Circuit Theory and Electronics Fundamentals

T2 Laboratory Report

Aerospace Engineering, Técnico, University of Lisbon

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Group 19

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Contents

1 Introduction

The objective of this laboratory assignment is to study a circuit containing a capacitor and a sinusoidal voltage source v_s.The value of this sinusoidal voltage source varies in time according to the following equations:

$$v_s(t) = V_s u(-t) + sin(2\pi f t)u(t) \tag{1}$$

with

$$u(t) = \begin{cases} 0 & \text{if } t < 0 \\ 1, & \text{if } t \geq 0 \end{cases} \tag{2}$$

In this circuit there are both a linearly dependent voltage and current source. The circuit also contains 7 resistors.

The nodes of the circuit were numbered arbitrarily (from V_0 to V_7), and it was considered that *node 0* was the ground node. The voltage-controlled current source I_b has a linear dependence on Voltage V_b, of constant K_b. The voltage V_b is the voltage drop at the ends of resistor R_3. The current-controlled voltage source V_d has a linear dependence on current I_d, of constant K_d. The control current I_d is the current that passes through the resistor R_6. The circuit can be seen in Figure 1.

These values for the capacitance, resistors and the constants for the dependent sources were obtained using the Python script provided by the Professor and using the number 95802 as the seed.The seed number can be altered in the top Makefile. By doing so, all figures and tables will be updated acording to the new values.

In Section 2, a theoretical analysis of the circuit is presented. Here the circuit is analysed for $t < 0$ using the nodal analysis and the equivalent resistance R_{eq} as seen from the capacitor terminals is determined. In this section both the natural and forced solutions for V_6 are also determined as well as the frequency responses for V_c, V_s and V_6. In Section 3, the circuit is analysed by simulation using the program Ngspice. An operating point analysis is used to analyse the circuit when $t < 0$ and another one to determine the time constant.A transient analysis is used to determine the natural and forced responses on node 6. A frequency analysis is also performed on node 6. The conclusions of this study are outlined in Section 4, where the theoretical results obtained in Section 2 are compared to the simulation results obtained in Section 3.

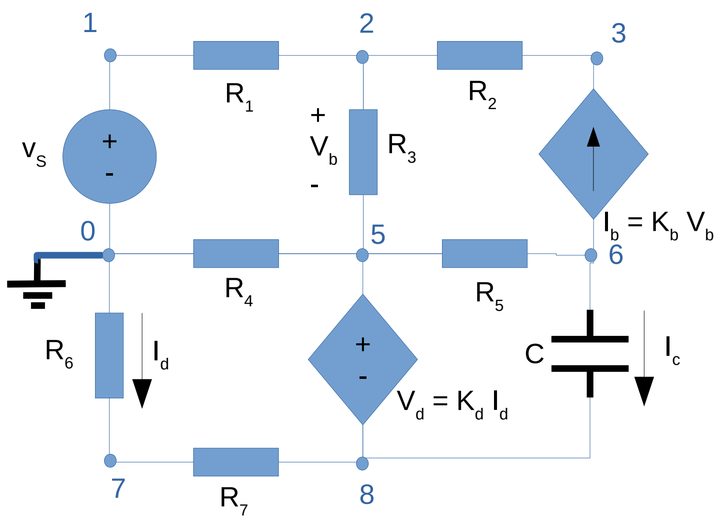

Figure 1: Circuit in study

2 Theoretical Analysis

In this section, although it isn't necessary, an imaginary node 4 was created between resistor 6 and node 7, inserting an independent voltage source, V_x, providing 0V to the circuit. The reason for this is so that comparisons between tables generated from Octave and NGSpice are easier, since the latter actually requires the creation of this virtual node.

2.1 Analysis for $t < 0$

When $t < 0$, the circuit is in an equilibrium state, and because of that, there's no current passing through the capacitor, making this component act like an open circuit. The computations of the values in this section are made using the nodal method and KVL principles.

In Table 1 are presented the theoretical values. All of the values are obtained from the Octave script.

$$V_0 = 0 \tag{3}$$

$$V_4 = V_7 \tag{4}$$

$$V_5 - V_8 = K_d \frac{V_0 - V_4}{R_6} \tag{5}$$

$$V_1 - V_0 = V_s \tag{6}$$

$$\frac{V_2 - V_1}{R_1} + \frac{V_2 - V_5}{R_3} + \frac{V_2 - V_3}{R_2} = 0 \tag{7}$$

$$\frac{V_3 - V_2}{R_2} - K_b(V_2 - V_5) = 0 \tag{8}$$

$$\frac{V_5 - V_2}{R_3} + \frac{V_5 - V_0}{R_4} + \frac{V_5 - V_6}{R_5} + \frac{V_8 - V_7}{R_7} = 0 \tag{9}$$

$$\frac{V_6 - V_5}{R_5} + K_b(V_2 - V_5) = 0 \tag{10}$$

$$\frac{V_4 - V_0}{R_6} + \frac{V_7 - V_8}{R_7} = 0 \tag{11}$$

Name	Nodal method
@c	0
@Gb	-2.241280e-04
@r1	2.143086302573204e-04
@r2	2.241279676947193e-04
@r3	-9.819337437399266e-06
@r4	-1.183453890132603e-03
@r5	-2.241279676947192e-04
@r6	9.691452598752827e-04
@r7	9.691452598752829e-04
v(1)	5.08211987776
v(2)	4.86425757452
v(3)	4.39791092949
v(4)	-2.00234226900
v(5)	4.89494572428
v(6)	5.57809723710
v(7)	-2.00234226900
v(8)	-2.98688786865

Table 1: A variable that starts with "@" is of type *current* and expressed in Ampere (A); all the other variables that start with a "V" are of the type *voltage* and expressed in Volt (V).

