# Improving Irrigation for Kidney Stone Removal

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Industrially Focused Mathematical Modelling
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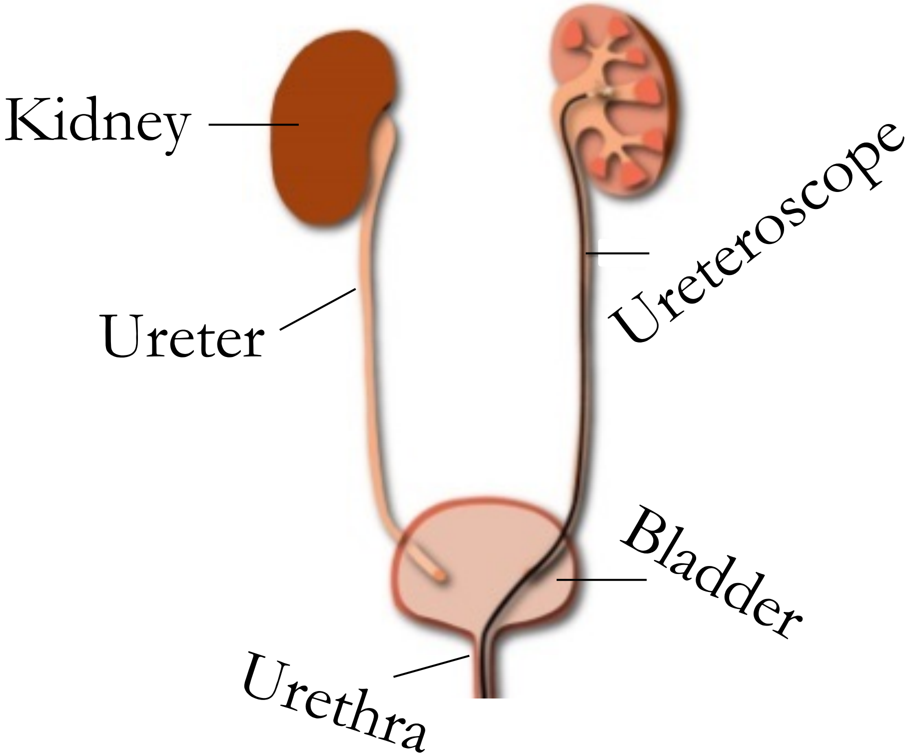
# Treatment Method



- **Ureteroscopy** - surgical procedure to remove kidney stones



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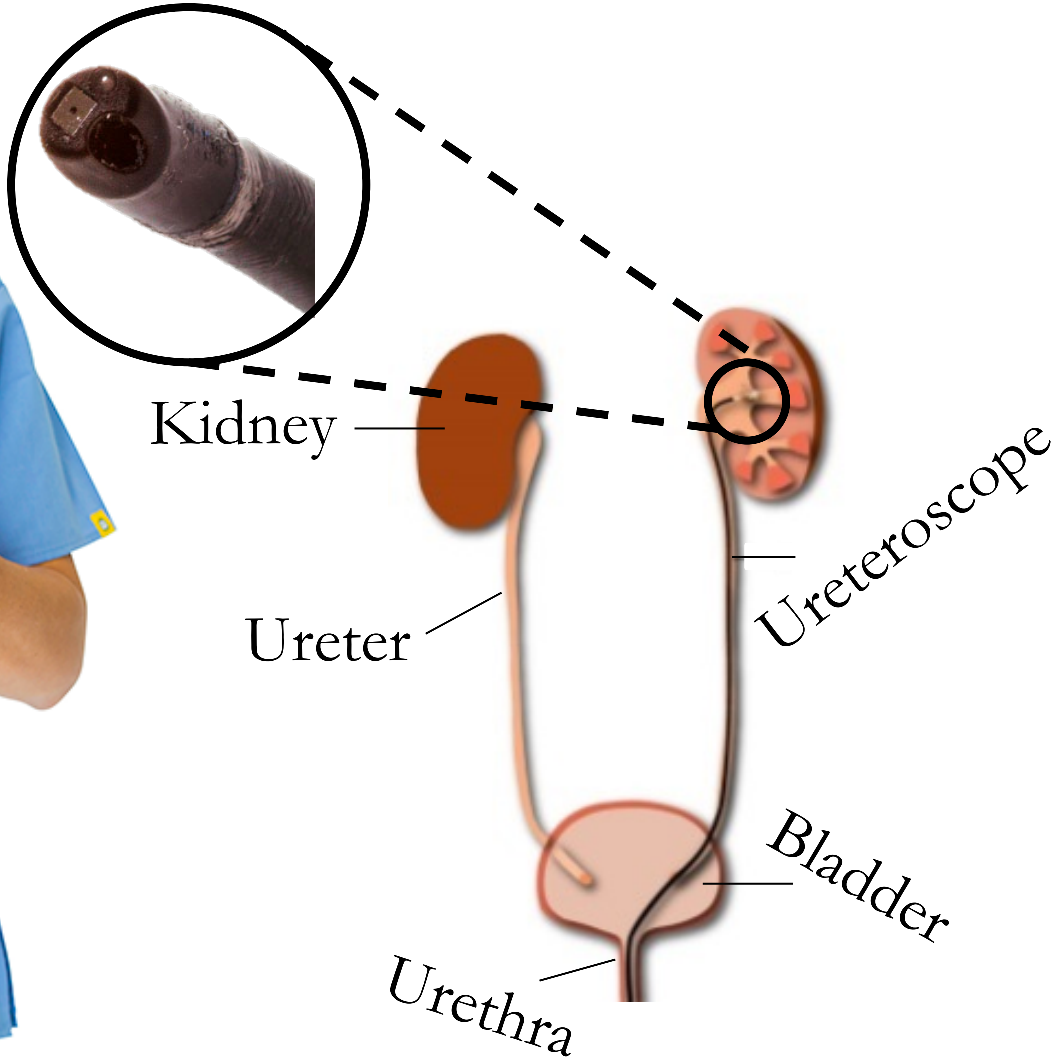
- **Ureteroscopy** - surgical procedure to remove kidney stones

# Irrigation Flow

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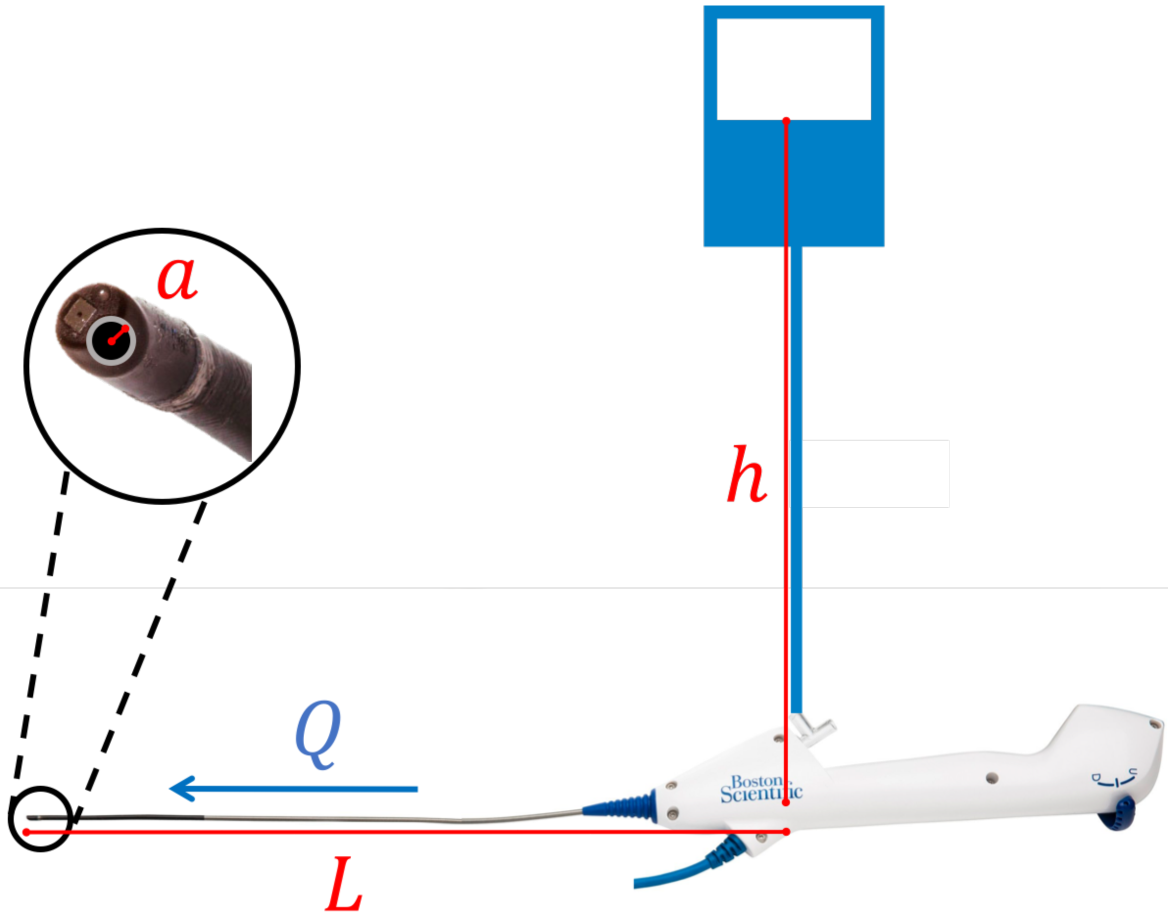
- **Irrigation** – opens up the ureter and improves visualisation

