# Module 1 – Linear Systems and Span
# Topic 1 – Systems of Linear Equations
# Lesson 1 – Solution Sets of Linear Equations

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## 1 Topics

We will explore the following concepts:

- Systems of Linear Equations
- Elementary Row Operations

## 2 Learning Objectives:

Students should be able to do the following after watching the video and completing the assigned homework:

- Apply elementary row operations to solve systems of linear equations.

## 3 A Single Linear Equation

A linear equation has the form

$$a_1x_1 + a_2x_2 + \cdots + a_nx_n = b$$

$a_1$, $a_2$, ..., $a_n$ and $b$ are the **coefficients**, $x_1$, $x_2$, ..., and $x_n$ are the **variables**, and $n$ is the **dimension**, or number of variables.
For example:

- $2x_1 + 4x_2 = 4$ is a line in 2 dimensions
- $3x_1 + 2x_2 + x_3 = 6$ is a plane in 3 dimensions

# 4 Systems of Linear Equations

When we have one or more linear equations, we have a **Linear System** of equations.
For example, a linear system with two equations is

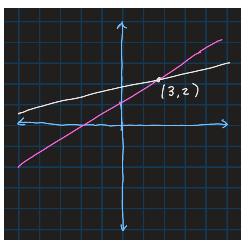
$$\begin{aligned} x_1 + 1.5x_2 + \pi x_3 &= 4 \\ 5x_1 \qquad\quad + 7x_3 &= 5 \end{aligned}$$

The set of all possible values of $x_1, x_2, \ldots, x_n$ that satisfy all equations is the **solution set**. One point in the solution set is a **solution**.

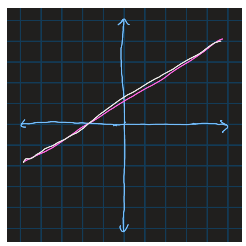
# 5 Two Variable Case

Consider the following systems. How are they different from each other?

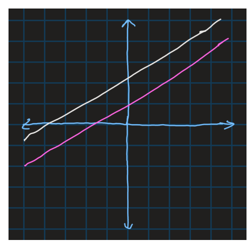
$$\begin{aligned} x_1 - 2x_2 &= -1 \\ -x_1 + 3x_2 &= 3 \end{aligned} \qquad \begin{aligned} x_1 - 2x_2 &= -1 \\ -x_1 + 2x_2 &= 1 \end{aligned} \qquad \begin{aligned} x_1 - 2x_2 &= -1 \\ -x_1 + 2x_2 &= 3 \end{aligned}$$




Non-Parallel Lines
Unique Solution



Identical Lines
Infinite Solutions

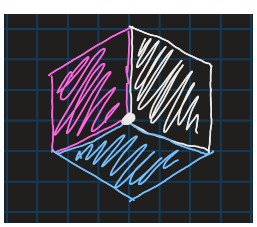


Parallel Lines
No Solutions

# 6 Three Variable Case

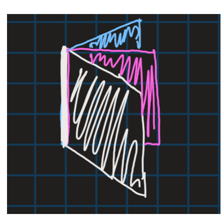
An equation $a_1x_1 + a_2x_2 + a_3x_3 = b$ defines a plane in $\mathbb{R}$. The **solution** to a system of **three equations** is the set of points where all three planes intersect:

Planes Intersect At a Point



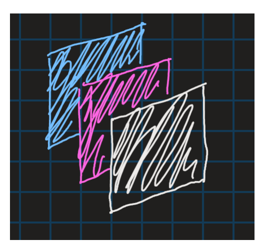
Unique Solution

Planes Intersect On a Line



Infinite Solutions

Parallel Planes



No Solutions

# 7 Row Reduction by Elementary Row Operations

How can we find the solution set to a set of linear equations?
We can manipulate equations in a linear system using **row operations**:

1. (Replacement/Addition) Add a multiple of one row to another
2. (Interchange) Interchange/swap two rows
3. (Scaling) Multiple a row by a non-zero scalar

Let's use these operations to solve a system of equations:

# 8 Summary

# 9 Practice 1

# 10 Practice 2