Search Models

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The exercise asks you to solve a version of the Diamond, Mortensen Pissarides. The details are in Shimer's book and I have attached the relevant chapters. Here I list the key equations that need to be solved numerically. The notation uses $c(s^t)$ for consumption, $n(s^t)$ for fraction of people employed,$v(s^t)$ for vacancies per employed person, $\theta(s^t)$ for the ratio of vacancies to unemployed, $k(s^t)$ for capital, $w(s^t)$ for wages. The job finding rate $f(\theta) = \bar{\mu}\theta^{\eta}$ and hiring rate per vacancy is given by $\mu(\theta) = \frac{f(\theta)}{\theta}$. The scalar x is the separation rate.

1. The first equation is the Euler equation for a HH

$$1 = \sum_{s_{t+1}} \pi(s^{t+1}|s^t)\beta \left(\frac{c(s^t)}{c(s^{t+1})}\right) \left[\underbrace{\alpha z(s^t) \left(\frac{k(s^{t+1})}{n(s^{t+1})(1-v(s^{t+1})}\right)^{1-\alpha} + 1 - \delta}_{MPK}\right]$$

   Next we have definition of $\theta(s^t)$:

2. The next equation is a condition that equates the return from recruiting and producing for the firm:

$$(1-\alpha)\left(\frac{k(s^t)}{n(s^t) - \theta(s^t)(1-n(s^t))}\right)^{\alpha} =$$

$$\beta\mu((\theta(s^t)) \sum_{s_{t+1}} \pi(s^{t+1}|s^t)\beta \left(\frac{c(s^t)}{c(s^{t+1})}\right) \left[(1-\alpha)z(s^{t+1}) \left(\frac{k(s^{t+1})}{n(s^{t+1}) - \theta(s^{t+1})(1-n(s^{t+1})}\right)^{\alpha} \left(1 + \frac{1-x}{\mu(\theta(s^{t+1}))}\right) - w(s^{t+}\right.$$

3. The next equation comes from Nash Bargaining. The surplus is split using bargaining weights ϕ.

$$(1-\tau)w(s^t) = \phi\left[1 + \theta(s^t)\right](1-\alpha)(1-\tau)z(s^t)\left(\frac{k(s^t)}{n(s^t)(1-v(s^t)}\right)^{\alpha} + (1-\phi)\gamma c(s^t)$$

4. The next two equations are the resource constraints and law of motion of employment when x fraction exogenously exit jobs.

$$k(s^{t+1}) = (1-\delta)k(s^t) + z(s^t)k(s^t)^{\alpha}(n(s^t)(1-v(s^t))^{1-\alpha} - c(s^t)$$

$$n(s^{t+1}) = (1-x)n(s^t) + f(\theta(s^t))(1-n(s^t))$$

5. Assume that the driving process for productivity has a stochastic trend given by

$$\log z(s^t) = \log z(s^t) + s_{t+1}$$

$$s_{t+1} = (1-\rho)\,\bar{s} + \rho s_t + \zeta \nu_{t+1}$$

Please answer the following questions based on the setup described above

1. First show that there exists an equilibrium in the following transformed variables: $\tilde{c}(s^t) = c(s^t) z(s^t)^{\frac{-1}{1-\alpha}}$, $\tilde{k}(s^t) = k(s^t) z(s^t)^{\frac{-1}{1-\alpha}}$ and $\tilde{w}(s^t) = w(s^t) z(s^t)^{\frac{-1}{1-\alpha}}$. Then show that there exists a Markovian solution with k, n, s as state variables. Boil down the equations below to two functional equations in two unknown function: $\theta(k, n, s)$ and $\tilde{c}(k, n, s)$

2. Compute the non stochastic steady state. Log linearize around the non stochastic steady state to get a system of linear expectational difference equations and solve for the policy rules. For example

$$\log(\theta_{t+1}) = \log \bar{\theta} + \theta_s (s_{t+1} - \bar{s}) + \theta_n (\log n_t - \log \bar{n}) + \theta_k (\log k_t - \log \bar{k})$$

3. Using the calibration in Table 3.2 to compute the IRF and the ergodic moments of all the relevant variables. Please check your results against figure 3.2 in the book.

4. Derive an expression for the labor wedge in this economy using the consumption output ratio and hours which here is the fraction of HH that are employed. Obtain a log linear expansion of the wedge using the log linear policy rules

5. What is the correlation of the labor wedge with output and employment? You can sign it by staring at the policy rules. What is the ergodic std. of the labor wedge? How does the correlations and volatility compare to data

6. Now extract a TFP series such that the simulated path of $y(s^t)$ is exactly as in the data. Start with the steady state level of capital and employment. Using the policy rule for output reverse engineer a shock such that detrended output matches exactly to the value in the data. Now update the state variables using ur policy rules and keep iterating this procedure. How does the TFP series extracted compare to a Solow residual in the data?