Common discrete distributions; descriptions, pmfs, means and variances and mgfs.

The Bernoulli(p).
$X = 1$ if a success occurs, $X = 0$ is a failure occurs.
$p(x) = p^x(1-p)^{1-x}$ for $x = 0, 1$.
$E(X) = p$, $Var(X) = p(1-p)$.
$M(s) = 1 - p + pe^s$ for all real s.

The binomial(n, p).
X counts the number of successes in n independent Berоulli(p) trials.
$p(x) = \binom{n}{x}p^x(1-p)^{n-x}$ for $x = 0, 1, \ldots, n$.
$E(X) = np$, $Var(X) = np(1-p)$.
$M(s) = (1 - p + pe^s)^n$ for all real s.
NB: if $X_1, X_2, \ldots, X_n$ are independent Bernoulli(p) random variables, then $X = \sum_{i=1}^n X_i$ has a binomial(n, p) distribution.

The geometric(p).
X returns the trial of the first success in repeated independent Bernoulli(p) trials.
$p(x) = p(1-p)^{x-1}$ for $x = 1, 2, 3, \ldots,$
$E(X) = 1/p$, $Var(X) = (1-p)/p^2$.
$M(s) = \frac{pe^s}{1-(1-p)e^s}$ for all real s.

The negative binomial(r, p) a.k.a. Pascal(r, p).
X returns the trial of the r-th success in repeated independent Bernoulli(p) trials.
$p(x) = \binom{x-1}{r-1}p^r(1-p)^{x-r}$ for $x = r, r+1, r+2, \ldots,$
$E(X) = r/p$, $Var(X) = r(1-p)/p^2$. $M(s) = \left(\frac{pe^s}{1-(1-p)e^s}\right)^r$ for all real s.
NB: If $X_1, X_2, \ldots, X_r$ are independent geometric(p) random variables, then $X = \sum_{i=1}^r X_i$ has a neg.binom(r, p) distribution.

The Poisson(λ).
X counts the number of events that happen in a fixed amount of time, where we expect λ events to happen in the amount of time.
$p(x) = e^{-\lambda}\lambda^x/x!$ for $x = 0, 1, 2, \ldots,$
$E(X) = \lambda$, $Var(X) = \lambda$.
$M(s) = e^{\lambda(e^s-1)}$ for all real s.

The hypergeometric(n, M, N).
X counts the number of successes in a random sample of size n drawn (without replacement) from a population of N objects of which M are successes, and therefore, $N - M$ are failures.
$p(x) = \binom{M}{x}\binom{N-M}{n-x}/\binom{N}{n}$ for $x = 0, 1, \ldots, n$, $x \le M$ and $n - x \le N - M$,
$E(X) = nM/N$, $Var(X) = n\frac{M}{N}(1 - \frac{M}{N})(\frac{N-n}{N-1})$. NB: in sampling without replacement if we let $X_i = 1$ if we draw a success and $X_i = 0$ if we draw a failure, then $X = \sum_{i=1}^n X_i$ has the hypergeometric(n, M, N) distribution...notice that unlike the binomial($n, M/N$) distribution these Bernoulli(M/N)'s are dependent.

Next page for continuous distributions

Common continuous distrib.; descriptions, pdfs, means and variances and mgfs.

The uniform(a, b)**.**
(a, b) represents a (finite) interval of the real line with $a < b$; this distribution is the generalization of equally-likely outcomes to a continuous interval of values.
$f(x) = \begin{cases} \frac{1}{b-a} & \text{if } a \le x \le b \\ 0 & \text{elsewhere} \end{cases}.$
$E(X) = \frac{b+a}{2}$, $Var(X) = \frac{(b-a)^2}{12}$.
$M(s) = \frac{e^{sb} - e^{sa}}{(b-a)s}$ for all real s.

The exponential(λ)**.**
$\lambda > 0$ represents a positive real parameter which is the reciprocal of the mean of X.
$f(x) = \begin{cases} \lambda e^{-\lambda x} & \text{if } x > 0 \\ 0 & \text{elsewhere} \end{cases}.$
$E(X) = \frac{1}{\lambda}$, $Var(X) = \frac{1}{\lambda^2}$.
$M(s) = \frac{\lambda}{\lambda - s} = (1 - \frac{s}{\lambda})^{-1}$ for $s < \lambda$.

The Gamma(α, β)**.**
$\alpha > 0$ represents the shape parameter, $\beta > 0$ is the scale parameter.
$f(x) = \begin{cases} \frac{x^{\alpha-1} e^{-x/\beta}}{\beta^\alpha \Gamma(\alpha)} & \text{if } x > 0 \\ 0 & \text{elsewhere} \end{cases}$, where $\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x}\, dx$ is the Euler Gamma defined for $\alpha > 0$.
$E(X) = \alpha\beta$, $Var(X) = \alpha\beta^2$.
$M(s) = (1 - \beta s)^{-\alpha}$ for $s < \frac{1}{\beta}$.

The Normal(μ, σ^2)**.**
μ is a parameter that represents the mean, $\sigma^2 > 0$ is a parameter that represents the variance.
$f(x) = \frac{e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2}}{\sigma\sqrt{2\pi}}$ for $-\infty < x < \infty$.
$E(X) = \mu$, $Var(X) = \sigma^2$.
$M(s) = e^{\mu s + \frac{\sigma^2 s^2}{2}}$ for all real s.
NB: When $\mu = 0$ and $\sigma^2 = 1$ we call this the **standard normal** distribution.

The χ^2_ν **also called the chi-square distribution with** ν **degrees of freedom.**
$\nu \ge 1$ represents the degrees of freedom.
$f(x) = \begin{cases} \frac{x^{\frac{\nu}{2}-1} e^{-x/2}}{2^{\frac{\nu}{2}} \Gamma(\frac{\nu}{2})} & \text{if } x > 0 \\ 0 & \text{elsewhere} \end{cases}.$
$E(X) = \nu$, $Var(X) = 2\nu$.
$M(s) = (1 - 2s)^{-\frac{\nu}{2}}$ for $x < \frac{1}{2}$.
NB: The χ^2_ν is just the Gamma(α, β) with $\alpha = \nu/2$ and $\beta = 2$.