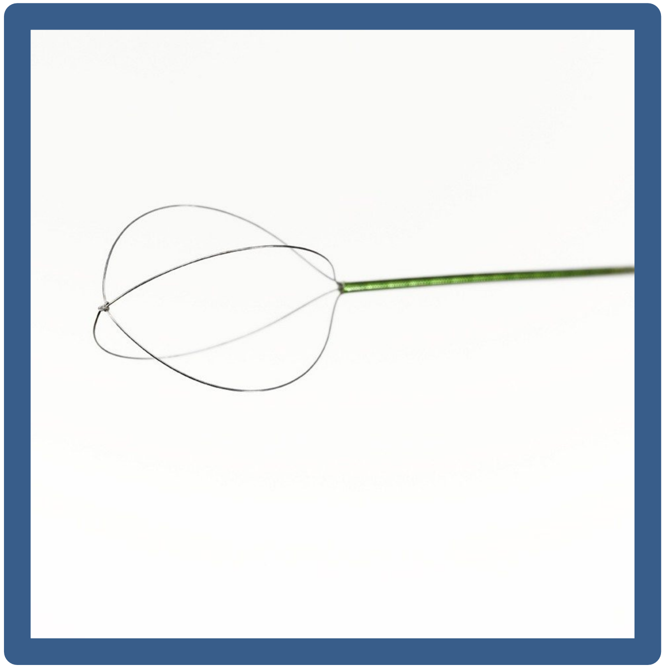


# Working Tools



- Working tools are inserted into the scope…

**Laser fibre**



**Stone basket**

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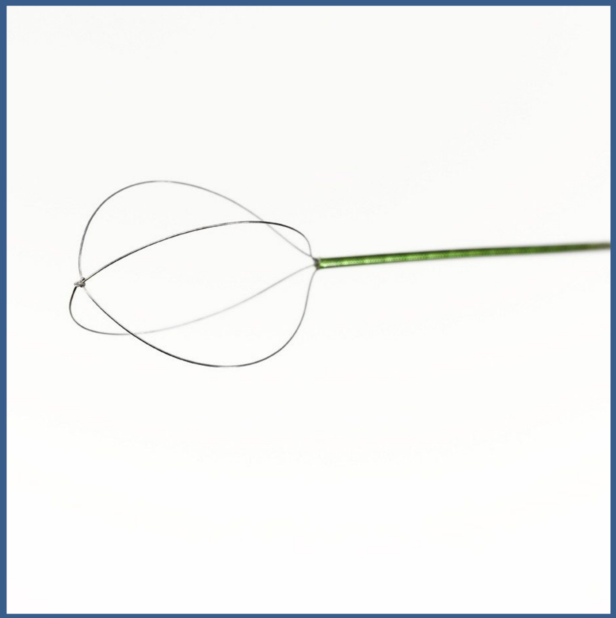


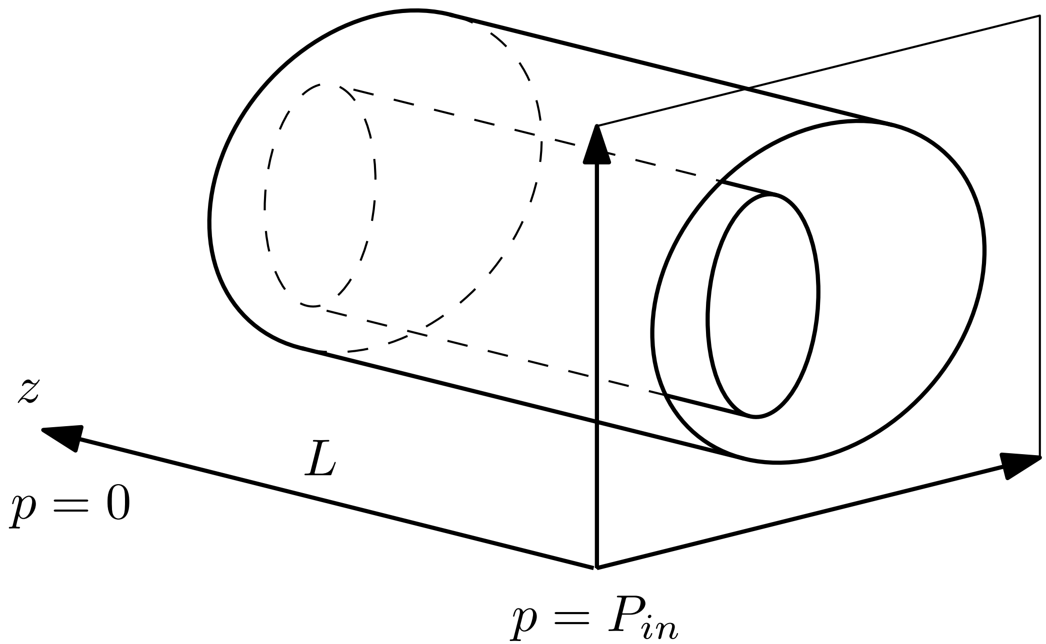
**Stone basket**

**Challenge:** Tools cause loss of irrigation

# Modelling Questions



1. Can we optimise irrigation flow by modifying the geometry?
    - Position of tool in channel
    - Cross-sectional shape of channel



