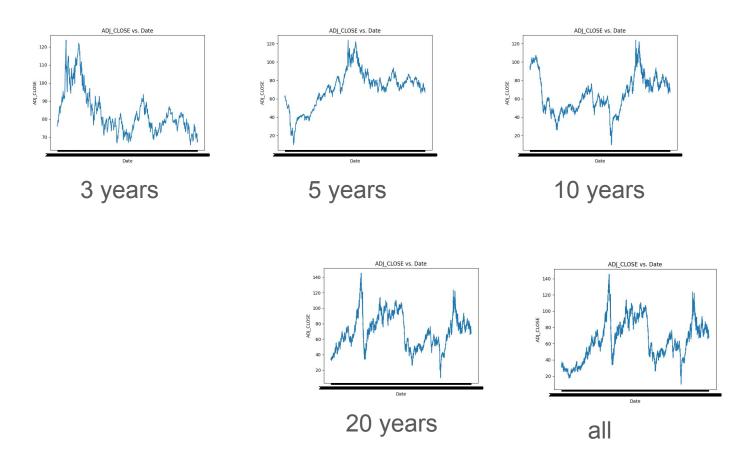
# Visualizations for EDA of Time Series and Results

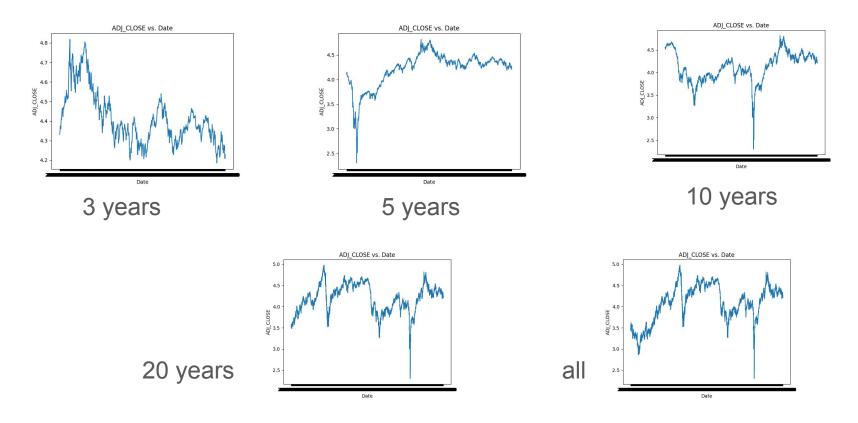
Julia Gugulski CSCI 1420 Capstone Project Spring 2025

# Raw data plot:



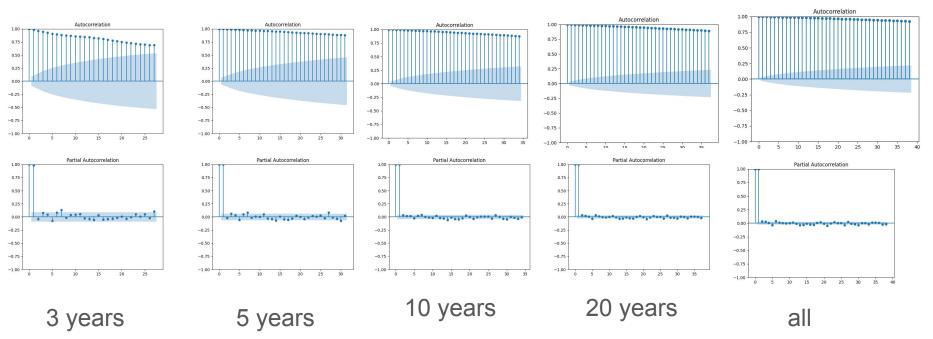
#### Transformed data plot:

The transformation = logarithm of each adjusted closing value



#### ACF and PACF plots:

Autocorrelation function = correlation of the time series with its lags; Partial autocorrelation function = partial correlation of time series with its lags after removing the effects of lower order lags between them (<a href="https://www.youtube.com/watch?v=DeORzP0go5l&t=1s">https://www.youtube.com/watch?v=DeORzP0go5l&t=1s</a>)



The ACF plot show that the correlations with the lags are high and positive with a slow decline. The PACF plot show a spike at 1 and then small or no spikes afterwards. These are signs of a simple random walk, a common time series, which is not stationary.

#### **ADF Test:**

Augmented Dickey-Fuller Test = tests for stationarity. The null hypothesis is that the data has a unit root and is non-stationary.

