STA 371H:
Scribing for Wednesday, April 29th

Three Applications of Monte Carlo Simulation:

1. Using Monte Carlo simulation and bootstrapping to compare risk return properties of various portfolios
2. Compute VaR (Value at risk)-standard metric for risk management of portfolio
    a. All major banks are required to compute this value
    b. With some probability (a percentage that you specify: 5%), how big of a loss am I likely to see?
3. Incorporating future sources of uncertainty into regression-based forecasting models
    a. Example: What's the weather going to be like two years into the future?

**Applications will be used in Exercises 10 due Wednesday, May 6th **

Key Idea in Monte Carlo Simulation (or Stochastic Simulation):

- Suppose we have multiple random variables $X_1, X_2,\ldots,X_D$ with joint distribution $P(X_1, X_2,\ldots,X_D)$:
    - Concrete example: think of these as the future prices or equivalently, implied interest rates or returns of "D" assets
    - Asset classes are almost always correlated and are very rarely completely independent
        - Sometimes positively correlated (commodities tend to go up and down depending on macroeconomic trends)
    - Stocks and bonds tend to have negative correlations or inverse relationships
        - Negative correlations between risky assets like stocks or less risky or safer assets like short term government bonds or corporate bonds
    - Joint distribution-these embed all of the correlation structure about the group of random variables
    - *Need to find a clever way to find joint distribution without using a big, ugly spreadsheet*

- Of interest:
    - Function of linear variables or linear combinations of random variables
    - Let $f(X_1, X_2,\ldots,X_D)$

        - Function could be any function of interest about the future such as utility, mean (expected value), etc.
    - What's the expected value of this function? (useful property that could be useful to describe distribution)
        - Has to do with joint distribution of random variables and will change if joint distribution changes

- We want:

    - E{f(X_1, X_2,…,X_D) under the joint distribution P(X_1, X_2,…,X_D)

- Strategy:

    1. Repeatedly sample $X^{(i)}$ =($X_1^{(i)}$, $X_2^{(i)}$,…, $X_D^{(i)}$), N times where N is large **(hard to do)**
        a. There is an arrow above the $X^{(i)}$ to connote that it is a vector
        b. $X1^{(i)}$ - first sample from joint distribution
        c. *Extremely hard part of the strategy*
        d. This is where bootstrapping sampling comes into play
            i. Resampling from our sample to calculate sample distribution that is close to the population distribution
            ii. We do an easy thing and treat the sample as the population
        e. Analogous to the need to draw from a sampling distribution of a statistical estimate

    2. For every sample, evaluate the function:
        a. f($X^{(i)}$ = f($X_1^{(i)}$ + $X_2^{(i)}$ … $X_D^{(i)}$), N times where N is large
            i. The larger N is, the better because the more accurate
            ii. 1,2, D - indicates which asset
            iii. i - indicates sample or draw from the Monte Carlo distribution

    3. Average the function evaluations
        a. E{f(X_1, X_2,…,X_D)} approx.. = (1/N) $\Sigma^N_{(i=0)}$ f($X_1^{(I)}$ + $X_2^{(i)}$ … $X_D^{(i)}$)
            i. Expected value is approximately equal to the sample mean
            ii. Sample mean will be approximately equal to population mean if we take enough samples

    4. We've considered simple univariate distributions

a. We can think of creating a joint distribution using either a table (similar to stock return question on Exercises 8) or using probabilistic theory
b. Sampling distribution of a statistical model-we imagine if we continuously took samples from the population and ran the least squares and ask how does my sample give rise to a different estimator everyday?
    i. This creates a sampling distribution, but no one would do this
c. Bootstrapping-resampling from sample in order to approximate day to day sampling distribution and uncertainty
    i. For joint distributions, we settle for approximation and draw from past observations
    ii. Pattern of returns we saw *x* number of days ago was a draw from the joint distribution (what you see when you take a sample)
    iii. Past data can be considered a draw from joint distribution and sample from past data is sampling from a sample (treat sample as the population)
    iv. Unable to sample from real distribution
    v. Recognize that past data does represent draws from joint distribution so treat sample as the population

R Script:

(Look at diversification and portfolio R scripts and Exercises 10 Problem 2B)

- First Application: Exercises 10 Problem B
    - Use script and function from Portfolio
    - Use Yahoo! Series Function to retrieve data
    - Gives you 5 tickers:
        - SPY-S&P 500-underlying price tracks value of S&P 500,
        - TLT-price tracks the value of government bonds
        - LQD-long-term corporate bonds
        - DBC-ETF for commodities (i.e. metal, oil, timber, water)
        - VNQ-ETF that tracks portfolio of real-estate investment trusts
            - Exchanged traded funds-like a mutual funds (managed by a professional investment manager and they decide where to invest money); a mutual fund where you can buy shares of it on the open market
            - Funds that give you exposure to five very broad and distinct asset classes

        - Designed to track different indexes of assets
  - Turning time series of prices and converting them into implied interest rates
  - **Goal:**
    - Characterize the risk return properties of the three portfolios?
    - Which of these portfolios maximizes expected utility after a year?
  - After everyday there is going to be a daily return and after that day you rebalance your portfolio (at the end of everyday you sell everything and then you rebuy at 50/50 or whatever desired investment proportion you decided on)

- Second Application:

  - Value at Risk (VaR)-the VaR of some portfolio at level alpha (between 0 & 1) for horizon T is defined as follows:
    - Let X_o is the known value of portfolio at the beginning of the horizon (not a random variable)
    - Let X is the (unknown/random) value of the portfolio at end of horizon
    - Let $Y=X-X_o$
      - Negative-loss
      - Positive-profit
    - The VaR of the portfolio is the alpha quantile of Y
      - Quantile-percentile of distribution
      - Have to specify: What's the portfolio, what's the level, and what's the horizon?
      - Example: the VaR is 5% of losing $5,000 or more money
        - Typically denoted as a positive value because it represents a loss