

Math 493 Project 1

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Sensitivity System 1

- We solved the following sensitivity system

$$D \begin{pmatrix} x \\ \partial_{\theta} x \end{pmatrix} = \begin{pmatrix} \theta x^2 \\ x^2 + 2\theta x(\partial_{\theta} x) \end{pmatrix}$$
$$\begin{pmatrix} x \\ \partial_{\theta} \end{pmatrix} (1) = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

Problem 1 Results

Method	θ	SSE
fminsearch	1.8432617	0.1313268
Gauss-Newton	1.8416666	0.1313271

Problem 1 Results

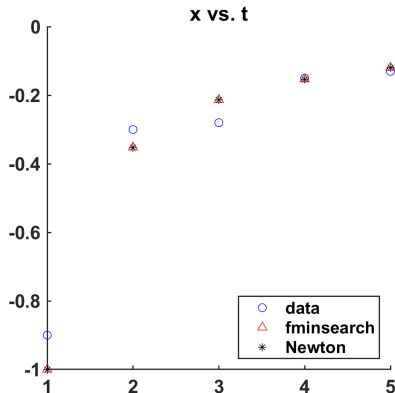


Figure: Approximations compared with data. It took fminsearch 16 steps while it took Gauss-Newton 12 steps to converge.

Problem 1 Results

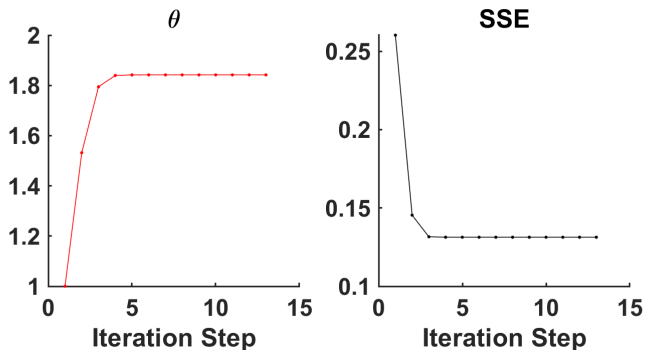


Figure: Convergence of Gauss-Newton

Sensitivity System 2

$$D \begin{pmatrix} x \\ \partial_{\theta} x \\ \partial_{x_0} x \end{pmatrix} = \begin{pmatrix} x^2 \theta \\ x^2 + 2x\theta \partial_{\theta} x \\ 2x\theta \partial_{x_0} x \end{pmatrix}$$

$$\begin{pmatrix} x \\ \partial_{\theta} x \\ \partial_{x_0} x \end{pmatrix} (1) = \begin{pmatrix} -0.9 \\ 0 \\ 1 \end{pmatrix}$$

Problem 2 Results

Method	θ	x_0	SSE
fminsearch	1.7553438	-0.8962938	0.0808069
Gauss-Newton	1.7543119	-0.8963022	0.0808071

Problem 2 Results

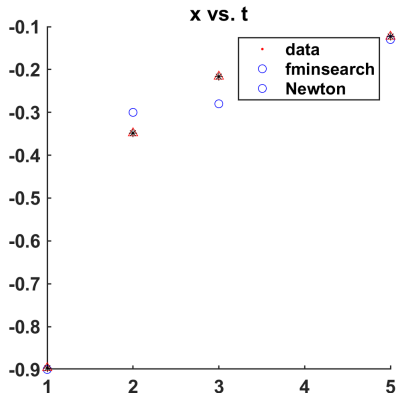


Figure: Approximations compared with data. It took fminsearch 41 steps while it took Gauss-Newton 13 steps to converge.

Problem 2 Results

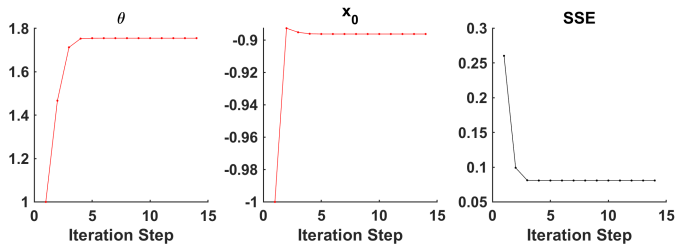


Figure: Convergence of Gauss-Newton

Sensitivity System 3

$$D \begin{pmatrix} x \\ y \\ \partial_a x \\ \partial_a y \\ \partial_b x \\ \partial_b y \end{pmatrix} = \begin{pmatrix} -axy \\ axy - by \\ -xy - ay\partial_a x - ayx\partial_a y \\ xy + ay\partial_a x + (ax - b)\partial_a y \\ -ay\partial_b x - ax\partial_b y - y + ay\partial_b x + (ax - b)\partial_b y \end{pmatrix}$$

$$D \begin{pmatrix} x \\ y \\ \partial_a x \\ \partial_a y \\ \partial_b x \\ \partial_b y \end{pmatrix} (0) = \begin{pmatrix} 0.9 \\ 0.1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Problem 3 Results

Method	a	b	SSE
fminsearch	0.5022775	0.1030516	0.0171071
Gauss-Newton	0.5132331	0.1091130	0.0173460

Problem 3 Results

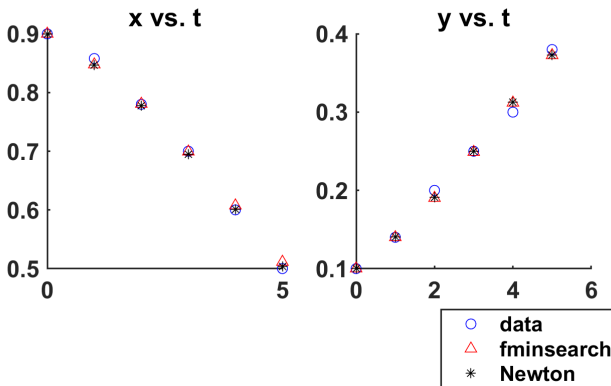


Figure: Approximations compared with data. It took fminsearch 71 steps while it took Gauss-Newton 49 steps to converge.

Problem 3 Results

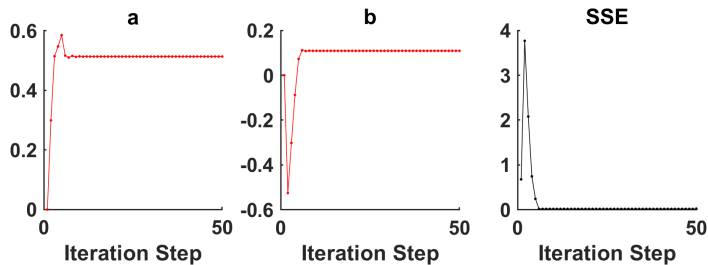


Figure: Convergence of Gauss-Newton

Population Dynamic

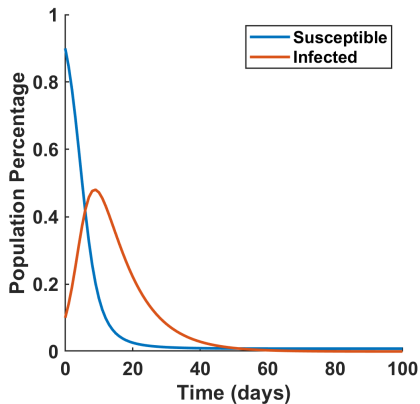


Figure: Population percentage of susceptible and infected population