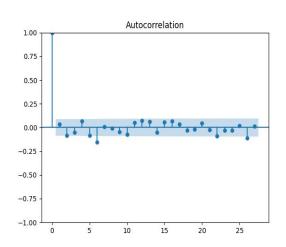
The results for each of the subsets of data:

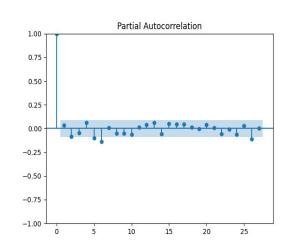
- Past 3 years = 0.53 ⇒ fail to reject null hypothesis (data is not stationary)
- Past 5 years = 0.63 ⇒ fail to reject null hypothesis (data is not stationary)
- Past 10 years =  $0.17 \Rightarrow$  fail to reject null hypothesis (data in not stationary)
- Past 20 years = 0.03 ⇒ can reject null hypothesis (data seems to be stationary)
- All years = 0.08 ⇒ fail to reject null hypothesis (data is not stationary)

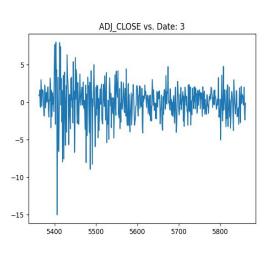
I will difference all data, and for the 20 years I will see both the behavior of differencing and not differencing (although it seems like it should still be differenced because the ACF and PACF do not indicate that it is stationary)

#### Differenced Data: 3 years

After one differencing it is clearer that the differenced data is more stationary. The p-value of the ADF test is 6.28e-23, so we can reject the null and confidently say that the differenced data is stationary.



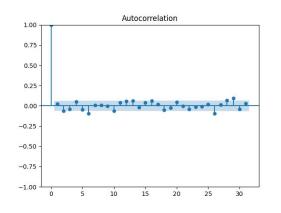


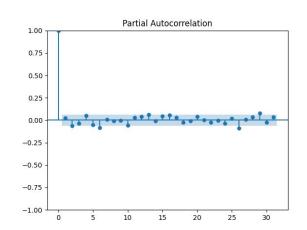


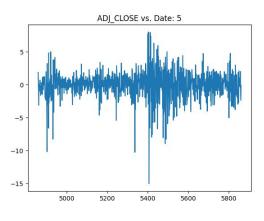
There is a significant spike at lag 6 in both the ACF and PACF plots. This implies p = 6/q = 0 and p = 0/q = 6, respectively. d = 1 because we only differenced once.

#### Differenced Data: 5 years

After one differencing it is clearer that the differenced data is more stationary. The p-value of the ADF test is 1.26e-20, so we can reject the null and confidently say that the differenced data is stationary.





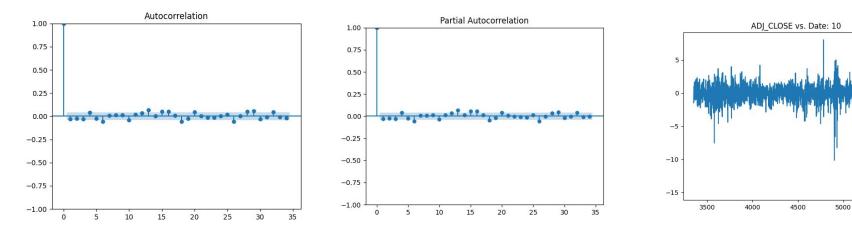


There is a significant spike at lag 6 in both the ACF and PACF plots. This implies p = 6/q = 0 and p = 0/q = 6, respectively. d = 1 because we only differenced once.

#### Differenced Data: 10 years

After one differencing it is clearer that the differenced data is more stationary. The p-value of the ADF test is 1.71e-15, so we can reject the null and confidently say that the differenced data is stationary.

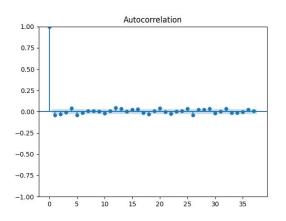
5500

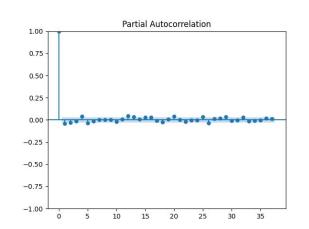


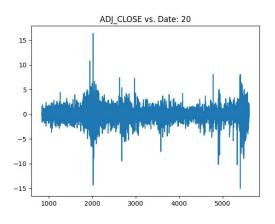
There is a significant spike at lag 6 in both the ACF and PACF plots. This implies p = 6/q = 0 and p = 0/q = 6, respectively. d = 1 because we only differenced once.

#### Differenced Data: 20 years

After one differencing it is clearer that the differenced data is more stationary. The p-value of the ADF test is 2.94e-27, so we can reject the null and confidently say that the differenced data is stationary.



