Monday April 13^{th}, 2015

Baye's Rule

P(A|B) = [P(A) P(B|A)] / P(B)
A: some statement/hypothesis
B: data/ "new" information

Example –
A: "accused is guilty"
B: evidence in trial - DNA is a match
P(A) = 1/10,000,000
P(B|~A) = 1/1,000,000 (chance of a false positive - the probability of B given *not* A is one in a million)
P(B|A) = 1 (assuming there is never a chance of a false negative)
P(B) = P(B and A) + P(B and ~A) ← adding up the two possibilities
  = P(A)P(B|A) + P(~A)P(B|~A)
  = (1/10,000,000)(1) + (9,999,999/10,000,000)(1/1,000,000)
  ~= (1/10,000,000) + (1)(10/10,000,000)
  = 11/10,000,000
Baye's Rule: P(A|B) = [P(A) P(B|A)] / P(B)
  P(A|B) = [(1/10,000)(1)]/(11/10,000,000) = 1/11
Prosecutor's fallacy: P(B|~A) = P(~A|B) - prosecutor wants to deny Baye's Rule

Think about Baye's Rule in terms of cohorts of people
  10,000 people in New York:
  1 guilty person → 1 positive test
  10,000,000 not guilty → 10 positive tests (one in a million times ten million)
  1 guilty / 11 total

Probability

- X: random variable – anything we are unsure about
- Ω (omega): space/probability space/support = set of possible outcomes
- x e Ω : possible outcomes
- Probability distribution: P(X)
  - Complete description of uncertainty
    - Brute-force list

x	P(X=x)
x1 $.99	.3
x2 $1.00	.5
x3 $1.01	.2

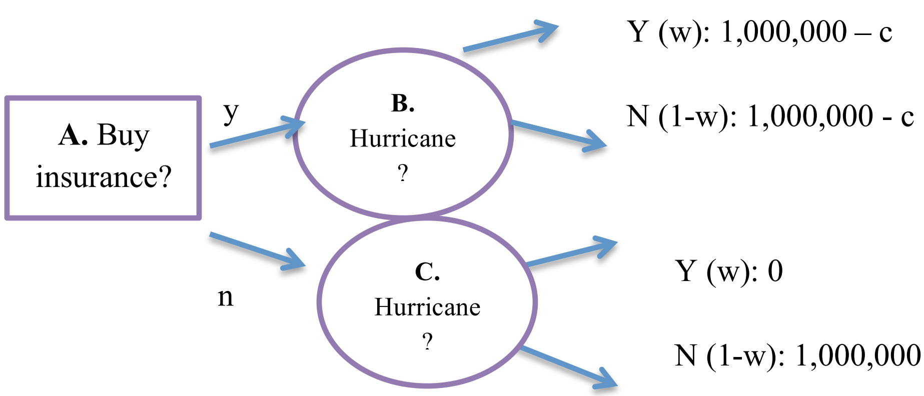
- Probability mass function
  - Large number of possibilities and probabilities
  - PMF: P(X=x) = f(x)
  - Examples: parametric probability models
    - Binomial distribution: X~Binomial(N,w)
    - Poisson: X~Poisson(ℓ)
    -
- Probability density function

- Expected value = "center" of your distribution
  - E(X) = summation$_{\text{x e omega}}$ P(X=x) * x
    - Summing all possible events (assuming omega is discrete)
  - X = stock price of AAPL (12 months from today)

x	P(X=x)
$100	.2
$120	.4
$140	.4

→ E(X) = .2*100+.14*120+.4*140 =
= 20+48+64 = 68+64 = 132

- Decision Trees
  - Ordered temporally left to right
  - How a sequence of events may unfold in time
  - Squares and circles are nodes
    - Square – decision node
    - Circle – stochastic node
  -



  - Solving a Decision Tree:
    - Work backward in time
    - Reduce every stochastic node to get expected value
    - At every decision node, take the option with the highest EV

Insurance Decision Tree example:

$E(B) = w(1M-c) + (1-w)(1M-c) = 1M - c$

E© $= w*0 + (1-w)*1M = 1M - w*1M$

Buy whenever $E(B) > E(C)$

$1M-c > 1M - w1M$

$-c > -w1M$

$c < w * 1,000,000$