# Modelling Questions

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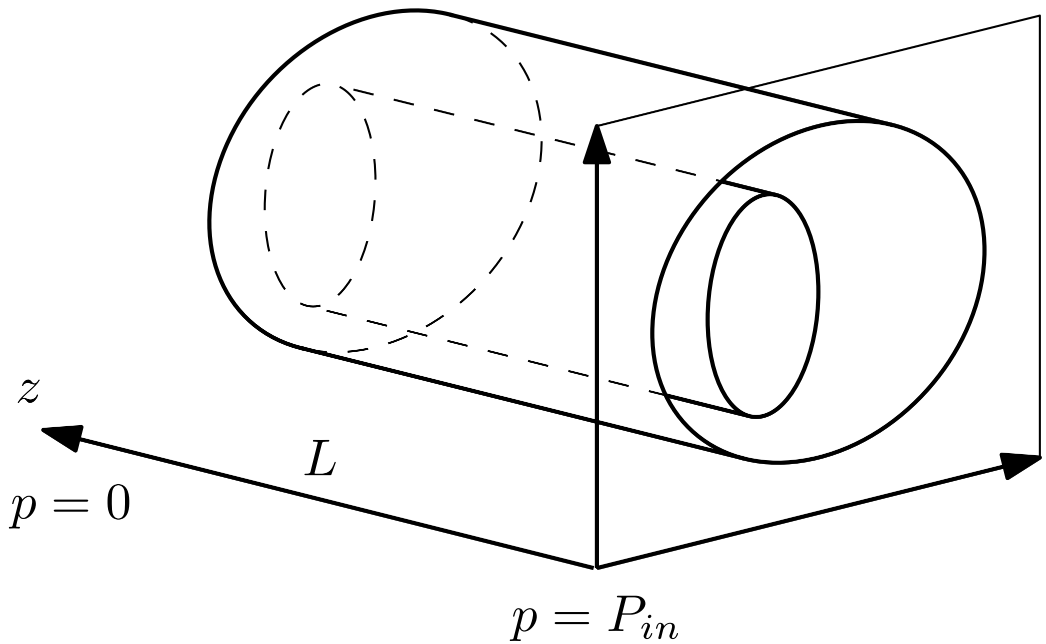
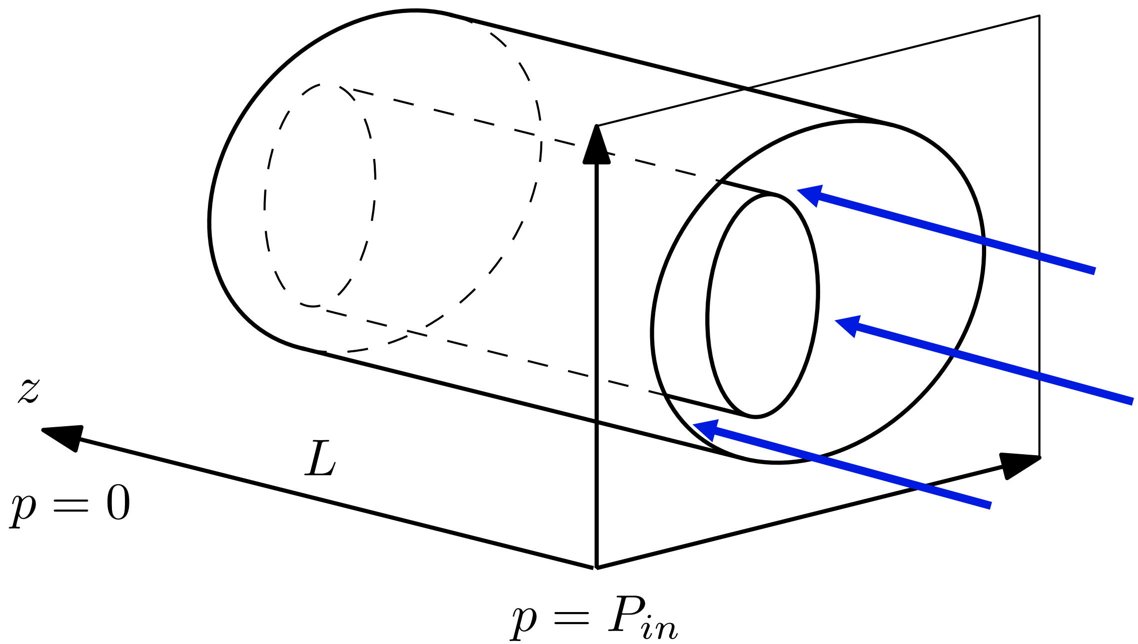
1. Can we optimise irrigation flow by modifying the geometry?
    - Position of tool in channel
    - Cross-sectional shape of channel

2. Where will a tool naturally sit in the channel?



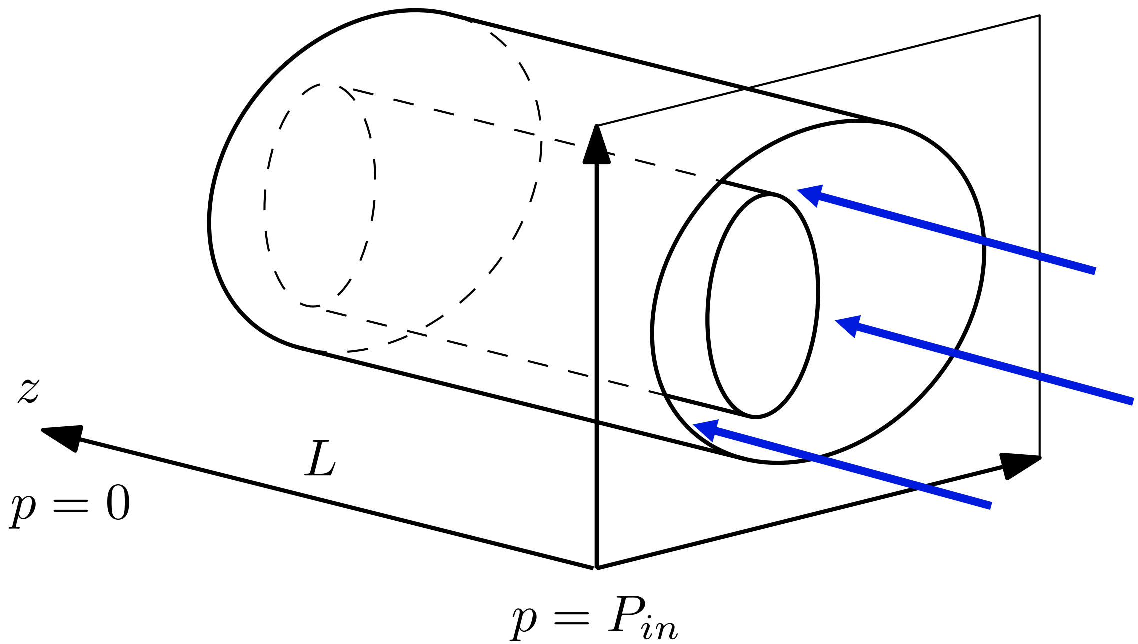
# Axial Flow

- Assumptions:
  - Long and thin



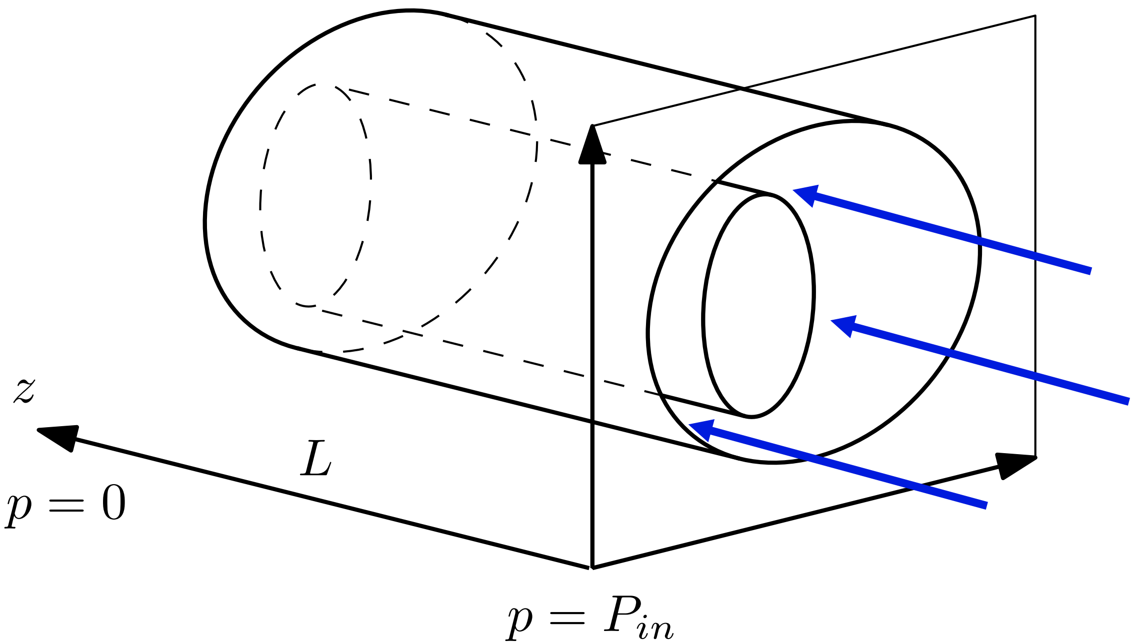
# Axial Flow

- Assumptions:
  - Long and thin
  - Co-axial tool and channel



# Axial Flow

- Assumptions:
  - Long and thin
  - Co-axial tool and channel
  - Uniform cross-sections