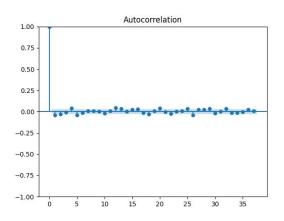


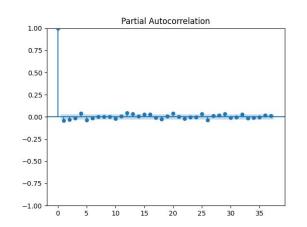


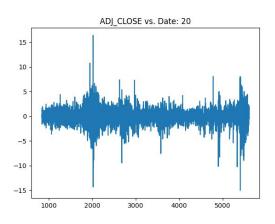
There is a significant spike at lag 4 in both the ACF and PACF plots. This implies p = 4/q = 0 and p = 0/q = 4, respectively. d = 1 because we only differenced once.

#### Differenced Data: Full dataset

After one differencing it is clearer that the differenced data is more stationary. The p-value of the ADF test is 1.19e-20, so we can reject the null and confidently say that the differenced data is stationary.







There is a significant spike at lag 4 in both the ACF and PACF plots. This implies p = 4/q = 0 and p = 0/q = 4, respectively. d = 1 because we only differenced once.

# Results

#### Methods

#### 3 types of models

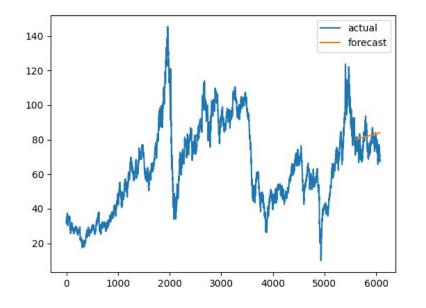
- Auto generated parameters
- Only AR and I parameters
- Only MA and I parameters

Comparing the performance of each type of model across the different sized subsets of data.

#### Auto generated parameters: full data

	SP	ARIIVIAA I	Results				
Dep. Variabl Model: Date: Time: Sample:	SARII Wed, ( 1	MAX(0, 1 )7 May 2 1:57:29	IQIC	Likelihood			
	pef std e	err 2	z P> z	[0.025	0.975]	======	
ma.L1 - ma.L2 - ma.L3 - ma.L4 ma.L5 - ma.L4	-0.0081 0.0366	0.008 0.008 0.008	4.811 -5.717	0.002 0.305 0.000	-0.051 -0.043 -0.024 0.022	0.051	
Ljung-Box (L Prob(Q): Heteroskeda Prob(H) (two	asticity (H)		0.00 Jar 7 Prob(Jl 2.55 Sk 0.00 Ku	B): kew:		20437.73 00 -0.37 12.32	
Warnings: [1] Covarian	ce matrix	calculate	ed using th	e outer pr	oduct of g	radients (comple	x-ste

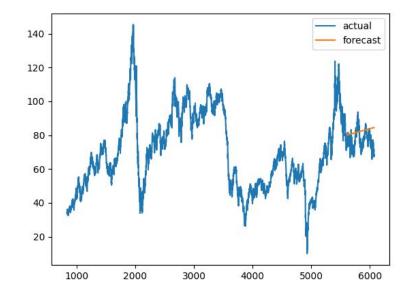
SARIMAY Results



mean absolute error: 6.0411036512809755 mean absolute percentage error: 0.0814159644326747 residual mean squared error: 7.3642166352007425

#### Auto generated parameters: 20 years

		SA	RIMAX	Results						
======						======			=====	======
Dep. Varia Model: Date: Time: Sample: Covarianc	V	Ved, ( 1 - 4	MAX(0, 1) 07 May 2 1:57:53 0 1 776	1, 5) Log 2025 AIC	ervations:   Likelihood 		88	=====		
==										
	coef	std e	err	z P> z	[0.025	0.975]				
	-0.03 -0.02 -0.00 0.03	392 265 097 384 420	0.008 0.010 0.009 0.009 0.008	-5.119 -2.767 -1.086 4.423 -4.976	0.665 0.000 0.006 0.278 0.000 0.000	-0.054 -0.045 -0.027 0.021	-0.024 -0.008 0.008 0.055			
Ljung-Box Prob(Q): Heteroske Prob(H) (t	dastic	ity (H)	0.9	0.00 Ja 97 Prob(, 0.90 S 0.04 K	Skew:		13322.27 .00 -0.34 11.15		=====	



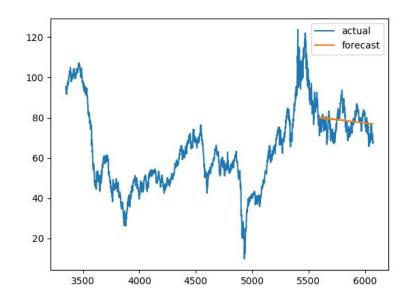
Warnings

[1] Covariance matrix calculated using the outer product of gradients (complex-step).

