ECON 8185 - Homework 1

João Lazzaro *

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Question 1

We must compute the equilibrium for the following model:

$$\max_{\{c_t,x_t,l_t\}} \sum_{t=0}^{\infty} \big(\beta(1+\gamma_n)\big)^t [log(c_t) + \varphi log(l_t)]$$

st:

$$c_t + x_t = k_t^\theta \big((1+\gamma_z)^t z_t h_t\big)^{1-\theta}$$

$$(1+\gamma_z)(1+\gamma_n)k_{t+1} = (1-\delta)k_t + x_t$$

$$\log z_t = \rho \log z_{t-1} + \epsilon_t,\ \epsilon - N(0,\sigma^2)$$

$$h_t + l_t = 1$$

$$c_t, x_t \geq 0$$

Note the model above is the detrended version of the question model (I did not use hats, but they should be there). I implemented the three methods in the files VFI.jl, Riccati.jl and Vaughan.jl.

For VFI, I used 100 and 40 gridpoints for capital and the exogenous process. No kind of interpolation was used hence the agent is constrained by choices on the grid. This is not the best method in terms of accuracy, it is simple to implement and here I wanted to focus on basics of each method. Vaughan and LQ methods are a translation of the lecture notes (with help of Ljunqvist Sargent textbook) to Julia.

*santo279@umn.edu

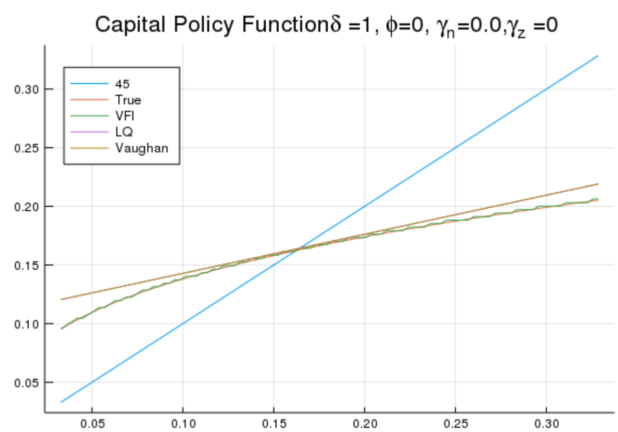
On average VFI took 10 times more to compute even with such a small capital grid, it also use a lot of memory and if more state variables were added, the problem would soon become too expensive computationally. LQ and Vaughan run in a fraction of a second and use almost no memory.

Question 2

For this section, I will change the parameters δ, γ_n and γ_z. The other parameters are held constant at $\theta = 1/3,\ \beta = 0.9,\ \rho = 0.5,\ \sigma = 0.5,\ \varphi = 0$. In order to compare each method, I will plot the policy functions for capital. Labor policy functions will not be plotted because they are uninteresting since they all are constant (as expected by theory) and equal to 1[1].

2.a) $\delta = 1, \gamma_n = 0$ and $\gamma_z = 0$

In this case, I also plot the true analytical solution for the policy function.



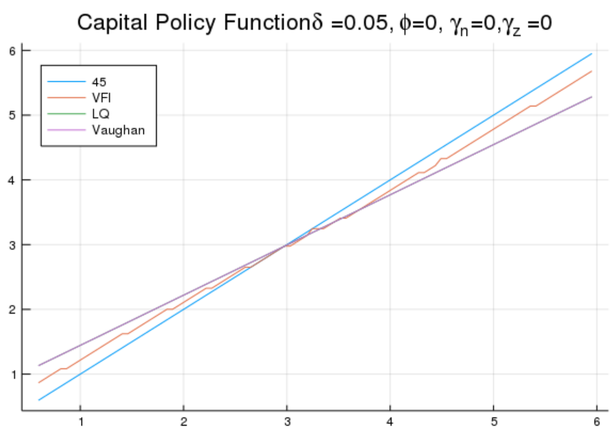
Note that the VFI policy function is exactly the same as the true function[2]. Vaughan and LQ gives us the exact same solution. Since it is a linear approximation around the Steady state, we observe that the solution is better closes to the SS. This is the case where the policy function is visually less linear, so probably the worst performance for the linear approximations.

[1]Note that if φ was different, than this result is not valid anymore. The codes are also able to accomodate endogenous labor choice.

[2]Except for numerical errors that can be easily decreased using interpolation and other techniques.

2.b) $\delta = 0.05, \gamma_n = 0$ and $\gamma_z = 0$

From now on, there is no analytical formula for the policy function.



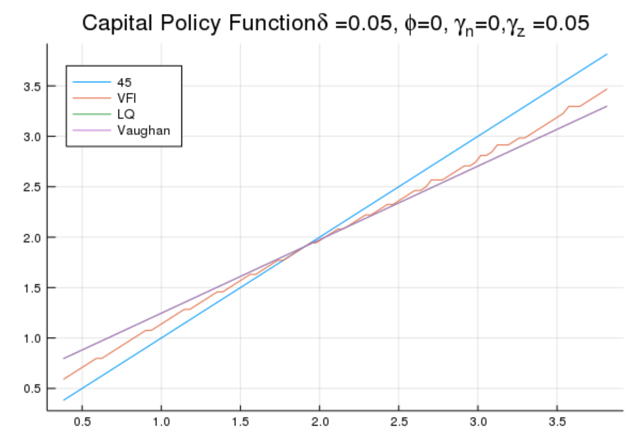
Note that the VFI policy function seems to be closer to linear, so the approximations are thought to be better. Since the capital grids were fixed from 1/10 to 3 times the Steady State value, the relative distance between the VFI solution to the linear approximation can be used as a measure of how good the approximation is. We we'll see, that at least visually, this dstance seems to be decreasing in the following exercises.

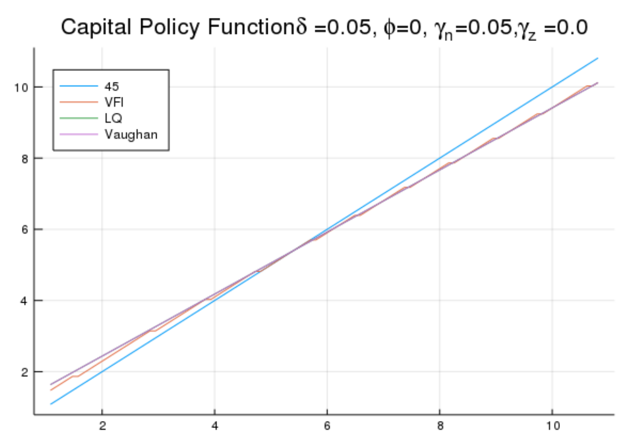
2.c)$\delta = 0.05, \gamma_n = 0$ and $\gamma_z = 0.05$

The analysis for this figure is pretty much the same as the previous one. Adding technology growth does not seem to matter much for the algorithms.

2.d) $\delta = 0.05, \gamma_n = 0.05$ and $\gamma_z = 0$

It is interesting to note that in this case the VFI solution is much closer to the linear approximations. Also, since an increase in population must lead to an adjustment of the discount parameter, the convergence of the VFI algorithm is much slower. So cases with patient agents and population growth are the best to us linear approximations.


Capital Policy Functionδ =0.05, ϕ=0, γ_n=0,γ_z =0.05
45
VFI
LQ
Vaughan
0.5
1.0
1.5
2.0
2.5
3.0
3.5


Capital Policy Functionδ =0.05, ϕ=0, γ_n=0.05,γ_z =0.0
45
VFI
LQ
Vaughan
2
4
6
8
10