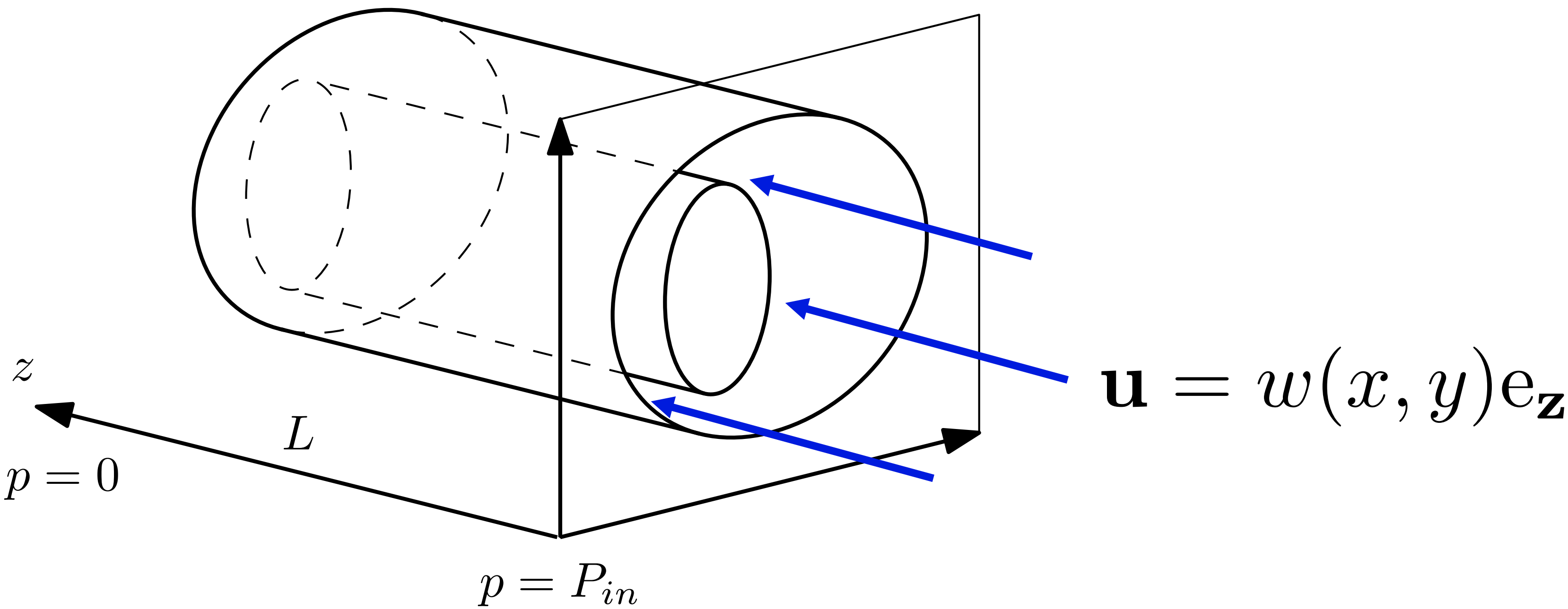


# Axial Flow Equation

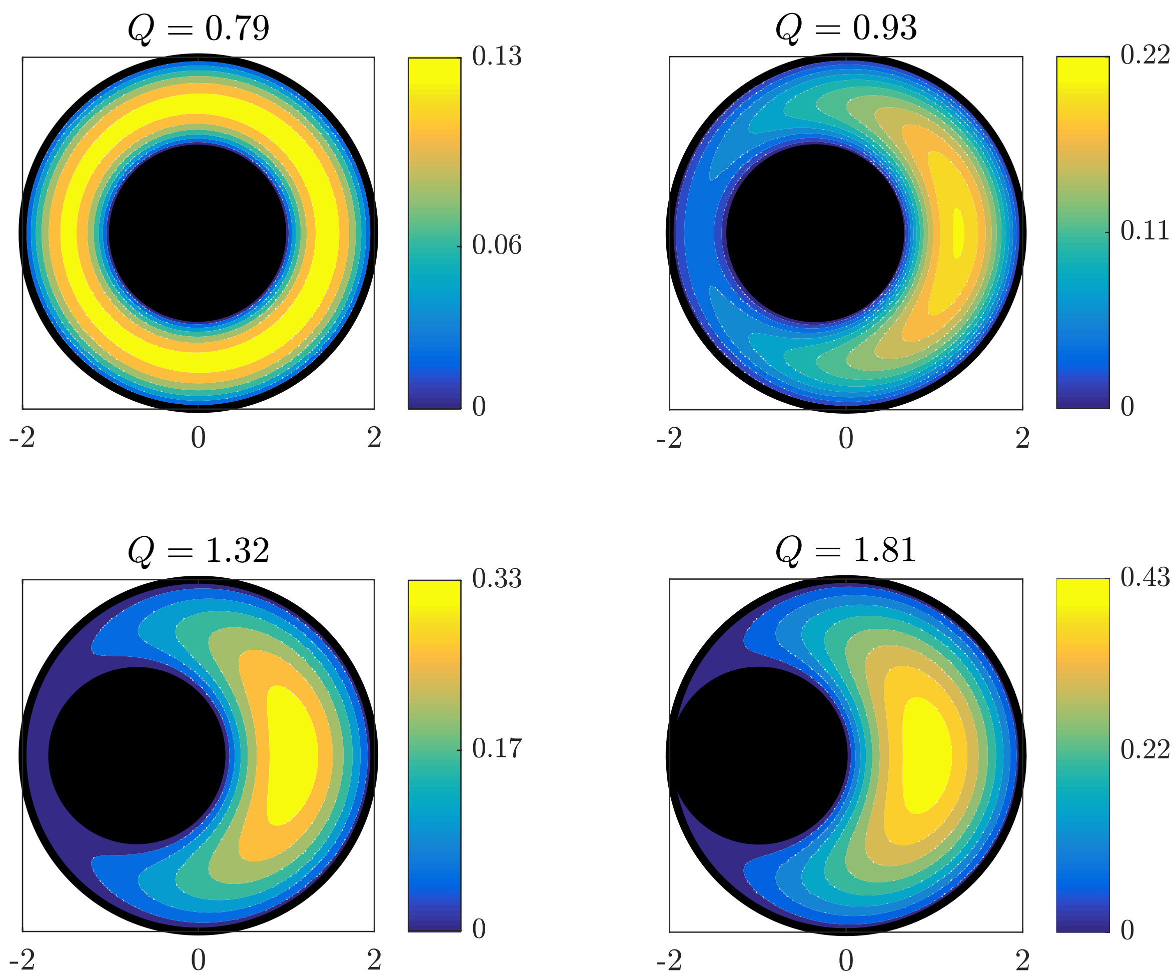




$$w_{xx} + w_{yy} = \frac{\mathrm{d}p}{\mathrm{d}z}$$



# Effect of Tool Position




- Moving tool from centre to edge minimises resistance

# Elliptical Cross-Sections

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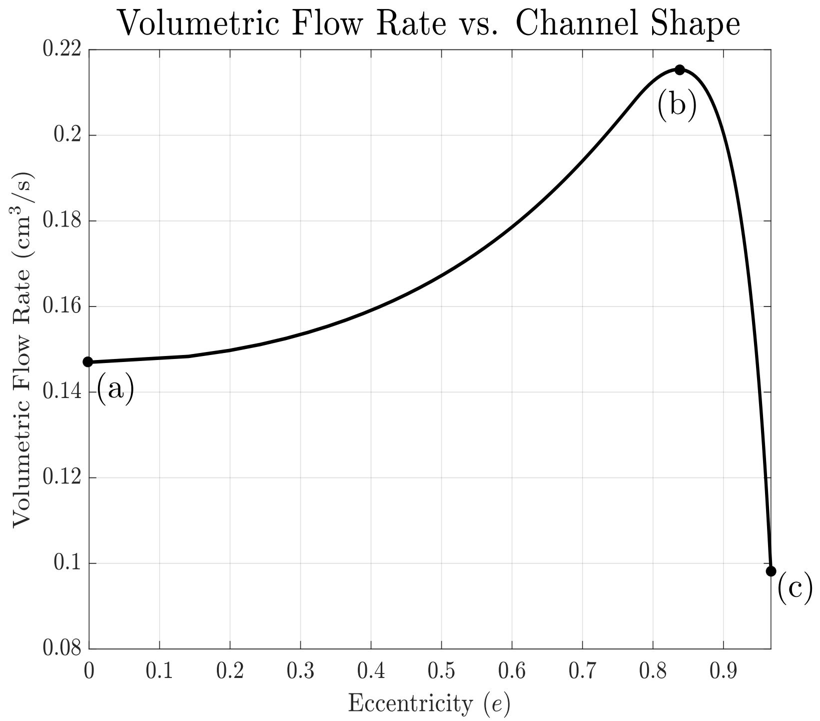
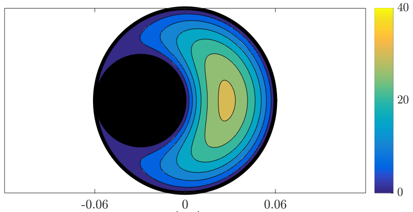
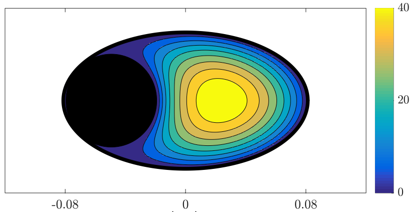
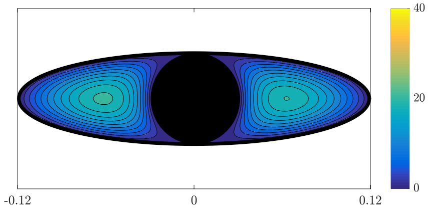


(a)

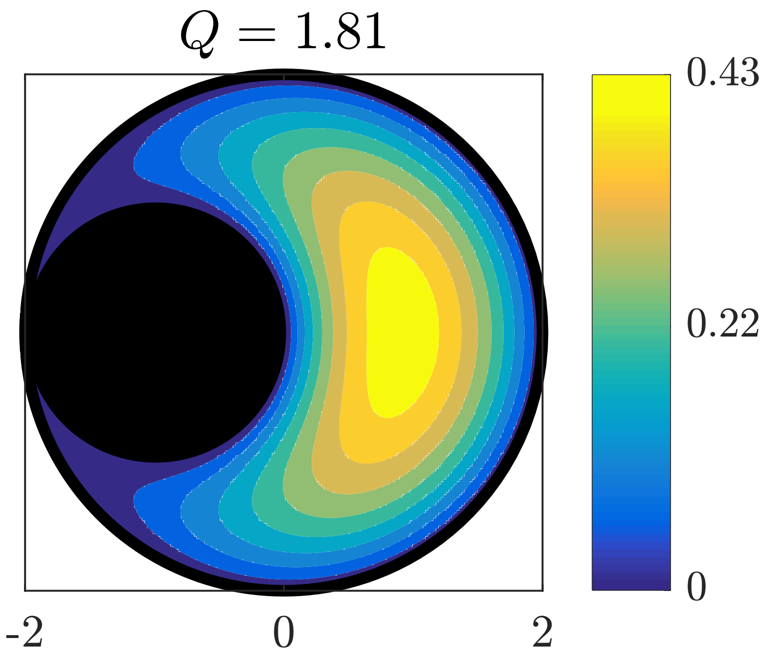


