

## Scribed Notes Jaan Bains

### Monte Carlo Risk/Return

- 1) Compare risk/return properties
- 2) Compute VaR (Value @Risk) of portfolio
- 3) Incorporating future sources of uncertainty into regression-based models

### Key Idea in Monte Carlo Simulation

- Suppose you have random variables;  $x_1, x_2, x_3, \dots, x_d$  representing returns on various assets
- You rarely have classes of assets that are totally uncorrelated
- Stocks and Bonds have negative correlation with risky assets
- With joint dist:  $p(x_1, x_2, \dots, x_d)$
- Of interest let  $f(x_1, \dots, x_d)$  example utility
- We want  $E(f(x_1, \dots, x_d))$  under the joint dist  $p(x_1, \dots, x_d)$

### Strategy

- 1) Repeatedly sample vector  $x_i = (x_1, x_2, \dots, x_d)$   $n$  times when  $n$  is large
- 2) For each sample, evaluate the function
  - a.  $F(x(i)) = f(x_1(i), x_2(i), \dots, x_d(i))$
- 3) Average the function evaluations
  - a.  $E(f(x_1, \dots, x_d)) = 1/n \text{ Summation } f(x_1(i), x_2(i), \dots, x_d(i))$
  - b. Says that pop. Mean is pretty close to the sample mean
  - c. So what's the hard part?
    - i. Taking repeated samples
    - ii. We can't take 10,000 real days, so we settle for an approximation by using the past and bootstrapping it
    - iii. All of the past data is a draw from the joint dist., so we say this is a sample of the pop., and then sample from it

### Value at risk (VaR)

The VaR of some portfolio at level  $\alpha$  (b/t 0+1) for time horizon  $T$  is defined as follows:

Let  $X_0$  is the known value of portfolio at beginning of horizon

Let  $X$  is (unknown/random) value of portfolio @ end of horizon

Let  $Y = x - x_0$  (The VaR of part is alpha quantile of Y)

The image shows a computer screen with an R console window and a whiteboard. The console window displays the following code and output:

```

Console -/Projects/
+ mynames = strsplit(colnames(percentreturn), '.', fixed=TRUE)
+ mynames = lapply(mynames, function(x) return(paste0(x[1], ".PctReturn")))
+ colnames(percentreturn) = mynames
+ na.omit(percentreturn)
+ }
> myreturns = computereturns(myprices)
>

```

The Global Environment window shows:

```

Data
+ myreturns 1510 obs. of 5 variables
Values
+ myprices Formal class timeSeries
+ mytickers chr [1:5] "SPY" "TLT" "LQD" "DBC" "..."
Functions

```

The whiteboard contains handwritten notes:

① Compare risk/return properties of portfolios.  
 ② Compute VaR (Value at risk) of portfolio.  
 ③ Incorporating future sources of uncertainty into regression-based forecasting models.

Key idea in MC simulation  
 Suppose  $x_1, x_2, \dots, x_D$   
 with joint distribution  $P(x_1, \dots, x_D)$

Of interest:  
 Let  $f(x_1, \dots, x_D)$   
 (example: utility)

We want  
 $E\{f(x_1, \dots, x_D)\}$   
 under the joint dist'n  
 $P(x_1, \dots, x_D)$

```
returns = computereturns(myprices)
```

#### Values

myprices  
mytickers

Formal class timeSeries

chr [1:5] "SPY" "TLT" "LQD" "DBC" "..."

#### Functions

### Strategy

① Repeatedly sample  $\vec{X}^{(i)} = (X_1^{(i)}, X_2^{(i)}, \dots, X_D^{(i)})$ ,  $N$  times  
where  $N$  is large.

② For each sample, evaluate the function:

$$f(\vec{X}^{(i)}) = f(X_1^{(i)}, X_2^{(i)}, \dots, X_D^{(i)})$$

③ Average the function evaluations

$$E\{f(X_1, \dots, X_D)\} \approx \frac{1}{N} \sum_{i=1}^N f(X_1^{(i)}, X_2^{(i)}, \dots, X_D^{(i)})$$