2.2 Equivalent resistor as seen from the capacitor terminals

Once found the solution for the node $t < 0$, it is necessary to compute the boundary conditions for the nodes in the circuit, since the voltage in the nodes is not necessarily continuous. To compute the equivalent resistance as seen by C the independent source V_c needs to be switched off. That's possible, creating a short circuit ($V_s = 0$). In addition, due to the presence of dependent sources, it is also needed to replace the capacitor with a voltage source $V_x = V_6 - V_8$. We use the V_6 and V_8 from the previous section because the voltage drop at the terminals of the capacitor needs to be a continuous function (there can not be an energy discontinuity in the capacitor). With this in mind, a nodal analysis is performed in order to determine the current I_x that is supplied by V_x. With these values we can now compute R_{eq} ($R_{eq} = V_x/I_x$). All these procedures were required in order to determine the time constant τ ($\tau = R_{eq} * C$). The time constant is crucial to determine the natural and forced solutions for V_6, which will be done in the next subsections. The equations considered for these calculations were 3, 4, 5, 6, 7, 8 and the following:

$$\frac{V_1 - V_2}{R_1} + \frac{V_0 - V_4}{R_6} + \frac{V_0 - V_5}{R_4} = 0 \tag{12}$$

$$K_b(V_2 - V_5) + \frac{V_6 - V_5}{R_5} + I_x = 0 \tag{13}$$

$$\frac{V_4 - V_0}{R_6} + \frac{V_7 - V_8}{R_7} = 0 \tag{14}$$

$$V_x = V_6 - V_8 \tag{15}$$

Name	Theoretical values
@Gb	0.00000000000
@r1	0
@r2	0
@r3	0
@r4	0
@r5	-2.809995541354671e-03
@r6	0
@r7	0
v(1)	0.00000000000
v(2)	0.00000000000
v(3)	0.00000000000
v(4)	0.00000000000
v(5)	0.00000000000
v(6)	8.56498510575
v(7)	0.00000000000
v(8)	0.00000000000
Ix	-0.00280999554
Vx	8.56498510575
Req	-3.048042e+03
τ	-3.122414e-03

Table 2: A variable that starts with a "V" is of type *voltage* and expressed in Volt (V). The variable R_{eq} is expressed in Ohm and the variable τ is expressed in seconds

2.3 Natural solution for V6

The natural solution depends on the initial charge (voltage). Using R_{eq} and V_6 it is computed the natural solution of $V_6(t)$ by removing all independent sources and applying KVL. To compute the Natural solution, the general formula derived in the theoretical classes was used: $V_{6n}(t) = Ae^{\frac{-t}{\tau}}$. In this formula τ is the time constant determined in the previous section and A is a constant that can be determined through the boundary conditions (when $t = 0$, $A = V_x$).

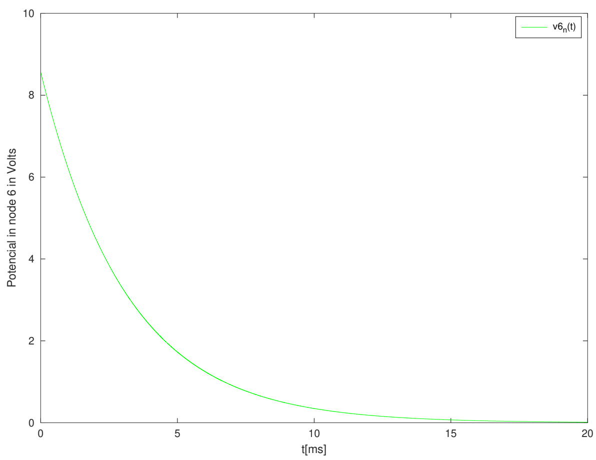

Figure 2: Natural response of V_6 as a function of time in the interval from [0,20] ms

2.4 Forced solution for V6 with f=1000Hz

Considering that V_s is a sinusoidal source in $t < 0$,it is easier to perform a complex analysis, replacing the components in the circuit for their impedance. It was also considered that the magnitude of the phasor representing the voltage source $\tilde{V_s}$ was 1 ($V_s = 1$), a result of expression 2. By taking all these steps the phasor voltages in all nodes were determined in accordance to the following equations:

$$Z = \frac{1}{jwC} \tag{16}$$

$$\tilde{V_s} = 1 \tag{17}$$

$$\tilde{V_0} = 0 \tag{18}$$

$$\tilde{V_4} = \tilde{V_7} \tag{19}$$

$$\tilde{V_5} - \tilde{V_8} = K_d \frac{\tilde{V_0} - \tilde{V_4}}{R_6} \tag{20}$$

$$\tilde{V_1} - \tilde{V_0} = \tilde{V_s} \tag{21}$$

$$\frac{\tilde{V_2} - \tilde{V_1}}{R_1} + \frac{\tilde{V_2} - \tilde{V_5}}{R_3} + \frac{\tilde{V_2} - \tilde{V_3}}{R_2} = 0 \tag{22}$$

