Incomplete Markets with Endogenous Labor

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This problem set asks you to compute an Aiyagari-style incomplete markets model that we reviewed in the class but now with endogenous labor and income taxes.Small letters are for individuals and capital letters are aggregates. The setup is as follows:

1. Continuum of ex-ante identical agents who have preferences over consumption and leisure using the following utility specification

$$\mathbb{E}_0 \sum \beta^t \left\{ \frac{\left(c_{i,t}^{\eta} l_{i,t}^{1-\eta}\right)^{1-\mu}}{1-\mu} \right\}$$

   Households supply labor, save in risk-free bonds subject to a debt limit. They also pay an income tax, τ_y and receive transfers T_t. Let w_t and r_t be the pre-tax wage rate and return on savings. Let budget constraint for a typical household would look like

$$c_{i,t} + a_{i,t+1} \leq (1-\tau_y) e_{i,t} w_t (1 - l_{i,t}) + T_t + (1 + (1-\tau_y) r_t) a_{i,t}$$

   where $a_{i,t+1} \geq \underline{a}$,$l_{i,t} \leq 1$ and $c_{i,t} \geq 0$. The skill process $e_{i,t}$

$$e_{i,t} = \rho e_{i,t-1} + \sigma \epsilon_{i,t}$$

   The aggregate asset supply $A_t = \int a_{i,t} di$, the labor supply $N_t = \int e_{i,t}(1 - l_{i,t})$

2. The government budget constraint is

$$G_t + T_t + (1-\tau_y) r_t B_t = B_{t+1} + \tau_y (w_t N_t + r A_t)$$

   where G_t is government spending and B_t is debt.

3. Technology: There is a representative firm that uses capital, $K_t$and labor L_t to operate a CRS technology that produces output

$$Y_t = F(K_t, N_t) = A K_t^{\theta} N_t^{1-\theta}$$

   This will pin down the rental rate $r_t = F_K - \delta$ and wage rate $w_t = F_N$.

4. Asset markets: Agents can trade claims to one-period risk-free bonds, capital and government bonds.

$$A_t = K_t + B_t$$

For your baseline calculations set $\beta = 0.98$, $\mu = 1.5$, and $\eta = 0.3$, $\tau_y = 0.4$, $\rho = 0.6$, $\sigma = 0.3$,$\delta = 0.075$, $\theta = 0.3$. It would be useful to set A such that steady state $Y = 1$. You want to set transfers T_t such that aggregate transfers to output is roughly 10% and G_t such government expenses are 20% of output. You can discretize the AR(1) process for skills using 5 states. Please answer the following questions

1. Compute the stationary equilibrium for this economy.
2. Plot the consumption and savings functions as a function of individual assets for a low, medium and high skill agent
3. Plot the supply and demand curves for assets as a function of the interest rate and explain how to pin down the market clearing prices.
4. Plot the distribution of agents across assets. Compute the mean, std, gini coefficients and Lorenz curve for wealth. In what respects do you think the model misses the wealth distribution in the data? Why?
5. Now lower the tax rate to 20%. Make a table that describes how ket aggregates and distributional moments change when you do this tax reform.