Exercises 8 · Making smart decisions

Due Monday, April 18, 2016

Problem 1: Stocks and bonds

Let (X_{t1}, X_{t2}) denote the return in year t on the stock market (X_1) and on government bonds (X_2), respectively. Suppose that (X_1, X_2) are independent and identically distributed (IID) and follow a bivariate normal distribution with parameters:

$$\begin{aligned} \mu_1 &= 0.065\,, \quad \mu_2 = 0.015 \\ \sigma_1 &= 0.20\,, \quad \sigma_2 = 0.10 \\ \rho &= -0.1\,. \end{aligned}$$

Set up a Monte Carlo simulation that assess which of the following portfolio results in the highest long-term growth rate. (1) 50% stocks, 50% bonds; (2) 80% stocks, 20% bonds; or (3) 100% stocks. Assume that each year, you rebalance your portfolio to the target mix of assets (since otherwise it will drift randomly away from the target over time). Remember two points:

- Following our discussion in class, maximizing a portfolio's expected growth rate is equivalent to maximizing $E[\log(1 + Y_t)]$ where Y_t is the per-period return on your investment in year t.
- The return Y_t of a mixed portfolio of stocks and bonds will depend both on the return of stocks X_{t1} and the return of bonds X_{t2}.

Describe your approach and your answer. Turn in your R code as a supplement to (but not a replacement of) this description. Make sure you print your R code in a fixed-width font, liked Courier.

Problem 2: decision trees

You are the owner of a small electronics company. In 6 months, a proposal is due for an electronic timing system for downhill skiing events in the 2018 Winter Olympic Games. For several years, your company has been developing a new form of "finish beam" that records the time at which a ski racer crosses the finish line. A finish beam is a critical component in any ski timing system. They must be accurate to the millisecond and immune to poor weather. They must also be able to distinguish between a skiier and a puff of snow crossing the beam, and they must maintain a wireless data link with the starting gate in conditions which make radio transmission difficult (e.g. high mountains). When completed, yours will be superior to anything currently on the market. But your progress in research and development has been slower than expected, and you are unsure about whether your staff can perfect the technology and produce the components in time to submit a proposal. You must make a decision about whether to press ahead with an accelerated schedule of research and development, taking into account your uncertainty about future events. Consider these facts:

- The Olympic contract pays $1 million to the winner.
- If your R&D effort succeeds in developing the new beam (which will happen with probability p_1), there is a good chance (probability p_2) that your company will win the contract.
- If you decide to accelerate the R&D schedule for the purposes of developing the new beam, you will need to invest $200,000 in extra funds. With probability $1 - p_1$, this effort will fail to develop the new beam in time.
- If you fail to develop the new beam, there is a small chance (probability p_3) that you will still be able to win the same contract with your older model of timing system that has already been developed. This involves no extra cost for research and development.
- Making a proposal requires developing a prototype timing system at an additional production cost, above and beyond R&D costs. This additional cost would be $50,000 for the new model of timing system, and $40,000 for the old model.
- Finally, if you win the contract, the finished product—whether the new or the old model—will cost an additional $250,000 to produce, above and beyond the cost of building a prototype.

The following page shows a decision tree for this problem. Each oval represents the outcome of a random event; this is called a *stochastic node*. Each rectangle represents a decision that you must make under a particular set of circumstances; this is called a *decision node*. These nodes are labeled with the letters A through I. Each terminal arrow of the tree corresponds to a particular set of outcomes and decisions. These are labelled 1 through 11. Remember the four basic rules for solving a decision problem using a tree:

1. Ensure that the tree includes all relevant decision and stochastic nodes, in temporal order from left to right.
2. Work *right to left*, i.e. backwards in time.
3. For every stochastic node, compute the expected value of the random outcome. This quantity becomes the value of that

node.