$$\frac{\tilde{V_3} - \tilde{V_2}}{R_2} - K_b(\tilde{V_2} - \tilde{V_5}) = 0 \tag{23}$$

$$\frac{\tilde{V_1} - \tilde{V_2}}{R_1} + \frac{\tilde{V_0} - \tilde{V_4}}{R_6} + \frac{\tilde{V_0} - \tilde{V_5}}{R_4} = 0 \tag{24}$$

$$K_b(\tilde{V_2} - \tilde{V_5}) + \frac{\tilde{V_6} - \tilde{V_5}}{R_5} + \frac{\tilde{V_6} - \tilde{V_8}}{Z} = 0 \tag{25}$$

$$\frac{\tilde{V_4} - \tilde{V_0}}{R_6} + \frac{\tilde{V_7} - \tilde{V_8}}{R_7} = 0 \tag{26}$$

$$\tilde{V_x} = \tilde{V_6} - \tilde{V_8} \tag{27}$$

The complex amplitudes of the phasors are presented in **Table 3**

Name	Complex amplitude voltage
V0	0
V1	1
V2	9.571316087616473e-01
V3	8.653693803521179e-01
V4	3.939974493247299e-01
V5	9.631700632846432e-01
V6	5.896161348830701e-01
V7	3.939974493247299e-01
V8	5.877247960493270e-01

Table 3: Complex amplitudes in all nodes in Volts

2.5 Final total solution V6(t)

In this section the final total solution V_6 for a frequency of 1KHz is determined using the natural and forced solutions determined in previous sections (V_6=V_{n6}+V_{6f}). In Figure: 3 the voltage of the independent source V_s and the voltage of V_6 were plotted for the time interval of [-5,20] ms.

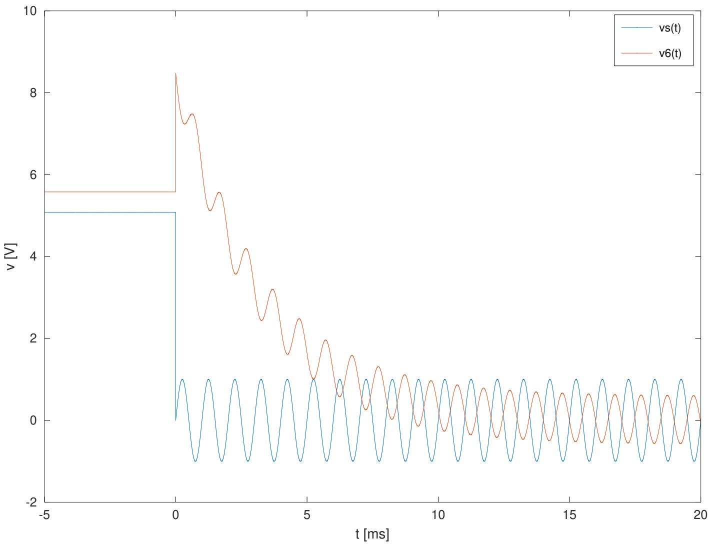

Figure 3: Voltage of $V_6(t)$ and $V_s(t)$ as functions of time from [-5,20] ms

2.6 Frequency responses $v_c(f)$, $v_s(f)$ and $v_6(f)$ for frequency range 0.1 Hz to 1 MHz

For this section, it was considered that $v_s(t) = sin(2\pi ft)$. For low frequencies, the capacitor has enough time to react to changes in the voltage source. Therefore, the phase difference is 0 or close to 0. Moreover, since the voltage source was switched on for a long period of time, the capacitor is fully charged when t=0, and thus it works as an open circuit. However when the frequencies are high, the capacitor only has a small time to charge up before the input changes direction, which means it will act approximately as a short-circuit. Therefore, there will be almost no voltage drop between nodes 6 and 8, causing the capacitor and source to fall out of phase, for frequencies greater that the cutoff frequency (f_c). This frequency can be calculated with the following formula. $f_c = \frac{1}{2.\pi.\tau}$. For the values provided this cutoff frequency is around 50Hz. This explains the steep drop in potential difference that we can see in the graph around the first and second decades. The phase difference between the capacitor voltage and the voltage source also begins to show at around this frequency as can be seen in Figure: 5.

Simplifying this circuit to something close to its Thevenan equivalent, using only V_{eq}, R_{eq} and a Capacitor leads to the following equations, allowing for a better understanding of the phase and magnitude decrease as the frequency increases:

$$V_c = \frac{V_s}{\sqrt{1 + (R_{eq}.C.2\pi.f)^2}} \tag{28}$$

$$\phi_{V_c} = -\frac{\pi}{2} + arctan(R_{eq}.C.2\pi.f) \tag{29}$$

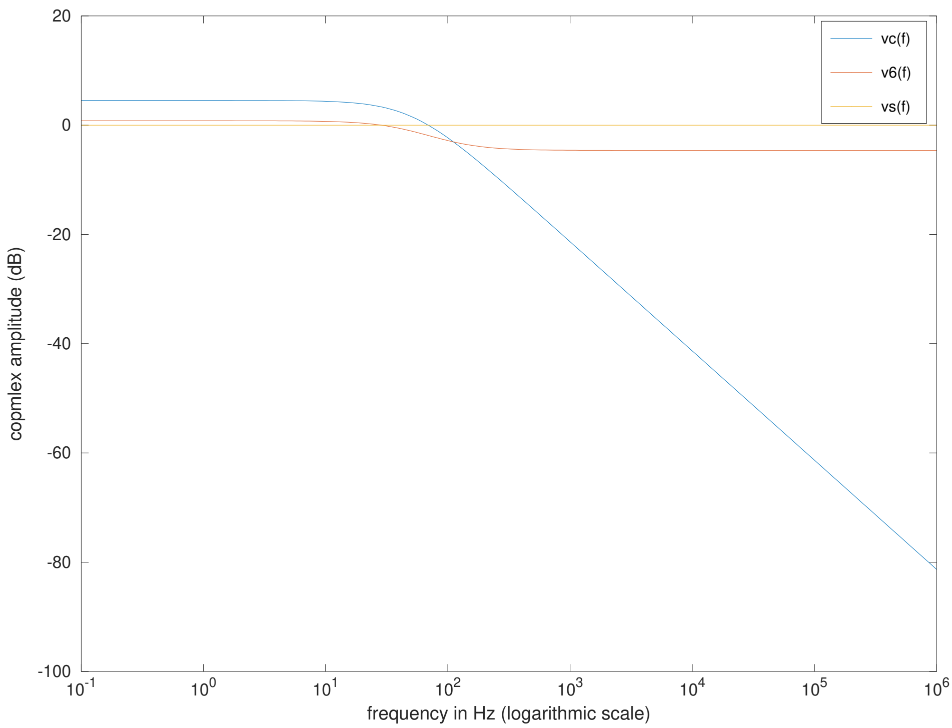

Figure 4: Graph for amplitude frequency response, in dB, of V_c, V_6 and V_s for frequencies ranging from 0.1Hz to 1MHz (logarithmic scale).

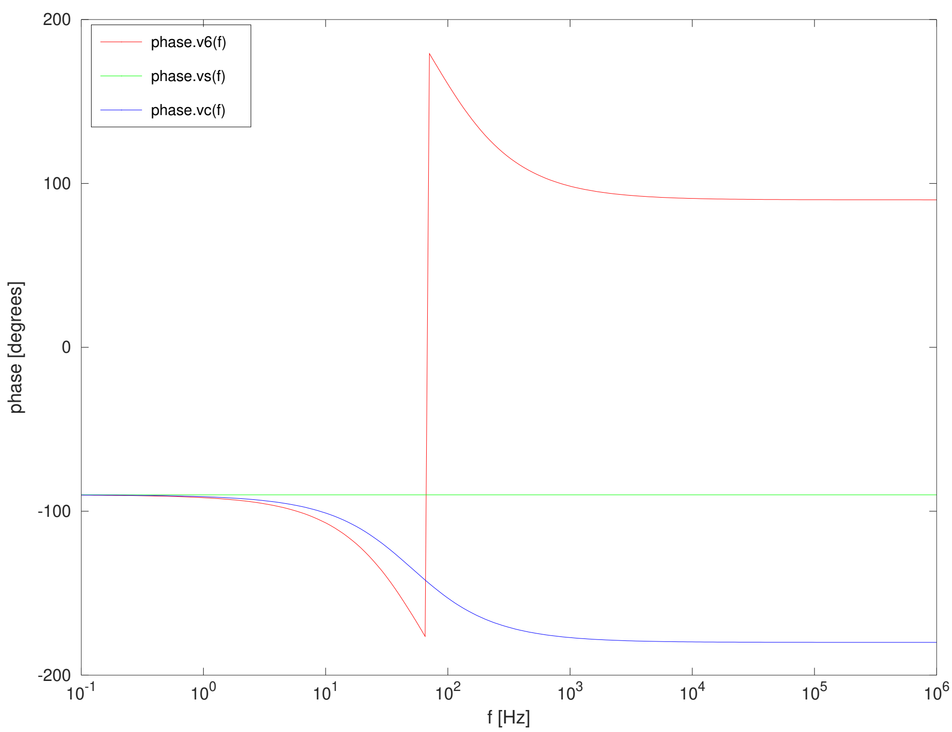

Figure 5: Graph for the phase response, in degrees of V_c, V_6 and V_s for frequencies ranging from 0.1Hz to 1MHz, displayed in a logarithmic scale. The phase of V_6 is actually continuous, and the reason for the apparent discontinuity in the graph is because of the domain of the arctan function - D_{arctan} =]-180, 180].