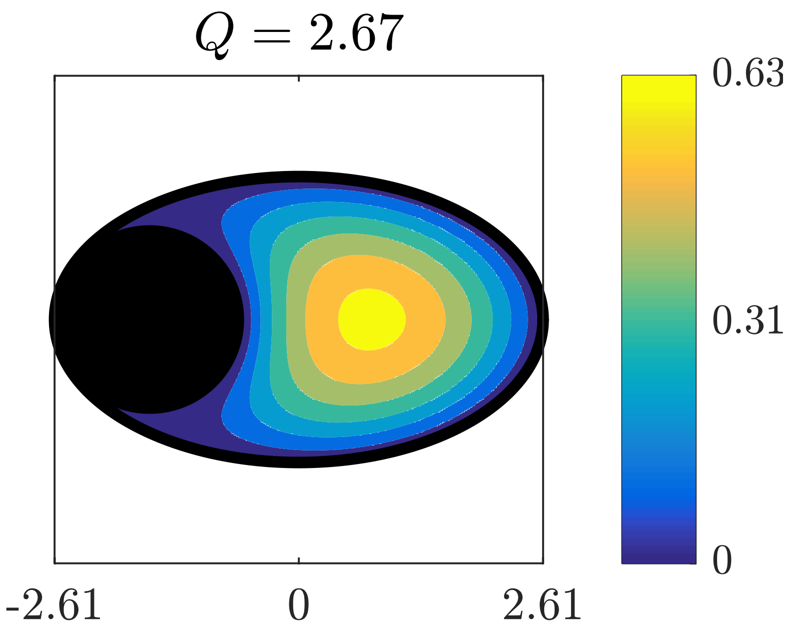
(b)



(c)

# Elliptical Cross-Sections




- 50% flow increase with elliptical channel
- Potential for design modifications
- This requires tool to be at the edge of the channel…

# Where will the tool naturally sit?

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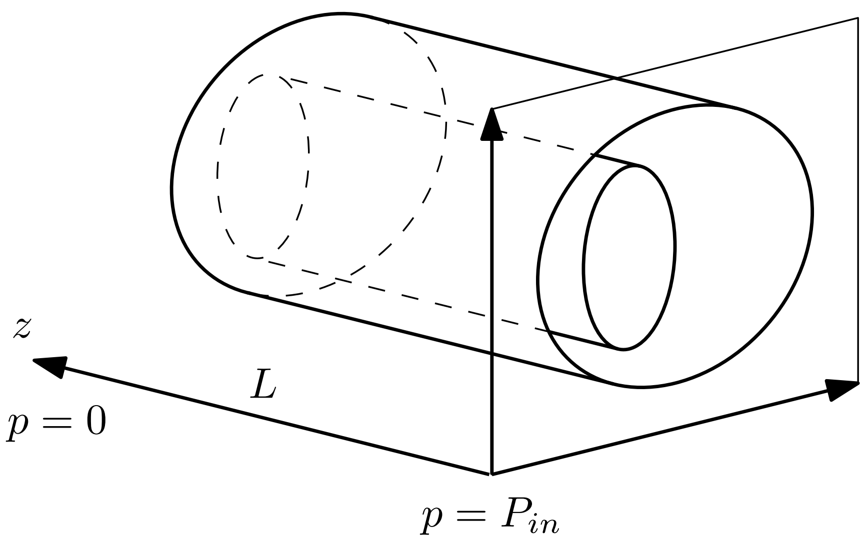
- Will the tool position itself so that the flow resistance is minimised?



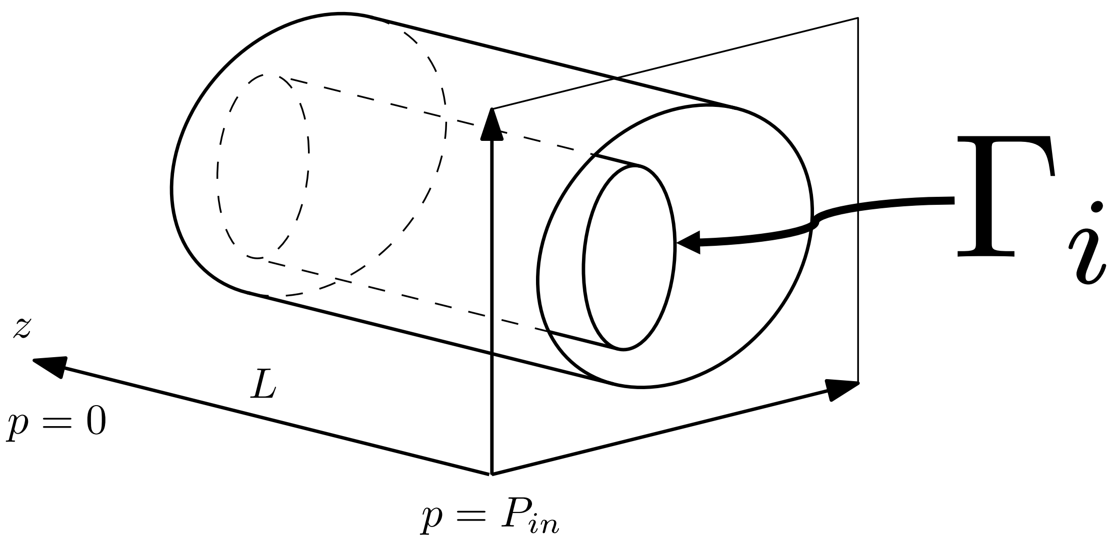
# Where will the tool naturally sit?