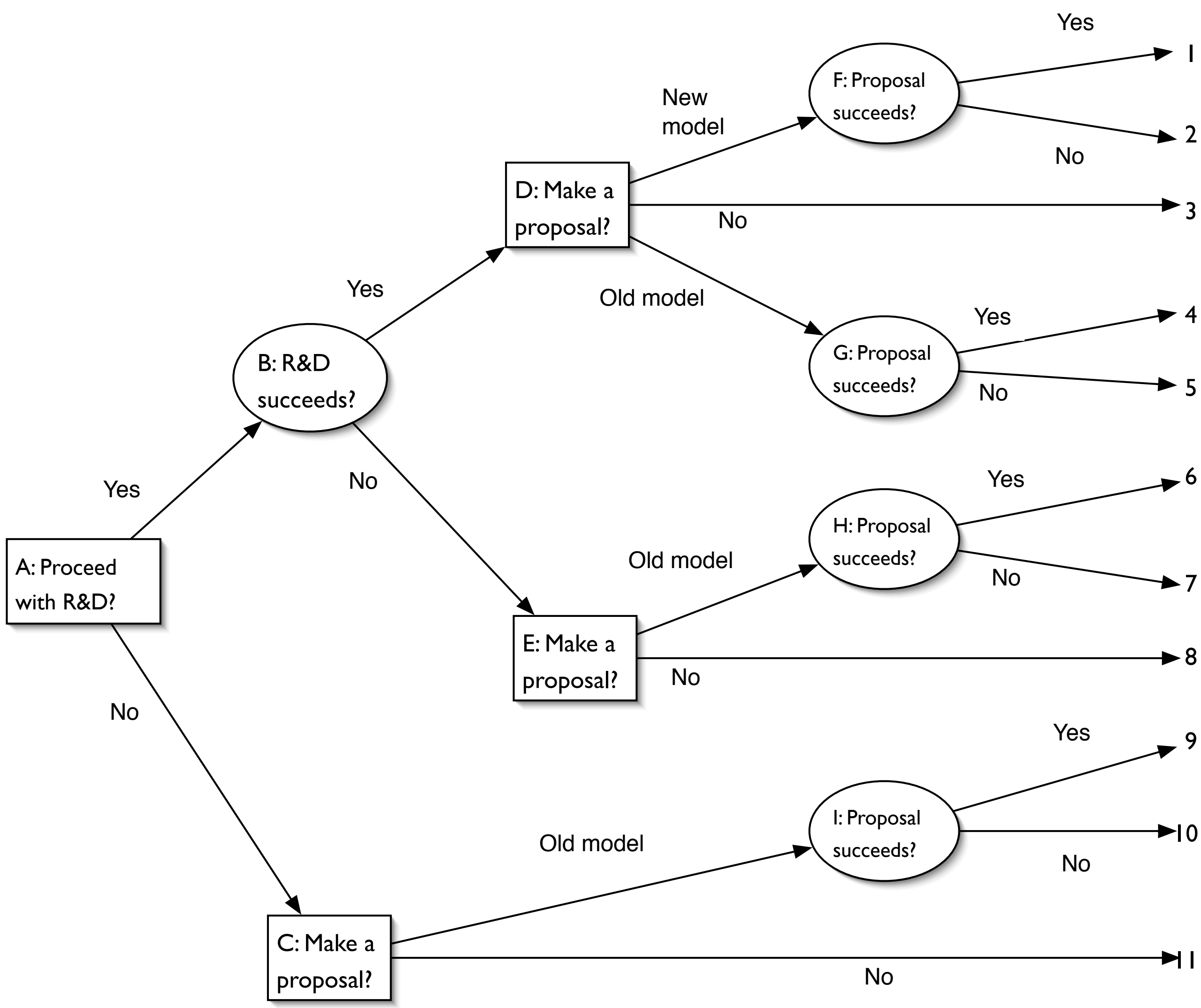
4. For every decision node, choose the course of action with the highest expected value. This quantity becomes the value of that node.

Part A: Each of the 11 terminal arrows of the tree, corresponds to a particular pattern of events that may or may not happen. For each node, fill in the amount of net profit you would make if those events actually happen. Make sure to account for all costs along each path of the tree.

Part B: For the right-most four stochastic nodes of the tree (F, G, H, and I), fill in the probabilities of the possible outcomes (two for each node), and then compute the expected value of the node in terms of p_1, p_2, and p_3. Remember, the outcome at that node is a random variable, and we know how to calculate expected values of random variables.

Part C: For decision nodes D, E, and C, write down a *decision rule*—that is, a criterion expressed in terms of the relevant probabilities (p_1, p_2, and p_3) that tells you which decision you should make to maximize utility in that set of circumstances. Derive these rules by hand, and show your work. You may use software to help with routine calculations, but not to derive the rules.

Part D: Suppose $p_2 = 0.8$ and $p_3 = 0.1$. Write down an expression for the expected value of Node B in terms of p_1. What value of p_1 makes you indifferent between proceeding with the accelerated R&D effort, or not?


A: Proceed with R&D?
Yes
No
B: R&D succeeds?
Yes
No
C: Make a proposal?
D: Make a proposal?
E: Make a proposal?
New model
No
Old model
F: Proposal succeeds?
G: Proposal succeeds?
H: Proposal succeeds?
I: Proposal succeeds?
Yes
No
1
2
3
4
5
6
7
8
9
10
11