3 Simulation Analysis

In this section the steps needed to simulate this circuit using the software Ngspice are described. The analysis (operating point, frequency response and phase response) are the following:

- operating point for $t < 0$;

- operating point for $V_s(0) = 0$, replacing the capacitor with a a voltage source $V_x = V_6 - V_8$, where V_6 and V_8 are the voltages in nodes 6 and 8 as obtained in the previous step (the reason for this is the fact that the equivalent resistance experienced by the capacitor must be calculated. Furthermore, because the energy discharge in the capacitor is continuous, it is required that the initial boundary conditions are computed);

- simulate the natural response of the circuit (using the boundary conditions V(6) and V(8) as obtained previously) using a transient analysis;

- repeating the third step, using V_s as given in **equation 2** and f = 1kHz in order to simulate for the total response on node 6

- simulate the frequency response in node 6 for a frequency range 0.1 Hz to 1MHz.

$$v_s(t) = V_s u(-t) + sin(2\pi f t)u(t) \tag{30}$$

with

$$u(t) = \begin{cases} 0 & \text{if } t < 0 \\ 1, & \text{if } t \geq 0 \end{cases} \tag{31}$$

3.1 Operating Point Analysis for $t < 0$

In this section, there was a need to create a Node 4, between the resistor R6 and Node 7, where an independent voltage source, V_x, providing 0V was inserted, to measure the voltage drop ($-V7$) felt by I_d. This was created in order to comply with NGSpice's requirements for defining the current controlled voltage source V_d. Table 4 shows the simulated operating point results for the circuit under analysis for $t < 0$.

Name	**Value [mA or V]**
@c[i]	0.000000e+00
@gb[i]	-2.24128e-04
@r1[i]	2.143087e-04
@r2[i]	2.241280e-04
@r3[i]	-9.81934e-06
@r4[i]	-1.18345e-03
@r5[i]	-2.24128e-04
@r6[i]	9.691454e-04
@r7[i]	9.691454e-04
v(1)	5.082120e+00
v(2)	4.864258e+00
v(3)	4.397911e+00
v(4)	-2.00234e+00
v(5)	4.894946e+00
v(6)	5.578097e+00
v(7)	-2.00234e+00
v(8)	-2.98689e+00

Table 4: Operating point for $t < 0$. A variable preceded by @ is of type *current* and expressed in milliAmpere; other variables are of type *voltage* and expressed in Volt.

3.2 Operating Point Analysis for $t = 0$

In this section the circuit is simulated using an operating point analysis with $V_s(0) = 0$ and with the capacitor replaced by a voltage source $V_x = V(6) - V(8)$, using the values obtained in the last step. This step was taken because we must compute the new initial conditions that guarantee continuity in the capacitor's discharge. Therefore, $V(6) - V(8)$ must be a continuous function, defined in branches (constant for $t < 0$ and varying in time for $t \geq 0$), as there can not be a energy discontinuity in the capacitor ($E_C = \frac{1}{2}CV^2$). However, that does not imply that that $V(6)$ and $V(8)$ are continuous functions in time. In **Table 5** the simulation results are presented.

Name	**Value [mA or V and Ohm]**
@gb[i]	2.127063e-18
@r1[i]	-2.03387e-18
@r2[i]	-2.12706e-18
@r3[i]	9.318941e-20
@r4[i]	-4.29471e-19
@r5[i]	-2.81000e-03
@r6[i]	4.336809e-19
@r7[i]	8.665622e-19
v(1)	0.000000e+00
v(2)	2.067600e-15
v(3)	6.493414e-15
v(4)	-8.96024e-16
v(5)	1.776357e-15
v(6)	8.564986e+00
v(7)	-8.96024e-16
v(8)	-1.77636e-15
Ix	-2.81000e-03
Vx	8.564986e+00
Req	-3.04804e+03

Table 5: Operating point for $v_s(0) = 0$. A variable preceded by @ is of type *current* and expressed in miliAmpere; variables are of type *voltage* and expressed in Volt. The equivalent resistance is in Ohms

3.3 Natural solution for V_6 using transient analysis

In this section the natural response of the circuit in the interval [0,20] ms was studied using a transient analysis simulation. Using the previous simulations, the initial conditions for V(6) and V(8) were defined, using NGSpice's diretive *.ic*.

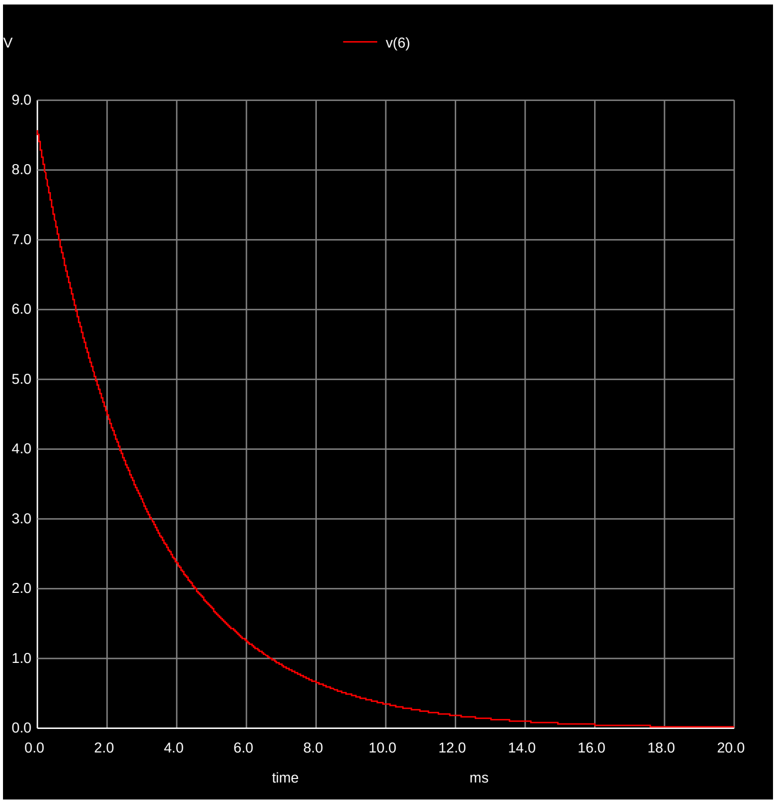

Figure 6: Simulated natural response of $V_6(t)$ in the interval [0,20] ms. The *x axis* represents the time in milliseconds and the *y axis* the Voltage in node 6 in Volts.

