Y) = 41.75$$