- Will the tool position itself so that the flow resistance is minimised?

$$\boldsymbol{F} = \oint_{\Gamma_i} \boldsymbol{\sigma} \cdot \boldsymbol{n}$$

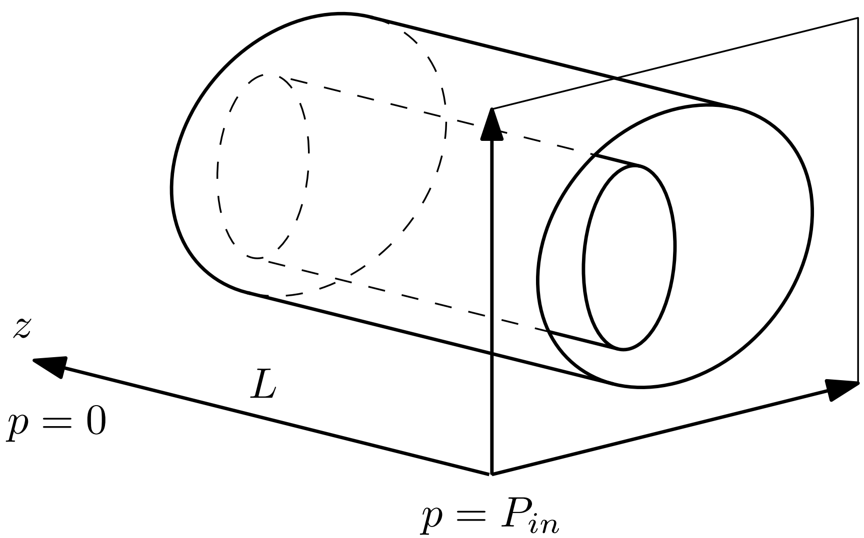
$$\boldsymbol{\sigma} = [-p\boldsymbol{I} + \mu(\nabla \boldsymbol{u} + \nabla \boldsymbol{u}^T)]$$



# Where will the tool naturally sit?

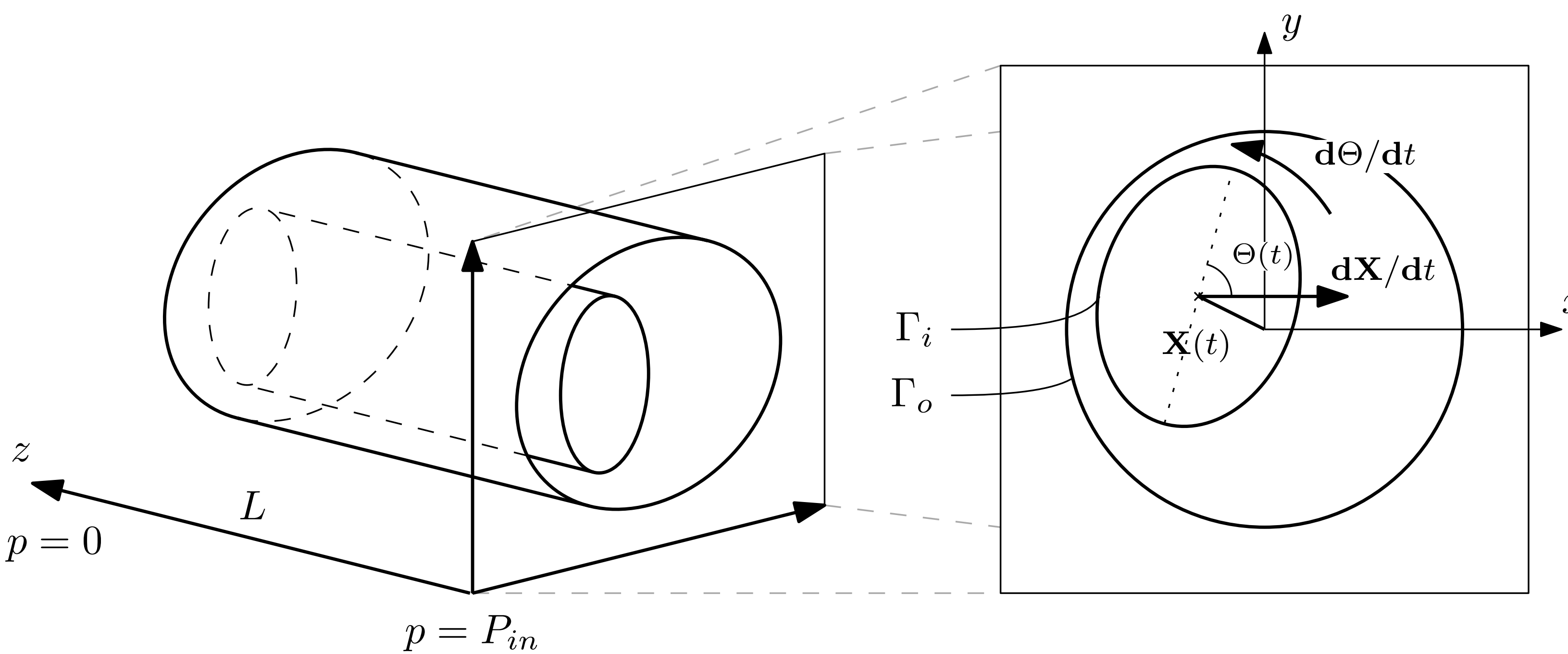


- Force on stationary tool is zero



# Provide instantaneous velocity

- Force on stationary tool is zero
- Perturb position, while keeping coaxial



# Problem decouples

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$$\hat{\boldsymbol{u}} = \hat{\boldsymbol{u}}_{\mathbf{0}} + \epsilon^2 \hat{\boldsymbol{u}}_{\mathbf{1}} + \cdots \,,$$

$$\hat{p} = \hat{p}_0 + \epsilon^2 \hat{p}_1 + \cdots \,,$$

$\hat{\nabla}_{\perp}^2 \hat{w}_0 = \hat{p}_{0\hat{z}}(\hat{z}),$ Unidirectional axial flow ✔

$$\left.\begin{aligned} \hat{\nabla}_{\perp} \cdot \hat{\boldsymbol{u}}_0^{\perp} &= 0, \\ \hat{\nabla}_{\perp}^2 \hat{u}_0 &= \hat{p}_{1\hat{x}}, \\ \hat{\nabla}_{\perp}^2 \hat{v}_0 &= \hat{p}_{1\hat{y}}, \end{aligned}\right\}$$

Stokes flow in the cross-section
driven by prescribed velocity

# Solve for trajectories

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$$\hat{\nabla}_{\perp}^{2}\hat{u}_0 = \hat{p}_{1\hat{x}},\qquad \hat{\nabla}_{\perp}^{2}\hat{v}_0 = \hat{p}_{1\hat{y}}$$

Solve for cross-sectional flows and pressure gradients (along with incompressibility)

$$\hat{\boldsymbol{F}} = \oint_{\hat{\Gamma}_i} \hat{\boldsymbol{\sigma}}\cdot\boldsymbol{n}\,\mathrm{d}s,$$

$$\hat{\tau} = \left(\oint_{\hat{\Gamma}_i} (\hat{\boldsymbol{x}} - \hat{\boldsymbol{X}}(\hat{t}) \wedge (\hat{\boldsymbol{\sigma}}\cdot\boldsymbol{n})\mathrm{d}s\right)\cdot\boldsymbol{k}$$

