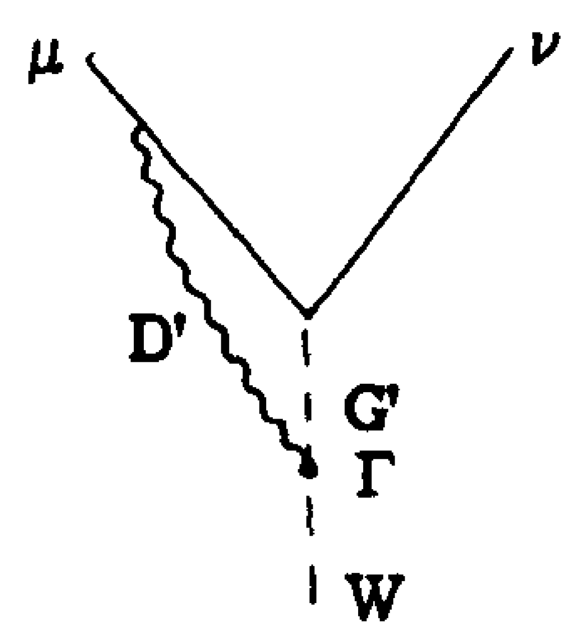

Fig. 1.

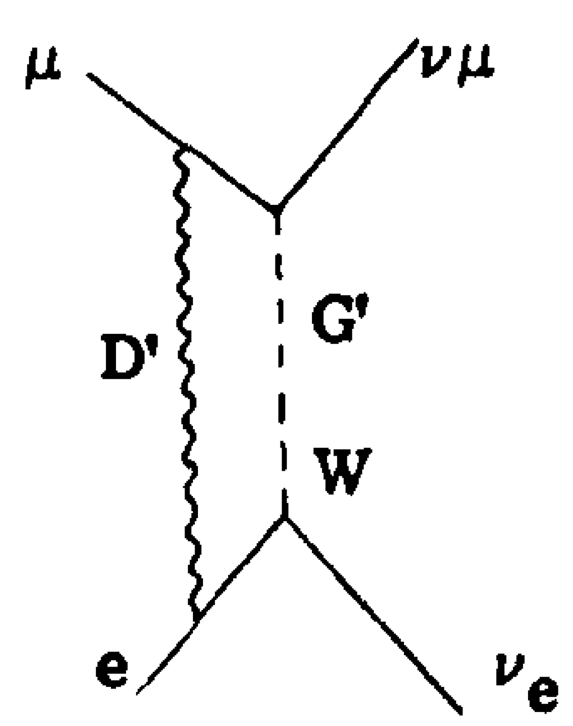

Fig. 2.

well into account the radiation correction to the β-decay constant found by Berman [3] and Kinoshita and Sirlin [4] we obtain for the muon life time

$$\frac{\tau_\mu}{\tau_\mu^{\,o}} = 1 - \frac{3e^2}{4\pi}\ln\frac{\Lambda^2}{\mu^2} + \frac{3e^2}{2\pi}\ln\frac{\Lambda_\beta}{2E} - \frac{3}{5}\frac{M_\mu^2}{\mu^2}, \qquad (1)$$

where $\tau_\mu^{\,o}$ is the muon life time calculated by means of universal theory of four fermion interaction with a constant taken from β-decay without any corrections, Λ_β is the cut off momentum due to the strong interactions, $\Lambda_\beta \sim M$, E is the energy of β-transition. According to experimental data $\tau_\mu/\tau_\mu^{\,o} = 0.988 \pm 0.004$.

Substituting the numbers into (1) we obtain $\tau_\mu/\tau_\mu^{\,o} = 1.003$ and the disagreement between the theory and experiment will be in our case $1.5 \pm 0.4\%$. When discussing this result one should take into consideration that in (1) only the terms $\sim e^2 \ln e^{-2}$ were correctly taken into account but the terms $\sim e^2$ were discarded.

It seems to us that the conclusion that in the theory of weak interaction with intermediate W-meson β- and μ-constants must be with good accuracy the same (taking into account the corrections due to the electromagnetic and weak interactions), is in favour of the weak interaction theory with W-meson unlike the four-fermion theory.

More detailed paper will be published elsewhere.