mean absolute error: 6.216849279120286 mean absolute percentage error: 0.08380943795302777 residual mean squared error: 7.5582893200684955

#### Auto generated parameters: 10 years

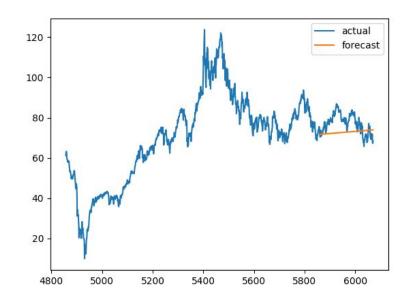
======= == Dep. Varial				======			 	
Model: Date: Time: Sample: Covariance	SAR Wed,	IMAX(2, 1 07 May 2 11:58:11 0 F 2263	, 2) Log 025 AIC	Likelihood	d			:===
==	coef std	err z	z P> z	[0.025	0.975	 ]	 	
ar.L2 ma.L1 ma.L2	0.0416 -0.9839 -0.0476 0.9628	0.007 0.007 - 0.010 0.010	5.877 138.059 -4.664 96.732	0.000	0.028 -0.998 -0.068 0.943	-0.970 -0.028		
======= Ljung-Box Prob(Q): Heterosked Prob(H) (tv	dasticity (F	0.2 H):	1.41 Ja 3 Prob(J 2.77 S 0.00 Ki	kéw:		7488.81 0.00 -0.84 11.76	 ======	:===



mean absolute error: 4.570801424542164 mean absolute percentage error: 0.06005604886165142 residual mean squared error: 5.633087283700272

#### Auto generated parameters: 5 years

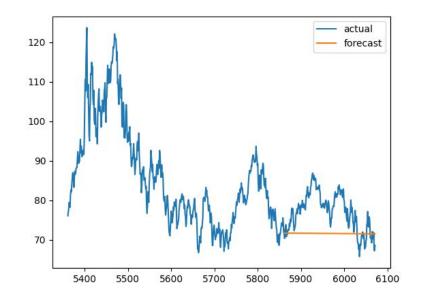
	S	ARIMAX	Results						
Dep. Varia Model: Date: Time: Sample:	-	IMAX(2, 7 07 May 2 11:58:21 0 F 1005	1, 2) Log 2025 AIC	Likelihoo	d 42' 4308.16 4289.89	-2133.346 78.691 32	=====		
==	coef std	orr	7 D\I7	. [0 025	. 0.075	1			
ar.L1 ar.L2 ma.L1 ma.L2	0.0202 0.0441 -0.9758 -0.0513 0.9436 4.1018	0.128 0.013 0.014 0.020 0.020	0.157 3.273 -68.180 -2.571 47.023	0.875 0.001 0.000 0.010 0.000	-0.231 0.018 -1.004 -0.090 0.904	0.272 0.071 -0.948 -0.012 0.983			
Prob(Q): Heteroske	x (L1) (Q): edasticity (F two-sided):	0.3 H):	6 Prob(	kew:		1488.5 0.00 -0.84 8.72	5		
Warnings [1] Covari		x calculate	ed using t	he outer p	roduct of (	gradients (c	omplex-s	tep).	



mean absolute error: 5.530824841047598 mean absolute percentage error: 0.06996323162062335 residual mean squared error: 6.546903792567009

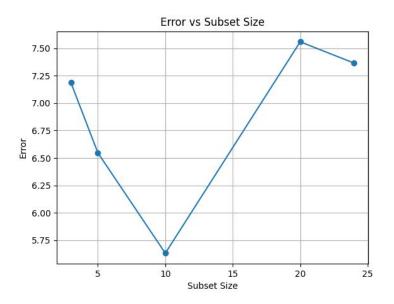
#### Auto generated parameters: 3 years

	S	SARIMAX	Results					
Model:	Wed,	RIMAX(2, 07 May : 11:58:29 0 - 501		Likelihoo	od			=====
======	coef std	err	z P> z	:====== :0 02	====== 5 0.975	======= 1	========	=======
ar.L1 ar.L2 ma.L1 ma.L2	-0.0016 0.0351 -0.9772 -0.0502 0.9562 5.7712	0.215 0.020 0.019 0.029 0.028	-0.007 1.754 -51.428 -1.717 33.959	0.994 0.079 0.000 0.086 0.000	-0.422 -0.004 -1.014 -0.108	0.419 0.074 -0.940 0.007 1.011		
Prob(Q): Heterosk	x (L1) (Q): edasticity (Itwo-sided):	0.: H):	1.44 Ja 23 Prob(J 0.24 S 0.00 Ki	IB): kew:		323.76 0.00 -0.56 6.78		



mean absolute error: 6.045010860403555 mean absolute percentage error: 0.07595413066603919 residual mean squared error: 7.186302468294149

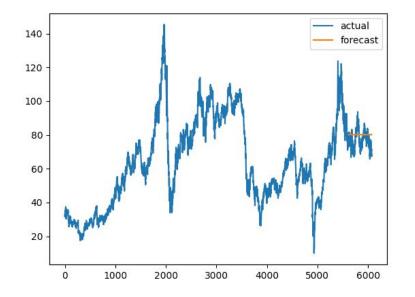
# Auto generated parameters: error vs subset size



10 years of data has the best residual mean squared error.