Compute forces and torque using viscous stress tensor

$$\mathrm{d}^2\hat{\boldsymbol{X}}/\mathrm{d}\hat{t}^2 = \alpha\hat{\boldsymbol{F}},\qquad \mathrm{d}^2\Theta/\mathrm{d}\hat{t}^2 = \alpha'\hat{\tau},$$

Drive motion of tool via Newton's equations

# Forces and Torques

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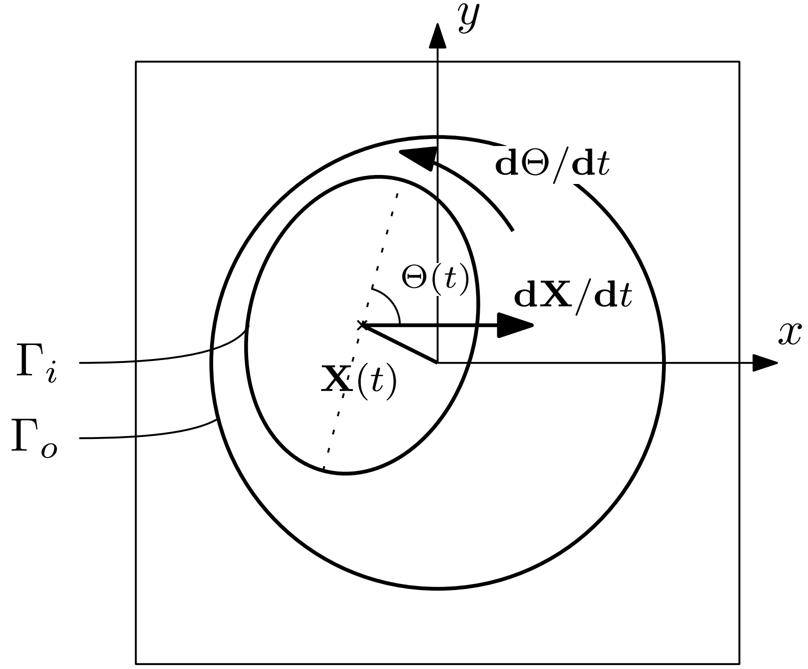
$$\begin{bmatrix} \hat{F}_{\hat{x}} \\ \hat{F}_{\hat{y}} \\ \hat{\tau}_{\hat{z}} \end{bmatrix} = - \begin{bmatrix} K_{\hat{x}\hat{x}} & K_{\hat{x}\hat{y}} & C_{\hat{x}} \\ K_{\hat{x}\hat{y}} & K_{\hat{y}\hat{y}} & C_{\hat{y}} \\ C_{\hat{x}} & C_{\hat{y}} & A_{\hat{z}\hat{z}} \end{bmatrix} \begin{bmatrix} \mathrm{d}\hat{X}/\mathrm{d}\hat{t} \\ \mathrm{d}\hat{Y}/\mathrm{d}\hat{t} \\ \mathrm{d}\Theta/\mathrm{d}\hat{t} \end{bmatrix}$$



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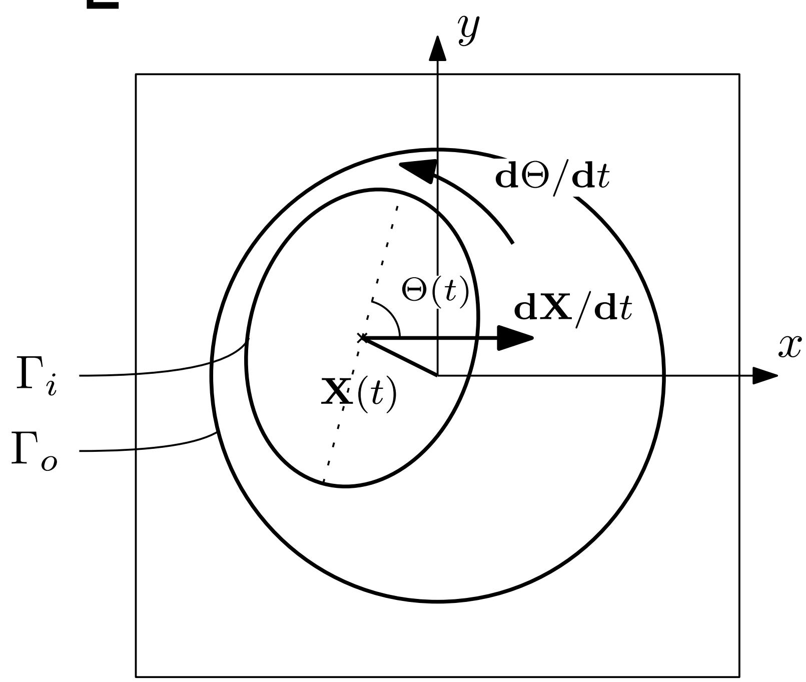
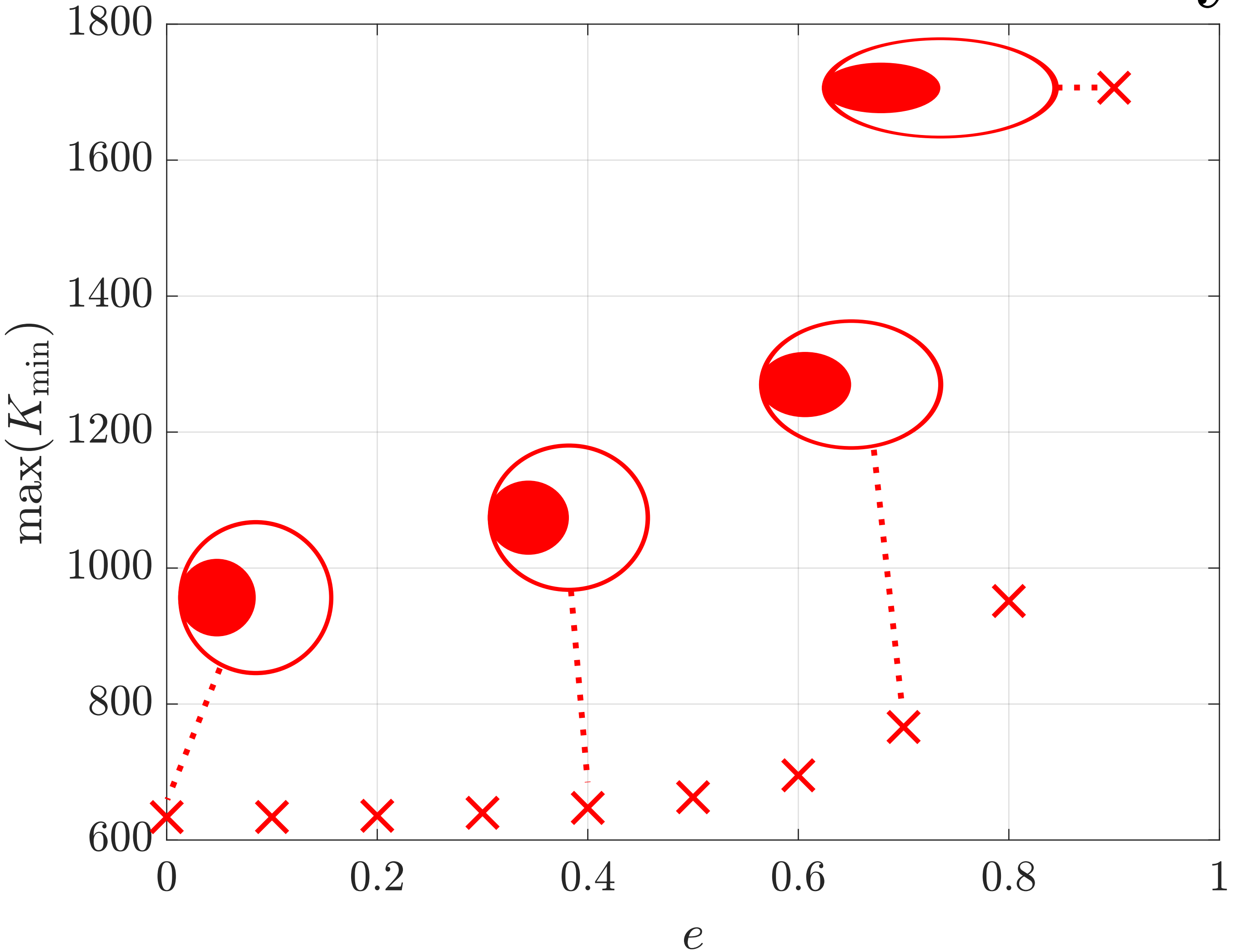