3.4 Total solution for V_6 using transient analysis

In this section the total response of V_6 (natural + forced) is simulated using NGSpice's transient analysis capabilities. This is done by repeating the previous section, but using V_s as given in 2 and f = 1kHz.

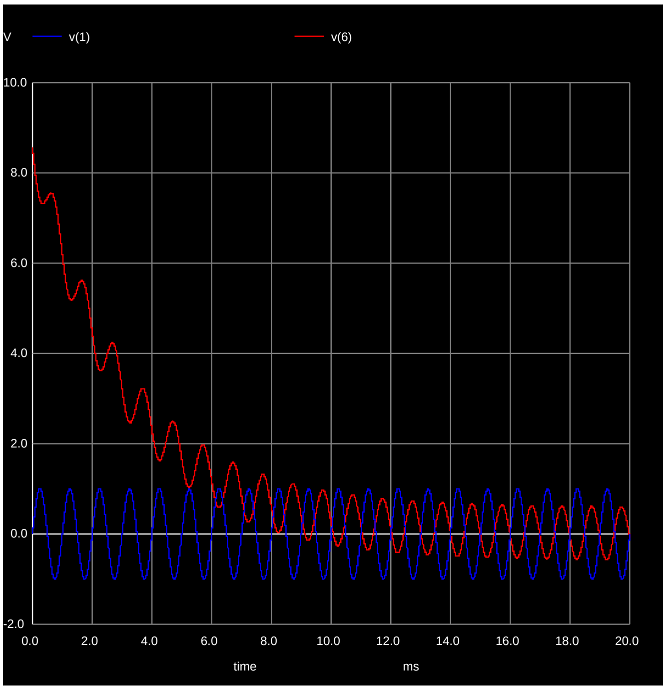

Figure 7: Simulated response of $V_6(t)$ and of the stimulus $V_s(t)$ as functions of time from [0,20] ms. The *x axis* represents the time in milliseconds and the *y axis* the Voltage in Volts.

3.5 Frequency response in node 6

In this section, the frequency response in node 6 is simulated for the frequency range from 0.1 Hz to 1 MHz, along with the phase response of the circuit. The reasons of how and why $V_6(t)$ and $V_s(t)$ differ have been covered in **subsection 2.6**.

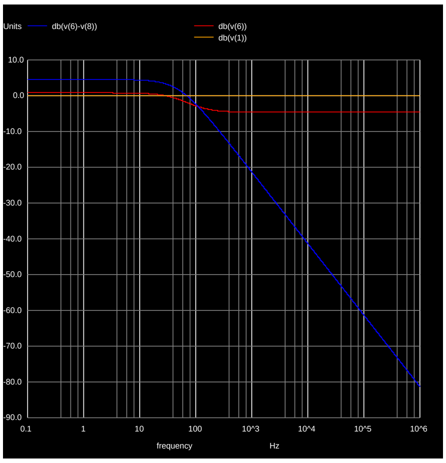

Figure 8: Magnitude of $V_s(f)$, $V_c(f)$ and of $V_6(f)$. The *x axis* represents the frequency in Hz, using a logarithmic scale and the *y axis* the magnitude in dB.

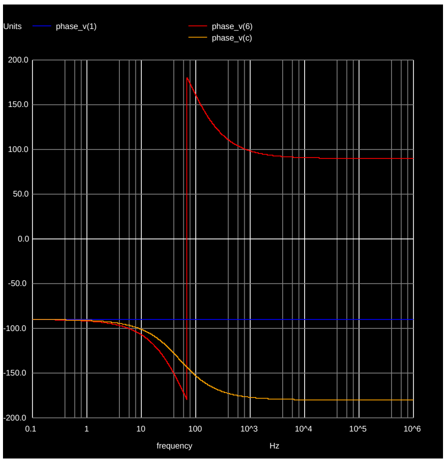

Figure 9: Phase of $V_s(f)$, $V_c(f)$ and of $V_6(f)$. The *x axis* represents the frequency in HZ, using a logarithmic scale and the *y axis* the phase in degrees.

4 Conclusion

In conclusion, for this second laboratory assignment, both a theoretical analysis and simulations have been presented.

By plotting and compiling the data obtained in tables, it is found that the simulations coincide with the theoretical predictions up until the last digit, where only some rounding errors may be found. This is due to the fact that a simple circuit is being analysed, consisted only of linear components, therefore, no approximation was required do solve the circuit. However, when analyzing more complex circuits, such as those including transistors, NGSpice may prove to be much more exact and precise.

In order to clearly compare the simulations from NGSpice with the theoretical predictions from octave, the tables containing the same information and the plots regarding the same events are shown side by side, where it can be seen that the results are extremely similar.

