POLYNOMIAL TRICK PROOF

1) PERSON CHOOSES A POLYNOMIAL OF ANY DEGREE WITH ALL POSITIVE INTEGER COEFFICIENTS:

WE DON'T KNOW ANYTHING ABOUT IT.

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x^1 + a_0 x^0 \quad \text{WHERE } a_n, a_{n-1}, \ldots, a_1, a_0 \in \mathbb{Z}^+ \text{ AND } n \geq 0$$

2) ASK PERSON FOR VALUE OF $f(1)$

$$f(1) = a_n \cdot 1^n + a_{n-1} \cdot 1^{n-1} + \cdots + a_1 1^1 + a_0 1^0$$
$$f(1) = a_n \cdot 1 + a_{n-1} \cdot 1 + \cdots + a_1 \cdot 1 + a_0 \cdot 1$$
$$f(1) = a_n + a_{n-1} + \cdots + a_1 + a_0$$

SO $f(1)+1 = a_n + a_{n-1} + \cdots + a_1 + a_0 + 1$, WHICH WE CAN GUARANTEE IS LARGER THAN ANY INDIVIDUAL COEFFICIENT AS THEY ARE ALL POSITIVE

3) ASK PERSON FOR VALUE OF $f(f(1)+1)$

$$f(f(1)+1) = a_n(a_n + a_{n-1} + \cdots + a_1 + a_0 + 1)^n + a_{n-1}(a_n + a_{n-1} + \cdots + a_1 + a_0 + 1)^{n-1} + \cdots + a_1(a_n + a_{n-1} + \cdots + a_1 + a_0 + 1)^1 + a_0(a_n + a_{n-1} + \cdots + a_1 + a_0 + 1)^0$$

FOR ANY POSITIVE INTEGER BASE b, WE CAN WRITE A BASE 10 (ANY NUMBER x IN) REPRESENTATION AS:

$$x = x_n \cdot b^n + x_{n-1} b^{n-1} + \cdots + x_1 b^1 + x_0 b^0$$

WE CAN SEE THAT OUR VALUE FOR $f(f(1)+1)$ IS THE BASE 10 REPRESENTATION OF A NUMBER IN BASE $(f(1)+1) = a_n + a_{n-1} + \cdots + a_1 + a_0 + 1$, SPECIFICALLY $a_n a_{n-1} \ldots a_1 a_0 {}_{a_n + a_{n-1} + \cdots + a_1 + a_0 + 1}$

TO DETERMINE THE COEFFICIENTS WE FIRST CHECK THE LARGEST INTEGER POWER ~~THAT~~ OF $a_n + a_{n-1} + \cdots + a_1 + a_0 + 1$ THAT $f(f(1)+1)$ IS GREATER THAN.

LOOKING AT THE BREAKDOWN OF $f(f(1)+1)$ IN ADDITION TO KNOWING THAT FOR ANY POSITIVE INTEGER x, $x^n > (x-1) \cdot \sum_{a=0}^{n-1} x^a$, WE CAN SEE THAT THE LARGEST POWER IS n, AS $f(f(1)+1)$ HAS A $a_n(a_n + a_{n-1} + \cdots + a_1 + a_0 + 1)^n$ TERM. THIS ALSO TELLS US THAT $f(x)$ IS A POLYNOMIAL OF DEGREE n.

NOW WE DETERMINE THE LEADING COEFFICIENT BY DETERMINING THE NUMBER OF TIMES $(a_n + a_{n-1} + \cdots + a_1 + a_0 + 1)^n$ CAN BE SUBTRACTED FROM $f(f(1)+1)$ WHILE KEEPING THE RESULT POSITIVE. FOR THIS FUNCTION THAT WILL BE a_n TIMES.

$$\begin{aligned} f(f(1)+1) - a_n(a_n + a_{n-1} + \cdots + a_1 + a_0 + 1)^n &= a_{n-1}(a_n + a_{n-1} + \cdots + a_1 + a_0 + 1)^{n-1} \\ &+ \cdots + a_1(a_n + a_{n-1} + \cdots + a_1 + a_0 + 1)^1 \\ &+ a_0(a_n + a_{n-1} + \cdots + a_1 + a_0 + 1)^0 \end{aligned}$$

FOR SIMPLICITY:

$$f(f(1)+1) - a_n(f(1)+1)^n = a_{n-1}(f(1)+1)^{n-1} + \cdots + a_1(f(1)+1)^1 + a_0(f(1)+1)^0$$

REPEATING THAT STEP FOR EACH DECREASING ~~POWER~~ DEGREE IN THE FUNCTION WILL YIELD CONSECUTIVE COEFFICIENTS OF THE ORIGINAL FUNCTION, TELLING US THE VALUES $a_n, a_{n-1}, \ldots, a_1, a_0$ FOR THE ORIGINAL FUNCTION:

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x^1 + a_0 x^0$$