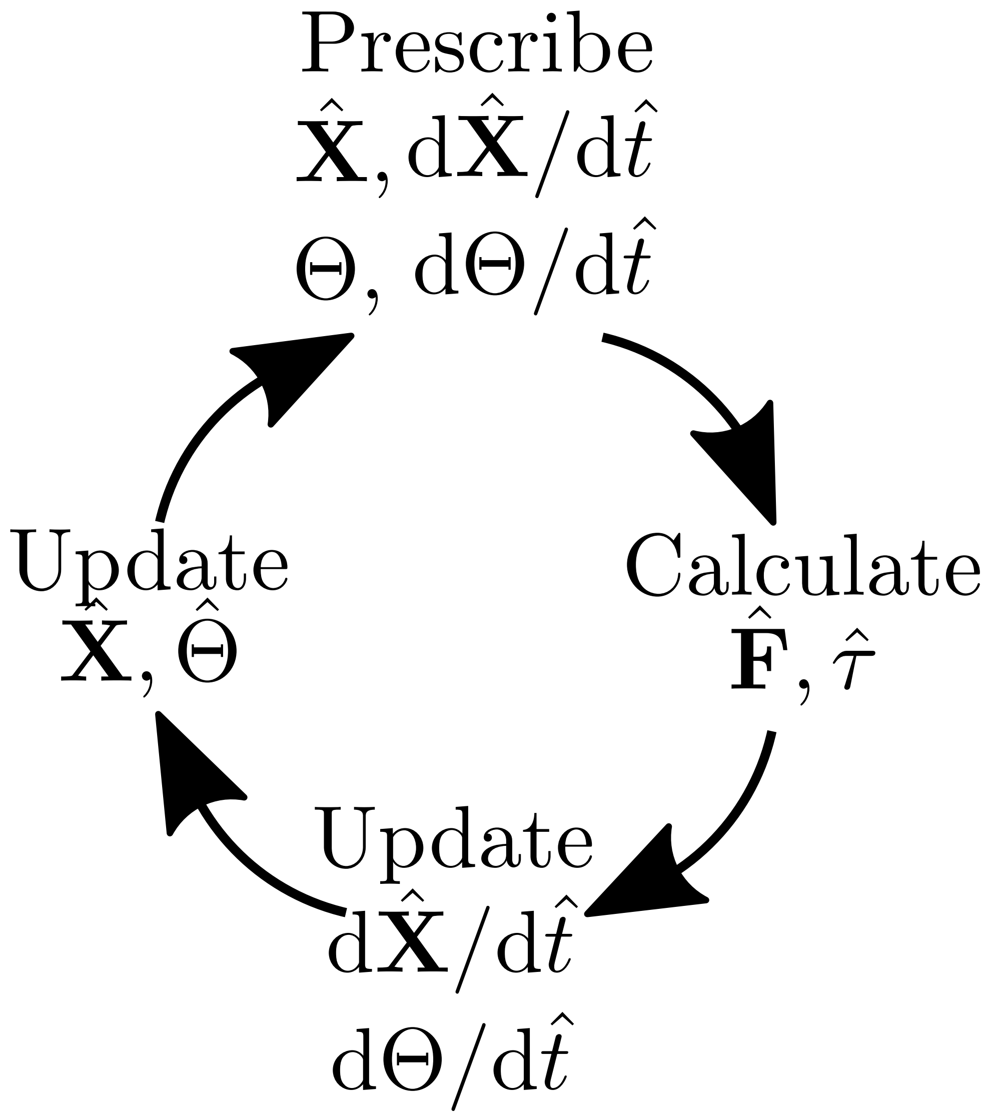
# Tool "Stability"

## "Most Stable" Geometry



# Trajectories





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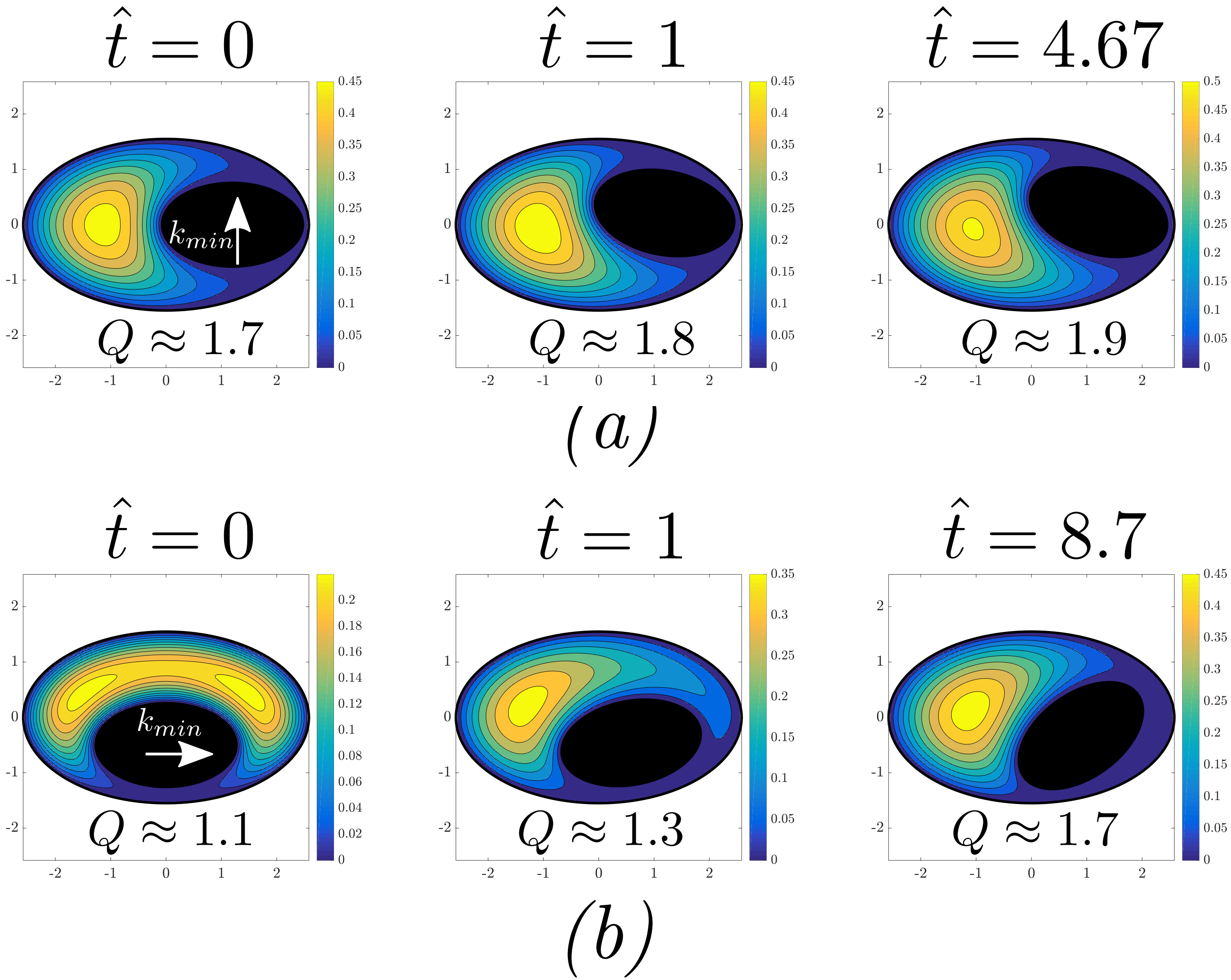

# Summary





- Tools provide resistance to flow
- Axial flow resistance minimised by
  - Having the tool at the edge
  - Having an elliptical cross-section
- Position where tool will move the least when perturbed in direction of minimal resistance is also at edge of channel

# Thank you!