#### Only AR parameter: full data

SARIMAX Results
Dep. Variable: adj_close No. Observations: 5612 Model: ARIMA(4, 1, 0) Log Likelihood -10391.573 Date: Wed, 07 May 2025 AIC 20793.145 Time: 11:57:30 BIC 20826.308 Sample: 0 HQIC 20804.701 -5612 Covariance Type: opg
=======================================
coef std err z P> z  [0.025 0.975]
ar.L1 -0.0380 0.007 -5.735 0.000 -0.051 -0.025 ar.L2 -0.0291 0.008 -3.545 0.000 -0.045 -0.013 ar.L3 -0.0087 0.008 -1.138 0.255 -0.024 0.006 ar.L4 0.0376 0.007 5.174 0.000 0.023 0.052 sigma2 2.3777 0.019 126.515 0.000 2.341 2.414
=======================================
Ljung-Box (L1) (Q): 0.01 Jarque-Bera (JB): 21488.91 Prob(Q): 0.92 Prob(JB): 0.00
Heteroskedasticity (H): 2.56 Skew: -0.36 Prob(H) (two-sided): 0.00 Kurtosis: 12.56
=======================================



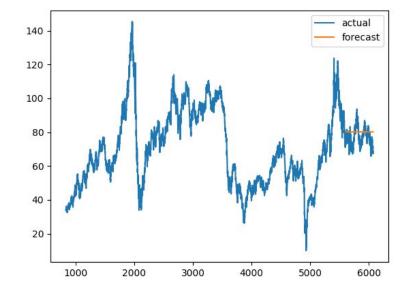
Warnings:

[1] Covariance matrix calculated using the outer product of gradients (complex-step).

mean absolute error: 5.048268230105487 mean absolute percentage error: 0.06746413019293145 residual mean squared error: 6.182928904804433

#### Only AR parameters: 20 years

SARIMAX Results
Dep. Variable: adj_close No. Observations: 4776 Model: ARIMA(4, 1, 0) Log Likelihood -9150.755 Date: Wed, 07 May 2025 AIC 18311.509 Time: 11:57:54 BIC 18343.865 Sample: 0 HQIC 18322.876 - 4776
Covariance Type: opg
coef std err z P> z  [0.025 0.975]
ar.L1 -0.0396
Ljung-Box (L1) (Q): 0.01 Jarque-Bera (JB): 14025.73  Prob(Q): 0.92 Prob(JB): 0.00  Heteroskedasticity (H): 0.90 Skew: -0.34  Prob(H) (two-sided): 0.03 Kurtosis: 11.37
Warnings:

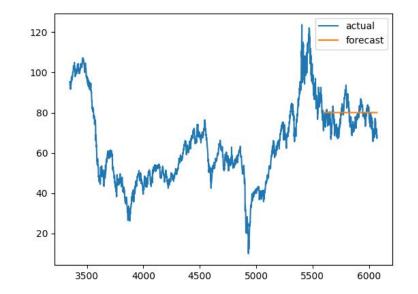


[1] Covariance matrix calculated using the outer product of gradients (complex-step).

mean absolute error: 5.047081085128184 mean absolute percentage error: 0.06744687959669475 residual mean squared error: 6.181587396291245

#### Only AR parameters: 10 years

SARIMAX Results
Dep. Variable: adj_close No. Observations: 2263  Model: ARIMA(6, 1, 0) Log Likelihood -4238.190  Date: Wed, 07 May 2025 AIC 8490.380  Time: 11:58:12 BIC 8530.448  Sample: 0 HQIC 8505.000  - 2263  Covariance Type: opg
==
coef std err z P> z  [0.025 0.975]
ar.L1
======  Ljung-Box (L1) (Q): 0.00 Jarque-Bera (JB): 6420.36  Prob(Q): 0.99 Prob(JB): 0.00  Heteroskedasticity (H): 2.82 Skew: -0.82  Prob(H) (two-sided): 0.00 Kurtosis: 11.09