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BROKEN SYMMETRIES, MASSLESS PARTICLES AND GAUGE FIELDS

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Recently a number of people have discussed the Goldstone theorem [1,2]: that any solution of a Lorentz-invariant theory which violates an internal symmetry operation of that theory must contain a massless scalar particle. Klein and Lee [3] showed that this theorem does not necessarily apply in non-relativistic theories and implied that their considerations would apply equally well to Lorentz-invariant field theories. Gilbert [4], however, gave a proof that the failure of the Goldstone theorem in the nonrelativistic case is of a type which cannot exist when Lorentz invariance is imposed on a theory. The purpose of this note is to show that Gilbert's argument fails for an important class of field theories, that in which the conserved currents are coupled to gauge fields.

Following the procedure used by Gilbert [4], let us consider a theory of two hermitian scalar fields

$\varphi_1(x)$, $\varphi_2(x)$ which is invariant under the phase transformation

$$\varphi_1 \to \varphi_1 \cos\alpha + \varphi_2 \sin\alpha \,, \qquad \varphi_2 \to -\varphi_1 \sin\alpha + \varphi_2 \cos\alpha \,. \tag{1}$$

Then there is a conserved current j_μ such that

$$\mathrm{i}\left[\int \mathrm{d}^3x\, j_0(x),\ \varphi_1(y)\right] = \varphi_2(y). \tag{2}$$

We assume that the Lagrangian is such that symmetry is broken by the nonvanishing of the vacuum expectation value of φ_2. Goldstone's theorem is proved by showing that the Fourier transform of $\mathrm{i}\langle[j_\mu(x),\ \varphi_1(y)]\rangle$ contains a term $2\pi\langle\varphi_2\rangle\epsilon(k_0)k_\mu\,\delta(k^2)$, where k_μ is the momentum, as a consequence of Lorentz-covariance, the conservation law and eq. (2).

Klein and Lee [3] avoided this result in the nonrelativistic case by showing that the most general form of this Fourier transform is now, in Gilbert's notation,

$$\mathrm{F.T.} = k_\mu\rho_1(k^2, nk) + n_\mu\rho_2(k^2, nk) + C_3 n_\mu\,\delta^4(k)\,, \tag{3}$$

where n_μ, which may be taken as (1, 0, 0, 0), picks out a special Lorentz frame. The conversation law then reduces eq. (3) to the less general form

$$\mathrm{F.T.} = k_\mu\,\delta(k^2)\rho_4(nk) + [k^2 n_\mu - k_\mu(nk)]\rho_5(k^2, nk) + C_3 n_\mu\,\delta^4(k)\,. \tag{4}$$

It turns out, on applying eq. (2), that all three terms in eq. (4) can contribute to $\langle\varphi_2\rangle$. Thus the Goldstone theorem fails if $\rho_4 = 0$, which is possible only if the other terms exist. Gilbert's remark that no special timelike vector n_μ is available in a Lorentz-covariant theory appears to rule out this possibility in such a theory.

There is however a class of relativistic field theories in which a vector n_μ does indeed play a part. This is the class of gauge theories, where an auxiliary unit timelike vector n_μ must be introduced in order to define a radiation gauge in which the vector gauge fields are well defined operators. Such theories are nevertheless Lorentz-covariant, as has been shown by Schwinger [5]. (This has, of course, long been known of the simplest such theory, quantum electrodynamics.) There seems to be no reason why the vector n_μ should not appear in the Fourier transform under consideration.

It is characteristic of gauge theories that the conservation laws hold in the strong sense, as a consequence of field equations of the form

$$j^\mu = \partial_\nu F'^{\mu\nu}\,, \qquad F_{\mu\nu}' = \partial_\mu A_\nu' - \partial_\nu A_\mu'\,. \tag{5}$$

Except in the case of abelian gauge theories, the fields A_μ', $F_{\mu\nu}'$ are not simply the gauge field variables A_μ, $F_{\mu\nu}$, but contain additional terms with combinations of the structure constants of the group as coefficients. Now the structure of the Fourier transform of $\mathrm{i}\langle[A_\mu'(x),\ \varphi_1(y)]\rangle$ must be given by eq. (3). Applying eq. (5) to this commutator gives us as the Fourier transform of $\mathrm{i}\langle[j_\mu(x),\ \varphi_1(y)]\rangle$ the single term $[k^2 n_\mu - k_\mu(nk)]\rho(k^2, nk)$. We have thus exorcised both Goldstone's zero-mass bosons and the "spurion" state (at $k_\mu = 0$) proposed by Klein and Lee.

In a subsequent note it will be shown, by considering some classical field theories which display broken symmetries, that the introduction of gauge fields may be expected to produce qualitative changes in the nature of the particles described by such theories after quantization.

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