Random variables

A random variable is any uncertain outcome. Examples:

- Whether a coin flip comes up heads or tails.
- The winning team in the NCAA basketball tournament.
- The value of Apple stock next year.
- The name of the next new person you meet.

A probability distribution describes a random variable as a list of two things.

- The sample space: the set of possible outcomes ("events") for the random variable
- The probability for each event in the sample space.

Examples

Soon, we'll study a few examples of common probability distributions.

- Binomial
- Poisson
- Normal

For now, let's focus on a simple example: the outcome of rolling two dice.

Examples

Suppose you roll two dice. Let X be the sum of the dice.

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Examples

Here the the probability distribution for X.

x_k	$P(X = x_k)$
2	$1/36$
3	$2/36$
4	$3/36$
5	$4/36$
6	$5/36$
7	$6/36$
8	$5/36$
9	$4/36$
10	$3/36$
11	$2/36$
12	$1/36$

Expected value

The expected value, or mean, is one measure for the center of a probability distribution. Suppose the random variable X has sample space $x_1, \ldots, x_M$. Then

$$E(X) = \sum_{k=1}^{M} x_k \cdot P(X = x_k)$$

It is the weighted average of the possible outcomes, where the weight on each event is that event's probability.

Dice example:

$$E(X) = 2 \cdot \frac{1}{36} + 3 \cdot \frac{2}{36} + \cdots + 11 \cdot \frac{2}{36} + 12 \cdot \frac{1}{36} = 7\,.$$

Variance

The variance measures the spread, or dispersion, of a probability distribution. Again, suppose the random variable X has sample space $x_1, \ldots, x_M$. Then

$$\mathrm{var}(X) = E([X - E(X)]^2) = \sum_{k=1}^{M} (x_k - E(X))^2 \cdot P(X = x_k)$$

It is the weighted average of the squared deviations from the mean, where the weight on each event is that event's probability.

Dice example:

$$\mathrm{var}(X) = (2-7)^2 \cdot \frac{1}{36} + (3-7)^2 \cdot \frac{2}{36} + \cdots + (12-7)^2 \cdot \frac{1}{36} \approx 5.83\,.$$