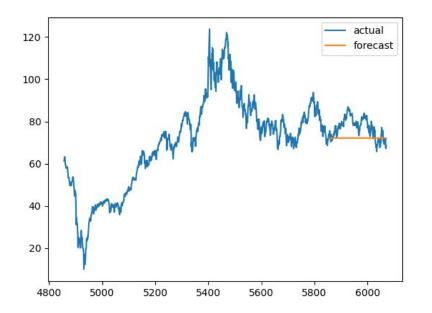
Warnings

[1] Covariance matrix calculated using the outer product of gradients (complex-step).

mean absolute error: 4.992347621530395 mean absolute percentage error: 0.06664481722083114 residual mean squared error: 6.117747640251688

#### Only AR parameters: 5 years

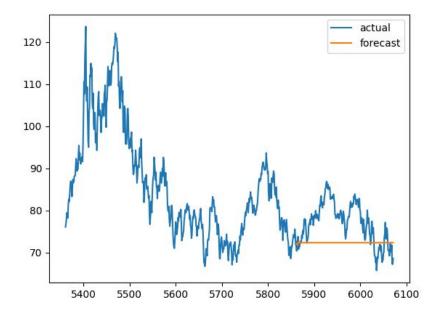
SARIMAX Results	
Dep. Variable: adj_close No. Observations: 1005 Model: ARIMA(6, 1, 0) Log Likelihood -2135.775 Date: Wed, 07 May 2025 AlC 4285.549 Time: 11:58:22 BlC 4319.931 Sample: 0 HQIC 4298.614 -1005 Covariance Type: opg	
==	
coef std err z P> z  [0.025 0.975]	
ar.L1 0.0200 0.024 0.821 0.412 -0.028 0.068 ar.L2 -0.0584 0.022 -2.638 0.008 -0.102 -0.015	
ar.L3 -0.0458 0.019 -2.358 0.018 -0.084 -0.008	
arl 4 0 0458 0 022 2 075 0 038 0 003 0 089	
ar.L5 -0.0522 0.022 -2.429 0.015 -0.094 -0.010	
ar.L6 -0.0864 0.023 -3.833 0.000 -0.131 -0.042 sigma2 4.1229 0.107 38.651 0.000 3.914 4.332	
======================================	
====== Ljung-Box (L1) (Q): 0.00 Jarque-Bera (JB): 1212.42 Prob(Q): 0.98 Prob(JB): 0.00 Heteroskedasticity (H): 1.20 Skew: -0.83 Prob(H) (two-sided): 0.10 Kurtosis: 8.12	
Warnings: [1] Covariance matrix calculated using the outer product of gradients (complex-step)	



mean absolute error: 5.710568337699619 mean absolute percentage error: 0.07183979191057999 residual mean squared error: 6.810863986801155

#### Only AR parameters: 3 years

SARIMAX Results
== Dep. Variable: adj_close No. Observations: 501 Model: ARIMA(6, 1, 0) Log Likelihood -1143.880 Date: Wed, 07 May 2025 AIC 2301.761 Time: 11:58:29 BIC 2331.263 Sample: 0 HQIC 2313.338 -501 Covariance Type: opg
=======================================
==
coef std err z P> z  [0.025 0.975]
ar.L1 0.0309 0.041 0.748 0.454 -0.050 0.112 ar.L2 -0.0736 0.035 -2.129 0.033 -0.141 -0.006 ar.L3 -0.0643 0.028 -2.274 0.023 -0.120 -0.009 ar.L4 0.0562 0.033 1.707 0.088 -0.008 0.121 ar.L5 -0.0927 0.032 -2.887 0.004 -0.156 -0.030 ar.L6 -0.1402 0.037 -3.784 0.000 -0.213 -0.068 sigma2 5.6815 0.267 21.290 0.000 5.158 6.205
=======  Ljung-Box (L1) (Q): 0.00 Jarque-Bera (JB): 190.45  Prob(Q): 0.97 Prob(JB): 0.00  Heteroskedasticity (H): 0.24 Skew: -0.45  Prob(H) (two-sided): 0.00 Kurtosis: 5.88  ==================================

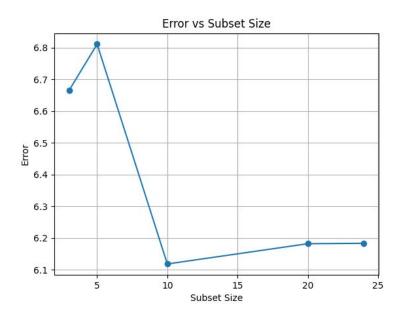


Warnings

[1] Covariance matrix calculated using the outer product of gradients (complex-step).

mean absolute error: 5.585222184360733 mean absolute percentage error: 0.07030817536113423 residual mean squared error: 6.665028566557918