5 Visual Data Gallery

Name	Value [A or V]
@c	0
@Gb	-2.241280e-04
@r1	2.143086302573204e-04
@r2	2.241279676947193e-04
@r3	-9.819337437399266e-06
@r4	-1.183453890132603e-03
@r5	-2.241279676947192e-04
@r6	9.691452598752827e-04
@r7	9.691452598752829e-04
v(1)	5.08211987776
v(2)	4.86425757452
v(3)	4.39791092949
v(4)	-2.00234226900
v(5)	4.89494572428
v(6)	5.57809723710
v(7)	-2.00234226900
v(8)	-2.98688786865

Figure 10: Theoretical Question 1

Name	Value [A or V]
@c[i]	0.000000e+00
@gb[i]	-2.24128e-04
@r1[i]	2.143087e-04
@r2[i]	2.241280e-04
@r3[i]	-9.81934e-06
@r4[i]	-1.18345e-03
@r5[i]	-2.24128e-04
@r6[i]	9.691454e-04
@r7[i]	9.691454e-04
v(1)	5.082120e+00
v(2)	4.864258e+00
v(3)	4.397911e+00
v(4)	-2.00234e+00
v(5)	4.894946e+00
v(6)	5.578097e+00
v(7)	-2.00234e+00
v(8)	-2.98689e+00

Figure 11: Simulation Question 1

Name	Value [A or V]
@Gb	0.00000000000
@r1	0
@r2	0
@r3	0
@r4	0
@r5	-2.809995541354671e-03
@r6	0
@r7	0
v(1)	0.00000000000
v(2)	0.00000000000
v(3)	0.00000000000
v(4)	0.00000000000
v(5)	0.00000000000
v(6)	8.56498510575
v(7)	0.00000000000
v(8)	0.00000000000
Ix	-0.00280999554
Vx	8.56498510575
Req	-3.048042e+03
τ	-3.122414e-03

Figure 12: Theoretical Question 2

Name	Value [A or V]
@gb[i]	2.127063e-18
@r1[i]	-2.03387e-18
@r2[i]	-2.12706e-18
@r3[i]	9.318941e-20
@r4[i]	-4.29471e-19
@r5[i]	-2.81000e-03
@r6[i]	4.336809e-19
@r7[i]	8.665622e-19
v(1)	0.000000e+00
v(2)	2.067600e-15
v(3)	6.493414e-15
v(4)	-8.96024e-16
v(5)	1.776357e-15
v(6)	8.564986e+00
v(7)	-8.96024e-16
v(8)	-1.77636e-15
Ix	-2.81000e-03
Vx	8.564986e+00
Req	-3.04804e+03

