STA371H April 15th

Topics for Today: Functions of random variables, variance, PDF (Probability Density Function), joint distributions, correlated random variables, co-variance (Co-Variance Hurts), linear combinations of random variables

Moment: some simplified description of uncertainty
X: random variable
x: some possible outcome
Ω: set of all possible outcomes
→ Expected value (a moment)
E(X) = $\sum_{x=\Omega} x * P(X = x)$
Say f(x) is some function, or a decision

E(f(x))= $\sum_{x=\Omega} f(x) * P(X = x)$

Example: x=stock price; f(x)=0.75x

X	P(X=x)
100	.2
150	.6
200	.2

E(f(X))=
f(100)(.2)+f(150)(.6)+f(200)(.2)
= 0.75*100*.2
+ 0.75*150*.6
+0.75*200*,2

Important rules

Y= f(X)= aX+ b
E(Y)= E(f(X))= aE(x)+ b
i.e E[f(x)]=f[E(x)]
** but only for linear functions
For all other functions E[f(x)]≠f[E(x)]
This is Jensen's Inequality

Y=f(X)
Z=(x-E(X))^2
u=E(X)
Z=f(X)=(x-u)^2
E(Z)= var (x)
Therefore variance is
Var(X)= E [X-E(X)]^2
Var(X)= $\sum_{x=\Omega}(x-u)^2 * P(X = x)$ where u=E(X)

Variance Example: In Excel spreadsheet shown below

	A	B	C	D	E	F	G
1	x	P(X=x)	x*P(X=x)	x-E(X)	(x-E(X))^2	[(x-E(X))^2]*P(X=x)	
2	2	0.2	0.4	-1	1	0.2	
3	3	0.6	1.8	0	0	0	
4	4	0.2	0.8	1	1	0.2	
5							
6		E(x)=	3		Variance	0.4	
7					sqrt of var	0.63245553	
8					st dev		

D1= A1-C6
E1=$(D1)^2$
F1= E1*B1

Joint Distribution/Multivariate distributions

Joint Distribution is P(X,Y)
X= % return on AAPL from now to next month
Y= % return on GOOG

X	Y	P(X=x and Y=y)
-1	-1	.3
1	-1	.2
-1	1	.2
1	1	.3

Some function f(X,Y)
^this could be a policy, decision, asset allocation, happiness function

E[f(X,y)] =$\sum_{all\ states\ of\ the\ world} f(x_i, y_i) * P(X = x_i, Y = y_i)$

<u>Example</u>
Z=f(X,Y)= 0.7X+ 0.3Y
What is E(Z)? E(Z) is column E in the excel spreadsheet below

	A	B	C	D	E	F
1	x	y	JointProb	Z=f(X,Y)	f(X,Y)*P(X=x, Y=y)	
2	-2.5	-2.5	0.00158754	-2.5	-0.0039689	
3	-2.5	-1.5	0.00711487	-2.2	-0.0156527	
4	-2.5	-0.5	0.00431539	-1.9	-0.0081992	
5	-2.5	0.5	0.00035423	-1.6	-0.0005668	
6	-2.5	1.5	3.94E-06	-1.3	-5.122E-06	
7	-2.5	2.5	1.00E-08	-1	-1E-08	
8	-2.5	3.5	0	-0.7	0	
9	-1.5	-2.5	0.00158754	-1.8	-0.0028576	
10	-1.5	-1.5	0.01934023	-1.5	-0.0290103	

This is calculated by multiplying Z by the joint probability
So take row 1
D1=.7A2+.3B2
E1= C2*D2