# Only AR parameters: error vs subset size

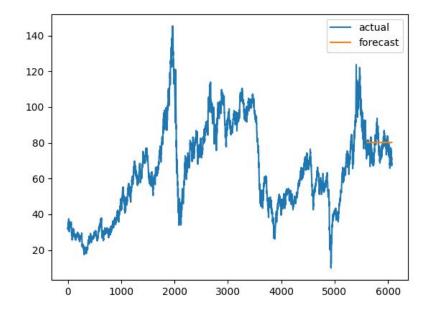


10 years of data has the best residual mean squared error, but not by a lot compared to the other subset sizes

#### Only MA parameter: full data

SARIMAY Regulte

SARIIVIAX RESUITS
Dep. Variable: adj_close No. Observations: 5612 Model: ARIMA(0, 1, 4) Log Likelihood -10392.232 Date: Wed, 07 May 2025 AIC 20794.464 Time: 11:57:31 BIC 20827.626 Sample: 0 HQIC 20806.019 -5612 Covariance Type: opg
=======================================
=======
coef std err z P> z  [0.025 0.975]
ma.L1     -0.0351     0.007     -5.349     0.000     -0.048     -0.022       ma.L2     -0.0276     0.008     -3.292     0.001     -0.044     -0.011       ma.L3     -0.0095     0.008     -1.221     0.222     -0.025     0.006       ma.L4     0.0354     0.007     4.771     0.000     0.021     0.050       sigma2     2.3782     0.019     126.466     0.000     2.341     2.415
Ljung-Box (L1) (Q): 0.01 Jarque-Bera (JB): 21555.56  Prob(Q): 0.91 Prob(JB): 0.00  Heteroskedasticity (H): 2.56 Skew: -0.36  Prob(H) (two-sided): 0.00 Kurtosis: 12.57
=======================================
Warnings:



Warnings

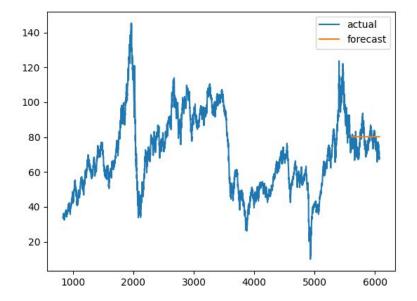
[1] Covariance matrix calculated using the outer product of gradients (complex-step).

mean absolute error: 5.051131467142908 mean absolute percentage error: 0.06750569786973963 residual mean squared error: 6.1861542860250305

#### Only MA parameter: 20 years

CADIMAN Describe

SARIMAX Results
Dep. Variable:       adj_close       No. Observations:       4776         Model:       ARIMA(0, 1, 4)       Log Likelihood       -9151.334         Date:       Wed, 07 May 2025       AIC       18312.668         Time:       11:57:55       BIC       18345.024         Sample:       0 HQIC       18324.035         - 4776
Covariance Type: opg
=======
coef std err z P> z  [0.025 0.975]
ma.L1     -0.0366     0.007     -4.909     0.000     -0.051     -0.022       ma.L2     -0.0275     0.010     -2.882     0.004     -0.046     -0.009       ma.L3     -0.0112     0.009     -1.270     0.204     -0.028     0.006       ma.L4     0.0370     0.008     4.377     0.000     0.020     0.054       sigma2     2.7051     0.024     110.457     0.000     2.657     2.753
Ljung-Box (L1) (Q): 0.01 Jarque-Bera (JB): 14072.90 Prob(Q): 0.91 Prob(JB): 0.00 Heteroskedasticity (H): 0.90 Skew: -0.34 Prob(H) (two-sided): 0.03 Kurtosis: 11.38
Warnings:

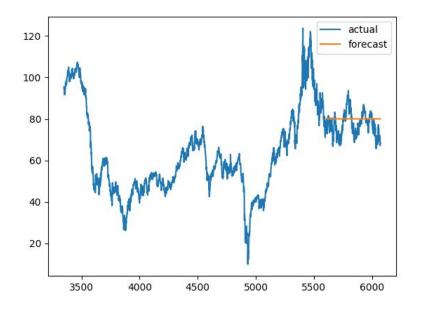


[1] Covariance matrix calculated using the outer product of gradients (complex-step).

mean absolute error: 5.0501230318969075 mean absolute percentage error: 0.06749105979849974 residual mean squared error: 6.185020670714847