Figure 13: Simulation Question 2

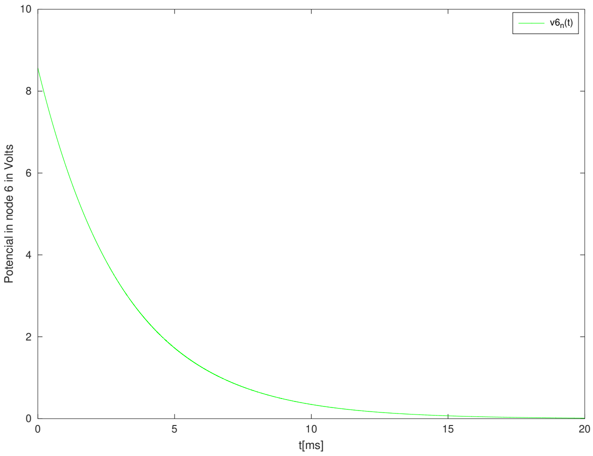

Figure 14: Theoretical Question 3

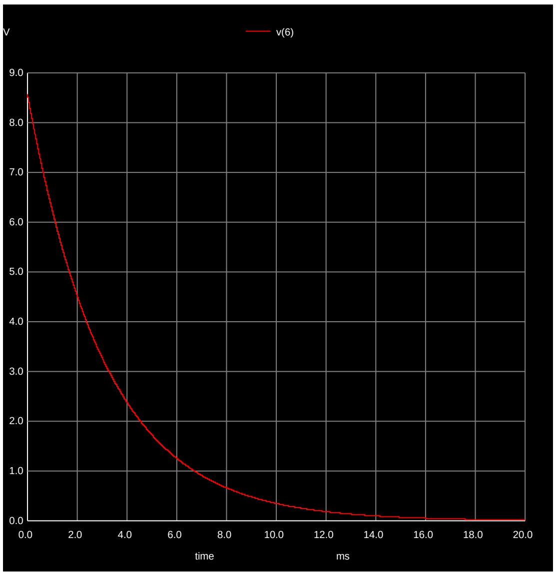

Figure 15: Simulation Question 3

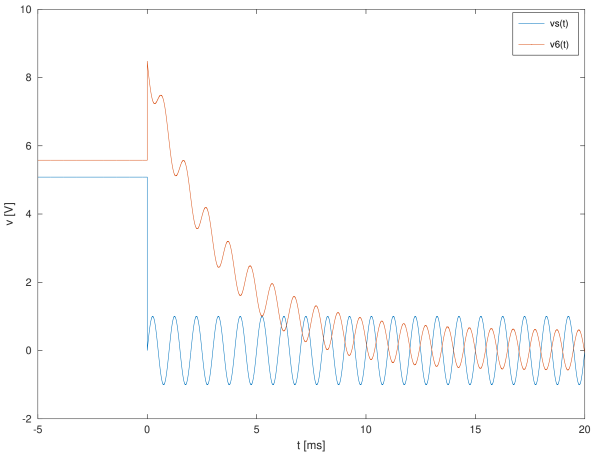

Figure 16: Theoretical Question 5

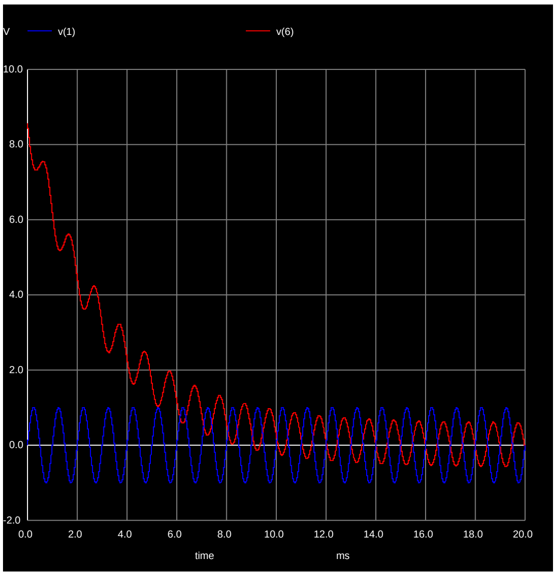

Figure 17: Simulation Question 4

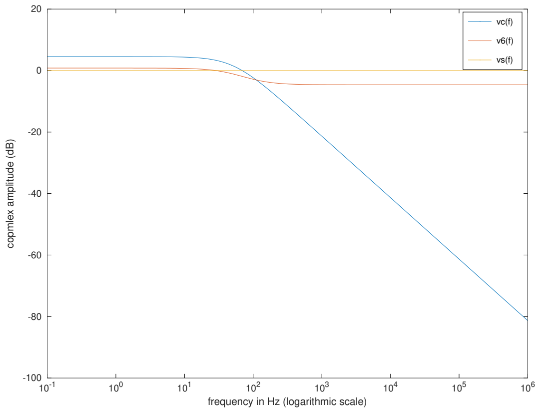

Figure 18: Theoretical Question 6 - Frequency Response

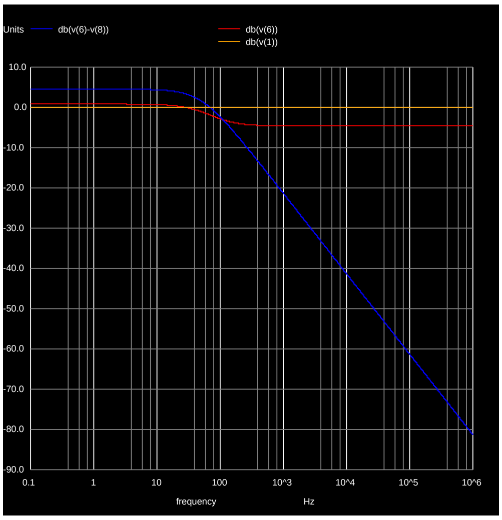

Figure 19: Simulation Question 5 - Frequency Response

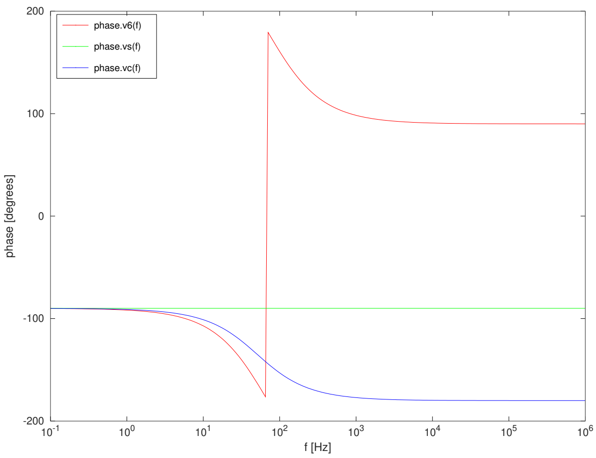

Figure 20: Theoretical Question 6 - Phase

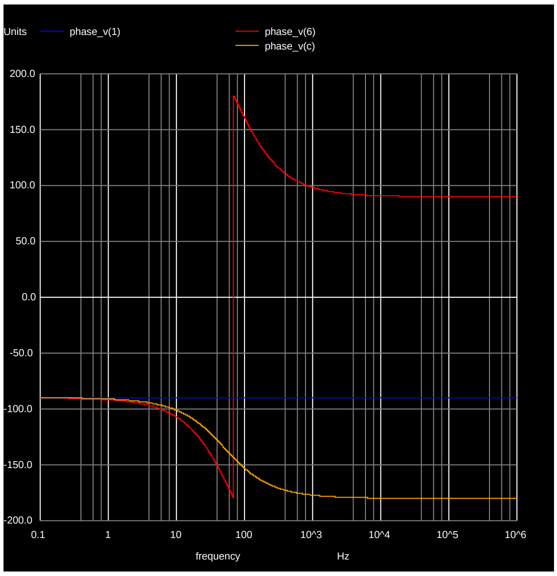

Figure 21: Simulation Question 6 - Phase