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Statistics 371H:
Scribing for Wednesday, April 8th

How do we Model Decisions?

- Begin with "A": an event (e.g., "Greece leaves the Eurozone")
    - P(A) = Probability that "A" happens
- How do we visualize the set of all future possible events?

Where "A" is some event happening within the set of all possible events.

What is Probability? (3 Interpretations)

1) Mathematician's Interpretation (Axiomatic):

(Kolmogorov's Axioms)

I. $0 \leq P(A) \leq 1$ for any event
II. If Ω is certain than $P(\Omega) = 1$
III. If A and B cannot both happen then: $P(A \cup B) = P(A) + P(B)$

These are unsatisfying because it's very helpful with real-world events.

2) Frequentist Interpretation (i.e., "The Casino Owner's Interpretation")
    - P(A) is the long-run frequency with which event A happens under repeated sampling.
        - The odds are that it goes towards this. This is the limiting frequency.
    - This goes hand and hand with the Law of Large Numbers (if you perform an experiment a very large number of times, the average frequency of the result will be very close to the expected frequency as calculated by probability theory.)
    - This is useful for events that could actually be repeated under the same circumstances/ replicated tests.
    - Still somewhat subjective as a practical matter because so many potential confounders are likely involved
        - Example: 63% chance of living 5 years with colon cancer. Ideally means that with 100 patients diagnosed at same time, 63 will live 5 years but think of all the potential confounders based on individual plus this is too hard to replicate to be of any practical use.
3) Subjectivist/ Bayesian Interpretation (i.e., "Wall Street Interpretation")
    - NOTE: These three mentioned positions aren't mutually exclusive of one-another. They all have the same underlying principles and underlying math, just vary in terms of applicability and perspective.
    - P(A) is you degree of personal belief, on a 0 to 1 scale, that A will happen.

- Other two probabilities aren't very helpful because stocks aren't repetitive; they don't yield the same continual yield with enough consistency for a strictly frequentist interpretation unlike something more predictable (such as coin-flips).
- P(A) = "market value of a $100 option on A
  - Say A is "Duke wins in 2016"
  - Chisam's P(A) = .1 means he'd be willing to bet $100(.1) = $10 that Duke wins
- Like bookies, but there must be a fair contact where bookies are willing to trade either side of the contract (idealized); must be indifferent between the two different sides of a bet. It's like putting subjective market forces at market equilibrium.

Odds and Probabilities

- P(A) can be written in the form of odds
  - Odds against A = (1-P(A))/(P(A))
  - P(A) = 1/(odds against A) +1
- Even though probabilities are subjective, you can't pick them in any way you'd like. Still must obey some rules identical to Kolmogorov's axioms
  - Obey the property of coherence: can't choose probabilities guaranteed to lose you money.

A: Sure Money	or	B: Event (pays of $100 if happens)
$35	or	#1 wins
$55	or	#2 wins

Example of arbitrage opportunity because could get both #1 and #2 and still be better off than the sure money bet. Arbitrage is an error that allows for the possibility of risk free profit. To avoid it both side of the equation ought to be equal. Instead, one side is worth $90 collectively, one worth $100 collectively. Isn't a fair bet. This is validated by "Dutch Book" Theorem

Some Probability Rules:

- "Probabilities for disjoint events add together"
  - $P(A \cup B) = P(A) + P(B)$ ← Kolmogorov's 3rd axiom
- Rules of Addition in Multiplication:
  - Addition Rule: $P(A \cup B) = P(A) + P(B)$
  - Multiplication Rule: $P(A, B) = P(A) * P(B \mid A)$

Bayes' Rule and Conditional Probability

Bayes' Rule: Describes conditional probability (conditional probability is assessment attempted from assimilating info to reach an informed decision). Is this case because realistically probability can't artificially be isolated from the world; it's contingent on what we know

Bayes' Rule is derived from the multiplication rule:

$$P(A,B) = P(A) * P(B \mid A) \rightarrow P(A \mid B) = (P(A) * P(B \mid A))/ P(B)$$

Also helpful transformation to know for calculations of Bayes' Theorem: $P(B) = P(B \mid A) * P(A) + P(B \mid {\sim}A) * P({\sim}A)$

Example 1: Longhorns and Aggies in East Texas

Say you're driving through East Texas and you don't know which way Austin is. In order to try to find out you ask someone around. Where you're at in East Texas is 60% Aggies and 40% Longhorns and whoever you ask is totally random to the point where you don't know which one they are. What if you've made it back home to Austin and now want to know the probability of knowing if the local who directed you was a Longhorn.

- The Longhorn will point you in the right direction 95% of the time, and only accidentally point you to college station 5% of the time.
- The Longhorn will point you in the right direction 70% of the time, and only maliciously point you to college station 30% of the time.

$$P(\text{Longhorn} \mid \text{Pointed to Austin}) = \\ (P(\text{longhorn}) * P(\text{Pointed to Austin} \mid \text{Longhorn})) / P(\text{Pointed to Austin})$$

$$.475 = ((.4)(.95)) / ((.6)(.7)) + ((.4)(.95))$$

Example 2: New York City Crime Example (from Mon. 4/13)

A: Some statement or hypothesis that the accused is guilty

B: data; new evidence at the trial (a DNA match for example)

$P(A) = 1/10{,}000{,}000 \leftarrow$ (assuming NYC has 10 million people exactly, what is the chance that this person is guilty. Chances of this person being more guilty than any random person they drug in off the street

$P(B \mid \sim A) \leftarrow$ (probability of B, given not A)

$P(B \mid A) = 1$

$P(B) = P(B \text{ and } A) + P(B \text{ and } \sim A)$

$= P(A) * P(B \mid A) + P(\sim A) * P(B \mid \sim A)$

$= (1/10{,}000{,}000) * 1 + (9{,}999{,}999/10{,}000{,}000) * (1/1{,}000{,}000)$

$= (1/10{,}000{,}000) + 1 * (10/10{,}000{,}000) \rightarrow (11/10{,}000{,}000)$

$P(A \mid B) = ((1/10{,}000{,}000) * 1) / 11/10{,}000{,}000$

$= 1/11$

Prosecutor's Fallacy: $P(B \mid \sim A)$ DOES NOT EQUAL $P(\sim A \mid B)$