#### Only MA parameter: 10 years

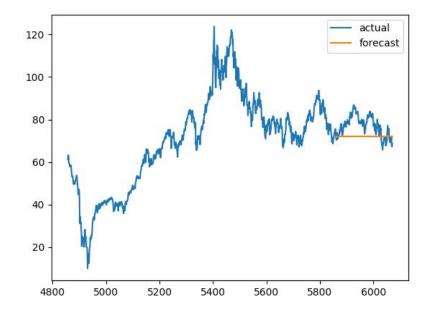
SARIMAX Results			
Dep. Variable: adj_close No. Observations: 2263 Model: ARIMA(0, 1, 6) Log Likelihood -4238.989 Date: Wed, 07 May 2025 AIC 8491.977 Time: 11:58:13 BIC 8532.045 Sample: 0 HQIC 8506.598 - 2263 Covariance Type: opg			
coef std err z P> z  [0.025 0.975]			
ma.L1       -0.0289       0.014       -2.140       0.032       -0.055       -0.002         ma.L2       -0.0208       0.013       -1.596       0.110       -0.046       0.005         ma.L3       -0.0278       0.012       -2.323       0.020       -0.051       -0.004         ma.L4       0.0375       0.013       2.847       0.004       0.012       0.063         ma.L5       -0.0242       0.012       -1.945       0.052       -0.049       0.000         ma.L6       -0.0475       0.013       -3.599       0.000       -0.073       -0.022         sigma2       2.4846       0.036       69.139       0.000       2.414       2.555			
=====  Ljung-Box (L1) (Q): 0.00 Jarque-Bera (JB): 6519.61  Prob(Q): 0.97 Prob(JB): 0.00  Heteroskedasticity (H): 2.82 Skew: -0.82  Prob(H) (two-sided): 0.00 Kurtosis: 11.15			
Warnings: [1] Covariance matrix calculated using the outer product of gradients (complex-step).			



mean absolute error: 4.9834340267599435 mean absolute percentage error: 0.06651331369804896 residual mean squared error: 6.107166987098335

#### Only MA parameter: 5 years

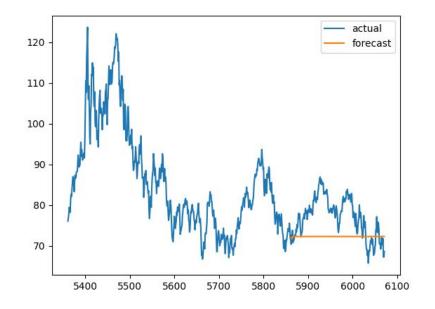
SARIMAX Results
Dep. Variable: adj_close No. Observations: 1005 Model: ARIMA(0, 1, 6) Log Likelihood -2136.603 Date: Wed, 07 May 2025 AIC 4287.207 Time: 11:58:23 BIC 4321.589 Sample: 0 HQIC 4300.272 -1005 Covariance Type: opg
coef std err z P> z  [0.025 0.975]
ma.L1
Ljung-Box (L1) (Q):  0.00 Jarque-Bera (JB):  1246.01  Prob(Q):  0.97 Prob(JB):  0.00  Heteroskedasticity (H):  1.20 Skew:  -0.85  Prob(H) (two-sided):  8.19



mean absolute error: 5.743960011185369 mean absolute percentage error: 0.07224958389212578 residual mean squared error: 6.84883727234121

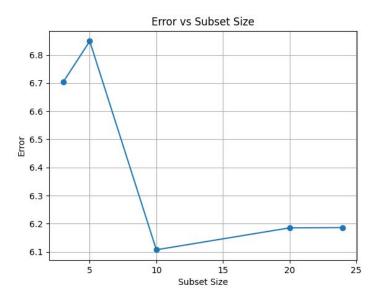
#### Only MA parameter: 3 years

SARIMAX Results				
	501 -1144.102 2302.204 31.706 13.781			
coef std err z P> z  [0.025 0				
ma.L1         0.0346         0.041         0.845         0.398         -0.0           ma.L2         -0.0818         0.035         -2.359         0.018         -0.1           ma.L3         -0.0869         0.030         -2.940         0.003         -0.1           ma.L4         0.0478         0.037         1.286         0.198         -0.0           ma.L5         -0.0712         0.032         -2.246         0.025         -0.1           ma.L6         -0.1373         0.037         -3.677         0.000         -0.2           sigma2         5.6868         0.264         21.503         0.000         5.	150 -0.014 145 -0.029 025 0.121 133 -0.009			
Ljung-Box (L1) (Q): 0.00 Jarque-Bera (JB): 194.29  Prob(Q): 0.96 Prob(JB): 0.00  Heteroskedasticity (H): 0.24 Skew: -0.46  Prob(H) (two-sided): 5.91				
Warnings: [1] Covariance matrix calculated using the outer product of gradients (complex-step).				



mean absolute error: 5.616602821308457 mean absolute percentage error: 0.07068858609460543 residual mean squared error: 6.702965772731589

# Only MA parameters: error vs subset size



10 years of data has the best residual mean squared error, but not by a lot compared to the other subset sizes

#### Takeaways

- 10 years of data is the sweet spot for not having too much or too little data (based on the errors)
- Manually chosen parameters performed as well, if not better, than the auto-generated parameters.
- Predictions tend to be linear.